# My big title

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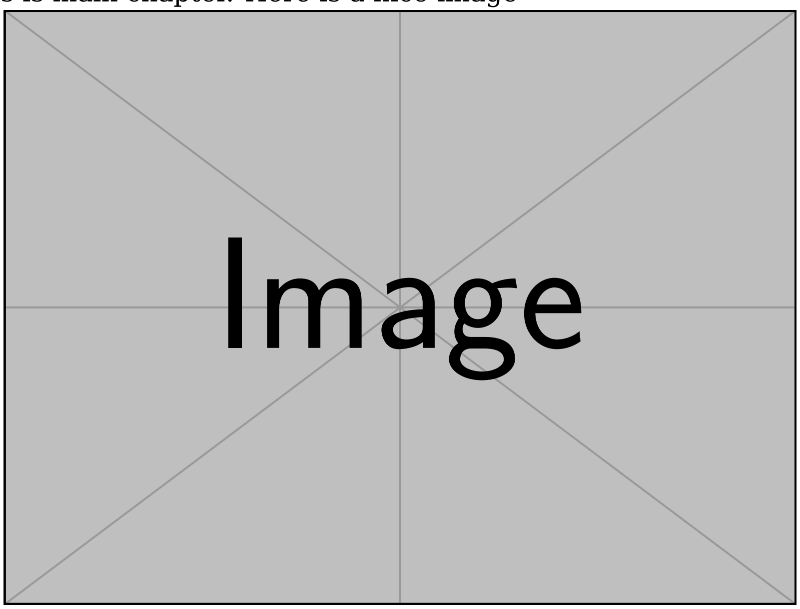
# Chapter 1

# Introduction

This is main chapter. Here is a nice image




## 1.1 some math

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**Theorem 1 (Residue Theorem)** *Let $f$ be analytic in the region $G$ except for the isolated singularities $a_1, a_2, \ldots, a_m$. If $\gamma$ is a closed rectifiable curve in $G$ which does* *not pass through any of the points* $a_k$ *and if* $\gamma \approx 0$ *in* $G$*, then*

$$\frac{1}{2\pi i}\int_\gamma f = \sum_{k=1}^{m} n(\gamma; a_k)\,\mathrm{Res}(f; a_k)\,.$$

### 1.1.1 this is a subsection

Here is some code in minipage

```
%check if we converged or not
if k>opt.MAX_ITER || gradientNormTol(k)<=opt.gradientNormTol ...
|| (k>1 && levelSets(k)>levelSets(k-1))% check for getting worst
   keepRunning = false;
else
   ....
end
```

Here is example using listings

```
%Evaluate J(u) at u
function f = objectiveFunc(u)
 u=u(:);
 N = size(u,1);
 f = 0;
 for i = 1:N-1
     f  = f + 100*(u(i+1)-u(i)^2)^2 + (1-u(i))^2;
 end
end
```

#### This is subsubsection with images

These two images should be side by side

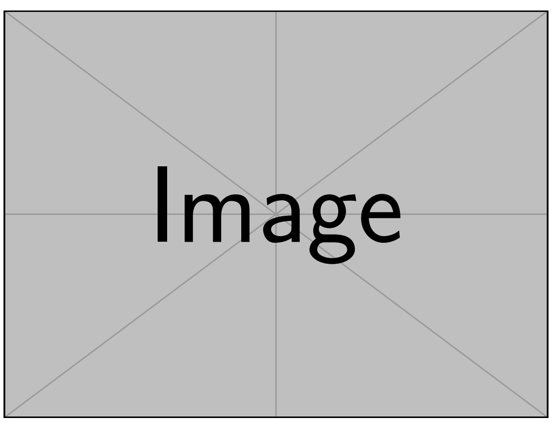


Figure 1.1: Contour $J(u)$

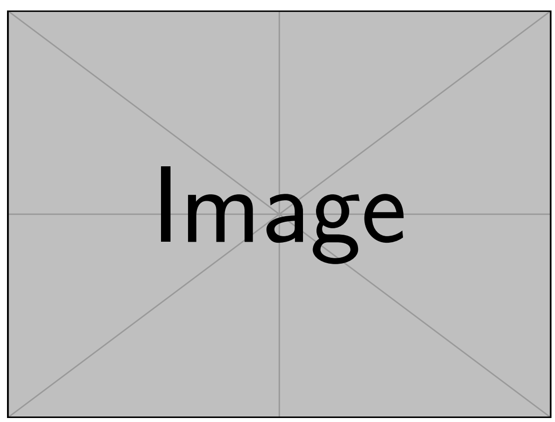


Figure 1.2: Zooming on $J(u)$

# Chapter 2

# This is a new chapter

Here is some verbatim

```
K>> gradientNormTol(end-6:end)
....
          16.1440020280613
           17.487837406306
           16.092991548592
          17.4442963174089
```

## 2.1 This is a section for more math

### 2.1.1 problem 1

problem Transform the following problem or system to set of first order ODE $t^2 x'' + tx' + \left(t^2 - 1\right) x = 0$

solution Since this is second order ODE, we need two state variables, say $x_1, x$
Let $x_1 = x, x_2 = x'$, hence

$$\left.\begin{array}{r} x_1 = x \\ x_2 = x' \end{array}\right\} \overset{\text{take derivative}}{\longrightarrow} \left.\begin{array}{r} x_1' = x' \\ x_2' = x'' \end{array}\right\} \overset{\text{replace RHS}}{\longrightarrow} \begin{array}{c} x_1' = x_2 \\ x_2' = -\frac{x'}{t} - \frac{(t^2-1)x}{t} = -\frac{x_2}{t} - \frac{(t^2-1)x_1}{t} \end{array}$$

Hence the two first order ODE's are (now coupled)

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= -\frac{x_2}{t} - \frac{\left(t^2 - 1\right) x_1}{t} \end{aligned}$$

The matrix form of the above is

$$\begin{aligned} \mathbf{x}' &= A\mathbf{x} \\ \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} &= \begin{pmatrix} 0 & 1 \\ -\frac{t^2-1}{t} & -\frac{1}{t} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \end{aligned}$$

### 2.1.2 Example on page 500, textbook (Edwards&Penny, 3rd edition)

problem This problem was solved in textbook using matrix exponential. Here is solved using the fundamental matrix only. Use the method of variation of parameters to solve

$\mathbf{x}' = A\mathbf{x} + \mathbf{f}(t)$.

$$A = \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix}$$
$$\bar{f}(t) = \begin{pmatrix} -15 \\ 4 \end{pmatrix} te^{-2t}$$
$$\bar{x}(0) = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$$

Solution

The homogeneous solution was found in the book as

$$\bar{x}_h = c_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{5t}$$

Following scalar case, the guess would be $\bar{x}_p = \left(\bar{b} + \bar{a}t\right) e^{-2t}$ but since $e^{-2t}$ is in the homogeneous, we have to adjust to be $\bar{x}_p = \left(\bar{b}t + \bar{a}t^2\right) e^{-2t} + \bar{c}e^{5t}$. Notice we had to add $\bar{c}e^{5t}$, else it will not work if we just guessed $\bar{x}_p = \left(\bar{b}t + \bar{a}t^2\right) e^{-2t}$ based on what we would do in scalar case, we will find we get $\bar{a} = \bar{b} = 0$. This seems to be a trial and error stage and one just have to try to find out. This is why undermined coefficients for systems is not as easy to use as with scalar case. Hence

$$\bar{x}_p = \left(\bar{b}t + \bar{a}t^2\right) e^{-2t} + \bar{c}e^{5t}$$

Now we plug-in this back into the ODE and solve for $\bar{a}, \bar{b}, \bar{c}$. But an easier method is to use Variation of parameters. The fundamental matrix is

$$\begin{aligned} \Phi &= \begin{pmatrix} \bar{x}_1 & \bar{x}_2 \end{pmatrix} \\ &= \begin{pmatrix} e^{-2t} & 2e^{5t} \\ -2e^{-2t} & e^{5t} \end{pmatrix} \end{aligned}$$

And

$$\Phi^{-1} = \frac{\begin{pmatrix} e^{5t} & 2e^{-2t} \\ -2e^{5t} & e^{-2t} \end{pmatrix}^T}{|\Phi|} = \frac{\begin{pmatrix} e^{5t} & -2e^{5t} \\ 2e^{-2t} & e^{-2t} \end{pmatrix}}{e^{3t} + 4e^{3t}} = \frac{1}{5} \begin{pmatrix} e^{2t} & -2e^{2t} \\ 2e^{-5t} & e^{-5t} \end{pmatrix}$$

Hence using

$$\begin{aligned} \bar{x}_p &= \Phi \int \Phi^{-1} \bar{f}(t) \, dt \\ &= \frac{1}{5}\Phi \int \begin{pmatrix} e^{2t} & -2e^{2t} \\ 2e^{-5t} & e^{-5t} \end{pmatrix} \begin{pmatrix} -15te^{-2t} \\ 4te^{-2t} \end{pmatrix} dt \\ &= \frac{1}{5}\Phi \int \begin{pmatrix} -23t \\ -26te^{-7t} \end{pmatrix} dt \end{aligned}$$

The integral of $\int -23t dt = \frac{-23}{2}t^2$ and $\int -26te^{-7t} dt = \frac{26}{49}e^{-7t}(7t+1)$ (using integration by parts) hence the above simplifies to

$$
\begin{aligned}
\bar{x}_p &= \Phi \begin{pmatrix} \frac{-23}{10} t^2 \\ \frac{26}{245} e^{-7t} + \frac{26}{35} t e^{-7t} \end{pmatrix} \\
&= \begin{pmatrix} e^{-2t} & 2e^{5t} \\ -2e^{-2t} & e^{5t} \end{pmatrix} \begin{pmatrix} \frac{-23}{10} t^2 \\ \frac{26}{245} e^{-7t} + \frac{26}{35} t e^{-7t} \end{pmatrix} \\
&= \begin{pmatrix} \frac{52}{245} e^{-2t} + \frac{52}{35} t e^{-2t} - \frac{23}{10} t^2 e^{-2t} \\ \frac{26}{245} e^{-2t} + \frac{26}{35} t e^{-2t} + \frac{23}{5} t^2 e^{-2t} \end{pmatrix} \\
&= \begin{pmatrix} \frac{1}{490} e^{-2t} \left( -1127t^2 + 728t + 104 \right) \\ \frac{1}{245} e^{-2t} \left( 1127t^2 + 182t + 26 \right) \end{pmatrix}
\end{aligned}
$$

Hence the complete solution is

$$
\begin{aligned}
\bar{x} &= \bar{x}_h + \bar{x}_p \\
&= c_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{5t} + \begin{pmatrix} \frac{1}{490} e^{-2t} \left( -1127t^2 + 728t + 104 \right) \\ \frac{1}{245} e^{-2t} \left( 1127t^2 + 182t + 26 \right) \end{pmatrix}
\end{aligned}
$$

To find the constants, we apply initial conditions. At $t = 0$

$$
\begin{aligned}
\begin{pmatrix} 7 \\ 3 \end{pmatrix} &= c_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{52}{245} \\ \frac{26}{245} \end{pmatrix} \\
c_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} &= \begin{pmatrix} 7 \\ 3 \end{pmatrix} - \begin{pmatrix} \frac{52}{245} \\ \frac{26}{245} \end{pmatrix} \\
\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} &= \begin{pmatrix} \frac{1663}{245} \\ \frac{709}{245} \end{pmatrix} \\
\begin{pmatrix} 1 & 2 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} &= \begin{pmatrix} \frac{1663}{245} \\ \frac{807}{49} \end{pmatrix}
\end{aligned}
$$

Hence $5c_2 = \frac{807}{49}$ or $c_2 = \frac{807}{245}$ and $c_1 + 2c_2 = \frac{1663}{245}$, hence $c_1 = \frac{1663}{245} - 2\left(\frac{807}{245}\right) = \frac{1}{5}$. Therefore the solution becomes

$$
\bar{x} = \frac{1}{5} \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-2t} + \frac{807}{245} \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{5t} + \begin{pmatrix} \frac{1}{490} e^{-2t} \left( -1127t^2 + 728t + 104 \right) \\ \frac{1}{245} e^{-2t} \left( 1127t^2 + 182t + 26 \right) \end{pmatrix}
$$