

Ten Hard Integrals

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The first 10 integrals from Kevin Charlwood's 2008 article "Integration on Computer Algebra Systems" are solved using different CAS systems.

The original post on this topic is sci.math.symbolic by Martin

These are the CAS systems used

1. Maple 18 on windows 7 (64 bit)
2. Mathematica 9.01 on windows 7
3. Rubi 4.1 on Mathematica 9.01 on windows 7
4. Sage 5.4 using the Sage web server notebook interface
5. Fricas 1.2 on Linux using sbcl lisp
6. wxMaxima 12.04.0 (Maxima 5.28.02) on windows 7
7. Axiom on windows 7 (May 2012) welcome screen image
8. sympy on linux (python 2.7.3 full installation. sympy 0.7.1.rc1-3) starting image
9. reduce `reduce-windows64-20110414` help screen message with the `algint` package loaded.
10. mupad engine in Matlab 2013a symbolic toolbox
11. xcas 2013 January, version 1.0 on windows 7

Optimal answer to each intergal taken from Rich's referenced PDF below.

Downloads, references and links

1. Mathematica downloads
 - Mathematica notebook of the first 20 integrals
 - Mathematica PDF of the first 20 integrals
 - Mathematica HTML of the first 20 integrals
2. Rubi downloads
 - Rubi notebook for the first 20 integrals
 - Rubi PDF for the first 20 integrals
 - Rubi HTML for the first 20 integrals
3. Maple downloads
 - Maple worksheet in wm format

- HTML worksheet in HTML
- PDF worksheet in PDF

Maple Problem 10 trace entered as `int(x^3*exp(1)^arcsin(x)/sqrt(1-x^2),x);`

- HTML
- PDF
- mw Maple worksheet

Maple Problem 10 trace entered as `int(x^3*exp(arcsin(x))/sqrt(1-x^2),x);`

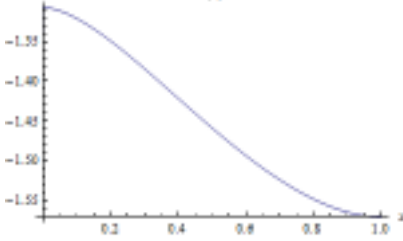
- HTML
- PDF
- mw Maple worksheet

- maxima.wxm Maxima notebook.
- copy of Kevin Charlwood’s 2008 paper in PDF
- <http://www.apmaths.uwo.ca/~arich/CharlwoodIntegrationProblems.pdf> Albert Rich pdf file showing 50 integrals and the best antiderivatives expected
- <http://www.apmaths.uwo.ca/~arich/CharlwoodProblems.m> The above is m format.
- <http://www.math.utah.edu/faq/reduce/>
- <http://www.reduce-algebra.com/packages.htm>
- <http://reduce-algebra.sourceforge.net/>
- <http://www.reduce-algebra.com/docs/reduce.pdf>
- xCAS web page
- <http://www.apmaths.uwo.ca/~arich/> Rubi Mathematica package home

The following is summary of results for each integral. Result with a () around it means the antiderivative contains nonelementary functions.

system	1	2	3	4	5	6	7	8	9	10	score
Mathematica 9.01	✓	✓	✓	✓	(✓)	(✓)	(✓)	(✓)	(✓)	✓	100%
Rubi 4.1	✓	✓	✗	✓	(✓)	✓	✓	✓	✓	✓	90%
Maple 18	✓	✓	✓	✓	(✓)	✓	✓	✓	✗	✓	90%
Axiom (May 2012)	✓	✓	✗	✓	✓	✓	✓	✓	✗	✓	80%
FriCAS 1.2	✓	✓	✓	✓	✗	✓	✓	✗	✓	✓	80%
Sage 5.4	✓	✓	✗	✗	✗	✗	✓	✗	✓	✗	40%
Maxima 5.28.02	✓	✓	✗	✗	✗	✗	✓	✗	✓	✗	40%
xcas 1.0	✗	✗	✗	✓	✗	✓	✗	✓	✓	✗	40%
Sympy 0.7.2	✗	✓	✗	✗	✗	✗	✗	✗	✗	✓	20%
Reduce 2008	✗	✓	✗	✓	✗	✗	✗	✗	✗	✗	20%
mupad 2013a	✗	✗	✗	✓	✗	✗	✗	✗	✓	✗	20%

1 $\int \arcsin(x) \ln(x) \, dx$

optimal 	$-2\sqrt{1-x^2} + \sqrt{1-x^2} \log(x) + \tanh^{-1}\left(\sqrt{1-x^2}\right) - x(1-\log(x)) \sin^{-1}(x)$
M 9.01	<pre>Integrate[ArcSin[x] Log[x], x]</pre> $-2 \sqrt{1-x^2} + x \operatorname{ArcSin}[x] \left(-1 + \operatorname{Log}[x]\right) + \left(-1 + \sqrt{1-x^2}\right) \operatorname{Log}[x] + \operatorname{Log}\left[1 + \sqrt{1-x^2}\right]$
Rubi 4.1	<pre>Int[ArcSin[x] Log[x], x]</pre> $-2 \sqrt{1-x^2} - x \operatorname{ArcSin}[x] + \operatorname{ArcTanh}\left[\sqrt{1-x^2}\right] + \sqrt{1-x^2} \operatorname{Log}[x]$
Maple 18	<pre>> restart; int(arcsin(x)*log(x), x);</pre> $\frac{1}{1 + \tan\left(\frac{1}{2} \arcsin(x)\right)^2} \left(-2 \arcsin(x) \tan\left(\frac{1}{2} \arcsin(x)\right) - 2 \tan\left(\frac{1}{2} \arcsin(x)\right)^2 \ln\left(\frac{2 \tan\left(\frac{1}{2} \arcsin(x)\right)}{1 + \tan\left(\frac{1}{2} \arcsin(x)\right)^2}\right) + 2 \arcsin(x) \tan\left(\frac{1}{2} \arcsin(x)\right) \ln\left(\frac{2 \tan\left(\frac{1}{2} \arcsin(x)\right)}{1 + \tan\left(\frac{1}{2} \arcsin(x)\right)^2} - 4\right) - \ln(1) \right)$
Sage 5.4	<pre>integral(arcsin(x)*ln(x), x)</pre> $(x \log(x) - x) \arcsin(x) + \sqrt{-x^2 + 1} \log(x) - 2 \sqrt{-x^2 + 1}$
Fricas 1.2	<pre>(8) -> integrate(asin(x)*log(x), x)</pre> <pre>(8)</pre> $\frac{\log(2\sqrt{-x^2 + 1} + 2) - \log(-2\sqrt{-x^2 + 1} + 2) + (-2x \log(x) + 2x) \operatorname{atan}\left(\frac{x\sqrt{-x^2 + 1}}{x^2 - 1}\right) + (2\log(x) - 4)\sqrt{-x^2 + 1}}{2}$ Type: Union(Expression(Integer), ...)

sympy 0.7.1	<pre>>>> integrate(asin(x)*log(x),x) Integral(log(x)*asin(x), x) >>></pre>
Axiom 5/12	<pre>(5) -> integrate(asin(x)*log(x),x) (5) -> (5)</pre> $\frac{\log(\sqrt{-x^2+1}+1) - \log(\sqrt{-x^2+1}-1) + (-x \log(x) + x) \operatorname{atan}\left(\frac{2x\sqrt{-x^2+1}}{2x^2-1}\right) + (2\log(x) - 4)\sqrt{-x^2+1}}{2}$ <pre>Type: Union(Expression Integer,...) (6) -></pre>
Maxima 5.28.02	<pre>integrate(asin(x)*log(x),x);</pre> $\log\left(\frac{2\sqrt{1-x^2}}{ x } + \frac{2}{ x }\right) + \operatorname{asin}(x)(x \log(x) - x) + \sqrt{1-x^2} \log(x) - 2\sqrt{1-x^2}$
reduce 2008	<pre>int(asin(x)*log(x),x);</pre> $\int \arcsin(x) \log(x) \, dx$ HTML of trace PDF of trace
mupad 2013a	<pre>evalin(symengine,'int(asin(x)*log(x),x)')</pre> <pre>int(asin(x)*log(x), x)</pre>
xcas	<pre>int(asin(x)*log(x),x)</pre> Simplification assuming x near 0+ Simplification assuming x near 0+ $\int \operatorname{asin}(x) * \ln(x) \, dx$

2

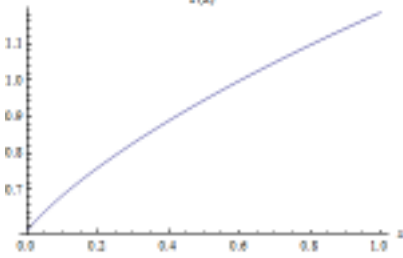
$$\int \frac{x \arcsin(x)}{\sqrt{1-x^2}} dx$$

optimal	$x - \sqrt{1-x^2} \sin^{-1}(x)$
M 9.01	$\text{Integrate}\left[\frac{x \text{ArcSin}[x]}{\sqrt{1-x^2}}, x\right]$ $x - \sqrt{1-x^2} \text{ArcSin}[x]$
Rubi 4.1	$\text{Int}\left[\frac{x \text{ArcSin}[x]}{\sqrt{1-x^2}}, x\right]$ $x - \sqrt{1-x^2} \text{ArcSin}[x]$
Maple 18	<pre>> restart; int(x*arcsin(x)/sqrt(1-x^2), x);</pre> $x - \arcsin(x) \sqrt{-x^2 + 1}$
Sage 5.4	integral(x*arcsin(x)/sqrt(1-x^2), x) $-\sqrt{-x^2 + 1} \arcsin(x) + x$
Fricas 1.2	<pre>(9) -> integrate(x*asin(x)/sqrt(1-x^2), x) +-----+ 2 x\ - x + 1 (9) \ - x + 1 atan(-----) + x 2 x - 1 Type: Union(Expression(Integer), ...)</pre>
sympy 0.7.1	<pre>>>> integrate(x*asin(x)/sqrt(1-x**2),x) x - (-x**2 + 1)**(1/2)*asin(x) >>></pre>
Axiom 5/12	<pre>-> integrate(x*asin(x)/sqrt(1-x^2),x) -></pre> $(6) \frac{\sqrt{1-x^2} \operatorname{atan}\left(\frac{2x\sqrt{1-x^2}}{2x^2-1}\right) + 2x}{2}$
Maxima 5.28.02	<pre>(%i5) integrate(x*asin(x)/sqrt(1-x^2), x); (%o5) x - sqrt(1-x^2) asin(x)</pre>

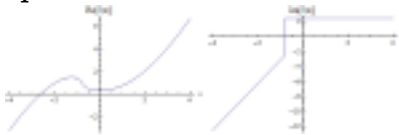
reduce 2008	$\int x \arcsin(x) \sqrt{1-x^2} \, dx$
mupad 2013a	$\int x \arcsin(x) \sqrt{1-x^2} \, dx$
xcas	$\int x \arcsin(x) \sqrt{1-x^2} \, dx$

3

$$\int \arcsin \left(\sqrt{x+1} - \sqrt{x} \right) dx$$

optimal 	$\frac{(\sqrt{x}+3\sqrt{x+1})\sqrt{\sqrt{x}\sqrt{x+1}-x}}{4\sqrt{2}} - \left(x+\frac{3}{8}\right)\sin^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)$
M 9.01	$\text{Integrate}\left[\text{ArcSin}\left[\sqrt{x+1}-\sqrt{x}\right],x\right]$ $-x\text{ArcSin}\left[\sqrt{x}-\sqrt{1+x}\right]-$ $\left(\left(1+x\right)\left(1+2x-2\sqrt{x}\sqrt{1+x}\right)^2\left(2\sqrt{-x+\sqrt{x}\sqrt{1+x}}\right.\right.$ $\left.\left(-3-2x+2\sqrt{x}\sqrt{1+x}\right)+3\sqrt{-2-4x+4\sqrt{x}\sqrt{1+x}}\right.$ $\left.\text{Log}\left[2\sqrt{-x+\sqrt{x}\sqrt{1+x}}+\sqrt{-2-4x+4\sqrt{x}\sqrt{1+x}}\right]\right)$ $\left(8\sqrt{2}\left(-\sqrt{x}+\sqrt{1+x}\right)^3\left(1+x-\sqrt{x}\sqrt{1+x}\right)^2\right)$
Rubi 4.1	$\text{Int}\left[\text{ArcSin}\left[\sqrt{x+1}-\sqrt{x}\right],x\right]$ $-x\text{ArcSin}\left[\sqrt{x}-\sqrt{1+x}\right]+\frac{\text{Subst}\left[\text{Int}\left[\sqrt{1-x^2+x}\sqrt{-1+x^2}\right],x,\sqrt{x}\right]}{\sqrt{2}}$
Maple 18	<pre>> restart; int(arcsin(sqrt(x+1)-sqrt(x)),x);</pre> $-\left(\frac{1}{16}\arcsin(-\sqrt{x+1}+\sqrt{x})+\frac{3}{8}\tan\left(\frac{1}{2}\arcsin(-\sqrt{x+1}+\sqrt{x})\right)^3-\frac{3}{8}\right.$ $\left.+\sqrt{x}\right)^5-\frac{1}{8}\tan\left(\frac{1}{2}\arcsin(-\sqrt{x+1}+\sqrt{x})\right)^7+\frac{1}{8}\arcsin(-\sqrt{x+1}$ $+\sqrt{x})\tan\left(\frac{1}{2}\arcsin(-\sqrt{x+1}+\sqrt{x})\right)^2+\frac{9}{8}\arcsin(-\sqrt{x+1}+\sqrt{x})\tan$ $(-\sqrt{x+1}+\sqrt{x})\right)^4+\frac{1}{8}\arcsin(-\sqrt{x+1}+\sqrt{x})\tan\left(\frac{1}{2}\arcsin(-\sqrt{x+1}$ $-\sqrt{x+1}+\sqrt{x})\tan\left(\frac{1}{2}\arcsin(-\sqrt{x+1}+\sqrt{x})\right)^8+\frac{1}{8}\tan\left(\frac{1}{2}\arcsin(-$ $\left.\left(\left(1+\tan\left(\frac{1}{2}\arcsin(-\sqrt{x+1}+\sqrt{x})\right)\right)^2\right)^2\tan\left(\frac{1}{2}\arcsin(-\sqrt{x+1}+\sqrt{x}\right.\right.$ $\left.\left.\right)\right)$
Sage 5.4	<pre>integral(arcsin(sqrt(x+1)-sqrt(x)), x) evaluate</pre>

$$4 \int \ln \left(1 + x \sqrt{1 + x^2} \right) dx$$

optimal 	$x \log \left(\sqrt{x^2 + 1} x + 1 \right) + \sqrt{2 \left(1 + \sqrt{5} \right)} \tan^{-1} \left(\sqrt{\sqrt{5} - 2} \left(\sqrt{x^2 + 1} + \sqrt{2 \left(\sqrt{5} - 1 \right)} \tanh^{-1} \left(\sqrt{2 + \sqrt{5}} \left(\sqrt{x^2 + 1} + x \right) \right) - 2 x \right.$
M 9.01	$\text{Integrate}\left[\text{Log}\left[1 + x \sqrt{1 + x^2}\right], x\right]$ $-2 x + \frac{\left(5 + \sqrt{5}\right) \text{ArcTan}\left[\sqrt{\frac{2}{1 + \sqrt{5}}} x\right]}{\sqrt{10 \left(1 + \sqrt{5}\right)}} +$ $\sqrt{\frac{2}{-1 + \sqrt{5}}} \text{ArcTan}\left[\sqrt{\frac{2}{-1 + \sqrt{5}}} \sqrt{1 + x^2}\right] - \frac{\left(-5 + \sqrt{5}\right) \text{ArcTan}\left[\sqrt{\frac{2}{-1 + \sqrt{5}}} \sqrt{1 + x^2}\right]}{\sqrt{10 \left(-1 + \sqrt{5}\right)}} +$ $\sqrt{\frac{2}{1 + \sqrt{5}}} \text{ArcTanh}\left[\sqrt{\frac{2}{1 + \sqrt{5}}} \sqrt{1 + x^2}\right] + x \text{Log}\left[1 + x \sqrt{1 + x^2}\right]$
Rubi 4.1	$\text{Int}\left[\text{Log}\left[1 + x \sqrt{1 + x^2}\right], x\right]$ $-2 x - \sqrt{\frac{1}{10} \left(1 + \sqrt{5}\right)} \text{ArcTan}\left[\sqrt{\frac{2}{1 + \sqrt{5}}} x\right] +$ $2 \sqrt{\frac{1}{5} \left(2 + \sqrt{5}\right)} \text{ArcTan}\left[\sqrt{\frac{2}{1 + \sqrt{5}}} x\right] + \sqrt{\frac{2}{5 \left(-1 + \sqrt{5}\right)}} \text{ArcTan}\left[\sqrt{\frac{2}{-1 + \sqrt{5}}} \sqrt{1 + x^2}\right] +$ $\sqrt{\frac{2}{5} \left(-1 + \sqrt{5}\right)} \text{ArcTan}\left[\sqrt{\frac{2}{-1 + \sqrt{5}}} \sqrt{1 + x^2}\right] + 2 \sqrt{\frac{1}{5} \left(-2 + \sqrt{5}\right)} \text{ArcTan}\left[\sqrt{\frac{2}{-1 + \sqrt{5}}} \sqrt{1 + x^2}\right] +$ $\sqrt{\frac{1}{10} \left(-1 + \sqrt{5}\right)} \text{ArcTanh}\left[\sqrt{\frac{2}{-1 + \sqrt{5}}} x\right] + \sqrt{\frac{2}{5 \left(1 + \sqrt{5}\right)}} \text{ArcTanh}\left[\sqrt{\frac{2}{1 + \sqrt{5}}} \sqrt{1 + x^2}\right] +$ $\sqrt{\frac{2}{5} \left(1 + \sqrt{5}\right)} \text{ArcTanh}\left[\sqrt{\frac{2}{1 + \sqrt{5}}} \sqrt{1 + x^2}\right] + x \text{Log}\left[1 + x \sqrt{1 + x^2}\right]$

Maple 18	<pre>> restart; int(log(1+x*sqrt(1+x^2)), x);</pre> $\ln(1+x\sqrt{x^2+1})x-2x+\frac{\sqrt{5}\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{-2+2\sqrt{5}}}-\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right)}{\sqrt{-2+2\sqrt{5}}}+\frac{\sqrt{5}\operatorname{arctan}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{\sqrt{2\sqrt{5}+2}}+\frac{\operatorname{arctan}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{\sqrt{2\sqrt{5}+2}}-\frac{2}{5}\sqrt{\sqrt{5}-2}\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{\sqrt{5}-2}}\right)+\frac{2}{5}\sqrt{2+\sqrt{5}}\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{\sqrt{5}-2}}\right)-\frac{1}{2}\frac{\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{2+\sqrt{5}}}\right)}{\sqrt{2+\sqrt{5}}}-\frac{1}{2}\frac{\sqrt{5}\operatorname{arctan}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{\sqrt{5}-2}}\right)}{\sqrt{\sqrt{5}-2}}-\frac{3}{10}\frac{\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{\sqrt{5}-2}}\right)}{\sqrt{\sqrt{5}-2}}+\frac{1}{2}\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{\sqrt{5}-2}}\right)}{\sqrt{\sqrt{5}-2}}-\frac{3}{10}\frac{\sqrt{5}\operatorname{arctan}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{2+\sqrt{5}}}\right)}{\sqrt{2+\sqrt{5}}}-\frac{1}{2}\frac{\operatorname{arctan}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{2+\sqrt{5}}}\right)}{\sqrt{2+\sqrt{5}}}-\frac{1}{2}\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{2+\sqrt{5}}}\right)}{\sqrt{2+\sqrt{5}}}+\frac{1}{2}\frac{\operatorname{arctan}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{\sqrt{5}-2}}\right)}{\sqrt{\sqrt{5}-2}}$
Sage 5.4	<pre>integral(ln(1+x*sqrt(1+x^2)), x)</pre> $x\log(\sqrt{x^2+1}x+1)-2x+\arctan(x)+\int\frac{2x^2+1}{x^2+\sqrt{x^2+1}(x^2+1)}dx$

Fricas 1.2

```
(12) -> integrate(log(1+x*sqrt(1+x^2)), x)

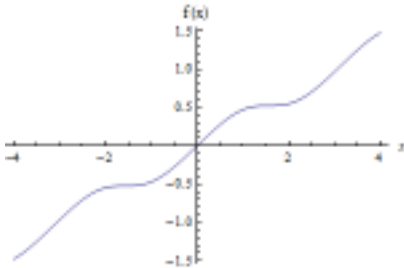
(12)
-
      +-----+
      | +-+
      4\|\|5  + 1
      *
      atan
      +-----+
      +-+ | 2 +-+ | 2 +-+
      ((\|5  - 1)\|x  + 1  - x\|5  + x)\|\|
      /
      ROOT
      +-----+
      +-+ 3 | 2
      (- 8x\|5  - 16x  - 8x)\|x  + 1
      +
      2
      16x  + 4
      +
      +-----+
      +-+ | 2 2 +-+
      - 2x\|2 \|x  + 1  + (2x  + 2)\|2
      +
      -
      +-----+
      | +-+
      \|\|5  - 1
      *
      log
      +-----+
      +-+ | 2 +-+ | 2 +-+
      ((\|5  + 1)\|x  + 1  - x\|5  - x)\|\|
      +
      2 +-+
      (2x  + 2)\|2
      +
      +-----+ +-----+
      | +-+ | +-+ +-+ +-+
      \|\|5  - 1 log(\|\|5  - 1  + x\|2 ) + 2x\|2 log
      +
      +-----+ +-----+
      | +-+ | +-+ +-+
      - \|\|5  - 1 log(- \|\|5  - 1  + x\|2 )
      +
      +-----+
      | +-+
      \|\|5  - 1
      *
      log
      +-----+
      +-+ | 2 +-+ | 2 +-+
      ((- \|5  - 1)\|x  + 1  + x\|5  + x)\|\|5  -
      +
      2 +-+
      (2x  + 2)\|2
      +
      +-----+
      | +-+
      - 4\|\|5  + 1 atan(-----) - 4x\|
      +-----+
      | +-+ 2 +-+
      \|\|5  + 1
```

sympy 0.7.1	<pre>>>> integrate(log(1+x* sqrt(1+x**2)),x) Integral(log(x*(x**2 + 1)**(1/2) + 1), x) >>> >>> (8) -> (8) -> integrate(log(1+x*sqrt(1+x^2)),x) (8) -> (8)</pre>
Axiom 5/12	<pre>- +-----+ +-+ \ \ 5 - 1 * log +-----+ +-+ 2 +-+ +-+ ((\ 5 + 1)\ x + 1 - x\ 5 - x)\ \ 5 - 1 - + 2 +-+ (2x + 2)\ 2 + +-----+ +-----+ +-----+ +-+ +-+ +-+ +-+ 2 \ \ 5 - 1 log(\ \ 5 - 1 + x\ 2) + 2x\ 2 log(x\ x + + +-----+ +-----+ +-+ +-+ +-+ - \ \ 5 - 1 log(- \ \ 5 - 1 + x\ 2) + +-----+ +-+ \ \ 5 - 1 * log +-----+ +-+ 2 +-+ +-+ ((- \ 5 - 1)\ x + 1 + x\ 5 + x)\ \ 5 - 1 - + 2 +-+ (2x + 2)\ 2 + +-----+ +-----+ +-----+ +-+ 2 +-+ +-+ 2\ \ 5 + 1 atan(-----) +-+ 2 2 +-+ 2x\ 2 \ x + 1 + (- 2x - 2)\ 2 + +-----+ +-----+ +-+ +-+ - 2\ \ 5 + 1 atan(-----) - 4x\ 2 +-+ x\ 2 / +-+ 2\ 2 Type: Union(Expression</pre>
Maxima 5.28.02	<pre>integrate(log(1+x*sqrt(1+x^2)),x); ∫^{2 x²+1}_{√x²+1 (x³+x)+x²+1}dx + x log(x√x²+1+1)+atan(x)-2 x</pre>

reduce 2008	<pre>14: int(log(1+x*sqrt(1+x^2)),x);</pre> $\left(2\sqrt{\sqrt{5}+2}\sqrt{5}\arctan\left(\frac{\sqrt{x^2+1}+x}{\sqrt{\sqrt{5}+2}}\right)-2\sqrt{\sqrt{5}+2}\arctan\left(\frac{\sqrt{x^2+1}+x}{\sqrt{\sqrt{5}+2}}\right)+\sqrt{\sqrt{5}-2}\right)$ $\sqrt{\sqrt{5}-2}\log\left(\sqrt{x^2+1}-\sqrt{\sqrt{5}-2}+x\right)-\sqrt{\sqrt{5}-2}\log\left(\sqrt{x^2+1}+\sqrt{\sqrt{5}-2}+x\right)+2\log$
mupad 2013a	<pre>evalin(symengine,'int(log(1+x*sqrt(1+x^2)),x)')</pre> <pre>result in mupad_4.txt</pre>
xcas	<pre>}int(log(1+x*sqrt(1+x^2)),x)[</pre> $\frac{\operatorname{rootof}([[1, -1, 3, -27], [1, 0, -6, -40, 29]])*\ln\left x-\sqrt{\frac{\operatorname{rootof}([[-1, 1, -3, 27], [1, 0, -6, -40, 29]])}{64}}\right }{64}$ $\frac{\operatorname{rootof}([[-1, 1, -3, 27], [1, 0, -6, -40, 29]])*\ln\left x+\sqrt{\frac{\operatorname{rootof}([[-1, 1, -3, 27], [1, 0, -6, -40, 29]])}{64}}\right }{64}$ $\frac{\operatorname{rootof}([[1, -1, -9, -23], [1, 0, -14, -40, -11]])*\operatorname{atan}\left(\frac{\operatorname{rootof}([[-1, 1, -3, 27], [1, 0, -6, -40, 29]])}{16}\right)}{16}$ $\frac{\operatorname{rootof}([[-1, 2, 7], [1, 0, -14, -40, -11]])*\ln\left \sqrt{x^2+1}-\frac{\operatorname{rootof}([[-1, 2, 7], [1, 0, -14, -40, -11]])}{32}\right }{32}$ $\frac{\operatorname{rootof}([[1, -2, -7], [1, 0, -14, -40, -11]])*\ln\left \sqrt{x^2+1}+\frac{\operatorname{rootof}([[-1, 2, 7], [1, 0, -14, -40, -11]])}{32}\right }{32}$ $\frac{\operatorname{rootof}([[1, 3, -5, -39], [1, 0, -6, -40, 29]])*\operatorname{atan}\left(\frac{\operatorname{rootof}([[-1, 1, -3, 27], [1, 0, -6, -40, 29]])}{64}\right)}{64}$ $-2*\left(\frac{\operatorname{rootof}([[-1, 1, -3, 27], [1, 0, -6, -40, 29]])}{64}\right)$

5

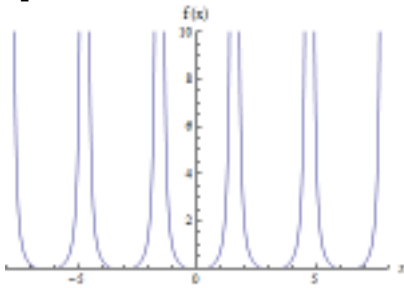
$$\int \frac{\cos^2(x)}{\sqrt{\cos^4(x) + \cos^2(x) + 1}} dx$$

optimal 	$\frac{x}{3} + \frac{1}{3} \tan^{-1} \left(\frac{\sin(x) \cos(x) (\cos^2(x)+1)}{\sqrt{\cos^4(x)+\cos^2(x)+1} \cos^2(x)+1} \right)$
M 9.01	$\text{Integrate}\left[\frac{\text{Cos}[x]^2}{\sqrt{\text{Cos}[x]^4 + \text{Cos}[x]^2 + 1}}, x\right]$ $-\left(2 i \text{Cos}[x]^2 \text{EllipticPi}\left[\frac{3}{2} + \frac{i \sqrt{3}}{2}, i \text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i + \sqrt{3}}} \text{Tan}[x]\right], \frac{3 i -}{3 i +}\right.\right.$ $\left.\left.\sqrt{1 - \frac{2 i \text{Tan}[x]^2}{-3 i + \sqrt{3}}} \sqrt{1 + \frac{2 i \text{Tan}[x]^2}{3 i + \sqrt{3}}}\right) / \left(\sqrt{-\frac{i}{-3 i + \sqrt{3}}} \sqrt{15 + 8 \text{Cos}[2 x]}\right.\right.$
Rubi 4.1	$\text{Int}\left[\frac{\text{Cos}[x]^2}{\sqrt{\text{Cos}[x]^4 + \text{Cos}[x]^2 + 1}}, x\right]$ $\frac{\text{EllipticPi}\left[\frac{1}{2} \left(3 - i \sqrt{3}\right), \text{ArcSin}\left[\sqrt{-\frac{2 i}{3 i + \sqrt{3}}} \text{Tan}[x]\right], \frac{3 i + \sqrt{3}}{3 i - \sqrt{3}}\right]}{3 \sqrt{-\frac{2 i}{3 i + \sqrt{3}}} \left(1 + \text{Tan}[x]^2\right) \sqrt{1 + \frac{2 \text{Tan}[x]^2}{3 - i \sqrt{3}}} \sqrt{1 + \frac{2 \text{Tan}[x]^2}{3 + i \sqrt{3}}}}$
Maple 18	<pre>> restart; int(cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1), x);</pre> $-\left(2 \sqrt{-\left(\cos(2 x)^2+4 \cos(2 x)+7\right)\left(\cos(2 x)^2-1\right)}\left(-3\right.\right.$ $\left.+I \sqrt{3}\right) \sqrt{\frac{\left(-1+I \sqrt{3}\right)\left(\cos(2 x)-1\right)}{\left(-3+I \sqrt{3}\right)\left(\cos(2 x)+1\right)}}\left(\cos(2 x)\right.$ $\left.+1\right)$ $2 \sqrt{\frac{\cos(2 x)+2+I \sqrt{3}}{\left(I \sqrt{3}+3\right)\left(\cos(2 x)+1\right)}} \sqrt{\frac{I \sqrt{3}-\cos(2 x)-2}{\left(-3+I \sqrt{3}\right)\left(\cos(2 x)+1\right)}} \text{EllipticPi}\left[\frac{1}{2}\left(3-i \sqrt{3}\right), \text{ArcSin}\left[\sqrt{-\frac{2 i}{3 i + \sqrt{3}}} \text{Tan}[x]\right], \frac{3 i + \sqrt{3}}{3 i - \sqrt{3}}\right]$ $\sqrt{\frac{\left(-1+I \sqrt{3}\right)\left(\cos(2 x)-1\right)}{\left(-3+I \sqrt{3}\right)\left(\cos(2 x)+1\right)}}, \frac{-3+I \sqrt{3}}{-1+I \sqrt{3}}, \sqrt{\frac{\left(1+I \sqrt{3}\right)\left(-3+I \sqrt{3}\right)}{\left(I \sqrt{3}+3\right)\left(-1+I \sqrt{3}\right)}}\left(I \sqrt{3}+3\right)$ $\sqrt{\left(\cos(2 x)-1\right)\left(\cos(2 x)+1\right)\left(\cos(2 x)+2+I \sqrt{3}\right)\left(I \sqrt{3}-\cos(2 x)-2\right)} \sqrt{\cos(2 x)^2+4 \cos(2 x)+7})$

Sage 5.4	<pre>integral(cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1), x)</pre> $\int \frac{\cos(x)^2}{\sqrt{\cos(x)^4 + \cos(x)^2 + 1}} dx$
Fricas 1.2	<pre>(16) -> integrate(cos(x)^2/sqrt(cos(x)^4 + cos(x)^2 +1), x) >> Error detected within library code: Imaginary part is nonzero. Cannot retract. (16) -> </pre>
sympy 0.7.1	<pre>(waited one hr) >>> >>> integrate(cos(x)**2/sqrt(cos(x)**4 + cos(x)**2 +1), x) ^C ^C ^C^C^C Killed > ></pre>
Axiom 5/12	<pre>(9) -> (9) -> integrate(cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1), x) (9) -> (9) atan 5 4 3 (cos(x) + 2cos(x) + cos(x))sin(x) * +-----+ 4 2 16cos(x) + 16cos(x) + 16 ----- 4 3 2 \cos(x) + 4cos(x) + 6cos(x) + 4cos(x) + 1 / 6 4cos(x) - 2 / 6 Type: Union(Expression Integer,...)</pre>
Maxima 5.28.02	<pre>integrate(cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1), x);</pre> $\int \frac{\cos(x)^2}{\sqrt{\cos(x)^4 + \cos(x)^2 + 1}} dx$
reduce 2008	<pre>int(cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1), x);</pre> $\int \frac{\sqrt{\cos(x)^4 + \cos(x)^2 + 1} \cos(x)}{\cos(x)^4 + \cos(x)^2 + 1} dx$
mupad 2013a	<pre>evalin(symengine,'int(cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1), x)')</pre> <pre>int(cos(x)^2/(cos(x)^4 + cos(x)^2 + 1)^(1/2), x)</pre>
xcas	<pre>int(cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1), x)</pre> $\int \frac{\cos(x)^2}{\sqrt{\cos(x)^4 + \cos(x)^2 + 1}} dx$

6

$$\int \tan(x) \sqrt{1 + \tan^4(x)} \, dx$$

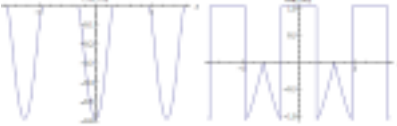
optimal 	$\frac{1}{2} \sqrt{\tan^4(x) + 1} - \frac{\tanh^{-1}\left(\frac{1 - \tan^2(x)}{\sqrt{2} \sqrt{\tan^4(x) + 1}}\right)}{\sqrt{2}} - \frac{1}{2} \sinh^{-1}(\tan^2(x))$
M 9.01	click to view
Rubi 4.1	$\text{Int}\left[\text{Tan}[\text{x}] \sqrt{1 + \text{Tan}[\text{x}]^4}, \text{x}\right]$ $-\frac{1}{2} \text{ArcSinh}\left[\text{Tan}[\text{x}]^2\right] - \frac{\text{ArcTanh}\left[\frac{1 - \text{Tan}[\text{x}]^2}{\sqrt{2} \sqrt{1 + \text{Tan}[\text{x}]^4}}\right]}{\sqrt{2}} + \frac{1}{2} \sqrt{1 + \text{Tan}[\text{x}]^4}$
Maple 18	<pre>> restart; int(tan(x)*sqrt(1+tan(x)^4), x);</pre> $\frac{1}{2} \sqrt{\left(1 + \tan(x)^2\right)^2 - 2 \tan(x)^2} - \frac{1}{2} \operatorname{arcsinh}\left(\tan(x)^2\right)$ $- \frac{1}{2} \sqrt{2} \operatorname{arctanh}\left(\frac{1}{4} \frac{\left(-2 \tan(x)^2 + 2\right) \sqrt{2}}{\sqrt{\left(1 + \tan(x)^2\right)^2 - 2 \tan(x)^2}}\right)$
Sage 5.4	$\text{integral}(\tan(x) \sqrt{1 + \tan(x)^4}, x)$ $\int \sqrt{\tan(x)^4 + 1} \tan(x) \, dx$
Fricas 1.2	<pre>(21) -> integrate(tan(x)*sqrt(1+tan(x)^4), x) (21) +-----+ 4 2 2log(\ tan(x) + 1 - tan(x)) + +-----+ 4 4 2 ++ (2\ 2 tan(x) - 2\ 2)\ tan(x) + 1 + 3tan(x) - 2tan(x) \ 2 log(-----) 4 2 tan(x) + 2tan(x) + 1 + +-----+ 4 2\ tan(x) + 1 / 4 Type: Union(Expression(Integer), ...)</pre>
sympy 0.7.1	<pre>>>> integrate(tan(x)* sqrt(1+tan(x)**4),x) Integral((tan(x)**4 + 1)**(1/2)*tan(x), x) >>> >>></pre>

Axiom 5/12	<pre>(10) -> (10) -> integrate(tan(x)*sqrt(1+tan(x)^4),x) (10) -> (10)</pre> $ \begin{aligned} & \frac{\sqrt{2} \log\left(\frac{(4\sqrt{2}\cos(x)^4 - 2\sqrt{2}\cos(x)^2) \sqrt{2\cos(x)^4 - 2\cos(x)^2 + 1}}{\cos(x)^4} + \frac{8\cos(x)^4 - 8\cos(x)^2 + 3}{\cos(x)^4}\right) + \log\left(\frac{(2\cos(x)^4 - 2\cos(x)^2) \sqrt{2\cos(x)^4 - 2\cos(x)^2 + 1}}{\cos(x)^4} + \frac{-4\cos(x)^2 + 2}{\cos(x)^4}\right)}{4} \\ & + \frac{2 \sqrt{2\cos(x)^4 - 2\cos(x)^2 + 1}}{\sqrt{\cos(x)^4}} \end{aligned} $ Type: Union(Expression Integer,...) <pre>(11) -> (11) -> (11) -></pre>
Maxima 5.28.02	<pre>integrate(tan(x)*sqrt(1+tan(x)^4),x);</pre> $\int \tan(x) \sqrt{\tan(x)^4 + 1} \, dx$
reduce 2008	<pre>int(tan(x)*sqrt(1+tan(x)^4),x);</pre> $\int \sqrt{\tan(x)^4 + 1} \tan(x) \, dx$
mupad 2013a	<pre>evalin(symengine,'int(tan(x)*sqrt(1+tan(x)^4),x)')</pre> $\text{int}(\tan(x) * (\tan(x)^4 + 1)^{(1/2)}, x)$

xcas	<code>int (tan (x) *sqrt (1+tan (x) ^4) , x)</code> $\frac{\sqrt{4*\left(\frac{\tan(x)^2}{2}\right)^2+1}}{2}+2*\left(\frac{\ln(\sqrt{4*\left(\frac{\tan(x)^2}{2}\right)^2+1}-\frac{2*\tan(x)^2}{2})}{4}+\frac{1}{2}*\right)$
------	---

7

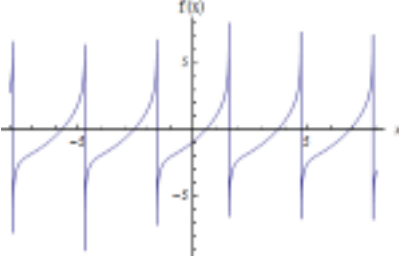
$$\int \frac{\tan(x)}{\sqrt{\sec^3(x)+1}} dx$$

optimal 	$-\frac{2}{3} \tanh^{-1}\left(\sqrt{\sec^3(x)+1}\right)$
M 9.01	<pre>In[23]:= Integrate[$\frac{\text{Tan}[x]}{\sqrt{\text{Sec}[x]^3+1}}$, x]</pre> $\text{Out[23]}= -\left(i \cos [x]^2 \left(\text{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{3} \sqrt{\frac{i \cos [x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3 i+\sqrt{3}}}\right], \frac{3 i-\sqrt{3}}{3 i+\sqrt{3}}\right]\right.\right.$ $\left.\text{EllipticPi}\left[\frac{1}{6}(3+i \sqrt{3}), i \operatorname{ArcSinh}\left[\sqrt{3} \sqrt{\frac{i \cos [x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3 i+\sqrt{3}}}\right],\right.\right.$ $\left.\sqrt{(4+3 \cos [x]+\cos [3 x]) \operatorname{Sec}[x]^3} \sqrt{\frac{\sqrt{3}-3 i \tan \left[\frac{x}{2}\right]^2}{-3 i+\sqrt{3}}}\sqrt{\frac{\sqrt{3}+3 i}{3 i+}}\right.$ $\left.\left(\sqrt{3} \sqrt{\frac{\cos [x] \operatorname{Sec}\left[\frac{x}{2}\right]^2}{-3-i \sqrt{3}}}\left(1+3 \tan \left[\frac{x}{2}\right]^4\right)\right)\right)$
Rubi 4.1	<pre>In[22]:= Int[$\frac{\text{Tan}[x]}{\sqrt{\text{Sec}[x]^3+1}}$, x]</pre> $\text{Out[22]}= -\frac{2}{3} \operatorname{ArcTanh}\left[\sqrt{1+\operatorname{Sec}[x]^3}\right]$
Maple 18	<pre>restart; int(tan(x)/sqrt(sec(x)^3+1), x);</pre> $-\frac{2}{3} \operatorname{arctanh}\left(\sqrt{\sec (x)^3+1}\right)$
Sage 5.4	<pre>integral(tan(x)/sqrt(sec(x)^3+1), x)</pre> $\frac{1}{3} \log \left(\sqrt{\frac{1}{\cos (x)^3}+1}-1\right)-\frac{1}{3} \log \left(\sqrt{\frac{1}{\cos (x)^3}+1}+1\right)$
Fricas 1.2	<pre>(17) -> (17) -> integrate(tan(x)/sqrt(sec(x)^3+1), x)</pre> $\log \left(\left(-\cos (x)^3-\cos (x)^2\right) \sqrt{\frac{4 \cos (x)^2-4 \cos (x)+4}{\cos (x)^2+\cos (x)}}-2 \cos (x)\right)$ <pre>(17) ----- Type: Union(Expression(Integer),...) (18) -> </pre>

sympy 0.7.1	<pre>>>> >>> integrate(tan(x)/sqrt(sec(x)**3+1),x) Traceback (most recent call last): File "<stdin>", line 1, in <module> NameError: name 'sec' is not defined >>></pre>
Axiom 5/12	<pre>(11) -> (11) -> integrate(tan(x)/sqrt(sec(x)^3+1),x) (11) -> (11) - log 6 5 3 2 (- 4cos(x) - 4cos(x) - 2cos(x) - 2cos(x)) * +-----+ 2 4cos(x) - 4cos(x) + 4 ----- 2 cos(x) + cos(x) +-----+ + 6 3 8cos(x) + 8cos(x) + 1 / 6 Type: Union(Expression Integer,...) (12) -> █</pre>
Maxima 5.28.02	<pre>integrate(tan(x)/sqrt(sec(x)^3+1),x);</pre> $\frac{\log\left(\sqrt{\frac{1}{\cos(x)^3}+1}-1\right)}{3}-\frac{\log\left(\sqrt{\frac{1}{\cos(x)^3}+1}+1\right)}{3}$
reduce 2008	<pre>int(tan(x)/sqrt(sec(x)^3+1),x);</pre> $\int \frac{\sqrt{\sec(x)^3+1} \tan(x)}{\sec(x)^3+1} dx$
mupad 2013a	<pre>evalin(symengine,'int(tan(x)/sqrt(sec(x)^3+1),x)') int(tan(x)/(1/cos(x)^3 + 1)^(1/2), x)</pre>
xcas	<pre>int(tan(x)/sqrt(sec(x)^3+1),x)</pre> Warning, integration of abs or sign assumes constant sign by intervals (c Check Vector [abs(cos(x))] Discontinuities at zeroes of cos(x) were not checked $\int \frac{1}{\sqrt{\cos(x)^4 + \cos(x)}} * \left(\frac{1}{\cos(x)}\right)^2 * \left(\frac{1}{\sin(x)}\right) dx$

8

$$\int \sqrt{\tan^2(x) + 2 \tan(x) + 2} \, dx$$

optimal 	$\sqrt{\frac{1}{2} \left(1 + \sqrt{5}\right) \tan^{-1}\left(\frac{\sqrt{1 + \sqrt{5} \tan(x)} - \sqrt{\sqrt{5} - 1}}{\sqrt{2} \sqrt{\tan(x) (\tan(x) + 2) + 2}}\right)} - \sqrt{\frac{1}{2} \left(\sqrt{5} - 1\right) \tanh^{-1}\left(\frac{\sqrt{\sqrt{5} - 1} \tanh(x)}{\sqrt{2} \sqrt{\tan(x) (\tan(x) + 2) + 2}}\right)} + \sinh^{-1}(\tan(x) + 1)$
M 9.01	click to view
Rubi 4.1	Int[$\sqrt{\text{Tan}[x]^2 + 2 \text{Tan}[x] + 2}$, x] $\text{ArcSinh}[1 + \text{Tan}[x]] - \frac{1}{2} i \sqrt{1 - 2 i} \text{ArcTanh}\left[\frac{(4 - 2 i) + (2 + 2 i) \text{Tan}[x]}{2 \sqrt{1 - 2 i} \sqrt{2 + 2 \text{Tan}[x] + \text{Tan}[x]^2}}\right] + \frac{1}{2} i \sqrt{1 + 2 i} \text{ArcTanh}\left[\frac{(4 + 2 i) + (2 + 2 i) \text{Tan}[x]}{2 \sqrt{1 + 2 i} \sqrt{2 + 2 \text{Tan}[x] + \text{Tan}[x]^2}}\right]$
Maple 18	image
Sage 5.4	<pre>integral(sqrt(tan(x)^2+2*tan(x)+2), x) Traceback (click to the left of this block) ... RuntimeError: ECL says: Error executing cannot be imaginary; found %i</pre>
Fricas 1.2	<pre>(18) -> (18) -> integrate(sqrt(tan(x)^2+2*tan(x)+2), x) x +-----+ ++ 2 (18) \ tan(%A) + 2tan(%A) + 2 d%A ++ Type: Union(Expression(Integer),...) (19) -> </pre>
sympy 0.7.1	<pre>>>> >>> integrate(sqrt(tan(x)**2 + 2 *tan(x)+2),x) Integral((tan(x)**2 + 2*tan(x) + 2)**(1/2), x) >>> </pre>
Axiom 5/12	HTML
Maxima 5.28.02	<pre>(%i12) integrate(sqrt(tan(x)^2+2*tan(x)+2),x); sign: argument cannot be imaginary; found %i -- an error. To debug this try: debugmode(true);</pre>
reduce 2008	<pre>int(sqrt(tan(x)^2+2*tan(x)+2),x);</pre> $\int \sqrt{\tan^2(x) + 2 \tan(x) + 2} \, dx$
mupad 2013a	<pre>evalin(symengine,'int(sqrt(tan(x)^2+2*tan(x)+2),x)') int((2*tan(x) + tan(x)^2 + 2)^(1/2), x)</pre>

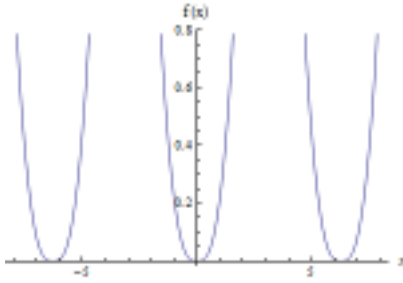
xcas

```
int(sqrt(tan(x)^2+2*tan(x)+2),x)
```

$$\frac{((8^{\frac{1}{4}})^3 + 8 * 8^{\frac{1}{4}}) * \ln((\sqrt{\tan(x)^2 + 2 * \tan(x) + 2} - \tan(x))^2 + (-8^{\frac{1}{4}} * (\sqrt{2}))^2)}{64}$$
$$(- (8^{\frac{1}{4}})^3 - 8 * (\sqrt{2}) + 8 * 8^{\frac{1}{4}}) * \operatorname{atan}\left(\frac{(\sqrt{\tan(x)^2 + 2 * \tan(x) + 2} - \tan(x)) - (-8^{\frac{1}{4}} * (\sqrt{2}))}{8^{\frac{1}{4}}}\right)$$
$$\frac{32}{2 * ((- (8^{\frac{1}{4}})^3 - 8 * 8^{\frac{1}{4}}) * \ln((\sqrt{\tan(x)^2 + 2 * \tan(x) + 2} - \tan(x))^2 + 8^{\frac{1}{4}} * (\sqrt{2})^2) + (- (8^{\frac{1}{4}})^3 + 8 * (\sqrt{2}) + 8 * 8^{\frac{1}{4}}) * \operatorname{atan}\left(\frac{(\sqrt{\tan(x)^2 + 2 * \tan(x) + 2} - \tan(x)) - (-8^{\frac{1}{4}} * (\sqrt{2}))}{8^{\frac{1}{4}}}\right))}$$

9

$$\int \sin(x) \arctan \left(\sqrt{\sec(x) - 1} \right) dx$$

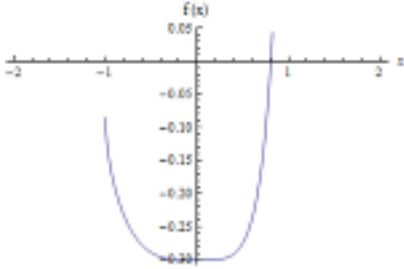
optimal 	$\frac{1}{2} \cos(x) \sqrt{\sec(x) - 1} + \frac{1}{2} \tan^{-1} \left(\sqrt{\sec(x) - 1} \right) - \cos(x) \tan^{-1} \left(\sqrt{\sec(x) - 1} \right)$
M 9.01	click to view
Rubi 4.1	Int[Sin[x] ArcTan[Sqrt[Sec[x] - 1]], x] $\frac{1}{2} \operatorname{ArcTan} \left[\sqrt{-1 + \operatorname{Sec}[x]} \right] - \operatorname{ArcTan} \left[\sqrt{-1 + \operatorname{Sec}[x]} \right] \operatorname{Cos}[x] + \frac{1}{2}$
Maple 18	<pre>restart; int(sin(x)*arctan(sqrt(sec(x)-1)), x);</pre> $\int \sin(x) \arctan(\sqrt{\sec(x) - 1}) \, dx$
Sage 5.4	integral(sin(x)*arctan(sqrt(sec(x)-1)), x) $-\cos(x) \arctan \left(\sqrt{-\frac{\cos(x)-1}{\cos(x)}} \right) - \frac{\sqrt{-\frac{\cos(x)-1}{\cos(x)}}}{2 \left(\frac{\cos(x)-1}{\cos(x)} - 1 \right)} + \frac{1}{2} \arctan$

Fricas 1.2	<pre>(19) -> (19) -> integrate(sin(x)*atan(sqrt(sec(x)-1)),x) >> Error detected within library code: failed - cannot handle that integrand (19) -> </pre> update: per post on sci.math.symbolic on June 2008 BTW: Current developement FriCAS can also do #9: <pre>(3) -> integrate(sin(x)*atan(sqrt(sec(x) - 1)), x) (3)</pre> $\frac{\cos(x) \sqrt{\cos(x)} \operatorname{atan}\left(\sqrt{\sec(x) - 1}\right) + \operatorname{atan}\left(\frac{\sqrt{\cos(x) - 1}}{\cos(x)}\right) - 2 \cos(x) \operatorname{atan}\left(\sqrt{\sec(x) - 1}\right) + \operatorname{atan}\left(\frac{\sqrt{\cos(x) - 1}}{\cos(x)}\right)}{2}$ Type: Union(Expression Integer,...)
sympy 0.7.1	<pre>>>> >>> integrate(sin(x)*atan(sqrt(sec(x)-1)),x) Traceback (most recent call last): File "<stdin>", line 1, in <module> NameError: name 'sec' is not defined >>></pre>
Axiom 5/12	<pre>(13) -> (13) -> integrate(sin(x)*atan(sqrt(sec(x)-1)),x) (13) -> >> Error detected within library code: failed - cannot handle that integrand (13) -> </pre>
Maxima 5.28.02	<pre>integrate(sin(x)*atan(sqrt(sec(x)-1)),x);</pre> $-\cos(x) \operatorname{atan}\left(\sqrt{\frac{\cos(x)-1}{\cos(x)}}\right) + \frac{\operatorname{atan}\left(\sqrt{\frac{\cos(x)-1}{\cos(x)}}\right)}{2} + \frac{\sqrt{\frac{\cos(x)-1}{\cos(x)}}}{2 \frac{\cos(x)-1}{\cos(x)}}$
reduce 2008	<pre>int(sin(x)*atan(sqrt(sec(x)-1)),x);</pre> $\int \arctan\left(\sqrt{\sec(x) - 1}\right) \sin(x) dx$

mupad 2013a	<pre>evalin(symengine,'int(sin(x)*atan(sqrt(sec(x)-1)),x)')</pre> <pre>pretty(ans)</pre> $-\frac{\operatorname{atan}\left(\frac{1}{\cos(x)}\right)}{\cos(x)} - \frac{\sqrt{\frac{1}{2}} \sqrt{\cos(x)}}{\sqrt{2 \cos(x)}}$
xcas	<pre>int(sin(x)*atan(sqrt(sec(x)-1)),x)</pre> Warning, integration of abs or sign assumes constant sign by intervals (c Check Vector [abs(cos(x))] Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Discontinuities at zeroes of cos(x) were not checked Evaluation time: 11.933 $-\operatorname{atan}\left(\frac{\sqrt{-(-\cos(x))^2+\cos(x)}\operatorname{sign}(\cos(x))\cos(x)}{\cos(x)^2}\right)\cos(x) + 2(-\operatorname{sign}(\cos(x)))^3 \cdot \frac{1}{(\operatorname{sign}(\cos(x)))^2-1} \cdot \frac{1}{2} \cdot \frac{1}{2}\operatorname{asin}(2 \cdot -\cos(x)+1) + 2(\operatorname{sign}(\cos(x)))^3 \cdot \frac{1}{(\operatorname{sign}(\cos(x)))^2-1}$ $\frac{1}{\operatorname{abs}(\operatorname{sign}(\cos(x)))}\operatorname{atan}\left(\frac{(\operatorname{sign}(\cos(x)))^2(2\sqrt{-(-\cos(x))^2+\cos(x)}-1)-(\operatorname{sign}(\cos(x)))^2+\frac{2\sqrt{-(-\cos(x))^2+\cos(x)}}{-2\cdot-\cos(x)-1}}{\operatorname{abs}(\operatorname{sign}(\cos(x)))\cdot 2}\right)$

10

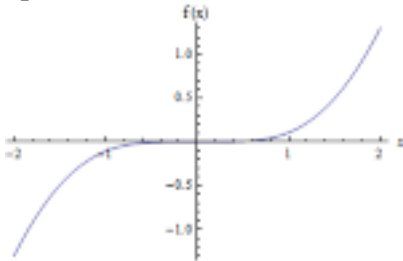
$$\int \frac{x^3 e^{\arcsin(x)}}{\sqrt{1-x^2}} dx$$

optimal 	$\frac{1}{10} \left(x^3 - 3\sqrt{1-x^2} x^2 - 3\sqrt{1-x^2} + 3x \right) e^{\sin^{-1}(x)}$
M 9.01	$\text{Integrate}\left[\frac{x^3 \text{Exp}[\text{ArcSin}[x]]}{\sqrt{1-x^2}}, x\right]$ $-\frac{1}{40} e^{\text{ArcSin}[x]} \left(15 \left(-x + \sqrt{1-x^2} \right) - 3 \cos[3 \text{ArcSin}[x]] + \sin[3 \text{ArcSin}[x]] \right)$
Rubi 4.1	$\text{Int}\left[\frac{x^3 \text{Exp}[\text{ArcSin}[x]]}{\sqrt{1-x^2}}, x\right]$ $\frac{3}{10} e^{\text{ArcSin}[x]} x + \frac{1}{10} e^{\text{ArcSin}[x]} x^3 - \frac{3}{10} e^{\text{ArcSin}[x]} \sqrt{1-x^2} - \frac{3}{10} e^{\text{ArcSin}[x]}$
Maple 18	<pre>> restart; int(x^3*exp(1)^arcsin(x)/sqrt(1-x^2), x); 1/10 (x-3 sqrt(-x^2+1)) e^arcsin(x) x^2 + 3/10 e^arcsin(x) (x-sqrt(-x^2+1))</pre>
Sage 5.4	integral(x^3*exp(arcsin(x))/sqrt(1-x^2), x) $\int \frac{x^3 e^{\arcsin(x)}}{\sqrt{-x^2+1}} dx$
Fricas 1.2	<pre>(19) -> (19) -> integrate(x^3*exp(asin(x))/sqrt(1-x^2),x) +-----+ 2 x\ - x + 1 - atan(-----) 2 x - 1 +-----+ 2 3 (- 3x - 3)\ - x + 1 + x + 3x)%e (19) ----- 10 Type: Union(Expression(Integer),...)</pre> <pre>(20) -> (20) -> █</pre>
sympy 0.7.1	<pre>>>> >>> integrate(x**3*exp(asin(x))/sqrt(1-x**2),x) x**3*exp(asin(x))/10 - 3*x**2*(-x**2 + 1)**(1/2)*exp(asin(x))/10 + 3*x*exp(asin(x))** (1/2)*exp(asin(x))/10 >>> >>> _</pre>

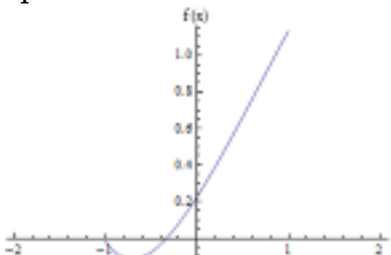
Axiom 5/12	<pre>(13) -> (13) -> integrate(x^3*exp(asin(x))/sqrt(1-x^2),x) (13) -></pre> $\frac{(((-3x^2 - 3)\sqrt{-x^2 + 1} + x^3 + 3x)e^{\operatorname{atan}\left(\frac{2x\sqrt{-x^2 + 1}}{2x^2 - 1}\right)}}{10}$ <pre>(14) -> </pre>
Maxima 5.28.02	<pre>integrate(x^3*exp(asin(x))/sqrt(1-x^2),x);</pre> $\int \frac{x^3 e^{\operatorname{asin}(x)}}{\sqrt{1-x^2}} dx$
reduce 2008	<pre>int(x^3*exp(asin(x))/sqrt(1-x^2),x);</pre> $\int \frac{e^{\operatorname{arcsin}(x)} x^3}{\sqrt{-x^2 + 1}} dx$
mupad 2013a	<pre>evalin(symengine,'int(x^3*exp(asin(x))/sqrt(1-x^2),x)')</pre> <pre>int((x^3*exp(asin(x)))/(1 - x^2)^(1/2), x)</pre>
xcas	<pre>int(x^3*exp(asin(x))/sqrt(1-x^2),x)</pre> Simplification assuming x near 0+ Simplification assuming x near 0+ $\int x^3 * \frac{1}{\sqrt{-(x^2 - 1)}}$

11

$$\int \frac{x \log(1+x^2) \log(x+\sqrt{1+x^2})}{\sqrt{1+x^2}} dx$$

optimal 	$4 x - 2 \operatorname{ArcTan}[x] - 2 \sqrt{1+x^2} \operatorname{Log}\left[x+\sqrt{1+x^2}\right] + \operatorname{Log}\left[1+x^2\right] \left($
M 9.01	<pre> Integrate[(x*Log[1+x^2]*Log[x+Sqrt[1+x^2]]) / Sqrt[1+x^2], x] 4 x - 2 ArcTan[x] - 2 sqrt(1+x^2) Log[x+sqrt(1+x^2)] + Log[1+x^2] (-x+sqrt(1+x^2) Log[x+sqrt(1+x^2)]) </pre>
Rubi 4.1	<pre> Int[(x*Log[1+x^2]*Log[x+Sqrt[1+x^2]]) / Sqrt[1+x^2], x] 4 x - 2 ArcTan[x] - x Log[1+x^2] - 2 sqrt(1+x^2) Log[x+sqrt(1+x^2)] + sqrt(1+x^2) Log[1+x^2] Log[x+ </pre>
Maple 18	<pre> > restart; intg:= x*log(1+x^2)*log(x+sqrt(1+x^2))/sqrt(1+x^2); x ln(x^2 + 1) ln(x + sqrt(x^2 + 1)) ----- sqrt(x^2 + 1) > int(intg,x); ∫ x ln(x^2 + 1) ln(x + sqrt(x^2 + 1)) ----- dx sqrt(x^2 + 1) </pre>

12 $\int \tan^{-1} \left(\sqrt{1-x^2} + x \right) dx$

optimal 	$-\frac{\text{ArcSin}[x]}{2} + \frac{1}{4} \sqrt{3} \text{ArcTan}\left[\frac{-1 + \sqrt{3} x}{\sqrt{1-x^2}}\right] + \frac{1}{4} \sqrt{3} \text{ArcTan}\left[\frac{1 + \sqrt{3} x}{\sqrt{1-x^2}}\right]$ $+ \frac{1}{4} \sqrt{3} \text{ArcTan}\left[\frac{-1 + 2 x^2}{\sqrt{3}}\right] + x \text{ArcTan}\left[x + \sqrt{1-x^2}\right] - \frac{1}{4} \text{ArcTan}\left[\frac{1 + 2 x^2}{\sqrt{3}}\right]$
M 9.01	<pre>Integrate[ArcTan[x + Sqrt[1 - x^2]], x]</pre> $x \text{ArcTan}\left[x + \sqrt{1-x^2}\right] + \frac{1}{16} \left(-8 \text{ArcSin}[x] + 2 \sqrt{2+2i\sqrt{3}} \right.$ $\text{ArcTan}\left[\frac{(1+i\sqrt{3}-2x^2)(-1+x^2)}{(-3i-\sqrt{3}+2\sqrt{3}x^4+x^3(-6-2i\sqrt{3}-2\sqrt{2-2i\sqrt{3}}\sqrt{1-x^2})+x(6+2i\sqrt{3}-2\sqrt{2-2i\sqrt{3}}\sqrt{1-x^2})+x^2(3i-\sqrt{3}+2\sqrt{6-6i\sqrt{3}}\sqrt{1-x^2}))}\right] - 2\sqrt{2+2i\sqrt{3}}$ $\text{ArcTan}\left[\frac{(1+i\sqrt{3}-2x^2)(-1+x^2)}{(-3i-\sqrt{3}+2\sqrt{3}x^4+2x(-3-i\sqrt{3}+\sqrt{2-2i\sqrt{3}}\sqrt{1-x^2})+2x^3(3+i\sqrt{3}+\sqrt{2-2i\sqrt{3}}\sqrt{1-x^2})+x^2(3i-\sqrt{3}+2\sqrt{6-6i\sqrt{3}}\sqrt{1-x^2}))}\right] - 2\sqrt{2-2i\sqrt{3}}$ $\text{ArcTan}\left[\frac{(-1+x^2)(-1-i\sqrt{3}+2x^2)}{(3i-\sqrt{3}+2\sqrt{3}x^4+x(6-2i\sqrt{3}-2\sqrt{2+2i\sqrt{3}}\sqrt{1-x^2})+x^3(-6+2i\sqrt{3}-2\sqrt{2+2i\sqrt{3}}\sqrt{1-x^2})+x^2(-3i-\sqrt{3}+2\sqrt{6+6i\sqrt{3}}\sqrt{1-x^2}))}\right] + 2\sqrt{2-2i\sqrt{3}}$ $\text{ArcTan}\left[\frac{(-1+x^2)(-1-i\sqrt{3}+2x^2)}{(3i-\sqrt{3}+2\sqrt{3}x^4+2x^3(3-i\sqrt{3}+\sqrt{2+2i\sqrt{3}}\sqrt{1-x^2})+2x(-3+i\sqrt{3}+\sqrt{2+2i\sqrt{3}}\sqrt{1-x^2})+x^2(-3i-\sqrt{3}+2\sqrt{6+6i\sqrt{3}}\sqrt{1-x^2}))}\right] -$ $2 \text{Log}\left[-\frac{1}{2} - \frac{i\sqrt{3}}{2} + x^2\right] + 2i\sqrt{3} \text{Log}\left[-\frac{1}{2} - \frac{i\sqrt{3}}{2} + x^2\right] - 2 \text{Log}\left[\frac{1}{2} i (i + \sqrt{3}) + x^2\right] -$ $2i\sqrt{3} \text{Log}\left[\frac{1}{2} i (i + \sqrt{3}) + x^2\right] -$ $i\sqrt{2-2i\sqrt{3}} \text{Log}\left[16(1+\sqrt{3}x+x^2)^2\right] +$ $i\sqrt{2+2i\sqrt{3}} \text{Log}\left[16(1+\sqrt{3}x+x^2)^2\right] +$ $i\sqrt{2-2i\sqrt{3}} \text{Log}\left[(4-4\sqrt{3}x+4x^2)^2\right] -$ $i\sqrt{2+2i\sqrt{3}} \text{Log}\left[(4-4\sqrt{3}x+4x^2)^2\right] -$ $i\sqrt{2+2i\sqrt{3}} \text{Log}\left[3i+\sqrt{3} - (-i+\sqrt{3})x^4 + 2i\sqrt{2-2i\sqrt{3}}\sqrt{1-x^2} + 5ix^2(2+\sqrt{2-2i\sqrt{3}}\sqrt{1-x^2}) + x(3-5i\sqrt{3}+3i\sqrt{6-6i\sqrt{3}}\sqrt{1-x^2}) + ix^3(3i+3\sqrt{3}+\sqrt{6-6i\sqrt{3}}\sqrt{1-x^2})\right] +$ $i\sqrt{2+2i\sqrt{3}} \text{Log}\left[3i+\sqrt{3} - (-i+\sqrt{3})x^4 + 2i\sqrt{2-2i\sqrt{3}}\sqrt{1-x^2} + 5ix^2(2+\sqrt{2-2i\sqrt{3}}\sqrt{1-x^2}) + x^3(3-3i\sqrt{3}-i\sqrt{6-6i\sqrt{3}}\sqrt{1-x^2}) - ix(-3i+5\sqrt{3}+3\sqrt{6-6i\sqrt{3}}\sqrt{1-x^2})\right] +$ $i\sqrt{2-2i\sqrt{3}} \text{Log}\left[-3i+\sqrt{3} - (i+\sqrt{3})x^4 - 2i\sqrt{2+2i\sqrt{3}}\sqrt{1-x^2} - 5ix^2(2+\sqrt{2+2i\sqrt{3}}\sqrt{1-x^2}) + x(3-5i\sqrt{3}-3i\sqrt{6+6i\sqrt{3}}\sqrt{1-x^2}) - ix^3(-3i+3\sqrt{3}+\sqrt{6+6i\sqrt{3}}\sqrt{1-x^2})\right] -$ $i\sqrt{2-2i\sqrt{3}} \text{Log}\left[-3i+\sqrt{3} - (i+\sqrt{3})x^4 - 2i\sqrt{2+2i\sqrt{3}}\sqrt{1-x^2} - 5ix^2(2+\sqrt{2+2i\sqrt{3}}\sqrt{1-x^2}) + x^3(3+3i\sqrt{3}+i\sqrt{6+6i\sqrt{3}}\sqrt{1-x^2}) + ix(3i+5\sqrt{3}+3\sqrt{6+6i\sqrt{3}}\sqrt{1-x^2})\right]$

Rubi 4.1	<code>Int[ArcTan[x + Sqrt[1 - x^2]], x]</code> $-\frac{\text{ArcSin}[x]}{2} + \frac{1}{4} \sqrt{3} \text{ArcTan}\left[\frac{1 - 2x^2}{\sqrt{3}}\right] + \frac{\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right]}{\sqrt{3}} +$ $\frac{1}{12} (3i - \sqrt{3}) \text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right] + \frac{\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}x}{\sqrt{1-x^2}}\right]}{\sqrt{3}} -$ $\frac{1}{12} (3i + \sqrt{3}) \text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}x}{\sqrt{1-x^2}}\right] + x \text{ArcTan}\left[x + \sqrt{1-x^2}\right] - \frac{1}{8} \text{Log}\left[1-x^2 + x\right]$
Maple 18	<pre>> restart; intg:= arctan(sqrt(1-x^2)+x); int(intg,x);</pre> $\text{intg} := \arctan(\sqrt{-x^2+1} + x)$ $x \arctan(\sqrt{-x^2+1} + x) - \frac{1}{8} \ln(x^4 - x^2 + 1) - \frac{1}{4} \sqrt{3} \arctan\left(\frac{1}{3} (2x^2 - 1) \sqrt{3}\right) + \frac{1}{8} \ln\left(\frac{(\sqrt{-x^2+1} - 1)^2}{x^2}\right)$ $+ \frac{(-1 - I\sqrt{3}) (\sqrt{-x^2+1} - 1)}{x} - 1\Big) - \frac{1}{8} I\sqrt{3} \ln\left(\frac{(\sqrt{-x^2+1} - 1)^2}{x^2} + \frac{(1 + I\sqrt{3}) (\sqrt{-x^2+1} - 1)}{x} - 1\right)$ $+ \frac{1}{8} \ln\left(\frac{(\sqrt{-x^2+1} - 1)^2}{x^2} + \frac{(-1 + I\sqrt{3}) (\sqrt{-x^2+1} - 1)}{x} - 1\right) - \frac{1}{8} I\sqrt{3} \ln\left(\frac{(\sqrt{-x^2+1} - 1)^2}{x^2} + \frac{(1 + I\sqrt{3}) (\sqrt{-x^2+1} - 1)}{x} - 1\right)$ $+ \frac{(-1 + I\sqrt{3}) (\sqrt{-x^2+1} - 1)}{x} - 1\Big) - \frac{1}{8} \ln\left(\frac{(\sqrt{-x^2+1} - 1)^2}{x^2} + \frac{(1 + I\sqrt{3}) (\sqrt{-x^2+1} - 1)}{x} - 1\right)$ $+ \frac{1}{8} I\sqrt{3} \ln\left(\frac{(\sqrt{-x^2+1} - 1)^2}{x^2} + \frac{(1 - I\sqrt{3}) (\sqrt{-x^2+1} - 1)}{x} - 1\right) - \frac{1}{8} \ln\left(\frac{(\sqrt{-x^2+1} - 1)^2}{x^2} + \frac{(1 - I\sqrt{3}) (\sqrt{-x^2+1} - 1)}{x} - 1\right)$ $+ \frac{(1 - I\sqrt{3}) (\sqrt{-x^2+1} - 1)}{x} - 1\Big) + \frac{1}{8} I\sqrt{3} \ln\left(\frac{(\sqrt{-x^2+1} - 1)^2}{x^2} + \frac{(-1 - I\sqrt{3}) (\sqrt{-x^2+1} - 1)}{x} - 1\right)$ $+ \arctan\left(\frac{\sqrt{-x^2+1} - 1}{x}\right)$