Ten Hard Integrals

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The first 10 integrals from Kevin Charlwood's 2008 article "Integration on Computer Algebra Systems" are solved using different CAS systems.

The original post on this topic is sci.math.symbolic by Martin

These are the CAS systems used

1. Maple 18 on windows 7 (64 bit)
2. Mathematica 9.01 on windows 7
3. Rubi 4.1 on Mathematica 9.01 on windows 7
4. Sage 5.4 using the Sage web server notebook interface
5. Fricas 1.2 on Linux using sbcl lisp
6. wxMaxima 12.04.0 (Maxima 5.28.02) on windows 7
7. Axiom on windows 7 (May 2012) welcome screen image
8. sympy on linux (python 2.7.3 full installation. sympy 0.7.1.rc1-3) starting image
9. reduce `reduce-windows64-20110414` help screen message with the `algint` package loaded.
10. mupad engine in Matlab 2013a symbolic toolbox
11. xcas 2013 January, version 1.0 on windows 7

Optimal answer to each intergal taken from Rich's referenced PDF below.

Downloads, references and links

1. Mathematica downloads

- Mathematica notebook of the first 20 integrals
- Mathematica PDF of the first 20 integrals
- Mathematica HTML of the first 20 integrals

2. Rubi downloads
   - Rubi notebook for the first 20 integrals
   - Rubi PDF for the first 20 integrals
   - Rubi HTML for the first 20 integrals
3. Maple downloads
   - Maple worksheet in wm format
   - HTML worksheet in HTML
   - PDF worksheet in PDF

   Maple Problem 10 trace entered as `int(x^3*exp(1)^arcsin(x)/sqrt(1-x^2),x);`
   - HTML
   - PDF
   - mw Maple worksheet

   Maple Problem 10 trace entered as `int(x^3*exp(arcsin(x))/sqrt(1-x^2),x);`
   - HTML
   - PDF
   - mw Maple worksheet
4. maxima.wxm Maxima notebook.
5. copy of Kevin Charlwood's 2008 paper in PDF
6. `http://www.apmaths.uwo.ca/~arich/CharlwoodIntegrationProblems.pdf` Albert Rich pdf file showing 50 integrals and the best antiderivatives expected
7. http://www.apmaths.uwo.ca/ arich/CharlwoodProblems.m The above is m format.
8. `http://www.math.utah.edu/faq/reduce/`
9. `http://www.reduce-algebra.com/packages.htm`

10. `http://reduce-algebra.sourceforge.net/`

11. `http://www.reduce-algebra.com/docs/reduce.pdf`

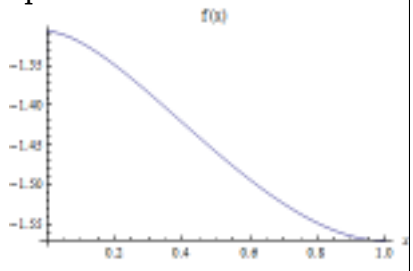
12. xCAS web page

13. `http://www.apmaths.uwo.ca/~arich/` Rubi Mathematica package home

The following is summary of results for each integral. Result with a `()` around it means the antiderivative contains nonelementary functions.

system	1	2	3	4	5	6	7	8	9	10	score
Mathematica 9.01	✓	✓	✓	✓	(✓)	(✓)	(✓)	(✓)	(✓)	✓	100%
Rubi 4.1	✓	✓	✗	✓	(✓)	✓	✓	✓	✓	✓	90%
Maple 18	✓	✓	✓	✓	(✓)	✓	✓	✓	✗	✓	90%
Axiom (May 2012)	✓	✓	✗	✓	✓	✓	✓	✓	✗	✓	80%
FriCAS 1.2	✓	✓	✓	✓	✗	✓	✓	✗	✓	✓	80%
Sage 5.4	✓	✓	✗	✗	✗	✗	✓	✗	✓	✗	40%
Maxima 5.28.02	✓	✓	✗	✗	✗	✗	✓	✗	✓	✗	40%
xcas 1.0	✗	✗	✗	✓	✗	✓	✗	✓	✓	✗	40%
Sympy 0.7.2	✗	✓	✗	✗	✗	✗	✗	✗	✗	✓	20%
Reduce 2008	✗	✓	✗	✓	✗	✗	✗	✗	✗	✗	20%
mupad 2013a	✗	✗	✗	✓	✗	✗	✗	✗	✓	✗	20%

1 $\int \arcsin(x)\ln(x)\,dx$

optimal 	$-2\sqrt{1-x^2}+\sqrt{1-x^2}\log(x)+\tanh^{-1}\left(\sqrt{1-x^2}\right)-x(1-\log(x))\sin^{-1}(x)$
M 9.01	`Integrate[ArcSin[x] Log[x], x]` $-2\sqrt{1-x^2}+x\,\text{ArcSin}[x]\,(-1+\text{Log}[x])+\left(-1+\sqrt{1-x^2}\right)\text{Log}[x]+\text{Log}\left[1+\sqrt{1-x^2}\right]$
Rubi 4.1	`Int[ArcSin[x] Log[x], x]` $-2\sqrt{1-x^2}-x\,\text{ArcSin}[x]+\text{ArcTanh}\left[\sqrt{1-x^2}\right]+\sqrt{1-x^2}\,\text{Log}[x]$

Maple 18	`> restart;`
	`int(arcsin(x)*log(x),x);`
	$\frac{1}{1+\tan\left(\frac{1}{2}\arcsin(x)\right)^2}\left(-2\arcsin(x)\tan\left(\frac{1}{2}\arcsin(x)\right)-2\tan\left(\frac{1}{2}\arcsin(x)\right)^2\ln\left(\frac{2\tan\left(\frac{1}{2}\arcsin(x)\right)}{1+\tan\left(\frac{1}{2}\arcsin(x)\right)^2}\right)+2\arcsin(x)\tan\left(\frac{1}{2}\arcsin(x)\right)\ln\left(\frac{2\tan\left(\frac{1}{2}\arcsin(x)\right)}{1+\tan\left(\frac{1}{2}\arcsin(x)\right)^2}\right)-4\right)-\ln\left(1\right.$
	=

Sage 5.4	`integral(arcsin(x)*ln(x), x)` $(x\log(x)-x)\arcsin(x)+\sqrt{-x^2+1}\log(x)-2\sqrt{-x^2}$

Fricas 1.2

```
(8) -> integrate(asin(x)*log(x), x)

   (8)
            +--------+                  +--------+
            |   2                       |   2
      log(2\|- x  + 1  + 2) - log(- 2\|- x  + 1  + 2)
     +
                                +--------+
                                |   2                          +----
                              x\|- x  + 1                      |   2
      (- 2x log(x) + 2x)atan(------------) + (2log(x) - 4)\|- x
                                   2
                                  x  - 1
  /
     2
                                                    Type: Union(Expression(I
```

sympy 0.7.1

```
>>> integrate(asin(x)*log(x),x)
Integral(log(x)*asin(x), x)
>>>
```

Axiom 5/12

```
(5) -> integrate(asin(x)*log(x),x)
(5) ->
   (5)
            +--------+                +--------+
            |   2                     |   2
      log(\|- x  + 1  + 1) - log(\|- x  + 1  - 1)
     +
                                +--------+
                                |   2                          +---
                             2x\|- x  + 1                      |
      (- x log(x) + x)atan(-------------) + (2log(x) - 4)\|- x
                                   2
                                 2x  - 1
  /
     2
                                                    Type: Union(Expressior
(6) ->
```

Maxima 5.28.02

```
integrate(asin(x)*log(x),x);
```

$$\log\left(\frac{2\sqrt{1-x^2}}{|x|}+\frac{2}{|x|}\right)+\operatorname{asin}(x)(x\log(x)-x)+\sqrt{1-x^2}\log(x)-2\sqrt{1-x^2}$$

reduce 2008

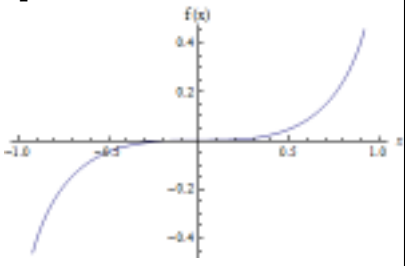
```
int(asin(x)*log(x),x);
```

$$\int \arcsin(x)\log(x)\,dx$$

HTML of trace
PDF of trace

mupad 2013a	evalin(symengine,'int(asin(x)*log(x),x)') int(asin(x)*log(x), x)
xcas	int(asin(x)*log(x),x) Simplification assuming x near 0+ Simplification assuming x near 0+ $\int \text{asin}(x)*\ln(x)\,dx$

2 $\int \frac{x \arcsin(x)}{\sqrt{1-x^2}}\,dx$

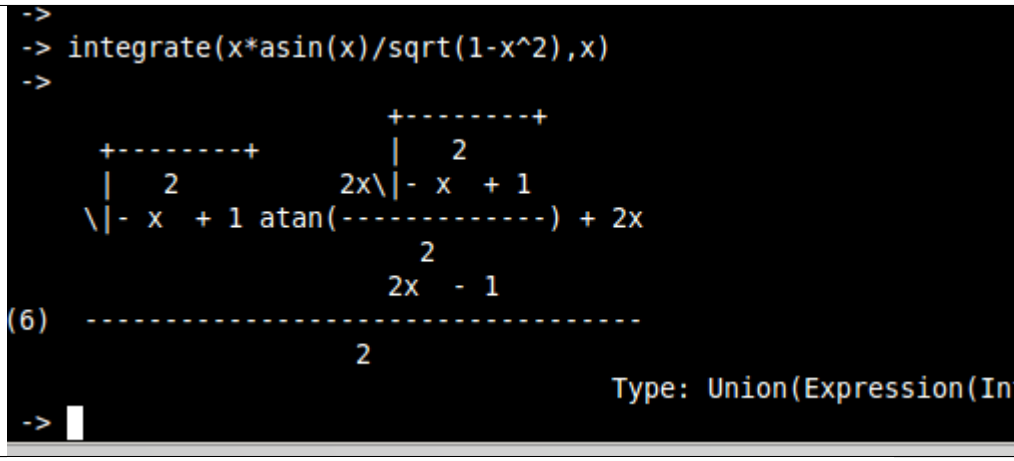
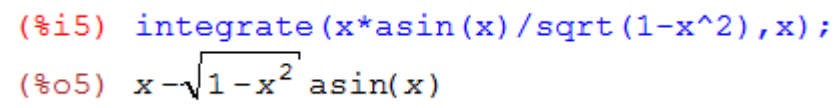
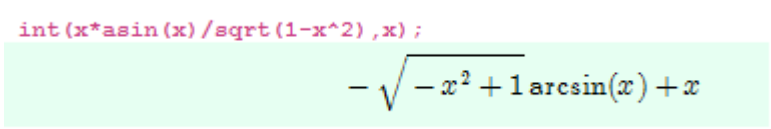
optimal 	$x - \sqrt{1-x^2}\sin^{-1}(x)$
M 9.01	$\int \frac{x \arcsin(x)}{\sqrt{1-x^2}}\,dx$ $x - \sqrt{1-x^2}\ \arcsin(x)$
Rubi 4.1	$\int \frac{x \arcsin(x)}{\sqrt{1-x^2}}\,dx$ $x - \sqrt{1-x^2}\ \arcsin(x)$

Maple 18	`> restart;`
	`int(x*arcsin(x)/sqrt(1-x^2),x);`
	$x - \arcsin(x)\sqrt{-x^2+1}$

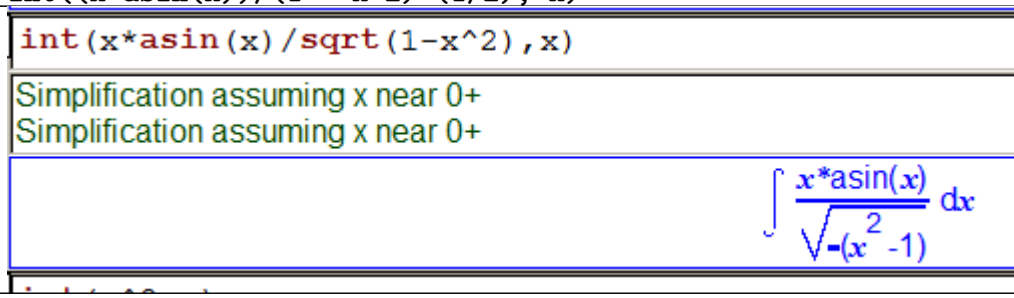
Sage 5.4	`integral(x*arcsin(x)/sqrt(1-x^2), x)` $-\sqrt{-x^2+1}\arcsin(x) + x$

Fricas 1.2	`(9) -> integrate(x*asin(x)/sqrt(1-x^2), x)`
	`(9)  |- x  + 1 atan(x|- x  + 1 / (x  - 1)) + x`
	`Type: Union(Expression(I`
	`(10) ->`

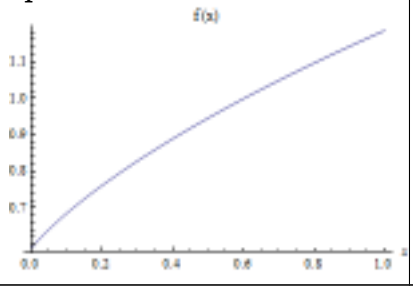
sympy 0.7.1	`>>> integrate(x*asin(x)/sqrt(1-x**2),x)`
	`x - (-x**2 + 1)**(1/2)*asin(x)`
	`>>>`
	`>>>`

Axiom 5/12	
Maxima 5.28.02	
reduce 2008	

mupad 2013a	`evalin(symengine,'int(x*asin(x)/sqrt(1-x^2),x)')`
	
	`int((x*asin(x))/(1 - x^2)^(1/2), x)`

xcas	

3 $\int \arcsin\left(\sqrt{x+1}-\sqrt{x}\right)\,dx$

optimal 	$\frac{(\sqrt{x}+3\sqrt{x+1})\sqrt{\sqrt{x}\sqrt{x+1}-x}}{4\sqrt{2}}-\left(x+\frac{3}{8}\right)\sin^{-1}\left(\sqrt{x}-\sqrt{x+1}\right)$
M 9.01	Integrate[ArcSin[$\sqrt{x+1}-\sqrt{x}$], x] $-x\,\text{ArcSin}\left[\sqrt{x}-\sqrt{1+x}\right]-$ $\left((1+x)\left(1+2x-2\sqrt{x}\sqrt{1+x}\right)^2\left(2\sqrt{-x+\sqrt{x}\sqrt{1+x}}\right.\right.$ $\left(-3-2x+2\sqrt{x}\sqrt{1+x}\right)+3\sqrt{-2-4x+4\sqrt{x}\sqrt{1+x}}$ $\left.\left.\text{Log}\left[2\sqrt{-x+\sqrt{x}\sqrt{1+x}}+\sqrt{-2-4x+4\sqrt{x}\sqrt{1+x}}\right]\right)\right)$ $\left(8\sqrt{2}\left(-\sqrt{x}+\sqrt{1+x}\right)^3\left(1+x-\sqrt{x}\sqrt{1+x}\right)^2\right)$
Rubi 4.1	Int[ArcSin[$\sqrt{x+1}-\sqrt{x}$], x] $-x\,\text{ArcSin}\left[\sqrt{x}-\sqrt{1+x}\right]+\frac{\text{Subst}\left[\text{Int}\left[\sqrt{1-x^2+x\sqrt{-1+x^2}}\right.\right.}{\sqrt{2}}$

Maple 18

```
> restart;
  int(arcsin(sqrt(x+1)-sqrt(x)),x);
```

$$-\left(\frac{1}{16}\arcsin\left(-\sqrt{x+1}+\sqrt{x}\right)+\frac{3}{8}\tan\left(\frac{1}{2}\arcsin\left(-\sqrt{x+1}+\sqrt{x}\right)\right)^3-\frac{3}{8}\right.$$
$$+\sqrt{x}\Big)\Big)^5-\frac{1}{8}\tan\left(\frac{1}{2}\arcsin\left(-\sqrt{x+1}+\sqrt{x}\right)\right)^7+\frac{1}{8}\arcsin\left(-\sqrt{x+1}\right.$$
$$+\sqrt{x}\Big)\tan\left(\frac{1}{2}\arcsin\left(-\sqrt{x+1}+\sqrt{x}\right)\right)^2+\frac{9}{8}\arcsin\left(-\sqrt{x+1}+\sqrt{x}\right)t$$
$$-\sqrt{x+1}+\sqrt{x}\Big)\Big)^4+\frac{1}{8}\arcsin\left(-\sqrt{x+1}+\sqrt{x}\right)\tan\left(\frac{1}{2}\arcsin\left(-\sqrt{x+1}\right.\right.$$
$$-\sqrt{x+1}+\sqrt{x}\Big)\tan\left(\frac{1}{2}\arcsin\left(-\sqrt{x+1}+\sqrt{x}\right)\right)^8+\frac{1}{8}\tan\left(\frac{1}{2}\arcsin\left(-\right.\right.$$
$$\left(\left(1+\tan\left(\frac{1}{2}\arcsin\left(-\sqrt{x+1}+\sqrt{x}\right)\right)^2\right)^2\tan\left(\frac{1}{2}\arcsin\left(-\sqrt{x+1}+\sqrt{x}\right.\right.\right.$$

=

Sage 5.4

```
integral(arcsin(sqrt(x+1)-sqrt(x)), x)
```

evaluate

Fricas 1.2

```
(10) -> integrate(asin(sqrt(x+1)-sqrt(x)), x)

   (10)
                    +-----+    +-+                          +-------
                   \|x + 1  - \|x          +-----+    +-+  |  +-+ +
   (8x + 3)atan(--------------------) + (3\|x + 1  + \|x )\|2\|x \|
                +------------------+
                |  +-+ +-----+
               \|2\|x \|x + 1  - 2x
   ------------------------------------------------------------------
                                      8
                                      Type: Union(Expression(Inte
(11) ->
```

sympy 0.7.1

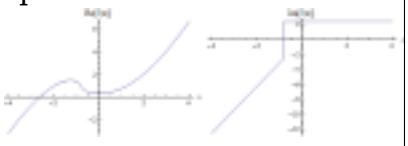
```
>>>
>>> integrate(asin( sqrt(x+1) - sqrt(x) ),x)
Integral(-asin(x**(1/2) - (x + 1)**(1/2)), x)
>>>
```

Axiom 5/12

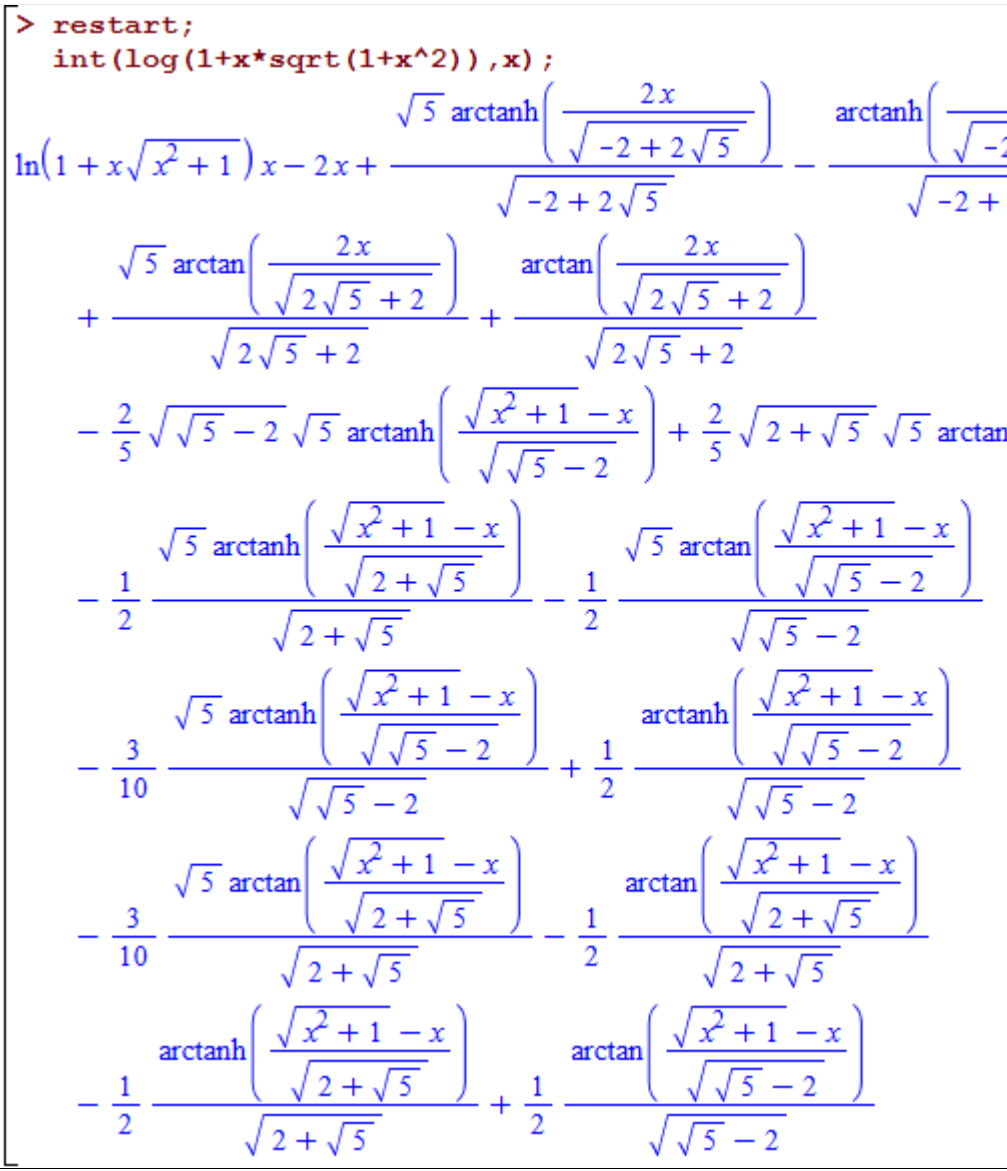
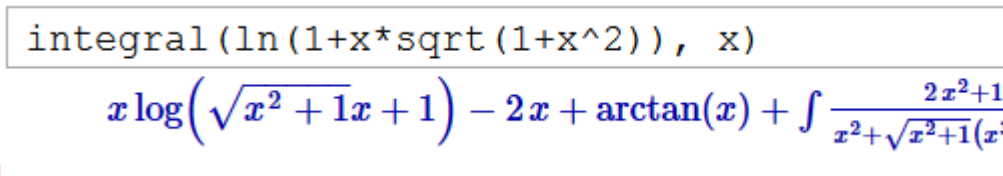
```
(7) ->
(7) -> integrate(asin(sqrt(x+1)-sqrt(x)),x)
 7) ->
   >> Error detected within library code:
   failed - cannot handle that integrand

(7) ->
```

Maxima 5.28.02	(%i6) integrate(asin(sqrt(x+1)-sqrt(x)),x); (%o6) $\frac{\pi x}{2}$
reduce 2008	int(asin(sqrt(x+1)-sqrt(x)),x); $\int \arcsin\left(\sqrt{x+1}-\sqrt{x}\right) dx$
mupad 2013a	evalin(symengine,'int(asin(sqrt(x+1)- sqrt(x)),x)') -int(asin(x^(1/2) - (x + 1)^(1/2)), x)
xcas	int(asin(sqrt(x+1)-sqrt(x)),x) Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0, Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0, Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0, Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0, Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0, Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0, Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0, Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0, Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0, sym2poly/r2sym(const gen & e,const index_m & i,const ve

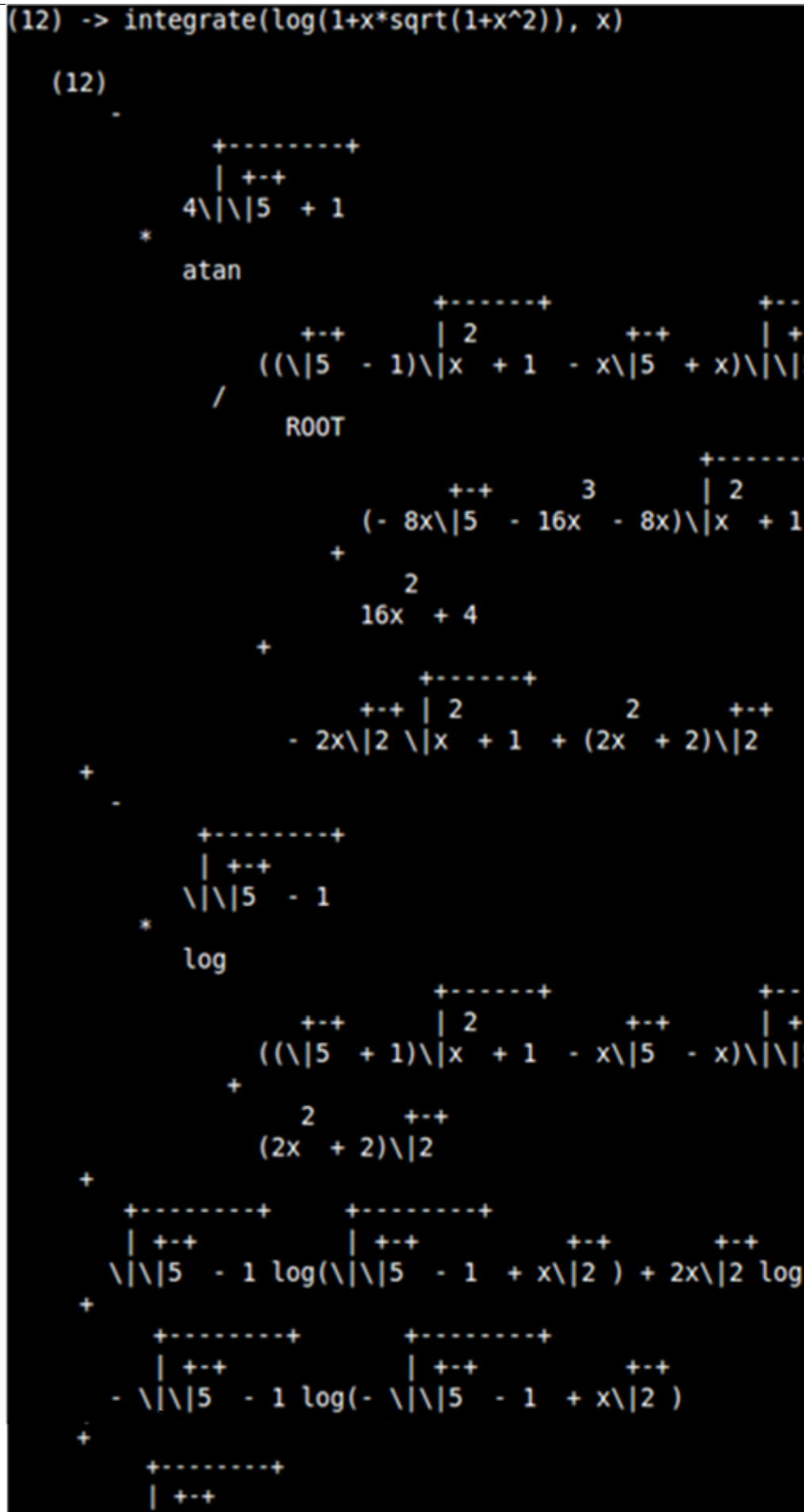
4 $\int \ln\left(1+x\sqrt{1+x^2}\right)\,dx$

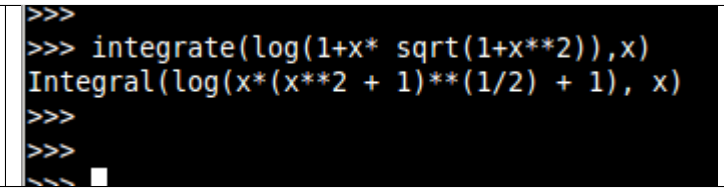
optimal 	$x\log\left(\sqrt{x^2+1}x+1\right) \quad + \quad \sqrt{2\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\sqrt{5}-2}(\sqrt{x^2+1}+$ $\sqrt{2\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\sqrt{2+\sqrt{5}}(\sqrt{x^2+1}+x)\right)-2x$
M 9.01	$\texttt{Integrate}\left[\texttt{Log}\left[1+x\sqrt{1+x^2}\right],\,x\right]$ $-2x+\frac{\left(5+\sqrt{5}\right)\texttt{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right]}{\sqrt{10\left(1+\sqrt{5}\right)}}+$ $\sqrt{\frac{2}{-1+\sqrt{5}}}\,\texttt{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\,\sqrt{1+x^2}\right]-\frac{\left(-5+\sqrt{5}\right)\texttt{ArcTa}}{\sqrt{10\left(-1\right.}}$ $\sqrt{\frac{2}{1+\sqrt{5}}}\,\texttt{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,\sqrt{1+x^2}\right]+x\,\texttt{Log}\left[1+x\sqrt{1+x^2}\right.$

Rubi 4.1	$$\text{Int}\left[\text{Log}\left[1+x\sqrt{1+x^2}\,\right],x\right]$$ $$-2x-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\,\text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right]+$$ $$2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\,\text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right]+\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\,\text{Arc}$$ $$\sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\,\text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\,\sqrt{1+x^2}\,\right]+2\sqrt{\frac{1}{5}\left(-2+\right.}$$ $$\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\,\text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right]+\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\,\text{A}$$ $$\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\,\text{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,\sqrt{1+x^2}\,\right]+x\,\text{Log}\left[1+x\sqrt{\right.}$$

Maple 18	
Sage 5.4	

Fricas 1.2



sympy 0.7.1	

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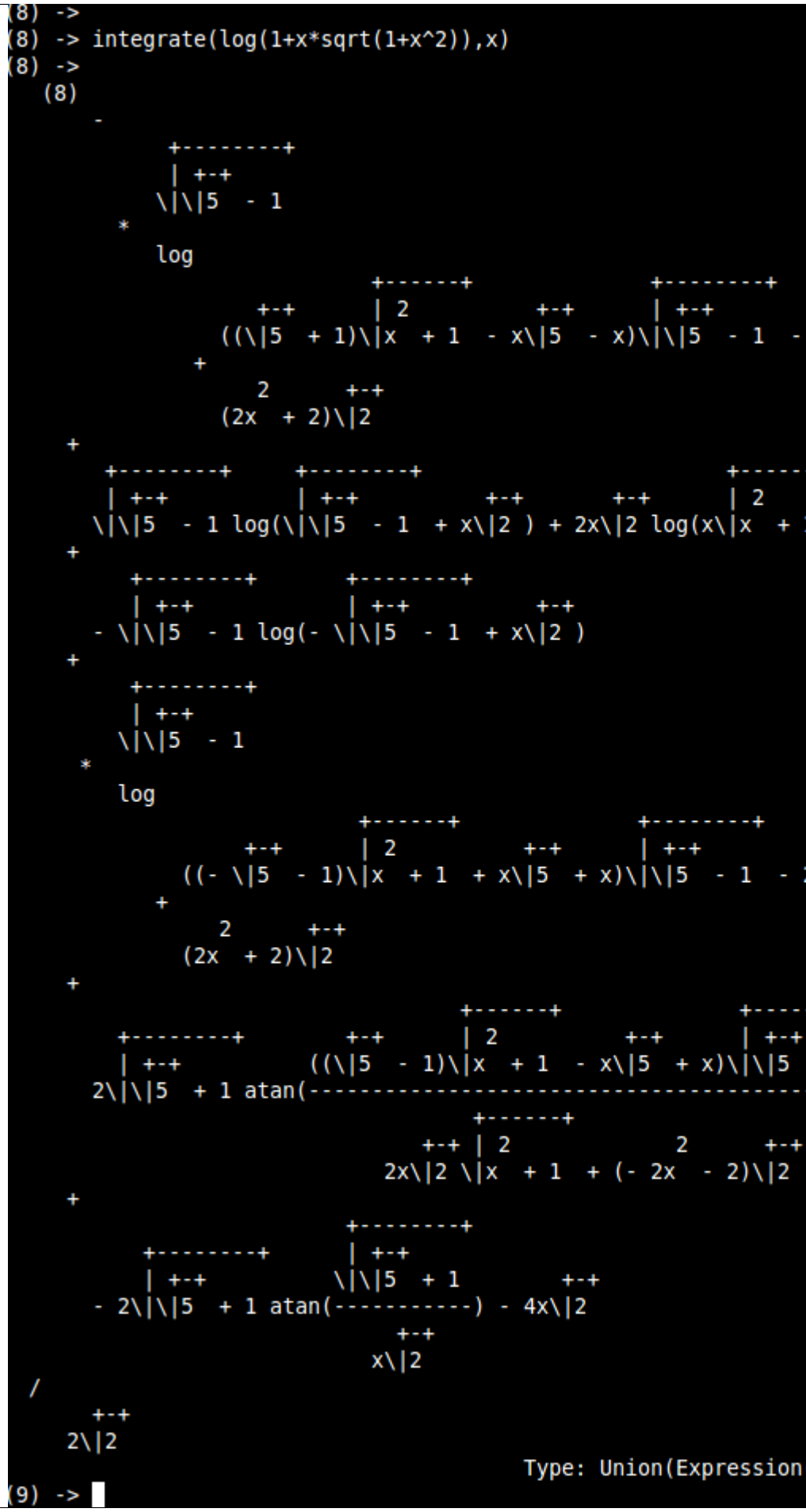
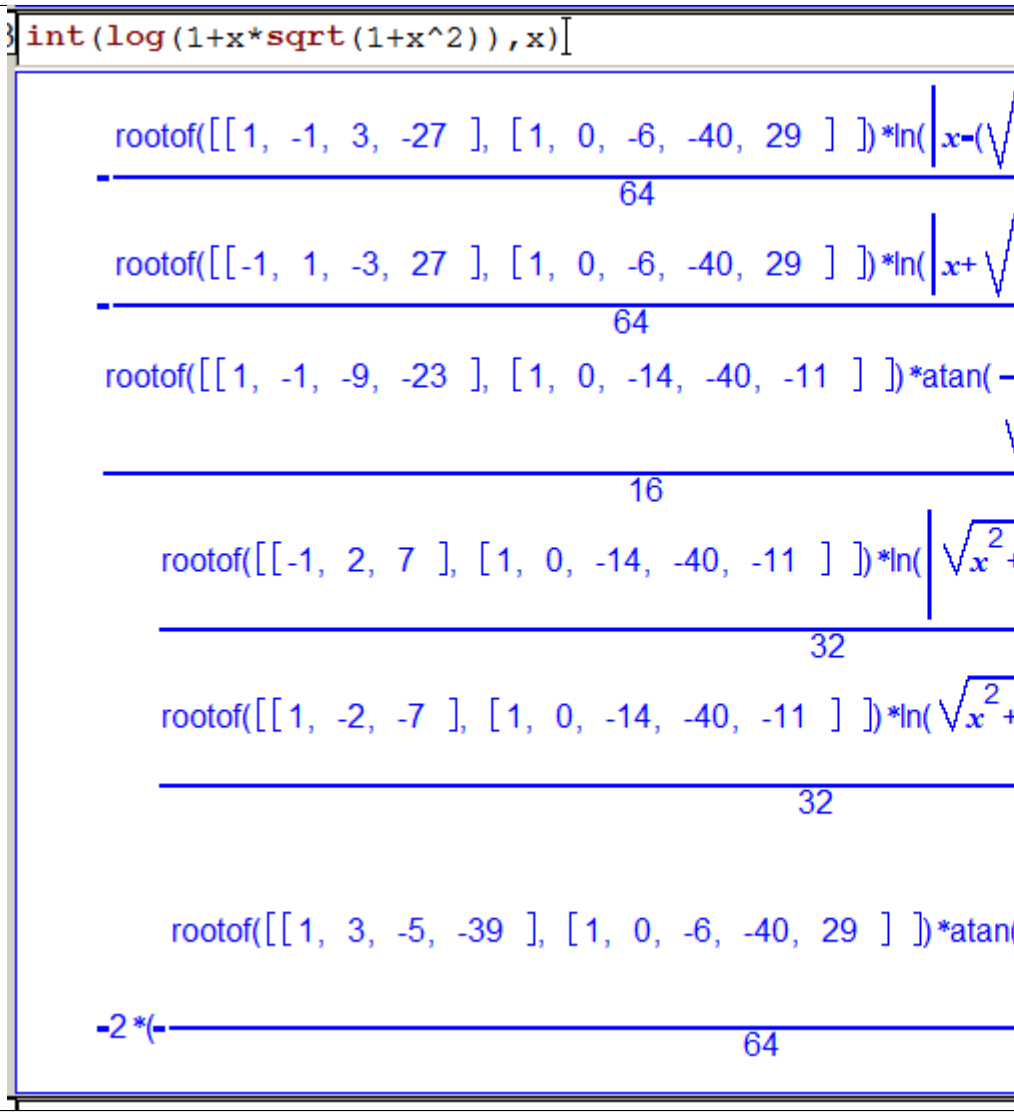
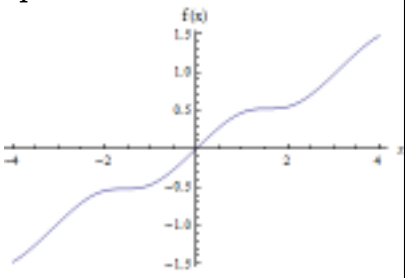

(8) ->
(8) -> integrate(log(1+x*sqrt(1+x^2)),x)
(8) ->
(8)
log
log
log
Type: Union(Expression
(9) ->

Maxima 5.28.02	`integrate(log(1+x*sqrt(1+x^2)),x);` $\int \frac{2x^2+1}{\sqrt{x^2+1}\,(x^3+x)+x^2+1}\,dx + x\log\left(x\sqrt{x^2+1}+1\right)+\text{atan}(x)-2x$
reduce 2008	`14: int(log(1+x*sqrt(1+x^2)),x);` $\left(2\sqrt{\sqrt{5}+2}\sqrt{5}\arctan\left(\frac{\sqrt{x^2+1}+x}{\sqrt{\sqrt{5}+2}}\right)-2\sqrt{\sqrt{5}+2}\arctan\left(\frac{\sqrt{x^2+1}+x}{\sqrt{\sqrt{5}+2}}\right)+\sqrt{\sqrt{5}-2}\right.$ $\left.\sqrt{\sqrt{5}-2}\log\left(\sqrt{x^2+1}-\sqrt{\sqrt{5}-2}+x\right)-\sqrt{\sqrt{5}-2}\log\left(\sqrt{x^2+1}+\sqrt{\sqrt{5}-2}+x\right)+2\log\right.$
mupad 2013a	`evalin(symengine,'int(log(1+x*sqrt(1+x^2)),x)')` result in `mupad_4.txt`

xcas	 int(log(1+x*sqrt(1+x^2)),x) rootof([[1, -1, 3, -27], [1, 0, -6, -40, 29]])*ln(64 rootof([[-1, 1, -3, 27], [1, 0, -6, -40, 29]])*ln(64 rootof([[1, -1, -9, -23], [1, 0, -14, -40, -11]])*atan(16 rootof([[-1, 2, 7], [1, 0, -14, -40, -11]])*ln(32 rootof([[1, -2, -7], [1, 0, -14, -40, -11]])*ln(32 rootof([[1, 3, -5, -39], [1, 0, -6, -40, 29]])*atan(-2*(- 64

5 $\int \dfrac{\cos^2(x)}{\sqrt{\cos^4(x)+\cos^2(x)+1}}\,dx$

optimal 	$\frac{x}{3}+\frac{1}{3}\tan^{-1}\left(\frac{\sin(x)\cos(x)(\cos^2(x)+1)}{\sqrt{\cos^4(x)+\cos^2(x)+1}\cos^2(x)+1}\right)$
M 9.01	$\text{Integrate}\left[\frac{\text{Cos}[x]^2}{\sqrt{\text{Cos}[x]^4+\text{Cos}[x]^2+1}},x\right]$ $-\left(2i\,\text{Cos}[x]^2\,\text{EllipticPi}\left[\frac{3}{2}+\frac{i\sqrt{3}}{2},i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{3}}}\,\text{Tan}[x]\right],\frac{3i-}{3i+}\right.\right.$ $\left.\left.\sqrt{1-\frac{2i\,\text{Tan}[x]^2}{-3i+\sqrt{3}}}\sqrt{1+\frac{2i\,\text{Tan}[x]^2}{3i+\sqrt{3}}}\right)\Big/\left(\sqrt{-\frac{i}{-3i+\sqrt{3}}}\sqrt{15+8\,\text{Cos}[2x]}\right.\right.$
Rubi 4.1	$\text{Int}\left[\frac{\text{Cos}[x]^2}{\sqrt{\text{Cos}[x]^4+\text{Cos}[x]^2+1}},x\right]$ $\text{EllipticPi}\left[\frac{1}{2}\left(3-i\sqrt{3}\right),\text{ArcSin}\left[\sqrt{-\frac{2i}{3i+\sqrt{3}}}\,\text{Tan}[x]\right],\frac{3i+\sqrt{}}{3i-\sqrt{}}\right.$ $\left.3\sqrt{-\frac{2i}{3i+\sqrt{3}}}\left(1+\text{Tan}[x]^2\right)\sqrt{1+\frac{2\text{Tan}[x]^2}{3-i\sqrt{3}}}\sqrt{1+\frac{2\text{Tan}[x]^2}{3+i\sqrt{3}}}\right.$

Maple 18

```
> restart;
  int(cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1),x);
```

$$-\left(2\sqrt{-\left(\cos(2x)^2+4\cos(2x)+7\right)\left(\cos(2x)^2-1\right)}\,(-3+\mathrm{I}\sqrt{3})\sqrt{\frac{(-1+\mathrm{I}\sqrt{3})(\cos(2x)-1)}{(-3+\mathrm{I}\sqrt{3})(\cos(2x)+1)}}\,(\cos(2x)+1)\right.$$

$$2\sqrt{\frac{\cos(2x)+2+\mathrm{I}\sqrt{3}}{(\mathrm{I}\sqrt{3}+3)(\cos(2x)+1)}}\sqrt{\frac{\mathrm{I}\sqrt{3}-\cos(2x)-2}{(-3+\mathrm{I}\sqrt{3})(\cos(2x)+1)}}\,\text{Elliptic}$$

$$\sqrt{\frac{(-1+\mathrm{I}\sqrt{3})(\cos(2x)-1)}{(-3+\mathrm{I}\sqrt{3})(\cos(2x)+1)}},\frac{-3+\mathrm{I}\sqrt{3}}{-1+\mathrm{I}\sqrt{3}},\sqrt{\frac{(1+\mathrm{I}\sqrt{3})(-3+\mathrm{I}\sqrt{}}{(\mathrm{I}\sqrt{3}+3)(-1+\mathrm{I}\sqrt{}}}$$

$$+\mathrm{I}\sqrt{3})$$

$$\sqrt{(\cos(2x)-1)(\cos(2x)+1)(\cos(2x)+2+\mathrm{I}\sqrt{3})(\mathrm{I}\sqrt{3}-\cos(2x)-}$$

$$\left.\sqrt{\cos(2x)^2+4\cos(2x)+7}\right)$$

Sage 5.4

```
integral(cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1
```

$$\int \frac{\cos(x)^2}{\sqrt{\cos(x)^4+\cos(x)^2+1}}\,dx$$

Fricas 1.2

```
(16) -> integrate( cos(x)^2/sqrt(cos(x)^4 + cos(x)^2 +1), x)

   >> Error detected within library code:
   Imaginary part is nonzero. Cannot retract.

(16) ->
```

sympy 0.7.1

(waited one hr)

```
>>>
>>> integrate(cos(x)**2/sqrt( cos(x)**4 + cos(x)**
^C
^C

^C^C^C
Killed
>
>
```

Axiom 5/12

```
(9) ->
(9) -> integrate( cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1),x)
(9) ->
   (9)
     atan
                     5          4         3
              (cos(x)  + 2cos(x)  + cos(x) )sin(x)
          *
               +-----------------------------------------+
               |                   4           2
               |          16cos(x)  + 16cos(x)  + 16
               |-----------------------------------------
               |      4          3          2
              \|cos(x)  + 4cos(x)  + 6cos(x)  + 4cos(x) + 1
       /
                    6
           4cos(x)  - 2
  /
     6
                                                Type: Union(Expression
(10) ->
(10) ->
```

Maxima 5.28.02

```
integrate(cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1),x);
```

$$\int \frac{\cos(x)^2}{\sqrt{\cos(x)^4+\cos(x)^2+1}}\,dx$$

reduce 2008

```
int( cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1),x);
```

$$\int \frac{\sqrt{\cos(x)^4+\cos(x)^2+1}\cos(}{\cos(x)^4+\cos(x)^2+1}$$

mupad 2013a

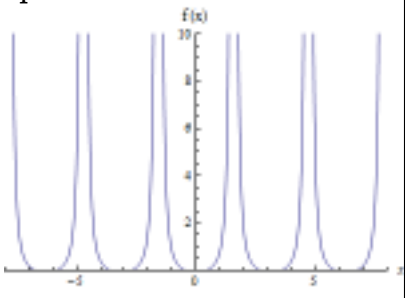
```
evalin(symengine,'int(cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1),x)')

int(cos(x)^2/(cos(x)^4 + cos(x)^2 + 1)^(1/2), x)
```

xcas

```
int(cos(x)^2/sqrt(cos(x)^4+cos(x)^2+1),x)
```

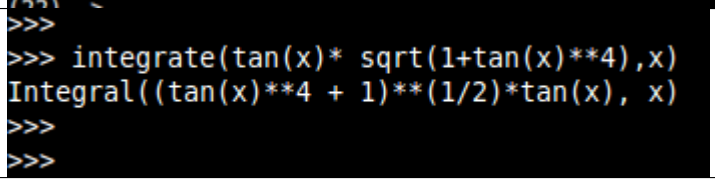
$$\int \frac{\cos(x)^2}{\sqrt{\cos(x)^4+\cos}}$$

6 $\int \tan(x)\sqrt{1+\tan^4(x)}\,dx$

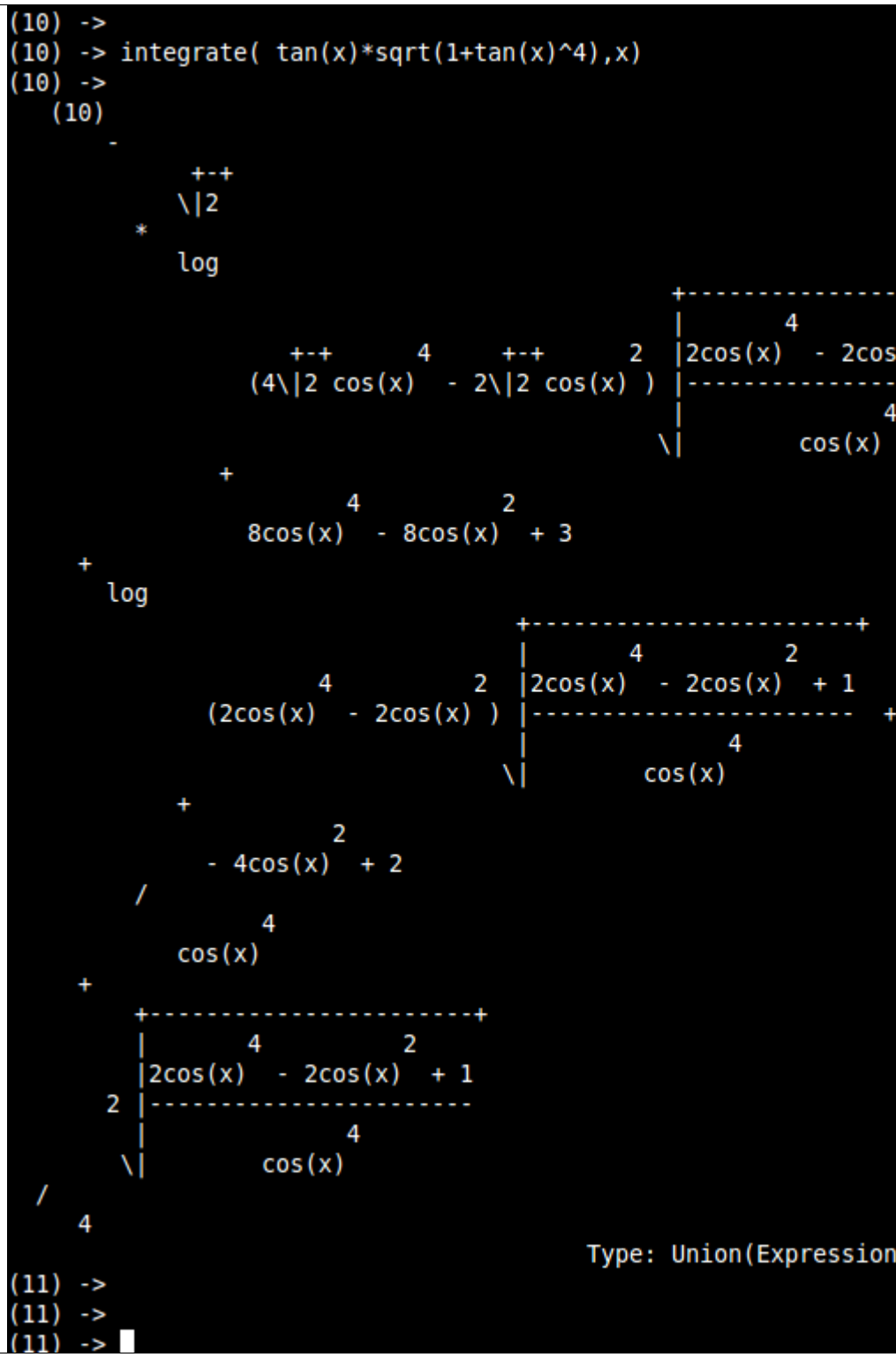
optimal 	$\frac{1}{2}\sqrt{\tan^4(x)+1}-\frac{\tanh^{-1}\left(\frac{1-\tan^2(x)}{\sqrt{2}\sqrt{\tan^4(x)+1}}\right)}{\sqrt{2}}-\frac{1}{2}\sinh^{-1}\left(\tan^2(x)\right)$
M 9.01	click to view
Rubi 4.1	$\text{Int}\left[\text{Tan}[x]\sqrt{1+\text{Tan}[x]^4},x\right]$ $-\frac{1}{2}\text{ArcSinh}\left[\text{Tan}[x]^2\right]-\frac{\text{ArcTanh}\left[\frac{1-\text{Tan}[x]^2}{\sqrt{2}\sqrt{1+\text{Tan}[x]^4}}\right]}{\sqrt{2}}+\frac{1}{2}\sqrt{1+\text{Tan}$

Maple 18	`> restart;`
	`int(tan(x)*sqrt(1+tan(x)^4),x);`
	$\frac{1}{2}\sqrt{\left(1+\tan(x)^2\right)^2-2\tan(x)^2}-\frac{1}{2}\operatorname{arcsinh}\left(\tan(x)^2\right)$
	$-\frac{1}{2}\sqrt{2}\operatorname{arctanh}\left(\frac{1}{4}\frac{\left(-2\tan(x)^2+2\right)\sqrt{2}}{\sqrt{\left(1+\tan(x)^2\right)^2-2\tan(x)^2}}\right)$

Sage 5.4	`integral(tan(x)*sqrt(1+tan(x)^4), x)` $\int\sqrt{\tan(x)^4+1}\tan(x)\,dx$

Fricas 1.2	
sympy 0.7.1	

Axiom 5/12

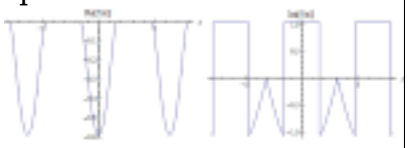

(10) ->
(10) -> integrate(tan(x)*sqrt(1+tan(x)^4),x)
(10) ->
(10)
-
+-+
\|2
*
log
+---------------
| 4
+-+ 4 +-+ 2 |2cos(x) - 2cos
(4\|2 cos(x) - 2\|2 cos(x)) |---------------
| 4
\| cos(x)
+
4 2
8cos(x) - 8cos(x) + 3
+
log
+----------------------+
| 4 2
4 2 |2cos(x) - 2cos(x) + 1
(2cos(x) - 2cos(x)) |---------------------- +
| 4
\| cos(x)
+
2
- 4cos(x) + 2
/
4
cos(x)
+
+----------------------+
| 4 2
|2cos(x) - 2cos(x) + 1
2 |----------------------
| 4
\| cos(x)
/
4
Type: Union(Expression
(11) ->
(11) ->
(11) >

Maxima 5.28.02	`integrate(tan(x)*sqrt(1+tan(x)^4),x);` $\int \tan(x)\sqrt{\tan(x)^4+1}\,dx$
reduce 2008	`int(tan(x)*sqrt(1+tan(x)^4),x);` $\int \sqrt{\tan(x)^4+1}\tan($

mupad 2013a	`evalin(symengine,'int(tan(x)*sqrt(1+tan(x)^4),x)')`
	
	`int(tan(x)*(tan(x)^4 + 1)^(1/2), x)`

xcas	`int(tan(x)*sqrt(1+tan(x)^4),x)` $\frac{\sqrt{4*(\frac{\tan(x)^{2\,2}}{2})+1}}{2}+2*($ $\frac{\ln(\sqrt{4*(\frac{\tan(x)^{2\,2}}{2})+1}-\frac{2*\tan(x)^{2}}{2})}{4}+\frac{1}{2}*$

7 $\int \frac{\tan(x)}{\sqrt{\sec^3(x)+1}}\,dx$

optimal 	$-\frac{2}{3}\tanh^{-1}\left(\sqrt{\sec^3(x)+1}\right)$
M 9.01	In[23]:= Integrate[Tan[x]/Sqrt[Sec[x]^3 + 1], x] Out[23]= - (i Cos[x]^2 (EllipticF[i ArcSinh[Sqrt[3] Sqrt[i Cos[x] Sec[x/2]^2 / (-3 i + Sqrt[3])]], (3 i - Sqrt[3])/(3 i + Sqrt[3])] EllipticPi[1/6 (3 + i Sqrt[3]), i ArcSinh[Sqrt[3] Sqrt[i Cos[x] Sec[x/2]^2 / (-3 i + Sqrt[3])]], Sqrt[(4 + 3 Cos[x] + Cos[3 x]) Sec[x]^3] Sqrt[(Sqrt[3] - 3 i Tan[x/2]^2)/(-3 i + Sqrt[3])] Sqrt[(Sqrt[3] + 3 i)/(3 i + (Sqrt[3] Sqrt[Cos[x] Sec[x/2]^2 / (-3 - i Sqrt[3])] (1 + 3 Tan[x/2]^4))
Rubi 4.1	In[22]:= Int[Tan[x]/Sqrt[Sec[x]^3 + 1], x] Out[22]= -2/3 ArcTanh[Sqrt[1 + Sec[x]^3]]
Maple 18	restart; int(tan(x)/sqrt(sec(x)^3+1),x); $-\frac{2}{3}\operatorname{arctanh}\left(\sqrt{\sec(x)^3+1}\right)$

Sage 5.4

```
integral(tan(x)/sqrt(sec(x)^3+1), x)
```

$$\frac{1}{3}\log\left(\sqrt{\frac{1}{\cos(x)^3}+1}-1\right)-\frac{1}{3}\log\left(\sqrt{\frac{1}{\cos(x)^3}+1}+1\right)$$

Fricas 1.2

```
(17) ->
(17) -> integrate( tan(x)/sqrt(sec(x)^3+1), x)

                                  +----------------------+
                                  |       2
                     3        2  |4cos(x)  - 4cos(x) + 4
        log((- cos(x)  - cos(x) ) |----------------------  - 2cos
                                  |         2
                                 \|   cos(x)  + cos(x)
   (17)  ------------------------------------------------------
                                        3
                                                Type: Union(Expression(I
(18) ->
```

sympy 0.7.1

```
>>>
>>> integrate(tan(x)/sqrt(sec(x)**3+1),x)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
NameError: name 'sec' is not defined
>>>
```

Axiom 5/12

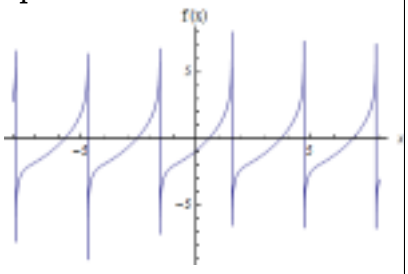
```
(11) ->
(11) -> integrate( tan(x)/sqrt(sec(x)^3+1),x)
(11) ->
   (11)
   -
        log
                          6          5          3          2
                 (- 4cos(x)  - 4cos(x)  - 2cos(x)  - 2cos(x) )
             *
                  +----------------------+
                  |       2
                  |4cos(x)  - 4cos(x) + 4
                  |----------------------
                  |         2
                 \|   cos(x)  + cos(x)
            +
                        6          3
                  8cos(x)  + 8cos(x)  + 1
      /
        6
                                                Type: Union(Expressior
(12) ->
```

Maxima 5.28.02	`integrate(tan(x)/sqrt(sec(x)^3+1),x);` $\frac{\log\left(\sqrt{\frac{1}{\cos(x)^3}+1}-1\right)}{3}-\frac{\log\left(\sqrt{\frac{1}{\cos(x)^3}+1}+1\right)}{3}$
reduce 2008	`int(tan(x)/sqrt(sec(x)^3+1),x);` $\int\frac{\sqrt{\sec(x)^3+1}\tan(x)}{\sec(x)^3+1}\,dx$

mupad 2013a	`evalin(symengine,'int(tan(x)/sqrt(sec(x)^3+1),x)')`
	
	`int(tan(x)/(1/cos(x)^3 + 1)^(1/2), x)`

xcas	`int(tan(x)/sqrt(sec(x)^3+1),x)` Warning, integration of abs or sign assumes constant sign by intervals (c Check Vector [abs(cos(x))] Discontinuities at zeroes of cos(x) were not checked $\int\frac{1}{\sqrt{\cos(x)^4+\cos(x)}}*\left(\frac{1}{\cos(x)}\right)^2*(\frac{}{\text{sig}}$

8 $\int \sqrt{\tan^2(x) + 2\tan(x) + 2}\, dx$

optimal 	$\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{1+\sqrt{5}}\tan(x)-\sqrt{\sqrt{5}-1}}{\sqrt{2}\sqrt{\tan(x)(\tan(x)+2)+2}}\right)-\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\sqrt{5}-1}\ta}{\sqrt{2}\sqrt{\tan(x}}\right.$ $\sinh^{-1}(\tan(x)+1)$
M 9.01	click to view
Rubi 4.1	$\texttt{Int}\left[\sqrt{\texttt{Tan[x]}^2+2\,\texttt{Tan[x]}+2}\,,\,\texttt{x}\right]$ $\texttt{ArcSinh[1+Tan[x]]}-\frac{1}{2}\,\texttt{i}\,\sqrt{1-2\,\texttt{i}}\,\texttt{ArcTanh}\left[\frac{(4-2\,\texttt{i})+(}{2\sqrt{1-2\,\texttt{i}}\,\sqrt{2+}}\right.$ $\frac{1}{2}\,\texttt{i}\,\sqrt{1+2\,\texttt{i}}\,\texttt{ArcTanh}\left[\frac{(4+2\,\texttt{i})+(2+2\,\texttt{i})\,\texttt{Tan[x]}}{2\sqrt{1+2\,\texttt{i}}\,\sqrt{2+2\,\texttt{Tan[x]}+\texttt{Tan[x]}^2}}\right]$
Maple 18	image

Sage 5.4	`integral(sqrt(tan(x)^2+2*tan(x)+2), x)`
	`Traceback (click to the left of this bl`
	`...`
	`RuntimeError: ECL says: Error executing`
	`cannot be imaginary; found %i`

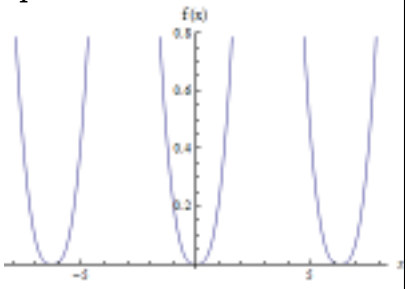
Fricas 1.2	`(18) ->`
	`(18) -> integrate( sqrt(tan(x)^2+2*tan(x)+2), x)`
	`x +----------------------+`
	`++ | 2`
	`(18) | |tan(%A) + 2tan(%A) + 2 d%A`
	`++`
	`Type: Union(Expression(Ir`
	`(19) ->`

sympy 0.7.1	`>>>`
	`>>> integrate( sqrt( tan(x)**2 + 2 *tan(x)+2),x)`
	`Integral((tan(x)**2 + 2*tan(x) + 2)**(1/2), x)`
	`>>>`

Axiom 5/12	HTML

Maxima 5.28.02	(%i12) integrate(sqrt(tan(x)^2+2*tan(x)+2),x); sign: argument cannot be imaginary; found %i -- an error. To debug this try: debugmode(true);
reduce 2008	int(sqrt(tan(x)^2+2*tan(x)+2),x); $\int \sqrt{\tan(x)^2+2\tan(x)+2}\,dx$
mupad 2013a	evalin(symengine,'int(sqrt(tan(x)^2+2*tan(x)+2),x)') int((2*tan(x) + tan(x)^2 + 2)^(1/2), x)
xcas	int(sqrt(tan(x)^2+2*tan(x)+2),x) $\frac{((8^{\frac{1}{4}})^3+8*8^{\frac{1}{4}})*\ln((\sqrt{\tan(x)^2+2*\tan(x)+2}-\tan(x))^2+(-8^{\frac{1}{4}}*(\sqrt{2}))}{64}$ $\frac{(-(8^{\frac{1}{4}})^3-8*(\sqrt{2})+8*8^{\frac{1}{4}})*\operatorname{atan}(\frac{(\sqrt{\tan(x)^2+2*\tan(x)+2}-\tan(x)-}{8^{\frac{1}{4}}}}{32}$ $\frac{(-(8^{\frac{1}{4}})^3-8*8^{\frac{1}{4}})*\ln((\sqrt{\tan(x)^2+2*\tan(x)+2}-\tan(x))^2+8^{\frac{1}{4}}*(\sqrt{2})*(}{64}$ $\frac{(-(8^{\frac{1}{4}})^3+8*(\sqrt{2})+8*8^{\frac{1}{4}})*\operatorname{atan}(\frac{(\sqrt{\tan(x)^2+2*\tan(x)+2}-\tan(x)-}{8^{\frac{1}{4}}}}{32}$ $2*($

9 $\int \sin(x) \arctan\left(\sqrt{\sec(x)-1}\right)\, dx$

optimal 	$\frac{1}{2}\cos(x)\sqrt{\sec(x)-1}+\frac{1}{2}\tan^{-1}\left(\sqrt{\sec(x)-1}\right)-\cos(x)\tan^{-1}\left(\sqrt{\sec(x)-$
M 9.01	click to view
Rubi 4.1	$\text{Int}\left[\text{Sin}[x]\ \text{ArcTan}\left[\sqrt{\text{Sec}[x]-1}\right], x\right]$ $\frac{1}{2}\text{ArcTan}\left[\sqrt{-1+\text{Sec}[x]}\right]-\text{ArcTan}\left[\sqrt{-1+\text{Sec}[x]}\right]\text{Cos}[x]+\frac{1}{2}$

Maple 18	`restart;`
	`int(sin(x)*arctan(sqrt(sec(x)-1)),x);`
	$\int \sin(x)\arctan\left(\sqrt{\sec(x)-1}\right)dx$

Sage 5.4	`integral(sin(x)*arctan(sqrt(sec(x)-1)), x)` $-\cos(x)\arctan\left(\sqrt{-\frac{\cos(x)-1}{\cos(x)}}\right)-\frac{\sqrt{-\frac{\cos(x)-1}{\cos(x)}}}{2\left(\frac{\cos(x)-1}{\cos(x)}-1\right)}+\frac{1}{2}\text{arct}$

Fricas 1.2

```
(19) ->
(19) -> integrate(sin(x)*atan(sqrt(sec(x)-1)),x)

   >> Error detected within library code:
   failed - cannot handle that integrand

(19) ->
```

update: per post on sci.math.symbolic on June 2

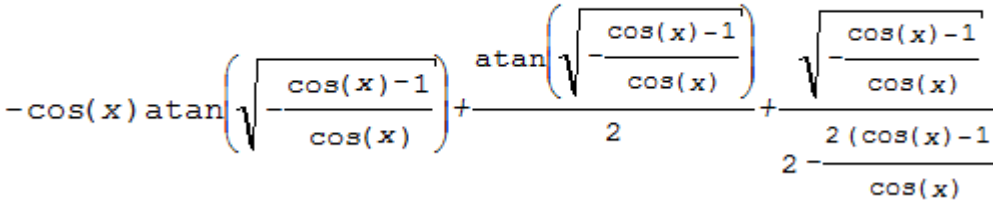
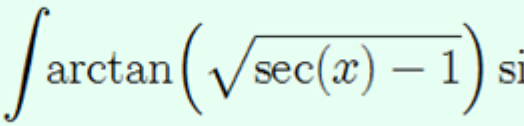
```
BTW: Current developement FriCAS can also do #9:

(3) -> integrate(sin(x)*atan(sqrt(sec(x) - 1)), x)

   (3)
                                                          +----------
+
                                                          |- cos(x) +
                                                  cos(x) |----------
                       +----------
+               \|   cos(x)
       - 2cos(x)atan(\|sec(x) - 1 ) + atan(------------------
)
                                                              cos(x) - 1
     +
               +------------+
               |- cos(x) + 1
       cos(x) |------------
             \|   cos(x)
  /
     2
                                                   Type: Union(Expressi
```

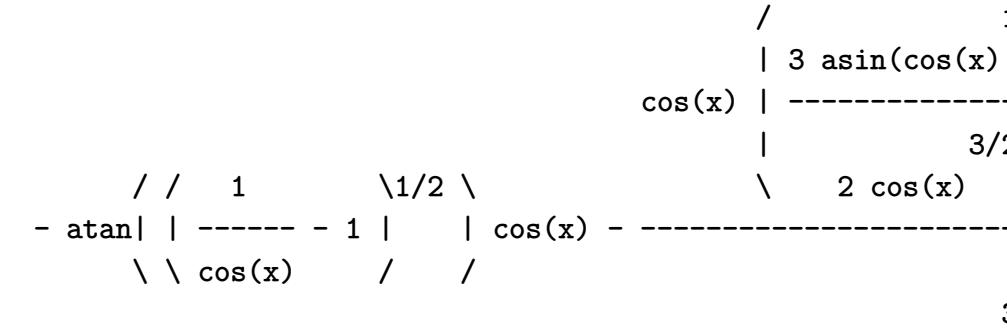
sympy 0.7.1

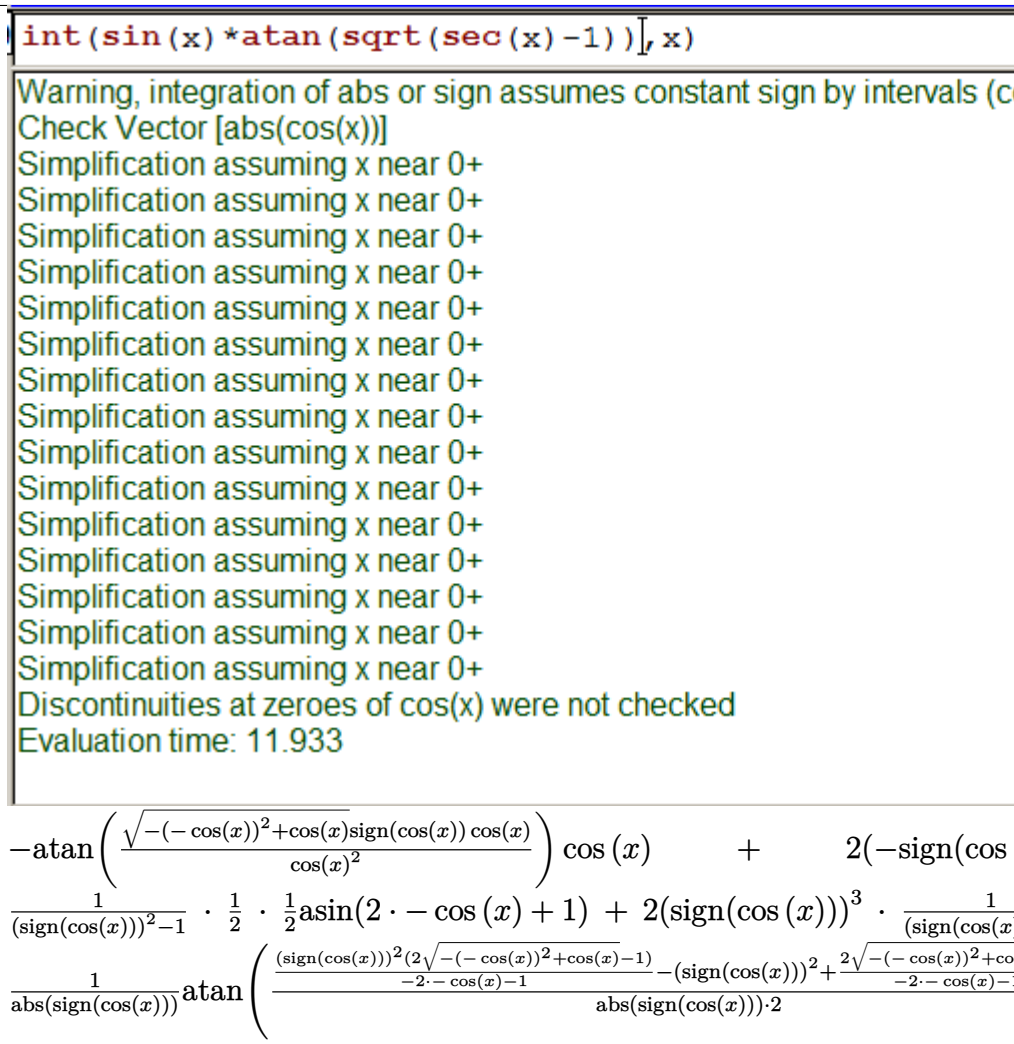
```
>>>
>>> integrate( sin(x)*atan(sqrt(sec(x)-1)),x)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
NameError: name 'sec' is not defined
>>>
```

Axiom 5/12

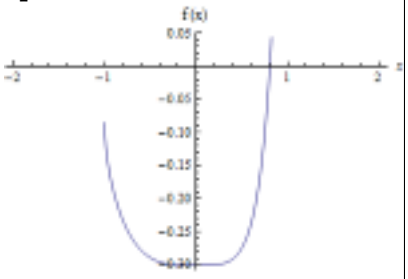
```
(13) ->
(13) -> integrate( sin(x)*atan(sqrt(sec(x)-1)),x)
 13) ->
   >> Error detected within library code:
   failed - cannot handle that integrand

(13) ->
```

Maxima 5.28.02	`integrate(sin(x)*atan(sqrt(sec(x)-1)),x);` 
reduce 2008	`int(sin(x)*atan(sqrt(sec(x)-1)),x);` 

mupad 2013a	`evalin(symengine,'int(sin(x)*atan(sqrt(sec(x)-1)),x)')`
	
	`pretty(ans)`
	 

xcas	 int(sin(x)*atan(sqrt(sec(x)-1)),x) Warning, integration of abs or sign assumes constant sign by intervals (c Check Vector [abs(cos(x))] Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Simplification assuming x near 0+ Discontinuities at zeroes of cos(x) were not checked Evaluation time: 11.933 $-\operatorname{atan}\left(\frac{\sqrt{-(-\cos(x))^2+\cos(x)}\operatorname{sign}(\cos(x))\cos(x)}{\cos(x)^2}\right)\cos(x) + 2(-\operatorname{sign}(\cos$ $\frac{1}{(\operatorname{sign}(\cos(x)))^2-1}\cdot\frac{1}{2}\cdot\frac{1}{2}\operatorname{asin}(2\cdot-\cos(x)+1) + 2(\operatorname{sign}(\cos(x)))^3\cdot\frac{1}{(\operatorname{sign}(\cos(x$ $\frac{1}{\operatorname{abs}(\operatorname{sign}(\cos(x)))}\operatorname{atan}\left(\frac{\frac{(\operatorname{sign}(\cos(x)))^2(2\sqrt{-(-\cos(x))^2+\cos(x)}-1)}{-2\cdot-\cos(x)-1}-(\operatorname{sign}(\cos(x)))^2+\frac{2\sqrt{-(-\cos(x))^2+\cos}}{-2\cdot-\cos(x)-}}{\operatorname{abs}(\operatorname{sign}(\cos(x)))\cdot 2}\right.$

10 $\displaystyle\int \frac{x^3 e^{\arcsin(x)}}{\sqrt{1-x^2}}\,dx$

optimal	$\frac{1}{10}\left(x^3-3\sqrt{1-x^2}x^2-3\sqrt{1-x^2}+3x\right)e^{\sin^{-1}(x)}$ 
M 9.01	$\int \frac{x^3\,\text{Exp}[\text{ArcSin}[x]]}{\sqrt{1-x^2}}\,dx$ $-\frac{1}{40}e^{\text{ArcSin}[x]}\left(15\left(-x+\sqrt{1-x^2}\right)-3\,\text{Cos}[3\,\text{ArcSin}[x]]+\text{Sin}[3\right.$
Rubi 4.1	$\int \frac{x^3\,\text{Exp}[\text{ArcSin}[x]]}{\sqrt{1-x^2}}\,dx$ $\frac{3}{10}e^{\text{ArcSin}[x]}x+\frac{1}{10}e^{\text{ArcSin}[x]}x^3-\frac{3}{10}e^{\text{ArcSin}[x]}\sqrt{1-x^2}-\frac{3}{10}e^{\text{Ar}}$

Maple 18	`> restart;`
	`int(x^3*exp(1)^arcsin(x)/sqrt(1-x^2),x);`
	$\frac{1}{10}\left(x-3\sqrt{-x^2+1}\right)e^{\arcsin(x)}x^2+\frac{3}{10}e^{\arcsin(x)}\left(x-\sqrt{-x^2}\right.$

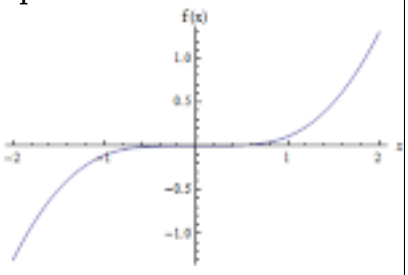
Sage 5.4	`integral(x^3*exp(arcsin(x))/sqrt(1-x^2), x` $\int \frac{x^3 e^{\arcsin(x)}}{\sqrt{-x^2+1}}\,dx$

Fricas 1.2	`(19) -> integrate(x^3*exp(asin(x))/sqrt(1-x^2),x)`
	$\frac{((-3x^2-3)\sqrt{-x^2+1}+x^3+3x)\%e^{-\arctan\left(\frac{x\sqrt{-x^2+1}}{x^2-1}\right)}}{10}$
	`Type: Union(Expression(Integer),...)`

<table>
<tr><td>sympy 0.7.1</td><td>>>>
>>> integrate(x**3*exp(asin(x))/sqrt(1-x**2),x)
x**3*exp(asin(x))/10 - 3*x**2*(-x**2 + 1)**(1/2)*exp(asin(x))/10 + 3*x*exp(asi
)**(1/2)*exp(asin(x))/10
>>>
>>> _</td></tr>
<tr><td>Axiom 5/12</td><td>(13) ->
(13) -> integrate(x^3*exp(asin(x))/sqrt(1-x^2),x)
(13) ->
$$(13)\quad \frac{\left((-3x^2-3)\sqrt{-x^2+1}+x^3+3x\right)\%e^{-\frac{\operatorname{atan}\left(\frac{2x\sqrt{-x^2+1}}{2x^2-1}\right)}{2}}}{10}$$
Type: Union(Expression(Integer),...)
(14) -></td></tr>
<tr><td>Maxima 5.28.02</td><td>integrate(x^3*exp(asin(x))/sqrt(1-x^2),x);
$$\int \frac{x^3\,\%e^{\operatorname{asin}(x)}}{\sqrt{1-x^2}}\,dx$$</td></tr>
<tr><td>reduce 2008</td><td>int(x^3*exp(asin(x))/sqrt(1-x^2),x);
$$\int \frac{e^{\arcsin(x)}\,x^3}{\sqrt{-x^2+1}}\,dx$$</td></tr>
<tr><td>mupad 2013a</td><td>evalin(symengine,'int(x^3*exp(asin(x))/sqrt(1-x^2),x)')

int((x^3*exp(asin(x)))/(1 - x^2)^(1/2), x)</td></tr>
<tr><td>xcas</td><td>int(x^3*exp(asin(x))/sqrt(1-x^2),x)
Simplification assuming x near 0+
Simplification assuming x near 0+
$\int x^3 * \frac{1}{\sqrt{-(x^2 -1}}$</td></tr>
</table>

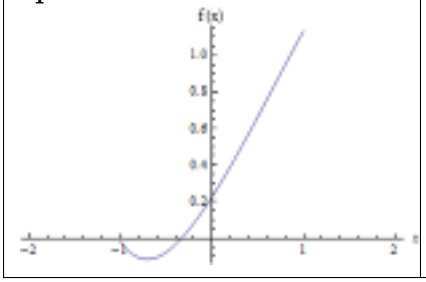
11 $\displaystyle\int \frac{x\log(1+x^2)\log(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}\,dx$

optimal 	$4x-2\arctan(x)-2\sqrt{1+x^2}\,\log\left[x+\sqrt{1+x^2}\right]+\log\left[1+x^2\right]\left($
M 9.01	`Integrate[(x*Log[1+x^2]*Log[x+Sqrt[1+x^2]])/Sqrt[1+x^2],x]` $4x-2\arctan(x)-2\sqrt{1+x^2}\,\log\left[x+\sqrt{1+x^2}\right]+\log\left[1+x^2\right]\left(-x+\sqrt{1+x^2}\,\log\left[x+\sqrt{1+x^2}\right]\right)$
Rubi 4.1	`Int[(x*Log[1+x^2]*Log[x+Sqrt[1+x^2]])/Sqrt[1+x^2],x]` $4x-2\arctan(x)-x\log\left[1+x^2\right]-2\sqrt{1+x^2}\,\log\left[x+\sqrt{1+x^2}\right]+\sqrt{1+x^2}\,\log\left[1+x^2\right]\log[x+$

Maple 18	`> restart;`
	`intg:= x*log(1+x^2)*log(x+sqrt(1+x^2))/sqrt(1+x^2);`
	$intg := \frac{x\ln(x^2+1)\ln\left(x+\sqrt{x^2+1}\right)}{\sqrt{x^2+1}}$
	`> int(intg,x);`
	$\int \frac{x\ln(x^2+1)\ln\left(x+\sqrt{x^2+1}\right)}{\sqrt{x^2+1}}\,dx$

12 $\int \tan^{-1}\left(\sqrt{1-x^2}+x\right)\,dx$

optimal



$$-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1+}{\sqrt{1}}\right.$$

$$\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+2x^2}{\sqrt{3}}\right]+x\,\text{ArcTan}\left[x+\sqrt{1-x^2}\,\right]-\frac{1}{4}\text{ArcTan}$$

M 9.01	Integrate[ArcTan[x + Sqrt[1 - x^2]], x] $x \operatorname{ArcTan}\left[x+\sqrt{1-x^2}\right]+\frac{1}{16}\left(-8 \operatorname{ArcSin}[x]+2 \sqrt{2+2 i \sqrt{3}}\right.$ $\operatorname{ArcTan}\left[\left(\left(1+i \sqrt{3}-2 x^2\right)\left(-1+x^2\right)\right) / \left(-3 i-\sqrt{3}+2 \sqrt{3} x^4+x^3\left(-6-2 i \sqrt{3}-2 \sqrt{2-2 i \sqrt{3}} \sqrt{1-x^2}\right)+\right.\right.$ $\left.\left.x\left(6+2 i \sqrt{3}-2 \sqrt{2-2 i \sqrt{3}} \sqrt{1-x^2}\right)+x^2\left(3 i-\sqrt{3}+2 \sqrt{6-6 i \sqrt{3}} \sqrt{1-x^2}\right)\right)\right]-2 \sqrt{2+2 i \sqrt{3}}$ $\operatorname{ArcTan}\left[\left(\left(1+i \sqrt{3}-2 x^2\right)\left(-1+x^2\right)\right) / \left(-3 i-\sqrt{3}+2 \sqrt{3} x^4+2 x\left(-3-i \sqrt{3}+\sqrt{2-2 i \sqrt{3}} \sqrt{1-x^2}\right)+\right.\right.$ $\left.\left.2 x^3\left(3+i \sqrt{3}+\sqrt{2-2 i \sqrt{3}} \sqrt{1-x^2}\right)+x^2\left(3 i-\sqrt{3}+2 \sqrt{6-6 i \sqrt{3}} \sqrt{1-x^2}\right)\right)\right]-2 \sqrt{2-2 i \sqrt{3}}$ $\operatorname{ArcTan}\left[\left(\left(-1+x^2\right)\left(-1+i \sqrt{3}+2 x^2\right)\right) / \left(3 i-\sqrt{3}+2 \sqrt{3} x^4+x\left(6-2 i \sqrt{3}-2 \sqrt{2+2 i \sqrt{3}} \sqrt{1-x^2}\right)+x^3\right.\right.$ $\left.\left.\left(-6+2 i \sqrt{3}-2 \sqrt{2+2 i \sqrt{3}} \sqrt{1-x^2}\right)+x^2\left(-3 i-\sqrt{3}+2 \sqrt{6+6 i \sqrt{3}} \sqrt{1-x^2}\right)\right)\right]+2 \sqrt{2-2 i \sqrt{3}}$ $\operatorname{ArcTan}\left[\left(\left(-1+x^2\right)\left(-1+i \sqrt{3}+2 x^2\right)\right) / \left(3 i-\sqrt{3}+2 \sqrt{3} x^4+2 x^3\left(3-i \sqrt{3}+\sqrt{2+2 i \sqrt{3}} \sqrt{1-x^2}\right)+\right.\right.$ $\left.\left.2 x\left(-3+i \sqrt{3}+\sqrt{2+2 i \sqrt{3}} \sqrt{1-x^2}\right)+x^2\left(-3 i-\sqrt{3}+2 \sqrt{6+6 i \sqrt{3}} \sqrt{1-x^2}\right)\right)\right]-$ $2 \operatorname{Log}\left[-\frac{1}{2}-\frac{i \sqrt{3}}{2}+x^2\right]+2 i \sqrt{3} \operatorname{Log}\left[-\frac{1}{2}-\frac{i \sqrt{3}}{2}+x^2\right]-2 \operatorname{Log}\left[\frac{1}{2} i\left(i+\sqrt{3}\right)+x^2\right]-$ $2 i \sqrt{3} \operatorname{Log}\left[\frac{1}{2} i\left(i+\sqrt{3}\right)+x^2\right]-$ $i \sqrt{2-2 i \sqrt{3}} \operatorname{Log}\left[16\left(1+\sqrt{3} x+x^2\right)^2\right]+$ $i \sqrt{2+2 i \sqrt{3}} \operatorname{Log}\left[16\left(1+\sqrt{3} x+x^2\right)^2\right]+$ $i \sqrt{2-2 i \sqrt{3}} \operatorname{Log}\left[\left(4-4 \sqrt{3} x+4 x^2\right)^2\right]-$ $i \sqrt{2+2 i \sqrt{3}} \operatorname{Log}\left[\left(4-4 \sqrt{3} x+4 x^2\right)^2\right]-$ $i \sqrt{2+2 i \sqrt{3}} \operatorname{Log}\left[3 i+\sqrt{3}-\left(-i+\sqrt{3}\right) x^4+2 i \sqrt{2-2 i \sqrt{3}} \sqrt{1-x^2}+5 i x^2\left(2+\sqrt{2-2 i \sqrt{3}} \sqrt{1-x^2}\right)+\right.$ $\left.x\left(3+5 i \sqrt{3}+3 i \sqrt{6-6 i \sqrt{3}} \sqrt{1-x^2}\right)+i x^3\left(3 i+3 \sqrt{3}+\sqrt{6-6 i \sqrt{3}} \sqrt{1-x^2}\right)\right]+$ $i \sqrt{2+2 i \sqrt{3}} \operatorname{Log}\left[3 i+\sqrt{3}-\left(-i+\sqrt{3}\right) x^4+2 i \sqrt{2-2 i \sqrt{3}} \sqrt{1-x^2}+5 i x^2\left(2+\sqrt{2-2 i \sqrt{3}} \sqrt{1-x^2}\right)+\right.$ $\left.x^3\left(3-3 i \sqrt{3}-i \sqrt{6-6 i \sqrt{3}} \sqrt{1-x^2}\right)-i x\left(-3 i+5 \sqrt{3}+3 \sqrt{6-6 i \sqrt{3}} \sqrt{1-x^2}\right)\right]+$ $i \sqrt{2-2 i \sqrt{3}} \operatorname{Log}\left[-3 i+\sqrt{3}-\left(i+\sqrt{3}\right) x^4-2 i \sqrt{2+2 i \sqrt{3}} \sqrt{1-x^2}-5 i x^2\left(2+\sqrt{2+2 i \sqrt{3}} \sqrt{1-x^2}\right)+\right.$ $\left.x\left(3-5 i \sqrt{3}-3 i \sqrt{6+6 i \sqrt{3}} \sqrt{1-x^2}\right)-i x^3\left(-3 i+3 \sqrt{3}+\sqrt{6+6 i \sqrt{3}} \sqrt{1-x^2}\right)\right]-$ $i \sqrt{2-2 i \sqrt{3}} \operatorname{Log}\left[-3 i+\sqrt{3}-\left(i+\sqrt{3}\right) x^4-2 i \sqrt{2+2 i \sqrt{3}} \sqrt{1-x^2}-5 i x^2\left(2+\sqrt{2+2 i \sqrt{3}} \sqrt{1-x^2}\right)+\right.$ $\left.\left.x^3\left(3+3 i \sqrt{3}+i \sqrt{6+6 i \sqrt{3}} \sqrt{1-x^2}\right)+i x\left(3 i+5 \sqrt{3}+3 \sqrt{6+6 i \sqrt{3}} \sqrt{1-x^2}\right)\right]\right)$

Rubi 4.1	Int[ArcTan[x + Sqrt[1 - x^2]], x] $-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1-2x^2}{\sqrt{3}}\right]+\frac{\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right]}{\sqrt{3}}+$ $\frac{1}{12}\left(3i-\sqrt{3}\right)\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right]+\frac{\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right]}{\sqrt{3}}-$ $\frac{1}{12}\left(3i+\sqrt{3}\right)\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right]+x\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]-\frac{1}{8}\text{Log}\left[1-x^2+x\right.$

Maple 18	> restart;
	intg:= arctan(sqrt(1-x^2)+x);
	int(intg,x);
	$intg := \arctan\left(\sqrt{-x^2+1}+x\right)$
	$x\arctan\left(\sqrt{-x^2+1}+x\right)-\frac{1}{8}\ln\left(x^4-x^2+1\right)-\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\left(2x^2-1\right)\sqrt{3}\right)+\frac{1}{8}\ln\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}\right.$
	$+\frac{\left(-1-I\sqrt{3}\right)\left(\sqrt{-x^2+1}-1\right)}{x}-1\Big)-\frac{1}{8}I\sqrt{3}\ln\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{\left(1+I\sqrt{3}\right)\left(\sqrt{-x^2+1}-1\right)}{x}-1\right)$
	$+\frac{1}{8}\ln\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{\left(-1+I\sqrt{3}\right)\left(\sqrt{-x^2+1}-1\right)}{x}-1\right)-\frac{1}{8}I\sqrt{3}\ln\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}\right.$
	$+\frac{\left(-1+I\sqrt{3}\right)\left(\sqrt{-x^2+1}-1\right)}{x}-1\Big)-\frac{1}{8}\ln\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{\left(1+I\sqrt{3}\right)\left(\sqrt{-x^2+1}-1\right)}{x}-1\right)$
	$+\frac{1}{8}I\sqrt{3}\ln\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{\left(1-I\sqrt{3}\right)\left(\sqrt{-x^2+1}-1\right)}{x}-1\right)-\frac{1}{8}\ln\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}\right.$
	$+\frac{\left(1-I\sqrt{3}\right)\left(\sqrt{-x^2+1}-1\right)}{x}-1\Big)+\frac{1}{8}I\sqrt{3}\ln\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{\left(-1-I\sqrt{3}\right)\left(\sqrt{-x^2+1}-1\right)}{x}-1\right)$
	$+\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$