Solving partial differential equations in Maple and Mathematica

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July 3, 2018 Compiled on July 3, 2018 at 8:51pm

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This report gives the result of running a number of partial differential equations in Mathematica and Maple.

The following systems were used at this time.

1. Mathematica 11.3 (64 bit).
2. Maple 2018.1 (64 bit) with Physics version MapleCloud 73.

No time limit was used.

All possible options, assumptions and HINTS were tried to obtain a solution. The command `DSolve` was used in Mathematica and the command `pdsolve` in Maple.

It is possible I missed some option, assumption or HINT, which could help make the CAS able to solve a given PDE now marked as unsolved. Will correct such a case if found. I have verified some, but not all, solutions returned by Maple or Mathematica.

Number of problems is [122]. Mathematica solved 82 or 67.21%. Maple solved 99 or 81.15%.

Table 1: Breakdown of results for each PDE

#	PDE	description	Mathematica	Maple
1	First order PDE	Linear PDE, the transport equation	Solved	Solved
2	First order PDE	Linear PDE	Solved	Solved
3	First order PDE	Linear PDE, initial value problem	Solved	Solved
4	First order PDE	Initial-boundary value problem	Solved	Did not solve
5	First order PDE	Linear PDE, the transport equation with initial conditions	Solved	Solved
6	First order PDE	General solution for a quasilinear first-order PDE	Solved	Solved
7	First order PDE	quasilinear first-order PDE, scalar conservation law	Solved, solution in implicit form	Solved
8	First order PDE	quasilinear first-order PDE, scalar conservation law with initial value	Solved	Solved
9	First order PDE	nonlinear first-order PDE, the Clairaut equation	Solved	Solved
10	First order PDE	nonlinear first-order PDE, the Clairaut equation with initial value	Solved	Solved
11	First order PDE	Another example of nonlinear Clairaut equation	Solved	Solved
12	First order PDE	Recover a function from its gradient vector	Solved	Solved
13	First order PDE	General solution of a first order nonlinear PDE	Did not solve	Solved
14	First order PDE	Nonlinear first order PDE	Solved	Solved
15	Heat PDE	Heat PDE on bar, homogeneous Dirichlet boundary conditions, No source.	Solved	Solved
16	Heat PDE	Heat PDE on bar, homogeneous Dirichlet boundary conditions, No source.	Solved	Solved
17	Heat PDE	Heat PDE on bar, homogeneous Dirichlet boundary conditions, No source.	Solved	Solved
18	Heat PDE	Heat PDE on bar, homogeneous Dirichlet boundary conditions, No source.	Solved	Solved
19	Heat PDE	Heat PDE on bar, homogeneous Neumann boundary conditions, No source.	Solved	Solved
20	Heat PDE	Heat PDE on bar, homogeneous Dirichlet boundary conditions with heat sink	Did not solve	Solved
21	Heat PDE	Heat PDE on bar, homogeneous Neumann boundary conditions, No source	Solved	Solved
22	Heat PDE	Heat PDE on bar, homogeneous Neumann boundary conditions, No source	Solved	Solved
23	Heat PDE	Heat PDE on bar, homogeneous Neumann boundary conditions, No source	Solved	Solved
24	Heat PDE	Heat PDE on bar, homogeneous Neumann boundary conditions, No source	Solved	Solved

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Table 1 – continued from previous page

#	PDE	description	Mathematica	Maple
25	Heat PDE	Heat PDE on bar, homogeneous Neumann on left and Dirichlet on right, No source	Solved	Solved
26	Heat PDE	Heat PDE on bar, semi-infinite domain, No source	Solved	Solved
27	Heat PDE	Heat PDE on bar, periodic boundary conditions, No source	Did not solve	Solved
28	Heat PDE	Heat PDE on bar, semi-infinite domain, zero initial condition, No source	Solved	Solved
29	Heat PDE	Heat PDE on bar, semi-infinite domain, non-zero initial condition, No source	Solved	Solved
30	Heat PDE	Heat PDE on bar, heat absorption radiation in bounded domain, No source	Did not solve	Solved
31	Heat PDE	Heat PDE infinite domain	Solved	Solved
32	Heat PDE	Heat PDE on bar, with domain from -1 to +1, no source	Did not solve	Solved
33	Heat PDE	Heat PDE on bar, Dirichlet nonhomogeneous BC, no source term	Solved	Solved
34	Heat PDE	Heat PDE on bar, nonhomogeneous Dirichlet BC, with constant source term	Did not solve	Solved
35	Heat PDE	Heat PDE on bar, homogeneous Dirichlet BC, non zero initial conditions, with extra term	Did not solve	Solved
36	Heat PDE	Heat PDE on bar with initial conditions sum of sine terms, homogeneous Dirichlet BC, no source	Solved	Solved
37	Heat PDE	Heat PDE on bar, homogeneous Dirichlet BC, initial condition is piecewise function, no source	Solved	Solved
38	Heat PDE	Heat PDE on bar, inhomogeneous Dirichlet BC, initial condition is piecewise function, no source	Solved	Solved
39	Heat PDE	Heat PDE on bar, inhomogeneous Dirichlet BC which depends on time. Zero initial condition, no source	Solved	Did not solve
40	Heat PDE	Heat PDE on bar, homogeneous Neumann boundary conditions, non zero initial conditions, with source as sin function that depends on space only.	Did not solve	Solved
41	Heat PDE	Heat PDE on bar, homogeneous Neumann boundary conditions, nonzero initial conditions, with source that depends on time only	Did not solve	Solved
42	Heat PDE	Heat PDE on bar, homogeneous Neumann boundary conditions, nonzero initial conditions, with source that depends on time and space	Did not solve	Solved
43	Heat PDE	Heat PDE on bar, non-homogeneous, time dependent, Neumann boundary conditions, with source that depends on time and space	Did not solve	Solved
44	Heat PDE	Heat PDE on bar, non-homogeneous, not time dependent Neumann boundary conditions, No source term	Did not solve	Solved
45	Heat PDE	Heat PDE on bar, homogeneous Neumann boundary conditions, Source term that depends on both time and space	Solved	Solved

Continued on next page

Table 1 – continued from previous page

#	PDE	description	Mathematica	Maple
46	Heat PDE	Heat PDE on bar, homogeneous Neumann boundary conditions, Source term that depends on both time and space	Solved	Solved
47	Heat PDE	Heat PDE on bar, Dirichlet boundary conditions that depends on time with source that depends on space only	Did not solve	Solved
48	Heat PDE	Heat PDE on bar, homogeneous Dirichlet boundary conditions, with source that depends on time and space	Did not solve	Solved
49	Heat PDE	Heat/Diffusion PDE in 2D, inside rectangle with initial and boundary conditions	Did not solve	Solved
50	Heat PDE	Heat/Diffusion PDE in 2D, inside rectangle with initial and boundary conditions with heat loss	Did not solve	Solved
51	Heat PDE	Heat PDE inside disk, with no θ dependency. initial and boundary conditions given	Solved	Did not solve
52	Heat PDE	Heat PDE on whole line with no intial nor boundary conditions specified	Did not solve	Solved, returning a solution that is not the most general one
53	Heat PDE	Heat PDE in 1D on the whole real line with initial position specified	Solved	Solved
54	Heat PDE	Heat PDE in 1D on the whole real line, with linear adevction	Solved	Solved
55	Heat PDE	Heat PDE in 1D on the whole real line with initial position as UnitBox	Solved	Solved
56	Heat PDE	Heat PDE on half the line with nonzero initial conditions and Dirichlet boundary conditions	Solved	Solved, but has unresolved inverse Laplace transforms
57	Heat PDE	Heat PDE on half the line with zero initial conditions and time dependent boundary conditions	Solved	Solved, but has unresolved inverse Laplace transforms
58	Heat PDE	Initial value problem for the heat PDE with a Neumann condition on the half-line	Solved	Did not solve
59	Laplace PDE	Laplace PDE inside quarter-circle	Did not solve	Did not solve
60	Laplace PDE	Laplace PDE inside semi-circle	Did not solve	Solved
61	Laplace PDE	Laplace PDE inside rectangle	Solved	Solved
62	Laplace PDE	Laplace PDE inside rectangle	Solved	Solved
63	Laplace PDE	Laplace PDE inside rectangle	Did not solve	Solved
64	Laplace PDE	Laplace PDE inside rectangle	Did not solve	Solved
65	Laplace PDE	Laplace PDE inside rectangle	Did not solve	Solved
66	Laplace PDE	Laplace PDE inside rectangle, top/bottom edges non-zero	Solved	Solved
67	Laplace PDE	Laplace PDE inside circular annulus, Neumann boundary conditions using unspecified functions	Did not solve	Did not solve
68	Laplace PDE	Laplace PDE inside circular annulus, Dirichlet boundary conditions using specified functions	Solved	Did not solve
69	Laplace PDE	Laplace PDE example 18 from Maple help page	Solved	Solved
70	Laplace PDE	Laplace PDE on rectangle with one edge at infinity	Did not solve	Solved
71	Laplace PDE	Laplace PDE inside a disk, periodic boundary conditions	Solved	Solved
72	Laplace PDE	Dirichlet problem for the Laplace equation in upper half plan	Solved	Did not solve
73	Laplace PDE	Dirichlet problem for the Laplace equation in right half-plane:	Solved	Did not solve
74	Laplace PDE	Dirichlet problem for the Laplace equation in the first quadrant	Solved	Did not solve

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Table 1 – continued from previous page

#	PDE	description	Mathematica	Maple
75	Laplace PDE	Neumann problem for the Laplace equation in the upper half-plane	Solved	Did not solve
76	Laplace PDE	Dirichlet problem for the Laplace equation in a rectangle	Solved	Solved
77	Laplace PDE	Laplace PDE outside a disk, periodic boundary conditions	Did not solve	Solved
78	Laplace PDE	Laplace equation in spherical coordinates	Did not solve	Solved, but not verified
79	Poisson PDE	Dirichlet problem for the Poisson equation in a rectangle	Solved	Solved
80	Helmholtz PDE	Dirichlet problem for the Helmholtz equation in a rectangle	Solved	Did not solve
81	Wave PDE	General solution for a second-order hyperbolic PDE on real line	Solved	Solved
82	Wave PDE	Hyperbolic PDE with non-rational coefficients	Solved	Did not solve. Tried all HINTS
83	Wave PDE	Inhomogeneous hyperbolic PDE with constant coefficients	Solved	Solved
84	Wave PDE	system of 2 inhomogeneous linear hyperbolic system with constant coefficients	Solved	Did not solve
85	Wave PDE	Wave PDE on string (finite domain) with zero initial position and velocity, and with source term	Solved	Solved
86	Wave PDE	Wave PDE on string, one end fixed, another free, both initial conduitions non zero, and source that depends on time and space	Did not solve	Solved
87	Wave PDE	Wave PDE on string (finite domain), fixed ends, no initial conduitions give and no source	Did not solve	Solved
88	Wave PDE	Wave PDE on string (finite domain), one fixed end, one free end, initial position not zero, initial velocity zero, no source	Did not solve	Solved
89	Wave PDE	Wave PDE on string (finite domain), both ends fixed end, initial conditions zero, with source as generic function that depends on time and space	Did not solve	Solved
90	Wave PDE	Wave PDE on string (finite domain), both ends fixed, initial conditions both not zero, No source	Solved	Solved
91	Wave PDE	Wave PDE on string (finite domain), both ends fixed end, initial conditions both not zero, and with constant source	Did not solve	Solved
92	Wave PDE	Wave PDE on string (finite domain), both ends fixed end, with source	Did not solve	Solved
93	Wave PDE	Wave PDE on semi-infinite domain, with one end having a moving boundary condition	Solved	Solved
94	Wave PDE	Telegraphy PDE, a wave PDE on string, both ends fixed with damping	Did not solve	Solved, But $n = 1$ should not be included.
95	Wave PDE	Wave PDE, on string, both ends fixed. Initial velocity zero. Dispersion term present	Did not solve due to adding dispersion term	Solved
96	Wave PDE	Wave PDE on string with fixed ends, non-zero initial position	Solved but sum should not include $n = 2$	Solved, but sum should not include $n = 2$
97	Wave PDE	Wave PDE homogeneous in square, given initial position but with zero initial velocity	Did not solve	Solved
98	Wave PDE	Wave PDE homogeneous in square with damping. Given zero initial position but with non-zero initial velocity	Did not solve	Solved

Continued on next page

Table 1 – continued from previous page

#	PDE	description	Mathematica	Maple
99	Wave PDE	Wave PDE inside rectangle. All 4 edges are fixed and given non-zero initial position with zero initial velocity	Solved	Solved
100	Wave PDE	Wave PDE inside disk. fixed edge of disk, no θ dependency, with initial position and velocity given	Solved	Did not solve
101	Wave PDE	Wave PDE inside disk. fixed edge of disk, with θ dependency, zero initial velocity	Did not solve	Did not solve
102	Wave PDE	Wave PDE on infinite domain with initial conditions specified, no source	Solved	Solved
103	Wave PDE	Wave PDE on infinite domain with initial conditions specified, with source term	Solved	Solved
104	Wave PDE	Wave PDE initial value with a Dirichlet condition on the half-line	Solved	Solved
105	Wave PDE	Wave PDE Initial value problem with a Neumann condition on the half-line	Solved	Did not solve
106	Wave PDE	non-linear wave PDE (Solitons)	Solved. build a special solution.	Solved. Returning a solution that is not the most general one
107	Schrodinger PDE	Schrodinger PDE with zero potential	Solved	Solved
108	Schrodinger PDE	Schrodinger PDE with initial and boundary conditions	Solved	Did not solve, hangs
109	Schrodinger PDE	Initial value problem with Dirichlet boundary conditions	Solved	Solved
110	Schrodinger PDE	Solve a Schrodinger equation with potential over the whole real line	Solved	Did not solve. Maple does not support ∞ in boundary conditions
111	Beam PDE	Beam PDE with zero initial velocity	Solved	Solved
112	Burger's PDE	viscous fluid flow with no initial conditions	Solved	Solved
113	Burger's PDE	viscous fluid flow with initial conditions	Solved	Solved, but has unresolved integrals
114	Burger's PDE	viscous fluid flow with initial conditions as UnitBox	Solved	Solved, but has unresolved integrals
115	Black Scholes PDE	classic Black Scholes model from finance	Solved	Did not solve
116	Black Scholes PDE	Boundary value problem for the Black Scholes equation	Solved	Did not solve
117	Korteweg-deVries PDE	Korteweg-deVries (waves on shallow water surfaces) with no initial conditions	Solved	Solved
118	Tricomi PDE	Boundary value problem for the Tricomi equation	Solved	Did not solve
119	Cauchy Riemann PDE's	Cauchy Riemann PDE with Prescribe the values of u and v on the x axis	Solved	Did not solve
120	Cauchy Riemann PDE's	Cauchy Riemann PDE With extra term on right side	Did not Solve	Solved
121	Hamilton-Jacobi PDE	Hamilton-Jacobi type PDE	Did not Solve	Solved
122	Other second order PDE's	A second order PDE	Did not Solve	Solved

1 First order PDE

1.1 Linear PDE, the transport equation

problem number 1

Taken from Mathematica Symbolic PDE document
Solve for $u(x,t)$

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=0$$

Mathematica

```
ClearAll[u, x, t];
 pde = D[u[x, t], {t}] + D[u[x, t], {x}] == 0;
 sol = DSolve[pde, u[x, t], {x, t}];
 ;
```

$$\{\{u(x,t)\to c_1(t-x)\}\}$$

Result Solved

Maple

```

interface(showassumed=0);
u:='u';x:='x';t:='t';
pde := diff(u(x, t), t) + diff(u(x, t),x) =0;
sol:=pdsolve(pde,u(x,t));
```

$$u(x,t)=\_F1(-x+t)$$

Result Solved

1.2 Linear PDE

problem number 2

Taken from Mathematica help pages
Solve for $u(x,y)$

$$3\frac{\partial u}{\partial x}+5\frac{\partial u}{\partial y}=0$$

Mathematica

```
ClearAll[u, x, y];
 sol = DSolve[3*D[u[x, y], x] + 5*D[u[x, y], y] == x, u[x, y], {x, y}];
 ;
```

$$\left\{\left\{u(x,y)\to\frac{1}{6}\left(6c_1\left(\frac{1}{3}(3y-5x)\right)+x^2\right)\right\}\right\}$$

Result Solved

Maple

```
interface(showassumed=0);
u:='u';x:='x';y:='y';
pde:=3*diff(u(x, y), x) + 5*diff(u(x, y), y) = x;
sol:=pdsolve(pde,u(x,y));
```

$$u\left(x,y\right)=1/6\,x^{2}+\_F1\left(-5/3\,x+y\right)$$

Result Solved

1.3 Linear PDE, initial value problem

problem number 3

Taken from Mathematica help pages
Solve for $u(x,y)$

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=-4xyu(x,y)$$

with initial value $u(x,0)=e^{-x^2}$

Mathematica

```
ClearAll[u, x, y];
 pde = x*D[u[x, y], y] + y*D[u[x, y], x] == -4*x*y*u[x, y];
 ic = u[x, 0] == E^(-x^2);
 sol = DSolve[{pde, ic}, u[x, y], {x, y}];
 ;
```

$$\left\{\left\{u(x,y)\to e^{-x^2-y^2}\right\}\right\}$$

Result Solved

Maple

```

interface(showassumed=0);
u:='u';x:='x';y:='y';
pde := x*diff(u(x, y), y) + y*diff(u(x, y), x) = -4*x*y*u(x, y);
ic := u(x, 0) = exp(-x^2);
sol:=pdsolve({pde, ic}, u(x, y));
```

$$u(x,y)=\mathrm{e}^{-x^2-y^2}$$

Result Solved

1.4 Initial-boundary value problem

problem number 4

Taken from Mathematica help pages
Solve for $u(x,t)$

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=0$$

with initial value $u(x,0)=\sin x$ and boundary value $u(0,t)=0$

Mathematica

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] + D[u[x, t], x] == 0;
 bc = u[0, t] == 0;
 ic = u[x, 0] == Sin[x];
 sol = DSolve[{pde, ic, bc}, u[x, t], {x, t}];
 ;
```

$$\{\{u(x,t)\to(\theta(t-x)-1)\sin(t-x)\}\}$$

Result Solved

Maple

```
u:='u';x:='x';t:='t';
pde:=diff(u(x,t),t)+diff(u(x,t),x)=0;
bc:=u(0,t)=0;
ic:=u(x,0)=sin(x);
sol:=pdsolve({pde,ic,bc},u(x,t));
```

$$\text{sol}=()$$

Result Did not solve

1.5 Linear PDE, the transport equation with initial conditions

problem number 5

Taken from Mathematica help pages
Solve for $u(x,t)$

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

With initial conditions $u(x,0) = e^{-x^2}$

Mathematica

```
ClearAll[u, x, t, c];
 ic = u[x, 0] == Exp[-x^2];
 pde = D[u[x, t], {t}] + c*D[u[x, t], {x}] == 0;
 sol = DSolve[{pde, ic}, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t) \to e^{-(x-ct)^2}\right\}\right\}$$

Result Solved

Maple

```
interface(showassumed=0);
u:='u';x:='x';t:='t';c:='c';
pde := diff(u(x, t), t) + c* diff(u(x, t),x) =0;
ic:=u(x,0)=exp(-x^2);
sol:=pdsolve({pde,ic},u(x,t));
```

$$u(x,t) = \mathrm{e}^{-(tc-x)^2}$$

Result Solved

1.6 General solution for a quasilinear first-order PDE

problem number 6

Taken from Mathematica help pages
Solve for $u(x,y)$

$$2\frac{\partial u}{\partial x}+5\frac{\partial u}{\partial y}=u^2(x,y)+1$$

Mathematica

```
ClearAll[u, x, y];
 pde = 2*D[u[x, y], x] + 5*D[u[x, y], y] == u[x, y]^2 + 1;
 sol = DSolve[pde, u[x, y], {x, y}];
 ;
```

$$\left\{\left\{u(x,y)\to\tan\left(\frac{1}{2}\left(2c_1\left(\frac{1}{2}(2y-5x)\right)+x\right)\right)\right\}\right\}$$

Result Solved

Maple

```
interface(showassumed=0);
u:='u';x:='x';y:='y';
pde := 2* diff(u(x, y), x) + 5*diff(u(x, y), y) = u(x, y)^2 + 1;
sol:=pdsolve(pde, u(x, y));
```

$$u(x,y)=\tan\left(x/2+1/2\_F1\left(-5/2\,x+y\right)\right)$$

Result Solved

1.7 quasilinear first-order PDE, scalar conservation law

problem number 7

Taken from Mathematica Symbolic PDE document
Solve for $u(x,y)$

$$\frac{\partial u}{\partial x} + u(x,y)\frac{\partial u}{\partial y} = 0$$

Mathematica

```
ClearAll[u, x, y];
 pde = D[u[x, y], {x}] + u[x, y]*D[u[x, y], {y}] == 0;
 sol = DSolve[pde, u[x, y], {x, y}];
 ;
```

$$\text{Solve}\left[u(x,y) = c_1\left(x - \frac{y}{u(x,y)}\right), u(x,y)\right]$$

Result Solved, solution in implicit form

Maple

```
interface(showassumed=0);
u:='u';x:='x';y:='y';
pde := diff(u(x, y), x) + u(x,y)*diff(u(x, y),y) =0;
sol:=pdsolve(pde,u(x,y));
sol:=DEtools:-remove_RootOf(sol);
```

$$-y + xu\left(x,y\right) + \_F1\left(u\left(x,y\right)\right) = 0$$

Result Solved

1.8 quasilinear first-order PDE, scalar conservation law with initial value

problem number 8

Taken from Mathematica Symbolic PDE document
Solve for $u(x,y)$

$$\frac{\partial u}{\partial x} + u(x,y)\frac{\partial u}{\partial y} = 0$$

With $u(x,0) = \frac{1}{x+1}$

Mathematica

```
ClearAll[u, x, y];
 pde = D[u[x, y], {x}] + u[x, y]*D[u[x, y], {y}] == 0;
 ic = u[x, 0] == 1/(x + 1);
 sol = DSolve[{pde, ic}, u[x, y], {x, y}];
 ;
```

$$\left\{\left\{u(x,y)\to \frac{y+1}{x+1}\right\}\right\}$$

Result Solved

Maple

```
interface(showassumed=0);
u:='u';x:='x';y:='y';
pde := diff(u(x, y), x) + u(x,y)*diff(u(x, y),y) =0;
ic:=u(x,0)=1/(x+1);
sol:=pdsolve({pde,ic},u(x,y));
```

$$u(x,y) = \frac{y+1}{x+1}$$

Result Solved

1.9 nonlinear first-order PDE, the Clairaut equation

problem number 9

Taken from Mathematica Symbolic PDE document
Solve for $u(x,y)$

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+\frac{1}{2}\left(\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right)=0$$

Mathematica

```
ClearAll[u, x, y];
 pde = u[x, y] == x*D[u[x, y], {x}] + y*D[u[x, y], {y}] + (1/2)*(D[u[x, y], {x}]^2 + D[u[x, y], {y}]^2);
 sol = DSolve[pde, u[x, y], {x, y}];
 ;
```

$$\left\{\left\{u(x,y)\to c_1x+c_2y+\frac{1}{2}\left(c_1^2+c_2^2\right)\right\}\right\}$$

Result Solved

Maple

```
interface(showassumed=0);
u:='u';x:='x';y:='y';
pde := x*diff(u(x, y), x) + y*diff(u(x, y),y) + 1/2 * ( diff(u(x, y), x)^2 + diff(u(x, y), y)^2)=0;
sol:=pdsolve(pde,u(x,y),'build');
```

$$u(x,y)=-1/2\,x^2-1/2\,x\sqrt{x^2+2\_c_1}-\_c_1\ln\left(x+\sqrt{x^2+2\_c_1}\right)+\_C1-1/2\,y^2-1/2\,y\sqrt{y^2-2\_c_1}+\_c_1\ln\left(y+\sqrt{y^2-2\_c_1}\right)+\_C2$$

Result Solved

1.10 nonlinear first-order PDE, the Clairaut equation with initial value

problem number 10

Taken from Mathematica Symbolic PDE document
Solve for $u(x,y)$

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+\frac{1}{2}\left(\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right)=0$$

With $u(x,0)=\frac{1}{2}(1-x^2)$

Mathematica

```
ClearAll[u, x, y];
 pde = u[x, y] == x*D[u[x, y], {x}] + y*D[u[x, y], {y}] + (1/2)*(D[u[x, y], {x}]^2 + D[u[x, y], {y}]^2);
 ic = u[x, 0] == (1*(1 - x^2))/2;
 sol = DSolve[{pde, ic}, u[x, y], {x, y}];
 ;
```

$$\left\{\left\{u(x,y)\to\frac{1}{2}\left(-x^2-2y+1\right)\right\}\right\}$$

Result Solved

Maple

```
interface(showassumed=0);
u:='u';x:='x';y:='y';
pde := x*diff(u(x, y), x) + y*diff(u(x, y),y) + 1/2 * ( diff(u(x, y), x)^2 + diff(u(x, y), y)^2)=0;
ic:=u(x,0)=1/2*(1-x^2);
sol:=pdsolve({pde,ic},u(x,y));
```

$$-1/2\left(x-y+1\right)\left(x-y-1\right)=-1/2\left(1+x+y\right)\left(x+y-1\right)$$

Result Solved

1.11 Another example of nonlinear Clairaut equation

problem number 11

Taken from Mathematica DSolve help pages
Solve for $u(x,y)$

$$u(x,y) = x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \sin\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right)$$

Mathematica

```
ClearAll[u, x, y];
 pde = u[x, y] == x*D[u[x, y], x] + y*D[u[x, y], y] + Sin[D[u[x, y], x] + D[u[x, y], y]];
 sol = DSolve[pde, u[x, y], {x, y}];
 ;
```

$$\{\{u(x,y) \to c_1 x + c_2 y + \sin(c_1 + c_2)\}\}$$

Result Solved

Maple

```
u:='u';x:='x';y:='y';
pde:= u(x,y)= x*diff(u(x,y),x)+y*diff(u(x,y),y)+sin(diff(u(x,y),x)+diff(u(x,y),y));
sol:=pdsolve(pde,u(x,y));
```

$$u(x,y) = x\_c_1 + y\_c_2 + \sin(\_c_1 + \_c_2)$$

Result Solved

1.12 Recover a function from its gradient vector

problem number 12

Taken from Mathematica DSolve help pages
Solve for $f(x,y)$

$$\frac{\partial f}{\partial x} = xy\cos(xy) + \sin(xy)$$
$$\frac{\partial f}{\partial y} = -e^{-y} + x^2\cos(xy)$$

Mathematica

```
ClearAll[f, x, y];
 eq1 = D[f[x, y], x] == x*y*Cos[x*y] + Sin[x*y];
 eq2 = D[f[x, y], y] == -E^(-y) + x^2*Cos[x*y];
 sol = DSolve[{eq1, eq2}, f[x, y], {x, y}];
 ;
```

$$\{\{f(x,y) \to c_1 + x\sin(xy) + e^{-y}\}\}$$

Result Solved

Maple

```
u:='u';x:='x';y:='y';
eq1:=diff(f(x,y),x)=x*y*cos(x*y)+sin(x*y);
eq2:=diff(f(x,y),y)=-exp(-y)+x^2*cos(x*y);
sol:=pdsolve({eq1,eq2},f(x,y));
```

$$\left\{f\left(x,y\right) = x\sin\left(yx\right) + \mathrm{e}^{-y} + \_C1\right\}$$

Result Solved

1.13 General solution of a first order nonlinear PDE

problem number 13

Taken from Maple pdsolve help pages
Solve for $f(x,y)$

$$x\frac{\partial f}{\partial y}-\frac{\partial f}{\partial x}=\frac{f^2(x,y)g(x)}{h(y)}$$

Mathematica

```
ClearAll[f, x, y, h, g];
 pde = x*D[f[x, y], y] - D[f[x, y], x] == (f[x, y]^2*g[x])/h[y];
 sol = DSolve[pde, f[x, y], {x, y}];
 ;
```

$$\text{DSolve}\left[xf^{(0,1)}(x,y)-f^{(1,0)}(x,y)=\frac{g(x)f(x,y)^2}{h(y)},f(x,y),\{x,y\}\right]$$

Result Did not solve

Maple

```
x:='x';y:='y';f:='f';g:='g';h:='h';
pde := x*diff(f(x,y),y)-diff(f(x,y),x)=f(x,y)^2*g(x)/h(y);
sol:=pdsolve(pde,f(x,y));
```

$$f(x,y)=\left(\int^{x}\frac{g(\_a)}{h\left(-1/2\_a^2+1/2\,x^2+y\right)}d\_a+\_F1\left(1/2\,x^2+y\right)\right)^{-1}$$

Result Solved

1.14 Nonlinear first order PDE

problem number 14

Taken from Maple pdsolve help pages, probem 5
Solve for $f(x,y,z)$

$$\frac{\partial f}{\partial y} + \left(\frac{\partial f}{\partial y}\right)^2 = f(x,y,z) + z$$

Mathematica

```
ClearAll[f, x, y, z];
 pde = D[f[x, y, z], x] + D[f[x, y, z], y]^2 == f[x, y, z] + z;
 sol = DSolve[pde, f[x, y, z], {x, y, z}];
 ;
```

$$\left\{\left\{f(x,y,z)\to\frac{1}{4}\left(c_1(z)^2\text{ProductLog}\left(-\frac{e^{\frac{y}{c_1(z)}+\frac{c_2(z)}{c_1(z)}+x-1}}{c_1(z)}\right)^2+2c_1(z)^2\text{ProductLog}\left(-\frac{e^{\frac{y}{c_1(z)}+\frac{c_2(z)}{c_1(z)}+x-1}}{c_1(z)}\right)-4z\right)\right\}\right\}$$

Result Solved

Maple

```

x:='x';y:='y';f:='f';z:='z';
pde := diff(f(x,y,z),x) + (diff(f(x,y,z),y))^2 = f(x,y,z)+z;
sol:=pdsolve(pde,'build');
```

$$f\left(x,y,z\right)=-\frac{\mathrm{e}^{-x}z\_C5^2+\mathrm{e}^{x}\_C3^2+\_C3\,y\_C5+z\_C4\_C5+\_C1\_C5}{\_C5^2\mathrm{e}^{-x}}$$

Result Solved

2 Heat PDE

2.1 Heat PDE on bar, homogeneous Dirichlet boundary conditions, No source.

problem number 15

This is problem 2.3.3, part (a) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

Subject to boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$ with the temperature initially $u(x,0) = 6\sin\left(\frac{3\pi x}{L}\right)$

Mathematica

```
ClearAll[u, t, k, x, L, n];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = u[x, 0] == 6*Sin[(9*Pi*x)/L];
 NumericQ[L] = True;
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t)\to 6e^{-\frac{81\pi^2kt}{L^2}}\sin\left(\frac{9\pi x}{L}\right)\right\}\right\}$$

Result Solved

Maple

```
interface(showassumed=0);
assume(L>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=6*sin(9*Pi*x/L);
sol:=pdsolve([pde,bc,ic],u(x,t));
```

$$u(x,t) = 6\sin\left(9\frac{\pi x}{L}\right)e^{-81\frac{k\pi^2t}{L^2}}$$

Result Solved

2.2 Heat PDE on bar, homogeneous Dirichlet boundary conditions, No source.

problem number 16

This is problem 2.3.3, part (b) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

Subject to boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$ with the temperature initially $u(x,0) = 3\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, n];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = u[x, 0] == 3*Sin[(Pi*x)/L] - Sin[(3*Pi*x)/L];
 NumericQ[L] = True;
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t) \to e^{-\frac{9\pi^2 kt}{L^2}}\sin\left(\frac{\pi x}{L}\right)\left(3e^{\frac{8\pi^2 kt}{L^2}} - 2\cos\left(\frac{2\pi x}{L}\right) - 1\right)\right\}\right\}$$

Result Solved

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
assume(L>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=3*sin(Pi*x/L)-sin(3*Pi*x/L);
sol:=pdsolve([pde,bc,ic],u(x,t));
```

$$u(x,t) = 3\sin\left(\frac{\pi x}{L}\right)e^{-\frac{k\pi^2 t}{L^2}} - \sin\left(3\frac{\pi x}{L}\right)e^{-9\frac{k\pi^2 t}{L^2}}$$

Result Solved

2.3 Heat PDE on bar, homogeneous Dirichlet boundary conditions, No source.

problem number 17

This is problem 2.3.3, part (c) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

Subject to boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$ with the temperature initially $u(x,0) = 2\cos\frac{3\pi x}{L}$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, L, n];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = u[x, 0] == 2*Cos[(3*Pi*x)/L];
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> L > 0];
 sol = sol /. K[1] -> n;
 ;
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}\frac{4\left(1+(-1)^n\right)e^{-\frac{kn^2\pi^2t}{L^2}}n\sin\left(\frac{n\pi x}{L}\right)}{\left(n^2-9\right)\pi}\right\}\right\}$$

Result Solved

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
assume(L>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=2*cos(3*Pi*x/L);
sol:=pdsolve([pde,bc,ic],u(x,t));
sol:=algsubs(_Z1=n,sol);
```

$$u(x,t)=\sum_{n=1}^{\infty}\begin{cases}0 & n=3\\ 4\,\frac{n\left((-1)^n+1\right)}{\pi\left(n^2-9\right)}\sin\left(\frac{n\pi x}{L}\right)\mathrm{e}^{-\frac{\pi^2kn^2t}{L^2}} & \text{otherwise}\end{cases}$$

Result Solved

2.4 Heat PDE on bar, homogeneous Dirichlet boundary conditions, No source.

problem number 18

This is problem 2.3.3, part (d) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

Subject to boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$ with the temperature initially $u(x,0) = \begin{cases} 1 & 0 < x \leq \frac{L}{2} \\ 2 & \frac{L}{2} < x \leq L \end{cases}$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, L, n];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = u[x, 0] == Piecewise[{{1, Inequality[0, Less, x, LessEqual, L/2]}, {2, L/2 < x < L}}];
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> L > 0];
 sol = sol /. K[1] -> n;
 ;
```

$$\left\{\left\{u(x,t) \to \sum_{n=1}^{\infty} \frac{4e^{-\frac{kn^2\pi^2 t}{L^2}}\left(4\cos\left(\frac{n\pi}{2}\right)+3\right)\sin^2\left(\frac{n\pi}{4}\right)\sin\left(\frac{n\pi x}{L}\right)}{n\pi}\right\}\right\}$$

Result Solved

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
assume(L>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=piecewise(0<x and x<=L/2,1,L/2<x and x<L,2);
#need to convert below, else it will not work;
ic:=convert(ic,piecewise,x);
sol:=pdsolve([pde,bc,ic],u(x,t)) assuming 0<L;
sol:=subs(_Z1=n,sol);
```

$$u(x,t) = \sum_{n=1}^{\infty} \frac{2\cos(1/2\, n\pi) + 2 + 4\,(-1)^{1+n}}{n\pi} \sin\left(\frac{n\pi\, x}{L}\right) e^{-\frac{\pi^2 k n^2 t}{L^2}}$$

Result Solved

2.5 Heat PDE on bar, homogeneous Neumann boundary conditions, No source.

problem number 19

This is problem 2.3.7, from Richard Haberman applied partial differential equations, 5th edition. Consider the heat equation

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$$

Subject to boundary conditions $\frac{\partial u}{\partial x}(0,t) = 0$ $\frac{\partial u}{\partial x}(L,t) = 0$ with the temperature initially $u(x,0) = f(x)$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, L, sol, n, f];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == f[x];
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {L > 0, k > 0, t > 0}];
 sol = sol /. {K[1] -> n, K[2] -> x};
 ;
```

$$\left\{\left\{u(x,t)\to \frac{2\sum_{n=1}^{\infty} e^{-\frac{kn^2\pi^2 t}{L^2}} \cos\left(\frac{n\pi x}{L}\right)\int_0^L \cos\left(\frac{n\pi x}{L}\right) f(x)\,dx}{L} + \frac{\int_0^L f(x)\,dx}{L}\right\}\right\}$$

Result Solved

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';
interface(showassumed=0);
assume(L>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic:=u(x,0)=f(x);
sol:=pdsolve([pde,bc,ic],u(x,t));
sol:=subs(_Z2=n,sol);
```

$$u(x,t) = \frac{1}{L}\left(\sum_{n=1}^{\infty}\left(2\frac{1}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)\mathrm{d}x\cos\left(\frac{n\pi x}{L}\right)\mathrm{e}^{-\frac{\pi^2 k n^2 t}{L^2}}\right)L + \int_0^L f(x)\,\mathrm{d}x\right)$$

Result Solved

2.6 Heat PDE on bar, homogeneous Dirichlet boundary conditions with heat sink

problem number 20

This is problem 2.3.8, from Richard Haberman applied partial differential equations, 5th edition. Consider the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} - \alpha u$$

This corresponds to a one-dimentional rod either with heat loss through the lateral sides with outside temperature zero degrees ($\alpha > 0$) or with insulated sides with a heat sink propertional to the temperature.

Suppose the boundary conditions are $u(0,t) = 0, u(L,t) = 0$, solve with the temperature initially $u(x,0) = f(x)$ if $\alpha > 0$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, L, a, f];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] - a*u[x, t];
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = u[x, 0] == f[x];
 NumericQ[L] = True;
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,1)}(x,t) = ku^{(2,0)}(x,t) - au(x,t), \{u(0,t) = 0, u(L,t) = 0\}, u(x,0) = f(x)\right\}, u(x,t), \{x,t\}\right]$$

Result Did not solve

Maple

```
L:='L'; u:='u'; t:='t'; x:='x'; a:='a';f:='f';
interface(showassumed=0);
assume(a>0);
assume(L>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2)-a*u(x,t);
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=f(x);
sol:=pdsolve([pde,bc,ic],u(x,t));
sol:=subs(_Z1=n,sol);
```

$$u(x,t) = \sum_{n=1}^{\infty} \left(2\frac{1}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right) \mathrm{d}x \sin\left(\frac{n\pi x}{L}\right) \mathrm{e}^{-\frac{t\left(\pi^2 k n^2 + L^2 a\right)}{L^2}}\right)$$

Result Solved

2.7 Heat PDE on bar, homogeneous Neumann boundary conditions, No source

problem number 21

This is problem 2.4.1 part(a) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

The boundary conditions are $\frac{\partial u}{\partial x}(0,t) = 0$ $\frac{\partial u}{\partial x}(L,t) = 0$ with the temperature initially $u(x,0) = \begin{cases} 0 & x < \frac{L}{2} \\ 1 & x > \frac{L}{2} \end{cases}$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, L];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == Piecewise[{{0, x < L/2}, {1, x > L/2}}];
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {L > 0 && k > 0}];
 sol = sol /. {K[1] -> n};
 ;
```

$$\left\{\left\{u(x,t)\to\frac{2\sum_{n=1}^{\infty}-\frac{e^{-\frac{kn^2\pi^2t}{L^2}}L\cos\left(\frac{n\pi x}{L}\right)\sin\left(\frac{n\pi}{2}\right)}{n\pi}}{L}+\frac{1}{2}\right\}\right\}$$

Result Solved

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
assume(L>0);
ic:=u(x,0)=piecewise(0<x and x<=L/2,0,L/2<x and x<L,1);
sol:=pdsolve([pde,bc,ic],u(x,t));
sol:=subs(_Z2=n,sol);
sol:=simplify(rhs(sol));
```

$$1/2+\sum_{n=1}^{\infty}-2\,\frac{\sin(1/2\,n\pi)}{n\pi}\cos\left(\frac{n\pi\,x}{L}\right)e^{-\frac{\pi^2kn^2t}{L^2}}$$

Result Solved

2.8 Heat PDE on bar, homogeneous Neumann boundary conditions, No source

problem number 22

This is problem 2.4.1 part(b) from Richard Haberman applied partial differential equations, 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

The boundary conditions are $\frac{\partial u}{\partial x}(0,t) = 0$ $\frac{\partial u}{\partial x}(L,t) = 0$ with the temperature initially $u(x,0) = 6 + 4\cos\left(\frac{3\pi x}{L}\right)$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, L];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == 6 + 4*Cos[(3*Pi*x)/L];
 NumericQ[L] = True;
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 sol = sol /. {K[1] -> n};
 ;
```

$$\left\{\left\{u(x,t)\to 4e^{-\frac{9\pi^2 kt}{L^2}}\cos\left(\frac{3\pi x}{L}\right)+6\right\}\right\}$$

Result Solved

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
assume(L>0 and k>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic:=u(x,0)=6+4*cos(3*Pi*x/L);
sol:=pdsolve([pde,bc,ic],u(x,t));
```

$$u(x,t) = 6+4\cos\left(3\frac{\pi x}{L}\right)e^{-9\frac{k\pi^2 t}{L^2}}$$

Result Solved

2.9 Heat PDE on bar, homogeneous Neumann boundary conditions, No source

problem number 23

This is problem 2.4.1 part(c) from Richard Haberman applied partial differential equations, 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

The boundary conditions are $\frac{\partial u}{\partial x}(0,t) = 0$ $\frac{\partial u}{\partial x}(L,t) = 0$ with the temperature initially $u(x,0) = -2\sin\frac{\pi x}{L}$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, L];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == -2*Sin[(Pi*x)/L];
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {L > 0 && k > 0}];
 sol = sol /. {K[1] -> n};
 ;
```

$$\left\{\left\{u(x,t) \to \frac{2\sum_{n=1}^{\infty}\frac{2(1+(-1)^n)e^{-\frac{kn^2\pi^2 t}{L^2}}L\cos\left(\frac{n\pi x}{L}\right)}{(n^2-1)\pi}}{L} - \frac{4}{\pi}\right\}\right\}$$

Result Solved

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
assume(L>0 and k>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic:=u(x,0)=-2*sin(Pi*x/L);
sol:=pdsolve([pde,bc,ic],u(x,t));
sol:=subs(_Z2=n,sol);
```

$$u(x,t) = \frac{1}{\pi}\left(\sum_{n=1}^{\infty}\begin{cases}0 & n\leq 1\\ 4\frac{(-1)^n+1}{\pi(n^2-1)}\cos\left(\frac{n\pi x}{L}\right)\mathrm{e}^{-\frac{k\pi^2 n^2 t}{L^2}} & 1<n\end{cases}\pi - 4\right)$$

Result Solved

2.10 Heat PDE on bar, homogeneous Neumann boundary conditions, No source

problem number 24

This is problem 2.4.1 part(d) from Richard Haberman applied partial differential equations, 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

The boundary conditions are $\frac{\partial u}{\partial x}(0,t) = 0$ and $\frac{\partial u}{\partial x}(L,t) = 0$ with the temperature initially $u(x,0) = -3\cos\frac{8\pi x}{L}$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, L];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == -3*Cos[(8*Pi*x)/L];
 NumericQ[L] = True;
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 sol = sol /. {K[1] -> n};
 ;
```

$$\left\{\left\{u(x,t)\to -3e^{-\frac{64\pi^2 kt}{L^2}}\cos\left(\frac{8\pi x}{L}\right)\right\}\right\}$$

Result Solved

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
assume(L>0 and k>0);
ic:=u(x,0)=-3*cos(8*Pi*x/L);
sol:=pdsolve([pde,bc,ic],u(x,t));
sol:=simplify(rhs(sol));
```

$$-3\cos\left(8\,\frac{\pi x}{L}\right)e^{-64\,\frac{k\pi^2 t}{L^2}}$$

Result Solved

2.11 Heat PDE on bar, homogeneous Neumann on left and Dirichlet on right, No source

problem number 25

This is problem 2.4.2 from Richard Haberman applied partial differential equations, 5th edition. Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

The boundary conditions are $\frac{\partial u}{\partial x}(0,t) = 0$ $u(L,t) = 0$ with the temperature initially $u(x,0) = f(x)$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, L, f];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, u[L, t] == 0};
 ic = u[x, 0] == f[x];
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {L > 0 && k > 0}];
 sol = sol /. {K[1] -> n, K[2] -> x};
 ;
```

$$\left\{\left\{u(x,t)\to \frac{2\sum_{n=0}^{\infty} e^{-\frac{k(2n+1)^2\pi^2 t}{4L^2}}\cos\left(\frac{(2n+1)\pi x}{2L}\right)\int_0^L \cos\left(\frac{(2n+1)\pi x}{2L}\right)f(x)\,dx}{L}\right\}\right\}$$

Result Solved

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,u(L,t)=0;
assume(L>0);
ic:=u(x,0)=f(x);
sol:=pdsolve([pde,bc,ic],u(x,t));
sol:=subs(_Z2=n,sol);
```

$$u(x,t) = \sum_{n=0}^{\infty}\left(2\frac{1}{L}\int_0^L f(x)\cos\left(1/2\,\frac{(1+2n)\pi x}{L}\right)\mathrm{d}x\cos\left(1/2\,\frac{(1+2n)\pi x}{L}\right)\mathrm{e}^{-1/4\,\frac{k\pi^2(1+2n)^2 t}{L^2}}\right)$$

Result Solved

2.12 Heat PDE on bar, semi-infinite domain, No source

problem number 26

This is problem at page 76 from David J Logan text book.
Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

The boundary conditions are $u(0,t) = f(t)$ and initial conditions $u(x,0) = 0$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, t, x, f];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 bc = u[0, t] == f[t];
 ic = u[x, 0] == 0;
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {t > 0, x > 0}];
 sol = sol /. {K[2] -> z};
```

$$\left\{\left\{u(x,t) \to \frac{x \int_0^t \frac{f(z) e^{-\frac{x^2}{4(t-z)}}}{(t-z)^{3/2}} \, dz}{2\sqrt{\pi}}\right\}\right\}$$

Result Solved

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,t),t)=diff(u(x,t),x$2);
ic:=u(x,0)=0;
bc:=u(0,t)=f(t);
sol:=pdsolve([pde,ic,bc],u(x,t),HINT = boundedseries) assuming t>0,x>0;
```

$$u(x,t) = 1/2\, \frac{x}{\sqrt{\pi}} \int_0^t \frac{f(\_U1)}{(t - \_U1)^{3/2}} e^{-\frac{x^2}{4t - 4\_U1}} \, d\_U1$$

Result Solved

2.13 Heat PDE on bar, periodic boundary conditions, No source

problem number 27

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

For $-L < x < L$ and $t > 0$. The boundary conditions are

$$u(-L,t) = u(L,t)$$
$$\frac{\partial u}{\partial x}(-L,t) = \frac{\partial u}{\partial x}(L,t)$$

And initial conditions $u(x,0) = f(x)$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, t, x, L, c, f, k];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {u[-L, t] == u[L, t], Derivative[1, 0][u][-L, t] == Derivative[1, 0][u][L, t]};
 ic = u[x, 0] == f[x];
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {-L <= x <= L, t > 0}];
 ;
```

$\text{DSolve}\left[\left\{u^{(0,1)}(x,t) = ku^{(2,0)}(x,t), \left\{u(-L,t) = u(L,t), u^{(1,0)}(-L,t) = u^{(1,0)}(L,t)\right\}, u(x,0) = f(x)\right\}, u(x,t), \{x,t\}, \text{Assumptions} \to \{-L \le x \le L, t > 0\}\right]$

Result Did not solve

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(-L,t)=u(L,t),D[1](u)(-L,t)=D[1](u)(L,t);
ic:=u(x,0)=f(x);
sol:=pdsolve([pde,bc,ic],u(x,t)) assuming x>=-L,x<=L;
```

$$u(x,t) = 1/2\,\frac{1}{L}\left(2\sum_{n=1}^{\infty}\left(\frac{1}{L}\left(\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)\mathrm{d}x\sin\left(\frac{n\pi x}{L}\right) + \int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)\mathrm{d}x\cos\left(\frac{n\pi x}{L}\right)\right)\mathrm{e}^{-\frac{k\pi^2 n^2 t}{L^2}}\right)L + \int_{-L}^{L} f(x)\,\mathrm{d}x\right)$$

Result Solved

2.14 Heat PDE on bar, semi-infinite domain, zero initial condition, No source

problem number 28

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

For $x > 0$ and $t > 0$. The boundary conditions is $u(0,t) = 1$ and And initial condition $u(x,0) = 0$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, t, x, k];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = u[0, t] == 1;
 ic = u[x, 0] == 0;
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {t > 0, k > 0, x > 0}];
 ;
```

$$\left\{\left\{u(x,t) \to \operatorname{erfc}\left(\frac{x}{2\sqrt{kt}}\right)\right\}\right\}$$

Result Solved

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic:=u(x,0)=0:
bc:=u(0,t)=1;
sol:=pdsolve([pde,ic,bc],u(x,t),HINT = boundedseries) assuming t>0,x>0,k>0;
```

$$u(x,t) = 1 - Erf\left(1/2\,\frac{x}{\sqrt{t}\sqrt{k}}\right)$$

Result Solved

2.15 Heat PDE on bar, semi-infinite domain, non-zero initial condition, No source

problem number 29

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

For $x > 0$ and $t > 0$. The boundary conditions is $u(0,t) = \mu$ and And initial condition $u(x,0) = \lambda$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, t, x, k, \[Lambda], \[Mu]];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = u[0, t] == \[Lambda];
 ic = u[x, 0] == \[Mu];
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {t > 0, k > 0, x > 0}];
 ;
```

$$\left\{\left\{u(x,t)\to \mu \operatorname{erf}\left(\frac{x}{2\sqrt{kt}}\right)+\lambda \operatorname{erfc}\left(\frac{x}{2\sqrt{kt}}\right)\right\}\right\}$$

Result Solved

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';mu:='mu';lambda:='lambda';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic:=u(x,0)=mu:
bc:=u(0,t)=lambda;
sol:=pdsolve([pde,ic,bc],u(x,t),HINT = boundedseries) assuming t>0,x>0,k>0;
```

$$u(x,t) = (-\lambda+\mu)\, Erf\left(1/2\,\frac{x}{\sqrt{t}\sqrt{k}}\right)+\lambda$$

Result Solved

2.16 Heat PDE on bar, heat absorption radiation in bounded domain, No source

problem number 30

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) + u(0,t) = 0$$
$$\frac{\partial u}{\partial x}(L,t) + u(L,t) = 0$$

And initial condition $u(x,0) = f(x)$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, t, x, k, L, f];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] + u[0, t] == 0, Derivative[1, 0][u][L, t] + u[L, t] == 0};
 ic = u[x, 0] == f[x];
 NumericQ[L] = True;
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {t >= 0, k > 0, x >= 0, x <= L}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,1)}(x,t) = ku^{(2,0)}(x,t), \left\{u^{(1,0)}(0,t) + u(0,t) = 0, u^{(1,0)}(L,t) + u(L,t) = 0\right\}, u(x,0) = f(x)\right\}, u(x,t), \{x,t\}, \text{Assumptions} \to \{t \geq 0, k > 0, x \geq 0, x \leq L\}\right]$$

Result Did not solve

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';mu:='mu';lambda:='lambda';f:='f';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic:=u(x,0)=f(x);
bc:=D[1](u)(0,t)+u(0,t)=0,D[1](u)(L,t)+u(L,t)=0;
sol:=pdsolve([pde,ic,bc],u(x,t)) assuming t>0,x>=0,x<=L;
sol:=subs(_Z1=n,sol);
```

$$u(x,t) = \sum_{n=1}^{\infty}\left(2\frac{1}{L\left(\pi^2 n^2 + L^2\right)}\int_0^L f(x)\left(-\pi n\cos\left(\frac{n\pi x}{L}\right) + \sin\left(\frac{n\pi x}{L}\right)L\right)\mathrm{d}x\left(-\pi n\cos\left(\frac{n\pi x}{L}\right) + \sin\left(\frac{n\pi x}{L}\right)L\right)\mathrm{e}^{-\frac{k\pi^2 n^2 t}{L^2}}\right)$$

Result Solved

2.17 Heat PDE infinite domain

problem number 31

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + m$$

For $-\infty < x < \infty$ and $t > 0$. The boundary conditions are
Initial condition is $u(x,0) = \sin(x)$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, t, x, m, k];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + m;
 ic = u[x, 0] == Sin[x];
 sol = DSolve[{pde, ic}, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t) \to e^{-kt}\sin(x) + mt\right\}\right\}$$

Result Solved

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';m:='m';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2)+m;
ic:=u(x,0)=sin(x);
sol:=pdsolve([pde,ic],u(x,t));
```

$$u(x,t) = \sin(x)\,\mathrm{e}^{-kt} + mt$$

Result Solved

2.18 Heat PDE on bar, with domain from -1 to +1, no source

problem number 32

Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

For $-1 < x < 1$ and $t > 0$. The boundary conditions are zero at both ends. Initial condition is $u(x,0) = f(x)$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, t, x, f];
 pde = D[u[x, t], {t, 1}] == D[u[x, t], {x, 2}];
 ic = u[x, 0] == f[x];
 bc = {u[-1, t] == 0, u[1, t] == 0};
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,1)}(x,t) = u^{(2,0)}(x,t), \{u(-1,t) = 0, u(1,t) = 0\}, u(x,0) = f(x)\right\}, u(x,t), \{x,t\}\right]$$

Result Did not solve

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';
interface(showassumed=0);
pde := diff(u(x,t),t) =diff(u(x,t),x$2);
ic := u(x,0) = f(x);
bc := u(-1,t)=0,  u(1,t)=0;
sol:=pdsolve([pde, ic, bc],u(x,t)) assuming t>0;
```

$$u(x,t) = \sum_{n=1}^{\infty} \left(\left(\left(\int_{-1}^{1} f(x) \sin(\pi n x)\, dx \sin(\pi n x) e^{1/4 \pi^2 (2n-1)^2 t} + \int_{-1}^{1} f(x) \cos(1/2 \pi x (2n-1))\, dx \cos(1/2 \pi x (2n-1)) e^{\pi^2 n^2 t} \right) e^{-1/4 \pi^2 t (8n^2 - 4n + 1)} \right) \right)$$

Result Solved

2.19 Heat PDE on bar, Dirichlet nonhomogeneous BC, no source term

problem number 33

Taken from Maple PDE help pages
Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = 20$$
$$u(1,t) = 50$$

Initial condition is $u(x,0) = 0$

Mathematica

```
ClearAll[u, x, t, n];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 bc = {u[0, t] == 20, u[1, t] == 50};
 ic = u[x, 0] == 0;
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t];
 sol = sol /. K[1] -> n;
 ;
```

$$\left\{\left\{u(x,t) \to -\frac{2\sum_{n=1}^{\infty} \frac{\left(20-50(-1)^n\right)e^{-n^2\pi^2 t}\sin(n\pi x)}{n}}{\pi} + 30x + 20\right\}\right\}$$

Result Solved

Maple

```
u:='u'; t:='t'; x:='x';
pde :=  diff(u(x,t),t)+k*diff(u(x,t),x$2);
ic  :=  u(x,0)=0;
bc  :=  u(0,t)=20, u(1,t)=50;
sol:=pdsolve({pde,ic,bc},u(x,t));
```

$$u(x,t) = 20 + 30x + \sum_{n=1}^{\infty} \frac{\left(100(-1)^n - 40\right)\sin(\pi n x)\,e^{k\pi^2 n^2 t}}{n\pi}$$

Result Solved

2.20 Heat PDE on bar, nonhomogeneous Dirichlet BC, with constant source term

problem number 34

This is problem 8.2.1 par(d) from Richard Haberman applied partial differential equations 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + k$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = A$$
$$u(L,t) = B$$

Initial condition is $u(x,0) = f(x)$

Mathematica

```
ClearAll[u, x, t, k, f, A0, B0, L0];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + k;
 bc = {u[0, t] == A0, u[L0, t] == B0};
 ic = u[x, 0] == f[x];
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,1)}(x,t) = k u^{(2,0)}(x,t) + k, u(x,0) = f(x), \{u(0,t) = \text{A0}, u(\text{L0},t) = \text{B0}\}\right\}, u(x,t), x, t\right]$$

Result Did not solve

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';A:='A';B:='B';
interface(showassumed=0);
pde :=  diff(u(x,t),t)+k*diff(u(x,t),x$2)+k;
ic  :=  u(x,0)=f(x);
bc  :=  u(0,t)=A, u(L,t)=B;
sol:=pdsolve({pde,ic,bc},u(x,t));
```

$$u(x,t) = 1/2\,\frac{1}{L}\left(2\sum_{n=1}^{\infty}\left(-\frac{1}{L^2}\int_0^L 2\sin\left(\frac{\pi n x}{L}\right)\left(-f(x)L + 1/2L^2x + \left(-1/2x^2 + A\right)L - x(A-B)\right)dx\sin\left(\frac{\pi n x}{L}\right)e^{\frac{k\pi^2 n^2 t}{L^2}}\right)L + L^2x + \left(-x^2 + 2A\right)L - 2x(A-B)\right)$$

Result Solved

2.21 Heat PDE on bar, homogeneous Dirichlet BC, non zero initial conditions, with extra term

problem number 35

Solve the heat equation

$$\frac{\partial u}{\partial t} + u(x,t) = k\frac{\partial^2 u}{\partial x^2}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[x, t, u, f, L];
 pde = D[u[x, t], t] + u[x, t] == D[u[x, t], {x, 2}];
 ic = u[x, 0] == f[x];
 bc = {u[0, t] == 0, u[L, t] == 0};
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,1)}(x,t) + u(x,t) = u^{(2,0)}(x,t), u(x,0) = f(x), \{u(0,t) = 0, u(L,t) = 0\}\right\}, u(x,t), x, t\right]$$

Result Did not solve

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';
interface(showassumed=0);
pde:=diff(u(x,t),t)+u(x,t)=diff(u(x,t),x$2);
ic:=u(x,0)=f(x);
bc:=u(0,t)=0,u(L,t)=0;
sol:=pdsolve({pde,ic,bc},u(x,t)) assuming L>0;
```

$$u(x,t) = \sum_{n=1}^{\infty}\left(2\frac{1}{L}\int_0^L f(x)\sin\left(\frac{\pi n x}{L}\right) dx \sin\left(\frac{\pi n x}{L}\right) e^{-\frac{t\left(\pi^2 n^2 + L^2\right)}{L^2}}\right)$$

Result Solved

2.22 Heat PDE on bar with initial conditions sum of sine terms, homogeneous Dirichlet BC, no source

problem number 36

added Feb 10, 2018.
Solve the heat equation

$$\frac{\partial u}{\partial t} + u(x,t) = 100\frac{\partial^2 u}{\partial x^2}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(1,t) = 0$$

Initial condition is $u(x,0) = \sin(2\pi x) - \sin(5\pi x)$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[x, t, u, f, L];
 f = Sin[2*Pi*x] - Sin[5*Pi*x];
 pde = D[u[x, t], t] == 100*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[1, t] == 0};
 ic = u[x, 0] == f;
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t][[1,1]];
 ;
```

$$u(x,t) \to e^{-400\pi^2 t}\sin(2\pi x) - e^{-2500\pi^2 t}\sin(5\pi x)$$

Result Solved

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,t),t)=100*diff(u(x,t),x$2);
ic:=u(x,0)=sin(2*Pi*x)-sin(5*Pi*x);
bc:=u(0,t)=0,u(1,t)=0;
sol:=pdsolve({pde,ic,bc},u(x,t));
```

$$u(x,t) = \sin(2\pi x)\,\mathrm{e}^{-400\pi^2 t} - \sin(5\pi x)\,\mathrm{e}^{-2500\pi^2 t}$$

Result Solved

2.23 Heat PDE on bar, homogeneous Dirichlet BC, initial condition is piecewise function, no source

problem number 37

added Feb 10, 2018.
Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(40,t) = 0$$

Initial condition is

$$u(x,0) = \begin{cases} x & 0 \le x < 20 \\ 40 - x & 20 \ge x \le 40 \end{cases}$$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[x, t, u, f, L, n];
 f = Piecewise[{{x, Inequality[0, LessEqual, x, Less, 20]}, {40 - x, 20 <= x <= 40}}];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[40, t] == 0};
 ic = u[x, 0] == f;
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t];
 sol = sol /. K[1] -> n;
 ;
```

$$\left\{\left\{u(x,t) \to \sum_{n=1}^{\infty} \frac{640 e^{-\frac{n^2\pi^2 t}{1600}} \cos\left(\frac{n\pi}{4}\right)\sin^3\left(\frac{n\pi}{4}\right)\sin\left(\frac{n\pi x}{40}\right)}{n^2\pi^2}\right\}\right\}$$

Result Solved

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
f:=piecewise(0<x and x<20,x,20<x and x<40,(40-x));
pde:=diff(u(x,t),t)=diff(u(x,t),x$2);
ic:=u(x,0)=f;
bc:=u(0,t)=0,u(40,t)=0;
sol:=pdsolve({pde,ic,bc},u(x,t));
```

$$u(x,t) = \sum_{n=1}^{\infty} 160 \frac{\sin(1/2\, n\pi)\sin(1/40\,\pi\, n x)}{\pi^2 n^2} e^{-\frac{\pi^2 n^2 t}{1600}}$$

Result Solved

2.24 Heat PDE on bar, inhomogeneous Dirichlet BC, initial condition is piecewise function, no source

problem number 38

Added July 2, 2018, taken from Maple 2018.1 improvement to PDE document.
Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(1,t) = 0$$

Initial condition is
$u(x,0) = \begin{cases} 1 & x = 1 \\ 0 & \text{otherwise} \end{cases}$

Mathematica

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[1, t] == 0};
 ic = u[x, 0] == Piecewise[{{0, x == 0}, {1, True}}];
 sol = DSolve[Flatten[{pde, ic, bc}], u[x, t], x, t];
 ;
```

$$\left\{\left\{u(x,t) \to \sum_{K[1]=1}^{\infty} -\frac{2\left(-1+(-1)^{K[1]}\right) e^{-\pi^2 t K[1]^2} \sin(\pi x K[1])}{\pi K[1]}\right\}\right\}$$

Result Solved

Maple

```

pde := diff(u(x,t), t) = diff(u(x,t), x, x);
bc  := u(0,t) = 0, u(1,t) = 1;
ic  := u(x,0) = piecewise(x = 0, 0, 1);
sol := pdsolve([pde, ic, bc]);
```

$$u(x,t) = x + \sum_{n=1}^{\infty} 2\,\frac{\sin(\pi\, n x)\,\mathrm{e}^{-\pi^2 n^2 t}}{n\pi}$$

Result Solved

2.25 Heat PDE on bar, inhomogeneous Dirichlet BC which depends on time. Zero initial condition, no source

problem number 39

added March 8, 2018. Exam problem
Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$u(0,t) = t$$
$$u(\pi,t) = 0$$

Initial condition is $u(x,0) = 0$.

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, t, x, n];
 pde = D[u[x, t], {t, 1}] == D[u[x, t], {x, 2}];
 bc = {u[0, t] == t, u[Pi, t] == 0};
 ic = u[x, 0] == 0;
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 sol = sol /. {K[1] -> n};
 ;
```

$$\left\{\left\{u(x,t) \to \sum_{n=1}^{\infty} -\frac{\left(2-2e^{-n^2t}\right)\sin(nx)}{n^3\pi} - \frac{tx}{\pi} + t\right\}\right\}$$

Result Solved

Maple

```
u:='u'; t:='t'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,t),t)=diff(u(x,t),x$2);
bc:=u(0,t)=t,u(Pi,t)=0:
ic:=u(x,0)=0:
sol:=pdsolve([pde,bc,ic],u(x,t));
```

$$u(x,t) = 1/6\,\frac{1}{\pi}\left(6\sum_{n=1}^{\infty} 2\,\frac{\sin(nx)\,\mathrm{e}^{-n^2t}}{\pi\,n^3}\pi - 6\,(-\pi+x)\left(1/6\,x^2 - 1/3\,\pi\,x + t\right)\right)$$

Result Did not solve

2.26 Heat PDE on bar, homogeneous Neumann boundary conditions, non zero initial conditions, with source as sin function that depends on space only.

problem number 40

added March 18, 2018.

This is problem 8.2.1, part(f) from Richard Haberman applied partial differential equations 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + \sin\left(\frac{2\pi x}{L}\right)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$.

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, x, t, k, f, L];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Sin[(2*Pi*x)/L];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == f[x];
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {L > 0, k > 0, t > 0}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,1)}(x,t) = k u^{(2,0)}(x,t) + \sin\left(\frac{2\pi x}{L}\right), u(x,0) = f(x), \left\{u^{(1,0)}(0,t) = 0, u^{(1,0)}(L,t) = 0\right\}\right\}, u(x,t), x, t, \text{Assumptions} \to \{L > 0, k > 0, t > 0\}\right]$$

Result Did not solve

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f'; k:='k';
interface(showassumed=0);
pde := diff(u(x,t),t)+k*diff(u(x,t),x$2)+sin(2*Pi*x/L);
ic  := u(x,0)=f(x);
bc  := D[1](u)(0,t)=0, D[1](u)(L,t)=0;
sol:=pdsolve({pde,ic,bc},u(x,t)) assuming L>0,t>0,k>0;
```

$$u(x,t) = 1/4\,\frac{1}{\pi^2 kL}\left(4\sum_{n=1}^{\infty}\left(-1/2\,\frac{1}{\pi^2 kL}\int_0^L L^2\sin\left(2\,\frac{\pi x}{L}\right)\cos\left(\frac{\pi nx}{L}\right) - 2\pi\left(-2\pi k\_C2 + 2\pi kf(x) + Lx\right)\cos\left(\frac{\pi nx}{L}\right)dx\cos\left(\frac{\pi nx}{L}\right)e^{\frac{\pi^2 kn^2 t}{L^2}}\right)\pi^2 kL + 4\_C2\pi^2 kL + L^3\sin\left(2\,\frac{\pi x}{L}\right) - 2L^2 x\pi - \int_0^L -4f(x)\pi^2 k + 4\_C2\pi^2 k + L^2\sin\left(2\,\frac{\pi x}{L}\right) - 2Lx\pi\,dx\right)$$

Result Solved

2.27 Heat PDE on bar, homogeneous Neumann boundary conditions, nonzero initial conditions, with source that depends on time only

problem number 41

Added July 2, 2018. Taken from Maple 2018.1 document, originally exercise 6.25 from Pinchover and Rubinstein.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + \cos(wt)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(L,t) = 0$$

Initial condition is $u(x,0) = x$.

Mathematica

```
ClearAll[u, x, t, k, L];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Cos[w*t];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == x;
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {L > 0, t > 0, k > 0}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,1)}(x,t) = ku^{(2,0)}(x,t) + \cos(tw), u(x,0) = x, \left\{u^{(1,0)}(0,t) = 0, u^{(1,0)}(L,t) = 0\right\}\right\}, u(x,t), x, t, \text{Assumptions} \to \{L > 0, t > 0, k > 0\}\right]$$

Result Did not solve

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';k:='k';
interface(showassumed=0);
pde := diff(u(x, t), t) = k*(diff(u(x, t), x, x))+cos(w*t);
bc := (D[1](u))(L, t) = 0, (D[1](u))(0, t) = 0;
ic:= u(x, 0) = x;
sol := pdsolve({pde,ic,bc},u(x,t)) assuming(L>0);
```

$$u(x,t) = \begin{cases} L/2 + \sum_{n=1}^{\infty} 2\,\frac{L\left((-1)^n - 1\right)}{n^2\pi^2}\cos\left(\frac{\pi n x}{L}\right)e^{-\frac{k\pi^2 n^2 t}{L^2}} + t & w = 0 \\ 1/2\,\frac{1}{w}\left(Lw + 2\sum_{n=1}^{\infty} 2\,\frac{L\left((-1)^n - 1\right)}{n^2\pi^2}\cos\left(\frac{\pi n x}{L}\right)e^{-\frac{k\pi^2 n^2 t}{L^2}}\,w + 2\sin(wt)\right) & \text{otherwise} \end{cases}$$

Result Solved

2.28 Heat PDE on bar, homogeneous Neumann boundary conditions, nonzero initial conditions, with source that depends on time and space

problem number 42

added March 18, 2018.
Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + \left(e^{-ct}\sin\left(\frac{2\pi x}{L}\right)\right)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$.

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, x, t, k, f, L, c];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Exp[-(c*t)]*Sin[(2*Pi*x)/L];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == f[x];
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {L > 0, k > 0, t > 0}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,1)}(x,t) = ku^{(2,0)}(x,t) + e^{-ct}\sin\left(\frac{2\pi x}{L}\right), u(x,0) = f(x), \left\{u^{(1,0)}(0,t) = 0, u^{(1,0)}(L,t) = 0\right\}\right\}, u(x,t), x, t, \text{Assumptions} \to \{L > 0, k > 0, t > 0\}\right]$$

Result Did not solve

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';c:='c';
interface(showassumed=0);
pde := diff(u(x,t),t)=k*diff(u(x,t),x$2)+(exp(-c*t)*sin(2*Pi*x/L));
ic  := u(x,0)=f(x);
bc  := D[1](u)(0,t)=0, D[1](u)(L,t)=0;
sol:=pdsolve({pde,ic,bc},u(x,t)) assuming L>0,t>0,k>0;
```

$$u(x,t) = \frac{1}{L}\left(\sum_{n=1}^{\infty}\left(2\frac{1}{L}\int_0^L f(\tau 1)\cos\left(\frac{n\pi\tau 1}{L}\right)d\tau 1\cos\left(\frac{n\pi x}{L}\right)e^{-\frac{k\pi^2 n^2 t}{L^2}}\right)L + \int_0^t 1/3\frac{1}{\pi}\left(8e^{\frac{-c\tau 1L^2 - k\pi^2(t-\tau 1)}{L^2}}\cos\left(\frac{\pi x}{L}\right) + 3\sum_{n1=3}^{\infty}4\frac{(-1)^{n1}-1}{\pi\left(n1^2-4\right)}e^{\frac{-\pi^2 k(t-\tau 1)n1^2 - c\tau 1L^2}{L^2}}\cos\left(\frac{n1\pi x}{L}\right)\pi\right)d\tau 1L + \int_0^L f(\tau 1)\,d\tau 1\right)$$

Result Solved

2.29 Heat PDE on bar, non-homogeneous, time dependent, Neumann boundary conditions, with source that depends on time and space

problem number 43

added July 2, 2018.

Pinchover and Rubinstein's exercise 6.17. Taken from Maple document for new improvements in Maple 2018.1

Solve the heat equation

$$\frac{\partial}{\partial t} u(x,t) - \frac{\partial^2}{\partial x^2} u(x,t) = 1 + x\cos(t)$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = \sin(t)$$
$$\frac{\partial u}{\partial x}(1,t) = \sin(t)$$

Initial condition is $u(x,0) = 1 + \cos(2\pi x)$.

Mathematica

```
pde = D[u[x, t], x] == D[u[x, t], {x, 2}] + 1 + x*Cos[t];
 bc = {Derivative[1, 0][u][0, t] == Sin[t], Derivative[1, 0][u][1, t] == Sin[t]};
 ic = u[x, 0] == 1 + Cos[2*Pi*x];
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t];
 sol = Simplify[sol];
 ;
```

$$\text{DSolve}\left[\left\{u^{(2,0)}(x,t) + x\cos(t) + 1 = u^{(1,0)}(x,t), \cos(2\pi x) + 1 = u(x,0), \left\{\sin(t) = u^{(1,0)}(0,t), \sin(t) = u^{(1,0)}(1,t)\right\}\right\}, u(x,t), x, t\right]$$

Result Did not solve

Maple

```
x:='x';t:='t';
pde := diff(u(x, t), t)= (diff(u(x, t), x, x)) + 1+x*cos(t);
bc := (D[1](u))(0, t) = sin(t), (D[1](u))(1, t) = sin(t);
ic := u(x, 0) = 1+cos(2*Pi*x);
sol:=pdsolve([pde, ic, bc]);
```

$$u(x,t) = 1 + \cos(2\pi x)\,e^{-4\pi^2 t} + t + x\sin(t)$$

Result Solved

2.30 Heat PDE on bar, non-homogeneous, not time dependent Neumann boundary conditions, No source term

problem number 44

added July 2, 2018.
Second example from Maple document for new improvements in Maple 2018.1
Solve the heat equation

$$\frac{\partial u}{\partial t} = 13\frac{\partial^2 u}{\partial x^2}$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(1,t) = 1$$

Initial condition is $u(x,0) = \frac{1}{2}x^2 + x$.

Mathematica

```
pde = D[u[x, t], x] == 13*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][1, t] == 1};
 ic = u[x, 0] == (1*x^2)/2 + x;
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t];
 ;
```

$$\text{DSolve}\left[\left\{u^{(1,0)}(x,t) = 13u^{(2,0)}(x,t), u(x,0) = \frac{x^2}{2} + x, \left\{u^{(1,0)}(0,t) = 0, u^{(1,0)}(1,t) = 1\right\}\right\}, u(x,t), x, t\right]$$

Result Did not solve

Maple

```
x:='x';t:='t';
pde := diff(u(x, t), t) = 13*(diff(u(x, t), x, x));
bc := D[1](u)(0,t)=0,D[1](u)(1,t)=1;
ic := u(x, 0) = 1/2*x^2+x;
sol:= simplify(pdsolve([pde, ic, bc]));
```

$$u(x,t) = 1/2 + \sum_{n=1}^{\infty} 2\,\frac{(-1+(-1)^n)\cos(n\pi x)\,e^{-13\pi^2 n^2 t}}{\pi^2 n^2} + 13\,t + 1/2\,x^2$$

Result Solved

2.31 Heat PDE on bar, homogeneous Neumann boundary conditions, Source term that depends on both time and space

problem number 45

added July 2, 2018.

4th example from Maple document for new improvements in Maple 2018.1, originally taken from Pinchover and Rubinstein's exercise 6.23 .

Solve the heat equation on bar

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + g(x,t)$$

Where $g(x,t) = e^{3t}\cos(17\pi x)$ for $0 < x < 1$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(1,t) = 0$$

Initial condition is $u(x,0) = f(x)$ where $f(x) = 3\cos(42\pi x)$.

Mathematica

```
ClearAll[u, t, x, f, g];
 f[x] := 3*Cos[42*Pi*x];
 g[x, t] := Exp[3*t]*Cos[17*x*Pi];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}] + g[x, t];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][1, t] == 0};
 ic = u[x, 0] == f[x];
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
```

$$\left\{\left\{u(x,t)\to \frac{e^{3t}\cos(17\pi x)}{3+289\pi^2} - \frac{e^{-289\pi^2 t}\cos(17\pi x)}{3+289\pi^2} + 3e^{-1764\pi^2 t}\cos(42\pi x)\right\}\right\}$$

Result Solved

Maple

```
f:='f';g:='g';x:='x';y:='y';t:='t';
f := x->3*cos(42*x*Pi);
g :=(x,t)->exp(3*t)*cos(17*x*Pi);
pde := diff(u(x, t), t)= (diff(u(x, t), x, x)) + g(x, t);
bc := (D[1](u))(0, t) = 0, (D[1](u))(1, t) = 0;
ic:= u(x, 0) = f(x);
sol:=pdsolve([pde, ic, bc]);
```

$$u(x,t) = \frac{\left(867\pi^2+9\right)\cos(42\pi x)\,\mathrm{e}^{-1764\pi^2 t} + \cos(17\pi x)\left(\mathrm{e}^{3t} - \mathrm{e}^{-289\pi^2 t}\right)}{289\pi^2+3}$$

Result Solved

2.32 Heat PDE on bar, homogeneous Neumann boundary conditions, Source term that depends on both time and space

problem number 46

added July 2, 2018.

Taken from Maple document for new improvements in Maple 2018.1, originally taken from Pinchover and Rubinstein's exercise 6.21

Solve the heat equation on bar

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + g(x,t)$$

Where $g(x,t) = t\cos(2001x)$ for $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(1,t) = 0$$

Initial condition is $u(x,0) = f(x)$ where $f(x) = \pi\cos(2x)$.

Mathematica

```
ClearAll[u, t, x];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}] + t*Cos[2001*x];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][Pi, t] == 0};
 ic = u[x, 0] == Pi*Cos[2*x];
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t)\to \pi e^{-4t}\cos(2x) + \frac{e^{-4004001t}\cos(2001x)}{16032024008001} + \frac{t\cos(2001x)}{4004001} - \frac{\cos(2001x)}{16032024008001}\right\}\right\}$$

Result Solved

Maple

```
x:='x';t:='t';
pde:= diff(u(x, t), t)= (diff(u(x, t), x, x)) + t*cos(2001*x);
bc := (D[1](u))(0, t) = 0, (D[1](u))(Pi, t) = 0;
ic:= u(x, 0) = Pi*cos(2*x);
sol:=pdsolve([pde, ic, bc]);
```

$$u(x,t) = \frac{\left(4004001\,t + \mathrm{e}^{-4004001\,t} - 1\right)\cos(2001\,x)}{16032024008001} + \pi\cos(2\,x)\,\mathrm{e}^{-4\,t}$$

Result Solved

2.33 Heat PDE on bar, Dirichlet boundary conditions that depends on time with source that depends on space only

problem number 47

added March 28, 2018. A problem from my PDE animation page.
Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + x$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$u(0,t) = \frac{t\sin t}{5}$$
$$u(\pi,t) = \frac{t\cos t}{10}$$

Initial condition is $u(x,0) = 60 - 20x$.

Mathematica

```
ClearAll[u, x, t, x];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}] + x;
 bc = {u[0, t] == (t*Sin[t])/5, u[Pi, t] == (t*Cos[t])/10};
 ic = u[x, 0] == 60 - 2*x;
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {t > 0, x > 0}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,1)}(x,t) = u^{(2,0)}(x,t) + x, u(x,0) = 60 - 2x, \left\{u(0,t) = \frac{1}{5}t\sin(t), u(\pi,t) = \frac{1}{10}t\cos(t)\right\}\right\}, u(x,t), x, t, \text{Assumptions} \to \{t > 0, x > 0\}\right]$$

Result Did not solve

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';c:='c';
interface(showassumed=0);
pde :=  diff(u(x,t),t)=diff(u(x,t),x$2)+x;
ic  :=  u(x,0)=(60-2*x);
bc  :=  u(0,t)=t/5*sin(t), u(Pi,t)=t/10*cos(t);
sol :=pdsolve({pde,ic,bc},u(x,t)) assuming t>0,x>0;
```

$$u(x,t) = 1/10\frac{1}{\pi}\left(\cos(t)\,tx + 2t(\pi - x)\sin(t) + 10\pi\left(\int_0^t \sum_{n1=1}^{\infty} -2\frac{\left((1/10\sin(\tau1)\tau1 + \pi - 1/10\cos(\tau1))(-1)^{n1} + 1/5\tau1\cos(\tau1) + 1/5\sin(\tau1)\right)e^{n1^2(-t+\tau1)}\sin(n1\,x)}{n1\pi}\,d\tau1 + \sum_{n=1}^{\infty} 4\frac{\left((-1)^n\pi + 30(-1)^{n+1} + 30\right)\sin(nx)e^{-n^2t}}{n\pi}\right)\right)$$

Result Solved

2.34 Heat PDE on bar, homogeneous Dirichlet boundary conditions, with source that depends on time and space

problem number 48

Taken from Maple PDE help pages
Solve the heat equation for $u(x,t)$

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + f(x,t)$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(1,t) = 0$$

Initial condition is $u(x,0) = g(x)$

Mathematica

```
ClearAll[u, x, t, k, f, g, c];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + f[x, t];
 bc = {u[0, t] == 0, u[1, t] == 0};
 ic = u[x, 0] == g[x];
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {k > 0, t > 0, x > 0, x < 1}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,1)}(x,t) = f(x,t) + ku^{(2,0)}(x,t), u(x,0) = g(x), \{u(0,t) = 0, u(1,t) = 0\}\right\}, u(x,t), x, t, \text{Assumptions} \to \{k > 0, t > 0, x > 0, x < 1\}\right]$$

Result Did not solve

Maple

```
u:='u'; t:='t'; x:='x';f:='f';k:='k';g:='g';
interface(showassumed=0);
pde:= diff(u(x, t), t) = k*(diff(u(x, t), x, x))+f(x, t);
bc := u(0, t) = 0, u(l, t) = 0;
ic:= u(x, 0) = g(x);
sol:=pdsolve([pde, bc,ic], u(x, t)) assuming 0 <= x and  x <= l and k>0 and t>0:
sol:=subs(n1=m,sol);
```

$$u(x,t) = \sum_{m=1}^{\infty} \left(2\frac{1}{l}\int_0^l g(x)\sin\left(\frac{\pi m x}{l}\right)dx \sin\left(\frac{\pi m x}{l}\right)e^{-\frac{k\pi^2 m^2 t}{l^2}}\right) + \int_0^t \sum_{n=1}^{\infty} \left(2\frac{1}{l}\int_0^l f(x,\tau 1)\sin\left(\frac{n\pi x}{l}\right)dx \sin\left(\frac{n\pi x}{l}\right)e^{\frac{k\pi^2 n^2(-t+\tau 1)}{l^2}}\right)d\tau 1$$

Result Solved

2.35 Heat/Diffusion PDE in 2D, inside rectangle with initial and boundary conditions

problem number 49

Taken from Maple help pages on PDE
Solve the heat equation for $u(x,y,t)$

$$\frac{\partial u}{\partial t} = 1/10\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

For $0 < x < 1$ and $0 < y < 1$ and $t > 0$. The boundary conditions are

$$\begin{aligned} u(0,y,t) &= 0 \\ u(1,y,t) &= 0 \\ u(x,0,t) &= 0 \\ u(x,1,t) &= 0 \end{aligned}$$

Initial condition is $u(x,y,0) = x(1-x)(1-y)y$.

Mathematica

```
ClearAll[x, y, t];
 pde = D[u[x, y, t], t] == (1*(D[u[x, y, t], {x, 2}] + D[u[x, y, t], {y, 2}]))/10;
 ic = u[x, y, 0] == x*(1 - x)*(1 - y)*y;
 bc = {u[0, y, t] == 0, u[1, y, t] == 0, u[x, 0, t] == 0, u[x, 1, t] == 0};
 sol = DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,0,1)}(x,y,t) = \frac{1}{10}\left(u^{(0,2,0)}(x,y,t) + u^{(2,0,0)}(x,y,t)\right), u(x,y,0) = (1-x)x(1-y)y, \{u(0,y,t) = 0, u(1,y,t) = 0, u(x,0,t) = 0, u(x,1,t) = 0\}\right\}, u(x,y,t), \{x,y,t\}\right]$$

Result Did not solve

Maple

```
x:='x'; u:='u'; t:='t'; y:='y';
pde  := diff(u(x, y, t), t) = 1/10*(diff(u(x, y, t), x$2)+diff(u(x, y, t), y$2));
bc := u(0, y, t) = 0, u(1, y, t) = 0, u(x, 0, t) = 0, u(x, 1, t) = 0;
ic:=u(x, y, 0) = x*(1-x)*(1-y)*y;
pdsolve({pde,ic,bc},u(x,y,t));
```

$$u(x,t) = \sum_{m=1}^{\infty}\left(2\frac{1}{l}\int_0^l g(x)\sin\left(\frac{\pi m x}{l}\right)dx\sin\left(\frac{\pi m x}{l}\right)e^{-\frac{k\pi^2 m^2 t}{l^2}}\right) + \int_0^t \sum_{n=1}^{\infty}\left(2\frac{1}{l}\int_0^l f(x,\tau 1)\sin\left(\frac{n\pi x}{l}\right)dx\sin\left(\frac{n\pi x}{l}\right)e^{\frac{k\pi^2 n^2(-t+\tau 1)}{l^2}}\right)d\tau 1$$

Result Solved

2.36 Heat/Diffusion PDE in 2D, inside rectangle with initial and boundary conditions with heat loss

problem number 50

Taken from Maple help pages on PDE
Solve the heat equation for $u(x, y, t)$

$$\frac{\partial u}{\partial t} = 1/10\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{5}u(x, y, t);$$

For $0 < x < 1$ and $0 < y < 1$ and $t > 0$. The boundary conditions are

$$\begin{aligned} \frac{\partial u}{\partial x}u(0, y, t) &= 0 \\ u(1, y, t) &= 0 \\ u(x, 0, t) &= 0 \\ \frac{\partial u}{\partial y}u(x, 1, t) &= 0 \end{aligned}$$

Initial condition is $u(x, y, 0) = (1 - x^2)(1 - \frac{1}{2}y)y$.

Mathematica

```
ClearAll[x, y, t];
 pde = D[u[x, y, t], t] == (1*(D[u[x, y, t], {x, 2}] + D[u[x, y, t], {y, 2}]))/10 - (1*u[x, y, t])/5;
 ic = u[x, y, 0] == (-x^2 + 1)*(1 - (1/2)*y)*y;
 bc = {Derivative[1, 0, 0][u][0, y, t] == 0, u[1, y, t] == 0, u[x, 0, t] == 0, Derivative[0, 1, 0][u][x, 1, t] == 0};
 sol = DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,0,1)}(x,y,t) = \frac{1}{10}\left(u^{(0,2,0)}(x,y,t) + u^{(2,0,0)}(x,y,t)\right) - \frac{1}{5}u(x,y,t), u(x,y,0) = \left(1-x^2\right)\left(1-\frac{y}{2}\right)y, \left\{u^{(1,0,0)}(0,y,t) = 0, u(1,y,t) = 0, u(x,0,t) = 0, u^{(0,1,0)}(x,1,t) = 0\right\}\right\}, u(x,y,t), \{x,y,t\}\right]$$

Result Did not solve

Maple

```
x:='x'; u:='u'; t:='t'; y:='y';
pde  := diff(u(x, y, t), t) = 1/10*(diff(u(x, y, t), x$2)+diff(u(x, y, t), y$2)) - 1/5 * u(x,y,t);
ic:= u(x, y, 0) = (-x^2+1)*(1-(1/2)*y)*y;
bc := (D[1](u))(0, y, t) = 0, u(1, y, t) = 0, u(x, 0, t) = 0, (D[2](u))(x, 1, t) = 0;
sol:=pdsolve([pde, ic,bc], u(x, y, t));
sol:=subs(n1=m,sol);
```

$$u\left(x, y, t\right) = \sum_{m=0}^{\infty}\left(\sum_{n=0}^{\infty} 512\,\frac{(-1)^n \cos\left(1/2\,(1+2\,n)\,\pi\,x\right)\sin\left(1/2\,(1+2\,m)\,\pi\,y\right)\mathrm{e}^{-1/10\left(2+\left(n^2+m^2+n+m+1/2\right)\pi^2\right)t}}{(1+2\,n)^3\,\pi^6\,(1+2\,m)^3}\right)$$

Result Solved

2.37 Heat PDE inside disk, with no θ dependency. initial and boundary conditions given

problem number 51

Taken from Mathematica DSolve help pages
Solve the heat equation in polar coordinates for $u(r,t)$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}$$

For $0 < r < 1$ and $t > 0$. The boundary conditions are

$$u(1,t) = 0$$

Initial condition is $u(r,0) = 1 - r$.

Mathematica

```
ClearAll[r, t, u];
 pde = D[u[r, t], t] == D[u[r, t], {r, 2}] + (1*D[u[r, t], r])/r;
 ic = u[r, 0] == 1 - r;
 bc = u[1, t] == 0;
 sol = DSolve[{pde, ic, bc}, u[r, t], {r, t}];
 sol = sol /. K[1] -> n;
 ;
```

$$\left\{\left\{u(r,t) \to \sum_{n=1}^{\infty} \frac{2e^{-t(j_{0,n})^2} J_0\left(r\, j_{0,n}\right)\left(\frac{J_1(j_{0,n})}{j_{0,n}} - \frac{1}{3}\,{}_1F_2\left(\frac{3}{2};1,\frac{5}{2};-\frac{1}{4}\left(j_{0,n}\right)^2\right)\right)}{J_0\left(j_{0,n}\right)^2 + J_1\left(j_{0,n}\right)^2}\right\}\right\}$$

Result Solved

Maple

```
r:='r'; u:='u'; t:='t';
pde:=diff(u(r,t),t)= diff(u(r,t),r$2)+ 1/r*diff(u(r,t),r);
ic:=u(r,0)=1-r;
bc := u(1,t) =0;
sol:=pdsolve([pde,ic,bc],u(r,t),HINT =boundedseries(r=0));
```

$$\text{sol} = ()$$

Result Did not solve

2.38 Heat PDE on whole line with no intial nor boundary conditions specified

problem number 52

Solve the heat equation for $u(x,t)$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

Mathematica

```
ClearAll[x, y, t];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 sol = DSolve[pde, u[x, t], {x, t}];
 ;
```

$$\text{DSolve}\left[u^{(0,1)}(x,t) = u^{(2,0)}(x,t), u(x,t), \{x,t\}\right]$$

Result Did not solve

Maple

```
x:='x'; u:='u'; t:='t';
pde := diff(u(x, t), t)=diff(u(x, t), x$2);
sol:= pdsolve(pde,u(x,t),'build') assuming t>0;
```

$$u(x,t) = \_C3\,e^{\_c_1 t}\,\_C1\,e^{\sqrt{\_c_1}\,x} + \frac{\_C3\,e^{\_c_1 t}\,\_C2}{e^{\sqrt{\_c_1}\,x}}$$

Result Solved, returning a solution that is not the most general one

2.39 Heat PDE in 1D on the whole real line with initial position specified

problem number 53

From Mathematica DSolve help pages. Solve the heat equation for $u(x,t)$ on real line with $t>0$

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

With initial condition

$$u(x,0)=e^{-x^2}$$

Mathematica

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 ic = u[x, 0] == E^(-x^2);
 sol = DSolve[{pde, ic}, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t)\to\frac{e^{-\frac{x^2}{4t+1}}}{\sqrt{4t+1}}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; u:='u'; t:='t';
pde := diff(u(x, t), t)=diff(u(x, t), x$2);
ic:=u(x,0)=exp(-x^2);
sol:= pdsolve({pde,ic},u(x,t)) assuming t>0;
```

$$u(x,t)=\frac{1}{\sqrt{4t+1}}\mathrm{e}^{-\frac{x^2}{4t+1}}$$

Result Solved

2.40 Heat PDE in 1D on the whole real line, with linear adevction

problem number 54

From Mathematica DSolve help pages. Solve the heat equation for $u(x,t)$ on real line with $t > 0$

$$\frac{\partial u}{\partial t} = 12\frac{\partial^2 u}{\partial x^2} + \sin t \frac{\partial u}{\partial x}$$

With initial condition

$$u(x,0) = x$$

Mathematica

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] == 12*D[u[x, t], {x, 2}] + Sin[t]*D[u[x, t], x];
 ic = u[x, 0] == x;
 sol = DSolve[{pde, ic}, u[x, t], {x, t}];
 ;
```

$$\{\{u(x,t) \to -\cos(t) + x + 1\}\}$$

Result Solved

Maple

```
x:='x'; u:='u'; t:='t';
pde:=diff(u(x,t),t)= 12* diff(u(x,t),x$2)+sin(t)*diff(u(x,t),x);
ic:=u(x,0)=x;
sol:=pdsolve([pde,ic],u(x,t));
```

$$u(x,t) = -\cos(t) + x + 1$$

Result Solved

2.41 Heat PDE in 1D on the whole real line with initial position as UnitBox

problem number 55

From Mathematica DSolve help pages. Solve the heat equation for $u(x,t)$ on real line with $t>0$

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

With initial condition

$$u(x,0)=\text{UnitBox[x]}$$

Where UnitBox is equal to 1 if $|x|\leq \frac{1}{2}$ and zero otherwise.

Mathematica

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 ic = u[x, 0] == UnitBox[x];
 sol = DSolve[{pde, ic}, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t)\to \frac{1}{2}\left(\operatorname{erf}\left(\frac{1-2x}{4\sqrt{t}}\right)+\operatorname{erf}\left(\frac{2x+1}{4\sqrt{t}}\right)\right)\right\}\right\}$$

Result Solved

Maple

```
x:='x'; u:='u'; t:='t';
pde := diff(u(x, t), t)=diff(u(x, t), x$2);
ic:= u(x,0)=piecewise( x< -1/2 or x>1/2,0, 1);
sol:= pdsolve({pde,ic},u(x,t)) assuming t>0;
```

$$u(x,t)=1/2\,Erf\left(1/4\,\frac{2\,x+1}{\sqrt{t}}\right)-1/2\,Erf\left(1/4\,\frac{2\,x-1}{\sqrt{t}}\right)$$

Result Solved

2.42 Heat PDE on half the line with non-zero initial conditions and Dirichlet boundary conditions

problem number 56

From Mathematica DSolve help pages.
Solve the heat equation for $u(x,t)$ on half the line $x>0$ and $t>0$

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

With initial condition

$$u(x,0)=\cos x$$

And boundary conditions

$$u(0,t)=1$$

Mathematica

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 ic = u[x, 0] == Cos[x];
 bc = u[0, t] == 1;
 sol = FullSimplify[DSolve[{pde, ic, bc}, u[x, t], {x, t}]];
 ;
```

$$\left\{\left\{u(x,t)\to\ \begin{cases}\operatorname{erfc}\left(\frac{x}{2\sqrt{t}}\right)-\frac{ie^{-\frac{x^2}{4t}}\left(F\left(\frac{2t+ix}{2\sqrt{t}}\right)-F\left(\sqrt{t}-\frac{ix}{2\sqrt{t}}\right)\right)}{\sqrt{\pi}} & x>0\\ \text{Indeterminate} & \text{True}\end{cases}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; u:='u'; t:='t';
pde := diff(u(x, t), t)=diff(u(x, t), x$2);
ic:=u(x,0)=cos(x);
bc:=u(0,t)=1;
sol:= pdsolve({pde,ic,bc},u(x,t)) assuming t>0 and x>0;
```

$$u(x,t)=-\mathit{invlaplace}\left(\mathrm{e}^{\sqrt{s}x}\_F1(s),s,t\right)-\mathit{invlaplace}\left(\frac{\mathrm{e}^{\sqrt{s}x}}{s+1},s,t\right)+\mathit{invlaplace}\left(\frac{\mathrm{e}^{\sqrt{s}x}}{s},s,t\right)+\mathit{invlaplace}\left(\mathrm{e}^{-\sqrt{s}x}\_F1(s),s,t\right)+\cos(x)\,\mathrm{e}^{-t}$$

Result Solved, but has unresolved inverse Laplace transforms

2.43 Heat PDE on half the line with zero initial conditions and time dependent boundary conditions

problem number 57

Solve the heat equation for $u(x,t)$ on half the line $x>0$ and $t>0$

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$$

With initial condition

$$u(x,0)=0$$

And boundary conditions

$$u(0,t)=\sin(t)\,u(\infty,t) \qquad \leq\infty$$

The last condition above means it is bounded at infinity.

Mathematica

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 ic = u[x, 0] == 0;
 bc = u[0, t] == t;
 sol = DSolve[{pde, ic, bc}, u[x, t], {x, t}, Assumptions -> {k > 0, x > 0, t > 0}];
 ;
```

$$\left\{\left\{u(x,t)\to\frac{(2kt+x^2)\operatorname{erfc}\left(\frac{x}{2\sqrt{kt}}\right)-\frac{2x\sqrt{kt}e^{-\frac{x^2}{4kt}}}{\sqrt{\pi}}}{2k}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; u:='u'; t:='t'; k:='k';
interface(showassumed=0);
pde := diff(u(x, t), t)=k*diff(u(x, t), x$2);
ic:=u(x,0)=0;
bc:=u(0,t)=t;
assume(x>0);
assume(t>0);
assume(k>0);
sol:= pdsolve({pde,ic,bc},u(x,t));
```

$$u(x,t)=1/2\,\frac{1}{\pi k}\left(-2\sqrt{\pi}\sqrt{t}\sqrt{k}x\mathrm{e}^{-1/4\,\frac{x^2}{kt}}-2\pi\left(\left(kt+1/2\,x^2\right)Erf\left(1/2\,\frac{x}{\sqrt{k}\sqrt{t}}\right)-invlaplace\left(\_F1(s)\,\mathrm{e}^{\frac{\sqrt{s}x}{\sqrt{k}}},s,t\right)k+invlaplace\left(\mathrm{e}^{-\frac{\sqrt{s}x}{\sqrt{k}}}\_F1(s),s,t\right)k-kt-1/2\,x^2\right)\right)$$

Result Solved, but has unresolved inverse Laplace transforms

2.44 Initial value problem for the heat PDE with a Neumann condition on the half-line

problem number 58

From Mathematica DSolve help pages.
Solve the heat equation for $u(x,t)$ on half the line $x>0$ and $t>0$

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

With initial condition

$$u(x,0)=\text{UnitTriagle[x-3]}$$

And boundary conditions

$$\frac{\partial u}{\partial x}(0,t)=0$$

Mathematica

```
ClearAll[u, x, t];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 ic = u[x, 0] == UnitTriangle[x - 3];
 bc = Derivative[1, 0][u][0, t] == 0;
 sol = DSolve[{pde, ic, bc}, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t)\to\begin{cases}\frac{1}{2}\left(\frac{\operatorname{erf}\left(\frac{|x-4|}{2\sqrt{t}}\right)(x-4)^2}{|4-x|}+(x+2)\operatorname{erf}\left(\frac{x+2}{2\sqrt{t}}\right)-2(x+3)\operatorname{erf}\left(\frac{x+3}{2\sqrt{t}}\right)+(x+4)\operatorname{erf}\left(\frac{x+4}{2\sqrt{t}}\right)-\frac{2(x-3)^2\operatorname{erf}\left(\frac{|x-3|}{2\sqrt{t}}\right)}{|3-x|}+\frac{(x-2)^2\operatorname{erf}\left(\frac{|x-2|}{2\sqrt{t}}\right)}{|2-x|}+\frac{2\left(e^{-\frac{(x-4)^2}{4t}}-2e^{-\frac{(x-3)^2}{4t}}+e^{-\frac{(x-2)^2}{4t}}+e^{-\frac{(x+2)^2}{4t}}-2e^{-\frac{(x+3)^2}{4t}}+e^{-\frac{(x+4)^2}{4t}}\right)\sqrt{t}}{\sqrt{\pi}}\right) & x>0\\ \text{Indeterminate} & \text{True}\end{cases}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; u:='u'; t:='t';
pde := diff(u(x, t), t)=diff(u(x, t), x$2);
ic:=u(x,0)=piecewise( x>2 and x<3,-2+x, x>3 and x<4, 4-x, 0);
bc:=(D[1](u))(0,t)=0;
sol:= pdsolve({pde,ic,bc},u(x,t)) assuming t>0 and x>0;
```

$$\text{sol}=()$$

Result Did not solve

3 Laplace PDE

3.1 Laplace PDE inside quarter-circle

problem number 59

This is problem 2.5.5 part (c) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

Inside quarter circle of radius 1 with $0 \leq \theta \leq \frac{\pi}{2}$ and $0 \leq r \leq 1$, with following boundary conditions

$$\begin{aligned} u(r,0) &= 0 \\ u(r,\frac{\pi}{2}) &= 0 \\ \frac{\partial u}{\partial r}(1,\theta) &= f(\theta) \end{aligned}$$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, theta, r, f];
 pde = D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r]*1*D[u[r, theta], {theta, 2}])/(r*r^2) == 0;
 bc = {Derivative[1, 0][u][1, theta] == f[theta], u[r, Pi/2] == 0, u[r, 0] == 0};
 sol = DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions -> {0 <= r <= 1 && 0 <= theta <= Pi/2}];
 ;
```

$$\text{DSolve}\left[\left\{\frac{u^{(0,2)}(r,\theta)u^{(1,0)}(r,\theta)}{r^3} + u^{(2,0)}(r,\theta) = 0, \left\{u^{(1,0)}(1,\theta) = f(\theta), u\left(r,\frac{\pi}{2}\right) = 0, u(r,0) = 0\right\}\right\}, u(r,\theta), \{r,\theta\}, \text{Assumptions} \rightarrow \left\{0 \leq r \leq 1 \land 0 \leq \theta \leq \frac{\pi}{2}\right\}\right]$$

Result Did not solve

Maple

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';
interface(showassumed=0);
pde:=diff(u(r,theta),r$2)+1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc:=u(r,0)=0,u(r,Pi/2)=0,D[1](u)(1,theta)=f(theta);
sol:=pdsolve([pde,bc],u(r,theta)) assuming 0<=theta,theta<=Pi/2,0<=r,r<=1;
```

$$\text{sol} = ()$$

Result Did not solve

3.2 Laplace PDE inside semi-circle

problem number 60

Solve Laplace equation

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=0$$

Inside semi-circle of radius 1 with $0 \leq \theta \leq \pi$ and $0 \leq r \leq 1$, with following boundary conditions

$$\begin{aligned} u(r,0) &= 0 \\ u(r,\pi) &= 0 \\ u(0,\theta) &= 0 \\ u(1,\theta) &= f(\theta) \end{aligned}$$

Mathematica

```
NumericQ[L] =. ;
 ClearAll[u, theta, r, f];
 pde = D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r]*1*D[u[r, theta], {theta, 2}])/(r*r^2) == 0;
 bc = {u[r, 0] == 0, u[r, Pi] == 0, u[0, theta] == 0, u[1, theta] == f[theta]};
 sol = DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions -> {0 <= r <= 1 && 0 <= theta <= Pi}];
 ;
```

$$\text{DSolve}\left[\left\{\frac{u^{(0,2)}(r,\theta)u^{(1,0)}(r,\theta)}{r^3}+u^{(2,0)}(r,\theta)=0,\{u(r,0)=0,u(r,\pi)=0,u(0,\theta)=0,u(1,\theta)=f(\theta)\}\right\},u(r,\theta),\{r,\theta\},\text{Assumptions}\to\{0\leq r\leq 1\land 0\leq\theta\leq\pi\}\right]$$

Result Did not solve

Maple

```
L:='L'; u:='u';f:='f';
interface(showassumed=0);
pde:=diff(u(r,theta),r$2)+1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc:=u(r,0)=0,u(r,Pi)=0,u(0,theta)=0,u(1,theta)=f(theta);
sol:=pdsolve([pde,bc],u(r,theta),HINT = boundedseries(r = 0)) assuming 0<theta,theta<Pi;
```

$$u(r,\theta)=\sum_{n=1}^{\infty}\left(2\,\frac{\int_0^{\pi}\sin(n\theta)\,f(\theta)\,\mathrm{d}\theta\, r^n\sin(n\theta)}{\pi}\right)$$

Result Solved

3.3 Laplace PDE inside rectangle

problem number 61

This is problem 2.5.1 part (a) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

inside a rectangle $0 \le x \le L, 0 \le y \le H$, with following boundary conditions

$$\begin{aligned}\frac{\partial u}{\partial x}(0, y) &= 0\\ \frac{\partial u}{\partial x}(L, y) &= 0\\ u(x,0) &= 0\\ u(x,H) &= f(x)\end{aligned}$$

Mathematica

```
NumericQ[L] =. ;
 NumericQ[H] =. ;
 ClearAll[u, t, k, x, L, H, f];
 pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = {Derivative[1, 0][u][0, y] == 0, Derivative[1, 0][u][L, y] == 0, u[x, 0] == 0, u[x, H] == f[x]};
 sol = DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <= x <= L && 0 <= y <= H}];
 sol = sol /. {K[1] -> n};
 ;
```

$$\left\{\left\{u(x,y) \to \sum_{n=1}^{\infty} \frac{2\cos\left(\frac{n\pi x}{L}\right)\operatorname{csch}\left(\frac{Hn\pi}{L}\right)\left(\int_0^L \cos\left(\frac{n\pi x}{L}\right) f(x)\,dx\right)\sinh\left(\frac{n\pi y}{L}\right)}{L} + \frac{y\int_0^L f(x)\,dx}{HL}\right\}\right\}$$

Result Solved

Maple

```
H:='H';L:='L'; u:='u'; y:='y'; x:='x';f:='f';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc:=D[1](u)(0,y)=0,D[1](u)(L,y)=0,u(x,0)=0,u(x,H)=f(x);
sol:=pdsolve([pde,bc],u(x,y)) assuming(0<=x and x<=L and 0<=y and y<=H);
#these simplifications below to convert answer to one that match standard;
sol:=convert(sol,trigh);
sol:=simplify(expand(sol));
```

$$u(x,y) = \frac{1}{HL}\left(2\sum_{n=1}^{\infty}\left(1\sinh\left(\frac{\pi y n}{L}\right)\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)\mathrm{d}x\cos\left(\frac{n\pi x}{L}\right)\left(\sinh\left(\frac{n\pi H}{L}\right)\right)^{-1}\right)H + \int_0^L f(x)\,\mathrm{d}x\,y\right)$$

Result Solved

3.4 Laplace PDE inside rectangle

problem number 62

This is problem 2.5.1 part (b) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

inside a rectangle $0 \leq x \leq L, 0 \leq y \leq H$, with following boundary conditions

$$\begin{aligned}
\frac{\partial u}{\partial x}(0, y) &= g(y) \\
\frac{\partial u}{\partial x}(L, y) &= 0 \\
u(x, 0) &= 0 \\
u(x, H) &= 0
\end{aligned}$$

Mathematica

```
NumericQ[L] =. ;
 NumericQ[H] =. ;
 ClearAll[u, t, k, x, L, H, g, f];
 pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = {Derivative[1, 0][u][0, y] == g[y], Derivative[1, 0][u][L, y] == 0, u[x, 0] == 0, u[x, H] == 0};
 sol = DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <= x <= L && 0 <= y <= H}];
 sol = sol /. {K[1] -> n};
 ;
```

$$\left\{\left\{u(x,y) \to \sum_{n=1}^{\infty} -\frac{2\cosh\left(\frac{n\pi(L-x)}{H}\right)\operatorname{csch}\left(\frac{Ln\pi}{H}\right)\left(\int_0^H g(y)\sin\left(\frac{n\pi y}{H}\right)dy\right)\sin\left(\frac{n\pi y}{H}\right)}{n\pi}\right\}\right\}$$

Result Solved

Maple

```
H:='H';L:='L'; u:='u'; y:='y'; x:='x';f:='f';g:='g';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0):
bc:=D[1](u)(0,y)=g(y),D[1](u)(L,y)=0,u(x,0)=0,u(x,H)=0:
sol:=pdsolve([pde,bc],u(x,y)) assuming(0<=x and x<=L and 0<=y and y<=H):
sol:=convert(sol,trigh);
```

$$u(x,y) = \sum_{n=1}^{\infty}\left(-2\frac{1}{n\pi}\sin\left(\frac{n\pi y}{H}\right)\int_0^H \sin\left(\frac{n\pi y}{H}\right)g(y)\,\mathrm{d}y\left(\cosh\left(\frac{n\pi(2L-x)}{H}\right)+\sinh\left(\frac{n\pi(2L-x)}{H}\right)+\cosh\left(\frac{n\pi x}{H}\right)+\sinh\left(\frac{n\pi x}{H}\right)\right)\left(\cosh\left(2\frac{n\pi L}{H}\right)+\sinh\left(2\frac{n\pi L}{H}\right)-1\right)^{-1}\right)$$

Result Solved

3.5 Laplace PDE inside rectangle

problem number 63

This is problem 2.5.1 part (c) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

inside a rectangle $0 \leq x \leq L, 0 \leq y \leq H$, with following boundary conditions

$$\begin{aligned}
\frac{\partial u}{\partial x}(0, y) &= 0 \\
u(L, y) &= g(y) \\
u(x, 0) &= 0 \\
u(x, H) &= 0
\end{aligned}$$

Mathematica

```
NumericQ[L] =. ;
 NumericQ[H] =. ;
 ClearAll[u, t, k, x, L, H, g, f];
 pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = {Derivative[1, 0][u][0, y] == 0, u[L, y] == g[y], u[x, 0] == 0, u[x, H] == 0};
 sol = DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <= x <= L && 0 <= y <= H}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,2)}(x,y)+u^{(2,0)}(x,y)=0,\left\{u^{(1,0)}(0,y)=0,u(L,y)=g(y),u(x,0)=0,u(x,H)=0\right\}\right\},u(x,y),\{x,y\},\text{Assumptions}\to\{0\leq x\leq L\land 0\leq y\leq H\}\right]$$

Result Did not solve

Maple

```
H:='H';L:='L'; u:='u'; y:='y'; x:='x';f:='f';g:='g';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc:=D[1](u)(0,y)=0,u(L,y)=g(y),u(x,0)=0,u(x,H)=0;
sol:=pdsolve([pde,bc],u(x,y)) assuming(0<=x and x<=L and 0<=y and y<=H);
sol:=convert(sol,trigh);
```

$$u(x,y)=\sum_{n=1}^{\infty}\left(2\frac{1}{H}\int_0^H\sin\left(\frac{n\pi y}{H}\right)g(y)\,\mathrm{d}y\sin\left(\frac{n\pi y}{H}\right)\left(\cosh\left(2\frac{n\pi x}{H}\right)+\sinh\left(2\frac{n\pi x}{H}\right)+1\right)\left(\cosh\left(\frac{\pi n(L-x)}{H}\right)+\sinh\left(\frac{\pi n(L-x)}{H}\right)\right)\left(\cosh\left(2\frac{n\pi L}{H}\right)+\sinh\left(2\frac{n\pi L}{H}\right)+1\right)^{-1}\right)$$

Result Solved

3.6 Laplace PDE inside rectangle

problem number 64

This is problem 2.5.1 part (d) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

inside a rectangle $0 \le x \le L, 0 \le y \le H$, with following boundary conditions

$$\begin{aligned} u(0,y) &= g(y) \\ u(L,y) &= 0 \\ \frac{\partial u}{\partial y} u(x,0) &= 0 \\ u(x,H) &= 0 \end{aligned}$$

Mathematica

```
NumericQ[L] =. ;
 NumericQ[H] =. ;
 ClearAll[u, x, L, H, g, f];
 pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = {u[0, y] == 0, u[L, y] == 0, Derivative[0, 1][u][x, 0] == 0, u[x, H] == 0};
 sol = DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <= x <= L && 0 <= y <= H}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,2)}(x,y) + u^{(2,0)}(x,y) = 0, \left\{u(0,y) = 0, u(L,y) = 0, u^{(0,1)}(x,0) = 0, u(x,H) = 0\right\}\right\}, u(x,y), \{x,y\}, \text{Assumptions} \to \{0 \le x \le L \land 0 \le y \le H\}\right]$$

Result Did not solve

Maple

```
H:='H';L:='L'; u:='u'; y:='y'; x:='x';f:='f';g:='g';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc:=u(0,y)=g(y),u(L,y)=0,D[2](u)(x,0)=0,u(x,H)=0;
sol:=pdsolve([pde,bc],u(x,y)) assuming(0<=x and x<=L and 0<=y and y<=H);
sol:=convert(sol,trigh);
```

$$u(x,y) = \sum_{n=0}^{\infty} \left(2 \frac{1}{H} \sin\left(1/2 \frac{\pi(2yn+H+y)}{H}\right) \int_0^H \sin\left(1/2 \frac{\pi(2yn+H+y)}{H}\right) g(y)\,dy \left(\cosh\left(\frac{(1+2n)\pi(L-x)}{H}\right) + \sinh\left(\frac{(1+2n)\pi(L-x)}{H}\right) - 1\right) \left(\cosh\left(1/2 \frac{(1+2n)\pi x}{H}\right) + \sinh\left(1/2 \frac{(1+2n)\pi x}{H}\right)\right) \left(\cosh\left(\frac{(1+2n)\pi L}{H}\right) + \sinh\left(\frac{(1+2n)\pi L}{H}\right) - 1\right)^{-1} \right)$$

Result Solved

3.7 Laplace PDE inside rectangle

problem number 65

This is problem 2.5.1 part (e) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

inside a rectangle $0 \le x \le L, 0 \le y \le H$, with following boundary conditions

$$\begin{aligned} u(0,y) &= 0 \\ u(L,y) &= 0 \\ u(x,0) - \frac{\partial u}{\partial y} u(x,0) &= 0 \\ u(x,H) &= f(x) \end{aligned}$$

Mathematica

```
NumericQ[L] =. ;
 NumericQ[H] =. ;
 ClearAll[u, x, L, H, g, f];
 pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = {u[0, y] == 0, u[L, y] == 0, u[x, 0] - Derivative[0, 1][u][x, 0] == 0, u[x, H] == f[x]};
 sol = DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <= x <= L && 0 <= y <= H}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,2)}(x,y) + u^{(2,0)}(x,y) = 0, \left\{u(0,y) = 0, u(L,y) = 0, u(x,0) - u^{(0,1)}(x,0) = 0, u(x,H) = f(x)\right\}\right\}, u(x,y), \{x,y\}, \text{Assumptions} \to \{0 \le x \le L \land 0 \le y \le H\}\right]$$

Result Did not solve

Maple

```
H:='H';L:='L'; u:='u'; y:='y'; x:='x';f:='f';g:='g';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc:=u(0,y)=0,u(L,y)=0,u(x,0)-D[2](u)(x,0)=0,u(x,H)=f(x);
sol:=pdsolve([pde,bc],u(x,y)) assuming(0<=x and x<=L and 0<=y and y<=H);
sol:=convert(sol,trigh);
```

$$u(x,y) = \sum_{n=1}^{\infty} \left(2 \frac{1}{L} \left(\sinh\left(2 \frac{\pi y n}{L}\right) \pi n + \cosh\left(2 \frac{\pi y n}{L}\right) \pi n + L \sinh\left(2 \frac{\pi y n}{L}\right) + L \cosh\left(2 \frac{\pi y n}{L}\right) + n\pi - L\right) \int_0^L \sin\left(\frac{n\pi x}{L}\right) f(x)\, dx \left(\cosh\left(\frac{\pi n (H-y)}{L}\right) + \sinh\left(\frac{\pi n (H-y)}{L}\right)\right) \sin\left(\frac{n\pi x}{L}\right) \left(\sinh\left(2 \frac{n\pi H}{L}\right) \pi n + \cosh\left(2 \frac{n\pi H}{L}\right) \pi n + L \sinh\left(2 \frac{n\pi H}{L}\right) + L \cosh\left(2 \frac{n\pi H}{L}\right) + n\pi - L\right)^{-1}\right)$$

Result Solved

3.8 Laplace PDE inside rectangle, top/bottom edges non-zero

problem number 66

Taken from Mathematica DSolve help pages.
Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

inside a rectangle $0 \le x \le 1, 0 \le y \le 2$, with following boundary conditions

$$\begin{aligned} u(0,y) &= 0 \\ u(1,y) &= 0 \\ u(x,0) &= \text{UnitTriagle(2 x-1)} \\ u(x,2) &= \text{UnitTriagle(2 x-1)} \end{aligned}$$

Mathematica

```
ClearAll[u, x, y];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 L0 = 1;
 H0 = 2;
 bc = DirichletCondition[u[x, y] == Piecewise[{{UnitTriangle[2*x - L0], y == 0 || y == H0}}, 0], True];
 domain = Rectangle[{0, 0}, {L0, H0}];
 sol = Simplify[DSolve[{pde, bc}, u[x, y], Element[{x, y}, domain]]];
 sol = sol /. K[1] -> n;
 ;
```

$$\left\{\left\{u(x,y)\to \sum_{n=1}^{\infty} \frac{8\operatorname{csch}(2n\pi)\sin\left(\frac{n\pi}{2}\right)\sin(n\pi x)(\sinh(n\pi(2-y))+\sinh(n\pi y))}{n^2\pi^2}\right\}\right\}$$

Result Solved

Maple

```
u:='u'; y:='y'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
f:=x-> piecewise(x>0 and x<1/2, 2*x, x>1/2 and x<1, 2-2*x);
bc:=u(0,y)=0,u(1,y)=0,u(x,0)=f(x),u(x,2)=f(x);
sol:=pdsolve([pde,bc],u(x,y)) assuming x>0,y>0;
```

$$u(x,y) = \sum_{n=1}^{\infty} -8\,\frac{\sin(1/2\,n\pi)\sin(n\pi\,x)\left(e^{\pi\,n(3\,y-4)} - e^{\pi\,n(3\,y-2)} + e^{\pi\,n(y-2)} - e^{\pi\,yn}\right)e^{-2\,\pi\,n(y-2)}}{\pi^2 n^2\left(e^{4\,n\pi}-1\right)}$$

Result Solved

3.9 Laplace PDE inside circular annulus, Neumann boundary conditions using unspecified functions

problem number 67

This is problem 2.5.8 part (b) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

Inside circular annulus $a < r < b$ subject to the following boundary conditions

$$\frac{\partial u}{\partial r}(a, \theta) = 0$$
$$u(b, 0) = g(\theta)$$

Mathematica

```
ClearAll[u, a, theta, r, g];
 pde = D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r])/r + (1*D[u[r, theta], {theta, 2}])/r^2 == 0;
 bc = {Derivative[1, 0][u][a, theta] == 0, u[b, theta] == g[theta]};
 sol = DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions -> Inequality[a, Less, r, LessEqual, b]];
 ;
```

$$\text{DSolve}\left[\left\{\frac{u^{(0,2)}(r,\theta)}{r^2} + \frac{u^{(1,0)}(r,\theta)}{r} + u^{(2,0)}(r,\theta) = 0, \left\{u^{(1,0)}(a,\theta) = 0, u(b,\theta) = g(\theta)\right\}\right\}, u(r,\theta), \{r,\theta\}, \text{Assumptions} \to a < r \leq b\right]$$

Result Did not solve

Maple

```
a:='a'; u:='u'; r:='r'; theta:='theta';g:='g';
interface(showassumed=0);
pde:=diff(u(r,theta),r$2)+1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc:=D[1](u)(a,theta)=0,u(b,theta)=g(theta);
sol:=pdsolve([pde,bc],u(r,theta)) assuming a<r,r<b;
```

$$\text{sol} = ()$$

Result Did not solve

3.10 Laplace PDE inside circular annulus, Dirichlet boundary conditions using specified functions

problem number 68

Solve Laplace equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

Inside circular annulus $1 < r < 2$ subject to the following boundary conditions

$$u(1,\theta) = 0$$
$$u(2,\theta) = \sin\theta$$

Mathematica

```
ClearAll[u, r, theta];
 pde = D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r])/r + (1*D[u[r, theta], {theta, 2}])/r^2 == 0;
 bc = {u[1, theta] == 0, u[2, theta] == Sin[theta]};
 sol = DSolve[{pde, bc}, u[r, theta], {r, theta}];
 ;
```

$$\left\{\left\{u(r,\theta) \to \begin{cases} \frac{2(r^2-1)\sin(\theta)}{3r} & 1 \le r \le 2 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Result Solved

Maple

```
u:='u'; r:='r'; theta:='theta';
pde:=diff(u(r,theta),r$2)+1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc:=u(1,theta)=0,u(2,theta)=sin(theta);
sol:=pdsolve([pde,bc],u(r,theta),HINT = boundedseries);
```

$$\text{sol} = ()$$

Result Did not solve

3.11 Laplace PDE example 18 from Maple help page

problem number 69

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

With boundary conditions

$$u(0, y) = \frac{\sin y}{y}$$

Mathematica

```
ClearAll[u, x, y];
 pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = u[0, y] == Sin[y]/y;
 sol = DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <= x && 0 <= y}];
 ;
```

$$\left\{\left\{u(x,y) \to \frac{(\sinh(x) - \cosh(x))(x\cos(y) - y\sin(y)) + x}{x^2 + y^2}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; y:='y'; u:='u';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
bc:=u(0,y)=sin(y)/y;
sol:=pdsolve([pde,bc],u(x,y));
```

$$u(x,y) = \frac{\sin(-y + ix) + \_F2(y - ix)(y - ix) + (-y + ix)\_F2(y + ix)}{-y + ix}$$

Result Solved

3.12 Laplace PDE on rectangle with one edge at infinity

problem number 70

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

With boundary conditions

$$\begin{aligned}u(0,y)&=\sin y\\ u(x,0)&=0\\ u(x,a)&=0\\ u(\infty,y)&=0\end{aligned}$$

Mathematica

```
ClearAll[x, y, a];
 ode = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = {u[x, 0] == 0, u[x, a] == 0, u[0, y] == Sin[y], u[Infinity, y] == 0};
 sol = DSolve[{ode, bc}, u[x, y], {x, y}, Assumptions -> a > 0];
 ;
```

$\text{DSolve}\left[\left\{u^{(0,2)}(x,y)+u^{(2,0)}(x,y)=0,\{u(x,0)=0,u(x,a)=0,u(0,y)=\sin(y),u(\infty,y)=0\}\right\},u(x,y),\{x,y\},\text{Assumptions}\to a>0\right]$

Result Did not solve

Maple

```
x:='x'; y:='y'; a:='a';
interface(showassumed=0);
ode:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
bc:=u(x,0)=0, u(x,a)=0, u(0,y)=sin(y), u(infinity,y)=0;
sol:=pdsolve({ode, bc}, u(x,y)) assuming a>0;
```

$$u(x,y)=\sum_{n=1}^{\infty}2\,\frac{(-1)^{1+n}\sin(a)\,\pi\, n}{\pi^2 n^2-a^2}\mathrm{e}^{-\frac{n\pi x}{a}}\sin\left(\frac{\pi y n}{a}\right)$$

Result Solved

3.13 Laplace PDE inside a disk, periodic boundary conditions

problem number 71

Solve Laplace equation in polar coordinates inside a disk
Solve for $u(r,\theta)$

$$
\begin{aligned}
\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} &= 0 \\
0 \leq r &\leq a \\
0 < \theta &\leq 2\pi
\end{aligned}
$$

Boundary conditions

$$
\begin{aligned}
u(a,\theta) &= f(\theta) \\
|u(0,\theta)| &< \infty \\
u(r,0) &= u(r,2\pi) \\
\frac{\partial u}{\partial \theta}(r,0) &= \frac{\partial u}{\partial \theta}(r,2\pi)
\end{aligned}
$$

Mathematica

```
ClearAll[u, theta, r, a, f];
 pde = D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r])/r + (1*D[u[r, theta], {theta, 2}])/r^2 == 0;
 bc = u[a, theta] == f[theta];
 sol = DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions -> a < r && a > 0 && Inequality[0, Less, theta, LessEqual, 2*Pi]];
 sol = sol /. K[1] -> n;
 ;
```

$$
\left\{\left\{u(r,\theta) \to \sum_{n=1}^{\infty} r^n \left(\frac{\cos(n\theta)\left(\int_{-\pi}^{\pi}\cos(n\theta) f(\theta)\,d\theta\right) a^{-n}}{\pi} + \frac{\left(\int_{-\pi}^{\pi} f(\theta)\sin(n\theta)\,d\theta\right)\sin(n\theta) a^{-n}}{\pi}\right) + \frac{\int_{-\pi}^{\pi} f(\theta)\,d\theta}{2\pi}\right\}\right\}
$$

Result Solved

Maple

```
r:='r'; theta:='theta'; a:='a'; r:='r';f:='f';
interface(showassumed=0);
pde := (diff(r*(diff(u(r, theta), r)), r))/r+(diff(u(r, theta), theta, theta))/r^2 = 0;
bc := u(a, theta) = f(theta), u(r, -Pi) = u(r, Pi), (D[2](u))(r, -Pi) = (D[2](u))(r, Pi);
sol:=pdsolve([pde, bc], u(r, theta), HINT = boundedseries(r=0));
```

$$
u(r,\theta) = 1/2\,\frac{1}{\pi}\left(2\sum_{n=1}^{\infty}\left(\frac{\int_{-\pi}^{\pi} f(\theta)\sin(n\theta)\,\mathrm{d}\theta\sin(n\theta) + \int_{-\pi}^{\pi} f(\theta)\cos(n\theta)\,\mathrm{d}\theta\cos(n\theta)}{\pi}\left(\frac{a}{r}\right)^{-n}\right)\pi + \int_{-\pi}^{\pi} f(\theta)\,\mathrm{d}\theta\right)
$$

Result Solved

3.14 Dirichlet problem for the Laplace equation in upper half plan

problem number 72

Taken from Mathematica DSolve help pages
Solve for $u(x, y)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Boundary conditions $u(x, 0) = 1$ for $-\frac{1}{2} \leq x \leq \frac{1}{2}$ and $x = 0$ otherwise. This is called UnitBox in Mathematica.

Mathematica

```
ClearAll[u, x, y];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 bc = u[x, 0] == UnitBox[x];
 sol = DSolve[{pde, bc}, u[x, y], {x, y}];
 ;
```

$$\left\{\left\{u(x,y) \to \begin{cases} \dfrac{\begin{cases} \tan^{-1}\left(\frac{\frac{1}{2}-x}{y}\right) + \tan^{-1}\left(\frac{x+\frac{1}{2}}{y}\right) & y > 0 \lor x > \frac{1}{2} \lor x < -\frac{1}{2} \\ 0 & \text{True} \end{cases}}{\pi} & y \geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; y:='y'; u:='u';
pde:=diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
bc := u(x,0) =piecewise( x< -1/2 or x>1/2,0, 1);
sol:=pdsolve([pde,bc],u(x,y));
```

$$u(x, y) = \begin{cases} -\_F2(-y + ix) & iy + x < -1/2 \\ -\_F2(-y + ix) + 1 & iy + x \leq 1/2 \\ -\_F2(-y + ix) & 1/2 < iy + x \end{cases} + \_F2(y + ix)$$

Result Did not solve

3.15 Dirichlet problem for the Laplace equation in right half-plane:

problem number 73

Taken from Mathematica DSolve help pages
Solve for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Boundary conditions $u(0,y) = \operatorname{sinc}(y)$.

Mathematica

```
ClearAll[u, x, y];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 bc = u[0, y] == Sinc[y];
 sol = DSolve[{pde, bc}, u[x, y], {x, y}];
 ;
```

$$\left\{\left\{u(x,y) \to \begin{cases} \frac{x+(x\cos(y)-y\sin(y))(\sinh(x)-\cosh(x))}{x^2+y^2} & x \geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; y:='y'; u:='u';
pde:=diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
sinc:= x -> piecewise( x=0, 1,   sin(x)/x );
bc := u(0,y) =sinc(x);
sol:=pdsolve([pde,bc],u(x,y));
```

$$\text{sol} = ()$$

Result Did not solve

3.16 Dirichlet problem for the Laplace equation in the first quadrant

problem number 74

Taken from Mathematica DSolve help pages
Solve for $u(x, y)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Boundary conditions $u(0, y) = \text{sinc}(y)$.

Mathematica

```
ClearAll[u, x, y];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 bc = {u[x, 0] == -((x - 2)^2 + 3)^(-1), u[0, y] == 1/((y - 3)^2 + 1)};
 sol = DSolve[{pde, bc}, u[x, y], {x, y}];
 ;
```

$$\left\{\left\{u(x,y)\to\begin{cases}\dfrac{2\left(\dfrac{3\left(y\left(3\pi(x+1)\left(x^4-4x^3+2\left(y^2+12\right)x^2-4\left(y^2+10\right)x+y^4-16y^2+100\right)+\ldots\right)\ldots\right)}{\left(x^2-2x+y^2-6y+10\right)\left(x^2+2x+y^2-6y+10\right)\left(x^2-2x+y^2+6y+10\right)\left(x^2+2x+y^2+6y+10\right)}-\ldots-140\sqrt{3}\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)\right)+3y\left(x^4+2\left(y^2+7\right)x^2+y^4-14y^2-63\right)\log\left(x^2+y^2\right)\right)}{3\pi} & x\geq 0\land y\geq 0 \\ \text{Indeterminate} & \text{True}\end{cases}\right\}\right\}$$ [illegible]

Result Solved

Maple

```
x:='x'; y:='y'; u:='u';
pde:=diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
bc:=u(x, 0) = (-1/((x - 2)^2 + 3)), u(0, y) = (1/((y - 3)^2 + 1));
sol:=pdsolve([pde,bc],u(x,y));
```

$$\text{sol} = ()$$

Result Did not solve

3.17 Neumann problem for the Laplace equation in the upper half-plane

problem number 75

Taken from Mathematica DSolve help pages
Solve for $u(x, y)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Boundary conditions $\frac{u}{y}(x, 0) = \text{UnitBox[x]}$ where UnitBox[x] is 1 for $-\frac{1}{2} \leq x \leq \frac{1}{2}$ and 0 otherwise. This is called UnitBox in Mathematica.

Mathematica

```
ClearAll[u, x, y];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 bc = Derivative[0, 1][u][x, 0] == UnitBox[x];
 sol = DSolve[{pde, bc}, u[x, y], {x, y}];
 ;
```

$$\left\{\left\{u(x,y) \to \begin{cases} \frac{4y\cot^{-1}\left(\frac{2y}{1-2x}\right)+4y\tan^{-1}\left(\frac{x+\frac{1}{2}}{y}\right)-2x\log\left((1-2x)^2+4y^2\right)+\log\left((1-2x)^2+4y^2\right)+2x\log\left((2x+1)^2+4y^2\right)+\log\left((2x+1)^2+4y^2\right)-2\log(4)-4}{4\pi} & y \geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; y:='y'; u:='u';
pde:=diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
bc:=(D[2](u))(x, 0) = piecewise( x< -1/2 or x>1/2,0, 1);
sol:=pdsolve([pde,bc],u(x,y));
```

$$\text{sol} = ()$$

Result Did not solve

3.18 Dirichlet problem for the Laplace equation in a rectangle

problem number 76

Taken from Mathematica DSolve help pages
Solve for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

Boundary conditions $u(x,0)=x^2(1-x), u(x,2)=0, u(0,y)=0, u(1,y)=0$.

Mathematica

```
ClearAll[u, x, y];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 bc = {u[x, 0] == x^2*(1 - x), u[x, 2] == 0, u[0, y] == 0, u[1, y] == 0};
 sol = DSolve[{pde, bc}, u[x, y], {x, y}];
 sol = sol /. K[1] -> n;
```

$$\left\{\left\{u(x,y)\to\sum_{n=1}^{\infty}-\frac{4\left(1+2(-1)^n\right)\operatorname{csch}(2n\pi)\sin(n\pi x)\sinh(n\pi(2-y))}{n^3\pi^3}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; y:='y'; u:='u';
pde:=diff(u(x,y),x$2)+ diff(u(x,y),y$2)=0;
bc:=u(x, 0) = x^2*(1 - x),u(x, 2) = 0, u(0, y) = 0, u(1, y) = 0;
sol:=pdsolve([pde,bc],u(x,y));
```

$$u(x,y)=\sum_{n=1}^{\infty}-8\frac{\left(-1/2+(-1)^{1+n}\right)\left(-\mathrm{e}^{-\pi n(y-4)}+\mathrm{e}^{\pi yn}\right)\sin(n\pi x)}{n^3\pi^3\left(\mathrm{e}^{4n\pi}-1\right)}$$

Result Solved

3.19 Laplace PDE outside a disk, periodic boundary conditions

problem number 77

Solve Laplace equation in polar coordinates outside a disk
Solve for $u(r,\theta)$

$$
\begin{aligned}
\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} &= 0\\
a &\leq r\\
0 &< \theta \leq 2\pi
\end{aligned}
$$

Boundary conditions

$$
\begin{aligned}
u(a,\theta) &= f(\theta)\\
|u(0,\theta)| &< \infty\\
u(r,0) &= u(r,2\pi)\\
\frac{\partial u}{\partial \theta}(r,0) &= \frac{\partial u}{\partial \theta}(r,2\pi)
\end{aligned}
$$

Mathematica

```
ClearAll[u, theta, r, a, f];
 pde = D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r])/r + (1*D[u[r, theta], {theta, 2}])/r^2 == 0;
 bc = {u[a, theta] == f[theta], u[r, -Pi] == u[r, Pi], Derivative[0, 1][u][r, -Pi] == Derivative[0, 1][u][r, Pi]};
 sol = DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions -> {a > 0, r > a}];
 ;
```

$$\text{DSolve}\left[\left\{\frac{u^{(0,2)}(r,\theta)}{r^2}+\frac{u^{(1,0)}(r,\theta)}{r}+u^{(2,0)}(r,\theta)=0,\left\{u(a,\theta)=f(\theta),u(r,-\pi)=u(r,\pi),u^{(0,1)}(r,-\pi)=u^{(0,1)}(r,\pi)\right\}\right\},u(r,\theta),\{r,\theta\},\text{Assumptions}\to\{a>0,r>a\}\right]$$

Result Did not solve

Maple

```
r:='r'; theta:='theta'; a:='a'; r:='r';f:='f';
interface(showassumed=0);
pde := (diff(r*(diff(u(r, theta), r)), r))/r+(diff(u(r, theta), theta, theta))/r^2 = 0;
bc := u(a, theta) = f(theta), u(r, -Pi) = u(r, Pi), (D[2](u))(r, -Pi) = (D[2](u))(r, Pi);
sol:=pdsolve([pde, bc], u(r, theta), HINT = boundedseries(r=infinity));
```

$$u(r,\theta) = 1/2\,\frac{1}{\pi}\left(2\sum_{n=1}^{\infty}\left(\frac{\int_{-\pi}^{\pi} f(\theta)\sin(n\theta)\,\mathrm{d}\theta\sin(n\theta)+\int_{-\pi}^{\pi} f(\theta)\cos(n\theta)\,\mathrm{d}\theta\cos(n\theta)}{\pi}\left(\frac{a}{r}\right)^{n}\right)\pi+\int_{-\pi}^{\pi} f(\theta)\,\mathrm{d}\theta\right)$$

Result Solved

3.20 Laplace equation in spherical coordinates

problem number 78

Taken from Maple pdsolve help pages
Solve for $u(r,\theta,\phi)$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2 u}{\partial\phi^2}=0$$

Mathematica

```
ClearAll[u, \[Theta], \[Phi], r];
 ClearAll[u, r, \[Theta], \[Phi]];
 lap = Laplacian[f[r, \[Theta], \[Phi]], {r, \[Theta], \[Phi]}, "Spherical"];
 sol = DSolve[lap == 0, f[r, \[Theta], \[Phi]], {r, \[Theta], \[Phi]}, Assumptions -> 0 <= \[Theta] <= Pi];
 ;
```

$$\text{DSolve}\left[\frac{\frac{f^{(0,2,0)}(r,\theta,\phi)}{r}+f^{(1,0,0)}(r,\theta,\phi)}{r}+f^{(2,0,0)}(r,\theta,\phi)+\frac{\csc(\theta)\left(\sin(\theta)f^{(1,0,0)}(r,\theta,\phi)+\frac{\cos(\theta)f^{(0,1,0)}(r,\theta,\phi)}{r}+\frac{\csc(\theta)f^{(0,0,2)}(r,\theta,\phi)}{r}\right)}{r}=0,f(r,\theta,\phi),\{r,\theta,\phi\},\text{Assumptions}\to 0\leq\theta\leq\pi\right]$$

Result Did not solve

Maple

```
r:='r'; theta:='theta'; phi:='phi'; r:='r';
PDE := Diff(r^2*diff(F(r,theta,phi),r),r)+ 1/sin(theta)*Diff(sin(theta)*diff(F(r,theta,phi),theta),theta)+ 1/sin(theta)^2*diff(F(r,theta,phi),phi,phi) = 0;
sol:=pdsolve(PDE,F(r,theta,phi),'build')  assuming 0 <= theta, theta <= Pi;
sol:=simplify(sol,size);
```

$$F(r,\theta,\phi)=\frac{(-1)^{1/2\sqrt{\_c_2}}(\sin(\theta))^{\sqrt{\_c_2}}\sqrt{2}\left(\_C5\sin\left(\sqrt{\_c_2}\phi\right)+\_C6\cos\left(\sqrt{\_c_2}\phi\right)\right)\left(\_C1\,r^{1/2\sqrt{1+4\_c_1}}+\_C2\,r^{-1/2\sqrt{1+4\_c_1}}\right)\left(\cos(\theta)\,{}_2F_1(1/2\sqrt{\_c_2}+1/4\sqrt{1+4\_c_1}+3/4,1/2\sqrt{\_c_2}-1/4\sqrt{1+4\_c_1}+3/4;3/2;1/2\cos(2\theta)+1/2)\_C3+\_C4\,{}_2F_1(1/2\sqrt{\_c_2}+1/4\sqrt{1+4\_c_1}+1/4,1/2\sqrt{\_c_2}-1/4\sqrt{1+4\_c_1}+1/4;1/2;1/2\cos(2\theta)+1/2)\right)}{\sqrt{r}}$$

Result Solved, but not verified

4 Poisson PDE

4.1 Dirichlet problem for the Poisson equation in a rectangle

problem number 79

Taken from Mathematica DSolve help pages.
Solve for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x - 6y$$

Boundary conditions

$$\begin{aligned} u(x,0) &= 1 + 11x + x^3 \\ u(x,2) &= -7 + 11x + x^3 \\ u(0,y) &= 1 - y^3 \\ u(4,y) &= 109 - y^3 \end{aligned}$$

Mathematica

```
ClearAll[u, x, y];
 pde = Laplacian[u[x, y], {x, y}] == 6*x - 6*y;
 bc = {u[x, 0] == 1 + 11*x + x^3, u[x, 2] == -7 + 11*x + x^3, u[0, y] == 1 - y^3, u[4, y] == 109 - y^3};
 sol = DSolve[{pde, bc}, u[x, y], {x, y}];
 ;
```

$$\{\{u(x,y) \to x^3 + 11x - y^3 + 1\}\}$$

Result Solved

Maple

```
x:='x'; y:='y'; u:='u';
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=6*x-6*y;
bc:=u(x,0)=1+11*x+x^3,u(x,2)=-7+11*x+x^3,u(0,y)=1-y^3,u(4,y)=109-y^3;
sol:=pdsolve([pde,bc],u(x,y));
```

$$u(x,y) = x^3 - y^3 + 11x + 1$$

Result Solved

5 Helmholtz PDE

5.1 Dirichlet problem for the Helmholtz equation in a rectangle

problem number 80

Taken from Mathematica DSolve help pages.
Solve for $u(x, y)$

$$\frac{\partial^2 u}{\partial x^2} + + \frac{\partial^2 u}{\partial y^2} + 5u(x, y) = 0$$

Boundary conditions

$$\begin{aligned} u(x,0) &= \text{UnitTriangle[x-2]} \\ u(x,2) &= 0 \\ u(0,y) &= 0 \\ u(4,y) &= 0 \end{aligned}$$

Mathematica

```
ClearAll[x, y, n, u];
 pde = {Laplacian[u[x, y], {x, y}] + 5*u[x, y] == 0};
 bc = {u[x, 0] == Piecewise[{{-1 + x, x > 1 && x < 2}, {3 - x, x > 2 && x < 3}}], u[x, 2] == 0, u[0, y] == 0, u[4, y] == 0};
 sol = DSolve[{pde, bc}, u[x, y], {x, y}];
 sol = sol /. K[1] -> n;
```

$$\left\{\left\{u(x,y) \to \frac{1}{2}\sum_{n=1}^{\infty} \frac{128\left(\cos\left(\frac{n\pi}{8}\right) + \cos\left(\frac{3n\pi}{8}\right)\right)\operatorname{csch}\left(\frac{1}{2}\sqrt{n^2\pi^2-80}\right)\sin^3\left(\frac{n\pi}{8}\right)\sin\left(\frac{n\pi x}{4}\right)\sinh\left(\frac{1}{4}\sqrt{n^2\pi^2-80}(2-y)\right)}{n^2\pi^2}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; y:='y'; u:='u';
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)+5*u(x,y)=0;
bc:=u(x,0)=piecewise( x>1 and x<2, -1+x,x>2 and x<3 ,3-x),u(x,2)=-7+11*x+x^3,u(0,y)=1-y^3,u(4,y)=109-y^3;
sol:=pdsolve([pde,bc],u(x,y));
```

$$\text{sol} = ()$$

Result Did not solve

6 Wave PDE

6.1 General solution for a second-order hyperbolic PDE on real line

problem number 81

From Mathematica DSolve help pages (slightly modified)
Solve for $u(x,t)$ with $t>0$ on real line

$$\frac{\partial^2 u}{\partial t^2}+\frac{\partial^2 u}{\partial t \partial x}=c^2\frac{\partial^2 u}{\partial x^2}$$

Mathematica

```
ClearAll[u, t, x, c];
 ode = D[u[x, t], {t, 2}] + D[u[x, t], x, t] == c^2*D[u[x, t], {x, 2}];
 sol = DSolve[ode, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t)\to c_1\left(t-\frac{\left(\sqrt{4c^2+1}-1\right)x}{2c^2}\right)+c_2\left(t-\frac{\left(-\sqrt{4c^2+1}-1\right)x}{2c^2}\right)\right\}\right\}$$

Result Solved

Maple

```
x:='x'; t:='t';c:='c';u:='u';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)+diff(u(x,t),t,x)=c^2*diff(u(x,t),x$2);
sol:=pdsolve(pde,u(x,t)) assuming t>0,x>0;
```

$$u(x,t)=\_F1\left(1/2\,\frac{2\,tc^2+x\sqrt{4\,c^2+1}+x}{c^2}\right)+\_F2\left(1/2\,\frac{2\,tc^2-x\sqrt{4\,c^2+1}+x}{c^2}\right)$$

Result Solved

6.2 Hyperbolic PDE with non-rational coefficients

problem number 82

From Mathematica DSolve help pages
Solve for $u(x, y)$

$$\frac{\partial^2 u}{\partial x^2} - 2\sin x \frac{\partial^2 u}{\partial x \partial y} - \cos^2 x \frac{\partial^2 u}{\partial y^2} - \cos x \frac{\partial u}{\partial y} = 0$$

Mathematica

```
ClearAll[u, x, y];
 ode = D[u[x, y], {x, 2}] - 2*Sin[x]*D[u[x, y], x, y] - Cos[x]^2*D[u[x, y], {y, 2}] - Cos[x]*D[u[x, y], y] == 0;
 sol = DSolve[ode, u[x, y], {x, y}];
 ;
```

$$\{\{u(x, y) \to c_1(x - \cos(x) + y) + c_2(-x - \cos(x) + y)\}\}$$

Result Solved

Maple

```
x:='x'; t:='t';c:='c';u:='u';
interface(showassumed=0);
ode := diff(u(x, y), x$2) - 2*sin(x)*diff(u(x, y),x,y)-cos(x)^2*diff(u(x, y), y$2) - cos(x)*diff(u(x, y), y) = 0;
sol:=pdsolve(ode, u(x, y));
```

$$\text{sol} = ()$$

Result Did not solve. Tried all HINTS

6.3 Inhomogeneous hyperbolic PDE with constant coefficients

problem number 83

From Mathematica DSolve help pages
Solve for $u(x,t)$

$$3\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} + \frac{\partial^2 u}{\partial x \partial t} = 1$$

Mathematica

```
ClearAll[u, x, t];
 ode = 3*D[u[x, t], {x, 2}] - D[u[x, t], {t, 2}] + D[u[x, t], x, t] == 1;
 sol = DSolve[ode, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t) \to c_1\left(t - \frac{1}{6}\left(1+\sqrt{13}\right)x\right) + c_2\left(t - \frac{1}{6}\left(1-\sqrt{13}\right)x\right) + \frac{x^2}{6}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; t:='t';y:='y';u:='u';
ode := 3*diff(u(x, t), x$2) - diff(u(x, t),t$2)+diff(u(x, t), x,t) =1;
sol:=pdsolve(ode, u(x, t));
```

$$u(x,t) = \_F2\left(1/6\left(-1+\sqrt{13}\right)x+t\right) + \_F1\left(1/2\left(1/13\sqrt{13}+1\right)x - 3/13\,t\sqrt{13}\right) + 1/13\sqrt{13}\left(1/6\left(-1+\sqrt{13}\right)x+t\right)\left(1/2\left(1/13\sqrt{13}+1\right)x - 3/13\,t\sqrt{13}\right)$$

Result Solved

6.4 system of 2 inhomogeneous linear hyperbolic system with constant coefficients

problem number 84

From Mathematica DSolve help pages
Solve for $u(x,t), v(x,t)$

$$\frac{\partial u}{\partial t} = \frac{\partial v}{\partial x} + 1$$
$$\frac{\partial v}{\partial t} = -\frac{\partial u}{\partial x} - 1$$

With initial conditions

$$u(x,0) = \cos^2 x$$
$$v(x,0) = \sin x$$

Mathematica

```
ClearAll[u, v, x, t];
 eqns = {D[u[x, t], t] == D[v[x, t], x] + 1, D[v[x, t], t] == -D[u[x, t], x] - 1};
 ic = {u[x, 0] == Cos[x]^2, v[x, 0] == Sin[x]};
 sol = FullSimplify[DSolve[{eqns, ic}, {u[x, t], v[x, t]}, {x, t}]];
 ;
```

$$\left\{\left\{u(x,t) \to \sinh(t)\cos(x) + \frac{1}{2}\cosh(2t)\cos(2x) + t + \frac{1}{2}, v(x,t) \to \cosh(t)\sin(x)(2\sinh(t)\cos(x)+1) - t\right\}\right\}$$

Result Solved

Maple

```
x:='x'; t:='t';v:='v';u:='u';
pde1 := diff(u(x, t), t) = diff(v(x, t), x) + 1;
pde2 := diff(v(x, t), t) = diff(u(x, t), x) - 1;
ic := u(x, 0) = (cos(x))^2, v(x, 0) = sin(x);
sol :=pdsolve({pde1,pde2, ic}, {u(x, t), v(x, t)});
```

$$\text{sol} = ()$$

Result Did not solve

6.5 Wave PDE on string (finite domain) with zero initial position and velocity, and with source term

problem number 85

This is problem at page 115, David J Logan textbook, applied PDE textbook.
Falling cable lying on a table that is suddenly removed.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} - g$$

With boundary condition

$$u(0,t) = 0$$

And initial conditions

$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

Mathematica

```
ClearAll[u, t, x, g, c];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] - g;
 bc = u[0, t] == 0;
 ic = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {t > 0 && c > 0 && x > 0}];
 ;
```

$$\left\{\left\{u(x,t) \to \frac{1}{2} g \left(\left(t - \frac{x}{c}\right)^2 \theta\left(t - \frac{x}{c}\right) - t^2\right) - c_1 \delta\left(t - \frac{x}{c}\right)\right\}\right\}$$

Result Solved

Maple

```
x:='x'; t:='t'; g:='g';c:='c';u:='u';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)-g;
ic:=D[2](u)(x,0)=0,u(0,t)=0,u(x,0)=0;
sol:=pdsolve([pde,ic],u(x,t),HINT = boundedseries) assuming t>0,x>0,c>0;
```

$$u(x,t) = 1/2 \frac{g}{c^2} \left(Heaviside\left(t - \frac{x}{c}\right)(tc - x)^2 - c^2 t^2\right)$$

Result Solved

6.6 Wave PDE on string, one end fixed, another free, both initial conduitions non zero, and source that depends on time and space

problem number 86

Added July 2, 2018. Taken from Maple 2018.1 improvement to PDE document.
Solve

$$-\frac{\partial^2 u}{\partial t^2} + u(x,t) = \frac{\partial^2 u}{\partial x^2} + 2e^{-t}\left(x - \frac{1}{2}x^2 + \frac{1}{2}t - 1\right)$$

With boundary condition

$$\begin{aligned} u(0,t) &= 0 \\ \frac{\partial u(1,t)}{\partial x} &= 0 \end{aligned}$$

And initial conditions

$$\begin{aligned} u(x,0) &= x^2 - 2x \\ u(x,1) &= u(x,\frac{1}{2}) + e^{-1}\left(\frac{1}{2}x^2 - x\right) \end{aligned}$$

Mathematica

```
ClearAll[u, x, t, k, L];
 pde = -D[u[x, t], {t, 2}] + u[x, t] == D[u[x, t], {x, 2}] + 2*Exp[-t]*(x - (1/2)*x^2 + (1/2)*t - 1);
 bc = {u[0, t] == 0, Derivative[1, 0][u][1, t] == 0};
 ic = {u[x, 0] == x^2 - 2*x, u[x, 1] == u[x, 1/2] + ((1/2)*x^2 - x)*Exp[-1] - ((3*x^2)/4 - (3/2)*x)*Exp[-2^(-1)]};
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t];
 ;
```

$$\text{DSolve}\left[\left\{u(x,t) - u^{(0,2)}(x,t) = u^{(2,0)}(x,t) + 2e^{-t}\left(\frac{t}{2} - \frac{x^2}{2} + x - 1\right), \left\{u(x,0) = x^2 - 2x, u(x,1) = u\left(x,\frac{1}{2}\right) + \frac{\frac{x^2}{2} - x}{e} - \frac{\frac{3x^2}{4} - \frac{3x}{2}}{\sqrt{e}}\right\}, \{u(0,t) = 0, u^{(1,0)}(1,t) = 0\}\right\}, u(x,t), x, t\right]$$

Result Did not solve

Maple

```
x:='x'; t:='t'; u:='u';
pde := -diff(u(x, t), t, t) + u(x, t) = diff(u(x, t), x, x)+ 2*exp(-t)*(x-(1/2)*x^2+(1/2)*t-1);
ic:= u(x, 0) = x^2-2*x, u(x, 1) = u(x, 1/2)+((1/2)*x^2-x)*exp(-1)-(3/4*(x^2)-3/2*x)*exp(-1/2);
bc:= u(0, t) = 0, eval(diff(u(x, t), x), {x = 1}) = 0;
sol:=pdsolve([pde, ic, bc]);
```

$$u(x,t) = -1/2\,e^{-t}x(-2+x)(t-2)$$

Result Solved

6.7 Wave PDE on string (finite domain), fixed ends, no initial conduitions give and no source

problem number 87

This is problem at page 28, David J Logan textbook, applied PDE textbook.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

With boundary condition

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Mathematica

```
ClearAll[u, t, x, L, c];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[L, t] == 0};
 sol = DSolve[{pde, bc}, u[x, t], {x, t}, Assumptions -> {L > 0}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,2)}(x,t) = c^2 u^{(2,0)}(x,t), \{u(0,t) = 0, u(L,t) = 0\}\right\}, u(x,t), \{x,t\}, \text{Assumptions} \to \{L > 0\}\right]$$

Result Did not solve

Maple

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0;
sol:=pdsolve([pde,bc],u(x,t)) assuming L>0;
sol:=subs(_Z1=n,sol);
```

$$u(x,t) = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi x}{L}\right)\left(\_C1(n)\sin\left(\frac{cn\pi t}{L}\right) + \_C5(n)\cos\left(\frac{cn\pi t}{L}\right)\right)$$

Result Solved

6.8 Wave PDE on string (finite domain), one fixed end, one free end, initial position not zero, initial velocity zero, no source

problem number 88

This is problem at page 130, David J Logan textbook, applied PDE textbook.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

With boundary conditions

$$\frac{\partial u}{\partial x}(L,0) = 0$$
$$u(0,t) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = f(x)$$

Mathematica

```
ClearAll[u, t, x, L, c, f];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f[x]};
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {0 <= x <= L}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,2)}(x,t) = c^2 u^{(2,0)}(x,t), \left\{u(0,t) = 0, u^{(1,0)}(L,t) = 0\right\}, \left\{u^{(0,1)}(x,0) = 0, u(x,0) = f(x)\right\}\right\}, u(x,t), \{x,t\}, \text{Assumptions} \to \{0 \le x \le L\}\right]$$

Result Did not solve

Maple

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';f:='f';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc:=u(0,t)=0,D[1](u)(L,t)=0;
ic:=D[2](u)(x,0)=0,u(x,0)=f(x);
sol:=pdsolve([pde,bc,ic],u(x,t)) assuming x>=0,x<=L;
sol:=subs(_Z1=n,sol);
```

$$u(x,t) = \sum_{n=0}^{\infty} \left(2 \frac{1}{L} \int_0^L \sin\left(1/2 \frac{(1+2n)\pi x}{L}\right) f(x)\, dx \sin\left(1/2 \frac{(1+2n)\pi x}{L}\right) \cos\left(1/2 \frac{c\pi (1+2n)t}{L}\right)\right)$$

Result Solved

6.9 Wave PDE on string (finite domain), both ends fixed end, initial conditions zero, with source as generic function that depends on time and space

problem number 89

This is problem at page 149, David J Logan textbook, applied PDE textbook.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + p(x,t)$$

With boundary conditions

$$u(\pi, 0) = 0$$
$$u(0,t) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = 0$$

Mathematica

```
ClearAll[u, t, x, c, p];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] + p[x, t];
 bc = {u[0, t] == 0, u[Pi, t] == 0};
 ic = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 ;
```

DSolve $\left[\left\{u^{(0,2)}(x,t) = c^2 u^{(2,0)}(x,t) + p(x,t), \{u(0,t) = 0, u(\pi,t) = 0\}, \left\{u(x,0) = 0, u^{(0,1)}(x,0) = 0\right\}\right\}, u(x,t), \{x,t\}\right]$

Result Did not solve

Maple

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';p:='p';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)+p(x,t);
bc:=u(0,t)=0,u(Pi,t)=0;
ic:=u(x,0)=0,D[2](u)(x,0)=0;
sol:=pdsolve([pde,bc,ic],u(x,t));
sol:=subs(_Z1=n,sol);
```

$$u(x,t) = \int_0^t \sum_{n=1}^{\infty} \left(2\, \frac{\int_0^{\pi} \sin(nx)\, p(x,\tau 1)\, \mathrm{d}x \sin(nx) \sin(c(t-\tau 1)\, n)}{n\pi c} \right) \mathrm{d}\tau 1$$

Result Solved

6.10 Wave PDE on string (finite domain), both ends fixed, initial conditions both not zero, No source

problem number 90

Added July 2, 2018.
Taken from Maple 2018.1 improvements to PDE's document.
Solve

$$\frac{\partial^2 v}{\partial t^2} = \frac{\partial^2 v}{\partial x^2}$$

For $t > 0$ and $0 < x < 1$. With boundary conditions

$$\begin{aligned} v(0,t) &= 0 \\ v(1,0) &= 0 \end{aligned}$$

With initial conditions

$$\begin{aligned} v(x,0) &= f(x) \\ \frac{\partial v}{\partial t}(x,0) &= g(x) \end{aligned}$$

Where $f(x) = -\frac{e^2 x - e^{x+1} - x + e^{1-x}}{e^2 - 1}$ and $g(x) = 1 + \frac{e^2 x - e^{x+1} - x + e^{1-x}}{e^2 - 1}$

Mathematica

```
ClearAll[v, t, t];
 pde = D[v[x, t], {t, 2}] == D[v[x, t], {x, 2}];
 bc = {v[0, t] == 0, v[1, t] == 0};
 ic = {v[x, 0] == -((Exp[2]*x - Exp[x + 1] - x + Exp[1 - x])/(Exp[2] - 1)), Derivative[0, 1][v][x, 0] == 1 + (Exp[2]*x - Exp[x + 1] - x + Exp[1 - x])/(Exp[2] - 1)};
 sol = DSolve[{pde, bc, ic}, v[x, t], {x, t}];
 sol = sol /. K[1] -> n;
 ;
```

$$\left\{\left\{v(x,t) \to \sum_{n=1}^{\infty} \left(\frac{2(-1)^n \cos(n\pi t)}{\pi^3 n^3 + \pi n} + \frac{\left(-2\left(-1+(-1)^n\right)\pi^2 n^2 - 4(-1)^n + 2\right)\sin(n\pi t)}{\pi^4 n^4 + \pi^2 n^2}\right)\sin(n\pi x)\right\}\right\}$$

Result Solved

Maple

```
v:='v';x:='x';t:='t';
pde := diff(v(x, t), t, t)=(diff(v(x, t), x, x));
bc := v(0, t) = 0, v(1, t) = 0;
ic:= v(x, 0) = -(exp(2)*x-exp(x+1)-x+exp(1-x))/(exp(2)-1), (D[2](v))(x, 0) = 1+(exp(2)*x-exp(x+1)-x+exp(1-x))/(exp(2)-1);
sol := pdsolve({pde,ic,bc},v(x,t));
```

$$v(x,t) = \sum_{n=1}^{\infty} -2\,\frac{\left(\left((-1)^n \pi^2 n^2 - \pi^2 n^2 + 2\,(-1)^n - 1\right)\sin(\pi\, t n) - (-1)^n \cos(\pi\, t n)\,\pi\, n\right)\sin(n\pi\, x)}{\pi^2 n^2 \left(\pi^2 n^2 + 1\right)}$$

Result Solved

6.11 Wave PDE on string (finite domain), both ends fixed end, initial conditions both not zero, and with constant source

problem number 91

Added July 2, 2018.
Third example, from Maple 2018.1 improvements to PDE's document.
Solve

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + 1$$

For $t > 0$ and $0 < x < L$. With boundary conditions

$$u(0,t) = 0$$
$$u(L,0) = 0$$

With initial conditions

$$u(x,0) = f(x)$$
$$\frac{\partial u}{\partial t}(x,0) = g(x)$$

Mathematica

```
ClearAll[u, t, x, c, L, f, g];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] + 1;
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == g[x]};
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> L > 0];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,2)}(x,t) = c^2 u^{(2,0)}(x,t) + 1, \{u(0,t) = 0, u(L,t) = 0\}, \left\{u(x,0) = f(x), u^{(0,1)}(x,0) = g(x)\right\}\right\}, u(x,t), \{x,t\}, \text{Assumptions} \to L > 0\right]$$

Result Did not solve

Maple

```
interface(showassumed=0);
x:='x';t:='t';a:='a';f='f';L:='L';g:='g';
pde :=diff(u(x, t), t, t) = c^2* diff(u(x, t), x, x) + 1 ;
bc := u(0, t) = 0, u(L, t) = 0;
ic:= u(x, 0) = f(x), (D[2](u))(x, 0) = g(x);
sol:=pdsolve([pde, ic, bc]) assuming L>0;
```

$$u(x,t) = 1/2\,\frac{1}{c^2}\left(2\sum_{n=1}^{\infty}\left(\frac{1}{n\pi\, c^2 L}\sin\left(\frac{n\pi\, x}{L}\right)\left(2L\int_0^L \sin\left(\frac{n\pi\, x}{L}\right)g(x)\,\mathrm{d}x \sin\left(\frac{n\pi\, c t}{L}\right)c - \int_0^L \sin\left(\frac{n\pi\, x}{L}\right)\left(-2f(x)c^2 + Lx - x^2\right)\mathrm{d}x \cos\left(\frac{n\pi\, c t}{L}\right)\pi\, n\right)\right)c^2 + Lx - x^2\right)$$

Result Solved

6.12 Wave PDE on string (finite domain), both ends fixed end, with source

problem number 92

This is problem at page 213, David J Logan textbook, applied PDE textbook.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + Ax$$

With boundary conditions

$$u(L,0) = 0$$
$$u(0,t) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = 0$$

Mathematica

```
ClearAll[u, t, x, c, A, L];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] + A*x;
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,2)}(x,t) = c^2 u^{(2,0)}(x,t) + Ax, \{u(0,t) = 0, u(L,t) = 0\}, \left\{u(x,0) = 0, u^{(0,1)}(x,0) = 0\right\}\right\}, u(x,t), \{x,t\}\right]$$

Result Did not solve

Maple

```
x:='x'; t:='t'; L:='L';c:='c';u:='u';A:='A';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)+A* x;
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=0,D[2](u)(x,0)=0;
sol:=pdsolve([pde,bc,ic],u(x,t));
sol:=subs(_Z1=n,sol);
```

$$u(x,t) = 1/6\,\frac{1}{c^2}\left(AL^2x - Ax^3 + 6\sum_{n=1}^{\infty} 2\,\frac{L^3(-1)^{n\,csgn(L^{-1})}A}{\pi^3n^3c^2}\sin\left(\frac{n\pi\,x}{L}\right)\cos\left(\frac{c\pi\,csgn\left(L^{-1}\right)tn}{L}\right)c^2\right)$$

Result Solved

6.13 Wave PDE on semi-infinite domain, with one end having a moving boundary condition

problem number 93

Solve for $u(x,t)$ with $t>0$ and $x>0$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

With boundary conditions

$$u(0,t) = g(t)$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = 0$$

Mathematica

```
ClearAll[u, t, x, g, c];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
 bc = u[0, t] == g[t];
 ic = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {t > 0 && c > 0 && x > 0}];
 ;
```

$$\left\{\left\{u(x,t) \to \begin{cases} 0 & x > ct \\ g\left(t - \frac{x}{c}\right) & x \le ct \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';g:='g';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
ic:=u(x,0)=0,D[2](u)(x,0)=0;
bc:=u(0,t)=g(t);
sol:=pdsolve([pde,ic,bc],u(x,t),HINT = boundedseries(x=0)) assuming t>0,x>0,c>0;
```

$$u(x,t) = \mathit{Heaviside}\left(t - \frac{x}{c}\right) g\left(\frac{tc - x}{c}\right)$$

Result Solved

6.14 Telegraphy PDE, a wave PDE on string, both ends fixed with damping

problem number 94

Solve

$$\frac{\partial^2 u}{\partial t^2} + 2\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

With boundary conditions

$$u(0,t) = 0$$
$$u(\pi,0) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = f(x)$$

Mathematica

```
pde = D[u[x, t], {t, 2}] + 2*D[u[x, t], t] == D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[Pi, t] == 0};
 ic = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f[x]};
 sol = DSolve[{pde, bc, ic}, u[x, t], x, t];
 ;
```

$\mathrm{DSolve}\left[\left\{2u^{(0,1)}(x,t) + u^{(0,2)}(x,t) = u^{(2,0)}(x,t), \{u(0,t) = 0, u(\pi,t) = 0\}, \left\{u^{(0,1)}(x,0) = 0, u(x,0) = f(x)\right\}\right\}, u(x,t), x, t\right]$

Result Did not solve

Maple

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';f:='f';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)+2*diff(u(x,t),t)=diff(u(x,t),x$2);
ic:=D[2](u)(x,0)=0,u(0,t)=0,u(x,0)=f(x);
bc:=u(0,t)=0,u(Pi,t)=0;
sol:=pdsolve([pde,ic,bc],u(x,t)) assuming t>0;
```

$$u(x,t) = \sum_{n=1}^{\infty} \left(\frac{\sin(nx)\left(\left(-1+\sqrt{-n^2+1}\right)\mathrm{e}^{-\left(\sqrt{-n^2+1}+1\right)t} + \mathrm{e}^{\left(-1+\sqrt{-n^2+1}\right)t}\left(\sqrt{-n^2+1}+1\right)\right)\int_0^{\pi}\sin(nx)\,f(x)\,\mathrm{d}x}{\sqrt{-n^2+1}\,\pi} \right)$$

Result Solved, But $n = 1$ should not be included.

6.15 Wave PDE, on string, both ends fixed. Initial velocity zero. Dispersion term present

problem number 95

Solve

$$\frac{1}{a^2}\frac{\partial^2 u}{\partial t^2}+\gamma^2 u(x,t)=c^2\frac{\partial^2 u}{\partial x^2}$$

Dispersion term $\gamma^2 u(x,t)$ causes the shape of the original wave to distort with time.
With $0<x<\pi$ and $t>0$ and with boundary conditions

$$\begin{aligned} u(0,t)&=0\\ u(\pi,0)&=0 \end{aligned}$$

With initial conditions

$$\begin{aligned} \frac{\partial u}{\partial t}(x,0)&=0\\ u(x,0)&=\sin^2(x) \end{aligned}$$

Mathematica

```
pde = (1*D[u[x, t], {t, 2}])/a^2 + \[Gamma]^2*u[x, t] == D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[Pi, t] == 0};
 ic = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == Sin[x]^2};
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 ;
```

$$\text{DSolve}\left[\left\{\frac{u^{(0,2)}(x,t)}{a^2}+\gamma^2 u(x,t)=u^{(2,0)}(x,t),\{u(0,t)=0,u(\pi,t)=0\},\left\{u^{(0,1)}(x,0)=0,u(x,0)=\sin^2(x)\right\}\right\},u(x,t),\{x,t\}\right]$$

Result Did not solve due to adding dispersion term

Maple

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';f:='f';a:='a';
interface(showassumed=0);
pde:=1/a^2*diff(u(x,t),t$2)+gamma^2*u(x,t)=diff(u(x,t),x$2);
bc:=u(0,t)=0,u(Pi,t)=0;
ic:=u(x,0)=sin(x)^2,(D[2](u))(x,0)=0;
sol:=pdsolve([pde,ic,bc],u(x,t));
```

$$u(x,t)=\sum_{n=1}^{\infty}4\,\frac{\left(-1+(-1)^n\right)\sin(nx)\cos\left(a\sqrt{\gamma^2+n^2}\,t\right)}{n\pi\left(n^2-4\right)}$$

Result Solved

6.16 Wave PDE on string with fixed ends, non-zero initial position

problem number 96

Added March 9, 2018.
Solve

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$$

With boundary conditions

$$u(0,t) = 0$$
$$u(\pi,0) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = \sin^2(x)$$

Mathematica

```
ClearAll[u, t, x, n];
 pde = D[u[x, t], {t, 2}] == 4*D[u[x, t], {x, 2}];
 ic = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == Sin[x]^2};
 bc = {u[0, t] == 0, u[Pi, t] == 0};
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 sol = sol /. K[1] -> n;
 ;
```

$$\left\{\left\{u(x,t) \to \sum_{n=1}^{\infty} \frac{4(\cos(n\pi)-1)\cos(2nt)\sin(nx)}{\left(n^3-4n\right)\pi}\right\}\right\}$$

Result Solved but sum should not include $n = 2$

Maple

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)= 4*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(Pi,t)=0;
ic:=u(x,0)=sin(x)^2,D[2](u)(x,0)=0;
sol:=pdsolve([pde,bc,ic],u(x,t));
```

$$u(x,t) = \sum_{n=1}^{\infty} 4\,\frac{(-1+(-1)^n)\sin(nx)\cos(2\,tn)}{n\pi\left(n^2-4\right)}$$

Result Solved, but sum should not include $n = 2$

6.17 Wave PDE homogeneous in square, given initial position but with zero initial velocity

problem number 97

Taken from Maple PDE help pages. This wave PDE inside square with free to move on left edge and right edge, and top and bottom edges are fixed. It has zero initial velocity, but given a non-zero initial position. Where $0 < x < \pi$ and $0 < y < \pi$ and $t > 0$.

Solve

$$\frac{\partial^2 u}{\partial t^2} = \frac{1}{4}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

With boundary conditions

$$\begin{aligned}\frac{\partial u}{\partial x} u(0, y, t) &= 0\\ \frac{\partial u}{\partial x} u(\pi, y, t) &= 0\\ u(x, 0, t) &= 0\\ u(x, \pi, 0) &= 0\end{aligned}$$

With initial conditions

$$\begin{aligned}\frac{\partial u}{\partial t}(x, y, 0) &= 0\\ u(x, 0) &= xy(\pi - y)\end{aligned}$$

Mathematica

```
ClearAll[u, t, y, x];
 pde = D[u[x, y, t], {t, 2}] == (1*(D[u[x, y, t], {x, 2}] + D[u[x, y, t], {y, 2}]))/4;
 ic = {Derivative[0, 0, 1][u][x, y, 0] == 0, u[x, y, 0] == x*y*(Pi - y)};
 bc = {Derivative[1, 0, 0][u][0, y, t] == 0, Derivative[1, 0, 0][u][Pi, y, t] == 0, u[x, 0, t] == 0, u[x, Pi, t] == 0};
 sol = DSolve[{pde, bc, ic}, u[x, y, t], {x, y, t}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,0,2)}(x,y,t) = \frac{1}{4}\left(u^{(0,2,0)}(x,y,t) + u^{(2,0,0)}(x,y,t)\right), \left\{u^{(1,0,0)}(0,y,t) = 0, u^{(1,0,0)}(\pi,y,t) = 0, u(x,0,t) = 0, u(x,\pi,t) = 0\right\}, \left\{u^{(0,0,1)}(x,y,0) = 0, u(x,y,0) = x(\pi - y)y\right\}\right\}, u(x,y,t), \{x,y,t\}\right]$$

Result Did not solve

Maple

```
x:='x'; t:='t'; y:='y'; u:='u';
pde := diff(u(x, y, t), t, t) = (1/4)*(diff(u(x, y, t), x, x))+(1/4)*(diff(u(x, y, t), y, y));
bc := (D[1](u))(0, y, t) = 0, (D[1](u))(Pi, y, t) = 0, u(x, 0, t) = 0, u(x, Pi, t) = 0;
ic:= u(x, y, 0) = x*y*(Pi-y),(D[3](u))(x, y, 0) = 0;
sol:=pdsolve([pde,bc,ic],u(x,y,t));
sol:=subs(n1=m,sol);
```

$$u(x,y,t) = \sum_{n=1}^{\infty} -2\,\frac{\left(-1 + (-1)^n\right)\sin(ny)\cos(1/2\,tn)}{n^3} + \sum_{n=1}^{\infty}\left(\sum_{m=1}^{\infty} 8\,\frac{\left(-(-1)^{n+m} + (-1)^n + (-1)^m - 1\right)\cos(mx)\sin(ny)\cos\left(1/2\sqrt{m^2+n^2}\,t\right)}{\pi^2 m^2 n^3}\right)$$

Result Solved

6.18 Wave PDE homogeneous in square with damping. Given zero initial position but with non-zero initial velocity

problem number 98

Taken from Maple PDE help pages. This wave PDE inside square with damping present.

Membrane is free to move on the right edge and also on top edge. But fixed at left edge and bottom edge.

It has zero initial position, but given a non-zero initial velocity. Where $0 < x < 1$ and $0 < y < 1$ and $t > 0$.

Solve

$$\frac{\partial^2 u}{\partial t^2} = \frac{1}{4}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{10}\frac{\partial u}{\partial t}$$

With boundary conditions

$$\begin{aligned} u(0,y,t) &= 0 \\ \frac{\partial u}{\partial x}u(1,y,t) &= 0 \\ u(x,0,t) &= 0 \\ \frac{\partial u}{\partial y}u(x,1,t) &= 0 \end{aligned}$$

With initial conditions

$$\begin{aligned} u(x,y,0) &= 0 \\ \frac{\partial u}{\partial t}(x,y,0) &= x(1-\frac{1}{2}x)(1-\frac{1}{2}y)y \end{aligned}$$

Mathematica

```
ClearAll[u, t, y, x];
 pde = D[u[x, y, t], {t, 2}] == (1*(D[u[x, y, t], {x, 2}] + D[u[x, y, t], {y, 2}]))/4 - (1*D[u[x, y, t], t])/10;
 ic = {u[x, y, 0] == 0, Derivative[0, 0, 1][u][x, y, 0] == x*(1 - (1/2)*x)*(1 - (1/2)*y)*y};
 bc = {u[0, y, t] == 0, Derivative[1, 0, 0][u][1, y, t] == 0, u[x, 0, t] == 0, Derivative[0, 1, 0][u][x, 1, t] == 0};
 sol = DSolve[{pde, bc, ic}, u[x, y, t], {x, y, t}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,0,2)}(x,y,t) = \frac{1}{4}\left(u^{(0,2,0)}(x,y,t) + u^{(2,0,0)}(x,y,t)\right) - \frac{1}{10}u^{(0,0,1)}(x,y,t), \{u(0,y,t) = 0, u^{(1,0,0)}(1,y,t) = 0, u(x,0,t) = 0, u^{(0,1,0)}(x,1,t) = 0\}, \left\{u(x,y,0) = 0, u^{(0,0,1)}(x,y,0) = \left(1-\frac{x}{2}\right)x\left(1-\frac{y}{2}\right)y\right\}\right\}, u(x,y,t), \{x,y,t\}\right]$$

Result Did not solve

Maple

```
x:='x'; t:='t'; y:='y'; u:='u';
pde := diff(u(x, y, t), t$2) = 1/4*(diff(u(x, y, t), x$2)+diff(u(x, y, t), y$2))-(1/10)*(diff(u(x, y, t), t));
bc := u(0, y, t) = 0, (D[1](u))(1, y, t) = 0, u(x, 0, t) = 0, (D[2](u))(x, 1, t) = 0;
ic:= u(x, y, 0) = 0, (D[3](u))(x, y, 0) = x*(1-(1/2)*x)*(1-(1/2)*y)*y;
sol:=pdsolve([pde, ic,bc], u(x, y, t));
sol:=subs(n1=m,sol);
```

$$u(x,y,t) = \sum_{m=0}^{\infty}\left(\sum_{n=0}^{\infty} 5120\,\frac{\sin(1/2\,(1+2\,n)\,\pi\,x)\sin\left(1/2\,(1+2\,m)\,\pi\,y\right)e^{-t/20}\sin\left(1/20\,t\sqrt{-1+\left(100\,m^2+100\,n^2+100\,m+100\,n+50\right)\pi^2}\right)}{\sqrt{-1+\left(100\,m^2+100\,n^2+100\,m+100\,n+50\right)\pi^2}\,\pi^6\,(1+2\,m)^3\,(1+2\,n)^3}\right)$$

Result Solved

6.19 Wave PDE inside rectangle. All 4 edges are fixed and given non-zero initial position with zero initial velocity

problem number 99

Taken from Mathematica helps pages on DSolve
Solve for $u(x,y,t)$ with $0<x<1$ and $0<y<2$ and $t>0$.
Solve

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

With boundary conditions

$$\begin{aligned} u(x,0,t) &= 0 \\ u(0,y,t) &= 0 \\ u(1,y,t) &= 0 \\ u(x,2,t) &= 0 \end{aligned}$$

With initial conditions

$$\begin{aligned} u(x,y,0) &= \frac{1}{10}(x-x^2)(2y-y^2) \\ \frac{\partial u}{\partial t}(x,y,0) &= 0 \end{aligned}$$

Mathematica

```
ClearAll[u, t, y, x, n, m];
 pde = D[u[x, y, t], {t, 2}] == Laplacian[u[x, y, t], {x, y}];
 ic = {u[x, y, 0] == (1/10)*(x - x^2)*(2*y - y^2), Derivative[0, 0, 1][u][x, y, 0] == 0};
 bc = {u[x, 0, t] == 0, u[0, y, t] == 0, u[1, y, t] == 0, u[x, 2, t] == 0};
 sol = DSolve[{pde, ic, bc}, u[x, y, t], {x, y, t}];
 sol = sol /. {K[1] -> n, K[2] -> m};
 sol = Assuming[Element[{n, m}, Integers], FullSimplify[sol]];
 ;
```

$$\left\{\left\{u(x,y,t)\to\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{32\left(-1+(-1)^m\right)\left(-1+(-1)^n\right)\cos\left(\sqrt{\frac{m^2}{4}+n^2}\pi t\right)\sin(n\pi x)\sin\left(\frac{m\pi y}{2}\right)}{5m^3n^3\pi^6}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; t:='t'; y:='y'; u:='u';
pde := diff(u(x, y, t), t$2) = diff(u(x, y, t), x$2)+diff(u(x, y, t), y$2);
ic:=u(x,y,0)=(1/10)*(x-x^2)*(2*y-y^2),(D[3](u))(x,y,0)=0;
bc:=u(x,0,t)=0,u(0,y,t)=0,u(1,y,t)=0,u(x,2,t)=0;
sol:=pdsolve([pde, ic,bc], u(x, y, t));
sol:=subs(n1=m,sol);
```

$$u(x,y,t)=\sum_{m=1}^{\infty}\left(\sum_{n=1}^{\infty}-\frac{\left(-32\,(-1)^{m+n}+32\,(-1)^m+32\,(-1)^n-32\right)\sin(n\pi x)\sin\left(1/2\,m\pi\,y\right)\cos\left(1/2\,\pi\sqrt{m^2+4\,n^2}\,t\right)}{5\,n^3\pi^6m^3}\right)$$

Result Solved

6.20 Wave PDE inside disk. fixed edge of disk, no θ dependency, with initial position and velocity given

problem number 100

Taken from Mathematica helps pages on DSolve
Solve for $u(r,t)$ with $0<r<1$ and $t>0$.

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right)$$

With boundary conditions

$$u(1,t) = 0$$

With initial conditions

$$u(r,0) = 1$$
$$\frac{\partial u}{\partial t}(r,0) = \frac{r}{3}$$

Mathematica

```
ClearAll[u, t, r, n];
 pde = D[u[r, t], {t, 2}] == c^2*(D[u[r, t], {r, 2}] + (1*D[u[r, t], r])/r);
 ic = {u[r, 0] == 1, Derivative[0, 1][u][r, 0] == r/3};
 bc = u[1, t] == 0;
 sol = DSolve[{pde, ic, bc}, u[r, t], {r, t}];
 sol = sol /. K[1] -> n;
 sol = FullSimplify[sol];
 ;
```

$$\left\{\left\{u(r,t)\to\sum_{n=1}^{\infty}\frac{2J_0\left(r\,j_{0,n}\right)\left(9\sqrt{c^2}J_1\left(j_{0,n}\right)\cos\left(ct\,j_{0,n}\right)+{}_1F_2\left(\frac{3}{2};1,\frac{5}{2};-\frac{1}{4}\left(j_{0,n}\right)^2\right)\sin\left(\sqrt{c^2}t\,j_{0,n}\right)\right)}{9\sqrt{c^2}\left(J_0\left(j_{0,n}\right)^2+J_1\left(j_{0,n}\right)^2\right)j_{0,n}}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; t:='t'; y:='y'; u:='u';
pde := diff(u(r, t), t$2) = c^2 * ( diff(u(r,t), r$2)  +(1/r)* diff(u(r,t),r));
ic:=u(r,0)=1,(D[2](u))(r,0)=r/3;
bc:=u(1,t)=0;
sol:=pdsolve([pde, ic,bc], u(r, t),HINT = boundedseries(r=0));
```

$$\mathrm{sol} = ()$$

Result Did not solve

6.21 Wave PDE inside disk. fixed edge of disk, with θ dependency, zero initial velocity

problem number 101

Solve for $u(r,\theta,t)$ with $0<r<a$ and $t>0$ and $-\pi<\theta<\pi$

$$\frac{\partial^2 u}{\partial t^2}=c^2\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right)$$

With boundary conditions

$$\begin{aligned} u(a,\theta,t)&=0\\ |u(0,\theta,t)|&<\infty\\ u(r,-\pi,t)&=u(r,\pi,t)\\ \frac{\partial u}{\partial \theta}(r,-\pi,t)&=\frac{\partial u}{\partial \theta}(r,\pi,t) \end{aligned}$$

With initial conditions

$$\begin{aligned} u(r,\theta,0)&=f(r,\theta)\\ \frac{\partial u}{\partial t}(r,\theta,0)&=0 \end{aligned}$$

Mathematica

```
ClearAll[u, t, r, n, theta, a, f];
 pde = D[u[r, theta, t], {t, 2}] == c^2*(D[u[r, theta, t], {r, 2}] + (1*D[u[r, theta, t], r])/r + (1*D[u[r, theta, t], {theta, 2}])/r^2);
 ic = {u[r, theta, 0] == f[r, theta], Derivative[0, 0, 1][u][r, theta, 0] == 0};
 bc = {u[a, theta, t] == 0, u[r, -Pi, t] == u[r, Pi, t], Derivative[0, 1, 0][u][r, -Pi, t] == Derivative[0, 1, 0][u][r, Pi, t]};
 sol = DSolve[{pde, ic, bc}, u[r, theta, t], {r, theta, t}, Assumptions -> {0 < r < a, a > 0, t > 0, -Pi < theta < Pi}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,0,2)}(r,\theta,t)=c^2\left(\frac{u^{(0,2,0)}(r,\theta,t)}{r^2}+\frac{u^{(1,0,0)}(r,\theta,t)}{r}+u^{(2,0,0)}(r,\theta,t)\right),\{u(r,\theta,0)=f(r,\theta),u^{(0,0,1)}(r,\theta,0)=0\},\{u(a,\theta,t)=0,u(r,-\pi,t)=u(r,\pi,t),u^{(0,1,0)}(r,-\pi,t)=u^{(0,1,0)}(r,\pi,t)\}\right\},u(r,\theta,t),\{r,\theta,t\},\text{Assumptions}\to\{0<r<a,a>0,t>0,-\pi<\theta<\pi\}\right]$$

Result Did not solve

Maple

```
x:='x'; t:='t'; y:='y'; u:='u';theta:='theta';
pde := diff(u(r, theta, t), t$2) = c^2*(diff(u(r, theta, t), r$2) + 1/r*diff(u(r, theta, t), r) + 1/r^2 *diff(u(r, theta, t), theta$2));
ic := u(r, theta, 0) = f(r, theta) , (D[3](u))(r, theta, 0) = 0;
bc := u(a, theta, t) = 0, u(r, -Pi, t) = u(r, Pi, t), (D[2](u))(r, -Pi, t) = (D[2](u))(r, Pi, t);
sol:=pdsolve([pde, ic,bc], u(r, theta ,t),HINT = boundedseries(r=0));
```

$$\text{sol} = ()$$

Result Did not solve

6.22 Wave PDE on infinite domain with initial conditions specified, no source

problem number 102

Taken from Mathematica DSolve help pages.
Solve initial value wave PDE on infinite domain

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

With initial conditions

$$u(x,0) = e^{-x^2}$$
$$\frac{\partial u}{\partial t}(x,0) = 1$$

Mathematica

```
ClearAll[u, t, x];
 pde = D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}];
 ic = {u[x, 0] == E^(-x^2), Derivative[0, 1][u][x, 0] == 1};
 sol = DSolve[{pde, ic}, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t) \to \frac{1}{2}\left(e^{-(x-t)^2} + e^{-(t+x)^2}\right) + t\right\}\right\}$$

Result Solved

Maple

```
x:='x'; t:='t'; u:='u';
pde := diff(u(x,t), t$2) = diff(u(x,t), x$2);
ic:= u(x, 0) = exp(-x^2), (D[2](u))(x,0) = 1;
sol:=pdsolve([pde, ic], u(x, t));
```

$$u(x,t) = 1/2\,\mathrm{e}^{-(-x+t)^2} + t + 1/2\,\mathrm{e}^{-(t+x)^2}$$

Result Solved

6.23 Wave PDE on infinite domain with initial conditions specified, with source term

problem number 103

Taken from Mathematica DSolve help pages.
Solve initial value wave PDE on infinite domain

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + m$$

With initial conditions

$$u(x,0) = \sin x - \frac{\cos 3x}{e^{\frac{abs(x)}{6}}}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

Mathematica

```
ClearAll[u, t, x];
 pde = {D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}] + m};
 ic = {u[x, 0] == Sin[x] - Cos[3*x]/E^(Abs[x]/6), Derivative[0, 1][u][x, 0] == 0};
 sol = DSolve[{pde, ic}, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t) \to \frac{1}{2}\left(-e^{-\frac{|x-t|}{6}}\cos(3(x-t)) - e^{-\frac{|t+x|}{6}}\cos(3(t+x)) - \sin(t-x) + \sin(t+x)\right) + \frac{mt^2}{2}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; t:='t'; u:='u';
pde:= diff(u(x, t), t$2) = diff(u(x, t), x$2) + m;
ic := u(x, 0) = sin(x) - cos(3*x)/exp(abs(x)/6), (D[2](u))(x, 0) =0;
sol := pdsolve({pde, ic}, u(x, t));
```

$$u(x,t) = 1/2\left(\left(mt^2 + \sin(t+x) - \sin(-x+t)\right)e^{1/6|t+x|+1/6|-x+t|} - e^{1/6|t+x|}\cos(3t-3x) - \cos(3t+3x)e^{1/6|-x+t|}\right)e^{-1/6|-x+t|-1/6|t+x|}$$

Result Solved

6.24 Wave PDE initial value with a Dirichlet condition on the half-line

problem number 104

Taken from Mathematica DSolve help pages.
Solve for $u(x,t)$ initial value wave PDE on infinite domain with $t>0$ and $x>0$.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

With initial conditions

$$u(x,0) = \sin^2(x) \qquad \pi < x < 2\pi$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

And boundary conditions $u(0,t) = 0$

Mathematica

```
ClearAll[u, t, x];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
 ic = {u[x, 0] == Piecewise[{{Sin[x]^2, Pi < x < 2*Pi}}], Derivative[0, 1][u][x, 0] == 0};
 bc = u[0, t] == 0;
 sol = DSolve[{pde, ic, bc}, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t)\to \begin{cases} \frac{1}{2}\left(\left(\begin{cases} \sin^2\left(\sqrt{c^2}t-x\right) & \pi < x-\sqrt{c^2}t < 2\pi \\ 0 & \text{True} \end{cases}\right)+\left(\begin{cases} \sin^2\left(\sqrt{c^2}t+x\right) & \pi < \sqrt{c^2}t+x < 2\pi \\ 0 & \text{True} \end{cases}\right)\right) & x > \sqrt{c^2}t \geq 0 \\ \frac{1}{2}\left(\left(\begin{cases} \sin^2\left(\sqrt{c^2}t+x\right) & \pi < \sqrt{c^2}t+x < 2\pi \\ 0 & \text{True} \end{cases}\right)-\left(\begin{cases} \sin^2\left(\sqrt{c^2}t-x\right) & \pi < \sqrt{c^2}t-x < 2\pi \\ 0 & \text{True} \end{cases}\right)\right) & 0 \leq x \leq \sqrt{c^2}t \\ \text{Indeterminate} & \text{True} \end{cases}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; t:='t'; u:='u';
pde:=diff(u(x, t), t$2) = c^2 * diff(u(x, t), x$2) ;
ic:=u(x,0)= piecewise(Pi<x and x<2*Pi,sin(x)^2),(D[2](u))(x,0)=0;
bc:=u(0,t)=0;
sol:=pdsolve({pde,ic,bc},u(x,t)) assuming t>0 and x>0;
```

$$u(x,t) = \begin{cases} \begin{cases} 0 & tc+x \leq \pi \\ 1/2\,(\sin(tc+x))^2 & tc+x < 2\pi \\ 0 & 2\pi \leq tc+x \end{cases} + \begin{cases} 0 & tc-x \leq \pi \\ -1/2\,(\sin(tc-x))^2 & tc-x < 2\pi \\ 0 & 2\pi \leq tc-x \end{cases} & x < tc \\ \begin{cases} 0 & tc+x \leq \pi \\ 1/2\,(\sin(tc+x))^2 & tc+x < 2\pi \\ 0 & 2\pi \leq tc+x \end{cases} + \begin{cases} 0 & -tc+x \leq \pi \\ 1/2\,(\sin(tc-x))^2 & -tc+x < 2\pi \\ 0 & 2\pi \leq -tc+x \end{cases} & tc < x \end{cases}$$

Result Solved

6.25 Wave PDE Initial value problem with a Neumann condition on the half-line

problem number 105

Taken from Mathematica DSolve help pages.
Solve initial value wave PDE on infinite domain

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

With initial conditions

$$u(x,0) = \sin^3(x)$$
$$\frac{\partial u}{\partial t}(x,0) = 1 - e^{-\frac{x}{10}}$$

And boundary conditions $\frac{\partial u}{\partial x}(0,t) = 1$

Mathematica

```
ClearAll[u, t, x];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
 ic = {u[x, 0] == Sin[x]^3, Derivative[0, 1][u][x, 0] == 1 - E^(-(x/10))};
 bc = Derivative[1, 0][u][0, t] == 1;
 sol = DSolveValue[{pde, ic, bc}, u[x, t], {x, t}];
 ;
```

$$\begin{cases} \frac{1}{2}\left(\sin^3\left(\sqrt{c^2}t + x\right) - \sin^3\left(\sqrt{c^2}t - x\right)\right) + \frac{2\sqrt{c^2}t - 20e^{-x/10}\sinh\left(\frac{\sqrt{c^2}t}{10}\right)}{2\sqrt{c^2}} & x > \sqrt{c^2}t \geq 0 \\ \frac{10e^{\frac{1}{10}\left(-\sqrt{c^2}t - x\right)} + 10e^{\frac{1}{10}\left(x - \sqrt{c^2}t\right)} + 2\sqrt{c^2}t - 20}{2\sqrt{c^2}} - \sqrt{c^2}\left(t - \frac{x}{\sqrt{c^2}}\right) + \frac{1}{2}\left(\sin^3\left(\sqrt{c^2}t - x\right) + \sin^3\left(\sqrt{c^2}t + x\right)\right) & 0 \leq x \leq \sqrt{c^2}t \end{cases}$$

Result Solved

Maple

```
x:='x'; t:='t'; u:='u';
pde:=diff(u(x, t), t$2) = c^2 * diff(u(x, t), x$2) ;
ic:=u(x,0)= sin(x)^3,(D[2](u))(x,0)=1-exp(-x/10);
bc:=(D[1](u))(0,t)=1;
sol:=pdsolve({pde,ic,bc},u(x,t)) assuming t>0 and x>0;
```

$$\text{sol} = ()$$

Result Did not solve

6.26 non-linear wave PDE (Solitons)

problem number 106

This was first solved analytically by (Krvskal, Zabrsky 1965).
Solve

$$\frac{\partial u}{\partial t} + 6u(x,t)\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0$$

Mathematica

```
ClearAll[u, t, x];
 pde = D[u[x, t], t] + 6*u[x, t]*D[u[x, t], x] + D[u[x, t], {x, 3}] == 0;
 sol = DSolve[pde, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t) \to -\frac{12c_1^3 \tanh^2(c_2 t + c_1 x + c_3) - 8c_1^3 + c_2}{6c_1}\right\}\right\}$$

Result Solved. build a special solution.

Maple

```
x:='x'; t:='t'; u:='u';
pde :=  diff(u(x,t),t)+6*u(x,t)*diff(u(x,t),x)+diff(u(x,t),x$3)=0;
sol:=pdsolve(pde,u(x,t)) assuming t>0,x>0;
;
```

$$u(x,t) = -2\_C2^2 (\tanh(\_C2\,x + \_C3\,t + \_C1))^2 + 1/6\,\frac{8\_C2^3 - \_C3}{\_C2}$$

Result Solved. Returning a solution that is not the most general one

7 Schrodinger PDE

7.1 Schrodinger PDE with zero potential

problem number 107

From page 30, David J Logan textbook, applied PDE textbook.
Solve

$$Ih\frac{\partial f}{\partial t} = -\frac{h^2}{2m}\frac{\partial^2 f}{\partial x^2}$$

With boundary conditions

$$f(0,t) = 0$$
$$f(L,0) = 0$$

Mathematica

```
ClearAll[f, t, x, L, m, h];
 pde = I*h*D[f[x, t], t] == -((h^2*D[f[x, t], {x, 2}])/(2*m));
 bc = {f[0, t] == 0, f[L, t] == 0};
 sol = DSolve[{pde, bc}, f[x, t], {x, t}, Assumptions -> L > 0];
 sol = sol /. K[1] -> n;
 ;
```

$$\left\{\left\{f(x,t)\to \sum_{n=1}^{\infty} e^{-\frac{ihn^2\pi^2 t}{2L^2 m}} c_n \sin\left(\frac{n\pi x}{L}\right)\right\}\right\}$$

Result Solved

Maple

```
x:='x'; t:='t'; L:='L'; c:='c';f:='f';
interface(showassumed=0);
pde:=I*h*diff(f(x,t),t)=-h^2/(2*m)*diff(f(x,t),x$2);
bc:=f(0,t)=0,f(L,t)=0;
sol:=pdsolve([pde,bc],f(x,t)) assuming L>0;
sol:=subs(_Z1=n,sol);
```

$$f(x,t) = \sum_{n=1}^{\infty} \_C1(n) \sin\left(\frac{n\pi x}{L}\right) e^{\frac{-i/2 h\pi^2 n^2 t}{mL^2}}$$

Result Solved

7.2 Schrodinger PDE with initial and boundary conditions

problem number 108

Solve for $f(x, y, t)$

$$I\frac{\partial f}{\partial t} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)$$

With boundary conditions

$$\begin{aligned} f(0, y, t) &= 0 \\ f(1, y, t) &= 0 \\ f(x, 1, t) &= 0 \\ f(x, 0, t) &= 0 \end{aligned}$$

And initial conditions $f(x, y, 0) = \sqrt{2}\left(\sin(2\pi x)\sin(\pi y) + \sin(\pi x)\sin(2\pi y)\right)$

Mathematica

```
ClearAll[f, t, x, y];
 pde = I*D[f[x, y, t], {t}] == -((hBar^2*Laplacian[f[x, y, t], {x, y}])/(2*m));
 initSum = f[x, y, 0] == Sqrt[2]*(Sin[2*Pi*x]*Sin[Pi*y] + Sin[Pi*x]*Sin[3*Pi*y]);
 bcs = {f[0, y, t] == 0, f[1, y, t] == 0, f[x, 1, t] == 0, f[x, 0, t] == 0};
 sol = DSolve[{pde, bcs, initSum}, f[x, y, t], {x, y, t}];
 ;
```

$$\left\{\left\{f(x,y,t) \to \sqrt{2}e^{-\frac{5i\pi^2 \mathrm{hBar}^2 t}{m}}\left(\sin(\pi x)\sin(3\pi y) + \sin(2\pi x)\sin(\pi y)e^{\frac{5i\pi^2\mathrm{hBar}^2 t}{2m}}\right)\right\}\right\}$$

Result Solved

Maple

```
x:='x'; t:='t'; y:='y'; hbar:='hbar';f:='f';
interface(showassumed=0);
pde:=  I* diff(f(x,y,t),t) = -hBar^2/(2*m) * (diff(f(x,y,t),x$2) +
diff(f(x,y,t),y$2));
ic := f(x, y, 0) = sqrt(2)*(sin(2*Pi*x)*sin(Pi*y) + sin(Pi*x)*sin(3*Pi*y));
bc := f(0, y, t) = 0,f(1, y, t) = 0, f(x, 1, t) = 0, f(x, 0, t) = 0;
sol:=pdsolve({pde,ic,bc},f(x,y,t));
```

$$f(x,t) = \sum_{n=1}^{\infty} \_C1(n)\sin\left(\frac{n\pi x}{L}\right)\mathrm{e}^{\frac{-i/2h\pi^2 n^2 t}{mL^2}}$$

Result Did not solve, hangs

7.3 Initial value problem with Dirichlet boundary conditions

problem number 109

Taken from Mathematica DSolve help pages
Solve for $f(x,t)$

$$I\frac{\partial f}{\partial t} = -2\frac{\partial^2 f}{\partial x^2}$$

With boundary conditions

$$f(5,t) = 0$$
$$f(10,t) = 0$$

And initial conditions $f(x,2) = f(x)$ where $f(x) = -350 + 155x - 22x^2 + x^3$

Mathematica

```
ClearAll[g, f, t, x];
 pde = I*D[f[x, t], t] == -2*D[f[x, t], {x, 2}];
 g[x_] := -350 + 155*x - 22*x^2 + x^3;
 ic = f[x, 2] == g[x];
 bc = {f[5, t] == 0, f[10, t] == 0};
 sol = DSolve[{pde, bc, ic}, f[x, t], {x, t}];
 sol = sol /. K[1] -> n;
 ;
```

$$\left\{\left\{f(x,t) \to \sum_{n=1}^{\infty} \frac{100\,(7+8(-1)^n)\,e^{-\frac{2}{25}in^2\pi^2(t-2)}\sin\left(\frac{1}{5}n\pi(x-5)\right)}{n^3\pi^3}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; t:='t'; y:='y'; f:='f';g:='g';
pde:=I*diff(f(x,t),t)=-2*diff(f(x,t),x$2);
bc:=f(5,t)=0,f(10,t)=0;
g:=x->-350+155*x-22*x^2+x^3;
ic:=f(x,2)=g(x);
sol:=pdsolve([pde,bc,ic],f(x,t));
```

$$f(x,t) = \sum_{n=1}^{\infty} \frac{(800+700\,(-1)^n)\sin(1/5\,n\pi\,x)\,\mathrm{e}^{-\frac{2}{25}\,i\pi^2n^2(-2+t)}}{n^3\pi^3}$$

Result Solved

7.4 Solve a Schrodinger equation with potential over the whole real line

problem number 110

Taken from Mathematica DSolve help pages
Solve for $f(x,t)$

$$I\frac{\partial f}{\partial t} = -\frac{\partial^2 f}{\partial x^2} + 2x^2 f(x,t)$$

With boundary conditions

$$f(-\infty,t) = 0$$
$$f(\infty,t) = 0$$

Mathematica

```
ClearAll[f, t, x];
 pde = I*D[f[x, t], t] == -D[f[x, t], {x, 2}] + 2*x^2*f[x, t];
 bc = {f[-Infinity, t] == 0, f[Infinity, t] == 0};
 sol = DSolve[{pde, bc}, f[x, t], {x, t}];
 sol = sol /. K[1] -> n;
 ;
```

$$\left\{\left\{f(x,t)\to\sum_{n=0}^{\infty}e^{-\frac{x^2}{\sqrt{2}}-2i\sqrt{2}\left(n+\frac{1}{2}\right)t}c_n\text{HermiteH}\left(n,\sqrt[4]{2}x\right)\right\}\right\}$$

Result Solved

Maple

```
x:='x'; t:='t'; y:='y'; f:='f';g:='g';
pde:=I*diff(f(x,t),t)=-diff(f(x,t),x$2)+2*x^2*f(x,t);
bc:=f(-infinity ,t)=0,f(infinity,t)=0;
try
  sol:=pdsolve([pde,bc],f(x,t));
catch:
  sol:=();
end try;
```

$$\text{sol} = ()$$

Result Did not solve. Maple does not support ∞ in boundary conditions

8 Beam PDE

8.1 Beam PDE with zero initial velocity

problem number 111

Added January 20, 2018.
Solve

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial^4 u}{\partial x^4} = 0$$

With boundary conditions

$$\begin{aligned} u(0,t) &= -12t^2 \\ f(1,t) &= 1 - 12t^2 \\ \frac{\partial^2 u}{\partial x^2} u(0,t) &= 0 \\ \frac{\partial^2 u}{\partial x^2} u(1,t) &= 12 \end{aligned}$$

And initial conditions

$$\begin{aligned} u(x,0) &= x^4 \\ \frac{\partial u}{\partial t} u(x,0) &= 0 \end{aligned}$$

Mathematica

```
ClearAll[u, x, t];
 pde = D[u[x, t], {t, 2}] + D[u[x, t], {x, 4}] == 0;
 bc = {u[0, t] == -12*t^2, u[1, t] == 1 - 12*t^2, Derivative[2, 0][u][0, t] == 0, Derivative[2, 0][u][1, t] == 12};
 ic = {u[x, 0] == x^4, Derivative[0, 1][u][x, 0] == 0};
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t];
 ;
```

$$\{\{u(x,t) \to x^4 - 12t^2\}\}$$

Result Solved

Maple

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)+diff(u(x,t),x$4)=0;
bc:=u(0,t)=-12*t^2,u(1,t)=1-12*t^2,D[1,1](u)(0,t)=0,D[1,1](u)(1,t)=12;
ic:=u(x,0)=x^4,D[2](u)(x,0)=0;
sol:=pdsolve({pde,ic,bc},u(x,t),HINT=`+`);
```

$$u(x,t) = x^4 - 12t^2$$

Result Solved

9 Burger's PDE

9.1 viscous fluid flow with no initial conditions

problem number 112

From Mathematica symbolic PDE document.
Solve for $u(x,t)$

$$\frac{\partial u}{\partial t} + u(x,t)\frac{\partial u}{\partial x} = \mu\frac{\partial^2 u}{\partial x^2}$$

Mathematica

```
ClearAll[u, x, t, mu];
 pde = D[u[x, t], {t}] + u[x, t]*D[u[x, t], {x}] == \[Mu]*D[u[x, t], {x, 2}];
 sol = DSolve[pde, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t)\to -\frac{2c_1^2\mu\tanh\left(c_2t+c_1x+c_3\right)+c_2}{c_1}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; t:='t'; y:='y'; mu:='mu';
interface(showassumed=0);
pde := diff(u(x, t), t) + u(x, t)*diff(u(x, t), x) = mu* diff(u(x,t),x$2);
sol := pdsolve(pde, u(x, t));
```

$$u(x,t) = -2\mu\_C2\tanh\left(\_C2\,x+\_C3\,t+\_C1\right)-\frac{\_C3}{\_C2}$$

Result Solved

9.2 viscous fluid flow with initial conditions

problem number 113

From Mathematica symbolic PDE document.
Solve for $u(x,t)$

$$\frac{\partial u}{\partial t}+u(x,t)\frac{\partial u}{\partial x}=\mu\frac{\partial^2 u}{\partial x^2}$$

With initial conditions
$u(x,0)=\begin{cases} 1 & x<0 \\ 0 & x\geq 0 \end{cases}$

Mathematica

```
ClearAll[u, x, y, mu];
 pde = D[u[x, t], {t}] + u[x, t]*D[u[x, t], {x}] == \[Mu]*D[u[x, t], {x, 2}];
 ic = u[x, 0] == Piecewise[{{1, x < 0}, {0, x >= 1}}];
 sol = DSolve[{pde, ic}, u[x, t], {x, t}, Assumptions -> mu > 0];
 ;
```

$$\left\{\left\{u(x,t)\to\frac{1}{\frac{e^{-\frac{t-2x}{4\mu}}\left(\operatorname{erf}\left(\frac{x}{2\sqrt{\mu t}}\right)+1\right)}{\operatorname{erf}\left(\frac{t-x}{2\sqrt{\mu t}}\right)+1}+1}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; y:='y'; mu:='mu';u:='u';
interface(showassumed=0);
pde := diff(u(x, t), t)+u(x, t)*(diff(u(x, t), x)) = mu*(diff(u(x, t), x$2));
ic := u(x, 0) =  PIECEWISE([0,x>=0],[1,x<0]);
sol:= pdsolve({pde, ic},u(x,t)) assuming mu > 0,t>0;
```

$$u(x,t)=1/2-1/2\,Erf\left(1/2\,\frac{x}{\sqrt{\mu}\sqrt{t}}\right)+\int_0^t\int_{-\infty}^{\infty}-1/2\,\frac{u(\zeta 1,\tau 1)\,\frac{\partial}{\partial\zeta 1}u(\zeta 1,\tau 1)}{\sqrt{\pi}\sqrt{\mu}\sqrt{t-\tau 1}}\,e^{1/4\,\frac{(x-\zeta 1)^2}{\mu(-t+\tau 1)}}\,d\zeta 1\,d\tau 1$$

Result Solved, but has unresolved integrals

9.3 viscous fluid flow with initial conditions as UnitBox

problem number 114

From Mathematica DSolve help pages.
Solve for $u(x,t)$

$$\frac{\partial u}{\partial t}+u(x,t)\frac{\partial u}{\partial x}=\mu\frac{\partial^2 u}{\partial x^2}$$

With initial conditions

$$u(x,0)=\begin{cases}1 & |x|\le\frac{1}{2}\\ 0 & \text{otherwise}\end{cases}$$

Mathematica

```
ClearAll[u, x, y, mu];
 pde = D[u[x, t], {t}] + u[x, t]*D[u[x, t], {x}] == \[Mu]*D[u[x, t], {x, 2}];
 ic = u[x, 0] == UnitBox[x];
 sol = DSolve[{pde, ic}, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t)\to\frac{e^{\frac{t+1}{4\mu}}\left(\operatorname{erf}\left(\frac{2t-2x+1}{4\sqrt{\mu t}}\right)-\operatorname{erf}\left(\frac{2t-2x-1}{4\sqrt{\mu t}}\right)\right)}{e^{\frac{t+1}{4\mu}}\left(\operatorname{erfc}\left(\frac{2t-2x-1}{4\sqrt{\mu t}}\right)-\operatorname{erfc}\left(\frac{2t-2x+1}{4\sqrt{\mu t}}\right)\right)+e^{\frac{x}{2\mu}}\left(\operatorname{erfc}\left(\frac{1-2x}{4\sqrt{\mu t}}\right)+e^{\frac{1}{2}/\mu}\operatorname{erfc}\left(\frac{2x+1}{4\sqrt{\mu t}}\right)\right)}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; y:='y'; mu:='mu';u:='u';
interface(showassumed=0);
pde := diff(u(x, t), t)+u(x, t)*(diff(u(x, t), x)) = mu*(diff(u(x, t), x$2));
ic:= u(x,0)=piecewise( x< -1/2 or x>1/2,0, 1);
sol:= pdsolve({pde, ic},u(x,t)) assuming mu > 0,t>0;
```

$$u(x,t)=1/2\,Erf\left(1/4\,\frac{2\,x+1}{\sqrt{\mu}\sqrt{t}}\right)-1/2\,Erf\left(1/4\,\frac{2\,x-1}{\sqrt{\mu}\sqrt{t}}\right)+\int_0^t\int_{-\infty}^{\infty}-1/2\,\frac{u(\zeta 1,\tau 1)\,\frac{\partial}{\partial\zeta 1}u(\zeta 1,\tau 1)}{\sqrt{\pi}\sqrt{\mu}\sqrt{t-\tau 1}}\mathrm{e}^{1/4\,\frac{(x-\zeta 1)^2}{\mu(-t+\tau 1)}}\,d\zeta 1\,d\tau 1$$

Result Solved, but has unresolved integrals

10 Black Scholes PDE

10.1 classic Black Scholes model from finance

problem number 115

From Mathematica symbolic PDE document.

Solve for $V(S,t)$ where V is the price of the option as a function of stock price S and time t. r is the risk-free interest rate, and σ is the volatility of the stock.

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}=rV-rS\frac{\partial V}{\partial S}$$

With boundary condition $V(S,T)=\max\{S-k,0\}$

Reference `https://en.wikipedia.org/wiki/Black%E2%80%93Scholes_equation`

Mathematica

```
ClearAll[V, S, t, r, sigma];
 pde = D[V[S, t], t] + (1*sigma^2*S^2*D[V[S, t], {S, 2}])/2 == r*V[S, t] - r*S*D[V[S, t], S];
 bc = V[S, T] == Max[S - k, 0];
 sol = DSolve[{pde, bc}, V[S, t], {S, t}, Assumptions -> t > 0];
 ;
```

$$\left\{\left\{V(S,t)\to\frac{1}{2}e^{-rT}\left(Se^{rT}\operatorname{erfc}\left(\frac{2\log(k)+\left(2r+\sigma^2\right)(t-T)-2\log(S)}{2\sqrt{2}\sigma\sqrt{T-t}}\right)-ke^{rt}\operatorname{erfc}\left(\frac{2\log(k)+\left(2r-\sigma^2\right)(t-T)-2\log(S)}{2\sqrt{2}\sigma\sqrt{T-t}}\right)\right)\right\}\right\}$$

Result Solved

Maple

```
x:='x'; y:='y'; sigma:='sigma';S:='S';V:='V';r:='r';
interface(showassumed=0);
pde :=diff(V(S,t),t)+1/2*sigma^2*S^2*diff(V(S,t),S$2)=r*V(S,t)- r*S*diff(V(S,t),S);
bc:=V(S,T)=max(S-k,0);
sol:=pdsolve({pde,bc},V(S,t));
```

$$\text{sol}=()$$

Result Did not solve

10.2 Boundary value problem for the Black Scholes equation

problem number 116

From Mathematica DSolve help pages.
Solve for $V(t,s)$

$$\frac{\partial v}{\partial t}+\frac{1}{2}\sigma^2 s^2 \frac{\partial^2 v}{\partial s^2}+(r-q)s\frac{\partial v}{\partial s}-r v(t,s)=0$$

With boundary condition $v(T,s)=\psi(s)$
Reference https://en.wikipedia.org/wiki/Black%E2%80%93Scholes_equation

Mathematica

```
pde = D[v[t, s], t] + (1*\[Sigma]^2*s^2*D[v[t, s], {s, 2}])/2 + (r - q)*s*D[v[t, s], s] - r*v[t, s] == 0;
 bc = v[T, s] == \[Psi][s];
 sol = DSolve[{pde, bc}, v[t, s], {t, s}];
 ;
```

$$\left\{\left\{v(t,s)\to \frac{e^{r(t-T)}\int_{-\infty}^{\infty}\psi\left(e^{K[1]}\right)\exp\left(-\frac{\left(-K[1]+(T-t)\left(-q+r-\frac{\sigma^2}{2}\right)+\log(s)\right)^2}{2\sigma^2(T-t)}\right)dK[1]}{\sqrt{2\pi}\sqrt{\sigma^2(T-t)}}\right\}\right\}$$

Result Solved

Maple

```
t:='t'; s:='s'; sigma:='sigma';v:='v';psi:='psi';
interface(showassumed=0);
pde:=diff(v(t, s), t)+s^2*(diff(v(t, s), s, s))/(2*sigma^2)+(r-q)*s*(diff(v(t, s), s))-r*v(t, s) = 0;
ic:=v(T, s) = psi(s);
sol:=pdsolve({pde,ic},v(t,s));
```

$$\text{sol} = ()$$

Result Did not solve

11 Korteweg-deVries PDE

11.1 Korteweg-deVries (waves on shallow water surfaces) with no initial conditions

problem number 117

From Mathematica symbolic PDE document.
Solve for $u(x,t)$

$$\frac{\partial^3 u}{\partial x^3}+\frac{\partial u}{\partial t}-6u(x,t)\frac{\partial u}{\partial x}=0$$

Reference `https://en.wikipedia.org/wiki/Korteweg%E2%80%93de_Vries_equation`

Mathematica

```
ClearAll[u, x, t];
 pde = D[u[x, t], {x, 3}] + D[u[x, t], {t}] - 6*u[x, t]*D[u[x, t], {x}] == 0;
 sol = DSolve[pde, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t)\to\frac{12c_1^3\tanh^2\left(c_2t+c_1x+c_3\right)-8c_1^3+c_2}{6c_1}\right\}\right\}$$

Result Solved

Maple

```
x:='x'; y:='y';u:='u';
pde:= diff(u(x,t),x$3)+ diff(u(x,t),t)-6*u(x,t)* diff(u(x,t),x)=0;
sol:=pdsolve(pde,u(x,t));
```

$$u(x,t)=2\_C2^2(\tanh(\_C2x+\_C3t+\_C1))^2-1/6\,\frac{8\_C2^3-\_C3}{\_C2}$$

Result Solved

12 Tricomi PDE

12.1 Boundary value problem for the Tricomi equation

problem number 118

From Mathematica DSolve helps pages.
Solve for $u(x,y)$

$$\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} = 0$$

With boundary conditions

$$u(x,0) = 0$$
$$\frac{\partial u}{\partial y}(x,0) = x^2$$

Mathematica

```
ClearAll[u, x, y];
 pde = D[u[x, y], {x, 2}] + y*D[u[x, y], {y, 2}] == 0;
 bc = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == x^2};
 sol = DSolve[{pde, bc}, u[x, y], {x, y}];
 ;
```

$$\{\{u(x,y) \to -y\left(y-x^2\right)\}\}$$

Result Solved

Maple

```
x:='x'; y:='y';u:='u';
pde:= diff(u(x,y),x$2)+ y*diff(u(x,y),y$2)=0;
bc:=u(x,0)=0,(D[2](u))(x,0)=x^2;
sol:=pdsolve([pde,bc],u(x,y));
```

$$u(x,y) = y\left(x^2-y\right)$$

Result Did not solve

13 Cauchy Riemann PDE's

13.1 Cauchy Riemann PDE with Prescribe the values of u and v on the x axis

problem number 119

From Mathematica DSolve helps pages.
Solve for $u(x,y), v(x,y$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$
$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

With boundary conditions

$$u(x,0) = x^3$$
$$v(x,0) = 0$$

Mathematica

```
ClearAll[u, v, x, y];
 pde1 = D[u[x, y], x] == D[v[x, y], y];
 pde2 = D[u[x, y], y] == -D[v[x, y], x];
 bc = {u[x, 0] == x^3, v[x, 0] == 0};
 sol = DSolve[{pde1, pde2, bc}, {u[x, y], v[x, y]}, {x, y}];
 ;
```

$$\left\{\left\{u(x,y)\to x^3-3xy^2, v(x,y)\to 3x^2y-y^3\right\}\right\}$$

Result Solved

Maple

```
x:='x'; y:='y';u:='u';
pde1:= diff(u(x,y),y)=diff(v(x,y),x);
pde2:= diff(u(x,y),x)=-diff(v(x,y),y);
bc:=u(x,0)=x^3,v(x,0)=0;
sol:=pdsolve({pde1,pde2,bc},{u(x,y),v(x,y)});
```

$$\mathrm{sol} = ()$$

Result Did not solve

13.2 Cauchy Riemann PDE With extra term on right side

problem number 120

Solve for $u(x,y), v(x,y$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$
$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} + y$$

Mathematica

```
ClearAll[u, v, x, y];
 pde1 = D[u[x, y], x] == D[v[x, y], y];
 pde2 = D[u[x, y], y] == -D[v[x, y], x] + y;
 sol = DSolve[{pde1, pde2}, {u[x, y], v[x, y]}, {x, y}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(1,0)}(x,y) = v^{(0,1)}(x,y), u^{(0,1)}(x,y) = y - v^{(1,0)}(x,y)\right\}, \{u(x,y), v(x,y)\}, \{x,y\}\right]$$

Result Did not Solve

Maple

```
x:='x'; y:='y';u:='u';
pde1:= diff(u(x,y),y)=diff(v(x,y),x);
pde2:= diff(u(x,y),x)=-diff(v(x,y),y)+y;
sol:=pdsolve({pde1,pde2},{u(x,y),v(x,y)});
```

$$\left\{u\left(x,y\right) = \_F1\left(y - ix\right) + \_F2\left(y + ix\right), v\left(x,y\right) = i\_F1\left(y - ix\right) - i\_F2\left(y + ix\right) + 1/2\, y^2 + \_C1\right\}$$

Result Solved

14 Hamilton-Jacobi PDE

14.1 Hamilton-Jacobi type PDE

problem number 121

Taken from Maple pdsolve help pages, which is taken from Landau, L.D. and Lifshitz, E.M. Translated by Sykes, J.B. and Bell, J.S. Mechanics. Oxford: Pergamon Press, 1969

Solve for $S(t,\xi,\eta,\phi)$

$$-\frac{\partial}{\partial t}S(t,\xi,\eta,\phi) = 1/2\,\frac{\left(\frac{\partial}{\partial \xi}S(t,\xi,\eta,\phi)\right)^2(\xi^2-1)}{\sigma^2 m(-\eta^2+\xi^2)} + 1/2\,\frac{\left(\frac{\partial}{\partial \eta}S(t,\xi,\eta,\phi)\right)^2(-\eta^2+1)}{\sigma^2 m(-\eta^2+\xi^2)} + 1/2\,\frac{\left(\frac{\partial}{\partial \phi}S(t,\xi,\eta,\phi)\right)^2}{\sigma^2 m(\xi^2-1)(-\eta^2+1)} + \frac{a(\xi)+b(\eta)}{-\eta^2+\xi^2}$$

Mathematica

```
ClearAll[t, \[Zeta], \[Eta], \[Phi], a, b, s];
 pde = -D[s[t, \[Zeta], \[Eta], \[Phi]], t] == ((\[Zeta]^2 - 1)*D[s[t, \[Zeta], \[Eta], \[Phi]], \[Zeta]]^2)/(2*\[Sigma]^2*m*(-\[Eta]^2 + \[Zeta]^2)) + ((-\[Eta]^2 - 1)*D[s[t, \[Zeta], \[Eta], \[Phi]], \[Eta]]^2)/(2*\[Sigma]^2*m*(-\[Eta]^2 + \[Zeta]^2)) + (1*D[s[t, \[Zeta], \[Eta], \[Phi]], \[Phi]]^2)/(2*\[Sigma]^2*m*(\[Zeta]^2 - 1)*(-\[Eta]^2 - 1)) + (a[\[Zeta]] + b[\[Zeta]])/(-\[Eta]^2 + \[Zeta]^2);
 sol = DSolve[pde, s[t, \[Zeta], \[Eta], \[Phi]], {t, \[Zeta], \[Eta], \[Phi]}];
 ;
```

$$\text{DSolve}\left[-s^{(1,0,0,0)}(t,\zeta,\eta,\phi) = \frac{s^{(0,0,0,1)}(t,\zeta,\eta,\phi)^2}{2(\zeta^2-1)(-\eta^2-1)m\sigma^2} + \frac{(-\eta^2-1)s^{(0,0,1,0)}(t,\zeta,\eta,\phi)^2}{2m\sigma^2(\zeta^2-\eta^2)} + \frac{(\zeta^2-1)s^{(0,1,0,0)}(t,\zeta,\eta,\phi)^2}{2m\sigma^2(\zeta^2-\eta^2)} + \frac{a(\zeta)+b(\zeta)}{\zeta^2-\eta^2}, s(t,\zeta,\eta,\phi), \{t,\zeta,\eta,\phi\}\right]$$

Result Did not Solve

Maple

```
S:='S';t:='t'; xi:='xi';eta:='eta';phi:='phi';
pde := -diff(S(t,xi,eta,phi),t) =
1/2*diff(S(t,xi,eta,phi),xi)^2*(xi^2-1)/sigma^2/m/(xi^2-eta^2)
+ 1/2*diff(S(t,xi,eta,phi),eta)^2*(1-eta^2)/m/sigma^2/(xi^2-eta^2)
+ 1/2*diff(S(t,xi,eta,phi),phi)^2/m/sigma^2/(xi^2-1)/(1-eta^2)
+ (a(xi)+b(eta))/(xi^2-eta^2);
sol:=pdsolve(pde,'build');
```

$$S(t,\xi,\eta,\phi) = \_c_4\phi + \_c_1 t + \_C1 + \_C2 + \_C3 + \_C4 - \int \frac{\sqrt{-2\eta^4 m\sigma^2\_c_1 + 2b(\eta)\eta^2 m\sigma^2 + 2\eta^2\_c_1\sigma^2 m - 2\eta^2\_c_3\sigma^2 m - 2b(\eta)\sigma^2 m + 2\_c_3\sigma^2 m - \_c_4{}^2}}{\eta^2-1}\,\mathrm{d}\eta - \int \frac{\sqrt{-2m\sigma^2\xi^4\_c_1 - 2a(\xi)m\sigma^2\xi^2 + 2\xi^2\_c_1\sigma^2 m - 2m\sigma^2\xi^2\_c_3 + 2a(\xi)\sigma^2 m + 2\_c_3\sigma^2 m - \_c_4{}^2}}{\xi^2-1}\,\mathrm{d}\xi$$

Result Solved

15 Other second order PDE's

15.1 A second order PDE

problem number 122

Taken from Maple pdsolve help pages, problem 4.
Solve for $S(x,y)$

$$S(x,y)\left(\frac{\partial^2 S}{\partial x\partial y}\right)+\frac{\partial S}{\partial x}\frac{\partial S}{\partial y}=1$$

Mathematica

```
ClearAll[s, x, y];
 pde = s[x, y]*D[s[x, y], x, y] + D[s[x, y], x]*D[s[x, y], y] == 1;
 sol = DSolve[pde, s[x, y], {x, y}];
 ;
```

$$\text{DSolve}\left[s^{(0,1)}(x,y)s^{(1,0)}(x,y)+s(x,y)s^{(1,1)}(x,y)=1,s(x,y),\{x,y\}\right]$$

Result Did not Solve

Maple

```
S:='S';x:='x'; y:='y';
pde := S(x,y)*diff(S(x,y),y,x) + diff(S(x,y),x)*diff(S(x,y),y) = 1;
sol:=pdsolve(pde,'build');
```

$$S(x,y)=\frac{\sqrt{2\_c_1x+\_C1}\sqrt{\_C2\_c_1{}^2+y\_c_1}}{\_c_1}$$

Result Solved