Analytical solutions to some textbooks PDE's using computer algebera systems

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Contents

1 Introduction and lookup table

This is a collection of PDE problems solved analytically using CAS. Current systems used are Maple 2017 and Mathematica 11.1.1. Problems are mostly taken from textbooks and help pages. A number of these I solved by hand as well for HW's.

For each problem, the CAS commands used are given, and a comment is given about the solution produced. This is work in progress.

It is clear from the results obtained, that CAS is still not very strong at solving PDE's analytically (compared to solving ODE's for example). All the PDE's that were tried below are basic ones, which all can be solved by hand analytically at the level of undergraduate or first year graduate level, but a number of them could not be solved, or the solution given was incomplete or wrong.

Notation used in the table below: T1 is textbook Richard Haberman applied partial differential equations 5th edition. T2 is David J. Logan's Applied Partial Differential Equations.

ToDo: Need to make summary of missing features or class of PDE's CAS can't solve.

In Maple the command `pdsolve` is used and in Mathematica the command `DSolve` is used.

Summary lookup table of result

Problem	Mathematica	Maple
T1, Problem 2.3.3(a) (heat PDE, 1D)	Solved	Solved
T1, Problem 2.3.3(b) (heat PDE, 1D)	Solved	Solved
T1, Problem 2.3.3(c) (heat PDE, 1D)	Solved	Solved
T1, Problem 2.3.3(d) (heat PDE, 1D)	Solved	Solved
T1, Problem 2.3.7 (heat PDE, 1D)	Solved	Partially correct. Missing zero eigenvalue
T1, Problem 2.3.8 (heat PDE, 1D)	Did not solve	Solved
T1, Problem 2.4.1(a) (heat PDE, 1D)	Solved	Partially correct. Missing zero eigenvalue
T1, Problem 2.4.1(b) (heat PDE, 1D)	Solved	Did not solve
T1, Problem 2.4.1(c) (heat PDE, 1D)	Solved	Partially correct. Missing zero eigenvalue
T1, Problem 2.4.1(d) (heat PDE, 1D)	Solved	Did not solve
T1, Problem 2.4.2 (heat PDE, 1D)	Solved	Partially correct. Used $2n+1$ instead of $2n-1$?
T1, Problem 2.5.1(a) (Laplace PDE, 2D)	Solved	Did not solve
T1, Problem 2.5.1(b) (Laplace PDE, 2D)	Solved	Did not solve
T1, Problem 2.5.1(c) (Laplace PDE, 2D)	Did not solve	Did not solve
T1, Problem 2.5.1(d) (Laplace PDE, 2D)	Did not solve	Did not solve
T1, Problem 2.5.1(e) (Laplace PDE, 2D)	Did not solve	Did not solve
T1, Problem 2.5.5(c) (Laplace PDE,quarter circle)	Did not solve	Error in input. Need to find why
T1, Problem 2.5.8(b) (Laplace on circular annulus)	Did not solve	Did not solve
T2, page 115 (Wave PDE 1D nonhomogeneous)	Solved	Solved
T2, page 76 (heat PDE 1D)	Solved	Solved
T2, page 28 (Wave PDE 1D)	Did not solve	Solved
T2, page 130 (Wave PDE 1D)	Did not solve	Partially correct. Used $2n+1$ instead of $2n-1$? as above.
T2, page 131 (heat PDE 1D, periodic BC)	Did not solve	Solved
T2, page 149 (wave PDE 1D, nonhomogeneous)	Did not solve	Solved
T2, page 213 (wave PDE 1D, nonhomogeneous)	Did not solve	Solved
T2, page 30 (Schrodinger PDE with zero potentia)	Solved	Solved
Problem 18 (Maple help page, p20, Laplace PDE, 1D)	Solved	Solved
PDE 2 (Maple help p14, wave PDE 1D)	Solved	Solved
PDE 3 (Maple help p14, heat PDE 1D)	Solved	Solved
PDE 4 (Maple help p15, heat PDE 1D)	Solved	Solved
heat absorption radiation in bounded domain	Did not solve	Solved
heat PDE infinite domain with initial conditions, nonhomogeneous	Solved	Solved
PDE 23, page 23, from Maple pdsolve help PDF file	Solved	Solved
heat PDE on 1D with boundary conditions from $-1\dots 1$	Did not solve	Did not solve
Laplace on 2D with one boundary condition at ∞	Did not solve	Solved
T1, 8.2.1 (d) Heat 1D with source, nonhomogeneous BC	Did not solve	Solved

2 Problem 2.3.3, Richard Haberman applied partial differential equations, 5th edition, (heat PDE, 1D)

> **2.3.3.** Consider the heat equation
>
> $$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2},$$
>
> subject to the boundary conditions
>
> $$u(0,t) = 0 \quad \text{and} \quad u(L,t) = 0.$$
>
> Solve the initial value problem if the temperature is initially
>
> (a) $u(x,0) = 6\sin\frac{9\pi x}{L}$ (b) $u(x,0) = 3\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}$
>
> * (c) $u(x,0) = 2\cos\frac{3\pi x}{L}$ (d) $u(x,0) = \begin{cases} 1 & 0 < x \le L/2 \\ 2 & L/2 < x < L \end{cases}$

2.1 part a

2.1.1 Maple

```
restart;
assume(L>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0:
ic:=u(x,0)=6*sin(9*Pi*x/L):
sol:=pdsolve([pde,bc,ic],u(x,t));
```

$$u(x,t) = 6\sin\left(9\frac{\pi x}{L}\right)e^{-81\frac{k\pi^2 t}{L^2}}$$

comment Solution is correct.

2.1.2 Mathematica

```
ClearAll[u,t,k,x,L];
pde=D[u[x,t],t]==k D[u[x,t],{x,2}];
bc={u[0,t]==0,u[L,t]==0}
ic=u[x,0]==6 Sin[9 Pi x/L]
NumericQ[L] = True; (*must do this, else it will not work*)
sol=DSolve[{pde,bc,ic},u[x,t],{x,t}]
```

$$u(x,t) \to 6e^{-\frac{81\pi^2 kt}{L^2}}\sin\left(\frac{9\pi x}{L}\right)$$

comment Solution is correct.

2.2 part b

2.2.1 Maple

```
restart;
assume(L>0):
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0:
ic:=u(x,0)=3*sin(Pi*x/L)-sin(3*Pi*x/L):
sol:=pdsolve([pde,bc,ic],u(x,t));
```

$$u(x,t) = \sin\left(\frac{\pi x}{L}\right)e^{-9\frac{k\pi^2 t}{L^2}}\left(-2\cos\left(2\frac{\pi x}{L}\right) + 3e^{8\frac{k\pi^2 t}{L^2}} - 1\right)$$

comment Verified OK against my analytical solution.

2.2.2 Mathematica

```
learAll[u,t,k,x,L];
pde=D[u[x,t],t]==k D[u[x,t],{x,2}];
bc={u[0,t]==0,u[L,t]==0}
ic=u[x,0]==3 Sin[Pi x/L]-Sin[3 Pi x/L];
NumericQ[L]=True; (*must do this*)
sol=DSolve[{pde,bc,ic},u[x,t],{x,t}]
```

$$u(x,t) \to e^{-\frac{9\pi^2 kt}{L^2}} \sin\left(\frac{\pi x}{L}\right)\left(3e^{\frac{8\pi^2 kt}{L^2}} - 2\cos\left(\frac{2\pi x}{L}\right) - 1\right)$$

comment Ok

2.3 part c

2.3.1 Maple

```
restart;
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0:
ic:=u(x,0)=2*cos(3*Pi*x/L):
sol:=pdsolve([pde,bc,ic],u(x,t)) assuming 0<L:
sol1:=algsubs(_Z1=n,sol);
latex(sol1,output=string);
```

$$u(x,t) = \sum_{n=1}^{\infty} 4\,\frac{\left((-1)^n + 1\right)n}{\pi\,(n^2-9)} \sin\left(\frac{\pi\,xn}{L}\right) e^{-\frac{\pi^2 ktn^2}{L^2}}$$

comment I did not solve this by hand. Assuming solution is correct.

2.3.2 Mathematica

```
ClearAll[u,t,k,x,L];
pde=D[u[x,t],t]==k D[u[x,t],{x,2}];
bc={u[0,t]==0,u[L,t]==0}
ic=u[x,0]==2*Cos[3 Pi x/L];
sol=DSolve[{pde,bc,ic},u[x,t],{x,t},Assumptions->L>0]
sol=sol/.K[1]->n
```

$$\left\{\left\{u(x,t) \to \sum_{n=1}^{\infty} \frac{4\left(1+(-1)^n\right)e^{-\frac{kn^2\pi^2 t}{L^2}} n \sin\left(\frac{n\pi x}{L}\right)}{(n^2-9)\,\pi}\right\}\right\}$$

comment Solved, and gives same solution as Maple, assuming correct solution.

2.4 part d

2.4.1 Maple

```
restart;
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0:
assume(L>0):
ic:=u(x,0)=piecewise(0<x and x<=L/2,1,L/2<x and x<L,2):
#need to convert below, else it will not work
ic:=convert(ic,piecewise,x):
sol:=pdsolve([pde,bc,ic],u(x,t)) assuming 0<L:
sol1:=subs(_Z1=n,sol);
latex(sol1,output=string);
```

$$u(x,t) = \sum_{n=1}^{\infty} \frac{2\cos(1/2\,\pi\,n) + 2 + 4\,(-1)^{1+n}}{\pi\,n} \sin\left(\frac{\pi\,nx}{L}\right) e^{-\frac{k\pi^2 n^2 t}{L^2}}$$

comment Verified correct solution, compared to my hand solution.

2.4.2 Mathematica

```
ClearAll[u, t, k, x, L];
pde = D[u[x, t], t] == k D[u[x, t], {x, 2}];
bc = {u[0, t] == 0, u[L, t] == 0}
ic = u[x, 0] == Piecewise[{{1, 0 < x <= L/2}, {2, L/2 < x < L}}];
sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> L > 0]
sol = sol /. K[1] -> n
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}\frac{4e^{-\frac{kn^2\pi^2t}{L^2}}\left(4\cos\left(\frac{n\pi}{2}\right)+3\right)\sin^2\left(\frac{n\pi}{4}\right)\sin\left(\frac{n\pi x}{L}\right)}{n\pi}\right\}\right\}$$

comment Solved, form is little different from Maple and my solution, but verified to be same by numerically evaluating at random times and locations.

3 Problem 2.3.7, Richard Haberman applied partial differential equations, 5th edition (heat PDE, 1D)

2.3.7. Consider the following boundary value problem (if necessary, see Sec. 2.4.1):

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2} \quad \text{with} \quad \frac{\partial u}{\partial x}(0,t)=0,\ \frac{\partial u}{\partial x}(L,t)=0, \quad \text{and} \quad u(x,0)=f(x).$$

(a) Give a one-sentence physical interpretation of this problem.

(b) Solve by the method of separation of variables. First show that there are no separated solutions which exponentially grow in time. [*Hint:* The answer is

$$u(x,t)=A_0+\sum_{n=1}^{\infty}A_n e^{-\lambda_n kt}\cos\frac{n\pi x}{L}.\Bigg]$$

What is λ_n?

3.1 Maple

```
restart;
assume(L>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0:
ic:=u(x,0)=f(x):
sol:=pdsolve([pde,bc,ic],u(x,t)):
sol1:=subs(_Z2=n,sol);
```

$$u\left(x,t\right)=\sum_{n=1}^{\infty}\left(2\,\frac{1}{L}\int_0^L f\left(x\right)\cos\left(\frac{n\pi\,x}{L}\right)\,\mathrm{d}x\cos\left(\frac{n\pi\,x}{L}\right)\mathrm{e}^{-\frac{k\pi^2n^2t}{L^2}}\right)$$

comment Answer is partially correct. This is a problem I found in Maple's solutions when both ends of the heat PDE are insulated. Maple overlooks the zero eigenvalue and seems to assume eigenvalues are all greater than zero. See my analytical solution. As the book also shows in the answer it gives.

3.2 Mathematica

```
ClearAll[u, t, k, x, L];
pde = D[u[x, t], t] == k D[u[x, t], {x, 2}];
bc = { Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0}
ic = u[x, 0] == f[x];
sol = DSolve[{pde, bc, ic}, u[x, t], {x, t},
   Assumptions -> {L > 0, k > 0, t > 0}];
sol = sol /. {K[1] -> n, K[2] -> x}
```

$$u(x,t)\to\frac{\text{Integrate}[f(x),\{x,0,L\}]}{L}+\frac{2}{L}\sum_{n=1}^{\infty}e^{-\frac{kn^2\pi^2t}{L^2}}\cos\left(\frac{n\pi x}{L}\right)\text{Integrate}\left[\cos\left(\frac{n\pi x}{L}\right)f(x),\{x,0,L\}\right]$$

comment Solved. Mathematica did better here than Maple as it accounted for the zero eigenvalue.

4 Problem 2.3.8, Richard Haberman applied partial differential equations, 5th edition (heat PDE, 1D)

***2.3.8.** Consider

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} - \alpha u.$$

This corresponds to a one-dimensional rod either with heat loss through the lateral sides with outside temperature 0° ($\alpha > 0$, see Exercise 1.2.4) or with insulated lateral sides with a heat sink proportional to the temperature. Suppose that the boundary conditions are

$$u(0,t) = 0 \quad \text{and} \quad u(L,t) = 0.$$

(a) What are the possible equilibrium temperature distributions if $\alpha > 0$?

(b) Solve the time-dependent problem $[u(x,0) = f(x)]$ if $\alpha > 0$. Analyze the temperature for large time ($t \to \infty$) and compare to part (a).

4.1 Maple

```
restart;
assume(alpha>0):
assume(L>0):
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2)-alpha*u(x,t);
bc:=u(0,t)=0,u(L,t)=0:
ic:=u(x,0)=f(x):
sol:=pdsolve([pde,bc,ic],u(x,t)):
sol1:=subs(_Z1=n,sol);
```

$$u(x,t) = \sum_{n=1}^{\infty}\left(2\frac{1}{L}\left[\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)\mathrm{d}x\right]\sin\left(\frac{n\pi x}{L}\right)\mathrm{e}^{-\frac{t\left(\pi^2 k n^2 + L^2 alpha\right)}{L^2}}\right)$$

comment Ok.

4.2 Mathematica

```
ClearAll[u, t, k, x, L, a];
pde = D[u[x, t], t] == k D[u[x, t], {x, 2}] - a u[x, t];
bc = { u[0, t] == 0, u[L, t] == 0}
ic = u[x, 0] == f[x];
NumericQ[L] = True;
sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}]
```

comment Did not solve. Input returned.

5 Problem 2.4.1, Richard Haberman applied partial differential equations, 5th edition (heat PDE, 1D)

***2.4.1.** Solve the heat equation $\partial u/\partial t = k\partial^2 u/\partial x^2$, $0 < x < L$, $t > 0$, subject to

$$\frac{\partial u}{\partial x}(0,t) = 0 \qquad t > 0$$

$$\frac{\partial u}{\partial x}(L,t) = 0 \qquad t > 0.$$

(a) $u(x,0) = \begin{cases} 0 & x < L/2 \\ 1 & x > L/2 \end{cases}$

(b) $u(x,0) = 6 + 4\cos\frac{3\pi x}{L}$

(c) $u(x,0) = -2\sin\frac{\pi x}{L}$

(d) $u(x,0) = -3\cos\frac{8\pi x}{L}$

5.1 part a

5.1.1 Maple

```
restart;
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0:
assume(L>0):
ic:=u(x,0)=piecewise(0<x and x<=L/2,0,L/2<x and x<L,1):
#ask why I need to convert below, else it will not work
ic:=convert(ic,piecewise,x):
sol:=pdsolve([pde,bc,ic],u(x,t)):
sol1:=subs(_Z2=n,sol);
```

$$u(x,t) = \sum_{n=1}^{\infty} -2\,\frac{\sin(1/2\,\pi\,n)}{\pi\,n}\cos\left(\frac{\pi\,nx}{L}\right)e^{-\frac{k\pi^2n^2t}{L^2}}$$

comment Partially correct. Same problem as mentioned earlier. Maple overlooks the zero eigenvalue. The correct answer is

$$u(x,t) = \frac{1}{2} + \sum_{n=1}^{\infty} -2\,\frac{\sin(1/2\,\pi\,n)}{\pi\,n}\cos\left(\frac{\pi\,nx}{L}\right)e^{-\frac{k\pi^2n^2t}{L^2}}$$

5.1.2 Mathematica

```
ClearAll[u, t, k, x, L];
pde = D[u[x, t], t] == k D[u[x, t], {x, 2}];
bc = { Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0}
ic = u[x, 0] == Piecewise[{{0, x < L/2}, {1, x > L/2}}];
sol = DSolve[{pde, bc, ic}, u[x, t], {x, t},
   Assumptions -> {L > 0 && k > 0}];
sol = sol /. {K[1] -> n}
```

$$\left\{\left\{u(x,t)\to \frac{2\sum_{n=1}^{\infty} -\frac{e^{-\frac{kn^2\pi^2t}{L^2}}L\cos\left(\frac{n\pi x}{L}\right)\sin\left(\frac{n\pi}{2}\right)}{n\pi}}{L}+\frac{1}{2}\right\}\right\}$$

comment Mathematica gives the correct answer. It accounts for the zero eigenvalue.

5.2 part b

5.2.1 Maple

```
restart;
assume(L>0 and k>0):
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0:
ic:=u(x,0)=6+4*cos(3*Pi*x/L);
sol:=pdsolve([pde,bc,ic],u(x,t));
```

comment Did not solve. Returned ().

5.2.2 Mathematica

```
learAll[u,t,k,x,L];
pde=D[u[x,t],t]==k D[u[x,t],{x,2}];
bc={ Derivative[1, 0][u][0, t] == 0,Derivative[1, 0][u][L, t] == 0}
ic=u[x,0]==6+4*Cos[3 Pi x/L];
NumericQ[L]=True;
sol=DSolve[{pde,bc,ic},u[x,t],{x,t}]
sol=sol/.{K[1]->n}
```

$$u(x,t)\to 4e^{-\frac{9\pi^2kt}{L^2}}\cos\left(\frac{3\pi x}{L}\right)+6$$

comment Ok
The above is verified using Maple

```
correct_solution:=u(x,t)=6+4*cos(3*Pi*x/L)*exp(-(3*Pi/L)^2*k*t);
pdetest(correct_solution,pde)

        0
```

5.3 part c

5.3.1 Maple

```
restart;
assume(L>0 and k>0):
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0:
ic:=u(x,0)=-2*sin(Pi*x/L);
sol:=pdsolve([pde,bc,ic],u(x,t)):
sol1:=subs(_Z2=n,sol);
```

$$u(x,t) = \sum_{n=1}^{\infty} 4\,\frac{(-1)^n + 1}{\pi\,(n^2-1)} \cos\left(\frac{n\pi\,x}{L}\right) \mathrm{e}^{-\frac{k\,\pi^2 n^2 t}{L^2}}$$

comment Partially correct. Overlooked the zero eigenvalue.

5.3.2 Mathematica

```
ClearAll[u, t, k, x, L];
pde = D[u[x, t], t] == k D[u[x, t], {x, 2}];
bc = { Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0}
ic = u[x, 0] == -2 Sin[Pi x/L];
sol = DSolve[{pde, bc, ic}, u[x, t], {x, t},
  Assumptions -> {L > 0 && k > 0}]
sol = sol /. {K[1] -> n}
TeXForm[sol]
```

$$\left\{\left\{u(x,t)\to \frac{2\sum_{n=1}^{\infty}\frac{2(1+(-1)^n)e^{-\frac{kn^2\pi^2 t}{L^2}}L\cos\left(\frac{n\pi x}{L}\right)}{(n^2-1)\pi}}{L}-\frac{4}{\pi}\right\}\right\}$$

comment Correct solution. Mathematica accounted for the zero eigenvalue.

5.4 part d

5.4.1 Maple

```
restart;
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0:
assume(L>0 and k>0):
ic:=u(x,0)=-3*cos(8*Pi*x/L);
sol:=pdsolve([pde,bc,ic],u(x,t));
```

comment Did not solve. Returned ().

5.4.2 Mathematica

```
ClearAll[u,t,k,x,L];
pde=D[u[x,t],t]==k D[u[x,t],{x,2}];
bc={ Derivative[1, 0][u][0, t] == 0,Derivative[1, 0][u][L, t] == 0}
ic=u[x,0]==-3 Cos[8 Pi x/L];
NumericQ[L]=True;
sol=DSolve[{pde,bc,ic},u[x,t],{x,t}]
sol=sol/.{K[1]->n}
```

$$u(x,t)\to -3e^{-\frac{64\pi^2 kt}{L^2}}\cos\left(\frac{8\pi x}{L}\right)$$

comment Ok
Verified by Maple:

```
correct_solution:=u(x,t)=-3*cos(8*Pi*x/L)*exp(-(8*Pi/L)^2*k*t);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
pdetest(correct_solution,pde)

         0
```

6 Problem 2.4.2, Richard Haberman applied partial differential equations, 5th edition (heat PDE, 1D)

***2.4.2. Solve**

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \quad \text{with} \quad \frac{\partial u}{\partial x}(0,t) = 0$$
$$u(L,t) = 0$$
$$u(x,0) = f(x).$$

For this problem you may assume that no solutions of the heat equation exponentially grow in time. You may also guess appropriate orthogonality conditions for the eigenfunctions.

6.1 Maple

```
restart;
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,u(L,t)=0:
assume(L>0):
ic:=u(x,0)=f(x):
sol:=pdsolve([pde,bc,ic],u(x,t)):
sol1:=subs(_Z2=n,sol);
```

$$u(x,t) = \sum_{n=1}^{\infty}\left(2\,\frac{1}{L}\mathrm{e}^{-1/4\,\frac{k\,\pi^2(1+2\,n)^2 t}{L^2}}\cos\left(1/2\,\frac{\pi\,(1+2\,n)\,x}{L}\right)\int_0^L f(x)\cos\left(1/2\,\frac{\pi\,(1+2\,n)\,x}{L}\right)\mathrm{d}x\right)$$

comment This seems to be wrong solution. It should be $2n-1$ and not $2n+1$. Otherwise, the sum should start from zero. See Mathematica solution below which also matches my hand solution.

6.2 Mathematica

```
ClearAll[u, t, k, x, L];
pde = D[u[x, t], t] == k D[u[x, t], {x, 2}];
bc = { Derivative[1, 0][u][0, t] == 0, u[L, t] == 0}
ic = u[x, 0] == f[x];
sol = DSolve[{pde, bc, ic}, u[x, t], {x, t},
  Assumptions -> {L > 0 && k > 0}]
sol = sol /. {K[1] -> n, K[2] -> x}
```

$$u(x,t) \to \frac{2}{L}\sum_{n=0}^{\infty} e^{-\frac{k(2n+1)^2\pi^2 t}{4L^2}}\cos\left(\frac{(2n+1)\pi x}{2L}\right)\text{Integrate}\left[\cos\left(\frac{(2n+1)\pi x}{2L}\right)f(x),\{x,0,L\}\right]$$

comment Correct solution.

7 Problem 2.5.1, Richard Haberman applied partial differential equations, 5th edition (Laplace PDE, 2D)

2.5.1. Solve Laplace's equation inside a rectangle $0 \le x \le L$, $0 \le y \le H$, with the following boundary conditions:

*(a)	$\frac{\partial u}{\partial x}(0,y)=0,$	$\frac{\partial u}{\partial x}(L,y)=0,$	$u(x,0)=0,$	$u(x,H)=f(x)$
(b)	$\frac{\partial u}{\partial x}(0,y)=g(y),$	$\frac{\partial u}{\partial x}(L,y)=0,$	$u(x,0)=0,$	$u(x,H)=0$
*(c)	$\frac{\partial u}{\partial x}(0,y)=0,$	$u(L,y)=g(y),$	$u(x,0)=0,$	$u(x,H)=0$
(d)	$u(0,y)=g(y),$	$u(L,y)=0,$	$\frac{\partial u}{\partial y}(x,0)=0,$	$u(x,H)=0$
*(e)	$u(0,y)=0,$	$u(L,y)=0,$	$u(x,0)-\frac{\partial u}{\partial y}(x,0)=0,$	$u(x,H)=f(x)$

7.1 part a

7.1.1 Maple

```
restart;
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0):
bc:=D[1](u)(0,y)=0,D[1](u)(L,y)=0,u(x,0)=0,u(x,H)=f(x):
sol:=pdsolve([pde,bc],u(x,y)) assuming(0<=x and x<=L and 0<=y and y<=H);
```

comment Did not solve. Returned ().

7.1.2 Mathematica

```
ClearAll[u, t, k, x, L, H];
pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
bc = { Derivative[1, 0][u][0, y] == 0,
       Derivative[1, 0][u][L, y] == 0,
       u[x, 0] == 0,
       u[x, H] == f[x]}
sol = DSolve[{pde, bc}, u[x, y], {x, y},
  Assumptions -> {0 <= x <= L && 0 <= y <= H}]
sol = sol /. {K[1] -> n}
```

$$\frac{y\int_0^L f(x)\,dx}{HL}+\sum_{n=1}^{\infty}\frac{2\cos\left(\frac{n\pi x}{L}\right)\operatorname{csch}\left(\frac{Hn\pi}{L}\right)\left(\int_0^L\cos\left(\frac{n\pi x}{L}\right)f(x)\,dx\right)\sinh\left(\frac{n\pi y}{L}\right)}{L}$$

comment Solved Ok.

7.2 part b

7.2.1 Maple

```
restart;
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0):
bc:=D[1](u)(0,y)=g(y),D[1](u)(L,y)=0,u(x,0)=0,u(x,H)=0:
sol:=pdsolve([pde,bc],u(x,y)) assuming(0<=x and x<=L and 0<=y and y<=H);
```

comment Did not solve. Returned ().

7.2.2 Mathematica

```
ClearAll[u,t,k,x,L,H,g,f];
pde=D[u[x,y],{x,2}]+ D[u[x,y],{y,2}]==0;
bc={ Derivative[1, 0][u][0, y] == g[y],
     Derivative[1, 0][u][L, y] == 0,
     u[x,0]==0,
     u[x,H]==0};
sol=DSolve[{pde,bc},u[x,y],{x,y},Assumptions->{0<=x<=L&& 0<=y<=H}];
sol=sol/.{K[1]->n}
```

$$u(x,y)\to\sum_{n=1}^{\infty}-\frac{2\cosh\left(\frac{n\pi(L-x)}{H}\right)\operatorname{csch}\left(\frac{Ln\pi}{H}\right)\left(\int_0^H g(y)\sin\left(\frac{n\pi y}{H}\right)\,dy\right)\sin\left(\frac{n\pi y}{H}\right)}{n\pi}$$

comment Solved Ok.

7.3 part c

7.3.1 Maple

```
restart;
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0):
bc:=D[1](u)(0,y)=0,u(L,y)=g(y),u(x,0)=0,u(x,H)=0:
sol:=pdsolve([pde,bc],u(x,y)) assuming(0<=x and x<=L and 0<=y and y<=H);
```

comment Did not solve. Returned ().

7.3.2 Mathematica

```
ClearAll[u,t,k,x,L,H,g,f];
pde=D[u[x,y],{x,2}]+ D[u[x,y],{y,2}]==0;
bc={ Derivative[1, 0][u][0, y] == 0,
     u[L, y] == g[y],
     u[x,0]==0,
     u[x,H]==0};

(*these added later, but still did not help. Could not solve*)
NumericQ[L] = True;
NumericQ[H] = True;

sol=DSolve[{pde,bc},u[x,y],{x,y},Assumptions->{0<=x<=L&& 0<=y<=H}]
TeXForm[sol]
```

comment Could not solve.
The solution is

$$u(x,y)=\sum_{n=1}^{\infty}\frac{2}{H}\frac{1}{\cosh\frac{n\pi L}{H}}\left[\int_0^H g(y)\sin\frac{n\pi y}{H}\,dy\right]\cosh\frac{n\pi x}{H}\sin\frac{n\pi y}{H}$$

7.4 part d

7.4.1 Maple

```
restart;
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0):
bc:=u(0,y)=g(y),u(L,y)=0,D[2](u)(x,0)=0,u(x,H)=0:
sol:=pdsolve([pde,bc],u(x,y)) assuming(0<=x and x<=L and 0<=y and y<=H);
```

comment Did not solve. Returned ().

7.4.2 Mathematica

```
ClearAll[u, t, k, x, L, H, g, f];
pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
bc = { u[0, y] == 0,
       u[L, y] == 0,
       Derivative[0, 1][u][x, 0] == 0,
       u[x, H] == 0};

(*these added later, but still did not help. Could not solve*)
NumericQ[L] = True;
NumericQ[H] = True;

sol = DSolve[{pde, bc}, u[x, y], {x, y},
              Assumptions -> {0 <= x <= L && 0 <= y <= H}]
```

comment Could not solve.

7.5 part e

7.5.1 Maple

```
restart;
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0):
bc:=u(0,y)=0,u(L,y)=0,u(x,0)-D[2](u)(x,0)=0,u(x,H)=f(x):
sol:=pdsolve([pde,bc],u(x,y)) assuming(0<=x and x<=L and 0<=y and y<=H);
```

comment Did not solve. Returned ().

7.5.2 Mathematica

```
ClearAll[u, t, k, x, L, H, g, f];
pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
bc = { u[0, y] == 0,
       u[L, y] == 0,
       u[x, 0] - Derivative[0, 1][u][x, 0] == 0,
       u[x, H] == f[x]};

(*these added later, but still did not help. Could not solve*)
NumericQ[L] = True;
NumericQ[H] = True;

sol = DSolve[{pde, bc}, u[x, y], {x, y},
  Assumptions -> {0 <= x <= L && 0 <= y <= H}]
```

comment Could not solve.

8 Problem 2.5.5 part(c), Richard Haberman applied partial differential equations, 5th edition (heat PDE, semi-disk)

2.5.5. Solve Laplace's equation inside the quarter-circle of radius 1 ($0 \le \theta \le \pi/2$, $0 \le r \le 1$) subject to the boundary conditions

* (a) $\frac{\partial u}{\partial \theta}(r,0) = 0,\quad u\left(r,\frac{\pi}{2}\right) = 0,\quad u(1,\theta) = f(\theta)$

(b) $\frac{\partial u}{\partial \theta}(r,0) = 0,\quad \frac{\partial u}{\partial \theta}\left(r,\frac{\pi}{2}\right) = 0,\quad u(1,\theta) = f(\theta)$

* (c) $u(r,0) = 0,\quad u\left(r,\frac{\pi}{2}\right) = 0,\quad \frac{\partial u}{\partial r}(1,\theta) = f(\theta)$

(d) $\frac{\partial u}{\partial \theta}(r,0) = 0,\quad \frac{\partial u}{\partial \theta}\left(r,\frac{\pi}{2}\right) = 0,\quad \frac{\partial u}{\partial r}(1,\theta) = g(\theta)$

Show that the solution [part (d)] exists only if $\int_0^{\pi/2} g(\theta)\, d\theta = 0$. Explain this condition physically.

8.1 Maple

```
restart;
pde:=diff(u(r,theta),r$2)+1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc:=u(r,0)=0,u(r,Pi/2)=0,D[1](u)(1,theta)=f(theta);
sol:=pdsolve([pde,bc],u(r,theta)) assuming 0<=theta,theta<=Pi/2,0<=r,r<=1;
```

comment I get an error which I do not understand. Here it is

```
Error, (in assuming) when calling 'PDEtools:-casesplit'.
Received: 'the input system cannot contain equations in the
arbitrary parameters alone; found equation depending only
on f(theta): -diff(f(theta),theta)'
```

8.2 Mathematica

```
ClearAll[u,theta,r];
pde=D[u[r,theta],{r,2}]+1/r  D[u[r,theta],r]1/r^2 D[u[r,theta],{theta,2}]==0;
bc={ Derivative[1, 0][u][1, theta] == f[theta],u[r,Pi/2] == 0,u[r,0]==0};
sol=DSolve[{pde,bc},u[r,theta],{r,theta},Assumptions->{0<=r<=1&& 0<=theta<=Pi/2}]
```

comment Did not solve.

9 Problem 2.5.8 part(b), Richard Haberman applied partial differential equations, 5th edition (Laplace on circular annulus)

2.5.8. **Solve Laplace's equation inside a circular annulus ($a < r < b$) subject to the boundary conditions**

* (a) $u(a, \theta) = f(\theta), \quad u(b, \theta) = g(\theta)$

(b) $\frac{\partial u}{\partial r}(a, \theta) = 0, \quad u(b, \theta) = g(\theta)$

(c) $\frac{\partial u}{\partial r}(a, \theta) = f(\theta), \quad \frac{\partial u}{\partial r}(b, \theta) = g(\theta)$

If there is a solvability condition, state it and explain it physically.

9.1 Maple

```
restart;
pde:=diff(u(r,theta),r$2)+1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc:=D[1](u)(a,theta)=0,u(b,theta)=g(theta);
sol:=pdsolve([pde,bc],u(r,theta)) assuming a<r,r<b;
```

comment Did not solve.

9.2 Mathematica

```
ClearAll[u,theta,r];
pde=D[u[r,theta],{r,2}]+1/r  D[u[r,theta],r]1/r^2 D[u[r,theta],{theta,2}]==0;
bc={ Derivative[1, 0][u][a, theta] == 0,u[b,theta] == g[theta]};
sol=DSolve[{pde,bc},u[r,theta],{r,theta},Assumptions->a<r<=b]
```

comment Did not solve.

10 page 115, David J Logan textbook, applied PDE (Wave PDE 1D nonhomogeneous)

Falling cable lying on a table that is suddenly removed.

$$\frac{\partial^2}{\partial t^2} u(x,t) = c^2 \frac{\partial^2}{\partial x^2} u(x,t) - g$$

With initial conditions

$$u(x,0) = 0$$
$$\frac{\partial u(x,0)}{\partial t} = 0$$

And boundary condition

$$u(0,t) = 0$$

10.1 Maple

```
restart;
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)-g;
ic:=D[2](u)(x,0)=0,u(0,t)=0,u(x,0)=0:
sol:=pdsolve([pde,ic],u(x,t),HINT = boundedseries) assuming t>0,x>0,c>0;
```

$$u(x,t) = 1/2 \frac{g}{c^2} \left(Heaviside\left(t - \frac{x}{c}\right)(ct-x)^2 - c^2 t^2 \right)$$

comment Solved.

10.2 Mathematica

```
ClearAll[u,t,x,g,c];
pde=D[u[x,t],{t,2}]==c^2 D[u[x,t],{x,2}]-g;
bc=u[0,t]==0;
ic={u[x,0]==0,Derivative[0, 1][u][x, 0] == 0};
sol=DSolve[{pde,bc,ic},u[x,t],{x,t},Assumptions->{t>0&&c>0&&x>0}]
```

$$u(x,t) \to \frac{1}{2} g \left(\left(t - \frac{x}{c} \right)^2 \theta \left(t - \frac{x}{c} \right) - t^2 \right) - c_1 \delta \left(t - \frac{x}{c} \right)$$

comment Solved.
Verify Mathematica solution using Maple

```
with(MmaTranslator); #load the package
mma_solution:=FromMma(`-C[1] DiracDelta[t-x/c]+1/2 g (-t^2+(t-x/c)^2 HeavisideTheta[t-x/c])`);
pdetest(u(x,t)=mma_solution,[pde,bc,ic]) assuming t>0,x>0,c>0;

   [0,0,0,0]
```

11 page 76, David J Logan textbook, applied PDE (heat PDE 1D)

$$\frac{\partial}{\partial t} u(x,t) = \frac{\partial^2}{\partial x^2} u(x,t)$$

With initial conditions

$$u(x,0) = 0$$

And boundary condition

$$u(0,t) = f(t)$$

11.1 Maple

```
restart;
pde:=diff(u(x,t),t)=diff(u(x,t),x$2);
ic:=u(x,0)=0:
bc:=u(0,t)=f(t);
sol:=pdsolve([pde,ic,bc],u(x,t),HINT = boundedseries) assuming t>0,x>0;
```

$$u(x,t) = 1/2 \frac{x}{\sqrt{\pi}} \int_0^t \frac{f(\_U1)}{(t - \_U1)^{3/2}} \mathrm{e}^{-\frac{x^2}{4t - 4\_U1}} \, \mathrm{d}\_U1$$

comment Solved.

11.2 Mathematica

```
ClearAll[u,t,x,g,c];
pde=D[u[x,t],t]== D[u[x,t],{x,2}];
bc=u[0,t]==f[t];
ic=u[x,0]==0;
sol=DSolve[{pde,bc,ic},u[x,t],{x,t},Assumptions->{t>0,x>0}]
```

$$u(x,t) \to \frac{x \text{Integrate}\left[\frac{f(K[2]) e^{-\frac{x^2}{4(t-K[2])}}}{(t-K[2])^{3/2}}, \{K[2], 0, t\} \right]}{2\sqrt{\pi}}$$

comment Solved.

12 page 28, David J Logan textbook, applied PDE (Wave PDE 1D)

$$\frac{\partial^2}{\partial t^2} u(x,t) = c^2 \frac{\partial^2}{\partial x^2} u(x,t)$$

With boundary conditions

$$u(x,0) = 0$$
$$u(L,0) = 0$$

12.1 Maple

```
restart;
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0;
sol:=pdsolve([pde,bc],u(x,t)) assuming L>0;
sol:=subs(_Z1=n,sol);
```

$$u(x,t) = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi x}{L}\right) \left(\_C1(n) \sin\left(\frac{cn\pi t}{L}\right) + \_C5(n) \cos\left(\frac{cn\pi t}{L}\right) \right)$$

comment Solved.

12.2 Mathematica

```
ClearAll[u,t,x,L,c];
pde=D[u[x,t],{t,2}]==c^2  D[u[x,t],{x,2}]
bc={u[0,t]==0,u[L,t]==0};
sol=DSolve[{pde,bc},u[x,t],{x,t},Assumptions->{L>0}]
```

comment Could not solve.

13 page 130, David J Logan textbook, applied PDE (Wave PDE 1D)

Solve

$$\frac{\partial^2}{\partial t^2} u(x,t) = c^2 \frac{\partial^2}{\partial x^2} u(x,t)$$

With initial conditions

$$\frac{\partial u(x,0)}{\partial t} = 0$$
$$u(x,0) = f(x)$$

And boundary conditions

$$\frac{\partial u(L,t)}{\partial x} = 0$$
$$u(0,t) = 0$$

13.1 Maple

```
restart;
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc:=u(0,t)=0,D[1](u)(L,t)=0;
ic:=D[2](u)(x,0)=0,u(x,0)=f(x);
sol:=pdsolve([pde,bc,ic],u(x,t)) assuming x>=0,x<=L;
sol:=subs(_Z1=n,sol);
```

$$u(x,t) = \sum_{n=1}^{\infty} \left(2\,\frac{1}{L} \int_0^L f(x) \sin\left(1/2\,\frac{\pi\,(1+2\,n)\,x}{L}\right) dx \sin\left(1/2\,\frac{\pi\,(1+2\,n)\,x}{L}\right) \cos\left(1/2\,\frac{c\pi\,(1+2\,n)\,t}{L}\right)\right)$$

comment Solved. But sum is wrong. It should be $2n-1$ and not $2n+1$. If $2n+1$ is to be used, then the sum index should start from zero and not one.

13.2 Mathematica

```
ClearAll[u, t, x, L, c, f];
pde = D[u[x, t], {t, 2}] == c^2  D[u[x, t], {x, 2}]
bc = {u[0, t] == 0, Derivative[1, 0][u][L, t] == 0};
ic = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f[x]};
sol = DSolve[{pde, bc, ic}, u[x, t], {x, t},
  Assumptions -> {0 <= x <= L}]
```

comment Did not solve.

14 page 131, David J Logan textbook, applied PDE (heat PDE 1D, periodic BC)

Solve

$$\frac{\partial}{\partial t} u(x,t) = k \frac{\partial^2}{\partial x^2} u(x,t)$$

With initial conditions

$$u(x,0) = f(x)$$

And boundary conditions

$$u(0,t) = u(2L,t)$$
$$\frac{\partial u(0,t)}{\partial x} = \frac{\partial u(2L,t)}{\partial x}$$

14.1 Maple

```
restart;
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=u(2*L,t),D[1](u)(0,t)=D[1](u)(2*L,t):
ic:=u(x,0)=f(x):
sol:=pdsolve([pde,bc,ic],u(x,t)) assuming x>=0,x<=2*L:
sol:=subs(_Z1=n,sol);
```

$$u(x,t) = \_C8/2+\sum_{n=1}^{\infty}\left(\frac{1}{L}\left(\int_0^{2L} f(x)\cos\left(\frac{n\pi x}{L}\right)\,\mathrm{d}x\cos\left(\frac{n\pi x}{L}\right)+\int_0^{2L} f(x)\sin\left(\frac{n\pi x}{L}\right)\,\mathrm{d}x\sin\left(\frac{n\pi x}{L}\right)\right)\mathrm{e}^{-\frac{k\pi^2 n^2 t}{L^2}}\right)$$

comment Solved.

14.2 Mathematica

```
ClearAll[u, t, x, L, c, f, k];
pde = D[u[x, t], t] == k  D[u[x, t], {x, 2}]
bc = {u[0, t] == u[2 L, t],
   Derivative[1, 0][u][0, t] == Derivative[1, 0][u][2 L, t] };
ic = {u[x, 0] == f[x]};
sol = DSolve[{pde, bc, ic}, u[x, t], {x, t},
  Assumptions -> {0 <= x <= 2 L}]
```

comment Did not solve.

15 page 149, David J Logan textbook, applied PDE (wave PDE 1D, nonhomogeneous)

Solve

$$\frac{\partial^2}{\partial t^2}u(x,t) = c^2\frac{\partial^2}{\partial x^2}u(x,t) + p(x,t)$$

With initial conditions

$$u(x,0) = 0$$
$$\frac{\partial u(x,0)}{\partial t} = 0$$

And boundary conditions

$$u(0,t) = 0$$
$$u(\pi,t) = 0$$

15.1 Maple

```
restart;
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)+p(x,t);
bc:=u(0,t)=0,u(Pi,t)=0;
ic:=u(x,0)=0,D[2](u)(x,0)=0:
sol:=pdsolve([pde,bc,ic],u(x,t));
sol:=subs(_Z1=n,sol);
```

$$u(x,t) = \int_0^t \sum_{n=1}^{\infty}\left(2\,\frac{\int_0^{\pi} p(x,\tau)\sin(nx)\,\mathrm{d}x\sin(nx)\sin(cn(t-\tau))}{\pi\,nc}\right)\mathrm{d}\tau$$

comment Solved.

15.2 Mathematica

```
ClearAll[u,t,x,c];
pde=D[u[x,t],{t,2}]==c^2  D[u[x,t],{x,2}]
bc={u[0,t]==0,u[Pi,t]==0};
ic={u[x,0]==0,Derivative[0, 1][u][x, 0] == 0};
sol=DSolve[{pde,bc,ic},u[x,t],{x,t}]
```

$$\left\{\left\{u(x,t)\to\sum_{K[1]=1}^{\infty}0\right\}\right\}$$

comment Did not solve. Gives only trivial solution.

16 page 213, David J Logan textbook, 3rd ed, applied PDE (nonhomogeneous wave PDE 1D)

Solve

$$\frac{\partial^2}{\partial t^2}u(x,t) = c^2\frac{\partial^2}{\partial x^2}u(x,t) + Ax$$

With initial conditions

$$u(x,0) = 0$$
$$\frac{\partial u(x,0)}{\partial t} = 0$$

And boundary conditions

$$u(0,t) = 0$$
$$u(L,t) = 0$$

16.1 Maple

```
restart;
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)+A* x;
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=0,D[2](u)(x,0)=0:
sol:=pdsolve([pde,bc,ic],u(x,t)):
sol:=subs(_Z1=n,sol);
```

$$u(x,t) = \int_0^t \sum_{n=1}^{\infty}\left(2\frac{A}{n\pi c}\int_0^L x\sin\left(\frac{n\pi x}{L}\right)\mathrm{d}x\sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{n\pi c(t-\tau)}{L}\right)\right)\mathrm{d}\tau$$

comment Solved.

16.2 Mathematica

```
ClearAll[u,t,x,c,A,L];
pde=D[u[x,t],{t,2}]==c^2  D[u[x,t],{x,2}] + A x
bc={u[0,t]==0,u[L,t]==0};
ic={u[x,0]==0,Derivative[0, 1][u][x, 0] == 0};

NumericQ[L] = True; (*this had no effect*)
sol=DSolve[{pde,bc,ic},u[x,t],{x,t}]
```

comment Did not solve.

17 page 30, David J Logan textbook, applied PDE (Schrodinger PDE with zero potential)

Solve

$$Ih\frac{\partial}{\partial t}f(x,t) = \frac{-h^2}{2m}\frac{\partial^2 f(x,t)}{\partial x^2}$$

With boundary conditions

$$f(0,t) = 0$$
$$f(L,t) = 0$$

17.1 Maple

```
restart;
pde:=I*h*diff(f(x,t),t)=-h^2/(2*m)*diff(f(x,t),x$2);
bc:=f(0,t)=0,f(L,t)=0;
sol:=pdsolve([pde,bc],f(x,t)) assuming L>0:
sol:=subs(_Z1=n,sol);
```

$$f(x,t) = \sum_{n=1}^{\infty}\_C1(n)\sin\left(\frac{n\pi x}{L}\right)\mathrm{e}^{\frac{-i/2h\pi^2n^2t}{mL^2}}$$

comment Solved.

17.2 Mathematica

```
ClearAll[f, t, x, L, m, h];
pde = I h D[f[x, t], t] == - h^2  D[f[x, t], {x, 2}]/(2 m);
bc = {f[0, t] == 0, f[L, t] == 0};
sol = DSolve[{pde, bc}, f[x, t], {x, t}, Assumptions -> L > 0];
sol = sol /. K[1] -> n
```

$$f(x,t) \to \sum_{n=1}^{\infty} e^{-\frac{ihn^2\pi^2 t}{2L^2 m}} c_n \sin\left(\frac{n\pi x}{L}\right)$$

comment Solved.

18 Problem 18, from Maple help page (Laplace PDE, 1D)

Solve

$$\frac{\partial^2}{\partial x^2} u(x,y) + \frac{\partial^2}{\partial y^2} u(x,y) = 0$$

With

$$u(0,y) = \frac{\sin(y)}{y}$$

18.1 Maple

```
restart;
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
bc:=u(0,y)=sin(y)/y;
sol:=pdsolve([pde,bc],u(x,y));
```

$$u(x,y) = \frac{\sin(-y+ix) + \_F2(y-ix)(y-ix) + (-y+ix)\_F2(y+ix)}{-y+ix}$$

comment Solved.

18.2 Mathematica

```
ClearAll[u,t,k,x,L,H,g,f];
pde=D[u[x,y],{x,2}]+ D[u[x,y],{y,2}]==0;
bc=u[0,y]==Sin[y]/y;
sol=DSolve[{pde,bc},u[x,y],{x,y},Assumptions->{0<=x&&0<=y}]
```

$$u(x,y) \to \frac{(\sinh(x) - \cosh(x))(x\cos(y) - y\sin(y)) + x}{x^2 + y^2}$$

comment Solved.
Verify Mathematica solution using Maple

```
restart;
mma_solution:=u(x,y)=((sinh(x)-cosh(x))*(x*cos(y)-y*sin(y))+x)/(x^2+y^2);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
bc:=u(0,y)=sin(y)/y;
pdetest(mma_solution,[pde,bc]);

   [0,0]
```

19 PDE 2 from Maple PDF help, page 14 (wave PDE 1D)

Solve

$$\frac{\partial^2}{\partial t^2} u(x,t) = c^2 \frac{\partial^2}{\partial x^2} u(x,t)$$

With initial conditions

$$u(x,0) = 0$$
$$\frac{\partial u(x,0)}{\partial t} = 0$$

And boundary condition

$$u(0,t) = g(t)$$

19.1 Maple

```
restart;
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
ic:=u(x,0)=0,D[2](u)(x,0)=0:
bc:=u(0,t)=g(t);
sol:=pdsolve([pde,ic,bc],u(x,t),HINT = boundedseries) assuming t>0,x>0,c>0;
```

$$u(x,t) = \mathit{Heaviside}\left(t - \frac{x}{c}\right) g\left(\frac{ct - x}{c}\right)$$

comment Solved.

19.2 Mathematica

```
ClearAll[u,t,x,g,c];
pde=D[u[x,t],{t,2}]==c^2 D[u[x,t],{x,2}];
bc=u[0,t]==g[t];
ic={u[x,0]==0,Derivative[0, 1][u][x, 0] == 0};
sol=DSolve[{pde,bc,ic},u[x,t],{x,t},Assumptions->{t>0&&c>0&&x>0}]
```

$$u(x,t) \to \begin{cases} 0 & x > ct \\ g\left(t - \frac{x}{c}\right) & x \le ct \\ \text{Indeterminate} & \text{True} \end{cases}$$

comment Solved.

20 PDE 3 from Maple PDF help, page 14 (heat PDE 1D)

Solve

$$\frac{\partial}{\partial t} u(x,t) = k \frac{\partial^2}{\partial x^2} u(x,t)$$

With initial conditions

$$u(x,0) = 0$$

And boundary condition

$$u(0,t) = 1$$

20.1 Maple

```
restart;
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic:=u(x,0)=0:
bc:=u(0,t)=1;
sol:=pdsolve([pde,ic,bc],u(x,t),HINT = boundedseries) assuming t>0,x>0,k>0;
```

$$u(x,t) = 1 - \operatorname{erf}\left(1/2 \frac{x}{\sqrt{t}\sqrt{k}}\right)$$

comment Solved.

20.2 Mathematica

```
ClearAll[u,t,x,k];
pde=D[u[x,t],t]==k D[u[x,t],{x,2}];
bc=u[0,t]==1;
ic=u[x,0]==0;
sol=DSolve[{pde,bc,ic},u[x,t],{x,t},Assumptions->{t>0,k>0,x>0}]
```

$$u(x,t) \to \operatorname{erfc}\left(\frac{x}{2\sqrt{kt}}\right)$$

comment Solved.

21 PDE 4 from Maple PDF help, page 15 (heat PDE 1D)

Solve

$$\frac{\partial}{\partial t} u(x,t) = k \frac{\partial^2}{\partial x^2} u(x,t)$$

With initial conditions

$$u(x,0) = \mu$$

And boundary condition

$$u(0,t) = \lambda$$

21.1 Maple

```
restart;
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic:=u(x,0)=mu:
bc:=u(0,t)=lambda;
sol:=pdsolve([pde,ic,bc],u(x,t),HINT = boundedseries) assuming t>0,x>0,k>0;
```

$$u(x,t) = (-\lambda + \mu)\operatorname{erf}\left(1/2\,\frac{x}{\sqrt{t}\sqrt{k}}\right) + \lambda$$

comment Solved.

21.2 Mathematica

```
ClearAll[u, t, x, k, \[Lambda], \[Mu]];
pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
bc = u[0, t] == \[Lambda];
ic = u[x, 0] == \[Mu];
sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {t > 0, k > 0, x > 0}]
```

$$u(x,t) \to \mu\text{erf}\left(\frac{x}{2\sqrt{kt}}\right) + \lambda\text{erfc}\left(\frac{x}{2\sqrt{kt}}\right)$$

comment Solved.

22 heat absorption radiation in bounded domain

Solve

$$\frac{\partial}{\partial t}u(x,t) = k\frac{\partial^2}{\partial x^2}u(x,t)$$

With initial conditions

$$u(x,0) = f(x)$$

And boundary condition

$$\frac{\partial u(0,t)}{\partial x} + u(0,t) = 0$$
$$\frac{\partial u(L,t)}{\partial x} + u(L,t) = 0$$

22.1 Maple

```
restart;
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic:=u(x,0)=f(x);
bc:=D[1](u)(0,t)+u(0,t)=0,D[1](u)(L,t)+u(L,t)=0;
sol:=pdsolve([pde,ic,bc],u(x,t)) assuming t>0,x>=0,x<=L;
sol:=subs(_Z1=n,sol);
```

$$u(x,t) = \_C8/2 + \sum_{n=1}^{\infty}\left(2\,\frac{1}{L}\left(\int_0^L f(x)\sin\left(\frac{n\pi\,x}{L}\right)\,\mathrm{d}x\sin\left(\frac{n\pi\,x}{L}\right) + \int_0^L f(x)\cos\left(\frac{n\pi\,x}{L}\right)\,\mathrm{d}x\cos\left(\frac{n\pi\,x}{L}\right)\right)\mathrm{e}^{-\frac{k\pi^2n^2t}{L^2}}\right)$$

comment Solved.

22.2 Mathematica

```
ClearAll[u, t, x, k, L,f];
pde = D[u[x, t], t] == k D[u[x, t], {x, 2}];
bc = {Derivative[1, 0][u][0, t] +u[0,t] == 0,Derivative[1, 0][u][L, t] +u[L,t] == 0};
ic = u[x, 0] == f[x];

NumericQ[L] = True; (*adding this had no effect*)

sol = DSolve[{pde, bc, ic}, u[x, t], {x, t},    Assumptions -> {t >=  0, k > 0, x
>=  0,x<=L}]
```

comment Did not solve.

23 heat PDE, infinite domain with initial conditions, nonhomogeneous

Solve

$$\frac{\partial}{\partial t}u\left(x,t\right)=k\frac{\partial^2}{\partial x^2}u\left(x,t\right)+m$$

With initial conditions

$$u\left(x,0\right)=\sin(x)$$

23.1 Maple

```
restart;
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2)+m;
ic:=u(x,0)=sin(x);
sol:=pdsolve([pde,ic],u(x,t));
latex(sol,output=string);
```

$$u\left(x,t\right)=\sin\left(x\right){\rm e}^{-kt}+mt$$

comment Solved.

23.2 Mathematica

```
ClearAll[u, t, x,m,k];
pde = D[u[x, t], t] == k  D[u[x, t], {x, 2}]+m;
ic = u[x, 0] == Sin[x];
sol = DSolve[{pde,  ic}, u[x, t], {x, t}]
```

$$u(x,t)\to e^{-kt}\sin(x)+mt$$

comment Solved

24 PDE 23, page 23, from Maple pdsolve help PDF file

Solve

$$\frac{\partial^2}{\partial r^2}u\left(r,t\right)+\frac{1}{r}\frac{\partial}{\partial r}u\left(r,t\right)+\frac{1}{r^2}\frac{\partial^2}{\partial t^2}u\left(r,t\right)=0$$

With boundary conditions

$$u\left(1,t\right)=0$$
$$u\left(2,t\right)=5$$

24.1 Maple

```
restart;
pde:=diff(u(r,t),r$2)+1/r*diff(u(r,t),r)+1/r^2*diff(u(r,t),t$2)=0;
bc:=u(1,t)=0,u(2,t)=5;
sol:=pdsolve([pde,bc],u(r,t));
```

$$u\left(r,t\right)=5\frac{\ln\left(r\right)}{\ln\left(2\right)}$$

comment Solved.

24.2 Mathematica

```
ClearAll[u, t, r];
pde = D[u[r, t], {r,2}] + 1/r  D[u[r, t], r]+ 1/r^2 D[u[r, t], {t,2}]==0;
bc = {u[1, t] ==0,u[2, t] ==5};
sol = DSolve[{pde,  bc}, u[r, t], {r, t}]
```

$$\left\{\left\{u(r,t)\to\left\{\begin{array}{cc}\frac{5\log(r)}{\log(2)} & 1\leq r\leq 2\\ \text{Indeterminate} & \text{True}\end{array}\right.\right\}\right\}$$

comment Solved

25 heat PDE on 1D with boundary conditions from $-1 \ldots 1$

Solve

$$\frac{\partial}{\partial t}u\left(x,t\right) = \frac{\partial^2}{\partial x^2}u\left(x,t\right)$$

With boundary conditions

$$\begin{aligned} u\left(-1,t\right) &= 0 \\ u\left(1,t\right) &= 0 \end{aligned}$$

And initial conditions

$$u\left(x,0\right) = f(x)$$

25.1 Maple

```
restart;
pde := diff(u(x,t),t) =diff(u(x,t),x$2);
ic := u(x,0) = f(x);
bc := u(-1,t)=0,  u(1,t)=0;
sol:=pdsolve([pde, ic, bc],u(x,t)) assuming t>0;
```

comment Did not solve. To solve numerically do

```
restart;
pde := diff(u(x,t),t) =0.1* diff(u(x,t),x,x);
ic := u(x,0) = sin(x);
bc := u(-1,t)=0,  u(1,t)=0;
sol:=pdsolve(pde, {ic, bc},numeric,time=t,range=-1..1);
sol:-animate(t=2.5,frames=50,title="time = %f");
```

25.2 Mathematica

```
ClearAll[u,t,x,f];
pde=D[u[x,t],{t,1}]== D[u[x,t],{x,2}]
ic=u[x,0]==f[x]
bc={u[-1,t]==0,u[1,t]==0};
sol=DSolve[{pde,bc,ic},u[x,t],{x,t}]
```

comment Did not solve.

26 heat PDE on 2D with one boundary condition at ∞

added December 27, 2017.

Solve

$$\frac{\partial^2}{\partial x^2}u\left(x,y\right) + \frac{\partial^2}{\partial y^2}u\left(x,y\right) = 0$$

With boundary conditions

$$\begin{aligned} u\left(0,y\right) &= \sin\left(y\right) \\ u\left(x,0\right) &= 0 \\ u\left(x,a\right) &= 0 \\ u\left(\infty,y\right) &= 0 \end{aligned}$$

26.1 Maple

```
restart;
ode:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
bc:=u(x,0)=0, u(x,a)=0, u(0,y)=sin(y), u(infinity,y)=0;
pdsolve({ode, bc}, u(x,y)) assuming a>0;
```

$$u\left(x,y\right) = \sum_{n=1}^{\infty} 2\,\frac{\left(-1\right)^{1+n}\pi\,\sin\left(a\right)n}{\pi^2 n^2 - a^2}\sin\left(\frac{\pi\,yn}{a}\right)\mathrm{e}^{-\frac{\pi\,xn}{a}}$$

comment Solved

26.2 Mathematica

```
ClearAll[x, y, a]
ode = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
bc = {u[x, 0] == 0, u[x, a] == 0, u[0, y] == Sin[y], u[Infinity, y] == 0};
Assuming[a > 0, DSolve[{ode, bc}, u[x, y], {x, y}]]
```

comment Did not solve.

27 T1 book, 8.2.1 (d). heat PDE. heat PDE with source, nonhomogeneous BC

added January 14, 2018.

Solve

$$\frac{\partial u(x,t)}{\partial t} = k\frac{\partial^2 u(x,t)}{\partial x^2} + k$$

With boundary conditions

$$u(0,t) = A$$
$$u(x,L) = B$$

And initial conditions

$$u(x,0) = f(x)$$

27.1 Maple

```
pde :=  diff(u(x,t),t)+k*diff(u(x,t),x$2)+k;
ic  :=  u(x,0)=f(x);
bc  :=  u(0,t)=A, u(L,t)=B;

pdsolve({pde,ic,bc},u(x,t));
```

$$u(x,t) = 1/2\,\frac{1}{L}\left(2\sum_{n=1}^{\infty}\left(-\frac{1}{L^2}\int_0^L 2\left(-f(x)\,L + 1/2\,L^2x + \left(-1/2\,x^2 + A\right)L - x\,(A-B)\right)\sin\left(\frac{\pi\,nx}{L}\right)\,dx\,\sin\left(\frac{\pi\,nx}{L}\right)e^{\frac{k\pi^2n^2t}{L^2}}\right)L + L^2x + \left(-x^2 + 2\,A\right)L - 2\,x\,(A-B)\right)$$

comment Solved

27.2 Mathematica

```
ClearAll[u,x,t,k,f,A0,B0,L0];

pde   = D[u[x,t],t] == k*D[u[x,t],{x,2}]+k;
bc    = {u[0,t]     == A0,  u[L0,t] == B0 };
ic    = u[x,0]      == f[x];

DSolve[{pde,ic,bc},u[x,t],x,t]
```

comment Did not solve.