Solving partial differential equations using Computer Algebra Systems (CAS)

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Contents

This report gives the result of running a number of partial differential equations on Mathematica and Maple.

The following systems were used at this time.

1. Mathematica 11.3 (64 bit). Windows 7.

2. Maple 2017.3 (64 bit). Windows 7. (With updated Physics Version 2018, March 9)

The PC used is an Intel i7-3930k running at 3.20 GHz with 16 GB memory.

Number of problems is [57]. Mathematica solved 37 or %64.91. Maple solved 51 or %89.47

Table 1: Breakdown of results for each PDE

#	PDE	description	Mathematica result	Maple result
1	First order PDE	Linear PDE, the transport equation	Solved	Solved
2	First order PDE	Linear PDE, the transport equation with initial conditions	Solved	Solved
3	First order PDE	quasilinear first-order PDE, scalar conservation law	Solved, solution in implicit form	Solved
4	First order PDE	quasilinear first-order PDE, scalar conservation law with initial value	Solved	Solved
5	First order PDE	nonlinear first-order PDE, the Clairaut equation	Solved	Solved
6	First order PDE	nonlinear first-order PDE, the Clairaut equation with initial value	Solved	Solved
7	Heat PDE	Heat PDE on bar, homogeneous Dirichlet boundary conditions, No source.	Solved	Solved
8	Heat PDE	Heat PDE on bar, homogeneous Dirichlet boundary conditions, No source.	Solved	Solved
9	Heat PDE	Heat PDE on bar, homogeneous Dirichlet boundary conditions, No source.	Solved	Solved
10	Heat PDE	Heat PDE on bar, homogeneous Dirichlet boundary conditions, No source.	Solved	Solved
11	Heat PDE	Heat PDE on bar, homogeneous Neumann boundary conditions, No source.	Solved	Solved
12	Heat PDE	Heat PDE on bar, homogeneous Dirichlet boundary conditions with heat sink	Did not solve	Solved
13	Heat PDE	Heat PDE on bar, homogeneous Neumann boundary conditions, No source	Solved	Solved
14	Heat PDE	Heat PDE on bar, homogeneous Neumann boundary conditions, No source	Solved	Solved
15	Heat PDE	Heat PDE on bar, homogeneous Neumann boundary conditions, No source	Solved	Solved
16	Heat PDE	Heat PDE on bar, homogeneous Neumann boundary conditions, No source	Solved	Solved
17	Heat PDE	Heat PDE on bar, homogeneous Neumann on left and Dirichlet on right, No source	Solved	Solved
18	Heat PDE	Heat PDE on bar, semi-infinite domain, No source	Solved	Solved
19	Heat PDE	Heat PDE on bar, periodic boundary conditions, No source	Did not solve	Solved
20	Heat PDE	Heat PDE on bar, semi-infinite domain, zero initial condition, No source	Solved	Solved

Continued on next page

Table 1 – continued from previous page

#	PDE	description	Maple result	Mathematica result
21	Heat PDE	Heat PDE on bar, semi-infinite domain, non-zero initial condition, No source	Solved	Solved
22	Heat PDE	Heat PDE on bar, heat absorption radiation in bounded domain, No source	Did not solve	Solved
23	Heat PDE	Heat PDE infinite domain	Solved	Solved
24	Heat PDE	Heat PDE on bar, with domain from -1 to +1, no source	Did not solve	Solved
25	Heat PDE	Heat PDE on bar, with source, nonhomogeneous BC	Did not solve	Solved
26	Heat PDE	Heat PDE on bar with extra term	Did not solve	Solved
27	Heat PDE	Heat PDE on bar with initial conditions sum of sine terms, no source	Solved	Solved
28	Heat PDE	Heat PDE on bar, with initial conditions as piecewise function, no source	Solved	Solved
29	Heat PDE	Heat PDE on bar, with time dependent B.C, no source	Solved	Did not solve
30	Heat PDE	Heat PDE on bar, homogeneous Neumann boundary conditions, with source as sin function	Did not solve	Solved
31	Heat PDE	Heat PDE on bar, homogeneous Neumann boundary conditions, with source that depends on time and space	Did not solve	Solved
32	Laplace PDE	Laplace PDE inside quarter-circle	Did not solve	Did not solve
33	Laplace PDE	Laplace PDE inside semi-circle	Did not solve	Gave a solution, but extra terms in solution is not correct. Therefore counted as not solved
34	Laplace PDE	Laplace PDE inside rectangle	Solved	Solved
35	Laplace PDE	Laplace PDE inside rectangle	Solved	Solved
36	Laplace PDE	Laplace PDE inside rectangle	Did not solve	Solved
37	Laplace PDE	Laplace PDE inside rectangle	Did not solve	Solved
38	Laplace PDE	Laplace PDE inside rectangle	Did not solve	Solved
39	Laplace PDE	Laplace PDE inside rectangle, top/bottom edges non-zero	Solved	Solved
40	Laplace PDE	Laplace PDE inside circular annulus	Did not solve	Did not solve
41	Laplace PDE	Laplace PDE example 18 from Maple help page	Solved	Solved
42	Laplace PDE	Laplace PDE on rectangle with one edge at infinity	Did not solve	Solved
43	Wave PDE	Wave PDE on string with source	Solved	Solved
44	Wave PDE	Wave PDE on string, fixed ends	Did not solve	Solved
45	Wave PDE	Wave PDE on string, one fixed end, one free end	Did not solve	Solved
46	Wave PDE	Wave PDE on string, both ends fixed end, with source	Did not solve	Solved
47	Wave PDE	Wave PDE on string, both ends fixed end, with source	Did not solve	Solved
48	Wave PDE	Wave PDE on string, both ends fixed end moving boundary condition	Solved	Solved
49	Wave PDE	Wave PDE on string, both ends fixed with damping	Did not solve	Solved, But $n = 1$ should not be included.

Continued on next page

Table 1 – continued from previous page

#	PDE	description	Maple result	Mathematica result
50	Wave PDE	Wave PDE on string with fixed ends, non-zero initial position	Solved but sum should not include $n=2$	Solved, but sum should not include $n=2$
51	Schrodinger PDE	Schrodinger PDE with zero potential	Solved	Solved
52	Schrodinger PDE	Schrodinger PDE with initial and boundary conditions	Solved	Did not solve
53	Beam PDE	Beam PDE with zero initial velocity	Solved	Solved
54	Burger's PDE	Burger's PDE for viscous fluid flow with no initial conditions	Solved	Solved
55	Burger's PDE	Burger's PDE for viscous fluid flow with initial conditions	Solved	Solved, but has unresolved integrals
56	Black Scholes PDE	classic Black Scholes model from finance	Solved	Did not solve
57	Korteweg-deVries PDE (waves on shallow water surfaces)	Korteweg-deVries Equation with no initial conditions	Solved	Solved

1 First order PDE

Linear PDE, the transport equation

Problem description

problem number 1

Solve for $u(x,t)$

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$$

Mathematica

Raw commands

```
ClearAll[u, x, t];
 pde = D[u[x, t], {t}] + D[u[x, t], {x}] == 0;
 sol = DSolve[pde, u[x, t], {x, t}];
 ;
```

$$\{\{u(x,t) \to c_1(t-x)\}\}$$

Result Solved

Maple

Raw commands

```

interface(showassumed=0);
u:='u';x:='x';t:='t';
pde := diff(u(x, t), t) + diff(u(x, t),x) =0;
sol:=pdsolve(pde,u(x,t));
```

$$u(x,t) = \_F1(-x+t)$$

Result Solved

Linear PDE, the transport equation with initial conditions

Problem description

problem number 2

Solve for $u(x,t)$

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$$

With initial conditions $u(x,0) = e^{-x^2}$

Mathematica

Raw commands

```
ClearAll[u, x, t];
 ic = u[x, 0] == Exp[-x^2];
 pde = D[u[x, t], {t}] + D[u[x, t], {x}] == 0;
 sol = DSolve[{pde, ic}, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t)\to e^{-(x-t)^2}\right\}\right\}$$

Result Solved

Maple

Raw commands

```

interface(showassumed=0);
u:='u';x:='x';t:='t';
pde := diff(u(x, t), t) + diff(u(x, t),x) =0;
ic:=u(x,0)=exp(-x^2);
sol:=pdsolve({pde,ic},u(x,t));
```

$$u(x,t) = \mathrm{e}^{-(-x+t)^2}$$

Result Solved

quasilinear first-order PDE, scalar conservation law

Problem description

problem number 3

Solve for $u(x,y)$

$$\frac{\partial u}{\partial x} + u(x,y)\frac{\partial u}{\partial y} = 0$$

Mathematica

Raw commands

```
ClearAll[u, x, y];
 pde = D[u[x, y], {x}] + u[x, y]*D[u[x, y], {y}] == 0;
 sol = DSolve[pde, u[x, y], {x, y}];
 ;
```

$$\text{Solve}\left[u(x,y) = c_1\left(x - \frac{y}{u(x,y)}\right), u(x,y)\right]$$

Result Solved, solution in implicit form

Maple

Raw commands

```

interface(showassumed=0);
u:='u';x:='x';y:='y';
pde := diff(u(x, y), x) + u(x,y)*diff(u(x, y),y) =0;
sol:=pdsolve(pde,u(x,y));
sol:=DEtools:-remove_RootOf(sol);
```

$$-y + u(x,y)\,x + \_F1\,(u(x,y)) = 0$$

Result Solved

quasilinear first-order PDE, scalar conservation law with initial value

Problem description

problem number 4

Solve for $u(x,y)$

$$\frac{\partial u}{\partial x} + u(x,y)\frac{\partial u}{\partial y} = 0$$

With $u(x,0) = \frac{1}{x+1}$

Mathematica

Raw commands

```
ClearAll[u, x, y];
 pde = D[u[x, y], {x}] + u[x, y]*D[u[x, y], {y}] == 0;
 ic = u[x, 0] == 1/(x + 1);
 sol = DSolve[{pde, ic}, u[x, y], {x, y}];
 ;
```

$$\left\{\left\{u(x,y)\to \frac{y+1}{x+1}\right\}\right\}$$

Result Solved

Maple

Raw commands

```

interface(showassumed=0);
u:='u';x:='x';y:='y';
pde := diff(u(x, y), x) + u(x,y)*diff(u(x, y),y) =0;
ic:=u(x,0)=1/(x+1);
sol:=pdsolve({pde,ic},u(x,y));
```

$$u\left(x,y\right)=\frac{y+1}{x+1}$$

Result Solved

nonlinear first-order PDE, the Clairaut equation

Problem description

problem number 5

Solve for $u(x,y)$

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+\frac{1}{2}\left(\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right)=0$$

Mathematica

Raw commands

```
ClearAll[u, x, y];
 pde = u[x, y] == x*D[u[x, y], {x}] + y*D[u[x, y], {y}] + (1/2)*(D[u[x, y], {x}]^2 + D[u[x, y], {y}]^2);
 sol = DSolve[pde, u[x, y], {x, y}];
 ;
```

$$\left\{\left\{u(x,y)\to c_1x+c_2y+\frac{1}{2}\left(c_1^2+c_2^2\right)\right\}\right\}$$

Result Solved

Maple

Raw commands

```

interface(showassumed=0);
u:='u';x:='x';y:='y';
pde := x*diff(u(x, y), x) + y*diff(u(x, y),y) + 1/2 * ( diff(u(x, y), x)^2 + diff(u(x, y), y)^2)=0;
sol:=pdsolve(pde,u(x,y),'build');
```

$$u\left(x,y\right)=-1/2\,x^2-1/2\,x\sqrt{x^2+2\_c_1}-\_c_1\ln\left(x+\sqrt{x^2+2\_c_1}\right)+\_C1-1/2\,y^2-1/2\,y\sqrt{y^2-2\_c_1}+\_c_1\ln\left(y+\sqrt{y^2-2\right.$$

Result Solved

nonlinear first-order PDE, the Clairaut equation with initial value

Problem description

problem number 6

Solve for $u(x,y)$

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+\frac{1}{2}\left(\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right)=0$$

With $u(x,0)=\frac{1}{2}(1-x^2)$

Mathematica

Raw commands

```
ClearAll[u, x, y];
 pde = u[x, y] == x*D[u[x, y], {x}] + y*D[u[x, y], {y}] + (1/2)*(D[u[x, y], {x}]^2 + D[u[x, y], {y}]^2);
 ic = u[x, 0] == (1*(1 - x^2))/2;
 sol = DSolve[{pde, ic}, u[x, y], {x, y}];
 ;
```

$$\left\{\left\{u(x,y)\to\frac{1}{2}\left(-x^2-2y+1\right)\right\}\right\}$$

Result Solved

Maple

Raw commands

```
interface(showassumed=0);
u:='u';x:='x';y:='y';
pde := x*diff(u(x, y), x) + y*diff(u(x, y),y) + 1/2 * ( diff(u(x, y), x)^2 + diff(u(x, y), y)^2)=0;
ic:=u(x,0)=1/2*(1-x^2);
sol:=pdsolve({pde,ic},u(x,y));
```

$$-1/2\ (x-y+1)\,(x-y-1)=-1/2\ (x+y+1)\,(x+y-1)$$

Result Solved

2 Heat PDE

Heat PDE on bar, homogeneous Dirichlet boundary conditions, No source.

Problem description

problem number 7

This is problem 2.3.3, part (a) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$$

Subject to boundary conditions $u(0,t)=0$ and $u(L,t)=0$ with the temperature initially $u(x,0)=6\sin\left(\frac{3\pi x}{L}\right)$

Mathematica

Raw commands

```
ClearAll[u, t, k, x, L, n];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = u[x, 0] == 6*Sin[(9*Pi*x)/L];
 NumericQ[L] = True;
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t)\to 6e^{-\frac{81\pi^2kt}{L^2}}\sin\left(\frac{9\pi x}{L}\right)\right\}\right\}$$

Result Solved

Maple

Raw commands

```
interface(showassumed=0);
assume(L>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=6*sin(9*Pi*x/L);
sol:=pdsolve([pde,bc,ic],u(x,t));
```

$$u\left(x,t\right)=6\ \sin\left(9\,\frac{\pi\,x}{L}\right)\mathrm{e}^{-81\,\frac{k\pi^2t}{L^2}}$$

Result Solved

Heat PDE on bar, homogeneous Dirichlet boundary conditions, No source.

Problem description

problem number 8

This is problem 2.3.3, part (b) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

Subject to boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$ with the temperature initially $u(x,0) = 3\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, n];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = u[x, 0] == 3*Sin[(Pi*x)/L] - Sin[(3*Pi*x)/L];
 NumericQ[L] = True;
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t) \to e^{-\frac{9\pi^2 kt}{L^2}} \sin\left(\frac{\pi x}{L}\right)\left(3e^{\frac{8\pi^2 kt}{L^2}} - 2\cos\left(\frac{2\pi x}{L}\right) - 1\right)\right\}\right\}$$

Result Solved

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
assume(L>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=3*sin(Pi*x/L)-sin(3*Pi*x/L);
sol:=pdsolve([pde,bc,ic],u(x,t));
```

$$u(x,t) = 3\sin\left(\frac{\pi x}{L}\right)\mathrm{e}^{-\frac{k\pi^2 t}{L^2}} - \sin\left(3\frac{\pi x}{L}\right)\mathrm{e}^{-9\frac{k\pi^2 t}{L^2}}$$

Result Solved

Heat PDE on bar, homogeneous Dirichlet boundary conditions, No source.

Problem description

problem number 9

This is problem 2.3.3, part (c) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

Subject to boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$ with the temperature initially $u(x,0) = 2\cos\frac{3\pi x}{L}$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, L, n];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = u[x, 0] == 2*Cos[(3*Pi*x)/L];
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> L > 0];
 sol = sol /. K[1] -> n;
 ;
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}\frac{4\left(1+(-1)^n\right)e^{-\frac{kn^2\pi^2t}{L^2}}n\sin\left(\frac{n\pi x}{L}\right)}{\left(n^2-9\right)\pi}\right\}\right\}$$

Result Solved

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
assume(L>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=2*cos(3*Pi*x/L);
sol:=pdsolve([pde,bc,ic],u(x,t));
sol:=algsubs(_Z1=n,sol);
```

$$u\left(x,t\right)=\sum_{n=1}^{\infty}4\,\frac{n\left(\left(-1\right)^n+1\right)}{\pi\,\left(n^2-9\right)}\sin\left(\frac{n\pi\,x}{L}\right)\mathrm{e}^{-\frac{\pi^2kn^2t}{L^2}}$$

Result Solved

Heat PDE on bar, homogeneous Dirichlet boundary conditions, No source.

Problem description

problem number 10

This is problem 2.3.3, part (d) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$\frac{\partial u}{\partial t}=k\frac{\partial^2u}{\partial x^2}$$

Subject to boundary conditions $u(0,t)=0$ and $u(L,t)=0$ with the temperature initially $u\left(x,0\right)=\begin{cases}1 & 0<x\leq\frac{L}{2}\\ 2 & \frac{L}{2}<x\leq L\end{cases}$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, L, n];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = u[x, 0] == Piecewise[{{1, Inequality[0, Less, x, LessEqual, L/2]}, {2, L/2 < x < L}}];
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> L > 0];
 sol = sol /. K[1] -> n;
 ;
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}\frac{4e^{-\frac{kn^2\pi^2t}{L^2}}\left(4\cos\left(\frac{n\pi}{2}\right)+3\right)\sin^2\left(\frac{n\pi}{4}\right)\sin\left(\frac{n\pi x}{L}\right)}{n\pi}\right\}\right\}$$

Result Solved

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
assume(L>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=piecewise(0<x and x<=L/2,1,L/2<x and x<L,2);
#need to convert below, else it will not work;
ic:=convert(ic,piecewise,x);
sol:=pdsolve([pde,bc,ic],u(x,t)) assuming 0<L;
sol:=subs(_Z1=n,sol);
```

$$u(x,t) = \sum_{n=1}^{\infty} \frac{2\cos(1/2\,\pi\, n) + 2 + 4\,(-1)^{1+n}}{\pi\, n} \sin\left(\frac{\pi\, nx}{L}\right) e^{-\frac{\pi^2 k n^2 t}{L^2}}$$

Result Solved

Heat PDE on bar, homogeneous Neumann boundary conditions, No source.

Problem description

problem number 11

This is problem 2.3.7, from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

Subject to boundary conditions $\frac{\partial u}{\partial x}(0,t) = 0$ $\frac{\partial u}{\partial x}(L,t) = 0$ with the temperature initially $u(x,0) = f(x)$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, L, sol, n, f];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == f[x];
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {L > 0, k > 0, t > 0}];
 sol = sol /. {K[1] -> n, K[2] -> x};
 ;
```

$$\left\{\left\{u(x,t) \to \frac{2\sum_{n=1}^{\infty} e^{-\frac{kn^2\pi^2 t}{L^2}} \cos\left(\frac{n\pi x}{L}\right) \int_0^L \cos\left(\frac{n\pi x}{L}\right) f(x)\,dx}{L} + \frac{\int_0^L f(x)\,dx}{L}\right\}\right\}$$

Result Solved

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';
interface(showassumed=0);
assume(L>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic:=u(x,0)=f(x);
sol:=pdsolve([pde,bc,ic],u(x,t));
sol:=subs(_Z2=n,sol);
```

$$u(x,t) = \frac{1}{L}\left(\sum_{n=1}^{\infty}\left(2\,\frac{1}{L}\int_0^L f(x)\cos\left(\frac{\pi\, nx}{L}\right)\,dx \cos\left(\frac{\pi\, nx}{L}\right) e^{-\frac{\pi^2 k n^2 t}{L^2}}\right) L + \int_0^L f(x)\,dx\right)$$

Result Solved

Heat PDE on bar, homogeneous Dirichlet boundary conditions with heat sink

Problem description

problem number 12

This is problem 2.3.8, from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} - \alpha u$$

This corresponds to a one-dimentional rod either with heat loss through the lateral sides with outside temperature zero degrees ($\alpha > 0$) or with insulated sides with a heat sink proportional to the temperature.

Suppose the boundary conditions are $u(0,t) = 0, u(L,t) = 0$, solve with the temperature initially $u(x,0) = f(x)$ if $\alpha > 0$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, L, a, f];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] - a*u[x, t];
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = u[x, 0] == f[x];
 NumericQ[L] = True;
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 ;
```

DSolve $\left[\left\{u^{(0,1)}(x,t)=ku^{(2,0)}(x,t)-au(x,t),\{u(0,t)=0,u(L,t)=0\},u(x,0)=f(x)\right\},u(x,t),\{x,t\}\right]$

Result Did not solve

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x'; a:='a';f:='f';
interface(showassumed=0);
assume(a>0);
assume(L>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2)-a*u(x,t);
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=f(x);
sol:=pdsolve([pde,bc,ic],u(x,t));
sol:=subs(_Z1=n,sol);
```

$$u(x,t)=\sum_{n=1}^{\infty}\left(2\,\frac{1}{L}\int_0^L f(x)\sin\left(\frac{\pi\,nx}{L}\right)\,\mathrm{d}x\sin\left(\frac{\pi\,nx}{L}\right)\mathrm{e}^{-\frac{t\left(\pi^2kn^2+L^2a\right)}{L^2}}\right)$$

Result Solved

Heat PDE on bar, homogeneous Neumann boundary conditions, No source

Problem description

problem number 13

This is problem 2.4.1 part(a) from Richard Haberman applied partial differential equations, 5th edition.

Consider the heat equation

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$$

The boundary conditions are $\frac{\partial u}{\partial x}(0,t) = 0$ $\frac{\partial u}{\partial x}(L,t) = 0$ with the temperature initially $u(x,0)=\begin{cases}0 & x<\frac{L}{2}\\ 1 & x>\frac{L}{2}\end{cases}$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, L];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == Piecewise[{{0, x < L/2}, {1, x > L/2}}];
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {L > 0 && k > 0}];
 sol = sol /. {K[1] -> n};
 ;
```

$$\left\{\left\{u(x,t)\to \frac{2\sum_{n=1}^{\infty} -\frac{e^{-\frac{kn^2\pi^2t}{L^2}} L\cos\left(\frac{n\pi x}{L}\right)\sin\left(\frac{n\pi}{2}\right)}{n\pi}}{L}+\frac{1}{2}\right\}\right\}$$

Result Solved

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
assume(L>0);
ic:=u(x,0)=piecewise(0<x and x<=L/2,0,L/2<x and x<L,1);
sol:=pdsolve([pde,bc,ic],u(x,t));
sol:=subs(_Z2=n,sol);
sol:=simplify(rhs(sol));
```

$$1/2+\sum_{n=1}^{\infty} -2\,\frac{\sin(1/2\,\pi\,n)}{\pi\,n}\cos\left(\frac{\pi\,nx}{L}\right)\mathrm{e}^{-\frac{\pi^2kn^2t}{L^2}}$$

Result Solved

Heat PDE on bar, homogeneous Neumann boundary conditions, No source

Problem description

problem number 14

This is problem 2.4.1 part(b) from Richard Haberman applied partial differential equations, 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$$

The boundary conditions are $\frac{\partial u}{\partial x}(0,t)=0$ $\frac{\partial u}{\partial x}(L,t)=0$ with the temperature initially $u(x,0)=6+4\cos\left(\frac{3\pi x}{L}\right)$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, L];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == 6 + 4*Cos[(3*Pi*x)/L];
 NumericQ[L] = True;
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 sol = sol /. {K[1] -> n};
 ;
```

$$\left\{\left\{u(x,t)\to 4e^{-\frac{9\pi^2kt}{L^2}}\cos\left(\frac{3\pi x}{L}\right)+6\right\}\right\}$$

Result Solved

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
assume(L>0 and k>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic:=u(x,0)=6+4*cos(3*Pi*x/L);
sol:=pdsolve([pde,bc,ic],u(x,t));
```

$$u(x,t) = 6 + 4\cos\left(3\frac{\pi x}{L}\right)e^{-9\frac{k\pi^2 t}{L^2}}$$

Result Solved

Heat PDE on bar, homogeneous Neumann boundary conditions, No source

Problem description

problem number 15

This is problem 2.4.1 part(c) from Richard Haberman applied partial differential equations, 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

The boundary conditions are $\frac{\partial u}{\partial x}(0,t) = 0$ $\frac{\partial u}{\partial x}(L,t) = 0$ with the temperature initially $u(x,0) = -2\sin\frac{\pi x}{L}$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, L];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == -2*Sin[(Pi*x)/L];
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {L > 0 && k > 0}];
 sol = sol /. {K[1] -> n};
 ;
```

$$\left\{\left\{u(x,t)\to\frac{2\sum_{n=1}^{\infty}\frac{2(1+(-1)^n)e^{-\frac{kn^2\pi^2 t}{L^2}}L\cos\left(\frac{n\pi x}{L}\right)}{(n^2-1)\pi}}{L}-\frac{4}{\pi}\right\}\right\}$$

Result Solved

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
assume(L>0 and k>0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
ic:=u(x,0)=-2*sin(Pi*x/L);
sol:=pdsolve([pde,bc,ic],u(x,t));
sol:=subs(_Z2=n,sol);
```

$$u(x,t) = \frac{1}{\pi}\left(\sum_{n=1}^{\infty}4\frac{(-1)^n+1}{\pi\,(n^2-1)}\cos\left(\frac{\pi nx}{L}\right)e^{-\frac{k\pi^2 n^2 t}{L^2}}\pi - 4\right)$$

Result Solved

Heat PDE on bar, homogeneous Neumann boundary conditions, No source

Problem description

problem number 16

This is problem 2.4.1 part(d) from Richard Haberman applied partial differential equations, 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

The boundary conditions are $\frac{\partial u}{\partial x}(0,t) = 0$ $\frac{\partial u}{\partial x}(L,t) = 0$ with the temperature initially $u(x,0) = -3\cos\frac{8\pi x}{L}$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, L];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == -3*Cos[(8*Pi*x)/L];
 NumericQ[L] = True;
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 sol = sol /. {K[1] -> n};
 ;
```

$$\left\{\left\{u(x,t)\to -3e^{-\frac{64\pi^2kt}{L^2}}\cos\left(\frac{8\pi x}{L}\right)\right\}\right\}$$

Result Solved

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,D[1](u)(L,t)=0;
assume(L>0 and k>0);
ic:=u(x,0)=-3*cos(8*Pi*x/L);
sol:=pdsolve([pde,bc,ic],u(x,t));
sol:=simplify(rhs(sol));
```

$$-3\cos\left(8\frac{\pi x}{L}\right)\mathrm{e}^{-64\frac{k\pi^2t}{L^2}}$$

Result Solved

Heat PDE on bar, homogeneous Neumann on left and Dirichlet on right, No source

Problem description

problem number 17

This is problem 2.4.2 from Richard Haberman applied partial differential equations, 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$$

The boundary conditions are $\frac{\partial u}{\partial x}(0,t)=0$ $u(L,t)=0$ with the temperature initially $u(x,0)=f(x)$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, t, k, x, L, f];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] == 0, u[L, t] == 0};
 ic = u[x, 0] == f[x];
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {L > 0 && k > 0}];
 sol = sol /. {K[1] -> n, K[2] -> x};
 ;
```

$$\left\{\left\{u(x,t)\to\frac{2\sum_{n=0}^{\infty}e^{-\frac{k(2n+1)^2\pi^2t}{4L^2}}\cos\left(\frac{(2n+1)\pi x}{2L}\right)\int_0^L\cos\left(\frac{(2n+1)\pi x}{2L}\right)f(x)\,dx}{L}\right\}\right\}$$

Result Solved

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=D[1](u)(0,t)=0,u(L,t)=0;
assume(L>0);
ic:=u(x,0)=f(x);
sol:=pdsolve([pde,bc,ic],u(x,t));
sol:=subs(_Z2=n,sol);
```

$$u(x,t)=\sum_{n=0}^{\infty}\left(2\,\frac{1}{L}\int_{0}^{L}f(x)\cos\left(1/2\,\frac{(1+2\,n)\,\pi\,x}{L}\right)\,dx\cos\left(1/2\,\frac{(1+2\,n)\,\pi\,x}{L}\right)e^{-1/4\,\frac{k\,\pi^2(1+2\,n)^2 t}{L^2}}\right)$$

Result Solved

Heat PDE on bar, semi-infinite domain, No source

Problem description

problem number 18

This is problem at page 76 from David J Logan text book.
Solve the heat equation

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

The boundary conditions are $u(0,t)=f(t)$ and initial conditions $u(x,0)=0$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, t, x, f];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 bc = u[0, t] == f[t];
 ic = u[x, 0] == 0;
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {t > 0, x > 0}];
 sol = sol /. {K[2] -> z};
```

$$\left\{\left\{u(x,t)\to\frac{x\int_0^t\frac{f(z)e^{-\frac{x^2}{4(t-z)}}}{(t-z)^{3/2}}\,dz}{2\sqrt{\pi}}\right\}\right\}$$

Result Solved

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,t),t)=diff(u(x,t),x$2);
ic:=u(x,0)=0;
bc:=u(0,t)=f(t);
sol:=pdsolve([pde,ic,bc],u(x,t),HINT = boundedseries) assuming t>0,x>0;
```

$$u(x,t)=1/2\,\frac{x}{\sqrt{\pi}}\int_0^t\frac{f(\_U1)}{(t-\_U1)^{3/2}}e^{-\frac{x^2}{4\,t-4\,\_U1}}\,d\_U1$$

Result Solved

Heat PDE on bar, periodic boundary conditions, No source

Problem description

problem number 19

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

For $-L < x < L$ and $t > 0$. The boundary conditions are

$$u(-L,t) = u(L,t)$$
$$\frac{\partial u}{\partial x}(-L,t) = \frac{\partial u}{\partial x}(L,t)$$

And initial conditions $u(x,0) = f(x)$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, t, x, L, c, f, k];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {u[-L, t] == u[L, t], Derivative[1, 0][u][-L, t] == Derivative[1, 0][u][L, t]};
 ic = u[x, 0] == f[x];
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {-L <= x <= L, t > 0}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,1)}(x,t) = ku^{(2,0)}(x,t), \left\{u(-L,t) = u(L,t), u^{(1,0)}(-L,t) = u^{(1,0)}(L,t)\right\}, u(x,0) = f(x)\right\}, u(x,t), \{x,t\}, \text{As}$$

Result Did not solve

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
bc:=u(-L,t)=u(L,t),D[1](u)(-L,t)=D[1](u)(L,t);
ic:=u(x,0)=f(x);
sol:=pdsolve([pde,bc,ic],u(x,t)) assuming x>=-L,x<=L;
```

$$u(x,t) = 1/2\,\frac{1}{L}\left(2\sum_{n=1}^{\infty}\left(\frac{1}{L}\left(\int_{-L}^{L} f(x)\cos\left(\frac{\pi\,nx}{L}\right)\,dx\cos\left(\frac{\pi\,nx}{L}\right) + \int_{-L}^{L} f(x)\sin\left(\frac{\pi\,nx}{L}\right)\,dx\sin\left(\frac{\pi\,nx}{L}\right)\right)e^{-\frac{k\,\pi^2 n^2 t}{L^2}}\right) L$$

Result Solved

Heat PDE on bar, semi-infinite domain, zero initial condition, No source

Problem description

problem number 20

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

For $x > 0$ and $t > 0$. The boundary conditions is $u(0,t) = 1$ and And initial condition $u(x,0) = 0$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, t, x, k];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = u[0, t] == 1;
 ic = u[x, 0] == 0;
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {t > 0, k > 0, x > 0}];
 ;
```

$$\left\{\left\{u(x,t) \to \text{erfc}\left(\frac{x}{2\sqrt{kt}}\right)\right\}\right\}$$

Result Solved

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic:=u(x,0)=0:
bc:=u(0,t)=1;
sol:=pdsolve([pde,ic,bc],u(x,t),HINT = boundedseries) assuming t>0,x>0,k>0;
```

$$u(x,t) = 1 - Erf\left(1/2\,\frac{x}{\sqrt{t}\sqrt{k}}\right)$$

Result Solved

Heat PDE on bar, semi-infinite domain, non-zero initial condition, No source

Problem description

problem number 21

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

For $x > 0$ and $t > 0$. The boundary conditions is $u(0,t) = \mu$ and And initial condition $u(x,0) = \lambda$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, t, x, k, \[Lambda], \[Mu]];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = u[0, t] == \[Lambda];
 ic = u[x, 0] == \[Mu];
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {t > 0, k > 0, x > 0}];
 ;
```

$$\left\{\left\{u(x,t)\to \mu\text{erf}\left(\frac{x}{2\sqrt{kt}}\right)+\lambda\text{erfc}\left(\frac{x}{2\sqrt{kt}}\right)\right\}\right\}$$

Result Solved

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';mu:='mu';lambda:='lambda';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic:=u(x,0)=mu:
bc:=u(0,t)=lambda;
sol:=pdsolve([pde,ic,bc],u(x,t),HINT = boundedseries) assuming t>0,x>0,k>0;
```

$$u(x,t) = (-\lambda+\mu)\,Erf\left(1/2\,\frac{x}{\sqrt{t}\sqrt{k}}\right)+\lambda$$

Result Solved

Heat PDE on bar, heat absorption radiation in bounded domain, No source

Problem description

problem number 22

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) + u(0,t) = 0$$
$$\frac{\partial u}{\partial x}(L,t) + u(L,t) = 0$$

And initial condition $u(x,0) = f(x)$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, t, x, k, L, f];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}];
 bc = {Derivative[1, 0][u][0, t] + u[0, t] == 0, Derivative[1, 0][u][L, t] + u[L, t] == 0};
 ic = u[x, 0] == f[x];
 NumericQ[L] = True;
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {t >= 0, k > 0, x >= 0, x <= L}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,1)}(x,t) = ku^{(2,0)}(x,t), \left\{u^{(1,0)}(0,t) + u(0,t) = 0, u^{(1,0)}(L,t) + u(L,t) = 0\right\}, u(x,0) = f(x)\right\}, u(x,t), \{x,t\right.$$

Result Did not solve

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';mu:='mu';lambda:='lambda';f:='f';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2);
ic:=u(x,0)=f(x);
bc:=D[1](u)(0,t)+u(0,t)=0,D[1](u)(L,t)+u(L,t)=0;
sol:=pdsolve([pde,ic,bc],u(x,t)) assuming t>0,x>=0,x<=L;
sol:=subs(_Z1=n,sol);
```

$$u(x,t) = \sum_{n=1}^{\infty} \left(2\,\frac{1}{L\,(\pi^2 n^2 + L^2)} \int_0^L f(x) \left(-\pi\, n \cos\left(\frac{\pi\, nx}{L}\right) + \sin\left(\frac{\pi\, nx}{L}\right) L\right) \mathrm{d}x \left(-\pi\, n \cos\left(\frac{\pi\, nx}{L}\right) + \sin\left(\frac{\pi\, nx}{L}\right) L\right) \mathrm{e}^{-\right.$$

Result Solved

Heat PDE infinite domain

Problem description

problem number 23

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + m$$

For $-\infty < x < \infty$ and $t > 0$. The boundary conditions are
Initial condition is $u(x,0) = \sin(x)$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, t, x, m, k];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + m;
 ic = u[x, 0] == Sin[x];
 sol = DSolve[{pde, ic}, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t) \to e^{-kt}\sin(x) + mt\right\}\right\}$$

Result Solved

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';m:='m';
interface(showassumed=0);
pde:=diff(u(x,t),t)=k*diff(u(x,t),x$2)+m;
ic:=u(x,0)=sin(x);
sol:=pdsolve([pde,ic],u(x,t));
```

$$u(x,t) = \sin(x)\,\mathrm{e}^{-tk} + mt$$

Result Solved

Heat PDE on bar, with domain from -1 to +1, no source

Problem description

problem number 24

Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

For $-1 < x < 1$ and $t > 0$. The boundary conditions are zero at both ends. Initial condition is $u(x,0) = f(x)$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, t, x, f];
 pde = D[u[x, t], {t, 1}] == D[u[x, t], {x, 2}];
 ic = u[x, 0] == f[x];
 bc = {u[-1, t] == 0, u[1, t] == 0};
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,1)}(x,t) = u^{(2,0)}(x,t), \{u(-1,t)=0, u(1,t)=0\}, u(x,0)=f(x)\right\}, u(x,t), \{x,t\}\right]$$

Result Did not solve

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';
interface(showassumed=0);
pde := diff(u(x,t),t) =diff(u(x,t),x$2);
ic := u(x,0) = f(x);
bc := u(-1,t)=0,  u(1,t)=0;
sol:=pdsolve([pde, ic, bc],u(x,t)) assuming t>0;
```

$$u(x,t) = \sum_{n=1}^{\infty} \left(\left(\left(\int_{-1}^{1} f(x) \sin(\pi n x)\, \mathrm{d}x \sin(\pi n x)\, \mathrm{e}^{1/4\,\pi^2 (2n-1)^2 t} + \int_{-1}^{1} f(x) \cos(1/2\, x\pi\, (2n-1))\, \mathrm{d}x \cos(1/2\, x\pi\, (2n -$$

Result Solved

Heat PDE on bar, with source, nonhomogeneous BC

Problem description

problem number 25

This is problem 8.2.1 par(d) from Richard Haberman applied partial differential equations 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + k$$

For $0 < x < 1$ and $t > 0$. The boundary conditions are

$$u(0,t) = A$$
$$u(x,L) = B$$

Initial condition is $u(x,0) = f(x)$

Mathematica

Raw commands

```
ClearAll[u, x, t, k, f, A0, B0, L0];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + k;
 bc = {u[0, t] == A0, u[L0, t] == B0};
 ic = u[x, 0] == f[x];
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,1)}(x,t) = ku^{(2,0)}(x,t) + k, u(x,0) = f(x), \{u(0,t) = \text{A0}, u(\text{L0},t) = \text{B0}\}\right\}, u(x,t), x, t\right]$$

Result Did not solve

Maple

Raw commands

```


L:='L'; u:='u'; t:='t'; x:='x';f:='f';
interface(showassumed=0);
pde :=  diff(u(x,t),t)+k*diff(u(x,t),x$2)+k;
ic  :=  u(x,0)=f(x);
bc  :=  u(0,t)=A, u(L,t)=B;
sol:=pdsolve({pde,ic,bc},u(x,t));
```

$$u\left(x,t\right) = 1/2\,\frac{1}{L}\left(2\sum_{n=1}^{\infty}\left(-\frac{1}{L^2}\int_0^L 2\,\sin\left(\frac{\pi\,nx}{L}\right)\left(-f\left(x\right)L + 1/2\,L^2x + \left(-1/2\,x^2 + A\right)L - x\left(A - B\right)\right)\,\mathrm{d}x\sin\left(\frac{\pi\,nx}{L}\right)\mathrm{e}$$

Result Solved

Heat PDE on bar with extra term

Problem description

problem number 26

Solve the heat equation

$$\frac{\partial u}{\partial t} + u(x,t) = k\frac{\partial^2 u}{\partial x^2}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(x,L) = 0$$

Initial condition is $u(x,0) = f(x)$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[x, t, u, f, L];
 pde = D[u[x, t], t] + u[x, t] == D[u[x, t], {x, 2}];
 ic = u[x, 0] == f[x];
 bc = {u[0, t] == 0, u[L, t] == 0};
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,1)}(x,t) + u(x,t) = u^{(2,0)}(x,t), u(x,0) = f(x), \{u(0,t) = 0, u(L,t) = 0\}\right\}, u(x,t), x, t\right]$$

Result Did not solve

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';
interface(showassumed=0);
pde:=diff(u(x,t),t)+u(x,t)=diff(u(x,t),x$2);
ic:=u(x,0)=f(x);
bc:=u(0,t)=0,u(L,t)=0;
sol:=pdsolve({pde,ic,bc},u(x,t)) assuming L>0;
```

$$u(x,t) = \sum_{n=1}^{\infty} \left(2\frac{1}{L} \int_0^L f(x) \sin\left(\frac{\pi n x}{L}\right) \mathrm{d}x \sin\left(\frac{\pi n x}{L}\right) \mathrm{e}^{-\frac{t(\pi^2 n^2 + L^2)}{L^2}} \right)$$

Result Solved

Heat PDE on bar with initial conditions sum of sine terms, no source

Problem description

problem number 27

added Feb 10, 2018.
Solve the heat equation

$$\frac{\partial u}{\partial t} + u(x,t) = 100\frac{\partial^2 u}{\partial x^2}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(x,L) = 0$$

Initial condition is $u(x,0) = \sin(2\pi x) - \sin(5\pi x)$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[x, t, u, f, L];
 f = Sin[2*Pi*x] - Sin[5*Pi*x];
 pde = D[u[x, t], t] == 100*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[1, t] == 0};
 ic = u[x, 0] == f;
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t][[1,1]];
 ;
```

$$u(x,t) \to e^{-400\pi^2 t} \sin(2\pi x) - e^{-2500\pi^2 t} \sin(5\pi x)$$

Result Solved

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,t),t)=100*diff(u(x,t),x$2);
ic:=u(x,0)=sin(2*Pi*x)-sin(5*Pi*x);
bc:=u(0,t)=0,u(1,t)=0;
sol:=pdsolve({pde,ic,bc},u(x,t));
```

$$u(x,t) = \sin(2\pi x) \mathrm{e}^{-400\pi^2 t} - \sin(5\pi x) \mathrm{e}^{-2500\pi^2 t}$$

Result Solved

Heat PDE on bar, with initial conditions as piecewise function, no source

Problem description

problem number 28

added Feb 10, 2018.

Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(x,L) = 0$$

Initial condition is
$u(x,0) = \begin{cases} x & 0 \leq x < 20 \\ 40 - x & 20 \geq x \leq 40 \end{cases}$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[x, t, u, f, L, n];
 f = Piecewise[{{x, Inequality[0, LessEqual, x, Less, 20]}, {40 - x, 20 <= x <= 40}}];
 pde = D[u[x, t], t] == D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[40, t] == 0};
 ic = u[x, 0] == f;
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t];
 sol = sol /. K[1] -> n;
 ;
```

$$\left\{\left\{u(x,t) \to \sum_{n=1}^{\infty} \frac{640 e^{-\frac{n^2 \pi^2 t}{1600}} \cos\left(\frac{n\pi}{4}\right) \sin^3\left(\frac{n\pi}{4}\right) \sin\left(\frac{n\pi x}{40}\right)}{n^2 \pi^2}\right\}\right\}$$

Result Solved

Maple

Raw commands

```


L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
f:=piecewise(0<x and x<20,x,20<x and x<40,(40-x));
pde:=diff(u(x,t),t)=diff(u(x,t),x$2);
ic:=u(x,0)=f;
bc:=u(0,t)=0,u(40,t)=0;
sol:=pdsolve({pde,ic,bc},u(x,t));
```

$$u(x,t) = \sum_{n=1}^{\infty} 160 \frac{\sin(1/2\,\pi\,n) \sin(1/40\,\pi\,nx)}{\pi^2 n^2} e^{-\frac{\pi^2 n^2 t}{1600}}$$

Result Solved

Heat PDE on bar, with time dependent B.C, no source

Problem description

problem number 29

added March 8, 2018.
Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

For $0 < x < \pi$ and $t > 0$. The boundary conditions are

$$u(0,t) = 0$$
$$u(\pi,t) = 0$$

Initial condition is $u(x,0) = 0$.

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, t, x, n];
 pde = D[u[x, t], {t, 1}] == D[u[x, t], {x, 2}];
 bc = {u[0, t] == t, u[Pi, t] == 0};
 ic = u[x, 0] == 0;
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 sol = sol /. {K[1] -> n};
 ;
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}-\frac{\left(2-2e^{-n^2t}\right)\sin(nx)}{n^3\pi}-\frac{tx}{\pi}+t\right\}\right\}$$

Result Solved

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,t),t)=diff(u(x,t),x$2);
bc:=u(0,t)=t,u(Pi,t)=0:
ic:=u(x,0)=0:
sol:=pdsolve([pde,bc,ic],u(x,t));
```

$$\text{sol}=()$$

Result Did not solve

Heat PDE on bar, homogeneous Neumann boundary conditions, with source as sin function

Problem description

problem number 30

added March 18, 2018.

This is problem 8.2.1, part(f) from Richard Haberman applied partial differential equations 5th edition.

Solve the heat equation

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}+\sin\left(\frac{2\pi x}{L}\right)$$

For $0<x<L$ and $t>0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t)=0$$
$$\frac{\partial u}{\partial x}(L,t)=0$$

Initial condition is $u(x,0)=f(x)$.

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, x, t, k, f, L];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Sin[(2*Pi*x)/L];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == f[x];
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {L > 0, k > 0, t > 0}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,1)}(x,t)=ku^{(2,0)}(x,t)+\sin\left(\frac{2\pi x}{L}\right),u(x,0)=f(x),\left\{u^{(1,0)}(0,t)=0,u^{(1,0)}(L,t)=0\right\}\right\},u(x,t),x,t,\text{Assu}$$

Result Did not solve

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';
interface(showassumed=0);
pde :=  diff(u(x,t),t)+k*diff(u(x,t),x$2)+sin(2*Pi*x/L);
ic  :=  u(x,0)=f(x);
bc  :=  D[1](u)(0,t)=0, D[1](u)(L,t)=0;
sol:=pdsolve({pde,ic,bc},u(x,t)) assuming L>0,t>0,k>0;
```

$$u(x,t) = \frac{1}{L}\left(\sum_{n=1}^{\infty}\left(2\,\frac{1}{L}\int_0^L f(\tau 1)\cos\left(\frac{\pi\, n \tau 1}{L}\right)\, \mathrm{d}\tau 1 \cos\left(\frac{\pi\, n x}{L}\right) \mathrm{e}^{\frac{k\,\pi^2 n^2 t}{L^2}}\right) L + \int_0^t \frac{1}{L}\left(\sum_{n1=1}^{\infty}\left(-2\,\frac{1}{L}\int_0^L \sin\left(2\,\frac{\pi\, x}{L}\right)\cos\left(\right.\right.\right.\right.$$

Result Solved

Heat PDE on bar, homogeneous Neumann boundary conditions, with source that depends on time and space

Problem description

problem number 31

added March 18, 2018.
Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + \left(e^{-ct}\sin\left(\frac{2\pi x}{L}\right)\right)$$

For $0 < x < L$ and $t > 0$. The boundary conditions are

$$\frac{\partial u}{\partial x}(0,t) = 0$$
$$\frac{\partial u}{\partial x}(L,t) = 0$$

Initial condition is $u(x,0) = f(x)$.

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, x, t, k, f, L, c];
 pde = D[u[x, t], t] == k*D[u[x, t], {x, 2}] + Exp[-(c*t)]*Sin[(2*Pi*x)/L];
 bc = {Derivative[1, 0][u][0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = u[x, 0] == f[x];
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t, Assumptions -> {L > 0, k > 0, t > 0}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,1)}(x,t) = ku^{(2,0)}(x,t) + e^{-ct}\sin\left(\frac{2\pi x}{L}\right), u(x,0) = f(x), \left\{u^{(1,0)}(0,t) = 0, u^{(1,0)}(L,t) = 0\right\}\right\}, u(x,t), x, t, \right.$$

Result Did not solve

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';c:='c';
interface(showassumed=0);
pde :=  diff(u(x,t),t)+k*diff(u(x,t),x$2)+(exp(-c*t)*sin(2*Pi*x/L));
ic  :=  u(x,0)=f(x);
bc  :=  D[1](u)(0,t)=0, D[1](u)(L,t)=0;
sol:=pdsolve({pde,ic,bc},u(x,t)) assuming L>0,t>0,k>0;
```

$$u(x,t) = \frac{1}{L}\left(\left(\int_0^t \frac{1}{L}\left(\sum_{n1=1}^{\infty}\left(-2\,\frac{1}{L}\mathrm{e}^{\frac{\pi^2 k\,(t-\tau 1)n1^2 - c\tau 1\, L^2}{L^2}}\int_0^L \sin\left(2\,\frac{\pi\, x}{L}\right)\cos\left(\frac{n1\,\pi\, x}{L}\right)\,\mathrm{d}x \cos\left(\frac{n1\,\pi\, x}{L}\right)\right)\right) L - \mathrm{e}^{-c\tau 1}\int_0^L \sin\right.\right.$$

Result Solved

3 Laplace PDE

Laplace PDE inside quarter-circle

Problem description

problem number 32

This is problem 2.5.5 part (c) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

Inside quarter circle of radius 1 with $0 \leq \theta \leq \frac{\pi}{2}$ and $0 \leq r \leq 1$, with following boundary conditions

$$u(r,0) = 0$$
$$u(r,\frac{\pi}{2}) = 0$$
$$\frac{\partial u}{\partial r}(1,\theta) = f(\theta)$$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, theta, r, f];
 pde = D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r]*1*D[u[r, theta], {theta, 2}])/(r*r^2) == 0;
 bc = {Derivative[1, 0][u][1, theta] == f[theta], u[r, Pi/2] == 0, u[r, 0] == 0};
 sol = DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions -> {0 <= r <= 1 && 0 <= theta <= Pi/2}];
 ;
```

$$\text{DSolve}\left[\left\{\frac{u^{(0,2)}(r,\theta)u^{(1,0)}(r,\theta)}{r^3} + u^{(2,0)}(r,\theta) = 0, \left\{u^{(1,0)}(1,\theta) = f(\theta), u\left(r,\frac{\pi}{2}\right) = 0, u(r,0) = 0\right\}\right\}, u(r,\theta), \{r,\theta\}, \text{Assu}$$

Result Did not solve

Maple

Raw commands

```
L:='L'; u:='u'; t:='t'; x:='x';f:='f';
interface(showassumed=0);
pde:=diff(u(r,theta),r$2)+1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc:=u(r,0)=0,u(r,Pi/2)=0,D[1](u)(1,theta)=f(theta);
sol:=pdsolve([pde,bc],u(r,theta)) assuming 0<=theta,theta<=Pi/2,0<=r,r<=1;
```

$$\text{sol} = ()$$

Result Did not solve

Laplace PDE inside semi-circle

Problem description

problem number 33

Solve Laplace equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

Inside semi-circle of radius 1 with $0 \leq \theta \leq \pi$ and $0 \leq r \leq 1$, with following boundary conditions

$$u(r,0) = 0$$
$$u(r,\pi) = 0$$
$$u(0,\theta) = 0$$
$$u(1,\theta) = f(\theta)$$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 ClearAll[u, theta, r, f];
 pde = D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r]*1*D[u[r, theta], {theta, 2}])/(r*r^2) == 0;
 bc = {u[r, 0] == 0, u[r, Pi] == 0, u[0, theta] == 0, u[1, theta] == f[theta]};
 sol = DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions -> {0 <= r <= 1 && 0 <= theta <= Pi}];
 ;
```

$$\text{DSolve}\left[\left\{\frac{u^{(0,2)}(r,\theta)u^{(1,0)}(r,\theta)}{r^3}+u^{(2,0)}(r,\theta)=0,\{u(r,0)=0,u(r,\pi)=0,u(0,\theta)=0,u(1,\theta)=f(\theta)\}\right\},u(r,\theta),\{r,\theta\},$$

Result Did not solve

Maple

Raw commands

```
L:='L'; u:='u';f:='f';
interface(showassumed=0);
pde:=diff(u(r,theta),r$2)+1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc:=u(r,0)=0,u(r,Pi)=0,u(0,theta)=0,u(1,theta)=f(theta);
sol:=pdsolve([pde,bc],u(r,theta)) assuming 0<theta,theta<Pi,0<=r,r<=1;
```

$$u\left(r,\theta\right)=\sum_{n=1}^{\infty}\left(-\frac{\left(-2\,r^n\int_0^{\pi}\sin\left(n\theta\right)f\left(\theta\right)\,\mathrm{d}\theta+\pi\,\_C5\left(n\right)\left(r^n-r^{-n}\right)\right)\sin\left(n\theta\right)}{\pi}\right)$$

Result Gave a solution, but extra terms in solution is not correct. Therefore counted as not solved

Laplace PDE inside rectangle

Problem description

problem number 34

This is problem 2.5.1 part (a) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

inside a rectangle $0\leq x\leq L, 0\leq y\leq H$, with following boundary conditions

$$\begin{aligned}\frac{\partial u}{\partial x}(0,y)&=0\\ \frac{\partial u}{\partial x}(L,y)&=0\\ u(x,0)&=0\\ u(x,H)&=f(x)\end{aligned}$$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 NumericQ[H] =. ;
 ClearAll[u, t, k, x, L, H, f];
 pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = {Derivative[1, 0][u][0, y] == 0, Derivative[1, 0][u][L, y] == 0, u[x, 0] == 0, u[x, H] == f[x]};
 sol = DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <= x <= L && 0 <= y <= H}];
 sol = sol /. {K[1] -> n};
 ;
```

$$\left\{\left\{u(x,y)\to\sum_{n=1}^{\infty}\frac{2\cos\left(\frac{n\pi x}{L}\right)\text{csch}\left(\frac{Hn\pi}{L}\right)\left(\int_0^L\cos\left(\frac{n\pi x}{L}\right)f(x)\,dx\right)\sinh\left(\frac{n\pi y}{L}\right)}{L}+\frac{y\int_0^L f(x)\,dx}{HL}\right\}\right\}$$

Result Solved

Maple

Raw commands

```
H:='H';L:='L'; u:='u'; y:='y'; x:='x';f:='f';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc:=D[1](u)(0,y)=0,D[1](u)(L,y)=0,u(x,0)=0,u(x,H)=f(x);
sol:=pdsolve([pde,bc],u(x,y)) assuming(0<=x and x<=L and 0<=y and y<=H);
#these simplifications below to convert answer to one that match standard;
sol:=convert(sol,trigh);
sol:=simplify(expand(sol));
```

$$u(x,y) = \frac{1}{H\,L}\left(2\sum_{n=1}^{\infty}\left(1\sinh\left(\frac{\pi\,yn}{L}\right)\cos\left(\frac{\pi\,nx}{L}\right)\int_0^L f(x)\cos\left(\frac{\pi\,nx}{L}\right)\,\mathrm{d}x\left(\sinh\left(\frac{n\pi\,H}{L}\right)\right)^{-1}\right)H + \int_0^L f(x)\,\mathrm{d}xy\right)$$

Result Solved

Laplace PDE inside rectangle

Problem description

problem number 35

This is problem 2.5.1 part (b) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

inside a rectangle $0 \le x \le L, 0 \le y \le H$, with following boundary conditions

$$\begin{aligned}\frac{\partial u}{\partial x}(0,y) &= g(y)\\ \frac{\partial u}{\partial x}(L,y) &= 0\\ u(x,0) &= 0\\ u(x,H) &= 0\end{aligned}$$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 NumericQ[H] =. ;
 ClearAll[u, t, k, x, L, H, g, f];
 pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = {Derivative[1, 0][u][0, y] == g[y], Derivative[1, 0][u][L, y] == 0, u[x, 0] == 0, u[x, H] == 0};
 sol = DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <= x <= L && 0 <= y <= H}];
 sol = sol /. {K[1] -> n};
 ;
```

$$\left\{\left\{u(x,y)\to\sum_{n=1}^{\infty}-\frac{2\cosh\left(\frac{n\pi(L-x)}{H}\right)\operatorname{csch}\left(\frac{Ln\pi}{H}\right)\left(\int_0^H g(y)\sin\left(\frac{n\pi y}{H}\right)\,dy\right)\sin\left(\frac{n\pi y}{H}\right)}{n\pi}\right\}\right\}$$

Result Solved

Maple

Raw commands

```
H:='H';L:='L'; u:='u'; y:='y'; x:='x';f:='f';g:='g';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0):
bc:=D[1](u)(0,y)=g(y),D[1](u)(L,y)=0,u(x,0)=0,u(x,H)=0:
sol:=pdsolve([pde,bc],u(x,y)) assuming(0<=x and x<=L and 0<=y and y<=H):
sol:=convert(sol,trigh);
```

$$u(x,y)=\sum_{n=1}^{\infty}\left(-2\,\frac{1}{\pi\,n}\int_0^H \sin\left(\frac{\pi\,ny}{H}\right)g(y)\,\mathrm{d}y\sin\left(\frac{\pi\,ny}{H}\right)\left(\cosh\left(\frac{\pi\,n\,(2\,L-x)}{H}\right)+\sinh\left(\frac{\pi\,n\,(2\,L-x)}{H}\right)+\cosh\left(\frac{\pi\,nx}{H}\right.\right.\right.$$

Result Solved

Laplace PDE inside rectangle

Problem description

problem number 36

This is problem 2.5.1 part (c) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

inside a rectangle $0\leq x\leq L, 0\leq y\leq H$, with following boundary conditions

$$\begin{aligned}\frac{\partial u}{\partial x}(0,y)&=0\\ u(L,y)&=g(y)\\ u(x,0)&=0\\ u(x,H)&=0\end{aligned}$$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 NumericQ[H] =. ;
 ClearAll[u, t, k, x, L, H, g, f];
 pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = {Derivative[1, 0][u][0, y] == 0, u[L, y] == g[y], u[x, 0] == 0, u[x, H] == 0};
 sol = DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <= x <= L && 0 <= y <= H}];
 ;
```

$$\mathrm{DSolve}\left[\left\{u^{(0,2)}(x,y)+u^{(2,0)}(x,y)=0,\left\{u^{(1,0)}(0,y)=0,u(L,y)=g(y),u(x,0)=0,u(x,H)=0\right\}\right\},u(x,y),\{x,y\},\mathrm{Ass}\right.$$

Result Did not solve

Maple

Raw commands

```


H:='H';L:='L'; u:='u'; y:='y'; x:='x';f:='f';g:='g';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc:=D[1](u)(0,y)=0,u(L,y)=g(y),u(x,0)=0,u(x,H)=0;
sol:=pdsolve([pde,bc],u(x,y)) assuming(0<=x and x<=L and 0<=y and y<=H);
sol:=convert(sol,trigh);
```

$$u(x,y)=\sum_{n=1}^{\infty}\left(2\,\frac{1}{H}\sin\left(\frac{\pi\,ny}{H}\right)\int_0^H \sin\left(\frac{\pi\,ny}{H}\right)g(y)\,\mathrm{d}y\left(\cosh\left(2\,\frac{\pi\,nx}{H}\right)+\sinh\left(2\,\frac{\pi\,nx}{H}\right)+1\right)\left(\cosh\left(\frac{\pi\,n\,(L-x)}{H}\right)+\mathrm{s}\right.\right.$$

Result Solved

Laplace PDE inside rectangle

Problem description

problem number 37

This is problem 2.5.1 part (d) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

inside a rectangle $0 \leq x \leq L, 0 \leq y \leq H$, with following boundary conditions

$$
\begin{aligned}
u(0,y) &= g(y)\\
u(L,y) &= 0\\
\frac{\partial u}{\partial y} u(x,0) &= 0\\
u(x,H) &= 0
\end{aligned}
$$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 NumericQ[H] =. ;
 ClearAll[u, x, L, H, g, f];
 pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = {u[0, y] == 0, u[L, y] == 0, Derivative[0, 1][u][x, 0] == 0, u[x, H] == 0};
 sol = DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <= x <= L && 0 <= y <= H}];
 ;
```

DSolve $\left[\left\{u^{(0,2)}(x,y)+u^{(2,0)}(x,y)=0,\left\{u(0,y)=0,u(L,y)=0,u^{(0,1)}(x,0)=0,u(x,H)=0\right\}\right\},u(x,y),\{x,y\},\right.$Assum

Result Did not solve

Maple

Raw commands

```


H:='H';L:='L'; u:='u'; y:='y'; x:='x';f:='f';g:='g';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc:=u(0,y)=g(y),u(L,y)=0,D[2](u)(x,0)=0,u(x,H)=0;
sol:=pdsolve([pde,bc],u(x,y)) assuming(0<=x and x<=L and 0<=y and y<=H);
sol:=convert(sol,trigh);
```

$$u\left(x,y\right)=\sum_{n=0}^{\infty}\left(2\,\frac{1}{H}\sin\left(1/2\,\frac{\pi\,\left(2\,yn+H+y\right)}{H}\right)\int_{0}^{H}\sin\left(1/2\,\frac{\pi\,\left(2\,yn+H+y\right)}{H}\right)g\left(y\right)\,\mathrm{d}y\left(\cosh\left(\frac{\left(1+2\,n\right)\left(L-x\right)\pi}{H}\right)\right.\right.$$

Result Solved

Laplace PDE inside rectangle

Problem description

problem number 38

This is problem 2.5.1 part (e) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

inside a rectangle $0 \leq x \leq L, 0 \leq y \leq H$, with following boundary conditions

$$
\begin{aligned}
u(0,y) &= 0\\
u(L,y) &= 0\\
u(x,0)-\frac{\partial u}{\partial y} u(x,0) &= 0\\
u(x,H) &= f(x)
\end{aligned}
$$

Mathematica

Raw commands

```
NumericQ[L] =. ;
 NumericQ[H] =. ;
 ClearAll[u, x, L, H, g, f];
 pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = {u[0, y] == 0, u[L, y] == 0, u[x, 0] - Derivative[0, 1][u][x, 0] == 0, u[x, H] == f[x]};
 sol = DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <= x <= L && 0 <= y <= H}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,2)}(x,y)+u^{(2,0)}(x,y)=0,\left\{u(0,y)=0,u(L,y)=0,u(x,0)-u^{(0,1)}(x,0)=0,u(x,H)=f(x)\right\}\right\},u(x,y),\{$$

Result Did not solve

Maple

Raw commands

```
H:='H';L:='L'; u:='u'; y:='y'; x:='x';f:='f';g:='g';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
assume(L>0 and H>0);
bc:=u(0,y)=0,u(L,y)=0,u(x,0)-D[2](u)(x,0)=0,u(x,H)=f(x);
sol:=pdsolve([pde,bc],u(x,y)) assuming(0<=x and x<=L and 0<=y and y<=H);
sol:=convert(sol,trigh);
```

$$u\left(x,y\right)=\sum_{n=1}^{\infty}\left(2\,\frac{1}{L}\sin\left(\frac{\pi\,nx}{L}\right)\left(\cosh\left(\frac{\pi\,n\left(H-y\right)}{L}\right)+\sinh\left(\frac{\pi\,n\left(H-y\right)}{L}\right)\right)\left(\pi\,\sinh\left(2\,\frac{\pi\,yn}{L}\right)n+\pi\,\cosh\left(2\,\frac{\pi\,yn}{L}\right)n$$

Result Solved

Laplace PDE inside rectangle, top/bottom edges non-zero

Problem description

problem number 39

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

inside a rectangle $0\leq x\leq 1, 0\leq y\leq 2$, with following boundary conditions

$$u(0,y)=0$$
$$u(1,y)=0$$
$$u(x,0)=\text{UnitTriagle(2 x-1)}$$
$$u(x,2)=\text{UnitTriagle(2 x-1)}$$

Mathematica

Raw commands

```
ClearAll[u, x, y];
 pde = Laplacian[u[x, y], {x, y}] == 0;
 L0 = 1;
 H0 = 2;
 bc = DirichletCondition[u[x, y] == Piecewise[{{UnitTriangle[2*x - L0], y == 0 || y == H0}}, 0], True];
 Null;
 domain = Rectangle[{0, 0}, {L0, H0}];
 sol = Simplify[DSolve[{pde, bc}, u[x, y], Element[{x, y}, domain]]];
 sol = sol /. K[1] -> n;
 ;
```

$$\left\{\left\{u(x,y)\to\sum_{n=1}^{\infty}\frac{8\text{csch}(2n\pi)\sin\left(\frac{n\pi}{2}\right)\sin(n\pi x)(\sinh(n\pi(2-y))+\sinh(n\pi y))}{n^2\pi^2}\right\}\right\}$$

Result Solved

Maple

Raw commands

```
u:='u'; y:='y'; x:='x';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
f:=x-> piecewise(x>0 and x<1/2, 2*x, x>1/2 and x<1, 2-2*x);
bc:=u(0,y)=0,u(1,y)=0,u(x,0)=f(x),u(x,2)=f(x);
sol:=pdsolve([pde,bc],u(x,y)) assuming x>0,y>0;
```

$$u(x,y) = \sum_{n=0}^{\infty} 8 \frac{\sin(1/2\,\pi\,n)\sin(\pi\,nx)\left(e^{\pi\,n(3\,y-2)} - e^{\pi\,n(3\,y-4)} + e^{\pi\,yn} - e^{\pi\,n(y-2)}\right)e^{-2\,\pi\,n(y-2)}}{\pi^2 n^2\left(e^{4\,\pi\,n} - 1\right)}$$

Result Solved

Laplace PDE inside circular annulus

Problem description

problem number 40

This is problem 2.5.8 part (b) from Richard Haberman applied partial differential equations, 5th edition

Solve Laplace equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

Inside circular annulus $a < r < b$ subject to the following boundary conditions

$$\frac{\partial u}{\partial r}(a,\theta) = 0$$
$$u(b,0) = g(\theta)$$

Mathematica

Raw commands

```
ClearAll[u, a, theta, r, g];
 pde = D[u[r, theta], {r, 2}] + (1*D[u[r, theta], r]*1*D[u[r, theta], {theta, 2}])/(r*r^2) == 0;
 bc = {Derivative[1, 0][u][a, theta] == 0, u[b, theta] == g[theta]};
 sol = DSolve[{pde, bc}, u[r, theta], {r, theta}, Assumptions -> Inequality[a, Less, r, LessEqual, b]];
 ;
```

$$\text{DSolve}\left[\left\{\frac{u^{(0,2)}(r,\theta)u^{(1,0)}(r,\theta)}{r^3} + u^{(2,0)}(r,\theta) = 0, \left\{u^{(1,0)}(a,\theta) = 0, u(b,\theta) = g(\theta)\right\}\right\}, u(r,\theta), \{r,\theta\}, \text{Assumptions} \to a$$

Result Did not solve

Maple

Raw commands

```
a:='a'; u:='u'; r:='r'; theta:='theta';g:='g';
interface(showassumed=0);
pde:=diff(u(r,theta),r$2)+1/r*diff(u(r,theta),r)+1/r^2*diff(u(r,theta),theta$2)=0;
bc:=D[1](u)(a,theta)=0,u(b,theta)=g(theta);
sol:=pdsolve([pde,bc],u(r,theta)) assuming a<r,r<b;
```

sol = ()

Result Did not solve

Laplace PDE example 18 from Maple help page

Problem description

problem number 41

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

With boundary conditions

$$u(0,y) = \frac{\sin y}{y}$$

Mathematica

Raw commands

```
ClearAll[u, x, y];
 pde = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = u[0, y] == Sin[y]/y;
 sol = DSolve[{pde, bc}, u[x, y], {x, y}, Assumptions -> {0 <= x && 0 <= y}];
 ;
```

$$\left\{\left\{u(x,y)\to\frac{(\sinh(x)-\cosh(x))(x\cos(y)-y\sin(y))+x}{x^2+y^2}\right\}\right\}$$

Result Solved

Maple

Raw commands

```
x:='x'; y:='y'; u:='u';
interface(showassumed=0);
pde:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
bc:=u(0,y)=sin(y)/y;
sol:=pdsolve([pde,bc],u(x,y));
```

$$u\left(x,y\right)=\frac{\sin\left(-y+ix\right)+\_F2\left(y-ix\right)\left(y-ix\right)+\left(-y+ix\right)\_F2\left(y+ix\right)}{-y+ix}$$

Result Solved

Laplace PDE on rectangle with one edge at infinity

Problem description

problem number 42

Solve Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

With boundary conditions

$$\begin{aligned}u(0,y)&=\sin y\\u(x,0)&=0\\u(x,a)&=0\\u(\infty,y)&=0\end{aligned}$$

Mathematica

Raw commands

```
ClearAll[x, y, a];
 ode = D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
 bc = {u[x, 0] == 0, u[x, a] == 0, u[0, y] == Sin[y], u[Infinity, y] == 0};
 sol = DSolve[{ode, bc}, u[x, y], {x, y}, Assumptions -> a > 0];
 ;
```

DSolve $\left[\left\{u^{(0,2)}(x,y)+u^{(2,0)}(x,y)=0,\{u(x,0)=0,u(x,a)=0,u(0,y)=\sin(y),u(\infty,y)=0\}\right\},u(x,y),\{x,y\},\text{Assum}\right.$

Result Did not solve

Maple

Raw commands

```
x:='x'; y:='y'; a:='a';
interface(showassumed=0);
ode:=diff(u(x,y),x$2)+diff(u(x,y),y$2)=0;
bc:=u(x,0)=0, u(x,a)=0, u(0,y)=sin(y), u(infinity,y)=0;
pdsolve({ode, bc}, u(x,y)) assuming a>0;
```

$$u\left(x,y\right)=\frac{\sin\left(-y+ix\right)+\_F2\left(y-ix\right)\left(y-ix\right)+\left(-y+ix\right)\_F2\left(y+ix\right)}{-y+ix}$$

Result Solved

4 Wave PDE

Wave PDE on string with source

Problem description

problem number 43

This is problem at page 115, David J Logan textbook, applied PDE textbook.
Falling cable lying on a table that is suddenly removed.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} - g$$

With boundary condition

$$u(0,t) = 0$$

And initial conditions

$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

Mathematica

Raw commands

```
ClearAll[u, t, x, g, c];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] - g;
 bc = u[0, t] == 0;
 ic = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {t > 0 && c > 0 && x > 0}];
 ;
```

$$\left\{\left\{u(x,t) \to \frac{1}{2} g \left(\left(t - \frac{x}{c}\right)^2 \theta\left(t - \frac{x}{c}\right) - t^2\right) - c_1 \delta\left(t - \frac{x}{c}\right)\right\}\right\}$$

Result Solved

Maple

Raw commands

```
x:='x'; t:='t'; g:='g';c:='c';u:='u';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)-g;
ic:=D[2](u)(x,0)=0,u(0,t)=0,u(x,0)=0;
sol:=pdsolve([pde,ic],u(x,t),HINT = boundedseries) assuming t>0,x>0,c>0;
```

$$u(x,t) = 1/2 \frac{g}{c^2} \left(Heaviside\left(t - \frac{x}{c}\right)(tc - x)^2 - c^2 t^2 \right)$$

Result Solved

Wave PDE on string, fixed ends

Problem description

problem number 44

This is problem at page 28, David J Logan textbook, applied PDE textbook.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

With boundary condition

$$u(0,t) = 0$$
$$u(L,t) = 0$$

Mathematica

Raw commands

```
ClearAll[u, t, x, L, c];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[L, t] == 0};
 sol = DSolve[{pde, bc}, u[x, t], {x, t}, Assumptions -> {L > 0}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,2)}(x,t)=c^2u^{(2,0)}(x,t),\{u(0,t)=0,u(L,t)=0\}\right\},u(x,t),\{x,t\},\text{Assumptions}\to\{L>0\}\right]$$

Result Did not solve

Maple

Raw commands

```

x:='x'; t:='t'; L:='L'; c:='c';u:='u';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(L,t)=0;
sol:=pdsolve([pde,bc],u(x,t)) assuming L>0;
sol:=subs(_Z1=n,sol);
```

$$u(x,t)=\sum_{n=1}^{\infty}\sin\left(\frac{\pi\,nx}{L}\right)\left(\_C1\,(n)\sin\left(\frac{cn\pi\,t}{L}\right)+\_C5\,(n)\cos\left(\frac{cn\pi\,t}{L}\right)\right)$$

Result Solved

Wave PDE on string, one fixed end, one free end

Problem description

problem number 45

This is problem at page 130, David J Logan textbook, applied PDE textbook.

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$$

With boundary conditions

$$\frac{\partial u}{\partial x}(L,0)=0$$
$$u(0,t)=0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0)=0$$
$$u(x,0)=f(x)$$

Mathematica

Raw commands

```
ClearAll[u, t, x, L, c, f];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, Derivative[1, 0][u][L, t] == 0};
 ic = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f[x]};
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {0 <= x <= L}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,2)}(x,t)=c^2u^{(2,0)}(x,t),\left\{u(0,t)=0,u^{(1,0)}(L,t)=0\right\},\left\{u^{(0,1)}(x,0)=0,u(x,0)=f(x)\right\}\right\},u(x,t),\{x,t\},\text{A}$$

Result Did not solve

Maple

Raw commands

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';f:='f';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
bc:=u(0,t)=0,D[1](u)(L,t)=0;
ic:=D[2](u)(x,0)=0,u(x,0)=f(x);
sol:=pdsolve([pde,bc,ic],u(x,t)) assuming x>=0,x<=L;
sol:=subs(_Z1=n,sol);
```

$$u(x,t)=\sum_{n=0}^{\infty}\left(2\,\frac{1}{L}\int_0^L \sin\left(1/2\,\frac{(1+2\,n)\,\pi\,x}{L}\right)f(x)\,\mathrm{d}x\sin\left(1/2\,\frac{(1+2\,n)\,\pi\,x}{L}\right)\cos\left(1/2\,\frac{c\pi\,(1+2\,n)\,t}{L}\right)\right)$$

Result Solved

Wave PDE on string, both ends fixed end, with source

Problem description

problem number 46

This is problem at page 149, David J Logan textbook, applied PDE textbook.

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}+p(x,t)$$

With boundary conditions

$$u(\pi,0)=0$$
$$u(0,t)=0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0)=0$$
$$u(x,0)=0$$

Mathematica

Raw commands

```
ClearAll[u, t, x, c, p];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] + p[x, t];
 bc = {u[0, t] == 0, u[Pi, t] == 0};
 ic = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,2)}(x,t)=c^2u^{(2,0)}(x,t)+p(x,t),\{u(0,t)=0,u(\pi,t)=0\},\left\{u(x,0)=0,u^{(0,1)}(x,0)=0\right\}\right\},u(x,t),\{x,t\}\right]$$

Result Did not solve

Maple

Raw commands

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';p:='p';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)+p(x,t);
bc:=u(0,t)=0,u(Pi,t)=0;
ic:=u(x,0)=0,D[2](u)(x,0)=0;
sol:=pdsolve([pde,bc,ic],u(x,t));
sol:=subs(_Z1=n,sol);
```

$$u(x,t)=\int_0^t\sum_{n=1}^{\infty}\left(2\,\frac{\int_0^{\pi}\sin(nx)\,p(x,\tau 1)\,\mathrm{d}x\sin(nx)\sin(c(t-\tau 1)\,n)}{\pi\,nc}\right)\mathrm{d}\tau 1$$

Result Solved

Wave PDE on string, both ends fixed end, with source

Problem description

problem number 47

This is problem at page 213, David J Logan textbook, applied PDE textbook.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + Ax$$

With boundary conditions

$$u(L,0) = 0$$
$$u(0,t) = 0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = 0$$

Mathematica

Raw commands

```
ClearAll[u, t, x, c, A, L];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}] + A*x;
 bc = {u[0, t] == 0, u[L, t] == 0};
 ic = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 ;
```

$$\text{DSolve}\left[\left\{u^{(0,2)}(x,t) = c^2 u^{(2,0)}(x,t) + Ax, \{u(0,t) = 0, u(L,t) = 0\}, \left\{u(x,0) = 0, u^{(0,1)}(x,0) = 0\right\}\right\}, u(x,t), \{x,t\}\right]$$

Result Did not solve

Maple

Raw commands

```
x:='x'; t:='t'; L:='L';c:='c';u:='u';A:='A';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2)+A* x;
bc:=u(0,t)=0,u(L,t)=0;
ic:=u(x,0)=0,D[2](u)(x,0)=0;
sol:=pdsolve([pde,bc,ic],u(x,t));
sol:=subs(_Z1=n,sol);
```

$$u(x,t) = \int_0^t \sum_{n=1}^{\infty} \left(2 \frac{A}{\pi\, nc} \int_0^L \sin\left(\frac{\pi\, nx}{L}\right) x \, dx \sin\left(\frac{\pi\, nx}{L}\right) \sin\left(\frac{c\pi\ (t-\tau 1)\, n}{L}\right)\right) d\tau 1$$

Result Solved

Wave PDE on string, both ends fixed end moving boundary condition

Problem description

problem number 48

Solve

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

With boundary conditions

$$u(0,t) = g(t)$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0) = 0$$
$$u(x,0) = 0$$

Mathematica

Raw commands

```
ClearAll[u, t, x, g, c];
 pde = D[u[x, t], {t, 2}] == c^2*D[u[x, t], {x, 2}];
 bc = u[0, t] == g[t];
 ic = {u[x, 0] == 0, Derivative[0, 1][u][x, 0] == 0};
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}, Assumptions -> {t > 0 && c > 0 && x > 0}];
 ;
```

$$\left\{\left\{u(x,t)\to \quad \left\{ \begin{array}{cc} 0 & x>ct \\ g\left(t-\frac{x}{c}\right) & x\leq ct \\ \text{Indeterminate} & \text{True} \end{array} \right.\right\}\right\}$$

Result Solved

Maple

Raw commands

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';g:='g';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)=c^2*diff(u(x,t),x$2);
ic:=u(x,0)=0,D[2](u)(x,0)=0;
bc:=u(0,t)=g(t);
sol:=pdsolve([pde,ic,bc],u(x,t),HINT = boundedseries) assuming t>0,x>0,c>0;
```

$$u\left(x,t\right)=Heaviside\left(t-\frac{x}{c}\right)g\left(\frac{tc-x}{c}\right)$$

Result Solved

Wave PDE on string, both ends fixed with damping

Problem description

problem number 49

Solve

$$\frac{\partial^2 u}{\partial t^2}+2\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$$

With boundary conditions

$$u(0,t)=0$$
$$u(\pi,0)=0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0)=0$$
$$u(x,0)=f(x)$$

Mathematica

Raw commands

```
pde = D[u[x, t], {t, 2}] + 2*D[u[x, t], t] == D[u[x, t], {x, 2}];
 bc = {u[0, t] == 0, u[Pi, t] == 0};
 ic = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == f[x]};
 sol = DSolve[{pde, bc, ic}, u[x, t], x, t];
 ;
```

DSolve $\left[\left\{2u^{(0,1)}(x,t)+u^{(0,2)}(x,t)=u^{(2,0)}(x,t),\{u(0,t)=0,u(\pi,t)=0\},\left\{u^{(0,1)}(x,0)=0,u(x,0)=f(x)\right\}\right\},u(x,t),

Maple

Raw commands

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';f:='f';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)+2*diff(u(x,t),t)=diff(u(x,t),x$2);
ic:=D[2](u)(x,0)=0,u(0,t)=0,u(x,0)=f(x);
bc:=u(0,t)=0,u(Pi,t)=0;
sol:=pdsolve([pde,ic,bc],u(x,t)) assuming t>0;
```

$$u(x,t)=\sum_{n=1}^{\infty}\left(\frac{\left(\left(-1+\sqrt{-n^2+1}\right)e^{-\left(1+\sqrt{-n^2+1}\right)t}+e^{\left(-1+\sqrt{-n^2+1}\right)t}\left(1+\sqrt{-n^2+1}\right)\right)\sin(nx)\int_0^{\pi}\sin(nx)f(x)\,dx}{\sqrt{-n^2+1}\pi}\right)$$

Result Solved, But $n=1$ should not be included.

Wave PDE on string with fixed ends, non-zero initial position

Problem description

problem number 50

Added March 9, 2018.
Solve

$$\frac{\partial^2 u}{\partial t^2}=4\frac{\partial^2 u}{\partial x^2}$$

With boundary conditions

$$u(0,t)=0$$
$$u(\pi,0)=0$$

With initial conditions

$$\frac{\partial u}{\partial t}(x,0)=0$$
$$u(x,0)=\sin^2(x)$$

Mathematica

Raw commands

```
ClearAll[u, t, x, n];
 pde = D[u[x, t], {t, 2}] == 4*D[u[x, t], {x, 2}];
 ic = {Derivative[0, 1][u][x, 0] == 0, u[x, 0] == Sin[x]^2};
 bc = {u[0, t] == 0, u[Pi, t] == 0};
 sol = DSolve[{pde, bc, ic}, u[x, t], {x, t}];
 sol = sol /. K[1] -> n;
 ;
```

$$\left\{\left\{u(x,t)\to\sum_{n=1}^{\infty}\frac{4(\cos(n\pi)-1)\cos(2nt)\sin(nx)}{(n^3-4n)\,\pi}\right\}\right\}$$

Result Solved but sum should not include $n=2$

Maple

Raw commands

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)= 4*diff(u(x,t),x$2);
bc:=u(0,t)=0,u(Pi,t)=0;
ic:=u(x,0)=sin(x)^2,D[2](u)(x,0)=0;
sol:=pdsolve([pde,bc,ic],u(x,t));
```

$$u(x,t)=\sum_{n=1}^{\infty}4\,\frac{\left((-1)^n-1\right)\sin(nx)\cos(2\,tn)}{\pi\,n\,(n^2-4)}$$

Result Solved, but sum should not include $n=2$

5 Schrodinger PDE

Schrodinger PDE with zero potential

Problem description

problem number 51

From page 30, David J Logan textbook, applied PDE textbook.
Solve

$$Ih\frac{\partial f}{\partial t} = -\frac{h^2}{2m}\frac{\partial^2 f}{\partial x^2}$$

With boundary conditions

$$f(0,t) = 0$$
$$f(L,0) = 0$$

Mathematica

Raw commands

```
ClearAll[f, t, x, L, m, h];
 pde = I*h*D[f[x, t], t] == -((h^2*D[f[x, t], {x, 2}])/(2*m));
 bc = {f[0, t] == 0, f[L, t] == 0};
 sol = DSolve[{pde, bc}, f[x, t], {x, t}, Assumptions -> L > 0];
 sol = sol /. K[1] -> n;
 ;
```

$$\left\{\left\{f(x,t) \to \sum_{n=1}^{\infty} e^{-\frac{ihn^2\pi^2 t}{2L^2 m}} c_n \sin\left(\frac{n\pi x}{L}\right)\right\}\right\}$$

Result Solved

Maple

Raw commands

```
x:='x'; t:='t'; L:='L'; c:='c';f:='f';
interface(showassumed=0);
pde:=I*h*diff(f(x,t),t)=-h^2/(2*m)*diff(f(x,t),x$2);
bc:=f(0,t)=0,f(L,t)=0;
sol:=pdsolve([pde,bc],f(x,t)) assuming L>0;
sol:=subs(_Z1=n,sol);
```

$$f(x,t) = \sum_{n=1}^{\infty} \_C1(n) \sin\left(\frac{\pi n x}{L}\right) e^{\frac{-i/2h\pi^2 n^2 t}{mL^2}}$$

Result Solved

Schrodinger PDE with initial and boundary conditions

Problem description

problem number 52

Solve for $f(x,y,t)$

$$I\frac{\partial f}{\partial t} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)$$

With boundary conditions

$$f(0,y,t) = 0$$
$$f(1,y,t) = 0$$
$$f(x,1,t) = 0$$
$$f(x,0,t) = 0$$

And initial conditions $f(x,y,0) = \sqrt{2}\left(\sin(2\pi x)\sin(\pi y) + \sin(\pi x)\sin(2\pi y)\right)$

Mathematica

Raw commands

```
ClearAll[f, t, x, y];
 pde = I*D[f[x, y, t], {t}] == -((hBar^2*Laplacian[f[x, y, t], {x, y}])/(2*m));
 initSum = f[x, y, 0] == Sqrt[2]*(Sin[2*Pi*x]*Sin[Pi*y] + Sin[Pi*x]*Sin[3*Pi*y]);
 bcs = {f[0, y, t] == 0, f[1, y, t] == 0, f[x, 1, t] == 0, f[x, 0, t] == 0};
 sol = DSolve[{pde, bcs, initSum}, f[x, y, t], {x, y, t}];
 ;
```

$$\left\{\left\{f(x,y,t)\to \sqrt{2}e^{-\frac{5i\pi^2\text{hBar}^2t}{m}}\left(\sin(\pi x)\sin(3\pi y)+\sin(2\pi x)\sin(\pi y)e^{\frac{5i\pi^2\text{hBar}^2t}{2m}}\right)\right\}\right\}$$

Result Solved

Maple

Raw commands

```
x:='x'; t:='t'; y:='y'; hbar:='hbar';f:='f';
interface(showassumed=0);
pde:=  I* diff(f(x,y,t),t) = -hBar^2/(2*m) * (diff(f(x,y,t),x$2) +  diff(f(x,y,t),y$2));
ic := f(x, y, 0) = sqrt(2)*(sin(2*Pi*x)*sin(Pi*y) + sin(Pi*x)*sin(3*Pi*y));
bc := f(0, y, t) = 0,f(1, y, t) = 0, f(x, 1, t) = 0, f(x, 0, t) = 0;
```

$$f(x,t)=\sum_{n=1}^{\infty}\_C1(n)\sin\left(\frac{\pi\, nx}{L}\right)\mathrm{e}^{\frac{-i/2h\pi^2n^2t}{mL^2}}$$

Result Did not solve

6 Beam PDE

Beam PDE with zero initial velocity

Problem description

problem number 53

Added January 20, 2018.
Solve

$$\frac{\partial^2 u}{\partial t^2}+\frac{\partial^4 u}{\partial x^4}=0$$

With boundary conditions

$$\begin{aligned}u(0,t)&=-12t^2\\ f(1,t)&=1-12t^2\\ \frac{\partial^2 u}{\partial x^2}u(0,t)&=0\\ \frac{\partial^2 u}{\partial x^2}u(1,t)&=12\end{aligned}$$

And initial conditions

$$\begin{aligned}u(x,0)&=x^4\\ \frac{\partial u}{\partial t}u(x,0)&=0\end{aligned}$$

Mathematica

Raw commands

```
ClearAll[u, x, t];
 pde = D[u[x, t], {t, 2}] + D[u[x, t], {x, 4}] == 0;
 bc = {u[0, t] == -12*t^2, u[1, t] == 1 - 12*t^2, Derivative[2, 0][u][0, t] == 0, Derivative[2, 0][u][1, t] == 12};
 ic = {u[x, 0] == x^4, Derivative[0, 1][u][x, 0] == 0};
 sol = DSolve[{pde, ic, bc}, u[x, t], x, t];
 ;
```

$$\left\{\left\{u(x,t)\to x^4-12t^2\right\}\right\}$$

Result Solved

Maple

Raw commands

```
x:='x'; t:='t'; L:='L'; c:='c';u:='u';
interface(showassumed=0);
pde:=diff(u(x,t),t$2)+diff(u(x,t),x$4)=0;
bc:=u(0,t)=-12*t^2,u(1,t)=1-12*t^2,D[1,1](u)(0,t)=0,D[1,1](u)(1,t)=12;
ic:=u(x,0)=x^4,D[2](u)(x,0)=0;
sol:=pdsolve({pde,ic,bc},u(x,t),HINT=`+`);
```

$$u(x,t) = x^4 - 12\,t^2$$

Result Solved

7 Burger's PDE

Burger's PDE for viscous fluid flow with no initial conditions

Problem description

problem number 54

Solve for $u(x,t)$

$$\frac{\partial u}{\partial t} + u(x,t)\frac{\partial u}{\partial x} = \mu\frac{\partial^2 u}{\partial x^2}$$

Mathematica

Raw commands

```
ClearAll[u, x, t, mu];
 pde = D[u[x, t], {t}] + u[x, t]*D[u[x, t], {x}] == mu*D[u[x, t], {x, 2}];
 sol = DSolve[pde, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t) \to -\frac{2c_1^2\mu\tanh\left(c_2 t + c_1 x + c_3\right) + c_2}{c_1}\right\}\right\}$$

Result Solved

Maple

Raw commands

```
x:='x'; t:='t'; y:='y'; mu:='mu';
interface(showassumed=0);
pde := diff(u(x, t), t) + u(x, t)*diff(u(x, t), x) = mu* diff(u(x,t),x$2);
sol := pdsolve(pde, u(x, t));
```

$$u(x,t) = -2\,\mu\,\_C2\,\tanh\left(\_C2\,x + \_C3\,t + \_C1\right) - \frac{\_C3}{\_C2}$$

Result Solved

Burger's PDE for viscous fluid flow with initial conditions

Problem description

problem number 55

Solve for $u(x,t)$

$$\frac{\partial u}{\partial t} + u(x,t)\frac{\partial u}{\partial x} = \mu\frac{\partial^2 u}{\partial x^2}$$

With initial conditions

$$u(x,0) = \begin{cases} 1 & x < 0 \\ 0 & x \geq 0 \end{cases}$$

Mathematica

Raw commands

```
ClearAll[u, x, y, mu];
 pde = D[u[x, t], {t}] + u[x, t]*D[u[x, t], {x}] == mu*D[u[x, t], {x, 2}];
 ic = u[x, 0] == Piecewise[{{1, x < 0}, {0, x >= 1}}];
 sol = DSolve[{pde, ic}, u[x, t], {x, t}, Assumptions -> mu > 0];
 ;
```

$$\left\{\left\{u(x,t)\to \frac{1}{\frac{e^{-\frac{t-2x}{4\mu}}\left(\operatorname{erf}\left(\frac{x}{2\sqrt{\mu}\sqrt{t}}\right)+1\right)}{\operatorname{erf}\left(\frac{t-x}{2\sqrt{\mu}\sqrt{t}}\right)+1}+1}\right\}\right\}$$

Result Solved

Maple

Raw commands

```
x:='x'; y:='y'; mu:='mu';u:='u';
interface(showassumed=0);
pde := diff(u(x, t), t)+u(x, t)*(diff(u(x, t), x)) = mu*(diff(u(x, t), x$2));
ic := u(x, 0) =  PIECEWISE([0,x>=0],[1,x<0]);
sol:= pdsolve({pde, ic},u(x,t)) assuming mu > 0,t>0;
sol:= sol[2];
```

$$1/2-1/2\,Erf\left(1/2\,\frac{x}{\sqrt{t}\sqrt{\mu}}\right)+\int_0^t\int_{-\infty}^{\infty}-1/2\,\frac{u\left(\zeta 1,\tau 1\right)\frac{\partial}{\partial \zeta 1}u\left(\zeta 1,\tau 1\right)}{\sqrt{\pi}\sqrt{\mu}\sqrt{t-\tau 1}}\mathrm{e}^{1/4\,\frac{(x-\zeta 1)^2}{\mu\,(-t+\tau 1)}}\,\mathrm{d}\zeta 1\,\mathrm{d}\tau 1$$

Result Solved, but has unresolved integrals

8 Black Scholes PDE

classic Black Scholes model from finance

Problem description

problem number 56

Solve for $V(S,t)$ where V is the price of the option as a function of stock price S and time t. r is the risk-free interest rate, and σ is the volatility of the stock.

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}=rV-rS\frac{\partial V}{\partial S}$$

With boundary condition $V(S,T)=\max\{S-k,0\}$

Reference `https://en.wikipedia.org/wiki/Black%E2%80%93Scholes_equation`

Mathematica

Raw commands

```
ClearAll[V, S, t, r, sigma];
 pde = D[V[S, t], t] + (1*sigma^2*S^2*D[V[S, t], {S, 2}])/2 == r*V[S, t] - r*S*D[V[S, t], S];
 bc = V[S, T] == Max[S - k, 0];
 sol = DSolve[{pde, bc}, V[S, t], {S, t}, Assumptions -> t > 0];
 ;
```

$$\left\{\left\{V(S,t)\to \frac{1}{2}e^{-rT}\left(Se^{rT}\operatorname{erfc}\left(\frac{2\log(k)+\left(2r+\sigma^2\right)(t-T)-2\log(S)}{2\sqrt{2}\sigma\sqrt{T-t}}\right)-ke^{rt}\operatorname{erfc}\left(\frac{2\log(k)+\left(2r-\sigma^2\right)(t-T)-}{2\sqrt{2}\sigma\sqrt{T-t}}\right.\right.\right.\right.$$

Result Solved

Maple

Raw commands

```
x:='x'; y:='y'; sigma:='sigma';S:='S';V:='V';r:='r';
interface(showassumed=0);
pde :=diff(V(S,t),t)+1/2*sigma^2*S^2*diff(V(S,t),S$2)=r*V(S,t)- r*S*diff(V(S,t),S);
bc:=V(S,T)=max(S-k,0);
sol:=pdsolve({pde,bc},V(S,t));
```

$$\text{sol} = ()$$

Result Did not solve

9 Korteweg-deVries PDE (waves on shallow water surfaces)

Korteweg-deVries Equation with no initial conditions

Problem description

problem number 57

Solve for $u(x,t)$

$$\frac{\partial^3 u}{\partial x^3} + \frac{\partial u}{\partial t} - 6u(x,t)\frac{\partial u}{\partial x} = 0$$

Reference `https://en.wikipedia.org/wiki/Korteweg%E2%80%93de_Vries_equation`

Mathematica

Raw commands

```
ClearAll[u, x, t];
 pde = D[u[x, t], {x, 3}] + D[u[x, t], {t}] - 6*u[x, t]*D[u[x, t], {x}] == 0;
 sol = DSolve[pde, u[x, t], {x, t}];
 ;
```

$$\left\{\left\{u(x,t) \to \frac{12c_1^3 \tanh^2\left(c_2 t + c_1 x + c_3\right) - 8c_1^3 + c_2}{6c_1}\right\}\right\}$$

Result Solved

Maple

Raw commands

```


x:='x'; y:='y';u:='u';
pde:= diff(u(x,t),x$3)+ diff(u(x,t),t)-6*u(x,t)* diff(u(x,t),x)=0;
sol:=pdsolve(pde,u(x,t));
```

$$u\left(x,t\right) = 2\_C2^2 \left(\tanh\left(\_C2\,x + \_C3\,t + \_C1\right)\right)^2 - 1/6\,\frac{8\_C2^3 - \_C3}{\_C2}$$

Result Solved