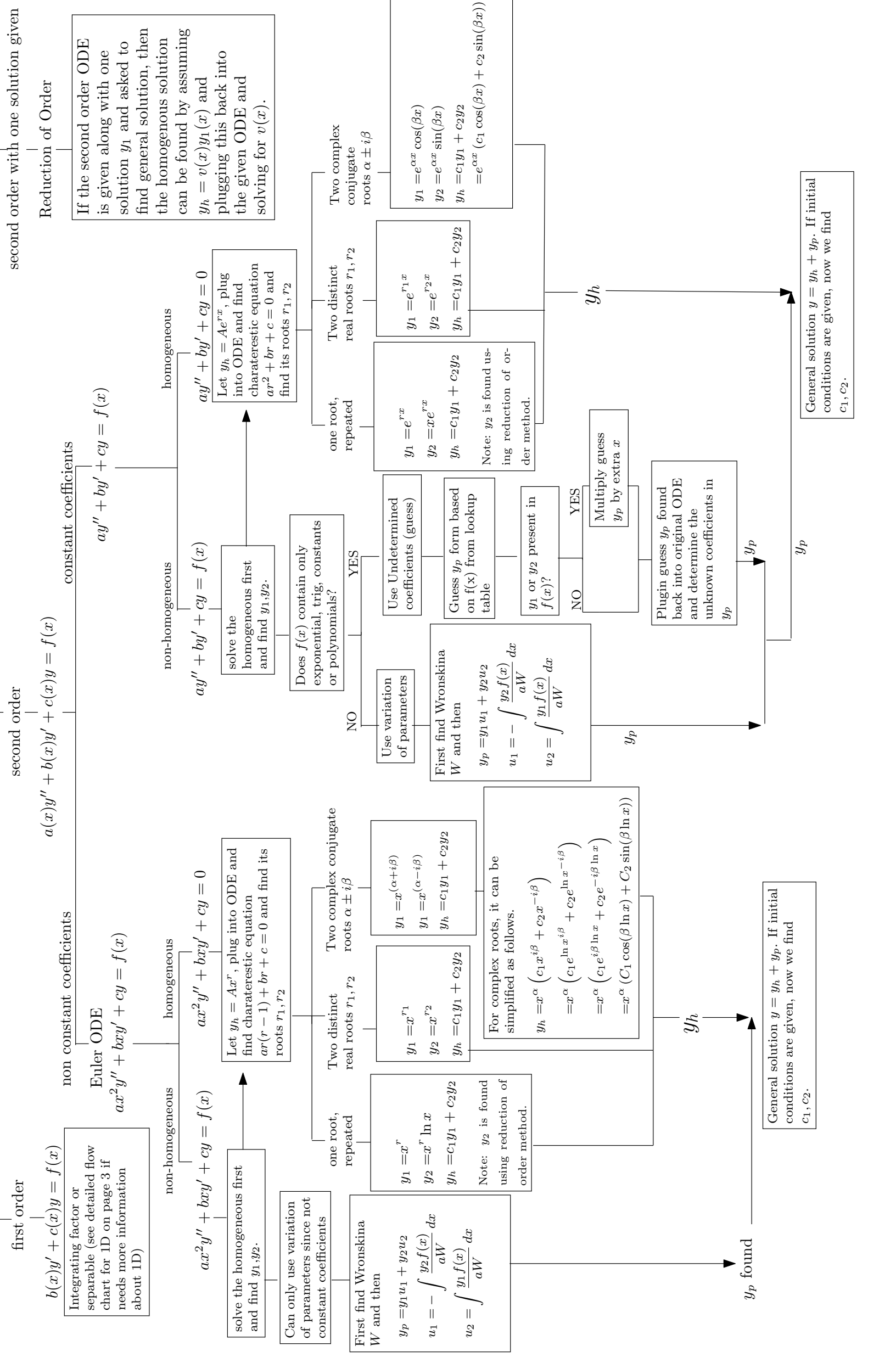
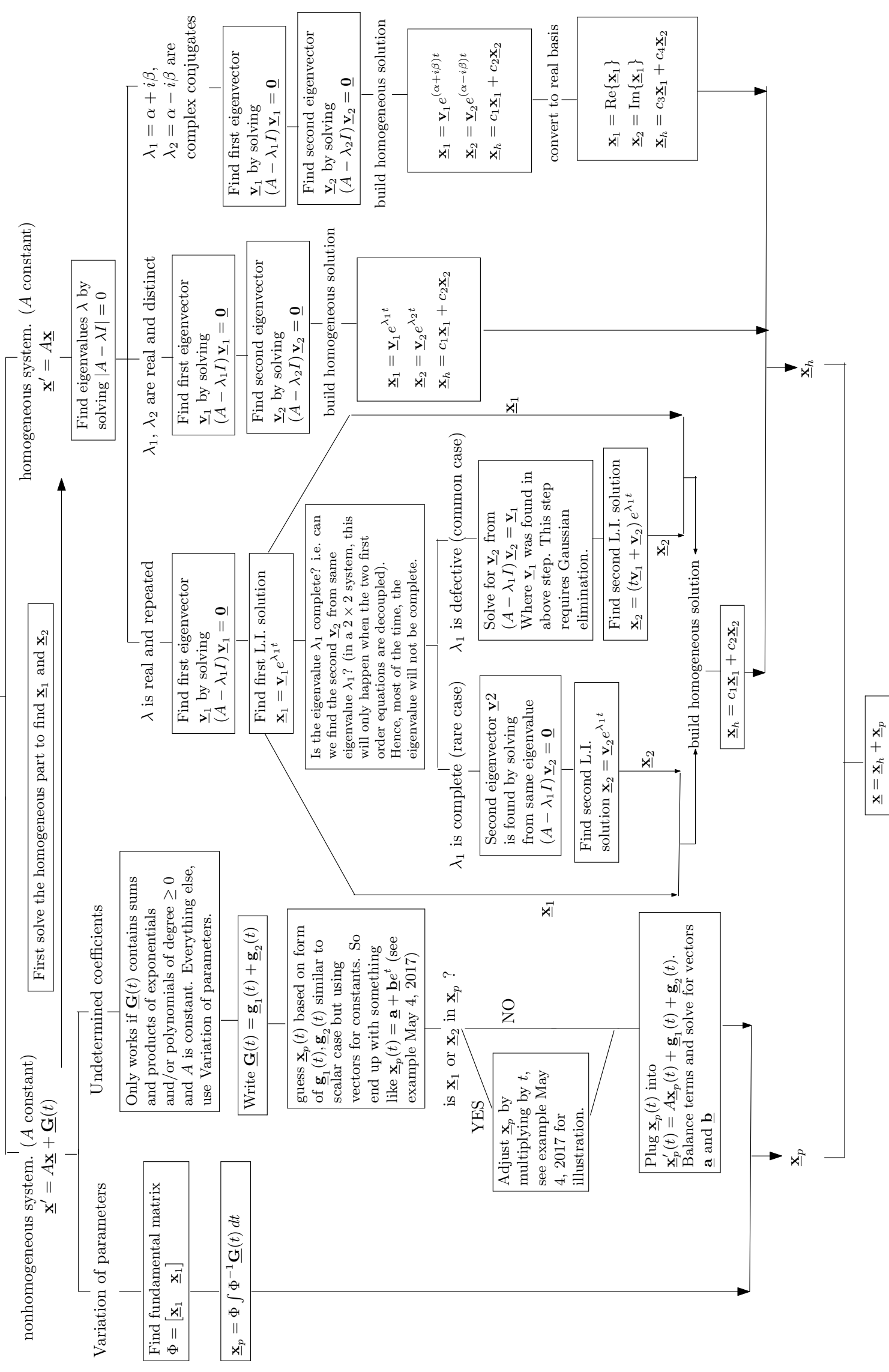


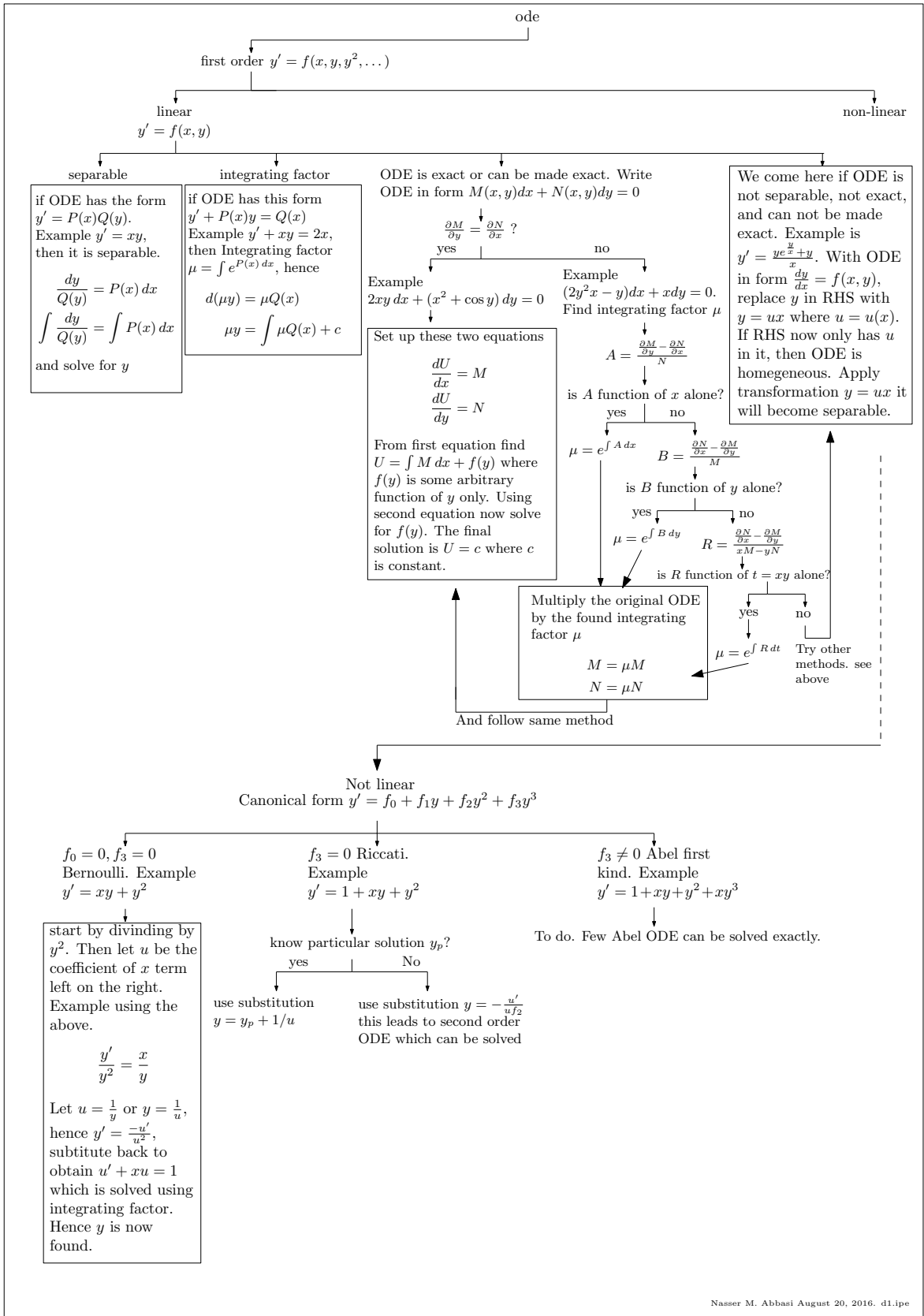
ODE classification

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July 26, 2017 compiled on — Wednesday July 26, 2017 at 11:34 PM







1 examples

1.1 second order, constant coeff.

1.1.1 second order, constant coeff. homogeneous

second order, constant coeff. homogeneous, one root repeated

$$y'' - 2y' + 1 = 0$$

Let $y = Ae^{rx}$ and plug into the above and simplify, we obtain the characteristic equation

$$\begin{aligned} r^2 - 2r + 1 &= 0 \\ (r - 1)^2 &= 0 \\ r &= 1 \end{aligned}$$

Repeated root. Hence the two L.I. basis solutions are

$$\begin{aligned} y_1 &= e^x \\ y_2 &= xe^x \end{aligned}$$

And the homogeneous solution is

$$\begin{aligned} y_h &= c_1y_1 + c_2y_2 \\ &= c_1e^x + c_2e^x \end{aligned}$$

second order, constant coeff. homogeneous, two real distinct roots

$$y'' + y' - 2y = 0$$

Let $y = Ae^{rx}$ and plug into the above and simplify, we obtain the characteristic equation

$$\begin{aligned} r^2 + r - 2 &= 0 \\ (r - 1)(r + 2) &= 0 \\ r_1 &= 1 \\ r_2 &= -2 \end{aligned}$$

Hence the L.I. basis solutions are

$$\begin{aligned} y_1 &= e^x \\ y_2 &= e^{-2x} \end{aligned}$$

And the homogeneous solution is

$$\begin{aligned} y_h &= c_1y_1 + c_2y_2 \\ &= c_1e^x + c_2e^{-2x} \end{aligned}$$

second order, constant coeff. homogeneous, two complex conjugate roots

$$y'' - 6y' + 13y = 0$$

Let $y = Ae^{rx}$ and plug into the above and simplify, we obtain the characteristic equation

$$r^2 - 6r + 13 = 0$$

Whose roots are

$$\begin{aligned} r_1 &= 3 + 2i \\ r_2 &= 3 - 2i \end{aligned}$$

Hence the L.I. basis solutions are

$$\begin{aligned}y_1 &= e^{(3+2i)x} \\y_2 &= e^{(3-2i)x}\end{aligned}$$

the homogeneous solution is

$$\begin{aligned}y_h &= c_1 y_1 + c_2 y_2 \\&= c_1 e^{(3+2i)x} + c_2 e^{(3-2i)x}\end{aligned}$$

This can be converted to real basis using Euler relation which results in

$$\begin{aligned}y_h &= C_1 e^{3x} \cos 2x + C_2 e^3 \sin 2x \\&= e^{3x} (C_1 \cos 2x + C_2 \sin 2x)\end{aligned}$$