How to use Mason rule to obtain transfer function of simple RLC electric circuit

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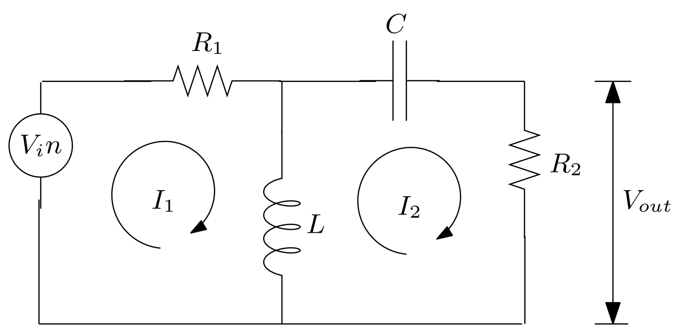
Contents

This is small example showing how to use Mason rule to find the transfer function $\frac{V_out(s)}{V_in(s)}$ of an RLC circuit.



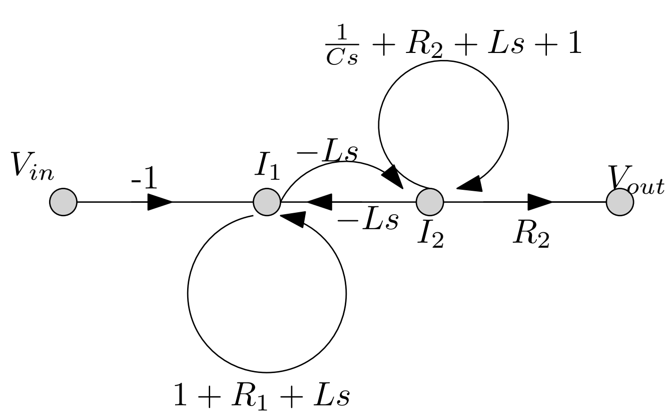
Solving the circuit loops ($V = Ri$) applied to each loop gives (all in done in Laplace domain)

$$
\begin{aligned}
(R_1 + sL) I_1 - I_2 Ls - V_{in}(s) &= 0 \\
\left(R_2 + \frac{1}{Cs}\right) I_2 + LsI_2 - I_1 Ls &= 0 \\
V_{out}(s) &= R_2 I_2
\end{aligned}
$$

The variables are I_1, I_2. In Mason, each variable goes to a node. Hence so we need to have each variable by on its own on the the LHS. To do this, do this trick: Add I_1 to each side of the first equation, and add I_2 to each side of the second equation, this gives

$$
\begin{aligned}
I_1 &= (R_1 + sL) I_1 - I_2 Ls - V_{in}(s) + I_1 \\
I_2 &= I_2 + \left(R_2 + \frac{1}{Cs}\right) I_2 + LsI_2 - I_1 Ls
\end{aligned}
$$

Now set up the signal graph, assign a node to each variable. The input and output go a node also. This is the result.



Now we Find $\frac{V_{out}}{V_{in}}$ for the above using Mason rule.

$$
\begin{aligned}
\frac{V_{out}}{V_{in}} &= \frac{\sum_{i=1}^{1} M_i \Delta_i}{1 - \sum \text{one at time} + \sum 2 \text{ at times}} \\
&= \frac{(-1)(-Ls)(R_2)}{1 - \sum (R_1 + Ls + 1) + \left(\frac{1}{Cs} + R_2 + Ls + 1\right) + \sum (R_1 + Ls + 1)\left(\frac{1}{Cs} + R_2 + Ls + 1\right)} \\
&= \frac{LsR_2}{1 - \left(R_1 + R_2 + \frac{1}{Cs} + 2Ls + 2\right) + (R_1 + Ls + 1)\left(R_2 + \frac{1}{Cs} + Ls + 1\right)} \\
&= \frac{LsR_2}{\frac{1}{Cs}(R_1 + Ls)\left(CLs^2 + CR_2 s + 1\right)}
\end{aligned}
$$