

How to use Mason rule to obtain transfer function of simple RLC electric circuit

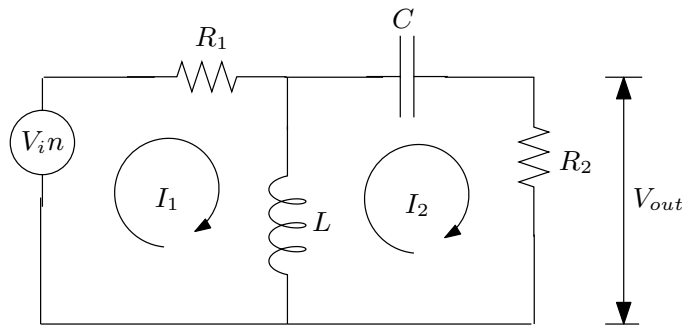
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This is small example showing how to use Mason rule to find the transfer function $\frac{V_{out}(s)}{V_{in}(s)}$ of an RLC circuit.



Solving the circuit loops ($V = Ri$) applied to each loop gives (all in done in Laplace domain)

$$(R_1 + sL) I_1 - I_2 L s - V_{in}(s) = 0$$

$$\left(R_2 + \frac{1}{Cs}\right) I_2 + Ls I_2 - I_1 L s = 0$$

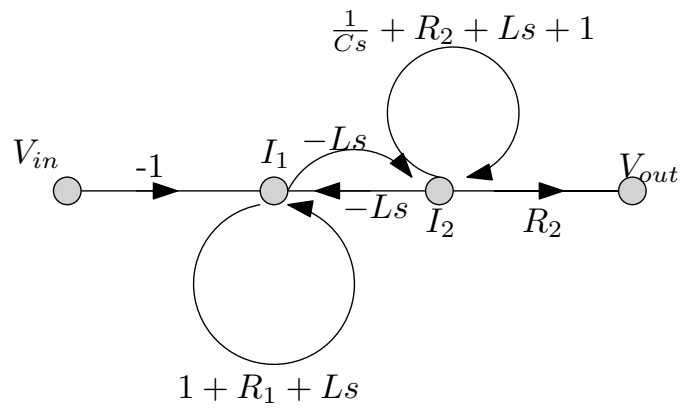
$$V_{out}(s) = R_2 I_2$$

The variables are I_1, I_2 . In Mason, each variable goes to a node. Hence so we need to have each variable by on its own on the the LHS. To do this, do this trick: Add I_1 to each side of the first equation, and add I_2 to each side of the second equation, this gives

$$I_1 = (R_1 + sL) I_1 - I_2 L s - V_{in}(s) + I_1$$

$$I_2 = I_2 + \left(R_2 + \frac{1}{Cs}\right) I_2 + Ls I_2 - I_1 L s$$

Now set up the signal graph, assign a node to each variable. The input and output go a node also. This is the result.



Now we Find $\frac{V_{out}}{V_{in}}$ for the above using Mason rule.

$$\begin{aligned}
 \frac{V_{out}}{V_{in}} &= \frac{\sum_{i=1}^1 M_i \Delta_i}{1 - \sum \text{one at time} + \sum 2 \text{ at times}} \\
 &= \frac{(-1)(-Ls)(R_2)}{1 - \sum (R_1 + Ls + 1) + \left(\frac{1}{Cs} + R_2 + Ls + 1\right) + \sum (R_1 + Ls + 1) \left(\frac{1}{Cs} + R_2 + Ls + 1\right)} \\
 &= \frac{LsR_2}{1 - \left(R_1 + R_2 + \frac{1}{Cs} + 2Ls + 2\right) + (R_1 + Ls + 1) \left(R_2 + \frac{1}{Cs} + Ls + 1\right)} \\
 &= \frac{LsR_2}{\frac{1}{Cs} (R_1 + Ls) (CLs^2 + CR_2s + 1)}
 \end{aligned}$$