

Table of Normalized Butterworth lowpass polynomials

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Using my own DSP mathematica package, the following table is generated for the normalized low pass Butterworth polynomials. In this table, the filter order N is give, then H(s) is given. The function used also returns the stable poles, but these are not displayed.

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In[159]:= << dsp`
          data = Table[dsp`getButterworthPolynomial [n, 1, s, form -> "quadratic"], {n, 1, 20}];
In[193]:= tbl = Table[{n, data[[n, 2]]}, {n, 1, Length[data]}];
          TableForm[N@tbl, TableHeadings -> {None, {"N", "H(s)"}, TableAlignments -> Left]
In[199]:= Do[{Print["N=", n, "\t", N@tbl[[n, 2]]]; Print[""]}, {n, 1, Length[data]}]
```

N=1
$$\frac{1.}{1. + s}$$

N=2
$$\frac{1.}{1. + 1.41421 s + s^2}$$

N=3
$$\frac{1.}{1. + s} - \frac{1. s}{1. + s + s^2}$$

N=4
$$\frac{-0.707107 - 0.92388 s}{1. + 0.765367 s + s^2} + \frac{1.70711 + 0.92388 s}{1. + 1.84776 s + s^2}$$

N=5
$$\frac{1.89443}{1. + s} + \frac{-1.78885 - 0.552786 s}{2. + 1.23607 s + 2. s^2} - \frac{3.23607 s}{2. + 3.23607 s + 2. s^2}$$

N=6
$$\frac{-2.1547 - 3.04721 s}{1. + 1.41421 s + s^2} + \frac{-1.1547 + 0.816497 s}{2. + 1.03528 s + 2. s^2} + \frac{7.4641 + 5.27792 s}{2. + 3.8637 s + 2. s^2}$$

N=7
$$\frac{4.31194}{1. + s} + \frac{0.736976 s}{1. + 0.445042 s + s^2} + \frac{-3.31194 - 2.06496 s}{1. + 1.24698 s + s^2} - \frac{2.98396 s}{1. + 1.80194 s + s^2}$$

$$N=8 \quad \frac{0.5 + 0.587938 s}{1. + 0.390181 s + s^2} + \frac{-6.06854 - 8.4173 s}{1. + 1.66294 s + s^2} + \frac{9.08221 + 7.13584 s}{1. + 1.96157 s + s^2} + \frac{-2.51367 + 0.69352 s}{1. + 1.11114 s + 1. s^2}$$

$$N=9 \quad \frac{10.7211}{1. + s} + \frac{0.666667 + 0.115765 s}{1. + 0.347296 s + s^2} + \frac{3.27432 s}{1. + s + s^2} - \frac{6.1537 s}{1. + 1.87939 s + s^2} + \frac{-10.3878 - 7.95753 s}{1. + 1.53209 s + 1. s^2}$$

$$N=10 \quad \frac{0.447214 - 0.371748 s}{1. + 0.312869 s + s^2} + \frac{2.8236 + 3.79773 s}{1. + 0.907981 s + s^2} - \frac{8.69013}{1. + 1.41421 s + s^2} + \frac{-17.0553 - 22.9394 s}{1. + 1.78201 s + s^2} + \frac{23.4747 + 19.5134 s}{1. + 1.97538 s + s^2}$$

$$N=11 \quad \frac{28.0831}{1. + s} - \frac{0.596885 s}{1. + 0.28463 s + s^2} + \frac{4.19411 + 1.7423 s}{1. + 0.83083 s + s^2} + \frac{10.795 s}{1. + 1.30972 s + s^2} + \frac{-31.2773 - 26.3121 s}{1. + 1.68251 s + s^2} - \frac{13.7115 s}{1. + 1.91899 s + s^2}$$

$$N=12 \quad \frac{-0.408248 - 0.458043 s}{1. + 0.261052 s + s^2} + \frac{3.10095 - 1.67822 s}{1. + 0.765367 s + s^2} + \frac{11.5729 + 16.2266 s}{1. + 1.21752 s + s^2} + \frac{-27.9395 - 5.1574 s}{1. + 1.58671 s + s^2} + \frac{-48.3926 - 63.228 s}{1. + 1.84776 s + s^2} + \frac{63.0665 + 54.2951 s}{1. + 1.98289 s + s^2}$$

$$N=13 \quad \frac{76.1374}{1. + s} + \frac{-0.5547 - 0.0668617 s}{1. + 0.241073 s + s^2} - \frac{4.2715 s}{1. + 0.70921 s + s^2} + \frac{18.5346 + 10.5288 s}{1. + 1.13613 s + s^2} + \frac{32.4079 s}{1. + 1.49702 s + s^2} + \frac{-93.1173 - 82.4512 s}{1. + 1.77091 s + s^2} - \frac{32.2846 s}{1. + 1.94188 s + s^2}$$

$$N=14 \quad \frac{-0.377964 + 0.333269 s}{1. + 0.223929 s + s^2} + \frac{-3.35453 - 4.27421 s}{1. + 0.660558 s + s^2} + \frac{14.6971 - 4.62508 s}{1. + 1.06406 s + s^2} + \frac{42.002 + 59.3998 s}{1. + 1.41421 s + s^2} + \frac{-87.2181 - 27.4469 s}{1. + 1.69345 s + s^2} + \frac{-138.807 - 176.862 s}{1. + 1.88777 s + s^2} + \frac{174.058 + 153.475 s}{1. + 1.98742 s + s^2}$$

$$N=15 \quad \frac{211.525}{1. + s} + \frac{0.513569 s}{1. + 0.209057 s + s^2} - \frac{20.018 s}{1. + s + s^2} + \frac{71.14 + 47.6019 s}{1. + 1.33826 s + s^2} - \frac{79.197 s}{1. + 1.9563 s + s^2} + \frac{-276.752 - 252.826 s}{1. + 1.82709 s + 1. s^2} + \frac{-9.82639 - 3.03652 s}{2. + 1.23607 s + 2. s^2} + \frac{187.836 s}{2. + 3.23607 s + 2. s^2}$$

$$N=16 \quad \frac{0.353553 + 0.386505 s}{1. + 0.196034 s + s^2} + \frac{-3.58969 + 2.39309 s}{1. + 0.580569 s + s^2} + \frac{-18.0466 - 24.4228 s}{1. + 0.942793 s + s^2} + \frac{59.4916 - 8.24656 s}{1. + 1.26879 s + s^2} + \frac{143.625 + 202.139 s}{1. + 1.54602 s + s^2} + \frac{-268.704 - 110.31 s}{1. + 1.76384 s + s^2} + \frac{-402.144 - 501.565 s}{1. + 1.91388 s + s^2} + \frac{490.014 + 439.625 s}{1. + 1.99037 s + s^2}$$

$$\begin{aligned}
 N=17 \quad & \frac{598.486}{1. + s} + \frac{0.485071 + 0.0447567 s}{1. + 0.184537 s + s^2} + \frac{5.03492 s}{1. + 0.547326 s + s^2} + \frac{-28.0035 - 12.4822 s}{1. + 0.891477 s + s^2} - \\
 & \frac{78.5424 s}{1. + 1.20527 s + s^2} + \frac{254.056 + 187.75 s}{1. + 1.47802 s + s^2} + \frac{268.593 s}{1. + 1.70043 s + s^2} + \frac{-824.024 - 768.379 s}{1. + 1.86494 s + s^2} - \frac{200.504 s}{1. + 1.96595 s + s^2}
 \end{aligned}$$

$$\begin{aligned}
 N=18 \quad & \frac{0.333333 - 0.303013 s}{1. + 0.174311 s + s^2} + \frac{-21.6077 + 10.4514 s}{1. + 0.845237 s + s^2} + \\
 & \frac{-80.641 - 112.311 s}{1. + 1.14715 s + s^2} + \frac{221.559}{1. + 1.41421 s + s^2} + \frac{475.135 + 661.735 s}{1. + 1.6383 s + s^2} + \frac{-822.959 - 398.056 s}{1. + 1.81262 s + s^2} + \\
 & \frac{1400.68 + 1273.27 s}{1. + 1.99239 s + s^2} + \frac{7.62003 + 9.3326 s}{2. + 1.03528 s + 2. s^2} + \frac{-2350.61 - 2878.9 s}{2. + 3.8637 s + 2. s^2}
 \end{aligned}$$

$$\begin{aligned}
 N=19 \quad & \frac{1717.48}{1. + s} - \frac{0.457264 s}{1. + 0.165159 s + s^2} + \frac{5.53727 + 1.35932 s}{1. + 0.490971 s + s^2} + \frac{30.3882 s}{1. + 0.803391 s + s^2} + \frac{-131.037 - 71.6704 s}{1. + 1.0939 s + s^2} - \\
 & \frac{280.824 s}{1. + 1.35456 s + s^2} + \frac{870.183 + 686.696 s}{1. + 1.57828 s + s^2} + \frac{765.303 s}{1. + 1.75895 s + s^2} + \frac{-2461.16 - 2327.81 s}{1. + 1.89163 s + s^2} - \frac{520.464 s}{1. + 1.97272 s + s^2}
 \end{aligned}$$

$$\begin{aligned}
 N=20 \quad & \frac{-0.316228 - 0.340064 s}{1. + 0.156918 s + s^2} + \frac{4.01805 - 2.96904 s}{1. + 0.466891 s + s^2} + \frac{25.369 + 33.1462 s}{1. + 0.765367 s + s^2} + \\
 & \frac{-105.669 + 34.8859 s}{1. + 1.045 s + s^2} + \frac{-325.217 - 458.509 s}{1. + 1.2989 s + s^2} + \frac{785.144 + 87.1179 s}{1. + 1.52081 s + s^2} + \\
 & \frac{1540.93 + 2118.99 s}{1. + 1.70528 s + s^2} + \frac{-2514.57 - 1360.88 s}{1. + 1.84776 s + s^2} + \frac{-3461.01 - 4173.34 s}{1. + 1.94474 s + s^2} + \frac{4052.32 + 3721.89 s}{1. + 1.99383 s + s^2}
 \end{aligned}$$