

# Kamke differential equations. Mathematica 12 and Maple 2019

Nasser M. Abbasi

April 22, 2019      Compiled on April 22, 2019 at 12:13am

## Contents

|          |                                            |           |
|----------|--------------------------------------------|-----------|
| <b>1</b> | <b>Introduction and summary of results</b> | <b>31</b> |
| <b>2</b> | <b>Problems table lookup</b>               | <b>34</b> |
| 2.1      | ODE No. 1 . . . . .                        | 71        |
| 2.2      | ODE No. 2 . . . . .                        | 71        |
| 2.3      | ODE No. 3 . . . . .                        | 72        |
| 2.4      | ODE No. 4 . . . . .                        | 72        |
| 2.5      | ODE No. 5 . . . . .                        | 73        |
| 2.6      | ODE No. 6 . . . . .                        | 73        |
| 2.7      | ODE No. 7 . . . . .                        | 74        |
| 2.8      | ODE No. 8 . . . . .                        | 74        |
| 2.9      | ODE No. 9 . . . . .                        | 75        |
| 2.10     | ODE No. 10 . . . . .                       | 76        |
| 2.11     | ODE No. 11 . . . . .                       | 76        |
| 2.12     | ODE No. 12 . . . . .                       | 77        |
| 2.13     | ODE No. 13 . . . . .                       | 78        |
| 2.14     | ODE No. 14 . . . . .                       | 79        |
| 2.15     | ODE No. 15 . . . . .                       | 80        |
| 2.16     | ODE No. 16 . . . . .                       | 81        |
| 2.17     | ODE No. 17 . . . . .                       | 83        |
| 2.18     | ODE No. 18 . . . . .                       | 84        |
| 2.19     | ODE No. 19 . . . . .                       | 85        |
| 2.20     | ODE No. 20 . . . . .                       | 85        |
| 2.21     | ODE No. 21 . . . . .                       | 86        |
| 2.22     | ODE No. 22 . . . . .                       | 87        |
| 2.23     | ODE No. 23 . . . . .                       | 89        |
| 2.24     | ODE No. 24 . . . . .                       | 90        |
| 2.25     | ODE No. 25 . . . . .                       | 92        |
| 2.26     | ODE No. 26 . . . . .                       | 92        |
| 2.27     | ODE No. 27 . . . . .                       | 93        |
| 2.28     | ODE No. 28 . . . . .                       | 95        |
| 2.29     | ODE No. 29 . . . . .                       | 96        |
| 2.30     | ODE No. 30 . . . . .                       | 97        |
| 2.31     | ODE No. 31 . . . . .                       | 100       |
| 2.32     | ODE No. 32 . . . . .                       | 101       |
| 2.33     | ODE No. 33 . . . . .                       | 102       |
| 2.34     | ODE No. 34 . . . . .                       | 103       |
| 2.35     | ODE No. 35 . . . . .                       | 104       |
| 2.36     | ODE No. 36 . . . . .                       | 105       |
| 2.37     | ODE No. 37 . . . . .                       | 106       |
| 2.38     | ODE No. 38 . . . . .                       | 107       |
| 2.39     | ODE No. 39 . . . . .                       | 108       |
| 2.40     | ODE No. 40 . . . . .                       | 108       |
| 2.41     | ODE No. 41 . . . . .                       | 108       |
| 2.42     | ODE No. 42 . . . . .                       | 108       |
| 2.43     | ODE No. 43 . . . . .                       | 109       |
| 2.44     | ODE No. 44 . . . . .                       | 109       |
| 2.45     | ODE No. 45 . . . . .                       | 110       |
| 2.46     | ODE No. 46 . . . . .                       | 110       |
| 2.47     | ODE No. 47 . . . . .                       | 110       |
| 2.48     | ODE No. 48 . . . . .                       | 110       |
| 2.49     | ODE No. 49 . . . . .                       | 111       |
| 2.50     | ODE No. 50 . . . . .                       | 111       |

|       |             |     |
|-------|-------------|-----|
| 2.51  | ODE No. 51  | 111 |
| 2.52  | ODE No. 52  | 111 |
| 2.53  | ODE No. 53  | 111 |
| 2.54  | ODE No. 54  | 112 |
| 2.55  | ODE No. 55  | 112 |
| 2.56  | ODE No. 56  | 112 |
| 2.57  | ODE No. 57  | 112 |
| 2.58  | ODE No. 58  | 112 |
| 2.59  | ODE No. 59  | 113 |
| 2.60  | ODE No. 60  | 113 |
| 2.61  | ODE No. 61  | 114 |
| 2.62  | ODE No. 62  | 114 |
| 2.63  | ODE No. 63  | 116 |
| 2.64  | ODE No. 64  | 116 |
| 2.65  | ODE No. 65  | 117 |
| 2.66  | ODE No. 66  | 117 |
| 2.67  | ODE No. 67  | 117 |
| 2.68  | ODE No. 68  | 117 |
| 2.69  | ODE No. 69  | 118 |
| 2.70  | ODE No. 70  | 118 |
| 2.71  | ODE No. 71  | 118 |
| 2.72  | ODE No. 72  | 118 |
| 2.73  | ODE No. 73  | 119 |
| 2.74  | ODE No. 74  | 119 |
| 2.75  | ODE No. 75  | 119 |
| 2.76  | ODE No. 76  | 120 |
| 2.77  | ODE No. 77  | 121 |
| 2.78  | ODE No. 78  | 122 |
| 2.79  | ODE No. 79  | 124 |
| 2.80  | ODE No. 80  | 124 |
| 2.81  | ODE No. 81  | 124 |
| 2.82  | ODE No. 82  | 124 |
| 2.83  | ODE No. 83  | 125 |
| 2.84  | ODE No. 84  | 125 |
| 2.85  | ODE No. 85  | 125 |
| 2.86  | ODE No. 86  | 125 |
| 2.87  | ODE No. 87  | 126 |
| 2.88  | ODE No. 88  | 126 |
| 2.89  | ODE No. 89  | 127 |
| 2.90  | ODE No. 90  | 128 |
| 2.91  | ODE No. 91  | 129 |
| 2.92  | ODE No. 92  | 129 |
| 2.93  | ODE No. 93  | 129 |
| 2.94  | ODE No. 94  | 129 |
| 2.95  | ODE No. 95  | 130 |
| 2.96  | ODE No. 96  | 131 |
| 2.97  | ODE No. 97  | 132 |
| 2.98  | ODE No. 98  | 133 |
| 2.99  | ODE No. 99  | 133 |
| 2.100 | ODE No. 100 | 133 |
| 2.101 | ODE No. 101 | 134 |
| 2.102 | ODE No. 102 | 135 |
| 2.103 | ODE No. 103 | 136 |
| 2.104 | ODE No. 104 | 138 |
| 2.105 | ODE No. 105 | 139 |
| 2.106 | ODE No. 106 | 139 |
| 2.107 | ODE No. 107 | 139 |
| 2.108 | ODE No. 108 | 140 |
| 2.109 | ODE No. 109 | 141 |
| 2.110 | ODE No. 110 | 142 |
| 2.111 | ODE No. 111 | 143 |
| 2.112 | ODE No. 112 | 143 |
| 2.113 | ODE No. 113 | 144 |
| 2.114 | ODE No. 114 | 144 |
| 2.115 | ODE No. 115 | 145 |
| 2.116 | ODE No. 116 | 146 |

|       |                       |     |
|-------|-----------------------|-----|
| 2.117 | ODE No. 117 . . . . . | 146 |
| 2.118 | ODE No. 118 . . . . . | 147 |
| 2.119 | ODE No. 119 . . . . . | 147 |
| 2.120 | ODE No. 120 . . . . . | 147 |
| 2.121 | ODE No. 121 . . . . . | 147 |
| 2.122 | ODE No. 122 . . . . . | 147 |
| 2.123 | ODE No. 123 . . . . . | 148 |
| 2.124 | ODE No. 124 . . . . . | 148 |
| 2.125 | ODE No. 125 . . . . . | 148 |
| 2.126 | ODE No. 126 . . . . . | 148 |
| 2.127 | ODE No. 127 . . . . . | 148 |
| 2.128 | ODE No. 128 . . . . . | 149 |
| 2.129 | ODE No. 129 . . . . . | 149 |
| 2.130 | ODE No. 130 . . . . . | 149 |
| 2.131 | ODE No. 131 . . . . . | 149 |
| 2.132 | ODE No. 132 . . . . . | 149 |
| 2.133 | ODE No. 133 . . . . . | 150 |
| 2.134 | ODE No. 134 . . . . . | 150 |
| 2.135 | ODE No. 135 . . . . . | 150 |
| 2.136 | ODE No. 136 . . . . . | 150 |
| 2.137 | ODE No. 137 . . . . . | 150 |
| 2.138 | ODE No. 138 . . . . . | 151 |
| 2.139 | ODE No. 139 . . . . . | 151 |
| 2.140 | ODE No. 140 . . . . . | 151 |
| 2.141 | ODE No. 141 . . . . . | 151 |
| 2.142 | ODE No. 142 . . . . . | 151 |
| 2.143 | ODE No. 143 . . . . . | 152 |
| 2.144 | ODE No. 144 . . . . . | 152 |
| 2.145 | ODE No. 145 . . . . . | 152 |
| 2.146 | ODE No. 146 . . . . . | 152 |
| 2.147 | ODE No. 147 . . . . . | 153 |
| 2.148 | ODE No. 148 . . . . . | 153 |
| 2.149 | ODE No. 149 . . . . . | 153 |
| 2.150 | ODE No. 150 . . . . . | 153 |
| 2.151 | ODE No. 151 . . . . . | 153 |
| 2.152 | ODE No. 152 . . . . . | 154 |
| 2.153 | ODE No. 153 . . . . . | 154 |
| 2.154 | ODE No. 154 . . . . . | 154 |
| 2.155 | ODE No. 155 . . . . . | 154 |
| 2.156 | ODE No. 156 . . . . . | 154 |
| 2.157 | ODE No. 157 . . . . . | 155 |
| 2.158 | ODE No. 158 . . . . . | 155 |
| 2.159 | ODE No. 159 . . . . . | 155 |
| 2.160 | ODE No. 160 . . . . . | 155 |
| 2.161 | ODE No. 161 . . . . . | 155 |
| 2.162 | ODE No. 162 . . . . . | 156 |
| 2.163 | ODE No. 163 . . . . . | 156 |
| 2.164 | ODE No. 164 . . . . . | 156 |
| 2.165 | ODE No. 165 . . . . . | 156 |
| 2.166 | ODE No. 166 . . . . . | 157 |
| 2.167 | ODE No. 167 . . . . . | 157 |
| 2.168 | ODE No. 168 . . . . . | 157 |
| 2.169 | ODE No. 169 . . . . . | 157 |
| 2.170 | ODE No. 170 . . . . . | 158 |
| 2.171 | ODE No. 171 . . . . . | 158 |
| 2.172 | ODE No. 172 . . . . . | 158 |
| 2.173 | ODE No. 173 . . . . . | 158 |
| 2.174 | ODE No. 174 . . . . . | 158 |
| 2.175 | ODE No. 175 . . . . . | 159 |
| 2.176 | ODE No. 176 . . . . . | 159 |
| 2.177 | ODE No. 177 . . . . . | 159 |
| 2.178 | ODE No. 178 . . . . . | 159 |
| 2.179 | ODE No. 179 . . . . . | 159 |
| 2.180 | ODE No. 180 . . . . . | 160 |
| 2.181 | ODE No. 181 . . . . . | 160 |
| 2.182 | ODE No. 182 . . . . . | 160 |

|       |                       |     |
|-------|-----------------------|-----|
| 2.183 | ODE No. 183 . . . . . | 160 |
| 2.184 | ODE No. 184 . . . . . | 161 |
| 2.185 | ODE No. 185 . . . . . | 161 |
| 2.186 | ODE No. 186 . . . . . | 161 |
| 2.187 | ODE No. 187 . . . . . | 161 |
| 2.188 | ODE No. 188 . . . . . | 162 |
| 2.189 | ODE No. 189 . . . . . | 162 |
| 2.190 | ODE No. 190 . . . . . | 162 |
| 2.191 | ODE No. 191 . . . . . | 162 |
| 2.192 | ODE No. 192 . . . . . | 162 |
| 2.193 | ODE No. 193 . . . . . | 163 |
| 2.194 | ODE No. 194 . . . . . | 163 |
| 2.195 | ODE No. 195 . . . . . | 163 |
| 2.196 | ODE No. 196 . . . . . | 163 |
| 2.197 | ODE No. 197 . . . . . | 164 |
| 2.198 | ODE No. 198 . . . . . | 164 |
| 2.199 | ODE No. 199 . . . . . | 164 |
| 2.200 | ODE No. 200 . . . . . | 164 |
| 2.201 | ODE No. 201 . . . . . | 164 |
| 2.202 | ODE No. 202 . . . . . | 165 |
| 2.203 | ODE No. 203 . . . . . | 165 |
| 2.204 | ODE No. 204 . . . . . | 165 |
| 2.205 | ODE No. 205 . . . . . | 165 |
| 2.206 | ODE No. 206 . . . . . | 165 |
| 2.207 | ODE No. 207 . . . . . | 165 |
| 2.208 | ODE No. 208 . . . . . | 166 |
| 2.209 | ODE No. 209 . . . . . | 166 |
| 2.210 | ODE No. 210 . . . . . | 166 |
| 2.211 | ODE No. 211 . . . . . | 166 |
| 2.212 | ODE No. 212 . . . . . | 166 |
| 2.213 | ODE No. 213 . . . . . | 167 |
| 2.214 | ODE No. 214 . . . . . | 167 |
| 2.215 | ODE No. 215 . . . . . | 167 |
| 2.216 | ODE No. 216 . . . . . | 167 |
| 2.217 | ODE No. 217 . . . . . | 168 |
| 2.218 | ODE No. 218 . . . . . | 168 |
| 2.219 | ODE No. 219 . . . . . | 168 |
| 2.220 | ODE No. 220 . . . . . | 168 |
| 2.221 | ODE No. 221 . . . . . | 168 |
| 2.222 | ODE No. 222 . . . . . | 169 |
| 2.223 | ODE No. 223 . . . . . | 169 |
| 2.224 | ODE No. 224 . . . . . | 169 |
| 2.225 | ODE No. 225 . . . . . | 169 |
| 2.226 | ODE No. 226 . . . . . | 169 |
| 2.227 | ODE No. 227 . . . . . | 170 |
| 2.228 | ODE No. 228 . . . . . | 170 |
| 2.229 | ODE No. 229 . . . . . | 170 |
| 2.230 | ODE No. 230 . . . . . | 170 |
| 2.231 | ODE No. 231 . . . . . | 171 |
| 2.232 | ODE No. 232 . . . . . | 171 |
| 2.233 | ODE No. 233 . . . . . | 171 |
| 2.234 | ODE No. 234 . . . . . | 171 |
| 2.235 | ODE No. 235 . . . . . | 171 |
| 2.236 | ODE No. 236 . . . . . | 172 |
| 2.237 | ODE No. 237 . . . . . | 172 |
| 2.238 | ODE No. 238 . . . . . | 172 |
| 2.239 | ODE No. 239 . . . . . | 172 |
| 2.240 | ODE No. 240 . . . . . | 173 |
| 2.241 | ODE No. 241 . . . . . | 173 |
| 2.242 | ODE No. 242 . . . . . | 173 |
| 2.243 | ODE No. 243 . . . . . | 173 |
| 2.244 | ODE No. 244 . . . . . | 173 |
| 2.245 | ODE No. 245 . . . . . | 174 |
| 2.246 | ODE No. 246 . . . . . | 174 |
| 2.247 | ODE No. 247 . . . . . | 174 |
| 2.248 | ODE No. 248 . . . . . | 174 |

|       |                       |     |
|-------|-----------------------|-----|
| 2.249 | ODE No. 249 . . . . . | 175 |
| 2.250 | ODE No. 250 . . . . . | 175 |
| 2.251 | ODE No. 251 . . . . . | 175 |
| 2.252 | ODE No. 252 . . . . . | 175 |
| 2.253 | ODE No. 253 . . . . . | 175 |
| 2.254 | ODE No. 254 . . . . . | 176 |
| 2.255 | ODE No. 255 . . . . . | 176 |
| 2.256 | ODE No. 256 . . . . . | 176 |
| 2.257 | ODE No. 257 . . . . . | 176 |
| 2.258 | ODE No. 258 . . . . . | 177 |
| 2.259 | ODE No. 259 . . . . . | 177 |
| 2.260 | ODE No. 260 . . . . . | 177 |
| 2.261 | ODE No. 261 . . . . . | 177 |
| 2.262 | ODE No. 262 . . . . . | 177 |
| 2.263 | ODE No. 263 . . . . . | 178 |
| 2.264 | ODE No. 264 . . . . . | 178 |
| 2.265 | ODE No. 265 . . . . . | 178 |
| 2.266 | ODE No. 266 . . . . . | 178 |
| 2.267 | ODE No. 267 . . . . . | 178 |
| 2.268 | ODE No. 268 . . . . . | 179 |
| 2.269 | ODE No. 269 . . . . . | 179 |
| 2.270 | ODE No. 270 . . . . . | 179 |
| 2.271 | ODE No. 271 . . . . . | 179 |
| 2.272 | ODE No. 272 . . . . . | 180 |
| 2.273 | ODE No. 273 . . . . . | 180 |
| 2.274 | ODE No. 274 . . . . . | 180 |
| 2.275 | ODE No. 275 . . . . . | 180 |
| 2.276 | ODE No. 276 . . . . . | 181 |
| 2.277 | ODE No. 277 . . . . . | 181 |
| 2.278 | ODE No. 278 . . . . . | 181 |
| 2.279 | ODE No. 279 . . . . . | 181 |
| 2.280 | ODE No. 280 . . . . . | 181 |
| 2.281 | ODE No. 281 . . . . . | 182 |
| 2.282 | ODE No. 282 . . . . . | 182 |
| 2.283 | ODE No. 283 . . . . . | 182 |
| 2.284 | ODE No. 284 . . . . . | 182 |
| 2.285 | ODE No. 285 . . . . . | 183 |
| 2.286 | ODE No. 286 . . . . . | 183 |
| 2.287 | ODE No. 287 . . . . . | 183 |
| 2.288 | ODE No. 288 . . . . . | 183 |
| 2.289 | ODE No. 289 . . . . . | 184 |
| 2.290 | ODE No. 290 . . . . . | 184 |
| 2.291 | ODE No. 291 . . . . . | 184 |
| 2.292 | ODE No. 292 . . . . . | 184 |
| 2.293 | ODE No. 293 . . . . . | 185 |
| 2.294 | ODE No. 294 . . . . . | 185 |
| 2.295 | ODE No. 295 . . . . . | 185 |
| 2.296 | ODE No. 296 . . . . . | 185 |
| 2.297 | ODE No. 297 . . . . . | 185 |
| 2.298 | ODE No. 298 . . . . . | 186 |
| 2.299 | ODE No. 299 . . . . . | 186 |
| 2.300 | ODE No. 300 . . . . . | 186 |
| 2.301 | ODE No. 301 . . . . . | 186 |
| 2.302 | ODE No. 302 . . . . . | 187 |
| 2.303 | ODE No. 303 . . . . . | 187 |
| 2.304 | ODE No. 304 . . . . . | 187 |
| 2.305 | ODE No. 305 . . . . . | 187 |
| 2.306 | ODE No. 306 . . . . . | 188 |
| 2.307 | ODE No. 307 . . . . . | 188 |
| 2.308 | ODE No. 308 . . . . . | 188 |
| 2.309 | ODE No. 309 . . . . . | 188 |
| 2.310 | ODE No. 310 . . . . . | 188 |
| 2.311 | ODE No. 311 . . . . . | 189 |
| 2.312 | ODE No. 312 . . . . . | 189 |
| 2.313 | ODE No. 313 . . . . . | 189 |
| 2.314 | ODE No. 314 . . . . . | 189 |

|       |                       |     |
|-------|-----------------------|-----|
| 2.315 | ODE No. 315 . . . . . | 190 |
| 2.316 | ODE No. 316 . . . . . | 190 |
| 2.317 | ODE No. 317 . . . . . | 190 |
| 2.318 | ODE No. 318 . . . . . | 190 |
| 2.319 | ODE No. 319 . . . . . | 191 |
| 2.320 | ODE No. 320 . . . . . | 191 |
| 2.321 | ODE No. 321 . . . . . | 191 |
| 2.322 | ODE No. 322 . . . . . | 191 |
| 2.323 | ODE No. 323 . . . . . | 192 |
| 2.324 | ODE No. 324 . . . . . | 192 |
| 2.325 | ODE No. 325 . . . . . | 192 |
| 2.326 | ODE No. 326 . . . . . | 192 |
| 2.327 | ODE No. 327 . . . . . | 193 |
| 2.328 | ODE No. 328 . . . . . | 193 |
| 2.329 | ODE No. 329 . . . . . | 193 |
| 2.330 | ODE No. 330 . . . . . | 193 |
| 2.331 | ODE No. 331 . . . . . | 193 |
| 2.332 | ODE No. 332 . . . . . | 194 |
| 2.333 | ODE No. 333 . . . . . | 194 |
| 2.334 | ODE No. 334 . . . . . | 194 |
| 2.335 | ODE No. 335 . . . . . | 194 |
| 2.336 | ODE No. 336 . . . . . | 194 |
| 2.337 | ODE No. 337 . . . . . | 195 |
| 2.338 | ODE No. 338 . . . . . | 195 |
| 2.339 | ODE No. 339 . . . . . | 195 |
| 2.340 | ODE No. 340 . . . . . | 195 |
| 2.341 | ODE No. 341 . . . . . | 195 |
| 2.342 | ODE No. 342 . . . . . | 196 |
| 2.343 | ODE No. 343 . . . . . | 196 |
| 2.344 | ODE No. 344 . . . . . | 196 |
| 2.345 | ODE No. 345 . . . . . | 196 |
| 2.346 | ODE No. 346 . . . . . | 196 |
| 2.347 | ODE No. 347 . . . . . | 197 |
| 2.348 | ODE No. 348 . . . . . | 197 |
| 2.349 | ODE No. 349 . . . . . | 197 |
| 2.350 | ODE No. 350 . . . . . | 197 |
| 2.351 | ODE No. 351 . . . . . | 197 |
| 2.352 | ODE No. 352 . . . . . | 198 |
| 2.353 | ODE No. 353 . . . . . | 198 |
| 2.354 | ODE No. 354 . . . . . | 198 |
| 2.355 | ODE No. 355 . . . . . | 198 |
| 2.356 | ODE No. 356 . . . . . | 198 |
| 2.357 | ODE No. 357 . . . . . | 199 |
| 2.358 | ODE No. 358 . . . . . | 199 |
| 2.359 | ODE No. 359 . . . . . | 199 |
| 2.360 | ODE No. 360 . . . . . | 199 |
| 2.361 | ODE No. 361 . . . . . | 199 |
| 2.362 | ODE No. 362 . . . . . | 200 |
| 2.363 | ODE No. 363 . . . . . | 200 |
| 2.364 | ODE No. 364 . . . . . | 200 |
| 2.365 | ODE No. 365 . . . . . | 200 |
| 2.366 | ODE No. 366 . . . . . | 200 |
| 2.367 | ODE No. 367 . . . . . | 201 |
| 2.368 | ODE No. 368 . . . . . | 201 |
| 2.369 | ODE No. 369 . . . . . | 201 |
| 2.370 | ODE No. 370 . . . . . | 201 |
| 2.371 | ODE No. 371 . . . . . | 201 |
| 2.372 | ODE No. 372 . . . . . | 201 |
| 2.373 | ODE No. 373 . . . . . | 202 |
| 2.374 | ODE No. 374 . . . . . | 202 |
| 2.375 | ODE No. 375 . . . . . | 202 |
| 2.376 | ODE No. 376 . . . . . | 202 |
| 2.377 | ODE No. 377 . . . . . | 202 |
| 2.378 | ODE No. 378 . . . . . | 203 |
| 2.379 | ODE No. 379 . . . . . | 203 |
| 2.380 | ODE No. 380 . . . . . | 203 |

|       |                       |     |
|-------|-----------------------|-----|
| 2.381 | ODE No. 381 . . . . . | 203 |
| 2.382 | ODE No. 382 . . . . . | 204 |
| 2.383 | ODE No. 383 . . . . . | 204 |
| 2.384 | ODE No. 384 . . . . . | 204 |
| 2.385 | ODE No. 385 . . . . . | 204 |
| 2.386 | ODE No. 386 . . . . . | 205 |
| 2.387 | ODE No. 387 . . . . . | 205 |
| 2.388 | ODE No. 388 . . . . . | 205 |
| 2.389 | ODE No. 389 . . . . . | 205 |
| 2.390 | ODE No. 390 . . . . . | 206 |
| 2.391 | ODE No. 391 . . . . . | 206 |
| 2.392 | ODE No. 392 . . . . . | 206 |
| 2.393 | ODE No. 393 . . . . . | 206 |
| 2.394 | ODE No. 394 . . . . . | 207 |
| 2.395 | ODE No. 395 . . . . . | 207 |
| 2.396 | ODE No. 396 . . . . . | 207 |
| 2.397 | ODE No. 397 . . . . . | 207 |
| 2.398 | ODE No. 398 . . . . . | 207 |
| 2.399 | ODE No. 399 . . . . . | 208 |
| 2.400 | ODE No. 400 . . . . . | 208 |
| 2.401 | ODE No. 401 . . . . . | 208 |
| 2.402 | ODE No. 402 . . . . . | 208 |
| 2.403 | ODE No. 403 . . . . . | 209 |
| 2.404 | ODE No. 404 . . . . . | 209 |
| 2.405 | ODE No. 405 . . . . . | 209 |
| 2.406 | ODE No. 406 . . . . . | 209 |
| 2.407 | ODE No. 407 . . . . . | 210 |
| 2.408 | ODE No. 408 . . . . . | 210 |
| 2.409 | ODE No. 409 . . . . . | 210 |
| 2.410 | ODE No. 410 . . . . . | 210 |
| 2.411 | ODE No. 411 . . . . . | 210 |
| 2.412 | ODE No. 412 . . . . . | 211 |
| 2.413 | ODE No. 413 . . . . . | 211 |
| 2.414 | ODE No. 414 . . . . . | 211 |
| 2.415 | ODE No. 415 . . . . . | 211 |
| 2.416 | ODE No. 416 . . . . . | 212 |
| 2.417 | ODE No. 417 . . . . . | 212 |
| 2.418 | ODE No. 418 . . . . . | 212 |
| 2.419 | ODE No. 419 . . . . . | 212 |
| 2.420 | ODE No. 420 . . . . . | 213 |
| 2.421 | ODE No. 421 . . . . . | 213 |
| 2.422 | ODE No. 422 . . . . . | 213 |
| 2.423 | ODE No. 423 . . . . . | 213 |
| 2.424 | ODE No. 424 . . . . . | 213 |
| 2.425 | ODE No. 425 . . . . . | 214 |
| 2.426 | ODE No. 426 . . . . . | 214 |
| 2.427 | ODE No. 427 . . . . . | 214 |
| 2.428 | ODE No. 428 . . . . . | 214 |
| 2.429 | ODE No. 429 . . . . . | 214 |
| 2.430 | ODE No. 430 . . . . . | 215 |
| 2.431 | ODE No. 431 . . . . . | 215 |
| 2.432 | ODE No. 432 . . . . . | 215 |
| 2.433 | ODE No. 433 . . . . . | 215 |
| 2.434 | ODE No. 434 . . . . . | 216 |
| 2.435 | ODE No. 435 . . . . . | 216 |
| 2.436 | ODE No. 436 . . . . . | 216 |
| 2.437 | ODE No. 437 . . . . . | 216 |
| 2.438 | ODE No. 438 . . . . . | 216 |
| 2.439 | ODE No. 439 . . . . . | 217 |
| 2.440 | ODE No. 440 . . . . . | 217 |
| 2.441 | ODE No. 441 . . . . . | 217 |
| 2.442 | ODE No. 442 . . . . . | 217 |
| 2.443 | ODE No. 443 . . . . . | 217 |
| 2.444 | ODE No. 444 . . . . . | 218 |
| 2.445 | ODE No. 445 . . . . . | 218 |
| 2.446 | ODE No. 446 . . . . . | 218 |

|       |                       |     |
|-------|-----------------------|-----|
| 2.447 | ODE No. 447 . . . . . | 218 |
| 2.448 | ODE No. 448 . . . . . | 218 |
| 2.449 | ODE No. 449 . . . . . | 219 |
| 2.450 | ODE No. 450 . . . . . | 219 |
| 2.451 | ODE No. 451 . . . . . | 219 |
| 2.452 | ODE No. 452 . . . . . | 219 |
| 2.453 | ODE No. 453 . . . . . | 219 |
| 2.454 | ODE No. 454 . . . . . | 220 |
| 2.455 | ODE No. 455 . . . . . | 220 |
| 2.456 | ODE No. 456 . . . . . | 220 |
| 2.457 | ODE No. 457 . . . . . | 220 |
| 2.458 | ODE No. 458 . . . . . | 221 |
| 2.459 | ODE No. 459 . . . . . | 221 |
| 2.460 | ODE No. 460 . . . . . | 221 |
| 2.461 | ODE No. 461 . . . . . | 221 |
| 2.462 | ODE No. 462 . . . . . | 221 |
| 2.463 | ODE No. 463 . . . . . | 222 |
| 2.464 | ODE No. 464 . . . . . | 222 |
| 2.465 | ODE No. 465 . . . . . | 222 |
| 2.466 | ODE No. 466 . . . . . | 222 |
| 2.467 | ODE No. 467 . . . . . | 222 |
| 2.468 | ODE No. 468 . . . . . | 223 |
| 2.469 | ODE No. 469 . . . . . | 223 |
| 2.470 | ODE No. 470 . . . . . | 223 |
| 2.471 | ODE No. 471 . . . . . | 224 |
| 2.472 | ODE No. 472 . . . . . | 224 |
| 2.473 | ODE No. 473 . . . . . | 224 |
| 2.474 | ODE No. 474 . . . . . | 224 |
| 2.475 | ODE No. 475 . . . . . | 224 |
| 2.476 | ODE No. 476 . . . . . | 225 |
| 2.477 | ODE No. 477 . . . . . | 225 |
| 2.478 | ODE No. 478 . . . . . | 225 |
| 2.479 | ODE No. 479 . . . . . | 225 |
| 2.480 | ODE No. 480 . . . . . | 226 |
| 2.481 | ODE No. 481 . . . . . | 226 |
| 2.482 | ODE No. 482 . . . . . | 226 |
| 2.483 | ODE No. 483 . . . . . | 226 |
| 2.484 | ODE No. 484 . . . . . | 226 |
| 2.485 | ODE No. 485 . . . . . | 226 |
| 2.486 | ODE No. 486 . . . . . | 227 |
| 2.487 | ODE No. 487 . . . . . | 227 |
| 2.488 | ODE No. 488 . . . . . | 227 |
| 2.489 | ODE No. 489 . . . . . | 227 |
| 2.490 | ODE No. 490 . . . . . | 227 |
| 2.491 | ODE No. 491 . . . . . | 228 |
| 2.492 | ODE No. 492 . . . . . | 228 |
| 2.493 | ODE No. 493 . . . . . | 228 |
| 2.494 | ODE No. 494 . . . . . | 228 |
| 2.495 | ODE No. 495 . . . . . | 228 |
| 2.496 | ODE No. 496 . . . . . | 229 |
| 2.497 | ODE No. 497 . . . . . | 229 |
| 2.498 | ODE No. 498 . . . . . | 229 |
| 2.499 | ODE No. 499 . . . . . | 229 |
| 2.500 | ODE No. 500 . . . . . | 229 |
| 2.501 | ODE No. 501 . . . . . | 230 |
| 2.502 | ODE No. 502 . . . . . | 230 |
| 2.503 | ODE No. 503 . . . . . | 230 |
| 2.504 | ODE No. 504 . . . . . | 230 |
| 2.505 | ODE No. 505 . . . . . | 230 |
| 2.506 | ODE No. 506 . . . . . | 231 |
| 2.507 | ODE No. 507 . . . . . | 231 |
| 2.508 | ODE No. 508 . . . . . | 231 |
| 2.509 | ODE No. 509 . . . . . | 231 |
| 2.510 | ODE No. 510 . . . . . | 231 |
| 2.511 | ODE No. 511 . . . . . | 232 |
| 2.512 | ODE No. 512 . . . . . | 232 |



|       |                       |     |
|-------|-----------------------|-----|
| 2.513 | ODE No. 513 . . . . . | 232 |
| 2.514 | ODE No. 514 . . . . . | 232 |
| 2.515 | ODE No. 515 . . . . . | 233 |
| 2.516 | ODE No. 516 . . . . . | 233 |
| 2.517 | ODE No. 517 . . . . . | 233 |
| 2.518 | ODE No. 518 . . . . . | 234 |
| 2.519 | ODE No. 519 . . . . . | 234 |
| 2.520 | ODE No. 520 . . . . . | 234 |
| 2.521 | ODE No. 521 . . . . . | 234 |
| 2.522 | ODE No. 522 . . . . . | 235 |
| 2.523 | ODE No. 523 . . . . . | 235 |
| 2.524 | ODE No. 524 . . . . . | 235 |
| 2.525 | ODE No. 525 . . . . . | 235 |
| 2.526 | ODE No. 526 . . . . . | 235 |
| 2.527 | ODE No. 527 . . . . . | 236 |
| 2.528 | ODE No. 528 . . . . . | 236 |
| 2.529 | ODE No. 529 . . . . . | 236 |
| 2.530 | ODE No. 530 . . . . . | 236 |
| 2.531 | ODE No. 531 . . . . . | 237 |
| 2.532 | ODE No. 532 . . . . . | 237 |
| 2.533 | ODE No. 533 . . . . . | 237 |
| 2.534 | ODE No. 534 . . . . . | 237 |
| 2.535 | ODE No. 535 . . . . . | 237 |
| 2.536 | ODE No. 536 . . . . . | 238 |
| 2.537 | ODE No. 537 . . . . . | 238 |
| 2.538 | ODE No. 538 . . . . . | 238 |
| 2.539 | ODE No. 539 . . . . . | 238 |
| 2.540 | ODE No. 540 . . . . . | 239 |
| 2.541 | ODE No. 541 . . . . . | 239 |
| 2.542 | ODE No. 542 . . . . . | 239 |
| 2.543 | ODE No. 543 . . . . . | 239 |
| 2.544 | ODE No. 544 . . . . . | 239 |
| 2.545 | ODE No. 545 . . . . . | 240 |
| 2.546 | ODE No. 546 . . . . . | 240 |
| 2.547 | ODE No. 547 . . . . . | 240 |
| 2.548 | ODE No. 548 . . . . . | 240 |
| 2.549 | ODE No. 549 . . . . . | 241 |
| 2.550 | ODE No. 550 . . . . . | 241 |
| 2.551 | ODE No. 551 . . . . . | 241 |
| 2.552 | ODE No. 552 . . . . . | 241 |
| 2.553 | ODE No. 553 . . . . . | 242 |
| 2.554 | ODE No. 554 . . . . . | 242 |
| 2.555 | ODE No. 555 . . . . . | 242 |
| 2.556 | ODE No. 556 . . . . . | 242 |
| 2.557 | ODE No. 557 . . . . . | 243 |
| 2.558 | ODE No. 558 . . . . . | 243 |
| 2.559 | ODE No. 559 . . . . . | 243 |
| 2.560 | ODE No. 560 . . . . . | 243 |
| 2.561 | ODE No. 561 . . . . . | 244 |
| 2.562 | ODE No. 562 . . . . . | 244 |
| 2.563 | ODE No. 563 . . . . . | 244 |
| 2.564 | ODE No. 564 . . . . . | 244 |
| 2.565 | ODE No. 565 . . . . . | 245 |
| 2.566 | ODE No. 566 . . . . . | 245 |
| 2.567 | ODE No. 567 . . . . . | 245 |
| 2.568 | ODE No. 568 . . . . . | 245 |
| 2.569 | ODE No. 569 . . . . . | 245 |
| 2.570 | ODE No. 570 . . . . . | 246 |
| 2.571 | ODE No. 571 . . . . . | 246 |
| 2.572 | ODE No. 572 . . . . . | 246 |
| 2.573 | ODE No. 573 . . . . . | 246 |
| 2.574 | ODE No. 574 . . . . . | 246 |
| 2.575 | ODE No. 575 . . . . . | 247 |
| 2.576 | ODE No. 576 . . . . . | 247 |
| 2.577 | ODE No. 577 . . . . . | 247 |
| 2.578 | ODE No. 578 . . . . . | 247 |

|       |                       |     |
|-------|-----------------------|-----|
| 2.579 | ODE No. 579 . . . . . | 247 |
| 2.580 | ODE No. 580 . . . . . | 248 |
| 2.581 | ODE No. 581 . . . . . | 248 |
| 2.582 | ODE No. 582 . . . . . | 248 |
| 2.583 | ODE No. 583 . . . . . | 248 |
| 2.584 | ODE No. 584 . . . . . | 249 |
| 2.585 | ODE No. 585 . . . . . | 249 |
| 2.586 | ODE No. 586 . . . . . | 249 |
| 2.587 | ODE No. 587 . . . . . | 249 |
| 2.588 | ODE No. 588 . . . . . | 250 |
| 2.589 | ODE No. 589 . . . . . | 250 |
| 2.590 | ODE No. 590 . . . . . | 250 |
| 2.591 | ODE No. 591 . . . . . | 250 |
| 2.592 | ODE No. 592 . . . . . | 251 |
| 2.593 | ODE No. 593 . . . . . | 251 |
| 2.594 | ODE No. 594 . . . . . | 251 |
| 2.595 | ODE No. 595 . . . . . | 251 |
| 2.596 | ODE No. 596 . . . . . | 252 |
| 2.597 | ODE No. 597 . . . . . | 252 |
| 2.598 | ODE No. 598 . . . . . | 252 |
| 2.599 | ODE No. 599 . . . . . | 252 |
| 2.600 | ODE No. 600 . . . . . | 253 |
| 2.601 | ODE No. 601 . . . . . | 253 |
| 2.602 | ODE No. 602 . . . . . | 253 |
| 2.603 | ODE No. 603 . . . . . | 253 |
| 2.604 | ODE No. 604 . . . . . | 254 |
| 2.605 | ODE No. 605 . . . . . | 254 |
| 2.606 | ODE No. 606 . . . . . | 254 |
| 2.607 | ODE No. 607 . . . . . | 254 |
| 2.608 | ODE No. 608 . . . . . | 255 |
| 2.609 | ODE No. 609 . . . . . | 255 |
| 2.610 | ODE No. 610 . . . . . | 255 |
| 2.611 | ODE No. 611 . . . . . | 255 |
| 2.612 | ODE No. 612 . . . . . | 256 |
| 2.613 | ODE No. 613 . . . . . | 256 |
| 2.614 | ODE No. 614 . . . . . | 256 |
| 2.615 | ODE No. 615 . . . . . | 256 |
| 2.616 | ODE No. 616 . . . . . | 257 |
| 2.617 | ODE No. 617 . . . . . | 257 |
| 2.618 | ODE No. 618 . . . . . | 257 |
| 2.619 | ODE No. 619 . . . . . | 257 |
| 2.620 | ODE No. 620 . . . . . | 258 |
| 2.621 | ODE No. 621 . . . . . | 258 |
| 2.622 | ODE No. 622 . . . . . | 258 |
| 2.623 | ODE No. 623 . . . . . | 258 |
| 2.624 | ODE No. 624 . . . . . | 259 |
| 2.625 | ODE No. 625 . . . . . | 259 |
| 2.626 | ODE No. 626 . . . . . | 259 |
| 2.627 | ODE No. 627 . . . . . | 259 |
| 2.628 | ODE No. 628 . . . . . | 260 |
| 2.629 | ODE No. 629 . . . . . | 260 |
| 2.630 | ODE No. 630 . . . . . | 260 |
| 2.631 | ODE No. 631 . . . . . | 260 |
| 2.632 | ODE No. 632 . . . . . | 261 |
| 2.633 | ODE No. 633 . . . . . | 261 |
| 2.634 | ODE No. 634 . . . . . | 261 |
| 2.635 | ODE No. 635 . . . . . | 261 |
| 2.636 | ODE No. 636 . . . . . | 261 |
| 2.637 | ODE No. 637 . . . . . | 262 |
| 2.638 | ODE No. 638 . . . . . | 262 |
| 2.639 | ODE No. 639 . . . . . | 262 |
| 2.640 | ODE No. 640 . . . . . | 262 |
| 2.641 | ODE No. 641 . . . . . | 262 |
| 2.642 | ODE No. 642 . . . . . | 263 |
| 2.643 | ODE No. 643 . . . . . | 263 |
| 2.644 | ODE No. 644 . . . . . | 263 |

|       |                       |     |
|-------|-----------------------|-----|
| 2.645 | ODE No. 645 . . . . . | 263 |
| 2.646 | ODE No. 646 . . . . . | 263 |
| 2.647 | ODE No. 647 . . . . . | 264 |
| 2.648 | ODE No. 648 . . . . . | 264 |
| 2.649 | ODE No. 649 . . . . . | 264 |
| 2.650 | ODE No. 650 . . . . . | 264 |
| 2.651 | ODE No. 651 . . . . . | 265 |
| 2.652 | ODE No. 652 . . . . . | 265 |
| 2.653 | ODE No. 653 . . . . . | 265 |
| 2.654 | ODE No. 654 . . . . . | 265 |
| 2.655 | ODE No. 655 . . . . . | 265 |
| 2.656 | ODE No. 656 . . . . . | 266 |
| 2.657 | ODE No. 657 . . . . . | 266 |
| 2.658 | ODE No. 658 . . . . . | 266 |
| 2.659 | ODE No. 659 . . . . . | 266 |
| 2.660 | ODE No. 660 . . . . . | 266 |
| 2.661 | ODE No. 661 . . . . . | 267 |
| 2.662 | ODE No. 662 . . . . . | 267 |
| 2.663 | ODE No. 663 . . . . . | 267 |
| 2.664 | ODE No. 664 . . . . . | 267 |
| 2.665 | ODE No. 665 . . . . . | 268 |
| 2.666 | ODE No. 666 . . . . . | 268 |
| 2.667 | ODE No. 667 . . . . . | 268 |
| 2.668 | ODE No. 668 . . . . . | 268 |
| 2.669 | ODE No. 669 . . . . . | 269 |
| 2.670 | ODE No. 670 . . . . . | 269 |
| 2.671 | ODE No. 671 . . . . . | 269 |
| 2.672 | ODE No. 672 . . . . . | 269 |
| 2.673 | ODE No. 673 . . . . . | 270 |
| 2.674 | ODE No. 674 . . . . . | 270 |
| 2.675 | ODE No. 675 . . . . . | 270 |
| 2.676 | ODE No. 676 . . . . . | 270 |
| 2.677 | ODE No. 677 . . . . . | 270 |
| 2.678 | ODE No. 678 . . . . . | 271 |
| 2.679 | ODE No. 679 . . . . . | 271 |
| 2.680 | ODE No. 680 . . . . . | 271 |
| 2.681 | ODE No. 681 . . . . . | 271 |
| 2.682 | ODE No. 682 . . . . . | 272 |
| 2.683 | ODE No. 683 . . . . . | 272 |
| 2.684 | ODE No. 684 . . . . . | 272 |
| 2.685 | ODE No. 685 . . . . . | 272 |
| 2.686 | ODE No. 686 . . . . . | 273 |
| 2.687 | ODE No. 687 . . . . . | 273 |
| 2.688 | ODE No. 688 . . . . . | 273 |
| 2.689 | ODE No. 689 . . . . . | 273 |
| 2.690 | ODE No. 690 . . . . . | 274 |
| 2.691 | ODE No. 691 . . . . . | 274 |
| 2.692 | ODE No. 692 . . . . . | 274 |
| 2.693 | ODE No. 693 . . . . . | 274 |
| 2.694 | ODE No. 694 . . . . . | 274 |
| 2.695 | ODE No. 695 . . . . . | 275 |
| 2.696 | ODE No. 696 . . . . . | 275 |
| 2.697 | ODE No. 697 . . . . . | 275 |
| 2.698 | ODE No. 698 . . . . . | 275 |
| 2.699 | ODE No. 699 . . . . . | 276 |
| 2.700 | ODE No. 700 . . . . . | 276 |
| 2.701 | ODE No. 701 . . . . . | 276 |
| 2.702 | ODE No. 702 . . . . . | 276 |
| 2.703 | ODE No. 703 . . . . . | 277 |
| 2.704 | ODE No. 704 . . . . . | 277 |
| 2.705 | ODE No. 705 . . . . . | 277 |
| 2.706 | ODE No. 706 . . . . . | 277 |
| 2.707 | ODE No. 707 . . . . . | 278 |
| 2.708 | ODE No. 708 . . . . . | 278 |
| 2.709 | ODE No. 709 . . . . . | 278 |
| 2.710 | ODE No. 710 . . . . . | 278 |

|       |                       |     |
|-------|-----------------------|-----|
| 2.711 | ODE No. 711 . . . . . | 279 |
| 2.712 | ODE No. 712 . . . . . | 279 |
| 2.713 | ODE No. 713 . . . . . | 279 |
| 2.714 | ODE No. 714 . . . . . | 279 |
| 2.715 | ODE No. 715 . . . . . | 280 |
| 2.716 | ODE No. 716 . . . . . | 280 |
| 2.717 | ODE No. 717 . . . . . | 280 |
| 2.718 | ODE No. 718 . . . . . | 280 |
| 2.719 | ODE No. 719 . . . . . | 281 |
| 2.720 | ODE No. 720 . . . . . | 281 |
| 2.721 | ODE No. 721 . . . . . | 281 |
| 2.722 | ODE No. 722 . . . . . | 281 |
| 2.723 | ODE No. 723 . . . . . | 282 |
| 2.724 | ODE No. 724 . . . . . | 282 |
| 2.725 | ODE No. 725 . . . . . | 282 |
| 2.726 | ODE No. 726 . . . . . | 282 |
| 2.727 | ODE No. 727 . . . . . | 283 |
| 2.728 | ODE No. 728 . . . . . | 283 |
| 2.729 | ODE No. 729 . . . . . | 283 |
| 2.730 | ODE No. 730 . . . . . | 283 |
| 2.731 | ODE No. 731 . . . . . | 284 |
| 2.732 | ODE No. 732 . . . . . | 284 |
| 2.733 | ODE No. 733 . . . . . | 284 |
| 2.734 | ODE No. 734 . . . . . | 284 |
| 2.735 | ODE No. 735 . . . . . | 284 |
| 2.736 | ODE No. 736 . . . . . | 285 |
| 2.737 | ODE No. 737 . . . . . | 285 |
| 2.738 | ODE No. 738 . . . . . | 285 |
| 2.739 | ODE No. 739 . . . . . | 285 |
| 2.740 | ODE No. 740 . . . . . | 286 |
| 2.741 | ODE No. 741 . . . . . | 286 |
| 2.742 | ODE No. 742 . . . . . | 286 |
| 2.743 | ODE No. 743 . . . . . | 286 |
| 2.744 | ODE No. 744 . . . . . | 287 |
| 2.745 | ODE No. 745 . . . . . | 287 |
| 2.746 | ODE No. 746 . . . . . | 287 |
| 2.747 | ODE No. 747 . . . . . | 287 |
| 2.748 | ODE No. 748 . . . . . | 288 |
| 2.749 | ODE No. 749 . . . . . | 288 |
| 2.750 | ODE No. 750 . . . . . | 288 |
| 2.751 | ODE No. 751 . . . . . | 288 |
| 2.752 | ODE No. 752 . . . . . | 289 |
| 2.753 | ODE No. 753 . . . . . | 289 |
| 2.754 | ODE No. 754 . . . . . | 289 |
| 2.755 | ODE No. 755 . . . . . | 289 |
| 2.756 | ODE No. 756 . . . . . | 289 |
| 2.757 | ODE No. 757 . . . . . | 290 |
| 2.758 | ODE No. 758 . . . . . | 290 |
| 2.759 | ODE No. 759 . . . . . | 290 |
| 2.760 | ODE No. 760 . . . . . | 290 |
| 2.761 | ODE No. 761 . . . . . | 291 |
| 2.762 | ODE No. 762 . . . . . | 291 |
| 2.763 | ODE No. 763 . . . . . | 291 |
| 2.764 | ODE No. 764 . . . . . | 291 |
| 2.765 | ODE No. 765 . . . . . | 291 |
| 2.766 | ODE No. 766 . . . . . | 292 |
| 2.767 | ODE No. 767 . . . . . | 292 |
| 2.768 | ODE No. 768 . . . . . | 292 |
| 2.769 | ODE No. 769 . . . . . | 292 |
| 2.770 | ODE No. 770 . . . . . | 293 |
| 2.771 | ODE No. 771 . . . . . | 293 |
| 2.772 | ODE No. 772 . . . . . | 293 |
| 2.773 | ODE No. 773 . . . . . | 293 |
| 2.774 | ODE No. 774 . . . . . | 294 |
| 2.775 | ODE No. 775 . . . . . | 294 |
| 2.776 | ODE No. 776 . . . . . | 294 |

|       |                       |     |
|-------|-----------------------|-----|
| 2.777 | ODE No. 777 . . . . . | 294 |
| 2.778 | ODE No. 778 . . . . . | 295 |
| 2.779 | ODE No. 779 . . . . . | 295 |
| 2.780 | ODE No. 780 . . . . . | 295 |
| 2.781 | ODE No. 781 . . . . . | 295 |
| 2.782 | ODE No. 782 . . . . . | 296 |
| 2.783 | ODE No. 783 . . . . . | 296 |
| 2.784 | ODE No. 784 . . . . . | 296 |
| 2.785 | ODE No. 785 . . . . . | 296 |
| 2.786 | ODE No. 786 . . . . . | 297 |
| 2.787 | ODE No. 787 . . . . . | 297 |
| 2.788 | ODE No. 788 . . . . . | 297 |
| 2.789 | ODE No. 789 . . . . . | 297 |
| 2.790 | ODE No. 790 . . . . . | 298 |
| 2.791 | ODE No. 791 . . . . . | 298 |
| 2.792 | ODE No. 792 . . . . . | 298 |
| 2.793 | ODE No. 793 . . . . . | 299 |
| 2.794 | ODE No. 794 . . . . . | 299 |
| 2.795 | ODE No. 795 . . . . . | 299 |
| 2.796 | ODE No. 796 . . . . . | 299 |
| 2.797 | ODE No. 797 . . . . . | 300 |
| 2.798 | ODE No. 798 . . . . . | 300 |
| 2.799 | ODE No. 799 . . . . . | 300 |
| 2.800 | ODE No. 800 . . . . . | 300 |
| 2.801 | ODE No. 801 . . . . . | 301 |
| 2.802 | ODE No. 802 . . . . . | 301 |
| 2.803 | ODE No. 803 . . . . . | 301 |
| 2.804 | ODE No. 804 . . . . . | 301 |
| 2.805 | ODE No. 805 . . . . . | 302 |
| 2.806 | ODE No. 806 . . . . . | 302 |
| 2.807 | ODE No. 807 . . . . . | 302 |
| 2.808 | ODE No. 808 . . . . . | 302 |
| 2.809 | ODE No. 809 . . . . . | 302 |
| 2.810 | ODE No. 810 . . . . . | 303 |
| 2.811 | ODE No. 811 . . . . . | 303 |
| 2.812 | ODE No. 812 . . . . . | 303 |
| 2.813 | ODE No. 813 . . . . . | 303 |
| 2.814 | ODE No. 814 . . . . . | 304 |
| 2.815 | ODE No. 815 . . . . . | 304 |
| 2.816 | ODE No. 816 . . . . . | 304 |
| 2.817 | ODE No. 817 . . . . . | 304 |
| 2.818 | ODE No. 818 . . . . . | 305 |
| 2.819 | ODE No. 819 . . . . . | 305 |
| 2.820 | ODE No. 820 . . . . . | 305 |
| 2.821 | ODE No. 821 . . . . . | 305 |
| 2.822 | ODE No. 822 . . . . . | 306 |
| 2.823 | ODE No. 823 . . . . . | 306 |
| 2.824 | ODE No. 824 . . . . . | 306 |
| 2.825 | ODE No. 825 . . . . . | 306 |
| 2.826 | ODE No. 826 . . . . . | 307 |
| 2.827 | ODE No. 827 . . . . . | 307 |
| 2.828 | ODE No. 828 . . . . . | 307 |
| 2.829 | ODE No. 829 . . . . . | 307 |
| 2.830 | ODE No. 830 . . . . . | 308 |
| 2.831 | ODE No. 831 . . . . . | 308 |
| 2.832 | ODE No. 832 . . . . . | 308 |
| 2.833 | ODE No. 833 . . . . . | 308 |
| 2.834 | ODE No. 834 . . . . . | 309 |
| 2.835 | ODE No. 835 . . . . . | 309 |
| 2.836 | ODE No. 836 . . . . . | 309 |
| 2.837 | ODE No. 837 . . . . . | 309 |
| 2.838 | ODE No. 838 . . . . . | 310 |
| 2.839 | ODE No. 839 . . . . . | 310 |
| 2.840 | ODE No. 840 . . . . . | 310 |
| 2.841 | ODE No. 841 . . . . . | 310 |
| 2.842 | ODE No. 842 . . . . . | 311 |

|       |                       |     |
|-------|-----------------------|-----|
| 2.843 | ODE No. 843 . . . . . | 311 |
| 2.844 | ODE No. 844 . . . . . | 311 |
| 2.845 | ODE No. 845 . . . . . | 311 |
| 2.846 | ODE No. 846 . . . . . | 312 |
| 2.847 | ODE No. 847 . . . . . | 312 |
| 2.848 | ODE No. 848 . . . . . | 312 |
| 2.849 | ODE No. 849 . . . . . | 312 |
| 2.850 | ODE No. 850 . . . . . | 313 |
| 2.851 | ODE No. 851 . . . . . | 313 |
| 2.852 | ODE No. 852 . . . . . | 313 |
| 2.853 | ODE No. 853 . . . . . | 313 |
| 2.854 | ODE No. 854 . . . . . | 314 |
| 2.855 | ODE No. 855 . . . . . | 314 |
| 2.856 | ODE No. 856 . . . . . | 314 |
| 2.857 | ODE No. 857 . . . . . | 314 |
| 2.858 | ODE No. 858 . . . . . | 314 |
| 2.859 | ODE No. 859 . . . . . | 315 |
| 2.860 | ODE No. 860 . . . . . | 315 |
| 2.861 | ODE No. 861 . . . . . | 315 |
| 2.862 | ODE No. 862 . . . . . | 315 |
| 2.863 | ODE No. 863 . . . . . | 316 |
| 2.864 | ODE No. 864 . . . . . | 316 |
| 2.865 | ODE No. 865 . . . . . | 316 |
| 2.866 | ODE No. 866 . . . . . | 316 |
| 2.867 | ODE No. 867 . . . . . | 317 |
| 2.868 | ODE No. 868 . . . . . | 317 |
| 2.869 | ODE No. 869 . . . . . | 317 |
| 2.870 | ODE No. 870 . . . . . | 317 |
| 2.871 | ODE No. 871 . . . . . | 318 |
| 2.872 | ODE No. 872 . . . . . | 318 |
| 2.873 | ODE No. 873 . . . . . | 318 |
| 2.874 | ODE No. 874 . . . . . | 318 |
| 2.875 | ODE No. 875 . . . . . | 319 |
| 2.876 | ODE No. 876 . . . . . | 319 |
| 2.877 | ODE No. 877 . . . . . | 319 |
| 2.878 | ODE No. 878 . . . . . | 319 |
| 2.879 | ODE No. 879 . . . . . | 320 |
| 2.880 | ODE No. 880 . . . . . | 320 |
| 2.881 | ODE No. 881 . . . . . | 320 |
| 2.882 | ODE No. 882 . . . . . | 320 |
| 2.883 | ODE No. 883 . . . . . | 321 |
| 2.884 | ODE No. 884 . . . . . | 321 |
| 2.885 | ODE No. 885 . . . . . | 321 |
| 2.886 | ODE No. 886 . . . . . | 321 |
| 2.887 | ODE No. 887 . . . . . | 322 |
| 2.888 | ODE No. 888 . . . . . | 322 |
| 2.889 | ODE No. 889 . . . . . | 322 |
| 2.890 | ODE No. 890 . . . . . | 322 |
| 2.891 | ODE No. 891 . . . . . | 323 |
| 2.892 | ODE No. 892 . . . . . | 323 |
| 2.893 | ODE No. 893 . . . . . | 323 |
| 2.894 | ODE No. 894 . . . . . | 323 |
| 2.895 | ODE No. 895 . . . . . | 324 |
| 2.896 | ODE No. 896 . . . . . | 324 |
| 2.897 | ODE No. 897 . . . . . | 324 |
| 2.898 | ODE No. 898 . . . . . | 324 |
| 2.899 | ODE No. 899 . . . . . | 325 |
| 2.900 | ODE No. 900 . . . . . | 325 |
| 2.901 | ODE No. 901 . . . . . | 325 |
| 2.902 | ODE No. 902 . . . . . | 325 |
| 2.903 | ODE No. 903 . . . . . | 326 |
| 2.904 | ODE No. 904 . . . . . | 326 |
| 2.905 | ODE No. 905 . . . . . | 326 |
| 2.906 | ODE No. 906 . . . . . | 326 |
| 2.907 | ODE No. 907 . . . . . | 327 |
| 2.908 | ODE No. 908 . . . . . | 327 |

|       |                       |     |
|-------|-----------------------|-----|
| 2.909 | ODE No. 909 . . . . . | 327 |
| 2.910 | ODE No. 910 . . . . . | 327 |
| 2.911 | ODE No. 911 . . . . . | 328 |
| 2.912 | ODE No. 912 . . . . . | 328 |
| 2.913 | ODE No. 913 . . . . . | 328 |
| 2.914 | ODE No. 914 . . . . . | 328 |
| 2.915 | ODE No. 915 . . . . . | 329 |
| 2.916 | ODE No. 916 . . . . . | 329 |
| 2.917 | ODE No. 917 . . . . . | 329 |
| 2.918 | ODE No. 918 . . . . . | 329 |
| 2.919 | ODE No. 919 . . . . . | 329 |
| 2.920 | ODE No. 920 . . . . . | 330 |
| 2.921 | ODE No. 921 . . . . . | 330 |
| 2.922 | ODE No. 922 . . . . . | 330 |
| 2.923 | ODE No. 923 . . . . . | 330 |
| 2.924 | ODE No. 924 . . . . . | 331 |
| 2.925 | ODE No. 925 . . . . . | 331 |
| 2.926 | ODE No. 926 . . . . . | 331 |
| 2.927 | ODE No. 927 . . . . . | 331 |
| 2.928 | ODE No. 928 . . . . . | 332 |
| 2.929 | ODE No. 929 . . . . . | 332 |
| 2.930 | ODE No. 930 . . . . . | 332 |
| 2.931 | ODE No. 931 . . . . . | 332 |
| 2.932 | ODE No. 932 . . . . . | 333 |
| 2.933 | ODE No. 933 . . . . . | 333 |
| 2.934 | ODE No. 934 . . . . . | 333 |
| 2.935 | ODE No. 935 . . . . . | 333 |
| 2.936 | ODE No. 936 . . . . . | 334 |
| 2.937 | ODE No. 937 . . . . . | 334 |
| 2.938 | ODE No. 938 . . . . . | 334 |
| 2.939 | ODE No. 939 . . . . . | 334 |
| 2.940 | ODE No. 940 . . . . . | 335 |
| 2.941 | ODE No. 941 . . . . . | 335 |
| 2.942 | ODE No. 942 . . . . . | 335 |
| 2.943 | ODE No. 943 . . . . . | 335 |
| 2.944 | ODE No. 944 . . . . . | 336 |
| 2.945 | ODE No. 945 . . . . . | 336 |
| 2.946 | ODE No. 946 . . . . . | 336 |
| 2.947 | ODE No. 947 . . . . . | 336 |
| 2.948 | ODE No. 948 . . . . . | 337 |
| 2.949 | ODE No. 949 . . . . . | 337 |
| 2.950 | ODE No. 950 . . . . . | 337 |
| 2.951 | ODE No. 951 . . . . . | 337 |
| 2.952 | ODE No. 952 . . . . . | 338 |
| 2.953 | ODE No. 953 . . . . . | 338 |
| 2.954 | ODE No. 954 . . . . . | 338 |
| 2.955 | ODE No. 955 . . . . . | 338 |
| 2.956 | ODE No. 956 . . . . . | 339 |
| 2.957 | ODE No. 957 . . . . . | 339 |
| 2.958 | ODE No. 958 . . . . . | 339 |
| 2.959 | ODE No. 959 . . . . . | 339 |
| 2.960 | ODE No. 960 . . . . . | 340 |
| 2.961 | ODE No. 961 . . . . . | 340 |
| 2.962 | ODE No. 962 . . . . . | 340 |
| 2.963 | ODE No. 963 . . . . . | 340 |
| 2.964 | ODE No. 964 . . . . . | 341 |
| 2.965 | ODE No. 965 . . . . . | 341 |
| 2.966 | ODE No. 966 . . . . . | 341 |
| 2.967 | ODE No. 967 . . . . . | 341 |
| 2.968 | ODE No. 968 . . . . . | 342 |
| 2.969 | ODE No. 969 . . . . . | 342 |
| 2.970 | ODE No. 970 . . . . . | 342 |
| 2.971 | ODE No. 971 . . . . . | 342 |
| 2.972 | ODE No. 972 . . . . . | 343 |
| 2.973 | ODE No. 973 . . . . . | 343 |
| 2.974 | ODE No. 974 . . . . . | 343 |

|        |                        |     |
|--------|------------------------|-----|
| 2.975  | ODE No. 975 . . . . .  | 343 |
| 2.976  | ODE No. 976 . . . . .  | 344 |
| 2.977  | ODE No. 977 . . . . .  | 344 |
| 2.978  | ODE No. 978 . . . . .  | 344 |
| 2.979  | ODE No. 979 . . . . .  | 344 |
| 2.980  | ODE No. 980 . . . . .  | 345 |
| 2.981  | ODE No. 981 . . . . .  | 345 |
| 2.982  | ODE No. 982 . . . . .  | 345 |
| 2.983  | ODE No. 983 . . . . .  | 345 |
| 2.984  | ODE No. 984 . . . . .  | 346 |
| 2.985  | ODE No. 985 . . . . .  | 346 |
| 2.986  | ODE No. 986 . . . . .  | 346 |
| 2.987  | ODE No. 987 . . . . .  | 346 |
| 2.988  | ODE No. 988 . . . . .  | 347 |
| 2.989  | ODE No. 989 . . . . .  | 347 |
| 2.990  | ODE No. 990 . . . . .  | 347 |
| 2.991  | ODE No. 991 . . . . .  | 347 |
| 2.992  | ODE No. 992 . . . . .  | 348 |
| 2.993  | ODE No. 993 . . . . .  | 348 |
| 2.994  | ODE No. 994 . . . . .  | 348 |
| 2.995  | ODE No. 995 . . . . .  | 348 |
| 2.996  | ODE No. 996 . . . . .  | 348 |
| 2.997  | ODE No. 997 . . . . .  | 349 |
| 2.998  | ODE No. 998 . . . . .  | 349 |
| 2.999  | ODE No. 999 . . . . .  | 349 |
| 2.1000 | ODE No. 1000 . . . . . | 349 |
| 2.1001 | ODE No. 1001 . . . . . | 349 |
| 2.1002 | ODE No. 1002 . . . . . | 350 |
| 2.1003 | ODE No. 1003 . . . . . | 350 |
| 2.1004 | ODE No. 1004 . . . . . | 352 |
| 2.1005 | ODE No. 1005 . . . . . | 354 |
| 2.1006 | ODE No. 1006 . . . . . | 356 |
| 2.1007 | ODE No. 1007 . . . . . | 356 |
| 2.1008 | ODE No. 1008 . . . . . | 357 |
| 2.1009 | ODE No. 1009 . . . . . | 357 |
| 2.1010 | ODE No. 1010 . . . . . | 358 |
| 2.1011 | ODE No. 1011 . . . . . | 360 |
| 2.1012 | ODE No. 1012 . . . . . | 362 |
| 2.1013 | ODE No. 1013 . . . . . | 362 |
| 2.1014 | ODE No. 1014 . . . . . | 363 |
| 2.1015 | ODE No. 1015 . . . . . | 363 |
| 2.1016 | ODE No. 1016 . . . . . | 363 |
| 2.1017 | ODE No. 1017 . . . . . | 363 |
| 2.1018 | ODE No. 1018 . . . . . | 363 |
| 2.1019 | ODE No. 1019 . . . . . | 364 |
| 2.1020 | ODE No. 1020 . . . . . | 364 |
| 2.1021 | ODE No. 1021 . . . . . | 364 |
| 2.1022 | ODE No. 1022 . . . . . | 364 |
| 2.1023 | ODE No. 1023 . . . . . | 364 |
| 2.1024 | ODE No. 1024 . . . . . | 365 |
| 2.1025 | ODE No. 1025 . . . . . | 365 |
| 2.1026 | ODE No. 1026 . . . . . | 365 |
| 2.1027 | ODE No. 1027 . . . . . | 365 |
| 2.1028 | ODE No. 1028 . . . . . | 365 |
| 2.1029 | ODE No. 1029 . . . . . | 366 |
| 2.1030 | ODE No. 1030 . . . . . | 366 |
| 2.1031 | ODE No. 1031 . . . . . | 366 |
| 2.1032 | ODE No. 1032 . . . . . | 366 |
| 2.1033 | ODE No. 1033 . . . . . | 366 |
| 2.1034 | ODE No. 1034 . . . . . | 367 |
| 2.1035 | ODE No. 1035 . . . . . | 368 |
| 2.1036 | ODE No. 1036 . . . . . | 368 |
| 2.1037 | ODE No. 1037 . . . . . | 368 |
| 2.1038 | ODE No. 1038 . . . . . | 369 |
| 2.1039 | ODE No. 1039 . . . . . | 369 |
| 2.1040 | ODE No. 1040 . . . . . | 369 |



|        |              |     |
|--------|--------------|-----|
| 2.1041 | ODE No. 1041 | 369 |
| 2.1042 | ODE No. 1042 | 369 |
| 2.1043 | ODE No. 1043 | 370 |
| 2.1044 | ODE No. 1044 | 371 |
| 2.1045 | ODE No. 1045 | 371 |
| 2.1046 | ODE No. 1046 | 371 |
| 2.1047 | ODE No. 1047 | 372 |
| 2.1048 | ODE No. 1048 | 373 |
| 2.1049 | ODE No. 1049 | 373 |
| 2.1050 | ODE No. 1050 | 373 |
| 2.1051 | ODE No. 1051 | 374 |
| 2.1052 | ODE No. 1052 | 374 |
| 2.1053 | ODE No. 1053 | 374 |
| 2.1054 | ODE No. 1054 | 374 |
| 2.1055 | ODE No. 1055 | 374 |
| 2.1056 | ODE No. 1056 | 375 |
| 2.1057 | ODE No. 1057 | 375 |
| 2.1058 | ODE No. 1058 | 375 |
| 2.1059 | ODE No. 1059 | 375 |
| 2.1060 | ODE No. 1060 | 375 |
| 2.1061 | ODE No. 1061 | 376 |
| 2.1062 | ODE No. 1062 | 376 |
| 2.1063 | ODE No. 1063 | 376 |
| 2.1064 | ODE No. 1064 | 376 |
| 2.1065 | ODE No. 1065 | 376 |
| 2.1066 | ODE No. 1066 | 377 |
| 2.1067 | ODE No. 1067 | 377 |
| 2.1068 | ODE No. 1068 | 377 |
| 2.1069 | ODE No. 1069 | 377 |
| 2.1070 | ODE No. 1070 | 377 |
| 2.1071 | ODE No. 1071 | 378 |
| 2.1072 | ODE No. 1072 | 378 |
| 2.1073 | ODE No. 1073 | 378 |
| 2.1074 | ODE No. 1074 | 378 |
| 2.1075 | ODE No. 1075 | 378 |
| 2.1076 | ODE No. 1076 | 378 |
| 2.1077 | ODE No. 1077 | 379 |
| 2.1078 | ODE No. 1078 | 379 |
| 2.1079 | ODE No. 1079 | 379 |
| 2.1080 | ODE No. 1080 | 379 |
| 2.1081 | ODE No. 1081 | 379 |
| 2.1082 | ODE No. 1082 | 380 |
| 2.1083 | ODE No. 1083 | 380 |
| 2.1084 | ODE No. 1084 | 380 |
| 2.1085 | ODE No. 1085 | 380 |
| 2.1086 | ODE No. 1086 | 380 |
| 2.1087 | ODE No. 1087 | 381 |
| 2.1088 | ODE No. 1088 | 381 |
| 2.1089 | ODE No. 1089 | 381 |
| 2.1090 | ODE No. 1090 | 381 |
| 2.1091 | ODE No. 1091 | 381 |
| 2.1092 | ODE No. 1092 | 382 |
| 2.1093 | ODE No. 1093 | 382 |
| 2.1094 | ODE No. 1094 | 382 |
| 2.1095 | ODE No. 1095 | 382 |
| 2.1096 | ODE No. 1096 | 382 |
| 2.1097 | ODE No. 1097 | 382 |
| 2.1098 | ODE No. 1098 | 383 |
| 2.1099 | ODE No. 1099 | 383 |
| 2.1100 | ODE No. 1100 | 383 |
| 2.1101 | ODE No. 1101 | 387 |
| 2.1102 | ODE No. 1102 | 390 |
| 2.1103 | ODE No. 1103 | 392 |
| 2.1104 | ODE No. 1104 | 392 |
| 2.1105 | ODE No. 1105 | 392 |
| 2.1106 | ODE No. 1106 | 392 |

|        |              |     |
|--------|--------------|-----|
| 2.1107 | ODE No. 1107 | 393 |
| 2.1108 | ODE No. 1108 | 393 |
| 2.1109 | ODE No. 1109 | 393 |
| 2.1110 | ODE No. 1110 | 393 |
| 2.1111 | ODE No. 1111 | 393 |
| 2.1112 | ODE No. 1112 | 394 |
| 2.1113 | ODE No. 1113 | 395 |
| 2.1114 | ODE No. 1114 | 395 |
| 2.1115 | ODE No. 1115 | 395 |
| 2.1116 | ODE No. 1116 | 395 |
| 2.1117 | ODE No. 1117 | 395 |
| 2.1118 | ODE No. 1118 | 396 |
| 2.1119 | ODE No. 1119 | 396 |
| 2.1120 | ODE No. 1120 | 396 |
| 2.1121 | ODE No. 1121 | 396 |
| 2.1122 | ODE No. 1122 | 396 |
| 2.1123 | ODE No. 1123 | 397 |
| 2.1124 | ODE No. 1124 | 397 |
| 2.1125 | ODE No. 1125 | 397 |
| 2.1126 | ODE No. 1126 | 397 |
| 2.1127 | ODE No. 1127 | 397 |
| 2.1128 | ODE No. 1128 | 398 |
| 2.1129 | ODE No. 1129 | 398 |
| 2.1130 | ODE No. 1130 | 398 |
| 2.1131 | ODE No. 1131 | 398 |
| 2.1132 | ODE No. 1132 | 398 |
| 2.1133 | ODE No. 1133 | 399 |
| 2.1134 | ODE No. 1134 | 399 |
| 2.1135 | ODE No. 1135 | 399 |
| 2.1136 | ODE No. 1136 | 399 |
| 2.1137 | ODE No. 1137 | 399 |
| 2.1138 | ODE No. 1138 | 400 |
| 2.1139 | ODE No. 1139 | 400 |
| 2.1140 | ODE No. 1140 | 400 |
| 2.1141 | ODE No. 1141 | 400 |
| 2.1142 | ODE No. 1142 | 400 |
| 2.1143 | ODE No. 1143 | 401 |
| 2.1144 | ODE No. 1144 | 401 |
| 2.1145 | ODE No. 1145 | 401 |
| 2.1146 | ODE No. 1146 | 401 |
| 2.1147 | ODE No. 1147 | 401 |
| 2.1148 | ODE No. 1148 | 402 |
| 2.1149 | ODE No. 1149 | 402 |
| 2.1150 | ODE No. 1150 | 402 |
| 2.1151 | ODE No. 1151 | 402 |
| 2.1152 | ODE No. 1152 | 402 |
| 2.1153 | ODE No. 1153 | 403 |
| 2.1154 | ODE No. 1154 | 403 |
| 2.1155 | ODE No. 1155 | 403 |
| 2.1156 | ODE No. 1156 | 403 |
| 2.1157 | ODE No. 1157 | 403 |
| 2.1158 | ODE No. 1158 | 404 |
| 2.1159 | ODE No. 1159 | 404 |
| 2.1160 | ODE No. 1160 | 404 |
| 2.1161 | ODE No. 1161 | 404 |
| 2.1162 | ODE No. 1162 | 404 |
| 2.1163 | ODE No. 1163 | 405 |
| 2.1164 | ODE No. 1164 | 405 |
| 2.1165 | ODE No. 1165 | 405 |
| 2.1166 | ODE No. 1166 | 405 |
| 2.1167 | ODE No. 1167 | 405 |
| 2.1168 | ODE No. 1168 | 406 |
| 2.1169 | ODE No. 1169 | 406 |
| 2.1170 | ODE No. 1170 | 406 |
| 2.1171 | ODE No. 1171 | 406 |
| 2.1172 | ODE No. 1172 | 406 |

|        |              |     |
|--------|--------------|-----|
| 2.1173 | ODE No. 1173 | 407 |
| 2.1174 | ODE No. 1174 | 407 |
| 2.1175 | ODE No. 1175 | 407 |
| 2.1176 | ODE No. 1176 | 407 |
| 2.1177 | ODE No. 1177 | 407 |
| 2.1178 | ODE No. 1178 | 408 |
| 2.1179 | ODE No. 1179 | 408 |
| 2.1180 | ODE No. 1180 | 408 |
| 2.1181 | ODE No. 1181 | 408 |
| 2.1182 | ODE No. 1182 | 408 |
| 2.1183 | ODE No. 1183 | 409 |
| 2.1184 | ODE No. 1184 | 409 |
| 2.1185 | ODE No. 1185 | 409 |
| 2.1186 | ODE No. 1186 | 409 |
| 2.1187 | ODE No. 1187 | 409 |
| 2.1188 | ODE No. 1188 | 410 |
| 2.1189 | ODE No. 1189 | 410 |
| 2.1190 | ODE No. 1190 | 410 |
| 2.1191 | ODE No. 1191 | 410 |
| 2.1192 | ODE No. 1192 | 410 |
| 2.1193 | ODE No. 1193 | 411 |
| 2.1194 | ODE No. 1194 | 411 |
| 2.1195 | ODE No. 1195 | 411 |
| 2.1196 | ODE No. 1196 | 411 |
| 2.1197 | ODE No. 1197 | 411 |
| 2.1198 | ODE No. 1198 | 412 |
| 2.1199 | ODE No. 1199 | 412 |
| 2.1200 | ODE No. 1200 | 412 |
| 2.1201 | ODE No. 1201 | 412 |
| 2.1202 | ODE No. 1202 | 412 |
| 2.1203 | ODE No. 1203 | 413 |
| 2.1204 | ODE No. 1204 | 413 |
| 2.1205 | ODE No. 1205 | 413 |
| 2.1206 | ODE No. 1206 | 413 |
| 2.1207 | ODE No. 1207 | 413 |
| 2.1208 | ODE No. 1208 | 414 |
| 2.1209 | ODE No. 1209 | 414 |
| 2.1210 | ODE No. 1210 | 414 |
| 2.1211 | ODE No. 1211 | 414 |
| 2.1212 | ODE No. 1212 | 414 |
| 2.1213 | ODE No. 1213 | 415 |
| 2.1214 | ODE No. 1214 | 415 |
| 2.1215 | ODE No. 1215 | 415 |
| 2.1216 | ODE No. 1216 | 415 |
| 2.1217 | ODE No. 1217 | 416 |
| 2.1218 | ODE No. 1218 | 416 |
| 2.1219 | ODE No. 1219 | 416 |
| 2.1220 | ODE No. 1220 | 416 |
| 2.1221 | ODE No. 1221 | 416 |
| 2.1222 | ODE No. 1222 | 417 |
| 2.1223 | ODE No. 1223 | 417 |
| 2.1224 | ODE No. 1224 | 417 |
| 2.1225 | ODE No. 1225 | 417 |
| 2.1226 | ODE No. 1226 | 417 |
| 2.1227 | ODE No. 1227 | 417 |
| 2.1228 | ODE No. 1228 | 418 |
| 2.1229 | ODE No. 1229 | 418 |
| 2.1230 | ODE No. 1230 | 418 |
| 2.1231 | ODE No. 1231 | 418 |
| 2.1232 | ODE No. 1232 | 418 |
| 2.1233 | ODE No. 1233 | 419 |
| 2.1234 | ODE No. 1234 | 419 |
| 2.1235 | ODE No. 1235 | 419 |
| 2.1236 | ODE No. 1236 | 419 |
| 2.1237 | ODE No. 1237 | 419 |
| 2.1238 | ODE No. 1238 | 420 |

|        |              |     |
|--------|--------------|-----|
| 2.1239 | ODE No. 1239 | 420 |
| 2.1240 | ODE No. 1240 | 420 |
| 2.1241 | ODE No. 1241 | 420 |
| 2.1242 | ODE No. 1242 | 420 |
| 2.1243 | ODE No. 1243 | 421 |
| 2.1244 | ODE No. 1244 | 421 |
| 2.1245 | ODE No. 1245 | 421 |
| 2.1246 | ODE No. 1246 | 421 |
| 2.1247 | ODE No. 1247 | 421 |
| 2.1248 | ODE No. 1248 | 422 |
| 2.1249 | ODE No. 1249 | 422 |
| 2.1250 | ODE No. 1250 | 422 |
| 2.1251 | ODE No. 1251 | 422 |
| 2.1252 | ODE No. 1252 | 422 |
| 2.1253 | ODE No. 1253 | 423 |
| 2.1254 | ODE No. 1254 | 423 |
| 2.1255 | ODE No. 1255 | 423 |
| 2.1256 | ODE No. 1256 | 423 |
| 2.1257 | ODE No. 1257 | 423 |
| 2.1258 | ODE No. 1258 | 424 |
| 2.1259 | ODE No. 1259 | 424 |
| 2.1260 | ODE No. 1260 | 424 |
| 2.1261 | ODE No. 1261 | 424 |
| 2.1262 | ODE No. 1262 | 424 |
| 2.1263 | ODE No. 1263 | 425 |
| 2.1264 | ODE No. 1264 | 425 |
| 2.1265 | ODE No. 1265 | 425 |
| 2.1266 | ODE No. 1266 | 425 |
| 2.1267 | ODE No. 1267 | 425 |
| 2.1268 | ODE No. 1268 | 426 |
| 2.1269 | ODE No. 1269 | 426 |
| 2.1270 | ODE No. 1270 | 426 |
| 2.1271 | ODE No. 1271 | 426 |
| 2.1272 | ODE No. 1272 | 426 |
| 2.1273 | ODE No. 1273 | 427 |
| 2.1274 | ODE No. 1274 | 427 |
| 2.1275 | ODE No. 1275 | 427 |
| 2.1276 | ODE No. 1276 | 427 |
| 2.1277 | ODE No. 1277 | 427 |
| 2.1278 | ODE No. 1278 | 428 |
| 2.1279 | ODE No. 1279 | 428 |
| 2.1280 | ODE No. 1280 | 428 |
| 2.1281 | ODE No. 1281 | 428 |
| 2.1282 | ODE No. 1282 | 428 |
| 2.1283 | ODE No. 1283 | 429 |
| 2.1284 | ODE No. 1284 | 429 |
| 2.1285 | ODE No. 1285 | 429 |
| 2.1286 | ODE No. 1286 | 429 |
| 2.1287 | ODE No. 1287 | 429 |
| 2.1288 | ODE No. 1288 | 430 |
| 2.1289 | ODE No. 1289 | 430 |
| 2.1290 | ODE No. 1290 | 430 |
| 2.1291 | ODE No. 1291 | 430 |
| 2.1292 | ODE No. 1292 | 430 |
| 2.1293 | ODE No. 1293 | 431 |
| 2.1294 | ODE No. 1294 | 431 |
| 2.1295 | ODE No. 1295 | 431 |
| 2.1296 | ODE No. 1296 | 431 |
| 2.1297 | ODE No. 1297 | 431 |
| 2.1298 | ODE No. 1298 | 432 |
| 2.1299 | ODE No. 1299 | 432 |
| 2.1300 | ODE No. 1300 | 432 |
| 2.1301 | ODE No. 1301 | 432 |
| 2.1302 | ODE No. 1302 | 432 |
| 2.1303 | ODE No. 1303 | 433 |
| 2.1304 | ODE No. 1304 | 433 |

|        |              |     |
|--------|--------------|-----|
| 2.1305 | ODE No. 1305 | 433 |
| 2.1306 | ODE No. 1306 | 433 |
| 2.1307 | ODE No. 1307 | 433 |
| 2.1308 | ODE No. 1308 | 434 |
| 2.1309 | ODE No. 1309 | 434 |
| 2.1310 | ODE No. 1310 | 434 |
| 2.1311 | ODE No. 1311 | 434 |
| 2.1312 | ODE No. 1312 | 434 |
| 2.1313 | ODE No. 1313 | 435 |
| 2.1314 | ODE No. 1314 | 435 |
| 2.1315 | ODE No. 1315 | 435 |
| 2.1316 | ODE No. 1316 | 435 |
| 2.1317 | ODE No. 1317 | 435 |
| 2.1318 | ODE No. 1318 | 436 |
| 2.1319 | ODE No. 1319 | 436 |
| 2.1320 | ODE No. 1320 | 436 |
| 2.1321 | ODE No. 1321 | 436 |
| 2.1322 | ODE No. 1322 | 436 |
| 2.1323 | ODE No. 1323 | 437 |
| 2.1324 | ODE No. 1324 | 437 |
| 2.1325 | ODE No. 1325 | 437 |
| 2.1326 | ODE No. 1326 | 437 |
| 2.1327 | ODE No. 1327 | 437 |
| 2.1328 | ODE No. 1328 | 438 |
| 2.1329 | ODE No. 1329 | 438 |
| 2.1330 | ODE No. 1330 | 438 |
| 2.1331 | ODE No. 1331 | 438 |
| 2.1332 | ODE No. 1332 | 438 |
| 2.1333 | ODE No. 1333 | 439 |
| 2.1334 | ODE No. 1334 | 439 |
| 2.1335 | ODE No. 1335 | 439 |
| 2.1336 | ODE No. 1336 | 439 |
| 2.1337 | ODE No. 1337 | 440 |
| 2.1338 | ODE No. 1338 | 440 |
| 2.1339 | ODE No. 1339 | 440 |
| 2.1340 | ODE No. 1340 | 440 |
| 2.1341 | ODE No. 1341 | 440 |
| 2.1342 | ODE No. 1342 | 441 |
| 2.1343 | ODE No. 1343 | 441 |
| 2.1344 | ODE No. 1344 | 441 |
| 2.1345 | ODE No. 1345 | 442 |
| 2.1346 | ODE No. 1346 | 442 |
| 2.1347 | ODE No. 1347 | 442 |
| 2.1348 | ODE No. 1348 | 442 |
| 2.1349 | ODE No. 1349 | 442 |
| 2.1350 | ODE No. 1350 | 443 |
| 2.1351 | ODE No. 1351 | 443 |
| 2.1352 | ODE No. 1352 | 443 |
| 2.1353 | ODE No. 1353 | 443 |
| 2.1354 | ODE No. 1354 | 443 |
| 2.1355 | ODE No. 1355 | 444 |
| 2.1356 | ODE No. 1356 | 444 |
| 2.1357 | ODE No. 1357 | 444 |
| 2.1358 | ODE No. 1358 | 444 |
| 2.1359 | ODE No. 1359 | 445 |
| 2.1360 | ODE No. 1360 | 445 |
| 2.1361 | ODE No. 1361 | 445 |
| 2.1362 | ODE No. 1362 | 445 |
| 2.1363 | ODE No. 1363 | 445 |
| 2.1364 | ODE No. 1364 | 446 |
| 2.1365 | ODE No. 1365 | 446 |
| 2.1366 | ODE No. 1366 | 446 |
| 2.1367 | ODE No. 1367 | 446 |
| 2.1368 | ODE No. 1368 | 446 |
| 2.1369 | ODE No. 1369 | 447 |
| 2.1370 | ODE No. 1370 | 447 |

|        |              |     |
|--------|--------------|-----|
| 2.1371 | ODE No. 1371 | 447 |
| 2.1372 | ODE No. 1372 | 447 |
| 2.1373 | ODE No. 1373 | 447 |
| 2.1374 | ODE No. 1374 | 448 |
| 2.1375 | ODE No. 1375 | 448 |
| 2.1376 | ODE No. 1376 | 448 |
| 2.1377 | ODE No. 1377 | 448 |
| 2.1378 | ODE No. 1378 | 449 |
| 2.1379 | ODE No. 1379 | 449 |
| 2.1380 | ODE No. 1380 | 449 |
| 2.1381 | ODE No. 1381 | 449 |
| 2.1382 | ODE No. 1382 | 450 |
| 2.1383 | ODE No. 1383 | 450 |
| 2.1384 | ODE No. 1384 | 450 |
| 2.1385 | ODE No. 1385 | 450 |
| 2.1386 | ODE No. 1386 | 451 |
| 2.1387 | ODE No. 1387 | 451 |
| 2.1388 | ODE No. 1388 | 451 |
| 2.1389 | ODE No. 1389 | 451 |
| 2.1390 | ODE No. 1390 | 452 |
| 2.1391 | ODE No. 1391 | 452 |
| 2.1392 | ODE No. 1392 | 452 |
| 2.1393 | ODE No. 1393 | 452 |
| 2.1394 | ODE No. 1394 | 452 |
| 2.1395 | ODE No. 1395 | 453 |
| 2.1396 | ODE No. 1396 | 453 |
| 2.1397 | ODE No. 1397 | 453 |
| 2.1398 | ODE No. 1398 | 453 |
| 2.1399 | ODE No. 1399 | 454 |
| 2.1400 | ODE No. 1400 | 454 |
| 2.1401 | ODE No. 1401 | 454 |
| 2.1402 | ODE No. 1402 | 454 |
| 2.1403 | ODE No. 1403 | 455 |
| 2.1404 | ODE No. 1404 | 455 |
| 2.1405 | ODE No. 1405 | 455 |
| 2.1406 | ODE No. 1406 | 455 |
| 2.1407 | ODE No. 1407 | 456 |
| 2.1408 | ODE No. 1408 | 456 |
| 2.1409 | ODE No. 1409 | 456 |
| 2.1410 | ODE No. 1410 | 456 |
| 2.1411 | ODE No. 1411 | 457 |
| 2.1412 | ODE No. 1412 | 457 |
| 2.1413 | ODE No. 1413 | 457 |
| 2.1414 | ODE No. 1414 | 457 |
| 2.1415 | ODE No. 1415 | 457 |
| 2.1416 | ODE No. 1416 | 458 |
| 2.1417 | ODE No. 1417 | 458 |
| 2.1418 | ODE No. 1418 | 458 |
| 2.1419 | ODE No. 1419 | 458 |
| 2.1420 | ODE No. 1420 | 458 |
| 2.1421 | ODE No. 1421 | 459 |
| 2.1422 | ODE No. 1422 | 459 |
| 2.1423 | ODE No. 1423 | 459 |
| 2.1424 | ODE No. 1424 | 459 |
| 2.1425 | ODE No. 1425 | 459 |
| 2.1426 | ODE No. 1426 | 460 |
| 2.1427 | ODE No. 1427 | 460 |
| 2.1428 | ODE No. 1428 | 460 |
| 2.1429 | ODE No. 1429 | 460 |
| 2.1430 | ODE No. 1430 | 461 |
| 2.1431 | ODE No. 1431 | 461 |
| 2.1432 | ODE No. 1432 | 461 |
| 2.1433 | ODE No. 1433 | 461 |
| 2.1434 | ODE No. 1434 | 461 |
| 2.1435 | ODE No. 1435 | 462 |
| 2.1436 | ODE No. 1436 | 462 |

|        |              |     |
|--------|--------------|-----|
| 2.1437 | ODE No. 1437 | 462 |
| 2.1438 | ODE No. 1438 | 462 |
| 2.1439 | ODE No. 1439 | 462 |
| 2.1440 | ODE No. 1440 | 463 |
| 2.1441 | ODE No. 1441 | 463 |
| 2.1442 | ODE No. 1442 | 463 |
| 2.1443 | ODE No. 1443 | 463 |
| 2.1444 | ODE No. 1444 | 463 |
| 2.1445 | ODE No. 1445 | 464 |
| 2.1446 | ODE No. 1446 | 464 |
| 2.1447 | ODE No. 1447 | 464 |
| 2.1448 | ODE No. 1448 | 464 |
| 2.1449 | ODE No. 1449 | 464 |
| 2.1450 | ODE No. 1450 | 465 |
| 2.1451 | ODE No. 1451 | 465 |
| 2.1452 | ODE No. 1452 | 465 |
| 2.1453 | ODE No. 1453 | 465 |
| 2.1454 | ODE No. 1454 | 465 |
| 2.1455 | ODE No. 1455 | 466 |
| 2.1456 | ODE No. 1456 | 466 |
| 2.1457 | ODE No. 1457 | 466 |
| 2.1458 | ODE No. 1458 | 466 |
| 2.1459 | ODE No. 1459 | 466 |
| 2.1460 | ODE No. 1460 | 467 |
| 2.1461 | ODE No. 1461 | 467 |
| 2.1462 | ODE No. 1462 | 467 |
| 2.1463 | ODE No. 1463 | 467 |
| 2.1464 | ODE No. 1464 | 467 |
| 2.1465 | ODE No. 1465 | 467 |
| 2.1466 | ODE No. 1466 | 468 |
| 2.1467 | ODE No. 1467 | 468 |
| 2.1468 | ODE No. 1468 | 468 |
| 2.1469 | ODE No. 1469 | 468 |
| 2.1470 | ODE No. 1470 | 468 |
| 2.1471 | ODE No. 1471 | 469 |
| 2.1472 | ODE No. 1472 | 469 |
| 2.1473 | ODE No. 1473 | 469 |
| 2.1474 | ODE No. 1474 | 469 |
| 2.1475 | ODE No. 1475 | 469 |
| 2.1476 | ODE No. 1476 | 469 |
| 2.1477 | ODE No. 1477 | 470 |
| 2.1478 | ODE No. 1478 | 470 |
| 2.1479 | ODE No. 1479 | 470 |
| 2.1480 | ODE No. 1480 | 470 |
| 2.1481 | ODE No. 1481 | 470 |
| 2.1482 | ODE No. 1482 | 471 |
| 2.1483 | ODE No. 1483 | 471 |
| 2.1484 | ODE No. 1484 | 471 |
| 2.1485 | ODE No. 1485 | 471 |
| 2.1486 | ODE No. 1486 | 471 |
| 2.1487 | ODE No. 1487 | 472 |
| 2.1488 | ODE No. 1488 | 472 |
| 2.1489 | ODE No. 1489 | 472 |
| 2.1490 | ODE No. 1490 | 472 |
| 2.1491 | ODE No. 1491 | 472 |
| 2.1492 | ODE No. 1492 | 473 |
| 2.1493 | ODE No. 1493 | 473 |
| 2.1494 | ODE No. 1494 | 473 |
| 2.1495 | ODE No. 1495 | 473 |
| 2.1496 | ODE No. 1496 | 473 |
| 2.1497 | ODE No. 1497 | 474 |
| 2.1498 | ODE No. 1498 | 474 |
| 2.1499 | ODE No. 1499 | 474 |
| 2.1500 | ODE No. 1500 | 474 |
| 2.1501 | ODE No. 1501 | 474 |
| 2.1502 | ODE No. 1502 | 475 |

|        |              |     |
|--------|--------------|-----|
| 2.1503 | ODE No. 1503 | 475 |
| 2.1504 | ODE No. 1504 | 475 |
| 2.1505 | ODE No. 1505 | 475 |
| 2.1506 | ODE No. 1506 | 475 |
| 2.1507 | ODE No. 1507 | 476 |
| 2.1508 | ODE No. 1508 | 476 |
| 2.1509 | ODE No. 1509 | 476 |
| 2.1510 | ODE No. 1510 | 476 |
| 2.1511 | ODE No. 1511 | 476 |
| 2.1512 | ODE No. 1512 | 477 |
| 2.1513 | ODE No. 1513 | 477 |
| 2.1514 | ODE No. 1514 | 477 |
| 2.1515 | ODE No. 1515 | 477 |
| 2.1516 | ODE No. 1516 | 477 |
| 2.1517 | ODE No. 1517 | 478 |
| 2.1518 | ODE No. 1518 | 478 |
| 2.1519 | ODE No. 1519 | 478 |
| 2.1520 | ODE No. 1520 | 478 |
| 2.1521 | ODE No. 1521 | 478 |
| 2.1522 | ODE No. 1522 | 479 |
| 2.1523 | ODE No. 1523 | 479 |
| 2.1524 | ODE No. 1524 | 479 |
| 2.1525 | ODE No. 1525 | 479 |
| 2.1526 | ODE No. 1526 | 479 |
| 2.1527 | ODE No. 1527 | 480 |
| 2.1528 | ODE No. 1528 | 480 |
| 2.1529 | ODE No. 1529 | 480 |
| 2.1530 | ODE No. 1530 | 480 |
| 2.1531 | ODE No. 1531 | 480 |
| 2.1532 | ODE No. 1532 | 481 |
| 2.1533 | ODE No. 1533 | 481 |
| 2.1534 | ODE No. 1534 | 481 |
| 2.1535 | ODE No. 1535 | 481 |
| 2.1536 | ODE No. 1536 | 481 |
| 2.1537 | ODE No. 1537 | 482 |
| 2.1538 | ODE No. 1538 | 482 |
| 2.1539 | ODE No. 1539 | 482 |
| 2.1540 | ODE No. 1540 | 482 |
| 2.1541 | ODE No. 1541 | 482 |
| 2.1542 | ODE No. 1542 | 483 |
| 2.1543 | ODE No. 1543 | 483 |
| 2.1544 | ODE No. 1544 | 483 |
| 2.1545 | ODE No. 1545 | 483 |
| 2.1546 | ODE No. 1546 | 483 |
| 2.1547 | ODE No. 1547 | 484 |
| 2.1548 | ODE No. 1548 | 484 |
| 2.1549 | ODE No. 1549 | 484 |
| 2.1550 | ODE No. 1550 | 484 |
| 2.1551 | ODE No. 1551 | 484 |
| 2.1552 | ODE No. 1552 | 485 |
| 2.1553 | ODE No. 1553 | 485 |
| 2.1554 | ODE No. 1554 | 485 |
| 2.1555 | ODE No. 1555 | 485 |
| 2.1556 | ODE No. 1556 | 485 |
| 2.1557 | ODE No. 1557 | 486 |
| 2.1558 | ODE No. 1558 | 486 |
| 2.1559 | ODE No. 1559 | 486 |
| 2.1560 | ODE No. 1560 | 486 |
| 2.1561 | ODE No. 1561 | 486 |
| 2.1562 | ODE No. 1562 | 487 |
| 2.1563 | ODE No. 1563 | 487 |
| 2.1564 | ODE No. 1564 | 487 |
| 2.1565 | ODE No. 1565 | 487 |
| 2.1566 | ODE No. 1566 | 487 |
| 2.1567 | ODE No. 1567 | 488 |
| 2.1568 | ODE No. 1568 | 488 |



|        |              |     |
|--------|--------------|-----|
| 2.1569 | ODE No. 1569 | 488 |
| 2.1570 | ODE No. 1570 | 488 |
| 2.1571 | ODE No. 1571 | 488 |
| 2.1572 | ODE No. 1572 | 489 |
| 2.1573 | ODE No. 1573 | 489 |
| 2.1574 | ODE No. 1574 | 489 |
| 2.1575 | ODE No. 1575 | 489 |
| 2.1576 | ODE No. 1576 | 489 |
| 2.1577 | ODE No. 1577 | 490 |
| 2.1578 | ODE No. 1578 | 490 |
| 2.1579 | ODE No. 1579 | 490 |
| 2.1580 | ODE No. 1580 | 490 |
| 2.1581 | ODE No. 1581 | 490 |
| 2.1582 | ODE No. 1582 | 491 |
| 2.1583 | ODE No. 1583 | 491 |
| 2.1584 | ODE No. 1584 | 491 |
| 2.1585 | ODE No. 1585 | 491 |
| 2.1586 | ODE No. 1586 | 492 |
| 2.1587 | ODE No. 1587 | 492 |
| 2.1588 | ODE No. 1588 | 492 |
| 2.1589 | ODE No. 1589 | 492 |
| 2.1590 | ODE No. 1590 | 492 |
| 2.1591 | ODE No. 1591 | 493 |
| 2.1592 | ODE No. 1592 | 493 |
| 2.1593 | ODE No. 1593 | 493 |
| 2.1594 | ODE No. 1594 | 493 |
| 2.1595 | ODE No. 1595 | 493 |
| 2.1596 | ODE No. 1596 | 493 |
| 2.1597 | ODE No. 1597 | 494 |
| 2.1598 | ODE No. 1598 | 494 |
| 2.1599 | ODE No. 1599 | 494 |
| 2.1600 | ODE No. 1600 | 494 |
| 2.1601 | ODE No. 1601 | 494 |
| 2.1602 | ODE No. 1602 | 495 |
| 2.1603 | ODE No. 1603 | 495 |
| 2.1604 | ODE No. 1604 | 495 |
| 2.1605 | ODE No. 1605 | 495 |
| 2.1606 | ODE No. 1606 | 495 |
| 2.1607 | ODE No. 1607 | 496 |
| 2.1608 | ODE No. 1608 | 496 |
| 2.1609 | ODE No. 1609 | 496 |
| 2.1610 | ODE No. 1610 | 496 |
| 2.1611 | ODE No. 1611 | 496 |
| 2.1612 | ODE No. 1612 | 497 |
| 2.1613 | ODE No. 1613 | 497 |
| 2.1614 | ODE No. 1614 | 497 |
| 2.1615 | ODE No. 1615 | 497 |
| 2.1616 | ODE No. 1616 | 497 |
| 2.1617 | ODE No. 1617 | 497 |
| 2.1618 | ODE No. 1618 | 498 |
| 2.1619 | ODE No. 1619 | 498 |
| 2.1620 | ODE No. 1620 | 498 |
| 2.1621 | ODE No. 1621 | 498 |
| 2.1622 | ODE No. 1622 | 498 |
| 2.1623 | ODE No. 1623 | 499 |
| 2.1624 | ODE No. 1624 | 499 |
| 2.1625 | ODE No. 1625 | 499 |
| 2.1626 | ODE No. 1626 | 499 |
| 2.1627 | ODE No. 1627 | 499 |
| 2.1628 | ODE No. 1628 | 499 |
| 2.1629 | ODE No. 1629 | 500 |
| 2.1630 | ODE No. 1630 | 500 |
| 2.1631 | ODE No. 1631 | 500 |
| 2.1632 | ODE No. 1632 | 500 |
| 2.1633 | ODE No. 1633 | 500 |
| 2.1634 | ODE No. 1634 | 501 |

|        |              |     |
|--------|--------------|-----|
| 2.1635 | ODE No. 1635 | 501 |
| 2.1636 | ODE No. 1636 | 501 |
| 2.1637 | ODE No. 1637 | 501 |
| 2.1638 | ODE No. 1638 | 501 |
| 2.1639 | ODE No. 1639 | 502 |
| 2.1640 | ODE No. 1640 | 502 |
| 2.1641 | ODE No. 1641 | 502 |
| 2.1642 | ODE No. 1642 | 502 |
| 2.1643 | ODE No. 1643 | 502 |
| 2.1644 | ODE No. 1644 | 502 |
| 2.1645 | ODE No. 1645 | 503 |
| 2.1646 | ODE No. 1646 | 503 |
| 2.1647 | ODE No. 1647 | 503 |
| 2.1648 | ODE No. 1648 | 503 |
| 2.1649 | ODE No. 1649 | 503 |
| 2.1650 | ODE No. 1650 | 504 |
| 2.1651 | ODE No. 1651 | 504 |
| 2.1652 | ODE No. 1652 | 504 |
| 2.1653 | ODE No. 1653 | 504 |
| 2.1654 | ODE No. 1654 | 505 |
| 2.1655 | ODE No. 1655 | 505 |
| 2.1656 | ODE No. 1656 | 505 |
| 2.1657 | ODE No. 1657 | 505 |
| 2.1658 | ODE No. 1658 | 506 |
| 2.1659 | ODE No. 1659 | 506 |
| 2.1660 | ODE No. 1660 | 506 |
| 2.1661 | ODE No. 1661 | 506 |
| 2.1662 | ODE No. 1662 | 506 |
| 2.1663 | ODE No. 1663 | 507 |
| 2.1664 | ODE No. 1664 | 507 |
| 2.1665 | ODE No. 1665 | 507 |
| 2.1666 | ODE No. 1666 | 507 |
| 2.1667 | ODE No. 1667 | 507 |
| 2.1668 | ODE No. 1668 | 508 |
| 2.1669 | ODE No. 1669 | 508 |
| 2.1670 | ODE No. 1670 | 508 |
| 2.1671 | ODE No. 1671 | 508 |
| 2.1672 | ODE No. 1672 | 508 |
| 2.1673 | ODE No. 1673 | 509 |
| 2.1674 | ODE No. 1674 | 509 |
| 2.1675 | ODE No. 1675 | 509 |
| 2.1676 | ODE No. 1676 | 509 |
| 2.1677 | ODE No. 1677 | 509 |
| 2.1678 | ODE No. 1678 | 510 |
| 2.1679 | ODE No. 1679 | 510 |
| 2.1680 | ODE No. 1680 | 510 |
| 2.1681 | ODE No. 1681 | 510 |
| 2.1682 | ODE No. 1682 | 510 |
| 2.1683 | ODE No. 1683 | 511 |
| 2.1684 | ODE No. 1684 | 511 |
| 2.1685 | ODE No. 1685 | 511 |
| 2.1686 | ODE No. 1686 | 511 |
| 2.1687 | ODE No. 1687 | 511 |
| 2.1688 | ODE No. 1688 | 512 |
| 2.1689 | ODE No. 1689 | 512 |
| 2.1690 | ODE No. 1690 | 512 |
| 2.1691 | ODE No. 1691 | 512 |
| 2.1692 | ODE No. 1692 | 512 |
| 2.1693 | ODE No. 1693 | 513 |
| 2.1694 | ODE No. 1694 | 513 |
| 2.1695 | ODE No. 1695 | 513 |
| 2.1696 | ODE No. 1696 | 513 |
| 2.1697 | ODE No. 1697 | 513 |
| 2.1698 | ODE No. 1698 | 514 |
| 2.1699 | ODE No. 1699 | 514 |
| 2.1700 | ODE No. 1700 | 514 |

|        |              |     |
|--------|--------------|-----|
| 2.1701 | ODE No. 1701 | 514 |
| 2.1702 | ODE No. 1702 | 514 |
| 2.1703 | ODE No. 1703 | 515 |
| 2.1704 | ODE No. 1704 | 515 |
| 2.1705 | ODE No. 1705 | 515 |
| 2.1706 | ODE No. 1706 | 515 |
| 2.1707 | ODE No. 1707 | 515 |
| 2.1708 | ODE No. 1708 | 516 |
| 2.1709 | ODE No. 1709 | 516 |
| 2.1710 | ODE No. 1710 | 516 |
| 2.1711 | ODE No. 1711 | 516 |
| 2.1712 | ODE No. 1712 | 517 |
| 2.1713 | ODE No. 1713 | 517 |
| 2.1714 | ODE No. 1714 | 517 |
| 2.1715 | ODE No. 1715 | 517 |
| 2.1716 | ODE No. 1716 | 517 |
| 2.1717 | ODE No. 1717 | 518 |
| 2.1718 | ODE No. 1718 | 518 |
| 2.1719 | ODE No. 1719 | 518 |
| 2.1720 | ODE No. 1720 | 518 |
| 2.1721 | ODE No. 1721 | 518 |
| 2.1722 | ODE No. 1722 | 519 |
| 2.1723 | ODE No. 1723 | 519 |
| 2.1724 | ODE No. 1724 | 519 |
| 2.1725 | ODE No. 1725 | 519 |
| 2.1726 | ODE No. 1726 | 519 |
| 2.1727 | ODE No. 1727 | 520 |
| 2.1728 | ODE No. 1728 | 520 |
| 2.1729 | ODE No. 1729 | 520 |
| 2.1730 | ODE No. 1730 | 520 |
| 2.1731 | ODE No. 1731 | 520 |
| 2.1732 | ODE No. 1732 | 521 |
| 2.1733 | ODE No. 1733 | 521 |
| 2.1734 | ODE No. 1734 | 521 |
| 2.1735 | ODE No. 1735 | 521 |
| 2.1736 | ODE No. 1736 | 521 |
| 2.1737 | ODE No. 1737 | 521 |
| 2.1738 | ODE No. 1738 | 522 |
| 2.1739 | ODE No. 1739 | 522 |
| 2.1740 | ODE No. 1740 | 522 |
| 2.1741 | ODE No. 1741 | 522 |
| 2.1742 | ODE No. 1742 | 522 |
| 2.1743 | ODE No. 1743 | 522 |
| 2.1744 | ODE No. 1744 | 523 |
| 2.1745 | ODE No. 1745 | 523 |
| 2.1746 | ODE No. 1746 | 523 |
| 2.1747 | ODE No. 1747 | 523 |
| 2.1748 | ODE No. 1748 | 523 |
| 2.1749 | ODE No. 1749 | 524 |
| 2.1750 | ODE No. 1750 | 524 |
| 2.1751 | ODE No. 1751 | 524 |
| 2.1752 | ODE No. 1752 | 524 |
| 2.1753 | ODE No. 1753 | 525 |
| 2.1754 | ODE No. 1754 | 525 |
| 2.1755 | ODE No. 1755 | 525 |
| 2.1756 | ODE No. 1756 | 525 |
| 2.1757 | ODE No. 1757 | 526 |
| 2.1758 | ODE No. 1758 | 526 |
| 2.1759 | ODE No. 1759 | 526 |
| 2.1760 | ODE No. 1760 | 526 |
| 2.1761 | ODE No. 1761 | 526 |
| 2.1762 | ODE No. 1762 | 527 |
| 2.1763 | ODE No. 1763 | 527 |
| 2.1764 | ODE No. 1764 | 527 |
| 2.1765 | ODE No. 1765 | 527 |
| 2.1766 | ODE No. 1766 | 527 |

|        |              |     |
|--------|--------------|-----|
| 2.1767 | ODE No. 1767 | 528 |
| 2.1768 | ODE No. 1768 | 528 |
| 2.1769 | ODE No. 1769 | 528 |
| 2.1770 | ODE No. 1770 | 528 |
| 2.1771 | ODE No. 1771 | 528 |
| 2.1772 | ODE No. 1772 | 529 |
| 2.1773 | ODE No. 1773 | 529 |
| 2.1774 | ODE No. 1774 | 529 |
| 2.1775 | ODE No. 1775 | 529 |
| 2.1776 | ODE No. 1776 | 529 |
| 2.1777 | ODE No. 1777 | 530 |
| 2.1778 | ODE No. 1778 | 530 |
| 2.1779 | ODE No. 1779 | 530 |
| 2.1780 | ODE No. 1780 | 531 |
| 2.1781 | ODE No. 1781 | 531 |
| 2.1782 | ODE No. 1782 | 531 |
| 2.1783 | ODE No. 1783 | 531 |
| 2.1784 | ODE No. 1784 | 531 |
| 2.1785 | ODE No. 1785 | 532 |
| 2.1786 | ODE No. 1786 | 532 |
| 2.1787 | ODE No. 1787 | 532 |
| 2.1788 | ODE No. 1788 | 532 |
| 2.1789 | ODE No. 1789 | 532 |
| 2.1790 | ODE No. 1790 | 533 |
| 2.1791 | ODE No. 1791 | 533 |
| 2.1792 | ODE No. 1792 | 533 |
| 2.1793 | ODE No. 1793 | 533 |
| 2.1794 | ODE No. 1794 | 534 |
| 2.1795 | ODE No. 1795 | 534 |
| 2.1796 | ODE No. 1796 | 534 |
| 2.1797 | ODE No. 1797 | 534 |
| 2.1798 | ODE No. 1798 | 535 |
| 2.1799 | ODE No. 1799 | 535 |
| 2.1800 | ODE No. 1800 | 535 |
| 2.1801 | ODE No. 1801 | 535 |
| 2.1802 | ODE No. 1802 | 535 |
| 2.1803 | ODE No. 1803 | 535 |
| 2.1804 | ODE No. 1804 | 536 |
| 2.1805 | ODE No. 1805 | 536 |
| 2.1806 | ODE No. 1806 | 536 |
| 2.1807 | ODE No. 1807 | 536 |
| 2.1808 | ODE No. 1808 | 536 |
| 2.1809 | ODE No. 1809 | 537 |
| 2.1810 | ODE No. 1810 | 537 |
| 2.1811 | ODE No. 1811 | 537 |
| 2.1812 | ODE No. 1812 | 537 |
| 2.1813 | ODE No. 1813 | 537 |
| 2.1814 | ODE No. 1814 | 538 |
| 2.1815 | ODE No. 1815 | 538 |
| 2.1816 | ODE No. 1816 | 538 |
| 2.1817 | ODE No. 1817 | 538 |
| 2.1818 | ODE No. 1818 | 538 |
| 2.1819 | ODE No. 1819 | 539 |
| 2.1820 | ODE No. 1820 | 539 |
| 2.1821 | ODE No. 1821 | 539 |
| 2.1822 | ODE No. 1822 | 539 |
| 2.1823 | ODE No. 1823 | 539 |
| 2.1824 | ODE No. 1824 | 540 |
| 2.1825 | ODE No. 1825 | 540 |
| 2.1826 | ODE No. 1826 | 540 |
| 2.1827 | ODE No. 1827 | 540 |
| 2.1828 | ODE No. 1828 | 540 |
| 2.1829 | ODE No. 1829 | 541 |
| 2.1830 | ODE No. 1830 | 541 |
| 2.1831 | ODE No. 1831 | 541 |
| 2.1832 | ODE No. 1832 | 541 |

|        |              |     |
|--------|--------------|-----|
| 2.1833 | ODE No. 1833 | 541 |
| 2.1834 | ODE No. 1834 | 542 |
| 2.1835 | ODE No. 1835 | 542 |
| 2.1836 | ODE No. 1836 | 542 |
| 2.1837 | ODE No. 1837 | 542 |
| 2.1838 | ODE No. 1838 | 542 |
| 2.1839 | ODE No. 1839 | 542 |
| 2.1840 | ODE No. 1840 | 543 |
| 2.1841 | ODE No. 1841 | 543 |
| 2.1842 | ODE No. 1842 | 543 |
| 2.1843 | ODE No. 1843 | 543 |
| 2.1844 | ODE No. 1844 | 543 |
| 2.1845 | ODE No. 1845 | 544 |
| 2.1846 | ODE No. 1846 | 544 |
| 2.1847 | ODE No. 1847 | 544 |
| 2.1848 | ODE No. 1848 | 544 |
| 2.1849 | ODE No. 1849 | 544 |
| 2.1850 | ODE No. 1850 | 545 |
| 2.1851 | ODE No. 1851 | 545 |
| 2.1852 | ODE No. 1852 | 545 |
| 2.1853 | ODE No. 1853 | 545 |
| 2.1854 | ODE No. 1854 | 545 |
| 2.1855 | ODE No. 1855 | 545 |
| 2.1856 | ODE No. 1856 | 546 |
| 2.1857 | ODE No. 1857 | 546 |
| 2.1858 | ODE No. 1858 | 546 |
| 2.1859 | ODE No. 1859 | 546 |
| 2.1860 | ODE No. 1860 | 546 |
| 2.1861 | ODE No. 1861 | 547 |
| 2.1862 | ODE No. 1862 | 547 |
| 2.1863 | ODE No. 1863 | 547 |
| 2.1864 | ODE No. 1864 | 547 |
| 2.1865 | ODE No. 1865 | 547 |
| 2.1866 | ODE No. 1866 | 548 |
| 2.1867 | ODE No. 1867 | 548 |
| 2.1868 | ODE No. 1868 | 548 |
| 2.1869 | ODE No. 1869 | 548 |
| 2.1870 | ODE No. 1870 | 548 |
| 2.1871 | ODE No. 1871 | 549 |
| 2.1872 | ODE No. 1872 | 549 |
| 2.1873 | ODE No. 1873 | 549 |
| 2.1874 | ODE No. 1874 | 549 |
| 2.1875 | ODE No. 1875 | 549 |
| 2.1876 | ODE No. 1876 | 550 |
| 2.1877 | ODE No. 1877 | 550 |
| 2.1878 | ODE No. 1878 | 550 |
| 2.1879 | ODE No. 1879 | 550 |
| 2.1880 | ODE No. 1880 | 550 |
| 2.1881 | ODE No. 1881 | 551 |
| 2.1882 | ODE No. 1882 | 551 |
| 2.1883 | ODE No. 1883 | 551 |
| 2.1884 | ODE No. 1884 | 551 |
| 2.1885 | ODE No. 1885 | 551 |
| 2.1886 | ODE No. 1886 | 552 |
| 2.1887 | ODE No. 1887 | 552 |
| 2.1888 | ODE No. 1888 | 552 |
| 2.1889 | ODE No. 1889 | 552 |
| 2.1890 | ODE No. 1890 | 552 |
| 2.1891 | ODE No. 1891 | 553 |
| 2.1892 | ODE No. 1892 | 553 |
| 2.1893 | ODE No. 1893 | 553 |
| 2.1894 | ODE No. 1894 | 553 |
| 2.1895 | ODE No. 1895 | 554 |
| 2.1896 | ODE No. 1896 | 554 |
| 2.1897 | ODE No. 1897 | 554 |
| 2.1898 | ODE No. 1898 | 554 |

|        |              |     |
|--------|--------------|-----|
| 2.1899 | ODE No. 1899 | 555 |
| 2.1900 | ODE No. 1900 | 555 |
| 2.1901 | ODE No. 1901 | 555 |
| 2.1902 | ODE No. 1902 | 555 |
| 2.1903 | ODE No. 1903 | 555 |
| 2.1904 | ODE No. 1904 | 556 |
| 2.1905 | ODE No. 1905 | 556 |
| 2.1906 | ODE No. 1906 | 556 |
| 2.1907 | ODE No. 1907 | 556 |
| 2.1908 | ODE No. 1908 | 556 |
| 2.1909 | ODE No. 1909 | 557 |
| 2.1910 | ODE No. 1910 | 557 |
| 2.1911 | ODE No. 1911 | 557 |
| 2.1912 | ODE No. 1912 | 557 |
| 2.1913 | ODE No. 1913 | 557 |
| 2.1914 | ODE No. 1914 | 558 |
| 2.1915 | ODE No. 1915 | 558 |
| 2.1916 | ODE No. 1916 | 558 |
| 2.1917 | ODE No. 1917 | 559 |
| 2.1918 | ODE No. 1918 | 559 |
| 2.1919 | ODE No. 1919 | 559 |
| 2.1920 | ODE No. 1920 | 559 |
| 2.1921 | ODE No. 1921 | 559 |
| 2.1922 | ODE No. 1922 | 560 |
| 2.1923 | ODE No. 1923 | 560 |
| 2.1924 | ODE No. 1924 | 560 |
| 2.1925 | ODE No. 1925 | 560 |
| 2.1926 | ODE No. 1926 | 560 |
| 2.1927 | ODE No. 1927 | 561 |
| 2.1928 | ODE No. 1928 | 561 |
| 2.1929 | ODE No. 1929 | 561 |
| 2.1930 | ODE No. 1930 | 561 |
| 2.1931 | ODE No. 1931 | 561 |
| 2.1932 | ODE No. 1932 | 562 |
| 2.1933 | ODE No. 1933 | 562 |
| 2.1934 | ODE No. 1934 | 562 |
| 2.1935 | ODE No. 1935 | 563 |
| 2.1936 | ODE No. 1936 | 563 |
| 2.1937 | ODE No. 1937 | 563 |
| 2.1938 | ODE No. 1938 | 563 |
| 2.1939 | ODE No. 1939 | 563 |
| 2.1940 | ODE No. 1940 | 564 |

# 1 Introduction and summary of results

This report gives the result of solving the 1,940 differential equations from Kamke book in Mathematica 12 and Maple 2019 on windows 10, 64 bit OS. PC with 64 GB RAM, using Intel I7-8086K at 4GHz.

The command `AboluteTiming[]` was used in Mathematica to obtain the CPU time. In Maple the following commands were used for this purpose

```
t0 := time[real]():
timeOut := 60*60;
result_of_solve := timelimit(timeOut,dsolve(ode[i]));
cpu_time := time[real]()-t0:
```

Both Maple and Mathematica had a CPU time limit of 5 minutes to complete each problem else the problem is considered not solved and marked as timed out.

When Mathematica returned `DifferentialRoot` as a solution to an ODE this was counted as not solved. Similarly, when Maple returned `DESol` this was also counted as not solved.

Table 1 below summarizes the performance of each CAS system

| system      | % solved | mean CPU (sec) | result mean leaf size | total CPU (minutes) | total leaf size |
|-------------|----------|----------------|-----------------------|---------------------|-----------------|
| Mathematica | 82.78    | 1.75           | 2690.99               | 46.97               | 4321725         |
| Maple       | 92.11    | 0.53           | 207.35                | 15.78               | 370541          |

Table 1: Summary of final results

Table 2 summarizes the Kamke equations used

| book chapter                                 | kamke equation numbers | This report numbers |
|----------------------------------------------|------------------------|---------------------|
| Chapter 1, linear first order                | 1.1—1.576              | 1—576               |
| Additional non-linear first order            | N/A                    | 577—1000            |
| Chapter 2, linear second order               | 2.1—2.448              | 1001—1448           |
| Chapter 3, linear third order                | 3.1—3.85               | 1449—1533           |
| Chapter 4, linear fourth order               | 4.1—4.44               | 1534—1577           |
| Chapter 5, linear fifth and higher order     | 5.1—5.13               | 1578—1590           |
| Chapter 6, non-linear second order           | 6.1—6.246              | 1591—1836           |
| Chapter 7, non-linear third and higher order | 7.1—7.19               | 1837—1855           |
| Chapter 8, system of ode, first order        | 8.1—8.57               | 1856—1912           |
| Chapter 9, system of ode, higher order       | 9.1—9.28               | 1913—1940           |

Table 2: Kamke equation numbering

The following summarizes which equations are solved by each system

**Not solved by Mathematica** 47, 48, 49, 50, 55, 56, 74, 79, 82, 87, 110, 121, 188, 202, 203, 205, 206, 219, 234, 237, 250, 253, 265, 266, 269, 331, 340, 367, 370, 394, 395, 413, 427, 428, 451, 460, 461, 465, 480, 482, 485, 489, 503, 506, 507, 510, 524, 531, 532, 537, 562, 572, 575, 576, 638, 639, 640, 730, 743, 746, 752, 759, 769, 807, 835, 837, 854, 855, 862, 885, 889, 894, 909, 916, 917, 919, 953, 996, 1015, 1019, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1038, 1072, 1073, 1074, 1075, 1076, 1077, 1080, 1081, 1082, 1083, 1084, 1085, 1099, 1126, 1157, 1205, 1212, 1216, 1236, 1248, 1261, 1268, 1278, 1303, 1306, 1323, 1329, 1330, 1343, 1348, 1362, 1367, 1372, 1373, 1398, 1402, 1403, 1407, 1408, 1419, 1427, 1439, 1440, 1441, 1443, 1444, 1445, 1450, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1471, 1472, 1473, 1474, 1476, 1482, 1484, 1489, 1500, 1505, 1507, 1510, 1515, 1516, 1520, 1526, 1527, 1529, 1530, 1531, 1540, 1541, 1542, 1543, 1544, 1547, 1552, 1569, 1572, 1573, 1574, 1575, 1576, 1581, 1586, 1590, 1593, 1595, 1596, 1598, 1599, 1601, 1603, 1605, 1606, 1608, 1609, 1611, 1612, 1613, 1614, 1615, 1616, 1617, 1618, 1619, 1622, 1623, 1624, 1625, 1626, 1627, 1628, 1629, 1631, 1633, 1634, 1636, 1637, 1639, 1642, 1643, 1644, 1645, 1648, 1649, 1652, 1658, 1659, 1660, 1662, 1663, 1664, 1665, 1666, 1667, 1672, 1673, 1675, 1677, 1678, 1680, 1681, 1682, 1684, 1685, 1686, 1690, 1691, 1692, 1693, 1695, 1696, 1702, 1704, 1708, 1709, 1710, 1713, 1719, 1720, 1721, 1729, 1732, 1734, 1735, 1737, 1738, 1739, 1742, 1746, 1751, 1757, 1761, 1762, 1777, 1779, 1780, 1788, 1789, 1797, 1798, 1801, 1802, 1806, 1807, 1809, 1811, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1823, 1825, 1827, 1831, 1832, 1833, 1834, 1836, 1837, 1838, 1839, 1840, 1841, 1844, 1845, 1850, 1851, 1853, 1854, 1855, 1875, 1880, 1885, 1890, 1905, 1911, 1912, 1915, 1918, 1919, 1920, 1921, 1922, 1925, 1926, 1927, 1928, 1929, 1932, 1933, 1934, 1935, 1936, 1937, 1939, 1940

**Not solved by Maple** 47, 48, 49, 50, 55, 56, 74, 79, 82, 87, 110, 121, 202, 203, 205, 206, 219, 234, 237, 250, 253, 265, 269, 331, 340, 367, 368, 370, 383, 395, 460, 461, 480, 482, 485, 503,

506, 507, 510, 531, 572, 575, 576, 733, 789, 790, 835, 837, 885, 894, 912, 920, 1015, 1019, 1026, 1028, 1030, 1031, 1038, 1072, 1073, 1075, 1076, 1077, 1081, 1157, 1205, 1212, 1216, 1234, 1236, 1278, 1408, 1439, 1440, 1441, 1443, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1473, 1474, 1476, 1484, 1489, 1510, 1515, 1531, 1540, 1541, 1542, 1543, 1581, 1582, 1586, 1593, 1595, 1596, 1598, 1599, 1606, 1608, 1609, 1617, 1619, 1623, 1625, 1628, 1634, 1642, 1643, 1645, 1649, 1675, 1685, 1698, 1702, 1704, 1705, 1706, 1721, 1729, 1732, 1734, 1735, 1737, 1738, 1739, 1751, 1757, 1761, 1788, 1789, 1797, 1801, 1802, 1807, 1811, 1835, 1851, 1854, 1855, 1890, 1905, 1921, 1922, 1927, 1928, 1940

**Solved by Mathematica but not by Maple** 368, 383, 733, 789, 790, 912, 920, 1234, 1582, 1698, 1705, 1706, 1835

**Solved by Maple but not by Mathematica** 188, 266, 394, 413, 427, 428, 451, 465, 489, 524, 532, 537, 562, 638, 639, 640, 730, 743, 746, 752, 759, 769, 807, 854, 855, 862, 889, 909, 916, 917, 919, 953, 996, 1027, 1029, 1032, 1074, 1080, 1082, 1083, 1084, 1085, 1099, 1126, 1248, 1261, 1268, 1303, 1306, 1323, 1329, 1330, 1343, 1348, 1362, 1367, 1372, 1373, 1398, 1402, 1403, 1407, 1419, 1427, 1444, 1445, 1450, 1471, 1472, 1482, 1500, 1505, 1507, 1516, 1520, 1526, 1527, 1529, 1530, 1544, 1547, 1552, 1569, 1572, 1573, 1574, 1575, 1576, 1590, 1601, 1603, 1605, 1611, 1612, 1613, 1614, 1615, 1616, 1618, 1622, 1624, 1626, 1627, 1629, 1631, 1633, 1636, 1637, 1639, 1644, 1648, 1652, 1658, 1659, 1660, 1662, 1663, 1664, 1665, 1666, 1667, 1672, 1673, 1677, 1678, 1680, 1681, 1682, 1684, 1686, 1690, 1691, 1692, 1693, 1695, 1696, 1708, 1709, 1710, 1713, 1719, 1720, 1742, 1746, 1762, 1777, 1779, 1780, 1798, 1806, 1809, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1823, 1825, 1827, 1831, 1832, 1833, 1834, 1836, 1837, 1838, 1839, 1840, 1841, 1844, 1845, 1850, 1853, 1875, 1880, 1885, 1911, 1912, 1915, 1918, 1919, 1920, 1925, 1926, 1929, 1932, 1933, 1934, 1935, 1936, 1937, 1939

**Solved by both Maple and Mathematica** 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 80, 81, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 235, 236, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 267, 268, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 369, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 452, 453, 454, 455, 456, 457, 458, 459, 462, 463, 464, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 481, 483, 484, 486, 487, 488, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 504, 505, 508, 509, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 525, 526, 527, 528, 529, 530, 533, 534, 535, 536, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 731, 732, 734, 735, 736, 737, 738, 739, 740, 741, 742, 744, 745, 747, 748, 749, 750, 751, 753, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 836, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 856, 857, 858, 859, 860, 861, 863, 864, 865,



866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 886, 887, 888, 890, 891, 892, 893, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 910, 911, 913, 914, 915, 918, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1020, 1021, 1022, 1023, 1024, 1025, 1033, 1034, 1035, 1036, 1037, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1078, 1079, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1206, 1207, 1208, 1209, 1210, 1211, 1213, 1214, 1215, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1235, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1262, 1263, 1264, 1265, 1266, 1267, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1304, 1305, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1324, 1325, 1326, 1327, 1328, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1344, 1345, 1346, 1347, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1363, 1364, 1365, 1366, 1368, 1369, 1370, 1371, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1399, 1400, 1401, 1404, 1405, 1406, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1442, 1446, 1447, 1448, 1449, 1451, 1452, 1453, 1454, 1455, 1456, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1475, 1477, 1478, 1479, 1480, 1481, 1483, 1485, 1486, 1487, 1488, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1501, 1502, 1503, 1504, 1506, 1508, 1509, 1511, 1512, 1513, 1514, 1517, 1518, 1519, 1521, 1522, 1523, 1524, 1525, 1528, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1545, 1546, 1548, 1549, 1550, 1551, 1553, 1554, 1555, 1556, 1557, 1558, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1570, 1571, 1577, 1578, 1579, 1580, 1583, 1584, 1585, 1587, 1588, 1589, 1591, 1592, 1594, 1597, 1600, 1602, 1604, 1607, 1610, 1620, 1621, 1630, 1632, 1635, 1638, 1640, 1641, 1646, 1647, 1650, 1651, 1653, 1654, 1655, 1656, 1657, 1661, 1668, 1669, 1670, 1671, 1674, 1676, 1679, 1683, 1687, 1688, 1689, 1694, 1697, 1699, 1700, 1701, 1703, 1707, 1711, 1712, 1714, 1715, 1716, 1717, 1718, 1722, 1723, 1724, 1725, 1726, 1727, 1728, 1730, 1731, 1733, 1736, 1740, 1741, 1743, 1744, 1745, 1747, 1748, 1749, 1750, 1752, 1753, 1754, 1755, 1756, 1758, 1759, 1760, 1763, 1764, 1765, 1766, 1767, 1768, 1769, 1770, 1771, 1772, 1773, 1774, 1775, 1776, 1778, 1781, 1782, 1783, 1784, 1785, 1786, 1787, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1799, 1800, 1803, 1804, 1805, 1808, 1810, 1812, 1813, 1814, 1822, 1824, 1826, 1828, 1829, 1830, 1842, 1843, 1846, 1847, 1848, 1849, 1852, 1856, 1857, 1858, 1859, 1860, 1861, 1862, 1863, 1864, 1865, 1866, 1867, 1868, 1869, 1870, 1871, 1872, 1873, 1874, 1876, 1877, 1878, 1879, 1881, 1882, 1883, 1884, 1886, 1887, 1888, 1889, 1891, 1892, 1893, 1894, 1895, 1896, 1897, 1898, 1899, 1900, 1901, 1902, 1903, 1904, 1906, 1907, 1908, 1909, 1910, 1913, 1914, 1916, 1917, 1923, 1924, 1930, 1931, 1938

**Both systems unable to solve** 47, 48, 49, 50, 55, 56, 74, 79, 82, 87, 110, 121, 202, 203, 205, 206, 219, 234, 237, 250, 253, 265, 269, 331, 340, 367, 370, 395, 460, 461, 480, 482, 485, 503, 506, 507, 510, 531, 572, 575, 576, 835, 837, 885, 894, 1015, 1019, 1026, 1028, 1030, 1031, 1038, 1072, 1073, 1075, 1076, 1077, 1081, 1157, 1205, 1212, 1216, 1236, 1278, 1408, 1439, 1440, 1441, 1443, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1473, 1474, 1476, 1484, 1489, 1510, 1515, 1531, 1540, 1541, 1542, 1543, 1581, 1586, 1593, 1595, 1596, 1598, 1599, 1606, 1608, 1609, 1617, 1619, 1623, 1625, 1628, 1634, 1642, 1643, 1645, 1649, 1675, 1685, 1702, 1704, 1721, 1729, 1732, 1734, 1735, 1737, 1738, 1739, 1751, 1757, 1761, 1788, 1789, 1797, 1801, 1802, 1807, 1811, 1851, 1854, 1855, 1890, 1905, 1921, 1922, 1927, 1928, 1940

## 2 Problems table lookup

Final conclusion table for each equation is given by table 3 below. Clicking on the problem opens a new page that shows the result and links to download each problem as well.

| Table 3: Breakdown of results for each Kamke differential equation |             |       |      |        |       |      |                                                                                                          |
|--------------------------------------------------------------------|-------------|-------|------|--------|-------|------|----------------------------------------------------------------------------------------------------------|
| #                                                                  | Mathematica |       |      | Maple  |       |      | note on solution                                                                                         |
|                                                                    | solved      | cpu   | leaf | solved | cpu   | leaf |                                                                                                          |
| Kamke 1                                                            | ✓           | 0.424 | 1117 | ✓      | 0.032 | 1089 | Linear first order, To Do                                                                                |
| Kamke 2                                                            | ✓           | 0.035 | 34   | ✓      | 0.013 | 25   | Linear first order, integrating factor $y'(x) + ay(x) = f(x)$                                            |
| Kamke 3                                                            | ✓           | 0.055 | 40   | ✓      | 0.026 | 37   | Linear first order, integrating factor $y'(x) + ay(x) = f(x)$                                            |
| Kamke 4                                                            | ✓           | 0.013 | 30   | ✓      | 0.004 | 18   | Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$                                         |
| Kamke 5                                                            | ✓           | 0.382 | 39   | ✓      | 0.081 | 21   | Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$                                         |
| Kamke 6                                                            | ✓           | 0.02  | 18   | ✓      | 0.02  | 15   | Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$                                         |
| Kamke 7                                                            | ✓           | 0.033 | 23   | ✓      | 0.007 | 13   | Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$                                         |
| Kamke 8                                                            | ✓           | 0.019 | 17   | ✓      | 0.007 | 13   | Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$                                         |
| Kamke 9                                                            | ✓           | 0.015 | 19   | ✓      | 0.013 | 14   | Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$                                         |
| Kamke 10                                                           | ✓           | 0.013 | 18   | ✓      | 0.01  | 15   | Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$                                         |
| Kamke 11                                                           | ✓           | 0.019 | 66   | ✓      | 0.018 | 24   | Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$                                         |
| Kamke 12                                                           | ✓           | 0.072 | 34   | ✓      | 0.033 | 8    | Non-linear first order, Riccati, separable $y'(x) + y^2(x) = 1$                                          |
| Kamke 13                                                           | ✓           | 0.074 | 79   | ✓      | 0.132 | 79   | Non-linear first order, Riccati, transform to second order Airy ODE using $y = -\frac{u'(x)}{uR(x)}$     |
| Kamke 14                                                           | ✓           | 0.085 | 254  | ✓      | 0.072 | 187  | Non-linear first order, Riccati, transform to second order Airy ODE using $y = -\frac{u'(x)}{uR(x)}$     |
| Kamke 15                                                           | ✓           | 0.067 | 25   | ✓      | 0.136 | 35   | Non-linear first order, Riccati, transform to separable first order                                      |
| Kamke 16                                                           | ✓           | 0.037 | 186  | ✓      | 0.068 | 49   | Non-linear first order, Riccati, transform to first order separable using $y = y_p + \frac{1}{u}$        |
| Kamke 17                                                           | ✓           | 0.053 | 34   | ✓      | 0.092 | 24   | Non-linear first order, Riccati, transform to second order standard ODE using $y = -\frac{u'(x)}{uR(x)}$ |
| Continued on next page                                             |             |       |      |        |       |      |                                                                                                          |

Table 3 – continued from previous page

| #        | Mathematica |       |      | Maple  |       |      | note on solution                                                                                                                                      |
|----------|-------------|-------|------|--------|-------|------|-------------------------------------------------------------------------------------------------------------------------------------------------------|
|          | solved      | cpu   | leaf | solved | cpu   | leaf |                                                                                                                                                       |
| Kamke 18 | ✓           | 0.073 | 50   | ✓      | 0.052 | 39   | Non-linear first order, Riccati, transform to first order ODE solved using integrating factor using transformation $y = y_p + \frac{1}{u}$            |
| Kamke 19 | ✓           | 0.057 | 30   | ✓      | 0.034 | 16   | Non-linear first order, Riccati, transform to first order separable                                                                                   |
| Kamke 20 | ✓           | 0.074 | 49   | ✓      | 0.05  | 34   | Non-linear first order, Riccati, transform to Bernoulli first order non-linear first order then solve Bernoulli                                       |
| Kamke 21 | ✓           | 0.322 | 7    | ✓      | 0.103 | 25   | Non-linear first order, Riccati, transform to first order ODE solved using integrating factor using transformation $y = y_p + \frac{1}{u}$            |
| Kamke 22 | ✓           | 0.425 | 113  | ✓      | 0.337 | 128  | Non-linear first order, Riccati, transform to first order ODE solved using integrating factor using transformation $y = y_p + \frac{1}{u}$            |
| Kamke 23 | ✓           | 0.063 | 43   | ✓      | 0.033 | 23   | Non-linear first order, Riccati, Separable                                                                                                            |
| Kamke 24 | ✓           | 0.097 | 277  | ✓      | 0.05  | 214  | Non-linear first order, Riccati, transform to second order Emden-Fowler ODE using $y = -\frac{u'(x)}{uR(x)}$ solution in terms of Bessel functions    |
| Kamke 25 | ✓           | 0.306 | 1835 | ✓      | 0.258 | 348  | Non-linear first order, Riccati. To do                                                                                                                |
| Kamke 26 | ✓           | 0.182 | 68   | ✓      | 0.057 | 45   | Non-Linear first order, Riccati, transform to second order standard ODE using $y = -\frac{u'(x)}{uR(x)}$                                              |
| Kamke 27 | ✓           | 0.158 | 24   | ✓      | 0.138 | 72   | Non-linear first order, Riccati, transform to Bernoulli first order non-linear first order then solve Bernoulli                                       |
| Kamke 28 | ✓           | 0.102 | 96   | ✓      | 0.073 | 51   | Non-linear first order, Riccati, transform to Bernoulli first order non-linear first order then solve Bernoulli                                       |
| Kamke 29 | ✓           | 0.074 | 39   | ✓      | 0.01  | 19   | Non-linear first order, Bernoulli                                                                                                                     |
| Kamke 30 | ✓           | 0.154 | 230  | ✓      | 0.06  | 54   | Non-Linear first order, Riccati, transform to second order Bessel like ODE using $y = -\frac{u'(x)}{uR(x)}$ , solution uses Bessel functions          |
| Kamke 31 | ✓           | 0.13  | 21   | ✓      | 0.036 | 23   | Non-Linear first order, Riccati, separable                                                                                                            |
| Kamke 32 | ✓           | 0.315 | 34   | ✓      | 0.224 | 28   | Non-Linear first order, Riccati, has particular solution, solution using $y = y_p + \frac{1}{u}$ leads to first order solved using integrating factor |
| Kamke 33 | ✓           | 0.359 | 160  | ✓      | 0.359 | 58   | Non-Linear first order, Riccati. Complicated algebra, will do later                                                                                   |

Continued on next page

Table 3 – continued from previous page

| #        | Mathematica |        |      | Maple  |       |      | note on solution                                                                                                                                                                                                                 |
|----------|-------------|--------|------|--------|-------|------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|          | solved      | cpu    | leaf | solved | cpu   | leaf |                                                                                                                                                                                                                                  |
| Kamke 34 | ✓           | 0.093  | 54   | ✓      | 0.02  | 28   | Non-Linear first order, Bernoulli. Standard method.                                                                                                                                                                              |
| Kamke 35 | ✓           | 0.116  | 61   | ✓      | 0.037 | 35   | Non-Linear first order, Riccati. Transform to second order ODE using $y(x) = -\frac{u'(x)}{u(x)R(x)}$                                                                                                                            |
| Kamke 36 | ✓           | 0.262  | 195  | ✓      | 0.068 | 62   | Non-Linear first order of Abel first kind with None constant invariant. Transform to a reverse Riccati then Solve the resulting second order Airy ODE, and transform solution back. Lots of algebra involved. Hardest ODE so far |
| Kamke 37 | ✓           | 0.755  | 78   | ✓      | 0.066 | 50   | To Do                                                                                                                                                                                                                            |
| Kamke 38 | ✓           | 0.157  | 99   | ✓      | 0.02  | 34   | Non-Linear first order. smart transformation makes it proper Abel first kind                                                                                                                                                     |
| Kamke 39 | ✓           | 0.121  | 54   | ✓      | 0.01  | 30   | To Do                                                                                                                                                                                                                            |
| Kamke 40 | ✓           | 0.32   | 185  | ✓      | 0.051 | 48   | To Do                                                                                                                                                                                                                            |
| Kamke 41 | ✓           | 0.163  | 103  | ✓      | 0.181 | 103  | To Do                                                                                                                                                                                                                            |
| Kamke 42 | ✓           | 0.843  | 485  | ✓      | 0.02  | 40   | To Do                                                                                                                                                                                                                            |
| Kamke 43 | ✓           | 5.718  | 490  | ✓      | 1.246 | 373  | To Do                                                                                                                                                                                                                            |
| Kamke 44 | ✓           | 0.074  | 72   | ✓      | 0.014 | 53   | Non-Linear first order Bernoulli. Solved using standard method of solving Bernoulli.                                                                                                                                             |
| Kamke 45 | ✓           | 0.5    | 133  | ✓      | 0.096 | 123  | To Do                                                                                                                                                                                                                            |
| Kamke 46 | ✓           | 0.254  | 228  | ✓      | 0.075 | 956  | To Do                                                                                                                                                                                                                            |
| Kamke 47 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do                                                                                                                                                                                                                            |
| Kamke 48 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do                                                                                                                                                                                                                            |
| Kamke 49 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do                                                                                                                                                                                                                            |
| Kamke 50 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do                                                                                                                                                                                                                            |
| Kamke 51 | ✓           | 1.542  | 355  | ✓      | 0.151 | 237  | To Do                                                                                                                                                                                                                            |
| Kamke 52 | ✓           | 0.231  | 117  | ✓      | 0.322 | 61   | To Do                                                                                                                                                                                                                            |
| Kamke 53 | ✓           | 0.414  | 96   | ✓      | 0.126 | 281  | To Do                                                                                                                                                                                                                            |
| Kamke 54 | ✓           | 0.317  | 74   | ✓      | 0.129 | 38   | To Do                                                                                                                                                                                                                            |
| Kamke 55 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do                                                                                                                                                                                                                            |
| Kamke 56 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do                                                                                                                                                                                                                            |
| Kamke 57 | ✓           | 0.053  | 26   | ✓      | 0.062 | 31   | To Do                                                                                                                                                                                                                            |
| Kamke 58 | ✓           | 0.223  | 118  | ✓      | 0.062 | 68   | To Do                                                                                                                                                                                                                            |
| Kamke 59 | ✓           | 0.229  | 96   | ✓      | 0.059 | 26   | To Do                                                                                                                                                                                                                            |
| Kamke 60 | ✓           | 0.116  | 173  | ✓      | 0.013 | 29   | Non-Linear first order, separable.                                                                                                                                                                                               |
| Kamke 61 | ✓           | 0.197  | 75   | ✓      | 0.01  | 50   | To Do                                                                                                                                                                                                                            |
| Kamke 62 | ✓           | 2.637  | 44   | ✓      | 0.29  | 34   | Non-Linear first order, special transformation makes it exact differential.                                                                                                                                                      |
| Kamke 63 | ✓           | 0.19   | 48   | ✓      | 5.093 | 35   | To Do                                                                                                                                                                                                                            |
| Kamke 64 | ✓           | 0.308  | 269  | ✓      | 0.073 | 124  | To Do                                                                                                                                                                                                                            |
| Kamke 65 | ✓           | 0.954  | 312  | ✓      | 0.038 | 47   | To Do                                                                                                                                                                                                                            |
| Kamke 66 | ✓           | 0.166  | 67   | ✓      | 0.087 | 40   | To Do                                                                                                                                                                                                                            |
| Kamke 67 | ✓           | 0.126  | 14   | ✓      | 0.013 | 51   | To Do                                                                                                                                                                                                                            |
| Kamke 68 | ✓           | 0.715  | 373  | ✓      | 0.048 | 77   | To Do                                                                                                                                                                                                                            |
| Kamke 69 | ✓           | 7.756  | 1163 | ✓      | 0.108 | 111  | To Do                                                                                                                                                                                                                            |
| Kamke 70 | ✓           | 12.691 | 81   | ✓      | 0.124 | 113  | To Do                                                                                                                                                                                                                            |
| Kamke 71 | ✓           | 2.762  | 2237 | ✓      | 0.095 | 113  | To Do                                                                                                                                                                                                                            |

Continued on next page

Table 3 – continued from previous page

| #        | Mathematica |       |      | Maple  |       |      | note on solution                                                                                                                                                                        |
|----------|-------------|-------|------|--------|-------|------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|          | solved      | cpu   | leaf | solved | cpu   | leaf |                                                                                                                                                                                         |
| Kamke 72 | ✓           | 0.161 | 89   | ✓      | 0.009 | 64   | To Do                                                                                                                                                                                   |
| Kamke 73 | ✓           | 1.339 | 733  | ✓      | 0.246 | 91   | To Do                                                                                                                                                                                   |
| Kamke 74 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do                                                                                                                                                                                   |
| Kamke 75 | ✓           | 0.153 | 20   | ✓      | 0.121 | 20   | Non-Linear first order, Separable                                                                                                                                                       |
| Kamke 76 | ✓           | 0.134 | 116  | ✓      | 0.036 | 41   | Non-Linear first order, Separable, integral requires the tangent half-angle substitution (Weierstrass substitution)                                                                     |
| Kamke 77 | ✓           | 0.295 | 124  | ✓      | 0.052 | 54   | Non-Linear first order, transform to Separable, becomes same as problem 76 above. Transform back after solution.                                                                        |
| Kamke 78 | ✓           | 0.662 | 1317 | ✓      | 0.078 | 89   | Non-Linear first order, transform to Separable, integral requires the tangent half-angle substitution (Weierstrass substitution). Kamke calls this d'Alembertsche differential equation |
| Kamke 79 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do                                                                                                                                                                                   |
| Kamke 80 | ✓           | 0.057 | 72   | ✓      | 0.841 | 41   | To Do                                                                                                                                                                                   |
| Kamke 81 | ✓           | 1.332 | 220  | ✓      | 0.839 | 78   | To Do                                                                                                                                                                                   |
| Kamke 82 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do                                                                                                                                                                                   |
| Kamke 83 | ✓           | 0.354 | 69   | ✓      | 0.283 | 44   | To Do                                                                                                                                                                                   |
| Kamke 84 | ✓           | 0.185 | 248  | ✓      | 0.032 | 37   | To Do                                                                                                                                                                                   |
| Kamke 85 | ✓           | 0.373 | 238  | ✓      | 0.348 | 152  | To Do                                                                                                                                                                                   |
| Kamke 86 | ✓           | 0.456 | 184  | ✓      | 0.385 | 52   | To Do                                                                                                                                                                                   |
| Kamke 87 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do                                                                                                                                                                                   |
| Kamke 88 | ✓           | 0.478 | 2831 | ✓      | 0.18  | 256  | Non-Linear first order, Riccati, Solved using $y = -\frac{u'}{uR(x)}$ substitution. Convert to second order Bessel ODE                                                                  |
| Kamke 89 | ✓           | 0.012 | 42   | ✓      | 0.013 | 56   | Linear first order, separable. Integration tricky. requires tangent half-angle substitution                                                                                             |
| Kamke 90 | ✓           | 0.013 | 24   | ✓      | 0.009 | 17   | Linear first order, separable. integrating factor                                                                                                                                       |
| Kamke 91 | ✓           | 0.011 | 15   | ✓      | 0.006 | 11   | To Do                                                                                                                                                                                   |
| Kamke 92 | ✓           | 0.012 | 15   | ✓      | 0.004 | 12   | Linear first order, separable. integrating factor                                                                                                                                       |
| Kamke 93 | ✓           | 0.03  | 16   | ✓      | 0.011 | 12   | To Do                                                                                                                                                                                   |
| Kamke 94 | ✓           | 0.016 | 25   | ✓      | 0.009 | 23   | Linear first order, separable. integrating factor                                                                                                                                       |
| Kamke 95 | ✓           | 0.065 | 32   | ✓      | 0.05  | 27   | Non-Linear first order, Riccati, conversion to second order linear Lienard ODE using $y = \frac{u'}{uR}$ . Solution in terms of Bessel functions                                        |
| Kamke 96 | ✓           | 0.066 | 33   | ✓      | 0.029 | 11   | Non-Linear first order, Riccati, but it is separable, so easy to solve by direct integration                                                                                            |

Continued on next page

Table 3 – continued from previous page

| #         | Mathematica |       |      | Maple  |       |      | note on solution                                                                                                                                                                                                                                           |
|-----------|-------------|-------|------|--------|-------|------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|           | solved      | cpu   | leaf | solved | cpu   | leaf |                                                                                                                                                                                                                                                            |
| Kamke 97  | ✓           | 0.101 | 46   | ✓      | 0.032 | 25   | Non-Linear first order, Riccati, smart substitution transfer it to separable first order, so easy to solve by direct integration                                                                                                                           |
| Kamke 98  | ✓           | 0.164 | 442  | ✓      | 0.045 | 38   | To Do                                                                                                                                                                                                                                                      |
| Kamke 99  | ✓           | 0.155 | 244  | ✓      | 0.089 | 171  | To Do                                                                                                                                                                                                                                                      |
| Kamke 100 | ✓           | 0.076 | 157  | ✓      | 0.056 | 59   | Non-Linear first order, Riccati, conversion to second order linear Lienard ODE using $y = \frac{u'}{uR}$ . Solution in terms of Bessel functions                                                                                                           |
| Kamke 101 | ✓           | 0.071 | 18   | ✓      | 0.009 | 16   | Non-Linear first order, Bernoulli, standard method of solving Bernoulli                                                                                                                                                                                    |
| Kamke 102 | ✓           | 0.085 | 36   | ✓      | 0.031 | 22   | Non-Linear first order, Riccati, but transformed using smart substitution $y = xv$ to separable first order which is easily solved                                                                                                                         |
| Kamke 103 | ✓           | 0.146 | 90   | ✓      | 0.027 | 29   | Non-Linear first order, Riccati, conversion to second order linear Sturm-Liouville ODE using $y = \frac{u'}{uR}$ , then smart substitution $t = \frac{x^2}{2}$ is used to solve Sturm-Liouville by converting it to constant coefficients second order ODE |
| Kamke 104 | ✓           | 0.11  | 43   | ✓      | 0.04  | 63   | Non-Linear first order, Riccati, conversion using smart transformation to separable first order                                                                                                                                                            |
| Kamke 105 | ✓           | 0.218 | 473  | ✓      | 0.196 | 844  | To Do                                                                                                                                                                                                                                                      |
| Kamke 106 | ✓           | 0.197 | 40   | ✓      | 0.038 | 41   | To Do                                                                                                                                                                                                                                                      |
| Kamke 107 | ✓           | 0.326 | 1415 | ✓      | 0.194 | 174  | To Do                                                                                                                                                                                                                                                      |
| Kamke 108 | ✓           | 0.066 | 15   | ✓      | 0.01  | 13   | Non-Linear first order, Bernoulli, standard method of solving Bernoulli, leads to first order separable                                                                                                                                                    |
| Kamke 109 | ✓           | 0.069 | 17   | ✓      | 0.01  | 15   | Non-Linear first order, Bernoulli, standard method of solving Bernoulli, leads to first order separable                                                                                                                                                    |
| Kamke 110 | ✗           | 0     | 0    | ✗      | 0     | 0    | Non-Linear first order, Riccati, has known particular solution. Using $y = y_p + \frac{1}{u}$ convert it to first order separable ODE                                                                                                                      |
| Kamke 111 | ✓           | 0.362 | 55   | ✓      | 0.105 | 54   | To Do                                                                                                                                                                                                                                                      |
| Kamke 112 | ✓           | 0.098 | 13   | ✓      | 0.03  | 27   | Non-Linear first order, smart transformation $y(x) = xv(x)$ makes it separable                                                                                                                                                                             |
| Kamke 113 | ✓           | 0.098 | 16   | ✓      | 0.023 | 33   | Non-Linear first order, very similar to 112. Smart transformation $y(x) = xv(x)$ makes it separable                                                                                                                                                        |

Continued on next page

Table 3 – continued from previous page

| #         | Mathematica |       |      | Maple  |       |      | note on solution                                                                                    |
|-----------|-------------|-------|------|--------|-------|------|-----------------------------------------------------------------------------------------------------|
|           | solved      | cpu   | leaf | solved | cpu   | leaf |                                                                                                     |
| Kamke 114 | ✓           | 0.088 | 12   | ✓      | 2.004 | 28   | Non-Linear first order, very similar to 113. Smart transformation $y(x) = xv(x)$ makes it separable |
| Kamke 115 | ✓           | 0.176 | 221  | ✓      | 0.171 | 49   | Non-Linear first order, Smart transformation $y(x) = xv(x)$ makes it separable                      |
| Kamke 116 | ✓           | 0.35  | 121  | ✓      | 0.174 | 86   | To Do                                                                                               |
| Kamke 117 | ✓           | 0.152 | 21   | ✓      | 0.089 | 20   | To Do                                                                                               |
| Kamke 118 | ✓           | 0.063 | 13   | ✓      | 0.046 | 8    | To Do                                                                                               |
| Kamke 119 | ✓           | 0.087 | 17   | ✓      | 0.05  | 14   | To Do                                                                                               |
| Kamke 120 | ✓           | 0.096 | 20   | ✓      | 0.143 | 17   | To Do                                                                                               |
| Kamke 121 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do                                                                                               |
| Kamke 122 | ✓           | 0.274 | 21   | ✓      | 0.296 | 16   | To Do                                                                                               |
| Kamke 123 | ✓           | 0.105 | 19   | ✓      | 0.043 | 44   | To Do                                                                                               |
| Kamke 124 | ✓           | 0.088 | 16   | ✓      | 0.023 | 12   | To Do                                                                                               |
| Kamke 125 | ✓           | 0.182 | 16   | ✓      | 0.05  | 14   | To Do                                                                                               |
| Kamke 126 | ✓           | 0.158 | 115  | ✓      | 0.018 | 29   | To Do                                                                                               |
| Kamke 127 | ✓           | 0.238 | 186  | ✓      | 0.086 | 39   | To Do                                                                                               |
| Kamke 128 | ✓           | 3.149 | 41   | ✓      | 0.21  | 33   | To Do                                                                                               |
| Kamke 129 | ✓           | 0.174 | 44   | ✓      | 0.023 | 33   | To Do                                                                                               |
| Kamke 130 | ✓           | 0.008 | 21   | ✓      | 0.006 | 15   | To Do                                                                                               |
| Kamke 131 | ✓           | 0.173 | 21   | ✓      | 0.155 | 31   | To Do                                                                                               |
| Kamke 132 | ✓           | 0.077 | 115  | ✓      | 0.025 | 153  | To Do                                                                                               |
| Kamke 133 | ✓           | 0.008 | 27   | ✓      | 0.008 | 16   | To Do                                                                                               |
| Kamke 134 | ✓           | 0.017 | 27   | ✓      | 0.007 | 17   | To Do                                                                                               |
| Kamke 135 | ✓           | 0.008 | 14   | ✓      | 0.004 | 11   | To Do                                                                                               |
| Kamke 136 | ✓           | 0.069 | 28   | ✓      | 0.014 | 18   | To Do                                                                                               |
| Kamke 137 | ✓           | 0.065 | 16   | ✓      | 0.012 | 14   | To Do                                                                                               |
| Kamke 138 | ✓           | 0.061 | 13   | ✓      | 0.026 | 11   | To Do                                                                                               |
| Kamke 139 | ✓           | 0.203 | 821  | ✓      | 0.11  | 219  | To Do                                                                                               |
| Kamke 140 | ✓           | 0.076 | 17   | ✓      | 0.034 | 20   | To Do                                                                                               |
| Kamke 141 | ✓           | 0.122 | 67   | ✓      | 0.046 | 51   | To Do                                                                                               |
| Kamke 142 | ✓           | 0.177 | 122  | ✓      | 0.065 | 52   | To Do                                                                                               |
| Kamke 143 | ✓           | 0.089 | 51   | ✓      | 0.04  | 41   | To Do                                                                                               |
| Kamke 144 | ✓           | 0.22  | 1787 | ✓      | 0.086 | 219  | To Do                                                                                               |
| Kamke 145 | ✓           | 0.476 | 267  | ✓      | 0.085 | 117  | To Do                                                                                               |
| Kamke 146 | ✓           | 0.65  | 78   | ✓      | 0.118 | 84   | To Do                                                                                               |
| Kamke 147 | ✓           | 0.648 | 343  | ✓      | 0.164 | 178  | To Do                                                                                               |
| Kamke 148 | ✓           | 0.012 | 30   | ✓      | 0.008 | 16   | To Do                                                                                               |
| Kamke 149 | ✓           | 0.012 | 27   | ✓      | 0.007 | 20   | To Do                                                                                               |
| Kamke 150 | ✓           | 0.01  | 30   | ✓      | 0.005 | 23   | To Do                                                                                               |
| Kamke 151 | ✓           | 0.476 | 203  | ✓      | 0.039 | 85   | To Do                                                                                               |
| Kamke 152 | ✓           | 0.364 | 40   | ✓      | 0.55  | 25   | To Do                                                                                               |
| Kamke 153 | ✓           | 0.012 | 21   | ✓      | 0.011 | 20   | To Do                                                                                               |
| Kamke 154 | ✓           | 0.013 | 26   | ✓      | 0.007 | 16   | To Do                                                                                               |
| Kamke 155 | ✓           | 0.124 | 46   | ✓      | 0.075 | 14   | To Do                                                                                               |
| Kamke 156 | ✓           | 0.078 | 21   | ✓      | 0.013 | 20   | To Do                                                                                               |
| Kamke 157 | ✓           | 0.184 | 158  | ✓      | 0.178 | 231  | To Do                                                                                               |
| Kamke 158 | ✓           | 0.14  | 31   | ✓      | 0.011 | 22   | To Do                                                                                               |
| Kamke 159 | ✓           | 0.096 | 22   | ✓      | 0.11  | 13   | To Do                                                                                               |
| Kamke 160 | ✓           | 0.137 | 27   | ✓      | 0.022 | 21   | To Do                                                                                               |
| Kamke 161 | ✓           | 0.014 | 53   | ✓      | 0.01  | 27   | To Do                                                                                               |
| Kamke 162 | ✓           | 0.426 | 133  | ✓      | 0.151 | 58   | To Do                                                                                               |
| Kamke 163 | ✓           | 0.094 | 43   | ✓      | 0.033 | 26   | To Do                                                                                               |
| Kamke 164 | ✓           | 0.129 | 131  | ✓      | 0.151 | 102  | To Do                                                                                               |
| Kamke 165 | ✓           | 0.111 | 22   | ✓      | 0.017 | 17   | To Do                                                                                               |
| Kamke 166 | ✓           | 0.141 | 71   | ✓      | 0.118 | 97   | To Do                                                                                               |
| Kamke 167 | ✓           | 0.075 | 35   | ✓      | 0.026 | 20   | To Do                                                                                               |

Continued on next page

Table 3 – continued from previous page

| #         | Mathematica |       |      | Maple  |       |      | note on solution |
|-----------|-------------|-------|------|--------|-------|------|------------------|
|           | solved      | cpu   | leaf | solved | cpu   | leaf |                  |
| Kamke 168 | ✓           | 0.173 | 234  | ✓      | 0.126 | 140  | To Do            |
| Kamke 169 | ✓           | 1.895 | 149  | ✓      | 0.119 | 153  | To Do            |
| Kamke 170 | ✓           | 0.073 | 43   | ✓      | 0.013 | 23   | To Do            |
| Kamke 171 | ✓           | 0.065 | 17   | ✓      | 0.007 | 15   | To Do            |
| Kamke 172 | ✓           | 0.076 | 35   | ✓      | 0.205 | 26   | To Do            |
| Kamke 173 | ✓           | 0.093 | 29   | ✓      | 0.033 | 27   | To Do            |
| Kamke 174 | ✓           | 0.008 | 17   | ✓      | 0.003 | 13   | To Do            |
| Kamke 175 | ✓           | 0.019 | 24   | ✓      | 0.014 | 20   | To Do            |
| Kamke 176 | ✓           | 0.187 | 82   | ✓      | 0.075 | 30   | To Do            |
| Kamke 177 | ✓           | 0.103 | 22   | ✓      | 0.019 | 17   | To Do            |
| Kamke 178 | ✓           | 0.149 | 49   | ✓      | 0.144 | 61   | To Do            |
| Kamke 179 | ✓           | 0.591 | 2833 | ✓      | 0.11  | 112  | To Do            |
| Kamke 180 | ✓           | 0.316 | 132  | ✓      | 0.047 | 58   | To Do            |
| Kamke 181 | ✓           | 0.091 | 347  | ✓      | 0.056 | 28   | To Do            |
| Kamke 182 | ✓           | 0.204 | 96   | ✓      | 0.102 | 18   | To Do            |
| Kamke 183 | ✓           | 0.014 | 22   | ✓      | 0.009 | 18   | To Do            |
| Kamke 184 | ✓           | 1.15  | 704  | ✓      | 0.302 | 493  | To Do            |
| Kamke 185 | ✓           | 0.372 | 123  | ✓      | 0.03  | 63   | To Do            |
| Kamke 186 | ✓           | 0.209 | 19   | ✓      | 0.032 | 17   | To Do            |
| Kamke 187 | ✓           | 0.2   | 328  | ✓      | 0.053 | 60   | To Do            |
| Kamke 188 | ✗           | 0     | 0    | ✓      | 0.018 | 32   | To Do            |
| Kamke 189 | ✓           | 0.384 | 91   | ✓      | 0.271 | 60   | To Do            |
| Kamke 190 | ✓           | 0.122 | 173  | ✓      | 0.006 | 29   | To Do            |
| Kamke 191 | ✓           | 0.114 | 39   | ✓      | 0.013 | 16   | To Do            |
| Kamke 192 | ✓           | 0.691 | 168  | ✓      | 0.015 | 36   | To Do            |
| Kamke 193 | ✓           | 0.014 | 16   | ✓      | 0.006 | 14   | To Do            |
| Kamke 194 | ✓           | 0.166 | 98   | ✓      | 0.017 | 23   | To Do            |
| Kamke 195 | ✓           | 0.151 | 27   | ✓      | 0.089 | 28   | To Do            |
| Kamke 196 | ✓           | 0.204 | 53   | ✓      | 0.103 | 29   | To Do            |
| Kamke 197 | ✓           | 0.108 | 98   | ✓      | 0.079 | 237  | To Do            |
| Kamke 198 | ✓           | 0.021 | 15   | ✓      | 0.01  | 13   | To Do            |
| Kamke 199 | ✓           | 0.196 | 15   | ✓      | 0.142 | 102  | To Do            |
| Kamke 200 | ✓           | 0.107 | 77   | ✓      | 0.044 | 53   | To Do            |
| Kamke 201 | ✓           | 0.134 | 39   | ✓      | 0.034 | 23   | To Do            |
| Kamke 202 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 203 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 204 | ✓           | 0.071 | 70   | ✓      | 0.228 | 92   | To Do            |
| Kamke 205 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 206 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 207 | ✓           | 0.082 | 47   | ✓      | 0.016 | 37   | To Do            |
| Kamke 208 | ✓           | 0.192 | 118  | ✓      | 0.049 | 106  | To Do            |
| Kamke 209 | ✓           | 0.097 | 84   | ✓      | 0.009 | 21   | To Do            |
| Kamke 210 | ✓           | 0.077 | 47   | ✓      | 0.014 | 33   | To Do            |
| Kamke 211 | ✓           | 0.206 | 41   | ✓      | 0.027 | 31   | To Do            |
| Kamke 212 | ✓           | 0.364 | 95   | ✓      | 0.095 | 30   | To Do            |
| Kamke 213 | ✓           | 0.097 | 71   | ✓      | 0.552 | 66   | To Do            |
| Kamke 214 | ✓           | 0.104 | 78   | ✓      | 0.139 | 48   | To Do            |
| Kamke 215 | ✓           | 0.106 | 80   | ✓      | 0.162 | 51   | To Do            |
| Kamke 216 | ✓           | 0.093 | 82   | ✓      | 0.153 | 51   | To Do            |
| Kamke 217 | ✓           | 0.016 | 29   | ✓      | 0.024 | 23   | To Do            |
| Kamke 218 | ✓           | 0.067 | 257  | ✓      | 0.162 | 57   | To Do            |
| Kamke 219 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 220 | ✓           | 0.069 | 57   | ✓      | 0.017 | 43   | To Do            |
| Kamke 221 | ✓           | 0.016 | 35   | ✓      | 0.039 | 21   | To Do            |
| Kamke 222 | ✓           | 0.05  | 65   | ✓      | 0.052 | 32   | To Do            |
| Kamke 223 | ✓           | 0.019 | 55   | ✓      | 0.139 | 51   | To Do            |
| Kamke 224 | ✓           | 0.015 | 29   | ✓      | 0.041 | 35   | To Do            |
| Kamke 225 | ✓           | 0.015 | 33   | ✓      | 0.036 | 20   | To Do            |
| Kamke 226 | ✓           | 0.015 | 35   | ✓      | 0.034 | 21   | To Do            |
| Kamke 227 | ✓           | 0.012 | 107  | ✓      | 0.15  | 33   | To Do            |

Continued on next page



Table 3 – continued from previous page

| #         | Mathematica |        |      | Maple  |       |      | note on solution |
|-----------|-------------|--------|------|--------|-------|------|------------------|
|           | solved      | cpu    | leaf | solved | cpu   | leaf |                  |
| Kamke 228 | ✓           | 0.251  | 3357 | ✓      | 0.336 | 271  | To Do            |
| Kamke 229 | ✓           | 0.015  | 121  | ✓      | 0.152 | 32   | To Do            |
| Kamke 230 | ✓           | 0.157  | 98   | ✓      | 0.031 | 100  | To Do            |
| Kamke 231 | ✓           | 1.92   | 252  | ✓      | 0.194 | 178  | To Do            |
| Kamke 232 | ✓           | 0.068  | 56   | ✓      | 0.013 | 39   | To Do            |
| Kamke 233 | ✓           | 0.137  | 38   | ✓      | 0.018 | 30   | To Do            |
| Kamke 234 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 235 | ✓           | 0.056  | 40   | ✓      | 0.037 | 30   | To Do            |
| Kamke 236 | ✓           | 0.115  | 114  | ✓      | 0.047 | 141  | To Do            |
| Kamke 237 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 238 | ✓           | 0.237  | 192  | ✓      | 0.056 | 93   | To Do            |
| Kamke 239 | ✓           | 0.098  | 54   | ✓      | 0.149 | 59   | To Do            |
| Kamke 240 | ✓           | 0.096  | 41   | ✓      | 0.017 | 34   | To Do            |
| Kamke 241 | ✓           | 0.084  | 41   | ✓      | 0.01  | 33   | To Do            |
| Kamke 242 | ✓           | 0.071  | 60   | ✓      | 0.01  | 39   | To Do            |
| Kamke 243 | ✓           | 11.189 | 487  | ✓      | 0.109 | 391  | To Do            |
| Kamke 244 | ✓           | 11.123 | 484  | ✓      | 0.094 | 391  | To Do            |
| Kamke 245 | ✓           | 0.424  | 1453 | ✓      | 3.007 | 31   | To Do            |
| Kamke 246 | ✓           | 0.123  | 80   | ✓      | 0.05  | 63   | To Do            |
| Kamke 247 | ✓           | 11.192 | 693  | ✓      | 0.194 | 517  | To Do            |
| Kamke 248 | ✓           | 0.173  | 106  | ✓      | 0.015 | 75   | To Do            |
| Kamke 249 | ✓           | 3.527  | 115  | ✓      | 0.171 | 202  | To Do            |
| Kamke 250 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 251 | ✓           | 0.12   | 60   | ✓      | 0.013 | 51   | To Do            |
| Kamke 252 | ✓           | 10.983 | 819  | ✓      | 0.636 | 1338 | To Do            |
| Kamke 253 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 254 | ✓           | 0.124  | 99   | ✓      | 0.021 | 59   | To Do            |
| Kamke 255 | ✓           | 3.21   | 30   | ✓      | 0.19  | 74   | To Do            |
| Kamke 256 | ✓           | 0.018  | 21   | ✓      | 0.039 | 31   | To Do            |
| Kamke 257 | ✓           | 0.37   | 39   | ✓      | 0.079 | 98   | To Do            |
| Kamke 258 | ✓           | 0.11   | 43   | ✓      | 0.016 | 33   | To Do            |
| Kamke 259 | ✓           | 0.23   | 50   | ✓      | 0.019 | 51   | To Do            |
| Kamke 260 | ✓           | 0.118  | 80   | ✓      | 0.023 | 59   | To Do            |
| Kamke 261 | ✓           | 0.762  | 32   | ✓      | 0.098 | 18   | To Do            |
| Kamke 262 | ✓           | 0.173  | 101  | ✓      | 0.243 | 65   | To Do            |
| Kamke 263 | ✓           | 0.09   | 181  | ✓      | 0.106 | 173  | To Do            |
| Kamke 264 | ✓           | 0.38   | 680  | ✓      | 0.753 | 574  | To Do            |
| Kamke 265 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 266 | ✗           | 0      | 0    | ✓      | 1.374 | 65   | To Do            |
| Kamke 267 | ✓           | 0.282  | 36   | ✓      | 0.021 | 32   | To Do            |
| Kamke 268 | ✓           | 0.165  | 146  | ✓      | 0.044 | 118  | To Do            |
| Kamke 269 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 270 | ✓           | 0.124  | 327  | ✓      | 0.02  | 319  | To Do            |
| Kamke 271 | ✓           | 0.174  | 370  | ✓      | 0.163 | 352  | To Do            |
| Kamke 272 | ✓           | 0.12   | 42   | ✓      | 0.125 | 43   | To Do            |
| Kamke 273 | ✓           | 0.174  | 297  | ✓      | 0.019 | 401  | To Do            |
| Kamke 274 | ✓           | 0.174  | 411  | ✓      | 0.029 | 657  | To Do            |
| Kamke 275 | ✓           | 0.118  | 18   | ✓      | 0.062 | 30   | To Do            |
| Kamke 276 | ✓           | 0.14   | 61   | ✓      | 0.046 | 47   | To Do            |
| Kamke 277 | ✓           | 0.11   | 53   | ✓      | 0.286 | 41   | To Do            |
| Kamke 278 | ✓           | 0.178  | 39   | ✓      | 0.04  | 28   | To Do            |
| Kamke 279 | ✓           | 0.604  | 107  | ✓      | 0.127 | 116  | To Do            |
| Kamke 280 | ✓           | 0.13   | 21   | ✓      | 0.041 | 24   | To Do            |
| Kamke 281 | ✓           | 0.186  | 75   | ✓      | 0.051 | 55   | To Do            |
| Kamke 282 | ✓           | 0.21   | 2129 | ✓      | 0.263 | 71   | To Do            |
| Kamke 283 | ✓           | 0.338  | 477  | ✓      | 0.056 | 407  | To Do            |
| Kamke 284 | ✓           | 0.108  | 59   | ✓      | 0.129 | 21   | To Do            |
| Kamke 285 | ✓           | 0.175  | 402  | ✓      | 0.059 | 432  | To Do            |
| Kamke 286 | ✓           | 0.243  | 3501 | ✓      | 1.18  | 1337 | To Do            |
| Kamke 287 | ✓           | 1.492  | 77   | ✓      | 0.052 | 56   | To Do            |

Continued on next page

Table 3 – continued from previous page

| #         | Mathematica |        |       | Maple  |       |      | note on solution |
|-----------|-------------|--------|-------|--------|-------|------|------------------|
|           | solved      | cpu    | leaf  | solved | cpu   | leaf |                  |
| Kamke 288 | ✓           | 0.227  | 534   | ✓      | 0.026 | 579  | To Do            |
| Kamke 289 | ✓           | 0.178  | 115   | ✓      | 0.026 | 115  | To Do            |
| Kamke 290 | ✓           | 0.446  | 831   | ✓      | 0.082 | 1388 | To Do            |
| Kamke 291 | ✓           | 0.842  | 39    | ✓      | 0.121 | 50   | To Do            |
| Kamke 292 | ✓           | 36.246 | 1716  | ✓      | 0.032 | 115  | To Do            |
| Kamke 293 | ✓           | 0.163  | 661   | ✓      | 0.284 | 35   | To Do            |
| Kamke 294 | ✓           | 0.232  | 71    | ✓      | 0.066 | 112  | To Do            |
| Kamke 295 | ✓           | 0.207  | 31    | ✓      | 0.149 | 29   | To Do            |
| Kamke 296 | ✓           | 0.47   | 102   | ✓      | 0.59  | 135  | To Do            |
| Kamke 297 | ✓           | 0.141  | 216   | ✓      | 0.313 | 29   | To Do            |
| Kamke 298 | ✓           | 0.083  | 72    | ✓      | 0.014 | 73   | To Do            |
| Kamke 299 | ✓           | 0.107  | 371   | ✓      | 0.174 | 276  | To Do            |
| Kamke 300 | ✓           | 0.08   | 99    | ✓      | 0.014 | 83   | To Do            |
| Kamke 301 | ✓           | 0.13   | 64    | ✓      | 0.183 | 25   | To Do            |
| Kamke 302 | ✓           | 0.092  | 70    | ✓      | 0.104 | 133  | To Do            |
| Kamke 303 | ✓           | 0.114  | 25    | ✓      | 0.138 | 34   | To Do            |
| Kamke 304 | ✓           | 0.333  | 44    | ✓      | 0.214 | 44   | To Do            |
| Kamke 305 | ✓           | 0.18   | 1277  | ✓      | 0.017 | 21   | To Do            |
| Kamke 306 | ✓           | 0.114  | 201   | ✓      | 0.299 | 231  | To Do            |
| Kamke 307 | ✓           | 0.229  | 149   | ✓      | 0.032 | 125  | To Do            |
| Kamke 308 | ✓           | 0.008  | 55    | ✓      | 0.009 | 37   | To Do            |
| Kamke 309 | ✓           | 0.099  | 151   | ✓      | 0.026 | 113  | To Do            |
| Kamke 310 | ✓           | 0.166  | 159   | ✓      | 0.088 | 125  | To Do            |
| Kamke 311 | ✓           | 0.224  | 2201  | ✓      | 0.067 | 50   | To Do            |
| Kamke 312 | ✓           | 0.617  | 204   | ✓      | 1.136 | 240  | To Do            |
| Kamke 313 | ✓           | 0.32   | 537   | ✓      | 0.155 | 748  | To Do            |
| Kamke 314 | ✓           | 0.147  | 188   | ✓      | 0.036 | 158  | To Do            |
| Kamke 315 | ✓           | 0.18   | 368   | ✓      | 0.075 | 376  | To Do            |
| Kamke 316 | ✓           | 0.125  | 48    | ✓      | 0.038 | 53   | To Do            |
| Kamke 317 | ✓           | 0.323  | 23    | ✓      | 0.104 | 29   | To Do            |
| Kamke 318 | ✓           | 0.227  | 4284  | ✓      | 0.016 | 28   | To Do            |
| Kamke 319 | ✓           | 0.143  | 302   | ✓      | 0.023 | 35   | To Do            |
| Kamke 320 | ✓           | 0.111  | 76    | ✓      | 0.07  | 78   | To Do            |
| Kamke 321 | ✓           | 0.327  | 47    | ✓      | 0.13  | 42   | To Do            |
| Kamke 322 | ✓           | 0.274  | 2077  | ✓      | 0.02  | 29   | To Do            |
| Kamke 323 | ✓           | 0.25   | 463   | ✓      | 0.092 | 630  | To Do            |
| Kamke 324 | ✓           | 0.115  | 723   | ✓      | 0.099 | 815  | To Do            |
| Kamke 325 | ✓           | 0.19   | 139   | ✓      | 0.469 | 124  | To Do            |
| Kamke 326 | ✓           | 3.015  | 13289 | ✓      | 0.388 | 160  | To Do            |
| Kamke 327 | ✓           | 0.33   | 669   | ✓      | 0.123 | 583  | To Do            |
| Kamke 328 | ✓           | 0.235  | 42    | ✓      | 0.145 | 33   | To Do            |
| Kamke 329 | ✓           | 0.671  | 102   | ✓      | 0.269 | 71   | To Do            |
| Kamke 330 | ✓           | 0.102  | 52    | ✓      | 0.024 | 22   | To Do            |
| Kamke 331 | ✗           | 0      | 0     | ✗      | 0     | 0    | To Do            |
| Kamke 332 | ✓           | 0.207  | 34    | ✓      | 0.011 | 33   | To Do            |
| Kamke 333 | ✓           | 0.311  | 72    | ✓      | 0.073 | 32   | To Do            |
| Kamke 334 | ✓           | 0.099  | 39    | ✓      | 0.021 | 19   | To Do            |
| Kamke 335 | ✓           | 0.196  | 75    | ✓      | 0.008 | 50   | To Do            |
| Kamke 336 | ✓           | 0.257  | 53    | ✓      | 0.024 | 41   | To Do            |
| Kamke 337 | ✓           | 0.225  | 161   | ✓      | 0.054 | 28   | To Do            |
| Kamke 338 | ✓           | 62.98  | 17681 | ✓      | 0.517 | 129  | To Do            |
| Kamke 339 | ✓           | 0.338  | 27    | ✓      | 0.134 | 27   | To Do            |
| Kamke 340 | ✗           | 0      | 0     | ✗      | 0     | 0    | To Do            |
| Kamke 341 | ✓           | 0.228  | 33    | ✓      | 0.049 | 33   | To Do            |
| Kamke 342 | ✓           | 0.443  | 163   | ✓      | 0.032 | 17   | To Do            |
| Kamke 343 | ✓           | 0.131  | 35    | ✓      | 0.049 | 27   | To Do            |
| Kamke 344 | ✓           | 0.189  | 23    | ✓      | 0.023 | 19   | To Do            |
| Kamke 345 | ✓           | 0.165  | 35    | ✓      | 0.056 | 36   | To Do            |
| Kamke 346 | ✓           | 0.359  | 24    | ✓      | 0.204 | 19   | To Do            |
| Kamke 347 | ✓           | 0.287  | 32    | ✓      | 0.132 | 16   | To Do            |

Continued on next page

Table 3 – continued from previous page

| #         | Mathematica |       |      | Maple  |       |      | note on solution |
|-----------|-------------|-------|------|--------|-------|------|------------------|
|           | solved      | cpu   | leaf | solved | cpu   | leaf |                  |
| Kamke 348 | ✓           | 0.179 | 17   | ✓      | 0.072 | 15   | To Do            |
| Kamke 349 | ✓           | 0.224 | 15   | ✓      | 0.026 | 17   | To Do            |
| Kamke 350 | ✓           | 0.548 | 53   | ✓      | 0.677 | 226  | To Do            |
| Kamke 351 | ✓           | 0.469 | 61   | ✓      | 0.333 | 55   | To Do            |
| Kamke 352 | ✓           | 0.527 | 43   | ✓      | 0.35  | 33   | To Do            |
| Kamke 353 | ✓           | 0.075 | 14   | ✓      | 0.066 | 12   | To Do            |
| Kamke 354 | ✓           | 0.134 | 145  | ✓      | 0.05  | 108  | To Do            |
| Kamke 355 | ✓           | 0.16  | 17   | ✓      | 0.069 | 15   | To Do            |
| Kamke 356 | ✓           | 0.227 | 21   | ✓      | 0.084 | 19   | To Do            |
| Kamke 357 | ✓           | 0.504 | 35   | ✓      | 0.378 | 13   | To Do            |
| Kamke 358 | ✓           | 0.144 | 29   | ✓      | 0.076 | 11   | To Do            |
| Kamke 359 | ✓           | 0.3   | 45   | ✓      | 0.046 | 28   | To Do            |
| Kamke 360 | ✓           | 4.334 | 369  | ✓      | 0.25  | 48   | To Do            |
| Kamke 361 | ✓           | 0.632 | 31   | ✓      | 0.165 | 22   | To Do            |
| Kamke 362 | ✓           | 0.356 | 23   | ✓      | 0.184 | 23   | To Do            |
| Kamke 363 | ✓           | 0.335 | 33   | ✓      | 0.063 | 35   | To Do            |
| Kamke 364 | ✓           | 0.456 | 31   | ✓      | 0.076 | 23   | To Do            |
| Kamke 365 | ✓           | 0.347 | 156  | ✓      | 0.261 | 42   | To Do            |
| Kamke 366 | ✓           | 0.269 | 91   | ✓      | 0.066 | 45   | To Do            |
| Kamke 367 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 368 | ✓           | 1.377 | 795  | ✗      | 0     | 0    | To Do            |
| Kamke 369 | ✓           | 0.058 | 107  | ✓      | 0.098 | 68   | To Do            |
| Kamke 370 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 371 | ✓           | 0.04  | 37   | ✓      | 0.059 | 20   | To Do            |
| Kamke 372 | ✓           | 0.003 | 27   | ✓      | 0.04  | 232  | To Do            |
| Kamke 373 | ✓           | 0.181 | 185  | ✓      | 0.183 | 49   | To Do            |
| Kamke 374 | ✓           | 0.068 | 73   | ✓      | 0.023 | 85   | To Do            |
| Kamke 375 | ✓           | 0.031 | 71   | ✓      | 0.018 | 49   | To Do            |
| Kamke 376 | ✓           | 0.23  | 110  | ✓      | 0.524 | 219  | To Do            |
| Kamke 377 | ✓           | 0.003 | 19   | ✓      | 0.013 | 24   | To Do            |
| Kamke 378 | ✓           | 0.003 | 18   | ✓      | 0.013 | 20   | To Do            |
| Kamke 379 | ✓           | 0.003 | 18   | ✓      | 0.013 | 22   | To Do            |
| Kamke 380 | ✓           | 0.379 | 1757 | ✓      | 0.032 | 619  | To Do            |
| Kamke 381 | ✓           | 0.381 | 1757 | ✓      | 0.024 | 579  | To Do            |
| Kamke 382 | ✓           | 0.177 | 201  | ✓      | 0.025 | 146  | To Do            |
| Kamke 383 | ✓           | 5.475 | 1118 | ✗      | 0     | 0    | To Do            |
| Kamke 384 | ✓           | 1.324 | 183  | ✓      | 0.018 | 50   | To Do            |
| Kamke 385 | ✓           | 0.683 | 6217 | ✓      | 0.576 | 169  | To Do            |
| Kamke 386 | ✓           | 0.154 | 119  | ✓      | 0.769 | 27   | To Do            |
| Kamke 387 | ✓           | 1.186 | 190  | ✓      | 1.402 | 115  | To Do            |
| Kamke 388 | ✓           | 0.493 | 53   | ✓      | 0.162 | 223  | To Do            |
| Kamke 389 | ✓           | 0.059 | 57   | ✓      | 2.641 | 71   | To Do            |
| Kamke 390 | ✓           | 1.481 | 142  | ✓      | 1.32  | 281  | To Do            |
| Kamke 391 | ✓           | 0.007 | 29   | ✓      | 0.052 | 22   | To Do            |
| Kamke 392 | ✓           | 0.194 | 27   | ✓      | 2.165 | 50   | To Do            |
| Kamke 393 | ✓           | 0.085 | 31   | ✓      | 0.246 | 77   | To Do            |
| Kamke 394 | ✗           | 0     | 0    | ✓      | 6.714 | 109  | To Do            |
| Kamke 395 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 396 | ✓           | 0.045 | 29   | ✓      | 0.031 | 20   | To Do            |
| Kamke 397 | ✓           | 0.419 | 143  | ✓      | 0.824 | 128  | To Do            |
| Kamke 398 | ✓           | 1.502 | 258  | ✓      | 2.255 | 138  | To Do            |
| Kamke 399 | ✓           | 0.003 | 20   | ✓      | 0.013 | 22   | To Do            |
| Kamke 400 | ✓           | 0.259 | 135  | ✓      | 0.332 | 74   | To Do            |
| Kamke 401 | ✓           | 0.308 | 1093 | ✓      | 0.112 | 580  | To Do            |
| Kamke 402 | ✓           | 5.291 | 1211 | ✓      | 0.199 | 101  | To Do            |
| Kamke 403 | ✓           | 0.228 | 116  | ✓      | 3.888 | 197  | To Do            |
| Kamke 404 | ✓           | 2.705 | 795  | ✓      | 0.503 | 389  | To Do            |
| Kamke 405 | ✓           | 0.845 | 53   | ✓      | 0.507 | 378  | To Do            |
| Kamke 406 | ✓           | 0.627 | 49   | ✓      | 0.193 | 262  | To Do            |
| Kamke 407 | ✓           | 0.021 | 51   | ✓      | 0.042 | 39   | To Do            |

Continued on next page

Table 3 – continued from previous page

| #         | Mathematica |         |       | Maple  |         |      | note on solution |
|-----------|-------------|---------|-------|--------|---------|------|------------------|
|           | solved      | cpu     | leaf  | solved | cpu     | leaf |                  |
| Kamke 408 | ✓           | 0.151   | 97    | ✓      | 0.102   | 73   | To Do            |
| Kamke 409 | ✓           | 30.571  | 66    | ✓      | 0.179   | 63   | To Do            |
| Kamke 410 | ✓           | 31.234  | 80    | ✓      | 0.186   | 64   | To Do            |
| Kamke 411 | ✓           | 0.141   | 99    | ✓      | 0.117   | 65   | To Do            |
| Kamke 412 | ✓           | 0.409   | 3229  | ✓      | 0.161   | 146  | To Do            |
| Kamke 413 | ✗           | 0       | 0     | ✓      | 0.262   | 269  | To Do            |
| Kamke 414 | ✓           | 0.075   | 107   | ✓      | 0.273   | 269  | To Do            |
| Kamke 415 | ✓           | 0.308   | 25    | ✓      | 0.447   | 95   | To Do            |
| Kamke 416 | ✓           | 1.272   | 383   | ✓      | 0.134   | 136  | To Do            |
| Kamke 417 | ✓           | 0.356   | 48    | ✓      | 0.023   | 35   | To Do            |
| Kamke 418 | ✓           | 0.679   | 220   | ✓      | 0.072   | 42   | To Do            |
| Kamke 419 | ✓           | 1.056   | 9073  | ✓      | 0.048   | 109  | To Do            |
| Kamke 420 | ✓           | 0.296   | 777   | ✓      | 0.182   | 689  | To Do            |
| Kamke 421 | ✓           | 0.038   | 27    | ✓      | 0.047   | 32   | To Do            |
| Kamke 422 | ✓           | 0.054   | 81    | ✓      | 0.049   | 30   | To Do            |
| Kamke 423 | ✓           | 0.063   | 59    | ✓      | 0.053   | 44   | To Do            |
| Kamke 424 | ✓           | 0.49    | 223   | ✓      | 0.285   | 193  | To Do            |
| Kamke 425 | ✓           | 0.192   | 59    | ✓      | 0.029   | 45   | To Do            |
| Kamke 426 | ✓           | 0.425   | 40    | ✓      | 0.029   | 51   | To Do            |
| Kamke 427 | ✗           | 0       | 0     | ✓      | 0.035   | 60   | To Do            |
| Kamke 428 | ✗           | 0       | 0     | ✓      | 0.07    | 66   | To Do            |
| Kamke 429 | ✓           | 0.032   | 26    | ✓      | 0.067   | 72   | To Do            |
| Kamke 430 | ✓           | 245.198 | 478   | ✓      | 1.983   | 1602 | To Do            |
| Kamke 431 | ✓           | 0.046   | 81    | ✓      | 0.296   | 62   | To Do            |
| Kamke 432 | ✓           | 1.151   | 64    | ✓      | 7.73    | 242  | To Do            |
| Kamke 433 | ✓           | 1.33    | 22    | ✓      | 0.343   | 34   | To Do            |
| Kamke 434 | ✓           | 0.038   | 27    | ✓      | 0.025   | 7    | To Do            |
| Kamke 435 | ✓           | 0.059   | 61    | ✓      | 0.54    | 22   | To Do            |
| Kamke 436 | ✓           | 0.055   | 26    | ✓      | 4.319   | 61   | To Do            |
| Kamke 437 | ✓           | 0.333   | 47    | ✓      | 0.089   | 36   | To Do            |
| Kamke 438 | ✓           | 0.011   | 21    | ✓      | 0.031   | 17   | To Do            |
| Kamke 439 | ✓           | 0.037   | 49    | ✓      | 0.119   | 33   | To Do            |
| Kamke 440 | ✓           | 0.01    | 19    | ✓      | 0.027   | 15   | To Do            |
| Kamke 441 | ✓           | 0.079   | 65    | ✓      | 5.179   | 83   | To Do            |
| Kamke 442 | ✓           | 0.014   | 28    | ✓      | 0.032   | 21   | To Do            |
| Kamke 443 | ✓           | 2.749   | 10121 | ✓      | 7.904   | 221  | To Do            |
| Kamke 444 | ✓           | 0.411   | 75    | ✓      | 4.934   | 120  | To Do            |
| Kamke 445 | ✓           | 0.052   | 49    | ✓      | 0.032   | 35   | To Do            |
| Kamke 446 | ✓           | 0.334   | 95    | ✓      | 0.037   | 57   | To Do            |
| Kamke 447 | ✓           | 0.009   | 89    | ✓      | 0.082   | 33   | To Do            |
| Kamke 448 | ✓           | 0.095   | 349   | ✓      | 300.181 | 166  | To Do            |
| Kamke 449 | ✓           | 0.014   | 27    | ✓      | 0.03    | 23   | To Do            |
| Kamke 450 | ✓           | 0.316   | 26    | ✓      | 0.676   | 51   | To Do            |
| Kamke 451 | ✗           | 0       | 0     | ✓      | 0.124   | 78   | To Do            |
| Kamke 452 | ✓           | 0.017   | 23    | ✓      | 1.358   | 37   | To Do            |
| Kamke 453 | ✓           | 0.583   | 327   | ✓      | 2.691   | 229  | To Do            |
| Kamke 454 | ✓           | 0.269   | 241   | ✓      | 0.26    | 106  | To Do            |
| Kamke 455 | ✓           | 0.969   | 123   | ✓      | 0.612   | 66   | To Do            |
| Kamke 456 | ✓           | 0.124   | 421   | ✓      | 0.72    | 33   | To Do            |
| Kamke 457 | ✓           | 0.252   | 118   | ✓      | 3.371   | 45   | To Do            |
| Kamke 458 | ✓           | 0.022   | 120   | ✓      | 0.123   | 90   | To Do            |
| Kamke 459 | ✓           | 2.772   | 271   | ✓      | 0.932   | 65   | To Do            |
| Kamke 460 | ✗           | 0       | 0     | ✗      | 0       | 0    | To Do            |
| Kamke 461 | ✗           | 0       | 0     | ✗      | 0       | 0    | To Do            |
| Kamke 462 | ✓           | 0.02    | 43    | ✓      | 0.058   | 27   | To Do            |
| Kamke 463 | ✓           | 0.031   | 47    | ✓      | 0.185   | 50   | To Do            |
| Kamke 464 | ✓           | 0.087   | 52    | ✓      | 1.424   | 70   | To Do            |
| Kamke 465 | ✗           | 0       | 0     | ✓      | 0.147   | 210  | To Do            |
| Kamke 466 | ✓           | 1.876   | 433   | ✓      | 1.201   | 71   | To Do            |
| Kamke 467 | ✓           | 0.246   | 226   | ✓      | 0.125   | 148  | To Do            |

Continued on next page

Table 3 – continued from previous page

| #         | Mathematica |         |       | Maple  |       |      | note on solution |
|-----------|-------------|---------|-------|--------|-------|------|------------------|
|           | solved      | cpu     | leaf  | solved | cpu   | leaf |                  |
| Kamke 468 | ✓           | 3.57    | 609   | ✓      | 0.146 | 181  | To Do            |
| Kamke 469 | ✓           | 0.278   | 157   | ✓      | 0.25  | 264  | To Do            |
| Kamke 470 | ✓           | 0.239   | 143   | ✓      | 0.661 | 87   | To Do            |
| Kamke 471 | ✓           | 0.008   | 47    | ✓      | 0.033 | 33   | To Do            |
| Kamke 472 | ✓           | 0.393   | 269   | ✓      | 1.371 | 121  | To Do            |
| Kamke 473 | ✓           | 0.267   | 165   | ✓      | 1.901 | 71   | To Do            |
| Kamke 474 | ✓           | 0.163   | 135   | ✓      | 1.726 | 152  | To Do            |
| Kamke 475 | ✓           | 0.096   | 57    | ✓      | 1.397 | 67   | To Do            |
| Kamke 476 | ✓           | 0.247   | 143   | ✓      | 0.634 | 87   | To Do            |
| Kamke 477 | ✓           | 0.214   | 146   | ✓      | 0.629 | 622  | To Do            |
| Kamke 478 | ✓           | 0.322   | 223   | ✓      | 0.348 | 88   | To Do            |
| Kamke 479 | ✓           | 9.049   | 552   | ✓      | 0.234 | 929  | To Do            |
| Kamke 480 | ✗           | 0       | 0     | ✗      | 0     | 0    | To Do            |
| Kamke 481 | ✓           | 0.012   | 49    | ✓      | 0.033 | 35   | To Do            |
| Kamke 482 | ✗           | 0       | 0     | ✗      | 0     | 0    | To Do            |
| Kamke 483 | ✓           | 0.177   | 71    | ✓      | 0.069 | 103  | To Do            |
| Kamke 484 | ✓           | 0.142   | 81    | ✓      | 0.052 | 115  | To Do            |
| Kamke 485 | ✗           | 0       | 0     | ✗      | 0     | 0    | To Do            |
| Kamke 486 | ✓           | 0.049   | 117   | ✓      | 0.228 | 54   | To Do            |
| Kamke 487 | ✓           | 0.286   | 157   | ✓      | 0.548 | 100  | To Do            |
| Kamke 488 | ✓           | 0.29    | 85    | ✓      | 0.626 | 111  | To Do            |
| Kamke 489 | ✗           | 0       | 0     | ✓      | 2.747 | 551  | To Do            |
| Kamke 490 | ✓           | 0.421   | 70    | ✓      | 0.625 | 145  | To Do            |
| Kamke 491 | ✓           | 0.722   | 79    | ✓      | 0.925 | 195  | To Do            |
| Kamke 492 | ✓           | 0.08    | 97    | ✓      | 1.684 | 122  | To Do            |
| Kamke 493 | ✓           | 7.377   | 553   | ✓      | 1.194 | 111  | To Do            |
| Kamke 494 | ✓           | 0.113   | 49    | ✓      | 0.237 | 161  | To Do            |
| Kamke 495 | ✓           | 0.154   | 83    | ✓      | 2.276 | 61   | To Do            |
| Kamke 496 | ✓           | 19.584  | 17831 | ✓      | 0.464 | 130  | To Do            |
| Kamke 497 | ✓           | 0.144   | 76    | ✓      | 0.787 | 203  | To Do            |
| Kamke 498 | ✓           | 0.078   | 107   | ✓      | 1.002 | 99   | To Do            |
| Kamke 499 | ✓           | 0.383   | 212   | ✓      | 0.253 | 189  | To Do            |
| Kamke 500 | ✓           | 0.889   | 100   | ✓      | 1.02  | 220  | To Do            |
| Kamke 501 | ✓           | 21.821  | 913   | ✓      | 5.392 | 215  | To Do            |
| Kamke 502 | ✓           | 1.197   | 100   | ✓      | 0.69  | 195  | To Do            |
| Kamke 503 | ✗           | 0       | 0     | ✗      | 0     | 0    | To Do            |
| Kamke 504 | ✓           | 0.332   | 51    | ✓      | 1.107 | 247  | To Do            |
| Kamke 505 | ✓           | 0.079   | 73    | ✓      | 0.042 | 52   | To Do            |
| Kamke 506 | ✗           | 0       | 0     | ✗      | 0     | 0    | To Do            |
| Kamke 507 | ✗           | 0       | 0     | ✗      | 0     | 0    | To Do            |
| Kamke 508 | ✓           | 2.03    | 88    | ✓      | 1.753 | 60   | To Do            |
| Kamke 509 | ✓           | 0.276   | 34    | ✓      | 1.148 | 212  | To Do            |
| Kamke 510 | ✗           | 0       | 0     | ✗      | 0     | 0    | To Do            |
| Kamke 511 | ✓           | 1.44    | 229   | ✓      | 3.776 | 199  | To Do            |
| Kamke 512 | ✓           | 5.926   | 713   | ✓      | 4.408 | 137  | To Do            |
| Kamke 513 | ✓           | 0.075   | 81    | ✓      | 3.5   | 1138 | To Do            |
| Kamke 514 | ✓           | 9.975   | 605   | ✓      | 0.257 | 87   | To Do            |
| Kamke 515 | ✓           | 2.04    | 1922  | ✓      | 0.943 | 113  | To Do            |
| Kamke 516 | ✓           | 0.478   | 253   | ✓      | 0.822 | 70   | To Do            |
| Kamke 517 | ✓           | 0.493   | 283   | ✓      | 0.842 | 78   | To Do            |
| Kamke 518 | ✓           | 0.517   | 236   | ✓      | 0.156 | 126  | To Do            |
| Kamke 519 | ✓           | 0.615   | 353   | ✓      | 0.342 | 197  | To Do            |
| Kamke 520 | ✓           | 187.208 | 1590  | ✓      | 0.115 | 245  | To Do            |
| Kamke 521 | ✓           | 0.003   | 14    | ✓      | 0.02  | 33   | To Do            |
| Kamke 522 | ✓           | 0.004   | 20    | ✓      | 0.029 | 44   | To Do            |
| Kamke 523 | ✓           | 145.521 | 392   | ✓      | 0.037 | 231  | To Do            |
| Kamke 524 | ✗           | 0       | 0     | ✓      | 0.045 | 261  | To Do            |
| Kamke 525 | ✓           | 0.074   | 135   | ✓      | 0.035 | 122  | To Do            |
| Kamke 526 | ✓           | 0.048   | 45    | ✓      | 0.012 | 32   | To Do            |
| Kamke 527 | ✓           | 0.015   | 20    | ✓      | 0.518 | 43   | To Do            |

Continued on next page

Table 3 – continued from previous page

| #         | Mathematica |        |      | Maple  |       |      | note on solution |
|-----------|-------------|--------|------|--------|-------|------|------------------|
|           | solved      | cpu    | leaf | solved | cpu   | leaf |                  |
| Kamke 528 | ✓           | 0.624  | 398  | ✓      | 0.068 | 86   | To Do            |
| Kamke 529 | ✓           | 39.37  | 1758 | ✓      | 0.037 | 1251 | To Do            |
| Kamke 530 | ✓           | 7.976  | 648  | ✓      | 0.086 | 432  | To Do            |
| Kamke 531 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 532 | ✗           | 0      | 0    | ✓      | 0.101 | 848  | To Do            |
| Kamke 533 | ✓           | 0.016  | 40   | ✓      | 0.025 | 76   | To Do            |
| Kamke 534 | ✓           | 0.05   | 114  | ✓      | 0.049 | 84   | To Do            |
| Kamke 535 | ✓           | 0.036  | 49   | ✓      | 0.036 | 51   | To Do            |
| Kamke 536 | ✓           | 0.012  | 64   | ✓      | 0.029 | 52   | To Do            |
| Kamke 537 | ✗           | 0      | 0    | ✓      | 3.105 | 250  | To Do            |
| Kamke 538 | ✓           | 9.067  | 110  | ✓      | 0.82  | 1625 | To Do            |
| Kamke 539 | ✓           | 0.023  | 45   | ✓      | 0.034 | 32   | To Do            |
| Kamke 540 | ✓           | 0.031  | 69   | ✓      | 0.052 | 109  | To Do            |
| Kamke 541 | ✓           | 0.029  | 39   | ✓      | 0.404 | 103  | To Do            |
| Kamke 542 | ✓           | 0.029  | 20   | ✓      | 0.385 | 107  | To Do            |
| Kamke 543 | ✓           | 0.024  | 55   | ✓      | 1.089 | 277  | To Do            |
| Kamke 544 | ✓           | 0.073  | 22   | ✓      | 0.697 | 4201 | To Do            |
| Kamke 545 | ✓           | 0.665  | 323  | ✓      | 0.122 | 144  | To Do            |
| Kamke 546 | ✓           | 0.029  | 113  | ✓      | 0.094 | 171  | To Do            |
| Kamke 547 | ✓           | 2.446  | 490  | ✓      | 0.192 | 118  | To Do            |
| Kamke 548 | ✓           | 0.969  | 479  | ✓      | 0.226 | 250  | To Do            |
| Kamke 549 | ✓           | 0.257  | 216  | ✓      | 0.348 | 553  | To Do            |
| Kamke 550 | ✓           | 0.73   | 488  | ✓      | 0.221 | 60   | To Do            |
| Kamke 551 | ✓           | 0.113  | 86   | ✓      | 0.309 | 55   | To Do            |
| Kamke 552 | ✓           | 0.031  | 41   | ✓      | 0.059 | 43   | To Do            |
| Kamke 553 | ✓           | 0.256  | 52   | ✓      | 0.042 | 36   | To Do            |
| Kamke 554 | ✓           | 0.097  | 49   | ✓      | 0.309 | 32   | To Do            |
| Kamke 555 | ✓           | 0.311  | 119  | ✓      | 0.019 | 15   | To Do            |
| Kamke 556 | ✓           | 4.464  | 60   | ✓      | 0.141 | 581  | To Do            |
| Kamke 557 | ✓           | 0.024  | 39   | ✓      | 0.091 | 74   | To Do            |
| Kamke 558 | ✓           | 0.561  | 327  | ✓      | 0.112 | 223  | To Do            |
| Kamke 559 | ✓           | 0.379  | 212  | ✓      | 0.203 | 215  | To Do            |
| Kamke 560 | ✓           | 13.956 | 110  | ✓      | 0.722 | 1120 | To Do            |
| Kamke 561 | ✓           | 1.817  | 2138 | ✓      | 1.358 | 50   | To Do            |
| Kamke 562 | ✗           | 0      | 0    | ✓      | 0.12  | 3306 | To Do            |
| Kamke 563 | ✓           | 0.136  | 59   | ✓      | 0.129 | 66   | To Do            |
| Kamke 564 | ✓           | 0.053  | 28   | ✓      | 0.018 | 32   | To Do            |
| Kamke 565 | ✓           | 0.017  | 25   | ✓      | 0.102 | 17   | To Do            |
| Kamke 566 | ✓           | 0.031  | 31   | ✓      | 0.023 | 16   | To Do            |
| Kamke 567 | ✓           | 0.042  | 42   | ✓      | 0.029 | 18   | To Do            |
| Kamke 568 | ✓           | 0.052  | 28   | ✓      | 0.046 | 32   | To Do            |
| Kamke 569 | ✓           | 0.093  | 59   | ✓      | 0.306 | 147  | To Do            |
| Kamke 570 | ✓           | 1.423  | 51   | ✓      | 0.04  | 30   | To Do            |
| Kamke 571 | ✓           | 0.087  | 116  | ✓      | 0.418 | 169  | To Do            |
| Kamke 572 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 573 | ✓           | 0.016  | 42   | ✓      | 0.097 | 24   | To Do            |
| Kamke 574 | ✓           | 0.024  | 102  | ✓      | 0.096 | 41   | To Do            |
| Kamke 575 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 576 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 577 | ✓           | 0.243  | 243  | ✓      | 0.025 | 28   | To Do            |
| Kamke 578 | ✓           | 0.158  | 100  | ✓      | 0.032 | 22   | To Do            |
| Kamke 579 | ✓           | 0.24   | 514  | ✓      | 0.035 | 35   | To Do            |
| Kamke 580 | ✓           | 0.245  | 203  | ✓      | 0.045 | 31   | To Do            |
| Kamke 581 | ✓           | 0.232  | 144  | ✓      | 0.075 | 32   | To Do            |
| Kamke 582 | ✓           | 0.248  | 142  | ✓      | 0.181 | 30   | To Do            |
| Kamke 583 | ✓           | 0.211  | 126  | ✓      | 0.102 | 31   | To Do            |
| Kamke 584 | ✓           | 0.273  | 115  | ✓      | 0.047 | 35   | To Do            |
| Kamke 585 | ✓           | 0.21   | 205  | ✓      | 0.344 | 120  | To Do            |
| Kamke 586 | ✓           | 0.564  | 975  | ✓      | 0.197 | 39   | To Do            |
| Kamke 587 | ✓           | 0.223  | 123  | ✓      | 0.098 | 29   | To Do            |

Continued on next page

Table 3 – continued from previous page

| #         | Mathematica |        |      | Maple  |       |      | note on solution |
|-----------|-------------|--------|------|--------|-------|------|------------------|
|           | solved      | cpu    | leaf | solved | cpu   | leaf |                  |
| Kamke 588 | ✓           | 0.2    | 113  | ✓      | 0.089 | 53   | To Do            |
| Kamke 589 | ✓           | 0.228  | 245  | ✓      | 0.107 | 38   | To Do            |
| Kamke 590 | ✓           | 0.228  | 94   | ✓      | 0.105 | 28   | To Do            |
| Kamke 591 | ✓           | 0.471  | 253  | ✓      | 0.141 | 108  | To Do            |
| Kamke 592 | ✓           | 0.518  | 241  | ✓      | 0.138 | 33   | To Do            |
| Kamke 593 | ✓           | 0.36   | 221  | ✓      | 0.247 | 35   | To Do            |
| Kamke 594 | ✓           | 0.34   | 236  | ✓      | 0.099 | 67   | To Do            |
| Kamke 595 | ✓           | 0.306  | 204  | ✓      | 0.098 | 72   | To Do            |
| Kamke 596 | ✓           | 0.25   | 156  | ✓      | 0.072 | 26   | To Do            |
| Kamke 597 | ✓           | 0.38   | 130  | ✓      | 0.333 | 37   | To Do            |
| Kamke 598 | ✓           | 0.169  | 37   | ✓      | 0.016 | 29   | To Do            |
| Kamke 599 | ✓           | 0.154  | 95   | ✓      | 0.07  | 57   | To Do            |
| Kamke 600 | ✓           | 0.262  | 246  | ✓      | 0.108 | 38   | To Do            |
| Kamke 601 | ✓           | 0.241  | 190  | ✓      | 0.098 | 61   | To Do            |
| Kamke 602 | ✓           | 0.524  | 167  | ✓      | 0.099 | 33   | To Do            |
| Kamke 603 | ✓           | 0.312  | 117  | ✓      | 0.108 | 27   | To Do            |
| Kamke 604 | ✓           | 0.388  | 143  | ✓      | 0.109 | 30   | To Do            |
| Kamke 605 | ✓           | 0.449  | 145  | ✓      | 0.158 | 29   | To Do            |
| Kamke 606 | ✓           | 0.538  | 361  | ✓      | 0.602 | 34   | To Do            |
| Kamke 607 | ✓           | 0.216  | 121  | ✓      | 0.075 | 22   | To Do            |
| Kamke 608 | ✓           | 0.414  | 274  | ✓      | 0.112 | 40   | To Do            |
| Kamke 609 | ✓           | 0.257  | 117  | ✓      | 0.121 | 22   | To Do            |
| Kamke 610 | ✓           | 0.089  | 25   | ✓      | 0.01  | 20   | To Do            |
| Kamke 611 | ✓           | 0.259  | 191  | ✓      | 0.065 | 28   | To Do            |
| Kamke 612 | ✓           | 0.331  | 199  | ✓      | 0.101 | 27   | To Do            |
| Kamke 613 | ✓           | 0.281  | 226  | ✓      | 0.078 | 23   | To Do            |
| Kamke 614 | ✓           | 0.358  | 177  | ✓      | 0.332 | 60   | To Do            |
| Kamke 615 | ✓           | 0.27   | 77   | ✓      | 0.148 | 26   | To Do            |
| Kamke 616 | ✓           | 0.457  | 177  | ✓      | 0.069 | 26   | To Do            |
| Kamke 617 | ✓           | 0.969  | 615  | ✓      | 0.207 | 47   | To Do            |
| Kamke 618 | ✓           | 0.249  | 25   | ✓      | 0.32  | 34   | To Do            |
| Kamke 619 | ✓           | 0.651  | 330  | ✓      | 0.323 | 81   | To Do            |
| Kamke 620 | ✓           | 0.759  | 210  | ✓      | 0.143 | 37   | To Do            |
| Kamke 621 | ✓           | 0.056  | 445  | ✓      | 0.237 | 59   | To Do            |
| Kamke 622 | ✓           | 0.27   | 140  | ✓      | 0.181 | 77   | To Do            |
| Kamke 623 | ✓           | 0.143  | 77   | ✓      | 0.227 | 49   | To Do            |
| Kamke 624 | ✓           | 28.927 | 9837 | ✓      | 1.898 | 46   | To Do            |
| Kamke 625 | ✓           | 0.295  | 56   | ✓      | 0.161 | 55   | To Do            |
| Kamke 626 | ✓           | 0.152  | 88   | ✓      | 0.264 | 115  | To Do            |
| Kamke 627 | ✓           | 0.811  | 25   | ✓      | 0.153 | 35   | To Do            |
| Kamke 628 | ✓           | 0.154  | 38   | ✓      | 0.158 | 23   | To Do            |
| Kamke 629 | ✓           | 0.838  | 47   | ✓      | 0.152 | 62   | To Do            |
| Kamke 630 | ✓           | 0.37   | 101  | ✓      | 0.247 | 98   | To Do            |
| Kamke 631 | ✓           | 0.174  | 38   | ✓      | 0.174 | 23   | To Do            |
| Kamke 632 | ✓           | 0.146  | 65   | ✓      | 0.214 | 54   | To Do            |
| Kamke 633 | ✓           | 0.159  | 85   | ✓      | 0.76  | 52   | To Do            |
| Kamke 634 | ✓           | 0.201  | 33   | ✓      | 0.175 | 26   | To Do            |
| Kamke 635 | ✓           | 0.172  | 33   | ✓      | 0.143 | 22   | To Do            |
| Kamke 636 | ✓           | 0.1    | 24   | ✓      | 0.138 | 19   | To Do            |
| Kamke 637 | ✓           | 11.739 | 59   | ✓      | 0.136 | 84   | To Do            |
| Kamke 638 | ✗           | 0      | 0    | ✓      | 0.138 | 35   | To Do            |
| Kamke 639 | ✗           | 0      | 0    | ✓      | 0.185 | 48   | To Do            |
| Kamke 640 | ✗           | 0      | 0    | ✓      | 0.27  | 47   | To Do            |
| Kamke 641 | ✓           | 0.201  | 35   | ✓      | 0.159 | 26   | To Do            |
| Kamke 642 | ✓           | 0.164  | 105  | ✓      | 0.259 | 286  | To Do            |
| Kamke 643 | ✓           | 0.174  | 31   | ✓      | 0.143 | 22   | To Do            |
| Kamke 644 | ✓           | 0.347  | 34   | ✓      | 0.324 | 27   | To Do            |
| Kamke 645 | ✓           | 0.082  | 20   | ✓      | 0.07  | 14   | To Do            |
| Kamke 646 | ✓           | 0.288  | 35   | ✓      | 0.198 | 23   | To Do            |
| Kamke 647 | ✓           | 0.363  | 115  | ✓      | 0.207 | 460  | To Do            |

Continued on next page

Table 3 – continued from previous page

| #         | Mathematica |        |      | Maple  |       |      | note on solution |
|-----------|-------------|--------|------|--------|-------|------|------------------|
|           | solved      | cpu    | leaf | solved | cpu   | leaf |                  |
| Kamke 648 | ✓           | 0.53   | 128  | ✓      | 0.47  | 41   | To Do            |
| Kamke 649 | ✓           | 0.255  | 37   | ✓      | 0.158 | 27   | To Do            |
| Kamke 650 | ✓           | 0.397  | 109  | ✓      | 0.182 | 28   | To Do            |
| Kamke 651 | ✓           | 0.1    | 16   | ✓      | 0.047 | 13   | To Do            |
| Kamke 652 | ✓           | 1.243  | 101  | ✓      | 0.146 | 27   | To Do            |
| Kamke 653 | ✓           | 0.51   | 94   | ✓      | 0.132 | 24   | To Do            |
| Kamke 654 | ✓           | 0.307  | 37   | ✓      | 0.182 | 23   | To Do            |
| Kamke 655 | ✓           | 15.253 | 82   | ✓      | 0.509 | 66   | To Do            |
| Kamke 656 | ✓           | 0.098  | 20   | ✓      | 0.048 | 15   | To Do            |
| Kamke 657 | ✓           | 0.248  | 37   | ✓      | 0.155 | 26   | To Do            |
| Kamke 658 | ✓           | 0.419  | 41   | ✓      | 0.232 | 28   | To Do            |
| Kamke 659 | ✓           | 0.689  | 164  | ✓      | 0.168 | 41   | To Do            |
| Kamke 660 | ✓           | 0.431  | 109  | ✓      | 0.169 | 29   | To Do            |
| Kamke 661 | ✓           | 0.685  | 164  | ✓      | 0.15  | 39   | To Do            |
| Kamke 662 | ✓           | 0.266  | 37   | ✓      | 0.161 | 26   | To Do            |
| Kamke 663 | ✓           | 1.541  | 101  | ✓      | 0.135 | 27   | To Do            |
| Kamke 664 | ✓           | 0.555  | 94   | ✓      | 0.128 | 25   | To Do            |
| Kamke 665 | ✓           | 0.449  | 41   | ✓      | 0.342 | 28   | To Do            |
| Kamke 666 | ✓           | 0.137  | 29   | ✓      | 0.129 | 24   | To Do            |
| Kamke 667 | ✓           | 0.927  | 90   | ✓      | 0.163 | 82   | To Do            |
| Kamke 668 | ✓           | 0.595  | 78   | ✓      | 0.549 | 58   | To Do            |
| Kamke 669 | ✓           | 0.654  | 264  | ✓      | 0.142 | 72   | To Do            |
| Kamke 670 | ✓           | 0.494  | 62   | ✓      | 0.25  | 70   | To Do            |
| Kamke 671 | ✓           | 0.329  | 192  | ✓      | 0.148 | 237  | To Do            |
| Kamke 672 | ✓           | 2.294  | 4512 | ✓      | 0.149 | 36   | To Do            |
| Kamke 673 | ✓           | 0.31   | 23   | ✓      | 0.314 | 17   | To Do            |
| Kamke 674 | ✓           | 0.608  | 91   | ✓      | 0.217 | 27   | To Do            |
| Kamke 675 | ✓           | 0.328  | 48   | ✓      | 0.044 | 37   | To Do            |
| Kamke 676 | ✓           | 0.436  | 144  | ✓      | 0.305 | 43   | To Do            |
| Kamke 677 | ✓           | 0.176  | 80   | ✓      | 0.04  | 48   | To Do            |
| Kamke 678 | ✓           | 0.325  | 101  | ✓      | 0.236 | 37   | To Do            |
| Kamke 679 | ✓           | 0.149  | 59   | ✓      | 0.036 | 37   | To Do            |
| Kamke 680 | ✓           | 0.736  | 89   | ✓      | 0.23  | 28   | To Do            |
| Kamke 681 | ✓           | 0.204  | 84   | ✓      | 0.049 | 45   | To Do            |
| Kamke 682 | ✓           | 0.25   | 39   | ✓      | 0.303 | 28   | To Do            |
| Kamke 683 | ✓           | 1.108  | 84   | ✓      | 0.082 | 152  | To Do            |
| Kamke 684 | ✓           | 0.108  | 20   | ✓      | 2.668 | 30   | To Do            |
| Kamke 685 | ✓           | 0.148  | 87   | ✓      | 0.058 | 48   | To Do            |
| Kamke 686 | ✓           | 11.719 | 68   | ✓      | 0.098 | 85   | To Do            |
| Kamke 687 | ✓           | 0.155  | 130  | ✓      | 0.082 | 39   | To Do            |
| Kamke 688 | ✓           | 0.351  | 82   | ✓      | 0.068 | 42   | To Do            |
| Kamke 689 | ✓           | 0.318  | 60   | ✓      | 0.034 | 25   | To Do            |
| Kamke 690 | ✓           | 0.512  | 105  | ✓      | 0.263 | 40   | To Do            |
| Kamke 691 | ✓           | 0.298  | 21   | ✓      | 0.474 | 17   | To Do            |
| Kamke 692 | ✓           | 0.104  | 20   | ✓      | 2.174 | 30   | To Do            |
| Kamke 693 | ✓           | 0.291  | 146  | ✓      | 0.074 | 40   | To Do            |
| Kamke 694 | ✓           | 0.404  | 66   | ✓      | 0.24  | 30   | To Do            |
| Kamke 695 | ✓           | 0.206  | 34   | ✓      | 0.034 | 39   | To Do            |
| Kamke 696 | ✓           | 0.682  | 51   | ✓      | 0.044 | 32   | To Do            |
| Kamke 697 | ✓           | 0.255  | 114  | ✓      | 0.076 | 40   | To Do            |
| Kamke 698 | ✓           | 0.254  | 108  | ✓      | 0.062 | 34   | To Do            |
| Kamke 699 | ✓           | 0.328  | 101  | ✓      | 0.235 | 36   | To Do            |
| Kamke 700 | ✓           | 0.151  | 76   | ✓      | 0.09  | 62   | To Do            |
| Kamke 701 | ✓           | 1.785  | 88   | ✓      | 4.746 | 71   | To Do            |
| Kamke 702 | ✓           | 2.54   | 37   | ✓      | 0.057 | 35   | To Do            |
| Kamke 703 | ✓           | 0.439  | 101  | ✓      | 0.178 | 44   | To Do            |
| Kamke 704 | ✓           | 9.778  | 66   | ✓      | 0.043 | 38   | To Do            |
| Kamke 705 | ✓           | 0.144  | 30   | ✓      | 0.137 | 24   | To Do            |
| Kamke 706 | ✓           | 40.219 | 610  | ✓      | 0.464 | 65   | To Do            |
| Kamke 707 | ✓           | 4.953  | 1391 | ✓      | 0.368 | 105  | To Do            |

Continued on next page



Table 3 – continued from previous page

| #         | Mathematica |        |      | Maple  |        |      | note on solution |
|-----------|-------------|--------|------|--------|--------|------|------------------|
|           | solved      | cpu    | leaf | solved | cpu    | leaf |                  |
| Kamke 708 | ✓           | 0.349  | 89   | ✓      | 13.651 | 229  | To Do            |
| Kamke 709 | ✓           | 2.778  | 217  | ✓      | 0.227  | 39   | To Do            |
| Kamke 710 | ✓           | 1.182  | 38   | ✓      | 1.887  | 31   | To Do            |
| Kamke 711 | ✓           | 0.203  | 28   | ✓      | 0.105  | 31   | To Do            |
| Kamke 712 | ✓           | 0.482  | 105  | ✓      | 0.274  | 38   | To Do            |
| Kamke 713 | ✓           | 0.086  | 649  | ✓      | 0.292  | 86   | To Do            |
| Kamke 714 | ✓           | 1.438  | 162  | ✓      | 0.139  | 96   | To Do            |
| Kamke 715 | ✓           | 0.482  | 104  | ✓      | 0.227  | 39   | To Do            |
| Kamke 716 | ✓           | 2.725  | 133  | ✓      | 0.25   | 37   | To Do            |
| Kamke 717 | ✓           | 0.593  | 106  | ✓      | 0.281  | 33   | To Do            |
| Kamke 718 | ✓           | 0.296  | 127  | ✓      | 0.045  | 44   | To Do            |
| Kamke 719 | ✓           | 0.37   | 49   | ✓      | 0.094  | 34   | To Do            |
| Kamke 720 | ✓           | 2.929  | 314  | ✓      | 0.166  | 48   | To Do            |
| Kamke 721 | ✓           | 0.105  | 27   | ✓      | 0.066  | 19   | To Do            |
| Kamke 722 | ✓           | 41.867 | 490  | ✓      | 0.246  | 70   | To Do            |
| Kamke 723 | ✓           | 0.215  | 663  | ✓      | 0.056  | 856  | To Do            |
| Kamke 724 | ✓           | 43.173 | 422  | ✓      | 0.035  | 18   | To Do            |
| Kamke 725 | ✓           | 0.314  | 19   | ✓      | 0.707  | 25   | To Do            |
| Kamke 726 | ✓           | 0.057  | 625  | ✓      | 0.213  | 83   | To Do            |
| Kamke 727 | ✓           | 0.415  | 29   | ✓      | 0.193  | 25   | To Do            |
| Kamke 728 | ✓           | 0.35   | 72   | ✓      | 0.25   | 50   | To Do            |
| Kamke 729 | ✓           | 0.282  | 327  | ✓      | 0.086  | 404  | To Do            |
| Kamke 730 | ✗           | 0      | 0    | ✓      | 1.31   | 41   | To Do            |
| Kamke 731 | ✓           | 0.34   | 47   | ✓      | 0.148  | 42   | To Do            |
| Kamke 732 | ✓           | 0.712  | 116  | ✓      | 0.285  | 43   | To Do            |
| Kamke 733 | ✓           | 12.536 | 73   | ✗      | 0      | 0    | To Do            |
| Kamke 734 | ✓           | 0.233  | 37   | ✓      | 0.228  | 39   | To Do            |
| Kamke 735 | ✓           | 11.549 | 573  | ✓      | 0.066  | 78   | To Do            |
| Kamke 736 | ✓           | 0.21   | 31   | ✓      | 0.165  | 43   | To Do            |
| Kamke 737 | ✓           | 0.025  | 36   | ✓      | 0.089  | 29   | To Do            |
| Kamke 738 | ✓           | 0.484  | 1347 | ✓      | 0.861  | 1054 | To Do            |
| Kamke 739 | ✓           | 0.288  | 39   | ✓      | 0.152  | 35   | To Do            |
| Kamke 740 | ✓           | 0.115  | 74   | ✓      | 0.075  | 72   | To Do            |
| Kamke 741 | ✓           | 2.178  | 175  | ✓      | 0.9    | 246  | To Do            |
| Kamke 742 | ✓           | 3.059  | 3913 | ✓      | 1.325  | 239  | To Do            |
| Kamke 743 | ✗           | 0      | 0    | ✓      | 0.322  | 296  | To Do            |
| Kamke 744 | ✓           | 0.163  | 510  | ✓      | 0.144  | 621  | To Do            |
| Kamke 745 | ✓           | 11.469 | 546  | ✓      | 0.047  | 78   | To Do            |
| Kamke 746 | ✗           | 0      | 0    | ✓      | 0.267  | 232  | To Do            |
| Kamke 747 | ✓           | 2.911  | 88   | ✓      | 0.283  | 75   | To Do            |
| Kamke 748 | ✓           | 0.292  | 285  | ✓      | 0.082  | 404  | To Do            |
| Kamke 749 | ✓           | 0.153  | 126  | ✓      | 0.095  | 192  | To Do            |
| Kamke 750 | ✓           | 0.32   | 72   | ✓      | 0.198  | 49   | To Do            |
| Kamke 751 | ✓           | 0.167  | 30   | ✓      | 0.069  | 26   | To Do            |
| Kamke 752 | ✗           | 0      | 0    | ✓      | 1.096  | 723  | To Do            |
| Kamke 753 | ✓           | 0.168  | 41   | ✓      | 0.102  | 38   | To Do            |
| Kamke 754 | ✓           | 0.109  | 47   | ✓      | 0.011  | 26   | To Do            |
| Kamke 755 | ✓           | 0.277  | 2633 | ✓      | 0.093  | 44   | To Do            |
| Kamke 756 | ✓           | 0.155  | 95   | ✓      | 0.022  | 37   | To Do            |
| Kamke 757 | ✓           | 0.023  | 36   | ✓      | 0.044  | 26   | To Do            |
| Kamke 758 | ✓           | 0.768  | 459  | ✓      | 0.165  | 41   | To Do            |
| Kamke 759 | ✗           | 0      | 0    | ✓      | 0.384  | 305  | To Do            |
| Kamke 760 | ✓           | 0.992  | 112  | ✓      | 0.785  | 136  | To Do            |
| Kamke 761 | ✓           | 0.022  | 33   | ✓      | 0.04   | 18   | To Do            |
| Kamke 762 | ✓           | 0.17   | 26   | ✓      | 0.086  | 22   | To Do            |
| Kamke 763 | ✓           | 0.151  | 22   | ✓      | 0.078  | 14   | To Do            |
| Kamke 764 | ✓           | 0.193  | 50   | ✓      | 0.08   | 36   | To Do            |
| Kamke 765 | ✓           | 0.342  | 138  | ✓      | 0.139  | 106  | To Do            |
| Kamke 766 | ✓           | 0.306  | 129  | ✓      | 0.109  | 89   | To Do            |
| Kamke 767 | ✓           | 0.024  | 38   | ✓      | 0.044  | 26   | To Do            |

Continued on next page

Table 3 – continued from previous page

| #         | Mathematica |        |      | Maple  |        |      | note on solution |
|-----------|-------------|--------|------|--------|--------|------|------------------|
|           | solved      | cpu    | leaf | solved | cpu    | leaf |                  |
| Kamke 768 | ✓           | 1.053  | 66   | ✓      | 0.059  | 26   | To Do            |
| Kamke 769 | ✗           | 0      | 0    | ✓      | 0.296  | 251  | To Do            |
| Kamke 770 | ✓           | 0.213  | 705  | ✓      | 0.105  | 1105 | To Do            |
| Kamke 771 | ✓           | 0.03   | 46   | ✓      | 0.076  | 84   | To Do            |
| Kamke 772 | ✓           | 0.155  | 21   | ✓      | 0.073  | 18   | To Do            |
| Kamke 773 | ✓           | 0.163  | 61   | ✓      | 0.159  | 48   | To Do            |
| Kamke 774 | ✓           | 0.026  | 45   | ✓      | 0.064  | 51   | To Do            |
| Kamke 775 | ✓           | 0.157  | 943  | ✓      | 0.059  | 44   | To Do            |
| Kamke 776 | ✓           | 0.376  | 133  | ✓      | 0.132  | 96   | To Do            |
| Kamke 777 | ✓           | 0.219  | 39   | ✓      | 0.142  | 51   | To Do            |
| Kamke 778 | ✓           | 0.148  | 95   | ✓      | 0.019  | 37   | To Do            |
| Kamke 779 | ✓           | 0.156  | 57   | ✓      | 0.079  | 50   | To Do            |
| Kamke 780 | ✓           | 0.155  | 15   | ✓      | 0.325  | 27   | To Do            |
| Kamke 781 | ✓           | 0.487  | 82   | ✓      | 0.234  | 61   | To Do            |
| Kamke 782 | ✓           | 4.142  | 115  | ✓      | 0.961  | 96   | To Do            |
| Kamke 783 | ✓           | 4.165  | 88   | ✓      | 0.122  | 75   | To Do            |
| Kamke 784 | ✓           | 19.904 | 27   | ✓      | 20.355 | 24   | To Do            |
| Kamke 785 | ✓           | 7.312  | 29   | ✓      | 66.315 | 24   | To Do            |
| Kamke 786 | ✓           | 3.621  | 61   | ✓      | 0.05   | 33   | To Do            |
| Kamke 787 | ✓           | 25.979 | 488  | ✓      | 0.267  | 191  | To Do            |
| Kamke 788 | ✓           | 28.549 | 348  | ✓      | 0.166  | 108  | To Do            |
| Kamke 789 | ✓           | 71.347 | 120  | ✗      | 0      | 0    | To Do            |
| Kamke 790 | ✓           | 86.116 | 127  | ✗      | 0      | 0    | To Do            |
| Kamke 791 | ✓           | 7.896  | 110  | ✓      | 13.26  | 306  | To Do            |
| Kamke 792 | ✓           | 3.032  | 157  | ✓      | 0.271  | 112  | To Do            |
| Kamke 793 | ✓           | 13.047 | 399  | ✓      | 0.115  | 32   | To Do            |
| Kamke 794 | ✓           | 0.192  | 67   | ✓      | 0.41   | 32   | To Do            |
| Kamke 795 | ✓           | 0.376  | 111  | ✓      | 0.019  | 37   | To Do            |
| Kamke 796 | ✓           | 12.283 | 102  | ✓      | 0.88   | 143  | To Do            |
| Kamke 797 | ✓           | 1.901  | 349  | ✓      | 0.258  | 168  | To Do            |
| Kamke 798 | ✓           | 0.584  | 27   | ✓      | 0.125  | 30   | To Do            |
| Kamke 799 | ✓           | 1.061  | 126  | ✓      | 0.236  | 147  | To Do            |
| Kamke 800 | ✓           | 0.409  | 128  | ✓      | 0.017  | 41   | To Do            |
| Kamke 801 | ✓           | 0.362  | 126  | ✓      | 0.043  | 63   | To Do            |
| Kamke 802 | ✓           | 0.122  | 101  | ✓      | 0.087  | 27   | To Do            |
| Kamke 803 | ✓           | 0.291  | 637  | ✓      | 0.299  | 65   | To Do            |
| Kamke 804 | ✓           | 0.609  | 43   | ✓      | 0.796  | 38   | To Do            |
| Kamke 805 | ✓           | 0.163  | 37   | ✓      | 0.539  | 42   | To Do            |
| Kamke 806 | ✓           | 0.432  | 22   | ✓      | 0.44   | 22   | To Do            |
| Kamke 807 | ✗           | 0      | 0    | ✓      | 0.45   | 43   | To Do            |
| Kamke 808 | ✓           | 1.469  | 149  | ✓      | 0.07   | 45   | To Do            |
| Kamke 809 | ✓           | 0.295  | 128  | ✓      | 0.014  | 41   | To Do            |
| Kamke 810 | ✓           | 0.106  | 40   | ✓      | 0.039  | 16   | To Do            |
| Kamke 811 | ✓           | 1.754  | 33   | ✓      | 1.545  | 32   | To Do            |
| Kamke 812 | ✓           | 0.392  | 78   | ✓      | 0.197  | 30   | To Do            |
| Kamke 813 | ✓           | 0.548  | 93   | ✓      | 0.372  | 40   | To Do            |
| Kamke 814 | ✓           | 0.129  | 72   | ✓      | 0.028  | 38   | To Do            |
| Kamke 815 | ✓           | 12.919 | 99   | ✓      | 0.604  | 168  | To Do            |
| Kamke 816 | ✓           | 0.239  | 74   | ✓      | 0.444  | 190  | To Do            |
| Kamke 817 | ✓           | 0.552  | 63   | ✓      | 0.518  | 27   | To Do            |
| Kamke 818 | ✓           | 0.186  | 34   | ✓      | 0.149  | 34   | To Do            |
| Kamke 819 | ✓           | 0.285  | 65   | ✓      | 0.183  | 30   | To Do            |
| Kamke 820 | ✓           | 0.53   | 63   | ✓      | 0.482  | 27   | To Do            |
| Kamke 821 | ✓           | 0.234  | 2093 | ✓      | 0.122  | 27   | To Do            |
| Kamke 822 | ✓           | 0.301  | 32   | ✓      | 0.092  | 25   | To Do            |
| Kamke 823 | ✓           | 0.419  | 39   | ✓      | 0.137  | 38   | To Do            |
| Kamke 824 | ✓           | 0.201  | 68   | ✓      | 0.341  | 61   | To Do            |
| Kamke 825 | ✓           | 0.412  | 148  | ✓      | 0.075  | 48   | To Do            |
| Kamke 826 | ✓           | 0.558  | 70   | ✓      | 0.255  | 51   | To Do            |
| Kamke 827 | ✓           | 0.19   | 221  | ✓      | 0.169  | 49   | To Do            |

Continued on next page

Table 3 – continued from previous page

| #         | Mathematica |        |      | Maple  |        |      | note on solution |
|-----------|-------------|--------|------|--------|--------|------|------------------|
|           | solved      | cpu    | leaf | solved | cpu    | leaf |                  |
| Kamke 828 | ✓           | 0.408  | 56   | ✓      | 0.217  | 54   | To Do            |
| Kamke 829 | ✓           | 0.423  | 74   | ✓      | 0.231  | 34   | To Do            |
| Kamke 830 | ✓           | 0.447  | 37   | ✓      | 0.122  | 38   | To Do            |
| Kamke 831 | ✓           | 2.988  | 145  | ✓      | 0.286  | 35   | To Do            |
| Kamke 832 | ✓           | 2.179  | 2497 | ✓      | 0.143  | 31   | To Do            |
| Kamke 833 | ✓           | 0.193  | 221  | ✓      | 0.124  | 49   | To Do            |
| Kamke 834 | ✓           | 0.651  | 90   | ✓      | 0.333  | 60   | To Do            |
| Kamke 835 | ✗           | 0      | 0    | ✗      | 0      | 0    | To Do            |
| Kamke 836 | ✓           | 10.852 | 379  | ✓      | 0.161  | 73   | To Do            |
| Kamke 837 | ✗           | 0      | 0    | ✗      | 0      | 0    | To Do            |
| Kamke 838 | ✓           | 0.155  | 31   | ✓      | 0.088  | 25   | To Do            |
| Kamke 839 | ✓           | 0.266  | 28   | ✓      | 0.052  | 19   | To Do            |
| Kamke 840 | ✓           | 0.286  | 30   | ✓      | 0.046  | 19   | To Do            |
| Kamke 841 | ✓           | 1.083  | 236  | ✓      | 0.217  | 97   | To Do            |
| Kamke 842 | ✓           | 0.21   | 186  | ✓      | 0.017  | 43   | To Do            |
| Kamke 843 | ✓           | 0.212  | 198  | ✓      | 0.02   | 43   | To Do            |
| Kamke 844 | ✓           | 13.021 | 386  | ✓      | 0.148  | 97   | To Do            |
| Kamke 845 | ✓           | 3.226  | 266  | ✓      | 0.184  | 44   | To Do            |
| Kamke 846 | ✓           | 0.892  | 365  | ✓      | 0.144  | 40   | To Do            |
| Kamke 847 | ✓           | 0.824  | 103  | ✓      | 0.215  | 34   | To Do            |
| Kamke 848 | ✓           | 0.205  | 157  | ✓      | 0.404  | 27   | To Do            |
| Kamke 849 | ✓           | 0.746  | 102  | ✓      | 0.211  | 33   | To Do            |
| Kamke 850 | ✓           | 0.228  | 1485 | ✓      | 0.783  | 32   | To Do            |
| Kamke 851 | ✓           | 0.343  | 145  | ✓      | 0.046  | 42   | To Do            |
| Kamke 852 | ✓           | 0.322  | 145  | ✓      | 0.047  | 42   | To Do            |
| Kamke 853 | ✓           | 0.156  | 76   | ✓      | 0.026  | 63   | To Do            |
| Kamke 854 | ✗           | 0      | 0    | ✓      | 0.158  | 51   | To Do            |
| Kamke 855 | ✗           | 0      | 0    | ✓      | 0.148  | 51   | To Do            |
| Kamke 856 | ✓           | 0.555  | 103  | ✓      | 0.207  | 65   | To Do            |
| Kamke 857 | ✓           | 0.607  | 107  | ✓      | 0.213  | 32   | To Do            |
| Kamke 858 | ✓           | 0.344  | 145  | ✓      | 0.046  | 42   | To Do            |
| Kamke 859 | ✓           | 0.813  | 105  | ✓      | 0.198  | 63   | To Do            |
| Kamke 860 | ✓           | 0.364  | 33   | ✓      | 1.57   | 29   | To Do            |
| Kamke 861 | ✓           | 1.364  | 158  | ✓      | 0.119  | 26   | To Do            |
| Kamke 862 | ✗           | 0      | 0    | ✓      | 0.159  | 27   | To Do            |
| Kamke 863 | ✓           | 0.176  | 30   | ✓      | 6.227  | 38   | To Do            |
| Kamke 864 | ✓           | 0.281  | 137  | ✓      | 0.06   | 162  | To Do            |
| Kamke 865 | ✓           | 0.311  | 87   | ✓      | 0.144  | 23   | To Do            |
| Kamke 866 | ✓           | 0.684  | 117  | ✓      | 0.236  | 37   | To Do            |
| Kamke 867 | ✓           | 0.201  | 77   | ✓      | 0.048  | 30   | To Do            |
| Kamke 868 | ✓           | 0.169  | 79   | ✓      | 0.044  | 28   | To Do            |
| Kamke 869 | ✓           | 0.032  | 42   | ✓      | 0.063  | 37   | To Do            |
| Kamke 870 | ✓           | 1.514  | 35   | ✓      | 0.682  | 30   | To Do            |
| Kamke 871 | ✓           | 0.23   | 22   | ✓      | 0.069  | 26   | To Do            |
| Kamke 872 | ✓           | 0.041  | 215  | ✓      | 0.046  | 49   | To Do            |
| Kamke 873 | ✓           | 0.512  | 53   | ✓      | 0.216  | 50   | To Do            |
| Kamke 874 | ✓           | 0.229  | 101  | ✓      | 0.037  | 40   | To Do            |
| Kamke 875 | ✓           | 0.24   | 497  | ✓      | 0.174  | 73   | To Do            |
| Kamke 876 | ✓           | 0.166  | 135  | ✓      | 0.036  | 41   | To Do            |
| Kamke 877 | ✓           | 0.186  | 49   | ✓      | 0.032  | 73   | To Do            |
| Kamke 878 | ✓           | 0.345  | 130  | ✓      | 10.437 | 75   | To Do            |
| Kamke 879 | ✓           | 0.243  | 239  | ✓      | 0.16   | 55   | To Do            |
| Kamke 880 | ✓           | 0.423  | 131  | ✓      | 0.049  | 41   | To Do            |
| Kamke 881 | ✓           | 0.17   | 75   | ✓      | 0.033  | 77   | To Do            |
| Kamke 882 | ✓           | 0.239  | 119  | ✓      | 0.047  | 41   | To Do            |
| Kamke 883 | ✓           | 1.135  | 164  | ✓      | 0.511  | 352  | To Do            |
| Kamke 884 | ✓           | 0.564  | 71   | ✓      | 0.296  | 107  | To Do            |
| Kamke 885 | ✗           | 0      | 0    | ✗      | 0      | 0    | To Do            |
| Kamke 886 | ✓           | 0.172  | 82   | ✓      | 0.023  | 42   | To Do            |
| Kamke 887 | ✓           | 0.225  | 106  | ✓      | 0.034  | 72   | To Do            |

Continued on next page

Table 3 – continued from previous page

| #         | Mathematica |        |      | Maple  |       |      | note on solution |
|-----------|-------------|--------|------|--------|-------|------|------------------|
|           | solved      | cpu    | leaf | solved | cpu   | leaf |                  |
| Kamke 888 | ✓           | 0.17   | 78   | ✓      | 0.043 | 79   | To Do            |
| Kamke 889 | ✗           | 0      | 0    | ✓      | 0.825 | 49   | To Do            |
| Kamke 890 | ✓           | 0.181  | 103  | ✓      | 0.519 | 34   | To Do            |
| Kamke 891 | ✓           | 0.175  | 135  | ✓      | 0.043 | 56   | To Do            |
| Kamke 892 | ✓           | 2.248  | 1283 | ✓      | 0.376 | 40   | To Do            |
| Kamke 893 | ✓           | 0.169  | 80   | ✓      | 0.019 | 41   | To Do            |
| Kamke 894 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 895 | ✓           | 0.222  | 81   | ✓      | 0.044 | 79   | To Do            |
| Kamke 896 | ✓           | 0.295  | 106  | ✓      | 0.39  | 63   | To Do            |
| Kamke 897 | ✓           | 0.212  | 79   | ✓      | 0.056 | 87   | To Do            |
| Kamke 898 | ✓           | 0.176  | 106  | ✓      | 0.033 | 87   | To Do            |
| Kamke 899 | ✓           | 0.203  | 106  | ✓      | 0.023 | 47   | To Do            |
| Kamke 900 | ✓           | 0.169  | 381  | ✓      | 0.049 | 48   | To Do            |
| Kamke 901 | ✓           | 0.514  | 33   | ✓      | 0.414 | 30   | To Do            |
| Kamke 902 | ✓           | 0.224  | 295  | ✓      | 0.18  | 183  | To Do            |
| Kamke 903 | ✓           | 0.117  | 19   | ✓      | 0.053 | 48   | To Do            |
| Kamke 904 | ✓           | 0.103  | 23   | ✓      | 0.035 | 64   | To Do            |
| Kamke 905 | ✓           | 0.239  | 85   | ✓      | 0.03  | 46   | To Do            |
| Kamke 906 | ✓           | 0.123  | 326  | ✓      | 0.244 | 37   | To Do            |
| Kamke 907 | ✓           | 0.272  | 22   | ✓      | 0.112 | 20   | To Do            |
| Kamke 908 | ✓           | 1.098  | 1269 | ✓      | 0.282 | 1742 | To Do            |
| Kamke 909 | ✗           | 0      | 0    | ✓      | 0.407 | 84   | To Do            |
| Kamke 910 | ✓           | 0.227  | 98   | ✓      | 0.022 | 42   | To Do            |
| Kamke 911 | ✓           | 0.732  | 106  | ✓      | 0.361 | 30   | To Do            |
| Kamke 912 | ✓           | 1.241  | 201  | ✗      | 0     | 0    | To Do            |
| Kamke 913 | ✓           | 0.78   | 716  | ✓      | 0.118 | 43   | To Do            |
| Kamke 914 | ✓           | 1.179  | 401  | ✓      | 3.299 | 71   | To Do            |
| Kamke 915 | ✓           | 0.865  | 724  | ✓      | 0.044 | 43   | To Do            |
| Kamke 916 | ✗           | 0      | 0    | ✓      | 0.293 | 73   | To Do            |
| Kamke 917 | ✗           | 0      | 0    | ✓      | 0.155 | 38   | To Do            |
| Kamke 918 | ✓           | 0.452  | 720  | ✓      | 0.961 | 41   | To Do            |
| Kamke 919 | ✗           | 0      | 0    | ✓      | 0.164 | 61   | To Do            |
| Kamke 920 | ✓           | 0.454  | 301  | ✗      | 0     | 0    | To Do            |
| Kamke 921 | ✓           | 0.232  | 92   | ✓      | 0.1   | 30   | To Do            |
| Kamke 922 | ✓           | 1.24   | 882  | ✓      | 0.102 | 47   | To Do            |
| Kamke 923 | ✓           | 1.394  | 432  | ✓      | 0.242 | 36   | To Do            |
| Kamke 924 | ✓           | 0.236  | 80   | ✓      | 0.107 | 46   | To Do            |
| Kamke 925 | ✓           | 2.039  | 228  | ✓      | 0.18  | 38   | To Do            |
| Kamke 926 | ✓           | 0.185  | 128  | ✓      | 0.044 | 67   | To Do            |
| Kamke 927 | ✓           | 0.566  | 112  | ✓      | 0.083 | 68   | To Do            |
| Kamke 928 | ✓           | 1.23   | 23   | ✓      | 0.273 | 20   | To Do            |
| Kamke 929 | ✓           | 0.807  | 683  | ✓      | 0.03  | 42   | To Do            |
| Kamke 930 | ✓           | 1.414  | 39   | ✓      | 0.419 | 36   | To Do            |
| Kamke 931 | ✓           | 0.17   | 80   | ✓      | 0.032 | 73   | To Do            |
| Kamke 932 | ✓           | 4.539  | 4323 | ✓      | 0.101 | 54   | To Do            |
| Kamke 933 | ✓           | 0.236  | 99   | ✓      | 0.026 | 39   | To Do            |
| Kamke 934 | ✓           | 0.269  | 102  | ✓      | 0.051 | 39   | To Do            |
| Kamke 935 | ✓           | 27.544 | 248  | ✓      | 0.153 | 55   | To Do            |
| Kamke 936 | ✓           | 0.247  | 99   | ✓      | 0.053 | 39   | To Do            |
| Kamke 937 | ✓           | 0.271  | 124  | ✓      | 0.043 | 79   | To Do            |
| Kamke 938 | ✓           | 0.239  | 108  | ✓      | 0.026 | 39   | To Do            |
| Kamke 939 | ✓           | 0.529  | 136  | ✓      | 0.105 | 70   | To Do            |
| Kamke 940 | ✓           | 0.206  | 80   | ✓      | 0.035 | 63   | To Do            |
| Kamke 941 | ✓           | 0.453  | 53   | ✓      | 0.04  | 35   | To Do            |
| Kamke 942 | ✓           | 4.315  | 349  | ✓      | 0.427 | 43   | To Do            |
| Kamke 943 | ✓           | 0.505  | 53   | ✓      | 0.039 | 40   | To Do            |
| Kamke 944 | ✓           | 1.65   | 233  | ✓      | 0.058 | 47   | To Do            |
| Kamke 945 | ✓           | 1.289  | 213  | ✓      | 0.046 | 41   | To Do            |
| Kamke 946 | ✓           | 0.606  | 150  | ✓      | 0.109 | 85   | To Do            |
| Kamke 947 | ✓           | 0.412  | 30   | ✓      | 0.193 | 44   | To Do            |

Continued on next page

Table 3 – continued from previous page

| #          | Mathematica |       |      | Maple  |       |      | note on solution                                                                          |
|------------|-------------|-------|------|--------|-------|------|-------------------------------------------------------------------------------------------|
|            | solved      | cpu   | leaf | solved | cpu   | leaf |                                                                                           |
| Kamke 948  | ✓           | 0.603 | 39   | ✓      | 0.185 | 68   | To Do                                                                                     |
| Kamke 949  | ✓           | 0.203 | 76   | ✓      | 0.037 | 81   | To Do                                                                                     |
| Kamke 950  | ✓           | 0.496 | 141  | ✓      | 0.066 | 42   | To Do                                                                                     |
| Kamke 951  | ✓           | 0.406 | 140  | ✓      | 0.054 | 41   | To Do                                                                                     |
| Kamke 952  | ✓           | 0.335 | 341  | ✓      | 0.266 | 62   | To Do                                                                                     |
| Kamke 953  | ✗           | 0     | 0    | ✓      | 0.273 | 145  | To Do                                                                                     |
| Kamke 954  | ✓           | 0.334 | 115  | ✓      | 0.062 | 53   | To Do                                                                                     |
| Kamke 955  | ✓           | 0.316 | 112  | ✓      | 0.075 | 101  | To Do                                                                                     |
| Kamke 956  | ✓           | 1.147 | 28   | ✓      | 0.092 | 79   | To Do                                                                                     |
| Kamke 957  | ✓           | 1.12  | 28   | ✓      | 0.049 | 79   | To Do                                                                                     |
| Kamke 958  | ✓           | 0.433 | 82   | ✓      | 0.033 | 40   | To Do                                                                                     |
| Kamke 959  | ✓           | 0.256 | 20   | ✓      | 0.059 | 15   | To Do                                                                                     |
| Kamke 960  | ✓           | 0.16  | 14   | ✓      | 0.04  | 11   | To Do                                                                                     |
| Kamke 961  | ✓           | 5.119 | 813  | ✓      | 0.264 | 45   | To Do                                                                                     |
| Kamke 962  | ✓           | 4.464 | 1191 | ✓      | 1.214 | 79   | To Do                                                                                     |
| Kamke 963  | ✓           | 0.523 | 108  | ✓      | 0.15  | 39   | To Do                                                                                     |
| Kamke 964  | ✓           | 4.06  | 264  | ✓      | 2.166 | 80   | To Do                                                                                     |
| Kamke 965  | ✓           | 0.287 | 29   | ✓      | 0.047 | 26   | To Do                                                                                     |
| Kamke 966  | ✓           | 0.739 | 292  | ✓      | 0.523 | 50   | To Do                                                                                     |
| Kamke 967  | ✓           | 0.51  | 151  | ✓      | 0.054 | 91   | To Do                                                                                     |
| Kamke 968  | ✓           | 0.355 | 30   | ✓      | 0.062 | 22   | To Do                                                                                     |
| Kamke 969  | ✓           | 0.281 | 19   | ✓      | 0.065 | 15   | To Do                                                                                     |
| Kamke 970  | ✓           | 0.588 | 66   | ✓      | 0.605 | 181  | To Do                                                                                     |
| Kamke 971  | ✓           | 0.199 | 157  | ✓      | 0.221 | 86   | To Do                                                                                     |
| Kamke 972  | ✓           | 0.023 | 32   | ✓      | 0.069 | 27   | To Do                                                                                     |
| Kamke 973  | ✓           | 0.338 | 146  | ✓      | 0.297 | 136  | To Do                                                                                     |
| Kamke 974  | ✓           | 0.083 | 39   | ✓      | 0.029 | 57   | To Do                                                                                     |
| Kamke 975  | ✓           | 0.081 | 47   | ✓      | 0.028 | 59   | To Do                                                                                     |
| Kamke 976  | ✓           | 0.163 | 101  | ✓      | 0.177 | 57   | To Do                                                                                     |
| Kamke 977  | ✓           | 0.373 | 139  | ✓      | 0.155 | 122  | To Do                                                                                     |
| Kamke 978  | ✓           | 0.139 | 60   | ✓      | 0.138 | 71   | To Do                                                                                     |
| Kamke 979  | ✓           | 0.099 | 37   | ✓      | 0.033 | 57   | To Do                                                                                     |
| Kamke 980  | ✓           | 0.114 | 43   | ✓      | 0.013 | 35   | To Do                                                                                     |
| Kamke 981  | ✓           | 0.148 | 49   | ✓      | 0.017 | 41   | To Do                                                                                     |
| Kamke 982  | ✓           | 0.348 | 132  | ✓      | 0.327 | 145  | To Do                                                                                     |
| Kamke 983  | ✓           | 0.486 | 238  | ✓      | 0.34  | 188  | To Do                                                                                     |
| Kamke 984  | ✓           | 5.841 | 428  | ✓      | 0.22  | 40   | To Do                                                                                     |
| Kamke 985  | ✓           | 0.329 | 103  | ✓      | 0.027 | 43   | To Do                                                                                     |
| Kamke 986  | ✓           | 0.127 | 44   | ✓      | 0.017 | 36   | To Do                                                                                     |
| Kamke 987  | ✓           | 0.152 | 41   | ✓      | 0.039 | 22   | To Do                                                                                     |
| Kamke 988  | ✓           | 0.204 | 107  | ✓      | 0.031 | 29   | To Do                                                                                     |
| Kamke 989  | ✓           | 0.163 | 56   | ✓      | 0.033 | 29   | To Do                                                                                     |
| Kamke 990  | ✓           | 0.203 | 58   | ✓      | 0.387 | 44   | To Do                                                                                     |
| Kamke 991  | ✓           | 0.193 | 104  | ✓      | 0.027 | 29   | To Do                                                                                     |
| Kamke 992  | ✓           | 0.116 | 43   | ✓      | 0.03  | 25   | To Do                                                                                     |
| Kamke 993  | ✓           | 1.403 | 73   | ✓      | 0.02  | 35   | To Do                                                                                     |
| Kamke 994  | ✓           | 0.206 | 198  | ✓      | 0.017 | 43   | To Do                                                                                     |
| Kamke 995  | ✓           | 0.164 | 17   | ✓      | 0.114 | 14   | To Do                                                                                     |
| Kamke 996  | ✗           | 0     | 0    | ✓      | 0.047 | 15   | To Do                                                                                     |
| Kamke 997  | ✓           | 0.099 | 18   | ✓      | 0.043 | 16   | To Do                                                                                     |
| Kamke 998  | ✓           | 0.462 | 27   | ✓      | 0.273 | 27   | To Do                                                                                     |
| Kamke 999  | ✓           | 0.183 | 24   | ✓      | 0.039 | 36   | To Do                                                                                     |
| Kamke 1000 | ✓           | 0.958 | 351  | ✓      | 0.107 | 19   | To Do                                                                                     |
| Kamke 1001 | ✓           | 0.003 | 12   | ✓      | 0.003 | 9    | Linear second order, homogeneous, constant coefficient. First problem. Direct integration |
| Kamke 1002 | ✓           | 0.003 | 16   | ✓      | 0.005 | 13   | Linear second order, homogeneous, constant coefficient. Direct method                     |

Continued on next page

Table 3 – continued from previous page

| #          | Mathematica |       |      | Maple  |       |      | note on solution                                                                                                              |
|------------|-------------|-------|------|--------|-------|------|-------------------------------------------------------------------------------------------------------------------------------|
|            | solved      | cpu   | leaf | solved | cpu   | leaf |                                                                                                                               |
| Kamke 1003 | ✓           | 0.098 | 45   | ✓      | 0.039 | 26   | Linear second order, Non-homogeneous, constant coefficient. variation of parameters                                           |
| Kamke 1004 | ✓           | 0.089 | 47   | ✓      | 0.033 | 27   | Linear second order, Non-homogeneous, constant coefficient. variation of parameters                                           |
| Kamke 1005 | ✓           | 0.414 | 1163 | ✓      | 0.069 | 82   | Linear second order, Non-homogeneous, constant coefficient. variation of parameters                                           |
| Kamke 1006 | ✓           | 0.003 | 20   | ✓      | 0.004 | 15   | Linear second order, homogeneous, constant coefficient                                                                        |
| Kamke 1007 | ✓           | 0.086 | 135  | ✓      | 0.014 | 26   | Linear second order, Non-homogeneous, constant coefficient. variation of parameters                                           |
| Kamke 1008 | ✓           | 0.035 | 48   | ✓      | 0.058 | 41   | To Do                                                                                                                         |
| Kamke 1009 | ✓           | 0.004 | 28   | ✓      | 0.012 | 21   | To Do                                                                                                                         |
| Kamke 1010 | ✓           | 0.005 | 46   | ✓      | 0.03  | 31   | Linear second order, homogeneous, variable coefficient. Airy ODE with plus sign, series solution                              |
| Kamke 1011 | ✓           | 0.005 | 33   | ✓      | 0.02  | 17   | Linear second order, homogeneous, variable coefficient. power series solution                                                 |
| Kamke 1012 | ✓           | 0.006 | 47   | ✓      | 0.072 | 29   | To Do                                                                                                                         |
| Kamke 1013 | ✓           | 0.013 | 43   | ✓      | 0.027 | 22   | To Do                                                                                                                         |
| Kamke 1014 | ✓           | 0.019 | 170  | ✓      | 0.078 | 63   | To Do                                                                                                                         |
| Kamke 1015 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do                                                                                                                         |
| Kamke 1016 | ✓           | 0.084 | 312  | ✓      | 0.15  | 91   | To Do                                                                                                                         |
| Kamke 1017 | ✓           | 0.019 | 46   | ✓      | 0.03  | 17   | To Do                                                                                                                         |
| Kamke 1018 | ✓           | 0.015 | 55   | ✓      | 0.039 | 39   | To Do                                                                                                                         |
| Kamke 1019 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do                                                                                                                         |
| Kamke 1020 | ✓           | 0.497 | 180  | ✓      | 0.141 | 58   | To Do                                                                                                                         |
| Kamke 1021 | ✓           | 0.024 | 44   | ✓      | 0.165 | 39   | To Do                                                                                                                         |
| Kamke 1022 | ✓           | 0.019 | 28   | ✓      | 0.158 | 21   | To Do                                                                                                                         |
| Kamke 1023 | ✓           | 0.01  | 44   | ✓      | 0.16  | 29   | To Do                                                                                                                         |
| Kamke 1024 | ✓           | 0.11  | 84   | ✓      | 0.095 | 30   | To Do                                                                                                                         |
| Kamke 1025 | ✓           | 0.656 | 615  | ✓      | 0.135 | 102  | To Do                                                                                                                         |
| Kamke 1026 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do                                                                                                                         |
| Kamke 1027 | ✗           | 0     | 0    | ✓      | 0.263 | 69   | To Do                                                                                                                         |
| Kamke 1028 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do                                                                                                                         |
| Kamke 1029 | ✗           | 0     | 0    | ✓      | 0.092 | 22   | To Do                                                                                                                         |
| Kamke 1030 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do                                                                                                                         |
| Kamke 1031 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do                                                                                                                         |
| Kamke 1032 | ✗           | 0     | 0    | ✓      | 0.082 | 48   | To Do                                                                                                                         |
| Kamke 1033 | ✓           | 0.012 | 37   | ✓      | 0.01  | 27   | Linear second order, homogeneous, variable coefficient. smart substitution turns it to second order with constant coefficient |
| Kamke 1034 | ✓           | 0.008 | 20   | ✓      | 0.009 | 15   | Linear second order, homogeneous, variable coefficient. smart substitution turns it to second order with constant coefficient |

Continued on next page

Table 3 – continued from previous page

| #          | Mathematica |       |      | Maple  |       |      | note on solution                                                              |
|------------|-------------|-------|------|--------|-------|------|-------------------------------------------------------------------------------|
|            | solved      | cpu   | leaf | solved | cpu   | leaf |                                                                               |
| Kamke 1035 | ✓           | 0.004 | 58   | ✓      | 0.01  | 41   | To Do                                                                         |
| Kamke 1036 | ✓           | 0.126 | 209  | ✓      | 0.071 | 124  | To Do                                                                         |
| Kamke 1037 | ✓           | 0.02  | 101  | ✓      | 0.069 | 64   | To Do                                                                         |
| Kamke 1038 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do                                                                         |
| Kamke 1039 | ✓           | 0.009 | 47   | ✓      | 0.007 | 25   | To Do                                                                         |
| Kamke 1040 | ✓           | 0.03  | 53   | ✓      | 0.029 | 33   | To Do                                                                         |
| Kamke 1041 | ✓           | 0.007 | 55   | ✓      | 0.055 | 41   | To Do                                                                         |
| Kamke 1042 | ✓           | 0.006 | 61   | ✓      | 0.052 | 41   | To Do                                                                         |
| Kamke 1043 | ✓           | 0.099 | 69   | ✓      | 0.164 | 39   | Linear second order, homogeneous, variable coefficient. power series solution |
| Kamke 1044 | ✓           | 0.006 | 39   | ✓      | 0.055 | 35   | To Do                                                                         |
| Kamke 1045 | ✓           | 0.028 | 39   | ✓      | 0.005 | 21   | To Do                                                                         |
| Kamke 1046 | ✓           | 0.006 | 31   | ✓      | 0.054 | 31   | To Do                                                                         |
| Kamke 1047 | ✓           | 0.01  | 27   | ✓      | 0.02  | 16   | Linear second order, homogeneous, variable coefficient. power series solution |
| Kamke 1048 | ✓           | 0.008 | 45   | ✓      | 0.052 | 37   | To Do                                                                         |
| Kamke 1049 | ✓           | 0.049 | 109  | ✓      | 0.128 | 66   | To Do                                                                         |
| Kamke 1050 | ✓           | 0.009 | 23   | ✓      | 0.017 | 14   | To Do                                                                         |
| Kamke 1051 | ✓           | 0.017 | 44   | ✓      | 0.023 | 27   | To Do                                                                         |
| Kamke 1052 | ✓           | 0.015 | 78   | ✓      | 0.058 | 58   | To Do                                                                         |
| Kamke 1053 | ✓           | 0.023 | 57   | ✓      | 0.032 | 35   | To Do                                                                         |
| Kamke 1054 | ✓           | 0.033 | 172  | ✓      | 0.034 | 98   | To Do                                                                         |
| Kamke 1055 | ✓           | 0.139 | 421  | ✓      | 0.136 | 262  | To Do                                                                         |
| Kamke 1056 | ✓           | 0.025 | 41   | ✓      | 0.069 | 47   | To Do                                                                         |
| Kamke 1057 | ✓           | 0.034 | 56   | ✓      | 0.118 | 50   | To Do                                                                         |
| Kamke 1058 | ✓           | 0.033 | 56   | ✓      | 0.088 | 29   | To Do                                                                         |
| Kamke 1059 | ✓           | 0.027 | 39   | ✓      | 0.052 | 56   | To Do                                                                         |
| Kamke 1060 | ✓           | 0.026 | 83   | ✓      | 0.122 | 81   | To Do                                                                         |
| Kamke 1061 | ✓           | 0.058 | 70   | ✓      | 0.066 | 28   | To Do                                                                         |
| Kamke 1062 | ✓           | 0.019 | 35   | ✓      | 0.02  | 19   | To Do                                                                         |
| Kamke 1063 | ✓           | 0.04  | 28   | ✓      | 0.224 | 34   | To Do                                                                         |
| Kamke 1064 | ✓           | 0.477 | 1400 | ✓      | 0.22  | 125  | To Do                                                                         |
| Kamke 1065 | ✓           | 0.122 | 114  | ✓      | 0.158 | 60   | To Do                                                                         |
| Kamke 1066 | ✓           | 0.023 | 18   | ✓      | 0.049 | 15   | To Do                                                                         |
| Kamke 1067 | ✓           | 0.023 | 21   | ✓      | 0.033 | 17   | To Do                                                                         |
| Kamke 1068 | ✓           | 0.105 | 20   | ✓      | 0.153 | 45   | To Do                                                                         |
| Kamke 1069 | ✓           | 0.025 | 19   | ✓      | 0.064 | 15   | To Do                                                                         |
| Kamke 1070 | ✓           | 0.235 | 143  | ✓      | 0.118 | 60   | To Do                                                                         |
| Kamke 1071 | ✓           | 0.06  | 44   | ✓      | 0.046 | 24   | To Do                                                                         |
| Kamke 1072 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do                                                                         |
| Kamke 1073 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do                                                                         |
| Kamke 1074 | ✗           | 0     | 0    | ✓      | 0.017 | 21   | To Do                                                                         |
| Kamke 1075 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do                                                                         |
| Kamke 1076 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do                                                                         |
| Kamke 1077 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do                                                                         |
| Kamke 1078 | ✓           | 0.031 | 76   | ✓      | 0.023 | 33   | To Do                                                                         |
| Kamke 1079 | ✓           | 0.192 | 315  | ✓      | 0.015 | 37   | To Do                                                                         |
| Kamke 1080 | ✗           | 0     | 0    | ✓      | 0.24  | 74   | To Do                                                                         |
| Kamke 1081 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do                                                                         |
| Kamke 1082 | ✗           | 0     | 0    | ✓      | 0.103 | 74   | To Do                                                                         |
| Kamke 1083 | ✗           | 0     | 0    | ✓      | 0.072 | 31   | To Do                                                                         |
| Kamke 1084 | ✗           | 0     | 0    | ✓      | 0.059 | 20   | To Do                                                                         |
| Kamke 1085 | ✗           | 0     | 0    | ✓      | 0.058 | 24   | To Do                                                                         |
| Kamke 1086 | ✓           | 0.004 | 42   | ✓      | 0.018 | 29   | To Do                                                                         |
| Kamke 1087 | ✓           | 0.006 | 36   | ✓      | 0.062 | 33   | To Do                                                                         |
| Kamke 1088 | ✓           | 0.068 | 180  | ✓      | 0.098 | 31   | To Do                                                                         |

Continued on next page

Table 3 – continued from previous page

| #          | Mathematica |       |      | Maple  |       |      | note on solution                                                                                                                                                 |
|------------|-------------|-------|------|--------|-------|------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|            | solved      | cpu   | leaf | solved | cpu   | leaf |                                                                                                                                                                  |
| Kamke 1089 | ✓           | 0.031 | 99   | ✓      | 0.036 | 58   | To Do                                                                                                                                                            |
| Kamke 1090 | ✓           | 0.025 | 50   | ✓      | 0.033 | 40   | To Do                                                                                                                                                            |
| Kamke 1091 | ✓           | 0.018 | 41   | ✓      | 0.032 | 35   | To Do                                                                                                                                                            |
| Kamke 1092 | ✓           | 0.066 | 72   | ✓      | 0.05  | 29   | To Do                                                                                                                                                            |
| Kamke 1093 | ✓           | 0.007 | 13   | ✓      | 0.004 | 10   | To Do                                                                                                                                                            |
| Kamke 1094 | ✓           | 0.016 | 41   | ✓      | 0.007 | 29   | To Do                                                                                                                                                            |
| Kamke 1095 | ✓           | 0.007 | 30   | ✓      | 0.022 | 23   | To Do                                                                                                                                                            |
| Kamke 1096 | ✓           | 0.009 | 61   | ✓      | 0.055 | 39   | To Do                                                                                                                                                            |
| Kamke 1097 | ✓           | 0.02  | 46   | ✓      | 0.007 | 31   | To Do                                                                                                                                                            |
| Kamke 1098 | ✓           | 0.007 | 41   | ✓      | 0.007 | 27   | To Do                                                                                                                                                            |
| Kamke 1099 | ✗           | 0     | 0    | ✓      | 0.038 | 25   | To Do                                                                                                                                                            |
| Kamke 1100 | ✓           | 0.014 | 44   | ✓      | 0.033 | 23   | Linear second order, non-homogeneous, variable coefficient. Singularity, Frobenius series solution and also using smart transformation. Variation of parameters. |
| Kamke 1101 | ✓           | 0.018 | 52   | ✓      | 0.027 | 29   | Linear second order, homogeneous, variable coefficient. Singularity, Frobenius series solution and also using smart transformation                               |
| Kamke 1102 | ✓           | 0.005 | 42   | ✓      | 0.028 | 33   | Linear second order, homogeneous, variable coefficient. Use smart transformation to convert to Emden-Fowler then use Power series solution                       |
| Kamke 1103 | ✓           | 0.02  | 64   | ✓      | 0.008 | 33   | To Do                                                                                                                                                            |
| Kamke 1104 | ✓           | 0.026 | 104  | ✓      | 0.01  | 41   | To Do                                                                                                                                                            |
| Kamke 1105 | ✓           | 0.015 | 64   | ✓      | 0.027 | 39   | To Do                                                                                                                                                            |
| Kamke 1106 | ✓           | 0.036 | 441  | ✓      | 0.072 | 71   | To Do                                                                                                                                                            |
| Kamke 1107 | ✓           | 0.02  | 40   | ✓      | 0.054 | 30   | To Do                                                                                                                                                            |
| Kamke 1108 | ✓           | 0.025 | 37   | ✓      | 0.056 | 26   | To Do                                                                                                                                                            |
| Kamke 1109 | ✓           | 0.119 | 45   | ✓      | 0.02  | 33   | To Do                                                                                                                                                            |
| Kamke 1110 | ✓           | 0.032 | 36   | ✓      | 0.039 | 23   | To Do                                                                                                                                                            |
| Kamke 1111 | ✓           | 0.016 | 20   | ✓      | 0.016 | 13   | Linear second order, homogeneous, variable coefficient. Solved using Laplace transform instead of series method                                                  |
| Kamke 1112 | ✓           | 0.021 | 30   | ✓      | 0.026 | 22   | To Do                                                                                                                                                            |
| Kamke 1113 | ✓           | 0.015 | 24   | ✓      | 0.05  | 17   | To Do                                                                                                                                                            |
| Kamke 1114 | ✓           | 0.035 | 39   | ✓      | 0.036 | 34   | To Do                                                                                                                                                            |
| Kamke 1115 | ✓           | 0.047 | 76   | ✓      | 0.736 | 47   | To Do                                                                                                                                                            |
| Kamke 1116 | ✓           | 0.041 | 43   | ✓      | 0.061 | 31   | To Do                                                                                                                                                            |
| Kamke 1117 | ✓           | 0.062 | 107  | ✓      | 0.082 | 82   | To Do                                                                                                                                                            |
| Kamke 1118 | ✓           | 0.061 | 51   | ✓      | 0.068 | 39   | To Do                                                                                                                                                            |
| Kamke 1119 | ✓           | 0.113 | 77   | ✓      | 0.03  | 20   | To Do                                                                                                                                                            |
| Kamke 1120 | ✓           | 0.041 | 168  | ✓      | 0.151 | 109  | To Do                                                                                                                                                            |
| Kamke 1121 | ✓           | 0.177 | 41   | ✓      | 0.019 | 23   | To Do                                                                                                                                                            |
| Kamke 1122 | ✓           | 0.18  | 57   | ✓      | 0.112 | 28   | To Do                                                                                                                                                            |
| Kamke 1123 | ✓           | 0.009 | 91   | ✓      | 0.039 | 45   | To Do                                                                                                                                                            |
| Kamke 1124 | ✓           | 0.053 | 65   | ✓      | 0.059 | 29   | To Do                                                                                                                                                            |
| Kamke 1125 | ✓           | 0.102 | 84   | ✓      | 0.033 | 36   | To Do                                                                                                                                                            |
| Kamke 1126 | ✗           | 0     | 0    | ✓      | 0.027 | 19   | To Do                                                                                                                                                            |
| Kamke 1127 | ✓           | 0.027 | 36   | ✓      | 0.013 | 21   | To Do                                                                                                                                                            |
| Kamke 1128 | ✓           | 0.018 | 40   | ✓      | 0.181 | 32   | To Do                                                                                                                                                            |
| Kamke 1129 | ✓           | 0.055 | 42   | ✓      | 0.024 | 30   | To Do                                                                                                                                                            |
| Kamke 1130 | ✓           | 0.008 | 46   | ✓      | 0.013 | 31   | To Do                                                                                                                                                            |

Continued on next page



Table 3 – continued from previous page

| #          | Mathematica |       |      | Maple  |       |      | note on solution |
|------------|-------------|-------|------|--------|-------|------|------------------|
|            | solved      | cpu   | leaf | solved | cpu   | leaf |                  |
| Kamke 1131 | ✓           | 0.008 | 58   | ✓      | 0.058 | 33   | To Do            |
| Kamke 1132 | ✓           | 0.008 | 48   | ✓      | 0.059 | 29   | To Do            |
| Kamke 1133 | ✓           | 0.047 | 52   | ✓      | 0.075 | 37   | To Do            |
| Kamke 1134 | ✓           | 0.069 | 78   | ✓      | 0.033 | 21   | To Do            |
| Kamke 1135 | ✓           | 0.007 | 27   | ✓      | 0.007 | 17   | To Do            |
| Kamke 1136 | ✓           | 0.018 | 30   | ✓      | 0.022 | 16   | To Do            |
| Kamke 1137 | ✓           | 0.069 | 74   | ✓      | 0.036 | 25   | To Do            |
| Kamke 1138 | ✓           | 0.021 | 38   | ✓      | 0.054 | 26   | To Do            |
| Kamke 1139 | ✓           | 0.01  | 74   | ✓      | 0.059 | 37   | To Do            |
| Kamke 1140 | ✓           | 0.034 | 190  | ✓      | 0.017 | 66   | To Do            |
| Kamke 1141 | ✓           | 0.091 | 79   | ✓      | 0.034 | 55   | To Do            |
| Kamke 1142 | ✓           | 0.033 | 108  | ✓      | 0.078 | 53   | To Do            |
| Kamke 1143 | ✓           | 0.03  | 93   | ✓      | 0.075 | 57   | To Do            |
| Kamke 1144 | ✓           | 0.027 | 88   | ✓      | 0.062 | 60   | To Do            |
| Kamke 1145 | ✓           | 0.259 | 386  | ✓      | 0.122 | 248  | To Do            |
| Kamke 1146 | ✓           | 0.003 | 18   | ✓      | 0.005 | 15   | To Do            |
| Kamke 1147 | ✓           | 0.003 | 18   | ✓      | 0.006 | 15   | To Do            |
| Kamke 1148 | ✓           | 0.007 | 77   | ✓      | 0.009 | 35   | To Do            |
| Kamke 1149 | ✓           | 0.046 | 212  | ✓      | 0.012 | 45   | To Do            |
| Kamke 1150 | ✓           | 0.007 | 53   | ✓      | 0.05  | 27   | To Do            |
| Kamke 1151 | ✓           | 0.012 | 129  | ✓      | 0.06  | 43   | To Do            |
| Kamke 1152 | ✓           | 0.013 | 114  | ✓      | 0.159 | 53   | To Do            |
| Kamke 1153 | ✓           | 0.022 | 56   | ✓      | 0.02  | 31   | To Do            |
| Kamke 1154 | ✓           | 0.014 | 88   | ✓      | 0.094 | 57   | To Do            |
| Kamke 1155 | ✓           | 0.035 | 225  | ✓      | 0.036 | 67   | To Do            |
| Kamke 1156 | ✓           | 0.09  | 33   | ✓      | 0.093 | 71   | To Do            |
| Kamke 1157 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1158 | ✓           | 0.116 | 43   | ✓      | 0.174 | 178  | To Do            |
| Kamke 1159 | ✓           | 0.006 | 24   | ✓      | 0.009 | 19   | To Do            |
| Kamke 1160 | ✓           | 0.007 | 30   | ✓      | 0.01  | 23   | To Do            |
| Kamke 1161 | ✓           | 0.034 | 78   | ✓      | 0.009 | 31   | To Do            |
| Kamke 1162 | ✓           | 0.041 | 18   | ✓      | 0.007 | 15   | To Do            |
| Kamke 1163 | ✓           | 0.107 | 72   | ✓      | 0.024 | 49   | To Do            |
| Kamke 1164 | ✓           | 0.015 | 30   | ✓      | 0.02  | 23   | To Do            |
| Kamke 1165 | ✓           | 0.054 | 26   | ✓      | 0.01  | 19   | To Do            |
| Kamke 1166 | ✓           | 0.01  | 23   | ✓      | 0.01  | 21   | To Do            |
| Kamke 1167 | ✓           | 0.055 | 326  | ✓      | 0.02  | 63   | To Do            |
| Kamke 1168 | ✓           | 0.007 | 15   | ✓      | 0.007 | 11   | To Do            |
| Kamke 1169 | ✓           | 0.049 | 236  | ✓      | 0.02  | 49   | To Do            |
| Kamke 1170 | ✓           | 0.016 | 58   | ✓      | 0.026 | 43   | To Do            |
| Kamke 1171 | ✓           | 0.036 | 142  | ✓      | 0.096 | 49   | To Do            |
| Kamke 1172 | ✓           | 0.047 | 158  | ✓      | 0.023 | 47   | To Do            |
| Kamke 1173 | ✓           | 0.071 | 74   | ✓      | 0.057 | 37   | To Do            |
| Kamke 1174 | ✓           | 0.007 | 33   | ✓      | 0.017 | 25   | To Do            |
| Kamke 1175 | ✓           | 0.198 | 38   | ✓      | 0.036 | 29   | To Do            |
| Kamke 1176 | ✓           | 0.014 | 33   | ✓      | 0.02  | 15   | To Do            |
| Kamke 1177 | ✓           | 0.751 | 141  | ✓      | 0.049 | 34   | To Do            |
| Kamke 1178 | ✓           | 0.037 | 74   | ✓      | 0.016 | 23   | To Do            |
| Kamke 1179 | ✓           | 0.017 | 38   | ✓      | 0.023 | 19   | To Do            |
| Kamke 1180 | ✓           | 0.059 | 75   | ✓      | 0.033 | 49   | To Do            |
| Kamke 1181 | ✓           | 0.031 | 37   | ✓      | 0.014 | 25   | To Do            |
| Kamke 1182 | ✓           | 0.011 | 24   | ✓      | 0.009 | 20   | To Do            |
| Kamke 1183 | ✓           | 0.007 | 27   | ✓      | 0.01  | 22   | To Do            |
| Kamke 1184 | ✓           | 0.007 | 38   | ✓      | 0.019 | 25   | To Do            |
| Kamke 1185 | ✓           | 0.026 | 67   | ✓      | 0.019 | 33   | To Do            |
| Kamke 1186 | ✓           | 0.014 | 42   | ✓      | 0.019 | 36   | To Do            |
| Kamke 1187 | ✓           | 0.008 | 99   | ✓      | 0.01  | 53   | To Do            |
| Kamke 1188 | ✓           | 0.108 | 266  | ✓      | 0.122 | 114  | To Do            |
| Kamke 1189 | ✓           | 0.049 | 445  | ✓      | 0.026 | 79   | To Do            |
| Kamke 1190 | ✓           | 0.022 | 122  | ✓      | 0.073 | 38   | To Do            |

Continued on next page

Table 3 – continued from previous page

| #          | Mathematica |        |      | Maple  |       |      | note on solution |
|------------|-------------|--------|------|--------|-------|------|------------------|
|            | solved      | cpu    | leaf | solved | cpu   | leaf |                  |
| Kamke 1191 | ✓           | 0.007  | 110  | ✓      | 0.011 | 23   | To Do            |
| Kamke 1192 | ✓           | 0.062  | 40   | ✓      | 0.103 | 51   | To Do            |
| Kamke 1193 | ✓           | 0.194  | 44   | ✓      | 0.026 | 38   | To Do            |
| Kamke 1194 | ✓           | 0.092  | 65   | ✓      | 0.034 | 48   | To Do            |
| Kamke 1195 | ✓           | 0.021  | 80   | ✓      | 0.083 | 93   | To Do            |
| Kamke 1196 | ✓           | 0.026  | 37   | ✓      | 0.026 | 31   | To Do            |
| Kamke 1197 | ✓           | 0.013  | 78   | ✓      | 0.031 | 43   | To Do            |
| Kamke 1198 | ✓           | 0.034  | 41   | ✓      | 0.028 | 37   | To Do            |
| Kamke 1199 | ✓           | 0.013  | 41   | ✓      | 0.016 | 35   | To Do            |
| Kamke 1200 | ✓           | 0.015  | 62   | ✓      | 0.012 | 27   | To Do            |
| Kamke 1201 | ✓           | 0.107  | 44   | ✓      | 0.015 | 34   | To Do            |
| Kamke 1202 | ✓           | 0.011  | 22   | ✓      | 0.023 | 14   | To Do            |
| Kamke 1203 | ✓           | 0.015  | 124  | ✓      | 0.015 | 28   | To Do            |
| Kamke 1204 | ✓           | 0.016  | 132  | ✓      | 0.036 | 35   | To Do            |
| Kamke 1205 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1206 | ✓           | 0.075  | 120  | ✓      | 0.044 | 76   | To Do            |
| Kamke 1207 | ✓           | 0.084  | 294  | ✓      | 0.159 | 110  | To Do            |
| Kamke 1208 | ✓           | 0.037  | 59   | ✓      | 0.033 | 35   | To Do            |
| Kamke 1209 | ✓           | 0.017  | 67   | ✓      | 0.036 | 40   | To Do            |
| Kamke 1210 | ✓           | 0.195  | 252  | ✓      | 0.146 | 81   | To Do            |
| Kamke 1211 | ✓           | 0.039  | 68   | ✓      | 0.039 | 36   | To Do            |
| Kamke 1212 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1213 | ✓           | 0.064  | 54   | ✓      | 0.038 | 53   | To Do            |
| Kamke 1214 | ✓           | 0.21   | 260  | ✓      | 0.112 | 71   | To Do            |
| Kamke 1215 | ✓           | 0.111  | 664  | ✓      | 0.114 | 148  | To Do            |
| Kamke 1216 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1217 | ✓           | 0.098  | 30   | ✓      | 0.026 | 24   | To Do            |
| Kamke 1218 | ✓           | 0.1    | 38   | ✓      | 0.027 | 30   | To Do            |
| Kamke 1219 | ✓           | 0.102  | 218  | ✓      | 0.066 | 69   | To Do            |
| Kamke 1220 | ✓           | 0.041  | 98   | ✓      | 0.013 | 40   | To Do            |
| Kamke 1221 | ✓           | 0.023  | 42   | ✓      | 0.016 | 35   | To Do            |
| Kamke 1222 | ✓           | 0.014  | 30   | ✓      | 0.013 | 23   | To Do            |
| Kamke 1223 | ✓           | 0.013  | 25   | ✓      | 0.011 | 19   | To Do            |
| Kamke 1224 | ✓           | 0.013  | 30   | ✓      | 0.01  | 23   | To Do            |
| Kamke 1225 | ✓           | 0.02   | 29   | ✓      | 0.027 | 23   | To Do            |
| Kamke 1226 | ✓           | 0.012  | 30   | ✓      | 0.029 | 25   | To Do            |
| Kamke 1227 | ✓           | 0.024  | 21   | ✓      | 0.01  | 16   | To Do            |
| Kamke 1228 | ✓           | 0.011  | 82   | ✓      | 0.063 | 53   | To Do            |
| Kamke 1229 | ✓           | 0.028  | 48   | ✓      | 0.012 | 31   | To Do            |
| Kamke 1230 | ✓           | 0.018  | 82   | ✓      | 0.079 | 36   | To Do            |
| Kamke 1231 | ✓           | 0.055  | 58   | ✓      | 0.036 | 52   | To Do            |
| Kamke 1232 | ✓           | 11.651 | 6628 | ✓      | 0.117 | 409  | To Do            |
| Kamke 1233 | ✓           | 0.549  | 6628 | ✓      | 0.098 | 409  | To Do            |
| Kamke 1234 | ✓           | 0.058  | 48   | ✗      | 0     | 0    | To Do            |
| Kamke 1235 | ✓           | 0.034  | 97   | ✓      | 0.059 | 45   | To Do            |
| Kamke 1236 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1237 | ✓           | 0.011  | 30   | ✓      | 0.026 | 20   | To Do            |
| Kamke 1238 | ✓           | 0.017  | 36   | ✓      | 0.019 | 26   | To Do            |
| Kamke 1239 | ✓           | 0.01   | 46   | ✓      | 0.058 | 35   | To Do            |
| Kamke 1240 | ✓           | 0.012  | 18   | ✓      | 0.033 | 15   | To Do            |
| Kamke 1241 | ✓           | 0.011  | 30   | ✓      | 0.036 | 24   | To Do            |
| Kamke 1242 | ✓           | 0.202  | 68   | ✓      | 0.066 | 41   | To Do            |
| Kamke 1243 | ✓           | 0.023  | 45   | ✓      | 0.046 | 21   | To Do            |
| Kamke 1244 | ✓           | 0.02   | 42   | ✓      | 0.055 | 27   | To Do            |
| Kamke 1245 | ✓           | 0.015  | 42   | ✓      | 0.033 | 27   | To Do            |
| Kamke 1246 | ✓           | 0.014  | 42   | ✓      | 0.146 | 28   | To Do            |
| Kamke 1247 | ✓           | 0.155  | 128  | ✓      | 0.02  | 27   | To Do            |
| Kamke 1248 | ✗           | 0      | 0    | ✓      | 0.177 | 134  | To Do            |
| Kamke 1249 | ✓           | 0.116  | 193  | ✓      | 0.052 | 134  | To Do            |
| Kamke 1250 | ✓           | 0.037  | 41   | ✓      | 0.02  | 41   | To Do            |

Continued on next page

Table 3 – continued from previous page

| #          | Mathematica |       |      | Maple  |       |      | note on solution |
|------------|-------------|-------|------|--------|-------|------|------------------|
|            | solved      | cpu   | leaf | solved | cpu   | leaf |                  |
| Kamke 1251 | ✓           | 0.025 | 25   | ✓      | 0.024 | 20   | To Do            |
| Kamke 1252 | ✓           | 0.107 | 151  | ✓      | 0.034 | 124  | To Do            |
| Kamke 1253 | ✓           | 0.019 | 34   | ✓      | 0.003 | 16   | To Do            |
| Kamke 1254 | ✓           | 0.139 | 69   | ✓      | 0.049 | 43   | To Do            |
| Kamke 1255 | ✓           | 0.635 | 360  | ✓      | 0.016 | 42   | To Do            |
| Kamke 1256 | ✓           | 0.017 | 26   | ✓      | 0.075 | 51   | To Do            |
| Kamke 1257 | ✓           | 0.029 | 33   | ✓      | 0.169 | 27   | To Do            |
| Kamke 1258 | ✓           | 0.117 | 146  | ✓      | 0.029 | 110  | To Do            |
| Kamke 1259 | ✓           | 0.1   | 120  | ✓      | 0.035 | 92   | To Do            |
| Kamke 1260 | ✓           | 0.202 | 65   | ✓      | 0.31  | 76   | To Do            |
| Kamke 1261 | ✗           | 0     | 0    | ✓      | 0.132 | 105  | To Do            |
| Kamke 1262 | ✓           | 0.227 | 88   | ✓      | 0.158 | 53   | To Do            |
| Kamke 1263 | ✓           | 0.373 | 276  | ✓      | 0.052 | 52   | To Do            |
| Kamke 1264 | ✓           | 0.053 | 23   | ✓      | 0.026 | 19   | To Do            |
| Kamke 1265 | ✓           | 0.032 | 64   | ✓      | 0.227 | 93   | To Do            |
| Kamke 1266 | ✓           | 0.022 | 22   | ✓      | 0.007 | 19   | To Do            |
| Kamke 1267 | ✓           | 0.345 | 166  | ✓      | 0.045 | 41   | To Do            |
| Kamke 1268 | ✗           | 0     | 0    | ✓      | 0.073 | 39   | To Do            |
| Kamke 1269 | ✓           | 0.064 | 60   | ✓      | 0.049 | 40   | To Do            |
| Kamke 1270 | ✓           | 1.853 | 58   | ✓      | 0.107 | 46   | To Do            |
| Kamke 1271 | ✓           | 0.008 | 27   | ✓      | 0.004 | 14   | To Do            |
| Kamke 1272 | ✓           | 0.008 | 32   | ✓      | 0.031 | 23   | To Do            |
| Kamke 1273 | ✓           | 0.012 | 20   | ✓      | 0.058 | 17   | To Do            |
| Kamke 1274 | ✓           | 0.031 | 38   | ✓      | 0.009 | 19   | To Do            |
| Kamke 1275 | ✓           | 0.025 | 120  | ✓      | 0.103 | 53   | To Do            |
| Kamke 1276 | ✓           | 0.028 | 55   | ✓      | 0.049 | 31   | To Do            |
| Kamke 1277 | ✓           | 0.02  | 51   | ✓      | 0.038 | 27   | To Do            |
| Kamke 1278 | ✗           | 0     | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1279 | ✓           | 0.109 | 74   | ✓      | 0.13  | 32   | To Do            |
| Kamke 1280 | ✓           | 0.044 | 52   | ✓      | 0.033 | 40   | To Do            |
| Kamke 1281 | ✓           | 0.015 | 28   | ✓      | 0.028 | 15   | To Do            |
| Kamke 1282 | ✓           | 0.016 | 39   | ✓      | 0.026 | 21   | To Do            |
| Kamke 1283 | ✓           | 0.06  | 90   | ✓      | 0.091 | 48   | To Do            |
| Kamke 1284 | ✓           | 0.029 | 47   | ✓      | 0.012 | 41   | To Do            |
| Kamke 1285 | ✓           | 1.284 | 269  | ✓      | 0.066 | 52   | To Do            |
| Kamke 1286 | ✓           | 0.089 | 101  | ✓      | 0.013 | 32   | To Do            |
| Kamke 1287 | ✓           | 0.013 | 83   | ✓      | 0.017 | 27   | To Do            |
| Kamke 1288 | ✓           | 0.025 | 47   | ✓      | 0.02  | 21   | To Do            |
| Kamke 1289 | ✓           | 0.104 | 53   | ✓      | 0.062 | 33   | To Do            |
| Kamke 1290 | ✓           | 0.111 | 103  | ✓      | 0.018 | 29   | To Do            |
| Kamke 1291 | ✓           | 0.057 | 92   | ✓      | 0.046 | 50   | To Do            |
| Kamke 1292 | ✓           | 0.033 | 42   | ✓      | 0.036 | 31   | To Do            |
| Kamke 1293 | ✓           | 0.249 | 44   | ✓      | 0.038 | 33   | To Do            |
| Kamke 1294 | ✓           | 0.051 | 44   | ✓      | 0.036 | 33   | To Do            |
| Kamke 1295 | ✓           | 0.186 | 310  | ✓      | 0.196 | 106  | To Do            |
| Kamke 1296 | ✓           | 0.358 | 356  | ✓      | 0.224 | 150  | To Do            |
| Kamke 1297 | ✓           | 0.024 | 52   | ✓      | 0.017 | 63   | To Do            |
| Kamke 1298 | ✓           | 0.054 | 162  | ✓      | 0.083 | 124  | To Do            |
| Kamke 1299 | ✓           | 0.014 | 19   | ✓      | 0.01  | 27   | To Do            |
| Kamke 1300 | ✓           | 0.01  | 41   | ✓      | 0.029 | 31   | To Do            |
| Kamke 1301 | ✓           | 0.023 | 31   | ✓      | 0.012 | 19   | To Do            |
| Kamke 1302 | ✓           | 0.059 | 243  | ✓      | 0.043 | 98   | To Do            |
| Kamke 1303 | ✗           | 0     | 0    | ✓      | 0.133 | 501  | To Do            |
| Kamke 1304 | ✓           | 0.037 | 50   | ✓      | 0.035 | 38   | To Do            |
| Kamke 1305 | ✓           | 0.062 | 47   | ✓      | 0.044 | 44   | To Do            |
| Kamke 1306 | ✗           | 0     | 0    | ✓      | 0.11  | 69   | To Do            |
| Kamke 1307 | ✓           | 0.09  | 54   | ✓      | 0.034 | 36   | To Do            |
| Kamke 1308 | ✓           | 0.013 | 41   | ✓      | 0.012 | 40   | To Do            |
| Kamke 1309 | ✓           | 0.067 | 84   | ✓      | 0.059 | 85   | To Do            |
| Kamke 1310 | ✓           | 0.008 | 31   | ✓      | 0.007 | 20   | To Do            |

Continued on next page

Table 3 – continued from previous page

| #          | Mathematica |       |      | Maple  |       |      | note on solution |
|------------|-------------|-------|------|--------|-------|------|------------------|
|            | solved      | cpu   | leaf | solved | cpu   | leaf |                  |
| Kamke 1311 | ✓           | 0.092 | 63   | ✓      | 0.098 | 52   | To Do            |
| Kamke 1312 | ✓           | 0.017 | 32   | ✓      | 0.013 | 19   | To Do            |
| Kamke 1313 | ✓           | 0.146 | 87   | ✓      | 0.052 | 35   | To Do            |
| Kamke 1314 | ✓           | 0.133 | 87   | ✓      | 0.045 | 33   | To Do            |
| Kamke 1315 | ✓           | 0.019 | 44   | ✓      | 0.013 | 45   | To Do            |
| Kamke 1316 | ✓           | 0.065 | 38   | ✓      | 0.026 | 18   | To Do            |
| Kamke 1317 | ✓           | 0.083 | 38   | ✓      | 0.024 | 13   | To Do            |
| Kamke 1318 | ✓           | 0.205 | 172  | ✓      | 0.067 | 122  | To Do            |
| Kamke 1319 | ✓           | 0.047 | 60   | ✓      | 0.072 | 31   | To Do            |
| Kamke 1320 | ✓           | 0.065 | 21   | ✓      | 0.027 | 17   | To Do            |
| Kamke 1321 | ✓           | 0.018 | 18   | ✓      | 0.014 | 15   | To Do            |
| Kamke 1322 | ✓           | 0.028 | 44   | ✓      | 0.016 | 44   | To Do            |
| Kamke 1323 | ✗           | 0     | 0    | ✓      | 0.01  | 17   | To Do            |
| Kamke 1324 | ✓           | 0.021 | 25   | ✓      | 0.017 | 18   | To Do            |
| Kamke 1325 | ✓           | 0.18  | 52   | ✓      | 0.07  | 86   | To Do            |
| Kamke 1326 | ✓           | 0.017 | 29   | ✓      | 0.017 | 22   | To Do            |
| Kamke 1327 | ✓           | 0.128 | 104  | ✓      | 0.197 | 81   | To Do            |
| Kamke 1328 | ✓           | 0.015 | 36   | ✓      | 0.016 | 27   | To Do            |
| Kamke 1329 | ✗           | 0     | 0    | ✓      | 0.184 | 64   | To Do            |
| Kamke 1330 | ✗           | 0     | 0    | ✓      | 0.707 | 1147 | To Do            |
| Kamke 1331 | ✓           | 0.032 | 55   | ✓      | 0.016 | 19   | To Do            |
| Kamke 1332 | ✓           | 0.018 | 26   | ✓      | 0.013 | 17   | To Do            |
| Kamke 1333 | ✓           | 0.076 | 70   | ✓      | 0.043 | 45   | To Do            |
| Kamke 1334 | ✓           | 0.142 | 114  | ✓      | 0.044 | 89   | To Do            |
| Kamke 1335 | ✓           | 0.226 | 893  | ✓      | 0.038 | 57   | To Do            |
| Kamke 1336 | ✓           | 0.035 | 70   | ✓      | 0.029 | 42   | To Do            |
| Kamke 1337 | ✓           | 0.052 | 62   | ✓      | 0.026 | 27   | To Do            |
| Kamke 1338 | ✓           | 0.04  | 40   | ✓      | 0.023 | 27   | To Do            |
| Kamke 1339 | ✓           | 0.188 | 66   | ✓      | 0.074 | 76   | To Do            |
| Kamke 1340 | ✓           | 0.026 | 32   | ✓      | 0.017 | 20   | To Do            |
| Kamke 1341 | ✓           | 61.86 | 2924 | ✓      | 0.114 | 201  | To Do            |
| Kamke 1342 | ✓           | 0.052 | 52   | ✓      | 0.036 | 31   | To Do            |
| Kamke 1343 | ✗           | 0     | 0    | ✓      | 0.069 | 58   | To Do            |
| Kamke 1344 | ✓           | 0.447 | 173  | ✓      | 0.043 | 23   | To Do            |
| Kamke 1345 | ✓           | 0.033 | 52   | ✓      | 0.032 | 25   | To Do            |
| Kamke 1346 | ✓           | 0.063 | 37   | ✓      | 0.033 | 25   | To Do            |
| Kamke 1347 | ✓           | 0.084 | 31   | ✓      | 0.017 | 19   | To Do            |
| Kamke 1348 | ✗           | 0     | 0    | ✓      | 0.125 | 73   | To Do            |
| Kamke 1349 | ✓           | 0.084 | 76   | ✓      | 0.046 | 85   | To Do            |
| Kamke 1350 | ✓           | 0.008 | 25   | ✓      | 0.007 | 21   | To Do            |
| Kamke 1351 | ✓           | 0.018 | 50   | ✓      | 0.02  | 24   | To Do            |
| Kamke 1352 | ✓           | 0.01  | 89   | ✓      | 0.034 | 43   | To Do            |
| Kamke 1353 | ✓           | 0.872 | 119  | ✓      | 0.18  | 67   | To Do            |
| Kamke 1354 | ✓           | 0.226 | 108  | ✓      | 0.234 | 33   | To Do            |
| Kamke 1355 | ✓           | 0.236 | 51   | ✓      | 0.08  | 30   | To Do            |
| Kamke 1356 | ✓           | 0.22  | 90   | ✓      | 0.045 | 29   | To Do            |
| Kamke 1357 | ✓           | 0.5   | 288  | ✓      | 0.069 | 97   | To Do            |
| Kamke 1358 | ✓           | 0.046 | 89   | ✓      | 0.039 | 20   | To Do            |
| Kamke 1359 | ✓           | 0.077 | 86   | ✓      | 0.052 | 57   | To Do            |
| Kamke 1360 | ✓           | 0.07  | 68   | ✓      | 0.04  | 47   | To Do            |
| Kamke 1361 | ✓           | 0.353 | 38   | ✓      | 0.022 | 33   | To Do            |
| Kamke 1362 | ✗           | 0     | 0    | ✓      | 0.136 | 109  | To Do            |
| Kamke 1363 | ✓           | 0.555 | 236  | ✓      | 0.076 | 161  | To Do            |
| Kamke 1364 | ✓           | 0.12  | 42   | ✓      | 0.065 | 25   | To Do            |
| Kamke 1365 | ✓           | 0.076 | 104  | ✓      | 0.04  | 59   | To Do            |
| Kamke 1366 | ✓           | 0.018 | 31   | ✓      | 0.007 | 17   | To Do            |
| Kamke 1367 | ✗           | 0     | 0    | ✓      | 0.14  | 88   | To Do            |
| Kamke 1368 | ✓           | 0.02  | 106  | ✓      | 0.05  | 71   | To Do            |
| Kamke 1369 | ✓           | 0.077 | 106  | ✓      | 0.033 | 55   | To Do            |
| Kamke 1370 | ✓           | 0.021 | 53   | ✓      | 0.007 | 19   | To Do            |

Continued on next page

Table 3 – continued from previous page

| #          | Mathematica |        |         | Maple  |       |      | note on solution |
|------------|-------------|--------|---------|--------|-------|------|------------------|
|            | solved      | cpu    | leaf    | solved | cpu   | leaf |                  |
| Kamke 1371 | ✓           | 0.016  | 48      | ✓      | 0.031 | 37   | To Do            |
| Kamke 1372 | ✗           | 0      | 0       | ✓      | 0.174 | 110  | To Do            |
| Kamke 1373 | ✗           | 0      | 0       | ✓      | 0.127 | 84   | To Do            |
| Kamke 1374 | ✓           | 0.025  | 32      | ✓      | 0.033 | 23   | To Do            |
| Kamke 1375 | ✓           | 0.034  | 54      | ✓      | 0.039 | 29   | To Do            |
| Kamke 1376 | ✓           | 0.033  | 69      | ✓      | 0.026 | 73   | To Do            |
| Kamke 1377 | ✓           | 0.178  | 163     | ✓      | 0.084 | 83   | To Do            |
| Kamke 1378 | ✓           | 0.038  | 65      | ✓      | 0.03  | 48   | To Do            |
| Kamke 1379 | ✓           | 0.053  | 99      | ✓      | 0.043 | 59   | To Do            |
| Kamke 1380 | ✓           | 0.199  | 132     | ✓      | 0.056 | 67   | To Do            |
| Kamke 1381 | ✓           | 0.27   | 589     | ✓      | 0.126 | 175  | To Do            |
| Kamke 1382 | ✓           | 0.467  | 154     | ✓      | 0.076 | 104  | To Do            |
| Kamke 1383 | ✓           | 0.094  | 50      | ✓      | 0.029 | 39   | To Do            |
| Kamke 1384 | ✓           | 0.024  | 110     | ✓      | 0.167 | 73   | To Do            |
| Kamke 1385 | ✓           | 0.015  | 78      | ✓      | 0.042 | 55   | To Do            |
| Kamke 1386 | ✓           | 0.063  | 108     | ✓      | 0.048 | 58   | To Do            |
| Kamke 1387 | ✓           | 0.028  | 50      | ✓      | 0.018 | 28   | To Do            |
| Kamke 1388 | ✓           | 0.227  | 235     | ✓      | 0.04  | 76   | To Do            |
| Kamke 1389 | ✓           | 0.273  | 217     | ✓      | 0.04  | 68   | To Do            |
| Kamke 1390 | ✓           | 0.026  | 41      | ✓      | 0.023 | 25   | To Do            |
| Kamke 1391 | ✓           | 0.041  | 27      | ✓      | 0.017 | 20   | To Do            |
| Kamke 1392 | ✓           | 62.053 | 1763961 | ✓      | 0.125 | 561  | To Do            |
| Kamke 1393 | ✓           | 13.835 | 413606  | ✓      | 0.099 | 299  | To Do            |
| Kamke 1394 | ✓           | 0.033  | 115     | ✓      | 0.067 | 79   | To Do            |
| Kamke 1395 | ✓           | 0.057  | 59      | ✓      | 0.04  | 39   | To Do            |
| Kamke 1396 | ✓           | 0.941  | 211     | ✓      | 0.116 | 178  | To Do            |
| Kamke 1397 | ✓           | 0.028  | 38      | ✓      | 0.066 | 27   | To Do            |
| Kamke 1398 | ✗           | 0      | 0       | ✓      | 0.111 | 69   | To Do            |
| Kamke 1399 | ✓           | 0.036  | 72      | ✓      | 0.04  | 34   | To Do            |
| Kamke 1400 | ✓           | 0.059  | 60      | ✓      | 0.029 | 35   | To Do            |
| Kamke 1401 | ✓           | 0.01   | 93      | ✓      | 0.032 | 45   | To Do            |
| Kamke 1402 | ✗           | 0      | 0       | ✓      | 0.166 | 58   | To Do            |
| Kamke 1403 | ✗           | 0      | 0       | ✓      | 0.52  | 298  | To Do            |
| Kamke 1404 | ✓           | 0.017  | 33      | ✓      | 0.027 | 19   | To Do            |
| Kamke 1405 | ✓           | 0.051  | 77      | ✓      | 0.041 | 42   | To Do            |
| Kamke 1406 | ✓           | 2.07   | 258     | ✓      | 0.112 | 44   | To Do            |
| Kamke 1407 | ✗           | 0      | 0       | ✓      | 1.261 | 2603 | To Do            |
| Kamke 1408 | ✗           | 0      | 0       | ✗      | 0     | 0    | To Do            |
| Kamke 1409 | ✓           | 0.019  | 43      | ✓      | 0.012 | 39   | To Do            |
| Kamke 1410 | ✓           | 0.093  | 481     | ✓      | 0.159 | 253  | To Do            |
| Kamke 1411 | ✓           | 0.272  | 42      | ✓      | 0.013 | 27   | To Do            |
| Kamke 1412 | ✓           | 0.011  | 29      | ✓      | 0.007 | 23   | To Do            |
| Kamke 1413 | ✓           | 0.046  | 16      | ✓      | 0.036 | 12   | To Do            |
| Kamke 1414 | ✓           | 0.781  | 231     | ✓      | 0.18  | 97   | To Do            |
| Kamke 1415 | ✓           | 0.671  | 273     | ✓      | 0.095 | 36   | To Do            |
| Kamke 1416 | ✓           | 0.152  | 46      | ✓      | 0.128 | 26   | To Do            |
| Kamke 1417 | ✓           | 0.124  | 52      | ✓      | 0.098 | 31   | To Do            |
| Kamke 1418 | ✓           | 0.121  | 15      | ✓      | 2.694 | 59   | To Do            |
| Kamke 1419 | ✗           | 0      | 0       | ✓      | 0.142 | 12   | To Do            |
| Kamke 1420 | ✓           | 0.328  | 134     | ✓      | 0.221 | 123  | To Do            |
| Kamke 1421 | ✓           | 0.15   | 81      | ✓      | 0.05  | 27   | To Do            |
| Kamke 1422 | ✓           | 0.077  | 58      | ✓      | 0.155 | 50   | To Do            |
| Kamke 1423 | ✓           | 0.055  | 70      | ✓      | 0.183 | 132  | To Do            |
| Kamke 1424 | ✓           | 0.128  | 90      | ✓      | 0.183 | 120  | To Do            |
| Kamke 1425 | ✓           | 1.254  | 385     | ✓      | 0.303 | 93   | To Do            |
| Kamke 1426 | ✓           | 4.25   | 4128    | ✓      | 0.37  | 549  | To Do            |
| Kamke 1427 | ✗           | 0      | 0       | ✓      | 1.004 | 203  | To Do            |
| Kamke 1428 | ✓           | 0.278  | 104     | ✓      | 0.202 | 183  | To Do            |
| Kamke 1429 | ✓           | 0.04   | 51      | ✓      | 0.024 | 25   | To Do            |
| Kamke 1430 | ✓           | 0.313  | 22      | ✓      | 0.23  | 101  | To Do            |

Continued on next page

Table 3 – continued from previous page

| #          | Mathematica |        |         | Maple  |       |      | note on solution |
|------------|-------------|--------|---------|--------|-------|------|------------------|
|            | solved      | cpu    | leaf    | solved | cpu   | leaf |                  |
| Kamke 1431 | ✓           | 0.152  | 80      | ✓      | 0.155 | 30   | To Do            |
| Kamke 1432 | ✓           | 0.063  | 37      | ✓      | 0.026 | 22   | To Do            |
| Kamke 1433 | ✓           | 0.139  | 46      | ✓      | 0.069 | 28   | To Do            |
| Kamke 1434 | ✓           | 70.893 | 1596424 | ✓      | 0.405 | 517  | To Do            |
| Kamke 1435 | ✓           | 0.104  | 70      | ✓      | 0.092 | 38   | To Do            |
| Kamke 1436 | ✓           | 0.364  | 42      | ✓      | 0.174 | 113  | To Do            |
| Kamke 1437 | ✓           | 0.2    | 42      | ✓      | 0.112 | 29   | To Do            |
| Kamke 1438 | ✓           | 0.636  | 615     | ✓      | 0.131 | 102  | To Do            |
| Kamke 1439 | ✗           | 0      | 0       | ✗      | 0     | 0    | To Do            |
| Kamke 1440 | ✗           | 0      | 0       | ✗      | 0     | 0    | To Do            |
| Kamke 1441 | ✗           | 0      | 0       | ✗      | 0     | 0    | To Do            |
| Kamke 1442 | ✓           | 0.052  | 46      | ✓      | 0.039 | 30   | To Do            |
| Kamke 1443 | ✗           | 0      | 0       | ✗      | 0     | 0    | To Do            |
| Kamke 1444 | ✗           | 0      | 0       | ✓      | 0.016 | 37   | To Do            |
| Kamke 1445 | ✗           | 0      | 0       | ✓      | 0.133 | 20   | To Do            |
| Kamke 1446 | ✓           | 0.028  | 33      | ✓      | 0.03  | 22   | To Do            |
| Kamke 1447 | ✓           | 0.027  | 29      | ✓      | 0.027 | 20   | To Do            |
| Kamke 1448 | ✓           | 0.211  | 149     | ✓      | 0.065 | 77   | To Do            |
| Kamke 1449 | ✓           | 0.005  | 53      | ✓      | 0.013 | 47   | To Do            |
| Kamke 1450 | ✗           | 0      | 0       | ✓      | 0.174 | 1616 | To Do            |
| Kamke 1451 | ✓           | 0.014  | 168     | ✓      | 0.066 | 114  | To Do            |
| Kamke 1452 | ✓           | 0.006  | 54      | ✓      | 0.006 | 35   | To Do            |
| Kamke 1453 | ✓           | 1.101  | 128     | ✓      | 0.109 | 122  | To Do            |
| Kamke 1454 | ✓           | 0.007  | 79      | ✓      | 0.036 | 55   | To Do            |
| Kamke 1455 | ✓           | 0.02   | 127     | ✓      | 0.079 | 71   | To Do            |
| Kamke 1456 | ✓           | 0.025  | 183     | ✓      | 0.042 | 73   | To Do            |
| Kamke 1457 | ✗           | 0      | 0       | ✗      | 0     | 0    | To Do            |
| Kamke 1458 | ✗           | 0      | 0       | ✗      | 0     | 0    | To Do            |
| Kamke 1459 | ✗           | 0      | 0       | ✗      | 0     | 0    | To Do            |
| Kamke 1460 | ✗           | 0      | 0       | ✗      | 0     | 0    | To Do            |
| Kamke 1461 | ✗           | 0      | 0       | ✗      | 0     | 0    | To Do            |
| Kamke 1462 | ✗           | 0      | 0       | ✗      | 0     | 0    | To Do            |
| Kamke 1463 | ✗           | 0      | 0       | ✗      | 0     | 0    | To Do            |
| Kamke 1464 | ✓           | 0.005  | 34      | ✓      | 0.004 | 27   | To Do            |
| Kamke 1465 | ✓           | 0.086  | 95      | ✓      | 0.121 | 89   | To Do            |
| Kamke 1466 | ✓           | 0.008  | 46      | ✓      | 0.011 | 27   | To Do            |
| Kamke 1467 | ✓           | 0.005  | 84      | ✓      | 0.02  | 590  | To Do            |
| Kamke 1468 | ✓           | 0.057  | 57      | ✓      | 0.081 | 59   | To Do            |
| Kamke 1469 | ✓           | 0.012  | 72      | ✓      | 0.02  | 37   | To Do            |
| Kamke 1470 | ✓           | 2.713  | 64      | ✓      | 0.062 | 36   | To Do            |
| Kamke 1471 | ✗           | 0      | 0       | ✓      | 0.088 | 36   | To Do            |
| Kamke 1472 | ✗           | 0      | 0       | ✓      | 0.103 | 33   | To Do            |
| Kamke 1473 | ✗           | 0      | 0       | ✗      | 0     | 0    | To Do            |
| Kamke 1474 | ✗           | 0      | 0       | ✗      | 0     | 0    | To Do            |
| Kamke 1475 | ✓           | 0.025  | 38      | ✓      | 0.013 | 23   | To Do            |
| Kamke 1476 | ✗           | 0      | 0       | ✗      | 0     | 0    | To Do            |
| Kamke 1477 | ✓           | 0.128  | 48      | ✓      | 0.012 | 41   | To Do            |
| Kamke 1478 | ✓           | 0.024  | 104     | ✓      | 0.066 | 48   | To Do            |
| Kamke 1479 | ✓           | 0.107  | 153     | ✓      | 0.131 | 92   | To Do            |
| Kamke 1480 | ✓           | 0.175  | 93      | ✓      | 0.138 | 35   | To Do            |
| Kamke 1481 | ✓           | 0.393  | 432     | ✓      | 0.042 | 44   | To Do            |
| Kamke 1482 | ✗           | 0      | 0       | ✓      | 0.228 | 1614 | To Do            |
| Kamke 1483 | ✓           | 0.112  | 112     | ✓      | 0.145 | 37   | To Do            |
| Kamke 1484 | ✗           | 0      | 0       | ✗      | 0     | 0    | To Do            |
| Kamke 1485 | ✓           | 0.161  | 64      | ✓      | 0.207 | 51   | To Do            |
| Kamke 1486 | ✓           | 0.317  | 65      | ✓      | 0.144 | 51   | To Do            |
| Kamke 1487 | ✓           | 0.679  | 87      | ✓      | 0.062 | 38   | To Do            |
| Kamke 1488 | ✓           | 0.401  | 102     | ✓      | 0.326 | 132  | To Do            |
| Kamke 1489 | ✗           | 0      | 0       | ✗      | 0     | 0    | To Do            |
| Kamke 1490 | ✓           | 0.025  | 33      | ✓      | 0.035 | 18   | To Do            |

Continued on next page

Table 3 – continued from previous page

| #          | Mathematica |       |       | Maple  |       |      | note on solution |
|------------|-------------|-------|-------|--------|-------|------|------------------|
|            | solved      | cpu   | leaf  | solved | cpu   | leaf |                  |
| Kamke 1491 | ✓           | 0.032 | 102   | ✓      | 0.054 | 88   | To Do            |
| Kamke 1492 | ✓           | 0.284 | 43    | ✓      | 0.071 | 39   | To Do            |
| Kamke 1493 | ✓           | 1.165 | 2585  | ✓      | 0.183 | 1033 | To Do            |
| Kamke 1494 | ✓           | 0.02  | 43    | ✓      | 0.01  | 32   | To Do            |
| Kamke 1495 | ✓           | 0.006 | 24    | ✓      | 0.007 | 16   | To Do            |
| Kamke 1496 | ✓           | 0.205 | 63    | ✓      | 0.014 | 57   | To Do            |
| Kamke 1497 | ✓           | 0.366 | 135   | ✓      | 0.152 | 77   | To Do            |
| Kamke 1498 | ✓           | 9.634 | 584   | ✓      | 0.141 | 53   | To Do            |
| Kamke 1499 | ✓           | 0.194 | 97    | ✓      | 0.146 | 25   | To Do            |
| Kamke 1500 | ✗           | 0     | 0     | ✓      | 0.126 | 55   | To Do            |
| Kamke 1501 | ✓           | 0.16  | 86    | ✓      | 0.127 | 37   | To Do            |
| Kamke 1502 | ✓           | 0.059 | 98    | ✓      | 0.27  | 103  | To Do            |
| Kamke 1503 | ✓           | 0.47  | 258   | ✓      | 0.014 | 67   | To Do            |
| Kamke 1504 | ✓           | 0.105 | 43    | ✓      | 0.11  | 18   | To Do            |
| Kamke 1505 | ✗           | 0     | 0     | ✓      | 0.087 | 79   | To Do            |
| Kamke 1506 | ✓           | 0.291 | 150   | ✓      | 0.035 | 43   | To Do            |
| Kamke 1507 | ✗           | 0     | 0     | ✓      | 0.415 | 1209 | To Do            |
| Kamke 1508 | ✓           | 0.639 | 143   | ✓      | 0.078 | 81   | To Do            |
| Kamke 1509 | ✓           | 0.008 | 34    | ✓      | 0.032 | 29   | To Do            |
| Kamke 1510 | ✗           | 0     | 0     | ✗      | 0     | 0    | To Do            |
| Kamke 1511 | ✓           | 0.049 | 51    | ✓      | 0.019 | 49   | To Do            |
| Kamke 1512 | ✓           | 0.033 | 29    | ✓      | 0.009 | 18   | To Do            |
| Kamke 1513 | ✓           | 0.077 | 25    | ✓      | 0.117 | 18   | To Do            |
| Kamke 1514 | ✓           | 0.551 | 102   | ✓      | 0.287 | 132  | To Do            |
| Kamke 1515 | ✗           | 0     | 0     | ✗      | 0     | 0    | To Do            |
| Kamke 1516 | ✗           | 0     | 0     | ✓      | 0.263 | 188  | To Do            |
| Kamke 1517 | ✓           | 0.303 | 30686 | ✓      | 0.326 | 866  | To Do            |
| Kamke 1518 | ✓           | 0.246 | 104   | ✓      | 0.24  | 60   | To Do            |
| Kamke 1519 | ✓           | 0.023 | 65    | ✓      | 0.082 | 19   | To Do            |
| Kamke 1520 | ✗           | 0     | 0     | ✓      | 0.298 | 288  | To Do            |
| Kamke 1521 | ✓           | 0.049 | 35    | ✓      | 0.237 | 28   | To Do            |
| Kamke 1522 | ✓           | 0.014 | 44    | ✓      | 0.024 | 34   | To Do            |
| Kamke 1523 | ✓           | 0.28  | 74    | ✓      | 0.231 | 23   | To Do            |
| Kamke 1524 | ✓           | 0.133 | 96    | ✓      | 0.302 | 98   | To Do            |
| Kamke 1525 | ✓           | 0.341 | 102   | ✓      | 0.363 | 287  | To Do            |
| Kamke 1526 | ✗           | 0     | 0     | ✓      | 0.131 | 19   | To Do            |
| Kamke 1527 | ✗           | 0     | 0     | ✓      | 0.337 | 437  | To Do            |
| Kamke 1528 | ✓           | 0.886 | 72    | ✓      | 0.125 | 71   | To Do            |
| Kamke 1529 | ✗           | 0     | 0     | ✓      | 0.046 | 25   | To Do            |
| Kamke 1530 | ✗           | 0     | 0     | ✓      | 0.153 | 113  | To Do            |
| Kamke 1531 | ✗           | 0     | 0     | ✗      | 0     | 0    | To Do            |
| Kamke 1532 | ✓           | 0.012 | 103   | ✓      | 0.058 | 58   | To Do            |
| Kamke 1533 | ✓           | 0.012 | 113   | ✓      | 0.05  | 58   | To Do            |
| Kamke 1534 | ✓           | 0.004 | 24    | ✓      | 0.008 | 21   | To Do            |
| Kamke 1535 | ✓           | 0.349 | 223   | ✓      | 0.013 | 36   | To Do            |
| Kamke 1536 | ✓           | 0.004 | 76    | ✓      | 0.01  | 50   | To Do            |
| Kamke 1537 | ✓           | 1.003 | 1722  | ✓      | 0.09  | 67   | To Do            |
| Kamke 1538 | ✓           | 0.17  | 66    | ✓      | 0.359 | 51   | To Do            |
| Kamke 1539 | ✓           | 0.006 | 44    | ✓      | 0.019 | 35   | To Do            |
| Kamke 1540 | ✗           | 0     | 0     | ✗      | 0     | 0    | To Do            |
| Kamke 1541 | ✗           | 0     | 0     | ✗      | 0     | 0    | To Do            |
| Kamke 1542 | ✗           | 0     | 0     | ✗      | 0     | 0    | To Do            |
| Kamke 1543 | ✗           | 0     | 0     | ✗      | 0     | 0    | To Do            |
| Kamke 1544 | ✗           | 0     | 0     | ✓      | 0.01  | 41   | To Do            |
| Kamke 1545 | ✓           | 0.726 | 40    | ✓      | 0.079 | 27   | To Do            |
| Kamke 1546 | ✓           | 0.539 | 300   | ✓      | 0.029 | 73   | To Do            |
| Kamke 1547 | ✗           | 0     | 0     | ✓      | 0.011 | 87   | To Do            |
| Kamke 1548 | ✓           | 0.082 | 50    | ✓      | 0.052 | 32   | To Do            |
| Kamke 1549 | ✓           | 0.012 | 34    | ✓      | 0.014 | 26   | To Do            |
| Kamke 1550 | ✓           | 3.082 | 270   | ✓      | 1.279 | 157  | To Do            |

Continued on next page

Table 3 – continued from previous page

| #          | Mathematica |        |      | Maple  |        |      | note on solution |
|------------|-------------|--------|------|--------|--------|------|------------------|
|            | solved      | cpu    | leaf | solved | cpu    | leaf |                  |
| Kamke 1551 | ✓           | 0.303  | 110  | ✓      | 0.144  | 62   | To Do            |
| Kamke 1552 | ✗           | 0      | 0    | ✓      | 0.049  | 89   | To Do            |
| Kamke 1553 | ✓           | 0.009  | 29   | ✓      | 0.007  | 17   | To Do            |
| Kamke 1554 | ✓           | 0.009  | 29   | ✓      | 0.009  | 18   | To Do            |
| Kamke 1555 | ✓           | 0.047  | 156  | ✓      | 0.097  | 61   | To Do            |
| Kamke 1556 | ✓           | 0.009  | 30   | ✓      | 0.009  | 19   | To Do            |
| Kamke 1557 | ✓           | 0.051  | 146  | ✓      | 0.07   | 61   | To Do            |
| Kamke 1558 | ✓           | 0.113  | 319  | ✓      | 0.102  | 67   | To Do            |
| Kamke 1559 | ✓           | 0.203  | 100  | ✓      | 0.102  | 33   | To Do            |
| Kamke 1560 | ✓           | 0.007  | 29   | ✓      | 0.006  | 18   | To Do            |
| Kamke 1561 | ✓           | 2.98   | 400  | ✓      | 0.167  | 69   | To Do            |
| Kamke 1562 | ✓           | 0.831  | 140  | ✓      | 0.228  | 77   | To Do            |
| Kamke 1563 | ✓           | 1.511  | 232  | ✓      | 0.16   | 87   | To Do            |
| Kamke 1564 | ✓           | 1.009  | 230  | ✓      | 0.138  | 88   | To Do            |
| Kamke 1565 | ✓           | 0.385  | 242  | ✓      | 0.231  | 71   | To Do            |
| Kamke 1566 | ✓           | 0.454  | 238  | ✓      | 0.207  | 35   | To Do            |
| Kamke 1567 | ✓           | 0.009  | 30   | ✓      | 0.009  | 19   | To Do            |
| Kamke 1568 | ✓           | 0.009  | 122  | ✓      | 0.019  | 89   | To Do            |
| Kamke 1569 | ✗           | 0      | 0    | ✓      | 0.29   | 63   | To Do            |
| Kamke 1570 | ✓           | 0.102  | 470  | ✓      | 0.062  | 49   | To Do            |
| Kamke 1571 | ✓           | 0.063  | 390  | ✓      | 0.144  | 143  | To Do            |
| Kamke 1572 | ✗           | 0      | 0    | ✓      | 0.251  | 35   | To Do            |
| Kamke 1573 | ✗           | 0      | 0    | ✓      | 0.026  | 41   | To Do            |
| Kamke 1574 | ✗           | 0      | 0    | ✓      | 0.508  | 252  | To Do            |
| Kamke 1575 | ✗           | 0      | 0    | ✓      | 0.519  | 638  | To Do            |
| Kamke 1576 | ✗           | 0      | 0    | ✓      | 0.016  | 67   | To Do            |
| Kamke 1577 | ✓           | 0.057  | 46   | ✓      | 0.004  | 21   | To Do            |
| Kamke 1578 | ✓           | 47.296 | 141  | ✓      | 0.287  | 89   | To Do            |
| Kamke 1579 | ✓           | 0.911  | 80   | ✓      | 0.283  | 69   | To Do            |
| Kamke 1580 | ✓           | 2.39   | 234  | ✓      | 0.659  | 79   | To Do            |
| Kamke 1581 | ✗           | 0      | 0    | ✗      | 0      | 0    | To Do            |
| Kamke 1582 | ✓           | 0.117  | 787  | ✗      | 0      | 0    | To Do            |
| Kamke 1583 | ✓           | 0.094  | 92   | ✓      | 0.023  | 40   | To Do            |
| Kamke 1584 | ✓           | 2.138  | 216  | ✓      | 0.142  | 118  | To Do            |
| Kamke 1585 | ✓           | 0.161  | 214  | ✓      | 0.027  | 679  | To Do            |
| Kamke 1586 | ✗           | 0      | 0    | ✗      | 0      | 0    | To Do            |
| Kamke 1587 | ✓           | 0.269  | 492  | ✓      | 0.306  | 154  | To Do            |
| Kamke 1588 | ✓           | 10.084 | 114  | ✓      | 0.082  | 90   | To Do            |
| Kamke 1589 | ✓           | 0.033  | 670  | ✓      | 11.161 | 4355 | To Do            |
| Kamke 1590 | ✗           | 0      | 0    | ✓      | 1.386  | 553  | To Do            |
| Kamke 1591 | ✓           | 0.029  | 26   | ✓      | 0.015  | 12   | To Do            |
| Kamke 1592 | ✓           | 0.022  | 14   | ✓      | 0.008  | 10   | To Do            |
| Kamke 1593 | ✗           | 0      | 0    | ✗      | 0      | 0    | To Do            |
| Kamke 1594 | ✓           | 0.332  | 373  | ✓      | 0.079  | 59   | To Do            |
| Kamke 1595 | ✗           | 0      | 0    | ✗      | 0      | 0    | To Do            |
| Kamke 1596 | ✗           | 0      | 0    | ✗      | 0      | 0    | To Do            |
| Kamke 1597 | ✓           | 1.603  | 242  | ✓      | 0.017  | 21   | To Do            |
| Kamke 1598 | ✗           | 0      | 0    | ✗      | 0      | 0    | To Do            |
| Kamke 1599 | ✗           | 0      | 0    | ✗      | 0      | 0    | To Do            |
| Kamke 1600 | ✓           | 1.693  | 1017 | ✓      | 0.056  | 89   | To Do            |
| Kamke 1601 | ✗           | 0      | 0    | ✓      | 2.981  | 151  | To Do            |
| Kamke 1602 | ✓           | 0.125  | 47   | ✓      | 0.181  | 73   | To Do            |
| Kamke 1603 | ✗           | 0      | 0    | ✓      | 25.336 | 8427 | To Do            |
| Kamke 1604 | ✓           | 0.041  | 34   | ✓      | 0.18   | 23   | To Do            |
| Kamke 1605 | ✗           | 0      | 0    | ✓      | 0.942  | 107  | To Do            |
| Kamke 1606 | ✗           | 0      | 0    | ✗      | 0      | 0    | To Do            |
| Kamke 1607 | ✓           | 0.08   | 79   | ✓      | 0.109  | 49   | To Do            |
| Kamke 1608 | ✗           | 0      | 0    | ✗      | 0      | 0    | To Do            |
| Kamke 1609 | ✗           | 0      | 0    | ✗      | 0      | 0    | To Do            |
| Kamke 1610 | ✓           | 3.011  | 754  | ✓      | 0.183  | 92   | To Do            |

Continued on next page



Table 3 – continued from previous page

| #          | Mathematica |        |      | Maple  |       |      | note on solution |
|------------|-------------|--------|------|--------|-------|------|------------------|
|            | solved      | cpu    | leaf | solved | cpu   | leaf |                  |
| Kamke 1611 | ✗           | 0      | 0    | ✓      | 0.526 | 57   | To Do            |
| Kamke 1612 | ✗           | 0      | 0    | ✓      | 0.983 | 57   | To Do            |
| Kamke 1613 | ✗           | 0      | 0    | ✓      | 0.017 | 27   | To Do            |
| Kamke 1614 | ✗           | 0      | 0    | ✓      | 0.052 | 33   | To Do            |
| Kamke 1615 | ✗           | 0      | 0    | ✓      | 3.733 | 91   | To Do            |
| Kamke 1616 | ✗           | 0      | 0    | ✓      | 0.85  | 63   | To Do            |
| Kamke 1617 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1618 | ✗           | 0      | 0    | ✓      | 1.241 | 56   | To Do            |
| Kamke 1619 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1620 | ✓           | 2.841  | 492  | ✓      | 0.086 | 253  | To Do            |
| Kamke 1621 | ✓           | 9.438  | 990  | ✓      | 1.922 | 1088 | To Do            |
| Kamke 1622 | ✗           | 0      | 0    | ✓      | 0.244 | 415  | To Do            |
| Kamke 1623 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1624 | ✗           | 0      | 0    | ✓      | 1.295 | 131  | To Do            |
| Kamke 1625 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1626 | ✗           | 0      | 0    | ✓      | 0.17  | 48   | To Do            |
| Kamke 1627 | ✗           | 0      | 0    | ✓      | 0.612 | 58   | To Do            |
| Kamke 1628 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1629 | ✗           | 0      | 0    | ✓      | 0.027 | 38   | To Do            |
| Kamke 1630 | ✓           | 7.461  | 3227 | ✓      | 0.424 | 783  | To Do            |
| Kamke 1631 | ✗           | 0      | 0    | ✓      | 0.04  | 38   | To Do            |
| Kamke 1632 | ✓           | 0.05   | 46   | ✓      | 0.075 | 23   | To Do            |
| Kamke 1633 | ✗           | 0      | 0    | ✓      | 0.231 | 97   | To Do            |
| Kamke 1634 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1635 | ✓           | 0.58   | 104  | ✓      | 0.154 | 79   | To Do            |
| Kamke 1636 | ✗           | 0      | 0    | ✓      | 0.667 | 59   | To Do            |
| Kamke 1637 | ✗           | 0      | 0    | ✓      | 0.401 | 58   | To Do            |
| Kamke 1638 | ✓           | 4.597  | 146  | ✓      | 0.161 | 115  | To Do            |
| Kamke 1639 | ✗           | 0      | 0    | ✓      | 2.365 | 56   | To Do            |
| Kamke 1640 | ✓           | 0.465  | 96   | ✓      | 0.171 | 70   | To Do            |
| Kamke 1641 | ✓           | 0.052  | 61   | ✓      | 0.026 | 29   | To Do            |
| Kamke 1642 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1643 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1644 | ✗           | 0      | 0    | ✓      | 0.381 | 56   | To Do            |
| Kamke 1645 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1646 | ✓           | 10.638 | 262  | ✓      | 0.149 | 94   | To Do            |
| Kamke 1647 | ✓           | 0.422  | 91   | ✓      | 0.432 | 60   | To Do            |
| Kamke 1648 | ✗           | 0      | 0    | ✓      | 1.137 | 205  | To Do            |
| Kamke 1649 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1650 | ✓           | 0.1    | 30   | ✓      | 0.224 | 16   | To Do            |
| Kamke 1651 | ✓           | 0.328  | 414  | ✓      | 0.135 | 31   | To Do            |
| Kamke 1652 | ✗           | 0      | 0    | ✓      | 0.226 | 36   | To Do            |
| Kamke 1653 | ✓           | 0.139  | 75   | ✓      | 0.104 | 41   | To Do            |
| Kamke 1654 | ✓           | 10.315 | 308  | ✓      | 0.181 | 38   | To Do            |
| Kamke 1655 | ✓           | 10.698 | 350  | ✓      | 0.197 | 84   | To Do            |
| Kamke 1656 | ✓           | 41.147 | 9706 | ✓      | 0.528 | 771  | To Do            |
| Kamke 1657 | ✓           | 0.299  | 192  | ✓      | 0.193 | 35   | To Do            |
| Kamke 1658 | ✗           | 0      | 0    | ✓      | 0.125 | 115  | To Do            |
| Kamke 1659 | ✗           | 0      | 0    | ✓      | 0.076 | 60   | To Do            |
| Kamke 1660 | ✗           | 0      | 0    | ✓      | 0.609 | 125  | To Do            |
| Kamke 1661 | ✓           | 0.069  | 92   | ✓      | 0.063 | 51   | To Do            |
| Kamke 1662 | ✗           | 0      | 0    | ✓      | 0.306 | 56   | To Do            |
| Kamke 1663 | ✗           | 0      | 0    | ✓      | 0.782 | 125  | To Do            |
| Kamke 1664 | ✗           | 0      | 0    | ✓      | 3.152 | 155  | To Do            |
| Kamke 1665 | ✗           | 0      | 0    | ✓      | 0.519 | 84   | To Do            |
| Kamke 1666 | ✗           | 0      | 0    | ✓      | 0.776 | 93   | To Do            |
| Kamke 1667 | ✗           | 0      | 0    | ✓      | 1.117 | 121  | To Do            |
| Kamke 1668 | ✓           | 0.055  | 60   | ✓      | 0.112 | 24   | To Do            |
| Kamke 1669 | ✓           | 0.453  | 160  | ✓      | 0.085 | 32   | To Do            |
| Kamke 1670 | ✓           | 7.363  | 51   | ✓      | 0.353 | 35   | To Do            |

Continued on next page

Table 3 – continued from previous page

| #          | Mathematica |        |      | Maple  |       |      | note on solution |
|------------|-------------|--------|------|--------|-------|------|------------------|
|            | solved      | cpu    | leaf | solved | cpu   | leaf |                  |
| Kamke 1671 | ✓           | 0.101  | 59   | ✓      | 0.061 | 35   | To Do            |
| Kamke 1672 | ✗           | 0      | 0    | ✓      | 0.8   | 65   | To Do            |
| Kamke 1673 | ✗           | 0      | 0    | ✓      | 0.61  | 60   | To Do            |
| Kamke 1674 | ✓           | 0.042  | 106  | ✓      | 0.049 | 25   | To Do            |
| Kamke 1675 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1676 | ✓           | 0.302  | 134  | ✓      | 0.211 | 72   | To Do            |
| Kamke 1677 | ✗           | 0      | 0    | ✓      | 1.272 | 101  | To Do            |
| Kamke 1678 | ✗           | 0      | 0    | ✓      | 0.203 | 60   | To Do            |
| Kamke 1679 | ✓           | 0.161  | 33   | ✓      | 0.159 | 27   | To Do            |
| Kamke 1680 | ✗           | 0      | 0    | ✓      | 0.54  | 103  | To Do            |
| Kamke 1681 | ✗           | 0      | 0    | ✓      | 0.037 | 31   | To Do            |
| Kamke 1682 | ✗           | 0      | 0    | ✓      | 0.828 | 94   | To Do            |
| Kamke 1683 | ✓           | 0.068  | 26   | ✓      | 0.042 | 23   | To Do            |
| Kamke 1684 | ✗           | 0      | 0    | ✓      | 1.04  | 100  | To Do            |
| Kamke 1685 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1686 | ✗           | 0      | 0    | ✓      | 0.987 | 128  | To Do            |
| Kamke 1687 | ✓           | 0.07   | 262  | ✓      | 0.083 | 21   | To Do            |
| Kamke 1688 | ✓           | 0.513  | 189  | ✓      | 0.076 | 32   | To Do            |
| Kamke 1689 | ✓           | 0.251  | 104  | ✓      | 0.105 | 37   | To Do            |
| Kamke 1690 | ✗           | 0      | 0    | ✓      | 0.414 | 99   | To Do            |
| Kamke 1691 | ✗           | 0      | 0    | ✓      | 0.69  | 254  | To Do            |
| Kamke 1692 | ✗           | 0      | 0    | ✓      | 3.769 | 156  | To Do            |
| Kamke 1693 | ✗           | 0      | 0    | ✓      | 0.217 | 68   | To Do            |
| Kamke 1694 | ✓           | 0.113  | 115  | ✓      | 0.109 | 54   | To Do            |
| Kamke 1695 | ✗           | 0      | 0    | ✓      | 0.55  | 103  | To Do            |
| Kamke 1696 | ✗           | 0      | 0    | ✓      | 0.457 | 100  | To Do            |
| Kamke 1697 | ✓           | 0.163  | 94   | ✓      | 0.03  | 39   | To Do            |
| Kamke 1698 | ✓           | 0.03   | 72   | ✗      | 0     | 0    | To Do            |
| Kamke 1699 | ✓           | 0.113  | 40   | ✓      | 0.076 | 33   | To Do            |
| Kamke 1700 | ✓           | 0.155  | 44   | ✓      | 0.246 | 86   | To Do            |
| Kamke 1701 | ✓           | 0.17   | 80   | ✓      | 0.211 | 42   | To Do            |
| Kamke 1702 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1703 | ✓           | 0.291  | 77   | ✓      | 0.048 | 21   | To Do            |
| Kamke 1704 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1705 | ✓           | 0.269  | 252  | ✗      | 0     | 0    | To Do            |
| Kamke 1706 | ✓           | 0.502  | 308  | ✗      | 0     | 0    | To Do            |
| Kamke 1707 | ✓           | 0.057  | 31   | ✓      | 0.056 | 39   | To Do            |
| Kamke 1708 | ✗           | 0      | 0    | ✓      | 0.806 | 73   | To Do            |
| Kamke 1709 | ✗           | 0      | 0    | ✓      | 1.599 | 84   | To Do            |
| Kamke 1710 | ✗           | 0      | 0    | ✓      | 2.415 | 91   | To Do            |
| Kamke 1711 | ✓           | 22.093 | 9987 | ✓      | 0.385 | 81   | To Do            |
| Kamke 1712 | ✓           | 0.058  | 75   | ✓      | 0.067 | 61   | To Do            |
| Kamke 1713 | ✗           | 0      | 0    | ✓      | 0.223 | 54   | To Do            |
| Kamke 1714 | ✓           | 0.123  | 25   | ✓      | 0.05  | 68   | To Do            |
| Kamke 1715 | ✓           | 0.114  | 26   | ✓      | 0.042 | 25   | To Do            |
| Kamke 1716 | ✓           | 0.538  | 172  | ✓      | 0.167 | 68   | To Do            |
| Kamke 1717 | ✓           | 1.116  | 49   | ✓      | 0.21  | 107  | To Do            |
| Kamke 1718 | ✓           | 1.272  | 744  | ✓      | 0.207 | 133  | To Do            |
| Kamke 1719 | ✗           | 0      | 0    | ✓      | 0.405 | 70   | To Do            |
| Kamke 1720 | ✗           | 0      | 0    | ✓      | 0.287 | 173  | To Do            |
| Kamke 1721 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1722 | ✓           | 11.209 | 697  | ✓      | 0.325 | 98   | To Do            |
| Kamke 1723 | ✓           | 0.697  | 130  | ✓      | 0.092 | 16   | To Do            |
| Kamke 1724 | ✓           | 0.289  | 38   | ✓      | 0.467 | 21   | To Do            |
| Kamke 1725 | ✓           | 2.241  | 75   | ✓      | 0.438 | 105  | To Do            |
| Kamke 1726 | ✓           | 0.252  | 77   | ✓      | 0.082 | 39   | To Do            |
| Kamke 1727 | ✓           | 0.354  | 166  | ✓      | 0.322 | 823  | To Do            |
| Kamke 1728 | ✓           | 0.004  | 31   | ✓      | 0.026 | 24   | To Do            |
| Kamke 1729 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1730 | ✓           | 0.726  | 135  | ✓      | 0.063 | 53   | To Do            |

Continued on next page

Table 3 – continued from previous page

| #          | Mathematica |        |      | Maple  |       |      | note on solution |
|------------|-------------|--------|------|--------|-------|------|------------------|
|            | solved      | cpu    | leaf | solved | cpu   | leaf |                  |
| Kamke 1731 | ✓           | 1.001  | 351  | ✓      | 0.085 | 61   | To Do            |
| Kamke 1732 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1733 | ✓           | 1.92   | 437  | ✓      | 0.069 | 71   | To Do            |
| Kamke 1734 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1735 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1736 | ✓           | 6.147  | 129  | ✓      | 0.069 | 49   | To Do            |
| Kamke 1737 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1738 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1739 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1740 | ✓           | 0.091  | 16   | ✓      | 0.029 | 13   | To Do            |
| Kamke 1741 | ✓           | 0.109  | 17   | ✓      | 0.056 | 34   | To Do            |
| Kamke 1742 | ✗           | 0      | 0    | ✓      | 0.158 | 60   | To Do            |
| Kamke 1743 | ✓           | 18.048 | 2761 | ✓      | 0.073 | 71   | To Do            |
| Kamke 1744 | ✓           | 0.533  | 155  | ✓      | 0.308 | 823  | To Do            |
| Kamke 1745 | ✓           | 0.704  | 251  | ✓      | 0.402 | 117  | To Do            |
| Kamke 1746 | ✗           | 0      | 0    | ✓      | 0.305 | 207  | To Do            |
| Kamke 1747 | ✓           | 0.099  | 20   | ✓      | 0.013 | 17   | To Do            |
| Kamke 1748 | ✓           | 0.179  | 43   | ✓      | 0.099 | 67   | To Do            |
| Kamke 1749 | ✓           | 0.457  | 153  | ✓      | 0.295 | 57   | To Do            |
| Kamke 1750 | ✓           | 3.565  | 2281 | ✓      | 0.313 | 87   | To Do            |
| Kamke 1751 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1752 | ✓           | 0.151  | 26   | ✓      | 0.074 | 33   | To Do            |
| Kamke 1753 | ✓           | 0.305  | 43   | ✓      | 0.105 | 147  | To Do            |
| Kamke 1754 | ✓           | 0.11   | 17   | ✓      | 0.032 | 15   | To Do            |
| Kamke 1755 | ✓           | 6.299  | 716  | ✓      | 0.217 | 418  | To Do            |
| Kamke 1756 | ✓           | 0.631  | 211  | ✓      | 0.115 | 75   | To Do            |
| Kamke 1757 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1758 | ✓           | 0.327  | 36   | ✓      | 0.05  | 42   | To Do            |
| Kamke 1759 | ✓           | 0.131  | 18   | ✓      | 0.023 | 31   | To Do            |
| Kamke 1760 | ✓           | 0.078  | 108  | ✓      | 0.056 | 114  | To Do            |
| Kamke 1761 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1762 | ✗           | 0      | 0    | ✓      | 0.703 | 108  | To Do            |
| Kamke 1763 | ✓           | 0.221  | 40   | ✓      | 0.045 | 148  | To Do            |
| Kamke 1764 | ✓           | 0.051  | 52   | ✓      | 0.135 | 18   | To Do            |
| Kamke 1765 | ✓           | 0.24   | 34   | ✓      | 0.025 | 27   | To Do            |
| Kamke 1766 | ✓           | 0.141  | 21   | ✓      | 0.029 | 64   | To Do            |
| Kamke 1767 | ✓           | 0.263  | 55   | ✓      | 0.276 | 50   | To Do            |
| Kamke 1768 | ✓           | 0.039  | 53   | ✓      | 0.04  | 43   | To Do            |
| Kamke 1769 | ✓           | 0.133  | 18   | ✓      | 0.033 | 21   | To Do            |
| Kamke 1770 | ✓           | 0.585  | 28   | ✓      | 0.045 | 26   | To Do            |
| Kamke 1771 | ✓           | 0.1    | 21   | ✓      | 0.128 | 22   | To Do            |
| Kamke 1772 | ✓           | 0.56   | 37   | ✓      | 0.116 | 37   | To Do            |
| Kamke 1773 | ✓           | 0.31   | 44   | ✓      | 0.026 | 30   | To Do            |
| Kamke 1774 | ✓           | 1.284  | 92   | ✓      | 0.191 | 136  | To Do            |
| Kamke 1775 | ✓           | 0.108  | 29   | ✓      | 0.071 | 31   | To Do            |
| Kamke 1776 | ✓           | 8.099  | 1743 | ✓      | 0.233 | 49   | To Do            |
| Kamke 1777 | ✗           | 0      | 0    | ✓      | 0.507 | 79   | To Do            |
| Kamke 1778 | ✓           | 0.192  | 65   | ✓      | 0.426 | 245  | To Do            |
| Kamke 1779 | ✗           | 0      | 0    | ✓      | 0.4   | 112  | To Do            |
| Kamke 1780 | ✗           | 0      | 0    | ✓      | 0.44  | 160  | To Do            |
| Kamke 1781 | ✓           | 0.18   | 46   | ✓      | 0.042 | 11   | To Do            |
| Kamke 1782 | ✓           | 0.156  | 93   | ✓      | 0.04  | 33   | To Do            |
| Kamke 1783 | ✓           | 1.035  | 26   | ✓      | 0.125 | 23   | To Do            |
| Kamke 1784 | ✓           | 0.259  | 74   | ✓      | 0.639 | 82   | To Do            |
| Kamke 1785 | ✓           | 0.315  | 95   | ✓      | 0.279 | 83   | To Do            |
| Kamke 1786 | ✓           | 0.06   | 53   | ✓      | 0.138 | 42   | To Do            |
| Kamke 1787 | ✓           | 0.908  | 170  | ✓      | 0.244 | 80   | To Do            |
| Kamke 1788 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1789 | ✗           | 0      | 0    | ✗      | 0     | 0    | To Do            |
| Kamke 1790 | ✓           | 0.616  | 186  | ✓      | 0.229 | 119  | To Do            |

Continued on next page

Table 3 – continued from previous page

| #          | Mathematica |        |       | Maple  |       |        | note on solution |
|------------|-------------|--------|-------|--------|-------|--------|------------------|
|            | solved      | cpu    | leaf  | solved | cpu   | leaf   |                  |
| Kamke 1791 | ✓           | 0.733  | 168   | ✓      | 0.246 | 90     | To Do            |
| Kamke 1792 | ✓           | 1.162  | 226   | ✓      | 0.351 | 194    | To Do            |
| Kamke 1793 | ✓           | 0.126  | 83    | ✓      | 0.049 | 40     | To Do            |
| Kamke 1794 | ✓           | 0.142  | 69    | ✓      | 0.062 | 46     | To Do            |
| Kamke 1795 | ✓           | 0.236  | 116   | ✓      | 1.019 | 529    | To Do            |
| Kamke 1796 | ✓           | 0.245  | 363   | ✓      | 0.185 | 51     | To Do            |
| Kamke 1797 | ✗           | 0      | 0     | ✗      | 0     | 0      | To Do            |
| Kamke 1798 | ✗           | 0      | 0     | ✓      | 0.179 | 166    | To Do            |
| Kamke 1799 | ✓           | 1.383  | 88    | ✓      | 0.099 | 46     | To Do            |
| Kamke 1800 | ✓           | 0.469  | 84    | ✓      | 0.036 | 60     | To Do            |
| Kamke 1801 | ✗           | 0      | 0     | ✗      | 0     | 0      | To Do            |
| Kamke 1802 | ✗           | 0      | 0     | ✗      | 0     | 0      | To Do            |
| Kamke 1803 | ✓           | 21.976 | 10387 | ✓      | 1.712 | 115620 | To Do            |
| Kamke 1804 | ✓           | 2.438  | 415   | ✓      | 0.026 | 31     | To Do            |
| Kamke 1805 | ✓           | 0.323  | 438   | ✓      | 0.033 | 34     | To Do            |
| Kamke 1806 | ✗           | 0      | 0     | ✓      | 2.356 | 733    | To Do            |
| Kamke 1807 | ✗           | 0      | 0     | ✗      | 0     | 0      | To Do            |
| Kamke 1808 | ✓           | 1.038  | 124   | ✓      | 0.105 | 72     | To Do            |
| Kamke 1809 | ✗           | 0      | 0     | ✓      | 0.342 | 336    | To Do            |
| Kamke 1810 | ✓           | 0.077  | 1677  | ✓      | 0.095 | 91     | To Do            |
| Kamke 1811 | ✗           | 0      | 0     | ✗      | 0     | 0      | To Do            |
| Kamke 1812 | ✓           | 0.264  | 29    | ✓      | 0.092 | 19     | To Do            |
| Kamke 1813 | ✓           | 31.532 | 176   | ✓      | 0.32  | 138    | To Do            |
| Kamke 1814 | ✓           | 0.345  | 120   | ✓      | 0.118 | 87     | To Do            |
| Kamke 1815 | ✗           | 0      | 0     | ✓      | 0.707 | 71     | To Do            |
| Kamke 1816 | ✗           | 0      | 0     | ✓      | 1.441 | 46     | To Do            |
| Kamke 1817 | ✗           | 0      | 0     | ✓      | 0.322 | 40     | To Do            |
| Kamke 1818 | ✗           | 0      | 0     | ✓      | 0.299 | 66     | To Do            |
| Kamke 1819 | ✗           | 0      | 0     | ✓      | 0.039 | 42     | To Do            |
| Kamke 1820 | ✗           | 0      | 0     | ✓      | 0.684 | 88     | To Do            |
| Kamke 1821 | ✗           | 0      | 0     | ✓      | 2.261 | 54     | To Do            |
| Kamke 1822 | ✓           | 0.861  | 371   | ✓      | 1.137 | 291    | To Do            |
| Kamke 1823 | ✗           | 0      | 0     | ✓      | 0.382 | 289    | To Do            |
| Kamke 1824 | ✓           | 0.721  | 347   | ✓      | 0.667 | 96     | To Do            |
| Kamke 1825 | ✗           | 0      | 0     | ✓      | 0.793 | 49     | To Do            |
| Kamke 1826 | ✓           | 0.796  | 201   | ✓      | 0.305 | 173    | To Do            |
| Kamke 1827 | ✗           | 0      | 0     | ✓      | 2.044 | 81     | To Do            |
| Kamke 1828 | ✓           | 0.008  | 32    | ✓      | 0.498 | 59     | To Do            |
| Kamke 1829 | ✓           | 0.006  | 24    | ✓      | 0.256 | 36     | To Do            |
| Kamke 1830 | ✓           | 0.023  | 24    | ✓      | 0.403 | 308    | To Do            |
| Kamke 1831 | ✗           | 0      | 0     | ✓      | 1.269 | 163    | To Do            |
| Kamke 1832 | ✗           | 0      | 0     | ✓      | 0.852 | 117    | To Do            |
| Kamke 1833 | ✗           | 0      | 0     | ✓      | 6.394 | 162    | To Do            |
| Kamke 1834 | ✗           | 0      | 0     | ✓      | 0.424 | 92     | To Do            |
| Kamke 1835 | ✓           | 0.08   | 143   | ✗      | 0     | 0      | To Do            |
| Kamke 1836 | ✗           | 0      | 0     | ✓      | 1.033 | 116    | To Do            |
| Kamke 1837 | ✗           | 0      | 0     | ✓      | 0.487 | 95     | To Do            |
| Kamke 1838 | ✗           | 0      | 0     | ✓      | 0.593 | 73     | To Do            |
| Kamke 1839 | ✗           | 0      | 0     | ✓      | 0.998 | 116    | To Do            |
| Kamke 1840 | ✗           | 0      | 0     | ✓      | 0.885 | 129    | To Do            |
| Kamke 1841 | ✗           | 0      | 0     | ✓      | 0.363 | 60     | To Do            |
| Kamke 1842 | ✓           | 0.16   | 286   | ✓      | 0.569 | 190    | To Do            |
| Kamke 1843 | ✓           | 2.038  | 409   | ✓      | 0.251 | 77     | To Do            |
| Kamke 1844 | ✗           | 0      | 0     | ✓      | 0.197 | 17     | To Do            |
| Kamke 1845 | ✗           | 0      | 0     | ✓      | 0.135 | 22     | To Do            |
| Kamke 1846 | ✓           | 0.036  | 51    | ✓      | 0.008 | 28     | To Do            |
| Kamke 1847 | ✓           | 0.173  | 95    | ✓      | 0.148 | 49     | To Do            |
| Kamke 1848 | ✓           | 0.403  | 187   | ✓      | 1.139 | 789    | To Do            |
| Kamke 1849 | ✓           | 0.608  | 426   | ✓      | 0.115 | 197    | To Do            |
| Kamke 1850 | ✗           | 0      | 0     | ✓      | 1.495 | 164    | To Do            |

Continued on next page

Table 3 – continued from previous page

| #          | Mathematica |        |       | Maple  |        |       | note on solution |
|------------|-------------|--------|-------|--------|--------|-------|------------------|
|            | solved      | cpu    | leaf  | solved | cpu    | leaf  |                  |
| Kamke 1851 | ✗           | 0      | 0     | ✗      | 0      | 0     | To Do            |
| Kamke 1852 | ✓           | 0.108  | 28    | ✓      | 0.128  | 28    | To Do            |
| Kamke 1853 | ✗           | 0      | 0     | ✓      | 0.589  | 110   | To Do            |
| Kamke 1854 | ✗           | 0      | 0     | ✗      | 0      | 0     | To Do            |
| Kamke 1855 | ✗           | 0      | 0     | ✗      | 0      | 0     | To Do            |
| Kamke 1856 | ✓           | 0.007  | 22    | ✓      | 0.029  | 19    | To Do            |
| Kamke 1857 | ✓           | 0.006  | 39    | ✓      | 0.038  | 35    | To Do            |
| Kamke 1858 | ✓           | 0.007  | 182   | ✓      | 0.04   | 64    | To Do            |
| Kamke 1859 | ✓           | 0.004  | 51    | ✓      | 0.029  | 38    | To Do            |
| Kamke 1860 | ✓           | 0.032  | 696   | ✓      | 0.057  | 177   | To Do            |
| Kamke 1861 | ✓           | 0.009  | 183   | ✓      | 0.077  | 152   | To Do            |
| Kamke 1862 | ✓           | 0.01   | 52    | ✓      | 0.027  | 39    | To Do            |
| Kamke 1863 | ✓           | 0.006  | 84    | ✓      | 0.024  | 35    | To Do            |
| Kamke 1864 | ✓           | 0.009  | 59    | ✓      | 0.029  | 44    | To Do            |
| Kamke 1865 | ✓           | 0.804  | 2062  | ✓      | 0.094  | 224   | To Do            |
| Kamke 1866 | ✓           | 0.037  | 132   | ✓      | 0.025  | 39    | To Do            |
| Kamke 1867 | ✓           | 0.088  | 124   | ✓      | 0.026  | 42    | To Do            |
| Kamke 1868 | ✓           | 0.077  | 162   | ✓      | 0.053  | 64    | To Do            |
| Kamke 1869 | ✓           | 0.084  | 118   | ✓      | 0.046  | 51    | To Do            |
| Kamke 1870 | ✓           | 0.138  | 122   | ✓      | 0.082  | 47    | To Do            |
| Kamke 1871 | ✓           | 0.317  | 180   | ✓      | 0.065  | 62    | To Do            |
| Kamke 1872 | ✓           | 0.114  | 162   | ✓      | 0.049  | 65    | To Do            |
| Kamke 1873 | ✓           | 0.091  | 322   | ✓      | 0.046  | 52    | To Do            |
| Kamke 1874 | ✓           | 0.007  | 115   | ✓      | 0.317  | 57    | To Do            |
| Kamke 1875 | ✗           | 0      | 0     | ✓      | 1.095  | 2606  | To Do            |
| Kamke 1876 | ✓           | 0.017  | 39    | ✓      | 0.105  | 18    | To Do            |
| Kamke 1877 | ✓           | 0.004  | 31    | ✓      | 0.024  | 31    | To Do            |
| Kamke 1878 | ✓           | 0.01   | 39    | ✓      | 0.042  | 39    | To Do            |
| Kamke 1879 | ✓           | 0.017  | 58    | ✓      | 0.035  | 54    | To Do            |
| Kamke 1880 | ✗           | 0      | 0     | ✓      | 0.059  | 23    | To Do            |
| Kamke 1881 | ✓           | 0.006  | 44    | ✓      | 0.021  | 48    | To Do            |
| Kamke 1882 | ✓           | 1.629  | 928   | ✓      | 0.062  | 99    | To Do            |
| Kamke 1883 | ✓           | 0.936  | 602   | ✓      | 0.086  | 80    | To Do            |
| Kamke 1884 | ✓           | 0.451  | 224   | ✓      | 0.073  | 69    | To Do            |
| Kamke 1885 | ✗           | 0      | 0     | ✓      | 0.075  | 47    | To Do            |
| Kamke 1886 | ✓           | 0.015  | 115   | ✓      | 0.041  | 49    | To Do            |
| Kamke 1887 | ✓           | 0.354  | 5748  | ✓      | 0.086  | 360   | To Do            |
| Kamke 1888 | ✓           | 16.103 | 35330 | ✓      | 0.141  | 457   | To Do            |
| Kamke 1889 | ✓           | 0.203  | 554   | ✓      | 0.033  | 60    | To Do            |
| Kamke 1890 | ✗           | 0      | 0     | ✗      | 0      | 0     | To Do            |
| Kamke 1891 | ✓           | 0.914  | 742   | ✓      | 0.033  | 64    | To Do            |
| Kamke 1892 | ✓           | 0.261  | 4815  | ✓      | 0.095  | 463   | To Do            |
| Kamke 1893 | ✓           | 0.443  | 5546  | ✓      | 0.711  | 1579  | To Do            |
| Kamke 1894 | ✓           | 0.45   | 3386  | ✓      | 0.506  | 1056  | To Do            |
| Kamke 1895 | ✓           | 0.295  | 7517  | ✓      | 0.151  | 1008  | To Do            |
| Kamke 1896 | ✓           | 0.496  | 1132  | ✓      | 0.039  | 67    | To Do            |
| Kamke 1897 | ✓           | 0.212  | 280   | ✓      | 0.118  | 86    | To Do            |
| Kamke 1898 | ✓           | 0.025  | 420   | ✓      | 0.046  | 71    | To Do            |
| Kamke 1899 | ✓           | 0.008  | 112   | ✓      | 0.056  | 52    | To Do            |
| Kamke 1900 | ✓           | 0.007  | 94    | ✓      | 0.058  | 50    | To Do            |
| Kamke 1901 | ✓           | 0.008  | 105   | ✓      | 0.049  | 43    | To Do            |
| Kamke 1902 | ✓           | 0.017  | 226   | ✓      | 0.047  | 51    | To Do            |
| Kamke 1903 | ✓           | 0.062  | 1304  | ✓      | 0.111  | 299   | To Do            |
| Kamke 1904 | ✓           | 0.046  | 1445  | ✓      | 0.071  | 257   | To Do            |
| Kamke 1905 | ✗           | 0      | 0     | ✗      | 0      | 0     | To Do            |
| Kamke 1906 | ✓           | 0.039  | 278   | ✓      | 0.055  | 120   | To Do            |
| Kamke 1907 | ✓           | 0.008  | 179   | ✓      | 0.046  | 66    | To Do            |
| Kamke 1908 | ✓           | 0.014  | 551   | ✓      | 0.417  | 1225  | To Do            |
| Kamke 1909 | ✓           | 0.041  | 1630  | ✓      | 14.222 | 33085 | To Do            |
| Kamke 1910 | ✓           | 0.007  | 39    | ✓      | 0.069  | 37    | To Do            |

Continued on next page

Table 3 – continued from previous page

| #          | Mathematica |         |       | Maple  |       |       | note on solution |
|------------|-------------|---------|-------|--------|-------|-------|------------------|
|            | solved      | cpu     | leaf  | solved | cpu   | leaf  |                  |
| Kamke 1911 | ✗           | 0       | 0     | ✓      | 0.106 | 309   | To Do            |
| Kamke 1912 | ✗           | 0       | 0     | ✓      | 1.556 | 2956  | To Do            |
| Kamke 1913 | ✓           | 0.019   | 64    | ✓      | 0.089 | 57    | To Do            |
| Kamke 1914 | ✓           | 0.249   | 204   | ✓      | 0.282 | 92    | To Do            |
| Kamke 1915 | ✗           | 0       | 0     | ✓      | 2.968 | 147   | To Do            |
| Kamke 1916 | ✓           | 0.34    | 557   | ✓      | 0.333 | 180   | To Do            |
| Kamke 1917 | ✓           | 213.617 | 3406  | ✓      | 0.696 | 109   | To Do            |
| Kamke 1918 | ✗           | 0       | 0     | ✓      | 1.627 | 184   | To Do            |
| Kamke 1919 | ✗           | 0       | 0     | ✓      | 2.577 | 203   | To Do            |
| Kamke 1920 | ✗           | 0       | 0     | ✓      | 2.559 | 205   | To Do            |
| Kamke 1921 | ✗           | 0       | 0     | ✗      | 0     | 0     | To Do            |
| Kamke 1922 | ✗           | 0       | 0     | ✗      | 0     | 0     | To Do            |
| Kamke 1923 | ✓           | 0.009   | 53    | ✓      | 0.03  | 35    | To Do            |
| Kamke 1924 | ✓           | 0.052   | 191   | ✓      | 0.24  | 180   | To Do            |
| Kamke 1925 | ✗           | 0       | 0     | ✓      | 0.207 | 194   | To Do            |
| Kamke 1926 | ✗           | 0       | 0     | ✓      | 0.105 | 96    | To Do            |
| Kamke 1927 | ✗           | 0       | 0     | ✗      | 0     | 0     | To Do            |
| Kamke 1928 | ✗           | 0       | 0     | ✗      | 0     | 0     | To Do            |
| Kamke 1929 | ✗           | 0       | 0     | ✓      | 2.447 | 116   | To Do            |
| Kamke 1930 | ✓           | 0.03    | 308   | ✓      | 0.04  | 45    | To Do            |
| Kamke 1931 | ✓           | 3.952   | 10101 | ✓      | 0.513 | 1117  | To Do            |
| Kamke 1932 | ✗           | 0       | 0     | ✓      | 0.741 | 383   | To Do            |
| Kamke 1933 | ✗           | 0       | 0     | ✓      | 1.892 | 17738 | To Do            |
| Kamke 1934 | ✗           | 0       | 0     | ✓      | 1.137 | 377   | To Do            |
| Kamke 1935 | ✗           | 0       | 0     | ✓      | 1.381 | 741   | To Do            |
| Kamke 1936 | ✗           | 0       | 0     | ✓      | 0.565 | 704   | To Do            |
| Kamke 1937 | ✗           | 0       | 0     | ✓      | 0.5   | 242   | To Do            |
| Kamke 1938 | ✓           | 0.007   | 137   | ✓      | 0.093 | 101   | To Do            |
| Kamke 1939 | ✗           | 0       | 0     | ✓      | 1.106 | 899   | To Do            |
| Kamke 1940 | ✗           | 0       | 0     | ✗      | 0     | 0     | To Do            |

## 2.1 ODE No. 1

---

$$y'(x) - \frac{1}{\sqrt{a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4}} = 0$$

✓ **Mathematica** : cpu = 0.424041 (sec), leaf count = 1117

$$\left\{ \left\{ y(x) \rightarrow c_1 - \frac{2F\left(\sin^{-1}\left(\sqrt{\frac{(x-\text{Root}[a_4\#1^4+a_3\#1^3+a_2\#1^2+a_1\#1+a_0\&,1])}((x-\text{Root}[a_4\#1^4+a_3\#1^3+a_2\#1^2+a_1\#1+a_0\&,2])\text{Root}[a_4\#1^4+a_3\#1^3+a_2\#1^2+a_1\#1+a_0\&,1])-\text{Root}[a_4\#1^4+a_3\#1^3+a_2\#1^2+a_1\#1+a_0\&,2])}\right)}{\left(\text{RootOf}\left(a_4\_Z^4+a_3\_Z^3+a_2\_Z^2+a_1\_Z+a_0, index=4\right)-\text{RootOf}\left(a_4\_Z^4+a_3\_Z^3+a_2\_Z^2+a_1\_Z+a_0, index=4\right)\right)}\right. \right.$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 1089

$$\left\{ y(x) = 2 \frac{(-\text{RootOf}(a_4\_Z^4 + a_3\_Z^3 + a_2\_Z^2 + a_1\_Z + a_0, index = 4) - \text{RootOf}(a_4\_Z^4 + a_3\_Z^3 + a_2\_Z^2 + a_1\_Z + a_0, index = 4))}{(\text{RootOf}(a_4\_Z^4 + a_3\_Z^3 + a_2\_Z^2 + a_1\_Z + a_0, index = 4) - \text{RootOf}(a_4\_Z^4 + a_3\_Z^3 + a_2\_Z^2 + a_1\_Z + a_0, index = 4))} \right.$$

Hand solution

$$y' - \frac{1}{\sqrt{a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4}} = 0 \tag{1}$$

To Do.

## 2.2 ODE No. 2

---

$$ay(x) + c \left(-e^{bx}\right) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0352105 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow \frac{ce^{x(a+b)-ax}}{a+b} + c_1e^{-ax} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 25

$$\left\{ y(x) = \left( \frac{ce^{(a+b)x}}{a+b} + \_C1 \right) e^{-ax} \right\}$$

Hand solution

$$\frac{dy}{dx} + ay(x) = ce^{bx} \tag{1}$$

Integrating factor  $\mu = e^{\int adx} = e^{ax}$ . Hence (1) becomes

$$\begin{aligned} \frac{d}{dx}(\mu y(x)) &= \mu ce^{bx} \\ \mu y(x) &= \int \mu ce^{bx} dx + C \end{aligned}$$

Replacing  $\mu$  by  $e^{ax}$

$$\begin{aligned} y(x) &= ce^{-ax} \int e^{(a+b)x} dx + Ce^{-ax} \\ &= ce^{-ax} \frac{e^{(a+b)x}}{a+b} + Ce^{-ax} \\ &= \frac{ce^{(a+b)x-ax}}{a+b} + Ce^{-ax} \end{aligned}$$

Can be reduced to

$$y(x) = c \frac{e^{bx}}{a+b} + Ce^{-ax}$$

## 2.3 ODE No. 3

---

$$ay(x) - b \sin(cx) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0547595 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow \frac{b(a \sin(cx) - c \cos(cx))}{a^2 + c^2} + c_1 e^{-ax} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 37

$$\left\{ y(x) = e^{-ax} \_C1 + \frac{b(\sin(cx) a - c \cos(cx))}{a^2 + c^2} \right\}$$

**Hand solution**

$$\frac{dy}{dx} + ay(x) = b \sin(cx) \quad (1)$$

Integrating factor  $\mu = e^{\int adx} = e^{ax}$ . Hence (1) becomes

$$\begin{aligned} \frac{d}{dx}(\mu y(x)) &= \mu b \sin(cx) \\ \mu y(x) &= b \int \mu \sin(cx) dx + C \end{aligned}$$

Replacing  $\mu$  by  $e^{ax}$

$$y(x) = b e^{-ax} \int e^{ax} \sin(cx) dx + C e^{-ax} \quad (2)$$

Using  $\sin(cx) = \frac{e^{icx} - e^{-icx}}{2i}$  then

$$\begin{aligned} \int e^{ax} \sin(cx) dx &= \int \frac{e^{(ic+a)x} - e^{(-ic+a)x}}{2i} dx \\ &= \frac{1}{2i} \left( \frac{e^{(ic+a)x}}{ic+a} - \frac{e^{(-ic+a)x}}{-ic+a} \right) \\ &= \frac{1}{2i} e^{ax} \left( \frac{e^{icx}}{ic+a} - \frac{e^{-icx}}{-ic+a} \right) \\ &= \frac{1}{2i} e^{ax} \left( \frac{e^{icx}(-ic+a) - e^{-icx}(ic+a)}{(ic+a)(-ic+a)} \right) \\ &= \frac{1}{2i} e^{ax} \left( \frac{-ice^{icx} + ae^{icx} - ice^{-icx} - ae^{-icx}}{(c^2 + a^2)} \right) \\ &= \frac{1}{2i} e^{ax} \left( \frac{-ic(e^{icx} + e^{-icx}) + a(e^{icx} - e^{-icx})}{(c^2 + a^2)} \right) \\ &= \frac{e^{ax}}{(c^2 + a^2)} \left( \frac{-ic(e^{icx} + e^{-icx})}{2i} + \frac{a(e^{icx} - e^{-icx})}{2i} \right) \\ &= \frac{e^{ax}}{(c^2 + a^2)} (-c \cos cx + a \sin cx) \end{aligned}$$

Therefore (2) becomes

$$\begin{aligned} y(x) &= b e^{-ax} \left[ \frac{e^{ax}}{(c^2 + a^2)} (-c \cos cx + a \sin cx) \right] + C e^{-ax} \\ &= \frac{b}{(c^2 + a^2)} (-c \cos cx + a \sin cx) + C e^{-ax} \end{aligned}$$

## 2.4 ODE No. 4

---

$$-e^{-x^2} x + y'(x) + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0134691 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-x^2} + \frac{1}{2} e^{-x^2} x^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.004 (sec), leaf count = 18

$$\left\{ y(x) = \left( \frac{x^2}{2} + \_C1 \right) e^{-x^2} \right\}$$



Hand solution

$$\frac{dy}{dx} + 2xy(x) = e^{-x^2}x \quad (1)$$

Integrating factor  $\mu = e^{\int 2xdx} = e^{x^2}$ . Hence (1) becomes

$$\begin{aligned} \frac{d}{dx} \left( e^{x^2} y(x) \right) &= e^{x^2} e^{-x^2} x \\ \frac{d}{dx} \left( e^{x^2} y(x) \right) &= x \end{aligned}$$

Integrating both sides

$$\begin{aligned} e^{x^2} y(x) &= \frac{x^2}{2} + C \\ y(x) &= e^{-x^2} \left( \frac{x^2}{2} + C \right) \end{aligned}$$

## 2.5 ODE No. 5

---

$$y'(x) + y(x) \cos(x) - e^{2x} = 0$$

✓ **Mathematica** : cpu = 0.381841 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow e^{-\sin(x)} \int_1^x e^{2K[1] + \sin(K[1])} dK[1] + c_1 e^{-\sin(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.081 (sec), leaf count = 21

$$\left\{ y(x) = e^{-\sin(x)} \left( \int e^{2x + \sin(x)} dx + \_C1 \right) \right\}$$

Hand solution

$$\frac{dy}{dx} + y(x) \cos(x) = e^{2x} \quad (1)$$

Integrating factor  $\mu = e^{\int \cos(x)dx} = e^{\sin(x)}$ . Hence (1) becomes

$$\frac{d}{dx} \left( e^{\sin(x)} y(x) \right) = e^{\sin(x)} e^{2x}$$

Integrating both sides

$$\begin{aligned} e^{\sin(x)} y(x) &= \int e^{\sin(x)} e^{2x} + C \\ y(x) &= e^{-\sin(x)} \int e^{2x + \sin(x)} + C e^{-\sin(x)} \end{aligned}$$

## 2.6 ODE No. 6

---

$$y'(x) + y(x) \cos(x) - \frac{1}{2} \sin(2x) = 0$$

✓ **Mathematica** : cpu = 0.0197433 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\sin(x)} + \sin(x) - 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 15

$$\left\{ y(x) = \sin(x) - 1 + e^{-\sin(x)} \_C1 \right\}$$

Hand solution

$$\frac{dy}{dx} + y(x) \cos(x) = \frac{1}{2} \sin(2x) \quad (1)$$

Integrating factor  $\mu = e^{\int \cos x dx} = e^{\sin(x)}$ . Therefore (1) becomes

$$\frac{d}{dx} \left( e^{\sin(x)} y(x) \right) = \frac{1}{2} e^{\sin(x)} \sin(2x)$$

Integrating

$$\begin{aligned} e^{\sin(x)} y(x) &= \frac{1}{2} \int e^{\sin(x)} \sin(2x) dx + C \\ y(x) &= \frac{e^{-\sin(x)}}{2} \int e^{\sin(x)} \sin(2x) dx + e^{-\sin(x)} C \end{aligned}$$

But  $e^{\sin(x)} \sin(2x)$  can be integrated by parts which gives  $e^{\sin(x)} (-2 + 2 \sin(x))$ . Hence the above becomes

$$\begin{aligned} y(x) &= \frac{e^{-\sin(x)}}{2} \left( e^{\sin(x)} (-2 + 2 \sin(x)) \right) + e^{-\sin(x)} C \\ &= -1 + \sin(x) + e^{-\sin(x)} C \end{aligned}$$

## 2.7 ODE No. 7

---

$$y'(x) + y(x) \cos(x) - e^{-\sin(x)} = 0$$

✓ **Mathematica** : cpu = 0.0329995 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\sin(x)} + x e^{-\sin(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 13

$$\left\{ y(x) = (x + \_C1) e^{-\sin(x)} \right\}$$

**Hand solution**

$$\frac{dy}{dx} + y(x) \cos(x) = e^{-\sin(x)} \quad (1)$$

Integrating factor  $\mu = e^{\int \cos x dx} = e^{\sin x}$ . Hence (1) becomes

$$\frac{d}{dx} (\mu y(x)) = \mu e^{-\sin(x)}$$

Replacing  $\mu$  by  $e^{\sin x}$  and integrating both sides

$$\begin{aligned} e^{\sin x} y(x) &= \int e^{\sin x} e^{-\sin(x)} dx + C \\ e^{\sin x} y(x) &= \int dx + C \\ e^{\sin x} y(x) &= x + C \\ y(x) &= x e^{-\sin x} + C e^{-\sin(x)} \end{aligned}$$

## 2.8 ODE No. 8

---

$$y'(x) + y(x) \tan(x) - \sin(2x) = 0$$

✓ **Mathematica** : cpu = 0.0186674 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow c_1 \cos(x) - 2 \cos^2(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 13

$$\{ y(x) = \cos(x) (-2 \cos(x) + \_C1) \}$$

**Hand solution**

$$\frac{dy}{dx} + y(x) \tan(x) = \sin(2x) \quad (1)$$

Integrating factor  $\mu = e^{\int \tan x dx} = e^{-\ln(\cos(x))} = \frac{1}{\cos(x)}$ . Hence (1) becomes

$$\frac{d}{dx} \left( y(x) \frac{1}{\cos(x)} \right) = \frac{1}{\cos(x)} \sin(2x)$$

Integrating both sides

$$\begin{aligned} y(x) \frac{1}{\cos(x)} &= \int \frac{1}{\cos(x)} \sin(2x) dx + C \\ y(x) &= \cos(x) \int \frac{\sin(2x)}{\cos(x)} dx + C \cos(x) \end{aligned}$$

But  $\sin(2x) = 2 \sin(x) \cos(x)$  hence

$$\begin{aligned} y(x) &= \cos(x) \int \frac{2 \sin(x) \cos(x)}{\cos(x)} dx + C \cos(x) \\ &= 2 \cos(x) \int \sin(x) dx + C \cos(x) \\ &= -2 \cos^2(x) + C \cos(x) \end{aligned}$$

## 2.9 ODE No. 9

---

$$y'(x) - y(x)(a + \sin(\log(x)) + \cos(\log(x))) = 0$$

✓ **Mathematica** : cpu = 0.015432 (sec), leaf count = 19

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{ax + x \sin(\log(x))} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 14

$$\left\{ y(x) = -C1 e^{x(\sin(\ln(x)) + a)} \right\}$$

**Hand solution**

$$\frac{dy}{dx} - y(x) [a + \sin(\log(x)) + \cos(\log(x))] = 0 \quad (1)$$

Integrating factor  $\mu = e^{-\int a - \sin(\log(x)) - \cos(\log(x)) dx} = e^{-ax} e^{-\int \sin(\log(x)) + \cos(\log(x)) dx}$ . To integrate  $\int \sin(\log(x)) + \cos(\log(x)) dx$ , let  $r = \log(x)$ ,  $\frac{dr}{dx} = \frac{1}{x}$ , then  $dx = x dr$ , But  $x = e^r$ , hence the integral becomes

$$\begin{aligned} \int \sin(\log(x)) + \cos(\log(x)) dx &= \int [\sin(r) + \cos(r)] e^r dr \\ &= \int e^r \sin(r) dr + \int e^r \cos(r) dr \end{aligned} \quad (2)$$

Integrating by parts  $\int e^r \cos(r) dr$ ,  $\int u dv = uv - \int v du$ , Let  $u = e^r \rightarrow du = e^r$  and  $dv = \cos(r) \rightarrow v = \sin(r)$ , hence (2) becomes

$$\begin{aligned} \int e^r \sin(r) dr + \int e^r \cos(r) dr &= \int e^r \sin(r) dr + e^r \sin(r) - \int \sin(r) e^r dr \\ &= e^r \sin(r) \end{aligned}$$

Therefore, substituting back  $r = \log(x)$  gives

$$\begin{aligned} \int \sin(\log(x)) + \cos(\log(x)) dx &= e^{\log(x)} \sin(\log(x)) \\ &= x \sin(\log(x)) \end{aligned}$$

Hence the integration factor is

$$\begin{aligned}\mu &= e^{-ax} e^{-\int \sin(\log(x)) + \cos(\log(x)) dx} \\ &= e^{-ax} e^{-x \sin(\log(x))}\end{aligned}$$

Therefore (1) becomes

$$\frac{d}{dx} (\mu y(x)) = 0$$

Integrating

$$\begin{aligned}y(x) e^{-ax} e^{-x \sin(\log(x))} &= C \\ y(x) &= C e^{ax} e^{x \sin(\log(x))} \\ &= C e^{ax + x \sin(\log(x))} \\ &= C e^{x(a + \sin(\log(x)))}\end{aligned}$$

## 2.10 ODE No. 10

---

$$y(x)f'(x) - f(x)f'(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0128871 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-f(x)} + f(x) - 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 15

$$\left\{ y(x) = f(x) - 1 + e^{-f(x)} C1 \right\}$$

**Hand solution**

$$\frac{dy}{dx} + y(x) \frac{df}{dx} = f(x) \frac{df}{dx} \tag{1}$$

Integrating factor  $\mu = e^{\int \frac{df}{dx} dx} = e^f$ . Therefore (1) becomes

$$\frac{d}{dx} \left( e^f y(x) \right) = e^f f(x) \frac{df}{dx}$$

Integrating

$$\begin{aligned}e^f y(x) &= \int e^f f(x) \frac{df}{dx} dx + C \\ y(x) &= e^{-f} \int e^f f df + e^{-f} C\end{aligned}$$

But  $\int e^f f df$  is the same as  $\int e^x x dx$  which by integration by parts gives  $e^x (x - 1)$  or in terms of  $f$ , gives  $e^f (f - 1)$ . Hence the above becomes

$$\begin{aligned}y(x) &= e^{-f} \left( e^f (f - 1) \right) + e^{-f} C \\ &= f - 1 + e^{-f} C\end{aligned}$$

## 2.11 ODE No. 11

---

$$f(x)y(x) - g(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0191528 (sec), leaf count = 66

$$\left\{ \left\{ y(x) \rightarrow c_1 \exp \left( \int_1^x -f(K[1]) dK[1] \right) + \exp \left( \int_1^x -f(K[1]) dK[1] \right) \int_1^x \exp \left( - \int_1^{K[2]} -f(K[1]) dK[1] \right) g(K[2] \right. \right.$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 24

$$\left\{ y(x) = \left( \int g(x) e^{\int f(x) dx} dx + C1 \right) e^{\int -f(x) dx} \right\}$$

**Hand solution**

$$\frac{dy}{dx} + y(x) f(x) = g(x) \quad (1)$$

Integrating factor  $\mu = e^{\int f(x)dx}$ . Therefore (1) becomes

$$\frac{d}{dx} \left( e^{\int f(x)dx} y(x) \right) = e^{\int f(x)dx} g(x)$$

Integrating

$$\begin{aligned} e^{\int f(x)dx} y(x) &= \int e^{\int f(x)dx} g(x) dx + C \\ y(x) &= e^{-\int f(x)dx} \int e^{\int f(x)dx} g(x) dx + e^{-\int f(x)dx} C \\ &= \left( \int e^{\int f(x)dx} g(x) dx + C \right) e^{-\int f(x)dx} \end{aligned}$$

## 2.12 ODE No. 12

---

$$y'(x) + y(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.0718015 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{2x} - e^{2C_1}}{e^{2C_1} + e^{2x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 8

$$\{y(x) = \tanh(x + \_C1)\}$$

**Hand solution**

$$\begin{aligned} \frac{dy}{dx} + y^2(x) - 1 &= 0 \\ \frac{dy}{dx} &= 1 - y^2(x) \end{aligned} \quad (1)$$

This is separable. Hence

$$\begin{aligned} \frac{dy}{dx} \frac{1}{1 - y^2(x)} &= 1 \\ \frac{dy}{1 - y^2(x)} &= dx \end{aligned}$$

Integrating

$$\int \frac{dy}{1 - y^2(x)} = x + C$$

Using  $\int \frac{1}{a+by^2} dy = \frac{\sqrt{-\frac{a}{b}} \tanh^{-1}\left(\frac{y}{\sqrt{-\frac{a}{b}}}\right)}{a}$  and since  $a = 1, b = -1$ , then  $\int \frac{dy}{1-y^2(x)} = \tanh^{-1}(y)$  and the above becomes

$$\tanh^{-1}(y) = x + C$$

Therefore

$$y = \tanh(x + C) \quad (2)$$

In terms of exponential, since  $\tanh(u) = \frac{e^u - e^{-u}}{e^u + e^{-u}}$  then (2) can also be written as

$$y = \frac{e^{x+C} - e^{-(x+C)}}{e^{x+C} + e^{-(x+C)}} = \frac{e^x e^C - e^{-x} e^{-C}}{e^x e^C + e^{-x} e^{-C}}$$

Multiplying numerator and denominator by  $e^{-C} e^x$

$$y = \frac{e^{2x} - e^{-2C}}{e^{2x} + e^{-2C}}$$

To get same answer as Mathematica, since  $C$  is constant, let  $C_1 = -C$ , then

$$y = \frac{e^{2x} - e^{2C_1}}{e^{2x} + e^{2C_1}}$$

## 2.13 ODE No. 13

---

$$-ax - b + y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0743389 (sec), leaf count = 79

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt[3]{a}c_1 \text{Ai}'\left(\frac{b+ax}{a^{2/3}}\right) + \sqrt[3]{a}\text{Bi}'\left(\frac{b+ax}{a^{2/3}}\right)}{-c_1 \text{Ai}\left(\frac{b+ax}{a^{2/3}}\right) - \text{Bi}\left(\frac{b+ax}{a^{2/3}}\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.132 (sec), leaf count = 79

$$\left\{ y(x) = -i\sqrt[3]{-ia} \left( \text{Ai}^{(1)}\left(-(ax+b)(-ia)^{-\frac{2}{3}}\right) C1 + \text{Bi}^{(1)}\left(-(ax+b)(-ia)^{-\frac{2}{3}}\right) \right) \left( \text{Ai}\left(-(ax+b)(-ia)^{-\frac{2}{3}}\right) C1 + \text{Bi}\left(-(ax+b)(-ia)^{-\frac{2}{3}}\right) \right) \right\}$$

**Hand solution**

$$\begin{aligned} y'(x) + y^2(x) - ax - b &= 0 \\ y'(x) &= b + ax - y^2(x) \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE of the form

$$y'(x) = P(x) + Q(x)y + R(x)y^2(x) \tag{2}$$

where in this case  $Q(x) = 0, R(x) = -1, P(x) = b + ax$ . We can solve this in two ways. If we know one particular solution  $y_p(x)$  for (1) then we use the substitution  $y = y_p + \frac{1}{u}$  and convert (1) to new associated linear ODE of the form  $u' + (Q(x) + 2R(x))y_p + R(x) = 0$ . If we do not know a particular solution, then we use the standard substitution  $y = \frac{-u'}{uR(x)} = \frac{u'}{u}$  since  $R(x) = -1$  and this is what we will do here.

Since  $u' = yu$  then

$$\begin{aligned} u'' &= yu' + y'u \\ &= y(yu) + (b + ax - y^2)u \\ &= y^2u + (b + ax)u - y^2u \\ &= (b + ax)u \end{aligned}$$

So we have new second order ODE

$$u'' - (b + ax)u = 0 \tag{3}$$

which we solve for  $u$ . This ODE is of the form  $u'' - q(x)u = 0$  which has solutions in terms of Airy function of first  $Ai(x)$  and second kind  $Bi(x)$ , where

$$\begin{aligned} Ai(x) &= \frac{1}{\pi} \int_0^\infty \cos\left(\frac{t^3}{3} + xt\right) dt \\ Bi(x) &= \frac{1}{\pi} \int_0^\infty \exp\left(-\frac{t^3}{3} + xt\right) + \sin\left(\frac{t^3}{3} + xt\right) dt \end{aligned}$$

Therefore the solution to (3) is

$$u(x) = c_1 Ai\left(\frac{b+ax}{a^{2/3}}\right) + c_2 Bi\left(\frac{b+ax}{a^{2/3}}\right)$$

We need to find  $u'(x)$  now. Using  $Ai'(x), Bi'(x)$  for derivative of Airy functions of first and second kind, then

$$u'(x) = c_1 Ai'\left(\frac{b+ax}{a^{2/3}}\right) a^{1/3} + c_2 Bi'\left(\frac{b+ax}{a^{2/3}}\right) a^{1/3}$$

Therefore since  $u' = yu$  then

$$\begin{aligned} y &= \frac{u'}{u} \\ &= \frac{c_1 Ai'\left(\frac{b+ax}{a^{2/3}}\right) a^{1/3} + c_2 Bi'\left(\frac{b+ax}{a^{2/3}}\right) a^{1/3}}{c_1 Ai\left(\frac{b+ax}{a^{2/3}}\right) + c_2 Bi\left(\frac{b+ax}{a^{2/3}}\right)} \end{aligned}$$

Let  $C_1 = \frac{c_1}{c_2}$  then the above can be written as

$$y = \frac{C_1 A' i \left( \frac{b+ax}{a^{\frac{2}{3}}} \right) a^{\frac{1}{3}} + B i' \left( \frac{b+ax}{a^{\frac{2}{3}}} \right) a^{\frac{1}{3}}}{C_1 A i \left( \frac{b+ax}{a^{\frac{2}{3}}} \right) + B i \left( \frac{b+ax}{a^{\frac{2}{3}}} \right)}$$

Reference: Airy function

## 2.14 ODE No. 14

$$ax^m + y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0853086 (sec), leaf count = 254

$$\left\{ \left\{ y(x) \rightarrow - \frac{i \sqrt{-ax}^{\frac{m+2}{2}} \left( c_1 J_{\frac{m+1}{m+2}} \left( \frac{2i \sqrt{-ax}^{\frac{m+1}{2}}}{m+2} \right) - c_1 J_{-\frac{m+3}{m+2}} \left( \frac{2i \sqrt{-ax}^{\frac{m+2}{2}}}{m+2} \right) - 2 J_{\frac{1}{m+2}-1} \left( \frac{2i \sqrt{-ax}^{\frac{m+2}{2}}}{m+2} \right) \right) - c_1 J_{-\frac{m+3}{m+2}} \left( \frac{2i \sqrt{-ax}^{\frac{m+2}{2}}}{m+2} \right)}{2x \left( c_1 J_{-\frac{1}{m+2}} \left( \frac{2i \sqrt{-ax}^{\frac{m+2}{2}}}{m+2} \right) + J_{\frac{1}{m+2}} \left( \frac{2i \sqrt{-ax}^{\frac{m+2}{2}}}{m+2} \right) \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 187

$$\left\{ y(x) = \frac{1}{x} \left( -J_{\frac{3+m}{m+2}} \left( 2 \frac{\sqrt{ax}^{m/2+1}}{m+2} \right) \sqrt{ax}^{\frac{m}{2}+1} - C_1 - Y_{\frac{3+m}{m+2}} \left( 2 \frac{\sqrt{ax}^{m/2+1}}{m+2} \right) \sqrt{ax}^{\frac{m}{2}+1} + C_1 J_{(m+2)^{-1}} \left( 2 \frac{\sqrt{ax}^{m/2+1}}{m+2} \right) \right) \right\}$$

**Hand solution**

$$\begin{aligned} y'(x) + y^2(x) + ax^m &= 0 \\ y'(x) &= -ax^m - y^2(x) \end{aligned} \quad (1)$$

This is Riccati first order non-linear ODE of the form

$$y'(x) = P(x) + Q(x)y + R(x)y^2(x) \quad (2)$$

where in this case  $Q(x) = 0, R(x) = -1, P(x) = -ax^m$ . We can solve this in two ways. If we know one particular solution  $y_p(x)$  for (1) then we use the substitution  $y = y_p + \frac{1}{u}$  and convert (1) to new associated linear ODE of the form  $u' + (Q(x) + 2R(x))y_p + R(x) = 0$ . If we do not know a particular solution, then we use the standard substitution  $y = \frac{-u'}{uR(x)} = \frac{u'}{u}$  since  $R(x) = -1$  and this is what we will do here.

Since  $u' = yu$  then

$$\begin{aligned} u'' &= yu' + y'u \\ &= y(yu) + (-ax^m - y^2)u \\ &= y^2u - ax^m u - y^2u \\ &= -ax^m u \end{aligned}$$

So we have new second order ODE

$$u'' + ax^m u = 0 \quad (3)$$

which we solve for  $u$ . This is Airy ODE but with a positive sign. Of the form  $u'' + q(x)u = 0$ . Recall that the solution to  $u'' - axu = 0$  is

$$u = c_1 Ai \left( a^{\frac{1}{3}} x \right) + c_2 Bi \left( a^{\frac{1}{3}} x \right)$$

When  $x$  has power on it (there are restriction on what values the power can take), the solution is written in terms of Bessel functions. The solution to  $u'' - ax^m u = 0$  is

$$u = c_1 \sqrt{x} BesselI \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) + c_2 \sqrt{x} BesselK \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right)$$

When the sign is positive, the solution to  $u'' + ax^m u = 0$  is

$$u(x) = c_1 \sqrt{x} BesselJ \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) + c_2 \sqrt{x} BesselY \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) \quad (4)$$

We need to find  $u'(x)$  now. From (4)

$$\frac{d}{dx} \left[ c_1 \sqrt{x} \text{Bessel}J \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) \right] = c_1 \frac{\text{Bessel}J \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) - \sqrt{ax}^{\frac{m+1}{2}} \text{Bessel}J \left( \frac{m+3}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right)}{\sqrt{x}}$$

And

$$\frac{d}{dx} \left[ c_2 \sqrt{x} \text{Bessel}Y \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) \right] = c_2 \frac{\text{Bessel}Y \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) - \sqrt{ax}^{\frac{m+1}{2}} \text{Bessel}Y \left( \frac{m+3}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right)}{\sqrt{x}}$$

Therefore

$$u'(x) = c_1 \frac{\text{Bessel}J \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) - \sqrt{ax}^{\frac{m+1}{2}} \text{Bessel}J \left( \frac{m+3}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right)}{\sqrt{x}} \\ + c_2 \frac{\text{Bessel}Y \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) - \sqrt{ax}^{\frac{m+1}{2}} \text{Bessel}Y \left( \frac{m+3}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right)}{\sqrt{x}}$$

Since  $u' = yu$  then

$$y = \frac{u'}{u} \\ = \frac{c_1 \frac{\text{Bessel}J \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) - \sqrt{ax}^{\frac{m+1}{2}} \text{Bessel}J \left( \frac{m+3}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right)}{\sqrt{x}} + c_2 \frac{\text{Bessel}Y \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) - \sqrt{ax}^{\frac{m+1}{2}} \text{Bessel}Y \left( \frac{m+3}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right)}{\sqrt{x}}}{c_1 \sqrt{x} \text{Bessel}J \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) + c_2 \sqrt{x} \text{Bessel}Y \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right)} \\ = \frac{c_1 \left[ \text{Bessel}J \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) - \sqrt{ax}^{\frac{m+1}{2}} \text{Bessel}J \left( \frac{m+3}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) \right] + c_2 \left[ \text{Bessel}Y \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) - \sqrt{ax}^{\frac{m+1}{2}} \text{Bessel}Y \left( \frac{m+3}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) \right]}{\sqrt{x} \left[ c_1 \sqrt{x} \text{Bessel}J \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) + c_2 \sqrt{x} \text{Bessel}Y \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) \right]} \\ = \frac{c_1 \left[ \text{Bessel}J \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) - \sqrt{ax}^{\frac{m+1}{2}} \text{Bessel}J \left( \frac{m+3}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) \right] + c_2 \left[ \text{Bessel}Y \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) - \sqrt{ax}^{\frac{m+1}{2}} \text{Bessel}Y \left( \frac{m+3}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) \right]}{c_1 x \text{Bessel}J \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) + c_2 x \text{Bessel}Y \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right)}$$

Let  $C_1 = \frac{c_1}{c_2}$  then the above can be written as

$$y = \frac{1}{x} \frac{C_1 \left[ \text{Bessel}J \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) - \sqrt{ax}^{\frac{m+1}{2}} \text{Bessel}J \left( \frac{m+3}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) \right] + \text{Bessel}Y \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) - \sqrt{ax}^{\frac{m+1}{2}} \text{Bessel}Y \left( \frac{m+3}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right)}{C_1 \text{Bessel}J \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right) + \text{Bessel}Y \left( \frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2} \right)}$$

## 2.15 ODE No. 15

$$x^4 - 2x^2 y(x) + y'(x) + y(x)^2 - 2x - 1 = 0$$

✓ **Mathematica** : cpu = 0.0665507 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 (-e^{2x}) - \frac{1}{2}} + x^2 + 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.136 (sec), leaf count = 35

$$\left\{ y(x) = \frac{x^2 (e^x)^2 - x^2 \_C1 + (e^x)^2 + \_C1}{(e^x)^2 - \_C1} \right\}$$

**Hand solution**

$$x^4 - 2x^2 y(x) + y'(x) + y^2(x) - 2x - 1 = 0$$

$$y'(x) = -x^4 + 2x + 1 + 2x^2 y(x) - y^2(x) \quad (1)$$

This is Riccati first order non-linear ODE of the form

$$y'(x) = P(x) + Q(x)y + R(x)y^2(x) \quad (2)$$



where in this case  $Q(x) = 2x^2$ ,  $R(x) = -1$ ,  $P(x) = -x^4 + 2x + 1$ .  
Let  $u = y - x^2$  or  $y = u + x^2$  then

$$\begin{aligned} u' &= y' - 2x \\ &= (-x^4 + 2x + 1 + 2x^2y - y^2) - 2x \\ &= \left(-x^4 + 2x + 1 + 2x^2(u + x^2) - (u + x^2)^2\right) - 2x \\ &= (-x^4 + 2x + 1 + 2x^2u + 2x^4 - (u^2 + x^4 + 2ux^2)) - 2x \\ &= -x^4 + 2x + 1 + 2x^2u + 2x^4 - u^2 - x^4 - 2ux^2 - 2x \\ &= 1 - u^2 \end{aligned}$$

Hence

$$u' = 1 - u^2$$

This is separable

$$\begin{aligned} \frac{du}{dx} &= 1 - u^2 \\ \frac{du}{1 - u^2} &= dx \end{aligned}$$

Integrating both sides

$$\begin{aligned} \tanh^{-1}(u) &= x + C \\ u(x) &= \tanh(x + C) \\ &= \frac{e^{x+C} - e^{-x-C}}{e^{x+C} + e^{-x-C}} \\ &= \frac{e^xe^C - e^{-x}e^{-C}}{e^xe^C + e^{-x}e^{-C}} \end{aligned}$$

Multiplying numerator and denominator by  $e^{-C}e^x$

$$u(x) = \frac{e^{2x} - e^{-2C}}{e^{2x} + e^{-2C}}$$

Let  $e^{-2C} = C_1$

$$u(x) = \frac{e^{2x} - C_1}{e^{2x} + C_1}$$

Since  $u = y - x^2$  then

$$\begin{aligned} y &= u + x^2 \\ &= \frac{e^{2x} - C_1}{e^{2x} + C_1} + x^2 \\ &= \frac{e^{2x} - C_1 + x^2e^{2x} + x^2C_1}{e^{2x} + C_1} \end{aligned}$$

To obtain same solution as Maple, we divide by  $C_1$

$$y = \frac{\frac{1}{C_1}e^{2x} - 1 + \frac{1}{C_1}x^2e^{2x} + x^2}{\frac{1}{C_1}e^{2x} + 1}$$

Let  $\frac{1}{C_1} = -C$  then

$$\begin{aligned} y &= \frac{-Ce^{2x} - 1 - Cx^2e^{2x} + x^2}{-Ce^{2x} + 1} \\ &= \frac{Ce^{2x} + 1 + Cx^2e^{2x} - x^2}{Ce^{2x} - 1} \end{aligned}$$

Which now agrees with the Maple solution form. Mathematica solution also verified to be correct.

## 2.16 ODE No. 16

$$f(x)(xy(x) - 1) + y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0366519 (sec), leaf count = 186

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 \exp\left(\int_1^x \frac{f(K[1])K[1]^2+2}{K[1]} dK[1]\right) + \exp\left(\int_1^x \frac{f(K[1])K[1]^2+2}{K[1]} dK[1]\right) \int_1^x \exp\left(-\int_1^{K[2]} \frac{f(K[1])K[1]^2+2}{K[1]} dK[1]\right)}{x \left(c_1 \exp\left(\int_1^x \frac{f(K[1])K[1]^2+2}{K[1]} dK[1]\right) + \exp\left(\int_1^x \frac{f(K[1])K[1]^2+2}{K[1]} dK[1]\right) \int_1^x \exp\left(-\int_1^{K[2]} \frac{f(K[1])K[1]^2+2}{K[1]} dK[1]\right)} \right. \right.$$

✓ **Maple** : cpu = 0.068 (sec), leaf count = 49

$$\left\{ y(x) = 1e^{\int \frac{-x^2 f(x) - 2}{x} dx} \left( -C1 + \int e^{\int \frac{-x^2 f(x) - 2}{x} dx} dx \right)^{-1} + x^{-1} \right\}$$

**Hand solution**

$$\begin{aligned} y' + y^2 + (xy - 1)f &= 0 \\ y'(x) &= (-xy + 1)f - y^2 \end{aligned} \quad (1)$$

This is Riccati first order non-linear ODE of the form. We can see a particular solution is  $y_p = \frac{1}{x}$ , therefore, we use the substitution

$$\begin{aligned} y(x) &= y_p(x) + \frac{1}{u(x)} \\ &= \frac{1}{x} + \frac{1}{u} \end{aligned}$$

Hence

$$\begin{aligned} y'(x) &= y_p'(x) - \frac{u'(x)}{u^2(x)} \\ &= \frac{-1}{x^2} - \frac{u'(x)}{u^2(x)} \end{aligned} \quad (2)$$

Equating (1) and (2) gives

$$\begin{aligned} (-xy + 1)f - y^2 &= \frac{-1}{x^2} - \frac{u'}{u^2} \\ \left( -x \left( \frac{1}{x} + \frac{1}{u} \right) + 1 \right) f - \left( \frac{1}{x} + \frac{1}{u} \right)^2 &= \frac{-1}{x^2} - \frac{u'}{u^2} \\ \left( \left( -1 - \frac{x}{u} \right) + 1 \right) f - \left( \frac{1}{x^2} + \frac{1}{u^2} + \frac{2}{xu} \right) &= \frac{-1}{x^2} - \frac{u'}{u^2} \\ -\frac{x}{u}f - \left( \frac{1}{x^2} + \frac{1}{u^2} + \frac{2}{xu} \right) &= \frac{-1}{x^2} - \frac{u'}{u^2} \\ -\frac{x}{u}f - \frac{1}{x^2} - \frac{1}{u^2} - \frac{2}{xu} &= \frac{-1}{x^2} - \frac{u'}{u^2} \\ -xuf - 1 - \frac{2u}{x} &= -u' \\ u' &= xuf + 1 + \frac{2u}{x} \end{aligned}$$

Hence

$$u' - \left( xf + \frac{2}{x} \right) u = 1$$

Integrating factor is  $\mu = e^{\int (xf + \frac{2}{x}) dx}$ , hence the solution is

$$d(\mu u) = \mu$$

Integrating both sides

$$\begin{aligned} \mu u &= \int \mu dx + C \\ u &= e^{-\int (xf + \frac{2}{x}) dx} \int e^{\int (xf + \frac{2}{x}) dx} dx + C e^{-\int (xf + \frac{2}{x}) dx} \\ &= e^{-\int (xf + \frac{2}{x}) dx} \left( \int e^{\int (xf + \frac{2}{x}) dx} dx + C \right) \end{aligned}$$

Hence

$$\begin{aligned} y &= y_p + \frac{1}{u} \\ &= \frac{1}{x} + \frac{1}{e^{-\int (xf + \frac{2}{x}) dx} \left( \int e^{\int (xf + \frac{2}{x}) dx} dx + C \right)} \end{aligned}$$

Hence

$$y(x) = \frac{1}{x} + e^{\int (xf + \frac{2}{x}) dx} \left( \int e^{\int (xf + \frac{2}{x}) dx} dx + C \right)^{-1}$$

## 2.17 ODE No. 17

---

$$y'(x) - y(x)^2 - 3y(x) + 4 = 0$$

✓ **Mathematica** : cpu = 0.0525925 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow \frac{-4e^{5c_1+5x} - 1}{e^{5c_1+5x} - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.092 (sec), leaf count = 24

$$\left\{ y(x) = \frac{-4e^{5x}C_1 - 1}{-1 + e^{5x}C_1} \right\}$$

**Hand solution**

$$\begin{aligned} y' - y^2 - 3y + 4 &= 0 \\ y' &= 3y - 4 + y^2 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE of the form. The general form is

$$y' = P(x) + Q(x)y + R(x)y^2$$

Where  $P(x) = -4$ ,  $Q(x) = 3$ ,  $R(x) = 1$ . Using the substitution  $y = -\frac{u'}{uR(x)} = \frac{-u'}{u}$  then

$$\begin{aligned} u' &= -yu \\ u'' &= -yu' - y'u \\ &= -y(-yu) - (3y - 4 + y^2)u \\ &= y^2u - 3\left(-\frac{u'}{u}\right)u + 4u - y^2u \\ &= 3u' + 4u \end{aligned}$$

Hence

$$u'' - 3u' - 4u = 0$$

This is standard second order ODE. The characteristic equation is  $\lambda^2 - 3\lambda - 4 = 0$ , with roots  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 + 16}}{2} = \frac{3 \pm 5}{2} = \{4, -1\}$ , hence

$$u(x) = c_1e^{4x} + c_2e^{-x}$$

And

$$u'(x) = c_14e^{4x} - c_2e^{-x}$$

Since  $y = \frac{-u'}{u}$  then

$$\begin{aligned} y(x) &= \frac{-c_14e^{4x} + c_2e^{-x}}{c_1e^{4x} + c_2e^{-x}} \\ &= \frac{-\frac{c_1}{c_2}4e^{4x} + e^{-x}}{\frac{c_1}{c_2}e^{4x} + e^{-x}} \end{aligned}$$

Let  $\frac{c_1}{c_2} = C_1$  then

$$y(x) = \frac{-4C_1e^{4x} + e^{-x}}{C_1e^{4x} + e^{-x}}$$

Dividing by  $e^{-x}$

$$y(x) = \frac{-4C_1e^{5x} + 1}{C_1e^{5x} + 1}$$

This is the same result given by CAS. To see it better, let  $C_2 = -C_1$  then the above becomes

$$\begin{aligned} y(x) &= \frac{4C_2e^{5x} + 1}{-C_2e^{5x} + 1} \\ &= -\frac{4C_2e^{5x} + 1}{C_2e^{5x} - 1} \end{aligned}$$

$$y'(x) - y(x)^2 - xy(x) - x + 1 = 0$$

✓ **Mathematica** : cpu = 0.0728682 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{\frac{x^2}{2}-2x}}{c_1 - \frac{\sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\frac{x-2}{\sqrt{2}}\right)}{e^2}} - 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 39

$$\left\{ y(x) = -1 + \frac{1}{-C1 + \frac{i}{2}\sqrt{\pi}e^{-2}\sqrt{2}\operatorname{Erf}\left(\frac{i}{2}\sqrt{2}(x-2)\right)} e^{\frac{x(x-4)}{2}} \right\}$$

**Hand solution**

$$\begin{aligned} y' - y^2 - xy - x + 1 &= 0 \\ y' &= x - 1 + xy + y^2 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE of the form. The general form is

$$y' = P(x) + Q(x)y + R(x)y^2$$

Where  $P(x) = x-1$ ,  $Q(x) = x$ ,  $R(x) = 1$ . We see that  $y_p = -1$  is a particular solution, therefore we use the substitution  $y = y_p + \frac{1}{u}$ , hence  $y' = -\frac{u'}{u^2}$  and equating this to (1) we obtain

$$\begin{aligned} -\frac{u'}{u^2} &= x - 1 + xy + y^2 \\ &= x - 1 + x\left(-1 + \frac{1}{u}\right) + \left(-1 + \frac{1}{u}\right)^2 \\ &= x - 1 - x + \frac{x}{u} + \left(1 + \frac{1}{u^2} - \frac{2}{u}\right) \\ &= \frac{x}{u} + \frac{1}{u^2} - \frac{2}{u} \end{aligned}$$

Hence

$$\begin{aligned} u' &= -u^2 \left( \frac{x}{u} + \frac{1}{u^2} - \frac{2}{u} \right) \\ &= -xu - 1 + 2u \\ u' + xu - 2u &= -1 \\ u' + u(x-2) &= -1 \end{aligned}$$

Integration factor is  $e^{\int (x-2)dx} = e^{\frac{x^2}{2}-2x} = e^{\frac{1}{2}x(x-4)}$ , therefore

$$d\left(e^{\frac{1}{2}x(x-4)}u\right) = -e^{\frac{1}{2}x(x-4)}$$

Integrating both sides

$$e^{\frac{1}{2}x(x-4)}u = -\int e^{\frac{1}{2}x(x-4)} + C$$

But

$$\int e^{\frac{1}{2}x(x-4)} = \frac{1}{e^2} \sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\frac{x-2}{\sqrt{2}}\right)$$

Hence

$$u(x) = e^{\frac{-1}{2}x(x-4)} \left( \frac{-1}{e^2} \sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\frac{x-2}{\sqrt{2}}\right) + C \right)$$

Since  $y = y_p + \frac{1}{u}$  then

$$y = -1 + \frac{1}{e^{\frac{-1}{2}x(x-4)} \left( \frac{-1}{e^2} \sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\frac{x-2}{\sqrt{2}}\right) + C \right)}$$

Or

$$y = \frac{e^{\frac{1}{2}x(x-4)}}{C - \frac{1}{e^2} \sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\frac{x-2}{\sqrt{2}}\right)} - 1$$

## 2.19 ODE No. 19

---

$$y'(x) - (y(x) + x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0565716 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 e^{2ix - \frac{i}{2}}} - x - i \right\} \right\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 16

$$\{y(x) = -x - \tan(-x + \_C1)\}$$

**Hand solution**

$$\begin{aligned} y' - (y + x)^2 &= 0 \\ y' &= (y + x)^2 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE of the form. Let  $u = y + x$ , then  $u' = y' + 1$  and (1) becomes

$$\begin{aligned} u' - 1 &= u^2 \\ u' &= 1 + u^2 \end{aligned}$$

This is separable

$$\begin{aligned} \frac{du}{dx} \frac{1}{1 + u^2} &= 1 \\ \int \frac{du}{1 + u^2} &= \int dx \\ \tan^{-1} u &= x + C \\ u &= \tan(x + C) \end{aligned}$$

Since  $u = y + x$  then

$$y = \tan(x + C) - x$$

## 2.20 ODE No. 20

---

$$(x^2 + 1) y(x) + y'(x) - y(x)^2 - 2x = 0$$

✓ **Mathematica** : cpu = 0.074442 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{\frac{x^3}{3} + x}}{c_1 - \int_1^x e^{\frac{K[1]^3}{3} + K[1]} dK[1]} + x^2 + 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 34

$$\left\{ y(x) = x^2 + 1 + 1e^{\frac{x^3}{3} + x} \left( \_C1 - \int e^{\frac{x^3}{3} + x} dx \right)^{-1} \right\}$$

**Hand solution**

$$\begin{aligned} (x^2 + 1) y + y' - y^2 - 2x &= 0 \\ y' &= -(x^2 + 1) y + y^2 + 2x \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE of the form of the general form  $y' = P(x) + Q(x)y + R(x)y^2$  where  $P(x) = 2x, Q(x) = -(x^2 + 1), R(x) = 1$ . We can convert this to Bernoulli first order ODE in  $u(x)$ , which is little easier to solve by using  $u = y - x^2 - 1$ . The difference between Bernoulli and Riccati is that the term  $P(x) = 0$  in Bernoulli. If  $P(x) \neq 0$  and  $R(x) \neq 0$  then it is called Riccati.

Using  $u = y - x^2 - 1$  gives

$$\begin{aligned}
 u' &= y' - 2x \\
 u' &= [- (x^2 + 1) y + y^2 + 2x] - 2x \\
 &= - (x^2 + 1) (u + x^2 + 1) + (u + x^2 + 1)^2 \\
 &= (u + x^2 + 1) [(u + x^2 + 1) - (x^2 + 1)] \\
 &= (u + x^2 + 1) u \\
 &= u^2 + u (1 + x^2)
 \end{aligned}$$

We see now this is Bernoulli since  $P(x) = 0$ . To solve Bernoulli we always start by dividing by  $u^2$  giving

$$\frac{u'}{u^2} = 1 + \frac{1}{u} (1 + x^2)$$

Next we let  $v = \frac{1}{u}$ , hence  $v' = -\frac{u'}{u^2}$  therefore the above becomes

$$\begin{aligned}
 -v' &= 1 + v (1 + x^2) \\
 v' + v (1 + x^2) &= -1
 \end{aligned}$$

Integrating factor is  $e^{\int (1+x^2)dx} = e^{\left(x+\frac{x^3}{2}\right)}$ , therefore

$$d\left(e^{\left(x+\frac{x^3}{2}\right)}v\right) = -e^{\left(x+\frac{x^3}{2}\right)}$$

Integrating

$$\begin{aligned}
 e^{\left(x+\frac{x^3}{2}\right)}v &= -\int e^{\left(x+\frac{x^3}{2}\right)}dx + C \\
 v(x) &= e^{-\left(x+\frac{x^3}{2}\right)}\left(C - \int e^{\left(x+\frac{x^3}{2}\right)}dx\right)
 \end{aligned}$$

Therefore

$$u = \frac{1}{v} = \frac{e^{\left(x+\frac{x^3}{2}\right)}}{\left(C - \int e^{\left(x+\frac{x^3}{2}\right)}dx\right)}$$

And since  $u = y - x^2 - 1$  then

$$\begin{aligned}
 y(x) &= u + 1 + x^2 \\
 &= \frac{e^{\left(x+\frac{x^3}{2}\right)}}{\left(C - \int e^{\left(x+\frac{x^3}{2}\right)}dx\right)} + 1 + x^2
 \end{aligned}$$

## 2.21 ODE No. 21

---

$$y'(x) - y(x)^2 + y(x) \sin(x) - \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.322139 (sec), leaf count = 7

$$\{\{y(x) \rightarrow \sin(x)\}\}$$

✓ **Maple** : cpu = 0.103 (sec), leaf count = 25

$$\left\{y(x) = -\frac{e^{-\cos(x)}}{-C1 + \int e^{-\cos(x)} dx} + \sin(x)\right\}$$

**Hand solution**

$$\begin{aligned}
 y' - y^2 + y \sin(x) - \cos(x) &= 0 \\
 y' &= y^2 - y \sin(x) + \cos(x)
 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE of the form of the general form  $y' = P(x) + Q(x)y + R(x)y^2$  where  $P(x) = \cos(x)$ ,  $Q(x) = -\sin(x)$ ,  $R(x) = 1$ . It is best to first try to spot a

particular solution  $y_p$  and use the transformation  $y = y_p + \frac{1}{u}$  otherwise we use  $y = -\frac{u'}{yR(x)}$  transformation. For this problem

$$y_p = \sin(x)$$

Therefore

$$y = \sin x + \frac{1}{u}$$

$$y' = \cos x - \frac{u'}{u^2}$$

Equating this to (1) gives

$$y^2 - y \sin(x) + \cos(x) = \cos x - \frac{u'}{u^2}$$

$$\left(\sin x + \frac{1}{u}\right)^2 - \left(\sin x + \frac{1}{u}\right) \sin x + \cos x = \cos x - \frac{u'}{u^2}$$

$$\sin^2 x + \frac{1}{u^2} + \frac{2}{u} \sin x - \sin^2 x - \frac{1}{u} \sin x = -\frac{u'}{u^2}$$

$$\frac{1}{u^2} + \frac{1}{u} \sin x = -\frac{u'}{u^2}$$

$$1 + u \sin x = -u'$$

$$u' + u \sin x = -1$$

Integrating factor is  $e^{\int \sin x} = e^{-\cos x}$ , hence

$$d(e^{-\cos x} u) = -e^{-\cos x}$$

Integrating both sides

$$e^{-\cos x} u = -\int e^{-\cos x} dx + C$$

$$u = e^{\cos x} \left( C - \int e^{-\cos x} dx \right)$$

Since  $y = \sin x + \frac{1}{u}$  then

$$y = \sin x + \frac{e^{-\cos x}}{C - \int e^{-\cos x} dx}$$

Or letting  $C_1 = -C$  to make match Maple form, we obtain

$$y = -\frac{e^{-\cos x}}{C_1 + \int e^{-\cos x} dx} + \sin x$$

## 2.22 ODE No. 22

$$y'(x) - y(x)^2 - y(x) \sin(2x) - \cos(2x) = 0$$

✓ **Mathematica** : cpu = 0.424576 (sec), leaf count = 113

$$\left\{ \left\{ y(x) \rightarrow -\frac{-\sin(x) \int_1^{\cos(x)} \frac{e^{-K[1]^2}}{K[1]^2 \sqrt{K[1]^2 - 1}} dK[1] - c_1 \sin(x) - \frac{e^{-\cos^2(x)} \tan(x)}{\sqrt{\cos^2(x) - 1}}}{\cos(x) \int_1^{\cos(x)} \frac{e^{-K[1]^2}}{K[1]^2 \sqrt{K[1]^2 - 1}} dK[1] + c_1 \cos(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.337 (sec), leaf count = 128

$$\left\{ y(x) = 2 \frac{\sin(2x)}{\sqrt{2 \cos(2x) + 2}} \left( -C1 (\cos(2x) + 1) \operatorname{HeunCPrime} \left( 1, 1/2, -1/2, -1, \frac{7}{8}, 1/2 \cos(2x) + 1/2 \right) + \operatorname{HeunCPrime} \left( 1, 1/2, -1/2, -1, \frac{7}{8}, 1/2 \cos(2x) + 1/2 \right) \right) \right\}$$

|                      |
|----------------------|
| <b>Hand solution</b> |
|----------------------|

$$y' - y^2 - y \sin(2x) - \cos(2x) = 0$$

$$y' = y^2 + y \sin(2x) + \cos(2x) \quad (1)$$

This is Riccati first order non-linear ODE of the form of the general form  $y' = P(x) + Q(x)y + R(x)y^2$  where  $P(x) = \cos(2x)$ ,  $Q(x) = \sin(2x)$ ,  $R(x) = 1$ . It is best to first try to spot a particular solution  $y_p$  and use the transformation  $y = y_p + \frac{1}{u}$  otherwise we use  $y = -\frac{u'}{yR(x)}$  transformation. For this problem

$$y_p = \tan(x)$$

To verify, since  $y_p' = \frac{1}{\cos^2 x}$  then plugging this particular in (1) gives

$$\frac{1}{\cos^2 x} - \tan^2(x) - \tan(x) \sin(2x) - \cos(2x) = 0$$

But  $\cos(2x) = \cos^2 x - \sin^2 x$  and  $\sin(2x) = 2 \sin x \cos x$  and  $\tan(x) = \frac{\sin x}{\cos x}$  therefore the above becomes

$$\begin{aligned} \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} - \frac{\sin x}{\cos x} (2 \sin x \cos x) - (\cos^2 x - \sin^2 x) &= 0 \\ \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} - 2 \sin^2 x - \cos^2 x + \sin^2 x &= 0 \\ \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} - \sin^2 x - \cos^2 x &= 0 \\ \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} - 1 &= 0 \\ \frac{1 - \sin^2 x}{\cos^2 x} - 1 &= 0 \\ \frac{\cos^2 x}{\cos^2 x} - 1 &= 0 \\ 1 - 1 &= 0 \\ 0 &= 0 \end{aligned}$$

Therefore we, we can use  $y = y_p + \frac{1}{u}$

$$y = \tan x + \frac{1}{u}$$

$$y' = \frac{1}{\cos^2 x} - \frac{u'}{u^2}$$

Equating this to (1) gives

$$-\frac{u'}{u^2} = y^2 + y \sin(2x) + \cos(2x)$$

$$-\frac{u'}{u^2} = -\frac{1}{\cos^2 x} + \left(\tan x + \frac{1}{u}\right)^2 + \left(\tan x + \frac{1}{u}\right) \sin(2x) + \cos(2x)$$

Using  $\sin(2x) = 2 \sin x \cos x$  and  $\cos 2x = \cos^2 x - \sin^2 x$  then above becomes

$$\begin{aligned} -\frac{u'}{u^2} &= -\frac{1}{\cos^2 x} + \left(\tan^2 x + \frac{1}{u^2} + \frac{2}{u} \tan x\right) + \left(\frac{\sin x}{\cos x} + \frac{1}{u}\right) 2 \sin x \cos x + (\cos^2 x - \sin^2 x) \\ u' &= \frac{u^2}{\cos^2 x} - \left(u^2 \frac{\sin^2 x}{\cos^2 x} + 1 + 2u \frac{\sin x}{\cos x}\right) - \left(u^2 \frac{\sin x}{\cos x} + u\right) 2 \sin x \cos x - u^2 \cos^2 x + u^2 \sin^2 x \\ &= \frac{u^2}{\cos^2 x} - u^2 \frac{\sin^2 x}{\cos^2 x} - 1 - 2u \frac{\sin x}{\cos x} - 2u^2 \frac{\sin x}{\cos x} \sin x \cos x - 2u \sin x \cos x - u^2 \cos^2 x + u^2 \sin^2 x \\ &= \frac{u^2}{\cos^2 x} - u^2 \frac{\sin^2 x}{\cos^2 x} - 1 - 2u \frac{\sin x}{\cos x} - 2u^2 \sin^2 x - 2u \sin x \cos x - u^2 \cos^2 x + u^2 \sin^2 x \\ &= \frac{u^2}{\cos^2 x} - u^2 \frac{\sin^2 x}{\cos^2 x} - 1 - 2u \frac{\sin x}{\cos x} - u^2 \sin^2 x - 2u \sin x \cos x - u^2 \cos^2 x \\ &= u^2 \left(\frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} - (\sin^2 x + \cos^2 x)\right) - 1 + u \left(-2 \frac{\sin x}{\cos x} - 2 \sin x \cos x\right) \\ &= u^2 \left(\frac{1 - \sin^2 x}{\cos^2 x} - 1\right) - 1 + u \left(-2 \frac{\sin x}{\cos x} - 2 \sin x \cos x\right) \\ &= u^2 \left(\frac{\cos^2 x}{\cos^2 x} - 1\right) - 1 + u \left(-2 \frac{\sin x}{\cos x} - 2 \sin x \cos x\right) \\ &= -1 + 2u \left(-\frac{\sin x}{\cos x} - \sin x \cos x\right) \end{aligned}$$



Hence

$$u' + 2u (\tan x + \sin x \cos x) = -1$$

Integrating factor is  $e^{2 \int \tan x + \sin x \cos x dx}$ . But

$$\int \tan x dx = -\ln(\cos x)$$

And

$$\int \sin x \cos x dx = \frac{-1}{2} \cos^2 x$$

Hence  $\mu = e^{-2 \ln \cos x} e^{-\cos^2 x} = \frac{1}{\cos^2 x} e^{-\cos^2 x}$ , therefore

$$d\left(\frac{1}{\cos^2 x} e^{-\cos^2 x} u\right) = \frac{-1}{\cos^2 x} e^{-\cos^2 x}$$

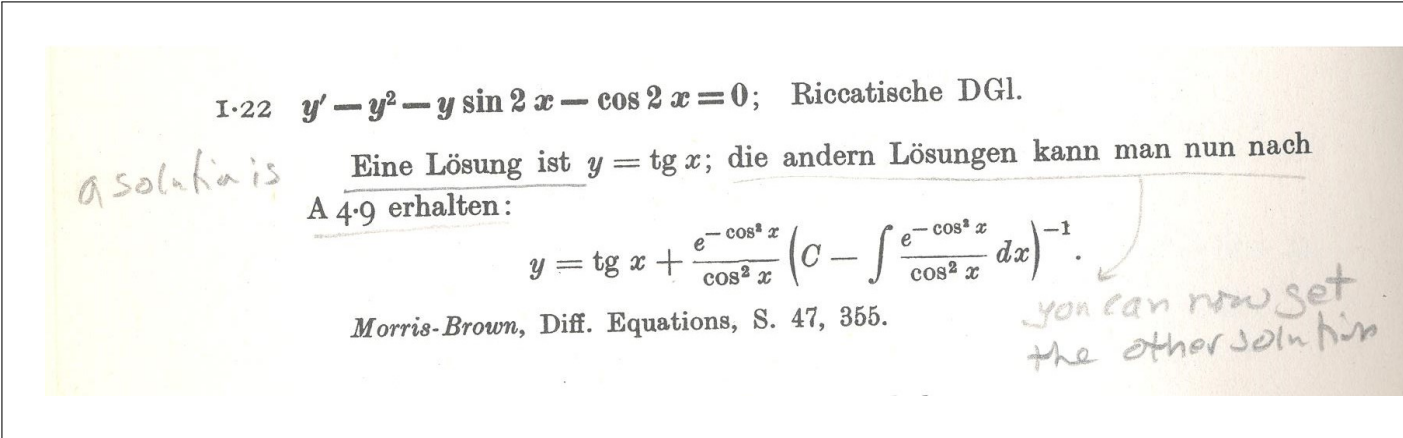
Integrating both sides

$$\begin{aligned} \frac{1}{\cos^2 x} e^{-\cos^2 x} u &= -\int \frac{e^{-\cos^2 x}}{\cos^2 x} dx + C \\ u &= \cos^2 x e^{\cos^2 x} \left( C - \int \frac{e^{-\cos^2 x}}{\cos^2 x} dx \right) \end{aligned}$$

Since  $y = \tan x + \frac{1}{u}$  then

$$\begin{aligned} y &= \tan x + \frac{1}{\cos^2 x e^{\cos^2 x} \left( C - \int \frac{e^{-\cos^2 x}}{\cos^2 x} dx \right)} \\ &= \tan x + \frac{e^{-\cos^2 x}}{\cos^2 x} \left( C - \int \frac{e^{-\cos^2 x}}{\cos^2 x} dx \right)^{-1} \end{aligned}$$

I do not know how Maple came up with the solution involving HeunC functions since  $\int \frac{e^{-\cos^2 x}}{\cos^2 x} dx$  has no closed form solution. I should ask CAS experts about this.  
Below is screen shot from Kamke book of the solution it gives, which matches the above result



## 2.23 ODE No. 23

$$ay(x)^2 - b + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.062718 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{b} \tanh\left(\sqrt{a}\sqrt{b}c_1 + \sqrt{a}\sqrt{b}x\right)}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 23

$$\left\{ y(x) = \frac{1}{a} \tanh\left(\sqrt{ab}(x + \_C1)\right) \sqrt{ab} \right\}$$

**Hand solution**

$$y' + ay^2 - b = 0$$

$$\frac{dy}{dx} = b - ay^2$$

Separable,

$$\frac{dy}{b - ay^2} = dx$$

$$\int \frac{dy}{b - ay^2} = \int dx$$

But

$$\int \frac{dy}{b - ay^2} = \frac{1}{\sqrt{ab}} \tanh^{-1} \left( \sqrt{\frac{a}{b}} y \right)$$

Hence

$$\frac{1}{\sqrt{ab}} \tanh^{-1} \left( \sqrt{\frac{a}{b}} y \right) = x + C$$

$$\tanh^{-1} \left( \sqrt{\frac{a}{b}} y \right) = \sqrt{ab} (x + C)$$

$$\sqrt{\frac{a}{b}} y = \tanh \left( \sqrt{ab} (x + C) \right)$$

$$y = \sqrt{\frac{b}{a}} \tanh \left( \sqrt{ab} (x + C) \right)$$

### 2.24 ODE No. 24

$$ay(x)^2 - bx^\nu + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0970147 (sec), leaf count = 277

$$\left\{\left\{y(x) \rightarrow -\frac{\sqrt{-a}\sqrt{b}x^{\frac{\nu+2}{2}}\left(c_1J_{\frac{\nu+1}{\nu+2}}\left(\frac{2\sqrt{-a}\sqrt{b}x^{\frac{\nu}{2}+1}}{\nu+2}\right)-c_1J_{-\frac{\nu+3}{\nu+2}}\left(\frac{2\sqrt{-a}\sqrt{b}x^{\frac{\nu+2}{2}}}{\nu+2}\right)-2J_{\frac{1}{\nu+2}-1}\left(\frac{2\sqrt{-a}\sqrt{b}x^{\frac{\nu+2}{2}}}{\nu+2}\right)\right)\right\}-c_1\right\}}{2ax\left(c_1J_{-\frac{1}{\nu+2}}\left(\frac{2\sqrt{-a}\sqrt{b}x^{\frac{\nu+2}{2}}}{\nu+2}\right)+J_{\frac{1}{\nu+2}}\left(\frac{2\sqrt{-a}\sqrt{b}x^{\frac{\nu+2}{2}}}{\nu+2}\right)\right)}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 214

$$\left\{y\left(x\right)=\frac{1}{ax}\left(-J_{\frac{3+\nu}{\nu+2}}\left(2\frac{\sqrt{-ab}x^{\nu/2+1}}{\nu+2}\right)\sqrt{-ab}x^{\frac{\nu}{2}+1}-C1-Y_{\frac{3+\nu}{\nu+2}}\left(2\frac{\sqrt{-ab}x^{\nu/2+1}}{\nu+2}\right)\sqrt{-ab}x^{\frac{\nu}{2}+1}+_C1J_{\left(\nu+2\right)^{-1}}\left(\frac{2\sqrt{-a}\sqrt{b}x^{\frac{\nu+2}{2}}}{\nu+2}\right)\right\}$$

Hand solution

$$y' + ay^2 - bx^v = 0$$

$$y' = bx^v - ay^2$$

$$= P\left(x\right) + Q\left(x\right)y + R\left(x\right)y^2$$

(1)

This is Riccati first order non-linear ODE with  $P\left(x\right) = bx^v, Q\left(x\right) = 0, R\left(x\right) = -a$ . Using the standard substitution

$$y = -\frac{u'}{uR\left(x\right)} = \frac{u'}{au}$$

Hence

$$y' = \frac{u''}{au} - \frac{\left(u'\right)^2}{au^2}$$

Therefore (1) becomes

$$\begin{aligned}
\frac{u''}{au} - \frac{(u')^2}{au^2} &= bx^v - ay^2 \\
&= bx^v - a \left( \frac{u'}{au} \right)^2 \\
&= bx^v - \frac{(u')^2}{au^2}
\end{aligned}$$

Hence

$$\begin{aligned}
\frac{u''}{au} &= bx^v \\
u'' - abx^v u &= 0
\end{aligned}$$

This is an Emden-Fowler equation, of the general form  $u'' = Ax^n u^m$ , where here  $m = 1$  and  $n = v$  and  $A = ab$ .

For any  $n$ , the solution uses Bessel functions and modified Bessel functions of first and second kind. From Handbook of exact solutions for ODE, page 237, equation 2.1.2.7 we see the solution is given as

$$u = \begin{cases} C_1 \sqrt{x} J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + C_2 \sqrt{x} Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) & ab < 0 \\ C_1 \sqrt{x} I_{\frac{1}{2q}} \left( \frac{\sqrt{ab}}{q} x^q \right) + C_2 \sqrt{x} K_{\frac{1}{2q}} \left( \frac{\sqrt{ab}}{q} x^q \right) & ab > 0 \end{cases}$$

Where  $q = \frac{n+1}{2}$ .  $J$  is Bessel function of first kind and  $Y$  is Bessel function of second kind.  $I$  is modified Besself function of first kind and  $K$  is modified Besself function of second kind. To find  $y$  we now use  $y = \frac{u'}{au}$ . Derivative of Bessel functions is given by

$$\begin{aligned}
J'_m(x) &= \frac{1}{2} (J_{m-1}(x) - J_{m+1}(x)) \\
Y'_m(x) &= \frac{1}{2} (Y_{m-1}(x) - Y_{m+1}(x)) \\
I'_m(x) &= \frac{1}{2} (I_{m-1}(x) + I_{m+1}(x)) \\
K'_m(x) &= -\frac{1}{2} (K_{m-1}(x) + K_{m+1}(x))
\end{aligned}$$

Using these, then

$$u' = \begin{cases} C_1 \left[ \frac{1}{2\sqrt{x}} J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + \sqrt{x} J'_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) \right] + C_2 \left[ \frac{1}{2\sqrt{x}} Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + \sqrt{x} Y'_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) \right] & ab < 0 \\ C_1 \left[ \frac{1}{2\sqrt{x}} I_{\frac{1}{2q}} \left( \frac{\sqrt{ab}}{q} x^q \right) + \sqrt{x} I'_{\frac{1}{2q}} \left( \frac{\sqrt{ab}}{q} x^q \right) \right] + C_2 \left[ \frac{1}{2\sqrt{x}} K_{\frac{1}{2q}} \left( \frac{\sqrt{ab}}{q} x^q \right) + \sqrt{x} K'_{\frac{1}{2q}} \left( \frac{\sqrt{ab}}{q} x^q \right) \right] & ab > 0 \end{cases}$$

Hence for  $ab < 0$

$$\begin{aligned}
y &= \frac{u'}{au} \\
&= \frac{C_1 \left[ \frac{1}{2\sqrt{x}} J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + \sqrt{x} J'_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) \right] + C_2 \left[ \frac{1}{2\sqrt{x}} Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + \sqrt{x} Y'_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) \right]}{aC_1 \sqrt{x} J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + aC_2 \sqrt{x} Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right)} \\
&= \frac{\sqrt{x} C_1 \left[ \frac{1}{2x} J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + J'_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) \right] + \sqrt{x} C_2 \left[ \frac{1}{2x} Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + Y'_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) \right]}{aC_1 \sqrt{x} J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + aC_2 \sqrt{x} Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right)} \\
&= \frac{C_1 \left[ \frac{1}{2x} J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + J'_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) \right] + C_2 \left[ \frac{1}{2x} Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + Y'_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) \right]}{aC_1 J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + aC_2 Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right)}
\end{aligned}$$

Using derivatives the above becomes

$$\begin{aligned}
y &= \frac{C_1 \left[ \frac{1}{2x} J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + \frac{1}{2} \left( J_{\frac{1}{2q}-1} \left( \frac{\sqrt{-ab}}{q} x^q \right) - J_{\frac{1}{2q}+1} \left( \frac{\sqrt{-ab}}{q} x^q \right) \right) \right]}{aC_1 J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + aC_2 Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right)} \\
&\quad + \frac{C_2 \left[ \frac{1}{2x} Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + \frac{1}{2} \left( Y_{\frac{1}{2q}-1} \left( \frac{\sqrt{-ab}}{q} x^q \right) - Y_{\frac{1}{2q}+1} \left( \frac{\sqrt{-ab}}{q} x^q \right) \right) \right]}{aC_1 J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + aC_2 Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right)}
\end{aligned}$$

Similar result can be found for  $ab > 0$

2.25 ODE No. 25

$$ay(x)^2 - bx^{2\nu} - cx^{\nu-1} + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.305905 (sec), leaf count = 1835

$$\left\{ \left\{ y(x) \rightarrow \frac{-2^{\frac{\nu}{2(\nu+1)}-1} e^{-\frac{\sqrt{a}\sqrt{b}x^{\nu+1}}{\sqrt{\nu^2+2\nu+1}}} \nu \left(x^{\nu+1}\right)^{\frac{\nu}{2(\nu+1)}} L^{\frac{\nu}{\nu+1}-1}_{\frac{\frac{\sqrt{a}\sqrt{b}\nu c}{\sqrt{(\nu+1)^2}} + \frac{\sqrt{a}\sqrt{b}c}{\sqrt{(\nu+1)^2}} + b\nu}{2(\nu b+b)}} \left(\frac{2\sqrt{a}\sqrt{b}x^{\nu+1}}{\sqrt{\nu^2+2\nu+1}}\right) x^{-\frac{\nu}{2}-1} - \frac{2^{\frac{\nu}{2(\nu+1)}} \sqrt{a}\sqrt{b}e^{-\frac{\sqrt{a}\sqrt{b}x^{\nu+1}}{\sqrt{\nu^2+2\nu+1}}}}{\sqrt{\nu^2+2\nu+1}} \right. \right. \right.$$

✓ **Maple** : cpu = 0.258 (sec), leaf count = 348

$$\left\{ y(x) = -\frac{1}{2ax} \left( \left( (-\nu-2)b^{\frac{3}{2}} + \sqrt{abc} \right) M_{-\frac{1}{2\nu+2}} \left( (-2\nu-2)\sqrt{b} + \sqrt{ac} \right) \frac{1}{\sqrt{b}}, (2\nu+2)^{-1} \left( 2 \frac{\sqrt{a}\sqrt{b}x^{\nu+1}}{\nu+1} \right) + 2b^{3/2} - C1 \right) (\nu+1) \right.$$

Hand solution

$$\begin{aligned} y' + ay^2 - bx^{2v} - cx^{v-1} &= 0 \\ y' &= bx^v + cx^{v-1} - ay^2 \\ &= P(x) + Q(x)y + R(x)y^2 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE with  $P(x) = bx^v + cx^{v-1}, Q(x) = 0, R(x) = -a$ .  
Need to do this later.

2.26 ODE No. 26

$$y'(x) - (Ay(x) - a)(By(x) - b) = 0$$

✓ **Mathematica** : cpu = 0.181778 (sec), leaf count = 68

$$\left\{ \left\{ y(x) \rightarrow \frac{ae^{Abc_1+Abx} - be^{aBc_1+aBx}}{Ae^{Abc_1+Abx} - Be^{aBc_1+aBx}} \right\} \right\}$$

✓ **Maple** : cpu = 0.057 (sec), leaf count = 45

$$\left\{ y(x) = \frac{e^{(x+C1)(Ab-aB)}a - b}{Ae^{(x+C1)(Ab-aB)} - B} \right\}$$

Hand solution

$$\begin{aligned} y' - (Ay - a)(By - b) &= 0 \\ y' &= (Ay - a)(By - b) \\ &= ab - y(Ab + Ba) + AB y^2 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE with  $P(x) = ab, Q(x) = -(Ab + Ba), R(x) = AB$ .  
Let  $y = -\frac{u'}{uR(x)} = -\frac{u'}{ABu}$ , hence

$$y' = \frac{-u''}{ABu} - \frac{(u')^2}{ABu^2}$$

Comparing to (1) results in

$$\begin{aligned} \frac{-u''}{ABu} - \frac{(u')^2}{ABu^2} &= ab - y(Ab + Ba) + AB y^2 \\ &= ab - \left(-\frac{u'}{ABu}\right)(Ab + Ba) + AB \left(-\frac{u'}{ABu}\right)^2 \\ &= ab + \frac{u'}{ABu}(Ab + Ba) + AB \frac{(u')^2}{(ABu)^2} \\ &= ab + \frac{u'}{ABu}(Ab + Ba) + \frac{(u')^2}{ABu^2} \end{aligned}$$

Hence

$$\begin{aligned} \frac{-u''}{ABu} &= ab + \frac{u'}{ABu}(Ab + Ba) \\ -u'' &= ABabu + u'(Ab + Ba) \\ u'' + u'(Ab + Ba) + u(ABab) &= 0 \end{aligned}$$

This is second order ODE with constant coefficient. Solution is

$$u = c_1e^{-aBx} + c_2e^{-Abx}$$

Therefore

$$u' = -aBc_1e^{-aBx} - c_2Abe^{-Abx}$$

And therefore the solution is

$$\begin{aligned} y &= -\frac{u'}{ABu} = -\frac{1}{AB} \frac{-aBc_1e^{-aBx} - c_2Abe^{-Abx}}{c_1e^{-aBx} + c_2e^{-Abx}} \\ &= \frac{aBc_1e^{-aBx} + c_2Abe^{-Abx}}{AB(c_1e^{-aBx} + c_2e^{-Abx})} \end{aligned}$$

Dividing by  $c_2$  and letting  $c = \frac{c_1}{c_2}$

$$y = \frac{aBce^{-aBx} + Abe^{-Abx}}{AB(ce^{-aBx} + e^{-Abx})}$$

Verification

```

1 eq:=diff(y(x),x)-(A*y(x)-a)*(B*y(x)-b) = 0;
2 sol:=(a*B*_C1*exp(-a*B*x)+A*b*exp(-A*b*x))/(A*B*(_C1*exp(-a*B*x)+exp(-A*b*x)));
3 odetest(y(x)=sol,eq);
4 0

```

## 2.27 ODE No. 27

$$ay(x)(y(x) - x) + y'(x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.157792 (sec), leaf count = 24

$$\left\{\left\{y(x) \rightarrow \frac{x}{2} - \frac{1}{a(c_1 - x)}\right\}\right\}$$

✓ **Maple** : cpu = 0.138 (sec), leaf count = 72

$$\left\{y(x)=1\left(2\sqrt{a}e^{-1/2ax^2}+x\left(\operatorname{Erf}\left(\frac{\sqrt{2}x}{2}\sqrt{a}\right)\sqrt{\pi}\sqrt{2}a+2a^{3/2}_{-C1}\right)\right)\left(\operatorname{Erf}\left(\frac{\sqrt{2}x}{2}\sqrt{a}\right)\sqrt{\pi}\sqrt{2}a+2a^{3/2}_{-C1}\right)\right\}$$

Hand solution

$$\begin{aligned} y' + ay(y - x) - 1 &= 0 \\ y' &= 1 - (ay^2 - ayx) \\ &= 1 + ayx - ay^2 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE  $y' = P(x) + A(x)y + R(x)y^2$  with  $P(x) = 1, Q(x) = -ax, R(x) = -a$ . We can convert Riccati to Bernoulli which is easier to solve using the substitution  $u = y - x$

$$\begin{aligned}
 u' &= y' - 1 \\
 &= (1 + ax - ay^2) - 1 \\
 &= (1 + a(u+x)x - a(u+x)^2) - 1 \\
 &= 1 + aux + ax^2 - a(u^2 + x^2 + 2ux) - 1 \\
 &= 1 + aux + ax^2 - au^2 - ax^2 - 2aux - 1 \\
 &= -aux - au^2 \\
 u' &= -aux - au^2
 \end{aligned}$$

This is of the form  $u' = P(x) + Q(x)u + R(x)u^2$  and since  $P(x) = 0$  then it is Bernoulli differential equation. (when  $P(x) \neq 0$  and  $R(x) \neq 0$  it is Riccati). To solve Bernoulli we always start by dividing by  $u^2$

$$\frac{u'}{u^2} = -\frac{ax}{u} - a$$

Then we let  $\zeta = \frac{1}{u}$ , hence  $\zeta' = -\frac{u'}{u^2}$ , therefore the above becomes

$$\begin{aligned}
 -\zeta' &= -ax\zeta - a \\
 \zeta' - ax\zeta &= a
 \end{aligned}$$

Integrating factor is  $e^{-\int axdx} = e^{-a\frac{x^2}{2}}$ , hence  $d\left(e^{-a\frac{x^2}{2}}\zeta\right) = ae^{-a\frac{x^2}{2}}$ . Integrating both sides gives

$$e^{-a\frac{x^2}{2}}\zeta = a \int e^{-a\frac{x^2}{2}} dx + C$$

But

$$\int e^{-a\frac{x^2}{2}} dx = \sqrt{\frac{\pi}{2a}} \operatorname{erf}\left(\sqrt{\frac{a}{2}}x\right)$$

Therefore

$$\begin{aligned}
 e^{-a\frac{x^2}{2}}\zeta &= a\sqrt{\frac{\pi}{2a}} \operatorname{erf}\left(\sqrt{\frac{a}{2}}x\right) + C \\
 \zeta &= e^{a\frac{x^2}{2}} \left(a\sqrt{\frac{\pi}{2a}} \operatorname{erf}\left(\sqrt{\frac{a}{2}}x\right) + C\right)
 \end{aligned}$$

Hence

$$\begin{aligned}
 u &= \frac{1}{\zeta} \\
 &= e^{-a\frac{x^2}{2}} \left(a\sqrt{\frac{\pi}{2a}} \operatorname{erf}\left(\sqrt{\frac{a}{2}}x\right) + C\right)^{-1}
 \end{aligned}$$

Since  $u = y - x$  then

$$\begin{aligned}
 y &= u + x \\
 &= e^{-a\frac{x^2}{2}} \left(a\sqrt{\frac{\pi}{2a}} \operatorname{erf}\left(\sqrt{\frac{a}{2}}x\right) + C\right)^{-1} + x \\
 &= \frac{e^{-a\frac{x^2}{2}}}{\sqrt{\frac{a\pi}{2}} \operatorname{erf}\left(\sqrt{\frac{a}{2}}x\right) + C} + x
 \end{aligned}$$

Verification

```

1 eq:=diff(y(x),x)+a*y(x)*(y(x)-x)-1 = 0;
2 sol:=exp(-a*x^2/2)/(sqrt(a*Pi/2)*erf(sqrt(a/2)*x)+_C1)+x;
3 odetest(y(x)=sol,eq);
4 0

```

$$x^3(-y(x)) + y'(x) + xy(x)^2 - 2x = 0$$

✓ **Mathematica** : cpu = 0.102271 (sec), leaf count = 96

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{\frac{x^4}{4}} x^3 + \frac{1}{2} \sqrt{\pi} e^{\frac{x^4}{4}} x^3 \operatorname{erf}\left(\frac{x^2}{2}\right) + x}{x \left( c_1 e^{\frac{x^4}{4}} + \frac{1}{2} \sqrt{\pi} e^{\frac{x^4}{4}} \operatorname{erf}\left(\frac{x^2}{2}\right) \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.073 (sec), leaf count = 51

$$\left\{ y(x) = \frac{1}{\sqrt{\pi}} \left( \operatorname{Erf}\left(\frac{x^2}{2}\right) \sqrt{\pi} - C_1 x^2 + x^2 \sqrt{\pi} + 2 e^{-1/4 x^4} - C_1 \right) \left( \operatorname{Erf}\left(\frac{x^2}{2}\right) - C_1 + 1 \right)^{-1} \right\}$$

|                      |
|----------------------|
| <b>Hand solution</b> |
|----------------------|

$$\begin{aligned} y' - yx^3 + xy^2 - 2x &= 0 \\ y' &= 2x + yx^3 - xy^2 \\ &= P(x) + Q(x)y + R(x)y^2 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE with  $P(x) = 2x, Q(x) = x^3, R(x) = -x$ . We can convert Riccati to Bernoulli which is easier to solve using the substitution  $u = x^2 - y$  or  $y = x^2 - u$

$$\begin{aligned} u' &= 2x - y' \\ &= 2x - (2x + yx^3 - xy^2) \\ &= 2x - (2x + (x^2 - u)x^3 - x(x^2 - u)^2) \\ &= 2x - (2x + (x^5 - ux^3) - x(x^4 + u^2 - 2x^2u)) \\ u' &= 2x - (2x + (x^5 - ux^3) - (x^5 + xu^2 - 2x^3u)) \\ &= 2x - 2x - (x^5 - ux^3) + (x^5 + xu^2 - 2x^3u) \\ &= -x^5 + ux^3 + x^5 + xu^2 - 2x^3u \\ &= -ux^3 + xu^2 \end{aligned}$$

This is of the form  $u' = P(x) + Q(x)u + R(x)u^2$  and since  $P(x) = 0$  then it is Bernoulli differential equation. (when  $P(x) \neq 0$  and  $R(x) \neq 0$  it is Riccati). To solve Bernoulli we always start by dividing by  $u^2$

$$\frac{u'}{u^2} = -\frac{1}{u}x^3 + x$$

Then we let  $\zeta = -\frac{1}{u}$ , hence  $\zeta' = \frac{u'}{u^2}$ , therefore the above becomes

$$\begin{aligned} \zeta' &= x^3\zeta + x \\ \zeta' - x^3\zeta &= x \end{aligned}$$

Integrating factor is  $e^{-\int x^3 dx} = e^{-\frac{x^4}{4}}$ , hence

$$d\left(e^{-\frac{x^4}{4}}\zeta\right) = xe^{-\frac{x^4}{4}}$$

Integrating both sides gives

$$e^{-\frac{x^4}{4}}\zeta = \int xe^{-\frac{x^4}{4}} dx + C$$

$\int xe^{-\frac{x^4}{4}} dx = \frac{\sqrt{\pi}}{2} \operatorname{erf}\left(\frac{x^2}{2}\right)$ , hence from above

$$\begin{aligned} e^{-\frac{x^4}{4}}\zeta &= \frac{\sqrt{\pi}}{2} \operatorname{erf}\left(\frac{x^2}{2}\right) + C \\ \zeta &= e^{\frac{x^4}{4}} \left( \frac{\sqrt{\pi}}{2} \operatorname{erf}\left(\frac{x^2}{2}\right) + C \right) \end{aligned}$$

Since  $\zeta = -\frac{1}{u}$  then

$$u = -e^{-\frac{x^4}{4}} \left( \frac{\sqrt{\pi}}{2} \operatorname{erf}\left(\frac{x^2}{2}\right) + C \right)^{-1}$$

And since  $y = x^2 - u$  then

$$y = x^2 + e^{-\frac{x^4}{4}} \left( \frac{\sqrt{\pi}}{2} \operatorname{erf} \left( \frac{x^2}{2} \right) + C \right)^{-1}$$

$$= x^2 + \frac{e^{-\frac{x^4}{4}}}{\frac{\sqrt{\pi}}{2} \operatorname{erf} \left( \frac{x^2}{2} \right) + C}$$

Verification

```
1 eq:=diff(y(x),x)+x*y(x)^2-x^3*y(x)-2*x = 0;
2 sol:=x^2+ exp(-x^4/4)/(_C1+ sqrt(Pi)/2*erf(x^2/2));
3 odetest(y(x)=sol,eq);
4 0
```

## 2.29 ODE No. 29

$$y'(x) - xy(x)^2 - 3xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0736526 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow -\frac{3e^{3c_1 + \frac{3x^2}{2}}}{e^{3c_1 + \frac{3x^2}{2}} - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 19

$$\left\{ y(x) = 3 \left( -1 + 3e^{-3/2x^2} _C1 \right)^{-1} \right\}$$

**Hand solution**

$$y' - xy^2 - 3xy = 0$$

$$y' = 3xy + xy^2$$

$$= P(x) + Q(x)y + R(x)y^2 \quad (1)$$

This is Bernoulli first order non-linear ODE since  $P(x) = 0$ . To solve Bernoulli we always start by dividing by  $y^2$

$$\frac{y'}{y^2} = \frac{3x}{y} + x$$

Then we let  $u = \frac{1}{y}$ , hence  $u' = \frac{-y'}{y^2}$ , therefore the above becomes

$$-u' = 3xu + x$$

$$u' + 3ux = -x$$

Integrating factor is  $e^{\int 3xdx} = e^{\frac{3x^2}{2}}$ , hence

$$d \left( e^{\frac{3x^2}{2}} u \right) = -xe^{\frac{3x^2}{2}}$$

Integrating both sides gives

$$e^{\frac{3x^2}{2}} u = \int -xe^{\frac{3x^2}{2}} dx + C$$

$$= -\frac{1}{3}e^{\frac{3x^2}{2}} + C$$

Hence from above

$$u = e^{\frac{-3x^2}{2}} \left( -\frac{1}{3}e^{\frac{3x^2}{2}} + C \right)$$

And since  $y = \frac{1}{u}$  then

$$y = \frac{e^{\frac{3x^2}{2}}}{C - \frac{1}{3}e^{\frac{3x^2}{2}}}$$

Verification

```
1 eq:=diff(y(x),x)-x*y(x)^2-3*x*y(x) = 0;
2 sol:=exp(3*x^2/2)/(_C1- 1/3*exp(3*x^2/2));
3 odetest(y(x)=sol,eq);
4 0
```



## 2.30 ODE No. 30

$$x^{-a-1}y(x)^2 - x^a + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.154188 (sec), leaf count = 230

$$\left\{ \left\{ y(x) \rightarrow \frac{x^{a+1} \left( c_1 \left( \frac{1}{2} x^{-\frac{a}{2}-\frac{1}{2}} \Gamma(a+1) (I_{a-1}(2\sqrt{x}) + I_{a+1}(2\sqrt{x})) - \frac{1}{2} a x^{-\frac{a}{2}-1} \Gamma(a+1) I_a(2\sqrt{x}) \right) - \frac{1}{2} (-1)^{-a} a x^{-\frac{a}{2}} \Gamma(a+1) I_a(2\sqrt{x}) \right)}{c_1 x^{-a/2} \Gamma(a+1) I_a(2\sqrt{x}) + (-1)^{-a} x^{-a/2} \Gamma(a+1) I_a(2\sqrt{x})} \right\} \right.$$

✓ **Maple** : cpu = 0.06 (sec), leaf count = 54

$$\left\{ y(x) = x^{a+1} (-K_{a+1}(2\sqrt{x})_C 1 + I_{a+1}(2\sqrt{x})) \frac{1}{\sqrt{x}} (K_a(2\sqrt{x})_C 1 + I_a(2\sqrt{x}))^{-1} \right\}$$

Hand solution

$$\begin{aligned} y' + x^{-a-1}y^2 - x^a &= 0 \\ y' &= x^a - x^{-a-1}y^2 \\ &= P(x) + Q(x)y + R(x)y^2 \end{aligned} \tag{1}$$

This is Ricatti first order non-linear ODE. Using standard transformation

$$y = -\frac{u'}{uR(x)} = x^{a+1} \frac{u'}{u}$$

Hence

$$y' = (a+1)x^a \frac{u'}{u} + x^{a+1} \frac{u''}{u} - x^{a+1} \frac{(u')^2}{u^2}$$

Comparing to (1) gives

$$\begin{aligned} x^a - x^{-a-1}y^2 &= (a+1)x^a \frac{u'}{u} + x^{a+1} \frac{u''}{u} - x^{a+1} \frac{(u')^2}{u^2} \\ x^a - x^{-a-1} \left( x^{a+1} \frac{u'}{u} \right)^2 &= (a+1)x^a \frac{u'}{u} + x^{a+1} \frac{u''}{u} - x^{a+1} \frac{(u')^2}{u^2} \\ 1 - \frac{x^{-a-1}}{x^a} x^{2a+2} \frac{(u')^2}{u^2} &= (a+1) \frac{u'}{u} + x \frac{u''}{u} - x \frac{(u')^2}{u^2} \\ 1 - x \frac{(u')^2}{u^2} &= (a+1) \frac{u'}{u} + x \frac{u''}{u} - x \frac{(u')^2}{u^2} \\ 1 &= (a+1) \frac{u'}{u} + x \frac{u''}{u} \\ xu'' + (1+a)u' - u &= 0 \end{aligned} \tag{2}$$

In standard form  $u'' + \frac{1}{x}(1+a)u' - \frac{1}{x}u = 0$  or  $u'' + p(x)(1+a)u' + q(x)u = 0$ . We see that  $p(x)$  is not analytic at  $x = 0$  (the expansion point). So we can't use power series solution, and will use Forbenius series. Power series, which is  $u = \sum_{n=0}^{\infty} c_n x^n$  is used when the expansion point is not singular point. (i.e.  $p(x)$  and  $q(x)$  are analytic there). Forbenius series  $u = x^r \sum_{n=0}^{\infty} c_n x^n$  is used when there is a removable singular point (called also regular singular point), as in this case. Starting with

$$u = x^r \sum_{n=0}^{\infty} c_n x^n = \sum_{n=0}^{\infty} c_n x^{n+r}$$

Hence

$$\begin{aligned} u' &= \sum_{n=0}^{\infty} (n+r) c_n x^{n+r-1} \\ u'' &= \sum_{n=0}^{\infty} (n+r)(n+r-1) c_n x^{n+r-2} \end{aligned}$$

Substituting in (2) gives

$$\begin{aligned} x \sum_{n=0}^{\infty} (n+r)(n+r-1) c_n x^{n+r-2} + (1+a) \sum_{n=0}^{\infty} (n+r) c_n x^{n+r-1} - \sum_{n=0}^{\infty} c_n x^{n+r} &= 0 \\ \sum_{n=0}^{\infty} (n+r)(n+r-1) c_n x^{n+r-1} + (1+a) \sum_{n=0}^{\infty} (n+r) c_n x^{n+r-1} - \sum_{n=0}^{\infty} c_n x^{n+r} &= 0 \end{aligned}$$

Dividing out  $x^r$

$$\sum_{n=0}^{\infty} (n+r)(n+r-1)c_n x^{n-1} + (1+a) \sum_{n=0}^{\infty} (n+r)c_n x^{n-1} - \sum_{n=0}^{\infty} c_n x^n = 0$$

Each term should have  $x^{n-1}$  in it. So we adjust the last term

$$\sum_{n=0}^{\infty} (n+r)(n+r-1)c_n x^{n-1} + (1+a) \sum_{n=0}^{\infty} (n+r)c_n x^{n-1} - \sum_{n=1}^{\infty} c_{n-1} x^{n-1} = 0$$

Expanding the second term

$$\sum_{n=0}^{\infty} (n+r)(n+r-1)c_n x^{n-1} + \sum_{n=0}^{\infty} (n+r)c_n x^{n-1} + \sum_{n=0}^{\infty} a(n+r)c_n x^{n-1} - \sum_{n=1}^{\infty} c_{n-1} x^{n-1} = 0$$

Hence for  $n = 0$

$$(n+r)(n+r-1)c_n x^{n-1} + (n+r)c_n x^{n-1} + a(n+r)c_n x^{n-1} = 0$$

$$r(r-1)c_0 + rc_0 + arc_0 = 0$$

Since  $c_0 \neq 0$  then

$$r(r-1) + r + ar = 0$$

Hence  $r = -a$  or  $r = 0$ . Now for  $n \geq 1$

$$(n+r)(n+r-1)c_n x^{n-1} + (n+r)c_n x^{n-1} + a(n+r)c_n x^{n-1} - c_{n-1} x^{n-1} = 0$$

$$(n+r)(n+r-1)c_n + (n+r)c_n + a(n+r)c_n - c_{n-1} = 0$$

$$((n+r)(n+r-1) + (n+r) + a(n+r))c_n = c_{n-1}$$

$$c_n = \frac{c_{n-1}}{(n+r)(n+r-1) + (n+r)}$$

For  $r = 0$ , we obtain

$$c_n = \frac{c_{n-1}}{n(n-1) + n + an} \quad (3)$$

For  $r = -a$

$$c_n = \frac{c_{n-1}}{(n-a)(n-a-1) + (n-a) + a(n-a)} \quad (4)$$

There are two solutions. Looking at (3) for now, for  $n = 1$

$$c_1 = \frac{c_0}{1+a}$$

For  $n = 2$

$$c_2 = \frac{c_1}{4+2a} = \frac{c_0}{1+a} \frac{1}{2(2+a)}$$

For  $n = 3$

$$c_3 = \frac{c_2}{3(2) + 3 + 3a} = \frac{c_2}{3(3+a)} = \frac{c_0}{1+a} \frac{1}{2(2+a)} \frac{1}{3(3+a)}$$

And so on. Since the solution is assumed to be  $x^r \sum_{n=0}^{\infty} c_n x^n$  and we are looking at case  $r = 0$  then

$$u_{r=0}(x) = \sum_{n=1}^{\infty} c_n x^n$$

$$= c_0 + c_1 x + c_2 x^2 + \dots$$

$$= c_0 x^0 + \frac{c_0}{1+a} x + \frac{c_0}{1+a} \frac{1}{2(2+a)} x^2 + \frac{c_0}{1+a} \frac{1}{2(2+a)} \frac{1}{3(3+a)} x^3 + \dots$$

$$= c_0 \left( x^0 + \frac{1}{1+a} x + \frac{1}{(1+a)} \frac{1}{2(2+a)} x^2 + \frac{1}{(1+a)} \frac{1}{2(2+a)} \frac{1}{3(3+a)} x^3 + \dots \right) \quad (5)$$

Since

$$\Gamma(n) = (n-1)!$$

and

$$a(1+a)(2+a) \dots (n+a) = \frac{\Gamma(a+n+1)}{\Gamma(a)}$$

Then

$$(1+a)(2+a) \dots (n+a) = \frac{\Gamma(a+n+1)}{a\Gamma(a)}$$

And (5) can now be written as

$$y_{r=0}(x) = c_0 \sum_{n=1}^{\infty} \frac{1}{n!} \frac{a\Gamma(a)}{\Gamma(a+n+1)} x^n \quad (6)$$

But modified Bessel function of first kind is

$$\text{BesselI}(a, z) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{\Gamma(a+n+1)} \left(\frac{z}{2}\right)^{2n+a}$$

So if we let  $z = 2\sqrt{x}$  we obtain

$$\begin{aligned} \text{BesselI}(a, 2\sqrt{x}) &= \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{\Gamma(a+n+1)} \left(\frac{2\sqrt{x}}{2}\right)^{2n+a} \\ &= \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{\Gamma(a+n+1)} (\sqrt{x})^{2n} (\sqrt{x})^a \\ &= \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{\Gamma(a+n+1)} x^n (\sqrt{x})^a \end{aligned}$$

Hence

$$\frac{1}{\sqrt{x}^a} \text{BesselI}(a, 2\sqrt{x}) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{\Gamma(a+n+1)} x^n \quad (7)$$

If we now compare (6) and (7), we see that if we set  $c_0$ , which is arbitrary, to be  $c_0 = \frac{1}{a\Gamma(a)}$ , then we obtain

$$\begin{aligned} u_{r=0}(x) &= \frac{1}{a\Gamma(a)} \sum_{n=0}^{\infty} \frac{1}{n!} \frac{a\Gamma(a)}{\Gamma(a+n+1)} x^n \\ &= \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{\Gamma(a+n+1)} x^n \end{aligned}$$

But this is (7). Hence we found the first solution, which is

$$u_{r=0}(x) = \frac{1}{\sqrt{x}^a} \text{BesselI}(a, 2\sqrt{x}) \quad (8)$$

The above was for  $r = 0$ . Now we find the second solution for  $r = -a$ . From (4)

$$c_n = \frac{c_{n-1}}{(n-a)(n-a-1) + (n-a) + a(n-a)}$$

For  $n = 1$

$$c_1 = \frac{c_0}{-a(1-a) + (1-a) + a(1-a)} = \frac{c_0}{(1-a)}$$

For  $n = 2$

$$c_2 = \frac{c_1}{(2-a)(1-a) + (2-a) + a(2-a)} = \frac{c_1}{4-2a} = \frac{c_0}{(1-a)} \frac{1}{2(2-a)}$$

For  $n = 3$

$$c_3 = \frac{c_2}{(3-a)(2-a) + (3-a) + a(3-a)} = \frac{c_2}{3(3-a)} = \frac{c_0}{(1-a)} \frac{1}{2(2-a)} \frac{1}{3(3-a)}$$

And so on. Since the solution is assumed to be  $x^r \sum_{n=0}^{\infty} c_n x^n$  then

$$\begin{aligned} u_{r=-a} &= x^{-a} \sum_{n=0}^{\infty} c_n x^n \\ &= \sum_{n=0}^{\infty} c_n x^{n-a} \\ &= c_0 x^{-a} \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{1}{(1-a)} \frac{1}{(2-a)} \frac{1}{(3-a)} \cdots \frac{1}{(n-a)} \right) x^{n-a} \end{aligned}$$

But as we found above, we obtain that  $(1-a)(2-a) \cdots (n-a) = \frac{\Gamma(-a+n+1)}{-a\Gamma(-a)}$ , therefore

$$u_{r=-a} = c_0 \sum_{n=0}^{\infty} \frac{1}{n!} \frac{-a \Gamma(-a)}{\Gamma(-a+n+1)} x^{n-a}$$

Modified Bessel function of second kind is  $\text{BesselK}(a, z) = \frac{\pi}{2 \sin(a\pi)} (\text{BesselI}(-a, z) - \text{BesselI}(a, z))$ . The above should result in  $\frac{1}{\sqrt{x^a}} \text{BesselK}(a, 2\sqrt{x})$  for  $z = 2\sqrt{x}$  by setting  $c_0$  to appropriate arbitrary value. I need to work out this final manipulation later. Hence we find  $u_{r=-a}(x) = \frac{1}{\sqrt{x^a}} \text{BesselK}(a, 2\sqrt{x})$ . Therefore, the solution is

$$u = C_1 \frac{1}{\sqrt{x^a}} \text{BesselI}(a, 2\sqrt{x}) + C_2 \frac{1}{\sqrt{x^a}} \text{BesselK}(a, 2\sqrt{x})$$

But

$$\begin{aligned} \frac{d}{dx} \frac{1}{\sqrt{x^a}} \text{BesselI}(a, 2\sqrt{x}) &= \frac{1}{\sqrt{x^{1+a}}} \text{BesselI}(1+a, 2\sqrt{x}) \\ \frac{d}{dx} \frac{1}{\sqrt{x^a}} \text{BesselI}(a, 2\sqrt{x}) &= -\frac{1}{\sqrt{x^{1+a}}} \text{BesselK}(1+a, 2\sqrt{x}) \end{aligned}$$

Hence

$$u' = C_1 \frac{1}{\sqrt{x^{1+a}}} \text{BesselI}(1+a, 2\sqrt{x}) - C_2 \frac{1}{\sqrt{x^{1+a}}} \text{BesselK}(1+a, 2\sqrt{x})$$

And from  $y = x^{a+1} \frac{u'}{u}$

$$y = x^{1+a} \frac{C_1 \frac{1}{\sqrt{x^{1+a}}} \text{BesselI}(1+a, 2\sqrt{x}) - C_2 \frac{1}{\sqrt{x^{1+a}}} \text{BesselK}(1+a, 2\sqrt{x})}{C_1 \frac{1}{\sqrt{x^a}} \text{BesselI}(a, 2\sqrt{x}) + C_2 \frac{1}{\sqrt{x^a}} \text{BesselK}(a, 2\sqrt{x})}$$

Let  $C = \frac{C_2}{C_1}$  hence

$$y = x^{1+a} \frac{\frac{1}{\sqrt{x^{1+a}}} \text{BesselI}(1+a, 2\sqrt{x}) - C \frac{1}{\sqrt{x^{1+a}}} \text{BesselK}(1+a, 2\sqrt{x})}{\frac{1}{\sqrt{x^a}} \text{BesselI}(a, 2\sqrt{x}) + C \frac{1}{\sqrt{x^a}} \text{BesselK}(a, 2\sqrt{x})}$$

Or

$$\begin{aligned} y &= x^{1+a} \frac{x^{-\frac{1}{2}} \text{BesselI}(1+a, 2\sqrt{x}) - C x^{-\frac{1}{2}} \text{BesselK}(1+a, 2\sqrt{x})}{\text{BesselI}(a, 2\sqrt{x}) + C \text{BesselK}(a, 2\sqrt{x})} \\ &= \frac{x^{\frac{1}{2}+a} \text{BesselI}(1+a, 2\sqrt{x}) - C x^{\frac{1}{2}+a} \text{BesselK}(1+a, 2\sqrt{x})}{\text{BesselI}(a, 2\sqrt{x}) + C \text{BesselK}(a, 2\sqrt{x})} \end{aligned}$$

Verification

```
1 eq:=diff(y(x),x)+x^(-a-1)*y(x)^2-x^a = 0;
2 num:=x^(1/2+a)*BesselI(1+a,2*sqrt(x))-_C1*x^(1/2+a)*BesselK(1+a,2*sqrt(x));
3 den:=BesselI(a,2*sqrt(x))+_C1*BesselK(a,2*sqrt(x));
4 my_sol:=num/den;
5 odetest(y(x)=my_sol,eq);
6 0
```

## 2.31 ODE No. 31

$$y'(x) - ax^n (y(x)^2 + 1) = 0$$

✓ **Mathematica** : cpu = 0.129904 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \tan \left( \frac{ax^{n+1}}{n+1} + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 23

$$\left\{ y(x) = \tan \left( \frac{(x^{n+1} + (n+1) \_C1) a}{n+1} \right) \right\}$$

|                      |
|----------------------|
| <b>Hand solution</b> |
|----------------------|

$$\begin{aligned}
 y' - ax^n (y^2 + 1) &= 0 \\
 y' &= ax^n + ax^n y^2 \\
 &= P(x) + Q(x)y + R(x)y^2
 \end{aligned}
 \tag{1}$$

This is Ricatti first order non-linear ODE.  $P(x) = ax^n, Q(x) = 0, R(x) = ax^n$ . But this is separable also. Hence

$$\begin{aligned}
 \frac{y'}{(y^2 + 1)} &= ax^n \\
 \frac{dy}{(y^2 + 1)} &= ax^n dx
 \end{aligned}$$

Integrating

$$\arctan (y(x)) = a \frac{x^{n+1}}{n+1} + C$$

Or

$$y(x) = \tan \left( a \frac{x^{n+1}}{n+1} + C \right)$$

Verification

```

1 restart;
2 eq:=diff(y(x),x)-a*x^n*(y(x)^2+1) = 0;
3 sol:=tan(a*x^(n+1)/(n+1)+_C1);
4 odetest(y(x)=sol,eq);
5 0

```

## 2.32 ODE No. 32

$$y'(x) + y(x)^2 \sin(x) - 2 \tan(x) \sec(x) = 0$$

✓ **Mathematica** : cpu = 0.31515 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow \frac{\csc(x)(c_1 \tan(x) \sec(x) - 2 \sin(x) \cos(x))}{c_1 \sec(x) + \cos^2(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.224 (sec), leaf count = 28

$$\left\{ y(x) = \frac{-2 (\cos(x))^3 _C1 - 2}{\left( (\cos(x))^3 _C1 - 2 \right) \cos(x)} \right\}$$

Hand solution

$$\begin{aligned}
 y' + y^2 \sin(x) - 2 \frac{\sin x}{\cos^2 x} &= 0 \\
 y' &= 2 \frac{\sin x}{\cos^2 x} - y^2 \sin(x) \\
 &= P(x) + Q(x)y + R(x)y^2
 \end{aligned}
 \tag{1}$$

This is Ricatti first order non-linear ODE.  $P(x) = 2 \frac{\sin x}{\cos^2 x}, Q(x) = 0, R(x) = -\sin(x)$ . A particular solution is  $y_p = \frac{1}{\cos x}$ , therefore the solution is

$$\begin{aligned}
 y &= y_p + \frac{1}{u} \\
 y &= \frac{1}{\cos x} + \frac{1}{u}
 \end{aligned}$$

Hence

$$y' = \frac{\sin x}{\cos^2 x} - \frac{u'}{u^2}$$

Equating this to RHS of (1) gives

$$\begin{aligned}
 \frac{\sin x}{\cos^2 x} - \frac{u'}{u^2} &= 2 \frac{\sin x}{\cos^2 x} - y^2 \sin(x) \\
 &= 2 \frac{\sin x}{\cos^2 x} - \left( \frac{1}{\cos x} + \frac{1}{u} \right)^2 \sin(x) \\
 &= 2 \frac{\sin x}{\cos^2 x} - \left( \frac{1}{\cos^2 x} + \frac{1}{u^2} + \frac{2}{u \cos x} \right) \sin(x)
 \end{aligned}$$

Hence

$$\begin{aligned} -\frac{u'}{u^2} &= -\frac{\sin x}{\cos^2 x} + 2\frac{\sin x}{\cos^2 x} - \frac{\sin(x)}{\cos^2 x} - \frac{\sin(x)}{u^2} - \frac{2\sin(x)}{u\cos x} \\ &= -\frac{\sin(x)}{u^2} - \frac{2\sin(x)}{u\cos x} \end{aligned}$$

Or

$$\begin{aligned} u' &= \sin(x) + \frac{2u\sin(x)}{\cos x} \\ u' - 2u\tan(x) &= \sin(x) \end{aligned}$$

Integrating factor is  $e^{-2\int \tan x dx} = e^{2\ln(\cos x)} = \cos^2(x)$ . Hence the above becomes

$$d(u\cos^2 x) = \cos^2(x)\sin(x)$$

Integrating both sides

$$\begin{aligned} u\cos^2 x &= \int \cos^2(x)\sin(x) dx + C \\ &= \frac{-1}{3}\cos^3(x) + C \end{aligned}$$

Hence

$$u = \frac{-1}{3}\cos(x) + \frac{C}{\cos^2 x}$$

Therefore

$$\begin{aligned} y &= y_p + \frac{1}{u} \\ &= \frac{1}{\cos x} + \frac{1}{\frac{-1}{3}\cos(x) + \frac{C}{\cos^2 x}} \\ &= \frac{1}{\cos x} + \frac{3\cos^2 x}{3C - \cos^3(x)} \end{aligned}$$

Let  $3C = C_1$

$$y = \frac{1}{\cos x} + \frac{3\cos^2 x}{C_2 - \cos^3(x)}$$

Verification

```
1 restart;
2 ode:=diff(y(x),x)+y(x)^2*sin(x)-2*sin(x)/cos(x)^2 = 0;
3 my_sol:=1/cos(x)+ 3*cos(x)^2/(_C1-cos(x)^3);
4 odetest(y(x)=my_sol,ode);
5 0
```

### 2.33 ODE No. 33

$$-\frac{y(x)^2 f'(x)}{g(x)} + \frac{g'(x)}{f(x)} + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.358938 (sec), leaf count = 160

Solve  $\left[ \int_1^{y(x)} \left( \frac{1}{(g(x) + f(x)K[2])^2} - \int_1^x \left( \frac{2(f(K[1])K[2]^2 f'(K[1]) - g(K[1])g'(K[1]))}{g(K[1])(g(K[1]) + f(K[1])K[2])^3} - \frac{2K[2]f'(K[1])}{g(K[1])(g(K[1]) + f(K[1])K[2])^3} \right) dx \right. \right.$

✓ **Maple** : cpu = 0.359 (sec), leaf count = 58

$$\left\{ y(x) = \frac{1}{(f(x))^2} \left( -g(x)f(x) \int \frac{\frac{d}{dx}f(x)}{g(x)(f(x))^2} dx - g(x)f(x) - C1 - 1 \right) \left( \int \frac{\frac{d}{dx}f(x)}{g(x)(f(x))^2} dx + -C1 \right)^{-1} \right\}$$

Hand solution

$$\begin{aligned} -\frac{f'}{g}y^2 + \frac{g'}{f} + y' &= 0 \\ y' &= -\frac{g'}{f} + \frac{f'}{g}y^2 \\ &= P(x) + Q(x)y + R(x)y^2 \end{aligned} \tag{1}$$

This is Ricatti first order non-linear ODE.  $P(x) = -\frac{g'}{f}, Q(x) = 0, R(x) = \frac{f'}{g}$ .  
To do.

2.34 ODE No. 34

$$f(x)y(x)^2 + g(x)y(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0927555 (sec), leaf count = 54

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp\left(\int_1^x -g(K[1])dK[1]\right)}{c_1 - \int_1^x -\exp\left(\int_1^{K[2]} -g(K[1])dK[1]\right) f(K[2])dK[2]} \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 28

$$\left\{ y\left(x\right) = \frac{{\rm e}^{\int -g\left(x\right){\rm d}x}}{\int {\rm e}^{\int -g\left(x\right){\rm d}x} f\left(x\right){\rm d}x + \_C1} \right\}$$

**Hand solution**

$$\begin{aligned} y^2 f + g y + y' &= 0 \\ y' &= -g y - y^2 f \\ &= P(x) + Q(x) y + R(x) y^2 \end{aligned} \tag{1}$$

This is Bernoulli first order non-linear ODE.  $P(x) = 0, Q(x) = -g, R(x) = f$ . First step is to divide by  $y^2$

$$\frac{y'}{y^2} = -g \frac{1}{y} - f \tag{2}$$

Let  $u = \frac{1}{y}$ , then  $u' = \frac{-y'}{y^2}$  and (2) becomes

$$\begin{aligned} -u' &= -g u - f \\ u' - g u &= f \end{aligned}$$

Integrating factor is  $e^{-\int g dx}$  hence

$$\begin{aligned} d\left(e^{-\int g dx} u\right) &= f e^{-\int g dx} \\ e^{-\int g dx} u &= \int f e^{-\int g dx} dx + C \\ u &= e^{\int g dx} \left(\int f e^{-\int g dx} dx + C\right) \end{aligned}$$

Hence

$$\begin{aligned} y &= \frac{1}{e^{\int g dx} \left(\int f e^{-\int g dx} dx + C\right)} \\ &= \frac{e^{-\int g dx}}{\int f e^{-\int g dx} dx + C} \end{aligned}$$

Let  $\beta = e^{-\int g dx}$  then

$$y = \frac{\beta}{\int f \beta dx + C}$$

Verification

```
1 restart;
2 eq:=diff(y(x),x)+f(x)*y(x)^2+g(x)*y(x) = 0;
3 beta:=exp(-Int(g(x),x)):
4 my_sol:=beta/(Int(f(x)*beta,x)+_C1);
5 odetest(y(x)=my_sol,eq);
6 0
```

## 2.35 ODE No. 35

$$f(x) (2ay(x) + b + y(x)^2) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.116346 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow \sqrt{b-a^2} \tan \left( \sqrt{b-a^2} \int_1^x -f(K[1]) dK[1] + c_1 \sqrt{b-a^2} \right) - a \right\} \right\}$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 35

$$\left\{ y(x) = \tanh \left( \sqrt{a^2-b} \left( -C1 + \int f(x) dx \right) \right) \sqrt{a^2-b} - a \right\}$$

**Hand solution**

$$\begin{aligned} y'(x) + f(x) (2ay(x) + b + y^2(x)) &= 0 \\ y'(x) &= -2af(x)y(x) - bf(x) - f(x)y^2(x) \\ &= P(x) + Q(x)y + R(x)y^2 \end{aligned} \quad (1)$$

This is Riccati first order non-linear ODE.  $P(x) = -bf(x)$ ,  $Q(x) = -2af(x)$ ,  $R(x) = -f(x)$ .  
Let

$$y(x) = -\frac{u'(x)}{u(x)R(x)} = \frac{u'(x)}{u(x)f(x)}$$

Hence

$$y'(x) = \frac{u''(x)}{u(x)f(x)} - \frac{(u'(x))^2}{u^2(x)f(x)} - \frac{u'(x)f'(x)}{u(x)f^2(x)}$$

Equating this to RHS of (1) gives

$$\begin{aligned} \frac{u''(x)}{u(x)f(x)} - \frac{(u'(x))^2}{u^2(x)f(x)} - \frac{u'(x)f'(x)}{u(x)f^2(x)} &= -2af(x)y(x) - bf(x) - f(x)y^2(x) \\ &= -2af(x) \left[ \frac{u'(x)}{u(x)f(x)} \right] - bf(x) - f(x) \left[ \frac{u'(x)}{u(x)f(x)} \right]^2 \\ &= -2a \frac{u'(x)}{u(x)} - bf(x) - \frac{u'(x)^2}{u^2(x)f(x)} \end{aligned}$$

Simplifying

$$\begin{aligned} u''(x) - \frac{(u'(x))^2}{u(x)} - \frac{u'(x)f'(x)}{f(x)} &= -2au'(x)f(x) - u(x)bf^2(x) - \frac{u'(x)^2}{u(x)} \\ u''(x) - \frac{u'(x)f'(x)}{f(x)} &= -2au'(x)f(x) - u(x)bf^2(x) \\ u''(x) + u'(x) \left( -\frac{f'(x)}{f(x)} + 2af(x) \right) + u(x)bf^2(x) &= 0 \end{aligned}$$

Second order ODE with variable coefficients. Since coefficients are variables and not constants, a power series method is the standard way to continue. When I tried solving this now pretending the coefficients are constants in time, using the standard auxiliary equation method, the solution did verify OK. I need to look more into this. For now, this is solved using standard method for solving second order ODE with constant coefficients.

$$u(x) = C_1 \exp \left( \frac{\int f(x) \sqrt{-b} dx \left( \sqrt{\frac{b-a^2}{b}} b + a\sqrt{-b} \right)}{b} \right) + C_2 \exp \left( \frac{\int f(x) \sqrt{-b} dx \left( -\sqrt{\frac{b-a^2}{b}} b + a\sqrt{-b} \right)}{b} \right)$$

Hence

$$\begin{aligned} u'(x) &= \frac{C_1 f(x) \sqrt{-b}}{b} \left( \sqrt{\frac{-a^2+b}{b}} b + \sqrt{-ba} \right) e^{\frac{\int f(x) \sqrt{-b} dx}{b} \left( \sqrt{\frac{-a^2+b}{b}} b + \sqrt{-ba} \right)} \\ &\quad + \frac{C_2 f(x) \sqrt{-b}}{b} \left( -\sqrt{\frac{-a^2+b}{b}} b + \sqrt{-ba} \right) e^{\frac{\int f(x) \sqrt{-b} dx}{b} \left( -\sqrt{\frac{-a^2+b}{b}} b + \sqrt{-ba} \right)} \end{aligned}$$



Therefore

$$y = \frac{u'(x)}{u(x)f(x)}$$

$$= \frac{\frac{C_1 f(x)\sqrt{-b}}{b} \left( \sqrt{\frac{-a^2+b}{b}} b + \sqrt{-ba} \right) e^{\frac{\int f(x)\sqrt{-b} dx}{b} \left( \sqrt{\frac{-a^2+b}{b}} b + \sqrt{-ba} \right)} + \frac{C_2 f(x)\sqrt{-b}}{b} \left( -\sqrt{\frac{-a^2+b}{b}} b + \sqrt{-ba} \right) e^{\frac{\int f(x)\sqrt{-b} dx}{b} \left( -\sqrt{\frac{-a^2+b}{b}} b + \sqrt{-ba} \right)}}{f(x) \left[ C_1 \exp \left( \frac{\int f(x)\sqrt{-b} dx}{b} \left( \sqrt{\frac{-a^2+b}{b}} b + \sqrt{-ba} \right) \right) + C_2 \exp \left( \frac{\int f(x)\sqrt{-b} dx}{b} \left( -\sqrt{\frac{-a^2+b}{b}} b + \sqrt{-ba} \right) \right) \right]}$$

Verification

```

1 restart;
2 book:=diff(y(x),x)+f(x)*(2*a*y(x)+b+y(x)^2)=0;
3 eqU:=diff(u(x),x$2)+diff(u(x),x)*(- diff(f(x),x)/f(x)+2*a*f(x))+u(x)*f(x)^2*b=0;
4 solU:=dsolve(eqU,u(x));
5 my_sol:=diff(rhs(solU),x)/(rhs(solU)*f(x));
6 odetest(y(x)=my_sol,book);
7 0

```

### 2.36 ODE No. 36

$$axy(x)^2 + y'(x) + y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.262492 (sec), leaf count = 195

$$\text{Solve} \left[ \frac{\text{Ai}' \left( \frac{\sqrt[3]{-\frac{1}{2}} \sqrt[3]{a}}{y(x)} - \frac{1}{2} \sqrt[3]{-\frac{1}{2}} a^{4/3} x^2 \right) - \left( -\frac{1}{2} \right)^{2/3} a^{2/3} x \text{Ai} \left( \frac{\sqrt[3]{-\frac{1}{2}} \sqrt[3]{a}}{y(x)} - \frac{1}{2} \sqrt[3]{-\frac{1}{2}} a^{4/3} x^2 \right)}{\text{Bi}' \left( \frac{\sqrt[3]{-\frac{1}{2}} \sqrt[3]{a}}{y(x)} - \frac{1}{2} \sqrt[3]{-\frac{1}{2}} a^{4/3} x^2 \right) - \left( -\frac{1}{2} \right)^{2/3} a^{2/3} x \text{Bi} \left( \frac{\sqrt[3]{-\frac{1}{2}} \sqrt[3]{a}}{y(x)} - \frac{1}{2} \sqrt[3]{-\frac{1}{2}} a^{4/3} x^2 \right)} + c_1 = 0, y(x) \right]$$

✓ **Maple** : cpu = 0.068 (sec), leaf count = 62

$$\left\{ y(x) = 2 \frac{a}{a^2 x^2 + 2 \text{RootOf} \left( \sqrt[3]{-2} a^2 \text{Bi}(\_Z) \_C1 x + \sqrt[3]{-2} a^2 x \text{Ai}(\_Z) + 2 \text{Bi}^{(1)}(\_Z) \_C1 + 2 \text{Ai}^{(1)}(\_Z) \right) \sqrt[3]{-2}} \right.$$

Hand solution

$$y'(x) = -axy^2 - y^3 \tag{1}$$

This is Abel first order non-linear. The general form is of Abel first kind is

$$y'(x) = f_0(x) + f_1(x)y(x) + f_2(x)y^2(x) + f_3(x)y^3(x)$$

In this case,  $f_0(x) = 0, f_1(x) = 0, f_2(x) = -ax, f_3(x) = -1$ . Note  $\left(\frac{f_3}{f_2}\right)' = \left(\frac{1}{ax}\right)' = -\frac{1}{a}$ . While Abel second kind has the form

$$(y + g(x))y'(x) = f_0(x) + f_1(x)y(x) + f_2(x)y^2(x)$$

For  $g(x) \neq 0$ .

Looking at (1) again, using the transformation suggested in Kamke  $u = \frac{1}{y} - \frac{1}{2}ax^2$  or  $y = \frac{1}{u + \frac{1}{2}ax^2}$  Then

$$y' = \frac{-u' - ax}{\left(u + \frac{1}{2}ax^2\right)^2}$$

Equating the above to the RHS of (1) gives

$$\frac{-u' - ax}{\left(u + \frac{1}{2}ax^2\right)^2} = -ax \left( \frac{1}{u + \frac{1}{2}ax^2} \right)^2 - \left( \frac{1}{u + \frac{1}{2}ax^2} \right)^3$$

$$-u' - ax = -ax - \frac{1}{u + \frac{1}{2}ax^2}$$

$$\frac{du}{dx} = \frac{1}{u + \frac{1}{2}ax^2}$$

Writing as

$$\frac{dx}{du} = u + \frac{1}{2}ax^2 \quad (2)$$

This can now be viewed as reverse Riccati in  $x$ . Using the standard transformation

$$x = -\frac{z'}{z\left(\frac{1}{2}a\right)} = -\frac{2z'}{az} \quad (3)$$

Hence

$$\frac{dx}{du} = -\frac{2}{a} \left( \frac{z''}{z} - \frac{(z')^2}{z^2} \right)$$

Equating this to RHS of (2) gives a second order Airy ODE where the dependent variable is  $z$  and the independent variable is  $u$

$$\begin{aligned} -\frac{2}{a} \left( \frac{z''}{z} - \frac{(z')^2}{z^2} \right) &= u + \frac{1}{2}a \left( -\frac{2z'}{az} \right)^2 \\ -\frac{2}{a} \frac{z''}{z} + \frac{2}{a} \frac{(z')^2}{z^2} &= u + \frac{1}{2}a \frac{4(z')^2}{a^2 z^2} \\ -\frac{2}{a} \frac{z''}{z} + \frac{2}{a} \frac{(z')^2}{z^2} &= u + \frac{2}{a} \frac{(z')^2}{z^2} \\ -\frac{2}{a} \frac{z''}{z} &= u \\ z''(u) + \frac{a}{2}uz(u) &= 0 \end{aligned}$$

This is Airy ODE whose solution is found using power series method. The solution is

$$z(u) = C_1 \text{AiryAI} \left( -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u \right) + C_2 \text{AiryBI} \left( -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u \right) \quad (4)$$

We now go back to (3) and find  $x$

$$x = -\frac{2z'}{az}$$

Since

$$\begin{aligned} \frac{d}{du} \text{AiryAI} \left( -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u \right) &= -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}} \text{AiryAI} \left( 1, -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u \right) \\ \frac{d}{du} \text{AiryBI} \left( -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u \right) &= -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}} \text{AiryBI} \left( 1, -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u \right) \end{aligned}$$

Then

$$x = -\frac{2}{a} \frac{-C_1 \frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}} \text{AiryAI} \left( 1, -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u \right) - C_2 \frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}} \text{AiryBI} \left( 1, -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u \right)}{C_1 \text{AiryAI} \left( -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u \right) + C_2 \text{AiryBI} \left( -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u \right)}$$

Therefore  $\frac{dx}{du}$  is now found from above. Once we find  $\frac{dx}{du}$  then  $\frac{du}{dx}$  is also found. Using  $\frac{du}{dx} = \frac{1}{u + \frac{1}{2}ax^2}$  now  $u(x)$  is found. Once  $u(x)$  is found then  $y(x)$  is found from the original transformation  $y = \frac{1}{u + \frac{1}{2}ax^2}$ . This is all now just algebra.

## 2.37 ODE No. 37

---

$$-ae^x y(x)^2 + y'(x) - y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.755059 (sec), leaf count = 78

$$\text{Solve} \left[ -iae^x = \frac{2e^{\frac{1}{2}(-iae^x - \frac{i}{y(x)})^2}}{2c_1 + \sqrt{2\pi} \text{erfi} \left( \frac{-iae^x - \frac{i}{y(x)}}{\sqrt{2}} \right)}, y(x) \right]$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 50

$$\left\{ -C1 + \frac{1}{e^x a} e^{-\frac{(e^x a + (y(x))^{-1})^2}{2}} + \frac{\sqrt{\pi} \sqrt{2}}{2} \text{Erf} \left( \frac{(e^x a + (y(x))^{-1}) \sqrt{2}}{2} \right) = 0 \right\}$$

## 2.38 ODE No. 38

$$-ay(x)^3 - \frac{b}{x^{3/2}} + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.15699 (sec), leaf count = 99

$$\text{Solve} \left[ -2\text{RootSum} \left[ -2\#1^3 + \#1 \sqrt[3]{-\frac{1}{ab^2}} - 2\&, \frac{\log \left( y(x) \sqrt[3]{\frac{ax^{3/2}}{b}} - \#1 \right)}{\sqrt[3]{-\frac{1}{ab^2}} - 6\#1^2} \& \right] = \frac{ax \log(x)}{\left( \frac{ax^{3/2}}{b} \right)^{2/3}} + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 34

$$\left\{ y(x) = \text{RootOf} \left( -\ln(x) + \_C1 + 2 \int^{-Z} (2a\_a^3 + \_a + 2b)^{-1} d\_a \right) \frac{1}{\sqrt{x}} \right\}$$

**Hand solution**

$$y'(x) = ay^3 + bx^{-\frac{3}{2}} \quad (1)$$

This can be transformed to Abel first order non-linear ode as follows. Let  $y(x) = x^{-\frac{1}{2}}\eta(\xi)$  where  $\xi = \ln x$  hence

$$\begin{aligned} \frac{dy}{dx} &= -\frac{1}{2}x^{-\frac{3}{2}}\eta(\xi) + x^{-\frac{1}{2}}\frac{d\eta}{d\xi}\frac{d\xi}{dx} \\ &= -\frac{1}{2}x^{-\frac{3}{2}}\eta(\xi) + x^{-\frac{1}{2}}\frac{d\eta}{d\xi}\frac{1}{x} \\ &= -\frac{1}{2}x^{-\frac{3}{2}}\eta(\xi) + x^{-\frac{3}{2}}\frac{d\eta}{d\xi} \end{aligned}$$

Substituting in (1) gives

$$\begin{aligned} -\frac{1}{2}x^{-\frac{3}{2}}\eta(\xi) + x^{-\frac{3}{2}}\frac{d\eta}{d\xi} &= a \left( x^{-\frac{1}{2}}\eta(\xi) \right)^3 + bx^{-\frac{3}{2}} \\ -\frac{1}{2}x^{-\frac{3}{2}}\eta(\xi) + x^{-\frac{3}{2}}\frac{d\eta}{d\xi} &= ax^{-\frac{3}{2}}\eta^3(\xi) + bx^{-\frac{3}{2}} \\ -\frac{1}{2}\eta + \eta' &= a\eta^3 + b \\ \eta' &= b + \frac{1}{2}\eta + a\eta^3 \end{aligned}$$

This is Abel first kind. In general form it is

$$\eta' = f_0 + f_1\eta + f_2\eta^2 + f_3\eta^3$$

Where in this case  $f_0 = b, f_1 = \frac{1}{2}, f_2 = 0, f_3 = a$ . Using Maple, the solution to the above is (I need to learn how to solve Able by hand more) is implicit, given as

$$\eta = \xi - \int^{\eta(\xi)} \frac{1}{b + \frac{1}{2}z + az^3} dz + C$$

Where  $C$  is constant of integration. Hence, since  $y(x) = x^{-\frac{1}{2}}\eta(\xi)$ , then  $\eta(\xi) = \sqrt{xy}$  and the above becomes

$$\begin{aligned} \sqrt{xy} &= \ln x - \int^{\sqrt{xy}} \frac{1}{b + \frac{1}{2}z + az^3} dz + C \\ y(x) &= \left( \ln x - \int^{\sqrt{xy}} \frac{1}{b + \frac{1}{2}z + az^3} dz + C \right) \frac{1}{\sqrt{x}} \end{aligned}$$

Did not verify. Need to look more into this later.

### 2.39 ODE No. 39

$$-a_0 - a_1 y(x) - a_2 y(x)^2 - a_3 y(x)^3 + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.121109 (sec), leaf count = 54

$$\text{Solve} \left[ \text{RootSum} \left[ \#1^3 a_3 + \#1^2 a_2 + \#1 a_1 + a_0 \&, \frac{\log(y(x) - \#1)}{3 \#1^2 a_3 + 2 \#1 a_2 + a_1} \& \right] = c_1 + x, y(x) \right]$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 30

$$\left\{ x - \int^{y(x)} \left( -a^3 a_3 + -a^2 a_2 + -a a_1 + a_0 \right)^{-1} d_- a + -C_1 = 0 \right\}$$

### 2.40 ODE No. 40

$$3ay(x)^3 + 6axy(x)^2 + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.320318 (sec), leaf count = 185

$$\text{Solve} \left[ \frac{\sqrt[3]{-3} \sqrt[3]{a} x \text{Ai} \left( (-3)^{2/3} a^{2/3} x^2 - \frac{(-1)^{2/3}}{\sqrt[3]{3} \sqrt[3]{a} y(x)} \right) + \text{Ai}' \left( (-3)^{2/3} a^{2/3} x^2 - \frac{(-1)^{2/3}}{\sqrt[3]{3} \sqrt[3]{a} y(x)} \right)}{\sqrt[3]{-3} \sqrt[3]{a} x \text{Bi} \left( (-3)^{2/3} a^{2/3} x^2 - \frac{(-1)^{2/3}}{\sqrt[3]{3} \sqrt[3]{a} y(x)} \right) + \text{Bi}' \left( (-3)^{2/3} a^{2/3} x^2 - \frac{(-1)^{2/3}}{\sqrt[3]{3} \sqrt[3]{a} y(x)} \right)} + c_1 = 0, y(x) \right]$$

✓ **Maple** : cpu = 0.051 (sec), leaf count = 48

$$\left\{ y(x) = \left( 3 a x^2 + \text{RootOf} \left( \sqrt[3]{-3} a \text{Bi}(-Z) - C_1 x + \sqrt[3]{-3} a x \text{Ai}(-Z) + \text{Bi}^{(1)}(-Z) - C_1 + \text{Ai}^{(1)}(-Z) \right) \sqrt[3]{-3} a \right)^{-1} \right\}$$

### 2.41 ODE No. 41

$$axy(x)^3 + by(x)^2 + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.163278 (sec), leaf count = 103

$$\text{Solve} \left[ -\frac{b^2 \left( \frac{2 \tan^{-1} \left( \frac{-2 a x y(x) - b}{b \sqrt{-\frac{4 a}{b^2} - 1}} \right)}{\sqrt{-\frac{4 a}{b^2} - 1}} - \log \left( \frac{a(-x) y(x) (-a x y(x) - b) - a}{a^2 x^2 y(x)^2} \right) \right)}{2 a} = c_1 - \frac{b^2 \log(x)}{a}, y(x) \right]$$

✓ **Maple** : cpu = 0.181 (sec), leaf count = 103

$$\left\{ y(x) = \frac{1}{x} e^{\text{RootOf} \left( 2 \sqrt{b^2 + 4} a b \text{Artanh} \left( \frac{2 a e^{-Z} + b}{\sqrt{b^2 + 4} a} \right) - \ln(x^2 (a e^{-Z} + b e^{-Z} - 1)) b^2 + 2 - C_1 b^2 + 2 - Z b^2 - 4 \ln(x^2 (a e^{-Z} + b e^{-Z} - 1)) a + 8 - C_1 \right)} \right\}$$

### 2.42 ODE No. 42

$$y'(x) - x(x+2)y(x)^3 - (x+3)y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.842533 (sec), leaf count = 485

$$\text{Solve} \left[ c_1 = -\frac{\frac{i \sqrt{\frac{2}{\pi}} \sqrt{\frac{1}{2y(x)} + \frac{1}{4}} (x+1)^2 - \frac{1}{4}} \left( \frac{\sinh \left( \sqrt{\frac{1}{2y(x)} + \frac{1}{4}} (x+1)^2 - \frac{1}{4} \right)}{\sqrt{\frac{1}{2y(x)} + \frac{1}{4}} (x+1)^2 - \frac{1}{4}} - \cosh \left( \sqrt{\frac{1}{2y(x)} + \frac{1}{4}} (x+1)^2 - \frac{1}{4} \right) \right)}{\sqrt{-i \sqrt{\frac{1}{2y(x)} + \frac{1}{4}} (x+1)^2 - \frac{1}{4}}} - \frac{i \sqrt{\frac{2}{\pi}} \left( \frac{x+1}{2} + \frac{1}{2} \right) \sinh \left( \sqrt{\frac{1}{2y(x)} + \frac{1}{4}} (x+1)^2 - \frac{1}{4} \right)}{\sqrt{-i \sqrt{\frac{1}{2y(x)} + \frac{1}{4}} (x+1)^2 - \frac{1}{4}}}}{\frac{i \sqrt{\frac{2}{\pi}} \sqrt{\frac{1}{2y(x)} + \frac{1}{4}} (x+1)^2 - \frac{1}{4}} \left( i \sinh \left( \sqrt{\frac{1}{2y(x)} + \frac{1}{4}} (x+1)^2 - \frac{1}{4} \right) - \frac{i \cosh \left( \sqrt{\frac{1}{2y(x)} + \frac{1}{4}} (x+1)^2 - \frac{1}{4} \right)}{\sqrt{\frac{1}{2y(x)} + \frac{1}{4}} (x+1)^2 - \frac{1}{4}} \right)}{\sqrt{-i \sqrt{\frac{1}{2y(x)} + \frac{1}{4}} (x+1)^2 - \frac{1}{4}}} - \frac{\sqrt{\frac{2}{\pi}} \left( \frac{x+1}{2} + \frac{1}{2} \right) \cosh \left( \sqrt{\frac{1}{2y(x)} + \frac{1}{4}} (x+1)^2 - \frac{1}{4} \right)}{\sqrt{-i \sqrt{\frac{1}{2y(x)} + \frac{1}{4}} (x+1)^2 - \frac{1}{4}}}} \right]$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 40

$$\left\{ -C_1 + \text{Artanh} \left( x \sqrt{y(x)} \frac{1}{\sqrt{x(x+2)y(x)+2}} \right) + \frac{1}{2} \sqrt{x(x+2)y(x)+2} \frac{1}{\sqrt{y(x)}} = 0 \right\}$$

### 2.43 ODE No. 43

$$y(x)^3 (4a^2x + 3ax^2 + b) + y'(x) + 3xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 5.71825 (sec), leaf count = 490

$$\text{Solve} \left[ c_1 = -\frac{i\sqrt{-\frac{4a^3-3b}{4a^3} - \frac{3}{2a^2y(x)} + \frac{(-2a-3x)^2}{4a^2}} J_{\frac{1}{2}\sqrt{\frac{4a^3-3b}{a^3}}+1} \left( -i\sqrt{\frac{(-2a-3x)^2}{4a^2} - \frac{4a^3-3b}{4a^3} - \frac{3}{2a^2y(x)}} \right) + \left( \frac{1}{2}\sqrt{\frac{4a^3-3b}{a^3}} - \frac{3}{2a^2y(x)} \right) \right. \\ \left. i\sqrt{-\frac{4a^3-3b}{4a^3} - \frac{3}{2a^2y(x)} + \frac{(-2a-3x)^2}{4a^2}} Y_{\frac{1}{2}\sqrt{\frac{4a^3-3b}{a^3}}+1} \left( -i\sqrt{\frac{(-2a-3x)^2}{4a^2} - \frac{4a^3-3b}{4a^3} - \frac{3}{2a^2y(x)}} \right) + \left( \frac{1}{2}\sqrt{\frac{4a^3-3b}{a^3}} - \frac{3}{2a^2y(x)} \right) \right]$$

✓ **Maple** : cpu = 1.246 (sec), leaf count = 373

$$\left\{ -C1 + 1 \left( -K_{\frac{1}{2}\sqrt{\frac{4a^3-3b}{a^3}}+1} \left( -\frac{\sqrt{3}}{2} \sqrt{\frac{4y(x)a^2x + 3ax^2y(x) + by(x) - 2a}{a^3y(x)}} \right) \sqrt{3} \sqrt{\frac{4y(x)a^2x + 3ax^2y(x) + by(x)}{a^3y(x)}} \right) \right\}$$

### 2.44 ODE No. 44

$$2ax^3y(x)^3 + y'(x) + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0735979 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{2}}{\sqrt{-2ax^2 - a + 2c_1e^{2x^2}}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{2}}{\sqrt{-2ax^2 - a + 2c_1e^{2x^2}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 53

$$\left\{ y(x) = -2 \frac{1}{\sqrt{-4ax^2 + 4e^{2x^2} - C1 - 2a}}, y(x) = 2 \frac{1}{\sqrt{-4ax^2 + 4e^{2x^2} - C1 - 2a}} \right\}$$

**Hand solution**

$$y' = -2xy - 2ax^3y^3 \tag{1}$$

This is of the form  $y' = f_0 + f_1y + f_2y^2 + f_3y^3$  where  $f_0 = 0, f_2 = 0$ . Hence this is Bernoulli first order non-linear ODE. We start by diving by  $y^3$

$$\frac{y'}{y^3} = -2x\frac{1}{y^2} - 2ax^3$$

Let  $u = \frac{1}{y^2}$ , hence  $u' = -2\frac{y'}{y^3}$  and the above becomes

$$\begin{aligned} -\frac{1}{2}u' &= -2xu - 2ax^3 \\ u' - 4xu &= 4ax^3 \end{aligned}$$

Integrating factor is  $e^{-4\int xdx} = e^{-2x^2}$  hence

$$\frac{d}{dx} \left( e^{-2x^2} u \right) = 4ax^3 e^{-2x^2}$$

Integrating

$$\begin{aligned} e^{-2x^2} u &= 4a \int x^3 e^{-2x^2} dx + C \\ &= 4a \left( \frac{-1}{8} (2x^2 + 1) e^{-2x^2} \right) + C \end{aligned}$$

Therefore

$$u = -\frac{1}{2}a(2x^2 + 1) + Ce^{2x^2}$$

Hence

$$y^2 = \frac{1}{u} = \frac{1}{-\frac{1}{2}a(2x^2 + 1) + Ce^{2x^2}}$$

Or

$$y = \pm \frac{\sqrt{2}}{\sqrt{-a(2x^2 + 1) + Ce^{2x^2}}}$$

Verification

```

1 ode:=2*a*x^3*y(x)^3+diff(y(x),x)+2*x*y(x)=0;
2 my_sol:=sqrt(2)/sqrt(-a*(2*x^2+1)+_C1*exp(2*x^2));
3 odetest(y(x)=my_sol,ode);
4 0
5 my_sol:=-sqrt(2)/sqrt(-a*(2*x^2+1)+_C1*exp(2*x^2));
6 odetest(y(x)=my_sol,ode);
7 0

```

### 2.45 ODE No. 45

$$2y(x)^3\left(a^2x^3-b^2x\right)+3by(x)^2+y'(x)=0$$

✓ **Mathematica** : cpu = 0.499621 (sec), leaf count = 133

$$\text{Solve}\left[c_1=\sqrt[4]{\left(\frac{b}{ax}-\frac{1}{ax^2y(x)}\right)^2-1}\left(-\frac{\left(\frac{b}{ax}-\frac{1}{ax^2y(x)}\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\left(\frac{b}{ax}-\frac{1}{ax^2y(x)}\right)^2\right)}{2\sqrt[4]{1-\left(\frac{b}{ax}-\frac{1}{ax^2y(x)}\right)^2}}-\frac{ax}{b}\right),y(x)\right]$$

✓ **Maple** : cpu = 0.096 (sec), leaf count = 123

$$\left\{-C1+1\sqrt[4]{\left(\frac{ax}{b}+\left(\frac{b^2y(x)}{a}-\frac{b}{ax}\right)^{-1}\right)^2-1}\left(\frac{b^2y(x)}{a}-\frac{b}{ax}\right)^{-1}\frac{1}{\sqrt{\frac{ax}{b}+\left(\frac{b^2y(x)}{a}-\frac{b}{ax}\right)^{-1}}}-\int^{\frac{ax^2y(x)}{bxy(x)^{-1}}}1\sqrt[4]{-a^2}\right.$$

### 2.46 ODE No. 46

$$-x^{-a}y(x)+ax^{-a-1}-x^{-2a}-x^ay(x)^3+y'(x)+3y(x)^2=0$$

✓ **Mathematica** : cpu = 0.254204 (sec), leaf count = 228

$$\left\{\left\{y(x)\rightarrow x^{-a}-\frac{e^{-\frac{2x^{1-a}}{1-a}}}{\sqrt{c_1-\frac{2^{\frac{2(a+1)}{a-1}+1}x^{a+1}\left(\frac{x^{1-a}}{1-a}\right)^{\frac{a+1}{a-1}}\Gamma\left(\frac{a+1}{1-a},-\frac{4x^{1-a}}{a-1}\right)}}}\right\},\left\{y(x)\rightarrow \frac{e^{-\frac{2x^{1-a}}{1-a}}}{\sqrt{c_1-\frac{2^{\frac{2(a+1)}{a-1}+1}x^{a+1}\left(\frac{x^{1-a}}{1-a}\right)^{\frac{a+1}{a-1}}\Gamma\left(\frac{a+1}{1-a},-\frac{4x^{1-a}}{a-1}\right)}}}\right\}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 956

$$\left\{y(x)=-1e^{2\frac{x}{(a-1)x^a}}\frac{1}{\sqrt{-C1-2\frac{1}{1-a}2^{-2\frac{a}{1-a}-2(1-a)^{-1}}\left((1-a)^{-1}\right)^{-\frac{a}{1-a}-(1-a)^{-1}}\left(-\frac{(a-1)(1-a)}{(a+1)(-3+a)}2^{-3+2\frac{a}{1-a}+2(1-a)}\right)}}}\right\}$$

### 2.47 ODE No. 47

$$-a\left(x^n-x\right)y(x)^3+y'(x)-y(x)^2=0$$

✗ **Mathematica** : cpu = 28.2865 (sec), leaf count = 0 , could not solve  
DSolve[-y[x]^2 - a\*(-x + x^n)\*y[x]^3 + Derivative[1][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x)-a\*(x^n-x)\*y(x)^3-y(x)^2 = 0,y(x))

### 2.48 ODE No. 48

$$y(x)^3\left(-\left(ax^n+bx\right)\right)-cy(x)^2+y'(x)=0$$

✗ **Mathematica** : cpu = 30.3707 (sec), leaf count = 0 , could not solve  
DSolve[-(c\*y[x]^2) - (b\*x + a\*x^n)\*y[x]^3 + Derivative[1][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x)-(a\*x^n+b\*x)\*y(x)^3-c\*y(x)^2 = 0,y(x))

2.49 ODE No. 49

---

$$ay(x)^3\phi'(x) + \frac{(2a+1)y(x)\phi''(x)}{\phi'(x)} + 6a\phi(x)y(x)^2 + 2a + y'(x) + 2 = 0$$

✗ **Mathematica** : cpu = 25.6358 (sec), leaf count = 0 , could not solve  
DSolve[2 + 2\*a + 6\*a\*phi[x]\*y[x]^2 + a\*y[x]^3\*Derivative[1][phi][x] + Derivative[1][y][x] +  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x)+a\*diff(phi(x),x)\*y(x)^3+6\*a\*phi(x)\*y(x)^2+(2\*a+1)\*y(x)\*diff(diff(phi(x),

2.50 ODE No. 50

---

$$-f_0(x) - f_1(x)y(x) - f_2(x)y(x)^2 - f_3(x)y(x)^3 + y'(x) = 0$$

✗ **Mathematica** : cpu = 300.075 (sec), leaf count = 0 , timed out  
\$Aborted  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x)-f3(x)\*y(x)^3-f2(x)\*y(x)^2-f1(x)\*y(x)-f0(x) = 0,y(x))

2.51 ODE No. 51

---

$$-h(x)(y(x)-f(x))(y(x)-g(x))\left(y(x)-\frac{af(x)+bg(x)}{a+b}\right)-\frac{f'(x)(y(x)-g(x))}{f(x)-g(x)}-\frac{(y(x)-f(x))g'(x)}{g(x)-f(x)}+y'(x)=0$$

✓ **Mathematica** : cpu = 1.54225 (sec), leaf count = 355

$$\text{Solve}\left[-\frac{1}{3}(a-b)^{2/3}(2a+b)^{2/3}(a+2b)^{2/3}\text{RootSum}\left[\#1^3(a-b)^{2/3}(2a+b)^{2/3}(a+2b)^{2/3}-3\#1a^2-3\#1ab-3\right],\right.$$

✓ **Maple** : cpu = 0.151 (sec), leaf count = 237

$$\left\{y(x)=\frac{1}{9a^3+18ba^2+18b^2a+9b^3}\left(2(a-b)(a+2b)(a+b/2)(f(x)-g(x))\text{RootOf}\left(-27\int^Z\frac{1}{(2-a)a^2-}\right.\right.\right.$$

2.52 ODE No. 52

---

$$-ay(x)^n - bx^{\frac{n}{1-n}} + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.23076 (sec), leaf count = 117

$$\text{Solve}\left[\int_1^{\left(\frac{ax^{-\frac{n}{1-n}}}{b}\right)^{\frac{1}{n}}y(x)}\frac{1}{K[1]^n-\left(\frac{(-1)^nb^{1-n}(n-1)^{-n}}{a}\right)^{\frac{1}{n}}K[1]+1}dK[1]=\int_1^xbK[2]^{\frac{n}{1-n}}\left(\frac{aK[2]^{-\frac{n}{1-n}}}{b}\right)^{\frac{1}{n}}dK[2]+c,\right.$$

✓ **Maple** : cpu = 0.322 (sec), leaf count = 61

$$\left\{-\int_{-b}^{y(x)}1x^{\frac{n}{n-1}}\left((ax(n-1)-a^n+{}_a)x^{\frac{n}{n-1}}+bx(n-1)\right)^{-1}d{}_a(n-1)+\ln(x)-{}_C1=0\right\}$$

2.53 ODE No. 53

---

$$f(x)^{1-n}g'(x)y(x)^n(-(ag(x)+b)^{-n})-\frac{y(x)f'(x)}{f(x)}-f(x)g'(x)+y'(x)=0$$

✓ **Mathematica** : cpu = 0.413774 (sec), leaf count = 96

$$\text{Solve}\left[\int_1^{(f(x)^{-n}(b+ag(x))^{-n})^{\frac{1}{n}}y(x)}\frac{1}{K[1]^n-(a^n)^{\frac{1}{n}}K[1]+1}dK[1]=\frac{f(x)(ag(x)+b)\log(ag(x)+b)(f(x)^{-n}(ag(x)-}$$

✓ **Maple** : cpu = 0.126 (sec), leaf count = 281

$$\left\{y(x)=\frac{(ag(x)+b)f(x)}{a}RootOf\left(-\int^{-Z}\frac{\left((ag(x)+b)^{-n}\left(\frac{d}{dx}g(x)\right)(f(x))^{1-n}\right)^{-n-1}\left(f(x)\frac{d}{dx}g(x)\right)^{-2n+1}}{\right.}\right.\right.$$

2.54 ODE No. 54

---

$$-a^nf(x)^{1-n}g'(x)y(x)^n-\frac{y(x)f'(x)}{f(x)}-f(x)g'(x)+y'(x)=0$$

✓ **Mathematica** : cpu = 0.316594 (sec), leaf count = 74

$$\text{Solve}\left[y(x)\left(a^nf(x)^{-n}\right)^{\frac{1}{n}}{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\left(\left(a^nf(x)^{-n}\right)^{\frac{1}{n}}y(x)\right)^n\right)=f(x)g(x)\left(a^nf(x)^{-n}\right)^{\frac{1}{n}}+c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.129 (sec), leaf count = 38

$$\left\{\frac{ay(x)}{nf(x)}LerchPhi\left(-\left(\frac{ay(x)}{f(x)}\right)^n,1,n^{-1}\right)-ag(x)+_C1=0\right\}$$

2.55 ODE No. 55

---

$$-f(x)y(x)^n-g(x)y(x)-h(x)+y'(x)=0$$

✗ **Mathematica** : cpu = 3.82748 (sec), leaf count = 0 , could not solve

`DSolve[-h[x] - g[x]*y[x] - f[x]*y[x]^n + Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x),x)-f(x)*y(x)^n-g(x)*y(x)-h(x) = 0,y(x))`

2.56 ODE No. 56

---

$$-f(x)y(x)^a-g(x)y(x)^b+y'(x)=0$$

✗ **Mathematica** : cpu = 1.5787 (sec), leaf count = 0 , could not solve

`DSolve[-(f[x]*y[x]^a) - g[x]*y[x]^b + Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x),x)-f(x)*y(x)^a-g(x)*y(x)^b = 0,y(x))`

2.57 ODE No. 57

---

$$y'(x)-\sqrt{|y(x)|}=0$$

✓ **Mathematica** : cpu = 0.0525326 (sec), leaf count = 26

$$\left\{\left\{y(x)\rightarrow\text{InverseFunction}\left[\int_1^{\#1}\frac{1}{\sqrt{|K[1]|}}dK[1]\&\right][c_1+x]\right\}\right\}$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 31

$$\left\{x-\begin{cases}-2\sqrt{-y(x)} & y(x)\leq 0 \\ 2\sqrt{y(x)} & 0<y(x)\end{cases}+_C1=0\right\}$$

2.58 ODE No. 58

---

$$a\left(-\sqrt{y(x)}\right)-bx+y'(x)=0$$

✓ **Mathematica** : cpu = 0.222643 (sec), leaf count = 118



Solve

$$\frac{a^2 \left( -\log \left( a^2 \left( \sqrt{\frac{a^2 y(x)}{b^2 x^2}} + 1 \right) - \frac{2a^2 y(x)}{bx^2} \right) - \frac{2a \tanh^{-1} \left( \frac{a \left( 1 - \frac{4b \sqrt{\frac{a^2 y(x)}{b^2 x^2}}}{a^2} \right)}{\sqrt{a^2 + 8b}} \right)}{\sqrt{a^2 + 8b}} \right)}{2b} = \frac{a^2 \log(x)}{b} + c_1, y(x)$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 68

$$\left\{-\frac{1}{2}\ln\left(\sqrt{y\left(x\right)}ax+bx^2-2\,y\left(x\right)\right)+a\sqrt{y\left(x\right)}Artanh\left(1\left(a\sqrt{y\left(x\right)}+2\,bx\right)\frac{1}{\sqrt{y\left(x\right)\left(a^2+8\,b\right)}}\right)\frac{1}{\sqrt{y\left(x\right)\left(a^2+8\,b\right)}}\right\}$$

### 2.59 ODE No. 59

$$a\left(-\sqrt{y(x)^2+1}\right)-b+y'(x)=0$$

✓ **Mathematica** : cpu = 0.229127 (sec), leaf count = 96

$$\left\{\left\{y(x)\rightarrow \text{InverseFunction}\left[\frac{b\tan^{-1}\left(\frac{\#1b}{\sqrt{\#1^2+1}\sqrt{a^2-b^2}}\right)-\frac{b\tan^{-1}\left(\frac{\#1a}{\sqrt{a^2-b^2}}\right)+\sinh^{-1}(\#1)}{a}\&[c_1+x]\right]\right\}\right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 26

$$\left\{x-\int^{y(x)}\left(a\sqrt{-a^2+1}+b\right)^{-1}d\_a+\_C1=0\right\}$$

### 2.60 ODE No. 60

$$y'(x)-\frac{\sqrt{y(x)^2-1}}{\sqrt{x^2-1}}=0$$

✓ **Mathematica** : cpu = 0.11609 (sec), leaf count = 173

$$\left\{\left\{y(x)\rightarrow -\frac{1}{2}e^{-c_1}\sqrt{2e^{4c_1}x^2+2e^{4c_1}\sqrt{(x-1)(x+1)}x+2e^{2c_1}-e^{4c_1}+2x^2-2\sqrt{(x-1)(x+1)}x-1}\right\},\left\{y(x)-\right.\right.$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 29

$$\left\{\ln\left(x+\sqrt{x^2-1}\right)-\ln\left(y\left(x\right)+\sqrt{\left(y\left(x\right)\right)^2-1}\right)+\_C1=0\right\}$$

Hand solution

$$y'=\pm\frac{\sqrt{y^2-1}}{\sqrt{x^2-1}}\tag{1}$$

Separable. For the positive case

$$\frac{dy}{dx} \frac{1}{\sqrt{y^2-1}} = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{dy}{(y^2-1)^{\frac{1}{2}}} = \frac{dx}{(x^2-1)^{\frac{1}{2}}}$$

Integrating

$$\int \frac{dy}{(y^2-1)^{\frac{1}{2}}} = \int \frac{dx}{(x^2-1)^{\frac{1}{2}}} + C$$

But  $\int \frac{dy}{(y^2-1)^{\frac{1}{2}}} = \tanh^{-1} \frac{y}{(y^2-1)^{\frac{1}{2}}} = \ln(y + \sqrt{y^2-1})$ , hence

$$\ln(y + \sqrt{y^2-1}) = \ln(x + \sqrt{x^2-1}) + C$$

For the negative case

$$\frac{dy}{dx} \frac{1}{\sqrt{y^2-1}} = -\frac{1}{\sqrt{x^2-1}}$$

$$\frac{dy}{(y^2-1)^{\frac{1}{2}}} = -\frac{dx}{(x^2-1)^{\frac{1}{2}}}$$

Integrating

$$\int \frac{dy}{(y^2-1)^{\frac{1}{2}}} = -\int \frac{dx}{(x^2-1)^{\frac{1}{2}}} + C$$

But  $\int \frac{dy}{(y^2-1)^{\frac{1}{2}}} = \tanh^{-1} \frac{y}{(y^2-1)^{\frac{1}{2}}} = \ln(y + \sqrt{y^2-1})$ , hence

$$\ln(y + \sqrt{y^2-1}) = -\ln(x + \sqrt{x^2-1}) + C$$

Therefore

$$\ln(y + \sqrt{y^2-1}) = \pm \ln(x + \sqrt{x^2-1}) + C$$

## 2.61 ODE No. 61

---

$$y'(x) - \frac{\sqrt{x^2-1}}{\sqrt{y(x)^2-1}} = 0$$

✓ **Mathematica** : cpu = 0.197434 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{1}{2} \#1 \sqrt{\#1^2-1} - \frac{1}{2} \log \left( \sqrt{\#1^2-1} + \#1 \right) \& \right] \left[ c_1 + \frac{1}{2} \sqrt{x^2-1} x - \frac{1}{2} \log \left( \sqrt{x^2-1} \right) \right] \right. \right.$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 50

$$\left\{ -C1 + x\sqrt{x^2-1} - \ln(x + \sqrt{x^2-1}) - y(x)\sqrt{(y(x))^2-1} + \ln(y(x) + \sqrt{(y(x))^2-1}) = 0 \right\}$$

## 2.62 ODE No. 62

---

$$y'(x) - \frac{y(x) - x^2 \sqrt{x^2 - y(x)^2}}{xy(x) \sqrt{x^2 - y(x)^2} + x} = 0$$

✓ **Mathematica** : cpu = 2.63674 (sec), leaf count = 44

$$\text{Solve} \left[ -\tan^{-1} \left( \frac{\sqrt{x^2 - y(x)^2}}{y(x)} \right) + \frac{x^2}{2} + \frac{y(x)^2}{2} = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.29 (sec), leaf count = 34

$$\left\{ \frac{(y(x))^2}{2} + \arctan \left( y(x) \frac{1}{\sqrt{x^2 - (y(x))^2}} \right) + \frac{x^2}{2} - C1 = 0 \right\}$$

**Hand solution**

$$y' = \frac{y - x^2\sqrt{x^2 - y^2}}{xy\sqrt{x^2 - y^2} + x} \quad (1)$$

Let  $y = ux$  then  $y' = u + xu'$  therefore

$$\begin{aligned} u + xu' &= \frac{y - x^2\sqrt{x^2 - y^2}}{xy\sqrt{x^2 - y^2} + x} \\ &= \frac{ux - x^2\sqrt{x^2 - (ux)^2}}{x(ux)\sqrt{x^2 - (ux)^2} + x} \\ &= \frac{ux - x^3\sqrt{1 - u^2}}{x^3u\sqrt{1 - u^2} + x} \\ &= \frac{u - x^2\sqrt{1 - u^2}}{x^2u\sqrt{1 - u^2} + 1} \end{aligned}$$

Hence

$$\begin{aligned} u(x^2u\sqrt{1 - u^2} + 1) + xu'(x^2u\sqrt{1 - u^2} + 1) &= u - x^2\sqrt{1 - u^2} \\ x^2u^2\sqrt{1 - u^2} + u + u'(x^3u\sqrt{1 - u^2} + x) &= u - x^2\sqrt{1 - u^2} \\ x^2u^2\sqrt{1 - u^2} + u' &= -x^2\sqrt{1 - u^2} \\ xu^2\sqrt{1 - u^2} + u'(x^2u\sqrt{1 - u^2} + 1) &= -x\sqrt{1 - u^2} \\ xu^2 + u' \left( x^2u + \frac{1}{\sqrt{1 - u^2}} \right) &= -x \\ x(1 + u^2) + u' \left( x^2u + \frac{1}{\sqrt{1 - u^2}} \right) &= 0 \end{aligned}$$

Hence

$$x(1 + u^2)dx + \left( x^2u + \frac{1}{\sqrt{1 - u^2}} \right) du = 0 \quad (2)$$

Let  $M = x(1 + u^2)$ ,  $N = \left( x^2u + \frac{1}{\sqrt{1 - u^2}} \right)$ .

$$\begin{aligned} \frac{\partial M}{\partial u} &= 2xu \\ \frac{\partial N}{\partial x} &= 2xu \end{aligned}$$

Therefore (2) is exact. Let

$$x(1 + u^2)dx + \left( x^2u + \frac{1}{\sqrt{1 - u^2}} \right) du = dU$$

Since  $dU = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial u}du$ . Comparing with the above, we see that

$$\frac{\partial U}{\partial x} = x(1 + u^2) \quad (3)$$

$$\frac{\partial U}{\partial u} = x^2u + \frac{1}{\sqrt{1 - u^2}} \quad (4)$$

From (3)

$$\begin{aligned} U &= \int x(1 + u^2)dx \\ &= \frac{x^2}{2}(1 + u^2) + f(u) \end{aligned} \quad (5)$$

From (4)

$$\begin{aligned} \frac{d}{du} \left( \frac{x^2}{2}(1 + u^2) + f(u) \right) &= x^2u + \frac{1}{\sqrt{1 - u^2}} \\ x^2u + f'(u) &= x^2u + \frac{1}{\sqrt{1 - u^2}} \\ f'(u) &= \frac{1}{\sqrt{1 - u^2}} \end{aligned}$$

Therefore

$$f(u) = \arcsin(u)$$

From (5) we find

$$U(x, u) = \frac{x^2}{2} (1 + u^2) + \arcsin(u)$$

Since  $dU = 0$  then

$$\begin{aligned} \frac{x^2}{2} (1 + u^2) + \arcsin(u) &= C \\ \frac{x^2}{2} (1 + u^2) + \arcsin(u) - C &= 0 \end{aligned}$$

Since  $y = ux$  then the above can be written as

$$\begin{aligned} \frac{x^2}{2} \left( 1 + \left( \frac{y}{x} \right)^2 \right) + \arcsin \left( \frac{y}{x} \right) - C &= 0 \\ \frac{x^2}{2} \left( \frac{x^2 + y^2}{x^2} \right) + \arcsin \left( \frac{y}{x} \right) - C &= 0 \\ \frac{1}{2} (x^2 + y^2) + \arcsin \left( \frac{y}{x} \right) - C &= 0 \\ \arcsin \left( \frac{y}{x} \right) &= C - \frac{1}{2} (x^2 + y^2) \end{aligned}$$

Hence

$$\begin{aligned} \frac{y}{x} &= \sin \left( C - \frac{1}{2} (x^2 + y^2) \right) \\ y(x) &= x \sin \left( C - \frac{1}{2} (x^2 + y^2) \right) \end{aligned}$$

## 2.63 ODE No. 63

---

$$y'(x) - \frac{y(x)^2 + 1}{(x+1)^{3/2} \left| y(x) + \sqrt{y(x)+1} \right|} = 0$$

✓ **Mathematica** : cpu = 0.189875 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \int_1^{\#1} \frac{\left| K[1] + \sqrt{K[1] + 1} \right|}{K[1]^2 + 1} dK[1] \& \right] \left[ c_1 - \frac{2}{\sqrt{x+1}} \right] \right\} \right\}$$

✓ **Maple** : cpu = 5.093 (sec), leaf count = 35

$$\left\{ -2 \frac{1}{\sqrt{1+x}} - \int^{y(x)} \frac{1}{-a^2+1} \left| -a + \sqrt{-a+1} \right| d\_a + \_C1 = 0 \right\}$$

## 2.64 ODE No. 64

---

$$y'(x) - \sqrt{\frac{ay(x)^2 + by(x) + c}{ax^2 + bx + c}} = 0$$

✓ **Mathematica** : cpu = 0.307698 (sec), leaf count = 269

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-\sqrt{a}c_1} \left( 8a^{3/2}ce^{2\sqrt{a}c_1}\sqrt{ax^2+bx+c} - 8a^{3/2}c\sqrt{ax^2+bx+c} + 8a^2cxe^{2\sqrt{a}c_1} + 8a^2cx + 2b^3e^{\sqrt{a}c_1} - b^3e \right)}{\sqrt{a(y(x))^2 + by(x) + c}} \right\} \right\}$$

✓ **Maple** : cpu = 0.073 (sec), leaf count = 124

$$\left\{ -1 \sqrt{\frac{a(y(x))^2 + by(x) + c}{ax^2 + bx + c}} \sqrt{ax^2 + bx + c} \ln \left( \frac{1}{2} \left( 2 \sqrt{ax^2 + bx + c} \sqrt{a} + 2ax + b \right) \frac{1}{\sqrt{a}} \right) \frac{1}{\sqrt{a(y(x))^2 + by(x) + c}} \right\}$$

## 2.65 ODE No. 65

---

$$y'(x) - \sqrt{\frac{y(x)^3 + 1}{x^3 + 1}} = 0$$

✓ **Mathematica** : cpu = 0.953975 (sec), leaf count = 312

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{i(\#1 + 1) \sqrt{1 + \frac{6i}{(\sqrt{3} - 3i)(\#1 + 1)}} \sqrt{\frac{2}{3} - \frac{4i}{(\sqrt{3} + 3i)(\#1 + 1)}} F \left( i \sinh^{-1} \left( \frac{\sqrt{-\frac{6i}{3i + \sqrt{3}}}}{\sqrt{\#1 + 1}} \right) \middle| \frac{3i + \sqrt{3}}{3i - \sqrt{3}} \right)}{\sqrt{-\frac{i}{\sqrt{3} + 3i}} \sqrt{\#1^2 - \#1 + 1}} \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 47

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{-a^3 + 1}} d\_a + \int^x -1 \sqrt{\frac{(y(x))^3 + 1}{-a^3 + 1}} \frac{1}{\sqrt{(y(x))^3 + 1}} d\_a + \_C1 = 0 \right\}$$

## 2.66 ODE No. 66

---

$$y'(x) - \frac{\sqrt{|(1 - y(x))y(x)(1 - ay(x))|}}{\sqrt{|(1 - x)x(1 - ax)|}} = 0$$

✓ **Mathematica** : cpu = 0.166086 (sec), leaf count = 67

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \int_1^{\#1} \frac{1}{\sqrt{|(1 - K[1])K[1](1 - aK[1])|}} dK[1] \& \right] \left[ \int_1^x \frac{1}{\sqrt{|(1 - K[2])K[2](1 - aK[2])|}} dK[2] \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.087 (sec), leaf count = 40

$$\left\{ \int \frac{1}{\sqrt{|x(x-1)(ax-1)|}} dx - \int^{y(x)} \frac{1}{\sqrt{|_a(_a-1)(_a a-1)|}} d\_a + \_C1 = 0 \right\}$$

## 2.67 ODE No. 67

---

$$y'(x) - \frac{\sqrt{1 - y(x)^4}}{\sqrt{1 - x^4}} = 0$$

✓ **Mathematica** : cpu = 0.125916 (sec), leaf count = 14

$$\{ \{ y(x) \rightarrow \text{sn} (c_1 + F (\sin^{-1}(x) | -1) | -1) \} \}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 51

$$\left\{ \text{EllipticF} (x, i) \sqrt{-x^2 + 1} \sqrt{x^2 + 1} \frac{1}{\sqrt{-x^4 + 1}} - \int^{y(x)} \frac{1}{\sqrt{-_a^4 + 1}} d\_a + \_C1 = 0 \right\}$$

## 2.68 ODE No. 68

---

$$y'(x) - \sqrt{\frac{ay(x)^4 + by(x)^2 + 1}{ax^4 + bx^2 + 1}} = 0$$

✓ **Mathematica** : cpu = 0.714945 (sec), leaf count = 373

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{i \sqrt{\frac{2\#1^2 a + \sqrt{b^2 - 4a} + b}{\sqrt{b^2 - 4a} + b}} \sqrt{\frac{2\#1^2 a}{b - \sqrt{b^2 - 4a}} + 1} F \left( i \sinh^{-1} \left( \sqrt{2} \sqrt{\frac{a}{b + \sqrt{b^2 - 4a}}} \#1 \right) \middle| \frac{b + \sqrt{b^2 - 4a}}{b - \sqrt{b^2 - 4a}} \right)}{\sqrt{2} \sqrt{\frac{a}{\sqrt{b^2 - 4a} + b}} \sqrt{\#1^4 a + \#1^2 b + 1}} \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.048 (sec), leaf count = 77

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{-a^4 a + -a^2 b + 1}} d\_a + \int^x -1 \sqrt{\frac{a(y(x))^4 + b(y(x))^2 + 1}{-a^4 a + -a^2 b + 1}} \frac{1}{\sqrt{a(y(x))^4 + b(y(x))^2 + 1}} d\_a + \_C1 = 0 \right\}$$

2.69 ODE No. 69

$$y'(x) - \sqrt{(a0 + a1x + a2x^2 + a3x^3 + a4x^4) (b0 + b1y(x) + b2y(x)^2 + b3y(x)^3 + b4y(x)^4)} = 0$$

✓ **Mathematica** : cpu = 7.75649 (sec), leaf count = 1163

Solve 
$$\left[ -\frac{2F\left(\sin^{-1}\left(\sqrt{\frac{(\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,2]-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,4])(y(x)-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,1])}{(\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,1]-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,4])(y(x)-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,1])}\right)}{2}\right)}{2F\left(\sin^{-1}\left(\sqrt{\frac{(\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,2]-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,4])(y(x)-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,1])}{(\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,1]-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,4])(y(x)-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,1])}\right)}{2}\right)}{2}$$

✓ **Maple** : cpu = 0.108 (sec), leaf count = 111

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{-a^4 b_4 + -a^3 b_3 + -a^2 b_2 + -a b_1 + b_0}} d_-a + \int^x -1 \sqrt{\left(b_4 (y(x))^4 + b_3 (y(x))^3 + b_2 (y(x))^2 + b_1 y(x) + b_0\right)} dx \right\}$$

2.70 ODE No. 70

$$y'(x) - \sqrt{\frac{a0 + a1x + a2x^2 + a3x^3 + a4x^4}{b0 + b1y(x) + b2y(x)^2 + b3y(x)^3 + b4y(x)^4}} = 0$$

✓ **Mathematica** : cpu = 12.6908 (sec), leaf count = 81

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \int_1^{\#1} \sqrt{b4K[1]^4 + b3K[1]^3 + b2K[1]^2 + b1K[1] + b0dK[1]\&} \right] \left[ \int_1^x \sqrt{a4K[2]^4 + a3K[2]^3 + a2K[2]^2 + a1K[2] + a0dK[2]\&} \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.124 (sec), leaf count = 113

$$\left\{ \int^{y(x)} \sqrt{-a^4 b_4 + -a^3 b_3 + -a^2 b_2 + -a b_1 + b_0} d_-a + \int^x -\sqrt{\frac{-a^4 a_4 + -a^3 a_3 + -a^2 a_2 + -a a_1 + a_0}{b_4 (y(x))^4 + b_3 (y(x))^3 + b_2 (y(x))^2 + b_1 y(x) + b_0}} dx \right\}$$

2.71 ODE No. 71

$$y'(x) - \sqrt{\frac{b0 + b1y(x) + b2y(x)^2 + b3y(x)^3 + b4y(x)^4}{a0 + a1x + a2x^2 + a3x^3 + a4x^4}} = 0$$

✓ **Mathematica** : cpu = 2.76243 (sec), leaf count = 2237

Solve 
$$\left[ -\frac{2F\left(\sin^{-1}\left(\sqrt{\frac{(\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,2]-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,4])(y(x)-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,1])}{(\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,1]-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,4])(y(x)-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,1])}\right)}{2}\right)}{2F\left(\sin^{-1}\left(\sqrt{\frac{(\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,2]-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,4])(y(x)-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,1])}{(\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,1]-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,4])(y(x)-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,1])}\right)}{2}\right)}{2}$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 113

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{-a^4 b_4 + -a^3 b_3 + -a^2 b_2 + -a b_1 + b_0}} d_-a + \int^x -1 \sqrt{\frac{b_4 (y(x))^4 + b_3 (y(x))^3 + b_2 (y(x))^2 + b_1 y(x) + b_0}{-a^4 a_4 + -a^3 a_3 + -a^2 a_2 + -a a_1 + a_0}} dx \right\}$$

2.72 ODE No. 72

$$y'(x) - R1\left(x, \sqrt{a0 + a1x + a2x^2 + a3x^3 + a4x^4}\right) R2\left(y(x), \sqrt{b0 + b1y(x) + b2y(x)^2 + b3y(x)^3 + b4y(x)^4}\right) = 0$$

✓ **Mathematica** : cpu = 0.160829 (sec), leaf count = 89

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \int_1^{\#1} \frac{1}{R2\left(K[1], \sqrt{b4K[1]^4 + b3K[1]^3 + b2K[1]^2 + b1K[1] + b0}\right)} dK[1]\& \right] \left[ \int_1^x R1\left(x, \sqrt{a4x^4 + a3x^3 + a2x^2 + a1x + a0}\right) dx \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 64

$$\left\{ \int R1\left(x, \sqrt{a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0}\right) dx - \int^{y(x)} \left( R2\left(-a, \sqrt{-a^4 b_4 + -a^3 b_3 + -a^2 b_2 + -a b_1 + b_0}\right) \right) dy \right\}$$

2.73 ODE No. 73

$$y'(x) - \left(\frac{a_0 + a_1x + a_2x^2 + a_3x^3}{a_0 + a_1y(x) + a_2y(x)^2 + a_3y(x)^3}\right)^{2/3} = 0$$

✓ **Mathematica** : cpu = 1.33889 (sec), leaf count = 733

Solve 
$$\left[\frac{3(a_0 + y(x)(a_1 + y(x)(a_2 + a_3y(x))))^{2/3} (y(x) - \text{Root}[\#1^3a_3 + \#1^2a_2 + \#1a_1 + a_0\&, 1]) F_1\left(\frac{5}{3}; -\frac{2}{3}, -\frac{2}{3}\right)}{5 \left(\frac{y(x) - \text{Root}[\#1^3a_3 + \#1^2a_2 + \#1a_1 + a_0\&, 2]}{\text{Root}[\#1^3a_3 + \#1^2a_2 + \#1a_1 + a_0\&, 1]} - \text{Root}[\#1^3a_3 + \#1^2a_2 + \#1a_1 + a_0\&, 1]}\right)^{2/3}}\right]$$

✓ **Maple** : cpu = 0.246 (sec), leaf count = 91

$$\left\{\int^{y(x)}\left(-a^3a_3+_a^2a_2+_a a_1+a_0\right)^{\frac{2}{3}}d_a+\int^x-\left(\frac{-a^3a_3+_a^2a_2+_a a_1+a_0}{a_3\left(y(x)\right)^3+a_2\left(y(x)\right)^2+a_1y(x)+a_0}\right)^{\frac{2}{3}}\left(a_3\left(y(x)\right)^3+a_2\left(y(x)\right)^2+a_1y(x)+a_0\right)^{\frac{2}{3}}dx\right\}$$

2.74 ODE No. 74

$$y'(x) - f(x)(y(x) - g(x))\sqrt{(y(x) - a)(y(x) - b)} = 0$$

✗ **Mathematica** : cpu = 2.14211 (sec), leaf count = 0 , could not solve  
DSolve[-(f[x]\*Sqrt[(-a + y[x])\*(-b + y[x])]\*(-g[x] + y[x])) + Derivative[1][y][x] == 0, y[x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x)-f(x)\*(y(x)-g(x))\*((y(x)-a)\*(y(x)-b))^(1/2) = 0,y(x))

2.75 ODE No. 75

$$y'(x) - e^{x-y(x)} + e^x = 0$$

✓ **Mathematica** : cpu = 0.15306 (sec), leaf count = 20

$$\left\{\left\{y(x) \rightarrow \log \left(1 - e^{c_1 - e^x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.121 (sec), leaf count = 20

$$\left\{y\left(x\right)=-{e}^x+\ln \left(-1+{e}^{{e}^x+_C1}\right)-_C1\right\}$$

**Hand solution**

$$\begin{aligned} y' &= e^{x-y} - e^x \\ y' &= e^x (e^{-y} - 1) \\ \frac{1}{e^{-y} - 1} dy &= e^x dx \end{aligned} \tag{1}$$

Integrating both sides.  $\int \frac{1}{e^{-y}-1} dy$ . Let  $e^{-y} = u$ , then  $\frac{du}{dy} = -e^{-y} = -u$ . Hence  $dy = -\frac{du}{u}$ , therefore the integral becomes

$$\int \frac{1}{u-1} \left(-\frac{du}{u}\right) = -\int \frac{1}{u(u-1)} du$$

But  $\frac{1}{u(u-1)} = -\left(\frac{1}{u} - \frac{1}{u-1}\right)$ , hence

$$\begin{aligned} -\int \frac{1}{u(u-1)} du &= \int \left(\frac{1}{u} - \frac{1}{u-1}\right) du \\ &= \ln u - \ln (u-1) \\ &= \ln e^{-y} - \ln (e^{-y} - 1) \\ &= -(\ln (e^{-y} - 1) - \ln e^{-y}) \end{aligned}$$

But  $\ln x - \ln y = \ln \left(\frac{x}{y}\right)$  and the above becomes

$$\begin{aligned} \int \frac{1}{e^{-y} - 1} dy &= -\left[\ln \left(\frac{e^{-y} - 1}{e^{-y}}\right)\right] \\ &= -\ln (1 - e^y) \end{aligned}$$

Back to (1), when we integrate both sides, and since  $\int e^x dx = e^x + C$

$$\begin{aligned} -\ln(1-e^y) &= e^x + C \\ \ln(1-e^y) &= -e^x + C_1 \end{aligned}$$

Hence

$$\begin{aligned} 1-e^y &= \exp(-e^x + C_1) \\ e^y &= 1-\exp(-e^x + C_1) \end{aligned}$$

Taking logs

$$y = \ln(1-\exp(-e^x + C_1))$$

Let  $e^{C_1} = C_2$  then

$$y = \ln(1-C_2e^{-e^x})$$

Verification

```

1 ode:=diff(y(x),x)=exp(x-y(x))-exp(x);
2 my_sol:=log(1-C1*exp(-exp(x)));
3 odetest(y(x)=my_sol,ode);
4 0

```

## 2.76 ODE No. 76

$$-a\cos(y(x))+b+y'(x)=0$$

✓ **Mathematica** : cpu = 0.13375 (sec), leaf count = 116

$$\left\{\left\{y(x)\rightarrow 2\tan^{-1}\left(\frac{a\tanh\left(\frac{1}{2}\left(x\sqrt{(a-b)(a+b)}-c_1\sqrt{(a-b)(a+b)}\right)\right)}{\sqrt{(a-b)(a+b)}}\right)-\frac{b\tanh\left(\frac{1}{2}\left(x\sqrt{(a-b)(a+b)}-c_1\sqrt{(a-b)(a+b)}\right)\right)}{\sqrt{(a-b)(a+b)}}}\right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 41

$$\left\{y(x)=2\arctan\left(\frac{\tanh\left(1/2\sqrt{a^2-b^2}\left(x+_C1\right)\right)\sqrt{a^2-b^2}}{a+b}\right)\right\}$$

Hand solution

$$y'=a\cos y+b$$

This is separable.

$$\begin{aligned} \frac{dy}{a\cos y+b} &= dx \\ \int \frac{dy}{a\cos y+b} &= x+C \end{aligned} \tag{1}$$

Using standard Tangent half-angle substitution, let  $t = \tan \frac{y}{2}$ ,  $\cos y = \frac{1-t^2}{1+t^2}$ ,  $dy = \frac{2}{1+t^2}dt$ , then the integral becomes

$$\begin{aligned} \int \frac{dy}{a\cos y+b} &= \int \frac{2}{1+t^2}\frac{1}{\left(a\frac{1-t^2}{1+t^2}+b\right)}dt \\ &= 2\int \frac{1+t^2}{(1+t^2)(a(1-t^2)+b(1+t^2))}dt \\ &= 2\int \frac{dt}{a-at^2+b+bt^2} \\ &= 2\int \frac{dt}{(a+b)+t^2(b-a)} \\ &= 2\int \frac{dt}{(a+b)\left(1+\frac{t^2(b-a)}{(a+b)}\right)} \\ &= \frac{2}{a+b}\int \frac{dt}{\left(1+\frac{t^2(b-a)}{(a+b)}\right)} \end{aligned}$$



Let  $z^2 = \frac{t^2(b-a)}{(a+b)}$ , or  $z = \frac{t\sqrt{b-a}}{\sqrt{a+b}}$ , then  $\frac{dz}{dt} = \frac{\sqrt{b-a}}{\sqrt{a+b}}$  and the above integral becomes

$$\begin{aligned} \frac{2}{a+b} \int \frac{dt}{\left(1 + \frac{t^2(b-a)}{(a+b)}\right)} &= \frac{2}{a+b} \int \frac{\sqrt{a+b}}{\sqrt{b-a}} \frac{dz}{(1+z^2)} \\ &= \frac{2}{a+b} \frac{\sqrt{a+b}}{\sqrt{b-a}} \int \frac{dz}{(1+z^2)} \\ &= \frac{2}{\sqrt{a+b}} \frac{1}{\sqrt{b-a}} \int \frac{dz}{(1+z^2)} \\ &= \frac{2}{\sqrt{(a+b)(b-a)}} \int \frac{dz}{(1+z^2)} \\ &= \frac{2}{\sqrt{b^2-a^2}} \int \frac{dz}{(1+z^2)} \end{aligned}$$

Now,  $\int \frac{dz}{(1+z^2)} = \arctan(z)$ , hence

$$\begin{aligned} \frac{2}{\sqrt{b^2-a^2}} \int \frac{dz}{(1+z^2)} &= \frac{2}{\sqrt{b^2-a^2}} \arctan(z) \\ &= \frac{2}{\sqrt{b^2-a^2}} \arctan\left(\frac{t\sqrt{b-a}}{\sqrt{a+b}}\right) \end{aligned}$$

But  $t = \tan \frac{y}{2}$  therefore

$$\frac{2}{\sqrt{b^2-a^2}} \arctan\left(\frac{t\sqrt{b-a}}{\sqrt{a+b}}\right) = \frac{2}{\sqrt{b^2-a^2}} \arctan\left(\frac{\tan\left(\frac{y}{2}\right)\sqrt{b-a}}{\sqrt{a+b}}\right)$$

Going back to (1)

$$\begin{aligned} \int \frac{dy}{a \cos y + b} &= x + C \\ \frac{2}{\sqrt{b^2-a^2}} \arctan\left(\frac{\tan\left(\frac{y}{2}\right)\sqrt{b-a}}{\sqrt{a+b}}\right) &= x + C \\ \arctan\left(\frac{\tan\left(\frac{y}{2}\right)\sqrt{b-a}}{\sqrt{a+b}}\right) &= \frac{1}{2}\sqrt{b^2-a^2}(x+C) \\ \frac{\tan\left(\frac{y}{2}\right)\sqrt{b-a}}{\sqrt{a+b}} &= \tan\left(\frac{1}{2}\sqrt{b^2-a^2}(x+C)\right) \\ \tan\left(\frac{y}{2}\right) &= \frac{\sqrt{a+b}}{\sqrt{b-a}} \tan\left(\frac{1}{2}\sqrt{b^2-a^2}(x+C)\right) \\ \frac{y}{2} &= \arctan\left(\frac{(a+b)}{\sqrt{(a+b)(b-a)}} \tan\left(\frac{1}{2}\sqrt{b^2-a^2}(x+C)\right)\right) \\ &= \arctan\left(\frac{(a+b)}{\sqrt{b^2-a^2}} \tan\left(\frac{1}{2}\sqrt{b^2-a^2}(x+C)\right)\right) \\ y &= 2 \arctan\left(\frac{a+b}{\sqrt{b^2-a^2}} \tan\left(\frac{1}{2}\sqrt{b^2-a^2}(x+C)\right)\right) \end{aligned}$$

Verification

```

1 ode:=diff(y(x),x)=a*cos(y(x))+b;
2 my_sol:=2*arctan((a+b)/sqrt(b^2-a^2) * tan(1/2*sqrt(b^2-a^2)*(x+_C1)));
3 odetest(y(x)=my_sol,ode);
4 0

```

## 2.77 ODE No. 77

$$y'(x) - \cos(ay(x) + bx) = 0$$

✓ **Mathematica** : cpu = 0.295221 (sec), leaf count = 124

$$\left\{\left\{y(x) \rightarrow \frac{-2 \tan ^{-1}\left(\frac{a \tanh \left(\frac{1}{2}\left(c_1 \sqrt{a^2-b^2}-x \sqrt{a^2-b^2}\right)\right)}{\sqrt{a^2-b^2}}+\frac{b \tanh \left(\frac{1}{2}\left(c_1 \sqrt{a^2-b^2}-x \sqrt{a^2-b^2}\right)\right)}{\sqrt{a^2-b^2}}\right)-b x}{a}\right\}\right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 54

$$\left\{y(x)=\frac{1}{a}\left(-bx+2\arctan\left(\frac{\tanh\left(1/2\sqrt{a^2-b^2}(x-C1)\right)\sqrt{a^2-b^2}}{a-b}\right)\right)\right\}$$

**Hand solution**

$$y'=\cos (ay+bx)$$

This is separable after transformation of  $u=ay+bx$ , hence  $u'=ay'+b$  or  $y'=\frac{1}{a}(u'-b)$ . Therefore the above becomes

$$\begin{aligned}\frac{1}{a}(u'-b) &= \cos(u) \\ u' &= a\cos u + b \\ \frac{du}{a\cos u + b} &= dx\end{aligned}$$

This is the same as Kamke 76 (the problem before this), which we solved using half angle tan transformation, and the answer is

$$u=2\arctan\left(\frac{a+b}{\sqrt{b^2-a^2}}\tan\left(\frac{1}{2}\sqrt{b^2-a^2}(x+C)\right)\right)$$

Since  $u=ay+bx$  then  $y=\frac{u-bx}{a}$ , hence

$$y=\frac{1}{a}\left(2\arctan\left(\frac{a+b}{\sqrt{b^2-a^2}}\tan\left(\frac{1}{2}\sqrt{b^2-a^2}(x+C)\right)\right)-bx\right)$$

Verification

```
1 ode:=diff(y(x),x)=cos(a*y(x)+b*x);
2 my_sol:=(1/a)*(2*arctan((a+b)/sqrt(b^2-a^2)*tan(1/2*sqrt(b^2-a^2)*(x+C1)))-b*x);
3 odetest(y(x)=my_sol,ode);
4 0
```

2.78 ODE No. 78

$$a\sin(\alpha y(x)+\beta x)+b+y'(x)=0$$

✓ **Mathematica** : cpu = 0.662485 (sec), leaf count = 1317

$$\left\{y(x)\rightarrow\frac{2\tan^{-1}\left(\frac{a^2\sqrt{-(a\alpha+b\alpha-\beta)(a\alpha-b\alpha+\beta)}\tan\left(\frac{1}{2}\left(\frac{a^2x\alpha^2}{\sqrt{-(a\alpha+b\alpha-\beta)(a\alpha-b\alpha+\beta)}}-\frac{b^2x\alpha^2}{\sqrt{-(a\alpha+b\alpha-\beta)(a\alpha-b\alpha+\beta)}}-\frac{a^2c_1\alpha^2}{\sqrt{-(a\alpha+b\alpha-\beta)(a\alpha-b\alpha+\beta)}}\right)}\right)}{b\alpha-\beta}\right\}$$

✓ **Maple** : cpu = 0.078 (sec), leaf count = 89

$$\left\{y(x)=\frac{1}{\alpha}\left(-\beta x+2\arctan\left(\frac{-\tan\left(1/2\sqrt{(-a^2+b^2)\alpha^2-2\alpha b\beta+\beta^2}(x-C1)\right)\sqrt{(-a^2+b^2)\alpha^2-2\alpha b\beta}}{b\alpha-\beta}\right)\right)\right\}$$

**Hand solution**

$$y'=-a\sin(\alpha y+\beta x)-b$$

This is separable after transformation of  $u = \alpha y + \beta x$ , hence  $u' = \alpha y' + \beta$  or  $y' = \frac{1}{\alpha}(u' - \beta)$ . Therefore the above becomes

$$\begin{aligned}\frac{1}{\alpha}(u' - \beta) &= -a \sin(u) - b \\ u' &= -\alpha(a \sin(u) + b) + \beta \\ \frac{du}{\beta - \alpha(a \sin(u) + b)} &= dx\end{aligned}\tag{1}$$

Using half angle tan transformation where  $\tan\left(\frac{u}{2}\right) = t$ ,  $\sin(u) = \frac{2t}{t^2+1}$ ,  $du = \frac{2}{1+t^2}dt$  then

$$\begin{aligned}\int \frac{du}{\beta - \alpha(a \sin(u) + b)} &= \int \frac{2}{1+t^2} \frac{dt}{\beta - \alpha\left(a \frac{2t}{t^2+1} + b\right)} \\ &= 2 \int \frac{dt}{\beta(t^2+1) - \alpha(a2t + b(t^2+1))} \\ &= 2 \int \frac{dt}{t\beta^2 + \beta - (\alpha a2t + t^2\alpha b + \alpha b)} \\ &= 2 \int \frac{dt}{(\beta - \alpha b) \left( \frac{t\beta^2 - \alpha a2t - t^2\alpha b}{(\beta - \alpha b)} + 1 \right)} \\ &= \frac{2}{(\beta - \alpha b)} \int \frac{dt}{\frac{t\beta^2 - \alpha a2t - t^2\alpha b}{(\beta - \alpha b)} + 1} \\ &= \frac{2}{(\beta - \alpha b)} \frac{-(\alpha b - \beta)}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}} \tanh^{-1} \left( \frac{\left(t + \frac{a\alpha}{b\alpha - \beta}\right)(b\alpha - \beta)}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}} \right) \\ &= \frac{2}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}} \tanh^{-1} \left( \frac{t(b\alpha - \beta) + a\alpha}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}} \right)\end{aligned}$$

But  $t = \tan\left(\frac{u}{2}\right)$  therefore

$$\int \frac{du}{\beta - \alpha(a \sin(u) + b)} = \frac{2}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}} \tanh^{-1} \left( \frac{\tan\left(\frac{u}{2}\right)(b\alpha - \beta) + a\alpha}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}} \right)$$

But  $u = \alpha y + \beta x$ , and the above becomes

$$\int \frac{du}{\beta - \alpha(a \sin(u) + b)} = \frac{2 \tanh^{-1} \left( \frac{\tan\left(\frac{\alpha y + \beta x}{2}\right)(b\alpha - \beta) + a\alpha}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}} \right)}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}}$$

Back to (1), therefore after integrating both sides

$$\frac{2 \tanh^{-1} \left( \frac{\tan\left(\frac{\alpha y + \beta x}{2}\right)(b\alpha - \beta) + a\alpha}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}} \right)}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}} = x + C$$

Let

$$A = \sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}$$

Then

$$\begin{aligned}\tanh^{-1} \left( \frac{\tan\left(\frac{\alpha y + \beta x}{2}\right)(b\alpha - \beta) + a\alpha}{A} \right) &= \frac{1}{2} A (x + C) \\ \frac{\tan\left(\frac{\alpha y + \beta x}{2}\right)(b\alpha - \beta) + a\alpha}{A} &= \tanh \left( \frac{1}{2} A (x + C) \right) \\ \tan \left( \frac{\alpha y + \beta x}{2} \right) (b\alpha - \beta) + a\alpha &= A \tanh \left( \frac{1}{2} A (x + C) \right) \\ \tan \left( \frac{\alpha y + \beta x}{2} \right) &= \frac{A}{(b\alpha - \beta)} \tanh \left( \frac{1}{2} A (x + C) \right) - \frac{a\alpha}{(b\alpha - \beta)} \\ \frac{\alpha y + \beta x}{2} &= \arctan \left( \frac{A}{(b\alpha - \beta)} \tanh \left( \frac{1}{2} A (x + C) \right) - \frac{a\alpha}{(b\alpha - \beta)} \right) \\ y &= \frac{2}{\alpha} \arctan \left( \frac{A}{(b\alpha - \beta)} \tanh \left( \frac{1}{2} A (x + C) \right) - \frac{a\alpha}{(b\alpha - \beta)} \right) - \frac{\beta x}{\alpha}\end{aligned}$$

Verification

```

1 ode:=diff(y(x),x)=-a*sin(alpha*y(x)+beta*x)-b;
2 A0:=sqrt(alpha^2*a^2-(alpha^2*b^2+beta^2-2*alpha*b*beta));
3 B0:=(alpha*b-beta);
4 my_sol:=2/alpha*arctan(A0/B0*tanh((1/2)*A0*(x+_C1))-a*alpha/(B0))-beta*x/alpha;
5 odetest(y(x)=my_sol,ode);
6 0

```

## 2.79 ODE No. 79

$$f(x)\cos(ay(x))+g(x)\sin(ay(x))+h(x)+y'(x)=0$$

✗ **Mathematica** : cpu = 24.6085 (sec), leaf count = 0 , could not solve  
DSolve[Cos[a\*y[x]]\*f[x] + h[x] + g[x]\*Sin[a\*y[x]] + Derivative[1][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x)+f(x)\*cos(a\*y(x))+g(x)\*sin(a\*y(x))+h(x) = 0,y(x))

## 2.80 ODE No. 80

$$(1-f'(x))\cos(y(x))-f'(x)+f(x)\sin(y(x))+y'(x)-1=0$$

✓ **Mathematica** : cpu = 0.0574641 (sec), leaf count = 72

$$\left\{\left\{y(x)\rightarrow 2\tan^{-1}\left(\frac{1}{c_1\exp\left(\int_1^x-f(K[1])dK[1]\right)+\exp\left(\int_1^x-f(K[1])dK[1]\right)\int_1^x-\exp\left(-\int_1^{K[2]}-f(K[1])dK[1]\right)}\right.}\right\}$$

✓ **Maple** : cpu = 0.841 (sec), leaf count = 41

$$\left\{y(x)=2\arctan\left(\frac{-e^{\int f(x)\,dx}+\int e^{\int f(x)\,dx}\,dx\,f(x)+f(x)-C1}{-C1+\int e^{\int f(x)\,dx}\,dx}\right)\right\}$$

## 2.81 ODE No. 81

$$y'(x)+2\tan(x)\tan(y(x))-1=0$$

✓ **Mathematica** : cpu = 1.33152 (sec), leaf count = 220

$$\text{Solve}\left[c_1=-\frac{\frac{1}{2}\left(\frac{1}{\frac{i\tan(x)}{\tan^2(x)+1}-\frac{i\tan^2(x)\tan(y(x))}{\tan^2(x)+1}}+i\cot(x)\right)\sqrt[4]{1-\left(\frac{1}{\frac{i\tan(x)}{\tan^2(x)+1}-\frac{i\tan^2(x)\tan(y(x))}{\tan^2(x)+1}}+i\cot(x)\right)^2}{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{3}{2};\left(\frac{i\tan(x)}{\tan^2(x)+1}-\frac{i\tan^2(x)\tan(y(x))}{\tan^2(x)+1}\right)^2\right)}{\sqrt[4]{-1+\left(\frac{1}{\frac{i\tan(x)}{\tan^2(x)+1}-\frac{i\tan^2(x)\tan(y(x))}{\tan^2(x)+1}}+i\cot(x)\right)^2}}\right]$$

✓ **Maple** : cpu = 0.839 (sec), leaf count = 78

$$\left\{-C1+\tan(x)\frac{1}{\sqrt[4]{\frac{\left(1+\tan(y(x))^2\right)\left(1+\tan(x)^2\right)}{\left(\tan(y(x))\tan(x)-1\right)^2}}}+\frac{\tan(y(x))+\tan(x)}{2\tan(y(x))\tan(x)-2}{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{3}{2};-\frac{\left(\tan(y(x))+\tan(x)\right)^2}{\left(\tan(y(x))\tan(x)-1\right)}\right)\right\}$$

## 2.82 ODE No. 82

$$-a\left(\tan^2(y(x))+1\right)+y'(x)+\tan(x)\tan(y(x))=0$$

✗ **Mathematica** : cpu = 46.2651 (sec), leaf count = 0 , could not solve  
DSolve[Tan[x]\*Tan[y[x]] - a\*(1 + Tan[y[x]]^2) + Derivative[1][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x)-a\*(1+tan(y(x))^2)+tan(y(x))\*tan(x) = 0,y(x))





$$\begin{aligned}
u'(x) &= C_1 a \exp(ax) \operatorname{BesselJ}\left(-1/2 \frac{\sqrt{4a^2-3b}}{a}, 1/2 \frac{\sqrt{3}\sqrt{c} \exp(-ax)}{a}\right) \\
&- 1/2 C_1 \exp(ax) \left( -\operatorname{BesselJ}\left(-1/2 \frac{\sqrt{4a^2-3b}}{a} + 1, 1/2 \frac{\sqrt{3}\sqrt{c} \exp(-ax)}{a}\right) - 1/3 \frac{\sqrt{3}\sqrt{4a^2-3b}}{\sqrt{c} \exp(-ax)} \operatorname{BesselJ}\left(-1/2 \frac{\sqrt{4a^2-3b}}{a}, 1/2 \frac{\sqrt{3}\sqrt{c} \exp(-ax)}{a}\right) \right. \\
&\quad \left. + C_2 a \exp(ax) \operatorname{BesselY}\left(-1/2 \frac{\sqrt{4a^2-3b}}{a}, 1/2 \frac{\sqrt{3}\sqrt{c} \exp(-ax)}{a}\right) \right) \\
&- 1/2 C_1 \exp(ax) \left( -\operatorname{BesselY}\left(-1/2 \frac{\sqrt{4a^2-3b}}{a} + 1, 1/2 \frac{\sqrt{3}\sqrt{c} \exp(-ax)}{a}\right) - 1/3 \frac{\sqrt{3}\sqrt{4a^2-3b}}{\sqrt{c} \exp(-ax)} \operatorname{BesselY}\left(-1/2 \frac{\sqrt{4a^2-3b}}{a}, 1/2 \frac{\sqrt{3}\sqrt{c} \exp(-ax)}{a}\right) \right)
\end{aligned}$$

Hence from  $y = \frac{-2}{3} \frac{u'}{u}$  the solution is now found.

Verification

```

1 ode:=2*diff(y(x),x)-3*y(x)^2-4*a*y(x)=b+c*exp(-2*a*x);
2 uode:=diff(u(x),x$2)-2*a*diff(u(x),x)+3/4*(b+c*(exp(-2*a*x)))*u(x)=0;
3 uSol:=dsolve(uode,u(x));
4 my_sol:=(-2/3)*diff(rhs(uSol),x)/rhs(uSol);
5 odetest(y(x)=my_sol,ode);
6 0

```

## 2.89 ODE No. 89

$$xy'(x) - \sqrt{a^2 - x^2} = 0$$

✓ **Mathematica** : cpu = 0.0124099 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow \sqrt{a^2 - x^2} - a \tanh^{-1} \left( \frac{\sqrt{a^2 - x^2}}{a} \right) + c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 56

$$\left\{ y(x) = \sqrt{a^2 - x^2} - a^2 \ln \left( \frac{1}{x} \left( 2a^2 + 2\sqrt{a^2} \sqrt{a^2 - x^2} \right) \right) \frac{1}{\sqrt{a^2}} + -C1 \right\}$$

**Hand solution**

$$xy' = \pm \sqrt{a^2 - x^2}$$

This is separable.  $y' = \frac{\pm \sqrt{a^2 - x^2}}{x}$  or  $dy = \frac{\pm \sqrt{a^2 - x^2}}{x} dx$ . Hence

$$y = \pm \int \frac{\sqrt{a^2 - x^2}}{x} dx + C$$

Let  $x = a \sin u$ , then  $dx = a \cos(u) du$  and the integral becomes

$$\begin{aligned}
\int \frac{\sqrt{a^2 - x^2}}{x} dx &= \int \frac{\sqrt{a^2 - a^2 \sin^2 u}}{a \sin u} a \cos(u) du \\
&= \int \frac{a \sqrt{1 - \sin^2 u}}{a \sin u} a \cos(u) du \\
&= a \int \frac{\cos u}{\sin u} \cos(u) du \\
&= a \int \frac{\cos^2 u}{\sin u} du \\
&= a \int \frac{1 - \sin^2 u}{\sin u} du \\
&= a \left( \int \frac{1}{\sin u} du - \int \sin u du \right) \\
&= a \left( \int \frac{1}{\sin u} du + \cos u \right)
\end{aligned} \tag{1}$$

For  $\int \frac{1}{\sin u} du$ , using half tan angle, let  $t = \tan\left(\frac{u}{2}\right)$ ,  $du = \frac{2}{1+t^2} dt$ ,  $\sin u = \frac{2t}{1+t^2}$ , therefore

$$\begin{aligned}
\int \frac{1}{\sin u} du &= \int \frac{1+t^2}{2t} \frac{2}{1+t^2} dt \\
&= \int \frac{1}{t} dt \\
&= \ln(t)
\end{aligned}$$

Hence  $\int \frac{1}{\sin u} du = \ln \left( \tan \left( \frac{u}{2} \right) \right)$  and from (1)

$$\begin{aligned} \int \frac{\sqrt{a^2 - x^2}}{x} dx &= a \left( \int \frac{1}{\sin u} du + \cos u \right) \\ &= a \left( \ln \left( \tan \left( \frac{u}{2} \right) \right) + \cos u \right) \end{aligned}$$

But  $x = a \sin u$ , hence  $u = \arcsin \left( \frac{x}{a} \right)$  and the integral becomes

$$\int \frac{\sqrt{a^2 - x^2}}{x} dx = a \left[ \ln \left( \tan \left( \frac{\arcsin \left( \frac{x}{a} \right)}{2} \right) \right) + \cos \left( \arcsin \left( \frac{x}{a} \right) \right) \right]$$

Hence the solution is

$$y = \pm a \left[ \ln \left( \tan \left( \frac{\arcsin \left( \frac{x}{a} \right)}{2} \right) \right) + \cos \left( \arcsin \left( \frac{x}{a} \right) \right) \right] + C$$

Maple do not verify the above, but I do not see what is wrong with the solution. Will investigate more later.

## 2.90 ODE No. 90

$$xy'(x) + y(x) - x \sin(x) = 0$$

✓ **Mathematica** : cpu = 0.0127658 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{x} + \frac{\sin(x) - x \cos(x)}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 17

$$\left\{ y(x) = \frac{\sin(x) - \cos(x)x + \_C1}{x} \right\}$$

**Hand solution**

$$xy' + y = x \sin x$$

Linear first order, exact, separable.  $y' + \frac{y}{x} = \sin x$ , integrating factor  $\mu = e^{\int \frac{1}{x} dx} = x$ , hence

$$\begin{aligned} d(\mu y) &= \mu \sin x \\ xy &= \int x \sin x dx + C \end{aligned}$$

Using integration by parts.  $\int u dv = uv - \int v du$ . Let  $u = x, dv = \sin x$ , hence  $du = 1, v = -\cos x$ , therefore

$$\begin{aligned} \int x \sin x dx &= -x \cos x + \int \cos x \\ &= -x \cos x + \sin x \end{aligned}$$

Hence

$$\begin{aligned} xy &= -x \cos x + \sin x + C \\ y &= \frac{\sin x}{x} - \cos x + \frac{C}{x} \end{aligned}$$

Verification

```
1 restart;
2 ode:=x*diff(y(x),x)+y(x)=x*sin(x);
3 my_sol:=sin(x)/x-cos(x)+_C1/x;
4 odetest(y(x)=my_sol,ode);
5 0
```



2.91 ODE No. 91

$$xy'(x) - y(x) - \frac{x}{\log(x)} = 0$$

✓ **Mathematica** : cpu = 0.0114882 (sec), leaf count = 15

$$\{\{y(x) \rightarrow c_1 x + x \log(\log(x))\}\}$$

✓ **Maple** : cpu = 0.006 (sec), leaf count = 11

$$\{y(x) = (\ln(\ln(x)) + \_C1)x\}$$

2.92 ODE No. 92

$$x^2(-\sin(x)) + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0119997 (sec), leaf count = 15

$$\{\{y(x) \rightarrow c_1 x - x \cos(x)\}\}$$

✓ **Maple** : cpu = 0.004 (sec), leaf count = 12

$$\{y(x) = x(\_C1 - \cos(x))\}$$

Hand solution

$$xy' - y = x^2 \sin x$$

Linear first order, exact, separable.  $y' - \frac{y}{x} = x \sin x$ , integrating factor  $\mu = e^{\int -\frac{1}{x} dx} = e^{-\ln x} = \frac{1}{x}$ , hence

$$\begin{aligned} d(\mu y) &= \mu \sin x \\ \frac{1}{x} y &= \int \sin x dx + C \\ y &= x(C - \cos x) \end{aligned}$$

Verification

```
1 restart;
2 ode:=x*diff(y(x),x)-y(x)=x^2*sin(x);
3 my_sol:=x*(\_C1-cos(x));
4 odetest(y(x)=my_sol,ode);
5 0
```

2.93 ODE No. 93

$$xy'(x) - y(x) - \frac{x \cos(\log(\log(x)))}{\log(x)} = 0$$

✓ **Mathematica** : cpu = 0.0297642 (sec), leaf count = 16

$$\{\{y(x) \rightarrow c_1 x + x \sin(\log(\log(x)))\}\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 12

$$\{y(x) = x(\sin(\ln(\ln(x))) + \_C1)\}$$

2.94 ODE No. 94

$$ay(x) + bx^n + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0162264 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow c_1 x^{-a} - \frac{bx^n}{a+n} \right\} \right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 23

$$\left\{ y(x) = -\frac{bx^n}{n+a} + x^{-a} \_C1 \right\}$$

Hand solution

$$xy' + ay + bx^n = 0$$

Linear first order, exact, separable.  $y' + \frac{ay}{x} = -bx^{n-1}$ , integrating factor  $\mu = e^{\int \frac{a}{x} dx} = e^{a \ln x} = x^a$ , hence

$$\begin{aligned} d(\mu y) &= -\mu bx^{n-1} \\ x^a y &= -\int bx^{a+n-1} + C \end{aligned}$$

If  $a = -n$  then

$$\begin{aligned} x^a y &= -\int bx^{-1} + C \\ y &= -x^{-a} b \ln(x) + x^{-a} C \\ &= x^{-a} (C - b \ln x) \end{aligned}$$

If  $a \neq -n$  then

$$\begin{aligned} x^a y &= -\frac{bx^{a+n}}{a+n} + C \\ y &= -b\frac{x^n}{a+n} + Cx^{-a} \end{aligned}$$

Verification

```
1 restart;
2 ode:=x*diff(y(x),x)+a*y(x)+b*x^n=0;
3 s1:=x^(-a)*(_C1-b*ln(x));
4 s2:=-b*(x^n/(a+n))+_C1*x^(-a);
5 odetest(y(x)=s2,ode);
6 0
```

## 2.95 ODE No. 95

$$x^2 + xy'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.065443 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow \frac{x(-c_1 J_1(x) - Y_1(x))}{c_1 J_0(x) + Y_0(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 27

$$\left\{ y(x) = -\frac{(_C1 Y_1(x) + J_1(x)) x}{_C1 Y_0(x) + J_0(x)} \right\}$$

Hand solution

$$xy' + y^2 + x^2 = 0$$

This is Riccati first order non-linear. Writing it in standard form and for  $x \neq 0$

$$\begin{aligned} y' &= -x - \frac{1}{x}y^2 \\ &= f_0 + f_1y + f_2y^2 \end{aligned} \tag{1}$$

Where  $f_0 = -x, f_1 = 0, f_2 = -\frac{1}{x}$ . Using standard substitution  $y = \frac{-u'}{uf_2}$  changes the ODE to second order linear ODE

$$y = \frac{xu'}{u} \tag{2}$$

Hence

$$y' = \frac{u'}{u} + x\frac{u''}{u} - \frac{x(u')^2}{u^2}$$

Equating this to RHS of (1) gives

$$\begin{aligned}\frac{u'}{u} + x \frac{u''}{u} - \frac{x(u')^2}{u^2} &= -x - \frac{1}{x} \left(\frac{xu'}{u}\right)^2 \\ \frac{u'}{u} + x \frac{u''}{u} &= -x \\ u'' + \frac{1}{x}u' + u &= 0\end{aligned}$$

This is Lienard ODE. Since it is not constant coefficient ODE, the solution will be in Bessel functions, using Power series method. The solution is

$$u = C_1 \operatorname{BesselJ}(0,x) + C_2 \operatorname{BesselY}(0,x)$$

But  $\frac{d}{dx} \operatorname{BesselJ}(0,x) = -\operatorname{BesselJ}(1,x)$  and  $\frac{d}{dx} \operatorname{BesselY}(0,x) = -\operatorname{BesselY}(1,x)$ , hence

$$u'(x) = -C_1 \operatorname{BesselJ}(1,x) - C_2 \operatorname{BesselY}(1,x)$$

And from (2) the solution is

$$\begin{aligned}y &= \frac{xu'}{u} \\ &= x \frac{[-C_1 \operatorname{BesselJ}(1,x) - C_2 \operatorname{BesselY}(1,x)]}{C_1 \operatorname{BesselJ}(0,x) + C_2 \operatorname{BesselY}(0,x)} \\ &= -x \frac{C_1 \operatorname{BesselJ}(1,x) + C_2 \operatorname{BesselY}(1,x)}{C_1 \operatorname{BesselJ}(0,x) + C_2 \operatorname{BesselY}(0,x)}\end{aligned}$$

Let  $C = \frac{C_1}{C_2}$  then

$$y = -x \frac{C \operatorname{BesselJ}(1,x) + \operatorname{BesselY}(1,x)}{C \operatorname{BesselJ}(0,x) + \operatorname{BesselY}(0,x)}$$

Verification

```

1 restart;
2 ode:=x*diff(y(x),x)+y(x)^2+x^2=0;
3 my_sol:=-x*(_C1*BesselJ(1, x)+BesselY(1,x))/(_C1*BesselJ(0, x)+BesselY(0,x));
4 odetest(y(x)=my_sol,ode);
5 0

```

## 2.96 ODE No. 96

$$xy'(x) - y(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 0.0659587 (sec), leaf count = 33

$$\left\{\left\{y(x) \rightarrow \frac{1 - e^{2c_1 x^2}}{e^{2c_1 x^2} + 1}\right\}\right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 11

$$\{y(x) = -\tanh(\ln(x) + \_C1)\}$$

Hand solution

$$xy' - y^2 + 1 = 0$$

This is Riccati first order non-linear. But it is separable. Hence

$$y' = \frac{y^2 - 1}{x} \tag{1}$$

Hence

$$\begin{aligned}\frac{dy}{dx} &= \frac{y^2 - 1}{x} \\ \frac{dy}{y^2 - 1} &= \frac{dx}{x}\end{aligned}$$

Integrating

$$\begin{aligned} -\tanh^{-1}(y) &= \ln x + C \\ y &= -\tanh(\ln x + C) \end{aligned}$$

Verification

```
1 restart;
2 ode:=x*diff(y(x),x)-y(x)^2+1=0;
3 my_sol:=-tanh(ln(x)+_C1);
4 odetest(y(x)=my_sol,ode);
5 0
```

2.97      ODE No. 97

---

$$ay(x)^2 + bx^2 + xy'(x) - y(x) = 0$$

✓    **Mathematica** : cpu = 0.100914 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{b}x \tan\left(\sqrt{a}\sqrt{b}x - \sqrt{a}\sqrt{b}c_1\right)}{\sqrt{a}} \right\} \right\}$$

✓    **Maple** : cpu = 0.032 (sec), leaf count = 25

$$\left\{ y(x) = -\frac{x}{a} \tan\left(\sqrt{ab}(x + \_C1)\right) \sqrt{ab} \right\}$$

Hand solution

$$ay^2 + bx^2 + xy' - y = 0$$

This is Riccati first order non-linear. Let  $y = ux$ , hence the above becomes

$$\begin{aligned} au^2x^2 + bx^2 + x(u'x + u) - ux &= 0 \\ au^2x + bx + u'x &= 0 \\ au^2 + b + u' &= 0 \\ u' &= -au^2 - b \end{aligned}$$

Which is separable, Hence

$$\frac{du}{au^2 + b} = -dx$$

Integrating

$$\begin{aligned} \frac{1}{\sqrt{ab}} \arctan\left(\frac{au}{\sqrt{ab}}\right) &= -x + C \\ \frac{au}{\sqrt{ab}} &= \tan\left(\sqrt{ab}(-x + C)\right) \\ u &= \frac{\sqrt{ab}}{a} \tan\left(\sqrt{ab}(-x + C)\right) \end{aligned}$$

Therefore

$$\begin{aligned} y &= ux \\ &= x \frac{\sqrt{ab}}{a} \tan\left(\sqrt{ab}(-x + C)\right) \end{aligned}$$

Verification

```
1 restart;
2 ode:=a*y(x)^2+b*x^2+x*diff(y(x),x)-y(x)=0;
3 my_sol:=x*sqrt(a*b)/a*tan(sqrt(a*b)*(-x+_C1));
4 odetest(y(x)=my_sol,ode);
5 0
```

## 2.98 ODE No. 98

$$ay(x)^2 + cx^{2b} - by(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.164148 (sec), leaf count = 442

$$\left\{ \left\{ y(x) \rightarrow - \frac{\sqrt{-a}\sqrt{-c}x^b \left( \frac{\sqrt{\frac{2}{\pi}}c_1 \sin\left(\frac{\sqrt{-a}\sqrt{-c}x^b}{b}\right)}{\sqrt{\sqrt{-a}\sqrt{-c}x^b}} - \frac{\sqrt{\frac{2}{\pi}}c_1 \left( -\sin\left(\frac{\sqrt{-a}\sqrt{-c}x^b}{b}\right) - \frac{\sqrt{-a}b\sqrt{-c}x^{-b} \cos\left(\frac{\sqrt{-a}\sqrt{-c}x^b}{b}\right)}{ac} \right)}{\sqrt{\sqrt{-a}\sqrt{-c}x^b}} - \frac{2\sqrt{\frac{2}{\pi}} \cos\left(\frac{\sqrt{-a}\sqrt{-c}x^b}{b}\right)}{\sqrt{\sqrt{-a}\sqrt{-c}x^b}} \right)}{2a \left( \frac{\sqrt{\frac{2}{\pi}}c_1 \cos\left(\frac{\sqrt{-a}\sqrt{-c}x^b}{b}\right)}{\sqrt{\sqrt{-a}\sqrt{-c}x^b}} + \frac{\sqrt{\frac{2}{\pi}} \sin\left(\frac{\sqrt{-a}\sqrt{-c}x^b}{b}\right)}{\sqrt{\sqrt{-a}\sqrt{-c}x^b}} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.045 (sec), leaf count = 38

$$\left\{ y(x) = -\frac{1}{x^{-b}} \tan\left(\frac{1}{b} \left(x^b \sqrt{c}\sqrt{a} + {}_{{}_-}C1\,b\right)\right) \sqrt{c}\frac{1}{\sqrt{a}} \right\}$$

## 2.99 ODE No. 99

$$ay(x)^2 - by(x) - cx^\beta + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.155337 (sec), leaf count = 244

$$\left\{ \left\{ y(x) \rightarrow - \frac{\sqrt{-a}\sqrt{c}x^{\beta/2} \left( c_1 J_{1-\frac{b}{\beta}} \left( \frac{2\sqrt{-a}\sqrt{c}x^{\beta/2}}{\beta} \right) - c_1 J_{-\frac{b+\beta}{\beta}} \left( \frac{2\sqrt{-a}\sqrt{c}x^{\beta/2}}{\beta} \right) - 2J_{\frac{b}{\beta}-1} \left( \frac{2\sqrt{-a}\sqrt{c}x^{\beta/2}}{\beta} \right) \right) - bc_1 J_{-\frac{b}{\beta}} \left( \frac{2\sqrt{-a}\sqrt{c}x^{\beta/2}}{\beta} \right)}{2a \left( c_1 J_{-\frac{b}{\beta}} \left( \frac{2\sqrt{-a}\sqrt{c}x^{\beta/2}}{\beta} \right) + J_{\frac{b}{\beta}} \left( \frac{2\sqrt{-a}\sqrt{c}x^{\beta/2}}{\beta} \right) \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 171

$$\left\{ y(x) = \frac{1}{a} \left( - \left( Y_{\frac{b+\beta}{\beta}} \left( 2 \frac{\sqrt{-a}cx^{\beta/2}}{\beta} \right) {}_{{}_-}C1 + J_{\frac{b+\beta}{\beta}} \left( 2 \frac{\sqrt{-a}cx^{\beta/2}}{\beta} \right) \right) \sqrt{-a}cx^{\frac{\beta}{2}} + b \left( Y_{\frac{b}{\beta}} \left( 2 \frac{\sqrt{-a}cx^{\beta/2}}{\beta} \right) {}_{{}_-}C1 + \right. \right.$$

## 2.100 ODE No. 100

$$a + xy'(x) + xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0755735 (sec), leaf count = 157

$$\left\{ \left\{ y(x) \rightarrow - \frac{c_1 J_1(2i\sqrt{-a}\sqrt{x}) + i\sqrt{-a}\sqrt{x} (c_1 J_0(2i\sqrt{-a}\sqrt{x}) - c_1 J_2(2i\sqrt{-a}\sqrt{x}) - 2J_0(2i\sqrt{-a}\sqrt{x}))}{2x (J_1(2i\sqrt{-a}\sqrt{x}) - c_1 J_1(2i\sqrt{-a}\sqrt{x}))} \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 59

$$\left\{ y(x) = 1\sqrt{a} \left( J_0(2\sqrt{a}\sqrt{x}) {}_{{}_-}C1 + Y_0(2\sqrt{a}\sqrt{x}) \right) \frac{1}{\sqrt{x}} \left( {}_{{}_-}C1\,J_1(2\sqrt{a}\sqrt{x}) + Y_1(2\sqrt{a}\sqrt{x}) \right)^{-1} \right\}$$

**Hand solution**

$$xy' + xy^2 + a = 0$$

$$y' = -\frac{a}{x} - y^2$$

This is Riccati first order non-linear. Let  $y = -\frac{u'}{uR} = \frac{u'}{u}$ , hence  $y' = \frac{u''}{u} - \frac{(u')^2}{u^2}$ . Equating this to RHS of the above gives

$$\frac{u''}{u} - \frac{(u')^2}{u^2} = -\frac{a}{x} - \left(\frac{u'}{u}\right)^2$$

$$\frac{u''}{u} = -\frac{a}{x}$$

$$u'' + \frac{a}{x}u = 0$$

This is linear second order, an Emden Fowler ODE, with removal singularity. Solved using power series method. The solution is

$$u = C_1\sqrt{x} \operatorname{BesselJ}\left(1,2\sqrt{ax}\right) + C_2\sqrt{x} \operatorname{BesselY}\left(1,2\sqrt{ax}\right)$$

But

$$\frac{d}{dx} \operatorname{BesselJ}\left(1,2\sqrt{ax}\right) = \frac{\sqrt{a}}{\sqrt{x}} \left( \operatorname{BesselJ}\left(0,2\sqrt{ax}\right) - \frac{1}{2} \frac{1}{\sqrt{ax}} \operatorname{BesselJ}\left(1,2\sqrt{ax}\right) \right)$$

And

$$\frac{d}{dx} \operatorname{BesselY}\left(1,2\sqrt{ax}\right) = \frac{\sqrt{a}}{\sqrt{x}} \left( \operatorname{BesselY}\left(0,2\sqrt{ax}\right) - \frac{1}{2} \frac{1}{\sqrt{ax}} \operatorname{BesselY}\left(1,2\sqrt{ax}\right) \right)$$

Therefore,

$$\begin{aligned} u' = & C_1 \left( \frac{1}{2\sqrt{x}} \operatorname{BesselJ}\left(1,2\sqrt{a}\sqrt{x}\right) + \sqrt{x} \frac{\sqrt{a}}{\sqrt{x}} \left( \operatorname{BesselJ}\left(0,2\sqrt{a}\sqrt{x}\right) - \frac{1}{2} \frac{1}{\sqrt{ax}} \operatorname{BesselJ}\left(1,2\sqrt{a}\sqrt{x}\right) \right) \right) \\ & + C_2 \left( \frac{1}{2\sqrt{x}} \operatorname{BesselY}\left(1,2\sqrt{a}\sqrt{x}\right) + \sqrt{x} \frac{\sqrt{a}}{\sqrt{x}} \left( \operatorname{BesselY}\left(0,2\sqrt{a}\sqrt{x}\right) - \frac{1}{2} \frac{1}{\sqrt{ax}} \operatorname{BesselY}\left(1,2\sqrt{a}\sqrt{x}\right) \right) \right) \end{aligned}$$

Which is simplified to

$$u' = C_1\sqrt{a} \operatorname{BesselJ}\left(0,2\sqrt{a}\sqrt{x}\right) + C_2\sqrt{a} \operatorname{BesselY}\left(0,2\sqrt{a}\sqrt{x}\right)$$

Therefore, from  $y = \frac{u'}{u}$ , the solution is

$$y = \frac{C_1\sqrt{a} \operatorname{BesselJ}\left(0,2\sqrt{a}\sqrt{x}\right) + C_2\sqrt{a} \operatorname{BesselY}\left(0,2\sqrt{a}\sqrt{x}\right)}{C_1\sqrt{x} \operatorname{BesselJ}\left(1,2\sqrt{a}\sqrt{x}\right) + C_2\sqrt{x} \operatorname{BesselY}\left(1,2\sqrt{a}\sqrt{x}\right)}$$

Let  $C = \frac{C_1}{C_2}$ , hence

$$y = \frac{C\sqrt{a} \operatorname{BesselJ}\left(0,2\sqrt{a}\sqrt{x}\right) + \sqrt{a} \operatorname{BesselY}\left(0,2\sqrt{a}\sqrt{x}\right)}{C\sqrt{x} \operatorname{BesselJ}\left(1,2\sqrt{a}\sqrt{x}\right) + \sqrt{x} \operatorname{BesselY}\left(1,2\sqrt{a}\sqrt{x}\right)}$$

Verification

```

1 restart;
2 ode:=x*diff(y(x),x)+x*y(x)^2+a=0;
3 num:=-C1*sqrt(a)*BesselJ(0,2*sqrt(a)*sqrt(x))+sqrt(a)*BesselY(0,2*sqrt(a)*sqrt(x));
4 den:=-C1*sqrt(x)*BesselJ(1,2*sqrt(a)*sqrt(x))+sqrt(x)*BesselY(1,2*sqrt(a)*sqrt(x));
5 my_solution:=num/den;
6 odetest(y(x)=my_solution,ode);
7 0

```

## 2.101 ODE No. 101

$$xy'(x)+xy(x)^2-y(x)=0$$

✓ **Mathematica** : cpu = 0.0707184 (sec), leaf count = 18

$$\left\{\left\{y(x)\rightarrow \frac{2x}{2c_1+x^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 16

$$\left\{y(x)=2\frac{x}{x^2+2\_C1}\right\}$$

Hand solution

$$\begin{aligned} xy' + xy^2 - y &= 0 \\ y' &= \frac{1}{x}y - y^2 \end{aligned} \tag{1}$$

This is of the form  $y' = f_0 + f_1y + f_2y^2$  with  $f_0 = 0, f_1 = \frac{1}{x}, f_2 = -1$ . Since  $f_0 = 0$  this is Bernoulli differential equation. We always start by dividing by  $y^2$

$$\frac{y'}{y^2} = \frac{1}{x} \frac{1}{y} - 1$$

Then  $u = \frac{1}{y}$  or  $y = \frac{1}{u}$ , therefore  $y' = -\frac{u'}{u^2}$ . Equating this to RHS of (1) gives

$$\begin{aligned} -\frac{u'}{u^2}u^2 &= \frac{1}{x}u - 1 \\ -u' &= \frac{u}{x} - 1 \\ u' + \frac{u}{x} &= 1 \end{aligned}$$

Integrating factor is  $e^{\int \frac{1}{x}dx} = x$  and the above becomes

$$d\left(xu\right)=x$$

Integrating

$$\begin{aligned} xu &= \frac{x^2}{2} + C \\ u &= \frac{x}{2} + \frac{C}{x} \\ &= \frac{x^2 + 2C}{2x} \end{aligned}$$

Hence

$$\begin{aligned} y &= \frac{1}{u} \\ &= \frac{2x}{x^2 + 2C} \end{aligned}$$

Verification

```

1 restart;
2 ode:=x*diff(y(x),x)+x*y(x)^2-y=0;
3 my_solution:=2*x/(x^2+2*_C1);
4 odetest(y(x)=my_solution,ode);
5 0

```

## 2.102      ODE No. 102

$$-ax^3+xy'(x)+xy(x)^2-y(x)=0$$

✓    **Mathematica** : cpu = 0.0847245 (sec), leaf count = 36

$$\left\{\left\{y(x)\rightarrow \sqrt{a}x\tanh\left(\frac{1}{2}\left(2\sqrt{a}c_1+\sqrt{a}x^2\right)\right)\right\}\right\}$$

✓    **Maple** : cpu = 0.031 (sec), leaf count = 22

$$\left\{y(x)=\tanh\left(\frac{x^2+2\_C1}{2}\sqrt{a}\right)x\sqrt{a}\right\}$$

**Hand solution**

$$xy'+xy^2-y-ax^3=0$$

This is Riccati non-linear first order. But using the transformation  $y=xv$  it is transformed to easily solved ODE

$$y'=v+xv'$$

Therefore the ODE becomes

$$\begin{aligned} x\left(v+xv'\right)+x\left(xv\right)^2-xv-ax^3 &=0 \\ xv+x^2v'+x^3v^2-xv-ax^3 &=0 \\ x^2v'+x^3v^2-ax^3 &=0 \\ v'+xv^2-ax &=0 \\ \frac{dv}{dx} &=x\left(a-v^2\right) \\ \frac{dv}{a-v^2} &=xdx \end{aligned}$$

Integrating

$$\begin{aligned}\frac{1}{\sqrt{a}} \tanh^{-1} \left( \frac{v}{\sqrt{a}} \right) &= \frac{x^2}{2} + C \\ \tanh^{-1} \left( \frac{v}{\sqrt{a}} \right) &= \sqrt{a} \left( \frac{x^2}{2} + C \right) \\ \frac{v}{\sqrt{a}} &= \tanh \left( \sqrt{a} \left( \frac{x^2}{2} + C \right) \right) \\ v &= \sqrt{a} \tanh \left( \sqrt{a} \left( \frac{x^2}{2} + C \right) \right)\end{aligned}$$

Therefore

$$\begin{aligned}y &= xv \\ &= x\sqrt{a} \tanh \left( \sqrt{a} \left( \frac{x^2}{2} + C \right) \right)\end{aligned}$$

Verification

```
1 restart;
2 ode:=x*diff(y(x),x)+x*y(x)^2-y(x)-a*x^3=0;
3 my_solution:=x*sqrt(a)*tanh(sqrt(a)*(x^2/2+_C1));
4 odetest(y(x)=my_solution,ode);
5 0
```

## 2.103 ODE No. 103

$$-x^3 - (2x^2 + 1)y(x) + xy'(x) + xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.146263 (sec), leaf count = 90

$$\left\{ \left\{ y(x) \rightarrow \frac{x \left( e^{2\sqrt{2}c_1} - \sqrt{2}e^{2\sqrt{2}c_1} + e^{\sqrt{2}x^2} + \sqrt{2}e^{\sqrt{2}x^2} \right)}{e^{2\sqrt{2}c_1} + e^{\sqrt{2}x^2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 29

$$\left\{ y(x) = \frac{\sqrt{2}x}{2} \left( \sqrt{2} + 2 \tanh \left( 1/2 (x^2 + 2\_C1) \sqrt{2} \right) \right) \right\}$$

**Hand solution**

$$xy' - xy^2 - (2x^2 + 1)y - x^3 = 0$$

This is Riccati non-linear first order. Converting it to standard form

$$\begin{aligned}y' &= x^2 + \frac{(2x^2 + 1)}{x}y + y^2 \\ &= f_0 + f_1y + f_2y^2\end{aligned}\tag{1}$$

Using standard transformation  $y = -\frac{u'}{uf_2} = -\frac{u'}{u}$ , therefore

$$y' = -\frac{u''}{u} + \frac{(u')^2}{u^2}$$

Equating the above to RHS of (1) gives

$$\begin{aligned}-\frac{u''}{u} + \frac{(u')^2}{u^2} &= x^2 - \frac{(2x^2 + 1)}{x} \frac{u'}{u} + \left( -\frac{u'}{u} \right)^2 \\ -\frac{u''}{u} &= x^2 - \frac{(2x^2 + 1)}{x} \frac{u'}{u} \\ -u'' &= ux^2 - \frac{(2x^2 + 1)}{x}u' \\ -u'' + \frac{(2x^2 + 1)}{x}u' - ux^2 &= 0 \\ xu'' - (2x^2 + 1)u' + ux^3 &= 0\end{aligned}\tag{2}$$



This is second order linear ODE (Sturm-Liouville). Using the transformation  $t = \frac{x^2}{2}$ , then  $\frac{dt}{dx} = x$  and

$$\frac{du}{dx} = \frac{du}{dt} \frac{dt}{dx} = x \frac{du}{dt} = \sqrt{2t} \frac{du}{dt}$$

And

$$\begin{aligned} \frac{d^2u}{dx^2} &= \frac{d}{dx} \left( \frac{du}{dx} \right) \\ &= \frac{d}{dx} \left( x \frac{du}{dt} \right) \\ &= \frac{du}{dt} + x \frac{d^2u}{dt^2} \frac{dt}{dx} \\ &= \frac{du}{dt} + x \frac{d^2u}{dt^2} x \\ &= \frac{du}{dt} + x^2 \frac{d^2u}{dt^2} \\ &= \frac{du}{dt} + 2t \frac{d^2u}{dt^2} \end{aligned}$$

Hence (2) can be written as

$$\begin{aligned} \sqrt{2t} \left( \frac{du}{dt} + 2t \frac{d^2u}{dt^2} \right) - (2(2t) + 1) \sqrt{2t} \frac{du}{dt} + u (2t)^{\frac{3}{2}} &= 0 \\ \sqrt{2t} \frac{du}{dt} + \sqrt{2t} 2t \frac{d^2u}{dt^2} - (4t + 1) \sqrt{2t} \frac{du}{dt} + u (\sqrt{2t})^3 &= 0 \\ \frac{du}{dt} + 2t \frac{d^2u}{dt^2} - (4t + 1) \frac{du}{dt} + 2tu &= 0 \\ \frac{du}{dt} + 2t \frac{d^2u}{dt^2} - 4t \frac{du}{dt} - \frac{du}{dt} + 2tu &= 0 \\ 2t \frac{d^2u}{dt^2} - 4t \frac{du}{dt} + 2tu &= 0 \\ 2t \left( \frac{d^2u}{dt^2} - 2 \frac{du}{dt} + u \right) &= 0 \end{aligned}$$

Hence

$$\frac{d^2u}{dt^2} - 2 \frac{du}{dt} + u = 0$$

This is linear second order with constant coefficients. The indicial equation is  $\lambda^2 - 2\lambda + 1 = 0$  with roots  $\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{4 - 4}}{2} = 1$  double root. Hence

$$u(t) = Ae^t + tBe^t$$

Since  $t = \frac{x^2}{2}$  then

$$u(x) = Ae^{\frac{x^2}{2}} + \frac{x^2}{2} Be^{\frac{x^2}{2}}$$

But  $y = -\frac{u'}{u}$  therefore

$$\begin{aligned} u' &= Axe^{\frac{x^2}{2}} + \left( xBe^{\frac{x^2}{2}} + \frac{x^2}{2} xBe^{\frac{x^2}{2}} \right) \\ &= Axe^{\frac{x^2}{2}} + xBe^{\frac{x^2}{2}} + \frac{x^3}{2} Be^{\frac{x^2}{2}} \end{aligned}$$

Hence

$$y = -\frac{Axe^{\frac{x^2}{2}} + xBe^{\frac{x^2}{2}} + \frac{x^3}{2} Be^{\frac{x^2}{2}}}{Ae^{\frac{x^2}{2}} + \frac{x^2}{2} Be^{\frac{x^2}{2}}} = -\frac{Axe^{\frac{x^2}{2}} + B \left( xe^{\frac{x^2}{2}} + \frac{x^3}{2} e^{\frac{x^2}{2}} \right)}{Ae^{\frac{x^2}{2}} + \frac{x^2}{2} Be^{\frac{x^2}{2}}}$$

Let  $C = \frac{A}{B}$

$$\begin{aligned} y &= -\frac{xe^{\frac{x^2}{2}} \left( C + 1 + \frac{x^2}{2} \right)}{e^{\frac{x^2}{2}} \left( C + \frac{x^2}{2} \right)} \\ &= -\frac{x \left( C + 1 + \frac{x^2}{2} \right)}{C + \frac{x^2}{2}} \\ &= -\frac{x(2C + 2 + x^2)}{2C + x^2} \\ &= -\frac{x(C_1 + 2 + x^2)}{C_1 + x^2} \end{aligned}$$

Verification

```

1 restart;
2 ode:=x*diff(y(x),x)-x*y(x)^2-(2*x^2+1)*y(x)-x^3;
3 my_solution:=- (x*(_C1+2*x^2))/(_C1+x^2);
4 odetest(y(x)=my_solution,ode);
5 0

```

## 2.104 ODE No. 104

$$axy(x)^2 + bx + xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.109763 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{\frac{b}{a}} \tan \left( ax \sqrt{\frac{b}{a}} - c_1 \right) - \frac{1}{ax} \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 63

$$\left\{ y(x) = -\frac{1}{a} \left( -\frac{1}{x} \left( i\sqrt{a}\sqrt{bx} - 1 \right) + 1e^{-2i\sqrt{a}\sqrt{bx}} \left( -C1 - \frac{i}{2}e^{-2i\sqrt{a}\sqrt{bx}} \frac{1}{\sqrt{a}} \frac{1}{\sqrt{b}} \right)^{-1} \right) \right\}$$

### Hand solution

$xy' + axy^2 + 2y + bx = 0$  This is Riccati non-linear first order. Converting it to standard form

$$\begin{aligned} y' &= -b - \frac{2}{x}y - ay^2 \\ &= f_0 + f_1y + f_2y^2 \end{aligned} \tag{1}$$

Using transformation suggested by Kamke  $y = u(x) - \frac{1}{ax}$  then  $y' = u' + \frac{1}{ax^2}$ . Equating this to RHS of (1) gives

$$\begin{aligned} u' + \frac{1}{ax^2} &= -b - \frac{2}{x} \left( u - \frac{1}{ax} \right) - a \left( u - \frac{1}{ax} \right)^2 \\ &= -b - \frac{2}{x}u + \frac{2}{ax^2} - a \left( u^2 + \frac{1}{a^2x^2} - \frac{2u}{ax} \right) \\ &= -b - \frac{2}{x}u + \frac{2}{ax^2} - au^2 - \frac{1}{ax^2} + \frac{2u}{x} \end{aligned}$$

Hence

$$\begin{aligned} u' &= -b - au^2 \\ \frac{du}{dx} &= -b - au^2 \end{aligned}$$

This is separable

$$\frac{du}{b + au^2} = -dx$$

Integrating

$$\begin{aligned} \int \frac{du}{b + au^2} &= -x + C \\ \frac{1}{\sqrt{ba}} \arctan \left( \frac{au}{\sqrt{ba}} \right) &= -x + C \\ \arctan \left( \frac{au}{\sqrt{ba}} \right) &= -\sqrt{ba}x + C \\ u &= \frac{\sqrt{ba}}{a} \tan \left( -\sqrt{ba}x + C \right) \end{aligned}$$

Hence

$$\begin{aligned} y &= u - \frac{1}{ax} \\ &= \frac{\sqrt{ba}}{a} \tan \left( -\sqrt{ba}x + C \right) - \frac{1}{ax} \end{aligned}$$

Verification

```

1 restart;
2 ode:=x*diff(y(x),x)+a*x*y(x)^2+2*y(x)+b*x = 0;
3 my_solution:=sqrt(b*a)/a*tan(-sqrt(b*a)*x+_C1)-1/(a*x);
4 odetest(y(x)=my_solution,ode);
5 0

```

2.105

ODE No. 105

---

$$axy(x)^2+by(x)+cx+d+xy'(x)=0$$

✓ **Mathematica** : cpu = 0.217699 (sec), leaf count = 473

$$\left\{\left\{y(x)\rightarrow \frac{c_1\left(i\sqrt{a}e^{-i\sqrt{a}\sqrt{c}x}\left(b\left(-\sqrt{c}\right)-i\sqrt{a}d\right)U\left(1-\frac{-\sqrt{c}b-i\sqrt{a}d}{2\sqrt{c}},b+1,2i\sqrt{a}\sqrt{c}x\right)-i\sqrt{a}\sqrt{c}e^{-i\sqrt{a}\sqrt{c}x}U\left(-\frac{-\sqrt{c}b-i\sqrt{a}d}{2\sqrt{c}},b,2i\sqrt{a}\sqrt{c}x\right)\right)}{a\left(c_1e^{-i\sqrt{a}\sqrt{c}x}U\left(-\frac{-\sqrt{c}b-i\sqrt{a}d}{2\sqrt{c}},b,2i\sqrt{a}\sqrt{c}x\right)+e^{-i\sqrt{a}\sqrt{c}x}U\left(\frac{-\sqrt{c}b-i\sqrt{a}d}{2\sqrt{c}},b,2i\sqrt{a}\sqrt{c}x\right)\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.196 (sec), leaf count = 844

$$\left\{y(x)=-4c^2\left(-1/4\_C1\left(a^3c^2d^2+a^2b^2c^3-2(-ac)^{3/2}abcd-2(-ac)^{5/2}bd\right)U\left(1/2\frac{(-ac)^{3/2}d+ca\left(2\sqrt{-ac}\right)}{c^2a}\right)\right)\right\}$$

2.106

ODE No. 106

---

$$\frac{1}{2}(a-b)y(x)+x^ay(x)^2+x^b+xy'(x)=0$$

✓ **Mathematica** : cpu = 0.196936 (sec), leaf count = 40

$$\left\{\left\{y(x)\rightarrow -x^{\frac{b-a}{2}}\tan\left(\frac{2x^{\frac{a+b}{2}}}{a+b}-c_1\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 41

$$\left\{y(x)=-1\tan\left(\frac{1}{a+b}\left(2x^{a/2+b/2}+_C1\left(a+b\right)\right)\right)\left(x^{\frac{a}{2}-\frac{b}{2}}\right)^{-1}\right\}$$

2.107

ODE No. 107

---

$$ax^\alpha y(x)^2+by(x)-cx^\beta+xy'(x)=0$$

✓ **Mathematica** : cpu = 0.326367 (sec), leaf count = 1415

$$\left\{\left\{y(x)\rightarrow \frac{x^{1-\alpha}\left((-1)^{\frac{\alpha-b}{\alpha+\beta}}a^{\frac{\alpha-b}{\alpha+\beta}+\frac{1}{2}\left(\frac{b}{\alpha+\beta}-\frac{\alpha}{\alpha+\beta}\right)}(\alpha+\beta)^{\frac{\alpha}{\alpha+\beta}-\frac{b}{\alpha+\beta}+1}(\alpha^2+2\beta\alpha+\beta^2)^{-\frac{\alpha-b}{\alpha+\beta}}\left(\frac{\alpha-b}{\alpha+\beta}+\frac{1}{2}\left(\frac{b}{\alpha+\beta}-\frac{\alpha}{\alpha+\beta}\right)\right)\right)}{J_{\frac{b+\beta}{\alpha+\beta}}\left(2\frac{\sqrt{-ac}x^{\alpha/2+\beta/2}}{\alpha+\beta}\right)+\_C1Y_{\frac{b+\beta}{\alpha+\beta}}\left(2\frac{\sqrt{-ac}x^{\alpha/2+\beta/2}}{\alpha+\beta}\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.194 (sec), leaf count = 174

$$\left\{y(x)=-\frac{x^{1-\alpha}}{ax}\left(Y_{\frac{b+\beta}{\alpha+\beta}}\left(2\frac{\sqrt{-ac}x^{\alpha/2+\beta/2}}{\alpha+\beta}\right)\_C1+J_{\frac{b+\beta}{\alpha+\beta}}\left(2\frac{\sqrt{-ac}x^{\alpha/2+\beta/2}}{\alpha+\beta}\right)\right)x^{\frac{\alpha}{2}+\frac{\beta}{2}}\sqrt{-ac}\left(Y_{\frac{b-\alpha}{\alpha+\beta}}\left(2\frac{\sqrt{-ac}x^{\alpha/2+\beta/2}}{\alpha+\beta}\right)+\_C1J_{\frac{b-\alpha}{\alpha+\beta}}\left(2\frac{\sqrt{-ac}x^{\alpha/2+\beta/2}}{\alpha+\beta}\right)\right)\right\}$$

2.108      ODE No. 108

---

$$xy'(x) + y(x) + y(x)^2(-\log(x)) = 0$$

✓ **Mathematica** : cpu = 0.0656555 (sec), leaf count = 15

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 x + \log(x) + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 13

$$\left\{ y(x) = (1 + \_C1 x + \ln(x))^{-1} \right\}$$

Hand solution

$xy' + axy^2 + 2y + bx = 0$  This is Riccati non-linear first order. Converting it to standard form

$$\begin{aligned} xy' - y^2 \ln x + y &= 0 \\ y' &= -\frac{1}{x}y + y^2 \frac{\ln x}{x} \\ &= f_0 + f_1 y + f_2 y^2 \end{aligned} \tag{1}$$

This is Bernoulli non-linear first order ODE since  $f_0 = 0$ . Dividing by  $y^2$  gives

$$\frac{y'}{y^2} = -\frac{1}{x} \frac{1}{y} + \frac{\ln x}{x}$$

Let  $u = \frac{1}{y}$ , hence  $u' = -\frac{y'}{y^2}$ , and the above becomes

$$\begin{aligned} -u' &= -\frac{1}{x}u + \frac{\ln x}{x} \\ u' - \frac{1}{x}u &= -\frac{\ln x}{x} \end{aligned}$$

Integrating factor is  $\mu = e^{\int -\frac{1}{x} dx} = e^{-\ln x} = \frac{1}{x}$ , hence

$$d(\mu u) = -\mu \frac{\ln x}{x}$$

Integrating

$$\begin{aligned} \frac{1}{x}u &= -\int \frac{1}{x^2} \ln x dx + C \\ &= -\left(-\frac{\ln x}{x} - \frac{1}{x}\right) + C \end{aligned}$$

Therefore

$$u = \ln x + 1 + Cx$$

Since  $u = \frac{1}{y}$  then

$$y = \frac{1}{\ln x + 1 + Cx}$$

Verification

```
1 restart;
2 ode:=x*diff(y(x),x)-y(x)^2*ln(x)+y(x)=0;
3 my_solution:=1/(ln(x)+1+_C1*x);
4 odetest(y(x)=my_solution,ode);
5 0
```

2.109 ODE No. 109

$xy'(x) - y(x)(2y(x)\log(x) - 1) = 0$

✓ **Mathematica** : cpu = 0.0685779 (sec), leaf count = 17

$\left\{\left\{y(x) \rightarrow \frac{1}{c_1 x + 2 \log(x) + 2}\right\}\right\}$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 15

$\left\{y(x) = (2 + \_C1 x + 2 \ln(x))^{-1}\right\}$

Hand solution

$xy' + axy^2 + 2y + bx = 0$ This is Riccati non-linear first order. Converting it to standard form

$$\begin{aligned} xy' - y(2y \ln x - 1) &= 0 \\ xy' &= y(2y \ln x - 1) \\ y' &= -\frac{1}{x}y + y^2 \frac{2}{x} \ln x \\ y' &= f_0 + f_1 y + f_2 y^2 \end{aligned} \tag{1}$$

This is Bernoulli non-linear first order ODE since  $f_0 = 0$ . Dividing by  $y^2$  gives

$$\frac{y'}{y^2} = -\frac{1}{x} \frac{1}{y} + \frac{2}{x} \ln x$$

Putting  $u = \frac{1}{y}$ , hence  $u' = -\frac{y'}{y^2}$ , and the above becomes

$$\begin{aligned} -u' &= -\frac{1}{x}u + 2\frac{\ln x}{x} \\ -u' + \frac{1}{x}u &= 2\frac{\ln x}{x} \\ u' - \frac{1}{x}u &= -2\frac{\ln x}{x} \end{aligned}$$

Integrating factor is  $\mu = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$ , hence

$$\begin{aligned} d(\mu u) &= -2\mu \frac{\ln x}{x} \\ d\left(\frac{1}{x}u\right) &= -2\frac{\ln x}{x^2} \end{aligned}$$

Integrating

$$\begin{aligned} \frac{1}{x}u &= -2 \int \frac{1}{x^2} \ln x dx + C \\ &= -2 \left(-\frac{\ln x}{x} - \frac{1}{x}\right) + C \end{aligned}$$

Therefore

$$\begin{aligned} u &= -2x \left(-\frac{\ln x}{x} - \frac{1}{x}\right) + Cx \\ &= 2(\ln x + 1) + Cx \end{aligned}$$

Since  $u = \frac{1}{y}$  then

$$y = \frac{1}{2(\ln x + 1) + Cx}$$

Verification

```
1 restart;
2 ode:=x*diff(y(x),x)-y(x)*(2*y(x)*ln(x)-1)=0;
3 my_solution:=1/(2*(ln(x)+1)+_C1*x);
4 odetest(y(x)=my_solution,ode);
5 0
```

## 2.110 ODE No. 110

$$f(x) (y(x)^2 - x^2) + xy'(x) = 0$$

✗ **Mathematica** : cpu = 11.582 (sec), leaf count = 0 , could not solve

DSolve[f[x]\*(-x^2 + y[x]^2) + x\*Derivative[1][y][x] == 0, y[x], x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(x\*diff(y(x),x)+f(x)\*(y(x)^2-x^2) = 0,y(x))

### Hand solution

$xy' + axy^2 + 2y + bx = 0$  This is Riccati non-linear first order. Converting it to standard form

$$\begin{aligned} xy' + f(x) (y^2 - x^2) - y &= 0 \\ xy' &= -f(x) (y^2 - x^2) + y \\ y' &= -\frac{f}{x} y^2 + fx + \frac{1}{x} y \end{aligned} \quad (1)$$

This is Riccati non-linear first order. There are two particular solutions  $y_p = \pm x$ . Using  $y_p = x$ , then using the transformation  $y = y_p + \frac{1}{u}$ , gives  $y' = 1 - \frac{u'}{u^2}$  and (1) becomes

$$\begin{aligned} 1 - \frac{u'}{u^2} &= -\frac{f}{x} \left(x + \frac{1}{u}\right)^2 + fx + \frac{1}{x} \left(x + \frac{1}{u}\right) \\ &= -\frac{f}{x} \left(x^2 + \frac{1}{u^2} + 2\frac{x}{u}\right) + fx + \left(1 + \frac{1}{ux}\right) \\ &= -fx - \frac{f}{x} \frac{1}{u^2} - 2\frac{f}{u} + fx + 1 + \frac{1}{ux} \\ &= -\frac{f}{x} \frac{1}{u^2} - 2\frac{f}{u} + 1 + \frac{1}{ux} \end{aligned}$$

Hence

$$\begin{aligned} u^2 - u' &= -\frac{f}{x} - 2fu + u^2 + \frac{u}{x} \\ -u' &= -\frac{f}{x} - 2fu + \frac{u}{x} \\ -u' - \frac{u}{x} + 2fu &= \frac{-f}{x} \\ u' + \frac{u}{x} - 2fu &= \frac{-f}{x} \\ u' + u \left(\frac{1}{x} - 2f\right) &= \frac{-f}{x} \end{aligned}$$

Integrating factor is  $\mu = e^{\int \frac{1}{x} - 2f} = e^{\ln x} e^{-2 \int f dx} = x e^{-2 \int f dx}$ , hence

$$\begin{aligned} d(\mu u) &= -\mu \frac{f}{x} \\ d\left(x e^{-2 \int f dx} u\right) &= -\left(x e^{-2 \int f dx}\right) \frac{f}{x} \\ d\left(x e^{-2 \int f dx} u\right) &= -f \left(x e^{-2 \int f dx}\right) \end{aligned}$$

Integrating

$$\begin{aligned} x e^{-2 \int f dx} u &= -\int f \left(x e^{-2 \int f dx}\right) + C \\ u &= -\frac{1}{x} e^{2 \int f dx} \int f \left(x e^{-2 \int f dx}\right) + C \frac{1}{x} e^{2 \int f dx} \end{aligned}$$

Since  $u = \frac{1}{y}$  then

$$\begin{aligned} y &= \frac{1}{-\frac{1}{x} e^{2 \int f dx} \int f \left(x e^{-2 \int f dx}\right) + C \frac{1}{x} e^{2 \int f dx}} \\ &= \frac{x e^{-2 \int f dx}}{-\int f e^{-2 \int f dx} dx + C} \end{aligned}$$

Verification (Maple does not verify it, need to look more into this)

```
1 ode:=x*diff(y(x),x)+f(x)*(y(x)^2-x^2) =0;
2 dsolve(ode,y(x));
3 fint:=Int(f(x),x);
4 my_solution:=x*exp(-2*fint)/(-Int(f*exp(-2*fint),x)+_C1);
5 odetest(y(x)=my_solution,ode);
6 not zero
```

2.111      ODE No. 111

---

$$xy'(x) + y(x)^3 + 3xy(x)^2 = 0$$

✓    **Mathematica** : cpu = 0.362089 (sec), leaf count = 55

$$\text{Solve}\left[-3x = \frac{2e^{\frac{1}{2}\left(\frac{1}{y(x)}-3x\right)^2}}{2c_1 + \sqrt{2\pi}\text{erfi}\left(\frac{\frac{1}{y(x)}-3x}{\sqrt{2}}\right)}, y(x)\right]$$

✓    **Maple** : cpu = 0.105 (sec), leaf count = 54

$$\left\{-C1 - \frac{i}{x}e^{\frac{(3xy(x)-1)^2}{2(y(x))^2}} + \frac{\sqrt{\pi}\sqrt{2}}{2}\text{Erf}\left(\frac{(-i+3iy(x)x)\sqrt{2}}{2y(x)}\right) = 0\right\}$$

2.112      ODE No. 112

---

$$-\sqrt{x^2 + y(x)^2} + xy'(x) - y(x) = 0$$

✓    **Mathematica** : cpu = 0.0981619 (sec), leaf count = 13

$$\{\{y(x) \rightarrow x \sinh(c_1 + \log(x))\}\}$$

✓    **Maple** : cpu = 0.03 (sec), leaf count = 27

$$\left\{\frac{1}{x^2}\sqrt{(y(x))^2 + x^2} + \frac{y(x)}{x^2} - C1 = 0\right\}$$

Hand solution

$$xy' = \sqrt{x^2 + y^2} + y$$

Let  $y = xv$ , then  $y' = v + xv'$  and the above becomes

$$\begin{aligned} x(v + xv') &= \sqrt{x^2 + (xv)^2} + xv \\ x(v + xv') &= x\sqrt{1 + v^2} + xv \\ (v + xv') &= \sqrt{1 + v^2} + v \\ xv' &= \sqrt{1 + v^2} \end{aligned}$$

Separable.

$$\frac{dv}{\sqrt{1 + v^2}} = \frac{1}{x}dx$$

Integrating

$$\begin{aligned} \operatorname{arcsinh}(v) &= \ln x + C \\ v &= \sinh(\ln x + C) \end{aligned}$$

Since  $y = xv$  then

$$y = x \sinh(\ln x + C)$$

Verification

```
1 ode:=x*diff(y(x),x)=sqrt(x^2+y(x)^2)+y(x);
2 y0:=x*sinh(ln(x)+_C1);
3 odetest(y(x)=y0,ode) assuming x>= 0;
4 0
```

2.113      ODE No. 113

---

$$a\sqrt{x^2+y(x)^2}+xy'(x)-y(x)=0$$

✓    **Mathematica** : cpu = 0.0978274 (sec), leaf count = 16

$$\{\{y(x)\rightarrow x\sinh(c_1-a\log(x))\}\}$$

✓    **Maple** : cpu = 0.023 (sec), leaf count = 33

$$\left\{\frac{x^a}{x}\sqrt{(y(x))^2+x^2}+\frac{x^ay(x)}{x}-\_C1=0\right\}$$

**Hand solution**

$$xy'=-a\sqrt{x^2+y^2}+y$$

Let  $y = xv$ , then  $y' = v + xv'$  and the above becomes

$$\begin{aligned}x(v+xv') &= -a\sqrt{x^2+(xv)^2}+xv \\ x(v+xv') &= -ax\sqrt{1+v^2}+xv \\ (v+xv') &= -a\sqrt{1+v^2}+v \\ xv' &= -a\sqrt{1+v^2}\end{aligned}$$

Separable.

$$\frac{dv}{\sqrt{1+v^2}}=\frac{-a}{x}dx$$

Integrating

$$\begin{aligned}\operatorname{arcsinh}(v) &= -a\ln x+C \\ v &= \sinh(C-a\ln x)\end{aligned}$$

Since  $y = xv$  then

$$y=x\sinh(C-a\ln x)$$

Verification

```
1 ode:=x*diff(y(x),x)=-a*sqrt(x^2+y(x)^2)+y(x);
2 y0:=x*sinh(_C1-a*ln(x));
3 odetest(y(x)=y0,ode) assuming x >=0;
4 0
```

2.114      ODE No. 114

---

$$-x\sqrt{x^2+y(x)^2}+xy'(x)-y(x)=0$$

✓    **Mathematica** : cpu = 0.0878615 (sec), leaf count = 12

$$\{\{y(x)\rightarrow x\sinh(c_1+x)\}\}$$

✓    **Maple** : cpu = 2.004 (sec), leaf count = 28

$$\left\{\ln\left(\sqrt{(y(x))^2+x^2}+y(x)\right)-x-\ln(x)-\_C1=0\right\}$$

**Hand solution**

$$xy'=x\sqrt{x^2+y^2}+y$$

Let  $y = xv$ , then  $y' = v + xv'$  and the above becomes



$$\begin{aligned}x\left(v+xv'\right) &=x\sqrt{x^2+\left(xv\right)^2}+xv\\ \left(v+xv'\right) &=x\sqrt{1+v^2}+v\\ xv' &=x\sqrt{1+v^2}\\ v' &=\sqrt{1+v^2}\end{aligned}$$

Separable.

$$\frac{dv}{\sqrt{1+v^2}}=dx$$

Integrating

$$\begin{aligned}\operatorname{arcsinh}(v) &=x+C\\ v &=\sinh\left(x+C\right)\end{aligned}$$

Since  $y=xv$  then

$$y=x\sinh\left(x+C\right)$$

Verification

```

1 ode:=x*diff(y(x),x)=x*sqrt(x^2+y(x)^2)+y(x);
2 y0:=x*sinh(x+_C1);
3 odetest(y(x)=y0,ode) assuming x>0;
4 0

```

## 2.115      ODE No. 115

$$-x(y(x)-x)\sqrt{x^2+y(x)^2}+xy'(x)-y(x)=0$$

✓    **Mathematica** : cpu = 0.175946 (sec), leaf count = 221

$$\left\{\left\{y(x)\rightarrow\frac{x-2\sqrt{x^2\tanh^2\left(\frac{1}{2}\left(-2\sqrt{2}c_1-\sqrt{2}x^2\right)\right)-x^2\tanh^4\left(\frac{1}{2}\left(-2\sqrt{2}c_1-\sqrt{2}x^2\right)\right)}{2\tanh^2\left(\frac{1}{2}\left(-2\sqrt{2}c_1-\sqrt{2}x^2\right)\right)-1}\right\},\left\{y(x)\rightarrow\frac{2\sqrt{x^2\tanh^2\left(\frac{1}{2}\left(-2\sqrt{2}c_1-\sqrt{2}x^2\right)\right)}}{2\tanh^2\left(\frac{1}{2}\left(-2\sqrt{2}c_1-\sqrt{2}x^2\right)\right)-1}\right\}\right\}$$

✓    **Maple** : cpu = 0.171 (sec), leaf count = 49

$$\left\{\ln\left(2\frac{x\left(\sqrt{2\left(y\left(x\right)\right)^2+2x^2}+y\left(x\right)+x\right)}{y\left(x\right)-x}\right)+\frac{\sqrt{2}x^2}{2}-\ln\left(x\right)-\_C1=0\right\}$$

Hand solution

$$xy'=x\left(y-x\right)\sqrt{y^2-x^2}+y$$

Let  $y=xu$ , then  $y'=u+xu'$  and the above becomes

$$\begin{aligned}x\left(u+xu'\right) &=x\left(xu-x\right)\sqrt{\left(xu\right)^2-x^2}+xu\\ \left(u+xu'\right) &=\left(xu-x\right)\sqrt{\left(xu\right)^2-x^2}+u\\ xu' &=\left(xu-x\right)x\sqrt{u^2-1}\\ u' &=x\left(u-1\right)\sqrt{u^2-1}\end{aligned}$$

Separable.

$$\begin{aligned}\frac{du}{\left(u-1\right)\sqrt{u^2-1}} &=xdx\\ \frac{-u-1}{\sqrt{u^2-1}} &=\frac{x^2}{2}+C\end{aligned}$$

But  $y = xu$ , hence

$$\frac{-\frac{y}{x}-1}{\sqrt{\left(\frac{y}{x}\right)^2-1}}=\frac{x^2}{2}+C$$

Let  $\frac{y}{x}=z$

$$\begin{aligned}\frac{-z-1}{\sqrt{z^2-1}} &= \frac{x^2}{2}+C \\ -z-1 &= \sqrt{z^2-1}\left(\frac{x^2}{2}+C\right) \\ (-z-1)^2 &= (z^2-1)\left(\frac{x^2}{2}+C\right)^2 \\ z^2+1+2z &= z^2\left(\frac{x^2}{2}+C\right)^2-\left(\frac{x^2}{2}+C\right)^2 \\ z^2\left(1-\left(\frac{x^2}{2}+C\right)^2\right) &+2z+1+\left(\frac{x^2}{2}+C\right)^2=0\end{aligned}$$

Solving for  $z$  (quadratic formula, some conditions apply), one of the solutions is

$$z=\frac{4Cx^2+4C^2+x^4+4}{4Cx^2+4C^2+x^4-4}$$

Hence

$$y=x\frac{4Cx^2+4C^2+x^4+4}{4Cx^2+4C^2+x^4-4}$$

Need to work on verification. Kamke gives the final solution as

$$y=x\frac{-2Cx^2+C^2+x^4+4}{-2Cx^2+C^2+x^4-4}$$

I am not sure where my error now is. Need to look at this again.

### 2.116 ODE No. 116

---


$$-x\sqrt{\left(y(x)^2-4x^2\right)\left(y(x)^2-x^2\right)}+xy'(x)-y(x)=0$$

✓ **Mathematica** : cpu = 0.350461 (sec), leaf count = 121

$$\text{Solve}\left[\frac{\sqrt{\frac{y(x)+2}{\frac{y(x)}{x}-1}}\sqrt{\frac{\frac{y(x)}{x}+1}{\frac{2y(x)}{x}+4}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}\sqrt{\frac{\frac{y(x)-2}{x}}{\frac{y(x)}{x}-1}}\right)\middle|\frac{9}{8}\right)}{\sqrt{\frac{\frac{y(x)}{x}+1}{\frac{\frac{y(x)}{x}}{x}-1}}}=c_1+\frac{x^2}{2},y(x)\right]$$

✓ **Maple** : cpu = 0.174 (sec), leaf count = 86

$$\left\{\int_{-b}^x1\left(-a\sqrt{4-a^4-5-a^2\left(y\left(x\right)\right)^2+\left(y\left(x\right)\right)^4}+y\left(x\right)\right)\frac{1}{\sqrt{4-a^4-5-a^2\left(y\left(x\right)\right)^2+\left(y\left(x\right)\right)^4}}d-a+\int^{y(x)}-b-\sqrt{\right\}$$

### 2.117 ODE No. 117

---


$$xy'(x)+x\left(-e^{\frac{y(x)}{x}}\right)-y(x)-x=0$$

✓ **Mathematica** : cpu = 0.152046 (sec), leaf count = 21

$$\left\{\left\{y(x)\rightarrow -x\log\left(\frac{e^{-c_1}}{x}-1\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 20

$$\left\{y\left(x\right)=\left(\ln\left(-\frac{x}{-1+x\mathrm{e}^{-CI}}\right)+-CI\right)x\right\}$$

**2.118      ODE No. 118**

---

$$xy'(x) - y(x) \log(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.0629596 (sec), leaf count = 13

$$\left\{ \left\{ y(x) \rightarrow e^{e^{c_1} x} \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 8

$$\left\{ y(x) = e^{-C_1 x} \right\}$$

**2.119      ODE No. 119**

---

$$xy'(x) - y(x)(\log(xy(x)) - 1) = 0$$

✓ **Mathematica** : cpu = 0.087087 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{e^{c_1} x}}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 14

$$\left\{ y(x) = \frac{1}{x} e^{-\frac{x}{C_1}} \right\}$$

**2.120      ODE No. 120**

---

$$xy'(x) - y(x) \left( x \log \left( \frac{x^2}{y(x)} \right) + 2 \right) = 0$$

✓ **Mathematica** : cpu = 0.0960763 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow x^2 e^{-2c_1 e^{-x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.143 (sec), leaf count = 17

$$\left\{ y(x) = x^2 \left( e^{\frac{C_1}{e^x}} \right)^{-1} \right\}$$

**2.121      ODE No. 121**

---

$$xy'(x) - \sin(x - y(x)) = 0$$

✗ **Mathematica** : cpu = 1.95426 (sec), leaf count = 0 , could not solve  
DSolve[-Sin[x - y[x]] + x\*Derivative[1][y][x] == 0, y[x], x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(x\*diff(y(x),x)-sin(x-y(x)) = 0,y(x))

**2.122      ODE No. 122**

---

$$\cos(y(x)) (\sin(y(x)) - 3x^2 \cos(y(x))) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.273764 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \tan^{-1} \left( \frac{c_1 + 2x^3}{2x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.296 (sec), leaf count = 16

$$\left\{ y(x) = \arctan \left( \frac{x^3 + 2_{C_1}}{x} \right) \right\}$$

**2.123**      **ODE No. 123**

$$xy'(x) - y(x) - x \sin\left(\frac{y(x)}{x}\right) = 0$$

✓ **Mathematica** : cpu = 0.105019 (sec), leaf count = 19

$$\left\{ \left\{ y(x) \rightarrow 2x \cot^{-1} \left( \frac{e^{-c_1}}{x} \right) \right\} \right\}$$

## 2.128 ODE No. 128

---

$$-f(x)g(x^a y(x)) + ay(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 3.14882 (sec), leaf count = 41

$$\text{Solve} \left[ \int_1^{x^a y(x)} \frac{1}{g(K[1])} dK[1] = \int_1^x f(K[2])K[2]^{a-1} dK[2] + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.21 (sec), leaf count = 33

$$\left\{ y(x) = \frac{\text{RootOf} \left( -\int f(x) x^{a-1} dx + \int^{-Z} (g(\_a))^{-1} d\_a + \_C1 \right)}{x^a} \right\}$$

## 2.129 ODE No. 129

---

$$(x+1)y'(x) + y(x)(y(x) - x) = 0$$

✓ **Mathematica** : cpu = 0.174285 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow -\frac{e^{x+1}}{-ec_1 x - ec_1 - x \text{Ei}(x+1) - \text{Ei}(x+1) + e^{x+1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 33

$$\left\{ y(x) = \frac{e^x}{-e^{-1}(1+x) \text{Ei}(1, -1-x) - e^x + \_C1 (1+x)} \right\}$$

## 2.130 ODE No. 130

---

$$-2x^3 + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0079278 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt{x} + \frac{2x^3}{5} \right\} \right\}$$

✓ **Maple** : cpu = 0.006 (sec), leaf count = 15

$$\left\{ y(x) = \frac{2x^3}{5} + \sqrt{x} \_C1 \right\}$$

## 2.131 ODE No. 131

---

$$(2x+1)y'(x) - 4e^{-y(x)} + 2 = 0$$

✓ **Mathematica** : cpu = 0.173295 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \log \left( 2 - \frac{e^{c_1}}{2x+1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.155 (sec), leaf count = 31

$$\left\{ y(x) = -\ln \left( \frac{2x+1}{-1+(4x+2)e^2 \_C1} \right) - 2 \_C1 \right\}$$

## 2.132 ODE No. 132

---

$$3xy'(x) - y(x) - 3xy(x)^4 \log(x) = 0$$

✓ **Mathematica** : cpu = 0.0771371 (sec), leaf count = 115

$$\left\{ \left\{ y(x) \rightarrow \frac{(-2)^{2/3} \sqrt[3]{x}}{\sqrt[3]{4c_1 + 3x^2 - 6x^2 \log(x)}} \right\}, \left\{ y(x) \rightarrow \frac{2^{2/3} \sqrt[3]{x}}{\sqrt[3]{4c_1 + 3x^2 - 6x^2 \log(x)}} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt[3]{-12}^{2/3} \sqrt[3]{x}}{\sqrt[3]{4c_1 + 3x^2 - 6x^2 \log(x)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.025 (sec), leaf count = 153

$$\left\{ y(x) = \frac{1}{6x^2 \ln(x) - 3x^2 - 4 \_C1} \sqrt[3]{-4x(6x^2 \ln(x) - 3x^2 - 4 \_C1)^2}, y(x) = \frac{i\sqrt{3} - 1}{12x^2 \ln(x) - 6x^2 - 8 \_C1} \sqrt[3]{-4x(6x^2 \ln(x) - 3x^2 - 4 \_C1)^2} \right\}$$

### 2.133 ODE No. 133

---

$$x^2 y'(x) + y(x) - x = 0$$

✓ **Mathematica** : cpu = 0.0078546 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{x}} - e^{\frac{1}{x}} \text{Ei} \left( -\frac{1}{x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.008 (sec), leaf count = 16

$$\left\{ y(x) = \left( \text{Ei} \left( 1, x^{-1} \right) + \_C1 \right) e^{x^{-1}} \right\}$$

### 2.134 ODE No. 134

---

$$x^2 y'(x) + e^{x^{-\frac{1}{x}}} x^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0171448 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-1/x} - e^{x^{-\frac{1}{x}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 17

$$\left\{ y(x) = (-e^x + \_C1) e^{-x^{-1}} \right\}$$

### 2.135 ODE No. 135

---

$$x^2 y'(x) - (x - 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0079896 (sec), leaf count = 14

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{x}} x \right\} \right\}$$

✓ **Maple** : cpu = 0.004 (sec), leaf count = 11

$$\left\{ y(x) = \_C1 x e^{x^{-1}} \right\}$$

### 2.136 ODE No. 136

---

$$x^2 y'(x) + x^2 + xy(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0688106 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow \frac{-c_1 x - x + x \log(x)}{c_1 - \log(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 18

$$\left\{ y(x) = -\frac{x (\ln(x) + \_C1 - 1)}{\ln(x) + \_C1} \right\}$$

### 2.137 ODE No. 137

---

$$x^2 y'(x) - xy(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0653976 (sec), leaf count = 16

$$\left\{ \left\{ y(x) \rightarrow \frac{x}{c_1 - \log(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 14

$$\left\{ y(x) = \frac{x}{-\ln(x) + \_C1} \right\}$$

2.138 ODE No. 138

---

$$x^2y'(x) - x^2 - xy(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.060511 (sec), leaf count = 13

$$\{\{y(x) \rightarrow x \tan(c_1 + \log(x))\}\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 11

$$\{y(x) = \tan(\ln(x) + \_C1)x\}$$

2.139 ODE No. 139

---

$$ax^k - (b-1)b + x^2(y'(x) + y(x)^2) = 0$$

✓ **Mathematica** : cpu = 0.202571 (sec), leaf count = 821

$$\left\{\left\{y(x) \rightarrow -\frac{a^{\frac{b}{k} + \frac{1}{2}}\left(\frac{1}{k} - \frac{2b}{k}\right)\left(\frac{b}{k} + \frac{1}{2}\left(\frac{1}{k} - \frac{2b}{k}\right)\right)x^{k-1}\left(x^k\right)^{\frac{b}{k} + \frac{1}{2}}\left(\frac{1}{k} - \frac{2b}{k}\right)^{-1}J_{\frac{2b-1}{k}}\left(\frac{2\sqrt{a}\sqrt{x^k}}{k}\right)\Gamma\left(\frac{2b}{k} - \frac{1}{k} + 1\right)k^{1-\frac{1}{k}} + \frac{1}{2}a^{\frac{b}{k} + \frac{1}{2}}}{\right.}\right.$$

✓ **Maple** : cpu = 0.11 (sec), leaf count = 219

$$\left\{y(x) = \frac{1}{2x}\left(-2J_{\frac{\sqrt{(-1+2b)^2+k}}{k}}\left(2\frac{\sqrt{a}x^{k/2}}{k}\right)\sqrt{a}x^{k/2} - 2x^{k/2}Y_{\frac{\sqrt{(-1+2b)^2+k}}{k}}\left(2\frac{\sqrt{a}x^{k/2}}{k}\right)\sqrt{a}\_C1 + 2\left(Y_{\frac{\sqrt{(-1+2b)^2}}{k}}\right.\right.\right.$$

2.140 ODE No. 140

---

$$x^2(y'(x) + y(x)^2) + 4xy(x) + 2 = 0$$

✓ **Mathematica** : cpu = 0.075577 (sec), leaf count = 17

$$\left\{\left\{y(x) \rightarrow \frac{1}{c_1 + x} - \frac{2}{x}\right\}\right\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 20

$$\left\{y(x) = \frac{-2\_C1 + x}{x(-x + \_C1)}\right\}$$

2.141 ODE No. 141

---

$$axy(x) + b + x^2(y'(x) + y(x)^2) = 0$$

✓ **Mathematica** : cpu = 0.122099 (sec), leaf count = 67

$$\left\{\left\{y(x) \rightarrow -\frac{\sqrt{a^2 - 2a - 4b + 1}\left(\frac{2c_1}{x\sqrt{a^2 - 2a - 4b + 1} + c_1} - 1\right) - 1}{2x} - \frac{a}{2x}\right\}\right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 51

$$\left\{y(x) = \frac{1}{2x}\left(-\tanh\left(\frac{-\ln(x) + \_C1}{2}\sqrt{a^2 - 2a - 4b + 1}\right)\sqrt{a^2 - 2a - 4b + 1} - a + 1\right)\right\}$$

2.142 ODE No. 142

---

$$-ax^2y(x) + ax + x^2(y'(x) - y(x)^2) + 2 = 0$$

✓ **Mathematica** : cpu = 0.177342 (sec), leaf count = 122

$$\left\{\left\{y(x) \rightarrow -\frac{\frac{e^{ax}(a^2x^2-2ax+2)}{a^2x} - \frac{e^{ax}(a^2x^2-2ax+2)}{a^3x^2} + \frac{e^{ax}(2a^2x-2a)}{a^3x} - \frac{c_1}{x^2}}{\frac{e^{ax}(a^2x^2-2ax+2)}{a^3x} + \frac{c_1}{x}}\right\}\right\}$$

✓ **Maple** : cpu = 0.065 (sec), leaf count = 52

$$\left\{y(x) = \frac{-(ax-1)(a^2x^2+2)e^{ax} + \_C1}{((a^2x^2-2ax+2)e^{ax} + \_C1)x}\right\}$$

**2.143      ODE No. 143**

---

$$x^2 \left( ay(x)^2 + y'(x) \right) - b = 0$$

✓ **Mathematica** : cpu = 0.089394 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{4ab+1} \left( \frac{2c_1}{x\sqrt{4ab+1}+c_1} - 1 \right) - 1}{2ax} \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 41

$$\left\{ y(x) = \frac{1}{2ax} \left( -\tanh \left( \frac{-\ln(x) + \frac{C1}{2} \sqrt{4ab+1}}{2} \right) \sqrt{4ab+1} + 1 \right) \right\}$$

**2.144      ODE No. 144**

---

$$x^2 \left( ay(x)^2 + y'(x) \right) + bx^\alpha + c = 0$$

✓ **Mathematica** : cpu = 0.21966 (sec), leaf count = 1787

$$\left\{ \left\{ y(x) \rightarrow \frac{a^{\frac{i\sqrt{4ac-1}\alpha+\alpha}{2\alpha^2} - \frac{\sqrt{\alpha^2-4a\alpha^2c}}{2\alpha^2}} \alpha^{-\frac{i\sqrt{4ac-1}\alpha+\alpha}{\alpha^2} + \frac{\sqrt{\alpha^2-4a\alpha^2c}}{\alpha^2}} + 1 b^{\frac{i\sqrt{4ac-1}\alpha+\alpha}{2\alpha^2} - \frac{\sqrt{\alpha^2-4a\alpha^2c}}{2\alpha^2}} \left( \frac{i\sqrt{4ac-1}\alpha+\alpha}{2\alpha^2} - \frac{\sqrt{\alpha^2-4a\alpha^2c}}{2\alpha^2} \right) (x \right. \right.$$

✓ **Maple** : cpu = 0.086 (sec), leaf count = 219

$$\left\{ y(x) = \frac{1}{2ax} \left( -2 \left( Y_{\frac{\sqrt{-4ac+1}+\alpha}{\alpha}} \left( 2 \frac{\sqrt{ab}x^{\alpha/2}}{\alpha} \right) - C1 + J_{\frac{\sqrt{-4ac+1}+\alpha}{\alpha}} \left( 2 \frac{\sqrt{ab}x^{\alpha/2}}{\alpha} \right) \right) \sqrt{ab}x^{\alpha/2} + (\sqrt{-4ac+1} + 1) \right) \right\}$$

**2.145      ODE No. 145**

---

$$-ax^2y(x)^2 + ay(x)^3 + x^2y'(x) = 0$$

✓ **Mathematica** : cpu = 0.476149 (sec), leaf count = 267

$$\text{Solve} \left[ \frac{\left( -\frac{1}{2^{2/3}a^{2/3}y(x)} - \frac{\sqrt[3]{ax}}{2^{2/3}} \right) \text{Ai} \left( \left( -\frac{\sqrt[3]{ax}}{2^{2/3}} - \frac{1}{2^{2/3}a^{2/3}y(x)} \right)^2 + \frac{1}{\sqrt[3]{2}\sqrt[3]{ax}} \right) + \text{Ai}' \left( \left( -\frac{\sqrt[3]{ax}}{2^{2/3}} - \frac{1}{2^{2/3}a^{2/3}y(x)} \right)^2 + \frac{1}{\sqrt[3]{2}\sqrt[3]{ax}} \right)}{\left( -\frac{1}{2^{2/3}a^{2/3}y(x)} - \frac{\sqrt[3]{ax}}{2^{2/3}} \right) \text{Bi} \left( \left( -\frac{\sqrt[3]{ax}}{2^{2/3}} - \frac{1}{2^{2/3}a^{2/3}y(x)} \right)^2 + \frac{1}{\sqrt[3]{2}\sqrt[3]{ax}} \right) + \text{Bi}' \left( \left( -\frac{\sqrt[3]{ax}}{2^{2/3}} - \frac{1}{2^{2/3}a^{2/3}y(x)} \right)^2 + \frac{1}{\sqrt[3]{2}\sqrt[3]{ax}} \right)} \right]$$

✓ **Maple** : cpu = 0.085 (sec), leaf count = 117

$$\left\{ y(x) = - \left( ax + (-2a)^{\frac{2}{3}} \text{RootOf} \left( \text{Bi} \left( \frac{1}{x} (-Z^2\sqrt[3]{-2ax} - 1) \frac{1}{\sqrt[3]{-2a}} \right) - C1 - Z + -Z \text{Ai} \left( \frac{1}{x} (-Z^2\sqrt[3]{-2ax} - 1) \right) \right) \right) \right\}$$

**2.146      ODE No. 146**

---

$$ay(x)^2 + x^2y'(x) + xy(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.649781 (sec), leaf count = 78

$$\text{Solve} \left[ -\frac{ia}{x} = \frac{2e^{\frac{1}{2} \left( -\frac{ia}{x} - \frac{i}{y(x)} \right)^2}}{2c_1 + \sqrt{2\pi} \text{erfi} \left( \frac{-\frac{ia}{x} - \frac{i}{y(x)}}{\sqrt{2}} \right)}, y(x) \right]$$

✓ **Maple** : cpu = 0.118 (sec), leaf count = 84

$$\left\{ \frac{1}{2} \left( a\sqrt{\pi}\sqrt{2} \text{Erf} \left( \frac{\sqrt{2}(ay(x) + x)}{2xy(x)} \right) e^{\frac{(ay(x)+x)^2}{2x^2(y(x))^2}} + 2x \right) e^{-\frac{((x+a)y(x)+x)((a-x)y(x)+x)}{2x^2(y(x))^2}} + -C1 = 0 \right\}$$



2.147 ODE No. 147

$$ax^2y(x)^3+by(x)^2+x^2y'(x)=0$$

✓ **Mathematica** : cpu = 0.648232 (sec), leaf count = 343

Solve 
$$\left[\frac{\left(\frac{b^{2/3}}{2^{2/3}\sqrt[3]{ax}}+\frac{1}{2^{2/3}\sqrt[3]{a}\sqrt[3]{by(x)}}\right)\text{Ai}\left(\left(\frac{b^{2/3}}{2^{2/3}\sqrt[3]{ax}}+\frac{1}{2^{2/3}\sqrt[3]{ay(x)}\sqrt[3]{b}}\right)^2-\frac{\sqrt[3]{ax}}{\sqrt[3]{2}b^{2/3}}\right)+\text{Ai}'\left(\left(\frac{b^{2/3}}{2^{2/3}\sqrt[3]{ax}}+\frac{1}{2^{2/3}\sqrt[3]{ay(x)}\sqrt[3]{b}}\right)^2-\frac{\sqrt[3]{ax}}{\sqrt[3]{2}b^{2/3}}\right)}{\left(\frac{b^{2/3}}{2^{2/3}\sqrt[3]{ax}}+\frac{1}{2^{2/3}\sqrt[3]{a}\sqrt[3]{by(x)}}\right)\text{Bi}\left(\left(\frac{b^{2/3}}{2^{2/3}\sqrt[3]{ax}}+\frac{1}{2^{2/3}\sqrt[3]{ay(x)}\sqrt[3]{b}}\right)^2-\frac{\sqrt[3]{ax}}{\sqrt[3]{2}b^{2/3}}\right)+\text{Bi}'\left(\left(\frac{b^{2/3}}{2^{2/3}\sqrt[3]{ax}}+\frac{1}{2^{2/3}\sqrt[3]{ay(x)}\sqrt[3]{b}}\right)^2-\frac{\sqrt[3]{ax}}{\sqrt[3]{2}b^{2/3}}\right)}\right]$$

✓ **Maple** : cpu = 0.164 (sec), leaf count = 178

$$\left\{y(x)=-\sqrt[3]{2}abx\left(\sqrt[3]{2}ab^2-2\left(a^2b^2\right)^{2/3}RootOf\left(\text{Bi}\left(-1/2\frac{a2^{2/3}x-2\_Z^2\sqrt[3]{a^2b^2}}{\sqrt[3]{a^2b^2}}\right)-C1\_Z+_Z\text{Ai}\left(-1/2\frac{a2^{2/3}x-2\_Z^2\sqrt[3]{a^2b^2}}{\sqrt[3]{a^2b^2}}\right)\right)\right\}$$

2.148 ODE No. 148

$$\left(x^2+1\right)y'(x)+xy(x)-1=0$$

✓ **Mathematica** : cpu = 0.0121163 (sec), leaf count = 30

$$\left\{\left\{y(x)\rightarrow \frac{c_1}{\sqrt{x^2+1}}+\frac{\sinh^{-1}(x)}{\sqrt{x^2+1}}\right\}\right\}$$

✓ **Maple** : cpu = 0.008 (sec), leaf count = 16

$$\left\{y(x)=\left(Arcsinh\left(x\right)+\_C1\right)\frac{1}{\sqrt{x^2+1}}\right\}$$

2.149 ODE No. 149

$$\left(x^2+1\right)y'(x)-x\left(x^2+1\right)+xy(x)=0$$

✓ **Mathematica** : cpu = 0.0121177 (sec), leaf count = 27

$$\left\{\left\{y(x)\rightarrow \frac{c_1}{\sqrt{x^2+1}}+\frac{1}{3}\left(x^2+1\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 20

$$\left\{y(x)=\frac{x^2}{3}+\frac{1}{3}+_C1\frac{1}{\sqrt{x^2+1}}\right\}$$

2.150 ODE No. 150

$$\left(x^2+1\right)y'(x)-2x^2+2xy(x)=0$$

✓ **Mathematica** : cpu = 0.0097509 (sec), leaf count = 30

$$\left\{\left\{y(x)\rightarrow \frac{c_1}{x^2+1}+\frac{2x^3}{3\left(x^2+1\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.005 (sec), leaf count = 23

$$\left\{y(x)=\frac{2x^3+3\_C1}{3x^2+3}\right\}$$

2.151 ODE No. 151

$$\left(x^2+1\right)y'(x)+\left(2xy(x)-1\right)\left(y(x)^2+1\right)=0$$

✓ **Mathematica** : cpu = 0.475523 (sec), leaf count = 203

Solve 
$$c_1=\frac{\frac{1}{2}\left(\frac{1}{\frac{ix}{x^2+1}-\frac{ix^2y(x)}{x^2+1}}+\frac{i}{x}\right)^4\sqrt[4]{1-\left(\frac{1}{\frac{ix}{x^2+1}-\frac{ix^2y(x)}{x^2+1}}+\frac{i}{x}\right)^2}{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{3}{2};\left(\frac{1}{\frac{ix}{x^2+1}-\frac{ix^2y(x)}{x^2+1}}+\frac{i}{x}\right)^2\right)+ix}{\sqrt[4]{-1+\left(\frac{1}{\frac{ix}{x^2+1}-\frac{ix^2y(x)}{x^2+1}}+\frac{i}{x}\right)^2}},y(x)$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 85

$$\left\{-C1+x\frac{1}{\sqrt[4]{\left(x^{-1}+x^2\left(\frac{y(x)x^4}{x^2+1}-\frac{x^3}{x^2+1}\right)^{-1}\right)^2}+1}}+\frac{y(x)+x}{2xy(x)-2}{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{3}{2};-\frac{(y(x)+x)^2}{(xy(x)-1)^2}\right)=0\right\}$$

### 2.152 ODE No. 152

$$(x^2+1)y'(x)-x(x^2+1)\cos^2(y(x))+x\sin(y(x))\cos(y(x))=0$$

✓ **Mathematica** : cpu = 0.363777 (sec), leaf count = 40

$$\left\{\left\{y(x)\rightarrow \tan^{-1}\left(\frac{-6c_1\sqrt{x^2+1}+x^4+2x^2+1}{3(x^2+1)}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.55 (sec), leaf count = 25

$$\left\{y(x)=\arctan\left(\frac{1}{3}\left((x^2+1)^{\frac{3}{2}}+3\_C1\right)\frac{1}{\sqrt{x^2+1}}\right)\right\}$$

### 2.153 ODE No. 153

$$a+(x^2-1)y'(x)-xy(x)=0$$

✓ **Mathematica** : cpu = 0.0123991 (sec), leaf count = 21

$$\left\{\left\{y(x)\rightarrow ax+c_1\sqrt{x^2-1}\right\}\right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 20

$$\left\{y(x)=\sqrt{1+x}\sqrt{x-1}\_C1+ax\right\}$$

### 2.154 ODE No. 154

$$(x^2-1)y'(x)+2xy(x)-\cos(x)=0$$

✓ **Mathematica** : cpu = 0.013492 (sec), leaf count = 26

$$\left\{\left\{y(x)\rightarrow \frac{c_1}{x^2-1}+\frac{\sin(x)}{x^2-1}\right\}\right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 16

$$\left\{y(x)=\frac{\sin(x)+\_C1}{x^2-1}\right\}$$

### 2.155 ODE No. 155

$$(x^2-1)y'(x)+y(x)^2-2xy(x)+1=0$$

✓ **Mathematica** : cpu = 0.12385 (sec), leaf count = 46

$$\left\{\left\{y(x)\rightarrow \frac{1}{c_1+\frac{1}{2}\log(1-x)-\frac{1}{2}\log(x+1)}-\frac{x(1-x^2)}{x^2-1}\right\}\right\}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 14

$$\left\{y(x)=x+(\_C1-Artanh(x))^{-1}\right\}$$

### 2.156 ODE No. 156

$$(x^2-1)y'(x)-y(x)(y(x)-x)=0$$

✓ **Mathematica** : cpu = 0.0775226 (sec), leaf count = 21

$$\left\{\left\{y(x)\rightarrow \frac{1}{c_1\sqrt{x^2-1}+x}\right\}\right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 20

$$\left\{y(x)=(\sqrt{x-1}\sqrt{1+x}\_C1+x)^{-1}\right\}$$

**2.157 ODE No. 157**

---

$$a \left( y(x)^2 - 2xy(x) + 1 \right) + (x^2 - 1) y'(x) = 0$$

✓ **Mathematica** : cpu = 0.184251 (sec), leaf count = 158

$$\left\{ \left\{ y(x) \rightarrow \frac{(x^2 - 1) \left( c_1 \left( ax (x^2 - 1)^{\frac{a}{2} - 1} P_{a-1}(x) + (x^2 - 1)^{\frac{a}{2} - 1} (aP_a(x) - axP_{a-1}(x)) \right) + ax (x^2 - 1)^{\frac{a}{2} - 1} Q_{a-1}(x) \right)}{a \left( c_1 (x^2 - 1)^{a/2} P_{a-1}(x) + (x^2 - 1)^{a/2} Q_{a-1}(x) \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.178 (sec), leaf count = 231

$$\left\{ y(x) = \frac{1}{4a(1+x)} \left( 8\_C1 ((a-1/2)x - a/2 + 1/2)(1+x) HeunC \left( 0, -2a+1, 0, 0, a^2 - a + 1/2, 2(1+x) \right) \right) \right\}$$

**2.158 ODE No. 158**

---

$$axy(x)^2 + (x^2 - 1) y'(x) + xy(x) = 0$$

✓ **Mathematica** : cpu = 0.139701 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow -\frac{e^{c_1}}{ae^{c_1} - \sqrt{x^2 - 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 22

$$\left\{ y(x) = (\sqrt{x-1}\sqrt{1+x} \_C1 - a)^{-1} \right\}$$

**2.159 ODE No. 159**

---

$$(x^2 - 1) y'(x) - 2xy(x) \log(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.0962497 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow e^{e^{c_1} x^2 - e^{c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.11 (sec), leaf count = 13

$$\left\{ y(x) = e^{-C1(x-1)(1+x)} \right\}$$

**2.160 ODE No. 160**

---

$$(x^2 - 4) y'(x) + (x + 2)y(x)^2 - 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.137486 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow \frac{2-x}{(x+2)(c_1 - \log(x+2))} \right\} \right\}$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 21

$$\left\{ y(x) = \frac{x-2}{(x+2)(\ln(x+2) + \_C1)} \right\}$$

**2.161 ODE No. 161**

---

$$(x^2 - 5x + 6) y'(x) + x^2 + 3xy(x) - 8y(x) = 0$$

✓ **Mathematica** : cpu = 0.0141827 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{(2-x)^2(3-x)} + \frac{\frac{x^4}{4} - \frac{2x^3}{3}}{(2-x)^2(3-x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 27

$$\left\{ y(x) = \frac{1}{(x-3)(x-2)^2} \left( -\frac{x^4}{4} + \frac{2x^3}{3} + \_C1 \right) \right\}$$

## 2.162 ODE No. 162

---

$$k(-a + y(x) + x)(-b + y(x) + x) + (x - a)(x - b)y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.425524 (sec), leaf count = 133

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \sqrt{\frac{-a^2 k^2 + 2abk^2 - b^2 k^2}{(k+1)^2}} \tan \left( \frac{(k+1) \sqrt{\frac{-a^2 k^2 + 2abk^2 - b^2 k^2}{(k+1)^2}} (\log(x-b) - \log(x-a))}{2(a-b)} + c_1 \right) - \frac{ak - b}{2(a-b)} \right\} \right\}$$

✓ **Maple** : cpu = 0.151 (sec), leaf count = 58

$$\left\{ y(x) = \frac{k \left( (a-x)(a-x)^k + \_C1 (b-x)^k (b-x) \right)}{(k+1) \left( \_C1 (b-x)^k + (a-x)^k \right)} \right\}$$

## 2.163 ODE No. 163

---

$$2a^2 x + 2x^2 y'(x) - 2y(x)^2 - xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0940973 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{-a^2} \sqrt{x} \tan \left( \frac{2\sqrt{-a^2}}{\sqrt{x}} - c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 26

$$\left\{ y(x) = i \tan \left( 1 (\_C1 \sqrt{x} - 2ia) \frac{1}{\sqrt{x}} \right) \sqrt{xa} \right\}$$

## 2.164 ODE No. 164

---

$$2a^2 x + 2x^2 y'(x) - 2y(x)^2 - 3xy(x) = 0$$

✓ **Mathematica** : cpu = 0.129144 (sec), leaf count = 131

$$\left\{ \left\{ y(x) \rightarrow -\frac{x^2 \left( c_1 \left( \frac{ae^{-\frac{2a}{\sqrt{x}}}}{x} + \frac{e^{-\frac{2a}{\sqrt{x}}}}{2\sqrt{x}} \right) - \frac{e^{\frac{2a}{\sqrt{x}}}}{4a\sqrt{x}} + \frac{e^{\frac{2a}{\sqrt{x}}}}{2x} \right)}{c_1 \sqrt{x} e^{-\frac{2a}{\sqrt{x}}} - \frac{\sqrt{x} e^{\frac{2a}{\sqrt{x}}}}{2a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.151 (sec), leaf count = 102

$$\left\{ y(x) = 1 \left( \left( -2x \_C1 \sqrt{-\frac{a^2}{x}} - x \right) \sin \left( 2 \sqrt{-\frac{a^2}{x}} \right) - x \left( \_C1 - 2 \sqrt{-\frac{a^2}{x}} \right) \cos \left( 2 \sqrt{-\frac{a^2}{x}} \right) \right) \left( 2 \cos \left( 2 \sqrt{-\frac{a^2}{x}} \right) \right) \right\}$$

## 2.165 ODE No. 165

---

$$x(2x - 1)y'(x) + y(x)^2 - (4x + 1)y(x) + 4x = 0$$

✓ **Mathematica** : cpu = 0.110846 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow \frac{(1-2x)x}{c_1 - x} + 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 17

$$\left\{ y(x) = \frac{2x^2 + \_C1}{x + \_C1} \right\}$$

2.166 ODE No. 166

---

$$2(x-1)xy'(x) + (x-1)y(x)^2 - x = 0$$

✓ **Mathematica** : cpu = 0.140976 (sec), leaf count = 71

$$\left\{ \left\{ y(x) \rightarrow \frac{2x \left( \frac{c_1(E(x)-K(x))}{\pi x} - G_{2,2}^{2,0} \left( x \left| \begin{matrix} -\frac{1}{2}, \frac{1}{2} \\ -1, 0 \end{matrix} \right) \right) \right)}{G_{2,2}^{2,0} \left( x \left| \begin{matrix} \frac{1}{2}, \frac{3}{2} \\ 0, 0 \end{matrix} \right) \right) + \frac{2c_1 E(x)}{\pi}} \right\} \right\}$$

✓ **Maple** : cpu = 0.118 (sec), leaf count = 97

$$\left\{ y(x) = \frac{x}{2x-2} \left( LegendreQ \left( -\frac{1}{2}, 1, \frac{2-x}{x} \right) - C1 - LegendreQ \left( \frac{1}{2}, 1, \frac{2-x}{x} \right) - C1 + LegendreP \left( -\frac{1}{2}, 1, \frac{2-x}{x} \right) \right) \right\}$$

2.167 ODE No. 167

---

$$3x^2y'(x) - x^2 - 3xy(x) - 7y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0746933 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow \frac{x \tan \left( \frac{1}{3} (3\sqrt{7}c_1 + \sqrt{7} \log(x)) \right)}{\sqrt{7}} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 20

$$\left\{ y(x) = \frac{x\sqrt{7}}{7} \tan \left( \frac{(\ln(x) - C1)\sqrt{7}}{3} \right) \right\}$$

2.168 ODE No. 168

---

$$3(x^2-4)y'(x) + y(x)^2 - xy(x) - 3 = 0$$

✓ **Mathematica** : cpu = 0.172918 (sec), leaf count = 234

$$\left\{ \left\{ y(x) \rightarrow \frac{3(x^2-4) \left( c_1 \left( \frac{xP_{-\frac{1}{6}}^{\frac{1}{3}}(\frac{x}{2})}{6(x^2-4)^{11/12}} + \frac{{}^{12}\sqrt{x^2-4} \left( \frac{1}{2}P_{\frac{5}{6}}^{\frac{1}{3}}(\frac{x}{2}) - \frac{5}{12}xP_{-\frac{1}{6}}^{\frac{1}{3}}(\frac{x}{2}) \right) \right)}{2(\frac{x^2}{4}-1)} \right) + \frac{xQ_{-\frac{1}{6}}^{\frac{1}{3}}(\frac{x}{2})}{6(x^2-4)^{11/12}} + \frac{{}^{12}\sqrt{x^2-4} \left( \frac{1}{2}Q_{\frac{5}{6}}^{\frac{1}{3}}(\frac{x}{2}) - \frac{5}{12}xQ_{-\frac{1}{6}}^{\frac{1}{3}}(\frac{x}{2}) \right)}{2(\frac{x^2}{4}-1)} \right)}{c_1 {}^{12}\sqrt{x^2-4}P_{-\frac{1}{6}}^{\frac{1}{3}}(\frac{x}{2}) + {}^{12}\sqrt{x^2-4}Q_{-\frac{1}{6}}^{\frac{1}{3}}(\frac{x}{2})} \right\} \right\}$$

✓ **Maple** : cpu = 0.126 (sec), leaf count = 140

$$\left\{ y(x) = -3(x+2) \left( HeunC \left( 0, 4/3, -1/3, 0, \frac{25}{36}, 4(x+2)^{-1} \right) - C1 - 1/3(-x/4 - 1/2)^{4/3} HeunC \left( 0, -4/3, -1/3, 0, \frac{25}{36}, 4(x+2)^{-1} \right) \right) \right\}$$

2.169 ODE No. 169

---

$$(ax+b)^2y'(x) + y(x)^3(ax+b) + cy(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.89509 (sec), leaf count = 149

$$\text{Solve} \left[ -\frac{c}{\sqrt{-a(ax+b)^2}} = \frac{2 \exp \left( \frac{1}{2} \left( -\frac{c}{\sqrt{-a(ax+b)^2}} - \frac{(-a(ax+b)^2)^{3/2}}{ay(x)(ax+b)^3} \right)^2 \right)}{\sqrt{2\pi} \operatorname{erfi} \left( \frac{-\frac{c}{\sqrt{-a(ax+b)^2}} - \frac{(-a(ax+b)^2)^{3/2}}{ay(x)(ax+b)^3}}{\sqrt{2}} \right) + 2c_1}, y(x) \right]$$

✓ **Maple** : cpu = 0.119 (sec), leaf count = 153

$$\left\{ \frac{1}{2} \left( \left( \sqrt{2}\sqrt{\pi} \operatorname{Erf} \left( \frac{\sqrt{2}(cy(x) + a(ax+b))}{2(ax+b)y(x)} \frac{1}{\sqrt{a}} \right) e^{\frac{(cy(x)+a(ax+b))^2}{2(y(x))^2(ax+b)^2a}} ac + 2(ax+b)a^{3/2}} \right) e^{-\frac{((ax+b+c)y(x)+a(ax+b))((-ax-b-c)y(x)+a(ax+b))}{2(y(x))^2(ax+b)^2}} \right) \right\}$$

## 2.170 ODE No. 170

---

$$-x^4 + x^3 y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.072893 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow -\frac{x^3 \left( -\frac{c_1}{x^2} + \frac{1}{x^2} - \frac{\log(x)}{x^2} \right)}{\frac{c_1}{x} + \frac{\log(x)}{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 23

$$\left\{ y(x) = \frac{x^2 (\ln(x) - \_C1 - 1)}{\ln(x) - \_C1} \right\}$$

## 2.171 ODE No. 171

---

$$x^3 y'(x) - x^2 y(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0649064 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow \frac{x^2}{c_1 x + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 15

$$\left\{ y(x) = \frac{x^2}{\_C1 x + 1} \right\}$$

## 2.172 ODE No. 172

---

$$x^4 (-y(x)^2) + x^3 y'(x) + x^2 y(x) + 20 = 0$$

✓ **Mathematica** : cpu = 0.0762753 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow -\frac{5x^4 - \frac{4c_1}{x^5}}{x \left( \frac{c_1}{x^4} + x^5 \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.205 (sec), leaf count = 26

$$\left\{ y(x) = \frac{5x^9 + 4\_C1}{(-x^9 + \_C1)x^2} \right\}$$

## 2.173 ODE No. 173

---

$$x^6 (-y(x)^2) + x^3 y'(x) - (2x - 3)x^2 y(x) + 3 = 0$$

✓ **Mathematica** : cpu = 0.093267 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{x^3 (c_1 e^{4x} + \frac{1}{4})} - \frac{3}{x^3} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 27

$$\left\{ y(x) = \frac{-3(e^x)^4 \_C1 - 3}{x^3 ((e^x)^4 \_C1 - 3)} \right\}$$

## 2.174 ODE No. 174

---

$$(x^2 + 1) x y'(x) + x^2 y(x) = 0$$

✓ **Mathematica** : cpu = 0.008127 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{\sqrt{x^2 + 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.003 (sec), leaf count = 13

$$\left\{ y(x) = \_C1 \frac{1}{\sqrt{x^2 + 1}} \right\}$$

2.175 ODE No. 175

---

$$ax^3 + (x^2 - 1)xy'(x) - (2x^2 - 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0186721 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow ax + c_1 \sqrt{1 - x^2} x \right\} \right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 20

$$\left\{ y(x) = x \left( \sqrt{x - 1} \sqrt{1 + x} C1 + a \right) \right\}$$

2.176 ODE No. 176

---

$$(x^2 - 1)xy'(x) + (x^2 - 1)y(x)^2 - x^2 = 0$$

✓ **Mathematica** : cpu = 0.186903 (sec), leaf count = 82

$$\left\{ \left\{ y(x) \rightarrow \frac{x \left( \frac{2c_1(E(x^2) - K(x^2))}{\pi x} - 2x G_{2,2}^{2,0} \left( x^2 \middle| \begin{matrix} -\frac{1}{2}, \frac{1}{2} \\ -1, 0 \end{matrix} \right) \right)}{G_{2,2}^{2,0} \left( x^2 \middle| \begin{matrix} \frac{1}{2}, \frac{3}{2} \\ 0, 0 \end{matrix} \right) + \frac{2c_1 E(x^2)}{\pi}} \right\} \right\}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 30

$$\left\{ y(x) = \frac{-C1 \operatorname{EllipticCE}(x) + \operatorname{EllipticE}(x) - \operatorname{EllipticK}(x)}{-C1 \operatorname{EllipticCE}(x) - C1 \operatorname{EllipticCK}(x) + \operatorname{EllipticE}(x)} \right\}$$

2.177 ODE No. 177

---

$$(x - 1)x^2y'(x) - (x - 2)xy(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.103326 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow -\frac{x^2}{c_1 x - c_1 - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 17

$$\left\{ y(x) = \frac{x^2}{1 + C1(x - 1)} \right\}$$

2.178 ODE No. 178

---

$$2(x^2 - 1)xy'(x) + 2(x^2 - 1)y(x)^2 - (3x^2 - 5)y(x) + x^2 - 3 = 0$$

✓ **Mathematica** : cpu = 0.149054 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{x}}{\sqrt{1 - x^2} (c_1 + 2\sqrt{x} {}_2F_1(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; x^2))} + 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.144 (sec), leaf count = 61

$$\left\{ y(x) = 1 - 2 \frac{\sqrt{x}}{\sqrt{x - 1} \sqrt{1 + x}} \left( -C1 - 2 \frac{\operatorname{EllipticF}(\sqrt{1 + x}, 1/2 \sqrt{2}) \sqrt{-x} \sqrt{2} \sqrt{1 - x}}{\sqrt{x - 1} \sqrt{x}} \right)^{-1} \right\}$$

2.179 ODE No. 179

---

$$3x(x^2 - 1)y'(x) - (x^2 + 1)y(x) + xy(x)^2 - 3x = 0$$

✓ **Mathematica** : cpu = 0.591139 (sec), leaf count = 2833

$$\left\{ \left\{ y(x) \rightarrow \frac{3(x^2 - 1) \left( c_1 \left( \frac{e^{\int_1^x \operatorname{Root}[125K[1]^8 - 164K[1]^6 + 70K[1]^4 - 20K[1]^2 + (1296K[1]^{12} - 5184K[1]^{10} + 7776K[1]^8 - 5184K[1]^6 + 1296K[1]^4) \#1^4 + 1296K[1]^2 - 1296K[1]}{1296K[1]^{12} - 5184K[1]^{10} + 7776K[1]^8 - 5184K[1]^6 + 1296K[1]^4} \right)}{1296K[1]^{12} - 5184K[1]^{10} + 7776K[1]^8 - 5184K[1]^6 + 1296K[1]^4} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.11 (sec), leaf count = 112

$$\left\{ y(x) = 35 \frac{1}{\sqrt[3]{x} (8x^{2/3} {}_2F_1(5/6, 7/6; 4/3; x^2) - C1 + 8 {}_2F_1(1/2, 5/6; 2/3; x^2))} \left( -C1 \left( \frac{8x^2}{7} - \frac{16}{35} \right) {}_2F_1(5/6, 7/6; 4/3; x^2) \right) \right\}$$

## 2.180 ODE No. 180

---

$$(xy'(x) - y(x))(ax^2 + bx + c) + x^2 - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.316472 (sec), leaf count = 132

$$\left\{ \left\{ y(x) \rightarrow -\frac{x \left( \exp \left( \frac{4 \tan^{-1} \left( \frac{2ax}{\sqrt{4ac-b^2}} + \frac{b}{\sqrt{4ac-b^2}} \right) + 2c_1 \right) - 1 \right)}{\exp \left( \frac{4 \tan^{-1} \left( \frac{2ax}{\sqrt{4ac-b^2}} + \frac{b}{\sqrt{4ac-b^2}} \right) + 2c_1 \right) + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.047 (sec), leaf count = 58

$$\left\{ y(x) = -\tanh \left( 1 \left( -C1 \sqrt{4ac-b^2} + 2 \arctan \left( \frac{2ax+b}{\sqrt{4ac-b^2}} \right) \right) \frac{1}{\sqrt{4ac-b^2}} \right) x \right\}$$

## 2.181 ODE No. 181

---

$$a + x^4 (y'(x) + y(x)^2) = 0$$

✓ **Mathematica** : cpu = 0.090572 (sec), leaf count = 347

$$\left\{ \left\{ y(x) \rightarrow -\frac{\frac{i\sqrt{\frac{2}{\pi}}c_1 \sinh\left(\frac{\sqrt{-a}}{x}\right)}{\sqrt{-\frac{i\sqrt{-a}}{x}}} + \frac{i\sqrt{-a} \left( -\frac{\sqrt{\frac{2}{\pi}}c_1 \cosh\left(\frac{\sqrt{-a}}{x}\right)}{\sqrt{-\frac{i\sqrt{-a}}{x}}} + \frac{\sqrt{\frac{2}{\pi}}c_1 \left( -\frac{\sqrt{-ax} \sinh\left(\frac{\sqrt{-a}}{x}\right)}{a} - \cosh\left(\frac{\sqrt{-a}}{x}\right) \right)}{\sqrt{-\frac{i\sqrt{-a}}{x}}} \right)}{x} - \frac{2\sqrt{\frac{2}{\pi}} \left( i \sinh\left(\frac{\sqrt{-a}}{x}\right) + \frac{i\sqrt{-ax}}{\sqrt{-\frac{i\sqrt{-a}}{x}}} \right)}{\sqrt{-\frac{i\sqrt{-a}}{x}}} \right)}{2x \left( \frac{\sqrt{\frac{2}{\pi}} \cosh\left(\frac{\sqrt{-a}}{x}\right)}{\sqrt{-\frac{i\sqrt{-a}}{x}}} - \frac{i\sqrt{\frac{2}{\pi}}c_1 \sinh\left(\frac{\sqrt{-a}}{x}\right)}{\sqrt{-\frac{i\sqrt{-a}}{x}}} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 28

$$\left\{ y(x) = \frac{1}{x^2} \left( -\tan \left( \frac{-C1 x - 1}{x} \sqrt{a} \right) \sqrt{a} + x \right) \right\}$$

## 2.182 ODE No. 182

---

$$(x^3 - 1)xy'(x) + x^2 - 2xy(x)^2 + y(x) = 0$$

✓ **Mathematica** : cpu = 0.20425 (sec), leaf count = 96

$$\left\{ \left\{ y(x) \rightarrow -\frac{(x^3 - 1) \left( \frac{2c_1 x^2}{(1-x^3)^{5/3}} + \frac{x}{(1-x^3)^{2/3}} + \frac{x^4}{(1-x^3)^{5/3}} \right)}{2 \left( \frac{c_1}{(1-x^3)^{2/3}} + \frac{x^2}{2(1-x^3)^{2/3}} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.102 (sec), leaf count = 18

$$\left\{ y(x) = \frac{x(x + -C1)}{-C1 x^2 + 1} \right\}$$

## 2.183 ODE No. 183

---

$$(2x^4 - x)y'(x) - 2(x^3 - 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0143799 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 x^2}{\sqrt[3]{1-2x^3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 18

$$\left\{ y(x) = -C1 x^2 \frac{1}{\sqrt[3]{2x^3 - 1}} \right\}$$



2.184 ODE No. 184

---

$$\left(y'(x)+y(x)^2\right)\left(ax^2+bx+c\right)^2+A=0$$

✓ **Mathematica** : cpu = 1.1501 (sec), leaf count = 704

$$\left\{\left\{\begin{array}{l}c_1\left(\frac{2a\sqrt{1-\frac{4A}{b^2-4ac}}\sqrt{x(ax+b)+c}\exp\left(\frac{\sqrt{4ac-b^2}\sqrt{1-\frac{4A}{b^2-4ac}}\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(\frac{(2ax+b)^2}{4ac-b^2}+1\right)}+\frac{(2ax+b)\exp\left(\frac{\sqrt{4ac-b^2}\sqrt{1-\frac{4A}{b^2-4ac}}\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{b^2-4ac}}\right)}{2\sqrt{x(ax+b)+c}}\right.\\ \left. c_1\sqrt{x(ax+b)+c}\left(-\exp\left(\frac{\sqrt{4ac-b^2}\sqrt{1-\frac{4A}{b^2-4ac}}\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{b^2-4ac}}\right)\right.\right.\end{array}\right.\\ \left. y(x)\rightarrow-\frac{\hspace{10cm}}{\hspace{10cm}}\right\}$$

✓ **Maple** : cpu = 0.302 (sec), leaf count = 493

$$\left\{y(x)=2\frac{a}{\sqrt{-4ac+b^2}\left(2ax+b+i\sqrt{4ac-b^2}\right)\left(i\sqrt{4ac-b^2}-2ax-b\right)}\left(\left(i\sqrt{\frac{-4ac+b^2-4A}{a^2}}a\sqrt{4ac-b^2}\right.\right.\right.$$

2.185 ODE No. 185

---

$$x^7y'(x)+5x^3y(x)^2+2\left(x^2+1\right)y(x)^3=0$$

✓ **Mathematica** : cpu = 0.371992 (sec), leaf count = 123

$$\text{Solve}\left[c_1=\frac{\frac{1}{2}\sqrt[4]{1-\left(\frac{ix^2}{y(x)}+\frac{i}{x}\right)^2}\left(\frac{ix^2}{y(x)}+\frac{i}{x}\right){}_2F_1\left(\frac{1}{2},\frac{5}{4},\frac{3}{2};\left(\frac{ix^2}{y(x)}+\frac{i}{x}\right)^2\right)+ix}{\sqrt[4]{-1+\left(\frac{ix^2}{y(x)}+\frac{i}{x}\right)^2}},y(x)\right]$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 63

$$\left\{-C1+x\frac{1}{\sqrt[4]{\left(x^{-1}+\frac{x^2}{y(x)}\right)^2+1}}+\frac{x^3+y(x)}{2xy(x)}{}_2F_1\left(\frac{1}{2},\frac{5}{4},\frac{3}{2};-\frac{\left(x^3+y(x)\right)^2}{x^2\left(y(x)\right)^2}\right)=0\right\}$$

2.186 ODE No. 186

---

$$-(n-1)x^{n-1}y(x)+x^{2n-2}+x^ny'(x)+y(x)^2=0$$

✓ **Mathematica** : cpu = 0.209146 (sec), leaf count = 19

$$\left\{\left\{y(x)\rightarrow x^{n-1}\tan(c_1-\log(x))\right\}\right\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 17

$$\left\{y(x)=\tan\left(-\ln(x)+\_C1\right)x^{n-1}\right\}$$

2.187 ODE No. 187

---

$$-ay(x)^2-bx^{2n-2}+x^ny'(x)=0$$

✓ **Mathematica** : cpu = 0.199822 (sec), leaf count = 328

$$\left\{\left\{y(x)\rightarrow-\frac{x^n\left(\frac{1}{2}\sqrt{a}\sqrt{b}c_1\left(-\frac{n-1}{\sqrt{a}\sqrt{b}}-\sqrt{\frac{(n-1)^2}{ab}}-4\right)x^{\frac{1}{2}\sqrt{a}\sqrt{b}\left(-\frac{n-1}{\sqrt{a}\sqrt{b}}-\sqrt{\frac{(n-1)^2}{ab}}-4\right)^{-1}}+\frac{1}{2}\sqrt{a}\sqrt{b}\left(\sqrt{\frac{(n-1)^2}{ab}}-4-\frac{n-1}{\sqrt{a}\sqrt{b}}\right)\right)}{a\left(c_1x^{\frac{1}{2}\sqrt{a}\sqrt{b}\left(-\frac{n-1}{\sqrt{a}\sqrt{b}}-\sqrt{\frac{(n-1)^2}{ab}}-4\right)}+x^{\frac{1}{2}\sqrt{a}\sqrt{b}\left(\sqrt{\frac{(n-1)^2}{ab}}-4-\frac{n-1}{\sqrt{a}\sqrt{b}}\right)}\right)}\right.\right.$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 60

$$\left\{y(x)=\frac{x^{n-1}}{2a}\left(-\sqrt{4ab-n^2+2n-1}\tan\left(\frac{-\ln(x)+\_C1}{2}\sqrt{4ab-n^2+2n-1}\right)+n-1\right)\right\}$$

2.188 ODE No. 188

---

$$-ay(x)^3 - bnx^3 + x^{2n+1}y'(x) = 0$$

✗ **Mathematica** : cpu = 14.0019 (sec), leaf count = 0 , could not solve

DSolve[-(b\*n\*x^3) - a\*y[x]^3 + x^(1 + 2\*n)\*Derivative[1][y][x] == 0, y[x], x]

✓ **Maple** : cpu = 0.018 (sec), leaf count = 32

$$\left\{y(x) = RootOf\left(-\ln(x) + \_C1 + \int^{-Z} (a\_a^3 - n\_a + b)^{-1} d\_a\right) x^n\right\}$$

2.189 ODE No. 189

---

$$-ay(x)^n - bx^{(m+1)n} + x^{m(n-1)+n}y'(x) = 0$$

✓ **Mathematica** : cpu = 0.38383 (sec), leaf count = 91

$$\text{Solve}\left[\int_1^{\left(\frac{ax^{-(m+1)n}}{b}\right)^{\frac{1}{n}}y(x)} \frac{1}{K[1]^n - \left(\frac{b^{1-n}(m+1)^n}{a}\right)^{\frac{1}{n}}K[1] + 1} dK[1] = bx^{m+1}\log(x) \left(\frac{ax^{-(m+1)n}}{b}\right)^{\frac{1}{n}} + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.271 (sec), leaf count = 60

$$\left\{\int_{-b}^{y(x)} -\frac{x^{mn}x^n}{(bx^mx - (m+1)\_a)x^nx^{mn} + a\_a^nx^mx} d\_a + \ln(x) - \_C1 = 0\right\}$$

2.190 ODE No. 190

---

$$\sqrt{x^2 - 1}y'(x) - \sqrt{y(x)^2 - 1} = 0$$

✓ **Mathematica** : cpu = 0.121696 (sec), leaf count = 173

$$\left\{\left\{y(x) \rightarrow -\frac{1}{2}e^{-c_1}\sqrt{2e^{4c_1}x^2 + 2e^{4c_1}\sqrt{(x-1)(x+1)}x + 2e^{2c_1} - e^{4c_1} + 2x^2 - 2\sqrt{(x-1)(x+1)}x - 1}\right\}, \left\{y(x) \rightarrow \frac{1}{2}e^{-c_1}\sqrt{2e^{4c_1}x^2 + 2e^{4c_1}\sqrt{(x-1)(x+1)}x + 2e^{2c_1} - e^{4c_1} + 2x^2 - 2\sqrt{(x-1)(x+1)}x - 1}\right\}\right\}$$

✓ **Maple** : cpu = 0.006 (sec), leaf count = 29

$$\left\{\ln\left(x + \sqrt{x^2 - 1}\right) - \ln\left(y(x) + \sqrt{y(x)^2 - 1}\right) + \_C1 = 0\right\}$$

2.191 ODE No. 191

---

$$\sqrt{1 - x^2}y'(x) - y(x)\sqrt{y(x)^2 - 1} = 0$$

✓ **Mathematica** : cpu = 0.114155 (sec), leaf count = 39

$$\left\{\left\{y(x) \rightarrow -\sqrt{\tan^2(c_1 + \sin^{-1}(x)) + 1}\right\}, \left\{y(x) \rightarrow \sqrt{\tan^2(c_1 + \sin^{-1}(x)) + 1}\right\}\right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 16

$$\left\{\arcsin(x) + \arctan\left(\frac{1}{\sqrt{y(x)^2 - 1}}\right) + \_C1 = 0\right\}$$

2.192 ODE No. 192

---

$$\sqrt{a^2 + x^2}y'(x) - \sqrt{a^2 + x^2} + y(x) + x = 0$$

✓ **Mathematica** : cpu = 0.691221 (sec), leaf count = 168

$$\left\{\left\{y(x) \rightarrow \frac{\sqrt{1 - \frac{x}{\sqrt{a^2 + x^2}}}\int_1^x \frac{\sqrt{\frac{K[1]}{\sqrt{a^2 + K[1]^2}} + 1}(\sqrt{a^2 + K[1]^2} - K[1])}{\sqrt{a^2 + K[1]^2}\sqrt{1 - \frac{K[1]}{\sqrt{a^2 + K[1]^2}}}} dK[1]}{\sqrt{\frac{x}{\sqrt{a^2 + x^2}} + 1}} + \frac{c_1\sqrt{1 - \frac{x}{\sqrt{a^2 + x^2}}}}{\sqrt{\frac{x}{\sqrt{a^2 + x^2}} + 1}}\right\}\right\}$$

✓ **Maple** : cpu = 0.015 (sec), leaf count = 36

$$\left\{ y(x) = 1 \left( a^2 \ln \left( x + \sqrt{a^2 + x^2} \right) + \_C1 \right) \left( x + \sqrt{a^2 + x^2} \right)^{-1} \right\}$$

## 2.193 ODE No. 193

---

$$-ax(\log(x) + 1) + x \log(x)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.013579 (sec), leaf count = 16

$$\left\{ \left\{ y(x) \rightarrow ax + \frac{c_1}{\log(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.006 (sec), leaf count = 14

$$\left\{ y(x) = ax + \frac{\_C1}{\ln(x)} \right\}$$

## 2.194 ODE No. 194

---

$$x \log(x)y'(x) - y(x) (2 \log^2(x) + 1) - y(x)^2 \log(x) - \log^3(x) = 0$$

✓ **Mathematica** : cpu = 0.165834 (sec), leaf count = 98

$$\left\{ \left\{ y(x) \rightarrow - \frac{x \left( \frac{c_1 e^{\frac{\log^2(x)}{2}} \log(x)}{x} + \frac{e^{\frac{\log^2(x)}{2}} \log(x)}{x} + \frac{e^{\frac{\log^2(x)}{2}} \log^3(x)}{2x} \right)}{c_1 e^{\frac{\log^2(x)}{2}} + \frac{1}{2} e^{\frac{\log^2(x)}{2}} \log^2(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 23

$$\left\{ y(x) = - \frac{\ln(x) \left( (\ln(x))^2 + \_C1 + 2 \right)}{(\ln(x))^2 + \_C1} \right\}$$

## 2.195 ODE No. 195

---

$$\sin(x)y'(x) + y(x)^2 (-\sin^2(x)) + y(x)(\cos(x) - 3\sin(x)) + 4 = 0$$

✓ **Mathematica** : cpu = 0.151458 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow \frac{\csc(x)}{c_1 e^{5x} + \frac{1}{5}} - 4 \csc(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 28

$$\left\{ y(x) = \frac{-4 (e^x)^5 \_C1 - 4}{\sin(x) \left( (e^x)^5 \_C1 - 4 \right)} \right\}$$

## 2.196 ODE No. 196

---

$$\cos(x)y'(x) + y(x) + (\sin(x) + 1) \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.20444 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-2 \tanh^{-1}(\tan(\frac{x}{2}))} + e^{-2 \tanh^{-1}(\tan(\frac{x}{2}))} \left( \sin(x) + 4 \log \left( \cos \left( \frac{x}{2} \right) - \sin \left( \frac{x}{2} \right) \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.103 (sec), leaf count = 29

$$\left\{ y(x) = \frac{\sin(x) + 2 \ln(\cos(x)) - 2 \ln(\sec(x) + \tan(x)) + \_C1}{\sec(x) + \tan(x)} \right\}$$

**2.197      ODE No. 197**

---

$$\cos(x)y'(x) - y(x)^4 - y(x)\sin(x) = 0$$

✓ **Mathematica** : cpu = 0.107937 (sec), leaf count = 98

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{\sqrt[3]{c_1 \cos^3(x) - \sin(x) - 2 \sin(x) \cos^2(x)}} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt[3]{-1}}{\sqrt[3]{c_1 \cos^3(x) - \sin(x) - 2 \sin(x) \cos^2(x)}} \right\}, \left\{ y \right.$$

✓ **Maple** : cpu = 0.079 (sec), leaf count = 237

$$\left\{ y(x) = \frac{1}{\_C1 (\sin(x))^4 + 2 \cos(x) (\sin(x))^3 - 2 \_C1 (\sin(x))^2 - 3 \sin(x) \cos(x) + \_C1} \sqrt[3]{\cos(x) (\_C1 (\sin(x))^4 + 2 \cos(x) (\sin(x))^3 - 2 \_C1 (\sin(x))^2 - 3 \sin(x) \cos(x) + \_C1)} \right\}$$

**2.198      ODE No. 198**

---

$$\sin(x)\cos(x)y'(x) - y(x) - \sin^3(x) = 0$$

✓ **Mathematica** : cpu = 0.0205003 (sec), leaf count = 15

$$\{\{y(x) \rightarrow c_1 \tan(x) - \sin(x)\}\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 13

$$\{y(x) = \tan(x) (-\cos(x) + \_C1)\}$$

**2.199      ODE No. 199**

---

$$\sin(2x)y'(x) + \sin(2y(x)) = 0$$

✓ **Mathematica** : cpu = 0.196318 (sec), leaf count = 15

$$\{\{y(x) \rightarrow \cot^{-1}(e^{-2c_1} \tan(x))\}\}$$

✓ **Maple** : cpu = 0.142 (sec), leaf count = 102

$$\left\{ y(x) = \frac{1}{2} \arctan \left( 2 \frac{\_C1 (2 \sin(2x) + \sin(4x))}{-\_C1^2 \cos(4x) + \_C1^2 + 4 \cos(2x) + \cos(4x) + 3}, \frac{\_C1^2 \cos(4x) - \_C1^2 + 4 \cos(2x)}{-\_C1^2 \cos(4x) - \_C1^2 - 4 \cos(2x)} \right) \right\}$$

**2.200      ODE No. 200**

---

$$Ax(a\sin^2(x) + c) + y'(x)(a\sin^2(x) + b) + ay(x)\sin(2x) = 0$$

✓ **Mathematica** : cpu = 0.106725 (sec), leaf count = 77

$$\left\{ \left\{ y(x) \rightarrow \frac{\frac{1}{2}aAx^2 - \frac{1}{2}aAx\sin(2x) - \frac{1}{4}aA\cos(2x) + Acx^2}{a\cos(2x) - a - 2b} + \frac{c_1}{a\cos(2x) - a - 2b} \right\} \right\}$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 53

$$\left\{ y(x) = \frac{-A\cos(2x)a - 2A\sin(2x)ax + 2x^2(a + 2c)A - 8\_C1}{4a\cos(2x) - 4a - 8b} \right\}$$

**2.201      ODE No. 201**

---

$$-y(x)f'(x) + 2f(x)y'(x) + 2f(x)y(x)^2 - 2f(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.13359 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow i\sqrt{f(x)} \tan \left( c_1 + i \int_1^x -\sqrt{f(K[1])} dK[1] \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 23

$$\left\{ y(x) = i \tan \left( -i \int \sqrt{f(x)} dx + \_C1 \right) \sqrt{f(x)} \right\}$$

2.202 ODE No. 202

---

$$f(x)y'(x) + g(x)\operatorname{tg}(y(x)) + h(x) = 0$$

✗ **Mathematica** : cpu = 20.3572 (sec), leaf count = 0 , could not solve  
DSolve[h[x] + g[x]\*tg[y[x]] + f[x]\*Derivative[1][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(f(x)\*diff(y(x),x)+g(x)\*tg(y(x))+h(x) = 0,y(x))

2.203 ODE No. 203

---

$$x^3 + y(x)y'(x) + y(x) = 0$$

✗ **Mathematica** : cpu = 2.01992 (sec), leaf count = 0 , could not solve  
DSolve[x^3 + y[x] + y[x]\*Derivative[1][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(y(x)\*diff(y(x),x)+y(x)+x^3 = 0,y(x))

2.204 ODE No. 204

---

$$ay(x) + y(x)y'(x) + x = 0$$

✓ **Mathematica** : cpu = 0.0711729 (sec), leaf count = 70

$$\operatorname{Solve}\left[\frac{1}{2}\log\left(\frac{ay(x)}{x} + \frac{y(x)^2}{x^2} + 1\right) - \frac{a \tan^{-1}\left(\frac{a + \frac{2y(x)}{x}}{\sqrt{4-a^2}}\right)}{\sqrt{4-a^2}} = c_1 - \log(x), y(x)\right]$$

✓ **Maple** : cpu = 0.228 (sec), leaf count = 92

$$\left\{y(x) = \operatorname{RootOf}\left(-Z^2 - e^{\operatorname{RootOf}\left(x^2\left(\left(\tanh\left(\frac{2-C1+Z+2\ln(x)}{2a}\right)\sqrt{(a-2)(a+2)}\right)\right)^2a^2-4\left(\tanh\left(1/2\frac{\sqrt{(a-2)(a+2)}(2-C1+Z+2\ln(x))}{a}\right)\right)}\right)}\right\}$$

2.205 ODE No. 205

---

$$\frac{1}{4}(a^2 - 1)x + ay(x) + bx^n + y(x)y'(x) = 0$$

✗ **Mathematica** : cpu = 15.5764 (sec), leaf count = 0 , could not solve  
DSolve[((-1 + a^2)\*x)/4 + b\*x^n + a\*y[x] + y[x]\*Derivative[1][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(y(x)\*diff(y(x),x)+a\*y(x)+1/4\*(a^2-1)\*x+b\*x^n = 0,y(x))

2.206 ODE No. 206

---

$$ay(x) - 2a + be^x + y(x)y'(x) = 0$$

✗ **Mathematica** : cpu = 25.9618 (sec), leaf count = 0 , could not solve  
DSolve[-2\*a + b\*E^x + a\*y[x] + y[x]\*Derivative[1][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(y(x)\*diff(y(x),x)+a\*y(x)+b\*exp(x)-2\*a = 0,y(x))

2.207 ODE No. 207

---

$$y(x)y'(x) + y(x)^2 + 4x(x + 1) = 0$$

✓ **Mathematica** : cpu = 0.0819105 (sec), leaf count = 47

$$\left\{\left\{y(x) \rightarrow -\sqrt{c_1 e^{-2x} - 4x^2}\right\}, \left\{y(x) \rightarrow \sqrt{c_1 e^{-2x} - 4x^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 37

$$\left\{y(x) = \sqrt{e^{-2x} C1 - 4x^2}, y(x) = -\sqrt{e^{-2x} C1 - 4x^2}\right\}$$

**2.208 ODE No. 208**

---

$$ay(x)^2 - b \cos(c + x) + y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.192063 (sec), leaf count = 118

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{4a^2c_1e^{-2ax} + 4ab \cos(c + x) + c_1e^{-2ax} + 2b \sin(c + x)}}{\sqrt{4a^2 + 1}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{4a^2c_1e^{-2ax} + 4ab \cos(c + x)}}{\sqrt{4a^2 + 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 106

$$\left\{ y(x) = \frac{1}{4a^2 + 1} \sqrt{16\_C1 (a^2 + 1/4)^2 e^{-2ax} + 16 (\cos(x + c) a + 1/2 \sin(x + c)) (a^2 + 1/4) b}, y(x) = -\frac{1}{4a^2 + 1} \sqrt{16\_C1 (a^2 + 1/4)^2 e^{-2ax} + 16 (\cos(x + c) a + 1/2 \sin(x + c)) (a^2 + 1/4) b} \right\}$$

**2.209 ODE No. 209**

---

$$y(x)y'(x) - \sqrt{ay(x)^2 + b} = 0$$

✓ **Mathematica** : cpu = 0.0968506 (sec), leaf count = 84

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{2a^2c_1x + a^2c_1^2 + a^2x^2 - b}}{\sqrt{a}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{2a^2c_1x + a^2c_1^2 + a^2x^2 - b}}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 21

$$\left\{ x - \frac{1}{a} \sqrt{a (y(x))^2 + b} +\_C1 = 0 \right\}$$

**2.210 ODE No. 210**

---

$$y(x)y'(x) + xy(x)^2 - 4x = 0$$

✓ **Mathematica** : cpu = 0.0767036 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{e^{2c_1-x^2} + 4} \right\}, \left\{ y(x) \rightarrow \sqrt{e^{2c_1-x^2} + 4} \right\} \right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt{e^{-x^2} \_C1 + 4}, y(x) = -\sqrt{e^{-x^2} \_C1 + 4} \right\}$$

**2.211 ODE No. 211**

---

$$y(x)y'(x) - xe^{\frac{x}{y(x)}} = 0$$

✓ **Mathematica** : cpu = 0.20608 (sec), leaf count = 41

$$\text{Solve} \left[ \int_1^{\frac{y(x)}{x}} \frac{K[1]}{K[1]^2 - e^{\frac{1}{K[1]}}} dK[1] = c_1 - \log(x), y(x) \right]$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 31

$$\left\{ y(x) = \text{RootOf} \left( -\int^{-Z} \frac{-a}{-a^2 + e^{-a^{-1}}} d\_a + \ln(x) + \_C1 \right) x \right\}$$

**2.212 ODE No. 212**

---

$$g(x)f(x^2 + y(x)^2) + y(x)y'(x) + x = 0$$

✓ **Mathematica** : cpu = 0.363581 (sec), leaf count = 95

$$\text{Solve} \left[ \int_1^{y(x)} \left( \frac{K[2]}{f(x^2 + K[2]^2)} - \int_1^x -\frac{2K[1]K[2]f'(K[1]^2 + K[2]^2)}{f(K[1]^2 + K[2]^2)^2} dK[1] \right) dK[2] + \int_1^x \left( g(K[1]) + \frac{K[1]}{f(K[1]^2 + K[2]^2)} \right) dx = 0 \right]$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 30

$$\left\{ \int_{\_b}^{y(x)} \frac{-a}{f(-a^2 + x^2)} d\_a + \int g(x) dx - \_C1 = 0 \right\}$$

## 2.213 ODE No. 213

---

$$(y(x) + 1)y'(x) - y(x) - x = 0$$

✓ **Mathematica** : cpu = 0.0969976 (sec), leaf count = 71

$$\text{Solve} \left[ \frac{1}{2} \log \left( \frac{x^2 - y(x)^2 + (x - 3)y(x) - x - 1}{(x - 1)^2} \right) + \log(1 - x) = c_1 + \frac{\tanh^{-1} \left( \frac{y(x) + 2x - 1}{\sqrt{5}(y(x) + 1)} \right)}{\sqrt{5}}, y(x) \right]$$

✓ **Maple** : cpu = 0.552 (sec), leaf count = 66

$$\left\{ -\frac{1}{2} \ln \left( \frac{(y(x))^2 + (-x + 3)y(x) - x^2 + x + 1}{(x - 1)^2} \right) - \frac{\sqrt{5}}{5} \text{Artanh} \left( \frac{(-2y(x) - 3 + x)\sqrt{5}}{5x - 5} \right) - \ln(x - 1) - C1 = \right.$$

## 2.214 ODE No. 214

---

$$(y(x) + x - 1)y'(x) - y(x) + 2x + 3 = 0$$

✓ **Mathematica** : cpu = 0.10375 (sec), leaf count = 78

$$\text{Solve} \left[ 2\sqrt{2} \tan^{-1} \left( \frac{-y(x) + 2x + 3}{\sqrt{2}(y(x) + x - 1)} \right) = 3c_1 + 2 \log \left( \frac{6x^2 + 3y(x)^2 - 10y(x) + 8x + 11}{(3x + 2)^2} \right) + 4 \log(3x + 2), y(x) \right]$$

✓ **Maple** : cpu = 0.139 (sec), leaf count = 48

$$\left\{ y(x) = \frac{5}{3} + \frac{(-3x - 2)\sqrt{2} \tan \left( \text{RootOf} \left( \sqrt{2} \ln \left( 2 \left( (\tan(\_Z))^2 + 1 \right) (3x + 2)^2 \right) + 2\sqrt{2}C1 - 2\_Z \right) \right)}{3} \right\}$$

## 2.215 ODE No. 215

---

$$(y(x) + 2x - 2)y'(x) - y(x) + x + 1 = 0$$

✓ **Mathematica** : cpu = 0.106376 (sec), leaf count = 80

$$\text{Solve} \left[ 6\sqrt{3} \tan^{-1} \left( \frac{4 - 3y(x)}{\sqrt{3}(y(x) + 2x - 2)} \right) = 2c_1 + 3 \log \left( \frac{3x^2 + 3y(x)^2 + 3(x - 3)y(x) - 6x + 7}{(1 - 3x)^2} \right) + 6 \log(3x - 1), \right.$$

✓ **Maple** : cpu = 0.162 (sec), leaf count = 51

$$\left\{ y(x) = \frac{3}{2} - \frac{x}{2} + \frac{\sqrt{3}(3x - 1)}{6} \tan \left( \text{RootOf} \left( \sqrt{3} \ln \left( \frac{(3(\tan(\_Z))^2 + 3)(3x - 1)^2}{4} \right) + 2\sqrt{3}C1 + 6\_Z \right) \right) \right\}$$

## 2.216 ODE No. 216

---

$$(y(x) - 2x + 1)y'(x) + y(x) + x = 0$$

✓ **Mathematica** : cpu = 0.0934687 (sec), leaf count = 82

$$\text{Solve} \left[ 6\sqrt{3} \tan^{-1} \left( \frac{3y(x) + 1}{\sqrt{3}(-y(x) + 2x - 1)} \right) = 2c_1 + 3 \log \left( \frac{3x^2 + 3y(x)^2 - 3(x - 1)y(x) - 3x + 1}{(1 - 3x)^2} \right) + 6 \log(3x - 1), \right.$$

✓ **Maple** : cpu = 0.153 (sec), leaf count = 51

$$\left\{ y(x) = \frac{(-3x + 1)\sqrt{3}}{6} \tan \left( \text{RootOf} \left( \sqrt{3} \ln \left( \frac{(3(\tan(\_Z))^2 + 3)(3x - 1)^2}{4} \right) + 2\sqrt{3}C1 + 6\_Z \right) \right) \right\} + \frac{x}{2} -$$

2.217 ODE No. 217

---

$$(y(x) - x^2) y'(x) - x = 0$$

✓ **Mathematica** : cpu = 0.0159122 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( W \left( -e^{c_1 - 2x^2 - 1} \right) + 1 \right) + x^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 23

$$\left\{ y(x) = x^2 + \frac{\text{lambertW} \left( -4 \_C1 e^{-2x^2 - 1} \right)}{2} + \frac{1}{2} \right\}$$

2.218 ODE No. 218

---

$$(y(x) - x^2) y'(x) + 4xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0673085 (sec), leaf count = 257

$$\left\{ \left\{ y(x) \rightarrow x^2 + \frac{1}{-\frac{1}{2x^2} - \frac{\frac{1}{2} - \frac{i}{2}}{\sqrt{2x^2} \sqrt{x^2 \sinh\left(\frac{2c_1}{9}\right) + x^2 \cosh\left(\frac{2c_1}{9}\right) - i}}} \right\} \right\}, \left\{ y(x) \rightarrow x^2 + \frac{1}{-\frac{1}{2x^2} + \frac{\frac{1}{2} - \frac{i}{2}}{\sqrt{2x^2} \sqrt{x^2 \sinh\left(\frac{2c_1}{9}\right) + x^2 \cosh\left(\frac{2c_1}{9}\right)}}} \right\}$$

✓ **Maple** : cpu = 0.162 (sec), leaf count = 57

$$\left\{ y(x) = -\frac{C1}{2} \sqrt{-C1^2 - 4x^2} + \frac{C1^2}{2} - x^2, y(x) = \frac{C1}{2} \sqrt{-C1^2 - 4x^2} + \frac{C1^2}{2} - x^2 \right\}$$

2.219 ODE No. 219

---

$$-f_0(x) - f_1(x)y(x) - f_2(x)y(x)^2 + (g(x) + y(x))y'(x) = 0$$

✗ **Mathematica** : cpu = 300.553 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve((y(x)+g(x))\*diff(y(x),x)-f2(x)\*y(x)^2-f1(x)\*y(x)-f0(x) = 0,y(x))

2.220 ODE No. 220

---

$$-x^3 + 2y(x)y'(x) - xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0691999 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{c_1 e^{\frac{x^2}{2}} - x^2 - 2} \right\}, \left\{ y(x) \rightarrow \sqrt{c_1 e^{\frac{x^2}{2}} - x^2 - 2} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 43

$$\left\{ y(x) = \sqrt{e^{\frac{x^2}{2}} \_C1 - x^2 - 2}, y(x) = -\sqrt{e^{\frac{x^2}{2}} \_C1 - x^2 - 2} \right\}$$

2.221 ODE No. 221

---

$$(2y(x) + x + 1)y'(x) - 2y(x) - x + 1 = 0$$

✓ **Mathematica** : cpu = 0.0162922 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow \frac{2}{3} \left( W \left( -e^{c_1 + \frac{9x}{4} - 1} \right) + 1 \right) + \frac{1}{2}(-x - 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 21

$$\left\{ y(x) = -\frac{x}{2} + \frac{2}{3} \text{lambertW} \left( \frac{e^{-\frac{1}{4}} \_C1}{4} e^{\frac{9x}{4}} \right) + \frac{1}{6} \right\}$$



**2.222            ODE No. 222**

---

$$(2y(x) + x + 7)y'(x) - y(x) + 2x + 4 = 0$$

✓ **Mathematica** : cpu = 0.0503561 (sec), leaf count = 65

$$\text{Solve} \left[ 5c_1 + 2 \log \left( \frac{4(x^2 + y(x)^2 + 4y(x) + 6x + 13)}{5(x+3)^2} \right) + 2 \tan^{-1} \left( \frac{y(x) - 2(x+2)}{2y(x) + x + 7} \right) + 4 \log(x+3) = 0, y(x) \right]$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 32

$$\left\{ y(x) = -2 + (-x - 3) \tan \left( \text{RootOf} \left( \ln \left( (\cos(\_Z))^{-2} \right) - \_Z + 2 \ln(x+3) + 2\_C1 \right) \right) \right\}$$

**2.223            ODE No. 223**

---

$$(2y(x) - x)y'(x) - y(x) - 2x = 0$$

✓ **Mathematica** : cpu = 0.0190492 (sec), leaf count = 55

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( x - \sqrt{5x^2 - 4e^{c_1}} \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \left( \sqrt{5x^2 - 4e^{c_1}} + x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.139 (sec), leaf count = 51

$$\left\{ y(x) = \frac{1}{2\_C1} \left( \_C1 x - \sqrt{5\_C1^2 x^2 + 4} \right), y(x) = \frac{1}{2\_C1} \left( \_C1 x + \sqrt{5\_C1^2 x^2 + 4} \right) \right\}$$

**2.224            ODE No. 224**

---

$$(2y(x) - 6x)y'(x) - y(x) + 3x + 2 = 0$$

✓ **Mathematica** : cpu = 0.0152286 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow 3x - \frac{2}{5} \left( W \left( -e^{c_1 + \frac{25x}{4} - 1} \right) + 1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.041 (sec), leaf count = 35

$$\left\{ y(x) = \frac{1}{5} e^{-\text{lambertW} \left( -\frac{e^{-1}}{2} e^{\frac{25x}{4}} e^{-\frac{25}{4} C1} \right) + \frac{25x}{4} - 1 - \frac{25}{4} C1} + 3x - \frac{2}{5} \right\}$$

**2.225            ODE No. 225**

---

$$(4y(x) + 2x + 3)y'(x) - 2y(x) - x - 1 = 0$$

✓ **Mathematica** : cpu = 0.0145789 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{8} \left( W \left( -e^{c_1 + 8x - 1} \right) + 1 \right) + \frac{1}{4}(-2x - 3) \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 20

$$\left\{ y(x) = -\frac{x}{2} + \frac{\text{lambertW} \left( e^5 (e^x)^8 \_C1 \right)}{8} - \frac{5}{8} \right\}$$

**2.226            ODE No. 226**

---

$$(4y(x) - 2x - 3)y'(x) + 2y(x) - x - 1 = 0$$

✓ **Mathematica** : cpu = 0.0147693 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{8} \left( -W \left( -e^{c_1 + 8x - 1} \right) - 1 \right) + \frac{1}{4}(2x + 3) \right\} \right\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 21

$$\left\{ y(x) = \frac{x}{2} - \frac{\text{lambertW} \left( -e^5 (e^x)^8 \_C1 \right)}{8} + \frac{5}{8} \right\}$$

2.227 ODE No. 227

---

$$(4y(x) - 3x - 5)y'(x) - 3y(x) + 7x + 2 = 0$$

✓ **Mathematica** : cpu = 0.0117344 (sec), leaf count = 107

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4}(3x + 5) - \frac{1}{2}i\sqrt{-4c_1 - 2\left(-\frac{7x^2}{2} - 2x\right) - \frac{1}{4}(3x + 5)^2} \right\}, \left\{ y(x) \rightarrow \frac{1}{4}(3x + 5) + \frac{1}{2}i\sqrt{-4c_1 - 2\left(-\frac{7x^2}{2} - 2x\right) - \frac{1}{4}(3x + 5)^2} \right\} \right.$$

✓ **Maple** : cpu = 0.15 (sec), leaf count = 33

$$\left\{ y(x) = \frac{1}{76\_C1} \left( -\sqrt{4 - 6859 \left( x - \frac{7}{19} \right)^2\_C1^2 + (57x + 95)\_C1} \right) \right\}$$

2.228 ODE No. 228

---

$$(4y(x) + 11x - 11)y'(x) - 25y(x) - 8x + 62 = 0$$

✓ **Mathematica** : cpu = 0.250611 (sec), leaf count = 3357

$$\left\{ \left\{ y(x) \rightarrow \frac{4 \left( -\frac{81(9x-1) \left( 81 \cosh\left(\frac{3c_1}{8}\right)x^2 + 81 \sinh\left(\frac{3c_1}{8}\right)x^2 - 18 \cosh\left(\frac{3c_1}{8}\right)x - 18 \sinh\left(\frac{3c_1}{8}\right)x + \cosh\left(\frac{3c_1}{8}\right) + \sinh\left(\frac{3c_1}{8}\right) - 1 \right)^3}{\sqrt{-25828032}} \right) \right\} \right.$$

✓ **Maple** : cpu = 0.336 (sec), leaf count = 271

$$\left\{ y(x) = 1 \left( (-76x + 28) \sqrt[3]{64 - 8748(9x - 1)^2\_C1 + 108\sqrt{43046721}\sqrt{(x - 1/9)^2\left(-\frac{32}{177147} + (x - 1/9)^2\_C1\right)}} \right) \right\}$$

2.229 ODE No. 229

---

$$(12y(x) - 5x - 8)y'(x) - 5y(x) + 2x + 3 = 0$$

✓ **Mathematica** : cpu = 0.01543 (sec), leaf count = 121

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{12}(5x + 8) - \frac{i\sqrt{-12c_1 - 24\left(-\frac{x^2}{12} - \frac{x}{4}\right) - \frac{1}{12}(5x + 8)^2}}{2\sqrt{3}} \right\}, \left\{ y(x) \rightarrow \frac{1}{12}(5x + 8) + \frac{i\sqrt{-12c_1 - 24\left(-\frac{x^2}{12} - \frac{x}{4}\right) - \frac{1}{12}(5x + 8)^2}}{2\sqrt{3}} \right\} \right.$$

✓ **Maple** : cpu = 0.152 (sec), leaf count = 32

$$\left\{ y(x) = \frac{1}{12\_C1} \left( -\sqrt{(x + 4)^2\_C1^2 + 24 + (5x + 8)\_C1} \right) \right\}$$

2.230 ODE No. 230

---

$$ay(x)y'(x) + by(x)^2 + f(x) = 0$$

✓ **Mathematica** : cpu = 0.157378 (sec), leaf count = 98

$$\left\{ \left\{ y(x) \rightarrow -e^{-\frac{bx}{a}} \sqrt{2 \int_1^x -\frac{e^{\frac{2bK[1]}{a}} f(K[1])}{a} dK[1] + c_1} \right\}, \left\{ y(x) \rightarrow e^{-\frac{bx}{a}} \sqrt{2 \int_1^x -\frac{e^{\frac{2bK[1]}{a}} f(K[1])}{a} dK[1] + c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 100

$$\left\{ y(x) = \frac{1}{a} \sqrt{e^{2\frac{bx}{a}} a \left( -\_C1 a - 2 \int \left( e^{\frac{bx}{a}} \right)^2 f(x) dx \right) \left( e^{2\frac{bx}{a}} \right)^{-1}}, y(x) = -\frac{1}{a} \sqrt{e^{2\frac{bx}{a}} a \left( -\_C1 a - 2 \int \left( e^{\frac{bx}{a}} \right)^2 f(x) dx \right) \left( e^{2\frac{bx}{a}} \right)^{-1}} \right\}$$

2.231 ODE No. 231

$y'(x)(ay(x)+bx+c)+\alpha y(x)+\beta x+\gamma=0$

✓ **Mathematica** : cpu = 1.91958 (sec), leaf count = 252

Solve 
$$\left[ \begin{array}{c} (\alpha-b)^2 \left( -\log \left( \frac{(ay(x)+bx+c)^2 \left( -\frac{(\alpha(bx+c)-a(\beta x+\gamma)) \left( a(\alpha-b)y(x)+a(\beta x+\gamma)+b^2(-x)-bc}{(ay(x)+bx+c)^2} + a\beta - \alpha b \right)}{(\alpha(bx+c)-a(\beta x+\gamma))^2} \right) - \frac{2 \tan^{-1} \left( \frac{\frac{2a(\beta x+\gamma)-2a(\alpha b)}{ay(x)+bx+c}}{(\alpha-b) \sqrt{\frac{4(a\beta-\alpha b)}{(\alpha-b)^2}}} \right)}{\sqrt{\frac{4(a\beta-\alpha b)}{(\alpha-b)^2}}} \right) \right. \\ \left. \frac{2(a\beta-\alpha b)}{2(a\beta-\alpha b)} \right] \end{array} \right.$$

✓ **Maple** : cpu = 0.194 (sec), leaf count = 178

$$\left\{ y(x) = \frac{1}{-a\beta + b\alpha} \left( -b\gamma + \beta c + \frac{x(a\beta - b\alpha) + a\gamma - \alpha c}{2a} \left( \sqrt{4a\beta - \alpha^2 - 2b\alpha - b^2} \tan \left( RootOf \left( \sqrt{4a\beta - \alpha^2 - 2b\alpha - b^2} \right) \right) \right) \right. \right.$$

2.232 ODE No. 232

$x^2+xy(x)y'(x)+y(x)^2=0$

✓ **Mathematica** : cpu = 0.0683742 (sec), leaf count = 56

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{2c_1-x^4}}{\sqrt{2x}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{2c_1-x^4}}{\sqrt{2x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 39

$$\left\{ y(x) = -\frac{1}{2x} \sqrt{-2x^4 + 4\_C1}, y(x) = \frac{1}{2x} \sqrt{-2x^4 + 4\_C1} \right\}$$

2.233 ODE No. 233

$ax^3\cos(x)+xy(x)y'(x)-y(x)^2=0$

✓ **Mathematica** : cpu = 0.1374 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow -x\sqrt{c_1-2a\sin(x)} \right\}, \left\{ y(x) \rightarrow x\sqrt{c_1-2a\sin(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 30

$$\left\{ y(x) = \sqrt{-2a\sin(x) + \_C1}x, y(x) = -\sqrt{-2a\sin(x) + \_C1}x \right\}$$

2.234 ODE No. 234

$x^3-2x^2+xy(x)y'(x)+xy(x)-y(x)^2=0$

✗ **Mathematica** : cpu = 25.5066 (sec), leaf count = 0 , could not solve

`DSolve[-2*x^2 + x^3 + x*y[x] - y[x]^2 + x*y[x]*Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(x*y(x)*diff(y(x),x)-y(x)^2+x*y(x)+x^3-2*x^2 = 0,y(x))`

2.235 ODE No. 235

$(a+xy(x))y'(x)+by(x)=0$

✓ **Mathematica** : cpu = 0.0556297 (sec), leaf count = 40

Solve 
$$\left[ x = c_1 e^{-\frac{y(x)}{b}} - \frac{a e^{-\frac{y(x)}{b}} \operatorname{Ei}\left(\frac{y(x)}{b}\right)}{b}, y(x) \right]$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 30

$$\left\{-C1+\left(-e^{\frac{y(x)}{b}}bx+aEi\left(1,-\frac{y(x)}{b}\right)\right)^{-1}=0\right\}$$

**2.236 ODE No. 236**

---

$$x(y(x)+4)y'(x)-y(x)^2-2y(x)-2x=0$$

✓ **Mathematica** : cpu = 0.114607 (sec), leaf count = 114

$$\left\{\left\{y(x)\rightarrow\frac{1}{x\left(\frac{1}{x^2+4x}-\frac{e^{-2\left(\frac{\log(x)}{4}+\frac{3}{4}\log(x+4)\right)}}{\sqrt{c_1-\frac{4}{x+4}}}\right)}-4\right\},\left\{y(x)\rightarrow\frac{1}{x\left(\frac{e^{-2\left(\frac{\log(x)}{4}+\frac{3}{4}\log(x+4)\right)}}{\sqrt{c_1-\frac{4}{x+4}}}+\frac{1}{x^2+4x}\right)}-4\right\}\right\}$$

✓ **Maple** : cpu = 0.047 (sec), leaf count = 141

$$\left\{y(x)=1\left(-(x+4)^{\frac{3}{2}}\sqrt{\frac{-C1(x+4)-4}{x+4}}x-16\sqrt{x}-4x^{3/2}\right)\left(-(x+4)^{\frac{3}{2}}\sqrt{\frac{-C1(x+4)-4}{x+4}}+4\sqrt{x}+x^{\frac{3}{2}}\right)\right\}$$

**2.237 ODE No. 237**

---

$$x(a+y(x))y'(x)+by(x)+cx=0$$

✗ **Mathematica** : cpu = 5.29817 (sec), leaf count = 0 , could not solve  
DSolve[c\*x + b\*y[x] + x\*(a + y[x])\*Derivative[1][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(x\*(y(x)+a)\*diff(y(x),x)+b\*y(x)+c\*x = 0,y(x))

**2.238 ODE No. 238**

---

$$(a+x(y(x)+x))y'(x)-b-y(x)(y(x)+x)=0$$

✓ **Mathematica** : cpu = 0.236927 (sec), leaf count = 192

$$\left\{\left\{y(x)\rightarrow\frac{1}{x\left(-\frac{x}{(a^2+ax^2+bx^2)^{3/2}}\sqrt{c_1-\frac{1}{(a+b)(a^2+ax^2+bx^2)}}-\frac{a}{-a^2-ax^2-bx^2}\right)}-\frac{a+x^2}{x}\right\},\left\{y(x)\rightarrow\frac{1}{x\left(\frac{1}{(a^2+ax^2+bx^2)^{3/2}}\sqrt{c_1-\frac{1}{(a+b)(a^2+ax^2+bx^2)}}-\frac{a}{-a^2-ax^2-bx^2}\right)}-\frac{a+x^2}{x}\right\}\right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 93

$$\left\{y(x)=\frac{1}{-a^2+_C1}\left(-abx-_C1x+\sqrt{-C1(a+b)(ax^2+bx^2+a^2-_C1)}\right),y(x)=\frac{1}{a^2-_C1}\left(abx+_C1x-\sqrt{C1(a+b)(ax^2+bx^2+a^2+_C1)}\right)\right\}$$

**2.239 ODE No. 239**

---

$$(xy(x)-x^2)y'(x)-2x^2-3xy(x)+y(x)^2=0$$

✓ **Mathematica** : cpu = 0.0977911 (sec), leaf count = 54

$$\left\{\left\{y(x)\rightarrow x-\frac{\sqrt{e^{2c_1}+2x^4}}{x}\right\},\left\{y(x)\rightarrow\frac{\sqrt{e^{2c_1}+2x^4}}{x}+x\right\}\right\}$$

✓ **Maple** : cpu = 0.149 (sec), leaf count = 59

$$\left\{y(x)=\frac{1}{-_C1x}\left(-C1x^2-\sqrt{2x^4-_C1^2+1}\right),y(x)=\frac{1}{-_C1x}\left(-C1x^2+\sqrt{2x^4-_C1^2+1}\right)\right\}$$

## 2.240 ODE No. 240

---

$$ax + 2xy(x)y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0955371 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{c_1 x - ax \log(x)} \right\}, \left\{ y(x) \rightarrow \sqrt{c_1 x - ax \log(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 34

$$\left\{ y(x) = \sqrt{-ax \ln(x) + \_C1 x}, y(x) = -\sqrt{-x(a \ln(x) - \_C1)} \right\}$$

## 2.241 ODE No. 241

---

$$ax^2 + 2xy(x)y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0837987 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{c_1 x - ax^2} \right\}, \left\{ y(x) \rightarrow \sqrt{c_1 x - ax^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt{-ax^2 + \_C1 x}, y(x) = -\sqrt{-ax^2 + \_C1 x} \right\}$$

## 2.242 ODE No. 242

---

$$2xy(x)y'(x) + 2y(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 0.071339 (sec), leaf count = 60

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{e^{4c_1} - x^2}}{\sqrt{2x}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{e^{4c_1} - x^2}}{\sqrt{2x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 39

$$\left\{ y(x) = -\frac{1}{2x} \sqrt{-2x^2 + 4\_C1}, y(x) = \frac{1}{2x} \sqrt{-2x^2 + 4\_C1} \right\}$$

## 2.243 ODE No. 243

---

$$x(2y(x) + x - 1)y'(x) - y(x)(y(x) + 2x + 1) = 0$$

✓ **Mathematica** : cpu = 11.1893 (sec), leaf count = 487

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt[3]{2}x}{\sqrt[3]{-27c_1^2x^2 + \sqrt{108c_1^3x^3 + (27c_1^2x - 27c_1^2x^2)^2} + 27c_1^2x}} + \frac{\sqrt[3]{-27c_1^2x^2 + \sqrt{108c_1^3x^3 + (27c_1^2x - 27c_1^2x^2)^2} + 27c_1^2x}}{3\sqrt[3]{2}c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.109 (sec), leaf count = 391

$$\left\{ y(x) = \frac{1}{80\_C1} \left( -3 \left( x \left( \sqrt{5} \sqrt{\frac{80(x-1)^2\_C1 - x}{\_C1}} + 20x - 20 \right) \_C1^2 \right)^{2/3} (1 + i\sqrt{3}) \sqrt[3]{5} + 3 \left( x (i\sqrt{3} - 1) \sqrt{5} \sqrt{\frac{80(x-1)^2\_C1 - x}{\_C1}} + 20x - 20 \right) \_C1^2 \right)^{2/3} (1 - i\sqrt{3}) \sqrt[3]{5} \right\}$$

## 2.244 ODE No. 244

---

$$x(2y(x) - x - 1)y'(x) + (-y(x) + 2x - 1)y(x) = 0$$

✓ **Mathematica** : cpu = 11.1231 (sec), leaf count = 484

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt[3]{2}x}{\sqrt[3]{27c_1^2x^2 + \sqrt{(27c_1^2x^2 + 27c_1^2x)^2 - 108c_1^3x^3} + 27c_1^2x}} - \frac{\sqrt[3]{27c_1^2x^2 + \sqrt{(27c_1^2x^2 + 27c_1^2x)^2 - 108c_1^3x^3} + 27c_1^2x}}{3\sqrt[3]{2}c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.094 (sec), leaf count = 391

$$\left\{y(x)=\frac{3}{80-C1}\left(\left(x\left(\sqrt{5}\sqrt{\frac{80\left(1+x\right)^2-C1-x}{-C1}}-20x-20\right)-C1^2\right)^{\frac{2}{3}}\left(i\sqrt{3}-1\right)\sqrt[3]{5}-\left(x\left(1+i\sqrt{3}\right)5^{\frac{2}{3}}\right.\right.$$

### 2.245 ODE No. 245

$$\left(4x^3+2xy(x)\right)y'(x)+112x^2y(x)+y(x)^2=0$$

✔ **Mathematica** : cpu = 0.42394 (sec), leaf count = 1453

$$\left\{\left\{y(x)\rightarrow \text{Root}\left[-1521681143169024\#1x^{22}-697437190619136\#1^2x^{20}-145299414712320\#1^3x^{18}-18162426\right.\right.$$

✔ **Maple** : cpu = 3.007 (sec), leaf count = 31

$$\left\{y(x)=\frac{-C1}{x^{28}\left(\text{RootOf}\left(x^{30}Z^{360}-24x^{30}Z^{330}-C1\right)\right)^{330}}\right\}$$

### 2.246 ODE No. 246

$$x(3y(x)+2x)y'(x)+3(y(x)+x)^2=0$$

✔ **Mathematica** : cpu = 0.122587 (sec), leaf count = 80

$$\left\{\left\{y(x)\rightarrow \frac{1}{6}\left(-\frac{\sqrt{2}\sqrt{3e^{4c_1}-x^4}}{x}-4x\right)\right\},\left\{y(x)\rightarrow \frac{1}{6}\left(\frac{\sqrt{2}\sqrt{3e^{4c_1}-x^4}}{x}-4x\right)\right\}\right\}$$

✔ **Maple** : cpu = 0.05 (sec), leaf count = 63

$$\left\{y(x)=\frac{1}{6-C1x}\left(-4-C1x^2-\sqrt{-2x^4-C1^2+6}\right),y(x)=\frac{1}{6-C1x}\left(-4-C1x^2+\sqrt{-2x^4-C1^2+6}\right)\right\}$$

### 2.247 ODE No. 247

$$-7x^2+(3x+2)(y(x)-2x-1)y'(x)+xy(x)-y(x)^2-9x-3=0$$

✔ **Mathematica** : cpu = 11.1923 (sec), leaf count = 693

$$\left\{\left\{y(x)\rightarrow \frac{\sqrt[3]{\sqrt{(-324e^{2c_1}x-216e^{2c_1}+1458x^3+2916x^2+1944x+432)^2+4(-81x^2-108x-36)^3}-324e^{2c_1}}}{6\sqrt[3]{2}}\right.\right.$$

✔ **Maple** : cpu = 0.194 (sec), leaf count = 517

$$\left\{y(x)=-\frac{1}{3}+\frac{3x+2}{6}\left(7\left(-1/4\sqrt[3]{2\left(3x+2\right)-C1}-27\left(3x+2\right)^3-C1^3+2\sqrt{-27\left(3x+2\right)^4-C1^4+(3x+2)^3}\right.\right.$$

### 2.248 ODE No. 248

$$\left(x^2+6xy(x)+3\right)y'(x)+3y(x)^2+2xy(x)+2x=0$$

✔ **Mathematica** : cpu = 0.172945 (sec), leaf count = 106

$$\left\{\left\{y(x)\rightarrow -\frac{\sqrt{6c_1x-2x^3+\frac{1}{6}\left(x^2+3\right)^2}}{\sqrt{6}x}-\frac{x^2+3}{6x}\right\},\left\{y(x)\rightarrow \frac{\sqrt{6c_1x-2x^3+\frac{1}{6}\left(x^2+3\right)^2}}{\sqrt{6}x}-\frac{x^2+3}{6x}\right\}\right\}$$

✔ **Maple** : cpu = 0.015 (sec), leaf count = 75

$$\left\{y(x)=\frac{1}{6x}\left(-x^2-3-\sqrt{x^4-12x^3-12-C1x+6x^2+9}\right),y(x)=\frac{1}{6x}\left(-x^2-3+\sqrt{x^4-12x^3-12-C1x+6x^2+9}\right)\right\}$$

2.249 ODE No. 249

---

$$y'(x) \left(axy(x) + bx^n\right) + \alpha y(x)^3 + \beta y(x)^2 = 0$$

✓ **Mathematica** : cpu = 3.52738 (sec), leaf count = 115

Solve  $\left[\frac{(a(-n) + a + \alpha y(x))y(x)^{\frac{a-an}{\beta}-1}(\alpha y(x) + \beta)^{\frac{a(n-1)}{\beta}}}{a^2(n-1)^2(a(n-1) + \beta)} + \frac{x^{1-n} \exp\left(-\frac{a(n-1)(\log(y(x)) - \log(\alpha y(x) + \beta))}{\beta}\right)}{ab(1-n)(n-1)} = c_1, y(x)\right]$

✓ **Maple** : cpu = 0.171 (sec), leaf count = 202

$$\left\{y(x) = \beta \left( \text{RootOf} \left( -x^{1-n} \_Z^{\frac{a(n-1)}{\beta}} a^2 \beta n + \_C1 a^2 b n^2 + x^{1-n} \_Z^{\frac{a(n-1)}{\beta}} a^2 \beta - x^{1-n} \_Z^{\frac{a(n-1)}{\beta}} a \beta^2 - \_Z^{\frac{an-a+\beta}{\beta}} \right) \right)$$

2.250 ODE No. 250

---

$$y'(x) \left(ax + Ax^2 + by(x) + Bxy(x) + c\right) + Axy(x) + \alpha x - By(x)^2 + \beta y(x) + \gamma = 0$$

✗ **Mathematica** : cpu = 300.895 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve((B\*x\*y(x)+A\*x^2+a\*x+b\*y(x)+c)\*diff(y(x),x)-B\*g(x)^2+A\*x\*y(x)+alpha\*x+beta\*y(x)+gamma

2.251 ODE No. 251

---

$$(x^2y(x) - 1) y'(x) + xy(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.120102 (sec), leaf count = 60

$$\left\{\left\{y(x) \rightarrow \frac{1}{x^2} - \frac{\sqrt{c_1 x^2 + 2 x^3 + 1}}{x^2}\right\}, \left\{y(x) \rightarrow \frac{\sqrt{c_1 x^2 + 2 x^3 + 1}}{x^2} + \frac{1}{x^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 51

$$\left\{y(x) = \frac{1}{x^2} \left(1 - \sqrt{-2 \_C1 x^2 + 2 x^3 + 1}\right), y(x) = \frac{1}{x^2} \left(1 + \sqrt{-2 \_C1 x^2 + 2 x^3 + 1}\right)\right\}$$

2.252 ODE No. 252

---

$$(x^2y(x) - 1) y'(x) - xy(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 10.9833 (sec), leaf count = 819

$$\left\{\left\{y(x) \rightarrow \frac{6xc_1 - x}{6c_1 - 1} + \frac{\sqrt[3]{-1944c_1^2x^3 + 648c_1x^3 - 54x^3 + 1944c_1^2 - 648c_1 + \sqrt{4(54x^2c_1 - 9x^2)^3 + (-1944c_1^2 - 648c_1 + 54x^3)}}{3\sqrt[3]{2}(6c_1 - 1)}\right\}\right\}$$

✓ **Maple** : cpu = 0.636 (sec), leaf count = 1338

$$\left\{y(x) = 1 \left( ((-\_C1 + 80) x^7 - 160 x^4 + 80 x) \sqrt[3]{4} \sqrt[3]{\left(-\frac{1}{4} + \sqrt{\frac{-5 x^6 + 10 x^3 - 5}{-80 + (\_C1 - 80) x^6 + 160 x^3}}\right)} (-80 + (\_C1 - 80) x^4 + 80 x) \right)\right\}$$

2.253 ODE No. 253

---

$$(x^2y(x) - 1) y'(x) + 8xy(x)^2 - 8 = 0$$

✗ **Mathematica** : cpu = 12.1714 (sec), leaf count = 0 , could not solve

DSolve[-8 + 8\*x\*y[x]^2 + (-1 + x^2\*y[x])\*Derivative[1][y][x] == 0, y[x], x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve((x^2\*y(x)-1)\*diff(y(x),x)+8\*x\*y(x)^2-8 = 0,y(x))

### 2.254 ODE No. 254

$$x^2 y(x)^3 + x(x y(x) - 2) y'(x) + x y(x)^2 - 2 y(x) = 0$$

✓ **Mathematica** : cpu = 0.123948 (sec), leaf count = 99

$$\left\{ \left\{ y(x) \rightarrow -\frac{2x}{\frac{\sqrt{2}\sqrt{-2x(c_1 - \log(x)) - \frac{x}{2}}}{\sqrt{-\frac{1}{x^3}}} - x^2} \right\}, \left\{ y(x) \rightarrow \frac{2x}{\frac{\sqrt{2}\sqrt{-2x(c_1 - \log(x)) - \frac{x}{2}}}{\sqrt{-\frac{1}{x^3}}} + x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 59

$$\left\{ y(x) = \frac{1}{(-2 \ln(x) + 2\_C1) x} \left( -1 + \sqrt{1 - 4 \ln(x) + 4\_C1} \right), y(x) = \frac{1}{(2 \ln(x) - 2\_C1) x} \left( 1 + \sqrt{1 - 4 \ln(x) + 4\_C1} \right) \right\}$$

### 2.255 ODE No. 255

$$x(x y(x) - 3) y'(x) + x y(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 3.21031 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow -\frac{3W\left(e^{\frac{9c_1}{2^{2/3}} - 1} x^{2/3}\right)}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.19 (sec), leaf count = 74

$$\left\{ y(x) = -3 \frac{\text{lambertW}\left(2/3 \sqrt[3]{-1/8 x^2} \_C1\right)}{x}, y(x) = -3 \frac{\text{lambertW}\left(-1/3 \sqrt[3]{-1/8 x^2} \_C1 (1 + i\sqrt{3})\right)}{x}, y(x) = -3 \frac{\text{lambertW}\left(-1/3 \sqrt[3]{-1/8 x^2} \_C1 (1 - i\sqrt{3})\right)}{x} \right\}$$

### 2.256 ODE No. 256

$$x^2(y(x) - 1)y'(x) + (x - 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0176331 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow -W\left(x \left(-e^{\frac{1}{x} - c_1}\right)\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 31

$$\left\{ y(x) = e^{\frac{-C1 x + x \ln(x) - \text{lambertW}\left(-x e^{-C1 + x^{-1}}\right)_{x+1}}{x}} \right\}$$

### 2.257 ODE No. 257

$$x(x^4 + x y(x) - 1) y'(x) - y(x)(-x^4 + x y(x) - 1) = 0$$

✓ **Mathematica** : cpu = 0.370278 (sec), leaf count = 39

$$\text{Solve} \left[ \frac{x \left( c_1 - 2 \log\left(\frac{1}{1 - x y(x)}\right) - 2 \right)}{y(x)} + 2x^2 + \frac{y(x)}{x} = 0, y(x) \right]$$

✓ **Maple** : cpu = 0.079 (sec), leaf count = 98

$$\left\{ y(x) = \frac{-\_C1 + e^{\frac{\text{RootOf}\left(-2\_Z x^4 (e^{-Z})^2 + 2 x^4 (e^{-Z})^2 - 2 e^{-Z} \_C1 x^4 + (e^{-Z})^2 - 2 e^{-Z} \_C1 + \_C1^2\right)}{x e^{\text{RootOf}\left(-2\_Z x^4 (e^{-Z})^2 + 2 x^4 (e^{-Z})^2 - 2 e^{-Z} \_C1 x^4 + (e^{-Z})^2 - 2 e^{-Z} \_C1 + \_C1^2\right)}}}}{x e^{\text{RootOf}\left(-2\_Z x^4 (e^{-Z})^2 + 2 x^4 (e^{-Z})^2 - 2 e^{-Z} \_C1 x^4 + (e^{-Z})^2 - 2 e^{-Z} \_C1 + \_C1^2\right)}}} \right\}$$



## 2.258 ODE No. 258

---

$$-2x^3 + 2x^2y(x)y'(x) - x^2 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.110327 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{c_1 e^{\frac{1}{x}} + x^2} \right\}, \left\{ y(x) \rightarrow \sqrt{c_1 e^{\frac{1}{x}} + x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt{e^{x^{-1}} - C1 + x^2}, y(x) = -\sqrt{e^{x^{-1}} - C1 + x^2} \right\}$$

## 2.259 ODE No. 259

---

$$2x^2y(x)y'(x) - e^{x-\frac{1}{x}}x^2 - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.230071 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow -e^{-\frac{1}{2}/x} \sqrt{c_1 + e^x} \right\}, \left\{ y(x) \rightarrow e^{-\frac{1}{2}/x} \sqrt{c_1 + e^x} \right\} \right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 51

$$\left\{ y(x) = \sqrt{e^{-x^{-1}} - C1 + e^{\frac{x^2-1}{x}}}, y(x) = -\sqrt{e^{-x^{-1}} - C1 + e^{\frac{x^2-1}{x}}} \right\}$$

## 2.260 ODE No. 260

---

$$(2x^2y(x) + x)y'(x) - x^2y(x)^3 + 2xy(x)^2 + y(x) = 0$$

✓ **Mathematica** : cpu = 0.118427 (sec), leaf count = 80

$$\left\{ \left\{ y(x) \rightarrow \frac{x}{\frac{\sqrt{x(c_1-2\log(x))+4x}}{\sqrt{\frac{1}{x^3}}} - 2x^2} \right\}, \left\{ y(x) \rightarrow -\frac{x}{\frac{\sqrt{x(c_1-2\log(x))+4x}}{\sqrt{\frac{1}{x^3}}} + 2x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 59

$$\left\{ y(x) = \frac{1}{(2\ln(x) - 2 - C1)x} \left( -2 + \sqrt{4 - 2\ln(x) + 2 - C1} \right), y(x) = \frac{1}{(-2\ln(x) + 2 - C1)x} \left( 2 + \sqrt{4 - 2\ln(x) + 2 - C1} \right) \right\}$$

## 2.261 ODE No. 261

---

$$(2x^2y(x) - x)y'(x) - 2xy(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.76152 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2xW\left(\frac{e^{\frac{9c_1}{2^{2/3}}-1}}{x^2}\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.098 (sec), leaf count = 18

$$\left\{ y(x) = -\frac{1}{2x} \left( \text{lambertW} \left( -\frac{C1}{2x^2} \right) \right)^{-1} \right\}$$

## 2.262 ODE No. 262

---

$$2x^3 + (2x^2y(x) - x^3)y'(x) - 4xy(x)^2 + y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.173331 (sec), leaf count = 101

$$\left\{ \left\{ y(x) \rightarrow \frac{2x^3 - \sqrt{e^{4c_1}x^2 - 3e^{2c_1}x^4}}{e^{2c_1} + x^2} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{e^{4c_1}x^2 - 3e^{2c_1}x^4} + 2x^3}{e^{2c_1} + x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.243 (sec), leaf count = 65

$$\left\{ y(x) = \frac{x}{-C1x^2 - 1} \left( 2 - C1x^2 - \sqrt{3 - C1x^2 + 1} \right), y(x) = \frac{x}{-C1x^2 - 1} \left( 2 - C1x^2 + \sqrt{3 - C1x^2 + 1} \right) \right\}$$

2.263 ODE No. 263

---

$$2x^3 + 3x^2y(x)^2 + y(x)y'(x) + 7 = 0$$

✓ **Mathematica** : cpu = 0.0901645 (sec), leaf count = 181

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{c_1 e^{-2x^3} + \frac{7 \, 2^{2/3} e^{-2x^3} x \Gamma\left(\frac{1}{3}, -2x^3\right)}{3 \sqrt[3]{-x^3}} - \frac{2^{2/3} e^{-2x^3} x \Gamma\left(\frac{4}{3}, -2x^3\right)}{3 \sqrt[3]{-x^3}}} \right\}, \left\{ y(x) \rightarrow \sqrt{c_1 e^{-2x^3} + \frac{7 \, 2^{2/3} e^{-2x^3} x \Gamma\left(\frac{1}{3}, -2x^3\right)}{3 \sqrt[3]{-x^3}} + \frac{2^{2/3} e^{-2x^3} x \Gamma\left(\frac{4}{3}, -2x^3\right)}{3 \sqrt[3]{-x^3}}} \right\} \right.$$

✓ **Maple** : cpu = 0.106 (sec), leaf count = 173

$$\left\{ y(x) = -\frac{2^{\frac{2}{3}}}{18 \Gamma(2/3)} \sqrt{-240 \sqrt[3]{-x^3} \left( \frac{\left( -\frac{27 e^{-2x^3} C1}{2} + 9 x \right) \sqrt[3]{2} \Gamma(2/3) \sqrt[3]{-x^3}}{40} + e^{-2x^3} x \left( \pi \sqrt{3} - 3/2 \Gamma(1/3, -2x^3) \right) \right)} \right.$$

2.264 ODE No. 264

---

$$2x \left( x^3 y(x) + 1 \right) y'(x) + y(x) \left( 3x^3 y(x) - 1 \right) = 0$$

✓ **Mathematica** : cpu = 0.379675 (sec), leaf count = 680

$$\left\{ \left\{ y(x) \rightarrow \text{Root} \left[ 81 \#1^7 e^{\frac{21 c_1}{2}} x^{12} + 756 \#1^6 e^{\frac{21 c_1}{2}} x^9 + 2646 \#1^5 e^{\frac{21 c_1}{2}} x^6 + 4116 \#1^4 e^{\frac{21 c_1}{2}} x^3 + 2401 \#1^3 e^{\frac{21 c_1}{2}} - x^{3/2} \right] \right\} \right.$$

✓ **Maple** : cpu = 0.753 (sec), leaf count = 574

$$\left\{ y(x) = \frac{-40353607 \left( \text{RootOf} \left( 9 x^7 \_Z^{98} - 49 \_C1 \_Z^{42} + 14 \_C1 \_Z^{21} - \_C1 \right) \right)^{91} \_C1 + 756315 \left( \text{RootOf} \left( 9 x^7 \_Z^{98} - 49 \_C1 \_Z^{42} + 14 \_C1 \_Z^{21} - \_C1 \right) \right)^{91} \_C1}{3 x^3 \left( \text{RootOf} \left( 9 x^7 \_Z^{98} - 49 \_C1 \_Z^{42} + 14 \_C1 \_Z^{21} - \_C1 \right) \right)^7 \left( 5764801 \_C1 \left( \text{RootOf} \left( 9 x^7 \_Z^{98} - 49 \_C1 \_Z^{42} + 14 \_C1 \_Z^{21} - \_C1 \right) \right)^7 \right)}$$

2.265 ODE No. 265

---

$$2(n+1)^2 x^{n-1} \left( x^{n^2} y(x)^2 - 1 \right) + \left( x^{n(n+1)} y(x) - 1 \right) y'(x) = 0$$

✗ **Mathematica** : cpu = 285.003 (sec), leaf count = 0 , could not solve  
DSolve[2\*(1 + n)^2\*x^(-1 + n)\*(-1 + x^n^2\*y[x]^2) + (-1 + x^(n\*(1 + n))\*y[x])\*Derivative[1][y[x]], x] == 0  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve((x^(n\*(n+1))\*y(x)-1)\*diff(y(x),x)+2\*(n+1)^2\*x^(n-1)\*(x^(n^2)\*y(x)^2-1) = 0,y(x))

2.266 ODE No. 266

---

$$\sqrt{x^2 + 1}(y(x) - x)y'(x) - a\sqrt{(y(x)^2 + 1)^3} = 0$$

✗ **Mathematica** : cpu = 300.012 (sec), leaf count = 0 , timed out  
\$Aborted

✓ **Maple** : cpu = 1.374 (sec), leaf count = 65

$$\left\{ y(x) = \tan \left( \text{RootOf} \left( -\arctan(x) + \int^{-\arctan(x) + \_Z} \frac{1}{2 a^2 + \cos(2 \_a) - 1} \left( \sqrt{2} \sqrt{\frac{a^2}{\cos(2 \_a) + 1}} \sin(2 \_a) - \right) \right) \right)$$

2.267 ODE No. 267

---

$$y(x) \sin^2(x)y'(x) + y(x)^2 \sin(x) \cos(x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.281698 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{c_1 + 2x \csc(x)} \right\}, \left\{ y(x) \rightarrow \sqrt{c_1 + 2x \csc(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 32

$$\left\{ y(x) = \frac{1}{\sin(x)} \sqrt{2x + \_C1}, y(x) = -\frac{1}{\sin(x)} \sqrt{2x + \_C1} \right\}$$

2.268 ODE No. 268

---

$$f(x)y(x)y'(x) + g(x)y(x)^2 + h(x) = 0$$

✓ **Mathematica** : cpu = 0.165151 (sec), leaf count = 146

$$\left\{ \left\{ y(x) \rightarrow -\exp\left(\int_1^x -\frac{g(K[1])}{f(K[1])}dK[1]\right) \sqrt{2\int_1^x -\frac{\exp\left(-2\int_1^{K[2]} -\frac{g(K[1])}{f(K[1])}dK[1]\right)h(K[2])}{f(K[2])}dK[2] + c_1} \right\}, \left\{ y(x) \rightarrow \right.$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 118

$$\left\{ y(x) = 1\sqrt{e^{2\int\frac{g(x)}{f(x)}dx}\left(-2\int\frac{h(x)}{f(x)}\left(e^{\int\frac{g(x)}{f(x)}dx}\right)^2dx + \_C1\right)\left(e^{2\int\frac{g(x)}{f(x)}dx}\right)^{-1}}, y(x) = -1\sqrt{e^{2\int\frac{g(x)}{f(x)}dx}\left(-2\int\frac{h(x)}{f(x)}\left(e^{\int\frac{g(x)}{f(x)}dx}\right)^2dx + \_C1\right)\left(e^{2\int\frac{g(x)}{f(x)}dx}\right)^{-1}} \right\}$$

2.269 ODE No. 269

---

$$-f0(x) - f1(x)y(x) - f2(x)y(x)^2 - f3(x)y(x)^3 + y'(x)(g0(x) + g1(x)y(x)) = 0$$

✗ **Mathematica** : cpu = 329.924 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve((g1(x)\*y(x)+g0(x))\*diff(y(x),x)-f1(x)\*y(x)-f2(x)\*y(x)^2-f3(x)\*y(x)^3-f0(x) = 0,y(x))

2.270 ODE No. 270

---

$$x^2 + (y(x)^2 - x)y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.124156 (sec), leaf count = 327

$$\left\{ \left\{ y(x) \rightarrow -\frac{3\sqrt[3]{2}x}{\sqrt[3]{\sqrt{(81c_1 + 27x^3)^2 - 2916x^3} + 81c_1 + 27x^3}} - \frac{\sqrt[3]{\sqrt{(81c_1 + 27x^3)^2 - 2916x^3} + 81c_1 + 27x^3}}{3\sqrt[3]{2}} \right\}, \left\{ y(x) \rightarrow \right.$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 319

$$\left\{ y(x) = \frac{1}{2} \left( \left( -4x^3 - 12\_C1 + 4\sqrt{x^6 + (6\_C1 - 4)x^3 + 9\_C1^2} \right)^{\frac{2}{3}} + 4x \right) \frac{1}{\sqrt[3]{-4x^3 - 12\_C1 + 4\sqrt{x^6 + (6\_C1 - 4)x^3 + 9\_C1^2}}} \right\}$$

2.271 ODE No. 271

---

$$(x^2 + y(x)^2)y'(x) + 2x(y(x) + 2x) = 0$$

✓ **Mathematica** : cpu = 0.174085 (sec), leaf count = 370

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{\sqrt{-8e^{3c_1}x^3 + e^{6c_1} + 20x^6} + e^{3c_1} - 4x^3}}{\sqrt[3]{2}} - \frac{\sqrt[3]{2}x^2}{\sqrt[3]{\sqrt{-8e^{3c_1}x^3 + e^{6c_1} + 20x^6} + e^{3c_1} - 4x^3}} \right\}, \left\{ y(x) \rightarrow \frac{1}{2\sqrt[3]{\sqrt{-8e^{3c_1}x^3 + e^{6c_1} + 20x^6} + e^{3c_1} - 4x^3}}} \right\}$$

✓ **Maple** : cpu = 0.163 (sec), leaf count = 352

$$\left\{ y(x) = 1 \left( \frac{1}{2} \sqrt[3]{4 - 16x^3\_C1^{3/2} + 4\sqrt{20\_C1^3x^6 - 8x^3\_C1^{3/2} + 1}} - 2 \frac{\_C1x^2}{\sqrt[3]{4 - 16x^3\_C1^{3/2} + 4\sqrt{20\_C1^3x^6 - 8x^3\_C1^{3/2} + 1}}} \right) \right\}$$

**2.272 ODE No. 272**

---

$$(x^2 + y(x)^2) y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.120297 (sec), leaf count = 42

$$\text{Solve} \left[ \log \left( \frac{y(x)}{x} \right) + \frac{2 \tan^{-1} \left( \frac{\frac{2y(x)}{x} - 1}{\sqrt{3}} \right)}{\sqrt{3}} = c_1 - \log(x), y(x) \right]$$

✓ **Maple** : cpu = 0.125 (sec), leaf count = 43

$$\left\{ y(x) = e^{\frac{2\sqrt{3}}{3} \text{RootOf}(-\sqrt{3}xe^{-C1} + 3 \tan(\_Z)xe^{-C1} + 2\sqrt{3}e^{2/3\sqrt{3}\_Z}) - C1} \right\}$$

**2.273 ODE No. 273**

---

$$(a + x^2 + y(x)^2) y'(x) + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.174362 (sec), leaf count = 297

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{\sqrt{2916(a+x^2)^3 + 6561c_1^2 + 81c_1}}}{3\sqrt[3]{2}} - \frac{3\sqrt[3]{2}(a+x^2)}{\sqrt[3]{\sqrt{2916(a+x^2)^3 + 6561c_1^2 + 81c_1}}} \right\}, \left\{ y(x) \rightarrow \frac{1}{2^{2/3}\sqrt[3]{\sqrt{2916(a+x^2)^3 + 6561c_1^2 + 81c_1}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 401

$$\left\{ y(x) = \frac{1}{2} \left( \left( -12\_C1 + 4\sqrt{4x^6 + 12ax^4 + 12a^2x^2 + 4a^3 + 9\_C1^2} \right)^{\frac{2}{3}} - 4x^2 - 4a \right) \frac{1}{\sqrt[3]{-12\_C1 + 4\sqrt{4x^6 + 12ax^4 + 12a^2x^2 + 4a^3 + 9\_C1^2}}} \right\}$$

**2.274 ODE No. 274**

---

$$(a + x^2 + y(x)^2) y'(x) + b + x^2 + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.1738 (sec), leaf count = 411

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{\sqrt{2916(a+x^2)^3 + (-81bx + 81c_1 - 27x^3)^2 - 81bx + 81c_1 - 27x^3}}}{3\sqrt[3]{2}} - \frac{(-81bx + 81c_1 - 27x^3)}{\sqrt[3]{\sqrt{2916(a+x^2)^3 + (-81bx + 81c_1 - 27x^3)^2 - 81bx + 81c_1 - 27x^3}}} \right\}, \left\{ y(x) \rightarrow \frac{1}{\sqrt[3]{\sqrt{2916(a+x^2)^3 + (-81bx + 81c_1 - 27x^3)^2 - 81bx + 81c_1 - 27x^3}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 657

$$\left\{ y(x) = \frac{1}{2} \left( \left( -4x^3 - 12bx - 12\_C1 + 4\sqrt{5x^6 + (12a + 6b)x^4 + 6x^3\_C1 + (12a^2 + 9b^2)x^2 + 18bx\_C1 + 4\_C1^2} \right)^{\frac{2}{3}} - 4x^2 - 4a - 4b \right) \frac{1}{\sqrt[3]{-4x^3 - 12bx - 12\_C1 + 4\sqrt{5x^6 + (12a + 6b)x^4 + 6x^3\_C1 + (12a^2 + 9b^2)x^2 + 18bx\_C1 + 4\_C1^2}}} \right\}$$

**2.275 ODE No. 275**

---

$$(x^2 + y(x)^2 + x) y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.118106 (sec), leaf count = 18

$$\text{Solve} \left[ y(x) - \tan^{-1} \left( \frac{x}{y(x)} \right) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 30

$$\left\{ -C1 + \frac{(ix + y(x))e^{-2iy(x)}}{2iy(x) + 2x} = 0 \right\}$$

## 2.276 ODE No. 276

---

$$(y(x)^2 - x^2) y'(x) + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.140184 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( e^{c_1} - \sqrt{e^{2c_1} - 4x^2} \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \left( \sqrt{e^{2c_1} - 4x^2} + e^{c_1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 47

$$\left\{ y(x) = \frac{1}{2\_C1} \left( 1 + \sqrt{-4\_C1^2 x^2 + 1} \right), y(x) = \frac{1}{2\_C1} \left( 1 - \sqrt{-4\_C1^2 x^2 + 1} \right) \right\}$$

## 2.277 ODE No. 277

---

$$(x^4 + y(x)^2) y'(x) - 4x^3 y(x) = 0$$

✓ **Mathematica** : cpu = 0.109954 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( c_1 - \sqrt{c_1^2 + 4x^4} \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \left( \sqrt{c_1^2 + 4x^4} + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.286 (sec), leaf count = 41

$$\left\{ y(x) = -\frac{1}{2} \sqrt{4x^4 + \_C1^2} + \frac{\_C1}{2}, y(x) = \frac{1}{2} \sqrt{4x^4 + \_C1^2} + \frac{\_C1}{2} \right\}$$

## 2.278 ODE No. 278

---

$$y'(x) (y(x)^2 + 4 \sin(x)) - \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.177675 (sec), leaf count = 39

$$\text{Solve} \left[ -\frac{1}{32} e^{-4y(x)} (8y(x)^2 + 4y(x) + 1) - e^{-4y(x)} \sin(x) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 28

$$\left\{ \frac{\left( -8 (y(x))^2 - 4 y(x) - 32 \sin(x) - 1 \right) e^{-4y(x)}}{32} + \_C1 = 0 \right\}$$

## 2.279 ODE No. 279

---

$$(y(x)^2 + 2y(x) + x) y'(x) + y(x)^2 (y(x) + x)^2 + y(x)(y(x) + 1) = 0$$

✓ **Mathematica** : cpu = 0.603734 (sec), leaf count = 107

$$\left\{ \left\{ y(x) \rightarrow \frac{-\sqrt{(-c_1 x + x^2 - 1)^2 + 4(x - c_1)} + c_1 x - x^2 + 1}{2(x - c_1)} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{(-c_1 x + x^2 - 1)^2 + 4(x - c_1)} + c_1 x - x^2 + 1}{2(x - c_1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.127 (sec), leaf count = 116

$$\left\{ y(x) = \frac{1}{-2\_C1 + 4x} \left( -2x^2 + \_C1 x + \sqrt{4x^4 - 4\_C1 x^3 + (\_C1^2 - 8)x^2 + (4\_C1 + 16)x - 8\_C1 + 4} \right) \right\}$$

## 2.280 ODE No. 280

---

$$(y(x) + x)^2 y'(x) - a^2 = 0$$

✓ **Mathematica** : cpu = 0.130415 (sec), leaf count = 21

$$\text{Solve} \left[ y(x) - a \tan^{-1} \left( \frac{y(x) + x}{a} \right) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.041 (sec), leaf count = 24

$$\{y(x) = a \text{RootOf}(\tan(\_Z) a - \_Z a + \_C1 - x) - \_C1\}$$

**2.281            ODE No. 281**

---

$$\left(-x^2+2xy(x)+y(x)^2\right)y'(x)+x^2+2xy(x)-y(x)^2=0$$

✓ **Mathematica** : cpu = 0.186076 (sec), leaf count = 75

$$\left\{\left\{y(x)\rightarrow\frac{1}{2}\left(e^{c_1}-\sqrt{4e^{c_1}x+e^{2c_1}-4x^2}\right)\right\},\left\{y(x)\rightarrow\frac{1}{2}\left(\sqrt{4e^{c_1}x+e^{2c_1}-4x^2}+e^{c_1}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.051 (sec), leaf count = 55

$$\left\{y(x)=\frac{1}{2\_C1}\left(1+\sqrt{-4\_C1^2x^2+4\_C1x+1}\right),y(x)=\frac{1}{2\_C1}\left(1-\sqrt{-4\_C1^2x^2+4\_C1x+1}\right)\right\}$$

**2.282            ODE No. 282**

---

$$\left(y(x)+3x-1\right)^2y'(x)-\left(2y(x)-1\right)\left(4y(x)+6x-3\right)=0$$

✓ **Mathematica** : cpu = 0.20991 (sec), leaf count = 2129

$$\left\{\left\{y(x)\rightarrow\frac{1}{6}\left(12x+4e^{c_1}+1\right)-\frac{1}{6}\sqrt{36x^2+96e^{c_1}x-12x-16e^{c_1}+16e^{2c_1}+3\sqrt[2]{3}\sqrt{-7776e^{c_1}x^5+6480e^{c_1}x^4-12e^{c_1}x^3-12e^{c_1}x^2-12e^{c_1}x-12e^{c_1}}}\right\}\right\}$$

✓ **Maple** : cpu = 0.263 (sec), leaf count = 71

$$\left\{-\ln\left(\frac{-6y(x)+4-6x}{6x-1}\right)+3\ln\left(\frac{-6y(x)+3}{6x-1}\right)-3\ln\left(\frac{-6y(x)+18x}{6x-1}\right)-\ln(6x-1)-\_C1=0\right\}$$

**2.283            ODE No. 283**

---

$$3\left(y(x)^2-x^2\right)y'(x)+2y(x)^3-6x(x+1)y(x)-3e^x=0$$

✓ **Mathematica** : cpu = 0.33837 (sec), leaf count = 477

$$\left\{\left\{y(x)\rightarrow-\frac{e^{-2x}\sqrt[3]{\sqrt{(27c_1e^{4x}-27e^{7x})^2-2916e^{12x}x^6}+27c_1e^{4x}-27e^{7x}}}{3\sqrt[3]{2}}-\frac{3\sqrt[3]{2}e^{2x}x^2}{\sqrt[3]{\sqrt{(27c_1e^{4x}-27e^{7x})^2-2916e^{12x}x^6}+27c_1e^{4x}-27e^{7x}}}}\right\}\right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 407

$$\left\{y(x)=\frac{1}{4e^{2x}}\left(-4x^2\left(1+i\sqrt{3}\right)\left(e^{2x}\right)^2+\left(i\sqrt{3}-1\right)\left(\left(4e^{3x}-4\_C1+4\sqrt{-4x^6\left(e^{2x}\right)^2+\left(e^{3x}\right)^2-2e^{3x}\_C1}\right)\right)\right)\right\}$$

**2.284            ODE No. 284**

---

$$\left(x^2+4y(x)^2\right)y'(x)-xy(x)=0$$

✓ **Mathematica** : cpu = 0.108416 (sec), leaf count = 59

$$\left\{\left\{y(x)\rightarrow-\frac{x}{2\sqrt{W\left(\frac{1}{4}e^{-\frac{c_1}{2}}x^2\right)}}\right\},\left\{y(x)\rightarrow\frac{x}{2\sqrt{W\left(\frac{1}{4}e^{-\frac{c_1}{2}}x^2\right)}}\right\}\right\}$$

✓ **Maple** : cpu = 0.129 (sec), leaf count = 21

$$\left\{y(x)=e^{\frac{1}{2} LambertW\left(\frac{\left(e^{-C1}\right)^2x^2}{4}\right)-\_C1}\right\}$$

**2.285 ODE No. 285**

---

$$(3x^2 + 2xy(x) + 4y(x)^2) y'(x) + 2x^2 + 6xy(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.174751 (sec), leaf count = 402

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{\sqrt{(432e^{3c_1} + 54x^3)^2 + 3881196x^6 + 432e^{3c_1} + 54x^3}}}{12\sqrt[3]{2}} - \frac{33x^2}{2^{22/3}\sqrt[3]{\sqrt{(432e^{3c_1} + 54x^3)^2 + 3881196x^6 + 432e^{3c_1} + 54x^3}}} \right. \right.$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 432

$$\left\{ y(x) = \frac{1}{-C1} \left( \frac{1}{4} \sqrt[3]{x^3 - C1^3 + 8 + 2\sqrt{333 - C1^6x^6 + 4x^3 - C1^3 + 16}} - \frac{11 - C1^2x^2}{4} \frac{1}{\sqrt[3]{x^3 - C1^3 + 8 + 2\sqrt{333 - C1^6x^6 + 4x^3 - C1^3 + 16}}} \right) \right.$$

**2.286 ODE No. 286**

---

$$(2y(x) - 3x + 1)^2 y'(x) - (3y(x) - 2x - 4)^2 = 0$$

✓ **Mathematica** : cpu = 0.243073 (sec), leaf count = 3501

✓ **Maple** : cpu = 1.18 (sec), leaf count = 1337

$$\left\{ y(x) = \frac{(5x + 3) \left( \text{RootOf} \left( (115330078125 - C1x^9 - 2283535546875 - C1x^8 + 20095112812500 - C1x^7 - 10312500000000 - C1x^6 - 100000000000000 - C1x^5 - 1000000000000000 - C1x^4 - 10000000000000000 - C1x^3 - 100000000000000000 - C1x^2 - 1000000000000000000 - C1x - 10000000000000000000) \right) \right)}{5 \left( \text{RootOf} \left( (115330078125 - C1x^9 - 2283535546875 - C1x^8 + 20095112812500 - C1x^7 - 10312500000000 - C1x^6 - 100000000000000 - C1x^5 - 1000000000000000 - C1x^4 - 10000000000000000 - C1x^3 - 100000000000000000 - C1x^2 - 1000000000000000000 - C1x - 10000000000000000000) \right) \right)}$$

**2.287 ODE No. 287**

---

$$(2y(x) - 4x + 1)^2 y'(x) - (y(x) - 2x)^2 = 0$$

✓ **Mathematica** : cpu = 1.49201 (sec), leaf count = 77

Solve  $\left[ \frac{y(x)}{2} + \frac{1}{196} \left( 14y(x) - (8 - 9\sqrt{2}) \log(-7y(x) + 14x + \sqrt{2} - 4) - (8 + 9\sqrt{2}) \log(7y(x) - 14x + \sqrt{2} + 4) \right) \right]$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 56

$$\left\{ -\frac{x}{7} - \frac{9\sqrt{2}}{98} \text{Artanh} \left( \frac{(7y(x) - 14x + 4)\sqrt{2}}{2} \right) - \frac{2 \ln(7(y(x) - 2x)^2 + 8y(x) - 16x + 2)}{49} + \frac{4y(x)}{7} - C1 \right.$$

**2.288 ODE No. 288**

---

$$(-3x^2y(x) + 6y(x)^2 + 1) y'(x) - 3xy(x)^2 + x = 0$$

✓ **Mathematica** : cpu = 0.227309 (sec), leaf count = 534

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt[3]{4\sqrt{3}\sqrt{-54c_1x^6 + 648c_1x^2 + 432c_1^2 - 27x^8 + 207x^4 + 32 + 144c_1 - 9x^6 + 108x^2}}}{4 \cdot 3^{2/3}} + \frac{1}{3\sqrt[3]{3}\sqrt[3]{4\sqrt{3}\sqrt{-54c_1x^6 + 648c_1x^2 + 432c_1^2 - 27x^8 + 207x^4 + 32 + 144c_1 - 9x^6 + 108x^2}}} \right. \right.$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 579

$$\left\{ y(x) = -\frac{1}{24} \left( -6x^2 \sqrt[3]{-324x^2 - 432 - C1 + 27x^6 + 12\sqrt{-81x^8 - 162 - C1x^6 + 621x^4 + 1944 - C1x^2 + 1296}} \right) \right.$$

2.289 ODE No. 289

---

$$a + (6y(x) - x)^2 y'(x) - 6y(x)^2 + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.178287 (sec), leaf count = 115

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{6} \left( \sqrt[3]{-18ax + 18c_1 - x^3} + x \right) \right\}, \left\{ y(x) \rightarrow \frac{x}{6} - \frac{1}{12} \left( 1 - i\sqrt{3} \right) \sqrt[3]{-18ax + 18c_1 - x^3} \right\}, \left\{ y(x) \rightarrow \frac{x}{6} - \frac{1}{12} \left( 1 + i\sqrt{3} \right) \sqrt[3]{-18ax + 18c_1 - x^3} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 115

$$\left\{ y(x) = \frac{1}{6} \sqrt[3]{-x^3 - 18ax - 18C_1} + \frac{x}{6}, y(x) = -\frac{1}{12} \sqrt[3]{-x^3 - 18ax - 18C_1} - \frac{i}{12} \sqrt{3} \sqrt[3]{-x^3 - 18ax - 18C_1}, y(x) = -\frac{1}{12} \sqrt[3]{-x^3 - 18ax - 18C_1} + \frac{i}{12} \sqrt{3} \sqrt[3]{-x^3 - 18ax - 18C_1} \right\}$$

2.290 ODE No. 290

---

$$y'(x) (ay(x)^2 + 2bxy(x) + cx^2) + by(x)^2 + 2cxy(x) + dx^2 = 0$$

✓ **Mathematica** : cpu = 0.445511 (sec), leaf count = 831

$$\left\{ \left\{ y(x) \rightarrow -\frac{bx}{a} + \frac{\sqrt[3]{-54b^3x^3 + 81abcx^3 - 27a^2dx^3 + 27a^2e^{3c_1} + \sqrt{4(9acx^2 - 9b^2x^2)^3 + (-54b^3x^3 + 81abcx^3 - 27a^2dx^3 + 27a^2e^{3c_1})^2}}}{3\sqrt[3]{2a}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 1388

$$\left\{ y(x) = \frac{1}{-C_1} \left( \frac{1}{2a} \sqrt[3]{-4C_1^3a^2dx^3 + 12cx^3C_1^3ba - 8b^3x^3C_1^3 + 4\sqrt{-C_1^6a^2d^2x^6 - 6C_1^6abcdx^6 + 4C_1^6a^2b^2d^2x^6}} \right) \right\}$$

2.291 ODE No. 291

---

$$y'(x) (b(\alpha x + \beta y(x))^2 - \beta(ax + by(x))) - \alpha(ax + by(x)) + a(\alpha x + \beta y(x))^2 = 0$$

✓ **Mathematica** : cpu = 0.842298 (sec), leaf count = 39

$$\text{Solve} \left[ \frac{a\beta \left( \log(ax + by(x)) + \frac{1}{\alpha x + \beta y(x)} \right)}{a\beta - \alpha b} = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.121 (sec), leaf count = 50

$$\left\{ y(x) = \frac{-ax + e^{\text{RootOf}(-C_1a\beta x - C_1\alpha bx - Za\beta x + Z\alpha bx - C_1\beta e^{-Z} + e^{-Z}Z\beta + b)}}{b} \right\}$$

2.292 ODE No. 292

---

$$y'(x)(ay(x) + bx + c)^2 + (\alpha y(x) + \beta x + \gamma)^2 = 0$$

✓ **Mathematica** : cpu = 36.2462 (sec), leaf count = 1716

$$\text{Solve} \left[ (\alpha b - a\beta) \text{RootSum} \left[ -cy(x)^2\alpha^3 - b\#1y(x)^2\alpha^3 + a\beta\#1y(x)^2\alpha^2 + a\gamma y(x)^2\alpha^2 - 2b\beta\#1^2y(x)\alpha^2 - 2\beta c\#1y(x)\alpha^2 + \beta^2\right] \right]$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 115

$$\left\{ y(x) = \frac{1}{a\beta - b\alpha} \left( ((bx + c)\alpha - a(\beta x + \gamma)) \text{RootOf} \left( \int^{-Z} \frac{(-aa - b)^2}{-a^3a^2 - 2a^2ab - a^2\alpha^2 + 2a\alpha\beta + ab^2 - \beta^2} \right) \right) \right\}$$



2.293 ODE No. 293

---

$$x \left( y(x)^2 - 3x \right) y'(x) + 2y(x)^3 - 5xy(x) = 0$$

✓ **Mathematica** : cpu = 0.162976 (sec), leaf count = 661

$$\left\{ \left\{ y(x) \rightarrow \text{Root} \left[ -\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}} \&, 1 \right] \right\}, \left\{ y(x) \rightarrow \text{Root} \left[ -\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}} \&, 2 \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.284 (sec), leaf count = 35

$$\left\{ \ln(x) - \_C1 - \frac{2}{65} \ln \left( \frac{5 (y(x))^2 - 13x}{x} \right) + \frac{6}{13} \ln \left( y(x) \frac{1}{\sqrt{x}} \right) = 0 \right\}$$

2.294 ODE No. 294

---

$$x \left( -a + x^2 + y(x)^2 \right) y'(x) - y(x) \left( a + x^2 + y(x)^2 \right) = 0$$

✓ **Mathematica** : cpu = 0.2323 (sec), leaf count = 71

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( c_1 x - \sqrt{-4a + c_1^2 x^2 + 4x^2} \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \left( \sqrt{-4a + c_1^2 x^2 + 4x^2} + c_1 x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 112

$$\left\{ \left( (y(x))^{-2} - (-x^2 + a)^{-1} \right)^{-1} = -x\sqrt{x^2 - a} \frac{1}{\sqrt{-C1 + 4 \frac{a}{x^2 - a}}} + \frac{x^2}{2} - \frac{a}{2}, \left( (y(x))^{-2} - (-x^2 + a)^{-1} \right)^{-1} = x\sqrt{x^2 - a} \frac{1}{\sqrt{-C1 + 4 \frac{a}{x^2 - a}}} + \frac{x^2}{2} - \frac{a}{2} \right\}$$

2.295 ODE No. 295

---

$$x \left( -x^2 + xy(x) + y(x)^2 \right) y'(x) + x^2 y(x) - y(x)^3 + xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.206849 (sec), leaf count = 31

$$\text{Solve} \left[ \frac{x}{y(x)} + \frac{y(x)}{x} + \log \left( \frac{y(x)}{x} \right) = c_1 - 2 \log(x), y(x) \right]$$

✓ **Maple** : cpu = 0.149 (sec), leaf count = 29

$$\left\{ y(x) = e^{\text{RootOf} \left( (e^{-Z})^2 + 2e^{-Z} \ln(x) + 2e^{-Z} \_C1 + \_Z e^{-Z} + 1 \right)} x \right\}$$

2.296 ODE No. 296

---

$$x^4 + x \left( x^2 y(x) + x^2 + y(x)^2 \right) y'(x) - 2x^2 y(x)^2 - 2y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.469764 (sec), leaf count = 102

$$\left\{ \left\{ y(x) \rightarrow -e^{-c_1} x^2 - e^{-c_1} \sqrt{-e^{c_1} x^4 + e^{2c_1} x^2 + x^4} \right\}, \left\{ y(x) \rightarrow e^{-c_1} \sqrt{-e^{c_1} x^4 + e^{2c_1} x^2 + x^4} - e^{-c_1} x^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.59 (sec), leaf count = 135

$$\left\{ y(x) = -x \left( -x^3 + \_C1 x + x^2 + \sqrt{-\_C1 x^4 + \_C1^2 x^2 + x^4} \right) \left( \_C1 x - x^2 + \sqrt{-\_C1 x^4 + \_C1^2 x^2 + x^4} \right) \right\}$$

2.297 ODE No. 297

---

$$2x \left( 5x^2 + y(x)^2 \right) y'(x) - x^2 y(x) + y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.140979 (sec), leaf count = 216

$$\left\{ \left\{ y(x) \rightarrow \text{Root} \left[ -\#1^5 + \frac{\#1^2 e^{3c_1}}{x^{3/2}} + 3e^{3c_1} \sqrt{x} \&, 1 \right] \right\}, \left\{ y(x) \rightarrow \text{Root} \left[ -\#1^5 + \frac{\#1^2 e^{3c_1}}{x^{3/2}} + 3e^{3c_1} \sqrt{x} \&, 2 \right] \right\}, \left\{ y(x) \rightarrow \text{Root} \left[ -\#1^5 + \frac{\#1^2 e^{3c_1}}{x^{3/2}} + 3e^{3c_1} \sqrt{x} \&, 3 \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.313 (sec), leaf count = 29

$$\left\{ y(x) = \left( \text{RootOf} \left( x^9 \_C1 \_Z^{45} - \_Z^{18} - 6 \_Z^9 - 9 \right) \right)^{\frac{9}{2}} x \right\}$$

## 2.298 ODE No. 298

---

$$3xy(x)^2y'(x) + y(x)^3 - 2x = 0$$

✓ **Mathematica** : cpu = 0.0834763 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{c_1 + x^2}}{\sqrt[3]{x}} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt[3]{-1}\sqrt[3]{c_1 + x^2}}{\sqrt[3]{x}} \right\}, \left\{ y(x) \rightarrow \frac{(-1)^{2/3}\sqrt[3]{c_1 + x^2}}{\sqrt[3]{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 73

$$\left\{ y(x) = \frac{1}{x} \sqrt[3]{(x^2 + \_C1)x^2}, y(x) = -\frac{1 + i\sqrt{3}}{2x} \sqrt[3]{(x^2 + \_C1)x^2}, y(x) = \frac{i\sqrt{3} - 1}{2x} \sqrt[3]{(x^2 + \_C1)x^2} \right\}$$

## 2.299 ODE No. 299

---

$$(3xy(x)^2 - x^2) y'(x) + y(x)^3 - 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.106773 (sec), leaf count = 371

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt[3]{\frac{2}{3}x^2}}{\sqrt[3]{9c_1x^2 + \sqrt{3}\sqrt{27c_1^2x^4 - 4x^9}}} - \frac{\sqrt[3]{9c_1x^2 + \sqrt{3}\sqrt{27c_1^2x^4 - 4x^9}}}{\sqrt[3]{23^{2/3}x}} \right\}, \left\{ y(x) \rightarrow \frac{(1 + i\sqrt{3})}{2^{2/3}\sqrt[3]{3}\sqrt[3]{9c_1x^2 + \sqrt{3}\sqrt{27c_1^2x^4 - 4x^9}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.174 (sec), leaf count = 276

$$\left\{ y(x) = -\frac{12^{\frac{2}{3}}}{144x} \left( \left( -12ix^3 + i \left( \left( 12\sqrt{-12x^5 + 81\_C1^2} + 108\_C1 \right) x^2 \right)^{\frac{2}{3}} \right) \sqrt{3} + 12x^3 + \left( \left( 12\sqrt{-12x^5 + 81\_C1^2} + 108\_C1 \right) x^2 \right)^{\frac{2}{3}} \right) \right\}$$

## 2.300 ODE No. 300

---

$$6xy(x)^2y'(x) + 2y(x)^3 + x = 0$$

✓ **Mathematica** : cpu = 0.0796077 (sec), leaf count = 99

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{4c_1 - x^2}}{2^{2/3}\sqrt[3]{x}} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt[3]{-1}\sqrt[3]{4c_1 - x^2}}{2^{2/3}\sqrt[3]{x}} \right\}, \left\{ y(x) \rightarrow \frac{(-1)^{2/3}\sqrt[3]{4c_1 - x^2}}{2^{2/3}\sqrt[3]{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 83

$$\left\{ y(x) = \frac{1}{2x} \sqrt[3]{-2(x^2 - 4\_C1)x^2}, y(x) = -\frac{1 + i\sqrt{3}}{4x} \sqrt[3]{-2(x^2 - 4\_C1)x^2}, y(x) = \frac{i\sqrt{3} - 1}{4x} \sqrt[3]{-2(x^2 - 4\_C1)x^2} \right\}$$

## 2.301 ODE No. 301

---

$$(x^2 + 6xy(x)^2) y'(x) - y(x) (3y(x)^2 - x) = 0$$

✓ **Mathematica** : cpu = 0.129724 (sec), leaf count = 64

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{x}\sqrt{W\left(\frac{6e^{3c_1}}{x^3}\right)}}{\sqrt{6}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{x}\sqrt{W\left(\frac{6e^{3c_1}}{x^3}\right)}}{\sqrt{6}} \right\} \right\}$$

✓ **Maple** : cpu = 0.183 (sec), leaf count = 25

$$\left\{ y(x) = \frac{1}{x} e^{-\frac{1}{2}\text{lambertW}\left(6\frac{e^{3\_C1}}{x^3}\right) + \frac{3\_C1}{2}} \right\}$$

**2.302 ODE No. 302**

---

$$\left(x^2 y(x)^2 + x\right) y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0916786 (sec), leaf count = 70

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 x - \sqrt{x} \sqrt{c_1^2 x + 4}}{2x} \right\}, \left\{ y(x) \rightarrow \frac{c_1 x + \sqrt{x} \sqrt{c_1^2 x + 4}}{2x} \right\} \right\}$$

✓ **Maple** : cpu = 0.104 (sec), leaf count = 133

$$\left\{ y(x) = -\frac{1}{2\_C1\,x} \sqrt{-2\,x\_C1\,\left(-2\_C1-x+\sqrt{x\,(4\_C1+x)}\right)}, y(x) = \frac{1}{2\_C1\,x} \sqrt{-2\,x\_C1\,\left(-2\_C1-x+\sqrt{x\,(4\_C1+x)}\right)} \right\}$$

**2.303 ODE No. 303**

---

$$y(x)\left(x^2 y(x)^2 + 1\right) + x(x y(x) - 1)^2 y'(x) = 0$$

✓ **Mathematica** : cpu = 0.113607 (sec), leaf count = 25

$$\text{Solve}\left[xy(x) - \frac{1}{xy(x)} - 2\log(y(x)) = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.138 (sec), leaf count = 34

$$\left\{ y(x) = \frac{\mathrm{e}^{RootOf\left(-\mathrm{e}^{2\_Z} - 2\mathrm{e}^{-Z}\ln(x) + 2\mathrm{e}^{-Z}\_C1 + 2\_Z\mathrm{e}^{-Z} + 1\right)}}{x} \right\}$$

**2.304 ODE No. 304**

---

$$5x^2 y(x)^3 + \left(10x^3 y(x)^2 + x^2 y(x) + 2x\right) y'(x) + x y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.333295 (sec), leaf count = 44

$$\text{Solve}\left[y(x)\sqrt{5x^2 y(x)^2 + 2}e^{\frac{\tan^{-1}\left(\sqrt{\frac{5}{2}}xy(x)\right)}{\sqrt{10}}} = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.214 (sec), leaf count = 44

$$\left\{ y(x) = \frac{\sqrt{10}}{5\,x} \tan\left(\text{RootOf}\left(\sqrt{10}\ln\left(\frac{4\left((\tan(\_Z))^2 + 1\right)(\tan(\_Z))^2}{5\,x^2}\right) + 2\sqrt{10}\_C1 + 2\_Z\right)\right)\right\}$$

**2.305 ODE No. 305**

---

$$x^2 + \left(y(x)^3 - 3x\right) y'(x) - 3y(x) = 0$$

✓ **Mathematica** : cpu = 0.179901 (sec), leaf count = 1277

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2} \sqrt{\frac{16\sqrt[3]{2}(x^3 + 3c_1)}{\sqrt[3]{104976x^2 - \sqrt{11019960576x^4 - 4(144x^3 + 432c_1)}^3}} + \frac{\sqrt[3]{104976x^2 - \sqrt{11019960576x^4 - 4(144x^3 + 432c_1)}^3}}{9\sqrt[3]{2}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 21

$$\left\{ \frac{x^3}{3} - 3xy(x) + \frac{(y(x))^4}{4} + \_C1 = 0 \right\}$$

2.306 ODE No. 306

---

$$(y(x)^3 - x^3) y'(x) - x^2 y(x) = 0$$

✓ **Mathematica** : cpu = 0.113539 (sec), leaf count = 201

$$\left\{ \left\{ y(x) \rightarrow \sqrt[3]{x^3 - \sqrt{x^6 - e^{6c_1}}} \right\}, \left\{ y(x) \rightarrow -\sqrt[3]{-1} \sqrt[3]{x^3 - \sqrt{x^6 - e^{6c_1}}} \right\}, \left\{ y(x) \rightarrow (-1)^{2/3} \sqrt[3]{x^3 - \sqrt{x^6 - e^{6c_1}}} \right\}, \right.$$

✓ **Maple** : cpu = 0.299 (sec), leaf count = 231

$$\left\{ y(x) = x \frac{1}{\sqrt[3]{-\left(-C1 x^3 - \sqrt{-C1^2 x^6 + 1}\right) x^3 - C1}}, y(x) = x \frac{1}{\sqrt[3]{-\left(-C1 x^3 + \sqrt{-C1^2 x^6 + 1}\right) x^3 - C1}}, y(x) = \dots \right.$$

2.307 ODE No. 307

---

$$y(x) (a + x^2 + y(x)^2) y'(x) + x (-a + x^2 + y(x)^2) = 0$$

✓ **Mathematica** : cpu = 0.228682 (sec), leaf count = 149

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{-\sqrt{a^2 + 4ax^2 + 4c_1} - a - x^2} \right\}, \left\{ y(x) \rightarrow \sqrt{-\sqrt{a^2 + 4ax^2 + 4c_1} - a - x^2} \right\}, \left\{ y(x) \rightarrow -\sqrt{\sqrt{a^2 + 4ax^2 + 4c_1} - a - x^2} \right\}, \right.$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 125

$$\left\{ y(x) = \sqrt{-x^2 - a - \sqrt{4ax^2 + a^2 - 4-C1}}, y(x) = \sqrt{-x^2 - a + \sqrt{4ax^2 + a^2 - 4-C1}}, y(x) = -\sqrt{-x^2 - a - \sqrt{4ax^2 + a^2 - 4-C1}}, \right.$$

2.308 ODE No. 308

---

$$2y(x)^3 y'(x) + xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0076606 (sec), leaf count = 55

$$\left\{ \{y(x) \rightarrow 0\}, \left\{ y(x) \rightarrow -\frac{\sqrt{4c_1 - x^2}}{\sqrt{2}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{4c_1 - x^2}}{\sqrt{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 37

$$\left\{ y(x) = 0, y(x) = -\frac{1}{2} \sqrt{-2x^2 + 4-C1}, y(x) = \frac{1}{2} \sqrt{-2x^2 + 4-C1} \right\}$$

2.309 ODE No. 309

---

$$-2x^3 + (2y(x)^3 + y(x)) y'(x) - x = 0$$

✓ **Mathematica** : cpu = 0.0992953 (sec), leaf count = 151

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{-\sqrt{8c_1 + 4x^4 + 4x^2 + 1} - 1}}{\sqrt{2}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{-\sqrt{8c_1 + 4x^4 + 4x^2 + 1} - 1}}{\sqrt{2}} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt{\sqrt{8c_1 + 4x^4 + 4x^2 + 1} - 1}}{\sqrt{2}} \right\}, \right.$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 113

$$\left\{ y(x) = -\frac{1}{2} \sqrt{-2 - 2\sqrt{4x^4 + 4x^2 + 8-C1} + 1}, y(x) = \frac{1}{2} \sqrt{-2 - 2\sqrt{4x^4 + 4x^2 + 8-C1} + 1}, y(x) = -\frac{1}{2} \sqrt{-2 - 2\sqrt{4x^4 + 4x^2 + 8-C1} + 1}, \right.$$

2.310 ODE No. 310

---

$$x^3 + (5x^2 y(x) + 2y(x)^3) y'(x) + 5xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.166148 (sec), leaf count = 159

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{-\sqrt{2e^{4c_1} + 23x^4} - 5x^2}}{\sqrt{2}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{-\sqrt{2e^{4c_1} + 23x^4} - 5x^2}}{\sqrt{2}} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt{\sqrt{2e^{4c_1} + 23x^4} - 5x^2}}{\sqrt{2}} \right\}, \right.$$

✓ **Maple** : cpu = 0.088 (sec), leaf count = 125

$$\left\{ y(x) = -\frac{1}{2} \sqrt{-10-C1 x^2 - 2\sqrt{23x^4-C1^2} + 2} \frac{1}{\sqrt{-C1}}, y(x) = \frac{1}{2} \sqrt{-10-C1 x^2 - 2\sqrt{23x^4-C1^2} + 2} \frac{1}{\sqrt{-C1}}, y(x) = -\frac{1}{2} \sqrt{-10-C1 x^2 - 2\sqrt{23x^4-C1^2} + 2} \frac{1}{\sqrt{-C1}}, \right.$$

### 2.311 ODE No. 311

$$4x^3 + 9x^2y(x) + (3x^3 + 6x^2y(x) - 3xy(x)^2 + 20y(x)^3) y'(x) + 6xy(x)^2 - y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.22382 (sec), leaf count = 2201

$$\left\{ \left\{ y(x) \rightarrow \frac{x}{20} + \frac{1}{2} \sqrt{-\frac{39x^2}{100} + \frac{\sqrt[3]{99x^6 + 351e^{c_1}x^2 + \sqrt{3}\sqrt{-67037x^{12} + 185406e^{c_1}x^8 - 83733e^{2c_1}x^4 + 32000e^{3c_1}}}}{5\sqrt[3]{23^2/3}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.067 (sec), leaf count = 50

$$\left\{ y(x) = \frac{\text{RootOf}(x^4\_C1^4 + 3x^3\_C1^3\_Z + 3\_C1^2\_Z^2x^2 -\_C1\_Z^3x + 5\_Z^4 - 1)}{\_C1} \right\}$$

### 2.312 ODE No. 312

$$(y(x)y'(x) + x) \left( \frac{x^2}{a} + \frac{y(x)^2}{b} \right) + \frac{(a-b)(y(x)y'(x) - x)}{a+b} = 0$$

✓ **Mathematica** : cpu = 0.617385 (sec), leaf count = 204

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{b} \sqrt{a^2 + 2a^2 W \left( \frac{c_1(a+b)e^{\frac{bx^2}{2a^2} - \frac{b}{2a} - \frac{x^2}{2b} - \frac{1}{2}}} \right) + ab - ax^2 - bx^2}}{\sqrt{a}\sqrt{a+b}} \right\} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{b} \sqrt{a^2 + 2a^2 W \left( \frac{c_1(a+b)e^{\frac{bx^2}{2a^2} - \frac{b}{2a} - \frac{x^2}{2b} - \frac{1}{2}}} \right) + ab - ax^2 - bx^2}}{\sqrt{a}\sqrt{a+b}} \right\}$$

✓ **Maple** : cpu = 1.136 (sec), leaf count = 240

$$\left\{ y(x) = \frac{1}{a} \sqrt{a \left( e^{\frac{1}{2ba^2} \left( -2 \text{lambertW} \left( \frac{1}{2} \frac{(a+b)e^{-1/2}}{ba^2} e^{-1/2} \frac{x^2}{b} e^{1/2} \frac{bx^2}{a^2} e^{-1/2} \frac{b}{a} \left( e^{-\frac{C1}{ab}} \right)^{-1} \right) ba^2 + (-x^2 - b)a^2 + (-b^2 - 2\_C1)a + b^2x^2 \right)} \right)} \right\}$$

### 2.313 ODE No. 313

$$y'(x) (3axy(x)^2 + 2ay(x)^3 - bx^3 + cx^2) - ay(x)^3 + 2bx^3 + 3bx^2y(x) + cy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.319681 (sec), leaf count = 537

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{2}(3acx + 3ac_1)}{3a\sqrt[3]{\sqrt{(27a^2bx^3 + 27a^2c_1x)^2 + 4(3acx + 3ac_1)^3 + 27a^2bx^3 + 27a^2c_1x}}} - \frac{\sqrt[3]{\sqrt{(27a^2bx^3 + 27a^2c_1x)^2 + 4(3acx + 3ac_1)^3 + 27a^2bx^3 + 27a^2c_1x}}}{\sqrt[3]{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.155 (sec), leaf count = 748

$$\left\{ y(x) = \frac{1}{6a} \left( (-12cx + 12\_C1)a + \left( \left( -108bx^3 + 108\_C1x + 12\sqrt{\frac{81ab^2x^6 - 162\_C1abx^4 + 12c^3x^3 + 8c^2x^2 - 16c_1cx + 16c_1^2}{4}} \right) \right) \right)$$

### 2.314 ODE No. 314

$$xy(x)^3y'(x) + y(x)^4 - x\sin(x) = 0$$

✓ **Mathematica** : cpu = 0.146554 (sec), leaf count = 188

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt[4]{c_1 - 4x^4 \cos(x) + 16x^3 \sin(x) + 48x^2 \cos(x) - 96x \sin(x) - 96 \cos(x)}}{x} \right\} \right\}, \left\{ y(x) \rightarrow -\frac{i\sqrt[4]{c_1 - 4x^4 \cos(x) + 16x^3 \sin(x) + 48x^2 \cos(x) - 96x \sin(x) - 96 \cos(x)}}{x} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 158

$$\left\{ y(x) = \frac{1}{x} \sqrt[4]{(-4x^4 + 48x^2 - 96) \cos(x) + (16x^3 - 96x) \sin(x) + \_C1}, y(x) = \frac{-i}{x} \sqrt[4]{(-4x^4 + 48x^2 - 96) \cos(x) + (16x^3 - 96x) \sin(x) + \_C1} \right\}$$

2.315 ODE No. 315

---

$$(2xy(x)^3 - x^4) y'(x) + 2x^3y(x) - y(x)^4 = 0$$

✓ **Mathematica** : cpu = 0.179613 (sec), leaf count = 368

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{\frac{2}{3}} e^{c_1} x}{\sqrt[3]{\sqrt{3} \sqrt{27x^6 - 4e^{3c_1} x^3 - 9x^3}}} + \frac{\sqrt[3]{\sqrt{3} \sqrt{27x^6 - 4e^{3c_1} x^3 - 9x^3}}}{\sqrt[3]{23^{2/3}}} \right\}, \left\{ y(x) \rightarrow -\frac{(1 + i\sqrt{3}) e^{c_1}}{2^{2/3} \sqrt[3]{3} \sqrt[3]{\sqrt{3} \sqrt{27x^6 - 4e^{3c_1} x^3 - 9x^3}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 376

$$\left\{ y(x) = \frac{\sqrt[3]{12}}{6 - C_1} \left( x \sqrt[3]{12} C_1 + \left( x \left( -9 - C_1 x^2 + \sqrt{3} \sqrt{\frac{27 - C_1^3 x^4 - 4x}{-C_1}} \right) - C_1^2 \right)^{\frac{2}{3}} \right) \frac{1}{\sqrt[3]{x \left( -9 - C_1 x^2 + \sqrt{3} \sqrt{\frac{27 - C_1^3 x^4 - 4x}{-C_1}} \right)}} \right\}$$

2.316 ODE No. 316

---

$$(2xy(x)^3 + y(x)) y'(x) + 2y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.125444 (sec), leaf count = 48

$$\left\{ \{y(x) \rightarrow 0\}, \text{Solve} \left[ x = c_1 e^{-\frac{1}{2}y(x)^2} - \frac{1}{4} e^{-\frac{1}{2}y(x)^2} \text{Ei} \left( \frac{y(x)^2}{2} \right), y(x) \right] \right\}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 53

$$\left\{ y(x) = 0, y(x) = \sqrt{-2 \text{RootOf} (e^{-Z} \text{Ei} (1, -Z) + 4e^{-Z} C_1 - 4x)}, y(x) = -\sqrt{-2 \text{RootOf} (e^{-Z} \text{Ei} (1, -Z) + 4e^{-Z} C_1 - 4x)} \right\}$$

2.317 ODE No. 317

---

$$(x^2 + 2xy(x)^3 + xy(x)) y'(x) + y(x)^2 - xy(x) = 0$$

✓ **Mathematica** : cpu = 0.322883 (sec), leaf count = 23

$$\text{Solve} \left[ y(x)^2 - \frac{x}{y(x)} + \log(y(x)) + \log(x) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.104 (sec), leaf count = 29

$$\left\{ y(x) = e^{\text{RootOf} \left( -(e^{-Z})^3 - e^{-Z} \ln(x) + e^{-Z} C_1 - Z e^{-Z} + x \right)} \right\}$$

2.318 ODE No. 318

---

$$(3xy(x)^3 - 4xy(x) + y(x)) y'(x) + (y(x)^2 - 2) y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.227473 (sec), leaf count = 4284

$$\left\{ \{y(x) \rightarrow 0\}, \left\{ y(x) \rightarrow -\sqrt{\frac{4\sqrt[3]{2}x^2}{3\sqrt[3]{16x^6 + 24x^5 - 27c_1^2x^4 + 12x^4 + 2x^3 + 3\sqrt{3}\sqrt{-32c_1^2x^{10} - 48c_1^2x^9 + 27c_1^4x^8 - 32c_1^2x^7 + 12c_1^2x^6 - 3c_1^2x^5 - 3c_1^2x^4 - 3c_1^2x^3 - 3c_1^2x^2 - 3c_1^2x - 3c_1^2}}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 28

$$\left\{ x + (y(x))^{-2} - \frac{-C_1}{(y(x))^2} \frac{1}{\sqrt{(y(x))^2 - 2}} = 0, y(x) = 0 \right\}$$

## 2.319 ODE No. 319

---

$$(7xy(x)^3 + y(x) - 5x) y'(x) + y(x)^4 - 5y(x) = 0$$

✓ **Mathematica** : cpu = 0.143065 (sec), leaf count = 302

$$\left\{ \left\{ y(x) \rightarrow \text{Root} \left[ 10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 1 \right] \right\}, \left\{ y(x) \rightarrow \text{Root} \left[ 10\#1^7x + 2\#1^5 \right. \right. \right.$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 35

$$\left\{ x + \frac{2 (y(x))^5 - 25 (y(x))^2 - 10\_C1}{10 \left( (y(x))^3 - 5 \right)^2 y(x)} = 0 \right\}$$

## 2.320 ODE No. 320

---

$$(x^2y(x)^3 + xy(x)) y'(x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.110936 (sec), leaf count = 76

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{2xW\left(c_1e^{\frac{1}{2x}-1}\right) + 2x - 1}}{\sqrt{x}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{2xW\left(c_1e^{\frac{1}{2x}-1}\right) + 2x - 1}}{\sqrt{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.07 (sec), leaf count = 78

$$\left\{ y(x) = \frac{1}{x} \sqrt{2x^2 \text{lambertW}\left(1/2\_C1 e^{-1/2 \frac{2x-1}{x}}\right) + 2x^2 - x}, y(x) = -\frac{1}{x} \sqrt{2x^2 \text{lambertW}\left(1/2\_C1 e^{-1/2 \frac{2x-1}{x}}\right)} \right\}$$

## 2.321 ODE No. 321

---

$$(2x^2y(x)^3 + x^2y(x)^2 - 2x) y'(x) - 2y(x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.326769 (sec), leaf count = 47

$$\text{Solve} \left[ \frac{1}{64} (-4y(x)^2 + 4y(x) - 2 \log(8y(x) + 4) + 3) - \frac{1}{4x(2y(x) + 1)} = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.13 (sec), leaf count = 42

$$\left\{ y(x) = \frac{e^{\text{RootOf}\left(x(e^{-Z})^3 - 4x(e^{-Z})^2 + 8\_C1xe^{-Z} + 2\_Zxe^{-Z} + 3xe^{-Z} + 16\right)}}{2} - \frac{1}{2} \right\}$$

## 2.322 ODE No. 322

---

$$(10x^2y(x)^3 - 3y(x)^2 - 2) y'(x) + 5xy(x)^4 + x = 0$$

✓ **Mathematica** : cpu = 0.273817 (sec), leaf count = 2077

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2} \sqrt{\frac{4\sqrt[3]{2}(5x^4 - 10c_1x^2 - 2)}{5x^2\sqrt[3]{2268x^2 - 216c_1} + \sqrt{(2160x^2 + 108(x^2 - 2c_1))^2 - 4(60x^4 - 120c_1x^2 - 24)^3}}} + \frac{\sqrt[3]{2268x^2}}{2} \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 29

$$\left\{ \frac{5x^2(y(x))^4}{2} - (y(x))^3 + \frac{x^2}{2} - 2y(x) + \_C1 = 0 \right\}$$





2.327 ODE No. 327

---

$$(2x^2y(x)^3 + xy(x)^4 + 2y(x) + x) y'(x) + y(x)^5 + y(x) = 0$$

✓ **Mathematica** : cpu = 0.329989 (sec), leaf count = 669

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{9c_1^2x^2 + 3\sqrt{3}\sqrt{-4c_1^3x^6 - c_1^4x^4 + 18c_1^2x^4 + 4c_1^3x^2 + 27x^4 + 2c_1^3 + 27x^2}}}{3\sqrt[3]{2}x} - \frac{3x\sqrt[3]{9c_1^2x^2 + 3\sqrt{3}\sqrt{-4c_1^3x^6 - c_1^4x^4 + 18c_1^2x^4 + 4c_1^3x^2 + 27x^4 + 2c_1^3 + 27x^2}}}{3\sqrt[3]{2}x} \right\} \right.$$

✓ **Maple** : cpu = 0.123 (sec), leaf count = 583

$$\left\{ y(x) = \frac{1}{12\_C1\,x} \left( \left( -12\,ix^2\_C1 - i \left( 108\_C1^3x^2 + 12\sqrt{3}\sqrt{27\_C1^4x^2 + 18\_C1^2x^2 + (4x^4 - 4)\_C1 - x} \right) \right) \right.$$

2.328 ODE No. 328

---

$$ax^2y(x)^ny'(x) - 2xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.235148 (sec), leaf count = 42

$$\text{Solve} \left[ \frac{n(\log(x) - \log(-axy(x)^n + n + 2))}{n + 2} - \frac{2n \log(y(x))}{n + 2} = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.145 (sec), leaf count = 33

$$\left\{ \frac{x^n}{((y(x))^n ax - n - 2)^n ((y(x))^n)^2} -\_C1 = 0 \right\}$$

2.329 ODE No. 329

---

$$x^ny(x)^m (axy'(x) + by(x)) + \alpha xy'(x) + \beta y(x) = 0$$

✓ **Mathematica** : cpu = 0.671128 (sec), leaf count = 102

$$\text{Solve} \left[ \frac{m((a\beta - \alpha b) \log(x^ny(x)^m(bm - an) - \alpha n + \beta m) + \beta \log(x)(bm - an))}{(bm - an)(\beta m - \alpha n)} + \frac{\alpha m \log(\beta my(x) - \alpha ny(x))}{\beta m - \alpha n} = \right.$$

✓ **Maple** : cpu = 0.269 (sec), leaf count = 71

$$\left\{ ((y(x))^m)^{\alpha(an-bm)} (x^n(an-bm)(y(x))^m - \beta m + \alpha n)^{-a\beta m + bm\alpha} x^{\beta m(an-bm)} -\_C1 = 0 \right\}$$

2.330 ODE No. 330

---

$$(f(y(x) + x) + 1)y'(x) + f(y(x) + x) = 0$$

✓ **Mathematica** : cpu = 0.101989 (sec), leaf count = 52

$$\text{Solve} \left[ \int_1^{y(x)} \left( f(x + K[2]) - \int_1^x f'(K[1] + K[2])dK[1] + 1 \right) dK[2] + \int_1^x f(K[1] + y(x))dK[1] = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 22

$$\left\{ y(x) = -x + \text{RootOf} \left( -x + \int^{-Z} 1 + f(\_a) d\_a +\_C1 \right) \right\}$$

2.331 ODE No. 331

---

$$y'(x) \left( \sum_{\nu=1}^p y(x)^\nu f(\nu)(x) \right) - \sum_{\nu=1}^q y(x)^\nu g(\nu)(x) = 0$$

✗ **Mathematica** : cpu = 49.4735 (sec), leaf count = 0 , could not solve  
DSolve[-Sum[y[x]^nu\*g[nu][x], {nu, 1, q}] + Sum[y[x]^nu\*f[nu][x], {nu, 1, p}]\*Derivative[1][y[x]], y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x)\*sum(f[nu](x)\*y(x)^nu,nu = 1 .. p)-sum(g[nu](x)\*y(x)^nu,nu = 1 .. q) = 0, y(x))

**2.332 ODE No. 332**

---

$$x \left( \sqrt{xy(x)} - 1 \right) y'(x) - y(x) \left( \sqrt{xy(x)} + 1 \right) = 0$$

✓ **Mathematica** : cpu = 0.20701 (sec), leaf count = 34

$$\text{Solve} \left[ \log(y(x)) - \frac{y(x) \log(x) - \frac{2y(x)}{\sqrt{xy(x)}}}{y(x)} = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 33

$$\left\{ -1 \left( 1 + \left( -C1 - \ln(x) + \frac{\ln(xy(x))}{2} \right) \sqrt{xy(x)} \right) \frac{1}{\sqrt{xy(x)}} = 0 \right\}$$

**2.333 ODE No. 333**

---

$$-x^{3/2}y(x)^{5/2} + \left( 2x^{5/2}y(x)^{3/2} + x^2y(x) - x \right) y'(x) + xy(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.311406 (sec), leaf count = 72

$$\text{Solve} \left[ \frac{2\sqrt{xy(x)} \log(y(x))}{\sqrt{x} \sqrt{y(x)}} - \frac{\sqrt{xy(x)} \left( 3x^{3/2}y(x)^{3/2} \log(x) + 6xy(x) - 2 \right)}{3x^2y(x)^2} = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.073 (sec), leaf count = 32

$$\left\{ \ln(y(x)) - 1 \frac{1}{\sqrt{x}} \frac{1}{\sqrt{y(x)}} + \frac{1}{3} (y(x))^{-\frac{3}{2}} x^{-\frac{3}{2}} - \frac{\ln(x)}{2} - C1 = 0 \right\}$$

**2.334 ODE No. 334**

---

$$\left( \sqrt{y(x) + x} + 1 \right) y'(x) + 1 = 0$$

✓ **Mathematica** : cpu = 0.0994207 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow -2\sqrt{c_1 + x + 1} + c_1 + 2 \right\}, \left\{ y(x) \rightarrow 2\sqrt{c_1 + x + 1} + c_1 + 2 \right\} \right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 19

$$\left\{ -y(x) - 2\sqrt{y(x) + x} - C1 = 0 \right\}$$

**2.335 ODE No. 335**

---

$$\sqrt{y(x)^2 - 1} y'(x) - \sqrt{x^2 - 1} = 0$$

✓ **Mathematica** : cpu = 0.196418 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{1}{2} \#1 \sqrt{\#1^2 - 1} - \frac{1}{2} \log \left( \sqrt{\#1^2 - 1} + \#1 \right) \& \right] \left[ c_1 + \frac{1}{2} \sqrt{x^2 - 1} x - \frac{1}{2} \log \left( \sqrt{x^2 - 1} \right) \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.008 (sec), leaf count = 50

$$\left\{ -C1 + x\sqrt{x^2 - 1} - \ln \left( x + \sqrt{x^2 - 1} \right) - y(x) \sqrt{(y(x))^2 - 1} + \ln \left( y(x) + \sqrt{(y(x))^2 - 1} \right) = 0 \right\}$$

**2.336 ODE No. 336**

---

$$\left( ax + \sqrt{y(x)^2 + 1} \right) y'(x) + ay(x) + \sqrt{x^2 + 1} = 0$$

✓ **Mathematica** : cpu = 0.257103 (sec), leaf count = 53

$$\text{Solve} \left[ axy(x) + \frac{1}{2} \sqrt{x^2 + 1} x + \frac{1}{2} \left( y(x) \sqrt{y(x)^2 + 1} + \sinh^{-1}(y(x)) \right) + \frac{1}{2} \sinh^{-1}(x) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 41

$$\left\{ \frac{x}{2} \sqrt{x^2 + 1} + \frac{\text{Arcsinh}(x)}{2} + axy(x) + \frac{y(x)}{2} \sqrt{(y(x))^2 + 1} + \frac{\text{Arcsinh}(y(x))}{2} + C1 = 0 \right\}$$

**2.337      ODE No. 337**

---

$$\left(\sqrt{x^2+y(x)^2}+x\right)y'(x)-y(x)=0$$

✓ **Mathematica** : cpu = 0.225348 (sec), leaf count = 161

$$\left\{\left\{y(x)\rightarrow-\sqrt{-2ix\sinh(c_1)-2ix\cosh(c_1)-\sinh(2c_1)-\cosh(2c_1)}\right\},\left\{y(x)\rightarrow\sqrt{-2ix\sinh(c_1)-2ix\cosh(c_1)-\sinh(2c_1)-\cosh(2c_1)}\right\}\right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 28

$$\left\{-\_C1+\frac{1}{(y(x))^2}\sqrt{(y(x))^2+x^2}+\frac{x}{(y(x))^2}=0\right\}$$

**2.338      ODE No. 338**

---

$$y'(x)\left(\sin(\alpha)\left(y(x)^2-x^2\right)-2x\cos(\alpha)y(x)+\sqrt{x^2+y(x)^2}y(x)\right)+\cos(\alpha)\left(y(x)^2-x^2\right)+2x\sin(\alpha)y(x)+x\sqrt{x^2+y(x)^2}=0$$

✓ **Mathematica** : cpu = 62.9797 (sec), leaf count = 17681

Too large to display

✓ **Maple** : cpu = 0.517 (sec), leaf count = 129

$$\left\{y(x)=RootOf\left(-\ln(x)+\int^{-Z}\frac{1}{(\_a^2+1)(\cos(2\alpha)\_a^2+2\_a\sin(2\alpha)+\_a^2-\cos(2\alpha)+1)}\right)(-\cos(2\alpha))\right\}$$

**2.339      ODE No. 339**

---

$$\left(x\sqrt{x^2+y(x)^2+1}-y(x)\left(x^2+y(x)^2\right)\right)y'(x)-\sqrt{x^2+y(x)^2+1}y(x)-x\left(x^2+y(x)^2\right)=0$$

✓ **Mathematica** : cpu = 0.338255 (sec), leaf count = 27

$$\text{Solve}\left[\sqrt{x^2+y(x)^2+1}+\tan^{-1}\left(\frac{x}{y(x)}\right)=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.134 (sec), leaf count = 27

$$\left\{\arctan\left(\frac{y(x)}{x}\right)-\sqrt{x^2+(y(x))^2+1}-\_C1=0\right\}$$

**2.340      ODE No. 340**

---

$$y'(x)\left(\frac{e1(a+x)}{((a+x)^2+y(x)^2)^{3/2}}+\frac{e2(x-a)}{((x-a)^2+y(x)^2)^{3/2}}\right)-y(x)\left(\frac{e1}{((a+x)^2+y(x)^2)^{3/2}}+\frac{e2}{((x-a)^2+y(x)^2)^{3/2}}\right)=0$$

✗ **Mathematica** : cpu = 84.0191 (sec), leaf count = 0 , could not solve

`DSolve[-(y[x]*(e2/((-a + x)^2 + y[x]^2)^(3/2) + e1/((a + x)^2 + y[x]^2)^(3/2))) + ((e2*(-a + x))/((-a + x)^2 + y[x]^2)^(3/2) + (e1*(a + x))/((a + x)^2 + y[x]^2)^(3/2))*Derivative[1][y[x]]]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception  
time expired

**2.341      ODE No. 341**

---

$$\left(xe^{y(x)}+e^x\right)y'(x)+e^xy(x)+e^{y(x)}=0$$

✓ **Mathematica** : cpu = 0.227931 (sec), leaf count = 33

$$\left\{\left\{y(x)\rightarrow c_1e^{-x}-W\left(xe^{c_1e^{-x}-x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 33

$$\left\{y(x)=\frac{1}{e^x}\left(-\text{lambertW}\left(\frac{x}{e^x}\left(e^{\frac{-C1}{e^x}}\right)^{-1}\right)e^x-\_C1\right)\right\}$$

**2.342      ODE No. 342**

---

$$x \left( 2e^{-xy(x)} + 3e^{xy(x)} \right) (xy'(x) + y(x)) + 1 = 0$$

✓ **Mathematica** : cpu = 0.443145 (sec), leaf count = 163

$$\left\{ \left\{ y(x) \rightarrow -\frac{\cosh^{-1} \left( \frac{1}{24} \left( -5\sqrt{\log^2 \left( \frac{c_1}{x} \right) + 24} - \log \left( \frac{c_1}{x} \right) \right) \right)}{x} \right\} \right\}, \left\{ y(x) \rightarrow \frac{\cosh^{-1} \left( \frac{1}{24} \left( -5\sqrt{\log^2 \left( \frac{c_1}{x} \right) + 24} - \log \left( \frac{c_1}{x} \right) \right) \right)}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 17

$$\left\{ y(x) = \frac{1}{x} \ln \left( -\frac{\ln(x)}{5} + \frac{-C1}{5} \right) \right\}$$

**2.343      ODE No. 343**

---

$$y'(x)(\log(y(x)) + x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.131187 (sec), leaf count = 35

$$\text{Solve} \left[ x = c_1 e^{y(x)} + e^{y(x)} \left( \text{Ei}(-y(x)) - e^{-y(x)} \log(y(x)) \right), y(x) \right]$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 27

$$\left\{ y(x) = e^{\text{RootOf} \left( -x - \_Z - \text{Ei}(1, e^{-Z}) e^{e^{-Z}} + e^{e^{-Z}} - C1 \right)} \right\}$$

**2.344      ODE No. 344**

---

$$y'(x)(\log(y(x)) + 2x - 1) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.188756 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow -\frac{W \left( -2c_1 e^{-2x} \right)}{2c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 19

$$\left\{ y(x) = e^{-\text{lambertW}(-2e^{-2x}C1) - 2x} \right\}$$

**2.345      ODE No. 345**

---

$$xy'(x) (2x^2 y(x) \log(y(x)) + 1) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.165102 (sec), leaf count = 35

$$\text{Solve} \left[ \frac{y(x)}{x^2} + 2 \left( \frac{1}{2} y(x)^2 \log(y(x)) - \frac{y(x)^2}{4} \right) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 36

$$\left\{ y(x) = e^{\text{RootOf} \left( 2\_Z x^2 (e^{-Z})^2 - x^2 (e^{-Z})^2 + 2\_C1 x^2 + 2e^{-Z} \right)} \right\}$$

**2.346      ODE No. 346**

---

$$xy'(x)(-ax + y(x) + y(x) \log(xy(x))) - y(x)(ax \log(xy(x)) + ax - y(x)) = 0$$

✓ **Mathematica** : cpu = 0.358939 (sec), leaf count = 24

$$\text{Solve}[ax \log(xy(x)) - y(x) \log(xy(x)) = c_1, y(x)]$$

✓ **Maple** : cpu = 0.204 (sec), leaf count = 19

$$\left\{ (xy(x))^{-ax+y(x)} - \_C1 = 0 \right\}$$

**2.347 ODE No. 347**

---

$$(\sin(x) + 1)y'(x) \sin(y(x)) + \cos(x)(\cos(y(x)) - 1) = 0$$

✓ **Mathematica** : cpu = 0.287271 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow 0 \right\}, \left\{ y(x) \rightarrow 2 \sin^{-1} \left( \frac{1}{4} c_1 \left( \sin \left( \frac{x}{2} \right) + \cos \left( \frac{x}{2} \right) \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.132 (sec), leaf count = 16

$$\{y(x) = \pi - \arccos(\sin(x) \_C1 + \_C1 - 1)\}$$

**2.348 ODE No. 348**

---

$$y'(x)(x \cos(y(x)) + \sin(x)) + \sin(y(x)) + y(x) \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.179223 (sec), leaf count = 17

$$\text{Solve}[x \sin(y(x)) + y(x) \sin(x) = c_1, y(x)]$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 15

$$\{y(x) \sin(x) + x \sin(y(x)) + \_C1 = 0\}$$

**2.349 ODE No. 349**

---

$$xy'(x) \cot\left(\frac{y(x)}{x}\right) + 2x \sin\left(\frac{y(x)}{x}\right) - y(x) \cot\left(\frac{y(x)}{x}\right) = 0$$

✓ **Mathematica** : cpu = 0.224452 (sec), leaf count = 15

$$\left\{ \left\{ y(x) \rightarrow x \csc^{-1}(2(c_1 + \log(x))) \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 17

$$\left\{ y(x) = \arcsin\left((2 \ln(x) + 2 \_C1)^{-1}\right) x \right\}$$

**2.350 ODE No. 350**

---

$$y'(x) \cos(y(x)) - \sin(y(x)) - \cos(x) \sin^2(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.547628 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow \csc^{-1} \left( \frac{1}{2} (-2c_1 e^{-x} - \sin(x) - \cos(x)) \right) \right\}, \left\{ y(x) \rightarrow -\csc^{-1} \left( \frac{1}{2} (2c_1 e^{-x} + \sin(x) + \cos(x)) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.677 (sec), leaf count = 226

$$\left\{ y(x) = \arctan \left( -2 \frac{e^x}{e^x (\cos(x) + \sin(x)) + 2 \_C1}, \frac{\sqrt{16}}{4 \_C1^2 + 4 e^x (\cos(x) + \sin(x)) \_C1 + (e^x)^2 (2 \sin(x) \cos(x))} \right) \right\}$$

**2.351 ODE No. 351**

---

$$y'(x) \cos(y(x)) - \sin^3(y(x)) + x \sin(y(x)) \cos^2(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.468626 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow -\cot^{-1} \left( \sqrt{e^{x^2} (4c_1 - \sqrt{\pi} \text{erf}(x))} \right) \right\}, \left\{ y(x) \rightarrow \cot^{-1} \left( \sqrt{e^{x^2} (4c_1 - \sqrt{\pi} \text{erf}(x))} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.333 (sec), leaf count = 55

$$\left\{ y(x) = -\arcsin \left( \frac{1}{\sqrt{1 - \sqrt{\pi} \text{Erf}(x) e^{x^2} - 2 \_C1 e^{x^2}}} \right), y(x) = \arcsin \left( \frac{1}{\sqrt{1 - \sqrt{\pi} \text{Erf}(x) e^{x^2} - 2 \_C1 e^{x^2}}} \right) \right\}$$

## 2.352 ODE No. 352

---

$$y'(x) \cos(y(x))(\cos(y(x)) - \sin(\alpha) \sin(x)) + \cos(x)(\cos(x) - \sin(\alpha) \sin(y(x))) = 0$$

✓ **Mathematica** : cpu = 0.526993 (sec), leaf count = 43

$$\text{Solve} \left[ 4 \sin(\alpha) \sin(x) \sin(y(x)) - 4 \left( \frac{y(x)}{2} + \frac{1}{4} \sin(2y(x)) \right) - 2x - \sin(2x) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.35 (sec), leaf count = 33

$$\left\{ \frac{(-2 \sin(\alpha) \sin(x) + \cos(y(x))) \sin(y(x))}{2} + \frac{\sin(x) \cos(x)}{2} + \frac{x}{2} + \_C1 + \frac{y(x)}{2} = 0 \right\}$$

## 2.353 ODE No. 353

---

$$xy'(x) \cos(y(x)) + \sin(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.0753412 (sec), leaf count = 14

$$\left\{ \left\{ y(x) \rightarrow \sin^{-1} \left( \frac{e^{c_1}}{x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 12

$$\left\{ y(x) = \arcsin \left( \frac{1}{\_C1 x} \right) \right\}$$

## 2.354 ODE No. 354

---

$$y'(x)(x \sin(y(x)) - 1) + \cos(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.134014 (sec), leaf count = 145

$$\left\{ \left\{ y(x) \rightarrow -\cos^{-1} \left( \frac{c_1 x - \sqrt{c_1^2 - x^2 + 1}}{c_1^2 + 1} \right) \right\}, \left\{ y(x) \rightarrow \cos^{-1} \left( \frac{c_1 x - \sqrt{c_1^2 - x^2 + 1}}{c_1^2 + 1} \right) \right\}, \left\{ y(x) \rightarrow -\cos^{-1} \left( \frac{\sqrt{c_1^2 - x^2 + 1}}{c_1^2 + 1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 108

$$\left\{ y(x) = \arctan \left( \frac{1}{\_C1^2 + 1} \left( -\_C1 \sqrt{\_C1^2 - x^2 + 1} + x \right), \frac{1}{\_C1^2 + 1} \left( -\_C1 x + \sqrt{\_C1^2 - x^2 + 1} \right) \right), y(x) = \right\}$$

## 2.355 ODE No. 355

---

$$y'(x)(x \cos(y(x)) + \cos(x)) + \sin(y(x)) - y(x) \sin(x) = 0$$

✓ **Mathematica** : cpu = 0.160493 (sec), leaf count = 17

$$\text{Solve}[x \sin(y(x)) + y(x) \cos(x) = c_1, y(x)]$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 15

$$\{y(x) \cos(x) + x \sin(y(x)) + \_C1 = 0\}$$

## 2.356 ODE No. 356

---

$$y'(x) (x^2 \cos(y(x)) + 2y(x) \sin(x)) + 2x \sin(y(x)) + y(x)^2 \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.227113 (sec), leaf count = 21

$$\text{Solve}[x^2 \sin(y(x)) + y(x)^2 \sin(x) = c_1, y(x)]$$

✓ **Maple** : cpu = 0.084 (sec), leaf count = 19

$$\left\{ (y(x))^2 \sin(x) + x^2 \sin(y(x)) + \_C1 = 0 \right\}$$

## 2.357 ODE No. 357

---

$$x \log(x) y'(x) \sin(y(x)) + \cos(y(x))(1 - x \cos(y(x))) = 0$$

✓ **Mathematica** : cpu = 0.503776 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow -\sec^{-1} \left( \frac{x - c_1}{\log(x)} \right) \right\}, \left\{ y(x) \rightarrow \sec^{-1} \left( \frac{x - c_1}{\log(x)} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.378 (sec), leaf count = 13

$$\left\{ y(x) = \arccos \left( \frac{\ln(x)}{x + \_C1} \right) \right\}$$

## 2.358 ODE No. 358

---

$$\cos(x) y'(x) \sin(y(x)) + \sin(x) \cos(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.144335 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow -\cos^{-1} \left( \frac{1}{2} c_1 \sec(x) \right) \right\}, \left\{ y(x) \rightarrow \cos^{-1} \left( \frac{1}{2} c_1 \sec(x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.076 (sec), leaf count = 11

$$\left\{ y(x) = \arccos \left( \frac{\_C1}{\cos(x)} \right) \right\}$$

## 2.359 ODE No. 359

---

$$3 \sin(x) y'(x) \sin(y(x)) + 5 y(x) \cos^4(x) = 0$$

✓ **Mathematica** : cpu = 0.300361 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow \text{SinIntegral}^{(-1)} \left( c_1 - \frac{5}{3} \left( \frac{5 \cos(x)}{4} + \frac{1}{12} \cos(3x) + \log \left( \sin \left( \frac{x}{2} \right) \right) - \log \left( \cos \left( \frac{x}{2} \right) \right) \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 28

$$\left\{ \frac{\cos(3x)}{12} + \frac{5 \cos(x)}{4} + \ln(\csc(x) - \cot(x)) + \frac{3 \text{Si}(y(x))}{5} + \_C1 = 0 \right\}$$

## 2.360 ODE No. 360

---

$$y'(x) \cos(ay(x)) - b(1 - c \cos(ay(x))) \sqrt{c \cos(ay(x)) + \cos^2(ay(x))} - 1 = 0$$

✓ **Mathematica** : cpu = 4.3337 (sec), leaf count = 369

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ - \frac{i(\cos(\#1a) + 1) \sqrt{\frac{2c \cos(\#1a) + \cos(2\#1a) - 1}{(\cos(\#1a) + 1)^2}} \sqrt{\frac{c \tan^2\left(\frac{\#1a}{2}\right) + \sqrt{c^2 + 4} + 2}}{\sqrt{c^2 + 4} + 2}} \sqrt{1 - \frac{c \tan^2\left(\frac{\#1a}{2}\right)}{\sqrt{c^2 + 4} - 2}}} \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.25 (sec), leaf count = 48

$$\left\{ x + \int^{y(x)} 2 \frac{\cos(\_a a)}{b(c \cos(\_a a) - 1) \sqrt{2 \cos(2\_a a) - 2 + 4 c \cos(\_a a)}} d\_a + \_C1 = 0 \right\}$$

## 2.361 ODE No. 361

---

$$y'(x)(-\sin(y(x)) + x \sin(xy(x)) + \cos(y(x) + x)) + y(x) \sin(xy(x)) + \cos(y(x) + x) + \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.631822 (sec), leaf count = 31

$$\text{Solve}[\cos(y(x)) - \cos(xy(x)) + \sin(x) \cos(y(x)) + \cos(x) \sin(y(x)) + \sin(x) = c_1, y(x)]$$

✓ **Maple** : cpu = 0.165 (sec), leaf count = 22

$$\{-\cos(xy(x)) + \sin(x) + \sin(y(x) + x) + \cos(y(x)) + \_C1 = 0\}$$

**2.362 ODE No. 362**

---

$$y'(x) \left( x^2 y(x) \sin(xy(x)) - 4x \right) - y(x) + xy(x)^2 \sin(xy(x)) = 0$$

✓ **Mathematica** : cpu = 0.355899 (sec), leaf count = 23

$$\text{Solve}[-4 \log(y(x)) - \cos(xy(x)) - \log(x) = c_1, y(x)]$$

✓ **Maple** : cpu = 0.184 (sec), leaf count = 23

$$\left\{ y(x) = \frac{1}{x} \text{RootOf} \left( -\_Z + e^{-\frac{\cos(\_Z)}{4}} \_C1 x^{\frac{3}{4}} \right) \right\}$$

**2.363 ODE No. 363**

---

$$(xy'(x) - y(x)) \cos^2 \left( \frac{y(x)}{x} \right) + x = 0$$

✓ **Mathematica** : cpu = 0.3346 (sec), leaf count = 33

$$\text{Solve} \left[ \frac{y(x)}{2x} + \frac{1}{4} \sin \left( \frac{2y(x)}{x} \right) = c_1 - \log(x), y(x) \right]$$

✓ **Maple** : cpu = 0.063 (sec), leaf count = 35

$$\left\{ -\frac{1}{2x} \left( \cos \left( \frac{y(x)}{x} \right) \sin \left( \frac{y(x)}{x} \right) x + y(x) \right) - \ln(x) - \_C1 = 0 \right\}$$

**2.364 ODE No. 364**

---

$$xy'(x) \left( y(x) \sin \left( \frac{y(x)}{x} \right) - x \cos \left( \frac{y(x)}{x} \right) \right) - y(x) \left( y(x) \sin \left( \frac{y(x)}{x} \right) + x \cos \left( \frac{y(x)}{x} \right) \right) = 0$$

✓ **Mathematica** : cpu = 0.456237 (sec), leaf count = 31

$$\text{Solve} \left[ -\log \left( \frac{y(x)}{x} \right) - \log \left( \cos \left( \frac{y(x)}{x} \right) \right) = c_1 + 2 \log(x), y(x) \right]$$

✓ **Maple** : cpu = 0.076 (sec), leaf count = 23

$$\left\{ y(x) = \frac{\_C1}{\cos(\text{RootOf}(-\_Z \cos(\_Z) x^2 + \_C1)) x} \right\}$$

**2.365 ODE No. 365**

---

$$(y(x)f(x^2 + y(x)^2) - x)y'(x) + xf(x^2 + y(x)^2) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.346849 (sec), leaf count = 156

$$\text{Solve} \left[ \int_1^{y(x)} \left( \frac{x - f(x^2 + K[2]^2) K[2]}{x^2 + K[2]^2} - \int_1^x \left( \frac{-2K[1]K[2]f'(K[1]^2 + K[2]^2) - 1}{K[1]^2 + K[2]^2} - \frac{2(-f(K[1]^2 + K[2]^2) K[2]}{(K[1]^2 + K[2]^2)} \right) dK[1] \right) dK[2] + \int_1^x (y(x)) \right]$$

✓ **Maple** : cpu = 0.261 (sec), leaf count = 42

$$\left\{ y(x) = x \left( \tan \left( \text{RootOf} \left( -2\_Z - \int \frac{x^2((\tan(\_Z))^2 + 1)}{(\tan(\_Z))^2} \frac{f(\_a)}{\_a} d\_a + 2\_C1 \right) \right) \right)^{-1} \right\}$$

**2.366 ODE No. 366**

---

$$f(ay(x)^2 + x^2)(ay(x)y'(x) + x) - xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.269191 (sec), leaf count = 91

$$\text{Solve} \left[ \int_1^{y(x)} \left( x - af(x^2 + aK[2]^2) K[2] - \int_1^x (1 - 2aK[1]K[2]f'(K[1]^2 + aK[2]^2)) dK[1] \right) dK[2] + \int_1^x (y(x)) \right]$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 45

$$\left\{ -ax(y(x))^2 \frac{1}{\sqrt{a^2(y(x))^2}} - \int^{-\frac{a(y(x))^2}{2} - \frac{x^2}{2}} f(-2\_a) d\_a + \_C1 = 0 \right\}$$



2.367 ODE No. 367

---

$$f\left(x^cy(x)\right)\left(bxy'(x)-a\right)-x^ay(x)^b\left(cy(x)+xy'(x)\right)=0$$

✗ **Mathematica** : cpu = 11.6919 (sec), leaf count = 0 , could not solve  
DSolve[-(x^a\*y[x]^b\*(c\*y[x] + x\*Derivative[1][y][x])) + f[x^c\*y[x]]\*(-a + b\*x\*Derivative[1][y][x]), y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(f(x^c\*y(x))\*(b\*x\*diff(y(x),x)-a)-x^a\*y(x)^b\*(x\*diff(y(x),x)+c\*y(x)) = 0,y(x))

2.368 ODE No. 368

---

$$ay(x)+bx^2+y'(x)^2=0$$

✓ **Mathematica** : cpu = 1.37749 (sec), leaf count = 795  
$$\left\{\text{Solve}\left[\int_1^x\left(\frac{2\sqrt{-bK[1]^2-ay(x)}y(x)}{bK[1]^4+ay(x)K[1]^2+4y(x)^2}+\frac{bK[1]^3+ay(x)K[1]}{bK[1]^4+ay(x)K[1]^2+4y(x)^2}\right)dK[1]+\int_1^{y(x)}\left(-\frac{\sqrt{-bx^2-ay(x)}}{bx^4+aK[2]^2}\right)dy(x)\right],y(x)\right\}$$
  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception  
time expired

2.369 ODE No. 369

---

$$-a^2+y'(x)^2+y(x)^2=0$$

✓ **Mathematica** : cpu = 0.0581839 (sec), leaf count = 107  
$$\left\{\left\{y(x)\rightarrow-\frac{a\tan(x-c_1)}{\sqrt{\tan^2(x-c_1)+1}}\right\},\left\{y(x)\rightarrow\frac{a\tan(x-c_1)}{\sqrt{\tan^2(x-c_1)+1}}\right\},\left\{y(x)\rightarrow-\frac{a\tan(c_1+x)}{\sqrt{\tan^2(c_1+x)+1}}\right\},\left\{y(x)\rightarrow\frac{a\tan(c_1+x)}{\sqrt{\tan^2(c_1+x)+1}}\right\}\right\}$$
  
✓ **Maple** : cpu = 0.098 (sec), leaf count = 68  
$$\left\{y(x)=a,y(x)=\tan(-x+_C1)\sqrt{\frac{a^2}{(\tan(-x+_C1))^2+1}},y(x)=-a,y(x)=-\tan(-x+_C1)\sqrt{\frac{a^2}{(\tan(-x+_C1))^2+1}}\right\}$$

2.370 ODE No. 370

---

$$-f(x)^2+y'(x)^2+y(x)^2=0$$

✗ **Mathematica** : cpu = 12.8881 (sec), leaf count = 0 , could not solve  
DSolve[-f[x]^2 + y[x]^2 + Derivative[1][y][x]^2 == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x)^2+y(x)^2-f(x)^2 = 0,y(x))

2.371 ODE No. 371

---

$$y'(x)^2-y(x)^3+y(x)^2=0$$

✓ **Mathematica** : cpu = 0.04002 (sec), leaf count = 37  
$$\left\{\left\{y(x)\rightarrow\tan^2\left(\frac{c_1-x}{2}\right)+1\right\},\left\{y(x)\rightarrow\tan^2\left(\frac{c_1+x}{2}\right)+1\right\}\right\}$$
  
✓ **Maple** : cpu = 0.059 (sec), leaf count = 20  
$$\left\{y(x)=1,y(x)=\left(\tan\left(-\frac{x}{2}+\frac{C1}{2}\right)\right)^2+1\right\}$$

2.372 ODE No. 372

---

$$ay(x)+b+y'(x)^2-4y(x)^3=0$$

✓ **Mathematica** : cpu = 0.0030306 (sec), leaf count = 27  
$$\{\{y(x)\rightarrow\wp(x-c_1;a,b)\},\{y(x)\rightarrow\wp(x+c_1;a,b)\}\}$$
  
✓ **Maple** : cpu = 0.04 (sec), leaf count = 232  
$$\left\{y(x)=-\frac{1}{12}\left(\left(i\left(27b+3\sqrt{-3a^3+81b^2}\right)^{\frac{2}{3}}-3ia\right)\sqrt{3}+\left(27b+3\sqrt{-3a^3+81b^2}\right)^{\frac{2}{3}}+3a\right)\frac{1}{\sqrt[3]{27b+3\sqrt{-3a^3+81b^2}}}\right\}$$

**2.373 ODE No. 373**

---

$$a^2y(x)^2(\log^2(y(x)) - 1) + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.181003 (sec), leaf count = 185

$$\left\{ \left\{ y(x) \rightarrow \exp \left( -\frac{1}{2} \sqrt{-e^{-2c_1+2iax} - e^{2c_1-2iax} + 2} \right) \right\}, \left\{ y(x) \rightarrow \exp \left( \frac{1}{2} \sqrt{-e^{-2c_1+2iax} - e^{2c_1-2iax} + 2} \right) \right\}, \left\{ y(x) \right.$$

✓ **Maple** : cpu = 0.183 (sec), leaf count = 49

$$\left\{ y(x) = \left( e^{-\sin(a(x-_{-}C1))} \right)^{-1}, y(x) = e^{-\sin(a(x-_{-}C1))}, y(x) = e^{\text{RootOf}\left(a^2(e^{-Z})^2(_{-}Z^2-1)\right)} \right\}$$

**2.374 ODE No. 374**

---

$$y'(x)^2 - 2y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.068401 (sec), leaf count = 73

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{\sqrt{\#1^2+1}}{\#1} - \frac{1}{\#1} + \sinh^{-1}(\#1) \& \right] [c_1 - x] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{\sqrt{\#1^2+1}}{\#1} - \frac{1}{\#1} + \sinh^{-1}(\#1) \& \right] [c_1 - x] \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 85

$$\left\{ x - (y(x))^{-1} - \frac{1}{y(x)} \left( (y(x))^2 + 1 \right)^{\frac{3}{2}} + y(x) \sqrt{(y(x))^2 + 1} + \text{Arcsinh}(y(x)) - _{-}C1 = 0, x + \frac{1}{y(x)} \left( (y(x))^2 + 1 \right)^{\frac{3}{2}} + y(x) \sqrt{(y(x))^2 + 1} + \text{Arcsinh}(y(x)) - _{-}C1 = 0 \right\}$$

**2.375 ODE No. 375**

---

$$ay'(x) + bx + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0310666 (sec), leaf count = 71

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( -\frac{(a^2 - 4bx)^{3/2}}{6b} - ax \right) + c_1 \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \left( \frac{(a^2 - 4bx)^{3/2}}{6b} - ax \right) + c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 49

$$\left\{ y(x) = -\frac{ax}{2} - \frac{1}{12b} (a^2 - 4bx)^{\frac{3}{2}} + _{-}C1, y(x) = -\frac{ax}{2} + \frac{1}{12b} (a^2 - 4bx)^{\frac{3}{2}} + _{-}C1 \right\}$$

**2.376 ODE No. 376**

---

$$ay'(x) + by(x) + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.229626 (sec), leaf count = 110

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{\sqrt{a^2 - 4\#1b} + a \log \left( a - \sqrt{a^2 - 4\#1b} \right)}{2b} \& \right] \left[ c_1 + \frac{x}{2} \right] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{\sqrt{a^2 - 4\#1b} + a \log \left( a - \sqrt{a^2 - 4\#1b} \right)}{2b} \& \right] \left[ c_1 + \frac{x}{2} \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.524 (sec), leaf count = 219

$$\left\{ y(x) = -\frac{1}{4b} e^{\frac{1}{2a}} \left( -2a \text{LambertW} \left( 2 \frac{e^{-1}}{a} e^{-\frac{C1b}{a}} \frac{1}{\sqrt{-b-1}} \left( e^{\frac{bx}{a}} \right)^{-1} \right) - a \ln \left( -\frac{1}{4b} \right) - 2a + (-2x + 2_{-}C1)b \right) \left( e^{\frac{1}{2a}} \left( -2a \text{LambertW} \left( 2 \frac{e^{-1}}{a} e^{-\frac{C1b}{a}} \frac{1}{\sqrt{-b-1}} \left( e^{\frac{bx}{a}} \right)^{-1} \right) - a \ln \left( -\frac{1}{4b} \right) - 2a + (-2x + 2_{-}C1)b \right) \right)^{-1} \right\}$$

**2.377 ODE No. 377**

---

$$y'(x)^2 + (x - 2)y'(x) - y(x) + 1 = 0$$

✓ **Mathematica** : cpu = 0.0031425 (sec), leaf count = 19

$$\left\{ \left\{ y(x) \rightarrow c_1x + c_1^2 - 2c_1 + 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 24

$$\left\{ y(x) = -\frac{x^2}{4} + x, y(x) = 1 + _{-}C1^2 + (x - 2)_{-}C1 \right\}$$

**2.378      ODE No. 378**

---

$$(a+x)y'(x) + y'(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0031092 (sec), leaf count = 18

$$\left\{\left\{y(x) \rightarrow a c_1 + c_1 x + c_1^2\right\}\right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 20

$$\left\{y(x) = \_C1 \left(\_C1 + a + x\right), y(x) = -\frac{(x+a)^2}{4}\right\}$$

**2.379      ODE No. 379**

---

$$y'(x)^2 - (x+1)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0030894 (sec), leaf count = 18

$$\left\{\left\{y(x) \rightarrow c_1 x - c_1^2 + c_1\right\}\right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 22

$$\left\{y(x) = \_C1 \left(\_C1 + x + 1\right), y(x) = \frac{(1+x)^2}{4}\right\}$$

**2.380      ODE No. 380**

---

$$y'(x)^2 + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.378949 (sec), leaf count = 1757

$$\left\{\left\{y(x) \rightarrow -\frac{x^2}{4} - \frac{1}{4}\sqrt[3]{x^6 - 20 \cosh(3c_1)x^3 - 20 \sinh(3c_1)x^3 - 8 \cosh(6c_1) - 8 \sinh(6c_1) + 8\sqrt{-\cosh(3c_1)x^9 - \sinh(3c_1)x^9}}\right\}\right\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 619

$$\left\{y(x) = \frac{1}{16} \left(i \left(6\_C1 - x^3 + 2\sqrt{-3\_C1 x^3 + 9\_C1^2}\right)^{\frac{2}{3}} \sqrt{3} - i\sqrt{3}x^2 - \left(6\_C1 - x^3 + 2\sqrt{-3\_C1 x^3 + 9\_C1^2}\right)^{\frac{2}{3}} \sqrt{3}\right)\right\}$$

**2.381      ODE No. 381**

---

$$y'(x)^2 - 2xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.380686 (sec), leaf count = 1757

$$\left\{\left\{y(x) \rightarrow \frac{x^2}{4} + \frac{1}{4}\sqrt[3]{x^6 - 20 \cosh(3c_1)x^3 - 20 \sinh(3c_1)x^3 - 8 \cosh(6c_1) - 8 \sinh(6c_1) + 8\sqrt{-\cosh(3c_1)x^9 - \sinh(3c_1)x^9}}\right\}\right\}$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 579

$$\left\{y(x) = -\frac{1}{16} \left(i \left(-6\_C1 + x^3 + 2\sqrt{-3\_C1 x^3 + 9\_C1^2}\right)^{\frac{2}{3}} \sqrt{3} - i\sqrt{3}x^2 - \left(-6\_C1 + x^3 + 2\sqrt{-3\_C1 x^3 + 9\_C1^2}\right)^{\frac{2}{3}} \sqrt{3}\right)\right\}$$

2.382 ODE No. 382

---

$$axy'(x) - bx^2 - c + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.17664 (sec), leaf count = 201

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( \frac{1}{2} x \sqrt{a^2 x^2 + 4 b x^2 + 4 c} + \frac{2 c \log \left( \sqrt{a^2 + 4 b} \sqrt{a^2 x^2 + 4 b x^2 + 4 c} + a^2 x + 4 b x \right)}{\sqrt{a^2 + 4 b}} - \frac{a x^2}{2} \right) + c_1 \right\}, \left\{ y \right. \right.$$

✓ **Maple** : cpu = 0.025 (sec), leaf count = 146

$$\left\{ y(x) = -\frac{x}{4} \sqrt{(a^2 + 4 b) x^2 + 4 c} - c \ln \left( \sqrt{a^2 + 4 b} x + \sqrt{(a^2 + 4 b) x^2 + 4 c} \right) \frac{1}{\sqrt{a^2 + 4 b}} - \frac{a x^2}{4} + \_C1, y(x) = \frac{x}{4} \right.$$

2.383 ODE No. 383

---

$$axy'(x) + by(x) + cx^2 + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 5.47535 (sec), leaf count = 1118

$$\left\{ \text{Solve} \left[ \int_1^x \left( \frac{\sqrt{a^2 K[1]^2 - 4 c K[1]^2 - 4 b y(x)} y(x)}{c K[1]^4 + 2 a y(x) K[1]^2 + b y(x) K[1]^2 + 4 y(x)^2} + \frac{c K[1]^3 + a y(x) K[1] + b y(x) K[1]}{c K[1]^4 + 2 a y(x) K[1]^2 + b y(x) K[1]^2 + 4 y(x)^2} \right) d \right. \right.$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception  
time expired

2.384 ODE No. 384

---

$$(ax + b)y'(x) - ay(x) + c + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.32355 (sec), leaf count = 183

$$\left\{ \left\{ y(x) \rightarrow \frac{-2 \sqrt{-a^4 e^{2 c_1} x^2 - 2 a^4 e^{2 c_1} x + a^4 (-e^{2 c_1})} + 2 a^3 x + a^3 - 2 a^2 b x - a b^2 + 4 a c - a e^{2 c_1}}{4 a^2} \right\}, \left\{ y(x) \rightarrow \frac{2 \sqrt{-a^4 e^{2 c_1} x^2 - 2 a^4 e^{2 c_1} x + a^4 (-e^{2 c_1})} + 2 a^3 x + a^3 - 2 a^2 b x - a b^2 + 4 a c - a e^{2 c_1}}{4 a^2} \right\} \right.$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 50

$$\left\{ y(x) = \frac{\_C1^2 + (a x + b) \_C1 + c}{a}, y(x) = \frac{-a^2 x^2 - 2 a b x - b^2 + 4 c}{4 a} \right\}$$

2.385 ODE No. 385

---

$$-2x^2y'(x) + y'(x)^2 + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.682529 (sec), leaf count = 6217

$$\left\{ \left\{ y(x) \rightarrow -x^3 \left( \frac{1}{2} \sqrt{\frac{2 (8 e^{6 c_1} x^{18} + e^{12 c_1} x^{12})}{9 x^{24} \sqrt[3]{64 \sqrt{e^{12 c_1} x^{60} - 3 e^{18 c_1} x^{54} + 3 e^{24 c_1} x^{48} - e^{30 c_1} x^{42}} + \frac{64 e^{6 c_1}}{x^6} + \frac{160 e^{12 c_1}}{x^{12}} - \frac{8 e^{18 c_1}}{x^{18}}}} + \frac{1}{18} \sqrt[3]{64 \sqrt{e^{12 c_1} x^{60} - 3 e^{18 c_1} x^{54} + 3 e^{24 c_1} x^{48} - e^{30 c_1} x^{42}} + \frac{64 e^{6 c_1}}{x^6} + \frac{160 e^{12 c_1}}{x^{12}} - \frac{8 e^{18 c_1}}{x^{18}}}} \right. \right.$$

✓ **Maple** : cpu = 0.576 (sec), leaf count = 169

$$\left\{ y(x) = \frac{x^4 - \left( \text{RootOf} \left( x^{16} - 12 \_Z^2 x^{12} - 16 \_Z^3 x^{10} + 30 \_Z^4 x^8 + 96 \_Z^5 x^6 + 100 \_Z^6 x^4 + 48 \_Z^7 x^2 + 9 \_Z^8 \right) \right)}{2 x} \right.$$

2.386 ODE No. 386

---

$$ax^3y'(x) - 2ax^2y(x) + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.154064 (sec), leaf count = 119

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}(\sinh(2c_1) + \cosh(2c_1)) \left( -\sqrt{2}\sqrt{a}x^2 + 2\sinh(2c_1) + 2\cosh(2c_1) \right) \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{a}x^2 \sinh(2c_1)}{\sqrt{2}} + \frac{\sqrt{a}}{2} \right\} \right.$$

✓ **Maple** : cpu = 0.769 (sec), leaf count = 27

$$\left\{ y(x) = -\frac{ax^4}{8}, y(x) = -C1x^2 + 2 - \frac{C1^2}{a} \right\}$$

2.387 ODE No. 387

---

$$y'(x)^2 + e^x (y'(x) - y(x)) = 0$$

✓ **Mathematica** : cpu = 1.18554 (sec), leaf count = 190

$$\left\{ \text{Solve} \left[ \log(y(x)) - \frac{-e^{x/2} \sqrt{4y(x) + e^x} - \frac{4\sqrt{\frac{e^x}{y(x)} + 4y(x)^{3/2} \sinh^{-1}\left(\frac{e^{x/2}}{2\sqrt{y(x)}}\right)} + e^x}{2y(x)}} = c_1, y(x) \right], \text{Solve} \left[ \log(y(x)) - \frac{e^x}{2y(x)} = c_1, y(x) \right] \right.$$

✓ **Maple** : cpu = 1.402 (sec), leaf count = 115

$$\left\{ \ln(y(x)) - \frac{1}{2y(x)} \sqrt{e^{2x} + 4y(x)e^x} - 2 \operatorname{Artanh} \left( \sqrt{e^{2x} + 4y(x)e^x} e^{-x} \right) - \frac{e^x}{2y(x)} - C1 = 0, \ln(y(x)) + \frac{1}{2y(x)} \sqrt{e^{2x} + 4y(x)e^x} - 2 \operatorname{Artanh} \left( \sqrt{e^{2x} + 4y(x)e^x} e^{-x} \right) - \frac{e^x}{2y(x)} - C1 = 0 \right.$$

2.388 ODE No. 388

---

$$y'(x)^2 - 2y(x)y'(x) - 2x = 0$$

✓ **Mathematica** : cpu = 0.493027 (sec), leaf count = 53

$$\text{Solve} \left[ \left\{ x = \frac{c_1 \sqrt{K\$305940^2 + 1}}{\sqrt{K\$305940^2 + 1}} + \frac{K\$305940 \sinh^{-1}(K\$305940)}{2\sqrt{K\$305940^2 + 1}}, y(x) = \frac{K\$305940}{2} - \frac{x}{K\$305940} \right\}, \{y(x), K\$305940\} \right]$$

✓ **Maple** : cpu = 0.162 (sec), leaf count = 223

$$\left\{ 1 \left( \left( -\frac{y(x)}{2} - \frac{1}{2} \sqrt{(y(x))^2 + 2x} \right) \operatorname{Arcsinh} \left( y(x) + \sqrt{(y(x))^2 + 2x} \right) + x \sqrt{2(y(x))^2 + 2x + 2y(x) \sqrt{(y(x))^2 + 2x}} \right) \right.$$

2.389 ODE No. 389

---

$$y'(x)^2 - (4y(x) + 1)y'(x) + y(x)(4y(x) + 1) = 0$$

✓ **Mathematica** : cpu = 0.0585123 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{4}e^{x-4c_1} (2e^{2c_1} - e^x) \right\}, \left\{ y(x) \rightarrow \frac{1}{4}e^{2c_1+x} (e^{2c_1+x} - 2) \right\} \right\}$$

✓ **Maple** : cpu = 2.641 (sec), leaf count = 71

$$\left\{ y(x) = -\frac{1}{4}, y(x) = \frac{1}{-C1} \left( - (e^x)^2 \sqrt{-\frac{C1}{(e^x)^2}} + -C1 \right) \frac{1}{\sqrt{-\frac{C1}{(e^x)^2}}}, y(x) = -\frac{1}{-C1} \left( (e^x)^2 \sqrt{-\frac{C1}{(e^x)^2}} + -C1 \right) \frac{1}{\sqrt{-\frac{C1}{(e^x)^2}}} \right.$$

2.390

ODE No. 390

$$ay(x)y'(x)-bx-c+y'(x)^2=0$$

✓ **Mathematica** : cpu = 1.48113 (sec), leaf count = 142

$$\text{Solve}\left[\left\{x=e^{b\left(\frac{\log(\text{K}\$306586)}{b}-\frac{\log\left(b-a\text{K}\$306586^2\right)}{2b}\right)}\left(\frac{\tan^{-1}\left(\frac{\sqrt{a}\text{K}\$306586}{\sqrt{b-a\text{K}\$306586^2}}\right)}{\sqrt{a}}-\frac{c\sqrt{b-a\text{K}\$306586^2}}{b\text{K}\$306586}\right)+c_1e^{b\left(\frac{\log(\text{K}\$306586)}{b}-\frac{\log\left(b-a\text{K}\$306586^2\right)}{2b}\right)}\right\}\right]$$

✓ **Maple** : cpu = 1.32 (sec), leaf count = 281

$$\left\{y\left(x\right)=2\frac{{\mathrm{e}}^{{\mathrm{RootOf}}\left(\sqrt{a}\_C1\,b{\mathrm{e}}^{2-Z}-a{\mathrm{e}}^{2-Z}bx+\sqrt{a}\_C1\,b^2-{\mathrm{e}}^{2-Z}\_Z\,b-a{\mathrm{e}}^{2-Z}c+ab^2x-\_Z\,b^2+abc\right)}\left(-1/4\left({\mathrm{e}}^{2\,{\mathrm{RootOf}}\left(\sqrt{a}\_C1\,b{\mathrm{e}}^{2-Z}-a{\mathrm{e}}^{2-Z}bx+\sqrt{a}\_C1\,b^2-{\mathrm{e}}^{2-Z}\_Z\,b-a{\mathrm{e}}^{2-Z}c+ab^2x-\_Z\,b^2+abc\right)}\right)\right)}{a^{3/2}\left({\mathrm{e}}^{2\,{\mathrm{RootOf}}\left(\sqrt{a}\_C1\,b{\mathrm{e}}^{2-Z}-a{\mathrm{e}}^{2-Z}bx+\sqrt{a}\_C1\,b^2-{\mathrm{e}}^{2-Z}\_Z\,b-a{\mathrm{e}}^{2-Z}c+ab^2x-\_Z\,b^2+abc\right)}\right)}\right\}$$

2.391

ODE No. 391

$$y'(x)(ay(x)+bx)+abxy(x)+y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.006914 (sec), leaf count = 29

$$\left\{\left\{y(x)\rightarrow c_1e^{-ax}\right\},\left\{y(x)\rightarrow c_1-\frac{bx^2}{2}\right\}\right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 22

$$\left\{y\left(x\right)=\_C1\,{\mathrm{e}}^{-ax},y\left(x\right)=-\frac{bx^2}{2}+\_C1\right\}$$

2.392

ODE No. 392

$$y(x)^2\log(ay(x))-xy(x)y'(x)+y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.193659 (sec), leaf count = 27

$$\left\{\left\{y(x)\rightarrow \frac{e^{\frac{c_1x}{2}-\frac{c_1^2}{4}}}{a}\right\}\right\}$$

✓ **Maple** : cpu = 2.165 (sec), leaf count = 50

$$\left\{y\left(x\right)=\frac{1}{a}{\mathrm{e}}^{\frac{x^2}{4}},y\left(x\right)=\frac{1}{{\mathrm{e}}^{-C1^2}{\mathrm{e}}^{-C1\,x}a},y\left(x\right)=\frac{{\mathrm{e}}^{-C1\,x}}{{\mathrm{e}}^{-C1^2}a}\right\}$$

2.393

ODE No. 393

$$y'(x)^2+2y(x)\cot(x)y'(x)-y(x)^2=0$$

✓ **Mathematica** : cpu = 0.0854382 (sec), leaf count = 31

$$\left\{\left\{y(x)\rightarrow c_1\csc^2\left(\frac{x}{2}\right)\right\},\left\{y(x)\rightarrow c_1\sec^2\left(\frac{x}{2}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.246 (sec), leaf count = 77

$$\left\{y\left(x\right)=\frac{-C1}{\tan\left(x\right)}\left(1+\sqrt{\left(\tan\left(x\right)\right)^2+1}\right)\frac{1}{\sqrt{\frac{\left(\tan\left(x\right)\right)^2}{\left(\tan\left(x\right)\right)^2+1}}},y\left(x\right)=\frac{-C1\left(\left(\tan\left(x\right)\right)^2+1\right)}{\tan\left(x\right)}\sqrt{\frac{\left(\tan\left(x\right)\right)^2}{\left(\tan\left(x\right)\right)^2+1}}\left(1+\sqrt{\left(\tan\left(x\right)\right)^2+1}\right)\right\}$$

2.394 ODE No. 394

---

$$-\left(g(x)-f(x)^2\right) e^{-2 \int_a^x f(x p) d x p}+2 f(x) y(x) y'(x)+g(x) y(x)^2+y'(x)^2=0$$

✗ **Mathematica** : cpu = 87.1359 (sec), leaf count = 0 , could not solve  
DSolve[-((-f[x]^2 + g[x])/E^(2\*Integrate[f[xp], {xp, a, x}]))) + g[x]\*y[x]^2 + 2\*f[x]\*y[x]\*Derivative[1][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 6.714 (sec), leaf count = 109

$$\left\{y(x)=\tan \left(-\int \sqrt{\frac{g(x)-(f(x))^2}{\left(e^{\int_a^x f(x p) d x p}\right)^4}}\left(e^{\int_a^x f(x p) d x p}\right)^2 d x+_C1\right) \sqrt{e^{-2 \int_a^x f(x p) d x p}\left(\tan \left(-\int \sqrt{\frac{g(x)-(f(x))^2}{\left(e^{\int_a^x f(x p) d x p}\right)^4}}\left(e^{\int_a^x f(x p) d x p}\right)^2 d x+_C1\right)^2\right)}\right\}$$

2.395 ODE No. 395

---

$$2 f(x) y(x) y'(x)+g(x) y(x)^2+h(x)+y'(x)^2=0$$

✗ **Mathematica** : cpu = 28.9063 (sec), leaf count = 0 , could not solve  
DSolve[h[x] + g[x]\*y[x]^2 + 2\*f[x]\*y[x]\*Derivative[1][y][x] + Derivative[1][y][x]^2 == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x)^2+2\*f(x)\*y(x)\*diff(y(x),x)+g(x)\*y(x)^2+h(x) = 0,y(x))

2.396 ODE No. 396

---

$$(y(x)-x) y(x) y'(x)+y'(x)^2-x y(x)^3=0$$

✓ **Mathematica** : cpu = 0.0448843 (sec), leaf count = 29

$$\left\{\left\{y(x) \rightarrow \frac{1}{x-c_1}\right\},\left\{y(x) \rightarrow c_1 e^{\frac{x^2}{2}}\right\}\right\}$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 20

$$\left\{y(x)=(x+_C1)^{-1}, y(x)=e^{\frac{x^2}{2}}+_C1\right\}$$

2.397 ODE No. 397

---

$$-2 x^3 y(x)^2 y'(x)-4 x^2 y(x)^3+y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.419396 (sec), leaf count = 143

$$\left\{\text{Solve}\left[-\frac{x \sqrt{x^4 y(x)+4} y(x)^{3 / 2} \sinh ^{-1}\left(\frac{1}{2} x^2 \sqrt{y(x)}\right)}{2 \sqrt{x^2 y(x)^3\left(x^4 y(x)+4\right)}}-\frac{1}{4} \log (y(x))=c_1, y(x)\right], \text{Solve}\left[\frac{x y(x)^{3 / 2} \sqrt{x^4 y(x)+4} \operatorname{arctanh}\left(\frac{x \sqrt{x^4 y(x)+4}}{2 \sqrt{x^2 y(x)^3\left(x^4 y(x)+4\right)}}\right)}{2 \sqrt{x^2 y(x)^3\left(x^4 y(x)+4\right)}}+\frac{1}{4} \log (y(x))=c_1, y(x)\right]\right\}$$

✓ **Maple** : cpu = 0.824 (sec), leaf count = 128

$$\left\{y(x)=\frac{-2 \sqrt{2} x^2-2+_C1}{2+_C1 x^4-+_C1^3}, y(x)=\frac{2 \sqrt{2} x^2-2+_C1}{2+_C1 x^4-+_C1^3}, y(x)=\frac{\left(\sqrt{2} x^2+_C1-2\right) _C1^2}{2+_C1^2 x^4-4}, y(x)=-4 x^{-4}, y(x)=\frac{1}{x^4}\right\}$$

2.398 ODE No. 398

---

$$y'(x)^2-3 x y(x)^{2 / 3} y'(x)+9 y(x)^{5 / 3}=0$$

✓ **Mathematica** : cpu = 1.50225 (sec), leaf count = 258

$$\left\{\text{Solve}\left[-\frac{\left(x^2-4 \sqrt[3]{y(x)}\right)^{3 / 2} y(x)^2 \log (y(x))}{6\left(\left(x^2-4 \sqrt[3]{y(x)}\right) y(x)^{4 / 3}\right)^{3 / 2}}+\frac{\sqrt{\left(x^2-4 \sqrt[3]{y(x)}\right) y(x)^{4 / 3}} \log \left(\sqrt{x^2-4 \sqrt[3]{y(x)}}+x\right)}{\sqrt{x^2-4 \sqrt[3]{y(x)}} y(x)^{2 / 3}}+\frac{1}{6} \log (y(x))=c_1, y(x)\right]\right\}$$

✓ **Maple** : cpu = 2.255 (sec), leaf count = 138

$$\left\{\ln (x)+\frac{1}{6} \ln \left(64 \frac{y(x)}{x^6}-1\right)-\frac{1}{6} \ln \left(4 \sqrt[3]{\frac{y(x)}{x^6}}-1\right)-\frac{1}{6} \ln \left(16\left(\frac{y(x)}{x^6}\right)^{2 / 3}+4 \sqrt[3]{\frac{y(x)}{x^6}}+1\right)+\frac{1}{6} \ln \left(\frac{y(x)}{x^6}\right)=c_1\right\}$$

**2.399                    ODE No. 399**

---

$$2y'(x)^2 + (x-1)y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0030674 (sec), leaf count = 20

$$\left\{\left\{y(x) \rightarrow c_1 x + 2 c_1^2 - c_1\right\}\right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 22

$$\left\{y(x) = -C1 \left(2 - C1 + x - 1\right), y(x) = -\frac{(x-1)^2}{8}\right\}$$

**2.400                    ODE No. 400**

---

$$-2x^2y'(x) + 2y'(x)^2 + 3xy(x) = 0$$

✓ **Mathematica** : cpu = 0.258605 (sec), leaf count = 135

$$\left\{\text{Solve}\left[\frac{1}{3}\log(y(x)) - \frac{2\sqrt{x^4 - 6xy(x)}\tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{x^3 - 6y(x)}}\right)}{3\sqrt{x}\sqrt{x^3 - 6y(x)}} = c_1, y(x)\right], \text{Solve}\left[\frac{2\sqrt{x^4 - 6xy(x)}\tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{x^3 - 6y(x)}}\right)}{3\sqrt{x}\sqrt{x^3 - 6y(x)}} = c_1, y(x)\right]\right\}$$

✓ **Maple** : cpu = 0.332 (sec), leaf count = 74

$$\left\{y(x) = \frac{x^3}{6}, y(x) = \frac{1}{3 - C1} \left(-\sqrt{-6 - C1} x x + 3\right), y(x) = \frac{1}{3 - C1} \left(\sqrt{-6 - C1} x x + 3\right), y(x) = -\frac{x}{3} \sqrt{-6 - C1} x\right\}$$

**2.401                    ODE No. 401**

---

$$3y'(x)^2 - 2xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.308421 (sec), leaf count = 1093

$$\left\{\left\{y(x) \rightarrow \text{Root}\left[-16e^{6c_1}x^6 + 3\#1^4x^4 + 144e^{6c_1}\#1x^4 - 24\#1^5x^2 - 378e^{6c_1}\#1^2x^2 + 243e^{12c_1} + 48\#1^6 + 216e^{6c_1}\right], y(x)\right\}\right\}$$

✓ **Maple** : cpu = 0.112 (sec), leaf count = 580

$$\left\{y(x) = -\frac{1}{48} \left(i \left(-54 - C1 + x^3 + 6 \sqrt{-3 - C1} x^3 + 81 - C1^2\right)^{\frac{2}{3}} \sqrt{3} - i \sqrt{3} x^2 - \left(-54 - C1 + x^3 + 6 \sqrt{-3 - C1} x^3 + 81 - C1^2\right)^{\frac{2}{3}} \sqrt{3}\right)\right\}$$

**2.402                    ODE No. 402**

---

$$x^2 + 4xy'(x) + 3y'(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 5.2915 (sec), leaf count = 1211

$$\left\{\text{Solve}\left[\frac{1}{2} \left(\frac{\tanh^{-1}\left(\frac{1-3x}{2\sqrt{1-9y(x)}}\right)}{\sqrt{1-9y(x)}} + \log(-3x^2 + 2x - 12y(x) + 1) + \frac{9 \tanh^{-1}\left(\frac{-2\sqrt{1-9y(x)}x + x + 9y(x)}{\sqrt{-9y(x)} - 4\sqrt{1-9y(x)} + 5\sqrt{x^2 + 3y(x)}}\right)}{\sqrt{1-9y(x)}\sqrt{-9y(x)} - 4\sqrt{1-9y(x)} + 5\sqrt{x^2 + 3y(x)}}\right) = c_1, y(x)\right]\right\}$$

✓ **Maple** : cpu = 0.199 (sec), leaf count = 101

$$\left\{y(x) = -\frac{x^2}{3}, y(x) = \frac{-3 - C1^2x^2 - 2\sqrt{3} - C1x + 3}{12 - C1^2}, y(x) = \frac{-3 - C1^2x^2 + 2\sqrt{3} - C1x + 3}{12 - C1^2}, y(x) = -\frac{\sqrt{3} - C1}{6}\right\}$$



2.403 ODE No. 403

---

$$ay'(x)^2 + by'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.227671 (sec), leaf count = 116

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{\sqrt{4 \# 1 a + b^2} + b \log \left( b - \sqrt{4 \# 1 a + b^2} \right)}{2 a} \& \right] \left[ \frac{x}{2 a} + c_1 \right] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{\sqrt{4 \# 1 a + b^2} + b \log \left( b - \sqrt{4 \# 1 a + b^2} \right)}{2 a} \& \right] \left[ \frac{x}{2 a} + c_1 \right] \right\} \right\}$$

✓ **Maple** : cpu = 3.888 (sec), leaf count = 197

$$\left\{ y(x) = \frac{1}{4a} e^{-\frac{1}{2b} \left( 2 \operatorname{blambert} W \left( 2 \frac{e^{-1}}{b \sqrt{a-1}} e^{\frac{x}{b}} \left( e^{-\frac{C1}{b}} \right)^{-1} \right) + b \ln \left( \frac{1}{4a} \right) + 2 - C1 + 2b - 2x \right)} \left( e^{-\frac{1}{2b} \left( 2 \operatorname{blambert} W \left( 2 \frac{e^{-1}}{b \sqrt{a-1}} e^{\frac{x}{b}} \left( e^{-\frac{C1}{b}} \right)^{-1} \right) + b \ln \left( \frac{1}{4a} \right) + 2 - C1 + 2b - 2x \right)} \right) \right\}$$

2.404 ODE No. 404

---

$$ay'(x)^2 + bx^2y'(x) + cxy(x) = 0$$

✓ **Mathematica** : cpu = 2.70508 (sec), leaf count = 795

$$\left\{ \text{Solve} \left[ \int_1^x \left( \frac{(3b+2c)K[1]^2}{2(3bK[1]^3+cK[1]^3+9ay(x))} + \frac{3\sqrt{K[1](b^2K[1]^3-4acy(x))}}{2(3bK[1]^3+cK[1]^3+9ay(x))} \right) dK[1] + \int_1^{y(x)} \left( \frac{9\sqrt{x(b^2x+2c)}}{2(3b+c)x^2(3b+c)} \right) dy \right], \{y(x), K[1]\} \right\}$$

✓ **Maple** : cpu = 0.503 (sec), leaf count = 389

$$\left\{ \int_{-b}^x 1 \left( -b - a^2 - \sqrt{-a^4b^2 - 4 - aacy(x)} \right) \left( b - a^3 + \sqrt{-a^4b^2 - 4 - aacy(x)} - a + 6ay(x) \right)^{-1} d - a + \int_1^{y(x)} -2 \frac{y}{b} dy \right\}$$

2.405 ODE No. 405

---

$$ay'(x)^2 + y(x)y'(x) - x = 0$$

✓ **Mathematica** : cpu = 0.84483 (sec), leaf count = 53

$$\text{Solve} \left[ \left\{ x = \frac{a \operatorname{K\$335905} \sin^{-1}(\operatorname{K\$335905})}{\sqrt{1 - \operatorname{K\$335905}^2}} + \frac{c_1 \operatorname{K\$335905}}{\sqrt{1 - \operatorname{K\$335905}^2}}, y(x) = \frac{x}{\operatorname{K\$335905}} - a \operatorname{K\$335905} \right\}, \{y(x), \operatorname{K\$335905}\} \right]$$

✓ **Maple** : cpu = 0.507 (sec), leaf count = 378

$$\left\{ -C1 \left( y(x) - \sqrt{4ax + (y(x))^2} \right) \frac{1}{\sqrt{\frac{1}{a} \left( -y(x) + \sqrt{4ax + (y(x))^2} - 2a \right)}} \frac{1}{\sqrt{\frac{1}{a} \left( -y(x) + \sqrt{4ax + (y(x))^2} + 2a \right)}} \right\}$$

2.406 ODE No. 406

---

$$ay'(x)^2 - y(x)y'(x) - x = 0$$

✓ **Mathematica** : cpu = 0.6267 (sec), leaf count = 49

$$\text{Solve} \left[ \left\{ x = \frac{a \operatorname{K\$336231} \sinh^{-1}(\operatorname{K\$336231})}{\sqrt{\operatorname{K\$336231}^2 + 1}} + \frac{c_1 \operatorname{K\$336231}}{\sqrt{\operatorname{K\$336231}^2 + 1}}, y(x) = a \operatorname{K\$336231} - \frac{x}{\operatorname{K\$336231}} \right\}, \{y(x), \operatorname{K\$336231}\} \right]$$

✓ **Maple** : cpu = 0.193 (sec), leaf count = 262

$$\left\{ 1 \left( \left( y(x) - \sqrt{4ax + (y(x))^2} \right) \operatorname{Arcsinh} \left( \frac{1}{2a} \left( -y(x) + \sqrt{4ax + (y(x))^2} \right) \right) + x \sqrt{-2 \frac{y(x) \sqrt{4ax + (y(x))^2}}{a}} \right) \right\}$$

2.407 ODE No. 407

---

$$xy'(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0214657 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} \left( -4c_1\sqrt{x} + c_1^2 + 4x \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{4} \left( 4c_1\sqrt{x} + c_1^2 + 4x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 39

$$\left\{ y(x) = 0, y(x) = \frac{1}{x} \left( -x + \sqrt{-C1\,x} \right)^2, y(x) = \frac{1}{x} \left( x + \sqrt{-C1\,x} \right)^2 \right\}$$

2.408 ODE No. 408

---

$$xy'(x)^2 - 2y(x) + x = 0$$

✓ **Mathematica** : cpu = 0.150539 (sec), leaf count = 97

$$\left\{ \text{Solve} \left[ -2 \left( \frac{1}{1 - \sqrt{\frac{2y(x)}{x}} - 1} + \log \left( 1 - \sqrt{\frac{2y(x)}{x}} - 1 \right) \right) = c_1 + \log(x), y(x) \right], \text{Solve} \left[ 2 \left( \frac{1}{\sqrt{\frac{2y(x)}{x}} - 1} + 1 \right) = c_1 + \log(x), y(x) \right] \right\}$$

✓ **Maple** : cpu = 0.102 (sec), leaf count = 73

$$\left\{ y(x) = \left( \frac{1}{2} \left( \text{lambertW} \left( \frac{1}{-C1} \sqrt{-C1\,x} \right) + 1 \right)^2 \left( \text{lambertW} \left( \frac{1}{-C1} \sqrt{-C1\,x} \right) \right)^{-2} + \frac{1}{2} \right) x, y(x) = \left( \frac{1}{2} \left( \text{lambertW} \left( \frac{1}{-C1} \sqrt{-C1\,x} \right) - 1 \right)^2 \left( \text{lambertW} \left( \frac{1}{-C1} \sqrt{-C1\,x} \right) \right)^{-2} + \frac{1}{2} \right) x \right\}$$

2.409 ODE No. 409

---

$$xy'(x)^2 - 2y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 30.5713 (sec), leaf count = 66

$$\text{Solve} \left[ \left\{ x = \frac{y(\text{K\$337105}) + 2\text{K\$337105}}{\text{K\$337105}^2}, y(x) = c_1 e^{2(\log(\text{K\$337105}) - \log(1 - \text{K\$337105}))} + e^{2(\log(\text{K\$337105}) - \log(1 - \text{K\$337105}))} \right\} \right]$$

✓ **Maple** : cpu = 0.179 (sec), leaf count = 63

$$\left\{ y(x) = x e^{2 \text{RootOf}(-x e^2 - Z + 2 x e^{-Z} + 2 e^{-Z} + C1 - 2 Z - x)} - 2 e^{\text{RootOf}(-x e^2 - Z + 2 x e^{-Z} + 2 e^{-Z} + C1 - 2 Z - x)} \right\}$$

2.410 ODE No. 410

---

$$xy'(x)^2 + 4y'(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 31.2338 (sec), leaf count = 80

$$\text{Solve} \left[ \left\{ x = -\frac{2(2\text{K\$337610} - y(\text{K\$337610}))}{\text{K\$337610}^2}, y(x) = c_1 e^{-4 \left( \frac{1}{2} \log(2 - \text{K\$337610}) - \frac{\log(\text{K\$337610})}{2} \right)} + 4 e^{-4 \left( \frac{1}{2} \log(2 - \text{K\$337610}) - \frac{\log(\text{K\$337610})}{2} \right)} \right\} \right]$$

✓ **Maple** : cpu = 0.186 (sec), leaf count = 64

$$\left\{ y(x) = \frac{x e^{2 \text{RootOf}(-x e^2 - Z + 4 x e^{-Z} - 4 e^{-Z} + C1 + 8 Z - 4 x)}}{2} + 2 e^{\text{RootOf}(-x e^2 - Z + 4 x e^{-Z} - 4 e^{-Z} + C1 + 8 Z - 4 x)} \right\}$$

2.411 ODE No. 411

---

$$xy'(x)^2 + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.141181 (sec), leaf count = 99

$$\left\{ \text{Solve} \left[ \frac{1}{\sqrt{\frac{4y(x)}{x}} + 1} - \log \left( 1 - \sqrt{\frac{4y(x)}{x}} + 1 \right) = c_1 + \frac{\log(x)}{2}, y(x) \right], \text{Solve} \left[ \frac{1}{\sqrt{\frac{4y(x)}{x}} + 1} + \log \left( \sqrt{\frac{4y(x)}{x}} + 1 \right) = c_1 + \frac{\log(x)}{2}, y(x) \right] \right\}$$

✓ **Maple** : cpu = 0.117 (sec), leaf count = 65

$$\left\{ y(x) = \frac{x}{4} \left( 1 + 2 \text{lambertW} \left( -1/2 \frac{1}{\sqrt{\frac{-C1}{x}}} \right) \right) \left( \text{lambertW} \left( -\frac{1}{2} \frac{1}{\sqrt{\frac{-C1}{x}}} \right) \right)^{-2}, y(x) = \frac{x}{4} \left( 1 + 2 \text{lambertW} \left( -\frac{1}{2} \frac{1}{\sqrt{\frac{-C1}{x}}} \right) \right)^{-2} \right\}$$

**2.412 ODE No. 412**

---

$$a + xy'(x)^2 + y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.409039 (sec), leaf count = 3229

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2} \sqrt{-\frac{16ax}{9} + \frac{\sqrt[3]{-131072a^6x^6 + 2560a^3e^{3c_1}x^3 + e^{6c_1}} + \sqrt{-1073741824a^9e^{3c_1}x^9 + 3145728a^6e^{6c_1}x^6}}{18 \cdot 2^{2/3}ax}} \right\} \right\}$$

✓ **Maple** : cpu = 0.161 (sec), leaf count = 146

$$\left\{ -C1 x^2 \left( \frac{1}{x} \left( -y(x) + \sqrt{(y(x))^2 - 4ax} \right) \right)^{\frac{3}{2}} \left( -y(x) + \sqrt{(y(x))^2 - 4ax} \right)^{-2} + x + \frac{4ax^2}{3} \left( -y(x) + \sqrt{(y(x))^2 - 4ax} \right) \right\}$$

**2.413 ODE No. 413**

---

$$-x^2 + xy'(x)^2 + y(x)y'(x) = 0$$

✗ **Mathematica** : cpu = 301.786 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.262 (sec), leaf count = 269

$$\left\{ \int_{-b}^x \frac{1}{-a} \left( -y(x) - \sqrt{4_-a^3 + (y(x))^2} \right) \left( \sqrt{4_-a^3 + (y(x))^2} + 4y(x) \right)^{-1} d_-a + \int^{y(x)} 1 \left( -2 + \left( -48_-f - 12 \right) \right) dy \right\}$$

**2.414 ODE No. 414**

---

$$x^3 + xy'(x)^2 + y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0749663 (sec), leaf count = 107

$$\left\{ \left\{ y(x) \rightarrow x^2 \text{InverseFunction} \left[ \int_1^{\#1} \frac{1}{5K[1] + \sqrt{K[1]^2 - 4}} dK[1] \& \right] \left[ \int_1^x -\frac{1}{2K[2]} dK[2] + c_1 \right] \right\}, \left\{ y(x) \rightarrow x^2 \text{InverseFunction} \left[ \int_1^{\#1} \frac{1}{5K[1] + \sqrt{K[1]^2 - 4}} dK[1] \& \right] \left[ \int_1^x -\frac{1}{2K[2]} dK[2] + c_1 \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.273 (sec), leaf count = 269

$$\left\{ \int_{-b}^x \frac{1}{-a} \left( -y(x) - \sqrt{-4_-a^4 + (y(x))^2} \right) \left( \sqrt{-4_-a^4 + (y(x))^2} + 5y(x) \right)^{-1} d_-a + \int^{y(x)} 1 \left( -2 + \left( 80_-f + 12 \right) \right) dy \right\}$$

**2.415 ODE No. 415**

---

$$y(x)y'(x) + xy'(x)^2 - y(x)^4 = 0$$

✓ **Mathematica** : cpu = 0.307596 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow \frac{2e^{\frac{c_1}{2}}}{e^{c_1} - 4x} \right\} \right\}$$

✓ **Maple** : cpu = 0.447 (sec), leaf count = 95

$$\left\{ y(x) = -\frac{1}{2} \frac{1}{\sqrt{-x}}, y(x) = \frac{1}{2} \frac{1}{\sqrt{-x}}, y(x) = -\frac{1}{2x} \sqrt{-\left( \tanh \left( -\frac{\ln(x)}{2} + \frac{C1}{2} \right) \right)^2 x + x} \left( \tanh \left( -\frac{\ln(x)}{2} + \frac{C1}{2} \right) \right) \right\}$$

### 2.416 ODE No. 416

$$xy'(x)^2 + (y(x) - 3x)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 1.27176 (sec), leaf count = 383

$$\left\{ \text{Solve} \left[ \frac{1}{8} \left( -\sqrt{\frac{\frac{y(x)}{x} - 9}{\frac{y(x)}{x} - 1}} \left( \frac{y(x)}{x} - 1 \right) + \sqrt{\frac{y(x)}{x} - 9} \sqrt{\frac{y(x)}{x} - 1} - 3 \log \left( \frac{y(x)}{x} \right) - \frac{10 \sqrt{\frac{y(x)}{x} - 9} \sin^{-1} \left( \frac{\sqrt{9 - \frac{y(x)}{x}}}{2\sqrt{2}} \right)}{\sqrt{9 - \frac{y(x)}{x}}} \right) \right] \right\}$$

✓ **Maple** : cpu = 0.134 (sec), leaf count = 136

$$\left\{ -\frac{C1}{x} \left( -y(x) + 5x + \sqrt{9x^2 - 10xy(x) + (y(x))^2} \right) \left( \frac{1}{x} \left( -y(x) + 3x + \sqrt{9x^2 - 10xy(x) + (y(x))^2} \right) \right)^{-\frac{3}{2}} \right\}$$

### 2.417 ODE No. 417

$$a + xy'(x)^2 - y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.356075 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow \frac{16a^2 - 8ae^{\frac{c_1}{2}} + 4ax + e^{c_1}}{2 \left( e^{\frac{c_1}{2}} - 4a \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 35

$$\left\{ y(x) = \frac{x_{-}C1^2 + a}{_{-}C1}, y(x) = -2\sqrt{ax}, y(x) = 2\sqrt{ax} \right\}$$

### 2.418 ODE No. 418

$$ay(x) + xy'(x)^2 - y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.679103 (sec), leaf count = 220

$$\left\{ \text{Solve} \left[ -\frac{\frac{4a^{3/2} \sqrt{4 - \frac{y(x)}{ax}} \sin^{-1} \left( \frac{\sqrt{\frac{y(x)}{x}}}{2\sqrt{a}} \right)}{\sqrt{\frac{y(x)}{x} - 4a}} + \sqrt{\frac{y(x)}{x}} \sqrt{\frac{y(x)}{x} - 4a} + \frac{y(x)}{x}}{4a} = c_1 + \frac{\log(x)}{2}, y(x) \right], \text{Solve} \left[ \frac{4a^{3/2} \sqrt{4 - \frac{y(x)}{ax}} \sin^{-1} \left( \frac{\sqrt{\frac{y(x)}{x}}}{2\sqrt{a}} \right)}{\sqrt{\frac{y(x)}{x} - 4a}} + \sqrt{\frac{y(x)}{x}} \sqrt{\frac{y(x)}{x} - 4a} + \frac{y(x)}{x} \right] \right\}$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 42

$$\left\{ y(x) = 0, y(x) = -ax \left( \text{lambertW} \left( -\frac{xe}{_{-}C1 a} \right) - 1 \right)^2 \left( \text{lambertW} \left( -\frac{xe}{_{-}C1 a} \right) \right)^{-1} \right\}$$

### 2.419 ODE No. 419

$$xy'(x)^2 + 2y(x)y'(x) - x = 0$$

✓ **Mathematica** : cpu = 1.05555 (sec), leaf count = 9073

✓ **Maple** : cpu = 0.048 (sec), leaf count = 109

$$\left\{ x + \frac{C1}{x} \left( y(x) - \sqrt{(y(x))^2 + x^2} \right) \left( \frac{1}{x^2} \left( 2x^2 + 6(y(x))^2 - 6y(x) \sqrt{(y(x))^2 + x^2} \right) \right)^{-\frac{2}{3}} = 0, \frac{C1}{x} \left( \sqrt{(y(x))^2 + x^2} - y(x) \right) \right\}$$

**2.420 ODE No. 420**

---

$$a + xy'(x)^2 - 2y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.296321 (sec), leaf count = 777

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4}a^2e^{-\frac{3c_1}{2}}x^2 + \frac{1}{4}e^{-\frac{3c_1}{2}}\sqrt[3]{a^6x^6 - 20a^3e^{3c_1}x^3 + 8\sqrt{-a^9e^{3c_1}x^9 + 3a^6e^{6c_1}x^6 - 3a^3e^{9c_1}x^3 + e^{12c_1}} - 8e^{6c_1}} \right. \right.$$

✓ **Maple** : cpu = 0.182 (sec), leaf count = 689

$$\left\{ y(x) = \frac{x}{12\_C1} \left( 4 \frac{x^2}{\sqrt[3]{-36\_a\_C1^2 + 8x^3 + 12\sqrt{a(9\_a\_C1^2 - 4x^3)}\_C1}} + 2x + \sqrt[3]{-36\_a\_C1^2 + 8x^3 + 12\sqrt{a(9\_a\_C1^2 - 4x^3)}} \right) \right.$$

**2.421 ODE No. 421**

---

$$xy'(x)^2 - 2y(x)y'(x) - x = 0$$

✓ **Mathematica** : cpu = 0.0375557 (sec), leaf count = 27

$$\{\{y(x) \rightarrow x \sinh(c_1 - \log(x))\}, \{y(x) \rightarrow x \sinh(c_1 + \log(x))\}\}$$

✓ **Maple** : cpu = 0.047 (sec), leaf count = 32

$$\left\{ y(x) = -ix, y(x) = ix, y(x) = -\frac{C1^2 - x^2}{2\_C1} \right\}$$

**2.422 ODE No. 422**

---

$$xy'(x)^2 - 2y(x)y'(x) + 4x = 0$$

✓ **Mathematica** : cpu = 0.054299 (sec), leaf count = 81

$$\left\{ \left\{ y(x) \rightarrow -\frac{2(x \tanh^2(\frac{1}{2}(c_1 - \log(x))) + x)}{\tanh^2(\frac{1}{2}(c_1 - \log(x))) - 1} \right\}, \left\{ y(x) \rightarrow -\frac{2(x \tanh^2(\frac{1}{2}(c_1 + \log(x))) + x)}{\tanh^2(\frac{1}{2}(c_1 + \log(x))) - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 30

$$\left\{ y(x) = -2x, y(x) = 2x, y(x) = \frac{4\_C1^2 + x^2}{2\_C1} \right\}$$

**2.423 ODE No. 423**

---

$$xy'(x)^2 - 2y(x)y'(x) + 2y(x) + x = 0$$

✓ **Mathematica** : cpu = 0.0625016 (sec), leaf count = 59

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}(-2e^{-c_1}x^2 - e^{c_1} + 2x) \right\}, \left\{ y(x) \rightarrow \frac{1}{2}(-e^{c_1}x^2 - 2e^{-c_1} + 2x) \right\} \right\}$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 44

$$\left\{ y(x) = (1 - \sqrt{2})x, y(x) = (1 + \sqrt{2})x, y(x) = \frac{2\_C1^2 + 2\_C1x + x^2}{2\_C1} \right\}$$

**2.424 ODE No. 424**

---

$$ay(x)y'(x) + bx + xy'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.489811 (sec), leaf count = 223

$$\left\{ \text{Solve} \left[ \frac{-2a \tan^{-1} \left( \frac{ay(x)}{x \sqrt{4b - \frac{a^2 y(x)^2}{x^2}}} \right) + (a+2) \left( 2 \tan^{-1} \left( \frac{(a+2)y(x)}{x \sqrt{4b - \frac{a^2 y(x)^2}{x^2}}} \right) - i \log \left( \frac{(a+1)y(x)^2}{x^2} + b \right) \right)}{8(a+1)} = c_1 + \frac{1}{2}i \log \right. \right.$$

✓ **Maple** : cpu = 0.285 (sec), leaf count = 193

$$\left\{\frac{1}{x}\left(-\_C1\left(ay\left(x\right)-\sqrt{a^2\left(y\left(x\right)\right)^2-4bx^2}\right)\left(\frac{a}{2x^2}\left(-y\left(x\right)\left(a+1\right)\sqrt{a^2\left(y\left(x\right)\right)^2-4bx^2}+\left(a^2+a\right)\left(y\left(x\right)\right)^2-2\right.\right.\right.\right.$$

**2.425 ODE No. 425**

---

$$(x+1)y'(x)^2-(y(x)+x)y'(x)+y(x)=0$$

✓ **Mathematica** : cpu = 0.191543 (sec), leaf count = 59

$$\left\{\left\{y(x)\rightarrow-\frac{e^{c_1}\left(e^{c_1}-2x\right)}{2\left(e^{c_1}+2\right)}\right\},\left\{y(x)\rightarrow-\frac{2e^{c_1}\left(2e^{c_1}-x\right)}{2e^{c_1}+1}\right\}\right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 45

$$\left\{y\left(x\right)=\frac{\_C1\left(\_C1x+\_C1-x\right)}{\_C1-1},y\left(x\right)=x+2-2\sqrt{1+x},y\left(x\right)=x+2+2\sqrt{1+x}\right\}$$

**2.426 ODE No. 426**

---

$$(3x+1)y'(x)^2-3(y(x)+2)y'(x)+9=0$$

✓ **Mathematica** : cpu = 0.424672 (sec), leaf count = 40

$$\left\{\left\{y(x)\rightarrow\frac{-28e^{\frac{c_1}{2}}+e^{c_1}+12x+196}{2\left(e^{\frac{c_1}{2}}-12\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 51

$$\left\{y\left(x\right)=\frac{9+\left(3x+1\right)\_C1^2-6\_C1}{3\_C1},y\left(x\right)=-2-2\sqrt{3x+1},y\left(x\right)=-2+2\sqrt{3x+1}\right\}$$

**2.427 ODE No. 427**

---

$$(3x+5)y'(x)^2-(3y(x)+x)y'(x)+y(x)=0$$

✗ **Mathematica** : cpu = 300.091 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.035 (sec), leaf count = 60

$$\left\{y\left(x\right)=\frac{\left(3x+5\right)\_C1^2-\_C1x}{3\_C1-1},y\left(x\right)=\frac{x}{3}+\frac{10}{9}-\frac{2}{9}\sqrt{15x+25},y\left(x\right)=\frac{x}{3}+\frac{10}{9}+\frac{2}{9}\sqrt{15x+25}\right\}$$

**2.428 ODE No. 428**

---

$$y'(x)(-ay(x)+bx+c)+axy'(x)^2-by(x)=0$$

✗ **Mathematica** : cpu = 300.063 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.07 (sec), leaf count = 66

$$\left\{y\left(x\right)=\frac{1}{a}\left(-bx+c-2\sqrt{-bcx}\right),y\left(x\right)=\frac{1}{a}\left(-bx+c+2\sqrt{-bcx}\right),y\left(x\right)=\frac{\_C1\left(\_C1ax+bx+c\right)}{\_C1a+b}\right\}$$

**2.429 ODE No. 429**

---

$$-y'(x)(ay(x)-a+bx-b)+axy'(x)^2+by(x)=0$$

✓ **Mathematica** : cpu = 0.0319439 (sec), leaf count = 26

$$\left\{\left\{y(x)\rightarrow\frac{c_1(a+b)}{ac_1-b}+c_1x\right\}\right\}$$

✓ **Maple** : cpu = 0.067 (sec), leaf count = 72

$$\left\{y\left(x\right)=\frac{1}{a}\left(bx+a+b-2\sqrt{bx\left(a+b\right)}\right),y\left(x\right)=\frac{1}{a}\left(bx+a+b+2\sqrt{bx\left(a+b\right)}\right),y\left(x\right)=\frac{\_C1\left(\_C1ax-bx+\right)}{\_C1a-b}$$

2.430 ODE No. 430

---

$$a0x + y'(x)(a1x + b1y(x) + c1) + (a2x + c2)y'(x)^2 + b0y(x) + c0 = 0$$

✓ **Mathematica** : cpu = 245.198 (sec), leaf count = 478

Solve  $\left\{ \left[ \begin{aligned} x &= (b0 + b1K\$363169) \exp \left( \frac{(b1(b0 - a1) + 2a2b0) \tan^{-1} \left( \frac{a1 + 2K\$363169(a2 + b1) + b0}{\sqrt{4a0(a2 + b1) - a1^2 - 2a1b0 - b0^2}} \right)}{(a2 + b1)\sqrt{4a0(a2 + b1) - a1^2 - 2a1b0 - b0^2}} - \frac{(2a2 + b1)}{\sqrt{4a0(a2 + b1) - a1^2 - 2a1b0 - b0^2}} \right) \right] \right\}$

✓ **Maple** : cpu = 1.983 (sec), leaf count = 1602

$$\left\{ \frac{1}{2a^2x + 2c^2} \left( -2 \left( -C1 - 1/2 \int \frac{-a1x - b1y(x) - c1 + \sqrt{b1^2(y(x))^2 + ((2b1a1 - 4a2b0)x - 4b0c2 + 2b1c1)y(x) + (-4a0a2 + a1^2)x^2 + (-4c2a1 + 4c1a2 - c0)x - c0^2}}{2a^2x + 2c^2} dx \right) \right) \right\}$$

2.431 ODE No. 431

---

$$x^2y'(x)^2 - y(x)^4 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0458095 (sec), leaf count = 81

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{\tan^2(c1 - \log(x)) + 1} \right\}, \left\{ y(x) \rightarrow \sqrt{\tan^2(c1 - \log(x)) + 1} \right\}, \left\{ y(x) \rightarrow -\sqrt{\tan^2(c1 + \log(x)) + 1} \right\}, \left\{ y(x) \rightarrow \sqrt{\tan^2(c1 + \log(x)) + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.296 (sec), leaf count = 62

$$\left\{ y(x) = -1, y(x) = 1, y(x) = \frac{1}{\tan(-\ln(x) + \_C1)} \sqrt{(\tan(-\ln(x) + \_C1))^2 + 1}, y(x) = -\frac{1}{\tan(-\ln(x) + \_C1)} \sqrt{(\tan(-\ln(x) + \_C1))^2 + 1} \right\}$$

2.432 ODE No. 432

---

$$(a + xy'(x))^2 - 2ay(x) + x^2 = 0$$

✓ **Mathematica** : cpu = 1.15123 (sec), leaf count = 64

Solve  $\left[ \left\{ y(x) = \frac{a^2 + 2aK\$365773x + K\$365773^2x^2 + x^2}{2a}, x = \frac{c1}{\sqrt{K\$365773^2 + 1}} - \frac{a \sinh^{-1}(K\$365773)}{\sqrt{K\$365773^2 + 1}} \right\}, \{y(x)\} \right]$

✓ **Maple** : cpu = 7.73 (sec), leaf count = 242

$$\left\{ y(x) = \frac{1}{2a \left( \left( \text{RootOf} \left( (\text{Arcsinh}(\_Z))^2 a^2 - \_Z^2 x^2 - 2 \text{Arcsinh}(\_Z) \_C1 a + \_C1^2 - x^2 \right) \right)^2 + 1 \right)} \left( -2aR \right) \right\}$$

2.433 ODE No. 433

---

$$-4a - 4x^2 + (xy'(x) + y(x) + 2x)^2 - 4xy(x) = 0$$

✓ **Mathematica** : cpu = 1.32983 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow \frac{-a - 2c1x + c1^2}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.343 (sec), leaf count = 34

$$\left\{ y(x) = \frac{-x^2 - a}{x}, y(x) = \_C1 + \frac{\_C1^2 - 4a}{4x} \right\}$$

## 2.434 ODE No. 434

---

$$x^2 y'(x)^2 - x^2 - 2xy(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0384961 (sec), leaf count = 27

$$\{\{y(x) \rightarrow x \sinh(c_1 - \log(x))\}, \{y(x) \rightarrow x \sinh(c_1 + \log(x))\}\}$$

✓ **Maple** : cpu = 0.025 (sec), leaf count = 7

$$\{y(x) = x + \_C1\}$$

## 2.435 ODE No. 435

---

$$x^2 y'(x)^2 - 2xy(x)y'(x) + y(x)(y(x) + 1) - x = 0$$

✓ **Mathematica** : cpu = 0.0593743 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} (c_1^2 x - 4ic_1 \sqrt{x} + 4x - 4) \right\}, \left\{ y(x) \rightarrow \frac{1}{4} (c_1^2 x + 4ic_1 \sqrt{x} + 4x - 4) \right\} \right\}$$

✓ **Maple** : cpu = 0.54 (sec), leaf count = 22

$$\left\{ y(x) = x, y(x) = \_C1 \sqrt{x} - \frac{x \_C1^2}{4} + x - 1 \right\}$$

## 2.436 ODE No. 436

---

$$-x^4 + x^2 y'(x)^2 + (1 - x^2) y(x)^2 - 2xy(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0546411 (sec), leaf count = 26

$$\{\{y(x) \rightarrow -x \sinh(x - c_1)\}, \{y(x) \rightarrow x \sinh(c_1 + x)\}\}$$

✓ **Maple** : cpu = 4.319 (sec), leaf count = 61

$$\left\{ y(x) = -ix, y(x) = ix, y(x) = -\frac{x \left( (e^x)^2 - \_C1^2 \right)}{2 e^x \_C1}, y(x) = \frac{x \left( (e^x)^2 \_C1^2 - 1 \right)}{2 e^x \_C1} \right\}$$

## 2.437 ODE No. 437

---

$$-(a + 2xy(x))y'(x) + x^2 y'(x)^2 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.333044 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow \frac{x - 2\sqrt{a}c_1}{4c_1^2} \right\}, \left\{ y(x) \rightarrow \frac{2\sqrt{a}c_1 + x}{4c_1^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 36

$$\left\{ y(x) = -\frac{a}{4x}, y(x) = \_C1 x - \sqrt{\_C1 a}, y(x) = \_C1 x + \sqrt{\_C1 a} \right\}$$

## 2.438 ODE No. 438

---

$$x^2 y'(x)^2 + 3xy(x)y'(x) + 2y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0108685 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{x^2} \right\}, \left\{ y(x) \rightarrow \frac{c_1}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 17

$$\left\{ y(x) = \frac{\_C1}{x^2}, y(x) = \frac{\_C1}{x} \right\}$$



**2.439                      ODE No. 439**

---

$$x^2 y'(x)^2 + 3xy(x)y'(x) + 3y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.037141 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow c_1 x^{\frac{1}{2}(-3-i\sqrt{3})} \right\}, \left\{ y(x) \rightarrow c_1 x^{\frac{1}{2}i(\sqrt{3}+3i)} \right\} \right\}$$

✓ **Maple** : cpu = 0.119 (sec), leaf count = 33

$$\left\{ y(x) = \_C1 x^{-\frac{i}{2}\sqrt{3}} x^{-\frac{3}{2}}, y(x) = \_C1 x^{\frac{i}{2}\sqrt{3}} x^{-\frac{3}{2}} \right\}$$

**2.440                      ODE No. 440**

---

$$x^2 y'(x)^2 + 4xy(x)y'(x) - 5y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0102833 (sec), leaf count = 19

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{x^5} \right\}, \left\{ y(x) \rightarrow c_1 x \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 15

$$\left\{ y(x) = \frac{\_C1}{x^5}, y(x) = \_C1 x \right\}$$

**2.441                      ODE No. 441**

---

$$x^2 y'(x)^2 - 4x(y(x) + 2)y'(x) + 4y(x)(y(x) + 2) = 0$$

✓ **Mathematica** : cpu = 0.0792302 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow -e^{-c_1 x} \left( 2\sqrt{2} e^{\frac{c_1}{2}} - x \right) \right\}, \left\{ y(x) \rightarrow e^{\frac{c_1}{2} x} \left( e^{\frac{c_1}{2} x} - 2\sqrt{2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 5.179 (sec), leaf count = 83

$$\left\{ y(x) = -2, y(x) = \frac{1}{\_C1} \left( x^2 - 2\sqrt{2}\sqrt{\_C1 x^2} \right), y(x) = \frac{1}{\_C1} \left( 2\sqrt{2}\sqrt{\_C1 x^2} + x^2 \right), y(x) = \frac{x(-2\sqrt{2}\_C1 + \_C1^2)}{\_C1^2} \right\}$$

**2.442                      ODE No. 442**

---

$$x^2 y'(x)^2 + (1-x)(y(x)^2 - x^2 y(x)) + (x^3 + x^2 y(x) - 2xy(x))y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0135592 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-x} \right\}, \left\{ y(x) \rightarrow c_1 x - x^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 21

$$\left\{ y(x) = (-x + \_C1)x, y(x) = \_C1 e^{-x} \right\}$$

**2.443                      ODE No. 443**

---

$$x(xy'(x) - y(x))^2 - y'(x) = 0$$

✓ **Mathematica** : cpu = 2.74856 (sec), leaf count = 10121

✓ **Maple** : cpu = 7.904 (sec), leaf count = 221

$$\left\{ y(x) = -\frac{2}{9x^2}, y(x) = \frac{(RootOf(-729\_C1 x^{12} + \_Z^8 - 12\_Z^7 + 60\_Z^6 - 160\_Z^5 + 240\_Z^4 - 192\_Z^3 + \_Z^2))}{9x^2} \right\}$$

## 2.444 ODE No. 444

---

$$x^2 y'(x)^2 - (y(x) - 2x)y(x)y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.411125 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow -\frac{\cosh(2c_1) - \sinh(2c_1)}{x \sinh(2c_1) + x \cosh(2c_1) - 1} \right\}, \left\{ y(x) \rightarrow -\frac{\cosh(2c_1) - \sinh(2c_1)}{x \sinh(2c_1) + x \cosh(2c_1) + 1} \right\} \right\}$$

✓ **Maple** : cpu = 4.934 (sec), leaf count = 120

$$\left\{ y(x) = \frac{-C1^3 \sqrt{2} - 2x \_C1^2}{-2 \_C1^2 + 4x^2}, y(x) = \frac{-C1^2 (\sqrt{2} \_C1 + 2x)}{2 \_C1^2 - 4x^2}, y(x) = 4x, y(x) = -2 \frac{-C1^2 (-\sqrt{2} \_C1 + x)}{-2 \_C1^2 + x^2}, y \right\}$$

## 2.445 ODE No. 445

---

$$y'(x) (ax^2 y(x)^3 + b) + aby(x)^3 + x^2 y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0518717 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{\sqrt{2ax - 2c_1}} \right\}, \left\{ y(x) \rightarrow \frac{1}{\sqrt{2ax - 2c_1}} \right\}, \left\{ y(x) \rightarrow \frac{b}{x} + c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 35

$$\left\{ y(x) = \frac{1}{\sqrt{2ax + \_C1}}, y(x) = -\frac{1}{\sqrt{2ax + \_C1}}, y(x) = \frac{b}{x} + \_C1 \right\}$$

## 2.446 ODE No. 446

---

$$(x^2 + 1) y'(x)^2 - 2xy(x)y'(x) + y(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.333571 (sec), leaf count = 95

$$\left\{ \left\{ y(x) \rightarrow \frac{x \sinh(2c_1) + x \cosh(2c_1) - 2 \sinh(c_1) - 2 \cosh(c_1) - x}{\sinh(2c_1) + \cosh(2c_1) + 1} \right\}, \left\{ y(x) \rightarrow \frac{x \sinh(2c_1) + x \cosh(2c_1) + 2 \sinh(c_1) + 2 \cosh(c_1) + x}{\sinh(2c_1) + \cosh(2c_1) + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 57

$$\left\{ y(x) = \sqrt{x^2 + 1}, y(x) = -\sqrt{x^2 + 1}, y(x) = \_C1 x - \sqrt{-\_C1^2 + 1}, y(x) = \_C1 x + \sqrt{-\_C1^2 + 1} \right\}$$

## 2.447 ODE No. 447

---

$$(x^2 - 1) y'(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.0094949 (sec), leaf count = 89

$$\left\{ \left\{ y(x) \rightarrow c_1 + \frac{1}{2} \log \left( 1 - \frac{x}{\sqrt{x^2 - 1}} \right) - \frac{1}{2} \log \left( \frac{x}{\sqrt{x^2 - 1}} + 1 \right) \right\}, \left\{ y(x) \rightarrow c_1 - \frac{1}{2} \log \left( 1 - \frac{x}{\sqrt{x^2 - 1}} \right) + \frac{1}{2} \log \left( \frac{x}{\sqrt{x^2 - 1}} + 1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 33

$$\left\{ y(x) = -\ln \left( x + \sqrt{x^2 - 1} \right) + \_C1, y(x) = \ln \left( x + \sqrt{x^2 - 1} \right) + \_C1 \right\}$$

## 2.448 ODE No. 448

---

$$(x^2 - 1) y'(x)^2 - y(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 0.0950027 (sec), leaf count = 349

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2} e^{-c_1} \sqrt{2e^{4c_1} x^2 + 2e^{4c_1} \sqrt{(x-1)(x+1)} x + 2e^{2c_1} - e^{4c_1} + 2x^2 - 2\sqrt{(x-1)(x+1)} x - 1} \right\}, \left\{ y(x) \rightarrow \frac{1}{2} e^{c_1} \sqrt{2e^{4c_1} x^2 + 2e^{4c_1} \sqrt{(x-1)(x+1)} x + 2e^{2c_1} - e^{4c_1} + 2x^2 - 2\sqrt{(x-1)(x+1)} x - 1} \right\} \right\}$$

✓ **Maple** : cpu = 300.181 (sec), leaf count = 166

$$\left\{ 1 \sqrt{(-1 + y(x)) (1 + y(x))} \ln \left( y(x) + \sqrt{(y(x))^2 - 1} \right) \frac{1}{\sqrt{-1 + y(x)}} \frac{1}{\sqrt{1 + y(x)}} + \int^x \frac{1}{\_a^2 - 1} \sqrt{(\_a^2 - 1)} \left( (y(x) - 1) \sqrt{(y(x))^2 - 1} \right) dx \right\}$$

2.449 ODE No. 449

---

$$(x^2 - a^2) y'(x)^2 + 2xy(x)y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.014141 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{a - x} \right\}, \left\{ y(x) \rightarrow \frac{c_1}{a + x} \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 23

$$\left\{ y(x) = \frac{-C1}{a - x}, y(x) = \frac{-C1}{x + a} \right\}$$

2.450 ODE No. 450

---

$$(x^2 - a^2) y'(x)^2 - x^2 - 2xy(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.315843 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow \frac{a^2 + c_1^2 - x^2}{2c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.676 (sec), leaf count = 51

$$\left\{ y(x) = \sqrt{a^2 - x^2}, y(x) = -\sqrt{a^2 - x^2}, y(x) = -C1 x^2 - C1 a^2 - \frac{1}{4 C1} \right\}$$

2.451 ODE No. 451

---

$$(a + x^2) y'(x)^2 + b - 2xy(x)y'(x) + y(x)^2 = 0$$

✗ **Mathematica** : cpu = 301.184 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.124 (sec), leaf count = 78

$$\left\{ y(x) = \frac{1}{a} \sqrt{-ab(x^2 + a)}, y(x) = -\frac{1}{a} \sqrt{-ab(x^2 + a)}, y(x) = -C1 x - \sqrt{-a C1^2 - b}, y(x) = -C1 x + \sqrt{-a C1^2 - b} \right\}$$

2.452 ODE No. 452

---

$$(2x^2 + 1) y'(x)^2 + (x^2 + 2xy(x) + y(x)^2 + 2) y'(x) + 2y(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 0.017351 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow \frac{-c_1 x + c_1^2 + 1}{c_1 + x} \right\} \right\}$$

✓ **Maple** : cpu = 1.358 (sec), leaf count = 37

$$\left\{ y(x) = -3x - 2\sqrt{2x^2 + 1}, y(x) = -3x + 2\sqrt{2x^2 + 1} \right\}$$

2.453 ODE No. 453

---

$$(a^2 - 1) x^2 y'(x)^2 + a^2 x^2 + 2xy(x)y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.582589 (sec), leaf count = 327

$$\left\{ \text{Solve} \left[ \frac{2i \tan^{-1} \left( \frac{y(x)}{x \sqrt{a^2 - \frac{y(x)^2}{x^2} - 1}} \right) + a \tanh^{-1} \left( \frac{-a^2 - \frac{iy(x)}{x} + 1}{a \sqrt{a^2 - \frac{y(x)^2}{x^2} - 1}} \right) - a \tanh^{-1} \left( \frac{-a^2 + \frac{iy(x)}{x} + 1}{a \sqrt{a^2 - \frac{y(x)^2}{x^2} - 1}} \right) + a \log \left( \frac{y(x)^2}{x^2} + 1 \right)}{2a^2 - 2} \right] \right\}$$

✓ **Maple** : cpu = 2.691 (sec), leaf count = 229

$$\left\{ \frac{1}{2a} \left( -2 C1 a + 2a \ln(x) + \ln \left( \frac{(y(x))^2 + x^2}{x^2} \right) a - 2 \sqrt{-a^2} \arctan \left( \frac{a^2 y(x)}{\sqrt{-a^2} x \sqrt{\frac{(y(x))^2 + (-a^2 + 1)x^2}{x^2}}} \right) + 2 \ln \left( \frac{1}{x} \right) \right) \right\}$$

**2.454 ODE No. 454**

---

$$ax^2y'(x)^2 - (a-1)ax^2 - 2axy(x)y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.268677 (sec), leaf count = 241

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}} \left( e^{2c_1} - x^2\sqrt{\frac{a-1}{a}} \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}} \left( e^{2c_1} - x^2\sqrt{\frac{a-1}{a}} \right) \right\}, \left\{ y(x) \rightarrow \dots \right\} \right\}$$

✓ **Maple** : cpu = 0.26 (sec), leaf count = 106

$$\left\{ y(x) = \sqrt{-a}x, y(x) = \text{RootOf} \left( -\ln(x) - \int^{-x} \frac{1}{(a-1)(-a^2+a)} \sqrt{(a-1)(-a^2+a)} ad_-a + -C1 \right) x, y(x) = \dots \right\}$$

**2.455 ODE No. 455**

---

$$a + x^3y'(x)^2 + x^2y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.968694 (sec), leaf count = 123

$$\left\{ \left\{ y(x) \rightarrow -\frac{e^{-\frac{c_1}{2}}(2ae^{c_1}+x)}{\sqrt{2}x} \right\}, \left\{ y(x) \rightarrow \frac{e^{-\frac{c_1}{2}}(2ae^{c_1}+x)}{\sqrt{2}x} \right\}, \left\{ y(x) \rightarrow -\frac{e^{-\frac{c_1}{2}}(2ax+e^{c_1})}{\sqrt{2}x} \right\}, \left\{ y(x) \rightarrow \frac{e^{-\frac{c_1}{2}}(2ax+e^{c_1})}{\sqrt{2}x} \right\} \right\}$$

✓ **Maple** : cpu = 0.612 (sec), leaf count = 66

$$\left\{ y(x) = -2\frac{\sqrt{ax}}{x}, y(x) = 2\frac{\sqrt{ax}}{x}, y(x) = \frac{C1^2+4ax}{2-C1x}, y(x) = \frac{x-C1^2+4a}{2-C1x} \right\}$$

**2.456 ODE No. 456**

---

$$2(1-x^2)y(x)y'(x) + x(x^2-1)y'(x)^2 + xy(x)^2 - x = 0$$

✓ **Mathematica** : cpu = 0.123681 (sec), leaf count = 421

$$\left\{ \left\{ y(x) \rightarrow \frac{-x - x \tanh^2 \left( \frac{1}{4} \left( 2c_1 - \frac{i\sqrt{x-1}\sqrt{x+1}\sqrt{(x-1)(x+1)}\tan^{-1}(\sqrt{x^2-1})}{1-x} - \frac{i\sqrt{x-1}\sqrt{(x-1)(x+1)}\tan^{-1}(\sqrt{x^2-1})}{\sqrt{x+1}} \right) \right)}{-1 + \tanh^2 \left( \frac{1}{4} \left( 2c_1 - \frac{i\sqrt{x-1}\sqrt{x+1}\sqrt{(x-1)(x+1)}\tan^{-1}(\sqrt{x^2-1})}{1-x} - \frac{i\sqrt{x-1}\sqrt{(x-1)(x+1)}\tan^{-1}(\sqrt{x^2-1})}{\sqrt{x+1}} \right) \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.72 (sec), leaf count = 33

$$\left\{ y(x) = x, y(x) = -x, y(x) = \sqrt{-C1^2+1} + \sqrt{x^2-1}-C1 \right\}$$

**2.457 ODE No. 457**

---

$$x^4y'(x)^2 - xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.251613 (sec), leaf count = 118

$$\left\{ \text{Solve} \left[ -\frac{x\sqrt{4x^2y(x)+1}\tanh^{-1}\left(\sqrt{4x^2y(x)+1}\right)}{\sqrt{4x^4y(x)+x^2}} - \frac{1}{2}\log(y(x)) = c_1, y(x) \right], \text{Solve} \left[ \frac{x\sqrt{4x^2y(x)+1}\tanh^{-1}\left(\sqrt{4x^2y(x)+1}\right)}{\sqrt{4x^4y(x)+x^2}} - \frac{1}{2}\log(y(x)) = c_1, y(x) \right] \right\}$$

✓ **Maple** : cpu = 3.371 (sec), leaf count = 45

$$\left\{ y(x) = \frac{i-C1-x}{x-C1^2}, y(x) = \frac{-i-C1-x}{x-C1^2}, y(x) = -\frac{1}{4x^2} \right\}$$

2.458 ODE No. 458

---

$$x^2 \left(x^2 - a^2\right) y'(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.0221442 (sec), leaf count = 120

$$\left\{ \left\{ y(x) \rightarrow c_1 - \frac{x \sqrt{x^2 - a^2} \tan^{-1} \left( \frac{\sqrt{x^2 - a^2}}{a} \right)}{a \sqrt{x^4 - a^2 x^2}} \right\}, \left\{ y(x) \rightarrow \frac{x \sqrt{x^2 - a^2} \tan^{-1} \left( \frac{\sqrt{x^2 - a^2}}{a} \right)}{a \sqrt{x^4 - a^2 x^2}} + c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.123 (sec), leaf count = 90

$$\left\{ y(x) = -1 \ln \left( \frac{1}{x} \left( -2 a^2 + 2 \sqrt{-a^2} \sqrt{-a^2 + x^2} \right) \right) \frac{1}{\sqrt{-a^2}} + \_C1, y(x) = 1 \ln \left( \frac{1}{x} \left( -2 a^2 + 2 \sqrt{-a^2} \sqrt{-a^2 + x^2} \right) \right) \frac{1}{\sqrt{-a^2}} + \_C1 \right\}$$

2.459 ODE No. 459

---

$$-\left(y'(x)-1\right)^2+e^{-2 x} y'(x)^2+e^{-2 y(x)}=0$$

✓ **Mathematica** : cpu = 2.77165 (sec), leaf count = 271

$$\left\{ \left\{ y(x) \rightarrow \log \left( -\frac{e^{-c_1} (e^x + 1) (e^{2c_1+x} - e^{2c_1} - e^x - 1)}{\sqrt{8e^x + 4e^{2x} + 4}} \right) \right\}, \left\{ y(x) \rightarrow \log \left( \frac{e^{-c_1} (e^x + 1) (e^{2c_1+x} - e^{2c_1} - e^x - 1)}{\sqrt{8e^x + 4e^{2x} + 4}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.932 (sec), leaf count = 65

$$\left\{ y(x) = x + \ln \left( \frac{1}{e^x} \left( -1 - \sqrt{(e^x)^2 - \frac{(e^x)^2}{(e^{-C1})^2}} \right) \right) + \_C1, y(x) = x + \ln \left( \frac{1}{e^x} \left( -1 + \sqrt{(e^x)^2 - \frac{(e^x)^2}{(e^{-C1})^2}} \right) \right) + \_C1 \right\}$$

2.460 ODE No. 460

---

$$\cos^4(x) \left(y'(x)^2 + y(x)^2\right) - a^2 = 0$$

✗ **Mathematica** : cpu = 49.7128 (sec), leaf count = 0 , could not solve

`DSolve[-a^2 + Cos[x]^4*(y[x]^2 + Derivative[1][y][x]^2) == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve((diff(y(x),x)^2+y(x)^2)*cos(x)^4-a^2 = 0,y(x))`

2.461 ODE No. 461

---

$$a(x) y'(x)^2 + 2b(x) y(x) y'(x) + c(x) y(x)^2 + 2d(x) y'(x) + 2e(x) y(x) + f(x) = 0$$

✗ **Mathematica** : cpu = 300.046 (sec), leaf count = 0 , timed out

`$Aborted`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(d0(x)*diff(y(x),x)^2+2*b0(x)*y(x)*diff(y(x),x)+c0(x)*y(x)^2+2*d0(x)*diff(y(x),x)+2*e0(x)*y(x)+f0(x)=0,y(x))`

2.462 ODE No. 462

---

$$y(x) y'(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.0203884 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow \left( \frac{3}{2} \right)^{2/3} (c_1 - x)^{2/3} \right\}, \left\{ y(x) \rightarrow \left( \frac{3}{2} \right)^{2/3} (c_1 + x)^{2/3} \right\} \right\}$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 27

$$\left\{ x - \frac{2}{3} (y(x))^{\frac{3}{2}} - \_C1 = 0, x + \frac{2}{3} (y(x))^{\frac{3}{2}} - \_C1 = 0 \right\}$$

2.463 ODE No. 463

---

$$y(x)y'(x)^2 - e^{2x} = 0$$

✓ **Mathematica** : cpu = 0.0307633 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow \left(\frac{3}{2}\right)^{2/3} (c_1 - e^x)^{2/3} \right\}, \left\{ y(x) \rightarrow \left(\frac{3}{2}\right)^{2/3} (c_1 + e^x)^{2/3} \right\} \right\}$$

✓ **Maple** : cpu = 0.185 (sec), leaf count = 50

$$\left\{ -1\sqrt{y(x)(e^x)^2}\frac{1}{\sqrt{y(x)}} + \frac{2}{3}(y(x))^{\frac{3}{2}} + \_C1 = 0, 1\sqrt{y(x)(e^x)^2}\frac{1}{\sqrt{y(x)}} + \frac{2}{3}(y(x))^{\frac{3}{2}} + \_C1 = 0 \right\}$$

2.464 ODE No. 464

---

$$y(x)y'(x)^2 + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0869564 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow -e^{\frac{c_1}{2}} \sqrt{e^{c_1} - 2x} \right\}, \left\{ y(x) \rightarrow e^{\frac{c_1}{2}} \sqrt{e^{c_1} - 2x} \right\} \right\}$$

✓ **Maple** : cpu = 1.424 (sec), leaf count = 70

$$\left\{ y(x) = \sqrt{\_C1^2 - 2\_C1 x}, y(x) = \sqrt{\_C1^2 + 2\_C1 x}, y(x) = -ix, y(x) = ix, y(x) = -\sqrt{\_C1(2x + \_C1)} \right\}$$

2.465 ODE No. 465

---

$$y(x)y'(x)^2 + 2xy'(x) - 9y(x) = 0$$

✗ **Mathematica** : cpu = 300.448 (sec), leaf count = 0 , timed out  
\$Aborted

✓ **Maple** : cpu = 0.147 (sec), leaf count = 210

$$\left\{ \_C1 x \left( x + \sqrt{x^2 + 9(y(x))^2} \right) \left( \frac{1}{y(x)} \left( -x - \sqrt{x^2 + 9(y(x))^2} \right) \right)^{\frac{2}{7}} \left( x\sqrt{x^2 + 9(y(x))^2} + x^2 + (y(x))^2 \right)^{-1} \right\}$$

2.466 ODE No. 466

---

$$y(x)y'(x)^2 - 2xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 1.87605 (sec), leaf count = 433

$$\left\{ \text{Solve} \left[ -\frac{i\sqrt{\frac{y(x)^2}{x^2} - 1} \tan^{-1} \left( \sqrt{\frac{y(x)^2}{x^2} - 1} \right)}{\sqrt{\frac{y(x)}{x} - 1} \sqrt{\frac{y(x)}{x} + 1}} - i\sqrt{\frac{\frac{y(x)}{x} - 1}{\frac{y(x)}{x} + 1}} \left( \frac{y(x)}{x} + 1 \right) + i\sqrt{\frac{y(x)}{x} - 1} \sqrt{\frac{y(x)}{x} + 1} + \log \left( \frac{y(x)}{x} \right) \right] \right\}$$

✓ **Maple** : cpu = 1.201 (sec), leaf count = 71

$$\left\{ y(x) = x, y(x) = \sqrt{\_C1^2 - 2ix\_C1}, y(x) = \sqrt{\_C1^2 + 2ix\_C1}, y(x) = -x, y(x) = -\sqrt{\_C1^2 - 2ix\_C1}, y(x) = -\sqrt{\_C1^2 + 2ix\_C1} \right\}$$

2.467 ODE No. 467

---

$$y(x)y'(x)^2 - 4xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.245728 (sec), leaf count = 226

$$\left\{ \left\{ y(x) \rightarrow \sqrt{\frac{2^{2/3}c_1^3x}{\sqrt[3]{-40c_1^3x^3 + \sqrt{-4096}c_1^3x^9 + 768c_1^6x^6 - 48c_1^9x^3 + c_1^{12} - c_1^6 + 32x^6}} + \frac{\sqrt[3]{-40c_1^3x^3 + \sqrt{-4096}}}{\sqrt[3]{-40c_1^3x^3 + \sqrt{-4096}c_1^3x^9 + 768c_1^6x^6 - 48c_1^9x^3 + c_1^{12} - c_1^6 + 32x^6}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.125 (sec), leaf count = 148

$$\left\{ -\frac{C1\,x}{y\left(x\right)}\frac{1}{\sqrt[3]{\frac{1}{y\left(x\right)}\left(2\,x-\sqrt{4\,x^2-\left(y\left(x\right)\right)^2}\right)}}\frac{1}{\sqrt[3]{\frac{1}{\left(y\left(x\right)\right)^2}\left(8\,x^2-4\,\left(y\left(x\right)\right)^2-4\,x\sqrt{4\,x^2-\left(y\left(x\right)\right)^2}\right)}}+x=0,-\frac{C1}{y\left(x\right)}\right.$$

**2.468 ODE No. 468**

---

$$-4a^2xy'(x)+a^2y(x)+y(x)y'(x)^2=0$$

✓ **Mathematica** : cpu = 3.57039 (sec), leaf count = 609

$$\left\{ \text{Solve}\left[\frac{-4\left(\frac{y(x)^2}{x^2}-4a^2\right)\tanh^{-1}\left(\frac{\sqrt{4a^2-\frac{y(x)^2}{x^2}}}{2a}\right)+2\left(\frac{y(x)^2}{x^2}-4a^2\right)\tanh^{-1}\left(\frac{\sqrt{4a^2-\frac{y(x)^2}{x^2}}}{a}\right)+\sqrt{2a-\frac{y(x)}{x}}\sqrt{4a^2-\frac{y(x)^2}{x^2}}}{6\sqrt{2a-\frac{y(x)}{x}}}\right],\right.$$

✓ **Maple** : cpu = 0.146 (sec), leaf count = 181

$$\left\{ -\frac{C1\,x}{ay\left(x\right)}\frac{1}{\sqrt[3]{\frac{a^2}{\left(y\left(x\right)\right)^2}\left(2\,a^2x^2+\sqrt{4\,a^2x^2-\left(y\left(x\right)\right)^2}ax-\left(y\left(x\right)\right)^2\right)}}\frac{1}{\sqrt[3]{\frac{a}{y\left(x\right)}\left(2\,ax+\sqrt{4\,a^2x^2-\left(y\left(x\right)\right)^2}\right)}}+x=0,-\frac{C1}{y\left(x\right)}\right.$$

**2.469 ODE No. 469**

---

$$axy'(x)+by(x)+y(x)y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.278258 (sec), leaf count = 157

$$\left\{ \text{Solve}\left[\frac{\left(a+2b\right)\log\left(-\sqrt{a^2-\frac{4by(x)^2}{x^2}}+a+2b\right)+a\log\left(\sqrt{a^2-\frac{4by(x)^2}{x^2}}+a\right)}{4\left(a+b\right)}=c_1-\frac{\log(x)}{2},y(x)\right],\text{Solve}\left[\frac{a\log\left(\sqrt{a^2-\frac{4by(x)^2}{x^2}}+a\right)}{4\left(a+b\right)}=c_1-\frac{\log(x)}{2},y(x)\right],\right.$$

✓ **Maple** : cpu = 0.25 (sec), leaf count = 264

$$\left\{ \frac{x}{\left(y\left(x\right)\right)^2}\left(-C1\left(-\frac{1}{2\,y\left(x\right)}\left(ax+\sqrt{a^2x^2-4\,b\left(y\left(x\right)\right)^2}\right)\right)^{-\frac{a}{a+b}}\left(ax+\sqrt{a^2x^2-4\,b\left(y\left(x\right)\right)^2}\right)\left(\frac{a}{2\,\left(y\left(x\right)\right)^2}\left(ax^2+\sqrt{a^2x^2-4\,b\left(y\left(x\right)\right)^2}x-\left(y\left(x\right)\right)^2\right)\right.\right.$$

**2.470 ODE No. 470**

---

$$x^3y'(x)-x^2y(x)+y(x)y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.239065 (sec), leaf count = 143

$$\left\{ \text{Solve}\left[\frac{\frac{1}{2}\log(y(x))-\frac{\sqrt{x^6+4x^2y(x)^2}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4+4y(x)^2}}\right)}{2x\sqrt{x^4+4y(x)^2}}=c_1,y(x)\right],\text{Solve}\left[\frac{\sqrt{x^6+4x^2y(x)^2}\tanh^{-1}\left(\frac{\sqrt{x^4+4y(x)^2}}{x}\right)}{2x\sqrt{x^4+4y(x)^2}}=c_1,y(x)\right],\right.$$

✓ **Maple** : cpu = 0.661 (sec), leaf count = 87

$$\left\{ y\left(x\right)=-\frac{i}{2}x^2,y\left(x\right)=\frac{i}{2}x^2,y\left(x\right)=-\frac{1}{4}\sqrt{-4\_C1\,x^2+\_C1^2},y\left(x\right)=\frac{1}{4}\sqrt{-4\_C1\,x^2+\_C1^2},y\left(x\right)=-2\frac{\sqrt{-4\_C1\,x^2+\_C1^2}}{\_C1},y\left(x\right)=2\frac{\sqrt{-4\_C1\,x^2+\_C1^2}}{\_C1}\right.$$

## 2.471 ODE No. 471

---

$$y(x)y'(x)^2 - (y(x) - x)y'(x) - x = 0$$

✓ **Mathematica** : cpu = 0.0084314 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow c_1 + x \right\}, \left\{ y(x) \rightarrow -\sqrt{2c_1 - x^2} \right\}, \left\{ y(x) \rightarrow \sqrt{2c_1 - x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt{-x^2 + \_C1}, y(x) = -\sqrt{-x^2 + \_C1}, y(x) = x + \_C1 \right\}$$

## 2.472 ODE No. 472

---

$$(y(x) + x)y'(x)^2 + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.39306 (sec), leaf count = 269

$$\left\{ \left\{ y(x) \rightarrow -\frac{2\sqrt{-\sqrt{3}x \sinh(c_1) - \sqrt{3}x \cosh(c_1) + \sinh(2c_1) + \cosh(2c_1)}}{\sqrt{3}} - \frac{\sinh(c_1)}{\sqrt{3}} - \frac{\cosh(c_1)}{\sqrt{3}} \right\}, \left\{ y(x) \rightarrow \frac{2\sqrt{-\sqrt{3}x \sinh(c_1) - \sqrt{3}x \cosh(c_1) + \sinh(2c_1) + \cosh(2c_1)}}{\sqrt{3}} + \frac{\sinh(c_1)}{\sqrt{3}} + \frac{\cosh(c_1)}{\sqrt{3}} \right\} \right\}$$

✓ **Maple** : cpu = 1.371 (sec), leaf count = 121

$$\left\{ \ln(x) - \operatorname{Artanh}\left(\frac{y(x) + 2x}{2x} \frac{1}{\sqrt{\frac{(y(x))^2 + xy(x) + x^2}{x^2}}}\right) + \ln\left(\frac{y(x)}{x}\right) - \_C1 = 0, \ln(x) + \operatorname{Artanh}\left(\frac{y(x) + 2x}{2x} \frac{1}{\sqrt{\frac{(y(x))^2 + xy(x) + x^2}{x^2}}}\right) - \ln\left(\frac{y(x)}{x}\right) - \_C1 = 0 \right\}$$

## 2.473 ODE No. 473

---

$$(y(x) - 2x)y'(x)^2 - 2(x - 1)y'(x) + y(x) - 2 = 0$$

✓ **Mathematica** : cpu = 0.266758 (sec), leaf count = 165

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( -\sqrt{-4e^{c_1}x + 4e^{c_1} - e^{2c_1}} - e^{c_1} + 4 \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \left( \sqrt{-4e^{c_1}x + 4e^{c_1} - e^{2c_1}} - e^{c_1} + 4 \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \left( -\sqrt{-4e^{c_1}x + 4e^{c_1} - e^{2c_1}} - e^{c_1} + 4 \right) \right\} \right\}$$

✓ **Maple** : cpu = 1.901 (sec), leaf count = 71

$$\left\{ y(x) = 2 + \_C1 - \sqrt{\_C1 (-\_C1 + 2x - 2)}, y(x) = 2 + \frac{\_C1}{2} - \frac{1}{2}\sqrt{\_C1 (-\_C1 + 4x - 4)}, y(x) = (x - 1) + \_C1 \right\}$$

## 2.474 ODE No. 474

---

$$2y(x)y'(x)^2 - (4x - 5)y'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.163376 (sec), leaf count = 135

$$\left\{ \left\{ y(x) \rightarrow -i\sqrt{2}e^{\frac{c_1}{2}}\sqrt{8e^{c_1} + 4x - 5} \right\}, \left\{ y(x) \rightarrow i\sqrt{2}e^{\frac{c_1}{2}}\sqrt{8e^{c_1} + 4x - 5} \right\}, \left\{ y(x) \rightarrow -\frac{1}{4}ie^{\frac{c_1}{2}}\sqrt{e^{c_1} + 8x - 10} \right\}, \left\{ y(x) \rightarrow \frac{1}{4}ie^{\frac{c_1}{2}}\sqrt{e^{c_1} + 8x - 10} \right\} \right\}$$

✓ **Maple** : cpu = 1.726 (sec), leaf count = 152

$$\left\{ \ln\left(x - \frac{5}{4}\right) - \frac{1}{2}\ln\left(4\frac{y(x)}{4x - 5} - 1\right) - \frac{1}{2}\ln\left(4\frac{y(x)}{4x - 5} + 1\right) + \ln\left(\frac{y(x)}{4x - 5}\right) + \sqrt{-16\frac{(y(x))^2}{(4x - 5)^2} + 1} - \operatorname{Artanh}\left(\frac{y(x)}{4x - 5}\right) = 0 \right\}$$

## 2.475 ODE No. 475

---

$$4y(x)y'(x)^2 + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.095783 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2}e^{2c_1}\sqrt{e^{4c_1} - 2x} \right\}, \left\{ y(x) \rightarrow \frac{1}{2}e^{2c_1}\sqrt{e^{4c_1} - 2x} \right\} \right\}$$

✓ **Maple** : cpu = 1.397 (sec), leaf count = 67

$$\left\{ y(x) = \sqrt{\_C1^2 - \_C1 x}, y(x) = \sqrt{\_C1^2 + \_C1 x}, y(x) = -\frac{i}{2}x, y(x) = \frac{i}{2}x, y(x) = -\sqrt{\_C1 (x + \_C1)}, y(x) = \sqrt{\_C1 (x + \_C1)} \right\}$$



2.476 ODE No. 476

---

$$4x^3y'(x) - 4x^2y(x) + 9y(x)y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.247332 (sec), leaf count = 143

$$\left\{ \text{Solve} \left[ \frac{1}{2} \log(y(x)) - \frac{\sqrt{x^6 + 9x^2y(x)^2} \tanh^{-1} \left( \frac{x^2}{\sqrt{x^4 + 9y(x)^2}} \right)}{2x\sqrt{x^4 + 9y(x)^2}} = c_1, y(x) \right], \text{Solve} \left[ \frac{\sqrt{x^6 + 9x^2y(x)^2} \tanh^{-1} \left( \frac{x^2}{\sqrt{x^4 + 9y(x)^2}} \right)}{2x\sqrt{x^4 + 9y(x)^2}} = c_1, y(x) \right] \right\}$$

✓ **Maple** : cpu = 0.634 (sec), leaf count = 87

$$\left\{ y(x) = -\frac{i}{3}x^2, y(x) = \frac{i}{3}x^2, y(x) = -\frac{1}{6}\sqrt{-4\_C1\,x^2 + \_C1^2}, y(x) = \frac{1}{6}\sqrt{-4\_C1\,x^2 + \_C1^2}, y(x) = -2\sqrt{-4\_C1\,x^2 + \_C1^2} \right\}$$

2.477 ODE No. 477

---

$$ay(x)y'(x)^2 + (2x - b)y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.213909 (sec), leaf count = 146

$$\left\{ \left\{ y(x) \rightarrow -\frac{e^{\frac{c_1}{2}} \sqrt{2b + e^{c_1} - 4x}}{2\sqrt{a}} \right\}, \left\{ y(x) \rightarrow \frac{e^{\frac{c_1}{2}} \sqrt{2b + e^{c_1} - 4x}}{2\sqrt{a}} \right\}, \left\{ y(x) \rightarrow -\sqrt{2}e^{\frac{c_1}{2}} \sqrt{2ae^{c_1} - b + 2x} \right\}, \left\{ y(x) \rightarrow \sqrt{2}e^{\frac{c_1}{2}} \sqrt{2ae^{c_1} - b + 2x} \right\} \right\}$$

✓ **Maple** : cpu = 0.629 (sec), leaf count = 622

$$\left\{ \int_{\_b}^x 1 \left( -4\_a + 2b - 2\sqrt{4a(y(x))^2 + (b - 2\_a)^2} \right) \left( (-b + 2\_a)\sqrt{4a(y(x))^2 + (b - 2\_a)^2} + 4a(y(x))^2 + \right) \right\}$$

2.478 ODE No. 478

---

$$(y'(x)^2 + 1)(ay(x) + b) - c = 0$$

✓ **Mathematica** : cpu = 0.322007 (sec), leaf count = 223

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{\frac{c\sqrt{-ac}\sqrt{\frac{\#1a+b}{e}} \sin^{-1} \left( \frac{a\sqrt{-\#1a-b+c}}{\sqrt{-a}\sqrt{-ac}} \right)}{\sqrt{-a}} - (\#1a + b)\sqrt{-\#1a - b + c}}{a\sqrt{\#1a + b}} \& \right] [c_1 - x] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{\frac{c\sqrt{-ac}\sqrt{\frac{\#1a+b}{e}} \sin^{-1} \left( \frac{a\sqrt{-\#1a-b+c}}{\sqrt{-a}\sqrt{-ac}} \right)}{\sqrt{-a}} - (\#1a + b)\sqrt{-\#1a - b + c}}{a\sqrt{\#1a + b}} \& \right] [c_1 - x] \right\} \right\}$$

✓ **Maple** : cpu = 0.348 (sec), leaf count = 88

$$\left\{ x - \int^{y(x)} (\_a a + b) \frac{1}{\sqrt{-(\_a a + b)(\_a a + b - c)}} d\_a - \_C1 = 0, x - \int^{y(x)} -(\_a a + b) \frac{1}{\sqrt{-(\_a a + b)(\_a a + b - c)}} d\_a - \_C1 = 0 \right\}$$

2.479 ODE No. 479

---

$$a0x + y'(x)(a1x + b1y(x) + c1) + y'(x)^2(a2x + b2y(x) + c2) + b0y(x) + c0 = 0$$

✓ **Mathematica** : cpu = 9.0486 (sec), leaf count = 552

$$\text{Solve} \left[ \left\{ x = -\frac{-(b0 + K\$401181(b1 + b2K\$401181)) \exp \left( \text{RootSum} \left[ \#1^3b2 + \#1^2a2 + \#1^2b1 + \#1a1 + \#1b0 \right]}{a2} \right)}{a1} \right\} \right]$$

✓ **Maple** : cpu = 0.234 (sec), leaf count = 929

$$\left\{ x - e^{\int \frac{1}{2b2y(x)+2a2x+2c2} \left( -a1x - b1y(x) - c1 + \sqrt{-4a0a2x^2 - 4a0b2xy(x) + a1^2x^2 + 2a1b1xy(x) - 4a2b0xy(x) - 4b0b2(y(x))^2 + b1^2(y(x))^2 - 4a0c2x + c1^2} \right) dx} \right\}$$

**2.480            ODE No. 480**

---

$$(ay(x) - x^2) y'(x)^2 + 2xy(x)y'(x)^2 - y(x)^2 = 0$$

✗ **Mathematica** : cpu = 26.8121 (sec), leaf count = 0 , could not solve

DSolve[-y[x]^2 + 2\*x\*y[x]\*Derivative[1][y][x]^2 + (-x^2 + a\*y[x])\*Derivative[1][y][x]^2 == 0

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve((a\*y(x)-x^2)\*diff(y(x),x)^2+2\*x\*y(x)\*diff(y(x),x)^2-y(x)^2 = 0,y(x))

**2.481            ODE No. 481**

---

$$(x^2 + y(x)^2) y'(x) + xy(x)y'(x)^2 + xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0117413 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{x} \right\}, \left\{ y(x) \rightarrow -\sqrt{2c_1 - x^2} \right\}, \left\{ y(x) \rightarrow \sqrt{2c_1 - x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 35

$$\left\{ y(x) = \sqrt{-x^2 + \_C1}, y(x) = \frac{-C1}{x}, y(x) = -\sqrt{-x^2 + \_C1} \right\}$$

**2.482            ODE No. 482**

---

$$(a + x^{22} - y(x)^2) y'(x) + xy(x)y'(x)^2 - xy(x) = 0$$

✗ **Mathematica** : cpu = 55.9359 (sec), leaf count = 0 , could not solve

DSolve[-(x\*y[x]) + (a + x^22 - y[x]^2)\*Derivative[1][y][x] + x\*y[x]\*Derivative[1][y][x]^2 == 0

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(x\*y(x)\*diff(y(x),x)^2+(x^22-y(x)^2+a)\*diff(y(x),x)-x\*y(x) = 0,y(x))

**2.483            ODE No. 483**

---

$$(2xy(x) - x^2) y'(x)^2 + 2xy(x)y'(x) - y(x)^2 + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.177428 (sec), leaf count = 71

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{c_1}{2}} - \sqrt{2e^{\frac{c_1}{2}} x - x^2} \right\}, \left\{ y(x) \rightarrow \sqrt{2e^{\frac{c_1}{2}} x - x^2} + e^{\frac{c_1}{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 103

$$\left\{ y(x) = 0, y(x) = RootOf \left( -2 \ln(x) + \int^{-Z} \frac{1}{\_a (\_a^2 + 1)} \left( -2 \_a^2 + \sqrt{2} \sqrt{\_a (\_a - 1)^2} \right) d\_a + 2 \_C1 \right) x, \right.$$

**2.484            ODE No. 484**

---

$$(2xy(x) - x^2) y'(x)^2 - 6xy(x)y'(x) - y(x)^2 + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.141932 (sec), leaf count = 81

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{3x^2 - 2e^{\frac{c_1}{2}} x - e^{\frac{c_1}{2}} + 2x} \right\}, \left\{ y(x) \rightarrow \sqrt{3x^2 - 2e^{\frac{c_1}{2}} x - e^{\frac{c_1}{2}} + 2x} \right\} \right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 115

$$\left\{ y(x) = 0, y(x) = RootOf \left( -2 \ln(x) + \int^{-Z} \frac{1}{\_a (\_a^2 - 4\_a + 1)} \left( -2 \_a^2 + \sqrt{2} \sqrt{\_a (\_a + 1)^2} + 4 \_a \right) d\_a \right. \right.$$

**2.485            ODE No. 485**

---

$$-y'(x) (ay(x)^2 + bx^2 + c) + axy(x)y'(x)^2 + bxy(x) = 0$$

✗ **Mathematica** : cpu = 300.024 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

time expired

**2.486            ODE No. 486**

---

$$-a^2 + y(x)^2 y'(x)^2 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0488184 (sec), leaf count = 117

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{a^2 - 2c_1x - c_1^2 - x^2} \right\}, \left\{ y(x) \rightarrow \sqrt{a^2 - 2c_1x - c_1^2 - x^2} \right\}, \left\{ y(x) \rightarrow -\sqrt{a^2 + 2c_1x - c_1^2 - x^2} \right\}, \right.$$

✓ **Maple** : cpu = 0.228 (sec), leaf count = 54

$$\left\{ y(x) = a, y(x) = \sqrt{-\_C1^2 + 2\_C1x + a^2 - x^2}, y(x) = -a, y(x) = -\sqrt{(a+x-\_C1)(\_C1+a-x)} \right\}$$

**2.487            ODE No. 487**

---

$$-6x^3 y'(x) + 4x^2 y(x) + y(x)^2 y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.285889 (sec), leaf count = 157

$$\left\{ \text{Solve} \left[ \frac{3}{4} \log(y(x)) - \frac{\sqrt{9x^6 - 4x^2 y(x)^3} \tanh^{-1} \left( \frac{3x^2}{\sqrt{9x^4 - 4y(x)^3}} \right)}{2x\sqrt{9x^4 - 4y(x)^3}} = c_1, y(x) \right], \text{Solve} \left[ \frac{\sqrt{9x^6 - 4x^2 y(x)^3} \tanh^{-1} \left( \frac{3x^2}{\sqrt{9x^4 - 4y(x)^3}} \right)}{2x\sqrt{9x^4 - 4y(x)^3}} = c_1, y(x) \right] \right\}$$

✓ **Maple** : cpu = 0.548 (sec), leaf count = 100

$$\left\{ y(x) = \text{RootOf} \left( -\ln(x) + \int^{-Z} -\frac{3}{4\_a(4\_a^3 - 9)} \left( 4\_a^3 + 3\sqrt{-4\_a^3 + 9} - 9 \right) d\_a + \_C1 \right) x^{\frac{4}{3}}, y(x) = \dots \right\}$$

**2.488            ODE No. 488**

---

$$4a^2 - 4ay(x)y'(x) - 4ax + y(x)^2 y'(x)^2 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.290123 (sec), leaf count = 85

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{16a^3x - 4a^2x^2 - 4ac_1x - c_1^2}}{2a} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{16a^3x - 4a^2x^2 - 4ac_1x - c_1^2}}{2a} \right\} \right\}$$

✓ **Maple** : cpu = 0.626 (sec), leaf count = 111

$$\left\{ y(x) = -2\sqrt{ax}, y(x) = 2\sqrt{ax}, y(x) = -\frac{1}{4a} \sqrt{-16a^4 + 32a^3x + (-16x^2 + 8\_C1)a^2 + 8\_C1ax - \_C1^2}, y(x) = \dots \right\}$$

**2.489            ODE No. 489**

---

$$ay(x)^2 + bx + c + y(x)^2 y'(x)^2 + 2xy(x)y'(x) = 0$$

✗ **Mathematica** : cpu = 300.427 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 2.747 (sec), leaf count = 551

$$\left\{ y(x) = -\frac{\sqrt{16}}{2a(a+1)} \sqrt{\left( a \left( ax - \frac{b}{2} + x \right)^2 (a+1)^2 \text{RootOf} \left( -b \ln(2ax - b + 2x) + 2 \int^{-Z} -1/4 \frac{1}{\_a(a+1)} d\_a + \_C1 \right) \right)} \right\}$$

**2.490            ODE No. 490**

---

$$a - x^2 - 2xy(x)y'(x) + y(x)^2 y'(x)^2 + 2y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.420531 (sec), leaf count = 70

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{-a + 8c_1x - 4c_1^2 - 2x^2}}{\sqrt{2}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{-a + 8c_1x - 4c_1^2 - 2x^2}}{\sqrt{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.625 (sec), leaf count = 145

$$\left\{ y(x) = \sqrt{-2\sqrt{a+2\_C1x-\_C1-x^2}-a}, y(x) = \sqrt{2\sqrt{a+2\_C1x-\_C1-x^2}-a}, y(x) = -\sqrt{-2\sqrt{a+2\_C1x-\_C1-x^2}-a} \right\}$$

2.491 ODE No. 491

---

$$(a-1)b+ax^2+2axy(x)y'(x)+(1-a)y(x)^2+y(x)^2y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.721782 (sec), leaf count = 79

$$\left\{\left\{y(x)\rightarrow-\sqrt{-2ac_1x+ac_1^2+b+2c_1x-c_1^2-x^2}\right\},\left\{y(x)\rightarrow\sqrt{-2ac_1x+ac_1^2+b+2c_1x-c_1^2-x^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.925 (sec), leaf count = 195

$$\left\{y(x)=\sqrt{-ax^2+b},y(x)=\frac{1}{a}\sqrt{a\left(-2x\sqrt{-a(b-C1)(a-1)}+(-x^2+b)a-b-C1\right)},y(x)=\frac{1}{a}\sqrt{\left(2x\sqrt{-a(b-C1)(a-1)}+(-x^2+b)a-b-C1\right)}\right\}$$

2.492 ODE No. 492

---

$$\left(y(x)^2-a^2\right)y'(x)^2+y(x)^2=0$$

✓ **Mathematica** : cpu = 0.0802164 (sec), leaf count = 97

$$\left\{\left\{y(x)\rightarrow\text{InverseFunction}\left[\sqrt{a^2-\#1^2}-a\tanh^{-1}\left(\frac{\sqrt{a^2-\#1^2}}{a}\right)\&][c_1-x]\right\},\left\{y(x)\rightarrow\text{InverseFunction}\left[\sqrt{a^2-\#1^2}+a\tanh^{-1}\left(\frac{\sqrt{a^2-\#1^2}}{a}\right)\&][c_1-x]\right\}\right\}$$

✓ **Maple** : cpu = 1.684 (sec), leaf count = 122

$$\left\{x-\sqrt{a^2-\left(y(x)\right)^2}+a^2\ln\left(\frac{1}{y(x)}\left(2a^2+2\sqrt{a^2}\sqrt{a^2-\left(y(x)\right)^2}\right)\right)\frac{1}{\sqrt{a^2}}-C1=0,x+\sqrt{a^2-\left(y(x)\right)^2}-a^2\ln\left(\frac{1}{y(x)}\left(2a^2+2\sqrt{a^2}\sqrt{a^2-\left(y(x)\right)^2}\right)\right)\frac{1}{\sqrt{a^2}}-C1=0\right\}$$

2.493 ODE No. 493

---

$$\left(a^2-2ax+y(x)^2\right)y'(x)^2+2ay(x)y'(x)+y(x)^2=0$$

✓ **Mathematica** : cpu = 7.37684 (sec), leaf count = 553

$$\left\{\text{Solve}\left[\left\{y(x)=\frac{-\sqrt{-aK^2\left(aK^2-2K^2x-2x\right)}-aK}{K^2+1},x=\frac{ac_1^2K^2-2K^2x-2x}{K^2+1}\right\}\right]\right\}$$

✓ **Maple** : cpu = 1.194 (sec), leaf count = 111

$$\left\{[x(-T)=\frac{1}{2a}\left(\sqrt{-T^2+1}\left(\text{Artanh}\left(\frac{1}{\sqrt{-T^2+1}}\right)\right)^2a^2+\left(-2a-C1\sqrt{-T^2+1}-2a^2\right)\text{Artanh}\left(\frac{1}{\sqrt{-T^2+1}}\right)\right)\right\}$$

2.494 ODE No. 494

---

$$\left(y(x)^2-a^2x^2\right)y'(x)^2+\left(1-a^2\right)x^2+2xy(x)y'(x)=0$$

✓ **Mathematica** : cpu = 0.112739 (sec), leaf count = 49

$$\left\{\left\{y(x)\rightarrow ac_1-\sqrt{c_1^2-x^2}\right\},\left\{y(x)\rightarrow ac_1+\sqrt{c_1^2-x^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.237 (sec), leaf count = 161

$$\left\{y(x)=\sqrt{a^2-1}x,y(x)=\text{RootOf}\left(-\ln(x)+\int^{-Z}\frac{1}{(-a^2+1)(-a^2-a^2+1)}\left(-a^3+a^2+\sqrt{-a^2a^2-a^4}\right)\right)\right\}$$

2.495 ODE No. 495

---

$$\left((1-a)x^2+y(x)^2\right)y'(x)^2+2axy(x)y'(x)+(1-a)y(x)^2+x^2=0$$

✓ **Mathematica** : cpu = 0.154434 (sec), leaf count = 83

$$\left\{\text{Solve}\left[\sqrt{a-1}\tan^{-1}\left(\frac{y(x)}{x}\right)-\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right)=c_1+\log(x),y(x)\right],\text{Solve}\left[\sqrt{a-1}\tan^{-1}\left(\frac{y(x)}{x}\right)+\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right)=c_1+\log(x),y(x)\right]\right\}$$

✓ **Maple** : cpu = 2.276 (sec), leaf count = 61

$$\left\{y(x)=\tan\left(\text{RootOf}\left(-2-Z\sqrt{a-1}-\ln\left(\frac{x^2}{(\cos(-Z))^2}+2-C1\right)\right)\right),y(x)=\tan\left(\text{RootOf}\left(2-Z\sqrt{a-1}-\ln\left(\frac{x^2}{(\cos(-Z))^2}+2-C1\right)\right)\right)\right\}$$

**2.496                      ODE No. 496**

---

$$(y(x) - x)^2 (y'(x)^2 + 1) - a^2 (y'(x) + 1)^2 = 0$$

✓ **Mathematica** : cpu = 19.5841 (sec), leaf count = 17831

Too large to display

✓ **Maple** : cpu = 0.464 (sec), leaf count = 130

$$\left\{ y(x) = x - \sqrt{2}a, y(x) = x + \sqrt{2}a, y(x) = x + \text{RootOf} \left( -x + \int^{-Z} -\frac{1}{2\_a^2 - 4a^2} \left( -a^2 - 2a^2 + \sqrt{-\_a^4 + 2} \right) \right) \right\}$$

**2.497                      ODE No. 497**

---

$$-x^2 - 2xy(x)y'(x) + 3y(x)^2y'(x)^2 + 4y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.143917 (sec), leaf count = 76

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{-4ie^{3c_1}x + e^{6c_1} - 3x^2}}{\sqrt{3}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{-4ie^{3c_1}x + e^{6c_1} - 3x^2}}{\sqrt{3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.787 (sec), leaf count = 203

$$\left\{ \ln(x) - \frac{\sqrt{3}}{6} \sqrt{\frac{(\sqrt{3}x - 3y(x))(\sqrt{3}x + 3y(x))}{x^2}} + \frac{1}{2} \sqrt{\frac{x^2 - 3(y(x))^2}{x^2}} - \text{Artanh} \left( \frac{1}{2} \sqrt{\frac{x^2 - 3(y(x))^2}{x^2}} \right) + \frac{1}{2} \ln \right\}$$

**2.498                      ODE No. 498**

---

$$(3y(x) - 2)y'(x)^2 + 4y(x) - 4 = 0$$

✓ **Mathematica** : cpu = 0.0775503 (sec), leaf count = 107

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\sqrt{1 - \#1} \sqrt{3\#1 - 2} - \frac{\sin^{-1}(\sqrt{3 - 3\#1})}{\sqrt{3}} \& \right] [c_1 - 2x] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \right\} \right\}$$

✓ **Maple** : cpu = 1.002 (sec), leaf count = 99

$$\left\{ y(x) = 1, y(x) = \frac{\sin \left( \text{RootOf} \left( -8\sqrt{3}\_C1\_Z + 8\sqrt{3}x\_Z - (\cos(\_Z))^2 + 48\_C1^2 - 96\_C1x + 48x^2 + \right) \right)}{6} \right\}$$

**2.499                      ODE No. 499**

---

$$a^2(-x^2) - 2a^2xy(x)y'(x) + (1 - a^2)y(x)^2y'(x)^2 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.382935 (sec), leaf count = 212

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{a^6(-x^2) + 3a^4x^2 + 2a^2xe^{a^2c_1 - c_1} - 2xe^{a^2c_1 - c_1} + e^{2a^2c_1 - 2c_1} - 3a^2x^2 + x^2}}{\sqrt{a^6 - 3a^4 + 3a^2 - 1}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{a^6(-x^2)}}{\sqrt{a^6 - 3a^4 + 3a^2 - 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.253 (sec), leaf count = 189

$$\left\{ y(x) = \text{RootOf} \left( -\ln(x) + \int^{-Z} \frac{-a}{(\_a^2a^2 - \_a^2 + a^2)(\_a^2 + 1)} \left( -\_a^2a^2 + \_a^2 - a^2 + \sqrt{\_a^2a^2 - \_a^2 + a^2} \right) \right) \right\}$$

**2.500                      ODE No. 500**

---

$$(a - b)y(x)^2y'(x)^2 - ab + ay(x)^2 - bx^2 - 2bxy(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.889223 (sec), leaf count = 100

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{-ab - 2ac_1x + ac_1^2 + ax^2 + b^2 - bx^2}}{\sqrt{b - a}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{-ab - 2ac_1x + ac_1^2 + ax^2 + b^2 - bx^2}}{\sqrt{b - a}} \right\} \right\}$$

✓ **Maple** : cpu = 1.02 (sec), leaf count = 220

$$\left\{ y(x) = \frac{1}{b} \sqrt{\left( -2x\sqrt{-ab(b - \_C1)} + (-x^2 + \_C1 + a)b - \_C1a \right)}, y(x) = \frac{1}{b} \sqrt{\left( 2x\sqrt{-ab(b - \_C1)} + (-x^2 + \_C1 + a)b - \_C1a \right)} \right\}$$

2.501 ODE No. 501

$$y'(x)^2 \left( a y(x)^2 + b x + c \right) - b y(x) y'(x) + d y(x)^2 = 0$$

✓ **Mathematica** : cpu = 21.8209 (sec), leaf count = 913

$$\left\{ \text{Solve} \left[ \left\{ y(x) = \frac{b \sqrt{-b^2 + 4 a \left( \sqrt{d} \sqrt{-T^2 a + d} + d \right)}}{2 \left( a \sqrt{d} \sqrt{-T^2 a + d} + d \right)}, x = \right. \right. \right.$$

✓ **Maple** : cpu = 5.392 (sec), leaf count = 215

$$\left\{ [x(\_T) = -\frac{1}{4bd} \left( \left( \ln \left( \frac{1}{\_T} \left( \sqrt{d} \sqrt{-T^2 a + d} + d \right) \right) \right)^2 \sqrt{-T^2 a + d} b^2 + \left( \left( 2 \ln(2) b^2 + 4 \sqrt{d} \_C1 b \right) \sqrt{-T^2 a} \right) \right] \right.$$

2.502 ODE No. 502

$$\left( a y(x) - b x \right)^2 \left( a^2 y'(x)^2 + b^2 \right) - c^2 \left( a y'(x) + b \right)^2 = 0$$

✓ **Mathematica** : cpu = 1.19749 (sec), leaf count = 100

$$\left\{ \left\{ y(x) \rightarrow \frac{b c_1}{a} - \frac{\sqrt{2 b^2 c_1 x - b^2 c_1^2 + b^2 (-x^2) + c^2}}{a} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{2 b^2 c_1 x - b^2 c_1^2 + b^2 (-x^2) + c^2}}{a} + \frac{b c_1}{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.69 (sec), leaf count = 195

$$\left\{ y(x) = \frac{b x - \sqrt{2} c}{a}, y(x) = \frac{b x + \sqrt{2} c}{a}, y(x) = \frac{1}{a} \left( \text{RootOf} \left( -x + \int^{-Z} \frac{a}{(2 \_a^2 a^2 - 4 c^2) b} \left( -\_a^2 a^2 + 2 c^2 + \sqrt{-} \right) \right) \right.$$

2.503 ODE No. 503

$$a0 + y'(x)(a1x + b1y(x) + c1) + y'(x)^2(a2x + b2y(x) + c2)^2 + b0y(x) + c0 = 0$$

✗ **Mathematica** : cpu = 300.057 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve((b2\*y(x)+a2\*x+c2)^2\*diff(y(x),x)^2+(a1\*x+b1\*y(x)+c1)\*diff(y(x),x)+b0\*y(x)+a0+c0=0,y(x))

2.504 ODE No. 504

$$- \left( -a + x^3 + y(x)^3 \right) y'(x) + x^2 y(x) + x y(x)^2 y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.332354 (sec), leaf count = 51

$$\left\{ y(x) \rightarrow -\frac{\sqrt[3]{-\frac{1}{2} \sqrt[3]{2 c_1 + 1} \sqrt[3]{2 a c_1 - a + 2 x^3}}}{\sqrt[3]{2 c_1 - 1}} \right\}$$

✓ **Maple** : cpu = 1.107 (sec), leaf count = 247

$$\left\{ \int_{-b}^{y(x)} \_a^2 \frac{1}{\sqrt{x^6 + (-2 \_a^3 - 2 a) x^3 + (-\_a^3 + a)^2}} d\_a - \frac{\ln(x)}{2} - \_C1 = 0, \int_{-b}^{y(x)} \_a^2 \frac{1}{\sqrt{x^6 + (-2 \_a^3 - 2 a) x^3 + (-\_a^3 + a)^2}} d\_a - \frac{\ln(x)}{2} - \_C1 = 0 \right.$$

2.505 ODE No. 505

$$-x^3 + x y(x)^2 y'(x)^2 - 2 y(x)^3 y'(x) + 2 x y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0785706 (sec), leaf count = 73

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{2 c_1 + x^2} \right\}, \left\{ y(x) \rightarrow \sqrt{2 c_1 + x^2} \right\}, \left\{ y(x) \rightarrow -\sqrt{c_1 x^4 + x^2} \right\}, \left\{ y(x) \rightarrow \sqrt{c_1 x^4 + x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 52

$$\left\{ y(x) = \sqrt{x^2 + \_C1}, y(x) = \sqrt{\_C1 x^2 + 1 x}, y(x) = -\sqrt{x^2 + \_C1}, y(x) = -\sqrt{\_C1 x^2 + 1 x} \right\}$$

2.506 ODE No. 506

---

$$2x^2(y(x) - x)y(x)^2y'(x) + x^2(xy(x)^2 - 1)y'(x)^2 - (x^2y(x) - 1)y(x)^2 = 0$$

✗ **Mathematica** : cpu = 55.6069 (sec), leaf count = 0 , could not solve  
DSolve[-(y[x]^2\*(-1 + x^2\*y[x])) + 2\*x^2\*y[x]^2\*(-x + y[x])\*Derivative[1][y][x] + x^2\*(-1 + x\*y[x]^2)\*Derivative[1][y][x]^2 == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(x^2\*(x\*y(x)^2-1)\*diff(y(x),x)^2+2\*x^2\*y(x)^2\*(y(x)-x)\*diff(y(x),x)-y(x)^2\*(x^2\*y(x)-1)=0,y(x))

2.507 ODE No. 507

---

$$(y(x)^4 - a^2x^2)y'(x)^2 + 2a^2xy(x)y'(x) + y(x)^2(y(x)^2 - a^2) = 0$$

✗ **Mathematica** : cpu = 44.0017 (sec), leaf count = 0 , could not solve  
DSolve[y[x]^2\*(-a^2 + y[x]^2) + 2\*a^2\*x\*y[x]\*Derivative[1][y][x] + (-a^2\*x^2 + y[x]^4)\*Derivative[1][y][x]^2 == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve((y(x)^4-a^2\*x^2)\*diff(y(x),x)^2+2\*a^2\*x\*y(x)\*diff(y(x),x)+y(x)^2\*(y(x)^2-a^2)=0,y(x))

2.508 ODE No. 508

---

$$(x^2y(x)^2 - x^2 + y(x)^4)y'(x)^2 + 2xy(x)y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 2.02951 (sec), leaf count = 88

$$\text{Solve} \left[ \frac{\sqrt{x^2 + y(x)^2}y(x) \left( \log \left( \frac{x}{\sqrt{x^2 + y(x)^2}} + 1 \right) - \log \left( 1 - \frac{x}{\sqrt{x^2 + y(x)^2}} \right) \right)}{2x^2 \sqrt{\frac{y(x)^2(x^2 + y(x)^2)}{x^4}}} + y(x) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 1.753 (sec), leaf count = 60

$$\left\{ y(x) = -ix, y(x) = ix, y(x) = -\text{Artanh} \left( \text{RootOf} \left( (\text{Artanh}(\_Z))^2\_Z^2 - 2\text{Artanh}(\_Z)\_C1\_Z^2 + \_C1^2 \right) \right) \right\}$$

2.509 ODE No. 509

---

$$9(x^2 - 1)y(x)^4y'(x)^2 - 4x^2 - 6xy(x)^5y'(x) = 0$$

✓ **Mathematica** : cpu = 0.276089 (sec), leaf count = 34

$$\left\{ y(x) \rightarrow -\frac{\sqrt[3]{-\frac{1}{2}}\sqrt[3]{c_1^2 - 4x^2 + 4}}{\sqrt[3]{c_1}} \right\}$$

✓ **Maple** : cpu = 1.148 (sec), leaf count = 212

$$\left\{ y(x) = \sqrt[6]{-4x^2 + 4}, y(x) = -\sqrt[6]{-4x^2 + 4}, y(x) = -\frac{i\sqrt{3} - 1}{2}\sqrt[6]{-4x^2 + 4}, y(x) = \frac{i\sqrt{3} - 1}{2}\sqrt[6]{-4x^2 + 4}, y(x) = \frac{i\sqrt{3} + 1}{2}\sqrt[6]{-4x^2 + 4}, y(x) = -\frac{i\sqrt{3} + 1}{2}\sqrt[6]{-4x^2 + 4} \right\}$$

2.510 ODE No. 510

---

$$-(x^4y(x)^2 - 1)y(x)^2 + x^2(x^2y(x)^4 - 1)y'(x)^2 + 2x^3(y(x)^2 - x^2)y(x)^3y'(x) = 0$$

✗ **Mathematica** : cpu = 52.9533 (sec), leaf count = 0 , could not solve  
DSolve[-(y[x]^2\*(-1 + x^4\*y[x]^2)) + 2\*x^3\*y[x]^3\*(-x^2 + y[x]^2)\*Derivative[1][y][x] + x^2\*(1 + x^2\*y[x]^4)\*Derivative[1][y][x]^2 == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(x^2\*(x^2\*y(x)^4-1)\*diff(y(x),x)^2+2\*x^3\*y(x)^3\*(y(x)^2-x^2)\*diff(y(x),x)-y(x)^2\*(x^4\*y(x)^2-1)=0,y(x))

**2.511 ODE No. 511**

---

$$\left(a^2\sqrt{x^2+y(x)^2}-x^2\right)y'(x)^2+a^2\sqrt{x^2+y(x)^2}+2xy(x)y'(x)-y(x)^2=0$$

✓ **Mathematica** : cpu = 1.43967 (sec), leaf count = 229

$$\left\{\text{Solve}\left[\tan^{-1}\left(\frac{x}{y(x)}\right)-\frac{2\sqrt{a^2\left(x^2+y(x)^2\right)\left(\sqrt{x^2+y(x)^2}-a^2\right)}\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+y(x)^2}-a^2}}{a}\right)}{a\sqrt{x^2+y(x)^2}\sqrt{\sqrt{x^2+y(x)^2}-a^2}}=c_1,y(x)\right],\text{Solve}\left[\right.\right.$$

✓ **Maple** : cpu = 3.776 (sec), leaf count = 199

$$\left\{\arctan\left(\frac{x}{y(x)}\right)-2\frac{\sqrt{a^2\left(\left(y(x)\right)^2+x^2\right)\left(-a^2+\sqrt{\left(y(x)\right)^2+x^2}\right)}}{a\sqrt{\left(y(x)\right)^2+x^2}\sqrt{-a^2+\sqrt{\left(y(x)\right)^2+x^2}}}\arctan\left(\frac{\sqrt{-a^2+\sqrt{\left(y(x)\right)^2+x^2}}}{a}\right)-\_C1\right.$$

**2.512 ODE No. 512**

---

$$\left(a\left(x^2+y(x)^2\right)^{3/2}-x^2\right)y'(x)^2+a\left(x^2+y(x)^2\right)^{3/2}+2xy(x)y'(x)-y(x)^2=0$$

✓ **Mathematica** : cpu = 5.92611 (sec), leaf count = 713

$$\left\{\text{Solve}\left[\tan^{-1}\left(\frac{x}{y(x)}\right)-\frac{i\sqrt{a\left(\left(x^2+y(x)^2\right)^{5/2}-a\left(x^2+y(x)^2\right)^3\right)}\left(\sqrt{2}\left(\log\left(\frac{a^{3/2}\left(3i\sqrt{2}a\sqrt{x^2+y(x)^2}+4\sqrt{a}\sqrt{\sqrt{x^2+y(x)^2}-a^2}\right)}{4a\sqrt{x^2+y(x)^2}}\right)\right)}{1}\right)=c_1,y(x)\right],\text{Solve}\left[\right.$$

✓ **Maple** : cpu = 4.408 (sec), leaf count = 137

$$\left\{y(x)=x\left(\tan\left(\text{RootOf}\left(-\_Z+\int\frac{x^2\left((\tan(\_Z))^2+1\right)}{(\tan(\_Z))^2}dx-\frac{1}{2\_a^2(\_a^2-1)}(a\sqrt{\_a}+1)\sqrt{-\_a^3a^2+\_a^{\frac{5}{2}}ad\_a}+c_1\right)\right)\right.\right.$$

**2.513 ODE No. 513**

---

$$y'(x)^2\sin(y(x))+2xy'(x)\cos^3(y(x))-\sin(y(x))\cos^4(y(x))=0$$

✓ **Mathematica** : cpu = 0.0750519 (sec), leaf count = 81

$$\left\{\left\{y(x)\rightarrow\tan^{-1}\left(2\left(-\frac{c_1^{3/2}}{\sqrt{c_1+x}}-\frac{\sqrt{c_1}x}{\sqrt{c_1+x}}\right)\right)\right\},\left\{y(x)\rightarrow\tan^{-1}\left(2\left(\frac{c_1^{3/2}}{\sqrt{c_1+x}}+\frac{\sqrt{c_1}x}{\sqrt{c_1+x}}\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 3.5 (sec), leaf count = 1138

$$\left\{[x(\_T)=\frac{1}{2\_T}\left(\cos\left(\frac{1}{2}\arctan\left(\frac{1}{3}\left(\_C1^2\_T^2-2\_C1\_T\sqrt[3]{-\_C1^3\_T^3+54\_C1\_T+6\sqrt{3}\sqrt{-\_C1^2\_T^2-2\_C1\_T}}\right)}\right)\right)\right.\right.$$

**2.514 ODE No. 514**

---

$$y'(x)^2(a\cos(y(x))+b)-c\cos(y(x))+d=0$$

✓ **Mathematica** : cpu = 9.97534 (sec), leaf count = 605



$$\left\{\left\{y(x) \rightarrow \text{InverseFunction}\left[\frac{4 \sin ^2\left(\frac{\# 1}{2}\right) \csc (\# 1) \sqrt{a \cos (\# 1)+b} \sqrt{\frac{\cot ^2\left(\frac{\# 1}{2}\right)(c-d)}{c+d}} \sqrt{\frac{\csc ^2\left(\frac{\# 1}{2}\right)(a+b)(d-c \cos (\# 1))}{a d+b c}}\right)}{c(a \cos (\_a)+b)}\right.\right.$$

✓ **Maple** : cpu = 0.257 (sec), leaf count = 87

$$\left\{x-\int^{y(x)}(a \cos (\_a)+b) \frac{1}{\sqrt{(a \cos (\_a)+b)(c \cos (\_a)-d)}} d\_a-\_C 1=0, x-\int^{y(x)}-(a \cos (\_a)+b) \frac{1}{\sqrt{(a \cos (\_a)+b)(c \cos (\_a)-d)}} d\_a-\_C 1=0\right.$$

### 2.515 ODE No. 515

$$f\left(x^2+y(x)^2\right)\left(y'(x)^2+1\right)-\left(x y'(x)-y(x)\right)^2=0$$

✓ **Mathematica** : cpu = 2.04037 (sec), leaf count = 1922

$$\left\{\text{Solve}\left[\int_1^x\left(\frac{\sqrt{f\left(K[1]^2+y(x)^2\right)\left(K[1]^2+y(x)^2-f\left(K[1]^2+y(x)^2\right)\right)} K[1]}{f\left(K[1]^2+y(x)^2\right)\left(K[1]^2+y(x)^2\right)}-\frac{\sqrt{f\left(K[1]^2+y(x)^2\right)\left(K[1]^2+y(x)^2-f\left(K[1]^2+y(x)^2\right)\right)} K[1]}{f\left(K[1]^2+y(x)^2\right)\left(K[1]^2+y(x)^2\right)}\right] d x=C 1\right.$$

✓ **Maple** : cpu = 0.943 (sec), leaf count = 113

$$\left\{y(x)=x\left(\tan \left(\text{RootOf}\left(-\_Z+\int^{\frac{x^2\left((\tan (\_Z))^2+1\right)}{(\tan (\_Z))^2}}-\frac{1}{2\_a\left(f(\_a)-\_a\right)} \sqrt{-\left(f(\_a)-\_a\right) f(\_a)} d\_a+\_C 1\right)}\right)\right.$$

### 2.516 ODE No. 516

$$\left(x^2+y(x)^2\right) f\left(\frac{x}{\sqrt{x^2+y(x)^2}}\right)\left(y'(x)^2+1\right)-\left(x y'(x)-y(x)\right)^2=0$$

✓ **Mathematica** : cpu = 0.477517 (sec), leaf count = 253

$$\left\{\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{f\left(\frac{1}{\sqrt{K[1]^2+1}}\right) K[1]^2+f\left(\frac{1}{\sqrt{K[1]^2+1}}\right)-1}{\sqrt{f\left(\frac{1}{\sqrt{K[1]^2+1}}\right)\left(K[1]-i\right)\left(K[1]+i\right)\left(\sqrt{f\left(\frac{1}{\sqrt{K[1]^2+1}}\right) K[1]}+i \sqrt{f\left(\frac{1}{\sqrt{K[1]^2+1}}\right)-1}\right)}} d K[1]=c_1\right]$$

✓ **Maple** : cpu = 0.822 (sec), leaf count = 70

$$\left\{y(x)=\text{RootOf}\left(-\ln (x)+\int^{-Z}-\frac{1}{\_a^2+1}\left(-a f\left(\frac{1}{\sqrt{\_a^2+1}}\right)+\sqrt{-\left(f\left(\frac{1}{\sqrt{\_a^2+1}}\right)\right)^2+f\left(\frac{1}{\sqrt{\_a^2+1}}\right)}\right) d\_Z\right.\right.$$

### 2.517 ODE No. 517

$$\left(x^2+y(x)^2\right) f\left(\frac{y(x)}{\sqrt{x^2+y(x)^2}}\right)\left(y'(x)^2+1\right)-\left(x y'(x)-y(x)\right)^2=0$$

✓ **Mathematica** : cpu = 0.492725 (sec), leaf count = 283

$$\left\{\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{f\left(\frac{K[1]}{\sqrt{K[1]^2+1}}\right) K[1]^2+f\left(\frac{K[1]}{\sqrt{K[1]^2+1}}\right)-1}{\sqrt{f\left(\frac{K[1]}{\sqrt{K[1]^2+1}}\right)\left(K[1]-i\right)\left(K[1]+i\right)\left(\sqrt{f\left(\frac{K[1]}{\sqrt{K[1]^2+1}}\right) K[1]}+i \sqrt{f\left(\frac{K[1]}{\sqrt{K[1]^2+1}}\right)-1}\right)}} d K[1]=c_1\right]$$

✓ **Maple** : cpu = 0.842 (sec), leaf count = 78

$$\left\{y(x)=\text{RootOf}\left(-\ln (x)+\int^{-Z}-\frac{1}{\_a^2+1}\left(-a f\left(-a \frac{1}{\sqrt{\_a^2+1}}\right)+\sqrt{-\left(f\left(-a \frac{1}{\sqrt{\_a^2+1}}\right)\right)^2+f\left(-a \frac{1}{\sqrt{\_a^2+1}}\right)}\right) d\_Z\right.\right.$$

## 2.518      ODE No. 518

$$y'(x)^3 - (y(x) - a)^2(y(x) - b)^2 = 0$$

✓ **Mathematica** : cpu = 0.517174 (sec), leaf count = 236

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{3\sqrt[3]{a-\#1} \left( \frac{\#1-b}{a-b} \right)^{2/3} {}_2F_1 \left( \frac{1}{3}, \frac{2}{3}; \frac{4}{3}; \frac{a-\#1}{a-b} \right)}{(b-\#1)^{2/3}} \& [c_1+x] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \right\} \right.$$

✓ **Maple** : cpu = 0.156 (sec), leaf count = 126

$$\left\{ x - \int^{y(x)} \frac{1}{\sqrt[3]{(-a-a)^2(-a-b)^2}} da - C1 = 0, x - \int^{y(x)} 2 \frac{1}{(i\sqrt{3}-1)\sqrt[3]{(-a+a)^2(-a+b)^2}} da - C \right.$$

2.519      ODE No. 519

$$y'(x)^3 - f(x) (ay(x)^2 + by(x) + c)^2 = 0$$

✓ **Mathematica** : cpu = 0.614992 (sec), leaf count = 353

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{\sqrt[3]{2}(2\#1a+b) \left( \frac{a(\#1(\#1a+b)+c)}{4ac-b^2} \right)^{2/3} {}_2F_1 \left( \frac{1}{2}, \frac{2}{3}; \frac{3}{2}; \frac{(b+2a\#1)^2}{b^2-4ac} \right)}{a(\#1(\#1a+b)+c)^{2/3}} \right] \& \left[ \int_1^x \sqrt[3]{f(K[1])} dK \right. \right. \right.$$

✓ **Maple** : cpu = 0.342 (sec), leaf count = 197

$$\left\{ \int^{y(x)} (-a^2 a + -a b + c)^{-\frac{2}{3}} d_- a + \int^x -1 \sqrt[3]{f(-a) \left( a(y(x))^2 + b y(x) + c \right)^2} \left( a(y(x))^2 + b y(x) + c \right)^{-\frac{2}{3}} d_- a + \right.$$

## 2.520      ODE No. 520

$$y'(x)^3 + y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 187.208 (sec), leaf count = 1590

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{(27\#1^2 + 4) \left( 27\#1^2 - 3\sqrt{81\#1^2 + 12}\#1 + 4 {}_2F_1 \left( \frac{2}{3}, 1; \frac{5}{3}; \frac{1}{12} \left( \sqrt{81\#1^2 + 12} - \right. \right. \right. \right. \right.}{48\sqrt[3]{3} \left( 27\#1^2 - 3\sqrt{81\#1^2 + 12} \right)} \right. \right. \right.$$

✓ **Maple** : cpu = 0.115 (sec), leaf count = 245

$$\left\{ x - \int^{y(x)} 6 \frac{\sqrt[3]{108 - a + 12 \sqrt{81 - a^2 + 12}}}{(108 - a + 12 \sqrt{81 - a^2 + 12})^{2/3} - 12} d_{-a} - C1 = 0, x - \int^{y(x)} -12 \frac{1}{(i\sqrt{3} - 1) \left( -\sqrt[3]{108 - a + 12 \sqrt{81 - a^2 + 12}} \right)} d_{-a} - C2 = 0 \right\}$$

2.521      ODE No. 521

$$y'(x)^3 + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0028396 (sec), leaf count = 14

$$\{\{y(x) \rightarrow c_1x + c_1^3\}\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 33

$$\left\{y(x) = -C1 \left(-C1^2 + x\right), y(x) = -\frac{2x}{9}\sqrt{-3x}, y(x) = \frac{2x}{9}\sqrt{-3x}\right\}$$

2.522 ODE No. 522

---

$$y'(x)^3 - (x + 5)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0035051 (sec), leaf count = 20

$$\left\{\left\{y(x) \rightarrow c_1 x - c_1^3 + 5 c_1\right\}\right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 44

$$\left\{y(x) = -C1 \left(-C1^2 + x + 5\right), y(x) = -\frac{2 x + 10}{9} \sqrt{3 x + 15}, y(x) = \frac{2 x + 10}{9} \sqrt{3 x + 15}\right\}$$

2.523 ODE No. 523

---

$$-a x y'(x) + x^3 + y'(x)^3 = 0$$

✓ **Mathematica** : cpu = 145.521 (sec), leaf count = 392

$$\left\{\left\{y(x) \rightarrow \int_1^x \left(\frac{\sqrt[3]{\frac{2}{3} a K[1]}}{\sqrt[3]{\sqrt{3} \sqrt{27 K[1]^6 - 4 a^3 K[1]^3 - 9 K[1]^3}}} + \frac{\sqrt[3]{\sqrt{3} \sqrt{27 K[1]^6 - 4 a^3 K[1]^3 - 9 K[1]^3}}}{\sqrt[3]{2} 3^{2/3}}\right) d K[1] + c_1\right\}\right\}$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 231

$$\left\{y(x) = \int i \left(\left(\frac{i}{12} - \frac{\sqrt{3}}{12}\right) \left(-108 x^3 + 12 \sqrt{-12 a^3 x^3 + 81 x^6}\right)^{\frac{2}{3}} + a \left(\sqrt{3} + i\right) x\right) \frac{1}{\sqrt[3]{-108 x^3 + 12 \sqrt{-12 a^3 x^3 - 81 x^6}}}\right\}$$

2.524 ODE No. 524

---

$$y'(x)^3 - 2y(x)y'(x) + y(x)^2 = 0$$

✗ **Mathematica** : cpu = 300.005 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.045 (sec), leaf count = 261

$$\left\{x - \int^{y(x)} 6 \frac{\sqrt[3]{-108 - a^2 + 12 \sqrt{3} \sqrt{27 - a^4 - 32 - a^3}}}{\left(-108 - a^2 + 12 \sqrt{3} \sqrt{27 - a^4 - 32 - a^3}\right)^{2/3} + 24 - a} d - a - C1 = 0, x - \int^{y(x)} 24 \frac{1}{\left(i \sqrt{3} - 1\right) \left(\left(i \sqrt{3} - 1\right) \sqrt[3]{-108 - a^2 + 12 \sqrt{3} \sqrt{27 - a^4 - 32 - a^3}}\right)}\right\}$$

2.525 ODE No. 525

---

$$-a x y(x) y'(x) + 2 a y(x)^2 + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0737801 (sec), leaf count = 135

$$\left\{\left\{y(x) \rightarrow c_1 \exp \left(\frac{1}{2} \left(\frac{a x^2}{2} + \frac{1}{2} \sqrt{a} x \sqrt{a x^2 - 8} - 4 \log \left(\sqrt{a} \sqrt{a x^2 - 8} + a x\right)\right)\right)\right\}, \left\{y(x) \rightarrow c_1 \exp \left(\frac{1}{2} \left(\frac{a x^2}{2} - \frac{1}{2} \sqrt{a} x \sqrt{a x^2 - 8} + 4 \log \left(\sqrt{a} \sqrt{a x^2 - 8} + a x\right)\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 122

$$\left\{y(x) = -C1 \left(a^2 x \frac{1}{\sqrt{a^2}} + \sqrt{a^2 x^2 - 8 a}\right)^{-2 \frac{a}{\sqrt{a^2}}} e^{\frac{x}{4} \left(a x + \sqrt{a^2 x^2 - 8 a}\right)}, y(x) = -C1 \left(a^2 x \frac{1}{\sqrt{a^2}} + \sqrt{a^2 x^2 - 8 a}\right)^{2 \frac{a}{\sqrt{a^2}}}\right\}$$

2.526 ODE No. 526

---

$$-x^3 y(x)^3 - \left(x^2 + x y(x) + y(x)^2\right) y'(x)^2 + \left(x^3 y(x) + x^2 y(x)^2 + x y(x)^3\right) y'(x) + y'(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.0477202 (sec), leaf count = 45

$$\left\{\left\{y(x) \rightarrow \frac{1}{-c_1 - x}\right\}, \left\{y(x) \rightarrow c_1 e^{\frac{x^2}{2}}\right\}, \left\{y(x) \rightarrow c_1 + \frac{x^3}{3}\right\}\right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 32

$$\left\{y(x) = (-x + -C1)^{-1}, y(x) = -C1 e^{\frac{x^2}{2}}, y(x) = \frac{x^3}{3} + -C1\right\}$$

2.527 ODE No. 527

---

$$-xy(x)^4y'(x)+y'(x)^3-y(x)^5=0$$

✓ **Mathematica** : cpu = 0.0147895 (sec), leaf count = 20

$$\left\{\left\{y(x)\rightarrow \frac{1}{c_1\left(x-c_1^2\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.518 (sec), leaf count = 43

$$\left\{y\left(x\right)=-CI\sqrt{\frac{-CI^{10}}{\left(-CI^4x-1\right)^2}},y\left(x\right)=-\frac{3\sqrt{3}}{2}x^{-\frac{3}{2}},y\left(x\right)=\frac{3\sqrt{3}}{2}x^{-\frac{3}{2}}\right\}$$

2.528 ODE No. 528

---

$$abx+ay'(x)^2+by(x)+y'(x)^3=0$$

✓ **Mathematica** : cpu = 0.623946 (sec), leaf count = 398

Solve  $\left[\left[\begin{array}{l} x=c_1-\frac{-a\left(\frac{\sqrt[3]{-2a^3+\sqrt{\left(-2a^3-27abx-27by(x)\right)^2-4a^6-27abx-27by(x)}}}{3\sqrt[3]{2}}+\frac{\sqrt[3]{2}a^2}{3\sqrt[3]{-2a^3+\sqrt{\left(-2a^3-27abx-27by(x)\right)^2-4a^6-27abx-27by(x)}}}\right]}{b}\right]\right]$

✓ **Maple** : cpu = 0.068 (sec), leaf count = 86

$$\left\{y\left(x\right)=-ax-\frac{\left({\rm e}^{RootOf\left(-2\_Za^2-3{\rm e}^2\_Z+8ae^{-Z}+2\_C1b-5a^2-2bx\right)}-a\right)^2{\rm e}^{RootOf\left(-2\_Za^2-3{\rm e}^2\_Z+8ae^{-Z}+2\_C1b-5a^2-2bx\right)}}{b}\right\}$$

2.529 ODE No. 529

---

$$y'(x)^3+xy'(x)^2-y(x)=0$$

✓ **Mathematica** : cpu = 39.3703 (sec), leaf count = 1758

$$\left\{\left\{y(x)\rightarrow \frac{1}{2}\left(\frac{4\,2^{2/3}x^4}{3\left(-16x^3-72x^2-108x+216c_1+\sqrt{4\left(-4x^2-12x-9\right)^3+\left(-16x^3-72x^2-108x+216c_1+54\right)^2}\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 1251

$$\left\{y\left(x\right)=0,y\left(x\right)=1\left(\left(4x-6\right)\sqrt[3]{-36x^2-54x+108\_C1-8x^3+27}+6\sqrt{-6\left(1+2\_C1\right)\left(4x^3+18x^2-24x+6\_C1\right)}\right)\right\}$$

2.530 ODE No. 530

---

$$y'(x)^3-y(x)y'(x)^2+y(x)^2=0$$

✓ **Mathematica** : cpu = 7.97625 (sec), leaf count = 648

$$\left\{\left\{y(x)\rightarrow \text{InverseFunction}\left[\int_1^{\#1}\frac{\sqrt[3]{2K[1]^3-27K[1]^2+3\sqrt{3}\sqrt{-K[1]^4(4K[1]-27)}}}{2\sqrt[3]{2}K[1]^2+2\sqrt[3]{2K[1]^3-27K[1]^2+3\sqrt{3}\sqrt{-K[1]^4(4K[1]-27)}}K[1]+2^{2/3}\left(2K[1]^2-3\sqrt{3}\sqrt{-K[1]^4(4K[1]-27)}\right)}\right]}{2K[1]^2-3\sqrt{3}\sqrt{-K[1]^4(4K[1]-27)}}}\right\}\right\}$$

✓ **Maple** : cpu = 0.086 (sec), leaf count = 432

$$\left\{x-\int^{y(x)}6\frac{\sqrt[3]{-108\_a^2+8\_a^3+12\sqrt{-12\_a^5+81\_a^4}}}{4\_a^2+2\_a\sqrt[3]{-108\_a^2+8\_a^3+12\sqrt{-12\_a^5+81\_a^4}}+\left(-108\_a^2+8\_a^3+12\sqrt{-12\_a^5+81\_a^4}\right)}\right\}$$

2.531 ODE No. 531

$$-x^3y(x)^6 - (x^2 + y(x)^4 + xy(x)^2) y'(x)^2 + (x^3y(x)^2 + x^2y(x)^4 + xy(x)^6) y'(x) + y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 61.0335 (sec), leaf count = 0 , could not solve  
DSolve[-(x^3\*y[x]^6) + (x^3\*y[x]^2 + x^2\*y[x]^4 + x\*y[x]^6)\*Derivative[1][y][x] + Derivative[1][y][x]^2,x] == 0  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x)^2-(y(x)^4+x\*y(x)^2+x^2)\*diff(y(x),x)^2+(x\*y(x)^6+x^2\*y(x)^4+x^3\*y(x)^2)\*x^3\*y(x)^6=0,y(x))

2.532 ODE No. 532

$$ay'(x)^3 + by'(x)^2 + cy'(x) - d - y(x) = 0$$

✗ **Mathematica** : cpu = 300.004 (sec), leaf count = 0 , timed out  
\$Aborted  
✓ **Maple** : cpu = 0.101 (sec), leaf count = 848

$$\left\{x-\int^{y(x)}6\frac{a\sqrt[3]{12\sqrt{3}\sqrt{27\left(d+_{-}a\right)^2a^2}}}{\left(12\sqrt{3}\sqrt{27\left(d+_{-}a\right)^2a^2}+18c\left(\left(d+_{-}a\right)b+2/9c^2\right)a+\left(-4d-4_{-}a\right)b^3-b^2c^2a+\left(108d+108_{-}ab+108_{-}a^2\right)c\right)}dy\right\}$$

2.533 ODE No. 533

$$a + xy'(x)^3 - y(x)y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0163515 (sec), leaf count = 40

$$\left\{\left\{y(x)\rightarrow\frac{3\left(\sqrt{a}x\right)^{2/3}}{2^{2/3}}\right\},\left\{y(x)\rightarrow\frac{a+c_1^3x}{c_1^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.025 (sec), leaf count = 76

$$\left\{y\left(x\right)=\frac{x_{-}C1^3+a}{_{-}C1^2},y\left(x\right)=\frac{3\sqrt[3]{2}\sqrt[3]{ax^2}}{2},y\left(x\right)=\frac{3\sqrt[3]{2}\left(i\sqrt{3}-1\right)\sqrt[3]{ax^2}}{4},y\left(x\right)=-\frac{3\sqrt[3]{2}\left(i\sqrt{3}+1\right)\sqrt[3]{ax^2}}{4}\right\}$$

2.534 ODE No. 534

$$4xy'(x)^3 - 6y(x)y'(x)^2 + 3y(x) - x = 0$$

✓ **Mathematica** : cpu = 0.049745 (sec), leaf count = 114

$$\left\{\left\{y(x)\rightarrow\frac{-\sqrt{2}\sqrt{c_1x^3+3c_1^2x^2+3c_1^3x+c_1^4}-c_1^2}{3c_1}\right\},\left\{y(x)\rightarrow\frac{\sqrt{2}\sqrt{c_1x^3+3c_1^2x^2+3c_1^3x+c_1^4}-c_1^2}{3c_1}\right\}\right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 84

$$\left\{y\left(x\right)=x,y\left(x\right)=-\frac{\left(1+\sqrt{3}\right)x}{2},y\left(x\right)=\frac{\left(\sqrt{3}-1\right)x}{2},y\left(x\right)=\frac{1}{3_{-}C1}\left(-\sqrt{2}\left(_{-}C1+x\right)\sqrt{_{-}C1\left(_{-}C1+x\right)}-_{-}C1\right)\right\}$$

2.535 ODE No. 535

$$8xy'(x)^3 - 12y(x)y'(x)^2 + 9y(x) = 0$$

✓ **Mathematica** : cpu = 0.0362683 (sec), leaf count = 49

$$\left\{\left\{y(x)\rightarrow-\frac{\left(3c_1+x\right)^{3/2}}{3\sqrt{c_1}}\right\},\left\{y(x)\rightarrow\frac{\left(3c_1+x\right)^{3/2}}{3\sqrt{c_1}}\right\}\right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 51

$$\left\{y\left(x\right)=0,y\left(x\right)=-\frac{3x}{2},y\left(x\right)=\frac{3x}{2},y\left(x\right)=-\frac{1}{3_{-}C1^2}\left(_{-}C1\left(3_{-}C1+x\right)\right)^{\frac{3}{2}},y\left(x\right)=\frac{1}{3_{-}C1^2}\left(_{-}C1\left(3_{-}C1+x\right)\right)^{\frac{3}{2}}\right\}$$

2.536 ODE No. 536

---

$$b x \left(x^2 - a^2\right) y'(x)^2 + \left(x^2 - a^2\right) y'(x)^3 + b x + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0122029 (sec), leaf count = 64

$$\left\{\left\{y(x) \rightarrow c_1 - \frac{b x^2}{2}\right\},\left\{y(x) \rightarrow c_1 - \tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right)\right\},\left\{y(x) \rightarrow \tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1\right\}\right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 52

$$\left\{y(x) = -\frac{b x^2}{2} + \_C1, y(x) = -\arctan\left(x \frac{1}{\sqrt{a^2 - x^2}}\right) + \_C1, y(x) = \arctan\left(x \frac{1}{\sqrt{a^2 - x^2}}\right) + \_C1\right\}$$

2.537 ODE No. 537

---

$$\left(x^6 + 3 x y(x)^2\right) y'(x) - 2 x^5 y(x) + x^3 y'(x)^3 - 3 x^2 y(x) y'(x)^2 - y(x)^3 = 0$$

✗ **Mathematica** : cpu = 300.004 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 3.105 (sec), leaf count = 250

$$\left\{y(x) = RootOf\left(-\ln(x) + \int^{-Z} \frac{2}{3\_a(27\_a^2 + 4)}\left(27\left(-4\frac{-9\_a + \sqrt{81\_a^2 + 12}}{(27\_a^2 + 4)\sqrt{81\_a^2 + 12}}\right)^{2/3} - a^2 - 27\_a^2\sqrt{\phantom{x}}\right)\right.\right.$$

2.538 ODE No. 538

---

$$2\left(x y'(x) + y(x)\right)^3 - y(x) y'(x) = 0$$

✓ **Mathematica** : cpu = 9.06706 (sec), leaf count = 110

$$\left\{\left\{y(x) \rightarrow \frac{\int_1^x \frac{\text{InverseFunction}\left[-\frac{2\sqrt{\#1^2 - 8\#1^3} \tan^{-1}\left(\frac{\sqrt{8\#1 - 1}}{\#1\sqrt{8\#1 - 1}}\right) + \log(8\#1 - 1) + \log\left(1 + \frac{1}{8\#1 - 1}\right) + \frac{3\sqrt{\#1^2 - 8\#1^3}}{\#1}\right]}{K\$517620}\right]}{x} dK\right.\right.$$

✓ **Maple** : cpu = 0.82 (sec), leaf count = 1625

$$\left\{\int_{-b}^x 1\left(-6^{\frac{2}{3}}\left(-9 y(x)\left(-1/9 \sqrt{3} \sqrt{\frac{27\_a(y(x))^2 - 2 y(x)}{-a}} + y(x)\right) - a^2\right)^{\frac{2}{3}} + 6\_a y(x)\left(\sqrt[3]{6} \sqrt[3]{-9 y(x)\left(-\right.}\right.\right.\right.$$

2.539 ODE No. 539

---

$$\sin(x) y'(x)^3 - y'(x)^2 \left(y(x) \sin(x) - \cos^2(x)\right) - y'(x) \left(y(x) \cos^2(x) + \sin(x)\right) + y(x) \sin(x) = 0$$

✓ **Mathematica** : cpu = 0.0233608 (sec), leaf count = 45

$$\left\{\left\{y(x) \rightarrow c_1 e^x\right\},\left\{y(x) \rightarrow c_1 - \cos(x)\right\},\left\{y(x) \rightarrow c_1 - \log\left(\sin\left(\frac{x}{2}\right)\right) + \log\left(\cos\left(\frac{x}{2}\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 32

$$\{y(x) = \_C1 e^x, y(x) = -\cos(x) + \_C1, y(x) = -\ln(\csc(x) - \cot(x)) + \_C1\}$$

## 2.540 ODE No. 540

---

$$2y(x)y'(x)^3 - y(x)y'(x)^2 + 2xy'(x) - x = 0$$

✓ **Mathematica** : cpu = 0.0314972 (sec), leaf count = 69

$$\left\{ \left\{ y(x) \rightarrow c_1 + \frac{x}{2} \right\}, \left\{ y(x) \rightarrow \frac{(3c_1 - 2ix^{3/2})^{2/3}}{2^{2/3}} \right\}, \left\{ y(x) \rightarrow \frac{(3c_1 + 2ix^{3/2})^{2/3}}{2^{2/3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 109

$$\left\{ x + \frac{x\_C1}{y(x)} \left( \frac{1}{y(x)} \left( -x - \sqrt{-xy(x)} + y(x) \right) \right)^{-\frac{2}{3}} \left( \frac{1}{y(x)} \left( \sqrt{-xy(x)} + y(x) \right) \right)^{-\frac{2}{3}} = 0, x + \frac{x\_C1}{y(x)} \left( \frac{1}{y(x)} \left( -x - \sqrt{-xy(x)} + y(x) \right) \right)^{-\frac{2}{3}} \left( \frac{1}{y(x)} \left( \sqrt{-xy(x)} + y(x) \right) \right)^{-\frac{2}{3}} = 0 \right\}$$

## 2.541 ODE No. 541

---

$$y(x)^2y'(x)^3 + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.029067 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{2c_1x + c_1^3} \right\}, \left\{ y(x) \rightarrow \sqrt{2c_1x + c_1^3} \right\} \right\}$$

✓ **Maple** : cpu = 0.404 (sec), leaf count = 103

$$\left\{ y(x) = \sqrt{-C1^3 + 2x\_C1}, y(x) = -\frac{2i}{3}\sqrt[4]{2}\sqrt[4]{3}\sqrt[4]{-x^3}, y(x) = \frac{2i}{3}\sqrt[4]{2}\sqrt[4]{3}\sqrt[4]{-x^3}, y(x) = -\sqrt{-C1^3 + 2x\_C1}, y(x) = \sqrt{-C1^3 + 2x\_C1} \right\}$$

## 2.542 ODE No. 542

---

$$16y(x)^2y'(x)^3 + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0290756 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow \sqrt{c_1x + 2c_1^3} \right\} \right\}$$

✓ **Maple** : cpu = 0.385 (sec), leaf count = 107

$$\left\{ y(x) = \sqrt{16\_C1^3 + 2x\_C1}, y(x) = -\frac{i}{3}\sqrt[4]{2}\sqrt[4]{3}\sqrt[4]{-x^3}, y(x) = \frac{i}{3}\sqrt[4]{2}\sqrt[4]{3}\sqrt[4]{-x^3}, y(x) = -\sqrt{16\_C1^3 + 2x\_C1}, y(x) = \sqrt{16\_C1^3 + 2x\_C1} \right\}$$

## 2.543 ODE No. 543

---

$$x(x^2 + 1)y'(x) - x^2y(x) + y(x)^3(-y'(x)^2) + xy(x)^2y'(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.0239505 (sec), leaf count = 55

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{c_1x^2 + \frac{c_1}{c_1^2 + 1}} \right\}, \left\{ y(x) \rightarrow \sqrt{c_1x^2 + \frac{c_1}{c_1^2 + 1}} \right\} \right\}$$

✓ **Maple** : cpu = 1.089 (sec), leaf count = 277

$$\left\{ y(x) = -\frac{i}{2}\sqrt[4]{-16x^4 + 40x^2 + 2 - 2\sqrt{-(8x^2 - 1)^3}}, y(x) = -\frac{i}{2}\sqrt[4]{-16x^4 + 40x^2 + 2 + 2\sqrt{-(8x^2 - 1)^3}}, y(x) = \frac{i}{2}\sqrt[4]{-16x^4 + 40x^2 + 2 - 2\sqrt{-(8x^2 - 1)^3}}, y(x) = \frac{i}{2}\sqrt[4]{-16x^4 + 40x^2 + 2 + 2\sqrt{-(8x^2 - 1)^3}} \right\}$$

## 2.544 ODE No. 544

---

$$x^7y(x)^2y'(x)^3 - (3x^6y(x)^3 - 1)y'(x)^2 + 3x^5y(x)^4y'(x) - x^4y(x)^5 = 0$$

✓ **Mathematica** : cpu = 0.0727555 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow \sqrt[3]{c_1x^3 + c_1^{2/3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.697 (sec), leaf count = 4201

$$\left\{ \int_{-b-a}^x \frac{1}{y(x)} \left( (-1 - i\sqrt{3}) \left( -108(y(x))^6\_a^{12} + 12\sqrt{3}\sqrt{\frac{27\_a^6(y(x))^3 - 4}{y(x)}}(y(x))^5\_a^9 + 72\_a^6(y(x))^3 - 8 \right) \right) \right\}$$

### 2.545 ODE No. 545

$$y'(x)^4 - (y(x) - a)^3(y(x) - b)^2 = 0$$

✓ **Mathematica** : cpu = 0.664956 (sec), leaf count = 323

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; \frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}} \& \right] [c_1 - \sqrt[4]{-1}x] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; \frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}} \& \right] [c_1 - \sqrt[4]{-1}x] \right\} \right\}$$

✓ **Maple** : cpu = 0.122 (sec), leaf count = 144

$$\left\{ x - \int^{y(x)} \frac{1}{\sqrt[4]{(-a-a)^3(-a-b)^2}} d\_a - \_C1 = 0, x - \int^{y(x)} -i \frac{1}{\sqrt[4]{-(-a+a)^3(-a+b)^2}} d\_a - \_C1 = 0, x - \int^{y(x)} \frac{1}{\sqrt[4]{(-a-a)^3(-a-b)^2}} d\_a - \_C1 = 0, x - \int^{y(x)} -i \frac{1}{\sqrt[4]{-(-a+a)^3(-a+b)^2}} d\_a - \_C1 = 0 \right\}$$

### 2.546 ODE No. 546

$$y'(x)^4 + 3(x-1)y'(x)^2 - 3(2y(x)-1)y'(x) + 3x = 0$$

✓ **Mathematica** : cpu = 0.0290142 (sec), leaf count = 113

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{12} \left( -\sqrt{48c_1^2x^2 + 12c_1^4x + c_1^6 + 64x^3} - 6c_1x - c_1^3 + 6c_1 + 6 \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{12} \left( \sqrt{48c_1^2x^2 + 12c_1^4x + c_1^6 + 64x^3} - 6c_1x - c_1^3 + 6c_1 + 6 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.094 (sec), leaf count = 171

$$\left\{ y(x) = 1 \left( (-6 + \_C1^3 + (6x-6)\_C1) \sqrt{\_C1^2 + 4x} - 2\_C1^4 + (-14x+6)\_C1^2 + \left( (\_C1^2 + 4x)^{\frac{3}{2}} + 6x \right) \right) \right\}$$

### 2.547 ODE No. 547

$$y'(x)^4 - 4y(x)(xy'(x) - 2y(x))^2 = 0$$

✓ **Mathematica** : cpu = 2.44581 (sec), leaf count = 490

$$\left\{ \text{Solve} \left[ \frac{\sqrt{(x^2 - 4\sqrt{y(x)})} y(x) \log\left(\sqrt{x^2 - 4\sqrt{y(x)}} + x\right)}{\sqrt{x^2 - 4\sqrt{y(x)}}\sqrt{y(x)}} - \frac{\sqrt{x^2 - 4\sqrt{y(x)}}\sqrt{y(x)} \log(y(x))}{4\sqrt{(x^2 - 4\sqrt{y(x)})} y(x)} + \frac{1}{4} \log(y(x)) = 0 \right] \right\}$$

✓ **Maple** : cpu = 0.192 (sec), leaf count = 118

$$\left\{ 1 \left( \sqrt{x^2 - 4\sqrt{y(x)}} - x \right)^{1\sqrt{x^2y(x)-4(y(x))^{3/2}} \frac{1}{\sqrt{x^2-4\sqrt{y(x)}}} \frac{1}{\sqrt{y(x)}}} \frac{1}{\sqrt{y(x)}} \left( \left( \sqrt{x^2 - 4\sqrt{y(x)}} + x \right)^{1\sqrt{x^2y(x)-4(y(x))^{3/2}} \frac{1}{\sqrt{x^2-4\sqrt{y(x)}}} \frac{1}{\sqrt{y(x)}}} \right) \right\}$$

### 2.548 ODE No. 548

$$y'(x)^6 - (y(x) - a)^4(y(x) - b)^3 = 0$$

✓ **Mathematica** : cpu = 0.969213 (sec), leaf count = 479

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}; \frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}} \& \right] [c_1 - ix] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}; \frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}} \& \right] [c_1 - ix] \right\} \right\}$$

✓ **Maple** : cpu = 0.226 (sec), leaf count = 250

$$\left\{ x - \int^{y(x)} \frac{1}{\sqrt[6]{(-a-a)^4(-a-b)^3}} d\_a - \_C1 = 0, x - \int^{y(x)} \frac{-2i}{-\sqrt{3}+i} \frac{1}{\sqrt[6]{-(-a+a)^4(-a+b)^3}} d\_a - \_C1 = 0, x - \int^{y(x)} \frac{1}{\sqrt[6]{(-a-a)^4(-a-b)^3}} d\_a - \_C1 = 0, x - \int^{y(x)} \frac{-2i}{-\sqrt{3}+i} \frac{1}{\sqrt[6]{-(-a+a)^4(-a+b)^3}} d\_a - \_C1 = 0 \right\}$$



## 2.549 ODE No. 549

$$x^2 \left( y'(x)^2 + 1 \right)^3 - a^2 = 0$$

✓ **Mathematica** : cpu = 0.256686 (sec), leaf count = 216

$$\left\{ \left\{ y(x) \rightarrow c_1 - x \left( \frac{a^{2/3}}{x^{2/3}} - 1 \right)^{3/2} \right\}, \left\{ y(x) \rightarrow x \left( \frac{a^{2/3}}{x^{2/3}} - 1 \right)^{3/2} + c_1 \right\}, \left\{ y(x) \rightarrow c_1 - x \left( -1 - \frac{i(\sqrt{3} - i)a^{2/3}}{2x^{2/3}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.348 (sec), leaf count = 553

$$\left\{ y(x) = 1 \sqrt{-\frac{1}{a^4} (a^2 x)^{\frac{4}{3}} \left( (a^2 x)^{\frac{2}{3}} - a^2 \right) \left( a^2 - (a^2 x)^{\frac{2}{3}} \right) (a^2 x)^{-\frac{2}{3}} + \_C1}, y(x) = 1 \sqrt{-\frac{1}{a^4} (a^2 x)^{\frac{4}{3}} \left( (a^2 x)^{\frac{2}{3}} - a^2 \right) \left( a^2 - (a^2 x)^{\frac{2}{3}} \right) (a^2 x)^{-\frac{2}{3}} + \_C1} \right\}$$

## 2.550 ODE No. 550

$$-ay(x)^s - bx^{\frac{rs}{r-s}} + y'(x)^r = 0$$

✓ **Mathematica** : cpu = 0.730378 (sec), leaf count = 488

$$\text{Solve} \left[ \int_1^{y(x)} \left( \frac{r}{-rx \left( aK[2]^s + bx^{\frac{rs}{r-s}} \right)^{\frac{1}{r}} + sx \left( aK[2]^s + bx^{\frac{rs}{r-s}} \right)^{\frac{1}{r}} + rK[2]} - \int_1^x \left( \frac{asK[2]^{s-1} \left( aK[2]^s + bK[1]^{\frac{rs}{r-s}} \right)^{\frac{1}{r}}}{rK[1] \left( aK[2]^s + bK[1]^{\frac{rs}{r-s}} \right)^{\frac{1}{r}}} \right) dx \right) dy(x) = 0 \right]$$

✓ **Maple** : cpu = 0.221 (sec), leaf count = 60

$$\left\{ (-r + s) \int_{-b}^{y(x)} \left( x(r - s) \sqrt[r]{a_{-}a^s + bx^{\frac{rs}{r-s}} - r_{-}a} \right)^{-1} d_{-}a + \ln(x) - \_C1 = 0 \right\}$$

## 2.551 ODE No. 551

$$y'(x)^n - f(x)^n(y(x) - a)^{n+1}(y(x) - b)^{n-1} = 0$$

✓ **Mathematica** : cpu = 0.113192 (sec), leaf count = 86

$$\left\{ \left\{ y(x) \rightarrow \frac{-a(a - b)^n \left( \int_1^x (-1)^{1+\frac{1}{n}} f(K[1]) dK[1] + c_1 \right)^n - bn^n}{-(a - b)^n \left( \int_1^x (-1)^{1+\frac{1}{n}} f(K[1]) dK[1] + c_1 \right)^n - n^n} \right\} \right\}$$

✓ **Maple** : cpu = 0.309 (sec), leaf count = 55

$$\left\{ y(x) = 1 \left( \left( -\frac{n}{(a - b) \left( \int f(x) dx + \_C1 \right)} \right)^n b - a \right) \left( -1 + \left( -\frac{n}{(a - b) \left( \int f(x) dx + \_C1 \right)} \right)^n \right)^{-1} \right\}$$

## 2.552 ODE No. 552

$$y'(x)^n - f(x)g(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.0313445 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \int_1^{\#1} g(K[1])^{-1/n} dK[1] \& \right] \left[ \int_1^x f(K[2])^{\frac{1}{n}} dK[2] + c_1 \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 43

$$\left\{ \int^{y(x)} (g(\_a))^{-n-1} d_{-}a + \int^x -\frac{\sqrt[n]{f(\_a)g(y(x))}}{\sqrt[n]{g(y(x))}} d_{-}a + \_C1 = 0 \right\}$$

2.553 ODE No. 553

---

$$ay'(x)^m + by'(x)^n - y(x) = 0$$

✓ **Mathematica** : cpu = 0.255821 (sec), leaf count = 52

$$\text{Solve}\left[\left\{x = \frac{\frac{amK\$536895^m}{m-1} + \frac{bnK\$536895^n}{n-1}}{K\$536895} + c_1, y(x) = aK\$536895^m + bK\$536895^n\right\}, \{y(x), K\$536895\}\right]$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 36

$$\left\{x - \int^{y(x)} (RootOf(-a\_Z^m - b\_Z^n + \_a))^{-1} d\_a - \_C1 = 0, y(x) = 0\right\}$$

2.554 ODE No. 554

---

$$x^{n-1}y'(x)^n - nxy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.096582 (sec), leaf count = 49

$$\text{Solve}\left[\left\{y(x) = \frac{K\$537162nx^2 - K\$537162^n x^n}{x}, x = c_1(K\$537162 - K\$537162n)^{\frac{n}{1-n}}\right\}, \{y(x), K\$537162\}\right]$$

✓ **Maple** : cpu = 0.309 (sec), leaf count = 32

$$\left\{y(x) = \frac{1}{\_C1} \left( \_C1^2 \sqrt[n]{\frac{x}{\_C1}}^n - (\_C1^{-1})^{-n} \right)\right\}$$

2.555 ODE No. 555

---

$$xy'(x) + \sqrt{y'(x)^2 + 1} - y(x) = 0$$

✓ **Mathematica** : cpu = 0.311398 (sec), leaf count = 119

$$\left\{\left\{y(x) \rightarrow \frac{-x \sin(2c_1) + ix \cos(2c_1) + 2i \sin(c_1) + 2 \cos(c_1) - ix}{i \sin(2c_1) + \cos(2c_1) + 1}\right\}, \left\{y(x) \rightarrow \frac{i(ix \sin(2c_1) + x \cos(2c_1) - 2 \sin(c_1) + 2 \cos(c_1) - ix)}{i \sin(2c_1) + \cos(2c_1) + 1}\right\}\right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 15

$$\left\{y(x) = \sqrt{\_C1^2 + 1} + x\_C1\right\}$$

2.556 ODE No. 556

---

$$xy'(x)^2 + \sqrt{y'(x)^2 + 1} + y(x) = 0$$

✓ **Mathematica** : cpu = 4.46358 (sec), leaf count = 60

$$\text{Solve}\left[\left\{x = \frac{c_1}{(K\$537938 + 1)^2} + \frac{-\sqrt{K\$537938^2 + 1} - \sinh^{-1}(K\$537938)}{(K\$537938 + 1)^2}, y(x) = K\$537938^2(-x) - \sqrt{K\$537938^2 + 1}\right\}\right]$$

✓ **Maple** : cpu = 0.141 (sec), leaf count = 581

$$\left\{ \_C1 x^2 \left( \sqrt{-4xy(x) + 2 + 2\sqrt{4x^2 - 4xy(x) + 1}} - 2x \right)^{-2} + x + 2 \frac{x^2}{\left( \sqrt{-4xy(x) + 2 + 2\sqrt{4x^2 - 4xy(x) + 1}} \right)^2} \right\}$$

## 2.557 ODE No. 557

---

$$x \left( y'(x) + \sqrt{y'(x)^2 + 1} \right) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0244663 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{c_1 x - x^2} \right\}, \left\{ y(x) \rightarrow \sqrt{c_1 x - x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.091 (sec), leaf count = 74

$$\left\{ 2xy(x) - C1 \frac{1}{\sqrt{\frac{((y(x))^2 + x^2)^2}{x^2(y(x))^2}}} \left( \sqrt{\frac{x^4 + 2x^2(y(x))^2 + (y(x))^4}{x^2(y(x))^2}} xy(x) + (y(x))^2 - x^2 \right)^{-1} + x = 0 \right\}$$

## 2.558 ODE No. 558

---

$$ax\sqrt{y'(x)^2 + 1} + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.561397 (sec), leaf count = 327

$$\left\{ \text{Solve} \left[ \frac{2i \tan^{-1} \left( \frac{y(x)}{x\sqrt{a^2 - \frac{y(x)^2}{x^2} - 1}} \right) + a \tanh^{-1} \left( \frac{-a^2 - \frac{iy(x)}{x} + 1}{a\sqrt{a^2 - \frac{y(x)^2}{x^2} - 1}} \right) - a \tanh^{-1} \left( \frac{-a^2 + \frac{iy(x)}{x} + 1}{a\sqrt{a^2 - \frac{y(x)^2}{x^2} - 1}} \right) + a \log \left( \frac{y(x)^2}{x^2} + 1 \right)}{2a^2 - 2} \right] \right\}$$

✓ **Maple** : cpu = 0.112 (sec), leaf count = 223

$$\left\{ x - C1 e^{\frac{1}{a} \text{Arcsinh} \left( \frac{1}{(a^2 - 1)x} \left( \sqrt{-a^2 x^2 + x^2 + (y(x))^2} a + y(x) \right) \right)} \frac{1}{\sqrt{\frac{1}{(a^2 - 1)^2 x^2} \left( -a^2 x^2 + a^2 (y(x))^2 + 2 \sqrt{-a^2 x^2 + x^2 + (y(x))^2} \right)}} \right\}$$

## 2.559 ODE No. 559

---

$$-ay(x)y'(x) - ax + y(x)\sqrt{y'(x)^2 + 1} = 0$$

✓ **Mathematica** : cpu = 0.378857 (sec), leaf count = 212

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{a^6(-x^2) + 3a^4x^2 + 2a^2xe^{a^2c_1 - c_1} - 2xe^{a^2c_1 - c_1} + e^{2a^2c_1 - 2c_1} - 3a^2x^2 + x^2}}{\sqrt{a^6 - 3a^4 + 3a^2 - 1}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{a^6(-x^2) + 3a^4x^2 + 2a^2xe^{a^2c_1 - c_1} - 2xe^{a^2c_1 - c_1} + e^{2a^2c_1 - 2c_1} - 3a^2x^2 + x^2}}{\sqrt{a^6 - 3a^4 + 3a^2 - 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.203 (sec), leaf count = 215

$$\left\{ -e^{\int \frac{1}{(a^2 - 1)y(x)} \left( -a^2 x - \sqrt{(a^2 - 1)(y(x))^2 + a^2 x^2} \right) dx} a \left( a\sqrt{-a^2 + 1} - a \right) \frac{1}{\sqrt{-a^2 + 1}} \left( -a a - \sqrt{-a^2 + 1} \right)^{-1} \left( -a^2 a - \sqrt{-a^2 + 1} - a + a \right)^{-1} d - a - C1 + x \right\}$$

## 2.560 ODE No. 560

---

$$ay(x)\sqrt{y'(x)^2 + 1} - x^2 - 2xy(x)y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 13.9561 (sec), leaf count = 110

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{a^2 c_1^2 (-x^2) - 4a^2 c_1 x - 4a^2 + 4x^2}}{\sqrt{a^2 c_1^2 - 4}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{a^2 c_1^2 (-x^2) - 4a^2 c_1 x - 4a^2 + 4x^2}}{\sqrt{a^2 c_1^2 - 4}} \right\} \right\}$$

✓ **Maple** : cpu = 0.722 (sec), leaf count = 1120

$$\left\{ \int_{-b}^x 1 \left( 2 - a^3 - 2(y(x))^2 - a + \sqrt{a^2 \left( -a^4 + 2 - a^2(y(x))^2 - a^2(y(x))^2 + (y(x))^4 \right)} \right) \left( -2a^2 - a(y(x))^2 + 2 \right) dx \right\}$$

2.561 ODE No. 561

---

$$f\left(x^2+y(x)^2\right)\sqrt{y'(x)^2+1}-xy'(x)+y(x)=0$$

✓ **Mathematica** : cpu = 1.81722 (sec), leaf count = 2138

$$\left\{\text{Solve}\left[\int_1^x\left(\frac{\sqrt{f\left(K[1]^2+y(x)^2\right)^2\left(-f\left(K[1]^2+y(x)^2\right)^2+K[1]^2+y(x)^2\right)}K[1]}{f\left(K[1]^2+y(x)^2\right)^2\left(K[1]^2+y(x)^2\right)}-\frac{\sqrt{f\left(K[1]^2+y(x)^2\right)^2\left(-f\left(K[1]^2+y(x)^2\right)^2+K[1]^2+y(x)^2\right)}}{f\left(K[1]^2+y(x)^2\right)^2\left(-f\left(K[1]^2+y(x)^2\right)^2+K[1]^2+y(x)^2\right)}\right]d_x\right\}$$

✓ **Maple** : cpu = 1.358 (sec), leaf count = 50

$$\left\{y\left(x\right)=x\left(\tan\left(\text{RootOf}\left(-2\_Z+\int\frac{x^2\left(\left(\tan\left(\_Z\right)\right)^2+1\right)}{\left(\tan\left(\_Z\right)\right)^2}\frac{f\left(\_a\right)}{-a}\frac{1}{\sqrt{-\left(f\left(\_a\right)\right)^2+\_a}}d\_a+2\_C1\right)\right)\right)^{-1}\right\}$$

2.562 ODE No. 562

---

$$a\sqrt[3]{y'(x)^3+1}+bxy'(x)-y(x)=0$$

✗ **Mathematica** : cpu = 300.01 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.12 (sec), leaf count = 3306

$$\left\{\left(-\int\frac{1}{2b^3x^3+2a^3}\left(2b^2x^2y(x)\sqrt[3]{-4b^6x^6-8a^3b^3x^3-4b^3x^3(y(x))^3+4\sqrt{b^6x^6+2a^3b^3x^3+2b^3x^3(y(x))^3+a^6-2(y(x))^3a^3+(y(x))^6b^3x^3-4a^6}}\right)dx\right)\right\}$$

2.563 ODE No. 563

---

$$ay(x)+b+xy'(x)+\log\left(y'(x)\right)=0$$

✓ **Mathematica** : cpu = 0.136293 (sec), leaf count = 59

$$\text{Solve}\left[a\left(\frac{\left(a+1\right)\log\left(1-aW\left(xe^{-ay(x)-b}\right)\right)}{a^2}+\frac{W\left(xe^{-ay(x)-b}\right)}{a}\right)+ay(x)=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.129 (sec), leaf count = 66

$$\left\{-\left(e^{-ay(x)-\text{lambertW}\left(xe^{-ay(x)-b}\right)-b}\right)^{-(a+1)^{-1}}\_C1+x-\frac{e^{ay(x)+\text{lambertW}\left(xe^{-ay(x)-b}\right)+b}}{a}=0\right\}$$

2.564 ODE No. 564

---

$$a\left(xy'(x)-y(x)\right)+\log\left(y'(x)\right)=0$$

✓ **Mathematica** : cpu = 0.0533459 (sec), leaf count = 28

$$\left\{\left\{y(x)\rightarrow-\frac{e^{-c_1}\left(e^{c_1}c_1-ax\right)}{a}\right\}\right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 32

$$\left\{y\left(x\right)=\frac{1}{a}\left(\ln\left(-\frac{1}{ax}\right)-1\right),y\left(x\right)=x\_C1+\frac{\ln\left(\_C1\right)}{a}\right\}$$

2.565 ODE No. 565

---

$$y'(x) + y(x) \log(y'(x)) - xy(x) - y(x) \log(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.0173293 (sec), leaf count = 25

$$\left\{\left\{y(x) \rightarrow c_1 e^{\frac{1}{2} W(e^x)^2 + W(e^x)}\right\}\right\}$$

✓ **Maple** : cpu = 0.102 (sec), leaf count = 17

$$\left\{y(x) = \_C1 e^{\frac{\text{lambert}W(e^x)(\text{lambert}W(e^x)+2)}{2}}\right\}$$

2.566 ODE No. 566

---

$$y'(x) + \sin(y'(x)) - x = 0$$

✓ **Mathematica** : cpu = 0.0311171 (sec), leaf count = 31

Solve  $\left[\left\{x = K\$548413 + \sin(K\$548413), y(x) = c_1 + \frac{K\$548413^2}{2} + K\$548413 \sin(K\$548413) + \cos(K\$548413)\right\}\right]$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 16

$$\left\{y(x) = \int \text{RootOf}(\sin(\_Z) + \_Z - x) \, dx + \_C1\right\}$$

2.567 ODE No. 567

---

$$a \cos(y'(x)) + by'(x) + x = 0$$

✓ **Mathematica** : cpu = 0.041835 (sec), leaf count = 42

Solve  $\left[\left\{y(x) = a \sin(K\$548647) - aK\$548647 \cos(K\$548647) - \frac{bK\$548647^2}{2} + c_1, x = -a \cos(K\$548647) - bK\$548647\right\}\right]$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 18

$$\left\{y(x) = \int \text{RootOf}(a \cos(\_Z) + b\_Z + x) \, dx + \_C1\right\}$$

2.568 ODE No. 568

---

$$y'(x)^2 \sin(y'(x)) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0522431 (sec), leaf count = 28

Solve  $\left[\left\{x = c_1 + K\$548887 \sin(K\$548887) - \cos(K\$548887), y(x) = K\$548887^2 \sin(K\$548887)\right\}, \{y(x), K\$548887\}\right]$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 32

$$\left\{x - \int^{y(x)} (\text{RootOf}(\sin(\_Z) \_Z^2 - \_a))^{-1} d\_a - \_C1 = 0, y(x) = 0\right\}$$

2.569 ODE No. 569

---

$$(y'(x)^2 + 1) \sin^2(y(x) - xy'(x)) - 1 = 0$$

✓ **Mathematica** : cpu = 0.0927792 (sec), leaf count = 59

$$\left\{\left\{y(x) \rightarrow c_1 x - \frac{1}{2} \cos^{-1}\left(\frac{c_1^2 - 1}{c_1^2 + 1}\right)\right\}, \left\{y(x) \rightarrow c_1 x + \frac{1}{2} \cos^{-1}\left(\frac{c_1^2 - 1}{c_1^2 + 1}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.306 (sec), leaf count = 147

$$\left\{y(x) = x\_C1 - \arcsin\left(\frac{1}{\sqrt{\_C1^2 + 1}}\right), y(x) = x\_C1 + \arcsin\left(\frac{1}{\sqrt{\_C1^2 + 1}}\right), y(x) = -x\sqrt{1-x}\sqrt{x^{-1}} - a\right\}$$

2.570 ODE No. 570

---

$$\left(y'(x)^2+1\right)\left(ax+\tan^{-1}\left(y'(x)\right)\right)+y'(x)=0$$

✓ **Mathematica** : cpu = 1.42318 (sec), leaf count = 51

Solve  $\left\{y(x)=\frac{1}{a\left(K\$549361^2+1\right)}+c_1,x=\frac{K\$549361^2\left(-\tan^{-1}\left(K\$549361\right)\right)-K\$549361-\tan^{-1}\left(K\$549361\right)}{a\left(K\$549361^2+1\right)}\right\}$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 30

$$\left\{y(x)=\int \tan \left(\text{RootOf}\left(ax\left(\tan \left(\_Z\right)\right)^2+\left(\tan \left(\_Z\right)\right)^2\_Z+ax+\tan \left(\_Z\right)+\_Z\right)\right) \mathrm{d} x+\_C1\right\}$$

2.571 ODE No. 571

---

$$ax^nf\left(y'(x)\right)+xy'(x)-y(x)=0$$

✓ **Mathematica** : cpu = 0.0874542 (sec), leaf count = 116

Solve  $\left\{y(x)=af(K\$549626)x^n+K\$549626x,x=\left(nf(K\$549626)^{\frac{1}{n}-1}\int_1^{K\$549626}-\frac{f(K[1])^{\frac{n-1}{n}-1}}{an}dK[1]-f(K[1])\right)^{\frac{1}{n}}\right\}$

✓ **Maple** : cpu = 0.418 (sec), leaf count = 169

$$\left\{[y(\_T)=a\left(\left(\frac{1}{af(\_T)n}\left((1-n)\int (f(\_T))^{-n-1}\mathrm{d}\_T+\_C1an\right)\right)^{(n-1)^{-1}}(f(\_T))^{\frac{1}{n(n-1)}}\right)^nf(\_T)+\_C2]\right\}$$

2.572 ODE No. 572

---

$$f\left(y'(x)\right)\left(xy'(x)-y(x)\right)^n+y(x)g\left(y'(x)\right)+xh\left(y'(x)\right)=0$$

✗ **Mathematica** : cpu = 0.0385672 (sec), leaf count = 0 , could not solve  
DSolve[x\*h[Derivative[1][y][x]] + g[Derivative[1][y][x]]\*y[x] + f[Derivative[1][y][x]]\*(-y[x] + x\*Derivative[1][y][x])^n == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve((x\*diff(y(x),x)-y(x))^n\*f(diff(y(x),x))+y(x)\*g(diff(y(x),x))+x\*h(diff(y(x),x))=0,y(x))

2.573 ODE No. 573

---

$$f\left(xy'(x)^2\right)+2xy'(x)-y(x)=0$$

✓ **Mathematica** : cpu = 0.0157483 (sec), leaf count = 42

$$\left\{\left\{y(x)\rightarrow f(c_1)-2\sqrt{c_1}\sqrt{x},y(x)\rightarrow f(c_1)+2\sqrt{c_1}\sqrt{x}\right\}\right\}$$

✓ **Maple** : cpu = 0.097 (sec), leaf count = 24

$$\left\{y(x)=\_C1+2\text{RootOf}\left(-f\left(\_Z^2\right)-2\_Z+\_C1+\_C2\right)\sqrt{x}\right\}$$

2.574 ODE No. 574

---

$$f\left(x-\frac{3}{2}y'(x)^2\right)+y'(x)^3-y(x)=0$$

✓ **Mathematica** : cpu = 0.023909 (sec), leaf count = 102

$$\left\{\left\{y(x)\rightarrow \frac{1}{9}\left(9f(c_1)+2\sqrt{6}x\sqrt{x-c_1}-2\sqrt{6}c_1\sqrt{x-c_1}\right),y(x)\rightarrow \frac{1}{9}\left(9f(c_1)-2\sqrt{6}x\sqrt{x-c_1}+2\sqrt{6}c_1\sqrt{x-c_1}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.096 (sec), leaf count = 41

$$\left\{y(x)=f(\_C1)-\frac{2\sqrt{6}}{9}\sqrt{(-\_C1+x)^3},y(x)=f(\_C1)+\frac{2\sqrt{6}}{9}\sqrt{(-\_C1+x)^3}\right\}$$

2.575 ODE No. 575

---

$$y'(x)f\left(xy(x)y'(x)-y(x)^2\right)+x^2\left(-y'(x)\right)+xy(x)=0$$

✗ **Mathematica** : cpu = 0.0250821 (sec), leaf count = 0 , could not solve  
DSolve[x\*y[x] - x^2\*Derivative[1][y][x] + f[-y[x]^2 + x\*y[x]\*Derivative[1][y][x]]\*Derivative[1][y][x], y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x)\*f(x\*y(x)\*diff(y(x),x)-y(x)^2)-x^2\*diff(y(x),x)+x\*y(x)=0,y(x))

2.576 ODE No. 576

---

$$\phi\left(f\left(x,y(x),y'(x)\right),g\left(x,y(x),y'(x)\right)\right)=0$$

✗ **Mathematica** : cpu = 0.009432 (sec), leaf count = 0 , could not solve  
DSolve[phi[f[x, y[x], Derivative[1][y][x]], g[x, y[x], Derivative[1][y][x]]] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(phi(f(x,y(x),diff(y(x),x)),g(x,y(x),diff(y(x),x)))=0,y(x))

2.577 ODE No. 577

---

$$y'(x)=F\left(\frac{y(x)}{a+x}\right)$$

✓ **Mathematica** : cpu = 0.242657 (sec), leaf count = 243

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{1}{-aF\left(\frac{K[2]}{a+x}\right)-xF\left(\frac{K[2]}{a+x}\right)+K[2]}-\int_1^x\left(\frac{F'\left(\frac{K[2]}{a+K[1]}\right)}{\left(a+K[1]\right)\left(aF\left(\frac{K[2]}{a+K[1]}\right)+K[1]F\left(\frac{K[2]}{a+K[1]}\right)-K[2]\right)}\right)dx\right)dx=C1\right]$$

✓ **Maple** : cpu = 0.025 (sec), leaf count = 28

$$\left\{y\left(x\right)=-RootOf\left(\int^{-Z}\left(F\left(-\_a\right)+\_a\right)^{-1}d\_a+\ln\left(x+a\right)+\_C1\right)\left(x+a\right)\right\}$$

2.578 ODE No. 578

---

$$y'(x)=F\left(y(x)-x^2\right)+2x$$

✓ **Mathematica** : cpu = 0.157726 (sec), leaf count = 100

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{F\left(K[2]-x^2\right)\int_1^x-\frac{2K[1]F'\left(K[2]-K[1]^2\right)}{F\left(K[2]-K[1]^2\right)}dK[1]+1}{F\left(K[2]-x^2\right)}dK[2]+\int_1^x\left(\frac{2K[1]}{F\left(y(x)-K[1]^2\right)}+1\right)dK[1]=c_1\right)\right]$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 22

$$\left\{y\left(x\right)=x^2+RootOf\left(-x+\int^{-Z}\left(F\left(\_a\right)\right)^{-1}d\_a+\_C1\right)\right\}$$

2.579 ODE No. 579

---

$$y'(x)=F\left(\frac{ax^2}{4}+\frac{bx}{2}+y(x)\right)-\frac{ax}{2}$$

✓ **Mathematica** : cpu = 0.239925 (sec), leaf count = 514

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{b\int_1^x\left(\frac{2aK[1]F'\left(\frac{1}{4}aK[1]^2+\frac{1}{2}bK[1]+K[2]\right)}{\left(b+2F\left(\frac{1}{4}aK[1]^2+\frac{1}{2}bK[1]+K[2]\right)\right)^2}+\frac{2F'\left(\frac{1}{4}aK[1]^2+\frac{1}{2}bK[1]+K[2]\right)}{b+2F\left(\frac{1}{4}aK[1]^2+\frac{1}{2}bK[1]+K[2]\right)}-\frac{4F\left(\frac{1}{4}aK[1]^2+\frac{1}{2}bK[1]+K[2]\right)F'\left(\frac{1}{4}aK[1]^2+\frac{1}{2}bK[1]+K[2]\right)}{\left(b+2F\left(\frac{1}{4}aK[1]^2+\frac{1}{2}bK[1]+K[2]\right)\right)^2}\right)dx=C1\right]$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 35

$$\left\{y\left(x\right)=-\frac{ax^2}{4}-\frac{bx}{2}+RootOf\left(-x+2\int^{-Z}\left(2F\left(\_a\right)+b\right)^{-1}d\_a+\_C1\right)\right\}$$

2.580 ODE No. 580

---

$$y'(x) = e^{bx} F\left(e^{-bx} y(x)\right)$$

✓ **Mathematica** : cpu = 0.244631 (sec), leaf count = 203

Solve  $\left[ \int_1^{y(x)} \left( \frac{1}{bK[2] - e^{bx} F(e^{-bx} K[2])} - \int_1^x \left( \frac{F'(e^{-bK[1]} K[2])}{e^{bK[1]} F(e^{-bK[1]} K[2]) - bK[2]} - \frac{e^{bK[1]} F(e^{-bK[1]} K[2]) (F'(e^{-bK[1]} K[2]) - bK[2])}{(e^{bK[1]} F(e^{-bK[1]} K[2]) - bK[2])^2} \right) dx \right) dy \right]$

✓ **Maple** : cpu = 0.045 (sec), leaf count = 31

$$\left\{ y(x) = \frac{\text{RootOf}\left(-x + \int^{-Z} (F(\_a) - \_a b)^{-1} d\_a + \_C1\right)}{e^{-bx}} \right\}$$

2.581 ODE No. 581

---

$$y'(x) = \frac{x F\left(\frac{x^2 y(x) + \frac{1}{4}}{x^2}\right) + \frac{1}{2}}{x^3}$$

✓ **Mathematica** : cpu = 0.232309 (sec), leaf count = 144

Solve  $\left[ \int_1^{y(x)} - \frac{F\left(\frac{K[2]x^2 + \frac{1}{4}}{x^2}\right) \int_1^x - \frac{F'\left(\frac{K[2]K[1]^2 + \frac{1}{4}}{K[1]^2}\right)}{2F\left(\frac{K[2]K[1]^2 + \frac{1}{4}}{K[1]^2}\right)^2 K[1]^3} dK[1] + 1}{F\left(\frac{K[2]x^2 + \frac{1}{4}}{x^2}\right)} dK[2] + \int_1^x \left( \frac{1}{K[1]^2} + \frac{1}{2K[1]^3 F\left(\frac{y(x)K[1]^2 + \frac{1}{4}}{K[1]^2}\right)} \right) dK[1] \right) dy \right]$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 32

$$\left\{ y(x) = \frac{4 \text{RootOf}\left(\int^{-Z} (F(\_a))^{-1} d\_a x + x\_C1 + 1\right) x^2 - 1}{4 x^2} \right\}$$

2.582 ODE No. 582

---

$$y'(x) = \frac{ax^2 F\left(\frac{axy(x)+1}{ax}\right) + 1}{ax^2}$$

✓ **Mathematica** : cpu = 0.247768 (sec), leaf count = 142

Solve  $\left[ \int_1^{y(x)} - \frac{F\left(\frac{axK[2]+1}{ax}\right) \int_1^x \frac{F'\left(\frac{aK[1]K[2]+1}{aK[1]}\right)}{aF\left(\frac{aK[1]K[2]+1}{aK[1]}\right)^2 K[1]^2} dK[1] - 1}{F\left(\frac{axK[2]+1}{ax}\right)} dK[2] + \int_1^x \left( -1 - \frac{1}{aK[1]^2 F\left(\frac{aK[1]y(x)+1}{aK[1]}\right)} \right) dK[1] \right) dy \right]$

✓ **Maple** : cpu = 0.181 (sec), leaf count = 30

$$\left\{ y(x) = \frac{\text{RootOf}\left(-x + \int^{-Z} (F(\_a))^{-1} d\_a + \_C1\right) xa - 1}{ax} \right\}$$

2.583 ODE No. 583

---

$$y'(x) = -\frac{1}{2}x \left( ax^2 - 2F\left(\frac{ax^4}{8} + y(x)\right) \right)$$

✓ **Mathematica** : cpu = 0.210888 (sec), leaf count = 126

Solve  $\left[ \int_1^{y(x)} - \frac{F\left(\frac{ax^4}{8} + K[2]\right) \int_1^x \frac{aK[1]^3 F'\left(\frac{1}{8}aK[1]^4 + K[2]\right)}{2F\left(\frac{1}{8}aK[1]^4 + K[2]\right)^2} dK[1] + 1}{F\left(\frac{ax^4}{8} + K[2]\right)} dK[2] + \int_1^x \left( K[1] - \frac{aK[1]^3}{2F\left(\frac{1}{8}aK[1]^4 + y(x)\right)} \right) dK[1] \right) dy \right]$

✓ **Maple** : cpu = 0.102 (sec), leaf count = 31

$$\left\{ y(x) = -\frac{ax^4}{8} + \text{RootOf}\left(-x^2 + 2 \int^{-Z} (F(\_a))^{-1} d\_a + 2\_C1\right) \right\}$$



2.584 ODE No. 584

---

$$y'(x) = \frac{2a}{2aF(y(x)^2 - 4ax) + y(x)}$$

✓ **Mathematica** : cpu = 0.273436 (sec), leaf count = 115

Solve  $\left[ \int_1^{y(x)} \left( \frac{K[2]}{4a^2 F(K[2]^2 - 4ax)} - \frac{2a \int_1^x \frac{K[2] F'(K[2]^2 - 4aK[1])}{a F(K[2]^2 - 4aK[1])^2} dK[1] - 1}{2a} \right) dK[2] + \int_1^x -\frac{1}{2a F(y(x)^2 - 4aK[1])} dK[1] \right]$

✓ **Maple** : cpu = 0.047 (sec), leaf count = 35

$$\left\{ \frac{y(x)}{2a} + \frac{\int^{(y(x))^2 - 4ax} (F(\_a))^{-1} d\_a}{8a^2} - \_C1 = 0 \right\}$$

2.585 ODE No. 585

---

$$y'(x) = y(x)F(\log(\log(y(x))) - \log(x))$$

✓ **Mathematica** : cpu = 0.210367 (sec), leaf count = 205

Solve  $\left[ \int_1^{y(x)} \left( \frac{1}{K[2](xF(\log(\log(K[2])) - \log(x)) - \log(K[2]))} - \int_1^x \left( \frac{F(\log(\log(K[2])) - \log(K[1])) \left( \frac{K[1] F'(\log(\log(K[2])) - \log(K[1]))}{F(\log(\log(K[2])) - \log(K[1]))} \right)}{(F(\log(\log(K[2])) - \log(K[1])) - \log(K[1]))} \right) dK[1] \right) dK[2] \right]$

✓ **Maple** : cpu = 0.344 (sec), leaf count = 120

$$\left\{ \int_{\_b}^x \frac{F(\ln(\ln(y(x))) - \ln(\_a))}{\_a F(\ln(\ln(y(x))) - \ln(\_a)) + \ln(y(x))} d\_a + \int^{y(x)} -\frac{1}{\_f (-xF(\ln(\ln(\_f)) - \ln(x)) + \ln(\_f))} d\_f \right\}$$

2.586 ODE No. 586

---

$$y'(x) = \frac{xF\left(\frac{y(x)}{\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}}$$

✓ **Mathematica** : cpu = 0.56363 (sec), leaf count = 975

Solve  $\left[ \int_1^x \left( -\frac{K[1] \sqrt{K[1]^2 + 1} F\left(\frac{y(x)}{\sqrt{K[1]^2 + 1}}\right)^3}{y(x) \left( K[1]^2 F\left(\frac{y(x)}{\sqrt{K[1]^2 + 1}}\right)^2 + F\left(\frac{y(x)}{\sqrt{K[1]^2 + 1}}\right)^2 - y(x)^2 \right)} - \frac{K[1] F\left(\frac{y(x)}{\sqrt{K[1]^2 + 1}}\right)^2}{K[1]^2 F\left(\frac{y(x)}{\sqrt{K[1]^2 + 1}}\right)^2 + F\left(\frac{y(x)}{\sqrt{K[1]^2 + 1}}\right)^2} \right) dK[1] \right]$

✓ **Maple** : cpu = 0.197 (sec), leaf count = 39

$$\left\{ y(x) = RootOf \left( -\ln(x^2 + 1) + 2 \int^{-Z} (F(\_a) - \_a)^{-1} d\_a + 2\_C1 \right) \sqrt{x^2 + 1} \right\}$$

2.587 ODE No. 587

---

$$y'(x) = \frac{1}{2} \sqrt{x} \left( 2F\left(y(x) - \frac{x^3}{6}\right) + x^{3/2} \right)$$

✓ **Mathematica** : cpu = 0.223263 (sec), leaf count = 123

Solve  $\left[ \int_1^{y(x)} -\frac{F\left(K[2] - \frac{x^3}{6}\right) \int_1^x -\frac{K[1]^2 F'\left(K[2] - \frac{K[1]^3}{6}\right)}{2F\left(K[2] - \frac{K[1]^3}{6}\right)^2} dK[1] + 1}{F\left(K[2] - \frac{x^3}{6}\right)} dK[2] + \int_1^x \left( \frac{K[1]^2}{2F\left(y(x) - \frac{K[1]^3}{6}\right)} + \sqrt{K[1]} \right) dK[1] \right]$

✓ **Maple** : cpu = 0.098 (sec), leaf count = 29

$$\left\{ \int_{\_b}^{y(x)} \left( F\left(\_a - \frac{x^3}{6}\right) \right)^{-1} d\_a - \frac{2}{3} x^{\frac{3}{2}} - \_C1 = 0 \right\}$$

**2.588 ODE No. 588**

---

$$y'(x) = \frac{F(-(x-y(x))(y(x)+x))+x}{y(x)}$$

✓ **Mathematica** : cpu = 0.200434 (sec), leaf count = 113

$$\text{Solve} \left[ \int_1^{y(x)} \left( -\frac{K[2]}{F(-(x-K[2])(x+K[2]))} - \int_1^x -\frac{2K[1]K[2]F'(-(K[1]-K[2])(K[1]+K[2]))}{F(-(K[1]-K[2])(K[1]+K[2]))^2} dK[1] \right) dK[2] + 1 \right]$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 53

$$\left\{ y(x) = \sqrt{x^2 + \text{RootOf} \left( -2x + \int^{-Z} (F(\_a))^{-1} d\_a + 2\_C1 \right)}, y(x) = -\sqrt{x^2 + \text{RootOf} \left( -2x + \int^{-Z} (F(\_a))^{-1} d\_a + 2\_C1 \right)} \right\}$$

**2.589 ODE No. 589**

---

$$y'(x) = \frac{y(x)^2 F\left(\frac{1-y(x)\log(x)}{y(x)}\right)}{x}$$

✓ **Mathematica** : cpu = 0.228083 (sec), leaf count = 245

$$\text{Solve} \left[ \int_1^{y(x)} \left( \frac{1}{\left( -F\left(\frac{1-K[2]\log(x)}{K[2]}\right) - 1 \right) K[2]^2} - \int_1^x \left( \frac{\left( -\frac{\log(K[1])}{K[2]} - \frac{1-K[2]\log(K[1])}{K[2]^2} \right) F'\left(\frac{1-K[2]\log(K[1])}{K[2]}\right)}{\left( F\left(\frac{1-K[2]\log(K[1])}{K[2]}\right) + 1 \right) K[1]} - \frac{F\left(\frac{1-K[2]\log(K[1])}{K[2]}\right)}{K[1]} \right) dK[1] \right) dK[2] + 1 \right]$$

✓ **Maple** : cpu = 0.107 (sec), leaf count = 38

$$\left\{ \int_{-b}^{y(x)} \frac{1}{-a^2} \left( F\left(\frac{1-\_a \ln(x)}{-a}\right) + 1 \right)^{-1} d\_a - \ln(x) - \_C1 = 0 \right\}$$

**2.590 ODE No. 590**

---

$$y'(x) = \frac{x}{F(x^2 + y(x)^2) - y(x)}$$

✓ **Mathematica** : cpu = 0.228438 (sec), leaf count = 94

$$\text{Solve} \left[ \int_1^{y(x)} \left( -\frac{K[2]}{F(x^2 + K[2]^2)} - \int_1^x \frac{2K[1]K[2]F'(K[1]^2 + K[2]^2)}{F(K[1]^2 + K[2]^2)^2} dK[1] + 1 \right) dK[2] + \int_1^x -\frac{K[1]}{F(K[1]^2 + y(x)^2)} dK[1] \right]$$

✓ **Maple** : cpu = 0.105 (sec), leaf count = 28

$$\left\{ -y(x) + \frac{\int^{(y(x))^2+x^2} (F(\_a))^{-1} d\_a}{2} - \_C1 = 0 \right\}$$

**2.591 ODE No. 591**

---

$$y'(x) = \frac{x F\left(\frac{ay(x)^2+bx^2}{a}\right)}{\sqrt{a}y(x)}$$

✓ **Mathematica** : cpu = 0.470861 (sec), leaf count = 253

$$\text{Solve} \left[ \int_1^{y(x)} \left( -\frac{bK[2]}{b + \sqrt{a}F\left(\frac{bx^2+aK[2]^2}{a}\right)} - \int_1^x \left( \frac{2bK[1]K[2]F'\left(\frac{bK[1]^2+aK[2]^2}{a}\right)}{\sqrt{a}\left(b + \sqrt{a}F\left(\frac{bK[1]^2+aK[2]^2}{a}\right)\right)} - \frac{2bF\left(\frac{bK[1]^2+aK[2]^2}{a}\right)K[1]K[2]}{\left(b + \sqrt{a}F\left(\frac{bK[1]^2+aK[2]^2}{a}\right)\right)} \right) dK[1] \right) dK[2] + 1 \right]$$

✓ **Maple** : cpu = 0.141 (sec), leaf count = 108

$$\left\{ y(x) = \frac{1}{a} \sqrt{a \left( -bx^2 + \text{RootOf} \left( \int^{-Z} (F(\_a) a + b\sqrt{a})^{-1} d\_a b a^{\frac{3}{2}} - bx^2 + 2\_C1 a \right) a \right)}, y(x) = -\frac{1}{a} \sqrt{a \left( -bx^2 + \text{RootOf} \left( \int^{-Z} (F(\_a) a + b\sqrt{a})^{-1} d\_a b a^{\frac{3}{2}} - bx^2 + 2\_C1 a \right) a \right)} \right\}$$

## 2.592 ODE No. 592

$$y'(x) = \frac{F\left(-\frac{2x^3}{5} + y(x) - 2\sqrt{x}\right) + \frac{6x^3}{5} + \sqrt{x}}{x}$$

✓ **Mathematica** : cpu = 0.517571 (sec), leaf count = 241

$$\text{Solve} \left[ \int_1^{y(x)} -\frac{F\left(-\frac{2x^3}{5} - 2\sqrt{x} + K[2]\right) \int_1^x \left(-\frac{6F'\left(-\frac{2}{5}K[1]^3 - 2\sqrt{K[1]} + K[2]\right)K[1]^2}{5F\left(-\frac{2}{5}K[1]^3 - 2\sqrt{K[1]} + K[2]\right)^2} - \frac{F'\left(-\frac{2}{5}K[1]^3 - 2\sqrt{K[1]} + K[2]\right)}{F\left(-\frac{2}{5}K[1]^3 - 2\sqrt{K[1]} + K[2]\right)^2 \sqrt{K[1]}}\right) dx}{F\left(-\frac{2x^3}{5} - 2\sqrt{x} + K[2]\right)} \right]$$

✓ **Maple** : cpu = 0.138 (sec), leaf count = 33

$$\left\{ \int_{-b}^{y(x)} \left( F\left(-a - \frac{2x^3}{5} - 2\sqrt{x}\right) \right)^{-1} d_{-a} - \ln(x) - \_C1 = 0 \right\}$$

## 2.593 ODE No. 593

$$y'(x) = \frac{e^x F\left(y(x)^{3/2} - \frac{3e^x}{2}\right)}{\sqrt{y(x)}}$$

✓ **Mathematica** : cpu = 0.359741 (sec), leaf count = 221

$$\text{Solve} \left[ \int_1^{y(x)} \left( \frac{\sqrt{K[2]}}{F\left(K[2]^{3/2} - \frac{3e^{K[1]}}{2}\right) - 1} - \int_1^x \left( \frac{3e^{K[1]} F\left(K[2]^{3/2} - \frac{3e^{K[1]}}{2}\right) \sqrt{K[2]} F'\left(K[2]^{3/2} - \frac{3e^{K[1]}}{2}\right)}{2\left(F\left(K[2]^{3/2} - \frac{3e^{K[1]}}{2}\right) - 1\right)^2} - \frac{3e^{K[1]}}{2\left(F\left(K[2]^{3/2} - \frac{3e^{K[1]}}{2}\right) - 1\right)} \right) dx \right]$$

✓ **Maple** : cpu = 0.247 (sec), leaf count = 35

$$\left\{ \int_{-b}^{y(x)} 1\sqrt{-a} \left( F\left(-a^{\frac{3}{2}} - \frac{3e^x}{2}\right) - 1 \right)^{-1} d_{-a} - e^x - \_C1 = 0 \right\}$$

## 2.594 ODE No. 594

$$y'(x) = \frac{x F\left(\frac{y(x)^2 - b}{x^2}\right)}{y(x)}$$

✓ **Mathematica** : cpu = 0.339759 (sec), leaf count = 236

$$\text{Solve} \left[ \int_1^{y(x)} \left( -\frac{K[2]}{-F\left(\frac{K[2]^2 - b}{x^2}\right) x^2 + K[2]^2 - b} - \int_1^x \left( \frac{F\left(\frac{K[2]^2 - b}{K[1]^2}\right) K[1] \left(2K[2] F'\left(\frac{K[2]^2 - b}{K[1]^2}\right) - 2K[2]\right)}{\left(F\left(\frac{K[2]^2 - b}{K[1]^2}\right) K[1]^2 - K[2]^2 + b\right)^2} - \frac{K[1]}{K[1]^2} \right) dx \right]$$

✓ **Maple** : cpu = 0.099 (sec), leaf count = 67

$$\left\{ y(x) = \sqrt{\text{RootOf}\left(-2\ln(x) + \int^{-Z} (F(\_a) - \_a)^{-1} d_{-a} + 2\_C1\right) x^2 + b}, y(x) = -\sqrt{\text{RootOf}\left(-2\ln(x) + \int^{-Z} (F(\_a) - \_a)^{-1} d_{-a} + 2\_C1\right) x^2 + b} \right\}$$

## 2.595 ODE No. 595

$$y'(x) = \frac{F\left(\frac{xy(x)^2 + 1}{x}\right)}{x^2 y(x)}$$

✓ **Mathematica** : cpu = 0.305996 (sec), leaf count = 204

$$\text{Solve} \left[ \int_1^{y(x)} \left( \frac{K[2]}{2F\left(\frac{xK[2]^2 + 1}{x}\right) - 1} - \int_1^x \left( \frac{4F\left(\frac{K[1]K[2]^2 + 1}{K[1]}\right) K[2] F'\left(\frac{K[1]K[2]^2 + 1}{K[1]}\right)}{\left(2F\left(\frac{K[1]K[2]^2 + 1}{K[1]}\right) - 1\right)^2 K[1]^2} - \frac{2K[2] F'\left(\frac{K[1]K[2]^2 + 1}{K[1]}\right)}{\left(2F\left(\frac{K[1]K[2]^2 + 1}{K[1]}\right) - 1\right) K[1]} \right) dx \right]$$

✓ **Maple** : cpu = 0.098 (sec), leaf count = 72

$$\left\{ y(x) = \frac{1}{x} \sqrt{x \left( \text{RootOf}\left(\int^{-Z} (-1 + 2F(\_a))^{-1} d_{-a} x + x\_C1 + 1\right) x - 1\right)}, y(x) = -\frac{1}{x} \sqrt{x \left( \text{RootOf}\left(\int^{-Z} (-1 + 2F(\_a))^{-1} d_{-a} x + x\_C1 + 1\right) x - 1\right)} \right\}$$

2.596 ODE No. 596

---

$$y'(x) = \frac{F(x^2 + y(x) - x) - 2x^2 + x}{x}$$

✓ **Mathematica** : cpu = 0.249787 (sec), leaf count = 156

$$\text{Solve} \left[ \int_1^{y(x)} -\frac{F(x^2 - x + K[2]) \int_1^x \left( \frac{2K[1]F'(K[1]^2 - K[1] + K[2])}{F(K[1]^2 - K[1] + K[2])^2} - \frac{F'(K[1]^2 - K[1] + K[2])}{F(K[1]^2 - K[1] + K[2])^2} \right) dK[1] + 1}{F(x^2 - x + K[2])} dK[2] + \int_1^x \left( -\frac{F(x^2 - x + K[2])}{F(x^2 - x + K[2])} \right) dK[2] \right]$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 26

$$\left\{ y(x) = -x^2 + \text{RootOf} \left( -\ln(x) + \int^{-Z} (F(\_a))^{-1} d\_a + \_C1 \right) + x \right\}$$

2.597 ODE No. 597

---

$$y'(x) = \frac{2a}{x^2 \left( 2aF\left(\frac{xy(x)^2 - 4a}{x}\right) - y(x) \right)}$$

✓ **Mathematica** : cpu = 0.38036 (sec), leaf count = 130

$$\text{Solve} \left[ \int_1^{y(x)} \left( -\frac{K[2]}{2aF\left(\frac{xK[2]^2 - 4a}{x}\right)} - \int_1^x \frac{2K[2]F'\left(\frac{K[1]K[2]^2 - 4a}{K[1]}\right)}{F\left(\frac{K[1]K[2]^2 - 4a}{K[1]}\right)^2 K[1]^2} dK[1] + 1 \right) dK[2] + \int_1^x -\frac{1}{F\left(\frac{K[1]y(x)^2 - 4a}{K[1]}\right) K[1]} dK[1] \right]$$

✓ **Maple** : cpu = 0.333 (sec), leaf count = 37

$$\left\{ -\frac{y(x)}{2a} + \frac{1}{8a^2} \int^{(y(x))^2 - 4\frac{a}{x}} (F(\_a))^{-1} d\_a - \_C1 = 0 \right\}$$

2.598 ODE No. 598

---

$$y'(x) = \frac{F\left(\frac{y(x)}{x}\right) + y(x)}{x - 1}$$

✓ **Mathematica** : cpu = 0.168609 (sec), leaf count = 37

$$\text{Solve} \left[ \int_1^{\frac{y(x)}{x}} \frac{1}{F(K[1]) + K[1]} dK[1] = c_1 + \log(1 - x) - \log(x), y(x) \right]$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 29

$$\left\{ y(x) = \text{RootOf} \left( -\int^{-Z} (F(\_a) + \_a)^{-1} d\_a + \ln(x - 1) - \ln(x) + \_C1 \right) x \right\}$$

2.599 ODE No. 599

---

$$y'(x) = \frac{F(x^2 + y(x)^2) - x}{y(x)}$$

✓ **Mathematica** : cpu = 0.154238 (sec), leaf count = 95

$$\text{Solve} \left[ \int_1^{y(x)} \left( -\frac{K[2]}{F(x^2 + K[2]^2)} - \int_1^x \frac{2K[1]K[2]F'(K[1]^2 + K[2]^2)}{F(K[1]^2 + K[2]^2)^2} dK[1] \right) dK[2] + \int_1^x \left( 1 - \frac{K[1]}{F(K[1]^2 + y(x)^2)} \right) dK[1] \right]$$

✓ **Maple** : cpu = 0.07 (sec), leaf count = 57

$$\left\{ y(x) = \sqrt{-x^2 + \text{RootOf} \left( -2x + \int^{-Z} (F(\_a))^{-1} d\_a + 2\_C1 \right)}, y(x) = -\sqrt{-x^2 + \text{RootOf} \left( -2x + \int^{-Z} (F(\_a))^{-1} d\_a + 2\_C1 \right)} \right\}$$

## 2.600 ODE No. 600

---

$$y'(x) = \frac{y(x)^2 F\left(\frac{1-2y(x)\log(x)}{y(x)}\right)}{x}$$

✓ **Mathematica** : cpu = 0.261703 (sec), leaf count = 246

$$\text{Solve} \left[ \int_1^{y(x)} \left( - \int_1^x \left( \frac{2 \left( -\frac{2\log(K[1])}{K[2]} - \frac{1-2K[2]\log(K[1])}{K[2]^2} \right) F' \left( \frac{1-2K[2]\log(K[1])}{K[2]} \right)}{\left( F \left( \frac{1-2K[2]\log(K[1])}{K[2]} \right) + 2 \right) K[1]} - \frac{2F \left( \frac{1-2K[2]\log(K[1])}{K[2]} \right) \left( -\frac{2\log(K[1])}{K[2]} \right)}{\left( F \left( \frac{1-2K[2]\log(K[1])}{K[2]} \right) + 2 \right) K[1]} \right) dK[1] \right]$$

✓ **Maple** : cpu = 0.108 (sec), leaf count = 38

$$\left\{ \int_{-b}^{y(x)} \frac{1}{-a^2} \left( F \left( \frac{-2-a \ln(x)+1}{-a} \right) + 2 \right)^{-1} d_{-a} - \ln(x) - _C1 = 0 \right\}$$

## 2.601 ODE No. 601

---

$$y'(x) = \frac{x F(-(x-y(x))(y(x)+x))}{y(x)}$$

✓ **Mathematica** : cpu = 0.241001 (sec), leaf count = 190

$$\text{Solve} \left[ \int_1^{y(x)} \left( \frac{K[2]}{F(-(x-K[2])(x+K[2]))-1} - \int_1^x \left( \frac{2F(-(K[1]-K[2])(K[1]+K[2]))K[1]K[2]F'(-(K[1]-K[2])(K[1]+K[2]))}{(F(-(K[1]-K[2])(K[1]+K[2]))-1)} \right) dK[1] \right) dK[2] \right]$$

✓ **Maple** : cpu = 0.098 (sec), leaf count = 61

$$\left\{ y(x) = \sqrt{x^2 + \text{RootOf} \left( -x^2 + \int^{-Z} (F(_a) - 1)^{-1} d_{-a} + 2_{-C1} \right)}, y(x) = -\sqrt{x^2 + \text{RootOf} \left( -x^2 + \int^{-Z} (F(_a) - 1)^{-1} d_{-a} + 2_{-C1} \right)} \right\}$$

## 2.602 ODE No. 602

---

$$y'(x) = \frac{y(x)^2 \left( x^2 F \left( \frac{x^2-y(x)}{x^2 y(x)} \right) + 2 \right)}{x^3}$$

✓ **Mathematica** : cpu = 0.524275 (sec), leaf count = 167

$$\text{Solve} \left[ \int_1^{y(x)} \left( - \int_1^x - \frac{2 \left( -\frac{K[1]^2-K[2]}{K[1]^2 K[2]^2} - \frac{1}{K[1]^2 K[2]} \right) F' \left( \frac{K[1]^2-K[2]}{K[1]^2 K[2]} \right)}{F \left( \frac{K[1]^2-K[2]}{K[1]^2 K[2]} \right)^2 K[1]^3} dK[1] - \frac{1}{F \left( \frac{x^2-K[2]}{x^2 K[2]} \right) K[2]^2} \right) dK[2] + \int_1^x \left( - \frac{1}{F \left( \frac{x^2-K[2]}{x^2 K[2]} \right) K[2]^2} \right) dK[2] \right]$$

✓ **Maple** : cpu = 0.099 (sec), leaf count = 33

$$\left\{ y(x) = \frac{x^2}{\text{RootOf} \left( -\ln(x) - \int^{-Z} (F(_a))^{-1} d_{-a} + _C1 \right) x^2 + 1} \right\}$$

## 2.603 ODE No. 603

---

$$y'(x) = \frac{2x F(y(x) + \log(2x+1)) + F(y(x) + \log(2x+1)) - 2}{2x+1}$$

✓ **Mathematica** : cpu = 0.311519 (sec), leaf count = 117

$$\text{Solve} \left[ \int_1^{y(x)} - \frac{F(K[2] + \log(2x+1)) \int_1^x - \frac{2F'(K[2] + \log(2K[1]+1))}{F(K[2] + \log(2K[1]+1))^2 (2K[1]+1)} dK[1] - 1}{F(K[2] + \log(2x+1))} dK[2] + \int_1^x \left( \frac{1}{F(\log(2K[1]+1))} \right) dK[2] \right]$$

✓ **Maple** : cpu = 0.108 (sec), leaf count = 27

$$\left\{ y(x) = -\ln(2x+1) + \text{RootOf} \left( -x + \int^{-Z} (F(_a))^{-1} d_{-a} + _C1 \right) \right\}$$

2.604 ODE No. 604

---

$$y'(x) = \frac{2y(x)^3}{2y(x)F\left(\frac{4xy(x)^2+1}{y(x)^2}\right) + 1}$$

✓ **Mathematica** : cpu = 0.388309 (sec), leaf count = 143

$$\text{Solve} \left[ \int_1^{y(x)} \left( - \int_1^x \frac{\left( \frac{8K[1]}{K[2]} - \frac{2(4K[1]K[2]^2+1)}{K[2]^3} \right) F' \left( \frac{4K[1]K[2]^2+1}{K[2]^2} \right)}{F \left( \frac{4K[1]K[2]^2+1}{K[2]^2} \right)^2} dK[1] + \frac{1}{K[2]^2} + \frac{1}{2F \left( \frac{4xK[2]^2+1}{K[2]^2} \right) K[2]^3} \right) dK[2] \right]$$

✓ **Maple** : cpu = 0.109 (sec), leaf count = 30

$$\left\{ -\_C1 - (y(x))^{-1} - \frac{\int^{4x+(y(x))^{-2}} (F(\_a))^{-1} d\_a}{4} = 0 \right\}$$

2.605 ODE No. 605

---

$$y'(x) = -\frac{y(x)^2 \left( 2x - F \left( \frac{1-\frac{1}{2}xy(x)}{y(x)} \right) \right)}{4x}$$

✓ **Mathematica** : cpu = 0.448653 (sec), leaf count = 145

$$\text{Solve} \left[ \int_1^{y(x)} \left( - \int_1^x \frac{2 \left( -\frac{K[1]}{2K[2]} - \frac{1-\frac{1}{2}K[1]K[2]}{K[2]^2} \right) F' \left( \frac{1-\frac{1}{2}K[1]K[2]}{K[2]} \right)}{F \left( \frac{1-\frac{1}{2}K[1]K[2]}{K[2]} \right)^2} dK[1] - \frac{4}{F \left( \frac{1-\frac{1}{2}xK[2]}{K[2]} \right) K[2]^2} \right) dK[2] + \int_1^x \left( \frac{K[1]}{K[2]} - \frac{1-\frac{1}{2}K[1]K[2]}{K[2]^2} \right) F' \left( \frac{1-\frac{1}{2}K[1]K[2]}{K[2]} \right) dK[1] \right]$$

✓ **Maple** : cpu = 0.158 (sec), leaf count = 29

$$\left\{ y(x) = 2 \left( 2 \text{RootOf} \left( -\ln(x) - 4 \int^{-Z} (F(\_a))^{-1} d\_a + \_C1 \right) + x \right)^{-1} \right\}$$

2.606 ODE No. 606

---

$$y'(x) = -x \left( -F \left( y(x) - \frac{1}{2}e^{-x^2}x^2 \right) + e^{-x^2}x^2 - e^{-x^2} \right)$$

✓ **Mathematica** : cpu = 0.537564 (sec), leaf count = 361

$$\text{Solve} \left[ \int_1^{y(x)} - \frac{F \left( K[2] - \frac{1}{2}e^{-x^2}x^2 \right) \int_1^x \left( \frac{e^{-K[1]^2} F' \left( K[2] - \frac{1}{2}e^{-K[1]^2} K[1]^2 \right) K[1]^3}{F \left( K[2] - \frac{1}{2}e^{-K[1]^2} K[1]^2 \right)^2} - \frac{e^{-K[1]^2} \left( e^{K[1]^2} F \left( K[2] - \frac{1}{2}e^{-K[1]^2} K[1]^2 \right) + 1 \right)}{F \left( K[2] - \frac{1}{2}e^{-K[1]^2} K[1]^2 \right)} \right) dK[1]}{F \left( K[2] - \frac{1}{2}e^{-x^2}x^2 \right)} dK[2] \right]$$

✓ **Maple** : cpu = 0.602 (sec), leaf count = 34

$$\left\{ y(x) = \frac{x^2 e^{-x^2}}{2} + \text{RootOf} \left( x^2 - 2 \int^{-Z} (F(\_a))^{-1} d\_a + 2\_C1 \right) \right\}$$

2.607 ODE No. 607

---

$$y'(x) = \frac{x^3 F \left( \frac{y(x)}{x^2} \right) + 2y(x)}{x}$$

✓ **Mathematica** : cpu = 0.215566 (sec), leaf count = 121

$$\text{Solve} \left[ \int_1^{y(x)} - \frac{F \left( \frac{K[2]}{x^2} \right) \int_1^x \left( \frac{2}{F \left( \frac{K[2]}{K[1]^2} \right) K[1]^3} - \frac{2K[2]F' \left( \frac{K[2]}{K[1]^2} \right)}{F \left( \frac{K[2]}{K[1]^2} \right)^2 K[1]^5} \right) dK[1] x^2 + 1}{x^2 F \left( \frac{K[2]}{x^2} \right)} dK[2] + \int_1^x \left( \frac{2y(x)}{F \left( \frac{y(x)}{K[1]^2} \right) K[1]^3} + 1 \right) dK[1] \right]$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 22

$$\left\{ y(x) = \text{RootOf} \left( -x + \int^{-Z} (F(\_a))^{-1} d\_a + \_C1 \right) x^2 \right\}$$

2.608 ODE No. 608

---

$$y'(x) = \frac{\sqrt{y(x)}}{F\left(\frac{x-y(x)}{\sqrt{y(x)}}\right) + \sqrt{y(x)}}$$

✓ **Mathematica** : cpu = 0.414467 (sec), leaf count = 274

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{F\left(\frac{x-K[2]}{\sqrt{K[2]}}\right)}{x\sqrt{K[2]}}-\int_1^x-\frac{F\left(\frac{K[1]-K[2]}{\sqrt{K[2]}}\right)}{\sqrt{K[2]}}-2\left(-\frac{K[1]-K[2]}{2K[2]^{3/2}}-\frac{1}{\sqrt{K[2]}}\right)\sqrt{K[2]}F'\left(\frac{K[1]-K[2]}{\sqrt{K[2]}}\right)-1}{\left(-2\sqrt{K[2]}F\left(\frac{K[1]-K[2]}{\sqrt{K[2]}}\right)+K[1]-K[2]\right)^2}dK[1]-\right.$$

✓ **Maple** : cpu = 0.112 (sec), leaf count = 40

$$\left\{\frac{\ln(y(x))}{2}-\int^x\frac{1}{\sqrt{y(x)}-\sqrt{y(x)}}(2F(\_a)-\_a)^{-1}d\_a-\_C1=0\right\}$$

2.609 ODE No. 609

---

$$y'(x) = \frac{F\left(x^3y(x)\right)-3x^2y(x)}{x^3}$$

✓ **Mathematica** : cpu = 0.256558 (sec), leaf count = 117

$$\text{Solve}\left[\int_1^{y(x)}-\frac{x^3+F\left(x^3K[2]\right)\int_1^x\left(\frac{3K[1]^5K[2]F'(K[1]^3K[2])}{F(K[1]^3K[2])^2}-\frac{3K[1]^2}{F(K[1]^3K[2])}\right)dK[1]}{F\left(x^3K[2]\right)}dK[2]+\int_1^x\left(1-\frac{3K[1]^2y(x)}{F\left(K[1]^3y(x)\right)}\right)$$

✓ **Maple** : cpu = 0.121 (sec), leaf count = 22

$$\left\{y(x)=\frac{RootOf\left(x-\int^{-Z}(F(\_a))^{-1}d\_a+\_C1\right)}{x^3}\right\}$$

2.610 ODE No. 610

---

$$y'(x) = \frac{x^2F\left(\frac{y(x)}{x}\right)+y(x)}{x}$$

✓ **Mathematica** : cpu = 0.0888794 (sec), leaf count = 25

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}}\frac{1}{F(K[1])}dK[1]=c_1+x,y(x)\right]$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 20

$$\left\{y(x)=RootOf\left(x-\int^{-Z}(F(\_a))^{-1}d\_a+\_C1\right)x\right\}$$

2.611 ODE No. 611

---

$$y'(x) = \frac{F(x(y(x)+x))-y(x)-2x}{x}$$

✓ **Mathematica** : cpu = 0.258779 (sec), leaf count = 191

$$\text{Solve}\left[\int_1^{y(x)}-\frac{x+F(x(x+K[2]))\int_1^x\left(\frac{2F'(K[1](K[1]+K[2]))K[1]^2}{F(K[1](K[1]+K[2]))^2}+\frac{(K[2]-F(K[1](K[1]+K[2])))F'(K[1](K[1]+K[2]))K[1]}{F(K[1](K[1]+K[2]))^2}\right)}{F(x(x+K[2]))}dK[2]+$$

✓ **Maple** : cpu = 0.065 (sec), leaf count = 28

$$\left\{y(x)=\frac{-x^2+RootOf\left(-x+\int^{-Z}(F(\_a))^{-1}d\_a+\_C1\right)}{x}\right\}$$

2.612 ODE No. 612

---

$$y'(x) = \frac{1}{2}e^{\frac{x^2}{4}} \left( 2F \left( e^{-\frac{x^2}{4}} y(x) \right) + e^{-\frac{x^2}{4}} xy(x) \right)$$

✓ **Mathematica** : cpu = 0.331484 (sec), leaf count = 199

$$\text{Solve} \left[ \int_1^{y(x)} - \frac{e^{-\frac{x^2}{4}} \left( e^{\frac{x^2}{4}} F \left( e^{-\frac{x^2}{4}} K[2] \right) \int_1^x \left( \frac{e^{-\frac{1}{4} K[1]^2} K[1]}{2F \left( e^{-\frac{1}{4} K[1]^2} K[2] \right)} - \frac{e^{-\frac{1}{2} K[1]^2} K[1] K[2] F' \left( e^{-\frac{1}{4} K[1]^2} K[2] \right)}{2F \left( e^{-\frac{1}{4} K[1]^2} K[2] \right)^2} \right) dK[1] + 1}{F \left( e^{-\frac{x^2}{4}} K[2] \right)} dK[1] + 1 \right]$$

✓ **Maple** : cpu = 0.101 (sec), leaf count = 27

$$\left\{ y(x) = \text{RootOf} \left( -x + \int^{-Z} (F(\_a))^{-1} d\_a + \_C1 \right) \left( e^{-\frac{x^2}{4}} \right)^{-1} \right\}$$

2.613 ODE No. 613

---

$$y'(x) = \frac{x^2 F \left( \frac{y(x) - x \log(x)}{x} \right) + y(x) + x}{x}$$

✓ **Mathematica** : cpu = 0.280517 (sec), leaf count = 226

$$\text{Solve} \left[ \int_1^{y(x)} - \frac{x F \left( \frac{K[2] - x \log(x)}{x} \right) \int_1^x \left( - \frac{K[2] F' \left( \frac{K[2] - K[1] \log(K[1])}{K[1]} \right)}{F \left( \frac{K[2] - K[1] \log(K[1])}{K[1]} \right)^2 K[1]^3} - \frac{F' \left( \frac{K[2] - K[1] \log(K[1])}{K[1]} \right)}{F \left( \frac{K[2] - K[1] \log(K[1])}{K[1]} \right)^2 K[1]^2} + \frac{1}{F \left( \frac{K[2] - K[1] \log(K[1])}{K[1]} \right)}}{x F \left( \frac{K[2] - x \log(x)}{x} \right)} dK[1] + 1 \right]$$

✓ **Maple** : cpu = 0.078 (sec), leaf count = 23

$$\left\{ y(x) = \left( \ln(x) + \text{RootOf} \left( -x + \int^{-Z} (F(\_a))^{-1} d\_a + \_C1 \right) \right) x \right\}$$

2.614 ODE No. 614

---

$$y'(x) = \frac{(a-1)(a+1)x}{a^2 F \left( -\frac{1}{2} a^2 x^2 + \frac{x^2}{2} + \frac{y(x)^2}{2} \right) - F \left( -\frac{1}{2} a^2 x^2 + \frac{x^2}{2} + \frac{y(x)^2}{2} \right) + y(x)}$$

✓ **Mathematica** : cpu = 0.358399 (sec), leaf count = 177

$$\text{Solve} \left[ \int_1^{y(x)} \left( \frac{K[2]}{(a-1)(a+1) F \left( -\frac{1}{2} a^2 x^2 + \frac{x^2}{2} + \frac{K[2]^2}{2} \right)} - \int_1^x \frac{K[1] K[2] F' \left( -\frac{1}{2} a^2 K[1]^2 + \frac{K[1]^2}{2} + \frac{K[2]^2}{2} \right)}{F \left( -\frac{1}{2} a^2 K[1]^2 + \frac{K[1]^2}{2} + \frac{K[2]^2}{2} \right)^2} dK[1] + 1 \right) dK[2] + 1 \right]$$

✓ **Maple** : cpu = 0.332 (sec), leaf count = 60

$$\left\{ \frac{y(x)}{(a-1)(a+1)} + \frac{1}{2a^4 - 4a^2 + 2} \int^{-a^2 x^2 + x^2 + (y(x))^2} \left( F \left( \frac{-a}{2} \right) \right)^{-1} d\_a - \_C1 = 0 \right\}$$

2.615 ODE No. 615

---

$$y'(x) = \frac{y(x)}{x(y(x)F(xy(x)) - 1)}$$

✓ **Mathematica** : cpu = 0.269893 (sec), leaf count = 77

$$\text{Solve} \left[ \int_1^{y(x)} \left( - \int_1^x \frac{F'(K[1]K[2])}{F(K[1]K[2])^2} dK[1] - \frac{1}{F(xK[2])K[2]} + 1 \right) dK[2] + \int_1^x - \frac{1}{F(K[1]y(x))K[1]} dK[1] = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.148 (sec), leaf count = 26

$$\left\{ -y(x) + \int^{xy(x)} \frac{1}{F(\_a)\_a} d\_a - \_C1 = 0 \right\}$$



2.616 ODE No. 616

---

$$y'(x) = \frac{F(xy(x) - 1) - 2x^3y(x) + x^2}{x^4}$$

✓ **Mathematica** : cpu = 0.457044 (sec), leaf count = 177

Solve  $\left[ \int_1^{y(x)} -\frac{x^2 + F(x(xK[2] - 1)) \int_1^x \left( \frac{2K[2]F'(K[1](K[1]K[2]-1))K[1]^3}{F(K[1](K[1]K[2]-1))^2} - \frac{F'(K[1](K[1]K[2]-1))K[1]^2}{F(K[1](K[1]K[2]-1))^2} - \frac{2K[1]}{F(K[1](K[1]K[2]-1))} \right) dx}{F(x(xK[2] - 1))} \right]$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 26

$$\left\{ y(x) = \frac{RootOf \left( \int^{-Z} (F(\_a))^{-1} d\_a x + x\_C1 + 1 \right) + x}{x^2} \right\}$$

2.617 ODE No. 617

---

$$y'(x) = \frac{1}{9} e^{-\frac{3x^2}{2}} xy(x)^2 F\left(\frac{e^{\frac{3x^2}{2}}(y(x) + 3)}{3y(x)}\right)$$

✓ **Mathematica** : cpu = 0.968605 (sec), leaf count = 615

Solve  $\left[ \int_1^{y(x)} \left( \frac{9e^{\frac{3x^2}{2}} - F\left(\frac{e^{\frac{3x^2}{2}}(K[2]+3)}{3K[2]}\right)}{3 \left( \left( 9e^{\frac{3x^2}{2}} - F\left(\frac{e^{\frac{3x^2}{2}}(K[2]+3)}{3K[2]}\right) \right) K[2] + 27e^{\frac{3x^2}{2}} \right)} - \int_1^x \left( -\frac{K[2] \left( \frac{e^{\frac{3K[1]^2}{2}}}{3K[2]} - \frac{e^{\frac{3K[1]^2}{2}}(K[2]+3)}{3K[2]^2} \right) F'\left(\frac{e^{\frac{3K[1]^2}{2}}(K[2]+3)}{3K[2]}\right)}{-9e^{\frac{3K[1]^2}{2}}K[2] + F\left(\frac{e^{\frac{3K[1]^2}{2}}(K[2]+3)}{3K[2]}\right)} \right) dx \right]$

✓ **Maple** : cpu = 0.207 (sec), leaf count = 47

$$\left\{ y(x) = -3 \frac{e^{3/2 x^2}}{e^{3/2 x^2} - 3 RootOf \left( -x^2 - 18 \int^{-Z} (F(\_a) - 27\_a)^{-1} d\_a + 2\_C1 \right)} \right\}$$

2.618 ODE No. 618

---

$$y'(x) = \frac{(y(x) + 1)(x(y(x) - \log(y(x) + 1) - \log(x)) + 1)}{xy(x)}$$

✓ **Mathematica** : cpu = 0.248992 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow -W\left(-\frac{e^{c_1 e^x - 1}}{x}\right) - 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.32 (sec), leaf count = 34

$$\left\{ y(x) = \frac{1}{x} \left( e^{-\text{lambertW}\left(-\frac{e^{-C1} e^x - 1}{x}\right) + -C1 e^x - 1} - x \right) \right\}$$

2.619 ODE No. 619

---

$$y'(x) = \frac{6y(x)}{-F\left(-\frac{1}{3}y(x)^4 - \frac{y(x)^3}{2} - y(x)^2 - y(x) + x\right) + 8y(x)^4 + 9y(x)^3 + 12y(x)^2 + 6y(x)}$$

✓ **Mathematica** : cpu = 0.650789 (sec), leaf count = 330

Solve  $\left[ \int_1^{y(x)} \left( -\frac{8K[2]^3}{F\left(-\frac{1}{3}K[2]^4 - \frac{K[2]^3}{2} - K[2]^2 - K[2] + x\right)} - \frac{9K[2]^2}{F\left(-\frac{1}{3}K[2]^4 - \frac{K[2]^3}{2} - K[2]^2 - K[2] + x\right)} - \frac{1}{F\left(-\frac{1}{3}K[2]^4 - \frac{K[2]^3}{2} - K[2]^2 - K[2] + x\right)} \right] dx$

✓ **Maple** : cpu = 0.323 (sec), leaf count = 81

$$\left\{ \int_{-b}^{y(x)} \frac{1}{-a} \left( -8\_a^4 - 9\_a^3 - 12\_a^2 + F\left(-\frac{a^4}{3} - \frac{a^3}{2} -\_a^2 -\_a + x\right) - 6\_a \right) \left( F\left(-\frac{a^4}{3} - \frac{a^3}{2} -\_a^2 -\_a + x\right) - 6\_a \right) dx \right\}$$

**2.620 ODE No. 620**

---

$$y'(x) = \frac{e^{2F(-(x-y(x))(y(x)+x))} + x^2 + 2xy(x) + y(x)^2}{-e^{2F(-(x-y(x))(y(x)+x))} + x^2 + 2xy(x) + y(x)^2}$$

✓ **Mathematica** : cpu = 0.759274 (sec), leaf count = 210

Solve  $\left[ \int_1^{y(x)} \left( -\frac{2K[2]}{-x^2 + e^{2F(-(x-K[2])(x+K[2]))} + K[2]^2} - \int_1^x \left( \frac{2K[1](-4\exp(2F(-(K[1]-K[2])(K[1]+K[2])))}{(K[1]^2 - \exp(2F(-(K[1]-K[2])(K[1]+K[2])))} \right) dx \right) dy \right]$

✓ **Maple** : cpu = 0.143 (sec), leaf count = 37

$$\left\{ y(x) = \text{e}^{\text{RootOf}\left(-\_Z + \int^{\left(\text{e}^{-Z}\right)^2 - 2\text{e}^{-Z}x}\left(\text{e}^{2F(\_a)} + \_a\right)^{-1}d\_a + \_C1\right)} - x \right\}$$

**2.621 ODE No. 621**

---

$$y'(x) = \frac{1}{y(x) + \sqrt{x}}$$

✓ **Mathematica** : cpu = 0.0561221 (sec), leaf count = 445

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{\text{Root}\left[\#1^6(16e^{12c_1} + 16x^3) - 24\#1^4x^2 + 8\#1^3x^{3/2} + 9\#1^2x - 6\#1\sqrt{x} + 1\&, 1\right]} - \sqrt{x} \right\}, \left\{ y(x) \rightarrow \frac{1}{\sqrt{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.237 (sec), leaf count = 59

$$\left\{ y(x) = 1 \left( \sqrt{x} \left( \text{RootOf}(\_Z^{18} \_C1 - 9x\_Z^6 - 6\sqrt{x}\_Z^3 - 1) \right)^3 + 1 \right) \left( \text{RootOf}(\_Z^{18} \_C1 - 9x\_Z^6 - 6\sqrt{x}\_Z^3 - 1) \right) \right\}$$

**2.622 ODE No. 622**

---

$$y'(x) = \frac{1}{y(x) + \sqrt{3x+1} + 2}$$

✓ **Mathematica** : cpu = 0.270061 (sec), leaf count = 140

Solve  $\left[ 44c_1 + 6\sqrt{33} \tanh^{-1} \left( \frac{3y(x) + 7\sqrt{3x+1} + 6}{\sqrt{33}(y(x) + \sqrt{3x+1} + 2)} \right) = 33 \left( \log \left( \frac{-3\sqrt{3x+1}y(x)^2 - 3(3x + 4\sqrt{3x+1} + 1)y(x) + 12}{2(3x + 4\sqrt{3x+1} + 1)} \right) \right) \right]$

✓ **Maple** : cpu = 0.181 (sec), leaf count = 77

$$\left\{ \ln \left( (3y(x) + 6)\sqrt{3x+1} + 3(y(x))^2 - 6x + 12y(x) + 10 \right) - 6 \frac{\sqrt{3x+1}}{\sqrt{99x+33}} \text{Artanh} \left( \frac{3\sqrt{3x+1} + 6y(x) + 12}{\sqrt{99x+33}} \right) \right\}$$

**2.623 ODE No. 623**

---

$$y'(x) = \frac{x^2}{x^{3/2} + y(x)}$$

✓ **Mathematica** : cpu = 0.143007 (sec), leaf count = 77

Solve  $\left[ 44c_1 + 6\sqrt{33} \tanh^{-1} \left( \frac{7x^{3/2} + 3y(x)}{\sqrt{33}(x^{3/2} + y(x))} \right) = 33 \left( \log \left( -\frac{3y(x)}{2x^{3/2}} - \frac{3y(x)^2}{2x^3} + 1 \right) + 3\log(x) \right), y(x) \right]$

✓ **Maple** : cpu = 0.227 (sec), leaf count = 49

$$\left\{ \ln \left( 3x^{3/2}y(x) - 2x^3 + 3(y(x))^2 \right) - \frac{2\sqrt{33}}{11} \text{Artanh} \left( \frac{\sqrt{33}}{11} \left( x^{\frac{3}{2}} + 2y(x) \right) x^{-\frac{3}{2}} \right) - \_C1 = 0 \right\}$$

2.624 ODE No. 624

---

$$y'(x) = \frac{x^{5/3}}{x^{4/3} + y(x)}$$

✓ **Mathematica** : cpu = 28.9272 (sec), leaf count = 9837

✓ **Maple** : cpu = 1.898 (sec), leaf count = 46

$$\left\{y(x) = \frac{1}{2} \left( \text{RootOf} \left( -Z^{192} + 12 x^{4/3} - Z^{176} + 48 x^{8/3} - Z^{160} + 64 x^4 - Z^{144} - C1 \right) \right)^{16} + \frac{1}{2} x^{\frac{4}{3}} \right\}$$

2.625 ODE No. 625

---

$$y'(x) = \frac{1}{2} i x^2 \left( -2 \sqrt{6 y(x) - x^3} + i \right)$$

✓ **Mathematica** : cpu = 0.295057 (sec), leaf count = 56

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{6} \left( -W \left( i e^{-6 c_1 - x^3 - 1} \right)^2 - 2 W \left( i e^{-6 c_1 - x^3 - 1} \right) + x^3 - 1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.161 (sec), leaf count = 55

$$\left\{ \ln \left( -x^3 + 6 y(x) + 1 \right) - 2 i \sqrt{-x^3 + 6 y(x)} + 2 i \arctan \left( \sqrt{-x^3 + 6 y(x)} \right) + 2 x^3 - C1 = 0 \right\}$$

2.626 ODE No. 626

---

$$y'(x) = \frac{x}{\sqrt{x^2 + 1} + y(x)}$$

✓ **Mathematica** : cpu = 0.151535 (sec), leaf count = 88

$$\text{Solve} \left[ \frac{1}{2} \left( \log \left( -\frac{y(x)^2}{x^2 + 1} - \frac{y(x)}{\sqrt{x^2 + 1}} + 1 \right) + \log(x^2 + 1) \right) = c_1 + \frac{\tanh^{-1} \left( \frac{3 \sqrt{x^2 + 1} + y(x)}{\sqrt{5} (\sqrt{x^2 + 1} + y(x))} \right)}{\sqrt{5}}, y(x) \right]$$

✓ **Maple** : cpu = 0.264 (sec), leaf count = 115

$$\left\{ \frac{2}{3} \ln \left( -\frac{1296}{11} \left( \sqrt{x^2 + 1} y(x) - x^2 + (y(x))^2 - 1 \right) \left( y(x) + \sqrt{x^2 + 1} \right)^{-2} \right) - \frac{4 \sqrt{5}}{15} \text{Artanh} \left( \sqrt{5} \left( 3 \sqrt{x^2 + 1} + y(x) \right) \right) \right\}$$

2.627 ODE No. 627

---

$$y'(x) = \frac{(y(x) \log(x) - 1)^2}{x}$$

✓ **Mathematica** : cpu = 0.811084 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow \frac{\tan(c_1 + \log(x))}{\log(x) \tan(c_1 + \log(x)) + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.153 (sec), leaf count = 35

$$\left\{y(x)=\frac{\sin(\ln(x))\_C1+\cos(\ln(x))}{(\_C1+\ln(x))\cos(\ln(x))+\sin(\ln(x))(\ln(x)\_C1-1)}\right\}$$

**2.628 ODE No. 628**

---

$$y'(x)=\frac{1}{3}x\left(3\sqrt{x^2+3y(x)}-2\right)$$

✓ **Mathematica** : cpu = 0.153931 (sec), leaf count = 38

$$\left\{\left\{y(x)\rightarrow\frac{1}{432}\left(-486c_1x^2+729c_1^2-216c_1+81x^4-72x^2+16\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.158 (sec), leaf count = 23

$$\left\{-C1+\frac{3x^2}{4}+\frac{2}{3}-\sqrt{x^2+3y(x)}=0\right\}$$

**2.629 ODE No. 629**

---

$$y'(x)=\frac{(2y(x)\log(x)-1)^2}{x}$$

✓ **Mathematica** : cpu = 0.838142 (sec), leaf count = 47

$$\left\{\left\{y(x)\rightarrow\frac{1}{\sqrt{2}\left(\sqrt{2}\log(x)-\tan\left(\frac{1}{2}\left(\sqrt{2}c_1+2\sqrt{2}\log(x)\right)\right)\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.152 (sec), leaf count = 62

$$\left\{y(x)=\frac{\sin(\ln(x)\sqrt{2})\_C1-\cos(\ln(x)\sqrt{2})}{\sin(\ln(x)\sqrt{2})\left(2\_C1\ln(x)+\sqrt{2}\right)+\left(\sqrt{2}\_C1-2\ln(x)\right)\cos(\ln(x)\sqrt{2})}\right\}$$

**2.630 ODE No. 630**

---

$$y'(x)=\frac{e^{bx}}{e^{-bx}y(x)+1}$$

✓ **Mathematica** : cpu = 0.370277 (sec), leaf count = 101

$$\text{Solve}\left[\frac{1}{2}b\left(\log\left(-be^{-2bx}y(x)^2-be^{-bx}y(x)+1\right)+2bx\right)=\frac{b\tan^{-1}\left(\frac{(b+2)(-e^{bx})-by(x)}{b\sqrt{-\frac{b+4}{b}}(e^{bx}+y(x))}\right)}{\sqrt{-\frac{b+4}{b}}}+c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.247 (sec), leaf count = 98

$$\left\{y(x)=\frac{1}{e^{-bx}}RootOf\left(-e^{RootOf\left(\left(\tanh\left(\frac{2\_C1b-2bx-\_Z}{2b}\sqrt{b^2+4b}\right)\right)^2b+4\left(\tanh\left(1/2\frac{\sqrt{b^2+4b}(2\_C1b-2bx-\_Z)}{b}\right)\right)^2-4e^{-Z}-b-4\right)}\right)}\right\}$$

**2.631 ODE No. 631**

---

$$y'(x)=\frac{1}{2}x^2\left(2\sqrt{x^3-6y(x)}+1\right)$$

✓ **Mathematica** : cpu = 0.174481 (sec), leaf count = 38

$$\left\{\left\{y(x)\rightarrow\frac{1}{96}\left(-192c_1x^3-576c_1^2-48c_1-16x^6+8x^3-1\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.174 (sec), leaf count = 23

$$\left\{-C1-x^3-\frac{1}{2}-\sqrt{x^3-6y(x)}=0\right\}$$

**2.632 ODE No. 632**

---

$$y'(x) = \frac{e^x}{e^{-x}y(x) + 1}$$

✓ **Mathematica** : cpu = 0.145545 (sec), leaf count = 65

$$\text{Solve} \left[ \frac{1}{2} \log \left( -e^{-2x}y(x)^2 - e^{-x}y(x) + 1 \right) + x = c_1 + \frac{\tanh^{-1} \left( \frac{y(x)+3e^x}{\sqrt{5}(y(x)+e^x)} \right)}{\sqrt{5}}, y(x) \right]$$

✓ **Maple** : cpu = 0.214 (sec), leaf count = 54

$$\left\{ x - \frac{\sqrt{5}}{5} \text{Artanh} \left( \frac{2y(x)\sqrt{5}e^{-x}}{5} + \frac{\sqrt{5}}{5} \right) + \frac{\ln \left( (y(x))^2 (e^{-x})^2 + y(x) e^{-x} - 1 \right)}{2} - \_C1 = 0 \right\}$$

**2.633 ODE No. 633**

---

$$y'(x) = \frac{e^{2x/3}}{e^{-2x/3}y(x) + 1}$$

✓ **Mathematica** : cpu = 0.159373 (sec), leaf count = 85

$$\text{Solve} \left[ 7 \left( -9c_1 + 3 \log \left( -\frac{2}{3}e^{-4x/3}y(x)^2 - \frac{2}{3}e^{-2x/3}y(x) + 1 \right) + 4x \right) = 6\sqrt{7} \tanh^{-1} \left( \frac{y(x) + 4e^{2x/3}}{\sqrt{7}(y(x) + e^{2x/3})} \right), y(x) \right]$$

✓ **Maple** : cpu = 0.76 (sec), leaf count = 52

$$\left\{ y(x) = 1 \text{RootOf} \left( -e^{\text{RootOf} \left( 343 - 343 \left( \tanh \left( 1/6 (4\_C1 - 4x - 3\_Z) \sqrt{7} \right) \right)^2 + 98e^{-Z} \right) - 3 + 2\_Z + 2\_Z^2} \right) \left( e^{-\frac{2x}{3}} \right)^{-1} \right\}$$

**2.634 ODE No. 634**

---

$$y'(x) = \frac{x^5 \sqrt{4x^2y(x) + 1} + \frac{1}{2}}{x^3}$$

✓ **Mathematica** : cpu = 0.201415 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow \frac{-8c_1x^6 + 16c_1^2x^2 + x^{10} - 4}{16x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.175 (sec), leaf count = 26

$$\left\{ \_C1 - \frac{1}{x} \sqrt{4x^2y(x) + 1} + \frac{x^4}{2} = 0 \right\}$$

**2.635 ODE No. 635**

---

$$y'(x) = \frac{1}{2}x \left( 2\sqrt{x^3 - 6y(x)} + x \right)$$

✓ **Mathematica** : cpu = 0.171741 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{24} (36c_1x^2 - 36c_1^2 - 9x^4 + 4x^3) \right\} \right\}$$

✓ **Maple** : cpu = 0.143 (sec), leaf count = 22

$$\left\{ \_C1 - \frac{3x^2}{2} - \sqrt{x^3 - 6y(x)} = 0 \right\}$$

**2.636 ODE No. 636**

---

$$y'(x) = y(x) \left( x^2 - \log(y(x)) \right)$$

✓ **Mathematica** : cpu = 0.0999756 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow e^{-2c_1e^{-x}+x^2-2x+2} \right\} \right\}$$

✓ **Maple** : cpu = 0.138 (sec), leaf count = 19

$$\left\{ y(x) = e^{\frac{-C1}{e^x}+x^2-2x+2} \right\}$$

2.637 ODE No. 637

---

$$y'(x) = \frac{e^{-x^2} x}{e^{x^2} y(x) + 1}$$

✓ **Mathematica** : cpu = 11.7394 (sec), leaf count = 59

$$\text{Solve} \left[ -\frac{1}{4} \log \left( 2e^{2x^2} y(x)^2 + 2e^{x^2} y(x) + 1 \right) - \frac{1}{2} \tan^{-1} \left( 2e^{x^2} y(x) + 1 \right) + \frac{x^2}{2} = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.136 (sec), leaf count = 84

$$\left\{ y(x) = -\frac{1}{e^{x^2}} \tan \left( \text{RootOf} \left( 2x^2 + 2 \ln(9/2 \tan(\_Z) - 9/2) - \ln \left( \frac{81 (\tan(\_Z))^2}{10} + \frac{81}{10} \right) + 6\_C1 - 2\_Z \right) \right) \right\}$$

2.638 ODE No. 638

---

$$y'(x) = y(x)(-(\log(x) - \log(\log(y(x))))))$$

✗ **Mathematica** : cpu = 1.68362 (sec), leaf count = 0 , could not solve

DSolve[Derivative[1][y][x] == -((Log[x] - Log[Log[y[x]]])\*y[x]), y[x], x]

✓ **Maple** : cpu = 0.138 (sec), leaf count = 35

$$\left\{ \int_{\_b}^{y(x)} \frac{1}{\_a (-\ln(\ln(\_a)) x + x \ln(x) + \ln(\_a))} d\_a + \ln(x) - \_C1 = 0 \right\}$$

2.639 ODE No. 639

---

$$y'(x) = y(x)(\log(x) - \log(\log(y(x))))^2$$

✗ **Mathematica** : cpu = 0.274732 (sec), leaf count = 0 , could not solve

DSolve[Derivative[1][y][x] == (Log[x] - Log[Log[y[x]]])^2\*y[x], y[x], x]

✓ **Maple** : cpu = 0.185 (sec), leaf count = 48

$$\left\{ \int_{\_b}^{y(x)} \frac{1}{\_a \left( -(\ln(\ln(\_a)))^2 x + 2 \ln(\ln(\_a)) \ln(x) x - x (\ln(x))^2 + \ln(\_a) \right)} d\_a + \ln(x) - \_C1 = 0 \right\}$$

2.640 ODE No. 640

---

$$y'(x) = \frac{y(x)}{\log(\log(y(x))) - \log(x) + 1}$$

✗ **Mathematica** : cpu = 2.37359 (sec), leaf count = 0 , could not solve

DSolve[Derivative[1][y][x] == y[x]/(1 - Log[x] + Log[Log[y[x]]]), y[x], x]

✓ **Maple** : cpu = 0.27 (sec), leaf count = 47

$$\left\{ \int_{\_b}^{y(x)} \frac{-\ln(\ln(\_a)) + \ln(x) - 1}{(\ln(\_a) \ln(x) - \ln(\_a) \ln(\ln(\_a)) + x - \ln(\_a)) \_a} d\_a - \_C1 = 0 \right\}$$

2.641 ODE No. 641

---

$$y'(x) = \frac{x^4 \sqrt{4x^2 y(x) + 1} + \frac{1}{2}}{x^3}$$

✓ **Mathematica** : cpu = 0.201112 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow \frac{-24c_1 x^5 + 36c_1^2 x^2 + 4x^8 - 9}{36x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.159 (sec), leaf count = 26

$$\left\{ \_C1 - \frac{1}{x} \sqrt{4x^2 y(x) + 1} + \frac{2x^3}{3} = 0 \right\}$$

2.642 ODE No. 642

---

$$y'(x) = \frac{(4ax - y(x)^2)^2}{y(x)}$$

✓ **Mathematica** : cpu = 0.16373 (sec), leaf count = 105

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{4ax - \sqrt{2}\sqrt{a} \tanh\left(\frac{2\sqrt{2}ax - \sqrt{2}c_1}{\sqrt{a}}\right)} \right\}, \left\{ y(x) \rightarrow \sqrt{4ax - \sqrt{2}\sqrt{a} \tanh\left(\frac{2\sqrt{2}ax - \sqrt{2}c_1}{\sqrt{a}}\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.259 (sec), leaf count = 286

$$\left\{ y(x) = \sqrt{4}\sqrt{\left(-C1\left(ax - \frac{\sqrt{2}}{4}\sqrt{a}\right)e^{2x(\sqrt{2}\sqrt{a}-2ax)} + e^{-2x(\sqrt{2}\sqrt{a}+2ax)}\left(ax + \frac{\sqrt{2}}{4}\sqrt{a}\right)\right)\left(-C1e^{2x(\sqrt{2}\sqrt{a}-2ax)}\right)} \right\}$$

2.643 ODE No. 643

---

$$y'(x) = \frac{1}{3}x\left(3x\sqrt{x^2 + 3y(x)} - 2\right)$$

✓ **Mathematica** : cpu = 0.173581 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{12}(-6c_1x^3 + 9c_1^2 + x^6 - 4x^2) \right\} \right\}$$

✓ **Maple** : cpu = 0.143 (sec), leaf count = 22

$$\left\{ -C1 + \frac{x^3}{2} - \sqrt{x^2 + 3y(x)} = 0 \right\}$$

2.644 ODE No. 644

---

$$y'(x) = -\frac{1}{2}x^2\left(ax - 2\sqrt{a(ax^4 + 8y(x))}\right)$$

✓ **Mathematica** : cpu = 0.347422 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{72}a(-96c_1x^3 + 144c_1^2 + 16x^6 - 9x^4) \right\} \right\}$$

✓ **Maple** : cpu = 0.324 (sec), leaf count = 27

$$\left\{ -C1 + \frac{4ax^3}{3} - \sqrt{a(ax^4 + 8y(x))} = 0 \right\}$$

2.645 ODE No. 645

---

$$y'(x) = y(x)(x - \log(y(x)))$$

✓ **Mathematica** : cpu = 0.0823529 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow e^{-e^{c_1-x}+x-1} \right\} \right\}$$

✓ **Maple** : cpu = 0.07 (sec), leaf count = 14

$$\left\{ y(x) = e^{\frac{-C1}{e^x}-1+x} \right\}$$

2.646 ODE No. 646

---

$$y'(x) = \frac{\sqrt{x^3 - 6y(x)} + \frac{x^3}{2} + \frac{x^2}{2}}{x + 1}$$

✓ **Mathematica** : cpu = 0.287999 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{6}(18c_1 \log(x+1) - 9c_1^2 + x^3 - 9 \log^2(x+1)) \right\} \right\}$$

✓ **Maple** : cpu = 0.198 (sec), leaf count = 23

$$\left\{ -C1 - 3 \ln(1+x) - \sqrt{x^3 - 6y(x)} = 0 \right\}$$

**2.647      ODE No. 647**

---

$$y'(x) = \frac{x \left( ay(x)^2 + bx^2 \right)^2}{a^{5/2} y(x)}$$

✓ **Mathematica** : cpu = 0.36253 (sec), leaf count = 115

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{\frac{\sqrt{b} \tan\left(\frac{a^{3/2}bx^2+2c_1}{a^{9/4}\sqrt{b}}\right)}{\sqrt[4]{a}}} - \frac{bx^2}{a} \right\}, \left\{ y(x) \rightarrow \sqrt{\frac{\sqrt{b} \tan\left(\frac{a^{3/2}bx^2+2c_1}{a^{9/4}\sqrt{b}}\right)}{\sqrt[4]{a}}} - \frac{bx^2}{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.207 (sec), leaf count = 460

$$\left\{ y(x) = \frac{1}{a} \sqrt{-\left(-C1 e^{\frac{x^2}{2} \left(2 \sqrt{-\frac{b}{a^{3/2}}} a^{3/2} + bx^2\right) a^{-\frac{3}{2}}} + e^{\frac{x^2}{2} \left(-2 \sqrt{-\frac{b}{a^{3/2}}} a^{3/2} + bx^2\right) a^{-\frac{3}{2}}}\right) a \left(\left(bx^2 - \sqrt{-ba^{-\frac{3}{2}}} a^{\frac{3}{2}}\right) e^{\frac{x^2}{2} \left(-2 \sqrt{-\frac{b}{a^{3/2}}} a^{3/2} + bx^2\right) a^{-\frac{3}{2}}}\right)} \right\}$$

**2.648      ODE No. 648**

---

$$y'(x) = -\frac{\sqrt{ax^3} \left(-2\sqrt{ax^4+8y(x)} + \sqrt{ax} + \sqrt{a}\right)}{2(x+1)}$$

✓ **Mathematica** : cpu = 0.530367 (sec), leaf count = 128

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{72} \left(-96ac_1x^3 + 144ac_1x^2 - 288ac_1x + 288ac_1 \log(x+1) + 144ac_1^2 - 432ac_1 + 16ax^6 - 48ax^5 + 123ax^4 - 48ax^3 + 16ax^2 - 48ax + 16a\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.47 (sec), leaf count = 41

$$\left\{ \frac{1}{4} \sqrt{ax^4+8y(x)} \frac{1}{\sqrt{a}} - \frac{x^3}{3} + \frac{x^2}{2} - x + \ln(1+x) - C1 = 0 \right\}$$

**2.649      ODE No. 649**

---

$$y'(x) = x\sqrt{x^2+8y(x)-2x+1} - \frac{x}{4} + \frac{1}{4}$$

✓ **Mathematica** : cpu = 0.255398 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{8} \left(-16c_1x^2 + 16c_1^2 + 4x^4 - x^2 + 2x - 1\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.158 (sec), leaf count = 27

$$\left\{ -C1 + 2x^2 + \frac{1}{4} - \sqrt{x^2 - 2x + 1 + 8y(x)} = 0 \right\}$$

**2.650      ODE No. 650**

---

$$y'(x) = x\sqrt{a^2+2ax+x^2+4y(x)} - \frac{a}{2} - \frac{x}{2}$$

✓ **Mathematica** : cpu = 0.396734 (sec), leaf count = 109

Solve  $\left[-\frac{1}{2} \sqrt{a^2+2ax+x^2+4y(x)} - \frac{1}{2} a \log\left(\sqrt{a^2+2ax+x^2+4y(x)} + a + x\right) + \frac{1}{2} a \tanh^{-1}\left(\frac{2a+2\sqrt{a^2+2ax+x^2+4y(x)}}{2\sqrt{a^2+2ax+x^2+4y(x)} + a + x}\right) - C1\right] = 0$

✓ **Maple** : cpu = 0.182 (sec), leaf count = 28

$$\left\{ -C1 + x^2 + \frac{1}{2} - \sqrt{x^2 + 2ax + a^2 + 4y(x)} = 0 \right\}$$



2.651 ODE No. 651

---

$$y'(x) = \frac{y(x) \left(x^2 + \log(y(x))\right)}{x}$$

✓ **Mathematica** : cpu = 0.0999202 (sec), leaf count = 16

$$\left\{\left\{y(x) \rightarrow e^{2c_1x+x^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.047 (sec), leaf count = 13

$$\left\{y\left(x\right) = \mathrm{e}^{x-C1}\mathrm{e}^{x^2}\right\}$$

2.652 ODE No. 652

---

$$y'(x) = \frac{x\sqrt{4ax - y(x)^2} + 2a}{y(x)}$$

✓ **Mathematica** : cpu = 1.24277 (sec), leaf count = 101

$$\left\{\left\{y(x) \rightarrow -\frac{\sqrt{4096a^5x - 256a^4x^4 + 32a^2e^{c_1}x^2 - e^{2c_1}}}{32a^2}\right\}, \left\{y(x) \rightarrow \frac{\sqrt{4096a^5x - 256a^4x^4 + 32a^2e^{c_1}x^2 - e^{2c_1}}}{32a^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.146 (sec), leaf count = 27

$$\left\{-\sqrt{-(y\left(x\right))^2+4ax}-\frac{x^2}{2}-\_C1=0\right\}$$

2.653 ODE No. 653

---

$$y'(x) = x\sqrt{x^2 + 4y(x) - 4x} - \frac{x}{2} + 1$$

✓ **Mathematica** : cpu = 0.509653 (sec), leaf count = 94

Solve  $\left[-\frac{1}{2}\sqrt{x^2 + 4y(x) - 4x} + \log\left(-\sqrt{x^2 + 4y(x) - 4x} - x + 2\right) - \tanh^{-1}\left(\frac{2x - 4}{2\sqrt{x^2 + 4y(x) - 4x}}\right) + \frac{x^2}{2} - \frac{1}{2}\log\left(\frac{2x - 4}{2\sqrt{x^2 + 4y(x) - 4x}}\right)\right] = 0$

✓ **Maple** : cpu = 0.132 (sec), leaf count = 24

$$\left\{-C1+x^2+\frac{1}{2}-\sqrt{x^2-4x+4y\left(x\right)}=0\right\}$$

2.654 ODE No. 654

---

$$y'(x) = \frac{\sqrt{x^2 + 3y(x)} - \frac{2x^2}{3} - \frac{2x}{3}}{x + 1}$$

✓ **Mathematica** : cpu = 0.307256 (sec), leaf count = 37

$$\left\{\left\{y(x) \rightarrow \frac{1}{12}\left(-18c_1\log(x+1)+9c_1^2-4x^2+9\log^2(x+1)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.182 (sec), leaf count = 23

$$\left\{-C1+\frac{3\ln\left(1+x\right)}{2}-\sqrt{x^2+3y\left(x\right)}=0\right\}$$

2.655 ODE No. 655

---

$$y'(x) = \frac{e^{-4x/3}y(x)^3}{e^{-2x/3}y(x) + 1}$$

✓ **Mathematica** : cpu = 15.2532 (sec), leaf count = 82

Solve  $\left[\frac{3}{2}\log(y(x)) + \frac{1}{28}\left(-21\log\left(-3y(x)^2 + 2e^{2x/3}y(x) + 2e^{4x/3}\right) + 6\sqrt{7}\tanh^{-1}\left(\frac{y(x) + 2e^{2x/3}}{\sqrt{7}y(x)}\right) + 28x\right) = 0\right]$

✓ **Maple** : cpu = 0.509 (sec), leaf count = 66

$$\left\{x+\frac{3\sqrt{7}}{14}Artanh\left(\frac{3y\left(x\right)\sqrt{7}}{7}\mathrm{e}^{-\frac{2x}{3}}-\frac{\sqrt{7}}{7}\right)-\frac{3}{4}\ln\left(3\left(y\left(x\right)\right)^2\left(\mathrm{e}^{-2/3x}\right)^2-2y\left(x\right)\mathrm{e}^{-2/3x}-2\right)+\frac{3}{2}\ln\left(y\left(x\right)\mathrm{e}^{-2/3x}\right)\right\}$$

2.656 ODE No. 656

---

$$y'(x) = \frac{y(x) \left(x^3 + \log(y(x))\right)}{x}$$

✓ **Mathematica** : cpu = 0.0979594 (sec), leaf count = 20

$$\left\{\left\{y(x) \rightarrow e^{3c_1x + \frac{x^3}{2}}\right\}\right\}$$

✓ **Maple** : cpu = 0.048 (sec), leaf count = 15

$$\left\{y\left(x\right) = \mathrm{e}^{\frac{x^3}{2}}\mathrm{e}^{x-C1}\right\}$$

2.657 ODE No. 657

---

$$y'(x) = x^2\sqrt{x^2 + 8y(x) - 2x + 1} - \frac{x}{4} + \frac{1}{4}$$

✓ **Mathematica** : cpu = 0.24826 (sec), leaf count = 37

$$\left\{\left\{y(x) \rightarrow \frac{1}{72} \left(-96c_1x^3 + 144c_1^2 + 16x^6 - 9x^2 + 18x - 9\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.155 (sec), leaf count = 26

$$\left\{-C1 + \frac{4x^3}{3} - \sqrt{x^2 - 2x + 1 + 8y(x)} = 0\right\}$$

2.658 ODE No. 658

---

$$y'(x) = \frac{\sqrt{x^2 + 8y(x) - 2x + 1} - \frac{x^2}{4} + \frac{1}{4}}{x + 1}$$

✓ **Mathematica** : cpu = 0.419295 (sec), leaf count = 41

$$\left\{\left\{y(x) \rightarrow \frac{1}{8} \left(-32c_1 \log(x + 1) + 16c_1^2 - x^2 + 2x + 16 \log^2(x + 1) - 1\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.232 (sec), leaf count = 28

$$\left\{-C1 + 4 \ln(1 + x) - \frac{1}{4} - \sqrt{x^2 - 2x + 1 + 8y(x)} = 0\right\}$$

2.659 ODE No. 659

---

$$y'(x) = x\sqrt{a^2x^2 + 2abx + 4ay(x) + b^2 - 4c} - \frac{ax}{2} - \frac{b}{2}$$

✓ **Mathematica** : cpu = 0.688532 (sec), leaf count = 164

Solve  $\left[-\frac{\sqrt{a^2x^2 + 2abx + 4ay(x) + b^2 - 4c}}{2a} - \frac{b \log\left(\sqrt{a^2x^2 + 2abx + 4ay(x) + b^2 - 4c} + ax + b\right)}{2a} + \frac{b \tanh^{-1}\left(\frac{2a + 2\sqrt{a^2x^2 + 2abx + 4ay(x) + b^2 - 4c}}{2\sqrt{a^2 + 2ax + x^2 + 4y(x)}}\right)}{2a}\right]$

✓ **Maple** : cpu = 0.168 (sec), leaf count = 41

$$\left\{-C1 + ax^2 + \frac{a}{2} - \sqrt{a^2x^2 + 2abx + b^2 + 4ay(x) - 4c} = 0\right\}$$

2.660 ODE No. 660

---

$$y'(x) = x^2\sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{a}{2} - \frac{x}{2}$$

✓ **Mathematica** : cpu = 0.430836 (sec), leaf count = 109

Solve  $\left[-\frac{1}{2}\sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{1}{2}a \log\left(\sqrt{a^2 + 2ax + x^2 + 4y(x)} + a + x\right) + \frac{1}{2}a \tanh^{-1}\left(\frac{2a + 2\sqrt{a^2 + 2ax + x^2 + 4y(x)}}{2\sqrt{a^2 + 2ax + x^2 + 4y(x)}}\right)\right]$

✓ **Maple** : cpu = 0.169 (sec), leaf count = 29

$$\left\{-C1 + \frac{2x^3}{3} - \sqrt{x^2 + 2ax + a^2 + 4y(x)} = 0\right\}$$

**2.661            ODE No. 661**

---

$$y'(x) = x^2 \sqrt{a^2 x^2 + 2abx + 4ay(x) + b^2 - 4c} - \frac{ax}{2} - \frac{b}{2}$$

✓ **Mathematica** : cpu = 0.684571 (sec), leaf count = 164

Solve  $\left[ -\frac{\sqrt{a^2 x^2 + 2abx + 4ay(x) + b^2 - 4c}}{2a} - \frac{b \log \left( \sqrt{a^2 x^2 + 2abx + 4ay(x) + b^2 - 4c} + ax + b \right)}{2a} + \frac{b \tanh^{-1} \left( \frac{\sqrt{a^2 x^2 + 2abx + 4ay(x) + b^2 - 4c} + ax + b}{2} \right)}{2a} \right]$

✓ **Maple** : cpu = 0.15 (sec), leaf count = 39

$$\left\{ -C1 + \frac{2ax^3}{3} - \sqrt{a^2 x^2 + 2abx + b^2 + 4ay(x) - 4c} = 0 \right\}$$

**2.662            ODE No. 662**

---

$$y'(x) = x^2 \sqrt{x^2 - 4y(x) + 2x + 1} + \frac{x}{2} + \frac{1}{2}$$

✓ **Mathematica** : cpu = 0.266385 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{36} (24c_1 x^3 - 36c_1^2 - 4x^6 + 9x^2 + 18x + 9) \right\} \right\}$$

✓ **Maple** : cpu = 0.161 (sec), leaf count = 26

$$\left\{ -C1 - \frac{2x^3}{3} - \sqrt{x^2 + 2x + 1 - 4y(x)} = 0 \right\}$$

**2.663            ODE No. 663**

---

$$y'(x) = \frac{x^2 \sqrt{4ax - y(x)^2} + 2a}{y(x)}$$

✓ **Mathematica** : cpu = 1.5413 (sec), leaf count = 101

$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{147456a^7 x - 4096a^6 x^6 + 128a^3 e^{c_1} x^3 - e^{2c_1}}}{192a^3} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{147456a^7 x - 4096a^6 x^6 + 128a^3 e^{c_1} x^3 - e^{2c_1}}}{192a^3} \right\} \right\}$

✓ **Maple** : cpu = 0.135 (sec), leaf count = 27

$$\left\{ -\sqrt{-(y(x))^2 + 4ax} - \frac{x^3}{3} - C1 = 0 \right\}$$

**2.664            ODE No. 664**

---

$$y'(x) = x^2 \sqrt{x^2 + 4y(x) - 4x} - \frac{x}{2} + 1$$

✓ **Mathematica** : cpu = 0.554889 (sec), leaf count = 94

Solve  $\left[ \frac{x^3}{3} - \frac{1}{2} \sqrt{x^2 + 4y(x) - 4x} + \log \left( -\sqrt{x^2 + 4y(x) - 4x} - x + 2 \right) - \tanh^{-1} \left( \frac{2x - 4}{2\sqrt{x^2 + 4y(x) - 4x}} \right) - \frac{1}{2} \log \left( \frac{2x - 4}{2\sqrt{x^2 + 4y(x) - 4x}} \right) \right]$

✓ **Maple** : cpu = 0.128 (sec), leaf count = 25

$$\left\{ -C1 + \frac{2x^3}{3} - \sqrt{x^2 - 4x + 4y(x)} = 0 \right\}$$

**2.665 ODE No. 665**

---

$$y'(x) = -\frac{\sqrt{a}\left(-2\sqrt{ax^4+8y(x)}+\sqrt{ax^4}+\sqrt{ax^3}\right)}{2(x+1)}$$

✓ **Mathematica** : cpu = 0.449272 (sec), leaf count = 41

$$\left\{\left\{y(x)\rightarrow\frac{1}{8}\left(-32ac_1\log(x+1)+16ac_1^2-ax^4+16a\log^2(x+1)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.342 (sec), leaf count = 28

$$\left\{-\frac{1}{4}\sqrt{ax^4+8y(x)}\frac{1}{\sqrt{a}}+\ln(1+x)-\_C1=0\right\}$$

**2.666 ODE No. 666**

---

$$y'(x) = y(x)\left(x^3+x^2-\log(y(x))+1\right)$$

✓ **Mathematica** : cpu = 0.13705 (sec), leaf count = 29

$$\left\{\left\{y(x)\rightarrow e^{-c_1e^{-x}+x^3-2x^2+4x-3}\right\}\right\}$$

✓ **Maple** : cpu = 0.129 (sec), leaf count = 24

$$\left\{y\left(x\right)=\mathrm{e}^{\frac{-C1}{\mathrm{e}^x}+x^3-2x^2+4x-3}\right\}$$

**2.667 ODE No. 667**

---

$$y'(x) = \frac{e^{-2bx}y(x)^3}{e^{-bx}y(x)+1}$$

✓ **Mathematica** : cpu = 0.926587 (sec), leaf count = 90

$$\text{Solve}\left[\frac{\log(y(x))}{b}+\frac{1}{2}\left(-\frac{\log\left(y(x)^2-be^{bx}\left(e^{bx}+y(x)\right)\right)}{b}+\frac{2\tanh^{-1}\left(\frac{\sqrt{\frac{b}{b+4}}\left(2e^{bx}+y(x)\right)}{y(x)}\right)}{\sqrt{b}\sqrt{b+4}}+2x\right)=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.163 (sec), leaf count = 82

$$\left\{bx-b\mathit{Artanh}\left(\left(-2y\left(x\right)\mathrm{e}^{-bx}+b\right)\frac{1}{\sqrt{b^2+4b}}\right)\frac{1}{\sqrt{b^2+4b}}+\ln\left(y\left(x\right)\mathrm{e}^{-bx}\right)-\frac{\ln\left(-by\left(x\right)\mathrm{e}^{-bx}+\left(y\left(x\right)\right)^2\left(\mathrm{e}^{-bx}\right)^2\right)}{2}\right\}$$

**2.668 ODE No. 668**

---

$$y'(x) = \frac{e^{-2x}y(x)^3}{e^{-x}y(x)+1}$$

✓ **Mathematica** : cpu = 0.595035 (sec), leaf count = 78

$$\text{Solve}\left[\log(y(x))+y(x)^2\left(\frac{x}{y(x)^2}-\frac{\log\left(-y(x)^2+e^xy(x)+e^{2x}\right)}{2y(x)^2}+\frac{\tanh^{-1}\left(\frac{y(x)+2e^x}{\sqrt{5}y(x)}\right)}{\sqrt{5}y(x)^2}\right)=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.549 (sec), leaf count = 58

$$\left\{y\left(x\right)=\mathrm{e}^{\mathit{RootOf}\left(2\sqrt{5}\mathit{Artanh}\left(\frac{1}{5}\frac{\left(-2\mathrm{e}^{-Z}+\mathrm{e}^x\right)\sqrt{5}}{\mathrm{e}^x}\right)+5\ln\left(-\left(\mathrm{e}^x\right)^2-\mathrm{e}^{-Z+x}+\left(\mathrm{e}^{-Z}\right)^2\right)+10\_C1-10\_Z-10x\right)}\right\}$$

**2.669 ODE No. 669**

---

$$y'(x) = \frac{e^x \left(3e^x - 2y(x)^{3/2}\right)^2}{4\sqrt{y(x)}}$$

✓ **Mathematica** : cpu = 0.653582 (sec), leaf count = 264

$$\left\{ \left\{ y(x) \rightarrow \frac{\left(3e^{3c_1+x} + 2e^{3c_1} - 2e^{3e^x} + 3e^{x+3e^x}\right)^{2/3}}{\sqrt[3]{8e^{3c_1+3e^x} + 4e^{6c_1} + 4e^{6e^x}}} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt[3]{-1} \left(3e^{3c_1+x} + 2e^{3c_1} - 2e^{3e^x} + 3e^{x+3e^x}\right)^{2/3}}{\sqrt[3]{8e^{3c_1+3e^x} + 4e^{6c_1} + 4e^{6e^x}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.142 (sec), leaf count = 72

$$\left\{ -C1 + 1e^{-\frac{3e^x}{2} - \frac{9e^{2x}}{8}} \left(2 \left(y(x)\right)^{3/2} e^x - 2e^x - 3e^{2x}\right) \left(e^{\frac{3e^x}{2} - \frac{9e^{2x}}{8}}\right)^{-1} \left(2 \left(y(x)\right)^{3/2} e^x + 2e^x - 3e^{2x}\right)^{-1} = 0 \right\}$$

**2.670 ODE No. 670**

---

$$y'(x) = \frac{1}{2}ixy(x) \left(-2\sqrt{4\log(a) - x^2 + 4\log(y(x))} + i\right)$$

✓ **Mathematica** : cpu = 0.494221 (sec), leaf count = 62

$$\left\{ \left\{ y(x) \rightarrow \exp\left(\frac{1}{4}\left(-4\log(a) - W\left(ie^{-4c_1-x^2-1}\right)^2 - 2W\left(ie^{-4c_1-x^2-1}\right) + x^2 - 1\right)\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.25 (sec), leaf count = 70

$$\left\{ \frac{1}{2}\sqrt{-x^2 + 4\ln(a) + 4\ln(y(x))} - \frac{1}{2}\arctan\left(\sqrt{-x^2 + 4\ln(a) + 4\ln(y(x))}\right) + \frac{i}{4}\ln(x^2 - 4\ln(a) - 4\ln(y(x))) \right\}$$

**2.671 ODE No. 671**

---

$$y'(x) = \frac{(xy(x)^2 + 1)^2}{x^4y(x)}$$

✓ **Mathematica** : cpu = 0.329054 (sec), leaf count = 192

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{\sqrt{2}e^{\frac{2\sqrt{2}(c_1x+1)}{x}} - \frac{2e^{\frac{2\sqrt{2}(c_1x+1)}{x}}}{x} - \frac{2}{x} - \sqrt{2}}}}{\sqrt{2e^{\frac{2\sqrt{2}(c_1x+1)}{x}} + 2}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{\sqrt{2}e^{\frac{2\sqrt{2}(c_1x+1)}{x}} - \frac{2e^{\frac{2\sqrt{2}(c_1x+1)}{x}}}{x} - \frac{2}{x} - \sqrt{2}}}}{\sqrt{2e^{\frac{2\sqrt{2}(c_1x+1)}{x}} + 2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.148 (sec), leaf count = 237

$$\left\{ y(x) = -\frac{\sqrt{2}}{2x}\sqrt{-\left(-C1e^{\frac{-1-\sqrt{2}x}{x^2}} + e^{\frac{-1+\sqrt{2}x}{x^2}}\right)x\left(-C1\left(\sqrt{2}x+2\right)e^{\frac{-1-\sqrt{2}x}{x^2}} + \left(2-\sqrt{2}x\right)e^{\frac{-1+\sqrt{2}x}{x^2}}\right)\left(-C1e^{\frac{-1-\sqrt{2}x}{x^2}} + e^{\frac{-1+\sqrt{2}x}{x^2}}\right)} \right\}$$

**2.672 ODE No. 672**

---

$$y'(x) = \frac{x^2 \left(\sqrt{4y(x)^3 - 9x^4} + 3x\right)}{y(x)^2}$$

✓ **Mathematica** : cpu = 2.29389 (sec), leaf count = 4512

Solve  $\left[ \int_1^x \left( -\frac{24\sqrt{4y(x)^3 - 9K[1]^4}y(x)^9}{(9K[1]^4 - 4y(x)^3)(4y(x)^9 - 729)} + \frac{16K[1]^2\sqrt{4y(x)^3 - 9K[1]^4}y(x)^9}{(4K[1]^6 + 9K[1]^4 + 16K[1]^3 - 4y(x)^3 + 16)(4y(x)^9 - 729)} \right) dx \right]$

✓ **Maple** : cpu = 0.149 (sec), leaf count = 36

$$\left\{ \int_{-b}^{y(x)} -a^2 \frac{1}{\sqrt{-9x^4 + 4-a^3}} d_{-a} - \frac{x^3}{3} - -C1 = 0 \right\}$$

**2.673            ODE No. 673**

---

$$y'(x) = \frac{\frac{1}{2}x^2 \cos(2y(x)) + \frac{x^2}{2} - \frac{1}{2} \sin(2y(x))}{x}$$

✓ **Mathematica** : cpu = 0.309837 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow \tan^{-1} \left( \frac{3c_1 + 2x^3}{6x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.314 (sec), leaf count = 17

$$\left\{ y(x) = \arctan \left( \frac{x^3 + 6\_C1}{3x} \right) \right\}$$

**2.674            ODE No. 674**

---

$$y'(x) = \frac{\sqrt{x^2 + 4y(x) - 4x} - \frac{x^2}{2} + \frac{x}{2} + 1}{x + 1}$$

✓ **Mathematica** : cpu = 0.607849 (sec), leaf count = 91

Solve  $\left[ -\frac{1}{2} \sqrt{x^2 + 4y(x) - 4x} + \log \left( -\sqrt{x^2 + 4y(x) - 4x} - x + 2 \right) - \tanh^{-1} \left( \frac{2x - 4}{2\sqrt{x^2 + 4y(x) - 4x}} \right) - \frac{1}{2} \log(2 - \right.$

✓ **Maple** : cpu = 0.217 (sec), leaf count = 27

$$\left\{ \_C1 + 2 \ln(1 + x) - 1 - \sqrt{x^2 - 4x + 4y(x)} = 0 \right\}$$

**2.675            ODE No. 675**

---

$$y'(x) = \frac{ax^4 + ae^x x^3 + ax^3 - x^2 y(x)^2 - e^x x y(x)^2 - x y(x)^2 + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.327613 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow \sqrt{ax} \tanh \left( \frac{1}{6} \sqrt{a} (6c_1 + 2x^3 + 3x^2 + 6e^x x - 6e^x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 37

$$\left\{ y(x) = \tanh \left( \frac{(6x - 6)e^x + 2x^3 + 3x^2 + 6\_C1}{6} \sqrt{a} \right) x \sqrt{a} \right\}$$

**2.676            ODE No. 676**

---

$$y'(x) = \frac{x^6 \sqrt{4x^2 y(x) + 1} + \frac{x}{2} + \frac{1}{2}}{x^3(x + 1)}$$

✓ **Mathematica** : cpu = 0.435673 (sec), leaf count = 144

$\left\{ \left\{ y(x) \rightarrow \frac{-72c_1 x^6 + 96c_1 x^5 - 144c_1 x^4 + 288c_1 x^3 + 144c_1^2 x^2 - 288c_1 x^2 \log(x + 1) + 9x^{10} - 24x^9 + 52x^8 - 120x^7 + 120x^6 \log(x + 1) - 120x^5 \log^2(x + 1) + 120x^4 \log^3(x + 1) - 120x^3 \log^4(x + 1) + 120x^2 \log^5(x + 1) - 120x \log^6(x + 1) + \log^7(x + 1)}{x^6} \right\} \right\}$

✓ **Maple** : cpu = 0.305 (sec), leaf count = 43

$$\left\{ \_C1 + 2 \ln(1 + x) - \frac{1}{x} \sqrt{4x^2 y(x) + 1} - 2x + x^2 - \frac{2x^3}{3} + \frac{x^4}{2} = 0 \right\}$$

**2.677            ODE No. 677**

---

$$y'(x) = \frac{ax^4 + ax^3 + ax^3 \log(x + 1) - x^2 y(x)^2 - x y(x)^2 + y(x) - x y(x)^2 \log(x + 1)}{x}$$

✓ **Mathematica** : cpu = 0.175903 (sec), leaf count = 80

$$\left\{ \left\{ y(x) \rightarrow \sqrt{ax} \tanh \left( \frac{1}{12} (12\sqrt{a}c_1 + 4\sqrt{a}x^3 + 3\sqrt{a}x^2 + 6\sqrt{a}x^2 \log(x + 1) + 6\sqrt{a}x - 6\sqrt{a} \log(x + 1)) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 48

$$\left\{ y(x) = \tanh \left( \frac{6 \ln(1 + x) x^2 + 4x^3 + 3x^2 - 6 \ln(1 + x) + 12\_C1 + 6x + 9}{12} \sqrt{a} \right) x \sqrt{a} \right\}$$

## 2.678 ODE No. 678

---

$$y'(x) = \frac{x^2 \left( 2x\sqrt{x^3 - 6y(x)} + x + 1 \right)}{2(x + 1)}$$

✓ **Mathematica** : cpu = 0.325121 (sec), leaf count = 101

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{24} \left( 24c_1x^3 - 36c_1x^2 + 72c_1x - 72c_1 \log(x + 1) - 36c_1^2 - 4x^6 + 12x^5 - 33x^4 + 40x^3 + 24x^3 \log(x + 1) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.236 (sec), leaf count = 37

$$\left\{ -C1 - x^3 + \frac{3x^2}{2} - 3x + 3 \ln(1 + x) - \frac{1}{2} - \sqrt{x^3 - 6y(x)} = 0 \right\}$$

## 2.679 ODE No. 679

---

$$y'(x) = \frac{x^4 + x^3 + x^3 \log(x) + 7x^2y(x)^2 + 7xy(x)^2 + y(x) + 7xy(x)^2 \log(x)}{x}$$

✓ **Mathematica** : cpu = 0.148911 (sec), leaf count = 59

$$\left\{ \left\{ y(x) \rightarrow \frac{x \tan\left(\frac{1}{12} (12\sqrt{7}c_1 + 4\sqrt{7}x^3 + 3\sqrt{7}x^2 + 6\sqrt{7}x^2 \log(x))\right)}{\sqrt{7}} \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 37

$$\left\{ y(x) = \frac{x\sqrt{7}}{7} \tan\left(\frac{(6x^2 \ln(x) + 4x^3 + 3x^2 + 12 - C1)\sqrt{7}}{12}\right) \right\}$$

## 2.680 ODE No. 680

---

$$y'(x) = \frac{\sqrt{x^2 - 4y(x) + 2x + 1} + \frac{x^2}{2} + x + \frac{1}{2}}{x + 1}$$

✓ **Mathematica** : cpu = 0.736331 (sec), leaf count = 89

$$\text{Solve} \left[ \frac{1}{2} \left( \sqrt{x^2 - 4y(x) + 2x + 1} + \log\left(\sqrt{x^2 - 4y(x) + 2x + 1} + x + 1\right) - \tanh^{-1}\left(\frac{2x + 2}{2\sqrt{x^2 - 4y(x) + 2x + 1}}\right) \right) \right]$$

✓ **Maple** : cpu = 0.23 (sec), leaf count = 28

$$\left\{ -C1 - 2 \ln(1 + x) - \frac{1}{2} - \sqrt{x^2 + 2x + 1 - 4y(x)} = 0 \right\}$$

## 2.681 ODE No. 681

---

$$y'(x) = \frac{ax^2y(x)^2 + axy(x)^2 + axy(x)^2 \log\left(\frac{1}{x}\right) + bx^4 + bx^3 + bx^3 \log\left(\frac{1}{x}\right) + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.203968 (sec), leaf count = 84

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{b}x \tan\left(\frac{1}{12} \left( 12\sqrt{a}\sqrt{b}c_1 + 4\sqrt{a}\sqrt{b}x^3 + 9\sqrt{a}\sqrt{b}x^2 - 6\sqrt{a}\sqrt{b}x^2 \log(x) \right) \right)}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 45

$$\left\{ y(x) = \frac{x}{a} \tan\left(\frac{4x^3 + 6x^2 \ln(x^{-1}) + 9x^2 + 12 - C1}{12} \sqrt{ab}\right) \sqrt{ab} \right\}$$

**2.682 ODE No. 682**

---

$$y'(x) = \frac{2a}{x(-8a^2 + 2axy(x)^2 - xy(x))}$$

✓ **Mathematica** : cpu = 0.24987 (sec), leaf count = 39

$$\text{Solve} \left[ \frac{y(x)^2 e^{-4ay(x)}}{8a} - \frac{e^{-4ay(x)}}{2x} = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.303 (sec), leaf count = 28

$$\left\{ -C1 + \frac{-x(y(x))^2 + 4a}{e^{4ay(x)}x} = 0 \right\}$$

**2.683 ODE No. 683**

---

$$y'(x) = \frac{y(x)(x^4 y(x) \log(x(x+1)) - x^3 \log(x(x+1)) - 1)}{x}$$

✓ **Mathematica** : cpu = 1.10817 (sec), leaf count = 84

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{\frac{2x^3}{9} + \frac{x}{3}}}{c_1 e^{\frac{x^2}{6}} x \sqrt[3]{x+1} (x(x+1))^{\frac{x^3}{3}} + e^{\frac{x^2}{6} + \frac{1}{18}(4x^2-3x+6)} x} \right\} \right\}$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 152

$$\left\{ y(x) = \frac{1}{x} (x(1+x))^{-\frac{x^3}{3}} e^{\frac{2x^3}{9}} e^{-\frac{x^2}{6}} e^{\frac{x}{3}} \left( (1+x)^{-\frac{x^3}{3}} x^{-\frac{x^3}{3}} e^{\frac{x}{6}} (ix^2 \pi (\text{csgn}(ix(1+x)))^3 - i(\text{csgn}(ix) + \text{csgn}(i+ix))x^2 \pi (\text{csgn}(ix(1+x)))) \right) \right\}$$

**2.684 ODE No. 684**

---

$$y'(x) = \frac{x^2 \sqrt{x^2 + y(x)^2} + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.107754 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow x \sinh \left( \frac{1}{2} (2c_1 + x^2) \right) \right\} \right\}$$

✓ **Maple** : cpu = 2.668 (sec), leaf count = 30

$$\left\{ \ln \left( \sqrt{(y(x))^2 + x^2} + y(x) \right) - \frac{x^2}{2} - \ln(x) - C1 = 0 \right\}$$

**2.685 ODE No. 685**

---

$$y'(x) = \frac{x^3 \log((x-1)(x+1)) + y(x) + 7xy(x)^2 \log((x-1)(x+1))}{x}$$

✓ **Mathematica** : cpu = 0.148037 (sec), leaf count = 87

$$\left\{ \left\{ y(x) \rightarrow \frac{x \tan \left( \frac{1}{2} (2\sqrt{7}c_1 - \sqrt{7}x^2 + \sqrt{7}x^2 \log(x-1) + \sqrt{7}x^2 \log(x+1) - \sqrt{7} \log(1-x) - \sqrt{7} \log(x+1)) \right)}{\sqrt{7}} \right\} \right\}$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 48

$$\left\{ y(x) = \frac{x\sqrt{7}}{7} \tan \left( \frac{(x^2 \ln((1+x)(x-1)) - x^2 - \ln((1+x)(x-1)) + 2C1 + 1)\sqrt{7}}{2} \right) \right\}$$



## 2.686 ODE No. 686

---

$$y'(x) = \frac{e^{2x^2}xy(x)^3}{e^{x^2}y(x) + 1}$$

✓ **Mathematica** : cpu = 11.7191 (sec), leaf count = 68

$$\text{Solve} \left[ \log(y(x)) - 2y(x)^2 \left( \frac{\log \left( e^{2x^2}y(x)^2 + 2e^{x^2}y(x) + 2 \right)}{4y(x)^2} - \frac{\tan^{-1} \left( e^{x^2}y(x) + 1 \right)}{2y(x)^2} \right) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.098 (sec), leaf count = 85

$$\left\{ y(x) = \frac{1}{e^{x^2}} \left( 1 - \tan \left( \text{RootOf} \left( -2x^2 - \ln \left( \frac{81 (\tan(\_Z))^2}{10} + \frac{81}{10} \right) + 2 \ln(9/2 \tan(\_Z) - 9/2) + 6\_C1 - 2 \right) \right) \right)$$

## 2.687 ODE No. 687

---

$$y'(x) = \frac{x^3 \left( -\log \left( \frac{x+1}{x-1} \right) \right) + y(x) + xy(x)^2 \log \left( \frac{x+1}{x-1} \right)}{x}$$

✓ **Mathematica** : cpu = 0.155263 (sec), leaf count = 130

$$\left\{ \left\{ y(x) \rightarrow \frac{-x^2(x+1)^{x^2}e^{2c_1+2x} + x(x+1)^{x^2}e^{2c_1+2x} - x^2(x-1)^{x^2} - x(x-1)^{x^2}}{-(x+1)^{x^2}e^{2c_1+2x} + x(x+1)^{x^2}e^{2c_1+2x} - x(x-1)^{x^2} - (x-1)^{x^2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 39

$$\left\{ y(x) = -\tanh \left( \frac{x^2}{2} \ln \left( \frac{1+x}{x-1} \right) - \frac{1}{2} \ln \left( \frac{1+x}{x-1} \right) + \_C1 + x - 1 \right) x \right\}$$

## 2.688 ODE No. 688

---

$$y'(x) = \frac{e^{\frac{x+1}{x-1}}x^3 + e^{\frac{x+1}{x-1}}xy(x)^2 + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.350856 (sec), leaf count = 82

$$\left\{ \left\{ y(x) \rightarrow x \tan \left( \frac{1}{2} \left( 2c_1 - 8e\text{Ei} \left( \frac{2}{x-1} \right) + e^{\frac{x}{x-1} + \frac{1}{x-1}}x^2 + 2e^{\frac{x}{x-1} + \frac{1}{x-1}}x - 3e^{\frac{x}{x-1} + \frac{1}{x-1}} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.068 (sec), leaf count = 42

$$\left\{ y(x) = \tan \left( \frac{x^2 + 2x - 3}{2} e^{\frac{1+x}{x-1}} + 4e\text{Ei} \left( 1, -2(x-1)^{-1} \right) + \_C1 \right) x \right\}$$

## 2.689 ODE No. 689

---

$$y'(x) = \frac{-e^{x+1}x^3 + e^{x+1}xy(x)^2 + xy(x) - y(x)}{(x-1)x}$$

✓ **Mathematica** : cpu = 0.31832 (sec), leaf count = 60

$$\left\{ \left\{ y(x) \rightarrow -\frac{x \left( e^{2c_1+2e^2\text{Ei}(x-1)+2e^{x+1}} - 1 \right)}{e^{2c_1+2e^2\text{Ei}(x-1)+2e^{x+1}} + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 25

$$\{ y(x) = -\tanh(e^{1+x} - e^2\text{Ei}(1, 1-x) + \_C1) x \}$$

2.690 ODE No. 690

---

$$y'(x) = \frac{-\frac{x^2}{4} + x^3 \sqrt{x^2 + 8y(x)} - 2x + 1 + \frac{1}{4}}{x + 1}$$

✓ **Mathematica** : cpu = 0.51182 (sec), leaf count = 105

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{72} \left( -96c_1x^3 + 144c_1x^2 - 288c_1x + 288c_1 \log(x+1) + 144c_1^2 + 16x^6 - 48x^5 + 132x^4 - 144x^3 - 96x^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.263 (sec), leaf count = 40

$$\left\{ -C1 + \frac{4x^3}{3} - 2x^2 + 4x - 4 \ln(1+x) - \sqrt{x^2 - 2x + 1 + 8y(x)} = 0 \right\}$$

2.691 ODE No. 691

---

$$y'(x) = \frac{\frac{1}{2}x^3 \cos(2y(x)) + \frac{x^3}{2} - \frac{1}{2} \sin(2y(x))}{x}$$

✓ **Mathematica** : cpu = 0.297619 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \tan^{-1} \left( \frac{2c_1 + x^4}{4x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.474 (sec), leaf count = 17

$$\left\{ y(x) = \arctan \left( \frac{x^4 + 8 - C1}{4x} \right) \right\}$$

2.692 ODE No. 692

---

$$y'(x) = \frac{x^3 \sqrt{x^2 + y(x)^2} + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.103883 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow x \sinh \left( \frac{1}{3} (3c_1 + x^3) \right) \right\} \right\}$$

✓ **Maple** : cpu = 2.174 (sec), leaf count = 30

$$\left\{ \ln \left( \sqrt{(y(x))^2 + x^2} + y(x) \right) - \frac{x^3}{3} - \ln(x) - C1 = 0 \right\}$$

2.693 ODE No. 693

---

$$y'(x) = e^{bx} \left( e^{-3bx} y(x)^3 + e^{-2bx} y(x)^2 + 1 \right)$$

✓ **Mathematica** : cpu = 0.291336 (sec), leaf count = 146

Solve  $\left[ -\frac{1}{3}(9b+29)^{2/3} \text{RootSum} \left[ \#1^3(9b+29)^{2/3} - 9\#1b - 3\#1 + (9b+29)^{2/3} \&, \frac{\log \left( \frac{3e^{-2bx}y(x)+e^{-bx}}{\sqrt[3]{(9b+29)e^{-3bx}}} - \#1 \right)}{\#1^2 \left( -(9b+29)^{2/3} \right) + 3b + 1} \right] \right]$

✓ **Maple** : cpu = 0.074 (sec), leaf count = 40

$$\left\{ y(x) = \frac{1}{e^{-bx}} \text{RootOf} \left( -x - \int^{-Z} - (a^3 + a^2 - ab + 1)^{-1} da + C1 \right) \right\}$$

2.694 ODE No. 694

---

$$y'(x) = \frac{x^3 \sqrt{4x^2 y(x) + 1} + \frac{x}{2} + \frac{1}{2}}{x^3(x+1)}$$

✓ **Mathematica** : cpu = 0.404363 (sec), leaf count = 66

$$\left\{ \left\{ y(x) \rightarrow \frac{-8c_1x^3 + 4c_1^2x^2 + 8c_1x^2 \log(x+1) + 4x^4 - 8x^3 \log(x+1) + 4x^2 \log^2(x+1) - 1}{4x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.24 (sec), leaf count = 30

$$\left\{ -2 \ln(1+x) - \frac{1}{x} \sqrt{4x^2 y(x) + 1} + 2x + C1 = 0 \right\}$$

**2.695            ODE No. 695**

---

$$y'(x) = \frac{x^4 + x^3 + x^2 y(x)^2 + x y(x)^2 + y(x) \log(x-1)}{x \log(x-1)}$$

✓ **Mathematica** : cpu = 0.206066 (sec), leaf count = 34

$$\{\{y(x) \rightarrow x \tan(c_1 + 2\text{Ei}(\log(x-1)) + 3\text{Ei}(2 \log(x-1)) + \text{Ei}(3 \log(x-1)))\}\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 39

$$\{y(x) = \tan(-\text{Ei}(1, -3 \ln(x-1)) - 3 \text{Ei}(1, -2 \ln(x-1)) - 2 \text{Ei}(1, -\ln(x-1)) + \_C1) x\}$$

**2.696            ODE No. 696**

---

$$y'(x) = \frac{e^{x+1} x^3 + 7 e^{x+1} x y(x)^2 + y(x) \log(x-1)}{x \log(x-1)}$$

✓ **Mathematica** : cpu = 0.681949 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow \frac{x \tan \left( \sqrt{7} \int_1^x \frac{e^{K[1]+1} K[1]}{\log(K[1]-1)} dK[1] + \sqrt{7} c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 32

$$\left\{ y(x) = \frac{x \sqrt{7}}{7} \tan \left( \left( e \int \frac{x e^x}{\ln(x-1)} dx + \_C1 \right) \sqrt{7} \right) \right\}$$

**2.697            ODE No. 697**

---

$$y'(x) = e^{2x/3} \left( e^{-2x} y(x)^3 + e^{-4x/3} y(x)^2 + 1 \right)$$

✓ **Mathematica** : cpu = 0.254794 (sec), leaf count = 114

$$\text{Solve} \left[ -\frac{35}{3} \text{RootSum} \left[ -35 \#1^3 + 9 \sqrt[3]{35} \#1 - 35 \&, \frac{\log \left( \frac{3 e^{-4x/3} y(x) + e^{-2x/3}}{\sqrt[3]{35} \sqrt[3]{e^{-2x}}} - \#1 \right)}{3 \sqrt[3]{35} - 35 \#1^2} \& \right] = c_1 + \frac{1}{9} 35^{2/3} e^{4x/3} (e^{-2x})^2, \right]$$

✓ **Maple** : cpu = 0.076 (sec), leaf count = 40

$$\left\{ y(x) = \text{RootOf} \left( -x + 3 \int^{-Z} (3 \_a^3 + 3 \_a^2 - 2 \_a + 3)^{-1} d\_a + \_C1 \right) \left( e^{-\frac{2x}{3}} \right)^{-1} \right\}$$

**2.698            ODE No. 698**

---

$$y'(x) = e^x \left( e^{-3x} y(x)^3 + e^{-2x} y(x)^2 + 1 \right)$$

✓ **Mathematica** : cpu = 0.254395 (sec), leaf count = 108

$$\text{Solve} \left[ -\frac{19}{3} \text{RootSum} \left[ -19 \#1^3 + 6 \sqrt[3]{38} \#1 - 19 \&, \frac{\log \left( \frac{3 e^{-2x} y(x) + e^{-x}}{\sqrt[3]{38} \sqrt[3]{e^{-3x}}} - \#1 \right)}{2 \sqrt[3]{38} - 19 \#1^2} \& \right] = c_1 + \frac{1}{9} 38^{2/3} e^{2x} (e^{-3x})^{2/3} x, y \right]$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 34

$$\left\{ y(x) = \frac{\text{RootOf} \left( -x + \int^{-Z} (\_a^3 + \_a^2 - \_a + 1)^{-1} d\_a + \_C1 \right)}{e^{-x}} \right\}$$

## 2.699 ODE No. 699

---

$$y'(x) = \frac{x \left( 3x^2 \sqrt{x^2 + 3y(x)} - 2x - 2 \right)}{3(x+1)}$$

✓ **Mathematica** : cpu = 0.328138 (sec), leaf count = 101

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{48} \left( -24c_1x^3 + 36c_1x^2 - 72c_1x + 72c_1 \log(x+1) + 36c_1^2 + 4x^6 - 12x^5 + 33x^4 - 36x^3 - 24x^3 \log(x+1) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.235 (sec), leaf count = 36

$$\left\{ -C1 + \frac{x^3}{2} - \frac{3x^2}{4} - \frac{3 \ln(1+x)}{2} + \frac{3x}{2} - \sqrt{x^2 + 3y(x)} = 0 \right\}$$

## 2.700 ODE No. 700

---

$$y'(x) = \frac{1}{xy(x)(xy(x)^2 + x + 1)}$$

✓ **Mathematica** : cpu = 0.15071 (sec), leaf count = 76

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{2xW\left(c_1e^{\frac{1}{2x}-\frac{1}{2}}\right) + x - 1}}{\sqrt{x}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{2xW\left(c_1e^{\frac{1}{2x}-\frac{1}{2}}\right) + x - 1}}{\sqrt{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.09 (sec), leaf count = 62

$$\left\{ y(x) = \frac{1}{x} \sqrt{x \left( 2 \operatorname{lambertW}\left(1/2 - C1 e^{-1/2 \frac{x-1}{x}}\right) x + x - 1 \right)}, y(x) = -\frac{1}{x} \sqrt{x \left( 2 \operatorname{lambertW}\left(1/2 - C1 e^{-1/2 \frac{x-1}{x}}\right) x + x - 1 \right)} \right\}$$

## 2.701 ODE No. 701

---

$$y'(x) = \frac{x^4 + x^4 \log(x) - 2x^2y(x) - 2x^2y(x) \log(x) + y(x)^2 + y(x)^2 \log(x) + 2e^x x - 2x - \log(x) - 1}{e^x - 1}$$

✓ **Mathematica** : cpu = 1.78463 (sec), leaf count = 88

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp\left(\int_1^x \frac{2(\log(K[5])+1)}{-1+e^{K[5]}} dK[5]\right)}{c_1 - \int_1^x \frac{\exp\left(\int_1^{K[6]} \frac{2(\log(K[5])+1)}{-1+e^{K[5]}} dK[5]\right)(\log(K[6])+1)}{-1+e^{K[6]}} dK[6]} + x^2 + 1 \right\} \right\}$$

✓ **Maple** : cpu = 4.746 (sec), leaf count = 71

$$\left\{ y(x) = 1 \left( -x^2 \left( e^{\int \frac{\ln(x)+1}{e^x-1} dx} \right)^2 + -C1 x^2 + \left( e^{\int \frac{\ln(x)+1}{e^x-1} dx} \right)^2 + -C1 \right) \left( - \left( e^{\int \frac{\ln(x)+1}{e^x-1} dx} \right)^2 + -C1 \right)^{-1} \right\}$$

## 2.702 ODE No. 702

---

$$y'(x) = \frac{-x^3 + x^3(-\log(x)) - xy(x)^2 + xy(x) - e^x y(x) - xy(x)^2 \log(x)}{x(x - e^x)}$$

✓ **Mathematica** : cpu = 2.54025 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow x \tan \left( \int_1^x \frac{K[1](\log(K[1]) + 1)}{e^{K[1]} - K[1]} dK[1] + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.057 (sec), leaf count = 35

$$\left\{ y(x) = \tan \left( \int \frac{x \ln(x)}{e^x - x} dx + \int \frac{x}{e^x - x} dx + -C1 \right) x \right\}$$

**2.703            ODE No. 703**

---

$$y'(x) = \frac{y(x) \left( x^3 y(x) + x^2 y(x) \log(x) - x^2 - x - x \log(x) + 1 \right)}{(x-1)x}$$

✓ **Mathematica** : cpu = 0.43904 (sec), leaf count = 101

$$\left\{ \left\{ y(x) \rightarrow - \frac{e^{-\text{Li}_2(x)-x} (1-x)^{-\log(x)}}{(x-1)x \left( c_1 - \int_1^x \frac{\exp(-K[1]-\log(1-K[1])(\log(K[1])+1)-\text{Li}_2(K[1]))(K[1]^3+\log(K[1])K[1]^2)}{(K[1]-1)K[1]^2} dK[1] \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.178 (sec), leaf count = 44

$$\left\{ y(x) = \frac{e^{dilog(x)}}{xe^x(x-1)} \left( \int - \frac{e^{dilog(x)}(x+\ln(x))}{e^x(x-1)^2} dx + \_C1 \right)^{-1} \right\}$$

**2.704            ODE No. 704**

---

$$y'(x) = \frac{2ax^3y(x)^2 + 2bx^5 - y(x) + xy(x) \log(x)}{x(x \log(x) - 1)}$$

✓ **Mathematica** : cpu = 9.77776 (sec), leaf count = 66

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{b}x \tan \left( \sqrt{a}\sqrt{b} \int_1^x \frac{2K[1]^3}{K[1] \log(K[1])-1} dK[1] + \sqrt{a}\sqrt{b}c_1 \right)}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 38

$$\left\{ y(x) = \frac{x}{a} \tan \left( 2\sqrt{ab} \left( \_C1 + \int \frac{x^3}{x \ln(x)-1} dx \right) \right) \sqrt{ab} \right\}$$

**2.705            ODE No. 705**

---

$$y'(x) = \frac{y(x) \left( x^4 + x^3 + \log(y(x)) + x \right)}{x}$$

✓ **Mathematica** : cpu = 0.14364 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow x^x e^{c_1 x + \frac{x^4}{3} + \frac{x^3}{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.137 (sec), leaf count = 24

$$\left\{ y(x) = e^{\frac{x^4}{3}} e^{\frac{x^3}{2}} e^{x\_C1} x^x \right\}$$

**2.706            ODE No. 706**

---

$$y'(x) = -\frac{1}{8}x(y(x)+1)^2(-\log(y(x)-1)+\log(y(x)+1)+2\log(x))$$

✓ **Mathematica** : cpu = 40.2192 (sec), leaf count = 610

Solve  $\left[ \int_1^{y(x)} \left( \frac{-2\log(x)x^2 + \log(K[2]-1)x^2 - \log(K[2]+1)x^2 - 8}{2(2\log(x)x^2 - \log(K[2]-1)x^2 + \log(K[2]+1)x^2 + K[2](2\log(x)x^2 - \log(K[2]-1)x^2 + \log(K[2]+1)x^2))} \right) \right]$

✓ **Maple** : cpu = 0.464 (sec), leaf count = 65

$$\left\{ \int_{\_b}^{y(x)} -\frac{1}{2\_a+2} \left( -\frac{x^2(\_a+1)\ln(\_a-1)}{2} + \frac{x^2(\_a+1)\ln(\_a+1)}{2} + x^2(\_a+1)\ln(x) + 4\_a-4 \right)^{-1} d\_x \right\}$$

2.707 ODE No. 707

---

$$y'(x) = \frac{1}{16}x(y(x) + 1)^2(-\log(y(x) - 1) + \log(y(x) + 1) + 2\log(x))^2$$

✓ **Mathematica** : cpu = 4.95341 (sec), leaf count = 1391

Solve  $\left[ \int_1^x -\frac{1}{4\log^2(K[1])K[1]^2 + \log^2(y(x) - 1)K[1]^2 + \log^2(y(x) + 1)K[1]^2 - 4\log(K[1])\log(y(x) - 1)K[1]^2 + 4\log(K[1])\log(y(x) + 1)K[1]^2} dx \right]$

✓ **Maple** : cpu = 0.368 (sec), leaf count = 105

$$\left\{ \int_{-b}^{y(x)} \frac{1}{4-a+4} \left( \frac{x^2(-a+1)(\ln(-a+1))^2}{4} + \left( -\frac{\ln(-a-1)}{2} + \ln(x) \right) x^2(-a+1)\ln(-a+1) + \frac{x^2(-a+1)}{4} \right) dx \right\}$$

2.708 ODE No. 708

---

$$y'(x) = \frac{(4ax - y(x)^2)^3}{y(x)(4ax - y(x)^2 - 1)}$$

✓ **Mathematica** : cpu = 0.349485 (sec), leaf count = 89

Solve  $\left[ 2a \left( x - \frac{\text{RootSum}\left[-\#1^3 + 2\#1a - 2a\&, \frac{\#1a\log(-\#1+4ax-y(x)^2)-a\log(-\#1+4ax-y(x)^2)}{2a-3\#1^2}\&\right]}{2a} \right) = c_1, y(x) \right]$

✓ **Maple** : cpu = 13.651 (sec), leaf count = 229

$$\left\{ \int_{-b}^x -\frac{(4-a-a-(y(x))^2)^3}{-(y(x))^6 + 12-a-a(y(x))^4 + (-48-a^2a^2+2a)(y(x))^2 + 64-a^3a^3-8-a-a^2+2a} dx + \int^{y(x)} \frac{1}{-f^6} df \right\}$$

2.709 ODE No. 709

---

$$y'(x) = \frac{x^3\sqrt{4ax - y(x)^2} + 2ax + 2a}{(x + 1)y(x)}$$

✓ **Mathematica** : cpu = 2.77764 (sec), leaf count = 217

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{6}\sqrt{144ax - 24c_1x^3 + 36c_1x^2 - 72c_1x + 72c_1\log(x+1) - 36c_1^2 - 4x^6 + 12x^5 - 33x^4 + 36x^3 + 24x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.227 (sec), leaf count = 39

$$\left\{ -\sqrt{-(y(x))^2 + 4ax} - \frac{x^3}{3} + \frac{x^2}{2} - x + \ln(1+x) - C1 = 0 \right\}$$

2.710 ODE No. 710

---

$$y'(x) = \frac{2x^3 + 4x^2y(x) + 2xy(x)^2 + 2x + e^{\frac{1}{x}} - \log(x)}{\log(x) - e^{\frac{1}{x}}}$$

✓ **Mathematica** : cpu = 1.18221 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow \tan \left( \int_1^x -\frac{2K[5]}{e^{\frac{1}{K[5]}} - \log(K[5])} dK[5] + c_1 \right) - x \right\} \right\}$$

✓ **Maple** : cpu = 1.887 (sec), leaf count = 31

$$\left\{ y(x) = -x + \tan \left( 2C1 - 2 \int -\frac{x}{\ln(x) - e^{x^{-1}}} dx \right) \right\}$$

2.711 ODE No. 711

---

$$y'(x) = -\frac{y(x)(x \log(y(x)) + \log(y(x)) - 1)}{x + 1}$$

✓ **Mathematica** : cpu = 0.202847 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow e^{c_1 e^{-x} + e^{-x-1} \text{Ei}(x+1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.105 (sec), leaf count = 31

$$\left\{ y(x) = 1 e^{\frac{C_1}{e^x}} \left( e^{\frac{\text{Ei}(1, -1-x)}{e^x e}} \right)^{-1} \right\}$$

2.712 ODE No. 712

---

$$y'(x) = \frac{\frac{x^2}{2} + x^3 \sqrt{x^2 - 4y(x) + 2x + 1} + x + \frac{1}{2}}{x + 1}$$

✓ **Mathematica** : cpu = 0.481971 (sec), leaf count = 105

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{36} (24c_1 x^3 - 36c_1 x^2 + 72c_1 x - 72c_1 \log(x + 1) - 36c_1^2 - 4x^6 + 12x^5 - 33x^4 + 36x^3 + 24x^3 \log(x + 1)) \right\} \right\}$$

✓ **Maple** : cpu = 0.274 (sec), leaf count = 38

$$\left\{ -C_1 - \frac{2x^3}{3} + x^2 - 2x + 2 \ln(1 + x) - \sqrt{x^2 + 2x + 1 - 4y(x)} = 0 \right\}$$

2.713 ODE No. 713

---

$$y'(x) = \frac{-a^2 - aby(x) - ab\sqrt{x} + ab + b^2 x + b^2}{a(a(-y(x)) - a\sqrt{x} + a + bx + b)}$$

✓ **Mathematica** : cpu = 0.0862676 (sec), leaf count = 649

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{a^2 \text{Root}\left[\#1^6 (16e^{12c_1} + 16x^3) - \frac{24\#1^4 x^2}{a^4} + \frac{8\#1^3 x^{3/2}}{a^6} + \frac{9\#1^2 x}{a^8} - \frac{6\#1\sqrt{x}}{a^{10}} + \frac{1}{a^{12}} \&, 1\right]} - \frac{a\sqrt{x} - a - bx}{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.292 (sec), leaf count = 86

$$\left\{ y(x) = \frac{1}{2a} \left( 3 \tanh\left(\text{RootOf}\left(729x^3 (\tanh(\_Z))^6 a^6 - 2187x^3 (\tanh(\_Z))^4 a^6 + 2187x^3 (\tanh(\_Z))^2 a^6 - 729\right)\right) \right)$$

2.714 ODE No. 714

---

$$y'(x) = -\frac{y(x)(x^3 y(x) + x^2 y(x) \log(x) - x^2 + e^x - x \log(x) - \log(\frac{1}{x}))}{x(e^x - \log(\frac{1}{x}))}$$

✓ **Mathematica** : cpu = 1.43792 (sec), leaf count = 162

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp\left(\int_1^x -\frac{-K[1]^2 - \log(K[1])K[1] + e^{K[1]} - \log\left(\frac{1}{K[1]}\right)}{K[1]\left(e^{K[1]} - \log\left(\frac{1}{K[1]}\right)\right)} dK[1]\right)}{c_1 - \int_1^x -\frac{\exp\left(\int_1^{K[2]} -\frac{-K[1]^2 - \log(K[1])K[1] + e^{K[1]} - \log\left(\frac{1}{K[1]}\right)}{K[1]\left(e^{K[1]} - \log\left(\frac{1}{K[1]}\right)\right)} dK[1]\right) (K[2]^3 + \log(K[2])K[2]^2)}{K[2]\left(e^{K[2]} - \log\left(\frac{1}{K[2]}\right)\right)} dK[2]} \right\} \right\}$$

✓ **Maple** : cpu = 0.139 (sec), leaf count = 96

$$\left\{ y(x) = 1 e^{\int \frac{x \ln(x) + x^2 - e^x + \ln(x^{-1})}{(-\ln(x^{-1}) + e^x)x} dx} \left( \int \frac{x(x + \ln(x))}{-\ln(x^{-1}) + e^x} e^{\int \frac{x \ln(x) + x^2 - e^x + \ln(x^{-1})}{(-\ln(x^{-1}) + e^x)x} dx} dx + \_C1 \right)^{-1} \right\}$$

2.715 ODE No. 715

---

$$y'(x) = \frac{-\frac{x^2}{2} + x^3 \sqrt{x^2 + 4y(x) - 4x} + \frac{x}{2} + 1}{x + 1}$$

✓ **Mathematica** : cpu = 0.482392 (sec), leaf count = 104

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{36} \left( -24c_1x^3 + 36c_1x^2 - 72c_1x + 72c_1 \log(x+1) + 36c_1^2 + 4x^6 - 12x^5 + 33x^4 - 36x^3 - 24x^3 \log(x+1) \right) \right\} \right.$$

✓ **Maple** : cpu = 0.227 (sec), leaf count = 39

$$\left\{ -C1 + \frac{2x^3}{3} - x^2 + 2x - 2 \ln(1+x) - \sqrt{x^2 - 4x + 4y(x)} = 0 \right\}$$

2.716 ODE No. 716

---

$$y'(x) = \frac{\sqrt{9x^4 - 4y(x)^3} + 3x^4 + 3x^3}{(x+1)y(x)^2}$$

✓ **Mathematica** : cpu = 2.72477 (sec), leaf count = 133

$$\left\{ \left\{ y(x) \rightarrow \left( -\frac{3}{2} \right)^{2/3} \sqrt[3]{8c_1 \log(x+1) - 4c_1^2 + x^4 - 4 \log^2(x+1)} \right\}, \left\{ y(x) \rightarrow \left( \frac{3}{2} \right)^{2/3} \sqrt[3]{8c_1 \log(x+1) - 4c_1^2 + x^4 - 4 \log^2(x+1)} \right\} \right.$$

✓ **Maple** : cpu = 0.25 (sec), leaf count = 37

$$\left\{ \int_{-b}^{y(x)} -a^2 \frac{1}{\sqrt{9x^4 - 4a^3}} d_a - \ln(1+x) - C1 = 0 \right\}$$

2.717 ODE No. 717

---

$$y'(x) = \frac{\sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{ax}{2} - \frac{a}{2} - \frac{x^2}{2} - \frac{x}{2}}{x + 1}$$

✓ **Mathematica** : cpu = 0.592844 (sec), leaf count = 106

$$\text{Solve} \left[ -\frac{1}{2} \sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{1}{2} a \log \left( \sqrt{a^2 + 2ax + x^2 + 4y(x)} + a + x \right) + \frac{1}{2} a \tanh^{-1} \left( \frac{2a + 2x}{2\sqrt{a^2 + 2ax + x^2 + 4y(x)}} \right) \right]$$

✓ **Maple** : cpu = 0.281 (sec), leaf count = 33

$$\left\{ -C1 + \frac{a}{2} + 2 \ln(1+x) - \sqrt{x^2 + 2ax + a^2 + 4y(x)} = 0 \right\}$$

2.718 ODE No. 718

---

$$y'(x) = e^{-x^2} x \left( e^{3x^2} y(x)^3 + e^{2x^2} y(x)^2 + 1 \right)$$

✓ **Mathematica** : cpu = 0.295692 (sec), leaf count = 127

$$\text{Solve} \left[ \frac{11}{3} \text{RootSum} \left[ 11\#1^3 + 15\sqrt[3]{11}\#1 + 11\&, \frac{\log \left( \frac{3e^{2x^2}xy(x)+e^{x^2}x}{\sqrt[3]{11}\sqrt[3]{e^{3x^2}x^3}} - \#1 \right)}{11\#1^2 + 5\sqrt[3]{11}} \& \right] = c_1 + \frac{11^{2/3}e^{x^2}x^3}{18\sqrt[3]{e^{3x^2}x^3}}, y(x) \right]$$

✓ **Maple** : cpu = 0.045 (sec), leaf count = 44

$$\left\{ y(x) = \frac{-11 \text{RootOf} \left( -5x^2 + 20250 \int^{-Z} (121a^3 + 3375a - 3375)^{-1} d_a + 6C1 \right) - 15}{45e^{x^2}} \right\}$$



2.719 ODE No. 719

---

$$y'(x) = \frac{e^{-x}y(x) \left(x^2y(x) \log(2x) - e^x - x \log(2x)\right)}{x}$$

✓ **Mathematica** : cpu = 0.37027 (sec), leaf count = 49

$$\left\{\left\{y(x) \rightarrow \frac{2e^{-x}x^{e^{-x}-1}}{c_1e^{\text{Ei}(-x)} + 2e^{-x}x^{e^{-x}}}\right\}\right\}$$

✓ **Maple** : cpu = 0.094 (sec), leaf count = 34

$$\left\{y(x) = \left(2^{-e^{-x}}\_C1\,x^{-e^{-x}+1}e^{-Ei(1,x)} + x\right)^{-1}\right\}$$

2.720 ODE No. 720

---

$$y'(x) = \frac{x^3 \left(\sqrt{9x^4 - 4y(x)^3} + 3x + 3\right)}{(x+1)y(x)^2}$$

✓ **Mathematica** : cpu = 2.92933 (sec), leaf count = 314

$$\left\{\left\{y(x) \rightarrow \sqrt[3]{6c_1x^3 - 9c_1x^2 + 18c_1x - 18c_1\log(x+1) - 9c_1^2 - x^6 + 3x^5 - 6x^4 + 9x^3 + 6x^3\log(x+1) - 9x^2 - 9}\right\}\right\}$$

✓ **Maple** : cpu = 0.166 (sec), leaf count = 48

$$\left\{\int_{\_b}^{y(x)}\_a^2\frac{1}{\sqrt{9x^4-4\_a^3}}\text{d\_a} - \frac{x^3}{3} + \frac{x^2}{2} - x + \ln(1+x) -\_C1 = 0\right\}$$

2.721 ODE No. 721

---

$$y'(x) = \frac{1}{36}\sqrt{x}\left(18x^{3/2} + x^6 - 12x^3y(x) + 36y(x)^2\right)$$

✓ **Mathematica** : cpu = 0.104903 (sec), leaf count = 27

$$\left\{\left\{y(x) \rightarrow \frac{1}{c_1 - \frac{2x^{3/2}}{3}} + \frac{x^3}{6}\right\}\right\}$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 19

$$\left\{y(x) = \frac{x^3}{6} + \left(-C1 - \frac{2}{3}x^{\frac{3}{2}}\right)^{-1}\right\}$$

2.722 ODE No. 722

---

$$y'(x) = -\frac{y(x)^3}{x(-y(x) + 2y(x)\log(x) - 1)}$$

✓ **Mathematica** : cpu = 41.867 (sec), leaf count = 490

$$\text{Solve}\left[-\frac{\sqrt[3]{-2}\left((-2)^{2/3}-\frac{(1-2\log(x))^2\left(-\frac{1}{(2\log(x)-1)^3}\right)^{2/3}(y(x)(5-4\log(x))+2)}{2\sqrt[3]{2}(y(x)(2\log(x)-1)-1)}\right)}{\left(\frac{\sqrt[3]{2}\sqrt[3]{-\frac{1}{(2\log(x)-1)^3}}(2\log(x)-1)(y(x)(2\log(x)-1)-1)}{y(x)(4\log(x)-5)-2}\right)}\right]$$

✓ **Maple** : cpu = 0.246 (sec), leaf count = 70

$$\left\{y(x) = 1e^{\text{RootOf}\left(-e^{-Z}\ln\left(\frac{e^{-Z}+2}{2x^4}\right)+3\_C1\,e^{-Z}+\_Z\,e^{-Z}+2\right)}\left(1+(2\ln(x)-1)e^{\text{RootOf}\left(-e^{-Z}\ln\left(\frac{e^{-Z}+2}{2x^4}\right)+3\_C1\,e^{-Z}+\_Z\,e^{-Z}+2\right)}\right)\right\}$$

2.723 ODE No. 723

---

$$y'(x) = \frac{2a}{32a^3x^2 - 16a^2xy(x)^2 + 2ay(x)^4 + y(x)}$$

✓ **Mathematica** : cpu = 0.215288 (sec), leaf count = 663

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt[3]{-1024a^6c_1^3 + 9216a^5c_1x - 432a^2 + \sqrt{4(-64a^4c_1^2 - 192a^3x)^3 + (-1024a^6c_1^3 + 9216a^5c_1x - 432a^2)}}}{12\sqrt[3]{2a}} \right. \right.$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 856

$$\left\{ y(x) = -\frac{1}{12a} \left( -8\_C1 a^2 \sqrt[3]{\left( 64\_C1^3 a^4 - 576\_C1 a^3 x + 3 \sqrt{-12288\_C1^4 a^7 x + 24576\_C1^2 a^6 x^2 - 12288\_C1^3 a^5 x^3} \right)} \right) \right.$$

2.724 ODE No. 724

---

$$y'(x) = -\frac{y(x)^3}{x(-y(x) + y(x) \log(x) - 1)}$$

✓ **Mathematica** : cpu = 43.1731 (sec), leaf count = 422

$$\text{Solve} \left[ -\frac{\sqrt[3]{-2} \left( \frac{1-y(x)(\log(x)-4)}{\sqrt[3]{2} \sqrt[3]{-\frac{1}{(\log(x)-1)^3} (\log(x)-1)(y(x)(\log(x)-1)-1)}} + (-2)^{2/3} \right) \left( \frac{2^{2/3}(y(x)(\log(x)-4)-1)}{\sqrt[3]{-\frac{1}{(\log(x)-1)^3} (\log(x)-1)(y(x)(\log(x)-1)-1)}} + (-2)^{2/3} \right)}{x(-y(x) + y(x) \log(x) - 1)} \right]$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 18

$$\left\{ y(x) = (-\text{lambertW}(\_C1 e^{-2x}) + \ln(x) - 2)^{-1} \right\}$$

2.725 ODE No. 725

---

$$y'(x) = \frac{x^2 \log(2x) + 2xy(x) \log(2x) + y(x)^2 \log(2x) - \log(x) + \log(2x)}{\log(x)}$$

✓ **Mathematica** : cpu = 0.313836 (sec), leaf count = 19

$$\{ \{ y(x) \rightarrow \tan(c_1 + \log(2)\text{li}(x) + x) - x \} \}$$

✓ **Maple** : cpu = 0.707 (sec), leaf count = 25

$$\{ y(x) = -x - \tan(\ln(2) Ei(1, -\ln(x)) + \_C1 - x) \}$$

2.726 ODE No. 726

---

$$y'(x) = \frac{a^2 - aby(x) - ab\sqrt{x} - b^2x + bc}{a(ay(x) + a\sqrt{x} + bx - c)}$$

✓ **Mathematica** : cpu = 0.0572307 (sec), leaf count = 625

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{a^2 \text{Root} \left[ \#1^6 (16e^{12c_1} + 16x^3) - \frac{24\#1^4 x^2}{a^4} + \frac{8\#1^3 x^{3/2}}{a^6} + \frac{9\#1^2 x}{a^8} - \frac{6\#1\sqrt{x}}{a^{10}} + \frac{1}{a^{12}} \&, 1 \right]} - \frac{a\sqrt{x} + bx - c}{a} \right. \right.$$

✓ **Maple** : cpu = 0.213 (sec), leaf count = 83

$$\left\{ y(x) = \frac{1}{2a} \left( 3 \tanh \left( \text{RootOf} \left( -729 x^3 (\tanh(\_Z))^6 a^6 + 2187 x^3 (\tanh(\_Z))^4 a^6 - 2187 x^3 (\tanh(\_Z))^2 a^6 + \right) \right) \right.$$

2.727 ODE No. 727

---

$$y'(x) = \frac{y(x)(y(x) + 2x + 2)}{(x + 1)(\log(y(x)) + 2x - 1)}$$

✓ **Mathematica** : cpu = 0.415327 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow \frac{W\left(e^{-2x}(c_1 + \log(x + 1))\right)}{c_1 + \log(x + 1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.193 (sec), leaf count = 25

$$\left\{ y(x) = e^{-\text{lambertW}((\ln(1+x) - C1)e^{-2x}) - 2x} \right\}$$

2.728 ODE No. 728

---

$$y'(x) = \frac{y(x)(x^3 + 3y(x)^2)}{x(6y(x)^2 + x)}$$

✓ **Mathematica** : cpu = 0.350217 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{x}\sqrt{W\left(\frac{6e^{2c_1+x^2}}{x}\right)}}{\sqrt{6}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{x}\sqrt{W\left(\frac{6e^{2c_1+x^2}}{x}\right)}}{\sqrt{6}} \right\} \right\}$$

✓ **Maple** : cpu = 0.25 (sec), leaf count = 50

$$\left\{ \left( (y(x))^{-2} + 6x^{-1} \right)^{-1} = \frac{x}{54} \left( e^{\text{RootOf}\left(x^2e^{-Z} - e^{-Z}\ln\left(\frac{(e^{-Z}+9)x}{2}\right) + 3C1e^{-Z} + Ze^{-Z} + 9\right)} + 9 \right) \right\}$$

2.729 ODE No. 729

---

$$y'(x) = \frac{(x - y(x))y(x)}{x(x - y(x)^3)}$$

✓ **Mathematica** : cpu = 0.28227 (sec), leaf count = 327

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{2}(6c_1 - 6\log(x))}{3\sqrt[3]{\sqrt{4(6c_1 - 6\log(x))^3 + 2916x^2 + 54x}}} - \frac{\sqrt[3]{\sqrt{4(6c_1 - 6\log(x))^3 + 2916x^2 + 54x}}}{3\sqrt[3]{2}} \right\}, \left\{ y(x) \rightarrow \frac{(1 - \sqrt[3]{\sqrt{4(6c_1 - 6\log(x))^3 + 2916x^2 + 54x}})}{3\sqrt[3]{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.086 (sec), leaf count = 404

$$\left\{ y(x) = \frac{1}{6} \left( \left( i \left( -27x + 3\sqrt{24C1^3 - 72C1^2\ln(x) + 72C1(\ln(x))^2 - 24(\ln(x))^3 + 81x^2} \right) \right)^{\frac{2}{3}} + 6iC1 \right) \right\}$$

2.730 ODE No. 730

---

$$y'(x) = \frac{e^x(2y(x)^{3/2} - 3e^x)^3}{4\sqrt{y(x)}(2y(x)^{3/2} - 3e^x + 2)}$$

✗ **Mathematica** : cpu = 47.2981 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x] == (E^x*(-3*E^x + 2*y[x]^(3/2))^3)/(4*Sqrt[y[x]]*(2 - 3*E^x + 2*y[x]^(3/2))), y[x], x]`

✓ **Maple** : cpu = 1.31 (sec), leaf count = 41

$$\left\{ e^x - \int^{(y(x))^{\frac{3}{2}} - \frac{3e^x}{2}} \frac{2 + 2a}{3a^3 - 3a - 3} da - C1 = 0 \right\}$$

**2.731      ODE No. 731**

---

$$y'(x) = \frac{2y(x) + 1}{x(2xy(x)^3 + xy(x)^2 - 2)}$$

✓ **Mathematica** : cpu = 0.339934 (sec), leaf count = 47

$$\text{Solve} \left[ \frac{1}{64} (-4y(x)^2 + 4y(x) - 2 \log(8y(x) + 4) + 3) - \frac{1}{4x(2y(x) + 1)} = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.148 (sec), leaf count = 42

$$\left\{ y(x) = \frac{e^{\text{RootOf}\left(x(e^{-Z})^3 - 4x(e^{-Z})^2 + 8x\_C1 e^{-Z} + 2\_Z e^{-Z}x + 3e^{-Z}x + 16\right)}}{2} - \frac{1}{2} \right\}$$

**2.732      ODE No. 732**

---

$$y'(x) = \frac{x^3 \sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{ax}{2} - \frac{a}{2} - \frac{x^2}{2} - \frac{x}{2}}{x + 1}$$

✓ **Mathematica** : cpu = 0.712008 (sec), leaf count = 116

$$\text{Solve} \left[ -\frac{1}{2} \sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{1}{2} a \log \left( \sqrt{a^2 + 2ax + x^2 + 4y(x)} + a + x \right) + \frac{1}{2} a \tanh^{-1} \left( \frac{a + x}{\sqrt{a^2 + 2ax + x^2}} \right) \right]$$

✓ **Maple** : cpu = 0.285 (sec), leaf count = 43

$$\left\{ -C1 + \frac{2x^3}{3} - x^2 + 2x - 2 \ln(1 + x) - \sqrt{x^2 + 2ax + a^2 + 4y(x)} = 0 \right\}$$

**2.733      ODE No. 733**

---

$$y'(x) = \csc(x) (x^4 \log(2x) - 2x^2 y(x) \log(2x) + y(x)^2 \log(2x) - \log(2x) + 2x \sin(x))$$

✓ **Mathematica** : cpu = 12.5361 (sec), leaf count = 73

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp \left( \int_1^x 2 \csc(K[5]) \log(2K[5]) dK[5] \right)}{c_1 - \int_1^x \exp \left( \int_1^{K[6]} 2 \csc(K[5]) \log(2K[5]) dK[5] \right) \csc(K[6]) \log(2K[6]) dK[6]} + x^2 + 1 \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x) = (2\*x\*sin(x)-ln(2\*x)+ln(2\*x)\*x^4-2\*ln(2\*x)\*x^2\*y(x)+ln(2\*x)\*y(x)^2)/sin(x),y(x))

**2.734      ODE No. 734**

---

$$y'(x) = \frac{y(x) (x^3 - x \log(y(x)) - \log(y(x)))}{x + 1}$$

✓ **Mathematica** : cpu = 0.232825 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow \exp \left( -c_1 e^{-x} - e^{-x-1} \text{Ei}(x + 1) + x^2 - 3x + 4 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.228 (sec), leaf count = 39

$$\left\{ y(x) = \frac{e^{x^2} e^4}{(e^x)^3} e^{\frac{-C1}{e^x}} e^{\frac{\text{Ei}(1, -1-x)}{e e^x}} \right\}$$

**2.735      ODE No. 735**

---

$$y'(x) = \frac{(2y(x) \log(x) - 1)^3}{x(-y(x) + 2y(x) \log(x) - 1)}$$

✓ **Mathematica** : cpu = 11.5491 (sec), leaf count = 573

$$\text{Solve} \left[ \int_1^{y(x)} \left( \frac{2(2 \log(x) K[1] - K[1] - 1)}{8 \log^3(x) K[1]^3 + 4 \log(x) K[1]^3 - 2 K[1]^3 - 12 \log^2(x) K[1]^2 - 2 K[1]^2 + 6 \log(x) K[1] - 1} + 2 \text{RootOf} \right) \right]$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 78

$$\left\{y(x)=\frac{71\,RootOf\left(-82944\int^{-Z}(5041\_a^3-27648\_a+27648)^{-1}d\_a-16\ln(x)+3\_C1\right)-}{(142\ln(x)-71)\,RootOf\left(-82944\int^{-Z}(5041\_a^3-27648\_a+27648)^{-1}d\_a-16\ln(x)+3\_C1\right)}-\right.$$

2.736 ODE No. 736

---

$$y'(x)=\frac{x^4-2x^2y(x)+2x^2+y(x)^2+2x-1}{x+1}$$

✓ **Mathematica** : cpu = 0.209685 (sec), leaf count = 31

$$\left\{\left\{y(x)\rightarrow\frac{(x+1)^2}{c_1-\frac{x^2}{2}-x}+x^2+1\right\}\right\}$$

✓ **Maple** : cpu = 0.165 (sec), leaf count = 43

$$\left\{y(x)=\frac{-C1\left(x^4+2x^3-x^2-2x-2\right)+x^2+1}{1+(x^2+2x)\_C1}\right\}$$

2.737 ODE No. 737

---

$$y'(x)=\frac{x\left(2x^3-2xy(x)+x-1\right)}{x^2-y(x)}$$

✓ **Mathematica** : cpu = 0.024763 (sec), leaf count = 36

$$\left\{\left\{y(x)\rightarrow\frac{1}{2}\left(W\left(-e^{c_1+\frac{4x^3}{3}-2x^2-1}\right)+1\right)+x^2\right\}\right\}$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 29

$$\left\{y(x)=x^2+\frac{1}{2}lambertW\left(-2\frac{{\rm e}^{4/3\,x^3}\_C1\,{\rm e}^{-1}}{\left({\rm e}^{x^2}\right)^2}\right)+\frac{1}{2}\right\}$$

2.738 ODE No. 738

---

$$y'(x)=\frac{2a}{32a^3-16a^2xy(x)^2+2ax^2y(x)^4-x^2y(x)}$$

✓ **Mathematica** : cpu = 0.484158 (sec), leaf count = 1347

$$\left\{\left\{y(x)\rightarrow-\frac{4a+e^{c_1}}{12a}+\frac{\sqrt[3]{4608x^2a^4-128x^3a^3+1152e^{c_1}x^2a^3-96e^{c_1}x^3a^2-432x^3a^2-24e^{2c_1}x^3a-2e^{3c_1}x^3+}}{12a}\right\}\right\}$$

✓ **Maple** : cpu = 0.861 (sec), leaf count = 1054

$$\left\{y(x)=-\frac{1}{24\_C1\,ax}\left(2x\sqrt[3]{-216\_C1^3a^2x^3+576\_C1^2a^3x^2+12a\_C1\,x^2\sqrt{(324\_C1^4a^2+3\_C1)\,x^3+(-}}\right.\right.$$

2.739 ODE No. 739

---

$$y'(x)=\frac{2y(x)+1}{x\left(2xy(x)^2+xy(x)-2\right)}$$

✓ **Mathematica** : cpu = 0.287598 (sec), leaf count = 39

$$\text{Solve}\left[\frac{1}{8}(-2y(x)+\log(4y(x)+2)-1)-\frac{1}{2x(2y(x)+1)}=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.152 (sec), leaf count = 35

$$\left\{y(x)=\frac{{\rm e}^{RootOf\left(x\left({\rm e}^{-Z}\right)^2+2\_C1\,xe^{-Z}-\_Z\,xe^{-Z}-xe^{-Z}+4\right)}}{2}-\frac{1}{2}\right\}$$

2.740 ODE No. 740

---

$$y'(x) = \frac{x^4 - 2x^2y(x)^2 + y(x)^4 + x}{y(x)}$$

✓ **Mathematica** : cpu = 0.114547 (sec), leaf count = 74

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{2c_1x^2 + 2x^3 - 1}}{\sqrt{2}\sqrt{c_1 + x}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{2c_1x^2 + 2x^3 - 1}}{\sqrt{2}\sqrt{c_1 + x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 72

$$\left\{ y(x) = \frac{\sqrt{2}}{2\_C1 + 2x} \sqrt{(\_C1 + x)(2\_C1x^2 + 2x^3 - 1)}, y(x) = -\frac{\sqrt{2}}{2\_C1 + 2x} \sqrt{(\_C1 + x)(2\_C1x^2 + 2x^3 - 1)} \right\}$$

2.741 ODE No. 741

---

$$y'(x) = \frac{x \left( ay(x)^2 + bx^2 \right)^3}{a^{5/2}y(x) \left( ay(x)^2 + a + bx^2 \right)}$$

✓ **Mathematica** : cpu = 2.17816 (sec), leaf count = 175

$$\text{Solve} \left[ \frac{1}{2} \left( x^2 - a^{3/2} \text{RootSum} \left[ \#1^3b^3 + 3\#1^2ab^2y(x)^2 + \#1a^{3/2}b^2 + 3\#1a^2by(x)^4 + a^{5/2}by(x)^2 + a^{5/2}b + a^3y(x)^2 \right] \right) \right]$$

✓ **Maple** : cpu = 0.9 (sec), leaf count = 246

$$\left\{ \int_{\_b}^x \frac{\left( \_a^2b + a \left( y(x)^2 \right)^2 \right)^3 \_a}{a^3} \left( b \left( \left( y(x)^2 \right)^2 + 1 \right) a^{\frac{5}{2}} + a^{\frac{3}{2}}b^2 \_a^2 + \left( \_a^2b + a \left( y(x)^2 \right)^2 \right)^3 \right)^{-1} d\_a + \int^{y(x)} 1 \left( \left( \_b^2 + a \left( y(x)^2 \right)^2 \right)^3 \_b \right)^{-1} dy(x) \right\}$$

2.742 ODE No. 742

---

$$y'(x) = -\frac{(-\cos(y(x)) + x + 1) \cos(y(x))}{(x + 1)(x \sin(y(x)) - 1)}$$

✓ **Mathematica** : cpu = 3.05891 (sec), leaf count = 3913

$$\left\{ \left\{ y(x) \rightarrow -\sec^{-1} \left( \frac{c_1x^3}{x^2 - 1} + \frac{\log(x + 1)x^3}{x^2 - 1} - \frac{c_1^3x^3}{(x^2 - 1)(c_1^2 + 2\log(x + 1)c_1 + \log^2(x + 1) + 1)} - \frac{c_1^3x^3}{(x^2 - 1)(c_1^2 + 2\log(x + 1)c_1 + \log^2(x + 1) + 1)} \right) \right\} \right\}$$

✓ **Maple** : cpu = 1.325 (sec), leaf count = 239

$$\left\{ y(x) = \arctan \left( \frac{1}{\_C1^2 - 2\_C1 \ln(1 + x) + (\ln(1 + x))^2 + 1} \left( (-\_C1 + \ln(1 + x)) \sqrt{(\ln(1 + x))^2 - 2\_C1 \ln(1 + x) + 1} \right) \right) \right\}$$

2.743 ODE No. 743

---

$$y'(x) = -\frac{i \left( x^4 + 8x^2y(x)^2 + 16y(x)^4 + 8ix \right)}{32y(x)}$$

✗ **Mathematica** : cpu = 45.9889 (sec), leaf count = 0 , could not solve

DSolve[Derivative[1][y][x] == ((-I/32)\*((8\*I)\*x + x^4 + 8\*x^2\*y[x]^2 + 16\*y[x]^4))/y[x], y[x], x]

✓ **Maple** : cpu = 0.322 (sec), leaf count = 296

$$\left\{ y(x) = \sqrt{2} \sqrt{\left( \text{Ai} \left( \frac{(-\sqrt{3} + i)x}{2} \right) \_C1 + \text{Bi} \left( \frac{(-\sqrt{3} + i)x}{2} \right) \right) \left( (i\sqrt{3} + 1) \_C1 \text{Ai}^{(1)} \left( \frac{(-\sqrt{3} + i)x}{2} \right) + (i\sqrt{3} - 1) \_C1 \text{Bi}^{(1)} \left( \frac{(-\sqrt{3} + i)x}{2} \right) \right)} \right\}$$

2.744 ODE No. 744

---

$$y'(x) = \frac{x}{x^4 + 2x^2y(x)^2 + y(x)^4 - y(x)}$$

✓ **Mathematica** : cpu = 0.162603 (sec), leaf count = 510

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{144c_1x^2 + \sqrt{4(12x^2 - 4c_1^2)^3 + (144c_1x^2 + 16c_1^3 - 108)^2 + 16c_1^3 - 108}}}{6\sqrt[3]{2}} - \frac{3^{2/3}\sqrt[3]{144c_1x^2 + \sqrt{4(12x^2 - 4c_1^2)^3 + (144c_1x^2 + 16c_1^3 - 108)^2 + 16c_1^3 - 108}}}{6\sqrt[3]{2}} \right\} \right.$$

✓ **Maple** : cpu = 0.144 (sec), leaf count = 621

$$\left\{ y(x) = \frac{1}{12} \left( -2\_C1 \sqrt[3]{-36\_C1 x^2 - 54 -\_C1^3} + 6 \sqrt{48 x^6 + 24 x^4\_C1^2 + (3\_C1^4 + 108\_C1) x^2 + 3\_C1^3} \right) \right.$$

2.745 ODE No. 745

---

$$y'(x) = \frac{(y(x) \log(x) - 1)^3}{x(-y(x) + y(x) \log(x) - 1)}$$

✓ **Mathematica** : cpu = 11.4691 (sec), leaf count = 546

$$\text{Solve} \left[ \int_1^{y(x)} \left( \frac{\log(x)K[1] - K[1] - 1}{\log^3(x)K[1]^3 + \log(x)K[1]^3 - K[1]^3 - 3\log^2(x)K[1]^2 - K[1]^2 + 3\log(x)K[1] - 1} + \text{RootSum} \left[ K[1]^3 - 3\log^2(x)K[1]^2 - K[1]^2 + 3\log(x)K[1] - 1, \# \right] \right) dx, y(x) \right]$$

✓ **Maple** : cpu = 0.047 (sec), leaf count = 78

$$\left\{ y(x) = \frac{47 \text{RootOf} \left( -27783 \int^{-Z} (2209\_a^3 - 9261\_a + 9261)^{-1} d\_a - 7 \ln(x) + 3\_C1 \right) - 84}{(47 \ln(x) - 47) \text{RootOf} \left( -27783 \int^{-Z} (2209\_a^3 - 9261\_a + 9261)^{-1} d\_a - 7 \ln(x) + 3\_C1 \right) - 84} \right.$$

2.746 ODE No. 746

---

$$y'(x) = -\frac{i(x^4 + 2x^2y(x)^2 + y(x)^4 + ix)}{y(x)}$$

✗ **Mathematica** : cpu = 44.9473 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[1][y][x] == ((-1)\*(I\*x + x^4 + 2\*x^2\*y[x]^2 + y[x]^4))/y[x], y[x], x]

✓ **Maple** : cpu = 0.267 (sec), leaf count = 232

$$\left\{ y(x) = \frac{\sqrt{2}}{2 \text{Ai}(-\sqrt[3]{-8ix})\_C1 + 2 \text{Bi}(-\sqrt[3]{-8ix})} \sqrt{\left( \text{Ai}(-\sqrt[3]{-8ix})\_C1 + \text{Bi}(-\sqrt[3]{-8ix}) \right) \left( (i\sqrt{3} + 1)\_C1 \text{Ai}(-\sqrt[3]{-8ix}) + \text{Bi}(-\sqrt[3]{-8ix}) \right)}}$$

2.747 ODE No. 747

---

$$y'(x) = -\frac{y(x) \cot(x) (x^2y(x)(-\log(2x)) + x \log(2x) + \tan(x))}{x}$$

✓ **Mathematica** : cpu = 2.91138 (sec), leaf count = 88

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp \left( \int_1^x \frac{-\cot(K[1])K[1] \log(2K[1]) - 1}{K[1]} dK[1] \right)}{c_1 - \int_1^x \exp \left( \int_1^{K[2]} \frac{-\cot(K[1])K[1] \log(2K[1]) - 1}{K[1]} dK[1] \right) \cot(K[2])K[2] \log(2K[2]) dK[2]} \right\} \right.$$

✓ **Maple** : cpu = 0.283 (sec), leaf count = 75

$$\left\{ y(x) = 1e^{\int \frac{-x \ln(x) - x \ln(2) - \tan(x)}{x \tan(x)} dx} \left( \int -\frac{x (\ln(2) + \ln(x))}{\tan(x)} e^{\int \frac{-x \ln(x) - x \ln(2) - \tan(x)}{x \tan(x)} dx} dx +\_C1 \right)^{-1} \right\}$$

2.748 ODE No. 748

---

$$y'(x) = \frac{y(x)(y(x) + x)}{x(y(x)^3 + x)}$$

✓ **Mathematica** : cpu = 0.292097 (sec), leaf count = 285

$$\left\{ \left\{ y(x) \rightarrow \frac{2\sqrt[3]{2}(c_1 + \log(x))}{\sqrt[3]{\sqrt{2916x^2 - 864(c_1 + \log(x))^3 + 54x}}} + \frac{\sqrt[3]{\sqrt{2916x^2 - 864(c_1 + \log(x))^3 + 54x}}}{3\sqrt[3]{2}} \right\}, \left\{ y(x) \rightarrow -\frac{1}{\sqrt[3]{\sqrt{2916x^2 - 864(c_1 + \log(x))^3 + 54x}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 404

$$\left\{ y(x) = \frac{1}{6} \left( \left( i \left( 27x + 3\sqrt{-24\_C1^3 - 72\_C1^2 \ln(x) - 72\_C1 (\ln(x))^2 - 24 (\ln(x))^3 + 81x^2} \right) \right)^{\frac{2}{3}} - 6i\_C1 \right) \right\}$$

2.749 ODE No. 749

---

$$y'(x) = \frac{x(x - y(x))^2(y(x) + x)^2}{y(x)}$$

✓ **Mathematica** : cpu = 0.153221 (sec), leaf count = 126

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{x^2 e^{4c_1+2x^2} - e^{4c_1+2x^2} + x^2 + 1}}{\sqrt{e^{4c_1+2x^2} + 1}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{x^2 e^{4c_1+2x^2} - e^{4c_1+2x^2} + x^2 + 1}}{\sqrt{e^{4c_1+2x^2} + 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 192

$$\left\{ y(x) = 1\sqrt{\left( (x^2 + 1)e^{-\frac{x^2(x^2+2)}{2}} + \_C1 (x^2 - 1)e^{-\frac{x^2(x^2-2)}{2}} \right) \left( \_C1 e^{-\frac{x^2(x^2-2)}{2}} + e^{-\frac{x^2(x^2+2)}{2}} \right) \left( \_C1 e^{-\frac{x^2(x^2-2)}{2}} - e^{-\frac{x^2(x^2+2)}{2}} \right) \right\}$$

2.750 ODE No. 750

---

$$y'(x) = \frac{y(x)(x^2 + 3y(x)^2)}{x(6y(x)^2 + x)}$$

✓ **Mathematica** : cpu = 0.32049 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{x}\sqrt{W\left(\frac{6e^{2c_1+2x}}{x}\right)}}{\sqrt{6}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{x}\sqrt{W\left(\frac{6e^{2c_1+2x}}{x}\right)}}{\sqrt{6}} \right\} \right\}$$

✓ **Maple** : cpu = 0.198 (sec), leaf count = 49

$$\left\{ \left( (y(x))^{-2} + 6x^{-1} \right)^{-1} = \frac{x}{54} \left( e^{\text{RootOf}\left(-e^{-Z} \ln\left(\frac{(e^{-Z}+9)x}{2}\right) + 3\_C1 e^{-Z} + \_Z e^{-Z} + 2xe^{-Z} + 9\right)} + 9 \right) \right\}$$

2.751 ODE No. 751

---

$$y'(x) = \frac{y(x)(x^4 + x \log(y(x)) + \log(y(x)))}{x(x + 1)}$$

✓ **Mathematica** : cpu = 0.167444 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow (x + 1)^x e^{c_1 x + \frac{x^3}{2} - x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 26

$$\left\{ y(x) = \frac{(1 + x)^x e^{-C1 x}}{e^{x^2}} e^{\frac{x^3}{2}} \right\}$$



2.752 ODE No. 752

---

$$y'(x) = \frac{\cos(y(x)) \left(x^3 \cos(y(x)) - x - 1\right)}{(x + 1)(x \sin(y(x)) - 1)}$$

✗ **Mathematica** : cpu = 28.6556 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[1][y][x] == (Cos[y[x]]\*(-1 - x + x^3\*Cos[y[x]]))/((1 + x)\*(-1 + x\*Sin[y[x]]))][x]  
✓ **Maple** : cpu = 1.096 (sec), leaf count = 723

$$\left\{y(x) = \arctan \left(\frac{1}{36 \left(\ln (1+x)\right)^2 + \left(-24 x^3 + 36 x^2 - 72 _C1 - 72 x\right) \ln (1+x) + 4 x^6 - 12 x^5 + 33 x^4 + \left(24 _C1 - 72 x\right) x^3 - 36 x^2 + 12 x - 1}\right)\right\}$$

2.753 ODE No. 753

---

$$y'(x) = \frac{y(x) \log(y(x)) \left(x^4 \log(y(x)) + x + 1\right)}{x(x + 1)}$$

✓ **Mathematica** : cpu = 0.167611 (sec), leaf count = 41

$$\left\{\left\{y(x) \rightarrow \exp \left(-\frac{12 x}{-12 c_1 + 3 x^4 - 4 x^3 + 6 x^2 - 12 x + 12 \log (x + 1)}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.102 (sec), leaf count = 38

$$\left\{y(x) = e^{-12 \frac{x}{3 x^4 - 4 x^3 + 6 x^2 + 12 \ln (1+x) - 12 _C1 - 12 x}}\right\}$$

2.754 ODE No. 754

---

$$y'(x) = \frac{x^3 + x y(x)^2 + x y(x) + y(x)^3}{x^2}$$

✓ **Mathematica** : cpu = 0.109114 (sec), leaf count = 47

$$\text{Solve}\left[\text{RootSum}\left[\#1^3 + \#1^2 + 1 \&, \frac{\log\left(\frac{y(x)}{x} - \#1\right)}{3 \#1^2 + 2 \#1} \&\right] = c_1 + x, y(x)\right]$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 26

$$\left\{y(x) = \text{RootOf}\left(-\int^{-Z} \left(_a^3 + _a^2 + 1\right)^{-1} d_a + x + _C1\right) x\right\}$$

2.755 ODE No. 755

---

$$y'(x) = \frac{y(x)^{3/2}}{x^2 - 2 x y(x) + y(x)^2 + y(x)^{3/2}}$$

✓ **Mathematica** : cpu = 0.277235 (sec), leaf count = 2633

$$\left\{\left\{y(x) \rightarrow \frac{2}{3} \left(x + e^{c_1} + 2 e^{2 c_1}\right) - \frac{1}{3} \sqrt[3]{x^3 + 3 e^{c_1} x^2 - 12 e^{2 c_1} x^2 + 3 e^{2 c_1} x + 12 e^{3 c_1} x + 48 e^{4 c_1} x + e^{3 c_1} - 30 e^{4 c_1} - 96 e^{5 c_1}}\right\}\right\}$$

✓ **Maple** : cpu = 0.093 (sec), leaf count = 44

$$\left\{2 \frac{\sqrt{y(x)}}{y(x) - x} + (y(x) - x)^{-1} - 2 \frac{x}{(y(x) - x) \sqrt{y(x)}} - _C1 = 0\right\}$$

2.756 ODE No. 756

---

$$y'(x) = \frac{x^6 + 2 x^3 y(x) + x^2 y(x)^2 + y(x)^3}{x^4}$$

✓ **Mathematica** : cpu = 0.154634 (sec), leaf count = 95

$$\text{Solve}\left[-\frac{29}{3} \text{RootSum}\left[-29 \#1^3 + 3 \sqrt[3]{29} \#1 - 29 \&, \frac{\log\left(\frac{\frac{3 y(x)}{x^4} + \frac{1}{x^2}}{\sqrt[3]{29} \sqrt[3]{\frac{1}{x^6}}} - \#1\right)}{\sqrt[3]{29} - 29 \#1^2} \&\right] = c_1 + \frac{1}{9} 29^{2/3} \left(\frac{1}{x^6}\right)^{2/3} x^5, y(x)\right]$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 37

$$\left\{y(x)=\frac{\left(-3+29\,RootOf\left(-81\int^{-Z}\left(841\_a^3-27\_a+27\right)^{-1}d\_a+x+3\_C1\right)\right)x^2}{9}\right\}$$

2.757 ODE No. 757

---

$$y'(x)=\frac{x^3+2x^2-4xy(x)-4x-8}{2x^2-8y(x)+4x-8}$$

✓ **Mathematica** : cpu = 0.022504 (sec), leaf count = 36

$$\left\{\left\{y(x)\rightarrow 2\left(W\left(-e^{c_1-\frac{x}{4}-1}\right)+1\right)+\frac{1}{4}\left(x^2+2x-4\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 26

$$\left\{y(x)=\frac{x^2}{4}+2\,lambertW\left(1/2\_C1\,e^{-x/4}e^{-1/2}\right)+\frac{x}{2}+1\right\}$$

2.758 ODE No. 758

---

$$y'(x)=\frac{y(x)\left(x^3y(x)+2x+2\right)}{(x+1)(\log(y(x))+2x-1)}$$

✓ **Mathematica** : cpu = 0.768166 (sec), leaf count = 459

$$\left\{\left\{y(x)\rightarrow \frac{6W\left(-\frac{1}{6}\sqrt[6]{e^{-12x}\left(6c_1+2x^3-3x^2+6x-6\log(x+1)\right)^6}\right)}{6c_1+2x^3-3x^2+6x-6\log(x+1)}\right\},\left\{y(x)\rightarrow \frac{6W\left(\frac{1}{6}\sqrt[6]{e^{-12x}\left(6c_1+2x^3-3x^2+6x-6\log(x+1)\right)^6}\right)}{6c_1+2x^3-3x^2+6x-6\log(x+1)}\right\}\right\}$$

✓ **Maple** : cpu = 0.165 (sec), leaf count = 41

$$\left\{y(x)=e^{-lambertW\left(-\frac{\left(-2\,x^3+3\,x^2+6\ln(1+x)+6\_C1-6\,x\right)e^{-2\,x}}{6}\right)-2\,x}\right\}$$

2.759 ODE No. 759

---

$$y'(x)=-\frac{ix\left(x^8+18x^4y(x)^2+54ix^2+81y(x)^4\right)}{243y(x)}$$

✗ **Mathematica** : cpu = 40.6015 (sec), leaf count = 0 , could not solve

DSolve[Derivative[1][y][x]==((-I/243)\*x\*((54\*I)\*x^2+x^8+18\*x^4\*y[x]^2+81\*y[x]^4))/y[x],y[x],x]

✓ **Maple** : cpu = 0.384 (sec), leaf count = 305

$$\left\{y(x)=-\frac{\sqrt{3}}{3x}\sqrt{\left(J_{\frac{1}{3}}\left(\left(\frac{2}{27}-\frac{2i}{27}\right)\sqrt{6}x^3\right)\right)\_C1+Y_{\frac{1}{3}}\left(\left(\frac{2}{27}-\frac{2i}{27}\right)\sqrt{6}x^3\right)\right)\left(-9\left(1/27\,x^6+i\right)\_C1\,J_{1/3}\left(\left(\frac{2}{27}-\frac{2i}{27}\right)\sqrt{6}x^3\right)\right)}$$

2.760 ODE No. 760

---

$$y'(x)=\frac{(xy(x)^2+1)^3}{x^4y(x)(xy(x)^2+x+1)}$$

✓ **Mathematica** : cpu = 0.992347 (sec), leaf count = 112

$$\text{Solve}\left[2\left(\frac{1}{10}\log\left(2x^2y(x)^4+2x^2y(x)^2+x^2+4xy(x)^2+2x+2\right)-\frac{1}{5}\log\left(xy(x)^2-x+1\right)-\frac{1}{10}\tan^{-1}\left(2xy(x)^4\right)\right),y(x)\right]$$

✓ **Maple** : cpu = 0.785 (sec), leaf count = 136

$$\left\{-\frac{(1+y(x))\left(2\left(y(x)\right)^4+2\left(y(x)\right)^2+1\right)(-1+y(x))\left(\ln\left(2\,x^2\left(y(x)\right)^4+\left(2\,x^2+4\,x\right)\left(y(x)\right)^2+x^2+2\,x+2\right)\right)}{5\,x\left(\frac{1}{10}\log\left(2\,x^2\left(y(x)\right)^4+2\,x^2\left(y(x)\right)^2+x^2+4\,x\,y(x)^2+2\,x+2\right)-\frac{1}{5}\log\left(x\,y(x)^2-x+1\right)-\frac{1}{10}\tan^{-1}\left(2\,x\,y(x)^4\right)\right)}\right\}$$

2.761 ODE No. 761

---

$$y'(x) = \frac{-x^3 + 4x^2 - 4xy(x) - 4x + 8}{2x^2 + 8y(x) - 8x + 8}$$

✓ **Mathematica** : cpu = 0.0222225 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow W\left(-e^{c_1-x-1}\right) + \frac{1}{4}\left(-x^2 + 4x - 4\right) + 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 18

$$\left\{ y\left(x\right) = -\frac{x^2}{4} + \text{lambert}W\left(\frac{-C1}{\mathrm{e}^x}\right) + x \right\}$$

2.762 ODE No. 762

---

$$y'(x) = -\frac{y(x)(x\log(y(x)) + \log(y(x)) - x)}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.170417 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow (x+1)^{-1/x} e^{1-\frac{c_1}{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.086 (sec), leaf count = 22

$$\left\{ y\left(x\right) = \frac{\mathrm{e}}{\sqrt[3]{1+x}} \mathrm{e}^{\frac{-C1}{x}} \right\}$$

2.763 ODE No. 763

---

$$y'(x) = \frac{y(x)(x\log(y(x)) + \log(y(x)) + x)}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.151148 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow x^x (x+1)^{-x} e^{c_1 x} \right\} \right\}$$

✓ **Maple** : cpu = 0.078 (sec), leaf count = 14

$$\left\{ y\left(x\right) = \left(\frac{-C1\,x}{1+x}\right)^x \right\}$$

2.764 ODE No. 764

---

$$y'(x) = \frac{y(x)\left(x^4 - x\log(y(x)) - \log(y(x))\right)}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.192848 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow (x+1)^{\frac{1}{x}} e^{-\frac{c_1}{x} + \frac{x^3}{4} - \frac{x^2}{3} + \frac{x}{2} - \frac{25}{12x} - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.08 (sec), leaf count = 36

$$\left\{ y\left(x\right) = \mathrm{e}^{\frac{x^3}{4}} \mathrm{e}^{-\frac{x^2}{3}} \mathrm{e}^{\frac{x}{2}} \sqrt[3]{1+x} \mathrm{e}^{\frac{-C1}{x}} \mathrm{e}^{-1} \right\}$$

2.765 ODE No. 765

---

$$y'(x) = \frac{y(x)\left(xy(x)\log\left(\frac{(x-1)(x+1)}{x}\right) - \log\left(\frac{(x-1)(x+1)}{x}\right) - 1\right)}{x}$$

✓ **Mathematica** : cpu = 0.342323 (sec), leaf count = 138

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp\left(-\mathrm{Li}_2(1-x) + \mathrm{Li}_2(-x) - \frac{1}{2}\log^2(x) + \log(x+1)\log(x) - \log\left(x - \frac{1}{x}\right)\log(x)\right)}{x\left(c_1 - \int_1^x \frac{\exp\left(-\frac{1}{2}\log^2(K[1]) + \log(K[1]+1)\log(K[1]) - \log\left(K[1] - \frac{1}{K[1]}\right)\log(K[1]) - \mathrm{Li}_2(1-K[1]) + \mathrm{Li}_2(-K[1])\right)\log\left(\frac{K[1]-1}{K}\right)}{K[1]} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.139 (sec), leaf count = 106

$$\left\{y\left(x\right)=\frac{{\rm e}^{dilog\left(1+x\right)}x^{\ln\left(1+x\right)}}{xe^{dilog\left(x\right)}}{\rm e}^{-\frac{\left(\ln\left(x\right)\right)^2}{2}}\left(\int-\frac{{\rm e}^{dilog\left(1+x\right)}x^{\ln\left(1+x\right)}}{xe^{dilog\left(x\right)}}{\rm e}^{-\frac{\left(\ln\left(x\right)\right)^2}{2}}\ln\left(\frac{\left(1+x\right)\left(x-1\right)}{x}\right)\left(x^{\ln\left(\frac{\left(1+x\right)\left(x-1\right)}{x}\right)}\right)^{-1}{\rm d}x\right.\right.$$

### 2.766 ODE No. 766

$$y'(x)=\frac{y(x)\left(x^2y(x)\log\left(\frac{(x-1)(x+1)}{x}\right)-x\log\left(\frac{(x-1)(x+1)}{x}\right)-\log(x)\right)}{x\log(x)}$$

✓ **Mathematica** : cpu = 0.306374 (sec), leaf count = 129

$$\left\{\left\{y(x)\rightarrow\frac{\exp\left(\int_1^x\frac{-\log(K[1])-K[1]\log\left(\frac{(K[1]-1)(K[1]+1)}{K[1]}\right)}{K[1]\log(K[1])}dK[1]\right)}{c_1-\int_1^x\frac{\exp\left(\int_1^{K[2]}\frac{-\log(K[1])-K[1]\log\left(\frac{(K[1]-1)(K[1]+1)}{K[1]}\right)}{K[1]\log(K[1])}dK[1]\right)K[2]\log\left(\frac{(K[2]-1)(K[2]+1)}{K[2]}\right)}{\log(K[2])}dK[2]}\right\}\right\}$$

✓ **Maple** : cpu = 0.109 (sec), leaf count = 89

$$\left\{y\left(x\right)=1{\rm e}^{\int\frac{1}{x\ln\left(x\right)}\left(-x\ln\left(\frac{\left(1+x\right)\left(x-1\right)}{x}\right)-\ln\left(x\right)\right){\rm d}x}\left(\int-\frac{x}{\ln\left(x\right)}{\rm e}^{\int\frac{1}{x\ln\left(x\right)}\left(-x\ln\left(\frac{\left(1+x\right)\left(x-1\right)}{x}\right)-\ln\left(x\right)\right){\rm d}x}\ln\left(\frac{\left(1+x\right)\left(x-1\right)}{x}\right){\rm d}x\right.\right.$$

### 2.767 ODE No. 767

$$y'(x)=\frac{-x^3+2x^2-8xy(x)-8x+32}{4x^2+32y(x)-8x+32}$$

✓ **Mathematica** : cpu = 0.0238803 (sec), leaf count = 38

$$\left\{\left\{y(x)\rightarrow 4\left(W\left(-e^{c_1-\frac{x}{16}-1}\right)+1\right)+\frac{1}{8}\left(-x^2+2x-8\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 26

$$\left\{y\left(x\right)=-\frac{x^2}{8}+4\,lambertW\left(1/4\_C1\,{\rm e}^{-x/16}{\rm e}^{-3/4}\right)+\frac{x}{4}+3\right\}$$

### 2.768 ODE No. 768

$$y'(x)=\frac{y(x)(y(x)+1)}{x(xy(x)-y(x)-1)}$$

✓ **Mathematica** : cpu = 1.05264 (sec), leaf count = 66

$$\text{Solve}\left[\frac{2^{2/3}\left(xy(x)\left(-\log\left(\frac{xy(x)}{(x-1)y(x)-1}\right)+\log\left(\frac{y(x)+1}{-xy(x)+y(x)+1}\right)+\log(x)+1\right)-1\right)}{9xy(x)}=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 26

$$\left\{y\left(x\right)=-\left(xlambertW\left(\frac{1}{xe^{x^{-1}}\_C1}\right)+1\right)^{-1}\right\}$$

### 2.769 ODE No. 769

$$y'(x)=-\frac{ix\left(x^8+8x^4y(x)^2+16ix^2+16y(x)^4\right)}{32y(x)}$$

✗ **Mathematica** : cpu = 42.1802 (sec), leaf count = 0 , could not solve

DSolve[Derivative[1][y][x]==((-I/32)\*x\*((16\*I)\*x^2+x^8+8\*x^4\*y[x]^2+16\*y[x]^4))/y[x]

✓ **Maple** : cpu = 0.296 (sec), leaf count = 251

$$\left\{y\left(x\right)=-\frac{\sqrt{4}}{2x}\sqrt{\left(J_{\frac{1}{3}}\left(\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)\_C1+Y_{\frac{1}{3}}\left(\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)\right)\left(-2\left(1/8\,x^6+i\right)\_C1\,J_{1/3}\left(\left(1/3-i/3\right)x^3\right)+\right.\right.$$

2.770 ODE No. 770

---

$$y'(x) = \frac{2y(x)^6}{32x^2y(x)^4 + y(x)^3 + 16xy(x)^2 + 2}$$

✓ **Mathematica** : cpu = 0.21323 (sec), leaf count = 705

$$\left\{\left\{y(x) \rightarrow \frac{\sqrt[3]{18432c_1^2x^2 + \sqrt{4(192c_1^2x - 12c_1 - 256x^2)^3 + (18432c_1^2x^2 - 2880c_1x + 8192x^3 + 108)^2} - 2880c_1}}{3\sqrt[3]{2}(1 - 16c_1x)}\right.\right.$$

✓ **Maple** : cpu = 0.105 (sec), leaf count = 1105

$$\left\{y(x) = \frac{1}{96x + 6 - C1} \left(32 - C1x \sqrt[3]{96(-C1/16 + x)\sqrt{3}\sqrt{(4096x^3 + 27) - C1^4 + 576x - C1^3 + 2048 - C1^2x^2}}\right)\right.$$

2.771 ODE No. 771

---

$$y'(x) = \frac{-a^2x^3 - 2abx^2 - 4axy(x) - 4ax + 8}{2ax^2 + 4bx + 8y(x) + 8}$$

✓ **Mathematica** : cpu = 0.0302527 (sec), leaf count = 46

$$\left\{\left\{y(x) \rightarrow \frac{1}{4}(-ax^2 - 2bx - 4) - \frac{2\left(W\left(-e^{-\frac{b^2x}{4} + c_1 - 1}\right) + 1\right)}{b}\right.\right.\right.$$

✓ **Maple** : cpu = 0.076 (sec), leaf count = 84

$$\left\{y(x) = \frac{1}{4b} \left(-ax^2b - 2b^2x - 4b + 4e^{\frac{1}{4}\frac{1}{a}\left(-4\operatorname{lambertW}\left(-\frac{1}{2}e^{-1/4b^2x}e^{-1/2 - \frac{C1b^2}{a}}e^{-b/2}e^{-1}\right)\right)}_{a+(-b^2x-2b-4)a-2-C1b^2}\right)\right.$$

2.772 ODE No. 772

---

$$y'(x) = \frac{y(x)\log(y(x))(x\log(y(x)) + x + 1)}{x(x + 1)}$$

✓ **Mathematica** : cpu = 0.155289 (sec), leaf count = 21

$$\left\{\left\{y(x) \rightarrow e^{\frac{x}{c_1 - x + \log(x+1)}}\right.\right\}$$

✓ **Maple** : cpu = 0.073 (sec), leaf count = 18

$$\left\{y(x) = e^{\frac{x}{\ln(1+x) + -C1 - x}}\right\}$$

2.773 ODE No. 773

---

$$y'(x) = \frac{y(x)^2 + xy(x) + x}{(x - 1)(y(x) + x)}$$

✓ **Mathematica** : cpu = 0.162875 (sec), leaf count = 61

$$\operatorname{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2} + \frac{y(x)}{x} + 1\right) + \frac{\tan^{-1}\left(\frac{\frac{2y(x)}{x} + 1}{\sqrt{3}}\right)}{\sqrt{3}} = c_1 + \log(1 - x) - \log(x), y(x)\right]$$

✓ **Maple** : cpu = 0.159 (sec), leaf count = 48

$$\left\{y(x) = -\frac{x}{2} + \frac{\sqrt{3}x}{2} \tan\left(\operatorname{RootOf}\left(-\sqrt{3}\ln\left(\frac{3x^2((\tan(-Z))^2 + 1)}{4(x - 1)^2}\right) + 2\sqrt{3} - C1 - 2 - Z\right)\right)\right\}$$

2.774 ODE No. 774

---

$$y'(x) = \frac{-2ax^2 - x^3 - 4xy(x) - 4x + 8}{4ax + 2x^2 + 8y(x) + 8}$$

✓ **Mathematica** : cpu = 0.0262517 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} (-2ax - x^2 - 4) - \frac{2 \left( W \left( -e^{-\frac{a^2x}{4} + c_1 - 1} \right) + 1 \right)}{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.064 (sec), leaf count = 51

$$\left\{ y(x) = \frac{1}{4a} \left( -2a^2x - ax^2 - 8 \operatorname{lambertW} \left( -1/2 e^{-1/4 a^2 x} e^{-a/2} e^{-1} e^{1/4 - C1 a^2} \right) - 4a - 8 \right) \right\}$$

2.775 ODE No. 775

---

$$y'(x) = \frac{-y(x) + \sqrt{y(x)} + x}{-y(x) + \sqrt{y(x)} + x + 1}$$

✓ **Mathematica** : cpu = 0.157292 (sec), leaf count = 943

$$\left\{ \left\{ y(x) \rightarrow \operatorname{Root} \left[ x^6 - 2e^{3c_1} x^3 + e^{6c_1} + \#1^6 + (-6x - 6)\#1^5 + (15x^2 + 24x + 9)\#1^4 + (-20x^3 - 36x^2 - 18x + \right. \right. \right.$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 44

$$\left\{ -2 (y(x))^{3/2} + (y(x))^3 + (-3x - 3)(y(x))^2 + (3x^2 + 3x)y(x) - x^3 - C1 = 0 \right\}$$

2.776 ODE No. 776

---

$$y'(x) = \frac{y(x) \left( x^2 y(x) \log \left( \frac{x^2 + 1}{x} \right) - x \log \left( \frac{x^2 + 1}{x} \right) - \log \left( \frac{1}{x} \right) \right)}{x \log \left( \frac{1}{x} \right)}$$

✓ **Mathematica** : cpu = 0.376417 (sec), leaf count = 133

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp \left( \int_1^x \frac{-\log \left( \frac{1}{K[1]} \right) - K[1] \log \left( \frac{K[1]^2 + 1}{K[1]} \right)}{K[1] \log \left( \frac{1}{K[1]} \right)} dK[1] \right)}{c_1 - \int_1^x \frac{\exp \left( \int_1^{K[2]} \frac{-\log \left( \frac{1}{K[1]} \right) - K[1] \log \left( \frac{K[1]^2 + 1}{K[1]} \right)}{K[1] \log \left( \frac{1}{K[1]} \right)} dK[1] \right) K[2] \log \left( \frac{K[2]^2 + 1}{K[2]} \right)}{\log \left( \frac{1}{K[2]} \right)} dK[2]} \right\} \right\}$$

✓ **Maple** : cpu = 0.132 (sec), leaf count = 96

$$\left\{ y(x) = 1e^{\int \frac{1}{x \ln(x^{-1})} \left( -\ln \left( \frac{x^2 + 1}{x} \right) x - \ln(x^{-1}) \right) dx} \left( \int -\frac{x}{\ln(x^{-1})} e^{\int \frac{1}{x \ln(x^{-1})} \left( -\ln \left( \frac{x^2 + 1}{x} \right) x - \ln(x^{-1}) \right) dx} \ln \left( \frac{x^2 + 1}{x} \right) dx + \right. \right.$$

2.777 ODE No. 777

---

$$y'(x) = \frac{y(x)(y(x) + 1)}{x(xy(x)^4 - y(x) - 1)}$$

✓ **Mathematica** : cpu = 0.219397 (sec), leaf count = 39

$$\operatorname{Solve} \left[ -\frac{1}{2}(y(x) + 1)^2 + 2(y(x) + 1) - \frac{1}{xy(x)} - \log(y(x) + 1) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.142 (sec), leaf count = 51

$$\left\{ y(x) = e^{\operatorname{RootOf} \left( (e^{-Z})^3 x - 5x(e^{-Z})^2 + 2 - C1 x e^{-Z} + 2 - Z x e^{-Z} + 7x e^{-Z} - 2 - C1 x - 2 - Z x - 3x + 2 \right) - 1} \right\}$$

## 2.778 ODE No. 778

---

$$y'(x) = \frac{x^9 y(x)^3 + x^6 y(x)^2 - 3x^2 y(x) + 1}{x^3}$$

✓ **Mathematica** : cpu = 0.148108 (sec), leaf count = 95

$$\text{Solve} \left[ -\frac{29}{3} \text{RootSum} \left[ -29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log \left( \frac{3x^6 y(x) + x^3}{\sqrt[3]{29}\sqrt[3]{x^9}} - \#1 \right)}{\sqrt[3]{29} - 29\#1^2} \& \right] = c_1 + \frac{29^{2/3} (x^9)^{2/3}}{9x^5}, y(x) \right]$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 37

$$\left\{ y(x) = \frac{-3 + 29 \text{RootOf} \left( -81 \int^{-Z} (841\_a^3 - 27\_a + 27)^{-1} d\_a + x + 3\_C1 \right)}{9x^3} \right\}$$

## 2.779 ODE No. 779

---

$$y'(x) = \frac{x^3 y(x) + x^3 + x y(x)^2 + y(x)^3}{(x-1)x^3}$$

✓ **Mathematica** : cpu = 0.15583 (sec), leaf count = 57

$$\text{Solve} \left[ -\frac{1}{4} \log \left( \frac{y(x)^2}{x^2} + 1 \right) + \frac{1}{2} \log \left( \frac{y(x)}{x} + 1 \right) + \frac{1}{2} \tan^{-1} \left( \frac{y(x)}{x} \right) = c_1 + \log(1-x) - \log(x), y(x) \right]$$

✓ **Maple** : cpu = 0.079 (sec), leaf count = 50

$$\left\{ -\frac{1}{4} \ln \left( \frac{(y(x))^2 + x^2}{x^2} \right) + \frac{1}{2} \arctan \left( \frac{y(x)}{x} \right) + \frac{1}{2} \ln \left( \frac{y(x) + x}{x} \right) - \ln(x-1) + \ln(x) -\_C1 = 0 \right\}$$

## 2.780 ODE No. 780

---

$$y'(x) = \frac{x\sqrt{x^2 + y(x)^2} + x y(x) + y(x)}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.154961 (sec), leaf count = 15

$$\{\{y(x) \rightarrow x \sinh(c_1 + \log(x+1))\}\}$$

✓ **Maple** : cpu = 0.325 (sec), leaf count = 27

$$\left\{ -C1 + \frac{1}{x(1+x)} \left( y(x) + \sqrt{(y(x))^2 + x^2} \right) = 0 \right\}$$

## 2.781 ODE No. 781

---

$$y'(x) = \frac{y(x)(x^4 + x^3 + 3y(x)^2 + x)}{x(6y(x)^2 + x)}$$

✓ **Mathematica** : cpu = 0.487278 (sec), leaf count = 82

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{x}\sqrt{W\left(6xe^{2c_1+\frac{2x^3}{3}+x^2}\right)}}{\sqrt{6}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{x}\sqrt{W\left(6xe^{2c_1+\frac{2x^3}{3}+x^2}\right)}}{\sqrt{6}} \right\} \right\}$$

✓ **Maple** : cpu = 0.234 (sec), leaf count = 61

$$\left\{ \left( (y(x))^{-2} + 6x^{-1} \right)^{-1} = \frac{x}{54} \left( e^{\text{RootOf} \left( 2x^3 e^{-Z} + 3x^2 e^{-Z} - 3e^{-Z} \ln \left( 1/2 \frac{e^{-Z} + 9}{x} \right) + 9\_C1 e^{-Z} + 3\_Z e^{-Z} + 27 \right)} + 9 \right) \right\}$$

**2.782                    ODE No. 782**

---

$$y'(x) = \frac{y(x) \coth\left(\frac{1}{x}\right) \left(x^2 y(x) \log\left(\frac{x^2+1}{x}\right) - x \log\left(\frac{x^2+1}{x}\right) - \tanh\left(\frac{1}{x}\right)\right)}{x}$$

✓ **Mathematica** : cpu = 4.14199 (sec), leaf count = 115

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp\left(\int_1^x \frac{-\coth\left(\frac{1}{K[1]}\right) K[1] \log\left(\frac{K[1]^2+1}{K[1]}\right) - 1}{K[1]} dK[1]\right)}{c_1 - \int_1^x \exp\left(\int_1^{K[2]} \frac{-\coth\left(\frac{1}{K[1]}\right) K[1] \log\left(\frac{K[1]^2+1}{K[1]}\right) - 1}{K[1]} dK[1]\right) \coth\left(\frac{1}{K[2]}\right) K[2] \log\left(\frac{K[2]^2+1}{K[2]}\right) dK[2]} \right\} \right\}$$

✓ **Maple** : cpu = 0.961 (sec), leaf count = 96

$$\left\{ y(x) = 1 e^{\int \frac{1}{x \tanh(x^{-1})} \left(-\ln\left(\frac{x^2+1}{x}\right) x - \tanh(x^{-1})\right) dx} \left( \int -\frac{x}{\tanh(x^{-1})} e^{\int \frac{1}{x \tanh(x^{-1})} \left(-\ln\left(\frac{x^2+1}{x}\right) x - \tanh(x^{-1})\right) dx} \ln\left(\frac{x^2+1}{x}\right) dx + C_1 \right) \right\}$$

**2.783                    ODE No. 783**

---

$$y'(x) = -\frac{y(x) \coth(x) \left(x^2 y(x) (-\log(2x)) + x \log(2x) + \tanh(x)\right)}{x}$$

✓ **Mathematica** : cpu = 4.16498 (sec), leaf count = 88

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp\left(\int_1^x \frac{-\coth(K[1]) K[1] \log(2K[1]) - 1}{K[1]} dK[1]\right)}{c_1 - \int_1^x \exp\left(\int_1^{K[2]} \frac{-\coth(K[1]) K[1] \log(2K[1]) - 1}{K[1]} dK[1]\right) \coth(K[2]) K[2] \log(2K[2]) dK[2]} \right\} \right\}$$

✓ **Maple** : cpu = 0.122 (sec), leaf count = 75

$$\left\{ y(x) = 1 e^{\int \frac{-x \ln(x) - x \ln(2) - \tanh(x)}{x \tanh(x)} dx} \left( \int -\frac{x (\ln(2) + \ln(x))}{\tanh(x)} e^{\int \frac{-x \ln(x) - x \ln(2) - \tanh(x)}{x \tanh(x)} dx} dx + C_1 \right)^{-1} \right\}$$

**2.784                    ODE No. 784**

---

$$y'(x) = \operatorname{csch}(x) \left(x^2 \log(x) + 2xy(x) \log(x) + y(x)^2 \log(x) + \log(x) - \sinh(x)\right)$$

✓ **Mathematica** : cpu = 19.9038 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow \tan\left(\int_1^x \operatorname{csch}(K[5]) \log(K[5]) dK[5] + c_1\right) - x \right\} \right\}$$

✓ **Maple** : cpu = 20.355 (sec), leaf count = 24

$$\left\{ y(x) = -x - \tan\left(-C_1 - \int \frac{\ln(x)}{\sinh(x)} dx\right) \right\}$$

**2.785                    ODE No. 785**

---

$$y'(x) = \frac{x^2 \sinh(x) + 2xy(x) \sinh(x) + y(x)^2 \sinh(x) - \log(x) + \sinh(x)}{\log(x)}$$

✓ **Mathematica** : cpu = 7.31226 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow \tan\left(\int_1^x \frac{\sinh(K[5])}{\log(K[5])} dK[5] + c_1\right) - x \right\} \right\}$$

✓ **Maple** : cpu = 66.315 (sec), leaf count = 24

$$\left\{ y(x) = -x - \tan\left(-C_1 - \int \frac{\sinh(x)}{\ln(x)} dx\right) \right\}$$



2.786 ODE No. 786

$$y'(x) = \frac{axy(x)^2 \cosh(x) + bx^3 \cosh(x) + y(x) \log(x)}{x \log(x)}$$

✓ **Mathematica** : cpu = 3.62055 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{b}x \tan \left( \sqrt{a}\sqrt{b} \int_1^x \frac{\cosh(K[1])K[1]}{\log(K[1])} dK[1] + \sqrt{a}\sqrt{b}c_1 \right)}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 33

$$\left\{ y(x) = \frac{x}{a} \tan \left( \sqrt{ab} \left( -C1 + \int \frac{x \cosh(x)}{\ln(x)} dx \right) \right) \sqrt{ab} \right\}$$

2.787 ODE No. 787

$$y'(x) = \frac{x \left( 2x^4 - 2x^2y(x) + x^2 - x - 1 \right)}{(x+1) \left( x^2 - y(x) \right)}$$

✓ **Mathematica** : cpu = 25.9792 (sec), leaf count = 488

Solve  $\left[ \left( 2 - \frac{x(x^2-x-1)(2x^2-2y(x)+3)}{\sqrt[3]{x^3(x^2-x-1)^3(x^2-y(x))}} \right) \left( \frac{x(x^2-x-1)(2x^2-2y(x)+3)}{\sqrt[3]{x^3(x^2-x-1)^3(x^2-y(x))}} + 4 \right) \left( \left( 1 - \frac{x(x^2-x-1)(2x^2-2y(x)+3)}{2\sqrt[3]{x^3(x^2-x-1)^3(x^2-y(x))}} \right) \log \left( \frac{2 - \frac{x(x^2-x-1)(2x^2-2y(x)+3)}{\sqrt[3]{x^3(x^2-x-1)^3(x^2-y(x))}}}{\sqrt[3]{x^3(x^2-x-1)^3(x^2-y(x))}} \right) - 18\sqrt[3]{2} \left( -\frac{(2x^2-2y(x)+3)^3}{8(x^2-y(x))^3} + \frac{3x(x^2-x-1)}{2\sqrt[3]{x^3(x^2-x-1)^3(x^2-y(x))}} \right) \right] \right.$

✓ **Maple** : cpu = 0.267 (sec), leaf count = 191

$$\left\{ y(x) = 1 \left( 4x^2 e^{\text{RootOf}\left(8x^3 e^{-Z} - 24x^2 e^{-Z} - 36x^3 + 6 \ln\left(\frac{2e^{-Z}-9}{(1+x)^4}\right) e^{-Z} + 18\_C1 e^{-Z} - 6\_Z e^{-Z} + 24x e^{-Z} + 108x^2 - 27 \ln\left(\frac{2e^{-Z}-9}{(1+x)^4}\right) - 81\right)} \right) \right\}$$

2.788 ODE No. 788

$$y'(x) = -\frac{y(x) \left( x^2y(x)(-\coth(x+1)) + \log(x-1) + x \coth(x+1) \right)}{x \log(x-1)}$$

✓ **Mathematica** : cpu = 28.5492 (sec), leaf count = 348

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp \left( \int_1^x \frac{-e^2 \cosh(K[1])K[1] - \cosh(K[1])K[1] - e^2 \sinh(K[1])K[1] + \sinh(K[1])K[1] - e^2 \cosh(K[1]) \log(K[1]) - 1}{K[1] \log(K[1]) - 1} (e^2 \cosh(K[1]) - \cosh(K[1]) + e^2 \sinh(K[1]) + \sinh(K[1])) dx \right)}{c_1 - \int_1^x \frac{\exp \left( \int_1^{K[2]} \frac{-e^2 \cosh(K[1])K[1] - \cosh(K[1])K[1] - e^2 \sinh(K[1])K[1] + \sinh(K[1])K[1] - e^2 \cosh(K[1]) \log(K[1]) - 1}{K[1] \log(K[1]) - 1} (e^2 \cosh(K[1]) - \cosh(K[1]) + e^2 \sinh(K[1]) + \sinh(K[1])) dK[1]}{K[2] \log(K[2]) - 1} (e^2 \cosh(K[2]) - \cosh(K[2]) + e^2 \sinh(K[2]) + \sinh(K[2])) dK[2]} dx \right)}{K[2] \log(K[2]) - 1} (e^2 \cosh(K[2]) - \cosh(K[2]) + e^2 \sinh(K[2]) + \sinh(K[2])) dK[2]} \right\} \right\}$$

✓ **Maple** : cpu = 0.166 (sec), leaf count = 108

$$\left\{ y(x) = 1 \left( e^{-\int \frac{-\ln(x-1) \sinh(1+x) - x \cosh(1+x)}{\sinh(1+x)x \ln(x-1)} dx} \right)^{-1} \left( -C1 + \int -\frac{x \cosh(1+x)}{\ln(x-1) \sinh(1+x)} e^{\int \frac{-\ln(x-1) \sinh(1+x) - x \cosh(1+x)}{\sinh(1+x)x \ln(x-1)} dx} dx \right) \right\}$$

2.789 ODE No. 789

$$y'(x) = \frac{x^2 \coth(x+1) + 2xy(x) \coth(x+1) + y(x)^2 \coth(x+1) - \log(x-1) + \coth(x+1)}{\log(x-1)}$$

✓ **Mathematica** : cpu = 71.3468 (sec), leaf count = 120

$$\left\{ \left\{ y(x) \rightarrow \tan \left( \int_1^x \frac{e^2 \cosh(K[5]) + \cosh(K[5]) + e^2 \sinh(K[5]) - \sinh(K[5])}{\log(K[5] - 1) (e^2 \cosh(K[5]) - \cosh(K[5]) + e^2 \sinh(K[5]) + \sinh(K[5]))} dK[5] + c_1 \right) - \frac{e^2 x}{e} \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x) = -(ln(x-1)-coth(1+x)\*x^2-2\*coth(1+x)\*x\*y(x)-coth(1+x)-coth(1+x)\*y(x)^2)/ln(x-1),y(x))

**2.790 ODE No. 790**

---

$$y'(x) = \frac{x^4 \coth\left(\frac{x+1}{x-1}\right) - 2x^2 y(x) \coth\left(\frac{x+1}{x-1}\right) + y(x)^2 \coth\left(\frac{x+1}{x-1}\right) + 2x \log\left(\frac{1}{x-1}\right) - \coth\left(\frac{x+1}{x-1}\right)}{\log\left(\frac{1}{x-1}\right)}$$

✓ **Mathematica** : cpu = 86.1163 (sec), leaf count = 127

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp\left(\int_1^x \frac{2 \coth\left(\frac{K[5]}{K[5]-1} + \frac{1}{K[5]-1}\right)}{\log\left(\frac{1}{K[5]-1}\right)} dK[5]\right)}{c_1 - \int_1^x \frac{\exp\left(\int_1^{K[6]} \frac{2 \coth\left(\frac{K[5]}{K[5]-1} + \frac{1}{K[5]-1}\right)}{\log\left(\frac{1}{K[5]-1}\right)} dK[5]\right) \coth\left(\frac{K[6]}{K[6]-1} + \frac{1}{K[6]-1}\right)}{\log\left(\frac{1}{K[6]-1}\right)} dK[6]} + x^2 + 1 \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x) = (2\*x\*ln(1/(x-1))-coth((1+x)/(x-1))+coth((1+x)/(x-1))\*y(x)^2-2\*coth((1+x)/(x-1))\*x^2\*y(x)+coth((1+x)/(x-1))\*x^4)/ln(1/(x-1)),y(x))

**2.791 ODE No. 791**

---

$$y'(x) = \frac{\operatorname{sech}\left(\frac{1}{x-1}\right) \left(x^5 + x^4 - 2x^3 y(x) - 2x^2 y(x) + 2x^2 \cosh\left(\frac{1}{x-1}\right) + xy(x)^2 + y(x)^2 - x - 2x \cosh\left(\frac{1}{x-1}\right) - 1\right)}{x - 1}$$

✓ **Mathematica** : cpu = 7.89596 (sec), leaf count = 110

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp\left(\int_1^x \frac{2(K[5]+1)\operatorname{sech}\left(\frac{1}{K[5]-1}\right)}{K[5]-1} dK[5]\right)}{c_1 - \int_1^x \frac{\exp\left(\int_1^{K[6]} \frac{2(K[5]+1)\operatorname{sech}\left(\frac{1}{K[5]-1}\right)}{K[5]-1} dK[5]\right) (K[6]+1)\operatorname{sech}\left(\frac{1}{K[6]-1}\right)}{K[6]-1} dK[6]} + \frac{x^3 + x^2}{x + 1} + 1 \right\} \right\}$$

✓ **Maple** : cpu = 13.26 (sec), leaf count = 306

$$\left\{ y(x) = 1 \left( (-x^2 + 1) \left( e^{\frac{1}{\left(e^{(x-1)^{-1}}\right)^2 + 1} \int \frac{e^{(x-1)^{-1}(1+x)}}{\left(\left(e^{(x-1)^{-1}}\right)^2 + 1\right)(x-1)} dx} \right)^4 \left( e^{\frac{1}{\left(e^{(x-1)^{-1}}\right)^2 + 1} \int \frac{e^{(x-1)^{-1}(1+x)}}{\left(\left(e^{(x-1)^{-1}}\right)^2 + 1\right)(x-1)} dx} e^{2(x-1)^{-1}} \right)^{-1} \right\}$$

**2.792 ODE No. 792**

---

$$y'(x) = \frac{y(x) \operatorname{sech}\left(\frac{1}{x+1}\right) \left(x^3 y(x) + x^2 y(x) - x^2 - x - x \cosh\left(\frac{1}{x+1}\right) + \cosh\left(\frac{1}{x+1}\right)\right)}{(x - 1)x}$$

✓ **Mathematica** : cpu = 3.03214 (sec), leaf count = 157

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp\left(\int_1^x \frac{-\operatorname{sech}\left(\frac{1}{K[1]+1}\right) K[1]^2 - \operatorname{sech}\left(\frac{1}{K[1]+1}\right) K[1] - K[1] + 1}{(K[1]-1)K[1]} dK[1]\right)}{c_1 - \int_1^x \frac{\exp\left(\int_1^{K[2]} \frac{-\operatorname{sech}\left(\frac{1}{K[1]+1}\right) K[1]^2 - \operatorname{sech}\left(\frac{1}{K[1]+1}\right) K[1] - K[1] + 1}{(K[1]-1)K[1]} dK[1]\right) \left(\operatorname{sech}\left(\frac{1}{K[2]+1}\right) K[2]^3 + \operatorname{sech}\left(\frac{1}{K[2]+1}\right) K[2]^2\right)}{(K[2]-1)K[2]} dK[2]} \right\} \right\}$$

✓ **Maple** : cpu = 0.271 (sec), leaf count = 112

$$\left\{ y(x) = 1 e^{\int \frac{(1-x) \cosh\left((1+x)^{-1}\right) - x^2 - x}{x(x-1) \cosh\left((1+x)^{-1}\right)} dx} \left( \int -\frac{x(1+x)}{(x-1) \cosh\left((1+x)^{-1}\right)} e^{\int \frac{(1-x) \cosh\left((1+x)^{-1}\right) - x^2 - x}{x(x-1) \cosh\left((1+x)^{-1}\right)} dx} dx + \_C1 \right)^{-1} \right\}$$

2.793 ODE No. 793

---

$$y'(x) = -\frac{y(x)(xy(x) + 1)}{x(xy(x) - y(x) + 1)}$$

✓ **Mathematica** : cpu = 13.0469 (sec), leaf count = 399

Solve 
$$\left[ -\frac{\sqrt[3]{-2} \left( \frac{2^{2/3}((x-1)y(x)-2)}{\sqrt[3]{-\frac{1}{(x-1)^3}(x-1)((x-1)y(x)+1)}} + (-2)^{2/3} \right) \left( \frac{-xy(x)+y(x)+2}{\sqrt[3]{2} \sqrt[3]{-\frac{1}{(x-1)^3}(x-1)((x-1)y(x)+1)}} + (-2)^{2/3} \right) \left( \left( \frac{\sqrt[3]{-1}(-\frac{1}{(x-1)^3}(x-1)((x-1)y(x)+1)}{\sqrt[3]{-\frac{1}{(x-1)^3}(x-1)((x-1)y(x)+1)}}} \right) \right)}{\right]$$

✓ **Maple** : cpu = 0.115 (sec), leaf count = 32

$$\left\{ y(x) = -2 \frac{1}{x} e^{-\frac{\text{lambertW}\left(-2 \frac{(x-1)(e^{-C1})^3 e^{-1}}{x}\right) + 3 - C1 - 1}{3}} \right\}$$

2.794 ODE No. 794

---

$$y'(x) = \frac{y(x)}{x(x^3y(x)^4 + x^2y(x)^3 + y(x) - 1)}$$

✓ **Mathematica** : cpu = 0.191681 (sec), leaf count = 67

Solve 
$$\left[ \text{RootSum} \left[ \#1^3 y(x)^3 + \#1^2 y(x)^2 + 1 \&, \frac{\#1 y(x) \log(x - \#1) + \log(x - \#1)}{3 \#1 y(x) + 2} \& \right] + y(x) - \log(x) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.41 (sec), leaf count = 32

$$\left\{ -y(x) + \int^{xy(x)} \frac{1}{-a(-a^3 - a^2 + 1)} da - C1 = 0 \right\}$$

2.795 ODE No. 795

---

$$y'(x) = \frac{a^3 + 3a^2x + 3ax^2 + ay(x)^2 + x^3 + y(x)^3 + xy(x)^2}{(a+x)^3}$$

✓ **Mathematica** : cpu = 0.376293 (sec), leaf count = 111

Solve 
$$\left[ -\frac{19}{3} \text{RootSum} \left[ -19 \#1^3 + 6 \sqrt[3]{38} \#1 - 19 \&, \frac{\log \left( \frac{\frac{3y(x)}{(a+x)^3} + \frac{1}{(a+x)^2}}{\sqrt[3]{38} \sqrt[3]{\frac{1}{(a+x)^6}}} - \#1 \right)}{2 \sqrt[3]{38} - 19 \#1^2} \& \right] = \frac{1}{9} 38^{2/3} \left( \frac{1}{(a+x)^6} \right)^{2/3} (a+x) \right]$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 37

$$\left\{ y(x) = -\text{RootOf} \left( -\int^{-Z} (-a^3 - a^2 - a - 1)^{-1} da + \ln(x+a) - C1 \right) (x+a) \right\}$$

2.796 ODE No. 796

---

$$y'(x) = \frac{e^{-\frac{3x^2}{2}} xy(x)^3}{3 \left( e^{\frac{3x^2}{2}} y(x) + 3e^{\frac{3x^2}{2}} + 3y(x) \right)}$$

✓ **Mathematica** : cpu = 12.2831 (sec), leaf count = 102

Solve 
$$\left[ \frac{1}{62} \left( -31 \log \left( 9e^{\frac{3x^2}{2}} (y(x) + 3) y(x) + 3e^{3x^2} (y(x) + 3)^2 - y(x)^2 \right) + 6\sqrt{93} \tanh^{-1} \left( \frac{\sqrt{\frac{3}{31}} \left( 2e^{\frac{3x^2}{2}} (y(x) + 3) \right)}{y(x)} \right) \right) \right]$$

✓ **Maple** : cpu = 0.88 (sec), leaf count = 143

$$\left\{ y(x) = \text{RootOf} \left( \left( 7e^{3x^2 + \text{RootOf} \left( (e^{3/2} x^2)^2 \left( 217 \left( \tanh \left( \frac{(-C1 - 5 - Z)\sqrt{93}}{90} \right) \right)^2 e^{3x^2 - Z} + 42 \tanh \left( \frac{(-C1 - 5 - Z)\sqrt{93}}{90} \right) \right) \sqrt{93} e^{3x^2 - Z} + 18 \right)} \right) \right)$$

2.797 ODE No. 797

$$y'(x) = \frac{y(x) \left( x^3 y(x) \cosh \left( \frac{x+1}{x-1} \right) + x^2 y(x) \cosh \left( \frac{x+1}{x-1} \right) - x^2 \cosh \left( \frac{x+1}{x-1} \right) - x \cosh \left( \frac{x+1}{x-1} \right) - 1 \right)}{x}$$

✓ **Mathematica** : cpu = 1.90146 (sec), leaf count = 349

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp \left( \frac{(3e^2-1)\text{Chi}\left(\frac{2}{x-1}\right)}{e} + \frac{(1+3e^2)\text{Shi}\left(\frac{2}{x-1}\right)}{e} - \frac{1}{4}ex^2 \sinh \left( \frac{2}{x-1} \right) + \frac{x^2 \sinh \left( \frac{2}{x-1} \right)}{4e} - \frac{1}{4}ex^2 \cosh \left( \frac{2}{x-1} \right) - \frac{x^2 \cosh \left( \frac{2}{x-1} \right)}{4e} \right)}{x \left( \exp \left( \frac{(3e^2-1)\text{Chi}\left(\frac{2}{x-1}\right)}{e} + \frac{(1+3e^2)\text{Shi}\left(\frac{2}{x-1}\right)}{e} - \frac{1}{4}ex^2 \sinh \left( \frac{2}{x-1} \right) + \frac{x^2 \sinh \left( \frac{2}{x-1} \right)}{4e} - \frac{1}{4}ex^2 \cosh \left( \frac{2}{x-1} \right) - \frac{x^2 \cosh \left( \frac{2}{x-1} \right)}{4e} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.258 (sec), leaf count = 168

$$\left\{ y(x) = \frac{1}{x} \left( e^{\frac{x^2-1}{4}} e^{\frac{-1-x}{x-1}} + \frac{x^2+4x-5}{4} e^{\frac{1+x}{x-1}} - Ei(1, 2(x-1)^{-1}) e^{-1+3} Ei(1, -2(x-1)^{-1}) e \right)^{-1} \left( -C1 + \int -\cosh \left( \frac{1+x}{x-1} \right) e^{\frac{-x^2}{4}} \right)$$

2.798 ODE No. 798

$$y'(x) = \frac{y(x)(y(x) + x + 1)}{(x + 1)(2y(x)^3 + y(x) + x)}$$

✓ **Mathematica** : cpu = 0.583794 (sec), leaf count = 27

$$\text{Solve} \left[ y(x)^2 - \frac{x}{y(x)} + \log(y(x)) - \log(x + 1) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.125 (sec), leaf count = 30

$$\left\{ y(x) = e^{\text{RootOf}\left(-\left(e^{-Z}\right)^3 + \ln(1+x)e^{-Z} + e^{-Z} - C1 - Z e^{-Z} + x\right)} \right\}$$

2.799 ODE No. 799

$$y'(x) = \frac{y(x) \left( e^{\frac{x+1}{x-1}} x^3 y(x) + e^{\frac{x+1}{x-1}} x^2 y(x) - e^{\frac{x+1}{x-1}} x^2 - e^{\frac{x+1}{x-1}} x - 1 \right)}{x}$$

✓ **Mathematica** : cpu = 1.06124 (sec), leaf count = 126

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp \left( 6e\text{Ei} \left( \frac{2}{x-1} \right) + \frac{1}{2}e^{\frac{x}{x-1} + \frac{1}{x-1}} \left( x^2 + 4x - 5 \right) - e^{\frac{2}{x-1}} \left( \frac{1}{2}e(x-1)^2 + 3e(x-1) \right) \right)}{x \left( c_1 e^{\frac{1}{2}e^{\frac{x}{x-1} + \frac{1}{x-1}} (x^2 + 4x - 5)} + e^{6e\text{Ei} \left( \frac{2}{x-1} \right)} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.236 (sec), leaf count = 147

$$\left\{ y(x) = \frac{1}{x} e^{\frac{5}{2}} e^{\frac{1+x}{x-1}} e^{-\frac{x^2}{2}} e^{\frac{1+x}{x-1}} \left( e^{Ei(1, -2(x-1)^{-1})} e \right)^{-6} \left( e^{xe^{\frac{1+x}{x-1}}} \right)^{-2} \left( \int -(1+x) e^{\frac{1+x}{x-1}} e^{\frac{5}{2}} e^{\frac{1+x}{x-1}} e^{-\frac{x^2}{2}} e^{\frac{1+x}{x-1}} \left( e^{Ei(1, -2(x-1)^{-1})} e \right)^{-6} \right)$$

2.800 ODE No. 800

$$y'(x) = \frac{-b^3 + 6b^2x - 12bx^2 - 4by(x)^2 + 8x^3 + 8y(x)^3 + 8xy(x)^2}{(2x - b)^3}$$

✓ **Mathematica** : cpu = 0.409432 (sec), leaf count = 128

$$\text{Solve} \left[ -\frac{19}{3} \text{RootSum} \left[ -19\#1^3 + 6\sqrt[3]{38}\#1 - 19\&, \frac{\log \left( \frac{\frac{4}{(b-2x)^2} - \frac{24y(x)}{(b-2x)^3}}{4\sqrt[3]{38}\sqrt[3]{\frac{1}{(b-2x)^6}}} - \#1 \right)}{2\sqrt[3]{38} - 19\#1^2} \right] \& \right] = \frac{1}{9} 38^{2/3} \left( \frac{1}{(b-2x)^6} \right)^{2/3} (b-2x)$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 41

$$\left\{ y(x) = \frac{\text{RootOf} \left( -\int^{-Z} (-a^3 - a^2 - a - 1)^{-1} d_a + \ln(-2x + b) + C1 \right) (-2x + b)}{2} \right\}$$

2.801 ODE No. 801

---

$$y'(x) = \frac{1}{2}e^{\frac{x^2}{4}} \left( 2e^{-\frac{3x^2}{4}}y(x)^3 + 2e^{-\frac{x^2}{2}}y(x)^2 + e^{-\frac{x^2}{4}}xy(x) + 2 \right)$$

✓ **Mathematica** : cpu = 0.361924 (sec), leaf count = 126

$$\text{Solve} \left[ -\frac{29}{3} \text{RootSum} \left[ -29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log \left( \frac{3e^{-\frac{x^2}{2}}y(x)+e^{-\frac{x^2}{4}}}{\sqrt[3]{29}\sqrt[3]{e^{-\frac{3x^2}{4}}}} - \#1 \right)}{\sqrt[3]{29} - 29\#1^2} \& \right] = c_1 + \frac{1}{9}29^{2/3}e^{\frac{x^2}{2}} \left( e^{-\frac{3x^2}{4}} \right)^2, \right]$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 63

$$\left\{ y(x) = \frac{1}{9} \left( -3e^{-1/4x^2}e^{1/4x^2} + 29 \text{RootOf} \left( -81 \int^{-Z} (841\_a^3 - 27\_a + 27)^{-1} d\_a + x + 3\_C1 \right) \right) \left( e^{\frac{x^2}{4}} \right)^{-} \right\}$$

2.802 ODE No. 802

---

$$y'(x) = \frac{-F1 \left( y(x) + \frac{1}{x} \right) + \frac{1}{x}}{x}$$

✓ **Mathematica** : cpu = 0.121728 (sec), leaf count = 101

$$\text{Solve} \left[ \int_1^{y(x)} \frac{-F1 \left( K[2] + \frac{1}{x} \right) \int_1^x -\frac{F1' \left( K[2] + \frac{1}{K[1]} \right)}{K[1]^2 \left( -F1 \left( K[2] + \frac{1}{K[1]} \right) \right)^2} dK[1] + 1}{-F1 \left( K[2] + \frac{1}{x} \right)} dK[2] + \int_1^x \left( \frac{1}{K[1]} + \frac{1}{-F1 \left( y(x) + \frac{1}{K[1]} \right) K[1]} \right) dK[1] = c_1, \right]$$

✓ **Maple** : cpu = 0.087 (sec), leaf count = 27

$$\left\{ y(x) = \frac{\text{RootOf} \left( -\ln(x) + \int^{-Z} (-F1(\_a))^{-1} d\_a + \_C1 \right) x - 1}{x} \right\}$$

2.803 ODE No. 803

---

$$y'(x) = \frac{-F1 \left( y(x)^2 - 2\log(x) \right)}{x\sqrt{y(x)^2}}$$

✓ **Mathematica** : cpu = 0.291198 (sec), leaf count = 637

$$\text{Solve} \left[ \int_1^{y(x)} \left( \frac{K[2]}{(-F1(K[2]^2 - 2\log(x)) - 1)(-F1(K[2]^2 - 2\log(x)) + 1)} - \int_1^x \left( \frac{2K[2]_F1'(K[2]^2 - 2\log(x))}{K[1](-F1(K[2]^2 - 2\log(K[1])) - 1)} \right) dK[1] \right) dK[2] = c_1, \right]$$

✓ **Maple** : cpu = 0.299 (sec), leaf count = 65

$$\left\{ y(x) = \sqrt{2\ln(x) + 2\text{RootOf} \left( \ln(x) - \int^{-Z} (-F1(2\_a) - 1)^{-1} d\_a + \_C1 \right)}, y(x) = -\sqrt{2\ln(x) + 2\text{RootOf} \left( \ln(x) - \int^{-Z} (-F1(2\_a) - 1)^{-1} d\_a + \_C1 \right)} \right\}$$

2.804 ODE No. 804

---

$$y'(x) = \frac{\frac{1}{2}x^4\cos(2y(x)) + \frac{x^4}{2} - \frac{1}{2}x\sin(2y(x)) - \frac{1}{2}\sin(2y(x))}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.608996 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow \tan^{-1} \left( \frac{-12c_1 + 3x^4 - 4x^3 + 6x^2 - 12x + 12\log(x+1) - 25}{12x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.796 (sec), leaf count = 38

$$\left\{ y(x) = \arctan \left( \frac{3x^4 - 4x^3 + 6x^2 + 12\ln(1+x) - 12\_C1 - 12x}{12x} \right) \right\}$$

2.805 ODE No. 805

---

$$y'(x) = \frac{x^4 \sqrt{x^2 + y(x)^2} + xy(x) + y(x)}{x(x + 1)}$$

✓ **Mathematica** : cpu = 0.163177 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow x \sinh \left( \frac{1}{6} (6c_1 + 2x^3 - 3x^2 + 6x - 6 \log(x + 1) + 11) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.539 (sec), leaf count = 42

$$\left\{ \ln \left( y(x) + \sqrt{(y(x))^2 + x^2} \right) - \frac{x^3}{3} + \frac{x^2}{2} - x + \ln(1 + x) - \ln(x) - C1 = 0 \right\}$$

2.806 ODE No. 806

---

$$y'(x) = \frac{-\frac{1}{2}x \sin(2y(x)) - \frac{1}{2} \sin(2y(x)) + \frac{1}{2}x \cos(2y(x)) + \frac{x}{2}}{x(x + 1)}$$

✓ **Mathematica** : cpu = 0.431649 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow \tan^{-1} \left( \frac{-c_1 + x - \log(x + 1)}{x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.44 (sec), leaf count = 22

$$\left\{ y(x) = -\arctan \left( \frac{\ln(1 + x) - x - C1}{x} \right) \right\}$$

2.807 ODE No. 807

---

$$y'(x) = -\frac{1}{-e^{y(x)}y(x)\_F1(y(x) - \log(x)) - x}$$

✗ **Mathematica** : cpu = 1.34888 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[1][y][x] == -(-x - E^y[x]\*y[x]\*\_F1[-Log[x] + y[x]])^(-1), y[x], x]  
✓ **Maple** : cpu = 0.45 (sec), leaf count = 43

$$\left\{ \frac{(\ln(x))^2}{2} - y(x) \ln(x) - \int^{y(x) - \ln(x)} \frac{\_F1(\_a) \_a + e^{-\_a}}{\_F1(\_a)} d\_a - C1 = 0 \right\}$$

2.808 ODE No. 808

---

$$y'(x) = \frac{(y(x) + 1)(2y(x) + 1)}{x(2xy(x) - 2y(x) + x - 2)}$$

✓ **Mathematica** : cpu = 1.46902 (sec), leaf count = 149

Solve 
$$\left[ \frac{2^{2/3} \left( x \log \left( -\frac{6 \cdot 2^{2/3}(y(x)+1)}{2(x-1)y(x)+x-2} \right) - x \log \left( \frac{3 \cdot 2^{2/3}(2xy(x)+x)}{2(x-1)y(x)+x-2} \right) + 2xy(x) \left( \log \left( -\frac{6 \cdot 2^{2/3}(y(x)+1)}{2(x-1)y(x)+x-2} \right) - \log \left( \frac{3 \cdot 2^{2/3}(2xy(x)+x)}{2(x-1)y(x)+x-2} \right) \right) \right)}{9(2xy(x) + x)} \right]$$

✓ **Maple** : cpu = 0.07 (sec), leaf count = 45

$$\left\{ y(x) = 1 \left( -x \operatorname{lambertW} \left( \frac{1}{xe^{x^{-1}} - C1} \right) - 2 \right) \left( 2x \operatorname{lambertW} \left( \frac{1}{xe^{x^{-1}} - C1} \right) + 2 \right)^{-1} \right\}$$

2.809 ODE No. 809

---

$$y'(x) = \frac{64x^3 - 240x^2 + 64xy(x)^2 + 64y(x)^3 - 80y(x)^2 + 300x - 125}{(4x - 5)^3}$$

✓ **Mathematica** : cpu = 0.294911 (sec), leaf count = 128

Solve 
$$\left[ -\frac{19}{3} \operatorname{RootSum} \left[ -19\#1^3 + 6\sqrt[3]{38}\#1 - 19\&, \frac{\log \left( \frac{\frac{192y(x)}{(4x-5)^3} + \frac{16}{(4x-5)^2}}{16\sqrt[3]{38}\sqrt[3]{\frac{1}{(4x-5)^6}}} - \#1 \right)}{2\sqrt[3]{38} - 19\#1^2} \& \right] = c_1 + \frac{1}{9} 38^{2/3} \left( \frac{1}{(5 - 4x)^6} \right)^2 \right]$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 41

$$\left\{y(x)=-\frac{RootOf\left(-\int^{-Z}(\_a^3-\_a^2-\_a-1)^{-1}d\_a+\ln(4x-5)+\_C1\right)(4x-5)}{4}\right\}$$

**2.810 ODE No. 810**

---

$$y'(x)=\frac{x^2\log^2(x)+y(x)^2+y(x)-2xy(x)\log(x)+x}{x}$$

✓ **Mathematica** : cpu = 0.10567 (sec), leaf count = 40

$$\left\{\left\{y(x)\rightarrow\frac{1}{\frac{c_1}{x}-1}-\frac{1}{2}x^2\left(\frac{1-2x\log(x)}{x^2}-\frac{1}{x^2}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 16

$$\left\{y(x)=\left(\ln(x)+(\_C1-x)^{-1}\right)x\right\}$$

**2.811 ODE No. 811**

---

$$y'(x)=\frac{x^4+x^3e^{y(x)}+xy(x)+e^{y(x)}y(x)-x\log\left(e^{y(x)}+x\right)-e^{y(x)}\log\left(e^{y(x)}+x\right)+x}{x^2}$$

✓ **Mathematica** : cpu = 1.75427 (sec), leaf count = 33

$$\left\{\left\{y(x)\rightarrow-\log\left(\frac{e^{-c_1x-\frac{x^3}{2}}}{x}-\frac{1}{x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 1.545 (sec), leaf count = 32

$$\left\{y(x)=\frac{x^3}{2}+x\_C1+\ln\left(-x\left(-1+\mathrm{e}^{\frac{x^3}{2}}\mathrm{e}^{x\_C1}\right)^{-1}\right)\right\}$$

**2.812 ODE No. 812**

---

$$y'(x)=x^3\sqrt{x^3-6y(x)}+\sqrt{x^3-6y(x)}+\frac{x^2}{2}+x^2\sqrt{x^3-6y(x)}$$

✓ **Mathematica** : cpu = 0.392131 (sec), leaf count = 78

$$\left\{\left\{y(x)\rightarrow\frac{1}{96}\left(72c_1x^4+96c_1x^3+288c_1x-144c_1^2+24c_1-9x^8-24x^7-16x^6-72x^5-102x^4+8x^3-144x^2-\right.\right.\right.$$

✓ **Maple** : cpu = 0.197 (sec), leaf count = 30

$$\left\{-\_C1-\frac{3x^4}{4}-x^3-3x-\sqrt{x^3-6y(x)}=0\right\}$$

**2.813 ODE No. 813**

---

$$y'(x)=\frac{1}{2}\sqrt{a}\left(2\sqrt{ax^4+8y(x)}-\sqrt{a}x^3+2x^3\sqrt{ax^4+8y(x)}+2x^2\sqrt{ax^4+8y(x)}\right)$$

✓ **Mathematica** : cpu = 0.547703 (sec), leaf count = 93

$$\left\{\left\{y(x)\rightarrow\frac{-1152ac_1x^4-1536ac_1x^3-4608ac_1x+2304ac_1^2-288ac_1+144ax^8+384ax^7+256ax^6+1152ax^5-\right.}{1152}\right.$$

✓ **Maple** : cpu = 0.372 (sec), leaf count = 40

$$\left\{\frac{1}{4}\sqrt{ax^4+8y(x)}+\frac{-3x^4-4x^3-12x}{12}\sqrt{a}-\_C1=0\right\}$$

2.814 ODE No. 814

---

$$y'(x) = \frac{y(x) \left( x^7 y(x)^2 - 3 x^3 y(x) - 3 \right)}{x \left( x^3 y(x) + 1 \right)}$$

✓ **Mathematica** : cpu = 0.128825 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow \frac{x}{\frac{\sqrt{x(c_1-2x)+x}}{\sqrt{\frac{1}{x^7}}} - x^4} \right\}, \left\{ y(x) \rightarrow -\frac{x}{\frac{\sqrt{x(c_1-2x)+x}}{\sqrt{\frac{1}{x^7}}} + x^4} \right\} \right\}$$

✓ **Maple** : cpu = 0.028 (sec), leaf count = 38

$$\left\{ y(x) = \frac{1}{x^3} \left( \sqrt{-C1 - 2x} - 1 \right)^{-1}, y(x) = -\frac{1}{x^3} \left( \sqrt{-C1 - 2x} + 1 \right)^{-1} \right\}$$

2.815 ODE No. 815

---

$$y'(x) = \frac{e^{3x^2} x (y(x) + 3)^3}{81 \left( e^{\frac{3x^2}{2}} y(x) + 3 e^{\frac{3x^2}{2}} + 3 y(x) \right)}$$

✓ **Mathematica** : cpu = 12.9188 (sec), leaf count = 99

Solve  $\left[ \frac{1}{186} \left( \left( 31 + 3\sqrt{93} \right) \log \left( 9 \left( 9 + \sqrt{93} \right) y(x) - 2 e^{\frac{3x^2}{2}} (y(x) + 3) \right) + \left( 31 - 3\sqrt{93} \right) \log \left( 2 e^{\frac{3x^2}{2}} (y(x) + 3) + 9 \right) \right) \right]$

✓ **Maple** : cpu = 0.604 (sec), leaf count = 168

$$\left\{ -10 \ln \left( \frac{10 e^{3/2 x^2} (3 + y(x))}{27 e^{3/2 x^2} + 9 e^{3/2 x^2} y(x) + 27 y(x)} \right) + 5 \ln \left( \frac{100 (3 + y(x))^2 \left( e^{3/2 x^2} \right)^2 + \left( -8100 (y(x))^2 - 24300 y(x) \right)}{189 \left( e^{3/2 x^2} (3 + y(x)) + 3 y(x) \right)} \right) \right\}$$

2.816 ODE No. 816

---

$$y'(x) = \frac{x(x - y(x))^3(y(x) + x)^3}{y(x)(x^2 - y(x)^2 - 1)}$$

✓ **Mathematica** : cpu = 0.239307 (sec), leaf count = 74

Solve  $\left[ \frac{1}{2} \left( \text{RootSum} \left[ \#1^3 - \#1 + 1 \&, \frac{\#1 \log(-\#1 + x^2 - y(x)^2) - \log(-\#1 + x^2 - y(x)^2)}{3\#1^2 - 1} \& \right] + x^2 \right) = c_1, y(x) \right]$

✓ **Maple** : cpu = 0.444 (sec), leaf count = 190

$$\left\{ \int_{-b}^x \frac{(\_a - y(x))^3 (y(x) + \_a)^3 \_a}{\_a^6 - 3\_a^4 (y(x))^2 + 3\_a^2 (y(x))^4 - (y(x))^6 - \_a^2 + (y(x))^2 + 1} d\_a + \int^{y(x)} -\frac{(\_f^2}{-\_f^6 + 3\_f^4 x^2 - 3} \right\}$$

2.817 ODE No. 817

---

$$y'(x) = \frac{\csc(y(x)) \left( \frac{1}{2} x^3 \log(x) \cos(2y(x)) + \frac{1}{2} x^3 \log(x) - \cos(y(x)) \right)}{x \log(x)}$$

✓ **Mathematica** : cpu = 0.55245 (sec), leaf count = 63

$$\left\{ \left\{ y(x) \rightarrow -\sec^{-1} \left( \frac{-9c_1 - x^3 + 3x^3 \log(x)}{9 \log(x)} \right) \right\}, \left\{ y(x) \rightarrow \sec^{-1} \left( \frac{-9c_1 - x^3 + 3x^3 \log(x)}{9 \log(x)} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.518 (sec), leaf count = 27

$$\left\{ y(x) = \arccos \left( 9 \frac{\ln(x)}{3 x^3 \ln(x) - x^3 + 9 \_C1} \right) \right\}$$



2.818 ODE No. 818

---

$$y'(x) = \frac{y(x)}{x(xy(x)^4 + xy(x)^3 + xy(x) - 1)}$$

✓ **Mathematica** : cpu = 0.18595 (sec), leaf count = 34

$$\text{Solve} \left[ \frac{y(x)^3}{3} + \frac{y(x)^2}{2} + \frac{1}{xy(x)} + \log(y(x)) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.149 (sec), leaf count = 34

$$\left\{ y(x) = e^{\text{RootOf}(-2x(e^{-Z})^4 - 3x(e^{-Z})^3 + 6 - C1xe^{-Z} - 6 - Ze^{-Z} - 6)} \right\}$$

2.819 ODE No. 819

---

$$y'(x) = x^2\sqrt{x^2 + 3y(x)} + \sqrt{x^2 + 3y(x)} + x^3\sqrt{x^2 + 3y(x)} - \frac{2x}{3}$$

✓ **Mathematica** : cpu = 0.285183 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{192} (-72c_1x^4 - 96c_1x^3 - 288c_1x + 144c_1^2 + 9x^8 + 24x^7 + 16x^6 + 72x^5 + 96x^4 + 80x^2) \right\} \right\}$$

✓ **Maple** : cpu = 0.183 (sec), leaf count = 30

$$\left\{ -C1 + \frac{3x^4}{8} + \frac{x^3}{2} + \frac{3x}{2} - \sqrt{x^2 + 3y(x)} = 0 \right\}$$

2.820 ODE No. 820

---

$$y'(x) = \frac{\csc(y(x)) \left( \frac{1}{2}x^2 \log(x) \cos(2y(x)) + \frac{1}{2}x^2 \log(x) - \cos(y(x)) \right)}{x \log(x)}$$

✓ **Mathematica** : cpu = 0.529854 (sec), leaf count = 63

$$\left\{ \left\{ y(x) \rightarrow -\sec^{-1} \left( \frac{-4c_1 - x^2 + 2x^2 \log(x)}{4 \log(x)} \right) \right\}, \left\{ y(x) \rightarrow \sec^{-1} \left( \frac{-4c_1 - x^2 + 2x^2 \log(x)}{4 \log(x)} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.482 (sec), leaf count = 27

$$\left\{ y(x) = \arccos \left( 4 \frac{\ln(x)}{2x^2 \ln(x) - x^2 + 4 - C1} \right) \right\}$$

2.821 ODE No. 821

---

$$y'(x) = \frac{y(x)(xy(x) + 1)}{x(x^3y(x)^4 - xy(x) - 1)}$$

✓ **Mathematica** : cpu = 0.234498 (sec), leaf count = 2093

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{4} - \frac{1}{2} \sqrt{\frac{c_1^2}{4} + \frac{\sqrt[3]{1944c_1^2x^6 + 1458x^5 + \sqrt{(1944c_1^2x^6 + 1458x^5)^2 - 4(54c_1x^4 + 144x^3)^3}}}{18\sqrt[3]{2}x^3} + \frac{\sqrt[3]{19}}{x^3\sqrt[3]{19}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.122 (sec), leaf count = 27

$$\left\{ -\frac{1}{2x^2(y(x))^2} - \frac{1}{3x^3(y(x))^3} - y(x) + -C1 = 0 \right\}$$

2.822 ODE No. 822

---

$$y'(x) = \frac{1}{4}x \left( -4e^{-x^2} x^2 y(x) - 4e^{-x^2} x^2 + 4e^{-x^2} + e^{-2x^2} x^4 + 4y(x)^2 \right)$$

✓ **Mathematica** : cpu = 0.301187 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 - \frac{x^2}{2}} + \frac{1}{2} e^{-x^2} x^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.092 (sec), leaf count = 25

$$\left\{ y(x) = \frac{x^2 e^{-x^2}}{2} + \left( -C1 - \frac{x^2}{2} \right)^{-1} \right\}$$

2.823 ODE No. 823

---

$$y'(x) = \frac{y(x)(y(x) + x)}{x(y(x)^4 + y(x)^3 + y(x) + x)}$$

✓ **Mathematica** : cpu = 0.418963 (sec), leaf count = 39

$$\text{Solve} \left[ \frac{y(x)^3}{3} + \frac{y(x)^2}{2} + \log(y(x)) - \frac{y(x) \log(x) + x}{y(x)} = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.137 (sec), leaf count = 38

$$\left\{ y(x) = e^{\text{RootOf} \left( -2(e^{-Z})^4 - 3(e^{-Z})^3 + 6e^{-Z} \ln(x) + 6e^{-Z} C1 - 6e^{-Z} + 6x \right)} \right\}$$

2.824 ODE No. 824

---

$$y'(x) = \frac{y(x)(x^3 + x^2 y(x) + y(x)^2)}{(x-1)x^2(y(x) + x)}$$

✓ **Mathematica** : cpu = 0.201346 (sec), leaf count = 68

$$\text{Solve} \left[ -\frac{1}{2} \log \left( \frac{y(x)^2}{x^2} + \frac{y(x)}{x} + 1 \right) + \log \left( \frac{y(x)}{x} \right) + \frac{\tan^{-1} \left( \frac{\frac{2y(x)}{x} + 1}{\sqrt{3}} \right)}{\sqrt{3}} = c_1 + \log(1-x) - \log(x), y(x) \right]$$

✓ **Maple** : cpu = 0.341 (sec), leaf count = 61

$$\left\{ \ln \left( \frac{y(x)}{x} \right) - \frac{1}{2} \ln \left( \frac{(y(x))^2 + xy(x) + x^2}{x^2} \right) + \frac{\sqrt{3}}{3} \arctan \left( \frac{\sqrt{3}(x + 2y(x))}{3x} \right) - \ln(x-1) + \ln(x) - C1 = 0 \right\}$$

2.825 ODE No. 825

---

$$y'(x) = \frac{x \left( x^2 y(x)^3 + (x^2 + 1)^{3/2} y(x)^2 + x^2 (x^2 + 1)^{3/2} + (x^2 + 1)^{3/2} + y(x)^3 \right)}{(x^2 + 1)^3}$$

✓ **Mathematica** : cpu = 0.412246 (sec), leaf count = 148

$$\text{Solve} \left[ -\frac{19}{3} \text{RootSum} \left[ -19\#1^3 + 6\sqrt[3]{38}\#1 - 19\&, \frac{\log \left( \frac{\frac{3xy(x)}{(x^2+1)^2} + \frac{x}{(x^2+1)^{3/2}}}{\sqrt[3]{38}\sqrt[3]{\frac{x^3}{(x^2+1)^{9/2}}}} - \#1 \right)}{2\sqrt[3]{38} - 19\#1^2} \& \right] = c_1 + \frac{19^{2/3} \left( \frac{x^3}{(x^2+1)^{9/2}} \right)^2}{9} \right]$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 48

$$\left\{ y(x) = \frac{19 \text{RootOf} \left( -1296 \int^{-Z} (361 - a^3 - 432 - a + 432)^{-1} d_- a + 2 \ln(x^2 + 1) + 3 - C1 \right) - 6}{18} \sqrt{x^2 + 1} \right\}$$

**2.826 ODE No. 826**

---

$$y'(x) = \frac{y(x) \left( 3xy(x)^2 + 3y(x)^2 + x \right)}{x(x+1) \left( 6y(x)^2 + x \right)}$$

✓ **Mathematica** : cpu = 0.557672 (sec), leaf count = 70

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{x} \sqrt{W\left(\frac{6e^{2c_1}x}{(x+1)^2}\right)}}{\sqrt{6}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{x} \sqrt{W\left(\frac{6e^{2c_1}x}{(x+1)^2}\right)}}{\sqrt{6}} \right\} \right\}$$

✓ **Maple** : cpu = 0.255 (sec), leaf count = 51

$$\left\{ \left( (y(x))^{-2} + 6x^{-1} \right)^{-1} = \frac{x}{54} \left( e^{\text{RootOf}\left(-e^{-Z} \ln\left(\frac{(1+x)^2(e^{-Z}+9)}{2x}\right) + 3e^{-Z} - C1 + -Ze^{-Z} + 9\right)} + 9 \right) \right\}$$

**2.827 ODE No. 827**

---

$$y'(x) = \frac{x^2y(x)\sqrt{x^2+y(x)^2} + x^3\left(-\sqrt{x^2+y(x)^2}\right) + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.189673 (sec), leaf count = 221

$$\left\{ \left\{ y(x) \rightarrow \frac{x - 2\sqrt{x^2 \tanh^2\left(\frac{1}{3}(-3\sqrt{2}c_1 - \sqrt{2}x^3)\right)} - x^2 \tanh^4\left(\frac{1}{3}(-3\sqrt{2}c_1 - \sqrt{2}x^3)\right)}{2 \tanh^2\left(\frac{1}{3}(-3\sqrt{2}c_1 - \sqrt{2}x^3)\right) - 1} \right\}, \left\{ y(x) \rightarrow \frac{2\sqrt{x^2 \tanh^2\left(\frac{1}{3}(-3\sqrt{2}c_1 - \sqrt{2}x^3)\right)}}{2 \tanh^2\left(\frac{1}{3}(-3\sqrt{2}c_1 - \sqrt{2}x^3)\right) - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.169 (sec), leaf count = 49

$$\left\{ \ln\left(2 \frac{x\left(\sqrt{2(y(x))^2 + 2x^2} + y(x) + x\right)}{y(x) - x}\right) + \frac{\sqrt{2}x^3}{3} - \ln(x) - C1 = 0 \right\}$$

**2.828 ODE No. 828**

---

$$y'(x) = \frac{(y(x)+1)(2y(x)+1)}{x(2xy(x)^4+xy(x)^3-2y(x)-2)}$$

✓ **Mathematica** : cpu = 0.407943 (sec), leaf count = 56

$$\text{Solve}\left[-\frac{1}{8}y(x)^2 + \frac{3y(x)}{8} - \frac{1}{2x(2y(x)+1)} - \frac{1}{2}\log(y(x)+1) + \frac{1}{16}\log(2y(x)+1) = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.217 (sec), leaf count = 54

$$\left\{ y(x) = \frac{1}{2}e^{\text{RootOf}\left(x(e^{-Z})^3 - 8x(e^{-Z})^2 + 16\ln(1/2e^{-Z}+1/2)xe^{-Z} + 8 - C1xe^{-Z} - 2Ze^{-Z} + 7e^{-Z}x + 16\right)} - \frac{1}{2} \right\}$$

**2.829 ODE No. 829**

---

$$y'(x) = \frac{x^6\sqrt{4x^2y(x)+1} + x^5\sqrt{4x^2y(x)+1} + x^3\sqrt{4x^2y(x)+1} + \frac{1}{2}}{x^3}$$

✓ **Mathematica** : cpu = 0.422835 (sec), leaf count = 74

$$\left\{ \left\{ y(x) \rightarrow \frac{-160c_1x^7 - 200c_1x^6 - 400c_1x^4 + 400c_1^2x^2 + 16x^{12} + 40x^{11} + 25x^{10} + 80x^9 + 100x^8 + 100x^6 - 100}{400x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.231 (sec), leaf count = 34

$$\left\{ -C1 - \frac{1}{x}\sqrt{4x^2y(x)+1} + x^2 + \frac{x^4}{2} + \frac{2x^5}{5} = 0 \right\}$$

## 2.830 ODE No. 830

---

$$y'(x) = \frac{(x - y(x))y(x)}{x(-y(x)^4 - y(x)^3 - y(x) + x)}$$

✓ **Mathematica** : cpu = 0.446631 (sec), leaf count = 37

$$\text{Solve} \left[ -\frac{1}{3}y(x)^3 - \frac{y(x)^2}{2} - \frac{x}{y(x)} - \log(y(x)) + \log(x) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.122 (sec), leaf count = 38

$$\left\{ y(x) = e^{\text{RootOf}\left(2(e^{-Z})^4 + 3(e^{-Z})^3 - 6e^{-Z}\ln(x) + 6e^{-Z}C1 + 6e^{-Z} + 6x\right)} \right\}$$

## 2.831 ODE No. 831

---

$$y'(x) = \frac{x^3\sqrt{4ax - y(x)^2} + x^2\sqrt{4ax - y(x)^2} + \sqrt{4ax - y(x)^2} + 2a}{y(x)}$$

✓ **Mathematica** : cpu = 2.9878 (sec), leaf count = 145

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{12}\sqrt{576ax - 72c_1x^4 - 96c_1x^3 - 288c_1x - 144c_1^2 - 9x^8 - 24x^7 - 16x^6 - 72x^5 - 96x^4 - 144x^2} \right\}, \left\{ \right. \right.$$

✓ **Maple** : cpu = 0.286 (sec), leaf count = 35

$$\left\{ -\sqrt{-(y(x))^2 + 4ax} - \frac{x^4}{4} - \frac{x^3}{3} - x - C1 = 0 \right\}$$

## 2.832 ODE No. 832

---

$$y'(x) = \frac{y(x)(y(x) + x + 1)}{(x + 1)(y(x)^4 + y(x)^3 + y(x)^2 + x)}$$

✓ **Mathematica** : cpu = 2.1788 (sec), leaf count = 2497

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2}\sqrt{\frac{3\sqrt[3]{2}(-8x + 3c_1 + 3\log(x))}{\sqrt[3]{1944(c_1 + \log(x + 1))^2 + 972(c_1 + \log(x + 1)) + 3726x} + \sqrt{(1944(c_1 + \log(x + 1))^2 + 972(c_1 + \log(x + 1)) + 3726x)}}} \right\}, \left\{ \right. \right.$$

✓ **Maple** : cpu = 0.143 (sec), leaf count = 31

$$\left\{ \ln(1 + x) + \frac{x}{y(x)} - \frac{(y(x))^3}{3} - \frac{(y(x))^2}{2} - y(x) + C1 = 0 \right\}$$

## 2.833 ODE No. 833

---

$$y'(x) = \frac{x^4(-\sqrt{x^2 + y(x)^2}) + x^3y(x)\sqrt{x^2 + y(x)^2} + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.19301 (sec), leaf count = 221

$$\left\{ \left\{ y(x) \rightarrow \frac{x - 2\sqrt{x^2 \tanh^2\left(\frac{1}{4}(-4\sqrt{2}c_1 - \sqrt{2}x^4)\right)} - x^2 \tanh^4\left(\frac{1}{4}(-4\sqrt{2}c_1 - \sqrt{2}x^4)\right)}{2 \tanh^2\left(\frac{1}{4}(-4\sqrt{2}c_1 - \sqrt{2}x^4)\right) - 1} \right\}, \left\{ y(x) \rightarrow \frac{2\sqrt{x^2 \tanh^2\left(\frac{1}{4}(-4\sqrt{2}c_1 - \sqrt{2}x^4)\right)}}{2 \tanh^2\left(\frac{1}{4}(-4\sqrt{2}c_1 - \sqrt{2}x^4)\right) - 1} \right\} \right.$$

✓ **Maple** : cpu = 0.124 (sec), leaf count = 49

$$\left\{ \ln\left(2 \frac{x\left(\sqrt{2(y(x))^2 + 2x^2} + y(x) + x\right)}{y(x) - x}\right) + \frac{\sqrt{2}x^4}{4} - \ln(x) - C1 = 0 \right\}$$

**2.834 ODE No. 834**

---

$$y'(x) = \frac{y(x) \left( x^4 + 3xy(x)^2 + 3y(x)^2 \right)}{x(x+1) \left( 6y(x)^2 + x \right)}$$

✓ **Mathematica** : cpu = 0.650709 (sec), leaf count = 90

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{x} \sqrt{W\left(\frac{6(x+1)^2 e^{2c_1 + x^2 - 2x - 3}}{x}\right)}}{\sqrt{6}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{x} \sqrt{W\left(\frac{6(x+1)^2 e^{2c_1 + x^2 - 2x - 3}}{x}\right)}}{\sqrt{6}} \right\} \right\}$$

✓ **Maple** : cpu = 0.333 (sec), leaf count = 60

$$\left\{ \left( (y(x))^{-2} + 6x^{-1} \right)^{-1} = \frac{x}{54} \left( e^{\operatorname{RootOf}\left(x^2 e^{-Z} - e^{-Z} \ln\left(\frac{x(e^{-Z} + 9)}{2(1+x)^2}\right) + 3e^{-Z} - C1 + Z e^{-Z} - 2e^{-Z} x + 9\right)} + 9 \right) \right\}$$

**2.835 ODE No. 835**

---

$$y'(x) = -\frac{1}{x \left( -\sqrt[3]{y(x)^3} \right) {}_2F1\left(y(x)^3 - 3\log(x) - x(y(x)^3)^{2/3}\right)}$$

✗ **Mathematica** : cpu = 1.6724 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[1][y][x] == -(-(x\*(y[x]^3)^(2/3)) - x\*(y[x]^3)^(1/3)\*\_F1[-3\*Log[x] + y[x]^3], y[x], x]  
1), y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x) = -1/(-(y(x)^3)^(2/3)\*x-\_F1(y(x)^3-3\*ln(x))\*(y(x)^3)^(1/3)\*x),y(x))

**2.836 ODE No. 836**

---

$$y'(x) = \frac{(x - y(x))y(x)(y(x) + 1)}{x(xy(x) - y(x) + x)}$$

✓ **Mathematica** : cpu = 10.8515 (sec), leaf count = 379

$$\text{Solve} \left[ \frac{1}{9} 2^{2/3} \left( \frac{\left( 1 - \frac{(x-1)^2 \left( \frac{x^6}{(x-1)^3} \right)^{2/3} ((x+2)y(x)+x)}{x^4 ((x-1)y(x)+x)} \right) \left( \frac{\left( \frac{x^6}{(x-1)^3} \right)^{2/3} (x-1)^2 ((x+2)y(x)+x)}{x^4 ((x-1)y(x)+x)} + 2 \right) \left( \left( 1 - \frac{(x-1)^2 \left( \frac{x^6}{(x-1)^3} \right)^{2/3} ((x+2)y(x)+x)}{x^4 ((x-1)y(x)+x)} \right) \right)}{1} \right]$$

✓ **Maple** : cpu = 0.161 (sec), leaf count = 73

$$\left\{ y(x) = -x e^{\operatorname{RootOf}\left(-\ln\left(\frac{e^{-Z}}{2} + \frac{9}{2}\right) e^{-Z} + 3e^{-Z} - C1 + Z e^{-Z} - e^{-Z} x + 9\right)} \left( -9 + (x-1) e^{\operatorname{RootOf}\left(-\ln\left(\frac{e^{-Z}}{2} + \frac{9}{2}\right) e^{-Z} + 3e^{-Z} - C1 + Z e^{-Z} - e^{-Z} x + 9\right)} \right) \right\}$$

**2.837 ODE No. 837**

---

$$y'(x) = -\frac{1}{-\sqrt[3]{y(x)^3} \log(x) {}_2F1\left(3\operatorname{Ei}(-\log(x)) + y(x)^3\right) - (y(x)^3)^{2/3} \log(x)}$$

✗ **Mathematica** : cpu = 2.0365 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[1][y][x] == -(-(Log[x]\*(y[x]^3)^(2/3)) - Log[x]\*(y[x]^3)^(1/3)\*\_F1[3\*ExpInLog[x]] + y[x]^3)]^(-1), y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x) = -1/(-ln(x)\*(y(x)^3)^(2/3)-\_F1(y(x)^3+3\*Ei(1,-ln(x)))\*ln(x)\*(y(x)^3)^(1/3)),y(x))

2.838 ODE No. 838

---

$$y'(x) = \frac{\frac{8x^{7/2}}{5} + \frac{4x^6}{25} - \frac{4}{5}x^3y(x) + \frac{6x^3}{5} - 4\sqrt{x}y(x) + y(x)^2 + 4x + \sqrt{x}}{x}$$

✓ **Mathematica** : cpu = 0.15508 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 - \log(x)} + \frac{2}{5}\sqrt{x} \left( x^{5/2} + 5 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.088 (sec), leaf count = 25

$$\left\{ y(x) = \frac{2x}{5} \left( x^2 + 5 \frac{1}{\sqrt{x}} \right) + (\_C1 - \ln(x))^{-1} \right\}$$

2.839 ODE No. 839

---

$$y'(x) = \frac{e^{\frac{y(x)}{x}} \left( x^2 + xe^{-\frac{y(x)}{x}} + e^{-\frac{y(x)}{x}} y(x) \right)}{x}$$

✓ **Mathematica** : cpu = 0.266065 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow -x \log \left( \frac{e^{2c_1}}{2x} - \frac{x}{2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 19

$$\left\{ y(x) = \ln \left( 2 \frac{x}{-x^2 + \_C1} \right) x \right\}$$

2.840 ODE No. 840

---

$$y'(x) = \frac{e^{\frac{y(x)}{x}} \left( x^3 + xe^{-\frac{y(x)}{x}} + e^{-\frac{y(x)}{x}} y(x) \right)}{x}$$

✓ **Mathematica** : cpu = 0.286069 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow -x \log \left( \frac{e^{3c_1}}{3x} - \frac{x^2}{3} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 19

$$\left\{ y(x) = \ln \left( 3 \frac{x}{-x^3 + \_C1} \right) x \right\}$$

2.841 ODE No. 841

---

$$y'(x) = \frac{-2a^{3/2}bx^2y(x)^2 + 2a^{3/2}cy(x)^2 + a^{5/2}y(x)^4 + \sqrt{a}b^2x^4 - 2\sqrt{a}bcx^2 + \sqrt{a}c^2 + bx^3}{ax^2y(x)}$$

✓ **Mathematica** : cpu = 1.08269 (sec), leaf count = 236

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{2a^{5/2}bx^2 - 2a^{5/2}c + 4a^3b^2x^3 - 4a^3bcx + a^2x + 4\sqrt{a}b^2c_1x^2 - 4\sqrt{a}bcc_1 + 2bc_1x}}{\sqrt{2}\sqrt{2a^{3/2}bc_1 + a^{7/2} + 2a^4bx}} \right\} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{2a^{5/2}bx^2 - 2a^{5/2}c + 4a^3b^2x^3 - 4a^3bcx + a^2x + 4\sqrt{a}b^2c_1x^2 - 4\sqrt{a}bcc_1 + 2bc_1x}}{\sqrt{2}\sqrt{2a^{3/2}bc_1 + a^{7/2} + 2a^4bx}} \right\}$$

✓ **Maple** : cpu = 0.217 (sec), leaf count = 97

$$\left\{ y(x) = \frac{1}{x\_C1 + 1} \sqrt{\left( (x\_C1 + 1)(bx^2 - c)\sqrt{a} + \frac{x}{2} \right) (x\_C1 + 1) a^{\frac{3}{2}} a^{-\frac{3}{2}}}, y(x) = -2 \frac{\sqrt{((x\_C1 + 1)(bx^2 - c)\sqrt{a} + \frac{x}{2}) (x\_C1 + 1) a^{\frac{3}{2}} a^{-\frac{3}{2}}}}{a^{3/2} (2x)} \right\}$$

2.842 ODE No. 842

---

$$y'(x) = \frac{2x^2y(x)\log^2(x) + x^2y(x)^2\log(x) + x^2\log^3(x) + y(x)}{x\log(x)}$$

✓ **Mathematica** : cpu = 0.209774 (sec), leaf count = 186

$$\left\{ \left\{ y(x) \rightarrow -\frac{c_1 e^{\frac{1}{4}x^2(2\log(x)-1)} \left( \frac{x}{2} + \frac{1}{2}x(2\log(x)-1) \right) + \frac{1}{4}x^2 e^{\frac{1}{4}x^2(2\log(x)-1)}(2\log(x)-1) \left( \frac{x}{2} + \frac{1}{2}x(2\log(x)-1) \right) + \frac{1}{4}x^2 e^{\frac{1}{4}x^2(2\log(x)-1)}(2\log(x)-1)^2 \left( \frac{x}{2} + \frac{1}{2}x(2\log(x)-1) \right) + \frac{1}{4}x^2 e^{\frac{1}{4}x^2(2\log(x)-1)}(2\log(x)-1)^3}{x \left( c_1 e^{\frac{1}{4}x^2(2\log(x)-1)} + \frac{1}{4}x^2 e^{\frac{1}{4}x^2(2\log(x)-1)}(2\log(x)-1) + \frac{1}{4}x^2 e^{\frac{1}{4}x^2(2\log(x)-1)}(2\log(x)-1)^2 + \frac{1}{4}x^2 e^{\frac{1}{4}x^2(2\log(x)-1)}(2\log(x)-1)^3 \right)} \right. \right.$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 43

$$\left\{ y(x) = -\frac{\ln(x) \left( 2x^2 \ln(x) - x^2 + 2\_C1 + 4 \right)}{2x^2 \ln(x) - x^2 + 2\_C1} \right\}$$

2.843 ODE No. 843

---

$$y'(x) = \frac{2x^3y(x)\log^2(x) + x^3y(x)^2\log(x) + x^3\log^3(x) + y(x)}{x\log(x)}$$

✓ **Mathematica** : cpu = 0.211597 (sec), leaf count = 198

$$\left\{ \left\{ y(x) \rightarrow -\frac{c_1 e^{\frac{1}{9}x^3(3\log(x)-1)} \left( \frac{x^2}{3} + \frac{1}{3}x^2(3\log(x)-1) \right) + \frac{1}{9}x^3 e^{\frac{1}{9}x^3(3\log(x)-1)}(3\log(x)-1) \left( \frac{x^2}{3} + \frac{1}{3}x^2(3\log(x)-1) \right) + \frac{1}{9}x^3 e^{\frac{1}{9}x^3(3\log(x)-1)}(3\log(x)-1)^2 \left( \frac{x^2}{3} + \frac{1}{3}x^2(3\log(x)-1) \right) + \frac{1}{9}x^3 e^{\frac{1}{9}x^3(3\log(x)-1)}(3\log(x)-1)^3}{x^2 \left( c_1 e^{\frac{1}{9}x^3(3\log(x)-1)} + \frac{1}{9}x^3 e^{\frac{1}{9}x^3(3\log(x)-1)}(3\log(x)-1) + \frac{1}{9}x^3 e^{\frac{1}{9}x^3(3\log(x)-1)}(3\log(x)-1)^2 + \frac{1}{9}x^3 e^{\frac{1}{9}x^3(3\log(x)-1)}(3\log(x)-1)^3 \right)} \right. \right.$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 43

$$\left\{ y(x) = -\frac{\ln(x) \left( 6x^3 \ln(x) - 2x^3 + 9\_C1 + 18 \right)}{6x^3 \ln(x) - 2x^3 + 9\_C1} \right\}$$

2.844 ODE No. 844

---

$$y'(x) = \frac{y(x)(y(x)+1)(y(x)+x)}{x(xy(x)+y(x)+x)}$$

✓ **Mathematica** : cpu = 13.0211 (sec), leaf count = 386

Solve 
$$\left[ \frac{2^{2/3} \left( 1 - \frac{\left( \frac{x^6}{(x+1)^3} \right)^{2/3} (x+1)^2 ((x-2)y(x)+x)}{x^4 ((x+1)y(x)+x)} \right) \left( \frac{\left( \frac{x^6}{(x+1)^3} \right)^{2/3} (x+1)^2 ((x-2)y(x)+x)}{x^4 ((x+1)y(x)+x)} + 2 \right) \left( \left( 1 - \frac{\left( \frac{x^6}{(x+1)^3} \right)^{2/3} (x+1)^2 ((x-2)y(x)+x)}{x^4 ((x+1)y(x)+x)} \right) \left( \frac{\left( \frac{x^6}{(x+1)^3} \right)^{2/3} (x+1)^2 ((x-2)y(x)+x)}{x^4 ((x+1)y(x)+x)} + 2 \right) + 9 \left( \frac{\left( \frac{x^6}{(x+1)^3} \right)^{2/3} (x+1)^2 ((x-2)y(x)+x)}{x^4 ((x+1)y(x)+x)} + 2 \right)^2 \right]}{x(xy(x)+y(x)+x)}$$

✓ **Maple** : cpu = 0.148 (sec), leaf count = 97

$$\left\{ y(x) = -x e^{\text{RootOf}\left(-\ln\left(\frac{e^{-Z}}{2} + \frac{9}{2}\right) e^{-Z} + 3 e^{-Z} \_C1 + \_Z e^{-Z} + e^{-Z} x + 9\right)} \left( e^{\text{RootOf}\left(-\ln\left(\frac{e^{-Z}}{2} + \frac{9}{2}\right) e^{-Z} + 3 e^{-Z} \_C1 + \_Z e^{-Z} + e^{-Z} x + 9\right)} \right)$$

2.845 ODE No. 845

---

$$y'(x) = \frac{\sqrt{4y(x)^3 - 9x^4} + 3x^3 + x^3 \sqrt{4y(x)^3 - 9x^4} + x^2 \sqrt{4y(x)^3 - 9x^4}}{y(x)^2}$$

✓ **Mathematica** : cpu = 3.22622 (sec), leaf count = 266

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2} \sqrt[3]{-\frac{1}{2} \sqrt[3]{72c_1x^4 + 96c_1x^3 + 288c_1x + 144c_1^2 - 72c_1 + 9x^8 + 24x^7 + 16x^6 + 72x^5 + 114x^4 - 24x^3 - 12x^2 + 4x + 4\_C1}} \right. \right.$$

✓ **Maple** : cpu = 0.184 (sec), leaf count = 44

$$\left\{ \int_{\_b}^{y(x)} -a^2 \frac{1}{\sqrt{-9x^4 + 4\_a^3}} d\_a - \frac{x^4}{4} - \frac{x^3}{3} - x - \_C1 = 0 \right\}$$

2.846 ODE No. 846

---

$$y'(x) = \frac{1}{x^2 \left( - \left( \frac{1}{y(x)} + 1 \right) \right) {}_2F_1 \left( x \left( \frac{1}{y(x)} + 1 \right) \right) + x^2 {}_2F_1 \left( x \left( \frac{1}{y(x)} + 1 \right) \right) + x \left( \frac{1}{y(x)} + 1 \right) - x}$$

✓ **Mathematica** : cpu = 0.892233 (sec), leaf count = 365

$$\text{Solve} \left[ \int_1^{y(x)} \left( \frac{x {}_2F_1 \left( x \left( 1 + \frac{1}{K[2]} \right) \right) - 1}{x {}_2F_1 \left( x \left( 1 + \frac{1}{K[2]} \right) \right) K[2] - K[2] + x {}_2F_1 \left( x \left( 1 + \frac{1}{K[2]} \right) \right)} - \int_1^x \left( \frac{{}_2F_1 \left( K[1] \left( 1 + \frac{1}{K[2]} \right) \right) - \frac{K[1]}{K[2]}}{K[1] \left( K[2] {}_2F_1 \left( K[1] \left( 1 + \frac{1}{K[2]} \right) \right) - K[2]} \right) dx \right) dy(x) = 0 \right]$$

✓ **Maple** : cpu = 0.144 (sec), leaf count = 40

$$\left\{ y(x) = e^{\int_1^x \frac{1}{t} dt} \left( -Z - \int \frac{e^{-Z} x}{e^{-Z} - 1} \frac{1}{({}_2F_1(\frac{1}{a}) - a - 1) - a} d_a + C1 \right) - 1 \right\}$$

2.847 ODE No. 847

---

$$y'(x) = x^2 \sqrt{x^2 - 4y(x) + 2x + 1} + \sqrt{x^2 - 4y(x) + 2x + 1} + x^3 \sqrt{x^2 - 4y(x) + 2x + 1} + \frac{x}{2} + \frac{1}{2}$$

✓ **Mathematica** : cpu = 0.823634 (sec), leaf count = 103

$$\text{Solve} \left[ \frac{x^4}{4} + \frac{x^3}{3} + \frac{1}{2} \sqrt{x^2 - 4y(x) + 2x + 1} + \frac{1}{2} \log \left( \sqrt{x^2 - 4y(x) + 2x + 1} + x + 1 \right) - \frac{1}{2} \tanh^{-1} \left( \frac{2x + 1}{2\sqrt{x^2 - 4y(x) + 2x + 1}} \right) = 0 \right]$$

✓ **Maple** : cpu = 0.215 (sec), leaf count = 34

$$\left\{ -C1 - \frac{x^4}{2} - \frac{2x^3}{3} - 2x - \sqrt{x^2 + 2x + 1 - 4y(x)} = 0 \right\}$$

2.848 ODE No. 848

---

$$y'(x) = {}_2F_1(y(x) - \log(\sinh(x))) + \coth(x)$$

✓ **Mathematica** : cpu = 0.205334 (sec), leaf count = 157

$$\text{Solve} \left[ \int_1^{y(x)} - \frac{{}_2F_1(K[2] - \log(\sinh(x))) \int_1^x \left( \frac{(\coth(K[1]) + {}_2F_1(K[2] - \log(\sinh(K[1]))) {}_2F_1'(K[2] - \log(\sinh(K[1])))}{({}_2F_1(K[2] - \log(\sinh(K[1])))^2} - \frac{{}_2F_1'(K[2] - \log(\sinh(K[1])))}{{}_2F_1(K[2] - \log(\sinh(K[1]))}) \right) dx}{{}_2F_1(K[2] - \log(\sinh(x)))} dy(x) = 0 \right]$$

✓ **Maple** : cpu = 0.404 (sec), leaf count = 27

$$\left\{ \int_b^{y(x)} ({}_2F_1(a - \ln(\sinh(x))))^{-1} d_a - x - C1 = 0 \right\}$$

2.849 ODE No. 849

---

$$y'(x) = x^2 \sqrt{x^2 + 4y(x) - 4x} + \sqrt{x^2 + 4y(x) - 4x} + x^3 \sqrt{x^2 + 4y(x) - 4x} - \frac{x}{2} + 1$$

✓ **Mathematica** : cpu = 0.746434 (sec), leaf count = 102

$$\text{Solve} \left[ \frac{x^4}{4} + \frac{x^3}{3} - \frac{1}{2} \sqrt{x^2 + 4y(x) - 4x} + \log \left( -\sqrt{x^2 + 4y(x) - 4x} - x + 2 \right) - \tanh^{-1} \left( \frac{2x - 4}{2\sqrt{x^2 + 4y(x) - 4x}} \right) = 0 \right]$$

✓ **Maple** : cpu = 0.211 (sec), leaf count = 33

$$\left\{ -C1 + \frac{x^4}{2} + \frac{2x^3}{3} + 2x - \sqrt{x^2 - 4x + 4y(x)} = 0 \right\}$$



2.850 ODE No. 850

---

$$y'(x) = \_F1(y(x) - \log(\sin(x)) + \log(\cos(x) + 1)) + \csc(x)$$

✓ **Mathematica** : cpu = 0.227882 (sec), leaf count = 1485

Solve  $\left[ \int_1^x -\frac{(\cot^2(K[1]) + \csc(K[1]) \cot(K[1]) + 1) \sin(K[1])(\csc(K[1]) + \_F1(\log(\cos(K[1]) + 1) - \log(\sin(K[1]))) + y(x)) \cot(K[1]) + \csc^2(K[1]) + \csc(K[1])}{- \cot^2(K[1]) + \_F1(\log(\cos(K[1]) + 1) - \log(\sin(K[1]))) + y(x)) \cot(K[1]) + \csc^2(K[1]) + \csc(K[1])} dx - x - \_C1 = 0 \right]$

✓ **Maple** : cpu = 0.783 (sec), leaf count = 32

$$\left\{ \int_{-b}^{y(x)} (\_F1(\_a - \ln(\sin(x)) + \ln(\cos(x) + 1)))^{-1} d\_a - x - \_C1 = 0 \right\}$$

2.851 ODE No. 851

---

$$y'(x) = \frac{a^3x^3 + 3a^2bx^2y(x) + a^2bx^2 + 3ab^2xy(x)^2 + 2ab^2xy(x) + b^3y(x)^3 + b^3y(x)^2 + b^3}{b^3}$$

✓ **Mathematica** : cpu = 0.343324 (sec), leaf count = 145

Solve  $\left[ -\frac{1}{3}(27a + 29b)^{2/3} \text{RootSum} \left[ \#1^3(27a + 29b)^{2/3} - 3\#1b^{2/3} + (27a + 29b)^{2/3} \&, \frac{\log \left( \frac{\frac{3ax+b}{b} + 3y(x)}{\sqrt[3]{\frac{27a+29b}{b}}} - \#1 \right)}{b^{2/3} - \#1^2(27a + 29b)^{2/3}} \right] \right]$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 42

$$\left\{ y(x) = \frac{\text{RootOf} \left( \int^{-Z} (\_a^3b + \_a^2b + a + b)^{-1} d\_ab - x + \_C1 \right) b - ax}{b} \right\}$$

2.852 ODE No. 852

---

$$y'(x) = \frac{\alpha^3y(x)^3 + \alpha^3y(x)^2 + \alpha^3 + 3\alpha^2\beta xy(x)^2 + 2\alpha^2\beta xy(x) + 3\alpha\beta^2x^2y(x) + \alpha\beta^2x^2 + \beta^3x^3}{\alpha^3}$$

✓ **Mathematica** : cpu = 0.32207 (sec), leaf count = 145

Solve  $\left[ -\frac{1}{3}(29\alpha + 27\beta)^{2/3} \text{RootSum} \left[ \#1^3(29\alpha + 27\beta)^{2/3} - 3\#1\alpha^{2/3} + (29\alpha + 27\beta)^{2/3} \&, \frac{\log \left( \frac{\frac{\alpha+3\beta x}{\alpha} + 3y(x)}{\sqrt[3]{\frac{29\alpha+27\beta}{\alpha}}} - \#1 \right)}{\alpha^{2/3} - \#1^2(29\alpha + 27\beta)^{2/3}} \right] \right]$

✓ **Maple** : cpu = 0.047 (sec), leaf count = 42

$$\left\{ y(x) = \frac{\text{RootOf} \left( \int^{-Z} (\_a^3\alpha + \_a^2\alpha + \alpha + \beta)^{-1} d\_a\alpha - x + \_C1 \right) \alpha - \beta x}{\alpha} \right\}$$

2.853 ODE No. 853

---

$$y'(x) = \frac{x^3y(x)^3 + 6x^2y(x)^2 + 14xy(x) + 2x + 12}{x^2(xy(x) + x + 2)}$$

✓ **Mathematica** : cpu = 0.155731 (sec), leaf count = 76

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{x^3 \left( \frac{1}{x^3} - \frac{1}{x^3\sqrt{c_1-2x}} \right)} - \frac{x+2}{x} \right\}, \left\{ y(x) \rightarrow \frac{1}{x^3 \left( \frac{1}{x^3\sqrt{c_1-2x}} + \frac{1}{x^3} \right)} - \frac{x+2}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 63

$$\left\{ y(x) = \frac{1}{x} \left( -2\sqrt{\_C1 - 2x} - x - 2 \right) \left( \sqrt{\_C1 - 2x} + 1 \right)^{-1}, y(x) = \frac{1}{x} \left( -2\sqrt{\_C1 - 2x} + x + 2 \right) \left( \sqrt{\_C1 - 2x} + 1 \right)^{-1} \right\}$$

2.854 ODE No. 854

$$y'(x) = \frac{y(x) \left(x^2 \log^2(y(x)) + 2x^2 \log(x) \log(y(x)) + x^2 \log^2(x) + \log(y(x)) + \log(x) - 1\right)}{x}$$

✗ **Mathematica** : cpu = 0.556538 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[1][y][x] == ((-1 + Log[x] + x^2\*Log[x]^2 + Log[y[x]] + 2\*x^2\*Log[x]\*Log[y[x]]), y[x], x]  
✓ **Maple** : cpu = 0.158 (sec), leaf count = 51

$$\left\{y(x) = 1 \left(x^{\frac{x^3}{x^3+3-C1}}\right)^{-1} \left(x^{\frac{C1}{x^3+3-C1}}\right)^{-3} \left(e^{\frac{x}{x^3+3-C1}}\right)^{-3}\right\}$$

2.855 ODE No. 855

$$y'(x) = \frac{y(x) \left(x^3 \log^2(y(x)) + 2x^3 \log(x) \log(y(x)) + x^3 \log^2(x) + \log(y(x)) + \log(x) - 1\right)}{x}$$

✗ **Mathematica** : cpu = 0.613281 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[1][y][x] == ((-1 + Log[x] + x^3\*Log[x]^2 + Log[y[x]] + 2\*x^3\*Log[x]\*Log[y[x]]), y[x], x]  
✓ **Maple** : cpu = 0.148 (sec), leaf count = 51

$$\left\{y(x) = 1 \left(x^{\frac{x^4}{x^4+4-C1}}\right)^{-1} \left(x^{\frac{C1}{x^4+4-C1}}\right)^{-4} \left(e^{\frac{x}{x^4+4-C1}}\right)^{-4}\right\}$$

2.856 ODE No. 856

$$y'(x) = -\frac{x \left(-\_F1\left(y(x)^2 - 2x\right) - \frac{1}{x}\right)}{\sqrt{y(x)^2}}$$

✓ **Mathematica** : cpu = 0.555036 (sec), leaf count = 103

Solve  $\left[\int_1^{y(x)} \left(\frac{\sqrt{K[2]^2}}{\_F1(K[2]^2 - 2x)} - \int_1^x \frac{2K[2]\_F1'(K[2]^2 - 2K[1])}{(\_F1(K[2]^2 - 2K[1]))^2} dK[1]\right) dK[2] + \int_1^x \left(-K[1] - \frac{1}{\_F1(y(x)^2 - 2x)}\right) dx\right] = 0$

✓ **Maple** : cpu = 0.207 (sec), leaf count = 65

$$\left\{y(x) = \sqrt{2 \operatorname{RootOf}\left(x^2 - 2 \int^{-Z} (\_F1(2\_a))^{-1} d\_a + 4\_C1\right)} + 2x, y(x) = -\sqrt{2 \operatorname{RootOf}\left(x^2 - 2 \int^{-Z} (\_F1(2\_a))^{-1} d\_a + 4\_C1\right)} + 2x\right\}$$

2.857 ODE No. 857

$$y'(x) = x^2 \sqrt{x^2 + 8y(x) - 2x + 1} + \sqrt{x^2 + 8y(x) - 2x + 1} + x^3 \sqrt{x^2 + 8y(x) - 2x + 1} - \frac{x}{4} + \frac{1}{4}$$

✓ **Mathematica** : cpu = 0.606804 (sec), leaf count = 107

Solve  $\left[\frac{x^4}{4} + \frac{x^3}{3} - \frac{1}{4} \sqrt{x^2 + 8y(x) - 2x + 1} + \frac{1}{4} \log\left(-\sqrt{x^2 + 8y(x) - 2x + 1} - x + 1\right) - \frac{1}{4} \tanh^{-1}\left(\frac{2x}{2\sqrt{x^2 + 8y(x) - 2x + 1} - x + 1}\right)\right] = 0$

✓ **Maple** : cpu = 0.213 (sec), leaf count = 32

$$\left\{-C1 + x^4 + \frac{4x^3}{3} + 4x - \sqrt{x^2 - 2x + 1 + 8y(x)} = 0\right\}$$

2.858 ODE No. 858

$$y'(x) = \frac{a^3 y(x)^3 + a^3 y(x)^2 + a^3 + 3a^2 b x y(x)^2 + 2a^2 b x y(x) + 3ab^2 x^2 y(x) + ab^2 x^2 + b^3 x^3}{a^3}$$

✓ **Mathematica** : cpu = 0.34358 (sec), leaf count = 145

Solve  $\left[-\frac{1}{3}(29a + 27b)^{2/3} \operatorname{RootSum}\left[\#1^3(29a + 27b)^{2/3} - 3\#1a^{2/3} + (29a + 27b)^{2/3}\&, \frac{\log\left(\frac{\frac{a+3bx}{a} + 3y(x)}{\sqrt[3]{\frac{29a+27b}{a}}}\right) - \#1}{a^{2/3} - \#1^2(29a + 27b)^{2/3}}\right]\right] = 0$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 42

$$\left\{y(x)=\frac{RootOf\left(\int^{-Z}(\_a^3a+\_a^2a+a+b)^{-1}d\_aa-x+\_C1\right)a-bx}{a}\right\}$$

2.859 ODE No. 859

---

$$y'(x)=\frac{\_F1\left(y(x)^2-2x\right)+x}{x\sqrt{y(x)^2}}$$

✓ **Mathematica** : cpu = 0.813357 (sec), leaf count = 105

Solve  $\left[\int_1^{y(x)}\left(\frac{\sqrt{K[2]^2}}{\_F1\left(K[2]^2-2x\right)}-\int_1^x\frac{2K[2]\_F1'\left(K[2]^2-2K[1]\right)}{\left(\_F1\left(K[2]^2-2K[1]\right)\right)^2}dK[1]\right)dK[2]+\int_1^x\left(-\frac{1}{\_F1\left(y(x)^2-2K[1]\right)}\right)dK[1]\right]$

✓ **Maple** : cpu = 0.198 (sec), leaf count = 63

$$\left\{y(x)=\sqrt{2\,RootOf\left(\ln(x)-\int^{-Z}(\_F1\left(2\_a\right))^{-1}d\_a+2\_C1\right)}+2x,y(x)=-\sqrt{2\,RootOf\left(\ln(x)-\int^{-Z}(\_F1\left(2\_a\right))^{-1}d\_a+2\_C1\right)}\right\}$$

2.860 ODE No. 860

---

$$y'(x)=\frac{\frac{1}{2}x^4\cos(2y(x))+\frac{x^4}{2}+\frac{1}{2}x^3\cos(2y(x))+\frac{x^3}{2}-\frac{1}{2}\sin(2y(x))+\frac{1}{2}x\cos(2y(x))+\frac{x}{2}}{x}$$

✓ **Mathematica** : cpu = 0.364498 (sec), leaf count = 33

$$\left\{\left\{y(x)\rightarrow \tan^{-1}\left(\frac{10c_1+4x^5+5x^4+10x^2}{20x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 1.57 (sec), leaf count = 29

$$\left\{y(x)=\arctan\left(\frac{4x^5+5x^4+10x^2+40\_C1}{20x}\right)\right\}$$

2.861 ODE No. 861

---

$$y'(x)=-\frac{e^{-1/x}\left(-\_F1\left(e^{\frac{1}{x}}y(x)\right)-\frac{e^{\frac{1}{x}}y(x)}{x}\right)}{x}$$

✓ **Mathematica** : cpu = 1.36388 (sec), leaf count = 158

Solve  $\left[\int_1^{y(x)}-\frac{\_F1\left(e^{\frac{1}{x}}K[2]\right)\int_1^x\left(\frac{e^{\frac{1}{K[1]}}}{K[1]^2\_F1\left(e^{\frac{1}{K[1]}}K[2]\right)}-\frac{e^{\frac{2}{K[1]}K[2]}\_F1'\left(e^{\frac{1}{K[1]}K[2]}\right)}{K[1]^2\left(\_F1\left(e^{\frac{1}{K[1]}K[2]}\right)\right)^2}\right)dK[1]+e^{\frac{1}{x}}}{\_F1\left(e^{\frac{1}{x}}K[2]\right)}dK[2]+\int_1^x\left(\frac{e^{\frac{1}{K[1]}}}{K[1]^2\_F1\left(e^{\frac{1}{K[1]}K[2]}\right)}-\frac{e^{\frac{2}{K[1]}K[2]}\_F1'\left(e^{\frac{1}{K[1]}K[2]}\right)}{K[1]^2\left(\_F1\left(e^{\frac{1}{K[1]}K[2]}\right)\right)^2}\right)dK[1]\right]$

✓ **Maple** : cpu = 0.119 (sec), leaf count = 26

$$\left\{y(x)=\frac{RootOf\left(-\ln(x)+\int^{-Z}(\_F1\left(\_a\right))^{-1}d\_a+\_C1\right)}{e^{x^{-1}}}\right\}$$

2.862 ODE No. 862

---

$$y'(x)=-\log(y(x)-1)\left(\frac{\text{Ei}(-\log(y(x)-1))}{x}-\_F1(x)\right)$$

✗ **Mathematica** : cpu = 0.752759 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x]==-(Log[-1+y[x]]*(ExpIntegralEi[-Log[-1+y[x]]])/x-\_F1[x]),y[x]]`

✓ **Maple** : cpu = 0.159 (sec), leaf count = 27

$$\left\{y(x)=e^{RootOf\left(\int\frac{\_F1(x)}{x}dx+x\_C1+Ei(1,-Z)\right)}+1\right\}$$

**2.863                    ODE No. 863**

---

$$y'(x) = \frac{x\sqrt{x^2 + y(x)^2} + x^4\sqrt{x^2 + y(x)^2} + x^3\sqrt{x^2 + y(x)^2} + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.176132 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow x \sinh \left( \frac{1}{12} (12c_1 + 3x^4 + 4x^3 + 12x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 6.227 (sec), leaf count = 38

$$\left\{ \ln \left( y(x) + \sqrt{(y(x))^2 + x^2} \right) - \frac{x^4}{4} - \frac{x^3}{3} - x - \ln(x) - \_C1 = 0 \right\}$$

**2.864                    ODE No. 864**

---

$$y'(x) = \frac{e^{\frac{x^2}{4}} y(x) \left( 2e^{-\frac{3x^2}{4}} y(x)^2 + e^{-\frac{x^2}{2}} xy(x) + e^{-\frac{x^2}{4}} x \right)}{2e^{-\frac{x^2}{4}} y(x) + 2}$$

✓ **Mathematica** : cpu = 0.280505 (sec), leaf count = 137

$$\left\{ \left\{ y(x) \rightarrow \frac{2e^{\frac{x^2}{2}}}{\sqrt{2}\sqrt{2e^{\frac{x^2}{2}}(c_1 - 2x) + 2e^{\frac{x^2}{2}} - 2e^{\frac{x^2}{4}}}} \right\}, \left\{ y(x) \rightarrow -\frac{2e^{\frac{x^2}{2}}}{\sqrt{2}\sqrt{2e^{\frac{x^2}{2}}(c_1 - 2x) + 2e^{\frac{x^2}{2}} + 2e^{\frac{x^2}{4}}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.06 (sec), leaf count = 162

$$\left\{ y(x) = 1 \left( e^{\frac{x^2}{2}} \left( \sqrt{\_C1 - 2x} - 1 \right) e^{-\frac{x^2}{4}} - e^{\frac{x^2}{4}} \sqrt{\_C1 - 2x} \right) \left( e^{-\frac{x^2}{4}} \right)^{-1} \left( e^{\frac{x^2}{4}} \sqrt{\_C1 - 2x} + e^{-\frac{x^2}{4}} e^{\frac{x^2}{2}} \right)^{-1}, y(x) \right\}$$

**2.865                    ODE No. 865**

---

$$y'(x) = (1 - y(x)) \left( -f(x) + \frac{y(x) \log(y(x) - 1)}{x(1 - y(x)) \log(x)} - \frac{\log(y(x) - 1)}{x(1 - y(x)) \log(x)} \right)$$

✓ **Mathematica** : cpu = 0.311079 (sec), leaf count = 87

Solve  $\left[ \int_1^x \left( -\frac{f(K[1])}{\log(K[1])} - \frac{\log(y(x) - 1)}{K[1] \log^2(K[1])} \right) dK[1] + \int_1^{y(x)} \left( \frac{1}{(K[2] - 1) \log(x)} - \int_1^x -\frac{1}{K[1](K[2] - 1) \log^2(K[1])} \right) dy(x) \right]$

✓ **Maple** : cpu = 0.144 (sec), leaf count = 23

$$\left\{ y(x) = e^{\int \frac{f(x)}{\ln(x)} dx \ln(x)} x^{-C1} + 1 \right\}$$

**2.866                    ODE No. 866**

---

$$y'(x) = x^2\sqrt{a^2 + 2ax + x^2 + 4y(x)} + \sqrt{a^2 + 2ax + x^2 + 4y(x)} + x^3\sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{a}{2} - \frac{x}{2}$$

✓ **Mathematica** : cpu = 0.683723 (sec), leaf count = 117

Solve  $\left[ -\frac{1}{2}\sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{1}{2}a \log \left( \sqrt{a^2 + 2ax + x^2 + 4y(x)} + a + x \right) + \frac{1}{2}a \tanh^{-1} \left( \frac{2a + 2\sqrt{a^2 + 2ax + x^2 + 4y(x)}}{2\sqrt{a^2 + 2ax + x^2 + 4y(x)}} \right) \right]$

✓ **Maple** : cpu = 0.236 (sec), leaf count = 37

$$\left\{ \_C1 + \frac{x^4}{2} + \frac{2x^3}{3} + 2x - \sqrt{x^2 + 2ax + a^2 + 4y(x)} = 0 \right\}$$

## 2.867 ODE No. 867

---

$$y'(x) = \frac{x^6}{27} + \frac{1}{3}x^4y(x) + \frac{x^4}{9} + x^2y(x)^2 + \frac{2}{3}x^2y(x) + y(x)^3 + y(x)^2 - \frac{2x}{3} + 1$$

✓ **Mathematica** : cpu = 0.201235 (sec), leaf count = 77

$$\text{Solve} \left[ -\frac{29}{3} \text{RootSum} \left[ -29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log \left( \frac{x^2+3y(x)+1}{\sqrt[3]{29}} - \#1 \right)}{\sqrt[3]{29} - 29\#1^2} \& \right] = c_1 + \frac{1}{9}29^{2/3}x, y(x) \right]$$

✓ **Maple** : cpu = 0.048 (sec), leaf count = 30

$$\left\{ y(x) = -\frac{x^2}{3} + \text{RootOf} \left( -x + \int^{-Z} (\_a^3 + \_a^2 + 1)^{-1} d\_a + \_C1 \right) \right\}$$

## 2.868 ODE No. 868

---

$$y'(x) = -x^6 + 3x^4y(x) + x^4 - 3x^2y(x)^2 - 2x^2y(x) + y(x)^3 + y(x)^2 + 2x + 1$$

✓ **Mathematica** : cpu = 0.16876 (sec), leaf count = 79

$$\text{Solve} \left[ -\frac{29}{3} \text{RootSum} \left[ -29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log \left( \frac{-3x^2+3y(x)+1}{\sqrt[3]{29}} - \#1 \right)}{\sqrt[3]{29} - 29\#1^2} \& \right] = c_1 + \frac{1}{9}29^{2/3}x, y(x) \right]$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 28

$$\left\{ y(x) = x^2 + \text{RootOf} \left( -x + \int^{-Z} (\_a^3 + \_a^2 + 1)^{-1} d\_a + \_C1 \right) \right\}$$

## 2.869 ODE No. 869

---

$$y'(x) = \frac{2x^5 + 2x^4 - 2x^3y(x) + x^3 - 2x^2y(x) + 3x^2 - 2y(x) - x + 1}{x^2 - y(x)}$$

✓ **Mathematica** : cpu = 0.0318906 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( W \left( -e^{c_1+x^4+\frac{4x^3}{3}-2x^2+4x-1} \right) + 1 \right) + x^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.063 (sec), leaf count = 37

$$\left\{ y(x) = x^2 + \frac{1}{2} \text{lambertW} \left( -2 \frac{e^{x^4} e^{4/3 x^3} \_C1 (e^x)^4 e^{-1}}{(e^{x^2})^2} \right) + \frac{1}{2} \right\}$$

## 2.870 ODE No. 870

---

$$y'(x) = \frac{e^{\frac{y(x)}{x}} \left( x^4 + x^3 + x e^{-\frac{y(x)}{x}} + e^{-\frac{y(x)}{x}} y(x) + x \right)}{x}$$

✓ **Mathematica** : cpu = 1.51374 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow -x \log \left( \frac{-c_1 - \frac{x^4}{4} - \frac{x^3}{3} - x}{x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.682 (sec), leaf count = 30

$$\left\{ y(x) = -\ln \left( -\frac{3x^4 + 4x^3 + 12\_C1 + 12x}{12x} \right) x \right\}$$

2.871 ODE No. 871

---

$$y'(x) = \frac{2xy(x)^2 + y(x)^2 + 4xy(x)\log(2x+1) + 2y(x)\log(2x+1) + 2x\log^2(2x+1) + \log^2(2x+1) - 2}{2x+1}$$

✓ **Mathematica** : cpu = 0.229556 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 - x} - \log(2x+1) \right\} \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 26

$$\left\{ y(x) = \frac{-1 + (-C1 - x)\ln(2x+1)}{-C1 + x} \right\}$$

2.872 ODE No. 872

---

$$y'(x) = \frac{14x^{7/2} + \frac{12x^6}{5} - 6x^3y(x) - 6x^3 - 5\sqrt{x}y(x) + 10x - 5\sqrt{x} - 5}{x(2x^3 - 5y(x) + 10\sqrt{x} - 5)}$$

✓ **Mathematica** : cpu = 0.0413268 (sec), leaf count = 215

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{5}(2x^3 + 10\sqrt{x} - 5) - \frac{\sqrt{-25c_1x - x(2x^3 + 10\sqrt{x} - 5)^2 - 50x\left(-\frac{4x^{7/2}}{5} - \frac{2x^6}{25} + \frac{2x^3}{5} - 2x + 2\sqrt{x} + 1\right)}}{5\sqrt{-\frac{1}{x}x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 49

$$\left\{ y(x) = \frac{2x^3}{5} + 2\sqrt{x} - \sqrt{-C1 + 2\ln(x)} - 1, y(x) = \frac{2x^3}{5} + 2\sqrt{x} + \sqrt{-C1 + 2\ln(x)} - 1 \right\}$$

2.873 ODE No. 873

---

$$y'(x) = \frac{2y(x) + 1}{x(2xy(x)^4 + 3xy(x)^3 + xy(x)^2 + 2xy(x) + x - 2)}$$

✓ **Mathematica** : cpu = 0.511711 (sec), leaf count = 53

$$\text{Solve} \left[ \frac{1}{192}(-16y(x)^3 - 12y(x)^2 + 12y(x) - 54\log(4y(x) + 2) + 7) - \frac{1}{2x(2y(x) + 1)} = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.216 (sec), leaf count = 50

$$\left\{ y(x) = \frac{e^{\text{RootOf}\left(2x(e^{-Z})^4 - 3x(e^{-Z})^3 - 6x(e^{-Z})^2 + 48\_C1xe^{-Z} + 54\_Zxe^{-Z} + 7e^{-Z}x + 96\right)}}{2} - \frac{1}{2} \right\}$$

2.874 ODE No. 874

---

$$y'(x) = \frac{1}{512}x(a^3x^{12} + 24a^2x^8y(x) + 8a^2x^8 + 192ax^4y(x)^2 + 128ax^4y(x) - 256ax^2 + 512y(x)^3 + 512y(x)^2 + 512y(x) - 512)$$

✓ **Mathematica** : cpu = 0.229077 (sec), leaf count = 101

$$\text{Solve} \left[ -\frac{29}{3}\text{RootSum} \left[ -29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{\frac{1}{8}(3ax^5+8x)+3xy(x)}{\sqrt[3]{29}\sqrt[3]{x^3}} - \#1\right)}{\sqrt[3]{29} - 29\#1^2} \& \right] = c_1 + \frac{1}{18}29^{2/3}(x^3)^{2/3}, y(x) \right]$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 40

$$\left\{ y(x) = -\frac{ax^4}{8} - \frac{1}{3} + \frac{29\text{RootOf}\left(x^2 - 162\int^{-Z}(841\_a^3 - 27\_a + 27)^{-1}d\_a + 6\_C1\right)}{9} \right\}$$

2.875 ODE No. 875

---

$$y'(x) = \frac{x^5 \left( -\sqrt{x^2 + y(x)^2} \right) + x^4 y(x) \sqrt{x^2 + y(x)^2} + x y(x) + y(x)}{x(x + 1)}$$

✓ **Mathematica** : cpu = 0.239628 (sec), leaf count = 497

$$\left\{ \left\{ y(x) \rightarrow \frac{x - 2 \sqrt{x^2 \tanh^2 \left( \frac{1}{12} \left( -12 \sqrt{2} c_1 - 3 \sqrt{2} x^4 + 4 \sqrt{2} x^3 - 6 \sqrt{2} x^2 + 12 \sqrt{2} x - 12 \sqrt{2} \log(x + 1) + 25 \sqrt{2} \right) \right)}{2 \tanh^2 \left( \frac{1}{12} \left( -12 \sqrt{2} c_1 - 3 \sqrt{2} x^4 + 4 \sqrt{2} x^3 - 6 \sqrt{2} x^2 + 12 \sqrt{2} x - 12 \sqrt{2} \log(x + 1) + 25 \sqrt{2} \right) \right)} \right. \right.$$

✓ **Maple** : cpu = 0.174 (sec), leaf count = 73

$$\left\{ \ln \left( 2 \frac{x \left( \sqrt{2 \left( y(x) \right)^2 + 2 x^2 + y(x) + x}}{y(x) - x} \right) + \sqrt{2} \ln(1 + x) + \frac{(3 x^4 - 4 x^3 + 6 x^2 - 12 x) \sqrt{2}}{12} - \_C1 - \ln(x) \right. \right.$$

2.876 ODE No. 876

---

$$y'(x) = -\frac{y(x)^2 \left( x^2 y(x) - 2 x y(x) + y(x) - 2 x \right)}{2 x (x y(x) - 2 y(x) - 2)}$$

✓ **Mathematica** : cpu = 0.165911 (sec), leaf count = 135

$$\left\{ \left\{ y(x) \rightarrow -\frac{4 x}{\frac{2 \sqrt{-4 x \left( c_1 - 2 \left( \frac{x^2}{8} - \frac{x}{2} + \frac{\log(x)}{4} \right) \right) - x (x - 2)^2}}{\sqrt{-\frac{1}{x}}} - 2 (x - 2) x} \right\}, \left\{ y(x) \rightarrow \frac{4 x}{\frac{2 \sqrt{-4 x \left( c_1 - 2 \left( \frac{x^2}{8} - \frac{x}{2} + \frac{\log(x)}{4} \right) \right) - x (x - 2)^2}}{\sqrt{-\frac{1}{x}}} + 2 (x - 2) x} \right\} \right.$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 41

$$\left\{ y(x) = -4 \left( \sqrt{\_C1 - 8 \ln(x)} - 2 x + 4 \right)^{-1}, y(x) = 4 \left( \sqrt{\_C1 - 8 \ln(x)} + 2 x - 4 \right)^{-1} \right\}$$

2.877 ODE No. 877

---

$$y'(x) = \frac{x^6 - 3 x^4 y(x) + 2 x^3 + 3 x^2 y(x)^2 - 2 x y(x) - y(x)^3 - 2 x}{x^2 - y(x) - 1}$$

✓ **Mathematica** : cpu = 0.186079 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{1 - \frac{1}{\sqrt{c_1 - 2 x}}} + x^2 - 1 \right\}, \left\{ y(x) \rightarrow \frac{1}{\frac{1}{\sqrt{c_1 - 2 x}} + 1} + x^2 - 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 73

$$\left\{ y(x) = \frac{1}{-2 \_C1 + 2 x} \left( -2 \_C1 x^2 + 2 x^3 + \sqrt{2 \_C1 - 2 x + 1} - 1 \right), y(x) = \frac{1}{-2 x + 2 \_C1} \left( 2 \_C1 x^2 - 2 x^3 + \sqrt{2 \_C1 - 2 x + 1} - 1 \right) \right\}$$

2.878 ODE No. 878

---

$$y'(x) = \frac{-64 a^3 x^3 + 48 a^2 x^2 y(x)^2 + 16 a^2 x^2 - 12 a x y(x)^4 - 8 a x y(x)^2 + y(x)^6 + y(x)^4 + 1}{y(x)}$$

✓ **Mathematica** : cpu = 0.345161 (sec), leaf count = 130

Solve  $\left[ 2 a \left( x - \frac{1}{2} \text{RootSum} \left[ 64 \#1^3 a^3 - 48 \#1^2 a^2 y(x)^2 - 16 \#1^2 a^2 + 12 \#1 a y(x)^4 + 8 \#1 a y(x)^2 + 2 a - y(x)^6 - y(x)^4 + 1 \right] \right) \right]$

✓ **Maple** : cpu = 10.437 (sec), leaf count = 75

$$\left\{ \int_{\_b}^{y(x)} \frac{\_a}{-\_a^6 + 12 \_a^4 a x - 48 \_a^2 a^2 x^2 + 64 a^3 x^3 - \_a^4 + 8 \_a^2 a x - 16 a^2 x^2 + 2 a - 1} d\_a + x - \_C1 = 0 \right\}$$

2.879 ODE No. 879

---

$$y'(x) = \frac{x^2 \left( -\sqrt{x^2 + y(x)^2} \right) + xy(x)\sqrt{x^2 + y(x)^2} + xy(x) + y(x)}{x(x + 1)}$$

✓ **Mathematica** : cpu = 0.242861 (sec), leaf count = 239

$$\left\{ \left\{ y(x) \rightarrow \frac{x - 2\sqrt{x^2 \tanh^2(\sqrt{2}c_1 + \sqrt{2}x - \sqrt{2}\log(x + 1))} - x^2 \tanh^4(\sqrt{2}c_1 + \sqrt{2}x - \sqrt{2}\log(x + 1))}{2 \tanh^2(\sqrt{2}c_1 + \sqrt{2}x - \sqrt{2}\log(x + 1)) - 1} \right\}, \left\{ y(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.16 (sec), leaf count = 55

$$\left\{ \ln \left( 2 \frac{x \left( \sqrt{2 (y(x))^2 + 2x^2} + y(x) + x \right)}{y(x) - x} \right) + \sqrt{2}x - \sqrt{2}\ln(1 + x) - \ln(x) - \_C1 = 0 \right\}$$

2.880 ODE No. 880

---

$$y'(x) = -\frac{2a}{128a^4x^3 - 96a^3x^2y(x)^2 - 32a^3x^2 + 24a^2xy(x)^4 + 16a^2xy(x)^2 - 2ay(x)^6 - 2ay(x)^4 - 2a - y(x)}$$

✓ **Mathematica** : cpu = 0.422608 (sec), leaf count = 131

$$\text{Solve} \left[ \frac{\text{RootSum} \left[ -64\#1^3a^3 + 48\#1^2a^2y(x)^2 + 16\#1^2a^2 - 12\#1ay(x)^4 - 8\#1ay(x)^2 + y(x)^6 + y(x)^4 + 1\&, \frac{\_a}{48\#1} \right]}{8a^2} \right]$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 41

$$\left\{ \frac{y(x)}{2a} + \frac{\int^{(y(x))^2-4ax} (\_a^3 + \_a^2 + 1)^{-1} d\_a}{8a^2} - \_C1 = 0 \right\}$$

2.881 ODE No. 881

---

$$y'(x) = \frac{x^6 + 9x^4y(x) - 6x^3 + 27x^2y(x)^2 - 18xy(x) + 27y(x)^3 - 18x}{9x^2 + 27y(x) + 27}$$

✓ **Mathematica** : cpu = 0.169906 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{27 \left( \frac{1}{27} - \frac{1}{\sqrt{c_1 - 1458x}} \right)} + \frac{1}{3}(-x^2 - 3) \right\}, \left\{ y(x) \rightarrow \frac{1}{27 \left( \frac{1}{\sqrt{c_1 - 1458x}} + \frac{1}{27} \right)} + \frac{1}{3}(-x^2 - 3) \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 77

$$\left\{ y(x) = \frac{1}{-6x + 6\_C1} \left( -2\_C1x^2 + 2x^3 - 3\sqrt{2\_C1 - 2x + 1} + 3 \right), y(x) = \frac{1}{-6x + 6\_C1} \left( -2\_C1x^2 + 2x^3 - 3\sqrt{2\_C1 - 2x + 1} + 3 \right) \right\}$$

2.882 ODE No. 882

---

$$y'(x) = -\frac{1}{216}\sqrt{x} \left( -108x^{3/2} + x^9 - 18x^6y(x) - 6x^6 + 108x^3y(x)^2 + 72x^3y(x) - 216y(x)^3 - 216y(x)^2 - 216 \right)$$

✓ **Mathematica** : cpu = 0.239443 (sec), leaf count = 119

$$\text{Solve} \left[ -\frac{29}{3} \text{RootSum} \left[ -29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log \left( \frac{\frac{1}{2}(2\sqrt{x} - x^{7/2}) + 3\sqrt{x}y(x)}{\sqrt[3]{29}\sqrt[3]{x^{3/2}}} - \#1 \right)}{\sqrt[3]{29} - 29\#1^2} \& \right] = c_1 + \frac{2}{27}29^{2/3}\sqrt{x} \left( x^{3/2} - 243 \int_{320}^{-Z} (841\_a^3 - 27\_a + 27)^{-1} d\_a + 9\_C1 \right) \right]$$

✓ **Maple** : cpu = 0.047 (sec), leaf count = 41

$$\left\{ y(x) = \frac{x^3}{6} - \frac{1}{3} + \frac{29}{9} \text{RootOf} \left( 2x^{3/2} - 243 \int_{320}^{-Z} (841\_a^3 - 27\_a + 27)^{-1} d\_a + 9\_C1 \right) \right\}$$



2.883 ODE No. 883

$$y'(x) = \frac{x \left( a^3 y(x)^6 + a^3 y(x)^4 + a^3 + 3 a^2 b x^2 y(x)^4 + 2 a^2 b x^2 y(x)^2 + 3 a b^2 x^4 y(x)^2 + a b^2 x^4 + b^3 x^6 \right)}{a^{7/2} y(x)}$$

✓ **Mathematica** : cpu = 1.13492 (sec), leaf count = 164

$$\text{Solve} \left[ \frac{x^2}{2} - \frac{1}{2} a^{5/2} \text{RootSum} \left[ \#1^3 b^3 + 3 \#1^2 a b^2 y(x)^2 + \#1^2 a b^2 + 3 \#1 a^2 b y(x)^4 + 2 \#1 a^2 b y(x)^2 + a^{5/2} b + a^3 y(x)^6 \right. \right.$$

✓ **Maple** : cpu = 0.511 (sec), leaf count = 352

$$\left\{ \int_{-b}^x \left( b^3 - a^6 + 3 (y(x))^2 a b^2 - a^4 + 3 (y(x))^4 a^2 b - a^2 + (y(x))^6 a^3 + a - a^4 b^2 + 2 a^2 (y(x))^2 b - a^2 + (y(x))^4 a^3 + \right. \right.$$

2.884 ODE No. 884

$$y'(x) = -\frac{x \left( x^6 - 3 x^4 y(x)^2 - x^4 + 3 x^2 y(x)^4 + 2 x^2 y(x)^2 - y(x)^6 - y(x)^4 - 1 \right)}{y(x)}$$

✓ **Mathematica** : cpu = 0.564047 (sec), leaf count = 71

$$\text{Solve} \left[ \frac{1}{4} \left( 2 \log(-x^2 + y(x)^2 + 1) - 2 x^2 - \frac{1}{y(x)(y(x) + x)} + \frac{1}{x y(x) - y(x)^2} - 2 \log(x - y(x)) - 2 \log(y(x) + x) \right) \right]$$

✓ **Maple** : cpu = 0.296 (sec), leaf count = 107

$$\left\{ y(x) = e^{\text{RootOf} \left( -3 x^2 (e^{-Z})^2 + 6 x^3 e^{-Z} + 3 (e^{-Z})^2 \ln \left( \frac{(e^{-Z})^2 - 2 e^{-Z} x + 1}{e^{-Z} - 2 x} \right) - 2 - C1 (e^{-Z})^2 - 3 - Z (e^{-Z})^2 - 6 e^{-Z} \ln \left( \frac{(e^{-Z})^2 - 2 e^{-Z} x + 1}{e^{-Z} - 2 x} \right) \right)} \right\}$$

2.885 ODE No. 885

$$y'(x) = -\frac{i \left( x^6 + 12 x^4 y(x)^2 + 4 x^4 + 48 x^2 y(x)^4 + 32 x^2 y(x)^2 + 64 y(x)^6 + 64 y(x)^4 + 32 i x + 64 \right)}{128 y(x)}$$

✗ **Mathematica** : cpu = 40.6342 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[1][y][x] == ((-I/128)\*(64 + (32\*I)\*x + 4\*x^4 + x^6 + 32\*x^2\*y[x]^2 + 12\*x^2\*y[x]^4 + 64\*y[x]^6 + 64\*y[x]^4 + 32\*I\*x + 64)/128/y[x]), x] == {{}}  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x) = -1/128\*I\*(32\*I\*x+64+64\*y(x)^4+32\*x^2\*y(x)^2+4\*x^4+64\*y(x)^6+48\*x^2\*y(x)^4+32\*x^2\*y(x)^2+12\*x^2\*y(x)^4+64\*y(x)^6+64\*y(x)^4+32\*I\*x+64)/128/y(x),x)

2.886 ODE No. 886

$$y'(x) = \frac{x^6 y(x)^3 - 3 x^5 y(x)^2 + x^4 y(x)^2 + 3 x^4 y(x) - 4 x^3 y(x) - x^3 + 2 x^2 + 1}{x^4}$$

✓ **Mathematica** : cpu = 0.171533 (sec), leaf count = 82

$$\text{Solve} \left[ -\frac{29}{3} \text{RootSum} \left[ -29 \#1^3 + 3 \sqrt[3]{29} \#1 - 29 \&, \frac{\log \left( \frac{3 x^2 y(x) - 3 x + 1}{\sqrt[3]{29}} - \#1 \right)}{\sqrt[3]{29} - 29 \#1^2} \& \right] = c_1 - \frac{29^{2/3}}{9 x}, y(x) \right]$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 42

$$\left\{ y(x) = \frac{9 x - 3 + 29 \text{RootOf} \left( -81 \int^{-Z} \left( 841 - a^3 - 27 - a + 27 \right)^{-1} d - a x + 3 x - C1 - 1 \right)}{9 x^2} \right\}$$

2.887 ODE No. 887

---

$$y'(x) = \frac{a^3 x^3 y(x)^3 + 3a^2 x^2 y(x)^2 + a^2 x y(x) + a^2 x + 3a x y(x) + a + 1}{a^2 x^2 (a x y(x) + a x + 1)}$$

✓ **Mathematica** : cpu = 0.224655 (sec), leaf count = 106

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{a^3 x^3 \left( \frac{1}{a^3 x^3} - \frac{1}{x^3 \sqrt{c_1 - 2a^6 x}} \right)} - \frac{ax + 1}{ax} \right\}, \left\{ y(x) \rightarrow \frac{1}{a^3 x^3 \left( \frac{1}{x^3 \sqrt{c_1 - 2a^6 x}} + \frac{1}{a^3 x^3} \right)} - \frac{ax + 1}{ax} \right\} \right\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 72

$$\left\{ y(x) = \frac{1}{ax} \left( -ax - \sqrt{-C1 - 2x} - 1 \right) \left( \sqrt{-C1 - 2x} + 1 \right)^{-1}, y(x) = \frac{1}{ax} \left( ax - \sqrt{-C1 - 2x} + 1 \right) \left( \sqrt{-C1 - 2x} - 1 \right)^{-1} \right\}$$

2.888 ODE No. 888

---

$$y'(x) = \frac{x^4 y(x)^3 - 5x^3 y(x)^2 + 6x^2 y(x) - 2x y(x) - 2x + 1}{x^2 (x^2 y(x) - x + 1)}$$

✓ **Mathematica** : cpu = 0.170295 (sec), leaf count = 78

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{x^4 \left( \frac{1}{x^2} - \frac{1}{x^2 \sqrt{c_1 + \frac{2}{x}}} \right)} + \frac{x - 1}{x^2} \right\}, \left\{ y(x) \rightarrow \frac{1}{x^4 \left( \frac{1}{x^2 \sqrt{c_1 + \frac{2}{x}}} + \frac{1}{x^2} \right)} + \frac{x - 1}{x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 79

$$\left\{ y(x) = \frac{1}{x^2} \left( \sqrt{\frac{x - C1 + 2}{x}} x - x + 1 \right) \left( \sqrt{\frac{x - C1 + 2}{x}} - 1 \right)^{-1}, y(x) = \frac{1}{x^2} \left( \sqrt{\frac{x - C1 + 2}{x}} x + x - 1 \right) \left( \sqrt{\frac{x - C1 + 2}{x}} + 1 \right)^{-1} \right\}$$

2.889 ODE No. 889

---

$$y'(x) = -\frac{e^x (-8y(x)^{9/2} + 36e^x y(x)^3 - 8y(x)^3 + 24e^x y(x)^{3/2} - 54e^{2x} y(x)^{3/2} - 18e^{2x} + 27e^{3x} - 8)}{8\sqrt{y(x)}}$$

✗ **Mathematica** : cpu = 300.373 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.825 (sec), leaf count = 49

$$\left\{ e^x - \frac{2}{3} \ln \left( (y(x))^{\frac{3}{2}} - \frac{3e^x}{2} + 1 \right) - 4 \left( -6 (y(x))^{3/2} + 9e^x \right)^{-1} + \frac{2}{3} \ln \left( (y(x))^{\frac{3}{2}} - \frac{3e^x}{2} \right) - C1 = 0 \right\}$$

2.890 ODE No. 890

---

$$y'(x) = \frac{x}{x^6 + 3x^4 y(x)^2 + x^4 + 3x^2 y(x)^4 + 2x^2 y(x)^2 + y(x)^6 + y(x)^4 - y(x) + 1}$$

✓ **Mathematica** : cpu = 0.180655 (sec), leaf count = 103

Solve  $\left[ y(x) - \frac{1}{2} \text{RootSum} \left[ \#1^3 + 3\#1^2 y(x)^2 + \#1^2 + 3\#1 y(x)^4 + 2\#1 y(x)^2 + y(x)^6 + y(x)^4 + 1 \&, \frac{\sqrt{-3\#1^2 + 6\#1 + 1}}{2} \right] \right]$

✓ **Maple** : cpu = 0.519 (sec), leaf count = 34

$$\left\{ -y(x) + \frac{\int^{(y(x))^2 + x^2} (-a^3 + -a^2 + 1)^{-1} d_a}{2} - C1 = 0 \right\}$$

2.891 ODE No. 891

$$y'(x) = \frac{y(x)^2 \left(x^4 y(x) + 2 x^2 y(x) + 2 x^2 - 2 y(x)\right)}{x^3 \left(x^2 y(x) + x^2 - y(x)\right)}$$

✓ **Mathematica** : cpu = 0.175207 (sec), leaf count = 135

$$\left\{ \left\{ y(x) \rightarrow \frac{x^5}{\sqrt{x^5 \left(c_1 - 2 \left(\frac{1}{2 x^4} - \frac{1}{x^2} + \log(x)\right)\right) + (x^2 - 1)^2 x} - x^3 (x^2 - 1)} \right\}, \left\{ y(x) \rightarrow -\frac{x^5}{\sqrt{x^5 \left(c_1 - 2 \left(\frac{1}{2 x^4} - \frac{1}{x^2} + \log(x)\right)\right) + (x^2 - 1)^2 x} + \sqrt{\frac{1}{x^5}}} \right\} \right.$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 56

$$\left\{ y(x) = x^2 \left(\sqrt{-C1 - 2 \ln(x)} x^2 - x^2 + 1\right)^{-1}, y(x) = -x^2 \left(\sqrt{-C1 - 2 \ln(x)} x^2 + x^2 - 1\right)^{-1} \right\}$$

2.892 ODE No. 892

$$y'(x) = \frac{e^{-\frac{2}{x^2 - y(x)^2 - 1}} + x^2 + 2 x y(x) + y(x)^2}{-e^{-\frac{2}{x^2 - y(x)^2 - 1}} + x^2 + 2 x y(x) + y(x)^2}$$

✓ **Mathematica** : cpu = 2.24762 (sec), leaf count = 1283

$$\text{Solve} \left[ \int_1^x \left( -e^{\int_1^{(K[2] - y(x))(K[2] + y(x))} \frac{2((K[1] - 3)K[1] + 1)}{\left(e^{-\frac{2}{K[1] - 1} - K[1]}\right)^{(K[1] - 1)^2}} dK[1] + \frac{2}{K[2]^2 - y(x)^2 - 1}} \int_1^{(K[2] - y(x))(K[2] + y(x))} \frac{2((K[1] - 3)K[1] + 1)}{\left(e^{-\frac{2}{K[1] - 1} - K[1]}\right)^{(K[1] - 1)^2}} dK[1] \right) K[2]^2 - 2e^{\int_1^{(K[2] - y(x))(K[2] + y(x))} \frac{2((K[1] - 3)K[1] + 1)}{\left(e^{-\frac{2}{K[1] - 1} - K[1]}\right)^{(K[1] - 1)^2}} dK[1]} \right]$$

✓ **Maple** : cpu = 0.376 (sec), leaf count = 40

$$\left\{ y(x) = e^{\text{RootOf}\left(-\_Z + \int^{\left(e^{-Z}\right)^2 - 2 e^{-Z} x} \left(e^{2(1 + \_a)^{-1} + \_a}\right)^{-1} d\_a + \_C1\right)} - x \right\}$$

2.893 ODE No. 893

$$y'(x) = \frac{x^3 y(x)^3 + x^3 y(x)^2 + x^3 + 6 x^2 y(x)^2 + 4 x^2 y(x) + 12 x y(x) + 6 x + 8}{x^3}$$

✓ **Mathematica** : cpu = 0.168645 (sec), leaf count = 80

$$\text{Solve} \left[ -\frac{29}{3} \text{RootSum} \left[ -29 \#1^3 + 3 \sqrt[3]{29} \#1 - 29 \&, \frac{\log \left( \frac{3 y(x) + \frac{x + 6}{x}}{\sqrt[3]{29}} - \#1 \right)}{\sqrt[3]{29} - 29 \#1^2} \& \right] = c_1 + \frac{1}{9} 29^{2/3} x, y(x) \right]$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 41

$$\left\{ y(x) = \frac{29 \text{RootOf}\left(-81 \int^{-Z} \left(841 \_a^3 - 27 \_a + 27\right)^{-1} d\_a + x + 3 \_C1\right) x - 3 x - 18}{9 x} \right\}$$

2.894 ODE No. 894

$$y'(x) = -\frac{i \left(x^6 + 3 x^4 y(x)^2 + x^4 + 3 x^2 y(x)^4 + 2 x^2 y(x)^2 + y(x)^6 + y(x)^4 + i x + 1\right)}{y(x)}$$

✗ **Mathematica** : cpu = 40.5839 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[1][y][x] == ((-I)\*(1 + I\*x + x^4 + x^6 + 2\*x^2\*y[x]^2 + 3\*x^4\*y[x]^2 + y[x]^6 + y[x]^4 + I\*x + 1))/y[x], y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x) = -I\*(I\*x+1+x^4+2\*x^2\*y(x)^2+y(x)^4+x^6+3\*x^4\*y(x)^2+3\*x^2\*y(x)^4+y(x)^6+y(x)^4+I\*x+1)/y(x),y(x))

**2.895 ODE No. 895**

---

$$y'(x) = \frac{x(a^3x^{12} + 24a^2x^8y(x) - 32a^2x^6 + 192ax^4y(x)^2 - 256ax^2y(x) - 256ax^2 + 512y(x)^3)}{64ax^4 + 512y(x) + 512}$$

✓ **Mathematica** : cpu = 0.222041 (sec), leaf count = 81

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{8}(-ax^4 - 8) + \frac{1}{512\left(\frac{1}{512} - \frac{1}{\sqrt{c_1 - 262144x^2}}\right)} \right\}, \left\{ y(x) \rightarrow \frac{1}{8}(-ax^4 - 8) + \frac{1}{512\left(\frac{1}{\sqrt{c_1 - 262144x^2}} + \frac{1}{512}\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 79

$$\left\{ y(x) = 1\left(-8 + \left(-\sqrt{-x^2 + \_C1} - 1\right)ax^4\right)\left(8 + 8\sqrt{-x^2 + \_C1}\right)^{-1}, y(x) = 1\left(8 + \left(-\sqrt{-x^2 + \_C1} + 1\right)a\right)\left(8 + 8\sqrt{-x^2 + \_C1}\right)^{-1} \right\}$$

**2.896 ODE No. 896**

---

$$y'(x) = \frac{-x^6 + 3x^4y(x)^2 + x^4 - 3x^2y(x)^4 - 2x^2y(x)^2 + y(x)^6 + y(x)^4 + x + 1}{y(x)}$$

✓ **Mathematica** : cpu = 0.294567 (sec), leaf count = 106

$$\text{Solve}\left[\frac{1}{2}\text{RootSum}\left[-\#1^3 + 3\#1^2y(x)^2 + \#1^2 - 3\#1y(x)^4 - 2\#1y(x)^2 + y(x)^6 + y(x)^4 + 1\&, \frac{\log\left(\frac{1}{3\#1^2 - 6\#1y(x)^2}\right)}{\#1}\right], y(x)\right]$$

✓ **Maple** : cpu = 0.39 (sec), leaf count = 63

$$\left\{ \int_{\_b}^{y(x)} \frac{\_a}{-\_a^6 + 3\_a^4x^2 - 3\_a^2x^4 + x^6 - \_a^4 + 2\_a^2x^2 - x^4 - 1} d\_a + x - \_C1 = 0 \right\}$$

**2.897 ODE No. 897**

---

$$y'(x) = \frac{\sqrt{x}(-108x^{3/2}y(x) + 18x^{9/2} - 108x^{3/2} + x^9 - 18x^6y(x) + 108x^3y(x)^2 - 216y(x)^3)}{36x^3 - 216y(x) - 216}$$

✓ **Mathematica** : cpu = 0.211747 (sec), leaf count = 79

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{6}(x^3 - 6) - \frac{1}{216\left(-\frac{1}{\sqrt{c_1 - 62208x^{3/2}}} - \frac{1}{216}\right)} \right\}, \left\{ y(x) \rightarrow \frac{1}{6}(x^3 - 6) - \frac{1}{216\left(\frac{1}{\sqrt{c_1 - 62208x^{3/2}}} - \frac{1}{216}\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 87

$$\left\{ y(x) = 1\left(\sqrt{9\_C1 - 12x^{3/2}x^3} - 3x^3 + 18\right)\left(6\sqrt{9\_C1 - 12x^{3/2}} - 18\right)^{-1}, y(x) = 1\left(\sqrt{9\_C1 - 12x^{3/2}x^3} + 3x^3 - 18\right)\left(6\sqrt{9\_C1 - 12x^{3/2}} + 18\right)^{-1} \right\}$$

**2.898 ODE No. 898**

---

$$y'(x) = \frac{4x^6y(x)^3 + 2x^5y(x) + 2x^5 + 3x^4y(x)^2 + \frac{x^3}{2} + \frac{3}{4}x^2y(x) + \frac{1}{16}}{x^6(4x^2y(x) + 4x^2 + 1)}$$

✓ **Mathematica** : cpu = 0.176434 (sec), leaf count = 106

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{64x^8\left(\frac{1}{64x^8} - \frac{1}{x^8\sqrt{c_1 + \frac{8192}{x}}}\right)} - \frac{4x^2 + 1}{4x^2} \right\}, \left\{ y(x) \rightarrow \frac{1}{64x^8\left(\frac{1}{x^8\sqrt{c_1 + \frac{8192}{x}}} + \frac{1}{64x^8}\right)} - \frac{4x^2 + 1}{4x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 87

$$\left\{ y(x) = \frac{1}{4x^2}\left(-4x^2 - \sqrt{\frac{x\_C1 + 2}{x}} - 1\right)\left(\sqrt{\frac{x\_C1 + 2}{x}} + 1\right)^{-1}, y(x) = \frac{1}{4x^2}\left(4x^2 - \sqrt{\frac{x\_C1 + 2}{x}} + 1\right)\left(\sqrt{\frac{x\_C1 + 2}{x}} + 1\right)^{-1} \right\}$$

**2.899 ODE No. 899**

---

$$y'(x) = \frac{x^6 y(x)^3 + x^6 y(x)^2 + x^6 + \frac{x^5}{2} + \frac{3}{4} x^4 y(x)^2 + \frac{1}{2} x^4 y(x) + \frac{3}{16} x^2 y(x) + \frac{x^2}{16} + \frac{1}{64}}{x^8}$$

✓ **Mathematica** : cpu = 0.203255 (sec), leaf count = 106

$$\text{Solve} \left[ -\frac{29}{3} \text{RootSum} \left[ -29 \#1^3 + 3 \sqrt[3]{29} \#1 - 29 \&, -\frac{\log \left( \frac{\frac{3y(x)}{x^2} + \frac{4x^2+3}{4x^4}}{\sqrt[3]{29} \sqrt[3]{\frac{1}{x^6}}} - \#1 \right)}{\sqrt[3]{29} - 29 \#1^2} \& \right] = c_1 - \frac{1}{9} 29^{2/3} \left( \frac{1}{x^6} \right)^{2/3} x^3, y(x) \right]$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 47

$$\left\{ y(x) = \frac{116 \text{RootOf} \left( -81 \int^{-Z} (841\_a^3 - 27\_a + 27)^{-1} d\_a x + 3 x\_C1 - 1 \right) x^2 - 12 x^2 - 9}{36 x^2} \right\}$$

**2.900 ODE No. 900**

---

$$y'(x) = \frac{2a \left( 4ax - y(x)^2 - 1 \right)}{128a^4 x^3 - 96a^3 x^2 y(x)^2 + 24a^2 x y(x)^4 - 2a y(x)^6 + 4a x y(x) - y(x)^3 - y(x)}$$

✓ **Mathematica** : cpu = 0.168641 (sec), leaf count = 381

$$\left\{ \left\{ y(x) \rightarrow \text{Root} \left[ 8 \#1^5 a - 16 \#1^4 a^2 c_1 - 64 \#1^3 a^2 x + \#1^2 \left( 128 a^3 c_1 x - 2 \right) + 128 \#1 a^3 x^2 - 256 a^4 c_1 x^2 + 8 a x - 1 \&, \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 48

$$\left\{ \frac{y(x)}{2a} - \frac{1}{16a^2 \left( (y(x))^2 - 4ax \right)^2} + \left( 32a^3 x - 8a^2 (y(x))^2 \right)^{-1} - \_C1 = 0 \right\}$$

**2.901 ODE No. 901**

---

$$y'(x) = \frac{y(x) \left( -ax \log(y(x)) + x^2 + y(x) \right)}{x(ax - y(x) - y(x) \log(x) - y(x) \log(y(x)))}$$

✓ **Mathematica** : cpu = 0.514009 (sec), leaf count = 33

$$\text{Solve} \left[ ax \log(y(x)) - \frac{x^2}{2} - y(x) \log(x) - y(x) \log(y(x)) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.414 (sec), leaf count = 30

$$\left\{ y(x) = e^{\text{RootOf}(-2\_Z ax + 2 e^{-Z} \ln(x) + 2\_Z e^{-Z} + 2\_C1 a + x^2)} \right\}$$

**2.902 ODE No. 902**

---

$$y'(x) = \frac{x^6 - 3x^4 y(x)^2 + x^3 + 3x^2 y(x)^4 - x y(x)^2 - y(x)^6 - x}{y(x) (x^2 - y(x)^2 - 1)}$$

✓ **Mathematica** : cpu = 0.223868 (sec), leaf count = 295

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2} \sqrt{\frac{4x^3}{x-c_1} - \frac{4c_1 x^2}{x-c_1} - \frac{\sqrt{4c_1-4x+1}}{x-c_1} - \frac{1}{x-c_1}} \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \sqrt{\frac{4x^3}{x-c_1} - \frac{4c_1 x^2}{x-c_1} - \frac{\sqrt{4c_1-4x+1}}{x-c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.18 (sec), leaf count = 183

$$\left\{ y(x) = \frac{1}{2\_C1 + 6x} \sqrt{(\_C1 + 3x) \left( 4\_C1 x^2 + 12x^3 - \sqrt{-12\_C1 - 36x + 9} - 3 \right)}, y(x) = \frac{1}{2\_C1 + 6x} \sqrt{(\_C1 + 3x) \left( 4\_C1 x^2 + 12x^3 + \sqrt{-12\_C1 - 36x + 9} - 3 \right)} \right\}$$

2.903 ODE No. 903

---

$$y'(x) = \frac{\sin\left(\frac{y(x)}{x}\right) \csc\left(\frac{y(x)}{2x}\right) \sec\left(\frac{y(x)}{2x}\right) \left(2x^2 \sin\left(\frac{y(x)}{2x}\right) \cos\left(\frac{y(x)}{2x}\right) + y(x)\right)}{2x}$$

✓ **Mathematica** : cpu = 0.117046 (sec), leaf count = 19

$$\left\{\left\{y(x) \rightarrow 2x \cot^{-1}\left(e^{-c_1-x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 48

$$\left\{y(x) = \arctan\left(2 \frac{-C1 e^x}{-C1^2 (e^x)^2 + 1}, \frac{-C1^2 (e^x)^2 + 1}{-C1^2 (e^x)^2 + 1}\right) x\right\}$$

2.904 ODE No. 904

---

$$y'(x) = \frac{\sin\left(\frac{y(x)}{x}\right) \csc\left(\frac{y(x)}{2x}\right) \sec\left(\frac{y(x)}{2x}\right) \left(2x^3 \sin\left(\frac{y(x)}{2x}\right) \cos\left(\frac{y(x)}{2x}\right) + y(x)\right)}{2x}$$

✓ **Mathematica** : cpu = 0.103184 (sec), leaf count = 23

$$\left\{\left\{y(x) \rightarrow 2x \cot^{-1}\left(e^{-c_1-\frac{x^2}{2}}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 64

$$\left\{y(x) = \arctan\left(2 \frac{e^{1/2 x^2} C1}{(e^{1/2 x^2})^2 - C1^2 + 1}, 1 \left(-\left(e^{\frac{x^2}{2}}\right)^2 - C1^2 + 1\right) \left(\left(e^{\frac{x^2}{2}}\right)^2 - C1^2 + 1\right)^{-1}\right) x\right\}$$

2.905 ODE No. 905

---

$$y'(x) = \frac{a^3 x^3 y(x)^3 + a^3 x^3 y(x)^2 + a^3 x^3 + 3a^2 x^2 y(x)^2 + 2a^2 x^2 y(x) + a^2 x + 3a x y(x) + a x + 1}{a^3 x^3}$$

✓ **Mathematica** : cpu = 0.239061 (sec), leaf count = 85

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{\frac{ax+3}{ax}+3y(x)}{\sqrt[3]{29}} - \#1\right)}{\sqrt[3]{29} - 29\#1^2}\&\right] = c_1 + \frac{1}{9}29^{2/3}x, y(x)\right]$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 46

$$\left\{y(x) = \frac{29 \text{RootOf}\left(-81 \int^{-Z} (841 \_a^3 - 27 \_a + 27)^{-1} d\_a + x + 3 \_C1\right) ax - 3 ax - 9}{9 ax}\right\}$$

2.906 ODE No. 906

---

$$y'(x) = \frac{x(x^2 + y(x)^2 + 1)}{x^6 + 3x^4 y(x)^2 + 3x^2 y(x)^4 - x^2 y(x) + y(x)^6 - y(x)^3 - y(x)}$$

✓ **Mathematica** : cpu = 0.12338 (sec), leaf count = 326

$$\left\{\left\{y(x) \rightarrow \text{Root}\left[4\#1^5 - 4\#1^4 c_1 + 8\#1^3 x^2 + \#1^2 (2 - 8c_1 x^2) + 4\#1 x^4 - 4c_1 x^4 + 2x^2 + 1\&, 1\right]\right\}, \left\{y(x) \rightarrow \text{Root}\left[4\#1^5 - 4\#1^4 c_1 + 8\#1^3 x^2 + \#1^2 (2 - 8c_1 x^2) + 4\#1 x^4 - 4c_1 x^4 + 2x^2 + 1\&, 1\right]\right\}\right\}$$

✓ **Maple** : cpu = 0.244 (sec), leaf count = 37

$$\left\{-\frac{1}{4 \left((y(x))^2 + x^2\right)^2} - \left(2 (y(x))^2 + 2 x^2\right)^{-1} - y(x) + \_C1 = 0\right\}$$

2.907 ODE No. 907

$$y'(x) = \frac{\frac{3x^2}{2} + x^2 \sin(x) - 2x^2 \cos(x) + \frac{1}{2}x^2 \cos(2x) - 2xy(x) + y(x)^2 + 2xy(x) \cos(x) + x - x \cos(x)}{x}$$

✓ **Mathematica** : cpu = 0.272028 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 - \log(x)} - x(\cos(x) - 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.112 (sec), leaf count = 20

$$\left\{ y(x) = -(-1 + \cos(x))x + (_C1 - \ln(x))^{-1} \right\}$$

2.908 ODE No. 908

$$y'(x) = \frac{4(a-1)(a+1)x}{a^6x^4 - 3a^4x^4 - 2a^4x^2y(x)^2 + 3a^2x^4 + 4a^2x^2y(x)^2 + a^2y(x)^4 - x^4 - 2x^2y(x)^2 - y(x)^4 + 4y(x)}$$

✓ **Mathematica** : cpu = 1.09792 (sec), leaf count = 1269

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{3(a^2-1)} + \frac{\sqrt[3]{-18x^2c_1a^6 + 54x^2c_1a^4 + 54a^4 - 54x^2c_1a^2 - 108a^2 + 2c_1^3 + 18x^2c_1 + \sqrt{4(-3x^2a^6 + \dots}}}{\dots} \right\} \right\}$$

✓ **Maple** : cpu = 0.282 (sec), leaf count = 1742

$$\left\{ y(x) = \frac{9^{\frac{2}{3}}}{27a^2 - 27} \left( (-_C1 a^2 + _C1) \sqrt[3]{9} \sqrt[3]{(a+1)^2(a-1)^2 \left( \frac{1}{3} \sqrt{-3(a-1)^5(a+1)^5x^6 + 6\_C1^2(a-1)^4} \right)} \right) \right\}$$

2.909 ODE No. 909

$$y'(x) = \frac{x^3y(x)^6 + x^3y(x)^4 + x^3 + 3x^2y(x)^4 + 2x^2y(x)^2 + 3xy(x)^2 + x + 1}{x^5y(x)}$$

✗ **Mathematica** : cpu = 40.6199 (sec), leaf count = 0 , could not solve

DSolve[Derivative[1][y][x] == (1 + x + x^3 + 3\*x\*y[x]^2 + 2\*x^2\*y[x]^2 + 3\*x^2\*y[x]^4 + x^3\*y[x]^6 + x^3 + 3\*x^2\*y[x]^4 + 2\*x^2\*y[x]^2 + 3\*x\*y[x]^2 + x + 1)/x^5\*y[x], y[x], x]

✓ **Maple** : cpu = 0.407 (sec), leaf count = 84

$$\left\{ y(x) = \frac{1}{x} \sqrt{x \left( RootOf \left( \int^{-Z} (2\_a^3 + 2\_a^2 + 1)^{-1} d\_ax + x\_C1 + 1 \right) x - 1 \right)}, y(x) = -\frac{1}{x} \sqrt{x \left( RootOf \left( \int^{-Z} (2\_a^3 + 2\_a^2 + 1)^{-1} d\_ax + x\_C1 + 1 \right) x - 1 \right)} \right\}$$

2.910 ODE No. 910

$$y'(x) = \frac{x^6 + 3x^5y(x) + 3x^4y(x)^2 + x^4 + x^3y(x)^3 + 2x^3y(x) + x^2y(x)^2 - y(x) - 2x + 1}{x}$$

✓ **Mathematica** : cpu = 0.226512 (sec), leaf count = 98

$$\text{Solve} \left[ -\frac{29}{3} \text{RootSum} \left[ -29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log \left( \frac{3x^3 + 3x^2y(x) + x}{\sqrt[3]{29}\sqrt[3]{x^3}} - \#1 \right)}{\sqrt[3]{29} - 29\#1^2} \& \right] = c_1 + \frac{29^{2/3} (x^3)^{2/3}}{9x}, y(x) \right]$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 42

$$\left\{ y(x) = \frac{-9x^2 + 29 RootOf \left( -81 \int^{-Z} (841\_a^3 - 27\_a + 27)^{-1} d\_a + x + 3\_C1 \right) - 3}{9x} \right\}$$

**2.911            ODE No. 911**

---

$$y'(x) = -y(x) \left( -\_F1(x) - \frac{\log(y(x))}{x} + \cot(x) \log(y(x)) \right)$$

✓ **Mathematica** : cpu = 0.731553 (sec), leaf count = 106

$$\text{Solve} \left[ \int_1^x \left( \frac{2 \log(y(x)) \sin(K[1])}{K[1]^2} - \frac{2(\cos(K[1]) \log(y(x)) - \sin(K[1])\_F1(K[1]))}{K[1]} \right) dK[1] + \int_1^{y(x)} \left( -\frac{2 \sin(x)}{xK[2]} \right) \right]$$

✓ **Maple** : cpu = 0.361 (sec), leaf count = 30

$$\left\{ y(x) = e^{\frac{x \_C1}{\sin(x)}} e^{\frac{x}{\sin(x)} \int \frac{-F1(x) \sin(x)}{x} dx} \right\}$$

**2.912            ODE No. 912**

---

$$y'(x) = \frac{2ax}{-128a^4 + 96a^3xy(x)^2 + 32a^3x - 24a^2x^2y(x)^4 - 16a^2x^2y(x)^2 + 2ax^3y(x)^6 + 2ax^3y(x)^4 + 2ax^3 - x^3y(x)}$$

✓ **Mathematica** : cpu = 1.24094 (sec), leaf count = 201

$$\text{Solve} \left[ -\text{RootSum} \left[ -\#1^3y(x)^6 - \#1^3y(x)^4 - \#1^3 + 12\#1^2ay(x)^4 + 8\#1^2ay(x)^2 - 48\#1a^2y(x)^2 - 16\#1a^2 + 6 \right] \right]$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x) = 2\*a\*x/(-x^3\*y(x)+2\*a\*x^3+2\*a\*y(x)^4\*x^3-16\*y(x)^2\*a^2\*x^2+32\*a^3\*x+2\*a^3-24\*y(x)^4\*a^2\*x^2+96\*y(x)^2\*x\*a^3-128\*a^4),y(x))

**2.913            ODE No. 913**

---

$$y'(x) = \frac{y(x)^3 + y(x) + y(x)^3 (-\log^3(x)) + y(x)^3 \log^2(x) + 3y(x)^2 \log^2(x) - 2y(x)^2 \log(x) - 3y(x) \log(x) + 1}{xy(x)}$$

✓ **Mathematica** : cpu = 0.779856 (sec), leaf count = 716

$$\text{Solve} \left[ \int_1^{y(x)} \left( 2\text{RootSum} \left[ \#1^3K[1]^3 - \#1^2K[1]^3 - 2K[1]^3 - 3\#1^2K[1]^2 + 2\#1K[1]^2 + 3\#1K[1] - K[1] - 1 \right] \right) \right]$$

✓ **Maple** : cpu = 0.118 (sec), leaf count = 43

$$\left\{ y(x) = 9 \left( 9 \ln(x) + 56 \text{RootOf} \left( -81 \int^{-Z} (3136\_a^3 - 27\_a + 27)^{-1} d\_a - \ln(x) + 3\_C1 \right) - 3 \right)^{-1} \right\}$$

**2.914            ODE No. 914**

---

$$y'(x) = \frac{2a(-4a + xy(x)^2 + x)}{-128a^4 + 96a^3xy(x)^2 - 24a^2x^2y(x)^4 + 2ax^3y(x)^6 + 4ax^2y(x) - x^3y(x)^3 - x^3y(x)}$$

✓ **Mathematica** : cpu = 1.1785 (sec), leaf count = 401

$$\{ \{ y(x) \rightarrow \text{Root} [8\#1^5ax^2 - 8\#1^4ac_1x^2 - 64\#1^3a^2x + \#1^2(64a^2c_1x + 2x^2) + 128\#1a^3 - 128a^3c_1 - 8ax + x^2] \}$$

✓ **Maple** : cpu = 3.299 (sec), leaf count = 71

$$\left\{ \frac{x(y(x))^4 + (-4a + x)(y(x))^2 - 2a}{2a(y(x))^4(-x(y(x))^2 + 4a)^2} + \frac{8a(y(x))^5 + 2(y(x))^2 + 1}{16a^2(y(x))^4} + \_C1 = 0 \right\}$$



2.915 ODE No. 915

$$y'(x) = \frac{y(x)^3 + y(x) - 8y(x)^3 \log^3(x) + 4y(x)^3 \log^2(x) + 12y(x)^2 \log^2(x) - 4y(x)^2 \log(x) - 6y(x) \log(x) + 1}{xy(x)}$$

✓ **Mathematica** : cpu = 0.865219 (sec), leaf count = 724

Solve  $\left[ \int_1^{y(x)} \left( 4\text{RootSum} \left[ 8\#1^3 K[1]^3 - 4\#1^2 K[1]^3 - 3K[1]^3 - 12\#1^2 K[1]^2 + 4\#1 K[1]^2 + 6\#1 K[1] - K[1] - 1 \right] \right) dy(x) \right]$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 43

$$\left\{ y(x) = 9 \left( 18 \ln(x) + 83 \text{RootOf} \left( -81 \int^{-Z} (6889\_a^3 - 27\_a + 27)^{-1} d\_a - \ln(x) + 3\_C1 \right) - 3 \right)^{-1} \right\}$$

2.916 ODE No. 916

$$y'(x) = \frac{y(x) (x^4 \log^2(y(x)) + 2x^4 \log(x) \log(y(x)) + x^4 \log^2(x) + x \log(y(x)) + \log(y(x)) - x + x \log(x) + \log(x) - 1)}{x(x+1)}$$

✗ **Mathematica** : cpu = 1.60191 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[1][y][x] == ((-1 - x + Log[x] + x\*Log[x] + x^4\*Log[x]^2 + Log[y[x]] + x\*Log[x])

✓ **Maple** : cpu = 0.293 (sec), leaf count = 73

$$\left\{ y(x) = e^{\frac{-12 \ln(1+x) \ln(x) + (-3x^4 + 4x^3 - 6x^2 + 12\_C1 + 12x) \ln(x) - 12x}{3x^4 - 4x^3 + 6x^2 + 12 \ln(1+x) - 12\_C1 - 12x}} \right\}$$

2.917 ODE No. 917

$$y'(x) = \frac{y(x) (x \log^2(y(x)) + 2x \log(x) \log(y(x)) + x \log(y(x)) + \log(y(x)) - x + x \log^2(x) + x \log(x) + \log(x) - 1)}{x(x+1)}$$

✗ **Mathematica** : cpu = 1.02715 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[1][y][x] == ((-1 - x + Log[x] + x\*Log[x] + x\*Log[x]^2 + Log[y[x]] + x\*Log[x])

✓ **Maple** : cpu = 0.155 (sec), leaf count = 38

$$\left\{ y(x) = e^{\frac{\ln(1+x) \ln(x) + (-x +\_C1) \ln(x) - x}{-\ln(1+x) -\_C1 + x}} \right\}$$

2.918 ODE No. 918

$$y'(x) = \frac{2y(x)^8}{128x^3y(x)^6 + 32x^2y(x)^6 + 96x^2y(x)^4 + 2y(x)^6 + y(x)^5 + 16xy(x)^4 + 24xy(x)^2 + 2y(x)^2 + 2}$$

✓ **Mathematica** : cpu = 0.45165 (sec), leaf count = 720

Solve  $\left[ \int_1^{y(x)} \left( \text{RootSum} \left[ 64\#1^3 K[1]^6 + 16\#1^2 K[1]^6 + K[1]^6 + 48\#1^2 K[1]^4 + 8\#1 K[1]^4 + 12\#1 K[1]^2 + K[1]^2 - 1 \right] \right) dy(x) \right]$

✓ **Maple** : cpu = 0.961 (sec), leaf count = 41

$$\left\{ x - \text{RootOf} \left( \int^{-Z} (64\_a^3 + 16\_a^2 + 1)^{-1} d\_a y(x) +\_C1 y(x) + 1 \right) + \frac{1}{4 (y(x))^2} = 0 \right\}$$

2.919 ODE No. 919

$$y'(x) = \frac{(-y(x) + \sqrt{y(x)} + x) y(x)^{3/2}}{x^3 - 3x^2y(x) + 3xy(x)^2 + xy(x)^{3/2} - y(x)^3 - y(x)^{5/2} + y(x)^2}$$

✗ **Mathematica** : cpu = 300.415 (sec), leaf count = 0 , timed out  
\$Aborted

✓ **Maple** : cpu = 0.164 (sec), leaf count = 61

$$\left\{ -1 \left( \left( x - y(x) - \sqrt{y(x)} \right) \sqrt{2y(x) - \sqrt{y(x)} - 2x +\_C1 (y(x) - x)^{\frac{3}{2}} (y(x))^{\frac{3}{4}}} \right) (y(x) - x)^{-\frac{3}{2}} (y(x))^{-\frac{3}{4}} = 0 \right\}$$

2.920 ODE No. 920

---

$$y'(x) = \frac{2y(x)^6 \left(4xy(x)^2 + y(x)^2 + 1\right)}{128x^3y(x)^6 + 96x^2y(x)^4 + 4xy(x)^5 + y(x)^5 + y(x)^3 + 24xy(x)^2 + 2}$$

✓ **Mathematica** : cpu = 0.453647 (sec), leaf count = 301

$\{ \{ y(x) \rightarrow \text{Root} \left[ \#1^5 (128c_1x^2 - 8x - 1) + 128\#1^4x^2 + \#1^3(64c_1x - 2) + 64\#1^2x + 8\#1c_1 + 8\&, 1 \right] \}, \{ y(x) \rightarrow$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x) = 2\*y(x)^6\*(1+4\*x\*y(x)^2+y(x)^2)/(y(x)^3+4\*y(x)^5\*x+y(x)^5+2+24\*x\*y(x)^2

2.921 ODE No. 921

---

$$y'(x) = -y(x) \left( -\_F1(x) - \frac{\log(y(x))}{x} + \frac{\log(y(x))}{x \log(x)} \right)$$

✓ **Mathematica** : cpu = 0.231542 (sec), leaf count = 92

Solve  $\left[ \int_1^x \left( \frac{\log(y(x)) - \log(K[1]) \log(y(x))}{K[1]^2} - \frac{\log(K[1])\_F1(K[1])}{K[1]} \right) dK[1] + \int_1^{y(x)} \left( \frac{\log(x)}{xK[2]} - \int_1^x \frac{1}{K[2]} - \frac{\log(K[2])}{K[1]^2} \right) dK[2] \right]$

✓ **Maple** : cpu = 0.1 (sec), leaf count = 30

$$\left\{ y(x) = e^{\frac{C1}{\ln(x)}} e^{\frac{x}{\ln(x)} \int \frac{-F1(x) \ln(x)}{x} dx} \right\}$$

2.922 ODE No. 922

---

$$y'(x) = \frac{y(x)^2}{x^3 - 3x^2y(x) + x^2\sqrt{y(x)} + 3xy(x)^2 - 2xy(x)^{3/2} - y(x)^3 + y(x)^{5/2} + y(x)^2 + y(x)^{3/2}}$$

✓ **Mathematica** : cpu = 1.24035 (sec), leaf count = 882

Solve  $\left[ \int_1^{y(x)} \left( \frac{-x - K[1]}{2 \left( -2x^3 + 6K[1]x^2 - 2\sqrt{K[1]}x^2 - 6K[1]^2x + 4K[1]^{3/2}x + K[1]x + 2K[1]^3 - 2K[1]^{5/2} - K[1]^2 - \right)} \right) dy(x) \right]$

✓ **Maple** : cpu = 0.102 (sec), leaf count = 47

$$\left\{ \frac{\ln(y(x))}{2} - \int^{x \frac{1}{\sqrt{y(x)}} - \sqrt{y(x)}} (2\_a^3 + 2\_a^2 - \_a + 2)^{-1} d\_a - \_C1 = 0 \right\}$$

2.923 ODE No. 923

---

$$y'(x) = \frac{x^2 + 2xy(x) + e^{-2(x-y(x))(y(x)+x)} + y(x)^2}{x^2 + 2xy(x) - e^{-2(x-y(x))(y(x)+x)} + y(x)^2}$$

✓ **Mathematica** : cpu = 1.39399 (sec), leaf count = 432

Solve  $\left[ \int_1^{y(x)} \left( -\frac{2e^{2(x-K[2])(x+K[2])}K[2]}{-e^{2(x-K[2])(x+K[2])}x^2 + e^{2(x-K[2])(x+K[2])}K[2]^2 + 1} - \int_1^x \left( -\frac{2e^{2(K[1]-K[2])(K[1]+K[2])}K[1](2(K[1]-K[2]) + K[2])}{e^{2(K[1]-K[2])(K[1]+K[2])}K[1]^2 - e^{2(K[1]-K[2])(K[1]+K[2])}K[2]^2 + 1} \right) dK[1] \right) dy(x) \right]$

✓ **Maple** : cpu = 0.242 (sec), leaf count = 36

$$\left\{ y(x) = e^{\text{RootOf}\left(-\_Z + \int^{(e^{-Z})^2 - 2e^{-Z}x(e^{2\_a} + \_a)^{-1}d\_a - \_C1}\right) - x} \right\}$$

2.924 ODE No. 924

$$y'(x) = -\frac{y(x) \left( -_F1(x) - \frac{\log^2(y(x))}{2x} \right)}{\log(y(x))}$$

✓ **Mathematica** : cpu = 0.235722 (sec), leaf count = 80

Solve  $\left[ \int_1^x \left( -\frac{\log^2(y(x))}{2K[1]^2} - \frac{_F1(K[1])}{K[1]} \right) dK[1] + \int_1^{y(x)} \left( \frac{\log(K[2])}{xK[2]} - \int_1^x -\frac{\log(K[2])}{K[1]^2 K[2]} dK[1] \right) dK[2] = c_1, y(x) \right]$

✓ **Maple** : cpu = 0.107 (sec), leaf count = 46

$$\left\{ y(x) = e^{\sqrt{2 \int \frac{F1(x)}{x} dx + 2\_C1} x}, y(x) = e^{-\sqrt{2 \int x \left( \int \frac{F1(x)}{x} dx +\_C1 \right)} x} \right\}$$

2.925 ODE No. 925

$$y'(x) = \frac{x^2 + 2xy(x) + e^{2(x-y(x))^2(y(x)+x)^2} + y(x)^2}{x^2 + 2xy(x) - e^{2(x-y(x))^2(y(x)+x)^2} + y(x)^2}$$

✓ **Mathematica** : cpu = 2.0394 (sec), leaf count = 228

Solve  $\left[ \int_1^{y(x)} \left( -\frac{2K[2]}{-x^2 + e^{2(x-K[2])^2(x+K[2])^2} + K[2]^2} - \int_1^x \left( \frac{2K[1] \left( -2K[2] - e^{2(K[1]-K[2])^2(K[1]+K[2])^2} \right) (4(K[1] - \right)}{(K[1]^2 - e^{2(K[1]-K[2])^2(K[1]+K[2])^2} + K[2]^2)} \right) dK[1] \right) dK[2] = c_1, y(x) \right]$

✓ **Maple** : cpu = 0.18 (sec), leaf count = 38

$$\left\{ y(x) = e^{\text{RootOf}\left(-\_Z + \int \left(e^{-Z}\right)^2 - 2e^{-Z} x \left(e^{2-a^2} +\_a\right)^{-1} d\_a +\_C1\right) - x} \right\}$$

2.926 ODE No. 926

$$y'(x) = \frac{\frac{1}{16}x^3y(x)^3 - \frac{1}{2}x^2y(x)^3 - \frac{3}{8}x^2y(x)^2 + xy(x)^3 + xy(x)^2 + \frac{3}{4}xy(x) - \frac{1}{2}}{x(xy(x) - 2y(x) - 2)}$$

✓ **Mathematica** : cpu = 0.184798 (sec), leaf count = 128

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{16x(x-2) \left( -\frac{e^{2\left(\frac{1}{2}\log(2-x) - \frac{\log(x)}{2}\right)}}{\sqrt{c_1 + 2048\log(x)}} - \frac{1}{64} \right)} + \frac{2}{x-2} \right\}, \left\{ y(x) \rightarrow \frac{1}{16x(x-2) \left( \frac{e^{2\left(\frac{1}{2}\log(2-x) - \frac{\log(x)}{2}\right)}}{\sqrt{c_1 + 2048\log(x)}} - \frac{1}{64} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 67

$$\left\{ y(x) = 1 \left( 2 \sqrt{\_C1 + 8 \ln(x)} - 8 \right) \left( x \sqrt{\_C1 + 8 \ln(x)} - 4x + 8 \right)^{-1}, y(x) = 1 \left( 2 \sqrt{\_C1 + 8 \ln(x)} + 8 \right) \left( x \sqrt{\_C1 + 8 \ln(x)} + 4x - 8 \right)^{-1} \right\}$$

2.927 ODE No. 927

$$y'(x) = -\frac{1}{8}x \left( 12e^{-x^2}x^2y(x)^2 + 8e^{-x^2}x^2y(x) + 8e^{-x^2}x^2 - 8e^{-x^2} + e^{-3x^2}x^6 - 6e^{-2x^2}x^4y(x) - 2e^{-2x^2}x^4 - 8y(x)^3 \right)$$

✓ **Mathematica** : cpu = 0.565879 (sec), leaf count = 112

Solve  $\left[ -\frac{29}{3}\text{RootSum}\left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{\frac{1}{2}e^{-x^2}x(2e^{x^2}-3x^2)+3xy(x)}{\sqrt[3]{29}\sqrt[3]{x^3}} - \#1\right)}{\sqrt[3]{29} - 29\#1^2}\&\right] = c_1 + \frac{1}{18}29^{2/3}(x^3) \right]$

✓ **Maple** : cpu = 0.083 (sec), leaf count = 68

$$\left\{ y(x) = \frac{58 \text{RootOf}\left(x^2 - 162 \int^{-Z} (841\_a^3 - 27\_a + 27)^{-1} d\_a + 6\_C1\right) + \left(9x^2 - 6e^{x^2}\right)e^{-x^2}}{18e^{-x^2}e^{x^2}} \right\}$$

## 2.928 ODE No. 928

---

$$y'(x) = \frac{e^{\frac{y(x)}{x}} \left( x^2 e^{-\frac{y(x)}{x}} + x e^{-\frac{y(x)}{x}} + x e^{-\frac{y(x)}{x}} y(x) + e^{-\frac{y(x)}{x}} y(x) + x \right)}{x(x+1)}$$

✓ **Mathematica** : cpu = 1.22982 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow -x \log \left( -\frac{\log(x+1) - c_1}{x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.273 (sec), leaf count = 20

$$\left\{ y(x) = -\ln \left( \frac{-\ln(1+x) + \_C1}{x} \right) x \right\}$$

## 2.929 ODE No. 929

---

$$y'(x) = \frac{-\frac{1}{32}x^3y(x)^3 + \frac{1}{16}x^2y(x)^3 + \frac{3}{16}x^2y(x)^2 - \frac{1}{2}xy(x)^3 + \frac{y(x)^3}{4} - \frac{1}{4}xy(x)^2 - \frac{3}{8}xy(x) + \frac{y(x)}{4} + \frac{1}{4}}{xy(x)}$$

✓ **Mathematica** : cpu = 0.806826 (sec), leaf count = 683

$$\text{Solve} \left[ \int_1^{y(x)} \left( -32 \text{RootSum} \left[ \#1^3 K[1]^3 - 2\#1^2 K[1]^3 - 8K[1]^3 - 6\#1^2 K[1]^2 + 8\#1 K[1]^2 + 12\#1 K[1] - 8K[1] \right] \right) \right]$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 42

$$\left\{ y(x) = 18 \left( 58 \text{RootOf} \left( -324 \int^{-Z} (841\_a^3 - 27\_a + 27)^{-1} d\_a - \ln(x) + 12\_C1 \right) + 9x - 6 \right)^{-1} \right\}$$

## 2.930 ODE No. 930

---

$$y'(x) = \frac{e^{\frac{y(x)}{x}} \left( x^4 + x^2 e^{-\frac{y(x)}{x}} + x e^{-\frac{y(x)}{x}} + x e^{-\frac{y(x)}{x}} y(x) + e^{-\frac{y(x)}{x}} y(x) \right)}{x(x+1)}$$

✓ **Mathematica** : cpu = 1.41358 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow -x \log \left( \frac{-c_1 - \frac{x^3}{3} + \frac{x^2}{2} - x + \log(x+1)}{x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.419 (sec), leaf count = 36

$$\left\{ y(x) = -\ln \left( \frac{-2x^3 + 3x^2 + 6 \ln(1+x) - 6\_C1 - 6x}{6x} \right) x \right\}$$

## 2.931 ODE No. 931

---

$$y'(x) = \frac{x^6 + 3x^5y(x) + 3x^4y(x)^2 + x^3y(x)^3 - 2x^3 - 3x^2y(x) - xy(x)^2 - y(x) - 2x}{x(x^2 + xy(x) + 1)}$$

✓ **Mathematica** : cpu = 0.169507 (sec), leaf count = 80

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{x^2 \left( \frac{1}{x} - \frac{1}{x\sqrt{c_1-2x}} \right)} - \frac{x^2+1}{x} \right\}, \left\{ y(x) \rightarrow \frac{1}{x^2 \left( \frac{1}{x\sqrt{c_1-2x}} + \frac{1}{x} \right)} - \frac{x^2+1}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 73

$$\left\{ y(x) = \frac{1}{x} \left( -\sqrt{-C1 - 2xx^2 - x^2 - 1} \right) \left( \sqrt{-C1 - 2x} + 1 \right)^{-1}, y(x) = \frac{1}{x} \left( -\sqrt{-C1 - 2xx^2 + x^2 + 1} \right) \left( \sqrt{-C1 - 2x} + 1 \right)^{-1} \right\}$$

2.932 ODE No. 932

$$y'(x) = \frac{e^{-\frac{3x^2}{2}} x \left( 3e^{3x^2} y(x)^3 + e^{\frac{9x^2}{2}} y(x)^3 + 18e^{3x^2} y(x)^2 + 9e^{\frac{9x^2}{2}} y(x)^2 + 27e^{3x^2} y(x) + 27e^{\frac{9x^2}{2}} y(x) + 27e^{\frac{9x^2}{2}} + 27y(x) \right)}{243y(x)}$$

✓ **Mathematica** : cpu = 4.53863 (sec), leaf count = 4323

$$\text{Solve} \left[ \frac{x^2}{2} + \frac{27}{2} \text{RootSum} \left[ y(x)^3 \#1^3 + 9y(x)^2 \#1^3 + 27y(x) \#1^3 + 27 \#1^3 + 3y(x)^3 \#1^2 + 18y(x)^2 \#1^2 + 27y(x) \#1^2 + 27 \#1^2 \right], \right]$$

✓ **Maple** : cpu = 0.101 (sec), leaf count = 54

$$\left\{ y(x) = -369 \frac{e^{3/2 x^2}}{123 + 123 e^{3/2 x^2} - 136 \text{RootOf} \left( -41 x^2 - 50243409 \int^{-Z} (9248 \_a^3 - 1860867 \_a + 1860867)^{-1} d\_a \right)} \right\}$$

2.933 ODE No. 933

$$y'(x) = \frac{x^3 + x^3 \left( -\log^3(x) \right) + x^3 \log^2(x) + 3x^2 y(x) \log^2(x) - 2x^2 y(x) \log(x) + x^2 + x y(x)^2 + x y(x) + y(x)^3 - 3x}{x^2}$$

✓ **Mathematica** : cpu = 0.235948 (sec), leaf count = 99

$$\text{Solve} \left[ -\frac{29}{3} \text{RootSum} \left[ -29 \#1^3 + 3 \sqrt[3]{29} \#1 - 29 \&, \frac{\log \left( \frac{\frac{3y(x)}{x^2} + \frac{1-3 \log(x)}{x}}{\sqrt[3]{29} \sqrt[3]{\frac{1}{x^3}}} - \#1 \right)}{\sqrt[3]{29} - 29 \#1^2} \& \right] = c_1 + \frac{29^{2/3}}{9 \sqrt[3]{\frac{1}{x^3}}}, y(x) \right]$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 39

$$\left\{ y(x) = \frac{x \left( 9 \ln(x) - 3 + 29 \text{RootOf} \left( -81 \int^{-Z} (841 \_a^3 - 27 \_a + 27)^{-1} d\_a + x + 3 \_C1 \right) \right)}{9} \right\}$$

2.934 ODE No. 934

$$y'(x) = -\frac{x^6}{64} - \frac{3x^5}{32} + \frac{3}{16} x^4 y(x) - \frac{x^4}{8} + \frac{3}{4} x^3 y(x) + \frac{x^3}{8} - \frac{3}{4} x^2 y(x)^2 + \frac{1}{4} x^2 y(x) + \frac{x^2}{4} - \frac{3}{2} x y(x)^2 - x y(x) + y(x)^3 + y(x)^2 + \frac{x}{2} + \frac{1}{2}$$

✓ **Mathematica** : cpu = 0.268849 (sec), leaf count = 102

$$\text{Solve} \left[ -\frac{31}{3} \text{RootSum} \left[ -31 \#1^3 + 3 2^{2/3} \sqrt[3]{31} \#1 - 31 \&, \frac{\log \left( \sqrt[3]{\frac{2}{31}} \left( \frac{1}{4} (-3x^2 - 6x + 4) + 3y(x) \right) - \#1 \right)}{2^{2/3} \sqrt[3]{31} - 31 \#1^2} \& \right] = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.051 (sec), leaf count = 39

$$\left\{ y(x) = \frac{x^2}{4} + \frac{x}{2} + \text{RootOf} \left( -x + 2 \int^{-Z} (2 \_a^3 + 2 \_a^2 + 1)^{-1} d\_a + \_C1 \right) \right\}$$

2.935 ODE No. 935

$$y'(x) = \frac{x^6}{64} - \frac{3x^5}{16} + \frac{3}{16} x^4 y(x) + \frac{13x^4}{16} - \frac{3}{2} x^3 y(x) - \frac{3x^3}{2} + \frac{3}{4} x^2 y(x)^2 + \frac{7}{2} x^2 y(x) + x^2 - 3x y(x)^2 - 2x y(x) + y(x)^3 + y(x)^2 - \frac{x}{2} + \frac{1}{2}$$

✓ **Mathematica** : cpu = 27.5437 (sec), leaf count = 248

$$\text{Solve} \left[ \frac{\sqrt[3]{2} \left( \frac{\frac{1}{4} (3x^2 - 12x + 4) + 3y(x)}{\sqrt[3]{2}} + 2^{2/3} \right) \left( 2^{2/3} - 2^{2/3} \left( \frac{1}{4} (3x^2 - 12x + 4) + 3y(x) \right) \right) \left( \left( \frac{1}{4} (-3x^2 + 12x - 4) - 3y(x) \right) \right)}{9 \left( - \left( \frac{1}{4} (3x^2 - 12x + 4) \right) \right)} \right]$$

✓ **Maple** : cpu = 0.153 (sec), leaf count = 55

$$\left\{y(x)=\frac{\mathrm{e}^{RootOf\left(\ln\left(\mathrm{e}^{-Z}-4\right)\mathrm{e}^{-Z}+_C1\,\mathrm{e}^{-Z}-_Z\,\mathrm{e}^{-Z}+\mathrm{e}^{-Z}x-4\,\ln\left(\mathrm{e}^{-Z}-4\right)-4\,_C1+4\,_Z-4\,x+4\right)}}{4}-1-\frac{x^2}{4}+x\right\}$$

**2.936 ODE No. 936**

---

$$y'(x)=\frac{x^6}{512}-\frac{3x^5}{256}+\frac{3}{64}x^4y(x)+\frac{5x^4}{128}-\frac{3}{16}x^3y(x)-\frac{5x^3}{64}+\frac{3}{8}x^2y(x)^2+\frac{7}{16}x^2y(x)+\frac{x^2}{16}-\frac{3}{4}xy(x)^2-\frac{1}{2}xy(x)+y(x)^3+y(x)$$

✓ **Mathematica** : cpu = 0.247281 (sec), leaf count = 99

$$\text{Solve}\left[-\frac{89}{3}\text{RootSum}\left[-89\#1^3+6\sqrt[3]{178}\#1-89\&, \frac{\log\left(\frac{2^{2/3}\left(\frac{1}{8}(3x^2-6x+8)+3y(x)\right)}{\sqrt[3]{89}}-\#1\right)}{2\sqrt[3]{178}-89\#1^2}\&\right]=c_1+\frac{89^{2/3}x}{18\sqrt[3]{2}},y(x)\right].$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 39

$$\left\{y(x)=-\frac{x^2}{8}+\frac{x}{4}+RootOf\left(-x+4\int^{-Z}\left(4\_a^3+4\_a^2+3\right)^{-1}d\_a+_C1\right)\right\}$$

**2.937 ODE No. 937**

---

$$y'(x)=\frac{2xy(x)^3+y(x)^3-2y(x)+6xy(x)\log^2(2x+1)+3y(x)\log^2(2x+1)+6xy(x)^2\log(2x+1)+3y(x)^2\log(2x+1)}{(2x+1)(y(x)+\log(2x+1)+1)}$$

✓ **Mathematica** : cpu = 0.270636 (sec), leaf count = 124

$$\left\{\left\{y(x)\rightarrow\frac{1}{(2x+1)\left(\frac{2x+1}{4x^2+4x+1}-\frac{1}{(2x+1)\sqrt{c_1-2x}}\right)}-\log(2x+1)-1\right\},\left\{y(x)\rightarrow\frac{1}{(2x+1)\left(\frac{1}{(2x+1)\sqrt{c_1-2x}}+\frac{2x+1}{4x^2+4x+1}\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 79

$$\left\{y(x)=1\left(-\sqrt{-C1-2x}\ln(2x+1)-\ln(2x+1)-1\right)\left(\sqrt{-C1-2x}+1\right)^{-1},y(x)=1\left(-\sqrt{-C1-2x}\ln(2x+1)-\ln(2x+1)-1\right)\left(\sqrt{-C1-2x}+1\right)^{-1}\right\}$$

**2.938 ODE No. 938**

---

$$y'(x)=\frac{x^6-3x^5+3x^4y(x)+4x^4-6x^3y(x)-3x^3+3x^2y(x)^2+5x^2y(x)-x^2-3xy(x)^2-2xy(x)+y(x)^3+y(x)}{x}$$

✓ **Mathematica** : cpu = 0.239283 (sec), leaf count = 108

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3+3\sqrt[3]{29}\#1-29\&, \frac{\log\left(\frac{\frac{3x^2-3x+1}{x}+\frac{3y(x)}{\sqrt[3]{29}\sqrt[3]{\frac{1}{x^3}}}}{\sqrt[3]{29}}-\#1\right)}{\sqrt[3]{29}-29\#1^2}\&\right]=c_1+\frac{1}{9}29^{2/3}\left(\frac{1}{x^3}\right)^{2/3}x^2\log\left(\frac{1}{x^3}\right)\right]$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 39

$$\left\{y(x)=-x^2+x-\frac{1}{3}+\frac{29\,RootOf\left(-81\int^{-Z}\left(841\_a^3-27\_a+27\right)^{-1}d\_a+\ln(x)+3\_C1\right)}{9}\right\}$$

**2.939 ODE No. 939**

---

$$y'(x)=\frac{x^6+6x^5-12x^4y(x)+12x^4-48x^3y(x)+16x^3+48x^2y(x)^2-48x^2y(x)+16x^2+96xy(x)^2-32xy(x)-16x^2-64y(x)+32x-64}{16x^2-64y(x)+32x-64}$$

✓ **Mathematica** : cpu = 0.528905 (sec), leaf count = 136

$$\text{Solve}\left[\frac{2}{5}\text{RootSum}\left[\#1^4+4\#1^3-8\#1^2y(x)-16\#1y(x)-8\#1+16y(x)^2+16y(x)+8\&,\frac{\#1^2(-\log(x-\#1))}{5}\right]=c_1+\frac{2}{5}\log(x-\#1),y(x)\right]$$

✓ **Maple** : cpu = 0.105 (sec), leaf count = 70

$$\left\{x+\frac{2}{5}\ln\left(2\left(y\left(x\right)-1/4\,x^2-x/2\right)^2+2\,y\left(x\right)-\frac{x^2}{2}-x+1\right)-\frac{2}{5}\arctan\left(-2\,y\left(x\right)+\frac{x^2}{2}+x-1\right)-\frac{4}{5}\ln\left(y\left(x\right)\right)\right\}$$

**2.940 ODE No. 940**

---

$$y'(x)=\frac{x^3\log^3(x)-3x^2y(x)\log^2(x)-x^2+x^2\log(x)-y(x)^3-y(x)^2-2xy(x)+3xy(x)^2\log(x)+xy(x)\log(x)}{x(-y(x)-x+x\log(x))}$$

✓ **Mathematica** : cpu = 0.205835 (sec), leaf count = 80

$$\left\{\left\{y(x)\rightarrow-\frac{1}{x\left(-\frac{1}{x^2\sqrt{c_1-2x}}-\frac{1}{x^2}\right)}-x+x\log(x)\right\},\left\{y(x)\rightarrow-\frac{1}{x\left(\frac{1}{x^2\sqrt{c_1-2x}}-\frac{1}{x^2}\right)}-x+x\log(x)\right\}\right\}$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 63

$$\left\{y\left(x\right)=x\left(\ln\left(x\right)\sqrt{-C1-2x}-\ln\left(x\right)+1\right)\left(\sqrt{-C1-2x}-1\right)^{-1},y\left(x\right)=x\left(\ln\left(x\right)\sqrt{-C1-2x}+\ln\left(x\right)-1\right)\right\}$$

**2.941 ODE No. 941**

---

$$y'(x)=\frac{x^6-12x^5+12x^4y(x)+48x^4-96x^3y(x)-72x^3+48x^2y(x)^2+192x^2y(x)+32x^2-192xy(x)^2-32xy(x)}{16x^2+64y(x)-64x+64}$$

✓ **Mathematica** : cpu = 0.45322 (sec), leaf count = 53

Solve  $\left[x-8\text{RootSum}\left[11776\#1^3-40\#1-1\&,\#1\log\left(17664\#1^2-1472\#1+11x^2+44y(x)-44x-40\right)\&\right]=\right.$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 35

$$\left\{y\left(x\right)=-\frac{x^2}{4}+x+RootOf\left(-x+\int^{-Z}\frac{-a+1}{-a^3--a-1}d-a+_C1\right)\right\}$$

**2.942 ODE No. 942**

---

$$y'(x)=\frac{-\exp\left(\frac{2(x-y(x))^3(y(x)+x)^3}{x^2-y(x)^2-1}\right)-x^2-2xy(x)-y(x)^2}{\exp\left(\frac{2(x-y(x))^3(y(x)+x)^3}{x^2-y(x)^2-1}\right)-x^2-2xy(x)-y(x)^2}$$

✓ **Mathematica** : cpu = 4.3152 (sec), leaf count = 349

Solve  $\left[\int_1^{y(x)}\left(-\frac{2K[2]}{-x^2+\exp\left(\frac{2(x-K[2])^3(x+K[2])^3}{x^2-K[2]^2-1}\right)}+K[2]^2\right)-\int_1^x\left(\frac{2K[1]\left(-2K[2]-\exp\left(\frac{2(K[1]-K[2])^3(K[1]+K[2])}{K[1]^2-K[2]^2-1}\right)\right)}{\left(K[1]^2-K[2]^2-1\right)}\right)dx\right]=0$

✓ **Maple** : cpu = 0.427 (sec), leaf count = 43

$$\left\{y\left(x\right)=e^{RootOf\left(-_Z+\int^{\left(e^{-Z}\right)^2-2e^{-Z}x}\left(e^2\frac{a^3}{-a+1}+_a\right)^{-1}d-a+_C1\right)}-x\right\}$$

**2.943 ODE No. 943**

---

$$y'(x)=\frac{x^6-6x^5+24x^4y(x)+12x^4-96x^3y(x)-24x^3+192x^2y(x)^2+96x^2y(x)+32x^2-384xy(x)^2-128xy(x)}{64x^2+512y(x)-128x+512}$$

✓ **Mathematica** : cpu = 0.50473 (sec), leaf count = 53

Solve  $\left[x-16\text{RootSum}\left[6656\#1^3-23\#1-1\&,\#1\log\left(79872\#1^2-18304\#1+181x^2+1448y(x)-362x-184\right)\&\right]=\right.$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 40

$$\left\{y\left(x\right)=-\frac{x^2}{8}+\frac{x}{4}+RootOf\left(-x+\int^{335}\frac{-a+1}{4-a^3--a-1}d-a+_C1\right)\right\}$$

**2.944 ODE No. 944**

---

$$y'(x) = \frac{a^3x^6 + 6a^2bx^5 + 12a^2x^4y(x) - 8a^2x^3 + 12ab^2x^4 + 48abx^3y(x) - 16abx^2 + 48ax^2y(x)^2 - 32axy(x) - 32a}{16ax^2 + 32bx + 64y(x) + 64}$$

✓ **Mathematica** : cpu = 1.64987 (sec), leaf count = 233

$$\text{Solve} \left[ x - 4\text{RootSum} \left[ \#1^6a^3 + 6\#1^5a^2b + 12\#1^4a^2y(x) + 12\#1^4ab^2 + 48\#1^3aby(x) + 8\#1^3b^3 + 8\#1^2ab + 48\#1^2a^2b^2 \right], x \right]$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 47

$$\left\{ y(x) = -\frac{ax^2}{4} - \frac{bx}{2} + \text{RootOf} \left( bx + 2 \int^{-Z} -\frac{b(\_a + 1)}{2\_a^3 + \_a b + b} d\_a + 2\_C1 \right) \right\}$$

**2.945 ODE No. 945**

---

$$y'(x) = \frac{8a^3x^3 + 12a^2x^4 + 48a^2x^2y(x) + 6ax^5 + 48ax^3y(x) - 16ax^2 + 96axy(x)^2 + x^6 + 12x^4y(x) - 8x^3 + 48x^2y(x)^2}{32ax + 16x^2 + 64y(x) + 64}$$

✓ **Mathematica** : cpu = 1.28946 (sec), leaf count = 213

$$\text{Solve} \left[ x - 4\text{RootSum} \left[ \#1^6 + 6\#1^5a + 12\#1^4a^2 + 12\#1^4y(x) + 8\#1^3a^3 + 48\#1^3ay(x) + 48\#1^2a^2y(x) + 8\#1^2a^2y(x)^2 \right], x \right]$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 41

$$\left\{ y(x) = -\frac{x^2}{4} - \frac{ax}{2} + \text{RootOf} \left( -x + \int^{-Z} 2 \frac{\_a + 1}{2\_a^3 + \_a a + a} d\_a + \_C1 \right) \right\}$$

**2.946 ODE No. 946**

---

$$y'(x) = \frac{x \left( 12e^{-x^2}x^2y(x)^2 + 8e^{-x^2}x^2y(x) - 8e^{-x^2}y(x) + 4e^{-2x^2}x^2 + 8e^{-x^2}x^2 - 8e^{-x^2} + e^{-3x^2}x^6 - 6e^{-2x^2}x^4y(x) \right)}{4e^{-x^2}x^2 - 8y(x) - 8}$$

✓ **Mathematica** : cpu = 0.605836 (sec), leaf count = 150

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-3x^2}}{8 \left( \frac{1}{8}e^{-3x^2} - \frac{e^{-3x^2}}{\sqrt{c_1 - 64x^2}} \right)} - \frac{1}{2}e^{-x^2} \left( 2e^{x^2} - x^2 \right) \right\}, \left\{ y(x) \rightarrow \frac{e^{-3x^2}}{8 \left( \frac{e^{-3x^2}}{\sqrt{c_1 - 64x^2}} + \frac{1}{8}e^{-3x^2} \right)} - \frac{1}{2}e^{-x^2} \left( 2e^{x^2} - x^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.109 (sec), leaf count = 85

$$\left\{ y(x) = 1 \left( -2 + x^2 \left( \sqrt{-x^2 + \_C1} + 1 \right) e^{-x^2} \right) \left( 2 \sqrt{-x^2 + \_C1} + 2 \right)^{-1}, y(x) = 1 \left( 2 + x^2 \left( \sqrt{-x^2 + \_C1} - 1 \right) e^{-x^2} \right) \left( 2 \sqrt{-x^2 + \_C1} - 2 \right)^{-1} \right\}$$

**2.947 ODE No. 947**

---

$$y'(x) = \frac{x^3 \sin(x) + x^2y(x)^2 + 2x^2y(x) \cos(x) + \frac{x^2}{2} + x^2 \cos(x) + \frac{1}{2}x^2 \cos(2x) + 2xy(x) - 2xy(x) \sin(x) + x - x \sin(x)}{x^3}$$

✓ **Mathematica** : cpu = 0.412224 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 - \log(x)} - \frac{-\sin(x) + x \cos(x) + 1}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.193 (sec), leaf count = 44

$$\left\{ y(x) = \frac{(\cos(x)x - \sin(x) + 1) \ln(x) - \cos(x) \_C1 x + \sin(x) \_C1 + x - \_C1}{x(\_C1 - \ln(x))} \right\}$$





**2.952 ODE No. 952**

---

$$y'(x) = \frac{-x^2\sqrt{x^2+y(x)^2} + xy(x)\sqrt{x^2+y(x)^2} + x^5\left(-\sqrt{x^2+y(x)^2}\right) + x^4y(x)\sqrt{x^2+y(x)^2} - x^4\sqrt{x^2+y(x)^2}}{x}$$

✓ **Mathematica** : cpu = 0.335186 (sec), leaf count = 341

$$\left\{\left\{y(x) \rightarrow \frac{x-2\sqrt{x^2\tanh^2\left(\frac{1}{20}\left(-20\sqrt{2}c_1-4\sqrt{2}x^5-5\sqrt{2}x^4-10\sqrt{2}x^2\right)\right)-x^2\tanh^4\left(\frac{1}{20}\left(-20\sqrt{2}c_1-4\sqrt{2}x^5-\right.\right.}{2\tanh^2\left(\frac{1}{20}\left(-20\sqrt{2}c_1-4\sqrt{2}x^5-5\sqrt{2}x^4-10\sqrt{2}x^2\right)\right)-1}\right.}\right.$$

✓ **Maple** : cpu = 0.266 (sec), leaf count = 62

$$\left\{\ln\left(2\frac{x\left(\sqrt{2\left(y\left(x\right)\right)^2+2x^2+y\left(x\right)+x}\right)}{y\left(x\right)-x}\right)+\frac{\left(4x^5+5x^4+10x^2\right)\sqrt{2}}{20}-\_C1-\ln\left(x\right)=0\right\}$$

**2.953 ODE No. 953**

---

$$y'(x) = \frac{y(x)\left(x^4\log^2(y(x))+2x^4\log(x)\log(y(x))+x^4\log^2(x)+x^3\log^2(y(x))+2x^3\log(x)\log(y(x))+x^3\log^2(x)\right)}{x}$$

✗ **Mathematica** : cpu = 1.24781 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[1][y][x] == ((-1 + Log[x] + x\*Log[x]^2 + x^3\*Log[x]^2 + x^4\*Log[x]^2 + Log

✓ **Maple** : cpu = 0.273 (sec), leaf count = 145

$$\left\{y\left(x\right)=1\left(x^{\frac{x^5}{4x^5+5x^4+10x^2+20\_C1}}\right)^{-4}\left(x^{\frac{x^4}{4x^5+5x^4+10x^2+20\_C1}}\right)^{-5}\left(x^{\frac{x^2}{4x^5+5x^4+10x^2+20\_C1}}\right)^{-10}\left(x^{\frac{C1}{4x^5+5x^4+10x^2+20\_C1}}\right)}$$

**2.954 ODE No. 954**

---

$$y'(x) = \frac{\frac{24}{5}x^{7/2}y(x) - \frac{24x^{13/2}}{25} + \frac{8x^{7/2}}{5} - 8x^{3/2} - \frac{8x^9}{125} + \frac{12}{25}x^6y(x) + \frac{4x^6}{25} - \frac{24x^4}{5} - \frac{6}{5}x^3y(x)^2 - \frac{4}{5}x^3y(x) + \frac{6x^3}{5} + 12xy(x)}{x}$$

✓ **Mathematica** : cpu = 0.334305 (sec), leaf count = 115

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3+3\sqrt[3]{29}\#1-29\&, \frac{\log\left(\frac{\frac{-6x^3-30\sqrt{x}+5}{5x}+\frac{3y(x)}{x}}{\sqrt[3]{29}\sqrt[3]{\frac{1}{x^3}}}-\#1\right)}{\sqrt[3]{29}-29\#1^2}\&\right]=c_1+\frac{1}{9}29^{2/3}\left(\frac{1}{x^3}\right)^{2/3}x^2\right]$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 53

$$\left\{y\left(x\right)=\frac{1}{45}\left(18x^{7/2}+145\text{RootOf}\left(-81\int^{-Z}\left(841\_a^3-27\_a+27\right)^{-1}d\_a+\ln\left(x\right)+3\_C1\right)\sqrt{x}-15\sqrt{x}\right.}$$

**2.955 ODE No. 955**

---

$$y'(x) = \frac{-24x^{7/2}y(x) + \frac{24x^{13/2}}{5} + 14x^{7/2} + 40x^{3/2} + \frac{8x^9}{25} - \frac{12}{5}x^6y(x) + \frac{12x^6}{5} + 24x^4 + 6x^3y(x)^2 - 6x^3y(x) - 6x^3}{x\left(2x^3-5y(x)+10\sqrt{x}-5\right)}$$

✓ **Mathematica** : cpu = 0.316397 (sec), leaf count = 112

$$\left\{\left\{y(x) \rightarrow \frac{1}{5}\left(2x^3+10\sqrt{x}-5\right)-\frac{1}{125x\left(-\frac{1}{x\sqrt{c_1-31250\log(x)}}-\frac{1}{125x}\right)}\right\},\left\{y(x) \rightarrow \frac{1}{5}\left(2x^3+10\sqrt{x}-5\right)-\frac{1}{125x\left(\right.}\right.}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 101

$$\left\{y\left(x\right)=1\left(\left(2x^3+10\sqrt{x}\right)\sqrt{\_C1-2\ln\left(x\right)}-2x^3-10\sqrt{x}+5\right)\left(5\sqrt{\_C1-2\ln\left(x\right)}-5\right)^{-1},y\left(x\right)=1\left(\left(2x^3\right.\right.}$$

2.956 ODE No. 956

$$y'(x) = \frac{y(x) \left( y(x) e^{\frac{2 \log^2(x)}{\log(x)+1}} x^{\frac{2}{\log(x)+1}+2} + y(x) e^{\frac{2 \log^2(x)}{\log(x)+1}} \log^2(x) x^{\frac{2}{\log(x)+1}+2} + 2y(x) e^{\frac{2 \log^2(x)}{\log(x)+1}} \log(x) x^{\frac{2}{\log(x)+1}+2} - e^{\frac{2 \log^2(x)}{\log(x)+1}} \right)}{x(\log(x) + 1)}$$

✓ **Mathematica** : cpu = 1.14726 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{\left( c_1 e^{\frac{x^4}{4}} + 1 \right) (\log(x) + 1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.092 (sec), leaf count = 79

$$\left\{ y(x) = \frac{1}{\ln(x) + 1} e^{-\frac{x^4}{4}} \left( x^{-2 \frac{\ln(x)}{\ln(x)+1}} (\ln(x) + 1) e^{\frac{(-4 \ln(x)-4) \ln(\ln(x)+1) - x^4 \ln(x) - x^4 + 8 (\ln(x))^2}{4 \ln(x)+4}} + \_C1 \right)^{-1} \right\}$$

2.957 ODE No. 957

$$y'(x) = \frac{y(x) \left( y(x) e^{\frac{2 \log^2(x)}{\log(x)+1}} x^{\frac{2}{\log(x)+1}+3} + y(x) e^{\frac{2 \log^2(x)}{\log(x)+1}} \log^2(x) x^{\frac{2}{\log(x)+1}+3} + 2y(x) e^{\frac{2 \log^2(x)}{\log(x)+1}} \log(x) x^{\frac{2}{\log(x)+1}+3} - e^{\frac{2 \log^2(x)}{\log(x)+1}} \right)}{x(\log(x) + 1)}$$

✓ **Mathematica** : cpu = 1.12027 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{\left( c_1 e^{\frac{x^5}{5}} + 1 \right) (\log(x) + 1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 79

$$\left\{ y(x) = \frac{1}{\ln(x) + 1} e^{-\frac{x^5}{5}} \left( x^{-2 \frac{\ln(x)}{\ln(x)+1}} (\ln(x) + 1) e^{\frac{(-5 \ln(x)-5) \ln(\ln(x)+1) - x^5 \ln(x) - x^5 + 10 (\ln(x))^2}{5 \ln(x)+5}} + \_C1 \right)^{-1} \right\}$$

2.958 ODE No. 958

$$y'(x) = \frac{2xy(x)^3 + y(x)^3 + 2xy(x)^2 + y(x)^2 + 6xy(x) \log^2(2x + 1) + 3y(x) \log^2(2x + 1) + 6xy(x)^2 \log(2x + 1) + y(x)^2 \log(2x + 1)}{x^2 \log^2(2x + 1)}$$

✓ **Mathematica** : cpu = 0.432636 (sec), leaf count = 82

$$\text{Solve} \left[ -\frac{29}{3} \text{RootSum} \left[ -29 \#1^3 + 3 \sqrt[3]{29} \#1 - 29 \&, \frac{\log \left( \frac{3y(x)+3 \log(2x+1)+1}{\sqrt[3]{29}} - \#1 \right)}{\sqrt[3]{29} - 29 \#1^2} \& \right] = c_1 + \frac{1}{9} 29^{2/3} x, y(x) \right]$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 40

$$\left\{ y(x) = -\ln(2x + 1) - \frac{1}{3} + \frac{29 \text{RootOf} \left( -81 \int^{-Z} (841 \_a^3 - 27 \_a + 27)^{-1} d\_a + x + 3 \_C1 \right)}{9} \right\}$$

2.959 ODE No. 959

$$y'(x) = \frac{\csc \left( \frac{y(x)}{2x} \right) \sec \left( \frac{y(x)}{2x} \right) \sec \left( \frac{y(x)}{x} \right) \left( x^3 \sin \left( \frac{y(x)}{2x} \right) \sin \left( \frac{y(x)}{x} \right) \cos \left( \frac{y(x)}{2x} \right) - \frac{1}{2} y(x) \sin \left( \frac{y(x)}{x} \right) + \frac{1}{2} y(x) \sin \left( \frac{y(x)}{2x} \right) \right)}{x}$$

✓ **Mathematica** : cpu = 0.255663 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow x \sin^{-1} \left( e^{c_1 + \frac{x^2}{2}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 15

$$\left\{ y(x) = \arcsin \left( e^{\frac{x^2}{2}} \_C1 \right) x \right\}$$





**2.968 ODE No. 968**

---

$$y'(x) = -\frac{\csc\left(\frac{y(x)}{2x}\right)\sec\left(\frac{y(x)}{2x}\right)\sec\left(\frac{y(x)}{x}\right)\left(x^4\sin\left(\frac{y(x)}{2x}\right)\sin\left(\frac{y(x)}{x}\right)\cos\left(\frac{y(x)}{2x}\right) - \frac{1}{2}xy(x)\sin\left(\frac{y(x)}{x}\right) - \frac{1}{2}y(x)\sin\left(\frac{y(x)}{x}\right)\right)}{}$$

✓ **Mathematica** : cpu = 0.354667 (sec), leaf count = 30

$$\left\{\left\{y(x) \rightarrow x \sin^{-1}\left((x+1)e^{c_1+\frac{x^2}{2}-x-\frac{3}{2}}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 22

$$\left\{y(x) = \arcsin\left(\frac{-C1(1+x)e^{\frac{x^2}{2}}}{e^x}\right)x\right\}$$

**2.969 ODE No. 969**

---

$$y'(x) = -\frac{\csc\left(\frac{y(x)}{2x}\right)\sec\left(\frac{y(x)}{2x}\right)\sec\left(\frac{y(x)}{x}\right)\left(-\frac{1}{2}xy(x)\sin\left(\frac{y(x)}{x}\right) - \frac{1}{2}y(x)\sin\left(\frac{y(x)}{x}\right) + x\sin\left(\frac{y(x)}{2x}\right)\sin\left(\frac{y(x)}{x}\right)\cos\left(\frac{y(x)}{2x}\right)\right)}{}$$

✓ **Mathematica** : cpu = 0.28074 (sec), leaf count = 19

$$\left\{\left\{y(x) \rightarrow x \sin^{-1}\left(\frac{e^{c_1}x}{x+1}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.065 (sec), leaf count = 15

$$\left\{y(x) = \arcsin\left(\frac{-C1x}{1+x}\right)x\right\}$$

**2.970 ODE No. 970**

---

$$y'(x) = -\frac{216x^3 - 216x^2y(x)^4 - 324x^2y(x)^3 - 648x^2y(x)^2 - 648x^2y(x) - 8y(x)^{12} - 36y(x)^{11} - 126y(x)^{10} - 315y(x)^9 - 315y(x)^8 - 126y(x)^7 - 36y(x)^6 - 8y(x)^5 - 8y(x)^4 - 8y(x)^3 - 8y(x)^2 - 8y(x) - 8}{}$$

✓ **Mathematica** : cpu = 0.587665 (sec), leaf count = 66

$$\text{Solve}\left[\frac{36(2y(x)^4 + 3y(x)^3 + 6y(x)^2 + 6y(x) - 6x - 3)}{(y(x)(2y(x)^3 + 3y(x)^2 + 6y(x) + 6) - 6x)^2} + \log(y(x)) = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.605 (sec), leaf count = 181

$$\left\{\frac{1}{6\_C1 - 6\ln(y(x))}\left(-6\sqrt{3\ln(y(x)) - 3\_C1 + 9} + \left(2(y(x))^4 + 3(y(x))^3 + 6(y(x))^2 - 6x + 6y(x)\right)\ln(y(x))\right)\right\}$$

**2.971 ODE No. 971**

---

$$y'(x) = \frac{(xy(x) + 1)^3}{x^5}$$

✓ **Mathematica** : cpu = 0.199348 (sec), leaf count = 157

$$\text{Solve}\left[\frac{1}{3}\log\left(\frac{\frac{3}{x^3} + \frac{3y(x)}{x^2}}{3\sqrt[3]{-\frac{1}{x^6}}} + 1\right) - \frac{1}{6}\log\left(\frac{\left(\frac{3}{x^3} + \frac{3y(x)}{x^2}\right)^2}{9\left(-\frac{1}{x^6}\right)^{2/3}} - \frac{\frac{3}{x^3} + \frac{3y(x)}{x^2}}{3\sqrt[3]{-\frac{1}{x^6}}} + 1\right) + \frac{\tan^{-1}\left(\frac{\frac{2\left(\frac{3}{x^3} + \frac{3y(x)}{x^2}\right)}{3\sqrt[3]{-\frac{1}{x^6}}} - 1}{\sqrt{3}}\right)}{\sqrt{3}} = c_1 - \right]$$

✓ **Maple** : cpu = 0.221 (sec), leaf count = 86

$$\left\{y(x) = \frac{\sqrt{3}}{6x}\left(3\tan\left(\text{RootOf}\left(-18x^3(-x^{-6})^{2/3} - 6\_Z\sqrt{3} - \ln\left(\frac{(\sqrt{3} + \tan(\_Z))^6}{((\tan(\_Z))^2 + 1)^3}\right) + 18\_C1\right)\right)x^3\sqrt[3]{\frac{1}{x^6}}\right)\right\}$$

**2.972            ODE No. 972**

---

$$y'(x) = \frac{x \left( -2x^4 + 2x^2 y(x) - x^2 + 1 \right)}{y(x) - x^2}$$

✓ **Mathematica** : cpu = 0.0233276 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( W \left( -e^{c_1 + x^4 - 2x^2 - 1} \right) + 1 \right) + x^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 27

$$\left\{ y(x) = x^2 + \frac{1}{2} \text{lambertW} \left( -2 \frac{e^{x^4} - C1 e^{-1}}{(e^{x^2})^2} \right) + \frac{1}{2} \right\}$$

**2.973            ODE No. 973**

---

$$y'(x) = e^{-2bx} y(x) \left( e^{bx} y(x) + e^{2bx} + y(x)^2 \right)$$

✓ **Mathematica** : cpu = 0.338295 (sec), leaf count = 146

$$\text{Solve} \left[ -\frac{1}{3} (9b - 7)^{2/3} \text{RootSum} \left[ \#1^3 (9b - 7)^{2/3} - 9\#1b + 6\#1 + (9b - 7)^{2/3} \&, \frac{\log \left( \frac{3e^{-2bx} y(x) + e^{-bx}}{\sqrt[3]{(9b - 7)e^{-3bx}}} - \#1 \right)}{\#1^2 \left( -(9b - 7)^{2/3} \right) + 3b - 2} \& \right] \right]$$

✓ **Maple** : cpu = 0.297 (sec), leaf count = 136

$$\left\{ y(x) = -\frac{e^{bx}}{2} + \frac{1}{2} \tan \left( \text{RootOf} \left( -2\_Z e^{bx} - \sqrt{-(e^{bx})^2 (4b - 3)} \ln \left( -(4 (\tan(\_Z))^2 b - 3 (\tan(\_Z))^2 + 4b) \right) \right) \right.$$

**2.974            ODE No. 974**

---

$$y'(x) = -x^6 + 3x^4 y(x) - 3x^2 y(x)^2 + y(x)^3 + 2x$$

✓ **Mathematica** : cpu = 0.0828564 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow x^2 - \frac{1}{\sqrt{c_1 - 2x}} \right\}, \left\{ y(x) \rightarrow \frac{1}{\sqrt{c_1 - 2x}} + x^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 57

$$\left\{ y(x) = 1 \left( x^2 \sqrt{2\_C1 - 2x} - 1 \right) \frac{1}{\sqrt{2\_C1 - 2x}}, y(x) = 1 \left( x^2 \sqrt{2\_C1 - 2x} + 1 \right) \frac{1}{\sqrt{2\_C1 - 2x}} \right\}$$

**2.975            ODE No. 975**

---

$$y'(x) = \frac{x^6}{27} + \frac{1}{3} x^4 y(x) + x^2 y(x)^2 + y(x)^3 - \frac{2x}{3}$$

✓ **Mathematica** : cpu = 0.0811559 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{\sqrt{c_1 - 2x}} - \frac{x^2}{3} \right\}, \left\{ y(x) \rightarrow \frac{1}{\sqrt{c_1 - 2x}} - \frac{x^2}{3} \right\} \right\}$$

✓ **Maple** : cpu = 0.028 (sec), leaf count = 59

$$\left\{ y(x) = -\frac{1}{3} \left( x^2 \sqrt{-54\_C1 - 2x} - 3 \right) \frac{1}{\sqrt{-54\_C1 - 2x}}, y(x) = -\frac{1}{3} \left( x^2 \sqrt{-54\_C1 - 2x} + 3 \right) \frac{1}{\sqrt{-54\_C1 - 2x}} \right\}$$

**2.976 ODE No. 976**

---

$$y'(x) = \frac{y(x) \left( x^7 y(x)^2 + x^4 y(x) + x - 3 \right)}{x}$$

✓ **Mathematica** : cpu = 0.162698 (sec), leaf count = 101

$$\text{Solve} \left[ -\frac{7}{3} \text{RootSum} \left[ -7 \#1^3 + 6 \sqrt[3]{-7} \#1 - 7 \&, \frac{\log \left( \frac{3 x^6 y(x) + x^3}{\sqrt[3]{7} \sqrt[3]{-x^9}} - \#1 \right)}{2 \sqrt[3]{-7} - 7 \#1^2} \& \right] = c_1 + \frac{7^{2/3} (-x^9)^{2/3}}{9 x^5}, y(x) \right]$$

✓ **Maple** : cpu = 0.177 (sec), leaf count = 57

$$\left\{ y(x) = \frac{1}{2 x^3} \left( \sqrt{3} \tan \left( \text{RootOf} \left( -\sqrt{3} \ln \left( \frac{9 (\tan(_Z))^2 + 9}{7 (-3 \tan(_Z) + \sqrt{3})^2} \right) + 3 \sqrt{3} _C1 - 2 \sqrt{3} x - 2 _Z \right) \right) - 1 \right) \right\}$$

**2.977 ODE No. 977**

---

$$y'(x) = e^{2x^2} x y(x) \left( e^{-x^2} y(x) + e^{-2x^2} + y(x)^2 \right)$$

✓ **Mathematica** : cpu = 0.373049 (sec), leaf count = 139

$$\text{Solve} \left[ -\frac{25}{3} \text{RootSum} \left[ -25 \#1^3 + 24 \sqrt[3]{-15}^{2/3} \#1 - 25 \&, \frac{\log \left( \frac{3 e^{2x^2} x y(x) + e^{x^2} x}{5^{2/3} \sqrt[3]{-e^{3x^2} x^3}} - \#1 \right)}{8 \sqrt[3]{-15}^{2/3} - 25 \#1^2} \& \right] = c_1 - \frac{5 \sqrt[3]{5} e^{x^2} x^3}{18 \sqrt[3]{-e^{3x^2} x^3}}, y(x) \right]$$

✓ **Maple** : cpu = 0.155 (sec), leaf count = 122

$$\left\{ y(x) = \frac{1}{2 e^{x^2}} \left( \sqrt{11} \tan \left( \text{RootOf} \left( -4 \sqrt{11} x^2 + 4 \sqrt{11} \ln(11) - 4 \sqrt{11} \ln \left( \frac{2592 \sqrt{11} (e^{x^2})^2 \tan(_Z)}{25} - \frac{2592 \sqrt{11}}{25} \right) \right) \right) \right\}$$

**2.978 ODE No. 978**

---

$$y'(x) = \frac{y(x) \left( x^2 + x y(x) + y(x)^2 + x \right)}{x^2}$$

✓ **Mathematica** : cpu = 0.138787 (sec), leaf count = 60

$$\text{Solve} \left[ -\frac{1}{2} \log \left( \frac{y(x)^2}{x^2} + \frac{y(x)}{x} + 1 \right) + \log \left( \frac{y(x)}{x} \right) - \frac{\tan^{-1} \left( \frac{\frac{2y(x)}{x} + 1}{\sqrt{3}} \right)}{\sqrt{3}} = c_1 + x, y(x) \right]$$

✓ **Maple** : cpu = 0.138 (sec), leaf count = 71

$$\left\{ y(x) = \frac{\sqrt{3} x}{2} \tan \left( \text{RootOf} \left( -\sqrt{3} \ln \left( \frac{4}{3 + 3 (\tan(_Z))^2} \right) - 2 \sqrt{3} \ln \left( -1/6 \sqrt{3} + 1/2 \tan(_Z) \right) - \sqrt{3} \ln(3) + \right) \right) \right\}$$

**2.979 ODE No. 979**

---

$$y'(x) = \frac{-x^3 + 3x^2 y(x) - 3x y(x)^2 + y(x)^3 + x}{x}$$

✓ **Mathematica** : cpu = 0.0990425 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow x - \frac{1}{\sqrt{c_1 - 2 \log(x)}} \right\}, \left\{ y(x) \rightarrow \frac{1}{\sqrt{c_1 - 2 \log(x)}} + x \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 57

$$\left\{ y(x) = 1 \left( x \sqrt{2 _C1 - 2 \ln(x)} - 1 \right) \frac{1}{\sqrt{2 _C1 - 2 \ln(x)}}, y(x) = 1 \left( x \sqrt{2 _C1 - 2 \ln(x)} + 1 \right) \frac{1}{\sqrt{2 _C1 - 2 \ln(x)}} \right\}$$



2.980 ODE No. 980

---

$$y'(x) = \frac{x^3y(x)^3 + 6x^2y(x)^2 + 12xy(x) + 2x + 8}{x^3}$$

✓ **Mathematica** : cpu = 0.113835 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{\sqrt{c_1 - 2x}} - \frac{2}{x} \right\}, \left\{ y(x) \rightarrow \frac{1}{\sqrt{c_1 - 2x}} - \frac{2}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 35

$$\left\{ y\left(x\right)=-\frac{1}{\sqrt{-C1-2x}}-2x^{-1},y\left(x\right)=\frac{1}{\sqrt{-C1-2x}}-2x^{-1} \right\}$$

2.981 ODE No. 981

---

$$y'(x) = \frac{a^3x^3y(x)^3 + 3a^2x^2y(x)^2 + a^2x + 3axy(x) + 1}{a^3x^3}$$

✓ **Mathematica** : cpu = 0.148372 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{ax} - \frac{1}{\sqrt{c_1 - 2x}} \right\}, \left\{ y(x) \rightarrow \frac{1}{\sqrt{c_1 - 2x}} - \frac{1}{ax} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 41

$$\left\{ y\left(x\right)=-\frac{1}{\sqrt{-C1-2x}}-\frac{1}{ax},y\left(x\right)=\frac{1}{\sqrt{-C1-2x}}-\frac{1}{ax} \right\}$$

2.982 ODE No. 982

---

$$y'(x) = \frac{1}{2}e^{-\frac{x^2}{2}}y(x)\left(2e^{\frac{x^2}{4}}y(x) + 2e^{\frac{x^2}{2}} + e^{\frac{x^2}{2}}x + 2y(x)^2\right)$$

✓ **Mathematica** : cpu = 0.347731 (sec), leaf count = 132

$$\text{Solve}\left[-\frac{7}{3}\text{RootSum}\left[-7\#1^3+6\sqrt[3]{-7}\#1-7\&,-\frac{\log\left(\frac{3e^{-\frac{x^2}{2}}y(x)+e^{-\frac{x^2}{4}}}{\sqrt[3]{7}\sqrt[3]{-e^{-\frac{3x^2}{4}}}}-\#1\right)}{2\sqrt[3]{-7}-7\#1^2}\&\right]=c_1+\frac{1}{9}7^{2/3}e^{\frac{x^2}{2}}\left(-e^{-\frac{3x^2}{4}}\right)^{2/3}\right]$$

✓ **Maple** : cpu = 0.327 (sec), leaf count = 145

$$\left\{-\frac{2}{3}\ln\left(-6+\left(18\,y\left(x\right)\mathrm{e}^{-1/2\,x^2}+6\,\mathrm{e}^{-1/4\,x^2}\right)\mathrm{e}^{\frac{x^2}{4}}\right)+\frac{1}{3}\ln\left(36+\frac{324}{7}\left(y\left(x\right)\mathrm{e}^{-\frac{x^2}{2}}+\frac{1}{3}\mathrm{e}^{-\frac{x^2}{4}}\right)^2\left(\mathrm{e}^{\frac{x^2}{4}}\right)^2+\frac{1}{7}\left(108\right)\right)\right\}$$

2.983 ODE No. 983

---

$$y'(x) = \frac{-x^3 + 3x^2y(x) + x^2 - 3xy(x)^2 + y(x)^3}{(x-1)(x+1)}$$

✓ **Mathematica** : cpu = 0.4862 (sec), leaf count = 238

$$\text{Solve}\left[\frac{1}{3}\log\left(\frac{\frac{3y(x)}{x^2-1}-\frac{3x}{x^2-1}}{3\sqrt[3]{\frac{1}{(x-1)^3(x+1)^3}}}+1\right)-\frac{1}{6}\log\left(\frac{\left(\frac{3y(x)}{x^2-1}-\frac{3x}{x^2-1}\right)^2}{9\left(\frac{1}{(x-1)^3(x+1)^3}\right)^{2/3}}-\frac{\frac{3y(x)}{x^2-1}-\frac{3x}{x^2-1}}{3\sqrt[3]{\frac{1}{(x-1)^3(x+1)^3}}}+1\right)+\frac{\tan^{-1}\left(\frac{\frac{2\left(\frac{3y(x)}{x^2-1}-\frac{3x}{x^2-1}\right)}{3\sqrt[3]{\frac{1}{(x-1)^3(x+1)^3}}}}{\sqrt{3}}\right)}{\sqrt{3}}\right]$$

✓ **Maple** : cpu = 0.34 (sec), leaf count = 188

$$\left\{ y\left(x\right)=\frac{\sqrt{3}}{2}\left(\frac{x^2-1}{3}\left(3\,\tan\left(RootOf\left(9\left(\frac{1}{\left(1+x\right)^3\left(x-1\right)^3}\right)^{2/3}\ln\left(\frac{x-1}{1+x}\right)x^4-18\left(\frac{1}{\left(1+x\right)^3\left(x-1\right)^3}\right)^{2/3}\right)\right)\right.\right.$$

345

2.984 ODE No. 984

---

$$y'(x) = \frac{e^{-2x}(x-1)y(x)\left(x^2y(x)^2 + e^xxy(x) + e^{2x}\right)}{x}$$

✓ **Mathematica** : cpu = 5.84137 (sec), leaf count = 428

Solve  $\left[ \frac{\sqrt[3]{2} \left( \frac{3e^{-2x}x(x-1)y(x)+e^{-x}(x-1)}{\sqrt[3]{2}\sqrt[3]{e^{-3x}(x-1)^3}} + 2^{2/3} \right) \left( 2^{2/3} - \frac{2^{2/3}(3e^{-2x}x(x-1)y(x)+e^{-x}(x-1))}{\sqrt[3]{e^{-3x}(x-1)^3}} \right) \left( \left( 1 - \frac{3e^{-2x}x(x-1)y(x)+e^{-x}(x-1)}{\sqrt[3]{e^{-3x}(x-1)^3}} \right) \right)}{9 \left( -\frac{e^{3x}(3e^{-2x}x(x-1)y(x)+e^{-x}(x-1))}{(x-1)^3} \right)} \right]$

✓ **Maple** : cpu = 0.22 (sec), leaf count = 40

$$\left\{ y(x) = \frac{1}{9x} e^{\text{RootOf}\left(-e^{-Z} \ln\left(\frac{x(e^{-Z}+9)}{2}\right) + 3\_C1 e^{-Z} + \_Z e^{-Z} + e^{-Z}x + 9\right) + x} \right\}$$

2.985 ODE No. 985

---

$$y'(x) = \frac{(xy(x) + 1)\left(x^2y(x)^2 + x^2y(x) + x^2 + 2xy(x) + x + 1\right)}{x^5}$$

✓ **Mathematica** : cpu = 0.32877 (sec), leaf count = 103

Solve  $\left[ -\frac{17}{3} \text{RootSum} \left[ -17\#1^3 + 3\sqrt[3]{-34}\#1 - 17\&, -\frac{\log\left(\frac{\frac{x+3}{x^3} + \frac{3y(x)}{x^2}}{\sqrt[3]{34}\sqrt[3]{-\frac{1}{x^6}}} - \#1\right)}{\sqrt[3]{-34} - 17\#1^2} \& \right] = c_1 - \frac{1}{9} 34^{2/3} \left( -\frac{1}{x^6} \right)^{2/3} x^3, y(x) \right]$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 43

$$\left\{ y(x) = \frac{17 \text{RootOf}\left(162 \int^{-Z} (289\_a^3 + 54\_a - 54)^{-1} d\_ax + 3\_C1 x + 2\right) x - 3x - 9}{9x} \right\}$$

2.986 ODE No. 986

---

$$y'(x) = \frac{-x^3 \log^3(x) + 3x^2y(x) \log^2(x) + x^2 + y(x)^3 + xy(x) - 3xy(x)^2 \log(x)}{x^2}$$

✓ **Mathematica** : cpu = 0.12713 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow x \log(x) - \frac{x}{\sqrt{c_1 - 2x}} \right\}, \left\{ y(x) \rightarrow \frac{x}{\sqrt{c_1 - 2x}} + x \log(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 36

$$\left\{ y(x) = -x \frac{1}{\sqrt{-C1 - 2x}} + x \ln(x), y(x) = x \frac{1}{\sqrt{-C1 - 2x}} + x \ln(x) \right\}$$

2.987 ODE No. 987

---

$$y'(x) = \frac{y(x)}{x} - F(x)\left(y(x)^2 - ax^2\right)$$

✓ **Mathematica** : cpu = 0.15248 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow \sqrt{a}x \tanh\left(\sqrt{a} \int_1^x F(K[1])K[1]dK[1] + \sqrt{a}c_1\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 22

$$\left\{ y(x) = \tanh\left(\sqrt{a}\left(-C1 + \int F(x)x \, dx\right)\right) x \sqrt{a} \right\}$$

**2.988      ODE No. 988**

---

$$y'(x) = \frac{y(x)}{x} - F(x) (-x^2 - 2xy(x) + y(x)^2)$$

✓ **Mathematica** : cpu = 0.204122 (sec), leaf count = 107

$$\left\{ \left\{ y(x) \rightarrow -\frac{x \left( -\exp \left( 2\sqrt{2} \left( \int_1^x -F(K[1])K[1]dK[1] + c_1 \right) \right) + \sqrt{2} \exp \left( 2\sqrt{2} \left( \int_1^x -F(K[1])K[1]dK[1] + c_1 \right) \right) - 1}{\exp \left( 2\sqrt{2} \left( \int_1^x -F(K[1])K[1]dK[1] + c_1 \right) \right) + 1} \right. \right.$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 29

$$\left\{ y(x) = \frac{x \left( \sqrt{2} + 2 \tanh \left( \left( -C1 + \int F(x) x dx \right) \sqrt{2} \right) \right) \sqrt{2}}{2} \right\}$$

**2.989      ODE No. 989**

---

$$y'(x) = \frac{y(x)}{x} - F(x) (-ay(x)^2 - bx^2)$$

✓ **Mathematica** : cpu = 0.16291 (sec), leaf count = 56

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{b}x \tan \left( \sqrt{a}\sqrt{b} \int_1^x F(K[1])K[1]dK[1] + \sqrt{a}\sqrt{b}c_1 \right)}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 29

$$\left\{ y(x) = \frac{x}{a} \tan \left( \sqrt{ab} \left( -C1 + \int F(x) x dx \right) \right) \sqrt{ab} \right\}$$

**2.990      ODE No. 990**

---

$$y'(x) = 2x - F(x) (-x^4 + 2x^2y(x) - y(x)^2 + 1)$$

✓ **Mathematica** : cpu = 0.202833 (sec), leaf count = 58

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp \left( \int_1^x 2F(K[5])dK[5] \right)}{c_1 - \int_1^x \exp \left( \int_1^{K[6]} 2F(K[5])dK[5] \right) F(K[6])dK[6]} + x^2 + 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.387 (sec), leaf count = 44

$$\left\{ y(x) = \frac{-x^2 \left( e^{\int F(x) dx} \right)^2 + -C1 x^2 + \left( e^{\int F(x) dx} \right)^2 + -C1}{-\left( e^{\int F(x) dx} \right)^2 + -C1} \right\}$$

**2.991      ODE No. 991**

---

$$y'(x) = \frac{y(x)}{x} - F(x) (x^2 + 2xy(x) - y(x)^2)$$

✓ **Mathematica** : cpu = 0.192807 (sec), leaf count = 104

$$\left\{ \left\{ y(x) \rightarrow -\frac{x \left( -\exp \left( 2\sqrt{2} \left( \int_1^x F(K[1])K[1]dK[1] + c_1 \right) \right) + \sqrt{2} \exp \left( 2\sqrt{2} \left( \int_1^x F(K[1])K[1]dK[1] + c_1 \right) \right) - 1 - v}{\exp \left( 2\sqrt{2} \left( \int_1^x F(K[1])K[1]dK[1] + c_1 \right) \right) + 1} \right. \right.$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 29

$$\left\{ y(x) = \frac{x \left( \sqrt{2} - 2 \tanh \left( \left( -C1 + \int F(x) x dx \right) \sqrt{2} \right) \right) \sqrt{2}}{2} \right\}$$

2.992 ODE No. 992

---

$$y'(x) = \frac{y(x)}{x} - F(x) \left(-x^3 - 7xy(x)^2\right)$$

✓ **Mathematica** : cpu = 0.11647 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow \frac{x \tan \left( \sqrt{7} \int_1^x F(K[1]) K[1]^2 dK[1] + \sqrt{7} c_1 \right)}{\sqrt{7}} \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 25

$$\left\{ y(x) = \frac{\tan \left( \left( \int x^2 F(x) \, dx + \_C1 \right) \sqrt{7} \right) x \sqrt{7}}{7} \right\}$$

2.993 ODE No. 993

---

$$y'(x) = \frac{y(x)}{x \log(x)} - F(x) \left(-y(x)^2 - 2y(x) \log(x) - \log^2(x)\right)$$

✓ **Mathematica** : cpu = 1.40307 (sec), leaf count = 73

$$\left\{ \left\{ y(x) \rightarrow \frac{\int_1^x \frac{F(K[5])}{\sqrt{\frac{1}{\log^2(K[5])}}} dK[5] + c_1 - 1}{\sqrt{\frac{1}{\log^2(x)}} \int_1^x \frac{F(K[5])}{\sqrt{\frac{1}{\log^2(K[5])}}} dK[5] + c_1 \sqrt{\frac{1}{\log^2(x)}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 35

$$\left\{ y(x) = -\frac{\ln(x) \left( \int -2 \ln(x) F(x) \, dx - \_C1 - 2 \right)}{\int -2 \ln(x) F(x) \, dx - \_C1} \right\}$$

2.994 ODE No. 994

---

$$y'(x) = \frac{y(x)}{x \log(x)} - x^3 \left(-y(x)^2 - 2y(x) \log(x) - \log^2(x)\right)$$

✓ **Mathematica** : cpu = 0.205535 (sec), leaf count = 198

$$\left\{ \left\{ y(x) \rightarrow -\frac{c_1 e^{\frac{1}{16} x^4 (4 \log(x) - 1)} \left( \frac{x^3}{4} + \frac{1}{4} x^3 (4 \log(x) - 1) \right) + \frac{1}{16} x^4 e^{\frac{1}{16} x^4 (4 \log(x) - 1)} (4 \log(x) - 1) \left( \frac{x^3}{4} + \frac{1}{4} x^3 (4 \log(x) - 1) \right)}{x^3 \left( c_1 e^{\frac{1}{16} x^4 (4 \log(x) - 1)} + \frac{1}{16} x^4 e^{\frac{1}{16} x^4 (4 \log(x) - 1)} (4 \log(x) - 1) \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 43

$$\left\{ y(x) = -\frac{\ln(x) \left( 4 x^4 \ln(x) - x^4 + 8 \_C1 + 16 \right)}{4 x^4 \ln(x) - x^4 + 8 \_C1} \right\}$$

2.995 ODE No. 995

---

$$y'(x) = (y(x) - e^x)^2 + e^x$$

✓ **Mathematica** : cpu = 0.164414 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 - x} + e^x \right\} \right\}$$

✓ **Maple** : cpu = 0.114 (sec), leaf count = 14

$$\left\{ y(x) = e^x + (\_C1 - x)^{-1} \right\}$$

2.996 ODE No. 996

---

$$y'(x) = \frac{(y(x) - \text{Si}(x))^2 + \sin(x)}{x}$$

✗ **Mathematica** : cpu = 26.1229 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[1][y][x] == (Sin[x] + (-SinIntegral[x] + y[x])^2)/x, y[x], x]

✓ **Maple** : cpu = 0.047 (sec), leaf count = 15

$$\left\{ y(x) = \text{Si}(x) + (\_C1 - \ln(x))^{-1} \right\}$$

2.997 ODE No. 997

---

$$y'(x) = (y(x) + \cos(x))^2 + \sin(x)$$

✓ **Mathematica** : cpu = 0.0987217 (sec), leaf count = 18

$$\left\{\left\{y(x) \rightarrow \frac{1}{c_1 - x} - \cos(x)\right\}\right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 16

$$\left\{y(x) = -\cos(x) + (\_C1 - x)^{-1}\right\}$$

2.998 ODE No. 998

---

$$y'(x) = \frac{(-\text{Ci}(x) + y(x) - \log(x))^2 + \cos(x)}{x}$$

✓ **Mathematica** : cpu = 0.462001 (sec), leaf count = 27

$$\left\{\left\{y(x) \rightarrow \text{Ci}(x) + \frac{x^2}{c_1 - \frac{x^2}{2}} + \log(x) + 1\right\}\right\}$$

✓ **Maple** : cpu = 0.273 (sec), leaf count = 27

$$\left\{y(x) = \ln(x) + \text{Ci}(x) + \frac{-\_C1 x^2 + 1}{-\_C1 x^2 + 1}\right\}$$

2.999 ODE No. 999

---

$$y'(x) = \frac{(y(x) - x + \log(x + 1))^2 + x}{x + 1}$$

✓ **Mathematica** : cpu = 0.182749 (sec), leaf count = 24

$$\left\{\left\{y(x) \rightarrow \frac{1}{c_1 - \log(x + 1)} + x - \log(x + 1)\right\}\right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 36

$$\left\{y(x) = \frac{-(\ln(1 + x))^2 + (x - \_C1) \ln(1 + x) + \_C1 x - 1}{\ln(1 + x) + \_C1}\right\}$$

2.1000 ODE No. 1000

---

$$y'(x) = \frac{x^3 + 2x^2y(x) - xy(x) - y(x)^2 + xy(x) \log(x)}{x^2(x + \log(x))}$$

✓ **Mathematica** : cpu = 0.95808 (sec), leaf count = 351

$$\left\{\left\{y(x) \rightarrow \frac{x^2(x + \log(x)) \left(c_1 \left(\frac{\exp\left(-\frac{1}{2} \int_1^x \frac{K[5] + \log(K[5]) + 2}{K[5]^2 + \log(K[5])K[5]} dK[5]\right) - \frac{\sqrt{x}(x + \log(x) + 2) \exp\left(-\frac{1}{2} \int_1^x \frac{K[5] + \log(K[5]) + 2}{K[5]^2 + \log(K[5])K[5]} dK[5]\right)}{2(x^2 + x \log(x))}\right)}{c_1 \sqrt{x} \exp\left(-\frac{1}{2} \int_1^x \frac{K[5] + \log(K[5]) + 2}{K[5]^2 + \log(K[5])K[5]} dK[5]\right)}\right.\right\}$$

✓ **Maple** : cpu = 0.107 (sec), leaf count = 19

$$\left\{y(x) = \frac{x(\_C1 x - 1)}{-\_C1 \ln(x) + 1}\right\}$$

2.1001 ODE No. 1001

---

$$y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0027672 (sec), leaf count = 12

$$\{\{y(x) \rightarrow c_2 x + c_1\}\}$$

✓ **Maple** : cpu = 0.003 (sec), leaf count = 9

$$\{y(x) = \_C1 x + \_C2\}$$

Hand solution

$$y'' = 0$$

Integration twice gives

$$y(x) = c_1x + c_2$$

2.1002 ODE No. 1002

---

$$y''(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0034822 (sec), leaf count = 16

$$\{\{y(x) \rightarrow c_2 \sin(x) + c_1 \cos(x)\}\}$$

✓ **Maple** : cpu = 0.005 (sec), leaf count = 13

$$\{y(x) = \_C1 \sin(x) + \_C2 \cos(x)\}$$

Hand solution

$$y'' + y = 0$$

Let  $y = e^{\lambda x}$ , substitution in above gives

$$\lambda^2 e^{\lambda x} + e^{\lambda x} = 0$$

$$\lambda^2 + 1 = 0$$

Hence  $\lambda = \pm i$ , therefore the solution is

$$\begin{aligned} y &= Ae^{ix} + Be^{-ix} \\ &= A(\cos x + i \sin x) + B(\cos x - i \sin x) \\ &= \cos x (A + B) + \sin x (Ai - iB) \\ &= \cos x (A + B) + \sin x (i(A - B)) \end{aligned}$$

Let  $A + B = c_1, i(A - B) = c_2$  hence

$$y = c_1 \cos x + c_2 \sin x$$

2.1003 ODE No. 1003

---

$$-\sin(nx) + y''(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0979911 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin(x) + c_1 \cos(x) + \frac{\cos^2(x)(-\sin(nx)) - \sin^2(x) \sin(nx)}{n^2 - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 26

$$\left\{ y(x) = \sin(x) \_C2 + \cos(x) \_C1 - \frac{\sin(nx)}{n^2 - 1} \right\}$$

Hand solution

$$y'' + y = \sin nx \tag{1}$$

We start by solving the homogeneous equation

$$y'' + y = 0$$

Let  $y = e^{\lambda x}$ , substitution in above gives

$$\lambda^2 e^{\lambda x} + e^{\lambda x} = 0$$

$$\lambda^2 + 1 = 0$$

Hence  $\lambda = \pm i$ , therefore the solution is

$$\begin{aligned}
 y_h &= Ae^{ix} + Be^{-ix} \\
 &= A(\cos x + i \sin x) + B(\cos x - i \sin x) \\
 &= \cos x (A + B) + \sin x (Ai - iB) \\
 &= \cos x (A + B) + \sin x (i(A - B))
 \end{aligned}$$

Let  $A + B = c_1, i(A - B) = c_2$  hence

$$y_h = c_1 \cos x + c_2 \sin x$$

Now we solve for the particular solution using variation of parameters. Let

$$\begin{aligned}
 y_p &= u_1(x) \cos x + u_2(x) \sin x \\
 y_p' &= u_1' \cos x - u_1 \sin x + u_2' \sin x + u_2 \cos x \\
 &= u_2 \cos x - u_1 \sin x + u_1' \cos x + u_2' \sin x
 \end{aligned}$$

Let first condition be

$$u_1' \cos x + u_2' \sin x = 0 \quad (2)$$

Hence

$$\begin{aligned}
 y_p' &= u_2 \cos x - u_1 \sin x \\
 y_p'' &= u_2' \cos x - u_2 \sin x - u_1' \sin x - u_1 \cos x
 \end{aligned}$$

Substituting in (1) gives

$$\begin{aligned}
 y_p'' + y_p &= \sin nx \\
 u_2' \cos x - u_2 \sin x - u_1' \sin x - u_1 \cos x + u_1 \cos x + u_2 \sin x &= \sin nx \\
 u_2' \cos x - u_1' \sin x &= \sin nx
 \end{aligned} \quad (3)$$

So we have two equations (1)(2) to solve for  $u_1, u_2$

$$\begin{aligned}
 u_1' \cos x + u_2' \sin x &= 0 \\
 u_2' \cos x - u_1' \sin x &= \sin nx
 \end{aligned}$$

From the first equation

$$u_1' = -u_2' \frac{\sin x}{\cos x} \quad (4)$$

Substituting in the second equation

$$\begin{aligned}
 u_2' \cos x - \left(-u_2' \frac{\sin x}{\cos x}\right) \sin x &= \sin nx \\
 u_2' \left(\cos x + \frac{\sin x}{\cos x} \sin x\right) &= \sin nx \\
 u_2' \left(\frac{\cos^2 x + \sin^2 x}{\cos x}\right) &= \sin nx \\
 u_2' &= \cos x \sin nx
 \end{aligned}$$

Hence

$$\begin{aligned}
 u_2 &= \int \cos x \sin(nx) dx \\
 &= \frac{-n \cos x \cos(nx) - \sin x \sin(nx)}{n^2 - 1}
 \end{aligned}$$

From (4)

$$\begin{aligned}
 u_1' &= -\cos x \sin nx \frac{\sin x}{\cos x} \\
 u_1 &= -\int \sin(nx) \sin x dx \\
 &= \frac{n \cos(nx) \sin x - \cos x \sin(nx)}{n^2 - 1}
 \end{aligned}$$

Since  $y_p = u_1(x) \cos x + u_2(x) \sin x$  then

$$\begin{aligned} y_p &= \left( \frac{n \cos(nx) \sin x - \cos x \sin(nx)}{n^2 - 1} \right) \cos x + \left( \frac{-n \cos x \cos(nx) - \sin x \sin(nx)}{n^2 - 1} \right) \sin x \\ &= \frac{n \cos(nx) \cos x \sin x - \cos^2 x \sin(nx) - n \cos x \sin x \cos(nx) - \sin^2 x \sin(nx)}{n^2 - 1} \\ &= \frac{-\sin(nx) (\cos^2 x + \sin^2 x)}{n^2 - 1} \\ &= \frac{\sin(nx)}{1 - n^2} \end{aligned}$$

Therefore, the full solution is (for  $n^2 \neq 1$ )

$$\begin{aligned} y &= y_h + y_p \\ &= c_1 \cos x + c_2 \sin x + \frac{\sin(nx)}{1 - n^2} \end{aligned}$$

Solution using undetermined coefficients: Since RHS is  $\sin nx$  we guess  $y_p = A \cos(nx) + B \sin(nx)$ , therefore

$$\begin{aligned} y_p' &= -An \sin(nx) + Bn \cos(nx) \\ y_p'' &= -An^2 \cos(nx) - Bn^2 \sin(nx) \end{aligned}$$

Plug into the ODE gives

$$\begin{aligned} y_p'' + y_p &= \sin nx \\ -An^2 \cos(nx) - Bn^2 \sin(nx) + A \cos(nx) + B \sin(nx) &= \sin nx \\ \cos(nx) (-An^2 + A) + \sin(nx) (-Bn^2 + B) &= \sin(nx) \end{aligned}$$

Hence  $-Bn^2 - B = 1$  and  $-An^2 + A = 0$ . Therefore  $A = 0$  and from the first equation

$$\begin{aligned} B(n^2 + 1) &= -1 \\ B &= \frac{-1}{n^2 + 1} \end{aligned}$$

Hence

$$\begin{aligned} y_p &= A \cos(nx) + B \sin(nx) \\ &= \frac{\sin(nx)}{1 - n^2} \end{aligned}$$

Which is the same as variation of parameters method.

Note: Full solution should also really consider the case for  $n = 1$ . Will update later.

```

1 restart;
2 ode:=diff(diff(y(x),x),x)+y(x)-sin(n*x)=0;
3 y0:=-C1*cos(x)+_C2*sin(x)+sin(n*x)/(1-n^2);
4 odetest(y(x)=y0,ode);
5 0

```

## 2.1004 ODE No. 1004

$$-a \cos(bx) + y''(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0888685 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow \frac{-a \cos^2(x) \cos(bx) - a \sin^2(x) \cos(bx)}{b^2 - 1} + c_2 \sin(x) + c_1 \cos(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 27

$$\left\{ y(x) = \sin(x) \_C2 + \cos(x) \_C1 - \frac{a \cos(bx)}{b^2 - 1} \right\}$$

**Hand solution**



$$y'' + y = a \cos bx \quad (1)$$

We start by solving the homogeneous equation

$$y'' + y = 0$$

Let  $y = e^{\lambda x}$ , substitution in above gives

$$\begin{aligned} \lambda^2 e^{\lambda x} + e^{\lambda x} &= 0 \\ \lambda^2 + 1 &= 0 \end{aligned}$$

Hence  $\lambda = \pm i$ , therefore the solution is

$$\begin{aligned} y_h &= Ae^{ix} + Be^{-ix} \\ &= A(\cos x + i \sin x) + B(\cos x - i \sin x) \\ &= \cos x (A + B) + \sin x (Ai - iB) \\ &= \cos x (A + B) + \sin x (i(A - B)) \end{aligned}$$

Let  $A + B = c_1, i(A - B) = c_2$  hence

$$y_h = c_1 \cos x + c_2 \sin x$$

Now we solve for the particular solution using variation of parameters. Let

$$\begin{aligned} y_p &= u_1(x) \cos x + u_2(x) \sin x \\ y'_p &= u'_1 \cos x - u_1 \sin x + u'_2 \sin x + u_2 \cos x \\ &= u_2 \cos x - u_1 \sin x + u'_1 \cos x + u'_2 \sin x \end{aligned}$$

Let first condition be

$$u'_1 \cos x + u'_2 \sin x = 0 \quad (2)$$

Hence

$$\begin{aligned} y'_p &= u_2 \cos x - u_1 \sin x \\ y''_p &= u'_2 \cos x - u_2 \sin x - u'_1 \sin x - u_1 \cos x \end{aligned}$$

Substituting in (1) gives

$$\begin{aligned} y''_p + y_p &= a \cos bx \\ u'_2 \cos x - u_2 \sin x - u'_1 \sin x - u_1 \cos x + u_1 \cos x + u_2 \sin x &= a \cos bx \\ u'_2 \cos x - u'_1 \sin x &= a \cos bx \end{aligned} \quad (3)$$

So we have two equations (1)(2) to solve for  $u_1, u_2$

$$\begin{aligned} u'_1 \cos x + u'_2 \sin x &= 0 \\ u'_2 \cos x - u'_1 \sin x &= a \cos bx \end{aligned}$$

From the first equation

$$u'_1 = -u'_2 \frac{\sin x}{\cos x} \quad (4)$$

Substituting in the second equation

$$\begin{aligned} u'_2 \cos x - \left(-u'_2 \frac{\sin x}{\cos x}\right) \sin x &= a \cos bx \\ u'_2 \left(\cos x + \frac{\sin x}{\cos x} \sin x\right) &= a \cos bx \\ u'_2 \left(\frac{\cos^2 x + \sin^2 x}{\cos x}\right) &= a \cos bx \\ u'_2 &= a \cos x \cos bx \end{aligned}$$

Hence

$$\begin{aligned} u_2 &= a \int \cos x \cos (bx) dx \\ &= a \frac{-\cos (bx) \sin x + b \cos x \sin (bx)}{b^2 - 1} \end{aligned}$$

From (4)

$$\begin{aligned}u_1' &= -a \cos (bx) \sin x \\ u_1 &= -a \int \cos (bx) \sin x dx \\ &= -a \frac{\cos (bx) \cos x + b \sin x \sin (bx)}{b^2 - 1}\end{aligned}$$

Since  $y_p = u_1(x) \cos x + u_2(x) \sin x$  then

$$\begin{aligned}y_p &= \left(-a \frac{\cos (bx) \cos x + b \sin x \sin (bx)}{b^2 - 1}\right) \cos x + \left(a \frac{-\cos (bx) \sin x + b \cos x \sin (bx)}{b^2 - 1}\right) \sin x \\ &= \frac{-a \cos (bx) \cos ^2 x - ab \cos x \sin x \sin (bx) - a \cos (bx) \sin ^2 x + ab \sin x \cos x \sin (bx)}{b^2 - 1} \\ &= \frac{-a \cos (bx) \cos ^2 x - a \cos (bx) \sin ^2 x}{b^2 - 1} \\ &= \frac{-a \cos (bx) \left(\cos ^2 x + \sin ^2 x\right)}{b^2 - 1} \\ &= \frac{-a \cos (bx)}{b^2 - 1} \\ &= \frac{a \cos (bx)}{1 - b^2}\end{aligned}$$

Therefore, the full solution is (for  $b^2 \neq 1$ )

$$\begin{aligned}y &= y_h + y_p \\ &= c_1 \cos x + c_2 \sin x + \frac{a \cos (bx)}{1 - b^2}\end{aligned}$$

```

1 restart;
2 ode:=diff(diff(y(x),x),x)+y(x)-a*cos(b*x)=0;
3 y0:=-C1*cos(x)+_C2*sin(x)+a*cos(b*x)/(1-b^2);
4 odetest(y(x)=y0,ode);
5 0

```

## 2.1005      ODE No. 1005

$$-\sin(ax)\sin(bx)+y''(x)+y(x)=0$$

✓    **Mathematica** : cpu = 0.413988 (sec), leaf count = 1163

$$\left\{\left\{y(x)\rightarrow c_1\cos(x)+c_2\sin(x)+\frac{-\cos(x)\cos((a-b-1)x)a^3+\cos(x)\cos((a-b+1)x)a^3+\cos(x)\cos((a+b-1)x)a^3}{2a^4+(-4b^2-4)a^2+2b^4-4b^2+2}\right.\right.$$

✓    **Maple** : cpu = 0.069 (sec), leaf count = 82

$$\left\{y(x)=\sin(x)\_C2+\cos(x)\_C1+\frac{-(a+b+1)(a+b-1)\cos(x(a-b))+\cos((a+b)x)(a-b+1)(a-b-1)}{2a^4+(-4b^2-4)a^2+2b^4-4b^2+2}\right.$$

Hand solution

$$y''+y=\sin ax\sin bx\tag{1}$$

We start by solving the homogeneous equation

$$y''+y=0$$

Let  $y=e^{\lambda x}$ , substitution in above gives

$$\begin{aligned}\lambda^2e^{\lambda x}+e^{\lambda x}&=0 \\ \lambda^2+1&=0\end{aligned}$$

Hence  $\lambda=\pm i$ , therefore the solution is

$$\begin{aligned}y_h &= Ae^{ix}+Be^{-ix} \\ &= A(\cos x+i\sin x)+B(\cos x-i\sin x) \\ &= \cos x(A+B)+\sin x(Ai-iB) \\ &= \cos x(A+B)+\sin x(i(A-B))\end{aligned}$$

Let  $A + B = c_1, i(A - B) = c_2$  hence

$$\begin{aligned} y_h &= c_1 \cos x + c_2 \sin x \\ &= y_h = c_1 y_1 + c_2 y_2 \end{aligned}$$

Now we solve for the particular solution using variation of parameters.

$$y_p = u_1 y_1 + u_2 y_2$$

Wronskian is

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

Hence, using  $f = \sin ax \sin bx$ , which is the RHS of the ODE, and noting that  $a_0$  is the coefficient of  $y''$  which is one here, then

$$\begin{aligned} u_1 &= \int \frac{-y_2}{W} \frac{f}{a_0} dx = - \int \sin x \sin(ax) \sin(bx) dx \\ &= -\frac{1}{4} \left( \frac{\cos((a-b-1)x)}{a-b-1} - \frac{\cos((a-b+1)x)}{a-b+1} - \frac{\cos((a+b-1)x)}{a+b-1} + \frac{\cos((a+b+1)x)}{a+b+1} \right) \end{aligned}$$

And

$$\begin{aligned} u_2 &= \int \frac{y_1}{W} \frac{f}{a_0} dx = - \int \cos x \sin(ax) \sin(bx) dx \\ &= \frac{1}{4} \left( \frac{\sin((a-b-1)x)}{a-b-1} + \frac{\sin((a-b+1)x)}{a-b+1} - \frac{\sin((a+b-1)x)}{a+b-1} - \frac{\sin((a+b+1)x)}{a+b+1} \right) \end{aligned}$$

Since  $y_p = u_1(x) \cos x + u_2(x) \sin x$  then

$$\begin{aligned} y_p &= -\frac{1}{4} \left( \frac{\cos((a-b-1)x)}{a-b-1} - \frac{\cos((a-b+1)x)}{a-b+1} - \frac{\cos((a+b-1)x)}{a+b-1} + \frac{\cos((a+b+1)x)}{a+b+1} \right) \cos x \\ &\quad + \frac{1}{4} \left( \frac{\sin((a-b-1)x)}{a-b-1} + \frac{\sin((a-b+1)x)}{a-b+1} - \frac{\sin((a+b-1)x)}{a+b-1} - \frac{\sin((a+b+1)x)}{a+b+1} \right) \sin x \\ &= \frac{1}{4} \left( -\frac{\cos^2((a-b-1)x)}{a-b-1} + \frac{\sin^2((a-b-1)x)}{a-b-1} \right) + \frac{1}{4} \left( \frac{\cos^2((a-b+1)x)}{a-b+1} + \frac{\sin^2((a-b+1)x)}{a-b+1} \right) \\ &\quad + \frac{1}{4} \left( \frac{\cos^2((a+b-1)x)}{a+b-1} - \frac{\sin^2((a+b-1)x)}{a+b-1} \right) + \frac{1}{4} \left( -\frac{\cos^2((a+b+1)x)}{a+b+1} - \frac{\sin^2((a+b+1)x)}{a+b+1} \right) \\ &= \frac{1}{4} \left( -\frac{\cos^2((a-b-1)x)}{a-b-1} + \frac{\sin^2((a-b-1)x)}{a-b-1} \right) + \frac{1}{4} \left( \frac{1}{a-b+1} \right) \\ &\quad + \frac{1}{4} \left( \frac{\cos^2((a+b-1)x)}{a+b-1} - \frac{\sin^2((a+b-1)x)}{a+b-1} \right) - \frac{1}{4} \left( \frac{1}{a+b+1} \right) \end{aligned}$$

Let  $a-b-1 = \alpha, a+b-1 = \beta$  then

$$\begin{aligned} y_p &= \frac{1}{4} \left( \frac{\sin^2(\alpha x) - \cos^2(\alpha x)}{\alpha} \right) + \frac{1}{4} \left( \frac{\cos^2(\beta x) - \sin^2(\beta x)}{\beta} \right) + \frac{1}{4} \left( \frac{1}{\alpha+2} \right) - \frac{1}{4} \left( \frac{1}{\beta+2} \right) \\ &= \frac{1}{4} \left( \frac{\sin^2(\alpha x) - \cos^2(\alpha x)}{\alpha} + \frac{\cos^2(\beta x) - \sin^2(\beta x)}{\beta} + \frac{1}{\alpha+2} - \frac{1}{\beta+2} \right) \end{aligned}$$

Therefore, the full solution is

$$\begin{aligned} y &= y_h + y_p \\ &= c_1 \cos x + c_2 \sin x + \frac{1}{4} \left( \frac{\sin^2(\alpha x) - \cos^2(\alpha x)}{\alpha} + \frac{\cos^2(\beta x) - \sin^2(\beta x)}{\beta} + \frac{1}{\alpha+2} - \frac{1}{\beta+2} \right) \end{aligned}$$

I made mistake. Need to go over it again. I do not see it now. Maple does not verify.

```

1 restart;
2 ode:=diff(diff(y(x),x),x)+y(x)-sin(a*x)*sin(b*x)=0;
3 alpha:=a-b-1;
4 beta:=a+b-1;
5 yp:=1/4*(1/alpha*(sin(alpha*x)^2-cos(alpha*x)^2)+1/beta*(cos(beta*x)^2-sin(beta*x)^2))+(1/4)*(1/(alpha+2)-1/(beta+2));
6 y0:=yp+_C1*sin(x)+_C2*cos(x);
7 not zero

```

2.1006

ODE No. 1006

---

$$y''(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0032838 (sec), leaf count = 20

$$\left\{\left\{y(x) \rightarrow c_1 e^x + c_2 e^{-x}\right\}\right\}$$

✓ **Maple** : cpu = 0.004 (sec), leaf count = 15

$$\left\{y\left(x\right) = \_C1\, \mathrm{e}^{-x} + \_C2\, \mathrm{e}^x\right\}$$

Hand solution

$$y'' - y = 0 \tag{1}$$

Let  $y = e^{\lambda x}$ , substitution in above gives

$$\begin{aligned}\lambda^2 e^{\lambda x} - e^{\lambda x} &= 0 \\ \lambda^2 - 1 &= 0\end{aligned}$$

Hence  $\lambda = \pm 1$ , therefore the solution is

$$y_h = Ae^x + Be^{-x}$$

2.1007

ODE No. 1007

---

$$-4e^{x^2}x^2 + y''(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0856191 (sec), leaf count = 135

$$\left\{\left\{y(x) \rightarrow c_1 e^{\sqrt{2}x} + c_2 e^{-\sqrt{2}x} + \frac{e^{-\sqrt{2}x}\left(-2e^{x\left(x+\sqrt{2}\right)}x + 2e^{\left(x-\sqrt{2}\right)x+2\sqrt{2}x}x + \sqrt{2}e^{x\left(x+\sqrt{2}\right)} + \sqrt{2}e^{\left(x-\sqrt{2}\right)x+2\sqrt{2}x}\right)}{2\sqrt{2}}\right\}\right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 26

$$\left\{y\left(x\right) = \mathrm{e}^{\sqrt{2}x}\_C2 + \mathrm{e}^{-\sqrt{2}x}\_C1 + \mathrm{e}^{x^2}\right\}$$

Hand solution

$$y'' - 2y = 4x^2e^{x^2} \tag{1}$$

We start by solving the homogeneous equation

$$y'' - 2y = 0$$

Let  $y = e^{\lambda x}$ , substitution in above gives

$$\begin{aligned}\lambda^2 e^{\lambda x} - 2e^{\lambda x} &= 0 \\ \lambda^2 - 2 &= 0\end{aligned}$$

Hence  $\lambda = \pm \sqrt{2}$ , therefore the solution is

$$\begin{aligned}y_h &= Ae^{\sqrt{2}x} + Be^{\sqrt[2]{2}x} \\ &= Ay_1 + By_2\end{aligned}$$

Now we solve for the particular solution using variation of parameters.

$$y_p = u_1y_1 + u_2y_2$$

wronskian is

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{\sqrt{2}x} & e^{\sqrt[2]{2}x} \\ \sqrt{2}e^{\sqrt{2}x} & -\sqrt{2}e^{-\sqrt{2}x} \end{vmatrix} = -\sqrt{2} - \sqrt{2} = -2\sqrt{2}$$

Hence, using  $f = 4x^2e^{x^2}$ , which is the RHS of the ODE, and noting that  $a_0$  is the coefficient of  $y''$  which is one here, then

$$\begin{aligned} u_1 &= \int \frac{-y_2}{W} \frac{f}{a_0} dx = - \int \frac{e^{\sqrt{2}x}}{-2\sqrt{2}} 4x^2 e^{x^2} dx \\ &= \frac{2}{\sqrt{2}} \int x^2 e^{x^2} \sqrt{2} x dx \\ &= \frac{2}{\sqrt{2}} \left( \frac{1}{4} e^{x(x-\sqrt{2})} (\sqrt{2} + 2x) \right) \end{aligned}$$

And

$$\begin{aligned} u_2 &= \int \frac{y_1}{W} \frac{f}{a_0} dx = \int \frac{e^{\sqrt{2}x}}{-2\sqrt{2}} 4x^2 e^{x^2} dx \\ &= \frac{-2}{\sqrt{2}} \int x^2 e^{x^2} \sqrt{2} x dx \\ &= \frac{-2}{\sqrt{2}} \left( -\frac{1}{4} e^{x(x+\sqrt{2})} (\sqrt{2} - 2x) \right) \end{aligned}$$

Since  $y_p = u_1 e^{\sqrt{2}x} + u_2 e^{-\sqrt{2}x}$  then

$$\begin{aligned} y_p &= \frac{2}{\sqrt{2}} \left( \frac{1}{4} e^{x(x-\sqrt{2})} (\sqrt{2} + 2x) \right) e^{\sqrt{2}x} - \frac{2}{\sqrt{2}} \left( -\frac{1}{4} e^{x(x+\sqrt{2})} (\sqrt{2} - 2x) \right) e^{-\sqrt{2}x} \\ &= \frac{2}{\sqrt{2}} \frac{1}{4} e^{x^2} (\sqrt{2} + 2x) + \frac{2}{\sqrt{2}} \frac{1}{4} e^{x^2} (\sqrt{2} - 2x) \\ &= \frac{1}{2} e^{x^2} + \frac{1}{2} e^{x^2} \\ &= e^{x^2} \end{aligned}$$

Therefore, the full solution is

$$\begin{aligned} y &= y_h + y_p \\ &= Ae^{\sqrt{2}x} + Be^{-\sqrt{2}x} + e^{x^2} \end{aligned}$$

```

1 restart;
2 ode:=diff(y(x),x$2)-2*y(x)=4*x^2*exp(x^2);
3 y0:=-C1*exp(sqrt(2)*x)+_C2*exp(-sqrt(2)*x)+exp(x^2);
4 odetest(y(x)=y0,ode);
5 0

```

2.1008

ODE No. 1008

---

$$a^2y(x) - \cot(ax) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0354348 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sin(ax) \left( \log \left( \cos \left( \frac{ax}{2} \right) \right) - \log \left( \sin \left( \frac{ax}{2} \right) \right) \right)}{a^2} + c_2 \sin(ax) + c_1 \cos(ax) \right\} \right\}$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 41

$$\left\{ y\left(x\right) = \sin\left(ax\right) \_C2 + \cos\left(ax\right) \_C1 + \frac{\sin\left(ax\right)}{a^2} \ln\left(\frac{1 - \cos\left(ax\right)}{\sin\left(ax\right)}\right) \right\}$$

2.1009

ODE No. 1009

---

$$ly(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0036165 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin \left( \sqrt{l} x \right) + c_1 \cos \left( \sqrt{l} x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 21

$$\left\{ y\left(x\right) = \_C1 \sin \left( \sqrt{l} x \right) + \_C2 \cos \left( \sqrt{l} x \right) \right\}$$

## 2.1010 ODE No. 1010

---

$$y(x)(ax + b) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0048403 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow c_1 \text{Ai} \left( \frac{-b - ax}{(-a)^{2/3}} \right) + c_2 \text{Bi} \left( \frac{-b - ax}{(-a)^{2/3}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 31

$$\left\{ y(x) = \_C1 \text{Ai} \left( -(ax + b)a^{-\frac{2}{3}} \right) + \_C2 \text{Bi} \left( -(ax + b)a^{-\frac{2}{3}} \right) \right\}$$

**Hand solution**

$$y'' + (ax + b)y = 0 \tag{1}$$

For  $a \neq 0$ . Let  $y = \eta(\xi)$ ,  $\xi = ax + b$ , hence

$$\begin{aligned} \frac{dy}{dx} &= \frac{d\eta}{d\xi} \frac{d\xi}{dx} \\ &= \frac{d\eta}{d\xi} a \end{aligned}$$

And

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{d}{dx} \left( \frac{d\eta}{d\xi} a \right) \\ &= a \frac{d}{dx} \left( \frac{d\eta}{d\xi} \right) \\ &= a \left( \frac{d^2\eta}{d\xi^2} \frac{d\xi}{dx} \right) \\ &= a^2 \frac{d^2\eta}{d\xi^2} \end{aligned}$$

Therefore (1) becomes

$$\begin{aligned} a^2 \frac{d^2\eta}{d\xi^2} + \xi\eta(\xi) &= 0 \\ a^2 \eta'' + \xi\eta &= 0 \end{aligned} \tag{2}$$

This is Airy ODE but with plus sign instead of the normal Airy  $\eta'' - \xi\eta = 0$ . Let

$$\begin{aligned} \eta &= \sum_{n=0}^{\infty} c_n \xi^n \\ \eta' &= \sum_{n=0}^{\infty} n c_n \xi^{n-1} = \sum_{n=1}^{\infty} n c_n \xi^{n-1} = \sum_{n=0}^{\infty} (n+1) c_{n+1} \xi^n \\ \eta'' &= \sum_{n=0}^{\infty} n(n+1) c_{n+1} \xi^{n-1} = \sum_{n=1}^{\infty} n(n+1) c_{n+1} \xi^{n-1} = \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} \xi^n \end{aligned}$$

Hence (2) becomes

$$\begin{aligned} a^2 \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} \xi^n + \xi \sum_{n=0}^{\infty} c_n \xi^n &= 0 \\ \sum_{n=0}^{\infty} a^2 (n+1)(n+2) c_{n+2} \xi^n + \sum_{n=0}^{\infty} c_n \xi^{n+1} &= 0 \\ \sum_{n=0}^{\infty} a^2 (n+1)(n+2) c_{n+2} \xi^n + \sum_{n=1}^{\infty} c_{n-1} \xi^n &= 0 \\ 2a^2 c_2 + \sum_{n=1}^{\infty} (a^2 (n+1)(n+2) c_{n+2} + c_{n-1}) \xi^n &= 0 \end{aligned}$$

Hence

$$2a^2 c_2 = 0 \quad (3)$$

$$a^2 (n+1) (n+2) c_{n+2} + c_{n-1} = 0 \quad n \geq 1 \quad (4)$$

From (3) and since  $a \neq 0$

$$c_2 = 0$$

From (4)

$$c_{n+2} = \frac{-c_{n-1}}{a^2 (n+1) (n+2)}$$

Hence for  $n = 3$ , we see from the above recurrence equation and because  $c_2 = 0$  that

$$c_5 = \frac{-c_2}{a^2 (4) (6)} = 0$$

Similarly, for  $n = 6$

$$c_8 = \frac{-c_5}{a^2 (7) (8)} = 0$$

Similarly, for  $n = 9$

$$c_{11} = \frac{-c_8}{a^2 (10) (11)} = 0$$

And so on. Hence we found so far that for  $n = 3, 6, 9, 12, \dots$  all terms generated which are  $c_5, c_8, c_{11}, \dots$  are zero.

Now, for  $n = 1$ , the recurrence equation gives

$$c_3 = \frac{-c_0}{a^2} \frac{1}{2 \cdot 3}$$

For  $n = 2$

$$c_4 = \frac{-c_1}{a^2} \frac{1}{3 \cdot 4}$$

For  $n = 4$

$$c_6 = \frac{-c_3}{a^2 (5) (6)} = c_0 \left( \frac{1}{a^2} \frac{1}{5 \cdot 6} \right) \left( \frac{1}{a^2} \frac{1}{2 \cdot 3} \right)$$

For  $n = 5$

$$c_7 = \frac{-c_4}{a^2 (6) (7)} = c_1 \left( \frac{1}{a^2} \frac{1}{6 \cdot 7} \right) \left( \frac{1}{a^2} \frac{1}{3 \cdot 4} \right)$$

For  $n = 7$

$$c_9 = \frac{-c_6}{a^2 (8) (9)} = -c_0 \left( \frac{1}{a^2} \frac{1}{8 \cdot 9} \right) \left( \frac{1}{a^2} \frac{1}{5 \cdot 6} \right) \left( \frac{1}{a^2} \frac{1}{2 \cdot 3} \right)$$

For  $n = 8$

$$c_{10} = \frac{-c_7}{a^2 (9) (10)} = -c_1 \left( \frac{1}{a^2} \frac{1}{9 \cdot 10} \right) \left( \frac{1}{a^2} \frac{1}{6 \cdot 7} \right) \left( \frac{1}{a^2} \frac{1}{3 \cdot 4} \right)$$

Therefore, in summary, this is what we have so far. For  $n = 3, 6, 9, 12, \dots$  all terms are zero. For  $n = 1, 4, 7, \dots$  all terms are expressed using  $c_0$  and for  $n = 2, 5, 8, \dots$  all terms are expressed using  $c_1$ . So there are two arbitrary constants  $c_0, c_1$ .

In other words,  $c_2, c_5, c_8, c_{11}, \dots = 0$  and  $c_3, c_6, c_9, c_{12}, \dots = f(c_0)$  and  $c_4, c_7, c_{10}, c_{13}, \dots = f(c_1)$ .

$$\begin{aligned} \eta &= \sum_{n=0}^{\infty} c_n \xi^n \\ &= c_0 + c_1 \xi^1 + c_2 \xi^2 + c_3 \xi^3 + \dots \\ &= c_0 + (c_1 \xi^1) + 0 - \left( c_0 \frac{\xi^3}{a^2} \frac{1}{2 \cdot 3} \right) - \left( c_1 \frac{\xi^4}{a^2} \frac{1}{3 \cdot 4} \right) + 0 + c_0 \frac{\xi^6}{a^4} \frac{1}{2 \cdot 3 \cdot 5 \cdot 6} + c_1 \frac{\xi^7}{a^4} \frac{1}{3 \cdot 4 \cdot 6 \cdot 7} + 0 \\ &= c_0 \left( 1 - \frac{\xi^3}{a^2} \frac{1}{2 \cdot 3} + \frac{\xi^6}{a^4} \frac{1}{2 \cdot 3 \cdot 5 \cdot 6} - \dots \right) + c_1 \left( \xi - \frac{\xi^4}{a^2} \frac{1}{3 \cdot 4} + \frac{\xi^7}{a^4} \frac{1}{3 \cdot 4 \cdot 6 \cdot 7} - \dots \right) \\ &= c_0 \left( 1 - \frac{1}{a^2} \frac{\xi^3}{3!} + \frac{1 \cdot 4}{a^4} \frac{\xi^6}{6!} - \frac{1 \cdot 4 \cdot 7}{a^6} \frac{\xi^9}{9!} + \dots \right) + c_1 \left( \xi - \frac{\xi^4}{a^2} \frac{2}{4!} + \frac{\xi^7}{a^4} \frac{2 \cdot 5}{7!} - \frac{\xi^{10}}{a^6} \frac{2 \cdot 5 \cdot 8}{10!} + \dots \right) \end{aligned}$$

Hence

$$\eta = c_0 \left( \sum_{n=0}^{\infty} 3^n \left( \frac{1}{3} \right)_n \frac{(-1)^n}{a^{2n}} \frac{\xi^{3n}}{(3n)!} \right) + c_1 \left( \sum_{n=0}^{\infty} 3^n \left( \frac{2}{3} \right)_n \frac{(-1)^n}{a^{2n}} \frac{\xi^{3n+1}}{(3n+1)!} \right) \tag{5}$$

Where

$$3^n \left( \frac{1}{3} \right)_n = (1) \cdot (4) \cdot (7) \dots (3n-2)$$

$$3^n \left( \frac{2}{3} \right)_n = (2) \cdot (5) \cdot (8) \dots (3n-1)$$

And

$$\left( \frac{1}{3} \right)_0 = \left( \frac{2}{3} \right)_0 = 1$$

Equation (5) can be simplified more by moving  $\frac{(-1)^n}{a^{2n}}$  into  $\xi$  as follows

$$\eta = c_0 \left( \sum_{n=0}^{\infty} 3^n \left( \frac{1}{3} \right)_n \frac{1}{(3n)!} \left( \frac{-\xi}{a^{\frac{2}{3}}} \right)^{3n} \right) + ac_1 \left( \sum_{n=0}^{\infty} 3^n \left( \frac{2}{3} \right)_n \frac{1}{(3n+1)!} \left( \frac{-\xi}{a^{\frac{2}{3}}} \right)^{3n+1} \right)$$

Let  $\left( \frac{-\xi}{a^{\frac{2}{3}}} \right) = z$  then the above is

$$\eta = c_0 \left( \sum_{n=0}^{\infty} 3^n \left( \frac{1}{3} \right)_n \frac{z^{3n}}{(3n)!} \right) + ac_1 \left( \sum_{n=0}^{\infty} 3^n \left( \frac{2}{3} \right)_n \frac{z^{3n+1}}{(3n+1)!} \right)$$

Let

$$f(\xi) = \sum_{n=0}^{\infty} 3^n \left( \frac{1}{3} \right)_n \frac{\xi^{3n}}{(3n)!}$$

$$g(\xi) = \sum_{n=0}^{\infty} 3^n \left( \frac{2}{3} \right)_n \frac{\xi^{3n+1}}{(3n+1)!}$$

Now looking at definition of AiryAI(z) we see

$$\text{AiryAI}(z) = r_1 f(z) - r_2 g(z)$$

$$\text{AiryBI}(z) = \sqrt{3} (r_1 f(z) + r_2 g(z))$$

These are Airy functions AiryAI and AiryBI with appropriate choice of  $c_0, c_1$ . See definition of these special functions. Converting back to  $x$  using  $\xi = ax + b$  should result in solution given by CAS. Need to write these final details to make sure. Will finish later.

## 2.1011 ODE No. 1011

$$y''(x) - (x^2 + 1) y(x) = 0$$

✓ **Mathematica** : cpu = 0.0049012 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow c_1 D_{-1} \left( \sqrt{2} x \right) + c_2 D_0 \left( i \sqrt{2} x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 17

$$\left\{ y(x) = e^{\frac{x^2}{2}} \left( \text{Erf}(x) \_C2 + \_C1 \right) \right\}$$

Hand solution

$$y'' - (x^2 + 1) y = 0 \tag{1}$$

Second order with varying coefficient. Using power series, let  $y = \sum_{n=0}^{\infty} c_n x^n$ , hence

$$y' = \sum_{n=0}^{\infty} n c_n x^{n-1} = \sum_{n=1}^{\infty} n c_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n$$

$$y'' = \sum_{n=0}^{\infty} n(n+1) c_{n+1} x^{n-1} = \sum_{n=1}^{\infty} n(n+1) c_{n+1} x^{n-1} = \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n$$



Substituting back in the original ODE gives

$$\begin{aligned}
& \sum_{n=0}^{\infty} (n+1)(n+2)c_{n+2}x^n - (x^2+1) \sum_{n=0}^{\infty} c_n x^n = 0 \\
& \sum_{n=0}^{\infty} (n+1)(n+2)c_{n+2}x^n - x^2 \sum_{n=0}^{\infty} c_n x^n - \sum_{n=0}^{\infty} c_n x^n = 0 \\
& \sum_{n=0}^{\infty} (n+1)(n+2)c_{n+2}x^n - \sum_{n=0}^{\infty} c_n x^{n+2} - \sum_{n=0}^{\infty} c_n x^n = 0 \\
& \sum_{n=0}^{\infty} (n+1)(n+2)c_{n+2}x^n - \sum_{n=2}^{\infty} c_{n-2}x^n - \sum_{n=0}^{\infty} c_n x^n = 0
\end{aligned}$$

For  $n = 0$

$$\begin{aligned}
(n+1)(n+2)c_{n+2} - c_n &= 0 \\
2c_2 - c_0 &= 0 \\
c_2 &= \frac{c_0}{2}
\end{aligned}$$

For  $n = 1$

$$\begin{aligned}
(n+1)(n+2)c_{n+2} - c_n &= 0 \\
(2)(3)c_3 - c_1 &= 0 \\
c_3 &= \frac{c_1}{6}
\end{aligned}$$

For  $n \geq 2$

$$\begin{aligned}
(n+1)(n+2)c_{n+2} - c_{n-2} - c_n &= 0 \\
c_{n+2} &= \frac{c_{n-2} + c_n}{(n+1)(n+2)}
\end{aligned}$$

Hence for  $n = 2$

$$c_4 = \frac{c_0 + c_2}{(3)(4)} = \frac{c_0 + \frac{c_0}{2}}{(3)(4)} = \frac{2c_0 + c_0}{(2)(3)(4)} = c_0 \frac{3}{(2)(3)(4)}$$

For  $n = 3$

$$c_5 = \frac{c_1 + c_3}{(4)(5)} = \frac{c_1 + \frac{c_1}{6}}{(4)(5)} = \frac{6c_1 + c_1}{(4)(5)(6)} = c_1 \frac{7}{(4)(5)(6)}$$

For  $n = 4$

$$c_6 = \frac{c_2 + c_4}{(5)(6)} = \frac{\frac{c_0}{2} + c_0 \frac{3}{(2)(3)(4)}}{(5)(6)} = \frac{c_0(3)(4) + 3c_0}{(2)(3)(4)(5)(6)} = c_0 \frac{15}{(2)(3)(4)(5)(6)}$$

For  $n = 5$

$$\begin{aligned}
c_7 &= \frac{c_3 + c_5}{(6)(7)} \\
&= \frac{\frac{c_1}{6} + c_1 \frac{7}{(4)(5)(6)}}{(6)(7)} = \frac{3}{560} c_1
\end{aligned}$$

And so on. Hence the series is

$$\begin{aligned}
y &= \sum_{n=0}^{\infty} c_n x^n \\
&= c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots \\
&= c_0 + c_1 x + \frac{c_0}{2} x^2 + \frac{c_1}{6} x^3 + c_0 \frac{3}{(2)(3)(4)} x^4 + c_1 \frac{7}{(4)(5)(6)} x^5 + c_0 \frac{15}{(2)(3)(4)(5)(6)} x^6 + c_1 \frac{3}{560} x^7 + \dots \\
&= c_0 + c_1 x + \frac{c_0}{2} x^2 + \frac{c_1}{6} x^3 + c_0 \frac{1}{8} x^4 + c_1 \frac{7}{120} x^5 + c_0 \frac{1}{48} x^6 + c_1 \frac{3}{560} x^7 + \dots \\
&= c_0 \left( 1 + \frac{x^2}{2} + \frac{1}{8} x^4 + \frac{1}{48} x^6 + \dots \right) + c_1 \left( x + \frac{1}{6} x^3 + \frac{7}{120} x^5 + \frac{3}{560} x^7 + \dots \right)
\end{aligned}$$

Now the power series for  $e^{\frac{x^2}{2}} = 1 + \frac{x^2}{2} + \frac{x^4}{8} + \frac{x^6}{48} + \dots$ , so we can convert the first term above (the expression for  $c_0$  to be  $e^{\frac{x^2}{2}}$ . Hence

$$c_0 \left( 1 + \frac{x^2}{2} + \frac{1}{8} x^4 + \frac{1}{48} x^6 + \dots \right) = c_0 e^{\frac{x^2}{2}}$$

So now we have to work on the second term (the expression for  $c_1$ )

$$c_1 \left( x + \frac{1}{6}x^3 + \frac{7}{120}x^5 + \frac{3}{560}x^7 + \cdots \right) = ?$$

Recall that series for error function is

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \left( x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{48} + \cdots \right)$$

Multiplying  $e^{\frac{x^2}{2}}$  by  $\operatorname{erf}(x)$  gives

$$\begin{aligned} e^{\frac{x^2}{2}} \operatorname{erf}(x) &= \frac{2}{\sqrt{\pi}} \left( 1 + \frac{x^2}{2} + \frac{x^4}{(2)(4)} + \frac{x^6}{(2)(4)(6)} + \cdots \right) \left( x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{48} + \cdots \right) \\ &= \frac{2}{\sqrt{\pi}} \left( x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{48} + \cdots + \frac{x^3}{2} - \frac{x^5}{6} + \frac{x^7}{20} - \frac{x^{15}}{96} + \cdots \right) \\ &= \frac{2}{\sqrt{\pi}} \left( x + \frac{x^3}{6} + \frac{7x^5}{120} + \frac{3}{560}x^7 + \cdots \right) \end{aligned}$$

Comparing the above to the term next to  $c_1$  above, we see they are the same with a multiplier  $\frac{2}{\sqrt{\pi}}$ , which can be absorbed into the constant  $c_1$ , Hence

$$\begin{aligned} y &= c_0 \left( 1 + \frac{x^2}{2} + \frac{1}{8}x^4 + \frac{1}{48}x^6 + \cdots \right) + c_1 \left( x + \frac{1}{6}x^3 + \frac{7}{120}x^5 + \frac{3}{560}x^7 + \cdots \right) \\ &= c_0 e^{\frac{x^2}{2}} + c_1 \left( e^{\frac{x^2}{2}} \operatorname{erf}(x) \right) \end{aligned}$$

Hence final solution is

$$y = c_0 e^{\frac{x^2}{2}} + c_2 \left( e^{\frac{x^2}{2}} \operatorname{erf}(x) \right)$$

Verification

```

1 restart;
2 ode:=diff(diff(y(x),x),x)-(x^2+1)*y(x)=0;
3 y0:=-C1*exp(x^2/2)+_C2*exp(x^2/2)*erf(x);
4 odetest(y(x)=y0,ode);
5 0

```

2.1012

ODE No. 1012

---

$$y''(x) - \left(a + x^2\right) y(x) = 0$$

✓ **Mathematica** : cpu = 0.0058376 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow c_1 D_{\frac{1}{2}(-a-1)}\left(\sqrt{2} x\right)+c_2 D_{\frac{a-1}{2}}\left(i \sqrt{2} x\right)\right\} \right\}$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 29

$$\left\{ y\left(x\right)=1\left(-C2\,W_{-\frac{a}{4},\frac{1}{4}}\left(x^2\right)+_C1\,M_{-\frac{a}{4},\frac{1}{4}}\left(x^2\right)\right)\frac{1}{\sqrt{x}}\right\}$$

2.1013

ODE No. 1013

---

$$y''(x) - \left(a^2 x^2 + a\right) y(x) = 0$$

✓ **Mathematica** : cpu = 0.0131644 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow c_1 D_{-1}\left(\sqrt{2} \sqrt{a} x\right)+c_2 D_0\left(i \sqrt{2} \sqrt{a} x\right)\right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 22

$$\left\{ y\left(x\right)=e^{\frac{a x^2}{2}}\left(Erf\left(\sqrt{a} x\right)_C2+_C1\right)\right\}$$

2.1014 ODE No. 1014

---

$$y''(x) - cx^a y(x) = 0$$

✓ **Mathematica** : cpu = 0.0189907 (sec), leaf count = 170

$$\left\{ \left\{ y(x) \rightarrow (a+2)^{-\frac{1}{a+2}} c_1 c^{\frac{1}{2(a+2)}} x^{\frac{\frac{a}{2}+1}{a+2}} \Gamma\left(1 - \frac{1}{a+2}\right) I_{-\frac{1}{a+2}}\left(\frac{2\sqrt{c}x^{\frac{a+2}{2}}}{a+2}\right) + (-1)^{\frac{1}{a+2}} (a+2)^{-\frac{1}{a+2}} c_2 c^{\frac{1}{2(a+2)}} x^{1-\frac{\frac{a}{2}+1}{a+2}} \Gamma\left(1 + \frac{1}{a+2}\right) K_{\frac{1}{a+2}}\left(\frac{2\sqrt{c}x^{\frac{a+2}{2}}}{a+2}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.078 (sec), leaf count = 63

$$\left\{ y(x) = \sqrt{x} \left( Y_{(a+2)^{-1}} \left( 2 \frac{\sqrt{-c} x^{a/2+1}}{a+2} \right) {}_2F_2 \left( \begin{matrix} - \\ -C2 \end{matrix} ; \begin{matrix} - \\ -C1 \end{matrix} ; - \right) + J_{(a+2)^{-1}} \left( 2 \frac{\sqrt{-c} x^{a/2+1}}{a+2} \right) {}_2F_2 \left( \begin{matrix} - \\ -C1 \end{matrix} ; \begin{matrix} - \\ -C2 \end{matrix} ; - \right) \right) \right\}$$

2.1015 ODE No. 1015

---

$$y''(x) - y(x) (a^2 x^{2n} - 1) = 0$$

✗ **Mathematica** : cpu = 0.272784 (sec), leaf count = 0 , could not solve

`DSolve[-((-1 + a^2*x^(2*n))*y[x]) + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^2}{dx^2} Y(x) + (-a^2 x^{2n} + 1) Y(x) \right\}, \{ Y(x) \} \right) \right\}$$

2.1016 ODE No. 1016

---

$$y(x) (ax^{2c} + bx^{c-1}) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.083916 (sec), leaf count = 312

$$\left\{ \left\{ y(x) \rightarrow 2^{\frac{c}{2(c+1)}} c_1 (x^{c+1})^{\frac{c}{2(c+1)}} x^{-c/2} e^{-\frac{\sqrt{a} x^{c+1}}{\sqrt{-c^2-2c-1}}} U \left( \frac{\frac{\sqrt{a} c b}{\sqrt{-(c+1)^2}} + \frac{\sqrt{a} b}{\sqrt{-(c+1)^2}} + a c}{2(c a + a)}, \frac{c}{c+1}, \frac{2\sqrt{a} x^{c+1}}{\sqrt{-c^2-2c-1}} \right) + 2^{\frac{c}{2(c+1)}} c_2 (x^{c+1})^{\frac{c}{2(c+1)}} x^{-c/2} e^{-\frac{\sqrt{a} x^{c+1}}{\sqrt{-c^2-2c-1}}} U \left( \frac{\frac{\sqrt{a} c b}{\sqrt{-(c+1)^2}} + \frac{\sqrt{a} b}{\sqrt{-(c+1)^2}} + a c}{2(c a + a)}, \frac{c}{c+1}, \frac{2\sqrt{a} x^{c+1}}{\sqrt{-c^2-2c-1}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.15 (sec), leaf count = 91

$$\left\{ y(x) = x^{-\frac{c}{2}} \left( M_{\frac{-ib}{2c+2}, \frac{1}{\sqrt{a}}, (2c+2)^{-1}} \left( \frac{2ix^{c+1}}{c+1} \sqrt{a} \right) {}_2F_2 \left( \begin{matrix} - \\ -C1 \end{matrix} ; \begin{matrix} - \\ -C2 \end{matrix} ; - \right) + W_{\frac{-ib}{2c+2}, \frac{1}{\sqrt{a}}, (2c+2)^{-1}} \left( \frac{2ix^{c+1}}{c+1} \sqrt{a} \right) {}_2F_2 \left( \begin{matrix} - \\ -C2 \end{matrix} ; \begin{matrix} - \\ -C1 \end{matrix} ; - \right) \right) \right\}$$

2.1017 ODE No. 1017

---

$$(e^{2x} - v^2) y(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.018978 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow c_1 \Gamma(1-v) J_{-v}(\sqrt{e^{2x}}) + c_2 \Gamma(v+1) J_v(\sqrt{e^{2x}}) \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 17

$$\{y(x) = {}_0F_1 \left( \begin{matrix} - \\ -C1 \end{matrix} ; - \right) J_v(e^x) + {}_0F_1 \left( \begin{matrix} - \\ -C2 \end{matrix} ; - \right) Y_v(e^x) \}$$

2.1018 ODE No. 1018

---

$$ae^{bx} y(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.015129 (sec), leaf count = 55

$$\left\{ \left\{ y(x) \rightarrow c_1 J_0 \left( \frac{2\sqrt{a}\sqrt{e^{bx}}}{b} \right) + 2c_2 Y_0 \left( \frac{2\sqrt{a}\sqrt{e^{bx}}}{b} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 39

$$\left\{ y(x) = {}_0F_1 \left( \begin{matrix} - \\ -C1 \end{matrix} ; - \right) J_0 \left( 2 \frac{\sqrt{a} e^{1/2 bx}}{b} \right) + {}_0F_1 \left( \begin{matrix} - \\ -C2 \end{matrix} ; - \right) Y_0 \left( 2 \frac{\sqrt{a} e^{1/2 bx}}{b} \right) \right\}$$

**2.1019      ODE No. 1019**

---

$$y''(x) - y(x) \left( 4a^2 b^2 x^2 e^{2bx^2} - 1 \right) = 0$$

✗ **Mathematica** : cpu = 0.57384 (sec), leaf count = 0 , could not solve  
DSolve[-((-1 + 4\*a^2\*b^2\*E^(2\*b\*x^2)\*x^2)\*y[x]) + Derivative[2][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^2}{dx^2} Y(x) + \left( -4 a^2 b^2 x^2 e^{2 b x^2} + 1 \right) Y(x) \right\}, \{ Y(x) \} \right) \right\}$$

**2.1020      ODE No. 1020**

---

$$y(x) \left( a e^{2x} + b e^x + c \right) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.497464 (sec), leaf count = 180

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{i(\sqrt{c} \log(e^x) - \sqrt{a} e^x)} U \left( \frac{i(b - i\sqrt{a} + 2\sqrt{a}\sqrt{c})}{2\sqrt{a}}, 2i\sqrt{c} + 1, 2i\sqrt{a} e^x \right) + c_2 e^{i(\sqrt{c} \log(e^x) - \sqrt{a} e^x)} L_{-\frac{i(b - i\sqrt{a} + 2\sqrt{a})}{2\sqrt{a}}}^{2i\sqrt{c}} \right\} \right\}$$

✓ **Maple** : cpu = 0.141 (sec), leaf count = 58

$$\left\{ y(x) = e^{-\frac{x}{2}} \left( M_{-\frac{i}{2}b\frac{1}{\sqrt{a}}, i\sqrt{c}}(2i\sqrt{a}e^x) {}_2C1 + W_{-\frac{i}{2}b\frac{1}{\sqrt{a}}, i\sqrt{c}}(2i\sqrt{a}e^x) {}_2C2 \right) \right\}$$

**2.1021      ODE No. 1021**

---

$$y(x) \left( a \cos^2(x) + b \right) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0238298 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow c_1 \text{MathieuC} \left[ \frac{1}{2}(a + 2b), -\frac{a}{4}, x \right] + c_2 \text{MathieuS} \left[ \frac{1}{2}(a + 2b), -\frac{a}{4}, x \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.165 (sec), leaf count = 39

$$\left\{ y(x) = {}_2C1 \text{MathieuC} \left( -\frac{a}{2} - b, \frac{a}{4}, ix \right) + {}_2C2 \text{MathieuS} \left( -\frac{a}{2} - b, \frac{a}{4}, ix \right) \right\}$$

**2.1022      ODE No. 1022**

---

$$y(x) \left( a \cos(2x) + b \right) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0192206 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow c_1 \text{MathieuC} \left[ b, -\frac{a}{2}, x \right] + c_2 \text{MathieuS} \left[ b, -\frac{a}{2}, x \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.158 (sec), leaf count = 21

$$\left\{ y(x) = {}_2C1 \text{MathieuC} \left( b, -\frac{a}{2}, x \right) + {}_2C2 \text{MathieuS} \left( b, -\frac{a}{2}, x \right) \right\}$$

**2.1023      ODE No. 1023**

---

$$y(x) \left( a \cos^2(x) + b \right) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0099684 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow c_1 \text{MathieuC} \left[ \frac{1}{2}(a + 2b), -\frac{a}{4}, x \right] + c_2 \text{MathieuS} \left[ \frac{1}{2}(a + 2b), -\frac{a}{4}, x \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.16 (sec), leaf count = 29

$$\left\{ y(x) = {}_2C1 \text{MathieuC} \left( \frac{a}{2} + b, -\frac{a}{4}, x \right) + {}_2C2 \text{MathieuS} \left( \frac{a}{2} + b, -\frac{a}{4}, x \right) \right\}$$

2.1024 ODE No. 1024

---

$$y''(x) - y(x) \left( 2 \tan^2(x) + 1 \right) = 0$$

✓ **Mathematica** : cpu = 0.109967 (sec), leaf count = 84

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 \sqrt[4]{1 - \cos^2(x)} \sec(x)}{\sqrt[4]{\cos^2(x) - 1}} - \frac{c_2 \sqrt[4]{1 - \cos^2(x)} \sec(x) \left( \cos(x) \sqrt{1 - \cos^2(x)} - \sin^{-1}(\cos(x)) \right)}{2 \sqrt[4]{\cos^2(x) - 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 30

$$\left\{ y(x) = \frac{i \cos(x) \sin(x) \_C2 + \ln(\cos(x) + i \sin(x)) \_C2 + \_C1}{\cos(x)} \right\}$$

2.1025 ODE No. 1025

---

$$y''(x) - y(x) \left( a + (m - 1)m \sec^2(x) + (n - 1)n \csc^2(x) \right) = 0$$

✓ **Mathematica** : cpu = 0.656162 (sec), leaf count = 615

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 (-1)^{\frac{1}{2}(-2m-1)+1} \cos^2(x)^{\frac{1}{4}(-2m-1)+1} (\cos^2(x) - 1)^{\frac{1}{2} \left( \frac{4am+4\sqrt{-a}n^2+4an-4\sqrt{-a}n+4(-a)^{3/2}+8\sqrt{-a}a+\sqrt{-a}+4mn^2-4a}{8a+8n^2-8n+2} \right)}}{\right.} \right.$$

✓ **Maple** : cpu = 0.135 (sec), leaf count = 102

$$\left\{ y(x) = (\sin(x))^n \left( (\cos(x))^{-m+1} {}_2F_1\left(\frac{n}{2} - \frac{m}{2} + \frac{i}{2}\sqrt{a} + \frac{1}{2}, \frac{n}{2} - \frac{m}{2} - \frac{i}{2}\sqrt{a} + \frac{1}{2}; \frac{3}{2} - m; (\cos(x))^2\right) \_C2 + (\cos(x))^{-m+1} \right) \right\}$$

2.1026 ODE No. 1026

---

$$y''(x) - y(x)(B + n(n + 1)\wp(x; g2, g3)) = 0$$

✗ **Mathematica** : cpu = 0.161726 (sec), leaf count = 0 , could not solve

`DSolve[-((B + n*(1 + n)*WeierstrassP[x, {g2, g3}])*y[x]) + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^2}{dx^2} \_Y(x) + (-n(n + 1) WeierstrassP(x, g2, g3) - B) \_Y(x) \right\}, \{ \_Y(x) \} \right) \right\}$$

2.1027 ODE No. 1027

---

$$y(x) \left( a \operatorname{sn}(x|k)^2 + b \right) + y''(x) = 0$$

✗ **Mathematica** : cpu = 1.03479 (sec), leaf count = 0 , could not solve

`DSolve[(b + a*JacobiSN[x, k]^2)*y[x] + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.263 (sec), leaf count = 69

$$\left\{ y(x) = \_C1 HeunG \left( k^{-2}, \frac{b}{4k^2}, -\frac{n}{2}, \frac{n}{2} + \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, (JacobiSN(x, k))^2 \right) + \_C2 HeunG \left( k^{-2}, \frac{k^2 + b + 1}{4k^2}, \frac{n}{2} + 1, \frac{n}{2} + 1, \frac{1}{2}, \frac{1}{2} \right) \right\}$$

2.1028 ODE No. 1028

---

$$y''(x) - y(x) \left( ap(x) + b + \frac{p^4(x)}{30} + \frac{7p''(x)}{3} \right) = 0$$

✗ **Mathematica** : cpu = 0.228032 (sec), leaf count = 0 , could not solve

`DSolve[-(y[x]*(b + a*p[x] + (p^4)[x]/30 + (7*Derivative[2][p][x])/3)) + Derivative[2][y][x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^2}{dx^2} \_Y(x) + \left( -\frac{d^4 p(x)}{30} - \frac{7 \frac{d^2}{dx^2} p(x)}{3} - ap(x) - b \right) \_Y(x) \right\}, \{ \_Y(x) \} \right) \right\}$$

2.1029 ODE No. 1029

---

$$y''(x) - y(x) \left( f'(x) + f(x)^2 \right) = 0$$

✗ **Mathematica** : cpu = 0.114724 (sec), leaf count = 0 , could not solve  
DSolve[-(y[x]\*(f[x]^2 + Derivative[1][f][x])) + Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.092 (sec), leaf count = 22

$$\left\{ y(x) = \left( \int e^{\int -2f(x) dx} dx + \_C1 \right) e^{\int f(x) dx} \_C2 \right\}$$

2.1030 ODE No. 1030

---

$$y(x)(l + P(x)) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.123922 (sec), leaf count = 0 , could not solve  
DSolve[(1 + P[x])\*y[x] + Derivative[2][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ (P(x) + l) \_Y(x) + \frac{d^2}{dx^2} \_Y(x) \right\}, \{ \_Y(x) \} \right) \right\}$$

2.1031 ODE No. 1031

---

$$y''(x) - f(x)y(x) = 0$$

✗ **Mathematica** : cpu = 0.0996267 (sec), leaf count = 0 , could not solve  
DSolve[-(f[x]\*y[x]) + Derivative[2][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ -f(x) \_Y(x) + \frac{d^2}{dx^2} \_Y(x) \right\}, \{ \_Y(x) \} \right) \right\}$$

2.1032 ODE No. 1032

---

$$y(x) \left( \frac{\left( \frac{1}{4} - v^2 \right) g'(x)^2}{g(x)} + g'(x)^2 + \frac{g^3(x)}{2g'(x)} - \frac{3g''(x)^2}{4g'(x)^2} \right) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.386794 (sec), leaf count = 0 , could not solve  
DSolve[y[x]\*((g^3)[x]/(2\*Derivative[1][g][x]) + Derivative[1][g][x]^2 + ((1/4 - v^2)\*Derivative[2][g][x])) == 0, y[x], x]  
✓ **Maple** : cpu = 0.082 (sec), leaf count = 48

$$\left\{ y(x) = 1 \left( \_C2 W_{\frac{i}{2}v^2 - \frac{i}{8}, \frac{1}{2}}(2ig(x)) + \_C1 M_{\frac{i}{2}v^2 - \frac{i}{8}, \frac{1}{2}}(2ig(x)) \right) \frac{1}{\sqrt{\frac{d}{dx}g(x)}} \right\}$$

2.1033 ODE No. 1033

---

$$ae^{-2x}y(x) + y''(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0121353 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow c_1 \cos \left( \sqrt{a} e^{-x} \right) - c_2 \sin \left( \sqrt{a} e^{-x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 27

$$\{ y(x) = \_C1 \sin \left( e^{-x} \sqrt{a} \right) + \_C2 \cos \left( e^{-x} \sqrt{a} \right) \}$$

Hand solution

$$y'' + y' + ae^{-2x}y = 0$$

Let  $y(x) = \eta(\xi)$  where  $\xi = e^{-x}$ , hence

$$\begin{aligned} \frac{dy}{dx} &= \frac{d\eta}{d\xi} \frac{d\xi}{dx} \\ &= \frac{d\eta}{d\xi} (-e^{-x}) \end{aligned}$$

And

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{d}{dx} \left( \frac{d\eta}{d\xi} (-e^{-x}) \right) \\ &= \frac{d^2\eta}{d\xi^2} \frac{d\xi}{dx} (-e^{-x}) + \frac{d\eta}{d\xi} (e^{-x}) \\ &= \frac{d^2\eta}{d\xi^2} (-e^{-x}) (-e^{-x}) + \frac{d\eta}{d\xi} (e^{-x}) \\ &= \frac{d^2\eta}{d\xi^2} (e^{-2x}) + \frac{d\eta}{d\xi} (e^{-x})\end{aligned}$$

Hence the original ODE becomes

$$\begin{aligned}\frac{d^2\eta}{d\xi^2} (e^{-2x}) + \frac{d\eta}{d\xi} (e^{-x}) + \frac{d\eta}{d\xi} (-e^{-x}) + ae^{-2x}\eta(\xi) &= 0 \\ \eta'' + a\eta &= 0\end{aligned}$$

This is standard second order with constant coefficients. The solution is

$$\eta = c_1 \cos(\sqrt{a}\xi) + c_2 \sin(\sqrt{a}\xi)$$

Substituting back

$$y(x) = c_1 \cos(\sqrt{a}e^{-x}) + c_2 \sin(\sqrt{a}e^{-x})$$

Verification

```
1 restart;
2 ode:=diff(diff(y(x),x),x)+diff(y(x),x)+a*exp(-2*x)*y(x)=0;
3 ys:=_C1*cos(sqrt(a)*exp(-x))+_C2*sin(sqrt(a)*exp(-x));
4 odetest(y(x)=ys,ode);
5 0
```

## 2.1034 ODE No. 1034

$$y''(x) - y'(x) + e^{2x}y(x) = 0$$

✓ **Mathematica** : cpu = 0.0084406 (sec), leaf count = 20

$$\{\{y(x) \rightarrow c_2 \sin(e^x) + c_1 \cos(e^x)\}\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 15

$$\{y(x) = \_C1 \sin(e^x) + \_C2 \cos(e^x)\}$$

Hand solution

$$y'' - y' + e^{2x}y = 0$$

Let  $y(x) = \eta(\xi)$  where  $\xi = e^x$ , hence

$$\begin{aligned}\frac{dy}{dx} &= \frac{d\eta}{d\xi} \frac{d\xi}{dx} \\ &= \frac{d\eta}{d\xi} e^x\end{aligned}$$

And

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{d}{dx} \left( \frac{d\eta}{d\xi} e^x \right) \\ &= \frac{d^2\eta}{d\xi^2} \frac{d\xi}{dx} (e^x) + \frac{d\eta}{d\xi} (e^x) \\ &= \frac{d^2\eta}{d\xi^2} (e^x) (e^x) + \frac{d\eta}{d\xi} (e^x) \\ &= \frac{d^2\eta}{d\xi^2} (e^{2x}) + \frac{d\eta}{d\xi} (e^x)\end{aligned}$$

Hence the original ODE becomes

$$\frac{d^2\eta}{d\xi^2}(e^{2x}) + \frac{d\eta}{d\xi}(e^x) - \frac{d\eta}{d\xi}(e^x) + e^{2x}\eta = 0$$
$$\eta'' + \eta = 0$$

This is standard second order with constant coefficients. The solution is

$$\eta = c_1 \cos(\xi) + c_2 \sin(\xi)$$

Substituting back

$$y(x) = c_1 \cos(e^x) + c_2 \sin(e^x)$$

Verification

```
1 restart;
2 ode:=diff(diff(y(x),x),x)-diff(y(x),x)+exp(2*x)*y(x)=0;
3 ys:=-C1*cos(exp(x))+_C2*sin(exp(x));
4 odetest(y(x)=ys,ode);
5 0
```

2.1035 ODE No. 1035

---

$$ay'(x) + by(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0041726 (sec), leaf count = 58

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{2}x(-\sqrt{a^2-4b}-a)} + c_2 e^{\frac{1}{2}x(\sqrt{a^2-4b}-a)} \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 41

$$\left\{ y(x) = \_C1 e^{-\frac{x}{2}(a-\sqrt{a^2-4b})} + \_C2 e^{-\frac{x}{2}(a+\sqrt{a^2-4b})} \right\}$$

2.1036 ODE No. 1036

---

$$ay'(x) + by(x) - f(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.126227 (sec), leaf count = 209

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{1}{2}x(-\sqrt{a^2-4b}-a)} \int_1^x \frac{e^{aK[1]+\frac{1}{2}(\sqrt{a^2-4b}-a)K[1]} f(K[1])}{\sqrt{a^2-4b}} dK[1] + e^{\frac{1}{2}x(\sqrt{a^2-4b}-a)} \int_1^x \frac{e^{aK[2]+\frac{1}{2}(-a-\sqrt{a^2-4b})K[2]} f(K[2])}{\sqrt{a^2-4b}} dK[2] \right\} \right\}$$

✓ **Maple** : cpu = 0.071 (sec), leaf count = 124

$$\left\{ y(x) = e^{-\frac{x}{2}(a-\sqrt{a^2-4b})} \_C2 + e^{-\frac{x}{2}(a+\sqrt{a^2-4b})} \_C1 + 1 \left( \int f(x) e^{-\frac{x}{2}(-a+\sqrt{a^2-4b})} dx e^{x\sqrt{a^2-4b}} - \int f(x) e^{\frac{x}{2}(a+\sqrt{a^2-4b})} dx e^{-x\sqrt{a^2-4b}} \right) \right\}$$

2.1037 ODE No. 1037

---

$$ay'(x) + y(x) (- (b^2x^2 + c)) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0198268 (sec), leaf count = 101

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\frac{ax}{2} - \frac{bx^2}{2}} H_{\frac{-a^2-4b-4c}{8b}} \left( \sqrt{bx} \right) + c_2 e^{-\frac{ax}{2} - \frac{bx^2}{2}} {}_1F_1 \left( -\frac{-a^2-4b-4c}{16b}; \frac{1}{2}; bx^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 64

$$\left\{ y(x) = e^{-\frac{x(bx+a)}{2}} x \left( U \left( \frac{a^2+12b+4c}{16b}, \frac{3}{2}, bx^2 \right) \_C2 + M \left( \frac{a^2+12b+4c}{16b}, \frac{3}{2}, bx^2 \right) \_C1 \right) \right\}$$



**2.1038      ODE No. 1038**

---

$$2ay'(x) + f(x)y(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.293256 (sec), leaf count = 0 , could not solve

DSolve[f[x]\*y[x] + 2\*a\*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x) = DESol\left(\left\{f(x) \_ Y(x) + 2a \frac{d}{dx} \_ Y(x) + \frac{d^2}{dx^2} \_ Y(x)\right\}, \{\_ Y(x)\}\right)\right\}$$

**2.1039      ODE No. 1039**

---

$$y''(x) + xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0086234 (sec), leaf count = 47

$$\left\{\left\{y(x) \rightarrow \sqrt{\frac{\pi}{2}}c_1e^{-\frac{x^2}{2}}\operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right) + c_2e^{-\frac{x^2}{2}}\right\}\right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 25

$$\left\{y(x) = (\operatorname{Erf}\left(\frac{i}{2}\sqrt{2}x\right) \_ C1 + \_ C2)\left(e^{\frac{x^2}{2}}\right)^{-1}\right\}$$

**2.1040      ODE No. 1040**

---

$$y''(x) + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0300953 (sec), leaf count = 53

$$\left\{\left\{y(x) \rightarrow c_1x - \frac{1}{2}c_2e^{-\frac{x^2}{2}}\left(\sqrt{2\pi}e^{\frac{x^2}{2}}x\operatorname{erf}\left(\frac{x}{\sqrt{2}}\right) + 2\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 33

$$\left\{y(x) = \sqrt{\pi}\sqrt{2}e^{-\frac{x^2}{2}} \_ C2 + x\left(\pi \_ C2 \operatorname{Erf}\left(\frac{\sqrt{2}x}{2}\right) + \_ C1\right)\right\}$$

**2.1041      ODE No. 1041**

---

$$(n+1)y(x) + y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0071444 (sec), leaf count = 55

$$\left\{\left\{y(x) \rightarrow c_1e^{-\frac{x^2}{2}}H_n\left(\frac{x}{\sqrt{2}}\right) + c_2e^{-\frac{x^2}{2}}{}_1F_1\left(-\frac{n}{2}; \frac{1}{2}; \frac{x^2}{2}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.055 (sec), leaf count = 41

$$\left\{y(x) = e^{-\frac{x^2}{2}}x\left(M\left(-\frac{n}{2} + \frac{1}{2}, \frac{3}{2}, \frac{x^2}{2}\right) \_ C1 + U\left(-\frac{n}{2} + \frac{1}{2}, \frac{3}{2}, \frac{x^2}{2}\right) \_ C2\right)\right\}$$

**2.1042      ODE No. 1042**

---

$$-ny(x) + y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0059692 (sec), leaf count = 61

$$\left\{\left\{y(x) \rightarrow c_1e^{-\frac{x^2}{2}}H_{-n-1}\left(\frac{x}{\sqrt{2}}\right) + c_2e^{-\frac{x^2}{2}}{}_1F_1\left(\frac{n+1}{2}; \frac{1}{2}; \frac{x^2}{2}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 41

$$\left\{y(x) = e^{-\frac{x^2}{2}}x\left(U\left(\frac{n}{2} + 1, \frac{3}{2}, \frac{x^2}{2}\right) \_ C2 + M\left(\frac{n}{2} + 1, \frac{3}{2}, \frac{x^2}{2}\right) \_ C1\right)\right\}$$

## 2.1043 ODE No. 1043

---

$$y''(x) - xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0991634 (sec), leaf count = 69

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} c_2 \left( \sqrt{2\pi} x^2 \operatorname{erfi} \left( \frac{x}{\sqrt{2}} \right) - \sqrt{2\pi} \operatorname{erfi} \left( \frac{x}{\sqrt{2}} \right) - 2e^{\frac{x^2}{2}} x \right) + c_1 (x^2 - 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.164 (sec), leaf count = 39

$$\left\{ y(x) = -2 e^{1/2 x^2} {}_0F_1 \left( x + (x-1)(1+x) \left( \sqrt{2} \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{2}x}{2} \right) {}_0F_1 + {}_0F_2 \right) \right) \right\}$$

**Hand solution**

$$y'' - xy' + 2y = 0 \tag{1}$$

Second order with varying coefficient. Using power series, let  $y = \sum_{n=0}^{\infty} c_n x^n$ , hence

$$\begin{aligned} y' &= \sum_{n=0}^{\infty} n c_n x^{n-1} = \sum_{n=1}^{\infty} n c_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n \\ y'' &= \sum_{n=0}^{\infty} n(n+1) c_{n+1} x^{n-1} = \sum_{n=1}^{\infty} n(n+1) c_{n+1} x^{n-1} = \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n \end{aligned}$$

Substituting back in the original ODE gives

$$\begin{aligned} \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n - x \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n + 2 \sum_{n=0}^{\infty} c_n x^n &= 0 \\ \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n - \sum_{n=0}^{\infty} (n+1) c_{n+1} x^{n+1} + \sum_{n=0}^{\infty} 2c_n x^n &= 0 \\ \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n - \sum_{n=1}^{\infty} n c_n x^n + \sum_{n=0}^{\infty} 2c_n x^n &= 0 \end{aligned}$$

For  $n = 0$

$$\begin{aligned} (n+1)(n+2) c_{n+2} + 2c_n &= 0 \\ (1)(2) c_2 + 2c_0 &= 0 \\ c_2 &= -c_0 \end{aligned}$$

For  $n \geq 1$

$$\begin{aligned} (n+1)(n+2) c_{n+2} - n c_n + 2c_n &= 0 \\ c_{n+2} &= \frac{c_n (n-2)}{(n+1)(n+2)} \end{aligned}$$

Hence for  $n = 1$

$$c_3 = \frac{-c_1}{(2)(3)}$$

For  $n = 2$

$$c_4 = \frac{c_2 (2-2)}{(3)(4)} = 0$$

For  $n = 3$

$$c_5 = \frac{c_3}{(4)(5)} = \frac{-c_1}{(2)(3)(4)(5)}$$

For  $n = 4$  and since  $c_4 = 0$  then

$$c_6 = \frac{c_4 (n-2)}{(n+1)(n+2)} = 0$$

For  $n = 5$

$$c_7 = \frac{3c_5}{(6)(7)} = -\frac{3c_1}{(2)(3)(4)(5)(6)(7)}$$

For  $n = 6$  and since  $c_6 = 0$  then

$$c_8 = \frac{c_6 (n-2)}{(n+1)(n+2)} = 0$$

For  $n = 7$

$$c_9 = \frac{5c_7}{(8)(9)} = -\frac{(3)(5)c_1}{(2)(3)(4)(5)(6)(7)(8)(9)}$$

And so on. Hence

$$\begin{aligned} y &= \sum_{n=0}^{\infty} c_n x^n \\ &= c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots \\ &= c_0 + c_1 x - c_0 x^2 - \frac{c_1}{(2)(3)} x^3 - \frac{c_1}{(2)(3)(4)(5)} x^5 - \frac{3c_1}{(2)(3)(4)(5)(6)(7)} x^7 - \frac{(3)(5)c_1}{(2)(3)(4)(5)(6)(7)(8)(9)} x^9 - \dots \\ &= c_0 (1 - x^2) + c_1 \left( x - \frac{1}{(2)(3)} x^3 - \frac{1}{(2)(3)(4)(5)} x^5 - \frac{3}{(2)(3)(4)(5)(6)(7)} x^7 - \frac{(3)(5)}{(2)(3)(4)(5)(6)(7)(8)(9)} x^9 - \dots \right) \\ &= c_0 (1 - x^2) + c_1 \left( x - \frac{1}{3!} x^3 - \frac{1}{5!} x^5 - \frac{3}{7!} x^7 - \frac{15}{9!} x^9 - \dots \right) \end{aligned}$$

Hence

$$y(x) = c_0 (1 - x^2) + c_1 \left( x - \frac{1}{6} x^3 - \frac{1}{120} x^5 - \frac{1}{1680} x^7 - \frac{1}{24192} x^9 - \dots \right)$$

Verification

```
1 restart;
2 Order:=10:
3 sol:=dsolve(ode,y(x),series):
4 subs({y(0)=c0,D(y)(0)=c1},rhs(sol)):
5 sol:=convert(%,polynom):
6
7 sol:=collect(sol,{c0,c1});
8 sol := (-x^2+1)*c0+(x-(1/6)*x^3-(1/120)*x^5-(1/1680)*x^7-(1/24192)*x^9)*c1
```

## 2.1044 ODE No. 1044

$$-ay(x) + y''(x) - xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0064759 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow c_1 H_{-a} \left( \frac{x}{\sqrt{2}} \right) + c_2 {}_1F_1 \left( \frac{a}{2}; \frac{1}{2}; \frac{x^2}{2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.055 (sec), leaf count = 35

$$\left\{ y(x) = x \left( U \left( \frac{1}{2} + \frac{a}{2}, \frac{3}{2}, \frac{x^2}{2} \right) {}_2C_2 + M \left( \frac{1}{2} + \frac{a}{2}, \frac{3}{2}, \frac{x^2}{2} \right) {}_2C_1 \right) \right\}$$

## 2.1045 ODE No. 1045

$$y''(x) - xy'(x) + (x - 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0280429 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow \sqrt{\frac{\pi}{2}} c_2 e^{x-2} \operatorname{erfi} \left( \frac{x-2}{\sqrt{2}} \right) + c_1 e^x \right\} \right\}$$

✓ **Maple** : cpu = 0.005 (sec), leaf count = 21

$$\left\{ y(x) = e^x \left( \operatorname{Erf} \left( \frac{i}{2} \sqrt{2} (x-2) \right) {}_2C_1 + {}_2C_2 \right) \right\}$$

## 2.1046 ODE No. 1046

$$ay(x) + y''(x) - 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0055442 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow c_1 H_{\frac{a}{2}}(x) + c_2 {}_1F_1 \left( -\frac{a}{4}; \frac{1}{2}; x^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 31

$$\left\{ y(x) = x \left( U \left( \frac{1}{2} - \frac{a}{4}, \frac{3}{2}, x^2 \right) {}_2C_2 + M \left( \frac{1}{2} - \frac{a}{4}, \frac{3}{2}, x^2 \right) {}_2C_1 \right) \right\}$$

## 2.1047 ODE No. 1047

---

$$(4x^2 + 2)y(x) + y''(x) + 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0095861 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-x^2} + c_2 e^{-x^2} x \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 16

$$\left\{ y(x) = e^{-x^2} (-C_2 x + -C_1) \right\}$$

**Hand solution**

$$y'' + 4xy' + (4x^2 + 2)y = 0 \quad (1)$$

Second order with varying coefficient. Using power series, let  $y = \sum_{n=0}^{\infty} c_n x^n$ , hence

$$\begin{aligned} y' &= \sum_{n=0}^{\infty} n c_n x^{n-1} = \sum_{n=1}^{\infty} n c_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n \\ y'' &= \sum_{n=0}^{\infty} n(n+1) c_{n+1} x^{n-1} = \sum_{n=1}^{\infty} n(n+1) c_{n+1} x^{n-1} = \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n \end{aligned}$$

Substituting back in the original ODE gives

$$\begin{aligned} \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n + 4x \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n + (4x^2 + 2) \sum_{n=0}^{\infty} c_n x^n &= 0 \\ \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n + \sum_{n=0}^{\infty} 4(n+1) c_{n+1} x^{n+1} + \sum_{n=0}^{\infty} 4c_n x^{n+2} + \sum_{n=0}^{\infty} 2c_n x^n &= 0 \\ \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n + \sum_{n=1}^{\infty} 4n c_n x^n + \sum_{n=2}^{\infty} 4c_{n-2} x^n + \sum_{n=0}^{\infty} 2c_n x^n &= 0 \end{aligned}$$

For  $n = 0$

$$\begin{aligned} (n+1)(n+2) c_{n+2} + 2c_n &= 0 \\ (1)(2) c_2 + 2c_0 &= 0 \\ c_2 &= -c_0 \end{aligned}$$

For  $n = 1$

$$\begin{aligned} (n+1)(n+2) c_{n+2} + 4n c_n + 2c_n &= 0 \\ (2)(3) c_3 + 4c_1 + 2c_1 &= 0 \\ c_3 &= -c_1 \end{aligned}$$

For  $n \geq 2$

$$\begin{aligned} (n+1)(n+2) c_{n+2} + 4n c_n + 4c_{n-2} + 2c_n &= 0 \\ c_{n+2} &= \frac{(-4n-2) c_n - 4c_{n-2}}{(n+1)(n+2)} \end{aligned}$$

Hence for  $n = 2$

$$c_4 = \frac{(-8-2) c_2 - 4c_0}{(3)(4)} = \frac{(-8-2)(-c_0) - 4c_0}{(3)(4)} = c_0 \frac{6}{(3)(4)}$$

For  $n = 3$

$$c_5 = \frac{(-12-2) c_3 - 4c_1}{(4)(5)} = \frac{(-12-2)(-c_1) - 4c_1}{(4)(5)} = c_1 \frac{10}{(4)(5)}$$

For  $n = 4$

$$c_6 = \frac{(-16-2) c_4 - 4c_2}{(5)(6)} = \frac{(-16-2) \left( c_0 \frac{6}{(3)(4)} \right) - 4(-c_0)}{(5)(6)} = c_0 \frac{-5}{(5)(6)}$$

For  $n = 5$

$$c_7 = \frac{(-20-2) c_5 - 4c_3}{(6)(7)} = \frac{(-20-2) \left( c_1 \frac{10}{(4)(5)} \right) - 4(-c_1)}{(6)(7)} = c_1 \frac{-7}{(6)(7)}$$

For  $n = 6$

$$c_8 = \frac{(-24 - 2) c_6 - 4c_4}{(7)(8)} = \frac{(-24 - 2) \left( c_0 \frac{-5}{(5)(6)} \right) - 4 \left( c_0 \frac{6}{(3)(4)} \right)}{(7)(8)} = c_0 \frac{7}{3} \frac{1}{(7)(8)}$$

And so on. Hence

$$\begin{aligned} y &= \sum_{n=0}^{\infty} c_n x^n \\ &= c_0 + c_1 x + c_2 x^2 + \cdots \\ &= c_0 + c_1 x - c_0 x^2 - c_1 x^3 + c_0 \frac{6}{(3)(4)} x^4 + c_1 \frac{10}{(4)(5)} x^5 - c_0 \frac{5}{(5)(7)} x^6 - c_1 \frac{7}{(6)(7)} x^7 + c_0 \frac{12}{7} \frac{1}{(7)(8)} x^8 + \cdots \\ &= c_0 \left( 1 - x^2 + \frac{6}{(3)(4)} x^4 - \frac{5}{(5)(6)} x^6 + \frac{7}{3} \frac{1}{(7)(8)} x^8 + \cdots \right) + c_1 \left( x - x^3 + \frac{10}{(4)(5)} x^5 - \frac{7}{(6)(7)} x^7 + \cdots \right) \\ &= c_0 \left( 1 - x^2 + \frac{1}{2} x^4 - \frac{1}{6} x^6 + \frac{1}{24} x^8 + \cdots \right) + c_1 \left( x - x^3 + \frac{1}{2} x^5 - \frac{1}{6} x^7 + \cdots \right) \end{aligned}$$

But Taylor series for  $e^{-x^2} = 1 - x^2 + \frac{1}{2} x^4 - \frac{x^6}{4} + \cdots$ , therefore the above becomes

$$y = c_0 e^{-x^2} + c_1 x e^{-x^2}$$

Verification

```
1 restart;
2 restart;
3 ode:=diff(diff(y(x),x),x)+4*x*diff(y(x),x)+(4*x^2+2)*y(x)=0;
4 y0:=-C0*exp(-x^2)+_C1*x*exp(-x^2);
5 odetest(y(x)=y0,ode);
6 0
```

## 2.1048 ODE No. 1048

$$(2n + 3x^2 - 1) y(x) + y''(x) - 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0077407 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{x^2}{2}} H_n(x) + c_2 e^{\frac{x^2}{2}} {}_1F_1 \left( -\frac{n}{2}; \frac{1}{2}; x^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 37

$$\left\{ y(x) = x e^{\frac{x^2}{2}} \left( U \left( -\frac{n}{2} + \frac{1}{2}, \frac{3}{2}, x^2 \right) {}_2C_2 + M \left( -\frac{n}{2} + \frac{1}{2}, \frac{3}{2}, x^2 \right) {}_2C_1 \right) \right\}$$

## 2.1049 ODE No. 1049

$$(4x^2 - 1) y(x) + y''(x) - 4xy'(x) - e^x = 0$$

✓ **Mathematica** : cpu = 0.04868 (sec), leaf count = 109

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{x(x-i)} - \frac{1}{2} i c_2 e^{(x-i)x+2ix} + \frac{1}{4} \sqrt{\pi} e^{x(x-i)-\frac{i}{2}} \left( e^{2ix} \operatorname{erfi} \left( \left( \frac{1}{2} + \frac{i}{2} \right) - ix \right) - i e^i \operatorname{erf} \left( -x + \left( \frac{1}{2} + \frac{i}{2} \right) \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.128 (sec), leaf count = 66

$$\left\{ y(x) = \frac{\left( (i \cos(x) + \sin(x)) \sqrt{\pi} e^{\frac{i}{2}} \operatorname{Erf} \left( x - \frac{1}{2} - \frac{i}{2} \right) - (i \cos(x) - \sin(x)) e^{-\frac{i}{2}} \sqrt{\pi} \operatorname{Erf} \left( x - \frac{1}{2} + \frac{i}{2} \right) + 4 \sin(x) - 4 \right) e^x}{4} \right\}$$

## 2.1050 ODE No. 1050

$$(4x^2 - 2) y(x) + y''(x) - 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0086054 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{x^2} + c_2 e^{x^2} x \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 14

$$\left\{ y(x) = e^{x^2} ({}_2C_2 x + {}_2C_1) \right\}$$

2.1051 ODE No. 1051

---

$$(4x^2 - 3)y(x) - e^{x^2} + y''(x) - 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0174905 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{(x-1)x} + \frac{1}{2} c_2 e^{(x-1)x+2x} - e^{(x-1)x+x} \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 27

$$\left\{ y(x) = e^{x(1+x)} \_C2 + e^{x(x-1)} \_C1 - e^{x^2} \right\}$$

2.1052 ODE No. 1052

---

$$axy'(x) + by(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0145954 (sec), leaf count = 78

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\frac{ax^2}{2}} H_{\frac{b-a}{a}} \left( \frac{\sqrt{ax}}{\sqrt{2}} \right) + c_2 e^{-\frac{ax^2}{2}} {}_1F_1 \left( -\frac{b-a}{2a}; \frac{1}{2}; \frac{ax^2}{2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 58

$$\left\{ y(x) = e^{-\frac{ax^2}{2}} x \left( M \left( \frac{2a-b}{2a}, \frac{3}{2}, \frac{ax^2}{2} \right) \_C1 + U \left( \frac{2a-b}{2a}, \frac{3}{2}, \frac{ax^2}{2} \right) \_C2 \right) \right\}$$

2.1053 ODE No. 1053

---

$$a^2 x^2 y(x) + 2axy'(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0227701 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\frac{ax^2}{2} - \sqrt{ax}} + \frac{c_2 e^{\sqrt{ax} - \frac{ax^2}{2}}}{2\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 35

$$\left\{ y(x) = \_C1 e^{-\frac{x}{2}(ax-2\sqrt{a})} + \_C2 e^{-\frac{x}{2}(ax+2\sqrt{a})} \right\}$$

2.1054 ODE No. 1054

---

$$(ax + b)y'(x) + y(x)(cx + d) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0332217 (sec), leaf count = 172

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{cx}{a} - \frac{ax^2}{2} - bx} H_{\frac{-a^3+da^2-bca+c^2}{a^3}} \left( \frac{ab-2c}{\sqrt{2}a^{3/2}} + \frac{\sqrt{ax}}{\sqrt{2}} \right) + c_2 e^{\frac{cx}{a} - \frac{ax^2}{2} - bx} {}_1F_1 \left( -\frac{-a^3+da^2-bca+c^2}{2a^3}; \frac{1}{2}; \left( \frac{ax}{2} \right)^{1/2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 98

$$\left\{ y(x) = e^{-\frac{cx}{a}} \left( U \left( \frac{da^2-abc+c^2}{2a^3}, \frac{1}{2}, -\frac{(a^2x+ab-2c)^2}{2a^3} \right) \_C2 + M \left( \frac{da^2-abc+c^2}{2a^3}, \frac{1}{2}, -\frac{(a^2x+ab-2c)^2}{2a^3} \right) \_C1 \right) \right\}$$

2.1055 ODE No. 1055

---

$$(ax + b)y'(x) + y(x)(a1x^2 + b1x + c1) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.138688 (sec), leaf count = 421

$$\left\{ \left\{ y(x) \rightarrow c_1 \exp \left( \frac{-bx\sqrt{a^2-4a1} - \frac{1}{2}ax^2\sqrt{a^2-4a1} - \frac{1}{2}a^2x^2 - abx + 2a1x^2 + 2b1x}{2\sqrt{a^2-4a1}} \right) H_{\frac{-a^3+2c1a^2-\sqrt{a^2-4a1}a^2+4a1a^2}{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.136 (sec), leaf count = 262

$$\left\{ y(x) = e^{-\frac{x}{4} \left( (ax+2b)(a^2-4a1)^{\frac{3}{2}} + (a^2-4a1)(a^2x+2ab-4a1x-4b1) \right) (a^2-4a1)^{-\frac{3}{2}}} \left( \_C2 (a^2x+ab-4a1x-2b1) {}_1F_1 \left( \frac{1}{2}; \frac{3}{2}; \frac{(a^2x+ab-4a1x-2b1)^2}{4(a^2-4a1)} \right) + \_C1 \right) \right\}$$

**2.1056      ODE No. 1056**

---

$$x^2 \left(-y'(x)\right) + y''(x) + xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0247726 (sec), leaf count = 41

$$\left\{\left\{y(x) \rightarrow c_1 x - \frac{c_2 \sqrt[3]{-x^3} \Gamma\left(-\frac{1}{3}, -\frac{x^3}{3}\right)}{3 \sqrt[3]{3}}\right\}\right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 47

$$\left\{y(x) = \frac{1}{x^2} \left(e^{\frac{x^3}{3}} (-x^3)^{\frac{2}{3}} \sqrt[3]{3} {}_2C_2 + x^3 \left(-{}_2C_2 \Gamma\left(\frac{2}{3}\right) + {}_2C_2 \Gamma\left(\frac{2}{3}, -\frac{x^3}{3}\right) + {}_2C_1\right)\right)\right\}$$

**2.1057      ODE No. 1057**

---

$$x^2 \left(-y'(x)\right) + y''(x) - (x+1)^2 y(x) = 0$$

✓ **Mathematica** : cpu = 0.0342996 (sec), leaf count = 56

$$\left\{\left\{y(x) \rightarrow c_2 e^{\frac{x^3}{3} + x} \int_1^x e^{-\frac{1}{3} K[1]^3 - 2 K[1]} dK[1] + c_1 e^{\frac{x^3}{3} + x}\right\}\right\}$$

✓ **Maple** : cpu = 0.118 (sec), leaf count = 50

$$\left\{y(x) = {}_2C_1 \operatorname{HeunT}\left(0, -3, 2 \sqrt[3]{3}, \frac{3^{\frac{2}{3}} x}{3}\right) e^{-x} + {}_2C_2 \operatorname{HeunT}\left(0, 3, 2 \sqrt[3]{3}, -\frac{3^{\frac{2}{3}} x}{3}\right) e^{\frac{x(x^2+3)}{3}}\right\}$$

**2.1058      ODE No. 1058**

---

$$(x^4 - 2) xy(x) - (x+1)x^2 y'(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0334336 (sec), leaf count = 56

$$\left\{\left\{y(x) \rightarrow c_2 e^{\frac{x^3}{3}} \int_1^x e^{\frac{K[1]^4}{4} - \frac{K[1]^3}{3}} dK[1] + c_1 e^{\frac{x^3}{3}}\right\}\right\}$$

✓ **Maple** : cpu = 0.088 (sec), leaf count = 29

$$\left\{y(x) = e^{\frac{x^3}{3}} \left(\int e^{\frac{x^4}{4} - \frac{x^3}{3}} dx {}_2C_2 + {}_2C_1\right)\right\}$$

**2.1059      ODE No. 1059**

---

$$x^4 y'(x) - x^3 y(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0272441 (sec), leaf count = 39

$$\left\{\left\{y(x) \rightarrow c_1 x - \frac{c_2 \sqrt[5]{x^5} \Gamma\left(-\frac{1}{5}, \frac{x^5}{5}\right)}{5 \sqrt[5]{5}}\right\}\right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 56

$$\left\{y(x) = \frac{1}{x^7} \left(9 {}_2C_2 e^{-1/10 x^5} (x^5 + 4) M_{7/5, \frac{9}{10}}(1/5 x^5) + x^8 \left(x^2 {}_2C_2 e^{-\frac{x^5}{10}} M_{\frac{2}{5}, \frac{9}{10}}\left(\frac{x^5}{5}\right) + {}_2C_1\right)\right)\right\}$$

**2.1060      ODE No. 1060**

---

$$ax^{q-1} y'(x) + bx^{q-2} y(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0257178 (sec), leaf count = 83

$$\left\{\left\{y(x) \rightarrow c_2 q^{-1/q} a^{\frac{1}{q}} (x^q)^{\frac{1}{q}} {}_1F_1\left(\frac{b}{aq} + \frac{1}{q}; 1 + \frac{1}{q}; -\frac{ax^q}{q}\right) + c_1 {}_1F_1\left(\frac{b}{aq}; 1 - \frac{1}{q}; -\frac{ax^q}{q}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.122 (sec), leaf count = 81

$$\left\{y(x) = e^{-\frac{ax^q}{q}} x \left(M\left(\frac{aq-b}{aq}, \frac{q+1}{q}, \frac{ax^q}{q}\right) {}_2C_1 + U\left(\frac{aq-b}{aq}, \frac{q+1}{q}, \frac{ax^q}{q}\right) {}_2C_2\right)\right\}$$

2.1061 ODE No. 1061

---

$$-e^{-\frac{x^{3/2}}{3}}x + y''(x) + \sqrt{x}y'(x) + \left(\frac{x}{4} + \frac{1}{4\sqrt{x}} - 9\right)y(x) = 0$$

✓ **Mathematica** : cpu = 0.057806 (sec), leaf count = 70

$$\left\{\left\{y(x) \rightarrow c_1 e^{-\frac{1}{3}(\sqrt{x}+9)x} + \frac{1}{6}c_2 e^{6x-\frac{1}{3}(\sqrt{x}+9)x} - \frac{1}{9}e^{3x-\frac{1}{3}(\sqrt{x}+9)x}x\right\}\right\}$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 28

$$\left\{y(x) = -\frac{-9 \cosh(3x) - C1 - 9 \sinh(3x) - C2 + x}{9}e^{-\frac{1}{3}x^{\frac{3}{2}}}\right\}$$

2.1062 ODE No. 1062

---

$$\frac{(x + \sqrt{x} - 8)y(x)}{4x^2} + y''(x) - \frac{y'(x)}{\sqrt{x}} = 0$$

✓ **Mathematica** : cpu = 0.0190285 (sec), leaf count = 35

$$\left\{\left\{y(x) \rightarrow \frac{1}{3}c_2 e^{\sqrt{x}}x^2 + \frac{c_1 e^{\sqrt{x}}}{x}\right\}\right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 19

$$\left\{y(x) = \frac{C2 x^3 + -C1}{x}e^{\sqrt{x}}\right\}$$

2.1063 ODE No. 1063

---

$$y''(x) - (2e^x + 1)y'(x) + e^{2x}y(x) - e^{3x} = 0$$

✓ **Mathematica** : cpu = 0.0404552 (sec), leaf count = 28

$$\left\{\left\{y(x) \rightarrow c_1 e^{e^x} + c_2 e^{x+e^x} + e^x + 2\right\}\right\}$$

✓ **Maple** : cpu = 0.224 (sec), leaf count = 34

$$\left\{y(x) = e^{\frac{x}{2}+e^x} \sinh\left(\frac{x}{2}\right) - C2 + e^{\frac{x}{2}+e^x} \cosh\left(\frac{x}{2}\right) - C1 + e^x + 2\right\}$$

2.1064 ODE No. 1064

---

$$ay'(x) + by(x) + y''(x) + \tan(x) = 0$$

✓ **Mathematica** : cpu = 0.476844 (sec), leaf count = 1400

$$\left\{\left\{y(x) \rightarrow e^{\frac{1}{2}(-a-\sqrt{a^2-4b})x}c_1 + e^{\frac{1}{2}(\sqrt{a^2-4b}-a)x}c_2 + \frac{8\left({}_2F_1\left(1, \frac{1}{4}i\left(\sqrt{a^2-4b}-a\right); \frac{1}{4}i\left(\sqrt{a^2-4b}-a\right) + 1; -e^2\right)\right)}{e^{\frac{1}{2}(-a-\sqrt{a^2-4b})x}}\right\}\right\}$$

✓ **Maple** : cpu = 0.22 (sec), leaf count = 125

$$\left\{y(x) = e^{-\frac{x}{2}(a-\sqrt{a^2-4b})} - C2 + e^{-\frac{x}{2}(a+\sqrt{a^2-4b})} - C1 - 1 \left(\int \tan(x) e^{-\frac{x}{2}(-a+\sqrt{a^2-4b})} dx e^{x\sqrt{a^2-4b}} - \int \tan(x) e^{\frac{x}{2}}\right)\right\}$$

2.1065 ODE No. 1065

---

$$(n^2 - a^2)y(x) + 2n \cot(x)y'(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.122119 (sec), leaf count = 114

$$\left\{\left\{y(x) \rightarrow c_1 (\cos^2(x) - 1)^{\frac{1}{4}(1-2n)} P^{\frac{1}{2}(2n-1)}_{\frac{1}{2}(2\sqrt{2n^2-a^2}-1)}(\cos(x)) + c_2 (\cos^2(x) - 1)^{\frac{1}{4}(1-2n)} Q^{\frac{1}{2}(2n-1)}_{\frac{1}{2}(2\sqrt{2n^2-a^2}-1)}(\cos(x))\right\}\right\}$$

✓ **Maple** : cpu = 0.158 (sec), leaf count = 60

$$\left\{y(x) = (\sin(x))^{-n+\frac{1}{2}} \left(\text{LegendreP}\left(-\frac{1}{2} + \sqrt{-a^2 + 2n^2}, n - \frac{1}{2}, \cos(x)\right) - C1 + \text{LegendreQ}\left(-\frac{1}{2} + \sqrt{-a^2 + 2n^2}, n - \frac{1}{2}, \cos(x)\right) - C2\right)\right\}$$



**2.1066            ODE No. 1066**

---

$$y''(x) + \tan(x)y'(x) + y(x)\cos^2(x) = 0$$

✓ **Mathematica** : cpu = 0.0230192 (sec), leaf count = 18

$$\{\{y(x) \rightarrow c_2 \sin(\sin(x)) + c_1 \cos(\sin(x))\}\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 15

$$\{y(x) = \_C1 \sin(\sin(x)) + \_C2 \cos(\sin(x))\}$$

**2.1067            ODE No. 1067**

---

$$y''(x) + \tan(x)y'(x) - y(x)\cos^2(x) = 0$$

✓ **Mathematica** : cpu = 0.0230378 (sec), leaf count = 21

$$\{\{y(x) \rightarrow c_1 \cosh(\sin(x)) + i c_2 \sinh(\sin(x))\}\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 17

$$\left\{y(x) = \_C1 e^{\sin(x)} + \_C2 e^{-\sin(x)}\right\}$$

**2.1068            ODE No. 1068**

---

$$v(v+1)y(x) + y''(x) + \cot(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.104674 (sec), leaf count = 20

$$\{\{y(x) \rightarrow c_1 P_v(\cos(x)) + c_2 Q_v(\cos(x))\}\}$$

✓ **Maple** : cpu = 0.153 (sec), leaf count = 45

$$\left\{y(x) = \_C1 {}_2F_1\left(-\frac{v}{2}, \frac{1}{2} + \frac{v}{2}; \frac{1}{2}; (\cos(x))^2\right) + \_C2 \cos(x) {}_2F_1\left(1 + \frac{v}{2}, \frac{1}{2} - \frac{v}{2}; \frac{3}{2}; (\cos(x))^2\right)\right\}$$

**2.1069            ODE No. 1069**

---

$$y''(x) - \cot(x)y'(x) + y(x)\sin^2(x) = 0$$

✓ **Mathematica** : cpu = 0.0252014 (sec), leaf count = 19

$$\{\{y(x) \rightarrow c_1 \cos(\cos(x)) - c_2 \sin(\cos(x))\}\}$$

✓ **Maple** : cpu = 0.064 (sec), leaf count = 15

$$\{y(x) = \_C1 \sin(\cos(x)) + \_C2 \cos(\cos(x))\}$$

**2.1070            ODE No. 1070**

---

$$a \tan(x)y'(x) + by(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.234951 (sec), leaf count = 143

$$\left\{\left\{y(x) \rightarrow c_1 {}_2F_1\left(-\frac{a}{4} - \frac{1}{4}\sqrt{a^2 + 4b}, \frac{1}{4}\sqrt{a^2 + 4b} - \frac{a}{4}; \frac{1}{2} - \frac{a}{2}; \cos^2(x)\right) + i^{a+1} c_2 \cos^{a+1}(x) {}_2F_1\left(\frac{a}{4} - \frac{1}{4}\sqrt{a^2 + 4b}, \frac{1}{4}\sqrt{a^2 + 4b} - \frac{a}{4}; \frac{1}{2} + \frac{a}{2}; \cos^2(x)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.118 (sec), leaf count = 60

$$\left\{y(x) = (\cos(x))^{\frac{1}{2} + \frac{a}{2}} \left( LegendreQ\left(\frac{1}{2}\sqrt{a^2 + 4b} - \frac{1}{2}, \frac{1}{2} + \frac{a}{2}, \sin(x)\right) \_C2 + LegendreP\left(\frac{1}{2}\sqrt{a^2 + 4b} - \frac{1}{2}, \frac{1}{2} + \frac{a}{2}, \sin(x)\right) \_C1 \right)\right\}$$

2.1071 ODE No. 1071

---

$$(b^2 - a^2) y(x) + 2a \cot(ax) y'(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0597223 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-ibx} \csc(ax) - \frac{ic_2 e^{ibx} \csc(ax)}{2b} \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 24

$$\left\{ y(x) = \frac{C2 \cos(bx) + C1 \sin(bx)}{\sin(ax)} \right\}$$

2.1072 ODE No. 1072

---

$$y(x) (-4anp(x)^2 + a + bp(x)) + ap''(x)y'(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.27131 (sec), leaf count = 0 , could not solve

`DSolve[(a + b*p[x] - 4*a*n*p[x]^2)*y[x] + a*Derivative[1][y][x]*Derivative[2][p][x] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^2}{dx^2} Y(x) + a \left( \frac{d^2}{dx^2} p(x) \right) \frac{d}{dx} Y(x) + (a + bp(x) - 4na(p(x))^2) Y(x) \right\}, \{ Y(x) \} \right) \right\}$$

2.1073 ODE No. 1073

---

$$\frac{y'(x) (-\wp(x; a, b) \wp'(x; a, b) + \wp(x; a, b)^3 - 6\wp(x; a, b)^2 + \frac{a}{2})}{\wp'(x; a, b) - \wp(x; a, b)^2} + \frac{y(x) (\wp(x; a, b)^2 (-\wp'(x; a, b)) - (6\wp(x; a, b)^2 - \frac{a}{2}) \wp'(x; a, b))}{\wp(x; a, b)^2 + \wp'(x; a, b)}$$

✗ **Mathematica** : cpu = 1.1033 (sec), leaf count = 0 , could not solve

`DSolve[(-(WeierstrassP[x, {a, b}]*(-a/2 + 6*WeierstrassP[x, {a, b}]^2)) - WeierstrassP[x, {a, b}]^3 + 6*WeierstrassP[x, {a, b}]^2 + WeierstrassPPrime[x, {a, b}]) + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left( \frac{d^2}{dx^2} Y(x) + \frac{\frac{d}{dx} Y(x)}{WeierstrassPPrime(x, a, b) + (WeierstrassP(x, a, b))^2} \right) \left( 11 WeierstrassP(x, a, b) \right) \right) \right\}$$

2.1074 ODE No. 1074

---

$$\frac{k^2 \operatorname{cn}(x|k) \operatorname{sn}(x|k) y'(x)}{\operatorname{dn}(x|k)} + n^2 y(x) \operatorname{dn}(x|k)^2 + y''(x) = 0$$

✗ **Mathematica** : cpu = 4.1035 (sec), leaf count = 0 , could not solve

`DSolve[n^2*JacobiDN[x, k]^2*y[x] + (k^2*JacobiCN[x, k]*JacobiSN[x, k]*Derivative[1][y][x])/JacobiDN[x, k] + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.017 (sec), leaf count = 21

$$\{y(x) = C1 \sin(n \operatorname{JacobiAM}(x, k)) + C2 \cos(n \operatorname{JacobiAM}(x, k))\}$$

2.1075 ODE No. 1075

---

$$f(x)y'(x) + g(x)y(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.200222 (sec), leaf count = 0 , could not solve

`DSolve[g[x]*y[x] + f[x]*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ g(x) Y(x) + f(x) \frac{d}{dx} Y(x) + \frac{d^2}{dx^2} Y(x) \right\}, \{ Y(x) \} \right) \right\}$$

2.1076 ODE No. 1076

---

$$y(x) (a + f'(x)) + f(x)y'(x) - g(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.194879 (sec), leaf count = 0 , could not solve

`DSolve[-g[x] + y[x]*(a + Derivative[1][f][x]) + f[x]*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^2}{dx^2} Y(x) + f(x) \frac{d}{dx} Y(x) + \left( \frac{d}{dx} f(x) + a \right) Y(x) - g(x) \right\}, \{ Y(x) \} \right) \right\}$$

2.1077 ODE No. 1077

---

$$y'(x)(af(x) + b) + y(x)(cf(x) + d) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.324164 (sec), leaf count = 0 , could not solve  
DSolve[(d + c\*f[x])\*y[x] + (b + a\*f[x])\*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x)=DESol\left(\left\{\frac{d^2}{dx^2}Y(x)+(af(x)+b)\frac{d}{dx}Y(x)+(cf(x)+d)Y(x)\right\},\{Y(x)\}\right)\right\}$$

2.1078 ODE No. 1078

---

$$y(x)\left(a+\frac{f'(x)}{2}+\frac{f(x)^2}{4}\right)+f(x)y'(x)+y''(x)=0$$

✓ **Mathematica** : cpu = 0.0305373 (sec), leaf count = 76

$$\left\{\left\{y(x)\rightarrow c_1\exp\left(-\frac{1}{2}\int_1^xf(K[1])dK[1]-i\sqrt{a}x\right)-\frac{ic_2\exp\left(-\frac{1}{2}\int_1^xf(K[1])dK[1]+i\sqrt{a}x\right)}{2\sqrt{a}}\right\}\right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 33

$$\left\{y(x)=e^{-\frac{\int f(x)dx}{2}}\left(\sinh\left(\sqrt{-a}x\right)C1+\cosh\left(\sqrt{-a}x\right)C2\right)\right\}$$

2.1079 ODE No. 1079

---

$$by(x)f(x)^{2a}-\frac{af'(x)y'(x)}{f(x)}+y''(x)=0$$

✓ **Mathematica** : cpu = 0.192337 (sec), leaf count = 315

$$\left\{\left\{y(x)\rightarrow-\frac{\sqrt{c_1}\exp\left(-c_2-\int_1^x-i\sqrt{b}f(K[1])^adK[1]\right)\left(-1+\exp\left(2c_2+2\int_1^x-i\sqrt{b}f(K[1])^adK[1]\right)\right)}{\sqrt{2}}\right\},\left\{y(x)\right\}\right\}$$

✓ **Maple** : cpu = 0.015 (sec), leaf count = 37

$$\left\{y(x)=C1e^{\int i(f(x))^a\sqrt{b}dx}+C2e^{-\int i(f(x))^a\sqrt{b}dx}\right\}$$

2.1080 ODE No. 1080

---

$$y(x)\left(a^2+\frac{af'(x)}{f(x)}-b^2f(x)^2\right)-y'(x)\left(2a+\frac{f'(x)}{f(x)}\right)+y''(x)=0$$

✗ **Mathematica** : cpu = 0.334611 (sec), leaf count = 0 , could not solve  
DSolve[y[x]\*(a^2 - b^2\*f[x]^2 + (a\*Derivative[1][f][x])/f[x]) - (2\*a + Derivative[1][f][x]/f[x])\*y'[x] + y''[x] == 0, y[x]  
✓ **Maple** : cpu = 0.24 (sec), leaf count = 74

$$\left\{y(x)=e^{\int-1\left(\frac{f(x)\left(e^{-C1b}\right)^2_b}{\left(e^{\int f(x)dx}b\right)^2}+bf(x)-\frac{\left(e^{-C1b}\right)^2_a}{\left(e^{\int f(x)dx}b\right)^2}+a\right)\left(\frac{\left(e^{-C1b}\right)^2}{\left(e^{\int f(x)dx}b\right)^2}-1\right)^{-1}dx}C2\right\}$$

2.1081 ODE No. 1081

---

$$-\frac{a^2y(x)f'(x)^2}{b^2+f(x)^2}+\frac{f(x)f^3(x)y'(x)}{b^2+f(x)^2}+y''(x)=0$$

✗ **Mathematica** : cpu = 0.676623 (sec), leaf count = 0 , could not solve  
DSolve[-((a^2\*y[x]\*Derivative[1][f][x]^2)/(b^2 + f[x]^2)) + (f[x]\*(f^3[x]\*Derivative[1][y][x]) - y[x]\*Derivative[2][f][x])/f[x]^2 == 0, y[x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x)=DESol\left(\left\{\frac{d^2}{dx^2}Y(x)+\frac{f(x)\left(\frac{d^3}{dx^3}f(x)\right)\frac{d}{dx}Y(x)}{\left(f(x)\right)^2+b^2}-\frac{\left(\frac{d}{dx}f(x)\right)^2a^2Y(x)}{\left(f(x)\right)^2+b^2}\right\},\{Y(x)\}\right)\right\}$$

**2.1082                    ODE No. 1082**

---

$$y(x) \left( \frac{(m^2 - v^2) g'(x)^2}{g(x)} + g'(x)^2 \right) - y'(x) \left( \frac{(2m - 1)g'(x)}{g(x)} + \frac{g''(x)}{g'(x)} \right) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.530661 (sec), leaf count = 0 , could not solve

`DSolve[y[x]*(Derivative[1][g][x]^2 + ((m^2 - v^2)*Derivative[1][g][x]^2)/g[x]) - Derivative[1][y][x]*(2*m*Derivative[1][g][x])/g[x] + Derivative[2][g][x]/Derivative[1][g][x]) + Derivative[2][y][x] == 0, {y, g}, x]`

✓ **Maple** : cpu = 0.103 (sec), leaf count = 74

$$\left\{ y(x) = e^{-ig(x)} (g(x))^{2m} \left( U \left( \frac{i}{2}m^2 - \frac{i}{2}v^2 + m + \frac{1}{2}, 1 + 2m, 2ig(x) \right) {}_2F_2 + M \left( \frac{i}{2}m^2 - \frac{i}{2}v^2 + m + \frac{1}{2}, 1 + 2m, 2ig(x) \right) {}_2F_2 \right) \right\}$$

**2.1083                    ODE No. 1083**

---

$$-\frac{f'(x)y'(x)}{f(x)} + y(x) \left( -\frac{f''(x)}{2f(x)} + \frac{3f'(x)^2}{4f(x)^2} + \frac{(\frac{1}{4} - v^2)g'(x)^2}{g(x)^2} + g'(x)^2 + \frac{g^3(x)}{2g'(x)} - \frac{3g''(x)^2}{4g'(x)^2} \right) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.68243 (sec), leaf count = 0 , could not solve

`DSolve[-((Derivative[1][f][x]*Derivative[1][y][x])/f[x]) + y[x]*((3*Derivative[1][f][x]^2)/f[x]^3 - Derivative[2][f][x]/f[x]) + Derivative[2][y][x] == 0, {y, f}, x]`

✓ **Maple** : cpu = 0.072 (sec), leaf count = 31

$$\left\{ y(x) = \sqrt{\frac{f(x)g(x)}{\frac{d}{dx}g(x)}} (Y_v(g(x)) {}_2F_2 + J_v(g(x)) {}_2F_2) \right\}$$

**2.1084                    ODE No. 1084**

---

$$-y'(x) \left( \frac{2f'(x)}{f(x)} - \frac{g'(x)}{g(x)} + \frac{g''(x)}{g'(x)} \right) + y(x) \left( -\frac{f''(x)}{f(x)} + \frac{f'(x) \left( \frac{2f'(x)}{f(x)} - \frac{g'(x)}{g(x)} + \frac{g''(x)}{g'(x)} \right)}{f(x)} - \frac{v^2g'(x)^2}{g(x)^2} + g'(x)^2 \right) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.729259 (sec), leaf count = 0 , could not solve

`DSolve[-(Derivative[1][y][x]*((2*Derivative[1][f][x])/f[x] - Derivative[1][g][x]/g[x] + Derivative[2][g][x]/Derivative[1][g][x])) + y[x]*(v^2*Derivative[1][g][x]^2/g[x]^2 - Derivative[2][f][x]/f[x]) + Derivative[2][y][x] == 0, {y, f, g}, x]`

✓ **Maple** : cpu = 0.059 (sec), leaf count = 20

$$\{y(x) = f(x) (Y_v(g(x)) {}_2F_2 + J_v(g(x)) {}_2F_2)\}$$

**2.1085                    ODE No. 1085**

---

$$-y'(x) \left( \frac{(2v - 1)g'(x)}{g(x)} + \frac{g''(x)}{g'(x)} + \frac{2h'(x)}{h(x)} \right) + y(x) \left( g'(x)^2 + \frac{h'(x) \left( \frac{(2v-1)g'(x)}{g(x)} + \frac{g''(x)}{g'(x)} + \frac{2h'(x)}{h(x)} \right)}{h(x)} - \frac{h''(x)}{h(x)} \right) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.760194 (sec), leaf count = 0 , could not solve

`DSolve[-(Derivative[1][y][x]*(((2*v-1)*Derivative[1][g][x])/g[x] + (2*Derivative[1][h][x])/h[x] + Derivative[2][g][x]/Derivative[1][g][x]) + y[x]*(h'(x)*(2*(2*v-1)*Derivative[1][g][x])/g[x]^2 + Derivative[2][h][x]/h[x]) + Derivative[2][y][x] == 0, {y, g, h}, x]`

✓ **Maple** : cpu = 0.058 (sec), leaf count = 24

$$\{y(x) = h(x) (g(x))^v (Y_v(g(x)) {}_2F_2 + J_v(g(x)) {}_2F_2)\}$$

**2.1086                    ODE No. 1086**

---

$$4y''(x) + 9xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0040121 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow c_1 \text{Ai} \left( \sqrt[3]{-1} \left( \frac{3}{2} \right)^{2/3} x \right) + c_2 \text{Bi} \left( \sqrt[3]{-1} \left( \frac{3}{2} \right)^{2/3} x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 29

$$\left\{ y(x) = {}_2F_2 \text{Ai} \left( -\frac{3^{2/3} \sqrt[3]{2} x}{2} \right) + {}_2F_2 \text{Bi} \left( -\frac{3^{2/3} \sqrt[3]{2} x}{2} \right) \right\}$$

## 2.1087 ODE No. 1087

---

$$4y''(x) - (a + x^2)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0062891 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow c_1 D_{\frac{1}{4}(-a-2)}(x) + c_2 D_{\frac{a-2}{4}}(ix) \right\} \right\}$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 33

$$\left\{ y(x) = 1 \left( -C2 W_{-\frac{a}{8}, \frac{1}{4}} \left( \frac{x^2}{2} \right) + -C1 M_{-\frac{a}{8}, \frac{1}{4}} \left( \frac{x^2}{2} \right) \right) \frac{1}{\sqrt{x}} \right\}$$

## 2.1088 ODE No. 1088

---

$$4y''(x) + 4 \tan(x)y'(x) + y(x) (- (5 \tan^2(x) + 2)) = 0$$

✓ **Mathematica** : cpu = 0.0683472 (sec), leaf count = 180

$$\left\{ \left\{ y(x) \rightarrow -\frac{(-1)^{7/8} 2^{5/8} c_1}{\sqrt[8]{-8 \cos^2(2x) - 16 \cos(2x) - 8}} + \frac{3(-1)^{5/8} c_2 \left( 4 \sqrt[4]{-1} 2^{3/4} \sinh^{-1} \left( \frac{1}{2} \sqrt[4]{-\frac{1}{2}} \sqrt[4]{-8 \cos^2(2x) - 16 \cos(2x) - 8} \right) \right)}{8 \sqrt[8]{2} \sqrt[8]{-8 \cos^2(2x) - 16 \cos(2x) - 8}} \right\} \right\}$$

✓ **Maple** : cpu = 0.098 (sec), leaf count = 31

$$\left\{ y(x) = (i \cos(x) \sin(x) - C2 - \ln(\sin(x) + i \cos(x)) - C2 + -C1) \frac{1}{\sqrt{\cos(x)}} \right\}$$

## 2.1089 ODE No. 1089

---

$$-y'(x)(ab + c + x) + ay''(x) + y(x)(b(c + x) + d) = 0$$

✓ **Mathematica** : cpu = 0.0311281 (sec), leaf count = 99

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{bx} H_d \left( \frac{x}{\sqrt{2}\sqrt{a}} - \frac{ab - c}{\sqrt{2}\sqrt{a}} \right) + c_2 e^{bx} {}_1F_1 \left( -\frac{d}{2}; \frac{1}{2}; \left( \frac{x}{\sqrt{2}\sqrt{a}} - \frac{ab - c}{\sqrt{2}\sqrt{a}} \right)^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 58

$$\left\{ y(x) = e^{bx} \left( U \left( -\frac{d}{2}, \frac{1}{2}, \frac{(ab - c - x)^2}{2a} \right) - C2 + M \left( -\frac{d}{2}, \frac{1}{2}, \frac{(ab - c - x)^2}{2a} \right) - C1 \right) \right\}$$

## 2.1090 ODE No. 1090

---

$$a(a^2 - 2be^{-ax})y'(x) + a^2y''(x) + b^2e^{-2ax}y(x) = 0$$

✓ **Mathematica** : cpu = 0.0247447 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\frac{be^{-ax}}{a^2}} - \frac{bc_2 e^{-\frac{be^{-ax}}{a^2} - ax}}{a^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 40

$$\left\{ y(x) = e^{-\frac{a^3x + 2be^{-ax}}{2a^2}} \left( \sinh \left( \frac{ax}{2} \right) - C1 + \cosh \left( \frac{ax}{2} \right) - C2 \right) \right\}$$

## 2.1091 ODE No. 1091

---

$$x(y''(x) + y(x)) - \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.0175922 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} (\text{Ci}(2x) \sin(x) - \text{Si}(2x) \cos(x) + \log(x) \sin(x)) + c_2 \sin(x) + c_1 \cos(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 35

$$\left\{ y(x) = \frac{\sin(x) \text{Ci}(2x)}{2} - \frac{\text{Si}(2x) \cos(x)}{2} + \frac{(2 - C2 + \ln(x)) \sin(x)}{2} + \cos(x) - C1 \right\}$$

**2.1092                    ODE No. 1092**

---

$$(a+x)y(x)+xy''(x)=0$$

✓ **Mathematica** : cpu = 0.0663236 (sec), leaf count = 72

$$\left\{\left\{y(x)\rightarrow c_2e^{-ix}{}_1F_1\left(1-\frac{1}{4}i(-2(a-2)-4);2;2ix\right)+c_1e^{-ix}xU\left(1-\frac{1}{4}i(-2(a-2)-4),2,2ix\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 29

$$\left\{y\left(x\right)=-C1\,M_{-\frac{i}{2}a,\frac{1}{2}}\left(2\,ix\right)+-C2\,W_{-\frac{i}{2}a,\frac{1}{2}}\left(2\,ix\right)\right\}$$

**2.1093                    ODE No. 1093**

---

$$xy''(x)+y'(x)=0$$

✓ **Mathematica** : cpu = 0.006742 (sec), leaf count = 13

$$\left\{\left\{y(x)\rightarrow c_1\log(x)+c_2\right\}\right\}$$

✓ **Maple** : cpu = 0.004 (sec), leaf count = 10

$$\left\{y\left(x\right)=-C2\,\ln\left(x\right)+-C1\right\}$$

**2.1094                    ODE No. 1094**

---

$$ay(x)+xy''(x)+y'(x)=0$$

✓ **Mathematica** : cpu = 0.0161235 (sec), leaf count = 41

$$\left\{\left\{y(x)\rightarrow c_1J_0\left(2\sqrt{a}\sqrt{x}\right)+2c_2Y_0\left(2\sqrt{a}\sqrt{x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 29

$$\left\{y\left(x\right)=-C1\,J_0\left(2\,\sqrt{a}\sqrt{x}\right)+-C2\,Y_0\left(2\,\sqrt{a}\sqrt{x}\right)\right\}$$

**2.1095                    ODE No. 1095**

---

$$lxy(x)+xy''(x)+y'(x)=0$$

✓ **Mathematica** : cpu = 0.0066534 (sec), leaf count = 30

$$\left\{\left\{y(x)\rightarrow c_1J_0\left(\sqrt{l}x\right)+c_2Y_0\left(\sqrt{l}x\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 23

$$\left\{y\left(x\right)=-C1\,J_0\left(\sqrt{l}x\right)+-C2\,Y_0\left(\sqrt{l}x\right)\right\}$$

**2.1096                    ODE No. 1096**

---

$$(a+x)y(x)+xy''(x)+y'(x)=0$$

✓ **Mathematica** : cpu = 0.0089645 (sec), leaf count = 61

$$\left\{\left\{y(x)\rightarrow c_1e^{-ix}U\left(\frac{1}{2}i(a-i),1,2ix\right)+c_2e^{-ix}L_{-\frac{1}{2}i(a-i)}(2ix)\right\}\right\}$$

✓ **Maple** : cpu = 0.055 (sec), leaf count = 39

$$\left\{y\left(x\right)=e^{-ix}\left(U\left(\frac{1}{2}+\frac{i}{2}a,1,2\,ix\right)-C2+M\left(\frac{1}{2}+\frac{i}{2}a,1,2\,ix\right)-C1\right)\right\}$$

**2.1097                    ODE No. 1097**

---

$$ay(x)+xy''(x)-y'(x)=0$$

✓ **Mathematica** : cpu = 0.0195839 (sec), leaf count = 46

$$\left\{\left\{y(x)\rightarrow 2ac_1xJ_2\left(2\sqrt{a}\sqrt{x}\right)-2ac_2xY_2\left(2\sqrt{a}\sqrt{x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 31

$$\left\{y\left(x\right)=x\left(Y_2\left(2\,\sqrt{a}\sqrt{x}\right)-C2+J_2\left(2\,\sqrt{a}\sqrt{x}\right)-C1\right)\right\}$$

**2.1098                    ODE No. 1098**

---

$$-ax^3y(x) + xy''(x) - y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0070218 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow c_1 \cosh \left( \frac{\sqrt{a}x^2}{2} \right) + ic_2 \sinh \left( \frac{\sqrt{a}x^2}{2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 27

$$\left\{ y(x) = \_C1 \sinh \left( \frac{x^2}{2} \sqrt{a} \right) + \_C2 \cosh \left( \frac{x^2}{2} \sqrt{a} \right) \right\}$$

**2.1099                    ODE No. 1099**

---

$$x^3 \left( e^{x^3} - v^2 \right) y(x) + xy''(x) - y'(x) = 0$$

✗ **Mathematica** : cpu = 0.88984 (sec), leaf count = 0 , could not solve

DSolve[(E^x^3 - v^2)\*x^3\*y[x] - Derivative[1][y][x] + x\*Derivative[2][y][x] == 0, y[x], x]

✓ **Maple** : cpu = 0.038 (sec), leaf count = 25

$$\left\{ y(x) = \_C1 J_v \left( e^{\frac{x^2}{2}} \right) + \_C2 Y_v \left( e^{\frac{x^2}{2}} \right) \right\}$$

**2.1100                    ODE No. 1100**

---

$$xy''(x) + 2y'(x) - xy(x) - e^x = 0$$

✓ **Mathematica** : cpu = 0.0142998 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-x}}{x} + \frac{c_2 e^x}{2x} + \frac{e^x(2x-1)}{4x} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 23

$$\left\{ y(x) = \frac{\sinh(x) \_C2}{x} + \frac{\cosh(x) \_C1}{x} + \frac{e^x}{2} \right\}$$

**Hand solution**

$$xy'' + 2y' - xy = e^x \tag{1}$$

First method, much shorter, using transformation. Let  $y_h = \frac{u(x)}{x}$ , hence (now we are solving only the homogeneous part).

$$\begin{aligned} y' &= \frac{u'}{x} - \frac{u}{x^2} \\ y'' &= \frac{u''}{x} - \frac{u'}{x^2} - \frac{u'}{x^2} + 2\frac{u}{x^3} \end{aligned}$$

And (1) becomes

$$\begin{aligned} x \left( \frac{u''}{x} - \frac{u'}{x^2} - \frac{u'}{x^2} + 2\frac{u}{x^3} \right) + 2 \left( \frac{u'}{x} - \frac{u}{x^2} \right) - x \left( \frac{u}{x} \right) &= 0 \\ u'' - 2\frac{u'}{x} + 2\frac{u}{x^2} + \frac{2u'}{x} - \frac{2u}{x^2} - u &= 0 \\ u'' - u &= 0 \end{aligned}$$

Hence the roots of the characteristic equation are  $\pm 1$  and the solution is

$$u = Ae^x + Be^{-x}$$

Hence

$$y_h = \frac{1}{x} (Ae^x + Be^{-x})$$

The particular solution is found below, and given in the second method. The transformation method is much simpler.

The second method, which is much longer, using series method. This is used if a transformation is not known or can not be found. There is singularity at  $x = 0$ . We need to check if the singularity is regular or not. Writing in standard form  $y'' + p(x)y' + q(x)y = 0$  gives (we are looking at the homogeneous part now only)

$$y'' + \frac{2}{x}y' - y = 0$$

Hence  $\lim_{x \rightarrow 0} xp(x) = \lim_{x \rightarrow 0} x \frac{2}{x} = 2$  which is analytic at  $x = 0$ . And  $\lim_{x \rightarrow 0} x^2 q(x) = \lim_{x \rightarrow 0} -x^2 = 0$  which is analytic. Hence the singularity is regular (removable). Using Frobenius series, assume that

$$y = \sum_{n=-\infty}^{\infty} c_n x^{n+r}$$

Where  $c_n = 0$  for  $n < 0$ . Hence

$$\begin{aligned} y' &= \sum (n+r) c_n x^{n+r-1} \\ y'' &= \sum (n+r)(n+r-1) c_n x^{n+r-2} \end{aligned}$$

Substituting back in the original ODE gives

$$\sum (n+r)(n+r-1) c_n x^{n+r-1} + \sum 2(n+r) c_n x^{n+r-1} - \sum c_n x^{n+r+1} = 0$$

Adjusting so that all have same power  $x^{n+r}$  gives

$$\sum (n+r+1)(n+r) c_{n+1} x^{n+r} + \sum 2(n+r+1) c_{n+1} x^{n+r} - \sum c_{n-1} x^{n+r} = 0$$

Hence

$$\begin{aligned} (n+r+1)(n+r) c_{n+1} + 2(n+r+1) c_{n+1} - c_{n-1} &= 0 \\ (n+r+1)(2+(n+r)) c_{n+1} - c_{n-1} &= 0 \end{aligned} \quad (2)$$

We want equation with  $c_0$  in it. Hence let  $n = -1$

$$(-1+r+1)(2+(-1+r)) c_0 - c_{-2} = 0$$

But  $c_n = 0$  for all  $n < 0$  hence

$$(-1+r+1)(2+(-1+r)) c_0 = 0$$

But  $c_0 \neq 0$ , as this is the basis for this method. Therefore, we obtain the indicial equation for  $r$

$$\begin{aligned} (-1+r+1)(2+(-1+r)) &= 0 \\ r(r+1) &= 0 \end{aligned}$$

Hence  $r = 0$  or  $r = -1$  are the roots. Now for each  $r$  we find a solution. Using  $r = 0$ , we go back the recurrence equation (2)

$$\begin{aligned} (n+1)(2+n) c_{n+1} - c_{n-1} &= 0 \\ c_{n+1} &= \frac{c_{n-1}}{(n+1)(2+n)} \end{aligned}$$

For  $n = 0$

$$c_1 = \frac{c_{-1}}{(n+1)(2+n)} = 0$$

For  $n = 1$

$$c_2 = \frac{c_0}{(2)(3)}$$

For  $n = 2$

$$c_3 = \frac{c_1}{(n+1)(2+n)} = 0$$

For  $n = 3$

$$c_4 = \frac{c_2}{(4)(5)} = \frac{c_0}{(2)(3)(4)(5)}$$

And so on. Hence, for  $r = 0$  we have

$$\begin{aligned} y_{r=0} &= \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x^1 + c_2 x^2 + c_3 x^3 + \dots \\ &= c_0 + \frac{c_0}{6} x^2 + \frac{c_0}{120} x^4 + \dots \\ &= A \left( 1 + \frac{1}{6} x^2 + \frac{1}{120} x^4 + \dots \right) \end{aligned} \quad (3)$$



Where  $A$  is used as arbitrary constant instead of  $a_0$ . Now we find the solution for  $r = -1$ . we go back the recurrence equation (2)

$$\begin{aligned}(n-1+1)(2+(n-1))c_{n+1} - c_{n-1} &= 0 \\ n(1+n)c_{n+1} - c_{n-1} &= 0 \\ c_{n+1} &= \frac{c_{n-1}}{n(1+n)}\end{aligned}$$

For  $n = 0$

$$c_1 = \frac{c_{-1}}{n(1+n)} = 0$$

For  $n = 1$

$$c_2 = \frac{c_0}{2}$$

For  $n = 2$

$$c_3 = \frac{c_1}{n(1+n)} = 0$$

For  $n = 3$

$$c_4 = \frac{c_2}{(3)(4)} = \frac{c_0}{(2)(3)(4)}$$

For  $n = 4$

$$c_5 = \frac{c_3}{n(1+n)} = 0$$

For  $n = 5$

$$c_6 = \frac{c_4}{(5)(6)} = \frac{c_0}{(2)(3)(4)(5)(6)}$$

And so on. Hence solution is

$$\begin{aligned}y_{r=-1} &= \frac{1}{x} \sum_{n=0}^{\infty} c_n x^n = \frac{1}{x} \left( c_0 + \frac{c_0}{2} x^2 + \frac{c_0}{(2)(3)(4)} x^4 + \frac{c_0}{(2)(3)(4)(5)(6)} x^6 + \dots \right) \\ &= \frac{B}{x} \left( 1 + \frac{1}{2} x^2 + \frac{1}{24} x^4 + \frac{1}{720} x^6 + \dots \right)\end{aligned}$$

Where  $B$  is used as arbitrary constant instead of  $a_0$ . Therefore, the homogeneous solution found is

$$\begin{aligned}y_h &= y_{r=0} + y_{r=-1} \\ &= A \left( 1 + \frac{1}{6} x^2 + \frac{1}{120} x^4 + \dots \right) + \frac{B}{x} \left( 1 + \frac{1}{2} x^2 + \frac{1}{24} x^4 + \frac{1}{720} x^6 + \dots \right)\end{aligned}$$

But

$$e^x = 1 + x + \frac{1}{2} x^2 + \frac{1}{6} x^3 + \frac{1}{24} x^4 + \frac{1}{120} x^5 + \dots \quad (3)$$

And

$$e^{-x} = 1 - x + \frac{1}{2} x^2 - \frac{1}{6} x^3 + \frac{1}{24} x^4 - \frac{1}{120} x^5 + \dots \quad (4)$$

Hence adding (3)+(4) gives

$$\begin{aligned}e^x + e^{-x} &= 2 + 2 \frac{1}{2} x^2 + 2 \frac{1}{24} x^4 + \dots \\ &= 2 \left( 1 + \frac{1}{2} x^2 + \frac{1}{24} x^4 + \frac{1}{720} x^6 \dots \right)\end{aligned}$$

But  $y_{r=-1} = \frac{B}{x} \left( 1 + \frac{1}{2} x^2 + \frac{1}{24} x^4 + \frac{1}{720} x^6 + \dots \right)$ , therefore comparing the result we found above, we see that we can write  $y_{r=-1}$  as

$$y_{r=-1} = \frac{B}{x} \left( \frac{e^x + e^{-x}}{2} \right)$$

Similarly, we obtain  $y_{r=0}$  expression

$$\begin{aligned}\frac{1}{x} e^x &= \frac{1}{x} \left( 1 + x + \frac{1}{2} x^2 + \frac{1}{6} x^3 + \frac{1}{24} x^4 + \frac{1}{120} x^5 + \dots \right) \\ &= \frac{1}{x} + 1 + \frac{1}{2} x + \frac{1}{6} x^2 + \frac{1}{24} x^3 + \frac{1}{120} x^4 + \dots\end{aligned} \quad (3A)$$

And

$$\begin{aligned}\frac{1}{x} e^{-x} &= \frac{1}{x} \left( 1 - x + \frac{1}{2} x^2 - \frac{1}{6} x^3 + \frac{1}{24} x^4 - \frac{1}{120} x^5 + \dots \right) \\ &= \frac{1}{x} - 1 + \frac{1}{2} x - \frac{1}{6} x^2 + \frac{1}{24} x^3 - \frac{1}{120} x^4 + \dots\end{aligned} \quad (4A)$$

Now (3A)-(4A) gives

$$\begin{aligned}\frac{1}{x}e^x - \frac{1}{x}e^{-x} &= \left(\frac{1}{x} + 1 + \frac{1}{2}x + \frac{1}{6}x^2 + \frac{1}{24}x^3 + \frac{1}{120}x^4 + \dots\right) - \left(\frac{1}{x} - 1 + \frac{1}{2}x - \frac{1}{6}x^2 + \frac{1}{24}x^3 - \frac{1}{120}x^4 + \dots\right) \\ &= 2 + 2\frac{1}{6}x^2 + 2\frac{1}{120}x^4 + \dots \\ &= 2\left(1 + \frac{1}{6}x^2 + \frac{1}{120}x^4 + \dots\right)\end{aligned}$$

Hence

$$\left(1 + \frac{1}{6}x^2 + \frac{1}{120}x^4 + \dots\right) = \frac{1}{2x}(e^x - e^{-x})$$

But  $y_{r=0} = A\left(1 + \frac{1}{6}x^2 + \frac{1}{120}x^4 + \dots\right)$ , therefore comparing the result we found above, we see that we can write  $y_{r=0}$  as

$$\begin{aligned}y_{r=0} &= A\left(\frac{1}{2x}(e^x - e^{-x})\right) \\ &= \frac{A}{2x}(e^x - e^{-x})\end{aligned}$$

Therefore

$$\begin{aligned}y_h &= y_{r=0} + y_{r=-1} \\ &= \frac{A}{2x}(e^x - e^{-x}) + \frac{B}{2x}(e^x + e^{-x}) \\ &= \frac{1}{x}\left(\frac{A}{2}e^x - \frac{A}{2}e^{-x} + \frac{B}{2}e^x + \frac{B}{2}e^{-x}\right) \\ &= \frac{1}{x}\left(e^x\left(\frac{A}{2} + \frac{B}{2}\right) + e^{-x}\left(-\frac{A}{2} + \frac{B}{2}\right)\right)\end{aligned}$$

Let  $\frac{A+B}{2} = A_0$ ,  $\frac{B-A}{2} = B_0$  hence the above becomes

$$y_h = \frac{1}{x}(A_0e^x + B_0e^{-x})$$

We see this is the same solution using the transformation method given above. We now need to find particular solution. Let  $y_1 = \frac{e^x}{x}$ ,  $y_2 = \frac{e^{-x}}{x}$ , hence  $y_1' = \frac{e^x}{x} - \frac{e^x}{x^2}$  and  $y_2' = \frac{-e^{-x}}{x^2} - \frac{e^{-x}}{x}$ , hence the Wronskian is

$$\begin{aligned}W &= \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \\ &= \begin{vmatrix} \frac{e^x}{x} & \frac{e^{-x}}{x} \\ \frac{e^x}{x} - \frac{e^x}{x^2} & \frac{-e^{-x}}{x^2} - \frac{e^{-x}}{x} \end{vmatrix} \\ &= \frac{e^x}{x}\left(\frac{-e^{-x}}{x^2} - \frac{e^{-x}}{x}\right) - \frac{e^{-x}}{x}\left(\frac{e^x}{x} - \frac{e^x}{x^2}\right) \\ &= \left(\frac{-1}{x^3} - \frac{1}{x^2}\right) - \left(\frac{1}{x^2} - \frac{1}{x^3}\right) \\ &= -\frac{2}{x^2}\end{aligned}$$

Therefore, let  $y_p = u_1y_1 + u_2y_2$  and hence

$$u_1 = -\int \frac{y_2}{aW}e^x dx$$

Where  $a = x$  since the original ODE is  $xy'' + 2y' - xy = e^x$ , and  $a$  is the coefficient of  $y''$  always. Hence the above becomes

$$u_1 = -\int \frac{\frac{e^{-x}}{x}}{x\left(-\frac{2}{x^2}\right)}e^x dx = \int \frac{e^{-x}}{2}e^x dx = \int \frac{1}{2}dx = \frac{x}{2}$$

And

$$u_2 = \int \frac{\frac{e^x}{x}}{x\left(-\frac{2}{x^2}\right)}e^x dx = -\int \frac{\frac{e^x}{x}}{\frac{2}{x}}e^x dx = -\frac{1}{2}\int e^{2x} dx = -\frac{1}{2}\frac{e^{2x}}{2} = -\frac{1}{4}e^{2x}$$

Hence

$$\begin{aligned}y_p &= u_1y_1 + u_2y_2 \\ &= \frac{x}{2}\frac{e^x}{x} - \frac{1}{4}e^{2x}\frac{e^{-x}}{x} \\ &= \frac{1}{2}e^x - \frac{1}{4x}e^x\end{aligned}$$

Therefore

$$y_p = e^x \left( \frac{1}{2} - \frac{1}{4x} \right)$$

Hence the general solution is

$$\begin{aligned} y &= y_h + y_p \\ &= \frac{1}{x} (A_0 e^x + B_0 e^{-x}) + e^x \left( \frac{1}{2} - \frac{1}{4x} \right) \end{aligned}$$

Verification

```
1 restart;
2 ode:=x*diff(diff(y(x),x),x)+2*diff(y(x),x)-x*y(x)=exp(x);
3 y0:=1/x*( _C1* exp(x)+ _C2*exp(-x))+ (1/2-1/(4*x))*exp(x);
4 odetest(y(x)=y0,ode);
5 0
```

2.1101 ODE No. 1101

$$axy(x) + xy''(x) + 2y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0179366 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-i\sqrt{a}x}}{x} - \frac{ic_2 e^{i\sqrt{a}x}}{2\sqrt{a}x} \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 29

$$\left\{ y(x) = \frac{1}{x} \left( \_C1 \sinh(\sqrt{-a}x) + \_C2 \cosh(\sqrt{-a}x) \right) \right\}$$

Hand solution

$$xy'' + 2y' + axy = 0 \tag{1}$$

First method much shorter, using transformation suggested by Kamke. Let  $y = \frac{u(x)}{x}$ , hence

$$\begin{aligned} y' &= \frac{u'}{x} - \frac{u}{x^2} \\ y'' &= \frac{u''}{x} - \frac{u'}{x^2} - \frac{u'}{x^2} + 2\frac{u}{x^3} \end{aligned}$$

And (1) becomes

$$\begin{aligned} x \left( \frac{u''}{x} - \frac{u'}{x^2} - \frac{u'}{x^2} + 2\frac{u}{x^3} \right) + 2 \left( \frac{u'}{x} - \frac{u}{x^2} \right) + ax \left( \frac{u}{x} \right) &= 0 \\ u'' - 2\frac{u'}{x} + 2\frac{u}{x^2} + \frac{2u'}{x} - \frac{2u}{x^2} + au &= 0 \\ u'' + au &= 0 \end{aligned}$$

Hence the roots of the characteristic equation are  $\pm\sqrt{-a}$  and the solution is

$$u = A \cos(\sqrt{a}x) + B \sin(\sqrt{a}x)$$

Hence

$$y = \frac{1}{x} (A \cos(\sqrt{a}x) + B \sin(\sqrt{a}x))$$

Second method Using series method.

There is singularity at  $x = 0$ . We need to check if it regular or not. Writing in standard form  $y'' + p(x)y' + q(x)y = 0$  gives (we are looking at the homogeneous part now only)

$$y'' + \frac{2}{x}y' + ay = 0$$

Hence  $\lim_{x \rightarrow 0} xp(x) = \lim_{x \rightarrow 0} x \frac{2}{x} = 2$  which is analytic at  $x = 0$ . And  $\lim_{x \rightarrow 0} x^2 q(x) = \lim_{x \rightarrow 0} -ax^2 = 0$  which is analytic. Hence the singularity is regular (removable). Using Frobenius series, assume that

$$y = \sum_{n=-\infty}^{\infty} c_n x^{n+r}$$

Where  $c_n = 0$  for  $n < 0$ . Hence

$$\begin{aligned} y' &= \sum (n+r) c_n x^{n+r-1} \\ y'' &= \sum (n+r)(n+r-1) c_n x^{n+r-2} \end{aligned}$$

Substituting back in the original ODE gives

$$\sum (n+r)(n+r-1) c_n x^{n+r-1} + \sum 2(n+r) c_n x^{n+r-1} + \sum a c_n x^{n+r+1} = 0$$

Adjusting so that all have same power  $x^{n+r}$  gives

$$\sum (n+r+1)(n+r) c_{n+1} x^{n+r} + \sum 2(n+r+1) c_{n+1} x^{n+r} + \sum a c_{n-1} x^{n+r} = 0$$

Hence

$$\begin{aligned} (n+r+1)(n+r) c_{n+1} + 2(n+r+1) c_{n+1} + a c_{n-1} &= 0 \\ (n+r+1)(2+(n+r)) c_{n+1} + a c_{n-1} &= 0 \end{aligned} \quad (2)$$

We want equation with  $c_0$  in it. Hence let  $n = -1$

$$(-1+r+1)(2+(-1+r)) c_0 + a c_{-2} = 0$$

But  $c_n = 0$  for all  $n < 0$  hence

$$(-1+r+1)(2+(-1+r)) c_0 = 0$$

But  $c_0 \neq 0$ , as this is the basis for this method. Therefore, we obtain the indicial equation for  $r$

$$\begin{aligned} (-1+r+1)(2+(-1+r)) &= 0 \\ r(r+1) &= 0 \end{aligned}$$

Hence  $r = 0$  or  $r = -1$  are the roots. Now for each  $r$  we find a solution. Using  $r = 0$ , we go back the recurrence equation (2)

$$\begin{aligned} (n+1)(2+n) c_{n+1} + a c_{n-1} &= 0 \\ c_{n+1} &= \frac{-a c_{n-1}}{(n+1)(2+n)} \end{aligned}$$

For  $n = 0$

$$c_1 = \frac{-a c_{-1}}{(n+1)(2+n)} = 0$$

For  $n = 1$

$$c_2 = \frac{-a c_0}{(2)(3)}$$

For  $n = 2$

$$c_3 = \frac{-a c_1}{(n+1)(2+n)} = 0$$

For  $n = 3$

$$c_4 = \frac{-a c_2}{(4)(5)} = \frac{-a(-a c_0)}{(2)(3)(4)(5)} = \frac{a^2 c_0}{(2)(3)(4)(5)}$$

And so on. Hence, for  $r = 0$  we have

$$\begin{aligned} y_{r=0} &= \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x^1 + c_2 x^2 + c_3 x^3 + \dots \\ &= c_0 - a \frac{c_0}{6} x^2 + a^2 \frac{c_0}{120} x^4 + \dots \\ &= A \left( 1 - a \frac{1}{6} x^2 + a^2 \frac{1}{120} x^4 + \dots \right) \end{aligned} \quad (3)$$

Where  $A$  is used as arbitrary constant instead of  $a_0$ . Now we find the solution for  $r = -1$ . we go back the recurrence equation (2)

$$\begin{aligned} (n-1+1)(2+(n-1)) c_{n+1} + a c_{n-1} &= 0 \\ n(1+n) c_{n+1} + a c_{n-1} &= 0 \\ c_{n+1} &= \frac{-a c_{n-1}}{n(1+n)} \end{aligned}$$

For  $n = 0$

$$c_1 = \frac{-ac_{-1}}{n(1+n)} = 0$$

For  $n = 1$

$$c_2 = \frac{-ac_0}{2}$$

For  $n = 2$

$$c_3 = \frac{-ac_1}{n(1+n)} = 0$$

For  $n = 3$

$$c_4 = \frac{-ac_2}{(3)(4)} = \frac{-a(-ac_0)}{(2)(3)(4)} = \frac{a^2c_0}{(2)(3)(4)}$$

For  $n = 4$

$$c_5 = \frac{-ac_3}{n(1+n)} = 0$$

For  $n = 5$

$$c_6 = \frac{-ac_4}{(5)(6)} = \frac{-a(a^2c_0)}{(2)(3)(4)(5)(6)} = \frac{-a^3c_0}{(2)(3)(4)(5)(6)}$$

And so on. Hence solution is

$$\begin{aligned} y_{r=-1} &= \frac{1}{x} \sum_{n=0}^{\infty} c_n x^n = \frac{1}{x} \left( c_0 - \frac{ac_0}{2} x^2 + \frac{a^2c_0}{(2)(3)(4)} x^4 - \frac{a^3c_0}{(2)(3)(4)(5)(6)} x^6 + \dots \right) \\ &= \frac{B}{x} \left( 1 - \frac{a}{2} x^2 + \frac{a^2}{24} x^4 - \frac{a^3}{720} x^6 + \dots \right) \end{aligned}$$

Where  $B$  is used as arbitrary constant instead of  $a_0$ . Therefore, the homogeneous solution found is

$$y_h = y_{r=0} + y_{r=-1}$$

$$\begin{aligned} &= A \left( 1 - \frac{a}{6} x^2 + \frac{a^2}{120} x^4 + \dots \right) + \frac{B}{x} \left( 1 - \frac{a}{2} x^2 + \frac{a^2}{24} x^4 - \frac{a^3}{720} x^6 + \dots \right) \\ &= A \left( 1 - \frac{1}{6} (\sqrt{ax})^2 + \frac{1}{120} (\sqrt{ax})^4 + \dots \right) + \frac{B}{x} \left( 1 - \frac{1}{2} (\sqrt{ax})^2 + \frac{1}{24} (\sqrt{ax})^4 - \frac{1}{720} (\sqrt{ax})^6 + \dots \right) \end{aligned}$$

But

$$\sin x = x - \frac{1}{6} x^3 + \frac{1}{120} x^5 - \dots \quad (3)$$

And

$$\cos x = 1 - \frac{1}{2} x^2 + \frac{1}{24} x^4 - \frac{1}{720} x^6 + \dots \quad (4)$$

Therefore

$$\begin{aligned} \sin(\sqrt{ax}) &= \sqrt{ax} - \frac{1}{6} (\sqrt{ax})^3 + \frac{1}{120} (\sqrt{ax})^5 - \dots \\ \cos(\sqrt{ax}) &= 1 - \frac{1}{2} (\sqrt{ax})^2 + \frac{1}{24} (\sqrt{ax})^4 - \frac{1}{720} (\sqrt{ax})^6 + \dots \end{aligned}$$

Therefore, using the above, we can write  $y_h$  as

$$\begin{aligned} y_h &= \frac{A}{\sqrt{ax}} \sqrt{ax} \left( 1 - \frac{1}{6} (\sqrt{ax})^2 + \frac{1}{120} (\sqrt{ax})^4 + \dots \right) + \frac{B}{x} \cos(\sqrt{ax}) \\ &= \frac{A}{\sqrt{ax}} \left( \sqrt{ax} - \frac{1}{6} (\sqrt{ax})^3 + \frac{1}{120} (\sqrt{ax})^5 + \dots \right) + \frac{B}{x} \cos(\sqrt{ax}) \\ &= \frac{A}{\sqrt{ax}} \sin(\sqrt{ax}) + \frac{B}{x} \cos(\sqrt{ax}) \end{aligned}$$

Let  $A_0 = \frac{A}{\sqrt{a}}$ , hence the above becomes the same solution found using the transformation method

$$y(x) = \frac{1}{x} (A_0 \sin(\sqrt{ax}) + B \cos(\sqrt{ax}))$$

Clearly, the transformation method is much faster and better. But the trick is to see the correct transformation needed and this is not always easy.

Third method Using Laplace transform. Using property  $\mathcal{L}xf(x) = -\frac{d}{ds}F(s)$  then the Laplace transform of the ODE becomes

$$\begin{aligned} \mathcal{L}(xy'' + 2y' + axy) &= 0 \\ -\frac{d}{ds}(\mathcal{L}y'') + 2\left(-\frac{d}{ds}(\mathcal{L}y')\right) + a\left(-\frac{d}{ds}Y\right) &= 0 \\ -\frac{d}{ds}(s^2Y - sA - B) + 2\left(-\frac{d}{ds}(sY - A)\right) + a(-Y') &= 0 \end{aligned}$$

Where  $A = y(0), B = y'(0)$ . Simplifying gives

$$\begin{aligned} - (2sY + s^2Y' - A) - 2(Y + sY') - aY' &= 0 \\ Y'(-s^2 - 2s - a) + Y(-2s - 2) + A &= 0 \\ Y' + Y \frac{2s + 2}{s^2 + 2s + a} &= \frac{A}{s^2 + 2s + a} \end{aligned}$$

This is first order ODE, which is solved easily using an integrating factor. Solving for  $Y(s)$  gives

$$Y(s) = \frac{As + c_1}{s^2 + a + 2s}$$

Where  $c_1$  is constant of integration. The inverse Laplace of the above is

$$y(x) = e^{-x} \left( A \cos(x\sqrt{1-a}) + \frac{(-A + c_1)}{\sqrt{1-a}} \sinh(x\sqrt{1-a}) \right)$$

I need to find why the above does not verify. May be I made a mistake somewhere. Verification of the result of the first two methods is below.

```

1 restart;
2 ode:=x*diff(diff(y(x),x),x)+2*diff(y(x),x)+a*x*y(x)=0;
3 y0:=1/x*( _C1* cos(sqrt(a)*x)+ _C2*sin(sqrt(a)*x));
4 odetest(y(x)=y0,ode);
5 0

```

## 2.1102 ODE No. 1102

$$ax^2y(x) + xy''(x) + 2y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0050941 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 \text{Ai}\left(-\frac{ax}{(-a)^{2/3}}\right)}{x} + \frac{c_2 \text{Bi}\left(-\frac{ax}{(-a)^{2/3}}\right)}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.028 (sec), leaf count = 33

$$\left\{ y(x) = 1 \left( -C2 Y_{\frac{1}{3}} \left( \frac{2}{3} \sqrt{a} x^{\frac{3}{2}} \right) + -C1 J_{\frac{1}{3}} \left( \frac{2}{3} \sqrt{a} x^{\frac{3}{2}} \right) \right) \frac{1}{\sqrt{x}} \right\}$$

Hand solution

$$xy'' + 2y' + ax^2y = 0 \tag{1}$$

Since there is a term  $2y$ , we can use  $y = \frac{u(x)}{x}$ , hence

$$\begin{aligned} y' &= \frac{u'}{x} - \frac{u}{x^2} \\ y'' &= \frac{u''}{x} - \frac{u'}{x^2} - \frac{u'}{x^2} + 2\frac{u}{x^3} \end{aligned}$$

And (1) becomes

$$\begin{aligned} x \left( \frac{u''}{x} - \frac{u'}{x^2} - \frac{u'}{x^2} + 2\frac{u}{x^3} \right) + 2 \left( \frac{u'}{x} - \frac{u}{x^2} \right) + ax^2 \left( \frac{u}{x} \right) &= 0 \\ u'' - 2\frac{u'}{x} + 2\frac{u}{x^2} + \frac{2u'}{x} - \frac{2u}{x^2} + axu &= 0 \\ u'' + axu &= 0 \end{aligned} \tag{2}$$

This is Emdon-Fowler. (form is  $u'' + x^n u = 0$ ) with  $n = 1$ . Assume that

$$u = \sum_{n=0}^{\infty} c_n x^n$$

Hence

$$\begin{aligned} u' &= \sum_{n=0} n c_n x^{n-1} = \sum_{n=1} n c_n x^{n-1} = \sum_{n=0} (n+1) c_{n+1} x^n \\ u'' &= \sum_{n=0} n(n+1) c_{n+1} x^{n-1} = \sum_{n=1} n(n+1) c_{n+1} x^{n-1} = \sum_{n=0} (n+1)(n+2) c_{n+2} x^n \end{aligned}$$

Substituting back in (2) gives

$$\begin{aligned}\sum_{n=0} (n+1)(n+2)c_{n+2}x^n + \sum_{n=0} ac_nx^{n+1} &= 0 \\ \sum_{n=0} (n+1)(n+2)c_{n+2}x^n + \sum_{n=1} ac_{n-1}x^n &= 0\end{aligned}$$

For  $n = 0$

$$(1)(2)c_2 = 0$$

Hence  $c_2 = 0$ . For  $n \geq 1$

$$\begin{aligned}(n+1)(n+2)c_{n+2} + ac_{n-1} &= 0 \\ c_{n+2} &= \frac{-ac_{n-1}}{(n+1)(n+2)}\end{aligned}\tag{3}$$

For  $n = 1$ , from (3)

$$c_3 = \frac{-ac_0}{(2)(3)}$$

For  $n = 2$ , from (3)

$$c_4 = \frac{-ac_1}{(3)(4)}$$

For  $n = 3$ , from (3)

$$c_5 = \frac{-ac_2}{(4)(5)} = 0$$

For  $n = 4$ , from (3)

$$c_6 = \frac{-ac_3}{(5)(6)} = \frac{-a}{(5)(6)} \left( \frac{-ac_0}{(2)(3)} \right) = \frac{a^2c_0}{(2)(3)(5)(6)}$$

For  $n = 5$ , from (3)

$$c_7 = \frac{-ac_4}{(6)(7)} = \frac{-a}{(6)(7)} \left( \frac{-ac_1}{(3)(4)} \right) = \frac{a^2c_1}{(3)(4)(6)(7)}$$

For  $n = 6$ , from (3)

$$c_8 = \frac{-ac_5}{(7)(8)} = 0$$

For  $n = 7$ , from (3)

$$c_9 = \frac{-ac_6}{(8)(9)} = \frac{-a}{(8)(9)} \left( \frac{a^2c_0}{(2)(3)(5)(6)} \right) = \frac{-a^3c_0}{(2)(3)(5)(6)(8)(9)}$$

For  $n = 8$ , from (3)

$$c_{10} = \frac{-ac_7}{(9)(10)} = \frac{-a}{(9)(10)} \left( \frac{a^2c_1}{(3)(4)(6)(7)} \right) = \frac{-a^3c_1}{(3)(4)(6)(7)(9)(10)}$$

And so on. Hence,

$$\begin{aligned}u &= \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x^1 + c_2 x^2 + c_3 x^3 + \dots \\ &= c_0 + c_1 x + 0 + c_3 x^3 + c_4 x^4 + 0 + c_6 x^6 + c_7 x^7 + 0 + c_9 x^9 + \dots \\ &= c_0 + c_1 x - \frac{ac_0}{(2)(3)} x^3 - \frac{ac_1}{(3)(4)} x^4 + \frac{a^2c_0}{(2)(3)(5)(6)} x^6 + \frac{a^2c_1}{(3)(4)(6)(7)} x^7 - \frac{a^3c_0}{(2)(3)(5)(6)(8)(9)} x^9 - \frac{a^3c_1}{(3)(4)(6)(7)(9)(10)} x^{10} + \dots \\ &= c_0 \left( 1 - \frac{a}{6} x^3 + \frac{a^2}{180} x^6 - \frac{a^3}{12960} x^9 + \dots \right) + xc_1 \left( 1 - \frac{a}{12} x^3 + \frac{a^2}{504} x^6 - \frac{a^3}{45360} x^9 + \dots \right) \\ &= c_0 \left( 1 - \frac{1}{6} \left( a^{\frac{1}{3}} x \right)^3 + \frac{1}{180} \left( a^{\frac{1}{3}} x \right)^6 - \frac{1}{12960} \left( a^{\frac{1}{3}} x \right)^9 + \dots \right) + xc_1 \left( 1 - \frac{1}{12} \left( a^{\frac{1}{3}} x \right)^3 + \frac{1}{504} \left( a^{\frac{1}{3}} x \right)^6 - \frac{1}{45360} \left( a^{\frac{1}{3}} x \right)^9 + \dots \right)\end{aligned}$$

Comparing the above to the series expansion of Airy functions, we see that

$$u = c_0 \text{AiryAI} \left( -a^{\frac{1}{3}} x \right) + c_1 \text{AiryBI} \left( -a^{\frac{1}{3}} x \right)$$

And since  $y = \frac{u(x)}{x}$  then

$$y = \frac{1}{x} \left( c_0 \text{AiryAI} \left( -a^{\frac{1}{3}} x \right) + c_1 \text{AiryBI} \left( -a^{\frac{1}{3}} x \right) \right)$$

Verification

```
1 restart;
2 #for series solution of u''+axu=0, use this:
3 Order:=10;
4 sol:=dsolve(ode,u(x),series);
5 sol:=convert(sol,polynom);
6 sol:=subs({u(0)=c0,D(u)(0)=c1},rhs(sol));
7 collect(sol,{c0,c1});
8 (1-(1/6)*a*x^3+(1/180)*a^2*x^6-(1/12960)*a^3*x^9)*c0+(x-(1/12)*a*x^4+(1/504)*a^2*x
   ^7)*c1
9 #to verify final solution use this:
10 ode:=x*diff(y(x),x$2)+2*diff(y(x),x)+a*x^2*y(x)=0;
11 y0:=(1/x)*(_C0*AiryAi(-a^(1/3)*x)+_C1*AiryBi(-a^(1/3)*x));
12 odetest(y(x)=y0,ode);
13 0
```

2.1103 ODE No. 1103

---

$ay(x) + xy''(x) - 2y'(x) = 0$

✓ **Mathematica** : cpu = 0.0199716 (sec), leaf count = 64

$$\left\{ \left\{ y(x) \rightarrow 6a^{3/2}c_1x^{3/2}J_3\left(2\sqrt{a}\sqrt{x}\right) - 2ia^{3/2}c_2x^{3/2}Y_3\left(2\sqrt{a}\sqrt{x}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.008 (sec), leaf count = 33

$$\left\{ y\left(x\right) = x^{\frac{3}{2}}\left(Y_3\left(2\sqrt{a}\sqrt{x}\right)_-C2 + J_3\left(2\sqrt{a}\sqrt{x}\right)_-C1\right) \right\}$$

2.1104 ODE No. 1104

---

$ay(x) + vy'(x) + xy''(x) = 0$

✓ **Mathematica** : cpu = 0.0262799 (sec), leaf count = 104

$$\left\{ \left\{ y(x) \rightarrow c_2a^{\frac{v-1}{2}-v+1}x^{\frac{v-1}{2}-v+1}\Gamma(2-v)J_{1-v}\left(2\sqrt{a}\sqrt{x}\right) + c_1a^{\frac{1-v}{2}}x^{\frac{1-v}{2}}\Gamma(v)J_{v-1}\left(2\sqrt{a}\sqrt{x}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 41

$$\left\{ y\left(x\right) = x^{\frac{1}{2}-\frac{v}{2}}\left(Y_{v-1}\left(2\sqrt{a}\sqrt{x}\right)_-C2 + J_{v-1}\left(2\sqrt{a}\sqrt{x}\right)_-C1\right) \right\}$$

2.1105 ODE No. 1105

---

$ay'(x) + bxy(x) + xy''(x) = 0$

✓ **Mathematica** : cpu = 0.0146605 (sec), leaf count = 64

$$\left\{ \left\{ y(x) \rightarrow c_1x^{\frac{1-a}{2}}J_{\frac{a-1}{2}}\left(\sqrt{bx}\right) + c_2x^{\frac{1-a}{2}}Y_{\frac{a-1}{2}}\left(\sqrt{bx}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 39

$$\left\{ y\left(x\right) = x^{-\frac{a}{2}+\frac{1}{2}}\left(Y_{\frac{a}{2}-\frac{1}{2}}\left(\sqrt{bx}\right)_-C2 + J_{\frac{a}{2}-\frac{1}{2}}\left(\sqrt{bx}\right)_-C1\right) \right\}$$

2.1106 ODE No. 1106

---

$ay'(x) + bx^{a1}y(x) + xy''(x) = 0$

✓ **Mathematica** : cpu = 0.0364709 (sec), leaf count = 441

$$\left\{ \left\{ y(x) \rightarrow c_1\left(\frac{1}{a1}+1\right)^{\frac{a}{\left(\frac{1}{a1}+1\right)a1}-\frac{1}{\left(\frac{1}{a1}+1\right)a1}}\frac{1}{a1}^{\frac{a}{\left(\frac{1}{a1}+1\right)a1}-\frac{1}{\left(\frac{1}{a1}+1\right)a1}}b^{\frac{1}{2}\left(\frac{1}{\left(\frac{1}{a1}+1\right)a1}-\frac{a}{\left(\frac{1}{a1}+1\right)a1}\right)}\left(x^{a1}\right)^{\frac{1}{2}\left(\frac{1}{a1}+1\right)}\left(\frac{1}{\left(\frac{1}{a1}+1\right)a1}-\frac{a}{\left(\frac{1}{a1}+1\right)a1}\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 71

$$\left\{ y\left(x\right) = x^{-\frac{a}{2}+\frac{1}{2}}\left(Y_{\frac{a-1}{a1+1}}\left(2\frac{\sqrt{bx}^{a1/2+1/2}}{a1+1}\right)_-C2 + J_{\frac{a-1}{a1+1}}\left(2\frac{\sqrt{bx}^{a1/2+1/2}}{a1+1}\right)_-C1\right) \right\}$$



**2.1107      ODE No. 1107**

---

$$ay(x) + (b+x)y'(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0204938 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-x} U(b-a, b, x) + c_2 e^{-x} L_{a-b}^{b-1}(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 30

$$\{y(x) = e^{-x} (M(-a+b, b, x) \_C1 + U(-a+b, b, x) \_C2)\}$$

**2.1108      ODE No. 1108**

---

$$(a+b+x)y'(x) + ay(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0249678 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-x} U(b, a+b, x) + c_2 e^{-x} L_{-b}^{a+b-1}(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 26

$$\{y(x) = e^{-x} (M(b, a+b, x) \_C1 + U(b, a+b, x) \_C2)\}$$

**2.1109      ODE No. 1109**

---

$$xy''(x) - xy'(x) - y(x) - e^x x(x+1) = 0$$

✓ **Mathematica** : cpu = 0.119487 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow c_2 (-e^x x \text{Ei}(-x) - 1) + c_1 e^x x + e^x (x^2 + x - x \log(-x) - 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 33

$$\{y(x) = e^x (-Ei(1, x) x \_C1 + x^2 + \_C2 x - x \ln(x) + e^{-x} \_C1 - 1)\}$$

**2.1110      ODE No. 1110**

---

$$-ay(x) + xy''(x) - xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0315684 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow c_2 G_{1,2}^{2,0} \left( -x \left| \begin{array}{c} 1-a \\ 0, 1 \end{array} \right. \right) + c_1 x {}_1F_1(a+1; 2; x) \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 23

$$\{y(x) = x (U(a+1, 2, x) \_C2 + M(a+1, 2, x) \_C1)\}$$

**2.1111      ODE No. 1111**

---

$$xy''(x) - (x+1)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0160403 (sec), leaf count = 20

$$\{\{y(x) \rightarrow c_1 e^x + c_2 (-x-1)\}\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 13

$$\{y(x) = \_C2 e^x + \_C1 x + \_C1\}$$

**Hand solution**

$$xy'' - (x+1)y' + y = 0 \tag{1}$$

Taking Laplace transform of each term and using property of  $\mathcal{L}(xf(x)) = -\frac{d}{ds}F(s)$  where  $F(s) = \mathcal{L}f(x)$ , then

$$\mathcal{L}(xy'') = -\frac{d}{ds}(\mathcal{L}y'')$$

Let  $\mathcal{L}y(x) = Y(s) \equiv Y$ . Now  $\mathcal{L}y'' = s^2Y - sy(0) - y'(0)$ . Assuming  $y(0) = A, y'(0) = B$  then

$$\begin{aligned}\mathcal{L}(xy'') &= -\frac{d}{ds}(s^2Y - As - B) \\ &= -(2sY + s^2Y' - A)\end{aligned}$$

And

$$\begin{aligned}\mathcal{L}((x+1)y') &= \mathcal{L}(xy' + y') \\ &= -\frac{d}{ds}(\mathcal{L}y') + \mathcal{L}y' \\ &= -\frac{d}{ds}(sY - y(0)) + (sY - y(0)) \\ &= -\frac{d}{ds}(sY - A) + (sY - y(0)) \\ &= -(Y + sY') + (sY - A) \\ &= -Y - sY' + sY - A\end{aligned}$$

Hence Laplace transform of the ODE becomes

$$\begin{aligned}-(2sY + s^2Y' - A) - (-Y - sY' + sY - A) + Y &= 0 \\ -2sY - s^2Y' + A + Y + sY' - sY + A + Y &= 0 \\ Y'(s - s^2) + Y(-2s + 1 - s + 1) &= -2A \\ Y'(s^2 - s) + Y(3s - 2) &= 2A \\ Y' + \frac{(3s - 2)}{s(s - 1)}Y &= \frac{2A}{s(s - 1)}\end{aligned}$$

The integrating factor is  $\mu = e^{\int \frac{(3s-2)}{s(s-1)}} = e^{\ln(s-1)+2\ln(s)} = (s-1)s^2$ , hence

$$\begin{aligned}d((s-1)s^2Y) &= (s-1)s^2\frac{2A}{s(s-1)} \\ (s-1)s^2Y &= 2A\int sds + c_1 \\ (s-1)s^2Y &= 2A\frac{s^2}{2} + c_1 \\ Y &= \frac{As^2 + c_1}{(s-1)s^2}\end{aligned}$$

Inverse Laplace transform gives

$$\begin{aligned}y(x) &= -c_1 + (A + c_1)e^x - c_1x \\ &= -c_1(1+x) + (A + c_1)e^x\end{aligned}$$

Let  $-c_1 = A_0, A + c_1 = B_0$ , hence

$$y(x) = A_0(1+x) + B_0e^x$$

Verification

```

1 rrestart;
2 ode:=x*diff(diff(y(x),x),x)-(1+x)*diff(y(x),x)+y(x)=0;
3 y0:=-C0*(1+x)+_C1*exp(x);
4 odetest(y(x)=y0,ode);
5 0

```

## 2.1112 ODE No. 1112

$$xy''(x) - (x+1)y'(x) - 2(x-1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0206862 (sec), leaf count = 30

$$\left\{\left\{y(x)\rightarrow c_1e^{2x}-\frac{1}{9}c_2e^{-x}(3x+1)\right\}\right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 22

$$\left\{y(x)=\_C1\,e^{2\,x}+\_C2\,e^{-x}\left(3\,x+1\right)\right\}$$

**2.1113      ODE No. 1113**

---

$$-ay(x) + (b-x)y'(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0146392 (sec), leaf count = 24

$$\{\{y(x) \rightarrow c_1 U(a, b, x) + c_2 L_{-a}^{b-1}(x)\}\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 17

$$\{y(x) = \_C1 M(a, b, x) + \_C2 U(a, b, x)\}$$

**2.1114      ODE No. 1114**

---

$$xy''(x) - 2(x-1)y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0351361 (sec), leaf count = 39

$$\left\{\left\{y(x) \rightarrow c_2 G_{1,2}^{2,0}\left(-2x \left| \begin{array}{c} \frac{1}{2} \\ -1, 0 \end{array} \right. \right) + c_1 e^x (I_0(x) - I_1(x))\right\}\right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 34

$$\{y(x) = (\_C2 K_1(-x) - \_C2 K_0(-x) + \_C1 (I_0(x) - I_1(x))) e^x\}$$

**2.1115      ODE No. 1115**

---

$$xy''(x) - (3x-2)y'(x) - (2x-3)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0469112 (sec), leaf count = 76

$$\left\{\left\{y(x) \rightarrow c_2 e^{-\frac{1}{2}(\sqrt{17}-3)x} {}_1F_1\left(1 - \frac{6}{\sqrt{17}}; 2; \sqrt{17}x\right) + c_1 e^{-\frac{1}{2}(\sqrt{17}-3)x} U\left(1 - \frac{6}{\sqrt{17}}, 2, \sqrt{17}x\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.736 (sec), leaf count = 47

$$\left\{y(x) = e^{-\frac{x(-3+\sqrt{17})}{2}} \left( U\left(1 - \frac{6\sqrt{17}}{17}, 2, \sqrt{17}x\right) \_C2 + M\left(1 - \frac{6\sqrt{17}}{17}, 2, \sqrt{17}x\right) \_C1 \right)\right\}$$

**2.1116      ODE No. 1116**

---

$$y'(x)(ax+b+n) + any(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0407504 (sec), leaf count = 43

$$\left\{\left\{y(x) \rightarrow c_1 e^{-ax} U(b, b+n, ax) + c_2 e^{-ax} L_{-b}^{b+n-1}(ax)\right\}\right\}$$

✓ **Maple** : cpu = 0.061 (sec), leaf count = 31

$$\{y(x) = e^{-ax} (U(b, b+n, ax) \_C2 + M(b, b+n, ax) \_C1)\}$$

**2.1117      ODE No. 1117**

---

$$-(x+1)(a+b)y'(x) + abxy(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0621021 (sec), leaf count = 107

$$\left\{\left\{y(x) \rightarrow c_1 U\left(-\frac{-a^2-ba-a+b}{a-b}, a+b+2, (a-b)x\right) e^{(a+b+1)\log(x)+bx} + c_2 L_{\frac{-a^2-ba-a+b}{a-b}}^{a+b+1}((a-b)x) e^{(a+b+1)\log(x)+bx}\right\}\right\}$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 82

$$\left\{y(x) = e^{bx} x^{a+b+1} \left( U\left(\frac{a^2+ab+a-b}{a-b}, a+b+2, x(a-b)\right) \_C2 + M\left(\frac{a^2+ab+a-b}{a-b}, a+b+2, x(a-b)\right) \_C1 \right)\right\}$$

**2.1118 ODE No. 1118**

---

$$y'(x)(x(a+b)+m+n)+y(x)(abx+an+bm)+xy''(x)=0$$

✓ **Mathematica** : cpu = 0.0614819 (sec), leaf count = 51

$$\left\{\left\{y(x) \rightarrow c_1 e^{-ax} U(m, m+n, (a-b)x) + c_2 e^{-ax} L_{-m}^{m+n-1}((a-b)x)\right\}\right\}$$

✓ **Maple** : cpu = 0.068 (sec), leaf count = 39

$$\left\{y(x)=e^{-ax}\left(U(m, m+n, x(a-b))\_C2+M(m, m+n, x(a-b))\_C1\right)\right\}$$

**2.1119 ODE No. 1119**

---

$$y(x)\left(a^2 x+2 a b\right)-2(a x+b) y'(x)+x y''(x)=0$$

✓ **Mathematica** : cpu = 0.11322 (sec), leaf count = 77

$$\left\{\left\{y(x) \rightarrow c_1 e^{a x} x^{b-\frac{1}{2} \sqrt{(2 b+1)^2+\frac{1}{2}}}+\frac{c_2 e^{a x} x^{b+\frac{1}{2} \sqrt{(2 b+1)^2+\frac{1}{2}}}}{\sqrt{(2 b+1)^2}}\right\}\right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 20

$$\left\{y(x)=e^{a x}\left(x^{2 b+1} \_C2+\_C1\right)\right\}$$

**2.1120 ODE No. 1120**

---

$$(a x+b) y'(x)+y(x)(c x+d)+x y''(x)=0$$

✓ **Mathematica** : cpu = 0.0412815 (sec), leaf count = 168

$$\left\{\left\{y(x) \rightarrow c_1 e^{-\frac{1}{2} x \sqrt{a^2-4 c}-\frac{a x}{2}} U\left(-\frac{-a b-\sqrt{a^2-4 c} b+2 d}{2 \sqrt{a^2-4 c}}, b, \sqrt{a^2-4 c} x\right)+c_2 e^{-\frac{1}{2} x \sqrt{a^2-4 c}-\frac{a x}{2}} L^{\frac{-a b-\sqrt{a^2-4 c} b+2 d}{2 \sqrt{a^2-4 c}}}\left(\sqrt{a^2-4 c} x\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.151 (sec), leaf count = 109

$$\left\{y(x)=e^{-\frac{x}{2}\left(a+\sqrt{a^2-4 c}\right)}\left(M\left(\frac{1}{2}\left(b \sqrt{a^2-4 c}+a b-2 d\right) \frac{1}{\sqrt{a^2-4 c}}, b, \sqrt{a^2-4 c} x\right) \_C1+U\left(\frac{1}{2}\left(b \sqrt{a^2-4 c}-a b\right) \frac{1}{\sqrt{a^2-4 c}}, b, \sqrt{a^2-4 c} x\right) \_C2\right)\right\}$$

**2.1121 ODE No. 1121**

---

$$-\left(x^2-x\right) y'(x)+x y''(x)+(x-1) y(x)=0$$

✓ **Mathematica** : cpu = 0.177073 (sec), leaf count = 41

$$\left\{\left\{y(x) \rightarrow c_2 x \int_1^x \frac{e^{\frac{K[1]^2}{2}-K[1]}}{K[1]^2} d K[1]+c_1 x\right\}\right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 23

$$\left\{y(x)=\left(\int \frac{1}{x^2} e^{\frac{x(x-2)}{2}} d x \_C1+\_C2\right) x\right\}$$

**2.1122 ODE No. 1122**

---

$$-\left(x^2-x-2\right) y'(x)+x y''(x)-x(x+3) y(x)=0$$

✓ **Mathematica** : cpu = 0.179816 (sec), leaf count = 57

$$\left\{\left\{y(x) \rightarrow c_2 e^{\frac{x^2}{2}} \int_1^x \frac{e^{-\frac{1}{2} K[1]^2-K[1]}}{K[1]^2} d K[1]+c_1 e^{\frac{x^2}{2}}\right\}\right\}$$

✓ **Maple** : cpu = 0.112 (sec), leaf count = 28

$$\left\{y(x)=e^{\frac{x^2}{2}}\left(\int \frac{1}{x^2} e^{-\frac{x(x+2)}{2}} d x \_C2+\_C1\right)\right\}$$

## 2.1123 ODE No. 1123

---

$$-(2ax^2 + 1)y'(x) + bx^3y(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0089976 (sec), leaf count = 91

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{2}\sqrt{b}x^2 \left( \frac{a}{\sqrt{b}} - \frac{\sqrt{a^2-b}}{\sqrt{b}} \right)} + c_2 e^{\frac{1}{2}\sqrt{b}x^2 \left( \frac{\sqrt{a^2-b}}{\sqrt{b}} + \frac{a}{\sqrt{b}} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 45

$$\left\{ y(x) = {}_1C1 e^{\frac{x^2}{2}(\sqrt{a^2-b}+a)} + {}_1C2 e^{-\frac{x^2}{2}(\sqrt{a^2-b}-a)} \right\}$$

## 2.1124 ODE No. 1124

---

$$-2(x^2 - a)y'(x) + 2nxy(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0526193 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_1F_1 \left( -\frac{n}{2}; a + \frac{1}{2}; x^2 \right) + i^{1-2a} c_2 x^{1-2a} {}_1F_1 \left( -a - \frac{n}{2} + \frac{1}{2}; \frac{3}{2} - a; x^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 29

$$\left\{ y(x) = {}_1C1 M \left( -\frac{n}{2}, \frac{1}{2} + a, x^2 \right) + {}_1C2 U \left( -\frac{n}{2}, \frac{1}{2} + a, x^2 \right) \right\}$$

## 2.1125 ODE No. 1125

---

$$-4x^5 - 4x^3y(x) + (4x^2 - 1)y'(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.102178 (sec), leaf count = 84

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-(1-\sqrt{2})x^2} + c_2 e^{-(1+\sqrt{2})x^2} - \frac{3\sqrt{2}x^2 + 4x^2 + 6\sqrt{2} + 8}{\sqrt{2}(3 + 2\sqrt{2})} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 36

$$\left\{ y(x) = e^{x^2(\sqrt{2}-1)} {}_1C2 + e^{-x^2(1+\sqrt{2})} {}_1C1 - x^2 - 2 \right\}$$

## 2.1126 ODE No. 1126

---

$$(a^2x^3 + a)y(x) + (2ax^3 - 1)y'(x) + xy''(x) = 0$$

✗ **Mathematica** : cpu = 0.793863 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{(a^2x^3 + a)y(x) + (2x^3a - 1)y'(x) + xy''(x) = 0, y(1) = c_1, y'(1) = c_2\}) \right\} \right\}(x)$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 19

$$\left\{ y(x) = e^{-\frac{ax^3}{3}} ({}_1C2 x^2 + {}_1C1) \right\}$$

## 2.1127 ODE No. 1127

---

$$y(x)(a^2x \log^2(x) + a \log(x) + a) + (2ax \log(x) + 1)y'(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.027028 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{ax} x^{-ax} + c_2 e^{ax} x^{-ax} \log(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 21

$$\left\{ y(x) = x^{-ax} e^{ax} (\ln(x) {}_1C2 + {}_1C1) \right\}$$

## 2.1128 ODE No. 1128

---

$$(xf(x) + 2)y'(x) + f(x)y(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0184435 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \int_1^x \exp\left(-\int_1^{K[2]} f(K[1]) dK[1]\right) dK[2]}{x} + \frac{c_1}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.181 (sec), leaf count = 32

$$\left\{ y(x) = \frac{1}{x} \left( -C2 \int e^{\int \frac{-xf(x)-2}{x} dx} x^2 dx + -C1 \right) \right\}$$

## 2.1129 ODE No. 1129

---

$$(x-3)y''(x) - (4x-9)y'(x) + (3x-6)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0554315 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{8} c_2 e^{3x-9} (4x^3 - 42x^2 + 150x - 183) + c_1 e^{x-3} \right\} \right\}$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 30

$$\{y(x) = e^x -C1 + -C2 e^{3x} (4x^3 - 42x^2 + 150x - 183)\}$$

## 2.1130 ODE No. 1130

---

$$ay(x) + 2xy''(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0084929 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin\left(\sqrt{2}\sqrt{a}\sqrt{x}\right) + c_1 \cos\left(\sqrt{2}\sqrt{a}\sqrt{x}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 31

$$\left\{ y(x) = -C1 \sin\left(\sqrt{x}\sqrt{2}\sqrt{a}\right) + -C2 \cos\left(\sqrt{x}\sqrt{2}\sqrt{a}\right) \right\}$$

## 2.1131 ODE No. 1131

---

$$ay(x) + 2xy''(x) - (x-1)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0078418 (sec), leaf count = 58

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt{x} U\left(\frac{1}{2}(1-2a), \frac{3}{2}, \frac{x}{2}\right) + c_2 \sqrt{x} L^{\frac{1}{2}}_{\frac{1}{2}(2a-1)}\left(\frac{x}{2}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt{x} \left( U\left(-a + \frac{1}{2}, \frac{3}{2}, \frac{x}{2}\right) -C2 + M\left(-a + \frac{1}{2}, \frac{3}{2}, \frac{x}{2}\right) -C1 \right) \right\}$$

## 2.1132 ODE No. 1132

---

$$ay(x) + 2xy''(x) - (2x-1)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0081265 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt{x} U\left(\frac{1-a}{2}, \frac{3}{2}, x\right) + c_2 \sqrt{x} L^{\frac{1}{2}}_{\frac{a-1}{2}}(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 29

$$\left\{ y(x) = \sqrt{x} \left( U\left(-\frac{a}{2} + \frac{1}{2}, \frac{3}{2}, x\right) -C2 + M\left(-\frac{a}{2} + \frac{1}{2}, \frac{3}{2}, x\right) -C1 \right) \right\}$$

**2.1133      ODE No. 1133**

---

$$(2x - 1)y''(x) - (3x - 4)y'(x) + (x - 3)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0466157 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow 2^{5/8} c_1 e^{x-\frac{1}{2}} - \frac{c_2 e^{x-\frac{1}{2}} \Gamma\left(-\frac{1}{4}, \frac{1}{4}(2x-1)\right)}{4\sqrt[8]{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 37

$$\left\{ y(x) = 1e^{\frac{x}{2}} \left( U\left(1, \frac{3}{4}, \frac{x}{2} - \frac{1}{4}\right) {}_2C2 + M\left(1, \frac{3}{4}, \frac{x}{2} - \frac{1}{4}\right) {}_2C1 \right) \frac{1}{\sqrt[4]{2x-1}} \right\}$$

**2.1134      ODE No. 1134**

---

$$4xy''(x) - (a + x)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0691336 (sec), leaf count = 78

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} c_2 e^{-x/2} {}_1F_1\left(\frac{1}{128}(-32(-a-4i)-128i)+1; 2; x\right) + \frac{1}{4} c_1 e^{-x/2} x U\left(\frac{1}{128}(-32(-a-4i)-128i)+1; \frac{3}{2}; x\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 21

$$\left\{ y(x) = {}_2C1 M_{-\frac{a}{4}, \frac{1}{2}}(x) + {}_2C2 W_{-\frac{a}{4}, \frac{1}{2}}(x) \right\}$$

**2.1135      ODE No. 1135**

---

$$4xy''(x) + 2y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0071604 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow c_1 \cosh(\sqrt{x}) + i c_2 \sinh(\sqrt{x}) \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 17

$$\left\{ y(x) = {}_2C1 \sinh(\sqrt{x}) + {}_2C2 \cosh(\sqrt{x}) \right\}$$

**2.1136      ODE No. 1136**

---

$$4xy''(x) + 4y'(x) - (x + 2)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0180427 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow c_2 e^{x/2} \text{Ei}(-x) + c_1 e^{x/2} \right\} \right\}$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 16

$$\left\{ y(x) = e^{\frac{x}{2}} \left( \text{Ei}(1, x) {}_2C2 + {}_2C1 \right) \right\}$$

**2.1137      ODE No. 1137**

---

$$ly(x) + 4xy''(x) - (x + 2)y(x) + 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.0693734 (sec), leaf count = 74

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} c_2 e^{-x/2} {}_1F_1\left(\frac{1}{128}(-32(l+(2-4i))-128i)+1; 2; x\right) + \frac{1}{4} c_1 e^{-x/2} x U\left(\frac{1}{128}(-32(l+(2-4i))-128i)+1; \frac{3}{2}; x\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 25

$$\left\{ y(x) = {}_2C1 M_{l+\frac{1}{2}, \frac{1}{2}}(x) + {}_2C2 W_{l+\frac{1}{2}, \frac{1}{2}}(x) \right\}$$

## 2.1138 ODE No. 1138

---

$$y(x)(-(-2m - 4n + x)) + 4my'(x) + 4xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0208002 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-x/2} U(-n, m, x) + c_2 e^{-x/2} L_n^{m-1}(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 26

$$\left\{ y(x) = e^{-\frac{x}{2}} (U(-n, m, x) C_2 + M(-n, m, x) C_1) \right\}$$

## 2.1139 ODE No. 1139

---

$$-(a + x)y(x) + 16xy''(x) + 8y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0097252 (sec), leaf count = 74

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{4}(2\log(x)-x)} U\left(\frac{a+6}{8}, \frac{3}{2}, \frac{x}{2}\right) + c_2 e^{\frac{1}{4}(2\log(x)-x)} L_{\frac{1}{8}(-a-6)}\left(\frac{x}{2}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 37

$$\left\{ y(x) = \sqrt{x} e^{-\frac{x}{4}} \left( U\left(\frac{a}{8} + \frac{3}{4}, \frac{3}{2}, \frac{x}{2}\right) C_2 + M\left(\frac{a}{8} + \frac{3}{4}, \frac{3}{2}, \frac{x}{2}\right) C_1 \right) \right\}$$

## 2.1140 ODE No. 1140

---

$$axy''(x) + by'(x) + cy(x) = 0$$

✓ **Mathematica** : cpu = 0.0341202 (sec), leaf count = 190

$$\left\{ \left\{ y(x) \rightarrow c_1 a^{\frac{1}{2}\left(\frac{b}{a}-1\right)} c^{\frac{1}{2}\left(1-\frac{b}{a}\right)} x^{\frac{1}{2}\left(1-\frac{b}{a}\right)} \Gamma\left(\frac{b}{a}\right) J_{\frac{b}{a}-1}\left(\frac{2\sqrt{c}\sqrt{x}}{\sqrt{a}}\right) + c_2 a^{\frac{1}{2}\left(1-\frac{b}{a}\right)-\frac{a-b}{a}} c^{\frac{a-b}{a}+\frac{1}{2}\left(\frac{b}{a}-1\right)} x^{\frac{a-b}{a}+\frac{1}{2}\left(\frac{b}{a}-1\right)} \Gamma\left(2\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 66

$$\left\{ y(x) = x^{\frac{a-b}{2a}} \left( Y_{-\frac{a+b}{a}}\left(2\sqrt{\frac{c}{a}}\sqrt{x}\right) C_2 + J_{-\frac{a+b}{a}}\left(2\sqrt{\frac{c}{a}}\sqrt{x}\right) C_1 \right) \right\}$$

## 2.1141 ODE No. 1141

---

$$(3a + bx)y'(x) + axy''(x) + 3by(x) = 0$$

✓ **Mathematica** : cpu = 0.091415 (sec), leaf count = 79

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\frac{bx}{a}} - \frac{c_2 e^{-\frac{bx}{a}} \left( a^2 e^{\frac{bx}{a}} - b^2 x^2 \text{Ei}\left(\frac{bx}{a}\right) + abx e^{\frac{bx}{a}} \right)}{2a^2 x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 55

$$\left\{ y(x) = \frac{1}{x^2} \left( \text{Ei}\left(1, -\frac{bx}{a}\right) e^{-\frac{bx}{a}} C_2 b^2 x^2 + C_1 e^{-\frac{bx}{a}} x^2 + a C_2 (bx + a) \right) \right\}$$

## 2.1142 ODE No. 1142

---

$$cy(x)\sqrt[5]{ax+b} + 5(ax+b)y''(x) + 8ay'(x) = 0$$

✓ **Mathematica** : cpu = 0.0330418 (sec), leaf count = 108

$$\left\{ \left\{ y(x) \rightarrow \frac{3ac_2 \sin\left(\frac{\sqrt{5}\sqrt{c}(ax+b)^{3/5}}{3a}\right)}{\sqrt{5}\sqrt{c}(ax+b)^{3/5}} + \frac{6ac_1 \cos\left(\frac{\sqrt{5}\sqrt{c}(ax+b)^{3/5}}{3a}\right)}{\sqrt{5}\sqrt{c}(ax+b)^{3/5}} \right\} \right\}$$

✓ **Maple** : cpu = 0.078 (sec), leaf count = 53

$$\left\{ y(x) = 1 \left( -C_2 \cosh\left(\frac{1}{3a}(ax+b)^{\frac{3}{5}}\sqrt{-5c}\right) + C_1 \sinh\left(\frac{1}{3a}(ax+b)^{\frac{3}{5}}\sqrt{-5c}\right) \right) (ax+b)^{-\frac{3}{5}} \right\}$$



**2.1143                    ODE No. 1143**

---

$$(a + bx)y'(x) + 2axy''(x) + cy(x) = 0$$

✓ **Mathematica** : cpu = 0.0300589 (sec), leaf count = 93

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{a \log(x) - bx}{2a}} U\left(-\frac{c-b}{b}, \frac{3}{2}, \frac{bx}{2a}\right) + c_2 e^{\frac{a \log(x) - bx}{2a}} L_{\frac{1}{\frac{c-b}{b}} \frac{1}{2}}\left(\frac{bx}{2a}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 57

$$\left\{ y(x) = \sqrt{x} e^{-\frac{bx}{2a}} \left( U\left(\frac{b-c}{b}, \frac{3}{2}, \frac{bx}{2a}\right) {}_2F_2 + M\left(\frac{b-c}{b}, \frac{3}{2}, \frac{bx}{2a}\right) {}_2F_1 \right) \right\}$$

**2.1144                    ODE No. 1144**

---

$$(3a + bx)y'(x) + 2axy''(x) + cy(x) = 0$$

✓ **Mathematica** : cpu = 0.0265059 (sec), leaf count = 88

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\frac{bx}{2a}} U\left(-\frac{2c-3b}{2b}, \frac{3}{2}, \frac{bx}{2a}\right) + c_2 e^{-\frac{bx}{2a}} L_{\frac{1}{\frac{2c-3b}{2b}} \frac{1}{2}}\left(\frac{bx}{2a}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 60

$$\left\{ y(x) = e^{-\frac{bx}{2a}} \left( U\left(\frac{3b-2c}{2b}, \frac{3}{2}, \frac{bx}{2a}\right) {}_2F_2 + M\left(\frac{3b-2c}{2b}, \frac{3}{2}, \frac{bx}{2a}\right) {}_2F_1 \right) \right\}$$

**2.1145                    ODE No. 1145**

---

$$y(x)(a_0x + b_0) + (a_1x + b_1)y'(x) + (a_2x + b_2)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.258906 (sec), leaf count = 386

$$\left\{ \left\{ y(x) \rightarrow c_1 U\left(-\frac{b_2 a_1^2 - a_2 b_1 a_1 - \sqrt{a_1^2 - 4 a_0 a_2} b_2 a_1 + 2 a_2^2 b_0 + a_2 \sqrt{a_1^2 - 4 a_0 a_2} b_1 - 2 a_0 a_2 b_2 - 2 a_2^2 \sqrt{a_1^2 - 4 a_0 a_2}}{2 a_2^2 \sqrt{a_1^2 - 4 a_0 a_2}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.122 (sec), leaf count = 248

$$\left\{ y(x) = (a_2 x + b_2)^{\frac{a_1 b_2 + a_2^2 - a_2 b_1}{a_2^2}} e^{-\frac{x}{2 a_2} (\sqrt{-4 a_0 a_2 + a_1^2} + a_1)} \left( M\left(\frac{1}{2 a_2^2} ((a_1 b_2 + 2 a_2^2 - a_2 b_1) \sqrt{-4 a_0 a_2} + \dots) \right) \right) \right\}$$

**2.1146                    ODE No. 1146**

---

$$x^2 y''(x) - 6y(x) = 0$$

✓ **Mathematica** : cpu = 0.0032943 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow c_2 x^3 + \frac{c_1}{x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.005 (sec), leaf count = 15

$$\left\{ y(x) = \frac{{}_2F_2 + {}_2F_1}{x^2} \right\}$$

**2.1147                    ODE No. 1147**

---

$$x^2 y''(x) - 12y(x) = 0$$

✓ **Mathematica** : cpu = 0.0034287 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow c_2 x^4 + \frac{c_1}{x^3} \right\} \right\}$$

✓ **Maple** : cpu = 0.006 (sec), leaf count = 15

$$\left\{ y(x) = \frac{{}_2F_2 + {}_2F_1}{x^3} \right\}$$

**2.1148      ODE No. 1148**

---

$$ay(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0071491 (sec), leaf count = 77

$$\left\{ \left\{ y(x) \rightarrow c_1 x^{\frac{1}{2}} \left( \frac{1}{\sqrt{a}} - \frac{\sqrt{1-4a}}{\sqrt{a}} \right) \sqrt{a} + c_2 x^{\frac{1}{2}} \left( \frac{\sqrt{1-4a}}{\sqrt{a}} + \frac{1}{\sqrt{a}} \right) \sqrt{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 35

$$\left\{ y(x) = \_C1 x^{\frac{1}{2} + \frac{1}{2} \sqrt{1-4a}} + \_C2 x^{\frac{1}{2} - \frac{1}{2} \sqrt{1-4a}} \right\}$$

**2.1149      ODE No. 1149**

---

$$y(x)(ax + b) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0458717 (sec), leaf count = 212

$$\left\{ \left\{ y(x) \rightarrow c_2 a^{\frac{1}{2}(\sqrt{1-4b}+1) - \frac{1}{2}\sqrt{1-4b}} x^{\frac{1}{2}(\sqrt{1-4b}+1) - \frac{1}{2}\sqrt{1-4b}} \Gamma\left(\sqrt{1-4b} + 1\right) J_{\sqrt{1-4b}}(2\sqrt{a}\sqrt{x}) + c_1 a^{\frac{1}{2}(1-\sqrt{1-4b}) + \frac{1}{2}\sqrt{1-4b}} \right\} \right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 45

$$\left\{ y(x) = \sqrt{x} \left( Y_{\sqrt{1-4b}}(2\sqrt{a}\sqrt{x}) \_C2 + J_{\sqrt{1-4b}}(2\sqrt{a}\sqrt{x}) \_C1 \right) \right\}$$

**2.1150      ODE No. 1150**

---

$$x^2 y''(x) + (x^2 - 2) y(x) = 0$$

✓ **Mathematica** : cpu = 0.006845 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow \sqrt{\frac{2}{\pi}} c_2 \left( -\sin(x) - \frac{\cos(x)}{x} \right) + \sqrt{\frac{2}{\pi}} c_1 \left( \frac{\sin(x)}{x} - \cos(x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 27

$$\left\{ y(x) = \frac{(\_C1 x + \_C2) \cos(x) + \sin(x) (\_C2 x - \_C1)}{x} \right\}$$

**2.1151      ODE No. 1151**

---

$$x^2 y''(x) - (ax^2 + 2) y(x) = 0$$

✓ **Mathematica** : cpu = 0.0123186 (sec), leaf count = 129

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{\frac{2}{\pi}} c_2 \sqrt{x} \left( i \sinh(\sqrt{ax}) - \frac{i \cosh(\sqrt{ax})}{\sqrt{ax}} \right)}{\sqrt{-i\sqrt{ax}}} + \frac{\sqrt{\frac{2}{\pi}} c_1 \sqrt{x} \left( \frac{\sinh(\sqrt{ax})}{\sqrt{ax}} - \cosh(\sqrt{ax}) \right)}{\sqrt{-i\sqrt{ax}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.06 (sec), leaf count = 43

$$\left\{ y(x) = \frac{1}{x} \left( \_C2 (ax + \sqrt{a}) e^{-\sqrt{ax}} - \_C1 e^{\sqrt{ax}} (ax - \sqrt{a}) \right) \right\}$$

**2.1152      ODE No. 1152**

---

$$(a^2 x^2 - 6) y(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0130603 (sec), leaf count = 114

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{\frac{2}{\pi}} c_1 \sqrt{x} \left( \frac{3 \sin(ax)}{a^2 x^2} - \sin(ax) - \frac{3 \cos(ax)}{ax} \right)}{\sqrt{ax}} + \frac{\sqrt{\frac{2}{\pi}} c_2 \sqrt{x} \left( -\frac{3 \cos(ax)}{a^2 x^2} - \frac{3 \sin(ax)}{ax} + \cos(ax) \right)}{\sqrt{ax}} \right\} \right\}$$

✓ **Maple** : cpu = 0.159 (sec), leaf count = 53

$$\left\{ y(x) = \frac{(\_C1 a^2 x^2 + 3 \_C2 ax - 3 \_C1) \cos(ax) + \sin(ax) (\_C2 a^2 x^2 - 3 \_C1 ax - 3 \_C2)}{x^2} \right\}$$

**2.1153      ODE No. 1153**

---

$$y(x) \left( ax^2 - (v-1)v \right) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0221002 (sec), leaf count = 56

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt{x} J_{\frac{1}{2}(2v-1)}(\sqrt{ax}) + c_2 \sqrt{x} Y_{\frac{1}{2}(2v-1)}(\sqrt{ax}) \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 31

$$\left\{ y(x) = \sqrt{x} \left( Y_{v-\frac{1}{2}}(\sqrt{ax}) {}_2C_2 + J_{v-\frac{1}{2}}(\sqrt{ax}) {}_2C_1 \right) \right\}$$

**2.1154      ODE No. 1154**

---

$$y(x) \left( ax^2 + bx + c \right) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0141277 (sec), leaf count = 88

$$\left\{ \left\{ y(x) \rightarrow c_1 M_{-\frac{ib}{2\sqrt{a}}, -\frac{1}{2}i\sqrt{4c-1}}(2i\sqrt{ax}) + c_2 W_{-\frac{ib}{2\sqrt{a}}, -\frac{1}{2}i\sqrt{4c-1}}(2i\sqrt{ax}) \right\} \right\}$$

✓ **Maple** : cpu = 0.094 (sec), leaf count = 57

$$\left\{ y(x) = {}_2C_1 M_{-\frac{i}{2}b\frac{1}{\sqrt{a}}, \frac{1}{2}\sqrt{1-4c}}(2i\sqrt{ax}) + {}_2C_2 W_{-\frac{i}{2}b\frac{1}{\sqrt{a}}, \frac{1}{2}\sqrt{1-4c}}(2i\sqrt{ax}) \right\}$$

**2.1155      ODE No. 1155**

---

$$y(x) \left( ax^k - (b-1)b \right) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0354846 (sec), leaf count = 225

$$\left\{ \left\{ y(x) \rightarrow c_1 k^{-\frac{2(1-b)}{k} - \frac{2b}{k} + \frac{1}{k}} a^{\frac{1-b}{k} + \frac{1}{2} \left( \frac{2b}{k} - \frac{1}{k} \right)} \left( x^k \right)^{\frac{1-b}{k} + \frac{1}{2} \left( \frac{2b}{k} - \frac{1}{k} \right)} \Gamma \left( -\frac{2b}{k} + \frac{1}{k} + 1 \right) J_{\frac{1-2b}{k}} \left( \frac{2\sqrt{a}\sqrt{x^k}}{k} \right) + c_2 k^{-1/k} a^{\frac{b}{k}} \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 67

$$\left\{ y(x) = \sqrt{x} \left( Y_{\frac{1}{k}\sqrt{(2b-1)^2}} \left( 2 \frac{\sqrt{ax^{k/2}}}{k} \right) {}_2C_2 + J_{\frac{1}{k}\sqrt{(2b-1)^2}} \left( 2 \frac{\sqrt{ax^{k/2}}}{k} \right) {}_2C_1 \right) \right\}$$

**2.1156      ODE No. 1156**

---

$$x^2 y''(x) + \frac{y(x)}{\log(x)} - e^x x (x \log(x) + 2) = 0$$

✓ **Mathematica** : cpu = 0.0896086 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow c_2 \log(x) \left( \text{Ei}(\log(x)) - \frac{x}{\log(x)} \right) + c_1 \log(x) + e^x \log(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.093 (sec), leaf count = 71

$$\left\{ y(x) = \ln(x) {}_2C_2 - (\text{Ei}(1, -\ln(x)) \ln(x) + x) {}_2C_1 - \left( - \int \frac{(\text{Ei}(1, -\ln(x)) \ln(x) + x) e^x (2 + x \ln(x))}{x} dx + \right) \right\}$$

**2.1157      ODE No. 1157**

---

$$ay'(x) + x^2 y''(x) - xy(x) = 0$$

✗ **Mathematica** : cpu = 0.467125 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{y''(x)x^2 - y(x)x + ay'(x) = 0, y(1) = c_1, y'(1) = c_2\}) (x) \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \text{DESol} \left( \left\{ \frac{d^2}{dx^2} Y(x) + \frac{a \frac{d}{dx} Y(x)}{x^2} - \frac{Y(x)}{x} \right\}, \{ {}_2Y(x) \} \right) \right\}$$

**2.1158 ODE No. 1158**

---

$$-y(x) \left( ab + b^2 x^2 \right) + ay'(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.116283 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow c_2 e^{bx} \int_1^x e^{\frac{a}{K[1]} - 2bK[1]} dK[1] + c_1 e^{bx} \right\} \right\}$$

✓ **Maple** : cpu = 0.174 (sec), leaf count = 178

$$\left\{ y(x) = \sqrt{x} \left( e^{bx} \operatorname{HeunD} \left( -4\sqrt{2}\sqrt{ab}, -1 - 4\sqrt{2}\sqrt{ab}, 8\sqrt{2}\sqrt{ab}, -4\sqrt{2}\sqrt{ab} + 1, 1 \left( \sqrt{2}\sqrt{ab}x - a \right) \left( \sqrt{2}\sqrt{ab}x + a \right) \right) \right) \right\}$$

**2.1159 ODE No. 1159**

---

$$-ax^2 + x^2 y''(x) + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0056152 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow \frac{ax^2}{3} + c_2 x + \frac{c_1}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 19

$$\left\{ y(x) = x\_C2 + \frac{ax^2}{3} + \frac{\_C1}{x} \right\}$$

**2.1160 ODE No. 1160**

---

$$ay(x) + x^2 y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0067497 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin \left( \sqrt{a} \log(x) \right) + c_1 \cos \left( \sqrt{a} \log(x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 23

$$\left\{ y(x) = \_C1 \sin \left( \sqrt{a} \ln(x) \right) + \_C2 \cos \left( \sqrt{a} \ln(x) \right) \right\}$$

**2.1161 ODE No. 1161**

---

$$-(a+x)y(x) + x^2 y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0343208 (sec), leaf count = 78

$$\left\{ \left\{ y(x) \rightarrow (-1)^{-\sqrt{a}} c_1 \Gamma \left( 1 - 2\sqrt{a} \right) I_{-2\sqrt{a}} \left( 2\sqrt{x} \right) + (-1)^{\sqrt{a}} c_2 \Gamma \left( 2\sqrt{a} + 1 \right) I_{2\sqrt{a}} \left( 2\sqrt{x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 31

$$\left\{ y(x) = \_C1 I_{2\sqrt{a}} \left( 2\sqrt{x} \right) + \_C2 K_{2\sqrt{a}} \left( 2\sqrt{x} \right) \right\}$$

**2.1162 ODE No. 1162**

---

$$\left( x^2 - v^2 \right) y(x) + x^2 y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0410505 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow c_1 J_v(x) + c_2 Y_v(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 15

$$\left\{ y(x) = \_C1 J_v(x) + \_C2 Y_v(x) \right\}$$

**2.1163      ODE No. 1163**

---

$$-f(x) + (x^2 - v^2) y(x) + x^2 y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.107017 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow J_v(x) \int_1^x -\frac{\pi Y_v(K[1]) f(K[1])}{2K[1]} dK[1] + Y_v(x) \int_1^x \frac{\pi J_v(K[2]) f(K[2])}{2K[2]} dK[2] + c_1 J_v(x) + c_2 Y_v(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 49

$$\left\{ y(x) = -\frac{J_v(x)\pi}{2} \int \frac{Y_v(x)f(x)}{x} dx + \frac{Y_v(x)\pi}{2} \int \frac{J_v(x)f(x)}{x} dx + Y_v(x) \_C1 + J_v(x) \_C2 \right\}$$

**2.1164      ODE No. 1164**

---

$$y(x) (lx^2 - v^2) + x^2 y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0153555 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow c_1 J_v(\sqrt{l}x) + c_2 Y_v(\sqrt{l}x) \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 23

$$\left\{ y(x) = \_C1 J_v(\sqrt{l}x) + \_C2 Y_v(\sqrt{l}x) \right\}$$

**2.1165      ODE No. 1165**

---

$$(a + x)y'(x) + x^2 y''(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0536049 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2(a+x)}{a^2} + c_1 x e^{a/x} \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 19

$$\left\{ y(x) = (x+a) \_C1 + \_C2 x e^{\frac{a}{x}} \right\}$$

**2.1166      ODE No. 1166**

---

$$-3x^3 + x^2 y''(x) - xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0099262 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow c_1 x + c_2 x \log(x) + \frac{3x^3}{4} \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 21

$$\left\{ y(x) = \frac{x \left( \_C1 \ln(x) + 3x^2 + 4 \_C2 \right)}{4} \right\}$$

**2.1167      ODE No. 1167**

---

$$y(x) (ax^m + b) + x^2 y''(x) - xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0548052 (sec), leaf count = 326

$$\left\{ \left\{ y(x) \rightarrow c_1 m^{-\frac{2(m-i\sqrt{b-1}m)}{m^2} - \frac{2i\sqrt{b-1}}{m}} a^{\frac{m-i\sqrt{b-1}m}{m^2} + \frac{i\sqrt{b-1}}{m}} (x^m)^{\frac{m-i\sqrt{b-1}m}{m^2} + \frac{i\sqrt{b-1}}{m}} \Gamma\left(1 - \frac{2i\sqrt{b-1}}{m}\right) J_{-\frac{2i\sqrt{b-1}}{m}}\left(\frac{2\sqrt{a}\sqrt{x^m}}{m}\right) \right. \right.$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 63

$$\left\{ y(x) = x \left( Y_{2\frac{\sqrt{1-b}}{m}} \left( 2 \frac{\sqrt{ax^{m/2}}}{m} \right) \_C2 + J_{2\frac{\sqrt{1-b}}{m}} \left( 2 \frac{\sqrt{ax^{m/2}}}{m} \right) \_C1 \right) \right\}$$

**2.1168                    ODE No. 1168**

---

$$x^2y''(x) + 2xy'(x) = 0$$

✓    **Mathematica** : cpu = 0.0068953 (sec), leaf count = 15

$$\left\{ \left\{ y(x) \rightarrow c_2 - \frac{c_1}{x} \right\} \right\}$$

✓    **Maple** : cpu = 0.007 (sec), leaf count = 11

$$\left\{ y(x) = -C1 + \frac{-C2}{x} \right\}$$

**2.1169                    ODE No. 1169**

---

$$y(x) (ax - b^2) + x^2y''(x) + 2xy'(x) = 0$$

✓    **Mathematica** : cpu = 0.049169 (sec), leaf count = 236

$$\left\{ \left\{ y(x) \rightarrow c_1 a^{\frac{1}{2}(-\sqrt{4b^2+1}-1)+\frac{1}{2}\sqrt{4b^2+1}} x^{\frac{1}{2}(-\sqrt{4b^2+1}-1)+\frac{1}{2}\sqrt{4b^2+1}} \Gamma\left(1-\sqrt{4b^2+1}\right) J_{-\sqrt{4b^2+1}}(2\sqrt{a}\sqrt{x}) + c_2 a^{\frac{1}{2}(\sqrt{4b^2+1}-1)+\frac{1}{2}\sqrt{4b^2+1}} x^{\frac{1}{2}(\sqrt{4b^2+1}-1)+\frac{1}{2}\sqrt{4b^2+1}} \Gamma\left(1+\sqrt{4b^2+1}\right) Y_{-\sqrt{4b^2+1}}(2\sqrt{a}\sqrt{x}) \right\} \right\}$$

✓    **Maple** : cpu = 0.02 (sec), leaf count = 49

$$\left\{ y(x) = 1 \left( -C2 Y_{\sqrt{4b^2+1}}(2\sqrt{a}\sqrt{x}) + -C1 J_{\sqrt{4b^2+1}}(2\sqrt{a}\sqrt{x}) \right) \frac{1}{\sqrt{x}} \right\}$$

**2.1170                    ODE No. 1170**

---

$$y(x) (ax^2 + b) + x^2y''(x) + 2xy'(x) = 0$$

✓    **Mathematica** : cpu = 0.0159677 (sec), leaf count = 58

$$\left\{ \left\{ y(x) \rightarrow c_1 j_{\frac{1}{2}(\sqrt{1-4b}-1)}(\sqrt{a}x) + c_2 y_{\frac{1}{2}(\sqrt{1-4b}-1)}(\sqrt{a}x) \right\} \right\}$$

✓    **Maple** : cpu = 0.026 (sec), leaf count = 43

$$\left\{ y(x) = 1 \left( -C2 Y_{\frac{1}{2}\sqrt{1-4b}}(\sqrt{a}x) + -C1 J_{\frac{1}{2}\sqrt{1-4b}}(\sqrt{a}x) \right) \frac{1}{\sqrt{x}} \right\}$$

**2.1171                    ODE No. 1171**

---

$$y(x) (ax + lx^2 - n(n+1)) + x^2y''(x) + 2xy'(x) = 0$$

✓    **Mathematica** : cpu = 0.0355618 (sec), leaf count = 142

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{n \log(x) - i\sqrt{l}x} U\left(\frac{i(a - 2i\sqrt{l}n - 2i\sqrt{l})}{2\sqrt{l}}, 2n+2, 2i\sqrt{l}x\right) + c_2 e^{n \log(x) - i\sqrt{l}x} L_{-\frac{i(a - 2i\sqrt{l}n - 2i\sqrt{l})}{2\sqrt{l}}}^{2n+1}(2i\sqrt{l}x) \right\} \right\}$$

✓    **Maple** : cpu = 0.096 (sec), leaf count = 49

$$\left\{ y(x) = \frac{1}{x} \left( -C1 M_{-\frac{i}{2}a\frac{1}{\sqrt{l}}, n+\frac{1}{2}}(2i\sqrt{l}x) + -C2 W_{-\frac{i}{2}a\frac{1}{\sqrt{l}}, n+\frac{1}{2}}(2i\sqrt{l}x) \right) \right\}$$

**2.1172                    ODE No. 1172**

---

$$ay(x) + x^2y''(x) + 2(x-1)y'(x) = 0$$

✓    **Mathematica** : cpu = 0.0470415 (sec), leaf count = 158

$$\left\{ \left\{ y(x) \rightarrow 2^{\frac{1}{2}(1-\sqrt{1-4a})} c_1 \left(\frac{1}{x}\right)^{\frac{1}{2}(1-\sqrt{1-4a})} {}_1F_1\left(\frac{1}{2}-\frac{1}{2}\sqrt{1-4a}; 1-\sqrt{1-4a}; -\frac{2}{x}\right) + 2^{\frac{1}{2}(\sqrt{1-4a}+1)} c_2 \left(\frac{1}{x}\right)^{\frac{1}{2}(\sqrt{1-4a}+1)} {}_1F_1\left(\frac{1}{2}+\frac{1}{2}\sqrt{1-4a}; 1+\sqrt{1-4a}; -\frac{2}{x}\right) \right\} \right\}$$

✓    **Maple** : cpu = 0.023 (sec), leaf count = 47

$$\left\{ y(x) = e^{-x^{-1}\sqrt{x^{-1}}} \left( K_{\frac{1}{2}\sqrt{1-4a}}(x^{-1}) - C2 + I_{\frac{1}{2}\sqrt{1-4a}}(x^{-1}) - C1 \right) \right\}$$

**2.1173      ODE No. 1173**

---

$$2(a+x)y'(x) - (b-1)by(x) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.07063 (sec), leaf count = 74

$$\left\{ \left\{ y(x) \rightarrow (-2)^{1-b} c_1 a^{1-b} \left( \frac{1}{x} \right)^{1-b} {}_1F_1 \left( 1-b; 2-2b; \frac{2a}{x} \right) + (-2)^b c_2 a^b \left( \frac{1}{x} \right)^b {}_1F_1 \left( b; 2b; \frac{2a}{x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.057 (sec), leaf count = 37

$$\left\{ y(x) = 1e^{\frac{a}{x}} \left( I_{b-\frac{1}{2}} \left( \frac{a}{x} \right) {}_-\!C1 + K_{b-\frac{1}{2}} \left( \frac{a}{x} \right) {}_-\!C2 \right) \frac{1}{\sqrt{x}} \right\}$$

**2.1174      ODE No. 1174**

---

$$x^5(-\log(x)) + x^2y''(x) - 2xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0066633 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow c_2 x^2 + c_1 x + \frac{1}{144} (12x^5 \log(x) - 7x^5) \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 25

$$\left\{ y(x) = \frac{x^5 \ln(x)}{12} - \frac{7x^5}{144} + x^2 {}_-\!C2 + {}_-\!C1 x \right\}$$

**2.1175      ODE No. 1175**

---

$$-(ax^2 + 12a + 4) \cos(x) + x^2y''(x) - 2xy'(x) - 4y(x) - x \sin(x) = 0$$

✓ **Mathematica** : cpu = 0.198414 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow \frac{-2a \sin(x) - ax \cos(x) - \sin(x)}{x} + c_2 x^4 + \frac{c_1}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 29

$$\left\{ y(x) = \frac{(-2a-1) \sin(x) + x^5 {}_-\!C2 - ax \cos(x) + {}_-\!C1}{x} \right\}$$

**2.1176      ODE No. 1176**

---

$$x^2y''(x) + (x^2 + 2)y(x) - 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0135326 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 15

$$\{y(x) = x(\cos(x) {}_-\!C2 + \sin(x) {}_-\!C1)\}$$

**2.1177      ODE No. 1177**

---

$$x^2y''(x) + (x^2 + 2)y'(x) + x^2(-\sec(x)) - 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.751451 (sec), leaf count = 141

$$\left\{ \left\{ y(x) \rightarrow c_2 \int_1^x e^{\frac{2}{K[1]} - K[1]} K[1]^2 dK[1] + \int_1^x e^{\frac{2}{K[1]} - K[1]} K[1]^2 dK[1] \int_1^x \frac{e^{K[3] - \frac{2}{K[3]}} \sec(K[3])}{K[3]^2} dK[3] + \int_1^x -\frac{e^{K[2] - \frac{2}{K[2]}} \sec(K[2])}{K[2]^2} dK[2] \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 34

$$\left\{ y(x) = x \left( -\cos(x) \int \frac{\sin(x)}{\cos(x)x} dx + \cos(x) {}_-\!C1 + \sin(x) ({}_-\!C2 + \ln(x)) \right) \right\}$$

**2.1178      ODE No. 1178**

---

$$x^3(-\sec(x)) + x^2 y''(x) + (x^2 + 2) y(x) - 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0369433 (sec), leaf count = 74

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x + \frac{1}{2} e^{-ix} x \left( e^{2ix} \log(1 + e^{-2ix}) + \log(1 + e^{2ix}) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 23

$$\{y(x) = x(\cos(x) \ln(\cos(x)) + \cos(x) \_C1 + \sin(x)(x + \_C2))\}$$

**2.1179      ODE No. 1179**

---

$$(a^2 x^2 + 2) y(x) + x^2 y''(x) - 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0168626 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow c_1 x e^{-iax} - \frac{ic_2 x e^{iax}}{2a} \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 19

$$\{y(x) = x(\cos(ax) \_C2 + \sin(ax) \_C1)\}$$

**2.1180      ODE No. 1180**

---

$$-f(x) + (-v^2 + x^2 + 1) y(x) + x^2 y''(x) + 3xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0587207 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow \frac{J_v(x) \int_1^x -\frac{1}{2} \pi Y_v(K[1]) f(K[1]) dK[1] + Y_v(x) \int_1^x \frac{1}{2} \pi J_v(K[2]) f(K[2]) dK[2]}{x} + \frac{c_1 J_v(x)}{x} + \frac{c_2 Y_v(x)}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 49

$$\left\{ y(x) = \frac{-J_v(x) \pi \int Y_v(x) f(x) \, dx + Y_v(x) \pi \int J_v(x) f(x) \, dx + 2 Y_v(x) \_C1 + 2 J_v(x) \_C2}{2 x} \right\}$$

**2.1181      ODE No. 1181**

---

$$x^2 y''(x) + (3x - 1) y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0308515 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-1/x}}{x} - \frac{c_2 e^{-1/x} \text{Ei}\left(\frac{1}{x}\right)}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 25

$$\left\{ y(x) = \frac{1}{x e^{x^{-1}}} (\_C1 \, Ei(1, -x^{-1}) + \_C2) \right\}$$

**2.1182      ODE No. 1182**

---

$$x^2 y''(x) - 3xy'(x) + 4y(x) - 5x = 0$$

✓ **Mathematica** : cpu = 0.0113202 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow c_1 x^2 + 2c_2 x^2 \log(x) + 5x \right\} \right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 20

$$\{y(x) = x^2 \_C2 + x^2 \ln(x) \_C1 + 5x\}$$



## 2.1183 ODE No. 1183

---

$$x^2 y''(x) + x^2(-\log(x)) - 3xy'(x) - 5y(x) = 0$$

✓ **Mathematica** : cpu = 0.0070076 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow c_2 x^5 + \frac{c_1}{x} - \frac{1}{9} x^2 \log(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 22

$$\left\{ y(x) = x^5 \_C2 + \frac{\_C1}{x} - \frac{x^2 \ln(x)}{9} \right\}$$

## 2.1184 ODE No. 1184

---

$$-x^4 + x^2 y''(x) + x^2 - 4xy'(x) + 6y(x) = 0$$

✓ **Mathematica** : cpu = 0.0065802 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow c_2 x^3 + c_1 x^2 + \frac{1}{2} (x^4 + 2x^2 + 2x^2 \log(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 25

$$\left\{ y(x) = \frac{x^2 (2x \_C2 + x^2 + 2 \ln(x) + 2 \_C1 + 2)}{2} \right\}$$

## 2.1185 ODE No. 1185

---

$$-(2x^3 - 4) y(x) + x^2 y''(x) + 5xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.026313 (sec), leaf count = 67

$$\left\{ \left\{ y(x) \rightarrow \frac{3 \sqrt[3]{6} c_2 K_0\left(\frac{2}{3} \sqrt{2} x^{3/2}\right)}{x^2} - \frac{3 \sqrt[3]{-3} c_1 I_0\left(\frac{2}{3} \sqrt{2} x^{3/2}\right)}{2^{2/3} x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 33

$$\left\{ y(x) = \frac{1}{x^2} \left( \_C2 K_0\left(\frac{2 \sqrt{2}}{3} x^{\frac{3}{2}}\right) + \_C1 I_0\left(\frac{2 \sqrt{2}}{3} x^{\frac{3}{2}}\right) \right) \right\}$$

## 2.1186 ODE No. 1186

---

$$x^3(-\sin(x)) + x^2 y''(x) - 5xy'(x) + 8y(x) = 0$$

✓ **Mathematica** : cpu = 0.0137269 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} (x^4 \text{Ci}(x) - x^3 \sin(x) + x^2 \cos(x)) + c_2 x^4 + c_1 x^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 36

$$\left\{ y(x) = \frac{x^4 \text{Ci}(x)}{2} - \frac{\sin(x) x^3}{2} + \frac{x^2 (2 \_C1 x^2 + 2 \_C2 + \cos(x))}{2} \right\}$$

## 2.1187 ODE No. 1187

---

$$axy'(x) + by(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0084673 (sec), leaf count = 99

$$\left\{ \left\{ y(x) \rightarrow c_1 x^{\frac{1}{2} \sqrt{b} \left( -\frac{\sqrt{a^2 - 2a - 4b + 1}}{\sqrt{b}} - \frac{a-1}{\sqrt{b}} \right)} + c_2 x^{\frac{1}{2} \sqrt{b} \left( \frac{\sqrt{a^2 - 2a - 4b + 1}}{\sqrt{b}} - \frac{a-1}{\sqrt{b}} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 53

$$\left\{ y(x) = \_C1 x^{-\frac{a}{2} + \frac{1}{2} + \frac{1}{2} \sqrt{a^2 - 2a - 4b + 1}} + \_C2 x^{-\frac{a}{2} + \frac{1}{2} - \frac{1}{2} \sqrt{a^2 - 2a - 4b + 1}} \right\}$$

2.1188 ODE No. 1188

---

$$(ax + b)y'(x) + cy(x) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.10789 (sec), leaf count = 266

$$\left\{ \left\{ y(x) \rightarrow c_1 i^{-\sqrt{a^2-2a-4c+1}+a-1} b^{\frac{1}{2}(-\sqrt{a^2-2a-4c+1}+a-1)} \left(\frac{1}{x}\right)^{\frac{1}{2}(-\sqrt{a^2-2a-4c+1}+a-1)} {}_1F_1\left(\frac{a}{2}-\frac{1}{2}\sqrt{a^2-2a-4c+1}, \frac{b}{x}\right) \right\} \right.$$

✓ **Maple** : cpu = 0.122 (sec), leaf count = 114

$$\left\{ y(x) = x^{-\frac{1}{2}\sqrt{a^2-2a-4c+1}-\frac{a}{2}+\frac{1}{2}} \left( M\left(-\frac{1}{2}+\frac{1}{2}\sqrt{a^2-2a-4c+1}+\frac{a}{2}, 1+\sqrt{a^2-2a-4c+1}, \frac{b}{x}\right) {}_2F_2\left(\frac{a}{2}, \frac{a}{2}, \frac{b}{x}, \frac{b}{x}\right) - C1 + U\left(\frac{a}{2}, \frac{a}{2}, \frac{b}{x}, \frac{b}{x}\right) \right) \right.$$

2.1189 ODE No. 1189

---

$$axy'(x) + y(x)(bx^m + c) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0492563 (sec), leaf count = 445

$$\left\{ \left\{ y(x) \rightarrow c_1 m^{-\frac{-\sqrt{a^2-2a-4c+1}-a+1}{m}-\frac{\sqrt{a^2-2a-4c+1}}{m}} b^{\frac{-\sqrt{a^2-2a-4c+1}-a+1}{2m}+\frac{\sqrt{a^2-2a-4c+1}}{2m}} (x^m)^{\frac{-\sqrt{a^2-2a-4c+1}-a+1}{2m}+\frac{\sqrt{a^2-2a-4c+1}}{2m}} \right\} \right.$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 79

$$\left\{ y(x) = x^{-\frac{a}{2}+\frac{1}{2}} \left( Y_{\frac{1}{m}\sqrt{a^2-2a-4c+1}}\left(2\frac{\sqrt{b}x^{m/2}}{m}\right) {}_2F_2\left(\frac{a}{2}, \frac{a}{2}, \frac{b}{x}, \frac{b}{x}\right) - C2 + J_{\frac{1}{m}\sqrt{a^2-2a-4c+1}}\left(2\frac{\sqrt{b}x^{m/2}}{m}\right) {}_2F_2\left(\frac{a}{2}, \frac{a}{2}, \frac{b}{x}, \frac{b}{x}\right) - C1 \right) \right\}$$

2.1190 ODE No. 1190

---

$$y(x)(ax + b) + x^2y''(x) + x^2y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0218274 (sec), leaf count = 122

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{2}((\sqrt{1-4b}+1)\log(x)-2x)} U\left(\frac{1}{2}(-2a+\sqrt{1-4b}+1), \sqrt{1-4b}+1, x\right) + c_2 e^{\frac{1}{2}((\sqrt{1-4b}+1)\log(x)-2x)} L_{\frac{1}{2}(\frac{x}{2})}^{\sqrt{1-4b}+1}\left(\frac{x}{2}\right) \right\} \right.$$

✓ **Maple** : cpu = 0.073 (sec), leaf count = 38

$$\left\{ y(x) = e^{-\frac{x}{2}} \left( W_{a, \frac{1}{2}\sqrt{1-4b}}(x) {}_2F_2\left(\frac{a}{2}, \frac{a}{2}, \frac{b}{x}, \frac{b}{x}\right) - C2 + M_{a, \frac{1}{2}\sqrt{1-4b}}(x) {}_2F_2\left(\frac{a}{2}, \frac{a}{2}, \frac{b}{x}, \frac{b}{x}\right) - C1 \right) \right\}$$

2.1191 ODE No. 1191

---

$$x^2y''(x) + x^2y'(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0071295 (sec), leaf count = 110

$$\left\{ \left\{ y(x) \rightarrow \frac{2c_2 e^{\frac{1}{2}(\log(x)-x)} \left( i \sinh\left(\frac{x}{2}\right) - \frac{2i \cosh\left(\frac{x}{2}\right)}{x} \right)}{\sqrt{\pi}\sqrt{-ix}} + \frac{2c_1 e^{\frac{1}{2}(\log(x)-x)} \left( \frac{2 \sinh\left(\frac{x}{2}\right)}{x} - \cosh\left(\frac{x}{2}\right) \right)}{\sqrt{\pi}\sqrt{-ix}} \right\} \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 23

$$\left\{ y(x) = \frac{{}_2F_2\left(\frac{a}{2}, \frac{a}{2}, \frac{b}{x}, \frac{b}{x}\right) - C2}{x} + \frac{{}_2F_2\left(\frac{a}{2}, \frac{a}{2}, \frac{b}{x}, \frac{b}{x}\right) - C1}{x} \right\}$$

2.1192 ODE No. 1192

---

$$x^2y''(x) + (x^2 - 1)y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0616836 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow c_2 e^{-x} \int_1^x e^{K[1]-\frac{1}{K[1]}} dK[1] + c_1 e^{-x} \right\} \right\}$$

✓ **Maple** : cpu = 0.103 (sec), leaf count = 51

$$\left\{ y(x) = \sqrt{x} \left( e^{-x} HeunD\left(4, 3, -8, 5, \frac{x-1}{1+x}\right) {}_2F_2\left(\frac{a}{2}, \frac{a}{2}, \frac{b}{x}, \frac{b}{x}\right) - C1 + e^{-x^{-1}} HeunD\left(-4, 3, -8, 5, \frac{x-1}{1+x}\right) {}_2F_2\left(\frac{a}{2}, \frac{a}{2}, \frac{b}{x}, \frac{b}{x}\right) - C2 \right) \right\}$$

## 2.1193 ODE No. 1193

---

$$x^2 y''(x) + (x+1)xy'(x) + (x-9)y(x) = 0$$

✓ **Mathematica** : cpu = 0.19406 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1((x-8)x+20)}{x^3} - \frac{c_2 e^{-x}(x^3+9x^2+36x+60)}{x^3} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 38

$$\left\{ y(x) = \frac{-C2(x^3+9x^2+36x+60)e^{-x} - C1(x^2-8x+20)}{x^3} \right\}$$

## 2.1194 ODE No. 1194

---

$$x^2 y''(x) + (x+1)xy'(x) + (3x-1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0924335 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-x}(x-3)x - \frac{c_2 e^{-x}(x^3(-\text{Ei}(x)) + 3x^2 \text{Ei}(x) + e^x x^2 - 2e^x x - e^x)}{6x} \right\} \right\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 48

$$\left\{ y(x) = \frac{x^2 - C2 e^{-x}(x-3) \text{Ei}(1, -x) - C1 x^2 (x-3) e^{-x} - C2 (x^2 - 2x - 1)}{x} \right\}$$

## 2.1195 ODE No. 1195

---

$$x^2 y''(x) + (x+3)xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0209772 (sec), leaf count = 80

$$\left\{ \left\{ y(x) \rightarrow c_1 U\left(2+\sqrt{2}, 1+2\sqrt{2}, x\right) e^{(\sqrt{2}-1)\log(x)-x} + c_2 L_{-2-\sqrt{2}}^{2\sqrt{2}}(x) e^{(\sqrt{2}-1)\log(x)-x} \right\} \right\}$$

✓ **Maple** : cpu = 0.083 (sec), leaf count = 93

$$\left\{ y(x) = -1e^{-\frac{x}{2}} \left( -C1 \left( \sqrt{2} + x + 1 \right) I_{-\frac{1}{2}+\sqrt{2}}\left(\frac{x}{2}\right) - C1 \left( -\sqrt{2} + x + 1 \right) I_{\frac{1}{2}+\sqrt{2}}\left(\frac{x}{2}\right) + C2 \left( (-x-\sqrt{2}-1) \right) \right) \right\}$$

## 2.1196 ODE No. 1196

---

$$x^2 y''(x) - (x-1)xy'(x) + (x-1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.02644 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2(x^2 \text{Ei}(x) - e^x x - e^x)}{2x} + c_1 x \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 31

$$\left\{ y(x) = \frac{\text{Ei}(1, -x) - C2 x^2 - C2 (1+x) e^x - C1 x^2}{x} \right\}$$

## 2.1197 ODE No. 1197

---

$$-(a+x)y(x) + x^2 y''(x) - (x^2 - 2x) y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0133549 (sec), leaf count = 78

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{2}(x-\log(x))} J_{\frac{1}{2}\sqrt{4a+1}}\left(-\frac{ix}{2}\right) + c_2 e^{\frac{1}{2}(x-\log(x))} Y_{\frac{1}{2}\sqrt{4a+1}}\left(-\frac{ix}{2}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 43

$$\left\{ y(x) = 1e^{\frac{x}{2}} \left( K_{\frac{1}{2}\sqrt{4a+1}}\left(\frac{x}{2}\right) - C2 + I_{\frac{1}{2}\sqrt{4a+1}}\left(\frac{x}{2}\right) - C1 \right) \frac{1}{\sqrt{x}} \right\}$$

**2.1198      ODE No. 1198**

---

$$x^2 y''(x) - (x^2 - 2x) y'(x) - (3x + 2)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0344521 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow c_1 e^x x - \frac{c_2 (e^x x^3 \text{Ei}(-x) + x^2 - x + 2)}{6x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.028 (sec), leaf count = 37

$$\left\{ y(x) = \frac{e^x \text{Ei}(1, x) - C2 x^3 + -C1 x^3 e^x - C2 (x^2 - x + 2)}{x^2} \right\}$$

**2.1199      ODE No. 1199**

---

$$x^2 y''(x) - (x + 4)xy'(x) + 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.0130047 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow c_2 e^x x^4 - \frac{1}{6} c_1 x (e^x x^3 \text{Ei}(-x) + x^2 - x + 2) \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 35

$$\{y(x) = x (e^x \text{Ei}(1, x) - C2 x^3 + -C1 x^3 e^x - C2 (x^2 - x + 2))\}$$

**2.1200      ODE No. 1200**

---

$$-(v - 1)vy(x) + x^2 y''(x) + 2x^2 y'(x) = 0$$

✓ **Mathematica** : cpu = 0.015292 (sec), leaf count = 62

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-x} \sqrt{x} J_{\frac{1}{2}(2v-1)}(-ix) + c_2 e^{-x} \sqrt{x} Y_{\frac{1}{2}(2v-1)}(-ix) \right\} \right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 27

$$\left\{ y(x) = \sqrt{x} e^{-x} \left( K_{v-\frac{1}{2}}(x) - C2 + I_{v-\frac{1}{2}}(x) - C1 \right) \right\}$$

**2.1201      ODE No. 1201**

---

$$x^2 y''(x) + (2x + 1)xy'(x) - 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.106754 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-2x} (2x + 3)}{2x^2} + \frac{c_2 (2x^2 - 4x + 3)}{4x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.015 (sec), leaf count = 34

$$\left\{ y(x) = \frac{-C1 (2x^2 - 4x + 3)}{x^2} + \frac{-C2 e^{-2x} (2x + 3)}{x^2} \right\}$$

**2.1202      ODE No. 1202**

---

$$x^2 y''(x) - 2(x + 1)xy'(x) + 2(x + 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0108188 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow c_1 x + \frac{1}{2} c_2 e^{2x} x \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 14

$$\{y(x) = x (e^{2x} - C2 + -C1)\}$$

2.1203 ODE No. 1203

---

$$ax^2y'(x) + x^2y''(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.015127 (sec), leaf count = 124

$$\left\{ \left\{ y(x) \rightarrow \frac{2c_2 e^{\frac{1}{2}(\log(x)-ax)} \left( i \sinh\left(\frac{ax}{2}\right) - \frac{2i \cosh\left(\frac{ax}{2}\right)}{ax} \right)}{\sqrt{\pi} \sqrt{-iax}} + \frac{2c_1 e^{\frac{1}{2}(\log(x)-ax)} \left( \frac{2 \sinh\left(\frac{ax}{2}\right)}{ax} - \cosh\left(\frac{ax}{2}\right) \right)}{\sqrt{\pi} \sqrt{-iax}} \right\} \right\}$$

✓ **Maple** : cpu = 0.015 (sec), leaf count = 28

$$\left\{ y(x) = \frac{{}_-C2 \, (ax+2) \, \mathrm{e}^{-ax} + {}_-C1 \, (ax-2)}{x} \right\}$$

2.1204 ODE No. 1204

---

$$x^2(a+2b)y'(x) + y(x) \left( bx^2(a+b) - 2 \right) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0159988 (sec), leaf count = 132

$$\left\{ \left\{ y(x) \rightarrow \frac{2c_2 e^{\frac{1}{2}(-ax-2bx+\log(x))} \left( i \sinh\left(\frac{ax}{2}\right) - \frac{2i \cosh\left(\frac{ax}{2}\right)}{ax} \right)}{\sqrt{\pi} \sqrt{-iax}} + \frac{2c_1 e^{\frac{1}{2}(-ax-2bx+\log(x))} \left( \frac{2 \sinh\left(\frac{ax}{2}\right)}{ax} - \cosh\left(\frac{ax}{2}\right) \right)}{\sqrt{\pi} \sqrt{-iax}} \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 35

$$\left\{ y(x) = \frac{{}_-C2 \, (ax+2) \, \mathrm{e}^{-(a+b)x} + {}_-C1 \, \mathrm{e}^{-bx} \, (ax-2)}{x} \right\}$$

2.1205 ODE No. 1205

---

$$ax^2y'(x) + f(x)y(x) + x^2y''(x) = 0$$

✗ **Mathematica** : cpu = 0.221293 (sec), leaf count = 0 , could not solve  
DSolve[f[x]\*y[x] + a\*x^2\*Derivative[1][y][x] + x^2\*Derivative[2][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{f(x) {}_ -Y(x)}{x^2} + a \frac{\mathrm{d}}{\mathrm{d}x} {}_ -Y(x) + \frac{\mathrm{d}^2}{\mathrm{d}x^2} {}_ -Y(x) \right\}, \{ {}_ -Y(x) \} \right) \right\}$$

2.1206 ODE No. 1206

---

$$y(x) \left( abx + cx^2 + d \right) + x(2ax + b)y'(x) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0752464 (sec), leaf count = 120

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{2}(-2ax-(b-1)\log(x))} J_{\frac{1}{2}\sqrt{b^2-2b-4d+1}} \left( -i\sqrt{a^2-cx} \right) + c_2 e^{\frac{1}{2}(-2ax-(b-1)\log(x))} Y_{\frac{1}{2}\sqrt{b^2-2b-4d+1}} \left( -i\sqrt{a^2-cx} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 76

$$\left\{ y(x) = \mathrm{e}^{-ax} x^{-\frac{b}{2}+\frac{1}{2}} \left( Y_{\frac{1}{2}\sqrt{b^2-2b-4d+1}} \left( \sqrt{-a^2+cx} \right) {}_ -C2 + J_{\frac{1}{2}\sqrt{b^2-2b-4d+1}} \left( \sqrt{-a^2+cx} \right) {}_ -C1 \right) \right\}$$

2.1207 ODE No. 1207

---

$$x(ax+b)y'(x) + y(x) \left( a1x^2 + b1x + c1 \right) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0837621 (sec), leaf count = 294

$$\left\{ \left\{ y(x) \rightarrow c_1 U \left( -\frac{-ab+2b1-\sqrt{a^2-4a1}-\sqrt{a^2-4a1}\sqrt{b^2-2b-4c1+1}}{2\sqrt{a^2-4a1}}, \sqrt{b^2-2b-4c1+1}+1, \sqrt{a^2-4a1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.159 (sec), leaf count = 110

$$\left\{ y(x) = \mathrm{e}^{-\frac{ax}{2}} x^{-\frac{b}{2}} \left( M_{-\frac{ab-2b1}{2}, \frac{1}{2}\sqrt{b^2-2b-4c1+1}} \left( \sqrt{a^2-4a1x} \right) {}_ -C1 + W_{-\frac{ab-2b1}{2}, \frac{1}{2}\sqrt{b^2-2b-4c1+1}} \left( \sqrt{a^2-4a1x} \right) \right) \right\}$$

2.1208 ODE No. 1208

---

$$x^3y'(x) + x^2y''(x) + (x^2 - 2)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0370824 (sec), leaf count = 59

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 e^{-\frac{x^2}{2}} \left( \sqrt{2\pi} e^{\frac{x^2}{2}} \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right) - 2x \right)}{2x} + \frac{c_1}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 35

$$\left\{ y(x) = \frac{1}{x} \left( \operatorname{Erf}\left(\frac{\sqrt{2}x}{2}\right) \sqrt{2}\sqrt{\pi} C2 - 2e^{-1/2x^2} C2 x + C1 \right) \right\}$$

2.1209 ODE No. 1209

---

$$x^2y''(x) + (x^2 + 2)xy'(x) + (x^2 - 2)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0174899 (sec), leaf count = 67

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-\frac{x^2}{2}} \left( e^{\frac{x^2}{2}} x - \sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right) \right)}{x^2} + \frac{c_2 e^{-\frac{x^2}{2}}}{x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 40

$$\left\{ y(x) = \frac{1}{x^2} \left( \left( \operatorname{Erf}\left(\frac{i}{2}\sqrt{2}x\right) \pi C2 + C1 \right) e^{-\frac{x^2}{2}} - i\sqrt{2}\sqrt{\pi} C2 x \right) \right\}$$

2.1210 ODE No. 1210

---

$$y(x) \left( a((-1)^n - 1) + 2nx^2 \right) - 2x(x^2 - a)y'(x) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.195225 (sec), leaf count = 252

$$\left\{ \left\{ y(x) \rightarrow c_1 (-1)^{\frac{1}{4}} \left( -\sqrt{4a^2 - 4a(-1)^{n+1} - 2a + 1} \right) x^{\frac{1}{2}} \left( -\sqrt{4a^2 - 4a(-1)^{n+1} - 2a + 1} \right) {}_1F_1\left(-\frac{a}{2} - \frac{n}{2} - \frac{1}{4}\sqrt{4a^2 - 4(-1)^n a + 1}, x\right) \right. \right.$$

✓ **Maple** : cpu = 0.146 (sec), leaf count = 81

$$\left. \left\{ y(x) = e^{\frac{x^2}{2}} x^{-\frac{1}{2}-a} \left( M_{\frac{n}{2}+\frac{a}{2}+\frac{1}{4}, \frac{1}{4}\sqrt{1-4(-1)^n a+4a^2}}(x^2) C1 + W_{\frac{n}{2}+\frac{a}{2}+\frac{1}{4}, \frac{1}{4}\sqrt{1-4(-1)^n a+4a^2}}(x^2) C2 \right) \right\} \right\}$$

2.1211 ODE No. 1211

---

$$4x^3y'(x) + x^2y''(x) + (4x^4 + 2x^2 + 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0391403 (sec), leaf count = 68

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-x^2} x^{\frac{1}{2}-\frac{i\sqrt{3}}{2}} - \frac{ic_2 e^{-x^2} x^{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}{\sqrt{3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 36

$$\left\{ y(x) = e^{-x^2} \left( x^{\frac{1}{2}-\frac{i}{2}\sqrt{3}} C2 + x^{\frac{1}{2}+\frac{i}{2}\sqrt{3}} C1 \right) \right\}$$

2.1212 ODE No. 1212

---

$$x(ax^2 + b)y'(x) + f(x)y(x) + x^2y''(x) = 0$$

✗ **Mathematica** : cpu = 0.404678 (sec), leaf count = 0 , could not solve  
DSolve[f[x]\*y[x] + x\*(b + a\*x^2)\*Derivative[1][y][x] + x^2\*Derivative[2][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol\left(\left\{\frac{f(x)Y(x)}{x^2} + \frac{(ax^2 + b)\frac{d}{dx}Y(x)}{x} + \frac{d^2}{dx^2}Y(x)\right\}, \{Y(x)\}\right)\right\}$$

**2.1213                    ODE No. 1213**

---

$$(x^3 + 1) x y'(x) + x^2 y''(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0639934 (sec), leaf count = 54

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{3} c_1 {}_1F_1\left(-\frac{1}{3}; \frac{1}{3}; -\frac{x^3}{3}\right)}{x} + \frac{c_2 x {}_1F_1\left(\frac{1}{3}; \frac{5}{3}; -\frac{x^3}{3}\right)}{\sqrt[3]{3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 53

$$\left\{ y(x) = \left( -C1 I_{-\frac{1}{6}}\left(\frac{x^3}{6}\right) + -C1 I_{\frac{5}{6}}\left(\frac{x^3}{6}\right) - C2 \left( K_{\frac{1}{6}}\left(\frac{x^3}{6}\right) - K_{\frac{5}{6}}\left(\frac{x^3}{6}\right) \right) \right) e^{-\frac{x^3}{6} x^{\frac{3}{2}}} \right\}$$

**2.1214                    ODE No. 1214**

---

$$y(x) \left( -a^2 + x^2(2a + 2n + 1) + a(-1)^n - x^4 \right) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.209819 (sec), leaf count = 260

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-\frac{x^2}{2}} 2^{\frac{1}{4}(\sqrt{4a^2 - 4a(-1)^{n+1}} + 2)} (x^2)^{\frac{1}{4}(\sqrt{4a^2 - 4a(-1)^{n+1}} + 2)} U\left(\frac{1}{4}(-2a - 2n + \sqrt{4a^2 - 4(-1)^n a + 1} + 1)\right)}{\sqrt{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.112 (sec), leaf count = 71

$$\left\{ y(x) = 1 \left( -C1 M_{\frac{n}{2} + \frac{a}{2} + \frac{1}{4}, \frac{1}{4}\sqrt{1-4(-1)^n a + 4a^2}}(x^2) + -C2 W_{\frac{n}{2} + \frac{a}{2} + \frac{1}{4}, \frac{1}{4}\sqrt{1-4(-1)^n a + 4a^2}}(x^2) \right) \frac{1}{\sqrt{x}} \right\}$$

**2.1215                    ODE No. 1215**

---

$$x y'(x) (a x^n + b) + y(x) (a_1 x^{2n} + b_1 x^n + c_1) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.111124 (sec), leaf count = 664

$$\left\{ \left\{ y(x) \rightarrow c_1 x^{\frac{1-n}{2}} 2^{\frac{\sqrt{b^2 n^2 - 2b n^2 - 4c_1 n^2 + n^2} + n^2}{2n^2}} (x^n)^{\frac{\sqrt{b^2 n^2 - 2b n^2 - 4c_1 n^2 + n^2} + n^2}{2n^2}} \exp\left(\frac{1}{2}\left(-\frac{a x^n}{n} - b \log(x)\right) - \frac{\sqrt{a^2 - 4a_1} x^n}{2n}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.114 (sec), leaf count = 148

$$\left\{ y(x) = e^{-\frac{a x^n}{2n} x^{-\frac{b}{2} - \frac{n}{2} + \frac{1}{2}}} \left( W_{-\frac{(b+n-1)a-2b_1}{2n} \frac{1}{\sqrt{a^2-4a_1}}, \frac{1}{2n}\sqrt{b^2-2b-4c_1+1}}\left(\frac{x^n}{n}\sqrt{a^2-4a_1}\right) - C2 + M_{-\frac{(b+n-1)a-2b_1}{2n} \frac{1}{\sqrt{a^2-4a_1}}} \right) \right\}$$

**2.1216                    ODE No. 1216**

---

$$x y'(x) (a x^{a_1} + b) + y(x) (A x^{2a_1} + B x^{a_1} + C x^{b_1} + DD) + x^2 y''(x) = 0$$

✗ **Mathematica** : cpu = 1.12322 (sec), leaf count = 0 , could not solve  
DSolve[(DD + B\*x^a1 + A\*x^(2\*a1) + C\*x^b1)\*y[x] + x\*(b + a\*x^a1)\*Derivative[1][y][x] + x^2\*D[y[x],x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol\left(\left\{\frac{d^2}{dx^2} Y(x) + \frac{(a x^{a_1} + b) \frac{d}{dx} Y(x)}{x} + \frac{(A x^{2a_1} + B x^{a_1} + C x^{b_1} + DD) Y(x)}{x^2}\right\}, \{Y(x)\}\right) \right\}$$

## 2.1217 ODE No. 1217

---

$$-y(x)(a + x \tan(x)) + x^2 y''(x) - (2x^2 \tan(x) - x) y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0978416 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow c_1 \sec(x) J_{\sqrt{a}}(x) + c_2 \sec(x) Y_{\sqrt{a}}(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 24

$$\left\{ y(x) = \frac{1}{\cos(x)} \left( -C1 J_{\sqrt{a}}(x) + -C2 Y_{\sqrt{a}}(x) \right) \right\}$$

## 2.1218 ODE No. 1218

---

$$y(x)(a + x \cot(x)) + x^2 y''(x) + (2x^2 \cot(x) + x) y'(x) = 0$$

✓ **Mathematica** : cpu = 0.100488 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow c_1 \csc(x) J_{i\sqrt{a}}(x) + c_2 \csc(x) Y_{i\sqrt{a}}(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 30

$$\left\{ y(x) = \frac{1}{\sin(x)} \left( -C2 Y_{i\sqrt{a}}(x) + -C1 J_{i\sqrt{a}}(x) \right) \right\}$$

## 2.1219 ODE No. 1219

---

$$y(x) (ax^2 + bx + c + x f'(x) + f(x)^2 - f(x)) + 2x f(x) y'(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.102068 (sec), leaf count = 218

$$\left\{ \left\{ y(x) \rightarrow c_1 U \left( -\frac{-ib - \sqrt{a} - \sqrt{a}\sqrt{1-4c}}{2\sqrt{a}}, \sqrt{1-4c} + 1, 2i\sqrt{ax} \right) \exp \left( \int_1^x \frac{-2f(K[1]) - 2i\sqrt{a}K[1] + \sqrt{1-4c}}{2K[1]} dx \right) \right. \right.$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 69

$$\left\{ y(x) = e^{-\int \frac{f(x)}{x} dx} \left( W_{-\frac{i}{2}b\frac{1}{\sqrt{a}}, \frac{1}{2}\sqrt{1-4c}}(2i\sqrt{ax}) - C2 + M_{-\frac{i}{2}b\frac{1}{\sqrt{a}}, \frac{1}{2}\sqrt{1-4c}}(2i\sqrt{ax}) - C1 \right) \right\}$$

## 2.1220 ODE No. 1220

---

$$y(x) (x^2 (a + f'(x) + f(x)^2) - (v-1)v) + 2x^2 f(x) y'(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.040686 (sec), leaf count = 98

$$\left\{ \left\{ y(x) \rightarrow c_1 J_{\frac{1}{2}(2v-1)}(\sqrt{ax}) \exp \left( \int_1^x \frac{1 - 2f(K[1])K[1]}{2K[1]} dK[1] \right) + c_2 Y_{\frac{1}{2}(2v-1)}(\sqrt{ax}) \exp \left( \int_1^x \frac{1 - 2f(K[1])K[1]}{2K[1]} dK[1] \right) \right. \right.$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 40

$$\left\{ y(x) = \sqrt{x} e^{-\frac{\int 2f(x) dx}{2}} \left( Y_{v-\frac{1}{2}}(\sqrt{ax}) - C2 + J_{v-\frac{1}{2}}(\sqrt{ax}) - C1 \right) \right\}$$

## 2.1221 ODE No. 1221

---

$$y(x) (x^2 (-f'(x) + f(x)^2 + 1) - x f(x) - v^2) + (x - 2x^2 f(x)) y'(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0232734 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow c_1 J_v(x) \exp \left( \int_1^x f(K[1]) dK[1] \right) + c_2 Y_v(x) \exp \left( \int_1^x f(K[1]) dK[1] \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 35

$$\left\{ y(x) = \sqrt{x} e^{-\frac{1}{2} \int \frac{-2xf(x)+1}{x} dx} (J_v(x) - C1 + Y_v(x) - C2) \right\}$$



**2.1222      ODE No. 1222**

---

$$(x^2 + 1) y''(x) + xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0143825 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin \left( \sqrt{2} \sinh^{-1}(x) \right) + c_1 \cos \left( \sqrt{2} \sinh^{-1}(x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 23

$$\left\{ y(x) = \_C1 \sin \left( \sqrt{2} \operatorname{Arcsinh}(x) \right) + \_C2 \cos \left( \sqrt{2} \operatorname{Arcsinh}(x) \right) \right\}$$

**2.1223      ODE No. 1223**

---

$$(x^2 + 1) y''(x) + xy'(x) - 9y(x) = 0$$

✓ **Mathematica** : cpu = 0.0131393 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow c_1 \cosh \left( 3 \sinh^{-1}(x) \right) + ic_2 \sinh \left( 3 \sinh^{-1}(x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 19

$$\{y(x) = \_C1 \sinh(3 \operatorname{Arcsinh}(x)) + \_C2 \cosh(3 \operatorname{Arcsinh}(x))\}$$

**2.1224      ODE No. 1224**

---

$$ay(x) + (x^2 + 1) y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0130039 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin \left( \sqrt{a} \sinh^{-1}(x) \right) + c_1 \cos \left( \sqrt{a} \sinh^{-1}(x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 23

$$\{y(x) = \_C1 \sin(\sqrt{a} \operatorname{Arcsinh}(x)) + \_C2 \cos(\sqrt{a} \operatorname{Arcsinh}(x))\}$$

**2.1225      ODE No. 1225**

---

$$(x^2 + 1) y''(x) - xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0197583 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow c_2 \left( x \sinh^{-1}(x) - \sqrt{x^2 + 1} \right) + c_1 x \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 23

$$\left\{ y(x) = -\sqrt{x^2 + 1} \_C2 + x (\_C2 \operatorname{Arcsinh}(x) + \_C1) \right\}$$

**2.1226      ODE No. 1226**

---

$$-(v - 1)vy(x) + (x^2 + 1) y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0124397 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow c_1 P_{v-1}(ix) + c_2 Q_{v-1}(ix) \right\} \right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 25

$$\{y(x) = \_C1 \operatorname{LegendreP}(v - 1, ix) + \_C2 \operatorname{LegendreQ}(v - 1, ix)\}$$

**2.1227      ODE No. 1227**

---

$$(x^2 + 1) y''(x) - 2xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0242813 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow c_2 x - c_1 (x - i)^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 16

$$\{y(x) = \_C2 x^2 + \_C1 x - \_C2\}$$

**2.1228                      ODE No. 1228**

---

$$ay(x) + (x^2 + 1) y''(x) + 3xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0111699 (sec), leaf count = 82

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 P_{\frac{1}{2}(2\sqrt{1-a}-1)}(ix)}{\sqrt[4]{x^2+1}} + \frac{c_2 Q_{\frac{1}{2}(2\sqrt{1-a}-1)}(ix)}{\sqrt[4]{x^2+1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.063 (sec), leaf count = 53

$$\left\{ y(x) = 1 \left( -C2 \left( x + \sqrt{x^2 + 1} \right)^{-\sqrt{1-a}} + -C1 \left( x + \sqrt{x^2 + 1} \right)^{\sqrt{1-a}} \right) \frac{1}{\sqrt{x^2 + 1}} \right\}$$

**2.1229                      ODE No. 1229**

---

$$(x^2 + 1) y''(x) + 4xy'(x) + 2y(x) + 2x - 2 \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.027938 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{x^2 + 1} + \frac{c_2 x}{x^2 + 1} + \frac{-x^3 - 6 \cos(x)}{3(x^2 + 1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 31

$$\left\{ y(x) = \frac{-x^3 + 3 - C1 x - 6 \cos(x) + 3 - C2}{3x^2 + 3} \right\}$$

**2.1230                      ODE No. 1230**

---

$$axy'(x) + (a - 2)y(x) + (x^2 + 1) y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0183465 (sec), leaf count = 82

$$\left\{ \left\{ y(x) \rightarrow c_1 (x^2 + 1)^{\frac{2-a}{4}} P_{\frac{a-2}{\frac{a-4}{2}}}(ix) + c_2 (x^2 + 1)^{\frac{2-a}{4}} Q_{\frac{a-2}{\frac{a-4}{2}}}(ix) \right\} \right\}$$

✓ **Maple** : cpu = 0.079 (sec), leaf count = 36

$$\left\{ y(x) = -C1 (x^2 + 1)^{1-\frac{a}{2}} + -C2 {}_2F_1\left(1, \frac{a}{2} - \frac{1}{2}; \frac{3}{2}; -x^2\right)x \right\}$$

**2.1231                      ODE No. 1231**

---

$$(x^2 - 1) y''(x) - v(v + 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0548795 (sec), leaf count = 58

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1\left(-\frac{v}{2} - \frac{1}{2}, \frac{v}{2}; \frac{1}{2}; x^2\right) + ic_2 x {}_2F_1\left(\frac{v}{2} + \frac{1}{2}, -\frac{v}{2}; \frac{3}{2}; x^2\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 52

$$\left\{ y(x) = -(x - 1)(1 + x) \left( {}_2F_1\left(1 - \frac{v}{2}, \frac{3}{2} + \frac{v}{2}; \frac{3}{2}; x^2\right) - C2 x + -C1 {}_2F_1\left(1 + \frac{v}{2}, \frac{1}{2} - \frac{v}{2}; \frac{1}{2}; x^2\right) \right) \right\}$$

**2.1232                      ODE No. 1232**

---

$$\frac{nxP_n(x) - nP_{n-1}(x)}{x^2 - 1} - n(n + 1)y(x) + (x^2 - 1) y''(x) = 0$$

✓ **Mathematica** : cpu = 11.6511 (sec), leaf count = 6628

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1\left(-\frac{n}{2} - \frac{1}{2}, \frac{n}{2}; \frac{1}{2}; x^2\right) + \int_1^x \left( \frac{1}{4 \left( {}_2F_1\left(\frac{1}{2}(-n - 1), \frac{n}{2}; \frac{1}{2}; K[1]^2\right) {}_2F_1\left(1 - \frac{n}{2}, \frac{n+3}{2}; \frac{5}{2}; K[1]^2\right) n^2 + 2 \right)} \right. \right.$$

✓ **Maple** : cpu = 0.117 (sec), leaf count = 409

$$\left\{ y(x) = -3(1 + x) \left( -{}_2F_1(n/2 + 1, -n/2 + 1/2; 1/2; x^2)(n + 1) \int -1/3 \frac{1}{(1 + x)^3 (({}_2F_1(n/2 + 1, -n/2 + 1/2;$$



**2.1238 ODE No. 1238**

---

$$-a + (x^2 - 1) y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0166395 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}(a + c_1) \log(1 - x) + \frac{1}{2}(a - c_1) \log(x + 1) + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 26

$$\left\{ y(x) = \frac{(a - \_C1) \ln(1 + x)}{2} + \frac{(a + \_C1) \ln(x - 1)}{2} + \_C2 \right\}$$

**2.1239 ODE No. 1239**

---

$$-ly(x) + (x^2 - 1) y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.009825 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow c_1 P_{\frac{1}{2}(\sqrt{4l+1}-1)}(x) + c_2 Q_{\frac{1}{2}(\sqrt{4l+1}-1)}(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 35

$$\left\{ y(x) = \_C1 \operatorname{LegendreP}\left(\frac{1}{2}\sqrt{1+4l}-\frac{1}{2}, x\right) + \_C2 \operatorname{LegendreQ}\left(\frac{1}{2}\sqrt{1+4l}-\frac{1}{2}, x\right) \right\}$$

**2.1240 ODE No. 1240**

---

$$-v(v+1)y(x) + (x^2 - 1) y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0124625 (sec), leaf count = 18

$$\{\{y(x) \rightarrow c_1 P_v(x) + c_2 Q_v(x)\}\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 15

$$\{y(x) = \_C1 \operatorname{LegendreP}(v, x) + \_C2 \operatorname{LegendreQ}(v, x)\}$$

**2.1241 ODE No. 1241**

---

$$-(v-1)(v+2)y(x) + (x^2 - 1) y''(x) - 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0109906 (sec), leaf count = 30

$$\{\{y(x) \rightarrow c_1 (x^2 - 1) P_v^2(x) + c_2 (x^2 - 1) Q_v^2(x)\}\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 24

$$\{y(x) = (1+x)(x-1)(\_C1 \operatorname{LegendreP}(v, 2, x) + \_C2 \operatorname{LegendreQ}(v, 2, x))\}$$

**2.1242 ODE No. 1242**

---

$$(x^2 - 1) y''(x) - (x^2 - x) y(x) - (3x + 1) y'(x) = 0$$

✓ **Mathematica** : cpu = 0.201538 (sec), leaf count = 68

$$\{\{y(x) \rightarrow c_1 e^{-x}(x+1)^2 - c_2 e^{-x-2}(x^2(-\operatorname{Ei}(2(x+1))) - 2x\operatorname{Ei}(2(x+1)) - \operatorname{Ei}(2(x+1)) + 2e^{2x+2})\}\}$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 41

$$\left\{ y(x) = \_C2 e^{-2-x}(1+x)^2 \operatorname{Ei}(1, -2x-2) + \_C1 e^{-x}(1+x)^2 + 2e^x \_C2 \right\}$$

## 2.1243 ODE No. 1243

---

$$(x^2 - 1) y''(x) + (x^2 + 1) y(x) + 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0227192 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-ix}}{x^2 - 1} - \frac{ic_2 e^{ix}}{2(x^2 - 1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 21

$$\left\{ y(x) = \frac{{}_-C2 \cos(x) + {}_C1 \sin(x)}{x^2 - 1} \right\}$$

## 2.1244 ODE No. 1244

---

$$-(v - n)(n + v + 1)y(x) + 2(n + 1)xy'(x) + (x^2 - 1) y''(x) = 0$$

✓ **Mathematica** : cpu = 0.019699 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow c_1 (x^2 - 1)^{-n/2} P_v^n(x) + c_2 (x^2 - 1)^{-n/2} Q_v^n(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.055 (sec), leaf count = 27

$$\left\{ y(x) = (x^2 - 1)^{-\frac{n}{2}} (LegendreP(v, n, x) {}_C1 + LegendreQ(v, n, x) {}_C2) \right\}$$

## 2.1245 ODE No. 1245

---

$$-(-n + v + 1)(n + v)y(x) - 2(n - 1)xy'(x) + (x^2 - 1) y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0152274 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow c_1 (x^2 - 1)^{n/2} P_v^n(x) + c_2 (x^2 - 1)^{n/2} Q_v^n(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 27

$$\left\{ y(x) = (x^2 - 1)^{\frac{n}{2}} (LegendreP(v, n, x) {}_C1 + LegendreQ(v, n, x) {}_C2) \right\}$$

## 2.1246 ODE No. 1246

---

$$-2(v - 1)xy'(x) - 2vy(x) + (x^2 - 1) y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0141261 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow c_1 (x^2 - 1)^{v/2} P_v^v(x) + c_2 (x^2 - 1)^{v/2} Q_v^v(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.146 (sec), leaf count = 28

$$\left\{ y(x) = (x^2 - 1)^v \left( {}_2F_1\left(\frac{1}{2}, v + 1; \frac{3}{2}; x^2\right) {}_C2 x + {}_C1 \right) \right\}$$

## 2.1247 ODE No. 1247

---

$$2axy'(x) + (a - 1)ay(x) + (x^2 - 1) y''(x) = 0$$

✓ **Mathematica** : cpu = 0.154895 (sec), leaf count = 128

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \sqrt{1 - x^2} (x + 1)^{\sqrt{(a-1)^2}} (x^2 - 1)^{-a/2} (1 - x)^{-\sqrt{(a-1)^2}} e^{-\sqrt{(a-1)^2} \tanh^{-1}(x)}}{2\sqrt{(a-1)^2}} + c_1 \sqrt{1 - x^2} (x^2 - 1)^{-a/2} \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 27

$$\left\{ y(x) = {}_C1 (x - 1)^{1-a} + {}_C2 (1 + x)^{1-a} \right\}$$

## 2.1248 ODE No. 1248

$$ax y'(x) + y(x) (bx^2 + cx + d) + (x^2 - 1) y''(x) = 0$$

**X Mathematica :** cpu = 1.57644 (sec), leaf count = 0 , DifferentialRoot result

$$\{ \{ y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{ (bx^2 + cx + d)y(x) + xay'(x) + (x^2 - 1)y''(x) = 0, y(0) = c_1, y'(0) = c_2 \})$$

✓ **Maple** : cpu = 0.177 (sec), leaf count = 134

$$\left\{ y(x) = e^{\sqrt{-b}x} (x^2 - 1)^{-\frac{a}{4}} \left( \left( \frac{1}{2} + \frac{x}{2} \right)^{-\frac{a}{4}+1} HeunC \left( 4\sqrt{-b}, 1 - \frac{a}{2}, \frac{a}{2} - 1, 2c, d - c - \frac{a^2}{8} + b + \frac{1}{2}, \frac{1}{2} + \frac{x}{2} \right) \left( -\frac{1}{2} \right. \right.$$

## 2.1249      ODE No. 1249

$$(ax + b)y'(x) + cy(x) + (x^2 - 1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.116317 (sec), leaf count = 193

$$\left\{ \left\{ y(x) \rightarrow c_2 2^{\frac{1}{2}(a+b-2)} (x-1)^{\frac{1}{2}(-a-b+2)} {}_2F_1 \left( -\frac{b}{2} - \frac{1}{2} \sqrt{a^2 - 2a - 4c + 1} + \frac{1}{2}, -\frac{b}{2} + \frac{1}{2} \sqrt{a^2 - 2a - 4c + 1} + \frac{1}{2}; - \right. \right. \right.$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 134

$$\left\{ y(x) = {}_2F_1\left(-\frac{1}{2} + \frac{a}{2} - \frac{1}{2}\sqrt{a^2 - 2a - 4c + 1}, -\frac{1}{2} + \frac{a}{2} + \frac{1}{2}\sqrt{a^2 - 2a - 4c + 1}; \frac{a}{2} - \frac{b}{2}; \frac{1}{2} + \frac{x}{2}\right) + {}_2F_1\left(-\frac{1}{2} + \frac{a}{2} - \frac{1}{2}\sqrt{a^2 - 2a - 4c + 1}, -\frac{1}{2} + \frac{a}{2} + \frac{1}{2}\sqrt{a^2 - 2a - 4c + 1}; \frac{a}{2} - \frac{b}{2}; \frac{1}{2} + \frac{x}{2}\right) \right.$$

2.1250      ODE No. 1250

$$(x^2 - a^2) y''(x) + 8xy'(x) + 12y(x) = 0$$

✓ **Mathematica** : cpu = 0.0368101 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 (a^2 + 3x^2)}{3(a-x)^3(a+x)^3} + \frac{c_1}{(a+x)^3} \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 41

$$\left\{ y(x) = \frac{3\_C2\,a^2x +\_C2\,x^3 +\_C1\,a^2 + 3\_C1\,x^2}{(a-x)^3(x+a)^3} \right\}$$

2.1251      ODE No. 1251

$$x(x+1)y''(x) - (x-1)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0253234 (sec), leaf count = 25

$$\{\{y(x) \rightarrow c_1(x-1) + c_2(x \log(x) - \log(x) - 4)\}\}$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 20

$$\{y(x) = -C_2(x-1)\ln(x) - 4C_2 + C_1(x-1)\}$$

## 2.1252      ODE No. 1252

$$(ax + b)y'(x) + cy(x) + x(x + 1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.10678 (sec), leaf count = 151

$$\left\{ \left\{ y(x) \rightarrow c_2 x^{1-b} {}_2F_1 \left( \frac{a}{2} - b - \frac{1}{2} \sqrt{a^2 - 2a - 4c + 1} + \frac{1}{2}, \frac{a}{2} - b + \frac{1}{2} \sqrt{a^2 - 2a - 4c + 1} + \frac{1}{2}; 2 - b; -x \right) + c_1 {}_2F_1 \right. \right.$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 124

$$\left\{ y(x) = {}_2F_1\left(-\frac{1}{2} + \frac{a}{2} - \frac{1}{2}\sqrt{a^2 - 2a - 4c + 1}, -\frac{1}{2} + \frac{a}{2} + \frac{1}{2}\sqrt{a^2 - 2a - 4c + 1}; a - b; 1 + x\right) + {}_2F_1\left(-\frac{1}{2} + \frac{a}{2} - \frac{1}{2}\sqrt{a^2 - 2a - 4c + 1}, -\frac{1}{2} + \frac{a}{2} - \frac{1}{2}\sqrt{a^2 - 2a - 4c + 1}; a - b; 1 + x\right) \right.$$

2.1253 ODE No. 1253

---

$$x(x+1)y''(x)+(3x+2)y'(x)+y(x)=0$$

✓ **Mathematica** : cpu = 0.0192418 (sec), leaf count = 34

$$\left\{\left\{y(x)\rightarrow\frac{\sqrt{2}c_1}{x}+\frac{c_2\log(2x+2)}{\sqrt{2}x}\right\}\right\}$$

✓ **Maple** : cpu = 0.003 (sec), leaf count = 16

$$\left\{y\left(x\right)=\frac{\ln\left(1+x\right)\_C1+\_C2}{x}\right\}$$

2.1254 ODE No. 1254

---

$$\left(x^2+x-2\right)y''(x)+\left(x^2-x\right)y'(x)-\left(6x^2+7x\right)y(x)=0$$

✓ **Mathematica** : cpu = 0.13895 (sec), leaf count = 69

$$\left\{\left\{y(x)\rightarrow\frac{1}{5}c_2e^{-3x-5}\left(195e^{5x}x\mathrm{Ei}(5-5x)-195e^{5x}\mathrm{Ei}(5-5x)+e^5x+44e^5\right)-c_1e^{2x}(x-1)\right\}\right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 43

$$\left\{y\left(x\right)=195\_C2\,\mathrm{e}^{-5+2\,x}\left(x-1\right)Ei\left(1,5\,x-5\right)-\_C2\,\left(x+44\right)\mathrm{e}^{-3\,x}+\_C1\,\mathrm{e}^{2\,x}\left(x-1\right)\right\}$$

2.1255 ODE No. 1255

---

$$ay'(x)+(x-1)xy''(x)-2y(x)=0$$

✓ **Mathematica** : cpu = 0.635284 (sec), leaf count = 360

$$\left\{\left\{y(x)\rightarrow\frac{c_2x^a\left(a^2+2ax-a+2x^2-2x\right)\left(1-x\right)^{-a}\left(-\frac{{}_a{}_2F_1\left(1,-a;1-a;\frac{\left(-a+\sqrt{1-a^2}+1\right)\left(x-1\right)}{\left(-a+\sqrt{1-a^2}-1\right)x}\right)}{\left(1-a^2\right)^{3/2}}+\frac{{}_a{}_2F_1\left(1,-a;1-a;\frac{\left(a+\sqrt{1-a^2}+1\right)\left(x-1\right)}{\left(a+\sqrt{1-a^2}-1\right)x}\right)}{\left(1-a^2\right)^{3/2}}\right)}{2a^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 42

$$\left\{y\left(x\right)=\left(a^2+a\left(2x-1\right)+2x^2-2x\right)\_C1+\frac{C2\,x^ax\left(x-1\right)}{\left(x-1\right)^a}\right\}$$

2.1256 ODE No. 1256

---

$$-v(v+1)y(x)+(x-1)xy''(x)+(2x-1)y'(x)=0$$

✓ **Mathematica** : cpu = 0.0167243 (sec), leaf count = 26

$$\{\{y(x)\rightarrow c_1P_v(2x-1)+c_2Q_v(2x-1)\}\}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 51

$$\left\{y\left(x\right)=\_C1\,{}_2F_1\left(-v,-v;-2\,v;x^{-1}\right)x^v+\_C2\,{}_2F_1\left(v+1,v+1;2\,v+2;x^{-1}\right)x^{-v-1}\right\}$$

2.1257 ODE No. 1257

---

$$\left((a+1)x+b\right)y'(x)+(x-1)xy''(x)=0$$

✓ **Mathematica** : cpu = 0.0288396 (sec), leaf count = 33

$$\left\{\left\{y(x)\rightarrow\frac{c_1x^{b+1}{}_2F_1(b+1,a+b+1;b+2;x)}{b+1}+c_2\right\}\right\}$$

✓ **Maple** : cpu = 0.169 (sec), leaf count = 27

$$\left\{y\left(x\right)=\_C1+{}_2F_1\left(b+1,a+b+1;b+2;x\right)x^{b+1}\_C2\right\}$$

2.1258 ODE No. 1258

$$(ax + b)y'(x) + cy(x) + (x - 1)xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.117177 (sec), leaf count = 146

$$\left\{ \left\{ y(x) \rightarrow (-1)^{b+1} c_2 x^{b+1} {}_2F_1 \left( \frac{a}{2} + b - \frac{1}{2} \sqrt{a^2 - 2a - 4c + 1} + \frac{1}{2}, \frac{a}{2} + b + \frac{1}{2} \sqrt{a^2 - 2a - 4c + 1} + \frac{1}{2}; b + 2; x \right) + \right. \right.$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 110

$$\left\{ y(x) = {}_2F_1 \left( -\frac{1}{2} + \frac{a}{2} - \frac{1}{2} \sqrt{a^2 - 2a - 4c + 1}, -\frac{1}{2} + \frac{a}{2} + \frac{1}{2} \sqrt{a^2 - 2a - 4c + 1}; -b; x \right) + {}_2F_1 \left( \frac{1}{2} \right.$$

2.1259 ODE No. 1259

$$((a + 1)x + b)y'(x) - ly(x) + (x - 1)xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0998341 (sec), leaf count = 120

$$\left\{ \left\{ y(x) \rightarrow (-1)^{b+1} c_2 x^{b+1} {}_2F_1 \left( \frac{a}{2} + b - \frac{1}{2} \sqrt{a^2 + 4l} + 1, \frac{a}{2} + b + \frac{1}{2} \sqrt{a^2 + 4l} + 1; b + 2; x \right) + c_1 {}_2F_1 \left( \frac{a}{2} - \frac{1}{2} \sqrt{a^2} \right. \right.$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 92

$$\left\{ y(x) = {}_2F_1 \left( \frac{a}{2} - \frac{1}{2} \sqrt{a^2 + 4l}, \frac{a}{2} + \frac{1}{2} \sqrt{a^2 + 4l}; -b; x \right) + {}_2F_1 \left( \frac{a}{2} - \frac{1}{2} \sqrt{a^2 + 4l} + b + 1, \frac{a}{2} + \frac{1}{2} \sqrt{a^2} \right.$$

2.1260 ODE No. 1260

$$y'(x)(x(a_1 + b_1 + 1) - d_1) + a_1 b_1 d_1 + (x - 1)xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.202342 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow a_1 b_1 x \Gamma(d_1 + 1) {}_3\tilde{F}_2(1, a_1 + b_1 + 1, 1; d_1 + 1, 2; x) - \frac{c_1 x^{1-d_1} {}_2F_1(1 - d_1, a_1 + b_1 - d_1 + 1; 2 - d_1; x)}{d_1 - 1} \right. \right.$$

✓ **Maple** : cpu = 0.31 (sec), leaf count = 76

$$\left\{ y(x) = \int \left( -a_1 b_1 (\text{signum}(x - 1))^{a_1 + b_1 - d_1} (-\text{signum}(x - 1))^{-a_1 - b_1 + d_1} {}_2F_1(d_1, -a_1 - b_1 + d_1; 1 + d_1; x) \right. \right.$$

2.1261 ODE No. 1261

$$y(x)(2lx(-n + p - 1) + 2lp + m) + 2(x(-2l + n + 1) - lx^2 + n + 1)y'(x) + x(x + 2)y''(x) = 0$$

✗ **Mathematica** : cpu = 1.90593 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{(-2xl - 2xnl + 2xpl + 2pl + m)y(x) + 2(-lx^2 - 2lx + nx + x + n + 1)y'(x) \right. \right.$$

✓ **Maple** : cpu = 0.132 (sec), leaf count = 105

$$\left\{ y(x) = (x + 2)^{-\frac{n}{2} - \frac{1}{2}} \left( -\frac{x}{2} - 1 \right)^{\frac{n}{2} + \frac{1}{2}} \left( \text{HeunC} \left( 4l, -n, n, -4pl, \frac{(4n + 4p + 4)l}{2} - \frac{n^2}{2} + m - n, -\frac{x}{2} \right) x^{-n} {}_2F_1 \left( \frac{1}{2}, \frac{n}{2} + 1; \frac{n}{2} + 2; x \right) \right. \right.$$

2.1262 ODE No. 1262

$$(x^2 + x - 1)y'(x) + (x + 1)^2 y''(x) - (x + 2)y(x) = 0$$

✓ **Mathematica** : cpu = 0.226536 (sec), leaf count = 88

$$\left\{ \left\{ y(x) \rightarrow c_2 e^{-x} \int_1^x \exp \left( -\frac{K[1]^2}{K[1] + 1} - \frac{K[1]}{K[1] + 1} + 2K[1] - \frac{1}{K[1] + 1} \right) (K[1] + 1)^{\frac{K[1]}{K[1] + 1} + \frac{1}{K[1] + 1}} dK[1] + c_1 e^{-x} \right. \right.$$

✓ **Maple** : cpu = 0.158 (sec), leaf count = 53

$$\left\{ y(x) = (1 + x) \left( {}_2F_1 \left( 4, 4, -8, 12, \frac{x}{x + 2} \right) + {}_2F_1 \left( -4, 4, -8, 12, \frac{x}{x + 2} \right) e^{\frac{x-1}{2x+2}} \right) \right\}$$



2.1263 ODE No. 1263

---

$$-(20x + 30) \left(x^2 + 3x\right)^{7/3} + x(x + 3)y''(x) + (3x - 1)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.373196 (sec), leaf count = 276

$$\left\{\left\{y(x) \rightarrow \frac{4x^{4/3} \int_1^x \left(45 \sqrt[3]{3} {}_2F_1\left(-\frac{4}{3}, -\frac{4}{3}; -\frac{1}{3}; -\frac{K[1]}{3}\right) \sqrt[3]{K[1](K[1] + 3)} K[1]^{11/3} + \frac{675}{2} \sqrt[3]{3} {}_2F_1\left(-\frac{4}{3}, -\frac{4}{3}; -\frac{1}{3}; -\frac{K[1]}{3}\right) \right.}{\right.}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 52

$$\left\{y(x) = 1 \left(-C2 + \int \frac{1}{x^2 + 3x} (x + 3)^{\frac{7}{3}} \left(-C1 + 3 \left(x^2 + 3x\right)^{7/3} x (x + 3)\right) x^{-\frac{4}{3}} \mathrm{d}x\right) x^{\frac{4}{3}} (x + 3)^{-\frac{7}{3}}\right\}$$

2.1264 ODE No. 1264

---

$$\left(x^2 + 3x + 4\right) y''(x) + \left(x^2 + x + 1\right) y'(x) - (2x + 3)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0532353 (sec), leaf count = 23

$$\left\{\left\{y(x) \rightarrow c_2 \left(x^2 + x + 3\right) + c_1 e^{-x}\right\}\right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 19

$$\left\{y(x) = -C1 \mathrm{e}^{-x} + -C2 \left(x^2 + x + 3\right)\right\}$$

2.1265 ODE No. 1265

---

$$(x - 2)(x - 1)y''(x) - (2x - 3)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0315748 (sec), leaf count = 64

$$\left\{\left\{y(x) \rightarrow c_1 \left(x^2 - 3x + 2\right) P_{\frac{1}{2}\left(-1 + \sqrt{5}\right)}^2(2x - 3) + c_2 \left(x^2 - 3x + 2\right) Q_{\frac{1}{2}\left(-1 + \sqrt{5}\right)}^2(2x - 3)\right\}\right\}$$

✓ **Maple** : cpu = 0.227 (sec), leaf count = 93

$$\left\{y(x) = (x - 2)^2 \left(-C1 {}_2F_1\left(\frac{5}{2} - \frac{\sqrt{5}}{2}, \frac{1}{2} - \frac{\sqrt{5}}{2}; -\sqrt{5} + 1; (x - 1)^{-1}\right) (x - 1)^{\frac{\sqrt{5}}{2} - \frac{1}{2}} + -C2 {}_2F_1\left(\frac{1}{2} + \frac{\sqrt{5}}{2}, \frac{5}{2} + \frac{\sqrt{5}}{2}\right)\right.\right.$$

2.1266 ODE No. 1266

---

$$(x - 2)^2 y''(x) - (x - 2)y'(x) - 3y(x) = 0$$

✓ **Mathematica** : cpu = 0.0217364 (sec), leaf count = 22

$$\left\{\left\{y(x) \rightarrow c_1 (x - 2)^3 + \frac{c_2}{x - 2}\right\}\right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 19

$$\left\{y(x) = \frac{-C1 \left(x - 2\right)^4 + -C2}{x - 2}\right\}$$

2.1267 ODE No. 1267

---

$$-\left(l + 2x^2 - 5x\right) y'(x) + 2x^2 y''(x) - (4x - 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.345119 (sec), leaf count = 166

$$\left\{\left\{y(x) \rightarrow \frac{c_1 e^{x - \frac{l}{2x}}}{\sqrt{x}} - \frac{\sqrt{\frac{\pi}{2}} c_2 e^{-\frac{l}{2x} - \sqrt{2}\sqrt{-l} + x} \left(\mathrm{erf}\left(\frac{\sqrt{-l}}{\sqrt{2}\sqrt{x}} - \sqrt{x}\right) + e^{2\sqrt{2}\sqrt{-l}} \mathrm{erf}\left(\frac{\sqrt{-l}}{\sqrt{2}\sqrt{x}} + \sqrt{x}\right) - e^{2\sqrt{2}\sqrt{-l}} + 1\right)}{\sqrt{-l}\sqrt{x}}\right\}\right\}$$

✓ **Maple** : cpu = 0.045 (sec), leaf count = 41

$$\left\{y(x) = \mathrm{e}^x \left(-C1 \int \frac{1}{2 \mathrm{e}^x} \mathrm{e}^{\frac{l}{2x}} x^{-\frac{3}{2}} \mathrm{d}x + -C2\right) \left(\mathrm{e}^{\frac{l}{2x}}\right)^{-1} \frac{1}{\sqrt{x}}\right\}$$

**2.1268                    ODE No. 1268**

---

$$y(x)(ax + b) + 2(x - 1)xy''(x) + (2x - 1)y'(x) = 0$$

✗ **Mathematica** : cpu = 1.17703 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot} \left( \{y, x\}, \left\{ (xa + b)y(x) + (2x - 1)y'(x) + 2(x - 1)xy''(x) = 0, y(2) = c_1, y'(2) = c_2 \right\} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.073 (sec), leaf count = 39

$$\left\{ y(x) = \_C1 \text{MathieuC} \left( -a - 2b, \frac{a}{2}, \arccos(\sqrt{x}) \right) + \_C2 \text{MathieuS} \left( -a - 2b, \frac{a}{2}, \arccos(\sqrt{x}) \right) \right\}$$

**2.1269                    ODE No. 1269**

---

$$((2v + 5)x - 2v - 3)y'(x) + (v + 1)y(x) + 2(x - 1)xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0643468 (sec), leaf count = 60

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1 \left( \frac{1}{2}, v + 1; v + \frac{3}{2}; x \right) + c_2 i^{-2v-1} x^{\frac{1}{2}(-2v-1)} {}_2F_1 \left( \frac{1}{2}, -v; \frac{1}{2} - v; x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 40

$$\left\{ y(x) = \_C1 {}_2F_1 \left( \frac{1}{2}, v + 1; \frac{3}{2} + v; x \right) + \_C2 x^{-\frac{1}{2}-v} {}_2F_1 \left( \frac{1}{2}, -v; \frac{1}{2} - v; x \right) \right\}$$

**2.1270                    ODE No. 1270**

---

$$(2x^2 + 6x + 4)y''(x) + (10x^2 + 21x + 8)y'(x) + (12x^2 + 17x + 8)y(x) = 0$$

✓ **Mathematica** : cpu = 1.85336 (sec), leaf count = 58

$$\left\{ \left\{ y(x) \rightarrow c_2 e^{-3x} (x + 2)^4 \int_1^x \frac{e^{K[1]} (K[1] + 1)^{3/2}}{(K[1] + 2)^5} dK[1] + c_1 e^{-3x} (x + 2)^4 \right\} \right\}$$

✓ **Maple** : cpu = 0.107 (sec), leaf count = 46

$$\left\{ y(x) = e^{-2x} (x + 2)^4 \left( \_C2 \text{HeunC} \left( -1, \frac{5}{2}, 4, -\frac{7}{4}, \frac{7}{2}, -1 - x \right) (1 + x)^{\frac{5}{2}} + \_C1 \text{HeunC} \left( -1, -\frac{5}{2}, 4, -\frac{7}{4}, \frac{7}{2}, -1 - x \right) \right) \right\}$$

**2.1271                    ODE No. 1271**

---

$$4x^2 y''(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0079645 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt{x} + \frac{1}{2} c_2 \sqrt{x} \log(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.004 (sec), leaf count = 14

$$\{y(x) = \sqrt{x} (\ln(x) \_C2 + \_C1)\}$$

**2.1272                    ODE No. 1272**

---

$$(4a^2 x^2 + 1)y(x) + 4x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0084318 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt{x} J_0(ax) + c_2 \sqrt{x} Y_0(ax) \right\} \right\}$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 23

$$\{y(x) = \sqrt{x} (Y_0(ax) \_C2 + J_0(ax) \_C1)\}$$

**2.1273      ODE No. 1273**

---

$$4x^2y''(x) - y(x) (-4kx + 4m^2 + x^2 - 1) = 0$$

✓ **Mathematica** : cpu = 0.0121755 (sec), leaf count = 20

$$\{\{y(x) \rightarrow c_1 M_{k,m}(x) + c_2 W_{k,m}(x)\}\}$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 17

$$\{y(x) = \_C1 M_{k,m}(x) + \_C2 W_{k,m}(x)\}$$

**2.1274      ODE No. 1274**

---

$$(x - v^2) y(x) + 4x^2 y''(x) + 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0306805 (sec), leaf count = 38

$$\{\{y(x) \rightarrow c_1 \Gamma(1 - v) J_{-v}(\sqrt{x}) + c_2 \Gamma(v + 1) J_v(\sqrt{x})\}\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 19

$$\{y(x) = \_C1 J_v(\sqrt{x}) + \_C2 Y_v(\sqrt{x})\}$$

**2.1275      ODE No. 1275**

---

$$y(x) (2x(2l - m + 1) - m^2 - x^2 + 1) + 4x^2 y''(x) + 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0245469 (sec), leaf count = 120

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{2}(\sqrt{m^2-1} \log(x)-x)} U\left(\frac{1}{2}(-2l+m+\sqrt{m^2-1}), \sqrt{m^2-1}+1, x\right) + c_2 e^{\frac{1}{2}(\sqrt{m^2-1} \log(x)-x)} L^{\frac{1}{2}(2l-m)}_{\sqrt{m^2-1}}(\sqrt{x}) \right\} \right\}$$

✓ **Maple** : cpu = 0.103 (sec), leaf count = 53

$$\left\{ y(x) = 1 \left( \_C1 M_{l-\frac{m}{2}+\frac{1}{2}, \frac{1}{2}\sqrt{m-1}\sqrt{m+1}}(x) + \_C2 W_{l-\frac{m}{2}+\frac{1}{2}, \frac{1}{2}\sqrt{m-1}\sqrt{m+1}}(x) \right) \frac{1}{\sqrt{x}} \right\}$$

**2.1276      ODE No. 1276**

---

$$-4e^x \sqrt{x^3} + 4x^2 y''(x) - (4x^2 + 1) y(x) + 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0284909 (sec), leaf count = 55

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-x}}{\sqrt{x}} + \frac{c_2 e^x}{2\sqrt{x}} + \frac{e^x \sqrt{x^3} (2x - 1)}{4x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 31

$$\left\{ y(x) = \sinh(x) \_C2 \frac{1}{\sqrt{x}} + \cosh(x) \_C1 \frac{1}{\sqrt{x}} + \frac{e^x}{2x} \sqrt{x^3} \right\}$$

**2.1277      ODE No. 1277**

---

$$-(ax^2 + 1) y(x) + 4x^2 y''(x) + 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0203516 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-\frac{\sqrt{a}x}}{\sqrt{x}} + \frac{c_2 e^{\frac{\sqrt{a}x}}{\sqrt{a}\sqrt{x}}}}{\sqrt{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 27

$$\left\{ y(x) = 1 \left( \_C2 \cosh\left(\frac{x}{2}\sqrt{a}\right) + \_C1 \sinh\left(\frac{x}{2}\sqrt{a}\right) \right) \frac{1}{\sqrt{x}} \right\}$$

**2.1278 ODE No. 1278**

---

$$f(x)y(x) + 4x^2y''(x) + 4xy'(x) = 0$$

✗ **Mathematica** : cpu = 0.336196 (sec), leaf count = 0 , could not solve  
DSolve[f[x]\*y[x] + 4\*x\*Derivative[1][y][x] + 4\*x^2\*Derivative[2][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x) = DESol\left(\left\{\frac{f(x) - Y(x)}{4x^2} + \frac{\frac{d}{dx}Y(x)}{x} + \frac{d^2}{dx^2}Y(x)\right\}, \{-Y(x)\}\right)\right\}$$

**2.1279 ODE No. 1279**

---

$$4x^2y''(x) + 5xy'(x) - y(x) - \log(x) = 0$$

✓ **Mathematica** : cpu = 0.108683 (sec), leaf count = 74

$$\left\{\left\{y(x) \rightarrow c_2x^{\frac{1}{2}\left(\frac{\sqrt{17}}{4}-\frac{1}{4}\right)} + c_1x^{\frac{1}{2}\left(-\frac{1}{4}-\frac{\sqrt{17}}{4}\right)} - \frac{256(\log(x)+1)}{(\sqrt{17}-1)^2(1+\sqrt{17})^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.13 (sec), leaf count = 32

$$\left\{y(x) = x^{-\frac{1}{8}+\frac{\sqrt{17}}{8}}\_C2 + x^{-\frac{1}{8}-\frac{\sqrt{17}}{8}}\_C1 - \ln(x) - 1\right\}$$

**2.1280 ODE No. 1280**

---

$$4x^2y''(x) - (4x^2 + 12x + 3)y(x) + 8xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0436703 (sec), leaf count = 52

$$\left\{\left\{y(x) \rightarrow \frac{c_2e^{-x}(4e^{2x}x^2\text{Ei}(-2x) + 2x - 1)}{2x^{3/2}} + c_1e^x\sqrt{x}\right\}\right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 40

$$\left\{y(x) = (4e^xEi(1,2x)\_C2x^2 + (-2x+1)\_C2e^{-x} + \_C1x^2e^x)x^{-\frac{3}{2}}\right\}$$

**2.1281 ODE No. 1281**

---

$$4x^2y''(x) + (4x^2 - 4x - 1)y(x) - 4(2x - 1)xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0145298 (sec), leaf count = 28

$$\left\{\left\{y(x) \rightarrow \frac{c_1e^x}{\sqrt{x}} + c_2e^x\sqrt{x}\right\}\right\}$$

✓ **Maple** : cpu = 0.028 (sec), leaf count = 15

$$\left\{y(x) = e^x(x\_C2 + \_C1)\frac{1}{\sqrt{x}}\right\}$$

**2.1282 ODE No. 1282**

---

$$4x^3y'(x) + 4x^2y''(x) + (x^2 - 4)(x^2 + 6)y(x) = 0$$

✓ **Mathematica** : cpu = 0.015557 (sec), leaf count = 39

$$\left\{\left\{y(x) \rightarrow \frac{c_1e^{-\frac{x^2}{4}}}{x^2} + \frac{1}{5}c_2e^{-\frac{x^2}{4}}x^3\right\}\right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 21

$$\left\{y(x) = \frac{\_C2x^5 + \_C1}{x^2}e^{-\frac{x^2}{4}}\right\}$$

**2.1283                    ODE No. 1283**

---

$$4x^2y''(x) + 4x^2\log(x)y'(x) + y(x)\left(x^2\log^2(x) + 2x - 8\right) - 4\sqrt{e^xx^{-x}}x^2 = 0$$

✓ **Mathematica** : cpu = 0.0597582 (sec), leaf count = 90

$$\left\{\left\{y(x) \rightarrow c_1e^{x/2}x^{-\frac{x}{2}-1} + \frac{1}{3}c_2e^{x/2}x^{2-\frac{x}{2}} + \frac{1}{9}\left(3x^2\sqrt{e^xx^{-x}}\log(x) - x^2\sqrt{e^xx^{-x}}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.091 (sec), leaf count = 48

$$\left\{y(x) = \frac{x^2}{3}\left(\ln(x) - \frac{1}{3}\right)\sqrt{x^{-x}e^x} + e^{\frac{x}{2}}\left(-C1\,x^{-\frac{x}{2}+2} + -C2\,x^{-\frac{x}{2}-1}\right)\right\}$$

**2.1284                    ODE No. 1284**

---

$$(2x + 1)^2y''(x) - 2(2x + 1)y'(x) - 12y(x) - 3x - 1 = 0$$

✓ **Mathematica** : cpu = 0.0291083 (sec), leaf count = 47

$$\left\{\left\{y(x) \rightarrow c_1(2x + 1)^3 + \frac{c_2}{2x + 1} + \frac{-72x^2 - 56x - 7}{192(2x + 1)}\right\}\right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 41

$$\left\{y(x) = \frac{-C1}{2x + 1} + (2x + 1)^3-C2 + \frac{-72\,x^2 - 56\,x - 7}{384\,x + 192}\right\}$$

**2.1285                    ODE No. 1285**

---

$$((4a + 2)x - a)y'(x) + (a - 1)ay(x) + x(4x - 1)y''(x) = 0$$

✓ **Mathematica** : cpu = 1.28375 (sec), leaf count = 269

$$\left\{\left\{y(x) \rightarrow c_2x^{-a/2}\left(\sqrt{4x-1}+i\right)^{\frac{1}{2}+\frac{1}{2}i\sqrt{-(a-1)^2}}\left(-\sqrt{4x-1}+i\right)^{\frac{1}{2}-\frac{1}{2}i\sqrt{-(a-1)^2}}\int_1^x-\frac{\left(i-\sqrt{4K[1]-1}\right)^{i\sqrt{-(a-1)^2}}}{\sqrt{1-4K[1]}}\left(\right.\right.$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 52

$$\left\{y(x) = -C1\,{}_2F_1\left(\frac{a}{2},\frac{a}{2}-\frac{1}{2};a;4x\right) + -C2\,x^{1-a}{}_2F_1\left(1-\frac{a}{2},-\frac{a}{2}+\frac{1}{2};-a+2;4x\right)\right\}$$

**2.1286                    ODE No. 1286**

---

$$(3x - 1)^2y''(x) + 3(3x - 1)y'(x) - 9y(x) - \log^2(3x - 1) = 0$$

✓ **Mathematica** : cpu = 0.0889154 (sec), leaf count = 101

$$\left\{\left\{y(x) \rightarrow \frac{c_1\left((1-3x)^2+1\right)}{2(1-3x)} + \frac{ic_2\left((1-3x)^2-1\right)}{2(1-3x)} + \frac{-6x-3x\log^2(3x-1)+\log^2(3x-1)+\log(3x-1)-\log(3x-1)}{9(3x-1)}\right\}\right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 32

$$\left\{y(x) = \frac{-C1}{3x-1} + (3x-1)-C2 - \frac{(\ln(3x-1))^2}{9} - \frac{2}{9}\right\}$$

**2.1287                    ODE No. 1287**

---

$$9(x - 1)xy''(x) + 3(2x - 1)y'(x) - 20y(x) = 0$$

✓ **Mathematica** : cpu = 0.0134879 (sec), leaf count = 83

$$\left\{\left\{y(x) \rightarrow c_2\sqrt[3]{1-x}\sqrt[3]{x}Q_1^{\frac{2}{3}}(2x-1) - \frac{c_1(2-2x)^{2/3}\sqrt[3]{1-xx}^{2/3}(6x-5)}{3\,2^{2/3}(x-1)\Gamma\left(\frac{4}{3}\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 27

$$\left\{y(x) = -C1\,(6\,x-5)\,x^{\frac{2}{3}} + -C2\,(6\,x-1)\,(x-1)^{\frac{2}{3}}\right\}$$

**2.1288      ODE No. 1288**

---

$$16x^2y''(x) + (4x + 3)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0247938 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{i\sqrt{x}} \sqrt[4]{x} + i c_2 e^{-i\sqrt{x}} \sqrt[4]{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 21

$$\{y(x) = \sqrt[4]{x} (\cos(\sqrt{x}) - C2 + \sin(\sqrt{x}) - C1)\}$$

**2.1289      ODE No. 1289**

---

$$16x^2y''(x) + 32xy'(x) - (4x + 5)y(x) = 0$$

✓ **Mathematica** : cpu = 0.104433 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 e^{-\sqrt{x}} (\sqrt{x} + 1)}{x^{5/4}} - \frac{c_1 e^{\sqrt{x}} (\sqrt{x} - 1)}{x^{5/4}} \right\} \right\}$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 33

$$\left\{ y(x) = 1 \left( -C2 (\sqrt{x} + 1) e^{-\sqrt{x}} + -C1 e^{\sqrt{x}} (\sqrt{x} - 1) \right) x^{-\frac{5}{4}} \right\}$$

**2.1290      ODE No. 1290**

---

$$(27x^2 + 4)y''(x) + 27xy'(x) - 3y(x) = 0$$

✓ **Mathematica** : cpu = 0.111139 (sec), leaf count = 103

$$\left\{ \left\{ y(x) \rightarrow c_1 \cosh \left( \frac{\sqrt{-27x^2 - 4} \tan^{-1} \left( \frac{3x}{\sqrt{-9x^2 - \frac{4}{3}}} \right)}{3\sqrt{27x^2 + 4}} \right) + i c_2 \sinh \left( \frac{\sqrt{-27x^2 - 4} \tan^{-1} \left( \frac{3x}{\sqrt{-9x^2 - \frac{4}{3}}} \right)}{3\sqrt{27x^2 + 4}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 29

$$\left\{ y(x) = -C1 \sinh \left( \frac{1}{3} \operatorname{Arcsinh} \left( \frac{3\sqrt{3}x}{2} \right) \right) + -C2 \cosh \left( \frac{1}{3} \operatorname{Arcsinh} \left( \frac{3\sqrt{3}x}{2} \right) \right) \right\}$$

**2.1291      ODE No. 1291**

---

$$48(x - 1)xy''(x) + (152x - 40)y'(x) + 53y(x) = 0$$

✓ **Mathematica** : cpu = 0.0573672 (sec), leaf count = 92

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1 \left( \frac{13}{12} - \frac{\sqrt{\frac{5}{2}}}{6}, \frac{13}{12} + \frac{\sqrt{\frac{5}{2}}}{6}; \frac{5}{6}; x \right) + \sqrt[6]{-1} c_2 \sqrt[6]{x} {}_2F_1 \left( \frac{5}{4} - \frac{\sqrt{\frac{5}{2}}}{6}, \frac{5}{4} + \frac{\sqrt{\frac{5}{2}}}{6}; \frac{7}{6}; x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 50

$$\left\{ y(x) = -C1 {}_2F_1 \left( \frac{13}{12} - \frac{\sqrt{10}}{12}, \frac{13}{12} + \frac{\sqrt{10}}{12}; \frac{5}{6}; x \right) + -C2 \sqrt[6]{x} {}_2F_1 \left( \frac{5}{4} - \frac{\sqrt{10}}{12}, \frac{5}{4} + \frac{\sqrt{10}}{12}; \frac{7}{6}; x \right) \right\}$$

**2.1292      ODE No. 1292**

---

$$50(x - 1)xy''(x) + 25(2x - 1)y'(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0325845 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin \left( \frac{2}{5} \sin^{-1}(\sqrt{1 - x}) \right) + c_1 \cos \left( \frac{2}{5} \sin^{-1}(\sqrt{1 - x}) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 31

$$\left\{ y(x) = 1 \left( -C1 (\sqrt{x} + \sqrt{x - 1})^{\frac{4}{5}} + -C2 \right) (\sqrt{x} + \sqrt{x - 1})^{-\frac{2}{5}} \right\}$$

**2.1293      ODE No. 1293**

---

$$144(x-1)xy''(x) + (120x-48)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.249076 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow (-1)^{2/3} c_2 x^{2/3} {}_2F_1 \left( \frac{7}{12}, \frac{7}{12}; \frac{5}{3}; x \right) + c_1 {}_2F_1 \left( -\frac{1}{12}, -\frac{1}{12}; \frac{1}{3}; x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt[3]{x} \left( LegendreQ \left( -\frac{1}{2}, \frac{2}{3}, \sqrt{1-x} \right) - C2 + LegendreP \left( -\frac{1}{2}, \frac{2}{3}, \sqrt{1-x} \right) - C1 \right) \right\}$$

**2.1294      ODE No. 1294**

---

$$144(x-1)xy''(x) + (168x-96)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.051031 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1 \left( \frac{1}{12}, \frac{1}{12}; \frac{2}{3}; x \right) + \sqrt[3]{-1} c_2 \sqrt[3]{x} {}_2F_1 \left( \frac{5}{12}, \frac{5}{12}; \frac{4}{3}; x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt[6]{x} \left( LegendreQ \left( -\frac{1}{2}, \frac{1}{3}, \sqrt{1-x} \right) - C2 + LegendreP \left( -\frac{1}{2}, \frac{1}{3}, \sqrt{1-x} \right) - C1 \right) \right\}$$

**2.1295      ODE No. 1295**

---

$$ax^2y''(x) + bxy'(x) + y(x) (cx^2 + dx + f) = 0$$

✓ **Mathematica** : cpu = 0.18624 (sec), leaf count = 310

$$\left\{ \left\{ y(x) \rightarrow c_1 U \left( -\frac{-\sqrt{ca} - id\sqrt{a} - \sqrt{c}\sqrt{a^2 - 2ba - 4fa + b^2}}{2a\sqrt{c}}, \frac{\sqrt{a^2 - 2ba - 4fa + b^2}}{a} + 1, \frac{2i\sqrt{cx}}{\sqrt{a}} \right) \exp \left( \frac{\log(x)}{\sqrt{a}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.196 (sec), leaf count = 106

$$\left\{ y(x) = x^{-\frac{b}{2a}} \left( M_{-\frac{i}{2}d\frac{1}{\sqrt{a}}\frac{1}{\sqrt{c}}, \frac{1}{2a}\sqrt{a^2+(-2b-4f)a+b^2}} \left( 2ix\sqrt{c}\frac{1}{\sqrt{a}} \right) - C1 + W_{-\frac{i}{2}d\frac{1}{\sqrt{a}}\frac{1}{\sqrt{c}}, \frac{1}{2a}\sqrt{a^2+(-2b-4f)a+b^2}} \left( 2ix\sqrt{c}\frac{1}{\sqrt{a}} \right) - C2 \right) \right\}$$

**2.1296      ODE No. 1296**

---

$$y(x) (a0x^2 + b0x + c0) + (a1x^2 + b1x) y'(x) + a2x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.3581 (sec), leaf count = 356

$$\left\{ \left\{ y(x) \rightarrow c_1 U \left( -\frac{2b0a2 - \sqrt{a1^2 - 4a0a2a2} - a1b1 - \sqrt{a1^2 - 4a0a2}\sqrt{a2^2 - 2b1a2 - 4c0a2 + b1^2}}{2a2\sqrt{a1^2 - 4a0a2}}, \frac{\sqrt{a2^2 - 2b1a2 - 4c0a2 + b1^2}}{\sqrt{a1^2 - 4a0a2}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.224 (sec), leaf count = 150

$$\left\{ y(x) = x^{-\frac{b1}{2a2}} e^{-\frac{a1x}{2a2}} \left( M_{-\frac{a1b1-2a2b0}{2a2}\frac{1}{\sqrt{-4a0a2+a1^2}}, \frac{1}{2a2}\sqrt{a2^2+(-2b1-4c0)a2+b1^2}} \left( \frac{x}{a2}\sqrt{-4a0a2+a1^2} \right) - C1 + W_{-\frac{a1b1-2a2b0}{2a2}\frac{1}{\sqrt{-4a0a2+a1^2}}, \frac{1}{2a2}\sqrt{a2^2+(-2b1-4c0)a2+b1^2}} \left( \frac{x}{a2}\sqrt{-4a0a2+a1^2} \right) - C2 \right) \right\}$$

**2.1297      ODE No. 1297**

---

$$(ax^2 + 1) y''(x) + axy'(x) + by(x) = 0$$

✓ **Mathematica** : cpu = 0.0241098 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin \left( \frac{\sqrt{b} \sinh^{-1}(\sqrt{ax})}{\sqrt{a}} \right) + c_1 \cos \left( \frac{\sqrt{b} \sinh^{-1}(\sqrt{ax})}{\sqrt{a}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 63

$$\left\{ y(x) = 1 \left( -C1 \left( \left( \sqrt{ax} + \sqrt{ax^2 + 1} \right)^{i\sqrt{b}\frac{1}{\sqrt{a}}} \right)^2 + -C2 \right) \left( \left( \sqrt{ax} + \sqrt{ax^2 + 1} \right)^{i\sqrt{b}\frac{1}{\sqrt{a}}} \right)^{-1} \right\}$$

2.1298 ODE No. 1298

---

$$(ax^2 + 1) y''(x) + bxy'(x) + cy(x) = 0$$

✓ **Mathematica** : cpu = 0.0537818 (sec), leaf count = 162

$$\left\{ \left\{ y(x) \rightarrow c_1 (ax^2 + 1)^{\frac{2a-b}{4a}} P_{\frac{b-2a}{2a}}^{\frac{2a}{\sqrt{a^2-2ba-4ca+b^2-a}}} (i\sqrt{ax}) + c_2 (ax^2 + 1)^{\frac{2a-b}{4a}} Q_{\frac{b-2a}{2a}}^{\frac{2a}{\sqrt{a^2-2ba-4ca+b^2-a}}} (i\sqrt{ax}) \right\} \right\}$$

✓ **Maple** : cpu = 0.083 (sec), leaf count = 124

$$\left\{ y(x) = (ax^2 + 1)^{\frac{2a-b}{4a}} \left( LegendreP \left( \frac{1}{2a} \left( \sqrt{a^2 + (-2b - 4c)a + b^2} - a \right), \frac{2a-b}{2a}, \sqrt{-ax} \right) _-C1 + LegendreQ \left( \right. \right. \right.$$

2.1299 ODE No. 1299

---

$$(a^2x^2 - 1) y''(x) + 2a^2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0143287 (sec), leaf count = 19

$$\left\{ \left\{ y(x) \rightarrow c_2 - \frac{c_1 \tanh^{-1}(ax)}{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 27

$$\left\{ y(x) = _-C1 - \frac{(-\ln(ax - 1) + \ln(ax + 1)) _-C2}{2a} \right\}$$

2.1300 ODE No. 1300

---

$$(a^2x^2 - 1) y''(x) + 2a^2xy'(x) - 2a^2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0104688 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow ac_1x + c_2 \left( ax \left( \frac{1}{2} \log(ax + 1) - \frac{1}{2} \log(1 - ax) \right) - 1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 31

$$\left\{ y(x) = \frac{_-C2 a \ln(ax - 1) x}{2} - \frac{_-C2 a \ln(ax + 1) x}{2} + _-C1 x + _-C2 \right\}$$

2.1301 ODE No. 1301

---

$$(ax^2 + bx) y''(x) - 2ay(x) + 2by'(x) = 0$$

✓ **Mathematica** : cpu = 0.0228979 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2(ax + b)^3}{3ax} + \frac{c_1}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 19

$$\left\{ y(x) = \frac{_-C1 + _-C2 (ax + b)^3}{x} \right\}$$

2.1302 ODE No. 1302

---

$$A0y(x)(ax + b) + A1(ax + b)y'(x) + A2(ax + b)^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.059428 (sec), leaf count = 243

$$\left\{ \left\{ y(x) \rightarrow c_1 \left( \frac{2b}{a} + 2x \right)^{\frac{A1}{2aA2}} (2aA2x + 2A2b)^{-\frac{A1}{2aA2}} \left( -\frac{A0 \left( \frac{b}{a} + x \right)}{aA2} \right)^{\frac{1}{2} - \frac{A1}{2aA2}} I_{\frac{A1}{aA2} - 1} \left( 2\sqrt{-\frac{A0 \left( \frac{b}{a} + x \right)}{aA2}} \right) + c_2 \left( \right. \right. \right.$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 98

$$\left\{ y(x) = (ax + b)^{-\frac{-aA2+A1}{2aA2}} \left( J_{\frac{aA2-A1}{aA2}} \left( 2\sqrt{A0}\sqrt{\frac{ax+b}{a^2A2}} \right) _-C1 + Y_{\frac{aA2-A1}{aA2}} \left( 2\sqrt{A0}\sqrt{\frac{ax+b}{a^2A2}} \right) _-C2 \right) \right\}$$



**2.1303                    ODE No. 1303**

---

$$y''(x) \left( ax^2 + bx + c \right) + (dx + f)y'(x) + gy(x) = 0$$

✗ **Mathematica** : cpu = 9.49692 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot} \left( \{y, x\}, \left\{ gy(x) + (xd + f)y'(x) + \left( ax^2 + bx + c \right) y''(x) = 0, y(0) = c_1, y'(0) = c_2 \right\} \right) (x) \right\} \right.$$

✓ **Maple** : cpu = 0.133 (sec), leaf count = 501

$$\left\{ y(x) = {}_2F_1\left(\frac{1}{2a} \left( -a + d + \sqrt{a^2 + (-2d - 4g)a + d^2} \right), -\frac{1}{2a} \left( a - d + \sqrt{a^2 + (-2d - 4g)a + d^2} \right); \frac{1}{2a^2} \right. \right.$$

**2.1304                    ODE No. 1304**

---

$$x^3y''(x) + xy'(x) - (2x + 3)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0374322 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \left( e^{\frac{1}{x}} \text{Ei} \left( -\frac{1}{x} \right) + 2x^3 - x^2 + x \right)}{6x} + \frac{c_1 e^{\frac{1}{x}}}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 38

$$\left\{ y(x) = \frac{1}{x} \left( \text{Ei} \left( 1, x^{-1} \right) e^{x^{-1}} {}_2F_2 \left( -C2 + {}_2F_2 \left( x^2 - x/2 + 1/2 \right) \right) \right) \right\}$$

**2.1305                    ODE No. 1305**

---

$$x^3y''(x) + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0624227 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow c_2 G_{1,2}^{2,0} \left( -\frac{2}{x} \middle| \begin{matrix} \frac{1}{2} \\ -1, 0 \end{matrix} \right) + c_1 e^{\frac{1}{x}} \left( I_0 \left( \frac{1}{x} \right) - I_1 \left( \frac{1}{x} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 44

$$\left\{ y(x) = e^{x^{-1}} \left( -C2 K_1 \left( -x^{-1} \right) + C2 K_0 \left( -x^{-1} \right) + C1 \left( I_0 \left( x^{-1} \right) - I_1 \left( x^{-1} \right) \right) \right) \right\}$$

**2.1306                    ODE No. 1306**

---

$$y(x) \left( ax^2 + a + bx \right) + x^3y''(x) + x^2y'(x) = 0$$

✗ **Mathematica** : cpu = 0.839426 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot} \left( \{y, x\}, \left\{ y''(x)x^3 + y'(x)x^2 + \left( ax^2 + bx + a \right) y(x) = 0, y(1) = c_1, y'(1) = c_2 \right\} \right) (x) \right\} \right\}$$

✓ **Maple** : cpu = 0.11 (sec), leaf count = 69

$$\left\{ y(x) = \text{HeunD} \left( 0, 8a + 4b, 0, 8a - 4b, \frac{1+x}{x-1} \right) \left( \int \frac{1}{x} \left( \text{HeunD} \left( 0, 8a + 4b, 0, 8a - 4b, \frac{1+x}{x-1} \right) \right)^{-2} dx - C2 + \right. \right.$$

**2.1307                    ODE No. 1307**

---

$$x^3y''(x) + (x + 1)xy'(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0899807 (sec), leaf count = 54

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{\frac{1}{x}} (x + 1)}{x} - \frac{c_2 \left( e^{\frac{1}{x}} x \text{Ei} \left( -\frac{1}{x} \right) + e^{\frac{1}{x}} \text{Ei} \left( -\frac{1}{x} \right) + x \right)}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 36

$$\left\{ y(x) = \frac{-C2 e^{x^{-1}} (1 + x) \text{Ei} \left( 1, x^{-1} \right) + C1 (1 + x) e^{x^{-1}} - C2 x}{x} \right\}$$

## 2.1308 ODE No. 1308

---

$$x^3 y''(x) - x^2 y'(x) + xy(x) - \log^3(x) = 0$$

✓ **Mathematica** : cpu = 0.0132192 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow c_1 x + c_2 x \log(x) + \frac{2 \log^3(x) + 6 \log^2(x) + 9 \log(x) + 6}{8x} \right\} \right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 40

$$\left\{ y(x) = \frac{2 (\ln(x))^3 + 6 (\ln(x))^2 + (8 - C1 x^2 + 9) \ln(x) + 8 - C2 x^2 + 6}{8x} \right\}$$

## 2.1309 ODE No. 1309

---

$$x^3 y''(x) - (x^2 - 1) y'(x) + xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0674043 (sec), leaf count = 84

$$\left\{ \left\{ y(x) \rightarrow c_2 G_{1,2}^{2,0} \left( -\frac{1}{2x^2} \middle| -\frac{1}{2}, -\frac{1}{2} \right) + \sqrt{2} c_1 e^{\frac{1}{4x^2}} x \left( \left( 1 - \frac{1}{2x^2} \right) I_0 \left( \frac{1}{4x^2} \right) + \frac{I_1 \left( \frac{1}{4x^2} \right)}{2x^2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 85

$$\left\{ y(x) = \frac{C1}{x} e^{\frac{1}{4x^2}} \left( 2x^2 I_0(1/4x^{-2}) + I_1 \left( \frac{1}{4x^2} \right) - I_0 \left( \frac{1}{4x^2} \right) \right) + \frac{C2}{x} e^{\frac{1}{4x^2}} \left( 2K_0(-1/4x^{-2})x^2 - K_0 \left( -\frac{1}{4x} \right) \right) \right\}$$

## 2.1310 ODE No. 1310

---

$$x^3 y''(x) + 3x^2 y'(x) + xy(x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.0078678 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{x} + \frac{c_2 \log(x)}{x} + \frac{\log^2(x)}{2x} \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 20

$$\left\{ y(x) = \frac{1}{x} \left( -C1 \ln(x) + \frac{(\ln(x))^2}{2} + -C2 \right) \right\}$$

## 2.1311 ODE No. 1311

---

$$-v(v+1)xy(x) + x(x^2+1)y''(x) + (2x^2+1)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0915081 (sec), leaf count = 63

$$\left\{ \left\{ y(x) \rightarrow c_2 G_{2,2}^{2,0} \left( -x^2 \middle| \frac{1-v}{2}, \frac{v+2}{2} \right) + c_1 {}_2F_1 \left( \frac{v}{2} + \frac{1}{2}, -\frac{v}{2}; 1; -x^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.098 (sec), leaf count = 52

$$\left\{ y(x) = -C1 {}_2F_1 \left( -\frac{v}{2}, \frac{1}{2} + \frac{v}{2}; \frac{1}{2}; x^2 + 1 \right) + -C2 \sqrt{x^2 + 1} {}_2F_1 \left( 1 + \frac{v}{2}, \frac{1}{2} - \frac{v}{2}; \frac{3}{2}; x^2 + 1 \right) \right\}$$

## 2.1312 ODE No. 1312

---

$$x(x^2+1)y''(x) + 2(x^2-1)y'(x) - 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0168132 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{x^2+1} + \frac{c_2 x^3}{3(x^2+1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 19

$$\left\{ y(x) = \frac{-C2 x^3 + -C1}{x^2 + 1} \right\}$$

### 2.1313 ODE No. 1313

---

$$x(-v-n)(n+v+1)y(x) + (2(n+1)x^2 + 2n+1)y'(x) + x(x^2+1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.14554 (sec), leaf count = 87

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1 \left( \frac{n}{2} - \frac{v}{2}, \frac{n}{2} + \frac{v}{2} + \frac{1}{2}; n+1; -x^2 \right) + c_2 x^{-2n} {}_2F_1 \left( -\frac{n}{2} - \frac{v}{2}, -\frac{n}{2} + \frac{v}{2} + \frac{1}{2}; 1-n; -x^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 35

$$\left\{ y(x) = x^{-n} \left( \text{LegendreQ} \left( v, n, \sqrt{x^2+1} \right) \_C2 + \text{LegendreP} \left( v, n, \sqrt{x^2+1} \right) \_C1 \right) \right\}$$

### 2.1314 ODE No. 1314

---

$$x(n-v-1)(n+v)y(x) - (2(n-1)x^2 + 2n-1)y'(x) + x(x^2+1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.133159 (sec), leaf count = 87

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1 \left( -\frac{n}{2} - \frac{v}{2}, -\frac{n}{2} + \frac{v}{2} + \frac{1}{2}; 1-n; -x^2 \right) + c_2 x^{2n} {}_2F_1 \left( \frac{n}{2} - \frac{v}{2}, \frac{n}{2} + \frac{v}{2} + \frac{1}{2}; n+1; -x^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.045 (sec), leaf count = 33

$$\left\{ y(x) = x^n \left( \text{LegendreQ} \left( v, n, \sqrt{x^2+1} \right) \_C2 + \text{LegendreP} \left( v, n, \sqrt{x^2+1} \right) \_C1 \right) \right\}$$

### 2.1315 ODE No. 1315

---

$$ax^3y(x) + (x^2-1)xy''(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0187241 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin \left( \sqrt{a} \sqrt{x^2-1} \right) + c_1 \cos \left( \sqrt{a} \sqrt{x^2-1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 45

$$\left\{ y(x) = \_C1 \sin \left( (1+x)(x-1) \sqrt{a} \frac{1}{\sqrt{x^2-1}} \right) + \_C2 \cos \left( (1+x)(x-1) \sqrt{a} \frac{1}{\sqrt{x^2-1}} \right) \right\}$$

### 2.1316 ODE No. 1316

---

$$x(x^2-1)y''(x) + (x^2-1)y'(x) - xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0653708 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow c_2 G_{2,2}^{2,0} \left( x^2 \middle| \begin{matrix} \frac{1}{2}, \frac{3}{2} \\ 0, 0 \end{matrix} \right) + \frac{2c_1 E(x^2)}{\pi} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 18

$$\{y(x) = \_C1 \text{EllipticE}(x) + \_C2 (\text{EllipticCE}(x) - \text{EllipticCK}(x))\}$$

### 2.1317 ODE No. 1317

---

$$x(x^2-1)y''(x) + (3x^2-1)y'(x) + xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0827074 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow c_2 G_{2,2}^{2,0} \left( x^2 \middle| \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, 0 \end{matrix} \right) + \frac{2c_1 K(x^2)}{\pi} \right\} \right\}$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 13

$$\{y(x) = \_C1 \text{EllipticK}(x) + \_C2 \text{EllipticCK}(x)\}$$

## 2.1318 ODE No. 1318

---

$$(ax^2 + b) y'(x) + cxy(x) + x(x^2 - 1) y''(x) = 0$$

✓ **Mathematica** : cpu = 0.205496 (sec), leaf count = 172

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1 \left( \frac{a}{4} - \frac{1}{4} \sqrt{a^2 - 2a - 4c + 1} - \frac{1}{4}, \frac{a}{4} + \frac{1}{4} \sqrt{a^2 - 2a - 4c + 1} - \frac{1}{4}; \frac{1}{2} - \frac{b}{2}; x^2 \right) + i^{b+1} c_2 x^{b+1} {}_2F_1 \left( \frac{a}{4} - \frac{1}{4} \sqrt{a^2 - 2a - 4c + 1} - \frac{1}{4}, \frac{a}{4} + \frac{1}{4} \sqrt{a^2 - 2a - 4c + 1} - \frac{1}{4}; \frac{1}{2} - \frac{b}{2}; x^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.067 (sec), leaf count = 122

$$\left\{ y(x) = {}_2F_1 \left( -\frac{1}{4} + \frac{a}{4} + \frac{1}{4} \sqrt{a^2 - 2a - 4c + 1}, -\frac{1}{4} + \frac{a}{4} - \frac{1}{4} \sqrt{a^2 - 2a - 4c + 1}; -\frac{b}{2} + \frac{1}{2}; x^2 \right) + {}_2F_1 \left( -\frac{1}{4} + \frac{a}{4} + \frac{1}{4} \sqrt{a^2 - 2a - 4c + 1}, -\frac{1}{4} + \frac{a}{4} - \frac{1}{4} \sqrt{a^2 - 2a - 4c + 1}; -\frac{b}{2} + \frac{1}{2}; x^2 \right) \right\}$$

## 2.1319 ODE No. 1319

---

$$x(x^2 + 2) y''(x) - y'(x) - 6xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0469194 (sec), leaf count = 60

$$\left\{ \left\{ y(x) \rightarrow c_1 x^{3/2} (x^2 + 2)^{3/4} - \frac{c_2 (x^2 + 2)^{3/4} {}_2F_1 \left( -\frac{3}{4}, \frac{7}{4}; \frac{1}{4}; -\frac{x^2}{2} \right)}{3 \cdot 2^{3/4}} \right\} \right\}$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 31

$$\left\{ y(x) = (x^2 + 2)^{\frac{3}{4}} \left( x^{\frac{3}{2}} {}_2F_1 \left( -\frac{3}{4}, \frac{7}{4}; \frac{1}{4}; -\frac{x^2}{2} \right) - C_2 \right) \right\}$$

## 2.1320 ODE No. 1320

---

$$x(x^2 - 2) y''(x) + (x^2 + 4x + 2) y(x) - (x^3 + 3x^2 - 2x - 2) y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0653842 (sec), leaf count = 21

$$\{ \{ y(x) \rightarrow c_1 e^x x^2 + c_2 (x - 1) \} \}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 17

$$\{ y(x) = {}_2F_1 \left( -\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; x \right) - C_2 e^x x^2 \}$$

## 2.1321 ODE No. 1321

---

$$(x + 1)x^2 y''(x) - (2x + 1)xy'(x) + (2x + 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0184189 (sec), leaf count = 18

$$\{ \{ y(x) \rightarrow c_1 x + c_2 x(x + \log(x)) \} \}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 15

$$\{ y(x) = x({}_2F_1 \left( -\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; x \right) - C_2 \ln(x) + C_1) \}$$

## 2.1322 ODE No. 1322

---

$$(x + 1)x^2 y''(x) + 2(3x + 2)xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0281078 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow c_1 \left( -\frac{1}{3x^3} + \frac{1}{x^2} - \frac{3}{x} - \frac{1}{x+1} - 4\log(x) + 4\log(x+1) \right) + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 44

$$\left\{ y(x) = {}_2F_1 \left( -\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; x \right) - C_2 \ln(x) + C_1 \right\}$$

**2.1323                    ODE No. 1323**

---

$$y''(x) = \frac{2(x+1)y(x)}{(x-1)x} - \frac{2(x-2)y'(x)}{(x-1)x}$$

✗ **Mathematica** : cpu = 0.649503 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot} \left( \{y, x\}, \{(-2x-2)y(x) + (2x-4)y'(x) + (x-1)xy''(x) = 0, y(2) = c_1, y'(2) = c_2\} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 17

$$\left\{ y(x) = \frac{-C1 + -C2 (x-1)^3}{x^2} \right\}$$

**2.1324                    ODE No. 1324**

---

$$y''(x) = \frac{(5x-4)y'(x)}{(x-1)x} - \frac{(9x-6)y(x)}{(x-1)x^2}$$

✓ **Mathematica** : cpu = 0.0206923 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow c_1 x^3 - c_2 x^2 (x \log(x) + 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 18

$$\{y(x) = x^2 (\ln(x) - C2 x + -C1 x + -C2)\}$$

**2.1325                    ODE No. 1325**

---

$$y''(x) = -\frac{y(x)(abx - \alpha\beta)}{(x-1)x^2} - \frac{y'(x)(x(a+b+1) + \alpha + \beta - 1)}{(x-1)x}$$

✓ **Mathematica** : cpu = 0.180463 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow (-1)^\alpha c_1 x^\alpha {}_2F_1(a+\alpha, \alpha+b; \alpha-\beta+1; x) + (-1)^\beta c_2 x^\beta {}_2F_1(a+\beta, b+\beta; -\alpha+\beta+1; x) \right\} \right\}$$

✓ **Maple** : cpu = 0.07 (sec), leaf count = 86

$$\left\{ y(x) = (x-1)^{1-a-\alpha-b-\beta} \left( {}_2F_1(1-b-\beta, 1-a-\beta; 1+\alpha-\beta; x)x^\alpha - C1 + {}_2F_1(1-a-\alpha, 1-\alpha-b; 1-\alpha+ \right. \right.$$

**2.1326                    ODE No. 1326**

---

$$y''(x) = -\frac{y'(x)}{x+1} - \frac{y(x)}{x(x+1)^2}$$

✓ **Mathematica** : cpu = 0.017492 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 x}{x+1} + \frac{c_2 (x \log(x) - 1)}{x+1} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 22

$$\left\{ y(x) = \frac{\ln(x) - C2 x + -C1 x - -C2}{1+x} \right\}$$

**2.1327                    ODE No. 1327**

---

$$y''(x) = \frac{2y'(x)}{(x-2)x} - \frac{y(x)}{(x-2)x^2}$$

✓ **Mathematica** : cpu = 0.127915 (sec), leaf count = 104

$$\left\{ \left\{ y(x) \rightarrow \left( -\frac{1}{2} \right)^{-\frac{1}{\sqrt{2}}} c_1 x^{-\frac{1}{\sqrt{2}}} {}_2F_1 \left( -\frac{1}{\sqrt{2}}, -1 - \frac{1}{\sqrt{2}}; 1 - \sqrt{2}; \frac{x}{2} \right) + \left( -\frac{1}{2} \right)^{\frac{1}{\sqrt{2}}} c_2 x^{\frac{1}{\sqrt{2}}} {}_2F_1 \left( \frac{1}{\sqrt{2}}, -1 + \frac{1}{\sqrt{2}}; 1 + \sqrt{2}; \frac{x}{2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.197 (sec), leaf count = 81

$$\left\{ y(x) = (x-2)^2 \left( -C2 {}_2F_1 \left( 2 + \frac{\sqrt{2}}{2}, 1 + \frac{\sqrt{2}}{2}; 1 + \sqrt{2}; \frac{x}{2} \right) x^{\frac{\sqrt{2}}{2}} + -C1 {}_2F_1 \left( 2 - \frac{\sqrt{2}}{2}, 1 - \frac{\sqrt{2}}{2}; 1 - \sqrt{2}; \frac{x}{2} \right) x^{-\frac{\sqrt{2}}{2}} \right) \right\}$$

**2.1328 ODE No. 1328**

---

$$y''(x) = \frac{2y(x)}{(x-1)^2x}$$

✓ **Mathematica** : cpu = 0.0153982 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 (-x^2 + 2x \log(x) + 1)}{x-1} - \frac{c_1 x}{x-1} \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 27

$$\left\{ y(x) = \frac{2 \ln(x) - C2 x - C2 x^2 + C1 x + C2}{x-1} \right\}$$

**2.1329 ODE No. 1329**

---

$$y''(x) = -\frac{y'(x) (-x(a(\delta + \text{gamma1}) + \alpha + \beta - \delta + 1) + a\text{gamma1} + x^2(\alpha + \beta + 1))}{(x-1)x(x-a)} - \frac{y(x)(\alpha\beta x - q)}{(x-1)x(x-a)}$$

✗ **Mathematica** : cpu = 4.56007 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{(x\alpha\beta - q)y(x) + (\alpha x^2 + \beta x^2 + x^2 - \alpha x - \beta x - a\delta x + \delta x - a\text{gamma1}x - x\alpha\beta x - q)\}) \right\} \right\}$$

✓ **Maple** : cpu = 0.184 (sec), leaf count = 64

$$\left\{ y(x) = C1 \text{HeunG}(a, q, \alpha, \beta, \gamma1, \delta, x) + C2 x^{1-\gamma1} \text{HeunG}(a, q - (-1 + \gamma1)(\delta(a-1) + \alpha + \beta - \gamma1 + 1), \beta, \gamma1, \delta, x) \right\}$$

**2.1330 ODE No. 1330**

---

$$y''(x) = -\frac{y'(x)(Ax^2 + Bx + C)}{(x-a)(x-b)(x-c)} - \frac{(DDx + e)y(x)}{(x-a)(x-b)(x-c)}$$

✗ **Mathematica** : cpu = 111.392 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{(xDD + e)y(x) + (Ax^2 + Bx + C)y'(x) - (a-x)(b-x)(c-x)y''(x) = 0, (DDx + e)y(x) - (Ax^2 + Bx + C)y'(x) - (a-x)(b-x)(c-x)y''(x) = 0\}) \right\} \right\}$$

✓ **Maple** : cpu = 0.707 (sec), leaf count = 1147

$$\left\{ y(x) = C1 \text{HeunG}\left(\frac{a-c}{a-b}, \frac{DDa + E}{a-b}, \frac{A}{2} - \frac{1}{2} + \frac{1}{2}\sqrt{A^2 - 2A - 4DD + 1}, 1, \left((A(b-c)a - Abc - Bc - C)\sqrt{a-b}\right)\right) \right\}$$

**2.1331 ODE No. 1331**

---

$$y''(x) = \frac{(x-4)y'(x)}{2(x-2)x} - \frac{(x-3)y(x)}{2(x-2)x^2}$$

✓ **Mathematica** : cpu = 0.0322935 (sec), leaf count = 55

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 \sqrt[4]{x-2}\sqrt{x}}{\sqrt[4]{2-x}} + \frac{2c_2(x-2)^{3/4}\sqrt{x}}{\sqrt[4]{2-x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 19

$$\left\{ y(x) = C1 \sqrt{x} + C2 \sqrt{x(x-2)} \right\}$$

**2.1332 ODE No. 1332**

---

$$y''(x) = \frac{y'(x)}{x+1} - \frac{(3x+1)y(x)}{4x^2(x+1)}$$

✓ **Mathematica** : cpu = 0.0176087 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt{x} + c_2 \sqrt{x}(x + \log(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 17

$$\left\{ y(x) = \sqrt{x}(-C2 \ln(x) + C2 x + C1) \right\}$$

## 2.1333 ODE No. 1333

---

$$y''(x) = \frac{v(v+1)y(x)}{4x^2} - \frac{(3x-1)y'(x)}{2(x-1)x}$$

✓ **Mathematica** : cpu = 0.0760837 (sec), leaf count = 70

$$\left\{ \left\{ y(x) \rightarrow c_1 i^{-v} x^{-v/2} {}_2F_1\left(\frac{1}{2}, -v; \frac{1}{2} - v; x\right) + c_2 i^{v+1} x^{\frac{v+1}{2}} {}_2F_1\left(\frac{1}{2}, v+1; v + \frac{3}{2}; x\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 45

$$\left\{ y(x) = {}_C1 x^{-\frac{v}{2}} {}_2F_1\left(\frac{1}{2}, -v; \frac{1}{2} - v; x\right) + {}_C2 x^{\frac{1}{2} + \frac{v}{2}} {}_2F_1\left(\frac{1}{2}, v+1; \frac{3}{2} + v; x\right) \right\}$$

## 2.1334 ODE No. 1334

---

$$y''(x) = -\frac{y(x)(x(a^2 - b^2) + c^2)}{4(x-1)x^2} - \frac{((a+1)x-1)y'(x)}{(x-1)x}$$

✓ **Mathematica** : cpu = 0.142146 (sec), leaf count = 114

$$\left\{ \left\{ y(x) \rightarrow i^{-c} c_1 x^{-c/2} {}_2F_1\left(\frac{a}{2} - \frac{b}{2} - \frac{c}{2}, \frac{a}{2} + \frac{b}{2} - \frac{c}{2}; 1 - c; x\right) + i^c c_2 x^{c/2} {}_2F_1\left(\frac{a}{2} - \frac{b}{2} + \frac{c}{2}, \frac{a}{2} + \frac{b}{2} + \frac{c}{2}; c+1; x\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 89

$$\left\{ y(x) = (x-1)^{1-a} \left( x^{\frac{c}{2}} {}_2F_1\left(-\frac{a}{2} - \frac{b}{2} + \frac{c}{2} + 1, -\frac{a}{2} + \frac{b}{2} + \frac{c}{2} + 1; 1 + c; x\right) {}_C1 + {}_2F_1\left(-\frac{a}{2} + \frac{b}{2} - \frac{c}{2} + 1, -\frac{a}{2} - \frac{b}{2} \right) \right. \right.$$

## 2.1335 ODE No. 1335

---

$$y''(x) = -\frac{y(x)(ax+b)}{4(x-1)^2x} - \frac{(3x-1)y'(x)}{2(x-1)x}$$

✓ **Mathematica** : cpu = 0.226111 (sec), leaf count = 893

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{1}{4}(-2\log(1-x) - \log(x))} \sqrt[4]{x} c_1 {}_2F_1\left(\frac{1}{4} \left( \sqrt{-8a-4b-4\sqrt{4a^2+4ba-a-b+1+1}} \right), \frac{(-8a-4b-4\sqrt{4a^2+4ba-a-b+1+1})}{4} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 57

$$\left\{ y(x) = {}_C1 \text{LegendreP}\left(\frac{1}{2}\sqrt{1-4a} - \frac{1}{2}, \sqrt{-a-b}, \sqrt{x}\right) + {}_C2 \text{LegendreQ}\left(\frac{1}{2}\sqrt{1-4a} - \frac{1}{2}, \sqrt{-a-b}, \sqrt{x}\right) \right\}$$

## 2.1336 ODE No. 1336

---

$$y''(x) = -\frac{(1-3x)y(x)}{(x-1)(2x-1)^2}$$

✓ **Mathematica** : cpu = 0.0351568 (sec), leaf count = 70

$$\left\{ \left\{ y(x) \rightarrow c_2 \sqrt{1-2x} (2x \log(2(x-1)+1) - 2 \log(2(x-1)+1) - 2x \log(x-1) + 2 \log(x-1) - 1) - c_1 \sqrt{1-2x} \right\} \right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 42

$$\left\{ y(x) = \sqrt{2x-1} (-2 {}_C2 (x-1) \ln(2x-1) + 2 {}_C2 (x-1) \ln(x-1) + {}_C1 x - {}_C1 + {}_C2) \right\}$$

**2.1337 ODE No. 1337**

---

$$y''(x) = -\frac{(a+2b+3x)y'(x)}{2(a+x)(b+x)} - \frac{(a-b)y(x)}{4(a+x)^2(b+x)}$$

✓ **Mathematica** : cpu = 0.0517003 (sec), leaf count = 62

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{\sqrt{\frac{b+x}{a-b}} + 1} + \frac{c_2 \sqrt{b+x}}{\sqrt{a-b} \sqrt{\frac{b+x}{a-b}} + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 27

$$\left\{ y(x) = 1 \left( \sqrt{x+b} \_C1 + \_C2 \right) \frac{1}{\sqrt{\frac{x+a}{a-b}}} \right\}$$

**2.1338 ODE No. 1338**

---

$$y''(x) = \frac{y(x)}{3(x-2)x^2} + \frac{(6x-1)y'(x)}{3(x-2)x}$$

✓ **Mathematica** : cpu = 0.0398033 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow \frac{3}{935} c_2 x (18x^2 - 102x + 187) + c_1 \sqrt[6]{x} (2-x)^{17/6} \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 27

$$\left\{ y(x) = \_C2 \sqrt[6]{x} (x-2)^{\frac{17}{6}} + 18x \left( x^2 - \frac{17x}{3} + \frac{187}{18} \right) \_C1 \right\}$$

**2.1339 ODE No. 1339**

---

$$y''(x) = -\frac{y'(x)(a(b+2)x^2 + x(c-d+1))}{x^2(ax+1)} - \frac{y(x)(abx-cd)}{x^2(ax+1)}$$

✓ **Mathematica** : cpu = 0.188441 (sec), leaf count = 66

$$\left\{ \left\{ y(x) \rightarrow c_1 a^{-c} x^{-c} {}_2F_1(1-c, b-c; -c-d+1; -ax) + c_2 a^d x^d {}_2F_1(d+1, b+d; c+d+1; -ax) \right\} \right\}$$

✓ **Maple** : cpu = 0.074 (sec), leaf count = 76

$$\left\{ y(x) = (ax+1)^{-b+c-d} \left( {}_2F_1(-d, 1-b-d; 1-d-c; -ax) x^{-c} \_C2 + {}_2F_1(c, 1-b+c; 1+d+c; -ax) x^d \_C1 \right) \right\}$$

**2.1340 ODE No. 1340**

---

$$y''(x) = \frac{2(ax+2b)y'(x)}{x(ax+b)} - \frac{y(x)(2ax+6b)}{x^2(ax+b)}$$

✓ **Mathematica** : cpu = 0.0255379 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 x^3}{ax+b} + \frac{c_1 x^2}{ax+b} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 20

$$\left\{ y(x) = \frac{x^2 (\_C2 x + \_C1)}{ax+b} \right\}$$

**2.1341 ODE No. 1341**

---

$$y''(x) = -\frac{y(x)(avx-b)}{x^2(ax+b)} - \frac{(2ax+b)y'(x)}{x(ax+b)} + Ax$$

✓ **Mathematica** : cpu = 61.8602 (sec), leaf count = 2924



$$\left\{\left\{y(x) \rightarrow \frac{axc_1 {}_2F_1\left(\frac{3}{2}-\frac{1}{2}\sqrt{1-4v}, \frac{1}{2}\sqrt{1-4v}+\frac{3}{2}; 3; -\frac{ax}{b}\right)}{b} + c_2 G_{2,2}^{2,0}\left(-\frac{ax}{b} \middle| \begin{matrix} \frac{1}{2}(1-\sqrt{1-4v}), \frac{1}{2}(\sqrt{1-4v}+1) \\ -1, 1 \end{matrix}\right)\right.\right.$$

✓ **Maple** : cpu = 0.114 (sec), leaf count = 201

$$\left\{y(x)=\frac{1}{a^2(v+6)(v+2)(v+12)}\left(x^{-\frac{1}{2}+\frac{1}{2}\sqrt{1-4v}}a^2-C2(v+6)(v+2)(v+12){}_2F_1\left(\frac{3}{2}-\frac{1}{2}\sqrt{1-4v},-\frac{1}{2}-\frac{1}{2}\sqrt{1-4v}\right)\right)\right\}$$

## 2.1342 ODE No. 1342

---

$$y''(x)=-\frac{ay(x)}{x^4}$$

✓ **Mathematica** : cpu = 0.0519943 (sec), leaf count = 52

$$\left\{\left\{y(x) \rightarrow c_1 x e^{\frac{i \sqrt{a}}{x}}-\frac{i c_2 x e^{-\frac{i \sqrt{a}}{x}}}{2 \sqrt{a}}\right\}\right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 31

$$\left\{y(x)=x\left(\cosh \left(\frac{1}{x} \sqrt{-a}\right)-C2+\sinh \left(\frac{1}{x} \sqrt{-a}\right)-C1\right)\right\}$$

## 2.1343 ODE No. 1343

---

$$y''(x)=-\frac{y(x)\left((1-a) a x^2-b(b+x)\right)}{x^4}$$

✗ **Mathematica** : cpu = 0.544853 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{\left\{y(x) \rightarrow \text{DifferentialRoot}\left(\left\{y, x\right\},\left\{y''(x) x^4+\left(-a^2 x^2+a x^2-b x-b^2\right) y(x)=0, y(1)=c_1, y'(1)=c_2\right\}\right)(x)\right\}\right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 58

$$\left\{y(x)=I_{a+1}\left(\frac{b}{x}\right)-C1\,b-K_{a+1}\left(\frac{b}{x}\right)-C2\,b+2\,(ax+b/2)\left(-C1\,I_a\left(\frac{b}{x}\right)+-C2\,K_a\left(\frac{b}{x}\right)\right)\right\}$$

## 2.1344 ODE No. 1344

---

$$y''(x)=-\frac{\left(e^{2/x}-v^2\right) y(x)}{x^4}$$

✓ **Mathematica** : cpu = 0.446925 (sec), leaf count = 173

$$\left\{\left\{y(x) \rightarrow \frac{c_1 2^{v+\frac{v+1}{2}}\left(e^{2/x}\right)^{\frac{v+1}{2}-\frac{1}{2}}\left(-e^{2/x}\right)^{\frac{1}{2}(-v-1)+\frac{1}{2}} I_v\left(\sqrt{-e^{2/x}}\right)}{\log \left(e^{2/x}\right)}+\frac{c_2(-1)^{-v} 2^{v+\frac{v+1}{2}}\left(e^{2/x}\right)^{\frac{v+1}{2}-\frac{1}{2}}\left(-e^{2/x}\right)^{\frac{1}{2}(-v-1)+\frac{1}{2}}}{\log \left(e^{2/x}\right)}\right.\right.$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 23

$$\left\{y(x)=x\left(J_v\left(e^{x^{-1}}\right)-C1+Y_v\left(e^{x^{-1}}\right)-C2\right)\right\}$$

2.1345 ODE No. 1345

---

$$y''(x) = \frac{2y(x)}{x^4} - \frac{y'(x)}{x^3}$$

✓ **Mathematica** : cpu = 0.0328943 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{2x^2}} x - \sqrt{\frac{\pi}{2}} c_2 e^{\frac{1}{2x^2}} x \operatorname{erf}\left(\frac{1}{\sqrt{2}x}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 25

$$\left\{ y(x) = x e^{\frac{1}{2x^2}} \left( \operatorname{Erf}\left(\frac{\sqrt{2}}{2x}\right) {}_2C_2 + {}_2C_1 \right) \right\}$$

2.1346 ODE No. 1346

---

$$y''(x) = \frac{(a+b)y'(x)}{x^2} - \frac{y(x)(x(a+b)+ab)}{x^4}$$

✓ **Mathematica** : cpu = 0.0630303 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 x e^{-\frac{a}{x}}}{a-b} + c_1 x e^{-\frac{b}{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 25

$$\left\{ y(x) = x \left( e^{-\frac{b}{x}} {}_2C_2 + e^{-\frac{a}{x}} {}_2C_1 \right) \right\}$$

2.1347 ODE No. 1347

---

$$y''(x) = -\frac{y(x)}{x^4} - \frac{y'(x)}{x}$$

✓ **Mathematica** : cpu = 0.0839685 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow c_2 J_0\left(\frac{1}{x}\right) + \frac{c_1 K_0\left(\frac{i}{x}\right)}{\sqrt{\pi}} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 19

$$\left\{ y(x) = {}_2C_1 J_0(x^{-1}) + {}_2C_2 Y_0(x^{-1}) \right\}$$

2.1348 ODE No. 1348

---

$$y''(x) = -\frac{y(x)(a(x^4+1)+bx^2)}{x^4} - \frac{y'(x)}{x}$$

✗ **Mathematica** : cpu = 1.12217 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \operatorname{DifferentialRoot}\left(\{y, x\}, \left\{ y''(x)x^4 + y'(x)x^3 + (ax^4 + bx^2 + a)y(x) = 0, y(1) = c_1, y'(1) = c_2 \right\}\right)(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.125 (sec), leaf count = 73

$$\left\{ y(x) = \operatorname{HeunD}\left(0, 2a+b, 0, 2a-b, \frac{x^2+1}{x^2-1}\right) \left( \int \frac{1}{x} \left( \operatorname{HeunD}\left(0, 2a+b, 0, 2a-b, \frac{x^2+1}{x^2-1}\right) \right)^{-2} dx {}_2C_2 + {}_2C_1 \right) \right\}$$

2.1349 ODE No. 1349

---

$$y''(x) = -\frac{y(x)}{x^4} - \frac{(x^2+1)y'(x)}{x^3}$$

✓ **Mathematica** : cpu = 0.0837524 (sec), leaf count = 76

$$\left\{ \left\{ y(x) \rightarrow c_2 G_{1,2}^{2,0}\left(-\frac{1}{2x^2} \middle| \begin{matrix} \frac{3}{2} \\ 0, 0 \end{matrix} \right) + c_1 e^{\frac{1}{4x^2}} \left( \left(1 - \frac{1}{2x^2}\right) I_0\left(\frac{1}{4x^2}\right) + \frac{I_1\left(\frac{1}{4x^2}\right)}{2x^2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 85

$$\left\{ y(x) = \frac{{}_2C_1}{x^2} e^{\frac{1}{4x^2}} \left( 2x^2 I_0(1/4x^{-2}) + I_1\left(\frac{1}{4x^2}\right) - I_0\left(\frac{1}{4x^2}\right) \right) + \frac{{}_2C_2}{x^2} e^{\frac{1}{4x^2}} \left( 2K_0(-1/4x^{-2})x^2 - K_0\left(-\frac{1}{4x^2}\right) \right) \right\}$$

**2.1350                    ODE No. 1350**

---

$$y''(x) = -\frac{a^2 y(x)}{x^4} - \frac{2y'(x)}{x}$$

✓ **Mathematica** : cpu = 0.0078448 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow c_1 \cos\left(\frac{a}{x}\right) - c_2 \sin\left(\frac{a}{x}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 21

$$\left\{ y(x) = \_C1 \sin\left(\frac{a}{x}\right) + \_C2 \cos\left(\frac{a}{x}\right) \right\}$$

**2.1351                    ODE No. 1351**

---

$$y''(x) = \frac{y(x)}{x^4} - \frac{(2x^2 + 1)y'(x)}{x^3}$$

✓ **Mathematica** : cpu = 0.0176943 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{2x^2}} - \sqrt{\frac{\pi}{2}} c_2 e^{\frac{1}{2x^2}} \operatorname{erf}\left(\frac{1}{\sqrt{2}x}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 24

$$\left\{ y(x) = e^{\frac{1}{2x^2}} \left( \operatorname{Erf}\left(\frac{\sqrt{2}}{2x}\right) \_C2 + \_C1 \right) \right\}$$

**2.1352                    ODE No. 1352**

---

$$y''(x) = -\frac{2(a+x)y'(x)}{x^2} - \frac{by(x)}{x^4}$$

✓ **Mathematica** : cpu = 0.0104785 (sec), leaf count = 89

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\frac{\sqrt{b}\left(-\frac{\sqrt{a^2-b}}{\sqrt{b}} - \frac{a}{\sqrt{b}}\right)}{x}} + c_2 e^{-\frac{\sqrt{b}\left(\frac{\sqrt{a^2-b}}{\sqrt{b}} - \frac{a}{\sqrt{b}}\right)}{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 43

$$\left\{ y(x) = \_C1 e^{\frac{1}{x}(a-\sqrt{a^2-b})} + \_C2 e^{\frac{1}{x}(a+\sqrt{a^2-b})} \right\}$$

**2.1353                    ODE No. 1353**

---

$$y''(x) = \frac{(2x^2 - 1)y'(x)}{x^3} - \frac{y(x)}{x^4}$$

✓ **Mathematica** : cpu = 0.87181 (sec), leaf count = 119

$$\left\{ \left\{ y(x) \rightarrow c_1 \left( x^3 + 2x - \frac{1}{x} \right) - \frac{c_2 \left( \sqrt{2}\pi x^4 \operatorname{erfi}\left(\frac{1}{\sqrt{2}x}\right) + 2\sqrt{2}\pi x^2 \operatorname{erfi}\left(\frac{1}{\sqrt{2}x}\right) - \sqrt{2}\pi \operatorname{erfi}\left(\frac{1}{\sqrt{2}x}\right) + 2e^{\frac{1}{2x^2}}x - 2e^{\frac{1}{2x^2}}x \right)}{16x} \right\} \right\}$$

✓ **Maple** : cpu = 0.18 (sec), leaf count = 67

$$\left\{ y(x) = \frac{1}{x} \left( \_C1 \sqrt{2}\sqrt{\pi} (x^4 + 2x^2 - 1) \operatorname{erfi}\left(\frac{\sqrt{2}}{2x}\right) + (2\_C1 x^3 - 2\_C1 x) e^{\frac{1}{2x^2}} + \_C2 (x^4 + 2x^2 - 1) \right) \right\}$$

**2.1354                    ODE No. 1354**

---

$$y''(x) = \frac{(2x^2 - 1)y'(x)}{x^3} - \frac{2y(x)}{x^4}$$

✓ **Mathematica** : cpu = 0.225552 (sec), leaf count = 108

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \left( -5\sqrt{2}\pi x^2 \operatorname{erfi}\left(\frac{1}{\sqrt{2}x}\right) + \sqrt{2}\pi \operatorname{erfi}\left(\frac{1}{\sqrt{2}x}\right) - 2e^{\frac{1}{2x^2}}x + 4e^{\frac{1}{2x^2}}x^5 + 8e^{\frac{1}{2x^2}}x^3 \right)}{12x^2} + c_1 \left( 1 - \frac{1}{5x^2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.234 (sec), leaf count = 33

$$\left\{y(x) = \frac{1}{x^2} \left( -C2 {}_1F_1\left(-\frac{5}{2}; -\frac{1}{2}; \frac{1}{2x^2}\right)x^5 + 5 {}_C1 x^2 - {}_C1 \right) \right\}$$

## 2.1355 ODE No. 1355

---

$$y''(x) = \frac{xy(x)}{x^3 + 1} - \frac{(x^3 - 1)y'(x)}{x(x^3 + 1)}$$

✓ **Mathematica** : cpu = 0.235685 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} c_2 \sqrt[3]{x^3 + 1} x^2 {}_2F_1\left(\frac{2}{3}, \frac{4}{3}; \frac{5}{3}; -x^3\right) + c_1 \sqrt[3]{x^3 + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.08 (sec), leaf count = 30

$$\left\{y(x) = \sqrt[3]{x^3 + 1} \left( {}_2F_1\left(\frac{2}{3}, \frac{4}{3}; \frac{5}{3}; -x^3\right) {}_C1 x^2 + {}_C2 \right) \right\}$$

## 2.1356 ODE No. 1356

---

$$y''(x) = -\frac{y(x)(-n^2 - v(v+1)x^2)}{x^2(x^2 + 1)} - \frac{(2x^2 + 1)y'(x)}{x(x^2 + 1)}$$

✓ **Mathematica** : cpu = 0.219516 (sec), leaf count = 90

$$\left\{ \left\{ y(x) \rightarrow c_1 x^{-n} {}_2F_1\left(-\frac{n}{2} - \frac{v}{2}, -\frac{n}{2} + \frac{v}{2} + \frac{1}{2}; 1 - n; -x^2\right) + c_2 x^n {}_2F_1\left(\frac{n}{2} - \frac{v}{2}, \frac{n}{2} + \frac{v}{2} + \frac{1}{2}; n + 1; -x^2\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.045 (sec), leaf count = 29

$$\left\{y(x) = {}_C1 \operatorname{LegendreP}\left(v, n, \sqrt{x^2 + 1}\right) + {}_C2 \operatorname{LegendreQ}\left(v, n, \sqrt{x^2 + 1}\right) \right\}$$

## 2.1357 ODE No. 1357

---

$$y''(x) = -\frac{(ax^2 + a - 1)y'(x)}{x(x^2 + 1)} - \frac{y(x)(bx^2 + c)}{x^2(x^2 + 1)}$$

✓ **Mathematica** : cpu = 0.500163 (sec), leaf count = 288

$$\left\{ \left\{ y(x) \rightarrow c_1 x^{\frac{1}{2}(-\sqrt{a^2 - 4a - 4c + 4} - a + 2)} {}_2F_1\left(-\frac{1}{4}\sqrt{a^2 - 2a - 4b + 1} - \frac{1}{4}\sqrt{a^2 - 4a - 4c + 4} + \frac{1}{4}, \frac{1}{4}\sqrt{a^2 - 2a - 4b + 1} + \frac{1}{4}\sqrt{a^2 - 4a - 4c + 4} + \frac{1}{4}; \frac{1}{4}\sqrt{a^2 - 2a - 4b + 1} + \frac{1}{4}\sqrt{a^2 - 4a - 4c + 4} + \frac{1}{4}; -x^2\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 97

$$\left\{y(x) = x^{1 - \frac{a}{2}} \left( \operatorname{LegendreQ}\left(-\frac{1}{2} + \frac{1}{2}\sqrt{a^2 - 2a - 4b + 1}, \frac{1}{2}\sqrt{a^2 - 4a - 4c + 4}, \sqrt{x^2 + 1}\right) {}_C2 + \operatorname{LegendreP}\left(-\frac{1}{2} + \frac{1}{2}\sqrt{a^2 - 2a - 4b + 1}, \frac{1}{2}\sqrt{a^2 - 4a - 4c + 4}, \sqrt{x^2 + 1}\right) {}_C1 \right) \right\}$$

## 2.1358 ODE No. 1358

---

$$y''(x) = \frac{(x^2 - 2)y'(x)}{x(x^2 - 1)} - \frac{(x^2 - 2)y(x)}{x^2(x^2 - 1)}$$

✓ **Mathematica** : cpu = 0.0464735 (sec), leaf count = 89

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 x \sqrt[4]{x^2 - 1}}{\sqrt[4]{1 - x^2}} - \frac{c_2 x \sqrt[4]{x^2 - 1} \left( \log\left(1 - \frac{x}{\sqrt{x^2 - 1}}\right) - \log\left(\frac{x}{\sqrt{x^2 - 1}} + 1\right) \right)}{2 \sqrt[4]{1 - x^2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 20

$$\left\{y(x) = x \left( \ln\left(x + \sqrt{x^2 - 1}\right) {}_C2 + {}_C1 \right) \right\}$$

2.1359 ODE No. 1359

---

$$y''(x) = -\frac{v(v+1)y(x)}{x^2(x^2-1)} - \frac{2xy'(x)}{x^2-1}$$

✓ **Mathematica** : cpu = 0.0769748 (sec), leaf count = 86

$$\left\{ \left\{ y(x) \rightarrow c_1 i^{-v} x^{-v} {}_2F_1\left(\frac{1}{2} - \frac{v}{2}, -\frac{v}{2}; \frac{1}{2} - v; x^2\right) + c_2 i^{v+1} x^{v+1} {}_2F_1\left(\frac{v}{2} + \frac{1}{2}, \frac{v}{2} + 1; v + \frac{3}{2}; x^2\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 57

$$\left\{ y(x) = {}_C1 {}_2F_1\left(-\frac{v}{2}, \frac{1}{2} - \frac{v}{2}; \frac{1}{2} - v; x^2\right) x^{-v} + {}_C2 {}_2F_1\left(1 + \frac{v}{2}, \frac{1}{2} + \frac{v}{2}; \frac{3}{2} + v; x^2\right) x^{v+1} \right\}$$

2.1360 ODE No. 1360

---

$$y''(x) = \frac{v(v+1)y(x)}{x^2} - \frac{2xy'(x)}{x^2-1}$$

✓ **Mathematica** : cpu = 0.0702283 (sec), leaf count = 68

$$\left\{ \left\{ y(x) \rightarrow c_1 i^{-v} x^{-v} {}_2F_1\left(\frac{1}{2}, -v; \frac{1}{2} - v; x^2\right) + c_2 i^{v+1} x^{v+1} {}_2F_1\left(\frac{1}{2}, v + 1; v + \frac{3}{2}; x^2\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 47

$$\left\{ y(x) = {}_C1 {}_2F_1\left(\frac{1}{2}, -v; \frac{1}{2} - v; x^2\right) x^{-v} + {}_C2 {}_2F_1\left(\frac{1}{2}, v + 1; \frac{3}{2} + v; x^2\right) x^{v+1} \right\}$$

2.1361 ODE No. 1361

---

$$y''(x) = \frac{2xy'(x)}{x^2-1} - \frac{(a(a+1) - a(a+3)x^2)y(x)}{x^2(x^2-1)}$$

✓ **Mathematica** : cpu = 0.352708 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow c_1 x^{-a} + c_2 (-2ax^2 + 2a - x^2 + 3) x^{a+1} \right\} \right\}$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 33

$$\left\{ y(x) = {}_C1 x^{-a} + {}_C2 x^{a+1} (2ax^2 + x^2 - 2a - 3) \right\}$$

2.1362 ODE No. 1362

---

$$y''(x) = \frac{2xy'(x)}{x^2-1} - \frac{y(x) ((x^2-1)x^2(a-n)(a+n+1) + 2ax^2 + n(n+1)(x^2-1))}{x^2(x^2-1)}$$

✗ **Mathematica** : cpu = 9.96565 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}\left(\{y, x\}, \left\{-2y'(x)x^3 + (a^2x^4 - n^2x^4 + ax^4 - nx^4 - a^2x^2 + 2n^2x^2 + ax^2 + 2nx^2 - n^2\right\}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.136 (sec), leaf count = 109

$$\left\{ y(x) = {}_C1 \text{HeunC}\left(0, -n - \frac{1}{2}, -2, -\frac{a^2}{4} + \frac{n^2}{4} - \frac{a}{4} + \frac{n}{4}, -\frac{n^2}{4} - \frac{n}{4} + \frac{3}{4} + \frac{a^2}{4} - \frac{a}{4}, x^2\right) x^{-n} + {}_C2 \text{HeunC}\left(0, -n - \frac{1}{2}, -2, -\frac{a^2}{4} + \frac{n^2}{4} - \frac{a}{4} + \frac{n}{4}, -\frac{n^2}{4} - \frac{n}{4} + \frac{3}{4} + \frac{a^2}{4} - \frac{a}{4}, x^2\right) x^{-n} \right\}$$

2.1363 ODE No. 1363

---

$$y''(x) = -\frac{(ax^2 + a - 2)y'(x)}{x(x^2-1)} - \frac{by(x)}{x^2}$$

✓ **Mathematica** : cpu = 0.555293 (sec), leaf count = 236

$$\left\{ \left\{ y(x) \rightarrow c_1 (-1)^{\frac{1}{4}(-\sqrt{a^2-2a-4b+1}+a-1)} x^{\frac{1}{2}(-\sqrt{a^2-2a-4b+1}+a-1)} {}_2F_1\left(\frac{a}{2} - \frac{1}{2}, \frac{a}{2} - \frac{1}{2}; \frac{a}{2} - \frac{1}{2}; \sqrt{a^2-2a-4b+1} - \frac{1}{2}; 1 - \frac{1}{2}\right) + c_2 (-1)^{\frac{1}{4}(-\sqrt{a^2-2a-4b+1}+a-1)} x^{\frac{1}{2}(-\sqrt{a^2-2a-4b+1}+a-1)} {}_2F_1\left(\frac{a}{2} - \frac{1}{2}, \frac{a}{2} - \frac{1}{2}; \frac{a}{2} - \frac{1}{2}; \sqrt{a^2-2a-4b+1} - \frac{1}{2}; 1 - \frac{1}{2}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.076 (sec), leaf count = 161

$$\left\{ y(x) = (x^2-1)^{-a+2} \left( {}_2F_1\left(-\frac{a}{2} + \frac{3}{2}, -\frac{a}{2} + \frac{3}{2} + \frac{1}{2}\sqrt{a^2-2a-4b+1}; 1 + \frac{1}{2}\sqrt{a^2-2a-4b+1}; x^2\right) x^{\frac{a}{2}-\frac{1}{2}+\frac{1}{2}\sqrt{a^2-2a-4b+1}} + {}_2F_1\left(-\frac{a}{2} + \frac{3}{2}, -\frac{a}{2} + \frac{3}{2} - \frac{1}{2}\sqrt{a^2-2a-4b+1}; 1 - \frac{1}{2}\sqrt{a^2-2a-4b+1}; x^2\right) x^{\frac{a}{2}-\frac{1}{2}-\frac{1}{2}\sqrt{a^2-2a-4b+1}} \right) \right\}$$

**2.1364 ODE No. 1364**

---

$$y''(x) = \frac{y'(x) \left(2(a-1)x^2 - 2a + 2bc \left(x^2 - 1\right) x^c\right)}{x \left(x^2 - 1\right)} - \frac{y(x) \left(bc(2a - c - 1)x^{c+2} - bc(2a - c + 1)x^c + x^2((a-1)a - bc)\right)}{x^2 \left(x^2 - 1\right)}$$

✓ **Mathematica** : cpu = 0.120261 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow c_1 P_v(x) e^{a \log(x) + bx^c} + c_2 Q_v(x) e^{a \log(x) + bx^c} \right\} \right\}$$

✓ **Maple** : cpu = 0.065 (sec), leaf count = 25

$$\left\{ y(x) = x^a e^{bx^c} \left( LegendreP(v, x) \_C1 + LegendreQ(v, x) \_C2 \right) \right\}$$

**2.1365 ODE No. 1365**

---

$$y''(x) = -\frac{ay(x)}{(x^2 + 1)^2}$$

✓ **Mathematica** : cpu = 0.0756184 (sec), leaf count = 104

$$\left\{ \left\{ y(x) \rightarrow \frac{ic_2 \sqrt{x^2 + 1} (1 - ix)^{\sqrt{a+1}} (1 + ix)^{-\sqrt{a+1}} e^{i\sqrt{a+1} \tan^{-1}(x)}}{2\sqrt{a+1}} + c_1 \sqrt{x^2 + 1} e^{i\sqrt{a+1} \tan^{-1}(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 59

$$\left\{ y(x) = \sqrt{x^2 + 1} \left( \left( \frac{x+i}{-x+i} \right)^{-\frac{1}{2}\sqrt{a+1}} \_C2 + \left( \frac{x+i}{-x+i} \right)^{\frac{1}{2}\sqrt{a+1}} \_C1 \right) \right\}$$

**2.1366 ODE No. 1366**

---

$$y''(x) = -\frac{2xy'(x)}{x^2 + 1} - \frac{y(x)}{(x^2 + 1)^2}$$

✓ **Mathematica** : cpu = 0.0176026 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{\sqrt{x^2 + 1}} + \frac{c_2 x}{\sqrt{x^2 + 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 17

$$\left\{ y(x) = (\_C1 x + \_C2) \frac{1}{\sqrt{x^2 + 1}} \right\}$$

**2.1367 ODE No. 1367**

---

$$y''(x) = -\frac{y(x) \left(a^2 \left(x^2 + 1\right)^2 + m^2 - n(n+1) \left(x^2 + 1\right)\right)}{\left(x^2 + 1\right)^2} - \frac{2xy'(x)}{x^2 + 1}$$

✗ **Mathematica** : cpu = 1.70092 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot} \left( \{y, x\}, \left\{ y''(x) \left(x^2 + 1\right)^2 + 2xy'(x) \left(x^2 + 1\right) + \left(a^2 x^4 + 2a^2 x^2 - n^2 x^2 - nx^2 + a^2 + m^2 - n(n+1) \left(x^2 + 1\right)\right) \right\} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.14 (sec), leaf count = 88

$$\left\{ y(x) = \left(x^2 + 1\right)^{\frac{m}{2}} \left( HeunC \left( 0, \frac{1}{2}, m, -\frac{a^2}{4}, \frac{1}{4} + \frac{a^2}{4} + \frac{m^2}{4} - \frac{n^2}{4} - \frac{n}{4}, -x^2 \right) \_C2 x + HeunC \left( 0, -\frac{1}{2}, m, -\frac{a^2}{4}, \frac{1}{4} + \frac{a^2}{4} + \frac{m^2}{4} - \frac{n^2}{4} - \frac{n}{4}, -x^2 \right) \_C1 \right) \right\}$$

**2.1368 ODE No. 1368**

---

$$y''(x) = -\frac{axy'(x)}{x^2 + 1} - \frac{by(x)}{(x^2 + 1)^2}$$

✓ **Mathematica** : cpu = 0.0198983 (sec), leaf count = 106

$$\left\{ \left\{ y(x) \rightarrow c_1 \left(x^2 + 1\right)^{\frac{2-a}{4}} P^{\frac{1}{2}\sqrt{a^2-4a+4b+4}}_{\frac{a-2}{2}}(ix) + c_2 \left(x^2 + 1\right)^{\frac{2-a}{4}} Q^{\frac{1}{2}\sqrt{a^2-4a+4b+4}}_{\frac{a-2}{2}}(ix) \right\} \right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 71

$$\left\{ y(x) = \left(x^2 + 1\right)^{\frac{1}{2}-\frac{a}{4}} \left( LegendreQ \left( \frac{a}{2} - 1, \frac{1}{2} \sqrt{a^2 - 4a + 4b + 4}, ix \right) \_C2 + LegendreP \left( \frac{a}{2} - 1, \frac{1}{2} \sqrt{a^2 - 4a + 4b + 4}, ix \right) \_C1 \right) \right\}$$

**2.1369 ODE No. 1369**

---

$$y''(x) = -\frac{ay(x)}{(x^2-1)^2}$$

✓ **Mathematica** : cpu = 0.0766993 (sec), leaf count = 106

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \sqrt{1-x^2} (x+1)^{\sqrt{1-a}} (1-x)^{-\sqrt{1-a}} e^{-\sqrt{1-a} \tanh^{-1}(x)}}{2\sqrt{1-a}} + c_1 \sqrt{1-x^2} e^{-\sqrt{1-a} \tanh^{-1}(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 55

$$\left\{ y(x) = \sqrt{x^2-1} \left( \left( \frac{x-1}{1+x} \right)^{-\frac{1}{2}\sqrt{1-a}} {}_2F_1\left( \begin{matrix} -\frac{1}{2}\sqrt{1-a} \\ 1 \end{matrix} ; -C2 \right) + \left( \frac{x-1}{1+x} \right)^{\frac{1}{2}\sqrt{1-a}} {}_2F_1\left( \begin{matrix} \frac{1}{2}\sqrt{1-a} \\ 1 \end{matrix} ; -C1 \right) \right) \right\}$$

**2.1370 ODE No. 1370**

---

$$y''(x) = \frac{a^2 y(x)}{(x^2-1)^2} - \frac{2xy'(x)}{x^2-1}$$

✓ **Mathematica** : cpu = 0.0208797 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow c_1 \cosh\left(\frac{1}{2}a(\log(1-x) - \log(x+1))\right) + ic_2 \sinh\left(\frac{1}{2}a(\log(1-x) - \log(x+1))\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 19

$$\{y(x) = {}_2F_1\left( \begin{matrix} -\frac{1}{2}a \\ 1 \end{matrix} ; -C1 \right) \sinh(a \operatorname{Artanh}(x)) + {}_2F_1\left( \begin{matrix} \frac{1}{2}a \\ 1 \end{matrix} ; -C2 \right) \cosh(a \operatorname{Artanh}(x))\}$$

**2.1371 ODE No. 1371**

---

$$y''(x) = -\frac{y(x)(-a^2 - \lambda(x^2-1))}{(x^2-1)^2} - \frac{2xy'(x)}{x^2-1}$$

✓ **Mathematica** : cpu = 0.0158983 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow c_1 P_{\frac{1}{2}(\sqrt{4\lambda+1}-1)}^a(x) + c_2 Q_{\frac{1}{2}(\sqrt{4\lambda+1}-1)}^a(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 37

$$\left\{ y(x) = {}_2F_1\left( \begin{matrix} \frac{1}{2}\sqrt{1+4\lambda} - \frac{1}{2} \\ 1 \end{matrix} ; -C1 \right) \operatorname{LegendreP}\left(\frac{1}{2}\sqrt{1+4\lambda} - \frac{1}{2}, a, x\right) + {}_2F_1\left( \begin{matrix} \frac{1}{2}\sqrt{1+4\lambda} - \frac{1}{2} \\ 1 \end{matrix} ; -C2 \right) \operatorname{LegendreQ}\left(\frac{1}{2}\sqrt{1+4\lambda} - \frac{1}{2}, a, x\right) \right\}$$

**2.1372 ODE No. 1372**

---

$$y''(x) = -\frac{y(x)((x^2-1)(ax^2+bx+c)-k^2)}{(x^2-1)^2} - \frac{2xy'(x)}{x^2-1}$$

✗ **Mathematica** : cpu = 2.95066 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \operatorname{DifferentialRoot}\left(\{y, x\}, \left\{ (ax^4+bx^3-ax^2+cx^2-bx-k^2-c)y(x) + (2x^3-2x)y'(x) + (x^4-2x^2) \right\} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.174 (sec), leaf count = 110

$$\left\{ y(x) = e^{\sqrt{-a}x} \left( \sqrt{2x-2} (1+x)^{-\frac{k}{2}} (x-1)^{\frac{k}{2}-\frac{1}{2}} \operatorname{HeunC}\left(4\sqrt{-a}, -k, k, 2b, \frac{k^2}{2} + a - b + c, \frac{1}{2} + \frac{x}{2}\right) - C2 \right) + (x^2-1) \right\}$$

**2.1373 ODE No. 1373**

---

$$y''(x) = -\frac{y(x)(-a^2(x^2-1)^2 - m^2 - n(n+1)(x^2-1))}{(x^2-1)^2} - \frac{2xy'(x)}{x^2-1}$$

✗ **Mathematica** : cpu = 1.73094 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \operatorname{DifferentialRoot}\left(\{y, x\}, \left\{ (-a^2x^4+2a^2x^2-n^2x^2-nx^2-a^2-m^2+n^2+n)y(x) + (2x^3-2x)y'(x) \right\} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.127 (sec), leaf count = 84

$$\left\{ y(x) = (x^2-1)^{\frac{m}{2}} \left( \operatorname{HeunC}\left(0, \frac{1}{2}, m, -\frac{a^2}{4}, \frac{1}{4} + \frac{a^2}{4} + \frac{m^2}{4} - \frac{n^2}{4} - \frac{n}{4}, x^2\right) - C2 \right) x + \operatorname{HeunC}\left(0, -\frac{1}{2}, m, -\frac{a^2}{4}, \frac{1}{4} - \frac{a^2}{4} - \frac{m^2}{4} + \frac{n^2}{4} + \frac{n}{4}, x^2\right) \right\}$$

## 2.1374 ODE No. 1374

---

$$y''(x) = \frac{2(2a-1)xy'(x)}{x^2-1} - \frac{y(x)(x^2(2a(2a-1)-v(v+1))+2a+v(v+1))}{(x^2-1)^2}$$

✓ **Mathematica** : cpu = 0.025258 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow c_1 (x^2-1)^a P_v(x) + c_2 (x^2-1)^a Q_v(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 23

$$\left\{ y(x) = (x^2-1)^a (\text{LegendreQ}(v, x) \_C2 + \text{LegendreP}(v, x) \_C1) \right\}$$

## 2.1375 ODE No. 1375

---

$$y''(x) = -\frac{y(x)(4ax^2(a-n)-(x^2-1)(2a+(v-n)(n+v+1)))}{(x^2-1)^2} - \frac{2x(-2a+n+1)y'(x)}{x^2-1}$$

✓ **Mathematica** : cpu = 0.0335379 (sec), leaf count = 54

$$\left\{ \left\{ y(x) \rightarrow c_1 (x^2-1)^{\frac{1}{2}(2a-n)} P_v^n(x) + c_2 (x^2-1)^{\frac{1}{2}(2a-n)} Q_v^n(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 29

$$\left\{ y(x) = (x^2-1)^{a-\frac{n}{2}} (\text{LegendreQ}(v, n, x) \_C2 + \text{LegendreP}(v, n, x) \_C1) \right\}$$

## 2.1376 ODE No. 1376

---

$$y''(x) = -\frac{by(x)}{x^2(a+x^2)} - \frac{(a+2x^2)y'(x)}{x(a+x^2)}$$

✓ **Mathematica** : cpu = 0.0326371 (sec), leaf count = 69

$$\left\{ \left\{ y(x) \rightarrow c_1 \cos \left( \frac{\sqrt{b} \tanh^{-1} \left( \frac{\sqrt{a+x^2}}{\sqrt{a}} \right)}{\sqrt{a}} \right) - c_2 \sin \left( \frac{\sqrt{b} \tanh^{-1} \left( \frac{\sqrt{a+x^2}}{\sqrt{a}} \right)}{\sqrt{a}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 73

$$\left\{ y(x) = 1 \left( \_C2 \left( \left( \frac{1}{x} (2a+2\sqrt{a}\sqrt{x^2+a}) \right)^{i\sqrt{b}\frac{1}{\sqrt{a}}} \right)^2 + \_C1 \right) \left( \left( \frac{1}{x} (2a+2\sqrt{a}\sqrt{x^2+a}) \right)^{i\sqrt{b}\frac{1}{\sqrt{a}}} \right)^{-1} \right\}$$

## 2.1377 ODE No. 1377

---

$$y''(x) = -\frac{b^2 y(x)}{(a^2+x^2)^2}$$

✓ **Mathematica** : cpu = 0.178233 (sec), leaf count = 163

$$\left\{ \left\{ y(x) \rightarrow \frac{ic_2 \sqrt{a^2+x^2} \left(1 - \frac{ix}{a}\right) \sqrt{\frac{a^2+b^2}{a^2}} \left(1 + \frac{ix}{a}\right)^{-\sqrt{\frac{a^2+b^2}{a^2}}} e^{i\sqrt{\frac{a^2+b^2}{a^2}} \tan^{-1}(\frac{x}{a})}}{2a\sqrt{\frac{a^2+b^2}{a^2}}} + c_1 \sqrt{a^2+x^2} e^{i\sqrt{\frac{b^2}{a^2}+1} \tan^{-1}(\frac{x}{a})} \right\} \right\}$$

✓ **Maple** : cpu = 0.084 (sec), leaf count = 83

$$\left\{ y(x) = \sqrt{a^2+x^2} \left( \left( \frac{ix-a}{ix+a} \right)^{-\frac{1}{2a}\sqrt{a^2+b^2}} \_C2 + \left( \frac{ix-a}{ix+a} \right)^{\frac{1}{2a}\sqrt{a^2+b^2}} \_C1 \right) \right\}$$



## 2.1378 ODE No. 1378

---

$$y''(x) = -\frac{2(x^2-1)y'(x)}{(x-1)^2x} - \frac{(-2x^2+2x+2)y(x)}{(x-1)^2x^2}$$

✓ **Mathematica** : cpu = 0.0380964 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 x^2}{1-x} + \frac{c_2 x (2x^2 \log(1-x) - 2x^2 \log(x) + 2x - 2x \log(1-x) + 2x \log(x) - 1)}{(x-1)^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 48

$$\left\{ y(x) = \frac{x}{(x-1)^2} \left( -\_C2 x (x-1) \ln(x-1) + \_C2 x (x-1) \ln(x) + \_C1 x^2 + (-\_C1 - \_C2) x + \frac{\_C2}{2} \right) \right\}$$

## 2.1379 ODE No. 1379

---

$$y''(x) = \frac{12y(x)}{(x+1)^2(x^2+2x+3)}$$

✓ **Mathematica** : cpu = 0.0528595 (sec), leaf count = 99

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \left( 2x^3 + 4x^2 - 3\sqrt{2}x^2 \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right) + 8x - 6\sqrt{2}x \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right) - 9\sqrt{2} \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right) + 2 \right)}{2(x+1)^2} + c_1 \left( \frac{x}{(x+1)^2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 59

$$\left\{ y(x) = \frac{1}{(1+x)^2} \left( -3\_C2 (x^2 + 2x + 3) \arctan\left(\frac{1}{2}(1+x)\sqrt{2}\right) + \_C2 (x^3 + 2x^2 + 4x + 1)\sqrt{2} + \_C1 (x^2 + 2x + 3) \right) \right\}$$

## 2.1380 ODE No. 1380

---

$$y''(x) = -\frac{by(x)}{x^2(x-a)^2}$$

✓ **Mathematica** : cpu = 0.198875 (sec), leaf count = 132

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 (x-a)^{\frac{1}{2}\sqrt{\frac{a^2-4b}{a^2}} + \frac{1}{2}x^{\frac{1}{2}-\frac{1}{2}\sqrt{\frac{a^2-4b}{a^2}}}}{a\sqrt{\frac{a^2-4b}{a^2}}} + c_1 (x-a)^{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{4b}{a^2}}} x^{\frac{1}{2}\sqrt{1-\frac{4b}{a^2}} + \frac{1}{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 67

$$\left\{ y(x) = \sqrt{x(a-x)} \left( \left( \frac{x}{a-x} \right)^{\frac{1}{2a}\sqrt{a^2-4b}} \_C2 + \left( \frac{a-x}{x} \right)^{\frac{1}{2a}\sqrt{a^2-4b}} \_C1 \right) \right\}$$

## 2.1381 ODE No. 1381

---

$$y''(x) = c - \frac{by(x)}{x^2(x-a)^2}$$

✓ **Mathematica** : cpu = 0.269888 (sec), leaf count = 589

$$\left\{ \left\{ y(x) \rightarrow -\frac{2cx^2(a-x) \left(1 - \frac{x}{a}\right)^{-\frac{1}{2}\sqrt{\frac{a^2-4b}{a^2}}} \left( \sqrt{\frac{a^2-4b}{a^2}} \left(1 - \frac{x}{a}\right)^{\sqrt{\frac{a^2-4b}{a^2}}} {}_2F_1\left(\frac{1}{2}\sqrt{1-\frac{4b}{a^2}} - \frac{1}{2}, \frac{1}{2}\sqrt{1-\frac{4b}{a^2}} + \frac{3}{2}; \frac{1}{2}\sqrt{1-\frac{4b}{a^2}}\right) \right)}{x^2(x-a)^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.126 (sec), leaf count = 175

$$\left\{ y(x) = 1 \left( \left( -\_C2 \sqrt{a^2-4b} - \int \sqrt{x(a-x)} \left( \frac{a-x}{x} \right)^{-\frac{1}{2a}\sqrt{a^2-4b}} dx \right) \left( \frac{a-x}{x} \right)^{\frac{1}{2a}\sqrt{a^2-4b}} + \left( \frac{x}{a-x} \right)^{\frac{1}{2a}\sqrt{a^2-4b}} \right) \right\}$$

## 2.1382 ODE No. 1382

---

$$y''(x) = \frac{cy(x)}{(x-a)^2(x-b)^2}$$

✓ **Mathematica** : cpu = 0.467329 (sec), leaf count = 154

$$\left\{ \left\{ y(x) \rightarrow c_1 (x-a)^{\frac{1}{2}} \left( \sqrt{\frac{4c}{(a-b)^2}+1}+1 \right) (x-b)^{\frac{1}{2}} \left( 1-\sqrt{\frac{4c}{(a-b)^2}+1} \right) - \frac{c_2 (x-a)^{\frac{1}{2}-\frac{1}{2}} \sqrt{\frac{4c}{(a-b)^2}+1} (x-b)^{\frac{1}{2}} \sqrt{\frac{4c}{(a-b)^2}+1+\frac{1}{2}}}{(a-b) \sqrt{\frac{4c}{(a-b)^2}+1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.076 (sec), leaf count = 104

$$\left\{ y(x) = \sqrt{(a-x)(b-x)} \left( \left( \frac{a-x}{b-x} \right)^{\frac{1}{2a-2b} \sqrt{a^2-2ab+b^2+4c}} - C1 + \left( \frac{a-x}{b-x} \right)^{-\frac{1}{2a-2b} \sqrt{a^2-2ab+b^2+4c}} - C2 \right) \right\}$$

## 2.1383 ODE No. 1383

---

$$y''(x) = -\frac{y'(x) \left( (x-a)^2(\alpha+\beta+1)(x-b) + (x-a)(-\alpha-\beta+1)(x-b)^2 \right)}{(x-a)^2(x-b)^2} - \frac{\alpha\beta(a-b)^2 y(x)}{(x-a)^2(x-b)^2}$$

✓ **Mathematica** : cpu = 0.0942452 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\alpha(\log(x-a)-\log(x-b))} + c_2 e^{\beta(\log(x-a)-\log(x-b))} \right\} \right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 39

$$\left\{ y(x) = -C1 \left( \frac{a-x}{b-x} \right)^{\beta} + -C2 \left( \frac{a-x}{b-x} \right)^{\alpha} \right\}$$

## 2.1384 ODE No. 1384

---

$$y''(x) = -\frac{y(x) \left( -(a^2-1)x^2 + 2(a+3)bx - b^2 \right)}{4x^2}$$

✓ **Mathematica** : cpu = 0.0238864 (sec), leaf count = 110

$$\left\{ \left\{ y(x) \rightarrow c_1 M_{\frac{(a+3)b}{2\sqrt{a^2-1}}, \frac{\sqrt{b(b^2+1)}}{2\sqrt{b}}} \left( \sqrt{a^2-1}x \right) + c_2 W_{\frac{(a+3)b}{2\sqrt{a^2-1}}, \frac{\sqrt{b(b^2+1)}}{2\sqrt{b}}} \left( \sqrt{a^2-1}x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.167 (sec), leaf count = 73

$$\left\{ y(x) = -C1 M_{\frac{b(a+3)}{2}, \frac{1}{\sqrt{a^2-1}}, \frac{1}{2}\sqrt{b^2+1}} \left( \sqrt{a^2-1}x \right) + -C2 W_{\frac{b(a+3)}{2}, \frac{1}{\sqrt{a^2-1}}, \frac{1}{2}\sqrt{b^2+1}} \left( \sqrt{a^2-1}x \right) \right\}$$

## 2.1385 ODE No. 1385

---

$$y''(x) = -\frac{(ax^2+a-3)y(x)}{4(x^2+1)^2}$$

✓ **Mathematica** : cpu = 0.0151598 (sec), leaf count = 78

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt{x^2+1} P_{\frac{1}{2}(\sqrt{1-a}-1)}^{\frac{1}{2}}(ix) + c_2 \sqrt{x^2+1} Q_{\frac{1}{2}(\sqrt{1-a}-1)}^{\frac{1}{2}}(ix) \right\} \right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 55

$$\left\{ y(x) = \sqrt[4]{x^2+1} \left( \left( x + \sqrt{x^2+1} \right)^{-\frac{1}{2}\sqrt{1-a}} - C2 + \left( x + \sqrt{x^2+1} \right)^{\frac{1}{2}\sqrt{1-a}} - C1 \right) \right\}$$

**2.1386 ODE No. 1386**

---

$$y''(x) = \frac{18y(x)}{(2x+1)^2(x^2+x+1)}$$

✓ **Mathematica** : cpu = 0.0633781 (sec), leaf count = 108

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1(x^2+x+1)}{(2x+1)^2} + \frac{c_2 \left( 16x^3 + 24x^2 - 12\sqrt{3}x^2 \tan^{-1} \left( \frac{2x+1}{\sqrt{3}} \right) + 30x - 12\sqrt{3}x \tan^{-1} \left( \frac{2x+1}{\sqrt{3}} \right) - 12\sqrt{3} \tan^{-1} \left( \frac{2x+1}{\sqrt{3}} \right) \right)}{(2x+1)^2} \right. \right.$$

✓ **Maple** : cpu = 0.048 (sec), leaf count = 58

$$\left\{ y(x) = \frac{1}{(2x+1)^2} \left( 36\_C2 (x^2+x+1) \arctan \left( \frac{1}{3} (2x+1) \sqrt{3} \right) - 16\_C2 \left( x^3 + x^2 + \frac{11x}{8} + 3/16 \right) \sqrt{3} + \right. \right.$$

**2.1387 ODE No. 1387**

---

$$y''(x) = \frac{3y(x)}{4(x^2+x+1)^2}$$

✓ **Mathematica** : cpu = 0.0276848 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt{x^2+x+1} + \frac{2c_2 \sqrt{x^2+x+1} \tan^{-1} \left( \frac{2x+1}{\sqrt{3}} \right)}{\sqrt{3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 28

$$\left\{ y(x) = \sqrt{x^2+x+1} \left( \arctan \left( \frac{(2x+1)\sqrt{3}}{3} \right) \_C2 + \_C1 \right) \right\}$$

**2.1388 ODE No. 1388**

---

$$y''(x) = -\frac{y(x)(v(v+1)(x-1)-a^2x)}{4(x-1)^2x^2} - \frac{(3x-1)y'(x)}{2(x-1)x}$$

✓ **Mathematica** : cpu = 0.226583 (sec), leaf count = 235

$$\left\{ \left\{ y(x) \rightarrow c_2(-1)^{\frac{1}{2}(-2v-3)+1} x^{\frac{1}{4}(-2v-3)+1} e^{\frac{1}{4}(-2\log(1-x)-\log(x))} (x-1)^{\frac{1}{2}(\frac{1}{2}(a+v+1)+\frac{1}{2}(a+v+2)+\frac{1}{2}(-2v-3)+1)} {}_2F_1 \left( \frac{1}{2}(-2v-3)+1, -\frac{1}{2}(-2v-3)+1; \frac{1}{2}(-2v-3)+1; -\frac{1}{2}(-2v-3)+1 \right) \right. \right.$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 76

$$\left\{ y(x) = (x-1)^{-\frac{a}{2}} \left( x^{-\frac{v}{2}} {}_2F_1 \left( -\frac{v}{2} - \frac{a}{2}, \frac{1}{2} - \frac{v}{2} - \frac{a}{2}; \frac{1}{2} - v; x \right) \_C1 + x^{\frac{1}{2}+\frac{v}{2}} {}_2F_1 \left( 1 + \frac{v}{2} - \frac{a}{2}, \frac{1}{2} + \frac{v}{2} - \frac{a}{2}; \frac{3}{2} + v; x \right) \_C2 \right) \right.$$

**2.1389 ODE No. 1389**

---

$$y''(x) = -\frac{y(x)(-4n^2x-v(v+1)(x-1)^2)}{4(x-1)^2x^2} - \frac{(3x-1)y'(x)}{2(x-1)x}$$

✓ **Mathematica** : cpu = 0.27325 (sec), leaf count = 217

$$\left\{ \left\{ y(x) \rightarrow c_2(-1)^{\frac{1}{2}(-2v-3)+1} x^{\frac{1}{4}(-2v-3)+1} e^{\frac{1}{4}(-2\log(1-x)-\log(x))} (x-1)^{\frac{1}{2}(n+\frac{1}{2}(2n+1)+\frac{1}{2}(-2v-3)+v+2)} {}_2F_1 \left( \frac{1}{2}(2n+1)+\frac{1}{2}(-2v-3)+v+2, -\frac{1}{2}(-2v-3)+v+2; \frac{1}{2}(-2v-3)+v+2; -\frac{1}{2}(-2v-3)+v+2 \right) \right. \right.$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 68

$$\left\{ y(x) = (x-1)^{-n} \left( x^{-\frac{v}{2}} {}_2F_1 \left( -v-n, \frac{1}{2} - n; \frac{1}{2} - v; x \right) \_C1 + x^{\frac{1}{2}+\frac{v}{2}} {}_2F_1 \left( v-n+1, \frac{1}{2} - n; \frac{3}{2} + v; x \right) \_C2 \right) \right\}$$

**2.1390 ODE No. 1390**

---

$$y''(x) = -\frac{3y(x)}{16(x-1)^2x^2}$$

✓ **Mathematica** : cpu = 0.0256193 (sec), leaf count = 41

$$\left\{\left\{y(x) \rightarrow 2c_2\sqrt[4]{1-xx^{3/4}}+c_1(1-x)^{3/4}\sqrt[4]{x}\right\}\right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 25

$$\left\{y\left(x\right)=-C1\,x^{\frac{3}{4}}\sqrt[4]{x-1}+_C2\sqrt[4]{x}\left(x-1\right)^{\frac{3}{4}}\right\}$$

**2.1391 ODE No. 1391**

---

$$y''(x) = \frac{(7ax^2+5)y'(x)}{x(ax^2+1)} - \frac{(15ax^2+5)y(x)}{x^2(ax^2+1)}$$

✓ **Mathematica** : cpu = 0.0405212 (sec), leaf count = 27

$$\left\{\left\{y(x) \rightarrow c_1x^5-\frac{1}{4}c_2x\left(2ax^2+1\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 20

$$\left\{y\left(x\right)=-C1\,x^5+2\_C2\,ax^3+_C2\,x\right\}$$

**2.1392 ODE No. 1392**

---

$$y''(x) = -\frac{bxy'(x)}{a(x^2-1)} - \frac{y(x)(cx^2+dx+e)}{a(x^2-1)^2}$$

✓ **Mathematica** : cpu = 62.053 (sec), leaf count = 1763961

Too large to display

✓ **Maple** : cpu = 0.125 (sec), leaf count = 561

$$\left\{y\left(x\right)=\left(-\frac{1}{2}+\frac{x}{2}\right)^{\frac{1}{4a}\left(2a+\sqrt{4a^2+(-4b-4c-4d-4e)a+b^2}\right)}\left(x^2-1\right)^{-\frac{b}{4a}}\left({}_2F_1\left(-\frac{1}{4a}\left(-\sqrt{4a^2+(-4b-4c-4d-4e)a+b^2}\right),-\frac{1}{4a}\right)\right)\right\}$$

**2.1393 ODE No. 1393**

---

$$y''(x) = -\frac{y(x)(bx^2+cx+d)}{a(x-1)^2x^2}$$

✓ **Mathematica** : cpu = 13.8347 (sec), leaf count = 413606

Too large to display

✓ **Maple** : cpu = 0.099 (sec), leaf count = 299

$$\left\{y\left(x\right)=-C1\,x^{\frac{1}{2}\left(\sqrt{a-4d}+\sqrt{a}\right)\frac{1}{\sqrt{a}}}\left(x-1\right)^{\frac{1}{2}\left(\sqrt{a}-\sqrt{a-4b-4c-4d}\right)\frac{1}{\sqrt{a}}}\,{}_2F_1\left(\frac{1}{2}\left(-\sqrt{a-4b-4c-4d}+\sqrt{a}+\sqrt{a-4d}\right),-\frac{1}{2}\right)\right\}$$

**2.1394 ODE No. 1394**

---

$$y''(x) = -\frac{cy(x)}{x^2(ax+b)^2} - \frac{2y'(x)}{x}$$

✓ **Mathematica** : cpu = 0.0331779 (sec), leaf count = 115

$$\left\{\left\{y(x) \rightarrow c_1 \exp\left(\frac{\sqrt{c}\left(-\frac{\sqrt{b^2-4c}}{\sqrt{c}}-\frac{b}{\sqrt{c}}\right)(\log(x)-\log(ax+b))}{2b}\right)+c_2 \exp\left(\frac{\sqrt{c}\left(\frac{\sqrt{b^2-4c}}{\sqrt{c}}-\frac{b}{\sqrt{c}}\right)(\log(x)-\log(ax+b))}{2b}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.067 (sec), leaf count = 79

$$\left\{y\left(x\right)=\sqrt{\frac{ax+b}{x}}\left(\left(\frac{x}{ax+b}\right)^{-\frac{a}{2b}\sqrt{\frac{b^2-4c}{a^2}}}-C2+\left(\frac{x}{ax+b}\right)^{\frac{a}{2b}\sqrt{\frac{b^2-4c}{a^2}}}-C1\right)\right\}$$

### 2.1395 ODE No. 1395

$$y''(x) = -\frac{y(x)}{(ax+b)^4}$$

✓ **Mathematica** : cpu = 0.0570896 (sec), leaf count = 59

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{i}{a(ax+b)}} (ax+b) - \frac{1}{2} i c_2 e^{-\frac{i}{a(ax+b)}} (ax+b) \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 39

$$\left\{ y(x) = (ax+b) \left( -C2 \cos\left(\frac{1}{a(ax+b)}\right) + -C1 \sin\left(\frac{1}{a(ax+b)}\right) \right) \right\}$$

### 2.1396 ODE No. 1396

$$y''(x) = -\frac{Ay(x)}{(ax^2+bx+c)^2}$$

✓ **Mathematica** : cpu = 0.940832 (sec), leaf count = 211

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \sqrt{ax^2+bx+c} \exp\left(-\frac{\sqrt{4ac-b^2} \sqrt{1-\frac{4A}{b^2-4ac}} \tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}\right)}{\sqrt{b^2-4ac} \sqrt{1-\frac{4A}{b^2-4ac}}} + c_1 \sqrt{x(ax+b)+c} \exp\left(-\frac{\sqrt{4ac-b^2} \sqrt{1-\frac{4A}{b^2-4ac}}}{\sqrt{-4ca+b^2}} \frac{1}{\sqrt{-4ca+b^2}}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.116 (sec), leaf count = 178

$$\left\{ y(x) = \sqrt{ax^2+bx+c} \left( \left( 1 \left( i \sqrt{4ca-b^2} - 2ax-b \right) \left( 2ax+b+i\sqrt{4ca-b^2} \right)^{-1} \right)^{\frac{a}{2} \sqrt{\frac{-4ca+b^2-4A}{a^2}} \frac{1}{\sqrt{-4ca+b^2}}} - C2 \right) \right\}$$

### 2.1397 ODE No. 1397

$$y''(x) = \frac{y(x)}{x^5} - \frac{y'(x)}{x^4}$$

✓ **Mathematica** : cpu = 0.0283605 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \Gamma\left(\frac{1}{3}, -\frac{1}{3x^3}\right)}{3^{2/3} \sqrt[3]{-\frac{1}{x^3}}} + c_1 x \right\} \right\}$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 27

$$\left\{ y(x) = x \left( -\sqrt{3} \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{1}{3}, -\frac{1}{3x^3}\right) - C2 + 2\pi - C2 + -C1 \right) \right\}$$

### 2.1398 ODE No. 1398

$$y''(x) = -\frac{(-(2v+1)^2+x^2-1)y(x)}{(x^2-1)^2} - \frac{(3x^2-1)y'(x)}{x(x^2-1)}$$

✗ **Mathematica** : cpu = 1.15923 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}\left(\{y, x\}, \left\{ x(x^2-4v^2-4v-2)y(x) + (3x^4-4x^2+1)y'(x) + (x^5-2x^3+x)y''(x) \right\} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.111 (sec), leaf count = 69

$$\left\{ y(x) = -C1 (x^2-1)^{-\frac{1}{2}-v} {}_2F_1(-v, -v; -2v; -x^2+1) + -C2 (x^2-1)^{v+\frac{1}{2}} {}_2F_1(v+1, v+1; 2v+2; -x^2+1) \right\}$$

**2.1399                      ODE No. 1399**

---

$$y''(x) = \frac{(3x+1)y'(x)}{(x-1)(x+1)} - \frac{36(x+1)^2y(x)}{(x-1)^2(3x+5)^2}$$

✓ **Mathematica** : cpu = 0.0363696 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{2}(3 \log(1-x) + \log(3x+5))} + \frac{1}{2} c_2 e^{\frac{1}{2}(3 \log(1-x) + \log(3x+5))} (3 \log(1-x) + \log(3x+5)) \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 34

$$\left\{ y(x) = (x-1)^{\frac{3}{2}} \sqrt{3x+5} \left( -C_2 \ln(3x+5) + 3 - C_2 \ln(x-1) + -C_1 \right) \right\}$$

**2.1400                      ODE No. 1400**

---

$$y''(x) = \frac{y'(x)}{x} - \frac{ay(x)}{x^6}$$

✓ **Mathematica** : cpu = 0.0589265 (sec), leaf count = 60

$$\left\{ \left\{ y(x) \rightarrow c_1 x^2 e^{\frac{i\sqrt{a}}{2x^2}} - \frac{ic_2 x^2 e^{-\frac{i\sqrt{a}}{2x^2}}}{2\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 35

$$\left\{ y(x) = x^2 \left( \cosh \left( \frac{1}{2x^2} \sqrt{-a} \right) - C_2 + \sinh \left( \frac{1}{2x^2} \sqrt{-a} \right) - C_1 \right) \right\}$$

**2.1401                      ODE No. 1401**

---

$$y''(x) = -\frac{(a+3x^2)y'(x)}{x^3} - \frac{by(x)}{x^6}$$

✓ **Mathematica** : cpu = 0.009739 (sec), leaf count = 93

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\frac{\sqrt{b} \left( -\frac{\sqrt{a^2-4b}}{\sqrt{b}} - \frac{a}{\sqrt{b}} \right)}{4x^2}} + c_2 e^{-\frac{\sqrt{b} \left( \frac{\sqrt{a^2-4b}}{\sqrt{b}} - \frac{a}{\sqrt{b}} \right)}{4x^2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 45

$$\left\{ y(x) = -C_1 e^{-\frac{1}{4x^2}(-a+\sqrt{a^2-4b})} + -C_2 e^{\frac{1}{4x^2}(a+\sqrt{a^2-4b})} \right\}$$

**2.1402                      ODE No. 1402**

---

$$y''(x) = -\frac{y(x) \left( 4a(a+1)x^4 - 2a(x^2-1)x^2 + (x^2-1)^2(x^2-v^2) \right)}{x^2(x^2-1)^2} - \frac{((1-4a)x^2-1)y'(x)}{x(x^2-1)}$$

✗ **Mathematica** : cpu = 3.74061 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{ (x^6 + 4a^2x^4 - v^2x^4 + 2ax^4 - 2x^4 + 2v^2x^2 + 2ax^2 + x^2 - v^2) y(x) + (-4ax^5 + \dots) \right\} \right\}$$

✓ **Maple** : cpu = 0.166 (sec), leaf count = 58

$$\left\{ y(x) = (x^2-1)^a (x^2-1) \left( -C_2 x^{-v} \text{HeunC} \left( 0, -v, 1, \frac{1}{4}, \frac{a}{2} + \frac{1}{4}, x^2 \right) + -C_1 x^v \text{HeunC} \left( 0, v, 1, \frac{1}{4}, \frac{a}{2} + \frac{1}{4}, x^2 \right) \right) \right\}$$

### 2.1403 ODE No. 1403

$$y''(x) = -y'(x) \left( \frac{-a1 - b1 + 1}{x - c1} + \frac{-a2 - b2 + 1}{x - c2} + \frac{-a3 - b3 + 1}{x - c3} \right) - \frac{y(x) \left( \frac{a1b1(c1-c2)(c1-c3)}{x-c1} + \frac{a2b2(c2-c1)(c2-c3)}{x-c2} + \frac{a3b3(c3-c1)(c3-c2)}{x-c3} \right)}{(x - c1)(x - c2)(x - c3)}$$

✗ **Mathematica** : cpu = 64.4706 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot} \left( \{y, x\}, \left\{ (c1 - x)^2 (c2 - x)^2 y''(x) (c3 - x)^2 + (c1 - x) (c2 - x) (a1 x^2 + a2 x^2 + a3 x^2 + \dots \right\} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.52 (sec), leaf count = 298

$$\left\{ y(x) = (x - c2)^{a2} (x - c3)^{b3} \left( (x - c1)^{b1} \text{HeunG} \left( \frac{c1 - c3}{c1 - c2}, \frac{((-2 a1 - a3 - b2 + 2) c1 + (a1 + a3 - 1) c2 + \dots)}{c1 - c2} \right) \right) \right\}$$

### 2.1404 ODE No. 1404

$$y''(x) = -\frac{(1 - 2x^2) y(x)}{4x^6} - \frac{(2x^2 + 1) y'(x)}{x^3}$$

✓ **Mathematica** : cpu = 0.0167071 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow \frac{c1 e^{\frac{1}{4x^2}}}{x} + c2 e^{\frac{1}{4x^2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 19

$$\left\{ y(x) = \frac{-C1 x + -C2}{x} e^{\frac{1}{4x^2}} \right\}$$

### 2.1405 ODE No. 1405

$$y''(x) = \frac{(2x^2 + 1) y'(x)}{x^3} - \frac{(ax^4 + 10x^2 + 1) y(x)}{4x^6}$$

✓ **Mathematica** : cpu = 0.0511717 (sec), leaf count = 77

$$\left\{ \left\{ y(x) \rightarrow c1 e^{-\frac{1}{4x^2}} x^{\frac{3}{2} - \frac{\sqrt{9-a}}{2}} + \frac{c2 e^{-\frac{1}{4x^2}} x^{\frac{\sqrt{9-a}}{2} + \frac{3}{2}}}{\sqrt{9-a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.041 (sec), leaf count = 42

$$\left\{ y(x) = e^{-\frac{1}{4x^2}} \left( x^{\frac{3}{2} - \frac{1}{2}\sqrt{-a+9}} - C2 + x^{\frac{3}{2} + \frac{1}{2}\sqrt{-a+9}} - C1 \right) \right\}$$

### 2.1406 ODE No. 1406

$$y''(x) = -\frac{27xy(x)}{16(x^3 - 1)^2}$$

✓ **Mathematica** : cpu = 2.06959 (sec), leaf count = 258

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{2}c2(1-x)^{3/4} \sqrt[4]{x^2+x+1} \int_1^x \frac{\sqrt{\sqrt{3}K[1]+\sqrt{2K[1]-i\sqrt{3}+1}\sqrt{2K[1]+i\sqrt{3}+1+\sqrt{3}}}}{2(1-K[1])^{3/2}\sqrt{K[1]^2+K[1]+1}} dK[1]}{\sqrt[4]{\sqrt{3}x+\sqrt{2x-i\sqrt{3}+1}\sqrt{2x+i\sqrt{3}+1}+\sqrt{3}}} + \frac{\sqrt{2}c1(1-x)^{3/4} \sqrt[4]{x^2+x+1} \int_1^x \frac{\sqrt{\sqrt{3}K[1]+\sqrt{2K[1]-i\sqrt{3}+1}\sqrt{2K[1]+i\sqrt{3}+1+\sqrt{3}}}}{2(1-K[1])^{3/2}\sqrt{K[1]^2+K[1]+1}} dK[1]}{\sqrt[4]{\sqrt{3}x+\sqrt{2x-i\sqrt{3}+1}\sqrt{2x+i\sqrt{3}+1}+\sqrt{3}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.112 (sec), leaf count = 44

$$\left\{ y(x) = \sqrt{x} \sqrt[4]{x^3 - 1} \left( \text{LegendreQ} \left( -\frac{1}{6}, \frac{1}{3}, \sqrt{-x^3 + 1} \right) - C2 + \text{LegendreP} \left( -\frac{1}{6}, \frac{1}{3}, \sqrt{-x^3 + 1} \right) - C1 \right) \right\}$$

### 2.1407 ODE No. 1407

$$y''(x) = -y'(x) \left( \frac{b1(-a11 - b11 + 1)}{b1x - a1} + \frac{b2(-a12 - b12 + 1)}{b2x - a2} + \frac{b3(-a13 - b13 + 1)}{b3x - a3} \right) - \frac{y(x) \left( \frac{a1b1b1(a1b2 - a2b1)(a3b1 - a1b3)}{b1x - a1} + \frac{a2b2b2(a2b3 - a3b2)(a1b2 - a2b1)}{b2x - a2} + \frac{a3b3b3(a3b1 - a1b3)(a2b3 - a3b2)}{b3x - a3} \right)}{b1x - a1 + b2x - a2 + b3x - a3}$$

✗ **Mathematica** : cpu = 280.612 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot} \left( \{y, x\}, \left\{ (a1 - xb1)^2(a2 - xb2)^2 y''(x)(a3 - xb3)^2 + (a1 - xb1)(a2 - xb2) (a1b1b2b3 - a1b1b3b2 - a1b2b3b1 - a2b1b3b2 - a2b2b3b1 - a3b1b2b3) y'(x) + (a1 - xb1)(a2 - xb2)(a3 - xb3) y(x) \right\} \right) \right\} \right\}$$

✓ **Maple** : cpu = 1.261 (sec), leaf count = 2603

$$\left\{ y(x) = (b2x - a2)^{\frac{a12}{2} + \frac{b12}{2} + \frac{1}{2}\sqrt{a12^2 + 6a12b12 + b12^2}} (b3x - a3)^{\frac{1}{2}\left((a13 + b13)\sqrt{a11^2 + (2a12 + 2a13 + 2b11 + 2b12 + 2b13 - 4)a11 + a12^2} + a11\right)} \right\}$$

### 2.1408 ODE No. 1408

$$y''(x) = -\frac{y(x)(Ax^2 + B)}{x(x^2 - a1)(x^2 - a2)(x^2 - a3)} - \frac{y'(x)(x^2((x^2 - a1)(x^2 - a2) + (x^2 - a1)(x^2 - a3) + (x^2 - a2)(x^2 - a3)) - (a1x^4 - a2x^4 - a3x^4 + a1a2a3)y'(x) - x(a1 - x^2))}{x(x^2 - a1)(x^2 - a2)(x^2 - a3)}$$

✗ **Mathematica** : cpu = 49.2965 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot} \left( \{y, x\}, \left\{ (Ax^2 + B)y(x) + (2x^6 - a1x^4 - a2x^4 - a3x^4 + a1a2a3)y'(x) - x(a1 - x^2) \right\} \right) \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \text{DESol} \left( \left( \frac{d^2}{dx^2} Y(x) + \frac{(x^2((x^2 - a1)(x^2 - a2) + (x^2 - a2)(x^2 - a3) + (x^2 - a3)(x^2 - a1)) - (a1x^4 - a2x^4 - a3x^4 + a1a2a3)y'(x) - x(a1 - x^2))}{(x^2 - a1)(x^2 - a2)x(x^2 - a3)} \right) \right) \right\}$$

### 2.1409 ODE No. 1409

$$y''(x) = -b^2x^{-2a}y(x) - \frac{ay'(x)}{x}$$

✓ **Mathematica** : cpu = 0.0187334 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow c1 \cos \left( \frac{bx^{1-a}}{a-1} \right) - c2 \sin \left( \frac{bx^{1-a}}{a-1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 39

$$\left\{ y(x) = \_C1 \sin \left( \frac{x^{1-a}b}{a-1} \right) + \_C2 \cos \left( \frac{x^{1-a}b}{a-1} \right) \right\}$$

### 2.1410 ODE No. 1410

$$y''(x) = -\frac{y'(x)(apx^b + q)}{x(ax^b - 1)} - \frac{y(x)(arx^b + s)}{x^2(ax^b - 1)}$$

✓ **Mathematica** : cpu = 0.0934959 (sec), leaf count = 481

$$\left\{ \left\{ y(x) \rightarrow c1 i^{\frac{-\sqrt{q^2 + 2q + 4s + 1} + q + 1}{b}} a^{\frac{-\sqrt{q^2 + 2q + 4s + 1} + q + 1}{2b}} (x^b)^{\frac{-\sqrt{q^2 + 2q + 4s + 1} + q + 1}{2b}} {}_2F_1 \left( \frac{p}{2b} + \frac{q}{2b} - \frac{\sqrt{p^2 - 2p - 4r + 1}}{2b} - \frac{\sqrt{q^2 + 2q + 4s + 1}}{2b}, \frac{p}{2b} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.159 (sec), leaf count = 253

$$\left\{ y(x) = \_C1 {}_2F_1 \left( \frac{1}{2b} \left( p + q + \sqrt{q^2 + 2q + 4s + 1} - \sqrt{p^2 - 2p - 4r + 1} \right), \frac{1}{2b} \left( p + q + \sqrt{q^2 + 2q + 4s + 1} + \sqrt{p^2 - 2p - 4r + 1} \right) \right), \frac{1}{2b} \left( p + q + \sqrt{q^2 + 2q + 4s + 1} - \sqrt{p^2 - 2p - 4r + 1} \right) \right\}$$



2.1411 ODE No. 1411

---

$$y''(x) = \frac{y(x)}{e^x + 1}$$

✓ **Mathematica** : cpu = 0.271635 (sec), leaf count = 42

$$\left\{\left\{y(x) \rightarrow c_1 \left(e^{-x} + 1\right) + c_2 e^{-x} \left(e^x \log \left(e^x + 1\right) + \log \left(e^x + 1\right) + 1\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 27

$$\left\{y\left(x\right) = \frac{{\_C1}\left({e}^x + 1\right)\ln\left({e}^x + 1\right) + \_C2\,{e}^x + \_C1 + \_C2}{{e}^x}\right\}$$

2.1412 ODE No. 1412

---

$$y''(x) = \frac{y'(x)}{x \log(x)} + y(x) \log^2(x)$$

✓ **Mathematica** : cpu = 0.0111821 (sec), leaf count = 29

$$\left\{\left\{y(x) \rightarrow c_1 \cosh(x(\log(x) - 1)) + i c_2 \sinh(x(\log(x) - 1))\right\}\right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 23

$$\left\{y\left(x\right) = \_C1\,\sinh\left(x\left(\ln\left(x\right) - 1\right)\right) + \_C2\,\cosh\left(x\left(\ln\left(x\right) - 1\right)\right)\right\}$$

2.1413 ODE No. 1413

---

$$y''(x) = \frac{y'(x)}{x(\log(x) - 1)} - \frac{y(x)}{x^2(\log(x) - 1)}$$

✓ **Mathematica** : cpu = 0.0457763 (sec), leaf count = 16

$$\left\{\left\{y(x) \rightarrow c_1 x - c_2 \log(x)\right\}\right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 12

$$\left\{y\left(x\right) = \_C1\,x + \_C2\,\ln\left(x\right)\right\}$$

2.1414 ODE No. 1414

---

$$y''(x) = y(x) \left(-\operatorname{csch}^2(x)\right) \left(-a^2 \sinh^2(x) - (n - 1)n\right)$$

✓ **Mathematica** : cpu = 0.781015 (sec), leaf count = 231

$$\left\{\left\{y(x) \rightarrow \frac{c_2 (-1)^{\frac{1}{2}(-2n-1)+1} \tanh^2(x)^{\frac{1}{4}(-2n-1)+1} (\tanh^2(x) - 1)^{\frac{1}{2}(\frac{a+n}{2} + \frac{1}{2}(a+n+1) + \frac{1}{2}(-2n-1)+1) - \frac{1}{2}} {}_2F_1\left(\frac{1}{2}(-2n - 1), \frac{1}{2}(-2n - 1) + 1; \frac{3}{2}(-2n - 1) + 1; \sqrt{\tanh(x)}\right)}{\sqrt{\tanh(x)}}\right\}\right\}$$

✓ **Maple** : cpu = 0.18 (sec), leaf count = 97

$$\left\{y\left(x\right) = \_C1\,\left(\sinh\left(x\right)\right)^n {}_2F_1\left(-\frac{a}{2} + \frac{n}{2}, \frac{a}{2} + \frac{n}{2}; \frac{1}{2}; \frac{\cosh\left(2\,x\right)}{2} + \frac{1}{2}\right) + \_C2\,\left(\sinh\left(x\right)\right)^n \left(2\,\cosh\left(2\,x\right) + 2\right)^{\frac{3}{4}} {}_2F_1\left(\frac{1}{2}(-2n - 1), \frac{1}{2}(-2n - 1) + 1; \frac{3}{2}(-2n - 1) + 1; \sqrt{\tanh(x)}\right)\right\}$$

2.1415 ODE No. 1415

---

$$y''(x) = -\left(n^2 - a^2\right) y(x) - 2n \coth(x) y'(x)$$

✓ **Mathematica** : cpu = 0.671113 (sec), leaf count = 273

$$\left\{\left\{y(x) \rightarrow \frac{c_2 (-1)^{\frac{1}{2}(-2n-1)+1} \tanh^2(x)^{\frac{1}{4}(-2n-1)+1} (\tanh^2(x) - 1)^{\frac{1}{2}(\frac{a+n}{2} + \frac{1}{2}(a+n+1) + \frac{1}{2}(-2n-1)+1)} {}_2F_1\left(\frac{1}{2}(-2n - 1), \frac{1}{2}(-2n - 1) + 1; \frac{3}{2}(-2n - 1) + 1; \sqrt{\tanh(x)}\right)}{\sqrt{\tanh(x)}}\right\}\right\}$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 36

$$\left\{y\left(x\right) = \left(\sinh\left(x\right)\right)^{\frac{1}{2}-n} \left(\operatorname{LegendreP}\left(-\frac{1}{2} + a, n - \frac{1}{2}, \cosh\left(x\right)\right) \_C1 + \operatorname{LegendreQ}\left(-\frac{1}{2} + a, n - \frac{1}{2}, \cosh\left(x\right)\right) \_C2\right)\right\}$$

**2.1416 ODE No. 1416**

---

$$y''(x) = -(v - n)(n + v + 1)y(x) - (2n + 1) \cot(x)y'(x)$$

✓ **Mathematica** : cpu = 0.151611 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow c_1 (\cos^2(x) - 1)^{-n/2} P_v^n(\cos(x)) + c_2 (\cos^2(x) - 1)^{-n/2} Q_v^n(\cos(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.128 (sec), leaf count = 26

$$\{y(x) = (\sin(x))^{-n} (LegendreP(v, n, \cos(x)) \_C1 + LegendreQ(v, n, \cos(x)) \_C2)\}$$

**2.1417 ODE No. 1417**

---

$$y''(x) = -\csc(x)y'(x) (\sin^2(x) - \cos(x)) - y(x) \sin^2(x)$$

✓ **Mathematica** : cpu = 0.123742 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{\cos(x)}{2}} \cos\left(\frac{1}{2}\sqrt{3}\cos(x)\right) + c_2 e^{\frac{\cos(x)}{2}} \sin\left(\frac{1}{2}\sqrt{3}\cos(x)\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.098 (sec), leaf count = 31

$$\left\{ y(x) = e^{\frac{\cos(x)}{2}} \left( \sin\left(\frac{\sqrt{3}\cos(x)}{2}\right) \_C1 + \cos\left(\frac{\sqrt{3}\cos(x)}{2}\right) \_C2 \right) \right\}$$

**2.1418 ODE No. 1418**

---

$$y''(x) = \frac{y(x) \sin(x)}{x \cos(x) - \sin(x)} - \frac{x \sin(x) y'(x)}{x \cos(x) - \sin(x)}$$

✓ **Mathematica** : cpu = 0.120646 (sec), leaf count = 15

$$\{\{y(x) \rightarrow c_1 x + c_2 \sin(x)\}\}$$

✓ **Maple** : cpu = 2.694 (sec), leaf count = 59

$$\left\{ y(x) = \sin(x) \left( \int e^{\int \frac{-2(\cos(x))^3 x + 3(\cos(x))^2 \sin(x) - \sin(x)}{\cos(x)(\cos(x)x - \sin(x)) \sin(x)} dx} \cos(x) dx \_C2 + \_C1 \right) \right\}$$

**2.1419 ODE No. 1419**

---

$$y''(x) = -\frac{\sec(x)y'(x) (x^2 \sin(x) - 2x \cos(x))}{x^2} - \frac{y(x) \sec(x)(2x \cos(x) - x \sin(x))}{x^2}$$

✗ **Mathematica** : cpu = 0.955153 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[2][y][x] == -(Sec[x]*(2*x*Cos[x] - x*Sin[x])*y[x])/x^2 - (Sec[x]*(2*x*Cos[x] + x^2*Sin[x])*Derivative[1][y][x])/x^2, y[x], x]`

✓ **Maple** : cpu = 0.142 (sec), leaf count = 12

$$\{y(x) = x (\sin(x) \_C2 + \_C1)\}$$

**2.1420 ODE No. 1420**

---

$$\cos^2(x)y''(x) - y(x) (a \cos^2(x) + (n - 1)n) = 0$$

✓ **Mathematica** : cpu = 0.328498 (sec), leaf count = 134

$$\left\{ \left\{ y(x) \rightarrow c_1 i^{1-n} \cos^{1-n}(x) {}_2F_1\left(-\frac{n}{2} - \frac{i\sqrt{a}}{2} + \frac{1}{2}, -\frac{n}{2} + \frac{i\sqrt{a}}{2} + \frac{1}{2}; \frac{3}{2} - n; \cos^2(x)\right) + c_2 i^n \cos^n(x) {}_2F_1\left(\frac{n}{2} - \frac{i\sqrt{a}}{2}, \frac{n}{2} + \frac{i\sqrt{a}}{2} - \frac{1}{2}; \frac{3}{2} - n; \cos^2(x)\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.221 (sec), leaf count = 123

$$\left\{ y(x) = \_C1 \sin(2x) (\cos(x))^{-n} {}_2F_1\left(1 + \frac{i}{2}\sqrt{a} - \frac{n}{2}, 1 - \frac{i}{2}\sqrt{a} - \frac{n}{2}; \frac{3}{2} - n; \frac{\cos(2x)}{2} + \frac{1}{2}\right) + \_C2 (\cos(x))^n (-1)^n \right\}$$

2.1421 ODE No. 1421

---

$$y''(x) = -a^2 n y(x) \sec^2(ax) \left( (n-1) \sin^2(ax) + \cos^2(ax) \right) - a(n-1) \sin(2ax) \sec^2(ax) y'(x)$$

✓ **Mathematica** : cpu = 0.14984 (sec), leaf count = 81

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-iax} \cos^{n-1}(ax) - \frac{ic_2 e^{2iax} \left( \frac{1}{2} e^{-iax} + \frac{1}{2} e^{iax} \right)^n}{a(1 + e^{2iax})} \right\} \right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 27

$$\left\{ y(x) = \_C1 (\cos(ax))^n + \_C2 (\cos(ax))^{n-1} \sin(ax) \right\}$$

2.1422 ODE No. 1422

---

$$y''(x) = 2y(x) \csc^2(x)$$

✓ **Mathematica** : cpu = 0.0769243 (sec), leaf count = 58

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 \cos(x)}{\sqrt{\cos^2(x) - 1}} + \frac{c_2 \left( \cos(x) (-\sin^{-1}(\cos(x))) - \sqrt{1 - \cos^2(x)} \right)}{\sqrt{1 - \cos^2(x)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.155 (sec), leaf count = 50

$$\left\{ y(x) = \frac{-i \sin(2x) \ln(\cos(2x) + i \sin(2x)) \_C2 + \_C1 \sin(2x) + 2 \_C2 (\cos(2x) - 1)}{\cos(2x) - 1} \right\}$$

2.1423 ODE No. 1423

---

$$y''(x) = -ay(x) \csc^2(x)$$

✓ **Mathematica** : cpu = 0.0552478 (sec), leaf count = 70

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt[4]{\cos^2(x) - 1} P_{-\frac{1}{2}}^{\frac{1}{2}\sqrt{1-4a}}(\cos(x)) + c_2 \sqrt[4]{\cos^2(x) - 1} Q_{-\frac{1}{2}}^{\frac{1}{2}\sqrt{1-4a}}(\cos(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.183 (sec), leaf count = 132

$$\left\{ y(x) = 1 \sqrt{-2 \cos(2x) + 2} \sqrt[4]{2 \cos(2x) + 2} \left( \frac{\cos(2x)}{2} - \frac{1}{2} \right)^{\frac{1}{4}\sqrt{1-4a}} \left( {}_2F_1\left(\frac{1}{4}\sqrt{1-4a} + \frac{3}{4}, \frac{1}{4}\sqrt{1-4a} + \frac{3}{4}; \frac{3}{2}; \frac{\cos(2x)}{2}\right) \right) \right\}$$

2.1424 ODE No. 1424

---

$$\sin^2(x) y''(x) - y(x) (a \sin^2(x) + (n-1)n) = 0$$

✓ **Mathematica** : cpu = 0.127731 (sec), leaf count = 90

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt[4]{\cos^2(x) - 1} P_{\frac{1}{2}i(2\sqrt{a}+i)}^{\frac{1}{2}(2n-1)}(\cos(x)) + c_2 \sqrt[4]{\cos^2(x) - 1} Q_{\frac{1}{2}i(2\sqrt{a}+i)}^{\frac{1}{2}(2n-1)}(\cos(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.183 (sec), leaf count = 120

$$\left\{ y(x) = 1 \left( \frac{\cos(2x)}{2} - \frac{1}{2} \right)^{\frac{n}{2}} \left( \sqrt[4]{-2 \cos(2x) + 2} (2 \cos(2x) + 2)^{\frac{3}{4}} {}_2F_1\left(\frac{1}{2} + \frac{i}{2}\sqrt{a} + \frac{n}{2}, \frac{1}{2} - \frac{i}{2}\sqrt{a} + \frac{n}{2}; \frac{3}{2}; \frac{\cos(2x)}{2}\right) \right) \right\}$$

2.1425 ODE No. 1425

---

$$y''(x) = y(x) \csc^2(x) \left( -(-a^2 \cos^2(x) - (3-2a) \cos(x) + 3a - 3) \right)$$

✓ **Mathematica** : cpu = 1.25375 (sec), leaf count = 385

$$\left\{ \left\{ y(x) \rightarrow \frac{(2a+1)c_2(-2a(\cos(x)-1) + \cos(x)-1)(-2a\cos(x) + \cos^2(x) - 1))}{(1-2a)^2 a (\cos(x)+1) \left( (3-2a)^2 \left( -F_1\left(2a+1; a-\frac{1}{2}, a+\frac{1}{2}; 2a+2; \frac{3-2a}{-2a\cos(x)+\cos(x)+2}, \frac{2a+1}{-2a\cos(x)+\cos^2(x)-1}\right) \right) \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.303 (sec), leaf count = 93

$$\left\{ y(x) = 1 \left( \_C2 {}_2F_1\left(-\frac{1}{2} + a, -a - \frac{1}{2}; -a + \frac{3}{2}; \frac{\cos(x)}{2} + \frac{1}{2}\right) (\cos(x) + 1)^{-\frac{1}{4} - \frac{a}{2}} \sqrt{2 \cos(x) + 2} (-\cos(x) + 1)^{-\frac{1}{4} - \frac{a}{2}} \right) \right\}$$

2.1426 ODE No. 1426

$$\sin^2(x)y''(x)-y(x)\left(a^2\cos^2(x)+\frac{b^2}{(2a-3)^2}+3a+b\cos(x)+2\right)=0$$

✓ **Mathematica** : cpu = 4.24952 (sec), leaf count = 4128

$$\left\{\left\{c_1(\cos(x)+1)\left(\frac{1}{2}\left(-\frac{8a^2}{-16a^2+48a-36}+\frac{24a}{-16a^2+48a-36}+a-\frac{-32a^2+96a+\sqrt{\left(32a^2-96a+72\right)^2-4\left(-16a^2+48a-36\right)\left(16a^4+16ba^2-88a^2-32a^2+96a+72\right)}}{2\left(-16a^2+48a-36\right)}\right)\right\}\right\}y(x)\rightarrow$$

✓ **Maple** : cpu = 0.37 (sec), leaf count = 549

$$\left\{y(x)=1\left(\frac{\cos(x)}{2}-\frac{1}{2}\right)^{\frac{1}{8a-12}\left(4a-6+\sqrt{4b^2+16(a-3/2)^2b+16a^4-72a^2+81}\right)}\left({}_2F_1\left(\frac{1}{8a-12}\left(8a^2+\sqrt{4b^2+16(a-3/2)^2b+16a^4-72a^2+81}\right),-\frac{b}{2\left(4a-6+\sqrt{4b^2+16(a-3/2)^2b+16a^4-72a^2+81}\right)}\right)\right)\right\}$$

2.1427 ODE No. 1427

$$y''(x)=y(x)\left(-\csc^2(x)\right)\left(-\left(a^2b^2-(a+1)^2\right)\sin^2(x)-a(a+1)b\sin(2x)-(a-1)a\right)$$

✗ **Mathematica** : cpu = 200.739 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[2][y][x]==-(Csc[x]^2\*(-((-1+a)\*a)-(-(1+a)^2+a^2\*b^2)\*Sin[x]^2-(a-1)\*a)],y,x]

✓ **Maple** : cpu = 1.004 (sec), leaf count = 203

$$\left\{y(x)=1e^{\int\frac{(ab^2-a-2)(\cos(2x))^2+(-2b(a+1)\sin(2x)-2a-1)\cos(2x)+(-2a-1)b\sin(2x)-ab^2-a+1}{(\cos(2x)+1)(b\cos(2x)-\sin(2x)-b)}dx}\left(\int-2e^{-2\int\frac{(ab^2-a-2)(\cos(2x))^2+(-2b(a+1)\sin(2x)-2a-1)\cos(2x)+(-2a-1)b\sin(2x)-ab^2-a+1}{(\cos(2x)+1)(b\cos(2x)-\sin(2x)-b)}dx}\right)\right\}$$

2.1428 ODE No. 1428

$$y''(x)=y(x)\left(-\csc^2(x)\right)\left(a\cos^2(x)+b\sin^2(x)+c\right)$$

✓ **Mathematica** : cpu = 0.277973 (sec), leaf count = 104

$$\left\{\left\{y(x)\rightarrow c_1\sqrt[4]{\cos^2(x)-1}P_{\frac{1}{2}(2\sqrt{b-a}-1)}^{\frac{1}{2}\sqrt{-4a-4c+1}}(\cos(x))+c_2\sqrt[4]{\cos^2(x)-1}Q_{\frac{1}{2}(2\sqrt{b-a}-1)}^{\frac{1}{2}\sqrt{-4a-4c+1}}(\cos(x))\right\}\right\}$$

✓ **Maple** : cpu = 0.202 (sec), leaf count = 183

$$\left\{y(x)=1\sqrt{-2\cos(2x)+2}\sqrt[4]{2\cos(2x)+2}\left(\frac{\cos(2x)}{2}-\frac{1}{2}\right)^{\frac{1}{4}\sqrt{-4a+1-4c}}\left({}_2F_1\left(\frac{1}{4}\sqrt{-4a+1-4c}+\frac{1}{2}\sqrt{-a+b},-\frac{b}{2\left(2\cos(2x)+2\right)\sqrt{-2\cos(2x)+2}}\right)\right)\right\}$$

2.1429 ODE No. 1429

$$y''(x)=y(x)\csc^2(x)-\cot(x)y'(x)$$

✓ **Mathematica** : cpu = 0.0399909 (sec), leaf count = 51

$$\left\{\left\{y(x)\rightarrow c_1\cosh\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)-ic_2\sinh\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 25

$$\left\{y(x)=\frac{\sin(x)-C1}{\cos(x)-1}+\frac{(\cos(x)-1)-C2}{\sin(x)}\right\}$$

**2.1430                    ODE No. 1430**

---

$$y''(x) = y(x) \csc^2(x) \left( - \left( v(v+1) \sin^2(x) - n^2 \right) \right) - \cot(x) y'(x)$$

✓ **Mathematica** : cpu = 0.312992 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow c_1 P_v^n(\cos(x)) + c_2 Q_v^n(\cos(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.23 (sec), leaf count = 101

$$\left\{ y(x) = 1 \left( \frac{\cos(2x)}{2} - \frac{1}{2} \right)^{\frac{n}{2}} \left( {}_2F_1 \left( 1 + \frac{v}{2} + \frac{n}{2}, \frac{1}{2} - \frac{v}{2} + \frac{n}{2}; \frac{3}{2}; \frac{\cos(2x)}{2} + \frac{1}{2} \right) \sin(2x) - C1 + {}_2F_1 \left( -\frac{v}{2} + \frac{n}{2}, \frac{1}{2} + \right. \right. \right.$$

**2.1431                    ODE No. 1431**

---

$$y''(x) = \cot(2x) y'(x) - 2y(x)$$

✓ **Mathematica** : cpu = 0.151661 (sec), leaf count = 80

$$\left\{ \left\{ y(x) \rightarrow c_1 \left( \cos^2(x) - \frac{1}{2} \right) - \frac{2}{3} c_2 \cos^{\frac{3}{2}}(x) \left( 2 \cos^2(x) {}_2F_1 \left( \frac{1}{4}, \frac{3}{4}; \frac{7}{4}; \cos^2(x) \right) - {}_2F_1 \left( \frac{1}{4}, \frac{3}{4}; \frac{7}{4}; \cos^2(x) \right) + 3 \left( 1 - \right. \right. \right.$$

✓ **Maple** : cpu = 0.155 (sec), leaf count = 30

$$\left\{ y(x) = (\sin(2x))^{\frac{3}{4}} \left( LegendreQ \left( \frac{1}{4}, \frac{3}{4}, \cos(2x) \right) - C2 + LegendreP \left( \frac{1}{4}, \frac{3}{4}, \cos(2x) \right) - C1 \right) \right\}$$

**2.1432                    ODE No. 1432**

---

$$y''(x) = -\cot(x) y'(x) - \frac{1}{4} y(x) \left( -17 \sin^2(x) - 1 \right) \csc^2(x)$$

✓ **Mathematica** : cpu = 0.0632961 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-2x}}{\sqrt{\sin(x)}} + \frac{c_2 e^{2x}}{4 \sqrt{\sin(x)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 22

$$\left\{ y(x) = (-C1 \sinh(2x) + -C2 \cosh(2x)) \frac{1}{\sqrt{\sin(x)}} \right\}$$

**2.1433                    ODE No. 1433**

---

$$y''(x) = -\frac{y(x) \sec^2(x) (2x^2 + x^2 \sin^2(x) - 24 \cos^2(x))}{4x^2} - \tan(x) y'(x) + \sqrt{\cos(x)}$$

✓ **Mathematica** : cpu = 0.138866 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{5} c_2 x^3 \sqrt{\cos(x)} + \frac{c_1 \sqrt{\cos(x)}}{x^2} - \frac{1}{4} x^2 \sqrt{\cos(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 28

$$\left\{ y(x) = \frac{4x^5 - C1 - x^4 + 4 - C2}{4x^2} \sqrt{\cos(x)} \right\}$$

**2.1434                    ODE No. 1434**

---

$$y''(x) = -\frac{b \cot(x) y'(x)}{a} - \frac{y(x) \csc^2(x) (c \cos^2(x) + d \cos(x) + e)}{a}$$

✓ **Mathematica** : cpu = 70.893 (sec), leaf count = 1596424

Too large to display

✓ **Maple** : cpu = 0.405 (sec), leaf count = 517

$$\left\{ y(x) = (\sin(x))^{-\frac{a+b}{2a}} \left( \frac{\cos(x)}{2} - \frac{1}{2} \right)^{\frac{1}{4a} \left( 2a + \sqrt{a^2 + (-2b - 4c - 4d - 4e)a + b^2} \right)} \left( {}_2F_1 \left( -\frac{1}{4a} \left( 2i \sqrt{4ca - b^2} + \sqrt{a^2 + (-2} \right. \right. \right.$$

## 2.1435      ODE No. 1435

$$y''(x) = -4y(x) \sin(3x) \csc^3(x)$$

✓ **Mathematica** : cpu = 0.104257 (sec), leaf count = 70

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt[4]{\cos^2(x) - 1} P_{-\frac{1}{2} + 4i}^{\frac{i\sqrt{47}}{2}}(\cos(x)) + c_2 \sqrt[4]{\cos^2(x) - 1} Q_{-\frac{1}{2} + 4i}^{\frac{i\sqrt{47}}{2}}(\cos(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.092 (sec), leaf count = 38

$$\left\{y(x) = \sqrt{\sin(x)} \left( LegendreQ\left(-\frac{1}{2} + 4i, \frac{i}{2}\sqrt{47}, \cos(x)\right)_{-C2} + LegendreP\left(-\frac{1}{2} + 4i, \frac{i}{2}\sqrt{47}, \cos(x)\right)_{-C1} \right) \right.$$

## 2.1436      ODE No. 1436

$$y''(x) = -\frac{1}{4}y(x) \csc^2(x) (-4n^2 + 4v(v+1) \sin^2(x) - \cos^2(x) + 2)$$

✓ **Mathematica** : cpu = 0.3637 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt[4]{\cos^2(x) - 1} P_v^n(\cos(x)) + c_2 \sqrt[4]{\cos^2(x) - 1} Q_v^n(\cos(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.174 (sec), leaf count = 113

$$\left\{ y(x) = 1\sqrt{-2\cos(2x)+2} \sqrt[4]{2\cos(2x)+2} \left( \frac{\cos(2x)}{2} - \frac{1}{2} \right)^{\frac{n}{2}} \left( \sqrt{2\cos(2x)+2} {}_2F_1\left(1+\frac{v}{2}+\frac{n}{2}, \frac{1}{2}-\frac{v}{2}+\frac{n}{2}; \frac{3}{2}-\frac{v}{2}+\frac{n}{2}; -\frac{\cos(2x)}{2}\right) \right. \right.$$

## 2.1437      ODE No. 1437

$$y''(x) = (3 \sin^2(x) + 1) \csc(x) \sec(x) y'(x) + y(x) \tan^2(x)$$

✓ **Mathematica** : cpu = 0.199781 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow c_2 \cos^{\frac{1}{2}}(\sqrt{13}-3)(x) + c_1 \cos^{\frac{1}{2}}(-3-\sqrt{13})(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.112 (sec), leaf count = 29

$$\left\{ y(x) = {}_{{}_-}C1 \, (\cos(x))^{-\frac{3}{2}+\frac{\sqrt{13}}{2}} + {}_{{}_-}C2 \, (\cos(x))^{-\frac{3}{2}-\frac{\sqrt{13}}{2}} \right\}$$

## 2.1438      ODE No. 1438

$$y''(x) = y(x) \left( -\csc^2(x) \right) \sec^2(x) \left( -a \sin^2(x) \cos^2(x) - (m-1)m \sin^2(x) - (n-1)n \cos^2(x) \right)$$

✓ **Mathematica** : cpu = 0.635893 (sec), leaf count = 615

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2(-1)^{\frac{1}{2}(-2m-1)+1} \cos^2(x)^{\frac{1}{4}(-2m-1)+1} (\cos^2(x) - 1)^{\frac{1}{2} \left( \frac{4am+4\sqrt{-a}n^2+4an-4\sqrt{-a}n+4(-a)^{3/2}+8\sqrt{-a}a+\sqrt{-a}+4mn^2-4}{8a+8n^2-8n+2} \right)} \right. \right.$$

✓ **Maple** : cpu = 0.131 (sec), leaf count = 102

$$\left\{ y(x) = (\sin(x))^n \left( (\cos(x))^{-m+1} {}_2F_1\left(\frac{n}{2} - \frac{m}{2} + \frac{i}{2}\sqrt{a} + \frac{1}{2}, \frac{n}{2} - \frac{m}{2} - \frac{i}{2}\sqrt{a} + \frac{1}{2}; \frac{3}{2} - m; (\cos(x))^2\right) C_2 + (\cos(x))^{-m+1} \right. \right.$$

## 2.1439      ODE No. 1439

$$y''(x) = \frac{\phi'(x)y'(x)}{\phi(x) - \phi(a)} - \frac{y(x)(\phi''(a) - n(n+1)(\phi(x) - \phi(a))^2)}{\phi(x) - \phi(a)}$$

**X Mathematica** : cpu = 0.644057 (sec), leaf count = 0 , could not solve

$$\text{DSolve}[\text{Derivative}[2][y][x] == (\text{Derivative}[1][\phi][x] * \text{Derivative}[1][y][x]) / (-\phi[a] + \phi[x]) \\ (n*(1 + n)*(-\phi[a] + \phi[x])^2) + \text{Derivative}[2][\phi][a]) / (-\phi[a] + \phi[x]), y[x], x]$$

**X Maple :** cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^2}{dx^2} Y(x) - \frac{\left( \frac{d}{dx} \phi(x) \right) \frac{d}{dx} Y(x)}{\phi(x) - \phi(a)} + \frac{\left( -n(n+1)(\phi(x) - \phi(a))^2 + \frac{d^2}{da^2} \phi(a) \right)}{\phi(x) - \phi(a)} \right\} Y(x) \right) \right\},$$

2.1440 ODE No. 1440

---

$$y''(x) = -\frac{y'(x) \left(-\phi''(x) - \phi(x)\phi'(x) + \phi\left(x^3\right)\right)}{\phi'(x) + \phi(x)^2} - \frac{y(x) \left(-\phi(x)\phi''(x) + \phi(x)^2 \left(-\phi'(x)\right) + \phi'(x)^2\right)}{\phi'(x) + \phi(x)^2}$$

✗ **Mathematica** : cpu = 0.677969 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[2][y][x] == -((Derivative[1][y][x]\*(phi[x^3] - phi[x]\*Derivative[1][phi][x] - (phi[x]^2\*Derivative[1][phi][x]) + Derivative[1][phi][x]^2 - phi[x]\*Derivative[2][phi][x])))/  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x)=DESol\left(\left(\frac{\left(\left(\frac{d}{dx}\phi(x)\right)^2-\left(\phi(x)\right)^2\frac{d}{dx}\phi(x)-\phi(x)\frac{d^2}{dx^2}\phi(x)\right)-Y(x)}{\frac{d}{dx}\phi(x)+\left(\phi(x)\right)^2}+\frac{\left(\phi\left(x^3\right)-\phi(x)\frac{d}{dx}\phi(x)-\frac{d^2}{dx^2}\phi(x)\right)}{\frac{d}{dx}\phi(x)+\left(\phi(x)\right)^2}\right)\right\}$$

2.1441 ODE No. 1441

---

$$y''(x) = -\frac{y'(x)(-\operatorname{cn}(x|k)\operatorname{dn}(x|k)-2\operatorname{sn}(x|k))}{\operatorname{sn}(x|k)^2-\operatorname{sn}(a|k)^2} - \frac{y(x)\left(6k^2\operatorname{sn}(a|k)^4-4\left(k^2+1\right)\operatorname{sn}(a|k)^2+2\right)}{\operatorname{sn}(x|k)^2-\operatorname{sn}(a|k)^2} - \frac{1}{\operatorname{sn}(x|k)^2-\operatorname{sn}(a|k)^2}$$

✗ **Mathematica** : cpu = 1.20813 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[2][y][x] == -(-JacobiSN[a, k]^2 + JacobiSN[x, k]^2)^(-1) - ((2 - 4\*(1 + k^2)\*JacobiSN[a, k]^2 + JacobiSN[x, k]^2) - ((- (JacobiCN[x, k]\*JacobiDN[x, k]) - 2\*JacobiSN[x, k]^2) + JacobiSN[a, k]^2 + JacobiSN[x, k]^2), y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x)=DESol\left(\left(\frac{d^2}{dx^2}-Y(x)-2\frac{JacobiSN(x,k)JacobiCN(x,k)JacobiDN(x,k)\frac{d}{dx}-Y(x)}{\left(JacobiSN(x,k)\right)^2-JacobiSN(a,k)}-\frac{\left(-2+4\left(k^2\right.\right.\right.\right.$$

2.1442 ODE No. 1442

---

$$y''(x) = \frac{y(x)}{f(x)} - \frac{xy'(x)}{f(x)}$$

✓ **Mathematica** : cpu = 0.0522746 (sec), leaf count = 46

$$\left\{\left\{y(x)\rightarrow c_2x\int_1^x\frac{\exp\left(-\int_1^{K[2]}\frac{K[1]}{f(K[1])}dK[1]\right)}{K[2]^2}dK[2]+c_1x\right\}\right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 30

$$\left\{y(x)=x\left(-C1\int e^{\int_x^1\left(-2-\frac{x^2}{f(x)}\right)dx}dx+_C2\right)\right\}$$

2.1443 ODE No. 1443

---

$$y''(x) = -\frac{f'(x)y'(x)}{2f(x)} - \frac{g(x)y(x)}{f(x)}$$

✗ **Mathematica** : cpu = 0.280083 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[2][y][x] == -((g[x]\*y[x])/f[x]) - (Derivative[1][f][x]\*Derivative[1][y][x])/f[x]^2,  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x)=DESol\left(\left(\left\{\frac{g(x)-Y(x)}{f(x)}+\frac{\left(\frac{d}{dx}f(x)\right)\frac{d}{dx}Y(x)}{2f(x)}+\frac{d^2}{dx^2}-Y(x)\right\},\left\{-Y(x)\right\}\right)\right)\right\}$$

2.1444 ODE No. 1444

---

$$y''(x) = -by(x)f(x)^{2a} - \frac{af'(x)y'(x)}{f(x)}$$

✗ **Mathematica** : cpu = 1.08393 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[2][y][x] == -(b\*f[x]^(2\*a)\*y[x]) - (a\*Derivative[1][f][x]\*Derivative[1][y][x])/f[x]^2,  
✓ **Maple** : cpu = 0.016 (sec), leaf count = 37

$$\left\{y(x)=-C1\,e^{\int i(f(x))^a\sqrt{b}\,dx}+_C2\,e^{-\int i(f(x))^a\sqrt{b}\,dx}\right\}$$

**2.1445                    ODE No. 1445**

---

$$y''(x) = -\frac{y'(x) \left(2f(x)g(x)g'(x)^2 - (g(x)^2 - 1) (2f'(x)g'(x) + f(x)g''(x))\right)}{f(x) (g(x)^2 - 1) g'(x)} - \frac{y(x) \left((g(x)^2 - 1) (f'(x) (2f'(x)g'(x) + f(x)g''(x)) - (g(x)^2 - 1) f''(x))\right)}{f(x) (g(x)^2 - 1) g'(x)}$$

✗ **Mathematica** : cpu = 1.04693 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[2][y][x] == -(Derivative[1][y][x]\*(2\*f[x]\*g[x]\*Derivative[1][g][x]^2 - (-1 + g[x]^2)\*(2\*Derivative[1][f][x]\*Derivative[1][g][x] + f[x]\*Derivative[2][g][x]))/(f[x]\*(1 + g[x]^2)\*Derivative[1][g][x])) - (y[x]\*(-(f[x]\*Derivative[1][g][x]^2\*(2\*g[x]\*Derivative[1][f][x] + g[x]^2)\*(-f[x]\*Derivative[1][g][x]\*Derivative[2][f][x]) + Derivative[1][f][x]\*(2\*Derivative[1][g][x]^2 + Derivative[2][f][x]\*g[x])) - y[x], x]  
✓ **Maple** : cpu = 0.133 (sec), leaf count = 20

$$\{y(x) = f(x) (LegendreQ(v, g(x))\_C2 + LegendreP(v, g(x))\_C1)\}$$

**2.1446                    ODE No. 1446**

---

$$y''(x) = -\frac{(x-1)y(x)}{x^4} - \frac{y'(x)}{x}$$

✓ **Mathematica** : cpu = 0.0276999 (sec), leaf count = 33

$$\left\{\left\{y(x) \rightarrow c_1 e^{-1/x} - c_2 e^{-1/x} \text{Ei}\left(\frac{2}{x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 22

$$\left\{y(x) = e^{-x^{-1}} \left(Ei(1, -2x^{-1})\_C2 + \_C1\right)\right\}$$

**2.1447                    ODE No. 1447**

---

$$y''(x) = -\frac{(-x-1)y(x)}{x^4} - \frac{y'(x)}{x}$$

✓ **Mathematica** : cpu = 0.0270002 (sec), leaf count = 29

$$\left\{\left\{y(x) \rightarrow c_1 e^{\frac{1}{x}} - c_2 e^{\frac{1}{x}} \text{Ei}\left(-\frac{2}{x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 20

$$\left\{y(x) = e^{x^{-1}} \left(Ei(1, 2x^{-1})\_C2 + \_C1\right)\right\}$$

**2.1448                    ODE No. 1448**

---

$$y''(x) = -\frac{b^2 y(x)}{(x^2 - a^2)^2}$$

✓ **Mathematica** : cpu = 0.210899 (sec), leaf count = 149

$$\left\{\left\{y(x) \rightarrow c_1 (x-a)^{\frac{1}{2}\sqrt{1-\frac{b^2}{a^2}}+\frac{1}{2}} (a+x)^{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{b^2}{a^2}}} - \frac{c_2 (x-a)^{\frac{1}{2}-\frac{1}{2}\sqrt{\frac{a^2-b^2}{a^2}}} (a+x)^{\frac{1}{2}\sqrt{\frac{a^2-b^2}{a^2}+\frac{1}{2}}}{2a\sqrt{\frac{a^2-b^2}{a^2}}}\right\}\right\}$$

✓ **Maple** : cpu = 0.065 (sec), leaf count = 77

$$\left\{y(x) = \sqrt{(a-x)(x+a)} \left(\left(\frac{a-x}{x+a}\right)^{-\frac{1}{2a}\sqrt{a^2-b^2}}\_C2 + \left(\frac{a-x}{x+a}\right)^{\frac{1}{2a}\sqrt{a^2-b^2}}\_C1\right)\right\}$$

**2.1449                    ODE No. 1449**

---

$$y^{(3)}(x) - \lambda y(x) = 0$$

✓ **Mathematica** : cpu = 0.005375 (sec), leaf count = 53

$$\left\{\left\{y(x) \rightarrow c_1 e^{(-1)^{2/3} \sqrt[3]{\lambda} x} + c_2 e^{-\sqrt[3]{-1} \sqrt[3]{\lambda} x} + c_3 e^{\sqrt[3]{\lambda} x}\right\}\right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 47

$$\left\{y(x) = \_C1 e^{-\frac{(i\sqrt{3}+1)x}{2} \sqrt[3]{\lambda}} + \_C2 e^{\frac{(i\sqrt{3}-1)x}{2} \sqrt[3]{\lambda}} + \_C3 e^{\sqrt[3]{\lambda} x}\right\}$$



2.1450      ODE No. 1450

---

$$ax^3y(x) - bx + y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0723516 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot} \left( \{y, x\}, \left\{ ay(x)x^3 - bx + y^{(3)}(x) = 0, y(0) = c_1, y'(0) = c_2, y''(0) = c_3 \right\} \right) (x) \right\} \right\}$$

✓ **Maple** : cpu = 0.174 (sec), leaf count = 1616

$$\left\{ y(x) = \int -11211200 \, b x^3 \left( \left( -5/8 x^6 {}_0F_2 \left( ; \frac{13}{6}, 7/3; -\frac{x^6 a}{216} \right) a + 35 {}_0F_2 \left( ; 7/6, 4/3; -\frac{x^6 a}{216} \right) \right) {}_0F_2 \left( ; 5/6, 7/6; -\frac{x^6 a}{216} \right) \right) dx \right\}$$

2.1451      ODE No. 1451

---

$$y^{(3)}(x) - ax^b y(x) = 0$$

✓ **Mathematica** : cpu = 0.0140444 (sec), leaf count = 168

$$\left\{ \left\{ y(x) \rightarrow (-1)^{\frac{1}{b+3}} (b+3)^{-\frac{3}{b+3}} c_2 x a^{\frac{1}{b+3}} {}_0F_2 \left( ; 1 - \frac{1}{b+3}, 1 + \frac{1}{b+3}; \frac{ax^{b+3}}{(b+3)^3} \right) + (-1)^{\frac{2}{b+3}} (b+3)^{-\frac{6}{b+3}} c_3 x^2 a^{\frac{2}{b+3}} {}_0F_2 \left( ; 1 - \frac{2}{b+3}, 1 + \frac{2}{b+3}; \frac{2ax^{b+3}}{(b+3)^3} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 114

$$\left\{ y(x) = {}_0F_2 \left( ; \frac{b+2}{b+3}, \frac{b+1}{b+3}; \frac{x^{b+3}a}{(b+3)^3} \right) + {}_0F_2 \left( ; \frac{b+4}{b+3}, \frac{b+2}{b+3}; \frac{x^{b+3}a}{(b+3)^3} \right) + {}_0F_2 \left( ; \frac{b+5}{b+3}, \frac{b+4}{b+3}; \frac{x^{b+3}a}{(b+3)^3} \right) \right\}$$

2.1452      ODE No. 1452

---

$$y^{(3)}(x) + 3y'(x) - 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.0056233 (sec), leaf count = 54

$$\left\{ \left\{ y(x) \rightarrow c_3 e^x + c_1 e^{-x/2} \sin \left( \frac{\sqrt{15}x}{2} \right) + c_2 e^{-x/2} \cos \left( \frac{\sqrt{15}x}{2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.006 (sec), leaf count = 35

$$\left\{ y(x) = c_1 e^x + c_2 e^{-\frac{x}{2}} \sin \left( \frac{\sqrt{15}x}{2} \right) + c_3 e^{-\frac{x}{2}} \cos \left( \frac{\sqrt{15}x}{2} \right) \right\}$$

2.1453      ODE No. 1453

---

$$a^2 (-y'(x)) - e^{2ax} \sin^2(x) + y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 1.10099 (sec), leaf count = 128

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-ax} (-9 (a^2 - 4) a^4 e^{3ax} \cos(2x) - 3 (11a^2 - 4) a^3 e^{3ax} \sin(2x) + (9a^6 + 49a^4 + 56a^2 + 16) (12a^2 c_1 e^{2ax} - 12a^3 c_2 e^{2ax}))}{12a^3 (9a^6 + 49a^4 + 56a^2 + 16)} \right\} \right\}$$

✓ **Maple** : cpu = 0.109 (sec), leaf count = 122

$$\left\{ y(x) = \frac{1}{108 a^9 + 588 a^7 + 672 a^5 + 192 a^3} \left( ((-9 a^6 + 36 a^4) \cos(2 x) + (-33 a^5 + 12 a^3) \sin(2 x) + 9 a^6 + 49 a^4) e^{2 a x} \right) \right\}$$

2.1454      ODE No. 1454

---

$$2axy'(x) + ay(x) + y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.0072917 (sec), leaf count = 79

$$\left\{ \left\{ y(x) \rightarrow c_1 \text{Ai} \left( \sqrt[3]{-\frac{1}{2}} \sqrt[3]{ax} \right)^2 + c_3 \text{Bi} \left( \sqrt[3]{-\frac{1}{2}} \sqrt[3]{ax} \right)^2 + c_2 \text{Ai} \left( \sqrt[3]{-\frac{1}{2}} \sqrt[3]{ax} \right) \text{Bi} \left( \sqrt[3]{-\frac{1}{2}} \sqrt[3]{ax} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 55

$$\left\{ y(x) = c_1 \left( \text{Ai} \left( -\frac{2^{\frac{2}{3}} x}{2} \sqrt[3]{a} \right) \right)^2 + c_2 \left( \text{Bi} \left( -\frac{2^{\frac{2}{3}} x}{2} \sqrt[3]{a} \right) \right)^2 + c_3 \text{Ai} \left( -\frac{2^{\frac{2}{3}} x}{2} \sqrt[3]{a} \right) \text{Bi} \left( -\frac{2^{\frac{2}{3}} x}{2} \sqrt[3]{a} \right) \right\}$$

2.1455 ODE No. 1455

---

$$x(a+b-1)y'(x)-aby(x)+x^2\left(-y''(x)\right)+y^{(3)}(x)=0$$

✓ **Mathematica** : cpu = 0.0204216 (sec), leaf count = 127

$$\left\{\left\{y(x)\rightarrow \sqrt[3]{-\frac{1}{3}}c_2x\,{}_2F_2\left(\frac{1}{3}-\frac{a}{3},\frac{1}{3}-\frac{b}{3};\frac{2}{3},\frac{4}{3};\frac{x^3}{3}\right)+c_1\,{}_2F_2\left(-\frac{a}{3},-\frac{b}{3};\frac{1}{3},\frac{2}{3};\frac{x^3}{3}\right)+\left(-\frac{1}{3}\right)^{2/3}c_3x^2\,{}_2F_2\left(\frac{2}{3}-\frac{a}{3},\frac{2}{3}-\frac{b}{3};\frac{4}{3},\frac{5}{3};\frac{x^3}{3}\right)\right\}\right.$$

✓ **Maple** : cpu = 0.079 (sec), leaf count = 71

$$\left\{y(x)=-C1\,{}_2F_2\left(-\frac{a}{3},-\frac{b}{3};\frac{1}{3},\frac{2}{3};\frac{x^3}{3}\right)-C2\,x^2{}_2F_2\left(-\frac{b}{3}+\frac{2}{3},-\frac{a}{3}+\frac{2}{3};\frac{4}{3},\frac{5}{3};\frac{x^3}{3}\right)-C3\,{}_2F_2\left(\frac{1}{3}-\frac{a}{3},\frac{1}{3}-\frac{b}{3};\frac{2}{3},\frac{4}{3};\frac{x^3}{3}\right)\right\}$$

2.1456 ODE No. 1456

---

$$x^{2c-2}y'(x)+(c-1)x^{2c-3}y(x)+y^{(3)}(x)=0$$

✓ **Mathematica** : cpu = 0.024968 (sec), leaf count = 183

$$\left\{\left\{y(x)\rightarrow c_1\,{}_1F_2\left(\frac{1}{2}-\frac{1}{2c};1-\frac{1}{c},1-\frac{1}{2c};-\frac{x^{2c}}{4c^2}\right)+4^{-1/c}c^{-2/c}c_3\left(x^{2c}\right)^{\frac{1}{c}}\,{}_1F_2\left(\frac{1}{2}+\frac{1}{2c};1+\frac{1}{2c},1+\frac{1}{c};-\frac{x^{2c}}{4c^2}\right)+2c_2x\right\}\right.$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 73

$$\left\{y(x)=x\left(\left(Y_{\frac{1}{2c}}\left(\frac{x^c}{2c}\right)\right)^2-C2+Y_{\frac{1}{2c}}\left(\frac{x^c}{2c}\right)J_{\frac{1}{2c}}\left(\frac{x^c}{2c}\right)-C3+\left(J_{\frac{1}{2c}}\left(\frac{x^c}{2c}\right)\right)^2-C1\right)\right\}$$

2.1457 ODE No. 1457

---

$$-3y'(x)(a+2\wp(x;g2,g3))+by(x)+y^{(3)}(x)=0$$

✗ **Mathematica** : cpu = 0.0107705 (sec), leaf count = 0 , could not solve  
DSolve[b\*y[x] - 3\*(a + 2\*WeierstrassP[x, {g2, g3}])\*Derivative[1][y][x] + Derivative[3][y][x], y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x)=DESol\left(\left\{\frac{d^3}{dx^3}{}_{{-}}Y(x)+(-6\,WeierstrassP(x,g2,g3)-3a)\frac{d}{dx}{}_{{-}}Y(x)+b{}_{{-}}Y(x)\right\},\{{}_{{-}}Y(x)\}\right)\right\}$$

2.1458 ODE No. 1458

---

$$\frac{1}{2}y(x)\left((1-n^2)\wp'(x;g2,g3)-a\right)+(1-n^2)y'(x)\wp(x;g2,g3)+y^{(3)}(x)=0$$

✗ **Mathematica** : cpu = 0.0140337 (sec), leaf count = 0 , could not solve  
DSolve[((-a + (1 - n^2)\*WeierstrassPPrime[x, {g2, g3}])\*y[x])/2 + (1 - n^2)\*WeierstrassP[x, {g2, g3}]\*y'[x] + y'''[x], y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x)=DESol\left(\left\{\frac{d^3}{dx^3}{}_{{-}}Y(x)+(-WeierstrassP(x,g2,g3)n^2+WeierstrassP(x,g2,g3))\frac{d}{dx}{}_{{-}}Y(x)+\left(-WeierstrassP(x,g2,g3)\right){}_{{-}}Y(x)\right\},\{{}_{{-}}Y(x)\}\right)\right\}$$

2.1459 ODE No. 1459

---

$$-y'(x)(a+4n(n+1)\wp(x;g2,g3))-2n(n+1)y(x)\wp'(x;g2,g3)+y^{(3)}(x)=0$$

✗ **Mathematica** : cpu = 0.0129924 (sec), leaf count = 0 , could not solve  
DSolve[-2\*n\*(1 + n)\*WeierstrassPPrime[x, {g2, g3}]\*y[x] - (a + 4\*n\*(1 + n)\*WeierstrassP[x, {g2, g3}])\*y'[x] + y'''[x], y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x)=\left(DESol\left(\left\{\frac{d^2}{dx^2}{}_{{-}}Y(x)+\left(-WeierstrassP(x,g2,g3)n^2-nWeierstrassP(x,g2,g3)-\frac{a}{4}\right){}_{{-}}Y(x)\right\},\{{}_{{-}}Y(x)\}\right)\right)\right\}$$

2.1460 ODE No. 1460

---

$$y'(x)(a + A\wp(x; g2, g3)) + By(x)\wp'(x; g2, g3) + y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0099156 (sec), leaf count = 0 , could not solve  
DSolve[B\*WeierstrassPPrime[x, {g2, g3}]\*y[x] + (a + A\*WeierstrassP[x, {g2, g3}])\*Derivative[3][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x) = DESol\left(\left\{\frac{d^3}{dx^3}Y(x) + (AWeierstrassP(x, g2, g3) + a)\frac{d}{dx}Y(x) + BWeierstrassPPrime(x, g2, g3)Y(x)\right\}, \{Y(x)\}\right)\right\}$$

2.1461 ODE No. 1461

---

$$-y'(x)(a + 3k^2\operatorname{sn}(z|x)^2) + y(x)(b + c\operatorname{sn}(z|x)^2 - 3k^2\operatorname{cn}(z|x)\operatorname{dn}(z|x)\operatorname{sn}(z|x)) + y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0205206 (sec), leaf count = 0 , could not solve  
DSolve[(b - 3\*k^2\*JacobiCN[z, x]\*JacobiDN[z, x]\*JacobiSN[z, x] + c\*JacobiSN[z, x]^2)\*y[x] - (a + 3\*k^2\*JacobiCN[z, x]^2)\*Derivative[1][y][x] + Derivative[3][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x) = DESol\left(\left\{\frac{d^3}{dx^3}Y(x) + \left(-3k^2(JacobiSN(z, x))^2 - a\right)\frac{d}{dx}Y(x) + \left(b + c(JacobiSN(z, x))^2 - 3k^2JacobiCN(z, x)^2\right)Y(x)\right\}, \{Y(x)\}\right)\right\}$$

2.1462 ODE No. 1462

---

$$-y'(x)(a + 6k^2\sin^2(x)) + by(x) + y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0156785 (sec), leaf count = 0 , could not solve  
DSolve[b\*y[x] - (a + 6\*k^2\*Sin[x]^2)\*Derivative[1][y][x] + Derivative[3][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x) = DESol\left(\left\{\frac{d^3}{dx^3}Y(x) + \left(-6k^2(\sin(x))^2 - a\right)\frac{d}{dx}Y(x) + bY(x)\right\}, \{Y(x)\}\right)\right\}$$

2.1463 ODE No. 1463

---

$$y(x)f'(x) + 2f(x)y'(x) + y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0453776 (sec), leaf count = 0 , could not solve  
DSolve[y[x]\*Derivative[1][f][x] + 2\*f[x]\*Derivative[1][y][x] + Derivative[3][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x) = \left(DESol\left(\left\{\frac{d^2}{dx^2}Y(x) + \frac{f(x)Y(x)}{2}\right\}, \{Y(x)\}\right)\right)^2\right\}$$

2.1464 ODE No. 1464

---

$$y^{(3)}(x) - 2y''(x) - 3y'(x) + 10y(x) = 0$$

✓ **Mathematica** : cpu = 0.0046187 (sec), leaf count = 34

$$\left\{\left\{y(x) \rightarrow c_3e^{-2x} + c_1e^{2x}\sin(x) + c_2e^{2x}\cos(x)\right\}\right\}$$

✓ **Maple** : cpu = 0.004 (sec), leaf count = 27

$$\left\{y(x) = \_C1e^{-2x} + \_C2e^{2x}\sin(x) + \_C3e^{2x}\cos(x)\right\}$$

2.1465 ODE No. 1465

---

$$-a^2y'(x) + 2a^2y(x) + y^{(3)}(x) - 2y''(x) - \sinh(x) = 0$$

✓ **Mathematica** : cpu = 0.0863302 (sec), leaf count = 95

$$\left\{\left\{y(x) \rightarrow \frac{e^{-x}(3a^2e^{2x} - a^2 - 3e^{2x} - 12e^x\sinh(x) - 6e^x\cosh(x) + 1)}{6(a-2)(a+2)(a^2-1)} + c_1e^{-ax} + c_3e^{ax} + c_2e^{2x}\right\}\right\}$$

✓ **Maple** : cpu = 0.121 (sec), leaf count = 89

$$\left\{y(x) = \frac{1}{6a^4 - 30a^2 + 24}(-9e^{ax}(e^x - 1/3e^{-x})e^{-ax} + 3(a+1)(a-1)(e^{-x} - 1/3\cosh(3x) + 1/3\sinh(3x)))\right\}$$

2.1466 ODE No. 1466

---

$$a^3(-y(x)) + 3a^2y'(x) - 3ay''(x) - e^{ax} + y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.0083792 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow c_3 x^2 e^{ax} + c_2 x e^{ax} + c_1 e^{ax} + \frac{1}{6} x^3 e^{ax} \right\} \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 27

$$\left\{ y(x) = \frac{e^{ax} (6 \_C3 x^2 + x^3 + 6 \_C2 x + 6 \_C1)}{6} \right\}$$

2.1467 ODE No. 1467

---

$$a0y(x) + a1y'(x) + a2y''(x) + y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.0048726 (sec), leaf count = 84

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{x \text{Root}[\#1^3 + \#1^2 a2 + \#1 a1 + a0 \&, 1]} + c_2 e^{x \text{Root}[\#1^3 + \#1^2 a2 + \#1 a1 + a0 \&, 2]} + c_3 e^{x \text{Root}[\#1^3 + \#1^2 a2 + \#1 a1 + a0 \&, 3]} \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 590

$$\left\{ y(x) = \_C1 e^{-x \left( \left( \frac{i}{12} \sqrt{3} + \frac{1}{12} \right) \left( 36 a1 a2 - 108 a0 - 8 a2^3 + 12 \sqrt{12 a0 a2^3 - 3 a1^2 a2^2 - 54 a1 a2 a0 + 12 a1^3 + 81 a0^2} \right)^{\frac{2}{3}} + \frac{a2}{3} \sqrt[3]{36 a1 a2 - 108 a0 - 8 a2^3 + 12 \sqrt{12 a0 a2^3 - 3 a1^2 a2^2 - 54 a1 a2 a0 + 12 a1^3 + 81 a0^2}} \right)} \right\}$$

2.1468 ODE No. 1468

---

$$2(2a + 4x^2 - 1)y'(x) - 8axy(x) + y^{(3)}(x) - 6xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0570111 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow c_2 H_{\frac{a}{2}}(x) {}_1F_1 \left( -\frac{a}{4}; \frac{1}{2}; x^2 \right) + c_1 H_{\frac{a}{2}}(x)^2 + c_3 {}_1F_1 \left( -\frac{a}{4}; \frac{1}{2}; x^2 \right)^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.081 (sec), leaf count = 59

$$\left\{ y(x) = x^2 \left( \left( U \left( \frac{1}{2} - \frac{a}{4}, \frac{3}{2}, x^2 \right) \right)^2 \_C2 + U \left( \frac{1}{2} - \frac{a}{4}, \frac{3}{2}, x^2 \right) M \left( \frac{1}{2} - \frac{a}{4}, \frac{3}{2}, x^2 \right) \_C3 + \left( M \left( \frac{1}{2} - \frac{a}{4}, \frac{3}{2}, x^2 \right) \right)^2 \right) \right\}$$

2.1469 ODE No. 1469

---

$$a^3x^3y(x) + 3a^2x^2y'(x) + 3axy''(x) + y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.0124438 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\frac{ax^2}{2}} + c_2 e^{-\frac{ax^2}{2} - \sqrt{3}\sqrt{ax}} + c_3 e^{\sqrt{3}\sqrt{ax} - \frac{ax^2}{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 37

$$\left\{ y(x) = e^{-\frac{ax^2}{2}} \left( \_C1 + \_C2 e^{\sqrt{3}\sqrt{ax}} + \_C3 e^{-\sqrt{3}\sqrt{ax}} \right) \right\}$$

2.1470 ODE No. 1470

---

$$y^{(3)}(x) - \sin(x)y''(x) - 2\cos(x)y'(x) + y(x)\sin(x) - \log(x) = 0$$

✓ **Mathematica** : cpu = 2.71327 (sec), leaf count = 64

$$\left\{ \left\{ y(x) \rightarrow e^{-\cos(x)} \int_1^x \frac{1}{4} e^{\cos(K[1])} (2 \log(K[1]) K[1]^2 - 3 K[1]^2 + 4 c_1 K[1] + 4 c_2) dK[1] + c_3 e^{-\cos(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 36

$$\left\{ y(x) = \left( \_C3 + \int \left( 2 \_C1 x + \_C2 - \frac{3x^2}{4} + \frac{x^2 \ln(x)}{2} \right) e^{\cos(x)} dx \right) e^{-\cos(x)} \right\}$$

2.1471 ODE No. 1471

---

$$f(x)y''(x) + f(x)y(x) + y^{(3)}(x) + y'(x) = 0$$

✗ **Mathematica** : cpu = 0.0523386 (sec), leaf count = 0 , could not solve  
DSolve[f[x]\*y[x] + Derivative[1][y][x] + f[x]\*Derivative[2][y][x] + Derivative[3][y][x] == 0  
✓ **Maple** : cpu = 0.088 (sec), leaf count = 36

$$\left\{y(x) = e^{ix} \left( \int e^{-2ix} \left( \int -C3 e^{i-f(x) dx} dx + -C2 \right) dx + -C1 \right) \right\}$$

2.1472 ODE No. 1472

---

$$f(x) \left( x^2 y''(x) - 2xy'(x) + 2y(x) \right) + y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0530117 (sec), leaf count = 0 , could not solve  
DSolve[f[x]\*(2\*y[x] - 2\*x\*Derivative[1][y][x] + x^2\*Derivative[2][y][x]) + Derivative[3][y][x] == 0  
✓ **Maple** : cpu = 0.103 (sec), leaf count = 33

$$\left\{y(x) = \left( \int -C1 + -C2 \int e^{-\int x^2 f(x) + 3x^{-1} dx} dx dx + -C3 \right) x \right\}$$

2.1473 ODE No. 1473

---

$$y(x) \left( f(x)g(x) + g'(x) \right) + f(x)y''(x) + g(x)y'(x) + y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0099427 (sec), leaf count = 0 , could not solve  
DSolve[y[x]\*(f[x]\*g[x] + Derivative[1][g][x]) + g[x]\*Derivative[1][y][x] + f[x]\*Derivative[2][y][x] + y[x]\*Derivative[3][y][x] == 0  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x) = DESol \left( \left\{ \frac{d^3}{dx^3} - Y(x) + f(x) \frac{d^2}{dx^2} - Y(x) + g(x) \frac{d}{dx} - Y(x) + \left( f(x)g(x) + \frac{d}{dx}g(x) \right) - Y(x) \right\}, \{ - Y(x) \} \right) \right\}$$

2.1474 ODE No. 1474

---

$$y'(x) \left( f'(x) + 2f(x)^2 + 4g(x) \right) + y(x) \left( 4f(x)g(x) + 2g'(x) \right) + 3f(x)y''(x) + y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0110377 (sec), leaf count = 0 , could not solve  
DSolve[y[x]\*(4\*f[x]\*g[x] + 2\*Derivative[1][g][x]) + (2\*f[x]^2 + 4\*g[x] + Derivative[1][f][x])\*y[x] + 3\*f[x]\*Derivative[2][y][x] + y[x]\*Derivative[3][y][x] == 0  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x) = \left( DESol \left( \left\{ \frac{d^2}{dx^2} - Y(x) + f(x) \frac{d}{dx} - Y(x) + g(x) - Y(x) \right\}, \{ - Y(x) \} \right) \right)^2 \right\}$$

2.1475 ODE No. 1475

---

$$4y^{(3)}(x) - 8y''(x) - 11y'(x) - 3y(x) + 18e^x = 0$$

✓ **Mathematica** : cpu = 0.0252897 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-x/2} + c_2 e^{-x/2} x + c_3 e^{3x} + e^x \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 23

$$\left\{y(x) = (-C3 x + -C2) e^{-\frac{x}{2}} + -C1 e^{3x} + e^x \right\}$$

2.1476 ODE No. 1476

---

$$-36n^2 y'(x) \wp(x; g2, g3) - 2(n+3)(4n-3)ny(x)\phi'(x) + 27y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0835904 (sec), leaf count = 0 , could not solve  
DSolve[-2\*n\*(3 + n)\*(-3 + 4\*n)\*y[x]\*Derivative[1][phi][x] - 36\*n^2\*WeierstrassP[x, {g2, g3}] + 27\*Derivative[3][y][x] == 0  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x) = DESol \left( \left\{ 27 \frac{d^3}{dx^3} - Y(x) - 36 WeierstrassP(x, g2, g3) n^2 \frac{d}{dx} - Y(x) + (-8 WeierstrassPPrime(x, g2, g3) + 2n \phi'(x)) Y(x) \right\} \right) \right\}$$

2.1477 ODE No. 1477

---

$$xy^{(3)}(x) + 3y''(x) + xy(x) = 0$$

✓ **Mathematica** : cpu = 0.128312 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-x}}{x} + \frac{c_2 e^{\sqrt[3]{-1}x}}{x} + \frac{c_3 e^{-(-1)^{2/3}x}}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 41

$$\left\{ y(x) = \frac{1}{x} \left( -C1 e^{-x} + -C2 e^{\frac{x}{2}} \sin\left(\frac{\sqrt{3}x}{2}\right) + -C3 e^{\frac{x}{2}} \cos\left(\frac{\sqrt{3}x}{2}\right) \right) \right\}$$

2.1478 ODE No. 1478

---

$$-ax^2y(x) + xy^{(3)}(x) + 3y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0243025 (sec), leaf count = 104

$$\left\{ \left\{ y(x) \rightarrow -\frac{2(-1)^{3/4}\sqrt{2}c_1 {}_0F_2\left(\begin{smallmatrix} \frac{1}{2}, \frac{3}{4}; \frac{ax^4}{64} \end{smallmatrix}\right)}{\sqrt[4]{ax}} + c_2 {}_0F_2\left(\begin{smallmatrix} \frac{3}{4}, \frac{5}{4}; \frac{ax^4}{64} \end{smallmatrix}\right) + \frac{\sqrt[4]{-1}\sqrt[4]{a}c_3 x {}_0F_2\left(\begin{smallmatrix} \frac{5}{4}, \frac{3}{2}; \frac{ax^4}{64} \end{smallmatrix}\right)}{2\sqrt{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 48

$$\left\{ y(x) = -C1 {}_0F_2\left(\begin{smallmatrix} \frac{3}{4}, \frac{5}{4}; \frac{ax^4}{64} \end{smallmatrix}\right) + \frac{C2}{x} {}_0F_2\left(\begin{smallmatrix} \frac{1}{2}, \frac{3}{4}; \frac{ax^4}{64} \end{smallmatrix}\right) + -C3 x {}_0F_2\left(\begin{smallmatrix} \frac{5}{4}, \frac{3}{2}; \frac{ax^4}{64} \end{smallmatrix}\right) \right\}$$

2.1479 ODE No. 1479

---

$$(a+b)y''(x) - ay(x) + xy^{(3)}(x) - xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.10718 (sec), leaf count = 153

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} i c_2 x {}_1F_2\left(\begin{smallmatrix} a, \frac{1}{2}; \frac{3}{2}, \frac{a}{2} + \frac{b}{2} + \frac{1}{2}; \frac{x^2}{4} \end{smallmatrix}\right) + c_1 {}_1F_2\left(\begin{smallmatrix} a; \frac{1}{2}, \frac{a}{2} + \frac{b}{2}; \frac{x^2}{4} \end{smallmatrix}\right) + c_3 \left(\frac{i}{2}\right)^{-a-b+2} x^{-a-b+2} {}_1F_2\left(1 - \frac{b}{2}; 2 - \frac{b}{2}; \frac{x^2}{4}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.131 (sec), leaf count = 92

$$\left\{ y(x) = -C1 {}_1F_2\left(\begin{smallmatrix} a, \frac{1}{2}; \frac{3}{2}, \frac{a}{2} + \frac{b}{2} + \frac{1}{2}; \frac{x^2}{4} \end{smallmatrix}\right) + -C2 x {}_1F_2\left(\begin{smallmatrix} a; \frac{1}{2}, \frac{a}{2} + \frac{b}{2}; \frac{x^2}{4} \end{smallmatrix}\right) + -C3 x^{-a-b+2} {}_1F_2\left(1 - \frac{b}{2}; 2 - \frac{b}{2}; \frac{x^2}{4}\right) \right\}$$

2.1480 ODE No. 1480

---

$$-(2v+x)y''(x) - (-2v+x-1)y'(x) + xy^{(3)}(x) + (x-1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.175109 (sec), leaf count = 93

$$\left\{ \left\{ y(x) \rightarrow \frac{c_3 e^x x^{2v+2} \Gamma\left(v + \frac{3}{2}\right) {}_1\tilde{F}_1\left(v + \frac{3}{2}; 2v+3; -2x\right)}{\Gamma\left(\frac{1}{2} - v\right)} + c_2 2^{-2v-2} e^x G_{2,3}^{2,1}\left(2x \left| \begin{smallmatrix} 1, v + \frac{3}{2} \\ 1, 2(v+1), 0 \end{smallmatrix} \right. \right) + c_1 e^x \right\} \right\}$$

✓ **Maple** : cpu = 0.138 (sec), leaf count = 35

$$\left\{ y(x) = -C1 e^x + -C2 x^{v+1} I_{-v-1}(x) + -C3 x^{v+1} K_{v+1}(x) \right\}$$

2.1481 ODE No. 1481

---

$$-f(x) + (x^2 - 3) y''(x) + xy^{(3)}(x) + 4xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.393147 (sec), leaf count = 432

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{240} e^{-\frac{x^2}{2}} \left( -240 x^5 \int_1^x \left( -\frac{1}{240} \left( 15 \operatorname{Ei}\left(\frac{K[1]^2}{2}\right) + 16 e^{\frac{K[1]^2}{2}} \right) f(K[1]) + \frac{1}{15} \sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\frac{K[1]}{\sqrt{2}}\right) K[1] f(K[1]) \right) \mathrm{d}x \right) e^{\frac{x^2}{2}} \mathrm{d}x \right\} \right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 44

$$\left\{ y(x) = \left( -C3 + \int \frac{2 - C1 x + -C2 - \iint -f(x) \mathrm{d}x \mathrm{d}x}{x^6} e^{\frac{x^2}{2}} \mathrm{d}x \right) e^{-\frac{x^2}{2}} x^5 \right\}$$

**2.1482                    ODE No. 1482**

---

$$axy(x) - b + 2xy^{(3)}(x) + 3y''(x) = 0$$

✗ **Mathematica** : cpu = 0.410708 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot} \left( \{y, x\}, \left\{ -b + xay(x) + 3y''(x) + 2xy^{(3)}(x) = 0, y(1) = c_1, y'(1) = c_2, y''(1) = c_3 \right\} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.228 (sec), leaf count = 1614

$$\left\{ y(x) = \int -2802800bx \left( \left( -5/8 {}_0F_2 \left( ; \frac{13}{6}, 7/3; -\frac{ax^3}{54} \right) ax^3 + \frac{35}{4} {}_0F_2 \left( ; 7/6, 4/3; -\frac{ax^3}{54} \right) {}_0F_2 \left( ; 5/6, 7/6; -\frac{ax^3}{54} \right) \right) \right. \right.$$

**2.1483                    ODE No. 1483**

---

$$-4(\nu + x - 1)y''(x) + (6\nu + 2x - 5)y'(x) + (1 - 2\nu)y(x) + 2xy^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.11159 (sec), leaf count = 112

$$\left\{ \left\{ y(x) \rightarrow \frac{c_3 e^x x \Gamma \left( \frac{5}{2} - 3\nu \right) \left( \frac{{}_2F_1 \left( \frac{3}{2} - 3\nu; 1 - 2\nu; -x \right)}{3(2\nu - 1)x} + \frac{2}{3x\Gamma(2 - 2\nu)} \right)}{\Gamma \left( \frac{3}{2} - \nu \right)} + c_2 e^x G_{2,3}^{2,1} \left( x \left| \begin{matrix} 1, 3\nu - \frac{1}{2} \\ 1, 2\nu, 0 \end{matrix} \right. \right) + c_1 e^x \right\} \right\}$$

✓ **Maple** : cpu = 0.145 (sec), leaf count = 37

$$\left\{ y(x) = \_C1 e^x + \_C2 e^{\frac{x}{2}} x^\nu I_\nu \left( \frac{x}{2} \right) + \_C3 e^{\frac{x}{2}} x^\nu K_\nu \left( \frac{x}{2} \right) \right\}$$

**2.1484                    ODE No. 1484**

---

$$6y'(x)(ak + bx) + 3(2ax + k)y''(x) + y(x)(3bk + 2cx) + 2xy^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 62.1639 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot} \left( \{y, x\}, \left\{ (2xc + 3bk)y(x) + (6xb + 6ak)y'(x) + (6xa + 3k)y''(x) + 2xy^{(3)}(x) = 0, y(1) = c_1, y'(1) = c_2, y''(1) = c_3 \right\} \right) \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \text{DESol} \left( \left\{ (3bk + 2cx) \_Y(x) + (6ak + 6bx) \frac{d}{dx} \_Y(x) + (6ax + 3k) \frac{d^2}{dx^2} \_Y(x) + 2x \frac{d^3}{dx^3} \_Y(x) \right\} \right) \right\}$$

**2.1485                    ODE No. 1485**

---

$$(x - 2)xy^{(3)}(x) - (x - 2)xy''(x) - 2y'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.161081 (sec), leaf count = 64

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} c_3 x^2 \left( -\frac{4e^{x-2} \text{Ei}(2-x)}{x^2} + \frac{2}{x^2} + \frac{2}{x} + \log(2-x) - \log(x) \right) + c_1 x^2 + c_2 e^x \right\} \right\}$$

✓ **Maple** : cpu = 0.207 (sec), leaf count = 51

$$\left\{ y(x) = \_C3 Ei(1, x-2) e^{x-2} + \frac{\_C3 x^2 \ln(x-2)}{4} + \_C2 e^x - \frac{\_C3 x^2 \ln(x)}{4} + \frac{(2x+2)\_C3}{4} + \_C1 x^2 \right\}$$

**2.1486                    ODE No. 1486**

---

$$(2x - 1)y^{(3)}(x) - 8xy'(x) + 8y(x) = 0$$

✓ **Mathematica** : cpu = 0.317132 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} c_3 x \left( \frac{e^{2x-2} \text{Ei}(2-4x)}{x} - \frac{2 \text{Ei}(1-2x)}{e} - \frac{e^{-2x}}{x} \right) + c_1 x - c_2 e^{2x} \right\} \right\}$$

✓ **Maple** : cpu = 0.144 (sec), leaf count = 51

$$\left\{ y(x) = \_C1 x + \_C2 e^{2x} - \frac{\_C3 (2xe^{-1} Ei(1, 2x-1) - Ei(1, 4x-2) e^{2x-2} - e^{-2x})}{4} \right\}$$

2.1487 ODE No. 1487

---

$$(2x-1)y^{(3)}(x)+(x+4)y''(x)+2y'(x)=0$$

✓ **Mathematica** : cpu = 0.679361 (sec), leaf count = 87

$$\left\{\left\{y(x)\rightarrow\int_1^x\left(\frac{e^{-\frac{K[1]}{2}}c_1\left(1+\frac{1}{4\left(\frac{K[1]}{2}-\frac{1}{4}\right)}\right)}+e^{-\frac{K[1]}{2}}c_2L_{-\frac{1}{4}}^{\frac{5}{4}}\left(\frac{K[1]}{2}-\frac{1}{4}\right)\right)dK[1]+c_3\right\}\right\}$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 38

$$\left\{y(x)=1\left(-C3+\int(2-C1\,x+_C2)\mathrm{e}^{\frac{x}{2}}(2x-1)^{-\frac{3}{4}}\mathrm{d}x\right)\mathrm{e}^{-\frac{x}{2}}\frac{1}{\sqrt[4]{2x-1}}\right\}$$

2.1488 ODE No. 1488

---

$$ax^2y(x)+x^2y^{(3)}(x)-6y'(x)=0$$

✓ **Mathematica** : cpu = 0.40061 (sec), leaf count = 102

$$\left\{\left\{y(x)\rightarrow\frac{c_1e^{-\sqrt[3]{ax}}\left(\sqrt[3]{ax}+2\right)}{x}+\frac{c_2e^{\sqrt[3]{-1}\sqrt[3]{ax}}\left(\sqrt[3]{ax}+2(-1)^{2/3}\right)}{x}+\frac{c_3e^{-(-1)^{2/3}\sqrt[3]{ax}}\left(\sqrt[3]{ax}-2\sqrt[3]{-1}\right)}{x}\right\}\right\}$$

✓ **Maple** : cpu = 0.326 (sec), leaf count = 132

$$\left\{y(x)=\frac{1}{x}\left(-_C2\left((-i-\sqrt{3})\left(-a^4\right)^{\frac{2}{3}}+ia^3x\right)\mathrm{e}^{\frac{-\frac{i}{2}(\sqrt{3}-i)x}{a}\sqrt[3]{-a^4}}+\left((\sqrt{3}-i)\left(-a^4\right)^{\frac{2}{3}}+ia^3x\right)_C3\mathrm{e}^{\frac{\frac{i}{2}(\sqrt{3}+i)x}{a}}\right)\right\}$$

2.1489 ODE No. 1489

---

$$x^2y^{(3)}(x)+(x+1)y''(x)-y(x)=0$$

✗ **Mathematica** : cpu = 0.571371 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{\left\{y(x)\rightarrow\text{DifferentialRoot}\left(\left\{y,x\right\},\left\{y^{(3)}(x)x^2-y(x)+(x+1)y''(x)=0,y(1)=c_1,y'(1)=c_2,y''(1)=c_3\right\}\right)(x)\right\}\right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x)=DESol\left(\left\{-_Y(x)+(1+x)\frac{\mathrm{d}^2}{\mathrm{d}x^2}_Y(x)+x^2\frac{\mathrm{d}^3}{\mathrm{d}x^3}_Y(x)\right\},\{_Y(x)\}\right)\right\}$$

2.1490 ODE No. 1490

---

$$x^2y^{(3)}(x)+(x^2+1)y'(x)-xy''(x)=0$$

✓ **Mathematica** : cpu = 0.0248448 (sec), leaf count = 33

$$\left\{\left\{y(x)\rightarrow\frac{1}{2}c_1x^2{}_0\tilde{F}_1\left(2;-\frac{x^2}{4}\right)+c_2xY_1(x)+c_3\right\}\right\}$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 18

$$\{y(x)=_C1+_C2\,xJ_1(x)+_C3\,xY_1(x)\}$$

2.1491 ODE No. 1491

---

$$(-4a^2\nu^2+4a^2x^{2a}+1)y'(x)+x^2y^{(3)}(x)+3xy''(x)=4a^3x^{2a-1}y(x)$$

✓ **Mathematica** : cpu = 0.0322949 (sec), leaf count = 102

$$\left\{\left\{y(x)\rightarrow c_2\left(x^{2a}\right)^{-\nu}{}_1F_2\left(-\nu-\frac{1}{2};1-2\nu,1-\nu;-x^{2a}\right)+c_3\left(x^{2a}\right)^\nu{}_1F_2\left(\nu-\frac{1}{2};\nu+1,2\nu+1;-x^{2a}\right)+c_1{}_1F_2\right\}\right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 88

$$\left\{y(x)=_C1{}_1F_2(-\frac{1}{2};\nu+1,-\nu+1;-x^{2a})+_C2\,x^{-2a\nu}{}_1F_2(-\frac{1}{2}-\nu;1-2\nu,-\nu+1;-x^{2a})+_C3\,x^{2a\nu}{}_1F_2\right\}$$



2.1492 ODE No. 1492

---

$$(4x(n-m) + m(2m-1) + 2x^2) y'(x) - 2n(-2m+2x+1)y(x) - 3x(x-m)y''(x) + x^2y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.284176 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow c_2 U(-n, m, x) L_n^{m-1}(x) + c_1 U(-n, m, x)^2 + c_3 L_n^{m-1}(x)^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.071 (sec), leaf count = 39

$$\left\{ y(x) = \_C1 \left( M(-n, m, x) \right)^2 + \_C2 \left( U(-n, m, x) \right)^2 + \_C3 M(-n, m, x) U(-n, m, x) \right\}$$

2.1493 ODE No. 1493

---

$$-f(x) + x^2y^{(3)}(x) + (x^2 + 2) y'(x) + 4xy''(x) + 3xy(x) = 0$$

✓ **Mathematica** : cpu = 1.16514 (sec), leaf count = 2585

$$\left\{ \left\{ y(x) \rightarrow J_0(x)c_1 + 2Y_0(x)c_2 + \frac{2c_3 {}_1F_2\left(1; \frac{1}{2}, \frac{1}{2}; -\frac{x^2}{4}\right)}{x} + \frac{xJ_0(x) \int_1^x \left(\frac{-16J_1(K[1])Y_0(K[1])^2 f(K[1]) {}_1F_2(3; \frac{5}{2}, \frac{5}{2}; -\frac{1}{4}K[1]^2)}{1}\right)}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.183 (sec), leaf count = 1033

$$\left\{ y(x) = \frac{1}{x} \left( \int -9 \frac{((-9x^2J_0(x) - 36xJ_1(x) + 27J_0(x)) {}_1F_2(1; 1/2, 1/2; -1/4x^2) + 4x^2(9/4J_0(x)(x^2+9)) {}_1F_2(1; 1/2, 1/2; -1/4x^2)}{(((-9x^2J_0(x) - 36xJ_1(x) + 27J_0(x)) {}_1F_2(1; 1/2, 1/2; -1/4x^2) + 4x^2(9/4J_0(x)(x^2+9)) {}_1F_2(1; 1/2, 1/2; -1/4x^2))} \right) \right\}$$

2.1494 ODE No. 1494

---

$$x^2y^{(3)}(x) + 5xy''(x) + 4y'(x) - \log(x) = 0$$

✓ **Mathematica** : cpu = 0.0198399 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow -\frac{c_1}{x} - \frac{2c_2}{x} - \frac{2c_2 \log(x)}{x} + c_3 - \frac{x}{2} + \frac{1}{4}x \log(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 32

$$\left\{ y(x) = \frac{(x^2 + 4\_C2) \ln(x) - 2x^2 + 4\_C1 x + 4\_C3}{4x} \right\}$$

2.1495 ODE No. 1495

---

$$x^2y^{(3)}(x) + 6xy''(x) + 6y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0059515 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow -\frac{c_1}{2x^2} - \frac{c_2}{x} + c_3 \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 16

$$\left\{ y(x) = \_C1 + \frac{C2}{x} + \frac{C3}{x^2} \right\}$$

2.1496 ODE No. 1496

---

$$ax^2y(x) + x^2y^{(3)}(x) + 6xy''(x) + 6y'(x) = 0$$

✓ **Mathematica** : cpu = 0.2048 (sec), leaf count = 63

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-\sqrt[3]{a}x}}{x^2} + \frac{c_2 e^{\sqrt[3]{-1}\sqrt[3]{a}x}}{x^2} + \frac{c_3 e^{-(-1)^{2/3}\sqrt[3]{a}x}}{x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 57

$$\left\{ y(x) = \frac{1}{x^2} \left( \_C1 e^{\frac{(i\sqrt{3}-1)x}{2}\sqrt[3]{-a}} + \_C2 e^{-\frac{(i\sqrt{3}+1)x}{2}\sqrt[3]{-a}} + \_C3 e^{\sqrt[3]{-a}x} \right) \right\}$$

2.1497 ODE No. 1497

---

$$-3x(p+q)y''(x)+3p(3q+1)y'(x)+x^2y^{(3)}(x)+x^2(-y(x))=0$$

✓ **Mathematica** : cpu = 0.365762 (sec), leaf count = 135

$$\left\{\left\{y(x)\rightarrow c_1{}_0F_2\left(\frac{2}{3}-p,\frac{1}{3}-q;\frac{x^3}{27}\right)+c_2(-1)^{\frac{1}{3}(3p+1)}3^{-3p-1}x^{3p+1}{}_0F_2\left(p+\frac{4}{3},p-q+\frac{2}{3};\frac{x^3}{27}\right)+c_3(-1)^{\frac{1}{3}(3q+2)}\right.\right.$$

✓ **Maple** : cpu = 0.152 (sec), leaf count = 77

$$\left\{y(x)=-C1{}_0F_2\left(-p+\frac{2}{3},-q+\frac{1}{3};\frac{x^3}{27}\right)+-C2x^{1+3p}{}_0F_2\left(p+\frac{4}{3},\frac{2}{3}-q+p;\frac{x^3}{27}\right)+-C3x^{2+3q}{}_0F_2\left(q+\frac{5}{3},\right.\right.$$

2.1498 ODE No. 1498

---

$$(ax^2+6n)y'(x)-2axy(x)-2(n+1)xy''(x)+x^2y^{(3)}(x)=0$$

✓ **Mathematica** : cpu = 9.6336 (sec), leaf count = 584

$$\left\{\left\{y(x)\rightarrow -\frac{\pi c_32^{-n-\frac{3}{2}}x(\sqrt{a}x)^{-n-\frac{1}{2}}\left(-a^{3/2}2^{2n}x^3\sec(\pi n)\Gamma\left(\frac{3}{2}-n\right)\Gamma\left(n+\frac{3}{2}\right)J_{\frac{1}{2}(2n+1)}(\sqrt{a}x){}_1\tilde{F}_2\left(\frac{3}{2}-n;\frac{1}{2}-n\right)\right.\right.\right.$$

✓ **Maple** : cpu = 0.141 (sec), leaf count = 53

$$\left\{y(x)=-C1x^{n+\frac{1}{2}}J_{-n-\frac{1}{2}}(\sqrt{a}x)+-C2x^{n+\frac{1}{2}}Y_{-n-\frac{1}{2}}(\sqrt{a}x)+-C3(ax^2+4n-2)\right\}$$

2.1499 ODE No. 1499

---

$$-\left(\nu^2+x^2-\frac{1}{4}\right)y'(x)+\left(\nu^2+x^2-2x-\frac{1}{4}\right)y(x)+x^2y^{(3)}(x)-(x^2-2x)y''(x)=0$$

✓ **Mathematica** : cpu = 0.19394 (sec), leaf count = 97

$$\left\{\left\{y(x)\rightarrow \frac{c_3e^xx^{\nu+\frac{1}{2}}\Gamma\left(\nu+\frac{1}{2}\right){}_1\tilde{F}_1\left(\nu+\frac{1}{2};2\nu+1;-2x\right)}{\Gamma\left(\frac{3}{2}-\nu\right)}+c_22^{-\nu-\frac{1}{2}}e^xG_{2,3}^{2,1}\left(2x\left|\begin{array}{c}1,0\\\frac{1}{2}-\nu,\nu+\frac{1}{2},0\end{array}\right.\right)+c_1e^x\right\}\right\}$$

✓ **Maple** : cpu = 0.146 (sec), leaf count = 25

$$\left\{y(x)=-C1e^x+-C2\sqrt{x}I_\nu(x)+-C3\sqrt{x}K_\nu(x)\right\}$$

2.1500 ODE No. 1500

---

$$\nu(2x+1)y'(x)-\nu(x+1)y(x)-x(v+x)y''(x)+x^2y^{(3)}(x)=0$$

✗ **Mathematica** : cpu = 32.0597 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{\left\{y(x)\rightarrow \text{DifferentialRoot}\left(\{y,x\},\left\{y^{(3)}(x)x^2-(x+v)y''(x)x-(x+1)\nu y(x)+(2x\nu+\nu)y'(x)=0,y(1)=c_1,\right.\right.\right.\right.$$

✓ **Maple** : cpu = 0.126 (sec), leaf count = 55

$$\left\{y(x)=-C1e^x+-C2x^{\frac{\nu}{2}+\frac{1}{2}}J_{-\nu-1}\left(2\sqrt{\nu}\sqrt{x}\right)+-C3x^{\frac{\nu}{2}+\frac{1}{2}}Y_{-\nu-1}\left(2\sqrt{\nu}\sqrt{x}\right)\right\}$$

2.1501 ODE No. 1501

---

$$\left(-\nu^2+x^2-2x+\frac{1}{4}\right)y'(x)+\left(\nu^2-\frac{1}{4}\right)y(x)+x^2y^{(3)}(x)-2(x^2-x)y''(x)=0$$

✓ **Mathematica** : cpu = 0.160389 (sec), leaf count = 86

$$\left\{\left\{y(x)\rightarrow \frac{c_3e^xx^{\nu+\frac{1}{2}}\Gamma\left(\nu+\frac{1}{2}\right){}_1\tilde{F}_1\left(\nu+\frac{1}{2};2\nu+1;-x\right)}{\Gamma\left(\frac{3}{2}-\nu\right)}+c_2e^xG_{2,3}^{2,1}\left(x\left|\begin{array}{c}1,0\\\frac{1}{2}-\nu,\nu+\frac{1}{2},0\end{array}\right.\right)+c_1e^x\right\}\right\}$$

✓ **Maple** : cpu = 0.127 (sec), leaf count = 37

$$\left\{y(x)=-C1e^x+-C2e^{\frac{x}{2}}\sqrt{x}I_\nu\left(\frac{x}{2}\right)+-C3e^{\frac{x}{2}}\sqrt{x}K_\nu\left(\frac{x}{2}\right)\right\}$$

2.1502 ODE No. 1502

---

$$-\left(x^4-6x\right)y''(x)-\left(2x^3-6\right)y'(x)+x^2y^{(3)}(x)+2x^2y(x)=0$$

✓ **Mathematica** : cpu = 0.0589441 (sec), leaf count = 98

$$\left\{\left\{y(x)\rightarrow \frac{c_2\Gamma\left(\frac{1}{3}\right){}_2F_2\left(-\frac{2}{3},\frac{1}{3};\frac{2}{3},\frac{4}{3};\frac{x^3}{3}\right)}{3x\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt[3]{-\frac{1}{3}}c_3\Gamma\left(\frac{2}{3}\right){}_2F_2\left(-\frac{1}{3},\frac{2}{3};\frac{4}{3},\frac{5}{3};\frac{x^3}{3}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{c_1}{x^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.27 (sec), leaf count = 103

$$\left\{y(x)=\frac{1}{x^2}\left(-C2\int e^{\frac{x^3}{6}}\sqrt{x}\left(I_{\frac{1}{6}}\left(-\frac{x^3}{6}\right)x^3+I_{-\frac{5}{6}}\left(-\frac{x^3}{6}\right)x^3-2I_{1/6}\left(-1/6x^3\right)\right)dx+_C3\int e^{\frac{x^3}{6}}\sqrt{x}\left(K_{\frac{1}{6}}\left(-\frac{x^3}{6}\right)\right)dx\right)\right\}$$

2.1503 ODE No. 1503

---

$$\left(x^2+1\right)y^{(3)}(x)+\frac{1}{x^2}+8xy''(x)+10y'(x)-2\log(x)-3=0$$

✓ **Mathematica** : cpu = 0.470428 (sec), leaf count = 258

$$\left\{\left\{y(x)\rightarrow c_3+\frac{1}{225}\left(-\frac{225c_2x}{x^2+1}-\frac{150c_2x}{\left(x^2+1\right)^2}-\frac{225c_1}{4\left(x^2+1\right)^2}+\frac{75c_2}{x-i}+\frac{75c_2}{x+i}+\frac{225}{2}ic_2\log\left(1-ix\right)-\frac{225}{2}ic_2\log\left(1+ix\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 67

$$\left\{y(x)=\frac{\left(45x^5+150x^3+225x\right)\ln(x)-9x^5+225\_C1x^4+\left(225\_C2-50\right)x^3+450\_C1x^2+\left(675\_C2-450\_C1\right)x+_C3}{225\left(x^2+1\right)^2}\right\}$$

2.1504 ODE No. 1504

---

$$\left(x^2+2\right)y^{(3)}(x)+\left(x^2+2\right)y'(x)-2xy''(x)-2xy(x)=0$$

✓ **Mathematica** : cpu = 0.10529 (sec), leaf count = 43

$$\left\{\left\{y(x)\rightarrow \frac{c_1x^2}{2}+\frac{1}{2}ic_2e^{-ix}-\frac{1}{4}c_3e^{ix}\right\}\right\}$$

✓ **Maple** : cpu = 0.11 (sec), leaf count = 18

$$\left\{y(x)=\_C1x^2+_C2\cos(x)+\_C3\sin(x)\right\}$$

2.1505 ODE No. 1505

---

$$\left(2ax+b\right)y'(x)+ay(x)+2\left(x-1\right)xy^{(3)}(x)+3\left(2x-1\right)y''(x)=0$$

✗ **Mathematica** : cpu = 61.5046 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{\left\{y(x)\rightarrow \text{DifferentialRoot}\left(\left\{y,x\right\},\left\{ay(x)+\left(2xa+b\right)y'(x)+\left(6x-3\right)y''(x)+2\left(x-1\right)xy^{(3)}(x)=0,y(2)=c_1,y'(2)=c_2\right\}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.087 (sec), leaf count = 79

$$\left\{y(x)=\_C1\left(\text{MathieuC}\left(-\frac{a}{2}-\frac{b}{2}+1,\frac{a}{4},\arccos(\sqrt{x})\right)\right)^2+_C2\left(\text{MathieuS}\left(-\frac{a}{2}-\frac{b}{2}+1,\frac{a}{4},\arccos(\sqrt{x})\right)\right)^2\right\}$$

2.1506 ODE No. 1506

---

$$4x^2y^{(3)}(x)+\left(x^2+14x-1\right)y''(x)+4\left(x+1\right)y'(x)+2y(x)=0$$

✓ **Mathematica** : cpu = 0.291299 (sec), leaf count = 150

$$\left\{\left\{y(x)\rightarrow c_2e^{\frac{1}{4}\left(-x-\frac{1}{x}+2\log(x)\right)}\int_1^xe^{\frac{K[1]^2-10\log(K[1])K[1]+1}{4K[1]}}dK[1]-\sqrt{\pi}c_3\left(\text{erfi}\left(\frac{1-x}{2\sqrt{x}}\right)+\text{erfi}\left(\frac{x+1}{2\sqrt{x}}\right)-i(e-1)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 43

$$\left\{y(x)=\left(-C3+\int\frac{2\_C1x+_C2}{4}e^{\frac{x}{4}}e^{\frac{1}{4x}}x^{-\frac{5}{2}}dx\right)e^{-\frac{x}{4}}e^{-\frac{1}{4x}}\sqrt{x}\right\}$$

2.1507 ODE No. 1507

---

$$xy^{(3)}(x)(ax+b) + (\alpha x + \beta)y''(x) - f(x) + xy'(x) + y(x) = 0$$

✗ **Mathematica** : cpu = 2.03112 (sec), leaf count = 0 , could not solve  
DSolve[-f[x] + y[x] + x\*Derivative[1][y][x] + (beta + alpha\*x)\*Derivative[2][y][x] + x\*(b +  
✓ **Maple** : cpu = 0.415 (sec), leaf count = 1209

$$\left\{y(x) = -\left(HeunC\left(0, \frac{2b-\beta}{b}, \frac{(2b+\beta)a-\alpha b}{ab}, -\frac{b}{a^2}, \frac{(4a-\alpha)b^2-\alpha\beta b+a\beta^2}{2ab^2}, -\frac{ax}{b}\right)\left(\int(-C1 + \int f(x) dx\right) + C1\right)\right\}$$

2.1508 ODE No. 1508

---

$$y(x)\left(ax^3+\nu^2-1\right)+\left(1-\nu^2\right)xy'(x)+x^3y^{(3)}(x)=0$$

✓ **Mathematica** : cpu = 0.638743 (sec), leaf count = 143

$$\left\{\left\{y(x) \rightarrow c_2 3^{\nu-1} a^{\frac{1-\nu}{3}} x^{1-\nu} {}_0F_2\left(\left;; 1-\frac{2\nu}{3}, 1-\frac{\nu}{3}; -\frac{ax^3}{27}\right) + c_3 3^{-\nu-1} a^{\frac{\nu+1}{3}} x^{\nu+1} {}_0F_2\left(\left;; \frac{\nu}{3}+1, \frac{2\nu}{3}+1; -\frac{ax^3}{27}\right) + \frac{1}{3}\right\}\right\}$$

✓ **Maple** : cpu = 0.078 (sec), leaf count = 81

$$\left\{y(x) = {}_0F_2\left(\left;; -\frac{\nu}{3}+1, \frac{\nu}{3}+1; -\frac{ax^3}{27}\right) + {}_0F_2\left(\left;; 1-\frac{2\nu}{3}, -\frac{\nu}{3}+1; -\frac{ax^3}{27}\right) + {}_0F_2\left(\left;; \frac{2\nu}{3}+1, \frac{\nu}{3}+1; -\frac{ax^3}{27}\right)\right\}$$

2.1509 ODE No. 1509

---

$$\left(\left(1-4\nu^2\right)x+4x^3\right)y'(x)+\left(4\nu^2-1\right)y(x)+x^3y^{(3)}(x)=0$$

✓ **Mathematica** : cpu = 0.0081141 (sec), leaf count = 34

$$\left\{\left\{y(x) \rightarrow c_1 x J_\nu(x)^2 + c_3 x Y_\nu(x)^2 + c_2 x J_\nu(x) Y_\nu(x)\right\}\right\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 29

$$\left\{y(x) = x\left(\left(Y_\nu(x)\right)^2 {}_0F_2\left(\left;; -\frac{\nu}{3}+1, \frac{\nu}{3}+1; -\frac{ax^3}{27}\right) + Y_\nu(x) J_\nu(x) {}_0F_2\left(\left;; 1-\frac{2\nu}{3}, -\frac{\nu}{3}+1; -\frac{ax^3}{27}\right) + \left(J_\nu(x)\right)^2 {}_0F_2\left(\left;; \frac{2\nu}{3}+1, \frac{\nu}{3}+1; -\frac{ax^3}{27}\right)\right)\right\}$$

2.1510 ODE No. 1510

---

$$y(x)\left(a\left(\nu-1\right)x^{2\nu}+bx^{3\nu}+\nu^2-1\right)+x\left(ax^{2\nu}-\nu^2+1\right)y'(x)+x^3y^{(3)}(x)=0$$

✗ **Mathematica** : cpu = 0.0603342 (sec), leaf count = 0 , could not solve  
DSolve[(-1 + nu^2 + a\*(-1 + nu)\*x^(2\*nu) + b\*x^(3\*nu))\*y[x] + x\*(1 - nu^2 + a\*x^(2\*nu))\*Derivative[1][y][x] + x^3\*Derivative[3][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x) = DESol\left(\left\{x^3 \frac{d^3}{dx^3} {}_0F_2\left(\left;; -\frac{\nu}{3}+1, \frac{\nu}{3}+1; -\frac{ax^3}{27}\right) + \left(x^{2\nu}ax - \nu^2x + x\right) \frac{d}{dx} {}_0F_2\left(\left;; 1-\frac{2\nu}{3}, -\frac{\nu}{3}+1; -\frac{ax^3}{27}\right) + \left(x^{2\nu}a\nu - ax^{2\nu} + bx^{3\nu} + \nu^2 - 1\right) {}_0F_2\left(\left;; \frac{2\nu}{3}+1, \frac{\nu}{3}+1; -\frac{ax^3}{27}\right)\right\}\right)\right\}$$

2.1511 ODE No. 1511

---

$$x^3y^{(3)}(x) + (x+8)x^3 - 6(x-1)x^3\log(x) + 3x^2y''(x) - 2xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0491939 (sec), leaf count = 51

$$\left\{\left\{y(x) \rightarrow \frac{c_1}{x^2} + c_2x + c_3x\log(x) + \frac{1}{450}\left(-50x^4 + 50x^4\log(x) - 18x^3 - 135x^3\log(x)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 49

$$\left\{y(x) = \frac{\left(50x^6 - 135x^5 + 450 {}_0F_3\left(\left;; -\frac{\nu}{3}+1, \frac{\nu}{3}+1; -\frac{ax^3}{27}\right)x^3 + 450 {}_0F_2\left(\left;; 1-\frac{2\nu}{3}, -\frac{\nu}{3}+1; -\frac{ax^3}{27}\right)x^3 + 450 {}_0F_2\left(\left;; \frac{2\nu}{3}+1, \frac{\nu}{3}+1; -\frac{ax^3}{27}\right)x^3\right)\ln(x) - 50x^6 - 18x^5 + 450 {}_0F_1\left(\left;; -\frac{\nu}{3}+1, \frac{\nu}{3}+1; -\frac{ax^3}{27}\right)x^3 + 450 {}_0F_1\left(\left;; 1-\frac{2\nu}{3}, -\frac{\nu}{3}+1; -\frac{ax^3}{27}\right)x^3 + 450 {}_0F_1\left(\left;; \frac{2\nu}{3}+1, \frac{\nu}{3}+1; -\frac{ax^3}{27}\right)x^3\right)}{450x^2}\right\}$$

2.1512 ODE No. 1512

---

$$(1 - a^2) x y'(x) + x^3 y^{(3)}(x) + 3 x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0328694 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow -\frac{c_1 x^{-a}}{a} + \frac{c_2 x^a}{a} + c_3 \right\} \right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 18

$$\left\{ y(x) = \_C1 + x^a \_C2 + \_C3 x^{-a} \right\}$$

2.1513 ODE No. 1513

---

$$x^3 y^{(3)}(x) - 4 x^2 y''(x) + (x^2 + 8) x y'(x) - 2 (x^2 + 4) y(x) = 0$$

✓ **Mathematica** : cpu = 0.0767326 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow c_1 x^2 - c_2 x \sin(x) + c_3 x \cos(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.117 (sec), leaf count = 18

$$\left\{ y(x) = x (\cos(x) \_C3 + \sin(x) \_C2 + \_C1 x) \right\}$$

2.1514 ODE No. 1514

---

$$(a x^3 - 12) y(x) + x^3 y^{(3)}(x) + 6 x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.550514 (sec), leaf count = 102

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-\sqrt[3]{a} x} (\sqrt[3]{a} x + 2)}{x^3} + \frac{c_2 e^{\sqrt[3]{-1} \sqrt[3]{a} x} (\sqrt[3]{a} x + 2 (-1)^{2/3})}{x^3} + \frac{c_3 e^{-(-1)^{2/3} \sqrt[3]{a} x} (\sqrt[3]{a} x - 2 \sqrt[3]{-1})}{x^3} \right\} \right\}$$

✓ **Maple** : cpu = 0.287 (sec), leaf count = 132

$$\left\{ y(x) = \frac{1}{x^3} \left( -\_C2 \left( (-i - \sqrt{3}) (-a^4)^{\frac{2}{3}} + i a^3 x \right) e^{\frac{-\frac{i}{2} (\sqrt{3} - i) x}{a} \sqrt[3]{-a^4}} + \left( (\sqrt{3} - i) (-a^4)^{\frac{2}{3}} + i a^3 x \right) \_C3 e^{\frac{i}{2} (\sqrt{3} + i) x}{a} \right) \right\}$$

2.1515 ODE No. 1515

---

$$y(x) \left( a \left( 4 c^2 \nu^2 - a^2 \right) + 4 b^2 c^2 (c - a) x^{2c} \right) + y'(x) \left( 3 (a - 1) a x + 4 b^2 c^2 x^{2c+1} - 4 c^2 \nu^2 + 1 \right) + 3 (1 - a) x^2 y''(x) + x^3 y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.144626 (sec), leaf count = 0 , could not solve

`DSolve[(a*(-a^2 + 4*c^2*nu^2) + 4*b^2*c^2*(-a + c)*x^(2*c))*y[x] + (1 - 4*c^2*nu^2 + 3*(-1 + a)*a*x + 4*b^2*c^2*x^(1 + 2*c))*Derivative[1][y][x] + 3*(1 - a)*x^2*Derivative[2][y][x] + x^3*Derivative[3][y][x] == 0,x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ x^3 \frac{d^3}{dx^3} \_Y(x) + (-3 a x^2 + 3 x^2) \frac{d^2}{dx^2} \_Y(x) + (4 b^2 c^2 x^{2c+1} - 4 \nu^2 c^2 + 3 a^2 x - 3 a x + 1) \frac{d}{dx} \_Y(x) + \_Y(x) \right\} \right) \right\}$$

2.1516 ODE No. 1516

---

$$x^3 y^{(3)}(x) + (x + 3) x^2 y''(x) + 5 (x - 6) x y'(x) + (4 x + 30) y(x) = 0$$

✗ **Mathematica** : cpu = 304.153 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.263 (sec), leaf count = 188

$$\left\{ y(x) = -\_C3 e^{-x} (x^8 + 28 x^7 + 450 x^6 + 5100 x^5 + 42900 x^4 + 267120 x^3 + 1179360 x^2 + 3326400 x + 4536000) \right\}$$

2.1517 ODE No. 1517

---

$$x^3y^{(3)}(x) - 2x^3 + x^2y''(x) + 2xy'(x) - y(x) + \log(x) = 0$$

✓ **Mathematica** : cpu = 0.302559 (sec), leaf count = 30686

Too large to display

✓ **Maple** : cpu = 0.326 (sec), leaf count = 866

$$\left\{ y(x) = - \int - \frac{\sqrt[3]{44 + 12 \sqrt{69}} \left( 3 \sqrt{69} \sqrt[3]{44 + 12 \sqrt{69}} - 11 \sqrt[3]{44 + 12 \sqrt{69}} + 100 \right) \left( x^{\frac{(11 - 3 \sqrt{69})(44 + 12 \sqrt{69})}{1200} \frac{2}{3}} + \frac{\sqrt[3]{44 + 12 \sqrt{69}}}{12} \right)}{13800 x^3} \right.$$

2.1518 ODE No. 1518

---

$$x \left( x^2 + 1 \right) y^{(3)}(x) + 3 \left( 2x^2 + 1 \right) y''(x) - 12y(x) = 0$$

✓ **Mathematica** : cpu = 0.246408 (sec), leaf count = 104

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{3} c_1 \left( 2x^2 + 1 \right) + \frac{1}{3} c_2 x \sqrt{x^2 + 1} - \frac{c_3 \left( 2x^2 + 1 \right) \left( 3 \left( x^4 + x^2 \right) \tanh^{-1} \left( \sqrt{x^2 + 1} \right) - \sqrt{x^2 + 1} \left( 3x^2 + 1 \right) \right)}{6 \sqrt{x^2 + 1} \left( 2x^3 + x \right)} \right\} \right.$$

✓ **Maple** : cpu = 0.24 (sec), leaf count = 60

$$\left\{ y(x) = \frac{1}{x} \left( 3 \sqrt{x^2 + 1} \operatorname{Artanh} \left( \frac{1}{\sqrt{x^2 + 1}} \right) - C2 x^2 + \_C1 x^2 \sqrt{x^2 + 1} + 2 \_C3 x^3 - 3 \_C2 x^2 + \_C3 x - \_C2 \right) \right.$$

2.1519 ODE No. 1519

---

$$(x + 3)x^2y^{(3)}(x) - 3(x + 2)xy''(x) + 6(x + 1)y'(x) - 6y(x) = 0$$

✓ **Mathematica** : cpu = 0.0233223 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} c_1 \left( x^3 - 3x^2 + 3x + 3 \right) + \frac{1}{2} c_2 \left( -x^3 + 3x^2 - x - 1 \right) + \frac{1}{8} c_3 \left( 3x^3 - 5x^2 + x + 1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 19

$$\left\{ y(x) = \_C2 x^3 + \_C1 x^2 + \_C3 x + \_C3 \right\}$$

2.1520 ODE No. 1520

---

$$y''(x) \left( -6x(a1 + a2 + a3) + 3a1a2 + 3a1a3 + 3a2a3 + 9x^2 \right) + 2(x - a1)(x - a2)(x - a3)y^{(3)}(x) - 2 \left( b + (n^2 + n - 3) \right)$$

✗ **Mathematica** : cpu = 68.0512 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \operatorname{DifferentialRoot} \left( \{y, x\}, \left\{ -n(n + 1)y(x) - 2 \left( xn^2 + xn - 3x + b \right) y'(x) + 3 \left( 3x^2 - 2a1x - 2a2x - 2a3 \right) \right\} \right) \right\} \right.$$

✓ **Maple** : cpu = 0.298 (sec), leaf count = 288

$$\left\{ y(x) = - \_C2 (x - a1) \left( \operatorname{HeunG} \left( \frac{-a3 + a1}{-a2 + a1}, \frac{(-n^2 - n + 3) a1 - b}{-4 a2 + 4 a1}, 1 + \frac{n}{2}, -\frac{n}{2} + \frac{1}{2}, \frac{3}{2}, \frac{1}{2}, \frac{-x + a1}{-a2 + a1} \right) \right)^2 + \right.$$

2.1521 ODE No. 1521

---

$$(x + 1)x^3y^{(3)}(x) - (4x + 2)x^2y''(x) + (10x + 4)xy'(x) - 4(3x + 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0487205 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow c_1 x^2 + c_3 x^2 \left( x + \frac{1}{x} + \log^2(x) \right) + c_2 x^2 \log(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.237 (sec), leaf count = 28

$$\left\{ y(x) = x \left( (\ln(x))^2 \_C3 x + \_C2 x \ln(x) + \_C3 x^2 + \_C1 x + \_C3 \right) \right\}$$

2.1522 ODE No. 1522

---

$$4x^4y^{(3)}(x) - 4x^3y''(x) + 4x^2y'(x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.0136424 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 x^2}{2} - \frac{c_2 x^2}{4} + \frac{1}{2} c_2 x^2 \log(x) + c_3 - \frac{1}{36 x} \right\} \right\}$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 34

$$\left\{ y(x) = \frac{18 x^3 \_C1 \ln(x) - 1 + (-9 \_C1 + 18 \_C2) x^3 + 36 \_C3 x}{36 x} \right\}$$

2.1523 ODE No. 1523

---

$$-(4x^2 + 2) x^2 y''(x) + (10x^2 + 4) x y'(x) - 4 (3x^2 + 1) y(x) + (x^2 + 1) x^3 y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.280317 (sec), leaf count = 74

$$\left\{ \left\{ y(x) \rightarrow c_1 (-x^3 + 3x^2 - x) + \frac{1}{2} c_2 (x^3 - 2x^2 + x) - \frac{c_3 x (-x^3 + 3x^2 - x) (\log(x) + 1)}{2 (x^2 - 3x + 1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.231 (sec), leaf count = 23

$$\{y(x) = x (\ln(x) \_C2 x + \_C3 x^2 + (\_C1 + \_C2) x + \_C3)\}$$

2.1524 ODE No. 1524

---

$$x^6 y^{(3)}(x) + x^2 y''(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.133262 (sec), leaf count = 96

$$\left\{ \left\{ y(x) \rightarrow -\frac{(-\frac{1}{3})^{2/3} c_2 x \Gamma(\frac{1}{3}) {}_2F_2(-\frac{2}{3}, \frac{1}{3}; \frac{2}{3}, \frac{4}{3}; \frac{1}{3x^3})}{3 \Gamma(\frac{4}{3})} + \frac{c_3 \Gamma(\frac{2}{3}) {}_2F_2(-\frac{1}{3}, \frac{2}{3}; \frac{4}{3}, \frac{5}{3}; \frac{1}{3x^3})}{9 \Gamma(\frac{5}{3})} + c_1 x^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.302 (sec), leaf count = 98

$$\left\{ y(x) = x^2 \left( \int 1 e^{\frac{1}{6 x^3}} \left( 2 x^3 I_{1/6}(-1/6 x^{-3}) - I_{\frac{1}{6}}\left(-\frac{1}{6 x^3}\right) - I_{-\frac{5}{6}}\left(-\frac{1}{6 x^3}\right) \right) x^{-\frac{11}{2}} dx \_C2 + \int 1 e^{\frac{1}{6 x^3}} \left( 2 x^3 K_{1/6} \right. \right. \right.$$

2.1525 ODE No. 1525

---

$$ay(x) + x^6 y^{(3)}(x) + 6x^5 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.341388 (sec), leaf count = 102

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{\sqrt[3]{a}}{x}} (2x - \sqrt[3]{a}) + c_2 e^{\frac{(-1)^{2/3} \sqrt[3]{a}}{x}} \left( x - \frac{1}{2} (-1)^{2/3} \sqrt[3]{a} \right) + c_3 e^{-\frac{\sqrt[3]{-1} \sqrt[3]{a}}{x}} \left( \frac{1}{2} \sqrt[3]{-1} \sqrt[3]{a} + x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.363 (sec), leaf count = 287

$$\left\{ y(x) = \_C1 (-8 x^3 + a)^4 e^{-\frac{1}{ax} \sqrt[3]{-a^4}} \left( 2 a x + \sqrt[3]{-a^4} \right)^{-3} \left( 4 a^2 x^2 - 2 x \sqrt[3]{-a^4} a + (-a^4)^{\frac{2}{3}} \right)^{-4} + \_C2 (-8 x^3 + a)^4 e^{-\frac{1}{ax} \sqrt[3]{-a^4}} \left( 2 a x + \sqrt[3]{-a^4} \right)^{-3} \left( 4 a^2 x^2 - 2 x \sqrt[3]{-a^4} a + (-a^4)^{\frac{2}{3}} \right)^{-4} + \_C3 (-8 x^3 + a)^4 e^{-\frac{1}{ax} \sqrt[3]{-a^4}} \left( 2 a x + \sqrt[3]{-a^4} \right)^{-3} \left( 4 a^2 x^2 - 2 x \sqrt[3]{-a^4} a + (-a^4)^{\frac{2}{3}} \right)^{-4} \right\}$$

2.1526 ODE No. 1526

---

$$(x^4 + 2x^2 + 2x + 1) x^2 y^{(3)}(x) - (2x^6 + 3x^4 - 6x^2 - 6x - 1) y''(x) + (x^6 - 6x^3 - 15x^2 - 12x - 2) y'(x) + (x^4 + 4x^3 + 4x^2 + 4x + 1) y(x) = 0$$

✗ **Mathematica** : cpu = 300.113 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.131 (sec), leaf count = 19

$$\left\{ y(x) = \_C2 e^{x^{-1}} + e^x (\_C3 x + \_C1) \right\}$$

2.1527 ODE No. 1527

---

$$(x-a)^3(x-b)^3y^{(3)}(x)-cy(x)=0$$

✗ **Mathematica** : cpu = 133.251 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{\left\{y(x)\rightarrow \text{DifferentialRoot}\left(\left\{y,x\right\},\left\{\left(a-x\right)^3\left(b-x\right)^3y^{(3)}(x)-cy(x)=0,y(0)=c_1,y'(0)=c_2,y''(0)=c_3\right\}\right)(x)\right.\right.$$

✓ **Maple** : cpu = 0.337 (sec), leaf count = 437

$$\left\{y\left(x\right)=\left(x-a\right)^{-2\frac{b}{a-b}}\left(x-b\right)^{2\frac{a}{a-b}}\left(\left(b-x\right)^{-\frac{\text{RootOf}\left(-Z^3+(-3\,a-3\,b)\_Z^2+\left(2\,a^2+8\,ab+2\,b^2\right)\_Z-4\,a^2b-4\,ab^2-c,\text{index}=1\right)}{a-b}}\left(a-x\right)^{\frac{\text{RootOf}\left(-Z^3+(-3\,a-3\,b)\_Z^2+\left(2\,a^2+8\,ab+2\,b^2\right)\_Z-4\,a^2b-4\,ab^2-c,\text{index}=1\right)}{a-b}}\right.\right.$$

2.1528 ODE No. 1528

---

$$y^{(3)}(x)\sin(x)+(2\cos(x)+1)y''(x)-\sin(x)y'(x)-\cos(x)=0$$

✓ **Mathematica** : cpu = 0.886252 (sec), leaf count = 72

$$\left\{\left\{y(x)\rightarrow \frac{\sin\left(\frac{x}{2}\right)\left(\sqrt{2}\left(c_2x\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\left(c_2\log(2(\cos(x)+1))+2c_1\right)\right)-2\cos\left(\frac{x}{2}\right)\sin^{-1}(\cos(x))\right)}{\cos(x)-1}+c_3\right\}\right\}$$

✓ **Maple** : cpu = 0.125 (sec), leaf count = 71

$$\left\{y\left(x\right)=\frac{1}{\sin\left(x\right)\left(\cos\left(x\right)-1\right)}\left(\left(\sin\left(x\right)\right)^2\ln\left(\frac{-\cos\left(x\right)+1}{\sin\left(x\right)}\right)\_C1-\ln\left(\sin\left(x\right)\right)\left(\sin\left(x\right)\right)^2\_C1+\left(\sin\left(x\right)\right)^2\_C3-\right.$$

2.1529 ODE No. 1529

---

$$y^{(3)}(x)(x+\sin(x))+3(\cos(x)+1)y''(x)-3\sin(x)y'(x)-y(x)\cos(x)+\sin(x)=0$$

✗ **Mathematica** : cpu = 0.0522392 (sec), leaf count = 0 , could not solve

`DSolve[Sin[x] - Cos[x]*y[x] - 3*Sin[x]*Derivative[1][y][x] + 3*(1 + Cos[x])*Derivative[2][y][x]`

✓ **Maple** : cpu = 0.046 (sec), leaf count = 25

$$\left\{y\left(x\right)=\frac{\_C3+\_C1\,x^2+\_C2\,x-\cos\left(x\right)}{\sin\left(x\right)+x}\right\}$$

2.1530 ODE No. 1530

---

$$y'(x)\left(4\nu(\nu+1)\sin^2(x)+\cos(2x)\right)+2\nu(\nu+1)y(x)\sin(2x)+y^{(3)}(x)\sin^2(x)+3\sin(x)\cos(x)y''(x)=0$$

✗ **Mathematica** : cpu = 0.1084 (sec), leaf count = 0 , could not solve

`DSolve[2*nu*(1 + nu)*Sin[2*x]*y[x] + (Cos[2*x] + 4*nu*(1 + nu)*Sin[x]^2)*Derivative[1][y][x]`

✓ **Maple** : cpu = 0.153 (sec), leaf count = 113

$$\left\{y\left(x\right)=\_C1\left({}_2F_1\left(-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{2};\frac{1}{2};\frac{\cos\left(2x\right)}{2}+\frac{1}{2}\right)\right)^2+\_C2\left(\cos\left(2x\right)+1\right)\left({}_2F_1\left(1+\frac{\nu}{2},\frac{1}{2}-\frac{\nu}{2};\frac{3}{2};\frac{\cos\left(2x\right)}{2}+\right.\right.$$

2.1531 ODE No. 1531

---

$$A(x)\left(f(x)y''(x)+g(x)y'(x)+h(x)y(x)\right)+f'(x)y''(x)+f(x)y^{(3)}(x)+g'(x)y'(x)+g(x)y''(x)+y(x)h'(x)+h(x)y'(x)$$

✗ **Mathematica** : cpu = 0.0208718 (sec), leaf count = 0 , could not solve

`DSolve[y[x]*Derivative[1][h][x] + h[x]*Derivative[1][y][x] + Derivative[1][g][x]*Derivative[1][y][x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y\left(x\right)=DESol\left(\left\{f\left(x\right)\frac{\mathrm{d}^3}{\mathrm{d}x^3}\_Y\left(x\right)+\left(\frac{\mathrm{d}}{\mathrm{d}x}f\left(x\right)+g\left(x\right)+A\left(x\right)f\left(x\right)\right)\frac{\mathrm{d}^2}{\mathrm{d}x^2}\_Y\left(x\right)+\left(\frac{\mathrm{d}}{\mathrm{d}x}g\left(x\right)+h\left(x\right)+A\left(x\right)\right.\right.\right.$$



**2.1532                    ODE No. 1532**

---

$$ny(x) + y^{(3)}(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0117751 (sec), leaf count = 103

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 x {}_1F_2\left(\frac{n}{3} + \frac{1}{3}; \frac{2}{3}, \frac{4}{3}; -\frac{x^3}{9}\right)}{3^{2/3}} + c_1 {}_1F_2\left(\frac{n}{3}; \frac{1}{3}, \frac{2}{3}; -\frac{x^3}{9}\right) + \frac{c_3 x^2 {}_1F_2\left(\frac{n}{3} + \frac{2}{3}; \frac{4}{3}, \frac{5}{3}; -\frac{x^3}{9}\right)}{3\sqrt[3]{3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 58

$$\left\{ y(x) = {}_C1 {}_1F_2\left(\frac{n}{3}; \frac{1}{3}, \frac{2}{3}; -\frac{x^3}{9}\right) + {}_C2 x {}_1F_2\left(\frac{1}{3} + \frac{n}{3}; \frac{2}{3}, \frac{4}{3}; -\frac{x^3}{9}\right) + {}_C3 x^2 {}_1F_2\left(\frac{2}{3} + \frac{n}{3}; \frac{4}{3}, \frac{5}{3}; -\frac{x^3}{9}\right) \right\}$$

**2.1533                    ODE No. 1533**

---

$$-ny(x) + y^{(3)}(x) - xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0120248 (sec), leaf count = 113

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{-1} c_2 x {}_1F_2\left(\frac{n}{3} + \frac{1}{3}; \frac{2}{3}, \frac{4}{3}; \frac{x^3}{9}\right)}{3^{2/3}} + c_1 {}_1F_2\left(\frac{n}{3}; \frac{1}{3}, \frac{2}{3}; \frac{x^3}{9}\right) + \frac{(-1)^{2/3} c_3 x^2 {}_1F_2\left(\frac{n}{3} + \frac{2}{3}; \frac{4}{3}, \frac{5}{3}; \frac{x^3}{9}\right)}{3\sqrt[3]{3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 58

$$\left\{ y(x) = {}_C1 {}_1F_2\left(\frac{n}{3}; \frac{1}{3}, \frac{2}{3}; \frac{x^3}{9}\right) + {}_C2 x {}_1F_2\left(\frac{1}{3} + \frac{n}{3}; \frac{2}{3}, \frac{4}{3}; \frac{x^3}{9}\right) + {}_C3 x^2 {}_1F_2\left(\frac{2}{3} + \frac{n}{3}; \frac{4}{3}, \frac{5}{3}; \frac{x^3}{9}\right) \right\}$$

**2.1534                    ODE No. 1534**

---

$$y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 0.0036549 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow c_4 x^3 + c_3 x^2 + c_2 x + c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.008 (sec), leaf count = 21

$$\left\{ y(x) = \frac{{}_C1 x^3}{6} + \frac{{}_C2 x^2}{2} + {}_C3 x + {}_C4 \right\}$$

**2.1535                    ODE No. 1535**

---

$$-f(x) + y^{(4)}(x) + 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.348502 (sec), leaf count = 223

$$\left\{ \left\{ y(x) \rightarrow e^{-x} \left( \cos(x) \int_1^x \frac{1}{8} e^{K[1]} f(K[1]) (\cos(K[1]) - \sin(K[1])) (\cos^2(K[1]) + \sin^2(K[1])) dK[1] + \sin(x) \int_1^x \frac{1}{8} \right. \right. \right.$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 36

$$\left\{ y(x) = \frac{f}{4} + {}_C1 e^x \cos(x) + {}_C2 e^x \sin(x) + {}_C3 e^{-x} \cos(x) + {}_C4 e^{-x} \sin(x) \right\}$$

**2.1536                    ODE No. 1536**

---

$$\lambda y(x) + y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 0.0037967 (sec), leaf count = 76

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{(-1)^{3/4} \sqrt[4]{\lambda} x} + c_2 e^{-\sqrt[4]{-1} \sqrt[4]{\lambda} x} + c_3 e^{-(-1)^{3/4} \sqrt[4]{\lambda} x} + c_4 e^{\sqrt[4]{-1} \sqrt[4]{\lambda} x} \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 50

$$\left\{ y(x) = {}_C1 e^{-i \sqrt[4]{-\lambda} x} + {}_C2 e^{i \sqrt[4]{-\lambda} x} + {}_C3 e^{-\sqrt[4]{-\lambda} x} + {}_C4 e^{\sqrt[4]{-\lambda} x} \right\}$$

**2.1537 ODE No. 1537**

---

$$-16e^{x^2}x^4 + y^{(4)}(x) - 12y''(x) + 12y(x) = 0$$

✓ **Mathematica** : cpu = 1.00329 (sec), leaf count = 1722

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{12} e^{-\left(\sqrt{2(3-\sqrt{6})}-x\right)x-\sqrt{2(3+\sqrt{6})}x-\sqrt{2(3-\sqrt{6})}x} \left( -2\sqrt{3+\sqrt{6}} e^{\sqrt{2(3+\sqrt{6})}x+2\sqrt{2(3-\sqrt{6})}x} x^3 + 2\sqrt{3+\sqrt{6}} \right) \right. \right.$$

✓ **Maple** : cpu = 0.09 (sec), leaf count = 67

$$\left\{ y(x) = e^{x^2} + \_C1 e^{\sqrt{6-2}\sqrt{6}x} + \_C2 e^{\sqrt{6+2}\sqrt{6}x} + \_C3 e^{-\sqrt{6-2}\sqrt{6}x} + \_C4 e^{-\sqrt{6+2}\sqrt{6}x} \right\}$$

**2.1538 ODE No. 1538**

---

$$a^4y(x) + 2a^2y''(x) - \cosh(ax) + y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 0.170429 (sec), leaf count = 66

$$\left\{ \left\{ y(x) \rightarrow \frac{\cos^2(ax) \cosh(ax) + \sin^2(ax) \cosh(ax)}{4a^4} + c_3 \sin(ax) + c_4 x \sin(ax) + c_1 \cos(ax) + c_2 x \cos(ax) \right\} \right\}$$

✓ **Maple** : cpu = 0.359 (sec), leaf count = 51

$$\left\{ y(x) = \frac{e^{-ax} + (8\_C3 x + 8\_C1) a^4 \cos(ax) + (8\_C4 x + 8\_C2) a^4 \sin(ax) + e^{ax}}{8 a^4} \right\}$$

**2.1539 ODE No. 1539**

---

$$a^4\lambda y(x) + a^2(\lambda + 1)y''(x) + y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 0.0062766 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin\left(a\sqrt{\lambda}x\right) + c_1 \cos\left(a\sqrt{\lambda}x\right) + c_4 \sin(ax) + c_3 \cos(ax) \right\} \right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 35

$$\left\{ y(x) = \_C1 \sin(ax) + \_C2 \cos(ax) + \_C3 \sin\left(a\sqrt{\lambda}x\right) + \_C4 \cos\left(a\sqrt{\lambda}x\right) \right\}$$

**2.1540 ODE No. 1540**

---

$$a(bx - 1)y''(x) + aby'(x) + \lambda y(x) + y^{(4)}(x) = 0$$

✗ **Mathematica** : cpu = 0.266418 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}\left(\{y, x\}, \left\{ \lambda y(x) + aby'(x) + a(xb - 1)y''(x) + y^{(4)}(x) = 0, y(0) = c_1, y'(0) = c_2, y''(0) = c_3 \right\} \right) \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \text{DESol}\left(\left\{ \lambda \_Y(x) + ab \frac{d}{dx} \_Y(x) + a(bx - 1) \frac{d^2}{dx^2} \_Y(x) + \frac{d^4}{dx^4} \_Y(x) \right\}, \{\_Y(x)\} \right) \right\}$$

**2.1541 ODE No. 1541**

---

$$y''(x)(ax^2 + b\lambda + c) + y(x)(ax^2 + \beta\lambda + \gamma) + y^{(4)}(x) = 0$$

✗ **Mathematica** : cpu = 80.1613 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}\left(\{y, x\}, \left\{ (ax^2 + \beta\lambda + \gamma)y(x) + (ax^2 + c + b\lambda)y''(x) + y^{(4)}(x) = 0, y(0) = c_1, y'(0) = c_2 \right\} \right) \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \text{DESol}\left(\left\{ (ax^2 + \beta\lambda + \gamma) \_Y(x) + (ax^2 + b\lambda + c) \frac{d^2}{dx^2} \_Y(x) + \frac{d^4}{dx^4} \_Y(x) \right\}, \{\_Y(x)\} \right) \right\}$$

2.1542 ODE No. 1542

---

$$ay''(x)\wp(x;g2,g3)+by'(x)\wp'(x;g2,g3)+y(x)\left(c\left(6\wp(x;g2,g3)^2-\frac{g2^2}{2}\right)+d\right)+y^{(4)}(x)=0$$

✘ **Mathematica** : cpu = 0.0201287 (sec), leaf count = 0 , could not solve  
DSolve[(d + c\*(-g2/2 + 6\*WeierstrassP[x, {g2, g3}]^2))\*y[x] + b\*WeierstrassPPrime[x, {g2, g3}]]  
✘ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x)=DESol\left(\left\{\frac{d^4}{dx^4}-Y(x)+aWeierstrassP(x,g2,g3)\frac{d^2}{dx^2}-Y(x)+bWeierstrassPPrime(x,g2,g3)\frac{d}{dx}-\right.\right.\right.$$

2.1543 ODE No. 1543

---

$$-y''(x)\left(a+12k^2\operatorname{sn}(z|x)^2\right)+y(x)\left(\alpha\operatorname{sn}(z|x)^2+\beta\right)+by'(x)+y^{(4)}(x)=0$$

✘ **Mathematica** : cpu = 0.0654751 (sec), leaf count = 0 , could not solve  
DSolve[(beta + alpha\*JacobiSN[z, x]^2)\*y[x] + b\*Derivative[1][y][x] - (a + 12\*k^2\*JacobiSN[z, x]^2)\*y[x] + y[x]\*(3\*f[x]^2 + 3\*Derivative[2][f][x])]  
✘ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x)=DESol\left(\left\{\frac{d^4}{dx^4}-Y(x)+\left(-12k^2(JacobiSN(z,x))^2-a\right)\frac{d^2}{dx^2}-Y(x)+b\frac{d}{dx}-Y(x)+\left(\alpha(JacobiSN(z,x))^2+\beta\right)Y(x)\right.\right.\right.$$

2.1544 ODE No. 1544

---

$$y(x)\left(3f''(x)+3f(x)^2\right)+10f'(x)y'(x)+10f(x)y''(x)+y^{(4)}(x)=0$$

✘ **Mathematica** : cpu = 0.0110015 (sec), leaf count = 0 , could not solve  
DSolve[10\*Derivative[1][f][x]\*Derivative[1][y][x] + y[x]\*(3\*f[x]^2 + 3\*Derivative[2][f][x])]  
✓ **Maple** : cpu = 0.01 (sec), leaf count = 41

$$\left\{y(x)=\sum_{a=1}^4e^{RootOf\left(\_Z^4+10f\_Z^2+10df\_Z+3f^2+3ddf,index=\_a\right)x}\_C\_a\right\}$$

2.1545 ODE No. 1545

---

$$y^{(4)}(x)+2y^{(3)}(x)-3y''(x)-4y'(x)+4y(x)-32\sin(2x)+24\cos(2x)=0$$

✓ **Mathematica** : cpu = 0.725697 (sec), leaf count = 40

$$\left\{\left\{y(x)\rightarrow c_1e^{-2x}+c_2e^{-2x}x+c_3e^x+c_4e^xx+\sin(2x)\right\}\right\}$$

✓ **Maple** : cpu = 0.079 (sec), leaf count = 27

$$\left\{y(x)=\left(\_C4x+\_C2\right)e^{-2x}+\sin(2x)+\left(\_C3x+\_C1\right)e^x\right\}$$

2.1546 ODE No. 1546

---

$$a^4x^4y(x)+4a^3x^3y'(x)+6a^2x^2y''(x)+4axy^{(3)}(x)+y^{(4)}(x)=0$$

✓ **Mathematica** : cpu = 0.538687 (sec), leaf count = 300

$$\left\{\left\{y(x)\rightarrow\frac{2\left(\sqrt{6}-3\right)\sqrt{-\left(\sqrt{6}-3\right)ac_3}\exp\left(-\frac{ax^2}{2}-\sqrt{-\left(\sqrt{6}-3\right)ax}-\frac{\left(-3+\sqrt{3}+\sqrt{6}\right)ax}{\sqrt{-\left(\sqrt{6}-3\right)a}}\right)}{\left(-3-\sqrt{3}+\sqrt{6}\right)\left(-3+\sqrt{3}+\sqrt{6}\right)a}-\frac{2\left(\sqrt{6}-3\right)\sqrt{-\left(\sqrt{6}-3\right)ac_3}}{\left(-3-\sqrt{3}+\sqrt{6}\right)\left(-3+\sqrt{3}+\sqrt{6}\right)a}\right.\right.$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 73

$$\left\{y(x)=e^{-\frac{ax^2}{2}}\left(-C2e^{\sqrt{-a(-3+\sqrt{6})}x}+_C4e^{\sqrt{(3+\sqrt{6})}ax}+_C1e^{-\sqrt{-a(-3+\sqrt{6})}x}+_C3e^{-\sqrt{(3+\sqrt{6})}ax}\right)\right\}$$

2.1547 ODE No. 1547

---

$3y(x)\left(2g(x)f'(x)+5f(x)g'(x)+6f(x)^2g(x)+g''(x)+3g(x)^2\right)+y''(x)\left(4f'(x)+11f(x)^2+10g(x)\right)+y'(x)\left(f''(x)+2f(x)g'(x)+g''(x)+3g(x)^2\right)$

✗ **Mathematica** : cpu = 0.0237531 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[1][y][x]\*(6\*f[x]^3 + 30\*f[x]\*g[x] + 7\*f[x]\*Derivative[1][f][x] + 10\*Derivative[1][g][x]) + y''[x]\*(4\*f'[x] + 11\*f[x]^2 + 10\*g[x]) + y'[x]\*(f''[x] + 2\*f[x]\*g'[x] + g''[x] + 3\*g[x]^2), y[x], x]

✓ **Maple** : cpu = 0.011 (sec), leaf count = 87

$$\left\{y(x)=\sum_{a=1}^4\mathrm{e}^{\mathrm{RootOf}\left(\_Z^4+6\,f\_Z^3+(11\,f^2+4\,df+10\,g)\_Z^2+(6\,f^3+7\,df\,f+30\,fg+ddf+10\,dg)\_Z+18\,f^2g+6\,df\,g+15\,dg\,f+9\,g^2+3\,ddg\right)}\right\}$$

2.1548 ODE No. 1548

---

$4y^{(4)}(x)-12y^{(3)}(x)+11y''(x)-3y'(x)-4\cos(x)=0$

✓ **Mathematica** : cpu = 0.0815867 (sec), leaf count = 50

$$\left\{\left\{y(x)\rightarrow 2c_1e^{x/2}+\frac{2}{3}c_2e^{3x/2}+c_3e^x+c_4+\frac{18\sin(x)}{65}-\frac{14\cos(x)}{65}\right\}\right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 32

$$\left\{y(x)=\_C1\,\mathrm{e}^x+2\_C2\,\mathrm{e}^{x/2}+\frac{2\_C3}{3}\mathrm{e}^{\frac{3x}{2}}+\frac{18\sin(x)}{65}-\frac{14\cos(x)}{65}+\_C4\right\}$$

2.1549 ODE No. 1549

---

$xy^{(4)}(x)+5y^{(3)}(x)-24=0$

✓ **Mathematica** : cpu = 0.0122743 (sec), leaf count = 34

$$\left\{\left\{y(x)\rightarrow c_4x^2-\frac{c_1}{24x^2}+c_3x+c_2+\frac{4x^3}{5}\right\}\right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 26

$$\left\{y(x)=\frac{-C2\,x^2}{2}+\frac{4\,x^3}{5}-\frac{C1}{24\,x^2}+\_C3\,x+\_C4\right\}$$

2.1550 ODE No. 1550

---

$12x^3y''(x)-(6x^2+1)y^{(3)}(x)-(9x^2-7)x^2y'(x)+2(x^2-3)x^3y(x)+xy^{(4)}(x)=0$

✓ **Mathematica** : cpu = 3.08225 (sec), leaf count = 270

$$\left\{\left\{y(x)\rightarrow c_3e^{\frac{x^2}{2}}\int_1^xe^{\frac{K[1]^2}{2}}\left(\int\frac{\exp\left(\frac{1}{4}\sqrt{5}K[1]^2+\frac{1}{2}\left(-\frac{1}{2}K[1]^2-2\log(K[1])\right)\right)U\left(-\frac{-9+\sqrt{5}}{4\sqrt{5}},-\frac{1}{2},-\frac{1}{2}\sqrt{5}K[1]^2\right)}{\sqrt{K[1]}^4\sqrt{K[1]}^2}dK[1]\right)\right.\right.\\ \left.\left.\frac{K[1]}{\sqrt[4]{2}}-dK[1]\right\}\right\}$$

✓ **Maple** : cpu = 1.279 (sec), leaf count = 157

$$\left\{y(x)=-\mathrm{e}^{x^2}\int 1W_{\frac{9\sqrt{5}}{20},\frac{3}{4}}\left(\frac{\sqrt{5}x^2}{2}\right)\mathrm{e}^{-\frac{x^2}{4}}x^{-\frac{3}{2}}\mathrm{d}x\_C4-\mathrm{e}^{x^2}\int 1M_{\frac{9\sqrt{5}}{20},\frac{3}{4}}\left(\frac{\sqrt{5}x^2}{2}\right)\mathrm{e}^{-\frac{x^2}{4}}x^{-\frac{3}{2}}\mathrm{d}x\_C3+\int 1W_{\frac{9\sqrt{5}}{20},\frac{3}{4}}\left(\frac{\sqrt{5}x^2}{2}\right)\mathrm{e}^{-\frac{x^2}{4}}x^{-\frac{3}{2}}\mathrm{d}x\_C4\right\}$$

2.1551 ODE No. 1551

---

$-2\left(\nu^2x^2+6\right)y''(x)+\nu^2\left(\nu^2x^2+4\right)y(x)+x^2y^{(4)}(x)=0$

✓ **Mathematica** : cpu = 0.30286 (sec), leaf count = 110

$$\left\{\left\{y(x)\rightarrow \frac{c_3(1-x)e^{-\nu x}\left(\nu^2x^2+\nu^2x+\nu^2+6\nu x+6\nu+15\right)}{x}+\frac{c_4(1-x)e^{\nu x}\left(\nu^2x^2+\nu^2x+\nu^2-6\nu x-6\nu+15\right)}{x}\right\}\right\}$$

✓ **Maple** : cpu = 0.144 (sec), leaf count = 62

$$\left\{y(x)=\frac{\left(-C4\,\nu^2x^3+6\,_{-}C4\,\nu\,x^2+15\,_{-}C4\,x+_{-}C2\right)e^{-\nu\,x}+e^{\nu\,x}\left(-C3\,\nu^2x^3-6\,_{-}C3\,\nu\,x^2+15\,_{-}C3\,x+_{-}C1\right)}{x}\right.$$

2.1552 ODE No. 1552

---

$$ay(x)-bx^2+x^2y^{(4)}(x)+2xy^{(3)}(x)=0$$

✗ **Mathematica** : cpu = 300.003 (sec), leaf count = 0 , timed out  
\$Aborted

✓ **Maple** : cpu = 0.049 (sec), leaf count = 89

$$\left\{y(x)=\frac{bx^2}{a}+_{-}C1\,\sqrt{x}J_1\left(2\sqrt[4]{-a}\sqrt{x}\right)+_{-}C2\,\sqrt{x}Y_1\left(2\sqrt[4]{-a}\sqrt{x}\right)+_{-}C3\,\sqrt{x}J_1\left(2\sqrt{-\sqrt{-a}}\sqrt{x}\right)+_{-}C4\,\sqrt{x}Y_1\left(2\sqrt{-\sqrt{-a}}\sqrt{x}\right)\right.$$

2.1553 ODE No. 1553

---

$$x^2y^{(4)}(x)+4xy^{(3)}(x)+2y''(x)=0$$

✓ **Mathematica** : cpu = 0.0092852 (sec), leaf count = 29

$$\left\{\left\{y(x)\rightarrow c_2(-x)+c_4x+c_2x\log(x)-c_1\log(x)+c_3\right\}\right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 17

$$\left\{y(x)=\left(_{-}C2\,x+_{-}C4\right)\ln(x)+_{-}C1\,x+_{-}C3\right\}$$

2.1554 ODE No. 1554

---

$$x^2y^{(4)}(x)+6xy^{(3)}(x)+6y''(x)=0$$

✓ **Mathematica** : cpu = 0.0088841 (sec), leaf count = 29

$$\left\{\left\{y(x)\rightarrow c_4x+\frac{1}{2}\left(\frac{c_1}{x}-2c_2\log(x)\right)+c_3\right\}\right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 18

$$\left\{y(x)=_{-}C1+_{-}C2\,\ln(x)+\frac{_{-}C3}{x}+_{-}C4\,x\right\}$$

2.1555 ODE No. 1555

---

$$\lambda^2(-y(x))+x^2y^{(4)}(x)+6xy^{(3)}(x)+6y''(x)=0$$

✓ **Mathematica** : cpu = 0.0473522 (sec), leaf count = 156

$$\left\{\left\{y(x)\rightarrow c_4G_{0,4}^{2,0}\left(\frac{\lambda^2x^2}{16}\middle|\begin{array}{l} -\frac{1}{2},\frac{1}{2},0,0 \end{array}\right)+c_2G_{0,4}^{2,0}\left(\frac{\lambda^2x^2}{16}\middle|\begin{array}{l} 0,0,-\frac{1}{2},\frac{1}{2} \end{array}\right)+\frac{c_1\left(J_1\left(2\sqrt{\lambda}\sqrt{x}\right)+I_1\left(2\sqrt{\lambda}\sqrt{x}\right)\right)}{2\sqrt{\lambda}\sqrt{x}}-_{-}C4\sqrt{x}Y_1\left(2\sqrt{\lambda}\sqrt{x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.097 (sec), leaf count = 61

$$\left\{y(x)=1\left(_{-}C4\,Y_1\left(2\sqrt{-\lambda}\sqrt{x}\right)+_{-}C3\,J_1\left(2\sqrt{-\lambda}\sqrt{x}\right)+_{-}C2\,Y_1\left(2\sqrt{\lambda}\sqrt{x}\right)+_{-}C1\,J_1\left(2\sqrt{\lambda}\sqrt{x}\right)\right)\frac{1}{\sqrt{x}}\right\}$$

2.1556 ODE No. 1556

---

$$x^2y^{(4)}(x)+8xy^{(3)}(x)+12y''(x)=0$$

✓ **Mathematica** : cpu = 0.0092124 (sec), leaf count = 30

$$\left\{\left\{y(x)\rightarrow\frac{1}{6}\left(\frac{c_1}{x^2}+\frac{3c_2}{x}\right)+c_4x+c_3\right\}\right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 19

$$\left\{y(x)=_{-}C1+\frac{_{-}C2}{x^2}+\frac{_{-}C3}{x}+_{-}C4\,x\right\}$$

2.1557 ODE No. 1557

---

$$\lambda^2(-y(x)) + x^2y^{(4)}(x) + 8xy^{(3)}(x) + 12y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0506758 (sec), leaf count = 146

$$\left\{ \left\{ y(x) \rightarrow c_4 G_{0,4}^{2,0} \left( \frac{\lambda^2 x^2}{16} \middle| -1, 0, -\frac{1}{2}, \frac{1}{2} \right) + c_2 G_{0,4}^{2,0} \left( \frac{\lambda^2 x^2}{16} \middle| -\frac{1}{2}, \frac{1}{2}, -1, 0 \right) - \frac{3ic_1 \left( I_2 \left( 2\sqrt{\lambda}\sqrt{x} \right) - J_2 \left( 2\sqrt{\lambda}\sqrt{x} \right) \right)}{4\lambda x} \right. \right.$$

✓ **Maple** : cpu = 0.07 (sec), leaf count = 61

$$\left\{ y(x) = \frac{1}{x} \left( \_C4 Y_2 \left( 2\sqrt{-\lambda}\sqrt{x} \right) + \_C3 J_2 \left( 2\sqrt{-\lambda}\sqrt{x} \right) + \_C2 Y_2 \left( 2\sqrt{\lambda}\sqrt{x} \right) + \_C1 J_2 \left( 2\sqrt{\lambda}\sqrt{x} \right) \right) \right\}$$

2.1558 ODE No. 1558

---

$$-\frac{1}{16}b^4y(x) + x(2n - 2\nu + 4)y^{(3)}(x) + (n - \nu + 1)(n - \nu + 2)y''(x) + x^2y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 0.113166 (sec), leaf count = 319

$$\left\{ \left\{ y(x) \rightarrow c_4 i^{-n+\nu+1} 2^{3n-3\nu-3} b^{2(-n+\nu+1)+n-\nu-2} x^{\frac{1}{2}(n-\nu-2)-n+\nu+1} \Gamma(-n+\nu+2) \left( I_{\nu-n}(b\sqrt{x}) - J_{\nu-n}(b\sqrt{x}) \right) + c \right. \right.$$

✓ **Maple** : cpu = 0.102 (sec), leaf count = 67

$$\left\{ y(x) = x^{-\frac{n}{2}+\frac{\nu}{2}} \left( Y_{n-\nu}(b\sqrt{x})\_C4 + J_{n-\nu}(b\sqrt{x})\_C2 + K_{n-\nu}(b\sqrt{x})\_C3 + I_{n-\nu}(b\sqrt{x})\_C1 \right) \right\}$$

2.1559 ODE No. 1559

---

$$a^4 \left( -x^3 \right) y(x) + x^3 y^{(4)}(x) + 2x^2 y^{(3)}(x) - xy''(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.203156 (sec), leaf count = 100

$$\left\{ \left\{ y(x) \rightarrow c_4 G_{0,4}^{2,0} \left( \frac{a^4 x^4}{256} \middle| 0, 0, \frac{1}{2}, \frac{1}{2} \right) + c_2 G_{0,4}^{2,0} \left( \frac{a^4 x^4}{256} \middle| \frac{1}{2}, \frac{1}{2}, 0, 0 \right) + \frac{1}{8} i c_1 (I_0(ax) - J_0(ax)) + \frac{1}{2} c_3 (J_0(ax) + I_0(ax)) \right. \right.$$

✓ **Maple** : cpu = 0.102 (sec), leaf count = 33

$$\{y(x) = \_C1 I_0(ax) + \_C2 J_0(ax) + \_C3 K_0(ax) + \_C4 Y_0(ax)\}$$

2.1560 ODE No. 1560

---

$$x^3y^{(4)}(x) + 6x^2y^{(3)}(x) + 6xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0069091 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow c_4 x + \frac{1}{2} \left( \frac{c_1}{x} - 2c_2 \log(x) \right) + c_3 \right\} \right\}$$

✓ **Maple** : cpu = 0.006 (sec), leaf count = 18

$$\left\{ y(x) = \_C1 + \_C2 \ln(x) + \frac{\_C3}{x} + \_C4 x \right\}$$

2.1561 ODE No. 1561

---

$$y(x) \left( ax^4 + (n-2)n(n+1)(n+3) \right) - 2n(n+1)x^2y''(x) + 4n(n+1)xy'(x) + x^4y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 2.9797 (sec), leaf count = 400

$$\left\{ \left\{ y(x) \rightarrow c_1 \left( -2^{n-\frac{5}{2}} \right) \sqrt{x} a^{\frac{2-n}{4}+\frac{1}{4}(n-\frac{3}{2})} \Gamma\left(\frac{3}{2}-n\right) \left( \cos\left(\frac{3}{4}\pi\left(\frac{3}{2}-n\right)\right) \operatorname{ber}_{-n-\frac{1}{2}}(\sqrt[4]{ax}) + \sin\left(\frac{3}{4}\pi\left(\frac{3}{2}-n\right)\right) \right) \right. \right.$$

✓ **Maple** : cpu = 0.167 (sec), leaf count = 69

$$\left\{ y(x) = \sqrt{x} \left( Y_{n+\frac{1}{2}}(\sqrt[4]{-ax})\_C2 + J_{n+\frac{1}{2}}(\sqrt[4]{-ax})\_C1 + Y_{n+\frac{1}{2}}(\sqrt{-\sqrt{-ax}})\_C4 + J_{n+\frac{1}{2}}(\sqrt{-\sqrt{-ax}})\_C3 \right) \right\}$$

**2.1562                    ODE No. 1562**

---

$$-\left(4 n^2-1\right) x^2 y''(x)+\left(4 n^2-1\right) x y'(x)+x^4 y^{(4)}(x)-4 x^4 y(x)+4 x^3 y^{(3)}(x)=0$$

✓    **Mathematica** : cpu = 0.831439 (sec), leaf count = 140

$$\left\{\left\{y(x) \rightarrow c_1 {}_0 F_3\left(\left(; \frac{1}{2}, 1-\frac{n}{2}, \frac{n}{2}+1 ; \frac{x^4}{64}\right)+\frac{1}{8} i c_2 x^2 {}_0 F_3\left(\left(; \frac{3}{2}, \frac{3}{2}-\frac{n}{2}, \frac{n}{2}+\frac{3}{2} ; \frac{x^4}{64}\right)+c_3\left(\frac{i}{2}\right)^{-n} \Gamma(1-n)^2\left(\operatorname{ber}_{-n}\left(\sqrt{2} x\right)\right)\right.\right.\right.$$

✓    **Maple** : cpu = 0.228 (sec), leaf count = 77

$$\left\{y(x)=\left(Y_n\left(\left(\left(\frac{1}{2}-\frac{i}{2}\right) \sqrt{2} x\right)_{-C 3}+J_n\left(\left(\left(\frac{1}{2}-\frac{i}{2}\right) \sqrt{2} x\right)_{-C 1}\right) J_n\left(\left(\left(\frac{1}{2}+\frac{i}{2}\right) \sqrt{2} x\right)+Y_n\left(\left(\left(\frac{1}{2}+\frac{i}{2}\right) \sqrt{2} x\right)\right.\right.\right.$$

**2.1563                    ODE No. 1563**

---

$$\left(4 n^2-4 x^4-1\right) y(x)-\left(4 n^2-1\right) x^2 y''(x)-\left(4 n^2-1\right) x y'(x)+x^4 y^{(4)}(x)+4 x^3 y^{(3)}(x)=0$$

✓    **Mathematica** : cpu = 1.51146 (sec), leaf count = 232

$$\left\{\left\{y(x) \rightarrow \frac{{}^4 \sqrt{-1} c_2 x {}_0 F_3\left(\left(; \frac{3}{2}, 1-\frac{n}{2}, \frac{n}{2}+1 ; \frac{x^4}{64}\right)-\frac{2(-1)^{3 / 4} \sqrt{2} c_1 {}_0 F_3\left(\left(; \frac{1}{2}, \frac{1}{2}-\frac{n}{2}, \frac{n}{2}+\frac{1}{2} ; \frac{x^4}{64}\right)}{x}+c_3(-1)^{\frac{1}{4}(1-2 n)} 2^{2 n+1}\right.\right.\right.$$

✓    **Maple** : cpu = 0.16 (sec), leaf count = 87

$$\left\{y(x)=\frac{1}{x}\left(-C 4 {}_0 F_3\left(\left(; \frac{1}{2},-\frac{n}{2}+\frac{1}{2}, \frac{n}{2}+\frac{1}{2} ; \frac{x^4}{64}\right)+\left(-C 3 {}_0 F_3\left(\left(; \frac{3}{2}, 1+\frac{n}{2}, 1-\frac{n}{2} ; \frac{x^4}{64}\right)+_{-C 2}\left(\operatorname{bei}_{-n}(x)\right)^2+\left(\operatorname{ber}_{-n}(x)\right)^2\right)\right.\right.$$

**2.1564                    ODE No. 1564**

---

$$-\left(12 n^2+4 x^4-3\right) y(x)-\left(4 n^2+3\right) x^2 y''(x)+\left(12 n^2-3\right) x y'(x)+x^4 y^{(4)}(x)+4 x^3 y^{(3)}(x)=0$$

✓    **Mathematica** : cpu = 1.00919 (sec), leaf count = 230

$$\left\{\left\{y(x) \rightarrow \frac{{}^4 \sqrt{-1} c_1 x {}_0 F_3\left(\left(; \frac{1}{2}, \frac{3}{2}-\frac{n}{2}, \frac{n}{2}+\frac{3}{2} ; \frac{x^4}{64}\right)}{2 \sqrt{2}}+c_3(-1)^{\frac{1}{4}(-2 n-1)} 2^{2 n+\frac{1}{2}(2 n+1)+1} x^{-2 n-1} {}_0 F_3\left(\left(; 1-n, \frac{1}{2}-\frac{n}{2},-\frac{n}{2} ; \frac{x^4}{64}\right)\right.\right.\right.$$

✓    **Maple** : cpu = 0.138 (sec), leaf count = 88

$$\left\{y(x)=\frac{1}{x}\left(-C 4 x^2 {}_0 F_3\left(\left(; \frac{1}{2}, \frac{3}{2}+\frac{n}{2}, \frac{3}{2}-\frac{n}{2} ; \frac{x^4}{64}\right)+_{-C 3} x^4 {}_0 F_3\left(\left(; \frac{3}{2}, 2-\frac{n}{2}, 2+\frac{n}{2} ; \frac{x^4}{64}\right)+_{-C 2}\left(\operatorname{bei}_{-n}(x)\right)^2+\left(\operatorname{ber}_{-n}(x)\right)^2\right)\right.\right.$$

**2.1565                    ODE No. 1565**

---

$$\left(x\left(-\rho^2-\sigma^2+1\right)+16 x^3\right) y'(x)+y(x)\left(\rho^2 \sigma^2+8 x^2\right)+\left(x^2\left(-\rho^2-\sigma^2+7\right)+4 x^4\right) y''(x)+x^4 y^{(4)}(x)+6 x^3 y^{(3)}(x)=0$$

✓    **Mathematica** : cpu = 0.385456 (sec), leaf count = 242

$$\left\{\left\{y(x) \rightarrow c_1 x^{-\rho} {}_2 F_3\left(\left(\frac{1}{2}-\frac{\rho}{2}, 1-\frac{\rho}{2} ; 1-\rho,-\frac{\rho}{2}-\frac{\sigma}{2}+1,-\frac{\rho}{2}+\frac{\sigma}{2}+1 ;-x^2\right)+c_3 x^{-\sigma} {}_2 F_3\left(\left(\frac{1}{2}-\frac{\sigma}{2}, 1-\frac{\sigma}{2} ; 1-\sigma,-\frac{\sigma}{2}-\frac{\rho}{2}+1\right)\right.\right.\right.$$

✓    **Maple** : cpu = 0.231 (sec), leaf count = 71

$$\left\{y(x)=\left(Y_{\frac{\rho}{2}-\frac{\sigma}{2}}(x)_{-C 2}+_{-C 1} J_{\frac{\rho}{2}-\frac{\sigma}{2}}(x)\right) J_{\frac{\rho}{2}+\frac{\sigma}{2}}(x)+Y_{\frac{\rho}{2}+\frac{\sigma}{2}}(x)\left(Y_{\frac{\rho}{2}-\frac{\sigma}{2}}(x)_{-C 4}+J_{\frac{\rho}{2}-\frac{\sigma}{2}}(x)_{-C 3}\right)\right\}$$

**2.1566                    ODE No. 1566**

---

$$\left(x\left(-2 \mu^2-2 \nu^2+1\right)+16 x^3\right) y'(x)+y(x)\left(\left(\mu^2-\nu^2\right)^2+8 x^2\right)+\left(x^2\left(-2 \mu^2-2 \nu^2+7\right)+4 x^4\right) y''(x)+x^4 y^{(4)}(x)+6 x^3 y^{(3)}(x)=0$$

✓    **Mathematica** : cpu = 0.454169 (sec), leaf count = 238

$$\left\{\left\{y(x) \rightarrow c_1 x^{-\mu-\nu} {}_2 F_3\left(\left(-\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2},-\frac{\mu}{2}-\frac{\nu}{2}+1 ; 1-\mu, 1-\nu,-\mu-\nu+1 ;-x^2\right)+c_2 x^{\mu-\nu} {}_2 F_3\left(\left(\frac{\mu}{2}-\frac{\nu}{2}+\frac{1}{2}, \frac{\mu}{2}-\frac{\nu}{2}+1\right)\right.\right.\right.$$

✓    **Maple** : cpu = 0.207 (sec), leaf count = 35

$$\left\{y(x)=\left(Y_{\mu}(x)_{-C 2}+J_{\mu}(x)_{-C 1}\right) J_{\nu}(x)+Y_{\nu}(x)\left(Y_{\mu}(x)_{-C 4}+_{-C 3} J_{\mu}(x)\right)\right\}$$

**2.1567                    ODE No. 1567**

---

$$x^4y^{(4)}(x)+8x^3y^{(3)}(x)+12x^2y''(x)=0$$

✓ **Mathematica** : cpu = 0.0094639 (sec), leaf count = 30

$$\left\{\left\{y(x)\rightarrow\frac{1}{6}\left(\frac{c_1}{x^2}+\frac{3c_2}{x}\right)+c_4x+c_3\right\}\right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 19

$$\left\{y\left(x\right)=-C1+\frac{C2}{x^2}+\frac{C3}{x}+_C4\,x\right\}$$

**2.1568                    ODE No. 1568**

---

$$ay(x)+x^4y^{(4)}(x)+8x^3y^{(3)}(x)+12x^2y''(x)=0$$

✓ **Mathematica** : cpu = 0.0087693 (sec), leaf count = 122

$$\left\{\left\{y(x)\rightarrow c_1x^{\frac{1}{2}\left(-\sqrt{5-4\sqrt{1-a}}-1\right)}+c_2x^{\frac{1}{2}\left(\sqrt{5-4\sqrt{1-a}}-1\right)}+c_3x^{\frac{1}{2}\left(-\sqrt{4\sqrt{1-a}+5}-1\right)}+c_4x^{\frac{1}{2}\left(\sqrt{4\sqrt{1-a}+5}-1\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 89

$$\left\{y\left(x\right)=-C1\,x^{-\frac{1}{2}-\frac{1}{2}\sqrt{5-4\sqrt{1-a}}}+_C2\,x^{-\frac{1}{2}+\frac{1}{2}\sqrt{5-4\sqrt{1-a}}}+_C3\,x^{-\frac{1}{2}-\frac{1}{2}\sqrt{5+4\sqrt{1-a}}}+_C4\,x^{-\frac{1}{2}+\frac{1}{2}\sqrt{5+4\sqrt{1-a}}}\right\}$$

**2.1569                    ODE No. 1569**

---

$$xy'(x)\left((2a-1)C0+4b^2B0c^2x^{2c}\right)+(6-4a)x^3y^{(3)}(x)+x^2y''(x)\left(A0+4b^2c^2x^{2c}\right)+y(x)\left(4b^2c^2D0x^{2c}+E0\right)+x^4y^{(4)}(x)=0$$

✗ **Mathematica** : cpu = 300.047 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.29 (sec), leaf count = 63

$$\{y\left(x\right)=\left(\left(J_{\mu}\left(x^cb\right)_-C2+Y_{\mu}\left(x^cb\right)_-C3\right)J_{\nu}\left(x^cb\right)+Y_{\nu}\left(x^cb\right)\left(_-C4\,Y_{\mu}\left(x^cb\right)+_-C1\,J_{\mu}\left(x^cb\right)\right)\right)x^a\}$$

**2.1570                    ODE No. 1570**

---

$$y(x)\left(\left(a^2-c^2\nu^2\right)\left(a^2+4ac-c^2\nu^2+4c^2\right)-b^4c^4x^{4c}\right)+x^2\left(2a^2+4(a+c-1)^2+4(a-1)(c-1)-2c^2\nu^2-1\right)y''(x)=0$$

✓ **Mathematica** : cpu = 0.101622 (sec), leaf count = 470

$$\left\{\left\{y(x)\rightarrow c_1\Gamma(1-\nu)(-1)^{\frac{a-c\nu}{4c}}2^{-\frac{2(a-c\nu)}{c}-\nu-1}b^{\frac{a-c\nu}{c}+\nu}\left(x^{4c}\right)^{\frac{a-c\nu}{4c}+\frac{\nu}{4}}\left(J_{-\nu}\left(b\sqrt[4]{x^{4c}}\right)+I_{-\nu}\left(b\sqrt[4]{x^{4c}}\right)\right)+c_2\Gamma(2-\nu)\right\}\right\}$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 49

$$\{y\left(x\right)=x^a\left(Y_{\nu}\left(x^cb\right)_-C2+Y_{\nu}\left(ibx^c\right)_-C4+J_{\nu}\left(ibx^c\right)_-C3+J_{\nu}\left(x^cb\right)_-C1\right)\}$$

**2.1571                    ODE No. 1571**

---

$$-\frac{1}{16}b^4x^{2/v}y(x)+\nu^4x^4y^{(4)}(x)+\nu^3(4\nu-2)x^3y^{(3)}(x)+(\nu-1)\nu^2(2\nu-1)x^2y''(x)=0$$

✓ **Mathematica** : cpu = 0.0633978 (sec), leaf count = 390

$$\left\{\left\{y(x)\rightarrow c_1{}_0F_3\left(\left(;1-\frac{v}{2},1-\frac{v}{2\nu},-\frac{v}{2\nu}-\frac{v}{2}+1;\frac{b^4v^4x^{2/v}}{256\nu^4}\right)+c_2\left(\frac{i}{16}\right)^v v^{2v}b^{2v}\nu^{-2v}\left(x^{2/v}\right)^{v/2}{}_0F_3\left(\left(\frac{v}{2}+1,1,1-\frac{v}{2};-\frac{b^4v^4x^{2/v}}{256\nu^4}\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.144 (sec), leaf count = 143

$$\left\{y\left(x\right)=\sqrt{x}\left(Y_{\left(\lfloor\nu^{-1}\rfloor\right)^{-1}}\left(\frac{1}{\left[\nu^{-1}\right]}\sqrt{\frac{b^2}{\nu^2}}x^{\frac{\left\lfloor\nu^{-1}\right\rfloor}{2}}\right)_-C2+J_{\left(\lfloor\nu^{-1}\rfloor\right)^{-1}}\left(\frac{1}{\left[\nu^{-1}\right]}\sqrt{\frac{b^2}{\nu^2}}x^{\frac{\left\lfloor\nu^{-1}\right\rfloor}{2}}\right)_-C1+Y_{\left(\lfloor\nu^{-1}\rfloor\right)^{-1}}\left(\frac{1}{\left[\nu^{-1}\right]}\sqrt{\frac{b^2}{\nu^2}}x^{\frac{\left\lfloor\nu^{-1}\right\rfloor}{2}}\right)_-C1\right)\}$$



**2.1572                    ODE No. 1572**

---

$(-2\left(x^2-1\right)\left(\mu (\mu +1)+\nu (\nu +1)\right)+24x^3-8)y''(x)-6x(\mu (\mu +1)+\nu (\nu +1)-2)y'(x)+((\mu (\mu +1)-\nu (\nu +1))^2-$

✕ **Mathematica** : cpu = 90.1062 (sec), leaf count = 0 , DifferentialRoot result

$\left\{\left\{y(x)\rightarrow \text{DifferentialRoot}\left(\left\{y,x\right\},\left\{\left(\mu -\nu -1\right)\left(\mu -\nu +1\right)\left(\mu +\nu \right)\left(\mu +\nu +2\right)y(x)-6x\left(\mu ^2+\mu +\nu ^2+\nu -2\right)\right\}\right)\right\}$

✓ **Maple** : cpu = 0.251 (sec), leaf count = 35

$\{y\left(x\right)=\left(\text{LegendreQ}\left(\mu ,x\right)\_C2+\text{LegendreP}\left(\mu ,x\right)\_C1\right)\text{LegendreP}\left(\nu ,x\right)+\text{LegendreQ}\left(\nu ,x\right)\left(\text{LegendreQ}\left(\mu ,x\right)-\text{LegendreP}\left(\mu ,x\right)\right)\}$

**2.1573                    ODE No. 1573**

---

$$-\frac{1}{x^5}+(2x+e^x)y^{(4)}(x)+4\left(e^x+2\right)y^{(3)}(x)+6e^xy''(x)+4e^xy'(x)+e^xy(x)=0$$

✕ **Mathematica** : cpu = 0.0395448 (sec), leaf count = 0 , could not solve

$\text{DSolve}[-x^{(-5)}+E^x*y[x]+4*E^x*Derivative[1][y][x]+6*E^x*Derivative[2][y][x]+4*(2+E^x)*Derivative[3][y][x],y[x],x]$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 41

$$\left\{y\left(x\right)=\frac{24\_C1\,x^4+24\_C2\,x^3+24\_C3\,x^2+24\_C4\,x+1}{\left(24\,e^x+48\,x\right)x}\right\}$$

**2.1574                    ODE No. 1574**

---

$y(x)\left(a^4\sin^4(x)-3\right)+y^{(4)}(x)\sin^4(x)+2y^{(3)}(x)\sin^3(x)\cos(x)+\left(\sin^2(x)-3\right)\sin^2(x)y''(x)+\left(2\sin^2(x)+3\right)\sin(x)\cos(x)y'(x)+\left(a^4-3\right)y(x)=0$

✕ **Mathematica** : cpu = 0.133596 (sec), leaf count = 0 , could not solve

$\text{DSolve}[(-3+a^4*\text{Sin}[x]^4)*y[x]+Cos[x]*\text{Sin}[x]*(3+2*\text{Sin}[x]^2)*Derivative[1][y][x]+\text{Sin}[x]*(3+2*\text{Sin}[x]^2)*Derivative[2][y][x]+2*Cos[x]*\text{Sin}[x]^3*Derivative[3][y][x]+\text{Sin}[x]^4*Derivative[4][y][x],y[x],x]$

✓ **Maple** : cpu = 0.508 (sec), leaf count = 252

$\left\{y\left(x\right)=\sin \left(x\right)\left({}_2F_1\left(\frac{3}{4}+\frac{1}{4}\sqrt{-4\sqrt{-\left(a-1\right)\left(a+1\right)\left(a^2+1\right)}+5},\frac{3}{4}-\frac{1}{4}\sqrt{-4\sqrt{-\left(a-1\right)\left(a+1\right)\left(a^2+1\right)}+5};-\frac{1}{4}\right)\right\}$

**2.1575                    ODE No. 1575**

---

$-f(x)+y^{(4)}(x)\sin^6(x)+4y^{(3)}(x)\sin^5(x)\cos(x)-6\sin^6(x)y''(x)-4\sin^5(x)\cos(x)y'(x)+y(x)\sin^6(x)=0$

✕ **Mathematica** : cpu = 0.0441519 (sec), leaf count = 0 , could not solve

$\text{DSolve}[-f[x]+\text{Sin}[x]^6*y[x]-4*Cos[x]*\text{Sin}[x]^5*Derivative[1][y][x]-6*\text{Sin}[x]^6*Derivative[2][y][x]+4*\text{Sin}[x]^5*\text{Cos}[x]*Derivative[3][y][x]+y[x]*\text{Sin}[x]^6,x,y]$

✓ **Maple** : cpu = 0.519 (sec), leaf count = 638

$\left\{y\left(x\right)=\frac{1}{48\left(e^{2ix}-1\right)^4\sin \left(x\right)}\left(12f\left(e^{6ix}-1/4e^{8ix}+e^{2ix}-3/2e^{4ix}-1/4\right)\left(x^2+\frac{20}{3}\right)x\ln \left(1-e^{ix}\right)-72if\left(e^{6ix}-1/4e^{8ix}+e^{2ix}-3/2e^{4ix}-1/4\right)\right)\right\}$

**2.1576                    ODE No. 1576**

---

$$2f'(x)\left(y^{(3)}(x)-a^2y'(x)\right)+f(x)\left(a^4y(x)-2a^2y''(x)+y^{(4)}(x)\right)=0$$

✕ **Mathematica** : cpu = 0.158001 (sec), leaf count = 0 , could not solve

$\text{DSolve}[2*Derivative[1][f][x]*(-(a^2*Derivative[1][y][x])+Derivative[3][y][x])+f[x]*(a^4*y[x]-2*a^2*Derivative[2][y][x]+Derivative[4][y][x]),y[x],x]$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 67

$$\left\{y\left(x\right)=\_C1\,e^{ax}+\_C2\,e^{-ax}+\_C3\,e^{\frac{x}{f}}\left(-df+\sqrt{a^2f^2+df^2}\right)+\_C4\,e^{-\frac{x}{f}}\left(df+\sqrt{a^2f^2+df^2}\right)\right\}$$

2.1577 ODE No. 1577

$$f''(x)y''(x)+2y^{(3)}(x)f'(x)+f(x)y^{(4)}(x)=0$$

✓ **Mathematica** : cpu = 0.0569354 (sec), leaf count = 46

$$\left\{\left\{y(x)\rightarrow \int_1^x\int_1^{K[2]}\left(\frac{c_1}{f(K[1])}+\frac{c_2K[1]}{f(K[1])}\right)dK[1]dK[2]+c_4x+c_3\right\}\right\}$$

✓ **Maple** : cpu = 0.004 (sec), leaf count = 21

$$\left\{y\left(x\right) = \frac{-C1\,x^3}{6} + \frac{-C2\,x^2}{2} + \_C3\,x + \_C4\right\}$$

2.1578 ODE No. 1578

$$a^4y(x)-\lambda(ax-b)\left(y''(x)-a^2y(x)\right)-2a^2y''(x)+y^{(4)}(x)=0$$

✓ **Mathematica** : cpu = 47.2958 (sec), leaf count = 141

$$\left\{\left\{y(x)\rightarrow c_3e^{-ax}\int_1^x2ae^{2aK[1]}\int e^{-aK[1]}\text{Ai}\left(\frac{a^2+\lambda K[1]a-b\lambda}{(a\lambda)^{2/3}}\right)dK[1]dK[1]+c_4e^{-ax}\int_1^x2ae^{2aK[2]}\int e^{-aK[2]}\text{Ei}\left(\frac{a^2+\lambda K[2]a-b\lambda}{(a\lambda)^{2/3}}\right)dK[2]dK[2]\right\}\right\}$$

✓ **Maple** : cpu = 0.287 (sec), leaf count = 89

$$\left\{y\left(x\right) = \text{e}^{ax}\left(\int \text{e}^{-2ax}\left(\int \text{e}^{ax}\left(-C4\,\text{Bi}\left(-\frac{\lambda\left(ax-b\right)+a^2}{a\lambda}\sqrt[3]{-a\lambda}\right)+\text{Ai}\left(-\frac{\lambda\left(ax-b\right)+a^2}{a\lambda}\sqrt[3]{-a\lambda}\right)-C3}\right)\text{d}x\right.\right.$$

2.1579 ODE No. 1579

$$-ax-b\sin(x)-c\cos(x)+y^{(n)}(x)+2y^{(3)}(x)+y'(x)=0$$

✓ **Mathematica** : cpu = 0.911319 (sec), leaf count = 80

$$\left\{\left\{y(x)\rightarrow \frac{1}{16}\left(8ax^2+\sin(x)\left(-6bx+c\left(13-2x^2\right)+16(c_2x+c_1+c_4)\right)+\cos(x)\left(b\left(2x^2-9\right)-2(5cx+8(c_4x-c_3))\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.283 (sec), leaf count = 69

$$\left\{y\left(x\right) = \frac{\left(bx^2+\left(-4\,c-8\_C4\right)x-6\,b-8\_C2+8\_C3\right)\cos\left(x\right)}{8} + \frac{\left(-cx^2+\left(-4\,b+8\_C3\right)x+6\,c+8\_C1\right)\sin\left(x\right)}{8}$$

2.1580 ODE No. 1580

$$y^{(6)}(x)+y(x)-\sin\left(\frac{x}{2}\right)\sin\left(\frac{3x}{2}\right)=0$$

✓ **Mathematica** : cpu = 2.38973 (sec), leaf count = 234

$$\left\{\left\{y(x)\rightarrow c_4e^{-\frac{\sqrt{3}x}{2}}\sin\left(\frac{x}{2}\right)+c_6e^{\frac{\sqrt{3}x}{2}}\sin\left(\frac{x}{2}\right)+c_5\sin(x)+c_1e^{\frac{\sqrt{3}x}{2}}\cos\left(\frac{x}{2}\right)+c_3e^{-\frac{\sqrt{3}x}{2}}\cos\left(\frac{x}{2}\right)+c_2\cos(x)+\frac{1}{504}\right\}\right\}$$

✓ **Maple** : cpu = 0.659 (sec), leaf count = 79

$$\left\{y\left(x\right) = \frac{1}{504}\left(504\cos\left(x/2\right)-C3+504\sin\left(x/2\right)-C4\right)\text{e}^{-\frac{\sqrt{3}x}{2}} + \frac{1}{504}\left(504\cos\left(x/2\right)-C5+504\sin\left(x/2\right)-C6\right)\text{e}^{\frac{\sqrt{3}x}{2}}$$

2.1581 ODE No. 1581

$$-axy(x)-b+y^{(5)}(x)=0$$

✗ **Mathematica** : cpu = 0.133685 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{\left\{y(x)\rightarrow \text{DifferentialRoot}\left(\left\{y,x\right\},\left\{-b-xay(x)+y^{(5)}(x)=0,y(0)=c_1,y'(0)=c_2,y''(0)=c_3,y^{(3)}(0)=c_4,y^{(4)}(0)=c_5\right\}\right)\right\}\right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(diff(diff(diff(diff(y(x),x),x),x),x)-a\*x\*y(x)-b=0,y(x))

2.1582 ODE No. 1582

---

$$a \nu x^{\nu-1} y(x) + a x^{\nu} y'(x) + y^{(5)}(x) = 0$$

✓ **Mathematica** : cpu = 0.117428 (sec), leaf count = 787

$$\left\{ \left\{ y(x) \rightarrow c_5 \left( \frac{4}{\nu} + 1 \right)^{-\frac{16}{\nu+4}} \nu^{-\frac{16}{\nu+4}} a^{\frac{4}{\nu+4}} (x^{\nu})^{\frac{4(\frac{4}{\nu}+1)}{\nu+4}} {}_1F_4 \left( \frac{4}{\nu(1+\frac{4}{\nu})} + \frac{1}{1+\frac{4}{\nu}}; 1 + \frac{1}{(1+\frac{4}{\nu})\nu}, 1 + \frac{2}{(1+\frac{4}{\nu})\nu}, 1 + \frac{1}{(1+\frac{4}{\nu})\nu}, 1 + \frac{2}{(1+\frac{4}{\nu})\nu} \right) \right. \right.$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^5}{dx^5} Y(x) + a x^{\nu} \frac{d}{dx} Y(x) + a \nu x^{\nu-1} Y(x) \right\}, \{ \_Y(x) \} \right) \right\}$$

2.1583 ODE No. 1583

---

$$a y^{(4)}(x) - f(x) + y^{(5)}(x) = 0$$

✓ **Mathematica** : cpu = 0.0937416 (sec), leaf count = 92

$$\left\{ \left\{ y(x) \rightarrow \int_1^x \int_1^{K[5]} \int_1^{K[4]} \int_1^{K[3]} \left( e^{-aK[2]} c_1 + e^{-aK[2]} \int_1^{K[2]} e^{aK[1]} f(K[1]) dK[1] \right) dK[2] dK[3] dK[4] dK[5] + c_5 \right. \right.$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 40

$$\left\{ y(x) = \frac{-C3 x^2}{2} + \frac{-C2 x^3}{6} + \frac{e^{-ax} C1}{a^4} + \frac{f x^4}{24 a} + \_C4 x + \_C5 \right\}$$

2.1584 ODE No. 1584

---

$$a x y(x) - 5 m y^{(4)}(x) + x y^{(5)}(x) = 0$$

✓ **Mathematica** : cpu = 2.13808 (sec), leaf count = 216

$$\left\{ \left\{ y(x) \rightarrow c_5 5^{-5m-4} a^{\frac{1}{5}(5m+4)} x^{5m+4} {}_0F_4 \left( ; m + \frac{6}{5}, m + \frac{7}{5}, m + \frac{8}{5}, m + \frac{9}{5}; -\frac{a x^5}{3125} \right) + \frac{1}{125} a^{3/5} c_4 x^3 {}_0F_4 \left( ; \frac{6}{5}, \frac{7}{5}, \frac{8}{5}, \frac{9}{5}; -\frac{a x^5}{3125} \right) \right. \right.$$

✓ **Maple** : cpu = 0.142 (sec), leaf count = 118

$$\left\{ y(x) = \_C1 {}_0F_4 \left( ; \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{5} - m; -\frac{x^5 a}{3125} \right) + \_C2 x {}_0F_4 \left( ; \frac{3}{5}, \frac{4}{5}, \frac{6}{5}, \frac{2}{5} - m; -\frac{x^5 a}{3125} \right) + \_C3 x^2 {}_0F_4 \left( ; \frac{4}{5}, \frac{6}{5}, \frac{7}{5}, \frac{3}{5} - m; -\frac{x^5 a}{3125} \right) \right.$$

2.1585 ODE No. 1585

---

$$x y(x) \left( a y'(x) + b y''(x) + c y^{(3)}(x) + e y^{(4)}(x) \right) = 0$$

✓ **Mathematica** : cpu = 0.160597 (sec), leaf count = 214

$$\left\{ \{ y(x) \rightarrow 0 \}, \left\{ y(x) \rightarrow \frac{c_1 e^{x \text{Root} \left[ \#1^3 + \frac{\#1^2 c}{e} + \frac{\#1 b}{e} + \frac{a}{e} \&, 1 \right]}}{\text{Root} \left[ \#1^3 + \frac{\#1^2 c}{e} + \frac{\#1 b}{e} + \frac{a}{e} \&, 1 \right]}} + \frac{c_2 e^{x \text{Root} \left[ \#1^3 + \frac{\#1^2 c}{e} + \frac{\#1 b}{e} + \frac{a}{e} \&, 2 \right]}}{\text{Root} \left[ \#1^3 + \frac{\#1^2 c}{e} + \frac{\#1 b}{e} + \frac{a}{e} \&, 2 \right]}} + \frac{c_3 e^{x \text{Root} \left[ \#1^3 + \frac{\#1^2 c}{e} + \frac{\#1 b}{e} + \frac{a}{e} \&, 3 \right]}}{\text{Root} \left[ \#1^3 + \frac{\#1^2 c}{e} + \frac{\#1 b}{e} + \frac{a}{e} \&, 3 \right]}} \right. \right.$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 679

$$\left\{ y(x) = 0, y(x) = \_C4 e^{\frac{x}{6e} \left( \left( 12 \sqrt{3} \sqrt{27 a^2 e^2 + (-18 a b c + 4 b^3) e + 4 c^3 a - b^2 c^2 e - 108 a e^2 + 36 b c e - 8 c^3} \right)^{\frac{2}{3}} - 2 c \sqrt[3]{12 \sqrt{3} \sqrt{27 a^2 e^2 + (-18 a b c + 4 b^3) e + 4 c^3 a - b^2 c^2 e - 108 a e^2 + 36 b c e - 8 c^3}} \right)} \right.$$

2.1586 ODE No. 1586

$$-y^{(4)}(x)(x(aA(5)-A(4))+A(5))-y^{(3)}(x)(x(aA(4)-A(3))+A(4))-(x(aA(3)-A(2))+A(3))y''(x)-(x(aA(2)-A(1))+A(2))y'(x)+x^2y(x)=0$$

✗ **Mathematica** : cpu = 83.0705 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{\left\{y(x) \rightarrow \text{DifferentialRoot}\left(\left\{y, x\right\},\left\{xA(0)-xaA(1)-A(1)+(xA(1)-xaA(2)-A(2))y'(x)+(xA(2)-xaA(3)-A(3))y''(x)+(xA(3)-xaA(4)-A(4))y'''(x)+(xA(4)-xaA(5)-A(5))y^{(4)}(x)\right\}\right)\right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x)=\int DESol\left(\left\{-\frac{(axA_2-xA_1+A_2)\_Y(x)}{x}-\frac{(axA_3-xA_2+A_3)\frac{d}{dx}\_Y(x)}{x}-\frac{(axA_4-xA_3+A_4)\frac{d^2}{dx^2}\_Y(x)}{x}\right\}\right)\right\}$$

2.1587 ODE No. 1587

$$x^5y^{(10)}(x)-ay(x)=0$$

✓ **Mathematica** : cpu = 0.268886 (sec), leaf count = 492

$$\left\{\left\{y(x) \rightarrow \frac{(-1)^{4/5}a^{9/5}c_1x^9{}_0F_9\left(\left(\frac{6}{5},\frac{7}{5},\frac{8}{5},\frac{9}{5},2,\frac{11}{5},\frac{12}{5},\frac{13}{5},\frac{14}{5};\frac{ax^5}{9765625}\right)\right)}{3814697265625}+\frac{(-1)^{3/5}a^{8/5}c_3x^8{}_0F_9\left(\left(\frac{4}{5},\frac{6}{5},\frac{7}{5},\frac{8}{5},\frac{9}{5},2,\frac{11}{5},\frac{12}{5},\frac{13}{5};\frac{ax^5}{9765625}\right)\right)}{152587890625}\right\}\right\}$$

✓ **Maple** : cpu = 0.306 (sec), leaf count = 154

$$\left\{y(x)=x^{\frac{5}{2}}\left(\_C1{}_0Y_5\left(2(-1)^{\frac{9}{10}}a^{1/10}\sqrt{x}\right)+\_C8Y_5\left(2(-1)^{3/10}a^{1/10}\sqrt{x}\right)+\_C9Y_5\left(2(-1)^{\frac{7}{10}}a^{1/10}\sqrt{x}\right)+\_C10Y_5\left(2(-1)^{\frac{1}{10}}a^{1/10}\sqrt{x}\right)\right)\right\}$$

2.1588 ODE No. 1588

$$x^{10}y^{(5)}(x)-ay(x)=0$$

✓ **Mathematica** : cpu = 10.0839 (sec), leaf count = 114

$$\left\{\left\{y(x) \rightarrow c_1x^4e^{-\frac{\sqrt[5]{a}}{x}}+c_2x^4e^{\frac{\sqrt[5]{-1}\sqrt[5]{a}}{x}}+c_3x^4e^{-\frac{(-1)^{2/5}\sqrt[5]{a}}{x}}+c_4x^4e^{\frac{(-1)^{3/5}\sqrt[5]{a}}{x}}+c_5x^4e^{-\frac{(-1)^{4/5}\sqrt[5]{a}}{x}}\right\}\right\}$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 90

$$\left\{y(x)=\_C1{}_0F_4\left(\left(\frac{6}{5},\frac{7}{5},\frac{8}{5},\frac{9}{5};-\frac{a}{3125x^5}\right)+\_C2{}_0F_4\left(\left(\frac{4}{5},\frac{6}{5},\frac{7}{5},\frac{8}{5};-\frac{a}{3125x^5}\right)+\_C3{}_0F_4\left(\left(\frac{3}{5},\frac{4}{5},\frac{6}{5},\frac{7}{5};-\frac{a}{3125x^5}\right)+\_C4{}_0F_4\left(\left(\frac{2}{5},\frac{3}{5},\frac{5}{5},\frac{6}{5};-\frac{a}{3125x^5}\right)\right)\right)\right\}$$

2.1589 ODE No. 1589

$$x^{11/2}y^{(11)}(x)-ay(x)=0$$

✓ **Mathematica** : cpu = 0.0327609 (sec), leaf count = 670

$$\left\{\left\{y(x) \rightarrow \frac{4}{121}(-1)^{2/11}a^{2/11}c_2x{}_0F_{10}\left(\left(-\frac{7}{11},-\frac{5}{11},-\frac{3}{11},-\frac{1}{11},\frac{1}{11},\frac{3}{11},\frac{5}{11},\frac{7}{11},\frac{9}{11},\frac{13}{11};\frac{2048ax^{11/2}}{285311670611}\right)\right)-\frac{1048576}{121}(-1)^{1/11}a^{1/11}c_3x{}_0F_{10}\left(\left(-\frac{7}{11},-\frac{5}{11},-\frac{3}{11},-\frac{1}{11},\frac{1}{11},\frac{3}{11},\frac{5}{11},\frac{7}{11},\frac{9}{11},\frac{13}{11};\frac{2048ax^{11/2}}{285311670611}\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 11.161 (sec), leaf count = 4355

2.1590 ODE No. 1590

$$(x-a)^5(x-b)^5y^{(5)}(x)-cy(x)=0$$

✗ **Mathematica** : cpu = 300.001 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 1.386 (sec), leaf count = 553

$$\left\{y(x)=ODESolStruc\left(e^{\int -4\frac{-g(\_f)\left((-b-\_f/4)e^{\left(\int -g(\_f)\frac{d\_f+\_C1}{(a-b)+a+\_f/4}\right)}d\_f+\_C2}{e^{\left(\int -g(\_f)\frac{d\_f+\_C1}{(a-b)}-1\right)}d\_f+\_C2}\right)},\left[\left\{\frac{1}{(\_g(\_f))^2}\left(\left(\frac{d^3}{d\_f^3}-g(\_f)\right)\right)\right\}\right]\right\}$$

2.1591 ODE No. 1591

---

$$y''(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0286731 (sec), leaf count = 26

$$\left\{\left\{y(x) \rightarrow \sqrt[3]{6} \wp\left(\frac{x+c_1}{\sqrt[3]{6}};0,c_2\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.015 (sec), leaf count = 12

$$\{y(x)=6\,WeierstrassP(x+_C1,0,_C2)\}$$

2.1592 ODE No. 1592

---

$$y''(x) - 6y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0220555 (sec), leaf count = 14

$$\{\{y(x) \rightarrow \wp(x+c_1;0,c_2)\}\}$$

✓ **Maple** : cpu = 0.008 (sec), leaf count = 10

$$\{y(x)=WeierstrassP(x+_C1,0,_C2)\}$$

2.1593 ODE No. 1593

---

$$y''(x) - 6y(x)^2 - x = 0$$

✗ **Mathematica** : cpu = 18.6699 (sec), leaf count = 0 , could not solve

`DSolve[-x - 6*y[x]^2 + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x),x),x)-6*y(x)^2-x=0,y(x))`

2.1594 ODE No. 1594

---

$$y''(x) - 6y(x)^2 + 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.331716 (sec), leaf count = 373

$$\text{Solve}\left[\frac{4\left(\text{Root}\left[4\#1^3-4\#1^2+c_1\&,2\right]-\text{Root}\left[4\#1^3-4\#1^2+c_1\&,3\right]\right)\left(y(x)-\text{Root}\left[4\#1^3-4\#1^2+c_1\&,1\right]\right)}{\left(c_1+4y(x)^3-4y(x)^2\right)\left(\text{Root}\left[4\#1^3-4\#1^2+c_1\&,1\right]\right)},y(x)\right]$$

✓ **Maple** : cpu = 0.079 (sec), leaf count = 59

$$\left\{\int^{y(x)}\frac{1}{\sqrt{4\_a^3-4\_a^2+_C1}}d\_a-x-_C2=0,\int^{y(x)}-\frac{1}{\sqrt{4\_a^3-4\_a^2+_C1}}d\_a-x-_C2=0\right\}$$

2.1595 ODE No. 1595

---

$$ay(x)^2 + bx + c + y''(x) = 0$$

✗ **Mathematica** : cpu = 20.0873 (sec), leaf count = 0 , could not solve

`DSolve[c + b*x + a*y[x]^2 + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x),x),x)+a*y(x)^2+b*x+c=0,y(x))`

2.1596 ODE No. 1596

---

$$a + y''(x) - 2y(x)^3 - xy(x) = 0$$

✗ **Mathematica** : cpu = 20.3806 (sec), leaf count = 0 , could not solve

`DSolve[a - x*y[x] - 2*y[x]^3 + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x),x),x)-2*y(x)^3-x*y(x)+a=0,y(x))`



2.1602 ODE No. 1602

---

$$(n+1)a^{2n}y(x)^{2n+1} + y''(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.124669 (sec), leaf count = 47

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{\sqrt{c_1 - K[1]^2 (a^{2n} K[1]^{2n} - 1)}} dK[1]^2 = (c_2 + x)^2, y(x)\right]$$

✓ **Maple** : cpu = 0.181 (sec), leaf count = 73

$$\left\{\int^{y(x)} \frac{1}{\sqrt{-a^{2n} - a^{2n+2} + -a^2 + -C1}} d -a - x - -C2 = 0, \int^{y(x)} - \frac{1}{\sqrt{-a^{2n} - a^{2n+2} + -a^2 + -C1}} d -a - x - -C\right.$$

2.1603 ODE No. 1603

---

$$y''(x) - \frac{1}{(ay(x)^2 + bxy(x) + cx^2 + dy(x) + ex + k)^{3/2}} = 0$$

✗ **Mathematica** : cpu = 60.3824 (sec), leaf count = 0 , could not solve  
DSolve[-(k + e\*x + c\*x^2 + d\*y[x] + b\*x\*y[x] + a\*y[x]^2)^(-3/2) + Derivative[2][y][x] == 0,  
✓ **Maple** : cpu = 25.336 (sec), leaf count = 8427

2.1604 ODE No. 1604

---

$$y''(x) - e^{y(x)} = 0$$

✓ **Mathematica** : cpu = 0.0408383 (sec), leaf count = 34

$$\left\{\left\{y(x) \rightarrow \log\left(\frac{1}{2}c_1\left(\tanh^2\left(\frac{1}{2}\sqrt{c_1(c_2+x)^2}\right) - 1\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.18 (sec), leaf count = 23

$$\left\{y\left(x\right)=\ln\left(\frac{1}{2-C1^2}\left(\left(\tan\left(\frac{-C2+x}{2-C1}\right)\right)^2+1\right)\right)\right\}$$

2.1605 ODE No. 1605

---

$$ae^x\sqrt{y(x)} + y''(x) = 0$$

✗ **Mathematica** : cpu = 20.1625 (sec), leaf count = 0 , could not solve  
DSolve[a\*E^x\*Sqrt[y[x]] + Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.942 (sec), leaf count = 107

$$\left\{y\left(x\right)=ODESolStruc\left(\frac{-a}{e^{-2\int -b(-a)d-a-2-C1}},\left[\left\{\frac{d}{d-a}-b(-a)=(-b(-a))^2(\sqrt{-a}b(-a)a+4-a-b(-a)-\right.\right.\right.\right.$$

2.1606 ODE No. 1606

---

$$y''(x) + e^x \sin(y(x)) = 0$$

✗ **Mathematica** : cpu = 20.5749 (sec), leaf count = 0 , could not solve  
DSolve[E^x\*Sin[y[x]] + Derivative[2][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(diff(y(x),x),x)+exp(x)\*sin(y(x))=0,y(x))

2.1607 ODE No. 1607

---

$$a \sin(y(x)) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0795254 (sec), leaf count = 79

$$\left\{ \left\{ y(x) \rightarrow -2 \operatorname{am} \left( \frac{1}{2} \sqrt{(2a + c_1)(x + c_2)^2} \middle| \frac{4a}{2a + c_1} \right) \right\}, \left\{ y(x) \rightarrow 2 \operatorname{am} \left( \frac{1}{2} \sqrt{(2a + c_1)(x + c_2)^2} \middle| \frac{4a}{2a + c_1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.109 (sec), leaf count = 49

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{2 \cos(\_a) a + \_C1}} d\_a - x - \_C2 = 0, \int^{y(x)} - \frac{1}{\sqrt{2 \cos(\_a) a + \_C1}} d\_a - x - \_C2 = 0 \right\}$$

2.1608 ODE No. 1608

---

$$a^2 \sin(y(x)) - b \sin(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.0344509 (sec), leaf count = 0 , could not solve  
DSolve[-(b\*Sin[x]) + a^2\*Sin[y[x]] + Derivative[2][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(diff(y(x),x),x)+a^2\*sin(y(x))-b\*sin(x)=0,y(x))

2.1609 ODE No. 1609

---

$$a^2 \sin(y(x)) - b f(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.0241782 (sec), leaf count = 0 , could not solve  
DSolve[-(b\*f[x]) + a^2\*Sin[y[x]] + Derivative[2][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(diff(y(x),x),x)+a^2\*sin(y(x))-b\*f(x)=0,y(x))

2.1610 ODE No. 1610

---

$$y''(x) - \frac{h\left(\frac{y(x)}{\sqrt{x}}\right)}{x^{3/2}} = 0$$

✓ **Mathematica** : cpu = 3.01079 (sec), leaf count = 754

$$\left\{ \operatorname{Solve} \left[ \int_1^{y(x)} \frac{2}{\sqrt{x} \sqrt{\frac{K[3]^2 + 4xc_1 + 8x \int_1^{\frac{K[3]}{\sqrt{x}}} h(K[2]) dK[2]}{x}}} dK[3] - \int_1^x \frac{2 \left( \frac{y(x)}{2\sqrt{K[4]}} - \frac{\sqrt{\frac{y(x)^2}{2K[4]} + 2c_1 + 4 \int_1^{\frac{y(x)}{\sqrt{K[4]}}} h(K[2]) dK[2]}}{\sqrt{2}}} \right)}{K[4] \sqrt{\frac{y(x)^2 + 4c_1 K[4] + 8K[4] \int_1^{\frac{y(x)}{\sqrt{K[4]}}} h(K[2]) dK[2]}{K[4]}}} dx \right] \right\}$$

✓ **Maple** : cpu = 0.183 (sec), leaf count = 92

$$\left\{ y(x) = \operatorname{RootOf} \left( -\ln(x) - 2 \int^{-Z} \frac{1}{\sqrt{\_C1 + 8 \int h(\_g) d\_g + \_g^2}} d\_g + 2 \_C2 \right) \sqrt{x}, y(x) = \operatorname{RootOf} \left( -\ln(x) - 2 \int^{-Z} \frac{1}{\sqrt{\_C1 + 8 \int h(\_g) d\_g + \_g^2}} d\_g + 2 \_C2 \right) \sqrt{x} \right\}$$

2.1611 ODE No. 1611

---

$$y''(x) - 3y'(x) - y(x)^2 - 2y(x) = 0$$

✗ **Mathematica** : cpu = 3.42827 (sec), leaf count = 0 , could not solve  
DSolve[-2\*y[x] - y[x]^2 - 3\*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.526 (sec), leaf count = 57

$$\left\{ y(x) = \operatorname{ODESolStruc} \left( \_a, \left[ \left\{ \left( \frac{d}{d\_a} \_b(\_a) \right) \_b(\_a) - 3 \_b(\_a) - \_a^2 - 2 \_a = 0 \right\}, \left\{ \_a = y(x), \_b(\_a) = y'(x) \right\} \right] \right) \right\}$$



2.1612 ODE No. 1612

---

$$y''(x) - 7y'(x) - y(x)^{3/2} + 12y(x) = 0$$

✗ **Mathematica** : cpu = 21.4916 (sec), leaf count = 0 , could not solve  
DSolve[12\*y[x] - y[x]^(3/2) - 7\*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.983 (sec), leaf count = 57

$$\left\{y(x) = ODESolStruc\left(-a, \left[\left(\frac{d}{d\_a} b(-a)\right) b(-a) - 7 b(-a) - a^{\frac{3}{2}} + 12 a = 0\right], \left\{-a = y(x), b(-a)\right.\right.\right.$$

2.1613 ODE No. 1613

---

$$6a^2y(x) + 5ay'(x) + y''(x) - 6y(x)^2 = 0$$

✗ **Mathematica** : cpu = 1.85929 (sec), leaf count = 0 , could not solve  
DSolve[6\*a^2\*y[x] - 6\*y[x]^2 + 5\*a\*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.017 (sec), leaf count = 27

$$\left\{y(x) = WeierstrassP\left(-\frac{e^{-ax}}{a} + C1, 0, C2\right) (e^{-ax})^2\right\}$$

2.1614 ODE No. 1614

---

$$2a^2y(x) + 3ay'(x) + y''(x) - 2y(x)^3 = 0$$

✗ **Mathematica** : cpu = 0.941164 (sec), leaf count = 0 , could not solve  
DSolve[2\*a^2\*y[x] - 2\*y[x]^3 + 3\*a\*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.052 (sec), leaf count = 33

$$\left\{y(x) = \frac{C2}{e^{ax}} JacobiSN\left(\left(-\frac{1}{a}\sqrt{-e^{-2ax}} + C1\right) C2, i\right)\right\}$$

2.1615 ODE No. 1615

---

$$-\frac{2(n+1)(n+2)y(x)\left(y(x)^{\frac{n}{n+1}}-1\right)}{n^2}-\frac{(3n+4)y'(x)}{n}+y''(x)=0$$

✗ **Mathematica** : cpu = 76.2803 (sec), leaf count = 0 , could not solve  
DSolve[(-2\*(1 + n)\*(2 + n)\*y[x]\*(-1 + y[x]^(n/(1 + n))))/n^2 - ((4 + 3\*n)\*Derivative[1][y][x] + Derivative[2][y][x]) == 0, y[x], x]  
✓ **Maple** : cpu = 3.733 (sec), leaf count = 91

$$\left\{y(x) = ODESolStruc\left(-a, \left[\frac{1}{n^2}\left(-2(n+2)(n+1) a a^{\frac{n}{n+1}} + \left(\frac{d}{d\_a} b(-a)\right) b(-a) n^2 + (-3 n^2 - 4 n)\right.\right.\right.$$

2.1616 ODE No. 1616

---

$$\frac{1}{4}\left(a^2-1\right)y(x)+ay'(x)+by(x)^n+y''(x)=0$$

✗ **Mathematica** : cpu = 16.6146 (sec), leaf count = 0 , could not solve  
DSolve[((-1 + a^2)\*y[x])/4 + b\*y[x]^n + a\*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.85 (sec), leaf count = 63

$$\left\{y(x) = ODESolStruc\left(-a, \left[\left(\frac{d}{d\_a} b(-a)\right) b(-a) + a b(-a) + b a^n + \frac{a a^2}{4} - \frac{a}{4} = 0\right], \left\{-a = y(x), b(-a)\right.\right.\right.$$

2.1617 ODE No. 1617

---

$$ay'(x) + bx^ry(x)^n + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.0320168 (sec), leaf count = 0 , could not solve  
DSolve[b\*x^r\*y[x]^n + a\*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(diff(y(x),x),x)+a\*diff(y(x),x)+b\*x^r\*y(x)^n=0,y(x))

2.1618 ODE No. 1618

$$ay'(x) - 2a + be^{y(x)} + y''(x) = 0$$

✗ **Mathematica** : cpu = 26.6012 (sec), leaf count = 0 , could not solve  
DSolve[-2\*a + b\*E^y[x] + a\*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 1.241 (sec), leaf count = 56

$$\left\{y(x) = ODESolStruc\left(\_a, \left[\left(\frac{d}{d\_a} \_b(\_a)\right) \_b(\_a) + a\_b(\_a) + be^{-a} - 2a = 0\right], \left\{\_a = y(x), \_b(\_a)\right.\right.\right.$$

2.1619 ODE No. 1619

$$ay'(x) + f(x) \sin(y(x)) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.0417926 (sec), leaf count = 0 , could not solve  
DSolve[f[x]\*Sin[y[x]] + a\*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(diff(y(x), x), x) + a\*diff(y(x), x) + f(x)\*sin(y(x)) = 0, y(x))

2.1620 ODE No. 1620

$$y''(x) + y(x)y'(x) - y(x)^3 = 0$$

✓ **Mathematica** : cpu = 2.8407 (sec), leaf count = 492

$$\left\{\left\{y(x) \rightarrow \text{InverseFunction}\left[\int_1^{\#1} \frac{2}{\frac{e^{6c_1} K[1]^4}{\sqrt[3]{e^{18c_1} K[1]^6 - 2e^{12c_1} + 2\sqrt{e^{24c_1} - e^{30c_1} K[1]^6}}} - K[1]^2 + e^{-6c_1} \sqrt[3]{e^{18c_1} K[1]^6 - 2e^{12c_1} + 2\sqrt{e^{24c_1} - e^{30c_1} K[1]^6}}}}\right.\right.$$

✓ **Maple** : cpu = 0.086 (sec), leaf count = 253

$$\left\{\int^{y(x)} \left(\frac{-a^2}{2} + \frac{1}{2} \left(\sqrt[3]{-C1 + \sqrt{-a^6 + -C1^2}} - a^2 \frac{1}{\sqrt[3]{-C1 + \sqrt{-a^6 + -C1^2}}}\right)^2\right)^{-1} d\_a - x - \_C2 = 0, \int^y$$

2.1621 ODE No. 1621

$$ay(x) + y''(x) + y(x)y'(x) - y(x)^3 = 0$$

✓ **Mathematica** : cpu = 9.43759 (sec), leaf count = 990

$$\left\{\left\{y(x) \rightarrow \text{InverseFunction}\left[\int_1^{\#1} \frac{e^{6c_1} (a - K[1]^2)^2}{2 \sqrt[3]{e^{18c_1} K[1]^6 - 3ae^{18c_1} K[1]^4 + 3a^2 e^{18c_1} K[1]^2 - 2e^{12c_1} - a^3 e^{18c_1} + 2\sqrt{-e^{30c_1} K[1]^6 + 3ae^{30c_1} K[1]^4}}}}\right.\right.$$

✓ **Maple** : cpu = 1.922 (sec), leaf count = 1088

$$\left\{\int^{y(x)} \frac{1}{-63\_a^2 + 63a} \left(\frac{\left(\frac{i}{2}\sqrt{3} - \frac{1}{2}\right)^3}{2} \left(126 \frac{1}{-\_a^6 + 3a\_a^4 - 3\_a^2 a^2 + 80\_C1^3 + a^3} \sqrt[3]{-4 \left(-C1 \sqrt{5} \sqrt{\frac{-C1}{-\_a^6 + -C1}} - 1\right)}\right)^{\frac{2}{3}}\right.$$

2.1622 ODE No. 1622

$$2a^2y(x) + (3a + y(x))y'(x) + ay(x)^2 + y''(x) - y(x)^3 = 0$$

✗ **Mathematica** : cpu = 24.0901 (sec), leaf count = 0 , could not solve  
DSolve[2\*a^2\*y[x] + a\*y[x]^2 - y[x]^3 + (3\*a + y[x])\*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.244 (sec), leaf count = 415

$$\left\{y(x) = \frac{1}{e^{ax}} \text{RootOf}\left(\int^{-Z} \frac{1}{-\_f^6 + -C1} \left(-f^8 - \_C1 \_f^2 + \left((-f^6 + -C1)^2 \left(\sqrt{\frac{-C1}{-\_f^6 + -C1}} - 1\right)\right)^{\frac{2}{3}}\right)\right.$$

**2.1623                    ODE No. 1623**

---

$$y(x)\left(f'(x)+2f(x)^2\right)+\left(3f(x)+y(x)\right)y'(x)+f(x)y(x)^2+y''(x)-y(x)^3=0$$

✗ **Mathematica** : cpu = 0.123667 (sec), leaf count = 0 , could not solve  
DSolve[f[x]\*y[x]^2 - y[x]^3 + y[x]\*(2\*f[x]^2 + Derivative[1][f][x])] + (3\*f[x] + y[x])\*Deriva  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(diff(y(x),x),x)+(y(x)+3\*f(x))\*diff(y(x),x)-y(x)^3+f(x)\*y(x)^2+y(x)\*(diff(f(x),x)

**2.1624                    ODE No. 1624**

---

$$y(x)\left(af(x)^2-\frac{f''(x)}{f(x)}+3f'(x)+\frac{3f'(x)^2}{f(x)^2}\right)+bf(x)^3-\left(\frac{f'(x)}{f(x)}+f(x)\right)\left(3y'(x)+y(x)^2\right)+y''(x)+y(x)y'(x)-y(x)^3=0$$

✗ **Mathematica** : cpu = 0.667678 (sec), leaf count = 0 , could not solve  
DSolve[b\*f[x]^3 - y[x]^3 + y[x]\*Derivative[1][y][x] - (f[x] + Derivative[1][f][x]/f[x])\*(y[x]  
✓ **Maple** : cpu = 1.295 (sec), leaf count = 131

$$\left\{y(x)=ODESolStruc\left(f\left(RootOf\left(\int\_b(\_a)\,d\_a+\_C1-\int^{-Z}f(\_f)\,d\_f\right)\right)\_a,\left[\frac{d}{d\_a}\_b(\_a)=(\_b\right.\right.\right.$$

**2.1625                    ODE No. 1625**

---

$$y'(x)\left(y(x)-\frac{3f'(x)}{2f(x)}\right)-\frac{y(x)^2f'(x)}{2f(x)}+y(x)\left(-\frac{f''(x)}{2f(x)}+\frac{f'(x)^2}{f(x)^2}+f(x)\right)+y''(x)-y(x)^3=0$$

✗ **Mathematica** : cpu = 0.388766 (sec), leaf count = 0 , could not solve  
DSolve[-y[x]^3 - (y[x]^2\*Derivative[1][f][x])/(2\*f[x]) + (y[x] - (3\*Derivative[1][f][x])/(2\*f[x]))  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(diff(y(x),x),x)+(y(x)-3/2\*diff(f(x),x)/f(x))\*diff(y(x),x)-y(x)^3-1/2\*diff(f(x),x)  
diff(diff(f(x),x),x))/f(x)\*y(x)=0,y(x))

**2.1626                    ODE No. 1626**

---

$$y(x)f'(x)+f(x)y'(x)+y''(x)+2y(x)y'(x)=0$$

✗ **Mathematica** : cpu = 28.1201 (sec), leaf count = 0 , could not solve  
DSolve[y[x]\*Derivative[1][f][x] + f[x]\*Derivative[1][y][x] + 2\*y[x]\*Derivative[1][y][x] + De  
✓ **Maple** : cpu = 0.17 (sec), leaf count = 48

$$\left\{y(x)=ODESolStruc\left(\_b(\_a),\left[\frac{d}{d\_a}\_b(\_a)=-f(\_a)\_b(\_a)-(\_b(\_a))^2-\_C1\right],\{\_a=x,\_b(\_a\right.\right.$$

**2.1627                    ODE No. 1627**

---

$$f(x)\left(y'(x)+y(x)^2\right)-g(x)+y''(x)+2y(x)y'(x)=0$$

✗ **Mathematica** : cpu = 0.128153 (sec), leaf count = 0 , could not solve  
DSolve[-g[x] + 2\*y[x]\*Derivative[1][y][x] + f[x]\*(y[x]^2 + Derivative[1][y][x]) + Derivative  
✓ **Maple** : cpu = 0.612 (sec), leaf count = 58

$$\left\{y(x)=ODESolStruc\left(\_b(\_a),\left\{-\int e^{\int f(\_a)\,d\_a}g(\_a)\,d\_a+\left((\_b(\_a))^2+\frac{d}{d\_a}\_b(\_a)\right)e^{\int f(\_a)\,d\_a}+\right.\right.\right.$$

**2.1628                    ODE No. 1628**

---

$$f(x)y(x)-g(x)+y''(x)+3y(x)y'(x)+y(x)^3=0$$

✗ **Mathematica** : cpu = 4.99384 (sec), leaf count = 0 , could not solve  
DSolve[-g[x] + f[x]\*y[x] + y[x]^3 + 3\*y[x]\*Derivative[1][y][x] + Derivative[2][y][x] == 0, y  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x)=\frac{\frac{d}{dx}DESol\left(\left\{-g(x)\_Y(x)+f(x)\frac{d}{dx}\_Y(x)+\frac{d^3}{dx^3}\_Y(x)\right\},\{\_Y(x)\}\right)}{DESol\left(\left\{-g(x)\_Y(x)+f(x)\frac{d}{dx}\_Y(x)+\frac{d^3}{dx^3}\_Y(x)\right\},\{\_Y(x)\}\right)}\right\}$$

2.1629 ODE No. 1629

---

$$(f(x)+3y(x))y'(x)+f(x)y(x)^2+y''(x)+y(x)^3=0$$

✗ **Mathematica** : cpu = 0.0240628 (sec), leaf count = 0 , could not solve  
DSolve[f[x]\*y[x]^2 + y[x]^3 + (f[x] + 3\*y[x])\*Derivative[1][y][x] + Derivative[2][y][x] == 0  
✓ **Maple** : cpu = 0.027 (sec), leaf count = 38

$$\left\{y(x)=\frac{\int\_C1\,\mathrm{e}^{-\int f(x)\,\mathrm{d}x}\,\mathrm{d}x+\_C2}{\iint\_C1\,\mathrm{e}^{-\int f(x)\,\mathrm{d}x}\,\mathrm{d}x\,\mathrm{d}x+\_C2\,x+1}\right\}$$

2.1630 ODE No. 1630

---

$$-4a^2y(x)-3ay(x)^2-b+y''(x)-3y(x)y'(x)=0$$

✓ **Mathematica** : cpu = 7.4608 (sec), leaf count = 3227

$$\left\{\left\{y(x)\rightarrow-\frac{2\left((-1)^{\frac{a^{3/2}\sqrt{4a^3-3b}-2a^3}{4a^3}}+12^{-\frac{3\left(a^{3/2}\sqrt{4a^3-3b}-2a^3\right)}{4a^3}+\frac{3\sqrt{4a^6-3a^3b}}{4a^3}+1\right)3^{\frac{a^{3/2}\sqrt{4a^3-3b}-2a^3}{4a^3}}-\frac{\sqrt{4a^6-3a^3b}}{4a^3}a^{-\frac{a^{3/2}\sqrt{4a^3-3b}-2a^3}{2a^3}}}{\right.}\right.$$

✓ **Maple** : cpu = 0.424 (sec), leaf count = 783

$$\left\{\int^{y(x)}-6\,a^2\left(-12\,a\,a^3-9\,a^2a^2+\left(RootOf\left(2\,K_{1/2}\frac{4\,a^3-3\,b}{\sqrt{4\,a^4-3\,a\,b\,a}}\left(-1/2\frac{Z}{a^2}\right)-C1\,a^2+3\,K_{1/2}\frac{4\,a^3-3\,b}{\sqrt{4\,a^4-3\,a\,b\,a}}\left(-1\right.\right.\right.\right.$$

2.1631 ODE No. 1631

---

$$-(f(x)+3y(x))y'(x)+f(x)y(x)^2+y''(x)+y(x)^3=0$$

✗ **Mathematica** : cpu = 0.0195061 (sec), leaf count = 0 , could not solve  
DSolve[f[x]\*y[x]^2 + y[x]^3 - (f[x] + 3\*y[x])\*Derivative[1][y][x] + Derivative[2][y][x] == 0  
✓ **Maple** : cpu = 0.04 (sec), leaf count = 38

$$\left\{y(x)=\frac{-\int\_C1\,\mathrm{e}^{\int f(x)\,\mathrm{d}x}\,\mathrm{d}x-\_C2}{\iint\_C1\,\mathrm{e}^{\int f(x)\,\mathrm{d}x}\,\mathrm{d}x\,\mathrm{d}x+\_C2\,x+1}\right\}$$

2.1632 ODE No. 1632

---

$$y''(x)-2ay(x)y'(x)=0$$

✓ **Mathematica** : cpu = 0.0502729 (sec), leaf count = 46

$$\left\{\left\{y(x)\rightarrow\frac{\sqrt{c_1}\tan\left(\sqrt{a}\sqrt{c_1}x+\sqrt{a}\sqrt{c_1}c_2\right)}{\sqrt{a}}\right\}\right\}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 23

$$\left\{y(x)=\frac{1}{a}\tan\left(\sqrt{a\_C1}\left(\_C2+x\right)\right)\sqrt{a\_C1}\right\}$$

2.1633 ODE No. 1633

---

$$ay(x)y'(x)+by(x)^3+y''(x)=0$$

✗ **Mathematica** : cpu = 33.2699 (sec), leaf count = 0 , could not solve  
DSolve[b\*y[x]^3 + a\*y[x]\*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.231 (sec), leaf count = 97

$$\left\{\int^{y(x)}\left(RootOf\left(-2\,a\_a^2\,Artanh\left(\frac{\_a^2a+4\_Z}{\sqrt{\_a^4\left(a^2-8\,b\right)}}\right)+\_C1\sqrt{\_a^4\left(a^2-8\,b\right)}-\ln\left(\_a^4b+\_Z\_a^2a+2\_Z\right.\right.\right.\right.$$

2.1634 ODE No. 1634

---

$$y'(x)h(x,y(x))+j(x,y(x))+y''(x)=0$$

✗ **Mathematica** : cpu = 0.107367 (sec), leaf count = 0 , could not solve  
DSolve[j[x, y[x]] + h[x, y[x]]\*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(diff(y(x),x),x)+h(x,y(x))\*diff(y(x),x)+j(x,y(x))=0,y(x))

2.1635 ODE No. 1635

---

$$ay'(x)^2+by(x)+y''(x)=0$$

✓ **Mathematica** : cpu = 0.579576 (sec), leaf count = 104

$$\left\{\left\{y(x)\rightarrow \text{InverseFunction}\left[\int_1^{\#1}-\frac{\sqrt{2}a}{\sqrt{2e^{-2aK[1]}c_1a^2-2bK[1]a+b}}dK[1]\&\right][c_2+x]\right\},\left\{y(x)\rightarrow \text{InverseFunction}\right.\right.$$

✓ **Maple** : cpu = 0.154 (sec), leaf count = 79

$$\left\{\int^{y(x)}-2\frac{a}{\sqrt{4e^{-2a}a}C1a^2-4b_aa+2b}d_a-x-C2=0,\int^{y(x)}2\frac{a}{\sqrt{4e^{-2a}a}C1a^2-4b_aa+2b}d_a-x-C2=0\right\}$$

2.1636 ODE No. 1636

---

$$ay'(x)\left|y'(x)\right|+by'(x)+cy(x)+y''(x)=0$$

✗ **Mathematica** : cpu = 8.63707 (sec), leaf count = 0 , could not solve  
DSolve[c\*y[x] + b\*Derivative[1][y][x] + a\*Abs[Derivative[1][y][x]]\*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.667 (sec), leaf count = 59

$$\left\{y(x)=ODESolStruc\left(-a,\left[\left\{\left(\frac{d}{d_a}b(-a)\right)-b(-a)+a_b(-a)|_b(-a)|+_b(-a)b+c_a=0\right\},\left\{-a=y(x)\right\}\right]\right\}$$

2.1637 ODE No. 1637

---

$$ay'(x)^2+by'(x)+cy(x)+y''(x)=0$$

✗ **Mathematica** : cpu = 19.505 (sec), leaf count = 0 , could not solve  
DSolve[c\*y[x] + b\*Derivative[1][y][x] + a\*Derivative[1][y][x]^2 + Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.401 (sec), leaf count = 58

$$\left\{y(x)=ODESolStruc\left(-a,\left[\left\{\left(\frac{d}{d_a}b(-a)\right)-b(-a)+a(-b(-a))^2+_b(-a)b+c_a=0\right\},\left\{-a=y(x)\right\}\right]\right\}$$

2.1638 ODE No. 1638

---

$$ay'(x)^2+b\sin(y(x))+y''(x)=0$$

✓ **Mathematica** : cpu = 4.59654 (sec), leaf count = 146

$$\left\{\left\{y(x)\rightarrow \text{InverseFunction}\left[\int_1^{\#1}-\frac{\sqrt{4a^2+1}}{\sqrt{4e^{-2aK[1]}c_1a^2-4b\sin(K[1])a+e^{-2aK[1]}c_1+2b\cos(K[1])}}dK[1]\&\right][c_2+x]\right\},\left\{y(x)\rightarrow \text{InverseFunction}\right.\right.$$

✓ **Maple** : cpu = 0.161 (sec), leaf count = 115

$$\left\{\int^{y(x)}(-4a^2-1)\frac{1}{\sqrt{16_C1(a^2+1/4)^2e^{-2a}a-16(a^2+1/4)(a\sin(-a)-1/2\cos(-a))b}}d_a-x-C2=0\right\}$$

2.1639 ODE No. 1639

---

$$ay'(x)|y'(x)| + b\sin(y(x)) + y''(x) = 0$$

✗ **Mathematica** : cpu = 40.5905 (sec), leaf count = 0 , could not solve  
DSolve[b\*Sin[y[x]] + a\*Abs[Derivative[1][y][x]]\*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 2.365 (sec), leaf count = 56

$$\left\{y(x) = ODESolStruc\left(-a, \left[\left(\frac{d}{d\_a} b(-a)\right) b(-a) + a b(-a) |b(-a)| + b \sin(-a) = 0\right], \left[-a = y(x)\right]\right.\right\}$$

2.1640 ODE No. 1640

---

$$ay(x)y'(x)^2 + by(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.464532 (sec), leaf count = 96

$$\left\{\left\{y(x) \rightarrow \text{InverseFunction}\left[\int_1^{\#1} -\frac{\sqrt{a}}{\sqrt{e^{2ac_1-aK[1]^2}-b}} dK[1] \& \right][c_2+x]\right\}, \left\{y(x) \rightarrow \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{\sqrt{e^{2ac_1-aK[1]^2}-b}} dK[1] \& \right][c_2+x]\right\}\right\}$$

✓ **Maple** : cpu = 0.171 (sec), leaf count = 70

$$\left\{\int^{y(x)} a \frac{1}{\sqrt{a(e^{-a^2a} C1 a - b)}} d\_a - x - \_C2 = 0, \int^{y(x)} -a \frac{1}{\sqrt{a(e^{-a^2a} C1 a - b)}} d\_a - x - \_C2 = 0\right\}$$

2.1641 ODE No. 1641

---

$$g(x)y'(x) + h(y(x))y'(x)^2 + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0523782 (sec), leaf count = 61

$$\left\{\left\{y(x) \rightarrow \text{InverseFunction}\left[\int_1^{\#1} \exp\left(-\int_1^{K[4]} -h(K[1])dK[1]\right) dK[4] \& \right]\left[\int_1^x -\exp\left(-\int_1^{K[5]} g(K[2])dK[2]\right) dK[5] \& \right]\right\}\right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 29

$$\left\{\int^{y(x)} e^{\int h(\_b) d\_b} d\_b - \_C1 \int e^{-\int g(x) dx} dx - \_C2 = 0\right\}$$

2.1642 ODE No. 1642

---

$$f(x)h(y(x)) + g(x)y'(x) - \frac{j(y(x))y'(x)^2}{h(y(x))} + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.323725 (sec), leaf count = 0 , could not solve  
DSolve[f[x]\*h[y[x]] + g[x]\*Derivative[1][y][x] - (j[y[x]]\*Derivative[1][y][x]^2)/h[y[x]] + Derivative[2][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(diff(y(x),x),x)-j(y(x))/h(y(x))\*diff(y(x),x)^2+g(x)\*diff(y(x),x)+f(x)\*h(y(x))=0, y(x), x)

2.1643 ODE No. 1643

---

$$f(x)y'(x) + g(x)j(y(x)) + h(y(x))y'(x)^2 + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.127049 (sec), leaf count = 0 , could not solve  
DSolve[g[x]\*j[y[x]] + f[x]\*Derivative[1][y][x] + h[y[x]]\*Derivative[1][y][x]^2 + Derivative[2][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve((1-D(j)(y(x)))/j(y(x))\*diff(y(x),x)^2+f(x)\*diff(y(x),x)+diff(diff(y(x),x),x)+g(x)\*j(y(x))=0, y(x), x)

2.1644 ODE No. 1644

---

$$h(y(x))y'(x)^2 + j(y(x))y'(x) + k(y(x)) + y''(x) = 0$$

✗ **Mathematica** : cpu = 82.1647 (sec), leaf count = 0 , could not solve  
DSolve[k[y[x]] + j[y[x]]\*Derivative[1][y][x] + h[y[x]]\*Derivative[1][y][x]^2 + Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.381 (sec), leaf count = 56

$$\left\{y(x) = ODESolStruc\left(-a, \left[\left(\frac{d}{d\_a} b(-a)\right) b(-a) + h(-a) (-b(-a))^2 + b(-a) + k(-a) = 0\right], \left[-a = y(x)\right]\right.\right\}$$



2.1650 ODE No. 1650

---

$$y''(x) - a\sqrt{y'(x)^2 + 1} = 0$$

✓ **Mathematica** : cpu = 0.0998381 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow \frac{\sinh(c_1) \sinh(ax)}{a} + \frac{\cosh(c_1) \cosh(ax)}{a} + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.224 (sec), leaf count = 16

$$\left\{ y(x) = \frac{\cosh(a(-C1 + x))}{a} + -C2 \right\}$$

2.1651 ODE No. 1651

---

$$a\left(-\sqrt{y'(x)^2 + 1}\right) - b + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.327977 (sec), leaf count = 414

$$\left\{ \left\{ y(x) \rightarrow \frac{a \operatorname{InverseFunction}\left[\frac{b \tan^{-1}\left(\frac{\#1 b}{\sqrt{\#1^2 + 1} \sqrt{a^2 - b^2}}\right) - \frac{b \tan^{-1}\left(\frac{\#1 a}{\sqrt{a^2 - b^2}}\right) + \sinh^{-1}(\#1)}{a}\right] \& [c_1 + x]^2 - b}{\sqrt{\dots}} \right\} \right\}$$

✓ **Maple** : cpu = 0.135 (sec), leaf count = 31

$$\left\{ y(x) = \int \operatorname{RootOf}\left(x - \int^{-Z} \left(a\sqrt{-f^2 + 1} + b\right)^{-1} d_-f + -C1\right) dx + -C2 \right\}$$

2.1652 ODE No. 1652

---

$$y''(x) - a\sqrt{by(x)^2 + y'(x)^2} = 0$$

✗ **Mathematica** : cpu = 0.291901 (sec), leaf count = 0 , could not solve  
DSolve[-(a\*Sqrt[b\*y[x]^2 + Derivative[1][y][x]^2]) + Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.226 (sec), leaf count = 36

$$\left\{ y(x) = e^{\int \operatorname{RootOf}\left(x - \int^{-Z} \left(-f^2 + a\sqrt{-f^2 + b}\right)^{-1} d_-f + -C1\right) dx + -C2} \right\}$$

2.1653 ODE No. 1653

---

$$y''(x) - a\left(y'(x)^2 + 1\right)^{3/2} = 0$$

✓ **Mathematica** : cpu = 0.139065 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow c_2 - \frac{i\sqrt{a^2 x^2 + 2ac_1 x + c_1^2 - 1}}{a} \right\}, \left\{ y(x) \rightarrow c_2 + \frac{i\sqrt{a^2 x^2 + 2ac_1 x + c_1^2 - 1}}{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.104 (sec), leaf count = 41

$$\left\{ y(x) = \frac{1}{a} \left( \left( -1 + (-C1 + x)^2 a^2 \right) \sqrt{-\left( -1 + (-C1 + x)^2 a^2 \right)^{-1} + -C2 a} \right) \right\}$$





2.1658 ODE No. 1658

---

$$y''(x) - h\left(y'(x), ax + by(x)\right) = 0$$

✗ **Mathematica** : cpu = 0.168427 (sec), leaf count = 0 , could not solve  
DSolve[-h[Derivative[1][y][x], a\*x + b\*y[x]] + Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.125 (sec), leaf count = 115

$$\left\{y(x) = ODESolStruc\left(-\frac{a\left(\int_{-a} b(\_a) d\_a + \_C1\right) - b\_a}{b}, \left[\frac{d}{d\_a} b(\_a) = -h\left(\frac{-a\_b(\_a) + b}{\_b(\_a)b}, b\_a\right)\right]\right.\right.$$

2.1659 ODE No. 1659

---

$$y''(x) - y(x)h\left(x, \frac{y'(x)}{y(x)}\right) = 0$$

✗ **Mathematica** : cpu = 7.71447 (sec), leaf count = 0 , could not solve  
DSolve[-(h[x, Derivative[1][y][x]/y[x]]\*y[x]) + Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.076 (sec), leaf count = 60

$$\left\{y(x) = ODESolStruc\left(e^{\int_{-a} b(\_a) d\_a + \_C1}, \left[\frac{d}{d\_a} b(\_a) = -(\_b(\_a))^2 + h(\_a, \_b(\_a))\right]\right), \left\{\_a = x, \_b\right.$$

2.1660 ODE No. 1660

---

$$y''(x) - x^{n-2}h\left(x^{-n}y(x), x^{1-n}y'(x)\right) = 0$$

✗ **Mathematica** : cpu = 2.38357 (sec), leaf count = 0 , could not solve  
DSolve[-(x^(-2 + n)\*h[y[x]/x^n, x^(1 - n)\*Derivative[1][y][x]]) + Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.609 (sec), leaf count = 125

$$\left\{y(x) = ODESolStruc\left(\frac{\_a}{e^{-(\int_{-a} b(\_a) d\_a + \_C1)n}}, \left[\frac{d}{d\_a} b(\_a) = (\_b(\_a))^2\left(-\_b(\_a)h\left(\_a, \frac{b(\_a) - a}{\_b(\_a)}\right)\right)\right]\right.\right.$$

2.1661 ODE No. 1661

---

$$8y''(x) + 9y'(x)^4 = 0$$

✓ **Mathematica** : cpu = 0.0694049 (sec), leaf count = 92

$$\left\{\left\{y(x) \rightarrow c_2 - \frac{1}{3}\sqrt[3]{-\frac{1}{3}(9x - 8c_1)^{2/3}}\right\}, \left\{y(x) \rightarrow \frac{(9x - 8c_1)^{2/3}}{3\sqrt[3]{3}} + c_2\right\}, \left\{y(x) \rightarrow \frac{(-1)^{2/3}(9x - 8c_1)^{2/3}}{3\sqrt[3]{3}} + c_2\right\}\right\}$$

✓ **Maple** : cpu = 0.063 (sec), leaf count = 51

$$\left\{y(x) = (\_C1 + x)^{\frac{2}{3}} + \_C2, y(x) = \frac{i\sqrt{3} - 1}{2}(\_C1 + x)^{\frac{2}{3}} + \_C2, y(x) = -\frac{i\sqrt{3} + 1}{2}(\_C1 + x)^{\frac{2}{3}} + \_C2\right\}$$

2.1662 ODE No. 1662

---

$$ay''(x) + cy(x) + h\left(y'(x)\right) = 0$$

✗ **Mathematica** : cpu = 0.846618 (sec), leaf count = 0 , could not solve  
DSolve[h[Derivative[1][y][x]] + c\*y[x] + a\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.306 (sec), leaf count = 56

$$\left\{y(x) = ODESolStruc\left(\_a, \left[\left(\frac{d}{d\_a} b(\_a)\right) \_b(\_a) + \frac{h(\_b(\_a)) + c\_a}{a} = 0\right]\right), \left\{\_a = y(x), \_b(\_a) = \right.$$

**2.1663                    ODE No. 1663**

---

$$-xy(x)^n + xy''(x) + 2y'(x) = 0$$

✗ **Mathematica** : cpu = 0.0231406 (sec), leaf count = 0 , could not solve  
DSolve[-(x\*y[x]^n) + 2\*Derivative[1][y][x] + x\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.782 (sec), leaf count = 125

$$\left\{y(x) = ODESolStruc \left( \_a e^{\int \_b(\_a) d\_a + \_C1}, \left[ \left\{ \frac{d}{d\_a} \_b(\_a) = -\frac{(\_b(\_a))^2 \left( \_b(\_a) (n-1)^2 \_a^n + 2 \_a \right)}{4} \right. \right. \right.$$

**2.1664                    ODE No. 1664**

---

$$ax^m y(x)^n + xy''(x) + 2y'(x) = 0$$

✗ **Mathematica** : cpu = 0.169789 (sec), leaf count = 0 , could not solve  
DSolve[a\*x^m\*y[x]^n + 2\*Derivative[1][y][x] + x\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 3.152 (sec), leaf count = 155

$$\left\{y(x) = ODESolStruc \left( \_a e^{\int \_b(\_a) d\_a + \_C1}, \left[ \left\{ \frac{d}{d\_a} \_b(\_a) = \frac{(\_b(\_a))^2 \left( a \_b(\_a) (n-1)^2 \_a^n + (\_a (n-1)^2 \_a^n + \_a \right)}{(m+1)}$$

**2.1665                    ODE No. 1665**

---

$$xy''(x) + 2y'(x) + xe^{y(x)} = 0$$

✗ **Mathematica** : cpu = 0.0944043 (sec), leaf count = 0 , could not solve  
DSolve[E^y[x]\*x + 2\*Derivative[1][y][x] + x\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.519 (sec), leaf count = 84

$$\left\{y(x) = ODESolStruc \left( \_a - 2 \int \_b(\_a) d\_a - 2 \_C1, \left[ \left\{ \frac{d}{d\_a} \_b(\_a) = (e^{-a} - 2) (\_b(\_a))^3 + (\_b(\_a)) \right. \right. \right.$$

**2.1666                    ODE No. 1666**

---

$$ay'(x) + bxe^{y(x)} + xy''(x) = 0$$

✗ **Mathematica** : cpu = 0.258859 (sec), leaf count = 0 , could not solve  
DSolve[b\*E^y[x]\*x + a\*Derivative[1][y][x] + x\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.776 (sec), leaf count = 93

$$\left\{y(x) = ODESolStruc \left( \_a - 2 \int \_b(\_a) d\_a - 2 \_C1, \left[ \left\{ \frac{d}{d\_a} \_b(\_a) = (be^{-a} - 2a + 2) (\_b(\_a))^3 + (a - 2) \_b(\_a) \right. \right. \right.$$

**2.1667                    ODE No. 1667**

---

$$bx^{5-2a}e^{y(x)} + ay'(x) + xy''(x) = 0$$

✗ **Mathematica** : cpu = 0.326493 (sec), leaf count = 0 , could not solve  
DSolve[b\*E^y[x]\*x^(5 - 2\*a) + a\*Derivative[1][y][x] + x\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 1.117 (sec), leaf count = 121

$$\left\{y(x) = ODESolStruc \left( (2a - 6) \int \_b(\_a) d\_a + 2a \_C1 + \_a - 6 \_C1, \left[ \left\{ \frac{d}{d\_a} \_b(\_a) = (be^{-a} + 2a^2 - 8a) \_b(\_a) \right. \right. \right.$$

2.1668 ODE No. 1668

---

$$xy''(x) - (1 - y(x))y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0545519 (sec), leaf count = 60

$$\left\{ \left\{ y(x) \rightarrow \sqrt{2}\sqrt{c_1+2} \tanh \left( \frac{1}{2} \left( \sqrt{2}\sqrt{c_1+2} \log(x) - 2\sqrt{2}\sqrt{c_1+2}c_2 \right) \right) + 2 \right\} \right\}$$

✓ **Maple** : cpu = 0.112 (sec), leaf count = 24

$$\left\{ y(x) = \frac{1}{-C1} \left( 2-C1 + \tanh \left( \frac{\ln(x) - C2}{2-C1} \right) \right) \right\}$$

2.1669 ODE No. 1669

---

$$-x^2y'(x)^2 + xy''(x) + 2y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.452957 (sec), leaf count = 160

Solve  $\left[ \int_1^{y(x)} -\frac{x}{e^{xK[1]c_1} + 2xK[1] + 1} dK[1] - \int_1^x \left( \int_1^{y(x)} \left( \frac{(e^{K[1]K[2]c_1}K[1] + 2K[1])K[2]}{(e^{K[1]K[2]c_1} + 2K[1]K[2] + 1)^2} - \frac{1}{e^{K[1]K[2]c_1} + 2K[1]K[2] + 1} \right) dK[2] \right) dx \right]$

✓ **Maple** : cpu = 0.085 (sec), leaf count = 32

$$\left\{ y(x) = \frac{1}{x} \text{RootOf} \left( -\ln(x) + C2 + \int^{-Z} -\left( e^{-f}C1 - 2f - 1 \right)^{-1} d_f \right) \right\}$$

2.1670 ODE No. 1670

---

$$a(xy'(x) - y(x))^2 - b + xy''(x) = 0$$

✓ **Mathematica** : cpu = 7.3633 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow x \left( \int_1^x \frac{\sqrt{-\frac{b}{a}} \tan \left( c_1 - a\sqrt{-\frac{b}{a}}K[2] \right)}{K[2]^2} dK[2] + c_2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.353 (sec), leaf count = 35

$$\left\{ y(x) = \left( \int \frac{i}{x^2} \tan \left( -i\sqrt{a}\sqrt{b}x + C1 \right) \sqrt{b} \frac{1}{\sqrt{a}} dx + C2 \right) x \right\}$$

2.1671 ODE No. 1671

---

$$2xy''(x) + y'(x)^3 + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.101041 (sec), leaf count = 59

$$\left\{ \left\{ y(x) \rightarrow c_2 - 2ie^{c_1} \sqrt{e^{2c_1} - x} \right\}, \left\{ y(x) \rightarrow c_2 + 2ie^{c_1} \sqrt{e^{2c_1} - x} \right\} \right\}$$

✓ **Maple** : cpu = 0.061 (sec), leaf count = 35

$$\left\{ y(x) = -2 \frac{\sqrt{-C1x-1}}{-C1} + C2, y(x) = 2 \frac{\sqrt{-C1x-1}}{-C1} + C2 \right\}$$

2.1672 ODE No. 1672

---

$$x^2y''(x) - a(y(x)^n - y(x)) = 0$$

✗ **Mathematica** : cpu = 11.9325 (sec), leaf count = 0 , could not solve  
DSolve[-(a\*(-y[x] + y[x]^n)) + x^2\*Derivative[2][y][x] == 0, y[x], x]

✓ **Maple** : cpu = 0.8 (sec), leaf count = 65

$$\left\{ y(x) = \text{ODESolStruc} \left( -a, \left[ \frac{d}{d\_a} b(-a) = (-b(-a))^2 (-a-b(-a))a - b(-a) - a^na - 1 \right] \right), \left\{ -a = y(x) \right\} \right\}$$

2.1673 ODE No. 1673

---

$$a\left(e^{y(x)}-1\right)+x^2y''(x)=0$$

✗ **Mathematica** : cpu = 25.4792 (sec), leaf count = 0 , could not solve  
DSolve[a\*(-1 + E^y[x]) + x^2\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.61 (sec), leaf count = 60

$$\left\{y(x)=ODESolStruc\left(-a,\left[\left\{\frac{d}{d\_a}b(\_a)=(\_b(\_a))^2(-1+a(e^{-a}-1)\_b(\_a))\right\},\left\{-a=y(x),\_b(\_a)\right\}\right]\right)\right\}$$

2.1674 ODE No. 1674

---

$$y(x)\left(a(a+b)+b^2c^2x^{2b}\right)-x(2a+b-1)y'(x)+x^2y''(x)=0$$

✓ **Mathematica** : cpu = 0.0415502 (sec), leaf count = 106

$$\left\{\left\{y(x)\rightarrow c_22^{-\frac{a+b}{b}}c^{\frac{a+b}{b}-1}\left(x^{2b}\right)^{\frac{a+b}{2b}-\frac{1}{2}}\sin\left(c\sqrt{x^{2b}}\right)+c_12^{-\frac{a}{b}}c^{a/b}\left(x^{2b}\right)^{\frac{a}{2b}}\cos\left(c\sqrt{x^{2b}}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 25

$$\left\{y(x)=x^a\left(\cos\left(x^bc\right)\_C2+\sin\left(x^bc\right)\_C1\right)\right\}$$

2.1675 ODE No. 1675

---

$$x^k\left(-h\left(x^ky(x),ky(x)+xy'(x)\right)\right)+(a+1)xy'(x)+x^2y''(x)=0$$

✗ **Mathematica** : cpu = 1.46913 (sec), leaf count = 0 , could not solve  
DSolve[-(x^k\*h[x^k\*y[x], k\*y[x] + x\*Derivative[1][y][x]]) + (1 + a)\*x\*Derivative[1][y][x] +  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(x^2\*diff(diff(y(x),x),x)+(a+1)\*x\*diff(y(x),x)-x^k\*h(x^k\*y(x),x\*diff(y(x),x)+k\*y(x))=0

2.1676 ODE No. 1676

---

$$a\left(xy'(x)-y(x)\right)^2-bx^2+x^2y''(x)=0$$

✓ **Mathematica** : cpu = 0.301902 (sec), leaf count = 134

$$\left\{\left\{y(x)\rightarrow x\left(c_2+\int_1^x\frac{i\sqrt{a}\sqrt{b}Y_1\left(-i\sqrt{a}\sqrt{b}K[2]\right)-i\sqrt{a}\sqrt{b}J_1\left(i\sqrt{a}\sqrt{b}K[2]\right)c_1}{a\left(Y_0\left(-i\sqrt{a}\sqrt{b}K[2]\right)+J_0\left(i\sqrt{a}\sqrt{b}K[2]\right)c_1\right)K[2]}dK[2]\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.211 (sec), leaf count = 72

$$\left\{y(x)=\left(\int-\frac{1}{ax}\sqrt{-ab}\left(\_C1Y_1\left(\sqrt{-abx}\right)+J_1\left(\sqrt{-abx}\right)\right)\left(\_C1Y_0\left(\sqrt{-abx}\right)+J_0\left(\sqrt{-abx}\right)\right)^{-1}dx+\_C2\right)\right\}$$

2.1677 ODE No. 1677

---

$$ay(x)y'(x)^2+bx+x^2y''(x)=0$$

✗ **Mathematica** : cpu = 46.8871 (sec), leaf count = 0 , could not solve  
DSolve[b\*x + a\*y[x]\*Derivative[1][y][x]^2 + x^2\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 1.272 (sec), leaf count = 101

$$\left\{y(x)=ODESolStruc\left(\_ae^{\int b(\_a)d\_a+\_C1},\left[\left\{\frac{d}{d\_a}b(\_a)=(a\_a^3+b)(\_b(\_a))^3+(2\_a^2a+1)(\_b(\_a)\right\}\right]\right)\right\}$$

2.1678 ODE No. 1678

---

$$x^2y''(x) - \sqrt{ax^2y'(x)^2 + by(x)^2} = 0$$

✗ **Mathematica** : cpu = 1.10214 (sec), leaf count = 0 , could not solve  
DSolve[-Sqrt[b\*y[x]^2 + a\*x^2\*Derivative[1][y][x]^2] + x^2\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.203 (sec), leaf count = 60

$$\left\{y(x) - e^{\int^{\ln(x)} \text{RootOf}\left(\int^{-Z-y(x)}\left(-a^2y(x)-_a y(x)-\sqrt{(y(x))^2(-a^2a+b)}\right)^{-1}d\_a-\_b+_C1\right)d\_b+_C2} = 0\right\}$$

2.1679 ODE No. 1679

---

$$(x^2 + 1) y''(x) + y'(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 0.16134 (sec), leaf count = 33

$$\left\{\left\{y(x) \rightarrow -x \cot(c_1) + \csc^2(c_1) \log(-x \sin(c_1) - \cos(c_1)) + c_2\right\}\right\}$$

✓ **Maple** : cpu = 0.159 (sec), leaf count = 27

$$\left\{y(x) = \frac{x}{\_C1} + \ln(\_C1 x - 1) + \frac{\ln(\_C1 x - 1)}{\_C1^2} + \_C2\right\}$$

2.1680 ODE No. 1680

---

$$x^4 \left(-y'(x)^2\right) + 4x^2y''(x) + 4y(x) = 0$$

✗ **Mathematica** : cpu = 7.12934 (sec), leaf count = 0 , could not solve  
DSolve[4\*y[x] - x^4\*Derivative[1][y][x]^2 + 4\*x^2\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.54 (sec), leaf count = 103

$$\left\{y(x) = ODESolStruc\left(\frac{\_a}{\left(e^{\int\_b(\_a)d\_a+_C1}\right)^2},\left[\left\{\frac{d}{d\_a}\_b(\_a) = \left(-\_a^2 + 7\_a\right)\left(\_b(\_a)\right)^3 + (\_a - 5)\left(\_b(\_a)\right)\right\}\right]\right)\right\}$$

2.1681 ODE No. 1681

---

$$ay(x)^3 + 9x^2y''(x) + 2y(x) = 0$$

✗ **Mathematica** : cpu = 2.67007 (sec), leaf count = 0 , could not solve  
DSolve[2\*y[x] + a\*y[x]^3 + 9\*x^2\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.037 (sec), leaf count = 31

$$\left\{y(x) = \_C2 \text{JacobiSN}\left(\left(\frac{\sqrt{2}}{2x^3}\sqrt{x^{\frac{20}{3}}a + \_C1}\right)\_C2,i\right)\sqrt[3]{x}\right\}$$

2.1682 ODE No. 1682

---

$$x^3 \left(y''(x) + y(x)y'(x) - y(x)^3\right) + 12xy(x) + 24 = 0$$

✗ **Mathematica** : cpu = 21.9063 (sec), leaf count = 0 , could not solve  
DSolve[24 + 12\*x\*y[x] + x^3\*(-y[x]^3 + y[x]\*Derivative[1][y][x] + Derivative[2][y][x]) == 0, y[x], x]  
✓ **Maple** : cpu = 0.828 (sec), leaf count = 94

$$\left\{y(x) = ODESolStruc\left(\_a e^{\int\_b(\_a)d\_a+_C1},\left[\left\{\frac{d}{d\_a}\_b(\_a) = -\left((\_a^3 + \_a^2 - 14\_a - 24)\_b(\_a) + \_a\right)\right\}\right]\right)\right\}$$

2.1683 ODE No. 1683

---

$$x^3 y''(x) - a \left( x y'(x) - y(x) \right)^2 = 0$$

✓ **Mathematica** : cpu = 0.0678282 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow -\frac{x \log \left( a \left( -\frac{c_1}{x} - c_2 \right) \right)}{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 23

$$\left\{ y(x) = -\frac{x}{a} \ln \left( \frac{a (-C1 x - C2)}{x} \right) \right\}$$

2.1684 ODE No. 1684

---

$$xy(x) \left( a - 2x^2 y(x)^2 + 3xy(x) \right) + b + 2x^3 y''(x) + x^2 (2xy(x) + 9) y'(x) = 0$$

✗ **Mathematica** : cpu = 47.1829 (sec), leaf count = 0 , could not solve

`DSolve[b + x*y[x]*(a + 3*x*y[x] - 2*x^2*y[x]^2) + x^2*(9 + 2*x*y[x])*Derivative[1][y][x] + 2`

✓ **Maple** : cpu = 1.04 (sec), leaf count = 100

$$\left\{ y(x) = ODESolStruc \left( \_a e^{\int \_b(\_a) d\_a + \_C1}, \left[ \left\{ \frac{d}{d\_a} \_b(\_a) = \frac{(\_b(\_a))^2 ((-2\_a^3 + \_a^2 + (a-5)\_a + b))}{2} \right. \right. \right. \right.$$

2.1685 ODE No. 1685

---

$$axy(x) + b - \left( kx^{k-1} - 12x^2 \right) \left( 3y'(x) + y(x)^2 \right) + 2 \left( 4x^3 - x^k \right) \left( y''(x) + y(x)y'(x) - y(x)^3 \right) = 0$$

✗ **Mathematica** : cpu = 2.80939 (sec), leaf count = 0 , could not solve

`DSolve[b + a*x*y[x] - (-12*x^2 + k*x^(-1 + k))*(y[x]^2 + 3*Derivative[1][y][x]) + 2*(4*x^3 - y[x]^3 + y[x]*Derivative[1][y][x] + Derivative[2][y][x]) == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(2*(-x^k+4*x^3)*(diff(diff(y(x),x),x)+y(x)*diff(y(x),x)-y(x)^3)-(k*x^(k-1)-12*x^2)*(3*diff(y(x),x)+y(x)^2)+a*x*y(x)+b=0,y(x))`

2.1686 ODE No. 1686

---

$$a^2 y(x)^n + x^4 y''(x) = 0$$

✗ **Mathematica** : cpu = 0.0226879 (sec), leaf count = 0 , could not solve

`DSolve[a^2*y[x]^n + x^4*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.987 (sec), leaf count = 128

$$\left\{ y(x) = ODESolStruc \left( \_a e^{\int \_b(\_a) d\_a + \_C1}, \left[ \left\{ \frac{d}{d\_a} \_b(\_a) = \frac{\left( a^2 \_b(\_a) (n-1)^2 \_a^n - 2 \_a (n-3) \_b(\_a) \right)}{4} \right. \right. \right.$$

2.1687 ODE No. 1687

---

$$x^4 y''(x) - x \left( x^2 + 2y(x) \right) y'(x) + 4y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0703762 (sec), leaf count = 262

$$\left\{ \left\{ y(x) \rightarrow -\frac{x^3 \left( i \left( -\frac{\sqrt{-c_1-1}}{\sqrt{c_1}} + \frac{i}{\sqrt{c_1}} \right) \sqrt{c_1} c_2 x^{-1+i \left( -\frac{\sqrt{-c_1-1}}{\sqrt{c_1}} + \frac{i}{\sqrt{c_1}} \right) \sqrt{c_1}} + i \left( \frac{\sqrt{-c_1-1}}{\sqrt{c_1}} + \frac{i}{\sqrt{c_1}} \right) \sqrt{c_1} x^{-1+i \left( \frac{\sqrt{-c_1-1}}{\sqrt{c_1}} + \frac{i}{\sqrt{c_1}} \right) \sqrt{c_1}} \right)}{c_2 x^{i \left( -\frac{\sqrt{-c_1-1}}{\sqrt{c_1}} + \frac{i}{\sqrt{c_1}} \right) \sqrt{c_1}} + x^{i \left( \frac{\sqrt{-c_1-1}}{\sqrt{c_1}} + \frac{i}{\sqrt{c_1}} \right) \sqrt{c_1}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.083 (sec), leaf count = 21

$$\{y(x) = x^2 (\tanh(\_C1 (\_C2 - \ln(x))) \_C1 + 1)\}$$

2.1688 ODE No. 1688

$$x^4y''(x) - x^2y'(x) \left(y'(x) + x\right) + 4y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.513437 (sec), leaf count = 189

Solve  $\left[\int_1^{y(x)} \frac{1}{-e^{\frac{K[1]}{x^2}} c_1 x^2 + 2x^2 + 4K[1]} dK[1] - \int_1^x \left(\frac{K[2] \left(e^{\frac{y(x)}{K[2]^2} c_1} + 2 \left(-\frac{y(x)}{K[2]^2} - 1\right)\right)}{-e^{\frac{y(x)}{K[2]^2} c_1} K[2]^2 + 2K[2]^2 + 4y(x)} + \int_1^{y(x)} -\frac{\frac{2e^{\frac{K[1]}{K[2]^2} c_1} K[1]}{K[2]}}{\left(-e^{\frac{K[1]}{K[2]^2} c_1} K[2]^2 + 2K[2]^2 + 4y(x)\right)} dK[2]\right) dy(x)\right]$

✓ **Maple** : cpu = 0.076 (sec), leaf count = 32

$$\left\{y(x) = RootOf\left(-\ln(x) + \_C2 - \int^{-Z}\left(e^{-f} \_C1 + 4\_f + 2\right)^{-1} d\_f\right) x^2\right\}$$

2.1689 ODE No. 1689

$$x^4y''(x) + \left(xy'(x) - y(x)\right)^3 = 0$$

✓ **Mathematica** : cpu = 0.251311 (sec), leaf count = 104

$$\left\{\left\{y(x) \rightarrow -ix \log\left(\frac{\frac{e^{c_2}}{x} - \frac{\sqrt{e^{2c_2} - 8ic_1x^2}}{x}}{4c_1}\right)\right\}, \left\{y(x) \rightarrow -ix \log\left(\frac{\frac{e^{c_2}}{x} + \frac{\sqrt{e^{2c_2} - 8ic_1x^2}}{x}}{4c_1}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.105 (sec), leaf count = 37

$$\left\{y(x) = \left(-\arctan\left(\frac{1}{\sqrt{\_C1} x^2 - 1}\right) + \_C2\right) x, y(x) = \left(\arctan\left(\frac{1}{\sqrt{\_C1} x^2 - 1}\right) + \_C2\right) x\right\}$$

2.1690 ODE No. 1690

$$\sqrt{x}y''(x) - y(x)^{3/2} = 0$$

✗ **Mathematica** : cpu = 21.7741 (sec), leaf count = 0 , could not solve  
DSolve[-y[x]^(3/2) + Sqrt[x]\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.414 (sec), leaf count = 99

$$\left\{y(x) = ODESolStruc\left(\frac{\_a}{\left(e^{\int \_b(\_a) d\_a + \_C1}\right)^3}, \left[\left\{\frac{d}{d\_a} \_b(\_a) = -(\_b(\_a))^3 \_a^{\frac{3}{2}} + 12(\_b(\_a))^3 \_a - 7(\_b(\_a))\right\}\right]\right)\right\}$$

2.1691 ODE No. 1691

$$y''(x) \left(ax^2 + bx + c\right)^{3/2} - f\left(\frac{y(x)}{\sqrt{ax^2 + bx + c}}\right) = 0$$

✗ **Mathematica** : cpu = 60.6763 (sec), leaf count = 0 , could not solve  
DSolve[-f[y[x]/Sqrt[c + b\*x + a\*x^2]] + (c + b\*x + a\*x^2)^(3/2)\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.69 (sec), leaf count = 254

$$\left\{y(x) = RootOf\left(-2 \int^{-Z} \frac{a}{\sqrt{4\_C1 a^2 - 4c\_g^2 a + b^2\_g^2 + 8 \int F(\_g) d\_g}} d\_g \sqrt{4ca - b^2} + \_C2 \sqrt{4ca - b^2}\right)\right\}$$

2.1692 ODE No. 1692

$$x^{\frac{n}{n+1}}y''(x) - y(x)^{\frac{2n+1}{n+1}} = 0$$

✗ **Mathematica** : cpu = 0.0583277 (sec), leaf count = 0 , could not solve  
DSolve[-y[x]^((1 + 2\*n)/(1 + n)) + x^(n/(1 + n))\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 3.769 (sec), leaf count = 156

$$\left\{y(x) = ODESolStruc\left(\_a \left(e^{\frac{(\int \_b(\_a) d\_a + \_C1)(n+2)}{n}}\right)^{-1}, \left[\left\{\frac{d}{d\_a} \_b(\_a) = 2 \frac{(\_b(\_a))^2}{n^2} \left(-1/2 \_b(\_a) \_a^{\frac{2n+1}{n+1}}\right)\right\}\right]\right)\right\}$$



**2.1693      ODE No. 1693**

---

$$-h\left(y(x), f(x) y'(x)\right)+f(x) f'(x) y'(x)+f(x)^2 y''(x)=0$$

✗ **Mathematica** : cpu = 0.485914 (sec), leaf count = 0 , could not solve

`DSolve[-h[y[x], f[x]*Derivative[1][y][x]] + f[x]*Derivative[1][f][x]*Derivative[1][y][x] + f`

✓ **Maple** : cpu = 0.217 (sec), leaf count = 68

$$\left\{y(x)=ODESolStruc\left(\_a, \left[\frac{\mathrm{d}}{\mathrm{d}\_a} \_b(\_a)=-h\left(\_a, (\_b(\_a))^{-1}\right)(\_b(\_a))^3\right], \left\{\_a=y(x), \_b(\_a)=\right.\right.\right.$$

**2.1694      ODE No. 1694**

---

$$y(x) y''(x)-a=0$$

✓ **Mathematica** : cpu = 0.113438 (sec), leaf count = 115

$$\left\{\left\{y(x) \rightarrow \exp \left(\frac{-c_1-2 \operatorname{erf}^{-1}\left(-i \sqrt{\frac{2}{\pi}} \sqrt{a e^{\frac{c_1}{a}}\left(c_2+x\right)^2}\right)^2}{2 a}\right)\right\},\left\{y(x) \rightarrow \exp \left(\frac{-c_1-2 \operatorname{erf}^{-1}\left(i \sqrt{\frac{2}{\pi}} \sqrt{a e^{\frac{c_1}{a}}\left(c_2+x\right)^2}\right)^2}{2 a}\right)\right\}\right.$$

✓ **Maple** : cpu = 0.109 (sec), leaf count = 54

$$\left\{\int^{y(x)} \frac{1}{\sqrt{2 \ln (\_a) a-2 \_a \_C1}} d\_a-x-\_C2=0, \int^{y(x)}-\frac{1}{\sqrt{-2 \_a(\_C1-\ln (\_a))}} d\_a-x-\_C2=0\right\}$$

**2.1695      ODE No. 1695**

---

$$y(x) y''(x)-a x=0$$

✗ **Mathematica** : cpu = 18.6101 (sec), leaf count = 0 , could not solve

`DSolve[-(a*x) + y[x]*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.55 (sec), leaf count = 103

$$\left\{y(x)=ODESolStruc\left(\_a\left(e^{\int-\_b(\_a) \mathrm{d}\_a+\_C1}\right)^{\frac{3}{2}},\left[\frac{\mathrm{d}}{\mathrm{d}\_a} \_b(\_a)=\frac{\left(3 \_a^2-4 \_a\right)(\_b(\_a))^3}{4 \_a}+2(\_b(\_a))^2\right]\right.$$

**2.1696      ODE No. 1696**

---

$$y(x) y''(x)-a x^2=0$$

✗ **Mathematica** : cpu = 17.1261 (sec), leaf count = 0 , could not solve

`DSolve[-(a*x^2) + y[x]*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.457 (sec), leaf count = 100

$$\left\{y(x)=ODESolStruc\left(\_a\left(e^{\int-\_b(\_a) \mathrm{d}\_a+\_C1}\right)^2,\left[\frac{\mathrm{d}}{\mathrm{d}\_a} \_b(\_a)=\frac{\left(2 \_a^2-\_a\right)(\_b(\_a))^3}{\_a}+3(\_b(\_a))^2\right]\right.$$

**2.1697      ODE No. 1697**

---

$$-a+y(x) y''(x)+y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.162683 (sec), leaf count = 94

$$\left\{\left\{y(x) \rightarrow-\frac{\sqrt{2 a^2 c_2 x+a^2 c_2^2+a^2 x^2-e^{2 c_1}}}{\sqrt{a}}\right\},\left\{y(x) \rightarrow \frac{\sqrt{2 a^2 c_2 x+a^2 c_2^2+a^2 x^2-e^{2 c_1}}}{\sqrt{a}}\right\}\right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 39

$$\left\{y(x)=\sqrt{a x^2-2 \_C1 x+2 \_C2}, y(x)=-\sqrt{a x^2-2 \_C1 x+2 \_C2}\right\}$$

**2.1698                    ODE No. 1698**

---

$$-ax - b + y(x)y''(x) + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0296774 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{ax^3 + 3bx^2 + 3c_2x + 6c_1}}{\sqrt{3}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{ax^3 + 3bx^2 + 3c_2x + 6c_1}}{\sqrt{3}} \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(diff(y(x),x),x)\*y(x)+y(x)^2-a\*x-b=0,y(x))

**2.1699                    ODE No. 1699**

---

$$y(x)y''(x) + y'(x)^2 - y'(x) = 0$$

✓ **Mathematica** : cpu = 0.112607 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow c_1 \left( -W \left( -\frac{e^{-\frac{x}{c_1} - \frac{c_2}{c_1} - 1}}{c_1} \right) \right) - c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.076 (sec), leaf count = 33

$$\left\{ y(x) = -\_C1 \left( \text{lambertW} \left( -\frac{e^{-1}}{-\_C1} \left( e^{\frac{-C2}{-C1}} \right)^{-1} \left( e^{\frac{x}{-C1}} \right)^{-1} \right) + 1 \right) \right\}$$

**2.1700                    ODE No. 1700**

---

$$y(x)y''(x) - y'(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 0.155016 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow -e^{-c_1} \sinh(e^{c_1}(c_2 + x)) \right\}, \left\{ y(x) \rightarrow e^{-c_1} \sinh(e^{c_1}(c_2 + x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.246 (sec), leaf count = 86

$$\left\{ y(x) = \frac{1}{2} \left( -\_C1 \left( e^{\frac{x}{-C1}} \right)^2 \left( e^{\frac{-C2}{-C1}} \right)^2 + \_C1 \right) \left( e^{\frac{-C2}{-C1}} \right)^{-1} \left( e^{\frac{x}{-C1}} \right)^{-1}, y(x) = \frac{1}{2} \left( -C1 \left( e^{\frac{x}{-C1}} \right)^2 \left( e^{\frac{-C2}{-C1}} \right)^2 - \_C1 \right) \right\}$$

**2.1701                    ODE No. 1701**

---

$$y(x)y''(x) - y'(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.169659 (sec), leaf count = 80

$$\left\{ \left\{ y(x) \rightarrow -\frac{e^{-c_1} \tanh(e^{c_1}(c_2 + x))}{\sqrt{\tanh^2(e^{c_1}(c_2 + x)) - 1}} \right\}, \left\{ y(x) \rightarrow \frac{e^{-c_1} \tanh(e^{c_1}(c_2 + x))}{\sqrt{\tanh^2(e^{c_1}(c_2 + x)) - 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.211 (sec), leaf count = 42

$$\left\{ y(x) = \frac{-C1}{2} \left( \left( e^{\frac{-C2}{-C1}} \right)^2 \left( e^{\frac{x}{-C1}} \right)^2 + 1 \right) \left( e^{\frac{-C2}{-C1}} \right)^{-1} \left( e^{\frac{x}{-C1}} \right)^{-1} \right\}$$

**2.1702                    ODE No. 1702**

---

$$e^{2x} (ay(x)^4 + b) + e^x y(x) (cy(x)^2 + d) + y(x)y''(x) - y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 1.04067 (sec), leaf count = 0 , could not solve  
DSolve[E^x\*y[x]\*(d + c\*y[x]^2) + E^(2\*x)\*(b + a\*y[x]^4) - Derivative[1][y][x]^2 + y[x]\*Derivative[2][y][x],y[x],x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(diff(y(x),x),x)\*y(x)-diff(y(x),x)^2+exp(x)\*y(x)\*(c\*y(x)^2+d)+exp(2\*x)\*(b+a\*y(x)^4),y(x))

2.1703 ODE No. 1703

$y(x)y''(x)-y'(x)^2+y(x)^2(-\log(y(x)))=0$

✓ **Mathematica** : cpu = 0.291192 (sec), leaf count = 77

$\left\{\left\{y(x)\rightarrow \exp\left(-\frac{1}{2}\sqrt{c_1}e^{-c_2-x}\left(e^{2c_2+2x}-1\right)\right)\right\},\left\{y(x)\rightarrow \exp\left(\frac{1}{2}\sqrt{c_1}e^{-c_2-x}\left(e^{2c_2+2x}-1\right)\right)\right\}\right\}$

✓ **Maple** : cpu = 0.048 (sec), leaf count = 21

$\left\{y\left(x\right)=\mathrm{e}^{\frac{\mathrm{e}^{-2\,x}\,C1}{2}}\mathrm{e}^{-\frac{C2}{2}}\mathrm{e}^x\right\}$

2.1704 ODE No. 1704

$y(x)^2\left(\frac{f''(x)}{f(x)}-\frac{f'(x)^2}{f(x)^2}\right)+f(x)y(x)^3+y(x)y''(x)-y'(x)^2-y'(x)=0$

✗ **Mathematica** : cpu = 21.3642 (sec), leaf count = 0 , could not solve  
DSolve[f[x]\*y[x]^3 - Derivative[1][y][x] - Derivative[1][y][x]^2 + y[x]^2\*(-Derivative[1][f[x]]/f[x]),y[x],x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(diff(y(x),x),x)\*y(x)-diff(y(x),x)^2-diff(y(x),x)+f(x)\*y(x)^3+y(x)^2\*(diff(diff(f(x),x),x)^2/f(x)^2)=0,y(x))

2.1705 ODE No. 1705

$-y(x)f'(x)+f(x)y'(x)+y(x)y''(x)-y'(x)^2-y(x)^3=0$

✓ **Mathematica** : cpu = 0.269456 (sec), leaf count = 252

$\left\{\left\{y(x)\rightarrow -\frac{\exp\left(c_2-\int_1^x\frac{y(K[2])^3+c_1^2y(K[2])^2+\int_1^{K[2]}\frac{-y(K[1])^3-f'(K[1])y(K[1])+f(K[1])y'(K[1])}{y(K[1])^2}dK[1]^2y(K[2])^2+2c_1\int_1^{K[2]}\frac{-y(K[1])^3-f'(K[1])y(K[1])+f(K[1])y'(K[1])}{y(K[1])^2}dK[1]}{y(K[2])^2\left(c_1+\int_1^{K[2]}\frac{-y(K[1])^3-f'(K[1])y(K[1])+f(K[1])y'(K[1])}{y(K[1])^2}dK[1]\right)}\right\}}{\int_1^x\frac{-y(K[1])^3-f'(K[1])y(K[1])+f(K[1])y'(K[1])}{y(K[1])^2}dK[1]}\right\}\right\}$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(diff(y(x),x),x)\*y(x)-diff(y(x),x)^2+f(x)\*diff(y(x),x)-diff(f(x),x)\*y(x)-y(x)^3=0,y(x))

2.1706 ODE No. 1706

$-y(x)f''(x)+f'(x)y'(x)+f(x)y(x)^3+y(x)y''(x)-y'(x)^2-y(x)^4=0$

✓ **Mathematica** : cpu = 0.50232 (sec), leaf count = 308

$\left\{\left\{y(x)\rightarrow -\frac{\exp\left(c_2-\int_1^x\frac{y(K[2])^4-f(K[2])y(K[2])^3+c_1^2y(K[2])^2+\int_1^{K[2]}\frac{-y(K[1])^4+f(K[1])y(K[1])^3-f''(K[1])y(K[1])+f'(K[1])y'(K[1])}{y(K[1])^2}dK[1]^2y(K[2])^2}{y(K[2])^2\left(c_1+\int_1^{K[2]}\frac{-y(K[1])^4+f(K[1])y(K[1])^3-f''(K[1])y(K[1])+f'(K[1])y'(K[1])}{y(K[1])^2}dK[1]\right)}\right\}}{\int_1^x\frac{-y(K[1])^4+f(K[1])y(K[1])^3-f''(K[1])y(K[1])+f'(K[1])y'(K[1])}{y(K[1])^2}dK[1]}\right\}\right\}$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(diff(y(x),x),x)\*y(x)-diff(y(x),x)^2+diff(f(x),x)\*diff(y(x),x)-diff(diff(f(x),x),x)\*y(x)^4=0,y(x))

2.1707 ODE No. 1707

$ay(x)y'(x)+by(x)^2+y(x)y''(x)-y'(x)^2=0$

✓ **Mathematica** : cpu = 0.057242 (sec), leaf count = 31

$\left\{\left\{y(x)\rightarrow c_2e^{-\frac{bx}{a}-\frac{c_1e^{-ax}}{a}}\right\}\right\}$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 39

$\left\{y\left(x\right)=1\mathrm{e}^{\frac{\mathrm{e}^{-ax}\,C1}{a}}\mathrm{e}^{\frac{b}{a^2}x}\left(\mathrm{e}^{\frac{bx}{a}}\right)^{-1}\left(\mathrm{e}^{\frac{C2}{a}}\right)^{-1}\right\}$



2.1712 ODE No. 1712

---

$$-f(x)y(x)y'(x)-g(x)y(x)^2+y(x)y''(x)-y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.0576697 (sec), leaf count = 75

$$\left\{\left\{y(x)\rightarrow c_2\exp\left(\int_1^x\left(\exp\left(\int_1^{K[3]}f(K[1])dK[1]\right)c_1+\exp\left(\int_1^{K[3]}f(K[1])dK[1]\right)\int_1^{K[3]}\exp\left(-\int_1^{K[2]}f(K[1])dK[1]\right)dK[2]\right)\right.\right.$$

✓ **Maple** : cpu = 0.067 (sec), leaf count = 61

$$\left\{y(x)=\frac{-C2}{e^{-C1}\int e^{\int f(x)dx}dx}e^{\int e^{\int f(x)dx}dx\int\frac{g(x)}{e^{\int f(x)dx}}dx}\left(e^{\int\frac{g(x)\int e^{\int f(x)dx}dx}{e^{\int f(x)dx}}dx}\right)^{-1}\right\}$$

2.1713 ODE No. 1713

---

$$-y(x)\left(g'(x)-y(x)^2f'(x)\right)+y'(x)\left(f(x)y(x)^2+g(x)\right)+y(x)y''(x)-y'(x)^2=0$$

✗ **Mathematica** : cpu = 20.7969 (sec), leaf count = 0 , could not solve  
DSolve[-(y[x]\*(-(y[x]^2\*Derivative[1][f][x]) + Derivative[1][g][x])) + (g[x] + f[x]\*y[x]^2)\*y'[x],y[x],x]  
✓ **Maple** : cpu = 0.223 (sec), leaf count = 54

$$\left\{y(x)=ODESolStruc\left(\_b(\_a),\left[\frac{f(\_a)(\_b(\_a))^2+\_C1\_b(\_a)+\frac{d}{d\_a}b(\_a)-g(\_a)}{\_b(\_a)}=0\right],\{\_a=\_b(\_a)\}\right.\right.$$

2.1714 ODE No. 1714

---

$$y(x)y''(x)+3y(x)y'(x)-3y'(x)^2-y(x)^2=0$$

✓ **Mathematica** : cpu = 0.123121 (sec), leaf count = 25

$$\left\{\left\{y(x)\rightarrow\frac{c_2e^x}{\sqrt{1-2e^{c_1+x}}}\right\}\right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 68

$$\left\{y(x)=\frac{\sqrt{2}}{2\_C1e^x-2\_C2}\sqrt{(\_C1e^x-\_C2)e^{2x}},y(x)=-\frac{\sqrt{2}}{2\_C1e^x-2\_C2}\sqrt{(\_C1e^x-\_C2)e^{2x}}\right\}$$

2.1715 ODE No. 1715

---

$$y(x)y''(x)-ay'(x)^2=0$$

✓ **Mathematica** : cpu = 0.113931 (sec), leaf count = 26

$$\left\{\left\{y(x)\rightarrow c_2(-ax-c_1+x)^{\frac{1}{1-a}}\right\}\right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 25

$$\left\{y(x)=\left(\frac{1}{(1-a)(\_C1x+\_C2)}\right)^{(a-1)^{-1}}\right\}$$

2.1716 ODE No. 1716

---

$$a\left(y'(x)^2+1\right)+y(x)y''(x)=0$$

✓ **Mathematica** : cpu = 0.537614 (sec), leaf count = 172

$$\left\{\left\{y(x)\rightarrow\text{InverseFunction}\left[-\frac{\#1\sqrt{1-e^{2c_1}\#1^{-2a}}{}_2F_1\left(\frac{1}{2},-\frac{1}{2a};1-\frac{1}{2a};e^{2c_1}\#1^{-2a}\right)}{\sqrt{e^{2c_1}\#1^{-2a}-1}}\&\right][c_2+x]\right\},\left\{y(x)\rightarrow\text{InverseFunction}\left[\frac{\#1\sqrt{1-e^{2c_1}\#1^{-2a}}{}_2F_1\left(\frac{1}{2},-\frac{1}{2a};1-\frac{1}{2a};e^{2c_1}\#1^{-2a}\right)}{\sqrt{e^{2c_1}\#1^{-2a}-1}}\&\right][c_2+x]\right\}\right\}$$

✓ **Maple** : cpu = 0.167 (sec), leaf count = 68

$$\left\{\int^{y(x)}\frac{1}{\_a^{-a}}\frac{1}{\sqrt{\_a^{2a}+\_C1}}d\_a-x-\_C2=0,\int^{y(x)}-\frac{1}{\_a^{-a}}\frac{1}{\sqrt{\_a^{2a}+\_C1}}d\_a-x-\_C2=0\right\}$$

## 2.1717 ODE No. 1717

$$ay'(x)^2 + by(x)^3 + y(x)y''(x) = 0$$

✓ **Mathematica** : cpu = 1.11553 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow 0^{\frac{1}{-a-1}} \right\}, \left\{ y(x) \rightarrow 2^{\frac{1}{-2a-3}} \left( \frac{b}{(2a+3)c_1} \right)^{\frac{1}{-2a-3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.21 (sec), leaf count = 107

$$\left\{ \int^{y(x)} (2a+3)_{-} a^{2a} \frac{1}{\sqrt{-(2a+3)_{-} a^{2a} (2b_{-} a^{2a+3} - C1)}} d_{-} a - x - C2 = 0, \int^{y(x)} (-2a-3)_{-} a^{2a} \frac{1}{\sqrt{-(2a+3)_{-} a^{2a} (2b_{-} a^{2a+3} - C1)}} d_{-} a - x - C2 = 0 \right\}$$

## 2.1718      ODE No. 1718

$$dy(x)^{1-a} + ay'(x)^2 + by(x)y'(x) + cy(x)^2 + y(x)y''(x) = 0$$

✓ **Mathematica** : cpu = 1.27197 (sec), leaf count = 744

$$\left\{ \left\{ y(x) \rightarrow \frac{abc_1 \exp \left( -\frac{x(b\sqrt{-4ac+b^2-4c}-4(a+1)c+b^2)}{\sqrt{-4ac+b^2-4c}+b} - \frac{2(a+1)cx}{\sqrt{-4ac+b^2-4c}+b} \right)}{b\sqrt{-4ac+b^2-4c}-4ac+b^2-4c} + \frac{bc_1 \exp \left( -\frac{x(b\sqrt{-4ac+b^2-4c}-4(a+1)c+b^2)}{\sqrt{-4ac+b^2-4c}+b} \right)}{b\sqrt{-4ac+b^2-4c}-4ac} \right. \right.$$

✓ **Maple** : cpu = 0.207 (sec), leaf count = 133

$$\left\{ y(x) = e^{-\frac{x}{2a+2}\sqrt{(-4a-4)c+b^2}} e^{-\frac{bx}{2a+2}} \left( ((-4a-4)c^3 + b^2c^2) \left( de^{\frac{x}{2}(b+\sqrt{(-4a-4)c+b^2})} \sqrt{(-4a-4)c+b^2} + (e^{x\sqrt{(-4a-4)c+b^2}} \right. \right. \right.$$

## 2.1719 ODE No. 1719

$$ay'(x)^2 + f(x)y(x)y'(x) + g(x)y(x)^2 + y(x)y''(x) = 0$$

**X Mathematica** : cpu = 41.8596 (sec), leaf count = 0 , could not solve

```
DSolve[g[x]*y[x]^2 + f[x]*y[x]*Derivative[1][y][x] + a*Derivative[1][y][x]^2 + y[x]*Derivati
```

✓ **Maple** : cpu = 0.405 (sec), leaf count = 70

$$\left\{y(x) = ODESolStruc\left(e^{\int -b(-a) d\_a + -C1}, \left[\left\{\frac{d}{d\_a} b(-a) = (-a - 1)(-b(-a))^2 - f(-a) - b(-a) - g(-a)\right\}\right.\right.$$

2.1720      ODE No. 1720

$$ay'(x)^2 + by(x)^2y'(x) + cy(x)^4 + y(x)y''(x) = 0$$

**X Mathematica** : cpu = 192.713 (sec), leaf count = 0 , could not solve

```
DSolve[c*y[x]^4 + b*y[x]^2*Derivative[1][y][x] + a*Derivative[1][y][x]^2 + y[x]*Derivative[2]
```

✓ **Maple** : cpu = 0.287 (sec), leaf count = 173

$$\left\{ \int^{y(x)} (2a+4) \left( \tan \left( RootOf \left( 2\_Z b\_a^2 - 2a \ln(\_a) \sqrt{4\_a^4 ac - b^2\_a^4 + 8c\_a^4} - \ln \left( \frac{-a^4 \left( (\tan(\_Z)) \right)}{4} \right) \right) \right) \right.$$

## 2.1721      ODE No. 1721

$$-\frac{ay(x)^3 f'(x)}{a+2} + \frac{af(x)^2 y(x)^4}{(a+2)^2} - \frac{(a-1)y'(x)^2}{a} - f(x)y(x)^2 y'(x) + y(x)y''(x) = 0$$

**X Mathematica** : cpu = 21.2417 (sec), leaf count = 0 , could not solve

$$\text{DSolve}[(a f[x]^2 y[x]^4)/(2 + a)^2 - (a y[x]^3 \text{Derivative}[1][f][x])/(2 + a) - f[x] y[x]^2 \text{Derivative}[1][f][x] + (1 + a) \text{Derivative}[1][y][x]^2)/a + y[x] \text{Derivative}[2][y][x] == 0, y[x], x]$$

**X Maple :** cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve(diff(diff(y(x),x),x)*y(x)-(a-1)/a*diff(y(x),x)^2-f(x)*y(x)^2*diff(y(x),x)+a/(a+2)^2*f(a/(a+2)*diff(f(x),x)*y(x)^3=0,y(x))
```

2.1722 ODE No. 1722

---

$$-2ay(x)\left(y'(x)^2+1\right)^{3/2}+y(x)y''(x)-y'(x)^2-1=0$$

✓ **Mathematica** : cpu = 11.2093 (sec), leaf count = 697

$$\left\{\left\{y(x)\rightarrow \text{InverseFunction}\left[-\frac{\sqrt{1-\frac{2\#1^2a^2}{-2ac_1+\sqrt{1-4ac_1}+1}}\sqrt{\frac{2\#1^2a^2}{2ac_1+\sqrt{1-4ac_1}-1}}+1\left((-2ac_1+\sqrt{1-4ac_1}+1\right)E\left(i\sin\left(\frac{2\sqrt{1-4ac_1}}{2\sqrt{1-4ac_1}-1}\right)\right)\right)}{2}\right.\right.$$

✓ **Maple** : cpu = 0.325 (sec), leaf count = 98

$$\left\{\int^{y(x)}(\_a^2a+\_C1)\frac{1}{\sqrt{-\_a^4a^2-2\_C1\_a^2a-\_C1^2+\_a^2}}d\_a-x-\_C2=0,\int^{y(x)}-(\_a^2a+\_C1)\frac{1}{\sqrt{-\_a^4a^2-2\_C1\_a^2a-\_C1^2+\_a^2}}d\_a-x-\_C2=0\right\}$$

2.1723 ODE No. 1723

---

$$(y(x)+x)y''(x)+y'(x)^2-y'(x)=0$$

✓ **Mathematica** : cpu = 0.697331 (sec), leaf count = 130

$$\left\{\left\{y(x)\rightarrow \frac{1}{2}\left(-\sqrt{2}e^{-2c_1}\sqrt{4e^{3c_1}x+e^{2c_1}+4e^{3c_1}c_2}+e^{-c_1}+4c_2+2x\right)\right\},\left\{y(x)\rightarrow \frac{1}{2}\left(\sqrt{2}e^{-2c_1}\sqrt{4e^{3c_1}x+e^{2c_1}+4e^{3c_1}c_2}+e^{-c_1}+4c_2+2x\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.092 (sec), leaf count = 16

$$\left\{y\left(x\right)=\sqrt{-C1+2\,x}\_C2+\_C1+x\right\}$$

2.1724 ODE No. 1724

---

$$(x-y(x))y''(x)+2y'(x)\left(y'(x)+1\right)=0$$

✓ **Mathematica** : cpu = 0.28907 (sec), leaf count = 38

$$\left\{\left\{y(x)\rightarrow -\frac{e^{-c_1}\left(e^{c_1}c_2x+e^{c_1}c_2^2+1\right)}{c_2+x}\right\}\right\}$$

✓ **Maple** : cpu = 0.467 (sec), leaf count = 21

$$\left\{y\left(x\right)=\frac{-C2^2-\_C2\,x+\_C1}{\_C2-x}\right\}$$

2.1725 ODE No. 1725

---

$$(x-y(x))y''(x)-(y'(x)+1)\left(y'(x)^2+1\right)=0$$

✓ **Mathematica** : cpu = 2.24135 (sec), leaf count = 75

$$\left\{\left\{y(x)\rightarrow -\sqrt{-2c_2x+e^{2c_1}-c_2^2-x^2}-c_2\right\},\left\{y(x)\rightarrow \sqrt{-2c_2x+e^{2c_1}-c_2^2-x^2}-c_2\right\}\right\}$$

✓ **Maple** : cpu = 0.438 (sec), leaf count = 105

$$\left\{y\left(x\right)=x+RootOf\left(-x+\int^{-Z}(\_C1^2\_f^2-1)\left(-\_C1^2\_f^2+\_C1\sqrt{-\_C1^2\_f^2+2\_f+2}\right)^{-1}d\_f+\_C2\right)\right\}$$

2.1726 ODE No. 1726

---

$$(x-y(x))y''(x)-h\left(y'(x)\right)=0$$

✓ **Mathematica** : cpu = 0.251773 (sec), leaf count = 77

$$\text{Solve}\left[\left\{x=\int\frac{\exp\left(-\int_1^{\text{K}\$11120816}\frac{K[3]-1}{h(K[3])}dK[3]-c_1\right)}{h(\text{K}\$11120816)}d\text{K}\$11120816+c_2,y(x)=x-\exp\left(-\int_1^{\text{K}\$11120816}\frac{K[3]}{h(K[3])}dK[3]-c_1\right)\right\}\right]$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 39

$$\left\{y\left(x\right)=x+RootOf\left(-x+\int^{-Z}\left(-1+RootOf\left(\int^{-Z}\frac{a-1}{h\left(\_a\right)}d\_a+\ln\left(-\_g\right)+\_C1\right)\right)^{-1}d\_g+\_C2\right)\right\}$$

2.1727 ODE No. 1727

---

$$2y(x)y''(x) + y'(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 0.353967 (sec), leaf count = 166

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{\sqrt{\#1} \left( \#1 - e^{2c_1} \right) + e^{3c_1} \sqrt{1 - \#1 e^{-2c_1}} \sin^{-1} \left( \sqrt{\#1} e^{-c_1} \right)}{\sqrt{e^{2c_1} - \#1}} \& \right] [c_2 + x] \right\}, \left\{ y(x) \rightarrow \right.$$

✓ **Maple** : cpu = 0.322 (sec), leaf count = 823

$$\left\{ y(x) = \frac{\left( -\text{RootOf} \left( (\tan(_Z))^2 _C1^2 _Z^2 - 4 (\tan(_Z))^2 _C1 _C2 _Z - 4 (\tan(_Z))^2 _C1 x _Z + 4 (\tan(_Z))^2 _C1^2 _Z^2 \right)}{4 _C2} + _C1 x + _C2 \right)$$

2.1728 ODE No. 1728

---

$$a + 2y(x)y''(x) - y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0043437 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow \frac{x^2 (c_1^2 - a)}{4c_2} + c_1 x + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 24

$$\left\{ y(x) = \frac{(_C1^2 - a) x^2}{4 _C2} + _C1 x + _C2 \right\}$$

2.1729 ODE No. 1729

---

$$a + f(x)y(x)^2 + 2y(x)y''(x) - y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 0.0157692 (sec), leaf count = 0 , could not solve  
DSolve[a + f[x]\*y[x]^2 - Derivative[1][y][x]^2 + 2\*y[x]\*Derivative[2][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(2\*diff(diff(y(x),x),x)\*y(x)-diff(y(x),x)^2+f(x)\*y(x)^2+a=0,y(x))

2.1730 ODE No. 1730

---

$$2y(x)y''(x) - y'(x)^2 - 8y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.726331 (sec), leaf count = 135

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{2\sqrt{\#1}\sqrt{\frac{4\#1^2}{c_1} + 1} {}_2F_1 \left( \frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{4\#1^2}{c_1} \right)}{\sqrt{4\#1^2 + c_1}} \& \right] [c_2 + x] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \right.$$

✓ **Maple** : cpu = 0.063 (sec), leaf count = 53

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{4 _a^3 + _a _C1}} d _a - x - _C2 = 0, \int^{y(x)} -\frac{1}{\sqrt{4 _a^3 + _a _C1}} d _a - x - _C2 = 0 \right\}$$

2.1731 ODE No. 1731

---

$$2y(x)y''(x) - y'(x)^2 - 8y(x)^3 - 4y(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.00095 (sec), leaf count = 351

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{i\#1\sqrt{\frac{2c_1}{\#1-\#1\sqrt{1-c_1}}} + 4\sqrt{\frac{c_1}{\#1\sqrt{1-c_1}+\#1}} + 2F \left( i \sinh^{-1} \left( \frac{\sqrt{\frac{c_1}{2\sqrt{1-c_1}+2}}}{\sqrt{\#1}} \right) \Big| \frac{\sqrt{1-c_1}+1}{1-\sqrt{1-c_1}} \right)}{\sqrt{\frac{c_1}{\sqrt{1-c_1}+1}} \sqrt{4\#1^2 + 4\#1 + c_1}} \& \right]$$

✓ **Maple** : cpu = 0.085 (sec), leaf count = 61

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{4 _a^3 + _a _C1 + 4 _a^2}} d _a - x - _C2 = 0, \int^{y(x)} -\frac{1}{\sqrt{(4 _a^2 + _C1 + 4 _a) _a}} d _a - x - _C2 = 0 \right\}$$



2.1732 ODE No. 1732

$$2y(x)y''(x) - y'(x)^2 - 4(2y(x) + x)y(x)^2 = 0$$

✗ **Mathematica** : cpu = 1.16667 (sec), leaf count = 0 , could not solve  
DSolve[-4\*y[x]^2\*(x + 2\*y[x]) - Derivative[1][y][x]^2 + 2\*y[x]\*Derivative[2][y][x] == 0, y[x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(2\*diff(diff(y(x),x),x)\*y(x)-diff(y(x),x)^2-4\*(x+2\*y(x))\*y(x)^2=0,y(x))

2.1733 ODE No. 1733

$$y(x)^2(ay(x) + b) + 2y(x)y''(x) - y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.91986 (sec), leaf count = 437

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{i\sqrt{2}\#1^{3/2} \sqrt{\frac{4c_1}{\#1(\sqrt{2ac_1+b^2}-b)}} + 2\sqrt{1 - \frac{2c_1}{\#1(\sqrt{2ac_1+b^2}+b)}} F\left(i \sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{c_1}{\sqrt{b^2+2ac_1}}}}{\sqrt{\#1}}\right)}{\sqrt{\frac{c_1}{\sqrt{2ac_1+b^2}-b}}} \sqrt{-\#1(\#1^2a + 2\#1b - 2c_1)}\right] \right\} \right.$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 71

$$\left\{ \int^{y(x)} -2 \frac{1}{\sqrt{-2a_-a^3 - 4b_-a^2 + 4_-a_-C1}} d_-a - x - _C2 = 0, \int^{y(x)} 2 \frac{1}{\sqrt{-2a_-a^3 - 4b_-a^2 + 4_-a_-C1}} d_-a - x - _C2 = 0 \right\}$$

2.1734 ODE No. 1734

$$ay(x)^3 + 2y(x)y''(x) - y'(x)^2 + 2xy(x)^2 + 1 = 0$$

✗ **Mathematica** : cpu = 2.19422 (sec), leaf count = 0 , could not solve  
DSolve[1 + 2\*x\*y[x]^2 + a\*y[x]^3 - Derivative[1][y][x]^2 + 2\*y[x]\*Derivative[2][y][x] == 0, y[x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(2\*diff(diff(y(x),x),x)\*y(x)-diff(y(x),x)^2+1+2\*x\*y(x)^2+a\*y(x)^3=0,y(x))

2.1735 ODE No. 1735

$$y(x)^2(ay(x) + bx) + 2y(x)y''(x) - y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 0.869864 (sec), leaf count = 0 , could not solve  
DSolve[y[x]^2\*(b\*x + a\*y[x]) - Derivative[1][y][x]^2 + 2\*y[x]\*Derivative[2][y][x] == 0, y[x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(2\*diff(diff(y(x),x),x)\*y(x)-diff(y(x),x)^2+(a\*y(x)+b\*x)\*y(x)^2=0,y(x))

2.1736 ODE No. 1736

$$2y(x)y''(x) - y'(x)^2 - 3y(x)^4 = 0$$

✓ **Mathematica** : cpu = 6.14685 (sec), leaf count = 129

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{2\sqrt{\#1}\sqrt{\frac{\#1^3}{c_1}} + 1 {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; -\frac{\#1^3}{c_1}\right)}{\sqrt{\#1^3 + c_1}} \right] \& [c_2 + x] \right\} , \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{2\sqrt{\#1}\sqrt{\frac{\#1^3}{c_1}} + 1 {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; -\frac{\#1^3}{c_1}\right)}{\sqrt{\#1^3 + c_1}} \right] \& [c_2 + x] \right\} \right.$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 49

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{-a^4 + _a_-C1}} d_-a - x - _C2 = 0, \int^{y(x)} -\frac{1}{\sqrt{-a^4 + _a_-C1}} d_-a - x - _C2 = 0 \right\}$$

2.1737 ODE No. 1737

$$-4(a + x^2)y(x)^2 + b + 2y(x)y''(x) - y'(x)^2 - 3y(x)^4 - 8xy(x)^3 = 0$$

✗ **Mathematica** : cpu = 20.6132 (sec), leaf count = 0 , could not solve  
DSolve[b - 4\*(a + x^2)\*y[x]^2 - 8\*x\*y[x]^3 - 3\*y[x]^4 - Derivative[1][y][x]^2 + 2\*y[x]\*Derivative[2][y][x] == 0, y[x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(2\*diff(diff(y(x),x),x)\*y(x)-diff(y(x),x)^2+b-4\*(x^2+a)\*y(x)^2-8\*x\*y(x)^3-3\*y(x)^4=0,y(x))



2.1744 ODE No. 1744

---

$$2y(x)y''(x) - y'(x)^2 (y'(x)^2 + 1) = 0$$

✓ **Mathematica** : cpu = 0.533004 (sec), leaf count = 155

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -ie^{-c_1} \sqrt{\#1 e^{2c_1} - 1} \left( \frac{e^{-c_1} \sin^{-1}(\sqrt{\#1} e^{c_1})}{\sqrt{1 - \#1 e^{2c_1}}} + \sqrt{\#1} \right) \& \right] [c_2 + x] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -ie^{-c_1} \sqrt{\#1 e^{2c_1} - 1} \left( \frac{e^{-c_1} \sin^{-1}(\sqrt{\#1} e^{c_1})}{\sqrt{1 - \#1 e^{2c_1}}} + \sqrt{\#1} \right) \& \right] [c_2 + x] \right\} \right\}$$

✓ **Maple** : cpu = 0.308 (sec), leaf count = 823

$$\left\{ y(x) = \frac{\left( -\text{RootOf} \left( (\tan(_Z))^2 _C1^2 _Z^2 - 4 (\tan(_Z))^2 _C1 _C2 _Z - 4 (\tan(_Z))^2 _C1 x _Z + 4 (\tan(_Z))^2 _C1^2 \right) \right)}{\dots} \right\}$$

2.1745 ODE No. 1745

---

$$2(y(x) - a)y''(x) + y'(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 0.704168 (sec), leaf count = 251

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{2\sqrt{a - \#1} (2\#1 - 2a + e^{2c_1}) - \sqrt{2} e^{3c_1} \sqrt{e^{-2c_1} (2\#1 - 2a + e^{2c_1})} \sin^{-1}(\sqrt{2} e^{-c_1} \sqrt{a - \#1})}{2\sqrt{2} \sqrt{2\#1 - 2a + e^{2c_1}}} \right] [c_2 + x] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{2\sqrt{a - \#1} (2\#1 - 2a + e^{2c_1}) - \sqrt{2} e^{3c_1} \sqrt{e^{-2c_1} (2\#1 - 2a + e^{2c_1})} \sin^{-1}(\sqrt{2} e^{-c_1} \sqrt{a - \#1})}{2\sqrt{2} \sqrt{2\#1 - 2a + e^{2c_1}}} \right] [c_2 + x] \right\} \right\}$$

✓ **Maple** : cpu = 0.402 (sec), leaf count = 117

$$\left\{ -\frac{C1}{2} \arctan \left( 1 \left( y(x) - a - \frac{C1}{2} \right) \frac{1}{\sqrt{-(-y(x) + a)(a + _C1 - y(x))}} \right) - x - _C2 + \sqrt{-(-y(x) + a)(a + _C1 - y(x))} \right\}$$

2.1746 ODE No. 1746

---

$$-ax^2 - bx - c + 3y(x)y''(x) - 2y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 0.0285032 (sec), leaf count = 0 , could not solve  
DSolve[-c - b\*x - a\*x^2 - 2\*Derivative[1][y][x]^2 + 3\*y[x]\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.305 (sec), leaf count = 207

$$\left\{ y(x) = \text{RootOf} \left( -2 \int^{-Z} \frac{b}{\sqrt{4 _f^{4/3} _C1 b^2 - 36 c _f^2 a + 9 b^2 _f^2 - 2}} d_f \sqrt{4 ca - b^2} - 2 b \arctan \left( \frac{2 ax + b}{\sqrt{4 ca - b^2}} \right) \right) \right\}$$

2.1747 ODE No. 1747

---

$$3y(x)y''(x) - 5y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0985333 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2}{(3c_1 + 2x)^{3/2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 17

$$\left\{ -\frac{3}{2} (y(x))^{-\frac{2}{3}} - _C1 x - _C2 = 0 \right\}$$

2.1748 ODE No. 1748

---

$$4y(x)y''(x) - 3y'(x)^2 + 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.179417 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow \frac{(c_1^2 x^2 + 2c_2 c_1^2 x + c_2^2 c_1^2 - 64)^2}{256 c_1^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.099 (sec), leaf count = 67

$$\left\{ -4 \frac{\sqrt{-C1 (y(x))^{3/2} + 4 y(x)}}{\sqrt{y(x)} _C1} - x - _C2 = 0, 4 \frac{\sqrt{-C1 (y(x))^{3/2} + 4 y(x)}}{\sqrt{y(x)} _C1} - x - _C2 = 0, y(x) = 0 \right\}$$

2.1749 ODE No. 1749

---

$$4y(x)y''(x) - 3y'(x)^2 - 12y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.457493 (sec), leaf count = 153

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{4\#1\sqrt{\frac{4\#1^{3/2}}{c_1}} + 1 {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; -\frac{4\#1^{3/2}}{c_1}\right)}{\sqrt{\#1^{3/2}\left(4\#1^{3/2} + c_1\right)}} \& \right] [c_2 + x] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \right.$$

✓ **Maple** : cpu = 0.295 (sec), leaf count = 57

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{-C1 - a^{\frac{3}{2}} + 4 - a^3}} d_{-}a - x - _{C2} = 0, \int^{y(x)} -\frac{1}{\sqrt{-C1 - a^{\frac{3}{2}} + 4 - a^3}} d_{-}a - x - _{C2} = 0 \right\}$$

2.1750 ODE No. 1750

---

$$ay(x)^3 + by(x)^2 + cy(x) + 4y(x)y''(x) - 3y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 3.56493 (sec), leaf count = 2281

$$\left\{ \text{Solve} \left[ -\frac{4F\left(\sin^{-1}\left(\sqrt{\frac{(\text{Root}[a\#1^4+3b\#1^2-3c_1\#1-3c\&,2]-\text{Root}[a\#1^4+3b\#1^2-3c_1\#1-3c\&,4])\left(\sqrt{y(x)}-\text{Root}[a\#1^4+3b\#1^2-3c_1\#1-3c\&,1]-\text{Root}[a\#1^4+3b\#1^2-3c_1\#1-3c\&,4])\right)}{(\text{Root}[a\#1^4+3b\#1^2-3c_1\#1-3c\&,1]-\text{Root}[a\#1^4+3b\#1^2-3c_1\#1-3c\&,4])\left(\sqrt{y(x)}-\text{Root}[a\#1^4+3b\#1^2-3c_1\#1-3c\&,1]-\text{Root}[a\#1^4+3b\#1^2-3c_1\#1-3c\&,4])\right)}}\right)}{\right. \right.$$

✓ **Maple** : cpu = 0.313 (sec), leaf count = 87

$$\left\{ \int^{y(x)} -3 \frac{1}{\sqrt{9 - C1 - a^{3/2} - 3 a - a^3 - 9 b - a^2 + 9 c - a}} d_{-}a - x - _{C2} = 0, \int^{y(x)} 3 \frac{1}{\sqrt{9 - C1 - a^{3/2} - 3 a - a^3 - 9 b - a^2 + 9 c - a}} d_{-}a - x - _{C2} = 0 \right\}$$

2.1751 ODE No. 1751

---

$$y'(x)\left(6y(x)^2 - \frac{2y(x)f'(x)}{f(x)}\right) + f(x)y(x) + g(x)y(x)^2 + 4y(x)y''(x) - 2y(x)^2y'(x) - 3y'(x)^2 + y(x)^4 = 0$$

✗ **Mathematica** : cpu = 9.28954 (sec), leaf count = 0 , could not solve  
DSolve[f[x]\*y[x] + g[x]\*y[x]^2 + y[x]^4 - 2\*y[x]^2\*Derivative[1][y][x] + (6\*y[x]^2 - (2\*y[x]^2\*f'[x])/f[x])\*Derivative[2][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(4\*diff(diff(y(x),x),x)\*y(x)-3\*diff(y(x),x)^2+(6\*y(x)^2-2\*diff(f(x),x)\*y(x)/f(x))\*diff(y(x),x)^2\*diff(y(x),x)+g(x)\*y(x)^2+f(x)\*y(x)=0,y(x))

2.1752 ODE No. 1752

---

$$ay(x)^2 + 4y(x)y''(x) - 5y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.150738 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow c_2 \text{sech}^4\left(\frac{1}{4}\sqrt{a}(x - 4c_1)\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.074 (sec), leaf count = 33

$$\left\{ y(x) = 16 \frac{\left(e^{1/4\sqrt{ax}}\right)^4 a^2}{\left(e^{1/2\sqrt{ax}} - C1 - C2\right)^4} \right\}$$

2.1753 ODE No. 1753

---

$$12y(x)y''(x)-15y'(x)^2+8y(x)^3=0$$

✓ **Mathematica** : cpu = 0.304864 (sec), leaf count = 43

$$\left\{\left\{y(x)\rightarrow \frac{2304c_1^2}{\left(3c_1^2x^2+6c_2c_1^2x+3c_2^2c_1^2+128\right)^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.105 (sec), leaf count = 147

$$\left\{-12\frac{y\left(x\right)\left(8\sqrt{y\left(x\right)}-C1\right)\sqrt{8y\left(x\right)-C1}\sqrt{y\left(x\right)}}{\sqrt{-24\left(y\left(x\right)\right)^3+3C1\left(y\left(x\right)\right)^{5/2}-C1\sqrt{\sqrt{y\left(x\right)}\left(8\sqrt{y\left(x\right)}-C1\right)}}-x-C2=0,12\frac{y\left(x\right)\left(8\sqrt{y\left(x\right)}-C1\right)\sqrt{8y\left(x\right)-C1}\sqrt{y\left(x\right)}}{\sqrt{-24\left(y\left(x\right)\right)^3+3C1\left(y\left(x\right)\right)^{5/2}-C1\sqrt{\sqrt{y\left(x\right)}\left(8\sqrt{y\left(x\right)}-C1\right)}}-x-C2=0\right\}$$

2.1754 ODE No. 1754

---

$$ny(x)y''(x)-(n-1)y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.110062 (sec), leaf count = 17

$$\{\{y(x)\rightarrow c_2(x-c_1n)^n\}\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 15

$$\left\{y\left(x\right)=\left(\frac{-C1\,x+C2}{n}\right)^n\right\}$$

2.1755 ODE No. 1755

---

$$ay(x)y''(x)+by'(x)^2+c0+c1y(x)+c2y(x)^2+c3y(x)^3+c4y(x)^4=0$$

✓ **Mathematica** : cpu = 6.29888 (sec), leaf count = 716

$$\left\{\left\{y(x)\rightarrow \text{InverseFunction}\left[\int_1^{\#1}-\frac{1}{\sqrt{4b^5c_1K[1]^{-\frac{2b}{a}}+20ab^4c_1K[1]^{-\frac{2b}{a}}+35a^2b^3c_1K[1]^{-\frac{2b}{a}}+25a^3b^2c_1K[1]^{-\frac{2b}{a}}+20a^4bK[1]^{-\frac{2b}{a}}+a^5K[1]^{-\frac{2b}{a}}}}}\right]\right\}\right\}$$

✓ **Maple** : cpu = 0.217 (sec), leaf count = 418

$$\left\{\int^{y(x)}(2a+b)(3a+2b)(a+b)(a+2b)b_-a^{2\frac{b}{a}}\frac{1}{\sqrt{-36_-a^{2\frac{b}{a}}(a+b/2)(a+b)\left(2/3(a+b/2)(a+b)(a+2b)a^{2\frac{b}{a}}+2a^2b(a+b/2)(a+b)(a+2b)a^{2\frac{b}{a}}+a^3b^2(a+b/2)(a+b)a^{2\frac{b}{a}}+a^4b^3a^{2\frac{b}{a}}+a^5b^4a^{2\frac{b}{a}}\right)}}}\right\}$$

2.1756 ODE No. 1756

---

$$ay(x)y''(x)+by'(x)^2-\frac{y(x)y'(x)}{\sqrt{c^2+x^2}}=0$$

✓ **Mathematica** : cpu = 0.631381 (sec), leaf count = 211

$$\left\{\left\{y(x)\rightarrow c_2\exp\left(\int_1^x-\frac{\left(\frac{K[2]}{\sqrt{c^2+K[2]^2}}+1\right)^{\frac{1}{2}/a}}{\left(1-\frac{K[2]}{\sqrt{c^2+K[2]^2}}\right)^{\frac{1}{2}/a}\int_1^{K[2]}\frac{\exp\left(\frac{\frac{1}{2}\log\left(\frac{K[1]}{\sqrt{c^2+K[1]^2}}+1\right)-\frac{1}{2}\log\left(1-\frac{K[1]}{\sqrt{c^2+K[1]^2}}\right)}{a}\right)\left(-\sqrt{c^2+K[1]^2}a-b\sqrt{c^2+K[1]^2}\right)}{a\sqrt{c^2+K[1]^2}}}\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.115 (sec), leaf count = 75

$$\left\{y\left(x\right)=\left(\left(\frac{a}{a+b}\left(\frac{C1\sqrt[3]{2}ax^{a^{-1}+1}}{a+1}{}_2F_1\left(-\frac{1}{2a},-\frac{1}{2a}-\frac{1}{2};1-a^{-1};-\frac{c^2}{x^2}\right)+C2\right)^{-1}\right)^{\frac{a}{a+b}}\right)^{-1}\right\}$$

**2.1757            ODE No. 1757**

---

$$(a+2)f(x)y(x)^2y'(x)+ay(x)y''(x)+ay(x)^3y'(x)-(a-1)y'(x)^2+f(x)^2y(x)^4=0$$

✗ **Mathematica** : cpu = 9.5957 (sec), leaf count = 0 , could not solve

DSolve[f[x]^2\*y[x]^4 + (2 + a)\*f[x]\*y[x]^2\*Derivative[1][y][x] + a\*y[x]^3\*Derivative[1][y][x] + 1 + a)\*Derivative[1][y][x]^2 + a\*y[x]\*Derivative[2][y][x] == 0, y[x], x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(a\*y(x)\*diff(diff(y(x),x),x)-(a-1)\*diff(y(x),x)^2+(a+2)\*f(x)\*y(x)^2\*diff(y(x),x)+f(x)^2\*y(x)^4)=0,y(x))

**2.1758            ODE No. 1758**

---

$$y''(x)(ay(x)+b)+cy'(x)^2=0$$

✓ **Mathematica** : cpu = 0.32749 (sec), leaf count = 36

$$\left\{\left\{y(x)\rightarrow\frac{(c_1(-(a+c))(-c_2-x))^{\frac{a}{a+c}}-b}{a}\right\}\right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 42

$$\left\{y(x)=\frac{1}{a}\left((\_C1\,x+\_C2)(a+c)\left(\frac{1}{(a+c)(\_C1\,x+\_C2)}\right)^{\frac{c}{a+c}}-b\right)\right\}$$

**2.1759            ODE No. 1759**

---

$$xy(x)y''(x)+xy'(x)^2-y(x)y'(x)=0$$

✓ **Mathematica** : cpu = 0.130957 (sec), leaf count = 18

$$\left\{\left\{y(x)\rightarrow c_2\sqrt{c_1+x^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 31

$$\left\{y(x)=\sqrt{\_C1\,x^2+2\_C2},y(x)=-\sqrt{\_C1\,x^2+2\_C2}\right\}$$

**2.1760            ODE No. 1760**

---

$$ay(x)y'(x)+f(x)+xy(x)y''(x)+xy'(x)^2=0$$

✓ **Mathematica** : cpu = 0.0778341 (sec), leaf count = 108

$$\left\{\left\{y(x)\rightarrow-\sqrt{2}\sqrt{\int_1^x-K[2]^{-a}\left(c_1+\int_1^{K[2]}f(K[1])K[1]^{a-1}dK[1]\right)dK[2]+c_2}\right\},\left\{y(x)\rightarrow\sqrt{2}\sqrt{\int_1^x-K[2]^{-a}\left(c_1+\int_1^{K[2]}f(K[1])K[1]^{a-1}dK[1]\right)dK[2]+c_2}\right\}\right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 114

$$\left\{y(x)=\frac{\sqrt{2}}{a-1}\sqrt{(a-1)\left(x^{1-a}\int\frac{x^af(x)}{x}dx+x^{1-a}\_C1-\int f(x)\,dx-\_C2\right)},y(x)=-\frac{\sqrt{2}}{a-1}\sqrt{(a-1)\left(x^{1-a}\int\frac{x^af(x)}{x}dx+x^{1-a}\_C1-\int f(x)\,dx-\_C2\right)}\right\}$$

**2.1761            ODE No. 1761**

---

$$x\left(ay(x)^4+d\right)+y(x)\left(by(x)^2+c\right)+xy(x)y''(x)-xy'(x)^2+y(x)y'(x)=0$$

✗ **Mathematica** : cpu = 1.08001 (sec), leaf count = 0 , could not solve

DSolve[y[x]\*(c + b\*y[x]^2) + x\*(d + a\*y[x]^4) + y[x]\*Derivative[1][y][x] - x\*Derivative[1][y][x]^2 + y(x)\*y'(x) = 0, y[x], x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(x\*y(x)\*diff(diff(y(x),x),x)-x\*diff(y(x),x)^2+y(x)\*diff(y(x),x)+x\*(d+a\*y(x)^4)+y(x)\*y'(x)=0,y(x))

2.1762 ODE No. 1762

---

$$ay(x)y'(x)+bxy(x)^3+xy(x)y''(x)-xy'(x)^2=0$$

✗ **Mathematica** : cpu = 48.1856 (sec), leaf count = 0 , could not solve  
DSolve[b\*x\*y[x]^3 + a\*y[x]\*Derivative[1][y][x] - x\*Derivative[1][y][x]^2 + x\*y[x]\*Derivative[1][y][x], y[x], x]  
✓ **Maple** : cpu = 0.703 (sec), leaf count = 108

$$\left\{y(x)=ODESolStruc\left(\frac{-a}{\left(e^{\int -b(-a) d_-a+_-C1}\right)^2},\left[\left\{\frac{d}{d_-a}-b(-a)=-2\frac{-b(-a)\left(1/2+_-a^2(-1/2b_-a+a-1)\right)}{-a}\right\}\right]\right)\right\}$$

2.1763 ODE No. 1763

---

$$ay(x)y'(x)+xy(x)y''(x)+2xy'(x)^2=0$$

✓ **Mathematica** : cpu = 0.220945 (sec), leaf count = 40

$$\left\{\left\{y(x)\rightarrow c_2\left(3x^{1-a}-\left(a-1\right)c_1\right)^{-\frac{a-1}{3(1-a)}}\right\}\right\}$$

✓ **Maple** : cpu = 0.045 (sec), leaf count = 148

$$\left\{y(x)=\frac{\sqrt[3]{3}}{(a-1)x^a}\sqrt[3]{(a-1)^2(x^a)^2(-C2(a-1)x^a-_-C1x)},y(x)=\frac{\sqrt[3]{3}(i\sqrt{3}-1)}{(2a-2)x^a}\sqrt[3]{(a-1)^2(x^a)^2(-C2(a-1)x^a-_-C1x)}\right\}$$

2.1764 ODE No. 1764

---

$$xy(x)y''(x)-2xy'(x)^2+(y(x)+1)y'(x)=0$$

✓ **Mathematica** : cpu = 0.0508407 (sec), leaf count = 52

$$\left\{\left\{y(x)\rightarrow\frac{\tan\left(\frac{1}{2}\left(\sqrt{2}\sqrt{c_1}\log(x)-\sqrt{2}\sqrt{c_1}c_2\right)\right)}{\sqrt{2}\sqrt{c_1}}\right\}\right\}$$

✓ **Maple** : cpu = 0.135 (sec), leaf count = 18

$$\left\{y(x)=_-C1\tanh\left(\frac{\ln(x)-_-C2}{2_-C1}\right)\right\}$$

2.1765 ODE No. 1765

---

$$ay(x)y'(x)+xy(x)y''(x)-2xy'(x)^2=0$$

✓ **Mathematica** : cpu = 0.2399 (sec), leaf count = 34

$$\left\{\left\{y(x)\rightarrow c_2\left((a-1)c_1+x^{1-a}\right)^{\frac{a-1}{1-a}}\right\}\right\}$$

✓ **Maple** : cpu = 0.025 (sec), leaf count = 27

$$\left\{y(x)=-\frac{(a-1)x^a}{_-C2(a-1)x^a-_-C1x}\right\}$$

2.1766 ODE No. 1766

---

$$xy(x)y''(x)-4xy'(x)^2+4y(x)y'(x)=0$$

✓ **Mathematica** : cpu = 0.140616 (sec), leaf count = 21

$$\left\{\left\{y(x)\rightarrow\frac{c_2x}{\sqrt[3]{c_1x^3+1}}\right\}\right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 64

$$\left\{y(x)=x\frac{1}{\sqrt[3]{-3_-C2x^3+_-C1}},y(x)=\frac{(i\sqrt{3}-1)x}{2}\frac{1}{\sqrt[3]{-3_-C2x^3+_-C1}},y(x)=-\frac{(i\sqrt{3}+1)x}{2}\frac{1}{\sqrt[3]{-3_-C2x^3+_-C1}}\right\}$$

**2.1767                      ODE No. 1767**

---

$$\left(\frac{ax}{\sqrt{b^2-x^2}}-x\right)y'(x)^2+xy(x)y''(x)-y(x)y'(x)=0$$

✓ **Mathematica** : cpu = 0.263143 (sec), leaf count = 55

$$\left\{\left\{y(x)\rightarrow c_2\exp\left(\frac{c_1\log\left(a\sqrt{b^2-x^2}-c_1\right)}{a^2}+\frac{\sqrt{b^2-x^2}}{a}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.276 (sec), leaf count = 50

$$\left\{y\left(x\right)=-C2\,\mathrm{e}^{\int -x\sqrt{b^2-x^2}\left(-C1\sqrt{b^2-x^2}+a\left(b^2-x^2\right)\right)^{-1}\mathrm{d}x}\right\}$$

**2.1768                      ODE No. 1768**

---

$$x(y(x)+x)y''(x)+xy'(x)^2+(x-y(x))y'(x)-y(x)=0$$

✓ **Mathematica** : cpu = 0.0388935 (sec), leaf count = 53

$$\left\{\left\{y(x)\rightarrow -\sqrt{2c_2x^2+c_1+x^2}-x\right\},\left\{y(x)\rightarrow \sqrt{2c_2x^2+c_1+x^2}-x\right\}\right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 43

$$\left\{y\left(x\right)=-x-\sqrt{\left(-C2+1\right)x^2+C1},y\left(x\right)=-x+\sqrt{\left(-C2+1\right)x^2+C1}\right\}$$

**2.1769                      ODE No. 1769**

---

$$2xy(x)y''(x)-xy'(x)^2+y(x)y'(x)=0$$

✓ **Mathematica** : cpu = 0.132861 (sec), leaf count = 18

$$\left\{\left\{y(x)\rightarrow c_2\left(c_1+\sqrt{x}\right)^2\right\}\right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 21

$$\left\{y\left(x\right)=-C1\sqrt{x}-C2+C1^2x+\frac{C2^2}{4}\right\}$$

**2.1770                      ODE No. 1770**

---

$$x^2(y(x)-1)y''(x)-2x^2y'(x)^2-2x(y(x)-1)y'(x)-2(y(x)-1)^2y(x)=0$$

✓ **Mathematica** : cpu = 0.584841 (sec), leaf count = 28

$$\left\{\left\{y(x)\rightarrow \frac{1}{x^2\left(-\frac{c_1}{x}+c_2-\frac{1}{x^2}\right)}+1\right\}\right\}$$

✓ **Maple** : cpu = 0.045 (sec), leaf count = 26

$$\left\{y\left(x\right)=\frac{x\left(-C1\,x-C2\right)}{-C1\,x^2-C2\,x-1}\right\}$$

**2.1771                      ODE No. 1771**

---

$$x^2(y(x)+x)y''(x)-\left(xy'(x)-y(x)\right)^2=0$$

✓ **Mathematica** : cpu = 0.100415 (sec), leaf count = 21

$$\left\{\left\{y(x)\rightarrow c_2xe^{\frac{c_1}{x}}-x\right\}\right\}$$

✓ **Maple** : cpu = 0.128 (sec), leaf count = 22

$$\left\{y\left(x\right)=-\frac{x}{-C1}\left(-\mathrm{e}^{\frac{C2}{x}}\mathrm{e}^{-1}+C1\right)\right\}$$



**2.1772      ODE No. 1772**

---

$$a \left( x y'(x) - y(x) \right)^2 + x^2 (x - y(x)) y''(x) = 0$$

✓ **Mathematica** : cpu = 0.559772 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow x \left( \left( (a-1) \left( \frac{(-1)^{a+1} c_1}{x} - c_2 \right) \right)^{\frac{1}{1-a}} + 1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.116 (sec), leaf count = 37

$$\left\{ \left( x^a y(x) - x^{a+1} \right) (x - y(x))^{-a} - x (a-1) \_C1 + \_C2 = 0 \right\}$$

**2.1773      ODE No. 1773**

---

$$2x^2 y(x) y''(x) + x^2 \left( - \left( y'(x)^2 + 1 \right) \right) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.30985 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{x \left( c_1^2 \log^2(x) - 2c_2 c_1^2 \log(x) + c_2^2 c_1^2 + 4 \right)}{4c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 30

$$\left\{ y(x) = \frac{x \left( 4 \_C2^2 (\ln(x))^2 + 4 \_C1 \ln(x) \_C2 + \_C1^2 + 1 \right)}{4 \_C2} \right\}$$

**2.1774      ODE No. 1774**

---

$$ax^2 y(x) y''(x) + bx^2 y'(x)^2 + cxy(x) y'(x) + dy(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.28417 (sec), leaf count = 92

$$\left\{ \left\{ y(x) \rightarrow c_2 \exp \left( - \frac{\log(x) \left( a \left( \sqrt{\frac{a^2 - 2a(c+2d) - 4bd + c^2}{a^2}} - 1 \right) + c \right) - 2a \log \left( x \sqrt{\frac{a^2 - 2a(c+2d) - 4bd + c^2}{a^2}} + c_1 \right)}{2(a+b)} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.191 (sec), leaf count = 136

$$\left\{ y(x) = x^{-\frac{1}{2a+2b} \sqrt{(-4a-4b)d+(a-c)^2}} x^{\frac{a}{2a+2b}} x^{-\frac{c}{2a+2b}} \left( \frac{a^2 + (-2c-4d)a-4db+c^2}{(a+b)^2} \left( x^{\frac{1}{a} \sqrt{(-4a-4b)d+(a-c)^2}} \_C1 \right. \right. \right.$$

**2.1775      ODE No. 1775**

---

$$-a(x+2)y(x)^2 + x(x+1)^2 y(x) y''(x) - x(x+1)^2 y'(x)^2 + 2(x+1)^2 y(x) y'(x) = 0$$

✓ **Mathematica** : cpu = 0.107727 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow c_2 e^{\frac{-a-c_1}{x} + a \log(x+1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.071 (sec), leaf count = 31

$$\left\{ y(x) = \frac{(1+x)^a}{\_C1 e^a} e^{\frac{\_C2}{x}} \left( e^{\frac{a}{x}} \right)^{-1} \right\}$$

**2.1776      ODE No. 1776**

---

$$8 \left( 1 - x^3 \right) y(x) y''(x) - 4 \left( 1 - x^3 \right) y'(x)^2 - 12x^2 y(x) y'(x) + 3xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 8.09905 (sec), leaf count = 1743

$$\left\{\left(\int_1^x-\frac{2\left(-\frac{3(1-K[2])^{3/4}\sqrt[4]{K[2]^2+K[2]+1}}{\sqrt[4]{\sqrt{3}K[1]+\sqrt{2K[1]-i\sqrt{3}+1}}\sqrt{2K[1]+i\sqrt{3}+1+\sqrt{3}}}}\frac{K[2]}{2(1-K[1])^{3/2}\sqrt{K[1]^2+K[1]+1}}dK[1]K[2]^2\right)+c_1\left(-\frac{3(1-K[2])^{3/4}\sqrt[4]{K[2]^2+K[2]+1}}{2\sqrt{2}\left(K[2]^3-1\right)^{5/4}\sqrt[4]{\sqrt{3}K[2]+\sqrt{2K[2]-i\sqrt{3}+1}}\sqrt{2K[2]+i\sqrt{3}+1+\sqrt{3}}}\right)}{2\sqrt{2}\left(K[2]^3-1\right)^{5/4}\sqrt[4]{\sqrt{3}K[2]+\sqrt{2K[2]-i\sqrt{3}+1}}\sqrt{2K[2]+i\sqrt{3}+1+\sqrt{3}}}}\right\}y(x)\rightarrow e$$

✓ **Maple** : cpu = 0.233 (sec), leaf count = 49

$$\left\{y\left(x\right)=-\frac{x}{-C1}\left(-C1\,LegendreQ\left(-\frac{1}{6},\frac{1}{3},\sqrt{-(x-1)\left(x^2+x+1\right)}\right)+\frac{C2}{2}LegendreP\left(-\frac{1}{6},\frac{1}{3},\sqrt{-(x-1)\left(x^2+x+1\right)}\right)\right)$$

### 2.1777 ODE No. 1777

$$f0(x)y(x)y''(x)+f1(x)y'(x)^2+f2(x)y(x)y'(x)+f3(x)y(x)^2=0$$

✗ **Mathematica** : cpu = 46.5017 (sec), leaf count = 0 , could not solve  
DSolve[f3[x]\*y[x]^2 + f2[x]\*y[x]\*Derivative[1][y][x] + f1[x]\*Derivative[1][y][x]^2 + f0[x]\*y[x]^2==0,y[x],x]

✓ **Maple** : cpu = 0.507 (sec), leaf count = 79

$$\left\{y\left(x\right)=ODESolStruc\left(e^{\int -b\left(\_a\right)d\_a+-C1},\left[\left\{\frac{d}{d\_a}-b\left(\_a\right)=\frac{\left(-f0\left(\_a\right)-f1\left(\_a\right)\right)\left(\_b\left(\_a\right)\right)^2-f2\left(\_a\right)\_b\left(\_a\right)}{f0\left(\_a\right)}\right.\right.\right.$$

### 2.1778 ODE No. 1778

$$y(x)^2y''(x)-a=0$$

✓ **Mathematica** : cpu = 0.19161 (sec), leaf count = 65

$$\text{Solve}\left[\left(\frac{y(x)\sqrt{c_1-\frac{2a}{y(x)}}}{c_1}+\frac{2a\,\tanh^{-1}\left(\frac{\sqrt{c_1-\frac{2a}{y(x)}}}{\sqrt{c_1}}\right)}{c_1^{3/2}}\right)^2=(c_2+x)^2,y(x)\right]$$

✓ **Maple** : cpu = 0.426 (sec), leaf count = 245

$$\left\{y\left(x\right)=-\frac{-C1\left(a\_C1+e^{RootOf\left(csgn\left(\_C1^{-1}\right)\_C1^4a^2-2\_Z\_C1^3ae^{-Z}-csgn\left(\_C1^{-1}\right)\left(e^{-Z}\right)^2\_C1^2-2\,csgn\left(\_C1^{-1}\right)e^{-Z}\_C2-2\,csgn\left(\_C1^{-1}\right)\right)}\right)}{2\,e^{RootOf\left(csgn\left(\_C1^{-1}\right)\_C1^4a^2-2\_Z\_C1^3ae^{-Z}-csgn\left(\_C1^{-1}\right)\left(e^{-Z}\right)^2\_C1^2-2\,csgn\left(\_C1^{-1}\right)e^{-Z}\_C2-2\,csgn\left(\_C1^{-1}\right)\right)}}$$

### 2.1779 ODE No. 1779

$$ax+y(x)^2y''(x)+y(x)y'(x)^2=0$$

✗ **Mathematica** : cpu = 22.2545 (sec), leaf count = 0 , could not solve  
DSolve[a\*x + y[x]\*Derivative[1][y][x]^2 + y[x]^2\*Derivative[2][y][x] == 0, y[x], x]

✓ **Maple** : cpu = 0.4 (sec), leaf count = 112

$$\left\{\ln\left(x\right)-\int^{\frac{y\left(x\right)}{x}}\frac{-g^2}{2\_g^3+2\,a}\left(\sqrt[3]{\frac{a}{-g^3}}\sqrt{3}\tan\left(RootOf\left(-2\_Z\sqrt{3}+\ln\left(\frac{\left(\tan\left(\_Z\right)\right)^2+1}{\left(\tan\left(\_Z\right)\right)^2+2\sqrt{3}\tan\left(\_Z\right)+3}\right)\right)+C\right)$$

2.1780 ODE No. 1780

---

$$-ax - b + y(x)^2 y''(x) + y(x) y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 20.3137 (sec), leaf count = 0 , could not solve  
DSolve[-b - a\*x + y[x]\*Derivative[1][y][x]^2 + y[x]^2\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.44 (sec), leaf count = 160

$$\left\{ \frac{b \ln(ax + b)}{a} - \int^{\frac{y(x)}{ax+b}} - \frac{g^2 b \sqrt{3}}{6 g^3 a^2 - 6} \left( -3 \tan \left( \text{RootOf} \left( 6 b^2 \int \frac{g^2}{g^3 a^2 - 1} \left( -\frac{a}{b^3 g^3} \right)^{2/3} d g - 2 Z \sqrt{3} + \ln \right) \right) \right.$$

2.1781 ODE No. 1781

---

$$(y(x)^2 + 1) y''(x) + (1 - 2y(x)) y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.179918 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow \frac{i (1 - c_1^{2i} (c_2 + x)^{2i})}{1 + c_1^{2i} (c_2 + x)^{2i}} \right\} \right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 11

$$\{y(x) = \tan(\ln(_C1 x + _C2))\}$$

2.1782 ODE No. 1782

---

$$(y(x)^2 + 1) y''(x) - 3y(x) y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.155657 (sec), leaf count = 93

$$\left\{ \left\{ y(x) \rightarrow -\frac{ic_1(c_2 + x)}{\sqrt{c_1^2 x^2 + 2c_2 c_1^2 x + c_2^2 c_1^2 - 1}} \right\}, \left\{ y(x) \rightarrow \frac{ic_1(c_2 + x)}{\sqrt{c_1^2 x^2 + 2c_2 c_1^2 x + c_2^2 c_1^2 - 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt{-( _C1^2 x^2 + 2 _C1 _C2 x + _C2^2 - 1)^{-1} ( _C1 x + _C2)} \right\}$$

2.1783 ODE No. 1783

---

$$(y(x)^2 + x) y''(x) - 2 (x - y(x)^2) y'(x)^3 + (4y(x) y'(x) + 1) y'(x) = 0$$

✓ **Mathematica** : cpu = 1.03469 (sec), leaf count = 26

$$\text{Solve} \left[ x = c_2 e^{e^{-c_1} y(x)} - y(x)^2, y(x) \right]$$

✓ **Maple** : cpu = 0.125 (sec), leaf count = 23

$$\left\{ \frac{-y(x) _C1 + \ln \left( x + (y(x))^2 \right) + _C2 + 2}{y(x)} = 0 \right\}$$

2.1784 ODE No. 1784

---

$$(x^2 + y(x)^2) y''(x) - (x y'(x) - y(x)) (y'(x)^2 + 1) = 0$$

✓ **Mathematica** : cpu = 0.259395 (sec), leaf count = 74

$$\text{Solve} \left[ \frac{1}{2} \left( i \cot(c_1) \left( \log \left( 1 - \frac{i y(x)}{x} \right) - \log \left( 1 + \frac{i y(x)}{x} \right) \right) + \log \left( 1 - \frac{i y(x)}{x} \right) + \log \left( 1 + \frac{i y(x)}{x} \right) \right) = c_2 - \log(x) \right]$$

✓ **Maple** : cpu = 0.639 (sec), leaf count = 82

$$\left\{ y(x) = \tan \left( \text{RootOf} \left( - \left( e^{\frac{i _C1 Z}{-1 + _C1}} \right)^2 \left( e^{\frac{C2 - C1}{-1 + _C1}} \right)^2 \left( x^{\frac{-C1}{-1 + _C1}} \right)^2 \left( e^{\frac{i Z}{-1 + _C1}} \right)^2 + (\cos(_C2))^2 \left( e^{\frac{-C2}{-1 + _C1}} \right)^2 \left( x^{(-1 + _C1)} \right)^2 \right) \right) \right\}$$

2.1785 ODE No. 1785

$$\left(x^2+y(x)^2\right)y''(x)-2\left(xy'(x)-y(x)\right)\left(y'(x)^2+1\right)=0$$

✓ **Mathematica** : cpu = 0.315481 (sec), leaf count = 95

$$\left\{\left\{y(x)\rightarrow\frac{1}{2}\left(-\sqrt{4x\left(e^{c_2}-x\right)+e^{2c_2}\cot^2\left(c_1\right)-e^{c_2}\cot\left(c_1\right)}\right)\right\},\left\{y(x)\rightarrow\frac{1}{2}\left(\sqrt{4x\left(e^{c_2}-x\right)+e^{2c_2}\cot^2\left(c_1\right)-e^{c_2}\cot\left(c_1\right)}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.279 (sec), leaf count = 83

$$y\left(x\right)=\frac{1}{2\_C2}\left(-\_C1+1-\sqrt{-\_C1^2+\left(4i\_C2x+2\right)\_C1-4\_C2^2x^2-4i\_C2x+1}\right),y\left(x\right)=\frac{1}{2\_C2}\left(-\_C1+1+\sqrt{-\_C1^2+\left(4i\_C2x+2\right)\_C1-4\_C2^2x^2-4i\_C2x+1}\right)$$

2.1786 ODE No. 1786

$$f(x)(1-y(x))y(x)y'(x)+2(1-y(x))y(x)y''(x)-(1-2y(x))y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.0597954 (sec), leaf count = 53

$$\left\{\left\{y(x)\rightarrow 1-\sin^2\left(\frac{1}{2}\left(-\int_1^x-\exp\left(-\int_1^{K[3]}\frac{1}{2}f(K[1])dK[1]\right)c_1dK[3]-c_2\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.138 (sec), leaf count = 42

$$y\left(x\right)=\frac{1}{8\_C2}\left(2\mathrm{e}^{-\_C1\int\mathrm{e}^{-1/2\int f\left(x\right)\mathrm{d}x}\mathrm{d}x}\_C2+1\right)^2\left(\mathrm{e}^{-\_C1\int\mathrm{e}^{-\frac{\int f\left(x\right)\mathrm{d}x}{2}}\mathrm{d}x}\right)^{-1}$$

2.1787 ODE No. 1787

$$h(y(x))+2(1-y(x))y(x)y''(x)-(1-3y(x))y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.908003 (sec), leaf count = 170

$$\left\{\left\{y(x)\rightarrow\text{InverseFunction}\left[\int_1^{\#1}-\frac{1}{\left(K[2]-1\right)\sqrt{K[2]}\sqrt{c_1+2\int_1^{K[2]}\frac{e^{-2\left(\log\left(1-K[1]\right)+\frac{1}{2}\log\left(K[1]\right)\right)}h\left(K[1]\right)}{2\left(K[1]-1\right)K[1]}dK[1]}\right]}dK[2]\&\right\}\right\}$$

✓ **Maple** : cpu = 0.244 (sec), leaf count = 80

$$\left\{\int^{y(x)}\frac{1}{\_b-1}\frac{1}{\sqrt{\_b\left(-\_C1+\int\frac{h(\_b)}{\_b^2(\_b-1)^3}\mathrm{d}\_b\right)}}\mathrm{d}\_b-x-\_C2=0,\int^{y(x)}-\frac{1}{\_b-1}\frac{1}{\sqrt{\_b\left(-\_C1+\int\frac{h(\_b)}{\_b^2(\_b-1)^3}\mathrm{d}\_b\right)}}\mathrm{d}\_b-x-\_C2=0\right\}$$

2.1788 ODE No. 1788

$$-4(1-y(x))y(x)^2\left(-f'(x)-f(x)^2-g'(x)+g(x)^2\right)+4y(x)y'(x)(f(x)y(x)+g(x))-2(1-y(x))y(x)y''(x)+(1-3y(x))y'(x)^2=0$$

✗ **Mathematica** : cpu = 1.36113 (sec), leaf count = 0 , could not solve

`DSolve[-4*(1 - y[x])*y[x]^2*(-f[x]^2 + g[x]^2 - Derivative[1][f][x] - Derivative[1][g][x]) + 4*y[x]*y'[x]*(f[x]*y[x] + g[x]) - 2*(1 - y[x])*y[x]*y''[x] + (1 - 3*y[x])*y'[x]^2, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{\sqrt{y(x)}-2\frac{\frac{\partial}{\partial x}DESol\left(\left\{-1/4\mathrm{e}^{2\int g(x)\mathrm{d}x-2\int f(x)\mathrm{d}x}\_C1^2\_Y(x)-2g(x)\frac{\mathrm{d}}{\mathrm{d}x}\_Y(x)+\frac{\mathrm{d}^2}{\mathrm{d}x^2}\_Y(x)\right\},\left\{\sqrt{y(x)}\right\}\right)}{DESol\left(\left\{-1/4\mathrm{e}^{2\int g(x)\mathrm{d}x-2\int f(x)\mathrm{d}x}\_C1^2\_Y(x)-2g(x)\frac{\mathrm{d}}{\mathrm{d}x}\_Y(x)+\frac{\mathrm{d}^2}{\mathrm{d}x^2}\_Y(x)\right\},\left\{\_Y(x)\right\}\right)}\mathrm{e}^{\int g(x)\mathrm{d}x}}\right\}$$

2.1789 ODE No. 1789

$$4y(x)^2(1-y(x))\left(-f'(x)+f(x)^2-g'(x)-g(x)^2\right)-4y(x)y'(x)(f(x)y(x)+g(x))+(1-y(x))^3\left(f_0(x)^2y(x)^2-f_1(x)^2y(x)^2\right)=0$$

✗ **Mathematica** : cpu = 2.34524 (sec), leaf count = 0 , could not solve

`DSolve[(1 - y[x])^3*(-f1[x]^2 + f0[x]^2*y[x]^2) + 4*(1 - y[x])*y[x]^2*(f[x]^2 - g[x]^2 - Derivative[1][f][x] - Derivative[1][g][x]) + 4*y[x]*y'[x]*(f[x]*y[x] + g[x]) - 2*(1 - y[x])*y[x]^3*(f0[x]^2*y[x]^2 - f1[x]^2*y[x]^2), y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(-2*y(x)*(1-y(x))*diff(diff(y(x),x),x)+(1-3*y(x))*diff(y(x),x)^2-4*y(x)*diff(y(x),x)*(f0(x)^2*y(x)^2-f1(x)^2*y(x)^2)+4*y(x)^2*(1-y(x))*(f(x)^2-g(x)^2-diff(g(x),x)-diff(f(x),x)),y(x),x)`

### 2.1790 ODE No. 1790

$$-h(y(x)) + 3(1 - y(x))y(x)y''(x) - 2(1 - 2y(x))y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.615835 (sec), leaf count = 186

$$\left\{\left\{y(x) \rightarrow \text{InverseFunction}\left[\int_1^{\#1} -\frac{1}{(1-K[2])^{2/3}K[2]^{2/3}\sqrt{c_1+2\int_1^{K[2]}-\frac{\exp(-2(\frac{2}{3}\log(1-K[1])+\frac{2}{3}\log(K[1]))h(K[1]))}{3(K[1]-1)K[1]}dK[1]}}\right.\right.$$

✓ **Maple** : cpu = 0.229 (sec), leaf count = 119

$$\left\{\int^{y(x)}-\frac{\sqrt{9}}{3}\frac{1}{\sqrt{(\_b-1)\_b\sqrt[3]{\_b(\_b-1)}\left(\_C1-\frac{2}{3}\int\frac{h(\_b)}{\_b(\_b-1)}(\_b^2-\_b)^{-\frac{4}{3}}d\_b\right)}}d\_b-x-\_C2=0,\int^{y(x)}$$

### 2.1791 ODE No. 1791

$$-h(y(x)) + (1 - y(x))y''(x) - 3(1 - 2y(x))y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.733166 (sec), leaf count = 168

$$\left\{\left\{y(x) \rightarrow \text{InverseFunction}\left[\int_1^{\#1} -\frac{e^{\frac{1}{2}(12-12K[2])}}{(K[2]-1)^3\sqrt{c_1+2\int_1^{K[2]}-\frac{\exp(-2(6(K[1]-1)+3\log(K[1]-1))h(K[1]))}{K[1]-1}dK[1]}}dK[2]\right.\right.$$

✓ **Maple** : cpu = 0.246 (sec), leaf count = 90

$$\left\{\int^{y(x)}\frac{1}{(\_b-1)^3(e^{-b})^6}\frac{1}{\sqrt{-2\int\frac{h(\_b)}{(e^{-b})^{12}(\_b-1)^7}d\_b+\_C1}}d\_b-x-\_C2=0,\int^{y(x)}-\frac{1}{(\_b-1)^3(e^{-b})^6}\frac{1}{\sqrt{-2\int\frac{h(\_b)}{(e^{-b})^{12}(\_b-1)^7}d\_b+\_C1}}d\_b-x-\_C2=0,\int^{y(x)}$$

### 2.1792 ODE No. 1792

$$a(y(x) - 1)y(x)y''(x) + y'(x)^2(by(x) + c) + h(y(x)) = 0$$

✓ **Mathematica** : cpu = 1.16179 (sec), leaf count = 226

$$\left\{\left\{y(x) \rightarrow \text{InverseFunction}\left[\int_1^{\#1} -\frac{(1-K[2])^{\frac{1}{2}\left(\frac{2b}{a}+\frac{2c}{a}\right)}K[2]^{-\frac{c}{a}}}{\sqrt{c_1+2\int_1^{K[2]}-\frac{\exp\left(-\frac{2(c\log(K[1])-(b+c)\log(1-K[1]))}{a}\right)h(K[1])}{a(K[1]-1)K[1]}dK[1]}}dK[2]\&\right]\left[c_2+x\right.\right.$$

✓ **Maple** : cpu = 0.351 (sec), leaf count = 194

$$\left\{\int^{y(x)}a\frac{1}{\sqrt{a\left(a\_C1-2\int\frac{h(\_b)}{\_b(\_b-1)}\left((\_b-1)^{\frac{b}{a}}\right)^2\left((\_b-1)^{\frac{c}{a}}\right)^2\left(\_b^{\frac{c}{a}}\right)^{-2}d\_b\right)}}\left(\_b^{\frac{c}{a}}\right)^{-1}\left((\_b-1)^{\frac{-b-c}{a}}\right)^{-1}d\_b$$

### 2.1793 ODE No. 1793

$$a(y(x) - 1)y(x)y''(x) - (a - 1)(2y(x) - 1)y'(x)^2 + f(x)(y(x) - 1)y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.126445 (sec), leaf count = 83

$$\left\{\left\{y(x) \rightarrow \text{InverseFunction}\left[a\#1^{-1/a}(-(\#1-1)\#1)^{\frac{1}{a}}{}_2F_1\left(\frac{1}{a},\frac{a-1}{a};1+\frac{1}{a};1-\#1\right)\&\right]\left[\int_1^x\exp\left(-\int_1^{K[3]}\frac{f(K[3])}{K[3]}dK[3]\right)dK[3]\right.\right.$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 40

$$\left\{-C1\mathrm{e}^{-\frac{fx}{a}}-\_C2+\int^{y(x)}\frac{\sqrt[3]{\_a(\_a-1)}}{\_a(\_a-1)}d\_a=0\right\}$$

2.1794 ODE No. 1794

$ab(y(x)-1)y(x)y''(x)+y'(x)^2(-(2ab-a-b)y(x)+(1-a)b))+f(x)(y(x)-1)y(x)y'(x)=0$

✓ **Mathematica** : cpu = 0.142216 (sec), leaf count = 69

$$\left\{\left\{y(x)\rightarrow \text{InverseFunction}\left[-a\#1^{\frac{1}{a}}{}_2F_1\left(\frac{1}{a},1-\frac{1}{b};1+\frac{1}{a};\#1\right)\&\right]\left[\int_1^x\exp\left(-\int_1^{K[3]}\frac{f(K[1])}{ab}dK[1]\right)c_1dK[3]\right.\right.$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 46

$$\left\{-C1\,e^{-\frac{fx}{ab}}-C2+\int^{y(x)}\frac{\sqrt[b]{-a-1}\sqrt[a]{-a}}{-a\,(-a-1)}d\_a=0\right\}$$

2.1795 ODE No. 1795

$xy(x)^2y''(x)-a=0$

✓ **Mathematica** : cpu = 0.235919 (sec), leaf count = 116

$$\text{Solve}\left[-\frac{\sqrt{-\frac{2ay(x)}{x}-\frac{2c_1y(x)^2}{x^2}}}{2c_1}-\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c_1}\left(\frac{a}{2c_1}+\frac{y(x)}{x}\right)}{\sqrt{-\frac{2ay(x)}{x}-\frac{2c_1y(x)^2}{x^2}}}\right)}{2\sqrt{2}c_1^{3/2}}-c_2-\frac{1}{x}=0,y(x)\right]$$

✓ **Maple** : cpu = 1.019 (sec), leaf count = 529

$$\left\{y(x)=\frac{\left(9a\_C1+e^{RootOf\left(243\,csgn(\_C1^{-1})\_C1^4a^2x-54\_Z\,e^{-Z}ax\_C1^3-3\,csgn(\_C1^{-1})(e^{-Z})^2\_C1^2x-6\,csgn(\_C1^{-1})e^{-Z}\right)}\right)}{2\,e^{RootOf\left(243\,csgn(\_C1^{-1})\_C1^4a^2x-54\_Z\,e^{-Z}ax\_C1^3-3\,csgn(\_C1^{-1})(e^{-Z})^2\_C1^2x-6\,csgn(\_C1^{-1})e^{-Z}\right)}}$$

2.1796 ODE No. 1796

$(a^2-x^2)\left(a^2-y(x)^2\right)y''(x)+(a^2-x^2)y(x)y'(x)^2-x\left(a^2-y(x)^2\right)y'(x)=0$

✓ **Mathematica** : cpu = 0.245113 (sec), leaf count = 363

$$\left\{\left\{y(x)\rightarrow -\frac{1}{2}e^{-c_2}\left(1-\frac{x}{\sqrt{x^2-a^2}}\right)^{-\frac{c_1}{2}}\left(\frac{x}{\sqrt{x^2-a^2}}+1\right)^{-\frac{c_1}{2}}\sqrt{2a^2e^{2c_2}\left(1-\frac{x}{\sqrt{x^2-a^2}}\right)^{c_1}\left(\frac{x}{\sqrt{x^2-a^2}}+1\right)^{c_1}}\right.\right.$$

✓ **Maple** : cpu = 0.185 (sec), leaf count = 51

$$\left\{y(x)=\frac{1}{2\_C2}\left(\left(\left(x+\sqrt{-a^2+x^2}\right)^{-C1}\right)^2\_C2^2+a^2\right)\left(\left(x+\sqrt{-a^2+x^2}\right)^{-C1}\right)^{-1}\right\}$$

2.1797 ODE No. 1797

$(y(x)-1)^3\left(ay(x)^2+b\right)+cxy(x)^2(y(x)-1)+dx^2y(x)^2(y(x)+1)+2x^2y(x)(y(x)-1)y''(x)-x^2(3y(x)-1)y'(x)^2+2xy(x)y'(x)^2-x^2y(x)^2y'(x)^2)=0$

✗ **Mathematica** : cpu = 11.1626 (sec), leaf count = 0 , could not solve  
DSolve[c\*x\*(-1 + y[x])\*y[x]^2 + d\*x^2\*y[x]^2\*(1 + y[x]) + (-1 + y[x])^3\*(b + a\*y[x]^2) + 2\*x^2\*y[x]^2\*y[x]\*Derivative[1][y][x] - x^2\*(-1 + 3\*y[x])\*Derivative[1][y][x]^2 + 2\*x^2\*(-1 + y[x])\*y[x]\*Derivative[2][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(2\*x^2\*y(x)\*(-1+y(x))\*diff(diff(y(x),x),x)-x^2\*(3\*y(x)-1)\*diff(y(x),x)^2+2\*x\*y(x)\*(-1+y(x))\*diff(y(x),x)+(a\*y(x)^2+b)\*(-1+y(x))^3+c\*x\*y(x)^2\*(-1+y(x))+d\*x^2\*y(x)^2\*(1+y(x))=0,y(x))



2.1804 ODE No. 1804

$$y''(x) \left(-ay(x) - b + 4y(x)^3\right) - \left(6y(x)^2 - \frac{a}{2}\right) y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 2.43793 (sec), leaf count = 415

Solve 
$$\left[ \frac{2 \sqrt{\frac{y(x) - \text{Root}[4 \# 1^3 - \# 1 a - b \&, 1]}{\text{Root}[4 \# 1^3 - \# 1 a - b \&, 3] - \text{Root}[4 \# 1^3 - \# 1 a - b \&, 1]}} \sqrt{\frac{y(x) - \text{Root}[4 \# 1^3 - \# 1 a - b \&, 2]}{\text{Root}[4 \# 1^3 - \# 1 a - b \&, 3] - \text{Root}[4 \# 1^3 - \# 1 a - b \&, 2]}} (y(x) - \text{Root}[4 \# 1^3 - \# 1 a - b \&, 1]}{c_1 \sqrt{2 a y(x) + 2 b - 8 y(x)^3} \sqrt{\text{Root}[4 \# 1^3 - \# 1 a - b \&, 3] - \text{Root}[4 \# 1^3 - \# 1 a - b \&, 1]}} \right]$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 31

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{4 \_a^3 - a \_a - b}} d\_a - \_C1 x - \_C2 = 0 \right\}$$

2.1805 ODE No. 1805

$$\left(-ay(x) - b + 4y(x)^3\right) \left(f(x)y'(x) + y''(x)\right) - \left(6y(x)^2 - \frac{a}{2}\right) y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.323416 (sec), leaf count = 438

Solve 
$$\left[ \frac{2 \sqrt{\frac{y(x) - \text{Root}[4 \# 1^3 - \# 1 a - b \&, 1]}{\text{Root}[4 \# 1^3 - \# 1 a - b \&, 3] - \text{Root}[4 \# 1^3 - \# 1 a - b \&, 1]}} \sqrt{\frac{y(x) - \text{Root}[4 \# 1^3 - \# 1 a - b \&, 2]}{\text{Root}[4 \# 1^3 - \# 1 a - b \&, 3] - \text{Root}[4 \# 1^3 - \# 1 a - b \&, 2]}} (y(x) - \text{Root}[4 \# 1^3 - \# 1 a - b \&, 1]}{\sqrt{a y(x) + b - 4 y(x)^3} \sqrt{\text{Root}[4 \# 1^3 - \# 1 a - b \&, 3] - \text{Root}[4 \# 1^3 - \# 1 a - b \&, 1]}} \right]$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 34

$$\left\{ \_C1 e^{-f x} - \_C2 + \int^{y(x)} \frac{1}{\sqrt{4 \_a^3 - a \_a - b}} d\_a = 0 \right\}$$

2.1806 ODE No. 1806

$$-f(x)((y(x)-1)y(x)(y(x)-x))^{3/2}+2(1-y(x))\left(x^2-2xy(x)+y(x)\right)y(x)y'(x)-2(1-x)x(1-y(x))(x-y(x))y(x)y'(x)$$

✗ **Mathematica** : cpu = 14.0866 (sec), leaf count = 0 , could not solve  
DSolve[-((1 - y[x])^2\*y[x]^2) - f[x]\*((-1 + y[x])\*y[x]\*(-x + y[x]))^(3/2) + 2\*(1 - y[x])\*y[x] y'(x)]

✓ **Maple** : cpu = 2.356 (sec), leaf count = 733

$$\left\{ -\frac{C1}{2} eval \left( \int \frac{1}{x-1} e^{\int \frac{1}{x(x-1)} EllipticE(\sqrt{x}) (EllipticK(\sqrt{x}))^{-1} dx} \int 1 \int \frac{1}{(x-y)^2 y (-1+y)} \sqrt{-y (-1+y) (x-y)} dy dx \right) \right\}$$

2.1807 ODE No. 1807

$$a(1-y(x))^2(x-y(x))^2y(x)^2+bx(1-y(x))^2(x-y(x))^2-c(1-x)(x-y(x))^2y(x)^2-d(1-x)x(1-y(x))^2y(x)^2+2(1-x)^2y(x)^2$$

✗ **Mathematica** : cpu = 23.6759 (sec), leaf count = 0 , could not solve  
DSolve[b\*x\*(1 - y[x])^2\*(x - y[x])^2 - d\*(1 - x)\*x\*(1 - y[x])^2\*y[x]^2 - c\*(1 - x)\*(x - y[x]) y'(x)]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(2\*x^2\*y(x)\*(1-x)^2\*(1-y(x))\*(x-y(x))\*diff(diff(y(x),x),x)-x^2\*(1-x)^2\*(x-2\*x\*y(x)-2\*y(x)+3\*y(x)^2)\*diff(y(x),x)^2-2\*x\*y(x)\*(1-x)\*(1-y(x))\*(x^2+y(x)-2\*x\*y(x))\*diff(y(x),x)^2\*(x-y(x))^2-c\*(1-x)\*y(x)^2\*(x-y(x))^2-d\*x\*y(x)^2\*(1-x)\*(1-y(x))^2+a\*y(x)^2\*(x-y(x))^2\*(1-y(x))^2=0,y(x))

2.1808 ODE No. 1808

$$b\sqrt{(1-y(x)^2)(1-a^2y(x)^2)}y'(x)^2+(y(x)^2-1)\left(a^2y(x)^2-1\right)y''(x)+y(x)\left(-2a^2y(x)^2+a^2+1\right)y'(x)^2=0$$

✓ **Mathematica** : cpu = 1.03828 (sec), leaf count = 124

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \int_1^{\#1} \frac{\exp \left( \frac{b \sqrt{1-K[1]^2} \sqrt{1-a^2 K[1]^2} F(\sin^{-1}(K[1])|a^2)}{\sqrt{(K[1]^2-1)(a^2 K[1]^2-1)}} + \frac{1}{2} (-\log(1-K[1])-\log(K[1]+1)) \right)}{c_1} \right] \right\} \right\}$$



✓ **Maple** : cpu = 0.105 (sec), leaf count = 72

$$\left\{ \int^{y(x)} \frac{\int \frac{1}{(-b^2-1)(-b^2a^2-1)} \left( -2-b^3a^2+a^2-b+b\sqrt{(-b^2-1)(-b^2a^2-1)}+b \right) d_b}{d_b-C1x-C2} = 0 \right\}$$

2.1809 ODE No. 1809

---

$$y''(x) \left( ax^2 + 2bx + c + y(x)^2 \right)^2 + dy(x) = 0$$

✗ **Mathematica** : cpu = 19.7812 (sec), leaf count = 0 , could not solve  
DSolve[d\*y[x] + (c + 2\*b\*x + a\*x^2 + y[x]^2)^2\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.342 (sec), leaf count = 336

$$\left\{ y(x) = RootOf \left( -a \arctan \left( (ax+b) \frac{1}{\sqrt{ca-b^2}} \right) - \int^{-Z} \frac{a}{-f^4ac + f^4b^2 + C1f^2a^2 - cf^2a + b^2f^2 + } \right. \right.$$

2.1810 ODE No. 1810

---

$$\sqrt{y(x)}y''(x) - a = 0$$

✓ **Mathematica** : cpu = 0.0773425 (sec), leaf count = 1677

$$\left\{ \left\{ y(x) \rightarrow \frac{3c_1^2}{16a^2} + \frac{\sqrt[3]{-\frac{221184c_1^6}{a^6} + \frac{159252480x^2c_1^3}{a^2} + \frac{159252480c_2^2c_1^3}{a^2} + \frac{318504960xc_2c_1^3}{a^2} + 2293235712a^2x^4 + 2293235}} \right\} \right.$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 91

$$\left\{ \frac{1}{12a^2} \left( -3C1 \sqrt{4a\sqrt{y(x)} - C1} - \left( 4a\sqrt{y(x)} - C1 \right)^{\frac{3}{2}} \right) - x - C2 = 0, \frac{1}{12a^2} \left( 3C1 \sqrt{4a\sqrt{y(x)} - } \right. \right.$$

2.1811 ODE No. 1811

---

$$\sqrt{x^2 + y(x)^2}y''(x) - a \left( y'(x)^2 + 1 \right)^{3/2} = 0$$

✗ **Mathematica** : cpu = 300.029 (sec), leaf count = 0 , timed out  
\$Aborted  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve((y(x)^2+x^2)^(1/2)\*diff(diff(y(x),x),x)-a\*(diff(y(x),x)^2+1)^(3/2)=0,y(x))

2.1812 ODE No. 1812

---

$$y(x)y''(x)(1 - \log(y(x))) + y'(x)^2(\log(y(x)) + 1) = 0$$

✓ **Mathematica** : cpu = 0.263911 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{c_1x+c_2c_1-1}{c_1(c_2+x)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.092 (sec), leaf count = 19

$$\left\{ y(x) = e^{\frac{C1x+C2-1}{-C1x+C2}} \right\}$$

2.1813 ODE No. 1813

---

$$Ay(x) \left( a \sin^2(y(x)) + c \right) + y''(x) \left( a \sin^2(y(x)) + b \right) + ay'(x)^2 \sin(y(x)) \cos(y(x)) = 0$$

✓ **Mathematica** : cpu = 31.5321 (sec), leaf count = 176

$$\left\{ \left\{ y(x) \rightarrow InverseFunction \left[ \int_1^{\#1} -\frac{\sqrt{2}\sqrt{\cos(2K[1])a-a-2b}}{\sqrt{2aAK[1]^2 + 4AcK[1]^2 - 2aA \sin(2K[1])K[1] + 2c_1 - aA \cos(2K[1])}} dK[1] \right. \right. \right.$$

✓ **Maple** : cpu = 0.32 (sec), leaf count = 138

$$\left\{\int^{y(x)}\sqrt{2}\left(b+a\left(\sin\left(\_a\right)\right)^2\right)\frac{1}{\sqrt{-\left(b+a\left(\sin\left(\_a\right)\right)^2\right)\left(Aa\left(\sin\left(\_a\right)\right)^2-2Aa\_a\cos\left(\_a\right)\sin\left(\_a\right)+\_a^2\left(a+\sin\left(\_a\right)\right)\right)}}d\_a\right\}$$

### 2.1814 ODE No. 1814

$$ah(y(x))y'(x)^2+h(y(x))y''(x)+j(y(x))=0$$

✓ **Mathematica** : cpu = 0.345112 (sec), leaf count = 120

$$\left\{\left\{y(x)\rightarrow \text{InverseFunction}\left[\int_1^{\#1}-\frac{e^{aK[2]}}{\sqrt{c_1+2\int_1^{K[2]}-\frac{e^{2aK[1]}j(K[1])}{h(K[1])}dK[1]}}dK[2]\&\right][c_2+x]\right\},\left\{y(x)\rightarrow \text{InverseFunction}\left[\int_1^{\#1}-\frac{e^{aK[2]}}{\sqrt{c_1+2\int_1^{K[2]}-\frac{e^{2aK[1]}j(K[1])}{h(K[1])}dK[1]}}dK[2]\&\right][c_2+x]\right\}\right\}$$

✓ **Maple** : cpu = 0.118 (sec), leaf count = 87

$$\left\{\int^{y(x)}\frac{1}{(h(\_b))^{-a}}\frac{1}{\sqrt{-2\int\frac{((h(\_b))^a)^2}{h(\_b)}d\_b+\_C1}}d\_b-x-\_C2=0,\int^{y(x)}-\frac{1}{(h(\_b))^{-a}}\frac{1}{\sqrt{-2\int2\frac{((h(\_b))^a)^2}{h(\_b)}d\_b+\_C1}}d\_b-x-\_C2=0\right\}$$

### 2.1815 ODE No. 1815

$$h(y(x))^2\left(-j\left(x,\frac{y'(x)}{h(y(x))}\right)\right)+h(y(x))y''(x)-h(y(x))y'(x)^2=0$$

✗ **Mathematica** : cpu = 0.966021 (sec), leaf count = 0 , could not solve

`DSolve[-(h[y[x]]^2*j[x, Derivative[1][y][x]/h[y[x]]]) - h[y[x]]*Derivative[1][y][x]^2 + h[y[x]]*Derivative[2][y][x], y[x], x]`

✓ **Maple** : cpu = 0.707 (sec), leaf count = 71

$$\left\{y(x)=ODESolStruc\left(\text{RootOf}\left(\int\_b(\_a)\,d\_a+\_C1-\int^{-Z}(h(\_f))^{-1}d\_f\right),\left[\left\{\frac{d}{d\_a}\_b(\_a)=1\right\},\left\{-\_b(\_a)=\text{RootOf}\left(\int\_b(\_a)\,d\_a+\_C1-\int^{-Z}(h(\_f))^{-1}d\_f\right)\right\}\right]\right)\right\}$$

### 2.1816 ODE No. 1816

$$x^2(-y(x))y'(x)+y'(x)y''(x)-xy(x)^2=0$$

✗ **Mathematica** : cpu = 55.0609 (sec), leaf count = 0 , could not solve

`DSolve[-(x*y[x]^2) - x^2*y[x]*Derivative[1][y][x] + Derivative[1][y][x]*Derivative[2][y][x], y[x], x]`

✓ **Maple** : cpu = 1.441 (sec), leaf count = 46

$$\left\{y(x)=ODESolStruc\left(\_b(\_a),\left[\left\{-\_a^2(\_b(\_a))^2+\left(\frac{d}{d\_a}\_b(\_a)\right)^2+\_C1=0\right\},\{\_a=x,\_b(\_a)=\text{RootOf}\left(\int\_b(\_a)\,d\_a+\_C1-\int^{-Z}(h(\_f))^{-1}d\_f\right)\}\right]\right)\right\}$$

### 2.1817 ODE No. 1817

$$4y'(x)^2+(xy'(x)-y(x))\,y''(x)=0$$

✗ **Mathematica** : cpu = 13.81 (sec), leaf count = 0 , could not solve

`DSolve[4*Derivative[1][y][x]^2 + (-y[x] + x*Derivative[1][y][x])*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.322 (sec), leaf count = 40

$$\left\{y(x)=e^{\int^{\ln(x)}e^{\text{RootOf}\left(\ln\left(e^{-Z}-1\right)e^{-Z}+\_C1\,e^{-Z}-\_Z\,e^{-Z}-\_b\,e^{-Z}+2\right)-1d\_b+\_C2}}d\_x}\right\}$$

### 2.1818 ODE No. 1818

$$\left(xy'(x)-y(x)\right)y''(x)-\left(y'(x)^2+1\right)^2=0$$

✗ **Mathematica** : cpu = 0.874707 (sec), leaf count = 0 , could not solve

`DSolve[-(1 + Derivative[1][y][x]^2)^2 + (-y[x] + x*Derivative[1][y][x])*Derivative[2][y][x], y[x], x]`

✓ **Maple** : cpu = 0.299 (sec), leaf count = 66

$$\left\{y(x)=\text{RootOf}\left(-\ln(x)+\int^{-Z}\frac{-\_f+\text{RootOf}\left(-\_C1\tan\left(\_Z^{-1}\right)\_Z+\_f\_C1\tan\left(\_Z^{-1}\right)+\_C1\_Z\right)}{\_f^2+1}d\_Z,\_Z\right)\right\}$$

2.1819 ODE No. 1819

$ax^3y'(x)y''(x)+by(x)^2=0$

✗ **Mathematica** : cpu = 21.1134 (sec), leaf count = 0 , could not solve  
DSolve[b\*y[x]^2 + a\*x^3\*Derivative[1][y][x]\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.039 (sec), leaf count = 42

$$\left\{y\left(x\right)=e^{\int^{\ln\left(x\right)}RootOf\left(-\int^{-Z}\frac{a-a}{-a^3a-a-a^2+b}d\_a-\_b+\_C1\right)d\_b+\_C2}\right\}$$

2.1820 ODE No. 1820

$y''(x)\left(f1(x)y'(x)+f2(x)y(x)\right)+f3(x)y'(x)^2+f4(x)y(x)y'(x)+f5(x)y(x)^2=0$

✗ **Mathematica** : cpu = 307.64 (sec), leaf count = 0 , timed out  
\$Aborted  
✓ **Maple** : cpu = 0.684 (sec), leaf count = 88

$$\left\{y\left(x\right)=ODESolStruc\left(e^{\int\_b(\_a)d\_a+\_C1},\left[\left\{\frac{d}{d\_a}\_b(\_a)=\frac{-\left(\_b(\_a)\right)^3f1+\left(-f2-f3\right)\left(\_b(\_a)\right)^2-f4(\_b(\_a))}{\_b(\_a)f1+f2}\right.\right.\right.$$

2.1821 ODE No. 1821

$(x^2+2y(x)^2y'(x))\,y''(x)+2y(x)y'(x)^3+3xy'(x)+y(x)=0$

✗ **Mathematica** : cpu = 41.9174 (sec), leaf count = 0 , could not solve  
DSolve[y[x] + 3\*x\*Derivative[1][y][x] + 2\*y[x]\*Derivative[1][y][x]^3 + (x^2 + 2\*y[x]^2\*Derivative[1][y][x])\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 2.261 (sec), leaf count = 54

$$\left\{y\left(x\right)=ODESolStruc\left(\_b(\_a),\left[\left(\_b(\_a)\right)^2\left(\frac{d}{d\_a}\_b(\_a)\right)^2+\_a^2\frac{d}{d\_a}\_b(\_a)+\_b(\_a)\_a+\_C1\right.\right.$$

2.1822 ODE No. 1822

$(y'(x)^2+y(x)^2)\,y''(x)+y(x)^3=0$

✓ **Mathematica** : cpu = 0.861177 (sec), leaf count = 371

$$\left\{\left\{y\left(x\right)\rightarrow c_2\exp\left(\frac{1}{12}\left(-2\sqrt{3}\tan^{-1}\left(\frac{1+2\mathrm{InverseFunction}\left[\frac{\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{\#1}{\sqrt{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{\#1}{\sqrt{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{\sqrt{6\left(1-i\sqrt{3}\right)}}+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{\#1}{\sqrt{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{\sqrt{6\left(1+i\sqrt{3}\right)}}\right)}{\sqrt{3}}\right.\right.\right.$$

✓ **Maple** : cpu = 1.137 (sec), leaf count = 291

$$\left\{y\left(x\right)=\left(\_C1+\tan\left(\sqrt{3}x\right)\right)^{\left(2\_C1^2+2\right)^{-1}}\_C2\left(\_C1+\tan\left(\sqrt{3}x\right)\right)^{\frac{-C1^2}{2\_C1^2+2}}\left(1+\left(\tan\left(\sqrt{3}x\right)\right)^2\right)^{-\frac{C1^2}{4\_C1^2+4}}\left(\right.$$

2.1823 ODE No. 1823

$y''(x)\left(a\left(xy'(x)-y(x)\right)+y'(x)^2\right)-b=0$

✗ **Mathematica** : cpu = 0.114387 (sec), leaf count = 0 , could not solve  
DSolve[-b + (Derivative[1][y][x]^2 + a\*(-y[x] + x\*Derivative[1][y][x]))\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.382 (sec), leaf count = 289

$$\left\{y\left(x\right)=-\frac{ax^2}{4}+RootOf\left(-x-\int^{-Z}\frac{1}{-f^2a^2-4\_fb+2\_C1}\sqrt{\left(-f^2a^2-4\_fb+2\_C1\right)\left(-fa+\sqrt{4\_fb-}\right)}d\_f+\_C2\right)\right\}$$

2.1824 ODE No. 1824

---

$$y''(x) \left( a \sqrt{y'(x)^2 + 1} - x y'(x) \right) - y'(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.720772 (sec), leaf count = 347

$$\left\{ \left\{ y(x) \rightarrow \frac{-2 \sqrt{x^2 (a^2 + c_1^2 - x^2)} + c_1 x \log \left( -c_1 \left( \sqrt{x^2 (a^2 + c_1^2 - x^2)} + c_1 x \right) + a^2 (-x) + a x^2 \right) + c_1 x \log \left( c_1 \right)}{a^3 - a x^2} \right. \right.$$

✓ **Maple** : cpu = 0.667 (sec), leaf count = 96

$$\left\{ y(x) = \int \frac{1}{a^3 - a x^2} \left( -\_C1 a^2 - x \sqrt{a^2 (\_C1^2 + a^2 - x^2)} \right) dx + \_C2, y(x) = \int \frac{1}{a^3 - a x^2} \left( -\_C1 a^2 + x \sqrt{a^2 (\_C1^2 + a^2 - x^2)} \right) dx + \_C2 \right\}$$

2.1825 ODE No. 1825

---

$$f(x) + y''(x) h(y'(x)) + j(y(x)) y'(x) = 0$$

✗ **Mathematica** : cpu = 0.0298853 (sec), leaf count = 0 , could not solve  
DSolve[f[x] + j[y[x]]\*Derivative[1][y][x] + h[Derivative[1][y][x]]\*Derivative[2][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.793 (sec), leaf count = 49

$$\left\{ y(x) = ODESolStruc \left( \_f(\_b), \left[ \int \_f(\_b) d\_a + \int \_f(\_b) h(\_a) d\_a + \_b f + \_C1 = 0 \right], \{ \_b = x, \_C1 = 0 \} \right) \right\}$$

2.1826 ODE No. 1826

---

$$-ay(x) - b + y''(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.796439 (sec), leaf count = 201

$$\left\{ \text{Solve} \left[ \frac{(ay(x) + b)^2 \left( 1 - \frac{4(ay(x) + b)^{3/2}}{3ac_1} \right) {}_2F_1 \left( \frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \frac{4(b + ay(x))^{3/2}}{3ac_1} \right)^2}{a^2 \left( c_1 - \frac{4(ay(x) + b)^{3/2}}{3a} \right)} = (c_2 + x)^2, y(x) \right], \text{Solve} \left[ \frac{(ay(x) + b)^2 \left( \frac{4}{3} - \frac{4(ay(x) + b)^{3/2}}{3ac_1} \right)}{a^2 \left( c_1 - \frac{4(ay(x) + b)^{3/2}}{3a} \right)} = (c_2 + x)^2, y(x) \right] \right\}$$

✓ **Maple** : cpu = 0.305 (sec), leaf count = 173

$$\left\{ \int \_a^{y(x)} \frac{1}{a \sqrt{3} \sqrt{a (4 \_a \sqrt{\_a a + b a} + 4 \sqrt{\_a a + b b} - \_C1)}} d\_a - x - \_C2 = 0, \int \_a^{y(x)} \frac{a}{\sqrt{-12 a ((\_a a + b)^{3/2} - \_C1)}} d\_a - x - \_C2 = 0 \right\}$$

2.1827 ODE No. 1827

---

$$a^2 y''(x)^2 - 2a x y''(x) + y'(x) = 0$$

✗ **Mathematica** : cpu = 0.899081 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[1][y][x] - 2\*a\*x\*Derivative[2][y][x] + a^2\*Derivative[2][y][x]^2 == 0, y[x], x]  
✓ **Maple** : cpu = 2.044 (sec), leaf count = 81

$$\left\{ y(x) = \int \text{RootOf} \left( - \int \_g^{\_Z} \left( x \sqrt{x^2 - \_f} - x^2 + 2 \_f a \right)^{-1} d\_f + \_C1 \right) dx + \_C2, y(x) = \int \text{RootOf} \left( - \int \_g^{\_Z} \left( x \sqrt{x^2 - \_f} - x^2 + 2 \_f a \right)^{-1} d\_f + \_C1 \right) dx + \_C2 \right\}$$

2.1828 ODE No. 1828

---

$$2(x^2 + 1) y''(x)^2 + 2y'(x) (y'(x) + x) - x (4y'(x) + x) y''(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.007598 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2} \sqrt{c_2 - c_1^2 x^2} + c_1 x + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.498 (sec), leaf count = 59

$$\left\{ y(x) = \frac{\_C1 x^2}{2} + \_C2 x + \_C1^2 + \_C2^2, y(x) = \frac{x}{2} \left( -\_C1 + \frac{\text{Arcsinh}(x)}{4} \right) \sqrt{x^2 + 1} - \frac{3 x^2}{16} + \_C1^2 + \_C2^2 \right\}$$

2.1829 ODE No. 1829

$$3x^2y''(x)^2 + 4y'(x)^2 - 2\left(3xy'(x) + y(x)\right)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0056858 (sec), leaf count = 24

$$\left\{\left\{y(x) \rightarrow \frac{c_1^2 x^2}{c_2} + c_1 x + c_2\right\}\right\}$$

✓ **Maple** : cpu = 0.256 (sec), leaf count = 36

$$\left\{y(x) = 0, y(x) = x^{\frac{2\sqrt{3}}{3}} \_C1 x, y(x) = \frac{\_C1^2 x^2}{\_C2} + \_C1 x + \_C2\right\}$$

2.1830 ODE No. 1830

$$(2 - 9x)x^2y''(x)^2 + 6y(x)y''(x) - 36xy'(x)^2 - 6(1 - 6x)xy'(x)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0233418 (sec), leaf count = 24

$$\left\{\left\{y(x) \rightarrow \frac{c_1^2 x^3}{c_2} + c_1 x + c_2\right\}\right\}$$

✓ **Maple** : cpu = 0.403 (sec), leaf count = 308

$$\left\{y(x) = 0, y(x) = \frac{27\_C1\sqrt{5}\sqrt{4x}}{4}\left((9x-1)\sqrt{9}+9\sqrt{9x^2-2x}\right)^{-\frac{2\sqrt{9}}{9}}\left((9x-1)\sqrt{9}+9\sqrt{9x^2-2x}\right)^{-\frac{5\sqrt{9}}{18}}\sqrt{\dots}\right\}$$

2.1831 ODE No. 1831

$$y(x)(xF(0,2)+xF(2,0))y''(x)+xF(2,2)y''(x)^2+xF(1,1)y''(x)+y'(x)\left((xF(1,2)+xF(2,1))y''(x)+y(x)(xF(0,1)+xF(1,2)+xF(2,1))y'(x)\right)=0$$

✗ **Mathematica** : cpu = 300.011 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 1.269 (sec), leaf count = 163

$$\left\{y(x) = ODESolStruc\left(e^{\int -b(\_a)d\_a + \_C1}, \left[\left\{\frac{d}{d\_a} \_b(\_a) = \frac{1}{2(F_{2,2})(\_a)}\left(\sqrt{\left((F_{2,1})(\_a)\right)^2 + 2(F_{2,1})(\_a)}\right)\right\}\right]\right)\right\}$$

2.1832 ODE No. 1832

$$y(x)y''(x)^2 - ae^{2x} = 0$$

✗ **Mathematica** : cpu = 20.2342 (sec), leaf count = 0 , could not solve

DSolve[-(a\*E^(2\*x)) + y[x]\*Derivative[2][y][x]^2 == 0, y[x], x]

✓ **Maple** : cpu = 0.852 (sec), leaf count = 117

$$\left\{y(x) = ODESolStruc\left(\_a\left(e^{-\frac{2\int \_b(\_a)d\_a}{3} - \frac{2\_C1}{3}}\right)^{-1}, \left[\left\{\frac{d}{d\_a} \_b(\_a) = -\frac{(\_b(\_a))^3}{9\_a}(-4\_a^2 + 9\sqrt{\_a a})\right\}\right]\right)\right\}$$

2.1833 ODE No. 1833

$$y''(x)^2\left(a^2y(x)^2 - b^2\right) + y'(x)^2\left(a^2y'(x)^2 - 1\right) - 2a^2y(x)y'(x)^2y''(x) = 0$$

✗ **Mathematica** : cpu = 300.035 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 6.394 (sec), leaf count = 162

$$\left\{y(x) = \_C1, y(x) = \frac{b}{a}, y(x) = b\left(e^{\frac{-C2+x}{b}\sqrt{-C1^2a^2-1}} - \_C1\right)\frac{1}{\sqrt{-C1^2a^2-1}}, y(x) = \frac{b}{a}\tan\left(\frac{-x + \_C1}{ab}\sqrt{a^2}\right)\right\}$$

2.1834 ODE No. 1834

---

$$\left(x^2y(x)y''(x)+x^2\left(-y'(x)^2\right)+y(x)^2\right)^2-4xy(x)\left(xy'(x)-y(x)\right)^3=0$$

✗ **Mathematica** : cpu = 16.0658 (sec), leaf count = 0 , could not solve  
DSolve[-4\*x\*y[x]\*(-y[x] + x\*Derivative[1][y][x])^3 + (y[x]^2 - x^2\*Derivative[1][y][x]^2 + x  
✓ **Maple** : cpu = 0.424 (sec), leaf count = 92

$$\left\{y\left(x\right)=0,y\left(x\right)=\_C1\,x,y\left(x\right)=ODESolStruc\left(e^{\int\_b\left(\_a\right)d\_a+\_C1},\left[\left\{\frac{d}{d\_a}\_b\left(\_a\right)=\frac{1}{\_a^2}\left(2\sqrt{\_a\left(-1+\_b\right)}\right.\right.\right.\right.\right.$$

2.1835 ODE No. 1835

---

$$32y''(x)\left(xy''(x)-y'(x)\right)^3+\left(2y(x)y''(x)-y'(x)^2\right)^3=0$$

✓ **Mathematica** : cpu = 0.0799617 (sec), leaf count = 143

$$\left\{\left\{y\left(x\right)\rightarrow\frac{1}{4}\left(-\frac{8c_1^3}{\sqrt[3]{3}\sqrt[3]{\sqrt{27}c_1^{10}c_2^{10}-64c_1^9c_2^9-9c_1^5c_2^5}}}+\frac{c_1^2}{c_2}-\frac{2\sqrt[3]{\sqrt{3}\sqrt{27}c_1^{10}c_2^{10}-64c_1^9c_2^9-9c_1^5c_2^5}}{3^{2/3}c_2^3}\right\}\right.$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve((2\*diff(diff(y(x),x),x)\*y(x)-diff(y(x),x)^2)^3+32\*diff(diff(y(x),x),x)\*(x\*diff(diff(y(x),x),x))^3=0,y(x))

2.1836 ODE No. 1836

---

$$\sqrt{ay''(x)^2+by'(x)^2}+cy(x)y''(x)+dy'(x)^2=0$$

✗ **Mathematica** : cpu = 9.51262 (sec), leaf count = 0 , could not solve  
DSolve[d\*Derivative[1][y][x]^2 + c\*y[x]\*Derivative[2][y][x] + Sqrt[b\*Derivative[1][y][x]^2 +  
✓ **Maple** : cpu = 1.033 (sec), leaf count = 116

$$\left\{y\left(x\right)=0,y\left(x\right)=\_C1,y\left(x\right)=\_C1\,x+\_C2,y\left(x\right)=ODESolStruc\left(\_a,\left[\left\{\frac{\_b\left(\_a\right)}{-c^2\_a^2+a}\left(\left(\frac{d}{d\_a}\_b\left(\_a\right)\right)\right.\right.\right.\right.\right.$$

2.1837 ODE No. 1837

---

$$y^{(3)}(x)-a^2\left(y'(x)^5+2y'(x)^3+y'(x)\right)=0$$

✗ **Mathematica** : cpu = 10.6967 (sec), leaf count = 0 , could not solve  
DSolve[-(a^2\*(Derivative[1][y][x] + 2\*Derivative[1][y][x]^3 + Derivative[1][y][x]^5)) + Deri  
✓ **Maple** : cpu = 0.487 (sec), leaf count = 95

$$\left\{y\left(x\right)=\int RootOf\left(-3\int^{-Z}\frac{1}{\sqrt{3a^2\_f^6+9a^2\_f^4+9a^2\_f^2+9\_C1}}d\_f+x+\_C2\right)dx+\_C3,y\left(x\right)=\int$$

2.1838 ODE No. 1838

---

$$y^{(3)}(x)+y(x)y''(x)-y'(x)^2+1=0$$

✗ **Mathematica** : cpu = 0.0176279 (sec), leaf count = 0 , could not solve  
DSolve[1 - Derivative[1][y][x]^2 + y[x]\*Derivative[2][y][x] + Derivative[3][y][x] == 0, y[x]  
✓ **Maple** : cpu = 0.593 (sec), leaf count = 73

$$\left\{y\left(x\right)=ODESolStruc\left(\_a,\left[\left\{\left(\frac{d^2}{d\_a^2}\_b\left(\_a\right)\right)\left(\_b\left(\_a\right)\right)^2+\left(\frac{d}{d\_a}\_b\left(\_a\right)\right)^2\_b\left(\_a\right)+\left(\frac{d}{d\_a}\_b\left(\_a\right)\right)\right.\right.\right.\right.$$

2.1839 ODE No. 1839

---

$$y^{(3)}(x)-y(x)y''(x)+y'(x)^2=0$$

✗ **Mathematica** : cpu = 0.0175349 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[1][y][x]^2 - y[x]\*Derivative[2][y][x] + Derivative[3][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.998 (sec), leaf count = 116

$$\left\{y\left(x\right)=ODESolStruc\left(e^{\int\_g\left(\_f\right)d\_f+\_C2},\left[\left\{\frac{d}{d\_f}\_g\left(\_f\right)=6\frac{\left(1/6+\left(\_f-1/6\right)\_g\left(\_f\right)\right)\left(\_g\left(\_f\right)\_f+1\right)}{\_f}\right.\right.\right.\right.$$

2.1840 ODE No. 1840

$ay(x)y''(x)+y^{(3)}(x)=0$

✗ **Mathematica** : cpu = 0.0199495 (sec), leaf count = 0 , could not solve  
DSolve[a\*y[x]\*Derivative[2][y][x] + Derivative[3][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.885 (sec), leaf count = 129

$$\left\{y(x)=ODESolStruc\left(e^{\int -g(\_f)\,d\_f+\_C2},\left[\frac{d}{d\_f}-g(\_f)=\frac{-g(\_f)\left(6(\_g(\_f))^2\_f^2+2(\_g(\_f))^2\_f\,a\right)}{\_f}\right]\right)\right\}$$

2.1841 ODE No. 1841

$-f(x)+x^2y^{(3)}(x)+xy''(x)+(2xy(x)-1)y'(x)+y(x)^2=0$

✗ **Mathematica** : cpu = 0.079572 (sec), leaf count = 0 , could not solve  
DSolve[-f[x] + y[x]^2 + (-1 + 2\*x\*y[x])\*Derivative[1][y][x] + x\*Derivative[2][y][x] + x^2\*Derivative[3][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.363 (sec), leaf count = 60

$$\left\{y(x)=ODESolStruc\left(\_b(\_a),\left[\_a^2\frac{d^2}{d\_a^2}-b(\_a)+\_a(\_b(\_a))^2-\left(\frac{d}{d\_a}-b(\_a)\right)\_a-\int f(\_a)\,d\_a\right]\right)\right\}$$

2.1842 ODE No. 1842

$x^2y^{(3)}(x)+x(y(x)-1)y''(x)+xy'(x)^2+(1-y(x))y'(x)=0$

✓ **Mathematica** : cpu = 0.160437 (sec), leaf count = 286

$$\left\{\left\{y(x)\rightarrow\frac{2x\left(c_3\left(J_{\frac{\sqrt{c_2+2}}{\sqrt{2}}}(-\frac{1}{2}ix\sqrt{c_1})-\frac{1}{4}i\sqrt{c_1}x\left(J_{\frac{\sqrt{c_2+2}}{\sqrt{2}}-1}(-\frac{1}{2}ix\sqrt{c_1})-J_{\frac{\sqrt{c_2+2}}{\sqrt{2}}+1}(-\frac{1}{2}ix\sqrt{c_1})\right)\right)+Y_{\frac{\sqrt{c_2+2}}{\sqrt{2}}}\right)}{c_3xJ_{\frac{\sqrt{c_2+2}}{\sqrt{2}}}(-\frac{1}{2}ix\sqrt{c_1})+xY_{\frac{\sqrt{c_2+2}}{\sqrt{2}}}(-\frac{1}{2}ix\sqrt{c_1})}\right\}\right\}$$

✓ **Maple** : cpu = 0.569 (sec), leaf count = 190

$$\left\{\ln(x)+2\int^{y(x)}\left(2\left(RootOf\left(-2\sqrt{4+\_C1}Y_{1/2\sqrt{4+\_C1}}\left(1/2\sqrt{2}\_Z\right)\_C2+2Y_{1/2\sqrt{4+\_C1}}\left(1/2\sqrt{2}\_Z\right)\_C2\right)\right)\right)\right\}$$

2.1843 ODE No. 1843

$y^{(3)}(x)y(x)+y(x)^3y'(x)-y'(x)y''(x)=0$

✓ **Mathematica** : cpu = 2.03794 (sec), leaf count = 409

$$\left\{\left\{y(x)\rightarrow\text{InverseFunction}\left[-\frac{2i\sqrt{\frac{\#1^2}{2(\sqrt{c_2^2-c_1-c_2})}}+1\sqrt{1-\frac{\#1^2}{2(c_2+\sqrt{c_2^2-c_1})}}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{c_2^2-c_1-c_2}}}\#1\right)}{\sqrt{2}}\right)\right|_{\frac{c_2-\sqrt{c_2^2-c_1-c_2}}{c_2+\sqrt{c_2^2-c_1-c_2}}}}{\sqrt{\frac{1}{\sqrt{c_2^2-c_1-c_2}}}\sqrt{-\frac{\#1^4}{2}+2\#1^2c_2-2c_1}}\right]\right\}\right\}$$

✓ **Maple** : cpu = 0.251 (sec), leaf count = 77

$$\left\{\int^{y(x)}-2\frac{1}{\sqrt{-\_a^4+4\_C2\_a^2-4\_C2^2+4\_C1}}d\_a-x-\_C3=0,\int^{y(x)}2\frac{1}{\sqrt{-\_a^4+4\_C2\_a^2-4\_C1}}d\_a-x-\_C3=0\right\}$$

2.1844 ODE No. 1844

$4y(x)^2y^{(3)}(x)+15y'(x)^3-18y(x)y'(x)y''(x)=0$

✗ **Mathematica** : cpu = 0.0808882 (sec), leaf count = 0 , could not solve  
DSolve[15\*Derivative[1][y][x]^3 - 18\*y[x]\*Derivative[1][y][x]\*Derivative[2][y][x] + 4\*y[x]^2\*Derivative[3][y][x] == 0, y[x], x]  
✓ **Maple** : cpu = 0.197 (sec), leaf count = 17

$$\left\{y(x)=\frac{\_C3}{\left(-4+(x+\_C2)^2\_C1\right)^2}\right\}$$

2.1845 ODE No. 1845

---

$$9y(x)^2y^{(3)}(x) + 40y'(x)^3 - 45y(x)y'(x)y''(x) = 0$$

✗ **Mathematica** : cpu = 0.0800461 (sec), leaf count = 0 , could not solve

DSolve[40\*Derivative[1][y][x]^3 - 45\*y[x]\*Derivative[1][y][x]\*Derivative[2][y][x] + 9\*y[x]^2

✓ **Maple** : cpu = 0.135 (sec), leaf count = 22

$$\left\{y(x) = \_C3 \left(\_C2^2 + 2\_C2 x + x^2 - 9\_C1\right)^{-\frac{3}{2}}\right\}$$

2.1846 ODE No. 1846

---

$$2y^{(3)}(x)y'(x) - 3y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0355457 (sec), leaf count = 51

$$\left\{\{y(x) \rightarrow c_1\}, \left\{y(x) \rightarrow \sqrt{\frac{2}{3}}e^{-\sqrt{\frac{3}{2}}x} \left(c_1e^{\sqrt{6}x} - c_2\right) + c_3\right\}\right\}$$

✓ **Maple** : cpu = 0.008 (sec), leaf count = 28

$$\left\{y(x) = \_C1, y(x) = \_C1 + \_C2 e^{\frac{\sqrt{6}x}{2}} + \_C3 e^{-\frac{\sqrt{6}x}{2}}\right\}$$

2.1847 ODE No. 1847

---

$$y^{(3)}(x) \left(y'(x)^2 + 1\right) - 3y'(x)y''(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.172683 (sec), leaf count = 95

$$\left\{\left\{y(x) \rightarrow c_3 - \frac{i\sqrt{c_1^2x^2 + 2c_2c_1^2x + c_2^2c_1^2 - 1}}{c_1}\right\}, \left\{y(x) \rightarrow c_3 + \frac{i\sqrt{c_1^2x^2 + 2c_2c_1^2x + c_2^2c_1^2 - 1}}{c_1}\right\}\right\}$$

✓ **Maple** : cpu = 0.148 (sec), leaf count = 49

$$\left\{y(x) = -\sqrt{-\_C2^2 - 2\_C2 x - x^2 + \_C1} + \_C3, y(x) = \sqrt{-\_C2^2 - 2\_C2 x - x^2 + \_C1} + \_C3\right\}$$

2.1848 ODE No. 1848

---

$$y^{(3)}(x) \left(y'(x)^2 + 1\right) - y''(x)^2 \left(a + 3y'(x)\right) = 0$$

✓ **Mathematica** : cpu = 0.403484 (sec), leaf count = 187

$$\left\{\left\{y(x) \rightarrow c_3 - \frac{\left(1 - i\text{InverseFunction}\left[\frac{(\#1 - a)e^{-a \tan^{-1}(\#1)}}{\sqrt{\#1^2 + 1}(a^2 + 1)c_1}\right][c_2 + x]\right)^{-\frac{1}{2} - \frac{ia}{2}} \left(1 + i\text{InverseFunction}\left[\frac{(\#1 - a)e^{-a \tan^{-1}(\#1)}}{\sqrt{\#1^2 + 1}(a^2 + 1)c_1}\right]\right)^{\frac{1}{2} + \frac{ia}{2}}}{(a^2 + 1)c_1}\right\}\right\}$$

✓ **Maple** : cpu = 1.139 (sec), leaf count = 789

$$\left\{y(x) = \int \frac{\sin\left(\text{RootOf}\left(e^{2a\_Z}\_C1^2\_C2^2a^4 + 2e^{2a\_Z}\_C1^2\_C2a^4x + e^{2a\_Z}\_C1^2a^4x^2 + 2e^{2a\_Z}\_C1^2\_C2a^4x + e^{2a\_Z}\_C1^2a^4x^2 + 2e^{2a\_Z}\_C1^2\_C2a^4x + e^{2a\_Z}\_C1^2a^4x^2\right)\right)}{\cos\left(\text{RootOf}\left(e^{2a\_Z}\_C1^2\_C2^2a^4 + 2e^{2a\_Z}\_C1^2\_C2a^4x + e^{2a\_Z}\_C1^2a^4x^2 + 2e^{2a\_Z}\_C1^2\_C2a^4x + e^{2a\_Z}\_C1^2a^4x^2 + 2e^{2a\_Z}\_C1^2\_C2a^4x + e^{2a\_Z}\_C1^2a^4x^2\right)\right)} dx\right\}$$

2.1849 ODE No. 1849

---

$$y^{(3)}(x)y''(x) - a\sqrt{b^2y''(x)^2 + 1} = 0$$

✓ **Mathematica** : cpu = 0.608399 (sec), leaf count = 426

$$\left\{\left\{y(x) \rightarrow \frac{(a^2b^4x^2 + 2ab^4c_1x + b^4c_1^2 - 1)^{3/2}}{3ab^2} + \frac{\sqrt{a^2b^4x^2 + 2ab^4c_1x + b^4c_1^2 - 1}}{ab^2} - \frac{c_1 \log\left(\sqrt{a^2b^4x^2 + 2ab^4c_1x + b^4c_1^2 - 1} + ab^2x + b^2c_1\right)}{a} - x \log\left(\sqrt{a^2b^4x^2 + 2ab^4c_1x + b^4c_1^2 - 1} + ab^2x + b^2c_1\right)}{2ab^3}\right\}\right\}$$

✓ **Maple** : cpu = 0.115 (sec), leaf count = 197

$$\left\{y(x) = \_C2 x + \int \frac{1}{2b} \left(-1 \ln\left(\sqrt{(1 + b^2(x + \_C1)a)(-1 + b^2(x + \_C1)a)} + (x + \_C1)b^4a^2 \frac{1}{\sqrt{a^2b^4}}\right) \frac{1}{\sqrt{a^2b^4}}\right) dx\right\}$$



2.1850 ODE No. 1850

---

$$y^{(4)}(x)y'(x)-y^{(3)}(x)y''(x)+y^{(3)}(x)y'(x)^3=0$$

✗ **Mathematica** : cpu = 0.0983595 (sec), leaf count = 0 , could not solve  
DSolve[Derivative[1][y][x]^3\*Derivative[3][y][x] - Derivative[2][y][x]\*Derivative[3][y][x] +  
✓ **Maple** : cpu = 1.495 (sec), leaf count = 164

$$\left\{y(x)=ODESolStruc\left(\int\frac{j(h)}{e^{f(h)}d_h+C2}dh+C3,\left[\frac{d}{d_h}j(h)=\frac{j(h)\left(12j(h)^2h^2+\right)}{\right.}\right.\right.$$

2.1851 ODE No. 1851

---

$$y'(x)^3\left(f'(x)y'(x)+f(x)y''(x)\right)-y''(x)\left(f''(x)y'(x)+2f'(x)y''(x)+f(x)y^{(3)}(x)\right)+y'(x)\left(f^{(3)}(x)y'(x)+3f''(x)y\right)$$

✗ **Mathematica** : cpu = 0.393106 (sec), leaf count = 0 , could not solve  
DSolve[2\*q[x]\*Sin[y[x]]\*Derivative[1][y][x]^2 + Derivative[1][y][x]^3\*(Derivative[1][f][x]\*D  
(Derivative[1][q][x]\*Derivative[1][y][x]) + q[x]\*Derivative[2][y][x]) - Derivative[2][y][x]\*  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve(diff(y(x),x)\*(diff(diff(diff(f(x),x),x),x)\*diff(y(x),x)+3\*diff(diff(f(x),x),x)\*diff(d  
diff(diff(y(x),x),x)\*f\*diff(diff(diff(y(x),x),x),x)+diff(y(x),x)^3\*(diff(f(x),x)\*diff(y(x),x  
diff(q(x),x)\*diff(y(x),x))\*cos(y(x))=0,y(x))

2.1852 ODE No. 1852

---

$$3y^{(4)}(x)y''(x)-5y^{(3)}(x)^2=0$$

✓ **Mathematica** : cpu = 0.107677 (sec), leaf count = 28  
$$\left\{\left\{y(x)\rightarrow c_2\left(-\sqrt{3c_1+2x}\right)+c_4x+c_3\right\}\right\}$$
  
✓ **Maple** : cpu = 0.128 (sec), leaf count = 28

$$\left\{y(x)=3\left(C2+x\right)\sqrt{6}C1\sqrt{-\frac{C1}{C2+x}}+C3x+C4\right\}$$

2.1853 ODE No. 1853

---

$$40y^{(3)}(x)^3+9y^{(5)}(x)y''(x)^2-45y^{(4)}(x)y^{(3)}(x)y''(x)=0$$

✗ **Mathematica** : cpu = 0.0777351 (sec), leaf count = 0 , could not solve  
DSolve[40\*Derivative[3][y][x]^3 - 45\*Derivative[2][y][x]\*Derivative[3][y][x]\*Derivative[4][y][x]  
✓ **Maple** : cpu = 0.589 (sec), leaf count = 110

$$\left\{y(x)=\iint RootOf\left(-\int^{-Z}\left(RootOf\left(-20\ln(f)+\int^{-Z}k\left(e^{RootOf(81k^2e^{-Z}-40e^{-Z}\ln(2)-20e^{-Z}\ln(5)+20e^{-Z}}\right)}\right.\right.\right.\right.$$

2.1854 ODE No. 1854

---

$$y^{(n)}(x)-f\left(\frac{\partial^{n-1}y(x)}{\partial x^{n-1}}\right)=0$$

✗ **Mathematica** : cpu = 0.007774 (sec), leaf count = 0 , could not solve  
DSolve[-f[D[y[x], {x, -1 + n}]] + Derivative[n][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception  
unable to handle ODEs of undefined differential order

2.1855 ODE No. 1855

---

$$y^{(n)}(x)-f\left(\frac{\partial^{n-2}y(x)}{\partial x^{n-2}}\right)=0$$

✗ **Mathematica** : cpu = 0.0031354 (sec), leaf count = 0 , could not solve  
DSolve[-f[D[y[x], {x, -2 + n}]] + Derivative[n][y][x] == 0, y[x], x]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception  
unable to handle ODEs of undefined differential order

**2.1856                    ODE No. 1856**

---

$$\{x'(t) = ax(t), y'(t) = b\}$$

✓ **Mathematica** : cpu = 0.0068391 (sec), leaf count = 22

$$\{\{x(t) \rightarrow c_1 e^{at}, y(t) \rightarrow bt + c_2\}\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 19

$$\{\{x(t) = \_C1 e^{at}, y(t) = bt + \_C2\}\}$$

**2.1857                    ODE No. 1857**

---

$$\{x'(t) = ay(t), y'(t) = -ax(t)\}$$

✓ **Mathematica** : cpu = 0.0057232 (sec), leaf count = 39

$$\{\{x(t) \rightarrow c_2 \sin(at) + c_1 \cos(at), y(t) \rightarrow c_2 \cos(at) - c_1 \sin(at)\}\}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 35

$$\{\{x(t) = \_C1 \sin(at) + \_C2 \cos(at), y(t) = -\sin(at) \_C2 + \cos(at) \_C1\}\}$$

**2.1858                    ODE No. 1858**

---

$$\{x'(t) = ay(t), y'(t) = bx(t)\}$$

✓ **Mathematica** : cpu = 0.0071788 (sec), leaf count = 182

$$\left\{\left\{x(t) \rightarrow \frac{1}{2} c_1 e^{-\sqrt{a}\sqrt{b}t} \left(e^{2\sqrt{a}\sqrt{b}t} + 1\right) + \frac{\sqrt{a} c_2 e^{-\sqrt{a}\sqrt{b}t} \left(e^{2\sqrt{a}\sqrt{b}t} - 1\right)}{2\sqrt{b}}, y(t) \rightarrow \frac{\sqrt{b} c_1 e^{-\sqrt{a}\sqrt{b}t} \left(e^{2\sqrt{a}\sqrt{b}t} - 1\right)}{2\sqrt{a}} + \frac{1}{2} c_2\right.\right.$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 64

$$\left\{\left\{x(t) = \_C1 e^{\sqrt{a}\sqrt{b}t} + \_C2 e^{-\sqrt{a}\sqrt{b}t}, y(t) = 1\sqrt{b} \left(\_C1 e^{\sqrt{a}\sqrt{b}t} - \_C2 e^{-\sqrt{a}\sqrt{b}t}\right) \frac{1}{\sqrt{a}}\right\}\right\}$$

**2.1859                    ODE No. 1859**

---

$$\{x'(t) = ax(t) - y(t), y'(t) = ay(t) + x(t)\}$$

✓ **Mathematica** : cpu = 0.0040766 (sec), leaf count = 51

$$\{\{x(t) \rightarrow c_1 e^{at} \cos(t) - c_2 e^{at} \sin(t), y(t) \rightarrow c_1 e^{at} \sin(t) + c_2 e^{at} \cos(t)\}\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 38

$$\{\{x(t) = e^{at} (\_C2 \cos(t) + \_C1 \sin(t)), y(t) = -e^{at} (\cos(t) \_C1 - \sin(t) \_C2)\}\}$$

**2.1860                    ODE No. 1860**

---

$$\{x'(t) = ax(t) + by(t), y'(t) = by(t) + cx(t)\}$$

✓ **Mathematica** : cpu = 0.0316199 (sec), leaf count = 696

$$\left\{\left\{x(t) \rightarrow \frac{c_1 \left(a \left(-e^{\frac{1}{2}t \left(-\sqrt{a^2-2ab+b^2+4bc+a+b}\right)}\right) + a e^{\frac{1}{2}t \left(\sqrt{a^2-2ab+b^2+4bc+a+b}\right)} + b e^{\frac{1}{2}t \left(-\sqrt{a^2-2ab+b^2+4bc+a+b}\right)} + \sqrt{a^2-2ab+b^2+4bc+a+b}\right)\right.\right.$$

✓ **Maple** : cpu = 0.057 (sec), leaf count = 177

$$\left\{\left\{x(t) = \_C1 e^{\frac{t}{2} \left(a+b+\sqrt{b^2+(-2a+4c)b+a^2}\right)} + \_C2 e^{\frac{t}{2} \left(a+b-\sqrt{b^2+(-2a+4c)b+a^2}\right)}, y(t) = \frac{1}{2b} \left(-\_C2 \left(a-b+\sqrt{b^2+(-2a+4c)b+a^2}\right) + \_C1 \left(a-b-\sqrt{b^2+(-2a+4c)b+a^2}\right)\right)\right\}\right\}$$

2.1861 ODE No. 1861

$\{ax'(t)+by'(t)=\alpha x(t)+\beta y(t),bx'(t)-ay'(t)=\beta x(t)-\alpha y(t)\}$

✓ **Mathematica** : cpu = 0.0090549 (sec), leaf count = 183

$\left\{\left\{x(t)\rightarrow c_2e^{\frac{t(a\alpha+b\beta)}{a^2+b^2}}\sin\left(\frac{t(a\beta-\alpha b)}{a^2+b^2}\right)+c_1e^{\frac{t(a\alpha+b\beta)}{a^2+b^2}}\cos\left(\frac{t(a\beta-\alpha b)}{a^2+b^2}\right),y(t)\rightarrow c_2e^{\frac{t(a\alpha+b\beta)}{a^2+b^2}}\cos\left(\frac{t(a\beta-\alpha b)}{a^2+b^2}\right)-c_1e^{\frac{t(a\alpha+b\beta)}{a^2+b^2}}\sin\left(\frac{t(a\beta-\alpha b)}{a^2+b^2}\right)\right\}\right\}$

✓ **Maple** : cpu = 0.077 (sec), leaf count = 152

$\left\{\left\{x\left(t\right)=-C1\,e^{\frac{\left(i\beta+\alpha\right)a-\left(i\alpha-\beta\right)b)t}{a^2+b^2}}+C2\,e^{-\frac{\left(i\beta-\alpha\right)a-\left(i\alpha+\beta\right)b)t}{a^2+b^2}},y\left(t\right)=i\left(-C1\,e^{\frac{\left(i\beta+\alpha\right)a-\left(i\alpha-\beta\right)b)t}{a^2+b^2}}-C2\,e^{-\frac{\left(i\beta-\alpha\right)a-\left(i\alpha+\beta\right)b)t}{a^2+b^2}}\right\}\right\}$

2.1862 ODE No. 1862

$\{x'(t)=-y(t),y'(t)=2x(t)+2y(t)\}$

✓ **Mathematica** : cpu = 0.009715 (sec), leaf count = 52

$\{\{x(t)\rightarrow c_1e^t(\cos(t)-\sin(t))-c_2e^t\sin(t),y(t)\rightarrow 2c_1e^t\sin(t)+c_2e^t(\sin(t)+\cos(t))\}\}$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 39

$\{\{x\left(t\right)=e^t\left(\cos\left(t\right)C2+\sin\left(t\right)C1\right),y\left(t\right)=-\left(\left(-C1+C2\right)\cos\left(t\right)+\sin\left(t\right)\left(-C1-C2\right)\right)e^t\}\}$

2.1863 ODE No. 1863

$\{x'(t)+3x(t)+4y(t)=0,2x(t)+y'(t)+5y(t)=0\}$

✓ **Mathematica** : cpu = 0.0055977 (sec), leaf count = 84

$\left\{\left\{x(t)\rightarrow \frac{1}{3}c_1e^{-7t}\left(2e^{6t}+1\right)-\frac{2}{3}c_2e^{-7t}\left(e^{6t}-1\right),y(t)\rightarrow \frac{1}{3}c_2e^{-7t}\left(e^{6t}+2\right)-\frac{1}{3}c_1e^{-7t}\left(e^{6t}-1\right)\right\}\right\}$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 35

$\left\{\left\{x\left(t\right)=-C1\,e^{-t}+C2\,e^{-7t},y\left(t\right)=-\frac{C1\,e^{-t}}{2}+C2\,e^{-7t}\right\}\right\}$

2.1864 ODE No. 1864

$\{x'(t)=-5x(t)-2y(t),y'(t)=x(t)-7y(t)\}$

✓ **Mathematica** : cpu = 0.0086381 (sec), leaf count = 59

$\{\{x(t)\rightarrow c_1e^{-6t}(\sin(t)+\cos(t))-2c_2e^{-6t}\sin(t),y(t)\rightarrow c_1e^{-6t}\sin(t)+c_2e^{-6t}(\cos(t)-\sin(t))\}\}$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 44

$\left\{\left\{x\left(t\right)=e^{-6t}\left(\cos\left(t\right)C2+\sin\left(t\right)C1\right),y\left(t\right)=-\frac{\left(\left(-C1-C2\right)\cos\left(t\right)-\sin\left(t\right)\left(-C1+C2\right)\right)e^{-6t}}{2}\right\}\right\}$

2.1865 ODE No. 1865

$\{x'(t)=a1x(t)+b1y(t)+c1,y'(t)=a2x(t)+b2y(t)+c2\}$

✓ **Mathematica** : cpu = 0.804293 (sec), leaf count = 2062

$\left\{\left\{x(t)\rightarrow -\frac{b1e^{-\frac{1}{2}\left(a1+b2+\sqrt{a1^2-2b2a1+b2^2+4a2b1}\right)t}\left(2\left(\left(a1-b2+\sqrt{a1^2-2b2a1+b2^2+4a2b1}\right)c2-2a2c1\right)e^{\sqrt{a1^2-2b2a1+b2^2+4a2b1}t}-\left(a1-b2-\sqrt{a1^2-2b2a1+b2^2+4a2b1}\right)c2+2a2c1\right)}{2\left(a1^2-2a1b2+b2^2+4a2b1\right)}\right\}\right\}$

✓ **Maple** : cpu = 0.094 (sec), leaf count = 224

$\left\{\left\{x\left(t\right)=e^{\frac{t}{2}\left(a1+b2+\sqrt{a1^2-2a1b2+4a2b1+b2^2}\right)}C2+e^{\frac{t}{2}\left(a1+b2-\sqrt{a1^2-2a1b2+4a2b1+b2^2}\right)}C1+\frac{c2b1-b2c1}{a1b2-a2b1},y\left(t\right)=e^{\frac{t}{2}\left(a1+b2+\sqrt{a1^2-2a1b2+4a2b1+b2^2}\right)}C1+e^{\frac{t}{2}\left(a1+b2-\sqrt{a1^2-2a1b2+4a2b1+b2^2}\right)}C2+\frac{c2b1-b2c1}{a1b2-a2b1}\right\}\right\}$

**2.1866                      ODE No. 1866**

---

$$\{x'(t) + 2y(t) = 3t, y'(t) - 2x(t) = 4\}$$

✓ **Mathematica** : cpu = 0.0373126 (sec), leaf count = 132

$$\left\{ \left\{ x(t) \rightarrow -c_2 \sin(2t) + c_1 \cos(2t) + \cos(2t) \left( \frac{3}{2}t \sin(2t) - \frac{5}{4} \cos(2t) \right) - \sin(2t) \left( \frac{5}{4} \sin(2t) + \frac{3}{2}t \cos(2t) \right), y(t) \rightarrow \right. \right.$$

✓ **Maple** : cpu = 0.025 (sec), leaf count = 39

$$\left\{ \left\{ x(t) = \sin(2t) \_C2 + \cos(2t) \_C1 - \frac{5}{4}, y(t) = -\cos(2t) \_C2 + \sin(2t) \_C1 + \frac{3t}{2} \right\} \right\}$$

**2.1867                      ODE No. 1867**

---

$$\{-t^2 + x'(t) + y(t) + 6t + 1 = 0, y'(t) - x(t) = -3t^2 + 3t + 1\}$$

✓ **Mathematica** : cpu = 0.0878752 (sec), leaf count = 124

$$\left\{ \left\{ x(t) \rightarrow -c_2 \sin(t) + c_1 \cos(t) + \cos(t) \left( (3t^2 - t - 13) \cos(t) + (t - 12)t \sin(t) \right) - \sin(t) \left( (-3t^2 + t + 13) \sin(t) \right) \right. \right.$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 42

$$\left\{ \left\{ x(t) = \sin(t) \_C2 + \cos(t) \_C1 + 3t^2 - t - 13, y(t) = t^2 - \cos(t) \_C2 + \sin(t) \_C1 - 12t \right\} \right\}$$

**2.1868                      ODE No. 1868**

---

$$\{x'(t) + 3x(t) - y(t) = e^{2t}, x(t) + y'(t) + 5y(t) = e^t\}$$

✓ **Mathematica** : cpu = 0.0767294 (sec), leaf count = 162

$$\left\{ \left\{ x(t) \rightarrow c_1 e^{-4t}(t+1) + c_2 e^{-4t}t - e^t(t+1) \left( \frac{t}{5} + \frac{1}{36}e^t(6t-7) - \frac{1}{25} \right) + e^t t \left( \frac{t}{5} + \frac{1}{36}e^t(6t-1) + \frac{4}{25} \right), y(t) \rightarrow \right. \right.$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 64

$$\left\{ \left\{ x(t) = e^{-4t} \_C2 + e^{-4t}t \_C1 + \frac{e^t}{25} + \frac{7e^{2t}}{36}, y(t) = -\frac{e^{2t}}{36} - e^{-4t} \_C2 - e^{-4t}t \_C1 + \frac{4e^t}{25} + e^{-4t} \_C1 \right\} \right\}$$

**2.1869                      ODE No. 1869**

---

$$\{x'(t) + 2x(t) + y'(t) + y(t) = t + e^{2t}, x'(t) - x(t) + y'(t) + 3y(t) = e^t - 1\}$$

✓ **Mathematica** : cpu = 0.0843591 (sec), leaf count = 118

$$\left\{ \left\{ x(t) \rightarrow \frac{5}{72} \left( c_1 e^{-7t/5} + \frac{12(5712t + 833e^t + 2352e^{2t} - 5508)}{20825} \right) + \frac{1}{5}(t - e^t + e^{2t} + 1), y(t) \rightarrow \frac{5}{48} \left( c_1 e^{-7t/5} + \right. \right.$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 51

$$\left\{ \left\{ x(t) = \frac{3t}{7} - \frac{1}{49} - \frac{e^t}{6} + \frac{5e^{2t}}{17} + e^{-\frac{7t}{5}} \_C1, y(t) = -\frac{e^{2t}}{17} + \frac{e^t}{4} + \frac{t}{7} - \frac{26}{49} + \frac{3\_C1}{2} e^{-\frac{7t}{5}} \right\} \right\}$$

**2.1870                      ODE No. 1870**

---

$$\{x'(t) + y'(t) - y(t) = e^t, 2x'(t) + y'(t) + 2y(t) = \cos(t)\}$$

✓ **Mathematica** : cpu = 0.138365 (sec), leaf count = 122

$$\left\{ \left\{ x(t) \rightarrow -\frac{3}{4}c_2(e^{4t} - 1) + c_1 + \frac{1}{68}e^{-4t}(e^{4t} - 1)(34e^t + 3\sin(t) - 12\cos(t)) + \frac{1}{4} \left( 2e^{-3t} + 2e^t + \frac{3}{17}e^{-4t}\sin(t) \right) \right. \right.$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 47

$$\left\{ \left\{ x(t) = \frac{\_C1 e^{4t}}{4} + \frac{5 \sin(t)}{17} - \frac{3 \cos(t)}{17} + e^t + \_C2, y(t) = -\frac{\_C1 e^{4t}}{3} + \frac{4 \cos(t)}{17} - \frac{\sin(t)}{17} - \frac{2e^t}{3} \right\} \right\}$$

2.1871 ODE No. 1871

$$\{4x'(t) + 2x(t) + 9y'(t) + 31y(t) = e^t, 3x'(t) + x(t) + 7y'(t) + 24y(t) = 3\}$$

✓ **Mathematica** : cpu = 0.317186 (sec), leaf count = 180

$$\left\{\left\{x(t) \rightarrow -c_2 e^{-4 t} \sin (t)+c_1 e^{-4 t}(\cos (t)-\sin (t))+\frac{1}{442}\left(3\left(153 e^t-754\right) \sin (t)+31\left(17 e^t-78\right) \cos (t)\right)(\cos (t)-\sin (t))\right.\right.$$

✓ **Maple** : cpu = 0.065 (sec), leaf count = 62

$$\left\{\left\{x(t)=e^{-4 t} \sin (t) C_2+e^{-4 t} \cos (t) C_1-\frac{93}{17}+\frac{31 e^t}{26}, y(t)=\frac{\left((-221 C_1-221 C_2) \cos (t)+221 \sin (t)\right)}{221}\right.\right.$$

2.1872 ODE No. 1872

$$\{4x'(t) + 11x(t) + 9y'(t) + 31y(t) = e^t, 3x'(t) + 8x(t) + 7y'(t) + 24y(t) = e^{2t}\}$$

✓ **Mathematica** : cpu = 0.114329 (sec), leaf count = 162

$$\left\{\left\{x(t) \rightarrow -c_1 e^{-4 t}(t-1)-c_2 e^{-4 t} t-e^t t\left(-\frac{4 t}{5}+\frac{1}{36} e^t(30 t+19)-\frac{11}{25}\right)-e^t(t-1)\left(\frac{4 t}{5}-\frac{1}{36} e^t(30 t+49)+\frac{31}{25}\right)\right.\right.$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 65

$$\left\{\left\{x(t)=e^{-4 t} C_2+e^{-4 t} t C_1+\frac{31 e^t}{25}-\frac{49 e^{2 t}}{36}, y(t)=\frac{19 e^{2 t}}{36}-\frac{11 e^t}{25}-e^{-4 t} C_2-e^{-4 t} t C_1-e^{-4 t} C_1\right.\right.$$

2.1873 ODE No. 1873

$$\{4x'(t) + 44x(t) + 9y'(t) + 49y(t) = t, 3x'(t) + 34x(t) + 7y'(t) + 38y(t) = e^t\}$$

✓ **Mathematica** : cpu = 0.090829 (sec), leaf count = 322

$$\left\{\left\{x(t) \rightarrow \frac{1}{5} c_1 e^{-6 t}\left(4 e^{5 t}+1\right)-\frac{1}{5} c_2 e^{-6 t}\left(e^{5 t}-1\right)-\frac{1}{5} e^{-6 t}\left(e^{5 t}-1\right)\left(\frac{16}{5} e^{6 t}\left(\frac{t}{6}-\frac{1}{36}\right)+4 e^{2 t}-\frac{4 e^{7 t}}{7}-\frac{31}{5} e^t\left(t-\frac{1}{5}\right)\right)\right.\right.$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 52

$$\left\{\left\{x(t)=e^{-6 t} C_2+C_1 e^{-t}-\frac{56}{9}-\frac{29 e^t}{7}+\frac{19 t}{3}, y(t)=4 e^{-6 t} C_2-C_1 e^{-t}+\frac{24 e^t}{7}+\frac{55}{9}-\frac{17 t}{3}\right.\right.\}$$

2.1874 ODE No. 1874

$$\{x'(t) = f(t)x(t) + g(t)y(t), y'(t) = f(t)y(t) - g(t)x(t)\}$$

✓ **Mathematica** : cpu = 0.0073409 (sec), leaf count = 115

$$\left\{\left\{x(t) \rightarrow c_2 \exp \left(\int_1^t f(K[2]) d K[2]\right) \sin \left(\int_1^t g(K[1]) d K[1]\right)+c_1 \exp \left(\int_1^t f(K[2]) d K[2]\right) \cos \left(\int_1^t g(K[1]) d K[1]\right)\right.\right.$$

✓ **Maple** : cpu = 0.317 (sec), leaf count = 57

$$\left\{\left\{x(t)=e^{\int \tan \left(C_1-\int g(t) \mathrm{d} t\right) g(t)+f(t) \mathrm{d} t} C_2, y(t)=\tan \left(-C_1-\int g(t) \mathrm{d} t\right) e^{\int \tan \left(C_1-\int g(t) \mathrm{d} t\right) g(t)+f(t) \mathrm{d} t} C_2\right.\right.\}$$

2.1875 ODE No. 1875

$$\{f(t)(ax(t) + by(t)) + x'(t) = g(t), f(t)(cx(t) + dy(t)) + y'(t) = h(t)\}$$

✗ **Mathematica** : cpu = 0.0055809 (sec), leaf count = 0 , could not solve

DSolve[{f[t]\*(a\*x[t] + b\*y[t]) + Derivative[1][x][t] == g[t], f[t]\*(c\*x[t] + d\*y[t]) + Derivative[1][y][t] == h[t]}, {x[t], y[t]}, t]

✓ **Maple** : cpu = 1.095 (sec), leaf count = 2606

$$\left\{\left\{x(t)=e^{\int-\frac{f(t)}{2(a+d)}\left(\tan \left(\frac{-1+(a+d) \int f(t) \mathrm{d} t}{2(a+d)^2} \sqrt{-a^4-4 a^2 b c+2 a^2 d^2-8 a b c d-4 b c d^2-d^4}\right) \sqrt{-a^4-4 a^2 b c+2 a^2 d^2-8 a b c d-4 b c d^2-d^4}+(a+d)^2\right)}\right.\right.$$

**2.1876      ODE No. 1876**

---

$$\left\{x'(t) = x(t) \cos(t), y'(t) = x(t)e^{-\sin(t)}\right\}$$

✓ **Mathematica** : cpu = 0.016993 (sec), leaf count = 39

$$\left\{\left\{x(t) \rightarrow c_1 e^{\sin(t)}, y(t) \rightarrow c_1 \int_1^t e^{\sin(K[1]) - \sin(K[1])} dK[1] + c_2\right\}\right\}$$

✓ **Maple** : cpu = 0.105 (sec), leaf count = 18

$$\left\{\left\{x(t) = \_C2 e^{\sin(t)}, y(t) = \_C2 t + \_C1\right\}\right\}$$

**2.1877      ODE No. 1877**

---

$$\{tx'(t) + y(t) = 0, x(t) + ty'(t) = 0\}$$

✓ **Mathematica** : cpu = 0.0041038 (sec), leaf count = 31

$$\left\{\left\{x(t) \rightarrow c_1 t + \frac{c_2}{t}, y(t) \rightarrow \frac{c_2}{t} - c_1 t\right\}\right\}$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 31

$$\left\{\left\{x(t) = \frac{\_C1 t^2 + \_C2}{t}, y(t) = \frac{\_C1 t^2 + \_C2}{t}\right\}\right\}$$

**2.1878      ODE No. 1878**

---

$$\{tx'(t) + 2x(t) = t, -(t+2)x(t) + ty'(t) - ty(t) = -t\}$$

✓ **Mathematica** : cpu = 0.0096072 (sec), leaf count = 39

$$\left\{\left\{x(t) \rightarrow \frac{c_1}{t^2} + \frac{t}{3}, y(t) \rightarrow -\frac{c_1}{t^2} + c_2 e^t - \frac{t}{3}\right\}\right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 39

$$\left\{\left\{x(t) = \frac{t}{3} + \frac{\_C2}{t^2}, y(t) = \frac{3\_C1 e^{t^2} - t^3 - 3\_C2}{3t^2}\right\}\right\}$$

**2.1879      ODE No. 1879**

---

$$\{tx'(t) + 2(x(t) - y(t)) = t, x(t) + ty'(t) + 5y(t) = t^2\}$$

✓ **Mathematica** : cpu = 0.0167366 (sec), leaf count = 58

$$\left\{\left\{x(t) \rightarrow \frac{c_1}{t^4} + \frac{c_2}{t^3} + \frac{1}{30}t(2t+9), y(t) \rightarrow -\frac{c_1}{t^4} - \frac{c_2}{2t^3} + \frac{1}{60}t(8t-3)\right\}\right\}$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 54

$$\left\{\left\{x(t) = \frac{2t^6 + 9t^5 + 30\_C2 t + 30\_C1}{30t^4}, y(t) = \frac{8t^6 - 3t^5 - 30\_C2 t - 60\_C1}{60t^4}\right\}\right\}$$

**2.1880      ODE No. 1880**

---

$$\{t^2(1 - \sin(t))x'(t) = t^2y(t) + tx(t)(1 - 2\sin(t)), t^2(1 - \sin(t))y'(t) = x(t)(t\cos(t) - \sin(t)) + ty(t)(1 - t\cos(t))\}$$

✗ **Mathematica** : cpu = 0.0154514 (sec), leaf count = 0 , could not solve

`DSolve[{t^2*(1 - Sin[t])*Derivative[1][x][t] == t*(1 - 2*Sin[t])*x[t] + t^2*y[t], t^2*(1 - Sin[t])*Derivative[1][y][t] == x[t]*(t Cos[t] - Sin[t]) + t*y[t]*(1 - t Cos[t])}]`

✓ **Maple** : cpu = 0.059 (sec), leaf count = 23

$$\{\{x(t) = t(\_C2 t + \_C1), y(t) = \sin(t)\_C1 + \_C2 t\}\}$$

2.1881 ODE No. 1881

---

$\{x'(t)+y'(t)+y(t)=f(t),x''(t)+x(t)+y''(t)+y'(t)+y(t)=g(t)\}$

✓ **Mathematica** : cpu = 0.0063628 (sec), leaf count = 44

$\{\{x(t)\rightarrow -f''(t)-f'(t)-f(t)+g'(t)+g(t),y(t)\rightarrow f''(t)+f(t)-g'(t)\}\}$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 48

$\left\{\left\{x\left(t\right)=-\frac{\mathrm{d}}{\mathrm{d} t}f\left(t\right)+g\left(t\right)-f\left(t\right)-\frac{\mathrm{d}^2}{\mathrm{d} t^2}f\left(t\right)+\frac{\mathrm{d}}{\mathrm{d} t}g\left(t\right),y\left(t\right)=f\left(t\right)+\frac{\mathrm{d}^2}{\mathrm{d} t^2}f\left(t\right)-\frac{\mathrm{d}}{\mathrm{d} t}g\left(t\right)\right\}\right\}$

2.1882 ODE No. 1882

---

$\{2x'(t)-3x(t)+y'(t)=0,x''(t)+y'(t)-2y(t)=e^{2t}\}$

✓ **Mathematica** : cpu = 1.62941 (sec), leaf count = 928

$\left\{\left\{x(t)\rightarrow \frac{1}{46}e^{t/2}c_1\left(23\cos\left(\frac{\sqrt{23}t}{2}\right)+23e^{t/2}-3\sqrt{23}\sin\left(\frac{\sqrt{23}t}{2}\right)\right)+\frac{e^{3t/2}\left(23e^{t/2}\cos\left(\frac{\sqrt{23}t}{2}\right)-7\sqrt{23}e^{t/2}\sin\left(\frac{\sqrt{23}t}{2}\right)\right)}{46},y(t)\rightarrow \frac{1}{46}e^{t/2}c_2\left(23\cos\left(\frac{\sqrt{23}t}{2}\right)+23e^{t/2}-3\sqrt{23}\sin\left(\frac{\sqrt{23}t}{2}\right)\right)+\frac{e^{3t/2}\left(23e^{t/2}\cos\left(\frac{\sqrt{23}t}{2}\right)-7\sqrt{23}e^{t/2}\sin\left(\frac{\sqrt{23}t}{2}\right)\right)}{46}\right\}\right\}$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 99

$\left\{\left\{x(t)=\frac{e^{2t}}{4}+_C1\,e^t+_C2\,e^{\frac{t}{2}}\cos\left(\frac{\sqrt{23}t}{2}\right)+_C3\,e^{\frac{t}{2}}\sin\left(\frac{\sqrt{23}t}{2}\right),y(t)=-\frac{7}{4}\left(\frac{=_C3\,\sqrt{23}}{7}+_C2\right)e^{\frac{t}{2}}\cos\left(\frac{\sqrt{23}t}{2}\right)+\frac{7}{4}\left(\frac{=_C3\,\sqrt{23}}{7}+_C2\right)e^{\frac{t}{2}}\sin\left(\frac{\sqrt{23}t}{2}\right)\right\}\right\}$

2.1883 ODE No. 1883

---

$\{x'(t)+x(t)-y'(t)=2t,x''(t)-9x(t)+y'(t)+3y(t)=\sin(2t)\}$

✓ **Mathematica** : cpu = 0.935835 (sec), leaf count = 602

$\left\{\left\{x(t)\rightarrow \frac{1}{16}c_1e^{-3t}\left(20e^{4t}t+7e^{4t}+9\right)+\frac{1}{16}c_2e^{-3t}\left(4e^{4t}t+3e^{4t}-3\right)-\frac{3}{16}c_3e^{-3t}\left(4e^{4t}t-e^{4t}+1\right)+\frac{e^{-4t}\left(20e^{4t}t+7e^{4t}+9\right)}{16},y(t)\rightarrow \frac{1}{16}c_1e^{-3t}\left(20e^{4t}t+7e^{4t}+9\right)+\frac{1}{16}c_2e^{-3t}\left(4e^{4t}t+3e^{4t}-3\right)-\frac{3}{16}c_3e^{-3t}\left(4e^{4t}t-e^{4t}+1\right)+\frac{e^{-4t}\left(20e^{4t}t+7e^{4t}+9\right)}{16}\right\}\right\}$

✓ **Maple** : cpu = 0.086 (sec), leaf count = 80

$\left\{\left\{x(t)=-\frac{2\cos(2t)}{325}+4-\frac{36\sin(2t)}{325}+2t+_C1\,e^t+_C2\,e^{-3t}+_C3\,e^tt,y(t)=\frac{16\cos(2t)}{325}-\frac{37\sin(2t)}{325}+\frac{2t}{325}+_C1\,e^t+_C2\,e^{-3t}+_C3\,e^tt\right\}\right\}$

2.1884 ODE No. 1884

---

$\{x'(t)-x(t)+2y(t)=0,x''(t)-2y'(t)=2t-\cos(2t)\}$

✓ **Mathematica** : cpu = 0.451006 (sec), leaf count = 224

$\left\{\left\{x(t)\rightarrow 7\left(c_2+t^2-\frac{1}{2}\sin(2t)\right)+8\left(c_1e^{t/2}+c_2\left(e^{t/2}-1\right)+\frac{1}{136}e^{-t/2}\left(2e^{t/2}\cos(2t)-4\left(34e^{t/2}t^2+17e^{t/2}t-1\right)\right)\right\}\right\}$

✓ **Maple** : cpu = 0.073 (sec), leaf count = 69

$\left\{\left\{x(t)=2e^{t/2}_C1-t^2+\frac{\sin(2t)}{34}+\frac{2\cos(2t)}{17}-4t+_C2,y(t)=\frac{_C1}{2}e^{\frac{t}{2}}-t+\frac{\cos(2t)}{34}+\frac{9\sin(2t)}{68}+2t+_C2\right\}\right\}$

2.1885 ODE No. 1885

---

$\{tx'(t)-ty'(t)-2y(t)=0,tx''(t)+2x'(t)+tx(t)=0\}$

✗ **Mathematica** : cpu = 0.0171677 (sec), leaf count = 0 , could not solve  
DSolve[{-2\*y[t] + t\*Derivative[1][x][t] - t\*Derivative[1][y][t] == 0, t\*x[t] + 2\*Derivative[1][x][t] == 0}, {x[t], y[t]}, t]

✓ **Maple** : cpu = 0.075 (sec), leaf count = 47

$\left\{\left\{x(t)=\frac{=_C3\,\cos(t)+\sin(t)_C2}{t},y(t)=\frac{(_C3\,t+2\_C2)\cos(t)+(_C2\,t-2\_C3)\sin(t)+_C1}{t^2}\right\}\right\}$

2.1886 ODE No. 1886

{ay(t) + x''(t) = 0, y''(t) - a^2y(t) = 0}

✓ Mathematica : cpu = 0.0150121 (sec), leaf count = 115

{ { x(t) -> - (c4 e^{-at} (-2ate^{at} + e^{2at} - 1) / (2a^2) - c3 e^{-at} (e^{at} - 1)^2 / (2a) + c2 t + c1, y(t) -> 1/2 c3 e^{-at} (e^{2at} + 1) + c4 e^{-at} (e^{2at} - 1) / (2a) }

✓ Maple : cpu = 0.041 (sec), leaf count = 49

{ { x(t) = (-C4 e^{-at} - C3 e^{at} + a (-C1 t + C2)) / a, y(t) = -C3 e^{at} + C4 e^{-at} }

2.1887 ODE No. 1887

{x''(t) = ax(t) + by(t), y''(t) = cx(t) + dy(t)}

✓ Mathematica : cpu = 0.354281 (sec), leaf count = 5748

{ { x(t) -> ( e^{- (sqrt(a+d-sqrt(a^2-2da+d^2+4bct))/sqrt(2)) - (sqrt(a+d+sqrt(a^2-2da+d^2+4bct))/sqrt(2))} ( e^{ (sqrt(a+d-sqrt(a^2-2da+d^2+4bct))/sqrt(2))} a - e^{ (sqrt(a+d+sqrt(a^2-2da+d^2+4bct))/sqrt(2))} a - e^{sqrt(2)(a+d)} ) )

✓ Maple : cpu = 0.086 (sec), leaf count = 360

{ { x(t) = -C1 e^{-t/2 sqrt(-2 sqrt(a^2-2ad+4bc+d^2)+2a+2d)} + -C2 e^{t/2 sqrt(-2 sqrt(a^2-2ad+4bc+d^2)+2a+2d)} + -C3 e^{-t/2 sqrt(2 sqrt(a^2-2ad+4bc+d^2)+2a+2d)}

2.1888 ODE No. 1888

{x''(t) = a1x(t) + b1y(t) + c1, y''(t) = a2x(t) + b2y(t) + c2}

✓ Mathematica : cpu = 16.1028 (sec), leaf count = 35330

Too large to display

✓ Maple : cpu = 0.141 (sec), leaf count = 457

{ { x(t) = -C4 e^{t/2 sqrt(2 sqrt(a1^2-2a1b2+4a2b1+b2^2)+2a1+2b2)} + -C3 e^{-t/2 sqrt(2 sqrt(a1^2-2a1b2+4a2b1+b2^2)+2a1+2b2)} + -C2 e^{t/2 sqrt(2 sqrt(a1^2-2a1b2+4a2b1+b2^2)+2a1+2b2)}

2.1889 ODE No. 1889

{x''(t) + x(t) + y(t) = -5, -4x(t) + y''(t) - 3y(t) = -3}

✓ Mathematica : cpu = 0.202563 (sec), leaf count = 554

{ { x(t) -> -1/4 c4 e^{-t} (e^{2t} t + t - e^{2t} + 1) - 1/2 c1 e^{-t} (e^{2t} t - t - e^{2t} - 1) - 1/2 c2 e^{-t} (e^{2t} t + t - 2e^{2t} + 2) - 1/4 c3 e^{-t} (e^{2t} t + t - 2e^{2t} + 2) }

✓ Maple : cpu = 0.033 (sec), leaf count = 60

{ { x(t) = (-C4 t + C2) e^{-t} + 18 + (-C3 t + C1) e^t, y(t) = ((-2t + 2) C4 - 2 C2) e^{-t} - 23 + ((-2t - 2) C3 + 2 C1) e^t }

2.1890 ODE No. 1890

{ x''(t) = c^2 x(t) (3 cos^2(at + b) - 1) + 3/2 c^2 y(t) sin(2abt), y''(t) = 3/2 c^2 x(t) sin(2abt) + c^2 y(t) (3 sin^2(at + b) - 1) }

✗ Mathematica : cpu = 0.0068577 (sec), leaf count = 0 , could not solve DSolve[{Derivative[2][x][t] == c^2\*(-1 + 3\*Cos[b + a\*t]^2)\*x[t] + (3\*c^2\*Sin[2\*a\*b\*t]\*y[t])/1 + 3\*Sin[b + a\*t]^2\*y[t]}, {x[t], y[t]}, t]

✗ Maple : cpu = 0. (sec), leaf count = 0 , result contains DESol

{ { x(t) = DESol ( { { d^4/dt^4 - Y(t) + ( 2 sin(atb) ab / cos(atb) - 2 ab cos(atb) / sin(atb) ) d^3/dt^3 - Y(t) + ( 2 (sin(atb))^2 b^2 a^2 / (cos(atb))^2 + 2 a^2 b^2 (c



2.1891 ODE No. 1891

$\{x''(t)+6x(t)+7y(t)=0,3x(t)+y''(t)+2y(t)=2t\}$

✓ **Mathematica** : cpu = 0.913797 (sec), leaf count = 742

$\left\{\left\{x(t)\rightarrow-\frac{7}{60}c_4e^{-t}\left(3e^{2t}-2e^t\sin(3t)-3\right)+\frac{1}{60}c_2e^{-t}\left(9e^{2t}+14e^t\sin(3t)-9\right)-\frac{7}{20}c_3e^{-t}\left(e^{2t}-2e^t\cos(3t)+1\right)\right.\right.$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 64

$\left\{\left\{x\left(t\right)=\frac{14\,t}{9}+\_C1\,{\rm e}^t+\_C2\,\cos\left(3\,t\right)+\_C3\,{\rm e}^{-t}+\_C4\,\sin\left(3\,t\right),y\left(t\right)=-\_C1\,{\rm e}^t+\frac{3\_C2\,\cos\left(3\,t\right)}{7}-\_C3\right.\right.$

2.1892 ODE No. 1892

$\{-ay'(t)+bx(t)+x''(t)=0,ax'(t)+by(t)+y''(t)=0\}$

✓ **Mathematica** : cpu = 0.261118 (sec), leaf count = 4815

$\left\{\left\{x(t)\rightarrow\frac{e^{-\frac{\sqrt{-a^2-2b}-\sqrt{a^2(a^2+4b)}}{\sqrt{2}}t}-\frac{\sqrt{-a^2-2b}+\sqrt{a^2(a^2+4b)}}{\sqrt{2}}t\left(e^{\frac{\sqrt{-a^2-2b}-\sqrt{a^2(a^2+4b)}}{\sqrt{2}}t}a^2-e^{\frac{\sqrt{-a^2-2b}+\sqrt{a^2(a^2+4b)}}{\sqrt{2}}t}a^2-e^{\sqrt{2}\sqrt{-a^2-2b}}e^{\frac{\sqrt{-a^2-2b}-\sqrt{a^2(a^2+4b)}}{\sqrt{2}}t}a^2-e^{\sqrt{2}\sqrt{-a^2-2b}}e^{\frac{\sqrt{-a^2-2b}+\sqrt{a^2(a^2+4b)}}{\sqrt{2}}t}a^2\right)}{2\sqrt{-a^2-2b}}\right.\right.$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 463

$\left\{\left\{x\left(t\right)=\_C1\,{\rm e}^{-\frac{t}{2}\sqrt{-2\,a^2-2\sqrt{a^2(a^2+4b)}-4b}}+\_C2\,{\rm e}^{\frac{t}{2}\sqrt{-2\,a^2-2\sqrt{a^2(a^2+4b)}-4b}}+\_C3\,{\rm e}^{-\frac{t}{2}\sqrt{-2\,a^2+2\sqrt{a^2(a^2+4b)}-4b}}+\_C4\,{\rm e}^{\frac{t}{2}\sqrt{-2\,a^2+2\sqrt{a^2(a^2+4b)}-4b}}\right.\right.$

2.1893 ODE No. 1893

$\{-A0y'(t)+a1x''(t)+b1x'(t)+c1x(t)=B0e^{i\omega t},A0x'(t)+a2y''(t)+b2y'(t)+c2y(t)=0\}$

✓ **Mathematica** : cpu = 0.442546 (sec), leaf count = 5546

$\left\{\left\{x(t)\rightarrow -A0a1^2c2c3\text{RootSum}\left[\#1^4+a2b1\#1^3+a1b2\#1^3+A0^2a1a2\#1^2+a1a2b1b2\#1^2+a1a2^2c1\#1^2+a1a2b1c2\#1^2+a1a2^2c1c2\#1^2+a1a2b1c2c3\#1^2+a1a2^2c1c2c3\#1^2\right]\right.\right.$

✓ **Maple** : cpu = 0.711 (sec), leaf count = 1579

$\left\{\left\{x(t)=\frac{\_C1\left(a2\,a1\left(\text{RootOf}\left(a1\,a2\,\_Z^4+\left(a1\,b2+a2\,b1\right)\_Z^3+\left(A^2+a1\,c2+a2\,c1+b1\,b2\right)\_Z^2+\left(a1\,a2\,b1\,b2\right)\_Z+a1\,a2\,c1\,b2\right)\right)}{a1\,a2\,b1\,b2\,c1\,c2}\right.\right.$

2.1894 ODE No. 1894

$\{a\left(x'(t)-y'(t)\right)+b1x(t)+x''(t)=c1e^{i\omega t},a\left(y'(t)-x'(t)\right)+b2y(t)+y''(t)=c2e^{i\omega t}\}$

✓ **Mathematica** : cpu = 0.449676 (sec), leaf count = 3386

$\left\{\left\{x(t)\rightarrow -ab2c3\text{RootSum}\left[\#1^4+2a\#1^3+b1\#1^2+b2\#1^2+ab1\#1+ab2\#1+b1b2\&, \frac{a^2\#1^3+6a\#1^2+2b\#1^2+2a\#1+b1\#1+b2\#1}{a^2\#1^3+6a\#1^2+2b\#1^2+2a\#1+b1\#1+b2\#1}\right]\right.\right.$

✓ **Maple** : cpu = 0.506 (sec), leaf count = 1056

$\left\{\left\{x\left(t\right)=\frac{{\rm e}^{i\omega t}\left(-c1\,\omega^2+i\left(c1+c2\right)a\omega+b2\,c1\right)}{\omega^4-2\,ia\omega^3+\left(-b1-b2\right)\omega^2+i\left(b1+b2\right)a\omega+b1\,b2}\right.\right.$



2.1899 ODE No. 1899

{x'(t) = 2x(t), y'(t) = 3x(t) - 2y(t), z'(t) = 2y(t) + 3z(t)}

✓ Mathematica : cpu = 0.0080303 (sec), leaf count = 112

{ { x(t) -> c1 e^{2t}, y(t) -> 3/4 c1 e^{-2t} (e^{4t} - 1) + c2 e^{-2t}, z(t) -> 3/10 c1 e^{-2t} (2e^t + 3e^{2t} + 4e^{3t} + 1) (e^t - 1)^2 + 2/5 c2 e^{-2t} }

✓ Maple : cpu = 0.056 (sec), leaf count = 52

{ { x(t) = -C3 e^{2t}, y(t) = 3 - C3 e^{2t} / 4 + e^{-2t} - C2, z(t) = -C1 e^{3t} - 3 - C3 e^{2t} / 2 - 2 e^{-2t} - C2 / 5 } }

2.1900 ODE No. 1900

{x'(t) = 4x(t), y'(t) = x(t) - 2y(t), z'(t) = x(t) - 4y(t) + z(t)}

✓ Mathematica : cpu = 0.0070964 (sec), leaf count = 94

{ { x(t) -> c1 e^{4t}, y(t) -> 1/6 c1 e^{-2t} (e^{6t} - 1) + c2 e^{-2t}, z(t) -> 1/9 c1 e^{-2t} (e^{3t} + e^{6t} - 2) - 4/3 c2 e^{-2t} (e^{3t} - 1) + c3 e^t } }

✓ Maple : cpu = 0.058 (sec), leaf count = 50

{ { x(t) = -C3 e^{4t}, y(t) = -C3 e^{4t} / 6 + e^{-2t} - C2, z(t) = -C3 e^{4t} / 9 + -C1 e^t + 4 e^{-2t} - C2 / 3 } }

2.1901 ODE No. 1901

{x'(t) = y(t) - z(t), y'(t) = x(t) + y(t), z'(t) = x(t) + z(t)}

✓ Mathematica : cpu = 0.0081381 (sec), leaf count = 105

{ { x(t) -> c2 (e^t - 1) + c3 (1 - e^t) + c1, y(t) -> c1 (e^t - 1) + c2 (e^t t + 1) + c3 (-e^t t + e^t - 1), z(t) -> c1 (e^t - 1) }

✓ Maple : cpu = 0.049 (sec), leaf count = 43

{ { x(t) = -C2 + -C3 e^t, y(t) = (-C3 t + -C1) e^t - C2, z(t) = ((t - 1) - C3 + -C1) e^t - C2 } }

2.1902 ODE No. 1902

{x'(t) - y(t) + z(t) = 0, -x(t) + y'(t) - y(t) = t, -x(t) + z'(t) - z(t) = t}

✓ Mathematica : cpu = 0.0167971 (sec), leaf count = 226

{ { x(t) -> c2 (e^t - 1) + c3 (1 - e^t) + c1 + e^{-t} (1 - e^t) (-t - 1) + e^{-t} (e^t - 1) (-t - 1), y(t) -> c3 (-e^t t + e^t - 1) }

✓ Maple : cpu = 0.047 (sec), leaf count = 51

{ { x(t) = -C2 + -C3 e^t, y(t) = (-C3 t + -C1) e^t - t - C2 - 1, z(t) = ((t - 1) - C3 + -C1) e^t - t - C2 - 1 }

2.1903 ODE No. 1903

{ax'(t) = bc(y(t) - z(t)), by'(t) = ac(z(t) - x(t)), cz'(t) = ab(x(t) - y(t))}

✓ Mathematica : cpu = 0.061549 (sec), leaf count = 1304

{ { x(t) -> e^{-i sqrt(a^2 + b^2 + c^2) t} (2 e^{i sqrt(a^2 + b^2 + c^2) t} a^2 + b^2 e^{2 i sqrt(a^2 + b^2 + c^2) t} + c^2 e^{2 i sqrt(a^2 + b^2 + c^2) t} + b^2 + c^2) c1 - b e^{-i sqrt(a^2 + b^2 + c^2) t} (-

✓ Maple : cpu = 0.111 (sec), leaf count = 299

{ { x(t) = -C1 + -C2 sin(sqrt(a^2 + b^2 + c^2) t) + -C3 cos(sqrt(a^2 + b^2 + c^2) t), y(t) = 1 / (b (b^2 + c^2)) (-C1 b^3 + ((-C2 cos(sqrt(a^2 + b^2 + c^2) t) - C3 sin(sqrt(a^2 + b^2 + c^2) t) - 1) b^2 + c^2) }

2.1904 ODE No. 1904

{x'(t) = cy(t) - bz(t), y'(t) = az(t) - cx(t), z'(t) = bx(t) - ay(t)}

✓ Mathematica : cpu = 0.0455427 (sec), leaf count = 1445

{ { x(t) -> (e^{-sqrt{-a^2-b^2-c^2}t} (2e^{sqrt{-a^2-b^2-c^2}t} a^2 + b^2 e^{2sqrt{-a^2-b^2-c^2}t} + c^2 e^{2sqrt{-a^2-b^2-c^2}t} + b^2 + c^2) c\_1 - e^{-sqrt{-a^2-b^2-c^2}t} ) / (2 (a^2 + b^2 + c^2))

✓ Maple : cpu = 0.071 (sec), leaf count = 257

{ { x(t) = \_C1 + \_C2 sin(sqrt(a^2 + b^2 + c^2)t) + \_C3 cos(sqrt(a^2 + b^2 + c^2)t), y(t) = 1/(a(b^2 + c^2)) ((-a^2 b \_C3 + a

2.1905 ODE No. 1905

{x'(t) = h(t)y(t) - g(t)z(t), y'(t) = f(t)z(t) - h(t)x(t), z'(t) = g(t)x(t) - f(t)y(t)}

✗ Mathematica : cpu = 0.0054547 (sec), leaf count = 0 , could not solve

DSolve[{Derivative[1][x][t] == h[t]\*y[t] - g[t]\*z[t], Derivative[1][y][t] == -(h[t]\*x[t]) +

✗ Maple : cpu = 0. (sec), leaf count = 0 , result contains DESol

{ { x(t) = DESol ( { (d^3/dt^3 - Y(t) + (-2 (h(t) (d/dt h(t)) f(t) / ((h(t))^2 f(t) + (g(t))^2 f(t) - h(t) d/dt g(t) + (d/dt h(t)) g(t) - 2 (h(t))^2 f(t)

2.1906 ODE No. 1906

{x'(t) = x(t) + y(t) - z(t), y'(t) = -x(t) + y(t) + z(t), z'(t) = x(t) - y(t) + z(t)}

✓ Mathematica : cpu = 0.0394689 (sec), leaf count = 278

{ { x(t) -> 1/3 c\_1 e^t (2 cos(sqrt(3)t) + 1) - 1/3 c\_2 e^t (-sqrt(3) sin(sqrt(3)t) + cos(sqrt(3)t) - 1) - 1/3 c\_3 e^t (sqrt(3) sin(sqrt(3)t) + cos(sqrt(3)t) +

✓ Maple : cpu = 0.055 (sec), leaf count = 120

{ { x(t) = e^t (sin(sqrt(3)t) \_C2 + cos(sqrt(3)t) \_C3 + \_C1), y(t) = (e^t (\_C2 sqrt(3) - \_C3) cos(sqrt(3)t) / 2 + e^t (-\_C3 v

2.1907 ODE No. 1907

{x'(t) = -3x(t) + 48y(t) - 28z(t), y'(t) = -4x(t) + 40y(t) - 22z(t), z'(t) = -6x(t) + 57y(t) - 31z(t)}

✓ Mathematica : cpu = 0.0082161 (sec), leaf count = 179

{ { x(t) -> c\_1 (-e^t) (2e^{2t} - 3) + 6c\_2 e^t (2e^t + 3e^{2t} - 5) - 2c\_3 e^t (4e^t + 5e^{2t} - 9), y(t) -> -2c\_1 e^t (e^{2t} - 1) + c\_2 e^t (3e^{2t} - 5) - 2c\_3 e^t (4e^t + 5e^{2t} - 9)

✓ Maple : cpu = 0.046 (sec), leaf count = 66

{ { x(t) = \_C1 e^t + \_C2 e^{3t} + \_C3 e^{2t}, y(t) = (2 \_C1 e^t / 3 + \_C2 e^{3t} + \_C3 e^{2t}) / 4, z(t) = \_C1 e^t + (3 \_C2 e^{3t} + 2 \_C3 e^{2t}) / 4

2.1908 ODE No. 1908

{x'(t) = 6x(t) - 72y(t) + 44z(t), y'(t) = 4x(t) - 4y(t) + 26z(t), z'(t) = 6x(t) - 63y(t) + 38z(t)}

✓ Mathematica : cpu = 0.0142877 (sec), leaf count = 551

{ { x(t) -> -36c\_2 RootSum[#1^3 - 40#1^2 + 1714#1 + 1404&, (2#1 e^{#1 t} + e^{#1 t}) / (3#1^2 - 80#1 + 1714) &] + 4c\_3 RootSum[#1^3 - 40#1^2 + 1714#1 + 1404&, (2#1 e^{#1 t} + e^{#1 t}) / (3#1^2 - 80#1 + 1714) &]

✓ Maple : cpu = 0.417 (sec), leaf count = 1225

{ { x(t) = \_C2 e^{(-3542 + (263474 + 18 sqrt(351406311))^{2/3} + 80 sqrt[3]{263474 + 18 sqrt(351406311)})t / (6 sqrt[3]{263474 + 18 sqrt(351406311)})} sin(sqrt(3)t (sqrt[3]{4} sqrt[3]{(131737 + 9 sqrt(351406311))^2} / (6 sqrt[3]{263474 + 18 sqrt(351406311)}))

**2.1909                    ODE No. 1909**

---

$$\{x'(t) = ax(t) + \beta z(t) + gy(t), y'(t) = \alpha z(t) + by(t) + gx(t), z'(t) = \alpha y(t) + \beta x(t) + cz(t)\}$$

✓ **Mathematica** : cpu = 0.0407861 (sec), leaf count = 1630

$$\left\{\left\{x(t) \rightarrow -c_2 \text{RootSum}\left[\#1^3 - a\#1^2 - b\#1^2 - c\#1^2 - \alpha^2\#1 - \beta^2\#1 - g^2\#1 + ab\#1 + ac\#1 + bc\#1 + a\alpha^2 + \right.\right.\right.$$

✓ **Maple** : cpu = 14.222 (sec), leaf count = 33085

Too large to display

**2.1910                    ODE No. 1910**

---

$$\{tx'(t) = 2x(t) - t, t^3y'(t) = t^2y(t) - x(t) + t, t^4z'(t) = t^3z(t) - t^2y(t) - x(t) + t\}$$

✓ **Mathematica** : cpu = 0.0072081 (sec), leaf count = 39

$$\left\{\left\{x(t) \rightarrow c_3t^2 + t, y(t) \rightarrow c_2t + c_3, z(t) \rightarrow c_1t + \frac{c_3}{t} + c_2\right\}\right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 37

$$\left\{\left\{x(t) = \_C3\,t^2 + t, y(t) = \_C2\,t + \_C3, z(t) = \frac{\_C1\,t^2 + \_C2\,t + \_C3}{t}\right\}\right\}$$

**2.1911                    ODE No. 1911**

---

$$\{atx'(t) = bc(y(t) - z(t)), bty'(t) = ac(z(t) - x(t)), ctz'(t) = ab(x(t) - y(t))\}$$

✗ **Mathematica** : cpu = 0.0223375 (sec), leaf count = 0 , could not solve

`DSolve[{a*t*Derivative[1][x][t] == b*c*(y[t] - z[t]), b*t*Derivative[1][y][t] == a*c*(-x[t] + z[t]), c*t*Derivative[1][z][t] == a*b*(x[t] - y[t])}, {x[t], y[t], z[t]}, t]`

✓ **Maple** : cpu = 0.106 (sec), leaf count = 309

$$\left\{\left\{x(t) = \_C1 + \_C2 \sin\left(\sqrt{a^2 + b^2 + c^2} \ln(t)\right) + \_C3 \cos\left(\sqrt{a^2 + b^2 + c^2} \ln(t)\right), y(t) = \frac{1}{b(b^2 + c^2)} \left(\cos\left(\sqrt{a^2 + b^2 + c^2} \ln(t)\right) - \sin\left(\sqrt{a^2 + b^2 + c^2} \ln(t)\right)\right)\right\}\right\}$$

**2.1912                    ODE No. 1912**

---

$$\{x1'(t) = ax2(t) + bx3(t) \cos(ct) + bx4(t) \sin(ct), x2'(t) = -ax1(t) + bx3(t) \sin(ct) - bx4(t) \cos(ct), x3'(t) = ax4(t) - bx1(t) \sin(ct) + bx2(t) \cos(ct)\}$$

✗ **Mathematica** : cpu = 0.0071159 (sec), leaf count = 0 , could not solve

`DSolve[{Derivative[1][x1][t] == a*x2[t] + b*Cos[c*t]*x3[t] + b*Sin[c*t]*x4[t], Derivative[1][x2][t] == -a*x1[t] + b*Sin[c*t]*x3[t] - b*Cos[c*t]*x4[t], Derivative[1][x3][t] == -(b*Cos[c*t]*x1[t] + b*Sin[c*t]*x2[t]) + b*Cos[c*t]*x4[t] - a*x3[t]}, {x1[t], x2[t], x3[t], x4[t]}, t]`

✓ **Maple** : cpu = 1.556 (sec), leaf count = 2956

$$\left\{\left\{x1(t) = \_C2 + \_C3 \sin(ct) + \_C4 \cos(ct), x2(t) = -\cos(ct) \_C3 + \sin(ct) \_C4 + \_C1, x3(t) = \frac{(\cos(ct) - \sin(ct))}{c}\right\}\right\}$$

**2.1913                    ODE No. 1913**

---

$$\{x'(t) = -x(t)(x(t) + y(t)), y'(t) = y(t)(x(t) + y(t))\}$$

✓ **Mathematica** : cpu = 0.0192355 (sec), leaf count = 64

$$\{\{y(t) \rightarrow -\sqrt{c_1} \cot(\sqrt{c_1}t - \sqrt{c_1}c_2), x(t) \rightarrow -\sqrt{c_1} \tan(\sqrt{c_1}t - \sqrt{c_1}c_2)\}\}$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 57

$$\left\{\left[\{x(t) = 0\}, \left\{y(t) = (-t + \_C1)^{-1}\right\}\right], \left[\left\{x(t) = \frac{1}{\_C1} \tanh\left(\frac{\_C2 + t}{\_C1}\right)\right\}, \left\{y(t) = \frac{-(x(t))^2 - \frac{d}{dt}x(t)}{x(t)}\right\}\right]\right\}$$



2.1917 ODE No. 1917

{x'(t) = y(t)^2 - cos(x(t)), y'(t) = y(t)(- sin(x(t)))}

✓ Mathematica : cpu = 213.617 (sec), leaf count = 3406

{ { { y(t) -> (3\*sqrt(2)\*cos(InverseFunction[Integrate[1/(2\*2^(2/3)\*cos^2(K[1])+2\*(3\*c1+sqrt(9\*c1^2-4\*cos^3(K[1]))^(2/3)\*cos(K[1])+3\*sqrt(2)\*c1), {t, 0, t}]^#1)&], {t, 0, t}])^(2/3)\*cos(K[1]) + 3\*sqrt(2)\*c1)/sqrt(81\*c1^3 + 6561\*c1^2 - 2916\*cos^3(InverseFunction[Integrate[1/(2\*2^(2/3)\*cos^2(K[1])+2\*(3\*c1+sqrt(9\*c1^2-4\*cos^3(K[1]))^(2/3)\*cos(K[1])+3\*sqrt(2)\*c1), {t, 0, t}]^#1)&], {t, 0, t}])^(2/3)\*cos(K[1]) + 3\*sqrt(2)\*c1)) } }

✓ Maple : cpu = 0.696 (sec), leaf count = 109

{ [ { x(t) = RootOf(2\*int(-Z\*tan(RootOf(-3\*sqrt(-(cos(f))^2)\*ln(9/4\*(cos(f))^2/(cos(Z))^2)) + 3\*C1\*sqrt(-(cos(f))^2)), t) } }

2.1918 ODE No. 1918

{x'(t) = -x(t)y(t)^2 + x(t) + y(t), y'(t) = x(t)^2y(t) - x(t) - y(t)}

✗ Mathematica : cpu = 0.298385 (sec), leaf count = 0 , could not solve
DSolve[{Derivative[1][x][t] == x[t] + y[t] - x[t]\*y[t]^2, Derivative[1][y][t] == -x[t] - y[t] + x[t]^2\*y[t]}, {x[t], y[t]}, t]
✓ Maple : cpu = 1.627 (sec), leaf count = 184

{ [{x(t) = 0}, {y(t) = 0}], [ { x(t) = ODESolStruc(-a, [1/(2\*a^2)\*(sqrt((4\*a^2-4\*a\*b(a)+1)\*(a^3+a-b(a))) - a^2)], t) } }

2.1919 ODE No. 1919

{x'(t) = x(t)(-(x(t)^2 + y(t)^2)) + x(t) + y(t), y'(t) = -y(t)(x(t)^2 + y(t)^2) - x(t) + y(t)}

✗ Mathematica : cpu = 0.34649 (sec), leaf count = 0 , could not solve
DSolve[{Derivative[1][x][t] == x[t] + y[t] - x[t]\*(x[t]^2 + y[t]^2), Derivative[1][y][t] == x[t] + y[t] - y[t]\*(x[t]^2 + y[t]^2)}, {x[t], y[t]}, t]
✓ Maple : cpu = 2.577 (sec), leaf count = 203

{ [{x(t) = 0}, {y(t) = 0}], [ { x(t) = ODESolStruc(-a, [1/(2\*a^3)\*(sqrt(-(4\*a^4+4\*a\*b(a)-4\*a^2-1)\*(2\*a^3+a-b(a)))) - a^2)], t) } }

2.1920 ODE No. 1920

{x'(t) = x(t)(x(t)^2 + y(t)^2 - 1) - y(t), y'(t) = y(t)(x(t)^2 + y(t)^2 - 1) + x(t)}

✗ Mathematica : cpu = 0.275551 (sec), leaf count = 0 , could not solve
DSolve[{Derivative[1][x][t] == -y[t] + x[t]\*(-1 + x[t]^2 + y[t]^2), Derivative[1][y][t] == x[t] + y[t] + x[t]^2 + y[t]^2)}, {x[t], y[t]}, t]
✓ Maple : cpu = 2.559 (sec), leaf count = 205

{ [{x(t) = 0}, {y(t) = 0}], [ { x(t) = ODESolStruc(-a, [1/(2\*a^3)\*(sqrt(-(4\*a^4-4\*a^2-4\*a\*b(a)-1)\*(4\*a^3+a-b(a)))) - a^2)], t) } }

2.1921 ODE No. 1921

{ x'(t) = -y(t)(x(t)^2 + y(t)^2), y'(t) = ( { (x(t)^2 + y(t)^2)^(x(t)^2 + y(t)^2 >= 2\*x(t)) \* (x(t)/2 - y(t)/(2\*x(t))) , True } ) }

✗ Mathematica : cpu = 2.13606 (sec), leaf count = 0 , could not solve
DSolve[{Derivative[1][x][t] == -(y[t]\*(x[t]^2 + y[t]^2)), Derivative[1][y][t] == Piecewise[{x(t)^2 + y(t)^2 >= 2\*x(t), (x(t)/2 - y(t)/(2\*x(t))), True]}], {x[t], y[t]}, t]
✗ Maple : cpu = 0. (sec), leaf count = 0 , could not solve
dsolve({diff(x(t),t) = -y(t)\*(x(t)^2+y(t)^2), diff(y(t),t) = piecewise(2\*x(t) <= x(t)^2+y(t)^2, 1/2\*y(t)^2/x(t))\*(x(t)^2+y(t)^2)})

**2.1922                    ODE No. 1922**

---

$$\left\{x'(t) = \left(\begin{array}{cc} \sin\left(\frac{1}{x(t)^2+y(t)^2}\right)x(t)\left(x(t)^2+y(t)^2-1\right) & x(t)^2+y(t)^2 \neq 1 \\ 0 & \text{True} \end{array}\right) - y(t), y'(t) = \left(\begin{array}{cc} \sin\left(\frac{1}{x(t)^2+y(t)^2}\right)y(t)\left(x(t)^2+y(t)^2-1\right) & x(t)^2+y(t)^2 \neq 1 \\ 0 & \text{True} \end{array}\right) + x(t)\right\}$$

✗ **Mathematica** : cpu = 8.96914 (sec), leaf count = 0 , could not solve  
DSolve[{Derivative[1][x][t] == Piecewise[{{Sin[(x[t]^2 + y[t]^2)^(-1)]\*x[t]\*(-1 + x[t]^2 + y[t]^2)}\*y[t]\*(-1 + x[t]^2 + y[t]^2), x[t]^2 + y[t]^2 != 1}}, 0] + x[t]}, {x[t], y[t]}, t]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve({diff(x(t),t) = -y(t)+piecewise(x(t)^2+y(t)^2 <> 1,x(t)\*(x(t)^2+y(t)^2-1)\*sin(1/(x(t)^2+y(t)^2))),t))})

**2.1923                    ODE No. 1923**

---

$$\{(t^2 + 1) x'(t) = y(t) - tx(t), (t^2 + 1) y'(t) = -x(t) - ty(t)\}$$

✓ **Mathematica** : cpu = 0.0094938 (sec), leaf count = 53

$$\left\{\left\{x(t) \rightarrow \frac{c_1}{t^2 + 1} + \frac{c_2 t}{t^2 + 1}, y(t) \rightarrow \frac{c_2}{t^2 + 1} - \frac{c_1 t}{t^2 + 1}\right\}\right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 35

$$\left\{\left\{x(t) = \frac{-C1\,t + \_C2}{t^2 + 1}, y(t) = \frac{-\_C2\,t + \_C1}{t^2 + 1}\right\}\right\}$$

**2.1924                    ODE No. 1924**

---

$$\{(-t^2 + x(t)^2 + y(t)^2) x'(t) = -2tx(t), (-t^2 + x(t)^2 + y(t)^2) y'(t) = -2ty(t)\}$$

✓ **Mathematica** : cpu = 0.0518002 (sec), leaf count = 191

$$\left\{\left\{y(t) \rightarrow \frac{c_1 \left(e^{c_2} - \sqrt{-4c_1^2 t^2 + e^{2c_2} - 4t^2}\right)}{2(c_1^2 + 1)}, x(t) \rightarrow \frac{e^{c_2} - \sqrt{-4c_1^2 t^2 + e^{2c_2} - 4t^2}}{2(c_1^2 + 1)}\right\}, \left\{y(t) \rightarrow \frac{c_1 \left(\sqrt{-4c_1^2 t^2 + e^{2c_2} - 4t^2}\right)}{2(c_1^2 + 1)}, x(t) \rightarrow \frac{e^{c_2} - \sqrt{-4c_1^2 t^2 + e^{2c_2} - 4t^2}}{2(c_1^2 + 1)}\right\}\right\}$$

✓ **Maple** : cpu = 0.24 (sec), leaf count = 180

$$\left\{\left[\left\{x(t) = 0\right\}, \left\{y(t) = \frac{1}{2\_C1} \left(1 + \sqrt{-4\_C1^2 t^2 + 1}\right)\right\}, \left\{y(t) = \frac{1}{2\_C1} \left(1 - \sqrt{-4\_C1^2 t^2 + 1}\right)\right\}\right], \left[\left\{x(t) = \frac{1}{2\_C1} \left(1 + \sqrt{-4\_C1^2 t^2 + 1}\right)\right\}, \left\{y(t) = 0\right\}, \left\{x(t) = \frac{1}{2\_C1} \left(1 - \sqrt{-4\_C1^2 t^2 + 1}\right)\right\}, \left\{y(t) = 0\right\}\right]\right\}$$

**2.1925                    ODE No. 1925**

---

$$\{ay'(t) + tx'(t) - x(t) + y'(t)^2 = 0, x'(t)y'(t) + ty'(t) - y(t) = 0\}$$

✗ **Mathematica** : cpu = 5.51649 (sec), leaf count = 0 , could not solve  
DSolve[{-x[t] + t\*Derivative[1][x][t] + a\*Derivative[1][y][t] + Derivative[1][y][t]^2 == 0, y[t] + t\*Derivative[1][y][t] + Derivative[1][x][t]\*Derivative[1][y][t] == 0}, {x[t], y[t]}, t]  
✓ **Maple** : cpu = 0.207 (sec), leaf count = 194

$$\left\{\left[\left\{x(t) = -\frac{t^2}{3}\right\}, \left\{y(t) = -\frac{t^3}{27a}\right\}\right], \left[\left\{x(t) = \_C1\,t + \_C2\right\}, \left\{y(t) = -\frac{\left(\frac{d}{dt}x(t) + t\right)\left(\left(\frac{d}{dt}x(t)\right)^2 + t\frac{d}{dt}x(t) - \frac{d^2}{dt^2}x(t)\right)}{a}\right\}\right]\right\}$$

**2.1926                    ODE No. 1926**

---

$$\{x(t) = f(x'(t), y'(t)) + tx'(t), y(t) = g(x'(t), y'(t)) + ty'(t)\}$$

✗ **Mathematica** : cpu = 0.0043313 (sec), leaf count = 0 , could not solve  
DSolve[{x[t] == f[Derivative[1][x][t], Derivative[1][y][t]] + t\*Derivative[1][x][t], y[t] == g[Derivative[1][x][t], Derivative[1][y][t]] + t\*Derivative[1][y][t]}, {x[t], y[t]}, t]  
✓ **Maple** : cpu = 0.105 (sec), leaf count = 96

$$\left\{\left[\int \text{RootOf}\left(t\frac{d}{dt}y(t) + g\left(-Z, \frac{d}{dt}y(t)\right) - y(t)\right) dt + \_C1 = t\text{RootOf}\left(t\frac{d}{dt}y(t) + g\left(-Z, \frac{d}{dt}y(t)\right) - y(t)\right)\right]\right\}$$



2.1927 ODE No. 1927

---

$$\left\{x''(t) = ae^{2x(t)} + e^{-2x(t)} \cos^2(y(t)) - e^{-x(t)}, y''(t) = e^{-2x(t)} \sin(y(t)) \cos(y(t)) - \tan(y(t)) \sec^2(y(t))\right\}$$

✗ **Mathematica** : cpu = 0.0069965 (sec), leaf count = 0 , could not solve  
DSolve[{Derivative[2][x][t] == -E^(-x[t]) + a\*E^(2\*x[t]) + Cos[y[t]]^2/E^(2\*x[t]), Derivative[2][y][t] == (k\*y[t])/(x[t]^2 + y[t]^2)^(3/2)}, {x[t], y[t]}, t]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve({diff(diff(x(t),t),t) = a\*exp(2\*x(t))-exp(-x(t))+exp(-2\*x(t))\*cos(y(t))^2, diff(diff(y(t),t),t) = k\*y(t)/(x(t)^2+y(t)^2)^(3/2)}, {x(t), y(t)})

2.1928 ODE No. 1928

---

$$\left\{x''(t) = \frac{kx(t)}{(x(t)^2 + y(t)^2)^{3/2}}, y''(t) = \frac{ky(t)}{(x(t)^2 + y(t)^2)^{3/2}}\right\}$$

✗ **Mathematica** : cpu = 0.0053015 (sec), leaf count = 0 , could not solve  
DSolve[{Derivative[2][x][t] == (k\*x[t])/(x[t]^2 + y[t]^2)^(3/2), Derivative[2][y][t] == (k\*y[t])/(x[t]^2 + y[t]^2)^(3/2)}, {x[t], y[t]}, t]  
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve({diff(diff(x(t),t),t) = k\*x(t)/(x(t)^2+y(t)^2)^(3/2), diff(diff(y(t),t),t) = k\*y(t)/(x(t)^2+y(t)^2)^(3/2)}, {x(t), y(t)})

2.1929 ODE No. 1929

---

$$\left\{x''(t) = -\frac{cy(t)x'(t)f\left(\sqrt{x'(t)^2 + y'(t)^2}\right)}{\sqrt{x'(t)^2 + y'(t)^2}}, y''(t) = -\frac{cy(t)y'(t)f\left(\sqrt{x'(t)^2 + y'(t)^2}\right)}{\sqrt{x'(t)^2 + y'(t)^2}} - g\right\}$$

✗ **Mathematica** : cpu = 0.0064354 (sec), leaf count = 0 , could not solve  
DSolve[{Derivative[2][x][t] == -((c\*f[Sqrt[Derivative[1][x][t]^2 + Derivative[1][y][t]^2]]\*y[t])/(x[t]^2 + y[t]^2)^(3/2)) - g, Derivative[2][y][t] == (c\*f[Sqrt[Derivative[1][x][t]^2 + Derivative[1][y][t]^2]]\*y[t]\*Derivative[1][y][t])/Sqrt[x[t]^2 + y[t]^2]}, {x[t], y[t]}, t]  
✓ **Maple** : cpu = 2.447 (sec), leaf count = 116

$$\left\{\left[y(t) = ODESolStruc\left(-a, \left[\left(\frac{d}{d\_a} b(\_a)\right) b(\_a) + 1 \left(C(\_a) f\left(\sqrt{(\_b(\_a))^2}\right) b(\_a) + g \sqrt{(\_b(\_a))^2}\right)\right]\right)\right]\right\}$$

2.1930 ODE No. 1930

---

$$\{x'(t) = y(t) - z(t), y'(t) = x(t)^2 + y(t), z'(t) = x(t)^2 + z(t)\}$$

✓ **Mathematica** : cpu = 0.0300623 (sec), leaf count = 308

$$\left\{\left\{x(t) \rightarrow e^{-c_3} \left(e^{c_3} c_1 + e^t\right), y(t) \rightarrow c_2 \left(e^{-c_3} \left(e^{c_3} c_1 + e^t\right) - c_1\right) + \left(e^{-c_3} \left(e^{c_3} c_1 + e^t\right) - c_1\right) \left(-\frac{c_1^2}{e^{-c_3} \left(e^{c_3} c_1 + e^t\right) - c_1}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 45

$$\left\{\left[x(t) = \_C2 + \_C3 e^t\right], \left[y(t) = \left(\int (x(t))^2 e^{-t} dt + \_C1\right) e^t\right], \left[z(t) = -\frac{d}{dt} x(t) + y(t)\right]\right\}$$

2.1931 ODE No. 1931

---

$$\{ax'(t) = (b - c)y(t)z(t), by'(t) = (c - a)x(t)z(t), cz'(t) = (a - b)x(t)y(t)\}$$

✓ **Mathematica** : cpu = 3.95197 (sec), leaf count = 10101

$$\left\{ \left\{ \begin{array}{l} x(t) \rightarrow \frac{\sqrt{2}b^2\sqrt{a(a-c)}c_1\sin\left(\frac{\frac{\sqrt{2}\sqrt{a}\sqrt{a-c}c_2t}{\sqrt{b}\sqrt{b-c}} - \frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a-c}\sqrt{c_2}t}{\sqrt{b-c}} - \frac{\sqrt{2}\sqrt{a}\sqrt{a-c}c_2c_3}{\sqrt{b}\sqrt{b-c}} + \frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a-c}\sqrt{c_2}c_3}{\sqrt{b-c}}\right) - \frac{(a-b)bc_1}{(a-c)cc_2}}{a} - \frac{\sqrt{2}b\sqrt{a(a-c)}}{(a-c)\sqrt{b(b-c)}c_1} \end{array} \right. \right.$$

✓ **Maple** : cpu = 0.513 (sec), leaf count = 1117

$$\left\{ \left[ \left\{ x\left( t\right) =0\right\} ,\left\{ y\left( t\right) =0\right\} ,\left\{ z\left( t\right) =\_C1\right\} \right],\left[ \left\{ x\left( t\right) =0\right\} ,\left\{ y\left( t\right) =\_C1\right\} ,\left\{ z\left( t\right) =0\right\} \right],\left[ \left\{ x\left( t\right) =\_C1\right\} ,\left\{ y\left( t\right) =0\right\} \right],\left[ \left\{ x\left( t\right) =0\right\} ,\left\{ y\left( t\right) =\_C1\right\} \right] \right\}$$

### 2.1932 ODE No. 1932

$$\left\{ x'(t)=x(t)(y(t)-z(t)),y'(t)=y(t)(z(t)-x(t)),z'(t)=z(t)(x(t)-y(t))\right\}$$

✗ **Mathematica** : cpu = 1.75962 (sec), leaf count = 0 , could not solve  
DSolve[{Derivative[1][x][t] == x[t]\*(y[t] - z[t]), Derivative[1][y][t] == y[t]\*(-x[t] + z[t]), Derivative[1][z][t] == (x[t] - y[t])\*z[t]}, {x[t], y[t], z[t]}, t]  
✓ **Maple** : cpu = 0.741 (sec), leaf count = 383

$$\left\{ \left[ \left\{ x\left( t\right) =0\right\} ,\left\{ y\left( t\right) =0\right\} ,\left\{ z\left( t\right) =\_C1\right\} \right],\left[ \left\{ x\left( t\right) =0\right\} ,\left\{ y\left( t\right) =\frac{\_C1\,\mathrm{e}^{-C2}\_C1\,\mathrm{e}^{-C1\,t}}{-1+\mathrm{e}^{-C2}\_C1\,\mathrm{e}^{-C1\,t}}\right\} ,\left\{ z\left( t\right) =\frac{\frac{\mathrm{d}}{\mathrm{d}t}y\left( t\right) }{y\left( t\right) }\right\} \right],\left[ \left\{ x\left( t\right) =\frac{\frac{\mathrm{d}}{\mathrm{d}t}y\left( t\right) }{y\left( t\right) },\left\{ y\left( t\right) =0\right\} ,\left\{ z\left( t\right) =\_C1\right\} \right] \right\}$$

### 2.1933 ODE No. 1933

$$\left\{ x'(t)+y'(t)=x(t)y(t),y'(t)+z'(t)=y(t)z(t),x'(t)+z'(t)=x(t)z(t)\right\}$$

✗ **Mathematica** : cpu = 125.041 (sec), leaf count = 0 , could not solve  
DSolve[{Derivative[1][x][t] + Derivative[1][y][t] == x[t]\*y[t], Derivative[1][y][t] + Derivative[1][z][t] == y[t]\*z[t], Derivative[1][x][t] + Derivative[1][z][t] == x[t]\*z[t]}, {x[t], y[t], z[t]}, t]  
✓ **Maple** : cpu = 1.892 (sec), leaf count = 17738

Too large to display

### 2.1934 ODE No. 1934

$$\left\{ x'(t)=\frac{x(t)^2}{2}-\frac{y(t)}{24},y'(t)=2x(t)y(t)-3z(t),z'(t)=3x(t)z(t)-\frac{y(t)^2}{6}\right\}$$

✗ **Mathematica** : cpu = 64.1656 (sec), leaf count = 0 , could not solve  
DSolve[{Derivative[1][x][t] == x[t]^2/2 - y[t]/24, Derivative[1][y][t] == 2\*x[t]\*y[t] - 3\*z[t], Derivative[1][z][t] == 3\*x[t]\*z[t]}, {x[t], y[t], z[t]}, t]  
✓ **Maple** : cpu = 1.137 (sec), leaf count = 377

$$\left\{ \left[ \left\{ y\left( t\right) =0\right\} ,\left\{ x\left( t\right) =-2\left( -2\_C1+t\right) ^{-1}\right\} ,\left\{ z\left( t\right) =0\right\} \right],\left[ \left\{ y\left( t\right) =256\left( \_C1\,t+\_C2\right) ^{-4}\right\} ,\left\{ x\left( t\right) =\frac{1}{6\,y\left( t\right) }\left( \frac{256\left( \_C1\,t+\_C2\right) ^{-4}}{6\,y\left( t\right) }-\frac{y\left( t\right) ^2}{6}\right) \right\} \right] \right\}$$

2.1935 ODE No. 1935

$\{x'(t) = x(t) \left(y(t)^2 - z(t)^2\right), y'(t) = y(t) \left(z(t)^2 - x(t)^2\right), z'(t) = z(t) \left(x(t)^2 - y(t)^2\right)\}$

✗ **Mathematica** : cpu = 0.0372397 (sec), leaf count = 0 , could not solve  
DSolve[{Derivative[1][x][t] == x[t]\*(y[t]^2 - z[t]^2), Derivative[1][y][t] == y[t]\*(-x[t]^2 + z[t]^2), Derivative[1][z][t] == (x[t]^2 - y[t]^2)\*z[t]}, {x[t], y[t], z[t]}, t]  
✓ **Maple** : cpu = 1.381 (sec), leaf count = 741

$$\left\{ \left\{ x(t) = 0 \right\}, \left\{ y(t) = 0 \right\}, \left\{ z(t) = -C1 \right\} \right\}, \left[ \left\{ x(t) = 0 \right\}, \left\{ y(t) = \frac{1}{(e^{-C2-C1})^2 (e^{-C1 t})^2 - 1} \sqrt{\left( (e^{-C2-C1})^2 (e^{-C1 t})^2 - 1 \right)} \right\} \right]$$

2.1936 ODE No. 1936

$\{x'(t) = x(t) \left(y(t)^2 - z(t)^2\right), y'(t) = -y(t) \left(x(t)^2 + z(t)^2\right), z'(t) = z(t) \left(x(t)^2 + y(t)^2\right)\}$

✗ **Mathematica** : cpu = 0.0352716 (sec), leaf count = 0 , could not solve  
DSolve[{Derivative[1][x][t] == x[t]\*(y[t]^2 - z[t]^2), Derivative[1][y][t] == -(y[t]\*(x[t]^2 + z[t]^2)), Derivative[1][z][t] == z[t]\*(x[t]^2 + y[t]^2)}, {x[t], y[t], z[t]}, t]  
✓ **Maple** : cpu = 0.565 (sec), leaf count = 704

$$\left\{ \left\{ x(t) = 0 \right\}, \left\{ y(t) = 0 \right\}, \left\{ z(t) = -C1 \right\} \right\}, \left[ \left\{ x(t) = 0 \right\}, \left\{ y(t) = \frac{1}{(e^{-C2-C1})^2 (e^{-C1 t})^2 - 1} \sqrt{-(e^{-C1 t})^4 - C1} \right\} \right]$$

2.1937 ODE No. 1937

$\{x'(t) = -x(t)y(t)^2 + x(t) + y(t), y'(t) = x(t)^2y(t) - x(t) - y(t), z'(t) = y(t)^2 - x(t)^2\}$

✗ **Mathematica** : cpu = 0.622445 (sec), leaf count = 0 , could not solve  
DSolve[{Derivative[1][x][t] == x[t] + y[t] - x[t]\*y[t]^2, Derivative[1][y][t] == -x[t] - y[t] + x[t]^2\*y[t], Derivative[1][z][t] == -x[t]^2 + y[t]^2}, {x[t], y[t], z[t]}, t]  
✓ **Maple** : cpu = 0.5 (sec), leaf count = 242

$$\left\{ \left\{ x(t) = 0 \right\}, \left\{ y(t) = 0 \right\}, \left\{ z(t) = -C1 \right\} \right\}, \left[ \left\{ x(t) = ODESolStruc \left( -a, \left[ \frac{1}{2-a^2} \left( \sqrt{(4-a^2-4-a-b(-a)+...} \right) \right] \right) \right\} \right]$$

2.1938 ODE No. 1938

$$\left\{ x''(t) = \frac{x(t)f'(r)}{r}, y''(t) = \frac{y(t)f'(r)}{r}, z''(t) = \frac{z(t)f'(r)}{r} \right\}$$

✓ **Mathematica** : cpu = 0.0066229 (sec), leaf count = 137

$$\left\{ \left\{ x(t) \rightarrow c_1 e^{-\frac{t\sqrt{f'(r)}}{\sqrt{r}}} + c_2 e^{\frac{t\sqrt{f'(r)}}{\sqrt{r}}}, y(t) \rightarrow c_3 e^{-\frac{t\sqrt{f'(r)}}{\sqrt{r}}} + c_4 e^{\frac{t\sqrt{f'(r)}}{\sqrt{r}}}, z(t) \rightarrow c_5 e^{-\frac{t\sqrt{f'(r)}}{\sqrt{r}}} + c_6 e^{\frac{t\sqrt{f'(r)}}{\sqrt{r}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.093 (sec), leaf count = 101

$$\left\{ \left\{ x(t) = -C5 e^{t\sqrt{\frac{d}{dr}F(r)}\frac{1}{\sqrt{r}}} + -C6 e^{-t\sqrt{\frac{d}{dr}F(r)}\frac{1}{\sqrt{r}}}, y(t) = -C3 e^{t\sqrt{\frac{d}{dr}F(r)}\frac{1}{\sqrt{r}}} + -C4 e^{-t\sqrt{\frac{d}{dr}F(r)}\frac{1}{\sqrt{r}}}, z(t) = -C1 e^{t\sqrt{\frac{d}{dr}F(r)}\frac{1}{\sqrt{r}}} \right\} \right\}$$

2.1939 ODE No. 1939

$\{(x(t)-y(t))(x(t)-z(t))x'(t)=f(t), (y(t)-x(t))(y(t)-z(t))y'(t)=f(t), (z(t)-x(t))(z(t)-y(t))z'(t)=f(t)\}$

✗ **Mathematica** : cpu = 0.0074859 (sec), leaf count = 0 , could not solve  
DSolve[{(x[t] - y[t])\*(x[t] - z[t])\*Derivative[1][x][t] == f[t], (-x[t] + y[t])\*(y[t] - z[t])\*Derivative[1][y][t] == f[t], (x[t] + z[t])\*(-y[t] + z[t])\*Derivative[1][z][t] == f[t]}, {x[t], y[t], z[t]}, t]  
✓ **Maple** : cpu = 1.106 (sec), leaf count = 899

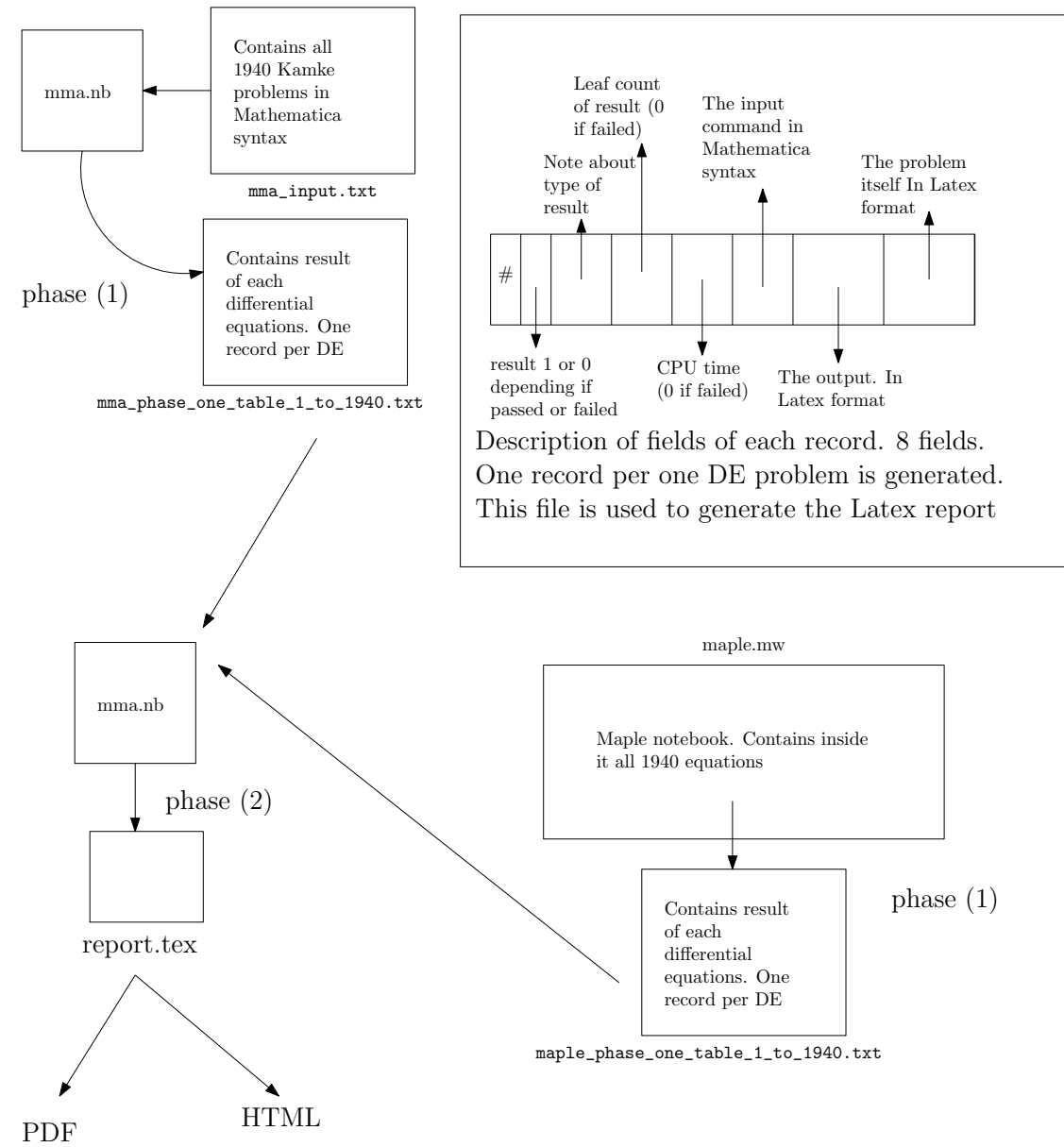
$$\left\{ \left[ \left\{ x(t) = \int 3 \frac{f(t)}{-C1^3 + 11664 - C2^2 - 23328 - C2 \int f(t) dt + 11664 (\int f(t) dt)^2} \right\} \right] \right\}$$

$\{x1'(t) \sin(x2(t)) = x4(t) \sin(x3(t)) + x5(t) \cos(x3(t)), x2'(t) = x4(t) \cos(x3(t)) - x5(t) \sin(x3(t)), x1'(t) \cos(x2(t)) = x4(t) \cos(x3(t)) + x5(t) \sin(x3(t))\}$

**X Mathematica** : cpu = 0.0067531 (sec), leaf count = 0 , could not solve  
DSolve[{Sin[x2[t]]\*Derivative[1][x1][t] == Sin[x3[t]]\*x4[t] + Cos[x3[t]]\*x5[t], Derivative[1][x2][t] == (a\*(1 - lambda)\*x5[t]) + Derivative[1][x4][t] == -(m\*Cos[x3[t]]\*Sin[x2[t]]), a\*(1 - lambda)\*x5[t] == m\*Ssin[x2[t]]\*Cos[x3[t]]}]

**X Maple** : cpu = 0. (sec), leaf count = 0 , could not solve  
dsolve({diff(x1(t),t)\*sin(x2(t)) = x4(t)\*sin(x3(t))+x5(t)\*cos(x3(t)), diff(x3(t),t)+diff(x1(t),t)\*cos(x2(t)) = x4(t)\*cos(x3(t)) - x5(t)\*sin(x3(t)), (1-lambda)\*a\*x5(t) = -m\*sin(x2(t))\*cos(x3(t)), diff(x5(t),t)+(1-lambda)\*a\*x4(t) = m\*sin(x2(t))\*cos(x3(t))})

3 Appendix



Kamke Differential equations build process

Nasser M. Abbasi (design.ipe)