

Kamke differential equations. Mathematica 12.3 and Maple 2021

Nasser M. Abbasi

May 24, 2021 Compiled on May 24, 2021 at 6:25am

Contents

1	Introduction and summary of results	2
2	Problems table lookup	5
2.1	ODE No. 1	87
2.2	ODE No. 2	87
2.3	ODE No. 3	88
2.4	ODE No. 4	89
2.5	ODE No. 5	89
2.6	ODE No. 6	90
2.7	ODE No. 7	90
2.8	ODE No. 8	91
2.9	ODE No. 9	92
2.10	ODE No. 10	93
2.11	ODE No. 11	93
2.12	ODE No. 12	94
2.13	ODE No. 13	95
2.14	ODE No. 14	96
2.15	ODE No. 15	98
2.16	ODE No. 16	99
2.17	ODE No. 17	101
2.18	ODE No. 18	102
2.19	ODE No. 19	103
2.20	ODE No. 20	103
2.21	ODE No. 21	105
2.22	ODE No. 22	106
2.23	ODE No. 23	108
2.24	ODE No. 24	109
2.25	ODE No. 25	111
2.26	ODE No. 26	112
2.27	ODE No. 27	113
2.28	ODE No. 28	115
2.29	ODE No. 29	116
2.30	ODE No. 30	117
2.31	ODE No. 31	121
2.32	ODE No. 32	122
2.33	ODE No. 33	123
2.34	ODE No. 34	123
2.35	ODE No. 35	124
2.36	ODE No. 36	126
2.37	ODE No. 37	128
2.38	ODE No. 38	128
2.39	ODE No. 39	129
2.40	ODE No. 40	129
2.41	ODE No. 41	130
2.42	ODE No. 42	130
2.43	ODE No. 43	130
2.44	ODE No. 44	131
2.45	ODE No. 45	132

2.46	ODE No. 46	132
2.47	ODE No. 47	132
2.48	ODE No. 48	133
2.49	ODE No. 49	133
2.50	ODE No. 50	133
2.51	ODE No. 51	133
2.52	ODE No. 52	133
2.53	ODE No. 53	134
2.54	ODE No. 54	134
2.55	ODE No. 55	134
2.56	ODE No. 56	135
2.57	ODE No. 57	135
2.58	ODE No. 58	135
2.59	ODE No. 59	136
2.60	ODE No. 60	136
2.61	ODE No. 61	137
2.62	ODE No. 62	137
2.63	ODE No. 63	139
2.64	ODE No. 64	139
2.65	ODE No. 65	140
2.66	ODE No. 66	140
2.67	ODE No. 67	140
2.68	ODE No. 68	140
2.69	ODE No. 69	141
2.70	ODE No. 70	141
2.71	ODE No. 71	141
2.72	ODE No. 72	142
2.73	ODE No. 73	142
2.74	ODE No. 74	142
2.75	ODE No. 75	142
2.76	ODE No. 76	143
2.77	ODE No. 77	145
2.78	ODE No. 78	146
2.79	ODE No. 79	148
2.80	ODE No. 80	148
2.81	ODE No. 81	148
2.82	ODE No. 82	149
2.83	ODE No. 83	149
2.84	ODE No. 84	149
2.85	ODE No. 85	150
2.86	ODE No. 86	150
2.87	ODE No. 87	150
2.88	ODE No. 88	151
2.89	ODE No. 89	152
2.90	ODE No. 90	153
2.91	ODE No. 91	154
2.92	ODE No. 92	154
2.93	ODE No. 93	155
2.94	ODE No. 94	155
2.95	ODE No. 95	156
2.96	ODE No. 96	157
2.97	ODE No. 97	158
2.98	ODE No. 98	159
2.99	ODE No. 99	159
2.100	ODE No. 100	159
2.101	ODE No. 101	161
2.102	ODE No. 102	162
2.103	ODE No. 103	163
2.104	ODE No. 104	165
2.105	ODE No. 105	166
2.106	ODE No. 106	166
2.107	ODE No. 107	166
2.108	ODE No. 108	167

2.109	ODE No. 109	168
2.110	ODE No. 110	169
2.111	ODE No. 111	170
2.112	ODE No. 112	171
2.113	ODE No. 113	171
2.114	ODE No. 114	172
2.115	ODE No. 115	173
2.116	ODE No. 116	174
2.117	ODE No. 117	175
2.118	ODE No. 118	175
2.119	ODE No. 119	175
2.120	ODE No. 120	175
2.121	ODE No. 121	175
2.122	ODE No. 122	176
2.123	ODE No. 123	176
2.124	ODE No. 124	176
2.125	ODE No. 125	176
2.126	ODE No. 126	177
2.127	ODE No. 127	177
2.128	ODE No. 128	177
2.129	ODE No. 129	177
2.130	ODE No. 130	178
2.131	ODE No. 131	178
2.132	ODE No. 132	178
2.133	ODE No. 133	178
2.134	ODE No. 134	179
2.135	ODE No. 135	179
2.136	ODE No. 136	179
2.137	ODE No. 137	179
2.138	ODE No. 138	179
2.139	ODE No. 139	180
2.140	ODE No. 140	180
2.141	ODE No. 141	180
2.142	ODE No. 142	180
2.143	ODE No. 143	181
2.144	ODE No. 144	181
2.145	ODE No. 145	181
2.146	ODE No. 146	182
2.147	ODE No. 147	182
2.148	ODE No. 148	182
2.149	ODE No. 149	183
2.150	ODE No. 150	183
2.151	ODE No. 151	183
2.152	ODE No. 152	183
2.153	ODE No. 153	184
2.154	ODE No. 154	184
2.155	ODE No. 155	184
2.156	ODE No. 156	184
2.157	ODE No. 157	185
2.158	ODE No. 158	185
2.159	ODE No. 159	185
2.160	ODE No. 160	185
2.161	ODE No. 161	186
2.162	ODE No. 162	186
2.163	ODE No. 163	186
2.164	ODE No. 164	186
2.165	ODE No. 165	187
2.166	ODE No. 166	187
2.167	ODE No. 167	187
2.168	ODE No. 168	187
2.169	ODE No. 169	188
2.170	ODE No. 170	188
2.171	ODE No. 171	188

2.172	ODE No. 172	189
2.173	ODE No. 173	189
2.174	ODE No. 174	189
2.175	ODE No. 175	189
2.176	ODE No. 176	190
2.177	ODE No. 177	190
2.178	ODE No. 178	190
2.179	ODE No. 179	190
2.180	ODE No. 180	191
2.181	ODE No. 181	191
2.182	ODE No. 182	191
2.183	ODE No. 183	192
2.184	ODE No. 184	192
2.185	ODE No. 185	193
2.186	ODE No. 186	193
2.187	ODE No. 187	193
2.188	ODE No. 188	193
2.189	ODE No. 189	194
2.190	ODE No. 190	194
2.191	ODE No. 191	194
2.192	ODE No. 192	194
2.193	ODE No. 193	195
2.194	ODE No. 194	195
2.195	ODE No. 195	195
2.196	ODE No. 196	195
2.197	ODE No. 197	196
2.198	ODE No. 198	196
2.199	ODE No. 199	196
2.200	ODE No. 200	196
2.201	ODE No. 201	197
2.202	ODE No. 202	197
2.203	ODE No. 203	197
2.204	ODE No. 204	197
2.205	ODE No. 205	197
2.206	ODE No. 206	198
2.207	ODE No. 207	198
2.208	ODE No. 208	198
2.209	ODE No. 209	198
2.210	ODE No. 210	199
2.211	ODE No. 211	199
2.212	ODE No. 212	199
2.213	ODE No. 213	199
2.214	ODE No. 214	200
2.215	ODE No. 215	200
2.216	ODE No. 216	200
2.217	ODE No. 217	200
2.218	ODE No. 218	201
2.219	ODE No. 219	201
2.220	ODE No. 220	201
2.221	ODE No. 221	201
2.222	ODE No. 222	202
2.223	ODE No. 223	202
2.224	ODE No. 224	202
2.225	ODE No. 225	202
2.226	ODE No. 226	203
2.227	ODE No. 227	203
2.228	ODE No. 228	203
2.229	ODE No. 229	204
2.230	ODE No. 230	204
2.231	ODE No. 231	204
2.232	ODE No. 232	205
2.233	ODE No. 233	205
2.234	ODE No. 234	205

2.235	ODE No. 235	205
2.236	ODE No. 236	205
2.237	ODE No. 237	206
2.238	ODE No. 238	206
2.239	ODE No. 239	206
2.240	ODE No. 240	206
2.241	ODE No. 241	207
2.242	ODE No. 242	207
2.243	ODE No. 243	207
2.244	ODE No. 244	207
2.245	ODE No. 245	208
2.246	ODE No. 246	208
2.247	ODE No. 247	208
2.248	ODE No. 248	209
2.249	ODE No. 249	209
2.250	ODE No. 250	209
2.251	ODE No. 251	210
2.252	ODE No. 252	210
2.253	ODE No. 253	210
2.254	ODE No. 254	210
2.255	ODE No. 255	211
2.256	ODE No. 256	211
2.257	ODE No. 257	211
2.258	ODE No. 258	211
2.259	ODE No. 259	212
2.260	ODE No. 260	212
2.261	ODE No. 261	212
2.262	ODE No. 262	212
2.263	ODE No. 263	213
2.264	ODE No. 264	213
2.265	ODE No. 265	213
2.266	ODE No. 266	213
2.267	ODE No. 267	214
2.268	ODE No. 268	214
2.269	ODE No. 269	214
2.270	ODE No. 270	215
2.271	ODE No. 271	215
2.272	ODE No. 272	215
2.273	ODE No. 273	216
2.274	ODE No. 274	216
2.275	ODE No. 275	216
2.276	ODE No. 276	217
2.277	ODE No. 277	217
2.278	ODE No. 278	217
2.279	ODE No. 279	217
2.280	ODE No. 280	218
2.281	ODE No. 281	218
2.282	ODE No. 282	218
2.283	ODE No. 283	218
2.284	ODE No. 284	219
2.285	ODE No. 285	219
2.286	ODE No. 286	219
2.287	ODE No. 287	219
2.288	ODE No. 288	220
2.289	ODE No. 289	220
2.290	ODE No. 290	220
2.291	ODE No. 291	221
2.292	ODE No. 292	221
2.293	ODE No. 293	221
2.294	ODE No. 294	221
2.295	ODE No. 295	222
2.296	ODE No. 296	222
2.297	ODE No. 297	222

2.298	ODE No. 298	222
2.299	ODE No. 299	223
2.300	ODE No. 300	223
2.301	ODE No. 301	223
2.302	ODE No. 302	223
2.303	ODE No. 303	224
2.304	ODE No. 304	224
2.305	ODE No. 305	224
2.306	ODE No. 306	225
2.307	ODE No. 307	225
2.308	ODE No. 308	225
2.309	ODE No. 309	225
2.310	ODE No. 310	226
2.311	ODE No. 311	226
2.312	ODE No. 312	226
2.313	ODE No. 313	227
2.314	ODE No. 314	227
2.315	ODE No. 315	227
2.316	ODE No. 316	228
2.317	ODE No. 317	228
2.318	ODE No. 318	228
2.319	ODE No. 319	229
2.320	ODE No. 320	229
2.321	ODE No. 321	229
2.322	ODE No. 322	229
2.323	ODE No. 323	230
2.324	ODE No. 324	230
2.325	ODE No. 325	230
2.326	ODE No. 326	231
2.327	ODE No. 327	231
2.328	ODE No. 328	231
2.329	ODE No. 329	231
2.330	ODE No. 330	232
2.331	ODE No. 331	232
2.332	ODE No. 332	232
2.333	ODE No. 333	232
2.334	ODE No. 334	233
2.335	ODE No. 335	233
2.336	ODE No. 336	233
2.337	ODE No. 337	233
2.338	ODE No. 338	234
2.339	ODE No. 339	234
2.340	ODE No. 340	234
2.341	ODE No. 341	234
2.342	ODE No. 342	234
2.343	ODE No. 343	235
2.344	ODE No. 344	235
2.345	ODE No. 345	235
2.346	ODE No. 346	235
2.347	ODE No. 347	236
2.348	ODE No. 348	236
2.349	ODE No. 349	236
2.350	ODE No. 350	236
2.351	ODE No. 351	237
2.352	ODE No. 352	237
2.353	ODE No. 353	237
2.354	ODE No. 354	237
2.355	ODE No. 355	238
2.356	ODE No. 356	238
2.357	ODE No. 357	238
2.358	ODE No. 358	238
2.359	ODE No. 359	238
2.360	ODE No. 360	239

2.361	ODE No. 361	239
2.362	ODE No. 362	239
2.363	ODE No. 363	239
2.364	ODE No. 364	240
2.365	ODE No. 365	240
2.366	ODE No. 366	240
2.367	ODE No. 367	240
2.368	ODE No. 368	241
2.369	ODE No. 369	241
2.370	ODE No. 370	241
2.371	ODE No. 371	241
2.372	ODE No. 372	241
2.373	ODE No. 373	242
2.374	ODE No. 374	242
2.375	ODE No. 375	242
2.376	ODE No. 376	242
2.377	ODE No. 377	243
2.378	ODE No. 378	243
2.379	ODE No. 379	243
2.380	ODE No. 380	244
2.381	ODE No. 381	244
2.382	ODE No. 382	244
2.383	ODE No. 383	245
2.384	ODE No. 384	245
2.385	ODE No. 385	245
2.386	ODE No. 386	245
2.387	ODE No. 387	246
2.388	ODE No. 388	246
2.389	ODE No. 389	246
2.390	ODE No. 390	246
2.391	ODE No. 391	247
2.392	ODE No. 392	247
2.393	ODE No. 393	247
2.394	ODE No. 394	247
2.395	ODE No. 395	248
2.396	ODE No. 396	248
2.397	ODE No. 397	248
2.398	ODE No. 398	249
2.399	ODE No. 399	249
2.400	ODE No. 400	249
2.401	ODE No. 401	249
2.402	ODE No. 402	250
2.403	ODE No. 403	250
2.404	ODE No. 404	250
2.405	ODE No. 405	251
2.406	ODE No. 406	251
2.407	ODE No. 407	251
2.408	ODE No. 408	252
2.409	ODE No. 409	252
2.410	ODE No. 410	252
2.411	ODE No. 411	252
2.412	ODE No. 412	253
2.413	ODE No. 413	253
2.414	ODE No. 414	253
2.415	ODE No. 415	254
2.416	ODE No. 416	254
2.417	ODE No. 417	254
2.418	ODE No. 418	254
2.419	ODE No. 419	255
2.420	ODE No. 420	255
2.421	ODE No. 421	255
2.422	ODE No. 422	256
2.423	ODE No. 423	256

2.424	ODE No. 424	256
2.425	ODE No. 425	256
2.426	ODE No. 426	257
2.427	ODE No. 427	257
2.428	ODE No. 428	257
2.429	ODE No. 429	257
2.430	ODE No. 430	258
2.431	ODE No. 431	258
2.432	ODE No. 432	258
2.433	ODE No. 433	258
2.434	ODE No. 434	259
2.435	ODE No. 435	259
2.436	ODE No. 436	259
2.437	ODE No. 437	259
2.438	ODE No. 438	259
2.439	ODE No. 439	260
2.440	ODE No. 440	260
2.441	ODE No. 441	260
2.442	ODE No. 442	260
2.443	ODE No. 443	260
2.444	ODE No. 444	261
2.445	ODE No. 445	261
2.446	ODE No. 446	261
2.447	ODE No. 447	262
2.448	ODE No. 448	262
2.449	ODE No. 449	262
2.450	ODE No. 450	262
2.451	ODE No. 451	262
2.452	ODE No. 452	263
2.453	ODE No. 453	263
2.454	ODE No. 454	263
2.455	ODE No. 455	264
2.456	ODE No. 456	264
2.457	ODE No. 457	264
2.458	ODE No. 458	264
2.459	ODE No. 459	265
2.460	ODE No. 460	265
2.461	ODE No. 461	265
2.462	ODE No. 462	265
2.463	ODE No. 463	265
2.464	ODE No. 464	266
2.465	ODE No. 465	266
2.466	ODE No. 466	266
2.467	ODE No. 467	266
2.468	ODE No. 468	267
2.469	ODE No. 469	267
2.470	ODE No. 470	267
2.471	ODE No. 471	268
2.472	ODE No. 472	268
2.473	ODE No. 473	268
2.474	ODE No. 474	268
2.475	ODE No. 475	269
2.476	ODE No. 476	269
2.477	ODE No. 477	269
2.478	ODE No. 478	270
2.479	ODE No. 479	270
2.480	ODE No. 480	270
2.481	ODE No. 481	270
2.482	ODE No. 482	271
2.483	ODE No. 483	271
2.484	ODE No. 484	271
2.485	ODE No. 485	271
2.486	ODE No. 486	271

2.487	ODE No. 487	272
2.488	ODE No. 488	272
2.489	ODE No. 489	272
2.490	ODE No. 490	272
2.491	ODE No. 491	273
2.492	ODE No. 492	273
2.493	ODE No. 493	273
2.494	ODE No. 494	273
2.495	ODE No. 495	274
2.496	ODE No. 496	274
2.497	ODE No. 497	274
2.498	ODE No. 498	274
2.499	ODE No. 499	275
2.500	ODE No. 500	275
2.501	ODE No. 501	275
2.502	ODE No. 502	275
2.503	ODE No. 503	276
2.504	ODE No. 504	276
2.505	ODE No. 505	276
2.506	ODE No. 506	276
2.507	ODE No. 507	277
2.508	ODE No. 508	277
2.509	ODE No. 509	277
2.510	ODE No. 510	277
2.511	ODE No. 511	278
2.512	ODE No. 512	278
2.513	ODE No. 513	278
2.514	ODE No. 514	279
2.515	ODE No. 515	279
2.516	ODE No. 516	280
2.517	ODE No. 517	280
2.518	ODE No. 518	280
2.519	ODE No. 519	281
2.520	ODE No. 520	281
2.521	ODE No. 521	281
2.522	ODE No. 522	282
2.523	ODE No. 523	282
2.524	ODE No. 524	282
2.525	ODE No. 525	282
2.526	ODE No. 526	283
2.527	ODE No. 527	283
2.528	ODE No. 528	283
2.529	ODE No. 529	283
2.530	ODE No. 530	284
2.531	ODE No. 531	284
2.532	ODE No. 532	284
2.533	ODE No. 533	285
2.534	ODE No. 534	285
2.535	ODE No. 535	285
2.536	ODE No. 536	285
2.537	ODE No. 537	286
2.538	ODE No. 538	286
2.539	ODE No. 539	286
2.540	ODE No. 540	286
2.541	ODE No. 541	287
2.542	ODE No. 542	287
2.543	ODE No. 543	287
2.544	ODE No. 544	287
2.545	ODE No. 545	288
2.546	ODE No. 546	288
2.547	ODE No. 547	288
2.548	ODE No. 548	289
2.549	ODE No. 549	289

2.550	ODE No. 550	289
2.551	ODE No. 551	290
2.552	ODE No. 552	290
2.553	ODE No. 553	290
2.554	ODE No. 554	290
2.555	ODE No. 555	291
2.556	ODE No. 556	291
2.557	ODE No. 557	291
2.558	ODE No. 558	291
2.559	ODE No. 559	292
2.560	ODE No. 560	292
2.561	ODE No. 561	292
2.562	ODE No. 562	293
2.563	ODE No. 563	293
2.564	ODE No. 564	293
2.565	ODE No. 565	293
2.566	ODE No. 566	294
2.567	ODE No. 567	294
2.568	ODE No. 568	294
2.569	ODE No. 569	294
2.570	ODE No. 570	295
2.571	ODE No. 571	295
2.572	ODE No. 572	295
2.573	ODE No. 573	295
2.574	ODE No. 574	296
2.575	ODE No. 575	296
2.576	ODE No. 576	296
2.577	ODE No. 577	296
2.578	ODE No. 578	297
2.579	ODE No. 579	297
2.580	ODE No. 580	297
2.581	ODE No. 581	297
2.582	ODE No. 582	298
2.583	ODE No. 583	298
2.584	ODE No. 584	299
2.585	ODE No. 585	299
2.586	ODE No. 586	299
2.587	ODE No. 587	300
2.588	ODE No. 588	300
2.589	ODE No. 589	300
2.590	ODE No. 590	301
2.591	ODE No. 591	301
2.592	ODE No. 592	301
2.593	ODE No. 593	302
2.594	ODE No. 594	302
2.595	ODE No. 595	302
2.596	ODE No. 596	303
2.597	ODE No. 597	303
2.598	ODE No. 598	303
2.599	ODE No. 599	303
2.600	ODE No. 600	304
2.601	ODE No. 601	304
2.602	ODE No. 602	304
2.603	ODE No. 603	305
2.604	ODE No. 604	305
2.605	ODE No. 605	305
2.606	ODE No. 606	306
2.607	ODE No. 607	306
2.608	ODE No. 608	306
2.609	ODE No. 609	307
2.610	ODE No. 610	307
2.611	ODE No. 611	307
2.612	ODE No. 612	307

2.613	ODE No. 613	308
2.614	ODE No. 614	308
2.615	ODE No. 615	308
2.616	ODE No. 616	309
2.617	ODE No. 617	309
2.618	ODE No. 618	309
2.619	ODE No. 619	310
2.620	ODE No. 620	310
2.621	ODE No. 621	310
2.622	ODE No. 622	311
2.623	ODE No. 623	311
2.624	ODE No. 624	311
2.625	ODE No. 625	312
2.626	ODE No. 626	312
2.627	ODE No. 627	312
2.628	ODE No. 628	312
2.629	ODE No. 629	313
2.630	ODE No. 630	313
2.631	ODE No. 631	313
2.632	ODE No. 632	313
2.633	ODE No. 633	314
2.634	ODE No. 634	314
2.635	ODE No. 635	314
2.636	ODE No. 636	314
2.637	ODE No. 637	315
2.638	ODE No. 638	315
2.639	ODE No. 639	315
2.640	ODE No. 640	315
2.641	ODE No. 641	316
2.642	ODE No. 642	316
2.643	ODE No. 643	316
2.644	ODE No. 644	316
2.645	ODE No. 645	317
2.646	ODE No. 646	317
2.647	ODE No. 647	317
2.648	ODE No. 648	317
2.649	ODE No. 649	318
2.650	ODE No. 650	318
2.651	ODE No. 651	318
2.652	ODE No. 652	318
2.653	ODE No. 653	319
2.654	ODE No. 654	319
2.655	ODE No. 655	319
2.656	ODE No. 656	319
2.657	ODE No. 657	320
2.658	ODE No. 658	320
2.659	ODE No. 659	320
2.660	ODE No. 660	320
2.661	ODE No. 661	321
2.662	ODE No. 662	321
2.663	ODE No. 663	321
2.664	ODE No. 664	321
2.665	ODE No. 665	322
2.666	ODE No. 666	322
2.667	ODE No. 667	322
2.668	ODE No. 668	322
2.669	ODE No. 669	323
2.670	ODE No. 670	323
2.671	ODE No. 671	323
2.672	ODE No. 672	324
2.673	ODE No. 673	324
2.674	ODE No. 674	324
2.675	ODE No. 675	324

2.676	ODE No. 676	325
2.677	ODE No. 677	325
2.678	ODE No. 678	325
2.679	ODE No. 679	325
2.680	ODE No. 680	326
2.681	ODE No. 681	326
2.682	ODE No. 682	326
2.683	ODE No. 683	326
2.684	ODE No. 684	327
2.685	ODE No. 685	327
2.686	ODE No. 686	327
2.687	ODE No. 687	328
2.688	ODE No. 688	328
2.689	ODE No. 689	328
2.690	ODE No. 690	328
2.691	ODE No. 691	329
2.692	ODE No. 692	329
2.693	ODE No. 693	329
2.694	ODE No. 694	329
2.695	ODE No. 695	330
2.696	ODE No. 696	330
2.697	ODE No. 697	330
2.698	ODE No. 698	330
2.699	ODE No. 699	331
2.700	ODE No. 700	331
2.701	ODE No. 701	331
2.702	ODE No. 702	331
2.703	ODE No. 703	332
2.704	ODE No. 704	332
2.705	ODE No. 705	332
2.706	ODE No. 706	333
2.707	ODE No. 707	333
2.708	ODE No. 708	333
2.709	ODE No. 709	333
2.710	ODE No. 710	334
2.711	ODE No. 711	334
2.712	ODE No. 712	334
2.713	ODE No. 713	334
2.714	ODE No. 714	335
2.715	ODE No. 715	335
2.716	ODE No. 716	335
2.717	ODE No. 717	336
2.718	ODE No. 718	336
2.719	ODE No. 719	336
2.720	ODE No. 720	336
2.721	ODE No. 721	337
2.722	ODE No. 722	337
2.723	ODE No. 723	337
2.724	ODE No. 724	338
2.725	ODE No. 725	338
2.726	ODE No. 726	338
2.727	ODE No. 727	338
2.728	ODE No. 728	339
2.729	ODE No. 729	339
2.730	ODE No. 730	339
2.731	ODE No. 731	340
2.732	ODE No. 732	340
2.733	ODE No. 733	340
2.734	ODE No. 734	340
2.735	ODE No. 735	341
2.736	ODE No. 736	341
2.737	ODE No. 737	341
2.738	ODE No. 738	341

2.739	ODE No. 739	342
2.740	ODE No. 740	342
2.741	ODE No. 741	342
2.742	ODE No. 742	343
2.743	ODE No. 743	343
2.744	ODE No. 744	343
2.745	ODE No. 745	344
2.746	ODE No. 746	344
2.747	ODE No. 747	344
2.748	ODE No. 748	345
2.749	ODE No. 749	345
2.750	ODE No. 750	345
2.751	ODE No. 751	346
2.752	ODE No. 752	346
2.753	ODE No. 753	346
2.754	ODE No. 754	346
2.755	ODE No. 755	347
2.756	ODE No. 756	347
2.757	ODE No. 757	347
2.758	ODE No. 758	347
2.759	ODE No. 759	348
2.760	ODE No. 760	348
2.761	ODE No. 761	348
2.762	ODE No. 762	349
2.763	ODE No. 763	349
2.764	ODE No. 764	349
2.765	ODE No. 765	349
2.766	ODE No. 766	350
2.767	ODE No. 767	350
2.768	ODE No. 768	350
2.769	ODE No. 769	351
2.770	ODE No. 770	351
2.771	ODE No. 771	351
2.772	ODE No. 772	352
2.773	ODE No. 773	352
2.774	ODE No. 774	352
2.775	ODE No. 775	352
2.776	ODE No. 776	353
2.777	ODE No. 777	353
2.778	ODE No. 778	353
2.779	ODE No. 779	354
2.780	ODE No. 780	354
2.781	ODE No. 781	354
2.782	ODE No. 782	355
2.783	ODE No. 783	355
2.784	ODE No. 784	355
2.785	ODE No. 785	356
2.786	ODE No. 786	356
2.787	ODE No. 787	356
2.788	ODE No. 788	357
2.789	ODE No. 789	357
2.790	ODE No. 790	357
2.791	ODE No. 791	357
2.792	ODE No. 792	358
2.793	ODE No. 793	358
2.794	ODE No. 794	359
2.795	ODE No. 795	359
2.796	ODE No. 796	359
2.797	ODE No. 797	360
2.798	ODE No. 798	360
2.799	ODE No. 799	360
2.800	ODE No. 800	361
2.801	ODE No. 801	361

2.802	ODE No. 802	361
2.803	ODE No. 803	362
2.804	ODE No. 804	362
2.805	ODE No. 805	362
2.806	ODE No. 806	362
2.807	ODE No. 807	363
2.808	ODE No. 808	363
2.809	ODE No. 809	363
2.810	ODE No. 810	363
2.811	ODE No. 811	364
2.812	ODE No. 812	364
2.813	ODE No. 813	364
2.814	ODE No. 814	365
2.815	ODE No. 815	365
2.816	ODE No. 816	365
2.817	ODE No. 817	366
2.818	ODE No. 818	366
2.819	ODE No. 819	366
2.820	ODE No. 820	366
2.821	ODE No. 821	367
2.822	ODE No. 822	367
2.823	ODE No. 823	367
2.824	ODE No. 824	367
2.825	ODE No. 825	368
2.826	ODE No. 826	368
2.827	ODE No. 827	368
2.828	ODE No. 828	369
2.829	ODE No. 829	369
2.830	ODE No. 830	369
2.831	ODE No. 831	370
2.832	ODE No. 832	370
2.833	ODE No. 833	370
2.834	ODE No. 834	370
2.835	ODE No. 835	371
2.836	ODE No. 836	371
2.837	ODE No. 837	371
2.838	ODE No. 838	372
2.839	ODE No. 839	372
2.840	ODE No. 840	372
2.841	ODE No. 841	372
2.842	ODE No. 842	373
2.843	ODE No. 843	373
2.844	ODE No. 844	373
2.845	ODE No. 845	374
2.846	ODE No. 846	374
2.847	ODE No. 847	374
2.848	ODE No. 848	374
2.849	ODE No. 849	375
2.850	ODE No. 850	375
2.851	ODE No. 851	375
2.852	ODE No. 852	376
2.853	ODE No. 853	376
2.854	ODE No. 854	376
2.855	ODE No. 855	376
2.856	ODE No. 856	377
2.857	ODE No. 857	377
2.858	ODE No. 858	377
2.859	ODE No. 859	378
2.860	ODE No. 860	378
2.861	ODE No. 861	378
2.862	ODE No. 862	378
2.863	ODE No. 863	379
2.864	ODE No. 864	379

2.865	ODE No. 865	379
2.866	ODE No. 866	380
2.867	ODE No. 867	380
2.868	ODE No. 868	380
2.869	ODE No. 869	380
2.870	ODE No. 870	381
2.871	ODE No. 871	381
2.872	ODE No. 872	381
2.873	ODE No. 873	381
2.874	ODE No. 874	382
2.875	ODE No. 875	382
2.876	ODE No. 876	382
2.877	ODE No. 877	383
2.878	ODE No. 878	383
2.879	ODE No. 879	383
2.880	ODE No. 880	383
2.881	ODE No. 881	384
2.882	ODE No. 882	384
2.883	ODE No. 883	384
2.884	ODE No. 884	384
2.885	ODE No. 885	385
2.886	ODE No. 886	385
2.887	ODE No. 887	385
2.888	ODE No. 888	386
2.889	ODE No. 889	386
2.890	ODE No. 890	386
2.891	ODE No. 891	386
2.892	ODE No. 892	387
2.893	ODE No. 893	387
2.894	ODE No. 894	387
2.895	ODE No. 895	388
2.896	ODE No. 896	388
2.897	ODE No. 897	388
2.898	ODE No. 898	389
2.899	ODE No. 899	389
2.900	ODE No. 900	389
2.901	ODE No. 901	390
2.902	ODE No. 902	390
2.903	ODE No. 903	390
2.904	ODE No. 904	390
2.905	ODE No. 905	391
2.906	ODE No. 906	391
2.907	ODE No. 907	391
2.908	ODE No. 908	391
2.909	ODE No. 909	392
2.910	ODE No. 910	392
2.911	ODE No. 911	392
2.912	ODE No. 912	393
2.913	ODE No. 913	393
2.914	ODE No. 914	393
2.915	ODE No. 915	393
2.916	ODE No. 916	394
2.917	ODE No. 917	394
2.918	ODE No. 918	394
2.919	ODE No. 919	395
2.920	ODE No. 920	395
2.921	ODE No. 921	395
2.922	ODE No. 922	395
2.923	ODE No. 923	396
2.924	ODE No. 924	396
2.925	ODE No. 925	396
2.926	ODE No. 926	396
2.927	ODE No. 927	397

2.928	ODE No. 928	397
2.929	ODE No. 929	397
2.930	ODE No. 930	398
2.931	ODE No. 931	398
2.932	ODE No. 932	398
2.933	ODE No. 933	399
2.934	ODE No. 934	399
2.935	ODE No. 935	399
2.936	ODE No. 936	399
2.937	ODE No. 937	400
2.938	ODE No. 938	400
2.939	ODE No. 939	400
2.940	ODE No. 940	401
2.941	ODE No. 941	401
2.942	ODE No. 942	401
2.943	ODE No. 943	402
2.944	ODE No. 944	402
2.945	ODE No. 945	402
2.946	ODE No. 946	402
2.947	ODE No. 947	403
2.948	ODE No. 948	403
2.949	ODE No. 949	403
2.950	ODE No. 950	403
2.951	ODE No. 951	404
2.952	ODE No. 952	404
2.953	ODE No. 953	404
2.954	ODE No. 954	404
2.955	ODE No. 955	405
2.956	ODE No. 956	405
2.957	ODE No. 957	405
2.958	ODE No. 958	406
2.959	ODE No. 959	406
2.960	ODE No. 960	406
2.961	ODE No. 961	407
2.962	ODE No. 962	407
2.963	ODE No. 963	407
2.964	ODE No. 964	407
2.965	ODE No. 965	408
2.966	ODE No. 966	408
2.967	ODE No. 967	408
2.968	ODE No. 968	409
2.969	ODE No. 969	409
2.970	ODE No. 970	409
2.971	ODE No. 971	409
2.972	ODE No. 972	410
2.973	ODE No. 973	410
2.974	ODE No. 974	411
2.975	ODE No. 975	411
2.976	ODE No. 976	411
2.977	ODE No. 977	411
2.978	ODE No. 978	412
2.979	ODE No. 979	412
2.980	ODE No. 980	412
2.981	ODE No. 981	413
2.982	ODE No. 982	413
2.983	ODE No. 983	413
2.984	ODE No. 984	414
2.985	ODE No. 985	414
2.986	ODE No. 986	414
2.987	ODE No. 987	415
2.988	ODE No. 988	415
2.989	ODE No. 989	415
2.990	ODE No. 990	415

2.991	ODE No. 991	416
2.992	ODE No. 992	416
2.993	ODE No. 993	416
2.994	ODE No. 994	417
2.995	ODE No. 995	417
2.996	ODE No. 996	417
2.997	ODE No. 997	417
2.998	ODE No. 998	418
2.999	ODE No. 999	418
2.1000	ODE No. 1000	418
2.1001	ODE No. 1001	418
2.1002	ODE No. 1002	419
2.1003	ODE No. 1003	419
2.1004	ODE No. 1004	422
2.1005	ODE No. 1005	423
2.1006	ODE No. 1006	425
2.1007	ODE No. 1007	426
2.1008	ODE No. 1008	427
2.1009	ODE No. 1009	427
2.1010	ODE No. 1010	428
2.1011	ODE No. 1011	431
2.1012	ODE No. 1012	433
2.1013	ODE No. 1013	433
2.1014	ODE No. 1014	433
2.1015	ODE No. 1015	434
2.1016	ODE No. 1016	434
2.1017	ODE No. 1017	434
2.1018	ODE No. 1018	434
2.1019	ODE No. 1019	435
2.1020	ODE No. 1020	435
2.1021	ODE No. 1021	435
2.1022	ODE No. 1022	435
2.1023	ODE No. 1023	435
2.1024	ODE No. 1024	436
2.1025	ODE No. 1025	436
2.1026	ODE No. 1026	436
2.1027	ODE No. 1027	436
2.1028	ODE No. 1028	437
2.1029	ODE No. 1029	437
2.1030	ODE No. 1030	437
2.1031	ODE No. 1031	437
2.1032	ODE No. 1032	437
2.1033	ODE No. 1033	438
2.1034	ODE No. 1034	439
2.1035	ODE No. 1035	440
2.1036	ODE No. 1036	440
2.1037	ODE No. 1037	440
2.1038	ODE No. 1038	440
2.1039	ODE No. 1039	441
2.1040	ODE No. 1040	441
2.1041	ODE No. 1041	441
2.1042	ODE No. 1042	441
2.1043	ODE No. 1043	442
2.1044	ODE No. 1044	443
2.1045	ODE No. 1045	443
2.1046	ODE No. 1046	444
2.1047	ODE No. 1047	444
2.1048	ODE No. 1048	445
2.1049	ODE No. 1049	446
2.1050	ODE No. 1050	446
2.1051	ODE No. 1051	446
2.1052	ODE No. 1052	446
2.1053	ODE No. 1053	447

2.1054	ODE No. 1054	447
2.1055	ODE No. 1055	447
2.1056	ODE No. 1056	447
2.1057	ODE No. 1057	448
2.1058	ODE No. 1058	448
2.1059	ODE No. 1059	448
2.1060	ODE No. 1060	448
2.1061	ODE No. 1061	449
2.1062	ODE No. 1062	449
2.1063	ODE No. 1063	449
2.1064	ODE No. 1064	449
2.1065	ODE No. 1065	450
2.1066	ODE No. 1066	450
2.1067	ODE No. 1067	450
2.1068	ODE No. 1068	450
2.1069	ODE No. 1069	450
2.1070	ODE No. 1070	451
2.1071	ODE No. 1071	451
2.1072	ODE No. 1072	451
2.1073	ODE No. 1073	451
2.1074	ODE No. 1074	452
2.1075	ODE No. 1075	452
2.1076	ODE No. 1076	452
2.1077	ODE No. 1077	452
2.1078	ODE No. 1078	452
2.1079	ODE No. 1079	453
2.1080	ODE No. 1080	453
2.1081	ODE No. 1081	453
2.1082	ODE No. 1082	453
2.1083	ODE No. 1083	454
2.1084	ODE No. 1084	454
2.1085	ODE No. 1085	454
2.1086	ODE No. 1086	454
2.1087	ODE No. 1087	455
2.1088	ODE No. 1088	455
2.1089	ODE No. 1089	455
2.1090	ODE No. 1090	455
2.1091	ODE No. 1091	456
2.1092	ODE No. 1092	456
2.1093	ODE No. 1093	456
2.1094	ODE No. 1094	456
2.1095	ODE No. 1095	456
2.1096	ODE No. 1096	457
2.1097	ODE No. 1097	457
2.1098	ODE No. 1098	457
2.1099	ODE No. 1099	457
2.1100	ODE No. 1100	457
2.1101	ODE No. 1101	462
2.1102	ODE No. 1102	465
2.1103	ODE No. 1103	467
2.1104	ODE No. 1104	468
2.1105	ODE No. 1105	468
2.1106	ODE No. 1106	468
2.1107	ODE No. 1107	468
2.1108	ODE No. 1108	468
2.1109	ODE No. 1109	469
2.1110	ODE No. 1110	469
2.1111	ODE No. 1111	469
2.1112	ODE No. 1112	470
2.1113	ODE No. 1113	470
2.1114	ODE No. 1114	471
2.1115	ODE No. 1115	471
2.1116	ODE No. 1116	471

2.1117	ODE No. 1117	471
2.1118	ODE No. 1118	471
2.1119	ODE No. 1119	472
2.1120	ODE No. 1120	472
2.1121	ODE No. 1121	472
2.1122	ODE No. 1122	472
2.1123	ODE No. 1123	473
2.1124	ODE No. 1124	473
2.1125	ODE No. 1125	473
2.1126	ODE No. 1126	473
2.1127	ODE No. 1127	474
2.1128	ODE No. 1128	474
2.1129	ODE No. 1129	474
2.1130	ODE No. 1130	474
2.1131	ODE No. 1131	474
2.1132	ODE No. 1132	475
2.1133	ODE No. 1133	475
2.1134	ODE No. 1134	475
2.1135	ODE No. 1135	475
2.1136	ODE No. 1136	476
2.1137	ODE No. 1137	476
2.1138	ODE No. 1138	476
2.1139	ODE No. 1139	476
2.1140	ODE No. 1140	476
2.1141	ODE No. 1141	477
2.1142	ODE No. 1142	477
2.1143	ODE No. 1143	477
2.1144	ODE No. 1144	477
2.1145	ODE No. 1145	478
2.1146	ODE No. 1146	478
2.1147	ODE No. 1147	478
2.1148	ODE No. 1148	478
2.1149	ODE No. 1149	479
2.1150	ODE No. 1150	479
2.1151	ODE No. 1151	479
2.1152	ODE No. 1152	479
2.1153	ODE No. 1153	480
2.1154	ODE No. 1154	480
2.1155	ODE No. 1155	480
2.1156	ODE No. 1156	480
2.1157	ODE No. 1157	481
2.1158	ODE No. 1158	481
2.1159	ODE No. 1159	481
2.1160	ODE No. 1160	481
2.1161	ODE No. 1161	481
2.1162	ODE No. 1162	482
2.1163	ODE No. 1163	482
2.1164	ODE No. 1164	482
2.1165	ODE No. 1165	482
2.1166	ODE No. 1166	483
2.1167	ODE No. 1167	483
2.1168	ODE No. 1168	483
2.1169	ODE No. 1169	483
2.1170	ODE No. 1170	484
2.1171	ODE No. 1171	484
2.1172	ODE No. 1172	484
2.1173	ODE No. 1173	484
2.1174	ODE No. 1174	485
2.1175	ODE No. 1175	485
2.1176	ODE No. 1176	485
2.1177	ODE No. 1177	485
2.1178	ODE No. 1178	486
2.1179	ODE No. 1179	486

2.1180	ODE No. 1180	486
2.1181	ODE No. 1181	486
2.1182	ODE No. 1182	487
2.1183	ODE No. 1183	487
2.1184	ODE No. 1184	487
2.1185	ODE No. 1185	487
2.1186	ODE No. 1186	488
2.1187	ODE No. 1187	488
2.1188	ODE No. 1188	488
2.1189	ODE No. 1189	488
2.1190	ODE No. 1190	489
2.1191	ODE No. 1191	489
2.1192	ODE No. 1192	489
2.1193	ODE No. 1193	489
2.1194	ODE No. 1194	490
2.1195	ODE No. 1195	490
2.1196	ODE No. 1196	490
2.1197	ODE No. 1197	490
2.1198	ODE No. 1198	491
2.1199	ODE No. 1199	491
2.1200	ODE No. 1200	491
2.1201	ODE No. 1201	491
2.1202	ODE No. 1202	492
2.1203	ODE No. 1203	492
2.1204	ODE No. 1204	492
2.1205	ODE No. 1205	492
2.1206	ODE No. 1206	493
2.1207	ODE No. 1207	493
2.1208	ODE No. 1208	493
2.1209	ODE No. 1209	493
2.1210	ODE No. 1210	494
2.1211	ODE No. 1211	494
2.1212	ODE No. 1212	494
2.1213	ODE No. 1213	494
2.1214	ODE No. 1214	495
2.1215	ODE No. 1215	495
2.1216	ODE No. 1216	495
2.1217	ODE No. 1217	495
2.1218	ODE No. 1218	496
2.1219	ODE No. 1219	496
2.1220	ODE No. 1220	496
2.1221	ODE No. 1221	496
2.1222	ODE No. 1222	497
2.1223	ODE No. 1223	497
2.1224	ODE No. 1224	497
2.1225	ODE No. 1225	497
2.1226	ODE No. 1226	497
2.1227	ODE No. 1227	498
2.1228	ODE No. 1228	498
2.1229	ODE No. 1229	498
2.1230	ODE No. 1230	498
2.1231	ODE No. 1231	499
2.1232	ODE No. 1232	499
2.1233	ODE No. 1233	499
2.1234	ODE No. 1234	499
2.1235	ODE No. 1235	500
2.1236	ODE No. 1236	500
2.1237	ODE No. 1237	500
2.1238	ODE No. 1238	500
2.1239	ODE No. 1239	501
2.1240	ODE No. 1240	501
2.1241	ODE No. 1241	501
2.1242	ODE No. 1242	501

2.1243	ODE No. 1243	501
2.1244	ODE No. 1244	502
2.1245	ODE No. 1245	502
2.1246	ODE No. 1246	502
2.1247	ODE No. 1247	502
2.1248	ODE No. 1248	503
2.1249	ODE No. 1249	503
2.1250	ODE No. 1250	503
2.1251	ODE No. 1251	503
2.1252	ODE No. 1252	504
2.1253	ODE No. 1253	504
2.1254	ODE No. 1254	504
2.1255	ODE No. 1255	504
2.1256	ODE No. 1256	505
2.1257	ODE No. 1257	505
2.1258	ODE No. 1258	505
2.1259	ODE No. 1259	505
2.1260	ODE No. 1260	506
2.1261	ODE No. 1261	506
2.1262	ODE No. 1262	506
2.1263	ODE No. 1263	506
2.1264	ODE No. 1264	507
2.1265	ODE No. 1265	507
2.1266	ODE No. 1266	507
2.1267	ODE No. 1267	507
2.1268	ODE No. 1268	508
2.1269	ODE No. 1269	508
2.1270	ODE No. 1270	508
2.1271	ODE No. 1271	508
2.1272	ODE No. 1272	508
2.1273	ODE No. 1273	509
2.1274	ODE No. 1274	509
2.1275	ODE No. 1275	509
2.1276	ODE No. 1276	509
2.1277	ODE No. 1277	510
2.1278	ODE No. 1278	510
2.1279	ODE No. 1279	510
2.1280	ODE No. 1280	510
2.1281	ODE No. 1281	511
2.1282	ODE No. 1282	511
2.1283	ODE No. 1283	511
2.1284	ODE No. 1284	511
2.1285	ODE No. 1285	512
2.1286	ODE No. 1286	512
2.1287	ODE No. 1287	512
2.1288	ODE No. 1288	512
2.1289	ODE No. 1289	513
2.1290	ODE No. 1290	513
2.1291	ODE No. 1291	513
2.1292	ODE No. 1292	513
2.1293	ODE No. 1293	514
2.1294	ODE No. 1294	514
2.1295	ODE No. 1295	514
2.1296	ODE No. 1296	514
2.1297	ODE No. 1297	515
2.1298	ODE No. 1298	515
2.1299	ODE No. 1299	515
2.1300	ODE No. 1300	515
2.1301	ODE No. 1301	516
2.1302	ODE No. 1302	516
2.1303	ODE No. 1303	516
2.1304	ODE No. 1304	516
2.1305	ODE No. 1305	517

2.1306	ODE No. 1306	517
2.1307	ODE No. 1307	517
2.1308	ODE No. 1308	517
2.1309	ODE No. 1309	518
2.1310	ODE No. 1310	518
2.1311	ODE No. 1311	518
2.1312	ODE No. 1312	518
2.1313	ODE No. 1313	519
2.1314	ODE No. 1314	519
2.1315	ODE No. 1315	519
2.1316	ODE No. 1316	519
2.1317	ODE No. 1317	520
2.1318	ODE No. 1318	520
2.1319	ODE No. 1319	520
2.1320	ODE No. 1320	520
2.1321	ODE No. 1321	521
2.1322	ODE No. 1322	521
2.1323	ODE No. 1323	521
2.1324	ODE No. 1324	521
2.1325	ODE No. 1325	521
2.1326	ODE No. 1326	522
2.1327	ODE No. 1327	522
2.1328	ODE No. 1328	522
2.1329	ODE No. 1329	522
2.1330	ODE No. 1330	523
2.1331	ODE No. 1331	523
2.1332	ODE No. 1332	523
2.1333	ODE No. 1333	523
2.1334	ODE No. 1334	524
2.1335	ODE No. 1335	524
2.1336	ODE No. 1336	524
2.1337	ODE No. 1337	524
2.1338	ODE No. 1338	525
2.1339	ODE No. 1339	525
2.1340	ODE No. 1340	525
2.1341	ODE No. 1341	525
2.1342	ODE No. 1342	526
2.1343	ODE No. 1343	526
2.1344	ODE No. 1344	526
2.1345	ODE No. 1345	527
2.1346	ODE No. 1346	527
2.1347	ODE No. 1347	527
2.1348	ODE No. 1348	527
2.1349	ODE No. 1349	528
2.1350	ODE No. 1350	528
2.1351	ODE No. 1351	528
2.1352	ODE No. 1352	528
2.1353	ODE No. 1353	529
2.1354	ODE No. 1354	529
2.1355	ODE No. 1355	529
2.1356	ODE No. 1356	529
2.1357	ODE No. 1357	530
2.1358	ODE No. 1358	530
2.1359	ODE No. 1359	530
2.1360	ODE No. 1360	530
2.1361	ODE No. 1361	531
2.1362	ODE No. 1362	531
2.1363	ODE No. 1363	531
2.1364	ODE No. 1364	531
2.1365	ODE No. 1365	532
2.1366	ODE No. 1366	532
2.1367	ODE No. 1367	532
2.1368	ODE No. 1368	532

2.1369	ODE No. 1369	533
2.1370	ODE No. 1370	533
2.1371	ODE No. 1371	533
2.1372	ODE No. 1372	533
2.1373	ODE No. 1373	534
2.1374	ODE No. 1374	534
2.1375	ODE No. 1375	534
2.1376	ODE No. 1376	534
2.1377	ODE No. 1377	535
2.1378	ODE No. 1378	535
2.1379	ODE No. 1379	535
2.1380	ODE No. 1380	535
2.1381	ODE No. 1381	536
2.1382	ODE No. 1382	536
2.1383	ODE No. 1383	536
2.1384	ODE No. 1384	537
2.1385	ODE No. 1385	537
2.1386	ODE No. 1386	537
2.1387	ODE No. 1387	537
2.1388	ODE No. 1388	538
2.1389	ODE No. 1389	538
2.1390	ODE No. 1390	538
2.1391	ODE No. 1391	538
2.1392	ODE No. 1392	539
2.1393	ODE No. 1393	539
2.1394	ODE No. 1394	539
2.1395	ODE No. 1395	539
2.1396	ODE No. 1396	540
2.1397	ODE No. 1397	540
2.1398	ODE No. 1398	540
2.1399	ODE No. 1399	541
2.1400	ODE No. 1400	541
2.1401	ODE No. 1401	541
2.1402	ODE No. 1402	541
2.1403	ODE No. 1403	542
2.1404	ODE No. 1404	542
2.1405	ODE No. 1405	542
2.1406	ODE No. 1406	542
2.1407	ODE No. 1407	543
2.1408	ODE No. 1408	543
2.1409	ODE No. 1409	543
2.1410	ODE No. 1410	544
2.1411	ODE No. 1411	544
2.1412	ODE No. 1412	544
2.1413	ODE No. 1413	544
2.1414	ODE No. 1414	545
2.1415	ODE No. 1415	545
2.1416	ODE No. 1416	545
2.1417	ODE No. 1417	545
2.1418	ODE No. 1418	546
2.1419	ODE No. 1419	546
2.1420	ODE No. 1420	546
2.1421	ODE No. 1421	546
2.1422	ODE No. 1422	547
2.1423	ODE No. 1423	547
2.1424	ODE No. 1424	547
2.1425	ODE No. 1425	547
2.1426	ODE No. 1426	548
2.1427	ODE No. 1427	548
2.1428	ODE No. 1428	548
2.1429	ODE No. 1429	549
2.1430	ODE No. 1430	549
2.1431	ODE No. 1431	549

2.1432	ODE No. 1432	549
2.1433	ODE No. 1433	550
2.1434	ODE No. 1434	550
2.1435	ODE No. 1435	550
2.1436	ODE No. 1436	550
2.1437	ODE No. 1437	551
2.1438	ODE No. 1438	551
2.1439	ODE No. 1439	551
2.1440	ODE No. 1440	551
2.1441	ODE No. 1441	552
2.1442	ODE No. 1442	552
2.1443	ODE No. 1443	552
2.1444	ODE No. 1444	552
2.1445	ODE No. 1445	553
2.1446	ODE No. 1446	553
2.1447	ODE No. 1447	553
2.1448	ODE No. 1448	553
2.1449	ODE No. 1449	554
2.1450	ODE No. 1450	554
2.1451	ODE No. 1451	554
2.1452	ODE No. 1452	554
2.1453	ODE No. 1453	555
2.1454	ODE No. 1454	555
2.1455	ODE No. 1455	555
2.1456	ODE No. 1456	555
2.1457	ODE No. 1457	556
2.1458	ODE No. 1458	556
2.1459	ODE No. 1459	556
2.1460	ODE No. 1460	556
2.1461	ODE No. 1461	556
2.1462	ODE No. 1462	557
2.1463	ODE No. 1463	557
2.1464	ODE No. 1464	557
2.1465	ODE No. 1465	557
2.1466	ODE No. 1466	557
2.1467	ODE No. 1467	558
2.1468	ODE No. 1468	558
2.1469	ODE No. 1469	558
2.1470	ODE No. 1470	558
2.1471	ODE No. 1471	559
2.1472	ODE No. 1472	559
2.1473	ODE No. 1473	559
2.1474	ODE No. 1474	559
2.1475	ODE No. 1475	560
2.1476	ODE No. 1476	560
2.1477	ODE No. 1477	560
2.1478	ODE No. 1478	560
2.1479	ODE No. 1479	561
2.1480	ODE No. 1480	561
2.1481	ODE No. 1481	561
2.1482	ODE No. 1482	561
2.1483	ODE No. 1483	562
2.1484	ODE No. 1484	562
2.1485	ODE No. 1485	562
2.1486	ODE No. 1486	562
2.1487	ODE No. 1487	563
2.1488	ODE No. 1488	563
2.1489	ODE No. 1489	563
2.1490	ODE No. 1490	563
2.1491	ODE No. 1491	564
2.1492	ODE No. 1492	564
2.1493	ODE No. 1493	564
2.1494	ODE No. 1494	564

2.1495	ODE No. 1495	565
2.1496	ODE No. 1496	565
2.1497	ODE No. 1497	565
2.1498	ODE No. 1498	565
2.1499	ODE No. 1499	566
2.1500	ODE No. 1500	566
2.1501	ODE No. 1501	566
2.1502	ODE No. 1502	566
2.1503	ODE No. 1503	567
2.1504	ODE No. 1504	567
2.1505	ODE No. 1505	567
2.1506	ODE No. 1506	567
2.1507	ODE No. 1507	568
2.1508	ODE No. 1508	568
2.1509	ODE No. 1509	568
2.1510	ODE No. 1510	568
2.1511	ODE No. 1511	569
2.1512	ODE No. 1512	569
2.1513	ODE No. 1513	569
2.1514	ODE No. 1514	569
2.1515	ODE No. 1515	570
2.1516	ODE No. 1516	570
2.1517	ODE No. 1517	570
2.1518	ODE No. 1518	570
2.1519	ODE No. 1519	571
2.1520	ODE No. 1520	571
2.1521	ODE No. 1521	571
2.1522	ODE No. 1522	571
2.1523	ODE No. 1523	572
2.1524	ODE No. 1524	572
2.1525	ODE No. 1525	572
2.1526	ODE No. 1526	572
2.1527	ODE No. 1527	573
2.1528	ODE No. 1528	573
2.1529	ODE No. 1529	573
2.1530	ODE No. 1530	573
2.1531	ODE No. 1531	574
2.1532	ODE No. 1532	574
2.1533	ODE No. 1533	574
2.1534	ODE No. 1534	574
2.1535	ODE No. 1535	575
2.1536	ODE No. 1536	575
2.1537	ODE No. 1537	575
2.1538	ODE No. 1538	575
2.1539	ODE No. 1539	576
2.1540	ODE No. 1540	576
2.1541	ODE No. 1541	576
2.1542	ODE No. 1542	576
2.1543	ODE No. 1543	576
2.1544	ODE No. 1544	577
2.1545	ODE No. 1545	577
2.1546	ODE No. 1546	577
2.1547	ODE No. 1547	577
2.1548	ODE No. 1548	577
2.1549	ODE No. 1549	578
2.1550	ODE No. 1550	578
2.1551	ODE No. 1551	578
2.1552	ODE No. 1552	579
2.1553	ODE No. 1553	579
2.1554	ODE No. 1554	579
2.1555	ODE No. 1555	579
2.1556	ODE No. 1556	579
2.1557	ODE No. 1557	580

2.1558	ODE No. 1558	580
2.1559	ODE No. 1559	580
2.1560	ODE No. 1560	580
2.1561	ODE No. 1561	581
2.1562	ODE No. 1562	581
2.1563	ODE No. 1563	581
2.1564	ODE No. 1564	581
2.1565	ODE No. 1565	582
2.1566	ODE No. 1566	582
2.1567	ODE No. 1567	582
2.1568	ODE No. 1568	582
2.1569	ODE No. 1569	583
2.1570	ODE No. 1570	583
2.1571	ODE No. 1571	583
2.1572	ODE No. 1572	583
2.1573	ODE No. 1573	584
2.1574	ODE No. 1574	584
2.1575	ODE No. 1575	584
2.1576	ODE No. 1576	584
2.1577	ODE No. 1577	585
2.1578	ODE No. 1578	585
2.1579	ODE No. 1579	585
2.1580	ODE No. 1580	585
2.1581	ODE No. 1581	586
2.1582	ODE No. 1582	586
2.1583	ODE No. 1583	586
2.1584	ODE No. 1584	586
2.1585	ODE No. 1585	587
2.1586	ODE No. 1586	587
2.1587	ODE No. 1587	587
2.1588	ODE No. 1588	587
2.1589	ODE No. 1589	588
2.1590	ODE No. 1590	588
2.1591	ODE No. 1591	588
2.1592	ODE No. 1592	588
2.1593	ODE No. 1593	589
2.1594	ODE No. 1594	589
2.1595	ODE No. 1595	589
2.1596	ODE No. 1596	589
2.1597	ODE No. 1597	589
2.1598	ODE No. 1598	590
2.1599	ODE No. 1599	590
2.1600	ODE No. 1600	590
2.1601	ODE No. 1601	590
2.1602	ODE No. 1602	590
2.1603	ODE No. 1603	591
2.1604	ODE No. 1604	591
2.1605	ODE No. 1605	591
2.1606	ODE No. 1606	591
2.1607	ODE No. 1607	592
2.1608	ODE No. 1608	592
2.1609	ODE No. 1609	592
2.1610	ODE No. 1610	592
2.1611	ODE No. 1611	593
2.1612	ODE No. 1612	593
2.1613	ODE No. 1613	593
2.1614	ODE No. 1614	593
2.1615	ODE No. 1615	593
2.1616	ODE No. 1616	594
2.1617	ODE No. 1617	594
2.1618	ODE No. 1618	594
2.1619	ODE No. 1619	594
2.1620	ODE No. 1620	594

2.1621	ODE No. 1621	595
2.1622	ODE No. 1622	595
2.1623	ODE No. 1623	595
2.1624	ODE No. 1624	596
2.1625	ODE No. 1625	596
2.1626	ODE No. 1626	596
2.1627	ODE No. 1627	596
2.1628	ODE No. 1628	596
2.1629	ODE No. 1629	597
2.1630	ODE No. 1630	597
2.1631	ODE No. 1631	597
2.1632	ODE No. 1632	597
2.1633	ODE No. 1633	598
2.1634	ODE No. 1634	598
2.1635	ODE No. 1635	598
2.1636	ODE No. 1636	598
2.1637	ODE No. 1637	599
2.1638	ODE No. 1638	599
2.1639	ODE No. 1639	599
2.1640	ODE No. 1640	599
2.1641	ODE No. 1641	600
2.1642	ODE No. 1642	600
2.1643	ODE No. 1643	600
2.1644	ODE No. 1644	600
2.1645	ODE No. 1645	600
2.1646	ODE No. 1646	601
2.1647	ODE No. 1647	601
2.1648	ODE No. 1648	601
2.1649	ODE No. 1649	601
2.1650	ODE No. 1650	602
2.1651	ODE No. 1651	602
2.1652	ODE No. 1652	602
2.1653	ODE No. 1653	603
2.1654	ODE No. 1654	603
2.1655	ODE No. 1655	603
2.1656	ODE No. 1656	603
2.1657	ODE No. 1657	604
2.1658	ODE No. 1658	604
2.1659	ODE No. 1659	604
2.1660	ODE No. 1660	604
2.1661	ODE No. 1661	605
2.1662	ODE No. 1662	605
2.1663	ODE No. 1663	605
2.1664	ODE No. 1664	605
2.1665	ODE No. 1665	606
2.1666	ODE No. 1666	606
2.1667	ODE No. 1667	606
2.1668	ODE No. 1668	606
2.1669	ODE No. 1669	607
2.1670	ODE No. 1670	607
2.1671	ODE No. 1671	607
2.1672	ODE No. 1672	607
2.1673	ODE No. 1673	608
2.1674	ODE No. 1674	608
2.1675	ODE No. 1675	608
2.1676	ODE No. 1676	608
2.1677	ODE No. 1677	608
2.1678	ODE No. 1678	609
2.1679	ODE No. 1679	609
2.1680	ODE No. 1680	609
2.1681	ODE No. 1681	609
2.1682	ODE No. 1682	610
2.1683	ODE No. 1683	610

2.1684	ODE No. 1684	610
2.1685	ODE No. 1685	610
2.1686	ODE No. 1686	611
2.1687	ODE No. 1687	611
2.1688	ODE No. 1688	611
2.1689	ODE No. 1689	611
2.1690	ODE No. 1690	612
2.1691	ODE No. 1691	612
2.1692	ODE No. 1692	612
2.1693	ODE No. 1693	612
2.1694	ODE No. 1694	613
2.1695	ODE No. 1695	613
2.1696	ODE No. 1696	613
2.1697	ODE No. 1697	613
2.1698	ODE No. 1698	614
2.1699	ODE No. 1699	614
2.1700	ODE No. 1700	614
2.1701	ODE No. 1701	614
2.1702	ODE No. 1702	615
2.1703	ODE No. 1703	615
2.1704	ODE No. 1704	615
2.1705	ODE No. 1705	615
2.1706	ODE No. 1706	615
2.1707	ODE No. 1707	616
2.1708	ODE No. 1708	616
2.1709	ODE No. 1709	616
2.1710	ODE No. 1710	617
2.1711	ODE No. 1711	617
2.1712	ODE No. 1712	617
2.1713	ODE No. 1713	617
2.1714	ODE No. 1714	618
2.1715	ODE No. 1715	618
2.1716	ODE No. 1716	618
2.1717	ODE No. 1717	618
2.1718	ODE No. 1718	619
2.1719	ODE No. 1719	619
2.1720	ODE No. 1720	619
2.1721	ODE No. 1721	620
2.1722	ODE No. 1722	620
2.1723	ODE No. 1723	620
2.1724	ODE No. 1724	620
2.1725	ODE No. 1725	621
2.1726	ODE No. 1726	621
2.1727	ODE No. 1727	621
2.1728	ODE No. 1728	621
2.1729	ODE No. 1729	622
2.1730	ODE No. 1730	622
2.1731	ODE No. 1731	622
2.1732	ODE No. 1732	622
2.1733	ODE No. 1733	623
2.1734	ODE No. 1734	623
2.1735	ODE No. 1735	623
2.1736	ODE No. 1736	623
2.1737	ODE No. 1737	624
2.1738	ODE No. 1738	624
2.1739	ODE No. 1739	624
2.1740	ODE No. 1740	624
2.1741	ODE No. 1741	624
2.1742	ODE No. 1742	625
2.1743	ODE No. 1743	625
2.1744	ODE No. 1744	625
2.1745	ODE No. 1745	625
2.1746	ODE No. 1746	626

2.1747	ODE No. 1747	626
2.1748	ODE No. 1748	626
2.1749	ODE No. 1749	626
2.1750	ODE No. 1750	627
2.1751	ODE No. 1751	627
2.1752	ODE No. 1752	627
2.1753	ODE No. 1753	627
2.1754	ODE No. 1754	628
2.1755	ODE No. 1755	628
2.1756	ODE No. 1756	628
2.1757	ODE No. 1757	629
2.1758	ODE No. 1758	629
2.1759	ODE No. 1759	629
2.1760	ODE No. 1760	629
2.1761	ODE No. 1761	630
2.1762	ODE No. 1762	630
2.1763	ODE No. 1763	630
2.1764	ODE No. 1764	630
2.1765	ODE No. 1765	630
2.1766	ODE No. 1766	631
2.1767	ODE No. 1767	631
2.1768	ODE No. 1768	631
2.1769	ODE No. 1769	631
2.1770	ODE No. 1770	632
2.1771	ODE No. 1771	632
2.1772	ODE No. 1772	632
2.1773	ODE No. 1773	632
2.1774	ODE No. 1774	633
2.1775	ODE No. 1775	633
2.1776	ODE No. 1776	633
2.1777	ODE No. 1777	634
2.1778	ODE No. 1778	634
2.1779	ODE No. 1779	634
2.1780	ODE No. 1780	635
2.1781	ODE No. 1781	635
2.1782	ODE No. 1782	636
2.1783	ODE No. 1783	636
2.1784	ODE No. 1784	636
2.1785	ODE No. 1785	636
2.1786	ODE No. 1786	637
2.1787	ODE No. 1787	637
2.1788	ODE No. 1788	637
2.1789	ODE No. 1789	637
2.1790	ODE No. 1790	638
2.1791	ODE No. 1791	638
2.1792	ODE No. 1792	638
2.1793	ODE No. 1793	639
2.1794	ODE No. 1794	639
2.1795	ODE No. 1795	639
2.1796	ODE No. 1796	640
2.1797	ODE No. 1797	640
2.1798	ODE No. 1798	640
2.1799	ODE No. 1799	640
2.1800	ODE No. 1800	641
2.1801	ODE No. 1801	641
2.1802	ODE No. 1802	641
2.1803	ODE No. 1803	641
2.1804	ODE No. 1804	641
2.1805	ODE No. 1805	642
2.1806	ODE No. 1806	642
2.1807	ODE No. 1807	642
2.1808	ODE No. 1808	642
2.1809	ODE No. 1809	643

2.1810	ODE No. 1810	643
2.1811	ODE No. 1811	643
2.1812	ODE No. 1812	643
2.1813	ODE No. 1813	644
2.1814	ODE No. 1814	644
2.1815	ODE No. 1815	644
2.1816	ODE No. 1816	645
2.1817	ODE No. 1817	645
2.1818	ODE No. 1818	645
2.1819	ODE No. 1819	645
2.1820	ODE No. 1820	645
2.1821	ODE No. 1821	646
2.1822	ODE No. 1822	646
2.1823	ODE No. 1823	646
2.1824	ODE No. 1824	647
2.1825	ODE No. 1825	647
2.1826	ODE No. 1826	647
2.1827	ODE No. 1827	647
2.1828	ODE No. 1828	648
2.1829	ODE No. 1829	648
2.1830	ODE No. 1830	648
2.1831	ODE No. 1831	648
2.1832	ODE No. 1832	649
2.1833	ODE No. 1833	649
2.1834	ODE No. 1834	649
2.1835	ODE No. 1835	649
2.1836	ODE No. 1836	650
2.1837	ODE No. 1837	650
2.1838	ODE No. 1838	650
2.1839	ODE No. 1839	650
2.1840	ODE No. 1840	651
2.1841	ODE No. 1841	651
2.1842	ODE No. 1842	651
2.1843	ODE No. 1843	651
2.1844	ODE No. 1844	652
2.1845	ODE No. 1845	652
2.1846	ODE No. 1846	652
2.1847	ODE No. 1847	652
2.1848	ODE No. 1848	653
2.1849	ODE No. 1849	653
2.1850	ODE No. 1850	653
2.1851	ODE No. 1851	653
2.1852	ODE No. 1852	654
2.1853	ODE No. 1853	654
2.1854	ODE No. 1854	654
2.1855	ODE No. 1855	654
2.1856	ODE No. 1856	654
2.1857	ODE No. 1857	655
2.1858	ODE No. 1858	655
2.1859	ODE No. 1859	655
2.1860	ODE No. 1860	655
2.1861	ODE No. 1861	656
2.1862	ODE No. 1862	656
2.1863	ODE No. 1863	656
2.1864	ODE No. 1864	656
2.1865	ODE No. 1865	657
2.1866	ODE No. 1866	657
2.1867	ODE No. 1867	657
2.1868	ODE No. 1868	657
2.1869	ODE No. 1869	658
2.1870	ODE No. 1870	658
2.1871	ODE No. 1871	658
2.1872	ODE No. 1872	658

2.1873	ODE No. 1873	659
2.1874	ODE No. 1874	659
2.1875	ODE No. 1875	659
2.1876	ODE No. 1876	659
2.1877	ODE No. 1877	660
2.1878	ODE No. 1878	660
2.1879	ODE No. 1879	660
2.1880	ODE No. 1880	660
2.1881	ODE No. 1881	660
2.1882	ODE No. 1882	661
2.1883	ODE No. 1883	661
2.1884	ODE No. 1884	661
2.1885	ODE No. 1885	661
2.1886	ODE No. 1886	662
2.1887	ODE No. 1887	662
2.1888	ODE No. 1888	662
2.1889	ODE No. 1889	662
2.1890	ODE No. 1890	663
2.1891	ODE No. 1891	663
2.1892	ODE No. 1892	663
2.1893	ODE No. 1893	664
2.1894	ODE No. 1894	664
2.1895	ODE No. 1895	664
2.1896	ODE No. 1896	664
2.1897	ODE No. 1897	665
2.1898	ODE No. 1898	665
2.1899	ODE No. 1899	665
2.1900	ODE No. 1900	665
2.1901	ODE No. 1901	666
2.1902	ODE No. 1902	666
2.1903	ODE No. 1903	666
2.1904	ODE No. 1904	666
2.1905	ODE No. 1905	667
2.1906	ODE No. 1906	667
2.1907	ODE No. 1907	667
2.1908	ODE No. 1908	667
2.1909	ODE No. 1909	667
2.1910	ODE No. 1910	668
2.1911	ODE No. 1911	668
2.1912	ODE No. 1912	668
2.1913	ODE No. 1913	668
2.1914	ODE No. 1914	669
2.1915	ODE No. 1915	669
2.1916	ODE No. 1916	670
2.1917	ODE No. 1917	670
2.1918	ODE No. 1918	670
2.1919	ODE No. 1919	671
2.1920	ODE No. 1920	671
2.1921	ODE No. 1921	671
2.1922	ODE No. 1922	671
2.1923	ODE No. 1923	672
2.1924	ODE No. 1924	672
2.1925	ODE No. 1925	672
2.1926	ODE No. 1926	672
2.1927	ODE No. 1927	673
2.1928	ODE No. 1928	673
2.1929	ODE No. 1929	673
2.1930	ODE No. 1930	673
2.1931	ODE No. 1931	674
2.1932	ODE No. 1932	674
2.1933	ODE No. 1933	674
2.1934	ODE No. 1934	675
2.1935	ODE No. 1935	675

2.1936	ODE No. 1936	675
2.1937	ODE No. 1937	675
2.1938	ODE No. 1938	676
2.1939	ODE No. 1939	676
2.1940	ODE No. 1940	676
3	Appendix	677

1 Introduction and summary of results

This report gives the result of solving the 1,940 differential equations from Kamke book in Mathematica 12.3 and Maple 2021 on windows 10, 64 bit OS. PC with 64 GB RAM, using Intel I7-8086K at 4GHz.

The command `AboluteTiming[]` was used in Mathematica to obtain the CPU time. In Maple the following commands were used for this purpose

```
t0 := time[real]():
timeOut := 5*60;
result_of_solve := timelimit(timeOut,dsolve(ode[i]));
cpu_time := time[real]()-t0:
```

Both Maple and Mathematica had a CPU time limit of 5 minutes to complete each problem else the problem is considered not solved and marked as timed out.

When Mathematica returned `DifferentialRoot` as a solution to an ODE this was counted as not solved. Similarly, when Maple returned `DESol` this was also counted as not solved.

Table 1 below summarizes the performance of each CAS system

system	% solved	mean CPU (sec)	result mean leaf size	total CPU (minutes)	total leaf size
Mathematica	88.04	2.66	2582.73	75.73	4411304
Maple	91.86	0.55	199.3	16.29	355148

Table 1: Summary of final results

Table 2 summarizes the Kamke equations used

book chapter	kamke equation numbers	This report numbers
Chapter 1, linear first order	1.1—1.576	1—576
Additional non-linear first order	N/A	577—1000
Chapter 2, linear second order	2.1—2.448	1001—1448
Chapter 3, linear third order	3.1—3.85	1449—1533
Chapter 4, linear fourth order	4.1—4.44	1534—1577
Chapter 5, linear fifth and higher order	5.1—5.13	1578—1590
Chapter 6, non-linear second order	6.1—6.246	1591—1836
Chapter 7, non-linear third and higher order	7.1—7.19	1837—1855
Chapter 8, system of ode, first order	8.1—8.57	1856—1912
Chapter 9, system of ode, higher order	9.1—9.28	1913—1940

Table 2: Kamke equation numbering

The following summarizes which equations are solved by each system

Not solved by Mathematica 47, 48, 49, 50, 55, 56, 74, 79, 82, 87, 110, 121, 188, 202, 203, 205, 206, 219, 234, 237, 250, 253, 265, 269, 331, 340, 367, 370, 395, 460, 461, 480, 482, 489, 503, 506, 507, 510, 531, 572, 575, 576, 835, 837, 862, 885, 894, 1015, 1019, 1026, 1028, 1030, 1031, 1032, 1038, 1072, 1073, 1074, 1075, 1076, 1077, 1081, 1082, 1083, 1099, 1126, 1157, 1205, 1212, 1216, 1236, 1278, 1306, 1323, 1362, 1408, 1419, 1439, 1440, 1441, 1443, 1444, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1473, 1474, 1476, 1484, 1489, 1500, 1515, 1531, 1540, 1541, 1542, 1543, 1544, 1547, 1552, 1569, 1572, 1576, 1581, 1586, 1593, 1595, 1596, 1598, 1599, 1601, 1603, 1605, 1606, 1608, 1609, 1611, 1612, 1615, 1616, 1617, 1618, 1619, 1623, 1624, 1625, 1626, 1627, 1628, 1634, 1636, 1637, 1639, 1642, 1643, 1644, 1645, 1648, 1649, 1658, 1659, 1660, 1662, 1663, 1664, 1665, 1666, 1667, 1672, 1673, 1675, 1677, 1678, 1680, 1684, 1685, 1686, 1690, 1692, 1693, 1695, 1696, 1702, 1704, 1708, 1710, 1713, 1719, 1729, 1732, 1734, 1735, 1737, 1738, 1739, 1742, 1751, 1757, 1761, 1762, 1777, 1779, 1780, 1788, 1789, 1797, 1801, 1802, 1806, 1807, 1811, 1815, 1816, 1818, 1819, 1820, 1821, 1825, 1827, 1831, 1832, 1836, 1838, 1839, 1840, 1841, 1850, 1851, 1854, 1855, 1890,

1905, 1915, 1918, 1919, 1920, 1921, 1922, 1927, 1928, 1929, 1932, 1933, 1934, 1935, 1936, 1937, 1940

Not solved by Maple 47, 48, 49, 50, 55, 56, 74, 79, 82, 87, 110, 121, 202, 203, 205, 206, 219, 234, 237, 250, 253, 265, 269, 340, 367, 368, 370, 383, 395, 448, 452, 460, 461, 480, 482, 485, 503, 506, 507, 510, 531, 543, 572, 575, 576, 708, 733, 789, 790, 835, 837, 885, 894, 912, 920, 1015, 1019, 1026, 1028, 1030, 1031, 1038, 1072, 1073, 1075, 1076, 1077, 1081, 1157, 1205, 1212, 1216, 1234, 1236, 1278, 1408, 1439, 1440, 1441, 1443, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1473, 1474, 1476, 1484, 1489, 1510, 1515, 1531, 1540, 1541, 1542, 1543, 1581, 1582, 1586, 1593, 1595, 1596, 1598, 1599, 1606, 1608, 1609, 1617, 1619, 1623, 1625, 1628, 1634, 1636, 1639, 1642, 1643, 1645, 1649, 1675, 1685, 1698, 1702, 1704, 1705, 1706, 1721, 1729, 1732, 1734, 1735, 1737, 1738, 1739, 1751, 1757, 1761, 1788, 1789, 1797, 1801, 1802, 1807, 1811, 1835, 1851, 1854, 1855, 1890, 1905, 1921, 1922, 1927, 1928, 1940

Solved by Mathematica but not by Maple 368, 383, 448, 452, 485, 543, 708, 733, 789, 790, 912, 920, 1234, 1510, 1582, 1698, 1705, 1706, 1721, 1835

Solved by Maple but not by Mathematica 188, 331, 489, 862, 1032, 1074, 1082, 1083, 1099, 1126, 1306, 1323, 1362, 1419, 1444, 1500, 1544, 1547, 1552, 1569, 1572, 1576, 1601, 1603, 1605, 1611, 1612, 1615, 1616, 1618, 1624, 1626, 1627, 1637, 1644, 1648, 1658, 1659, 1660, 1662, 1663, 1664, 1665, 1666, 1667, 1672, 1673, 1677, 1678, 1680, 1684, 1686, 1690, 1692, 1693, 1695, 1696, 1708, 1710, 1713, 1719, 1742, 1762, 1777, 1779, 1780, 1806, 1815, 1816, 1818, 1819, 1820, 1821, 1825, 1827, 1831, 1832, 1836, 1838, 1839, 1840, 1841, 1850, 1915, 1918, 1919, 1920, 1929, 1932, 1933, 1934, 1935, 1936, 1937

Solved by both Maple and Mathematica 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 80, 81, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 235, 236, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 266, 267, 268, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 369, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 449, 450, 451, 453, 454, 455, 456, 457, 458, 459, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 481, 483, 484, 486, 487, 488, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 504, 505, 508, 509, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 836, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 863, 864, 865, 866, 867, 868, 869, 870,

871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 886, 887, 888, 889, 890, 891, 892, 893, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 913, 914, 915, 916, 917, 918, 919, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1020, 1021, 1022, 1023, 1024, 1025, 1027, 1029, 1033, 1034, 1035, 1036, 1037, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1078, 1079, 1080, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1206, 1207, 1208, 1209, 1210, 1211, 1213, 1214, 1215, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1235, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1442, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1475, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1485, 1486, 1487, 1488, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1511, 1512, 1513, 1514, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1545, 1546, 1548, 1549, 1550, 1551, 1553, 1554, 1555, 1556, 1557, 1558, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1570, 1571, 1573, 1574, 1575, 1577, 1578, 1579, 1580, 1583, 1584, 1585, 1587, 1588, 1589, 1590, 1591, 1592, 1594, 1597, 1600, 1602, 1604, 1607, 1610, 1613, 1614, 1620, 1621, 1622, 1629, 1630, 1631, 1632, 1633, 1635, 1638, 1640, 1641, 1646, 1647, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1661, 1668, 1669, 1670, 1671, 1674, 1676, 1679, 1681, 1682, 1683, 1687, 1688, 1689, 1691, 1694, 1697, 1699, 1700, 1701, 1703, 1707, 1709, 1711, 1712, 1714, 1715, 1716, 1717, 1718, 1720, 1722, 1723, 1724, 1725, 1726, 1727, 1728, 1730, 1731, 1733, 1736, 1740, 1741, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1752, 1753, 1754, 1755, 1756, 1758, 1759, 1760, 1763, 1764, 1765, 1766, 1767, 1768, 1769, 1770, 1771, 1772, 1773, 1774, 1775, 1776, 1778, 1781, 1782, 1783, 1784, 1785, 1786, 1787, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1798, 1799, 1800, 1803, 1804, 1805, 1808, 1809, 1810, 1812, 1813, 1814, 1817, 1822, 1823, 1824, 1826, 1828, 1829, 1830, 1833, 1834, 1837, 1842, 1843, 1844, 1845, 1846, 1847, 1848, 1849, 1852, 1853, 1856, 1857, 1858, 1859, 1860, 1861, 1862, 1863, 1864, 1865, 1866, 1867, 1868, 1869, 1870, 1871, 1872, 1873, 1874, 1875, 1876, 1877, 1878, 1879, 1880, 1881, 1882, 1883, 1884, 1885, 1886, 1887, 1888, 1889, 1891, 1892, 1893, 1894, 1895, 1896, 1897, 1898, 1899, 1900, 1901, 1902, 1903, 1904, 1906, 1907, 1908, 1909, 1910, 1911, 1912, 1913, 1914, 1916, 1917, 1923, 1924, 1925, 1926, 1930, 1931, 1938, 1939

Both systems unable to solve 47, 48, 49, 50, 55, 56, 74, 79, 82, 87, 110, 121, 202, 203, 205, 206, 219, 234, 237, 250, 253, 265, 269, 340, 367, 370, 395, 460, 461, 480, 482, 503, 506, 507, 510, 531, 572, 575, 576, 835, 837, 885, 894, 1015, 1019, 1026, 1028, 1030, 1031, 1038, 1072, 1073, 1075, 1076, 1077, 1081, 1157, 1205, 1212, 1216, 1236, 1278, 1408, 1439, 1440, 1441, 1443, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1473, 1474, 1476, 1484, 1489, 1515, 1531, 1540, 1541, 1542, 1543, 1581, 1586, 1593, 1595, 1596, 1598, 1599, 1606, 1608, 1609, 1617, 1619, 1623, 1625, 1628, 1634, 1636, 1639, 1642, 1643, 1645, 1649, 1675, 1685, 1702, 1704, 1729, 1732, 1734, 1735, 1737, 1738, 1739, 1751, 1757, 1761, 1788, 1789, 1797, 1801, 1802, 1807, 1811, 1851, 1854, 1855, 1890, 1905, 1921, 1922, 1927, 1928, 1940

2 Problems table lookup

Final conclusion table for each equation is given by table 3 below. Clicking on the problem opens a new page that shows the result and links to download each problem as well.

Table 3: Breakdown of results for each Kamke differential equation

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 1	✓	0.466	1117	✓	0.056	1089	Linear first order, To Do
Kamke 2	✓	0.062	34	✓	0.021	25	Linear first order, integrating factor $y'(x)+ay(x) = f(x)$
Kamke 3	✓	0.073	40	✓	0.035	38	Linear first order, integrating factor $y'(x)+ay(x) = f(x)$
Kamke 4	✓	0.042	30	✓	0.007	18	Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$
Kamke 5	✓	0.446	39	✓	0.242	21	Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$
Kamke 6	✓	0.031	18	✓	0.037	15	Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$
Kamke 7	✓	0.098	23	✓	0.011	13	Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$
Kamke 8	✓	0.035	17	✓	0.022	13	Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$
Kamke 9	✓	0.031	19	✓	0.027	14	Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$
Kamke 10	✓	0.026	18	✓	0.022	15	Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$
Kamke 11	✓	0.033	66	✓	0.025	24	Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$
Kamke 12	✓	0.091	34	✓	0.056	8	Non-linear first order, Riccati, separable $y'(x)+y^2(x) = 1$
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 52	✓	0.307	117	✓	0.418	61	To Do
Kamke 53	✓	0.679	96	✓	0.181	281	To Do
Kamke 54	✓	0.476	74	✓	0.204	38	To Do
Kamke 55	✗	0	0	✗	0	0	To Do
Kamke 56	✗	0	0	✗	0	0	To Do
Kamke 57	✓	0.075	26	✓	0.12	31	To Do
Kamke 58	✓	0.381	118	✓	0.077	68	To Do
Kamke 59	✓	0.326	96	✓	0.058	26	To Do
Kamke 60	✓	0.178	173	✓	0.019	29	Non-Linear first order, sep- arable.
Kamke 61	✓	0.243	75	✓	0.015	50	To Do
Kamke 62	✓	3.205	44	✓	0.384	34	Non-Linear first order, spe- cial transformation makes it exact differential.
Kamke 63	✓	0.188	48	✓	5.726	35	To Do
Kamke 64	✓	0.381	269	✓	0.117	124	To Do
Kamke 65	✓	1.144	312	✓	0.059	47	To Do
Kamke 66	✓	0.213	67	✓	0.258	40	To Do
Kamke 67	✓	0.245	14	✓	0.022	51	To Do
Kamke 68	✓	1.076	373	✓	0.073	77	To Do
Kamke 69	✓	8.367	1163	✓	0.182	111	To Do
Kamke 70	✓	12.877	81	✓	0.177	113	To Do
Kamke 71	✓	3.749	2237	✓	0.129	113	To Do
Kamke 72	✓	0.205	89	✓	0.019	64	To Do
Kamke 73	✓	1.717	733	✓	0.501	91	To Do
Kamke 74	✗	0	0	✗	0	0	To Do
Kamke 75	✓	0.215	20	✓	0.16	20	Non-Linear first order, Sep- arable
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 115	✓	0.305	221	✓	0.221	49	Non-Linear first order, Smart transformation $y(x) = xv(x)$ makes it separable
Kamke 116	✓	0.632	121	✓	0.23	86	To Do
Kamke 117	✓	0.189	21	✓	0.116	20	To Do
Kamke 118	✓	0.086	13	✓	0.089	8	To Do
Kamke 119	✓	0.108	17	✓	0.088	14	To Do
Kamke 120	✓	0.118	20	✓	0.173	17	To Do
Kamke 121	✗	0	0	✗	0	0	To Do
Kamke 122	✓	0.314	21	✓	0.403	16	To Do
Kamke 123	✓	0.134	19	✓	0.079	44	To Do
Kamke 124	✓	0.111	16	✓	0.04	12	To Do
Kamke 125	✓	0.198	16	✓	0.086	14	To Do
Kamke 126	✓	0.253	115	✓	0.03	29	To Do
Kamke 127	✓	0.317	186	✓	0.127	39	To Do
Kamke 128	✓	3.409	41	✓	0.279	33	To Do
Kamke 129	✓	0.208	44	✓	0.036	33	To Do
Kamke 130	✓	0.019	21	✓	0.007	15	To Do
Kamke 131	✓	0.232	21	✓	0.193	31	To Do
Kamke 132	✓	0.129	115	✓	0.042	153	To Do
Kamke 133	✓	0.018	27	✓	0.015	16	To Do
Kamke 134	✓	0.07	27	✓	0.01	17	To Do
Kamke 135	✓	0.02	14	✓	0.006	11	To Do
Kamke 136	✓	0.096	28	✓	0.016	18	To Do
Kamke 137	✓	0.099	16	✓	0.014	14	To Do
Kamke 138	✓	0.124	13	✓	0.034	11	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 189	✓	0.488	91	✓	0.339	60	To Do
Kamke 190	✓	0.149	173	✓	0.006	29	To Do
Kamke 191	✓	0.141	39	✓	0.018	16	To Do
Kamke 192	✓	0.764	168	✓	0.018	36	To Do
Kamke 193	✓	0.031	16	✓	0.007	14	To Do
Kamke 194	✓	0.214	98	✓	0.025	23	To Do
Kamke 195	✓	0.184	27	✓	0.118	28	To Do
Kamke 196	✓	0.234	53	✓	0.157	29	To Do
Kamke 197	✓	0.134	98	✓	0.158	237	To Do
Kamke 198	✓	0.034	15	✓	0.037	13	To Do
Kamke 199	✓	0.232	15	✓	0.181	102	To Do
Kamke 200	✓	0.143	77	✓	0.096	53	To Do
Kamke 201	✓	0.164	39	✓	0.052	23	To Do
Kamke 202	✗	0	0	✗	0	0	To Do
Kamke 203	✗	0	0	✗	0	0	To Do
Kamke 204	✓	0.134	70	✓	0.325	92	To Do
Kamke 205	✗	0	0	✗	0	0	To Do
Kamke 206	✗	0	0	✗	0	0	To Do
Kamke 207	✓	0.101	47	✓	0.027	37	To Do
Kamke 208	✓	0.238	118	✓	0.088	106	To Do
Kamke 209	✓	0.134	84	✓	0.012	21	To Do
Kamke 210	✓	0.098	47	✓	0.026	33	To Do
Kamke 211	✓	0.261	41	✓	0.053	31	To Do
Kamke 212	✓	0.398	95	✓	0.141	30	To Do
Kamke 213	✓	0.146	71	✓	0.823	66	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 214	✓	0.179	78	✓	0.18	48	To Do
Kamke 215	✓	0.169	80	✓	0.204	51	To Do
Kamke 216	✓	0.148	82	✓	0.204	51	To Do
Kamke 217	✓	0.028	29	✓	0.036	23	To Do
Kamke 218	✓	0.152	257	✓	0.169	57	To Do
Kamke 219	✗	0	0	✗	0	0	To Do
Kamke 220	✓	0.127	57	✓	0.028	43	To Do
Kamke 221	✓	0.033	35	✓	0.117	21	To Do
Kamke 222	✓	0.131	65	✓	0.091	32	To Do
Kamke 223	✓	0.043	55	✓	0.184	51	To Do
Kamke 224	✓	0.034	29	✓	0.121	35	To Do
Kamke 225	✓	0.073	33	✓	0.108	20	To Do
Kamke 226	✓	0.035	35	✓	0.102	21	To Do
Kamke 227	✓	0.028	107	✓	0.178	33	To Do
Kamke 228	✓	0.463	3357	✓	0.416	271	To Do
Kamke 229	✓	0.036	121	✓	0.171	32	To Do
Kamke 230	✓	0.312	98	✓	0.053	100	To Do
Kamke 231	✓	3.808	252	✓	0.254	178	To Do
Kamke 232	✓	0.147	56	✓	0.019	39	To Do
Kamke 233	✓	0.221	38	✓	0.03	30	To Do
Kamke 234	✗	0	0	✗	0	0	To Do
Kamke 235	✓	0.21	40	✓	0.056	30	To Do
Kamke 236	✓	0.238	114	✓	0.065	141	To Do
Kamke 237	✗	0	0	✗	0	0	To Do
Kamke 238	✓	0.399	192	✓	0.079	93	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 289	✓	0.209	115	✓	0.039	115	To Do
Kamke 290	✓	0.518	831	✓	0.148	1388	To Do
Kamke 291	✓	1.276	39	✓	0.161	50	To Do
Kamke 292	✓	51.552	1716	✓	0.043	115	To Do
Kamke 293	✓	0.208	661	✓	0.295	35	To Do
Kamke 294	✓	0.287	71	✓	0.093	112	To Do
Kamke 295	✓	0.276	31	✓	0.269	29	To Do
Kamke 296	✓	0.537	102	✓	0.599	135	To Do
Kamke 297	✓	0.19	216	✓	0.293	29	To Do
Kamke 298	✓	0.121	72	✓	0.024	73	To Do
Kamke 299	✓	0.14	371	✓	0.233	276	To Do
Kamke 300	✓	0.1	99	✓	0.025	83	To Do
Kamke 301	✓	0.162	64	✓	0.221	25	To Do
Kamke 302	✓	0.122	70	✓	0.134	133	To Do
Kamke 303	✓	0.166	25	✓	0.24	34	To Do
Kamke 304	✓	0.483	44	✓	0.256	44	To Do
Kamke 305	✓	0.207	1277	✓	0.021	21	To Do
Kamke 306	✓	0.155	201	✓	0.263	231	To Do
Kamke 307	✓	0.27	149	✓	0.052	125	To Do
Kamke 308	✓	0.016	55	✓	0.018	37	To Do
Kamke 309	✓	0.118	151	✓	0.043	113	To Do
Kamke 310	✓	0.191	159	✓	0.151	125	To Do
Kamke 311	✓	0.251	2201	✓	0.117	50	To Do
Kamke 312	✓	0.763	204	✓	1.648	240	To Do
Kamke 313	✓	0.391	537	✓	0.202	748	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 339	✓	0.416	27	✓	0.222	27	To Do
Kamke 340	✗	0	0	✗	0	0	To Do
Kamke 341	✓	0.275	33	✓	0.069	33	To Do
Kamke 342	✓	0.624	163	✓	0.047	17	To Do
Kamke 343	✓	0.176	35	✓	0.114	27	To Do
Kamke 344	✓	0.203	23	✓	0.063	19	To Do
Kamke 345	✓	0.23	35	✓	0.097	36	To Do
Kamke 346	✓	0.456	24	✓	0.281	19	To Do
Kamke 347	✓	0.339	32	✓	0.158	12	To Do
Kamke 348	✓	0.213	17	✓	0.099	15	To Do
Kamke 349	✓	0.255	15	✓	0.044	17	To Do
Kamke 350	✓	0.654	53	✓	0.872	226	To Do
Kamke 351	✓	0.556	61	✓	0.434	55	To Do
Kamke 352	✓	0.607	43	✓	0.43	33	To Do
Kamke 353	✓	0.089	14	✓	0.089	12	To Do
Kamke 354	✓	0.192	145	✓	0.073	108	To Do
Kamke 355	✓	0.191	17	✓	0.096	15	To Do
Kamke 356	✓	0.275	21	✓	0.112	19	To Do
Kamke 357	✓	0.605	35	✓	0.606	13	To Do
Kamke 358	✓	0.163	29	✓	0.096	11	To Do
Kamke 359	✓	0.355	45	✓	0.074	28	To Do
Kamke 360	✓	4.649	369	✓	0.375	48	To Do
Kamke 361	✓	0.961	31	✓	0.198	22	To Do
Kamke 362	✓	0.384	23	✓	0.241	23	To Do
Kamke 363	✓	0.403	33	✓	0.081	35	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 364	✓	0.483	31	✓	0.199	23	To Do
Kamke 365	✓	0.393	156	✓	0.355	42	To Do
Kamke 366	✓	0.315	91	✓	0.137	45	To Do
Kamke 367	✗	0	0	✗	0	0	To Do
Kamke 368	✓	1.475	795	✗	0	0	To Do
Kamke 369	✓	0.088	107	✓	0.114	68	To Do
Kamke 370	✗	0	0	✗	0	0	To Do
Kamke 371	✓	0.057	37	✓	0.126	20	To Do
Kamke 372	✓	0.004	27	✓	0.05	232	To Do
Kamke 373	✓	0.326	185	✓	0.238	49	To Do
Kamke 374	✓	0.087	73	✓	0.036	85	To Do
Kamke 375	✓	0.034	71	✓	0.032	49	To Do
Kamke 376	✓	0.262	110	✓	0.768	279	To Do
Kamke 377	✓	0.004	19	✓	0.021	24	To Do
Kamke 378	✓	0.004	18	✓	0.025	20	To Do
Kamke 379	✓	0.004	18	✓	0.022	22	To Do
Kamke 380	✓	0.405	1757	✓	0.04	619	To Do
Kamke 381	✓	0.424	1757	✓	0.035	579	To Do
Kamke 382	✓	0.185	201	✓	0.037	146	To Do
Kamke 383	✓	5.636	1118	✗	0	0	To Do
Kamke 384	✓	0.005	24	✓	0.03	50	To Do
Kamke 385	✓	0.735	6217	✓	0.263	169	To Do
Kamke 386	✓	0.194	119	✓	0.556	27	To Do
Kamke 387	✓	2.393	190	✓	0.673	115	To Do
Kamke 388	✓	0.715	61	✓	0.084	223	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 389	✓	0.081	57	✓	0.529	71	To Do
Kamke 390	✓	1.871	154	✓	0.418	281	To Do
Kamke 391	✓	0.016	29	✓	0.01	22	To Do
Kamke 392	✓	0.224	27	✓	0.502	50	To Do
Kamke 393	✓	0.104	31	✓	0.175	77	To Do
Kamke 394	✓	0.374	89	✓	5.127	109	To Do
Kamke 395	✗	0	0	✗	0	0	To Do
Kamke 396	✓	0.07	29	✓	0.017	20	To Do
Kamke 397	✓	0.751	143	✓	0.343	128	To Do
Kamke 398	✓	1.756	258	✓	2.125	138	To Do
Kamke 399	✓	0.004	20	✓	0.026	22	To Do
Kamke 400	✓	0.482	135	✓	0.186	74	To Do
Kamke 401	✓	0.33	1093	✓	0.044	580	To Do
Kamke 402	✓	6.552	1211	✓	0.154	101	To Do
Kamke 403	✓	0.253	116	✓	0.815	247	To Do
Kamke 404	✓	2.58	795	✓	0.26	389	To Do
Kamke 405	✓	1.312	61	✓	0.19	375	To Do
Kamke 406	✓	0.866	57	✓	0.094	262	To Do
Kamke 407	✓	0.036	51	✓	0.036	39	To Do
Kamke 408	✓	0.288	97	✓	0.059	73	To Do
Kamke 409	✓	2.373	50	✓	0.119	63	To Do
Kamke 410	✓	30.857	90	✓	0.125	64	To Do
Kamke 411	✓	0.264	99	✓	0.059	65	To Do
Kamke 412	✓	0.45	3229	✓	0.052	146	To Do
Kamke 413	✓	0.13	673	✓	0.184	269	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 414	✓	0.152	107	✓	0.19	269	To Do
Kamke 415	✓	0.343	25	✓	0.237	95	To Do
Kamke 416	✓	2.465	383	✓	0.075	136	To Do
Kamke 417	✓	0.004	16	✓	0.04	35	To Do
Kamke 418	✓	0.993	220	✓	0.051	42	To Do
Kamke 419	✓	1.145	9073	✓	0.046	109	To Do
Kamke 420	✓	0.308	777	✓	0.053	689	To Do
Kamke 421	✓	0.051	27	✓	0.046	32	To Do
Kamke 422	✓	0.085	81	✓	0.043	30	To Do
Kamke 423	✓	0.095	59	✓	0.049	44	To Do
Kamke 424	✓	0.892	223	✓	0.115	193	To Do
Kamke 425	✓	0.005	21	✓	0.041	45	To Do
Kamke 426	✓	0.005	28	✓	0.043	51	To Do
Kamke 427	✓	0.005	24	✓	0.045	60	To Do
Kamke 428	✓	0.006	22	✓	0.075	66	To Do
Kamke 429	✓	0.006	30	✓	0.072	72	To Do
Kamke 430	✓	254.175	507	✓	2.898	1602	To Do
Kamke 431	✓	0.059	81	✓	0.208	62	To Do
Kamke 432	✓	1.278	70	✓	10.385	242	To Do
Kamke 433	✓	1.208	22	✓	0.216	34	To Do
Kamke 434	✓	0.053	27	✓	0.007	7	To Do
Kamke 435	✓	0.084	61	✓	0.221	22	To Do
Kamke 436	✓	0.071	26	✓	2.895	61	To Do
Kamke 437	✓	0.366	47	✓	0.05	36	To Do
Kamke 438	✓	0.028	21	✓	0.013	17	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 439	✓	0.051	49	✓	0.062	33	To Do
Kamke 440	✓	0.027	19	✓	0.012	15	To Do
Kamke 441	✓	0.101	65	✓	1.013	137	To Do
Kamke 442	✓	0.034	28	✓	0.026	21	To Do
Kamke 443	✓	2.977	10121	✓	1.482	221	To Do
Kamke 444	✓	0.43	75	✓	0.837	120	To Do
Kamke 445	✓	0.071	49	✓	0.018	35	To Do
Kamke 446	✓	0.045	45	✓	0.054	57	To Do
Kamke 447	✓	0.011	89	✓	0.035	33	To Do
Kamke 448	✓	0.133	349	✗	0	0	To Do
Kamke 449	✓	0.035	27	✓	0.017	23	To Do
Kamke 450	✓	0.369	26	✓	0.727	51	To Do
Kamke 451	✓	0.057	51	✓	0.065	78	To Do
Kamke 452	✓	0.018	23	✗	0	0	To Do
Kamke 453	✓	1.062	327	✓	2.323	229	To Do
Kamke 454	✓	0.423	241	✓	0.154	106	To Do
Kamke 455	✓	0.959	123	✓	0.445	66	To Do
Kamke 456	✓	0.15	421	✓	0.395	33	To Do
Kamke 457	✓	0.453	118	✓	0.957	45	To Do
Kamke 458	✓	0.023	120	✓	0.061	90	To Do
Kamke 459	✓	3.251	271	✓	0.625	65	To Do
Kamke 460	✗	0	0	✗	0	0	To Do
Kamke 461	✗	0	0	✗	0	0	To Do
Kamke 462	✓	0.032	43	✓	0.044	27	To Do
Kamke 463	✓	0.052	47	✓	0.112	50	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 514	✓	10.673	605	✓	0.266	87	To Do
Kamke 515	✓	2.193	1922	✓	1.44	117	To Do
Kamke 516	✓	0.573	253	✓	0.979	139	To Do
Kamke 517	✓	0.606	283	✓	0.972	78	To Do
Kamke 518	✓	0.591	236	✓	0.462	126	To Do
Kamke 519	✓	0.702	353	✓	0.537	197	To Do
Kamke 520	✓	0.025	1115	✓	0.336	249	To Do
Kamke 521	✓	0.004	14	✓	0.036	33	To Do
Kamke 522	✓	0.009	20	✓	0.2	44	To Do
Kamke 523	✓	145.417	392	✓	0.223	231	To Do
Kamke 524	✓	0.02	422	✓	0.207	243	To Do
Kamke 525	✓	0.089	135	✓	0.237	122	To Do
Kamke 526	✓	0.077	45	✓	0.016	32	To Do
Kamke 527	✓	0.017	20	✓	0.929	43	To Do
Kamke 528	✓	0.989	398	✓	0.124	86	To Do
Kamke 529	✓	43.474	1758	✓	0.195	1251	To Do
Kamke 530	✓	8.896	648	✓	0.25	424	To Do
Kamke 531	✗	0	0	✗	0	0	To Do
Kamke 532	✓	0.029	1124	✓	0.478	874	To Do
Kamke 533	✓	0.005	16	✓	0.197	76	To Do
Kamke 534	✓	0.063	114	✓	0.059	84	To Do
Kamke 535	✓	0.05	49	✓	0.052	51	To Do
Kamke 536	✓	0.016	64	✓	0.04	52	To Do
Kamke 537	✓	0.036	16	✓	4.172	250	To Do
Kamke 538	✓	9.984	113	✓	1.191	1625	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 539	✓	0.043	45	✓	0.056	32	To Do
Kamke 540	✓	0.038	69	✓	0.076	109	To Do
Kamke 541	✓	0.031	39	✓	0.533	103	To Do
Kamke 542	✓	0.034	20	✓	0.522	107	To Do
Kamke 543	✓	0.034	55	✗	0	0	To Do
Kamke 544	✓	0.083	22	✓	0.822	4201	To Do
Kamke 545	✓	0.926	323	✓	0.214	144	To Do
Kamke 546	✓	0.042	113	✓	0.099	171	To Do
Kamke 547	✓	3.173	490	✓	0.254	118	To Do
Kamke 548	✓	1.095	479	✓	0.413	250	To Do
Kamke 549	✓	0.252	216	✓	0.758	553	To Do
Kamke 550	✓	0.832	488	✓	0.273	60	To Do
Kamke 551	✓	0.185	86	✓	0.622	55	To Do
Kamke 552	✓	0.053	41	✓	0.087	43	To Do
Kamke 553	✓	0.271	56	✓	0.063	36	To Do
Kamke 554	✓	0.171	54	✓	0.39	32	To Do
Kamke 555	✓	0.007	20	✓	0.033	15	To Do
Kamke 556	✓	5.329	67	✓	0.213	581	To Do
Kamke 557	✓	0.049	39	✓	0.26	74	To Do
Kamke 558	✓	1.131	327	✓	0.341	223	To Do
Kamke 559	✓	0.4	212	✓	0.248	215	To Do
Kamke 560	✓	17.17	110	✓	0.833	1120	To Do
Kamke 561	✓	2.054	2138	✓	2.26	50	To Do
Kamke 562	✓	0.157	84	✓	0.185	3306	To Do
Kamke 563	✓	0.232	59	✓	0.225	66	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 564	✓	0.005	17	✓	0.029	32	To Do
Kamke 565	✓	0.03	25	✓	0.257	17	To Do
Kamke 566	✓	0.045	38	✓	0.041	16	To Do
Kamke 567	✓	0.09	49	✓	0.055	18	To Do
Kamke 568	✓	0.105	34	✓	0.067	32	To Do
Kamke 569	✓	0.111	59	✓	0.355	147	To Do
Kamke 570	✓	1.935	58	✓	0.065	30	To Do
Kamke 571	✓	0.163	124	✓	0.517	169	To Do
Kamke 572	✗	0	0	✗	0	0	To Do
Kamke 573	✓	0.029	42	✓	0.125	24	To Do
Kamke 574	✓	0.038	102	✓	0.141	41	To Do
Kamke 575	✗	0	0	✗	0	0	To Do
Kamke 576	✗	0	0	✗	0	0	To Do
Kamke 577	✓	0.309	243	✓	0.039	28	To Do
Kamke 578	✓	0.257	100	✓	0.049	22	To Do
Kamke 579	✓	0.322	514	✓	0.052	35	To Do
Kamke 580	✓	0.33	203	✓	0.07	31	To Do
Kamke 581	✓	0.304	144	✓	0.101	32	To Do
Kamke 582	✓	0.31	142	✓	0.237	30	To Do
Kamke 583	✓	0.278	126	✓	0.148	31	To Do
Kamke 584	✓	0.331	115	✓	0.107	35	To Do
Kamke 585	✓	0.275	205	✓	0.451	120	To Do
Kamke 586	✓	0.728	975	✓	0.251	39	To Do
Kamke 587	✓	0.299	123	✓	0.119	29	To Do
Kamke 588	✓	0.235	109	✓	0.121	53	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 589	✓	0.3	245	✓	0.155	38	To Do
Kamke 590	✓	0.286	94	✓	0.132	28	To Do
Kamke 591	✓	0.593	253	✓	0.226	108	To Do
Kamke 592	✓	0.712	241	✓	0.168	33	To Do
Kamke 593	✓	0.607	221	✓	0.274	35	To Do
Kamke 594	✓	0.48	236	✓	0.129	67	To Do
Kamke 595	✓	0.457	204	✓	0.165	72	To Do
Kamke 596	✓	0.35	156	✓	0.092	26	To Do
Kamke 597	✓	0.432	130	✓	0.421	37	To Do
Kamke 598	✓	0.189	37	✓	0.02	29	To Do
Kamke 599	✓	0.191	95	✓	0.095	57	To Do
Kamke 600	✓	0.323	246	✓	0.137	38	To Do
Kamke 601	✓	0.297	182	✓	0.124	61	To Do
Kamke 602	✓	0.596	167	✓	0.121	33	To Do
Kamke 603	✓	0.41	117	✓	0.138	27	To Do
Kamke 604	✓	0.456	143	✓	0.158	30	To Do
Kamke 605	✓	0.557	145	✓	0.121	29	To Do
Kamke 606	✓	0.69	361	✓	0.615	34	To Do
Kamke 607	✓	0.267	121	✓	0.096	22	To Do
Kamke 608	✓	0.596	274	✓	0.157	40	To Do
Kamke 609	✓	0.318	117	✓	0.177	22	To Do
Kamke 610	✓	0.106	25	✓	0.011	20	To Do
Kamke 611	✓	0.333	191	✓	0.083	28	To Do
Kamke 612	✓	0.452	199	✓	0.125	27	To Do
Kamke 613	✓	0.35	226	✓	0.105	23	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 614	✓	0.463	177	✓	0.393	60	To Do
Kamke 615	✓	0.311	77	✓	0.125	26	To Do
Kamke 616	✓	0.583	177	✓	0.095	26	To Do
Kamke 617	✓	1.237	615	✓	0.243	47	To Do
Kamke 618	✓	0.235	25	✓	0.513	34	To Do
Kamke 619	✓	0.837	330	✓	0.373	81	To Do
Kamke 620	✓	0.937	205	✓	0.227	37	To Do
Kamke 621	✓	0.074	445	✓	0.244	59	To Do
Kamke 622	✓	0.368	140	✓	0.214	77	To Do
Kamke 623	✓	0.307	77	✓	0.255	49	To Do
Kamke 624	✓	32.167	9837	✓	33.316	46	To Do
Kamke 625	✓	0.348	56	✓	0.299	53	To Do
Kamke 626	✓	0.245	88	✓	0.437	115	To Do
Kamke 627	✓	0.955	25	✓	0.253	35	To Do
Kamke 628	✓	0.197	38	✓	0.31	23	To Do
Kamke 629	✓	0.989	47	✓	0.251	62	To Do
Kamke 630	✓	0.502	101	✓	0.386	98	To Do
Kamke 631	✓	0.237	38	✓	0.249	23	To Do
Kamke 632	✓	0.243	65	✓	0.331	54	To Do
Kamke 633	✓	0.267	85	✓	1.121	52	To Do
Kamke 634	✓	0.246	33	✓	0.338	26	To Do
Kamke 635	✓	0.226	33	✓	0.277	22	To Do
Kamke 636	✓	0.143	24	✓	0.212	19	To Do
Kamke 637	✓	11.405	59	✓	0.197	84	To Do
Kamke 638	✓	0.146	41	✓	0.196	35	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 639	✓	0.193	53	✓	0.247	50	To Do
Kamke 640	✓	0.31	53	✓	0.367	47	To Do
Kamke 641	✓	0.251	35	✓	0.339	26	To Do
Kamke 642	✓	0.191	105	✓	0.275	286	To Do
Kamke 643	✓	0.23	31	✓	0.24	22	To Do
Kamke 644	✓	0.41	34	✓	0.476	27	To Do
Kamke 645	✓	0.096	20	✓	0.156	14	To Do
Kamke 646	✓	0.33	35	✓	0.379	23	To Do
Kamke 647	✓	0.418	115	✓	0.302	460	To Do
Kamke 648	✓	0.589	128	✓	0.843	41	To Do
Kamke 649	✓	0.306	37	✓	0.296	27	To Do
Kamke 650	✓	0.577	109	✓	0.312	28	To Do
Kamke 651	✓	0.112	16	✓	0.118	13	To Do
Kamke 652	✓	1.348	101	✓	0.177	27	To Do
Kamke 653	✓	0.843	94	✓	0.262	24	To Do
Kamke 654	✓	0.371	37	✓	0.372	23	To Do
Kamke 655	✓	14.457	82	✓	0.76	66	To Do
Kamke 656	✓	0.141	20	✓	0.132	15	To Do
Kamke 657	✓	0.298	37	✓	0.271	26	To Do
Kamke 658	✓	0.531	41	✓	0.422	28	To Do
Kamke 659	✓	0.989	164	✓	0.351	41	To Do
Kamke 660	✓	0.594	109	✓	0.322	29	To Do
Kamke 661	✓	0.955	164	✓	0.349	39	To Do
Kamke 662	✓	0.316	37	✓	0.301	26	To Do
Kamke 663	✓	1.71	101	✓	0.169	27	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 664	✓	1.024	94	✓	0.243	25	To Do
Kamke 665	✓	0.533	41	✓	0.694	28	To Do
Kamke 666	✓	0.168	29	✓	0.249	24	To Do
Kamke 667	✓	1.159	90	✓	0.303	82	To Do
Kamke 668	✓	0.756	78	✓	0.86	58	To Do
Kamke 669	✓	0.834	264	✓	0.195	72	To Do
Kamke 670	✓	0.591	62	✓	0.365	70	To Do
Kamke 671	✓	0.409	192	✓	0.273	237	To Do
Kamke 672	✓	2.205	4512	✓	0.195	36	To Do
Kamke 673	✓	0.363	23	✓	0.422	17	To Do
Kamke 674	✓	1.012	91	✓	0.44	27	To Do
Kamke 675	✓	0.395	48	✓	0.073	37	To Do
Kamke 676	✓	0.533	144	✓	0.643	43	To Do
Kamke 677	✓	0.232	80	✓	0.085	48	To Do
Kamke 678	✓	0.437	101	✓	0.475	37	To Do
Kamke 679	✓	0.195	59	✓	0.061	37	To Do
Kamke 680	✓	1.234	89	✓	0.436	28	To Do
Kamke 681	✓	0.269	84	✓	0.095	45	To Do
Kamke 682	✓	0.346	39	✓	0.356	28	To Do
Kamke 683	✓	1.225	84	✓	0.168	114	To Do
Kamke 684	✓	0.137	20	✓	4.142	30	To Do
Kamke 685	✓	0.194	87	✓	0.097	48	To Do
Kamke 686	✓	11.212	68	✓	0.198	85	To Do
Kamke 687	✓	0.197	130	✓	0.119	39	To Do
Kamke 688	✓	0.43	82	✓	0.119	42	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 689	✓	0.395	60	✓	0.069	25	To Do
Kamke 690	✓	0.613	105	✓	0.527	40	To Do
Kamke 691	✓	0.345	21	✓	0.626	17	To Do
Kamke 692	✓	0.131	20	✓	3.915	30	To Do
Kamke 693	✓	0.424	146	✓	0.11	40	To Do
Kamke 694	✓	0.49	66	✓	0.527	30	To Do
Kamke 695	✓	0.253	34	✓	0.066	39	To Do
Kamke 696	✓	1.115	51	✓	0.077	32	To Do
Kamke 697	✓	0.365	114	✓	0.125	40	To Do
Kamke 698	✓	0.377	108	✓	0.094	34	To Do
Kamke 699	✓	0.403	101	✓	0.485	36	To Do
Kamke 700	✓	0.171	76	✓	0.128	62	To Do
Kamke 701	✓	2.112	88	✓	8.82	71	To Do
Kamke 702	✓	2.86	37	✓	0.097	35	To Do
Kamke 703	✓	0.609	101	✓	0.234	44	To Do
Kamke 704	✓	9.054	66	✓	0.072	38	To Do
Kamke 705	✓	0.168	30	✓	0.214	24	To Do
Kamke 706	✓	40.533	610	✓	0.648	65	To Do
Kamke 707	✓	5.565	1391	✓	0.543	105	To Do
Kamke 708	✓	0.408	89	✗	0	0	To Do
Kamke 709	✓	3.549	217	✓	0.279	39	To Do
Kamke 710	✓	1.32	38	✓	3.191	31	To Do
Kamke 711	✓	0.263	28	✓	0.266	31	To Do
Kamke 712	✓	0.515	105	✓	0.493	38	To Do
Kamke 713	✓	0.143	649	✓	0.385	86	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 714	✓	1.585	162	✓	0.276	96	To Do
Kamke 715	✓	0.609	104	✓	0.449	39	To Do
Kamke 716	✓	3.305	133	✓	0.294	37	To Do
Kamke 717	✓	0.748	106	✓	0.513	33	To Do
Kamke 718	✓	0.383	127	✓	0.069	44	To Do
Kamke 719	✓	0.501	49	✓	0.147	34	To Do
Kamke 720	✓	3.743	314	✓	0.2	48	To Do
Kamke 721	✓	0.134	27	✓	0.089	19	To Do
Kamke 722	✓	32.239	490	✓	0.493	70	To Do
Kamke 723	✓	0.254	663	✓	0.122	856	To Do
Kamke 724	✓	20.114	422	✓	0.065	18	To Do
Kamke 725	✓	0.361	19	✓	0.942	25	To Do
Kamke 726	✓	0.135	625	✓	0.296	83	To Do
Kamke 727	✓	0.57	29	✓	0.282	25	To Do
Kamke 728	✓	0.364	72	✓	0.388	50	To Do
Kamke 729	✓	0.317	327	✓	0.121	404	To Do
Kamke 730	✓	0.339	83	✓	1.445	41	To Do
Kamke 731	✓	0.542	47	✓	0.255	42	To Do
Kamke 732	✓	0.885	116	✓	0.551	43	To Do
Kamke 733	✓	13.337	73	✗	0	0	To Do
Kamke 734	✓	0.282	37	✓	0.481	39	To Do
Kamke 735	✓	1.609	573	✓	0.119	78	To Do
Kamke 736	✓	0.236	31	✓	0.256	43	To Do
Kamke 737	✓	0.041	36	✓	0.313	29	To Do
Kamke 738	✓	0.53	1347	✓	1.059	1054	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 739	✓	0.448	39	✓	0.25	35	To Do
Kamke 740	✓	0.125	74	✓	0.096	72	To Do
Kamke 741	✓	2.06	175	✓	1.13	246	To Do
Kamke 742	✓	3.777	3913	✓	1.543	239	To Do
Kamke 743	✓	0.108	406	✓	0.521	296	To Do
Kamke 744	✓	0.175	510	✓	0.18	621	To Do
Kamke 745	✓	1.495	546	✓	0.085	78	To Do
Kamke 746	✓	0.095	274	✓	0.469	232	To Do
Kamke 747	✓	3.219	88	✓	0.31	75	To Do
Kamke 748	✓	0.348	285	✓	0.105	404	To Do
Kamke 749	✓	0.191	126	✓	0.123	192	To Do
Kamke 750	✓	0.346	72	✓	0.335	49	To Do
Kamke 751	✓	0.191	30	✓	0.162	26	To Do
Kamke 752	✓	0.243	849	✓	1.611	723	To Do
Kamke 753	✓	0.187	41	✓	0.258	38	To Do
Kamke 754	✓	0.14	47	✓	0.022	26	To Do
Kamke 755	✓	0.364	2633	✓	0.134	44	To Do
Kamke 756	✓	0.199	95	✓	0.036	37	To Do
Kamke 757	✓	0.043	36	✓	0.1	26	To Do
Kamke 758	✓	0.913	459	✓	0.278	41	To Do
Kamke 759	✓	0.104	498	✓	0.589	305	To Do
Kamke 760	✓	1.044	112	✓	2.026	137	To Do
Kamke 761	✓	0.049	33	✓	0.09	18	To Do
Kamke 762	✓	0.225	26	✓	0.176	22	To Do
Kamke 763	✓	0.178	22	✓	0.122	14	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 789	✓	76.965	120	✗	0	0	To Do
Kamke 790	✓	92.886	127	✗	0	0	To Do
Kamke 791	✓	8.216	110	✓	6.811	306	To Do
Kamke 792	✓	3.101	157	✓	0.437	112	To Do
Kamke 793	✓	14.854	399	✓	2.147	32	To Do
Kamke 794	✓	0.23	67	✓	0.59	32	To Do
Kamke 795	✓	0.505	111	✓	0.029	37	To Do
Kamke 796	✓	11.607	102	✓	1.647	143	To Do
Kamke 797	✓	2.137	349	✓	0.696	168	To Do
Kamke 798	✓	0.624	27	✓	0.208	30	To Do
Kamke 799	✓	1.296	126	✓	0.309	147	To Do
Kamke 800	✓	0.49	128	✓	0.024	41	To Do
Kamke 801	✓	0.416	126	✓	0.07	63	To Do
Kamke 802	✓	0.149	101	✓	0.272	27	To Do
Kamke 803	✓	0.488	637	✓	0.442	65	To Do
Kamke 804	✓	0.677	43	✓	0.986	38	To Do
Kamke 805	✓	0.197	37	✓	0.761	42	To Do
Kamke 806	✓	0.48	22	✓	0.587	22	To Do
Kamke 807	✓	0.322	59	✓	0.522	43	To Do
Kamke 808	✓	2.348	149	✓	0.299	45	To Do
Kamke 809	✓	0.393	128	✓	0.022	41	To Do
Kamke 810	✓	0.135	40	✓	0.065	16	To Do
Kamke 811	✓	2.043	33	✓	1.769	32	To Do
Kamke 812	✓	0.44	78	✓	0.389	30	To Do
Kamke 813	✓	0.66	93	✓	0.707	40	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 814	✓	0.152	72	✓	0.039	38	To Do
Kamke 815	✓	12.858	99	✓	0.975	168	To Do
Kamke 816	✓	0.288	74	✓	0.611	190	To Do
Kamke 817	✓	0.678	63	✓	0.824	27	To Do
Kamke 818	✓	0.266	34	✓	0.317	34	To Do
Kamke 819	✓	0.354	65	✓	0.429	30	To Do
Kamke 820	✓	0.573	63	✓	0.737	27	To Do
Kamke 821	✓	0.242	2093	✓	0.177	27	To Do
Kamke 822	✓	0.364	32	✓	0.146	25	To Do
Kamke 823	✓	0.486	39	✓	0.234	38	To Do
Kamke 824	✓	0.281	68	✓	0.392	61	To Do
Kamke 825	✓	0.588	148	✓	0.126	48	To Do
Kamke 826	✓	0.58	70	✓	0.426	51	To Do
Kamke 827	✓	0.306	221	✓	0.341	49	To Do
Kamke 828	✓	0.687	56	✓	0.318	54	To Do
Kamke 829	✓	0.532	74	✓	0.431	34	To Do
Kamke 830	✓	0.514	37	✓	0.23	38	To Do
Kamke 831	✓	3.378	145	✓	0.386	35	To Do
Kamke 832	✓	2.375	2497	✓	0.189	31	To Do
Kamke 833	✓	0.305	221	✓	0.301	49	To Do
Kamke 834	✓	0.687	90	✓	0.736	60	To Do
Kamke 835	✗	0	0	✗	0	0	To Do
Kamke 836	✓	14.782	379	✓	0.301	73	To Do
Kamke 837	✗	0	0	✗	0	0	To Do
Kamke 838	✓	0.197	31	✓	0.118	25	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 839	✓	0.346	28	✓	0.076	19	To Do
Kamke 840	✓	0.341	30	✓	0.069	19	To Do
Kamke 841	✓	1.31	236	✓	0.295	97	To Do
Kamke 842	✓	0.266	186	✓	0.033	43	To Do
Kamke 843	✓	0.24	198	✓	0.03	43	To Do
Kamke 844	✓	15.811	386	✓	0.304	97	To Do
Kamke 845	✓	4.348	266	✓	0.234	44	To Do
Kamke 846	✓	0.927	365	✓	0.306	40	To Do
Kamke 847	✓	1.442	103	✓	0.384	34	To Do
Kamke 848	✓	0.268	157	✓	0.545	27	To Do
Kamke 849	✓	1.277	102	✓	0.388	33	To Do
Kamke 850	✓	0.406	1485	✓	1.058	32	To Do
Kamke 851	✓	0.439	145	✓	0.061	42	To Do
Kamke 852	✓	0.417	145	✓	0.069	42	To Do
Kamke 853	✓	0.198	76	✓	0.04	63	To Do
Kamke 854	✓	0.168	24	✓	0.272	51	To Do
Kamke 855	✓	0.16	24	✓	0.244	51	To Do
Kamke 856	✓	0.634	103	✓	0.318	65	To Do
Kamke 857	✓	1.042	107	✓	0.362	32	To Do
Kamke 858	✓	0.442	145	✓	0.056	42	To Do
Kamke 859	✓	0.895	105	✓	0.265	63	To Do
Kamke 860	✓	0.404	33	✓	1.744	29	To Do
Kamke 861	✓	1.56	158	✓	0.158	26	To Do
Kamke 862	✗	0	0	✓	0.207	27	To Do
Kamke 863	✓	0.244	30	✓	14.23	38	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 889	✓	1.83	68	✓	0.911	49	To Do
Kamke 890	✓	0.213	103	✓	0.655	34	To Do
Kamke 891	✓	0.21	135	✓	0.065	56	To Do
Kamke 892	✓	2.491	1283	✓	0.635	40	To Do
Kamke 893	✓	0.212	80	✓	0.029	41	To Do
Kamke 894	✗	0	0	✗	0	0	To Do
Kamke 895	✓	0.315	81	✓	0.062	79	To Do
Kamke 896	✓	0.35	106	✓	0.511	63	To Do
Kamke 897	✓	0.256	79	✓	0.086	87	To Do
Kamke 898	✓	0.236	106	✓	0.043	87	To Do
Kamke 899	✓	0.266	106	✓	0.035	47	To Do
Kamke 900	✓	0.237	381	✓	0.074	48	To Do
Kamke 901	✓	0.7	33	✓	0.547	30	To Do
Kamke 902	✓	0.245	295	✓	0.224	183	To Do
Kamke 903	✓	0.14	19	✓	0.168	48	To Do
Kamke 904	✓	0.123	23	✓	0.177	64	To Do
Kamke 905	✓	0.263	85	✓	0.044	46	To Do
Kamke 906	✓	0.161	326	✓	0.298	37	To Do
Kamke 907	✓	0.286	22	✓	0.168	20	To Do
Kamke 908	✓	1.143	1269	✓	0.343	1742	To Do
Kamke 909	✓	0.186	64	✓	0.559	84	To Do
Kamke 910	✓	0.237	98	✓	0.029	42	To Do
Kamke 911	✓	0.932	106	✓	0.796	30	To Do
Kamke 912	✓	1.37	201	✗	0	0	To Do
Kamke 913	✓	0.908	716	✓	0.086	43	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 914	✓	1.301	401	✓	3.685	71	To Do
Kamke 915	✓	0.985	724	✓	0.083	43	To Do
Kamke 916	✓	0.251	43	✓	0.318	73	To Do
Kamke 917	✓	0.25	28	✓	0.218	38	To Do
Kamke 918	✓	0.591	720	✓	1.016	41	To Do
Kamke 919	✓	0.356	251	✓	0.167	61	To Do
Kamke 920	✓	0.468	301	✗	0	0	To Do
Kamke 921	✓	0.296	92	✓	0.196	30	To Do
Kamke 922	✓	1.428	882	✓	0.18	47	To Do
Kamke 923	✓	1.629	432	✓	0.413	36	To Do
Kamke 924	✓	0.313	80	✓	0.128	46	To Do
Kamke 925	✓	2.255	228	✓	0.312	38	To Do
Kamke 926	✓	0.234	128	✓	0.059	67	To Do
Kamke 927	✓	0.737	112	✓	0.114	68	To Do
Kamke 928	✓	1.269	23	✓	0.294	20	To Do
Kamke 929	✓	0.94	683	✓	0.038	42	To Do
Kamke 930	✓	1.598	39	✓	0.434	36	To Do
Kamke 931	✓	0.212	80	✓	0.038	73	To Do
Kamke 932	✓	5.822	4323	✓	0.133	54	To Do
Kamke 933	✓	0.279	99	✓	0.061	39	To Do
Kamke 934	✓	0.324	102	✓	0.068	39	To Do
Kamke 935	✓	62.194	248	✓	0.212	55	To Do
Kamke 936	✓	0.353	99	✓	0.068	39	To Do
Kamke 937	✓	0.33	124	✓	0.092	79	To Do
Kamke 938	✓	0.277	108	✓	0.029	39	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 939	✓	0.594	136	✓	0.085	70	To Do
Kamke 940	✓	0.284	80	✓	0.069	63	To Do
Kamke 941	✓	0.509	53	✓	0.055	35	To Do
Kamke 942	✓	4.933	349	✓	0.71	43	To Do
Kamke 943	✓	0.594	53	✓	0.056	40	To Do
Kamke 944	✓	1.92	233	✓	0.08	47	To Do
Kamke 945	✓	1.459	213	✓	0.059	41	To Do
Kamke 946	✓	0.748	150	✓	0.15	85	To Do
Kamke 947	✓	0.422	30	✓	0.273	44	To Do
Kamke 948	✓	0.751	39	✓	0.323	68	To Do
Kamke 949	✓	0.235	76	✓	0.048	81	To Do
Kamke 950	✓	0.56	141	✓	0.085	42	To Do
Kamke 951	✓	0.46	140	✓	0.07	41	To Do
Kamke 952	✓	0.419	341	✓	0.263	62	To Do
Kamke 953	✓	0.285	36	✓	0.41	145	To Do
Kamke 954	✓	0.386	115	✓	0.09	53	To Do
Kamke 955	✓	0.365	112	✓	0.109	101	To Do
Kamke 956	✓	1.338	28	✓	0.181	79	To Do
Kamke 957	✓	1.324	28	✓	0.112	79	To Do
Kamke 958	✓	0.481	82	✓	0.071	40	To Do
Kamke 959	✓	0.257	20	✓	0.154	15	To Do
Kamke 960	✓	0.198	14	✓	0.152	11	To Do
Kamke 961	✓	5.859	813	✓	0.437	45	To Do
Kamke 962	✓	5.323	1191	✓	1.338	79	To Do
Kamke 963	✓	0.546	108	✓	0.196	39	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 1049	✓	0.06	109	✓	0.182	66	To Do
Kamke 1050	✓	0.011	23	✓	0.026	14	To Do
Kamke 1051	✓	0.023	44	✓	0.036	27	To Do
Kamke 1052	✓	0.016	78	✓	0.164	58	To Do
Kamke 1053	✓	0.026	57	✓	0.054	35	To Do
Kamke 1054	✓	0.045	172	✓	0.122	98	To Do
Kamke 1055	✓	0.14	421	✓	0.289	262	To Do
Kamke 1056	✓	0.052	41	✓	0.101	52	To Do
Kamke 1057	✓	0.056	56	✓	0.264	50	To Do
Kamke 1058	✓	0.064	56	✓	0.261	29	To Do
Kamke 1059	✓	0.088	39	✓	0.085	56	To Do
Kamke 1060	✓	0.031	83	✓	0.293	81	To Do
Kamke 1061	✓	0.067	70	✓	0.104	28	To Do
Kamke 1062	✓	0.021	35	✓	0.023	19	To Do
Kamke 1063	✓	0.049	28	✓	0.31	34	To Do
Kamke 1064	✓	0.519	1400	✓	0.445	134	To Do
Kamke 1065	✓	0.158	114	✓	0.271	60	To Do
Kamke 1066	✓	0.026	18	✓	0.151	15	To Do
Kamke 1067	✓	0.028	21	✓	0.053	17	To Do
Kamke 1068	✓	0.13	20	✓	0.288	45	To Do
Kamke 1069	✓	0.031	19	✓	0.151	15	To Do
Kamke 1070	✓	0.285	143	✓	0.231	60	To Do
Kamke 1071	✓	0.074	44	✓	0.077	24	To Do
Kamke 1072	✗	0	0	✗	0	0	To Do
Kamke 1073	✗	0	0	✗	0	0	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 1166	✓	0.011	23	✓	0.013	21	To Do
Kamke 1167	✓	0.059	326	✓	0.177	63	To Do
Kamke 1168	✓	0.022	15	✓	0.007	11	To Do
Kamke 1169	✓	0.061	236	✓	0.189	49	To Do
Kamke 1170	✓	0.017	58	✓	0.142	43	To Do
Kamke 1171	✓	0.04	142	✓	0.221	49	To Do
Kamke 1172	✓	0.067	158	✓	0.154	47	To Do
Kamke 1173	✓	0.131	74	✓	0.213	37	To Do
Kamke 1174	✓	0.008	33	✓	0.026	25	To Do
Kamke 1175	✓	0.228	38	✓	0.087	29	To Do
Kamke 1176	✓	0.017	33	✓	0.03	15	To Do
Kamke 1177	✓	0.83	141	✓	0.123	34	To Do
Kamke 1178	✓	0.037	74	✓	0.033	23	To Do
Kamke 1179	✓	0.018	38	✓	0.033	19	To Do
Kamke 1180	✓	0.066	75	✓	0.065	49	To Do
Kamke 1181	✓	0.047	37	✓	0.021	25	To Do
Kamke 1182	✓	0.011	24	✓	0.013	20	To Do
Kamke 1183	✓	0.01	27	✓	0.013	22	To Do
Kamke 1184	✓	0.008	38	✓	0.026	25	To Do
Kamke 1185	✓	0.03	67	✓	0.024	33	To Do
Kamke 1186	✓	0.016	42	✓	0.036	36	To Do
Kamke 1187	✓	0.01	99	✓	0.011	53	To Do
Kamke 1188	✓	0.169	266	✓	0.273	114	To Do
Kamke 1189	✓	0.058	445	✓	0.164	79	To Do
Kamke 1190	✓	0.023	122	✓	0.192	38	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 1191	✓	0.008	110	✓	0.015	23	To Do
Kamke 1192	✓	0.089	40	✓	0.22	51	To Do
Kamke 1193	✓	0.213	44	✓	0.033	38	To Do
Kamke 1194	✓	0.1	65	✓	0.048	48	To Do
Kamke 1195	✓	0.029	80	✓	0.23	93	To Do
Kamke 1196	✓	0.029	37	✓	0.034	31	To Do
Kamke 1197	✓	0.015	78	✓	0.132	43	To Do
Kamke 1198	✓	0.047	41	✓	0.034	37	To Do
Kamke 1199	✓	0.015	41	✓	0.017	35	To Do
Kamke 1200	✓	0.016	62	✓	0.017	27	To Do
Kamke 1201	✓	0.12	44	✓	0.019	34	To Do
Kamke 1202	✓	0.012	22	✓	0.031	14	To Do
Kamke 1203	✓	0.016	124	✓	0.02	28	To Do
Kamke 1204	✓	0.016	132	✓	0.043	35	To Do
Kamke 1205	✗	0	0	✗	0	0	To Do
Kamke 1206	✓	0.088	120	✓	0.081	76	To Do
Kamke 1207	✓	0.09	294	✓	0.333	110	To Do
Kamke 1208	✓	0.047	59	✓	0.056	35	To Do
Kamke 1209	✓	0.018	67	✓	0.073	41	To Do
Kamke 1210	✓	0.223	252	✓	0.281	81	To Do
Kamke 1211	✓	0.054	68	✓	0.065	36	To Do
Kamke 1212	✗	0	0	✗	0	0	To Do
Kamke 1213	✓	0.088	54	✓	0.234	53	To Do
Kamke 1214	✓	0.227	260	✓	0.26	71	To Do
Kamke 1215	✓	0.123	664	✓	0.318	148	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 1216	✗	0	0	✗	0	0	To Do
Kamke 1217	✓	0.121	30	✓	0.045	24	To Do
Kamke 1218	✓	0.125	38	✓	0.047	30	To Do
Kamke 1219	✓	0.143	218	✓	0.231	69	To Do
Kamke 1220	✓	0.048	98	✓	0.019	40	To Do
Kamke 1221	✓	0.029	42	✓	0.026	35	To Do
Kamke 1222	✓	0.016	30	✓	0.02	23	To Do
Kamke 1223	✓	0.016	25	✓	0.018	19	To Do
Kamke 1224	✓	0.015	30	✓	0.016	23	To Do
Kamke 1225	✓	0.022	29	✓	0.039	23	To Do
Kamke 1226	✓	0.012	30	✓	0.167	25	To Do
Kamke 1227	✓	0.027	21	✓	0.013	16	To Do
Kamke 1228	✓	0.014	82	✓	0.253	53	To Do
Kamke 1229	✓	0.032	48	✓	0.02	31	To Do
Kamke 1230	✓	0.02	82	✓	0.214	36	To Do
Kamke 1231	✓	0.067	58	✓	0.19	52	To Do
Kamke 1232	✓	8.04	6628	✓	0.28	409	To Do
Kamke 1233	✓	0.542	6628	✓	0.255	409	To Do
Kamke 1234	✓	0.079	48	✗	0	0	To Do
Kamke 1235	✓	0.041	97	✓	0.036	45	To Do
Kamke 1236	✗	0	0	✗	0	0	To Do
Kamke 1237	✓	0.025	30	✓	0.017	20	To Do
Kamke 1238	✓	0.031	36	✓	0.026	26	To Do
Kamke 1239	✓	0.011	46	✓	0.168	35	To Do
Kamke 1240	✓	0.014	18	✓	0.15	15	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 1241	✓	0.013	30	✓	0.16	24	To Do
Kamke 1242	✓	0.226	68	✓	0.074	41	To Do
Kamke 1243	✓	0.037	45	✓	0.038	21	To Do
Kamke 1244	✓	0.024	42	✓	0.161	27	To Do
Kamke 1245	✓	0.02	42	✓	0.166	27	To Do
Kamke 1246	✓	0.015	42	✓	0.224	28	To Do
Kamke 1247	✓	0.184	128	✓	0.039	27	To Do
Kamke 1248	✓	0.295	238	✓	0.293	134	To Do
Kamke 1249	✓	0.141	193	✓	0.18	134	To Do
Kamke 1250	✓	0.042	41	✓	0.022	41	To Do
Kamke 1251	✓	0.038	25	✓	0.024	20	To Do
Kamke 1252	✓	0.13	151	✓	0.145	124	To Do
Kamke 1253	✓	0.022	34	✓	0.007	16	To Do
Kamke 1254	✓	0.151	69	✓	0.074	43	To Do
Kamke 1255	✓	0.715	360	✓	0.017	42	To Do
Kamke 1256	✓	0.018	26	✓	0.258	51	To Do
Kamke 1257	✓	0.043	33	✓	0.219	27	To Do
Kamke 1258	✓	0.134	146	✓	0.155	110	To Do
Kamke 1259	✓	0.117	120	✓	0.16	92	To Do
Kamke 1260	✓	0.246	65	✓	0.712	77	To Do
Kamke 1261	✓	0.476	148	✓	0.302	105	To Do
Kamke 1262	✓	0.294	88	✓	0.273	53	To Do
Kamke 1263	✓	0.429	276	✓	0.138	52	To Do
Kamke 1264	✓	0.057	23	✓	0.041	19	To Do
Kamke 1265	✓	0.036	64	✓	1.132	93	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 1266	✓	0.023	22	✓	0.01	19	To Do
Kamke 1267	✓	0.414	166	✓	0.134	41	To Do
Kamke 1268	✓	0.112	50	✓	0.248	39	To Do
Kamke 1269	✓	0.074	60	✓	0.221	40	To Do
Kamke 1270	✓	2.046	58	✓	0.243	46	To Do
Kamke 1271	✓	0.01	27	✓	0.007	14	To Do
Kamke 1272	✓	0.011	32	✓	0.038	23	To Do
Kamke 1273	✓	0.016	20	✓	0.151	17	To Do
Kamke 1274	✓	0.043	38	✓	0.01	19	To Do
Kamke 1275	✓	0.028	120	✓	0.262	53	To Do
Kamke 1276	✓	0.033	55	✓	0.181	31	To Do
Kamke 1277	✓	0.024	51	✓	0.168	27	To Do
Kamke 1278	✗	0	0	✗	0	0	To Do
Kamke 1279	✓	0.116	74	✓	0.169	32	To Do
Kamke 1280	✓	0.048	52	✓	0.148	40	To Do
Kamke 1281	✓	0.016	28	✓	0.066	15	To Do
Kamke 1282	✓	0.018	39	✓	0.034	21	To Do
Kamke 1283	✓	0.069	90	✓	0.141	48	To Do
Kamke 1284	✓	0.032	47	✓	0.019	41	To Do
Kamke 1285	✓	1.232	269	✓	0.11	52	To Do
Kamke 1286	✓	0.107	101	✓	0.02	32	To Do
Kamke 1287	✓	0.015	83	✓	0.033	27	To Do
Kamke 1288	✓	0.028	47	✓	0.023	21	To Do
Kamke 1289	✓	0.125	53	✓	0.111	33	To Do
Kamke 1290	✓	0.125	103	✓	0.033	29	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 1316	✓	0.075	38	✓	0.074	18	To Do
Kamke 1317	✓	0.085	38	✓	0.086	13	To Do
Kamke 1318	✓	0.23	172	✓	0.182	122	To Do
Kamke 1319	✓	0.062	60	✓	0.131	31	To Do
Kamke 1320	✓	0.074	21	✓	0.039	17	To Do
Kamke 1321	✓	0.021	18	✓	0.023	15	To Do
Kamke 1322	✓	0.052	44	✓	0.022	44	To Do
Kamke 1323	✗	0	0	✓	0.014	17	To Do
Kamke 1324	✓	0.025	25	✓	0.023	18	To Do
Kamke 1325	✓	0.211	52	✓	0.139	86	To Do
Kamke 1326	✓	0.02	29	✓	0.027	22	To Do
Kamke 1327	✓	0.146	104	✓	0.811	81	To Do
Kamke 1328	✓	0.018	36	✓	0.025	27	To Do
Kamke 1329	✓	0.723	67	✓	0.328	64	To Do
Kamke 1330	✓	4.171	1176	✓	0.917	1147	To Do
Kamke 1331	✓	0.033	55	✓	0.021	19	To Do
Kamke 1332	✓	0.021	26	✓	0.014	17	To Do
Kamke 1333	✓	0.094	70	✓	0.122	45	To Do
Kamke 1334	✓	0.17	114	✓	0.124	89	To Do
Kamke 1335	✓	0.259	893	✓	0.102	57	To Do
Kamke 1336	✓	0.043	70	✓	0.039	44	To Do
Kamke 1337	✓	0.062	62	✓	0.038	27	To Do
Kamke 1338	✓	0.045	40	✓	0.023	27	To Do
Kamke 1339	✓	0.211	66	✓	0.167	76	To Do
Kamke 1340	✓	0.029	32	✓	0.022	20	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 1341	✓	61.979	2924	✓	0.2	201	To Do
Kamke 1342	✓	0.084	52	✓	0.027	31	To Do
Kamke 1343	✓	0.189	73	✓	0.132	58	To Do
Kamke 1344	✓	0.51	173	✓	0.089	23	To Do
Kamke 1345	✓	0.059	52	✓	0.05	25	To Do
Kamke 1346	✓	0.093	37	✓	0.046	25	To Do
Kamke 1347	✓	0.089	31	✓	0.071	19	To Do
Kamke 1348	✓	0.38	34	✓	0.253	73	To Do
Kamke 1349	✓	0.105	76	✓	0.118	85	To Do
Kamke 1350	✓	0.008	25	✓	0.009	21	To Do
Kamke 1351	✓	0.019	50	✓	0.026	24	To Do
Kamke 1352	✓	0.012	89	✓	0.046	43	To Do
Kamke 1353	✓	0.839	119	✓	0.249	66	To Do
Kamke 1354	✓	0.244	108	✓	0.298	33	To Do
Kamke 1355	✓	0.289	51	✓	0.13	30	To Do
Kamke 1356	✓	0.246	90	✓	0.116	29	To Do
Kamke 1357	✓	0.578	288	✓	0.145	97	To Do
Kamke 1358	✓	0.049	89	✓	0.044	20	To Do
Kamke 1359	✓	0.088	86	✓	0.148	57	To Do
Kamke 1360	✓	0.079	68	✓	0.124	47	To Do
Kamke 1361	✓	0.48	38	✓	0.036	33	To Do
Kamke 1362	✗	0	0	✓	0.231	109	To Do
Kamke 1363	✓	0.601	236	✓	0.171	161	To Do
Kamke 1364	✓	0.123	42	✓	0.115	25	To Do
Kamke 1365	✓	0.081	104	✓	0.062	59	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 1366	✓	0.019	31	✓	0.011	17	To Do
Kamke 1367	✓	0.236	229	✓	0.23	88	To Do
Kamke 1368	✓	0.021	106	✓	0.115	71	To Do
Kamke 1369	✓	0.094	106	✓	0.053	55	To Do
Kamke 1370	✓	0.021	53	✓	0.013	19	To Do
Kamke 1371	✓	0.023	48	✓	0.106	37	To Do
Kamke 1372	✓	0.305	202	✓	0.286	110	To Do
Kamke 1373	✓	0.235	113	✓	0.253	84	To Do
Kamke 1374	✓	0.033	32	✓	0.099	23	To Do
Kamke 1375	✓	0.038	54	✓	0.109	29	To Do
Kamke 1376	✓	0.034	69	✓	0.06	73	To Do
Kamke 1377	✓	0.187	163	✓	0.128	83	To Do
Kamke 1378	✓	0.039	65	✓	0.039	48	To Do
Kamke 1379	✓	0.057	99	✓	0.059	60	To Do
Kamke 1380	✓	0.203	132	✓	0.089	67	To Do
Kamke 1381	✓	0.252	589	✓	0.192	175	To Do
Kamke 1382	✓	0.468	154	✓	0.13	104	To Do
Kamke 1383	✓	0.119	50	✓	0.056	39	To Do
Kamke 1384	✓	0.022	110	✓	0.26	73	To Do
Kamke 1385	✓	0.014	78	✓	0.064	55	To Do
Kamke 1386	✓	0.065	108	✓	0.054	58	To Do
Kamke 1387	✓	0.029	50	✓	0.026	28	To Do
Kamke 1388	✓	0.263	235	✓	0.112	76	To Do
Kamke 1389	✓	0.33	217	✓	0.124	68	To Do
Kamke 1390	✓	0.025	41	✓	0.026	25	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 1416	✓	0.181	46	✓	0.19	26	To Do
Kamke 1417	✓	0.142	52	✓	0.153	31	To Do
Kamke 1418	✓	0.13	15	✓	3.614	59	To Do
Kamke 1419	✗	0	0	✓	0.227	12	To Do
Kamke 1420	✓	0.412	134	✓	0.369	123	To Do
Kamke 1421	✓	0.162	81	✓	0.127	27	To Do
Kamke 1422	✓	0.081	58	✓	0.242	50	To Do
Kamke 1423	✓	0.062	70	✓	0.325	132	To Do
Kamke 1424	✓	0.213	90	✓	0.322	120	To Do
Kamke 1425	✓	0.899	236	✓	0.484	91	To Do
Kamke 1426	✓	5.044	4128	✓	0.624	549	To Do
Kamke 1427	✓	0.861	129	✓	1.349	203	To Do
Kamke 1428	✓	0.408	104	✓	0.341	183	To Do
Kamke 1429	✓	0.042	51	✓	0.048	25	To Do
Kamke 1430	✓	0.415	22	✓	0.344	85	To Do
Kamke 1431	✓	0.163	80	✓	0.22	30	To Do
Kamke 1432	✓	0.07	37	✓	0.046	22	To Do
Kamke 1433	✓	0.161	46	✓	0.109	28	To Do
Kamke 1434	✓	71.913	1596424	✓	0.656	517	To Do
Kamke 1435	✓	0.151	70	✓	0.159	38	To Do
Kamke 1436	✓	0.586	42	✓	0.299	113	To Do
Kamke 1437	✓	0.317	42	✓	0.272	29	To Do
Kamke 1438	✓	1.33	615	✓	0.23	102	To Do
Kamke 1439	✗	0	0	✗	0	0	To Do
Kamke 1440	✗	0	0	✗	0	0	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 1666	✗	0	0	✓	1.038	93	To Do
Kamke 1667	✗	0	0	✓	1.448	121	To Do
Kamke 1668	✓	0.059	60	✓	1.286	24	To Do
Kamke 1669	✓	0.501	160	✓	1.308	32	To Do
Kamke 1670	✓	7.643	51	✓	1.733	35	To Do
Kamke 1671	✓	0.117	59	✓	1.225	35	To Do
Kamke 1672	✗	0	0	✓	1.155	65	To Do
Kamke 1673	✗	0	0	✓	0.797	60	To Do
Kamke 1674	✓	0.046	106	✓	0.146	25	To Do
Kamke 1675	✗	0	0	✗	0	0	To Do
Kamke 1676	✓	0.364	134	✓	1.323	72	To Do
Kamke 1677	✗	0	0	✓	2.769	101	To Do
Kamke 1678	✗	0	0	✓	2.408	60	To Do
Kamke 1679	✓	0.201	33	✓	1.277	27	To Do
Kamke 1680	✗	0	0	✓	1.809	103	To Do
Kamke 1681	✓	2.762	41	✓	1.154	31	To Do
Kamke 1682	✓	22.24	41	✓	1.964	94	To Do
Kamke 1683	✓	0.089	26	✓	1.171	23	To Do
Kamke 1684	✗	0	0	✓	2.791	100	To Do
Kamke 1685	✗	0	0	✗	0	0	To Do
Kamke 1686	✗	0	0	✓	1.433	128	To Do
Kamke 1687	✓	0.079	262	✓	1.247	21	To Do
Kamke 1688	✓	0.594	189	✓	1.247	32	To Do
Kamke 1689	✓	0.309	104	✓	1.296	37	To Do
Kamke 1690	✗	0	0	✓	2.916	99	To Do
Continued on next page							

Table 3 – continued from previous page

	Mathematica			Maple			
#	solved	cpu	leaf	solved	cpu	leaf	note on solution
Kamke 1841	✗	0	0	✓	0.73	60	To Do
Kamke 1842	✓	0.208	286	✓	0.821	190	To Do
Kamke 1843	✓	2.926	409	✓	0.518	77	To Do
Kamke 1844	✓	0.115	20	✓	0.833	17	To Do
Kamke 1845	✓	0.113	22	✓	0.652	22	To Do
Kamke 1846	✓	0.036	51	✓	0.018	28	To Do
Kamke 1847	✓	0.247	95	✓	0.424	49	To Do
Kamke 1848	✓	0.493	187	✓	1.487	789	To Do
Kamke 1849	✓	0.645	426	✓	0.388	197	To Do
Kamke 1850	✗	0	0	✓	1.645	164	To Do
Kamke 1851	✗	0	0	✗	0	0	To Do
Kamke 1852	✓	0.13	28	✓	0.416	28	To Do
Kamke 1853	✓	0.142	43	✓	1.053	110	To Do
Kamke 1854	✗	0	0	✗	0	0	To Do
Kamke 1855	✗	0	0	✗	0	0	To Do
Kamke 1856	✓	0.018	22	✓	0.055	19	To Do
Kamke 1857	✓	0.01	39	✓	0.067	35	To Do
Kamke 1858	✓	0.009	182	✓	0.073	64	To Do
Kamke 1859	✓	0.005	51	✓	0.054	37	To Do
Kamke 1860	✓	0.035	696	✓	0.102	177	To Do
Kamke 1861	✓	0.011	183	✓	0.113	152	To Do
Kamke 1862	✓	0.013	52	✓	0.05	39	To Do
Kamke 1863	✓	0.007	84	✓	0.054	35	To Do
Kamke 1864	✓	0.01	59	✓	0.051	44	To Do
Kamke 1865	✓	0.837	2062	✓	0.144	224	To Do
Continued on next page							

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1916	✓	0.399	557	✓	0.655	180	To Do
Kamke 1917	✓	216.129	3406	✓	1.252	108	To Do
Kamke 1918	✗	0	0	✓	2.908	184	To Do
Kamke 1919	✗	0	0	✓	4.266	200	To Do
Kamke 1920	✗	0	0	✓	3.939	202	To Do
Kamke 1921	✗	0	0	✗	0	0	To Do
Kamke 1922	✗	0	0	✗	0	0	To Do
Kamke 1923	✓	0.011	53	✓	0.055	35	To Do
Kamke 1924	✓	0.072	191	✓	0.391	180	To Do
Kamke 1925	✓	0.019	31	✓	0.354	194	To Do
Kamke 1926	✓	0.006	28	✓	0.273	96	To Do
Kamke 1927	✗	0	0	✗	0	0	To Do
Kamke 1928	✗	0	0	✗	0	0	To Do
Kamke 1929	✗	0	0	✓	4.415	116	To Do
Kamke 1930	✓	0.034	308	✓	0.092	45	To Do
Kamke 1931	✓	4.138	10101	✓	1.4	1117	To Do
Kamke 1932	✗	0	0	✓	1.242	383	To Do
Kamke 1933	✗	0	0	✓	1.618	4258	To Do
Kamke 1934	✗	0	0	✓	1.512	377	To Do
Kamke 1935	✗	0	0	✓	2.276	741	To Do
Kamke 1936	✗	0	0	✓	1.076	704	To Do
Kamke 1937	✗	0	0	✓	1.039	242	To Do
Kamke 1938	✓	0.008	137	✓	0.211	101	To Do
Kamke 1939	✓	0.291	2168	✓	1.798	899	To Do
Kamke 1940	✗	0	0	✗	0	0	To Do

2.1 ODE No. 1

$$y'(x) - \frac{1}{\sqrt{a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4}} = 0$$

✓ **Mathematica** : cpu = 0.465968 (sec), leaf count = 1117

$$\left\{ \left\{ \left\{ y(x) \rightarrow c_1 - \frac{2F\left(\sin^{-1}\left(\sqrt{\frac{(x-\text{Root}[a_4\#1^4+a_3\#1^3+a_2\#1^2+a_1\#1+a_0\&,1])\left(\text{Root}[a_4\#1^4+a_3\#1^3+a_2\#1^2+a_1\#1+a_0\&,2] - \text{Root}[a_4\#1^4+a_3\#1^3+a_2\#1^2+a_1\#1+a_0\&,1])}{(x-\text{Root}[a_4\#1^4+a_3\#1^3+a_2\#1^2+a_1\#1+a_0\&,2])\left(\text{Root}[a_4\#1^4+a_3\#1^3+a_2\#1^2+a_1\#1+a_0\&,1] - \text{Root}[a_4\#1^4+a_3\#1^3+a_2\#1^2+a_1\#1+a_0\&,2])}\right)}{2}\right)}{2}\right\} \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 1089

Expression too large to display

HAND SOLUTION

$$y' - \frac{1}{\sqrt{a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4}} = 0 \tag{1}$$

To Do.

2.2 ODE No. 2

$$ay(x) + c(-e^{bx}) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0621456 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow \frac{ce^{x(a+b)-ax}}{a+b} + c_1e^{-ax} \right\} \right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 25

$$\left\{ y(x) = \left(\frac{ce^{(a+b)x}}{a+b} + c_1 \right) e^{-ax} \right\}$$

HAND SOLUTION

$$\frac{dy}{dx} + ay(x) = ce^{bx} \tag{1}$$

Integrating factor $\mu = e^{\int adx} = e^{ax}$. Hence (1) becomes

$$\begin{aligned} \frac{d}{dx}(\mu y(x)) &= \mu ce^{bx} \\ \mu y(x) &= \int \mu ce^{bx} dx + C \end{aligned}$$

Replacing μ by e^{ax}

$$\begin{aligned} y(x) &= ce^{-ax} \int e^{(a+b)x} dx + Ce^{-ax} \\ &= ce^{-ax} \frac{e^{(a+b)x}}{a+b} + Ce^{-ax} \\ &= \frac{ce^{(a+b)x-ax}}{a+b} + Ce^{-ax} \end{aligned}$$

Can be reduced to

$$y(x) = c \frac{e^{bx}}{a+b} + Ce^{-ax}$$

2.3 ODE No. 3

$$ay(x) - b \sin(cx) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0729384 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow \frac{b(a \sin(cx) - c \cos(cx))}{a^2 + c^2} + c_1 e^{-ax} \right\} \right\}$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 38

$$\left\{ y(x) = c_1 e^{-ax} - \frac{(-a \sin(cx) + c \cos(cx))b}{a^2 + c^2} \right\}$$

HAND SOLUTION

$$\frac{dy}{dx} + ay(x) = b \sin(cx) \quad (1)$$

Integrating factor $\mu = e^{\int a dx} = e^{ax}$. Hence (1) becomes

$$\begin{aligned} \frac{d}{dx}(\mu y(x)) &= \mu b \sin(cx) \\ \mu y(x) &= b \int \mu \sin(cx) dx + C \end{aligned}$$

Replacing μ by e^{ax}

$$y(x) = b e^{-ax} \int e^{ax} \sin(cx) dx + C e^{-ax} \quad (2)$$

Using $\sin(cx) = \frac{e^{icx} - e^{-icx}}{2i}$ then

$$\begin{aligned} \int e^{ax} \sin(cx) dx &= \int \frac{e^{(ic+a)x} - e^{(-ic+a)x}}{2i} dx \\ &= \frac{1}{2i} \left(\frac{e^{(ic+a)x}}{ic+a} - \frac{e^{(-ic+a)x}}{-ic+a} \right) \\ &= \frac{1}{2i} e^{ax} \left(\frac{e^{icx}}{ic+a} - \frac{e^{-icx}}{-ic+a} \right) \\ &= \frac{1}{2i} e^{ax} \left(\frac{e^{icx}(-ic+a) - e^{-icx}(ic+a)}{(ic+a)(-ic+a)} \right) \\ &= \frac{1}{2i} e^{ax} \left(\frac{-ice^{icx} + ae^{icx} - ice^{-icx} - ae^{-icx}}{(c^2 + a^2)} \right) \\ &= \frac{1}{2i} e^{ax} \left(\frac{-ic(e^{icx} + e^{-icx}) + a(e^{icx} - e^{-icx})}{(c^2 + a^2)} \right) \\ &= \frac{e^{ax}}{(c^2 + a^2)} \left(\frac{-ic(e^{icx} + e^{-icx})}{2i} + \frac{a(e^{icx} - e^{-icx})}{2i} \right) \\ &= \frac{e^{ax}}{(c^2 + a^2)} (-c \cos cx + a \sin cx) \end{aligned}$$

Therefore (2) becomes

$$\begin{aligned} y(x) &= be^{-ax} \left[\frac{e^{ax}}{(c^2 + a^2)} (-c \cos cx + a \sin cx) \right] + Ce^{-ax} \\ &= \frac{b}{(c^2 + a^2)} (-c \cos cx + a \sin cx) + Ce^{-ax} \end{aligned}$$

2.4 ODE No. 4

$$-e^{-x^2}x + y'(x) + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0419564 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} e^{-x^2} x^2 + c_1 e^{-x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 18

$$\left\{ y(x) = \left(\frac{x^2}{2} + c_1 \right) e^{-x^2} \right\}$$

HAND SOLUTION

$$\frac{dy}{dx} + 2xy(x) = e^{-x^2}x \tag{1}$$

Integrating factor $\mu = e^{\int 2x dx} = e^{x^2}$. Hence (1) becomes

$$\begin{aligned} \frac{d}{dx} (e^{x^2} y(x)) &= e^{x^2} e^{-x^2} x \\ \frac{d}{dx} (e^{x^2} y(x)) &= x \end{aligned}$$

Integrating both sides

$$\begin{aligned} e^{x^2} y(x) &= \frac{x^2}{2} + C \\ y(x) &= e^{-x^2} \left(\frac{x^2}{2} + C \right) \end{aligned}$$

2.5 ODE No. 5

$$y'(x) + y(x) \cos(x) - e^{2x} = 0$$

✓ **Mathematica** : cpu = 0.446427 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow e^{-\sin(x)} \int_1^x e^{2K[1] + \sin(K[1])} dK[1] + c_1 e^{-\sin(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.242 (sec), leaf count = 21

$$\{y(x) = \left(c_1 + \int e^{2x + \sin(x)} dx \right) e^{-\sin(x)} \}$$

HAND SOLUTION

$$\frac{dy}{dx} + y(x) \cos(x) = e^{2x} \tag{1}$$

Integrating factor $\mu = e^{\int \cos(x) dx} = e^{\sin(x)}$. Hence (1) becomes

$$\frac{d}{dx}(e^{\sin(x)}y(x)) = e^{\sin(x)}e^{2x}$$

Integrating both sides

$$\begin{aligned} e^{\sin(x)}y(x) &= \int e^{\sin(x)}e^{2x} + C \\ y(x) &= e^{-\sin(x)} \int e^{2x+\sin(x)} + Ce^{-\sin(x)} \end{aligned}$$

2.6 ODE No. 6

$$y'(x) + y(x) \cos(x) - \frac{1}{2} \sin(2x) = 0$$

✓ **Mathematica** : cpu = 0.0313337 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow \sin(x) + c_1 e^{-\sin(x)} - 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 15

$$\{y(x) = c_1 e^{-\sin(x)} + \sin(x) - 1\}$$

HAND SOLUTION

$$\frac{dy}{dx} + y(x) \cos(x) = \frac{1}{2} \sin(2x) \tag{1}$$

Integrating factor $\mu = e^{\int \cos x dx} = e^{\sin(x)}$. Therefore (1) becomes2

$$\frac{d}{dx}(e^{\sin(x)}y(x)) = \frac{1}{2} e^{\sin(x)} \sin(2x)$$

Integrating

$$\begin{aligned} e^{\sin(x)}y(x) &= \frac{1}{2} \int e^{\sin(x)} \sin(2x) + C \\ y(x) &= \frac{e^{-\sin(x)}}{2} \int e^{\sin(x)} \sin(2x) + e^{-\sin(x)} C \end{aligned}$$

But $e^{\sin(x)} \sin(2x)$ can be integrated by parts which gives $e^{\sin(x)}(-2 + 2 \sin(x))$. Hence the above becomes

$$\begin{aligned} y(x) &= \frac{e^{-\sin(x)}}{2} (e^{\sin(x)}(-2 + 2 \sin(x))) + e^{-\sin(x)} C \\ &= -1 + \sin(x) + e^{-\sin(x)} C \end{aligned}$$

2.7 ODE No. 7

$$y'(x) + y(x) \cos(x) - e^{-\sin(x)} = 0$$

✓ **Mathematica** : cpu = 0.0982518 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow x e^{-\sin(x)} + c_1 e^{-\sin(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 13

$$\{y(x) = (c_1 + x)e^{-\sin(x)}\}$$

HAND SOLUTION

$$\frac{dy}{dx} + y(x) \cos(x) = e^{-\sin(x)} \quad (1)$$

Integrating factor $\mu = e^{\int \cos x dx} = e^{\sin x}$. Hence (1) becomes

$$\frac{d}{dx}(\mu y(x)) = \mu e^{-\sin(x)}$$

Replacing μ by $e^{\sin x}$ and integrating both sides

$$e^{\sin x} y(x) = \int e^{\sin x} e^{-\sin(x)} dx + C$$

$$e^{\sin x} y(x) = \int dx + C$$

$$e^{\sin x} y(x) = x + C$$

$$y(x) = x e^{-\sin x} + C e^{-\sin(x)}$$

2.8 ODE No. 8

$$y'(x) + y(x) \tan(x) - \sin(2x) = 0$$

✓ **Mathematica** : cpu = 0.0353108 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow -2 \cos^2(x) + c_1 \cos(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 13

$$\{y(x) = (c_1 - 2 \cos(x)) \cos(x)\}$$

HAND SOLUTION

$$\frac{dy}{dx} + y(x) \tan(x) = \sin(2x) \quad (1)$$

Integrating factor $\mu = e^{\int \tan x dx} = e^{-\ln(\cos(x))} = \frac{1}{\cos(x)}$. Hence (1) becomes

$$\frac{d}{dx} \left(y(x) \frac{1}{\cos(x)} \right) = \frac{1}{\cos(x)} \sin(2x)$$

Integrating both sides

$$y(x) \frac{1}{\cos(x)} = \int \frac{1}{\cos(x)} \sin(2x) dx + C$$

$$y(x) = \cos(x) \int \frac{\sin(2x)}{\cos(x)} dx + C \cos(x)$$

But $\sin(2x) = 2 \sin(x) \cos(x)$ hence

$$y(x) = \cos(x) \int \frac{2 \sin(x) \cos(x)}{\cos(x)} dx + C \cos(x)$$

$$= 2 \cos(x) \int \sin(x) dx + C \cos(x)$$

$$= -2 \cos^2(x) + C \cos(x)$$

2.9 ODE No. 9

$$y'(x) - y(x)(a + \sin(\log(x)) + \cos(\log(x))) = 0$$

✓ **Mathematica** : cpu = 0.0308109 (sec), leaf count = 19

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{ax+x \sin(\log(x))} \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 14

$$\{y(x) = c_1 e^{(a+\sin(\ln(x)))x}\}$$

HAND SOLUTION

$$\frac{dy}{dx} - y(x)[a + \sin(\log(x)) + \cos(\log(x))] = 0 \quad (1)$$

Integrating factor $\mu = e^{-\int a - \sin(\log(x)) - \cos(\log(x)) dx} = e^{-ax} e^{-\int \sin(\log(x)) + \cos(\log(x)) dx}$. To integrate $\int \sin(\log(x)) + \cos(\log(x)) dx$, let $r = \log(x)$, $\frac{dr}{dx} = \frac{1}{x}$, then $dx = x dr$, But $x = e^r$, hence the integral becomes

$$\begin{aligned} \int \sin(\log(x)) + \cos(\log(x)) dx &= \int [\sin(r) + \cos(r)] e^r dr \\ &= \int e^r \sin(r) dr + \int e^r \cos(r) dr \end{aligned} \quad (2)$$

Integrating by parts $\int e^r \cos(r) dr$, $\int u dv = uv - \int v du$, Let $u = e^r \rightarrow du = e^r$ and $dv = \cos(r) \rightarrow v = \sin(r)$, hence (2) becomes

$$\begin{aligned} \int e^r \sin(r) dr + \int e^r \cos(r) dr &= \int e^r \sin(r) dr + e^r \sin(r) - \int \sin(r) e^r dr \\ &= e^r \sin(r) \end{aligned}$$

Therefore, substituting back $r = \log(x)$ gives

$$\begin{aligned} \int \sin(\log(x)) + \cos(\log(x)) dx &= e^{\log(x)} \sin(\log(x)) \\ &= x \sin(\log(x)) \end{aligned}$$

Hence the integration factor is

$$\begin{aligned} \mu &= e^{-ax} e^{-\int \sin(\log(x)) + \cos(\log(x)) dx} \\ &= e^{-ax} e^{-x \sin(\log(x))} \end{aligned}$$

Therefore (1) becomes

$$\frac{d}{dx}(\mu y(x)) = 0$$

Integrating

$$\begin{aligned} y(x) e^{-ax} e^{-x \sin(\log(x))} &= C \\ y(x) &= C e^{ax} e^{x \sin(\log(x))} \\ &= C e^{ax+x \sin(\log(x))} \\ &= C e^{x(a+\sin(\log(x)))} \end{aligned}$$

2.10 ODE No. 10

$$y(x)f'(x) - f(x)f'(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.02596 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow f(x) + c_1 e^{-f(x)} - 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 15

$$\{y(x) = c_1 e^{-f(x)} + f(x) - 1\}$$

HAND SOLUTION

$$\frac{dy}{dx} + y(x) \frac{df}{dx} = f(x) \frac{df}{dx} \quad (1)$$

Integrating factor $\mu = e^{\int \frac{df}{dx} dx} = e^f$. Therefore (1) becomes

$$\frac{d}{dx} \left(e^f y(x) \right) = e^f f(x) \frac{df}{dx}$$

Integrating

$$e^f y(x) = \int e^f f(x) \frac{df}{dx} dx + C$$

$$y(x) = e^{-f} \int e^f f df + e^{-f} C$$

But $\int e^f f df$ is the same as $\int e^x x dx$ which by integration by parts gives $e^x(x-1)$ or in terms of f , gives $e^f(f-1)$. Hence the above becomes

$$\begin{aligned} y(x) &= e^{-f} \left(e^f (f-1) \right) + e^{-f} C \\ &= f-1 + e^{-f} C \end{aligned}$$

2.11 ODE No. 11

$$f(x)y(x) - g(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0328135 (sec), leaf count = 66

$$\left\{ \left\{ y(x) \rightarrow \exp \left(\int_1^x -f(K[1]) dK[1] \right) \int_1^x \exp \left(- \int_1^{K[2]} -f(K[1]) dK[1] \right) g(K[2]) dK[2] + c_1 \exp \left(\int_1^x -f(K[1]) dK[1] \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.025 (sec), leaf count = 24

$$\{y(x) = \left(c_1 + \int e^{\int f(x) dx} g(x) dx \right) e^{-\int f(x) dx} \}$$

HAND SOLUTION

$$\frac{dy}{dx} + y(x)f(x) = g(x) \quad (1)$$

Integrating factor $\mu = e^{\int f(x) dx}$. Therefore (1) becomes

$$\frac{d}{dx} \left(e^{\int f(x) dx} y(x) \right) = e^{\int f(x) dx} g(x)$$

Integrating

$$e^{\int f(x) dx} y(x) = \int e^{\int f(x) dx} g(x) dx + C$$

$$\begin{aligned} y(x) &= e^{-\int f(x) dx} \int e^{\int f(x) dx} g(x) dx + e^{-\int f(x) dx} C \\ &= \left(\int e^{\int f(x) dx} g(x) dx + C \right) e^{-\int f(x) dx} \end{aligned}$$

2.12 ODE No. 12

$$y'(x) + y(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.0910503 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{2x} - e^{2c_1}}{e^{2x} + e^{2c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 8

$$\{y(x) = \tanh(c_1 + x)\}$$

HAND SOLUTION

$$\frac{dy}{dx} + y^2(x) - 1 = 0$$

$$\frac{dy}{dx} = 1 - y^2(x) \quad (1)$$

This is separable. Hence

$$\begin{aligned} \frac{dy}{dx} \frac{1}{1 - y^2(x)} &= 1 \\ \frac{dy}{1 - y^2(x)} &= dx \end{aligned}$$

Integrating

$$\int \frac{dy}{1 - y^2(x)} = x + C$$

Using $\int \frac{1}{a+by^2} dy = \frac{\sqrt{-\frac{a}{b}} \tanh^{-1}\left(\frac{y}{\sqrt{-\frac{a}{b}}}\right)}{a}$ and since $a = 1, b = -1$, then $\int \frac{dy}{1-y^2(x)} = \tanh^{-1}(y)$ and the above becomes

$$\tanh^{-1}(y) = x + C$$

Therefore

$$y = \tanh(x + C) \quad (2)$$

In terms of exponential, since $\tanh(u) = \frac{e^u - e^{-u}}{e^u + e^{-u}}$ then (2) can also be written as

$$y = \frac{e^{x+C} - e^{-(x+C)}}{e^{x+C} + e^{-(x+C)}} = \frac{e^x e^C - e^{-x} e^{-C}}{e^x e^C + e^{-x} e^{-C}}$$

Multiplying numerator and denominator by $e^{-C} e^x$

$$y = \frac{e^{2x} - e^{-2C}}{e^{2x} + e^{-2C}}$$

To get same answer as Mathematica, since C is constant, let $C_1 = -C$, then

$$y = \frac{e^{2x} - e^{2C_1}}{e^{2x} + e^{2C_1}}$$

2.13 ODE No. 13

$$-ax - b + y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0996128 (sec), leaf count = 79

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt[3]{a} \operatorname{Bi}'\left(\frac{b+ax}{a^{2/3}}\right) + \sqrt[3]{a} c_1 \operatorname{Ai}'\left(\frac{b+ax}{a^{2/3}}\right)}{-\operatorname{Bi}\left(\frac{b+ax}{a^{2/3}}\right) - c_1 \operatorname{Ai}\left(\frac{b+ax}{a^{2/3}}\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.206 (sec), leaf count = 79

$$\left\{ y(x) = -\frac{i(-ia)^{\frac{1}{3}} \left(c_1 \operatorname{AiryAi}\left(1, -\frac{ax+b}{(-ia)^{\frac{2}{3}}}\right) + \operatorname{AiryBi}\left(1, -\frac{ax+b}{(-ia)^{\frac{2}{3}}}\right) \right)}{c_1 \operatorname{AiryAi}\left(-\frac{ax+b}{(-ia)^{\frac{2}{3}}}\right) + \operatorname{AiryBi}\left(-\frac{ax+b}{(-ia)^{\frac{2}{3}}}\right)} \right\}$$

HAND SOLUTION

$$\begin{aligned} y'(x) + y^2(x) - ax - b &= 0 \\ y'(x) &= b + ax - y^2(x) \end{aligned} \quad (1)$$

This is Riccati first order non-linear ODE of the form

$$y'(x) = P(x) + Q(x)y + R(x)y^2(x) \quad (2)$$

where in this case $Q(x) = 0, R(x) = -1, P(x) = b + ax$. We can solve this in two ways. If we know one particular solution $y_p(x)$ for (1) then we use the substitution $y = y_p + \frac{1}{u}$ and convert (1) to new associated linear ODE of the form $u' + (Q(x) + 2R(x))y_p + R(x) = 0$. If we do not know a particular solution, then we use the standard substitution $y = \frac{-u'}{uR(x)} = \frac{u'}{u}$ since $R(x) = -1$ and this is what we will do here.

Since $u' = yu$ then

$$\begin{aligned} u'' &= yu' + y'u \\ &= y(yu) + (b + ax - y^2)u \\ &= y^2u + (b + ax)u - y^2u \\ &= (b + ax)u \end{aligned}$$

So we have new second order ODE

$$u'' - (b + ax)u = 0 \quad (3)$$

which we solve for u . This ODE is of the form $u'' - q(x)u = 0$ which has solutions in terms of Airy function of first $Ai(x)$ and second kind $Bi(x)$, where

$$\begin{aligned} Ai(x) &= \frac{1}{\pi} \int_0^\infty \cos\left(\frac{t^3}{3} + xt\right) dt \\ Bi(x) &= \frac{1}{\pi} \int_0^\infty \exp\left(-\frac{t^3}{3} + xt\right) + \sin\left(\frac{t^3}{3} + xt\right) dt \end{aligned}$$

Therefore the solution to (3) is

$$u(x) = c_1 Ai\left(\frac{b+ax}{a^{\frac{2}{3}}}\right) + c_2 Bi\left(\frac{b+ax}{a^{\frac{2}{3}}}\right)$$

We need to find $u'(x)$ now. Using $Ai'(x), Bi'(x)$ for derivative of Airy functions of first and second kind, then

$$u'(x) = c_1 Ai' \left(\frac{b+ax}{a^{\frac{2}{3}}} \right) a^{\frac{1}{3}} + c_2 Bi' \left(\frac{b+ax}{a^{\frac{2}{3}}} \right) a^{\frac{1}{3}}$$

Therefore since $u' = yu$ then

$$\begin{aligned} y &= \frac{u'}{u} \\ &= \frac{c_1 Ai' \left(\frac{b+ax}{a^{\frac{2}{3}}} \right) a^{\frac{1}{3}} + c_2 Bi' \left(\frac{b+ax}{a^{\frac{2}{3}}} \right) a^{\frac{1}{3}}}{c_1 Ai \left(\frac{b+ax}{a^{\frac{2}{3}}} \right) + c_2 Bi \left(\frac{b+ax}{a^{\frac{2}{3}}} \right)} \end{aligned}$$

Let $C_1 = \frac{c_1}{c_2}$ then the above can be written as

$$y = \frac{C_1 Ai' \left(\frac{b+ax}{a^{\frac{2}{3}}} \right) a^{\frac{1}{3}} + Bi' \left(\frac{b+ax}{a^{\frac{2}{3}}} \right) a^{\frac{1}{3}}}{C_1 Ai \left(\frac{b+ax}{a^{\frac{2}{3}}} \right) + Bi \left(\frac{b+ax}{a^{\frac{2}{3}}} \right)}$$

Reference: Airy function

2.14 ODE No. 14

$$ax^m + y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.126584 (sec), leaf count = 254

$$\left\{ \left\{ y(x) \rightarrow - \frac{i\sqrt{-a} x^{\frac{m+2}{2}} \left(c_1 J_{\frac{m+1}{m+2}} \left(\frac{2i\sqrt{-a} x^{\frac{m+2}{2}}}{m+2} \right) - 2J_{\frac{1}{m+2}-1} \left(\frac{2i\sqrt{-a} x^{\frac{m+2}{2}}}{m+2} \right) - c_1 J_{-\frac{m+3}{m+2}} \left(\frac{2i\sqrt{-a} x^{\frac{m+2}{2}}}{m+2} \right) \right) - c_1 J_{-\frac{1}{m+2}} \left(\frac{2i\sqrt{-a} x^{\frac{m+2}{2}}}{m+2} \right)}{2x \left(J_{\frac{1}{m+2}} \left(\frac{2i\sqrt{-a} x^{\frac{m+2}{2}}}{m+2} \right) + c_1 J_{-\frac{1}{m+2}} \left(\frac{2i\sqrt{-a} x^{\frac{m+2}{2}}}{m+2} \right) \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.105 (sec), leaf count = 187

$$\left\{ y(x) = \frac{-c_1 \sqrt{a} x^{\frac{m}{2}+1} \text{BesselJ} \left(\frac{m+3}{m+2}, \frac{2\sqrt{a} x^{\frac{m}{2}+1}}{m+2} \right) - \sqrt{a} x^{\frac{m}{2}+1} \text{BesselY} \left(\frac{m+3}{m+2}, \frac{2\sqrt{a} x^{\frac{m}{2}+1}}{m+2} \right) + c_1 \text{BesselJ} \left(\frac{1}{m+2}, \frac{2\sqrt{a} x^{\frac{m}{2}+1}}{m+2} \right) + \text{BesselY} \left(\frac{1}{m+2}, \frac{2\sqrt{a} x^{\frac{m}{2}+1}}{m+2} \right)}{\left(c_1 \text{BesselJ} \left(\frac{1}{m+2}, \frac{2\sqrt{a} x^{\frac{m}{2}+1}}{m+2} \right) + \text{BesselY} \left(\frac{1}{m+2}, \frac{2\sqrt{a} x^{\frac{m}{2}+1}}{m+2} \right) \right) x}$$

HAND SOLUTION

$$\begin{aligned} y'(x) + y^2(x) + ax^m &= 0 \\ y'(x) &= -ax^m - y^2(x) \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE of the form

$$y'(x) = P(x) + Q(x)y + R(x)y^2(x) \tag{2}$$

where in this case $Q(x) = 0, R(x) = -1, P(x) = -ax^m$. We can solve this in two ways. If we know one particular solution $y_p(x)$ for (1) then we use the substitution $y = y_p + \frac{1}{u}$ and convert (1) to new associated linear ODE of the form $u' + (Q(x) + 2R(x))y_p + R(x) = 0$. If we do not know a particular solution, then we use the standard substitution $y = \frac{-u'}{uR(x)} = \frac{u'}{u}$ since $R(x) = -1$ and this is what we will do here.

Since $u' = yu$ then

$$\begin{aligned}
 u'' &= yu' + y'u \\
 &= y(yu) + (-ax^m - y^2)u \\
 &= y^2u - ax^m u - y^2u \\
 &= -ax^m u
 \end{aligned}$$

So we have new second order ODE

$$u'' + ax^m u = 0 \quad (3)$$

which we solve for u . This is Airy ODE but with a positive sign. Of the form $u'' + q(x)u = 0$.

Recall that the solution to $u'' - axu = 0$ is

$$u = c_1 Ai\left(a^{\frac{1}{3}}x\right) + c_2 Bi\left(a^{\frac{1}{3}}x\right)$$

When x has power on it (there are restriction on what values the power can take), the solution is written in terms of Bessel functions. The solution to $u'' - ax^m u = 0$ is

$$u = c_1 \sqrt{x} \text{Bessel}I\left(\frac{1}{m+2}, \frac{2\sqrt{a}x^{\frac{m+1}{2}}}{m+2}\right) + c_2 \sqrt{x} \text{Bessel}K\left(\frac{1}{m+2}, \frac{2\sqrt{a}x^{\frac{m+1}{2}}}{m+2}\right)$$

When the sign is positive, the solution to $u'' + ax^m u = 0$ is

$$u(x) = c_1 \sqrt{x} \text{Bessel}J\left(\frac{1}{m+2}, \frac{2\sqrt{a}x^{\frac{m+1}{2}}}{m+2}\right) + c_2 \sqrt{x} \text{Bessel}Y\left(\frac{1}{m+2}, \frac{2\sqrt{a}x^{\frac{m+1}{2}}}{m+2}\right) \quad (4)$$

We need to find $u'(x)$ now. From (4)

$$\frac{d}{dx} \left[c_1 \sqrt{x} \text{Bessel}J\left(\frac{1}{m+2}, \frac{2\sqrt{a}x^{\frac{m+1}{2}}}{m+2}\right) \right] = c_1 \frac{\text{Bessel}J\left(\frac{1}{m+2}, \frac{2\sqrt{a}x^{\frac{m+1}{2}}}{m+2}\right) - \sqrt{a}x^{\frac{m+1}{2}} \text{Bessel}J\left(\frac{m+3}{m+2}, \frac{2\sqrt{a}x^{\frac{m+1}{2}}}{m+2}\right)}{\sqrt{x}}$$

And

$$\frac{d}{dx} \left[c_2 \sqrt{x} \text{Bessel}Y\left(\frac{1}{m+2}, \frac{2\sqrt{a}x^{\frac{m+1}{2}}}{m+2}\right) \right] = c_2 \frac{\text{Bessel}Y\left(\frac{1}{m+2}, \frac{2\sqrt{a}x^{\frac{m+1}{2}}}{m+2}\right) - \sqrt{a}x^{\frac{m+1}{2}} \text{Bessel}Y\left(\frac{m+3}{m+2}, \frac{2\sqrt{a}x^{\frac{m+1}{2}}}{m+2}\right)}{\sqrt{x}}$$

Therefore

$$\begin{aligned}
 u'(x) &= c_1 \frac{\text{Bessel}J\left(\frac{1}{m+2}, \frac{2\sqrt{a}x^{\frac{m+1}{2}}}{m+2}\right) - \sqrt{a}x^{\frac{m+1}{2}} \text{Bessel}J\left(\frac{m+3}{m+2}, \frac{2\sqrt{a}x^{\frac{m+1}{2}}}{m+2}\right)}{\sqrt{x}} \\
 &\quad + c_2 \frac{\text{Bessel}Y\left(\frac{1}{m+2}, \frac{2\sqrt{a}x^{\frac{m+1}{2}}}{m+2}\right) - \sqrt{a}x^{\frac{m+1}{2}} \text{Bessel}Y\left(\frac{m+3}{m+2}, \frac{2\sqrt{a}x^{\frac{m+1}{2}}}{m+2}\right)}{\sqrt{x}}
 \end{aligned}$$

Since $u' = yu$ then

$$\begin{aligned}
y &= \frac{u'}{u} \\
&= \frac{c_1 \frac{BesselJ\left(\frac{1}{m+2}, \frac{2\sqrt{a}x}{2}\right) - \sqrt{a}x^{\frac{m+1}{2}} BesselJ\left(\frac{m+3}{m+2}, \frac{2\sqrt{a}x}{2}\right)}{\sqrt{x}} + c_2 \frac{BesselY\left(\frac{1}{m+2}, \frac{2\sqrt{a}x}{2}\right) - \sqrt{a}x^{\frac{m+1}{2}} BesselY\left(\frac{m+3}{m+2}, \frac{2\sqrt{a}x}{2}\right)}{\sqrt{x}}}{c_1 \sqrt{x} BesselJ\left(\frac{1}{m+2}, \frac{2\sqrt{a}x}{2}\right) + c_2 \sqrt{x} BesselY\left(\frac{1}{m+2}, \frac{2\sqrt{a}x}{2}\right)} \\
&= \frac{c_1 \left[BesselJ\left(\frac{1}{m+2}, \frac{2\sqrt{a}x}{2}\right) - \sqrt{a}x^{\frac{m+1}{2}} BesselJ\left(\frac{m+3}{m+2}, \frac{2\sqrt{a}x}{2}\right) \right] + c_2 \left[BesselY\left(\frac{1}{m+2}, \frac{2\sqrt{a}x}{2}\right) - \sqrt{a}x^{\frac{m+1}{2}} BesselY\left(\frac{m+3}{m+2}, \frac{2\sqrt{a}x}{2}\right) \right]}{\sqrt{x} \left[c_1 \sqrt{x} BesselJ\left(\frac{1}{m+2}, \frac{2\sqrt{a}x}{2}\right) + c_2 \sqrt{x} BesselY\left(\frac{1}{m+2}, \frac{2\sqrt{a}x}{2}\right) \right]} \\
&= \frac{c_1 \left[BesselJ\left(\frac{1}{m+2}, \frac{2\sqrt{a}x}{2}\right) - \sqrt{a}x^{\frac{m+1}{2}} BesselJ\left(\frac{m+3}{m+2}, \frac{2\sqrt{a}x}{2}\right) \right] + c_2 \left[BesselY\left(\frac{1}{m+2}, \frac{2\sqrt{a}x}{2}\right) - \sqrt{a}x^{\frac{m+1}{2}} BesselY\left(\frac{m+3}{m+2}, \frac{2\sqrt{a}x}{2}\right) \right]}{c_1 x BesselJ\left(\frac{1}{m+2}, \frac{2\sqrt{a}x}{2}\right) + c_2 x BesselY\left(\frac{1}{m+2}, \frac{2\sqrt{a}x}{2}\right)}
\end{aligned}$$

Let $C_1 = \frac{c_1}{c_2}$ then the above can be written as

$$y = \frac{\frac{1}{x} \left[C_1 \left[BesselJ\left(\frac{1}{m+2}, \frac{2\sqrt{a}x}{2}\right) - \sqrt{a}x^{\frac{m+1}{2}} BesselJ\left(\frac{m+3}{m+2}, \frac{2\sqrt{a}x}{2}\right) \right] + BesselY\left(\frac{1}{m+2}, \frac{2\sqrt{a}x}{2}\right) - \sqrt{a}x^{\frac{m+1}{2}} BesselY\left(\frac{m+3}{m+2}, \frac{2\sqrt{a}x}{2}\right) \right]}{C_1 BesselJ\left(\frac{1}{m+2}, \frac{2\sqrt{a}x}{2}\right) + BesselY\left(\frac{1}{m+2}, \frac{2\sqrt{a}x}{2}\right)}$$

2.15 ODE No. 15

$$x^4 - 2x^2y(x) + y'(x) + y(x)^2 - 2x - 1 = 0$$

✓ **Mathematica** : cpu = 0.0922518 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow x^2 + \frac{1}{-\frac{1}{2} + c_1(-e^{2x})} + 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.181 (sec), leaf count = 33

$$\left\{ y(x) = \frac{c_1(x^2 + 1)e^{2x} - x^2 + 1}{c_1e^{2x} - 1} \right\}$$

HAND SOLUTION

$$\begin{aligned}
x^4 - 2x^2y(x) + y'(x) + y^2(x) - 2x - 1 &= 0 \\
y'(x) &= -x^4 + 2x + 1 + 2x^2y(x) - y^2(x)
\end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE of the form

$$y'(x) = P(x) + Q(x)y + R(x)y^2(x) \tag{2}$$

where in this case $Q(x) = 2x^2, R(x) = -1, P(x) = -x^4 + 2x + 1$.

Let $u = y - x^2$ or $y = u + x^2$ then

$$\begin{aligned}
u' &= y' - 2x \\
&= (-x^4 + 2x + 1 + 2x^2y - y^2) - 2x \\
&= (-x^4 + 2x + 1 + 2x^2(u + x^2) - (u + x^2)^2) - 2x \\
&= (-x^4 + 2x + 1 + 2x^2u + 2x^4 - (u^2 + x^4 + 2ux^2)) - 2x \\
&= -x^4 + 2x + 1 + 2x^2u + 2x^4 - u^2 - x^4 - 2ux^2 - 2x \\
&= 1 - u^2
\end{aligned}$$

Hence

$$u' = 1 - u^2$$

This is separable

$$\frac{du}{dx} = 1 - u^2$$

$$\frac{du}{1 - u^2} = dx$$

Integrating both sides

$$\tanh^{-1}(u) = x + C$$

$$u(x) = \tanh(x + C)$$

$$= \frac{e^{x+C} - e^{-x-C}}{e^{x+C} + e^{-x-C}}$$

$$= \frac{e^x e^C - e^{-x} e^{-C}}{e^x e^C + e^{-x} e^{-C}}$$

Multiplying numerator and denominator by $e^{-C}e^x$

$$u(x) = \frac{e^{2x} - e^{-2C}}{e^{2x} + e^{-2C}}$$

Let $e^{-2C} = C_1$

$$u(x) = \frac{e^{2x} - C_1}{e^{2x} + C_1}$$

Since $u = y - x^2$ then

$$y = u + x^2$$

$$= \frac{e^{2x} - C_1}{e^{2x} + C_1} + x^2$$

$$= \frac{e^{2x} - C_1 + x^2 e^{2x} + x^2 C_1}{e^{2x} + C_1}$$

To obtain same solution as Maple, we divide by C_1

$$y = \frac{\frac{1}{C_1} e^{2x} - 1 + \frac{1}{C_1} x^2 e^{2x} + x^2}{\frac{1}{C_1} e^{2x} + 1}$$

Let $\frac{1}{C_1} = -C$ then

$$y = \frac{-C e^{2x} - 1 - C x^2 e^{2x} + x^2}{-C e^{2x} + 1}$$

$$= \frac{C e^{2x} + 1 + C x^2 e^{2x} - x^2}{C e^{2x} - 1}$$

Which now agrees with the Maple solution form. Mathematica solution also verified to be correct.

2.16 ODE No. 16

$$f(x)(xy(x) - 1) + y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0926893 (sec), leaf count = 186

$$\left\{ \left(y(x) \rightarrow \frac{\exp\left(\int_1^x \frac{f(K[1])K[1]^2+2}{K[1]} dK[1]\right) \int_1^x \exp\left(-\int_1^{K[2]} \frac{f(K[1])K[1]^2+2}{K[1]} dK[1]\right) dK[2] + c_1 \exp\left(\int_1^x \frac{f(K[1])K[1]^2+2}{K[1]} dK[1]\right) + x}{x\left(\exp\left(\int_1^x \frac{f(K[1])K[1]^2+2}{K[1]} dK[1]\right) \int_1^x \exp\left(-\int_1^{K[2]} \frac{f(K[1])K[1]^2+2}{K[1]} dK[1]\right) dK[2] + c_1 \exp\left(\int_1^x \frac{f(K[1])K[1]^2+2}{K[1]} dK[1]\right)\right)} \right\}$$

✓ **Maple** : cpu = 0.117 (sec), leaf count = 49

$$\left\{ y(x) = \frac{e^{\int \frac{-x^2 f(x)-2}{x} dx}}{-c_1 + \int e^{\int \frac{-x^2 f(x)-2}{x} dx} dx} + \frac{1}{x} \right\}$$

HAND SOLUTION

$$\begin{aligned}
 y' + y^2 + (xy - 1)f &= 0 \\
 y'(x) &= (-xy + 1)f - y^2
 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE of the form. We can see a particular solution is $y_p = \frac{1}{x}$, therefore, we use the substitution

$$\begin{aligned}
 y(x) &= y_p(x) + \frac{1}{u(x)} \\
 &= \frac{1}{x} + \frac{1}{u}
 \end{aligned}$$

Hence

$$\begin{aligned}
 y'(x) &= y_p'(x) - \frac{u'(x)}{u^2(x)} \\
 &= \frac{-1}{x^2} - \frac{u'(x)}{u^2(x)}
 \end{aligned} \tag{2}$$

Equating (1) and (2) gives

$$\begin{aligned}
 (-xy + 1)f - y^2 &= \frac{-1}{x^2} - \frac{u'}{u^2} \\
 \left(-x\left(\frac{1}{x} + \frac{1}{u}\right) + 1\right)f - \left(\frac{1}{x} + \frac{1}{u}\right)^2 &= \frac{-1}{x^2} - \frac{u'}{u^2} \\
 \left(\left(-1 - \frac{x}{u}\right) + 1\right)f - \left(\frac{1}{x^2} + \frac{1}{u^2} + \frac{2}{xu}\right) &= \frac{-1}{x^2} - \frac{u'}{u^2} \\
 -\frac{x}{u}f - \left(\frac{1}{x^2} + \frac{1}{u^2} + \frac{2}{xu}\right) &= \frac{-1}{x^2} - \frac{u'}{u^2} \\
 -\frac{x}{u}f - \frac{1}{x^2} - \frac{1}{u^2} - \frac{2}{xu} &= \frac{-1}{x^2} - \frac{u'}{u^2} \\
 -xuf - 1 - \frac{2u}{x} &= -u' \\
 u' &= xuf + 1 + \frac{2u}{x}
 \end{aligned}$$

Hence

$$u' - \left(xf + \frac{2}{x}\right)u = 1$$

Integrating factor is $\mu = e^{\int \left(xf + \frac{2}{x}\right)dx}$, hence the solution is

$$d(\mu u) = \mu$$

Integrating both sides

$$\begin{aligned}
 \mu u &= \int \mu dx + C \\
 u &= e^{-\int \left(xf + \frac{2}{x}\right)dx} \int e^{\int \left(xf + \frac{2}{x}\right)dx} dx + C e^{-\int \left(xf + \frac{2}{x}\right)dx} \\
 &= e^{-\int \left(xf + \frac{2}{x}\right)dx} \left(\int e^{\int \left(xf + \frac{2}{x}\right)dx} dx + C \right)
 \end{aligned}$$

Hence

$$\begin{aligned}
 y &= y_p + \frac{1}{u} \\
 &= \frac{1}{x} + \frac{1}{e^{-\int \left(xf + \frac{2}{x}\right)dx} \left(\int e^{\int \left(xf + \frac{2}{x}\right)dx} dx + C \right)}
 \end{aligned}$$

Hence

$$y(x) = \frac{1}{x} + e^{\int \left(xf + \frac{2}{x}\right)dx} \left(\int e^{\int \left(xf + \frac{2}{x}\right)dx} dx + C \right)^{-1}$$

2.17 ODE No. 17

$$y'(x) - y(x)^2 - 3y(x) + 4 = 0$$

✓ **Mathematica** : cpu = 0.0826272 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow \frac{-1 - 4e^{5x+5c_1}}{-1 + e^{5x+5c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.126 (sec), leaf count = 24

$$\left\{ y(x) = \frac{-4c_1 e^{5x} - 1}{c_1 e^{5x} - 1} \right\}$$

HAND SOLUTION

$$\begin{aligned} y' - y^2 - 3y + 4 &= 0 \\ y' &= 3y - 4 + y^2 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE of the form. The general form is

$$y' = P(x) + Q(x)y + R(x)y^2$$

Where $P(x) = -4, Q(x) = 3, R(x) = 1$. Using the substitution $y = -\frac{u'}{uR(x)} = \frac{-u'}{u}$ then

$$\begin{aligned} u' &= -yu \\ u'' &= -yu' - y'u \\ &= -y(-yu) - (3y - 4 + y^2)u \\ &= y^2u - 3\left(-\frac{u'}{u}\right)u + 4u - y^2u \\ &= 3u' + 4u \end{aligned}$$

Hence

$$u'' - 3u' - 4u = 0$$

This is standard second order ODE. The characteristic equation is $\lambda^2 - 3\lambda - 4 = 0$, with roots $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9+16}}{2} = \frac{3 \pm 5}{2} = \{4, -1\}$, hence

$$u(x) = c_1 e^{4x} + c_2 e^{-x}$$

And

$$u'(x) = c_1 4e^{4x} - c_2 e^{-x}$$

Since $y = \frac{-u'}{u}$ then

$$\begin{aligned} y(x) &= \frac{-c_1 4e^{4x} + c_2 e^{-x}}{c_1 e^{4x} + c_2 e^{-x}} \\ &= \frac{-\frac{c_1}{c_2} 4e^{4x} + e^{-x}}{\frac{c_1}{c_2} e^{4x} + e^{-x}} \end{aligned}$$

Let $\frac{c_1}{c_2} = C_1$ then

$$y(x) = \frac{-4C_1 e^{4x} + e^{-x}}{C_1 e^{4x} + e^{-x}}$$

Dividing by e^{-x}

$$y(x) = \frac{-4C_1 e^{5x} + 1}{C_1 e^{5x} + 1}$$

This is the same result given by CAS. To see it better, let $C_2 = -C_1$ then the above becomes

$$\begin{aligned} y(x) &= \frac{4C_2 e^{5x} + 1}{-C_2 e^{5x} + 1} \\ &= -\frac{4C_2 e^{5x} + 1}{C_2 e^{5x} - 1} \end{aligned}$$

2.18 ODE No. 18

$$y'(x) - y(x)^2 - xy(x) - x + 1 = 0$$

✓ **Mathematica** : cpu = 0.104324 (sec), leaf count = 50

$$\left\{ \left(y(x) \rightarrow -1 + \frac{e^{\frac{x^2}{2}-2x}}{-\frac{\sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\frac{x-2}{\sqrt{2}}\right)}{e^2} + c_1} \right) \right\}$$

✓ **Maple** : cpu = 0.07 (sec), leaf count = 39

$$\left\{ y(x) = \frac{e^{\frac{(x-4)x}{2}}}{c_1 + \frac{i\sqrt{\pi} e^{-2}\sqrt{2} \operatorname{erf}\left(\frac{i\sqrt{2}(x-2)}{2}\right)}{2}} - 1 \right\}$$

HAND SOLUTION

$$\begin{aligned} y' - y^2 - xy - x + 1 &= 0 \\ y' &= x - 1 + xy + y^2 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE of the form. The general form is

$$y' = P(x) + Q(x)y + R(x)y^2$$

Where $P(x) = x - 1$, $Q(x) = x$, $R(x) = 1$. We see that $y_p = -1$ is a particular solution, therefore we use the substitution $y = y_p + \frac{1}{u}$, hence $y' = -\frac{u'}{u^2}$ and equating this to (1) we obtain

$$\begin{aligned} -\frac{u'}{u^2} &= x - 1 + xy + y^2 \\ &= x - 1 + x\left(-1 + \frac{1}{u}\right) + \left(-1 + \frac{1}{u}\right)^2 \\ &= x - 1 - x + \frac{x}{u} + \left(1 + \frac{1}{u^2} - \frac{2}{u}\right) \\ &= \frac{x}{u} + \frac{1}{u^2} - \frac{2}{u} \end{aligned}$$

Hence

$$\begin{aligned} u' &= -u^2\left(\frac{x}{u} + \frac{1}{u^2} - \frac{2}{u}\right) \\ &= -xu - 1 + 2u \\ u' + xu - 2u &= -1 \\ u' + u(x - 2) &= -1 \end{aligned}$$

Integration factor is $e^{\int (x-2)dx} = e^{\frac{x^2}{2}-2x} = e^{\frac{1}{2}x(x-4)}$, therefore

$$d\left(e^{\frac{1}{2}x(x-4)}u\right) = -e^{\frac{1}{2}x(x-4)}$$

Integrating both sides

$$e^{\frac{1}{2}x(x-4)}u = -\int e^{\frac{1}{2}x(x-4)} + C$$

But

$$\int e^{\frac{1}{2}x(x-4)} = \frac{1}{e^2}\sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\frac{x-2}{\sqrt{2}}\right)$$

Hence

$$u(x) = e^{-\frac{1}{2}x(x-4)}\left(\frac{-1}{e^2}\sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\frac{x-2}{\sqrt{2}}\right) + C\right)$$

Since $y = y_p + \frac{1}{u}$ then

$$y = -1 + \frac{1}{e^{\frac{-1}{2}x(x-4)}\left(\frac{-1}{e^2}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{x-2}{\sqrt{2}}\right) + C\right)}$$

Or

$$y = \frac{e^{\frac{1}{2}x(x-4)}}{C - \frac{1}{e^2}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{x-2}{\sqrt{2}}\right)} - 1$$

2.19 ODE No. 19

$$y'(x) - (y(x) + x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0950924 (sec), leaf count = 14

$$\{\{y(x) \rightarrow -x + \tan(x + c_1)\}\}$$

✓ **Maple** : cpu = 0.045 (sec), leaf count = 16

$$\{y(x) = -x - \tan(c_1 - x)\}$$

HAND SOLUTION

$$\begin{aligned} y' - (y + x)^2 &= 0 \\ y' &= (y + x)^2 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE of the form. Let $u = y + x$, then $u' = y' + 1$ and (1) becomes

$$\begin{aligned} u' - 1 &= u^2 \\ u' &= 1 + u^2 \end{aligned}$$

This is separable

$$\begin{aligned} \frac{du}{dx} \frac{1}{1 + u^2} &= 1 \\ \int \frac{du}{1 + u^2} &= \int dx \\ \tan^{-1} u &= x + C \\ u &= \tan(x + C) \end{aligned}$$

Since $u = y + x$ then

$$y = \tan(x + C) - x$$

2.20 ODE No. 20

$$(x^2 + 1)y(x) + y'(x) - y(x)^2 - 2x = 0$$

✓ **Mathematica** : cpu = 0.0997771 (sec), leaf count = 49

$$\left\{ \left(y(x) \rightarrow \frac{e^{\frac{x^3}{3} + x}}{-\int_1^x e^{\frac{K[1]^3}{3} + K[1]} dK[1] + c_1} + x^2 + 1 \right) \right\}$$

✓ **Maple** : cpu = 0.077 (sec), leaf count = 34

$$\left\{ y(x) = x^2 + \frac{e^{\frac{1}{3}x^3 + x}}{c_1 - \left(\int e^{\frac{1}{3}x^3 + x} dx\right)} + 1 \right\}$$

HAND SOLUTION

$$\begin{aligned}(x^2 + 1)y + y' - y^2 - 2x &= 0 \\ y' &= -(x^2 + 1)y + y^2 + 2x\end{aligned}\tag{1}$$

This is Riccati first order non-linear ODE of the form of the general form $y' = P(x) + Q(x)y + R(x)y^2$ where $P(x) = 2x$, $Q(x) = -(x^2 + 1)$, $R(x) = 1$. We can convert this to Bernoulli first order ODE in $u(x)$, which is little easier to solve by using $u = y - x^2 - 1$. The difference between Bernoulli and Riccati is that the term $P(x) = 0$ in Bernoulli. If $P(x) \neq 0$ and $R(x) \neq 0$ then it is called Riccati.

Using $u = y - x^2 - 1$ gives

$$\begin{aligned}u' &= y' - 2x \\ u' &= [-(x^2 + 1)y + y^2 + 2x] - 2x \\ &= -(x^2 + 1)(u + x^2 + 1) + (u + x^2 + 1)^2 \\ &= (u + x^2 + 1)[(u + x^2 + 1) - (x^2 + 1)] \\ &= (u + x^2 + 1)u \\ &= u^2 + u(1 + x^2)\end{aligned}$$

We see now this is Bernoulli since $P(x) = 0$. To solve Bernoulli we always start by dividing by u^2 giving

$$\frac{u'}{u^2} = 1 + \frac{1}{u}(1 + x^2)$$

Next we let $v = \frac{1}{u}$, hence $v' = -\frac{u'}{u^2}$ therefore the above becomes

$$\begin{aligned}-v' &= 1 + v(1 + x^2) \\ v' + v(1 + x^2) &= -1\end{aligned}$$

Integrating factor is $e^{\int(1+x^2)dx} = e^{\left(x+\frac{x^3}{2}\right)}$, therefore

$$d\left(e^{\left(x+\frac{x^3}{2}\right)}v\right) = -e^{\left(x+\frac{x^3}{2}\right)}$$

Integrating

$$\begin{aligned}e^{\left(x+\frac{x^3}{2}\right)}v &= -\int e^{\left(x+\frac{x^3}{2}\right)}dx + C \\ v(x) &= e^{-\left(x+\frac{x^3}{2}\right)}\left(C - \int e^{\left(x+\frac{x^3}{2}\right)}dx\right)\end{aligned}$$

Therefore

$$u = \frac{1}{v} = \frac{e^{\left(x+\frac{x^3}{2}\right)}}{\left(C - \int e^{\left(x+\frac{x^3}{2}\right)}dx\right)}$$

And since $u = y - x^2 - 1$ then

$$\begin{aligned}y(x) &= u + 1 + x^2 \\ &= \frac{e^{\left(x+\frac{x^3}{2}\right)}}{\left(C - \int e^{\left(x+\frac{x^3}{2}\right)}dx\right)} + 1 + x^2\end{aligned}$$

2.21 ODE No. 21

$$y'(x) - y(x)^2 + y(x) \sin(x) - \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.374642 (sec), leaf count = 7

$$\{\{y(x) \rightarrow \sin(x)\}\}$$

✓ **Maple** : cpu = 0.159 (sec), leaf count = 25

$$\left\{ y(x) = -\frac{e^{-\cos(x)}}{c_1 + \int e^{-\cos(x)} dx} + \sin(x) \right\}$$

HAND SOLUTION

$$\begin{aligned} y' - y^2 + y \sin(x) - \cos(x) &= 0 \\ y' &= y^2 - y \sin(x) + \cos(x) \end{aligned} \quad (1)$$

This is Riccati first order non-linear ODE of the form of the general form $y' = P(x) + Q(x)y + R(x)y^2$ where $P(x) = \cos(x)$, $Q(x) = -\sin(x)$, $R(x) = 1$. It is best to first try to spot a particular solution y_p and use the transformation $y = y_p + \frac{1}{u}$ otherwise we use $y = -\frac{u'}{yR(x)}$ transformation. For this problem

$$y_p = \sin(x)$$

Therefore

$$\begin{aligned} y &= \sin x + \frac{1}{u} \\ y' &= \cos x - \frac{u'}{u^2} \end{aligned}$$

Equating this to (1) gives

$$\begin{aligned} y^2 - y \sin(x) + \cos(x) &= \cos x - \frac{u'}{u^2} \\ \left(\sin x + \frac{1}{u} \right)^2 - \left(\sin x + \frac{1}{u} \right) \sin x + \cos x &= \cos x - \frac{u'}{u^2} \\ \sin^2 x + \frac{1}{u^2} + \frac{2}{u} \sin x - \sin^2 x - \frac{1}{u} \sin x &= -\frac{u'}{u^2} \\ \frac{1}{u^2} + \frac{1}{u} \sin x &= -\frac{u'}{u^2} \\ 1 + u \sin x &= -u' \\ u' + u \sin x &= -1 \end{aligned}$$

Integrating factor is $e^{\int \sin x} = e^{-\cos x}$, hence

$$d(e^{-\cos x} u) = -e^{-\cos x}$$

Integrating both sides

$$\begin{aligned} e^{-\cos x} u &= -\int e^{-\cos x} dx + C \\ u &= e^{\cos x} \left(C - \int e^{-\cos x} dx \right) \end{aligned}$$

Since $y = \sin x + \frac{1}{u}$ then

$$y = \sin x + \frac{e^{-\cos x}}{C - \int e^{-\cos x} dx}$$

Or letting $C_1 = -C$ to make match Maple form, we obtain

$$y = -\frac{e^{-\cos x}}{C_1 + \int e^{-\cos x} dx} + \sin x$$

2.22 ODE No. 22

$$y'(x) - y(x)^2 - y(x) \sin(2x) - \cos(2x) = 0$$

✓ **Mathematica** : cpu = 0.59381 (sec), leaf count = 113

$$\left\{ \left\{ y(x) \rightarrow -\frac{-\sin(x) \int_1^{\cos(x)} \frac{e^{-K[1]^2}}{K[1]^2 \sqrt{K[1]^2 - 1}} dK[1] - \frac{e^{-\cos^2(x)} \tan(x)}{\sqrt{\cos^2(x) - 1}} - c_1 \sin(x)}{\cos(x) \int_1^{\cos(x)} \frac{e^{-K[1]^2}}{K[1]^2 \sqrt{K[1]^2 - 1}} dK[1] + c_1 \cos(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.438 (sec), leaf count = 128

$$\left\{ y(x) = \frac{2 \left(c_1 \operatorname{HeunC}\left(1, \frac{1}{2}, -\frac{1}{2}, -1, \frac{7}{8}, \frac{\cos(2x)}{2} + \frac{1}{2}\right) + c_1 (\cos(2x) + 1) \operatorname{HeunCPrime}\left(1, \frac{1}{2}, -\frac{1}{2}, -1, \frac{7}{8}, \frac{\cos(2x)}{2} + \frac{1}{2}\right) + \frac{\sqrt{2 \cos(2x)}}{\sqrt{2 \cos(2x) + 2}} \right)}{\sqrt{2 \cos(2x) + 2} \left(c_1 \sqrt{2 \cos(2x) + 2} \operatorname{HeunC}\left(1, \frac{1}{2}, -\frac{1}{2}, -1, \frac{7}{8}, \frac{\cos(2x)}{2} + \frac{1}{2}\right) + \operatorname{HeunC}\left(1, -\frac{1}{2}, -1, \frac{7}{8}, \frac{\cos(2x)}{2} + \frac{1}{2}\right) \right)}$$

HAND SOLUTION

$$\begin{aligned} y' - y^2 - y \sin(2x) - \cos(2x) &= 0 \\ y' &= y^2 + y \sin(2x) + \cos(2x) \end{aligned} \quad (1)$$

This is Riccati first order non-linear ODE of the form of the general form $y' = P(x) + Q(x)y + R(x)y^2$ where $P(x) = \cos(2x)$, $Q(x) = \sin(2x)$, $R(x) = 1$. It is best to first try to spot a particular solution y_p and use the transformation $y = y_p + \frac{1}{u}$ otherwise we use $y = -\frac{u'}{yR(x)}$ transformation. For this problem

$$y_p = \tan(x)$$

To verify, since $y'_p = \frac{1}{\cos^2 x}$ then plugging this particular in (1) gives

$$\frac{1}{\cos^2 x} - \tan^2(x) - \tan(x) \sin(2x) - \cos(2x) = 0$$

But $\cos(2x) = \cos^2 x - \sin^2 x$ and $\sin(2x) = 2 \sin x \cos x$ and $\tan(x) = \frac{\sin x}{\cos x}$ therefore the above becomes

$$\begin{aligned}
\frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} - \frac{\sin x}{\cos x}(2 \sin x \cos x) - (\cos^2 x - \sin^2 x) &= 0 \\
\frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} - 2 \sin^2 x - \cos^2 x + \sin^2 x &= 0 \\
\frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} - \sin^2 x - \cos^2 x &= 0 \\
\frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} - 1 &= 0 \\
\frac{1 - \sin^2 x}{\cos^2 x} - 1 &= 0 \\
\frac{\cos^2 x}{\cos^2 x} - 1 &= 0 \\
1 - 1 &= 0 \\
0 &= 0
\end{aligned}$$

Therefore we, we can use $y = y_p + \frac{1}{u}$

$$\begin{aligned}
y &= \tan x + \frac{1}{u} \\
y' &= \frac{1}{\cos^2 x} - \frac{u'}{u^2}
\end{aligned}$$

Equating this to (1) gives

$$\begin{aligned}
-\frac{u'}{u^2} &= y^2 + y \sin(2x) + \cos(2x) \\
-\frac{u'}{u^2} &= -\frac{1}{\cos^2 x} + \left(\tan x + \frac{1}{u}\right)^2 + \left(\tan x + \frac{1}{u}\right) \sin(2x) + \cos(2x)
\end{aligned}$$

Using $\sin(2x) = 2 \sin x \cos x$ and $\cos 2x = \cos^2 x - \sin^2 x$ then above becomes

$$\begin{aligned}
-\frac{u'}{u^2} &= -\frac{1}{\cos^2 x} + \left(\tan^2 x + \frac{1}{u^2} + \frac{2}{u} \tan x\right) + \left(\frac{\sin x}{\cos x} + \frac{1}{u}\right) 2 \sin x \cos x + (\cos^2 x - \sin^2 x) \\
u' &= \frac{u^2}{\cos^2 x} - \left(u^2 \frac{\sin^2 x}{\cos^2 x} + 1 + 2u \frac{\sin x}{\cos x}\right) - \left(u^2 \frac{\sin x}{\cos x} + u\right) 2 \sin x \cos x - u^2 \cos^2 x + u^2 \sin^2 x \\
&= \frac{u^2}{\cos^2 x} - u^2 \frac{\sin^2 x}{\cos^2 x} - 1 - 2u \frac{\sin x}{\cos x} - 2u^2 \frac{\sin x}{\cos x} \sin x \cos x - 2u \sin x \cos x - u^2 \cos^2 x + u^2 \sin^2 x \\
&= \frac{u^2}{\cos^2 x} - u^2 \frac{\sin^2 x}{\cos^2 x} - 1 - 2u \frac{\sin x}{\cos x} - 2u^2 \sin^2 x - 2u \sin x \cos x - u^2 \cos^2 x + u^2 \sin^2 x \\
&= \frac{u^2}{\cos^2 x} - u^2 \frac{\sin^2 x}{\cos^2 x} - 1 - 2u \frac{\sin x}{\cos x} - u^2 \sin^2 x - 2u \sin x \cos x - u^2 \cos^2 x \\
&= u^2 \left(\frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} - (\sin^2 x + \cos^2 x) \right) - 1 + u \left(-2 \frac{\sin x}{\cos x} - 2 \sin x \cos x \right) \\
&= u^2 \left(\frac{1 - \sin^2 x}{\cos^2 x} - 1 \right) - 1 + u \left(-2 \frac{\sin x}{\cos x} - 2 \sin x \cos x \right) \\
&= u^2 \left(\frac{\cos^2 x}{\cos^2 x} - 1 \right) - 1 + u \left(-2 \frac{\sin x}{\cos x} - 2 \sin x \cos x \right) \\
&= -1 + 2u \left(-\frac{\sin x}{\cos x} - \sin x \cos x \right)
\end{aligned}$$

Hence

$$u' + 2u(\tan x + \sin x \cos x) = -1$$

Integrating factor is $e^{2 \int \tan x + \sin x \cos x dx}$. But

$$\int \tan x dx = -\ln(\cos x)$$

And

$$\int \sin x \cos x dx = \frac{-1}{2} \cos^2 x$$

Hence $\mu = e^{-2 \ln \cos x} e^{-\cos^2 x} = \frac{1}{\cos^2 x} e^{-\cos^2 x}$, therefore

$$d\left(\frac{1}{\cos^2 x} e^{-\cos^2 x} u\right) = \frac{-1}{\cos^2 x} e^{-\cos^2 x}$$

Integrating both sides

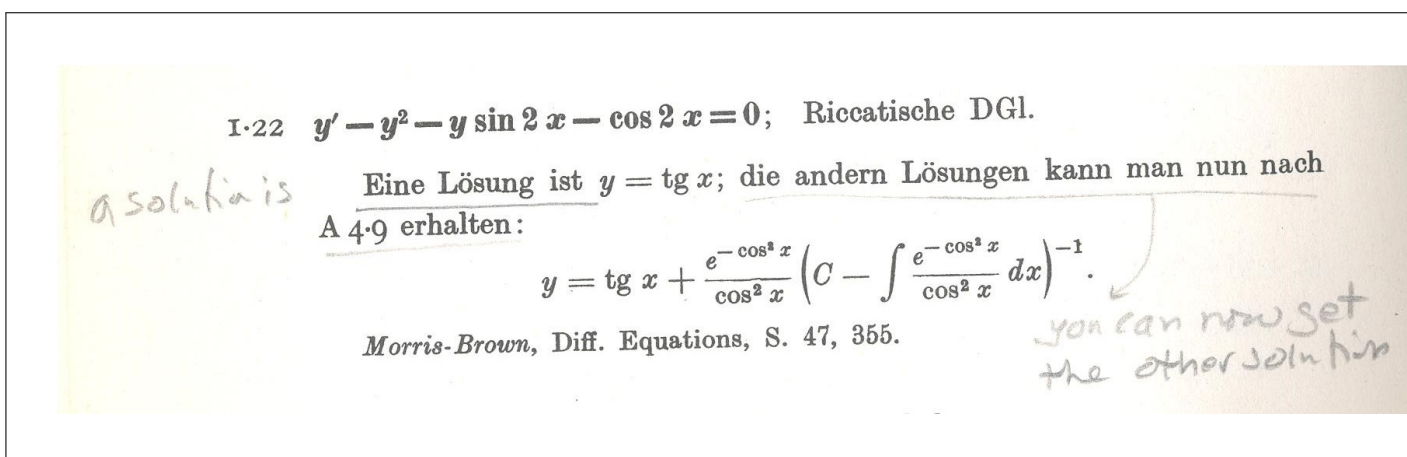
$$\begin{aligned} \frac{1}{\cos^2 x} e^{-\cos^2 x} u &= -\int \frac{e^{-\cos^2 x}}{\cos^2 x} dx + C \\ u &= \cos^2 x e^{\cos^2 x} \left(C - \int \frac{e^{-\cos^2 x}}{\cos^2 x} dx \right) \end{aligned}$$

Since $y = \tan x + \frac{1}{u}$ then

$$\begin{aligned} y &= \tan x + \frac{1}{\cos^2 x e^{\cos^2 x} \left(C - \int \frac{e^{-\cos^2 x}}{\cos^2 x} dx \right)} \\ &= \tan x + \frac{e^{-\cos^2 x}}{\cos^2 x} \left(C - \int \frac{e^{-\cos^2 x}}{\cos^2 x} dx \right)^{-1} \end{aligned}$$

I do not know how Maple came up with the solution involving HeunC functions since $\int \frac{e^{-\cos^2 x}}{\cos^2 x} dx$ has no closed form solution. I should ask CAS experts about this.

Below is screen shot from Kamke book of the solution it gives, which matches the above result



2.23 ODE No. 23

$$ay(x)^2 - b + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0867575 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{b} \tanh(\sqrt{a} \sqrt{b} x + \sqrt{a} \sqrt{b} c_1)}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 23

$$\left\{ y(x) = \frac{\sqrt{ab} \tanh(\sqrt{ab} (c_1 + x))}{a} \right\}$$

HAND SOLUTION

$$y' + ay^2 - b = 0$$
$$\frac{dy}{dx} = b - ay^2$$

Separable,

$$\frac{dy}{b - ay^2} = dx$$
$$\int \frac{dy}{b - ay^2} = \int dx$$

But

$$\int \frac{dy}{b - ay^2} = \frac{1}{\sqrt{ab}} \tanh^{-1}\left(\sqrt{\frac{a}{b}} y\right)$$

Hence

$$\frac{1}{\sqrt{ab}} \tanh^{-1}\left(\sqrt{\frac{a}{b}} y\right) = x + C$$
$$\tanh^{-1}\left(\sqrt{\frac{a}{b}} y\right) = \sqrt{ab} (x + C)$$
$$\sqrt{\frac{a}{b}} y = \tanh\left(\sqrt{ab} (x + C)\right)$$
$$y = \sqrt{\frac{b}{a}} \tanh\left(\sqrt{ab} (x + C)\right)$$

2.24 ODE No. 24

$$ay(x)^2 - bx^\nu + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.147869 (sec), leaf count = 277

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{-a} \sqrt{b} x^{\frac{\nu+2}{2}} \left(c_1 J_{\frac{\nu+1}{\nu+2}} \left(\frac{2\sqrt{-a} \sqrt{b} x^{\frac{\nu+1}{2}}}{\nu+2} \right) - 2 J_{\frac{1}{\nu+2}-1} \left(\frac{2\sqrt{-a} \sqrt{b} x^{\frac{\nu+2}{2}}}{\nu+2} \right) - c_1 J_{-\frac{\nu+3}{\nu+2}} \left(\frac{2\sqrt{-a} \sqrt{b} x^{\frac{\nu+2}{2}}}{\nu+2} \right) \right) - c_1 J_{-\frac{1}{\nu+2}} \left(\frac{2\sqrt{-a} \sqrt{b} x^{\frac{\nu+2}{2}}}{\nu+2} \right)}{2ax \left(J_{\frac{1}{\nu+2}} \left(\frac{2\sqrt{-a} \sqrt{b} x^{\frac{\nu+2}{2}}}{\nu+2} \right) + c_1 J_{-\frac{1}{\nu+2}} \left(\frac{2\sqrt{-a} \sqrt{b} x^{\frac{\nu+2}{2}}}{\nu+2} \right) \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 214

$$\left\{ y(x) = \frac{-c_1 \sqrt{-ab} x^{\frac{\nu}{2}+1} \text{BesselJ}\left(\frac{\nu+3}{\nu+2}, \frac{2\sqrt{-ab} x^{\frac{\nu}{2}+1}}{\nu+2}\right) + c_1 \text{BesselJ}\left(\frac{1}{\nu+2}, \frac{2\sqrt{-ab} x^{\frac{\nu}{2}+1}}{\nu+2}\right) - \sqrt{-ab} x^{\frac{\nu}{2}+1} \text{BesselY}\left(\frac{\nu+3}{\nu+2}, \frac{2\sqrt{-ab} x^{\frac{\nu}{2}+1}}{\nu+2}\right) + \left(c_1 \text{BesselJ}\left(\frac{1}{\nu+2}, \frac{2\sqrt{-ab} x^{\frac{\nu}{2}+1}}{\nu+2}\right) + \text{BesselY}\left(\frac{1}{\nu+2}, \frac{2\sqrt{-ab} x^{\frac{\nu}{2}+1}}{\nu+2}\right) \right) ax}{\left(c_1 \text{BesselJ}\left(\frac{1}{\nu+2}, \frac{2\sqrt{-ab} x^{\frac{\nu}{2}+1}}{\nu+2}\right) + \text{BesselY}\left(\frac{1}{\nu+2}, \frac{2\sqrt{-ab} x^{\frac{\nu}{2}+1}}{\nu+2}\right) \right) ax}$$

HAND SOLUTION

$$y' + ay^2 - bx^\nu = 0$$
$$y' = bx^\nu - ay^2 \tag{1}$$
$$= P(x) + Q(x)y + R(x)y^2$$

This is Riccati first order non-linear ODE with $P(x) = bx^\nu$, $Q(x) = 0$, $R(x) = -a$. Using the standard substitution

$$y = -\frac{u'}{uR(x)} = \frac{u'}{au}$$

Hence

$$y' = \frac{u''}{au} - \frac{(u')^2}{au^2}$$

Therefore (1) becomes

$$\begin{aligned} \frac{u''}{au} - \frac{(u')^2}{au^2} &= bx^v - ay^2 \\ &= bx^v - a \left(\frac{u'}{au} \right)^2 \\ &= bx^v - \frac{(u')^2}{au^2} \end{aligned}$$

Hence

$$\begin{aligned} \frac{u''}{au} &= bx^v \\ u'' - abx^v u &= 0 \end{aligned}$$

This is an Emden-Fowler equation, of the general form $u'' = Ax^n u^m$, where here $m = 1$ and $n = v$ and $A = ab$.

For any n , the solution uses Bessel functions and modified Bessel functions of first and second kind. From Handbook of exact solutions for ODE, page 237, equation 2.1.2.7 we see the solution is given as

$$u = \begin{cases} C_1 \sqrt{x} J_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) + C_2 \sqrt{x} Y_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) & ab < 0 \\ C_1 \sqrt{x} I_{\frac{1}{2q}} \left(\frac{\sqrt{ab}}{q} x^q \right) + C_2 \sqrt{x} K_{\frac{1}{2q}} \left(\frac{\sqrt{ab}}{q} x^q \right) & ab > 0 \end{cases}$$

Where $q = \frac{n+1}{2}$. J is Bessel function of first kind and Y is Bessel function of second kind. I is modified Besself function of first kind and K is modified Besself function of second kind. To find y we now use $y = \frac{u'}{au}$. Derivative of Bessel functions is given by

$$\begin{aligned} J'_m(x) &= \frac{1}{2}(J_{m-1}(x) - J_{m+1}(x)) \\ Y'_m(x) &= \frac{1}{2}(Y_{m-1}(x) - Y_{m+1}(x)) \\ I'_m(x) &= \frac{1}{2}(I_{m-1}(x) + I_{m+1}(x)) \\ K'_m(x) &= -\frac{1}{2}(K_{m-1}(x) + K_{m+1}(x)) \end{aligned}$$

Using these, then

$$u' = \begin{cases} C_1 \left[\frac{1}{2\sqrt{x}} J_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) + \sqrt{x} J'_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) \right] + C_2 \left[\frac{1}{2\sqrt{x}} Y_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) + \sqrt{x} Y'_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) \right] & ab < 0 \\ C_1 \left[\frac{1}{2\sqrt{x}} I_{\frac{1}{2q}} \left(\frac{\sqrt{ab}}{q} x^q \right) + \sqrt{x} I'_{\frac{1}{2q}} \left(\frac{\sqrt{ab}}{q} x^q \right) \right] + C_2 \left[\frac{1}{2\sqrt{x}} K_{\frac{1}{2q}} \left(\frac{\sqrt{ab}}{q} x^q \right) + \sqrt{x} K'_{\frac{1}{2q}} \left(\frac{\sqrt{ab}}{q} x^q \right) \right] & ab > 0 \end{cases}$$

Hence for $ab < 0$

$$\begin{aligned} y &= \frac{u'}{au} \\ &= \frac{C_1 \left[\frac{1}{2\sqrt{x}} J_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) + \sqrt{x} J'_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) \right] + C_2 \left[\frac{1}{2\sqrt{x}} Y_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) + \sqrt{x} Y'_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) \right]}{aC_1 \sqrt{x} J_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) + aC_2 \sqrt{x} Y_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right)} \\ &= \frac{\sqrt{x} C_1 \left[\frac{1}{2x} J_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) + J'_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) \right] + \sqrt{x} C_2 \left[\frac{1}{2x} Y_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) + Y'_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) \right]}{aC_1 \sqrt{x} J_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) + aC_2 \sqrt{x} Y_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right)} \\ &= \frac{C_1 \left[\frac{1}{2x} J_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) + J'_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) \right] + C_2 \left[\frac{1}{2x} Y_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) + Y'_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) \right]}{aC_1 J_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) + aC_2 Y_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right)} \end{aligned}$$

Using derivatives the above becomes

$$\begin{aligned} y &= \frac{C_1 \left[\frac{1}{2x} J_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) + \frac{1}{2} \left(J_{\frac{1}{2q}-1} \left(\frac{\sqrt{-ab}}{q} x^q \right) - J_{\frac{1}{2q}+1} \left(\frac{\sqrt{-ab}}{q} x^q \right) \right) \right]}{aC_1 J_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) + aC_2 Y_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right)} \\ &+ \frac{C_2 \left[\frac{1}{2x} Y_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) + \frac{1}{2} \left(Y_{\frac{1}{2q}-1} \left(\frac{\sqrt{-ab}}{q} x^q \right) - Y_{\frac{1}{2q}+1} \left(\frac{\sqrt{-ab}}{q} x^q \right) \right) \right]}{aC_1 J_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right) + aC_2 Y_{\frac{1}{2q}} \left(\frac{\sqrt{-ab}}{q} x^q \right)} \end{aligned}$$

Similar result can be found for $ab > 0$

2.25

ODE No. 25

$$ay(x)^2 - bx^{2\nu} - cx^{\nu-1} + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.408475 (sec), leaf count = 1835

$$\left\{ \left(\begin{aligned} &2^{\frac{\nu}{2(\nu+1)}} \sqrt{a} \sqrt{b} e^{-\frac{\sqrt{a} \sqrt{b} x^{\nu+1}}{\sqrt{\nu^2+2\nu+1}}} (\nu+1) (x^{\nu+1})^{\frac{\nu}{2(\nu+1)}} \\ &- 2^{\frac{\nu}{2(\nu+1)}-1} e^{-\frac{\sqrt{a} \sqrt{b} x^{\nu+1}}{\sqrt{\nu^2+2\nu+1}}} \nu (x^{\nu+1})^{\frac{\nu}{2(\nu+1)}} L^{\frac{\nu}{\nu+1}-1}_{\frac{\frac{\sqrt{a} \sqrt{b} \nu c}{\sqrt{(\nu+1)^2}} + \frac{\sqrt{a} \sqrt{b} c}{\sqrt{(\nu+1)^2}} + b\nu} \left(\frac{2\sqrt{a} \sqrt{b} x^{\nu+1}}{\sqrt{\nu^2+2\nu+1}} \right) x^{-\frac{\nu}{2}-1} - \frac{\phantom{2^{\frac{\nu}{2(\nu+1)}} \sqrt{a} \sqrt{b} e^{-\frac{\sqrt{a} \sqrt{b} x^{\nu+1}}{\sqrt{\nu^2+2\nu+1}}} (\nu+1) (x^{\nu+1})^{\frac{\nu}{2(\nu+1)}}}}{\sqrt{\nu^2+2\nu+1}} \end{aligned} \right) \right\} y(x) \rightarrow \phantom{2^{\frac{\nu}{2(\nu+1)}} \sqrt{a} \sqrt{b} e^{-\frac{\sqrt{a} \sqrt{b} x^{\nu+1}}{\sqrt{\nu^2+2\nu+1}}} (\nu+1) (x^{\nu+1})^{\frac{\nu}{2(\nu+1)}}}$$

✓ **Maple** : cpu = 0.325 (sec), leaf count = 348

$$\left\{ y(x) = -\frac{2c_1(\nu+1)b^{\frac{3}{2}} \operatorname{WhittakerW}\left(-\frac{\sqrt{a} c+(-2\nu-2)\sqrt{b}}{(2\nu+2)\sqrt{b}}, \frac{1}{2\nu+2}, \frac{2\sqrt{a} \sqrt{b} x^{\nu+1}}{\nu+1}\right) + \left(\sqrt{a} b c + (-\nu-2)b^{\frac{3}{2}}\right) \operatorname{WhittakerM}\left(-\frac{\sqrt{a} c+(-2\nu-2)\sqrt{b}}{(2\nu+2)\sqrt{b}}, \frac{1}{2\nu+2}, \frac{2\sqrt{a} \sqrt{b} x^{\nu+1}}{\nu+1}\right)}{2\left(c_1 \operatorname{WhittakerW}\left(-\frac{\sqrt{a} c}{(2\nu+2)\sqrt{b}}, \frac{1}{2\nu+2}, \frac{2\sqrt{a} \sqrt{b} x^{\nu+1}}{\nu+1}\right) + \left(\sqrt{a} b c + (-\nu-2)b^{\frac{3}{2}}\right) \operatorname{WhittakerM}\left(-\frac{\sqrt{a} c}{(2\nu+2)\sqrt{b}}, \frac{1}{2\nu+2}, \frac{2\sqrt{a} \sqrt{b} x^{\nu+1}}{\nu+1}\right)}\right\}$$

$$\begin{aligned}
y' + ay^2 - bx^{2v} - cx^{v-1} &= 0 \\
y' &= bx^v + cx^{v-1} - ay^2 \\
&= P(x) + Q(x)y + R(x)y^2
\end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE with $P(x) = bx^v + cx^{v-1}$, $Q(x) = 0$, $R(x) = -a$.

Need to do this later.

2.26 ODE No. 26

$$y'(x) - (Ay(x) - a)(By(x) - b) = 0$$

✓ **Mathematica** : cpu = 0.263709 (sec), leaf count = 68

$$\left\{ \left\{ y(x) \rightarrow \frac{ae^{Abx+Abc_1} - be^{aBx+aBc_1}}{Ae^{Abx+Abc_1} - Be^{aBx+aBc_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.283 (sec), leaf count = 45

$$\left\{ y(x) = \frac{a e^{(c_1+x)(Ab-aB)} - b}{A e^{(c_1+x)(Ab-aB)} - B} \right\}$$

HAND SOLUTION

$$\begin{aligned}
y' - (Ay - a)(By - b) &= 0 \\
y' &= (Ay - a)(By - b) \\
&= ab - y(Ab + Ba) + AB y^2
\end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE with $P(x) = ab$, $Q(x) = -(Ab + Ba)$, $R(x) = AB$. Let $y = -\frac{u'}{uR(x)} = -\frac{u'}{ABu}$, hence

$$y' = \frac{-u''}{ABu} - \frac{(u')^2}{ABu^2}$$

Comparing to (1) results in

$$\begin{aligned}
\frac{-u''}{ABu} - \frac{(u')^2}{ABu^2} &= ab - y(Ab + Ba) + AB y^2 \\
&= ab - \left(-\frac{u'}{ABu} \right) (Ab + Ba) + AB \left(-\frac{u'}{ABu} \right)^2 \\
&= ab + \frac{u'}{ABu} (Ab + Ba) + AB \frac{(u')^2}{(ABu)^2} \\
&= ab + \frac{u'}{ABu} (Ab + Ba) + \frac{(u')^2}{ABu^2}
\end{aligned}$$

Hence

$$\begin{aligned}
\frac{-u''}{ABu} &= ab + \frac{u'}{ABu} (Ab + Ba) \\
-u'' &= ABabu + u'(Ab + Ba) \\
u'' + u'(Ab + Ba) + u(ABab) &= 0
\end{aligned}$$

This is second order ODE with constant coefficient. Solution is

$$u = c_1 e^{-aBx} + c_2 e^{-Abx}$$

Therefore

$$u' = -aBc_1e^{-aBx} - c_2Abe^{-Abx}$$

And therefore the solution is

$$\begin{aligned} y &= -\frac{u'}{ABu} = -\frac{1}{AB} \frac{-aBc_1e^{-aBx} - c_2Abe^{-Abx}}{c_1e^{-aBx} + c_2e^{-Abx}} \\ &= \frac{aBc_1e^{-aBx} + c_2Abe^{-Abx}}{AB(c_1e^{-aBx} + c_2e^{-Abx})} \end{aligned}$$

Dividing by c_2 and letting $c = \frac{c_1}{c_2}$

$$y = \frac{aBce^{-aBx} + Abe^{-Abx}}{AB(ce^{-aBx} + e^{-Abx})}$$

Verification

```
eq:=diff(y(x),x)-(A*y(x)-a)*(B*y(x)-b) = 0;
sol:=(a*B*_C1*exp(-a*B*x)+A*b*exp(-A*b*x))/(A*B*( _C1*exp(-a*B*x)+exp(-A*b*x)));
odetest(y(x)=sol,eq);
0
```

2.27 ODE No. 27

$$ay(x)(y(x) - x) + y'(x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.272532 (sec), leaf count = 120

$$\left\{ \left\{ y(x) \rightarrow \frac{axe^{\frac{ax^2}{2}} + c_1 \left(\sqrt{\frac{\pi}{2}} \sqrt{a} xe^{\frac{ax^2}{2}} \operatorname{erf}\left(\frac{\sqrt{a}x}{\sqrt{2}}\right) + 1 \right)}{a \left(e^{\frac{ax^2}{2}} + \frac{\sqrt{\frac{\pi}{2}} c_1 e^{\frac{ax^2}{2}} \operatorname{erf}\left(\frac{\sqrt{a}x}{\sqrt{2}}\right)}{\sqrt{a}} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.184 (sec), leaf count = 72

$$\left\{ y(x) = \frac{2\sqrt{a} e^{-\frac{ax^2}{2}} + \left(2c_1a^{\frac{3}{2}} + \sqrt{\pi} \sqrt{2} a \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{a} x}{2}\right) \right) x}{2c_1a^{\frac{3}{2}} + \sqrt{\pi} \sqrt{2} a \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{a} x}{2}\right)} \right\}$$

HAND SOLUTION

$$\begin{aligned} y' + ay(y - x) - 1 &= 0 \\ y' &= 1 - (ay^2 - ayx) \\ &= 1 + ayx - ay^2 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE $y' = P(x) + A(x)y + R(x)y^2$ with $P(x) = 1, Q(x) = -ax, R(x) = -a$. We can convert Riccati to Bernoulli which is easier to solve using the substitution

$$u = y - x$$

$$\begin{aligned}
 u' &= y' - 1 \\
 &= (1 + ayx - ay^2) - 1 \\
 &= (1 + a(u+x)x - a(u+x)^2) - 1 \\
 &= 1 + aux + ax^2 - a(u^2 + x^2 + 2ux) - 1 \\
 &= 1 + aux + ax^2 - au^2 - ax^2 - 2aux - 1 \\
 &= -aux - au^2 \\
 u' &= -aux - au^2
 \end{aligned}$$

This is of the form $u' = P(x) + Q(x)u + R(x)u^2$ and since $P(x) = 0$ then it is Bernoulli differential equation. (when $P(x) \neq 0$ and $R(x) \neq 0$ it is Riccati). To solve Bernoulli we always start by dividing by u^2

$$\frac{u'}{u^2} = -\frac{ax}{u} - a$$

Then we let $\zeta = \frac{1}{u}$, hence $\zeta' = -\frac{u'}{u^2}$, therefore the above becomes

$$\begin{aligned}
 -\zeta' &= -ax\zeta - a \\
 \zeta' - ax\zeta &= a
 \end{aligned}$$

Integrating factor is $e^{-\int ax dx} = e^{-a\frac{x^2}{2}}$, hence $d\left(e^{-a\frac{x^2}{2}}\zeta\right) = ae^{-a\frac{x^2}{2}}$. Integrating both sides gives

$$e^{-a\frac{x^2}{2}}\zeta = a \int e^{-a\frac{x^2}{2}} dx + C$$

But

$$\int e^{-a\frac{x^2}{2}} dx = \sqrt{\frac{\pi}{2a}} \operatorname{erf}\left(\sqrt{\frac{a}{2}}x\right)$$

Therefore

$$\begin{aligned}
 e^{-a\frac{x^2}{2}}\zeta &= a\sqrt{\frac{\pi}{2a}} \operatorname{erf}\left(\sqrt{\frac{a}{2}}x\right) + C \\
 \zeta &= e^{a\frac{x^2}{2}}\left(a\sqrt{\frac{\pi}{2a}} \operatorname{erf}\left(\sqrt{\frac{a}{2}}x\right) + C\right)
 \end{aligned}$$

Hence

$$\begin{aligned}
 u &= \frac{1}{\zeta} \\
 &= e^{-a\frac{x^2}{2}}\left(a\sqrt{\frac{\pi}{2a}} \operatorname{erf}\left(\sqrt{\frac{a}{2}}x\right) + C\right)^{-1}
 \end{aligned}$$

Since $u = y - x$ then

$$\begin{aligned}
 y &= u + x \\
 &= e^{-a\frac{x^2}{2}}\left(a\sqrt{\frac{\pi}{2a}} \operatorname{erf}\left(\sqrt{\frac{a}{2}}x\right) + C\right)^{-1} + x \\
 &= \frac{e^{-a\frac{x^2}{2}}}{\sqrt{\frac{a\pi}{2}} \operatorname{erf}\left(\sqrt{\frac{a}{2}}x\right) + C} + x
 \end{aligned}$$

Verification

```

eq:=diff(y(x),x)+a*y(x)*(y(x)-x)-1 = 0;
sol:=exp(-a*x^2/2)/(sqrt(a*Pi/2)*erf(sqrt(a/2)*x)+_C1)+x;
odetest(y(x)=sol,eq);
0

```

2.28 ODE No. 28

$$x^3(-y(x)) + y'(x) + xy(x)^2 - 2x = 0$$

✓ **Mathematica** : cpu = 0.162688 (sec), leaf count = 96

$$\left\{ \left(y(x) \rightarrow \frac{\frac{1}{2}\sqrt{\pi} e^{\frac{x^4}{4}} x^3 \operatorname{erf}\left(\frac{x^2}{2}\right) + c_1 e^{\frac{x^4}{4}} x^3 + x}{x \left(\frac{1}{2}\sqrt{\pi} e^{\frac{x^4}{4}} \operatorname{erf}\left(\frac{x^2}{2}\right) + c_1 e^{\frac{x^4}{4}} \right)} \right) \right\}$$

✓ **Maple** : cpu = 0.096 (sec), leaf count = 51

$$\left\{ y(x) = \frac{c_1 \sqrt{\pi} x^2 \operatorname{erf}\left(\frac{x^2}{2}\right) + 2c_1 e^{-\frac{x^4}{4}} + \sqrt{\pi} x^2}{\sqrt{\pi} \left(c_1 \operatorname{erf}\left(\frac{x^2}{2}\right) + 1 \right)} \right\}$$

HAND SOLUTION

$$\begin{aligned} y' - yx^3 + xy^2 - 2x &= 0 \\ y' &= 2x + yx^3 - xy^2 \\ &= P(x) + Q(x)y + R(x)y^2 \end{aligned} \quad (1)$$

This is Riccati first order non-linear ODE with $P(x) = 2x, Q(x) = x^3, R(x) = -x$. We can convert Riccati to Bernoulli which is easier to solve using the substitution $u = x^2 - y$ or $y = x^2 - u$

$$\begin{aligned} u' &= 2x - y' \\ &= 2x - (2x + yx^3 - xy^2) \\ &= 2x - (2x + (x^2 - u)x^3 - x(x^2 - u)^2) \\ &= 2x - (2x + (x^5 - ux^3) - x(x^4 + u^2 - 2x^2u)) \\ u' &= 2x - (2x + (x^5 - ux^3) - (x^5 + xu^2 - 2x^3u)) \\ &= 2x - 2x - (x^5 - ux^3) + (x^5 + xu^2 - 2x^3u) \\ &= -x^5 + ux^3 + x^5 + xu^2 - 2x^3u \\ &= -ux^3 + xu^2 \end{aligned}$$

This is of the form $u' = P(x) + Q(x)u + R(x)u^2$ and since $P(x) = 0$ then it is Bernoulli differential equation. (when $P(x) \neq 0$ and $R(x) \neq 0$ it is Riccati). To solve Bernoulli we always start by dividing by u^2

$$\frac{u'}{u^2} = -\frac{1}{u}x^3 + x$$

Then we let $\zeta = -\frac{1}{u}$, hence $\zeta' = \frac{u'}{u^2}$, therefore the above becomes

$$\begin{aligned} \zeta' &= x^3\zeta + x \\ \zeta' - x^3\zeta &= x \end{aligned}$$

Integrating factor is $e^{-\int x^3 dx} = e^{-\frac{x^4}{4}}$, hence

$$d\left(e^{-\frac{x^4}{4}} \zeta\right) = xe^{-\frac{x^4}{4}}$$

Integrating both sides gives

$$e^{-\frac{x^4}{4}} \zeta = \int xe^{-\frac{x^4}{4}} dx + C$$

$\int xe^{-\frac{x^4}{4}} dx = \frac{\sqrt{\pi}}{2} \operatorname{erf}\left(\frac{x^2}{2}\right)$, hence from above

$$\begin{aligned} e^{-\frac{x^4}{4}} \zeta &= \frac{\sqrt{\pi}}{2} \operatorname{erf}\left(\frac{x^2}{2}\right) + C \\ \zeta &= e^{\frac{x^4}{4}} \left(\frac{\sqrt{\pi}}{2} \operatorname{erf}\left(\frac{x^2}{2}\right) + C \right) \end{aligned}$$

Since $\zeta = -\frac{1}{u}$ then

$$u = -e^{-\frac{x^4}{4}} \left(\frac{\sqrt{\pi}}{2} \operatorname{erf}\left(\frac{x^2}{2}\right) + C \right)^{-1}$$

And since $y = x^2 - u$ then

$$\begin{aligned} y &= x^2 + e^{-\frac{x^4}{4}} \left(\frac{\sqrt{\pi}}{2} \operatorname{erf}\left(\frac{x^2}{2}\right) + C \right)^{-1} \\ &= x^2 + \frac{e^{-\frac{x^4}{4}}}{\frac{\sqrt{\pi}}{2} \operatorname{erf}\left(\frac{x^2}{2}\right) + C} \end{aligned}$$

Verification

```
eq:=diff(y(x),x)+x*y(x)^2-x^3*y(x)-2*x = 0;
sol:=x^2+ exp(-x^4/4)/(_C1+ sqrt(Pi)/2*erf(x^2/2));
odetest(y(x)=sol,eq);
0
```

2.29 ODE No. 29

$$y'(x) - xy(x)^2 - 3xy(x) = 0$$

✓ **Mathematica** : cpu = 0.115562 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow -\frac{3e^{\frac{3x^2}{2}+3c_1}}{-1+e^{\frac{3x^2}{2}+3c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 19

$$\left\{ y(x) = \frac{3}{3c_1 e^{-\frac{3x^2}{2}} - 1} \right\}$$

HAND SOLUTION

$$\begin{aligned} y' - xy^2 - 3xy &= 0 \\ y' &= 3xy + xy^2 \\ &= P(x) + Q(x)y + R(x)y^2 \end{aligned} \tag{1}$$

This is Bernoulli first order non-linear ODE since $P(x) = 0$. To solve Bernoulli we always start by dividing by y^2

$$\frac{y'}{y^2} = \frac{3x}{y} + x$$

Then we let $u = \frac{1}{y}$, hence $u' = \frac{-y'}{y^2}$, therefore the above becomes

$$\begin{aligned} -u' &= 3xu + x \\ u' + 3ux &= -x \end{aligned}$$

Integrating factor is $e^{\int 3x dx} = e^{\frac{3x^2}{2}}$, hence

$$d\left(e^{\frac{3x^2}{2}} u\right) = -xe^{\frac{3x^2}{2}}$$

Integrating both sides gives

$$\begin{aligned} e^{\frac{3x^2}{2}} u &= \int -xe^{\frac{3x^2}{2}} dx + C \\ &= -\frac{1}{3}e^{\frac{3x^2}{2}} + C \end{aligned}$$

Hence from above

$$u = e^{\frac{-3x^2}{2}} \left(-\frac{1}{3} e^{\frac{3x^2}{2}} + C \right)$$

And since $y = \frac{1}{u}$ then

$$y = \frac{e^{\frac{3x^2}{2}}}{C - \frac{1}{3} e^{\frac{3x^2}{2}}}$$

Verification

```
eq:=diff(y(x),x)-x*y(x)^2-3*x*y(x) = 0;
sol:=exp(3*x^2/2)/(_C1- 1/3*exp(3*x^2/2));
odetest(y(x)=sol,eq);
0
```

2.30 ODE No. 30

$$x^{-a-1}y(x)^2 - x^a + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.200589 (sec), leaf count = 230

$$\left\{ \left\{ y(x) \rightarrow \frac{x^{a+1} \left(-\frac{1}{2} (-1)^{-a} a x^{-\frac{a}{2}-1} \Gamma(1-a) L_{-a}(2\sqrt{x}) + \frac{1}{2} (-1)^{-a} x^{-\frac{a}{2}-\frac{1}{2}} \Gamma(1-a) (L_{-a-1}(2\sqrt{x}) + I_{1-a}(2\sqrt{x})) \right) + c_1 \left(\frac{1}{2} x^{-\frac{a}{2}-\frac{1}{2}} \Gamma(a+1) L_a(2\sqrt{x}) - \frac{1}{2} (-1)^{-a} x^{-a/2} \Gamma(1-a) L_{-a}(2\sqrt{x}) + c_1 x^{-a/2} \Gamma(a+1) I_a(2\sqrt{x}) \right)}{(-1)^{-a} x^{-a/2} \Gamma(1-a) L_{-a}(2\sqrt{x}) + c_1 x^{-a/2} \Gamma(a+1) I_a(2\sqrt{x})} \right\} \right\}$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 54

$$\left\{ y(x) = \frac{\left(-c_1 \operatorname{BesselK}(a+1, 2\sqrt{x}) + \operatorname{BesselI}(a+1, 2\sqrt{x}) \right) x^{a+1}}{\left(c_1 \operatorname{BesselK}(a, 2\sqrt{x}) + \operatorname{BesselI}(a, 2\sqrt{x}) \right) \sqrt{x}} \right\}$$

HAND SOLUTION

$$\begin{aligned} y' + x^{-a-1}y^2 - x^a &= 0 \\ y' &= x^a - x^{-a-1}y^2 \\ &= P(x) + Q(x)y + R(x)y^2 \end{aligned} \tag{1}$$

This is Ricatti first order non-linear ODE. Using standard transformation

$$y = -\frac{u'}{uR(x)} = x^{a+1} \frac{u'}{u}$$

Hence

$$y' = (a+1)x^a \frac{u'}{u} + x^{a+1} \frac{u''}{u} - x^{a+1} \frac{(u')^2}{u^2}$$

Comparing to (1) gives

$$\begin{aligned} x^a - x^{-a-1}y^2 &= (a+1)x^a \frac{u'}{u} + x^{a+1} \frac{u''}{u} - x^{a+1} \frac{(u')^2}{u^2} \\ x^a - x^{-a-1} \left(x^{a+1} \frac{u'}{u} \right)^2 &= (a+1)x^a \frac{u'}{u} + x^{a+1} \frac{u''}{u} - x^{a+1} \frac{(u')^2}{u^2} \\ 1 - \frac{x^{-a-1}}{x^a} x^{2a+2} \frac{(u')^2}{u^2} &= (a+1) \frac{u'}{u} + x \frac{u''}{u} - x \frac{(u')^2}{u^2} \\ 1 - x \frac{(u')^2}{u^2} &= (a+1) \frac{u'}{u} + x \frac{u''}{u} - x \frac{(u')^2}{u^2} \\ 1 &= (a+1) \frac{u'}{u} + x \frac{u''}{u} \\ xu'' + (1+a)u' - u &= 0 \end{aligned} \tag{2}$$

In standard form $u'' + \frac{1}{x}(1+a)u' - \frac{1}{x}u = 0$ or $u'' + p(x)(1+a)u' + q(x)u = 0$. We see that $p(x)$ is not analytic at $x = 0$ (the expansion point). So we can't use power series solution, and will use Forbenius series. Power series, which is $u = \sum_{n=0}^{\infty} c_n x^n$ is used when the expansion point is not singular point. (i.e. $p(x)$ and $q(x)$ are analytic there). Forbenius series $u = x^r \sum_{n=0}^{\infty} c_n x^n$ is used when there is a removable singular point (called also regular singular point), as in this case. Starting with

$$u = x^r \sum_{n=0}^{\infty} c_n x^n = \sum_{n=0}^{\infty} c_n x^{n+r}$$

Hence

$$\begin{aligned} u' &= \sum_{n=0}^{\infty} (n+r) c_n x^{n+r-1} \\ u'' &= \sum_{n=0}^{\infty} (n+r)(n+r-1) c_n x^{n+r-2} \end{aligned}$$

Substituting in (2) gives

$$\begin{aligned} x \sum_{n=0}^{\infty} (n+r)(n+r-1) c_n x^{n+r-2} + (1+a) \sum_{n=0}^{\infty} (n+r) c_n x^{n+r-1} - \sum_{n=0}^{\infty} c_n x^{n+r} &= 0 \\ \sum_{n=0}^{\infty} (n+r)(n+r-1) c_n x^{n+r-1} + (1+a) \sum_{n=0}^{\infty} (n+r) c_n x^{n+r-1} - \sum_{n=0}^{\infty} c_n x^{n+r} &= 0 \end{aligned}$$

Dividing out x^r

$$\sum_{n=0}^{\infty} (n+r)(n+r-1) c_n x^{n-1} + (1+a) \sum_{n=0}^{\infty} (n+r) c_n x^{n-1} - \sum_{n=0}^{\infty} c_n x^n = 0$$

Each term should have x^{n-1} in it. So we adjust the last term

$$\sum_{n=0}^{\infty} (n+r)(n+r-1) c_n x^{n-1} + (1+a) \sum_{n=0}^{\infty} (n+r) c_n x^{n-1} - \sum_{n=1}^{\infty} c_{n-1} x^{n-1} = 0$$

Expanding the second term

$$\sum_{n=0}^{\infty} (n+r)(n+r-1) c_n x^{n-1} + \sum_{n=0}^{\infty} (n+r) c_n x^{n-1} + \sum_{n=0}^{\infty} a(n+r) c_n x^{n-1} - \sum_{n=1}^{\infty} c_{n-1} x^{n-1} = 0$$

Hence for $n = 0$

$$\begin{aligned} (n+r)(n+r-1) c_n x^{n-1} + (n+r) c_n x^{n-1} + a(n+r) c_n x^{n-1} &= 0 \\ r(r-1) c_0 + r c_0 + a r c_0 &= 0 \end{aligned}$$

Since $c_0 \neq 0$ then

$$r(r-1) + r + ar = 0$$

Hence $r = -a$ or $r = 0$. Now for $n \geq 1$

$$\begin{aligned} (n+r)(n+r-1) c_n x^{n-1} + (n+r) c_n x^{n-1} + a(n+r) c_n x^{n-1} - c_{n-1} x^{n-1} &= 0 \\ (n+r)(n+r-1) c_n + (n+r) c_n + a(n+r) c_n - c_{n-1} &= 0 \\ ((n+r)(n+r-1) + (n+r) + a(n+r)) c_n &= c_{n-1} \\ c_n &= \frac{c_{n-1}}{(n+r)(n+r-1) + (n+r) + a(n+r)} \end{aligned}$$

For $r = 0$, we obtain

$$c_n = \frac{c_{n-1}}{n(n-1) + n + an} \quad (3)$$

For $r = -a$

$$c_n = \frac{c_{n-1}}{(n-a)(n-a-1) + (n-a) + a(n-a)} \quad (4)$$

There are two solutions. Looking at (3) for now, for $n = 1$

$$c_1 = \frac{c_0}{1+a}$$

For $n = 2$

$$c_2 = \frac{c_1}{4 + 2a} = \frac{c_0}{1 + a} \frac{1}{2(2 + a)}$$

For $n = 3$

$$c_3 = \frac{c_2}{3(2) + 3 + 3a} = \frac{c_2}{3(3 + a)} = \frac{c_0}{1 + a} \frac{1}{2(2 + a)} \frac{1}{3(3 + a)}$$

And so on. Since the solution is assumed to be $x^r \sum_{n=0}^{\infty} c_n x^n$ and we are looking at case $r = 0$ then

$$\begin{aligned} u_{r=0}(x) &= \sum_{n=1}^{\infty} c_n x^n \\ &= c_0 + c_1 x + c_2 x^2 + \dots \\ &= c_0 x^0 + \frac{c_0}{1 + a} x + \frac{c_0}{1 + a} \frac{1}{2(2 + a)} x^2 + \frac{c_0}{1 + a} \frac{1}{2(2 + a)} \frac{1}{3(3 + a)} x^3 + \dots \\ &= c_0 \left(x^0 + \frac{1}{1 + a} x + \frac{1}{(1 + a)} \frac{1}{2(2 + a)} x^2 + \frac{1}{(1 + a)} \frac{1}{2(2 + a)} \frac{1}{3(3 + a)} x^3 + \dots \right) \end{aligned} \quad (5)$$

Since

$$\Gamma(n) = (n - 1)!$$

and

$$a(1 + a)(2 + a) \dots (n + a) = \frac{\Gamma(a + n + 1)}{\Gamma(a)}$$

Then

$$(1 + a)(2 + a) \dots (n + a) = \frac{\Gamma(a + n + 1)}{a\Gamma(a)}$$

And (5) can now be written as

$$y_{r=0}(x) = c_0 \sum_{n=1}^{\infty} \frac{1}{n!} \frac{a\Gamma(a)}{\Gamma(a + n + 1)} x^n \quad (6)$$

But modified Bessel function of first kind is

$$\text{BesselI}(a, z) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{\Gamma(a + n + 1)} \left(\frac{z}{2}\right)^{2n+a}$$

So if we let $z = 2\sqrt{x}$ we obtain

$$\begin{aligned} \text{BesselI}(a, 2\sqrt{x}) &= \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{\Gamma(a + n + 1)} \left(\frac{2\sqrt{x}}{2}\right)^{2n+a} \\ &= \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{\Gamma(a + n + 1)} (\sqrt{x})^{2n} (\sqrt{x})^a \\ &= \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{\Gamma(a + n + 1)} x^n (\sqrt{x})^a \end{aligned}$$

Hence

$$\frac{1}{\sqrt{x}^a} \text{BesselI}(a, 2\sqrt{x}) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{\Gamma(a + n + 1)} x^n \quad (7)$$

If we now compare (6) and (7), we see that if we set c_0 , which is arbitrary, to be $c_0 = \frac{1}{a\Gamma(a)}$, then we obtain

$$\begin{aligned} u_{r=0}(x) &= \frac{1}{a\Gamma(a)} \sum_{n=0}^{\infty} \frac{1}{n!} \frac{a\Gamma(a)}{\Gamma(a + n + 1)} x^n \\ &= \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{\Gamma(a + n + 1)} x^n \end{aligned}$$

But this is (7). Hence we found the first solution, which is

$$u_{r=0}(x) = \frac{1}{\sqrt{x}^a} \text{BesselI}(a, 2\sqrt{x}) \quad (8)$$

The above was for $r = 0$. Now we find the second solution for $r = -a$. From (4)

$$c_n = \frac{c_{n-1}}{(n-a)(n-a-1) + (n-a) + a(n-a)}$$

For $n = 1$

$$c_1 = \frac{c_0}{-a(1-a) + (1-a) + a(1-a)} = \frac{c_0}{(1-a)}$$

For $n = 2$

$$c_2 = \frac{c_1}{(2-a)(1-a) + (2-a) + a(2-a)} = \frac{c_1}{4-2a} = \frac{c_0}{(1-a)} \frac{1}{2(2-a)}$$

For $n = 3$

$$c_3 = \frac{c_2}{(3-a)(2-a) + (3-a) + a(3-a)} = \frac{c_2}{3(3-a)} = \frac{c_0}{(1-a)} \frac{1}{2(2-a)} \frac{1}{3(3-a)}$$

And so on. Since the solution is assumed to be $x^r \sum_{n=0}^{\infty} c_n x^n$ then

$$\begin{aligned} u_{r=-a} &= x^{-a} \sum_{n=0}^{\infty} c_n x^n \\ &= \sum_{n=0}^{\infty} c_n x^{n-a} \\ &= c_0 x^{-a} \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{1}{(1-a)} \frac{1}{(2-a)} \frac{1}{(3-a)} \cdots \frac{1}{(n-a)} \right) x^{n-a} \end{aligned}$$

But as we found above, we obtain that $(1-a)(2-a) \cdots (n-a) = \frac{\Gamma(-a+n+1)}{-a\Gamma(-a)}$, therefore

$$u_{r=-a} = c_0 \sum_{n=0}^{\infty} \frac{1}{n!} \frac{-a\Gamma(-a)}{\Gamma(-a+n+1)} x^{n-a}$$

Modified Bessel function of second kind is $\text{BesselK}(a, z) = \frac{\pi}{2} \frac{1}{\sin(a\pi)} (\text{BesselI}(-a, z) - \text{BesselI}(a, z))$.

The above should result in $\frac{1}{\sqrt{x^a}} \text{BesselK}(a, 2\sqrt{x})$ for $z = 2\sqrt{x}$ by setting c_0 to appropriate arbitrary value. I need to work out this final manipulation later. Hence we find $u_{r=-a}(x) = \frac{1}{\sqrt{x^a}} \text{BesselK}(a, 2\sqrt{x})$. Therefore, the solution is

$$u = C_1 \frac{1}{\sqrt{x^a}} \text{BesselI}(a, 2\sqrt{x}) + C_2 \frac{1}{\sqrt{x^a}} \text{BesselK}(a, 2\sqrt{x})$$

But

$$\begin{aligned} \frac{d}{dx} \frac{1}{\sqrt{x^a}} \text{BesselI}(a, 2\sqrt{x}) &= \frac{1}{\sqrt{x^{1+a}}} \text{BesselI}(1+a, 2\sqrt{x}) \\ \frac{d}{dx} \frac{1}{\sqrt{x^a}} \text{BesselK}(a, 2\sqrt{x}) &= -\frac{1}{\sqrt{x^{1+a}}} \text{BesselK}(1+a, 2\sqrt{x}) \end{aligned}$$

Hence

$$u' = C_1 \frac{1}{\sqrt{x^{1+a}}} \text{BesselI}(1+a, 2\sqrt{x}) - C_2 \frac{1}{\sqrt{x^{1+a}}} \text{BesselK}(1+a, 2\sqrt{x})$$

And from $y = x^{a+1} \frac{u'}{u}$

$$y = x^{1+a} \frac{C_1 \frac{1}{\sqrt{x^{1+a}}} \text{BesselI}(1+a, 2\sqrt{x}) - C_2 \frac{1}{\sqrt{x^{1+a}}} \text{BesselK}(1+a, 2\sqrt{x})}{C_1 \frac{1}{\sqrt{x^a}} \text{BesselI}(a, 2\sqrt{x}) + C_2 \frac{1}{\sqrt{x^a}} \text{BesselK}(a, 2\sqrt{x})}$$

Let $C = \frac{C_2}{C_1}$ hence

$$y = x^{1+a} \frac{\frac{1}{\sqrt{x^{1+a}}} \text{BesselI}(1+a, 2\sqrt{x}) - C \frac{1}{\sqrt{x^{1+a}}} \text{BesselK}(1+a, 2\sqrt{x})}{\frac{1}{\sqrt{x^a}} \text{BesselI}(a, 2\sqrt{x}) + C \frac{1}{\sqrt{x^a}} \text{BesselK}(a, 2\sqrt{x})}$$

Or

$$y = x^{1+a} \frac{x^{-\frac{1}{2}} \text{BesselI}\left(1+a, 2\sqrt{x}\right) - Cx^{-\frac{1}{2}} \text{BesselK}\left(1+a, 2\sqrt{x}\right)}{\text{BesselI}\left(a, 2\sqrt{x}\right) + C \text{BesselK}\left(a, 2\sqrt{x}\right)}$$
$$= \frac{x^{\frac{1}{2}+a} \text{BesselI}\left(1+a, 2\sqrt{x}\right) - Cx^{\frac{1}{2}+a} \text{BesselK}\left(1+a, 2\sqrt{x}\right)}{\text{BesselI}\left(a, 2\sqrt{x}\right) + C \text{BesselK}\left(a, 2\sqrt{x}\right)}$$

Verification

```
eq:=diff(y(x),x)+x^(-a-1)*y(x)^2-x^a = 0;
num:=x^(1/2+a)*BesselI(1+a,2*sqrt(x))-_C1*x^(1/2+a)*BesselK(1+a,2*sqrt(x));
den:=BesselI(a,2*sqrt(x))+_C1*BesselK(a,2*sqrt(x));
my_sol:=num/den;
odetest(y(x)=my_sol,eq);
0
```

2.31 ODE No. 31

$$y'(x) - ax^n(y(x)^2 + 1) = 0$$

✓ **Mathematica** : cpu = 0.218208 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \tan\left(\frac{ax^{n+1}}{n+1} + c_1\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 23

$$\left\{ y(x) = \tan\left(\frac{\left(c_1(n+1) + x^{n+1}\right)a}{n+1}\right) \right\}$$

HAND SOLUTION

$$y' - ax^n(y^2 + 1) = 0$$
$$y' = ax^n + ax^ny^2$$
$$= P(x) + Q(x)y + R(x)y^2 \tag{1}$$

This is Ricatti first order non-linear ODE. $P(x) = ax^n, Q(x) = 0, R(x) = ax^n$. But this is separable also. Hence

$$\frac{y'}{(y^2 + 1)} = ax^n$$
$$\frac{dy}{(y^2 + 1)} = ax^ndx$$

Integrating

$$\arctan(y(x)) = a \frac{x^{n+1}}{n+1} + C$$

Or

$$y(x) = \tan\left(a \frac{x^{n+1}}{n+1} + C\right)$$

Verification

```
restart;
eq:=diff(y(x),x)-a*x^n*(y(x)^2+1) = 0;
sol:=tan(a*x^(n+1)/(n+1)+_C1);
odetest(y(x)=sol,eq);
0
```


2.32 ODE No. 32

$$y'(x) + y(x)^2 \sin(x) - 2 \tan(x) \sec(x) = 0$$

✓ **Mathematica** : cpu = 0.364816 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow \frac{\csc(x)(-2 \sin(x) \cos(x) + c_1 \tan(x) \sec(x))}{\cos^2(x) + c_1 \sec(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.262 (sec), leaf count = 28

$$\left\{ y(x) = \frac{-2c_1(\cos^3(x)) - 2}{(c_1(\cos^3(x)) - 2) \cos(x)} \right\}$$

HAND SOLUTION

$$\begin{aligned} y' + y^2 \sin(x) - 2 \frac{\sin x}{\cos^2 x} &= 0 \\ y' &= 2 \frac{\sin x}{\cos^2 x} - y^2 \sin(x) \\ &= P(x) + Q(x)y + R(x)y^2 \end{aligned} \quad (1)$$

This is Ricatti first order non-linear ODE. $P(x) = 2 \frac{\sin x}{\cos^2 x}$, $Q(x) = 0$, $R(x) = -\sin(x)$. A particular solution is $y_p = \frac{1}{\cos x}$, therefore the solution is

$$\begin{aligned} y &= y_p + \frac{1}{u} \\ y &= \frac{1}{\cos x} + \frac{1}{u} \end{aligned}$$

Hence

$$y' = \frac{\sin x}{\cos^2 x} - \frac{u'}{u^2}$$

Equating this to RHS of (1) gives

$$\begin{aligned} \frac{\sin x}{\cos^2 x} - \frac{u'}{u^2} &= 2 \frac{\sin x}{\cos^2 x} - y^2 \sin(x) \\ &= 2 \frac{\sin x}{\cos^2 x} - \left(\frac{1}{\cos x} + \frac{1}{u} \right)^2 \sin(x) \\ &= 2 \frac{\sin x}{\cos^2 x} - \left(\frac{1}{\cos^2 x} + \frac{1}{u^2} + \frac{2}{u \cos x} \right) \sin(x) \end{aligned}$$

Hence

$$\begin{aligned} -\frac{u'}{u^2} &= -\frac{\sin x}{\cos^2 x} + 2 \frac{\sin x}{\cos^2 x} - \frac{\sin(x)}{\cos^2 x} - \frac{\sin(x)}{u^2} - \frac{2 \sin(x)}{u \cos x} \\ &= -\frac{\sin(x)}{u^2} - \frac{2 \sin(x)}{u \cos x} \end{aligned}$$

Or

$$\begin{aligned} u' &= \sin(x) + \frac{2u \sin(x)}{\cos x} \\ u' - 2u \tan(x) &= \sin(x) \end{aligned}$$

Integrating factor is $e^{-2 \int \tan x dx} = e^{2 \ln(\cos x)} = \cos^2(x)$. Hence the above becomes

$$d(u \cos^2 x) = \cos^2(x) \sin(x)$$

Integrating both sides

$$\begin{aligned} u \cos^2 x &= \int \cos^2(x) \sin(x) dx + C \\ &= \frac{-1}{3} \cos^3(x) + C \end{aligned}$$

Hence

$$u = \frac{-1}{3} \cos(x) + \frac{C}{\cos^2 x}$$

Therefore

$$\begin{aligned} y &= y_p + \frac{1}{u} \\ &= \frac{1}{\cos x} + \frac{1}{\frac{-1}{3} \cos(x) + \frac{C}{\cos^2 x}} \\ &= \frac{1}{\cos x} + \frac{3 \cos^2 x}{3C - \cos^3(x)} \end{aligned}$$

Let $3C = C_1$

$$y = \frac{1}{\cos x} + \frac{3 \cos^2 x}{C_2 - \cos^3(x)}$$

Verification

```
restart;
ode:=diff(y(x),x)+y(x)^2*sin(x)-2*sin(x)/cos(x)^2 = 0;
my_sol:=1/cos(x)+ 3*cos(x)^2/(_C1-cos(x)^3);
odetest(y(x)=my_sol,ode);
0
```

2.33 ODE No. 33

$$-\frac{y(x)^2 f'(x)}{g(x)} + \frac{g'(x)}{f(x)} + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.48339 (sec), leaf count = 160

Solve $\left[\int_1^{y(x)}\left(\frac{1}{(g(x)+f(x)K[2])^2}-\int_1^x\left(\frac{2\left(f(K[1])K[2]^2f'(K[1])-g(K[1])g'(K[1])\right)}{g(K[1])(g(K[1])+f(K[1])K[2])^3}-\frac{2K[2]f'(K[1])}{g(K[1])(g(K[1])+f(K[1])K[2])^2}\right)dx\right)dy\right]$

✓ **Maple** : cpu = 0.457 (sec), leaf count = 58

$$\left\{y(x)=\frac{-c_1 f(x) g(x)-f(x) g(x)\left(\int \frac{\frac{d}{d x} f(x)}{f(x)^2 g(x)} d x\right)-1}{\left(c_1+\int \frac{\frac{d}{d x} f(x)}{f(x)^2 g(x)} d x\right) f(x)^2}\right\}$$

HAND SOLUTION

$$\begin{aligned} -\frac{f'}{g}y^2 + \frac{g'}{f} + y' &= 0 \\ y' &= -\frac{g'}{f} + \frac{f'}{g}y^2 \\ &= P(x) + Q(x)y + R(x)y^2 \end{aligned} \tag{1}$$

This is Ricatti first order non-linear ODE. $P(x) = -\frac{g'}{f}, Q(x) = 0, R(x) = \frac{f'}{g}$.

To do.

2.34 ODE No. 34

$$f(x)y(x)^2 + g(x)y(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.116418 (sec), leaf count = 54

$$\left\{\left\{y(x) \rightarrow \frac{\exp \left(\int_1^x-g(K[1]) d K[1]\right)}{-\int_1^x-\exp \left(\int_1^{K[2]}-g(K[1]) d K[1]\right) f(K[2]) d K[2]+c_1}\right\}\right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 28

$$\left\{ y(x) = \frac{e^{\int -g(x)dx}}{c_1 + \int e^{\int -g(x)dx} f(x)dx} \right\}$$

HAND SOLUTION

$$\begin{aligned} y^2 f + gy + y' &= 0 \\ y' &= -gy - y^2 f \\ &= P(x) + Q(x)y + R(x)y^2 \end{aligned} \tag{1}$$

This is Bernoulli first order non-linear ODE. $P(x) = 0, Q(x) = -g, R(x) = f$. First step is to divide by y^2

$$\frac{y'}{y^2} = -g\frac{1}{y} - f \tag{2}$$

Let $u = \frac{1}{y}$, then $u' = \frac{-y'}{y^2}$ and (2) becomes

$$\begin{aligned} -u' &= -gu - f \\ u' - gu &= f \end{aligned}$$

Integrating factor is $e^{-\int gdx}$ hence

$$\begin{aligned} d\left(e^{-\int gdx}u\right) &= fe^{-\int gdx} \\ e^{-\int gdx}u &= \int fe^{-\int gdx}dx + C \\ u &= e^{\int gdx}\left(\int fe^{-\int gdx}dx + C\right) \end{aligned}$$

Hence

$$\begin{aligned} y &= \frac{1}{e^{\int gdx}\left(\int fe^{-\int gdx}dx + C\right)} \\ &= \frac{e^{-\int gdx}}{\int fe^{-\int gdx}dx + C} \end{aligned}$$

Let $\beta = e^{-\int gdx}$ then

$$y = \frac{\beta}{\int f\beta dx + C}$$

Verification

```
restart;
eq:=diff(y(x),x)+f(x)*y(x)^2+g(x)*y(x) = 0;
beta:=exp(-Int(g(x),x)):
my_sol:=beta/(Int(f(x)*beta,x)+_C1);
odetest(y(x)=my_sol,eq);
0
```

2.35 ODE No. 35

$$f(x)(2ay(x) + b + y(x)^2) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.137875 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow -a + \sqrt{b - a^2} \tan\left(\sqrt{b - a^2} \int_1^x -f(K[1])dK[1] + c_1\sqrt{b - a^2}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 35

$$\left\{ y(x) = -a + \sqrt{a^2 - b} \tanh\left(\sqrt{a^2 - b} \left(c_1 + \int f(x)dx\right)\right) \right\}$$

HAND SOLUTION

$$\begin{aligned}
y'(x) + f(x)(2ay(x) + b + y^2(x)) &= 0 \\
y'(x) &= -2af(x)y(x) - bf(x) - f(x)y^2(x) \\
&= P(x) + Q(x)y + R(x)y^2
\end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE. $P(x) = -bf(x)$, $Q(x) = -2af(x)$, $R(x) = -f(x)$.

Let

$$y(x) = -\frac{u'(x)}{u(x)R(x)} = \frac{u'(x)}{u(x)f(x)}$$

Hence

$$y'(x) = \frac{u''(x)}{u(x)f(x)} - \frac{(u'(x))^2}{u^2(x)f(x)} - \frac{u'(x)f'(x)}{u(x)f^2(x)}$$

Equating this to RHS of (1) gives

$$\begin{aligned}
\frac{u''(x)}{u(x)f(x)} - \frac{(u'(x))^2}{u^2(x)f(x)} - \frac{u'(x)f'(x)}{u(x)f^2(x)} &= -2af(x)y(x) - bf(x) - f(x)y^2(x) \\
&= -2af(x)\left[\frac{u'(x)}{u(x)f(x)}\right] - bf(x) - f(x)\left[\frac{u'(x)}{u(x)f(x)}\right]^2 \\
&= -2a\frac{u'(x)}{u(x)} - bf(x) - \frac{u'(x)^2}{u^2(x)f(x)}
\end{aligned}$$

Simplifying

$$\begin{aligned}
u''(x) - \frac{(u'(x))^2}{u(x)} - \frac{u'(x)f'(x)}{f(x)} &= -2au'(x)f(x) - u(x)bf^2(x) - \frac{u'(x)^2}{u(x)} \\
u''(x) - \frac{u'(x)f'(x)}{f(x)} &= -2au'(x)f(x) - u(x)bf^2(x) \\
u''(x) + u'(x)\left(-\frac{f'(x)}{f(x)} + 2af(x)\right) + u(x)bf^2(x) &= 0
\end{aligned}$$

Second order ODE with variable coefficients. Since coefficients are variables and not constants, a power series method is the standard way to continue. When I tried solving this now pretending the coefficients are constants in time, using the standard auxiliary equation method, the solution did verify OK. I need to look more into this. For now, this is solved using standard method for solving second order ODE with constant coefficients.

$$u(x) = C_1 \exp\left(\frac{\int f(x)\sqrt{-b} dx \left(\sqrt{\frac{b-a^2}{b}} b + a\sqrt{-b}\right)}{b}\right) + C_2 \exp\left(\frac{\int f(x)\sqrt{-b} dx \left(-\sqrt{\frac{b-a^2}{b}} b + a\sqrt{-b}\right)}{b}\right)$$

Hence

$$\begin{aligned}
u'(x) &= \frac{C_1 f(x)\sqrt{-b}}{b} \left(\sqrt{\frac{-a^2+b}{b}} b + \sqrt{-b} a\right) e^{\frac{\int f(x)\sqrt{-b} dx}{b} \left(\sqrt{\frac{-a^2+b}{b}} b + \sqrt{-b} a\right)} \\
&\quad + \frac{C_2 f(x)\sqrt{-b}}{b} \left(-\sqrt{\frac{-a^2+b}{b}} b + \sqrt{-b} a\right) e^{\frac{\int f(x)\sqrt{-b} dx}{b} \left(-\sqrt{\frac{-a^2+b}{b}} b + \sqrt{-b} a\right)}
\end{aligned}$$

Therefore

$$y = \frac{u'(x)}{u(x)f(x)}$$
$$= \frac{\frac{C_1 f(x)\sqrt{-b}}{b} \left(\sqrt{\frac{-a^2+b}{b}} b + \sqrt{-b} a \right) e^{\frac{\int f(x)\sqrt{-b} \, dx}{b} \left(\sqrt{\frac{-a^2+b}{b}} b + \sqrt{-b} a \right)} + \frac{C_2 f(x)\sqrt{-b}}{b} \left(-\sqrt{\frac{-a^2+b}{b}} b + \sqrt{-b} a \right) e^{\frac{\int f(x)\sqrt{-b} \, dx}{b} \left(-\sqrt{\frac{-a^2+b}{b}} b + \sqrt{-b} a \right)}}{f(x) \left[C_1 \exp \left(\frac{\int f(x)\sqrt{-b} \, dx \left(\sqrt{\frac{b-a^2}{b}} b + a\sqrt{-b} \right)}{b} \right) + C_2 \exp \left(\frac{\int f(x)\sqrt{-b} \, dx \left(-\sqrt{\frac{b-a^2}{b}} b + a\sqrt{-b} \right)}{b} \right) \right]}$$

Verification

```
restart;
book:=diff(y(x),x)+f(x)*(2*a*y(x)+b+y(x)^2)=0;
eqU:=diff(u(x),x$2)+diff(u(x),x)*(- diff(f(x),x)/f(x)+2*a*f(x))+u(x)*f(x)^2*b
=0;
solU:=dsolve(eqU,u(x));
my_sol:=diff(rhs(solU),x)/(rhs(solU)*f(x));
odetest(y(x)=my_sol,book);
0
```

2.36

ODE No. 36

$$axy(x)^2 + y'(x) + y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.405404 (sec), leaf count = 195

$$\text{Solve} \left[\frac{\text{Ai}' \left(\frac{\sqrt[3]{-\frac{1}{2}} \sqrt[3]{a}}{y(x)} - \frac{1}{2} \sqrt[3]{-\frac{1}{2}} a^{4/3} x^2 \right) - \left(-\frac{1}{2} \right)^{2/3} a^{2/3} x \text{Ai} \left(\frac{\sqrt[3]{-\frac{1}{2}} \sqrt[3]{a}}{y(x)} - \frac{1}{2} \sqrt[3]{-\frac{1}{2}} a^{4/3} x^2 \right)}{\text{Bi}' \left(\frac{\sqrt[3]{-\frac{1}{2}} \sqrt[3]{a}}{y(x)} - \frac{1}{2} \sqrt[3]{-\frac{1}{2}} a^{4/3} x^2 \right) - \left(-\frac{1}{2} \right)^{2/3} a^{2/3} x \text{Bi} \left(\frac{\sqrt[3]{-\frac{1}{2}} \sqrt[3]{a}}{y(x)} - \frac{1}{2} \sqrt[3]{-\frac{1}{2}} a^{4/3} x^2 \right)} + c_1 = 0, y(x) \right]$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 62

$$\left\{ y(x) = \frac{2a}{a^2 x^2 + 2(-2a^2)^{\frac{1}{3}} \text{RootOf} \left(c_1 (-2a^2)^{\frac{1}{3}} x \text{AiryBi}(\_Z) + 2c_1 \text{AiryBi}(1, \_Z) + (-2a^2)^{\frac{1}{3}} x \text{AiryAi}(\_Z) + 2 \text{AiryAi}(1, \_Z) \right)} \right.$$

HAND SOLUTION

$$y'(x) = -axy^2 - y^3$$

(1)

This is Abel first order non-linear. The general form is of Abel first kind is

$$y'(x) = f_0(x) + f_1(x)y(x) + f_2(x)y^2(x) + f_3(x)y^3(x)$$

In this case, $f_0(x) = 0, f_1(x) = 0, f_2(x) = -ax, f_3(x) = -1$. Note $\left(\frac{f_3}{f_2}\right)' = \left(\frac{1}{ax}\right)' = -\frac{1}{a}$. While Abel second kind has the form

$$(y + g(x))y'(x) = f_0(x) + f_1(x)y(x) + f_2(x)y^2(x)$$

For $g(x) \neq 0$.

Looking at (1) again, using the transformation suggested in Kamke $u = \frac{1}{y} - \frac{1}{2}ax^2$ or $y = \frac{1}{u + \frac{1}{2}ax^2}$ Then

$$y' = \frac{-u' - ax}{\left(u + \frac{1}{2}ax^2\right)^2}$$

Equating the above to the RHS of (1) gives

$$\begin{aligned}\frac{-u' - ax}{\left(u + \frac{1}{2}ax^2\right)^2} &= -ax \left(\frac{1}{u + \frac{1}{2}ax^2}\right)^2 - \left(\frac{1}{u + \frac{1}{2}ax^2}\right)^3 \\ -u' - ax &= -ax - \frac{1}{u + \frac{1}{2}ax^2} \\ \frac{du}{dx} &= \frac{1}{u + \frac{1}{2}ax^2}\end{aligned}$$

Writing as

$$\frac{dx}{du} = u + \frac{1}{2}ax^2 \quad (2)$$

This can now be viewed as reverse Riccati in x . Using the standard transformation

$$x = -\frac{z'}{z\left(\frac{1}{2}a\right)} = -\frac{2z'}{az} \quad (3)$$

Hence

$$\frac{dx}{du} = -\frac{2}{a} \left(\frac{z''}{z} - \frac{(z')^2}{z^2} \right)$$

Equating this to RHS of (2) gives a second order Airy ODE where the dependent variable is z and the independent variable is u

$$\begin{aligned}-\frac{2}{a} \left(\frac{z''}{z} - \frac{(z')^2}{z^2} \right) &= u + \frac{1}{2}a \left(-\frac{2z'}{az} \right)^2 \\ -\frac{2}{a} \frac{z''}{z} + \frac{2}{a} \frac{(z')^2}{z^2} &= u + \frac{1}{2}a \frac{4(z')^2}{a^2 z^2} \\ -\frac{2}{a} \frac{z''}{z} + \frac{2}{a} \frac{(z')^2}{z^2} &= u + \frac{2}{a} \frac{(z')^2}{z^2} \\ -\frac{2}{a} \frac{z''}{z} &= u \\ \frac{2}{a} \frac{z''}{z} &= -u \\ z''(u) + \frac{a}{2}uz(u) &= 0\end{aligned}$$

This is Airy ODE whose solution is found using power series method. The solution is

$$z(u) = C_1 \text{AiryAI}\left(-\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u\right) + C_2 \text{AiryBI}\left(-\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u\right) \quad (4)$$

We now go back to (3) and find x

$$x = -\frac{2z'}{az}$$

Since

$$\begin{aligned}\frac{d}{du} \text{AiryAI}\left(-\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u\right) &= -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}} \text{AiryAI}\left(1, -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u\right) \\ \frac{d}{du} \text{AiryBI}\left(-\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u\right) &= -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}} \text{AiryBI}\left(1, -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u\right)\end{aligned}$$

Then

$$x = -\frac{2}{a} \frac{-C_1 \frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}} \text{AiryAI}\left(1, -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u\right) - C_2 \frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}} \text{AiryBI}\left(1, -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u\right)}{C_1 \text{AiryAI}\left(-\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u\right) + C_2 \text{AiryBI}\left(-\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u\right)}$$

Therefore $\frac{dx}{du}$ is now found from above. Once we find $\frac{dx}{du}$ then $\frac{du}{dx}$ is also found. Using $\frac{du}{dx} = \frac{1}{u + \frac{1}{2}ax^2}$ now $u(x)$ is found. Once $u(x)$ is found then $y(x)$ is found from the original transformation $y = \frac{1}{u + \frac{1}{2}ax^2}$. This is all now just algebra.

2.37 ODE No. 37

$$-ae^x y(x)^2 + y'(x) - y(x)^3 = 0$$

✓ **Mathematica** : cpu = 1.05781 (sec), leaf count = 78

$$\text{Solve} \left[-iae^x = \frac{2e^{\frac{1}{2} \left(-iae^x - \frac{i}{y(x)} \right)^2}}{\sqrt{2\pi} \operatorname{erfi} \left(\frac{-iae^x - \frac{i}{y(x)}}{\sqrt{2}} \right)} + 2c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 50

$$\left\{ c_1 + \frac{e^{-x} e^{-\frac{\left(ae^x + \frac{1}{y(x)} \right)^2}{2}}}{a} + \frac{\sqrt{2} \sqrt{\pi} \operatorname{erf} \left(\frac{\left(ae^x + \frac{1}{y(x)} \right) \sqrt{2}}{2} \right)}{2} = 0 \right\}$$

2.38 ODE No. 38

$$-ay(x)^3 - \frac{b}{x^{3/2}} + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.203189 (sec), leaf count = 99

$$\text{Solve} \left[-2\text{RootSum} \left[-2\#1^3 + \#1 \sqrt[3]{-\frac{1}{ab^2}} - 2\&, \frac{\log \left(y(x) \sqrt[3]{\frac{ax^{3/2}}{b}} - \#1 \right)}{\sqrt[3]{-\frac{1}{ab^2}} - 6\#1^2} \& \right] = \frac{ax \log(x)}{\left(\frac{ax^{3/2}}{b} \right)^{2/3}} + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 34

$$\left\{ y(x) = \frac{\operatorname{RootOf} \left(c_1 + 2 \left(\int^{-Z} \frac{1}{2a\_a^3 + \_a + 2b} d\_a \right) - \ln(x) \right)}{\sqrt{x}} \right\}$$

HAND SOLUTION

$$y'(x) = ay^3 + bx^{-\frac{3}{2}} \quad (1)$$

This can be transformed to Abel first order non-linear ode as follows. Let $y(x) = x^{-\frac{1}{2}} \eta(\xi)$ where $\xi = \ln x$ hence

$$\begin{aligned} \frac{dy}{dx} &= -\frac{1}{2} x^{-\frac{3}{2}} \eta(\xi) + x^{-\frac{1}{2}} \frac{d\eta}{d\xi} \frac{d\xi}{dx} \\ &= -\frac{1}{2} x^{-\frac{3}{2}} \eta(\xi) + x^{-\frac{1}{2}} \frac{d\eta}{d\xi} \frac{1}{x} \\ &= -\frac{1}{2} x^{-\frac{3}{2}} \eta(\xi) + x^{-\frac{3}{2}} \frac{d\eta}{d\xi} \end{aligned}$$

Substituting in (1) gives

$$\begin{aligned}
-\frac{1}{2}x^{-\frac{3}{2}}\eta(\xi) + x^{-\frac{3}{2}}\frac{d\eta}{d\xi} &= a\left(x^{-\frac{1}{2}}\eta(\xi)\right)^3 + bx^{-\frac{3}{2}} \\
-\frac{1}{2}x^{-\frac{3}{2}}\eta(\xi) + x^{-\frac{3}{2}}\frac{d\eta}{d\xi} &= ax^{-\frac{3}{2}}\eta^3(\xi) + bx^{-\frac{3}{2}} \\
-\frac{1}{2}\eta + \eta' &= a\eta^3 + b \\
\eta' &= b + \frac{1}{2}\eta + a\eta^3
\end{aligned}$$

This is Abel first kind. In general form it is

$$\eta' = f_0 + f_1\eta + f_2\eta^2 + f_3\eta^3$$

Where in this case $f_0 = b, f_1 = \frac{1}{2}, f_2 = 0, f_3 = a$. Using Maple, the solution to the above is (I need to learn how to solve Able by hand more) is implicit, given as

$$\eta = \xi - \int^{\eta(\xi)} \frac{1}{b + \frac{1}{2}z + az^3} dz + C$$

Where C is constant of integration. Hence, since $y(x) = x^{-\frac{1}{2}}\eta(\xi)$, then $\eta(\xi) = \sqrt{x}y$ and the above becomes

$$\begin{aligned}
\sqrt{x}y &= \ln x - \int^{\sqrt{x}y} \frac{1}{b + \frac{1}{2}z + az^3} dz + C \\
y(x) &= \left(\ln x - \int^{\sqrt{x}y} \frac{1}{b + \frac{1}{2}z + az^3} dz + C \right) \frac{1}{\sqrt{x}}
\end{aligned}$$

Did not verify. Need to look more into this later.

2.39 ODE No. 39

$$-a_0 - a_1y(x) - a_2y(x)^2 - a_3y(x)^3 + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.142328 (sec), leaf count = 54

$$\text{Solve}\left[\text{RootSum}\left[\#1^3a_3 + \#1^2a_2 + \#1a_1 + a_0\&, \frac{\log(y(x) - \#1)}{3\#1^2a_3 + 2\#1a_2 + a_1}\&\right] = x + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 30

$$\left\{ c_1 + x - \left(\int^{y(x)} \frac{1}{\_a^3a_3 + \_a^2a_2 + \_aa_1 + a_0} d\_a \right) = 0 \right\}$$

2.40 ODE No. 40

$$3ay(x)^3 + 6axy(x)^2 + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.461629 (sec), leaf count = 185

$$\text{Solve}\left[\frac{\sqrt[3]{-3}\sqrt[3]{a}x\text{Ai}\left((-3)^{2/3}a^{2/3}x^2 - \frac{(-1)^{2/3}}{\sqrt[3]{3}\sqrt[3]{a}y(x)}\right) + \text{Ai}'\left((-3)^{2/3}a^{2/3}x^2 - \frac{(-1)^{2/3}}{\sqrt[3]{3}\sqrt[3]{a}y(x)}\right)}{\sqrt[3]{-3}\sqrt[3]{a}x\text{Bi}\left((-3)^{2/3}a^{2/3}x^2 - \frac{(-1)^{2/3}}{\sqrt[3]{3}\sqrt[3]{a}y(x)}\right) + \text{Bi}'\left((-3)^{2/3}a^{2/3}x^2 - \frac{(-1)^{2/3}}{\sqrt[3]{3}\sqrt[3]{a}y(x)}\right)} + c_1 = 0, y(x)\right]$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 48

$$\left\{ y(x) = \frac{1}{3ax^2 + (-3a)^{\frac{1}{3}}\text{RootOf}\left(c_1(-3a)^{\frac{1}{3}}x\text{AiryBi}(\_Z) + c_1\text{AiryBi}(1, \_Z) + (-3a)^{\frac{1}{3}}x\text{AiryAi}(\_Z) + \text{AiryAi}(1, \_Z)\right)} \right\}$$

2.41 ODE No. 41

$$axy(x)^3 + by(x)^2 + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.279243 (sec), leaf count = 103

Solve
$$\left[\frac{b^2 \left(\frac{2 \tan^{-1} \left(\frac{-2axy(x)-b}{b \sqrt{-\frac{4a}{b^2}-1}} \right)}{\sqrt{-\frac{4a}{b^2}-1}} - \log \left(\frac{a(-x)y(x)(-axy(x)-b)-a}{a^2x^2y(x)^2} \right) \right)}{2a} = -\frac{b^2 \log(x)}{a} + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.31 (sec), leaf count = 103

$$\left\{ y(x) = \frac{e^{\text{RootOf}\left(2c_1b^2+2\_Zb^2-b^2\ln\left((ae^{2-Z}+be^{-Z}-1)x^2\right)+8c_1a+8\_Za-4a\ln\left((ae^{2-Z}+be^{-Z}-1)x^2\right)+2\sqrt{b^2+4a}b\operatorname{arctanh}\left(\frac{2ae^{-Z}+b}{\sqrt{b^2+4a}}\right)\right)}}{x} \right\}$$

2.42 ODE No. 42

$$y'(x) - x(x + 2)y(x)^3 - (x + 3)y(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.21032 (sec), leaf count = 485

Solve
$$c_1 = -\frac{\frac{i\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}\left(\frac{\sinh\left(\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}\right)}{\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}}-\cosh\left(\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}\right)\right)}{\sqrt{-i\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}}}-\frac{i\sqrt{\frac{2}{\pi}}\left(\frac{x+1}{2}+\frac{1}{2}\right)\sinh\left(\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}\right)}{\sqrt{-i\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}}}}{\frac{i\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}\left(i\sinh\left(\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}\right)-\frac{i\cosh\left(\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}\right)}{\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}}\right)}{\sqrt{-i\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}}}-\frac{\sqrt{\frac{2}{\pi}}\left(\frac{x+1}{2}+\frac{1}{2}\right)\cosh\left(\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}\right)}{\sqrt{-i\sqrt{\frac{1}{2y(x)}+\frac{1}{4}(x+1)^2-\frac{1}{4}}}}}, y(x)$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 40

$$\left\{ c_1 + \operatorname{arctanh}\left(\frac{x\sqrt{y(x)}}{\sqrt{(x+2)xy(x)+2}}\right) + \frac{\sqrt{(x+2)xy(x)+2}}{2\sqrt{y(x)}} = 0 \right\}$$

2.43 ODE No. 43

$$y(x)^3(4a^2x + 3ax^2 + b) + y'(x) + 3xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 7.115 (sec), leaf count = 490

Solve
$$c_1 = -\frac{i\sqrt{-\frac{4a^3-3b}{4a^3}-\frac{3}{2a^2y(x)}+\frac{(-2a-3x)^2}{4a^2}}J_{\frac{1}{2}\sqrt{\frac{4a^3-3b}{a^3}}+1}\left(-i\sqrt{\frac{(-2a-3x)^2}{4a^2}-\frac{4a^3-3b}{4a^3}-\frac{3}{2a^2y(x)}}\right)+\left(\frac{1}{2}\sqrt{\frac{4a^3-3b}{a^3}}+\frac{-2a-3x}{2a}\right)J_{\frac{1}{2}\sqrt{\frac{4a^3-3b}{a^3}}+1}\left(i\sqrt{\frac{(-2a-3x)^2}{4a^2}-\frac{4a^3-3b}{4a^3}-\frac{3}{2a^2y(x)}}\right)}{i\sqrt{-\frac{4a^3-3b}{4a^3}-\frac{3}{2a^2y(x)}+\frac{(-2a-3x)^2}{4a^2}}Y_{\frac{1}{2}\sqrt{\frac{4a^3-3b}{a^3}}+1}\left(-i\sqrt{\frac{(-2a-3x)^2}{4a^2}-\frac{4a^3-3b}{4a^3}-\frac{3}{2a^2y(x)}}\right)+\left(\frac{1}{2}\sqrt{\frac{4a^3-3b}{a^3}}+\frac{-2a-3x}{2a}\right)Y_{\frac{1}{2}\sqrt{\frac{4a^3-3b}{a^3}}+1}\left(i\sqrt{\frac{(-2a-3x)^2}{4a^2}-\frac{4a^3-3b}{4a^3}-\frac{3}{2a^2y(x)}}\right)}$$

✓ **Maple** : cpu = 1.662 (sec), leaf count = 373

$$\left\{ \begin{array}{l} -\sqrt{3} \sqrt{\frac{4a^2xy(x)+3ax^2y(x)+by(x)-2a}{a^3y(x)}} a \operatorname{BesselK}\left(\frac{\sqrt{\frac{4a^3-3b}{a^3}}}{2} + 1, -\frac{\sqrt{3} \sqrt{\frac{4a^2xy(x)+3ax^2y(x)+by(x)-2a}{a^3y(x)}}}{2}\right) - \left(\sqrt{\frac{4a^3-3b}{a^3}} a - 2a - 3x\right) \operatorname{BesselI}\left(\frac{\sqrt{\frac{4a^3-3b}{a^3}}}{2} + 1, -\frac{\sqrt{3} \sqrt{\frac{4a^2xy(x)+3ax^2y(x)+by(x)-2a}{a^3y(x)}}}{2}\right) \\ \sqrt{3} \sqrt{\frac{4a^2xy(x)+3ax^2y(x)+by(x)-2a}{a^3y(x)}} a \operatorname{BesselI}\left(\frac{\sqrt{\frac{4a^3-3b}{a^3}}}{2} + 1, -\frac{\sqrt{3} \sqrt{\frac{4a^2xy(x)+3ax^2y(x)+by(x)-2a}{a^3y(x)}}}{2}\right) - \left(\sqrt{\frac{4a^3-3b}{a^3}} a - 2a - 3x\right) \operatorname{BesselK}\left(\frac{\sqrt{\frac{4a^3-3b}{a^3}}}{2} + 1, -\frac{\sqrt{3} \sqrt{\frac{4a^2xy(x)+3ax^2y(x)+by(x)-2a}{a^3y(x)}}}{2}\right) \end{array} \right\} c_1 +$$

2.44 ODE No. 44

$$2ax^3y(x)^3 + y'(x) + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.08882 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{2}}{\sqrt{-2ax^2 - a + 2c_1e^{2x^2}}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{2}}{\sqrt{-2ax^2 - a + 2c_1e^{2x^2}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.015 (sec), leaf count = 53

$$\left\{ y(x) = -\frac{2}{\sqrt{-4ax^2 + 4c_1e^{2x^2} - 2a}}, y(x) = \frac{2}{\sqrt{-4ax^2 + 4c_1e^{2x^2} - 2a}} \right\}$$

Hand solution

$$y' = -2xy - 2ax^3y^3 \tag{1}$$

This is of the form $y' = f_0 + f_1y + f_2y^2 + f_3y^3$ where $f_0 = 0, f_2 = 0$. Hence this is Bernoulli first order non-linear ODE. We start by diving by y^3

$$\frac{y'}{y^3} = -2x\frac{1}{y^2} - 2ax^3$$

Let $u = \frac{1}{y^2}$, hence $u' = -2\frac{y'}{y^3}$ and the above becomes

$$\begin{aligned} -\frac{1}{2}u' &= -2xu - 2ax^3 \\ u' - 4xu &= 4ax^3 \end{aligned}$$

Integrating factor is $e^{-4\int xdx} = e^{-2x^2}$ hence

$$\frac{d}{dx}\left(e^{-2x^2}u\right) = 4ax^3e^{-2x^2}$$

Integrating

$$\begin{aligned} e^{-2x^2}u &= 4a \int x^3e^{-2x^2}dx + C \\ &= 4a\left(\frac{-1}{8}(2x^2 + 1)e^{-2x^2}\right) + C \end{aligned}$$

Therefore

$$u = -\frac{1}{2}a(2x^2 + 1) + Ce^{2x^2}$$

Hence

$$y^2 = \frac{1}{u} = \frac{1}{-\frac{1}{2}a(2x^2 + 1) + Ce^{2x^2}}$$

Or

$$y = \pm \frac{\sqrt{2}}{\sqrt{-a(2x^2 + 1) + Ce^{2x^2}}}$$

Verification

```
ode:=2*a*x^3*y(x)^3+diff(y(x),x)+2*x*y(x)=0;
my_sol:=sqrt(2)/sqrt(-a*(2*x^2+1)+_C1*exp(2*x^2));
odetest(y(x)=my_sol,ode);
0
my_sol:=-sqrt(2)/sqrt(-a*(2*x^2+1)+_C1*exp(2*x^2));
odetest(y(x)=my_sol,ode);
0
```

2.45 ODE No. 45

$2y(x)^3(a^2x^3 - b^2x) + 3by(x)^2 + y'(x) = 0$

✓ **Mathematica** : cpu = 0.682554 (sec), leaf count = 133

$$\text{Solve}\left[c_1 = \sqrt[4]{\left(\frac{b}{ax} - \frac{1}{ax^2y(x)}\right)^2 - 1} \left(-\frac{\left(\frac{b}{ax} - \frac{1}{ax^2y(x)}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \left(\frac{b}{ax} - \frac{1}{ax^2y(x)}\right)^2\right)}{2\sqrt[4]{1 - \left(\frac{b}{ax} - \frac{1}{ax^2y(x)}\right)^2}} - \frac{ax}{b} \right), y(x) \right]$$

✓ **Maple** : cpu = 0.165 (sec), leaf count = 123

$$\left\{c_1 - \left(\int^{\frac{ax^2y(x)}{bxy(x)-1}} \frac{(-a^2-1)^{\frac{1}{4}}}{\sqrt{-a}} d_a\right) + \frac{\left(\left(\frac{ax}{b} + \frac{1}{\frac{b^2y(x)}{a} - \frac{b}{ax}}\right)^2 - 1\right)^{\frac{1}{4}}}{\left(\frac{b^2y(x)}{a} - \frac{b}{ax}\right)\sqrt{\frac{ax}{b} + \frac{1}{\frac{b^2y(x)}{a} - \frac{b}{ax}}}} = 0\right\}$$

2.46 ODE No. 46

$-x^{-a}y(x) + ax^{-a-1} - x^{-2a} - x^ay(x)^3 + y'(x) + 3y(x)^2 = 0$

✓ **Mathematica** : cpu = 0.313116 (sec), leaf count = 228

$$\left\{\left\{y(x) \rightarrow x^{-a} - \frac{e^{-\frac{2x^{1-a}}{1-a}}}{\sqrt{-\frac{2^{\frac{2(a+1)}{a-1}+1}x^{a+1}\left(\frac{x^{1-a}}{1-a}\right)^{\frac{a+1}{a-1}}\Gamma\left(\frac{a+1}{1-a}, -\frac{4x^{1-a}}{a-1}\right)} + c_1}}\right\}, \left\{y(x) \rightarrow x^{-a} + \frac{e^{-\frac{2x^{1-a}}{1-a}}}{\sqrt{-\frac{2^{\frac{2(a+1)}{a-1}+1}x^{a+1}\left(\frac{x^{1-a}}{1-a}\right)^{\frac{a+1}{a-1}}\Gamma\left(\frac{a+1}{1-a}, -\frac{4x^{1-a}}{a-1}\right)} + c_1}}\right\}\right\}$$

✓ **Maple** : cpu = 0.119 (sec), leaf count = 956

$$\left\{y(x) = x^{-a} - \frac{2^{\frac{2(a+1)}{a-1}+1}x^{a+1}\left(\frac{x^{1-a}}{1-a}\right)^{\frac{a+1}{a-1}}\Gamma\left(\frac{a+1}{1-a}, -\frac{4x^{1-a}}{a-1}\right)}{\sqrt{-\frac{(a-1)\left(-\frac{4a^2x^{-a+1}}{-a+1} + \frac{8ax^{-a+1}}{-a+1} + 2a - \frac{4x^{-a+1}}{-a+1} - 2\right)(-a+1)2^{\frac{2a}{-a+1}-3} + \frac{2}{-a+1} + \frac{2}{a-1}x - \frac{a^2}{-a+1} + a + \frac{1}{-a+1} - 1\left(\frac{1}{-a+1}\right)^{-\frac{a+1}{a-1}}\left(\frac{x^{-a+1}}{-a+1}\right)^{\frac{1}{a-1}}\text{WhittakerM}\left(-\frac{a}{a-1}, -\frac{1}{a-1}, \frac{x^{-a+1}}{-a+1}\right)}{(a+1)(a-3)}}}\right\}$$

2.47 ODE No. 47

$-a(x^n - x)y(x)^3 + y'(x) - y(x)^2 = 0$

✗ **Mathematica** : cpu = 29.1809 (sec), leaf count = 0 , could not solve

```
DSolve[-y[x]^2 - a*(-x + x^n)*y[x]^3 + Derivative[1][y][x] == 0, y[x], x]
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve(diff(y(x),x)-a*(x^n-x)*y(x)^3-y(x)^2 = 0,y(x))
```

2.48 ODE No. 48

$$-\left(y(x)^3(ax^n + bx)\right) - cy(x)^2 + y'(x) = 0$$

✗ **Mathematica** : cpu = 31.186 (sec), leaf count = 0 , could not solve

```
DSolve[-(c*y[x]^2) - (b*x + a*x^n)*y[x]^3 + Derivative[1][y][x] == 0, y[x], x]
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve(diff(y(x),x)-(a*x^n+b*x)*y(x)^3-c*y(x)^2 = 0,y(x))
```

2.49 ODE No. 49

$$ay(x)^3\phi'(x) + \frac{(2a + 1)y(x)\phi''(x)}{\phi'(x)} + 6a\phi(x)y(x)^2 + 2a + y'(x) + 2 = 0$$

✗ **Mathematica** : cpu = 26.192 (sec), leaf count = 0 , could not solve

```
DSolve[2 + 2*a + 6*a*phi[x]*y[x]^2 + a*y[x]^3*Derivative[1][phi][x] + Derivative[1][y][x] +
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve(diff(y(x),x)+a*diff(phi(x),x)*y(x)^3+6*a*phi(x)*y(x)^2+(2*a+1)*y(x)*diff(diff(phi(x),
```

2.50 ODE No. 50

$$-f_0(x) - f_1(x)y(x) - f_2(x)y(x)^2 - f_3(x)y(x)^3 + y'(x) = 0$$

✗ **Mathematica** : cpu = 68.3933 (sec), leaf count = 0 , could not solve

```
DSolve[-f0[x] - f1[x]*y[x] - f2[x]*y[x]^2 - f3[x]*y[x]^3 + Derivative[1][y][x] == 0, y[x], x]
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve(diff(y(x),x)-f3(x)*y(x)^3-f2(x)*y(x)^2-f1(x)*y(x)-f0(x) = 0,y(x))
```

2.51 ODE No. 51

$$-h(x)(y(x)-f(x))(y(x)-g(x))\left(y(x)-\frac{af(x)+bg(x)}{a+b}\right)-\frac{f'(x)(y(x)-g(x))}{f(x)-g(x)}-\frac{(y(x)-f(x))g'(x)}{g(x)-f(x)}+y'(x)=0$$

✓ **Mathematica** : cpu = 2.21423 (sec), leaf count = 355

Solve

$$\left[-\frac{1}{3}(a-b)^{2/3}(2a+b)^{2/3}(a+2b)^{2/3}\text{RootSum}\left[\#1^3(a-b)^{2/3}(2a+b)^{2/3}(a+2b)^{2/3}-3\#1a^2-3\#1ab-3\#1b^2+(a-b)^2\right],\right.$$

✓ **Maple** : cpu = 0.334 (sec), leaf count = 237

$$\left\{y(x)=\frac{2(a+2b)\left(a+\frac{b}{2}\right)(f(x)-g(x))(a-b)\text{RootOf}\left(c_1-27\left(\int^{-Z}\frac{(a^2+ab+b^2)^3}{(-_a^2+_aab-2_a^2b^2+3a^2+3ab+3b^2)(2_a^2-_aab-_a^2b^2-3a^2-3ab-3b^2)}\right)}{9a^3+18a^2b+9ab^2+b^3}\right.\right.$$

2.52 ODE No. 52

$$-ay(x)^n - bx^{\frac{n}{1-n}} + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.307046 (sec), leaf count = 117

Solve $\left[\int_1^{\left(\frac{ax^{-\frac{n}{1-n}}}{b}\right)^{\frac{1}{n}} y(x)} \frac{1}{K[1]^n - \left(\frac{(-1)^n b^{1-n} (n-1)^{-n}}{a}\right)^{\frac{1}{n}} K[1] + 1} dK[1] = \int_1^x b K[2]^{\frac{n}{1-n}} \left(\frac{a K[2]^{-\frac{n}{1-n}}}{b}\right)^{\frac{1}{n}} dK[2] + c_1, y(x) \right]$

✓ **Maple** : cpu = 0.418 (sec), leaf count = 61

$$\left\{ -c_1 - (n-1) \left(\int_{-b}^{y(x)} \frac{x^{\frac{n}{n-1}}}{(n-1)bx + ((n-1)ax - a^n + -a)x^{\frac{n}{n-1}}} d_a \right) + \ln(x) = 0 \right\}$$

2.53 ODE No. 53

$$f(x)^{1-n} g'(x) y(x)^n \left(-(ag(x) + b)^{-n} \right) - \frac{y(x) f'(x)}{f(x)} - f(x) g'(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.679161 (sec), leaf count = 96

Solve $\left[\int_1^{\left(f(x)^{-n} (b+ag(x))^{-n}\right)^{\frac{1}{n}} y(x)} \frac{1}{K[1]^n - (a^n)^{\frac{1}{n}} K[1] + 1} dK[1] = \frac{f(x)(ag(x) + b) \log(ag(x) + b) \left(f(x)^{-n} (ag(x) + b)^{-n}\right)^{\frac{1}{n}}}{a} + c_1, y(x) \right]$

✓ **Maple** : cpu = 0.181 (sec), leaf count = 281

$$\left\{ y(x) = \frac{(ag(x) + b) \text{RootOf}\left(c_1 - \left(\int^{-Z} \frac{n^{-n} \left(\left(\frac{d}{dx}g(x)\right)f(x)\right)^{-2n+1} \left((ag(x)+b)^{-n} f(x)^{-n+1} \left(\frac{d}{dx}g(x)\right)^n\right)}{-a n^{-n} \left(\left(\frac{d}{dx}g(x)\right)f(x)\right)^{-2n+1} \left((ag(x)+b)^{-n} f(x)^{-n+1} \left(\frac{d}{dx}g(x)\right)\right)^{-n-1} \left(an(ag(x)+b)^{-n-1} f(x)^{-n+2} \left(\frac{d}{dx}g(x)\right)^3\right)^n}\right)}{a} \right\}$$

2.54 ODE No. 54

$$-a^n f(x)^{1-n} g'(x) y(x)^n - \frac{y(x) f'(x)}{f(x)} - f(x) g'(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.476437 (sec), leaf count = 74

Solve $\left[y(x) \left(a^n f(x)^{-n}\right)^{\frac{1}{n}} {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\left(\left(a^n f(x)^{-n}\right)^{\frac{1}{n}} y(x)\right)^n\right) = f(x) g(x) \left(a^n f(x)^{-n}\right)^{\frac{1}{n}} + c_1, y(x) \right]$

✓ **Maple** : cpu = 0.204 (sec), leaf count = 38

$$\left\{ -ag(x) + c_1 + \frac{ay(x)\Phi\left(-\left(\frac{ay(x)}{f(x)}\right)^n, 1, \frac{1}{n}\right)}{nf(x)} = 0 \right\}$$

2.55 ODE No. 55

$$-f(x)y(x)^n - g(x)y(x) - h(x) + y'(x) = 0$$

✗ **Mathematica** : cpu = 3.80204 (sec), leaf count = 0 , could not solve

```
DSolve[-h[x] - g[x]*y[x] - f[x]*y[x]^n + Derivative[1][y][x] == 0, y[x], x]
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve(diff(y(x),x)-f(x)*y(x)^n-g(x)*y(x)-h(x) = 0,y(x))
```

2.56 ODE No. 56

$$-f(x)y(x)^a - g(x)y(x)^b + y'(x) = 0$$

✗ **Mathematica** : cpu = 1.54805 (sec), leaf count = 0 , could not solve

```
DSolve[-(f[x]*y[x]^a) - g[x]*y[x]^b + Derivative[1][y][x] == 0, y[x], x]
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve(diff(y(x),x)-f(x)*y(x)^a-g(x)*y(x)^b = 0,y(x))
```

2.57 ODE No. 57

$$y'(x) - \sqrt{|y(x)|} = 0$$

✓ **Mathematica** : cpu = 0.074623 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[\int_1^{\#1} \frac{1}{\sqrt{|K[1]|}} dK[1] \& \right] [x + c_1] \right\} \right\}$$

✓ **Maple** : cpu = 0.12 (sec), leaf count = 31

$$\left\{ c_1 + x - \left(\begin{array}{ll} -2\sqrt{-y(x)} & y(x) \leq 0 \\ 2\sqrt{y(x)} & 0 < y(x) \end{array} \right) = 0 \right\}$$

2.58 ODE No. 58

$$a(-\sqrt{y(x)}) - bx + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.381228 (sec), leaf count = 118

Solve

$$\frac{a^2 - \log\left(a^2\left(\sqrt{\frac{a^2 y(x)}{b^2 x^2}} + 1\right) - \frac{2a^2 y(x)}{bx^2}\right) - \frac{2a \tanh^{-1}\left(\frac{\left(a \left(1 - \frac{4b \sqrt{\frac{a^2 y(x)}{b^2 x^2}}\right)\right)}{a^2}\right)}{\sqrt{a^2 + 8b}}}{2b} = \frac{a^2 \log(x)}{b} + c_1, y(x)$$

✓ **Maple** : cpu = 0.077 (sec), leaf count = 68

$$\left\{ \frac{a \operatorname{arctanh}\left(\frac{2bx+a\sqrt{y(x)}}{\sqrt{(a^2+8b)y(x)}}\right)\sqrt{y(x)}}{\sqrt{(a^2+8b)y(x)}} + c_1 - \frac{\ln\left(bx^2+ax\sqrt{y(x)}-2y(x)\right)}{2} = 0 \right\}$$

2.59 ODE No. 59

$$a\left(-\sqrt{y(x)^2+1}\right)-b+y'(x)=0$$

✓ **Mathematica** : cpu = 0.326393 (sec), leaf count = 96

$$\left\{\left\{y(x) \rightarrow \text{InverseFunction}\left[\frac{b \tan^{-1}\left(\frac{\#1 b}{\sqrt{\#1^2+1} \sqrt{a^2-b^2}}\right)-\frac{b \tan^{-1}\left(\frac{\#1 a}{\sqrt{a^2-b^2}}\right)+\sinh^{-1}(\#1)}{a}\right] \& \left[x+c_1\right]\right\}\right\}$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 26

$$\left\{c_1+x-\left(\int^{y(x)} \frac{1}{\sqrt{-a^2+1} a+b} d\_a\right)=0\right\}$$

2.60 ODE No. 60

$$y'(x)-\frac{\sqrt{y(x)^2-1}}{\sqrt{x^2-1}}=0$$

✓ **Mathematica** : cpu = 0.177511 (sec), leaf count = 173

$$\left\{\left\{y(x) \rightarrow -\frac{1}{2} e^{-c_1} \sqrt{2 x^2+2 e^{4 c_1} x^2-2 \sqrt{(x-1)(x+1)} x+2 e^{4 c_1} \sqrt{(x-1)(x+1)} x-1+2 e^{2 c_1}-e^{4 c_1}}\right\},\left\{y(x) \rightarrow \frac{1}{2} e^{-c_1} \sqrt{2 x^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 29

$$\left\{c_1+\ln \left(x+\sqrt{x^2-1}\right)-\ln \left(y(x)+\sqrt{y(x)^2-1}\right)=0\right\}$$

HAND SOLUTION

$$y'=\pm \frac{\sqrt{y^2-1}}{\sqrt{x^2-1}} \tag{1}$$

Separable. For the positive case

$$\begin{aligned} \frac{dy}{dx} \frac{1}{\sqrt{y^2-1}} &= \frac{1}{\sqrt{x^2-1}} \\ \frac{dy}{(y^2-1)^{\frac{1}{2}}} &= \frac{dx}{(x^2-1)^{\frac{1}{2}}} \end{aligned}$$

Integrating

$$\int \frac{dy}{(y^2-1)^{\frac{1}{2}}} = \int \frac{dx}{(x^2-1)^{\frac{1}{2}}} + C$$

But $\int \frac{dy}{(y^2-1)^{\frac{1}{2}}} = \tanh^{-1} \frac{y}{(y^2-1)^{\frac{1}{2}}} = \ln(y + \sqrt{y^2-1})$, hence

$$\ln(y + \sqrt{y^2-1}) = \ln(x + \sqrt{x^2-1}) + C$$

For the negative case

$$\frac{dy}{dx} \frac{1}{\sqrt{y^2-1}} = -\frac{1}{\sqrt{x^2-1}}$$

$$\frac{dy}{(y^2-1)^{\frac{1}{2}}} = -\frac{dx}{(x^2-1)^{\frac{1}{2}}}$$

Integrating

$$\int \frac{dy}{(y^2-1)^{\frac{1}{2}}} = -\int \frac{dx}{(x^2-1)^{\frac{1}{2}}} + C$$

But $\int \frac{dy}{(y^2-1)^{\frac{1}{2}}} = \tanh^{-1} \frac{y}{(y^2-1)^{\frac{1}{2}}} = \ln(y + \sqrt{y^2-1})$, hence

$$\ln(y + \sqrt{y^2-1}) = -\ln(x + \sqrt{x^2-1}) + C$$

Therefore

$$\ln(y + \sqrt{y^2-1}) = \pm \ln(x + \sqrt{x^2-1}) + C$$

2.61 ODE No. 61

$$y'(x) - \frac{\sqrt{x^2-1}}{\sqrt{y(x)^2-1}} = 0$$

✓ **Mathematica** : cpu = 0.243064 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[\frac{1}{2} \#1 \sqrt{\#1^2-1} - \frac{1}{2} \log(\sqrt{\#1^2-1} + \#1) \& \left\| \frac{1}{2} \sqrt{x^2-1} x - \frac{1}{2} \log(\sqrt{x^2-1} + x) + c_1 \right\| \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.015 (sec), leaf count = 50

$$\left\{ c_1 + \sqrt{x^2-1} x - \ln(x + \sqrt{x^2-1}) + \ln(y(x) + \sqrt{y(x)^2-1}) - \sqrt{y(x)^2-1} y(x) = 0 \right\}$$

2.62 ODE No. 62

$$y'(x) - \frac{y(x) - x^2 \sqrt{x^2 - y(x)^2}}{xy(x) \sqrt{x^2 - y(x)^2} + x} = 0$$

✓ **Mathematica** : cpu = 3.20455 (sec), leaf count = 44

$$\text{Solve} \left[-\tan^{-1} \left(\frac{\sqrt{x^2 - y(x)^2}}{y(x)} \right) + \frac{x^2}{2} + \frac{y(x)^2}{2} = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.384 (sec), leaf count = 34

$$\left\{ \frac{x^2}{2} + \frac{y(x)^2}{2} - c_1 + \arctan \left(\frac{y(x)}{\sqrt{x^2 - y(x)^2}} \right) = 0 \right\}$$

HAND SOLUTION

$$y' = \frac{y - x^2 \sqrt{x^2 - y^2}}{xy \sqrt{x^2 - y^2} + x} \quad (1)$$

Let $y = ux$ then $y' = u + xu'$ therefore

$$\begin{aligned}
 u + xu' &= \frac{y - x^2\sqrt{x^2 - y^2}}{xy\sqrt{x^2 - y^2} + x} \\
 &= \frac{ux - x^2\sqrt{x^2 - (ux)^2}}{x(ux)\sqrt{x^2 - (ux)^2} + x} \\
 &= \frac{ux - x^3\sqrt{1 - u^2}}{x^3u\sqrt{1 - u^2} + x} \\
 &= \frac{u - x^2\sqrt{1 - u^2}}{x^2u\sqrt{1 - u^2} + 1}
 \end{aligned}$$

Hence

$$\begin{aligned}
 u(x^2u\sqrt{1 - u^2} + 1) + xu'(x^2u\sqrt{1 - u^2} + 1) &= u - x^2\sqrt{1 - u^2} \\
 x^2u^2\sqrt{1 - u^2} + u + u'(x^3u\sqrt{1 - u^2} + x) &= u - x^2\sqrt{1 - u^2} \\
 x^2u^2\sqrt{1 - u^2} + u' &= -x^2\sqrt{1 - u^2} \\
 xu^2\sqrt{1 - u^2} + u'(x^2u\sqrt{1 - u^2} + 1) &= -x\sqrt{1 - u^2} \\
 xu^2 + u' &= -x \\
 x(1 + u^2) + u' &= 0
 \end{aligned}$$

Hence

$$x(1 + u^2)dx + \left(x^2u + \frac{1}{\sqrt{1 - u^2}}\right)du = 0 \quad (2)$$

Let $M = x(1 + u^2), N = \left(x^2u + \frac{1}{\sqrt{1 - u^2}}\right)$.

$$\begin{aligned}
 \frac{\partial M}{\partial u} &= 2xu \\
 \frac{\partial N}{\partial x} &= 2xu
 \end{aligned}$$

Therefore (2) is exact. Let

$$x(1 + u^2)dx + \left(x^2u + \frac{1}{\sqrt{1 - u^2}}\right)du = dU$$

Since $dU = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial u}du$. Comparing with the above, we see that

$$\frac{\partial U}{\partial x} = x(1 + u^2) \quad (3)$$

$$\frac{\partial U}{\partial u} = x^2u + \frac{1}{\sqrt{1 - u^2}} \quad (4)$$

From (3)

$$\begin{aligned}
 U &= \int x(1 + u^2)dx \\
 &= \frac{x^2}{2}(1 + u^2) + f(u)
 \end{aligned} \quad (5)$$

From (4)

$$\begin{aligned}
 \frac{d}{du} \left(\frac{x^2}{2}(1 + u^2) + f(u) \right) &= x^2u + \frac{1}{\sqrt{1 - u^2}} \\
 x^2u + f'(u) &= x^2u + \frac{1}{\sqrt{1 - u^2}} \\
 f'(u) &= \frac{1}{\sqrt{1 - u^2}}
 \end{aligned}$$

Therefore

$$f(u) = \arcsin(u)$$

From (5) we find

$$U(x, u) = \frac{x^2}{2}(1 + u^2) + \arcsin(u)$$

Since $dU = 0$ then

$$\begin{aligned} \frac{x^2}{2}(1 + u^2) + \arcsin(u) &= C \\ \frac{x^2}{2}(1 + u^2) + \arcsin(u) - C &= 0 \end{aligned}$$

Since $y = ux$ then the above can be written as

$$\begin{aligned} \frac{x^2}{2}\left(1 + \left(\frac{y}{x}\right)^2\right) + \arcsin\left(\frac{y}{x}\right) - C &= 0 \\ \frac{x^2}{2}\left(\frac{x^2 + y^2}{x^2}\right) + \arcsin\left(\frac{y}{x}\right) - C &= 0 \\ \frac{1}{2}(x^2 + y^2) + \arcsin\left(\frac{y}{x}\right) - C &= 0 \\ \arcsin\left(\frac{y}{x}\right) &= C - \frac{1}{2}(x^2 + y^2) \end{aligned}$$

Hence

$$\begin{aligned} \frac{y}{x} &= \sin\left(C - \frac{1}{2}(x^2 + y^2)\right) \\ y(x) &= x \sin\left(C - \frac{1}{2}(x^2 + y^2)\right) \end{aligned}$$

2.63 ODE No. 63

$$y'(x) - \frac{y(x)^2 + 1}{(x + 1)^{3/2}[y(x) + \sqrt{y(x) + 1}]} = 0$$

✓ **Mathematica** : cpu = 0.188141 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[\int_1^{\#1} \frac{|K[1] + \sqrt{K[1] + 1}|}{K[1]^2 + 1} dK[1] \& \right] \left[-\frac{2}{\sqrt{x + 1}} + c_1 \right] \right\} \right\}$$

✓ **Maple** : cpu = 5.726 (sec), leaf count = 35

$$\left\{ c_1 - \left(\int^{y(x)} \frac{|\_a + \sqrt{\_a + 1}|}{\_a^2 + 1} d\_a \right) - \frac{2}{\sqrt{x + 1}} = 0 \right\}$$

2.64 ODE No. 64

$$y'(x) - \sqrt{\frac{ay(x)^2 + by(x) + c}{ax^2 + bx + c}} = 0$$

✓ **Mathematica** : cpu = 0.380561 (sec), leaf count = 269

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-\sqrt{a}c_1} \left(-8a^{3/2}c\sqrt{ax^2 + bx + c} + 8a^{3/2}ce^{2\sqrt{a}c_1}\sqrt{ax^2 + bx + c} + 8a^2cx + 8a^2cxe^{2\sqrt{a}c_1} + 2b^3e^{\sqrt{a}c_1} - b^3e^{2\sqrt{a}c_1} + 2 \right)}{a(16 \right.$$

✓ **Maple** : cpu = 0.117 (sec), leaf count = 124

$$\left\{ c_1 - \frac{\sqrt{\frac{ay(x)^2 + by(x) + c}{ax^2 + bx + c}} \sqrt{ax^2 + bx + c} \ln\left(\frac{2ax + b + 2\sqrt{ax^2 + bx + c} \sqrt{a}}{2\sqrt{a}}\right)}{\sqrt{ay(x)^2 + by(x) + c} \sqrt{a}} + \frac{\ln\left(\frac{2ay(x) + b}{2\sqrt{a}} + \sqrt{ay(x)^2 + by(x) + c}\right)}{\sqrt{a}} = 0 \right\}$$

2.65 ODE No. 65

$$y'(x) - \sqrt{\frac{y(x)^3 + 1}{x^3 + 1}} = 0$$

✓ **Mathematica** : cpu = 1.14392 (sec), leaf count = 312

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[\frac{i(\#1 + 1) \sqrt{1 + \frac{6i}{(\sqrt{3} - 3i)(\#1 + 1)}} \sqrt{\frac{2}{3} - \frac{4i}{(\sqrt{3} + 3i)(\#1 + 1)}} F \left(i \sinh^{-1} \left(\frac{\sqrt{-\frac{6i}{3i + \sqrt{3}}}}{\sqrt{\#1 + 1}} \right) \frac{3i + \sqrt{3}}{3i - \sqrt{3}} \right)}{\sqrt{-\frac{i}{\sqrt{3} + 3i}}} \sqrt{\#1^2 - \#1 + 1}} \right] \& \left[i(x + 1) \sqrt{\frac{y(x)^3 + 1}{x^3 + 1}} \right] \right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 47

$$\left\{ c_1 + \int^{y(x)} \frac{1}{\sqrt{-a^3 + 1}} d\_a + \int^x -\frac{\sqrt{\frac{y(x)^3 + 1}{-a^3 + 1}}}{\sqrt{y(x)^3 + 1}} d\_a = 0 \right\}$$

2.66 ODE No. 66

$$y'(x) - \frac{\sqrt{|(1 - y(x))y(x)(1 - ay(x))|}}{\sqrt{|(1 - x)x(1 - ax)|}} = 0$$

✓ **Mathematica** : cpu = 0.212899 (sec), leaf count = 67

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[\int_1^{\#1} \frac{1}{\sqrt{|(1 - K[1])K[1](1 - aK[1])|}} dK[1] \& \left[\int_1^x \frac{1}{\sqrt{|(1 - K[2])K[2](1 - aK[2])|}} dK[2] + c_1 \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.258 (sec), leaf count = 40

$$\left\{ c_1 - \left(\int^{y(x)} \frac{1}{\sqrt{|(-a - 1)(-aa - 1)_a|}} d\_a \right) + \int \frac{1}{\sqrt{|(x - 1)(ax - 1)x|}} dx = 0 \right\}$$

2.67 ODE No. 67

$$y'(x) - \frac{\sqrt{1 - y(x)^4}}{\sqrt{1 - x^4}} = 0$$

✓ **Mathematica** : cpu = 0.245416 (sec), leaf count = 14

$$\left\{ \left\{ y(x) \rightarrow \text{sn} \left(c_1 + F \left(\sin^{-1}(x) \right) - 1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 51

$$\left\{ c_1 + \frac{\sqrt{-x^2 + 1} \sqrt{x^2 + 1} \text{EllipticF}(x, i)}{\sqrt{-x^4 + 1}} - \left(\int^{y(x)} \frac{1}{\sqrt{-a^4 + 1}} d\_a \right) = 0 \right\}$$

2.68 ODE No. 68

$$y'(x) - \sqrt{\frac{ay(x)^4 + by(x)^2 + 1}{ax^4 + bx^2 + 1}} = 0$$

✓ **Mathematica** : cpu = 1.07621 (sec), leaf count = 373

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[\frac{i \sqrt{\frac{2\#1^2 a + \sqrt{b^2 - 4a} + b}{\sqrt{b^2 - 4a} + b}} \sqrt{\frac{2\#1^2 a}{b - \sqrt{b^2 - 4a}}} + 1 F \left(i \sinh^{-1} \left(\sqrt{2} \sqrt{\frac{a}{b + \sqrt{b^2 - 4a}}} \#1 \right) \right) \frac{b + \sqrt{b^2 - 4a}}{b - \sqrt{b^2 - 4a}} \right)}{\sqrt{2} \sqrt{\frac{a}{\sqrt{b^2 - 4a} + b}}} \sqrt{\#1^4 a + \#1^2 b + 1}} \right] \& \left[c_1 - \frac{i \sqrt{\frac{y(x)^4 + 1}{ax^4 + bx^2 + 1}}}{\sqrt{ax^4 + bx^2 + 1}} \right] \right\}$$

✓ **Maple** : cpu = 0.073 (sec), leaf count = 77

$$\left\{c_1 + \int^{y(x)} \frac{1}{\sqrt{-a^4a + -a^2b + 1}} d\_a + \int^x - \frac{\sqrt{\frac{ay(x)^4 + by(x)^2 + 1}{-a^4a + -a^2b + 1}}}{\sqrt{ay(x)^4 + by(x)^2 + 1}} d\_a = 0 \right\}$$

2.69 ODE No. 69

$$y'(x) - \sqrt{(a0 + a1x + a2x^2 + a3x^3 + a4x^4)(b0 + b1y(x) + b2y(x)^2 + b3y(x)^3 + b4y(x)^4)} = 0$$

✓ **Mathematica** : cpu = 8.3668 (sec), leaf count = 1163

Solve

$$\left[2F\left(\sin^{-1}\left(\sqrt{\frac{(\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,2]-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,4])(y(x)-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,1])}{(\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,1]-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,4])(y(x)-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,1])}}\right)}\right]$$

✓ **Maple** : cpu = 0.182 (sec), leaf count = 111

$$\left\{c_1 + \int^{y(x)} \frac{1}{\sqrt{-a^4b4 + -a^3b3 + -a^2b2 + -ab1 + b0}} d\_a + \int^x - \frac{\sqrt{(b4y(x)^4 + b3y(x)^3 + b2y(x)^2 + b1y(x) + b0)(-a^4a4 + -a^3a3 + -a^2a2 + -aa1 + a0)}}{\sqrt{b4y(x)^4 + b3y(x)^3 + b2y(x)^2 + b1y(x) + b0}} \sqrt{b4y(x)^4 + b3y(x)^3 + b2y(x)^2 + b1y(x) + b0} d\_a = 0 \right\}$$

2.70 ODE No. 70

$$y'(x) - \sqrt{\frac{a0 + a1x + a2x^2 + a3x^3 + a4x^4}{b0 + b1y(x) + b2y(x)^2 + b3y(x)^3 + b4y(x)^4}} = 0$$

✓ **Mathematica** : cpu = 12.877 (sec), leaf count = 81

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction}\left[\int_1^{\#1} \sqrt{b4K[1]^4 + b3K[1]^3 + b2K[1]^2 + b1K[1] + b0} dK[1] \& \right] \left[\int_1^x \sqrt{a4K[2]^4 + a3K[2]^3 + a2K[2]^2 + a1K[2] + a0} dK[2] \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.177 (sec), leaf count = 113

$$\left\{c_1 + \int^{y(x)} \sqrt{-a^4b4 + -a^3b3 + -a^2b2 + -ab1 + b0} d\_a + \int^x - \sqrt{\frac{-a^4a4 + -a^3a3 + -a^2a2 + -aa1 + a0}{b4y(x)^4 + b3y(x)^3 + b2y(x)^2 + b1y(x) + b0}} \sqrt{b4y(x)^4 + b3y(x)^3 + b2y(x)^2 + b1y(x) + b0} d\_a = 0 \right\}$$

2.71 ODE No. 71

$$y'(x) - \sqrt{\frac{b0 + b1y(x) + b2y(x)^2 + b3y(x)^3 + b4y(x)^4}{a0 + a1x + a2x^2 + a3x^3 + a4x^4}} = 0$$

✓ **Mathematica** : cpu = 3.74941 (sec), leaf count = 2237

Solve

$$\left[2F\left(\sin^{-1}\left(\sqrt{\frac{(\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,2]-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,4])(y(x)-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,1])}{(\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,1]-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,4])(y(x)-\text{Root}[b4\#1^4+b3\#1^3+b2\#1^2+b1\#1+b0\&,1])}}\right)}\right]$$

✓ **Maple** : cpu = 0.129 (sec), leaf count = 113

$$\left\{c_1 + \int^{y(x)} \frac{1}{\sqrt{-a^4b4 + -a^3b3 + -a^2b2 + -ab1 + b0}} d\_a + \int^x - \frac{\sqrt{\frac{b4y(x)^4 + b3y(x)^3 + b2y(x)^2 + b1y(x) + b0}{-a^4a4 + -a^3a3 + -a^2a2 + -aa1 + a0}}}{\sqrt{b4y(x)^4 + b3y(x)^3 + b2y(x)^2 + b1y(x) + b0}} d\_a = 0 \right\}$$

2.72 ODE No. 72

$$y'(x)-R1\left(x,\sqrt{a0+a1x+a2x^2+a3x^3+a4x^4}\right)R2\left(y(x),\sqrt{b0+b1y(x)+b2y(x)^2+b3y(x)^3+b4y(x)^4}\right)=0$$

✓ **Mathematica** : cpu = 0.20499 (sec), leaf count = 89

$$\left\{\left\{y(x)\rightarrow \text{InverseFunction}\left[\int_1^{\#1}\frac{1}{R2\left(K[1],\sqrt{b4K[1]^4+b3K[1]^3+b2K[1]^2+b1K[1]+b0}\right)}dK[1]\&\right]\left[\int_1^xR1\left(K[2],\sqrt{a4\right.\right.\right.$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 64

$$\left\{c_1-\left(\int^{\boldsymbol{y(x)}}\frac{1}{R2\left(\boldsymbol{-a},\sqrt{\boldsymbol{-a^4b4+a^3b3+a^2b2+ab1+b0}}\right)}d\boldsymbol{-a}\right)+\int R1\left(x,\sqrt{\boldsymbol{a4x^4+a3x^3+a2x^2+a1x+a0}}\right)dx=0\right\}$$

2.73 ODE No. 73

$$y'(x)-\left(\frac{a0+a1x+a2x^2+a3x^3}{a0+a1y(x)+a2y(x)^2+a3y(x)^3}\right)^{2/3}=0$$

✓ **Mathematica** : cpu = 1.71741 (sec), leaf count = 733

$$\text{Solve}\left[\frac{3(a0+y(x)(a1+y(x)(a2+a3y(x))))^{2/3}\left(y(x)-\text{Root}\left[\#1^3a3+\#1^2a2+\#1a1+a0\&,1\right]\right)F_1\left(\frac{5}{3};-\frac{2}{3},-\frac{2}{3};\frac{8}{3};\frac{}{\text{Root}\left[a3\#1^3+\right.}\right.}{5\left(\frac{y(x)-\text{Root}\left[\#1^3a3+\#1^2a2+\#1a1+a0\&,2\right]}{\text{Root}\left[\#1^3a3+\#1^2a2+\#1a1+a0\&,1\right]}-\text{Root}\left[\#1^3a3+\#1^2a2+\#1a1+a0\&,2\right]}\right)^{2/3}\left(\frac{}{\text{Root}\left[\#1^3a3+\#1^2a2+\#1a1+a0\&,1\right]}\right)^{2/3}\left(\frac{}{\text{Root}\left[\#1^3a3+\#1^2a2+\#1a1+a0\&,2\right]}\right)^{2/3}}{}\right]$$

✓ **Maple** : cpu = 0.501 (sec), leaf count = 91

$$\left\{c_1+\int^{\boldsymbol{y(x)}}\left(\boldsymbol{-a^3a3+a^2a2+aa1+a0}\right)^{\frac{2}{3}}d\boldsymbol{-a}+\int^x-\left(\frac{\boldsymbol{-a^3a3+a^2a2+aa1+a0}}{\boldsymbol{a3y(x)^3+a2y(x)^2+a1y(x)+a0}}\right)^{\frac{2}{3}}\left(\boldsymbol{a3y(x)^3+a2y(x)^2+a1y(x)+a0}\right)^{\frac{2}{3}}dx=0\right\}$$

2.74 ODE No. 74

$$y'(x)-f(x)(y(x)-g(x))\sqrt{(y(x)-a)(y(x)-b)}=0$$

✗ **Mathematica** : cpu = 2.36593 (sec), leaf count = 0 , could not solve

$$\text{DSolve}\left[-\left(\text{f}\left[\text{x}\right]\text{Sqrt}\left[\left(-\text{a}+\text{y}\left[\text{x}\right]\right)\left(-\text{b}+\text{y}\left[\text{x}\right]\right)\right]\left(-\text{g}\left[\text{x}\right]+\text{y}\left[\text{x}\right]\right)\right)+\text{Derivative}\left[1\right]\left[\text{y}\right]\left[\text{x}\right]==0,\text{y}\left[\text{x}\right]\right]$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

$$\text{dsolve}\left(\text{diff}\left(\text{y}\left(\text{x}\right),\text{x}\right)-\text{f}\left(\text{x}\right)\left(\text{y}\left(\text{x}\right)-\text{g}\left(\text{x}\right)\right)\left(\left(\text{y}\left(\text{x}\right)-\text{a}\right)\left(\text{y}\left(\text{x}\right)-\text{b}\right)\right)^{\frac{1}{2}}=0,\text{y}\left(\text{x}\right)\right)$$

2.75 ODE No. 75

$$y'(x)-e^{x-y(x)}+e^x=0$$

✓ **Mathematica** : cpu = 0.214679 (sec), leaf count = 20

$$\left\{\left\{y(x)\rightarrow \log\left(1-e^{-e^x+c_1}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.16 (sec), leaf count = 20

$$\{y(x)=-c_1-e^x+\ln\left(e^{c_1+e^x}-1\right)\}$$

HAND SOLUTION

$$\begin{aligned}y' &= e^{x-y} - e^x \\y' &= e^x(e^{-y} - 1) \\ \frac{1}{e^{-y} - 1} dy &= e^x dx\end{aligned}\tag{1}$$

Integrating both sides. $\int \frac{1}{e^{-y}-1} dy$. Let $e^{-y} = u$, then $\frac{du}{dy} = -e^{-y} = -u$. Hence $dy = -\frac{du}{u}$, therefore the integral becomes

$$\int \frac{1}{u-1} \left(-\frac{du}{u}\right) = -\int \frac{1}{u(u-1)} du$$

But $\frac{1}{u(u-1)} = -\left(\frac{1}{u} - \frac{1}{u-1}\right)$, hence

$$\begin{aligned}-\int \frac{1}{u(u-1)} du &= \int \left(\frac{1}{u} - \frac{1}{u-1}\right) du \\ &= \ln u - \ln(u-1) \\ &= \ln e^{-y} - \ln(e^{-y} - 1) \\ &= -(\ln(e^{-y} - 1) - \ln e^{-y})\end{aligned}$$

But $\ln x - \ln y = \ln\left(\frac{x}{y}\right)$ and the above becomes

$$\begin{aligned}\int \frac{1}{e^{-y}-1} dy &= -\left[\ln\left(\frac{e^{-y}-1}{e^{-y}}\right)\right] \\ &= -\ln(1 - e^y)\end{aligned}$$

Back to (1), when we integrate both sides, and since $\int e^x dx = e^x + C$

$$\begin{aligned}-\ln(1 - e^y) &= e^x + C \\ \ln(1 - e^y) &= -e^x + C_1\end{aligned}$$

Hence

$$\begin{aligned}1 - e^y &= \exp(-e^x + C_1) \\ e^y &= 1 - \exp(-e^x + C_1)\end{aligned}$$

Taking logs

$$y = \ln(1 - \exp(-e^x + C_1))$$

Let $e^{C_1} = C_2$ then

$$y = \ln(1 - C_2 e^{-e^x})$$

Verification

```
ode:=diff(y(x),x)=exp(x-y(x))-exp(x);
my_sol:=log(1-C1*exp(-exp(x)));
odetest(y(x)=my_sol,ode);
0
```

2.76 ODE No. 76

$$-a \cos(y(x)) + b + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.269774 (sec), leaf count = 116

$$\left\{\left\{y(x) \rightarrow 2 \tan^{-1}\left(\frac{a \tanh\left(\frac{1}{2}\left(x \sqrt{(a-b)(a+b)} - c_1 \sqrt{(a-b)(a+b)}\right)\right)}{\sqrt{(a-b)(a+b)}} - \frac{b \tanh\left(\frac{1}{2}\left(x \sqrt{(a-b)(a+b)} - c_1 \sqrt{(a-b)(a+b)}\right)\right)}{\sqrt{(a-b)(a+b)}}\right.\right.\right.$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 41

$$\left\{y(x) = 2 \arctan\left(\frac{\sqrt{(a-b)(a+b)} \tanh\left(\frac{\sqrt{(a-b)(a+b)} (c_1+x)}{2}\right)}{a+b}\right)\right\}$$

HAND SOLUTION

$$y' = a \cos y + b$$

This is separable.

$$\begin{aligned} \frac{dy}{a \cos y + b} &= dx \\ \int \frac{dy}{a \cos y + b} &= x + C \end{aligned} \tag{1}$$

Using standard Tangent half-angle substitution, let $t = \tan \frac{y}{2}$, $\cos y = \frac{1-t^2}{1+t^2}$, $dy = \frac{2}{1+t^2} dt$, then the integral becomes

$$\begin{aligned} \int \frac{dy}{a \cos y + b} &= \int \frac{2}{1+t^2} \frac{1}{\left(a \frac{1-t^2}{1+t^2} + b\right)} dt \\ &= 2 \int \frac{1+t^2}{(1+t^2)(a(1-t^2) + b(1+t^2))} dt \\ &= 2 \int \frac{dt}{a - at^2 + b + bt^2} \\ &= 2 \int \frac{dt}{(a+b) + t^2(b-a)} \\ &= 2 \int \frac{dt}{(a+b) \left(1 + \frac{t^2(b-a)}{(a+b)}\right)} \\ &= \frac{2}{a+b} \int \frac{dt}{\left(1 + \frac{t^2(b-a)}{(a+b)}\right)} \end{aligned}$$

Let $z^2 = \frac{t^2(b-a)}{(a+b)}$, or $z = \frac{t\sqrt{b-a}}{\sqrt{a+b}}$, then $\frac{dz}{dt} = \frac{\sqrt{b-a}}{\sqrt{a+b}}$ and the above integral becomes

$$\begin{aligned} \frac{2}{a+b} \int \frac{dt}{\left(1 + \frac{t^2(b-a)}{(a+b)}\right)} &= \frac{2}{a+b} \int \frac{\sqrt{a+b}}{\sqrt{b-a}} \frac{dz}{(1+z^2)} \\ &= \frac{2}{a+b} \frac{\sqrt{a+b}}{\sqrt{b-a}} \int \frac{dz}{(1+z^2)} \\ &= \frac{2}{\sqrt{a+b}} \frac{1}{\sqrt{b-a}} \int \frac{dz}{(1+z^2)} \\ &= \frac{2}{\sqrt{(a+b)(b-a)}} \int \frac{dz}{(1+z^2)} \\ &= \frac{2}{\sqrt{b^2 - a^2}} \int \frac{dz}{(1+z^2)} \end{aligned}$$

Now, $\int \frac{dz}{(1+z^2)} = \arctan(z)$, hence

$$\begin{aligned} \frac{2}{\sqrt{b^2 - a^2}} \int \frac{dz}{(1+z^2)} &= \frac{2}{\sqrt{b^2 - a^2}} \arctan(z) \\ &= \frac{2}{\sqrt{b^2 - a^2}} \arctan\left(\frac{t\sqrt{b-a}}{\sqrt{a+b}}\right) \end{aligned}$$

But $t = \tan \frac{y}{2}$ therefore

$$\frac{2}{\sqrt{b^2 - a^2}} \arctan\left(\frac{t\sqrt{b-a}}{\sqrt{a+b}}\right) = \frac{2}{\sqrt{b^2 - a^2}} \arctan\left(\frac{\tan\left(\frac{y}{2}\right)\sqrt{b-a}}{\sqrt{a+b}}\right)$$

Going back to (1)

$$\begin{aligned} \int \frac{dy}{a \cos y + b} &= x + C \\ \frac{2}{\sqrt{b^2 - a^2}} \arctan\left(\frac{\tan\left(\frac{y}{2}\right)\sqrt{b-a}}{\sqrt{a+b}}\right) &= x + C \\ \arctan\left(\frac{\tan\left(\frac{y}{2}\right)\sqrt{b-a}}{\sqrt{a+b}}\right) &= \frac{1}{2} \sqrt{b^2 - a^2} (x + C) \\ \frac{\tan\left(\frac{y}{2}\right)\sqrt{b-a}}{\sqrt{a+b}} &= \tan\left(\frac{1}{2} \sqrt{b^2 - a^2} (x + C)\right) \\ \tan\left(\frac{y}{2}\right) &= \frac{\sqrt{a+b}}{\sqrt{b-a}} \tan\left(\frac{1}{2} \sqrt{b^2 - a^2} (x + C)\right) \\ \frac{y}{2} &= \arctan\left(\frac{(a+b)}{\sqrt{(a+b)(b-a)}} \tan\left(\frac{1}{2} \sqrt{b^2 - a^2} (x + C)\right)\right) \\ &= \arctan\left(\frac{(a+b)}{\sqrt{b^2 - a^2}} \tan\left(\frac{1}{2} \sqrt{b^2 - a^2} (x + C)\right)\right) \\ y &= 2 \arctan\left(\frac{a+b}{\sqrt{b^2 - a^2}} \tan\left(\frac{1}{2} \sqrt{b^2 - a^2} (x + C)\right)\right) \end{aligned}$$

Verification

```
ode:=diff(y(x),x)=a*cos(y(x))+b;
my_sol:=2*arctan( (a+b)/sqrt(b^2-a^2) * tan(1/2*sqrt(b^2-a^2)*(x+_C1)));
odetest(y(x)=my_sol,ode);
0
```

2.77 ODE No. 77

$$y'(x) - \cos(ay(x) + bx) = 0$$

✓ **Mathematica** : cpu = 0.39693 (sec), leaf count = 124

$$\left\{ \left\{ y(x) \rightarrow \frac{-bx - 2 \tan^{-1}\left(\frac{a \tanh\left(\frac{1}{2}\left(-x\sqrt{a^2-b^2} + c_1\sqrt{a^2-b^2}\right)\right)}{\sqrt{a^2-b^2}} + \frac{b \tanh\left(\frac{1}{2}\left(-x\sqrt{a^2-b^2} + c_1\sqrt{a^2-b^2}\right)\right)}{\sqrt{a^2-b^2}}\right)}{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.083 (sec), leaf count = 54

$$\left\{ y(x) = \frac{-bx + 2 \arctan\left(\frac{\sqrt{(a-b)(a+b)} \tanh\left(\frac{\sqrt{(a-b)(a+b)} (-c_1+x)}{2}\right)}{a-b}\right)}{a} \right\}$$

HAND SOLUTION

$$y' = \cos(ay + bx)$$

This is separable after transformation of $u = ay + bx$, hence $u' = ay' + b$ or $y' = \frac{1}{a}(u' - b)$. Therefore the above becomes

$$\begin{aligned}\frac{1}{a}(u' - b) &= \cos(u) \\ u' &= a \cos u + b \\ \frac{du}{a \cos u + b} &= dx\end{aligned}$$

This is the same as Kamke 76 (the problem before this), which we solved using half angle tan transformation, and the answer is

$$u = 2 \arctan\left(\frac{a + b}{\sqrt{b^2 - a^2}} \tan\left(\frac{1}{2}\sqrt{b^2 - a^2} (x + C)\right)\right)$$

Since $u = ay + bx$ then $y = \frac{u-bx}{a}$, hence

$$y = \frac{1}{a}\left(2 \arctan\left(\frac{a + b}{\sqrt{b^2 - a^2}} \tan\left(\frac{1}{2}\sqrt{b^2 - a^2} (x + C)\right)\right) - bx\right)$$

Verification

```
ode:=diff(y(x),x)=cos(a*y(x)+b*x);
my_sol:=(1/a)*(2*arctan((a+b)/sqrt(b^2-a^2) * tan(1/2*sqrt(b^2-a^2)*(x+C1)))-b*x);
odetest(y(x)=my_sol,ode);
0
```

2.78 ODE No. 78

$$a \sin(\alpha y(x) + \beta x) + b + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.942125 (sec), leaf count = 1317

$$\left\{ \left\{ \left\{ \begin{aligned} &2 \tan^{-1}\left(\frac{a^2 \sqrt{-((a \alpha + b \alpha - \beta)(a \alpha - b \alpha + \beta))} \tan\left(\frac{1}{2}\left(\frac{a^2 x \alpha^2}{\sqrt{-((a \alpha + b \alpha - \beta)(a \alpha - b \alpha + \beta))}} - \frac{b^2 x \alpha^2}{\sqrt{-((a \alpha + b \alpha - \beta)(a \alpha - b \alpha + \beta))}} - \frac{a^2 c_1 \alpha^2}{\sqrt{-((a \alpha + b \alpha - \beta)(a \alpha - b \alpha + \beta))}} + \frac{b^2 c_1 \alpha^2}{\sqrt{-((a \alpha + b \alpha - \beta)(a \alpha - b \alpha + \beta))}} + \sqrt{\frac{a^2 \sqrt{-((a \alpha + b \alpha - \beta)(a \alpha - b \alpha + \beta))}}{(a \alpha + b \alpha - \beta)(a \alpha - b \alpha + \beta)}}\right)}\right)}{b \alpha - \beta}\right)}{a}\right\} \right\} y(x) \rightarrow \end{aligned} \right.$$

✓ **Maple** : cpu = 0.112 (sec), leaf count = 89

$$\left\{ y(x) = \frac{-\beta x + 2 \arctan\left(\frac{-a \alpha - \sqrt{-2 \alpha b \beta + (-a^2 + b^2) \alpha^2 + \beta^2} \tan\left(\frac{\sqrt{-2 \alpha b \beta + (-a^2 + b^2) \alpha^2 + \beta^2} (-c_1 + x)}{2}\right)}{b \alpha - \beta}\right)}{\alpha} \right\}$$

HAND SOLUTION

$$y' = -a \sin(\alpha y + \beta x) - b$$

This is separable after transformation of $u = \alpha y + \beta x$, hence $u' = \alpha y' + \beta$ or $y' = \frac{1}{\alpha}(u' - \beta)$.

Therefore the above becomes

$$\begin{aligned}\frac{1}{\alpha}(u' - \beta) &= -a \sin(u) - b \\ u' &= -\alpha(a \sin(u) + b) + \beta \\ \frac{du}{\beta - \alpha(a \sin(u) + b)} &= dx\end{aligned}\tag{1}$$

Using half angle tan transformation where $\tan\left(\frac{u}{2}\right) = t$, $\sin(u) = \frac{2t}{t^2+1}$, $du = \frac{2}{1+t^2}dt$ then

$$\begin{aligned}\int \frac{du}{\beta - \alpha(a \sin(u) + b)} &= \int \frac{2}{1+t^2} \frac{dt}{\beta - \alpha\left(a \frac{2t}{t^2+1} + b\right)} \\ &= 2 \int \frac{dt}{\beta(t^2+1) - \alpha(a2t + b(t^2+1))} \\ &= 2 \int \frac{dt}{t\beta^2 + \beta - (\alpha a2t + t^2\alpha b + \alpha b)} \\ &= 2 \int \frac{dt}{(\beta - \alpha b)\left(\frac{t\beta^2 - \alpha a2t - t^2\alpha b}{(\beta - \alpha b)} + 1\right)} \\ &= \frac{2}{(\beta - \alpha b)} \int \frac{dt}{\frac{t\beta^2 - \alpha a2t - t^2\alpha b}{(\beta - \alpha b)} + 1} \\ &= \frac{2}{(\beta - \alpha b)} \frac{-(\alpha b - \beta)}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}} \tanh^{-1} \left(\frac{\left(t + \frac{a\alpha}{b\alpha - \beta}\right)(b\alpha - \beta)}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}} \right) \\ &= \frac{2}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}} \tanh^{-1} \left(\frac{t(b\alpha - \beta) + a\alpha}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}} \right)\end{aligned}$$

But $t = \tan\left(\frac{u}{2}\right)$ therefore

$$\int \frac{du}{\beta - \alpha(a \sin(u) + b)} = \frac{2}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}} \tanh^{-1} \left(\frac{\tan\left(\frac{u}{2}\right)(b\alpha - \beta) + a\alpha}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}} \right)$$

But $u = \alpha y + \beta x$, and the above becomes

$$\int \frac{du}{\beta - \alpha(a \sin(u) + b)} = \frac{2 \tanh^{-1} \left(\frac{\tan\left(\frac{\alpha y + \beta x}{2}\right)(b\alpha - \beta) + a\alpha}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}} \right)}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}}$$

Back to (1), therefore after integrating both sides

$$\frac{2 \tanh^{-1} \left(\frac{\tan\left(\frac{\alpha y + \beta x}{2}\right)(b\alpha - \beta) + a\alpha}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}} \right)}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}} = x + C$$

Let

$$A = \sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b\beta)}$$

Then

$$\begin{aligned}\tanh^{-1}\left(\frac{\tan\left(\frac{\alpha y+\beta x}{2}\right)(b\alpha-\beta)+a\alpha}{A}\right) &= \frac{1}{2}A(x+C) \\ \frac{\tan\left(\frac{\alpha y+\beta x}{2}\right)(b\alpha-\beta)+a\alpha}{A} &= \tanh\left(\frac{1}{2}A(x+C)\right) \\ \tan\left(\frac{\alpha y+\beta x}{2}\right)(b\alpha-\beta)+a\alpha &= A \tanh\left(\frac{1}{2}A(x+C)\right) \\ \tan\left(\frac{\alpha y+\beta x}{2}\right) &= \frac{A}{(b\alpha-\beta)} \tanh\left(\frac{1}{2}A(x+C)\right) - \frac{a\alpha}{(b\alpha-\beta)} \\ \frac{\alpha y+\beta x}{2} &= \arctan\left(\frac{A}{(b\alpha-\beta)} \tanh\left(\frac{1}{2}A(x+C)\right) - \frac{a\alpha}{(b\alpha-\beta)}\right) \\ y &= \frac{2}{\alpha} \arctan\left(\frac{A}{(b\alpha-\beta)} \tanh\left(\frac{1}{2}A(x+C)\right) - \frac{a\alpha}{(b\alpha-\beta)}\right) - \frac{\beta x}{\alpha}\end{aligned}$$

Verification

```
ode:=diff(y(x),x)=-a*sin(alpha*y(x)+beta*x)-b;
A0:=sqrt(alpha^2*a^2-(alpha^2*b^2+beta^2-2*alpha*b*beta));
B0:=(alpha*b-beta);
my_sol:=2/alpha*arctan(A0/B0*tanh((1/2)*A0*(x+_C1))-a*alpha/(B0))-beta*x/alpha;
odetest(y(x)=my_sol,ode);
0
```

2.79 ODE No. 79

$$f(x)\cos(ay(x))+g(x)\sin(ay(x))+h(x)+y'(x)=0$$

✗ **Mathematica** : cpu = 24.9646 (sec), leaf count = 0 , could not solve
DSolve[Cos[a*y[x]]*f[x] + h[x] + g[x]*Sin[a*y[x]] + Derivative[1][y][x] == 0, y[x], x]
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve
dsolve(diff(y(x),x)+f(x)*cos(a*y(x))+g(x)*sin(a*y(x))+h(x) = 0,y(x))

2.80 ODE No. 80

$$(1-f'(x))\cos(y(x))-f'(x)+f(x)\sin(y(x))+y'(x)-1=0$$

✓ **Mathematica** : cpu = 0.0737126 (sec), leaf count = 72

$$\left\{\left\{y(x)\rightarrow 2\tan^{-1}\left(f(x)+\frac{1}{\exp\left(\int_1^x-f(K[1])dK[1]\right)\int_1^x-\exp\left(-\int_1^{K[2]}-f(K[1])dK[1]\right)dK[2]+c_1\exp\left(\int_1^x-f(K[1])dK[1]\right)}\right\}\right\}$$

✓ **Maple** : cpu = 1.2 (sec), leaf count = 41

$$\left\{y(x)=2\arctan\left(\frac{c_1f(x)+f(x)\left(\int e^{\int f(x)dx}dx\right)-e^{\int f(x)dx}}{c_1+\int e^{\int f(x)dx}dx}\right)\right\}$$

2.81 ODE No. 81

$$y'(x)+2\tan(x)\tan(y(x))-1=0$$

✓ **Mathematica** : cpu = 1.87795 (sec), leaf count = 220

Solve
$$c_1 = \frac{\frac{1}{2} \left(\frac{1}{\frac{i \tan(x)}{\tan^2(x)+1} - \frac{i \tan^2(x) \tan(y(x))}{\tan^2(x)+1}} + i \cot(x) \right)^4 \sqrt{1 - \left(\frac{1}{\frac{i \tan(x)}{\tan^2(x)+1} - \frac{i \tan^2(x) \tan(y(x))}{\tan^2(x)+1}} + i \cot(x) \right)^2} {}_2F_1 \left(\frac{1}{2}, \frac{5}{4}; \frac{3}{2}; \left(i \cot(x) + \frac{1}{\frac{i \tan(x)}{\tan^2(x)+1} - \frac{i \tan^2(x) \tan(y(x))}{\tan^2(x)+1}} \right)^2 \right)}{\sqrt{-1 + \left(\frac{1}{\frac{i \tan(x)}{\tan^2(x)+1} - \frac{i \tan^2(x) \tan(y(x))}{\tan^2(x)+1}} + i \cot(x) \right)^2}}$$

✓ **Maple** : cpu = 1.29 (sec), leaf count = 78

$$\left\{ c_1 + \frac{(\tan(x) + \tan(y(x))) \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{5}{4}\right], \left[\frac{3}{2}\right], -\frac{(\tan(x) + \tan(y(x)))^2}{(\tan(x) \tan(y(x)) - 1)^2}\right)}{2 \tan(x) \tan(y(x)) - 2} + \frac{\tan(x)}{\left(\frac{(\tan^2(y(x)) + 1)(\tan^2(x) + 1)}{(\tan(x) \tan(y(x)) - 1)^2}\right)^{\frac{1}{4}}} = 0 \right\}$$

2.82 ODE No. 82

$$-a(\tan^2(y(x)) + 1) + y'(x) + \tan(x) \tan(y(x)) = 0$$

✗ **Mathematica** : cpu = 47.4088 (sec), leaf count = 0 , could not solve

`DSolve[Tan[x]*Tan[y[x]] - a*(1 + Tan[y[x]]^2) + Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x), x) - a*(tan(y(x))^2 + 1) + tan(x)*tan(y(x)) = 0, y(x))`

2.83 ODE No. 83

$$y'(x) - \tan(xy(x)) = 0$$

✓ **Mathematica** : cpu = 0.429712 (sec), leaf count = 69

$$\text{Solve}\left[\frac{1}{2}\sqrt{\frac{\pi}{2}}e^{\frac{x^2}{2}}\left(\operatorname{erfi}\left(\frac{y(x)-ix}{\sqrt{2}}\right)+\operatorname{erfi}\left(\frac{y(x)+ix}{\sqrt{2}}\right)\right)=c_1e^{\frac{x^2}{2}},y(x)\right]$$

✓ **Maple** : cpu = 0.356 (sec), leaf count = 44

$$\left\{y(x)=-i\operatorname{RootOf}\left(c_1\sqrt{2}-\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{2}\left(\_Z+x\right)}{2}\right)-\sqrt{\pi}\operatorname{erf}\left(\frac{\left(\_Z-x\right)\sqrt{2}}{2}\right)\right)\right\}$$

2.84 ODE No. 84

$$y'(x) - f(ax + by(x)) = 0$$

✓ **Mathematica** : cpu = 0.236357 (sec), leaf count = 248

$$\text{Solve}\left[\int_1^{y(x)}\frac{f(ax+bK[2])}{a+b f(aK[1]+bK[2])}\int_1^x\left(\frac{b^2f'(aK[1]+bK[2])}{a+b f(aK[1]+bK[2])}-\frac{b^3f(aK[1]+bK[2])f'(aK[1]+bK[2])}{(a+b f(aK[1]+bK[2]))^2}\right)dK[1]b+b+a\int_1^x\left(\frac{b^2f'(aK[1]+bK[2])}{a+b f(aK[1]+bK[2])}-\frac{b^3f(aK[1]+bK[2])f'(aK[1]+bK[2])}{(a+b f(aK[1]+bK[2]))^2}\right)dK[1]b+b+a\right)dx=a+b f(ax+bK[2])\right]$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 37

$$\left\{y(x)=\frac{-ax+b\operatorname{RootOf}\left(b\left(\int^{-Z}\frac{1}{bf(\_ab)+a}d\_a\right)+c_1-x\right)}{b}\right\}$$

2.88 ODE No. 88

$$-ce^{-2ax} - 4ay(x) - b + 2y'(x) - 3y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.551086 (sec), leaf count = 2831

$$\left\{ \left\{ \left\{ y(x) \rightarrow -2 \left(-2^{-\frac{a\sqrt{4a^2-3b}-2a^2}{a^2} + \frac{\sqrt{4a^4-3a^2b}}{a^2} + 1 \right)^{\frac{a\sqrt{4a^2-3b}-2a^2}{4a^2} - \frac{\sqrt{4a^4-3a^2b}}{4a^2}} \frac{a\sqrt{4a^2-3b}-2a^2}{2a^2} + \frac{\sqrt{4a^4-3a^2b}}{2a^2} + 1 \right)^{\frac{a\sqrt{4a^2-3b}-2a^2}{4a^2} - \frac{\sqrt{4a^4-3a^2b}}{4a^2}} - 1 \left(\frac{a\sqrt{4a^2-3b}}{4a^2} \right) \right. \right. \right.$$

✓ **Maple** : cpu = 0.243 (sec), leaf count = 256

$$\left\{ y(x) = \frac{-\sqrt{3} \left(c_1 \text{BesselY} \left(-\frac{-2a+\sqrt{4a^2-3b}}{2a}, \frac{\sqrt{3}\sqrt{c}e^{-ax}}{2a} \right) + \text{BesselJ} \left(-\frac{-2a+\sqrt{4a^2-3b}}{2a}, \frac{\sqrt{3}\sqrt{c}e^{-ax}}{2a} \right) \right) \sqrt{c}e^{-ax} - (2a + \sqrt{4a^2-3b}) \left(\frac{3c_1 \text{BesselY} \left(-\frac{\sqrt{4a^2-3b}}{2a}, \frac{\sqrt{3}\sqrt{c}e^{-ax}}{2a} \right) + 3 \text{BesselJ} \left(-\frac{\sqrt{4a^2-3b}}{2a} \right)}{3} \right)}{3c_1 \text{BesselY} \left(-\frac{\sqrt{4a^2-3b}}{2a}, \frac{\sqrt{3}\sqrt{c}e^{-ax}}{2a} \right) + 3 \text{BesselJ} \left(-\frac{\sqrt{4a^2-3b}}{2a} \right)}$$

HAND SOLUTION

$$y' = \frac{1}{2}b + \frac{1}{2}ce^{-2ax} + 2ay + \frac{3}{2}y^2$$

This is of the form $y' = f_0 + f_1y + f_2y^2$ with $f_0 = \frac{1}{2}b + \frac{1}{2}ce^{-2ax}$, $f_1 = 2a$, $f_2 = \frac{3}{2}$. Hence it is Riccati non-linear first order. Transforming to second order ODE using

$$\begin{aligned} y &= -\frac{u'}{uf_2} \\ &= \frac{-2}{3} \frac{u'}{u} \end{aligned}$$

Hence $y' = \frac{-2}{3} \left(\frac{u''}{u} - \frac{(u')^2}{u^2} \right)$ and equating this to RHS of the ODE gives

$$\begin{aligned} \frac{-2}{3} \left(\frac{u''}{u} - \frac{(u')^2}{u^2} \right) &= \frac{1}{2}b + \frac{1}{2}ce^{-2ax} + 2a \left(\frac{-2}{3} \frac{u'}{u} \right) + \frac{3}{2} \left(\frac{-2}{3} \frac{u'}{u} \right)^2 \\ \frac{-2}{3} \frac{u''}{u} + \frac{2}{3} \frac{(u')^2}{u^2} &= \frac{1}{2}b + \frac{1}{2}ce^{-2ax} - \frac{4}{3}a \frac{u'}{u} + \frac{2}{3} \frac{(u')^2}{u^2} \\ \frac{-2}{3} \frac{u''}{u} &= \frac{1}{2}b + \frac{1}{2}ce^{-2ax} - \frac{4}{3}a \frac{u'}{u} \\ \frac{u''}{u} &= -\frac{3}{4}b - \frac{3}{4}ce^{-2ax} + 2a \frac{u'}{u} \\ u'' &= -\left(\frac{3}{4}b + \frac{3}{4}ce^{-2ax} \right)u + 2au' \\ u'' - 2au' + \frac{3}{4}(b + ce^{-2ax})u &= 0 \end{aligned}$$

This is second order linear ODE with varying coefficient. Solved using power series method giving solutions using special functions (Bessel functions). Let $A = \frac{\sqrt{4a^2-3b}}{a}$, $B = \frac{\sqrt{3c}e^{-ax}}{a}$ then

$$u(x) = C_1 e^{ax} \text{BesselJ} \left(-\frac{1}{2} \frac{\sqrt{4a^2-3b}}{a}, \frac{1}{2} \frac{\sqrt{3c}e^{-ax}}{a} \right) + C_2 e^{ax} \text{BesselY} \left(-\frac{1}{2} \frac{\sqrt{4a^2-3b}}{a}, \frac{1}{2} \frac{\sqrt{3c}e^{-ax}}{a} \right)$$

But

$$u'(x) = C_1 a \exp(ax) \operatorname{BesselJ}\left(-1/2 \frac{\sqrt{4 a^2 - 3 b}}{a}, 1/2 \frac{\sqrt{3} \sqrt{c} \exp(-ax)}{a}\right) \\ - 1/2 C_1 \exp(ax) \left(- \operatorname{BesselJ}\left(-1/2 \frac{\sqrt{4 a^2 - 3 b}}{a} + 1, 1/2 \frac{\sqrt{3} \sqrt{c} \exp(-ax)}{a}\right) - 1/3 \frac{\sqrt{3} \sqrt{4 a^2 - 3 b}}{\sqrt{c} \exp(-ax)} \operatorname{BesselJ}\left(-1/2 \frac{\sqrt{4 a^2 - 3 b}}{a}, 1/2 \frac{\sqrt{3} \sqrt{c} e^{-ax}}{a}\right) \right) \sqrt{3} \sqrt{c} \\ + C_2 a \exp(ax) \operatorname{BesselY}\left(-1/2 \frac{\sqrt{4 a^2 - 3 b}}{a}, 1/2 \frac{\sqrt{3} \sqrt{c} \exp(-ax)}{a}\right) \\ - 1/2 C_1 \exp(ax) \left(- \operatorname{BesselY}\left(-1/2 \frac{\sqrt{4 a^2 - 3 b}}{a} + 1, 1/2 \frac{\sqrt{3} \sqrt{c} \exp(-ax)}{a}\right) - 1/3 \frac{\sqrt{3} \sqrt{4 a^2 - 3 b}}{\sqrt{c} \exp(-ax)} \operatorname{BesselY}\left(-1/2 \frac{\sqrt{4 a^2 - 3 b}}{a}, 1/2 \frac{\sqrt{3} \sqrt{c} e^{-ax}}{a}\right) \right) \sqrt{3} \sqrt{c}$$

Hence from $y = \frac{-2}{3} \frac{u'}{u}$ the solution is now found.

Verification

```
ode:=2*diff(y(x),x)-3*y(x)^2-4*a*y(x)=b+c*exp(-2*a*x);
uode:=diff(u(x),x$2)-2*a*diff(u(x),x)+3/4*(b+c*(exp(-2*a*x)))*u(x)=0;
uSol:=dsolve(uode,u(x));
my_sol:=(-2/3)*diff(rhs(uSol),x)/rhs(uSol);
odetest(y(x)=my_sol,ode);
0
```

2.89

ODE No. 89

$$xy'(x) - \sqrt{a^2 - x^2} = 0$$

✓ **Mathematica** : cpu = 0.0136913 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow \sqrt{a^2 - x^2} - a \tanh^{-1}\left(\frac{\sqrt{a^2 - x^2}}{a}\right) + c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 56

$$\left\{ y(x) = -\frac{a^2 \ln\left(\frac{2a^2+2\sqrt{a^2} \sqrt{a^2-x^2}}{x}\right)}{\sqrt{a^2}} + c_1 + \sqrt{a^2 - x^2} \right\}$$

HAND SOLUTION

$$xy' = \pm \sqrt{a^2 - x^2}$$

This is separable. $y' = \frac{\pm \sqrt{a^2 - x^2}}{x}$ or $dy = \frac{\pm \sqrt{a^2 - x^2}}{x} dx$. Hence

$$y = \pm \int \frac{\sqrt{a^2 - x^2}}{x} dx + C$$

Let $x = a \sin u$, then $dx = a \cos(u) du$ and the integral becomes

$$\begin{aligned}
\int \frac{\sqrt{a^2 - x^2}}{x} dx &= \int \frac{\sqrt{a^2 - a^2 \sin^2 u}}{a \sin u} a \cos(u) du \\
&= \int \frac{a \sqrt{1 - \sin^2 u}}{a \sin u} a \cos(u) du \\
&= a \int \frac{\cos u}{\sin u} \cos(u) du \\
&= a \int \frac{\cos^2 u}{\sin u} du \\
&= a \int \frac{1 - \sin^2 u}{\sin u} du \\
&= a \left(\int \frac{1}{\sin u} du - \int \sin u du \right) \\
&= a \left(\int \frac{1}{\sin u} du + \cos u \right)
\end{aligned} \tag{1}$$

For $\int \frac{1}{\sin u} du$, using half tan angle, let $t = \tan\left(\frac{u}{2}\right)$, $du = \frac{2}{1+t^2} dt$, $\sin u = \frac{2t}{1+t^2}$, therefore

$$\begin{aligned}
\int \frac{1}{\sin u} du &= \int \frac{1+t^2}{2t} \frac{2}{1+t^2} dt \\
&= \int \frac{1}{t} dt \\
&= \ln(t)
\end{aligned}$$

Hence $\int \frac{1}{\sin u} du = \ln\left(\tan\left(\frac{u}{2}\right)\right)$ and from (1)

$$\begin{aligned}
\int \frac{\sqrt{a^2 - x^2}}{x} dx &= a \left(\int \frac{1}{\sin u} du + \cos u \right) \\
&= a \left(\ln\left(\tan\left(\frac{u}{2}\right)\right) + \cos u \right)
\end{aligned}$$

But $x = a \sin u$, hence $u = \arcsin\left(\frac{x}{a}\right)$ and the integral becomes

$$\int \frac{\sqrt{a^2 - x^2}}{x} dx = a \left[\ln\left(\tan\left(\frac{\arcsin\left(\frac{x}{a}\right)}{2}\right)\right) + \cos\left(\arcsin\left(\frac{x}{a}\right)\right) \right]$$

Hence the solution is

$$y = \pm a \left[\ln\left(\tan\left(\frac{\arcsin\left(\frac{x}{a}\right)}{2}\right)\right) + \cos\left(\arcsin\left(\frac{x}{a}\right)\right) \right] + C$$

Maple do not verify the above, but I do not see what is wrong with the solution. Will investigate more later.

2.90 ODE No. 90

$$xy'(x) + y(x) - x \sin(x) = 0$$

✓ **Mathematica** : cpu = 0.0244245 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow \frac{\sin(x) - x \cos(x)}{x} + \frac{c_1}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 17

$$\left\{ y(x) = \frac{-x \cos(x) + c_1 + \sin(x)}{x} \right\}$$

HAND SOLUTION

$$xy' + y = x \sin x$$

Linear first order, exact, separable. $y' + \frac{y}{x} = \sin x$, integrating factor $\mu = e^{\int \frac{1}{x} dx} = x$, hence

$$\begin{aligned} d(\mu y) &= \mu \sin x \\ xy &= \int x \sin x dx + C \end{aligned}$$

Using integration by parts. $\int u dv = uv - \int v du$. Let $u = x, dv = \sin x$, hence $du = 1, v = -\cos x$, therefore

$$\begin{aligned} \int x \sin x dx &= -x \cos x + \int \cos x \\ &= -x \cos x + \sin x \end{aligned}$$

Hence

$$\begin{aligned} xy &= -x \cos x + \sin x + C \\ y &= \frac{\sin x}{x} - \cos x + \frac{C}{x} \end{aligned}$$

Verification

```
restart;
ode:=x*diff(y(x),x)+y(x)=x*sin(x);
my_sol:=sin(x)/x-cos(x)+_C1/x;
odetest(y(x)=my_sol,ode);
0
```

2.91 ODE No. 91

$$xy'(x) - y(x) - \frac{x}{\log(x)} = 0$$

✓ **Mathematica** : cpu = 0.0249662 (sec), leaf count = 15

$$\{ \{ y(x) \rightarrow x \log(\log(x)) + c_1 x \} \}$$

✓ **Maple** : cpu = 0.008 (sec), leaf count = 11

$$\{ y(x) = (c_1 + \ln(\ln(x)))x \}$$

2.92 ODE No. 92

$$x^2(-\sin(x)) + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0237564 (sec), leaf count = 15

$$\{ \{ y(x) \rightarrow -x \cos(x) + c_1 x \} \}$$

✓ **Maple** : cpu = 0.006 (sec), leaf count = 12

$$\{ y(x) = (c_1 - \cos(x))x \}$$

HAND SOLUTION

$$xy' - y = x^2 \sin x$$

Linear first order, exact, separable. $y' - \frac{y}{x} = x \sin x$, integrating factor $\mu = e^{\int -\frac{1}{x} dx} = e^{-\ln x} = \frac{1}{x}$, hence

$$\begin{aligned} d(\mu y) &= \mu \sin x \\ \frac{1}{x} y &= \int \sin x dx + C \\ y &= x(C - \cos x) \end{aligned}$$

Verification

```
restart;
ode:=x*diff(y(x),x)-y(x)=x^2*sin(x);
my_sol:=x*(C1-cos(x));
odetest(y(x)=my_sol,ode);
0
```

2.93 ODE No. 93

$$xy'(x) - y(x) - \frac{x \cos(\log(\log(x)))}{\log(x)} = 0$$

✓ **Mathematica** : cpu = 0.0516383 (sec), leaf count = 16

$$\{ \{ y(x) \rightarrow x \sin(\log(\log(x))) + c_1 x \} \}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 12

$$\{ y(x) = (c_1 + \sin(\ln(\ln(x))))x \}$$

2.94 ODE No. 94

$$ay(x) + bx^n + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.042067 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow -\frac{bx^n}{a+n} + c_1 x^{-a} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 23

$$\left\{ y(x) = c_1 x^{-a} - \frac{bx^n}{a+n} \right\}$$

HAND SOLUTION

$$xy' + ay + bx^n = 0$$

Linear first order, exact, separable. $y' + \frac{ay}{x} = -bx^{n-1}$, integrating factor $\mu = e^{\int \frac{a}{x} dx} = e^{a \ln x} = x^a$, hence

$$\begin{aligned} d(\mu y) &= -\mu bx^{n-1} \\ x^a y &= - \int bx^{a+n-1} + C \end{aligned}$$

If $a = -n$ then

$$\begin{aligned} x^a y &= - \int bx^{-1} + C \\ y &= -x^{-a} b \ln(x) + x^{-a} C \\ &= x^{-a} (C - b \ln x) \end{aligned}$$

If $a \neq -n$ then

$$x^a y = -\frac{bx^{a+n}}{a+n} + C$$
$$y = -b\frac{x^n}{a+n} + Cx^{-a}$$

Verification

```
restart;
ode:=x*diff(y(x),x)+a*y(x)+b*x^n=0;
s1:=x^(-a)*(_C1-b*ln(x));
s2:=-b*(x^n/(a+n))+_C1*x^(-a);
odetest(y(x)=s2,ode);
0
```

2.95 ODE No. 95

$$x^2 + xy'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0818114 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow \frac{x(-Y_1(x) - c_1 J_1(x))}{Y_0(x) + c_1 J_0(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.071 (sec), leaf count = 27

$$\left\{ y(x) = -\frac{(c_1 \text{BesselY}(1, x) + \text{BesselJ}(1, x))x}{c_1 \text{BesselY}(0, x) + \text{BesselJ}(0, x)} \right\}$$

HAND SOLUTION

$$xy' + y^2 + x^2 = 0$$

This is Riccati first order non-linear. Writing it in standard form and for $x \neq 0$

$$y' = -x - \frac{1}{x}y^2$$
$$= f_0 + f_1y + f_2y^2$$

(1)

Where $f_0 = -x, f_1 = 0, f_2 = -\frac{1}{x}$. Using standard substitution $y = \frac{-u'}{uf_2}$ changes the ODE to second order linear ODE

$$y = \frac{xu'}{u}$$

(2)

Hence

$$y' = \frac{u'}{u} + x\frac{u''}{u} - \frac{x(u')^2}{u^2}$$

Equating this to RHS of (1) gives

$$\frac{u'}{u} + x\frac{u''}{u} - \frac{x(u')^2}{u^2} = -x - \frac{1}{x}\left(\frac{xu'}{u}\right)^2$$
$$\frac{u'}{u} + x\frac{u''}{u} = -x$$
$$u'' + \frac{1}{x}u' + u = 0$$

This is Lienard ODE. Since it is not constant coefficient ODE, the solution will be in Bessel functions, using Power series method. The solution is

$$u = C_1 \operatorname{BesselJ}(0, x) + C_2 \operatorname{BesselY}(0, x)$$

But $\frac{d}{dx} \operatorname{BesselJ}(0, x) = -\operatorname{BesselJ}(1, x)$ and $\frac{d}{dx} \operatorname{BesselY}(0, x) = -\operatorname{BesselY}(1, x)$, hence

$$u'(x) = -C_1 \operatorname{BesselJ}(1, x) - C_2 \operatorname{BesselY}(1, x)$$

And from (2) the solution is

$$\begin{aligned} y &= \frac{xu'}{u} \\ &= x \frac{[-C_1 \operatorname{BesselJ}(1, x) - C_2 \operatorname{BesselY}(1, x)]}{C_1 \operatorname{BesselJ}(0, x) + C_2 \operatorname{BesselY}(0, x)} \\ &= -x \frac{C_1 \operatorname{BesselJ}(1, x) + C_2 \operatorname{BesselY}(1, x)}{C_1 \operatorname{BesselJ}(0, x) + C_2 \operatorname{BesselY}(0, x)} \end{aligned}$$

Let $C = \frac{C_1}{C_2}$ then

$$y = -x \frac{C \operatorname{BesselJ}(1, x) + \operatorname{BesselY}(1, x)}{C \operatorname{BesselJ}(0, x) + \operatorname{BesselY}(0, x)}$$

Verification

```
restart;
ode:=x*diff(y(x),x)+y(x)^2+x^2=0;
my_sol:=-x*(C1*BesselJ(1, x)+BesselY(1,x))/(C1*BesselJ(0, x)+BesselY(0,x));
odetest(y(x)=my_sol,ode);
0
```

2.96 ODE No. 96

$$xy'(x) - y(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 0.0727398 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow \frac{1 - e^{2c_1 x^2}}{1 + e^{2c_1 x^2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 11

$$\{y(x) = -\tanh(c_1 + \ln(x))\}$$

HAND SOLUTION

$$xy' - y^2 + 1 = 0$$

This is Riccati first order non-linear. But it is separable. Hence

$$y' = \frac{y^2 - 1}{x} \tag{1}$$

Hence

$$\begin{aligned} \frac{dy}{dx} &= \frac{y^2 - 1}{x} \\ \frac{dy}{y^2 - 1} &= \frac{dx}{x} \end{aligned}$$

Integrating

$$\begin{aligned} -\tanh^{-1}(y) &= \ln x + C \\ y &= -\tanh(\ln x + C) \end{aligned}$$

Verification

```
restart;
ode:=x*diff(y(x),x)-y(x)^2+1=0;
my_sol:=-tanh(ln(x)+_C1);
odetest(y(x)=my_sol,ode);
0
```

2.97 ODE No. 97

$$ay(x)^2 + bx^2 + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.119685 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{b} x \tan\left(\sqrt{a} \sqrt{b} x - \sqrt{a} \sqrt{b} c_1\right)}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 25

$$\left\{ y(x) = -\frac{\sqrt{ab} x \tan\left(\sqrt{ab} (c_1 + x)\right)}{a} \right\}$$

HAND SOLUTION

$$ay^2 + bx^2 + xy' - y = 0$$

This is Riccati first order non-linear. Let $y = ux$, hence the above becomes

$$\begin{aligned} au^2x^2 + bx^2 + x(u'x + u) - ux &= 0 \\ au^2x + bx + u'x &= 0 \\ au^2 + b + u' &= 0 \\ u' &= -au^2 - b \end{aligned}$$

Which is separable, Hence

$$\frac{du}{au^2 + b} = -dx$$

Integrating

$$\begin{aligned} \frac{1}{\sqrt{ab}} \arctan\left(\frac{au}{\sqrt{ab}}\right) &= -x + C \\ \frac{au}{\sqrt{ab}} &= \tan\left(\sqrt{ab}(-x + C)\right) \\ u &= \frac{\sqrt{ab}}{a} \tan\left(\sqrt{ab}(-x + C)\right) \end{aligned}$$

Therefore

$$\begin{aligned} y &= ux \\ &= x \frac{\sqrt{ab}}{a} \tan\left(\sqrt{ab}(-x + C)\right) \end{aligned}$$

Verification

```
restart;
ode:=a*y(x)^2+b*x^2+x*diff(y(x),x)-y(x)=0;
my_sol:=x*sqrt(a*b)/a*tan(sqrt(a*b)*(-x+_C1));
odetest(y(x)=my_sol,ode);
0
```

2.98 ODE No. 98

$ay(x)^2 + cx^{2b} - by(x) + xy'(x) = 0$

✓ **Mathematica** : cpu = 0.204037 (sec), leaf count = 442

$$\left\{ \left\{ y(x) \rightarrow - \frac{\sqrt{-a} \sqrt{-c} x^b \left(- \frac{2 \sqrt{\frac{2}{\pi}} \cos\left(\frac{\sqrt{-a} \sqrt{-c} x^b}{b}\right)}{\sqrt{\frac{\sqrt{-a} \sqrt{-c} x^b}{b}}} + \frac{\sqrt{\frac{2}{\pi}} c_1 \sin\left(\frac{\sqrt{-a} \sqrt{-c} x^b}{b}\right)}{\sqrt{\frac{\sqrt{-a} \sqrt{-c} x^b}{b}}} - \frac{\sqrt{\frac{2}{\pi}} c_1 \left(- \sin\left(\frac{\sqrt{-a} \sqrt{-c} x^b}{b}\right) - \frac{\sqrt{-a} b \sqrt{-c} x^{-b} \cos\left(\frac{\sqrt{-a} \sqrt{-c} x^b}{b}\right)}{ac} \right)}{\sqrt{\frac{\sqrt{-a} \sqrt{-c} x^b}{b}}} \right) - \sqrt{\frac{2}{\pi}}}{2a \left(\frac{\sqrt{\frac{2}{\pi}} \sin\left(\frac{\sqrt{-a} \sqrt{-c} x^b}{b}\right)}{\sqrt{\frac{\sqrt{-a} \sqrt{-c} x^b}{b}}} + \frac{\sqrt{\frac{2}{\pi}} c_1 \cos\left(\frac{\sqrt{-a} \sqrt{-c} x^b}{b}\right)}{\sqrt{\frac{\sqrt{-a} \sqrt{-c} x^b}{b}}} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 38

$$\left\{ y(x) = - \frac{\sqrt{c} x^b \tan\left(\frac{c_1 b + \sqrt{a} \sqrt{c} x^b}{b}\right)}{\sqrt{a}} \right\}$$

2.99 ODE No. 99

$ay(x)^2 - by(x) - cx^\beta + xy'(x) = 0$

✓ **Mathematica** : cpu = 0.194934 (sec), leaf count = 244

$$\left\{ \left\{ y(x) \rightarrow - \frac{\sqrt{-a} \sqrt{c} x^{\beta/2} \left(- 2 J_{\frac{b}{\beta} - 1} \left(\frac{2 \sqrt{-a} \sqrt{c} x^{\beta/2}}{\beta} \right) + c_1 J_{1 - \frac{b}{\beta}} \left(\frac{2 \sqrt{-a} \sqrt{c} x^{\beta/2}}{\beta} \right) - c_1 J_{-\frac{b+\beta}{\beta}} \left(\frac{2 \sqrt{-a} \sqrt{c} x^{\beta/2}}{\beta} \right) \right) - b c_1 J_{-\frac{b}{\beta}} \left(\frac{2 \sqrt{-a} \sqrt{c} x^{\beta/2}}{\beta} \right)}{2a \left(J_{\frac{b}{\beta}} \left(\frac{2 \sqrt{-a} \sqrt{c} x^{\beta/2}}{\beta} \right) + c_1 J_{-\frac{b}{\beta}} \left(\frac{2 \sqrt{-a} \sqrt{c} x^{\beta/2}}{\beta} \right) \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.111 (sec), leaf count = 171

$$\left\{ y(x) = \frac{\left(c_1 \operatorname{BesselY}\left(\frac{b}{\beta}, \frac{2 \sqrt{-ac} x^{\frac{\beta}{2}}}{\beta}\right) + \operatorname{BesselJ}\left(\frac{b}{\beta}, \frac{2 \sqrt{-ac} x^{\frac{\beta}{2}}}{\beta}\right) \right) b - \left(c_1 \operatorname{BesselY}\left(\frac{b+\beta}{\beta}, \frac{2 \sqrt{-ac} x^{\frac{\beta}{2}}}{\beta}\right) + \operatorname{BesselJ}\left(\frac{b+\beta}{\beta}, \frac{2 \sqrt{-ac} x^{\frac{\beta}{2}}}{\beta}\right) \right) \sqrt{-a}}{\left(c_1 \operatorname{BesselY}\left(\frac{b}{\beta}, \frac{2 \sqrt{-ac} x^{\frac{\beta}{2}}}{\beta}\right) + \operatorname{BesselJ}\left(\frac{b}{\beta}, \frac{2 \sqrt{-ac} x^{\frac{\beta}{2}}}{\beta}\right) \right) a}$$

2.100 ODE No. 100

$a + xy'(x) + xy(x)^2 = 0$

✓ **Mathematica** : cpu = 0.0920759 (sec), leaf count = 157

$$\left\{ \left\{ y(x) \rightarrow - \frac{c_1 J_1(2i \sqrt{-a} \sqrt{x}) + i \sqrt{-a} \sqrt{x} \left(- 2 J_0(2i \sqrt{-a} \sqrt{x}) + c_1 J_0(2i \sqrt{-a} \sqrt{x}) - c_1 J_2(2i \sqrt{-a} \sqrt{x}) \right)}{2x \left(J_1(2i \sqrt{-a} \sqrt{x}) - c_1 J_1(2i \sqrt{-a} \sqrt{x}) \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.079 (sec), leaf count = 59

$$\left\{ y(x) = \frac{(c_1 \operatorname{BesselJ}(0, 2\sqrt{a} \sqrt{x}) + \operatorname{BesselY}(0, 2\sqrt{a} \sqrt{x}))\sqrt{a}}{(c_1 \operatorname{BesselJ}(1, 2\sqrt{a} \sqrt{x}) + \operatorname{BesselY}(1, 2\sqrt{a} \sqrt{x}))\sqrt{x}} \right\}$$

HAND SOLUTION

$$\begin{aligned} xy' + xy^2 + a &= 0 \\ y' &= -\frac{a}{x} - y^2 \end{aligned}$$

This is Riccati first order non-linear. Let $y = -\frac{u'}{uR} = \frac{u'}{u}$, hence $y' = \frac{u''}{u} - \frac{(u')^2}{u^2}$. Equating this to RHS of the above gives

$$\begin{aligned} \frac{u''}{u} - \frac{(u')^2}{u^2} &= -\frac{a}{x} - \left(\frac{u'}{u}\right)^2 \\ \frac{u''}{u} &= -\frac{a}{x} \\ u'' + \frac{a}{x}u &= 0 \end{aligned}$$

This is linear second order, an Emden Fowler ODE, with removal singularity. Solved using power series method. The solution is

$$u = C_1\sqrt{x} \operatorname{BesselJ}(1, 2\sqrt{ax}) + C_2\sqrt{x} \operatorname{BesselY}(1, 2\sqrt{ax})$$

But

$$\frac{d}{dx} \operatorname{BesselJ}(1, 2\sqrt{ax}) = \frac{\sqrt{a}}{\sqrt{x}} \left(\operatorname{BesselJ}(0, 2\sqrt{ax}) - \frac{1}{2} \frac{1}{\sqrt{ax}} \operatorname{BesselJ}(1, 2\sqrt{ax}) \right)$$

And

$$\frac{d}{dx} \operatorname{BesselY}(1, 2\sqrt{ax}) = \frac{\sqrt{a}}{\sqrt{x}} \left(\operatorname{BesselY}(0, 2\sqrt{ax}) - \frac{1}{2} \frac{1}{\sqrt{ax}} \operatorname{BesselY}(1, 2\sqrt{ax}) \right)$$

Therefore,

$$\begin{aligned} u' &= C_1 \left(\frac{1}{2\sqrt{x}} \operatorname{BesselJ}(1, 2\sqrt{a} \sqrt{x}) + \sqrt{x} \frac{\sqrt{a}}{\sqrt{x}} \left(\operatorname{BesselJ}(0, 2\sqrt{a} \sqrt{x}) - \frac{1}{2} \frac{1}{\sqrt{ax}} \operatorname{BesselJ}(1, 2\sqrt{a} \sqrt{x}) \right) \right) \\ &\quad + C_2 \left(\frac{1}{2\sqrt{x}} \operatorname{BesselY}(1, 2\sqrt{a} \sqrt{x}) + \sqrt{x} \frac{\sqrt{a}}{\sqrt{x}} \left(\operatorname{BesselY}(0, 2\sqrt{a} \sqrt{x}) - \frac{1}{2} \frac{1}{\sqrt{ax}} \operatorname{BesselY}(1, 2\sqrt{a} \sqrt{x}) \right) \right) \end{aligned}$$

Which is simplified to

$$u' = C_1\sqrt{a} \operatorname{BesselJ}(0, 2\sqrt{a} \sqrt{x}) + C_2\sqrt{a} \operatorname{BesselY}(0, 2\sqrt{a} \sqrt{x})$$

Therefore, from $y = \frac{u'}{u}$, the solution is

$$y = \frac{C_1\sqrt{a} \operatorname{BesselJ}(0, 2\sqrt{a} \sqrt{x}) + C_2\sqrt{a} \operatorname{BesselY}(0, 2\sqrt{a} \sqrt{x})}{C_1\sqrt{x} \operatorname{BesselJ}(1, 2\sqrt{a} \sqrt{x}) + C_2\sqrt{x} \operatorname{BesselY}(1, 2\sqrt{a} \sqrt{x})}$$

Let $C = \frac{C_1}{C_2}$, hence

$$y = \frac{C\sqrt{a} \operatorname{BesselJ}(0, 2\sqrt{a} \sqrt{x}) + \sqrt{a} \operatorname{BesselY}(0, 2\sqrt{a} \sqrt{x})}{C\sqrt{x} \operatorname{BesselJ}(1, 2\sqrt{a} \sqrt{x}) + \sqrt{x} \operatorname{BesselY}(1, 2\sqrt{a} \sqrt{x})}$$

Verification

```
restart;
ode:=x*diff(y(x),x)+x*y(x)^2+a=0;
num:=-C1*sqrt(a)*BesselJ(0,2*sqrt(a)*sqrt(x))+sqrt(a)*BesselY(0,2*sqrt(a)*sqrt(x));
den:=-C1*sqrt(x)*BesselJ(1,2*sqrt(a)*sqrt(x))+sqrt(x)*BesselY(1,2*sqrt(a)*sqrt(x));
my_solution:=num/den;
odetest(y(x)=my_solution,ode);
0
```

2.101 ODE No. 101

$$xy'(x) + xy(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0902297 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow \frac{2x}{x^2 + 2c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.015 (sec), leaf count = 16

$$\left\{ y(x) = \frac{2x}{x^2 + 2c_1} \right\}$$

HAND SOLUTION

$$\begin{aligned} xy' + xy^2 - y &= 0 \\ y' &= \frac{1}{x}y - y^2 \end{aligned} \tag{1}$$

This is of the form $y' = f_0 + f_1y + f_2y^2$ with $f_0 = 0, f_1 = \frac{1}{x}, f_2 = -1$. Since $f_0 = 0$ this is Bernoulli differential equation. We always start by dividing by y^2

$$\frac{y'}{y^2} = \frac{1}{x} \frac{1}{y} - 1$$

Then $u = \frac{1}{y}$ or $y = \frac{1}{u}$, therefore $y' = -\frac{u'}{u^2}$. Equating this to RHS of (1) gives

$$\begin{aligned} -\frac{u'}{u^2}u^2 &= \frac{1}{x}u - 1 \\ -u' &= \frac{u}{x} - 1 \\ u' + \frac{u}{x} &= 1 \end{aligned}$$

Integrating factor is $e^{\int \frac{1}{x}dx} = x$ and the above becomes

$$d(xu) = x$$

Integrating

$$\begin{aligned} xu &= \frac{x^2}{2} + C \\ u &= \frac{x}{2} + \frac{C}{x} \\ &= \frac{x^2 + 2C}{2x} \end{aligned}$$

Hence

$$\begin{aligned} y &= \frac{1}{u} \\ &= \frac{2x}{x^2 + 2C} \end{aligned}$$

Verification

```
restart;
ode:=x*diff(y(x),x)+x*y(x)^2-y=0;
my_solution:=2*x/(x^2+2*_C1);
odetest(y(x)=my_solution,ode);
0
```


2.102 ODE No. 102

$$-ax^3 + xy'(x) + xy(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0997935 (sec), leaf count = 36

$$\left\{\left\{y(x) \rightarrow \sqrt{a} x \tanh\left(\frac{1}{2}\left(\sqrt{a} x^2 + 2\sqrt{a} c_1\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 22

$$\left\{y(x) = \sqrt{a} x \tanh\left(\frac{\left(x^2 + 2c_1\right)\sqrt{a}}{2}\right)\right\}$$

HAND SOLUTION

$$xy' + xy^2 - y - ax^3 = 0$$

This is Riccati non-linear first order. But using the transformation $y = xv$ it is transformed to easily solved ODE

$$y' = v + xv'$$

Therefore the ODE becomes

$$\begin{aligned} x(v + xv') + x(xv)^2 - xv - ax^3 &= 0 \\ xv + x^2v' + x^3v^2 - xv - ax^3 &= 0 \\ x^2v' + x^3v^2 - ax^3 &= 0 \\ v' + xv^2 - ax &= 0 \\ \frac{dv}{dx} &= x(a - v^2) \\ \frac{dv}{a - v^2} &= xdx \end{aligned}$$

Integrating

$$\begin{aligned} \frac{1}{\sqrt{a}} \tanh^{-1}\left(\frac{v}{\sqrt{a}}\right) &= \frac{x^2}{2} + C \\ \tanh^{-1}\left(\frac{v}{\sqrt{a}}\right) &= \sqrt{a}\left(\frac{x^2}{2} + C\right) \\ \frac{v}{\sqrt{a}} &= \tanh\left(\sqrt{a}\left(\frac{x^2}{2} + C\right)\right) \\ v &= \sqrt{a} \tanh\left(\sqrt{a}\left(\frac{x^2}{2} + C\right)\right) \end{aligned}$$

Therefore

$$\begin{aligned} y &= xv \\ &= x\sqrt{a} \tanh\left(\sqrt{a}\left(\frac{x^2}{2} + C\right)\right) \end{aligned}$$

Verification

```
restart;
ode:=x*diff(y(x),x)+x*y(x)^2-y(x)-a*x^3=0;
my_solution:=x*sqrt(a)*tanh(sqrt(a)*(x^2/2+_C1));
odetest(y(x)=my_solution,ode);
0
```

2.103 ODE No. 103

$$-x^3 - (2x^2 + 1)y(x) + xy'(x) + xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.175563 (sec), leaf count = 90

$$\left\{ \left\{ y(x) \rightarrow \frac{x \left(e^{\sqrt{2}x^2} + \sqrt{2} e^{\sqrt{2}x^2} + e^{2\sqrt{2}c_1} - \sqrt{2} e^{2\sqrt{2}c_1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 29

$$\left\{ y(x) = \frac{\left(2 \tanh\left(\frac{(x^2+2c_1)\sqrt{2}}{2} \right) + \sqrt{2} \right) \sqrt{2} x}{2} \right\}$$

HAND SOLUTION

$$xy' - xy^2 - (2x^2 + 1)y - x^3 = 0$$

This is Riccati non-linear first order. Converting it to standard form

$$\begin{aligned} y' &= x^2 + \frac{(2x^2 + 1)}{x}y + y^2 \\ &= f_0 + f_1y + f_2y^2 \end{aligned} \quad (1)$$

Using standard transformation $y = -\frac{u'}{uf_2} = -\frac{u'}{u}$, therefore

$$y' = -\frac{u''}{u} + \frac{(u')^2}{u^2}$$

Equating the above to RHS of (1) gives

$$\begin{aligned} -\frac{u''}{u} + \frac{(u')^2}{u^2} &= x^2 - \frac{(2x^2 + 1)}{x} \frac{u'}{u} + \left(-\frac{u'}{u} \right)^2 \\ -\frac{u''}{u} &= x^2 - \frac{(2x^2 + 1)}{x} \frac{u'}{u} \\ -u'' &= ux^2 - \frac{(2x^2 + 1)}{x} u' \\ -u'' + \frac{(2x^2 + 1)}{x} u' - ux^2 &= 0 \\ xu'' - (2x^2 + 1)u' + ux^3 &= 0 \end{aligned} \quad (2)$$

This is second order linear ODE (Sturm-Liouville). Using the transformation $t = \frac{x^2}{2}$, then $\frac{dt}{dx} = x$ and

$$\frac{du}{dx} = \frac{du}{dt} \frac{dt}{dx} = x \frac{du}{dt} = \sqrt{2t} \frac{du}{dt}$$

And

$$\begin{aligned} \frac{d^2u}{dx^2} &= \frac{d}{dx} \left(\frac{du}{dx} \right) \\ &= \frac{d}{dx} \left(x \frac{du}{dt} \right) \\ &= \frac{du}{dt} + x \frac{d^2u}{dt^2} \frac{dt}{dx} \\ &= \frac{du}{dt} + x \frac{d^2u}{dt^2} x \\ &= \frac{du}{dt} + x^2 \frac{d^2u}{dt^2} \\ &= \frac{du}{dt} + 2t \frac{d^2u}{dt^2} \end{aligned}$$

Hence (2) can be written as

$$\begin{aligned}\sqrt{2t}\left(\frac{du}{dt} + 2t\frac{d^2u}{dt^2}\right) - (2(2t) + 1)\sqrt{2t}\frac{du}{dt} + u(2t)^{\frac{3}{2}} &= 0 \\ \sqrt{2t}\frac{du}{dt} + \sqrt{2t}2t\frac{d^2u}{dt^2} - (4t + 1)\sqrt{2t}\frac{du}{dt} + u(\sqrt{2t})^3 &= 0 \\ \frac{du}{dt} + 2t\frac{d^2u}{dt^2} - (4t + 1)\frac{du}{dt} + 2tu &= 0 \\ \frac{du}{dt} + 2t\frac{d^2u}{dt^2} - 4t\frac{du}{dt} - \frac{du}{dt} + 2tu &= 0 \\ 2t\frac{d^2u}{dt^2} - 4t\frac{du}{dt} + 2tu &= 0 \\ 2t\left(\frac{d^2u}{dt^2} - 2\frac{du}{dt} + u\right) &= 0\end{aligned}$$

Hence

$$\frac{d^2u}{dt^2} - 2\frac{du}{dt} + u = 0$$

This is linear second order with constant coefficients. The indicial equation is $\lambda^2 - 2\lambda + 1 = 0$ with roots $\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{4 - 4}}{2} = 1$ double root. Hence

$$u(t) = Ae^t + tBe^t$$

Since $t = \frac{x^2}{2}$ then

$$u(x) = Ae^{\frac{x^2}{2}} + \frac{x^2}{2}Be^{\frac{x^2}{2}}$$

But $y = -\frac{u'}{u}$ therefore

$$\begin{aligned}u' &= Axe^{\frac{x^2}{2}} + \left(xBe^{\frac{x^2}{2}} + \frac{x^2}{2}xBe^{\frac{x^2}{2}}\right) \\ &= Axe^{\frac{x^2}{2}} + xBe^{\frac{x^2}{2}} + \frac{x^3}{2}Be^{\frac{x^2}{2}}\end{aligned}$$

Hence

$$y = -\frac{Axe^{\frac{x^2}{2}} + xBe^{\frac{x^2}{2}} + \frac{x^3}{2}Be^{\frac{x^2}{2}}}{Ae^{\frac{x^2}{2}} + \frac{x^2}{2}Be^{\frac{x^2}{2}}} = -\frac{Axe^{\frac{x^2}{2}} + B\left(xe^{\frac{x^2}{2}} + \frac{x^3}{2}e^{\frac{x^2}{2}}\right)}{Ae^{\frac{x^2}{2}} + \frac{x^2}{2}Be^{\frac{x^2}{2}}}$$

Let $C = \frac{A}{B}$

$$\begin{aligned}y &= -\frac{xe^{\frac{x^2}{2}}\left(C + 1 + \frac{x^2}{2}\right)}{e^{\frac{x^2}{2}}\left(C + \frac{x^2}{2}\right)} \\ &= -\frac{x\left(C + 1 + \frac{x^2}{2}\right)}{C + \frac{x^2}{2}} \\ &= -\frac{x(2C + 2 + x^2)}{2C + x^2} \\ &= -\frac{x(C_1 + 2 + x^2)}{C_1 + x^2}\end{aligned}$$

Verification

```
restart;
ode:=x*diff(y(x),x)-x*y(x)^2-(2*x^2+1)*y(x)-x^3;
my_solution:=- (x*(C1+2+x^2))/(C1+x^2);
odetest(y(x)=my_solution,ode);
0
```

2.104 ODE No. 104

$$axy(x)^2 + bx + xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.129029 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{ax} - \sqrt{\frac{b}{a}} \tan \left(ax \sqrt{\frac{b}{a}} - c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 63

$$\left\{ y(x) = -\frac{\frac{e^{-2i\sqrt{a}\sqrt{b}x}}{c_1 - \frac{ie^{-2i\sqrt{a}\sqrt{b}x}}{2\sqrt{a}\sqrt{b}}} - \frac{i\sqrt{a}\sqrt{b}x-1}{x}}{a} \right\}$$

HAND SOLUTION

$xy' + axy^2 + 2y + bx = 0$ This is Riccati non-linear first order. Converting it to standard form

$$\begin{aligned} y' &= -b - \frac{2}{x}y - ay^2 \\ &= f_0 + f_1y + f_2y^2 \end{aligned} \quad (1)$$

Using transformation suggested by Kamke $y = u(x) - \frac{1}{ax}$ then $y' = u' + \frac{1}{ax^2}$. Equating this to RHS of (1) gives

$$\begin{aligned} u' + \frac{1}{ax^2} &= -b - \frac{2}{x} \left(u - \frac{1}{ax} \right) - a \left(u - \frac{1}{ax} \right)^2 \\ &= -b - \frac{2}{x}u + \frac{2}{ax^2} - a \left(u^2 + \frac{1}{a^2x^2} - \frac{2u}{ax} \right) \\ &= -b - \frac{2}{x}u + \frac{2}{ax^2} - au^2 - \frac{1}{ax^2} + \frac{2u}{x} \end{aligned}$$

Hence

$$\begin{aligned} u' &= -b - au^2 \\ \frac{du}{dx} &= -b - au^2 \end{aligned}$$

This is separable

$$\frac{du}{b + au^2} = -dx$$

Integrating

$$\begin{aligned} \int \frac{du}{b + au^2} &= -x + C \\ \frac{1}{\sqrt{ba}} \arctan \left(\frac{au}{\sqrt{ba}} \right) &= -x + C \\ \arctan \left(\frac{au}{\sqrt{ba}} \right) &= -\sqrt{ba}x + C \\ u &= \frac{\sqrt{ba}}{a} \tan \left(-\sqrt{ba}x + C \right) \end{aligned}$$

Hence

$$\begin{aligned} y &= u - \frac{1}{ax} \\ &= \frac{\sqrt{ba}}{a} \tan\left(-\sqrt{ba} x + C\right) - \frac{1}{ax} \end{aligned}$$

Verification

```
restart;
ode:=x*diff(y(x),x)+a*x*y(x)^2+2*y(x)+b*x = 0;
my_solution:=sqrt(b*a)/a*tan(-sqrt(b*a)*x+_C1)-1/(a*x);
odetest(y(x)=my_solution,ode);
0
```

2.105 ODE No. 105

$$axy(x)^2 + by(x) + cx + d + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.243652 (sec), leaf count = 473

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 \left(i \sqrt{a} e^{-i \sqrt{a} \sqrt{c} x} \left(b \left(-\sqrt{c} \right) - i \sqrt{a} d \right) U \left(1 - \frac{-\sqrt{c} b - i \sqrt{a} d}{2 \sqrt{c}}, b + 1, 2 i \sqrt{a} \sqrt{c} x \right) - i \sqrt{a} \sqrt{c} e^{-i \sqrt{a} \sqrt{c} x} U \left(-\frac{-\sqrt{c} b - i \sqrt{a} d}{2 \sqrt{c}}, b, 2 i \sqrt{a} \sqrt{c} x \right) + e^{-i \sqrt{a} \sqrt{c} x} \right)}{a \left(c_1 e^{-i \sqrt{a} \sqrt{c} x} U \left(-\frac{-\sqrt{c} b - i \sqrt{a} d}{2 \sqrt{c}}, b, 2 i \sqrt{a} \sqrt{c} x \right) + e^{-i \sqrt{a} \sqrt{c} x} \right)} \right. \right.$$

✓ **Maple** : cpu = 0.245 (sec), leaf count = 844

$$\left\{ y(x) = - \frac{4 \left(\left(ad + \sqrt{-ac} b \right) a c^3 \operatorname{KummerM} \left(\frac{ab c^2 + 2 \sqrt{-ac} a c d + (-ac)^{\frac{3}{2}} d}{2 a c^2}, \frac{\left((b+1) c + \sqrt{-ac} d \right) a c + (-ac)^{\frac{3}{2}} d}{a c^2}, 2 \sqrt{-ac} x \right) + \left(ad - \sqrt{-ac} b \right) a c^3 \operatorname{KummerM} \left(\frac{ab c^2 + 2 \sqrt{-ac} a c d + (-ac)^{\frac{3}{2}} d}{2 a c^2}, \frac{\left((b+1) c + \sqrt{-ac} d \right) a c + (-ac)^{\frac{3}{2}} d}{a c^2}, 2 \sqrt{-ac} x \right) + \left(bc - \sqrt{-ac} d \right) a^2 c^2 \operatorname{KummerM} \left(\frac{ab c^2 + 2 \sqrt{-ac} a c d + (-ac)^{\frac{3}{2}} d}{2 a c^2}, \frac{\left((b+1) c + \sqrt{-ac} d \right) a c + (-ac)^{\frac{3}{2}} d}{a c^2}, 2 \sqrt{-ac} x \right) + \left(bc + \sqrt{-ac} d \right) a^2 c^2 \operatorname{KummerM} \left(\frac{ab c^2 + 2 \sqrt{-ac} a c d + (-ac)^{\frac{3}{2}} d}{2 a c^2}, \frac{\left((b+1) c + \sqrt{-ac} d \right) a c + (-ac)^{\frac{3}{2}} d}{a c^2}, 2 \sqrt{-ac} x \right)}{4 \left(\left(bc - \sqrt{-ac} d \right) a^2 c^2 \operatorname{KummerM} \left(\frac{ab c^2 + 2 \sqrt{-ac} a c d + (-ac)^{\frac{3}{2}} d}{2 a c^2}, \frac{\left((b+1) c + \sqrt{-ac} d \right) a c + (-ac)^{\frac{3}{2}} d}{a c^2}, 2 \sqrt{-ac} x \right) + \left(bc + \sqrt{-ac} d \right) a^2 c^2 \operatorname{KummerM} \left(\frac{ab c^2 + 2 \sqrt{-ac} a c d + (-ac)^{\frac{3}{2}} d}{2 a c^2}, \frac{\left((b+1) c + \sqrt{-ac} d \right) a c + (-ac)^{\frac{3}{2}} d}{a c^2}, 2 \sqrt{-ac} x \right)} \right.$$

2.106 ODE No. 106

$$\frac{1}{2}(a - b)y(x) + x^a y(x)^2 + x^b + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.231483 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow -x^{\frac{b-a}{2}} \tan\left(\frac{2x^{\frac{a+b}{2}}}{a+b} - c_1\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 41

$$\left\{ y(x) = -x^{-\frac{a}{2} + \frac{b}{2}} \tan\left(\frac{c_1(a+b) + 2x^{\frac{a}{2} + \frac{b}{2}}}{a+b}\right) \right\}$$

2.107 ODE No. 107

$$ax^\alpha y(x)^2 + by(x) - cx^\beta + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.377808 (sec), leaf count = 1415

$$\left\{\left\{y(x) \rightarrow x^{1-\alpha}\left((-1)^{\frac{\alpha-b}{\alpha+\beta}} a^{\frac{\alpha-b}{\alpha+\beta}+\frac{1}{2}\left(\frac{b}{\alpha+\beta}-\frac{\alpha}{\alpha+\beta}\right)}(\alpha+\beta)^{\frac{\alpha}{\alpha+\beta}-\frac{b}{\alpha+\beta}+1}\left(\alpha^2+2 \beta \alpha+\beta^2\right)^{-\frac{\alpha-b}{\alpha+\beta}}\left(\frac{\alpha-b}{\alpha+\beta}+\frac{1}{2}\left(\frac{b}{\alpha+\beta}-\frac{\alpha}{\alpha+\beta}\right)\right) c^{\frac{\alpha-b}{\alpha+\beta}+\frac{1}{2}\left(\frac{b}{\alpha+\beta}-\frac{\alpha}{\alpha+\beta}\right)}\left(x^{\alpha}\right.\right.\right.\right.$$

✓ **Maple** : cpu = 0.271 (sec), leaf count = 174

$$\left\{y(x)=-\frac{\left(c_1 \operatorname{BesselY}\left(\frac{b+\beta}{\alpha+\beta}, \frac{2 \sqrt{-a c} x^{\frac{\alpha}{2}+\frac{\beta}{2}}\right)+\operatorname{BesselJ}\left(\frac{b+\beta}{\alpha+\beta}, \frac{2 \sqrt{-a c} x^{\frac{\alpha}{2}+\frac{\beta}{2}}\right)\right) \sqrt{-a c} x^{-\alpha+1} x^{\frac{\alpha}{2}+\frac{\beta}{2}}}{\left(c_1 \operatorname{BesselY}\left(\frac{-\alpha+b}{\alpha+\beta}, \frac{2 \sqrt{-a c} x^{\frac{\alpha}{2}+\frac{\beta}{2}}\right)+\operatorname{BesselJ}\left(\frac{-\alpha+b}{\alpha+\beta}, \frac{2 \sqrt{-a c} x^{\frac{\alpha}{2}+\frac{\beta}{2}}\right)\right) a x}\right\}$$

2.108 ODE No. 108

$$x y'(x)+y(x)+y(x)^2(-\log (x))=0$$

✓ **Mathematica** : cpu = 0.0826525 (sec), leaf count = 15

$$\left\{\left\{y(x) \rightarrow \frac{1}{\log (x)+c_1 x+1}\right\}\right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 13

$$\left\{y(x)=\frac{1}{c_1 x+\ln (x)+1}\right\}$$

HAND SOLUTION

$x y'+a x y^2+2 y+b x=0$ This is Riccati non-linear first order. Converting it to standard form

$$\begin{aligned} x y'-y^2 \ln x+y &=0 \\ y' &=-\frac{1}{x} y+y^2 \frac{\ln x}{x} \\ &=f_0+f_1 y+f_2 y^2 \end{aligned} \tag{1}$$

This is Bernoulli non-linear first order ODE since $f_0=0$. Dividing by y^2 gives

$$\frac{y'}{y^2}=-\frac{1}{x} \frac{1}{y}+\frac{\ln x}{x}$$

Let $u=\frac{1}{y}$, hence $u'=-\frac{y'}{y^2}$, and the above becomes

$$\begin{aligned} -u' &=-\frac{1}{x} u+\frac{\ln x}{x} \\ u'-\frac{1}{x} u &=-\frac{\ln x}{x} \end{aligned}$$

Integrating factor is $\mu=e^{\int-\frac{1}{x} d x}=e^{-\ln x}=\frac{1}{x}$, hence

$$d(\mu u)=-\mu \frac{\ln x}{x}$$

Integrating

$$\begin{aligned}\frac{1}{x}u &= -\int \frac{1}{x^2} \ln x dx + C \\ &= -\left(-\frac{\ln x}{x} - \frac{1}{x}\right) + C\end{aligned}$$

Therefore

$$u = \ln x + 1 + Cx$$

Since $u = \frac{1}{y}$ then

$$y = \frac{1}{\ln x + 1 + Cx}$$

Verification

```
restart;
ode:=x*diff(y(x),x)-y(x)^2*ln(x)+y(x)=0;
my_solution:=1/(ln(x)+1+_C1*x);
odetest(y(x)=my_solution,ode);
0
```

2.109 ODE No. 109

$$xy'(x) - y(x)(2y(x) \log(x) - 1) = 0$$

✓ **Mathematica** : cpu = 0.0814038 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2 \log(x) + c_1 x + 2} \right\} \right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 15

$$\left\{ y(x) = \frac{1}{c_1 x + 2 \ln(x) + 2} \right\}$$

HAND SOLUTION

$xy' + axy^2 + 2y + bx = 0$ This is Riccati non-linear first order. Converting it to standard form

$$\begin{aligned}xy' - y(2y \ln x - 1) &= 0 \\ xy' &= y(2y \ln x - 1) \\ y' &= -\frac{1}{x}y + y^2 \frac{2}{x} \ln x \\ y' &= f_0 + f_1 y + f_2 y^2\end{aligned}\tag{1}$$

This is Bernoulli non-linear first order ODE since $f_0 = 0$. Dividing by y^2 gives

$$\frac{y'}{y^2} = -\frac{1}{x} \frac{1}{y} + \frac{2}{x} \ln x$$

Putting $u = \frac{1}{y}$, hence $u' = -\frac{y'}{y^2}$, and the above becomes

$$\begin{aligned}-u' &= -\frac{1}{x}u + 2\frac{\ln x}{x} \\ -u' + \frac{1}{x}u &= 2\frac{\ln x}{x} \\ u' - \frac{1}{x}u &= -2\frac{\ln x}{x}\end{aligned}$$

Integrating factor is $\mu = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$, hence

$$\begin{aligned}d(\mu u) &= -2\mu \frac{\ln x}{x} \\d\left(\frac{1}{x}u\right) &= -2\frac{\ln x}{x^2}\end{aligned}$$

Integrating

$$\begin{aligned}\frac{1}{x}u &= -2 \int \frac{1}{x^2} \ln x dx + C \\&= -2\left(-\frac{\ln x}{x} - \frac{1}{x}\right) + C\end{aligned}$$

Therefore

$$\begin{aligned}u &= -2x\left(-\frac{\ln x}{x} - \frac{1}{x}\right) + Cx \\&= 2(\ln x + 1) + Cx\end{aligned}$$

Since $u = \frac{1}{y}$ then

$$y = \frac{1}{2(\ln x + 1) + Cx}$$

Verification

```
restart;
ode:=x*diff(y(x),x)-y(x)*(2*y(x)*ln(x)-1)=0;
my_solution:=1/(2*(ln(x)+1)+_C1*x);
odetest(y(x)=my_solution,ode);
0
```

2.110 ODE No. 110

$$f(x)(y(x)^2 - x^2) + xy'(x) = 0$$

✗ **Mathematica** : cpu = 13.4693 (sec), leaf count = 0 , could not solve

```
DSolve[f[x]*(-x^2 + y[x]^2) + x*Derivative[1][y][x] == 0, y[x], x]
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve(x*diff(y(x),x)+f(x)*(y(x)^2-x^2) = 0,y(x))
```

HAND SOLUTION

$xy' + axy^2 + 2y + bx = 0$ This is Riccati non-linear first order. Converting it to standard form

$$\begin{aligned}xy' + f(x)(y^2 - x^2) - y &= 0 \\xy' &= -f(y^2 - x^2) + y \\y' &= -\frac{f}{x}y^2 + fx + \frac{1}{x}y\end{aligned}\tag{1}$$

This is Riccati non-linear first order oder. There are two particular solutions $y_p = \pm x$. Using $y_p = x$, then using the transformation $y = y_p + \frac{1}{u}$, gives $y' = 1 - \frac{u'}{u^2}$ and (1) becomes

$$\begin{aligned}1 - \frac{u'}{u^2} &= -\frac{f}{x}\left(x + \frac{1}{u}\right)^2 + fx + \frac{1}{x}\left(x + \frac{1}{u}\right) \\&= -\frac{f}{x}\left(x^2 + \frac{1}{u^2} + 2\frac{x}{u}\right) + fx + \left(1 + \frac{1}{ux}\right) \\&= -fx - \frac{f}{x}\frac{1}{u^2} - 2\frac{f}{u} + fx + 1 + \frac{1}{ux} \\&= -\frac{f}{x}\frac{1}{u^2} - 2\frac{f}{u} + 1 + \frac{1}{ux}\end{aligned}$$

Hence

$$\begin{aligned}u^2 - u' &= -\frac{f}{x} - 2fu + u^2 + \frac{u}{x} \\-u' &= -\frac{f}{x} - 2fu + \frac{u}{x} \\-u' - \frac{u}{x} + 2fu &= \frac{-f}{x} \\u' + \frac{u}{x} - 2fu &= \frac{-f}{x} \\u' + u\left(\frac{1}{x} - 2f\right) &= \frac{-f}{x}\end{aligned}$$

Integrating factor is $\mu = e^{\int \frac{1}{x} - 2f} = e^{\ln x} e^{-2 \int f dx} = x e^{-2 \int f dx}$, hence

$$\begin{aligned}d(\mu u) &= -\mu \frac{f}{x} \\d\left(x e^{-2 \int f dx} u\right) &= -\left(x e^{-2 \int f dx}\right) \frac{f}{x} \\d\left(x e^{-2 \int f dx} u\right) &= -f\left(e^{-2 \int f dx}\right)\end{aligned}$$

Integrating

$$\begin{aligned}x e^{-2 \int f dx} u &= -\int f\left(e^{-2 \int f dx}\right) + C \\u &= -\frac{1}{x} e^{2 \int f dx} \int f\left(e^{-2 \int f dx}\right) + C \frac{1}{x} e^{2 \int f dx}\end{aligned}$$

Since $u = \frac{1}{y}$ then

$$\begin{aligned}y &= \frac{1}{-\frac{1}{x} e^{2 \int f dx} \int f\left(e^{-2 \int f dx}\right) + C \frac{1}{x} e^{2 \int f dx}} \\&= \frac{x e^{-2 \int f dx}}{-\int f e^{-2 \int f dx} dx + C}\end{aligned}$$

Verification (Maple does not verify it, need to look more into this)

```
ode:=x*diff(y(x),x)+f(x)*(y(x)^2-x^2) =0;
dsolve(ode,y(x));
fint:=Int(f(x),x);
my_solution:=x*exp(-2*fint)/(-Int(f*exp(-2*fint),x)+_C1);
odetest(y(x)=my_solution,ode);
not zero
```

2.111 ODE No. 111

$$xy'(x) + y(x)^3 + 3xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.508279 (sec), leaf count = 55

$$\text{Solve}\left[-3x = \frac{2e^{\frac{1}{2}\left(\frac{1}{y(x)}-3x\right)^2}}{\sqrt{2\pi} \operatorname{erfi}\left(\frac{\frac{1}{y(x)}-3x}{\sqrt{2}}\right)} + 2c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.128 (sec), leaf count = 54

$$\left\{c_1 + \frac{\sqrt{2} \sqrt{\pi} \operatorname{erf}\left(\frac{(3ixy(x)-i)\sqrt{2}}{2y(x)}\right)}{2} - \frac{ie^{\frac{(3xy(x)-1)^2}{2y(x)^2}}}{3x} = 0\right\}$$

2.112 ODE No. 112

$$-\sqrt{x^2 + y(x)^2} + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.124761 (sec), leaf count = 13

$$\{\{y(x) \rightarrow x \sinh(\log(x) + c_1)\}\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 27

$$\left\{-c_1 + \frac{y(x)}{x^2} + \frac{\sqrt{x^2 + y(x)^2}}{x^2} = 0\right\}$$

HAND SOLUTION

$$xy' = \sqrt{x^2 + y^2} + y$$

Let $y = xv$, then $y' = v + xv'$ and the above becomes

$$\begin{aligned} x(v + xv') &= \sqrt{x^2 + (xv)^2} + xv \\ x(v + xv') &= x\sqrt{1 + v^2} + xv \\ (v + xv') &= \sqrt{1 + v^2} + v \\ xv' &= \sqrt{1 + v^2} \end{aligned}$$

Separable.

$$\frac{dv}{\sqrt{1 + v^2}} = \frac{1}{x}dx$$

Integrating

$$\begin{aligned} \operatorname{arcsinh}(v) &= \ln x + C \\ v &= \sinh(\ln x + C) \end{aligned}$$

Since $y = xv$ then

$$y = x \sinh(\ln x + C)$$

Verification

```
ode:=x*diff(y(x),x)=sqrt(x^2+y(x)^2)+y(x);
y0:=x*sinh(ln(x)+_C1);
odetest(y(x)=y0,ode) assuming x>= 0;
0
```

2.113 ODE No. 113

$$a\sqrt{x^2 + y(x)^2} + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.124043 (sec), leaf count = 16

$$\{\{y(x) \rightarrow x \sinh(-a \log(x) + c_1)\}\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 33

$$\left\{-c_1 + \frac{x^a y(x)}{x} + \frac{\sqrt{x^2 + y(x)^2} x^a}{x} = 0\right\}$$

HAND SOLUTION

$$xy' = -a\sqrt{x^2 + y^2} + y$$

Let $y = xv$, then $y' = v + xv'$ and the above becomes

$$\begin{aligned} x(v + xv') &= -a\sqrt{x^2 + (xv)^2} + xv \\ x(v + xv') &= -ax\sqrt{1 + v^2} + xv \\ (v + xv') &= -a\sqrt{1 + v^2} + v \\ xv' &= -a\sqrt{1 + v^2} \end{aligned}$$

Separable.

$$\frac{dv}{\sqrt{1 + v^2}} = \frac{-a}{x}dx$$

Integrating

$$\begin{aligned} \operatorname{arcsinh}(v) &= -a \ln x + C \\ v &= \sinh(C - a \ln x) \end{aligned}$$

Since $y = xv$ then

$$y = x \sinh(C - a \ln x)$$

Verification

```
ode:=x*diff(y(x),x)=-a*sqrt(x^2+y(x)^2)+y(x);
y0:=x*sinh(_C1-a*ln(x));
odetest(y(x)=y0,ode) assuming x >=0;
0
```

2.114 ODE No. 114

$$-x\sqrt{x^2 + y(x)^2} + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.120998 (sec), leaf count = 12

$$\{\{y(x) \rightarrow x \sinh(x + c_1)\}\}$$

✓ **Maple** : cpu = 3.354 (sec), leaf count = 28

$$\left\{-c_1 - x - \ln(x) + \ln\left(y(x) + \sqrt{x^2 + y(x)^2}\right) = 0\right\}$$

HAND SOLUTION

$$xy' = x\sqrt{x^2 + y^2} + y$$

Let $y = xv$, then $y' = v + xv'$ and the above becomes

$$\begin{aligned} x(v + xv') &= x\sqrt{x^2 + (xv)^2} + xv \\ (v + xv') &= x\sqrt{1 + v^2} + v \\ xv' &= x\sqrt{1 + v^2} \\ v' &= \sqrt{1 + v^2} \end{aligned}$$

Separable.

$$\frac{dv}{\sqrt{1+v^2}} = dx$$

Integrating

$$\begin{aligned}\operatorname{arcsinh}(v) &= x + C \\ v &= \sinh(x + C)\end{aligned}$$

Since $y = xv$ then

$$y = x \sinh(x + C)$$

Verification

```
ode:=x*diff(y(x),x)=x*sqrt(x^2+y(x)^2)+y(x);
y0:=x*sinh(x+C1);
odetest(y(x)=y0,ode) assuming x>0;
0
```

2.115 ODE No. 115

$$-x(y(x)-x)\sqrt{x^2+y(x)^2}+xy'(x)-y(x)=0$$

✓ **Mathematica** : cpu = 0.30524 (sec), leaf count = 221

$$\left\{\left\{y(x) \rightarrow \frac{x-2\sqrt{x^2 \tanh^2\left(\frac{1}{2}(-\sqrt{2}x^2-2\sqrt{2}c_1)\right)}-x^2 \tanh^4\left(\frac{1}{2}(-\sqrt{2}x^2-2\sqrt{2}c_1)\right)}{-1+2 \tanh^2\left(\frac{1}{2}(-\sqrt{2}x^2-2\sqrt{2}c_1)\right)}\right\},\left\{y(x) \rightarrow \frac{x+2\sqrt{x^2 \tanh^2\left(\frac{1}{2}(-\sqrt{2}x^2-2\sqrt{2}c_1)\right)}}{-1+2 \tanh^2\left(\frac{1}{2}(-\sqrt{2}x^2-2\sqrt{2}c_1)\right)}\right\}\right.$$

✓ **Maple** : cpu = 0.221 (sec), leaf count = 49

$$\left\{\frac{\sqrt{2}x^2}{2}-c_1-\ln(x)+\ln\left(\frac{2\left(x+y(x)+\sqrt{2x^2+2y(x)^2}\right)x}{-x+y(x)}\right)=0\right\}$$

HAND SOLUTION

$$xy' = x(y-x)\sqrt{y^2-x^2} + y$$

Let $y = xu$, then $y' = u + xu'$ and the above becomes

$$\begin{aligned}x(u+xu') &= x(xu-x)\sqrt{(xu)^2-x^2} + xu \\ (u+xu') &= (xu-x)\sqrt{(xu)^2-x^2} + u \\ xu' &= (xu-x)x\sqrt{u^2-1} \\ u' &= x(u-1)\sqrt{u^2-1}\end{aligned}$$

Separable.

$$\begin{aligned}\frac{du}{(u-1)\sqrt{u^2-1}} &= xdx \\ \frac{-u-1}{\sqrt{u^2-1}} &= \frac{x^2}{2} + C\end{aligned}$$

But $y = xu$, hence

$$\frac{-\frac{y}{x}-1}{\sqrt{\left(\frac{y}{x}\right)^2-1}}=\frac{x^2}{2}+C$$

Let $\frac{y}{x}=z$

$$\begin{aligned}\frac{-z-1}{\sqrt{z^2-1}}&=\frac{x^2}{2}+C\\-z-1&=\sqrt{z^2-1}\left(\frac{x^2}{2}+C\right)\\(-z-1)^2&=\left(z^2-1\right)\left(\frac{x^2}{2}+C\right)^2\\z^2+1+2z&=z^2\left(\frac{x^2}{2}+C\right)^2-\left(\frac{x^2}{2}+C\right)^2\\z^2\left(1-\left(\frac{x^2}{2}+C\right)^2\right)+2z+1+\left(\frac{x^2}{2}+C\right)^2&=0\end{aligned}$$

Solving for z (quadratic formula, some conditions apply), one of the solutions is

$$z=\frac{4Cx^2+4C^2+x^4+4}{4Cx^2+4C^2+x^4-4}$$

Hence

$$y=x\frac{4Cx^2+4C^2+x^4+4}{4Cx^2+4C^2+x^4-4}$$

Need to work on verification. Kamke gives the final solution as

$$y=x\frac{-2Cx^2+C^2+x^4+4}{-2Cx^2+C^2+x^4-4}$$

I am not sure where my error now is. Need to look at this again.

2.116 ODE No. 116

$$-x\sqrt{\left(y(x)^2-4x^2\right)\left(y(x)^2-x^2\right)}+xy'(x)-y(x)=0$$

✓ **Mathematica** : cpu = 0.631941 (sec), leaf count = 121

$$\text{Solve}\left[\frac{\sqrt{\frac{\frac{y(x)}{x}+2}{\frac{y(x)}{x}-1}}\sqrt{\frac{\frac{y(x)}{x}+1}{\frac{2y(x)}{x}+4}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}\sqrt{\frac{\frac{y(x)}{x}-2}{\frac{y(x)}{x}-1}}\right)\middle|\frac{9}{8}\right)}{\sqrt{\frac{\frac{y(x)}{x}+1}{\frac{y(x)}{x}-1}}}=\frac{x^2}{2}+c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.23 (sec), leaf count = 86

$$\left\{c_1+\int^{y(x)}-\frac{b}{\sqrt{4_b^4-5_b^2_f^2+f^4}}d_f+\int_{-b}^x\frac{\sqrt{4_a^4-5_a^2y(x)^2+y(x)^4}-a+y(x)}{\sqrt{4_a^4-5_a^2y(x)^2+y(x)^4}}d_a=0\right\}$$

2.117 ODE No. 117

$$xy'(x) + x\left(-e^{\frac{y(x)}{x}}\right) - y(x) - x = 0$$

✓ **Mathematica** : cpu = 0.188868 (sec), leaf count = 21

$$\left\{\left\{y(x) \rightarrow -x \log\left(-1 + \frac{e^{-c_1}}{x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.116 (sec), leaf count = 20

$$\left\{y(x) = \left(c_1 + \ln\left(-\frac{x}{x e^{c_1} - 1}\right)\right)x\right\}$$

2.118 ODE No. 118

$$xy'(x) - y(x) \log(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.086483 (sec), leaf count = 13

$$\left\{\left\{y(x) \rightarrow e^{e^{c_1} x}\right\}\right\}$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 8

$$\{y(x) = e^{c_1 x}\}$$

2.119 ODE No. 119

$$xy'(x) - y(x)(\log(xy(x)) - 1) = 0$$

✓ **Mathematica** : cpu = 0.108007 (sec), leaf count = 17

$$\left\{\left\{y(x) \rightarrow \frac{e^{e^{c_1} x}}{x}\right\}\right\}$$

✓ **Maple** : cpu = 0.088 (sec), leaf count = 14

$$\left\{y(x) = \frac{e^{\frac{x}{c_1}}}{x}\right\}$$

2.120 ODE No. 120

$$xy'(x) - y(x)\left(x \log\left(\frac{x^2}{y(x)}\right) + 2\right) = 0$$

✓ **Mathematica** : cpu = 0.117749 (sec), leaf count = 20

$$\left\{\left\{y(x) \rightarrow x^2 e^{-2c_1 e^{-x}}\right\}\right\}$$

✓ **Maple** : cpu = 0.173 (sec), leaf count = 17

$$\{y(x) = x^2 e^{-c_1 e^{-x}}\}$$

2.121 ODE No. 121

$$xy'(x) - \sin(x - y(x)) = 0$$

✗ **Mathematica** : cpu = 2.16794 (sec), leaf count = 0 , could not solve

`DSolve[-Sin[x - y[x]] + x*Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(x*diff(y(x),x)-sin(x-y(x)) = 0,y(x))`

2.122 ODE No. 122

$$\cos(y(x))(\sin(y(x)) - 3x^2 \cos(y(x))) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.313522 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \tan^{-1} \left(\frac{2x^3 + c_1}{2x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.403 (sec), leaf count = 16

$$\left\{ y(x) = \arctan \left(\frac{x^3 + 2c_1}{x} \right) \right\}$$

2.123 ODE No. 123

$$xy'(x) - y(x) - x \sin \left(\frac{y(x)}{x} \right) = 0$$

✓ **Mathematica** : cpu = 0.134421 (sec), leaf count = 19

$$\left\{ \left\{ y(x) \rightarrow 2x \cot^{-1} \left(\frac{e^{-c_1}}{x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.079 (sec), leaf count = 44

$$\left\{ y(x) = x \arctan \left(\frac{2c_1 x}{c_1^2 x^2 + 1}, \frac{-c_1^2 x^2 + 1}{c_1^2 x^2 + 1} \right) \right\}$$

2.124 ODE No. 124

$$xy'(x) - y(x) + x \cos \left(\frac{y(x)}{x} \right) + x = 0$$

✓ **Mathematica** : cpu = 0.11128 (sec), leaf count = 16

$$\left\{ \left\{ y(x) \rightarrow 2x \tan^{-1}(-\log(x) + c_1) \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 12

$$\{y(x) = -2x \arctan(c_1 + \ln(x))\}$$

2.125 ODE No. 125

$$xy'(x) - y(x) + x \tan \left(\frac{y(x)}{x} \right) = 0$$

✓ **Mathematica** : cpu = 0.198305 (sec), leaf count = 16

$$\left\{ \left\{ y(x) \rightarrow x \sin^{-1} \left(\frac{e^{c_1}}{x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.086 (sec), leaf count = 14

$$\left\{ y(x) = x \arcsin \left(\frac{1}{c_1 x} \right) \right\}$$

2.126 ODE No. 126

$$xy'(x) - y(x)f(xy(x)) = 0$$

✓ **Mathematica** : cpu = 0.253086 (sec), leaf count = 115

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{1}{(-f(xK[2]) - 1)K[2]} - \int_1^x\left(\frac{f'(K[1]K[2])}{f(K[1]K[2]) + 1} - \frac{f(K[1]K[2])f'(K[1]K[2])}{(f(K[1]K[2]) + 1)^2}\right)dK[1]\right)dK[2] + \int_1^x \frac{f(K[1]y(x))}{(f(K[1]y(x)))}\right]$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 29

$$\left\{y(x) = \frac{\text{RootOf}\left(c_1 + \int_{-Z}^{\frac{1}{(f(-a)+1)-a}}d_{-a} - \ln(x)\right)}{x}\right\}$$

2.127 ODE No. 127

$$xy'(x) - y(x)f\left(x^ay(x)^b\right) = 0$$

✓ **Mathematica** : cpu = 0.317384 (sec), leaf count = 186

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{b}{\left(a + bf\left(x^aK[2]^b\right)\right)K[2]} - \int_1^x\left(\frac{b^2K[1]^{a-1}K[2]^{b-1}f'\left(K[1]^aK[2]^b\right)}{a + bf\left(K[1]^aK[2]^b\right)} - \frac{b^3f\left(K[1]^aK[2]^b\right)K[1]^{a-1}K[2]^{b-1}f'\left(K[1]^aK[2]^b\right)}{\left(a + bf\left(K[1]^aK[2]^b\right)\right)^2}\right)\right]$$

✓ **Maple** : cpu = 0.127 (sec), leaf count = 39

$$\left\{-c_1 + \int_{-b}^{y(x)} \frac{1}{\left(bf\left(-a^bx^a\right) + a\right)-a}d_{-a} - \frac{\ln(x)}{b} = 0\right\}$$

2.128 ODE No. 128

$$-f(x)g\left(x^ay(x)\right) + ay(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 3.40912 (sec), leaf count = 41

$$\text{Solve}\left[\int_1^{x^ay(x)} \frac{1}{g(K[1])}dK[1] = \int_1^x f(K[2])K[2]^{a-1}dK[2] + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.279 (sec), leaf count = 33

$$\left\{y(x) = x^{-a} \text{RootOf}\left(c_1 + \int_{-Z}^{\frac{1}{g(-a)}}d_{-a} - \left(\int x^{a-1}f(x)dx\right)\right)\right\}$$

2.129 ODE No. 129

$$(x + 1)y'(x) + y(x)(y(x) - x) = 0$$

✓ **Mathematica** : cpu = 0.207714 (sec), leaf count = 44

$$\left\{\left\{y(x) \rightarrow -\frac{e^{x+1}}{-x\text{Ei}(x + 1) - \text{Ei}(x + 1) + e^{x+1} - ec_1x - ec_1}\right\}\right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 33

$$\left\{y(x) = \frac{e^x}{c_1(x + 1) - e^{-1}(x + 1)\text{Ei}(1, -x - 1) - e^x}\right\}$$

2.130 ODE No. 130

$$-2x^3 + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0189719 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \frac{2x^3}{5} + c_1 \sqrt{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 15

$$\left\{ y(x) = \frac{2x^3}{5} + c_1 \sqrt{x} \right\}$$

2.131 ODE No. 131

$$(2x + 1)y'(x) - 4e^{-y(x)} + 2 = 0$$

✓ **Mathematica** : cpu = 0.23194 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \log \left(2 - \frac{e^{c_1}}{2x + 1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.193 (sec), leaf count = 31

$$\left\{ y(x) = -2c_1 - \ln \left(\frac{2x + 1}{(4x + 2)e^{2c_1} - 1} \right) \right\}$$

2.132 ODE No. 132

$$3xy'(x) - y(x) - 3xy(x)^4 \log(x) = 0$$

✓ **Mathematica** : cpu = 0.128854 (sec), leaf count = 115

$$\left\{ \left\{ y(x) \rightarrow \frac{(-2)^{2/3} \sqrt[3]{x}}{\sqrt[3]{3x^2 - 6x^2 \log(x) + 4c_1}} \right\}, \left\{ y(x) \rightarrow \frac{2^{2/3} \sqrt[3]{x}}{\sqrt[3]{3x^2 - 6x^2 \log(x) + 4c_1}} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt[3]{-1} 2^{2/3} \sqrt[3]{x}}{\sqrt[3]{3x^2 - 6x^2 \log(x) + 4c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 153

$$\left\{ y(x) = \frac{\left(-4(6x^2 \ln(x) - 3x^2 - 4c_1)^2 x \right)^{\frac{1}{3}}}{6x^2 \ln(x) - 3x^2 - 4c_1}, y(x) = \frac{\left(-4(6x^2 \ln(x) - 3x^2 - 4c_1)^2 x \right)^{\frac{1}{3}} (i\sqrt{3} - 1)}{12x^2 \ln(x) - 6x^2 - 8c_1}, y(x) = -\frac{\left(-4(6x^2 \ln(x) - 3x^2 - 4c_1)^2 x \right)^{\frac{1}{3}} (i\sqrt{3} + 1)}{12x^2 \ln(x) - 6x^2 - 8c_1} \right\}$$

2.133 ODE No. 133

$$x^2 y'(x) + y(x) - x = 0$$

✓ **Mathematica** : cpu = 0.017806 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow -e^{\frac{1}{x}} \text{Ei} \left(-\frac{1}{x} \right) + c_1 e^{\frac{1}{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.015 (sec), leaf count = 16

$$\left\{ y(x) = \left(c_1 + \text{Ei} \left(1, \frac{1}{x} \right) \right) e^{\frac{1}{x}} \right\}$$

2.134 ODE No. 134

$$x^2 y'(x) + e^{x^{-\frac{1}{x}}} x^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.069632 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow -e^{x^{-\frac{1}{x}}} + c_1 e^{-1/x} \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 17

$$\left\{ y(x) = (c_1 - e^x) e^{-\frac{1}{x}} \right\}$$

2.135 ODE No. 135

$$x^2 y'(x) - (x - 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0196931 (sec), leaf count = 14

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{x}} x \right\} \right\}$$

✓ **Maple** : cpu = 0.006 (sec), leaf count = 11

$$\left\{ y(x) = c_1 x e^{\frac{1}{x}} \right\}$$

2.136 ODE No. 136

$$x^2 y'(x) + x^2 + xy(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0964434 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow \frac{-x + x \log(x) - c_1 x}{-\log(x) + c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 18

$$\left\{ y(x) = -\frac{(c_1 + \ln(x) - 1)x}{c_1 + \ln(x)} \right\}$$

2.137 ODE No. 137

$$x^2 y'(x) - xy(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0992509 (sec), leaf count = 16

$$\left\{ \left\{ y(x) \rightarrow \frac{x}{-\log(x) + c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 14

$$\left\{ y(x) = \frac{x}{c_1 - \ln(x)} \right\}$$

2.138 ODE No. 138

$$x^2 y'(x) - x^2 - xy(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.124071 (sec), leaf count = 13

$$\{ \{ y(x) \rightarrow x \tan(\log(x) + c_1) \} \}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 11

$$\{ y(x) = x \tan(c_1 + \ln(x)) \}$$

2.139 ODE No. 139

$$ax^k - (b-1)b + x^2(y'(x) + y(x)^2) = 0$$

✓ **Mathematica** : cpu = 0.256101 (sec), leaf count = 821

$$\left\{ \left\{ y(x) \rightarrow - \frac{a^{\frac{b}{k} + \frac{1}{2} \left(\frac{1}{k} - \frac{2b}{k} \right)} \left(\frac{b}{k} + \frac{1}{2} \left(\frac{1}{k} - \frac{2b}{k} \right) \right) x^{k-1} (x^k)^{\frac{b}{k} + \frac{1}{2} \left(\frac{1}{k} - \frac{2b}{k} \right) - 1} J_{\frac{2b-1}{k}} \left(\frac{2\sqrt{a} \sqrt{x^k}}{k} \right) \Gamma \left(\frac{2b}{k} - \frac{1}{k} + 1 \right) k^{1 - \frac{1}{k}} + \frac{1}{2} a^{\frac{b}{k} + \frac{1}{2} \left(\frac{1}{k} - \frac{2b}{k} \right) + \frac{1}{2}} x^{k-1} (x^k)^{\frac{b}{k}}}{\dots} \right\} \right\}$$

✓ **Maple** : cpu = 0.135 (sec), leaf count = 219

$$\left\{ y(x) = \frac{-2c_1 \sqrt{a} x^{\frac{k}{2}} \text{BesselY} \left(\frac{k + \sqrt{(2b-1)^2}}{k}, \frac{2\sqrt{a} x^{\frac{k}{2}}}{k} \right) - 2\sqrt{a} x^{\frac{k}{2}} \text{BesselJ} \left(\frac{k + \sqrt{(2b-1)^2}}{k}, \frac{2\sqrt{a} x^{\frac{k}{2}}}{k} \right) + 2 \left(c_1 \text{BesselY} \left(\frac{\sqrt{(2b-1)^2}}{k}, \frac{2\sqrt{a} x^{\frac{k}{2}}}{k} \right) + \text{BesselJ} \left(\frac{\sqrt{(2b-1)^2}}{k}, \frac{2\sqrt{a} x^{\frac{k}{2}}}{k} \right) \right)}{2 \left(c_1 \text{BesselY} \left(\frac{\sqrt{(2b-1)^2}}{k}, \frac{2\sqrt{a} x^{\frac{k}{2}}}{k} \right) + \text{BesselJ} \left(\frac{\sqrt{(2b-1)^2}}{k}, \frac{2\sqrt{a} x^{\frac{k}{2}}}{k} \right) \right)}$$

2.140 ODE No. 140

$$x^2(y'(x) + y(x)^2) + 4xy(x) + 2 = 0$$

✓ **Mathematica** : cpu = 0.129566 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow -\frac{2}{x} + \frac{1}{x + c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.112 (sec), leaf count = 20

$$\left\{ y(x) = \frac{-2c_1 + x}{(c_1 - x)x} \right\}$$

2.141 ODE No. 141

$$axy(x) + b + x^2(y'(x) + y(x)^2) = 0$$

✓ **Mathematica** : cpu = 0.169393 (sec), leaf count = 67

$$\left\{ \left\{ y(x) \rightarrow -\frac{a}{2x} - \frac{-1 + \sqrt{a^2 - 2a - 4b + 1} \left(-1 + \frac{2c_1}{x \sqrt{a^2 - 2a - 4b + 1} + c_1} \right)}{2x} \right\} \right\}$$

✓ **Maple** : cpu = 0.063 (sec), leaf count = 51

$$\left\{ y(x) = \frac{-a - \sqrt{a^2 - 2a - 4b + 1} \tanh \left(\frac{\sqrt{a^2 - 2a - 4b + 1} (c_1 - \ln(x))}{2} \right) + 1}{2x} \right\}$$

2.142 ODE No. 142

$$-ax^2y(x) + ax + x^2(y'(x) - y(x)^2) + 2 = 0$$

✓ **Mathematica** : cpu = 0.218494 (sec), leaf count = 122

$$\left\{ \left\{ y(x) \rightarrow - \frac{\frac{e^{ax}(a^2x^2 - 2ax + 2)}{a^2x} - \frac{e^{ax}(a^2x^2 - 2ax + 2)}{a^3x^2} + \frac{e^{ax}(2a^2x - 2a)}{a^3x} - \frac{c_1}{x^2}}{\frac{e^{ax}(a^2x^2 - 2ax + 2)}{a^3x} + \frac{c_1}{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.091 (sec), leaf count = 52

$$\left\{y(x) = \frac{c_1 - (ax - 1)(a^2x^2 + 2)e^{ax}}{(c_1 + (a^2x^2 - 2ax + 2)e^{ax})x}\right\}$$

2.143 ODE No. 143

$$x^2\big(ay(x)^2 + y'(x)\big) - b = 0$$

✓ **Mathematica** : cpu = 0.111509 (sec), leaf count = 51

$$\left\{\left\{y(x) \rightarrow -\frac{-1 + \sqrt{4ab + 1}\left(-1 + \frac{2c_1}{x\sqrt{4ab + 1} + c_1}\right)}{2ax}\right\}\right\}$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 41

$$\left\{y(x) = \frac{-\sqrt{4ab + 1} \tanh\left(\frac{\sqrt{4ab + 1}(c_1 - \ln(x))}{2}\right) + 1}{2ax}\right\}$$

2.144 ODE No. 144

$$x^2\big(ay(x)^2 + y'(x)\big) + bx^\alpha + c = 0$$

✓ **Mathematica** : cpu = 0.321428 (sec), leaf count = 1787

$$\left\{\left\{y(x) \rightarrow \frac{\frac{i\sqrt{4ac-1}\alpha+\alpha}{a\frac{2}{2\alpha^2}} - \frac{\sqrt{\alpha^2-4aa^2c}}{2\alpha^2} \alpha - \frac{i\sqrt{4ac-1}\alpha+\alpha}{a^2} + \frac{\sqrt{\alpha^2-4aa^2c}}{a^2} + 1}{b\frac{i\sqrt{4ac-1}\alpha+\alpha}{2\alpha^2} - \frac{\sqrt{\alpha^2-4aa^2c}}{2\alpha^2}}\left(\frac{i\sqrt{4ac-1}\alpha+\alpha}{2\alpha^2} - \frac{\sqrt{\alpha^2-4aa^2c}}{2\alpha^2}\right)(x^\alpha)^{\frac{i\sqrt{4ac-1}\alpha+\alpha}{2\alpha^2} - \frac{\sqrt{\alpha^2-4aa^2c}}{2\alpha^2}}\right)}{\right\}\right\}$$

✓ **Maple** : cpu = 0.113 (sec), leaf count = 219

$$\left\{y(x) = \frac{-2\sqrt{ab}\left(c_1 \operatorname{BesselY}\left(\frac{\alpha + \sqrt{-4ac+1}}{\alpha}, \frac{2\sqrt{ab}x^{\frac{\alpha}{2}}}{\alpha}\right) + \operatorname{BesselJ}\left(\frac{\alpha + \sqrt{-4ac+1}}{\alpha}, \frac{2\sqrt{ab}x^{\frac{\alpha}{2}}}{\alpha}\right)\right)x^{\frac{\alpha}{2}} + (\sqrt{-4ac+1} + 1)\left(c_1 \operatorname{BesselY}\left(\frac{\sqrt{-4ac+1}}{\alpha}, \frac{2\sqrt{ab}x^{\frac{\alpha}{2}}}{\alpha}\right) + \operatorname{BesselJ}\left(\frac{\sqrt{-4ac+1}}{\alpha}, \frac{2\sqrt{ab}x^{\frac{\alpha}{2}}}{\alpha}\right)\right)ax}{2\left(c_1 \operatorname{BesselY}\left(\frac{\sqrt{-4ac+1}}{\alpha}, \frac{2\sqrt{ab}x^{\frac{\alpha}{2}}}{\alpha}\right) + \operatorname{BesselJ}\left(\frac{\sqrt{-4ac+1}}{\alpha}, \frac{2\sqrt{ab}x^{\frac{\alpha}{2}}}{\alpha}\right)\right)ax}\right\}$$

2.145 ODE No. 145

$$-ax^2y(x)^2 + ay(x)^3 + x^2y'(x) = 0$$

✓ **Mathematica** : cpu = 0.704306 (sec), leaf count = 267

$$\text{Solve}\left[\frac{\left(-\frac{1}{2^{2/3}a^{2/3}y(x)} - \frac{\sqrt[3]{a}x}{2^{2/3}}\right)\operatorname{Ai}\left(\left(-\frac{\sqrt[3]{a}x}{2^{2/3}} - \frac{1}{2^{2/3}a^{2/3}y(x)}\right)^2 + \frac{1}{\sqrt[3]{2}\sqrt[3]{a}x}\right) + \operatorname{Ai}'\left(\left(-\frac{\sqrt[3]{a}x}{2^{2/3}} - \frac{1}{2^{2/3}a^{2/3}y(x)}\right)^2 + \frac{1}{\sqrt[3]{2}\sqrt[3]{a}x}\right)}{\left(-\frac{1}{2^{2/3}a^{2/3}y(x)} - \frac{\sqrt[3]{a}x}{2^{2/3}}\right)\operatorname{Bi}\left(\left(-\frac{\sqrt[3]{a}x}{2^{2/3}} - \frac{1}{2^{2/3}a^{2/3}y(x)}\right)^2 + \frac{1}{\sqrt[3]{2}\sqrt[3]{a}x}\right) + \operatorname{Bi}'\left(\left(-\frac{\sqrt[3]{a}x}{2^{2/3}} - \frac{1}{2^{2/3}a^{2/3}y(x)}\right)^2 + \frac{1}{\sqrt[3]{2}\sqrt[3]{a}x}\right)} + c_1 = 0, y(x)\right]$$

✓ **Maple** : cpu = 0.103 (sec), leaf count = 117

$$\left\{y(x) = -\frac{1}{ax + (-2a)^{\frac{2}{3}} \operatorname{RootOf}\left(c_1\_Z \operatorname{AiryBi}\left(\frac{(-2a)^{\frac{1}{3}}\_Z^2x-1}{(-2a)^{\frac{1}{3}}x}\right) + c_1 \operatorname{AiryBi}\left(1, \frac{(-2a)^{\frac{1}{3}}\_Z^2x-1}{(-2a)^{\frac{1}{3}}x}\right) + \_Z \operatorname{AiryAi}\left(\frac{(-2a)^{\frac{1}{3}}\_Z^2x-1}{(-2a)^{\frac{1}{3}}x}\right) + \operatorname{AiryAi}\left(1, \frac{(-2a)^{\frac{1}{3}}\_Z^2x-1}{(-2a)^{\frac{1}{3}}x}\right)\right)}\right\}$$

2.146 ODE No. 146

$$ay(x)^2 + x^2y'(x) + xy(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.991355 (sec), leaf count = 78

$$\text{Solve}\left[-\frac{ia}{x} = \frac{2e^{\frac{1}{2}\left(-\frac{ia}{x} - \frac{i}{y(x)}\right)^2}}{\sqrt{2\pi} \operatorname{erfi}\left(\frac{-\frac{ia}{x} - \frac{i}{y(x)}}{\sqrt{2}}\right)} + 2c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.144 (sec), leaf count = 84

$$\left\{c_1 + \frac{\left(\sqrt{\pi} \sqrt{2} a \operatorname{erf}\left(\frac{\sqrt{2} (ay(x)+x)}{2xy(x)}\right) e^{\frac{(ay(x)+x)^2}{2x^2y(x)^2}} + 2x\right) e^{-\frac{(x+(a+x)y(x))(x+(a-x)y(x))}{2x^2y(x)^2}}}{2} = 0\right\}$$

2.147 ODE No. 147

$$ax^2y(x)^3 + by(x)^2 + x^2y'(x) = 0$$

✓ **Mathematica** : cpu = 0.956268 (sec), leaf count = 343

$$\text{Solve}\left[\frac{\left(\frac{b^{2/3}}{2^{2/3} \sqrt[3]{a} x} + \frac{1}{2^{2/3} \sqrt[3]{a} \sqrt[3]{b} y(x)}\right) \operatorname{Ai}\left(\left(\frac{b^{2/3}}{2^{2/3} \sqrt[3]{a} x} + \frac{1}{2^{2/3} \sqrt[3]{a} y(x) \sqrt[3]{b}}\right)^2 - \frac{\sqrt[3]{a} x}{\sqrt[3]{2} b^{2/3}}\right) + \operatorname{Ai}'\left(\left(\frac{b^{2/3}}{2^{2/3} \sqrt[3]{a} x} + \frac{1}{2^{2/3} \sqrt[3]{a} y(x) \sqrt[3]{b}}\right)^2 - \frac{\sqrt[3]{a} x}{\sqrt[3]{2} b^{2/3}}\right)}{\left(\frac{b^{2/3}}{2^{2/3} \sqrt[3]{a} x} + \frac{1}{2^{2/3} \sqrt[3]{a} \sqrt[3]{b} y(x)}\right) \operatorname{Bi}\left(\left(\frac{b^{2/3}}{2^{2/3} \sqrt[3]{a} x} + \frac{1}{2^{2/3} \sqrt[3]{a} y(x) \sqrt[3]{b}}\right)^2 - \frac{\sqrt[3]{a} x}{\sqrt[3]{2} b^{2/3}}\right) + \operatorname{Bi}'\left(\left(\frac{b^{2/3}}{2^{2/3} \sqrt[3]{a} x} + \frac{1}{2^{2/3} \sqrt[3]{a} y(x) \sqrt[3]{b}}\right)^2 - \frac{\sqrt[3]{a} x}{\sqrt[3]{2} b^{2/3}}\right)} + c_1 = 0\right]$$

✓ **Maple** : cpu = 0.206 (sec), leaf count = 178

$$\left\{y(x) = -\frac{2^{\frac{1}{3}} abx}{2^{\frac{1}{3}} a b^2 - 2(a^2 b^2)^{\frac{2}{3}} x \operatorname{RootOf}\left(c_1 \operatorname{AiryBi}\left(-\frac{-2(a^2 b^2)^{\frac{1}{3}} \_Z^2 + 2^{\frac{2}{3}} ax}{2(a^2 b^2)^{\frac{1}{3}}}\right) + c_1 \operatorname{AiryBi}\left(1, -\frac{-2(a^2 b^2)^{\frac{1}{3}} \_Z^2 + 2^{\frac{2}{3}} ax}{2(a^2 b^2)^{\frac{1}{3}}}\right) + \_Z \operatorname{AiryAi}\right)}\right\}$$

2.148 ODE No. 148

$$(x^2 + 1)y'(x) + xy(x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.0232917 (sec), leaf count = 30

$$\left\{\left\{y(x) \rightarrow \frac{\sinh^{-1}(x)}{\sqrt{x^2 + 1}} + \frac{c_1}{\sqrt{x^2 + 1}}\right\}\right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 16

$$\left\{y(x) = \frac{c_1 + \operatorname{arcsinh}(x)}{\sqrt{x^2 + 1}}\right\}$$

2.149 ODE No. 149

$$(x^2 + 1)y'(x) - x(x^2 + 1) + xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0242506 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{3}(x^2 + 1) + \frac{c_1}{\sqrt{x^2 + 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 20

$$\left\{ y(x) = \frac{x^2}{3} + \frac{c_1}{\sqrt{x^2 + 1}} + \frac{1}{3} \right\}$$

2.150 ODE No. 150

$$(x^2 + 1)y'(x) - 2x^2 + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0234024 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow \frac{2x^3}{3(x^2 + 1)} + \frac{c_1}{x^2 + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.006 (sec), leaf count = 23

$$\left\{ y(x) = \frac{2x^3 + 3c_1}{3x^2 + 3} \right\}$$

2.151 ODE No. 151

$$(x^2 + 1)y'(x) + (2xy(x) - 1)(y(x)^2 + 1) = 0$$

✓ **Mathematica** : cpu = 0.636967 (sec), leaf count = 203

$$\text{Solve} \left[c_1 = \frac{\frac{1}{2} \left(\frac{1}{\frac{ix}{x^2+1} - \frac{ix^2 y(x)}{x^2+1}} + \frac{i}{x} \right)^4 \sqrt{1 - \left(\frac{1}{\frac{ix}{x^2+1} - \frac{ix^2 y(x)}{x^2+1}} + \frac{i}{x} \right)^2} {}_2F_1 \left(\frac{1}{2}, \frac{5}{4}; \frac{3}{2}; \left(\frac{1}{\frac{ix}{x^2+1} - \frac{ix^2 y(x)}{x^2+1}} + \frac{i}{x} \right)^2 \right) + ix}{\sqrt[4]{-1 + \left(\frac{1}{\frac{ix}{x^2+1} - \frac{ix^2 y(x)}{x^2+1}} + \frac{i}{x} \right)^2}}, y(x) \right]$$

✓ **Maple** : cpu = 0.047 (sec), leaf count = 85

$$\left\{ c_1 + \frac{x}{\left(\left(\frac{x^2}{\frac{x^4 y(x)}{x^2+1} - \frac{x^3}{x^2+1}} + \frac{1}{x} \right)^2 + 1 \right)^{\frac{1}{4}}} + \frac{(x + y(x)) \operatorname{hypergeom} \left(\left[\frac{1}{2}, \frac{5}{4} \right], \left[\frac{3}{2} \right], -\frac{(x + y(x))^2}{(xy(x) - 1)^2} \right)}{2xy(x) - 2} = 0 \right\}$$

2.152 ODE No. 152

$$(x^2 + 1)y'(x) - x(x^2 + 1)\cos^2(y(x)) + x\sin(y(x))\cos(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.416217 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow \tan^{-1} \left(\frac{x^4 + 2x^2 - 6c_1 \sqrt{x^2 + 1} + 1}{3(x^2 + 1)} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.733 (sec), leaf count = 159

$$\left\{y(x) = \frac{\arctan\left(\frac{6\sqrt{x^2+1}\left(\sqrt{x^2+1}x^2+3c_1+\sqrt{x^2+1}\right)}{x^6+3x^4+9c_1^2+12x^2+(6c_1x^2+6c_1)\sqrt{x^2+1}+10}\right) - \frac{-x^6-3x^4-9c_1^2+6x^2+(-6c_1x^2-6c_1)\sqrt{x^2+1}+8}{x^6+3x^4+9c_1^2+12x^2+(6c_1x^2+6c_1)\sqrt{x^2+1}+10}}{2}\right\}$$

2.153 ODE No. 153

$$a + (x^2 - 1)y'(x) - xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0240529 (sec), leaf count = 21

$$\left\{\left\{y(x) \rightarrow ax + c_1\sqrt{x^2-1}\right\}\right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 20

$$\left\{y(x) = ax + c_1\sqrt{x+1}\sqrt{x-1}\right\}$$

2.154 ODE No. 154

$$(x^2 - 1)y'(x) + 2xy(x) - \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.0250227 (sec), leaf count = 26

$$\left\{\left\{y(x) \rightarrow \frac{\sin(x)}{x^2-1} + \frac{c_1}{x^2-1}\right\}\right\}$$

✓ **Maple** : cpu = 0.008 (sec), leaf count = 19

$$\left\{y(x) = \frac{c_1 + \sin(x)}{(x-1)(x+1)}\right\}$$

2.155 ODE No. 155

$$(x^2 - 1)y'(x) + y(x)^2 - 2xy(x) + 1 = 0$$

✓ **Mathematica** : cpu = 0.155791 (sec), leaf count = 46

$$\left\{\left\{y(x) \rightarrow -\frac{x(1-x^2)}{x^2-1} + \frac{1}{\frac{1}{2}\log(1-x) - \frac{1}{2}\log(x+1) + c_1}\right\}\right\}$$

✓ **Maple** : cpu = 0.097 (sec), leaf count = 14

$$\left\{y(x) = x + \frac{1}{c_1 - \operatorname{arctanh}(x)}\right\}$$

2.156 ODE No. 156

$$(x^2 - 1)y'(x) - y(x)(y(x) - x) = 0$$

✓ **Mathematica** : cpu = 0.097899 (sec), leaf count = 21

$$\left\{\left\{y(x) \rightarrow \frac{1}{x + c_1\sqrt{x^2-1}}\right\}\right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 20

$$\left\{y(x) = \frac{1}{c_1\sqrt{x-1}\sqrt{x+1} + x}\right\}$$

2.157 ODE No. 157

$$a\big(y(x)^2 - 2xy(x) + 1\big) + \big(x^2 - 1\big)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.241046 (sec), leaf count = 158

$$\left\{ \left\{ y(x) \rightarrow \frac{\big(x^2 - 1\big)\left(c_1\left(ax\big(x^2 - 1\big)^{\frac{a}{2}-1}P_{a-1}(x) + \big(x^2 - 1\big)^{\frac{a}{2}-1}(aP_a(x) - axP_{a-1}(x))\right) + ax\big(x^2 - 1\big)^{\frac{a}{2}-1}Q_{a-1}(x) + \big(x^2 - 1\big)^{\frac{a}{2}-1}(aQ_a(x) - axQ_{a-1}(x))}{a\left(\big(x^2 - 1\big)^{a/2}Q_{a-1}(x) + c_1\big(x^2 - 1\big)^{a/2}P_{a-1}(x)\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.252 (sec), leaf count = 231

$$\left\{ y(x) = \frac{-\left(x + 1\right)a\left(-\frac{x}{2} - \frac{1}{2}\right)^{-2a+1} \operatorname{HeunC}\left(0, 2a - 1, 0, 0, a^2 - a + \frac{1}{2}, \frac{2}{x+1}\right) + 8c_1\left(-\frac{a}{2} + \left(a - \frac{1}{2}\right)x + \frac{1}{2}\right)\left(x + 1\right) \operatorname{HeunC}\left(0, -2a + 1, 0, 0, a^2 - a + \frac{1}{2}, \frac{2}{x+1}\right)}{4\left(x + 1\right)\left(c_1 \operatorname{HeunC}\left(0, -2a + 1, 0, 0, a^2 - a + \frac{1}{2}, \frac{2}{x+1}\right) + \left(a - \frac{1}{2}\right)x + \frac{1}{2}\right)}$$

2.158 ODE No. 158

$$axy(x)^2 + \big(x^2 - 1\big)y'(x) + xy(x) = 0$$

✓ **Mathematica** : cpu = 0.162529 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow -\frac{e^{c_1}}{-\sqrt{x^2 - 1} + ae^{c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 22

$$\left\{ y(x) = \frac{1}{c_1\sqrt{x-1}\sqrt{x+1}-a} \right\}$$

2.159 ODE No. 159

$$\big(x^2 - 1\big)y'(x) - 2xy(x)\log(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.106119 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow e^{e^{c_1}x^2 - e^{c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.145 (sec), leaf count = 13

$$\{y(x) = e^{c_1(x-1)(x+1)}\}$$

2.160 ODE No. 160

$$\big(x^2 - 4\big)y'(x) + \big(x + 2\big)y(x)^2 - 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.139202 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow \frac{2 - x}{(x + 2)(-\log(x + 2) + c_1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 21

$$\left\{ y(x) = \frac{x - 2}{(c_1 + \ln(x + 2))(x + 2)} \right\}$$

2.161 ODE No. 161

$$(x^2 - 5x + 6)y'(x) + x^2 + 3xy(x) - 8y(x) = 0$$

✓ **Mathematica** : cpu = 0.0313999 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow \frac{\frac{x^4}{4} - \frac{2x^3}{3}}{(2-x)^2(3-x)} + \frac{c_1}{(2-x)^2(3-x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 27

$$\left\{ y(x) = \frac{-\frac{1}{4}x^4 + \frac{2}{3}x^3 + c_1}{(x-2)^2(x-3)} \right\}$$

2.162 ODE No. 162

$$k(-a + y(x) + x)(-b + y(x) + x) + (x - a)(x - b)y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.534002 (sec), leaf count = 133

$$\left\{ \left\{ y(x) \rightarrow -\frac{-ak - bk + 2kx}{2(k+1)} + \frac{1}{2} \sqrt{\frac{-a^2k^2 + 2abk^2 - b^2k^2}{(k+1)^2}} \tan\left(\frac{(k+1)\sqrt{\frac{-a^2k^2 + 2abk^2 - b^2k^2}{(k+1)^2}}(\log(x-b) - \log(x-a))}{2(a-b)} + c_1\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.196 (sec), leaf count = 58

$$\left\{ y(x) = \frac{(c_1(b-x)(b-x)^k + (a-x)(a-x)^k)k}{(k+1)(c_1(b-x)^k + (a-x)^k)} \right\}$$

2.163 ODE No. 163

$$2a^2x + 2x^2y'(x) - 2y(x)^2 - xy(x) = 0$$

✓ **Mathematica** : cpu = 0.124331 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{-a^2} \sqrt{x} \tan\left(\frac{2\sqrt{-a^2}}{\sqrt{x}} - c_1\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 27

$$\left\{ y(x) = -ia\sqrt{x} \tan\left(\frac{-c_1\sqrt{x} + 2ia}{\sqrt{x}}\right) \right\}$$

2.164 ODE No. 164

$$2a^2x + 2x^2y'(x) - 2y(x)^2 - 3xy(x) = 0$$

✓ **Mathematica** : cpu = 0.152179 (sec), leaf count = 131

$$\left\{ \left\{ y(x) \rightarrow -\frac{x^2 \left(-\frac{e^{\frac{2a}{\sqrt{x}}}}{4a\sqrt{x}} + \frac{e^{\frac{2a}{\sqrt{x}}}}{2x} + c_1 \left(\frac{ae^{-\frac{2a}{\sqrt{x}}}}{x} + \frac{e^{-\frac{2a}{\sqrt{x}}}}{2\sqrt{x}} \right) \right)}{-\frac{\sqrt{x}e^{\frac{2a}{\sqrt{x}}}}{2a} + c_1\sqrt{x}e^{-\frac{2a}{\sqrt{x}}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.191 (sec), leaf count = 100

$$\left\{ y(x) = \frac{-\left(c_1 - 2\sqrt{-\frac{a^2}{x}}\right)x \cos\left(2\sqrt{-\frac{a^2}{x}}\right) - 2\left(c_1\sqrt{-\frac{a^2}{x}} + \frac{1}{2}\right)x \sin\left(2\sqrt{-\frac{a^2}{x}}\right)}{2c_1 \cos\left(2\sqrt{-\frac{a^2}{x}}\right) + 2 \sin\left(2\sqrt{-\frac{a^2}{x}}\right)} \right\}$$

2.165 ODE No. 165

$$x(2x-1)y'(x) + y(x)^2 - (4x+1)y(x) + 4x = 0$$

✓ **Mathematica** : cpu = 0.137435 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow 1 + \frac{(1-2x)x}{-x+c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 17

$$\left\{ y(x) = \frac{2x^2 + c_1}{c_1 + x} \right\}$$

2.166 ODE No. 166

$$2(x-1)xy'(x) + (x-1)y(x)^2 - x = 0$$

✓ **Mathematica** : cpu = 0.170933 (sec), leaf count = 71

$$\left\{ \left(\left(\left(2x \left(-G_{2,2}^{2,0} \left(x \left| \begin{array}{c} -\frac{1}{2}, \frac{1}{2} \\ -1, 0 \end{array} \right. \right) + \frac{c_1(E(x)-K(x))}{\pi x} \right) \right) \right) \right) y(x) \rightarrow \frac{\left(\left(\left(G_{2,2}^{2,0} \left(x \left| \begin{array}{c} \frac{1}{2}, \frac{3}{2} \\ 0, 0 \end{array} \right. \right) + \frac{2c_1 E(x)}{\pi} \right) \right) \right)}{\left(\left(\left(2x \left(-G_{2,2}^{2,0} \left(x \left| \begin{array}{c} -\frac{1}{2}, \frac{1}{2} \\ -1, 0 \end{array} \right. \right) + \frac{c_1(E(x)-K(x))}{\pi x} \right) \right) \right) \right)} \right\}$$

✓ **Maple** : cpu = 0.148 (sec), leaf count = 97

$$\left\{ y(x) = \frac{\left(c_1 \operatorname{LegendreQ}\left(-\frac{1}{2}, 1, \frac{-x+2}{x}\right) - c_1 \operatorname{LegendreQ}\left(\frac{1}{2}, 1, \frac{-x+2}{x}\right) + \operatorname{LegendreP}\left(-\frac{1}{2}, 1, \frac{-x+2}{x}\right) - \operatorname{LegendreP}\left(\frac{1}{2}, 1, \frac{-x+2}{x}\right) \right)}{2\left(c_1 \operatorname{LegendreQ}\left(-\frac{1}{2}, 1, \frac{-x+2}{x}\right) + \operatorname{LegendreP}\left(-\frac{1}{2}, 1, \frac{-x+2}{x}\right) \right)}(x-1) \right\}$$

2.167 ODE No. 167

$$3x^2y'(x) - x^2 - 3xy(x) - 7y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.102134 (sec), leaf count = 35

$$\left\{ \left(\left(y(x) \rightarrow \frac{x \tan\left(\frac{1}{3}(\sqrt{7} \log(x) + 3\sqrt{7} c_1)\right)}{\sqrt{7}} \right) \right) \right\}$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 20

$$\left\{ y(x) = \frac{\sqrt{7} x \tan\left(\frac{(c_1 + \ln(x))\sqrt{7}}{3}\right)}{7} \right\}$$

2.168 ODE No. 168

$$3(x^2-4)y'(x) + y(x)^2 - xy(x) - 3 = 0$$

✓ **Mathematica** : cpu = 0.212471 (sec), leaf count = 234

$$\left\{ \left(\left(\left(3(x^2-4) \left(c_1 \left(\frac{x P_{-\frac{1}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right)}{6(x^2-4)^{11/12}} + \frac{{}^{12}\sqrt{x^2-4} \left(\frac{1}{2} P_{\frac{5}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right) - \frac{5}{12} x P_{-\frac{1}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right) \right)}{2\left(\frac{x^2}{4}-1\right)} \right) + \frac{x Q_{-\frac{1}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right)}{6(x^2-4)^{11/12}} + \frac{{}^{12}\sqrt{x^2-4} \left(\frac{1}{2} Q_{\frac{5}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right) - \frac{5}{12} x Q_{-\frac{1}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right) \right)}{2\left(\frac{x^2}{4}-1\right)} \right) \right) \right) y(x) \rightarrow \frac{\left(\left(\left(3(x^2-4) \left(c_1 \left(\frac{x P_{-\frac{1}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right)}{6(x^2-4)^{11/12}} + \frac{{}^{12}\sqrt{x^2-4} \left(\frac{1}{2} P_{\frac{5}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right) - \frac{5}{12} x P_{-\frac{1}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right) \right)}{2\left(\frac{x^2}{4}-1\right)} \right) + \frac{x Q_{-\frac{1}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right)}{6(x^2-4)^{11/12}} + \frac{{}^{12}\sqrt{x^2-4} \left(\frac{1}{2} Q_{\frac{5}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right) - \frac{5}{12} x Q_{-\frac{1}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right) \right)}{2\left(\frac{x^2}{4}-1\right)} \right) \right) \right)}{{}^{12}\sqrt{x^2-4} Q_{-\frac{1}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right) + c_1 {}^{12}\sqrt{x^2-4} P_{-\frac{1}{6}}^{\frac{1}{3}}\left(\frac{x}{2}\right)} \right\}$$

✓ **Maple** : cpu = 0.163 (sec), leaf count = 140

$$\left\{ y(x) = - \frac{3(x+2) \left(c_1 \operatorname{HeunC}\left(0, \frac{4}{3}, -\frac{1}{3}, 0, \frac{25}{36}, \frac{4}{x+2}\right) - \frac{\left(-\frac{x}{4} - \frac{1}{2}\right)^{\frac{4}{3}} \operatorname{HeunC}\left(0, -\frac{4}{3}, -\frac{1}{3}, 0, \frac{25}{36}, \frac{4}{x+2}\right)}{\left(-\frac{x}{4} - \frac{1}{2}\right)^{\frac{4}{3}}} \right)}{4c_1 \left(x - \frac{5}{4}\right)(x+2) \operatorname{HeunC}\left(0, \frac{4}{3}, -\frac{1}{3}, 0, \frac{25}{36}, \frac{4}{x+2}\right) - \left(-\frac{x}{4} - \frac{1}{2}\right)^{\frac{4}{3}} (x+2) \operatorname{HeunC}\left(0, -\frac{4}{3}, -\frac{1}{3}, 0, \frac{25}{36}, \frac{4}{x+2}\right) + 12(x-2) \left(c_1 \operatorname{HeunC}\left(0, \frac{4}{3}, -\frac{1}{3}, 0, \frac{25}{36}, \frac{4}{x+2}\right) - \frac{\left(-\frac{x}{4} - \frac{1}{2}\right)^{\frac{4}{3}} \operatorname{HeunC}\left(0, -\frac{4}{3}, -\frac{1}{3}, 0, \frac{25}{36}, \frac{4}{x+2}\right)}{\left(-\frac{x}{4} - \frac{1}{2}\right)^{\frac{4}{3}}} \right)} \right.$$

2.169 ODE No. 169

$$(ax + b)^2 y'(x) + y(x)^3 (ax + b) + cy(x)^2 = 0$$

✓ **Mathematica** : cpu = 2.32392 (sec), leaf count = 149

$$\text{Solve} \left[-\frac{c}{\sqrt{-a(ax+b)^2}} = \frac{2 \exp \left(\frac{1}{2} \left(-\frac{c}{\sqrt{-a(ax+b)^2}} - \frac{(-a(ax+b)^2)^{3/2}}{ay(x)(ax+b)^3} \right)^2 \right)}{\sqrt{2\pi} \operatorname{erfi} \left(\frac{-\frac{c}{\sqrt{-a(ax+b)^2}} - \frac{(-a(ax+b)^2)^{3/2}}{ay(x)(ax+b)^3}}{\sqrt{2}} \right) + 2c_1}, y(x) \right]$$

✓ **Maple** : cpu = 0.142 (sec), leaf count = 153

$$\left\{ \frac{2c_1 a^{\frac{5}{2}} + \left(\sqrt{2} \sqrt{\pi} a c \operatorname{erf} \left(\frac{\sqrt{2} (cy(x) + (ax+b)a)}{2(ax+b) \sqrt{a} y(x)} \right) e^{\frac{(cy(x) + (ax+b)a)^2}{2(ax+b)^2 ay(x)^2}} + 2(ax+b)a^{\frac{3}{2}} \right) e^{-\frac{((ax+b)a + (ax+b+c)y(x))((ax+b)a + (-ax-b+c)y(x))}{2(ax+b)^2 ay(x)^2}}}{2a^{\frac{5}{2}}} = 0 \right\}$$

2.170 ODE No. 170

$$-x^4 + x^3 y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0937589 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow -\frac{x^3 \left(\frac{1}{x^2} - \frac{\log(x)}{x^2} - \frac{c_1}{x^2} \right)}{\frac{\log(x)}{x} + \frac{c_1}{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 23

$$\left\{ y(x) = \frac{(-c_1 + \ln(x) - 1)x^2}{-c_1 + \ln(x)} \right\}$$

2.171 ODE No. 171

$$x^3 y'(x) - x^2 y(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0959222 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow \frac{x^2}{1 + c_1 x} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 15

$$\left\{ y(x) = \frac{x^2}{c_1 x + 1} \right\}$$

2.172 ODE No. 172

$$x^4(-y(x)^2) + x^3y'(x) + x^2y(x) + 20 = 0$$

✓ **Mathematica** : cpu = 0.0945504 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow -\frac{5x^4 - \frac{4c_1}{x^5}}{x\left(x^5 + \frac{c_1}{x^4}\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.3 (sec), leaf count = 26

$$\left\{ y(x) = \frac{5x^9 + 4c_1}{(-x^9 + c_1)x^2} \right\}$$

2.173 ODE No. 173

$$x^6(-y(x)^2) + x^3y'(x) - (2x - 3)x^2y(x) + 3 = 0$$

✓ **Mathematica** : cpu = 0.106558 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow -\frac{3}{x^3} + \frac{1}{x^3\left(\frac{1}{4} + c_1e^{4x}\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.123 (sec), leaf count = 27

$$\left\{ y(x) = \frac{-3c_1e^{4x} - 3}{(c_1e^{4x} - 3)x^3} \right\}$$

2.174 ODE No. 174

$$(x^2 + 1)xy'(x) + x^2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0201961 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{\sqrt{x^2 + 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.006 (sec), leaf count = 13

$$\left\{ y(x) = \frac{c_1}{\sqrt{x^2 + 1}} \right\}$$

2.175 ODE No. 175

$$ax^3 + (x^2 - 1)xy'(x) - (2x^2 - 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0326612 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow ax + c_1\sqrt{1 - x^2}x \right\} \right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 20

$$\left\{ y(x) = \left(c_1\sqrt{x-1}\sqrt{x+1} + a \right) x \right\}$$

2.176 ODE No. 176

$$(x^2 - 1)xy'(x) + (x^2 - 1)y(x)^2 - x^2 = 0$$

✓ **Mathematica** : cpu = 0.212384 (sec), leaf count = 82

$$\left\{ \left\{ y(x) \rightarrow \frac{x \left(-2x G_{2,2}^{2,0} \left(x^2 \middle| \begin{matrix} -\frac{1}{2}, \frac{1}{2} \\ -1, 0 \end{matrix} \right) + \frac{2c_1(E(x^2) - K(x^2))}{\pi x} \right)}{G_{2,2}^{2,0} \left(x^2 \middle| \begin{matrix} \frac{1}{2}, \frac{3}{2} \\ 0, 0 \end{matrix} \right) + \frac{2c_1 E(x^2)}{\pi}} \right\} \right\}$$

✓ **Maple** : cpu = 0.097 (sec), leaf count = 30

$$\left\{ y(x) = \frac{c_1 \operatorname{EllipticCE}(x) + \operatorname{EllipticE}(x) - \operatorname{EllipticK}(x)}{c_1 \operatorname{EllipticCE}(x) - c_1 \operatorname{EllipticCK}(x) + \operatorname{EllipticE}(x)} \right\}$$

2.177 ODE No. 177

$$(x-1)x^2y'(x) - (x-2)xy(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.166339 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow -\frac{x^2}{c_1 x - 1 - c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 17

$$\left\{ y(x) = \frac{x^2}{c_1(x-1) + 1} \right\}$$

2.178 ODE No. 178

$$2(x^2 - 1)xy'(x) + 2(x^2 - 1)y(x)^2 - (3x^2 - 5)y(x) + x^2 - 3 = 0$$

✓ **Mathematica** : cpu = 0.179559 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow 1 + \frac{\sqrt{x}}{\sqrt{1-x^2} \left(2\sqrt{x} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; x^2\right) + c_1 \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.104 (sec), leaf count = 61

$$\left\{ y(x) = -\frac{2\sqrt{x}}{\sqrt{x-1}\sqrt{x+1}\left(c_1 - \frac{2\sqrt{-x}\sqrt{2}\sqrt{-x+1}\operatorname{EllipticF}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)}{\sqrt{x-1}\sqrt{x}}\right)} + 1 \right\}$$

2.179 ODE No. 179

$$3x(x^2 - 1)y'(x) - (x^2 + 1)y(x) + xy(x)^2 - 3x = 0$$

✓ **Mathematica** : cpu = 0.661833 (sec), leaf count = 2833

$$\left\{ \left\{ y(x) \rightarrow \frac{3(x^2 - 1) \left(c_1 \left(\frac{\int_1^x \text{Root}[125K[1]^8 - 164K[1]^6 + 70K[1]^4 - 20K[1]^2 + (1296K[1]^{12} - 5184K[1]^{10} + 7776K[1]^8 - 5184K[1]^6 + 1296K[1]^4)\#1^4 + (-3456K[1]^{11} + 12096K[1]^9 - 15552K[1]^7 + 5184K[1]^5 - 576K[1]^3 + 48K[1])}{125K[1]^8 - 164K[1]^6 + 70K[1]^4 - 20K[1]^2 + (1296K[1]^{12} - 5184K[1]^{10} + 7776K[1]^8 - 5184K[1]^6 + 1296K[1]^4)\#1^4 + (-3456K[1]^{11} + 12096K[1]^9 - 15552K[1]^7 + 5184K[1]^5 - 576K[1]^3 + 48K[1])} \right) \right\} \right.$$

✓ **Maple** : cpu = 0.145 (sec), leaf count = 112

$$\left\{ y(x) = \frac{24x^{\frac{4}{3}} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{5}{6}\right], \left[\frac{2}{3}\right], x^2\right) + 35c_1\left(\frac{8x^2}{7} - \frac{16}{35}\right) \operatorname{hypergeom}\left(\left[\frac{5}{6}, \frac{7}{6}\right], \left[\frac{4}{3}\right], x^2\right) + 35c_1(x^4 - x^2) \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{5}{6}\right], \left[\frac{2}{3}\right], x^2\right)}{\left(8c_1x^{\frac{2}{3}} \operatorname{hypergeom}\left(\left[\frac{5}{6}, \frac{7}{6}\right], \left[\frac{4}{3}\right], x^2\right) + 8 \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{5}{6}\right], \left[\frac{2}{3}\right], x^2\right)\right)}$$

2.180 ODE No. 180

$$(xy'(x) - y(x))(ax^2 + bx + c) + x^2 - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.411871 (sec), leaf count = 132

$$\left\{ \left\{ y(x) \rightarrow -\frac{x \left(-1 + \exp\left(\frac{4 \tan^{-1}\left(\frac{2ax}{\sqrt{4ac-b^2}} + \frac{b}{\sqrt{4ac-b^2}} \right)}{\sqrt{4ac-b^2}} + 2c_1 \right) \right)}{1 + \exp\left(\frac{4 \tan^{-1}\left(\frac{2ax}{\sqrt{4ac-b^2}} + \frac{b}{\sqrt{4ac-b^2}} \right)}{\sqrt{4ac-b^2}} + 2c_1 \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.078 (sec), leaf count = 58

$$\left\{ y(x) = -x \tanh\left(\frac{c_1 \sqrt{4ac-b^2} + 2 \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}} \right)}{\sqrt{4ac-b^2}} \right) \right\}$$

2.181 ODE No. 181

$$a + x^4(y'(x) + y(x)^2) = 0$$

✓ **Mathematica** : cpu = 0.103347 (sec), leaf count = 347

$$\left\{ \left\{ y(x) \rightarrow -\frac{\frac{i\sqrt{\frac{2}{\pi}}c_1 \sinh\left(\frac{\sqrt{-a}}{x}\right)}{\sqrt{-\frac{i\sqrt{-a}}{x}}} + \frac{i\sqrt{-a} \left(-\frac{2\sqrt{\frac{2}{\pi}} \left(i \sinh\left(\frac{\sqrt{-a}}{x}\right) + \frac{i\sqrt{-a}x \cosh\left(\frac{\sqrt{-a}}{x}\right)}{a} \right)}{\sqrt{-\frac{i\sqrt{-a}}{x}}} - \frac{\sqrt{\frac{2}{\pi}}c_1 \cosh\left(\frac{\sqrt{-a}}{x}\right)}{\sqrt{-\frac{i\sqrt{-a}}{x}}} + \frac{\sqrt{\frac{2}{\pi}}c_1 \left(-\frac{\sqrt{-a}x \sinh\left(\frac{\sqrt{-a}}{x}\right)}{a} - \cosh\left(\frac{\sqrt{-a}}{x}\right) \right)}{\sqrt{-\frac{i\sqrt{-a}}{x}}} \right)}{x} \right)}{2x \left(\frac{\sqrt{\frac{2}{\pi}} \cosh\left(\frac{\sqrt{-a}}{x}\right)}{\sqrt{-\frac{i\sqrt{-a}}{x}}} - \frac{i\sqrt{\frac{2}{\pi}}c_1 \sinh\left(\frac{\sqrt{-a}}{x}\right)}{\sqrt{-\frac{i\sqrt{-a}}{x}}} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 28

$$\left\{ y(x) = \frac{-\sqrt{a} \tan\left(\frac{(c_1x-1)\sqrt{a}}{x} \right) + x}{x^2} \right\}$$

2.182 ODE No. 182

$$(x^3 - 1)xy'(x) + x^2 - 2xy(x)^2 + y(x) = 0$$

✓ **Mathematica** : cpu = 0.240075 (sec), leaf count = 96

$$\left\{ \left\{ y(x) \rightarrow - \frac{(x^3 - 1) \left(\frac{x}{(1-x^3)^{2/3}} + \frac{x^4}{(1-x^3)^{5/3}} + \frac{2c_1 x^2}{(1-x^3)^{5/3}} \right)}{2 \left(\frac{x^2}{2(1-x^3)^{2/3}} + \frac{c_1}{(1-x^3)^{2/3}} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.119 (sec), leaf count = 18

$$\left\{ y(x) = \frac{(c_1 + x)x}{c_1 x^2 + 1} \right\}$$

2.183 ODE No. 183

$$(2x^4 - x)y'(x) - 2(x^3 - 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0287992 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 x^2}{\sqrt[3]{1 - 2x^3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.015 (sec), leaf count = 18

$$\left\{ y(x) = \frac{c_1 x^2}{(2x^3 - 1)^{\frac{1}{3}}} \right\}$$

2.184 ODE No. 184

$$(y'(x) + y(x)^2)(ax^2 + bx + c)^2 + A = 0$$

✓ **Mathematica** : cpu = 1.29124 (sec), leaf count = 704

$$\left\{ \left\{ y(x) \rightarrow - \frac{2a\sqrt{ax^2+bx+c} \exp\left(-\frac{\sqrt{4ac-b^2} \sqrt{1-\frac{4A}{b^2-4ac}} \tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)\left(\frac{(2ax+b)^2}{4ac-b^2}+1\right)} + \frac{(2ax+b) \exp\left(-\frac{\sqrt{4ac-b^2} \sqrt{1-\frac{4A}{b^2-4ac}} \tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac} \sqrt{1-\frac{4A}{b^2-4ac}} \sqrt{ax^2+bx+c}} + c_1 \left(\frac{2a\sqrt{1-\frac{4A}{b^2-4ac}} \sqrt{ax^2+bx+c}}{\sqrt{b^2-4ac}} - \frac{\sqrt{ax^2+bx+c} \exp\left(-\frac{\sqrt{4ac-b^2} \sqrt{1-\frac{4A}{b^2-4ac}} \tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac} \sqrt{1-\frac{4A}{b^2-4ac}}} \right) + c_1 \sqrt{x(ax+b)+c} \right. \right. \\ \left. \left. - \frac{\sqrt{ax^2+bx+c} \exp\left(-\frac{\sqrt{4ac-b^2} \sqrt{1-\frac{4A}{b^2-4ac}} \tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac} \sqrt{1-\frac{4A}{b^2-4ac}}} + c_1 \sqrt{x(ax+b)+c} \right\} \right\}$$

✓ **Maple** : cpu = 0.422 (sec), leaf count = 493

$$\left\{ y(x) = \frac{2 \left(c_1 \left(i \sqrt{\frac{-4ac+b^2-4A}{a^2}} \sqrt{4ac-b^2} a - 2\sqrt{-4ac+b^2} \left(ax + \frac{b}{2} \right) \right) \left(\frac{-2ax-b+i\sqrt{4ac-b^2}}{2ax+b+i\sqrt{4ac-b^2}} \right)^{-\frac{\sqrt{\frac{-4ac+b^2-4A}{a^2}} a}{2\sqrt{-4ac+b^2}}} - \left(i \sqrt{\frac{-4ac+b^2-4A}{a^2}} \sqrt{4ac-b^2} a - 2\sqrt{-4ac+b^2} \left(ax + \frac{b}{2} \right) \right) \left(\frac{-2ax-b-i\sqrt{4ac-b^2}}{2ax+b-i\sqrt{4ac-b^2}} \right)^{-\frac{\sqrt{\frac{-4ac+b^2-4A}{a^2}} a}{2\sqrt{-4ac+b^2}}} \right)}{\sqrt{-4ac+b^2} \left(2ax+b+i\sqrt{4ac-b^2} \right) \left(-2ax-b+i\sqrt{4ac-b^2} \right)} c_1 \left(\frac{-2ax-b+i\sqrt{4ac-b^2}}{2ax+b+i\sqrt{4ac-b^2}} \right)^{-\frac{\sqrt{\frac{-4ac+b^2-4A}{a^2}} a}{2\sqrt{-4ac+b^2}}} - \left(i \sqrt{\frac{-4ac+b^2-4A}{a^2}} \sqrt{4ac-b^2} a - 2\sqrt{-4ac+b^2} \left(ax + \frac{b}{2} \right) \right) \left(\frac{-2ax-b-i\sqrt{4ac-b^2}}{2ax+b-i\sqrt{4ac-b^2}} \right)^{-\frac{\sqrt{\frac{-4ac+b^2-4A}{a^2}} a}{2\sqrt{-4ac+b^2}}} \right)}$$

2.185 ODE No. 185

$$x^7y'(x) + 5x^3y(x)^2 + 2(x^2 + 1)y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.514557 (sec), leaf count = 123

$$\text{Solve}\left[c_1 = \frac{\frac{1}{2} \sqrt[4]{1 - \left(\frac{ix^2}{y(x)} + \frac{i}{x}\right)^2} \left(\frac{ix^2}{y(x)} + \frac{i}{x}\right) {}_2F_1\left(\frac{1}{2}, \frac{5}{4}; \frac{3}{2}; \left(\frac{ix^2}{y(x)} + \frac{i}{x}\right)^2\right) + ix}{\sqrt[4]{-1 + \left(\frac{ix^2}{y(x)} + \frac{i}{x}\right)^2}}, y(x)\right]$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 63

$$\left\{c_1 + \frac{x}{\left(\left(\frac{x^2}{y(x)} + \frac{1}{x}\right)^2 + 1\right)^{\frac{1}{4}}} + \frac{(x^3 + y(x)) \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{5}{4}\right], \left[\frac{3}{2}\right], -\frac{(x^3 + y(x))^2}{x^2 y(x)^2}\right)}{2xy(x)} = 0\right\}$$

2.186 ODE No. 186

$$-((n-1)x^{n-1}y(x)) + x^{2n-2} + x^ny'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.322511 (sec), leaf count = 19

$$\left\{\left\{y(x) \rightarrow x^{n-1} \tan(-\log(x) + c_1)\right\}\right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 17

$$\{y(x) = x^{n-1} \tan(c_1 - \ln(x))\}$$

2.187 ODE No. 187

$$-ay(x)^2 - bx^{2n-2} + x^ny'(x) = 0$$

✓ **Mathematica** : cpu = 0.25037 (sec), leaf count = 328

$$\left\{\left\{y(x) \rightarrow -\frac{x^n \left(\frac{1}{2} \sqrt{a} \sqrt{b} \left(\sqrt{\frac{(n-1)^2}{ab}} - 4 - \frac{n-1}{\sqrt{a} \sqrt{b}}\right) x^{\frac{1}{2} \sqrt{a} \sqrt{b} \left(\sqrt{\frac{(n-1)^2}{ab}} - 4 - \frac{n-1}{\sqrt{a} \sqrt{b}}\right)^{-1}} + \frac{1}{2} \sqrt{a} \sqrt{b} c_1 \left(-\frac{n-1}{\sqrt{a} \sqrt{b}} - \sqrt{\frac{(n-1)^2}{ab}} - 4\right) x^{\frac{1}{2} \sqrt{a} \sqrt{b} \left(\sqrt{\frac{(n-1)^2}{ab}} - 4 - \frac{n-1}{\sqrt{a} \sqrt{b}}\right)}\right)}{a \left(x^{\frac{1}{2} \sqrt{a} \sqrt{b} \left(\sqrt{\frac{(n-1)^2}{ab}} - 4 - \frac{n-1}{\sqrt{a} \sqrt{b}}\right)} + c_1 x^{\frac{1}{2} \sqrt{a} \sqrt{b} \left(-\frac{n-1}{\sqrt{a} \sqrt{b}} - \sqrt{\frac{(n-1)^2}{ab}} - 4\right)}\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 60

$$\left\{y(x) = \frac{\left(n - \sqrt{4ab - n^2 + 2n - 1} \tan\left(\frac{\sqrt{4ab - n^2 + 2n - 1} (c_1 - \ln(x))}{2}\right) - 1\right) x^{n-1}}{2a}\right\}$$

2.188 ODE No. 188

$$-ay(x)^3 - bnx^3 + x^{2n+1}y'(x) = 0$$

✗ **Mathematica** : cpu = 14.0359 (sec), leaf count = 0 , could not solve

`DSolve[-(b*n*x^3) - a*y[x]^3 + x^(1 + 2*n)*Derivative[1][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.027 (sec), leaf count = 32

$$\left\{y(x) = x^n \operatorname{RootOf}\left(c_1 + \int^{-Z} \frac{1}{-a^3 a - n_a + b} d_a - \ln(x)\right)\right\}$$

2.189 ODE No. 189

$$-ay(x)^n - bx^{(m+1)n} + x^{m(n-1)+n}y'(x) = 0$$

✓ **Mathematica** : cpu = 0.488293 (sec), leaf count = 91

$$\text{Solve}\left[\int_1^{\left(\frac{ax^{-(m+1)n}}{b}\right)^{\frac{1}{n}}y(x)}\frac{1}{K[1]^n-\left(\frac{b^{1-n}(m+1)^n}{a}\right)^{\frac{1}{n}}K[1]}dK[1]=bx^{m+1}\log(x)\left(\frac{ax^{-(m+1)n}}{b}\right)^{\frac{1}{n}}+c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.339 (sec), leaf count = 60

$$\left\{-c_1+\int_{-b}^{y(x)}-\frac{x^nx^{mn}}{ax\_a^nx^m+(bx\,x^m-(m+1)\_a)x^nx^{mn}}d\_a+\ln(x)=0\right\}$$

2.190 ODE No. 190

$$\sqrt{x^2-1}y'(x)-\sqrt{y(x)^2-1}=0$$

✓ **Mathematica** : cpu = 0.149295 (sec), leaf count = 173

$$\left\{\left\{y(x)\rightarrow-\frac{1}{2}e^{-c_1}\sqrt{2x^2+2e^{4c_1}x^2-2\sqrt{(x-1)(x+1)}x+2e^{4c_1}\sqrt{(x-1)(x+1)}x-1+2e^{2c_1}-e^{4c_1}}\right\},\left\{y(x)\rightarrow\frac{1}{2}e^{-c_1}\sqrt{2x^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.006 (sec), leaf count = 29

$$\left\{c_1+\ln\left(x+\sqrt{x^2-1}\right)-\ln\left(y(x)+\sqrt{y(x)^2-1}\right)=0\right\}$$

2.191 ODE No. 191

$$\sqrt{1-x^2}y'(x)-y(x)\sqrt{y(x)^2-1}=0$$

✓ **Mathematica** : cpu = 0.14113 (sec), leaf count = 39

$$\left\{\left\{y(x)\rightarrow-\sqrt{1+\tan^2\left(\sin^{-1}(x)+c_1\right)}\right\},\left\{y(x)\rightarrow\sqrt{1+\tan^2\left(\sin^{-1}(x)+c_1\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 16

$$\left\{c_1+\arcsin(x)+\arctan\left(\frac{1}{\sqrt{y(x)^2-1}}\right)=0\right\}$$

2.192 ODE No. 192

$$\sqrt{a^2+x^2}y'(x)-\sqrt{a^2+x^2}+y(x)+x=0$$

✓ **Mathematica** : cpu = 0.764372 (sec), leaf count = 168

$$\left(\left(y(x)\rightarrow\frac{\sqrt{1-\frac{x}{\sqrt{a^2+x^2}}}\int_1^x\frac{\sqrt{\frac{K[1]}{\sqrt{a^2+K[1]^2}}+1}\left(\sqrt{a^2+K[1]^2}-K[1]\right)}{\sqrt{a^2+K[1]^2}\sqrt{1-\frac{K[1]}{\sqrt{a^2+K[1]^2}}}}dK[1]}{\sqrt{\frac{x}{\sqrt{a^2+x^2}}+1}}+\frac{c_1\sqrt{1-\frac{x}{\sqrt{a^2+x^2}}}}{\sqrt{\frac{x}{\sqrt{a^2+x^2}}+1}}\right)\right)$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 36

$$\left\{y(x)=\frac{a^2\ln\left(x+\sqrt{a^2+x^2}\right)+c_1}{x+\sqrt{a^2+x^2}}\right\}$$

2.193 ODE No. 193

$$-ax(\log(x) + 1) + x \log(x)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0308073 (sec), leaf count = 16

$$\left\{ \left\{ y(x) \rightarrow ax + \frac{c_1}{\log(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 14

$$\left\{ y(x) = ax + \frac{c_1}{\ln(x)} \right\}$$

2.194 ODE No. 194

$$x \log(x)y'(x) - y(x)(2 \log^2(x) + 1) - y(x)^2 \log(x) - \log^3(x) = 0$$

✓ **Mathematica** : cpu = 0.214417 (sec), leaf count = 98

$$\left\{ \left(\left(y(x) \rightarrow - \frac{x \left(\frac{e^{\frac{\log^2(x)}{2}} \log(x)}{x} + \frac{e^{\frac{\log^2(x)}{2}} \log^3(x)}{2x} + \frac{c_1 e^{\frac{\log^2(x)}{2}} \log(x)}{x} \right)}{\frac{1}{2} e^{\frac{\log^2(x)}{2}} \log^2(x) + c_1 e^{\frac{\log^2(x)}{2}}} \right) \right) \right\}$$

✓ **Maple** : cpu = 0.025 (sec), leaf count = 23

$$\left\{ y(x) = - \frac{(\ln(x)^2 + c_1 + 2) \ln(x)}{\ln(x)^2 + c_1} \right\}$$

2.195 ODE No. 195

$$\sin(x)y'(x) + y(x)^2(-\sin^2(x)) + y(x)(\cos(x) - 3 \sin(x)) + 4 = 0$$

✓ **Mathematica** : cpu = 0.183558 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow -4 \csc(x) + \frac{\csc(x)}{\frac{1}{5} + c_1 e^{5x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.118 (sec), leaf count = 28

$$\left\{ y(x) = \frac{-4c_1 e^{5x} - 4}{(c_1 e^{5x} - 4) \sin(x)} \right\}$$

2.196 ODE No. 196

$$\cos(x)y'(x) + y(x) + (\sin(x) + 1) \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.23401 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow e^{-2 \tanh^{-1}(\tan(\frac{x}{2}))} \left(\sin(x) + 4 \log \left(\cos \left(\frac{x}{2} \right) - \sin \left(\frac{x}{2} \right) \right) \right) + c_1 e^{-2 \tanh^{-1}(\tan(\frac{x}{2}))} \right\} \right\}$$

✓ **Maple** : cpu = 0.157 (sec), leaf count = 29

$$\left\{ y(x) = \frac{c_1 - 2 \ln(\sec(x) + \tan(x)) + 2 \ln(\cos(x)) + \sin(x)}{\sec(x) + \tan(x)} \right\}$$

2.197 ODE No. 197

$$\cos(x)y'(x) - y(x)^4 - y(x)\sin(x) = 0$$

✓ **Mathematica** : cpu = 0.134098 (sec), leaf count = 98

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{\sqrt[3]{-\sin(x) + c_1 \cos^3(x) - 2 \sin(x) \cos^2(x)}} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt[3]{-1}}{\sqrt[3]{-\sin(x) + c_1 \cos^3(x) - 2 \sin(x) \cos^2(x)}} \right\}, \left\{ y(x) \rightarrow \right.$$

✓ **Maple** : cpu = 0.158 (sec), leaf count = 237

$$\left\{ y(x) = \frac{\left((c_1(\sin^4(x)) + 2 \cos(x)(\sin^3(x)) - 2c_1(\sin^2(x)) - 3 \cos(x) \sin(x) + c_1)^2 \cos(x) \right)^{\frac{1}{3}}}{c_1(\sin^4(x)) + 2 \cos(x)(\sin^3(x)) - 2c_1(\sin^2(x)) - 3 \cos(x) \sin(x) + c_1}, y(x) = \frac{\left((c_1(\sin^4(x)) + 2 \cos(x)(\sin^3(x)) - 2c_1(\sin^2(x)) - 3 \cos(x) \sin(x) + c_1)^2 \cos(x) \right)^{\frac{1}{3}}}{2c_1(\sin^4(x)) + 2 \cos(x)(\sin^3(x)) - 2c_1(\sin^2(x)) - 3 \cos(x) \sin(x) + c_1} \right\}$$

2.198 ODE No. 198

$$\sin(x) \cos(x)y'(x) - y(x) - \sin^3(x) = 0$$

✓ **Mathematica** : cpu = 0.0340805 (sec), leaf count = 15

$$\{ \{ y(x) \rightarrow -\sin(x) + c_1 \tan(x) \} \}$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 13

$$\{ y(x) = (c_1 - \cos(x)) \tan(x) \}$$

2.199 ODE No. 199

$$\sin(2x)y'(x) + \sin(2y(x)) = 0$$

✓ **Mathematica** : cpu = 0.232108 (sec), leaf count = 15

$$\{ \{ y(x) \rightarrow \cot^{-1}(e^{-2c_1} \tan(x)) \} \}$$

✓ **Maple** : cpu = 0.181 (sec), leaf count = 102

$$\left\{ y(x) = \frac{\arctan\left(\frac{2c_1(2 \sin(2x) + \sin(4x))}{-c_1^2 \cos(4x) + c_1^2 + 4 \cos(2x) + \cos(4x) + 3} \right)}{2} \right\}$$

2.200 ODE No. 200

$$Ax(a \sin^2(x) + c) + y'(x)(a \sin^2(x) + b) + ay(x) \sin(2x) = 0$$

✓ **Mathematica** : cpu = 0.143451 (sec), leaf count = 77

$$\left\{ \left\{ y(x) \rightarrow \frac{\frac{1}{2}aAx^2 - \frac{1}{2}aAx \sin(2x) - \frac{1}{4}aA \cos(2x) + Acx^2}{a \cos(2x) - a - 2b} + \frac{c_1}{a \cos(2x) - a - 2b} \right\} \right\}$$

✓ **Maple** : cpu = 0.096 (sec), leaf count = 53

$$\left\{ y(x) = \frac{-2Aax \sin(2x) - Aa \cos(2x) + 2(a + 2c)A x^2 - 8c_1}{4a \cos(2x) - 4a - 8b} \right\}$$

2.201 ODE No. 201

$$-y(x)f'(x) + 2f(x)y'(x) + 2f(x)y(x)^2 - 2f(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.163848 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow i\sqrt{f(x)} \tan\left(i \int_1^x -\sqrt{f(K[1])} dK[1] + c_1\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 23

$$\left\{ y(x) = i\sqrt{f(x)} \tan\left(c_1 - i\left(\int \sqrt{f(x)} dx\right)\right) \right\}$$

2.202 ODE No. 202

$$f(x)y'(x) + g(x)\text{tg}(y(x)) + h(x) = 0$$

✗ **Mathematica** : cpu = 20.3047 (sec), leaf count = 0 , could not solve

`DSolve[h[x] + g[x]*tg[y[x]] + f[x]*Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(f(x)*diff(y(x),x)+g(x)*tg(y(x))+h(x) = 0,y(x))`

2.203 ODE No. 203

$$x^3 + y(x)y'(x) + y(x) = 0$$

✗ **Mathematica** : cpu = 1.82444 (sec), leaf count = 0 , could not solve

`DSolve[x^3 + y[x] + y[x]*Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(y(x)*diff(y(x),x)+y(x)+x^3 = 0,y(x))`

2.204 ODE No. 204

$$ay(x) + y(x)y'(x) + x = 0$$

✓ **Mathematica** : cpu = 0.13388 (sec), leaf count = 70

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{ay(x)}{x} + \frac{y(x)^2}{x^2} + 1\right) - \frac{a \tan^{-1}\left(\frac{a + \frac{2y(x)}{x}}{\sqrt{4-a^2}}\right)}{\sqrt{4-a^2}} = -\log(x) + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.325 (sec), leaf count = 92

$$\left\{ y(x) = x \text{RootOf}\left(-Z^2 + _Za - e^{\text{RootOf}\left(\left(a^2 \left(\tanh^2\left(\frac{\sqrt{(a-2)(a+2)}(2c_1+_Z+2\ln(x))}{2a}\right)\right) - a^2 - 4 \left(\tanh^2\left(\frac{\sqrt{(a-2)(a+2)}(2c_1+_Z+2\ln(x))}{2a}\right)\right) - 4e^{-Z} + 4\right)x^2}\right) + 1} \right\}$$

2.205 ODE No. 205

$$\frac{1}{4}(a^2 - 1)x + ay(x) + bx^n + y(x)y'(x) = 0$$

✗ **Mathematica** : cpu = 16.3552 (sec), leaf count = 0 , could not solve

`DSolve[((-1 + a^2)*x)/4 + b*x^n + a*y[x] + y[x]*Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(y(x)*diff(y(x),x)+a*y(x)+1/4*(a^2-1)*x+b*x^n = 0,y(x))`

2.206 ODE No. 206

$$ay(x) - 2a + be^x + y(x)y'(x) = 0$$

✗ **Mathematica** : cpu = 26.7588 (sec), leaf count = 0 , could not solve

`DSolve[-2*a + b*E^x + a*y[x] + y[x]*Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(y(x)*diff(y(x),x)+a*y(x)+b*exp(x)-2*a = 0,y(x))`

2.207 ODE No. 207

$$y(x)y'(x) + y(x)^2 + 4x(x + 1) = 0$$

✓ **Mathematica** : cpu = 0.100815 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{-4x^2 + c_1 e^{-2x}} \right\}, \left\{ y(x) \rightarrow \sqrt{-4x^2 + c_1 e^{-2x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 37

$$\left\{ y(x) = \sqrt{c_1 e^{-2x} - 4x^2}, y(x) = -\sqrt{c_1 e^{-2x} - 4x^2} \right\}$$

2.208 ODE No. 208

$$ay(x)^2 - b \cos(c + x) + y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.238141 (sec), leaf count = 118

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{4a^2 c_1 e^{-2ax} + 4ab \cos(c + x) + c_1 e^{-2ax} + 2b \sin(c + x)}}{\sqrt{4a^2 + 1}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{4a^2 c_1 e^{-2ax} + 4ab \cos(c + x) + c_1 e^{-2ax} + 2b \sin(c + x)}}{\sqrt{4a^2 + 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.088 (sec), leaf count = 106

$$\left\{ y(x) = \frac{\sqrt{16c_1 \left(a^2 + \frac{1}{4}\right)^2 e^{-2ax} + 16 \left(a^2 + \frac{1}{4}\right) \left(a \cos(c + x) + \frac{\sin(c+x)}{2}\right) b}}{4a^2 + 1}, y(x) = -\frac{\sqrt{16c_1 \left(a^2 + \frac{1}{4}\right)^2 e^{-2ax} + 16 \left(a^2 + \frac{1}{4}\right) \left(a \cos(c + x) + \frac{\sin(c+x)}{2}\right) b}}{4a^2 + 1} \right\}$$

2.209 ODE No. 209

$$y(x)y'(x) - \sqrt{ay(x)^2 + b} = 0$$

✓ **Mathematica** : cpu = 0.133764 (sec), leaf count = 84

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{a^2 x^2 + 2a^2 c_1 x + a^2 c_1^2 - b}}{\sqrt{a}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{a^2 x^2 + 2a^2 c_1 x + a^2 c_1^2 - b}}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 21

$$\left\{ c_1 + x - \frac{\sqrt{ay(x)^2 + b}}{a} = 0 \right\}$$

2.210 ODE No. 210

$$y(x)y'(x) + xy(x)^2 - 4x = 0$$

✓ **Mathematica** : cpu = 0.0976656 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{4 + e^{-x^2+2c_1}} \right\}, \left\{ y(x) \rightarrow \sqrt{4 + e^{-x^2+2c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt{c_1 e^{-x^2} + 4}, y(x) = -\sqrt{c_1 e^{-x^2} + 4} \right\}$$

2.211 ODE No. 211

$$y(x)y'(x) - xe^{\frac{x}{y(x)}} = 0$$

✓ **Mathematica** : cpu = 0.261051 (sec), leaf count = 41

$$\text{Solve} \left[\int_1^{\frac{y(x)}{x}} \frac{K[1]}{K[1]^2 - e^{\frac{1}{K[1]}}} dK[1] = -\log(x) + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 31

$$\left\{ y(x) = x \text{RootOf} \left(c_1 - \left(\int^{-Z} \frac{-a}{-a^2 + e^{\frac{1}{a}}} d_a \right) + \ln(x) \right) \right\}$$

2.212 ODE No. 212

$$g(x)f(x^2 + y(x)^2) + y(x)y'(x) + x = 0$$

✓ **Mathematica** : cpu = 0.39767 (sec), leaf count = 95

$$\text{Solve} \left[\int_1^{y(x)} \left(\frac{K[2]}{f(x^2 + K[2]^2)} - \int_1^x \frac{2K[1]K[2]f'(K[1]^2 + K[2]^2)}{f(K[1]^2 + K[2]^2)^2} dK[1] \right) dK[2] + \int_1^x \left(g(K[1]) + \frac{K[1]}{f(K[1]^2 + y(x)^2)} \right) dK[1] \right]$$

✓ **Maple** : cpu = 0.141 (sec), leaf count = 30

$$\left\{ -c_1 + \int_{-b}^{y(x)} \frac{-a}{f(-a^2 + x^2)} d_a + \int g(x) dx = 0 \right\}$$

2.213 ODE No. 213

$$(y(x) + 1)y'(x) - y(x) - x = 0$$

✓ **Mathematica** : cpu = 0.14623 (sec), leaf count = 71

$$\text{Solve} \left[\frac{1}{2} \log \left(\frac{x^2 - y(x)^2 + (x-3)y(x) - x - 1}{(x-1)^2} \right) + \log(1-x) = \frac{\tanh^{-1} \left(\frac{y(x)+2x-1}{\sqrt{5}(y(x)+1)} \right)}{\sqrt{5}} + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.823 (sec), leaf count = 66

$$\left\{ -c_1 - \frac{\sqrt{5} \operatorname{arctanh} \left(\frac{(x-2y(x)-3)\sqrt{5}}{5x-5} \right)}{5} - \frac{\ln \left(\frac{-x^2+y(x)^2+x+(-x+3)y(x)+1}{(x-1)^2} \right)}{2} - \ln(x-1) = 0 \right\}$$

2.214 ODE No. 214

$$(y(x) + x - 1)y'(x) - y(x) + 2x + 3 = 0$$

✓ **Mathematica** : cpu = 0.179265 (sec), leaf count = 78

$$\text{Solve}\left[2\sqrt{2} \tan^{-1}\left(\frac{-y(x) + 2x + 3}{\sqrt{2}(y(x) + x - 1)}\right) = 2 \log\left(\frac{6x^2 + 3y(x)^2 - 10y(x) + 8x + 11}{(3x + 2)^2}\right) + 4 \log(3x + 2) + 3c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.18 (sec), leaf count = 48

$$\left\{y(x) = \frac{(-3x - 2)\sqrt{2} \tan\left(\text{RootOf}\left(2c_1\sqrt{2} - 2\_Z + \sqrt{2} \ln\left(2\left(\tan^2(\_Z) + 1\right)(3x + 2)^2\right)\right)\right)}{3} + \frac{5}{3}\right\}$$

2.215 ODE No. 215

$$(y(x) + 2x - 2)y'(x) - y(x) + x + 1 = 0$$

✓ **Mathematica** : cpu = 0.16912 (sec), leaf count = 80

$$\text{Solve}\left[6\sqrt{3} \tan^{-1}\left(\frac{4 - 3y(x)}{\sqrt{3}(y(x) + 2x - 2)}\right) = 3 \log\left(\frac{3x^2 + 3y(x)^2 + 3(x - 3)y(x) - 6x + 7}{(1 - 3x)^2}\right) + 6 \log(3x - 1) + 2c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.204 (sec), leaf count = 51

$$\left\{y(x) = -\frac{x}{2} + \frac{\sqrt{3} (3x - 1) \tan\left(\text{RootOf}\left(2c_1\sqrt{3} + 6\_Z + \sqrt{3} \ln\left(\frac{3(\tan^2(\_Z) + 1)(3x - 1)^2}{4}\right)\right)\right)}{6} + \frac{3}{2}\right\}$$

2.216 ODE No. 216

$$(y(x) - 2x + 1)y'(x) + y(x) + x = 0$$

✓ **Mathematica** : cpu = 0.147666 (sec), leaf count = 82

$$\text{Solve}\left[6\sqrt{3} \tan^{-1}\left(\frac{3y(x) + 1}{\sqrt{3}(-y(x) + 2x - 1)}\right) = 3 \log\left(\frac{3x^2 + 3y(x)^2 - 3(x - 1)y(x) - 3x + 1}{(1 - 3x)^2}\right) + 6 \log(3x - 1) + 2c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.204 (sec), leaf count = 51

$$\left\{y(x) = \frac{x}{2} + \frac{(-3x + 1)\sqrt{3} \tan\left(\text{RootOf}\left(2c_1\sqrt{3} + 6\_Z + \sqrt{3} \ln\left(\frac{3(\tan^2(\_Z) + 1)(3x - 1)^2}{4}\right)\right)\right)}{6} - \frac{1}{2}\right\}$$

2.217 ODE No. 217

$$(y(x) - x^2)y'(x) - x = 0$$

✓ **Mathematica** : cpu = 0.028449 (sec), leaf count = 29

$$\left\{\left\{y(x) \rightarrow x^2 + \frac{1}{2}\left(1 + W\left(-e^{-2x^2 - 1 + c_1}\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 23

$$\left\{y(x) = x^2 + \frac{\text{LambertW}\left(-4c_1 e^{-2x^2 - 1}\right)}{2} + \frac{1}{2}\right\}$$

2.218 ODE No. 218

$$(y(x) - x^2)y'(x) + 4xy(x) = 0$$

✓ **Mathematica** : cpu = 0.152185 (sec), leaf count = 257

$$\left\{ \left\{ y(x) \rightarrow x^2 + \frac{1}{-\frac{1}{2x^2} - \frac{\frac{1}{2} - i}{\sqrt{2}x^2 \sqrt{x^2 \cosh\left(\frac{2c_1}{9}\right) + x^2 \sinh\left(\frac{2c_1}{9}\right) - i}}} \right\}, \left\{ y(x) \rightarrow x^2 + \frac{1}{-\frac{1}{2x^2} + \frac{\frac{1}{2} - i}{\sqrt{2}x^2 \sqrt{x^2 \cosh\left(\frac{2c_1}{9}\right) + x^2 \sinh\left(\frac{2c_1}{9}\right) - i}}} \right\}, y(x) \rightarrow \right.$$

✓ **Maple** : cpu = 0.169 (sec), leaf count = 57

$$\left\{ y(x) = \frac{c_1^2}{2} - x^2 - \frac{c_1 \sqrt{-4x^2 + c_1^2}}{2}, y(x) = \frac{c_1^2}{2} - x^2 + \frac{c_1 \sqrt{-4x^2 + c_1^2}}{2} \right\}$$

2.219 ODE No. 219

$$-f_0(x) - f_1(x)y(x) - f_2(x)y(x)^2 + (g(x) + y(x))y'(x) = 0$$

✗ **Mathematica** : cpu = 300.23 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve((y(x)+g(x))*diff(y(x),x)-f2(x)*y(x)^2-f1(x)*y(x)-f0(x) = 0,y(x))`

2.220 ODE No. 220

$$-x^3 + 2y(x)y'(x) - xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.127086 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{-x^2 + c_1 e^{\frac{x^2}{2}} - 2} \right\}, \left\{ y(x) \rightarrow \sqrt{-x^2 + c_1 e^{\frac{x^2}{2}} - 2} \right\} \right\}$$

✓ **Maple** : cpu = 0.028 (sec), leaf count = 43

$$\left\{ y(x) = \sqrt{c_1 e^{\frac{x^2}{2}} - x^2 - 2}, y(x) = -\sqrt{c_1 e^{\frac{x^2}{2}} - x^2 - 2} \right\}$$

2.221 ODE No. 221

$$(2y(x) + x + 1)y'(x) - 2y(x) - x + 1 = 0$$

✓ **Mathematica** : cpu = 0.0325724 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}(-x - 1) + \frac{2}{3} \left(1 + W \left(-e^{\frac{9x}{4} - 1 + c_1} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.117 (sec), leaf count = 21

$$\left\{ y(x) = -\frac{x}{2} + \frac{2 \operatorname{LambertW} \left(\frac{c_1 e^{-\frac{1}{4}} e^{\frac{9x}{4}}}{4} \right)}{3} + \frac{1}{6} \right\}$$

2.222 ODE No. 222

$$(2y(x) + x + 7)y'(x) - y(x) + 2x + 4 = 0$$

✓ **Mathematica** : cpu = 0.130989 (sec), leaf count = 65

$$\text{Solve}\left[2\log\left(\frac{4(x^2 + y(x)^2 + 4y(x) + 6x + 13)}{5(x + 3)^2}\right) + 2\tan^{-1}\left(\frac{y(x) - 2(x + 2)}{2y(x) + x + 7}\right) + 4\log(x + 3) + 5c_1 = 0, y(x)\right]$$

✓ **Maple** : cpu = 0.091 (sec), leaf count = 32

$$\left\{y(x) = (-x - 3) \tan\left(\text{RootOf}\left(2c_1 - \_Z + \ln\left(\frac{1}{\cos(\_Z)^2}\right) + 2\ln(x + 3)\right)\right) - 2\right\}$$

2.223 ODE No. 223

$$(2y(x) - x)y'(x) - y(x) - 2x = 0$$

✓ **Mathematica** : cpu = 0.0426833 (sec), leaf count = 55

$$\left\{\left\{y(x) \rightarrow \frac{1}{2}\left(x - \sqrt{5x^2 - 4e^{c_1}}\right)\right\}, \left\{y(x) \rightarrow \frac{1}{2}\left(x + \sqrt{5x^2 - 4e^{c_1}}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.184 (sec), leaf count = 51

$$\left\{y(x) = \frac{c_1x - \sqrt{5c_1^2x^2 + 4}}{2c_1}, y(x) = \frac{c_1x + \sqrt{5c_1^2x^2 + 4}}{2c_1}\right\}$$

2.224 ODE No. 224

$$(2y(x) - 6x)y'(x) - y(x) + 3x + 2 = 0$$

✓ **Mathematica** : cpu = 0.0343946 (sec), leaf count = 29

$$\left\{\left\{y(x) \rightarrow 3x - \frac{2}{5}\left(1 + W\left(-e^{\frac{25x}{4} - 1 + c_1}\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.121 (sec), leaf count = 35

$$\left\{y(x) = 3x + \frac{e^{-\frac{25c_1}{4} + \frac{25x}{4} - \text{LambertW}\left(-\frac{e^{-1}e^{-\frac{25c_1}{4}}e^{\frac{25x}{4}}}{2}\right) - 1}}{5} - \frac{2}{5}\right\}$$

2.225 ODE No. 225

$$(4y(x) + 2x + 3)y'(x) - 2y(x) - x - 1 = 0$$

✓ **Mathematica** : cpu = 0.0733567 (sec), leaf count = 33

$$\left\{\left\{y(x) \rightarrow \frac{1}{4}(-2x - 3) + \frac{1}{8}\left(1 + W\left(-e^{8x - 1 + c_1}\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.108 (sec), leaf count = 20

$$\left\{y(x) = -\frac{x}{2} + \frac{\text{LambertW}(c_1e^5e^{8x})}{8} - \frac{5}{8}\right\}$$

2.226 ODE No. 226

$(4y(x) - 2x - 3)y'(x) + 2y(x) - x - 1 = 0$

✓ **Mathematica** : cpu = 0.0351428 (sec), leaf count = 35

$$\left\{\left\{y(x) \rightarrow \frac{1}{4}(2x + 3) + \frac{1}{8}\left(-1 - W\left(-e^{8x-1+c_1}\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.102 (sec), leaf count = 21

$$\left\{y(x) = \frac{x}{2} - \frac{\text{LambertW}\left(-c_1 e^5 e^{8x}\right)}{8} + \frac{5}{8}\right\}$$

2.227 ODE No. 227

$(4y(x) - 3x - 5)y'(x) - 3y(x) + 7x + 2 = 0$

✓ **Mathematica** : cpu = 0.0278379 (sec), leaf count = 107

$$\left\{\left\{y(x) \rightarrow \frac{1}{4}(3x + 5) - \frac{1}{2}i\sqrt{-2\left(-\frac{7x^2}{2} - 2x\right) - \frac{1}{4}(3x + 5)^2 - 4c_1}\right\}, \left\{y(x) \rightarrow \frac{1}{4}(3x + 5) + \frac{1}{2}i\sqrt{-2\left(-\frac{7x^2}{2} - 2x\right) - \frac{1}{4}(3x + 5)^2 - 4c_1}\right\}\right\}$$

✓ **Maple** : cpu = 0.178 (sec), leaf count = 33

$$\left\{y(x) = \frac{c_1(57x + 95) - \sqrt{-6859\left(x - \frac{7}{19}\right)^2 c_1^2 + 4}}{76c_1}\right\}$$

2.228 ODE No. 228

$(4y(x) + 11x - 11)y'(x) - 25y(x) - 8x + 62 = 0$

✓ **Mathematica** : cpu = 0.462663 (sec), leaf count = 3357

$$\left\{\left\{y(x) \rightarrow \frac{4\left(-\frac{81(9x-1)\left(81\cosh\left(\frac{3c_1}{8}\right)x^2+81\sinh\left(\frac{3c_1}{8}\right)x^2-18\cosh\left(\frac{3c_1}{8}\right)x-18\sinh\left(\frac{3c_1}{8}\right)x+\cosh\left(\frac{3c_1}{8}\right)+\sinh\left(\frac{3c_1}{8}\right)-1\right)^3\sqrt{-258280326\cosh\left(\frac{3c_1}{4}\right)x^4-258280326\cosh\left(\frac{3c_1}{4}\right)x^3-258280326\cosh\left(\frac{3c_1}{4}\right)x^2-258280326\cosh\left(\frac{3c_1}{4}\right)x-258280326}}{81(9x-1)\left(81\cosh\left(\frac{3c_1}{8}\right)x^2+81\sinh\left(\frac{3c_1}{8}\right)x^2-18\cosh\left(\frac{3c_1}{8}\right)x-18\sinh\left(\frac{3c_1}{8}\right)x+\cosh\left(\frac{3c_1}{8}\right)+\sinh\left(\frac{3c_1}{8}\right)-1\right)^3\sqrt{-258280326\cosh\left(\frac{3c_1}{4}\right)x^4-258280326\cosh\left(\frac{3c_1}{4}\right)x^3-258280326\cosh\left(\frac{3c_1}{4}\right)x^2-258280326\cosh\left(\frac{3c_1}{4}\right)x-258280326}}\right\}\right\}$$

✓ **Maple** : cpu = 0.416 (sec), leaf count = 271

$$\left\{y(x) = \frac{(-76x + 28)\left(-8748c_1(9x - 1)^2 + 64 + 108\sqrt{43046721}\sqrt{c_1\left(c_1\left(x - \frac{1}{9}\right)^2 - \frac{32}{177147}\right)\left(x - \frac{1}{9}\right)^2}\right)^{\frac{1}{3}} + 4\left(x + \frac{1}{2}\right)\left(i\left(-8748c_1(9x - 1)^2 + 64 + 108\sqrt{43046721}\sqrt{c_1\left(c_1\left(x - \frac{1}{9}\right)^2 - \frac{32}{177147}\right)\left(x - \frac{1}{9}\right)^2}\right)^{\frac{1}{3}} + i\left(-8748c_1(9x - 1)^2 + 64 + 108\sqrt{43046721}\sqrt{c_1\left(c_1\left(x - \frac{1}{9}\right)^2 - \frac{32}{177147}\right)\left(x - \frac{1}{9}\right)^2}\right)^{\frac{1}{3}}\right)}{8\left(-8748c_1(9x - 1)^2 + 64 + 108\sqrt{43046721}\sqrt{c_1\left(c_1\left(x - \frac{1}{9}\right)^2 - \frac{32}{177147}\right)\left(x - \frac{1}{9}\right)^2}\right)^{\frac{1}{3}} + i\left(-8748c_1(9x - 1)^2 + 64 + 108\sqrt{43046721}\sqrt{c_1\left(c_1\left(x - \frac{1}{9}\right)^2 - \frac{32}{177147}\right)\left(x - \frac{1}{9}\right)^2}\right)^{\frac{1}{3}} + i\left(-8748c_1(9x - 1)^2 + 64 + 108\sqrt{43046721}\sqrt{c_1\left(c_1\left(x - \frac{1}{9}\right)^2 - \frac{32}{177147}\right)\left(x - \frac{1}{9}\right)^2}\right)^{\frac{1}{3}}\right)}$$

2.229 ODE No. 229

$$(12y(x) - 5x - 8)y'(x) - 5y(x) + 2x + 3 = 0$$

✓ **Mathematica** : cpu = 0.0361257 (sec), leaf count = 121

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{12}(5x + 8) - \frac{i\sqrt{-24\left(-\frac{x^2}{12} - \frac{x}{4}\right) - \frac{1}{12}(5x + 8)^2 - 12c_1}}{2\sqrt{3}} \right\}, \left\{ y(x) \rightarrow \frac{1}{12}(5x + 8) + \frac{i\sqrt{-24\left(-\frac{x^2}{12} - \frac{x}{4}\right) - \frac{1}{12}(5x + 8)^2 - 12c_1}}{2\sqrt{3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.171 (sec), leaf count = 32

$$\left\{ y(x) = \frac{c_1(5x + 8) - \sqrt{(x + 4)^2 c_1^2 + 24}}{12c_1} \right\}$$

2.230 ODE No. 230

$$ay(x)y'(x) + by(x)^2 + f(x) = 0$$

✓ **Mathematica** : cpu = 0.312155 (sec), leaf count = 98

$$\left\{ \left\{ y(x) \rightarrow -e^{-\frac{bx}{a}} \sqrt{2 \int_1^x -\frac{e^{\frac{2bK[1]}}{a}} f(K[1]) dK[1] + c_1} \right\}, \left\{ y(x) \rightarrow e^{-\frac{bx}{a}} \sqrt{2 \int_1^x -\frac{e^{\frac{2bK[1]}}{a}} f(K[1]) dK[1] + c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 100

$$\left\{ y(x) = \frac{\sqrt{\left(c_1 a - 2\left(\int e^{\frac{2bx}{a}} f(x) dx\right) a e^{\frac{2bx}{a}}\right) e^{-\frac{2bx}{a}}}}{a}, y(x) = -\frac{\sqrt{\left(c_1 a - 2\left(\int e^{\frac{2bx}{a}} f(x) dx\right) a e^{\frac{2bx}{a}}\right) e^{-\frac{2bx}{a}}}}{a} \right\}$$

2.231 ODE No. 231

$$y'(x)(ay(x) + bx + c) + \alpha y(x) + \beta x + \gamma = 0$$

✓ **Mathematica** : cpu = 3.80824 (sec), leaf count = 252

Solve
$$\left[(\alpha - b)^2 \left(-\log \left(\frac{(ay(x) + bx + c)^2 \left(-\frac{(\alpha(bx + c) - a(\beta x + \gamma)) (a(\alpha - b)y(x) + a(\beta x + \gamma) + b^2(-x) - bc)}{(ay(x) + bx + c)^2} + a\beta - \alpha b \right)}{(\alpha(bx + c) - a(\beta x + \gamma))^2} \right) - \frac{2 \tan^{-1} \left(\frac{\frac{2a(\beta x + \gamma) - 2\alpha(bx + c)}{ay(x) + bx + c} + \alpha - b}{(\alpha - b) \sqrt{\frac{4(a\beta - \alpha b)}{(\alpha - b)^2} - 1}} \right)}{\sqrt{\frac{4(a\beta - \alpha b)}{(\alpha - b)^2} - 1}} \right] = \frac{(\alpha - b)^2 \log}{2(a\beta - \alpha b)}$$

✓ **Maple** : cpu = 0.254 (sec), leaf count = 178

$$\left\{ y(x) = \frac{\beta c - \gamma b + \frac{(-ac + \gamma a + (a\beta - \alpha b)x) \left(\alpha + b + \sqrt{4a\beta - \alpha^2 - 2\alpha b - b^2} \tan \left(\text{RootOf} \left(2\_Z\alpha - 2\_Zb + 2c_1 \sqrt{4a\beta - \alpha^2 - 2\alpha b - b^2} + \sqrt{4a\beta - \alpha^2 - 2\alpha b - b^2} \ln \left(\frac{a\beta x - \alpha bx - ac + \dots}{\dots} \right) \right)}{2a}}{-a\beta + \alpha b} \right. \right\}$$

2.232 ODE No. 232

$$x^2 + xy(x)y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.146583 (sec), leaf count = 56

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{-x^4 + 2c_1}}{\sqrt{2}x} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{-x^4 + 2c_1}}{\sqrt{2}x} \right\} \right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 39

$$\left\{ y(x) = -\frac{\sqrt{-2x^4 + 4c_1}}{2x}, y(x) = \frac{\sqrt{-2x^4 + 4c_1}}{2x} \right\}$$

2.233 ODE No. 233

$$ax^3 \cos(x) + xy(x)y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.220998 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow -x\sqrt{-2a \sin(x) + c_1} \right\}, \left\{ y(x) \rightarrow x\sqrt{-2a \sin(x) + c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 30

$$\left\{ y(x) = \sqrt{-2a \sin(x) + c_1} x, y(x) = -\sqrt{-2a \sin(x) + c_1} x \right\}$$

2.234 ODE No. 234

$$x^3 - 2x^2 + xy(x)y'(x) + xy(x) - y(x)^2 = 0$$

✗ **Mathematica** : cpu = 28.5568 (sec), leaf count = 0 , could not solve

`DSolve[-2*x^2 + x^3 + x*y[x] - y[x]^2 + x*y[x]*Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(x*y(x)*diff(y(x),x)-y(x)^2+x*y(x)+x^3-2*x^2 = 0,y(x))`

2.235 ODE No. 235

$$(a + xy(x))y'(x) + by(x) = 0$$

✓ **Mathematica** : cpu = 0.209539 (sec), leaf count = 40

$$\text{Solve} \left[x = -\frac{ae^{-\frac{y(x)}{b}} \text{Ei}\left(\frac{y(x)}{b}\right)}{b} + c_1 e^{-\frac{y(x)}{b}}, y(x) \right]$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 30

$$\left\{ c_1 + \frac{1}{-bx e^{\frac{y(x)}{b}} + a \text{Ei}\left(1, -\frac{y(x)}{b}\right)} = 0 \right\}$$

2.236 ODE No. 236

$$x(y(x) + 4)y'(x) - y(x)^2 - 2y(x) - 2x = 0$$

✓ **Mathematica** : cpu = 0.238047 (sec), leaf count = 114

$$\left(\left(\left(y(x) \rightarrow -4 + \frac{1}{x \left(\frac{1}{x^2 + 4x} - \frac{e^{-2\left(\frac{\log(x)}{4} + \frac{3}{4} \log(x+4)\right)}}{\sqrt{-\frac{4}{x+4} + c_1}} \right)} \right) \right) \right(\left(y(x) \rightarrow -4 + \frac{1}{x \left(\frac{1}{x^2 + 4x} + \frac{e^{-2\left(\frac{\log(x)}{4} + \frac{3}{4} \log(x+4)\right)}}{\sqrt{-\frac{4}{x+4} + c_1}} \right)} \right) \right)$$

✓ **Maple** : cpu = 0.065 (sec), leaf count = 141

$$\left\{ y(x) = \frac{-4x^{\frac{3}{2}} - (x+4)^{\frac{3}{2}} \sqrt{\frac{c_1(x+4)-4}{x+4}} x - 16\sqrt{x}}{x^{\frac{3}{2}} + 4\sqrt{x} - (x+4)^{\frac{3}{2}} \sqrt{\frac{c_1(x+4)-4}{x+4}}}, y(x) = \frac{-4x^{\frac{3}{2}} + (x+4)^{\frac{3}{2}} \sqrt{\frac{c_1(x+4)-4}{x+4}} x - 16\sqrt{x}}{x^{\frac{3}{2}} + 4\sqrt{x} + (x+4)^{\frac{3}{2}} \sqrt{\frac{c_1(x+4)-4}{x+4}}} \right\}$$

2.237 ODE No. 237

$$x(a + y(x))y'(x) + by(x) + cx = 0$$

✗ **Mathematica** : cpu = 6.10842 (sec), leaf count = 0 , could not solve

`DSolve[c*x + b*y[x] + x*(a + y[x])*Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(x*(y(x)+a)*diff(y(x),x)+b*y(x)+c*x = 0,y(x))`

2.238 ODE No. 238

$$(a + x(y(x) + x))y'(x) - b - y(x)(y(x) + x) = 0$$

✓ **Mathematica** : cpu = 0.398748 (sec), leaf count = 192

$$\left\{ \left\{ y(x) \rightarrow -\frac{a+x^2}{x} + \frac{1}{x \left(-\frac{a}{-a^2-ax^2-bx^2} - \frac{x}{(a^2+ax^2+bx^2)^{3/2} \sqrt{-\frac{1}{(a+b)(a^2+ax^2+bx^2)} + c_1}} \right)} \right\}, \left\{ y(x) \rightarrow -\frac{a+x^2}{x} + \frac{1}{x \left(-\frac{a}{-a^2-ax^2-bx^2} + \frac{x}{(a^2+ax^2+bx^2)^{3/2} \sqrt{-\frac{1}{(a+b)(a^2+ax^2+bx^2)} + c_1}} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.079 (sec), leaf count = 93

$$\left\{ y(x) = \frac{-abx - c_1x + \sqrt{c_1(a+b)(ax^2 + bx^2 + a^2 - c_1)}}{-a^2 + c_1}, y(x) = \frac{abx + c_1x + \sqrt{c_1(a+b)(ax^2 + bx^2 + a^2 - c_1)}}{a^2 - c_1} \right\}$$

2.239 ODE No. 239

$$(xy(x) - x^2)y'(x) - 2x^2 - 3xy(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.165871 (sec), leaf count = 54

$$\left\{ \left\{ y(x) \rightarrow x - \frac{\sqrt{2x^4 + e^{2c_1}}}{x} \right\}, \left\{ y(x) \rightarrow x + \frac{\sqrt{2x^4 + e^{2c_1}}}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.171 (sec), leaf count = 59

$$\left\{ y(x) = \frac{c_1x^2 - \sqrt{2x^4c_1^2 + 1}}{c_1x}, y(x) = \frac{c_1x^2 + \sqrt{2x^4c_1^2 + 1}}{c_1x} \right\}$$

2.240 ODE No. 240

$$ax + 2xy(x)y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.132447 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{-ax \log(x) + c_1x} \right\}, \left\{ y(x) \rightarrow \sqrt{-ax \log(x) + c_1x} \right\} \right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 34

$$\left\{ y(x) = \sqrt{-ax \ln(x) + c_1x}, y(x) = -\sqrt{-(a \ln(x) - c_1)x} \right\}$$

2.241 ODE No. 241

$$ax^2 + 2xy(x)y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.123503 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{-ax^2 + c_1x} \right\}, \left\{ y(x) \rightarrow \sqrt{-ax^2 + c_1x} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt{-a x^2 + c_1x}, y(x) = -\sqrt{-a x^2 + c_1x} \right\}$$

2.242 ODE No. 242

$$2xy(x)y'(x) + 2y(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 0.113008 (sec), leaf count = 60

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{-x^2 + e^{4c_1}}}{\sqrt{2}x} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{-x^2 + e^{4c_1}}}{\sqrt{2}x} \right\} \right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 39

$$\left\{ y(x) = -\frac{\sqrt{-2x^2 + 4c_1}}{2x}, y(x) = \frac{\sqrt{-2x^2 + 4c_1}}{2x} \right\}$$

2.243 ODE No. 243

$$x(2y(x) + x - 1)y'(x) - y(x)(y(x) + 2x + 1) = 0$$

✓ **Mathematica** : cpu = 14.9924 (sec), leaf count = 487

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt[3]{2}x}{\sqrt[3]{-27c_1^2x^2 + \sqrt{108c_1^3x^3 + (27c_1^2x - 27c_1^2x^2)^2} + 27c_1^2x}} + \frac{\sqrt[3]{-27c_1^2x^2 + \sqrt{108c_1^3x^3 + (27c_1^2x - 27c_1^2x^2)^2} + 27c_1^2x}}{3\sqrt[3]{2}c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.132 (sec), leaf count = 391

$$\left\{ y(x) = \frac{-3c_1 \left((1 + i\sqrt{3})5^{\frac{2}{3}}x - \frac{80 \left(\left(20x + \sqrt{5} \sqrt{\frac{80c_1(x-1)^2 - x}{c_1}} - 20 \right) c_1^2 x \right)^{\frac{1}{3}}}{3} \right)^{\frac{1}{3}}}{80 \left(\left(20x + \sqrt{5} \sqrt{\frac{80c_1(x-1)^2 - x}{c_1}} - 20 \right) c_1^2 x \right)^{\frac{1}{3}} c_1} + 3 \left(\left(20x + \sqrt{5} \sqrt{\frac{80c_1(x-1)^2 - x}{c_1}} - 20 \right) c_1^2 x \right)^{\frac{2}{3}} (i\sqrt{3} - 1) 5^{\frac{1}{3}} \right\}$$

2.244 ODE No. 244

$$x(2y(x) - x - 1)y'(x) + (-y(x) + 2x - 1)y(x) = 0$$

✓ **Mathematica** : cpu = 14.3797 (sec), leaf count = 484

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt[3]{2}x}{\sqrt[3]{27c_1^2x^2 + \sqrt{(27c_1^2x^2 + 27c_1^2x)^2 - 108c_1^3x^3} + 27c_1^2x}} - \frac{\sqrt[3]{27c_1^2x^2 + \sqrt{(27c_1^2x^2 + 27c_1^2x)^2 - 108c_1^3x^3} + 27c_1^2x}}{3\sqrt[3]{2}c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.109 (sec), leaf count = 391

$$\left\{ \begin{aligned} y(x) = & -\frac{3c_1 \left((1+i\sqrt{3})5^{\frac{2}{3}x} + \frac{80 \left(-20 \left(x - \frac{\sqrt{5} \sqrt{\frac{80c_1(x+1)^2-x}{c_1}}}{20} + 1 \right) c_1^2 x \right)^{\frac{1}{3}}}{3} \right)^{\frac{1}{3}}}{80} + \frac{3(i\sqrt{3}-1) \left(\left(-20x + \sqrt{5} \sqrt{\frac{80c_1(x+1)^2-x}{c_1}} - 20 \right) c_1^2 x \right)^{\frac{2}{3}}}{80} 5^{\frac{1}{3}} \\ & - \frac{\left(-20 \left(x - \frac{\sqrt{5} \sqrt{\frac{80c_1(x+1)^2-x}{c_1}}}{20} + 1 \right) c_1^2 x \right)^{\frac{1}{3}}}{c_1} \end{aligned} \right\}, y(x) = -\frac{3 \left(-c_1 \left(i \sqrt{3} \right) \right)^{\frac{1}{3}}}{80}$$

2.245 ODE No. 245

$$(4x^3 + 2xy(x))y'(x) + 112x^2y(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.554292 (sec), leaf count = 1453

$$\left\{ \left\{ y(x) \rightarrow \text{Root} \left[-1521681143169024 \#1 x^{22} - 697437190619136 \#1^2 x^{20} - 145299414712320 \#1^3 x^{18} - 18162426839040 \#1^4 x^{16} - 18162426839040 \#1^5 x^{14} - 18162426839040 \#1^6 x^{12} - 18162426839040 \#1^7 x^{10} - 18162426839040 \#1^8 x^8 - 18162426839040 \#1^9 x^6 - 18162426839040 \#1^{10} x^4 - 18162426839040 \#1^{11} x^2 - 18162426839040 \#1^{12} \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.898 (sec), leaf count = 31

$$\left\{ y(x) = \frac{c_1}{x^{28} \text{RootOf}(x^{30} \_Z^{360} - 24x^{30} \_Z^{330} - c_1)^{330}} \right\}$$

2.246 ODE No. 246

$$x(3y(x) + 2x)y'(x) + 3(y(x) + x)^2 = 0$$

✓ **Mathematica** : cpu = 0.199788 (sec), leaf count = 80

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{6} \left(-4x - \frac{\sqrt{2} \sqrt{-x^4 + 3e^{4c_1}}}{x} \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{6} \left(-4x + \frac{\sqrt{2} \sqrt{-x^4 + 3e^{4c_1}}}{x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.079 (sec), leaf count = 63

$$\left\{ y(x) = \frac{-4c_1x^2 - \sqrt{-2x^4c_1^2 + 6}}{6c_1x}, y(x) = \frac{-4c_1x^2 + \sqrt{-2x^4c_1^2 + 6}}{6c_1x} \right\}$$

2.247 ODE No. 247

$$-7x^2 + (3x + 2)(y(x) - 2x - 1)y'(x) + xy(x) - y(x)^2 - 9x - 3 = 0$$

✓ **Mathematica** : cpu = 13.318 (sec), leaf count = 693

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{1458x^3 + 2916x^2 + \sqrt{4(-81x^2 - 108x - 36)^3 + (1458x^3 + 2916x^2 + 1944x - 324e^{2c_1}x + 432 - 216e^{2c_1})^2}}}{6\sqrt[3]{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.214 (sec), leaf count = 517

$$\left\{ \begin{array}{l} (3x+2) \left(7 - \frac{9(3x+2)^2 c_1^2}{4 \left(-27(3x+2)^3 c_1^3 + 2c_1(3x+2) + 2\sqrt{-27(3x+2)^4 c_1^4 + (3x+2)^2 c_1^2} \right)^{\frac{1}{3}}} - \frac{3c_1(3x+2)}{2} - \frac{\left(-27(3x+2)^3 c_1^3 + 2c_1(3x+2) + 2\sqrt{-27(3x+2)^4 c_1^4 + (3x+2)^2 c_1^2} \right)}{4} \right) \\ y(x) = -\frac{1}{3} + \frac{6 \left(-\frac{9(3x+2)^2 c_1^2}{4 \left(-27(3x+2)^3 c_1^3 + 2c_1(3x+2) + 2\sqrt{-27(3x+2)^4 c_1^4 + (3x+2)^2 c_1^2} \right)^{\frac{1}{3}}} - \frac{3c_1(3x+2)}{2} - \frac{\left(-27(3x+2)^3 c_1^3 + 2c_1(3x+2) + 2\sqrt{-27(3x+2)^4 c_1^4 + (3x+2)^2 c_1^2} \right)}{4} \right)}{1} \end{array} \right.$$

2.248 ODE No. 248

$$(x^2 + 6xy(x) + 3)y'(x) + 3y(x)^2 + 2xy(x) + 2x = 0$$

✓ **Mathematica** : cpu = 0.254448 (sec), leaf count = 110

$$\left\{ \left(y(x) \rightarrow \frac{-x^2 - 3}{6x} - \frac{\sqrt{-2x^3 + \frac{1}{6}(x^2 + 3)^2 + 6c_1x}}{\sqrt{6}x} \right), \left(y(x) \rightarrow \frac{-x^2 - 3}{6x} + \frac{\sqrt{-2x^3 + \frac{1}{6}(x^2 + 3)^2 + 6c_1x}}{\sqrt{6}x} \right) \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 75

$$\left\{ y(x) = \frac{-x^2 - 3 - \sqrt{x^4 - 12x^3 + 6x^2 - 12c_1x + 9}}{6x}, y(x) = \frac{-x^2 - 3 + \sqrt{x^4 - 12x^3 + 6x^2 - 12c_1x + 9}}{6x} \right\}$$

2.249 ODE No. 249

$$y'(x)(axy(x) + bx^n) + \alpha y(x)^3 + \beta y(x)^2 = 0$$

✓ **Mathematica** : cpu = 5.65868 (sec), leaf count = 115

$$\text{Solve} \left[\frac{(a(-n) + a + \alpha y(x))y(x)^{\frac{a-an}{\beta}-1}(\alpha y(x) + \beta)^{\frac{a(n-1)}{\beta}}}{a^2(n-1)^2(a(n-1) + \beta)} + \frac{x^{1-n} \exp\left(-\frac{a(n-1)(\log(y(x)) - \log(\alpha y(x) + \beta))}{\beta}\right)}{ab(1-n)(n-1)} = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.255 (sec), leaf count = 202

$$\left\{ y(x) = \frac{\beta}{\beta \text{RootOf}\left(c_1 a^2 b n^2 - a^2 \beta n \_Z^{\frac{(n-1)a}{\beta}} x^{-n+1} - 2c_1 a^2 b n + c_1 a b \beta n + a^2 \beta \_Z^{\frac{(n-1)a}{\beta}} x^{-n+1} + a \alpha b n \_Z^{\frac{(n-1)a}{\beta}} - a b \beta n \_Z^{\frac{an-a}{\beta}}\right)} \right.$$

2.250 ODE No. 250

$$y'(x)(ax + Ax^2 + by(x) + Bxy(x) + c) + Axy(x) + ax - By(x)^2 + \beta y(x) + \gamma = 0$$

✗ **Mathematica** : cpu = 300.652 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve((B*x*y(x)+A*x^2+a*x+b*y(x)+c)*diff(y(x),x)-B*g(x)^2+A*x*y(x)+alpha*x+beta*y(x)+gamma

2.251 ODE No. 251

$$(x^2y(x) - 1)y'(x) + xy(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.148873 (sec), leaf count = 60

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{x^2} - \frac{\sqrt{2x^3 + c_1x^2 + 1}}{x^2} \right\}, \left\{ y(x) \rightarrow \frac{1}{x^2} + \frac{\sqrt{2x^3 + c_1x^2 + 1}}{x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 51

$$\left\{ y(x) = \frac{1 - \sqrt{2x^3 - 2c_1x^2 + 1}}{x^2}, y(x) = \frac{1 + \sqrt{2x^3 - 2c_1x^2 + 1}}{x^2} \right\}$$

2.252 ODE No. 252

$$(x^2y(x) - 1)y'(x) - xy(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 10.0071 (sec), leaf count = 819

$$\left\{ \left\{ y(x) \rightarrow \frac{6xc_1 - x}{6c_1 - 1} + \frac{\sqrt[3]{-1944c_1^2x^3 + 648c_1x^3 - 54x^3 + 1944c_1^2 - 648c_1 + \sqrt{4(54x^2c_1 - 9x^2)^3 + (-1944c_1^2x^3 + 648c_1)}}{3\sqrt[3]{2}(6c_1 - 1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.662 (sec), leaf count = 1338

$$\left\{ y(x) = \frac{(c_1^2 - 80c_1)x^8 + 160c_1x^5 - 80c_1x^2 + ((-c_1 + 80)x^7 - 160x^4 + 80x)4^{\frac{1}{3}}\left(c_1\left(-\frac{1}{4} + \sqrt{\frac{-5x^6 + 10x^3 - 5}{(c_1 - 80)x^6 + 160x^3 - 80}}\right)((c_1 - 80)x\right.\right.}{\left.\left.\left((c_1^2 - 80c_1)x^8 + 160c_1x^5 - 80c_1x^2 + \left(c_1\left(-1 + 4\sqrt{\frac{-5x^6 + 10x^3 - 5}{(c_1 - 80)x^6 + 160x^3 - 80}}\right)(c_1x^6 - 80x^6 + 160x^3 - 80)\right)^{\frac{2}{3}}\right)x^2 + ((-c_1\right.}}{x^2 + ((-c_1$$

2.253 ODE No. 253

$$(x^2y(x) - 1)y'(x) + 8xy(x)^2 - 8 = 0$$

✗ **Mathematica** : cpu = 12.4471 (sec), leaf count = 0 , could not solve

`DSolve[-8 + 8*x*y[x]^2 + (-1 + x^2*y[x])*Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve((x^2*y(x)-1)*diff(y(x),x)+8*x*y(x)^2-8 = 0,y(x))`

2.254 ODE No. 254

$$x^2y(x)^3 + x(xy(x) - 2)y'(x) + xy(x)^2 - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.148685 (sec), leaf count = 99

$$\left\{ \left\{ y(x) \rightarrow -\frac{2x}{-x^2 + \frac{\sqrt{2}\sqrt{-\frac{x}{2} - 2x(-\log(x) + c_1)}}{\sqrt{-\frac{1}{x^3}}}} \right\}, \left\{ y(x) \rightarrow \frac{2x}{x^2 + \frac{\sqrt{2}\sqrt{-\frac{x}{2} - 2x(-\log(x) + c_1)}}{\sqrt{-\frac{1}{x^3}}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 59

$$\left\{ y(x) = \frac{-1 + \sqrt{4c_1 - 4\ln(x) + 1}}{2(c_1 - \ln(x))x}, y(x) = \frac{1 + \sqrt{4c_1 - 4\ln(x) + 1}}{2(-c_1 + \ln(x))x} \right\}$$

2.255 ODE No. 255

$$x(xy(x) - 3)y'(x) + xy(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 6.01949 (sec), leaf count = 30

$$\left\{ \left\{ \left(\frac{3W\left(e^{-1+\frac{9c_1}{2^{2/3}}x^{2/3}}\right)}{x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.214 (sec), leaf count = 74

$$\left\{ y(x) = -\frac{3 \operatorname{LambertW}\left(\frac{2c_1\left(-\frac{x^2}{8}\right)^{\frac{1}{3}}}{3}\right)}{x}, y(x) = -\frac{3 \operatorname{LambertW}\left(-\frac{c_1\left(-\frac{x^2}{8}\right)^{\frac{1}{3}}(1+i\sqrt{3})}{3}\right)}{x}, y(x) = -\frac{3 \operatorname{LambertW}\left(\frac{c_1\left(-\frac{x^2}{8}\right)^{\frac{1}{3}}(i\sqrt{3}-1)}{3}\right)}{x} \right\}$$

2.256 ODE No. 256

$$x^2(y(x) - 1)y'(x) + (x - 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0334352 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow -W\left(x\left(-e^{\frac{1}{x}-c_1}\right)\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.154 (sec), leaf count = 31

$$\left\{ y(x) = e^{\frac{c_1 x - x \operatorname{LambertW}\left(-x e^{c_1 + \frac{1}{x}}\right) + x \ln(x) + 1}{x}} \right\}$$

2.257 ODE No. 257

$$x(x^4 + xy(x) - 1)y'(x) - y(x)(-x^4 + xy(x) - 1) = 0$$

✓ **Mathematica** : cpu = 0.537836 (sec), leaf count = 39

$$\operatorname{Solve}\left[2x^2 + \frac{y(x)}{x} + \frac{x\left(-2\log\left(\frac{1}{1-xy(x)}\right) - 2 + c_1\right)}{y(x)} = 0, y(x)\right]$$

✓ **Maple** : cpu = 0.119 (sec), leaf count = 98

$$\left\{ y(x) = \frac{\left(-c_1 + e^{\operatorname{RootOf}\left(-2c_1x^4e^{-Z}-2_Zx^4e^{2-Z}+2x^4e^{2-Z}+c_1^2-2c_1e^{-Z}+e^{2-Z}\right)}\right)e^{-\operatorname{RootOf}\left(-2c_1x^4e^{-Z}-2_Zx^4e^{2-Z}+2x^4e^{2-Z}+c_1^2-2c_1e^{-Z}+e^{2-Z}\right)}}{x} \right\}$$

2.258 ODE No. 258

$$-2x^3 + 2x^2y(x)y'(x) - x^2 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.141946 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{x^2 + c_1e^{\frac{1}{x}}} \right\}, \left\{ y(x) \rightarrow \sqrt{x^2 + c_1e^{\frac{1}{x}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.028 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt{c_1 e^{\frac{1}{x}} + x^2}, y(x) = -\sqrt{c_1 e^{\frac{1}{x}} + x^2} \right\}$$

2.259 ODE No. 259

$$2x^2 y(x) y'(x) - e^{x - \frac{1}{x}} x^2 - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.282333 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow -e^{-\frac{1}{2}/x} \sqrt{e^x + c_1} \right\}, \left\{ y(x) \rightarrow e^{-\frac{1}{2}/x} \sqrt{e^x + c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 51

$$\left\{ y(x) = \sqrt{c_1 e^{-\frac{1}{x}} + e^{\frac{x^2-1}{x}}}, y(x) = -\sqrt{c_1 e^{-\frac{1}{x}} + e^{\frac{x^2-1}{x}}} \right\}$$

2.260 ODE No. 260

$$(2x^2 y(x) + x) y'(x) - x^2 y(x)^3 + 2x y(x)^2 + y(x) = 0$$

✓ **Mathematica** : cpu = 0.145263 (sec), leaf count = 80

$$\left\{ \left\{ y(x) \rightarrow \frac{x}{-2x^2 + \frac{\sqrt{4x+x(-2\log(x)+c_1)}}{\sqrt{\frac{1}{x^3}}}} \right\}, \left\{ y(x) \rightarrow -\frac{x}{2x^2 + \frac{\sqrt{4x+x(-2\log(x)+c_1)}}{\sqrt{\frac{1}{x^3}}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 59

$$\left\{ y(x) = \frac{-2 + \sqrt{2c_1 - 2\ln(x) + 4}}{2(-c_1 + \ln(x))x}, y(x) = \frac{2 + \sqrt{2c_1 - 2\ln(x) + 4}}{2(c_1 - \ln(x))x} \right\}$$

2.261 ODE No. 261

$$(2x^2 y(x) - x) y'(x) - 2x y(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.763326 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2x W\left(\frac{e^{-1+\frac{9c_1}{2^{2/3}}}}{x^2}\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.171 (sec), leaf count = 18

$$\left\{ y(x) = -\frac{1}{2x \operatorname{LambertW}\left(-\frac{c_1}{2x^2}\right)} \right\}$$

2.262 ODE No. 262

$$2x^3 + (2x^2 y(x) - x^3) y'(x) - 4x y(x)^2 + y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.204838 (sec), leaf count = 101

$$\left\{ \left\{ y(x) \rightarrow \frac{2x^3 - \sqrt{e^{4c_1} x^2 - 3e^{2c_1} x^4}}{x^2 + e^{2c_1}} \right\}, \left\{ y(x) \rightarrow \frac{2x^3 + \sqrt{e^{4c_1} x^2 - 3e^{2c_1} x^4}}{x^2 + e^{2c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.25 (sec), leaf count = 65

$$\left\{y(x)=\frac{\left(2c_1x^2-\sqrt{3c_1x^2+1}\right)x}{c_1x^2-1},y(x)=\frac{\left(2c_1x^2+\sqrt{3c_1x^2+1}\right)x}{c_1x^2-1}\right\}$$

2.263 ODE No. 263

$$2x^3+3x^2y(x)^2+y(x)y'(x)+7=0$$

✓ **Mathematica** : cpu = 0.109689 (sec), leaf count = 181

$$\left\{\left\{y(x)\rightarrow-\sqrt{\frac{7\,2^{2/3}e^{-2x^3}x\Gamma\left(\frac{1}{3},-2x^3\right)}{3\sqrt[3]{-x^3}}-\frac{2^{2/3}e^{-2x^3}x\Gamma\left(\frac{4}{3},-2x^3\right)}{3\sqrt[3]{-x^3}}+c_1e^{-2x^3}}\right\},\left\{y(x)\rightarrow\sqrt{\frac{7\,2^{2/3}e^{-2x^3}x\Gamma\left(\frac{1}{3},-2x^3\right)}{3\sqrt[3]{-x^3}}-\frac{2^{2/3}e^{-2x^3}x\Gamma\left(\frac{4}{3},-2x^3\right)}{3\sqrt[3]{-x^3}}+c_1e^{-2x^3}}\right\}\right\}$$

✓ **Maple** : cpu = 0.162 (sec), leaf count = 173

$$\left\{y(x)=-\frac{2^{\frac{2}{3}}\sqrt{-240\left(\left(-\frac{3\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{1}{3},-2x^3\right)}{2}+\pi\sqrt{3}\right)x\,e^{-2x^3}+\frac{9\left(-\frac{3c_1e^{-2x^3}}{2}+x\right)2^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)\left(-x^3\right)^{\frac{1}{3}}}{40}\right)2^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)\left(-x^3\right)^{\frac{1}{3}}}}{18\left(-x^3\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)},y(x)=\frac{2^{\frac{2}{3}}\sqrt{-240\left(\left(-\frac{3\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{1}{3},-2x^3\right)}{2}+\pi\sqrt{3}\right)x\,e^{-2x^3}+\frac{9\left(-\frac{3c_1e^{-2x^3}}{2}+x\right)2^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)\left(-x^3\right)^{\frac{1}{3}}}{40}\right)2^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)\left(-x^3\right)^{\frac{1}{3}}}}{18\left(-x^3\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)}\right\}$$

2.264 ODE No. 264

$$2x\big(x^3y(x)+1\big)y'(x)+y(x)\big(3x^3y(x)-1\big)=0$$

✓ **Mathematica** : cpu = 0.445037 (sec), leaf count = 680

$$\left\{\left\{y(x)\rightarrow\text{Root}\left[81\#1^7e^{\frac{21c_1}{2}}x^{12}+756\#1^6e^{\frac{21c_1}{2}}x^9+2646\#1^5e^{\frac{21c_1}{2}}x^6+4116\#1^4e^{\frac{21c_1}{2}}x^3+2401\#1^3e^{\frac{21c_1}{2}}-x^{3/2}\&,1\right]\right\},\left\{y(x)\rightarrow-\text{Root}\left[81\#1^7e^{\frac{21c_1}{2}}x^{12}+756\#1^6e^{\frac{21c_1}{2}}x^9+2646\#1^5e^{\frac{21c_1}{2}}x^6+4116\#1^4e^{\frac{21c_1}{2}}x^3+2401\#1^3e^{\frac{21c_1}{2}}-x^{3/2}\&,1\right]\right\}\right\}$$

✓ **Maple** : cpu = 0.433 (sec), leaf count = 37

$$\left\{y(x)=\frac{\text{RootOf}\left(c_1Z^{98}-14c_1Z^{77}+49c_1Z^{56}-9x^7\right)^{21}-7}{3x^3}\right\}$$

2.265 ODE No. 265

$$2(n+1)^2x^{n-1}\big(x^{n^2}y(x)^2-1\big)+\big(x^{n(n+1)}y(x)-1\big)y'(x)=0$$

✗ **Mathematica** : cpu = 287.249 (sec), leaf count = 0 , could not solve

`DSolve[2*(1 + n)^2*x^(-1 + n)*(-1 + x^n^2*y[x]^2) + (-1 + x^(n*(1 + n)))*y[x]]*Derivative[1][y[x]] == 0, y[x]]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve((x^(n*(n+1)))*y(x)-1)*diff(y(x),x)+2*(n+1)^2*x^(n-1)*(x^(n^2)*y(x)^2-1) = 0,y(x))`

2.266 ODE No. 266

$$\sqrt{x^2+1}\big(y(x)-x\big)y'(x)-a\sqrt{\big(y(x)^2+1\big)^3}=0$$

✓ **Mathematica** : cpu = 2.49879 (sec), leaf count = 69

Solve $\left[\left[\frac{2a \tan^{-1}\left(\frac{1-a \tan\left(\frac{K[1]}{2}\right)}{\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}}+K[1]+\tan^{-1}(x)=c_1, y(x)=\frac{\tan(K[1])+x}{1-x \tan(K[1])}\right],\{K[1], y(x)\}\right]$

✓ **Maple** : cpu = 2.078 (sec), leaf count = 65

$\left\{y(x)=\tan\left(\text{RootOf}\left(c_1+\int^{-Z-\arctan(x)}\frac{-\cos(2\_a)+\sqrt{2}\sqrt{\frac{a^2}{\cos(2\_a)+1}}\sin(2\_a)+1}{2a^2+\cos(2\_a)-1}d\_a-\arctan(x)\right)\right\}$

2.267 ODE No. 267

$y(x)\sin^2(x)y'(x)+y(x)^2\sin(x)\cos(x)-1=0$

✓ **Mathematica** : cpu = 0.315005 (sec), leaf count = 36

$\{\{y(x)\rightarrow-\sqrt{2x+c_1}\csc(x)\},\{y(x)\rightarrow\sqrt{2x+c_1}\csc(x)\}\}$

✓ **Maple** : cpu = 0.041 (sec), leaf count = 32

$\left\{y(x)=\frac{\sqrt{c_1+2x}}{\sin(x)},y(x)=-\frac{\sqrt{c_1+2x}}{\sin(x)}\right\}$

2.268 ODE No. 268

$f(x)y(x)y'(x)+g(x)y(x)^2+h(x)=0$

✓ **Mathematica** : cpu = 0.198925 (sec), leaf count = 146

$\left\{\left\{y(x)\rightarrow-\exp\left(\int_1^x-\frac{g(K[1])}{f(K[1])}dK[1]\right)\sqrt{2\int_1^x-\frac{\exp\left(-2\int_1^{K[2]}\frac{g(K[1])}{f(K[1])}dK[1]\right)h(K[2])}{f(K[2])}dK[2]+c_1}\right\},\left\{y(x)\rightarrow\exp\left(\int_1^x-\frac{g(K[1])}{f(K[1])}dK[1]\right)\sqrt{2\int_1^x-\frac{\exp\left(-2\int_1^{K[2]}\frac{g(K[1])}{f(K[1])}dK[1]\right)h(K[2])}{f(K[2])}dK[2]+c_1}\right\}\right\}$

✓ **Maple** : cpu = 0.09 (sec), leaf count = 118

$\left\{y(x)=\sqrt{\left(c_1-2\left(\int\frac{e^{\int\frac{2g(x)}{f(x)}dx}h(x)}{f(x)}dx\right)}e^{2\left(\int\frac{g(x)}{f(x)}dx\right)}e^{\int-\frac{2g(x)}{f(x)}dx},y(x)=-\sqrt{\left(c_1-2\left(\int\frac{e^{\int\frac{2g(x)}{f(x)}dx}h(x)}{f(x)}dx\right)}e^{2\left(\int\frac{g(x)}{f(x)}dx\right)}e^{\int-\frac{2g(x)}{f(x)}dx}\right\}$

2.269 ODE No. 269

$-f0(x)-f1(x)y(x)-f2(x)y(x)^2-f3(x)y(x)^3+y'(x)(g0(x)+g1(x)y(x))=0$

✗ **Mathematica** : cpu = 306.031 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve((g1(x)*y(x)+g0(x))*diff(y(x),x)-f1(x)*y(x)-f2(x)*y(x)^2-f3(x)*y(x)^3-f0(x) = 0,y(x))`

2.270 ODE No. 270

$$x^2 + (y(x)^2 - x)y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.15161 (sec), leaf count = 327

$$\left\{ \left\{ y(x) \rightarrow -\frac{3\sqrt[3]{2}x}{\sqrt[3]{27x^3 + \sqrt{-2916x^3 + (27x^3 + 81c_1)^2} + 81c_1}} - \frac{\sqrt[3]{27x^3 + \sqrt{-2916x^3 + (27x^3 + 81c_1)^2} + 81c_1}}{3\sqrt[3]{2}} \right\}, \left\{ y(x) \rightarrow \right.$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 319

$$\left\{ y(x) = \frac{4x + \left(-4x^3 - 12c_1 + 4\sqrt{x^6 + (6c_1 - 4)x^3 + 9c_1^2} \right)^{\frac{2}{3}}}{2\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + (6c_1 - 4)x^3 + 9c_1^2} \right)^{\frac{1}{3}}}, y(x) = -\frac{4x + \left(-4ix + i\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + (6c_1 - 4)x^3 + 9c_1^2} \right)^{\frac{2}{3}} \right)}{4\left(-4x^3 - 12c_1 + 4\sqrt{x^6 + (6c_1 - 4)x^3 + 9c_1^2} \right)^{\frac{1}{3}}} \right\}$$

2.271 ODE No. 271

$$(x^2 + y(x)^2)y'(x) + 2x(y(x) + 2x) = 0$$

✓ **Mathematica** : cpu = 0.300769 (sec), leaf count = 370

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{-4x^3 + \sqrt{20x^6 - 8e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}}{\sqrt[3]{2}} - \frac{\sqrt[3]{2}x^2}{\sqrt[3]{-4x^3 + \sqrt{20x^6 - 8e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt[3]{2}x^2}{2^{2/3}\sqrt[3]{-4x^3 + \sqrt{20x^6 - 8e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.191 (sec), leaf count = 352

$$\left\{ y(x) = \frac{-\frac{2c_1x^2}{\left(-16c_1^{\frac{3}{2}}x^3 + 4 + 4\sqrt{20c_1^3x^6 - 8c_1^{\frac{3}{2}}x^3 + 1} \right)^{\frac{1}{3}}} + \frac{\left(-16c_1^{\frac{3}{2}}x^3 + 4 + 4\sqrt{20c_1^3x^6 - 8c_1^{\frac{3}{2}}x^3 + 1} \right)^{\frac{1}{3}}}{2}}{\sqrt{c_1}}, y(x) = -\frac{-4c_1x^2 + \left(4ic_1x^2 + i\left(-16c_1^{\frac{3}{2}}x^3 + 4 + 4\sqrt{20c_1^3x^6 - 8c_1^{\frac{3}{2}}x^3 + 1} \right)^{\frac{1}{3}} \right)}{4\left(-16c_1^{\frac{3}{2}}x^3 + 4 + 4\sqrt{20c_1^3x^6 - 8c_1^{\frac{3}{2}}x^3 + 1} \right)^{\frac{1}{3}}} \right\}$$

2.272 ODE No. 272

$$(x^2 + y(x)^2)y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.164964 (sec), leaf count = 42

$$\text{Solve} \left[\log\left(\frac{y(x)}{x}\right) + \frac{2 \tan^{-1}\left(\frac{\frac{2y(x)}{x} - 1}{\sqrt{3}}\right)}{\sqrt{3}} = -\log(x) + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.155 (sec), leaf count = 43

$$\left\{ y(x) = e^{-c_1 + \frac{2\sqrt{3} \operatorname{RootOf}\left(3xe^{c_1} \tan(\_Z) - \sqrt{3}xe^{c_1} + 2\sqrt{3}e^{\frac{2\sqrt{3}\_Z}{3}} \right)}{3}} \right\}$$

2.273 ODE No. 273

$$(a + x^2 + y(x)^2)y'(x) + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.193793 (sec), leaf count = 297

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{\sqrt{2916(a+x^2)^3 + 6561c_1^2 + 81c_1}}}{3\sqrt[3]{2}} - \frac{3\sqrt[3]{2}(a+x^2)}{\sqrt[3]{\sqrt{2916(a+x^2)^3 + 6561c_1^2 + 81c_1}}} \right\}, \left\{ y(x) \rightarrow \frac{3(1+iy)}{2^{2/3}\sqrt[3]{\sqrt{2916(a+x^2)^3 + 6561c_1^2 + 81c_1}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 401

$$\left\{ y(x) = \frac{-4x^2 - 4a + \left(-12c_1 + 4\sqrt{4x^6 + 12ax^4 + 12a^2x^2 + 4a^3 + 9c_1^2}\right)^{\frac{2}{3}}}{2\left(-12c_1 + 4\sqrt{4x^6 + 12ax^4 + 12a^2x^2 + 4a^3 + 9c_1^2}\right)^{\frac{1}{3}}}, y(x) = -\frac{-4x^2 - 4a + \left(4ix^2 + 4ia + i(-12c_1 + 4\sqrt{4x^6 + 12ax^4 + 12a^2x^2 + 4a^3 + 9c_1^2})\right)^{\frac{2}{3}}}{2\left(-12c_1 + 4\sqrt{4x^6 + 12ax^4 + 12a^2x^2 + 4a^3 + 9c_1^2}\right)^{\frac{1}{3}}} \right\}$$

2.274 ODE No. 274

$$(a + x^2 + y(x)^2)y'(x) + b + x^2 + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.201124 (sec), leaf count = 411

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{\sqrt{2916(a+x^2)^3 + (-81bx - 27x^3 + 81c_1)^2 - 81bx - 27x^3 + 81c_1}}}{3\sqrt[3]{2}} - \frac{3\sqrt[3]{2}(a+x^2)}{\sqrt[3]{\sqrt{2916(a+x^2)^3 + (-81bx - 27x^3 + 81c_1)^2 - 81bx - 27x^3 + 81c_1}}} \right\}, \left\{ y(x) \rightarrow \frac{3(1+iy)}{2^{2/3}\sqrt[3]{\sqrt{2916(a+x^2)^3 + (-81bx - 27x^3 + 81c_1)^2 - 81bx - 27x^3 + 81c_1}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 657

$$\left\{ y(x) = \frac{-4x^2 - 4a + \left(-4x^3 - 12bx - 12c_1 + 4\sqrt{5x^6 + 6c_1x^3 + (12a + 6b)x^4 + 18c_1bx + 4a^3 + 9c_1^2 + (12a^2 + 9b^2)x^2}\right)^{\frac{2}{3}}}{2\left(-4x^3 - 12bx - 12c_1 + 4\sqrt{5x^6 + 6c_1x^3 + (12a + 6b)x^4 + 18c_1bx + 4a^3 + 9c_1^2 + (12a^2 + 9b^2)x^2}\right)^{\frac{1}{3}}}, y(x) = -\frac{-4x^2 - 4a + \left(4ix^2 + 4ia + i(-12c_1 + 4\sqrt{5x^6 + 6c_1x^3 + (12a + 6b)x^4 + 18c_1bx + 4a^3 + 9c_1^2 + (12a^2 + 9b^2)x^2})\right)^{\frac{2}{3}}}{2\left(-4x^3 - 12bx - 12c_1 + 4\sqrt{5x^6 + 6c_1x^3 + (12a + 6b)x^4 + 18c_1bx + 4a^3 + 9c_1^2 + (12a^2 + 9b^2)x^2}\right)^{\frac{1}{3}}} \right\}$$

2.275 ODE No. 275

$$(x^2 + y(x)^2 + x)y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.149678 (sec), leaf count = 18

$$\text{Solve}\left[y(x) - \tan^{-1}\left(\frac{x}{y(x)}\right) = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 30

$$\left\{ c_1 + \frac{(ix + y(x))e^{-2iy(x)}}{2x + 2iy(x)} = 0 \right\}$$

2.276 ODE No. 276

$$(y(x)^2 - x^2)y'(x) + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.15576 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left(e^{c_1} - \sqrt{-4x^2 + e^{2c_1}} \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \left(\sqrt{-4x^2 + e^{2c_1}} + e^{c_1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 47

$$\left\{ y(x) = \frac{1 - \sqrt{-4c_1^2x^2 + 1}}{2c_1}, y(x) = \frac{1 + \sqrt{-4c_1^2x^2 + 1}}{2c_1} \right\}$$

2.277 ODE No. 277

$$(x^4 + y(x)^2)y'(x) - 4x^3y(x) = 0$$

✓ **Mathematica** : cpu = 0.118486 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left(c_1 - \sqrt{4x^4 + c_1^2} \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \left(\sqrt{4x^4 + c_1^2} + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.312 (sec), leaf count = 41

$$\left\{ y(x) = \frac{c_1}{2} - \frac{\sqrt{4x^4 + c_1^2}}{2}, y(x) = \frac{c_1}{2} + \frac{\sqrt{4x^4 + c_1^2}}{2} \right\}$$

2.278 ODE No. 278

$$y'(x)(y(x)^2 + 4\sin(x)) - \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.246344 (sec), leaf count = 39

$$\text{Solve} \left[-\frac{1}{32} e^{-4y(x)} (8y(x)^2 + 4y(x) + 1) - e^{-4y(x)} \sin(x) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.061 (sec), leaf count = 28

$$\left\{ c_1 + \frac{(-8y(x)^2 - 32\sin(x) - 4y(x) - 1)e^{-4y(x)}}{32} = 0 \right\}$$

2.279 ODE No. 279

$$(y(x)^2 + 2y(x) + x)y'(x) + y(x)^2(y(x) + x)^2 + y(x)(y(x) + 1) = 0$$

✓ **Mathematica** : cpu = 0.643367 (sec), leaf count = 107

$$\left\{ \left\{ y(x) \rightarrow \frac{-x^2 - \sqrt{(x^2 - c_1x - 1)^2 + 4(x - c_1) + c_1x + 1}}{2(x - c_1)} \right\}, \left\{ y(x) \rightarrow \frac{-x^2 + \sqrt{(x^2 - c_1x - 1)^2 + 4(x - c_1) + c_1x + 1}}{2(x - c_1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.161 (sec), leaf count = 116

$$\left\{ y(x) = \frac{-c_1x + 2x^2 + \sqrt{-4c_1x^3 + 4x^4 + (c_1^2 - 8)x^2 - 8c_1 + (4c_1 + 16)x + 4} - 2}{2c_1 - 4x}, y(x) = \frac{c_1x - 2x^2 + \sqrt{-4c_1x^3 + 4x^4 + (c_1^2 - 8)x^2 - 8c_1 + (4c_1 + 16)x + 4} - 2}{-2} \right\}$$

2.280 ODE No. 280

$$(y(x) + x)^2 y'(x) - a^2 = 0$$

✓ **Mathematica** : cpu = 0.150665 (sec), leaf count = 21

$$\text{Solve}\left[y(x) - a \tan^{-1}\left(\frac{y(x) + x}{a}\right) = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.067 (sec), leaf count = 24

$$\{y(x) = a \text{RootOf}(-\_Za + a \tan(\_Z) + c_1 - x) - c_1\}$$

2.281 ODE No. 281

$$(-x^2 + 2xy(x) + y(x)^2)y'(x) + x^2 + 2xy(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.224416 (sec), leaf count = 75

$$\left\{\left\{y(x) \rightarrow \frac{1}{2}\left(e^{c_1} - \sqrt{-4x^2 + 4e^{c_1}x + e^{2c_1}}\right)\right\}, \left\{y(x) \rightarrow \frac{1}{2}\left(\sqrt{-4x^2 + 4e^{c_1}x + e^{2c_1}} + e^{c_1}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 55

$$\left\{y(x) = \frac{1 - \sqrt{-4c_1^2x^2 + 4c_1x + 1}}{2c_1}, y(x) = \frac{1 + \sqrt{-4c_1^2x^2 + 4c_1x + 1}}{2c_1}\right\}$$

2.282 ODE No. 282

$$(y(x) + 3x - 1)^2 y'(x) - (2y(x) - 1)(4y(x) + 6x - 3) = 0$$

✓ **Mathematica** : cpu = 0.256761 (sec), leaf count = 2129

$$\left\{\left\{y(x) \rightarrow \frac{1}{6}(12x + 4e^{c_1} + 1) - \frac{1}{6}\sqrt{36x^2 + 96e^{c_1}x - 12x - 16e^{c_1} + 16e^{2c_1} + 3^{2/3}\sqrt{-7776e^{c_1}x^5 + 6480e^{c_1}x^4 - 1296e^{2c_1}x^3 - 1296e^{3c_1}x^2 + 1296e^{4c_1}x - 1296e^{5c_1}}}\right\}\right\}$$

✓ **Maple** : cpu = 0.337 (sec), leaf count = 71

$$\left\{-c_1 + 3 \ln\left(\frac{-6y(x) + 3}{6x - 1}\right) - 3 \ln\left(\frac{18x - 6y(x)}{6x - 1}\right) - \ln\left(\frac{-6x - 6y(x) + 4}{6x - 1}\right) - \ln(6x - 1) = 0\right\}$$

2.283 ODE No. 283

$$3(y(x)^2 - x^2)y'(x) + 2y(x)^3 - 6x(x + 1)y(x) - 3e^x = 0$$

✓ **Mathematica** : cpu = 0.390417 (sec), leaf count = 477

$$\left\{\left\{y(x) \rightarrow -\frac{e^{-2x}\sqrt[3]{\sqrt{-2916e^{12x}x^6 + (-27e^{7x} + 27c_1e^{4x})^2} - 27e^{7x} + 27c_1e^{4x}}}{3\sqrt[3]{2}} - \frac{3\sqrt[3]{2}e^{2x}x^2}{\sqrt[3]{\sqrt{-2916e^{12x}x^6 + (-27e^{7x} + 27c_1e^{4x})^2} - 27e^{7x} + 27c_1e^{4x}}}\right\}\right\}$$

✓ **Maple** : cpu = 0.079 (sec), leaf count = 407

$$\left\{y(x) = \frac{\left(4x^2e^{4x} + \left((-4c_1 + 4e^{3x} + 4\sqrt{-4x^6e^{4x} + c_1^2 - 2c_1e^{3x} + e^{6x}})e^{4x}\right)^{\frac{2}{3}}\right)e^{-2x}}{2\left(-4\left(c_1 - e^{3x} - \sqrt{-4x^6e^{4x} + c_1^2 - 2c_1e^{3x} + e^{6x}}\right)e^{4x}\right)^{\frac{1}{3}}}, y(x) = \frac{\left(-4(1 + i\sqrt{3})x^2e^{4x} + (i\sqrt{3} - 1)e^{4x}\right)}{4\left(-4\left(c_1 - e^{3x} - \sqrt{-4x^6e^{4x} + c_1^2 - 2c_1e^{3x} + e^{6x}}\right)e^{4x}\right)^{\frac{1}{3}}}\right\}$$

2.284 ODE No. 284

$$(x^2 + 4y(x)^2)y'(x) - xy(x) = 0$$

✓ **Mathematica** : cpu = 0.124961 (sec), leaf count = 59

$$\left\{ \left\{ y(x) \rightarrow -\frac{x}{2\sqrt{W\left(\frac{1}{4}e^{-\frac{c_1}{2}}x^2\right)}} \right\}, \left\{ y(x) \rightarrow \frac{x}{2\sqrt{W\left(\frac{1}{4}e^{-\frac{c_1}{2}}x^2\right)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.12 (sec), leaf count = 21

$$\left\{ y(x) = e^{-c_1 + \frac{\text{LambertW}\left(\frac{x^2 e^{2c_1}}{4}\right)}{2}} \right\}$$

2.285 ODE No. 285

$$(3x^2 + 2xy(x) + 4y(x)^2)y'(x) + 2x^2 + 6xy(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.185592 (sec), leaf count = 402

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{54x^3 + \sqrt{3881196x^6 + (54x^3 + 432e^{3c_1})^2} + 432e^{3c_1}}}{12\sqrt[3]{2}} - \frac{33x^2}{2^{2/3}\sqrt[3]{54x^3 + \sqrt{3881196x^6 + (54x^3 + 432e^{3c_1})^2} + 432e^{3c_1}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 432

$$\left\{ y(x) = \frac{-\frac{11c_1^2x^2}{4\left(c_1^3x^3+8+2\sqrt{333c_1^6x^6+4c_1^3x^3+16}\right)^{\frac{1}{3}}}-\frac{c_1x}{4}+\frac{\left(c_1^3x^3+8+2\sqrt{333c_1^6x^6+4c_1^3x^3+16}\right)^{\frac{1}{3}}}{4}}{c_1}, y(x) = -\frac{-11c_1^2x^2+2c_1\left(c_1^3x^3+8+2\sqrt{333c_1^6x^6+4c_1^3x^3+16}\right)^{\frac{1}{3}}}{c_1} \right\}$$

2.286 ODE No. 286

$$(2y(x) - 3x + 1)^2y'(x) - (3y(x) - 2x - 4)^2 = 0$$

✓ **Mathematica** : cpu = 0.291813 (sec), leaf count = 3501

✓ **Maple** : cpu = 1.959 (sec), leaf count = 1337

$$\left\{ y(x) = \frac{(5x+3)\text{RootOf}\left(\left(115330078125c_1x^9-2283535546875c_1x^8+20095112812500c_1x^7-103154912437500c_1x^6+103154912437500c_1x^5-103154912437500c_1x^4+103154912437500c_1x^3-103154912437500c_1x^2+103154912437500c_1x-103154912437500\right)\right)}{5\text{RootOf}\left(\left(115330078125c_1x^9-2283535546875c_1x^8+20095112812500c_1x^7-103154912437500c_1x^6+103154912437500c_1x^5-103154912437500c_1x^4+103154912437500c_1x^3-103154912437500c_1x^2+103154912437500c_1x-103154912437500\right)\right)} \right\}$$

2.287 ODE No. 287

$$(2y(x) - 4x + 1)^2y'(x) - (y(x) - 2x)^2 = 0$$

✓ **Mathematica** : cpu = 3.1145 (sec), leaf count = 77

Solve $\left[\frac{y(x)}{2} + \frac{1}{196}(14y(x) - (8 - 9\sqrt{2})\log(-7y(x) + 14x + \sqrt{2} - 4) - (8 + 9\sqrt{2})\log(7y(x) - 14x + \sqrt{2} + 4) - 28x) = \right.$

✓ **Maple** : cpu = 0.074 (sec), leaf count = 56

$$\left\{-c_1-\frac{x}{7}-\frac{9\sqrt{2}\arctanh\left(\frac{(-14x+7y(x)+4)\sqrt{2}}{2}\right)}{98}-\frac{2\ln\left(-16x+8y(x)+7\left(-2x+y(x)\right)^2+2\right)}{49}+\frac{4y(x)}{7}=0\right\}$$

2.288 ODE No. 288

$$\left(-3x^2y(x)+6y(x)^2+1\right)y'(x)-3xy(x)^2+x=0$$

✓ **Mathematica** : cpu = 0.276407 (sec), leaf count = 534

$$\left\{\left\{y(x)\rightarrow\frac{x^2}{4}-\frac{\sqrt[3]{-9x^6+108x^2+4\sqrt{3}\sqrt{-27x^8-54c_1x^6+207x^4+648c_1x^2+32+432c_1^2+144c_1}}{4\,3^{2/3}}+\frac{\sqrt[3]{3}\sqrt[3]{-9x^6+108x^2+4\sqrt{3}\sqrt{-27x^8-54c_1x^6+207x^4+648c_1x^2+32+432c_1^2+144c_1}}{3\sqrt[3]{3}\sqrt[3]{-9x^6+108x^2+4\sqrt{3}\sqrt{-27x^8-54c_1x^6+207x^4+648c_1x^2+32+432c_1^2+144c_1}}}{24}\right\}\right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 579

$$\left\{y(x)=\frac{-9x^4+6\left(27x^6-324x^2-432c_1+12\sqrt{-81x^8-162c_1x^6+621x^4+1944c_1x^2+1296c_1^2+96}\right)^{\frac{1}{3}}x^2+\left(-9ix^4+i\left(2\sqrt[3]{-9x^6+108x^2+4\sqrt{3}\sqrt{-27x^8-54c_1x^6+207x^4+648c_1x^2+32+432c_1^2+144c_1}}\right)\sqrt[3]{3}\sqrt[3]{-9x^6+108x^2+4\sqrt{3}\sqrt{-27x^8-54c_1x^6+207x^4+648c_1x^2+32+432c_1^2+144c_1}}\right)}{24\left(27x^6-324x^2-432c_1+12\sqrt{-81x^8-162c_1x^6+621x^4+1944c_1x^2+1296c_1^2+96}\right)^{\frac{1}{3}}}$$

2.289 ODE No. 289

$$a+(6y(x)-x)^2y'(x)-6y(x)^2+2xy(x)=0$$

✓ **Mathematica** : cpu = 0.208898 (sec), leaf count = 115

$$\left\{\left\{y(x)\rightarrow\frac{1}{6}\left(x+\sqrt[3]{-18ax-x^3+18c_1}\right)\right\},\left\{y(x)\rightarrow\frac{x}{6}-\frac{1}{12}\left(1-i\sqrt{3}\right)\sqrt[3]{-18ax-x^3+18c_1}\right\},\left\{y(x)\rightarrow\frac{x}{6}-\frac{1}{12}\left(1+i\sqrt{3}\right)\sqrt[3]{-18ax-x^3+18c_1}\right\}\right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 115

$$\left\{y(x)=\frac{x}{6}+\frac{\left(-x^3-18ax-18c_1\right)^{\frac{1}{3}}}{6},y(x)=\frac{x}{6}-\frac{\left(-x^3-18ax-18c_1\right)^{\frac{1}{3}}}{12}-\frac{i\sqrt{3}\left(-x^3-18ax-18c_1\right)^{\frac{1}{3}}}{12},y(x)=\frac{x}{6}-\frac{\left(-x^3-18ax-18c_1\right)^{\frac{1}{3}}}{12}+\frac{i\sqrt{3}\left(-x^3-18ax-18c_1\right)^{\frac{1}{3}}}{12}\right\}$$

2.290 ODE No. 290

$$y'(x)\left(ay(x)^2+2bxy(x)+cx^2\right)+by(x)^2+2cxy(x)+dx^2=0$$

✓ **Mathematica** : cpu = 0.518176 (sec), leaf count = 831

$$\left\{\left\{y(x)\rightarrow-\frac{bx}{a}+\frac{\sqrt[3]{-54b^3x^3+81abcx^3-27a^2dx^3+27a^2e^{3c_1}+\sqrt{4\left(9acx^2-9b^2x^2\right)^3+\left(-54b^3x^3+81abcx^3-27a^2dx^3+27a^2e^{3c_1}\right)^2}}}{3\sqrt[3]{2}\,a}\right\}\right\}$$

✓ **Maple** : cpu = 0.148 (sec), leaf count = 1388

$$\left\{y(x)=\frac{-\frac{2\left(ac-b^2\right)c_1^2x^2}{\left(-4c_1^3a^2d\,x^3+12c_1^2abc\,x^3-8c_1^3b^3x^3+4a^2+4\sqrt{c_1^6a^2d^2x^6-6c_1^6abcd\,x^6+4c_1^6a\,c^3x^6+4c_1^6b^3d\,x^6-3c_1^6b^2c^2x^6-2c_1^3a^2d\,x^3+6c_1^3abc\,x^3-4c_1^3b^3x^3+a^2}\,a\right)^{\frac{1}{3}}a}-\frac{c_1b}{a}}{c_1}$$

2.291 ODE No. 291

$$y'(x)\big(b(ax+\beta y(x))^2-\beta(ax+by(x))\big)-\alpha(ax+by(x))+a(ax+\beta y(x))^2=0$$

✓ **Mathematica** : cpu = 1.27606 (sec), leaf count = 39

$$\text{Solve}\left[\frac{a\beta\left(\log(ax+by(x))+\frac{1}{ax+\beta y(x)}\right)}{a\beta-\alpha b}=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.161 (sec), leaf count = 50

$$\left\{y(x)=\frac{-ax+e^{\text{RootOf}\left(c_1a\beta x-c_1abx-Za\beta x-Zabx-c_1\beta e^{-Z}+Z\beta e^{-Z}+b\right)}}{b}\right\}$$

2.292 ODE No. 292

$$y'(x)(ay(x)+bx+c)^2+(\alpha y(x)+\beta x+\gamma)^2=0$$

✓ **Mathematica** : cpu = 51.5524 (sec), leaf count = 1716

$$\text{Solve}\left[(ab-a\beta)\text{RootSum}\left[-cy(x)^2\alpha^3-b\#1y(x)^2\alpha^3+a\beta\#1y(x)^2\alpha^2+a\gamma y(x)^2\alpha^2-2b\beta\#1^2y(x)\alpha^2-2\beta c\#1y(x)\alpha^2-2b\gamma\#1\right.\right.$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 115

$$\left\{y(x)=\frac{-\beta c+\gamma b+\left(-\left(\beta x+\gamma\right)a+(bx+c)\alpha\right)\text{RootOf}\left(c_1+\int^{-Z}\frac{(\_aa-b)^2}{\_a^3a^2-2\_a^2ab-\_a^2\alpha^2+2\_a\alpha\beta+\_ab^2-\beta^2}d\_a+\ln\left(a\beta x-abx-\alpha c\right)\right)}{a\beta-\alpha b}\right\}$$

2.293 ODE No. 293

$$x\big(y(x)^2-3x\big)y'(x)+2y(x)^3-5xy(x)=0$$

✓ **Mathematica** : cpu = 0.207643 (sec), leaf count = 661

$$\left\{\left\{y(x)\rightarrow\text{Root}\left[-\#1^{15}-\frac{25\#1^2e^{\frac{65c_1}{2}}}{x^{26}}+\frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&,1\right]\right\},\left\{y(x)\rightarrow\text{Root}\left[-\#1^{15}-\frac{25\#1^2e^{\frac{65c_1}{2}}}{x^{26}}+\frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&,2\right]\right\},\left\{y(x)\rightarrow\text{Root}\left[-\#1^{15}-\frac{25\#1^2e^{\frac{65c_1}{2}}}{x^{26}}+\frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&,3\right]\right\}\right\}$$

✓ **Maple** : cpu = 0.295 (sec), leaf count = 35

$$\left\{-c_1+\ln(x)-\frac{2\ln\left(\frac{5y(x)^2-13x}{x}\right)}{65}+\frac{6\ln\left(\frac{y(x)}{\sqrt{x}}\right)}{13}=0\right\}$$

2.294 ODE No. 294

$$x\big(-a+x^2+y(x)^2\big)y'(x)-y(x)\big(a+x^2+y(x)^2\big)=0$$

✓ **Mathematica** : cpu = 0.287296 (sec), leaf count = 71

$$\left\{\left\{y(x)\rightarrow\frac{1}{2}\left(c_1x-\sqrt{-4a+4x^2+c_1^2x^2}\right)\right\},\left\{y(x)\rightarrow\frac{1}{2}\left(\sqrt{-4a+4x^2+c_1^2x^2}+c_1x\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.093 (sec), leaf count = 112

$$\left\{\frac{1}{\frac{1}{y(x)^2}-\frac{1}{-x^2+a}}=\frac{x^2}{2}-\frac{a}{2}-\frac{\sqrt{x^2-a}\ x}{\sqrt{c_1+\frac{4a}{x^2-a}}},\frac{1}{\frac{1}{y(x)^2}-\frac{1}{-x^2+a}}=\frac{x^2}{2}-\frac{a}{2}+\frac{\sqrt{x^2-a}\ x}{\sqrt{c_1+\frac{4a}{x^2-a}}}\right\}$$

2.295 ODE No. 295

$$x(-x^2 + xy(x) + y(x)^2)y'(x) + x^2y(x) - y(x)^3 + xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.275585 (sec), leaf count = 31

$$\text{Solve}\left[\frac{x}{y(x)} + \frac{y(x)}{x} + \log\left(\frac{y(x)}{x}\right) = -2\log(x) + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.269 (sec), leaf count = 29

$$\{y(x) = x e^{\text{RootOf}(2c_1 e^{-Z} + Z e^{-Z} + 2 e^{-Z} \ln(x) + e^2 Z + 1)}\}$$

2.296 ODE No. 296

$$x^4 + x(x^2y(x) + x^2 + y(x)^2)y'(x) - 2x^2y(x)^2 - 2y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.536607 (sec), leaf count = 102

$$\left\{\left\{y(x) \rightarrow -e^{-c_1}x^2 - e^{-c_1}\sqrt{x^4 - e^{c_1}x^4 + e^{2c_1}x^2}\right\}, \left\{y(x) \rightarrow e^{-c_1}\sqrt{x^4 - e^{c_1}x^4 + e^{2c_1}x^2} - e^{-c_1}x^2\right\}\right\}$$

✓ **Maple** : cpu = 0.599 (sec), leaf count = 135

$$\left\{y(x) = -\frac{\left(-x^3 + c_1x + x^2 + \sqrt{-c_1x^4 + x^4 + c_1^2x^2}\right)x}{c_1x - x^2 + \sqrt{-c_1x^4 + x^4 + c_1^2x^2}}, y(x) = -\frac{\left(x^3 - c_1x - x^2 + \sqrt{-c_1x^4 + x^4 + c_1^2x^2}\right)x}{-c_1x + x^2 + \sqrt{-c_1x^4 + x^4 + c_1^2x^2}}\right\}$$

2.297 ODE No. 297

$$2x(5x^2 + y(x)^2)y'(x) - x^2y(x) + y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.19034 (sec), leaf count = 216

$$\left\{\left\{y(x) \rightarrow \text{Root}\left[-\#1^5 + \frac{\#1^2 e^{3c_1}}{x^{3/2}} + 3e^{3c_1}\sqrt{x} \&, 1\right]\right\}, \left\{y(x) \rightarrow \text{Root}\left[-\#1^5 + \frac{\#1^2 e^{3c_1}}{x^{3/2}} + 3e^{3c_1}\sqrt{x} \&, 2\right]\right\}, \left\{y(x) \rightarrow \text{Root}\left[-\#1^5 + \frac{\#1^2 e^{3c_1}}{x^{3/2}} + 3e^{3c_1}\sqrt{x} \&, 3\right]\right\}\right\}$$

✓ **Maple** : cpu = 0.293 (sec), leaf count = 29

$$\left\{y(x) = x \text{RootOf}(c_1x^9 - Z^{45} - Z^{18} - 6Z^9 - 9)^{\frac{9}{2}}\right\}$$

2.298 ODE No. 298

$$3xy(x)^2y'(x) + y(x)^3 - 2x = 0$$

✓ **Mathematica** : cpu = 0.120878 (sec), leaf count = 72

$$\left\{\left\{y(x) \rightarrow \frac{\sqrt[3]{x^2 + c_1}}{\sqrt[3]{x}}\right\}, \left\{y(x) \rightarrow -\frac{\sqrt[3]{-1}\sqrt[3]{x^2 + c_1}}{\sqrt[3]{x}}\right\}, \left\{y(x) \rightarrow \frac{(-1)^{2/3}\sqrt[3]{x^2 + c_1}}{\sqrt[3]{x}}\right\}\right\}$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 73

$$\left\{y(x) = \frac{\left((x^2 + c_1)x^2\right)^{\frac{1}{3}}}{x}, y(x) = -\frac{\left((x^2 + c_1)x^2\right)^{\frac{1}{3}}(1 + i\sqrt{3})}{2x}, y(x) = \frac{\left((x^2 + c_1)x^2\right)^{\frac{1}{3}}(i\sqrt{3} - 1)}{2x}\right\}$$

2.299 ODE No. 299

$$(3xy(x)^2 - x^2)y'(x) + y(x)^3 - 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.140321 (sec), leaf count = 371

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt[3]{\frac{2}{3}} x^2}{\sqrt[3]{9c_1 x^2 + \sqrt{3} \sqrt{-4x^9 + 27c_1^2 x^4}}} - \frac{\sqrt[3]{9c_1 x^2 + \sqrt{3} \sqrt{-4x^9 + 27c_1^2 x^4}}}{\sqrt[3]{2} 3^{2/3} x} \right\}, \left\{ y(x) \rightarrow \frac{(1 + i\sqrt{3})x^2}{2^{2/3} \sqrt[3]{3} \sqrt[3]{9c_1 x^2 + \sqrt{3} \sqrt{-4x^9 + 27c_1^2 x^4}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.233 (sec), leaf count = 276

$$\left\{ y(x) = -\frac{12^{\frac{2}{3}} \left(12x^3 + \left(-12ix^3 + i \left((108c_1 + 12\sqrt{-12x^5 + 81c_1^2})x^2 \right)^{\frac{2}{3}} \right) \sqrt{3} + \left((108c_1 + 12\sqrt{-12x^5 + 81c_1^2})x^2 \right)^{\frac{2}{3}} \right)}{144 \left((9c_1 + \sqrt{-12x^5 + 81c_1^2})x^2 \right)^{\frac{1}{3}} x}, y(x) = \dots \right\}$$

2.300 ODE No. 300

$$6xy(x)^2y'(x) + 2y(x)^3 + x = 0$$

✓ **Mathematica** : cpu = 0.100385 (sec), leaf count = 99

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{-x^2 + 4c_1}}{2^{2/3} \sqrt[3]{x}} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt[3]{-1} \sqrt[3]{-x^2 + 4c_1}}{2^{2/3} \sqrt[3]{x}} \right\}, \left\{ y(x) \rightarrow \frac{(-1)^{2/3} \sqrt[3]{-x^2 + 4c_1}}{2^{2/3} \sqrt[3]{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.025 (sec), leaf count = 83

$$\left\{ y(x) = \frac{(-2(x^2 - 4c_1)x^2)^{\frac{1}{3}}}{2x}, y(x) = -\frac{(-2(x^2 - 4c_1)x^2)^{\frac{1}{3}}(1 + i\sqrt{3})}{4x}, y(x) = \frac{(-2(x^2 - 4c_1)x^2)^{\frac{1}{3}}(i\sqrt{3} - 1)}{4x} \right\}$$

2.301 ODE No. 301

$$(x^2 + 6xy(x)^2)y'(x) - y(x)(3y(x)^2 - x) = 0$$

✓ **Mathematica** : cpu = 0.161957 (sec), leaf count = 64

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{x} \sqrt{W\left(\frac{6e^{3c_1}}{x^3}\right)}}{\sqrt{6}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{x} \sqrt{W\left(\frac{6e^{3c_1}}{x^3}\right)}}{\sqrt{6}} \right\} \right\}$$

✓ **Maple** : cpu = 0.221 (sec), leaf count = 25

$$\left\{ y(x) = \frac{e^{\frac{3c_1}{2} - \frac{\text{LambertW}\left(\frac{6e^{3c_1}}{x^3}\right)}{2}}}{x} \right\}$$

2.302 ODE No. 302

$$(x^2y(x)^2 + x)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.122219 (sec), leaf count = 70

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 x - \sqrt{x} \sqrt{4 + c_1^2 x}}{2x} \right\}, \left\{ y(x) \rightarrow \frac{c_1 x + \sqrt{x} \sqrt{4 + c_1^2 x}}{2x} \right\} \right\}$$

✓ **Maple** : cpu = 0.134 (sec), leaf count = 133

$$\left\{y(x)=-\frac{\sqrt{-2c_1\left(-2c_1-x+\sqrt{(4c_1+x)x}\right)}x}{2c_1x},y(x)=\frac{\sqrt{-2c_1\left(-2c_1-x+\sqrt{(4c_1+x)x}\right)}x}{2c_1x},y(x)=-\frac{\sqrt{2}\sqrt{c_1\left(2c_1+x+\sqrt{(4c_1+x)x}\right)}}{2c_1x}\right\}$$

2.303 ODE No. 303

$$y(x)\big(x^2y(x)^2+1\big)+x(xy(x)-1)^2y'(x)=0$$

✓ **Mathematica** : cpu = 0.166001 (sec), leaf count = 25

$$\text{Solve}\left[xy(x)-\frac{1}{xy(x)}-2\log(y(x))=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.24 (sec), leaf count = 34

$$\left\{y(x)=\frac{\mathrm{e}^{\mathrm{RootOf}\left(2c_1\mathrm{e}^{-Z}+2\_Z\mathrm{e}^{-Z}-2\mathrm{e}^{-Z}\ln(x)-\mathrm{e}^{2\_Z}+1\right)}}{x}\right\}$$

2.304 ODE No. 304

$$5x^2y(x)^3+\left(10x^3y(x)^2+x^2y(x)+2x\right)y'(x)+xy(x)^2=0$$

✓ **Mathematica** : cpu = 0.482891 (sec), leaf count = 44

$$\text{Solve}\left[y(x)\sqrt{5x^2y(x)^2+2}e^{\frac{\tan^{-1}\left(\sqrt{\frac{5}{2}}xy(x)\right)}{\sqrt{10}}}=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.256 (sec), leaf count = 44

$$\left\{y(x)=\frac{\sqrt{10}\tan\left(\mathrm{RootOf}\left(2c_1\sqrt{10}+2\_Z+\sqrt{10}\ln\left(\frac{4\left(\tan^2(\_Z)+1\right)\left(\tan^2(\_Z)\right)}{5x^2}\right)\right)\right)}{5x}\right\}$$

2.305 ODE No. 305

$$x^2+\left(y(x)^3-3x\right)y'(x)-3y(x)=0$$

✓ **Mathematica** : cpu = 0.206983 (sec), leaf count = 1277

$$\left\{\left\{\left\{y(x)\rightarrow-\frac{1}{2}\sqrt{\frac{16\sqrt[3]{2}\left(x^3+3c_1\right)}{\sqrt[3]{104976x^2-\sqrt{11019960576x^4-4\left(144x^3+432c_1\right)^3}}}}+\frac{\sqrt[3]{104976x^2-\sqrt{11019960576x^4-4\left(144x^3+432c_1\right)^3}}}{9\sqrt[3]{2}}}\right\}\right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 21

$$\left\{\frac{y(x)^4}{4}+\frac{x^3}{3}-3xy(x)+c_1=0\right\}$$

2.306 ODE No. 306

$$(y(x)^3 - x^3)y'(x) - x^2y(x) = 0$$

✓ **Mathematica** : cpu = 0.155274 (sec), leaf count = 201

$$\left\{ \left\{ y(x) \rightarrow \sqrt[3]{x^3 - \sqrt{x^6 - e^{6c_1}}} \right\}, \left\{ y(x) \rightarrow -\sqrt[3]{-1} \sqrt[3]{x^3 - \sqrt{x^6 - e^{6c_1}}} \right\}, \left\{ y(x) \rightarrow (-1)^{2/3} \sqrt[3]{x^3 - \sqrt{x^6 - e^{6c_1}}} \right\}, \left\{ y(x) \rightarrow \sqrt[3]{x^3} \right\} \right.$$

✓ **Maple** : cpu = 0.263 (sec), leaf count = 231

$$\left\{ y(x) = \frac{x}{\left(-c_1\left(c_1x^3 - \sqrt{c_1^2x^6 + 1}\right)x^3\right)^{\frac{1}{3}}}, y(x) = \frac{x}{\left(-c_1\left(c_1x^3 + \sqrt{c_1^2x^6 + 1}\right)x^3\right)^{\frac{1}{3}}}, y(x) = \frac{4x}{(1 + i\sqrt{3})^2\left(-c_1\left(c_1x^3 - \sqrt{c_1^2x^6 + 1}\right)\right)} \right.$$

2.307 ODE No. 307

$$y(x)(a + x^2 + y(x)^2)y'(x) + x(-a + x^2 + y(x)^2) = 0$$

✓ **Mathematica** : cpu = 0.270129 (sec), leaf count = 149

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{-\sqrt{a^2 + 4ax^2 + 4c_1} - a - x^2} \right\}, \left\{ y(x) \rightarrow \sqrt{-\sqrt{a^2 + 4ax^2 + 4c_1} - a - x^2} \right\}, \left\{ y(x) \rightarrow -\sqrt{\sqrt{a^2 + 4ax^2 + 4c_1} - a - x^2} \right\} \right.$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 125

$$\left\{ y(x) = \sqrt{-x^2 - a - \sqrt{4a x^2 + a^2 - 4c_1}}, y(x) = \sqrt{-x^2 - a + \sqrt{4a x^2 + a^2 - 4c_1}}, y(x) = -\sqrt{-x^2 - a - \sqrt{4a x^2 + a^2 - 4c_1}} \right.$$

2.308 ODE No. 308

$$2y(x)^3y'(x) + xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.016404 (sec), leaf count = 55

$$\left\{ \left\{ y(x) \rightarrow 0 \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt{-x^2 + 4c_1}}{\sqrt{2}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{-x^2 + 4c_1}}{\sqrt{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 37

$$\left\{ y(x) = 0, y(x) = -\frac{\sqrt{-2x^2 + 4c_1}}{2}, y(x) = \frac{\sqrt{-2x^2 + 4c_1}}{2} \right\}$$

2.309 ODE No. 309

$$-2x^3 + (2y(x)^3 + y(x))y'(x) - x = 0$$

✓ **Mathematica** : cpu = 0.117654 (sec), leaf count = 151

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{-1 - \sqrt{4x^4 + 4x^2 + 1 + 8c_1}}}{\sqrt{2}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{-1 - \sqrt{4x^4 + 4x^2 + 1 + 8c_1}}}{\sqrt{2}} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt{-1 + \sqrt{4x^4 + 4x^2 + 1 + 8c_1}}}{\sqrt{2}} \right\} \right.$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 113

$$\left\{ y(x) = -\frac{\sqrt{-2 - 2\sqrt{4x^4 + 4x^2 + 8c_1 + 1}}}{2}, y(x) = \frac{\sqrt{-2 - 2\sqrt{4x^4 + 4x^2 + 8c_1 + 1}}}{2}, y(x) = -\frac{\sqrt{-2 + 2\sqrt{4x^4 + 4x^2 + 8c_1 + 1}}}{2} \right.$$

2.310 ODE No. 310

$$x^3 + \left(5x^2y(x) + 2y(x)^3\right)y'(x) + 5xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.191035 (sec), leaf count = 159

$$\left\{\left\{y(x) \rightarrow -\frac{\sqrt{-5x^2 - \sqrt{23x^4 + 2e^{4c_1}}}}{\sqrt{2}}\right\}, \left\{y(x) \rightarrow \frac{\sqrt{-5x^2 - \sqrt{23x^4 + 2e^{4c_1}}}}{\sqrt{2}}\right\}, \left\{y(x) \rightarrow -\frac{\sqrt{-5x^2 + \sqrt{23x^4 + 2e^{4c_1}}}}{\sqrt{2}}\right\}, \left\{y(x) \rightarrow \frac{\sqrt{-5x^2 + \sqrt{23x^4 + 2e^{4c_1}}}}{\sqrt{2}}\right\}\right\}$$

✓ **Maple** : cpu = 0.151 (sec), leaf count = 125

$$\left\{y(x) = -\frac{\sqrt{-10c_1x^2 - 2\sqrt{23x^4c_1^2 + 2}}}{2\sqrt{c_1}}, y(x) = \frac{\sqrt{-10c_1x^2 - 2\sqrt{23x^4c_1^2 + 2}}}{2\sqrt{c_1}}, y(x) = -\frac{\sqrt{-10c_1x^2 + 2\sqrt{23x^4c_1^2 + 2}}}{2\sqrt{c_1}}, y(x) = \frac{\sqrt{-10c_1x^2 + 2\sqrt{23x^4c_1^2 + 2}}}{2\sqrt{c_1}}\right\}$$

2.311 ODE No. 311

$$4x^3 + 9x^2y(x) + \left(3x^3 + 6x^2y(x) - 3xy(x)^2 + 20y(x)^3\right)y'(x) + 6xy(x)^2 - y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.250914 (sec), leaf count = 2201

$$\left\{\left\{y(x) \rightarrow \frac{x}{20} + \frac{1}{2} \sqrt{-\frac{39x^2}{100} + \frac{\sqrt[3]{99x^6 + 351e^{c_1}x^2 + \sqrt{3} \sqrt{-67037x^{12} + 185406e^{c_1}x^8 - 83733e^{2c_1}x^4 + 32000e^{3c_1}}}}{5\sqrt[3]{2}3^{2/3}} + \frac{1}{5\sqrt[3]{99}}}\right\}\right\}$$

✓ **Maple** : cpu = 0.117 (sec), leaf count = 50

$$\left\{y(x) = \frac{\text{RootOf}\left(x^4c_1^4 + 3x^3c_1^3\_Z + 3c_1^2\_Z^2x^2 - c_1\_Z^3x + 5\_Z^4 - 1\right)}{c_1}\right\}$$

2.312 ODE No. 312

$$\left(y(x)y'(x) + x\right)\left(\frac{x^2}{a} + \frac{y(x)^2}{b}\right) + \frac{(a-b)(y(x)y'(x) - x)}{a+b} = 0$$

✓ **Mathematica** : cpu = 0.763158 (sec), leaf count = 204

$$\left\{\left\{y(x) \rightarrow -\frac{\sqrt{b} \sqrt{a^2 + 2a^2W\left(\frac{c_1(a+b)e^{\frac{bx^2}{2a^2} - \frac{b}{2a} - \frac{x^2}{2b} - \frac{1}{2}}\right)}}{2a^3b^2} + ab - ax^2 - bx^2}}{\sqrt{a} \sqrt{a+b}}\right\}, \left\{y(x) \rightarrow \frac{\sqrt{b} \sqrt{a^2 + 2a^2W\left(\frac{c_1(a+b)e^{\frac{bx^2}{2a^2} - \frac{b}{2a} - \frac{x^2}{2b} - \frac{1}{2}}\right)}}{2a^3b^2} + a}}{\sqrt{a} \sqrt{a+b}}\right\}\right\}$$

✓ **Maple** : cpu = 1.648 (sec), leaf count = 240

$$\left\{ y(x) = \frac{\sqrt{\left((-x^2 + a)b + e^{\frac{-2a^2b \operatorname{LambertW}\left(\frac{(a+b)e^{-\frac{1}{2}}e^{-\frac{x^2}{2b}}e^{-\frac{b}{2a}}e^{\frac{bx^2}{2a^2}}e^{-\frac{c_1}{ab}}\right) + b^2x^2 + (-x^2 - b)a^2 + (-b^2 - 2c_1)a}}{2a^2b}} \right)}{a}}, y(x) = -\sqrt{\left((-x^2 + a)b + e^{\frac{-2a^2b \operatorname{LambertW}\left(\frac{(a+b)e^{-\frac{1}{2}}e^{-\frac{x^2}{2b}}e^{-\frac{b}{2a}}e^{\frac{bx^2}{2a^2}}e^{-\frac{c_1}{ab}}\right) + b^2x^2 + (-x^2 - b)a^2 + (-b^2 - 2c_1)a}}{2a^2b}} \right)}{a} \right.$$

2.313 ODE No. 313

$$y'(x)\big(3axy(x)^2 + 2ay(x)^3 - bx^3 + cx^2\big) - ay(x)^3 + 2bx^3 + 3bx^2y(x) + cy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.391069 (sec), leaf count = 537

$$\left\{ y(x) \rightarrow \frac{\sqrt[3]{2}(3acx + 3ac_1)}{3a\sqrt[3]{\sqrt{(27a^2bx^3 + 27a^2c_1x)^2 + 4(3acx + 3ac_1)^3} + 27a^2bx^3 + 27a^2c_1x}} - \frac{\sqrt[3]{\sqrt{(27a^2bx^3 + 27a^2c_1x)^2 + 4(3acx + 3ac_1)^3} - 27a^2bx^3 - 27a^2c_1x}}{3\sqrt[3]{2}} \right.$$

✓ **Maple** : cpu = 0.202 (sec), leaf count = 748

$$\left\{ y(x) = \frac{(-12cx + 12c_1)a + \left(\left(-108bx^3 + 108c_1x + 12\sqrt{\frac{81ab^2x^6 - 162c_1abx^4 + 12c^3x^3 + 81c_1^2ax^2 - 36c_1c^2x^2 + 36c_1^2cx - 12c_1^3}{a}} \right) a^2 \right)^{\frac{2}{3}}}{6\left(\left(-108bx^3 + 108c_1x + 12\sqrt{\frac{81ab^2x^6 - 162c_1abx^4 + 12c^3x^3 + 81c_1^2ax^2 - 36c_1c^2x^2 + 36c_1^2cx - 12c_1^3}{a}} \right) a^2 \right)^{\frac{1}{3}} a}, y(x) = \dots \right.$$

2.314 ODE No. 314

$$xy(x)^3y'(x) + y(x)^4 - x\sin(x) = 0$$

✓ **Mathematica** : cpu = 0.17139 (sec), leaf count = 188

$$\left\{ y(x) \rightarrow -\frac{\sqrt[4]{-4x^4\cos(x) + 16x^3\sin(x) + 48x^2\cos(x) - 96x\sin(x) - 96\cos(x) + c_1}}{x} \right\}, \left\{ y(x) \rightarrow -\frac{i\sqrt[4]{-4x^4\cos(x) + 16x^3\sin(x) + 48x^2\cos(x) - 96x\sin(x) - 96\cos(x) + c_1}}{x} \right.$$

✓ **Maple** : cpu = 0.07 (sec), leaf count = 158

$$\left\{ y(x) = \frac{(c_1 + (-4x^4 + 48x^2 - 96)\cos(x) + (16x^3 - 96x)\sin(x))^{\frac{1}{4}}}{x}, y(x) = \frac{i(c_1 + (-4x^4 + 48x^2 - 96)\cos(x) + (16x^3 - 96x)\sin(x))^{\frac{1}{4}}}{x} \right.$$

2.315 ODE No. 315

$$(2xy(x)^3 - x^4)y'(x) + 2x^3y(x) - y(x)^4 = 0$$

✓ **Mathematica** : cpu = 0.340547 (sec), leaf count = 368

$$\left\{\left\{y(x) \rightarrow \frac{\sqrt[3]{\frac{2}{3}} e^{c_1 x}}{\sqrt[3]{-9 x^3+\sqrt{3} \sqrt{27 x^6-4 e^{3 c_1 x^3}}}}+\frac{\sqrt[3]{-9 x^3+\sqrt{3} \sqrt{27 x^6-4 e^{3 c_1 x^3}}}}{\sqrt[3]{2} 3^{2 / 3}}\right\},\left\{y(x) \rightarrow-\frac{\left(1+i \sqrt{3}\right) e^{c_1 x}}{2^{2 / 3} \sqrt[3]{3} \sqrt[3]{-9 x^3+\sqrt{3} \sqrt{27 x^6-4 e^{3 c_1 x^3}}}}\right\}\right\}$$

✓ **Maple** : cpu = 0.074 (sec), leaf count = 370

$$\left\{y(x)=-\frac{12 c_1 x+\left(-12 i c_1 x+i\left(-12\left(9 c_1 x^2-\sqrt{3} \sqrt{\frac{27 c_1^3 x^4-4 x}{c_1}}\right) c_1^2 x\right)^{\frac{2}{3}}\right) \sqrt{3}+\left(-12\left(9 c_1 x^2-\sqrt{3} \sqrt{\frac{27 c_1^3 x^4-4 x}{c_1}}\right) c_1^2 x\right)^{\frac{2}{3}}}{12\left(-108\left(c_1 x^2-\frac{\sqrt{3} \sqrt{\frac{27 c_1^3 x^4-4 x}{c_1}}}{9}\right) c_1^2 x\right)^{\frac{1}{3}}} c_1}, y(x)=\right\}$$

2.316 ODE No. 316

$$\left(2 x y(x)^3+y(x)\right) y'(x)+2 y(x)^2=0$$

✓ **Mathematica** : cpu = 0.211073 (sec), leaf count = 48

$$\left\{\{y(x) \rightarrow 0\}, \text { Solve }\left[x=-\frac{1}{4} e^{-\frac{1}{2} y(x)^2} \text {Ei}\left(\frac{y(x)^2}{2}\right)+c_1 e^{-\frac{1}{2} y(x)^2}, y(x)\right]\right\}$$

✓ **Maple** : cpu = 0.071 (sec), leaf count = 53

$$\left\{y(x)=0, y(x)=\sqrt{-2 \text {RootOf}\left(4 c_1 e^{-Z}+\text {Ei}\left(1,-Z\right) e^{-Z}-4 x\right)}, y(x)=-\sqrt{-2 \text {RootOf}\left(4 c_1 e^{-Z}+\text {Ei}\left(1,-Z\right) e^{-Z}-4 x\right)}\right\}$$

2.317 ODE No. 317

$$\left(x^2+2 x y(x)^3+x y(x)\right) y'(x)+y(x)^2-x y(x)=0$$

✓ **Mathematica** : cpu = 0.36513 (sec), leaf count = 23

$$\text {Solve}\left[y(x)^2-\frac{x}{y(x)}+\log (y(x))+\log (x)=c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.168 (sec), leaf count = 29

$$\{y(x)=e^{\text {RootOf}\left(c_1 e^{-Z}-Z e^{-Z}-e^{-Z} \ln (x)+x-e^{3 Z}\right)}\}$$

2.318 ODE No. 318

$$\left(3 x y(x)^3-4 x y(x)+y(x)\right) y'(x)+\left(y(x)^2-2\right) y(x)^2=0$$

✓ **Mathematica** : cpu = 0.26889 (sec), leaf count = 4284

$$\left\{\{y(x) \rightarrow 0\},\left\{y(x) \rightarrow-\sqrt[3]{\frac{4 \sqrt[3]{2} x^2}{3 \sqrt[3]{16 x^6+24 x^5-27 c_1^2 x^4+12 x^4+2 x^3+3 \sqrt{3} \sqrt{-32 c_1^2 x^{10}-48 c_1^2 x^9+27 c_1^4 x^8-24 c_1^2 x^8-4 c_1^2 x^7+3 c_1^2 x^6-3 c_1^2 x^5-3 c_1^2 x^4-3 c_1^2 x^3-3 c_1^2 x^2-3 c_1^2 x-3 c_1^2}}}\right\}\right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 28

$$\left\{x-\frac{c_1}{\sqrt{y(x)^2-2} y(x)^2}+\frac{1}{y(x)^2}=0, y(x)=0\right\}$$

2.319 ODE No. 319

$$(7xy(x)^3 + y(x) - 5x)y'(x) + y(x)^4 - 5y(x) = 0$$

✓ **Mathematica** : cpu = 0.177194 (sec), leaf count = 302

$$\left\{ \left\{ y(x) \rightarrow \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 1\right] \right\}, \left\{ y(x) \rightarrow \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\right] \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 35

$$\left\{ x + \frac{2y(x)^5 - 25y(x)^2 - 10c_1}{10(y(x)^3 - 5)^2 y(x)} = 0 \right\}$$

2.320 ODE No. 320

$$(x^2y(x)^3 + xy(x))y'(x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.131532 (sec), leaf count = 76

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{2xW\left(c_1e^{\frac{1}{2x}-1}\right) + 2x - 1}}{\sqrt{x}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{2xW\left(c_1e^{\frac{1}{2x}-1}\right) + 2x - 1}}{\sqrt{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.108 (sec), leaf count = 78

$$\left\{ y(x) = \frac{\sqrt{2x^2 \text{LambertW}\left(\frac{c_1e^{-\frac{2x-1}{2x}}}{2}\right) + 2x^2 - x}}{x}, y(x) = -\frac{\sqrt{2x^2 \text{LambertW}\left(\frac{c_1e^{-\frac{2x-1}{2x}}}{2}\right) + 2x^2 - x}}{x} \right\}$$

2.321 ODE No. 321

$$(2x^2y(x)^3 + x^2y(x)^2 - 2x)y'(x) - 2y(x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.525617 (sec), leaf count = 47

$$\text{Solve}\left[\frac{1}{64}\left(-4y(x)^2 + 4y(x) - 2\log(8y(x) + 4) + 3\right) - \frac{1}{4x(2y(x) + 1)} = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.231 (sec), leaf count = 42

$$\left\{ y(x) = \frac{e^{\text{RootOf}(8c_1xe^{-Z} + 2_Zxe^{-Z} + 3xe^{-Z} - 4xe^{2_Z} + xe^{3_Z} + 16)}}{2} - \frac{1}{2} \right\}$$

2.322 ODE No. 322

$$(10x^2y(x)^3 - 3y(x)^2 - 2)y'(x) + 5xy(x)^4 + x = 0$$

✓ **Mathematica** : cpu = 0.317412 (sec), leaf count = 2077

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2} \sqrt{\frac{4\sqrt[3]{2}(5x^4 - 10c_1x^2 - 2)}{5x^2\sqrt[3]{2268x^2 - 216c_1} + \sqrt{(2160x^2 + 108(x^2 - 2c_1))^2 - 4(60x^4 - 120c_1x^2 - 24)^3}}} + \frac{\sqrt[3]{2268x^2 - 216c_1}}{\sqrt[3]{2268x^2 - 216c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 29

$$\left\{ \frac{5x^2y(x)^4}{2} - y(x)^3 + \frac{x^2}{2} + c_1 - 2y(x) = 0 \right\}$$

2.323 ODE No. 323

$$xy'(x)(axy(x)^3 + c) + y(x)(bx^3y(x) + c) = 0$$

✓ **Mathematica** : cpu = 0.277495 (sec), leaf count = 463

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{54a^2cx^2 + \sqrt{2916a^4c^2x^4 + 108a^3x^3(bx^3 - 2c_1x)^3}}}{3\sqrt[3]{2}ax} - \frac{\sqrt[3]{2}(bx^3 - 2c_1x)}{\sqrt[3]{54a^2cx^2 + \sqrt{2916a^4c^2x^4 + 108a^3x^3(bx^3 - 2c_1x)^3}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.116 (sec), leaf count = 630

$$\left\{ y(x) = \frac{3^{\frac{2}{3}} \left((3x^4b - 6c_1x^2)a + \left(3i(bx^2 - 2c_1)a x^2 + i \left(\left(27c + 3\sqrt{\frac{3b^3x^8 - 18c_1b^2x^6 + 36c_1^2bx^4 - 24c_1^3x^2 + 81c^2a}{a}} \right) a^2x^2 \right)^{\frac{2}{3}} \right) \sqrt{3} - \left(\left(27c + 3\sqrt{\frac{3b^3x^8 - 18c_1b^2x^6 + 36c_1^2bx^4 - 24c_1^3x^2 + 81c^2a}{a}} \right) a^2x^2 \right)^{\frac{2}{3}} \right)}{18 \left(\left(9c + \sqrt{\frac{3b^3x^8 - 18c_1b^2x^6 + 36c_1^2bx^4 - 24c_1^3x^2 + 81c^2a}{a}} \right) a^2x^2 \right)^{\frac{1}{3}} ax}$$

2.324 ODE No. 324

$$(2x^3y(x)^3 - x)y'(x) + 2x^3y(x)^3 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.134385 (sec), leaf count = 723

$$\left\{ \left\{ y(x) \rightarrow -\frac{2x^3 - c_1x^2}{6x^2} + \frac{\sqrt[3]{-8x^9 + 12c_1x^8 - 6c_1^2x^7 + c_1^3x^6 - 27x^4 + 3\sqrt{3}\sqrt{16x^{13} - 24c_1x^{12} + 12c_1^2x^{11} - 2c_1^3x^{10} + 27x^4}}}{6x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.128 (sec), leaf count = 815

$$\left\{ y(x) = \frac{-c_1^2x^2 + 4c_1x^3 - 4x^4 + (-4x^2 + 2c_1x) \left(\left(c_1^3x^2 - 6c_1^2x^3 + 12c_1x^4 - 8x^5 + 3\sqrt{48x^5 - 72c_1x^4 + 36c_1^2x^3 - 6c_1^3x^2 + 81} \right) \right)}{\dots}$$

2.325 ODE No. 325

$$y(x)(y(x)^3 - 2x^3)y'(x) + x(2y(x)^3 - x^3) = 0$$

✓ **Mathematica** : cpu = 0.225761 (sec), leaf count = 139

$$\text{Solve} \left[\frac{1}{7} \text{RootSum} \left[\#1^4 + \#1^3 + 3\#1^2 + \#1 + 1 \&, -\frac{8\#1^3 \log\left(\frac{y(x)}{x} - \#1\right) + 9\#1^2 \log\left(\frac{y(x)}{x} - \#1\right) + 12\#1 \log\left(\frac{y(x)}{x} - \#1\right) - \log\left(\frac{y(x)}{x}\right)}{4\#1^3 + 3\#1^2 + 6\#1 + 1} \right] \right]$$

✓ **Maple** : cpu = 0.691 (sec), leaf count = 124

$$\left\{ -c_1 + \frac{2\sqrt{3} \arctan\left(\frac{\sqrt{3}(x^3 + 4x^2y(x) + 2xy(x)^2 + 2y(x)^3)}{3x^3}\right)}{7} - \frac{2\sqrt{3} \arctan\left(\frac{(x + 2y(x))\sqrt{3}}{3x}\right)}{7} - \ln(x) + \frac{\ln\left(\frac{-x + y(x)}{x}\right)}{7} - \frac{2\ln\left(\frac{4x^4 + 4x^3y(x) + 12x^2y(x)^2 + 8xy(x)^3 + y(x)^4}{x^4}\right)}{7} \right\}$$

2.326 ODE No. 326

$$y(x)y'(x)\big((ay(x)+bx)^3+bx^3\big)+x\big((ay(x)+bx)^3+ay(x)^3\big)=0$$

✓ **Mathematica** : cpu = 3.45581 (sec), leaf count = 13289

Too large to display

✓ **Maple** : cpu = 0.537 (sec), leaf count = 160

$$\left\{y(x)=\frac{\left(c_1x-b\operatorname{RootOf}\left(c_1^4x^4-2c_1^3Zbx-2c_1Z^3bx+_Z^4b^2+\left(a^2x^2c_1^2+b^2x^2c_1^2+c_1^2x^2-a^2\right)_Z^2\right)\right)x}{a\operatorname{RootOf}\left(c_1^4x^4-2c_1^3Zbx-2c_1Z^3bx+_Z^4b^2+\left(a^2x^2c_1^2+b^2x^2c_1^2+c_1^2x^2-a^2\right)_Z^2\right)}\right\}$$

2.327 ODE No. 327

$$\left(2x^2y(x)^3+xy(x)^4+2y(x)+x\right)y'(x)+y(x)^5+y(x)=0$$

✓ **Mathematica** : cpu = 0.35088 (sec), leaf count = 669

$$\left\{y(x)\rightarrow\frac{\sqrt[3]{27x^2+9c_1^2x^2+3\sqrt{3}\sqrt{-4c_1^3x^6+27x^4-c_1^4x^4+18c_1^2x^4+4c_1^3x^2+2c_1^3}}}{3\sqrt[3]{2}x}-\frac{\sqrt[3]{27x^2+9c_1^2x^2+3\sqrt{3}\sqrt{-4c_1^3x^6+27x^4-c_1^4x^4+18c_1^2x^4+4c_1^3x^2+2c_1^3}}}{3x\sqrt[3]{27x^2+9c_1^2x^2+3\sqrt{3}\sqrt{-4c_1^3x^6+27x^4-c_1^4x^4+18c_1^2x^4+4c_1^3x^2+2c_1^3}}}\right\}$$

✓ **Maple** : cpu = 0.158 (sec), leaf count = 583

$$\left\{y(x)=\frac{12c_1x^2+\left(-12ic_1x^2-i\left(108c_1^3x^2+36c_1x^2+12c_1\sqrt{3}\sqrt{27c_1^4x^2+18c_1^2x^2-x^2+c_1(4x^4-4)}x-8\right)^{\frac{2}{3}}+4i\right)\sqrt{3}-\left(12\left(108c_1^3x^2+36c_1x^2+12c_1\sqrt{3}\sqrt{27c_1^4x^2+18c_1^2x^2-x^2+c_1(4x^4-4)}x-8\right)^{\frac{2}{3}}+4i\right)\sqrt{3}}{12\left(108c_1^3x^2+36c_1x^2+12c_1\sqrt{3}\sqrt{27c_1^4x^2+18c_1^2x^2-x^2+c_1(4x^4-4)}x-8\right)^{\frac{2}{3}}+4i}\right\}$$

2.328 ODE No. 328

$$ax^2y(x)^ny'(x)-2xy'(x)+y(x)=0$$

✓ **Mathematica** : cpu = 0.314464 (sec), leaf count = 42

$$\operatorname{Solve}\left[\frac{n\left(\log(x)-\log\left(-axy(x)^n+n+2\right)\right)}{n+2}-\frac{2n\log(y(x))}{n+2}=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.224 (sec), leaf count = 33

$$\{x^{-n}\big(axy(x)^n-n-2\big)^ny(x)^{2n}-c_1=0\}$$

2.329 ODE No. 329

$$x^ny(x)^m\big(axy'(x)+by(x)\big)+axy'(x)+\beta y(x)=0$$

✓ **Mathematica** : cpu = 0.832767 (sec), leaf count = 102

$$\operatorname{Solve}\left[\frac{m\left((a\beta-ab)\log\left(x^ny(x)^m(bm-an)-\alpha n+\beta m\right)+\beta\log(x)(bm-an)\right)}{(bm-an)(\beta m-\alpha n)}+\frac{\alpha m\log(\beta my(x)-\alpha ny(x))}{\beta m-\alpha n}=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.466 (sec), leaf count = 73

$$\{x^{-a\beta mn+b\beta m^2}\big(y(x)^m\big)^{-an\alpha+bma}\big(\alpha n-\beta m+(an-bm)x^ny(x)^m\big)^{(a\beta-ab)m}-c_1=0\}$$

2.330 ODE No. 330

$$(f(y(x) + x) + 1)y'(x) + f(y(x) + x) = 0$$

✓ **Mathematica** : cpu = 0.127981 (sec), leaf count = 52

$$\text{Solve}\left[\int_1^{y(x)}\left(f(x+K[2])-\int_1^x f'(K[1]+K[2])dK[1]+1\right)dK[2]+\int_1^x f(K[1]+y(x))dK[1]=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 22

$$\{y(x) = -x + \text{RootOf}\left(c_1 - x + \int^{-Z} (f(_a) + 1) d_a\right)\}$$

2.331 ODE No. 331

$$y'(x)\left(\sum_{\nu=1}^p y(x)^\nu f(\nu)(x)\right) - \sum_{\nu=1}^q y(x)^\nu g(\nu)(x) = 0$$

✗ **Mathematica** : cpu = 55.8894 (sec), leaf count = 0 , could not solve

DSolve[-Sum[y[x]^nu*g[nu][x], {nu, 1, q}] + Sum[y[x]^nu*f[nu][x], {nu, 1, p}]*Derivative[1][y[x]], x]

✓ **Maple** : cpu = 0.241 (sec), leaf count = 78

$$\left\{\frac{y(x)^{p+1}\Phi\left(-(-1)^{\text{csgn}\left(iy(x)^q\right)}y(x)^q,1,\frac{p+1}{q}\right)-y(x)\Phi\left(-(-1)^{\text{csgn}\left(iy(x)^q\right)}y(x)^q,1,\frac{1}{q}\right)+\left(c_1+\int\frac{g_\nu(x)}{f_\nu(x)}dx\right)q}{q}=0\right\}$$

2.332 ODE No. 332

$$x(\sqrt{xy(x)} - 1)y'(x) - y(x)(\sqrt{xy(x)} + 1) = 0$$

✓ **Mathematica** : cpu = 0.345587 (sec), leaf count = 34

$$\text{Solve}\left[\log(y(x)) - \frac{y(x)\log(x) - \frac{2y(x)}{\sqrt{xy(x)}}}{y(x)} = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 33

$$\left\{-\frac{1+\left(c_1-\ln(x)+\frac{\ln(xy(x))}{2}\right)\sqrt{xy(x)}}{\sqrt{xy(x)}}=0\right\}$$

2.333 ODE No. 333

$$-x^{3/2}y(x)^{5/2} + (2x^{5/2}y(x)^{3/2} + x^2y(x) - x)y'(x) + xy(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.492765 (sec), leaf count = 72

$$\text{Solve}\left[\frac{2\sqrt{xy(x)}\log(y(x))}{\sqrt{x}\sqrt{y(x)}} - \frac{\sqrt{xy(x)}(3x^{3/2}y(x)^{3/2}\log(x) + 6xy(x) - 2)}{3x^2y(x)^2} = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.104 (sec), leaf count = 32

$$\left\{-c_1 - \frac{\ln(x)}{2} + \ln(y(x)) - \frac{1}{\sqrt{x}\sqrt{y(x)}} + \frac{1}{3x^{\frac{3}{2}}y(x)^{\frac{3}{2}}} = 0\right\}$$

2.334 ODE No. 334

$$\left(\sqrt{y(x)} + x + 1\right)y'(x) + 1 = 0$$

✓ **Mathematica** : cpu = 0.130494 (sec), leaf count = 39

$$\left\{\left\{y(x) \rightarrow -2\sqrt{x+1+c_1} + 2 + c_1\right\}, \left\{y(x) \rightarrow 2\sqrt{x+1+c_1} + 2 + c_1\right\}\right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 19

$$\{-c_1 - y(x) - 2\sqrt{x+y(x)} = 0\}$$

2.335 ODE No. 335

$$\sqrt{y(x)^2 - 1} y'(x) - \sqrt{x^2 - 1} = 0$$

✓ **Mathematica** : cpu = 0.199083 (sec), leaf count = 75

$$\left\{\left\{y(x) \rightarrow \text{InverseFunction}\left[\frac{1}{2}\#1\sqrt{\#1^2-1} - \frac{1}{2}\log\left(\sqrt{\#1^2-1} + \#1\right)\&\left[\frac{1}{2}\sqrt{x^2-1}x - \frac{1}{2}\log\left(\sqrt{x^2-1} + x\right) + c_1\right]\right\}\right\}\right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 50

$$\left\{c_1 + \sqrt{x^2-1}x - \ln\left(x + \sqrt{x^2-1}\right) + \ln\left(y(x) + \sqrt{y(x)^2-1}\right) - \sqrt{y(x)^2-1}y(x) = 0\right\}$$

2.336 ODE No. 336

$$\left(ax + \sqrt{y(x)^2 + 1}\right)y'(x) + ay(x) + \sqrt{x^2 + 1} = 0$$

✓ **Mathematica** : cpu = 0.317301 (sec), leaf count = 53

$$\text{Solve}\left[axy(x) + \frac{1}{2}\sqrt{x^2+1}x + \frac{1}{2}\left(y(x)\sqrt{y(x)^2+1} + \sinh^{-1}(y(x))\right) + \frac{1}{2}\sinh^{-1}(x) = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 41

$$\left\{axy(x) + c_1 + \frac{\sqrt{x^2+1}x}{2} + \frac{\text{arcsinh}(x)}{2} + \frac{\text{arcsinh}(y(x))}{2} + \frac{\sqrt{y(x)^2+1}y(x)}{2} = 0\right\}$$

2.337 ODE No. 337

$$\left(\sqrt{x^2 + y(x)^2} + x\right)y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.25801 (sec), leaf count = 161

$$\left\{\left\{y(x) \rightarrow -\sqrt{-2ix \cosh(c_1) - 2ix \sinh(c_1) - \cosh(2c_1) - \sinh(2c_1)}\right\}, \left\{y(x) \rightarrow \sqrt{-2ix \cosh(c_1) - 2ix \sinh(c_1) - \cosh(2c_1) - \sinh(2c_1)}\right\}\right\}$$

✓ **Maple** : cpu = 0.108 (sec), leaf count = 28

$$\left\{-c_1 + \frac{x}{y(x)^2} + \frac{\sqrt{x^2 + y(x)^2}}{y(x)^2} = 0\right\}$$

2.338 ODE No. 338

$$y'(x)\Big(\sin(\alpha)\big(y(x)^2-x^2\big)-2x\cos(\alpha)y(x)+\sqrt{x^2+y(x)^2}y(x)\Big)+\cos(\alpha)\big(y(x)^2-x^2\big)+2x\sin(\alpha)y(x)+x\sqrt{x^2+y(x)^2}=0$$

✓ **Mathematica** : cpu = 96.4769 (sec), leaf count = 17681

Too large to display

✓ **Maple** : cpu = 0.688 (sec), leaf count = 129

$$\left\{y(x)=x\operatorname{RootOf}\left(c_1+\int^x\frac{-_a^3\cos(2\alpha)-_a^3-3\_a^2\sin(2\alpha)+3\_a\cos(2\alpha)-\_a+\sin(2\alpha)+\sqrt{2}\sqrt{(-_a^2+1)(\_a^2\cos(2\alpha)+\_a^2+2\_a\sin(2\alpha)-\cos(2\alpha))}}{(-_a^2+1)(\_a^2\cos(2\alpha)+\_a^2+2\_a\sin(2\alpha)-\cos(2\alpha))}dx\right)\right\}$$

2.339 ODE No. 339

$$\left(x\sqrt{x^2+y(x)^2+1}-y(x)\left(x^2+y(x)^2\right)\right)y'(x)-\sqrt{x^2+y(x)^2+1}y(x)-x\left(x^2+y(x)^2\right)=0$$

✓ **Mathematica** : cpu = 0.415714 (sec), leaf count = 27

$$\operatorname{Solve}\left[\sqrt{x^2+y(x)^2+1}+\tan^{-1}\left(\frac{x}{y(x)}\right)=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.222 (sec), leaf count = 27

$$\left\{-c_1+\arctan\left(\frac{y(x)}{x}\right)-\sqrt{x^2+y(x)^2+1}=0\right\}$$

2.340 ODE No. 340

$$y'(x)\left(\frac{e1(a+x)}{\left((a+x)^2+y(x)^2\right)^{3/2}}+\frac{e2(x-a)}{\left((x-a)^2+y(x)^2\right)^{3/2}}\right)-y(x)\left(\frac{e1}{\left((a+x)^2+y(x)^2\right)^{3/2}}+\frac{e2}{\left((x-a)^2+y(x)^2\right)^{3/2}}\right)=0$$

✗ **Mathematica** : cpu = 87.1637 (sec), leaf count = 0 , could not solve

`DSolve[-(y[x]*(e2/((-a + x)^2 + y[x]^2)^(3/2) + e1/((a + x)^2 + y[x]^2)^(3/2))) + ((e2*(-a + x))/((-a + x)^2 + y[x]^2)^(3/2) + (e1*(a + x))/((a + x)^2 + y[x]^2)^(3/2))*Derivative[1][y[x]]]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

time expired

2.341 ODE No. 341

$$\left(xe^{y(x)}+e^x\right)y'(x)+e^xy(x)+e^{y(x)}=0$$

✓ **Mathematica** : cpu = 0.275273 (sec), leaf count = 33

$$\left\{\left\{y(x)\rightarrow c_1e^{-x}-W\left(xe^{-x+c_1e^{-x}}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 33

$$\{y(x)=\left(-\operatorname{LambertW}\left(xe^{-x}e^{-c_1e^{-x}}\right)e^x-c_1\right)e^{-x}\}$$

2.342 ODE No. 342

$$x\left(2e^{-xy(x)}+3e^{xy(x)}\right)\left(xy'(x)+y(x)\right)+1=0$$

✓ **Mathematica** : cpu = 0.623731 (sec), leaf count = 163

$$\left\{\left\{y(x)\rightarrow-\frac{\cosh^{-1}\left(\frac{1}{24}\left(-5\sqrt{24+\log^2\left(\frac{c_1}{x}\right)}-\log\left(\frac{c_1}{x}\right)\right)\right)}{x}\right\}\right\},\left\{y(x)\rightarrow\frac{\cosh^{-1}\left(\frac{1}{24}\left(-5\sqrt{24+\log^2\left(\frac{c_1}{x}\right)}-\log\left(\frac{c_1}{x}\right)\right)\right)}{x}\right\}\right\},\left\{y(x)\rightarrow\frac{\cosh^{-1}\left(\frac{1}{24}\left(-5\sqrt{24+\log^2\left(\frac{c_1}{x}\right)}-\log\left(\frac{c_1}{x}\right)\right)\right)}{x}\right\}\right\}$$

✓ **Maple** : cpu = 0.047 (sec), leaf count = 17

$$\left\{y(x) = \frac{\ln\left(\frac{c_1}{5} - \frac{\ln(x)}{5}\right)}{x}\right\}$$

2.343 ODE No. 343

$$y'(x)(\log(y(x)) + x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.176106 (sec), leaf count = 35

$$\text{Solve}\left[x = e^{y(x)}\left(\text{Ei}(-y(x)) - e^{-y(x)} \log(y(x))\right) + c_1 e^{y(x)}, y(x)\right]$$

✓ **Maple** : cpu = 0.114 (sec), leaf count = 27

$$\{y(x) = e^{\text{RootOf}\left(c_1 e^{e^{-Z}} - \text{Ei}(1, e^{-Z}) e^{e^{-Z}} - Z - x\right)}\}$$

2.344 ODE No. 344

$$y'(x)(\log(y(x)) + 2x - 1) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.202776 (sec), leaf count = 23

$$\left\{\left\{y(x) \rightarrow -\frac{W\left(-2c_1 e^{-2x}\right)}{2c_1}\right\}\right\}$$

✓ **Maple** : cpu = 0.063 (sec), leaf count = 19

$$\{y(x) = e^{-2x - \text{LambertW}\left(-2c_1 e^{-2x}\right)}\}$$

2.345 ODE No. 345

$$xy'(x)\left(2x^2 y(x) \log(y(x)) + 1\right) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.230414 (sec), leaf count = 35

$$\text{Solve}\left[\frac{y(x)}{x^2} + 2\left(\frac{1}{2}y(x)^2 \log(y(x)) - \frac{y(x)^2}{4}\right) = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.097 (sec), leaf count = 36

$$\{y(x) = e^{\text{RootOf}\left(2_Z x^2 e^{2-Z} + 2c_1 x^2 - x^2 e^{2-Z} + 2e^{-Z}\right)}\}$$

2.346 ODE No. 346

$$xy'(x)(-ax + y(x) + y(x) \log(xy(x))) - y(x)(ax \log(xy(x)) + ax - y(x)) = 0$$

✓ **Mathematica** : cpu = 0.455704 (sec), leaf count = 24

$$\text{Solve}[ax \log(xy(x)) - y(x) \log(xy(x)) = c_1, y(x)]$$

✓ **Maple** : cpu = 0.281 (sec), leaf count = 19

$$\{-c_1 + \left(xy(x)\right)^{-ax+y(x)} = 0\}$$

2.347 ODE No. 347

$$(\sin(x) + 1)y'(x) \sin(y(x)) + \cos(x)(\cos(y(x)) - 1) = 0$$

✓ **Mathematica** : cpu = 0.338686 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow 0 \right\}, \left\{ y(x) \rightarrow 2 \sin^{-1} \left(\frac{1}{4} c_1 \left(\sin \left(\frac{x}{2} \right) + \cos \left(\frac{x}{2} \right) \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.158 (sec), leaf count = 12

$$\{y(x) = \arccos(c_1 \sin(x) + c_1 + 1)\}$$

2.348 ODE No. 348

$$y'(x)(x \cos(y(x)) + \sin(x)) + \sin(y(x)) + y(x) \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.212954 (sec), leaf count = 17

$$\text{Solve}[x \sin(y(x)) + y(x) \sin(x) = c_1, y(x)]$$

✓ **Maple** : cpu = 0.099 (sec), leaf count = 15

$$\{x \sin(y(x)) + \sin(x)y(x) + c_1 = 0\}$$

2.349 ODE No. 349

$$xy'(x) \cot\left(\frac{y(x)}{x}\right) + 2x \sin\left(\frac{y(x)}{x}\right) - y(x) \cot\left(\frac{y(x)}{x}\right) = 0$$

✓ **Mathematica** : cpu = 0.254555 (sec), leaf count = 15

$$\left\{ \left\{ y(x) \rightarrow x \csc^{-1}(2(\log(x) + c_1)) \right\} \right\}$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 17

$$\left\{ y(x) = x \arcsin\left(\frac{1}{2c_1 + 2 \ln(x)}\right) \right\}$$

2.350 ODE No. 350

$$y'(x) \cos(y(x)) - \sin(y(x)) - \cos(x) \sin^2(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.654024 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow \csc^{-1} \left(\frac{1}{2} (-\sin(x) - \cos(x) - 2c_1 e^{-x}) \right) \right\}, \left\{ y(x) \rightarrow -\csc^{-1} \left(\frac{1}{2} (\sin(x) + \cos(x) + 2c_1 e^{-x}) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.872 (sec), leaf count = 226

$$\left\{ y(x) = \arctan \left(-\frac{2e^x}{2c_1 + (\cos(x) + \sin(x))e^x}, \frac{\sqrt{16} \sqrt{\left(c_1^2 + c_1(\cos(x) + \sin(x))e^x + \left(\frac{\cos(x) \sin(x)}{2} - \frac{3}{4} \right) e^{2x} \right) \left(c_1^2 + c_1(\cos(x) + \sin(x))e^x + \left(\frac{\cos(x) \sin(x)}{2} - \frac{3}{4} \right) e^{2x} \right)}\right)}{4c_1^2 + 4c_1(\cos(x) + \sin(x))e^x + (2 \cos(x) \sin(x) - 3)e^{2x}} \right) \right\}$$

2.351 ODE No. 351

$$y'(x) \cos(y(x)) - \sin^3(y(x)) + x \sin(y(x)) \cos^2(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.556262 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow -\cot^{-1} \left(\sqrt{e^{x^2} (-\sqrt{\pi} \operatorname{erf}(x) + 4c_1)} \right) \right\}, \left\{ y(x) \rightarrow \cot^{-1} \left(\sqrt{e^{x^2} (-\sqrt{\pi} \operatorname{erf}(x) + 4c_1)} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.434 (sec), leaf count = 55

$$\left\{ y(x) = -\arcsin \left(\frac{1}{\sqrt{-2c_1 e^{x^2} - \sqrt{\pi} \operatorname{erf}(x) e^{x^2} + 1}} \right), y(x) = \arcsin \left(\frac{1}{\sqrt{-2c_1 e^{x^2} - \sqrt{\pi} \operatorname{erf}(x) e^{x^2} + 1}} \right) \right\}$$

2.352 ODE No. 352

$$y'(x) \cos(y(x)) (\cos(y(x)) - \sin(\alpha) \sin(x)) + \cos(x) (\cos(x) - \sin(\alpha) \sin(y(x))) = 0$$

✓ **Mathematica** : cpu = 0.607159 (sec), leaf count = 43

$$\text{Solve} \left[4 \sin(\alpha) \sin(x) \sin(y(x)) - 4 \left(\frac{y(x)}{2} + \frac{1}{4} \sin(2y(x)) \right) - 2x - \sin(2x) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.43 (sec), leaf count = 33

$$\left\{ \frac{\cos(x) \sin(x)}{2} + c_1 + \frac{x}{2} + \frac{(-2 \sin(\alpha) \sin(x) + \cos(y(x))) \sin(y(x))}{2} + \frac{y(x)}{2} = 0 \right\}$$

2.353 ODE No. 353

$$xy'(x) \cos(y(x)) + \sin(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.0885775 (sec), leaf count = 14

$$\left\{ \left\{ y(x) \rightarrow \sin^{-1} \left(\frac{e^{c_1}}{x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 12

$$\left\{ y(x) = \arcsin \left(\frac{1}{c_1 x} \right) \right\}$$

2.354 ODE No. 354

$$y'(x) (x \sin(y(x)) - 1) + \cos(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.191955 (sec), leaf count = 145

$$\left\{ \left\{ y(x) \rightarrow -\cos^{-1} \left(\frac{c_1 x - \sqrt{-x^2 + 1 + c_1^2}}{1 + c_1^2} \right) \right\}, \left\{ y(x) \rightarrow \cos^{-1} \left(\frac{c_1 x - \sqrt{-x^2 + 1 + c_1^2}}{1 + c_1^2} \right) \right\}, \left\{ y(x) \rightarrow -\cos^{-1} \left(\frac{\sqrt{-x^2 + 1 + c_1^2}}{1 + c_1^2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.073 (sec), leaf count = 108

$$\left\{ y(x) = \arctan \left(\frac{-c_1 \sqrt{-x^2 + c_1^2 + 1} + x}{c_1^2 + 1}, \frac{c_1 x + \sqrt{-x^2 + c_1^2 + 1}}{c_1^2 + 1} \right), y(x) = \arctan \left(\frac{c_1 \sqrt{-x^2 + c_1^2 + 1} + x}{c_1^2 + 1}, \frac{c_1 x - \sqrt{-x^2 + c_1^2 + 1}}{c_1^2 + 1} \right) \right\}$$

2.355 ODE No. 355

$$y'(x)(x \cos(y(x)) + \cos(x)) + \sin(y(x)) - y(x) \sin(x) = 0$$

✓ **Mathematica** : cpu = 0.191022 (sec), leaf count = 17

$$\text{Solve}[x \sin(y(x)) + y(x) \cos(x) = c_1, y(x)]$$

✓ **Maple** : cpu = 0.096 (sec), leaf count = 15

$$\{x \sin(y(x)) + \cos(x)y(x) + c_1 = 0\}$$

2.356 ODE No. 356

$$y'(x)(x^2 \cos(y(x)) + 2y(x) \sin(x)) + 2x \sin(y(x)) + y(x)^2 \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.275337 (sec), leaf count = 21

$$\text{Solve}[x^2 \sin(y(x)) + y(x)^2 \sin(x) = c_1, y(x)]$$

✓ **Maple** : cpu = 0.112 (sec), leaf count = 19

$$\{x^2 \sin(y(x)) + \sin(x)y(x)^2 + c_1 = 0\}$$

2.357 ODE No. 357

$$x \log(x)y'(x) \sin(y(x)) + \cos(y(x))(1 - x \cos(y(x))) = 0$$

✓ **Mathematica** : cpu = 0.605152 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow -\sec^{-1}\left(\frac{x - c_1}{\log(x)}\right) \right\}, \left\{ y(x) \rightarrow \sec^{-1}\left(\frac{x - c_1}{\log(x)}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.606 (sec), leaf count = 13

$$\left\{ y(x) = \arccos\left(\frac{\ln(x)}{c_1 + x}\right) \right\}$$

2.358 ODE No. 358

$$\cos(x)y'(x) \sin(y(x)) + \sin(x) \cos(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.163464 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow -\cos^{-1}\left(\frac{1}{2}c_1 \sec(x)\right) \right\}, \left\{ y(x) \rightarrow \cos^{-1}\left(\frac{1}{2}c_1 \sec(x)\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.096 (sec), leaf count = 11

$$\left\{ y(x) = \arccos\left(\frac{c_1}{\cos(x)}\right) \right\}$$

2.359 ODE No. 359

$$3 \sin(x)y'(x) \sin(y(x)) + 5y(x) \cos^4(x) = 0$$

✓ **Mathematica** : cpu = 0.354584 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow \text{SinIntegral}^{(-1)}\left(-\frac{5}{3}\left(\frac{5 \cos(x)}{4} + \frac{1}{12} \cos(3x) + \log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right)\right) + c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.074 (sec), leaf count = 28

$$\left\{ c_1 + \frac{3 \text{Si}(y(x))}{5} + \frac{5 \cos(x)}{4} + \frac{\cos(3x)}{12} + \ln(-\cot(x) + \csc(x)) = 0 \right\}$$

2.360 ODE No. 360

$$y'(x)\cos(ay(x))-b(1-c\cos(ay(x)))\sqrt{c\cos(ay(x))+\cos^2(ay(x))-1}=0$$

✓ **Mathematica** : cpu = 4.64854 (sec), leaf count = 369

$$\left\{\left\{y(x)\rightarrow \text{InverseFunction}\left[-\frac{i(\cos(\#1a)+1)\sqrt{\frac{2c\cos(\#1a)+\cos(2\#1a)-1}{(\cos(\#1a)+1)^2}}\sqrt{\frac{c\tan^2\left(\frac{\#1a}{2}\right)+\sqrt{c^2+4}+2}{\sqrt{c^2+4}+2}}\sqrt{1-\frac{c\tan^2\left(\frac{\#1a}{2}\right)}{\sqrt{c^2+4}-2}}\left((c-1)F\left(i\sin\left(\frac{\#1a}{2}\right)\right)\right)}{a\left(c^2-1\right)\sqrt{\frac{c}{4-2\sqrt{c^2+4}}}\sqrt{2c\cos(\#1a)+\cos^2(\#1a)-1}}\right]\right\}$$

✓ **Maple** : cpu = 0.375 (sec), leaf count = 48

$$\left\{c_1+x+\int^{y(x)}\frac{2\cos(_aa)}{(c\cos(_aa)-1)\sqrt{4c\cos(_aa)+2\cos(2_aa)-2}b}d_a=0\right\}$$

2.361 ODE No. 361

$$y'(x)(-\sin(y(x))+x\sin(xy(x))+\cos(y(x)+x))+y(x)\sin(xy(x))+\cos(y(x)+x)+\cos(x)=0$$

✓ **Mathematica** : cpu = 0.960836 (sec), leaf count = 31

$$\text{Solve}[\cos(y(x))-\cos(xy(x))+\sin(x)\cos(y(x))+\cos(x)\sin(y(x))+\sin(x)=c_1,y(x)]$$

✓ **Maple** : cpu = 0.198 (sec), leaf count = 22

$$\{c_1-\cos(xy(x))+\cos(y(x))+\sin(x)+\sin(x+y(x))=0\}$$

2.362 ODE No. 362

$$y'(x)\big(x^2y(x)\sin(xy(x))-4x\big)-y(x)+xy(x)^2\sin(xy(x))=0$$

✓ **Mathematica** : cpu = 0.384123 (sec), leaf count = 23

$$\text{Solve}[-4\log(y(x))-\cos(xy(x))-\log(x)=c_1,y(x)]$$

✓ **Maple** : cpu = 0.241 (sec), leaf count = 23

$$\left\{y(x)=\frac{\text{RootOf}\left(c_1x^4\text{e}^{-\frac{\cos(_Z)}{4}}-_Z\right)}{x}\right\}$$

2.363 ODE No. 363

$$\big(xy'(x)-y(x)\big)\cos^2\left(\frac{y(x)}{x}\right)+x=0$$

✓ **Mathematica** : cpu = 0.402693 (sec), leaf count = 33

$$\text{Solve}\left[\frac{y(x)}{2x}+\frac{1}{4}\sin\left(\frac{2y(x)}{x}\right)=-\log(x)+c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.081 (sec), leaf count = 35

$$\left\{-c_1-\ln(x)-\frac{x\cos\left(\frac{y(x)}{x}\right)\sin\left(\frac{y(x)}{x}\right)+y(x)}{2x}=0\right\}$$

2.364 ODE No. 364

$$xy'(x)\left(y(x)\sin\left(\frac{y(x)}{x}\right)-x\cos\left(\frac{y(x)}{x}\right)\right)-y(x)\left(y(x)\sin\left(\frac{y(x)}{x}\right)+x\cos\left(\frac{y(x)}{x}\right)\right)=0$$

✓ **Mathematica** : cpu = 0.483019 (sec), leaf count = 31

$$\text{Solve}\left[-\log\left(\frac{y(x)}{x}\right)-\log\left(\cos\left(\frac{y(x)}{x}\right)\right)=2\log(x)+c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.199 (sec), leaf count = 23

$$\left\{y(x)=\frac{c_1}{x\cos(\text{RootOf}(-\_Zx^2\cos(\_Z)+c_1))}\right\}$$

2.365 ODE No. 365

$$(y(x)f(x^2+y(x)^2)-x)y'(x)+xf(x^2+y(x)^2)+y(x)=0$$

✓ **Mathematica** : cpu = 0.392581 (sec), leaf count = 156

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{x-f(x^2+K[2]^2)K[2]}{x^2+K[2]^2}-\int_1^x\left(\frac{-2K[1]K[2]f'(K[1]^2+K[2]^2)-1}{K[1]^2+K[2]^2}-\frac{2(-f(K[1]^2+K[2]^2)K[1]-K[2])K[2]}{(K[1]^2+K[2]^2)^2}\right)dx\right)dy(x)\right]$$

✓ **Maple** : cpu = 0.355 (sec), leaf count = 42

$$\left\{y(x)=\frac{x}{\tan\left(\text{RootOf}\left(2c_1-2\_Z-\left(\int\frac{(\tan^2(\_Z)+1)x^2}{\tan(\_Z)^2}\frac{f(\_a)}{\_a}d\_a\right)\right)\right)}\right\}$$

2.366 ODE No. 366

$$f(ay(x)^2+x^2)(ay(x)y'(x)+x)-xy'(x)-y(x)=0$$

✓ **Mathematica** : cpu = 0.314976 (sec), leaf count = 91

$$\text{Solve}\left[\int_1^{y(x)}\left(x-af(x^2+aK[2]^2)K[2]-\int_1^x(1-2aK[1]K[2]f'(K[1]^2+aK[2]^2))dK[1]\right)dK[2]+\int_1^x(y(x)-f(K[1]^2+aK[2]^2))dK[2]\right]$$

✓ **Maple** : cpu = 0.137 (sec), leaf count = 45

$$\left\{-\frac{axy(x)^2}{\sqrt{a^2y(x)^2}}+c_1-\left(\int^{-\frac{ay(x)^2}{2}-\frac{x^2}{2}}f(-2\_a)d\_a\right)=0\right\}$$

2.367 ODE No. 367

$$f(x^cy(x))(bxy'(x)-a)-x^ay(x)^b(cy(x)+xy'(x))=0$$

✗ **Mathematica** : cpu = 10.8087 (sec), leaf count = 0 , could not solve

`DSolve[-(x^a*y[x]^b*(c*y[x] + x*Derivative[1][y][x])) + f[x^c*y[x]]*(-a + b*x*Derivative[1][y][x]),y[x]]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(f(x^c*y(x))*(b*x*diff(y(x),x)-a)-x^a*y(x)^b*(x*diff(y(x),x)+c*y(x)) = 0,y(x))`

2.368 ODE No. 368

$$ay(x) + bx^2 + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.47476 (sec), leaf count = 795

$$\left\{ \text{Solve} \left[\int_1^x \left(\frac{2\sqrt{-bK[1]^2 - ay(x)y(x)}}{bK[1]^4 + ay(x)K[1]^2 + 4y(x)^2} + \frac{bK[1]^3 + ay(x)K[1]}{bK[1]^4 + ay(x)K[1]^2 + 4y(x)^2} \right) dK[1] + \int_1^{y(x)} \left(-\frac{\sqrt{-bx^2 - aK[2]x}}{bx^4 + aK[2]x^2 + 4K[2]^2} - \right. \right. \right.$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

time expired

2.369 ODE No. 369

$$-a^2 + y'(x)^2 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0877192 (sec), leaf count = 107

$$\left\{ \left\{ y(x) \rightarrow -\frac{a \tan(x - c_1)}{\sqrt{1 + \tan^2(x - c_1)}} \right\}, \left\{ y(x) \rightarrow \frac{a \tan(x - c_1)}{\sqrt{1 + \tan^2(x - c_1)}} \right\}, \left\{ y(x) \rightarrow -\frac{a \tan(x + c_1)}{\sqrt{1 + \tan^2(x + c_1)}} \right\}, \left\{ y(x) \rightarrow \frac{a \tan(x + c_1)}{\sqrt{1 + \tan^2(x + c_1)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.114 (sec), leaf count = 68

$$\left\{ y(x) = a, y(x) = \sqrt{\frac{a^2}{\tan^2(c_1 - x) + 1}} \tan(c_1 - x), y(x) = -a, y(x) = -\sqrt{\frac{a^2}{\tan^2(c_1 - x) + 1}} \tan(c_1 - x) \right\}$$

2.370 ODE No. 370

$$-f(x)^2 + y'(x)^2 + y(x)^2 = 0$$

✗ **Mathematica** : cpu = 14.8204 (sec), leaf count = 0 , could not solve

`DSolve[-f[x]^2 + y[x]^2 + Derivative[1][y][x]^2 == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x),x)^2+y(x)^2-f(x)^2 = 0,y(x))`

2.371 ODE No. 371

$$y'(x)^2 - y(x)^3 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0565052 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow 1 + \tan^2\left(\frac{-x + c_1}{2}\right) \right\}, \left\{ y(x) \rightarrow 1 + \tan^2\left(\frac{x + c_1}{2}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.126 (sec), leaf count = 20

$$\left\{ y(x) = 1, y(x) = \tan^2\left(-\frac{c_1}{2} + \frac{x}{2}\right) + 1 \right\}$$

2.372 ODE No. 372

$$ay(x) + b + y'(x)^2 - 4y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.0038499 (sec), leaf count = 27

$$\{\{y(x) \rightarrow \wp(x - c_1; a, b)\}, \{y(x) \rightarrow \wp(x + c_1; a, b)\}\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 232

$$\left\{y(x)=-\frac{3a+\left(-3ia+i\left(27b+3\sqrt{-3a^3+81b^2}\right)^{\frac{2}{3}}\right)\sqrt{3}+\left(27b+3\sqrt{-3a^3+81b^2}\right)^{\frac{2}{3}}}{12\left(27b+3\sqrt{-3a^3+81b^2}\right)^{\frac{1}{3}}},y(x)=\frac{3a+\left(27b+3\sqrt{-3a^3+81b^2}\right)^{\frac{2}{3}}}{6\left(27b+3\sqrt{-3a^3+81b^2}\right)^{\frac{1}{3}}}\right\}$$

2.373 ODE No. 373

$$a^2y(x)^2\big(\log^2(y(x))-1\big)+y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.325995 (sec), leaf count = 185

$$\left\{\left\{y(x)\rightarrow\exp\left(-\frac{1}{2}\sqrt{-e^{2iax-2c_1}-e^{2c_1-2iax}+2}\right)\right\},\left\{y(x)\rightarrow\exp\left(\frac{1}{2}\sqrt{-e^{2iax-2c_1}-e^{2c_1-2iax}+2}\right)\right\},\left\{y(x)\rightarrow\exp\left(-\frac{1}{2}\sqrt{-e^{-2iax-2c_1}-e^{2c_1-2iax}+2}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.238 (sec), leaf count = 49

$$\{y(x)=e^{-\sin((-c_1+x)a)},y(x)=e^{\text{RootOf}\left((\_Z^2-1)a^2e^{2\_Z}\right)},y(x)=e^{\sin(ax-c_1a)}\}$$

2.374 ODE No. 374

$$y'(x)^2-2y'(x)-y(x)^2=0$$

✓ **Mathematica** : cpu = 0.0867985 (sec), leaf count = 73

$$\left\{\left\{y(x)\rightarrow\text{InverseFunction}\left[-\frac{\sqrt{\#1^2+1}}{\#1}-\frac{1}{\#1}+\sinh^{-1}(\#1)\&[-x+c_1]\right\},\left\{y(x)\rightarrow\text{InverseFunction}\left[-\frac{\sqrt{\#1^2+1}}{\#1}+\frac{1}{\#1}+\sinh^{-1}(\#1)\&[-x+c_1]\right\}\right\}\right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 85

$$\left\{-c_1+x-\operatorname{arcsinh}\big(y(x)\big)-\sqrt{y(x)^2+1}\,y(x)+\frac{\big(y(x)^2+1\big)^{\frac{3}{2}}}{y(x)}-\frac{1}{y(x)}=0,-c_1+x+\operatorname{arcsinh}\big(y(x)\big)+\sqrt{y(x)^2+1}\,y(x)-\frac{\big(y(x)^2+1\big)^{\frac{3}{2}}}{y(x)}+\frac{1}{y(x)}=0\right\}$$

2.375 ODE No. 375

$$ay'(x)+bx+y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.0336034 (sec), leaf count = 71

$$\left\{\left\{y(x)\rightarrow\frac{1}{2}\left(-\frac{\left(a^2-4bx\right)^{3/2}}{6b}-ax\right)+c_1\right\},\left\{y(x)\rightarrow\frac{1}{2}\left(\frac{\left(a^2-4bx\right)^{3/2}}{6b}-ax\right)+c_1\right\}\right\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 49

$$\left\{y(x)=-\frac{ax}{2}+c_1-\frac{\left(a^2-4bx\right)^{\frac{3}{2}}}{12b},y(x)=-\frac{ax}{2}+c_1+\frac{\left(a^2-4bx\right)^{\frac{3}{2}}}{12b}\right\}$$

2.376 ODE No. 376

$$ay'(x)+by(x)+y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.262035 (sec), leaf count = 110

$$\left\{\left\{y(x) \rightarrow \text{InverseFunction}\left[-\frac{\sqrt{a^2-4\#1b}+a\log\left(a-\sqrt{a^2-4\#1b}\right)}{2b}\right]\&\left[\frac{x}{2}+c_1\right]\right\},\left\{y(x) \rightarrow \text{InverseFunction}\left[-\frac{\sqrt{a^2-4\#1b}+a\log\left(a-\sqrt{a^2-4\#1b}\right)}{2b}\right]\&\left[\frac{x}{2}+c_1\right]\right\}\right\}$$

✓ **Maple** : cpu = 0.768 (sec), leaf count = 279

$$\left\{y(x)=-\frac{\left(2a+e^{\frac{-2a\operatorname{LambertW}\left(\frac{2e^{-1}e^{\frac{c_1b}{a}}e^{-\frac{bx}{a}}}{\sqrt{-\frac{1}{b}}a}\right)-a\ln\left(-\frac{1}{4b}\right)-2a+(2c_1-2x)b}}{2a}}\right)e^{\frac{-2a\operatorname{LambertW}\left(\frac{2e^{-1}e^{\frac{c_1b}{a}}e^{-\frac{bx}{a}}}{\sqrt{-\frac{1}{b}}a}\right)-a\ln\left(-\frac{1}{4b}\right)-2a+(2c_1-2x)b}}{2a}}}{4b},y(x)=-\frac{\left(\operatorname{LambertW}\left(\frac{2e^{-1}e^{\frac{c_1b}{a}}e^{-\frac{bx}{a}}}{\sqrt{-\frac{1}{b}}a}\right)-a\ln\left(-\frac{1}{4b}\right)-2a+(2c_1-2x)b\right)}{4b}\right\}$$

2.377 ODE No. 377

$$y'(x)^2+(x-2)y'(x)-y(x)+1=0$$

✓ **Mathematica** : cpu = 0.0038089 (sec), leaf count = 19

$$\left\{\left\{y(x) \rightarrow c_1 x+1+c_1^2-2 c_1\right\}\right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 24

$$\left\{y(x)=-\frac{1}{4} x^2+x, y(x)=c_1^2+c_1(x-2)+1\right\}$$

2.378 ODE No. 378

$$(a+x)y'(x)+y'(x)^2-y(x)=0$$

✓ **Mathematica** : cpu = 0.0038321 (sec), leaf count = 18

$$\left\{\left\{y(x) \rightarrow a c_1+c_1 x+c_1^2\right\}\right\}$$

✓ **Maple** : cpu = 0.025 (sec), leaf count = 20

$$\left\{y(x)=c_1\left(c_1+a+x\right), y(x)=-\frac{(a+x)^2}{4}\right\}$$

2.379 ODE No. 379

$$y'(x)^2-(x+1)y'(x)+y(x)=0$$

✓ **Mathematica** : cpu = 0.0037739 (sec), leaf count = 18

$$\left\{\left\{y(x) \rightarrow c_1 x-c_1^2+c_1\right\}\right\}$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 22

$$\left\{y(x)=c_1\left(-c_1+x+1\right), y(x)=\frac{(x+1)^2}{4}\right\}$$

2.380 ODE No. 380

$$y'(x)^2 + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.404607 (sec), leaf count = 1757

$$\left\{ \left\{ y(x) \rightarrow -\frac{x^2}{4} - \frac{1}{4} \sqrt[3]{x^6 - 20 \cosh(3c_1)x^3 - 20 \sinh(3c_1)x^3 - 8 \cosh(6c_1) - 8 \sinh(6c_1) + 8 \sqrt{-\cosh(3c_1)x^9 - \sinh(3c_1)x^9}} \right. \right.$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 619

$$\left\{ y(x) = \frac{\left(-i\sqrt{3} x^2 - x^2 - 2\left(-x^3 + 6c_1 + 2\sqrt{-3c_1x^3 + 9c_1^2} \right)^{\frac{1}{3}} x + i\left(-x^3 + 6c_1 + 2\sqrt{-3c_1x^3 + 9c_1^2} \right)^{\frac{2}{3}} \sqrt{3} - \left(-x^3 + 6c_1 + 2\sqrt{-3c_1x^3 + 9c_1^2} \right)^{\frac{1}{3}} \right)}{\dots}$$

2.381 ODE No. 381

$$y'(x)^2 - 2xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.423756 (sec), leaf count = 1757

$$\left\{ \left\{ y(x) \rightarrow \frac{x^2}{4} + \frac{1}{4} \sqrt[3]{x^6 - 20 \cosh(3c_1)x^3 - 20 \sinh(3c_1)x^3 - 8 \cosh(6c_1) - 8 \sinh(6c_1) + 8 \sqrt{-\cosh(3c_1)x^9 - \sinh(3c_1)x^9}} \right. \right.$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 579

$$\left\{ y(x) = -\frac{\left(-i\sqrt{3} x^2 - x^2 + 2\left(x^3 - 6c_1 + 2\sqrt{-3c_1x^3 + 9c_1^2} \right)^{\frac{1}{3}} x + i\left(x^3 - 6c_1 + 2\sqrt{-3c_1x^3 + 9c_1^2} \right)^{\frac{2}{3}} \sqrt{3} - \left(x^3 - 6c_1 + 2\sqrt{-3c_1x^3 + 9c_1^2} \right)^{\frac{1}{3}} \right)}{\dots}$$

16

2.382 ODE No. 382

$$axy'(x) - bx^2 - c + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.185046 (sec), leaf count = 201

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left(\frac{1}{2} x \sqrt{a^2x^2 + 4bx^2 + 4c} + \frac{2c \log\left(\sqrt{a^2 + 4b} \sqrt{a^2x^2 + 4bx^2 + 4c} + a^2x + 4bx \right)}{\sqrt{a^2 + 4b}} - \frac{ax^2}{2} \right) + c_1 \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \left(-\frac{1}{2} x \sqrt{a^2x^2 + 4bx^2 + 4c} + \frac{2c \log\left(\sqrt{a^2 + 4b} \sqrt{a^2x^2 + 4bx^2 + 4c} + a^2x + 4bx \right)}{\sqrt{a^2 + 4b}} - \frac{ax^2}{2} \right) + c_1 \right\} \right.$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 146

$$\left\{ y(x) = -\frac{ax^2}{4} - \frac{c \ln\left(\sqrt{a^2 + 4b} x + \sqrt{(a^2 + 4b)x^2 + 4c} \right)}{\sqrt{a^2 + 4b}} + c_1 - \frac{\sqrt{(a^2 + 4b)x^2 + 4c} x}{4}, y(x) = -\frac{ax^2}{4} + \frac{c \ln\left(\sqrt{a^2 + 4b} x - \sqrt{(a^2 + 4b)x^2 + 4c} \right)}{\sqrt{a^2 + 4b}} + c_1 - \frac{\sqrt{(a^2 + 4b)x^2 + 4c} x}{4} \right.$$

2.383 ODE No. 383

$$axy'(x) + by(x) + cx^2 + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 5.63628 (sec), leaf count = 1118

$$\left\{ \text{Solve} \left[\int_1^x \left(\frac{\sqrt{a^2 K[1]^2 - 4cK[1]^2 - 4by(x)y(x)}}{cK[1]^4 + 2ay(x)K[1]^2 + by(x)K[1]^2 + 4y(x)^2} + \frac{cK[1]^3 + ay(x)K[1] + by(x)K[1]}{cK[1]^4 + 2ay(x)K[1]^2 + by(x)K[1]^2 + 4y(x)^2} \right) dK[1] + \int_1^{y(x)} \right. \right.$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

time expired

2.384 ODE No. 384

$$(ax + b)y'(x) - ay(x) + c + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0049734 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow \frac{bc_1 + c + c_1^2}{a} + c_1x \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 50

$$\left\{ y(x) = \frac{c_1^2 + c_1(ax + b) + c}{a}, y(x) = \frac{-a^2x^2 - 2abx - b^2 + 4c}{4a} \right\}$$

2.385 ODE No. 385

$$-2x^2y'(x) + y'(x)^2 + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.73493 (sec), leaf count = 6217

$$\left\{ \left\{ y(x) \rightarrow -x^3 \left(\frac{1}{2} \sqrt{\frac{2(8e^{6c_1}x^{18} + e^{12c_1}x^{12})}{9x^{24} \sqrt[3]{\frac{64\sqrt{e^{12c_1}x^{60} - 3e^{18c_1}x^{54} + 3e^{24c_1}x^{48} - e^{30c_1}x^{42}}}{x^{36}} + \frac{64e^{6c_1}}{x^6} + \frac{160e^{12c_1}}{x^{12}} - \frac{8e^{18c_1}}{x^{18}}}} + \frac{1}{18} \sqrt[3]{\frac{64\sqrt{e^{12c_1}x^{60} - 3e^{18c_1}x^{54}}}{x^{36}}}} \right. \right.$$

✓ **Maple** : cpu = 0.263 (sec), leaf count = 169

$$\left\{ y(x) = \frac{x^4 - \text{RootOf}(x^{16} - 12\_Z^2x^{12} - 16\_Z^3x^{10} + 30\_Z^4x^8 + 96\_Z^5x^6 + 100\_Z^6x^4 + 48\_Z^7x^2 + 9\_Z^8 - 16c_1x^4)^2}{2x}, y(x) \right.$$

2.386 ODE No. 386

$$ax^3y'(x) - 2ax^2y(x) + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.193919 (sec), leaf count = 119

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}(\cosh(2c_1) + \sinh(2c_1))(-\sqrt{2}\sqrt{a}x^2 + 2\cosh(2c_1) + 2\sinh(2c_1)) \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{a}x^2 \cosh(2c_1)}{\sqrt{2}} + \frac{\sqrt{a}x^2 \sinh(2c_1)}{\sqrt{2}} \right\} \right.$$

✓ **Maple** : cpu = 0.556 (sec), leaf count = 27

$$\left\{ y(x) = -\frac{ax^4}{8}, y(x) = c_1x^2 + \frac{2c_1^2}{a} \right\}$$

2.387 ODE No. 387

$$y'(x)^2 + e^x(y'(x) - y(x)) = 0$$

✓ **Mathematica** : cpu = 2.39279 (sec), leaf count = 190

$$\left\{ \text{Solve} \left[\log(y(x)) - \frac{-e^{x/2} \sqrt{4y(x) + e^x} - \frac{4 \sqrt{\frac{e^x}{y(x)} + 4} y(x)^{3/2} \sinh^{-1} \left(\frac{e^{x/2}}{2 \sqrt{y(x)}} \right)}{\sqrt{4y(x) + e^x}} + e^x}{2y(x)} = c_1, y(x) \right], \text{Solve} \left[\log(y(x)) - \frac{e^{x/2} \sqrt{4y(x) + e^x}}{2y(x)} = c_1, y(x) \right] \right\}$$

✓ **Maple** : cpu = 0.673 (sec), leaf count = 115

$$\left\{ -c_1 - 2 \operatorname{arctanh} \left(\sqrt{4 e^x y(x) + e^{2x}} e^{-x} \right) + \ln(y(x)) - \frac{e^x}{2y(x)} - \frac{\sqrt{4 e^x y(x) + e^{2x}}}{2y(x)} = 0, -c_1 + 2 \operatorname{arctanh} \left(\sqrt{4 e^x y(x) + e^{2x}} e^{-x} \right) + \ln(y(x)) - \frac{e^x}{2y(x)} - \frac{\sqrt{4 e^x y(x) + e^{2x}}}{2y(x)} = 0 \right\}$$

2.388 ODE No. 388

$$y'(x)^2 - 2y(x)y'(x) - 2x = 0$$

✓ **Mathematica** : cpu = 0.715414 (sec), leaf count = 61

$$\text{Solve} \left[\left\{ x = \frac{K[1] \sinh^{-1}(K[1])}{2 \sqrt{K[1]^2 + 1}} + \frac{c_1 K[1]}{\sqrt{K[1]^2 + 1}}, y(x) = \frac{K[1]}{2} - \frac{x}{K[1]} \right\}, \{y(x), K[1]\} \right]$$

✓ **Maple** : cpu = 0.084 (sec), leaf count = 223

$$\left\{ \frac{-2c_1 y(x) + 2c_1 \sqrt{y(x)^2 + 2x} + \sqrt{2y(x)^2 + 2x - 2\sqrt{y(x)^2 + 2x} y(x) + 1} x + \left(\frac{y(x)}{2} - \frac{\sqrt{y(x)^2 + 2x}}{2} \right) \operatorname{arcsinh}(-y(x) + \sqrt{y(x)^2 + 2x})}{\sqrt{2y(x)^2 + 2x - 2\sqrt{y(x)^2 + 2x} y(x) + 1}} = c_1 \right\}$$

2.389 ODE No. 389

$$y'(x)^2 - (4y(x) + 1)y'(x) + y(x)(4y(x) + 1) = 0$$

✓ **Mathematica** : cpu = 0.0806726 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{4} e^{x-4c_1} (-e^x + 2e^{2c_1}) \right\}, \left\{ y(x) \rightarrow \frac{1}{4} e^{x+2c_1} (-2 + e^{x+2c_1}) \right\} \right\}$$

✓ **Maple** : cpu = 0.529 (sec), leaf count = 71

$$\left\{ y(x) = -\frac{1}{4}, y(x) = \frac{c_1 - \sqrt{-c_1 e^{-2x}} e^{2x}}{\sqrt{-c_1 e^{-2x}} c_1}, y(x) = -\frac{c_1 + \sqrt{-c_1 e^{-2x}} e^{2x}}{\sqrt{-c_1 e^{-2x}} c_1} \right\}$$

2.390 ODE No. 390

$$ay(x)y'(x) - bx - c + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.87123 (sec), leaf count = 154

$$\text{Solve} \left[\left[x = \left(\frac{\tan^{-1} \left(\frac{\sqrt{a} K[1]}{\sqrt{b - aK[1]^2}} \right)}{\sqrt{a}} - \frac{c \sqrt{b - aK[1]^2}}{bK[1]} \right) \exp \left(b \left(\frac{\log(K[1])}{b} - \frac{\log(b - aK[1]^2)}{2b} \right) \right) + c_1 \exp \left(b \left(\frac{\log(K[1])}{b} - \frac{\log(b - aK[1]^2)}{2b} \right) \right) \right] \right]$$

✓ **Maple** : cpu = 0.418 (sec), leaf count = 281

$$\left\{y(x)=\frac{2\left((bx+c)a-\frac{\left(b+e^{2\operatorname{RootOf}\left(a\,b^2x-abxe^{2-Z}+c_1\sqrt{a}\,b^2+c_1\sqrt{a}\,be^{2-Z}-Zb^2-Zbe^{2-Z}+abc-ac\,e^{2-Z}\right)}\right)^2}{4}e^{-2\operatorname{RootOf}\left(a\,b^2x-abxe^{2-Z}+c_1\sqrt{a}\,b^2+c_1\sqrt{a}\,be^{2-Z}-Zb^2-Zbe^{2-Z}+abc-ac\,e^{2-Z}\right)}\right)}{b+e^{2\operatorname{RootOf}\left(a\,b^2x-abxe^{2-Z}+c_1\sqrt{a}\,b^2+c_1\sqrt{a}\,be^{2-Z}-Zb^2-Zbe^{2-Z}+abc-ac\,e^{2-Z}\right)}}$$

2.391 ODE No. 391

$$y'(x)(ay(x)+bx)+abxy(x)+y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.015656 (sec), leaf count = 29

$$\left\{\left\{y(x)\rightarrow c_1e^{-ax}\right\},\left\{y(x)\rightarrow -\frac{bx^2}{2}+c_1\right\}\right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 22

$$\left\{y(x)=c_1e^{-ax},y(x)=-\frac{bx^2}{2}+c_1\right\}$$

2.392 ODE No. 392

$$y(x)^2\log(ay(x))-xy(x)y'(x)+y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.223711 (sec), leaf count = 27

$$\left\{\left(\left\{y(x)\rightarrow \frac{e^{\frac{c_1x}{2}-\frac{c_1^2}{4}}}{a}\right\}\right)\right\}$$

✓ **Maple** : cpu = 0.502 (sec), leaf count = 50

$$\left\{y(x)=\frac{e^{\frac{x^2}{4}}}{a},y(x)=\frac{e^{c_1x}e^{-c_1^2}}{a},y(x)=\frac{e^{-c_1^2}e^{-c_1x}}{a}\right\}$$

2.393 ODE No. 393

$$y'(x)^2+2y(x)\cot(x)y'(x)-y(x)^2=0$$

✓ **Mathematica** : cpu = 0.104328 (sec), leaf count = 31

$$\left\{\left\{y(x)\rightarrow c_1\csc^2\left(\frac{x}{2}\right)\right\},\left\{y(x)\rightarrow c_1\sec^2\left(\frac{x}{2}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.175 (sec), leaf count = 77

$$\left\{y(x)=\frac{c_1\left(1+\sqrt{\tan^2(x)+1}\right)}{\sqrt{\frac{\tan^2(x)}{\tan^2(x)+1}}\tan(x)},y(x)=\frac{c_1\left(\tan^2(x)+1\right)\sqrt{\frac{\tan^2(x)}{\tan^2(x)+1}}}{\left(1+\sqrt{\tan^2(x)+1}\right)\tan(x)}\right\}$$

2.394 ODE No. 394

$$-\left(g(x)-f(x)^2\right)e^{-2\int_a^xf(xp)dxp}+2f(x)y(x)y'(x)+g(x)y(x)^2+y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.373914 (sec), leaf count = 89

$$\left(\left(\left(y(x) \rightarrow e^{-\int_a^x f(K[1]) \, dK[1]} \left\{ \begin{array}{ll} \sin\left(c_1 + \int_a^x \sqrt{g(K[1]) - f(K[1])^2} \, dK[1]\right) & g(x) > f(x)^2 \\ \cosh\left(c_1 + \int_a^x \sqrt{f(K[1])^2 - g(K[1])} \, dK[1]\right) & g(x) < f(x)^2 \\ c_1 & \text{True} \end{array} \right\} \right)\right)\right)$$

✓ **Maple** : cpu = 5.127 (sec), leaf count = 109

$$\left\{y(x) = \frac{e^{-2\left(\int_a^x f(xp)dxp\right)}}{\sqrt{\tan^2\left(c_1 - \left(\int \sqrt{(-f(x)^2 + g(x))e^{\int_a^x -4f(xp)dxp}} e^{\int_a^x 2f(xp)dxp} dx\right)\right)} + 1} \tan\left(c_1 - \left(\int \sqrt{(-f(x)^2 + g(x))e^{\int_a^x -4f(xp)dxp}} e^{\int_a^x 2f(xp)dxp} dx\right)\right)\right\}$$

2.395 ODE No. 395

$$2f(x)y(x)y'(x) + g(x)y(x)^2 + h(x) + y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 29.1152 (sec), leaf count = 0 , could not solve

`DSolve[h[x] + g[x]*y[x]^2 + 2*f[x]*y[x]*Derivative[1][y][x] + Derivative[1][y][x]^2 == 0, y[x]]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x),x)^2+2*f(x)*y(x)*diff(y(x),x)+g(x)*y(x)^2+h(x) = 0,y(x))`

2.396 ODE No. 396

$$(y(x) - x)y(x)y'(x) + y'(x)^2 - xy(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.0703115 (sec), leaf count = 29

$$\left\{\left\{y(x) \rightarrow \frac{1}{x - c_1}\right\}, \left\{y(x) \rightarrow c_1 e^{\frac{x^2}{2}}\right\}\right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 20

$$\left\{y(x) = \frac{1}{c_1 + x}, y(x) = c_1 e^{\frac{x^2}{2}}\right\}$$

2.397 ODE No. 397

$$-2x^3y(x)^2y'(x) - 4x^2y(x)^3 + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.751131 (sec), leaf count = 143

$$\left\{\text{Solve}\left[-\frac{x\sqrt{x^4y(x)+4}y(x)^{3/2}\sinh^{-1}\left(\frac{1}{2}x^2\sqrt{y(x)}\right)}{2\sqrt{x^2y(x)^3(x^4y(x)+4)}} - \frac{1}{4}\log(y(x)) = c_1, y(x)\right], \text{Solve}\left[\frac{xy(x)^{3/2}\sqrt{x^4y(x)+4}\sinh^{-1}\left(\frac{1}{2}x^2\sqrt{y(x)}\right)}{2\sqrt{x^2y(x)^3(x^4y(x)+4)}} = c_1, y(x)\right]\right\}$$

✓ **Maple** : cpu = 0.343 (sec), leaf count = 128

$$\left\{y(x) = \frac{-2\sqrt{2} \, x^2 - 2c_1}{2c_1x^4 - c_1^3}, y(x) = \frac{2\sqrt{2} \, x^2 - 2c_1}{2c_1x^4 - c_1^3}, y(x) = \frac{(c_1\sqrt{2} \, x^2 - 2)c_1^2}{2c_1^2x^4 - 4}, y(x) = -\frac{4}{x^4}, y(x) = -\frac{(c_1\sqrt{2} \, x^2 + 2)c_1^2}{2c_1^2x^4 - 4}\right\}$$

2.398 ODE No. 398

$$y'(x)^2 - 3xy(x)^{2/3}y'(x) + 9y(x)^{5/3} = 0$$

✓ **Mathematica** : cpu = 1.75629 (sec), leaf count = 258

$$\left\{ \text{Solve} \left[-\frac{\left(x^2 - 4\sqrt[3]{y(x)}\right)^{3/2} y(x)^2 \log(y(x))}{6\left(\left(x^2 - 4\sqrt[3]{y(x)}\right) y(x)^{4/3}\right)^{3/2}} + \frac{\sqrt{\left(x^2 - 4\sqrt[3]{y(x)}\right) y(x)^{4/3}} \log\left(\sqrt{x^2 - 4\sqrt[3]{y(x)}} + x\right)}{\sqrt{x^2 - 4\sqrt[3]{y(x)}} y(x)^{2/3}} + \frac{1}{6} \log(y(x)) = c_1, y(x) \right] \right\}$$

✓ **Maple** : cpu = 2.125 (sec), leaf count = 138

$$\left\{ -c_1 - \frac{\sqrt{-4\left(\frac{y(x)}{x^6}\right)^{\frac{5}{3}} + \left(\frac{y(x)}{x^6}\right)^{\frac{4}{3}}}}{\left(\frac{y(x)}{x^6}\right)^{\frac{2}{3}} \sqrt{-4\left(\frac{y(x)}{x^6}\right)^{\frac{1}{3}} + 1}} \operatorname{arctanh}\left(\sqrt{-4\left(\frac{y(x)}{x^6}\right)^{\frac{1}{3}} + 1}\right) + \ln(x) + \frac{\ln\left(\frac{y(x)}{x^6}\right)}{6} - \frac{\ln\left(4\left(\frac{y(x)}{x^6}\right)^{\frac{1}{3}} - 1\right)}{6} + \frac{\ln\left(\frac{64y(x)}{x^6} - 1\right)}{6} - \frac{\ln\left(\frac{64y(x)}{x^6} - 1\right)}{6} \right\}$$

2.399 ODE No. 399

$$2y'(x)^2 + (x - 1)y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0037386 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow c_1 x + 2c_1^2 - c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 22

$$\left\{ y(x) = c_1(2c_1 + x - 1), y(x) = -\frac{(x - 1)^2}{8} \right\}$$

2.400 ODE No. 400

$$-2x^2y'(x) + 2y'(x)^2 + 3xy(x) = 0$$

✓ **Mathematica** : cpu = 0.482087 (sec), leaf count = 135

$$\left\{ \text{Solve} \left[\frac{1}{3} \log(y(x)) - \frac{2\sqrt{x^4 - 6xy(x)} \tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{x^3 - 6y(x)}}\right)}{3\sqrt{x} \sqrt{x^3 - 6y(x)}} = c_1, y(x) \right], \text{Solve} \left[\frac{2\sqrt{x^4 - 6xy(x)} \tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{x^3 - 6y(x)}}\right)}{3\sqrt{x} \sqrt{x^3 - 6y(x)}} + \frac{1}{3} \log(y(x)) = c_1, y(x) \right] \right\}$$

✓ **Maple** : cpu = 0.186 (sec), leaf count = 74

$$\left\{ y(x) = \frac{x^3}{6}, y(x) = \frac{-\sqrt{-6c_1x} x + 3}{3c_1}, y(x) = \frac{\sqrt{-6c_1x} x + 3}{3c_1}, y(x) = c_1 - \frac{\sqrt{-6c_1x} x}{3}, y(x) = c_1 + \frac{\sqrt{-6c_1x} x}{3} \right\}$$

2.401 ODE No. 401

$$3y'(x)^2 - 2xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.330346 (sec), leaf count = 1093

$$\left\{ \left\{ y(x) \rightarrow \text{Root}\left[-16e^{6c_1}x^6 + 3\#1^4x^4 + 144e^{6c_1}\#1x^4 - 24\#1^5x^2 - 378e^{6c_1}\#1^2x^2 + 243e^{12c_1} + 48\#1^6 + 216e^{6c_1}\#1^3\&, 1\right] \right\}, \left\{ y(x) \rightarrow \text{Root}\left[-16e^{6c_1}x^6 + 3\#1^4x^4 + 144e^{6c_1}\#1x^4 - 24\#1^5x^2 - 378e^{6c_1}\#1^2x^2 + 243e^{12c_1} + 48\#1^6 + 216e^{6c_1}\#1^3\&, 1\right] \right\} \right\}$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 580

$$\left\{y(x)=-\frac{\left(-i\sqrt{3}\,x^2-x^2+2\left(x^3-54c_1+6\sqrt{-3c_1x^3+81c_1^2}\right)^{\frac{1}{3}}x+i\sqrt{3}\left(x^3-54c_1+6\sqrt{-3c_1x^3+81c_1^2}\right)^{\frac{2}{3}}-\left(x^3-54c_1+\right.\right.}{\left.\left.\right)}\right.$$

2.402

ODE No. 402

$$x^2+4xy'(x)+3y'(x)^2-y(x)=0$$

✓ **Mathematica** : cpu = 6.5522 (sec), leaf count = 1211

$$\left\{\text{Solve}\left[\frac{1}{2}\left(\frac{\tanh^{-1}\left(\frac{1-3x}{2\sqrt{1-9y(x)}}\right)}{\sqrt{1-9y(x)}}+\log\left(-3x^2+2x-12y(x)+1\right)+\frac{9\tanh^{-1}\left(\frac{-2\sqrt{1-9y(x)}x+x+9y(x)}{\sqrt{-9y(x)-4\sqrt{1-9y(x)}+5}\sqrt{x^2+3y(x)}}\right)y(x)}{\sqrt{1-9y(x)}\sqrt{-9y(x)-4\sqrt{1-9y(x)}+5}}-\frac{9\tanh^{-1}\left(\frac{-2\sqrt{1-9y(x)}x+x+9y(x)}{\sqrt{-9y(x)-4\sqrt{1-9y(x)}+5}\sqrt{x^2+3y(x)}}\right)}{\sqrt{1-9y(x)}}\right],y(x)\right\}$$

✓ **Maple** : cpu = 0.154 (sec), leaf count = 101

$$\left\{y(x)=-\frac{x^2}{3},y(x)=\frac{-3c_1^2x^2-2c_1\sqrt{3}\,x+3}{12c_1^2},y(x)=\frac{-3c_1^2x^2+2c_1\sqrt{3}\,x+3}{12c_1^2},y(x)=\frac{c_1^2}{4}-\frac{c_1\sqrt{3}\,x}{6}-\frac{x^2}{4},y(x)=\frac{c_1^2}{4}+\frac{c_1\sqrt{3}}{6}\right\}$$

2.403

ODE No. 403

$$ay'(x)^2+by'(x)-y(x)=0$$

✓ **Mathematica** : cpu = 0.252719 (sec), leaf count = 116

$$\left\{\left\{y(x)\rightarrow\text{InverseFunction}\left[\frac{\sqrt{4\#1a+b^2}+b\log\left(b-\sqrt{4\#1a+b^2}\right)}{2a}\&\left[\frac{x}{2a}+c_1\right]\right\},\left\{y(x)\rightarrow\text{InverseFunction}\left[\frac{\sqrt{4\#1a+b^2}-b\log\left(b+\sqrt{4\#1a+b^2}\right)}{2a}\&\left[\frac{x}{2a}+c_1\right]\right\}\right\}$$

✓ **Maple** : cpu = 0.815 (sec), leaf count = 247

$$\left\{y(x)=\frac{\left(2b+e^{-\frac{2b\,\text{LambertW}\left(\frac{2e^{-1}e^{\frac{x}{b}}e^{-\frac{c_1}{b}}}{\sqrt{\frac{1}{a}}\,b}\right)+b\ln\left(\frac{1}{4a}\right)+2c_1+2b-2x}}{2b}}\right)}{4a},y(x)=\frac{\left(2b+e^{-\frac{2b\,\text{LambertW}\left(\frac{2e^{-1}e^{\frac{x}{b}}e^{-\frac{c_1}{b}}}{\sqrt{\frac{1}{a}}\,b}\right)+b\ln\left(\frac{1}{4a}\right)+2c_1+2b-2x}}{2b}}\right)}{4a},y(x)=\frac{\left(\text{LambertW}\left(-\frac{2e^{-1}e^{\frac{x}{b}}e^{-\frac{c_1}{b}}}{\sqrt{\frac{1}{a}}\,b}\right)+b\ln\left(\frac{1}{4a}\right)+2c_1+2b-2x\right)}{4a}\right.$$

2.404

ODE No. 404

$$ay'(x)^2+bx^2y'(x)+cxy(x)=0$$

✓ **Mathematica** : cpu = 2.57967 (sec), leaf count = 795

$$\left\{ \text{Solve} \left[\int_1^x \left(\frac{(3b+2c)K[1]^2}{2(3bK[1]^3+cK[1]^3+9ay(x))} + \frac{3\sqrt{K[1](b^2K[1]^3-4acy(x))}}{2(3bK[1]^3+cK[1]^3+9ay(x))} \right) dK[1] + \int_1^{y(x)} \left(\frac{9\sqrt{x(b^2x^3-4acK[2])}}{2(3b+c)x^2(3bx^3+cx^3+9ay(x))} \right) dy(x) \right] \right\}$$

✓ **Maple** : cpu = 0.26 (sec), leaf count = 389

$$\left\{ c_1 + \int_b^x \frac{-a^2b - \sqrt{-a^4b^2 - 4_aacy(x)}}{-a^3b + 6ay(x) + \sqrt{-a^4b^2 - 4_aacy(x)}} d_a + \int^{y(x)} \left(-\frac{2a}{bx^3 + 6_fa + \sqrt{b^2x^4 - 4_facx}} x - \left(\int_b^x \frac{6\left(\frac{2_a_fac}{\sqrt{-a^4b^2 - 4_aacy(x)}}\right)}{(-a^3b + 6_fa)} dx \right) \right) dy(x) \right\}$$

2.405 ODE No. 405

$$ay'(x)^2 + y(x)y'(x) - x = 0$$

✓ **Mathematica** : cpu = 1.31218 (sec), leaf count = 61

$$\text{Solve} \left[\left\{ x = \frac{aK[1] \sin^{-1}(K[1])}{\sqrt{1-K[1]^2}} + \frac{c_1K[1]}{\sqrt{1-K[1]^2}}, y(x) = \frac{x}{K[1]} - aK[1] \right\}, \{y(x), K[1]\} \right]$$

✓ **Maple** : cpu = 0.19 (sec), leaf count = 375

$$\left\{ \frac{c_1(y(x) - \sqrt{4ax + y(x)^2})}{\sqrt{\frac{2a-y(x)+\sqrt{4ax+y(x)^2}}{a}} \sqrt{\frac{-2a-y(x)+\sqrt{4ax+y(x)^2}}{a}}} + x + \frac{(-y(x) + \sqrt{4ax + y(x)^2}) \ln \left(\frac{\sqrt{\frac{-4a^2+4ax+2y(x)^2-2\sqrt{4ax+y(x)^2} y(x)}{a^2}} a - y(x) + \sqrt{4ax+y(x)^2}}{2a} \right)}{\sqrt{-\frac{2(2a^2-2ax-y(x)^2+\sqrt{4ax+y(x)^2} y(x))}{a^2}}} \right\}$$

2.406 ODE No. 406

$$ay'(x)^2 - y(x)y'(x) - x = 0$$

✓ **Mathematica** : cpu = 0.865931 (sec), leaf count = 57

$$\text{Solve} \left[\left\{ x = \frac{aK[1] \sinh^{-1}(K[1])}{\sqrt{K[1]^2+1}} + \frac{c_1K[1]}{\sqrt{K[1]^2+1}}, y(x) = aK[1] - \frac{x}{K[1]} \right\}, \{y(x), K[1]\} \right]$$

✓ **Maple** : cpu = 0.094 (sec), leaf count = 262

$$\left\{ \frac{-c_1y(x) + c_1\sqrt{4ax + y(x)^2} + \sqrt{-\frac{2(-2a^2-2ax-y(x)^2+\sqrt{4ax+y(x)^2} y(x))}{a^2}} x + (y(x) - \sqrt{4ax + y(x)^2}) \operatorname{arcsinh}\left(\frac{-y(x)+\sqrt{4ax+y(x)^2}}{2a}\right)}{\sqrt{\frac{2y(x)^2+4(a+x)a-2\sqrt{4ax+y(x)^2} y(x)}{a^2}}} \right\}$$

2.407 ODE No. 407

$$xy'(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0355723 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4}(4x - 4c_1\sqrt{x} + c_1^2) \right\}, \left\{ y(x) \rightarrow \frac{1}{4}(4x + 4c_1\sqrt{x} + c_1^2) \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 39

$$\left\{ y(x) = 0, y(x) = \frac{(-x + \sqrt{c_1x})^2}{x}, y(x) = \frac{(x + \sqrt{c_1x})^2}{x} \right\}$$

2.408 ODE No. 408

$$xy'(x)^2 - 2y(x) + x = 0$$

✓ **Mathematica** : cpu = 0.288166 (sec), leaf count = 97

$$\left\{ \text{Solve} \left[-2 \left(\frac{1}{1 - \sqrt{\frac{2y(x)}{x}} - 1} + \log \left(1 - \sqrt{\frac{2y(x)}{x}} - 1 \right) \right) = \log(x) + c_1, y(x) \right], \text{Solve} \left[2 \left(\frac{1}{\sqrt{\frac{2y(x)}{x}} - 1} + 1 + \log \left(\sqrt{\frac{2y(x)}{x}} - 1 + 1 \right) \right) = \log(x) + c_1, y(x) \right] \right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 73

$$\left\{ y(x) = \left(\frac{\left(\text{LambertW} \left(\frac{\sqrt{c_1 x}}{c_1} \right) + 1 \right)^2}{2 \text{LambertW} \left(\frac{\sqrt{c_1 x}}{c_1} \right)^2} + \frac{1}{2} \right) x, y(x) = \left(\frac{\left(\text{LambertW} \left(-\frac{\sqrt{c_1 x}}{c_1} \right) + 1 \right)^2}{2 \text{LambertW} \left(-\frac{\sqrt{c_1 x}}{c_1} \right)^2} + \frac{1}{2} \right) x \right\}$$

2.409 ODE No. 409

$$xy'(x)^2 - 2y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 2.37288 (sec), leaf count = 50

$$\text{Solve} \left[\left\{ x = \frac{2K[1] - 2 \log(K[1])}{(K[1] - 1)^2} + \frac{c_1}{(K[1] - 1)^2}, y(x) = xK[1]^2 - 2K[1] \right\}, \{y(x), K[1]\} \right]$$

✓ **Maple** : cpu = 0.119 (sec), leaf count = 63

$$\{y(x) = x e^{2 \text{RootOf}(2x e^{-Z} - x e^{2-Z} + c_1 - 2_Z - x + 2 e^{-Z})} - 2 e^{\text{RootOf}(2x e^{-Z} - x e^{2-Z} + c_1 - 2_Z - x + 2 e^{-Z})}\}$$

2.410 ODE No. 410

$$xy'(x)^2 + 4y'(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 30.8569 (sec), leaf count = 90

$$\text{Solve} \left[\left\{ x = -\frac{2(2K[1] - y(K[1]))}{K[1]^2}, y(x) = 4 \left(\frac{2}{K[1]} + \log(K[1]) \right) \exp \left(-4 \left(\frac{1}{2} \log(2 - K[1]) - \frac{1}{2} \log(K[1]) \right) \right) + c_1 \exp \left(-4 \left(\frac{1}{2} \log(2 - K[1]) - \frac{1}{2} \log(K[1]) \right) \right) \right\}, \{y(x), K[1]\} \right]$$

✓ **Maple** : cpu = 0.125 (sec), leaf count = 64

$$\left\{ y(x) = \frac{x e^{2 \text{RootOf}(4x e^{-Z} - x e^{2-Z} + c_1 + 8_Z - 4x - 4 e^{-Z})}}{2} + 2 e^{\text{RootOf}(4x e^{-Z} - x e^{2-Z} + c_1 + 8_Z - 4x - 4 e^{-Z})} \right\}$$

2.411 ODE No. 411

$$xy'(x)^2 + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.264361 (sec), leaf count = 99

$$\left\{ \text{Solve} \left[\frac{1}{\sqrt{\frac{4y(x)}{x}} + 1} - \log \left(1 - \sqrt{\frac{4y(x)}{x}} + 1 \right) = \frac{\log(x)}{2} + c_1, y(x) \right], \text{Solve} \left[\frac{1}{\sqrt{\frac{4y(x)}{x}} + 1} + \log \left(\sqrt{\frac{4y(x)}{x}} + 1 + 1 \right) = \frac{\log(x)}{2} + c_1, y(x) \right] \right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 65

$$\left\{ y(x) = \frac{\left(2 \text{LambertW} \left(-\frac{1}{2 \sqrt{\frac{c_1}{x}}} \right) + 1 \right) x}{4 \text{LambertW} \left(-\frac{1}{2 \sqrt{\frac{c_1}{x}}} \right)^2}, y(x) = \frac{\left(2 \text{LambertW} \left(\frac{1}{2 \sqrt{\frac{c_1}{x}}} \right) + 1 \right) x}{4 \text{LambertW} \left(\frac{1}{2 \sqrt{\frac{c_1}{x}}} \right)^2} \right\}$$

2.412 ODE No. 412

$$a + xy'(x)^2 + y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.449911 (sec), leaf count = 3229

$$\left\{\left\{y(x) \rightarrow -\frac{1}{2}\sqrt{-\frac{16ax}{9} + \frac{\sqrt[3]{-131072a^6x^6 + 2560a^3e^{3c_1}x^3 + e^{6c_1}} + \sqrt{-1073741824a^9e^{3c_1}x^9 + 3145728a^6e^{6c_1}x^6 - 3072a^3e^{3c_1}}}{18\,2^{2/3}ax}}\right.\right.$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 146

$$\left\{-\frac{c_1\left(\frac{-y(x)+\sqrt{-4ax+y(x)^2}}{x}\right)^{\frac{3}{2}}x^2}{\left(-y(x)+\sqrt{-4ax+y(x)^2}\right)^2} + \frac{4ax^2}{3\left(-y(x)+\sqrt{-4ax+y(x)^2}\right)^2} + x = 0, \frac{c_1\left(\frac{-2y(x)-2\sqrt{-4ax+y(x)^2}}{x}\right)^{\frac{3}{2}}x^2}{\left(y(x)+\sqrt{-4ax+y(x)^2}\right)^2} + \frac{4ax^2}{3\left(y(x)+\sqrt{-4ax+y(x)^2}\right)^2} = 0\right\}$$

2.413 ODE No. 413

$$-x^2 + xy'(x)^2 + y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.130333 (sec), leaf count = 673

$$\left\{\text{Solve}\left[\int\left(\frac{4\sqrt{4x^3+y(x)^2}x^2}{5y(x)(4x^3-15y(x)^2)} + \frac{16x^2}{5(4x^3-15y(x)^2)} - \frac{\sqrt{4x^3+y(x)^2}}{5y(x)x} + \frac{1}{5x}\right)dx + \int\left(\frac{8y(x)}{15y(x)^2-4x^3} - \int\left(-\frac{4\sqrt{4x^3+y(x)^2}}{5y(x)^2(4x^3-15y(x)^2)}\right)dx\right)dx\right]\right\}$$

✓ **Maple** : cpu = 0.184 (sec), leaf count = 269

$$\left\{c_1 + \int_{-b}^x \frac{-y(x) - \sqrt{4a^3 + y(x)^2}}{(4y(x) + \sqrt{4a^3 + y(x)^2})a} d_a + \int^{y(x)} \frac{\left(-48f - 12\sqrt{4x^3 + f^2}\right)\left(\int_{-b}^x \frac{a^2}{(4f + \sqrt{4a^3 + f^2})^2 \sqrt{4a^3 + f^2}} d_a\right) - 2}{4f + \sqrt{4x^3 + f^2}} d_f = 0\right\}$$

2.414 ODE No. 414

$$x^3 + xy'(x)^2 + y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.151708 (sec), leaf count = 107

$$\left\{\left\{y(x) \rightarrow x^2\text{InverseFunction}\left[\int_1^{\#1} \frac{1}{5K[2] + \sqrt{K[2]^2 - 4}} dK[2] \& \left[\int_1^x -\frac{1}{2K[3]} dK[3] + c_1\right]\right\}, \left\{y(x) \rightarrow x^2\text{InverseFunction}\left[\int_1^{\#1} \frac{1}{5K[2] + \sqrt{K[2]^2 - 4}} dK[2] \& \left[\int_1^x -\frac{1}{2K[3]} dK[3] + c_1\right]\right\}\right\}$$

✓ **Maple** : cpu = 0.19 (sec), leaf count = 269

$$\left\{c_1 + \int_{-b}^x \frac{-y(x) - \sqrt{4a^4 + y(x)^2}}{(5y(x) + \sqrt{4a^4 + y(x)^2})a} d_a + \int^{y(x)} \frac{\left(80f + 16\sqrt{-4x^4 + f^2}\right)\left(\int_{-b}^x \frac{a^3}{(5f + \sqrt{-4a^4 + f^2})^2 \sqrt{-4a^4 + f^2}} d_a\right) - 2}{5f + \sqrt{-4x^4 + f^2}} d_f = 0\right\}$$

2.415 ODE No. 415

$$y(x)y'(x) + xy'(x)^2 - y(x)^4 = 0$$

✓ **Mathematica** : cpu = 0.343127 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow \frac{2e^{\frac{c_1}{2}}}{-4x + e^{c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.237 (sec), leaf count = 95

$$\left\{ y(x) = -\frac{1}{2\sqrt{-x}}, y(x) = \frac{1}{2\sqrt{-x}}, y(x) = -\frac{\sqrt{-x}\left(\tanh^2\left(\frac{c_1}{2} - \frac{\ln(x)}{2}\right)\right) + x}{2x \tanh\left(\frac{c_1}{2} - \frac{\ln(x)}{2}\right)}, y(x) = \frac{\sqrt{-x}\left(\tanh^2\left(\frac{c_1}{2} - \frac{\ln(x)}{2}\right)\right) + x}{2x \tanh\left(\frac{c_1}{2} - \frac{\ln(x)}{2}\right)} \right\}$$

2.416 ODE No. 416

$$xy'(x)^2 + (y(x) - 3x)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 2.46506 (sec), leaf count = 383

$$\left\{ \text{Solve} \left[\frac{1}{8} \left(-\sqrt{\frac{\frac{y(x)}{x} - 9}{\frac{y(x)}{x} - 1}} \left(\frac{y(x)}{x} - 1 \right) + \sqrt{\frac{y(x)}{x} - 9} \sqrt{\frac{y(x)}{x} - 1} - 3 \log\left(\frac{y(x)}{x}\right) - \frac{10\sqrt{\frac{y(x)}{x} - 9} \sin^{-1}\left(\frac{\sqrt{9 - \frac{y(x)}{x}}}{2\sqrt{2}}\right)}{\sqrt{9 - \frac{y(x)}{x}}} + 6 \tanh^{-1}\left(\frac{1}{3}\right) \right) \right] \right\}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 136

$$\left\{ x - \frac{c_1(5x - y(x) + \sqrt{9x^2 - 10xy(x) + y(x)^2})}{\left(\frac{3x - y(x) + \sqrt{9x^2 - 10xy(x) + y(x)^2}}{x}\right)^{\frac{3}{2}}x} = 0, x + \frac{c_1(-5x + y(x) + \sqrt{9x^2 - 10xy(x) + y(x)^2})}{\left(\frac{6x - 2y(x) - 2\sqrt{9x^2 - 10xy(x) + y(x)^2}}{x}\right)^{\frac{3}{2}}x} = 0, y(x) = x \right\}$$

2.417 ODE No. 417

$$a + xy'(x)^2 - y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0042745 (sec), leaf count = 16

$$\left\{ \left\{ y(x) \rightarrow \frac{a}{c_1} + c_1x \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 35

$$\left\{ y(x) = \frac{xc_1^2 + a}{c_1}, y(x) = -2\sqrt{ax}, y(x) = 2\sqrt{ax} \right\}$$

2.418 ODE No. 418

$$ay(x) + xy'(x)^2 - y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.992978 (sec), leaf count = 220

$$\left\{ \text{Solve} \left[-\frac{\frac{4a^{3/2}\sqrt{4 - \frac{y(x)}{ax}} \sin^{-1}\left(\frac{\sqrt{\frac{y(x)}{x}}}{2\sqrt{a}}\right)}{\sqrt{\frac{y(x)}{x} - 4a}} + \sqrt{\frac{y(x)}{x}} \sqrt{\frac{y(x)}{x} - 4a} + \frac{y(x)}{x}}{4a} = \frac{\log(x)}{2} + c_1, y(x) \right], \text{Solve} \left[\frac{\frac{4a^{3/2}\sqrt{4 - \frac{y(x)}{ax}} \sin^{-1}\left(\frac{\sqrt{\frac{y(x)}{x}}}{2\sqrt{a}}\right)}{\sqrt{\frac{y(x)}{x} - 4a}} - \sqrt{\frac{y(x)}{x}}}{4a} \right] \right\}$$

✓ **Maple** : cpu = 0.051 (sec), leaf count = 42

$$\left\{y(x)=0, y(x)=-\frac{\left(\operatorname{LambertW}\left(-\frac{\mathrm{e} x}{c_1 a}\right)-1\right)^2 a x}{\operatorname{LambertW}\left(-\frac{\mathrm{e} x}{c_1 a}\right)}\right\}$$

2.419 ODE No. 419

$$x y'(x)^2+2 y(x) y'(x)-x=0$$

✓ **Mathematica** : cpu = 1.14464 (sec), leaf count = 9073

✓ **Maple** : cpu = 0.046 (sec), leaf count = 109

$$\left\{x+\frac{c_1\left(y(x)-\sqrt{x^2+y(x)^2}\right)}{\left(\frac{2 x^2+6 y(x)^2-6 \sqrt{x^2+y(x)^2} y(x)}{x^2}\right)^{\frac{2}{3}} x}=0, x+\frac{c_1\left(y(x)+\sqrt{x^2+y(x)^2}\right)}{\left(\frac{x^2+3 y(x)^2+3 \sqrt{x^2+y(x)^2} y(x)}{x^2}\right)^{\frac{2}{3}} x}=0\right\}$$

2.420 ODE No. 420

$$a+x y'(x)^2-2 y(x) y'(x)=0$$

✓ **Mathematica** : cpu = 0.308459 (sec), leaf count = 777

$$\left\{\left\{y(x) \rightarrow \frac{1}{4} a^2 e^{-\frac{3 c_1}{2}} x^2+\frac{1}{4} e^{-\frac{3 c_1}{2}} \sqrt[3]{a^6 x^6-20 a^3 e^{3 c_1} x^3+8 \sqrt{-a^9 e^{3 c_1} x^9+3 a^6 e^{6 c_1} x^6-3 a^3 e^{9 c_1} x^3+e^{12 c_1}}-8 e^{6 c_1}}-\frac{\sqrt[3]{a^6 x^6-20 a^3 e^{3 c_1} x^3+8 \sqrt{-a^9 e^{3 c_1} x^9+3 a^6 e^{6 c_1} x^6-3 a^3 e^{9 c_1} x^3+e^{12 c_1}}-8 e^{6 c_1}}}{36 \sqrt[3]{a^6 x^6-20 a^3 e^{3 c_1} x^3+8 \sqrt{-a^9 e^{3 c_1} x^9+3 a^6 e^{6 c_1} x^6-3 a^3 e^{9 c_1} x^3+e^{12 c_1}}-8 e^{6 c_1}}}\right.\right.$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 689

$$\left\{y(x)=\frac{3 c_1 a}{\frac{4 x^2}{\left(-36 c_1^2 a+8 x^3+12 c_1 \sqrt{\left(9 c_1^2 a-4 x^3\right) a}\right)^{\frac{1}{3}}}+2 x+\left(-36 c_1^2 a+8 x^3+12 c_1 \sqrt{\left(9 c_1^2 a-4 x^3\right) a}\right)^{\frac{1}{3}}}+\frac{\left(\frac{4 x^2}{\left(-36 c_1^2 a+8 x^3+12 c_1 \sqrt{\left(9 c_1^2 a-4 x^3\right) a}\right)^{\frac{1}{3}}}\right)^{\frac{1}{3}}}{\left(-36 c_1^2 a+8 x^3+12 c_1 \sqrt{\left(9 c_1^2 a-4 x^3\right) a}\right)^{\frac{1}{3}}}+\frac{\left(\frac{4 x^2}{\left(-36 c_1^2 a+8 x^3+12 c_1 \sqrt{\left(9 c_1^2 a-4 x^3\right) a}\right)^{\frac{1}{3}}}\right)^{\frac{1}{3}}}{\left(-36 c_1^2 a+8 x^3+12 c_1 \sqrt{\left(9 c_1^2 a-4 x^3\right) a}\right)^{\frac{1}{3}}}\right\}$$

2.421 ODE No. 421

$$x y'(x)^2-2 y(x) y'(x)-x=0$$

✓ **Mathematica** : cpu = 0.050507 (sec), leaf count = 27

$$\{\{y(x) \rightarrow x \sinh (-\log (x)+c_1)\},\{y(x) \rightarrow x \sinh (\log (x)+c_1)\}\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 32

$$\left\{y(x)=i x, y(x)=-i x, y(x)=-\frac{-x^2+c_1^2}{2 c_1}\right\}$$

2.422 ODE No. 422

$$xy'(x)^2 - 2y(x)y'(x) + 4x = 0$$

✓ **Mathematica** : cpu = 0.0849954 (sec), leaf count = 81

$$\left\{ \left\{ y(x) \rightarrow -\frac{2\left(x + x \tanh^2\left(\frac{1}{2}(-\log(x) + c_1)\right)\right)}{-1 + \tanh^2\left(\frac{1}{2}(-\log(x) + c_1)\right)} \right\} \right\}, \left\{ \left\{ y(x) \rightarrow -\frac{2\left(x + x \tanh^2\left(\frac{1}{2}(\log(x) + c_1)\right)\right)}{-1 + \tanh^2\left(\frac{1}{2}(\log(x) + c_1)\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 30

$$\left\{ y(x) = -2x, y(x) = 2x, y(x) = \frac{x^2 + 4c_1^2}{2c_1} \right\}$$

2.423 ODE No. 423

$$xy'(x)^2 - 2y(x)y'(x) + 2y(x) + x = 0$$

✓ **Mathematica** : cpu = 0.0952307 (sec), leaf count = 59

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}(-2e^{-c_1}x^2 + 2x - e^{c_1}) \right\}, \left\{ y(x) \rightarrow \frac{1}{2}(-e^{c_1}x^2 + 2x - 2e^{-c_1}) \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 44

$$\left\{ y(x) = (1 - \sqrt{2})x, y(x) = (1 + \sqrt{2})x, y(x) = \frac{x^2 + 2c_1x + 2c_1^2}{2c_1} \right\}$$

2.424 ODE No. 424

$$ay(x)y'(x) + bx + xy'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.89173 (sec), leaf count = 223

$$\left\{ \text{Solve} \left[\frac{-2a \tan^{-1}\left(\frac{ay(x)}{x\sqrt{4b - \frac{a^2y(x)^2}{x^2}}}\right) + (a+2) \left(2 \tan^{-1}\left(\frac{(a+2)y(x)}{x\sqrt{4b - \frac{a^2y(x)^2}{x^2}}}\right) - i \log\left(\frac{(a+1)y(x)^2}{x^2} + b\right) \right)}{8(a+1)} = \frac{1}{2}i \log(x) + c_1, y(x) \right], \text{Solve} \left[\frac{-2a \tan^{-1}\left(\frac{ay(x)}{x\sqrt{4b - \frac{a^2y(x)^2}{x^2}}}\right) + (a+2) \left(2 \tan^{-1}\left(\frac{(a+2)y(x)}{x\sqrt{4b - \frac{a^2y(x)^2}{x^2}}}\right) - i \log\left(\frac{(a+1)y(x)^2}{x^2} + b\right) \right)}{8(a+1)} = \frac{1}{2}i \log(x) + c_1, y(x) \right] \right\}$$

✓ **Maple** : cpu = 0.115 (sec), leaf count = 193

$$\left\{ \frac{-c_1\left(ay(x) - \sqrt{a^2y(x)^2 - 4bx^2}\right)\left(\frac{(-2bx^2 + (a^2+a)y(x)^2 - (a+1)\sqrt{a^2y(x)^2 - 4bx^2}y(x))^a}{2x^2}\right)^{\frac{-a-2}{2a+2}} + x^2}{x} = 0, \frac{c_1\left(ay(x) + \sqrt{a^2y(x)^2 - 4bx^2}\right)\left(\frac{(-2bx^2 + (a^2+a)y(x)^2 - (a+1)\sqrt{a^2y(x)^2 - 4bx^2}y(x))^a}{2x^2}\right)^{\frac{-a-2}{2a+2}} + x^2}{x} = 0 \right\}$$

2.425 ODE No. 425

$$(x+1)y'(x)^2 - (y(x)+x)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0049452 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow c_1x + \frac{c_1^2}{-1 + c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.041 (sec), leaf count = 45

$$\left\{ y(x) = \frac{c_1(c_1x - x + c_1)}{c_1 - 1}, y(x) = x + 2 - 2\sqrt{x+1}, y(x) = x + 2 + 2\sqrt{x+1} \right\}$$

2.426 ODE No. 426

$$(3x+1)y'(x)^2 - 3(y(x)+2)y'(x) + 9 = 0$$

✓ **Mathematica** : cpu = 0.0047 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow c_1x + \frac{9 + c_1^2 - 6c_1}{3c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 51

$$\left\{ y(x) = \frac{(3x+1)c_1^2 - 6c_1 + 9}{3c_1}, y(x) = -2 - 2\sqrt{3x+1}, y(x) = -2 + 2\sqrt{3x+1} \right\}$$

2.427 ODE No. 427

$$(3x+5)y'(x)^2 - (3y(x)+x)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0053878 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow c_1x + \frac{5c_1^2}{-1 + 3c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.045 (sec), leaf count = 60

$$\left\{ y(x) = \frac{(3x+5)c_1^2 - c_1x}{3c_1 - 1}, y(x) = \frac{x}{3} + \frac{10}{9} - \frac{2\sqrt{15x+25}}{9}, y(x) = \frac{x}{3} + \frac{10}{9} + \frac{2\sqrt{15x+25}}{9} \right\}$$

2.428 ODE No. 428

$$y'(x)(-ay(x) + bx + c) + axy'(x)^2 - by(x) = 0$$

✓ **Mathematica** : cpu = 0.0055729 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow \frac{cc_1}{b + ac_1} + c_1x \right\} \right\}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 66

$$\left\{ y(x) = \frac{-bx + c - 2\sqrt{-bcx}}{a}, y(x) = \frac{-bx + c + 2\sqrt{-bcx}}{a}, y(x) = \frac{c_1(c_1ax + bx + c)}{c_1a + b} \right\}$$

2.429 ODE No. 429

$$-y'(x)(ay(x) - a + bx - b) + axy'(x)^2 + by(x) = 0$$

✓ **Mathematica** : cpu = 0.0061346 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow \frac{ac_1 + bc_1}{-b + ac_1} + c_1x \right\} \right\}$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 72

$$\left\{ y(x) = \frac{bx + a + b - 2\sqrt{(a+b)bx}}{a}, y(x) = \frac{bx + a + b + 2\sqrt{(a+b)bx}}{a}, y(x) = \frac{c_1(c_1ax - bx + a + b)}{c_1a - b} \right\}$$

2.430 ODE No. 430

$$a_0x + y'(x)(a_1x + b_1y(x) + c_1) + (a_2x + c_2)y'(x)^2 + b_0y(x) + c_0 = 0$$

✓ **Mathematica** : cpu = 254.175 (sec), leaf count = 507

$$\text{Solve} \left\{ \left\{ x = (b_1 K[1] + b_0) \exp \left(\frac{(b_1(b_0 - a_1) + 2a_2b_0) \tan^{-1} \left(\frac{2(a_2+b_1)K[1]+a_1+b_0}{\sqrt{4a_0(a_2+b_1)-a_1^2-2a_1b_0-b_0^2}} \right)}{(a_2+b_1)\sqrt{4a_0(a_2+b_1)-a_1^2-2a_1b_0-b_0^2}} \right) - \frac{(2a_2+b_1) \log(K[1])((a_2+b_1)K[1]+b_0)}{2(a_2+b_1)} \right\} \right\}$$

✓ **Maple** : cpu = 2.898 (sec), leaf count = 1602

Expression too large to display

2.431 ODE No. 431

$$x^2y'(x)^2 - y(x)^4 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.058878 (sec), leaf count = 81

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{1 + \tan^2(-\log(x) + c_1)} \right\}, \left\{ y(x) \rightarrow \sqrt{1 + \tan^2(-\log(x) + c_1)} \right\}, \left\{ y(x) \rightarrow -\sqrt{1 + \tan^2(\log(x) + c_1)} \right\}, \left\{ y(x) \rightarrow \sqrt{1 + \tan^2(\log(x) + c_1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.208 (sec), leaf count = 62

$$\left\{ y(x) = -1, y(x) = 1, y(x) = \frac{\sqrt{\tan^2(c_1 - \ln(x)) + 1}}{\tan(c_1 - \ln(x))}, y(x) = -\frac{\sqrt{\tan^2(c_1 - \ln(x)) + 1}}{\tan(c_1 - \ln(x))} \right\}$$

2.432 ODE No. 432

$$\left(a + xy'(x)\right)^2 - 2ay(x) + x^2 = 0$$

✓ **Mathematica** : cpu = 1.27785 (sec), leaf count = 70

$$\text{Solve} \left[\left\{ y(x) = \frac{2axK[1] + x^2K[1]^2 + a^2 + x^2}{2a}, x = -\frac{a \sinh^{-1}(K[1])}{\sqrt{K[1]^2 + 1}} + \frac{c_1}{\sqrt{K[1]^2 + 1}} \right\}, \{y(x), K[1]\} \right]$$

✓ **Maple** : cpu = 10.385 (sec), leaf count = 242

$$\left\{ y(x) = \frac{-2\left(a \operatorname{arcsinh}\left(\operatorname{RootOf}\left(-_Z^2x^2 + a^2 \operatorname{arcsinh}(_Z)^2 - 2c_1a \operatorname{arcsinh}(_Z) + c_1^2 - x^2\right)\right) - c_1\right)\sqrt{\operatorname{RootOf}\left(-_Z^2x^2 + a^2 \operatorname{arcsinh}(_Z)^2 - 2c_1a \operatorname{arcsinh}(_Z) + c_1^2 - x^2\right)}}{2a} \right\}$$

2.433 ODE No. 433

$$-4a - 4x^2 + \left(xy'(x) + y(x) + 2x\right)^2 - 4xy(x) = 0$$

✓ **Mathematica** : cpu = 1.20771 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow \frac{-a - 2c_1x + c_1^2}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.216 (sec), leaf count = 34

$$\left\{ y(x) = \frac{-x^2 - a}{x}, y(x) = c_1 + \frac{c_1^2 - 4a}{4x} \right\}$$

2.434 ODE No. 434

$$x^2 y'(x)^2 - x^2 - 2xy(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0526407 (sec), leaf count = 27

$$\{\{y(x) \rightarrow x \sinh(-\log(x) + c_1)\}, \{y(x) \rightarrow x \sinh(\log(x) + c_1)\}\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 7

$$\{y(x) = c_1 + x\}$$

2.435 ODE No. 435

$$x^2 y'(x)^2 - 2xy(x)y'(x) + y(x)(y(x) + 1) - x = 0$$

✓ **Mathematica** : cpu = 0.0839341 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} (4x + c_1^2 x - 4ic_1 \sqrt{x} - 4) \right\}, \left\{ y(x) \rightarrow \frac{1}{4} (4x + c_1^2 x + 4ic_1 \sqrt{x} - 4) \right\} \right\}$$

✓ **Maple** : cpu = 0.221 (sec), leaf count = 22

$$\left\{ y(x) = x, y(x) = -\frac{c_1^2 x}{4} + c_1 \sqrt{x} + x - 1 \right\}$$

2.436 ODE No. 436

$$-x^4 + x^2 y'(x)^2 + (1 - x^2)y(x)^2 - 2xy(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0711217 (sec), leaf count = 26

$$\{\{y(x) \rightarrow -x \sinh(x - c_1)\}, \{y(x) \rightarrow x \sinh(x + c_1)\}\}$$

✓ **Maple** : cpu = 2.895 (sec), leaf count = 61

$$\left\{ y(x) = ix, y(x) = -ix, y(x) = -\frac{(-c_1^2 + e^{2x})x e^{-x}}{2c_1}, y(x) = \frac{(c_1^2 e^{2x} - 1)x e^{-x}}{2c_1} \right\}$$

2.437 ODE No. 437

$$-(a + 2xy(x))y'(x) + x^2 y'(x)^2 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.365575 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow \frac{x - 2\sqrt{a} c_1}{4c_1^2} \right\}, \left\{ y(x) \rightarrow \frac{x + 2\sqrt{a} c_1}{4c_1^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 36

$$\left\{ y(x) = -\frac{a}{4x}, y(x) = c_1 x - \sqrt{c_1 a}, y(x) = c_1 x + \sqrt{c_1 a} \right\}$$

2.438 ODE No. 438

$$x^2 y'(x)^2 + 3xy(x)y'(x) + 2y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0282465 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{x^2} \right\}, \left\{ y(x) \rightarrow \frac{c_1}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 17

$$\left\{ y(x) = \frac{c_1}{x^2}, y(x) = \frac{c_1}{x} \right\}$$

2.439 ODE No. 439

$$x^2y'(x)^2 + 3xy(x)y'(x) + 3y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0513211 (sec), leaf count = 49

$$\left\{\left\{y(x) \rightarrow c_1x^{\frac{1}{2}(-3-i\sqrt{3})}\right\},\left\{y(x) \rightarrow c_1x^{\frac{1}{2}i(\sqrt{3}+3i)}\right\}\right\}$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 33

$$\left\{y(x) = \frac{c_1x^{-\frac{i\sqrt{3}}{2}}}{x^{\frac{3}{2}}}, y(x) = \frac{c_1x^{\frac{i\sqrt{3}}{2}}}{x^{\frac{3}{2}}}\right\}$$

2.440 ODE No. 440

$$x^2y'(x)^2 + 4xy(x)y'(x) - 5y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0273368 (sec), leaf count = 19

$$\left\{\left\{y(x) \rightarrow \frac{c_1}{x^5}\right\},\left\{y(x) \rightarrow c_1x\right\}\right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 15

$$\left\{y(x) = \frac{c_1}{x^5}, y(x) = c_1x\right\}$$

2.441 ODE No. 441

$$x^2y'(x)^2 - 4x(y(x) + 2)y'(x) + 4y(x)(y(x) + 2) = 0$$

✓ **Mathematica** : cpu = 0.101443 (sec), leaf count = 65

$$\left\{\left\{y(x) \rightarrow -e^{-c_1x}\left(-x + 2\sqrt{2}e^{\frac{c_1}{2}}\right)\right\},\left\{y(x) \rightarrow e^{\frac{c_1}{2}x}\left(-2\sqrt{2} + e^{\frac{c_1}{2}x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 1.013 (sec), leaf count = 137

$$\left\{y(x) = -2, y(x) = \frac{x^2 - 2\sqrt{2}\sqrt{c_1x^2}}{c_1}, y(x) = \frac{x^2 + 2\sqrt{2}\sqrt{c_1x^2}}{c_1}, y(x) = \frac{(x^2 - 8c_1^2)(-2c_1\sqrt{2} + x)x}{(8c_1^2 - 4c_1\sqrt{2}x + x^2)c_1^2}, y(x) = \frac{(x^2 - 8c_1^2)(2c_1 + x)}{(8c_1^2 + 4c_1\sqrt{2}x + x^2)c_1^2}\right\}$$

2.442 ODE No. 442

$$x^2y'(x)^2 + (1 - x)(y(x)^2 - x^2y(x)) + (x^3 + x^2y(x) - 2xy(x))y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0342572 (sec), leaf count = 28

$$\left\{\left\{y(x) \rightarrow c_1e^{-x}x\right\},\left\{y(x) \rightarrow -x^2 + c_1x\right\}\right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 21

$$\{y(x) = (c_1 - x)x, y(x) = c_1xe^{-x}\}$$

2.443 ODE No. 443

$$x(xy'(x) - y(x))^2 - y'(x) = 0$$

✓ **Mathematica** : cpu = 2.97667 (sec), leaf count = 10121

✓ **Maple** : cpu = 1.482 (sec), leaf count = 221

$$\left\{y(x) = -\frac{2}{9x^2}, y(x) = \frac{\left(\text{RootOf}\left(-729c_1x^{12} + \_Z^8 - 12\_Z^7 + 60\_Z^6 - 160\_Z^5 + 240\_Z^4 - 192\_Z^3 + 64\_Z^2\right) - 2\right)\left(\text{RootOf}\left(-729c_1x^{12} + \_Z^8 - 12\_Z^7 + 60\_Z^6 - 160\_Z^5 + 240\_Z^4 - 192\_Z^3 + 64\_Z^2\right) - 2\right)}{9x^2}\right\}$$

2.444 ODE No. 444

$$x^2y'(x)^2 - (y(x) - 2x)y(x)y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.429728 (sec), leaf count = 75

$$\left\{\left\{y(x) \rightarrow -\frac{\cosh(2c_1) - \sinh(2c_1)}{x \cosh(2c_1) + x \sinh(2c_1) - 1}\right\}, \left\{y(x) \rightarrow -\frac{\cosh(2c_1) - \sinh(2c_1)}{x \cosh(2c_1) + x \sinh(2c_1) + 1}\right\}\right\}$$

✓ **Maple** : cpu = 0.837 (sec), leaf count = 120

$$\left\{y(x) = \frac{\sqrt{2} c_1^3 - 2c_1^2x}{4x^2 - 2c_1^2}, y(x) = \frac{(c_1\sqrt{2} + 2x)c_1^2}{-4x^2 + 2c_1^2}, y(x) = 4x, y(x) = -\frac{2(-c_1\sqrt{2} + x)c_1^2}{x^2 - 2c_1^2}, y(x) = -\frac{2(c_1\sqrt{2} + x)c_1^2}{x^2 - 2c_1^2}\right\}$$

2.445 ODE No. 445

$$y'(x)(ax^2y(x)^3 + b) + aby(x)^3 + x^2y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.070667 (sec), leaf count = 49

$$\left\{\left\{y(x) \rightarrow -\frac{1}{\sqrt{2ax - 2c_1}}\right\}, \left\{y(x) \rightarrow \frac{1}{\sqrt{2ax - 2c_1}}\right\}, \left\{y(x) \rightarrow \frac{b}{x} + c_1\right\}\right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 35

$$\left\{y(x) = \frac{1}{\sqrt{2ax + c_1}}, y(x) = -\frac{1}{\sqrt{2ax + c_1}}, y(x) = c_1 + \frac{b}{x}\right\}$$

2.446 ODE No. 446

$$(x^2 + 1)y'(x)^2 - 2xy(x)y'(x) + y(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.0447207 (sec), leaf count = 45

$$\left\{\left\{y(x) \rightarrow c_1x - \sqrt{1 - c_1^2}\right\}, \left\{y(x) \rightarrow c_1x + \sqrt{1 - c_1^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 57

$$\left\{y(x) = \sqrt{x^2 + 1}, y(x) = -\sqrt{x^2 + 1}, y(x) = c_1x - \sqrt{-c_1^2 + 1}, y(x) = c_1x + \sqrt{-c_1^2 + 1}\right\}$$

2.447 ODE No. 447

$$(x^2 - 1)y'(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.0109582 (sec), leaf count = 89

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \log \left(1 - \frac{x}{\sqrt{x^2 - 1}} \right) - \frac{1}{2} \log \left(\frac{x}{\sqrt{x^2 - 1}} + 1 \right) + c_1 \right\}, \left\{ y(x) \rightarrow -\frac{1}{2} \log \left(1 - \frac{x}{\sqrt{x^2 - 1}} \right) + \frac{1}{2} \log \left(\frac{x}{\sqrt{x^2 - 1}} + 1 \right) + c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 33

$$\left\{ y(x) = c_1 - \ln(x + \sqrt{x^2 - 1}), y(x) = c_1 + \ln(x + \sqrt{x^2 - 1}) \right\}$$

2.448 ODE No. 448

$$(x^2 - 1)y'(x)^2 - y(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 0.132928 (sec), leaf count = 349

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2} e^{-c_1} \sqrt{2x^2 + 2e^{4c_1}x^2 - 2\sqrt{(x-1)(x+1)}x + 2e^{4c_1}\sqrt{(x-1)(x+1)}x - 1 + 2e^{2c_1} - e^{4c_1}} \right\}, \left\{ y(x) \rightarrow \frac{1}{2} e^{-c_1} \sqrt{2x^2 + 2e^{4c_1}x^2 - 2\sqrt{(x-1)(x+1)}x + 2e^{4c_1}\sqrt{(x-1)(x+1)}x - 1 + 2e^{2c_1} - e^{4c_1}} \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

time expired

2.449 ODE No. 449

$$(x^2 - a^2)y'(x)^2 + 2xy(x)y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0350793 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{a - x} \right\}, \left\{ y(x) \rightarrow \frac{c_1}{a + x} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 23

$$\left\{ y(x) = \frac{c_1}{a - x}, y(x) = \frac{c_1}{a + x} \right\}$$

2.450 ODE No. 450

$$(x^2 - a^2)y'(x)^2 - x^2 - 2xy(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.369148 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow \frac{a^2 - x^2 + c_1^2}{2c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.727 (sec), leaf count = 51

$$\left\{ y(x) = \sqrt{a^2 - x^2}, y(x) = -\sqrt{a^2 - x^2}, y(x) = -c_1a^2 + c_1x^2 - \frac{1}{4c_1} \right\}$$

2.451 ODE No. 451

$$(a + x^2)y'(x)^2 + b - 2xy(x)y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.056729 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow c_1x - \sqrt{-b - ac_1^2} \right\}, \left\{ y(x) \rightarrow \sqrt{-b - ac_1^2} + c_1x \right\} \right\}$$

✓ **Maple** : cpu = 0.065 (sec), leaf count = 78

$$\left\{y(x)=\frac{\sqrt{-(x^2+a)ab}}{a},y(x)=-\frac{\sqrt{-(x^2+a)ab}}{a},y(x)=c_1x-\sqrt{-ac_1^2-b},y(x)=c_1x+\sqrt{-ac_1^2-b}\right\}$$

2.452 ODE No. 452

$$(2x^2+1)y'(x)^2+(x^2+2xy(x)+y(x)^2+2)y'(x)+2y(x)^2+1=0$$

✓ **Mathematica** : cpu = 0.0177797 (sec), leaf count = 23

$$\left\{\left\{y(x)\rightarrow\frac{-c_1x+1+c_1^2}{x+c_1}\right\}\right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

$$\text{dsolve}((2*x^2+1)*\text{diff}(y(x),x)^2+(y(x)^2+2*x*y(x)+x^2+2)*\text{diff}(y(x),x)+2*y(x)^2+1=0,y(x))$$

2.453 ODE No. 453

$$(a^2-1)x^2y'(x)^2+a^2x^2+2xy(x)y'(x)-y(x)^2=0$$

✓ **Mathematica** : cpu = 1.062 (sec), leaf count = 327

$$\left\{\text{Solve}\left[\frac{2i\tan^{-1}\left(\frac{y(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+a\tanh^{-1}\left(\frac{-a^2-\frac{iy(x)}{x}+1}{a\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)-a\tanh^{-1}\left(\frac{-a^2+\frac{iy(x)}{x}+1}{a\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+a\log\left(\frac{y(x)^2}{x^2}+1\right)}{2a^2-2}=\frac{a\log(x-a^2x)}{1-a^2}\right.]\right\}$$

✓ **Maple** : cpu = 2.323 (sec), leaf count = 229

$$\left\{\frac{-2c_1a+2a\ln(x)+a\ln\left(\frac{x^2+y(x)^2}{x^2}\right)-2\sqrt{-a^2}\arctan\left(\frac{a^2y(x)}{\sqrt{-a^2}\sqrt{\frac{(-a^2+1)x^2+y(x)^2}{x^2}}x}\right)+2\ln\left(\frac{\sqrt{\frac{-a^2x^2+x^2+y(x)^2}{x^2}}x+y(x)}{x}\right)}{2a}=0,\frac{-2c_1a+2a\ln(x)+a\ln\left(\frac{x^2+y(x)^2}{x^2}\right)-2\sqrt{-a^2}\arctan\left(\frac{a^2y(x)}{\sqrt{-a^2}\sqrt{\frac{(-a^2+1)x^2+y(x)^2}{x^2}}x}\right)+2\ln\left(\frac{\sqrt{\frac{-a^2x^2+x^2+y(x)^2}{x^2}}x+y(x)}{x}\right)}{2a}=0}\right\}$$

2.454 ODE No. 454

$$ax^2y'(x)^2-(a-1)ax^2-2axy(x)y'(x)+y(x)^2=0$$

✓ **Mathematica** : cpu = 0.422821 (sec), leaf count = 241

$$\left\{\left\{y(x)\rightarrow-\frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(-x^2\sqrt{\frac{a-1}{a}}+e^{2c_1}\right)\right\},\left\{y(x)\rightarrow\frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(-x^2\sqrt{\frac{a-1}{a}}+e^{2c_1}\right)\right\},\left\{y(x)\rightarrow-\frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(-x^2\sqrt{\frac{a-1}{a}}+e^{2c_1}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.154 (sec), leaf count = 106

$$\left\{y(x)=x\text{RootOf}\left(c_1-\left(\int^{-x}\frac{\sqrt{(a-1)(-a^2+a)}a}{(a-1)(-a^2+a)}d\_a\right)-\ln(x)\right),y(x)=x\text{RootOf}\left(c_1+\int^{-x}\frac{\sqrt{(a-1)(-a^2+a)}a}{(a-1)(-a^2+a)}d\_a-\ln(x)\right)\right\}$$

2.455 ODE No. 455

$$a + x^3 y'(x)^2 + x^2 y(x) y'(x) = 0$$

✓ **Mathematica** : cpu = 0.959435 (sec), leaf count = 123

$$\left\{ \left\{ y(x) \rightarrow -\frac{e^{-\frac{c_1}{2}}(x + 2ae^{c_1})}{\sqrt{2}x} \right\}, \left\{ y(x) \rightarrow \frac{e^{-\frac{c_1}{2}}(x + 2ae^{c_1})}{\sqrt{2}x} \right\}, \left\{ y(x) \rightarrow -\frac{e^{-\frac{c_1}{2}}(2ax + e^{c_1})}{\sqrt{2}x} \right\}, \left\{ y(x) \rightarrow \frac{e^{-\frac{c_1}{2}}(2ax + e^{c_1})}{\sqrt{2}x} \right\} \right\}$$

✓ **Maple** : cpu = 0.445 (sec), leaf count = 66

$$\left\{ y(x) = -\frac{2\sqrt{ax}}{x}, y(x) = \frac{2\sqrt{ax}}{x}, y(x) = \frac{4ax + c_1^2}{2c_1x}, y(x) = \frac{c_1^2x + 4a}{2c_1x} \right\}$$

2.456 ODE No. 456

$$2(1 - x^2)y(x)y'(x) + x(x^2 - 1)y'(x)^2 + xy(x)^2 - x = 0$$

✓ **Mathematica** : cpu = 0.150413 (sec), leaf count = 421

$$\left\{ \left(\left(y(x) \rightarrow \frac{-x - x \tanh^2 \left(\frac{1}{4} \left(-\frac{i\sqrt{x-1}\sqrt{x+1}\sqrt{(x-1)(x+1)}\tan^{-1}(\sqrt{x^2-1})}{1-x} - \frac{i\sqrt{x-1}\sqrt{(x-1)(x+1)}\tan^{-1}(\sqrt{x^2-1})}{\sqrt{x+1}} + 2c_1 \right) \right)}{-1 + \tanh^2 \left(\frac{1}{4} \left(-\frac{i\sqrt{x-1}\sqrt{x+1}\sqrt{(x-1)(x+1)}\tan^{-1}(\sqrt{x^2-1})}{1-x} - \frac{i\sqrt{x-1}\sqrt{(x-1)(x+1)}\tan^{-1}(\sqrt{x^2-1})}{\sqrt{x+1}} + 2c_1 \right) \right)} \right) \right\}, \left(y(x) \rightarrow \frac{-x - x \tanh^2 \left(\frac{1}{4} \left(-\frac{i\sqrt{x-1}\sqrt{x+1}\sqrt{(x-1)(x+1)}\tan^{-1}(\sqrt{x^2-1})}{1-x} - \frac{i\sqrt{x-1}\sqrt{(x-1)(x+1)}\tan^{-1}(\sqrt{x^2-1})}{\sqrt{x+1}} + 2c_1 \right) \right)}{-1 + \tanh^2 \left(\frac{1}{4} \left(-\frac{i\sqrt{x-1}\sqrt{x+1}\sqrt{(x-1)(x+1)}\tan^{-1}(\sqrt{x^2-1})}{1-x} - \frac{i\sqrt{x-1}\sqrt{(x-1)(x+1)}\tan^{-1}(\sqrt{x^2-1})}{\sqrt{x+1}} + 2c_1 \right) \right)} \right) \right\}$$

✓ **Maple** : cpu = 0.395 (sec), leaf count = 33

$$\left\{ y(x) = x, y(x) = -x, y(x) = c_1\sqrt{x^2 - 1} + \sqrt{-c_1^2 + 1} \right\}$$

2.457 ODE No. 457

$$x^4 y'(x)^2 - xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.453479 (sec), leaf count = 118

$$\left\{ \text{Solve} \left[-\frac{x\sqrt{4x^2y(x)+1}\tanh^{-1}(\sqrt{4x^2y(x)+1})}{\sqrt{4x^4y(x)+x^2}} - \frac{1}{2}\log(y(x)) = c_1, y(x) \right], \text{Solve} \left[\frac{x\sqrt{4x^2y(x)+1}\tanh^{-1}(\sqrt{4x^2y(x)+1})}{\sqrt{4x^4y(x)+x^2}} - \frac{1}{2}\log(y(x)) = c_1, y(x) \right] \right\}$$

✓ **Maple** : cpu = 0.957 (sec), leaf count = 45

$$\left\{ y(x) = \frac{-ic_1 - x}{c_1^2x}, y(x) = \frac{ic_1 - x}{c_1^2x}, y(x) = -\frac{1}{4x^2} \right\}$$

2.458 ODE No. 458

$$x^2(x^2 - a^2)y'(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.0230423 (sec), leaf count = 120

$$\left\{ \left(y(x) \rightarrow -\frac{x\sqrt{x^2 - a^2}\tan^{-1}\left(\frac{\sqrt{x^2 - a^2}}{a}\right)}{a\sqrt{x^4 - a^2x^2}} + c_1 \right), \left(y(x) \rightarrow \frac{x\sqrt{x^2 - a^2}\tan^{-1}\left(\frac{\sqrt{x^2 - a^2}}{a}\right)}{a\sqrt{x^4 - a^2x^2}} + c_1 \right) \right\}$$

✓ **Maple** : cpu = 0.061 (sec), leaf count = 90

$$\left\{ y(x) = c_1 - \frac{\ln\left(\frac{-2a^2 + 2\sqrt{-a^2}\sqrt{-a^2 + x^2}}{x}\right)}{\sqrt{-a^2}}, y(x) = c_1 + \frac{\ln\left(\frac{-2a^2 + 2\sqrt{-a^2}\sqrt{-a^2 + x^2}}{x}\right)}{\sqrt{-a^2}} \right\}$$

2.459 ODE No. 459

$$-\left(y'(x)-1\right)^2+e^{-2x}y'(x)^2+e^{-2y(x)}=0$$

✓ **Mathematica** : cpu = 3.25094 (sec), leaf count = 271

$$\left\{\left\{y(x) \rightarrow \log \left(-\frac{e^{-c_1}\left(e^x+1\right)\left(-e^x+e^{x+2 c_1}-1-e^{2 c_1}\right)}{\sqrt{8 e^x+4 e^{2 x}+4}}\right)\right\},\left\{y(x) \rightarrow \log \left(\frac{e^{-c_1}\left(e^x+1\right)\left(-e^x+e^{x+2 c_1}-1-e^{2 c_1}\right)}{\sqrt{8 e^x+4 e^{2 x}+4}}\right)\right\}, \text {Solve}\left[-\frac{1}{2}\right]$$

✓ **Maple** : cpu = 0.625 (sec), leaf count = 65

$$\left\{y(x)=c_1+x+\ln \left(\left(-1-\sqrt{-e^{-2 c_1} e^{2 x}+e^{2 x}}\right) e^{-x}\right), y(x)=c_1+x+\ln \left(\left(-1+\sqrt{-e^{-2 c_1} e^{2 x}+e^{2 x}}\right) e^{-x}\right)\right\}$$

2.460 ODE No. 460

$$\cos ^4(x)\left(y'(x)^2+y(x)^2\right)-a^2=0$$

✗ **Mathematica** : cpu = 53.665 (sec), leaf count = 0 , could not solve

$$\text{DSolve}[-a^2+\text{Cos}[x]^4*(y[x]^2+\text{Derivative}[1][y][x]^2)==0,y[x],x]$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

$$\text{dsolve}((\text{diff}(y(x),x)^2+y(x)^2)*\cos(x)^4-a^2=0,y(x))$$

2.461 ODE No. 461

$$a(x)y'(x)^2+2b(x)y(x)y'(x)+c(x)y(x)^2+2d(x)y'(x)+2e(x)y(x)+f(x)=0$$

✗ **Mathematica** : cpu = 300.052 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

$$\text{dsolve}(d0(x)*\text{diff}(y(x),x)^2+2*b0(x)*y(x)*\text{diff}(y(x),x)+c0(x)*y(x)^2+2*d0(x)*\text{diff}(y(x),x)+2*e0(x)*y(x)+f0(x)=0,y(x))$$

2.462 ODE No. 462

$$y(x)y'(x)^2-1=0$$

✓ **Mathematica** : cpu = 0.0319725 (sec), leaf count = 43

$$\left\{\left\{y(x) \rightarrow \left(\frac{3}{2}\right)^{2/3}\left(-x+c_1\right)^{2/3}\right\},\left\{y(x) \rightarrow \left(\frac{3}{2}\right)^{2/3}\left(x+c_1\right)^{2/3}\right\}\right\}$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 27

$$\left\{-\frac{2y(x)^{\frac{3}{2}}}{3}-c_1+x=0,\frac{2y(x)^{\frac{3}{2}}}{3}-c_1+x=0\right\}$$

2.463 ODE No. 463

$$y(x)y'(x)^2-e^{2x}=0$$

✓ **Mathematica** : cpu = 0.0519976 (sec), leaf count = 47

$$\left\{\left\{y(x) \rightarrow \left(\frac{3}{2}\right)^{2/3}\left(-e^x+c_1\right)^{2/3}\right\},\left\{y(x) \rightarrow \left(\frac{3}{2}\right)^{2/3}\left(e^x+c_1\right)^{2/3}\right\}\right\}$$

✓ **Maple** : cpu = 0.112 (sec), leaf count = 50

$$\left\{\frac{2y(x)^{\frac{3}{2}}}{3}+c_1-\frac{\sqrt{e^{2x}y(x)}}{\sqrt{y(x)}}=0,\frac{2y(x)^{\frac{3}{2}}}{3}+c_1+\frac{\sqrt{e^{2x}y(x)}}{\sqrt{y(x)}}=0\right\}$$

2.464 ODE No. 464

$$y(x)y'(x)^2 + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.106651 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow -e^{\frac{c_1}{2}} \sqrt{-2x + e^{c_1}} \right\}, \left\{ y(x) \rightarrow e^{\frac{c_1}{2}} \sqrt{-2x + e^{c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.706 (sec), leaf count = 70

$$\left\{ y(x) = \sqrt{-2c_1x + c_1^2}, y(x) = \sqrt{2c_1x + c_1^2}, y(x) = ix, y(x) = -\sqrt{c_1(c_1 + 2x)}, y(x) = -\sqrt{-2c_1x + c_1^2}, y(x) = -ix \right\}$$

2.465 ODE No. 465

$$y(x)y'(x)^2 + 2xy'(x) - 9y(x) = 0$$

✓ **Mathematica** : cpu = 0.0584254 (sec), leaf count = 107

$$\left\{ \text{Solve} \left[\int \frac{y(x)}{x \left(\frac{y(x)^2}{x^2} - \sqrt{\frac{9y(x)^2}{x^2} + 1} + 1 \right)} d \frac{y(x)}{x} = -\log(x) + c_1, y(x) \right], \text{Solve} \left[\int \frac{y(x)}{x \left(\frac{y(x)^2}{x^2} + \sqrt{\frac{9y(x)^2}{x^2} + 1} + 1 \right)} d \frac{y(x)}{x} = -\log(x) \right] \right\}$$

✓ **Maple** : cpu = 0.085 (sec), leaf count = 210

$$\left\{ -\frac{c_1 \left(x - \sqrt{x^2 + 9y(x)^2} \right) \left(\frac{-x + \sqrt{x^2 + 9y(x)^2}}{y(x)} \right)^{\frac{2}{7}} x}{\left(\frac{2x^2 + 2y(x)^2 - 2\sqrt{x^2 + 9y(x)^2} x}{y(x)^2} \right)^{\frac{1}{7}} (2x^2 + 2y(x)^2 - 2\sqrt{x^2 + 9y(x)^2} x)} + x = 0, \frac{c_1 \left(x + \sqrt{x^2 + 9y(x)^2} \right) \left(\frac{-x - \sqrt{x^2 + 9y(x)^2}}{y(x)} \right)^{\frac{2}{7}} x}{(x^2 + y(x)^2 + \sqrt{x^2 + 9y(x)^2} x) \left(\frac{x^2 + y(x)^2 + \sqrt{x^2 + 9y(x)^2} x}{y(x)^2} \right)} \right\}$$

2.466 ODE No. 466

$$y(x)y'(x)^2 - 2xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 3.6073 (sec), leaf count = 433

$$\left\{ \text{Solve} \left[-\frac{i \sqrt{\frac{y(x)^2}{x^2} - 1} \tan^{-1} \left(\sqrt{\frac{y(x)^2}{x^2} - 1} \right)}{\sqrt{\frac{y(x)}{x} - 1} \sqrt{\frac{y(x)}{x} + 1}} - i \sqrt{\frac{\frac{y(x)}{x} - 1}{\frac{y(x)}{x} + 1}} \left(\frac{y(x)}{x} + 1 \right) + i \sqrt{\frac{y(x)}{x} - 1} \sqrt{\frac{y(x)}{x} + 1} + \log \left(\frac{y(x)}{x} \right) + \frac{2i \sqrt{\frac{y(x)}{x} - 1}}{\sqrt{1}} \right] \right\}$$

✓ **Maple** : cpu = 0.668 (sec), leaf count = 71

$$\left\{ y(x) = x, y(x) = \sqrt{-2ic_1x + c_1^2}, y(x) = \sqrt{2ic_1x + c_1^2}, y(x) = -x, y(x) = -\sqrt{-2ic_1x + c_1^2}, y(x) = -\sqrt{2ic_1x + c_1^2} \right\}$$

2.467 ODE No. 467

$$y(x)y'(x)^2 - 4xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.29527 (sec), leaf count = 226

$$\left\{ \left\{ y(x) \rightarrow \sqrt{x^2 + \frac{2^{2/3} c_1^3 x}{\sqrt[3]{32x^6 - 40c_1^3 x^3 + \sqrt{-4096c_1^3 x^9 + 768c_1^6 x^6 - 48c_1^9 x^3 + c_1^{12}} - c_1^6}}} + \frac{\sqrt[3]{32x^6 - 40c_1^3 x^3 + \sqrt{-4096c_1^3 x^9 + 768c_1^6 x^6 - 48c_1^9 x^3 + c_1^{12}} - c_1^6}}{\sqrt[3]{32x^6 - 40c_1^3 x^3 + \sqrt{-4096c_1^3 x^9 + 768c_1^6 x^6 - 48c_1^9 x^3 + c_1^{12}} - c_1^6}} \right\} \right\}$$

✓ **Maple** : cpu = 0.084 (sec), leaf count = 148

$$\left\{ -\frac{c_1 x}{\left(\frac{8x^2-4y(x)^2-4\sqrt{4x^2-y(x)^2}}{y(x)^2} x\right)^{\frac{1}{3}}\left(\frac{2x-\sqrt{4x^2-y(x)^2}}{y(x)}\right)^{\frac{1}{3}}} y(x) + x = 0, -\frac{c_1 x}{\left(\frac{2x^2-y(x)^2+\sqrt{4x^2-y(x)^2}}{y(x)^2} x\right)^{\frac{1}{3}}\left(\frac{2x+\sqrt{4x^2-y(x)^2}}{y(x)}\right)^{\frac{1}{3}}} y(x) + x = 0 \right\}$$

2.468 ODE No. 468

$$-4a^2xy'(x) + a^2y(x) + y(x)y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 8.59932 (sec), leaf count = 753

$$\left\{ \text{Solve} \left[\frac{8\left(4a^2 - \frac{y(x)^2}{x^2}\right)^{3/2} \sinh^{-1}\left(\frac{\sqrt{\frac{y(x)}{x}-2a}}{2\sqrt{a}}\right) + \sqrt{a} \sqrt{\frac{y(x)}{ax}} + 2\left(4\sqrt{\frac{y(x)}{x}} - 2a\left(\frac{y(x)^2}{x^2} - 4a^2\right) \tanh^{-1}\left(\frac{\sqrt{4a^2 - \frac{y(x)^2}{x^2}}}{2a}\right) - 2\sqrt{\frac{y(x)}{x}} - 2a}{6\sqrt{a} \sqrt{-\left(\frac{y(x)}{x} - 2a\right)}}, \right. \right.$$

✓ **Maple** : cpu = 0.094 (sec), leaf count = 181

$$\left\{ x - \frac{c_1 x}{\left(\frac{(2a^2x^2 + \sqrt{4a^2x^2 - y(x)^2} ax - y(x)^2)a^2}{y(x)^2}\right)^{\frac{1}{3}}\left(\frac{(2ax + \sqrt{4a^2x^2 - y(x)^2})a}{y(x)}\right)^{\frac{1}{3}}} ay(x) = 0, x - \frac{c_1 x}{\left(-\frac{4(-2a^2x^2 + \sqrt{4a^2x^2 - y(x)^2} ax + y(x)^2)a^2}{y(x)^2}\right)^{\frac{1}{3}}\left(-\frac{(-2ax + \sqrt{4a^2x^2 - y(x)^2})a}{y(x)}\right)^{\frac{1}{3}}} ay(x) = 0 \right\}$$

2.469 ODE No. 469

$$axy'(x) + by(x) + y(x)y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.422724 (sec), leaf count = 157

$$\left\{ \text{Solve} \left[\frac{(a+2b) \log\left(-\sqrt{a^2 - \frac{4by(x)^2}{x^2}} + a + 2b\right) + a \log\left(\sqrt{a^2 - \frac{4by(x)^2}{x^2}} + a\right)}{4(a+b)} = -\frac{\log(x)}{2} + c_1, y(x) \right], \text{Solve} \left[\frac{a \log\left(a - \sqrt{a^2 - \frac{4by(x)^2}{x^2}}\right)}{4(a+b)} = -\frac{\log(x)}{2} + c_1, y(x) \right] \right\}$$

✓ **Maple** : cpu = 0.14 (sec), leaf count = 264

$$\left\{ \frac{\left(c_1\left(ax + \sqrt{a^2x^2 - 4by(x)^2}\right)\left(-\frac{ax + \sqrt{a^2x^2 - 4by(x)^2}}{2y(x)}\right)^{-\frac{a}{a+b}}\left(\frac{(ax^2 + 2y(x)^2 + \sqrt{a^2x^2 - 4by(x)^2}x)a}{2y(x)^2}\right)^{\frac{-a-2b}{2a+2b}} + y(x)^2\right)x}{y(x)^2} = 0, -\frac{\left(c_1\left(ax - \sqrt{a^2x^2 - 4by(x)^2}\right)\left(\frac{ax - \sqrt{a^2x^2 - 4by(x)^2}}{2y(x)}\right)^{-\frac{a}{a+b}}\left(\frac{(ax^2 + 2y(x)^2 - \sqrt{a^2x^2 - 4by(x)^2}x)a}{2y(x)^2}\right)^{\frac{-a-2b}{2a+2b}} + y(x)^2\right)x}{y(x)^2} = 0 \right\}$$

2.470 ODE No. 470

$$x^3y'(x) - x^2y(x) + y(x)y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.464024 (sec), leaf count = 143

$$\left\{ \text{Solve} \left[\frac{1}{2} \log(y(x)) - \frac{\sqrt{x^6 + 4x^2y(x)^2} \tanh^{-1}\left(\frac{x^2}{\sqrt{x^4 + 4y(x)^2}}\right)}{2x\sqrt{x^4 + 4y(x)^2}} = c_1, y(x) \right], \text{Solve} \left[\frac{\sqrt{x^6 + 4x^2y(x)^2} \tanh^{-1}\left(\frac{x^2}{\sqrt{x^4 + 4y(x)^2}}\right)}{2x\sqrt{x^4 + 4y(x)^2}} + \frac{1}{2} \log(y(x)) = c_1, y(x) \right] \right\}$$

✓ **Maple** : cpu = 0.473 (sec), leaf count = 87

$$\left\{y(x) = -\frac{\sqrt{-4c_1x^2 + c_1^2}}{4}, y(x) = \frac{\sqrt{-4c_1x^2 + c_1^2}}{4}, y(x) = -\frac{ix^2}{2}, y(x) = \frac{ix^2}{2}, y(x) = -\frac{2\sqrt{c_1x^2 + 4}}{c_1}, y(x) = \frac{2\sqrt{c_1x^2 + 4}}{c_1}\right\}$$

2.471 ODE No. 471

$$y(x)y'(x)^2 - (y(x) - x)y'(x) - x = 0$$

✓ **Mathematica** : cpu = 0.017416 (sec), leaf count = 47

$$\left\{y(x) \rightarrow x + c_1, \left\{y(x) \rightarrow -\sqrt{-x^2 + 2c_1}\right\}, \left\{y(x) \rightarrow \sqrt{-x^2 + 2c_1}\right\}\right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 33

$$\left\{y(x) = \sqrt{-x^2 + c_1}, y(x) = -\sqrt{-x^2 + c_1}, y(x) = c_1 + x\right\}$$

2.472 ODE No. 472

$$(y(x) + x)y'(x)^2 + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.592004 (sec), leaf count = 269

$$\left\{\left\{y(x) \rightarrow -\frac{2\sqrt{-\sqrt{3}x \cosh(c_1) - \sqrt{3}x \sinh(c_1) + \cosh(2c_1) + \sinh(2c_1)}}{\sqrt{3}} - \frac{\cosh(c_1)}{\sqrt{3}} - \frac{\sinh(c_1)}{\sqrt{3}}\right\}, \left\{y(x) \rightarrow \frac{2\sqrt{-\sqrt{3}x \cosh(c_1) - \sqrt{3}x \sinh(c_1) + \cosh(2c_1) + \sinh(2c_1)}}{\sqrt{3}} + \frac{\cosh(c_1)}{\sqrt{3}} + \frac{\sinh(c_1)}{\sqrt{3}}\right\}\right\}$$

✓ **Maple** : cpu = 0.487 (sec), leaf count = 121

$$\left\{-c_1 - \operatorname{arctanh}\left(\frac{2x + y(x)}{2\sqrt{\frac{x^2 + xy(x) + y(x)^2}{x^2}}x}\right) + \ln(x) + \ln\left(\frac{y(x)}{x}\right) = 0, -c_1 + \operatorname{arctanh}\left(\frac{2x + y(x)}{2\sqrt{\frac{x^2 + xy(x) + y(x)^2}{x^2}}x}\right) + \ln(x) + \ln\left(\frac{y(x)}{x}\right) = 0\right\}$$

2.473 ODE No. 473

$$(y(x) - 2x)y'(x)^2 - 2(x - 1)y'(x) + y(x) - 2 = 0$$

✓ **Mathematica** : cpu = 0.34837 (sec), leaf count = 165

$$\left\{\left\{y(x) \rightarrow \frac{1}{2}\left(-\sqrt{-4e^{c_1}x + 4e^{c_1} - e^{2c_1}} + 4 - e^{c_1}\right)\right\}, \left\{y(x) \rightarrow \frac{1}{2}\left(\sqrt{-4e^{c_1}x + 4e^{c_1} - e^{2c_1}} + 4 - e^{c_1}\right)\right\}, \left\{y(x) \rightarrow -\sqrt{-2e^{c_1}x + 2e^{c_1}}\right\}\right\}$$

✓ **Maple** : cpu = 0.63 (sec), leaf count = 71

$$\left\{y(x) = c_1 + 2 - \sqrt{c_1(-c_1 + 2x - 2)}, y(x) = \frac{c_1}{2} + 2 - \frac{\sqrt{c_1(-c_1 + 4x - 4)}}{2}, y(x) = x + (x - 1)\sqrt{2} + 1, y(x) = -\sqrt{2}x + x + 1\right\}$$

2.474 ODE No. 474

$$2y(x)y'(x)^2 - (4x - 5)y'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.193005 (sec), leaf count = 135

$$\left\{\left\{y(x) \rightarrow -i\sqrt{2}e^{\frac{c_1}{2}}\sqrt{4x - 5 + 8e^{c_1}}\right\}, \left\{y(x) \rightarrow i\sqrt{2}e^{\frac{c_1}{2}}\sqrt{4x - 5 + 8e^{c_1}}\right\}, \left\{y(x) \rightarrow -\frac{1}{4}ie^{\frac{c_1}{2}}\sqrt{8x - 10 + e^{c_1}}\right\}, \left\{y(x) \rightarrow \frac{1}{4}ie^{\frac{c_1}{2}}\sqrt{8x - 10 + e^{c_1}}\right\}\right\}$$

✓ **Maple** : cpu = 0.999 (sec), leaf count = 152

$$\left\{-c_1 - \operatorname{arctanh}\left(\frac{1}{\sqrt{-\frac{16y(x)^2}{(4x-5)^2} + 1}}\right) + \ln\left(\frac{y(x)}{4x-5}\right) + \ln\left(x - \frac{5}{4}\right) - \frac{\ln\left(\frac{4y(x)}{4x-5} - 1\right)}{2} - \frac{\ln\left(\frac{4y(x)}{4x-5} + 1\right)}{2} + \frac{\ln\left(\frac{16y(x)^2}{(4x-5)^2} - 1\right)}{2} - \frac{\sqrt{4}\sqrt{-\frac{16y(x)^2}{(4x-5)^2} + 1}}{2}\right\}$$

2.475 ODE No. 475

$$4y(x)y'(x)^2 + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.118849 (sec), leaf count = 57

$$\left\{\left\{y(x) \rightarrow -\frac{1}{2}e^{2c_1}\sqrt{-2x + e^{4c_1}}\right\}, \left\{y(x) \rightarrow \frac{1}{2}e^{2c_1}\sqrt{-2x + e^{4c_1}}\right\}\right\}$$

✓ **Maple** : cpu = 0.74 (sec), leaf count = 67

$$\left\{y(x) = \sqrt{-c_1x + c_1^2}, y(x) = \sqrt{c_1x + c_1^2}, y(x) = -\sqrt{c_1(c_1 + x)}, y(x) = -\sqrt{-c_1x + c_1^2}, y(x) = -\frac{ix}{2}, y(x) = \frac{ix}{2}\right\}$$

2.476 ODE No. 476

$$4x^3y'(x) - 4x^2y(x) + 9y(x)y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.459249 (sec), leaf count = 143

$$\left\{\operatorname{Solve}\left[\frac{1}{2}\log(y(x)) - \frac{\sqrt{x^6 + 9x^2y(x)^2}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4 + 9y(x)^2}}\right)}{2x\sqrt{x^4 + 9y(x)^2}} = c_1, y(x)\right], \operatorname{Solve}\left[\frac{\sqrt{x^6 + 9x^2y(x)^2}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4 + 9y(x)^2}}\right)}{2x\sqrt{x^4 + 9y(x)^2}} + \frac{1}{2}\log(y(x)) = c_1, y(x)\right]\right\}$$

✓ **Maple** : cpu = 0.494 (sec), leaf count = 87

$$\left\{y(x) = -\frac{\sqrt{-4c_1x^2 + c_1^2}}{6}, y(x) = \frac{\sqrt{-4c_1x^2 + c_1^2}}{6}, y(x) = -\frac{ix^2}{3}, y(x) = \frac{ix^2}{3}, y(x) = -\frac{2\sqrt{c_1x^2 + 9}}{c_1}, y(x) = \frac{2\sqrt{c_1x^2 + 9}}{c_1}\right\}$$

2.477 ODE No. 477

$$ay(x)y'(x)^2 + (2x - b)y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.245232 (sec), leaf count = 146

$$\left\{\left\{y(x) \rightarrow -\frac{e^{\frac{c_1}{2}}\sqrt{2b - 4x + e^{c_1}}}{2\sqrt{a}}\right\}, \left\{y(x) \rightarrow \frac{e^{\frac{c_1}{2}}\sqrt{2b - 4x + e^{c_1}}}{2\sqrt{a}}\right\}, \left\{y(x) \rightarrow -\sqrt{2}e^{\frac{c_1}{2}}\sqrt{2ae^{c_1} - b + 2x}\right\}, \left\{y(x) \rightarrow \sqrt{2}e^{\frac{c_1}{2}}\sqrt{2ae^{c_1} - b + 2x}\right\}\right\}$$

✓ **Maple** : cpu = 0.49 (sec), leaf count = 622

$$\left\{c_1 + \int_{-b}^x \frac{-4\_a + 2b - 2\sqrt{4ay(x)^2 + (-2\_a + b)^2}}{4ay(x)^2 + (2\_a - b)\sqrt{4ay(x)^2 + (-2\_a + b)^2} + (-2\_a + b)^2} d\_a + \int^{y(x)} \frac{-4\_f^2 a - b^2 + 4bx - 4x^2 + \sqrt{4\_f^2 a + b^2}}{4ay(x)^2 + (2\_a - b)\sqrt{4ay(x)^2 + (-2\_a + b)^2} + (-2\_a + b)^2} dy(x)\right\}$$

2.478 ODE No. 478

$$(y'(x)^2 + 1)(ay(x) + b) - c = 0$$

✓ **Mathematica** : cpu = 0.34926 (sec), leaf count = 223

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[\frac{c \sqrt{-ac} \sqrt{\frac{\#1a+b}{c}} \sin^{-1} \left(\frac{a \sqrt{-\#1a-b+c}}{\sqrt{-a} \sqrt{-ac}} \right) - (\#1a+b) \sqrt{-\#1a-b+c}}{\sqrt{-a}}, \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.118 (sec), leaf count = 88

$$\left\{ -c_1 + x - \left(\int^{y(x)} \frac{-aa + b}{\sqrt{-(aa + b)(aa + b - c)}} d_a \right) = 0, -c_1 + x - \left(\int^{y(x)} - \frac{-aa + b}{\sqrt{-(aa + b)(aa + b - c)}} d_a \right) = 0, y(x) = \frac{-b}{a} \right\}$$

2.479 ODE No. 479

$$a0x + y'(x)(a1x + b1y(x) + c1) + y'(x)^2(a2x + b2y(x) + c2) + b0y(x) + c0 = 0$$

✓ **Mathematica** : cpu = 8.24177 (sec), leaf count = 576

$$\text{Solve} \left\{ x = - \frac{-(K[2](b2K[2] + b1) + b0) \exp \left(\text{RootSum} \left[\#1^3 b2 + \#1^2 a2 + \#1^2 b1 + \#1 a1 + \#1 b0 + a0 \&, \frac{\#1^2 b2 \log(K[2] - \#1)}{3 \#1^2} \right] \right)}{2 a2 x + 2 b2 y(x) + 2 c2} \right\}$$

✓ **Maple** : cpu = 0.375 (sec), leaf count = 929

$$\left\{ x - \left(c_1 + \int \frac{-a1x - b1y(x) - c1 + \sqrt{-4a0a2x^2 - 4a0b2xy(x) + a1^2x^2 + 2a1b1xy(x) - 4a2b0xy(x) - 4b0b2y(x)^2 + b1^2y(x)^2 - 4a0c2x + 2a1c1x - 4a2c0x - 4b0c2y(x) + 2b1c1y(x) - 4b2c0y(x) - 4c0c2 + c1^2}}{2a2x + 2b2y(x) + 2c2} \right) \right\}$$

2.480 ODE No. 480

$$(ay(x) - x^2)y'(x)^2 + 2xy(x)y'(x)^2 - y(x)^2 = 0$$

✗ **Mathematica** : cpu = 28.1144 (sec), leaf count = 0 , could not solve

$$\text{DSolve}[-y[x]^2 + 2*x*y[x]*\text{Derivative}[1][y][x]^2 + (-x^2 + a*y[x])* \text{Derivative}[1][y][x]^2 == 0, y[x], x]$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

$$\text{dsolve}((a*y(x)-x^2)*\text{diff}(y(x),x)^2+2*x*y(x)*\text{diff}(y(x),x)^2-y(x)^2 = 0,y(x))$$

2.481 ODE No. 481

$$(x^2 + y(x)^2)y'(x) + xy(x)y'(x)^2 + xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0308551 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{x} \right\}, \left\{ y(x) \rightarrow -\sqrt{-x^2 + 2c_1} \right\}, \left\{ y(x) \rightarrow \sqrt{-x^2 + 2c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 35

$$\left\{ y(x) = \sqrt{-x^2 + c_1}, y(x) = \frac{c_1}{x}, y(x) = -\sqrt{-x^2 + c_1} \right\}$$

2.482 ODE No. 482

$$(a + x^{22} - y(x)^2)y'(x) + xy(x)y'(x)^2 - xy(x) = 0$$

✗ **Mathematica** : cpu = 58.335 (sec), leaf count = 0 , could not solve

`DSolve[-(x*y[x]) + (a + x^22 - y[x]^2)*Derivative[1][y][x] + x*y[x]*Derivative[1][y][x]^2 ==`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(x*y(x)*diff(y(x),x)^2+(x^22-y(x)^2+a)*diff(y(x),x)-x*y(x) = 0,y(x))`

2.483 ODE No. 483

$$(2xy(x) - x^2)y'(x)^2 + 2xy(x)y'(x) - y(x)^2 + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.229679 (sec), leaf count = 71

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{c_1}{2}} - \sqrt{-x^2 + 2e^{\frac{c_1}{2}}x} \right\}, \left\{ y(x) \rightarrow \sqrt{-x^2 + 2e^{\frac{c_1}{2}}x} + e^{\frac{c_1}{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.081 (sec), leaf count = 103

$$\left\{ y(x) = 0, y(x) = x \operatorname{RootOf} \left(2c_1 + \int^x \frac{-2a^2 + \sqrt{2} \sqrt{(a-1)^2 a}}{(a^2 + 1)a} d_a - 2 \ln(x) \right), y(x) = x \operatorname{RootOf} \left(2c_1 - \left(\int^x \frac{2a^2 + \sqrt{2} \sqrt{(a+1)^2 a}}{(a^2 - 4a + 1)a} d_a - 2 \ln(x) \right) \right) \right\}$$

2.484 ODE No. 484

$$(2xy(x) - x^2)y'(x)^2 - 6xy(x)y'(x) - y(x)^2 + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.196225 (sec), leaf count = 81

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{3x^2 - 2e^{\frac{c_1}{2}}x} + 2x - e^{\frac{c_1}{2}} \right\}, \left\{ y(x) \rightarrow \sqrt{3x^2 - 2e^{\frac{c_1}{2}}x} + 2x - e^{\frac{c_1}{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.08 (sec), leaf count = 115

$$\left\{ y(x) = 0, y(x) = x \operatorname{RootOf} \left(2c_1 + \int^x \frac{-2a^2 + 4a + \sqrt{2} \sqrt{(a+1)^2 a}}{(a^2 - 4a + 1)a} d_a - 2 \ln(x) \right), y(x) = x \operatorname{RootOf} \left(2c_1 - \left(\int^x \frac{2a^2 + \sqrt{2} \sqrt{(a-1)^2 a}}{(a^2 + 4a + 1)a} d_a - 2 \ln(x) \right) \right) \right\}$$

2.485 ODE No. 485

$$-y'(x)(ay(x)^2 + bx^2 + c) + axy(x)y'(x)^2 + bxy(x) = 0$$

✓ **Mathematica** : cpu = 0.0586116 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow \sqrt{\frac{cc_1}{b - ac_1} + c_1 x^2} \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

time expired

2.486 ODE No. 486

$$-a^2 + y(x)^2 y'(x)^2 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.065371 (sec), leaf count = 117

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{a^2 - x^2 - 2c_1 x - c_1^2} \right\}, \left\{ y(x) \rightarrow \sqrt{a^2 - x^2 - 2c_1 x - c_1^2} \right\}, \left\{ y(x) \rightarrow -\sqrt{a^2 - x^2 + 2c_1 x - c_1^2} \right\}, \left\{ y(x) \rightarrow \sqrt{a^2 - x^2 + 2c_1 x - c_1^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.246 (sec), leaf count = 54

$$\left\{ y(x) = a, y(x) = \sqrt{a^2 - x^2 + 2c_1 x - c_1^2}, y(x) = -a, y(x) = -\sqrt{(-c_1 + a + x)(c_1 + a - x)} \right\}$$

2.487 ODE No. 487

$$-6x^3y'(x) + 4x^2y(x) + y(x)^2y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.507485 (sec), leaf count = 157

$$\left\{ \text{Solve} \left[\frac{3}{4} \log(y(x)) - \frac{\sqrt{9x^6 - 4x^2y(x)^3} \tanh^{-1}\left(\frac{3x^2}{\sqrt{9x^4 - 4y(x)^3}}\right)}{2x\sqrt{9x^4 - 4y(x)^3}} = c_1, y(x) \right], \text{Solve} \left[\frac{\sqrt{9x^6 - 4x^2y(x)^3} \tanh^{-1}\left(\frac{3x^2}{\sqrt{9x^4 - 4y(x)^3}}\right)}{2x\sqrt{9x^4 - 4y(x)^3}} + \right.$$

✓ **Maple** : cpu = 0.464 (sec), leaf count = 100

$$\left\{ y(x) = x^{\frac{4}{3}} \text{RootOf} \left(c_1 + \int^{-Z} -\frac{3(4\_a^3 + 3\sqrt{-4\_a^3 + 9} - 9)}{4(4\_a^3 - 9)\_a} d\_a - \ln(x) \right), y(x) = \frac{18^{\frac{1}{3}}x^{\frac{4}{3}}}{2}, y(x) = -\frac{18^{\frac{1}{3}}(1 + i\sqrt{3})x^{\frac{4}{3}}}{4}, y(x) \right.$$

2.488 ODE No. 488

$$4a^2 - 4ay(x)y'(x) - 4ax + y(x)^2y'(x)^2 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.345834 (sec), leaf count = 85

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{16a^3x - 4a^2x^2 - 4ac_1x - c_1^2}}{2a} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{16a^3x - 4a^2x^2 - 4ac_1x - c_1^2}}{2a} \right\} \right\}$$

✓ **Maple** : cpu = 0.5 (sec), leaf count = 111

$$\left\{ y(x) = -2\sqrt{ax}, y(x) = 2\sqrt{ax}, y(x) = -\frac{\sqrt{-16a^4 + 32a^3x + 8c_1ax - c_1^2 + (-16x^2 + 8c_1)a^2}}{4a}, y(x) = \frac{\sqrt{-16a^4 + 32a^3x + 8c_1ax - c_1^2 + (-16x^2 + 8c_1)a^2}}{4a} \right.$$

2.489 ODE No. 489

$$ay(x)^2 + bx + c + y(x)^2y'(x)^2 + 2xy(x)y'(x) = 0$$

✗ **Mathematica** : cpu = 300.229 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 2.316 (sec), leaf count = 551

$$\left\{ y(x) = -\frac{\sqrt{16} \sqrt{\left(-\frac{bx}{4} - \frac{c}{4} \right) a^2 + \left(ax - \frac{1}{2}b + x \right)^2 (a+1)^2 a} \text{RootOf} \left(2c_1a + 2 \int^{-Z} \frac{(-4\_a a^2 + \sqrt{-(4\_a a^3 + 8\_a a^2 + 4\_a a - 1)} e^{\frac{4a+4}{b}} e^{-\frac{2(a+1)}{b}})}{4\_a(4\_a a^2 + 8\_a a + 4\_a a + a + 2)(a+1)} d\_a}{\sqrt{16} \sqrt{\left(-\frac{bx}{4} - \frac{c}{4} \right) a^2 + \left(ax - \frac{1}{2}b + x \right)^2 (a+1)^2 a} \right. \right.$$

2.490 ODE No. 490

$$a - x^2 - 2xy(x)y'(x) + y(x)^2y'(x)^2 + 2y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.499205 (sec), leaf count = 70

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{-a - 2x^2 + 8c_1x - 4c_1^2}}{\sqrt{2}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{-a - 2x^2 + 8c_1x - 4c_1^2}}{\sqrt{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.479 (sec), leaf count = 145

$$\left\{ y(x) = \sqrt{-x^2 - c_1 - a - 2\sqrt{2c_1 + a}} x, y(x) = \sqrt{-x^2 - c_1 - a + 2\sqrt{2c_1 + a}} x, y(x) = -\sqrt{-x^2 - c_1 - a - 2\sqrt{2c_1 + a}} x, y(x) \right.$$

2.491 ODE No. 491

$$(a-1)b+ax^2+2axy(x)y'(x)+(1-a)y(x)^2+y(x)^2y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.873793 (sec), leaf count = 79

$$\left\{\left\{y(x)\rightarrow-\sqrt{-2ac_1x+ac_1^2+b-x^2+2c_1x-c_1^2}\right\},\left\{y(x)\rightarrow\sqrt{-2ac_1x+ac_1^2+b-x^2+2c_1x-c_1^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.664 (sec), leaf count = 195

$$\left\{y(x)=\sqrt{-ax^2+b},y(x)=\frac{\sqrt{\left(c_1+(-x^2+b)a-b-2\sqrt{-(-c_1+b)(a-1)a}x\right)a}}{a},y(x)=\frac{\sqrt{\left(c_1+(-x^2+b)a-b+2\sqrt{-(-c_1+b)(a-1)a}x\right)a}}{a}\right\}$$

2.492 ODE No. 492

$$\left(y(x)^2-a^2\right)y'(x)^2+y(x)^2=0$$

✓ **Mathematica** : cpu = 0.0981847 (sec), leaf count = 97

$$\left\{\left\{y(x)\rightarrow\text{InverseFunction}\left[\sqrt{a^2-\#1^2}-a\tanh^{-1}\left(\frac{\sqrt{a^2-\#1^2}}{a}\right)\&[-x+c_1]\right\},\left\{y(x)\rightarrow\text{InverseFunction}\left[\sqrt{a^2-\#1^2}+a\tanh^{-1}\left(\frac{\sqrt{a^2-\#1^2}}{a}\right)\&[-x+c_1]\right\}\right\}$$

✓ **Maple** : cpu = 0.56 (sec), leaf count = 122

$$\left\{-\frac{a^2\ln\left(\frac{2a^2+2\sqrt{a^2}\sqrt{a^2-y(x)^2}}{y(x)}\right)}{\sqrt{a^2}}-c_1+x+\sqrt{a^2-y(x)^2}=0,\frac{a^2\ln\left(\frac{2a^2+2\sqrt{a^2}\sqrt{a^2-y(x)^2}}{y(x)}\right)}{\sqrt{a^2}}-c_1+x-\sqrt{a^2-y(x)^2}=0\right\}$$

2.493 ODE No. 493

$$\left(a^2-2ax+y(x)^2\right)y'(x)^2+2ay(x)y'(x)+y(x)^2=0$$

✓ **Mathematica** : cpu = 8.48613 (sec), leaf count = 615

$$\left\{\text{Solve}\left[\left\{y(x)=\frac{-\sqrt{-aK[1]^2\left(aK[1]^2-2xK[1]^2-2x\right)}-aK[1]}{K[1]^2+1},x=\frac{aK[1]^2+aK[1]^2\log^2(K[1])+a\log^2(K[1])+aK[1]^2}{K[1]^2+1}\right\}\right]\right\}$$

✓ **Maple** : cpu = 1.067 (sec), leaf count = 111

$$\left[\left[x(_T)=\frac{\sqrt{-T^2+1}a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-T^2+1}}\right)+2c_1a+\left(-2c_1\sqrt{-T^2+1}a-2a^2\right)\operatorname{arctanh}\left(\frac{1}{\sqrt{-T^2+1}}\right)+\left(a^2+c_1^2\right)\sqrt{-T^2+1}}{2\sqrt{-T^2+1}a},y(_T)=\frac{\sqrt{-T^2+1}a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-T^2+1}}\right)+2c_1a+\left(-2c_1\sqrt{-T^2+1}a-2a^2\right)\operatorname{arctanh}\left(\frac{1}{\sqrt{-T^2+1}}\right)+\left(a^2+c_1^2\right)\sqrt{-T^2+1}}{2\sqrt{-T^2+1}a}\right]$$

2.494 ODE No. 494

$$\left(y(x)^2-a^2x^2\right)y'(x)^2+\left(1-a^2\right)x^2+2xy(x)y'(x)=0$$

✓ **Mathematica** : cpu = 0.132597 (sec), leaf count = 49

$$\left\{\left\{y(x)\rightarrow ac_1-\sqrt{-x^2+c_1^2}\right\},\left\{y(x)\rightarrow ac_1+\sqrt{-x^2+c_1^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.193 (sec), leaf count = 161

$$\left\{y(x)=x\operatorname{RootOf}\left(c_1+\int^{-Z}\frac{-a^3+_aa^2-_a+\sqrt{-a^2a^2-a^4+a^2}}{(-a^2+1)(-a^2-a^2+1)}d_a-\ln(x)\right),y(x)=x\operatorname{RootOf}\left(c_1-\left(\int^{-Z}\frac{-a^3-_aa^2+_a+\sqrt{-a^2a^2-a^4+a^2}}{(-a^2+1)(-a^2-a^2+1)}d_a-\ln(x)\right)\right)\right\}$$

2.495 ODE No. 495

$$\left((1-a)x^2+y(x)^2\right)y'(x)^2+2axy(x)y'(x)+(1-a)y(x)^2+x^2=0$$

✓ **Mathematica** : cpu = 0.238946 (sec), leaf count = 83

$$\left\{\text{Solve}\left[\sqrt{a-1}\tan^{-1}\left(\frac{y(x)}{x}\right)-\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right)=\log(x)+c_1,y(x)\right],\text{Solve}\left[\sqrt{a-1}\tan^{-1}\left(\frac{y(x)}{x}\right)+\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right)=\right.\right.$$

✓ **Maple** : cpu = 0.668 (sec), leaf count = 61

$$\left\{y(x)=x\tan\left(\text{RootOf}\left(2c_1-2\sqrt{a-1}\_Z-\ln\left(\frac{x^2}{\cos(\_Z)^2}\right)\right)\right),y(x)=x\tan\left(\text{RootOf}\left(2c_1+2\sqrt{a-1}\_Z-\ln\left(\frac{x^2}{\cos(\_Z)^2}\right)\right)\right)\right\}$$

2.496 ODE No. 496

$$\left(y(x)-x\right)^2\left(y'(x)^2+1\right)-a^2\left(y'(x)+1\right)^2=0$$

✓ **Mathematica** : cpu = 26.125 (sec), leaf count = 17831

Too large to display

✓ **Maple** : cpu = 0.257 (sec), leaf count = 130

$$\left\{y(x)=-\sqrt{2}\_a+x,y(x)=\sqrt{2}\_a+x,y(x)=x+\text{RootOf}\left(c_1-x+\int^{-Z}-\frac{-a^2-2a^2+\sqrt{-a^4+2\_a^2a^2}}{2(-a^2-2a^2)}d\_a\right),y(x)=x+\right.$$

2.497 ODE No. 497

$$-x^2-2xy(x)y'(x)+3y(x)^2y'(x)^2+4y(x)^2=0$$

✓ **Mathematica** : cpu = 0.171909 (sec), leaf count = 76

$$\left\{\left\{y(x)\rightarrow-\frac{\sqrt{-3x^2-4ie^{3c_1}x+e^{6c_1}}}{\sqrt{3}}\right\},\left\{y(x)\rightarrow\frac{\sqrt{-3x^2-4ie^{3c_1}x+e^{6c_1}}}{\sqrt{3}}\right\}\right\}$$

✓ **Maple** : cpu = 0.528 (sec), leaf count = 203

$$\left\{-c_1-\operatorname{arctanh}\left(\frac{\sqrt{\frac{x^2-3y(x)^2}{x^2}}}{2}\right)+\ln(x)+\frac{\ln\left(\frac{x^2+y(x)^2}{x^2}\right)}{2}+\frac{\sqrt{\frac{x^2-3y(x)^2}{x^2}}}{2}-\frac{\sqrt{3}\sqrt{\frac{(\sqrt{3}\_x+3y(x))(\sqrt{3}\_x-3y(x))}{x^2}}}{6}=0,-c_1+\operatorname{arctanh}\left(\frac{\sqrt{\frac{x^2-3y(x)^2}{x^2}}}{2}\right)+\ln(x)+\frac{\ln\left(\frac{x^2+y(x)^2}{x^2}\right)}{2}+\frac{\sqrt{\frac{x^2-3y(x)^2}{x^2}}}{2}-\frac{\sqrt{3}\sqrt{\frac{(\sqrt{3}\_x+3y(x))(\sqrt{3}\_x-3y(x))}{x^2}}}{6}=0\right\}$$

2.498 ODE No. 498

$$(3y(x)-2)y'(x)^2+4y(x)-4=0$$

✓ **Mathematica** : cpu = 0.133399 (sec), leaf count = 155

$$\left\{\left\{y(x)\rightarrow\text{InverseFunction}\left[-\sqrt{3(\#1-1)+1}\sqrt{1-\#1}-\frac{\sqrt{1-\#1}\sinh^{-1}\left(\sqrt{3}\sqrt{\#1-1}\right)}{\sqrt{3}\sqrt{\#1-1}}\right]\&[-2x+c_1]\right\},\left\{y(x)\rightarrow\text{InverseFunction}\left[-\sqrt{3(\#1-1)+1}\sqrt{1-\#1}+\frac{\sqrt{1-\#1}\sinh^{-1}\left(\sqrt{3}\sqrt{\#1-1}\right)}{\sqrt{3}\sqrt{\#1-1}}\right]\&[-2x+c_1]\right\}\right\}$$

✓ **Maple** : cpu = 0.388 (sec), leaf count = 99

$$\left\{y(x)=1,y(x)=\frac{\sin\left(\text{RootOf}\left(48c_1^2-8c_1\sqrt{3}\_Z-96c_1x+\_Z^2+8\sqrt{3}\_Zx+48x^2-\left(\cos^2(\_Z)\right)\right)\right)}{6}+\frac{5}{6},y(x)=\frac{\sin\left(\text{RootOf}\left(48c_1^2-8c_1\sqrt{3}\_Z-96c_1x+\_Z^2+8\sqrt{3}\_Zx+48x^2-\left(\cos^2(\_Z)\right)\right)\right)}{6}-\frac{5}{6}\right\}$$

2.499 ODE No. 499

$$a^2(-x^2) - 2a^2xy(x)y'(x) + (1 - a^2)y(x)^2y'(x)^2 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.40395 (sec), leaf count = 212

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{a^6(-x^2) + 3a^4x^2 - 3a^2x^2 + 2a^2xe^{a^2c_1-c_1} - 2xe^{a^2c_1-c_1} + e^{2a^2c_1-2c_1} + x^2}}{\sqrt{a^6 - 3a^4 + 3a^2 - 1}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{a^6(-x^2) + 3a^4x^2 - 3a^2x^2 + 2a^2xe^{a^2c_1-c_1} - 2xe^{a^2c_1-c_1} + e^{2a^2c_1-2c_1} + x^2}}{\sqrt{a^6 - 3a^4 + 3a^2 - 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.254 (sec), leaf count = 189

$$\left\{ y(x) = x \operatorname{RootOf} \left(c_1 + \int^{-x} \frac{(-a^2a^2 + a^2 - a^2 + \sqrt{-a^2a^2 - a^2 + a^2})_a}{(-a^2a^2 - a^2 + a^2)(-a^2 + 1)} d_a - \ln(x) \right), y(x) = x \operatorname{RootOf} \left(c_1 - \left(\int^{-x} \frac{(-a^2a^2 + a^2 - a^2 + \sqrt{-a^2a^2 - a^2 + a^2})_a}{(-a^2a^2 - a^2 + a^2)(-a^2 + 1)} d_a - \ln(x) \right) \right) \right\}$$

2.500 ODE No. 500

$$(a - b)y(x)^2y'(x)^2 - ab + ay(x)^2 - bx^2 - 2bxy(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 1.05828 (sec), leaf count = 100

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{-ab + ax^2 - 2ac_1x + ac_1^2 + b^2 - bx^2}}{\sqrt{b - a}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{-ab + ax^2 - 2ac_1x + ac_1^2 + b^2 - bx^2}}{\sqrt{b - a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.978 (sec), leaf count = 220

$$\left\{ y(x) = \frac{\sqrt{(-c_1a + (-x^2 + a + c_1)b - 2\sqrt{(-c_1 + b)ab} x)b}}{b}, y(x) = \frac{\sqrt{(-c_1a + (-x^2 + a + c_1)b + 2\sqrt{(-c_1 + b)ab} x)b}}{b} \right\}$$

2.501 ODE No. 501

$$y'(x)^2(ay(x)^2 + bx + c) - by(x)y'(x) + dy(x)^2 = 0$$

✓ **Mathematica** : cpu = 22.4866 (sec), leaf count = 975

$$\left\{ \operatorname{Solve} \left[\left\{ y(x) = \frac{bK[1] - \sqrt{-K[1]^2(-b^2 + 4axK[1]^2b + 4dxb + 4acK[1]^2 + 4cd)}}{2(aK[1]^2 + d)}, x = \frac{-b^2c_1^2d^4 - ab^2c_1^2K[1]^2d^3 + 2b^2c_1^2K[1]^2d^2 - b^2c_1^2K[1]^2d - b^2c_1^2K[1]^2}{4b^2c_1^2K[1]^2d^2 - b^2c_1^2K[1]^2d - b^2c_1^2K[1]^2} \right\} \right] \right\}$$

✓ **Maple** : cpu = 4.77 (sec), leaf count = 215

$$\left\{ \left[x(-T) = -\frac{\sqrt{-T^2a + d} b^2 \ln \left(\frac{d + \sqrt{-T^2a + d} \sqrt{d}}{-T} \right)^2 - 4c_1bd - 2 \ln(2)b^2\sqrt{d} + (-2b^2\sqrt{d} + (4c_1b\sqrt{d} + 2 \ln(2)b^2)\sqrt{-T^2a + d}) \ln \left(\frac{d + \sqrt{-T^2a + d} \sqrt{d}}{-T} \right)}{4\sqrt{-T^2a + d} bd} \right] \right\}$$

2.502 ODE No. 502

$$(ay(x) - bx)^2(a^2y'(x)^2 + b^2) - c^2(ay'(x) + b)^2 = 0$$

✓ **Mathematica** : cpu = 1.27299 (sec), leaf count = 100

$$\left\{ \left\{ y(x) \rightarrow \frac{bc_1}{a} - \frac{\sqrt{b^2(-x^2) + 2b^2c_1x - b^2c_1^2 + c^2}}{a} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{b^2(-x^2) + 2b^2c_1x - b^2c_1^2 + c^2}}{a} + \frac{bc_1}{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.354 (sec), leaf count = 195

$$\left\{ \begin{array}{l} y(x) = \frac{a \operatorname{RootOf}\left(c_1 - x + \int^{-Z} \frac{(-_a^2 a^2 + 2c^2 + \sqrt{(-_a^2 a^2 - 2c^2)} _a^2 a^2)}{2(_a^2 a^2 - 2c^2)b} d\_a\right) + bx}{a}, y(x) = \frac{a \operatorname{RootOf}\left(c_1 - x + \int^{-Z} - \frac{(_a^2 a^2 - 2c^2 + \sqrt{(-_a^2 a^2 - 2c^2)} _a^2 a^2)}{2(_a^2 a^2 - 2c^2)b} d\_a\right) + bx}{a} \end{array} \right.$$

2.503 ODE No. 503

$$a0 + y'(x)(a1x + b1y(x) + c1) + y'(x)^2(a2x + b2y(x) + c2)^2 + b0y(x) + c0 = 0$$

✗ **Mathematica** : cpu = 300.047 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve((b2*y(x)+a2*x+c2)^2*diff(y(x),x)^2+(a1*x+b1*y(x)+c1)*diff(y(x),x)+b0*y(x)+a0+c0=0,y(x))

2.504 ODE No. 504

$$-\left(-a + x^3 + y(x)^3\right)y'(x) + x^2y(x) + xy(x)^2y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0746197 (sec), leaf count = 36

$$\left\{ y(x) \rightarrow \frac{\sqrt[3]{c_1} \sqrt[3]{a - x^3 + c_1 x^3}}{\sqrt[3]{-1 + c_1}} \right\}$$

✓ **Maple** : cpu = 0.822 (sec), leaf count = 247

$$\left\{ -c_1 + \int_{-b}^{y(x)} \frac{\_a^2}{\sqrt{x^6 + (-2\_a^3 - 2a)x^3 + (-\_a^3 + a)^2}} d\_a - \frac{\ln(x)}{2} = 0, -c_1 + \int_{-b}^{y(x)} \frac{\_a^2}{\sqrt{x^6 + (-2\_a^3 - 2a)x^3 + (-\_a^3 + a)^2}} d\_a - \frac{\ln(x)}{2} = 0 \right.$$

2.505 ODE No. 505

$$-x^3 + xy(x)^2y'(x)^2 - 2y(x)^3y'(x) + 2xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.109843 (sec), leaf count = 73

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{x^2 + 2c_1} \right\}, \left\{ y(x) \rightarrow \sqrt{x^2 + 2c_1} \right\}, \left\{ y(x) \rightarrow -\sqrt{x^2 + c_1 x^4} \right\}, \left\{ y(x) \rightarrow \sqrt{x^2 + c_1 x^4} \right\} \right\}$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 52

$$\left\{ y(x) = \sqrt{x^2 + c_1}, y(x) = \sqrt{c_1 x^2 + 1} x, y(x) = -\sqrt{x^2 + c_1}, y(x) = -\sqrt{c_1 x^2 + 1} x \right\}$$

2.506 ODE No. 506

$$2x^2(y(x) - x)y(x)^2y'(x) + x^2(xy(x)^2 - 1)y'(x)^2 - ((x^2y(x) - 1)y(x)^2) = 0$$

✗ **Mathematica** : cpu = 57.3959 (sec), leaf count = 0 , could not solve

DSolve[-(y[x]^2*(-1 + x^2*y[x])) + 2*x^2*y[x]^2*(-x + y[x])*Derivative[1][y][x] + x^2*(-1 + x*y[x]^2)*Derivative[1][y][x]^2 == 0, y[x], x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(x^2*(x*y(x)^2-1)*diff(y(x),x)^2+2*x^2*y(x)^2*(-x+y(x))*diff(y(x),x)-y(x)^2*(x^2*y(x)-1)=0,y(x))

2.507 ODE No. 507

$$\left(y(x)^4 - a^2 x^2\right) y'(x)^2 + 2a^2 x y(x) y'(x) + y(x)^2 \left(y(x)^2 - a^2\right) = 0$$

✗ **Mathematica** : cpu = 44.2337 (sec), leaf count = 0 , could not solve

`DSolve[y[x]^2*(-a^2 + y[x]^2) + 2*a^2*x*y[x]*Derivative[1][y][x] + (-a^2*x^2) + y[x]^4]*Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve((y(x)^4-a^2*x^2)*diff(y(x),x)^2+2*a^2*x*y(x)*diff(y(x),x)+y(x)^2*(y(x)^2-a^2)=0,y(x))`

2.508 ODE No. 508

$$\left(x^2 y(x)^2 - x^2 + y(x)^4\right) y'(x)^2 + 2x y(x) y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 2.6248 (sec), leaf count = 88

$$\text{Solve}\left[\frac{\sqrt{x^2 + y(x)^2} y(x) \left(\log\left(\frac{x}{\sqrt{x^2 + y(x)^2}} + 1\right) - \log\left(1 - \frac{x}{\sqrt{x^2 + y(x)^2}}\right)\right)}{2x^2 \sqrt{\frac{y(x)^2 (x^2 + y(x)^2)}{x^4}}} + y(x) = c_1, y(x)\right]$$

✓ **Maple** : cpu = 2.451 (sec), leaf count = 60

$$\{y(x) = ix, y(x) = -ix, y(x) = c_1 - \operatorname{arctanh}\left(\operatorname{RootOf}\left(c_1^2 Z^2 - 2c_1 Z^2 \operatorname{arctanh}(Z) + Z^2 x^2 + Z^2 \operatorname{arctanh}(Z)^2 - x^2\right)\right)\}$$

2.509 ODE No. 509

$$9\left(x^2 - 1\right) y(x)^4 y'(x)^2 - 4x^2 - 6x y(x)^5 y'(x) = 0$$

✓ **Mathematica** : cpu = 0.332651 (sec), leaf count = 34

$$\left\{y(x) \rightarrow -\frac{\sqrt[3]{-\frac{1}{2}} \sqrt[3]{-4x^2 + 4 + c_1^2}}{\sqrt[3]{c_1}}\right\}$$

✓ **Maple** : cpu = 1.554 (sec), leaf count = 212

$$\left\{y(x) = \left(-4x^2 + 4\right)^{\frac{1}{6}}, y(x) = -\left(-4x^2 + 4\right)^{\frac{1}{6}}, y(x) = -\frac{\left(1 + i\sqrt{3}\right)\left(-4x^2 + 4\right)^{\frac{1}{6}}}{2}, y(x) = \frac{\left(1 + i\sqrt{3}\right)\left(-4x^2 + 4\right)^{\frac{1}{6}}}{2}, y(x) = -\frac{\left(i\sqrt{3} - 1\right)\left(-4x^2 + 4\right)^{\frac{1}{6}}}{2}, y(x) = \frac{\left(i\sqrt{3} - 1\right)\left(-4x^2 + 4\right)^{\frac{1}{6}}}{2}\right\}$$

2.510 ODE No. 510

$$-\left(x^4 y(x)^2 - 1\right) y(x)^2 + x^2 \left(x^2 y(x)^4 - 1\right) y'(x)^2 + 2x^3 \left(y(x)^2 - x^2\right) y(x)^3 y'(x) = 0$$

✗ **Mathematica** : cpu = 53.9006 (sec), leaf count = 0 , could not solve

`DSolve[-(y[x]^2*(-1 + x^4*y[x]^2)) + 2*x^3*y[x]^3*(-x^2 + y[x]^2)*Derivative[1][y][x] + x^2*(x^2*y[x]^4)*Derivative[1][y][x]^2 == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(x^2*(x^2*y(x)^4-1)*diff(y(x),x)^2+2*x^3*y(x)^3*(y(x)^2-x^2)*diff(y(x),x)-y(x)^2*(x^4*y(x)^2-1)=0,y(x))`

2.511 ODE No. 511

$$\left(a^2\sqrt{x^2+y(x)^2}-x^2\right)y'(x)^2+a^2\sqrt{x^2+y(x)^2}+2xy(x)y'(x)-y(x)^2=0$$

✓ **Mathematica** : cpu = 1.79877 (sec), leaf count = 229

$$\left\{\text{Solve}\left[\tan^{-1}\left(\frac{x}{y(x)}\right)-\frac{2\sqrt{a^2\left(x^2+y(x)^2\right)\left(\sqrt{x^2+y(x)^2}-a^2\right)}\tan^{-1}\left(\frac{\sqrt{\sqrt{x^2+y(x)^2}-a^2}}{a}\right)}{a\sqrt{x^2+y(x)^2}\sqrt{\sqrt{x^2+y(x)^2}-a^2}}=c_1,y(x)\right],\text{Solve}\left[\frac{2\sqrt{a^2\left(x^2+y(x)^2\right)}}{a\sqrt{x^2+y(x)^2}}=c_1,y(x)\right]\right\}$$

✓ **Maple** : cpu = 5.067 (sec), leaf count = 199

$$\left\{-c_1+\arctan\left(\frac{x}{y(x)}\right)-\frac{2\sqrt{\left(x^2+y(x)^2\right)\left(-a^2+\sqrt{x^2+y(x)^2}\right)a^2}\arctan\left(\frac{\sqrt{-a^2+\sqrt{x^2+y(x)^2}}}{a}\right)}{\sqrt{x^2+y(x)^2}\sqrt{-a^2+\sqrt{x^2+y(x)^2}}a}=0,-c_1+\arctan\left(\frac{x}{y(x)}\right)+\frac{2\sqrt{a^2\left(x^2+y(x)^2\right)}}{a\sqrt{x^2+y(x)^2}}=c_1,y(x)\right\}$$

2.512 ODE No. 512

$$\left(a\left(x^2+y(x)^2\right)^{3/2}-x^2\right)y'(x)^2+a\left(x^2+y(x)^2\right)^{3/2}+2xy(x)y'(x)-y(x)^2=0$$

✓ **Mathematica** : cpu = 7.25726 (sec), leaf count = 713

$$\left\{\text{Solve}\left[\tan^{-1}\left(\frac{x}{y(x)}\right)-\frac{i\sqrt{a\left(\left(x^2+y(x)^2\right)^{5/2}-a\left(x^2+y(x)^2\right)^3\right)}\left(\sqrt{2}\left(\log\left(\frac{a^{3/2}\left(3i\sqrt{2}a\sqrt{x^2+y(x)^2}+4\sqrt{a}\sqrt{\sqrt{x^2+y(x)^2}-a\left(x^2+y(x)^2\right)}-i\sqrt{2}\right)}{4a\sqrt{x^2+y(x)^2}+4}\right)\right)}{2\sqrt{a}\left(x^2+y(x)^2\right)}\right]=c_1,y(x)\right]\right\}$$

✓ **Maple** : cpu = 6.647 (sec), leaf count = 135

$$\left\{y(x)=\frac{x}{\tan\left(\text{RootOf}\left(c_1-\_Z+\int\frac{(\tan^2(_Z)+1)x^2}{\tan(_Z)^2}-\frac{\sqrt{-(\sqrt{a}-a-1)}a^{\frac{5}{2}}a\left(\sqrt{a}-a+1\right)}{2(-aa^2-1)-a^2}d\_a\right)\right)},y(x)=\frac{x}{\tan\left(\text{RootOf}\left(c_1-\_Z+\int\frac{(\tan^2(_Z)+1)x^2}{\tan(_Z)^2}-\frac{\sqrt{-(\sqrt{a}-a-1)}a^{\frac{5}{2}}a\left(\sqrt{a}-a+1\right)}{2(-aa^2-1)-a^2}d\_a\right)\right)}\right\}$$

2.513 ODE No. 513

$$y'(x)^2\sin(y(x))+2xy'(x)\cos^3(y(x))-\sin(y(x))\cos^4(y(x))=0$$

✓ **Mathematica** : cpu = 0.0825941 (sec), leaf count = 81

$$\left\{\left\{y(x)\rightarrow\tan^{-1}\left(2\left(-\frac{c_1^{3/2}}{\sqrt{x+c_1}}-\frac{\sqrt{c_1}x}{\sqrt{x+c_1}}\right)\right)\right\},\left\{y(x)\rightarrow\tan^{-1}\left(2\left(\frac{c_1^{3/2}}{\sqrt{x+c_1}}+\frac{\sqrt{c_1}x}{\sqrt{x+c_1}}\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 1.872 (sec), leaf count = 1134

$$\left\{\left\{\left\{\left\{\left\{\arctan\left(\frac{-T^2c_1^2-2c_1\left(c_1^3-T^3+54c_{1-T}+6\sqrt{3}\sqrt{\left(-T^2c_1^2+27\right)c_{1-T}^2}\right)^{\frac{1}{3}}-T+\left(c_1^3-T^3+54c_{1-T}+6\sqrt{3}\sqrt{\left(-T^2c_1^2+27\right)c_{1-T}^2}\right)^{\frac{2}{3}}}{\left(c_1^3-T^3+54c_{1-T}+6\sqrt{3}\sqrt{\left(-T^2c_1^2+27\right)c_{1-T}^2}\right)^{\frac{1}{3}}}\right\},-\left(\frac{\left(c_1^3-T^3+54c_{1-T}+6\sqrt{3}\sqrt{\left(-T^2c_1^2+27\right)c_{1-T}^2}\right)^{\frac{1}{3}}}{\left(c_1^3-T^3+54c_{1-T}+6\sqrt{3}\sqrt{\left(-T^2c_1^2+27\right)c_{1-T}^2}\right)^{\frac{1}{3}}}\right)^9\right\}\right\}\right\}\right\}x(_T)=\frac{\cos^4\left(\frac{\arctan\left(\frac{-T^2c_1^2-2c_1\left(c_1^3-T^3+54c_{1-T}+6\sqrt{3}\sqrt{\left(-T^2c_1^2+27\right)c_{1-T}^2}\right)^{\frac{1}{3}}-T+\left(c_1^3-T^3+54c_{1-T}+6\sqrt{3}\sqrt{\left(-T^2c_1^2+27\right)c_{1-T}^2}\right)^{\frac{2}{3}}}{\left(c_1^3-T^3+54c_{1-T}+6\sqrt{3}\sqrt{\left(-T^2c_1^2+27\right)c_{1-T}^2}\right)^{\frac{1}{3}}}\right)}{2}\right)}{2_T\cos\left(\frac{\arctan\left(\frac{-T^2c_1^2-2c_1\left(c_1^3-T^3+54c_{1-T}+6\sqrt{3}\sqrt{\left(-T^2c_1^2+27\right)c_{1-T}^2}\right)^{\frac{1}{3}}-T+\left(c_1^3-T^3+54c_{1-T}+6\sqrt{3}\sqrt{\left(-T^2c_1^2+27\right)c_{1-T}^2}\right)^{\frac{2}{3}}}{\left(c_1^3-T^3+54c_{1-T}+6\sqrt{3}\sqrt{\left(-T^2c_1^2+27\right)c_{1-T}^2}\right)^{\frac{1}{3}}}\right)}{2}\right)}$$

2.514 ODE No. 514

$$y'(x)^2(a\cos(y(x))+b)-c\cos(y(x))+d=0$$

✓ **Mathematica** : cpu = 10.673 (sec), leaf count = 605

$$\left\{\left\{\left\{y(x)\rightarrow\text{InverseFunction}\left[\frac{4\sin^2\left(\frac{\#1}{2}\right)\csc(\#1)\sqrt{a\cos(\#1)+b}\sqrt{\frac{\cot^2\left(\frac{\#1}{2}\right)(c-d)}{c+d}}\sqrt{\frac{\csc^2\left(\frac{\#1}{2}\right)(a+b)(d-c\cos(\#1))}{ad+bc}}\left(c(a+b)F\left(\sin^{-1}\left(\frac{1}{\sqrt{c\cos(\#1)-d}}\right)\right)}{c(a+b)\sqrt{c\cos(\#1)-d}}\right]\right\}\right\}\right\}$$

✓ **Maple** : cpu = 0.266 (sec), leaf count = 87

$$\left\{-c_1+x-\left(\int^{y(x)}\frac{a\cos(_a)+b}{\sqrt{(a\cos(_a)+b)(c\cos(_a)-d)}}d_a\right)=0,-c_1+x-\left(\int^{y(x)}-\frac{a\cos(_a)+b}{\sqrt{(a\cos(_a)+b)(c\cos(_a)-d)}}d_a\right)=0\right\}$$

2.515 ODE No. 515

$$f\left(x^2+y(x)^2\right)\left(y'(x)^2+1\right)-\left(x y'(x)-y(x)\right)^2=0$$

✓ **Mathematica** : cpu = 2.19266 (sec), leaf count = 1922

$$\left\{\text{Solve}\left[\int_1^x\left(\frac{\sqrt{f\left(K[1]^2+y(x)^2\right)\left(K[1]^2+y(x)^2-f\left(K[1]^2+y(x)^2\right)\right)}K[1]}{f\left(K[1]^2+y(x)^2\right)\left(K[1]^2+y(x)^2\right)}-\frac{\sqrt{f\left(K[1]^2+y(x)^2\right)\left(K[1]^2+y(x)^2-f\left(K[1]^2+y(x)^2\right)\right)}}{f\left(K[1]^2+y(x)^2\right)\left(K[1]^2+y(x)^2-f\left(K[1]^2+y(x)^2\right)\right)}\right]\right\}$$

✓ **Maple** : cpu = 1.44 (sec), leaf count = 117

$$\left\{ y(x) = \frac{x}{\tan\left(\text{RootOf}\left(2c_1 - 2\_Z - \left(\int \frac{(\tan^2(\_Z)+1)x^2}{\tan(\_Z)^2} \frac{\sqrt{-(-\_a+f(\_a))f(\_a)}}{(-\_a+f(\_a))\_a} d\_a\right)\right)\right)}, y(x) = \frac{x}{\tan\left(\text{RootOf}\left(2c_1 - 2\_Z + \int \frac{(\tan^2(\_Z)+1)x^2}{\tan(\_Z)^2}\right)\right)}$$

2.516 ODE No. 516

$$(x^2 + y(x)^2)f\left(\frac{x}{\sqrt{x^2 + y(x)^2}}\right)(y'(x)^2 + 1) - (xy'(x) - y(x))^2 = 0$$

✓ **Mathematica** : cpu = 0.573276 (sec), leaf count = 253

$$\left\{ \text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{f\left(\frac{1}{\sqrt{K[1]^2+1}}\right)K[1]^2 + f\left(\frac{1}{\sqrt{K[1]^2+1}}\right) - 1}{\sqrt{f\left(\frac{1}{\sqrt{K[1]^2+1}}\right)}(K[1] - i)(K[1] + i)\left(\sqrt{f\left(\frac{1}{\sqrt{K[1]^2+1}}\right)}K[1] + i\sqrt{f\left(\frac{1}{\sqrt{K[1]^2+1}}\right)} - 1\right)} dK[1] = -\log(x) + c_1, y(x)\right.$$

✓ **Maple** : cpu = 0.979 (sec), leaf count = 139

$$\left\{ y(x) = x \text{RootOf}\left(c_1 + \int^{-Z} \frac{-\_af\left(\frac{1}{\sqrt{\_a^2+1}}\right) + \sqrt{-f\left(\frac{1}{\sqrt{\_a^2+1}}\right)^2 + f\left(\frac{1}{\sqrt{\_a^2+1}}\right)}}{(\_a^2 + 1)f\left(\frac{1}{\sqrt{\_a^2+1}}\right)} d\_a - \ln(x)\right), y(x) = x \text{RootOf}\left(c_1 + \int^{-Z} \frac{-\_af\left(\frac{1}{\sqrt{\_a^2+1}}\right) + \sqrt{-f\left(\frac{1}{\sqrt{\_a^2+1}}\right)^2 + f\left(\frac{1}{\sqrt{\_a^2+1}}\right)}}{(\_a^2 + 1)f\left(\frac{1}{\sqrt{\_a^2+1}}\right)} d\_a - \ln(x)\right)$$

2.517 ODE No. 517

$$(x^2 + y(x)^2)f\left(\frac{y(x)}{\sqrt{x^2 + y(x)^2}}\right)(y'(x)^2 + 1) - (xy'(x) - y(x))^2 = 0$$

✓ **Mathematica** : cpu = 0.605819 (sec), leaf count = 283

$$\left\{ \text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{f\left(\frac{K[1]}{\sqrt{K[1]^2+1}}\right)K[1]^2 + f\left(\frac{K[1]}{\sqrt{K[1]^2+1}}\right) - 1}{\sqrt{f\left(\frac{K[1]}{\sqrt{K[1]^2+1}}\right)}(K[1] - i)(K[1] + i)\left(\sqrt{f\left(\frac{K[1]}{\sqrt{K[1]^2+1}}\right)}K[1] + i\sqrt{f\left(\frac{K[1]}{\sqrt{K[1]^2+1}}\right)} - 1\right)} dK[1] = -\log(x) + c_1, y(x)\right.$$

✓ **Maple** : cpu = 0.972 (sec), leaf count = 78

$$\left\{ y(x) = x \text{RootOf}\left(c_1 + \int^{-Z} \frac{-\_af\left(\frac{\_a}{\sqrt{\_a^2+1}}\right) + \sqrt{-f\left(\frac{\_a}{\sqrt{\_a^2+1}}\right)^2 + f\left(\frac{\_a}{\sqrt{\_a^2+1}}\right)}}{(\_a^2 + 1)f\left(\frac{\_a}{\sqrt{\_a^2+1}}\right)} d\_a - \ln(x)\right)\right\}$$

2.518 ODE No. 518

$$y'(x)^3 - (y(x) - a)^2(y(x) - b)^2 = 0$$

✓ **Mathematica** : cpu = 0.59083 (sec), leaf count = 236

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; \frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\right] \& [x + c_1] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; \frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\right] \& [x + c_1] \right\}$$

✓ **Maple** : cpu = 0.462 (sec), leaf count = 126

$$\left\{-c_1+x-\left(\int^{y(x)}\frac{1}{\left(\left(_a-a\right)^2\left(_a-b\right)^2\right)^{\frac{1}{3}}}d_a\right)=0,-c_1+x-\left(\int^{y(x)}-\frac{2}{\left(1+i\sqrt{3}\right)\left(\left(-_a+a\right)^2\left(-_a+b\right)^2\right)^{\frac{1}{3}}}d_a\right)=0,-c_1+$$

2.519 ODE No. 519

$$y'(x)^3-f(x)\big(ay(x)^2+by(x)+c\big)^2=0$$

✓ **Mathematica** : cpu = 0.702302 (sec), leaf count = 353

$$\left\{\left(y(x)\rightarrow \text{InverseFunction}\left[\frac{\sqrt[3]{2}\left(2\#1a+b\right)\left(\frac{a\left(\#1\left(\#1a+b\right)+c\right)}{4ac-b^2}\right)^{2/3}{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{\left(b+2a\#1\right)^2}{b^2-4ac}\right)}{a\left(\#1\left(\#1a+b\right)+c\right)^{2/3}}\&\left[\int_1^x\sqrt[3]{f(K[1])}dK[1]+c_1\right]\right\},\left\{y(x)\right\}$$

✓ **Maple** : cpu = 0.537 (sec), leaf count = 197

$$\left\{c_1+\int^{y(x)}\frac{1}{\left(_a^2a+_ab+c\right)^{\frac{2}{3}}}d_a+\int^x-\frac{\left(\left(ay(x)^2+by(x)+c\right)^2f(_a)\right)^{\frac{1}{3}}}{\left(ay(x)^2+by(x)+c\right)^{\frac{2}{3}}}d_a=0,c_1+\int^{y(x)}\frac{1}{\left(_a^2a+_ab+c\right)^{\frac{2}{3}}}d_a+\int$$

2.520 ODE No. 520

$$y'(x)^3+y'(x)-y(x)=0$$

✓ **Mathematica** : cpu = 0.0247629 (sec), leaf count = 1115

$$\left\{\left(y(x)\rightarrow \text{InverseFunction}\left[\int\frac{\sqrt[3]{-271^2\#1+2\,0^3-0\,27\,1^2}}{0\,2\sqrt[3]{-271^2\#1+2\,0^3-0\,27\,1^2}+\sqrt{4\left(-0^2+3\,1\,1\right)^3+\left(-271^2\#1+2\,0^3-0\,27\,1^2-0\,9\right)^2}}\right]\right\}$$

✓ **Maple** : cpu = 0.336 (sec), leaf count = 249

$$\left\{-c_1+x-\left(\int^{y(x)}\frac{6\left(108_a+12\sqrt{81_a^2+12}\right)^{\frac{1}{3}}}{\left(108_a+12\sqrt{81_a^2+12}\right)^{\frac{2}{3}}-12}d_a\right)=0,-c_1+x-\left(\int^{y(x)}\frac{12\left(108_a\right)}{\left(1+i\sqrt{3}\right)\left(-\left(108_a+12\sqrt{81_a^2+12}\right)^{\frac{1}{3}}\right)}d_a\right)=0,-c_1+$$

2.521 ODE No. 521

$$y'(x)^3+xy'(x)-y(x)=0$$

✓ **Mathematica** : cpu = 0.0035012 (sec), leaf count = 14

$$\left\{\left\{y(x)\rightarrow c_1x+c_1^3\right\}\right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 33

$$\left\{y(x)=c_1\left(c_1^2+x\right),y(x)=-\frac{2\sqrt{-3x}\,x}{9},y(x)=\frac{2\sqrt{-3x}\,x}{9}\right\}$$

2.522 ODE No. 522

$$y'(x)^3 - (x + 5)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0089831 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow c_1 x - c_1^3 + 5c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.2 (sec), leaf count = 44

$$\left\{ y(x) = c_1(-c_1^2 + x + 5), y(x) = -\frac{2\sqrt{3x+15}(x+5)}{9}, y(x) = \frac{2\sqrt{3x+15}(x+5)}{9} \right\}$$

2.523 ODE No. 523

$$-axy'(x) + x^3 + y'(x)^3 = 0$$

✓ **Mathematica** : cpu = 145.417 (sec), leaf count = 392

$$\left\{ \left\{ y(x) \rightarrow \int_1^x \left(\frac{\sqrt[3]{\frac{2}{3}} a K[1]}{\sqrt[3]{\sqrt{3} \sqrt{27 K[1]^6 - 4 a^3 K[1]^3} - 9 K[1]^3}} + \frac{\sqrt[3]{\sqrt{3} \sqrt{27 K[1]^6 - 4 a^3 K[1]^3} - 9 K[1]^3}}{\sqrt[3]{2} 3^{2/3}} \right) dK[1] + c_1 \right\}, \left\{ y(x) \rightarrow \int_1^x \left(\frac{\sqrt[3]{\frac{2}{3}} a K[1]}{\sqrt[3]{\sqrt{3} \sqrt{27 K[1]^6 - 4 a^3 K[1]^3} - 9 K[1]^3}} + \frac{\sqrt[3]{\sqrt{3} \sqrt{27 K[1]^6 - 4 a^3 K[1]^3} - 9 K[1]^3}}{\sqrt[3]{2} 3^{2/3}} \right) dK[1] + c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.223 (sec), leaf count = 231

$$\left\{ y(x) = c_1 + \int \frac{i \left((i - \sqrt{3}) a x + \left(\frac{i}{12} + \frac{\sqrt{3}}{12} \right) (-108 x^3 + 12 \sqrt{-12 a^3 x^3 + 81 x^6})^{\frac{2}{3}} \right)}{(-108 x^3 + 12 \sqrt{-12 a^3 x^3 + 81 x^6})^{\frac{1}{3}}} dx, y(x) = c_1 + \int \frac{i \left((\sqrt{3} + i) a x + \left(\frac{i}{12} - \frac{\sqrt{3}}{12} \right) (-108 x^3 + 12 \sqrt{-12 a^3 x^3 + 81 x^6})^{\frac{2}{3}} \right)}{(-108 x^3 + 12 \sqrt{-12 a^3 x^3 + 81 x^6})^{\frac{1}{3}}} dx \right\}$$

2.524 ODE No. 524

$$y'(x)^3 - 2y(x)y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.020245 (sec), leaf count = 422

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[\int \frac{\sqrt[3]{\sqrt{3} \sqrt{\#1^3 (27 \#1 - 32) - 9 \#1^2}}}{\sqrt[3]{2} \left(\sqrt{3} \sqrt{\#1^3 (27 \#1 - 32) - 9 \#1^2} \right)^{2/3} + 4 \sqrt[3]{2} \#1} d\#1 \& \left[\frac{x}{6^{2/3}} + c_1 \right] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[\int \frac{\sqrt[3]{\sqrt{3} \sqrt{\#1^3 (27 \#1 - 32) - 9 \#1^2}}}{\sqrt[3]{2} \left(\sqrt{3} \sqrt{\#1^3 (27 \#1 - 32) - 9 \#1^2} \right)^{2/3} + 4 \sqrt[3]{2} \#1} d\#1 \& \left[\frac{x}{6^{2/3}} + c_1 \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.207 (sec), leaf count = 243

$$\left\{ -c_1 + x - \left(\int^{y(x)} \frac{12(-108\_a^2 + 12\sqrt{81\_a^4 - 96\_a^3})^{\frac{1}{3}}}{12(i\sqrt{3} - 1)^2\_a + (i\sqrt{3} - 1)(-108\_a^2 + 12\sqrt{81\_a^4 - 96\_a^3})^{\frac{2}{3}}} d\_a \right) = 0, -c_1 + x - \left(\int^{y(x)} \frac{6(-108\_a^2 + 12\sqrt{81\_a^4 - 96\_a^3})^{\frac{1}{3}}}{24\_a + (-108\_a^2 + 12\sqrt{81\_a^4 - 96\_a^3})^{\frac{2}{3}}} d\_a \right) = 0 \right\}$$

2.525 ODE No. 525

$$-axy(x)y'(x) + 2ay(x)^2 + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0889798 (sec), leaf count = 135

$$\left\{ \left\{ y(x) \rightarrow c_1 \exp \left(\frac{1}{2} \left(\frac{ax^2}{2} + \frac{1}{2} \sqrt{a} x \sqrt{ax^2 - 8} - 4 \log \left(\sqrt{a} \sqrt{ax^2 - 8} + ax \right) \right) \right) \right\}, \left\{ y(x) \rightarrow c_1 \exp \left(\frac{1}{2} \left(\frac{ax^2}{2} - \frac{1}{2} \sqrt{a} x \sqrt{ax^2 - 8} - 4 \log \left(\sqrt{a} \sqrt{ax^2 - 8} - ax \right) \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.237 (sec), leaf count = 122

$$\left\{ y(x) = c_1 \left(\frac{a^2 x}{\sqrt{a^2}} + \sqrt{a^2 x^2 - 8a} \right)^{-\frac{2a}{\sqrt{a^2}}} e^{\frac{(ax + \sqrt{a^2 x^2 - 8a})x}{4}}, y(x) = c_1 \left(\frac{a^2 x}{\sqrt{a^2}} + \sqrt{a^2 x^2 - 8a} \right)^{\frac{2a}{\sqrt{a^2}}} e^{\frac{ax^2}{4} - \frac{\sqrt{a^2 x^2 - 8a} x}{4}} \right\}$$

2.526 ODE No. 526

$$-x^3y(x)^3 - \left(x^2 + xy(x) + y(x)^2\right)y'(x)^2 + \left(x^3y(x) + x^2y(x)^2 + xy(x)^3\right)y'(x) + y'(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.0770687 (sec), leaf count = 45

$$\left\{\left\{y(x) \rightarrow \frac{1}{-x - c_1}\right\}, \left\{y(x) \rightarrow c_1 e^{\frac{x^2}{2}}\right\}, \left\{y(x) \rightarrow \frac{x^3}{3} + c_1\right\}\right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 32

$$\left\{y(x) = \frac{1}{c_1 - x}, y(x) = c_1 e^{\frac{x^2}{2}}, y(x) = \frac{x^3}{3} + c_1\right\}$$

2.527 ODE No. 527

$$-xy(x)^4y'(x) + y'(x)^3 - y(x)^5 = 0$$

✓ **Mathematica** : cpu = 0.0167807 (sec), leaf count = 20

$$\left\{\left\{y(x) \rightarrow \frac{1}{c_1(x - c_1^2)}\right\}\right\}$$

✓ **Maple** : cpu = 0.929 (sec), leaf count = 43

$$\left\{y(x) = c_1 \sqrt{\frac{c_1^{10}}{(c_1^4 x - 1)^2}}, y(x) = -\frac{3\sqrt{3}}{2x^{\frac{3}{2}}}, y(x) = \frac{3\sqrt{3}}{2x^{\frac{3}{2}}}\right\}$$

2.528 ODE No. 528

$$abx + ay'(x)^2 + by(x) + y'(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.988976 (sec), leaf count = 398

Solve

$$\left\{x = -\frac{-a\left(\frac{\sqrt[3]{-2a^3 + \sqrt{(-2a^3 - 27abx - 27by(x))^2 - 4a^6 - 27abx - 27by(x)}}}{3\sqrt[3]{2}} + \frac{\sqrt[3]{2}a^2}{3\sqrt[3]{-2a^3 + \sqrt{(-2a^3 - 27abx - 27by(x))^2 - 4a^6 - 27abx - 27by(x)}}} - \frac{a}{3}\right) + \frac{3}{2}}{\right\}$$

✓ **Maple** : cpu = 0.124 (sec), leaf count = 86

$$\left\{y(x) = -ax - \frac{\left(-a + e^{\text{RootOf}\left(-2\_Z a^2 + 2c_1 b - 5a^2 + 8a e^{-Z} - 2bx - 3 e^{2-Z}\right)}\right)^2 e^{\text{RootOf}\left(-2\_Z a^2 + 2c_1 b - 5a^2 + 8a e^{-Z} - 2bx - 3 e^{2-Z}\right)}}{b}\right\}$$

2.529 ODE No. 529

$$y'(x)^3 + xy'(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 43.4744 (sec), leaf count = 1758

$$\left\{\left\{y(x) \rightarrow \frac{1}{2}\left(\frac{4 \cdot 2^{2/3} x^4}{3\left(-16x^3 - 72x^2 - 108x + 216c_1 + \sqrt{4\left(-4x^2 - 12x - 9\right)^3 + \left(-16x^3 - 72x^2 - 108x + 216c_1 + 54\right)^2 + 54}\right)}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.195 (sec), leaf count = 1251

$$\left\{ \begin{array}{l} y(x) = 0, y(x) = \frac{\left(4x^2 + 12x + (4x - 6)\left(-8x^3 - 36x^2 + 108c_1 - 54x + 27 + 6\sqrt{-6(2c_1 + 1)(4x^3 + 18x^2 + 27x - 27c_1)}\right)\right)^{\frac{1}{3}}}{\dots} \end{array} \right.$$

2.530 ODE No. 530

$$y'(x)^3 - y(x)y'(x)^2 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 8.89628 (sec), leaf count = 648

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[\int_1^{\#1} \frac{\sqrt[3]{2K[1]^3 - 27K[1]^2 + 3\sqrt{3}\sqrt{-K[1]^4(4K[1] - 27)}}}{2\sqrt[3]{2}K[1]^2 + 2\sqrt[3]{2K[1]^3 - 27K[1]^2 + 3\sqrt{3}\sqrt{-K[1]^4(4K[1] - 27)}}K[1] + 2^{2/3}(2K[1]^3 - 27)} \right] \right. \right.$$

✓ **Maple** : cpu = 0.25 (sec), leaf count = 424

$$\left\{ -c_1 + x - \left(\int^{y(x)} - \frac{12(8\_a^3 - 108\_a^2 + 12\sqrt{-12\_a^5 + 81\_a^4})^{\frac{1}{3}}}{4i\sqrt{3}\_a^2 + 4\_a^2 - 4(8\_a^3 - 108\_a^2 + 12\sqrt{-12\_a^5 + 81\_a^4})^{\frac{1}{3}}\_a - i\sqrt{3}(8\_a^3 - 108\_a^2 + 12\sqrt{-12\_a^5 + 81\_a^4})^{\frac{1}{3}}} \right) \right.$$

2.531 ODE No. 531

$$-x^3y(x)^6 - (x^2 + y(x)^4 + xy(x)^2)y'(x)^2 + (x^3y(x)^2 + x^2y(x)^4 + xy(x)^6)y'(x) + y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 66.2943 (sec), leaf count = 0 , could not solve

DSolve[-(x^3*y[x]^6) + (x^3*y[x]^2 + x^2*y[x]^4 + x*y[x]^6)*Derivative[1][y][x] + Derivative[1][y][x]^2,x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(diff(y(x),x)^2-(y(x)^4+x*y(x)^2+x^2)*diff(y(x),x)^2+(x*y(x)^6+x^2*y(x)^4+x^3*y(x)^2)*x^3*y(x)^6=0,y(x))

2.532 ODE No. 532

$$ay'(x)^3 + by'(x)^2 + cy'(x) - d - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0291744 (sec), leaf count = 1124

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[\int \frac{\sqrt[3]{2b^3 - 9acb - 27a^2d - 27a^2\#1}}{2b\sqrt[3]{2b^3 - 9acb - 27a^2d - 27a^2\#1} + \sqrt{4(3ac - b^2)^3 + (2b^3 - 9acb - 27a^2d - 27a^2\#1)^2}} \right] \right. \right.$$

✓ **Maple** : cpu = 0.478 (sec), leaf count = 874

$$\left\{ -c_1 + x - \left(\int^{y(x)} \frac{6\sqrt[3]{36abc - 8b^3 + (108\_a + 108d)a^2 + 12\sqrt{-b^2c^2 + (-4\_a - 4d)b^3 + 27(\_a + d)^2a^2 + 18\left(\frac{2c^2}{9} + (\_a + d)b\right)ac\_a}}}{-12\sqrt[3]{6}ac + 4\sqrt[3]{6}b^2 - 4\sqrt[3]{27}\left(\sqrt{3}\left(\frac{\sqrt{-b^2c^2 + (-4\_a - 4d)b^3 + 27(\_a + d)^2a^2 + 18\left(\frac{2c^2}{9} + (\_a + d)b\right)ac\_a}}{3} + \left(\frac{abc}{3} - \frac{2b^3}{27} + (\_a + d)\right)\right)\right)} \right. \right.$$

2.533 ODE No. 533

$$a + xy'(x)^3 - y(x)y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0049101 (sec), leaf count = 16

$$\left\{ \left\{ y(x) \rightarrow \frac{a}{c_1^2} + c_1 x \right\} \right\}$$

✓ **Maple** : cpu = 0.197 (sec), leaf count = 76

$$\left\{ y(x) = \frac{xc_1^3 + a}{c_1^2}, y(x) = \frac{3 \cdot 2^{\frac{1}{3}} (a x^2)^{\frac{1}{3}}}{2}, y(x) = -\frac{3 \cdot 2^{\frac{1}{3}} (a x^2)^{\frac{1}{3}} (1 + i\sqrt{3})}{4}, y(x) = \frac{3 \cdot 2^{\frac{1}{3}} (a x^2)^{\frac{1}{3}} (i\sqrt{3} - 1)}{4} \right\}$$

2.534 ODE No. 534

$$4xy'(x)^3 - 6y(x)y'(x)^2 + 3y(x) - x = 0$$

✓ **Mathematica** : cpu = 0.0629499 (sec), leaf count = 114

$$\left\{ \left\{ y(x) \rightarrow \frac{-\sqrt{2} \sqrt{c_1 x^3 + 3c_1^2 x^2 + 3c_1^3 x + c_1^4} - c_1^2}{3c_1} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{2} \sqrt{c_1 x^3 + 3c_1^2 x^2 + 3c_1^3 x + c_1^4} - c_1^2}{3c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 84

$$\left\{ y(x) = x, y(x) = -\frac{(1 + \sqrt{3})x}{2}, y(x) = \frac{(\sqrt{3} - 1)x}{2}, y(x) = \frac{-c_1^2 - \sqrt{2} (c_1 + x) \sqrt{c_1(c_1 + x)}}{3c_1}, y(x) = \frac{-c_1^2 + \sqrt{2} (c_1 + x) \sqrt{c_1(c_1 + x)}}{3c_1} \right\}$$

2.535 ODE No. 535

$$8xy'(x)^3 - 12y(x)y'(x)^2 + 9y(x) = 0$$

✓ **Mathematica** : cpu = 0.0503043 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow -\frac{(x + 3c_1)^{3/2}}{3\sqrt{c_1}} \right\}, \left\{ y(x) \rightarrow \frac{(x + 3c_1)^{3/2}}{3\sqrt{c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 51

$$\left\{ y(x) = 0, y(x) = -\frac{3x}{2}, y(x) = \frac{3x}{2}, y(x) = -\frac{(c_1(3c_1 + x))^{\frac{3}{2}}}{3c_1^2}, y(x) = \frac{(c_1(3c_1 + x))^{\frac{3}{2}}}{3c_1^2} \right\}$$

2.536 ODE No. 536

$$bx(x^2 - a^2)y'(x)^2 + (x^2 - a^2)y'(x)^3 + bx + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0157011 (sec), leaf count = 64

$$\left\{ \left\{ y(x) \rightarrow -\frac{bx^2}{2} + c_1 \right\}, \left\{ y(x) \rightarrow -\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1 \right\}, \left\{ y(x) \rightarrow \tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 52

$$\left\{ y(x) = c_1 - \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right), y(x) = c_1 + \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right), y(x) = -\frac{bx^2}{2} + c_1 \right\}$$

2.537 ODE No. 537

$$(x^6 + 3xy(x)^2)y'(x) - 2x^5y(x) + x^3y'(x)^3 - 3x^2y(x)y'(x)^2 - y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.0358459 (sec), leaf count = 16

$$\left\{\left\{y(x) \rightarrow x\left(c_1 x+c_1^3\right)\right\}\right\}$$

✓ **Maple** : cpu = 4.172 (sec), leaf count = 250

$$\left\{y(x)=x^{\frac{5}{2}}\operatorname{RootOf}\left(c_1+\int^{-Z}\frac{18\left(-\frac{4\left(-9\_a+\sqrt{81\_a^2+12}\right)}{\left(27\_a^2+4\right)\sqrt{81\_a^2+12}}\right)^{\frac{2}{3}}\_a^2-18\left(-\frac{4\left(-9\_a+\sqrt{81\_a^2+12}\right)}{\left(27\_a^2+4\right)\sqrt{81\_a^2+12}}\right)^{\frac{1}{3}}\_a^2+\frac{8\left(-\frac{4\left(-9\_a+\sqrt{81\_a^2+12}\right)}{\left(27\_a^2+4\right)\sqrt{81\_a^2+12}}\right)^{\frac{2}{3}}}{3}-\frac{8\left(-\frac{4\left(-9\_a+\sqrt{81\_a^2+12}\right)}{\left(27\_a^2+4\right)\sqrt{81\_a^2+12}}\right)^{\frac{1}{3}}}{\left(-\frac{4\left(-9\_a+\sqrt{81\_a^2+12}\right)}{\left(27\_a^2+4\right)\sqrt{81\_a^2+12}}\right)^{\frac{1}{3}}\left(27\_a^2+4\right)}\_a\right.\right.$$

2.538 ODE No. 538

$$2\big(xy'(x)+y(x)\big)^3-y(x)y'(x)=0$$

✓ **Mathematica** : cpu = 9.98446 (sec), leaf count = 113

$$\left\{\left\{y(x) \rightarrow \frac{\int_1^x \frac{\operatorname{InverseFunction}\left[-\frac{2 \sqrt{1^2-8 \# 1^3} \tan ^{-1}\left(\sqrt{8 \# 1-1}\right)}{\# 1 \sqrt{8 \# 1-1}}+\log (8 \# 1-1)+\log \left(1+\frac{1}{8 \# 1-1}\right)+\frac{3 \sqrt{1^2-8 \# 1^3}}{\# 1}\right) \& \left[c_1+2 \log (K[1])\right]}{K[1]} d K[1]}{x}\right.\right.\right\}$$

✓ **Maple** : cpu = 1.191 (sec), leaf count = 1625

Expression too large to display

2.539 ODE No. 539

$$\sin (x) y'(x)^3-y'(x)^2\left(y(x) \sin (x)-\cos ^2(x)\right)-y'(x)\left(y(x) \cos ^2(x)+\sin (x)\right)+y(x) \sin (x)=0$$

✓ **Mathematica** : cpu = 0.0430559 (sec), leaf count = 45

$$\left\{\left\{y(x) \rightarrow c_1 e^x\right\},\left\{y(x) \rightarrow-\cos (x)+c_1\right\},\left\{y(x) \rightarrow-\log \left(\sin \left(\frac{x}{2}\right)\right)+\log \left(\cos \left(\frac{x}{2}\right)\right)+c_1\right\}\right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 32

$$\{y(x)=c_1e^x,y(x)=c_1-\cos(x),y(x)=c_1-\ln(-\cot(x)+\csc(x))\}$$

2.540 ODE No. 540

$$2y(x)y'(x)^3-y(x)y'(x)^2+2xy'(x)-x=0$$

✓ **Mathematica** : cpu = 0.0377344 (sec), leaf count = 69

$$\left\{\left\{y(x) \rightarrow \frac{x}{2}+c_1\right\},\left\{y(x) \rightarrow \frac{\left(3 c_1-2 i x^{3 / 2}\right)^{2 / 3}}{2^{2 / 3}}\right\},\left\{y(x) \rightarrow \frac{\left(2 i x^{3 / 2}+3 c_1\right)^{2 / 3}}{2^{2 / 3}}\right\}\right\}$$

✓ **Maple** : cpu = 0.076 (sec), leaf count = 109

$$\left\{\frac{c_1 x}{\left(\frac{-x+y(x)-\sqrt{-x y(x)}}{y(x)}\right)^{\frac{2}{3}}\left(\frac{y(x)+\sqrt{-x y(x)}}{y(x)}\right)^{\frac{2}{3}} y(x)}+x=0, \frac{c_1 x}{\left(\frac{-x+y(x)+\sqrt{-x y(x)}}{y(x)}\right)^{\frac{2}{3}}\left(\frac{y(x)-\sqrt{-x y(x)}}{y(x)}\right)^{\frac{2}{3}} y(x)}+x=0, y(x)=c_1+\frac{x}{2}\right\}$$

2.541 ODE No. 541

$$y(x)^2y'(x)^3 + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0312306 (sec), leaf count = 39

$$\left\{\left\{y(x) \rightarrow -\sqrt{2c_1x + c_1^3}\right\}, \left\{y(x) \rightarrow \sqrt{2c_1x + c_1^3}\right\}\right\}$$

✓ **Maple** : cpu = 0.533 (sec), leaf count = 103

$$\left\{y(x) = \sqrt{c_1^3 + 2c_1x}, y(x) = -\sqrt{c_1^3 + 2c_1x}, y(x) = -\frac{22^{\frac{1}{4}}3^{\frac{1}{4}}(-x^3)^{\frac{1}{4}}}{3}, y(x) = \frac{22^{\frac{1}{4}}3^{\frac{1}{4}}(-x^3)^{\frac{1}{4}}}{3}, y(x) = -\frac{2i2^{\frac{1}{4}}3^{\frac{1}{4}}(-x^3)^{\frac{1}{4}}}{3}, y(x) = \frac{2i2^{\frac{1}{4}}3^{\frac{1}{4}}(-x^3)^{\frac{1}{4}}}{3}\right\}$$

2.542 ODE No. 542

$$16y(x)^2y'(x)^3 + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0342954 (sec), leaf count = 20

$$\left\{\left\{y(x) \rightarrow \sqrt{c_1x + 2c_1^3}\right\}\right\}$$

✓ **Maple** : cpu = 0.522 (sec), leaf count = 107

$$\left\{y(x) = \sqrt{16c_1^3 + 2c_1x}, y(x) = -\sqrt{16c_1^3 + 2c_1x}, y(x) = -\frac{2^{\frac{1}{4}}3^{\frac{1}{4}}(-x^3)^{\frac{1}{4}}}{3}, y(x) = \frac{2^{\frac{1}{4}}3^{\frac{1}{4}}(-x^3)^{\frac{1}{4}}}{3}, y(x) = -\frac{i2^{\frac{1}{4}}3^{\frac{1}{4}}(-x^3)^{\frac{1}{4}}}{3}, y(x) = \frac{i2^{\frac{1}{4}}3^{\frac{1}{4}}(-x^3)^{\frac{1}{4}}}{3}\right\}$$

2.543 ODE No. 543

$$x(x^2 + 1)y'(x) - x^2y(x) + y(x)^3(-y'(x)^2) + xy(x)^2y'(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.0341982 (sec), leaf count = 55

$$\left\{\left\{y(x) \rightarrow -\sqrt{c_1x^2 + \frac{c_1}{1 + c_1^2}}\right\}, \left\{y(x) \rightarrow \sqrt{c_1x^2 + \frac{c_1}{1 + c_1^2}}\right\}\right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(x*y(x)^2*diff(y(x),x)^3-y(x)^3*diff(y(x),x)^2+x*(x^2+1)*diff(y(x),x)-x^2*y(x)=0,y(x))

2.544 ODE No. 544

$$x^7y(x)^2y'(x)^3 - (3x^6y(x)^3 - 1)y'(x)^2 + 3x^5y(x)^4y'(x) - x^4y(x)^5 = 0$$

✓ **Mathematica** : cpu = 0.0830218 (sec), leaf count = 22

$$\left\{\left\{y(x) \rightarrow \sqrt[3]{c_1x^3 + c_1^{2/3}}\right\}\right\}$$

✓ **Maple** : cpu = 0.822 (sec), leaf count = 4201

Expression too large to display

2.545 ODE No. 545

$$y'(x)^4 - (y(x) - a)^3(y(x) - b)^2 = 0$$

✓ **Mathematica** : cpu = 0.925847 (sec), leaf count = 323

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[-\frac{4\sqrt[4]{a-\#1} \sqrt{\frac{\#1-b}{a-b}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}} \right] \& \left[-\sqrt[4]{-1} x + c_1 \right] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[-\frac{4\sqrt[4]{a-\#1} \sqrt{\frac{\#1-b}{a-b}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}} \right] \& \left[-\sqrt[4]{-1} x + c_1 \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.214 (sec), leaf count = 144

$$\left\{ -c_1 + x - \left(\int^{y(x)} \frac{1}{(\_a - a)^3 (\_a - b)^2} d\_a \right) = 0, -c_1 + x - \left(\int^{y(x)} \frac{i}{(-(-\_a + a)^3 (-\_a + b)^2)} d\_a \right) = 0, -c_1 + x - \left(\int^{y(x)} \frac{1}{(\_a - a)^3 (\_a - b)^2} d\_a \right) = 0 \right\}$$

2.546 ODE No. 546

$$y'(x)^4 + 3(x-1)y'(x)^2 - 3(2y(x)-1)y'(x) + 3x = 0$$

✓ **Mathematica** : cpu = 0.0423178 (sec), leaf count = 113

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{12} \left(-\sqrt{64x^3 + 48c_1^2x^2 + 12c_1^4x + c_1^6} - 6c_1x + 6 - c_1^3 + 6c_1 \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{12} \left(\sqrt{64x^3 + 48c_1^2x^2 + 12c_1^4x + c_1^6} - 6c_1x + 6 - c_1^3 + 6c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.099 (sec), leaf count = 171

$$\left\{ y(x) = \frac{-2c_1^4 + (-14x + 6)c_1^2 - 16x^2 + c_1 \left(-(c_1^2 + 4x)^{\frac{3}{2}} + 6 \right) + (-c_1^3 + c_1(-6x + 6) + 6) \sqrt{c_1^2 + 4x}}{12c_1 + 12\sqrt{c_1^2 + 4x}}, y(x) = \frac{-2c_1^4 + (-14x + 6)c_1^2 - 16x^2 + c_1 \left(-(c_1^2 + 4x)^{\frac{3}{2}} + 6 \right) + (-c_1^3 + c_1(-6x + 6) + 6) \sqrt{c_1^2 + 4x}}{12c_1 + 12\sqrt{c_1^2 + 4x}} \right\}$$

2.547 ODE No. 547

$$y'(x)^4 - 4y(x)(xy'(x) - 2y(x))^2 = 0$$

✓ **Mathematica** : cpu = 3.17277 (sec), leaf count = 490

$$\left\{ \text{Solve} \left[\frac{\sqrt{(x^2 - 4\sqrt{y(x)})y(x)} \log(\sqrt{x^2 - 4\sqrt{y(x)}} + x)}{\sqrt{x^2 - 4\sqrt{y(x)}} \sqrt{y(x)}} - \frac{\sqrt{x^2 - 4\sqrt{y(x)}} \sqrt{y(x)} \log(y(x))}{4\sqrt{(x^2 - 4\sqrt{y(x)})y(x)}} + \frac{1}{4} \log(y(x)) = c_1, y(x) \right], \text{Solve} \left[\frac{\sqrt{(x^2 - 4\sqrt{y(x)})y(x)} \log(\sqrt{x^2 - 4\sqrt{y(x)}} + x)}{\sqrt{x^2 - 4\sqrt{y(x)}} \sqrt{y(x)}} - \frac{\sqrt{x^2 - 4\sqrt{y(x)}} \sqrt{y(x)} \log(y(x))}{4\sqrt{(x^2 - 4\sqrt{y(x)})y(x)}} + \frac{1}{4} \log(y(x)) = c_1, y(x) \right] \right\}$$

✓ **Maple** : cpu = 0.254 (sec), leaf count = 118

$$\left\{ \frac{\left(-x + \sqrt{x^2 - 4\sqrt{y(x)}} \right) \frac{\sqrt{x^2y(x) - 4y(x)^{\frac{3}{2}}}}{\sqrt{x^2 - 4\sqrt{y(x)}} \sqrt{y(x)}} \left(x + \sqrt{x^2 - 4\sqrt{y(x)}} \right) - \frac{\sqrt{x^2y(x) - 4y(x)^{\frac{3}{2}}}}{\sqrt{x^2 - 4\sqrt{y(x)}} \sqrt{y(x)}}}{\sqrt{y(x)}} - c_1 = 0, y(x) = \frac{x^4}{16} \right\}$$

2.548 ODE No. 548

$$y'(x)^6 - (y(x) - a)^4(y(x) - b)^3 = 0$$

✓ **Mathematica** : cpu = 1.09469 (sec), leaf count = 479

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[-\frac{3\sqrt[3]{a-\#1} \sqrt{\frac{\#1-b}{a-b}} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}} \& \right] [c_1 - ix] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[-\frac{3\sqrt[3]{a-\#1} \sqrt{\frac{\#1-b}{a-b}} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}} \& \right] [c_1 - ix] \right\} \right\}$$

✓ **Maple** : cpu = 0.413 (sec), leaf count = 250

$$\left\{ -c_1 + x - \left(\int^{y(x)} \frac{1}{(\_a - a)^4 (\_a - b)^3}^{\frac{1}{6}} d\_a \right) = 0, -c_1 + x - \left(\int^{y(x)} \frac{1}{(-(-\_a + a)^4 (-\_a + b)^3)^{\frac{1}{6}}} d\_a \right) = 0, -c_1 + x - \left(\int^{y(x)} \frac{1}{(-(-\_a + a)^4 (-\_a + b)^3)^{\frac{1}{6}}} d\_a \right) = 0 \right\}$$

2.549 ODE No. 549

$$x^2(y'(x)^2 + 1)^3 - a^2 = 0$$

✓ **Mathematica** : cpu = 0.252285 (sec), leaf count = 216

$$\left\{ \left\{ y(x) \rightarrow -x \left(\frac{a^{2/3}}{x^{2/3}} - 1 \right)^{3/2} + c_1 \right\}, \left\{ y(x) \rightarrow x \left(\frac{a^{2/3}}{x^{2/3}} - 1 \right)^{3/2} + c_1 \right\}, \left\{ y(x) \rightarrow c_1 - x \left(-1 - \frac{i(\sqrt{3} - i)a^{2/3}}{2x^{2/3}} \right)^{3/2} \right\}, \left\{ y(x) \rightarrow x \left(-1 - \frac{i(\sqrt{3} - i)a^{2/3}}{2x^{2/3}} \right)^{3/2} \right\} \right\}$$

✓ **Maple** : cpu = 0.758 (sec), leaf count = 553

$$\left\{ y(x) = c_1 + \frac{\sqrt{-\frac{(a^2x)^{\frac{4}{3}} \left(-a^2 + (a^2x)^{\frac{2}{3}} \right)}{a^4}} \left(a^2 - (a^2x)^{\frac{2}{3}} \right)}{(a^2x)^{\frac{2}{3}}}, y(x) = c_1 + \frac{\sqrt{-\frac{(a^2x)^{\frac{4}{3}} \left(-a^2 + (a^2x)^{\frac{2}{3}} \right)}{a^4}} \left(-a^2 + (a^2x)^{\frac{2}{3}} \right)}{(a^2x)^{\frac{2}{3}}}, y(x) = c_1 - \frac{i \left(\sqrt{3} - i \right) a^{2/3}}{2x^{2/3}} \right\}$$

2.550 ODE No. 550

$$-ay(x)^s - bx^{\frac{rs}{r-s}} + y'(x)^r = 0$$

✓ **Mathematica** : cpu = 0.831968 (sec), leaf count = 488

$$\text{Solve} \left[\int_1^{y(x)} \left(\frac{r}{-rx \left(aK[2]^s + bx^{\frac{rs}{r-s}} \right)^{\frac{1}{r}} + sx \left(aK[2]^s + bx^{\frac{rs}{r-s}} \right)^{\frac{1}{r}} + rK[2]} - \int_1^x \frac{asK[2]^{s-1} \left(aK[2]^s + bK[1]^{\frac{rs}{r-s}} \right)}{rK[1] \left(aK[2]^s + bK[1]^{\frac{rs}{r-s}} \right)^{\frac{1}{r}} - sK[1] \left(aK[2]^s + bK[1]^{\frac{rs}{r-s}} \right)^{\frac{1}{r}}} dx \right) dy = 0 \right]$$

✓ **Maple** : cpu = 0.273 (sec), leaf count = 60

$$\left\{ -c_1 + (-r + s) \left(\int_{-b}^{y(x)} \frac{1}{-\_ar + (r - s)x \left(a\_a^s + b x^{\frac{rs}{r-s}} \right)^{\frac{1}{r}}} d\_a \right) + \ln(x) = 0 \right\}$$

2.551 ODE No. 551

$$y'(x)^n - f(x)^n(y(x) - a)^{n+1}(y(x) - b)^{n-1} = 0$$

✓ **Mathematica** : cpu = 0.184622 (sec), leaf count = 86

$$\left\{ \left\{ y(x) \rightarrow \frac{-bn^n - a(a-b)^n \left(\int_1^x (-1)^{1+\frac{1}{n}} f(K[1]) dK[1] + c_1 \right)^n}{-n^n - (a-b)^n \left(\int_1^x (-1)^{1+\frac{1}{n}} f(K[1]) dK[1] + c_1 \right)^n} \right\} \right\}$$

✓ **Maple** : cpu = 0.622 (sec), leaf count = 55

$$\left\{ y(x) = \frac{b \left(-\frac{n}{(a-b)(c_1 + \int f(x) dx)} \right)^n - a}{\left(-\frac{n}{(a-b)(c_1 + \int f(x) dx)} \right)^n - 1} \right\}$$

2.552 ODE No. 552

$$y'(x)^n - f(x)g(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.0533426 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[\int_1^{\#1} g(K[1])^{-1/n} dK[1] \& \right] \left[\int_1^x f(K[2])^{\frac{1}{n}} dK[2] + c_1 \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.087 (sec), leaf count = 43

$$\left\{ c_1 + \int^{y(x)} g(\_a)^{-\frac{1}{n}} d\_a + \int^x -\left(f(\_a)g(y(x)) \right)^{\frac{1}{n}} g(y(x))^{-\frac{1}{n}} d\_a = 0 \right\}$$

2.553 ODE No. 553

$$ay'(x)^m + by'(x)^n - y(x) = 0$$

✓ **Mathematica** : cpu = 0.270773 (sec), leaf count = 56

$$\text{Solve} \left[\left\{ x = \frac{amK[1]^{m-1}}{m-1} + \frac{bnK[1]^{n-1}}{n-1} + c_1, y(x) = aK[1]^m + bK[1]^n \right\}, \{y(x), K[1]\} \right]$$

✓ **Maple** : cpu = 0.063 (sec), leaf count = 36

$$\left\{ -c_1 + x - \left(\int^{y(x)} \frac{1}{\text{RootOf}(-a\_Z^m - b\_Z^n + \_a)} d\_a \right) = 0, y(x) = 0 \right\}$$

2.554 ODE No. 554

$$x^{n-1}y'(x)^n - nxy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.170876 (sec), leaf count = 54

$$\text{Solve} \left[\left\{ y(x) = \frac{nx^2K[1] - x^nK[1]^n}{x}, x = c_1(K[1] - nK[1])^{\frac{n}{1-n}} \right\}, \{y(x), K[1]\} \right]$$

✓ **Maple** : cpu = 0.39 (sec), leaf count = 32

$$\left\{ y(x) = \frac{c_1^2 n \left(\frac{x}{c_1} \right)^{\frac{1}{n}} - \left(\frac{1}{c_1} \right)^{-n}}{c_1} \right\}$$

2.555 ODE No. 555

$$xy'(x) + \sqrt{y'(x)^2 + 1} - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0073882 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow c_1 x + \sqrt{1 + c_1^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 15

$$\left\{ y(x) = c_1 x + \sqrt{c_1^2 + 1} \right\}$$

2.556 ODE No. 556

$$xy'(x)^2 + \sqrt{y'(x)^2 + 1} + y(x) = 0$$

✓ **Mathematica** : cpu = 5.32891 (sec), leaf count = 67

$$\text{Solve} \left[\left\{ x = \frac{-\sqrt{K[1]^2 + 1} - \sinh^{-1}(K[1])}{(K[1] + 1)^2} + \frac{c_1}{(K[1] + 1)^2}, y(x) = -xK[1]^2 - \sqrt{K[1]^2 + 1} \right\}, \{y(x), K[1]\} \right]$$

✓ **Maple** : cpu = 0.213 (sec), leaf count = 581

$$\left\{ \frac{c_1 x^2}{\left(-2x + \sqrt{-4xy(x) + 2 + 2\sqrt{4x^2 - 4xy(x) + 1}} \right)^2} + \frac{2 \left(-2 \operatorname{arcsinh} \left(\frac{\sqrt{-4xy(x) + 2 + 2\sqrt{4x^2 - 4xy(x) + 1}}}{2x} \right) + \sqrt{2} \sqrt{\frac{2x^2 - 2xy(x) + \sqrt{4x^2 - 4xy(x) + 1}}{x^2}} \right)}{\left(-2x + \sqrt{-4xy(x) + 2 + 2\sqrt{4x^2 - 4xy(x) + 1}} \right)^2} \right\}$$

2.557 ODE No. 557

$$x \left(y'(x) + \sqrt{y'(x)^2 + 1} \right) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0489829 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{-x^2 + c_1 x} \right\}, \left\{ y(x) \rightarrow \sqrt{-x^2 + c_1 x} \right\} \right\}$$

✓ **Maple** : cpu = 0.26 (sec), leaf count = 74

$$\left\{ \frac{2c_1 xy(x)}{\sqrt{\frac{(x^2 + y(x)^2)^2}{x^2 y(x)^2}} \left(-x^2 + \sqrt{\frac{x^4 + 2x^2 y(x)^2 + y(x)^4}{x^2 y(x)^2}} xy(x) + y(x)^2 \right)} + x = 0 \right\}$$

2.558 ODE No. 558

$$ax\sqrt{y'(x)^2 + 1} + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 1.13073 (sec), leaf count = 327

$$\left\{ \text{Solve} \left[\frac{2i \tan^{-1} \left(\frac{y(x)}{x \sqrt{a^2 - \frac{y(x)^2}{x^2} - 1}} \right) + a \tanh^{-1} \left(\frac{-a^2 - \frac{iy(x)}{x} + 1}{a \sqrt{a^2 - \frac{y(x)^2}{x^2} - 1}} \right) - a \tanh^{-1} \left(\frac{-a^2 + \frac{iy(x)}{x} + 1}{a \sqrt{a^2 - \frac{y(x)^2}{x^2} - 1}} \right) + a \log \left(\frac{y(x)^2}{x^2} + 1 \right)}{2a^2 - 2} = \frac{a \log(x - a^2 x)}{1 - a^2} \right] \right\}$$

✓ **Maple** : cpu = 0.341 (sec), leaf count = 223

$$\left\{ -\frac{c_1 e^{\frac{\operatorname{arcsinh}\left(\frac{\sqrt{-a^2 x^2 + x^2 + y(x)^2} a + y(x)}{(a^2 - 1)x}\right)}{a}}}{\sqrt{\frac{-a^2 x^2 + a^2 y(x)^2 + 2\sqrt{-a^2 x^2 + x^2 + y(x)^2} a y(x) + x^2 + y(x)^2}{(a^2 - 1)^2 x^2}}} + x = 0, -\frac{c_1 e^{\frac{\operatorname{arcsinh}\left(\frac{\sqrt{-a^2 x^2 + x^2 + y(x)^2} a - y(x)}{(a^2 - 1)x}\right)}{a}}}{\sqrt{\frac{-a^2 x^2 - a^2 y(x)^2 + 2\sqrt{-a^2 x^2 + x^2 + y(x)^2} a y(x) - x^2 - y(x)^2}{(a^2 - 1)^2 x^2}}} + x = 0 \right\}$$

2.559 ODE No. 559

$$-ay(x)y'(x) - ax + y(x)\sqrt{y'(x)^2 + 1} = 0$$

✓ **Mathematica** : cpu = 0.399837 (sec), leaf count = 212

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{a^6(-x^2) + 3a^4x^2 - 3a^2x^2 + 2a^2xe^{a^2c_1-c_1} - 2xe^{a^2c_1-c_1} + e^{2a^2c_1-2c_1} + x^2}}{\sqrt{a^6 - 3a^4 + 3a^2 - 1}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{a^6(-x^2) + 3a^4x^2 - 3a^2x^2 + 2a^2xe^{a^2c_1-c_1} - 2xe^{a^2c_1-c_1} + e^{2a^2c_1-2c_1} + x^2}}{\sqrt{a^6 - 3a^4 + 3a^2 - 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.248 (sec), leaf count = 215

$$\left\{ -c_1 e^{\int \frac{-a^2x - \sqrt{a^2x^2 + (a^2-1)y(x)^2}}{(a^2-1)y(x)} \frac{(-a + \sqrt{a^2+1}a)a}{\sqrt{a^2+1}(-aa - \sqrt{a^2+1})(a^2a - \sqrt{a^2+1}a + a)} d_a} + x = 0, -c_1 e^{\int \frac{-a^2x + \sqrt{a^2x^2 + (a^2-1)y(x)^2}}{(a^2-1)y(x)} \frac{(-a + \sqrt{a^2+1}a)a}{\sqrt{a^2+1}(-aa - \sqrt{a^2+1})(a^2a - \sqrt{a^2+1}a + a)} d_a} + x = 0 \right\}$$

2.560 ODE No. 560

$$ay(x)\sqrt{y'(x)^2 + 1} - x^2 - 2xy(x)y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 17.1699 (sec), leaf count = 110

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{a^2c_1^2(-x^2) - 4a^2c_1x - 4a^2 + 4x^2}}{\sqrt{-4 + a^2c_1^2}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{a^2c_1^2(-x^2) - 4a^2c_1x - 4a^2 + 4x^2}}{\sqrt{-4 + a^2c_1^2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.833 (sec), leaf count = 1120

$$\left\{ c_1 + \int_{-b}^x \frac{2a^3 - 2ay(x)^2 + \sqrt{(-a^4 + 2a^2y(x)^2 - a^2y(x)^2 + y(x)^4)a^2}}{2a^5 + 4a^3y(x)^2 - 2a^2y(x)^2 + 2ay(x)^4 + \sqrt{(-a^4 + 2a^2y(x)^2 - a^2y(x)^2 + y(x)^4)a^2}a^2 - \sqrt{(-a^4 + 2a^2y(x)^2 - a^2y(x)^2 + y(x)^4)a^2}} dx \right\}$$

2.561 ODE No. 561

$$f(x^2 + y(x)^2)\sqrt{y'(x)^2 + 1} - xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 2.05415 (sec), leaf count = 2138

$$\left\{ \text{Solve} \left[\int_1^x \left(\frac{\sqrt{f(K[1]^2 + y(x)^2)^2 (-f(K[1]^2 + y(x)^2)^2 + K[1]^2 + y(x)^2)} K[1]}{f(K[1]^2 + y(x)^2)^2 (K[1]^2 + y(x)^2)} - \frac{\sqrt{f(K[1]^2 + y(x)^2)^2 (-f(K[1]^2 + y(x)^2)^2 + K[1]^2 + y(x)^2)}}{f(K[1]^2 + y(x)^2)^2 (-f(K[1]^2 + y(x)^2)^2 + K[1]^2 + y(x)^2)} \right) dx \right] \right\}$$

✓ **Maple** : cpu = 2.26 (sec), leaf count = 50

$$\left\{ y(x) = \frac{x}{\tan\left(\text{RootOf}\left(2c_1 - 2\_Z + \int \frac{(\tan^2(\_Z)+1)x^2}{\tan(\_Z)^2} \frac{f(\_a)}{\sqrt{-f(\_a)^2 + \_a \_a}} d\_a\right)\right)} \right\}$$

2.562 ODE No. 562

$$a\sqrt[3]{y'(x)^3 + 1} + bxy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.156753 (sec), leaf count = 84

$$\text{Solve}\left[\left\{x = K[1]^{\frac{b}{1-b}} \left(\frac{a \int \frac{K[1]^{\frac{2b-1}{b-1}}}{(K[1]^3+1)^{2/3}} dK[1]}{1-b} + c_1\right), y(x) = a\sqrt[3]{K[1]^3 + 1} + bxK[1]\right\}, \{K[1], y(x)\}\right]$$

✓ **Maple** : cpu = 0.185 (sec), leaf count = 3306

Expression too large to display

2.563 ODE No. 563

$$ay(x) + b + xy'(x) + \log(y'(x)) = 0$$

✓ **Mathematica** : cpu = 0.231577 (sec), leaf count = 59

$$\text{Solve}\left[a\left(\frac{(a+1)\log(1 - aW(xe^{-ay(x)-b}))}{a^2} + \frac{W(xe^{-ay(x)-b})}{a}\right) + ay(x) = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.225 (sec), leaf count = 66

$$\left\{-c_1\left(e^{-ay(x)-b-\text{LambertW}(xe^{-ay(x)-b})}\right)^{-\frac{1}{a+1}} + x - \frac{e^{ay(x)+b+\text{LambertW}(xe^{-ay(x)-b})}}{a} = 0\right\}$$

2.564 ODE No. 564

$$a(xy'(x) - y(x)) + \log(y'(x)) = 0$$

✓ **Mathematica** : cpu = 0.0048774 (sec), leaf count = 17

$$\left\{\left\{y(x) \rightarrow \frac{\log(c_1)}{a} + c_1x\right\}\right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 32

$$\left\{y(x) = \frac{\ln\left(-\frac{1}{ax}\right) - 1}{a}, y(x) = c_1x + \frac{\ln(c_1)}{a}\right\}$$

2.565 ODE No. 565

$$y'(x) + y(x) \log(y'(x)) - xy(x) - y(x) \log(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.0301469 (sec), leaf count = 25

$$\left\{\left\{y(x) \rightarrow c_1e^{\frac{1}{2}W(e^x)^2+W(e^x)}\right\}\right\}$$

✓ **Maple** : cpu = 0.257 (sec), leaf count = 17

$$\left\{y(x)=c_1e^{\frac{(\operatorname{LambertW}(e^x)+2)\operatorname{LambertW}(e^x)}{2}}\right\}$$

2.566 ODE No. 566

$$y'(x)+\sin(y'(x))-x=0$$

✓ **Mathematica** : cpu = 0.0453682 (sec), leaf count = 38

$$\operatorname{Solve}\left[\left\{x=K[1]+\sin(K[1]),y(x)=\frac{K[1]^2}{2}+K[1]\sin(K[1])+\cos(K[1])+c_1\right\},\{y(x),K[1]\}\right]$$

✓ **Maple** : cpu = 0.041 (sec), leaf count = 16

$$\{y(x)=c_1+\int \operatorname{RootOf}(_Z-x+\sin(_Z))dx\}$$

2.567 ODE No. 567

$$a\cos(y'(x))+by'(x)+x=0$$

✓ **Mathematica** : cpu = 0.0902049 (sec), leaf count = 49

$$\operatorname{Solve}\left[\left\{y(x)=a\sin(K[1])-aK[1]\cos(K[1])-\frac{1}{2}bK[1]^2+c_1,x=-a\cos(K[1])-bK[1]\right\},\{y(x),K[1]\}\right]$$

✓ **Maple** : cpu = 0.055 (sec), leaf count = 18

$$\{y(x)=c_1+\int \operatorname{RootOf}(_Zb+a\cos(_Z)+x)dx\}$$

2.568 ODE No. 568

$$y'(x)^2\sin(y'(x))-y(x)=0$$

✓ **Mathematica** : cpu = 0.104988 (sec), leaf count = 34

$$\operatorname{Solve}\left[\left\{x=K[1]\sin(K[1])-\cos(K[1])+c_1,y(x)=K[1]^2\sin(K[1])\right\},\{y(x),K[1]\}\right]$$

✓ **Maple** : cpu = 0.067 (sec), leaf count = 32

$$\left\{-c_1+x-\left(\int^{y(x)}\frac{1}{\operatorname{RootOf}(_Z^2\sin(_Z)-_a)}d\_a\right)=0,y(x)=0\right\}$$

2.569 ODE No. 569

$$(y'(x)^2+1)\sin^2(y(x)-xy'(x))-1=0$$

✓ **Mathematica** : cpu = 0.111325 (sec), leaf count = 59

$$\left\{\left\{y(x)\rightarrow c_1x-\frac{1}{2}\cos^{-1}\left(\frac{-1+c_1^2}{1+c_1^2}\right)\right\},\left\{y(x)\rightarrow c_1x+\frac{1}{2}\cos^{-1}\left(\frac{-1+c_1^2}{1+c_1^2}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.355 (sec), leaf count = 147

$$\left\{y(x)=c_1x-\arcsin\left(\frac{1}{\sqrt{c_1^2+1}}\right),y(x)=c_1x+\arcsin\left(\frac{1}{\sqrt{c_1^2+1}}\right),y(x)=-\sqrt{-x+1}\sqrt{\frac{1}{x}}x-\arcsin\left(\sqrt{\frac{1}{x}}x\right),y(x)=\sqrt{-x+1}\sqrt{\frac{1}{x}}x+\arcsin\left(\sqrt{\frac{1}{x}}x\right)\right\}$$

2.570 ODE No. 570

$$\left(y'(x)^2 + 1\right)\left(ax + \tan^{-1}\left(y'(x)\right)\right) + y'(x) = 0$$

✓ **Mathematica** : cpu = 1.93517 (sec), leaf count = 58

$$\text{Solve}\left[\left\{y(x) = \frac{1}{a\left(K[1]^2 + 1\right)} + c_1, x = \frac{-K[1] + K[1]^2\left(-\tan^{-1}(K[1])\right) - \tan^{-1}(K[1])}{a\left(K[1]^2 + 1\right)}\right\}, \{y(x), K[1]\}\right]$$

✓ **Maple** : cpu = 0.065 (sec), leaf count = 30

$$\{y(x) = c_1 + \int \tan\left(\text{RootOf}\left(ax\left(\tan^2(\_Z)\right) + \_Z\left(\tan^2(\_Z)\right) + ax + \_Z + \tan(\_Z)\right)\right)dx\}$$

2.571 ODE No. 571

$$ax^n f\left(y'(x)\right) + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.163189 (sec), leaf count = 124

$$\text{Solve}\left[\left\{y(x) = ax^n f(K[1]) + xK[1], x = \left(nf(K[1])^{\frac{1}{n}-1} \int_1^{K[1]} -\frac{f(K[2])^{\frac{n-1}{n}-1}}{an} dK[2] - f(K[1])^{\frac{1}{n}-1} \int_1^{K[1]} -\frac{f(K[2])^{\frac{n-1}{n}-1}}{an} dK[2]\right)\right\}\right]$$

✓ **Maple** : cpu = 0.517 (sec), leaf count = 169

$$\left[\left[y(\_T) = \_T\left(\frac{c_1 an + (-n + 1)\left(\int f(\_T)^{-\frac{1}{n}} d\_T\right)}{an f(\_T)}\right)^{\frac{1}{n-1}} f(\_T)^{\frac{1}{(n-1)n}} + a\left(\frac{c_1 an + (-n + 1)\left(\int f(\_T)^{-\frac{1}{n}} d\_T\right)}{an f(\_T)}\right)^{\frac{1}{n-1}} f(\_T)^{\frac{1}{(n-1)n}}\right]^n\right]$$

2.572 ODE No. 572

$$f\left(y'(x)\right)\left(xy'(x) - y(x)\right)^n + y(x)g\left(y'(x)\right) + xh\left(y'(x)\right) = 0$$

✗ **Mathematica** : cpu = 0.0746323 (sec), leaf count = 0 , could not solve

`DSolve[x*h[Derivative[1][y][x]] + g[Derivative[1][y][x]]*y[x] + f[Derivative[1][y][x]]*(-y[x] + x*Derivative[1][y][x])^n == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve((x*diff(y(x),x)-y(x))^n*f(diff(y(x),x))+y(x)*g(diff(y(x),x))+x*h(diff(y(x),x))=0,y(x))`

2.573 ODE No. 573

$$f\left(xy'(x)^2\right) + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0293208 (sec), leaf count = 42

$$\left\{y(x) \rightarrow f(c_1) - 2\sqrt{c_1} \sqrt{x}, y(x) \rightarrow f(c_1) + 2\sqrt{c_1} \sqrt{x}\right\}$$

✓ **Maple** : cpu = 0.125 (sec), leaf count = 24

$$\{y(x) = 2\sqrt{x} \text{ RootOf}\left(c_1 + c_2 - 2\_Z - f(\_Z^2)\right) + c_1\}$$

2.574 ODE No. 574

$$f\left(x-\frac{3}{2}y'(x)^2\right)+y'(x)^3-y(x)=0$$

✓ **Mathematica** : cpu = 0.0377411 (sec), leaf count = 102

$$\left\{\left\{y(x)\rightarrow\frac{1}{9}\left(9f(c_1)+2\sqrt{6}x\sqrt{x-c_1}-2\sqrt{6}c_1\sqrt{x-c_1}\right),y(x)\rightarrow\frac{1}{9}\left(9f(c_1)-2\sqrt{6}x\sqrt{x-c_1}+2\sqrt{6}c_1\sqrt{x-c_1}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.141 (sec), leaf count = 41

$$\left\{y(x)=f(c_1)-\frac{2\sqrt{6}}{9}\frac{\sqrt{(-c_1+x)^3}}{9},y(x)=f(c_1)+\frac{2\sqrt{6}}{9}\frac{\sqrt{(-c_1+x)^3}}{9}\right\}$$

2.575 ODE No. 575

$$y'(x)f\left(xy(x)y'(x)-y(x)^2\right)+x^2\left(-y'(x)\right)+xy(x)=0$$

✗ **Mathematica** : cpu = 0.0385072 (sec), leaf count = 0 , could not solve

`DSolve[x*y[x] - x^2*Derivative[1][y][x] + f[-y[x]^2 + x*y[x]*Derivative[1][y][x]]*Derivative`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x),x)*f(x*y(x)*diff(y(x),x)-y(x)^2)-x^2*diff(y(x),x)+x*y(x)=0,y(x))`

2.576 ODE No. 576

$$\phi\left(f\left(x,y(x),y'(x)\right),g\left(x,y(x),y'(x)\right)\right)=0$$

✗ **Mathematica** : cpu = 0.017354 (sec), leaf count = 0 , could not solve

`DSolve[phi[f[x, y[x], Derivative[1][y][x]], g[x, y[x], Derivative[1][y][x]]] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(phi(f(x,y(x),diff(y(x),x)),g(x,y(x),diff(y(x),x)))=0,y(x))`

2.577 ODE No. 577

$$y'(x)=F\left(\frac{y(x)}{a+x}\right)$$

✓ **Mathematica** : cpu = 0.309148 (sec), leaf count = 243

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{1}{-aF\left(\frac{K[2]}{a+x}\right)-xF\left(\frac{K[2]}{a+x}\right)+K[2]}\right)-\int_1^x\left(\frac{F'\left(\frac{K[2]}{a+K[1]}\right)}{(a+K[1])\left(aF\left(\frac{K[2]}{a+K[1]}\right)+K[1]F\left(\frac{K[2]}{a+K[1]}\right)-K[2]\right)}-\frac{F\left(\frac{K[2]}{a+K[1]}\right)\left(\frac{aF'\left(\frac{K[2]}{a+K[1]}\right)}{a+K[1]}\right)}{\left(aF\left(\frac{K[2]}{a+K[1]}\right)+K[1]F\left(\frac{K[2]}{a+K[1]}\right)-K[2]\right)}\right)dx=0,y(x)\right]$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 28

$$\left\{y(x)=-(a+x)\text{RootOf}\left(c_1+\int_{-a}^{-Z}\frac{1}{-a+F(-_a)}d_{-a}+\ln(a+x)\right)\right\}$$

2.578 ODE No. 578

$$y'(x) = F(y(x) - x^2) + 2x$$

✓ **Mathematica** : cpu = 0.256811 (sec), leaf count = 100

$$\text{Solve} \left[\int_1^{y(x)} - \frac{F(K[2] - x^2) \int_1^x - \frac{2K[1]F'(K[2]-K[1]^2)}{F(K[2]-K[1]^2)^2} dK[1] + 1}{F(K[2] - x^2)} dK[2] + \int_1^x \left(\frac{2K[1]}{F(y(x) - K[1]^2)} + 1 \right) dK[1] = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 22

$$\left\{ y(x) = x^2 + \text{RootOf} \left(c_1 - x + \int^{-Z} \frac{1}{F(\_a)} d\_a \right) \right\}$$

2.579 ODE No. 579

$$y'(x) = F\left(\frac{ax^2}{4} + \frac{bx}{2} + y(x)\right) - \frac{ax}{2}$$

✓ **Mathematica** : cpu = 0.322084 (sec), leaf count = 514

$$\text{Solve} \left[\int_1^{y(x)} - \frac{b \int_1^x \left(\frac{2aK[1]F'(\frac{1}{4}aK[1]^2 + \frac{1}{2}bK[1]+K[2])}{(b+2F(\frac{1}{4}aK[1]^2 + \frac{1}{2}bK[1]+K[2]))^2} + \frac{2F'(\frac{1}{4}aK[1]^2 + \frac{1}{2}bK[1]+K[2])}{b+2F(\frac{1}{4}aK[1]^2 + \frac{1}{2}bK[1]+K[2])} - \frac{4F(\frac{1}{4}aK[1]^2 + \frac{1}{2}bK[1]+K[2])F'(\frac{1}{4}aK[1]^2 + \frac{1}{2}bK[1]+K[2])}{(b+2F(\frac{1}{4}aK[1]^2 + \frac{1}{2}bK[1]+K[2]))^2} \right) dK[2]}{F(K[2] - x^2)} dK[2] + \int_1^x \left(\frac{2K[1]}{F(y(x) - K[1]^2)} + 1 \right) dK[1] = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 35

$$\left\{ y(x) = -\frac{a x^2}{4} - \frac{bx}{2} + \text{RootOf} \left(c_1 - x + 2 \left(\int^{-Z} \frac{1}{b + 2F(\_a)} d\_a \right) \right) \right\}$$

2.580 ODE No. 580

$$y'(x) = e^{bx} F(e^{-bx} y(x))$$

✓ **Mathematica** : cpu = 0.330247 (sec), leaf count = 203

$$\text{Solve} \left[\int_1^{y(x)} \left(\frac{1}{bK[2] - e^{bx} F(e^{-bx} K[2])} - \int_1^x \left(\frac{F'(e^{-bK[1]} K[2])}{e^{bK[1]} F(e^{-bK[1]} K[2]) - bK[2]} - \frac{e^{bK[1]} F(e^{-bK[1]} K[2]) (F'(e^{-bK[1]} K[2]) - b)}{(e^{bK[1]} F(e^{-bK[1]} K[2]) - bK[2])^2} \right) dK[1] \right) dK[2] = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.07 (sec), leaf count = 31

$$\left\{ y(x) = \text{RootOf} \left(c_1 - x + \int^{-Z} \frac{1}{-\_ab + F(\_a)} d\_a \right) e^{bx} \right\}$$

2.581 ODE No. 581

$$y'(x) = \frac{x F\left(\frac{x^2 y(x) + \frac{1}{4}}{x^2}\right) + \frac{1}{2}}{x^3}$$

✓ **Mathematica** : cpu = 0.303825 (sec), leaf count = 144

Solve
$$\left[\int_1^{y(x)} - \frac{F\left(\frac{K[2]x^2 + \frac{1}{4}}{x^2}\right) \int_1^x - \frac{F'\left(\frac{K[2]K[1]^2 + \frac{1}{4}}{K[1]^2}\right)}{2F\left(\frac{K[2]K[1]^2 + \frac{1}{4}}{K[1]^2}\right)^2} dK[1] + 1}{F\left(\frac{K[2]x^2 + \frac{1}{4}}{x^2}\right)} dK[2] + \int_1^x \left(\frac{1}{K[1]^2} + \frac{1}{2K[1]^3 F\left(\frac{y(x)K[1]^2 + \frac{1}{4}}{K[1]^2}\right)} \right) dK[1] = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.101 (sec), leaf count = 32

$$\left\{ y(x) = \frac{4x^2 \operatorname{RootOf}\left(c_1 x + x \left(\int^{-Z} \frac{1}{F(-a)} d_a \right) + 1 \right) - 1}{4x^2} \right\}$$

2.582 ODE No. 582

$$y'(x) = \frac{ax^2 F\left(\frac{axy(x)+1}{ax}\right) + 1}{ax^2}$$

✓ **Mathematica** : cpu = 0.309711 (sec), leaf count = 142

Solve
$$\left[\int_1^{y(x)} - \frac{F\left(\frac{axK[2]+1}{ax}\right) \int_1^x \frac{F'\left(\frac{aK[1]K[2]+1}{aK[1]}\right)}{aF\left(\frac{aK[1]K[2]+1}{aK[1]}\right)^2} dK[1] - 1}{F\left(\frac{axK[2]+1}{ax}\right)} dK[2] + \int_1^x \left(-1 - \frac{1}{aK[1]^2 F\left(\frac{aK[1]y(x)+1}{aK[1]}\right)} \right) dK[1] = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.237 (sec), leaf count = 30

$$\left\{ y(x) = \frac{ax \operatorname{RootOf}\left(c_1 - x + \int^{-Z} \frac{1}{F(-a)} d_a \right) - 1}{ax} \right\}$$

2.583 ODE No. 583

$$y'(x) = -\frac{1}{2}x \left(ax^2 - 2F\left(\frac{ax^4}{8} + y(x)\right) \right)$$

✓ **Mathematica** : cpu = 0.278387 (sec), leaf count = 126

Solve
$$\left[\int_1^{y(x)} - \frac{F\left(\frac{ax^4}{8} + K[2]\right) \int_1^x \frac{aK[1]^3 F'\left(\frac{1}{8}aK[1]^4 + K[2]\right)}{2F\left(\frac{1}{8}aK[1]^4 + K[2]\right)^2} dK[1] + 1}{F\left(\frac{ax^4}{8} + K[2]\right)} dK[2] + \int_1^x \left(K[1] - \frac{aK[1]^3}{2F\left(\frac{1}{8}aK[1]^4 + y(x)\right)} \right) dK[1] = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.148 (sec), leaf count = 31

$$\left\{ y(x) = -\frac{ax^4}{8} + \operatorname{RootOf}\left(-x^2 + 2c_1 + 2 \left(\int^{-Z} \frac{1}{F(-a)} d_a \right) \right) \right\}$$

2.584 ODE No. 584

$$y'(x) = \frac{2a}{2aF(y(x)^2 - 4ax) + y(x)}$$

✓ **Mathematica** : cpu = 0.330916 (sec), leaf count = 115

$$\text{Solve} \left[\int_1^{y(x)} \left(\frac{K[2]}{4a^2F(K[2]^2 - 4ax)} - \frac{2a \int_1^x \frac{K[2]F'(K[2]^2 - 4aK[1])}{aF(K[2]^2 - 4aK[1])^2} dK[1] - 1}{2a} \right) dK[2] + \int_1^x -\frac{1}{2aF(y(x)^2 - 4aK[1])} dK[1] = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.107 (sec), leaf count = 35

$$\left\{ -c_1 + \frac{y(x)}{2a} + \frac{\int^{-4ax+y(x)^2} \frac{1}{F(\_a)} d\_a}{8a^2} = 0 \right\}$$

2.585 ODE No. 585

$$y'(x) = y(x)F(\log(\log(y(x))) - \log(x))$$

✓ **Mathematica** : cpu = 0.274639 (sec), leaf count = 205

$$\text{Solve} \left[\int_1^{y(x)} \left(\frac{1}{K[2](xF(\log(\log(K[2])) - \log(x)) - \log(K[2]))} - \int_1^x \left(\frac{F(\log(\log(K[2])) - \log(K[1])) \left(\frac{K[1]F'(\log(\log(K[2])) - \log(K[1]))}{K[2]\log(K[2])} \right)}{(F(\log(\log(K[2])) - \log(K[1]))K[1] - \log(K[1]))} \right) dK[1] \right) dK[2] = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.451 (sec), leaf count = 120

$$\left\{ c_1 + \int_{\_b}^x \frac{F(-\ln(\_a) + \ln(\ln(y(x))))}{\_aF(-\ln(\_a) + \ln(\ln(y(x)))) - \ln(y(x))} d\_a + \int^{y(x)} \left(- \left(\int_{\_b}^x \frac{F(-\ln(\_a) + \ln(\ln(\_f))) - D(F)(-\ln(\_a) + \ln(\ln(\_f)))}{(\_aF(-\ln(\_a) + \ln(\ln(\_f))) - \ln(\_f))^2 \_f} d\_f \right) dK[1] \right) = 0 \right\}$$

2.586 ODE No. 586

$$y'(x) = \frac{x F\left(\frac{y(x)}{\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}}$$

✓ **Mathematica** : cpu = 0.728045 (sec), leaf count = 975

$$\text{Solve} \left[\int_1^x \left(-\frac{K[1]\sqrt{K[1]^2+1} F\left(\frac{y(x)}{\sqrt{K[1]^2+1}}\right)^3}{y(x) \left(K[1]^2 F\left(\frac{y(x)}{\sqrt{K[1]^2+1}}\right)^2 + F\left(\frac{y(x)}{\sqrt{K[1]^2+1}}\right)^2 - y(x)^2 \right)} - \frac{K[1] F\left(\frac{y(x)}{\sqrt{K[1]^2+1}}\right)^2}{K[1]^2 F\left(\frac{y(x)}{\sqrt{K[1]^2+1}}\right)^2 + F\left(\frac{y(x)}{\sqrt{K[1]^2+1}}\right)^2 - y(x)^2} + \frac{K[1] F\left(\frac{y(x)}{\sqrt{K[1]^2+1}}\right)}{\sqrt{K[1]^2+1}} \right) dK[1] = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.251 (sec), leaf count = 39

$$\left\{ y(x) = \sqrt{x^2+1} \text{ RootOf} \left(2c_1 + 2 \left(\int^{-Z} \frac{1}{-\_a + F(\_a)} d\_a \right) - \ln(x^2+1) \right) \right\}$$

2.587 ODE No. 587

$$y'(x) = \frac{1}{2}\sqrt{x}\left(2F\left(y(x) - \frac{x^3}{6}\right) + x^{3/2}\right)$$

✓ **Mathematica** : cpu = 0.298573 (sec), leaf count = 123

$$\text{Solve}\left[\int_1^{y(x)} -\frac{F\left(K[2] - \frac{x^3}{6}\right)\int_1^x -\frac{K[1]^2 F'\left(K[2] - \frac{K[1]^3}{6}\right)}{2F\left(K[2] - \frac{K[1]^3}{6}\right)^2} dK[1] + 1}{F\left(K[2] - \frac{x^3}{6}\right)} dK[2] + \int_1^x \left(\frac{K[1]^2}{2F\left(y(x) - \frac{K[1]^3}{6}\right)} + \sqrt{K[1]}\right) dK[1] = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.119 (sec), leaf count = 29

$$\left\{-\frac{2x^{\frac{3}{2}}}{3} - c_1 + \int_{-b}^{y(x)} \frac{1}{F\left(-\frac{x^3}{6} + _a\right)} d_a = 0\right\}$$

2.588 ODE No. 588

$$y'(x) = \frac{F((y(x) - x)(y(x) + x)) + x}{y(x)}$$

✓ **Mathematica** : cpu = 0.234941 (sec), leaf count = 109

$$\text{Solve}\left[\int_1^{y(x)} \left(-\frac{K[2]}{F((K[2] - x)(x + K[2]))} - \int_1^x -\frac{2K[1]K[2]F'((K[2] - K[1])(K[1] + K[2]))}{F((K[2] - K[1])(K[1] + K[2]))^2} dK[1]\right) dK[2] + \int_1^x \left(\frac{1}{F((y(x) - K[1])(y(x) + K[1]))}\right) dK[1] = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.121 (sec), leaf count = 53

$$\left\{y(x) = \sqrt{x^2 + \text{RootOf}\left(2c_1 - 2x + \int^Z \frac{1}{F(_a)} d_a\right)}, y(x) = -\sqrt{x^2 + \text{RootOf}\left(2c_1 - 2x + \int^Z \frac{1}{F(_a)} d_a\right)}\right\}$$

2.589 ODE No. 589

$$y'(x) = \frac{y(x)^2 F\left(\frac{1-y(x)\log(x)}{y(x)}\right)}{x}$$

✓ **Mathematica** : cpu = 0.299659 (sec), leaf count = 245

$$\text{Solve}\left[\int_1^{y(x)} \left(\frac{1}{\left(-F\left(\frac{1-K[2]\log(x)}{K[2]}\right) - 1\right)K[2]^2} - \int_1^x \left(\frac{\left(-\frac{\log(K[1])}{K[2]} - \frac{1-K[2]\log(K[1])}{K[2]^2}\right)F'\left(\frac{1-K[2]\log(K[1])}{K[2]}\right)}{\left(F\left(\frac{1-K[2]\log(K[1])}{K[2]}\right) + 1\right)K[1]} - \frac{F\left(\frac{1-K[2]\log(K[1])}{K[2]}\right)\left(-\frac{\log(K[1])}{K[2]}\right)}{\left(F\left(\frac{1-K[2]\log(K[1])}{K[2]}\right) + 1\right)K[1]}\right) dK[2] = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.155 (sec), leaf count = 38

$$\left\{-c_1 + \int_{-b}^{y(x)} \frac{1}{\left(F\left(\frac{-_a\ln(x)+1}{-_a}\right) + 1\right)_a^2} d_a - \ln(x) = 0\right\}$$

2.590 ODE No. 590

$$y'(x) = \frac{x}{F(x^2 + y(x)^2) - y(x)}$$

✓ **Mathematica** : cpu = 0.286124 (sec), leaf count = 94

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{K[2]}{F(x^2 + K[2]^2)} - \int_1^x \frac{2K[1]K[2]F'(K[1]^2 + K[2]^2)}{F(K[1]^2 + K[2]^2)^2}dK[1] + 1\right)dK[2] + \int_1^x -\frac{K[1]}{F(K[1]^2 + y(x)^2)}dK[1] = c_1, y\right]$$

✓ **Maple** : cpu = 0.132 (sec), leaf count = 28

$$\left\{-c_1 + \frac{\left(\int^{x^2+y(x)^2}\frac{1}{F(_a)}d_a\right)}{2} - y(x) = 0\right\}$$

2.591 ODE No. 591

$$y'(x) = \frac{x F\left(\frac{a y(x)^2 + b x^2}{a}\right)}{\sqrt{a} y(x)}$$

✓ **Mathematica** : cpu = 0.593163 (sec), leaf count = 253

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{b K[2]}{b + \sqrt{a} F\left(\frac{b x^2 + a K[2]^2}{a}\right)} - \int_1^x \left(\frac{2 b K[1] K[2] F'\left(\frac{b K[1]^2 + a K[2]^2}{a}\right)}{\sqrt{a} \left(b + \sqrt{a} F\left(\frac{b K[1]^2 + a K[2]^2}{a}\right)\right)} - \frac{2 b F\left(\frac{b K[1]^2 + a K[2]^2}{a}\right) K[1] K[2] F'\left(\frac{b K[1]^2 + a K[2]^2}{a}\right)}{\left(b + \sqrt{a} F\left(\frac{b K[1]^2 + a K[2]^2}{a}\right)\right)^2}\right) dK[1] + 1\right) dK[2] = c_1, y\right]$$

✓ **Maple** : cpu = 0.226 (sec), leaf count = 108

$$\left\{y(x) = \frac{\sqrt{\left(-b x^2 + a \text{RootOf}\left(a^{\frac{3}{2}} b \left(\int^{-Z} \frac{1}{a F(_a) + \sqrt{a} b} d_a\right) - b x^2 + 2 c_1 a\right)\right) a}}{a}, y(x) = -\frac{\sqrt{\left(-b x^2 + a \text{RootOf}\left(a^{\frac{3}{2}} b \left(\int^{-Z} \frac{1}{a F(_a) + \sqrt{a} b} d_a\right) - b x^2 + 2 c_1 a\right)\right) a}}{a}\right\}$$

2.592 ODE No. 592

$$y'(x) = \frac{F\left(-\frac{2x^3}{5} + y(x) - 2\sqrt{x}\right) + \frac{6x^3}{5} + \sqrt{x}}{x}$$

✓ **Mathematica** : cpu = 0.711994 (sec), leaf count = 241

$$\text{Solve}\left[\int_1^{y(x)} -\frac{F\left(-\frac{2x^3}{5} - 2\sqrt{x} + K[2]\right) \int_1^x \left(-\frac{6F'\left(-\frac{2}{5}K[1]^3 - 2\sqrt{K[1]} + K[2]\right)K[1]^2}{5F\left(-\frac{2}{5}K[1]^3 - 2\sqrt{K[1]} + K[2]\right)^2} - \frac{F'\left(-\frac{2}{5}K[1]^3 - 2\sqrt{K[1]} + K[2]\right)}{F\left(-\frac{2}{5}K[1]^3 - 2\sqrt{K[1]} + K[2]\right)^2 \sqrt{K[1]}}\right) dK[1] + 1}{F\left(-\frac{2x^3}{5} - 2\sqrt{x} + K[2]\right)} dK[2] = c_1, y\right]$$

✓ **Maple** : cpu = 0.168 (sec), leaf count = 33

$$\left\{-c_1 + \int_{-b}^{y(x)} \frac{1}{F\left(-\frac{2x^3}{5} + _a - 2\sqrt{x}\right)} d_a - \ln(x) = 0\right\}$$

2.593 ODE No. 593

$$y'(x) = \frac{e^x F\left(y(x)^{3/2} - \frac{3e^x}{2}\right)}{\sqrt{y(x)}}$$

✓ **Mathematica** : cpu = 0.607303 (sec), leaf count = 221

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{\sqrt{K[2]}}{F\left(K[2]^{3/2}-\frac{3e^x}{2}\right)-1}-\int_1^x\left(\frac{3e^{K[1]}F\left(K[2]^{3/2}-\frac{3e^{K[1]}}{2}\right)\sqrt{K[2]}F'\left(K[2]^{3/2}-\frac{3e^{K[1]}}{2}\right)}{2\left(F\left(K[2]^{3/2}-\frac{3e^{K[1]}}{2}\right)-1\right)^2}-\frac{3e^{K[1]}\sqrt{K[2]}F'\left(K[2]^{3/2}-\frac{3e^{K[1]}}{2}\right)}{2\left(F\left(K[2]^{3/2}-\frac{3e^{K[1]}}{2}\right)-1\right)}\right)dx\right]$$

✓ **Maple** : cpu = 0.274 (sec), leaf count = 35

$$\left\{-c_1+\int_{-b}^{y(x)}\frac{\sqrt{-a}}{F\left(-a^{\frac{3}{2}}-\frac{3e^x}{2}\right)-1}d_{-a}-e^x=0\right\}$$

2.594 ODE No. 594

$$y'(x) = \frac{x F\left(\frac{y(x)^2-b}{x^2}\right)}{y(x)}$$

✓ **Mathematica** : cpu = 0.479769 (sec), leaf count = 236

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{K[2]}{-F\left(\frac{K[2]^2-b}{x^2}\right)x^2+K[2]^2-b}-\int_1^x\left(\frac{F\left(\frac{K[2]^2-b}{K[1]^2}\right)K[1]\left(2K[2]F'\left(\frac{K[2]^2-b}{K[1]^2}\right)-2K[2]\right)}{\left(F\left(\frac{K[2]^2-b}{K[1]^2}\right)K[1]^2-K[2]^2+b\right)^2}-\frac{2K[2]F'\left(\frac{K[2]^2-b}{K[1]^2}\right)}{K[1]\left(F\left(\frac{K[2]^2-b}{K[1]^2}\right)K[1]^2-K[2]^2+b\right)}\right)dx\right]$$

✓ **Maple** : cpu = 0.129 (sec), leaf count = 67

$$\left\{y(x)=\sqrt{x^2\text{RootOf}\left(2c_1+\int^{-Z}\frac{1}{-_{a}+F(_a)}d_{-a}-2\ln(x)\right)+b},y(x)=-\sqrt{x^2\text{RootOf}\left(2c_1+\int^{-Z}\frac{1}{-_{a}+F(_a)}d_{-a}-2\ln(x)\right)+b}\right\}$$

2.595 ODE No. 595

$$y'(x) = \frac{F\left(\frac{xy(x)^2+1}{x}\right)}{x^2 y(x)}$$

✓ **Mathematica** : cpu = 0.456962 (sec), leaf count = 204

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{K[2]}{2F\left(\frac{xK[2]^2+1}{x}\right)-1}-\int_1^x\left(\frac{4F\left(\frac{K[1]K[2]^2+1}{K[1]}\right)K[2]F'\left(\frac{K[1]K[2]^2+1}{K[1]}\right)}{\left(2F\left(\frac{K[1]K[2]^2+1}{K[1]}\right)-1\right)^2K[1]^2}-\frac{2K[2]F'\left(\frac{K[1]K[2]^2+1}{K[1]}\right)}{\left(2F\left(\frac{K[1]K[2]^2+1}{K[1]}\right)-1\right)K[1]^2}\right)dx\right]$$

✓ **Maple** : cpu = 0.165 (sec), leaf count = 72

$$\left\{y(x)=\frac{\sqrt{\left(x\text{RootOf}\left(c_1x+x\left(\int^{-Z}\frac{1}{2F(_a)-1}d_{-a}\right)+1\right)-1\right)x}}{x},y(x)=-\frac{\sqrt{\left(x\text{RootOf}\left(c_1x+x\left(\int^{-Z}\frac{1}{2F(_a)-1}d_{-a}\right)+1\right)-1\right)x}}{x}\right\}$$

2.596 ODE No. 596

$$y'(x) = \frac{F(x^2 + y(x) - x) - 2x^2 + x}{x}$$

✓ **Mathematica** : cpu = 0.350221 (sec), leaf count = 156

$$\text{Solve} \left[\int_1^{y(x)} -\frac{F(x^2 - x + K[2]) \int_1^x \left(\frac{2K[1]F'(K[1]^2 - K[1] + K[2])}{F(K[1]^2 - K[1] + K[2])^2} - \frac{F'(K[1]^2 - K[1] + K[2])}{F(K[1]^2 - K[1] + K[2])^2} \right) dK[1] + 1}{F(x^2 - x + K[2])} dK[2] + \int_1^x \left(-\frac{2K[1]}{F(K[1]^2 - K[1] + K[2])} \right) dK[1] = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.092 (sec), leaf count = 26

$$\left\{ y(x) = -x^2 + x + \text{RootOf} \left(c_1 + \int^{-Z} \frac{1}{F(_a)} d_a - \ln(x) \right) \right\}$$

2.597 ODE No. 597

$$y'(x) = \frac{2a}{x^2 \left(2aF\left(\frac{xy(x)^2 - 4a}{x}\right) - y(x) \right)}$$

✓ **Mathematica** : cpu = 0.431551 (sec), leaf count = 130

$$\text{Solve} \left[\int_1^{y(x)} \left(-\frac{K[2]}{2aF\left(\frac{xK[2]^2 - 4a}{x}\right)} - \int_1^x \frac{2K[2]F'\left(\frac{K[1]K[2]^2 - 4a}{K[1]}\right)}{F\left(\frac{K[1]K[2]^2 - 4a}{K[1]}\right)^2 K[1]^2} dK[1] + 1 \right) dK[2] + \int_1^x -\frac{1}{F\left(\frac{K[1]y(x)^2 - 4a}{K[1]}\right) K[1]^2} dK[1] = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.421 (sec), leaf count = 37

$$\left\{ -c_1 - \frac{y(x)}{2a} + \frac{\int^{y(x)^2 - \frac{4a}{x}} \frac{1}{F(_a)} d_a}{8a^2} = 0 \right\}$$

2.598 ODE No. 598

$$y'(x) = \frac{F\left(\frac{y(x)}{x}\right) + y(x)}{x - 1}$$

✓ **Mathematica** : cpu = 0.189332 (sec), leaf count = 37

$$\text{Solve} \left[\int_1^{\frac{y(x)}{x}} \frac{1}{F(K[1]) + K[1]} dK[1] = \log(1 - x) - \log(x) + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 29

$$\left\{ y(x) = x \text{RootOf} \left(c_1 - \left(\int^{-Z} \frac{1}{_a + F(_a)} d_a \right) - \ln(x) + \ln(x - 1) \right) \right\}$$

2.599 ODE No. 599

$$y'(x) = \frac{F(x^2 + y(x)^2) - x}{y(x)}$$

✓ **Mathematica** : cpu = 0.191244 (sec), leaf count = 95

$$\text{Solve} \left[\int_1^{y(x)} \left(-\frac{K[2]}{F(x^2 + K[2]^2)} - \int_1^x \frac{2K[1]K[2]F'(K[1]^2 + K[2]^2)}{F(K[1]^2 + K[2]^2)^2} dK[1] \right) dK[2] + \int_1^x \left(1 - \frac{K[1]}{F(K[1]^2 + y(x)^2)} \right) dK[1] = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 57

$$\left\{y(x)=\sqrt{-x^2+\text{RootOf}\left(2c_1-2x+\int^{\textstyle Z}\frac{1}{F(\textstyle _a)}d\textstyle _a\right)},y(x)=-\sqrt{-x^2+\text{RootOf}\left(2c_1-2x+\int^{\textstyle Z}\frac{1}{F(\textstyle _a)}d\textstyle _a\right)}\right\}$$

2.600 ODE No. 600

$$y'(x)=\frac{y(x)^2F\left(\frac{1-2y(x)\log(x)}{y(x)}\right)}{x}$$

✓ **Mathematica** : cpu = 0.322721 (sec), leaf count = 246

$$\text{Solve}\left[\int_1^{y(x)}\left(-\int_1^x\left(\frac{2\left(-\frac{2\log(K[1])}{K[2]}-\frac{1-2K[2]\log(K[1])}{K[2]^2}\right)F'\left(\frac{1-2K[2]\log(K[1])}{K[2]}\right)}{\left(F\left(\frac{1-2K[2]\log(K[1])}{K[2]}\right)+2\right)K[1]}-\frac{2F\left(\frac{1-2K[2]\log(K[1])}{K[2]}\right)\left(-\frac{2\log(K[1])}{K[2]}-\frac{1-2K[2]\log(K[1])}{K[2]^2}\right)}{\left(F\left(\frac{1-2K[2]\log(K[1])}{K[2]}\right)+2\right)^2K[1]}\right)dx\right]$$

✓ **Maple** : cpu = 0.137 (sec), leaf count = 38

$$\left\{-c_1+\int_{-b}^{y(x)}\frac{1}{\left(F\left(\frac{-2\_a\ln(x)+1}{\_a}\right)+2\right)\_a^2}d\_a-\ln(x)=0\right\}$$

2.601 ODE No. 601

$$y'(x)=\frac{x F((y(x)-x)(y(x)+x))}{y(x)}$$

✓ **Mathematica** : cpu = 0.297143 (sec), leaf count = 182

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{K[2]}{F((K[2]-x)(x+K[2]))-1}-\int_1^x\left(\frac{2F((K[2]-K[1])(K[1]+K[2]))K[1]K[2]F'((K[2]-K[1])(K[1]+K[2]))}{(F((K[2]-K[1])(K[1]+K[2]))-1)^2}\right)dx\right)dy\right]$$

✓ **Maple** : cpu = 0.124 (sec), leaf count = 61

$$\left\{y(x)=\sqrt{x^2+\text{RootOf}\left(-x^2+2c_1+\int^{\textstyle Z}\frac{1}{F(\textstyle _a)-1}d\textstyle _a\right)},y(x)=-\sqrt{x^2+\text{RootOf}\left(-x^2+2c_1+\int^{\textstyle Z}\frac{1}{F(\textstyle _a)-1}d\textstyle _a\right)}\right\}$$

2.602 ODE No. 602

$$y'(x)=\frac{y(x)^2\left(x^2F\left(\frac{x^2-y(x)}{x^2y(x)}\right)+2\right)}{x^3}$$

✓ **Mathematica** : cpu = 0.595777 (sec), leaf count = 167

$$\text{Solve}\left[\int_1^{y(x)}\left(-\int_1^x-\frac{2\left(-\frac{K[1]^2-K[2]}{K[1]^2K[2]^2}-\frac{1}{K[1]^2K[2]}\right)F'\left(\frac{K[1]^2-K[2]}{K[1]^2K[2]}\right)}{F\left(\frac{K[1]^2-K[2]}{K[1]^2K[2]}\right)^2K[1]^3}dK[1]-\frac{1}{F\left(\frac{x^2-K[2]}{x^2K[2]}\right)K[2]^2}dK[2]+\int_1^x\left(\frac{1}{K[1]}+\frac{2}{K[1]^3F\left(\frac{K[1]^2}{K[1]^2}\right)}\right)dx\right]$$

✓ **Maple** : cpu = 0.121 (sec), leaf count = 33

$$\left\{y(x)=\frac{x^2}{x^2\text{RootOf}\left(c_1-\left(\int^{\textstyle Z}\frac{1}{F(\textstyle _a)}d\textstyle _a\right)-\ln(x)\right)+1}\right\}$$

2.603 ODE No. 603

$$y'(x) = \frac{2xF(y(x) + \log(2x + 1)) + F(y(x) + \log(2x + 1)) - 2}{2x + 1}$$

✓ **Mathematica** : cpu = 0.410144 (sec), leaf count = 117

$$\text{Solve} \left[\int_1^{y(x)} -\frac{F(K[2] + \log(2x + 1)) \int_1^x -\frac{2F'(K[2] + \log(2K[1] + 1))}{F(K[2] + \log(2K[1] + 1))^2(2K[1] + 1)} dK[1] - 1}{F(K[2] + \log(2x + 1))} dK[2] + \int_1^x \left(\frac{2}{F(\log(2K[1] + 1) + y(x))(2K[1] + 1)} \right) dK[1] \right]$$

✓ **Maple** : cpu = 0.138 (sec), leaf count = 27

$$\left\{ y(x) = \text{RootOf} \left(c_1 - x + \int^{-Z} \frac{1}{F(-a)} d_{-a} \right) - \ln(2x + 1) \right\}$$

2.604 ODE No. 604

$$y'(x) = \frac{2y(x)^3}{2y(x)F\left(\frac{4xy(x)^2+1}{y(x)^2}\right) + 1}$$

✓ **Mathematica** : cpu = 0.45576 (sec), leaf count = 143

$$\text{Solve} \left[\int_1^{y(x)} \left(-\int_1^x \frac{\left(\frac{8K[1]}{K[2]} - \frac{2(4K[1]K[2]^2+1)}{K[2]^3} \right) F' \left(\frac{4K[1]K[2]^2+1}{K[2]^2} \right)}{F \left(\frac{4K[1]K[2]^2+1}{K[2]^2} \right)^2} dK[1] + \frac{1}{K[2]^2} + \frac{1}{2F \left(\frac{4xK[2]^2+1}{K[2]^2} \right) K[2]^3} \right) dK[2] + \int_1^x -\frac{1}{F \left(\frac{4K[1]y(x)^2+1}{y(x)^2} \right)} dy(x) \right]$$

✓ **Maple** : cpu = 0.158 (sec), leaf count = 30

$$\left\{ -c_1 - \frac{\left(\int^{4x+\frac{1}{y(x)^2}} \frac{1}{F(-a)} d_{-a} \right)}{4} - \frac{1}{y(x)} = 0 \right\}$$

2.605 ODE No. 605

$$y'(x) = -\frac{y(x)^2 \left(2x - F \left(\frac{1-\frac{1}{2}xy(x)}{y(x)} \right) \right)}{4x}$$

✓ **Mathematica** : cpu = 0.556547 (sec), leaf count = 145

$$\text{Solve} \left[\int_1^{y(x)} \left(-\int_1^x \frac{2 \left(-\frac{K[1]}{2K[2]} - \frac{1-\frac{1}{2}K[1]K[2]}{K[2]^2} \right) F' \left(\frac{1-\frac{1}{2}K[1]K[2]}{K[2]} \right)}{F \left(\frac{1-\frac{1}{2}K[1]K[2]}{K[2]} \right)^2} dK[1] - \frac{4}{F \left(\frac{1-\frac{1}{2}xK[2]}{K[2]} \right) K[2]^2} \right) dK[2] + \int_1^x \left(\frac{1}{K[1]} - \frac{2}{F \left(\frac{1-\frac{1}{2}K[1]y(x)}{y(x)} \right)} \right) dy(x) \right]$$

✓ **Maple** : cpu = 0.121 (sec), leaf count = 29

$$\left\{ y(x) = \frac{2}{x + 2 \text{RootOf} \left(c_1 - 4 \left(\int^{-Z} \frac{1}{F(-a)} d_{-a} \right) - \ln(x) \right) \right\}$$

2.606 ODE No. 606

$$y'(x) = x \left(F \left(y(x) - \frac{1}{2} e^{-x^2} x^2 \right) - e^{-x^2} x^2 + e^{-x^2} \right)$$

✓ **Mathematica** : cpu = 0.689518 (sec), leaf count = 361

$$\text{Solve} \left[\int_1^{y(x)} - \frac{F \left(K[2] - \frac{1}{2} e^{-x^2} x^2 \right) \int_1^x \left(\frac{e^{-K[1]^2} F' \left(K[2] - \frac{1}{2} e^{-K[1]^2} K[1]^2 \right) K[1]^3}{F \left(K[2] - \frac{1}{2} e^{-K[1]^2} K[1]^2 \right)^2} - \frac{e^{-K[1]^2} \left(e^{K[1]^2} F \left(K[2] - \frac{1}{2} e^{-K[1]^2} K[1]^2 \right) + 1 \right) F' \left(K[2] - \frac{1}{2} e^{-K[1]^2} K[1]^2 \right) K[1]^3}{F \left(K[2] - \frac{1}{2} e^{-K[1]^2} K[1]^2 \right)^2} \right) dK[1]}{F \left(K[2] - \frac{1}{2} e^{-x^2} x^2 \right)} \right]$$

✓ **Maple** : cpu = 0.615 (sec), leaf count = 34

$$\left\{ y(x) = \frac{x^2 e^{-x^2}}{2} + \text{RootOf} \left(x^2 + 2c_1 - 2 \left(\int^{-Z} \frac{1}{F(-a)} d_{-a} \right) \right) \right\}$$

2.607 ODE No. 607

$$y'(x) = \frac{x^3 F \left(\frac{y(x)}{x^2} \right) + 2y(x)}{x}$$

✓ **Mathematica** : cpu = 0.267311 (sec), leaf count = 121

$$\text{Solve} \left[\int_1^{y(x)} - \frac{F \left(\frac{K[2]}{x^2} \right) \int_1^x \left(\frac{2}{F \left(\frac{K[2]}{K[1]^2} \right) K[1]^3} - \frac{2K[2]F' \left(\frac{K[2]}{K[1]^2} \right)}{F \left(\frac{K[2]}{K[1]^2} \right)^2 K[1]^5} \right) dK[1] x^2 + 1}{x^2 F \left(\frac{K[2]}{x^2} \right)} dK[2] + \int_1^x \left(\frac{2y(x)}{F \left(\frac{y(x)}{K[1]^2} \right) K[1]^3} + 1 \right) dK[1] = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.096 (sec), leaf count = 22

$$\left\{ y(x) = x^2 \text{RootOf} \left(c_1 - x + \int^{-Z} \frac{1}{F(-a)} d_{-a} \right) \right\}$$

2.608 ODE No. 608

$$y'(x) = \frac{\sqrt{y(x)}}{F \left(\frac{x-y(x)}{\sqrt{y(x)}} \right) + \sqrt{y(x)}}$$

✓ **Mathematica** : cpu = 0.59644 (sec), leaf count = 274

$$\text{Solve} \left[\int_1^{y(x)} \left(- \frac{F \left(\frac{x-K[2]}{\sqrt{K[2]}} \right)}{x \sqrt{K[2]}} - \int_1^x - \frac{- \frac{F \left(\frac{K[1]-K[2]}{\sqrt{K[2]}} \right)}{\sqrt{K[2]}} - 2 \left(- \frac{K[1]-K[2]}{2K[2]^{3/2}} - \frac{1}{\sqrt{K[2]}} \right) \sqrt{K[2]} F' \left(\frac{K[1]-K[2]}{\sqrt{K[2]}} \right) - 1}{\left(-2\sqrt{K[2]} F \left(\frac{K[1]-K[2]}{\sqrt{K[2]}} \right) + K[1] - K[2] \right)^2} dK[1] + \frac{2F \left(\frac{x-K[2]}{\sqrt{K[2]}} \right)^2 + \sqrt{K[2]}}{x \left(-x + K[2] + 2F \left(\frac{x-K[2]}{\sqrt{K[2]}} \right) \right)} \right]$$

✓ **Maple** : cpu = 0.157 (sec), leaf count = 40

$$\left\{ -c_1 - \left(\int^{\frac{x}{\sqrt{y(x)}} - \sqrt{y(x)}} \frac{1}{-_{-a} + 2F(-a)} d_{-a} \right) + \frac{\ln(y(x))}{2} = 0 \right\}$$

2.609 ODE No. 609

$$y'(x) = \frac{F\left(x^3 y(x)\right) - 3x^2 y(x)}{x^3}$$

✓ **Mathematica** : cpu = 0.317916 (sec), leaf count = 117

$$\text{Solve}\left[\int_1^{y(x)} \frac{x^3 + F\left(x^3 K[2]\right) \int_1^x \left(\frac{3K[1]^5 K[2] F'(K[1]^3 K[2])}{F(K[1]^3 K[2])^2} - \frac{3K[1]^2}{F(K[1]^3 K[2])}\right) dK[1]}{F\left(x^3 K[2]\right)} dK[2] + \int_1^x \left(1 - \frac{3K[1]^2 y(x)}{F(K[1]^3 y(x))}\right) dK[1] = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.177 (sec), leaf count = 22

$$\left\{y(x) = \frac{\text{RootOf}\left(c_1 + x - \left(\int^{-Z} \frac{1}{F(\_a)} d\_a\right)\right)}{x^3}\right\}$$

2.610 ODE No. 610

$$y'(x) = \frac{x^2 F\left(\frac{y(x)}{x}\right) + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.106308 (sec), leaf count = 25

$$\text{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{1}{F(K[1])} dK[1] = x + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 20

$$\left\{y(x) = x \text{RootOf}\left(c_1 + x - \left(\int^{-Z} \frac{1}{F(\_a)} d\_a\right)\right)\right\}$$

2.611 ODE No. 611

$$y'(x) = \frac{F(x(y(x) + x)) - y(x) - 2x}{x}$$

✓ **Mathematica** : cpu = 0.333128 (sec), leaf count = 191

$$\text{Solve}\left[\int_1^{y(x)} \frac{x + F(x(x + K[2])) \int_1^x \left(\frac{2F'(K[1](K[1] + K[2]))K[1]^2}{F(K[1](K[1] + K[2]))^2} + \frac{(K[2] - F(K[1](K[1] + K[2])))F'(K[1](K[1] + K[2]))K[1]}{F(K[1](K[1] + K[2]))^2} - \frac{1 - K[1]F'(K[1](K[1] + K[2]))}{F(K[1](K[1] + K[2]))}\right) dK[1]}{F(x(x + K[2]))} dK[2] = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.083 (sec), leaf count = 28

$$\left\{y(x) = \frac{-x^2 + \text{RootOf}\left(c_1 - x + \int^{-Z} \frac{1}{F(\_a)} d\_a\right)}{x}\right\}$$

2.612 ODE No. 612

$$y'(x) = \frac{1}{2} e^{\frac{x^2}{4}} \left(2F\left(e^{-\frac{x^2}{4}} y(x)\right) + e^{-\frac{x^2}{4}} x y(x)\right)$$

✓ **Mathematica** : cpu = 0.452239 (sec), leaf count = 199

Solve
$$\left[\int_1^{y(x)} - \frac{e^{-\frac{x^2}{4}} \left(e^{\frac{x^2}{4}} F\left(e^{-\frac{x^2}{4}} K[2]\right) \int_1^x \left(\frac{e^{-\frac{1}{4} K[1]^2} K[1]}{2F\left(e^{-\frac{1}{4} K[1]^2} K[2]\right)} - \frac{e^{-\frac{1}{2} K[1]^2} K[1] K[2] F'\left(e^{-\frac{1}{4} K[1]^2} K[2]\right)}{2F\left(e^{-\frac{1}{4} K[1]^2} K[2]\right)^2} \right) dK[1] + 1 \right)}{F\left(e^{-\frac{x^2}{4}} K[2]\right)} dK[2] + \int_1^x \left(\frac{e^{-\frac{1}{4} K[1]^2} K[1]}{2F\left(e^{-\frac{1}{4} K[1]^2} K[2]\right)} \right) dK[1] \right]$$

✓ **Maple** : cpu = 0.125 (sec), leaf count = 27

$$\left\{ y(x) = \text{RootOf}\left(c_1 - x + \int^{-Z} \frac{1}{F(-a)} d_{-a}\right) e^{\frac{x^2}{4}} \right\}$$

2.613 ODE No. 613

$$y'(x) = \frac{x^2 F\left(\frac{y(x)-x \log(x)}{x}\right) + y(x) + x}{x}$$

✓ **Mathematica** : cpu = 0.349883 (sec), leaf count = 226

Solve
$$\left[\int_1^{y(x)} - \frac{x F\left(\frac{K[2]-x \log(x)}{x}\right) \int_1^x \left(-\frac{K[2] F'\left(\frac{K[2]-K[1] \log(K[1])}{K[1]}\right)}{F\left(\frac{K[2]-K[1] \log(K[1])}{K[1]}\right)^2 K[1]^3} - \frac{F'\left(\frac{K[2]-K[1] \log(K[1])}{K[1]}\right)}{F\left(\frac{K[2]-K[1] \log(K[1])}{K[1]}\right)^2 K[1]^2} + \frac{1}{F\left(\frac{K[2]-K[1] \log(K[1])}{K[1]}\right) K[1]^2} \right) dK[1] + 1}{x F\left(\frac{K[2]-x \log(x)}{x}\right)} dK[2] \right]$$

✓ **Maple** : cpu = 0.105 (sec), leaf count = 23

$$\left\{ y(x) = \left(\text{RootOf}\left(c_1 - x + \int^{-Z} \frac{1}{F(-a)} d_{-a}\right) + \ln(x) \right) x \right\}$$

2.614 ODE No. 614

$$y'(x) = \frac{(a-1)(a+1)x}{a^2 F\left(-\frac{1}{2} a^2 x^2 + \frac{x^2}{2} + \frac{y(x)^2}{2}\right) - F\left(-\frac{1}{2} a^2 x^2 + \frac{x^2}{2} + \frac{y(x)^2}{2}\right) + y(x)}$$

✓ **Mathematica** : cpu = 0.463008 (sec), leaf count = 177

Solve
$$\left[\int_1^{y(x)} \left(\frac{K[2]}{(a-1)(a+1) F\left(-\frac{1}{2} a^2 x^2 + \frac{x^2}{2} + \frac{K[2]^2}{2}\right)} - \int_1^x \frac{K[1] K[2] F'\left(-\frac{1}{2} a^2 K[1]^2 + \frac{K[1]^2}{2} + \frac{K[2]^2}{2}\right)}{F\left(-\frac{1}{2} a^2 K[1]^2 + \frac{K[1]^2}{2} + \frac{K[2]^2}{2}\right)^2} dK[1] + 1 \right) dK[2] + \int_1^x \frac{K[1] K[2] F'\left(-\frac{1}{2} a^2 K[1]^2 + \frac{K[1]^2}{2} + \frac{K[2]^2}{2}\right)}{F\left(-\frac{1}{2} a^2 K[1]^2 + \frac{K[1]^2}{2} + \frac{K[2]^2}{2}\right)^2} dK[1] \right]$$

✓ **Maple** : cpu = 0.393 (sec), leaf count = 60

$$\left\{ -c_1 + \frac{\int^{-a^2 x^2 + x^2 + y(x)^2} \frac{1}{F\left(\frac{-a}{2}\right)} d_{-a}}{2a^4 - 4a^2 + 2} + \frac{y(x)}{(a-1)(a+1)} = 0 \right\}$$

2.615 ODE No. 615

$$y'(x) = \frac{y(x)}{x(y(x)F(xy(x)) - 1)}$$

✓ **Mathematica** : cpu = 0.310851 (sec), leaf count = 77

Solve
$$\left[\int_1^{y(x)} \left(- \int_1^x \frac{F'(K[1]K[2])}{F(K[1]K[2])^2} dK[1] - \frac{1}{F(xK[2])K[2]} + 1 \right) dK[2] + \int_1^x - \frac{1}{F(K[1]y(x))K[1]} dK[1] = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.125 (sec), leaf count = 26

$$\left\{-c_1+\int^{xy(x)}\frac{1}{\_aF(\_a)}d\_a-y(x)=0\right\}$$

2.616 ODE No. 616

$$y'(x)=\frac{F(x(xy(x)-1))-2x^3y(x)+x^2}{x^4}$$

✓ **Mathematica** : cpu = 0.58307 (sec), leaf count = 177

$$\text{Solve}\left[\int_1^{y(x)}-\frac{x^2+F(x(xK[2]-1))\int_1^x\left(\frac{2K[2]F'(K[1](K[1]K[2]-1))K[1]^3}{F(K[1](K[1]K[2]-1))^2}-\frac{F'(K[1](K[1]K[2]-1))K[1]^2}{F(K[1](K[1]K[2]-1))^2}-\frac{2K[1]}{F(K[1](K[1]K[2]-1))}\right)dK[1]}{F(x(xK[2]-1))}dK[2]$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 26

$$\left\{y(x)=\frac{x+\text{RootOf}\left(c_1x+x\left(\int^{-Z}\frac{1}{F(\_a)}d\_a\right)+1\right)}{x^2}\right\}$$

2.617 ODE No. 617

$$y'(x)=\frac{1}{9}e^{-\frac{3x^2}{2}}xy(x)^2F\left(\frac{e^{\frac{3x^2}{2}}(y(x)+3)}{3y(x)}\right)$$

✓ **Mathematica** : cpu = 1.23709 (sec), leaf count = 615

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{9e^{\frac{3x^2}{2}}-F\left(\frac{e^{\frac{3x^2}{2}}(K[2]+3)}{3K[2]}\right)}{3\left(\left(9e^{\frac{3x^2}{2}}-F\left(\frac{e^{\frac{3x^2}{2}}(K[2]+3)}{3K[2]}\right)\right)K[2]+27e^{\frac{3x^2}{2}}\right)}-\int_1^x\left(-\frac{K[2]\left(\frac{e^{\frac{3K[1]^2}{2}}}{3K[2]}-\frac{e^{\frac{3K[1]^2}{2}}(K[2]+3)}{3K[2]^2}\right)F'\left(\frac{e^{\frac{3K[1]^2}{2}}(K[2]+3)}{3K[2]}\right)K[1]}{-9e^{\frac{3K[1]^2}{2}}K[2]+F\left(\frac{e^{\frac{3K[1]^2}{2}}(K[2]+3)}{3K[2]}\right)K[2]-27e^{\frac{3K[1]^2}{2}}}}+F\left(\frac{e^{\frac{3K[1]^2}{2}}(K[2]+3)}{3K[2]}\right)K[1]\right)dK[2]$$

✓ **Maple** : cpu = 0.243 (sec), leaf count = 47

$$\left\{y(x)=-\frac{3e^{\frac{3x^2}{2}}}{-3\text{RootOf}\left(-x^2+2c_1-18\left(\int^{-Z}\frac{1}{-27\_a+F(\_a)}d\_a\right)\right)+e^{\frac{3x^2}{2}}}\right\}$$

2.618 ODE No. 618

$$y'(x)=\frac{(y(x)+1)(x(y(x)-\log(y(x)+1)-\log(x))+1)}{xy(x)}$$

✓ **Mathematica** : cpu = 0.235213 (sec), leaf count = 25

$$\left\{\left\{y(x)\rightarrow-1-W\left(-\frac{e^{-1+c_1e^x}}{x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.513 (sec), leaf count = 34

$$\left\{y(x)=\frac{-x+e^{c_1e^x-\text{LambertW}\left(-\frac{e^{c_1e^x-1}}{x}\right)-1}}{x}\right\}$$

2.619 ODE No. 619

$$y'(x) = \frac{6y(x)}{-F\left(-\frac{1}{3}y(x)^4 - \frac{y(x)^3}{2} - y(x)^2 - y(x) + x\right) + 8y(x)^4 + 9y(x)^3 + 12y(x)^2 + 6y(x)}$$

✓ **Mathematica** : cpu = 0.837047 (sec), leaf count = 330

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{8K[2]^3}{F\left(-\frac{1}{3}K[2]^4 - \frac{K[2]^3}{2} - K[2]^2 - K[2] + x\right)} - \frac{9K[2]^2}{F\left(-\frac{1}{3}K[2]^4 - \frac{K[2]^3}{2} - K[2]^2 - K[2] + x\right)} - \frac{12K[2]}{F\left(-\frac{1}{3}K[2]^4 - \frac{K[2]^3}{2} - K[2]^2 - K[2] + x\right)}\right]d_a = 0\right]$$

✓ **Maple** : cpu = 0.373 (sec), leaf count = 81

$$\left\{-c_1 + \int_{-b}^{y(x)} \frac{-8\_a^4 - 9\_a^3 - 12\_a^2 - 6\_a + F\left(-\frac{1}{3}\_a^4 - \frac{1}{2}\_a^3 - \_a^2 - \_a + x\right)}{\_aF\left(-\frac{1}{3}\_a^4 - \frac{1}{2}\_a^3 - \_a^2 - \_a + x\right)}d\_a = 0\right\}$$

2.620 ODE No. 620

$$y'(x) = \frac{e^{2F((y(x)-x)(y(x)+x))} + x^2 + 2xy(x) + y(x)^2}{-e^{2F((y(x)-x)(y(x)+x))} + x^2 + 2xy(x) + y(x)^2}$$

✓ **Mathematica** : cpu = 0.936624 (sec), leaf count = 205

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{2K[2]}{-x^2 + e^{2F((K[2]-x)(x+K[2]))} + K[2]^2} - \int_1^x\left(\frac{2K[1](-4e^{2F((K[2]-K[1])(K[1]+K[2]))}F'((K[2]-K[1])(K[1]+K[2]))K[2]}{(K[1]^2 - e^{2F((K[2]-K[1])(K[1]+K[2]))} - K[2]^2)^2}\right)dx\right]d_a = 0\right]$$

✓ **Maple** : cpu = 0.227 (sec), leaf count = 37

$$\left\{y(x) = -x + e^{\text{RootOf}\left(c_1 - \int_1^x e^{-2x\_Z + e^{2\_Z}} \frac{1}{\_a + e^{2F(\_a)}} d\_a\right)}\right\}$$

2.621 ODE No. 621

$$y'(x) = \frac{1}{y(x) + \sqrt{x}}$$

✓ **Mathematica** : cpu = 0.0742469 (sec), leaf count = 445

$$\left\{\left\{y(x) \rightarrow -\sqrt{x} + \frac{1}{\text{Root}\left[\#1^6(16x^3 + 16e^{12c_1}) - 24\#1^4x^2 + 8\#1^3x^{3/2} + 9\#1^2x - 6\#1\sqrt{x} + 1\&, 1\right]}\right\}, \left\{y(x) \rightarrow -\sqrt{x} + \frac{1}{\text{Root}\left[\#1^6(16x^3 + 16e^{12c_1}) - 24\#1^4x^2 + 8\#1^3x^{3/2} + 9\#1^2x - 6\#1\sqrt{x} + 1\&, 1\right]}\right\}\right\}$$

✓ **Maple** : cpu = 0.244 (sec), leaf count = 59

$$\left\{y(x) = \frac{\sqrt{x} \text{RootOf}\left(c_1 - Z^{18} - 9\_Z^6x - 6\_Z^3\sqrt{x} - 1\right)^3 + 1}{\text{RootOf}\left(c_1 - Z^{18} - 9\_Z^6x - 6\_Z^3\sqrt{x} - 1\right)^3}\right\}$$

2.622 ODE No. 622

$$y'(x) = \frac{1}{y(x) + \sqrt{3x+1} + 2}$$

✓ **Mathematica** : cpu = 0.367691 (sec), leaf count = 140

$$\text{Solve}\left[6\sqrt{33} \tanh^{-1}\left(\frac{3y(x) + 7\sqrt{3x+1} + 6}{\sqrt{33}(y(x) + \sqrt{3x+1} + 2)}\right) + 44c_1 = 33\left(\log\left(\frac{-3\sqrt{3x+1}y(x)^2 - 3(3x + 4\sqrt{3x+1} + 1)y(x) + 6x}{2(3x+1)^{3/2}}\right)\right.\right.$$

✓ **Maple** : cpu = 0.214 (sec), leaf count = 77

$$\left\{-c_1 - \frac{6\sqrt{3x+1} \operatorname{arctanh}\left(\frac{6y(x)+3\sqrt{3x+1}+12}{\sqrt{99x+33}}\right)}{\sqrt{99x+33}} + \ln\left(3y(x)^2 - 6x + 12y(x) + (3y(x) + 6)\sqrt{3x+1} + 10\right) = 0\right\}$$

2.623 ODE No. 623

$$y'(x) = \frac{x^2}{x^{3/2} + y(x)}$$

✓ **Mathematica** : cpu = 0.306994 (sec), leaf count = 77

$$\text{Solve}\left[6\sqrt{33} \tanh^{-1}\left(\frac{7x^{3/2} + 3y(x)}{\sqrt{33}(x^{3/2} + y(x))}\right) + 44c_1 = 33\left(\log\left(-\frac{3y(x)}{2x^{3/2}} - \frac{3y(x)^2}{2x^3} + 1\right) + 3\log(x)\right), y(x)\right]$$

✓ **Maple** : cpu = 0.255 (sec), leaf count = 49

$$\left\{-c_1 - \frac{2\sqrt{33} \operatorname{arctanh}\left(\frac{\left(x^{\frac{3}{2}}+2y(x)\right)\sqrt{33}}{11x^{\frac{3}{2}}}\right)}{11} + \ln\left(-2x^3 + 3x^{\frac{3}{2}}y(x) + 3y(x)^2\right) = 0\right\}$$

2.624 ODE No. 624

$$y'(x) = \frac{x^{5/3}}{x^{4/3} + y(x)}$$

✓ **Mathematica** : cpu = 32.1671 (sec), leaf count = 9837

✓ **Maple** : cpu = 33.316 (sec), leaf count = 46

$$\left\{y(x) = \frac{\operatorname{RootOf}\left(-Z^{192} + 12_Z^{176}x^{\frac{4}{3}} + 48_Z^{160}x^{\frac{8}{3}} + 64_Z^{144}x^4 - c_1\right)^{16}}{2} + \frac{x^{\frac{4}{3}}}{2}\right\}$$

2.625 ODE No. 625

$$y'(x) = \frac{1}{2}ix^2\left(-2\sqrt{6y(x)-x^3}+i\right)$$

✓ **Mathematica** : cpu = 0.347914 (sec), leaf count = 56

$$\left\{\left\{y(x) \rightarrow \frac{1}{6}\left(-W\left(ie^{-x^3-1-6c_1}\right)^2-2W\left(ie^{-x^3-1-6c_1}\right)+x^3-1\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.299 (sec), leaf count = 53

$$\left\{2x^3 - c_1 + 2i \arctan\left(\sqrt{-x^3 + 6y(x)}\right) + \ln(x^3 - 6y(x) - 1) - 2i\sqrt{-x^3 + 6y(x)} = 0\right\}$$

2.626 ODE No. 626

$$y'(x) = \frac{x}{\sqrt{x^2+1}+y(x)}$$

✓ **Mathematica** : cpu = 0.244762 (sec), leaf count = 88

$$\text{Solve}\left[\frac{1}{2}\left(\log\left(-\frac{y(x)^2}{x^2+1}-\frac{y(x)}{\sqrt{x^2+1}}+1\right)+\log(x^2+1)\right)=\frac{\tanh^{-1}\left(\frac{3\sqrt{x^2+1}+y(x)}{\sqrt{5}\left(\sqrt{x^2+1}+y(x)\right)}\right)}{\sqrt{5}}+c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.437 (sec), leaf count = 115

$$\left\{-c_1 - \frac{4\sqrt{5} \operatorname{arctanh}\left(\frac{(y(x)+3\sqrt{x^2+1})\sqrt{5}}{5y(x)+5\sqrt{x^2+1}}\right)}{15} - \frac{4\ln\left(\frac{36\sqrt{x^2+1}}{y(x)+\sqrt{x^2+1}}\right)}{3} + \frac{2\ln\left(-\frac{1296(-x^2+y(x)^2+\sqrt{x^2+1})y(x)-1}{11(y(x)+\sqrt{x^2+1})^2}\right)}{3} + \frac{2\ln(x^2+1)}{3} = 0\right\}$$

2.627 ODE No. 627

$$y'(x) = \frac{(y(x)\log(x)-1)^2}{x}$$

✓ **Mathematica** : cpu = 0.954642 (sec), leaf count = 25

$$\left\{\left\{y(x) \rightarrow \frac{\tan(\log(x)+c_1)}{1+\log(x)\tan(\log(x)+c_1)}\right\}\right\}$$

✓ **Maple** : cpu = 0.253 (sec), leaf count = 35

$$\left\{y(x) = \frac{c_1 \sin(\ln(x)) + \cos(\ln(x))}{(c_1 + \ln(x)) \cos(\ln(x)) + (c_1 \ln(x) - 1) \sin(\ln(x))}\right\}$$

2.628 ODE No. 628

$$y'(x) = \frac{1}{3}x\left(3\sqrt{x^2+3y(x)}-2\right)$$

✓ **Mathematica** : cpu = 0.197082 (sec), leaf count = 38

$$\left\{\left\{y(x) \rightarrow \frac{1}{432}\left(81x^4-72x^2-486c_1x^2+16+729c_1^2-216c_1\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.31 (sec), leaf count = 23

$$\left\{\frac{3x^2}{4}+c_1+\frac{2}{3}-\sqrt{x^2+3y(x)}=0\right\}$$

2.629 ODE No. 629

$$y'(x) = \frac{(2y(x)\log(x) - 1)^2}{x}$$

✓ **Mathematica** : cpu = 0.989133 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{\sqrt{2} \left(\sqrt{2} \log(x) - \tan\left(\frac{1}{2}(2\sqrt{2} \log(x) + \sqrt{2} c_1)\right) \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.251 (sec), leaf count = 62

$$\left\{ y(x) = \frac{c_1 \sin(\sqrt{2} \ln(x)) + \cos(\sqrt{2} \ln(x))}{(c_1 \sqrt{2} + 2 \ln(x)) \cos(\sqrt{2} \ln(x)) + (2c_1 \ln(x) - \sqrt{2}) \sin(\sqrt{2} \ln(x))} \right\}$$

2.630 ODE No. 630

$$y'(x) = \frac{e^{bx}}{e^{-bx}y(x) + 1}$$

✓ **Mathematica** : cpu = 0.501585 (sec), leaf count = 101

$$\text{Solve} \left[\frac{1}{2} b \left(\log(-b e^{-2bx} y(x)^2 - b e^{-bx} y(x) + 1) + 2bx \right) = \frac{b \tan^{-1} \left(\frac{(b+2)(-e^{bx}) - b y(x)}{b \sqrt{-\frac{b+4}{b}} (e^{bx} + y(x))} \right)}{\sqrt{-\frac{b+4}{b}}} + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.386 (sec), leaf count = 98

$$\left\{ y(x) = \text{RootOf} \left(\_Z^2 b + \_Z b - e^{\text{RootOf} \left(b \left(\tanh^2 \left(\frac{\sqrt{b^2+4b} (-2bx+2c_1b-\_Z)}{2b} \right) \right) + 4 \left(\tanh^2 \left(\frac{\sqrt{b^2+4b} (-2bx+2c_1b-\_Z)}{2b} \right) \right) - b - 4 e^{-Z-4} \right) - 1} e^{bx} \right\}$$

2.631 ODE No. 631

$$y'(x) = \frac{1}{2} x^2 \left(2\sqrt{x^3 - 6y(x)} + 1 \right)$$

✓ **Mathematica** : cpu = 0.237145 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{96} (-16x^6 + 8x^3 - 192c_1x^3 - 1 - 576c_1^2 - 48c_1) \right\} \right\}$$

✓ **Maple** : cpu = 0.249 (sec), leaf count = 23

$$\left\{ -x^3 + c_1 - \frac{1}{2} - \sqrt{x^3 - 6y(x)} = 0 \right\}$$

2.632 ODE No. 632

$$y'(x) = \frac{e^x}{e^{-x}y(x) + 1}$$

✓ **Mathematica** : cpu = 0.243452 (sec), leaf count = 65

$$\text{Solve} \left[\frac{1}{2} \log(-e^{-2x} y(x)^2 - e^{-x} y(x) + 1) + x = \frac{\tanh^{-1} \left(\frac{y(x) + 3e^x}{\sqrt{5} (y(x) + e^x)} \right)}{\sqrt{5}} + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.331 (sec), leaf count = 54

$$\left\{ -c_1 + x - \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{2\sqrt{5} e^{-x}y(x)}{5} + \frac{\sqrt{5}}{5}\right)}{5} + \frac{\ln(e^{-2x}y(x)^2 + e^{-x}y(x) - 1)}{2} = 0 \right\}$$

2.633 ODE No. 633

$$y'(x) = \frac{e^{2x/3}}{e^{-2x/3}y(x) + 1}$$

✓ **Mathematica** : cpu = 0.267088 (sec), leaf count = 85

$$\text{Solve}\left[7\left(3\log\left(-\frac{2}{3}e^{-4x/3}y(x)^2 - \frac{2}{3}e^{-2x/3}y(x) + 1\right) + 4x - 9c_1\right) = 6\sqrt{7} \tanh^{-1}\left(\frac{y(x) + 4e^{2x/3}}{\sqrt{7}(y(x) + e^{2x/3})}\right), y(x)\right]$$

✓ **Maple** : cpu = 1.121 (sec), leaf count = 52

$$\left\{y(x) = \text{RootOf}\left(2\_Z^2 + 2\_Z - e^{\text{RootOf}\left(-343\left(\tanh^2\left(\frac{(4c_1-3\_Z-4x)\sqrt{7}}{6}\right)\right)+98e^{-Z}+343}\right)-3}\right)e^{\frac{2x}{3}}\right\}$$

2.634 ODE No. 634

$$y'(x) = \frac{x^5\sqrt{4x^2y(x) + 1} + \frac{1}{2}}{x^3}$$

✓ **Mathematica** : cpu = 0.245539 (sec), leaf count = 33

$$\left\{\left\{y(x) \rightarrow \frac{x^{10} - 8c_1x^6 + 16c_1^2x^2 - 4}{16x^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.338 (sec), leaf count = 26

$$\left\{\frac{x^4}{2} + c_1 - \frac{\sqrt{4x^2y(x) + 1}}{x} = 0\right\}$$

2.635 ODE No. 635

$$y'(x) = \frac{1}{2}x\left(2\sqrt{x^3 - 6y(x)} + x\right)$$

✓ **Mathematica** : cpu = 0.226266 (sec), leaf count = 33

$$\left\{\left\{y(x) \rightarrow \frac{1}{24}(-9x^4 + 4x^3 + 36c_1x^2 - 36c_1^2)\right\}\right\}$$

✓ **Maple** : cpu = 0.277 (sec), leaf count = 22

$$\left\{-\frac{3x^2}{2} + c_1 - \sqrt{x^3 - 6y(x)} = 0\right\}$$

2.636 ODE No. 636

$$y'(x) = y(x)(x^2 - \log(y(x)))$$

✓ **Mathematica** : cpu = 0.142582 (sec), leaf count = 24

$$\left\{\left\{y(x) \rightarrow e^{x^2-2x-2c_1e^{-x}+2}\right\}\right\}$$

✓ **Maple** : cpu = 0.212 (sec), leaf count = 19

$$\{y(x) = e^{c_1e^{-x}+x^2-2x+2}\}$$

2.637 ODE No. 637

$$y'(x) = \frac{e^{-x^2}x}{e^{x^2}y(x) + 1}$$

✓ **Mathematica** : cpu = 11.4054 (sec), leaf count = 59

$$\text{Solve}\left[-\frac{1}{4}\log\left(2e^{2x^2}y(x)^2 + 2e^{x^2}y(x) + 1\right) - \frac{1}{2}\tan^{-1}\left(2e^{x^2}y(x) + 1\right) + \frac{x^2}{2} = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.197 (sec), leaf count = 84

$$\left\{y(x) = -\frac{e^{-x^2}\tan\left(\text{RootOf}\left(2x^2 + 6c_1 - 2\_Z - \ln\left(\frac{81(\tan^2(\_Z))}{10} + \frac{81}{10}\right) + 2\ln\left(\frac{9\tan(\_Z)}{2} - \frac{9}{2}\right)\right)\right)}{\tan\left(\text{RootOf}\left(2x^2 + 6c_1 - 2\_Z - \ln\left(\frac{81(\tan^2(\_Z))}{10} + \frac{81}{10}\right) + 2\ln\left(\frac{9\tan(\_Z)}{2} - \frac{9}{2}\right)\right)\right) - 1}\right\}$$

2.638 ODE No. 638

$$y'(x) = y(x)(\log(\log(y(x))) - \log(x))$$

✓ **Mathematica** : cpu = 0.146057 (sec), leaf count = 41

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{K[1](x\log(x) + \log(K[1]) - x\log(\log(K[1])))} dK[1] = -\log(x) + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.196 (sec), leaf count = 35

$$\left\{-c_1 + \int_{-b}^{y(x)} \frac{1}{(x\ln(x) - x\ln(\ln(\_a)) + \ln(\_a))\_a} d\_a + \ln(x) = 0\right\}$$

2.639 ODE No. 639

$$y'(x) = y(x)(\log(x) - \log(\log(y(x))))^2$$

✓ **Mathematica** : cpu = 0.192856 (sec), leaf count = 53

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{K[1](x\log^2(x) - 2x\log(\log(K[1]))\log(x) + x\log^2(\log(K[1])) - \log(K[1]))} dK[1] = \log(x) + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.247 (sec), leaf count = 50

$$\left\{-c_1 + \int_{-b}^{y(x)} \frac{1}{(x\ln(x)^2 - 2x\ln(x)\ln(\ln(\_a)) + x\ln(\ln(\_a))^2 - \ln(\_a))\_a} d\_a - \ln(x) = 0\right\}$$

2.640 ODE No. 640

$$y'(x) = \frac{y(x)}{\log(\log(y(x))) - \log(x) + 1}$$

✓ **Mathematica** : cpu = 0.310486 (sec), leaf count = 53

$$\text{Solve}\left[\int_1^{y(x)} \frac{\log(x) - \log(\log(K[1])) - 1}{K[1](x + \log(x)\log(K[1]) - \log(K[1]) - \log(K[1])\log(\log(K[1])))} dK[1] = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.367 (sec), leaf count = 47

$$\left\{-c_1 + \int_{-b}^{y(x)} \frac{\ln(x) - \ln(\ln(\_a)) - 1}{(\ln(\_a)\ln(x) - \ln(\_a)\ln(\ln(\_a)) + x - \ln(\_a))\_a} d\_a = 0\right\}$$

2.641 ODE No. 641

$$y'(x) = \frac{x^4 \sqrt{4x^2 y(x) + 1} + \frac{1}{2}}{x^3}$$

✓ **Mathematica** : cpu = 0.250966 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow \frac{4x^8 - 24c_1 x^5 + 36c_1^2 x^2 - 9}{36x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.339 (sec), leaf count = 26

$$\left\{ \frac{2x^3}{3} + c_1 - \frac{\sqrt{4x^2 y(x) + 1}}{x} = 0 \right\}$$

2.642 ODE No. 642

$$y'(x) = \frac{(4ax - y(x)^2)^2}{y(x)}$$

✓ **Mathematica** : cpu = 0.191234 (sec), leaf count = 105

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{4ax - \sqrt{2} \sqrt{a} \tanh\left(\frac{2\sqrt{2} ax - \sqrt{2} c_1}{\sqrt{a}}\right)} \right\}, \left\{ y(x) \rightarrow \sqrt{4ax - \sqrt{2} \sqrt{a} \tanh\left(\frac{2\sqrt{2} ax - \sqrt{2} c_1}{\sqrt{a}}\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.275 (sec), leaf count = 286

$$\left\{ y(x) = \frac{\sqrt{4} \sqrt{\left(c_1 e^{2(-2ax + \sqrt{2} \sqrt{a})x} + e^{-2(2ax + \sqrt{2} \sqrt{a})x}\right)\left(c_1 \left(ax - \frac{\sqrt{2} \sqrt{a}}{4}\right) e^{2(-2ax + \sqrt{2} \sqrt{a})x} + \left(ax + \frac{\sqrt{2} \sqrt{a}}{4}\right) e^{-2(2ax + \sqrt{2} \sqrt{a})x}\right)}}{c_1 e^{2(-2ax + \sqrt{2} \sqrt{a})x} + e^{-2(2ax + \sqrt{2} \sqrt{a})x}}, y \right\}$$

2.643 ODE No. 643

$$y'(x) = \frac{1}{3}x(3x\sqrt{x^2 + 3y(x)} - 2)$$

✓ **Mathematica** : cpu = 0.230038 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{12}(x^6 - 6c_1 x^3 - 4x^2 + 9c_1^2) \right\} \right\}$$

✓ **Maple** : cpu = 0.24 (sec), leaf count = 22

$$\left\{ \frac{x^3}{2} + c_1 - \sqrt{x^2 + 3y(x)} = 0 \right\}$$

2.644 ODE No. 644

$$y'(x) = -\frac{1}{2}x^2\left(ax - 2\sqrt{a(ax^4 + 8y(x))}\right)$$

✓ **Mathematica** : cpu = 0.410196 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{72}a(16x^6 - 9x^4 - 96c_1 x^3 + 144c_1^2) \right\} \right\}$$

✓ **Maple** : cpu = 0.476 (sec), leaf count = 27

$$\left\{ \frac{4a x^3}{3} + c_1 - \sqrt{(a x^4 + 8y(x))a} = 0 \right\}$$

2.645 ODE No. 645

$$y'(x) = y(x)(x - \log(y(x)))$$

✓ **Mathematica** : cpu = 0.0958501 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow e^{x - e^{-x + c_1 - 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.156 (sec), leaf count = 14

$$\{y(x) = e^{c_1 e^{-x} + x - 1}\}$$

2.646 ODE No. 646

$$y'(x) = \frac{\sqrt{x^3 - 6y(x)} + \frac{x^3}{2} + \frac{x^2}{2}}{x + 1}$$

✓ **Mathematica** : cpu = 0.330144 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{6} \left(x^3 - 9 \log^2(x + 1) + 18c_1 \log(x + 1) - 9c_1^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.379 (sec), leaf count = 23

$$\left\{ c_1 - 3 \ln(x + 1) - \sqrt{x^3 - 6y(x)} = 0 \right\}$$

2.647 ODE No. 647

$$y'(x) = \frac{x \left(ay(x)^2 + bx^2 \right)^2}{a^{5/2} y(x)}$$

✓ **Mathematica** : cpu = 0.417613 (sec), leaf count = 115

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{-\frac{bx^2}{a} + \frac{\sqrt{b} \tan\left(\frac{a^{3/2}bx^2 + 2c_1}{a^{9/4}\sqrt{b}}\right)}{\sqrt[4]{a}}}\right\}, \left\{ y(x) \rightarrow \sqrt{-\frac{bx^2}{a} + \frac{\sqrt{b} \tan\left(\frac{a^{3/2}bx^2 + 2c_1}{a^{9/4}\sqrt{b}}\right)}{\sqrt[4]{a}}}\right\} \right\}$$

✓ **Maple** : cpu = 0.302 (sec), leaf count = 460

$$\left\{ y(x) = \frac{\sqrt{\left(-c_1 \left(bx^2 + \sqrt{-\frac{b}{3}} a^{\frac{3}{2}} \right) e^{\frac{\left(bx^{2+2} \sqrt{-\frac{b}{3}} a^{\frac{3}{2}} \right) x^2}}{2a^{\frac{3}{2}}}} + \left(bx^2 - \sqrt{-\frac{b}{3}} a^{\frac{3}{2}} \right) e^{\frac{\left(bx^{2-2} \sqrt{-\frac{b}{3}} a^{\frac{3}{2}} \right) x^2}}{2a^{\frac{3}{2}}}} \right) c_1 e^{\frac{\left(bx^{2+2} \sqrt{-\frac{b}{3}} a^{\frac{3}{2}} \right) x^2}}{2a^{\frac{3}{2}}}} + e^{\frac{\left(bx^{2-2} \sqrt{-\frac{b}{3}} a^{\frac{3}{2}} \right) x^2}}{2a^{\frac{3}{2}}}}}{\left(c_1 e^{\frac{\left(bx^{2+2} \sqrt{-\frac{b}{3}} a^{\frac{3}{2}} \right) x^2}}{2a^{\frac{3}{2}}}} + e^{\frac{\left(bx^{2-2} \sqrt{-\frac{b}{3}} a^{\frac{3}{2}} \right) x^2}}{2a^{\frac{3}{2}}}} \right) a}}$$

2.648 ODE No. 648

$$y'(x) = -\frac{\sqrt{a} x^3 \left(-2\sqrt{ax^4 + 8y(x)} + \sqrt{a} x + \sqrt{a} \right)}{2(x + 1)}$$

✓ **Mathematica** : cpu = 0.58899 (sec), leaf count = 128

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{72} \left(16ax^6 - 48ax^5 + 123ax^4 - 96ax^3 \log(x + 1) - 96ac_1x^3 - 72ax^2 + 144ax^2 \log(x + 1) + 144ac_1x^2 + 432ax + \dots \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.843 (sec), leaf count = 41

$$\left\{-\frac{x^3}{3}+\frac{x^2}{2}-c_1-x+\ln(x+1)+\frac{\sqrt{a\,x^4+8y(x)}}{4\sqrt{a}}=0\right\}$$

2.649 ODE No. 649

$$y'(x)=x\sqrt{x^2+8y(x)-2x+1}-\frac{x}{4}+\frac{1}{4}$$

✓ **Mathematica** : cpu = 0.305663 (sec), leaf count = 37

$$\left\{\left\{y(x)\rightarrow\frac{1}{8}\left(4x^4-x^2-16c_1x^2+2x-1+16c_1^2\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.296 (sec), leaf count = 27

$$\left\{2x^2+c_1+\frac{1}{4}-\sqrt{x^2-2x+8y(x)+1}=0\right\}$$

2.650 ODE No. 650

$$y'(x)=x\sqrt{a^2+2ax+x^2+4y(x)}-\frac{a}{2}-\frac{x}{2}$$

✓ **Mathematica** : cpu = 0.577284 (sec), leaf count = 109

Solve $\left[-\frac{1}{2}\sqrt{a^2+2ax+x^2+4y(x)}-\frac{1}{2}a\log\left(\sqrt{a^2+2ax+x^2+4y(x)}+a+x\right)+\frac{1}{2}a\tanh^{-1}\left(\frac{2a+2x}{2\sqrt{a^2+2ax+x^2+4y(x)}}\right)\right]$

✓ **Maple** : cpu = 0.312 (sec), leaf count = 28

$$\left\{x^2+c_1+\frac{1}{2}-\sqrt{a^2+2ax+x^2+4y(x)}=0\right\}$$

2.651 ODE No. 651

$$y'(x)=\frac{y(x)\left(x^2+\log(y(x))\right)}{x}$$

✓ **Mathematica** : cpu = 0.112369 (sec), leaf count = 16

$$\left\{\left\{y(x)\rightarrow e^{x^2+2c_1x}\right\}\right\}$$

✓ **Maple** : cpu = 0.118 (sec), leaf count = 13

$$\{y(x)=e^{x^2}e^{c_1x}\}$$

2.652 ODE No. 652

$$y'(x)=\frac{x\sqrt{4ax-y(x)^2}+2a}{y(x)}$$

✓ **Mathematica** : cpu = 1.34762 (sec), leaf count = 101

$$\left\{\left\{y(x)\rightarrow-\frac{\sqrt{4096a^5x-256a^4x^4+32a^2e^{c_1}x^2-e^{2c_1}}}{32a^2}\right\},\left\{y(x)\rightarrow\frac{\sqrt{4096a^5x-256a^4x^4+32a^2e^{c_1}x^2-e^{2c_1}}}{32a^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.177 (sec), leaf count = 27

$$\left\{-\frac{x^2}{2}-c_1-\sqrt{4ax-y(x)^2}=0\right\}$$

2.653 ODE No. 653

$$y'(x) = x\sqrt{x^2 + 4y(x) - 4x} - \frac{x}{2} + 1$$

✓ **Mathematica** : cpu = 0.843046 (sec), leaf count = 94

$$\text{Solve}\left[-\frac{1}{2}\sqrt{x^2 + 4y(x) - 4x} + \log\left(-\sqrt{x^2 + 4y(x) - 4x} - x + 2\right) - \tanh^{-1}\left(\frac{2x - 4}{2\sqrt{x^2 + 4y(x) - 4x}}\right) + \frac{x^2}{2} - \frac{1}{2}\log(2 - 2y(x))\right]$$

✓ **Maple** : cpu = 0.262 (sec), leaf count = 24

$$\left\{x^2 + c_1 + \frac{1}{2} - \sqrt{x^2 - 4x + 4y(x)} = 0\right\}$$

2.654 ODE No. 654

$$y'(x) = \frac{\sqrt{x^2 + 3y(x)} - \frac{2x^2}{3} - \frac{2x}{3}}{x + 1}$$

✓ **Mathematica** : cpu = 0.371279 (sec), leaf count = 37

$$\left\{\left\{y(x) \rightarrow \frac{1}{12}\left(-4x^2 + 9\log^2(x + 1) - 18c_1\log(x + 1) + 9c_1^2\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.372 (sec), leaf count = 23

$$\left\{c_1 + \frac{3\ln(x + 1)}{2} - \sqrt{x^2 + 3y(x)} = 0\right\}$$

2.655 ODE No. 655

$$y'(x) = \frac{e^{-4x/3}y(x)^3}{e^{-2x/3}y(x) + 1}$$

✓ **Mathematica** : cpu = 14.4572 (sec), leaf count = 82

$$\text{Solve}\left[\frac{3}{2}\log(y(x)) + \frac{1}{28}\left(-21\log(-3y(x)^2 + 2e^{2x/3}y(x) + 2e^{4x/3}) + 6\sqrt{7}\tanh^{-1}\left(\frac{y(x) + 2e^{2x/3}}{\sqrt{7}y(x)}\right) + 28x\right) = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.76 (sec), leaf count = 66

$$\left\{-c_1 + x + \frac{3\sqrt{7}\operatorname{arctanh}\left(\frac{3\sqrt{7}e^{-\frac{2x}{3}}y(x)}{7} - \frac{\sqrt{7}}{7}\right)}{14} + \frac{3\ln\left(e^{-\frac{2x}{3}}y(x)\right)}{2} - \frac{3\ln\left(3e^{-\frac{4x}{3}}y(x)^2 - 2e^{-\frac{2x}{3}}y(x) - 2\right)}{4} = 0\right\}$$

2.656 ODE No. 656

$$y'(x) = \frac{y(x)(x^3 + \log(y(x)))}{x}$$

✓ **Mathematica** : cpu = 0.140765 (sec), leaf count = 20

$$\left\{\left\{y(x) \rightarrow e^{\frac{x^3}{2} + 3c_1x}\right\}\right\}$$

✓ **Maple** : cpu = 0.132 (sec), leaf count = 15

$$\left\{y(x) = e^{c_1x}e^{\frac{x^3}{2}}\right\}$$

2.657 ODE No. 657

$$y'(x) = x^2 \sqrt{x^2 + 8y(x) - 2x + 1} - \frac{x}{4} + \frac{1}{4}$$

✓ **Mathematica** : cpu = 0.297964 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{72} (16x^6 - 96c_1x^3 - 9x^2 + 18x - 9 + 144c_1^2) \right\} \right\}$$

✓ **Maple** : cpu = 0.271 (sec), leaf count = 26

$$\left\{ \frac{4x^3}{3} + c_1 - \sqrt{x^2 - 2x + 8y(x) + 1} = 0 \right\}$$

2.658 ODE No. 658

$$y'(x) = \frac{\sqrt{x^2 + 8y(x) - 2x + 1} - \frac{x^2}{4} + \frac{1}{4}}{x + 1}$$

✓ **Mathematica** : cpu = 0.53068 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{8} (-x^2 + 2x + 16 \log^2(x + 1) - 32c_1 \log(x + 1) - 1 + 16c_1^2) \right\} \right\}$$

✓ **Maple** : cpu = 0.422 (sec), leaf count = 28

$$\left\{ c_1 + 4 \ln(x + 1) - \frac{1}{4} - \sqrt{x^2 - 2x + 8y(x) + 1} = 0 \right\}$$

2.659 ODE No. 659

$$y'(x) = x \sqrt{a^2x^2 + 2abx + 4ay(x) + b^2 - 4c} - \frac{ax}{2} - \frac{b}{2}$$

✓ **Mathematica** : cpu = 0.989316 (sec), leaf count = 164

Solve $\left[-\frac{\sqrt{a^2x^2 + 2abx + 4ay(x) + b^2 - 4c}}{2a} - \frac{b \log(\sqrt{a^2x^2 + 2abx + 4ay(x) + b^2 - 4c} + ax + b)}{2a} + \frac{b \tanh^{-1}\left(\frac{2a^2x}{2a\sqrt{a^2x^2 + 2abx + 4ay(x) + b^2 - 4c}}\right)}{2a} \right]$

✓ **Maple** : cpu = 0.351 (sec), leaf count = 41

$$\left\{ ax^2 + c_1 + \frac{a}{2} - \sqrt{a^2x^2 + 2abx + 4ay(x) + b^2 - 4c} = 0 \right\}$$

2.660 ODE No. 660

$$y'(x) = x^2 \sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{a}{2} - \frac{x}{2}$$

✓ **Mathematica** : cpu = 0.593841 (sec), leaf count = 109

Solve $\left[-\frac{1}{2} \sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{1}{2} a \log(\sqrt{a^2 + 2ax + x^2 + 4y(x)} + a + x) + \frac{1}{2} a \tanh^{-1}\left(\frac{2a + 2x}{2\sqrt{a^2 + 2ax + x^2 + 4y(x)}}\right) \right]$

✓ **Maple** : cpu = 0.322 (sec), leaf count = 29

$$\left\{ \frac{2x^3}{3} + c_1 - \sqrt{a^2 + 2ax + x^2 + 4y(x)} = 0 \right\}$$

2.661 ODE No. 661

$$y'(x) = x^2 \sqrt{a^2 x^2 + 2abx + 4ay(x) + b^2 - 4c} - \frac{ax}{2} - \frac{b}{2}$$

✓ **Mathematica** : cpu = 0.954937 (sec), leaf count = 164

$$\text{Solve} \left[-\frac{\sqrt{a^2 x^2 + 2abx + 4ay(x) + b^2 - 4c}}{2a} - \frac{b \log(\sqrt{a^2 x^2 + 2abx + 4ay(x) + b^2 - 4c} + ax + b)}{2a} + \frac{b \tanh^{-1}\left(\frac{2a^2 x}{2a \sqrt{a^2 x^2 + 2abx + 4ay(x) + b^2 - 4c}}\right)}{2a} \right]$$

✓ **Maple** : cpu = 0.349 (sec), leaf count = 39

$$\left\{ \frac{2a x^3}{3} + c_1 - \sqrt{a^2 x^2 + 2abx + 4ay(x) + b^2 - 4c} = 0 \right\}$$

2.662 ODE No. 662

$$y'(x) = x^2 \sqrt{x^2 - 4y(x) + 2x + 1} + \frac{x}{2} + \frac{1}{2}$$

✓ **Mathematica** : cpu = 0.31609 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{36} (-4x^6 + 24c_1 x^3 + 9x^2 + 18x + 9 - 36c_1^2) \right\} \right\}$$

✓ **Maple** : cpu = 0.301 (sec), leaf count = 26

$$\left\{ -\frac{2x^3}{3} + c_1 - \sqrt{x^2 + 2x - 4y(x) + 1} = 0 \right\}$$

2.663 ODE No. 663

$$y'(x) = \frac{x^2 \sqrt{4ax - y(x)^2} + 2a}{y(x)}$$

✓ **Mathematica** : cpu = 1.7102 (sec), leaf count = 101

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{147456a^7 x - 4096a^6 x^6 + 128a^3 e^{c_1} x^3 - e^{2c_1}}}{192a^3} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{147456a^7 x - 4096a^6 x^6 + 128a^3 e^{c_1} x^3 - e^{2c_1}}}{192a^3} \right\} \right\}$$

✓ **Maple** : cpu = 0.169 (sec), leaf count = 27

$$\left\{ -\frac{x^3}{3} - c_1 - \sqrt{4ax - y(x)^2} = 0 \right\}$$

2.664 ODE No. 664

$$y'(x) = x^2 \sqrt{x^2 + 4y(x) - 4x} - \frac{x}{2} + 1$$

✓ **Mathematica** : cpu = 1.02378 (sec), leaf count = 94

$$\text{Solve} \left[\frac{x^3}{3} - \frac{1}{2} \sqrt{x^2 + 4y(x) - 4x} + \log(-\sqrt{x^2 + 4y(x) - 4x} - x + 2) - \tanh^{-1}\left(\frac{2x - 4}{2\sqrt{x^2 + 4y(x) - 4x}}\right) - \frac{1}{2} \log(2 - 2y(x)) = 0 \right]$$

✓ **Maple** : cpu = 0.243 (sec), leaf count = 25

$$\left\{ \frac{2x^3}{3} + c_1 - \sqrt{x^2 - 4x + 4y(x)} = 0 \right\}$$

2.665 ODE No. 665

$$y'(x) = -\frac{\sqrt{a}(-2\sqrt{ax^4 + 8y(x)} + \sqrt{a}x^4 + \sqrt{a}x^3)}{2(x+1)}$$

✓ **Mathematica** : cpu = 0.532765 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{8}(-ax^4 + 16a \log^2(x+1) - 32ac_1 \log(x+1) + 16ac_1^2) \right\} \right\}$$

✓ **Maple** : cpu = 0.694 (sec), leaf count = 28

$$\left\{ -c_1 + \ln(x+1) - \frac{\sqrt{ax^4 + 8y(x)}}{4\sqrt{a}} = 0 \right\}$$

2.666 ODE No. 666

$$y'(x) = y(x)(x^3 + x^2 - \log(y(x)) + 1)$$

✓ **Mathematica** : cpu = 0.167724 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow e^{x^3 - 2x^2 + 4x - c_1 e^{-x} - 3} \right\} \right\}$$

✓ **Maple** : cpu = 0.249 (sec), leaf count = 24

$$\{y(x) = e^{x^3 + c_1 e^{-x} - 2x^2 + 4x - 3}\}$$

2.667 ODE No. 667

$$y'(x) = \frac{e^{-2bx}y(x)^3}{e^{-bx}y(x) + 1}$$

✓ **Mathematica** : cpu = 1.15861 (sec), leaf count = 90

$$\text{Solve} \left[\frac{\log(y(x))}{b} + \frac{1}{2} \left(-\frac{\log(y(x)^2 - be^{bx}(e^{bx} + y(x)))}{b} + \frac{2 \tanh^{-1} \left(\frac{\sqrt{\frac{b}{b+4}}(2e^{bx} + y(x))}{y(x)} \right)}{\sqrt{b}\sqrt{b+4}} + 2x \right) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.303 (sec), leaf count = 82

$$\left\{ bx - \frac{b \operatorname{arctanh} \left(\frac{-2e^{-bx}y(x)+b}{\sqrt{b^2+4b}} \right)}{\sqrt{b^2+4b}} - c_1 + \ln(e^{-bx}y(x)) - \frac{\ln(-be^{-bx}y(x) + e^{-2bx}y(x)^2 - b)}{2} = 0 \right\}$$

2.668 ODE No. 668

$$y'(x) = \frac{e^{-2x}y(x)^3}{e^{-x}y(x) + 1}$$

✓ **Mathematica** : cpu = 0.756247 (sec), leaf count = 78

$$\text{Solve} \left[\log(y(x)) + y(x)^2 \left(\frac{x}{y(x)^2} - \frac{\log(-y(x)^2 + e^x y(x) + e^{2x})}{2y(x)^2} + \frac{\tanh^{-1} \left(\frac{y(x) + 2e^x}{\sqrt{5}y(x)} \right)}{\sqrt{5}y(x)^2} \right) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.86 (sec), leaf count = 58

$$\left\{ y(x) = e^{\operatorname{RootOf} \left(10c_1 - 10_Z - 10x + 2\sqrt{5} \operatorname{arctanh} \left(\frac{(-2e^{-Z} + e^x)\sqrt{5}e^{-x}}{5} \right) + 5 \ln(e^{2-Z} - e^{2x} - e^{-Z+x}) \right)} \right\}$$

2.669 ODE No. 669

$$y'(x) = \frac{e^x \left(3e^x - 2y(x)^{3/2} \right)^2}{4\sqrt{y(x)}}$$

✓ **Mathematica** : cpu = 0.833809 (sec), leaf count = 264

$$\left\{ \left\{ y(x) \rightarrow \frac{\left(-2e^{3e^x} + 3e^{x+3e^x} + 3e^{x+3c_1} + 2e^{3c_1} \right)^{2/3}}{\sqrt[3]{4e^{6e^x} + 8e^{3e^x+3c_1} + 4e^{6c_1}}} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt[3]{-1} \left(-2e^{3e^x} + 3e^{x+3e^x} + 3e^{x+3c_1} + 2e^{3c_1} \right)^{2/3}}{\sqrt[3]{4e^{6e^x} + 8e^{3e^x+3c_1} + 4e^{6c_1}}} \right\}, \left\{ y(x) \rightarrow \frac{\left(-2e^{3e^x} + 3e^{x+3e^x} + 3e^{x+3c_1} + 2e^{3c_1} \right)^{2/3}}{\sqrt[3]{4e^{6e^x} + 8e^{3e^x+3c_1} + 4e^{6c_1}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.195 (sec), leaf count = 72

$$\left\{ \frac{\left(2e^x y(x)^{\frac{3}{2}} - 2e^x - 3e^{2x} \right) e^{-\frac{3e^x}{2} - \frac{9e^{2x}}{8}} e^{-\frac{3e^x}{2} + \frac{9e^{2x}}{8}}}{2e^x y(x)^{\frac{3}{2}} + 2e^x - 3e^{2x}} + c_1 = 0 \right\}$$

2.670 ODE No. 670

$$y'(x) = \frac{1}{2}ixy(x) \left(-2\sqrt{4\log(a) - x^2 + 4\log(y(x))} + i \right)$$

✓ **Mathematica** : cpu = 0.590861 (sec), leaf count = 62

$$\left\{ \left\{ y(x) \rightarrow \exp \left(\frac{1}{4} \left(-4\log(a) - W \left(ie^{-x^2-1-4c_1} \right)^2 - 2W \left(ie^{-x^2-1-4c_1} \right) + x^2 - 1 \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.365 (sec), leaf count = 70

$$\left\{ -\frac{ix^2}{2} - c_1 + \frac{\arctan \left(\sqrt{-x^2 + 4\log(a) + 4\log(y(x))} \right)}{2} - \frac{i\ln \left(x^2 - 4\log(a) - 4\log(y(x)) - 1 \right)}{4} - \frac{\sqrt{-x^2 + 4\log(a) + 4\log(y(x))}}{2} \right\}$$

2.671 ODE No. 671

$$y'(x) = \frac{\left(xy(x)^2 + 1 \right)^2}{x^4 y(x)}$$

✓ **Mathematica** : cpu = 0.409398 (sec), leaf count = 192

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{-\frac{2}{x} + \sqrt{2}} e^{\frac{2\sqrt{2}(1+c_1x)}{x}} - \frac{2e^{\frac{2\sqrt{2}(1+c_1x)}{x}}}{x} - \sqrt{2}}{\sqrt{2 + 2e^{\frac{2\sqrt{2}(1+c_1x)}{x}}}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{-\frac{2}{x} + \sqrt{2}} e^{\frac{2\sqrt{2}(1+c_1x)}{x}} - \frac{2e^{\frac{2\sqrt{2}(1+c_1x)}{x}}}{x} - \sqrt{2}}{\sqrt{2 + 2e^{\frac{2\sqrt{2}(1+c_1x)}{x}}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.273 (sec), leaf count = 237

$$\left\{ y(x) = -\frac{\sqrt{2} \sqrt{-\left(c_1(\sqrt{2}x + 2)e^{\frac{-\sqrt{2}x-1}{x^2}} + (-\sqrt{2}x + 2)e^{\frac{\sqrt{2}x-1}{x^2}} \right) \left(c_1e^{\frac{-\sqrt{2}x-1}{x^2}} + e^{\frac{\sqrt{2}x-1}{x^2}} \right) x}}{2 \left(c_1e^{\frac{-\sqrt{2}x-1}{x^2}} + e^{\frac{\sqrt{2}x-1}{x^2}} \right) x}, y(x) = \frac{\sqrt{2} \sqrt{-\left(c_1(\sqrt{2}x + 2)e^{\frac{-\sqrt{2}x-1}{x^2}} + (-\sqrt{2}x + 2)e^{\frac{\sqrt{2}x-1}{x^2}} \right) \left(c_1e^{\frac{-\sqrt{2}x-1}{x^2}} + e^{\frac{\sqrt{2}x-1}{x^2}} \right) x}}{2 \left(c_1e^{\frac{-\sqrt{2}x-1}{x^2}} + e^{\frac{\sqrt{2}x-1}{x^2}} \right) x} \right\}$$

2.672 ODE No. 672

$$y'(x) = \frac{x^2(\sqrt{4y(x)^3 - 9x^4} + 3x)}{y(x)^2}$$

✓ **Mathematica** : cpu = 2.20507 (sec), leaf count = 4512

Solve

$$\left[\int_1^x \left(-\frac{24\sqrt{4y(x)^3 - 9K[1]^4} y(x)^9}{(9K[1]^4 - 4y(x)^3)(4y(x)^9 - 729)} + \frac{16K[1]^2\sqrt{4y(x)^3 - 9K[1]^4} y(x)^9}{(4K[1]^6 + 9K[1]^4 + 16K[1]^3 - 4y(x)^3 + 16)(4y(x)^9 - 729)} + \frac{1}{(4K[1]^6 + 9K[1]^4 + 16K[1]^3 - 4y(x)^3 + 16)} \right) dx \right]$$

✓ **Maple** : cpu = 0.195 (sec), leaf count = 36

$$\left\{ -\frac{x^3}{3} - c_1 + \int_{-b}^{y(x)} \frac{-a^2}{\sqrt{-9x^4 + 4a^3}} d_{-a} = 0 \right\}$$

2.673 ODE No. 673

$$y'(x) = \frac{\frac{1}{2}x^2 \cos(2y(x)) + \frac{x^2}{2} - \frac{1}{2} \sin(2y(x))}{x}$$

✓ **Mathematica** : cpu = 0.362642 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow \tan^{-1} \left(\frac{2x^3 + 3c_1}{6x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.422 (sec), leaf count = 17

$$\left\{ y(x) = \arctan \left(\frac{x^3 + 6c_1}{3x} \right) \right\}$$

2.674 ODE No. 674

$$y'(x) = \frac{\sqrt{x^2 + 4y(x) - 4x} - \frac{x^2}{2} + \frac{x}{2} + 1}{x + 1}$$

✓ **Mathematica** : cpu = 1.01233 (sec), leaf count = 91

Solve

$$\left[-\frac{1}{2}\sqrt{x^2 + 4y(x) - 4x} + \log \left(-\sqrt{x^2 + 4y(x) - 4x} - x + 2 \right) - \tanh^{-1} \left(\frac{2x - 4}{2\sqrt{x^2 + 4y(x) - 4x}} \right) - \frac{1}{2} \log(2 - 2y(x)) + \log(2) \right]$$

✓ **Maple** : cpu = 0.44 (sec), leaf count = 27

$$\left\{ c_1 + 2 \ln(x + 1) - 1 - \sqrt{x^2 - 4x + 4y(x)} = 0 \right\}$$

2.675 ODE No. 675

$$y'(x) = \frac{ax^4 + ae^x x^3 + ax^3 - x^2 y(x)^2 - e^x x y(x)^2 - x y(x)^2 + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.39486 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow \sqrt{a} x \tanh \left(\frac{1}{6} \sqrt{a} (2x^3 + 3x^2 + 6e^x x - 6e^x + 6c_1) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.073 (sec), leaf count = 37

$$\left\{ y(x) = \sqrt{a} x \tanh \left(\frac{(2x^3 + 3x^2 + 6c_1 + (6x - 6)e^x) \sqrt{a}}{6} \right) \right\}$$

2.676 ODE No. 676

$$y'(x) = \frac{x^6 \sqrt{4x^2 y(x) + 1} + \frac{x}{2} + \frac{1}{2}}{x^3(x+1)}$$

✓ **Mathematica** : cpu = 0.533355 (sec), leaf count = 144

$$\left\{ \left\{ y(x) \rightarrow \frac{9x^{10} - 24x^9 + 52x^8 - 120x^7 + 132x^6 + 72x^6 \log(x+1) - 72c_1x^6 - 144x^5 - 96x^5 \log(x+1) + 96c_1x^5 + 144x^4}{x^3(x+1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.643 (sec), leaf count = 43

$$\left\{ \frac{x^4}{2} - \frac{2x^3}{3} + x^2 + c_1 - 2x + 2 \ln(x+1) - \frac{\sqrt{4x^2 y(x) + 1}}{x} = 0 \right\}$$

2.677 ODE No. 677

$$y'(x) = \frac{ax^4 + ax^3 + ax^3 \log(x+1) - x^2 y(x)^2 - xy(x)^2 + y(x) - xy(x)^2 \log(x+1)}{x}$$

✓ **Mathematica** : cpu = 0.232307 (sec), leaf count = 80

$$\left\{ \left\{ y(x) \rightarrow \sqrt{a} x \tanh \left(\frac{1}{12} (4\sqrt{a} x^3 + 3\sqrt{a} x^2 + 6\sqrt{a} x^2 \log(x+1) + 6\sqrt{a} x - 6\sqrt{a} \log(x+1) + 12\sqrt{a} c_1) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.085 (sec), leaf count = 48

$$\left\{ y(x) = \sqrt{a} x \tanh \left(\frac{(4x^3 + 6x^2 \ln(x+1) + 3x^2 + 12c_1 + 6x - 6 \ln(x+1) + 9)\sqrt{a}}{12} \right) \right\}$$

2.678 ODE No. 678

$$y'(x) = \frac{x^2(2x\sqrt{x^3 - 6y(x)} + x + 1)}{2(x+1)}$$

✓ **Mathematica** : cpu = 0.437452 (sec), leaf count = 101

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{24} (-4x^6 + 12x^5 - 33x^4 + 40x^3 + 24x^3 \log(x+1) + 24c_1x^3 - 36x^2 - 36x^2 \log(x+1) - 36c_1x^2 - 36 \log^2(x+1)) \right\} \right\}$$

✓ **Maple** : cpu = 0.475 (sec), leaf count = 37

$$\left\{ -x^3 + \frac{3x^2}{2} + c_1 - 3x + 3 \ln(x+1) - \frac{1}{2} - \sqrt{x^3 - 6y(x)} = 0 \right\}$$

2.679 ODE No. 679

$$y'(x) = \frac{x^4 + x^3 + x^3 \log(x) + 7x^2 y(x)^2 + 7xy(x)^2 + y(x) + 7xy(x)^2 \log(x)}{x}$$

✓ **Mathematica** : cpu = 0.195112 (sec), leaf count = 59

$$\left\{ \left\{ y(x) \rightarrow \frac{x \tan \left(\frac{1}{12} (4\sqrt{7} x^3 + 3\sqrt{7} x^2 + 6\sqrt{7} x^2 \log(x) + 12\sqrt{7} c_1) \right)}{\sqrt{7}} \right\} \right\}$$

✓ **Maple** : cpu = 0.061 (sec), leaf count = 37

$$\left\{ y(x) = \frac{\sqrt{7} x \tan \left(\frac{(4x^3 + 6x^2 \ln(x) + 3x^2 + 12c_1)\sqrt{7}}{12} \right)}{7} \right\}$$

2.680 ODE No. 680

$$y'(x) = \frac{\sqrt{x^2 - 4y(x) + 2x + 1} + \frac{x^2}{2} + x + \frac{1}{2}}{x + 1}$$

✓ **Mathematica** : cpu = 1.23419 (sec), leaf count = 89

$$\text{Solve}\left[\frac{1}{2}\left(\sqrt{x^2 - 4y(x) + 2x + 1} + \log\left(\sqrt{x^2 - 4y(x) + 2x + 1} + x + 1\right) - \tanh^{-1}\left(\frac{2x + 2}{2\sqrt{x^2 - 4y(x) + 2x + 1}}\right) + 2\log(x + 1)\right)\right]$$

✓ **Maple** : cpu = 0.436 (sec), leaf count = 28

$$\left\{c_1 - 2\ln(x + 1) - \frac{1}{2} - \sqrt{x^2 + 2x - 4y(x) + 1} = 0\right\}$$

2.681 ODE No. 681

$$y'(x) = \frac{ax^2y(x)^2 + axy(x)^2 + axy(x)^2\log\left(\frac{1}{x}\right) + bx^4 + bx^3 + bx^3\log\left(\frac{1}{x}\right) + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.268603 (sec), leaf count = 84

$$\left\{\left\{y(x) \rightarrow \frac{\sqrt{b}x \tan\left(\frac{1}{12}\left(4\sqrt{a}\sqrt{b}x^3 + 9\sqrt{a}\sqrt{b}x^2 - 6\sqrt{a}\sqrt{b}x^2\log(x) + 12\sqrt{a}\sqrt{b}c_1\right)\right)}{\sqrt{a}}\right\}\right\}$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 45

$$\left\{y(x) = \frac{\sqrt{ab}x \tan\left(\frac{\sqrt{ab}\left(4x^3 + 6x^2\ln\left(\frac{1}{x}\right) + 9x^2 + 12c_1\right)}{12}\right)}{a}\right\}$$

2.682 ODE No. 682

$$y'(x) = \frac{2a}{x(-8a^2 + 2axy(x)^2 - xy(x))}$$

✓ **Mathematica** : cpu = 0.34566 (sec), leaf count = 39

$$\text{Solve}\left[\frac{y(x)^2 e^{-4ay(x)}}{8a} - \frac{e^{-4ay(x)}}{2x} = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.356 (sec), leaf count = 28

$$\left\{c_1 + \frac{(-xy(x)^2 + 4a)e^{-4ay(x)}}{x} = 0\right\}$$

2.683 ODE No. 683

$$y'(x) = \frac{y(x)(x^4y(x)\log(x(x + 1)) - x^3\log(x(x + 1)) - 1)}{x}$$

✓ **Mathematica** : cpu = 1.22539 (sec), leaf count = 84

$$\left\{\left\{y(x) \rightarrow \frac{e^{\frac{2x^3}{9} + \frac{x}{3}}}{e^{\frac{x^2}{6} + \frac{1}{18}(4x^2 - 3x + 6)}x + c_1 e^{\frac{x^2}{6}}x\sqrt[3]{x + 1}(x(x + 1))^{\frac{x^3}{3}}}\right\}\right\}$$

✓ **Maple** : cpu = 0.168 (sec), leaf count = 114

$$\left\{ y(x) = \frac{((x+1)x)^{-\frac{x^3}{3}}}{c_1(x+1)^{\frac{1}{3}}x e^{-\frac{2}{9}x^3 + \frac{1}{6}x^2 - \frac{1}{3}x} + x^{-\frac{x^3}{3}+1}(x+1)^{-\frac{x^3}{3}} e^{\frac{i(\operatorname{csgn}(ix) - \operatorname{csgn}(i(x+1)x))(-\operatorname{csgn}(i(x+1)x) + \operatorname{csgn}(ix+i))\pi x^3 \operatorname{csgn}(i(x+1)x))}{6}}} \right\}$$

2.684 ODE No. 684

$$y'(x) = \frac{x^2 \sqrt{x^2 + y(x)^2} + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.137231 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow x \sinh \left(\frac{1}{2} (x^2 + 2c_1) \right) \right\} \right\}$$

✓ **Maple** : cpu = 4.142 (sec), leaf count = 30

$$\left\{ -\frac{x^2}{2} - c_1 - \ln(x) + \ln \left(y(x) + \sqrt{x^2 + y(x)^2} \right) = 0 \right\}$$

2.685 ODE No. 685

$$y'(x) = \frac{x^3 \log((x-1)(x+1)) + y(x) + 7xy(x)^2 \log((x-1)(x+1))}{x}$$

✓ **Mathematica** : cpu = 0.193616 (sec), leaf count = 87

$$\left\{ \left(y(x) \rightarrow \frac{x \tan \left(\frac{1}{2} (-\sqrt{7} x^2 + \sqrt{7} x^2 \log(x-1) + \sqrt{7} x^2 \log(x+1) - \sqrt{7} \log(1-x) - \sqrt{7} \log(x+1) + 2\sqrt{7} c_1) \right)}{\sqrt{7}} \right) \right\}$$

✓ **Maple** : cpu = 0.097 (sec), leaf count = 48

$$\left\{ y(x) = \frac{\sqrt{7} x \tan \left(\frac{(x^2 \ln((x-1)(x+1)) - x^2 + 2c_1 - \ln((x-1)(x+1)) + 1) \sqrt{7}}{2} \right)}{7} \right\}$$

2.686 ODE No. 686

$$y'(x) = \frac{e^{2x^2} xy(x)^3}{e^{x^2} y(x) + 1}$$

✓ **Mathematica** : cpu = 11.2119 (sec), leaf count = 68

$$\text{Solve} \left[\log(y(x)) - 2y(x)^2 \left(\frac{\log(e^{2x^2} y(x)^2 + 2e^{x^2} y(x) + 2)}{4y(x)^2} - \frac{\tan^{-1}(e^{x^2} y(x) + 1)}{2y(x)^2} \right) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.198 (sec), leaf count = 85

$$\left\{ y(x) = \frac{\left(-\tan \left(\text{RootOf} \left(-2x^2 + 6c_1 - 2\_Z - \ln \left(\frac{81(\tan^2(\_Z))}{10} + \frac{81}{10} \right) + 2 \ln \left(\frac{9 \tan(\_Z)}{2} - \frac{9}{2} \right) \right) \right) + 1 \right) e^{-x^2}}{\tan \left(\text{RootOf} \left(-2x^2 + 6c_1 - 2\_Z - \ln \left(\frac{81(\tan^2(\_Z))}{10} + \frac{81}{10} \right) + 2 \ln \left(\frac{9 \tan(\_Z)}{2} - \frac{9}{2} \right) \right) \right)} \right\}$$

2.687 ODE No. 687

$$y'(x) = \frac{x^3 \left(-\log\left(\frac{x+1}{x-1}\right) \right) + y(x) + xy(x)^2 \log\left(\frac{x+1}{x-1}\right)}{x}$$

✓ **Mathematica** : cpu = 0.197236 (sec), leaf count = 130

$$\left\{ \left\{ y(x) \rightarrow \frac{-x^2(x-1)^{x^2} - x(x-1)^{x^2} - x^2(x+1)^{x^2} e^{2x+2c_1} + x(x+1)^{x^2} e^{2x+2c_1}}{-x(x-1)^{x^2} - (x-1)^{x^2} - (x+1)^{x^2} e^{2x+2c_1} + x(x+1)^{x^2} e^{2x+2c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.119 (sec), leaf count = 39

$$\left\{ y(x) = -x \tanh\left(\frac{x^2 \ln\left(\frac{x+1}{x-1}\right)}{2} + c_1 + x - \frac{\ln\left(\frac{x+1}{x-1}\right)}{2} - 1 \right) \right\}$$

2.688 ODE No. 688

$$y'(x) = \frac{e^{\frac{x+1}{x-1}} x^3 + e^{\frac{x+1}{x-1}} xy(x)^2 + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.429828 (sec), leaf count = 82

$$\left\{ \left\{ y(x) \rightarrow x \tan\left(\frac{1}{2} \left(-8e \operatorname{Ei}\left(\frac{2}{x-1} \right) + e^{\frac{x}{x-1} + \frac{1}{x-1}} x^2 + 2e^{\frac{x}{x-1} + \frac{1}{x-1}} x - 3e^{\frac{x}{x-1} + \frac{1}{x-1}} + 2c_1 \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.119 (sec), leaf count = 42

$$\left\{ y(x) = x \tan\left(c_1 + 4e \operatorname{Ei}\left(1, -\frac{2}{x-1} \right) + \frac{(x^2 + 2x - 3)e^{\frac{x+1}{x-1}}}{2} \right) \right\}$$

2.689 ODE No. 689

$$y'(x) = \frac{-e^{x+1} x^3 + e^{x+1} xy(x)^2 + xy(x) - y(x)}{(x-1)x}$$

✓ **Mathematica** : cpu = 0.39464 (sec), leaf count = 60

$$\left\{ \left\{ y(x) \rightarrow -\frac{x(-1 + e^{2e^2 \operatorname{Ei}(x-1) + 2e^{x+1} + 2c_1})}{1 + e^{2e^2 \operatorname{Ei}(x-1) + 2e^{x+1} + 2c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 25

$$\{y(x) = -x \tanh(c_1 - e^2 \operatorname{Ei}(1, -x + 1) + e^{x+1})\}$$

2.690 ODE No. 690

$$y'(x) = \frac{-\frac{x^2}{4} + x^3 \sqrt{x^2 + 8y(x) - 2x + 1} + \frac{1}{4}}{x + 1}$$

✓ **Mathematica** : cpu = 0.61275 (sec), leaf count = 105

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{72} (16x^6 - 48x^5 + 132x^4 - 144x^3 - 96x^3 \log(x+1) - 96c_1 x^3 + 135x^2 + 144x^2 \log(x+1) + 144c_1 x^2 + 18x + 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.527 (sec), leaf count = 40

$$\left\{ \frac{4x^3}{3} - 2x^2 + c_1 + 4x - 4 \ln(x+1) - \sqrt{x^2 - 2x + 8y(x) + 1} = 0 \right\}$$

2.691 ODE No. 691

$$y'(x) = \frac{\frac{1}{2}x^3 \cos(2y(x)) + \frac{x^3}{2} - \frac{1}{2} \sin(2y(x))}{x}$$

✓ **Mathematica** : cpu = 0.345301 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \tan^{-1} \left(\frac{x^4 + 2c_1}{4x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.626 (sec), leaf count = 17

$$\left\{ y(x) = \arctan \left(\frac{x^4 + 8c_1}{4x} \right) \right\}$$

2.692 ODE No. 692

$$y'(x) = \frac{x^3 \sqrt{x^2 + y(x)^2} + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.131146 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow x \sinh \left(\frac{1}{3} (x^3 + 3c_1) \right) \right\} \right\}$$

✓ **Maple** : cpu = 3.915 (sec), leaf count = 30

$$\left\{ -\frac{x^3}{3} - c_1 - \ln(x) + \ln \left(y(x) + \sqrt{x^2 + y(x)^2} \right) = 0 \right\}$$

2.693 ODE No. 693

$$y'(x) = e^{bx} (e^{-3bx} y(x)^3 + e^{-2bx} y(x)^2 + 1)$$

✓ **Mathematica** : cpu = 0.423984 (sec), leaf count = 146

$$\text{Solve} \left[-\frac{1}{3} (9b + 29)^{2/3} \text{RootSum} \left[\#1^3 (9b + 29)^{2/3} - 9\#1b - 3\#1 + (9b + 29)^{2/3} \&, \frac{\log \left(\frac{3e^{-2bx} y(x) + e^{-bx}}{\sqrt[3]{(9b+29)e^{-3bx}}} - \#1 \right)}{\#1^2 (- (9b + 29)^{2/3}) + 3b + 1} \& \right] = \frac{1}{9} x e^{2bx} \right]$$

✓ **Maple** : cpu = 0.11 (sec), leaf count = 40

$$\left\{ y(x) = \text{RootOf} \left(c_1 - x - \left(\int^{-x} -\frac{1}{-a^3 + -a^2 - ab + 1} d_{-a} \right) e^{bx} \right) \right\}$$

2.694 ODE No. 694

$$y'(x) = \frac{x^3 \sqrt{4x^2 y(x) + 1} + \frac{x}{2} + \frac{1}{2}}{x^3 (x + 1)}$$

✓ **Mathematica** : cpu = 0.490293 (sec), leaf count = 66

$$\left\{ \left\{ y(x) \rightarrow \frac{4x^4 - 8x^3 \log(x + 1) - 8c_1 x^3 + 4x^2 \log^2(x + 1) + 4c_1^2 x^2 + 8c_1 x^2 \log(x + 1) - 1}{4x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.527 (sec), leaf count = 30

$$\left\{ c_1 + 2x - 2 \ln(x + 1) - \frac{\sqrt{4x^2 y(x) + 1}}{x} = 0 \right\}$$

2.695 ODE No. 695

$$y'(x) = \frac{x^4 + x^3 + x^2 y(x)^2 + x y(x)^2 + y(x) \log(x-1)}{x \log(x-1)}$$

✓ **Mathematica** : cpu = 0.253253 (sec), leaf count = 34

$$\{ \{ y(x) \rightarrow x \tan(2 \operatorname{Ei}(\log(x-1))) + 3 \operatorname{Ei}(2 \log(x-1)) + \operatorname{Ei}(3 \log(x-1)) + c_1 \} \}$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 39

$$\{ y(x) = x \tan(c_1 - \operatorname{Ei}(1, -3 \ln(x-1))) - 3 \operatorname{Ei}(1, -2 \ln(x-1)) - 2 \operatorname{Ei}(1, -\ln(x-1)) \}$$

2.696 ODE No. 696

$$y'(x) = \frac{e^{x+1} x^3 + 7 e^{x+1} x y(x)^2 + y(x) \log(x-1)}{x \log(x-1)}$$

✓ **Mathematica** : cpu = 1.11497 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow \frac{x \tan\left(\sqrt{7} \int_1^x \frac{e^{K[1]+1} K[1]}{\log(K[1]-1)} dK[1] + \sqrt{7} c_1\right)}{\sqrt{7}} \right\} \right\}$$

✓ **Maple** : cpu = 0.077 (sec), leaf count = 32

$$\left\{ y(x) = \frac{\sqrt{7} x \tan\left(\left(c_1 + e\left(\int \frac{x e^x}{\ln(x-1)} dx\right)\right)\sqrt{7}\right)}{7} \right\}$$

2.697 ODE No. 697

$$y'(x) = e^{2x/3} (e^{-2x} y(x)^3 + e^{-4x/3} y(x)^2 + 1)$$

✓ **Mathematica** : cpu = 0.365443 (sec), leaf count = 114

$$\text{Solve}\left[-\frac{35}{3} \text{RootSum}\left[-35 \#1^3 + 9 \sqrt[3]{35} \#1 - 35 \&, \frac{\log\left(\frac{3 e^{-4x/3} y(x) + e^{-2x/3}}{\sqrt[3]{35} \sqrt[3]{e^{-2x}}} - \#1\right)}{3 \sqrt[3]{35} - 35 \#1^2} \&\right] = \frac{1}{9} 35^{2/3} e^{4x/3} (e^{-2x})^{2/3} x + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.125 (sec), leaf count = 40

$$\left\{ y(x) = \text{RootOf}\left(c_1 - x + 3 \left(\int^{-Z} \frac{1}{3 \_a^3 + 3 \_a^2 - 2 \_a + 3} d\_a\right)\right) e^{\frac{2x}{3}} \right\}$$

2.698 ODE No. 698

$$y'(x) = e^x (e^{-3x} y(x)^3 + e^{-2x} y(x)^2 + 1)$$

✓ **Mathematica** : cpu = 0.377099 (sec), leaf count = 108

$$\text{Solve}\left[-\frac{19}{3} \text{RootSum}\left[-19 \#1^3 + 6 \sqrt[3]{38} \#1 - 19 \&, \frac{\log\left(\frac{3 e^{-2x} y(x) + e^{-x}}{\sqrt[3]{38} \sqrt[3]{e^{-3x}}} - \#1\right)}{2 \sqrt[3]{38} - 19 \#1^2} \&\right] = \frac{1}{9} 38^{2/3} e^{2x} (e^{-3x})^{2/3} x + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.094 (sec), leaf count = 34

$$\left\{ y(x) = \text{RootOf}\left(c_1 - x + \int^{-Z} \frac{1}{\_a^3 + \_a^2 - \_a + 1} d\_a\right) e^x \right\}$$

2.699 ODE No. 699

$$y'(x) = \frac{x(3x^2\sqrt{x^2+3y(x)} - 2x - 2)}{3(x+1)}$$

✓ **Mathematica** : cpu = 0.402997 (sec), leaf count = 101

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{48} (4x^6 - 12x^5 + 33x^4 - 36x^3 - 24x^3 \log(x+1) - 24c_1x^3 + 20x^2 + 36x^2 \log(x+1) + 36c_1x^2 + 36 \log^2(x+1)) \right. \right.$$

✓ **Maple** : cpu = 0.485 (sec), leaf count = 36

$$\left\{ \frac{x^3}{2} - \frac{3x^2}{4} + c_1 + \frac{3x}{2} - \frac{3 \ln(x+1)}{2} - \sqrt{x^2 + 3y(x)} = 0 \right\}$$

2.700 ODE No. 700

$$y'(x) = \frac{1}{xy(x)(xy(x)^2 + x + 1)}$$

✓ **Mathematica** : cpu = 0.170722 (sec), leaf count = 76

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{2xW\left(c_1e^{\frac{1}{2x}-\frac{1}{2}}\right) + x - 1}}{\sqrt{x}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{2xW\left(c_1e^{\frac{1}{2x}-\frac{1}{2}}\right) + x - 1}}{\sqrt{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.128 (sec), leaf count = 62

$$\left\{ y(x) = \frac{\sqrt{\left(2x \operatorname{LambertW}\left(\frac{c_1e^{-\frac{x-1}{2x}}}{2}\right) + x - 1\right)x}}{x}, y(x) = -\frac{\sqrt{\left(2x \operatorname{LambertW}\left(\frac{c_1e^{-\frac{x-1}{2x}}}{2}\right) + x - 1\right)x}}{x} \right\}$$

2.701 ODE No. 701

$$y'(x) = \frac{x^4 + x^4 \log(x) - 2x^2 y(x) - 2x^2 y(x) \log(x) + y(x)^2 + y(x)^2 \log(x) + 2e^x x - 2x - \log(x) - 1}{e^x - 1}$$

✓ **Mathematica** : cpu = 2.112 (sec), leaf count = 88

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp\left(\int_1^x \frac{2(\log(K[5])+1)}{-1+e^{K[5]}} dK[5]\right)}{-\int_1^x \frac{\exp\left(\int_1^{K[6]} \frac{2(\log(K[5])+1)}{-1+e^{K[5]}} dK[5]\right)(\log(K[6])+1)}{-1+e^{K[6]}} dK[6] + c_1} + x^2 + 1 \right\} \right\}$$

✓ **Maple** : cpu = 8.82 (sec), leaf count = 71

$$\left\{ y(x) = \frac{c_1 x^2 - x^2 e^{\int \frac{2 \ln(x)+2}{e^x-1} dx} + c_1 + e^{\int \frac{2 \ln(x)+2}{e^x-1} dx}}{c_1 - e^{\int \frac{2 \ln(x)+2}{e^x-1} dx}} \right\}$$

2.702 ODE No. 702

$$y'(x) = \frac{-x^3 + x^3(-\log(x)) - xy(x)^2 + xy(x) - e^x y(x) - xy(x)^2 \log(x)}{x(x - e^x)}$$

✓ **Mathematica** : cpu = 2.86037 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow x \tan\left(\int_1^x \frac{K[1](\log(K[1]) + 1)}{e^{K[1]} - K[1]} dK[1] + c_1\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.097 (sec), leaf count = 35

$$\left\{y(x) = x \tan\left(c_1 + \int \frac{x}{-x + e^x} dx + \int \frac{x \ln(x)}{-x + e^x} dx\right)\right\}$$

2.703 ODE No. 703

$$y'(x) = \frac{y(x)(x^3 y(x) + x^2 y(x) \log(x) - x^2 - x - x \log(x) + 1)}{(x-1)x}$$

✓ **Mathematica** : cpu = 0.609317 (sec), leaf count = 101

$$\left\{\left\{y(x) \rightarrow -\frac{e^{-\text{Li}_2(x)-x}(1-x)^{-\log(x)}}{(x-1)x\left(-\int_1^x \frac{\exp(-K[1]-\log(1-K[1])(\log(K[1])+1)-\text{Li}_2(K[1]))(K[1]^3+\log(K[1])K[1]^2)}{(K[1]-1)K[1]^2} dK[1] + c_1\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.234 (sec), leaf count = 44

$$\left\{y(x) = \frac{e^{-x} e^{\text{dilog}(x)}}{\left(c_1 + \int -\frac{(x+\ln(x))e^{-x} e^{\text{dilog}(x)}}{(x-1)^2} dx\right)(x-1)x}\right\}$$

2.704 ODE No. 704

$$y'(x) = \frac{2ax^3 y(x)^2 + 2bx^5 - y(x) + xy(x) \log(x)}{x(x \log(x) - 1)}$$

✓ **Mathematica** : cpu = 9.0543 (sec), leaf count = 66

$$\left\{\left\{y(x) \rightarrow \frac{\sqrt{b} x \tan\left(\sqrt{a} \sqrt{b} \int_1^x \frac{2K[1]^3}{K[1] \log(K[1])-1} dK[1] + \sqrt{a} \sqrt{b} c_1\right)}{\sqrt{a}}\right\}\right\}$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 38

$$\left\{y(x) = \frac{\sqrt{ab} x \tan\left(2\sqrt{ab} \left(c_1 + \int \frac{x^3}{x \ln(x)-1} dx\right)\right)}{a}\right\}$$

2.705 ODE No. 705

$$y'(x) = \frac{y(x)(x^4 + x^3 + \log(y(x)) + x)}{x}$$

✓ **Mathematica** : cpu = 0.167566 (sec), leaf count = 30

$$\left\{\left\{y(x) \rightarrow x^x e^{\frac{x^4}{3} + \frac{x^3}{2} + c_1 x}\right\}\right\}$$

✓ **Maple** : cpu = 0.214 (sec), leaf count = 24

$$\left\{y(x) = x^x e^{c_1 x} e^{\frac{x^3}{2}} e^{\frac{x^4}{3}}\right\}$$

2.706 ODE No. 706

$$y'(x) = -\frac{1}{8}x(y(x) + 1)^2(-\log(y(x) - 1) + \log(y(x) + 1) + 2\log(x))$$

✓ **Mathematica** : cpu = 40.5333 (sec), leaf count = 610

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{-2\log(x)x^2+\log(K[2]-1)x^2-\log(K[2]+1)x^2-8}{2\left(2\log(x)x^2-\log(K[2]-1)x^2+\log(K[2]+1)x^2+K[2]\left(2\log(x)x^2-\log(K[2]-1)x^2+\log(K[2]+1)x^2-\right.\right.\right.\right.$$

✓ **Maple** : cpu = 0.648 (sec), leaf count = 65

$$\left\{-c_1+\int_{-b}^{y(x)}\frac{1}{2\left(\left(\_a+1\right)x^2\ln(x)-\frac{\left(\_a+1\right)x^2\ln(\_a-1)}{2}+\frac{\left(\_a+1\right)x^2\ln(\_a+1)}{2}+4\_a-4\right)\left(\_a+1\right)}d\_a+\frac{\ln(x)}{8}=0\right\}$$

2.707 ODE No. 707

$$y'(x) = \frac{1}{16}x(y(x) + 1)^2(-\log(y(x) - 1) + \log(y(x) + 1) + 2\log(x))^2$$

✓ **Mathematica** : cpu = 5.56484 (sec), leaf count = 1391

$$\text{Solve}\left[\int_1^x-\frac{1}{4\log^2(K[1])K[1]^2+\log^2(y(x)-1)K[1]^2+\log^2(y(x)+1)K[1]^2-4\log(K[1])\log(y(x)-1)K[1]^2+4\log(K[1])\log(y(x)+1)K[1]^2}\right]$$

✓ **Maple** : cpu = 0.543 (sec), leaf count = 105

$$\left\{-c_1+\int_{-b}^{y(x)}\frac{1}{4\left(\left(\_a+1\right)x^2\ln(x)^2-\left(\_a+1\right)x^2\ln(x)\ln(\_a-1)+\frac{\left(\_a+1\right)x^2\ln(\_a-1)^2}{4}+\frac{\left(\_a+1\right)x^2\ln(\_a+1)^2}{4}+\left(\_a+1\right)\left(\ln(x)-\frac{\ln(\_a-1)}{2}\right)\right)}d\_a\right\}$$

2.708 ODE No. 708

$$y'(x) = \frac{\left(4ax - y(x)^2\right)^3}{y(x)\left(4ax - y(x)^2 - 1\right)}$$

✓ **Mathematica** : cpu = 0.40781 (sec), leaf count = 89

$$\text{Solve}\left[2a\left(x-\frac{\text{RootSum}\left[-\#1^3+2\#1a-2a\&,\frac{\#1a\log\left(-\#1+4ax-y(x)^2\right)-a\log\left(-\#1+4ax-y(x)^2\right)}{2a-3\#1^2}\right]}{2a}\right)=c_1,y(x)\right]$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(diff(y(x),x) = (4*a*x-y(x)^2)^3/(-y(x)^2+4*a*x-1)/y(x),y(x))

2.709 ODE No. 709

$$y'(x) = \frac{x^3\sqrt{4ax - y(x)^2} + 2ax + 2a}{(x + 1)y(x)}$$

✓ **Mathematica** : cpu = 3.54939 (sec), leaf count = 217

$$\left\{\left\{y(x)\rightarrow-\frac{1}{6}\sqrt{144ax-4x^6+12x^5-33x^4+36x^3+24x^3\log(x+1)-24c_1x^3-36x^2-36x^2\log(x+1)+36c_1x^2-36\log(x+1)}\right.\right.$$

✓ **Maple** : cpu = 0.279 (sec), leaf count = 39

$$\left\{-\frac{x^3}{3}+\frac{x^2}{2}-c_1-x+\ln(x+1)-\sqrt{4ax-y(x)^2}=0\right\}$$

2.710 ODE No. 710

$$y'(x)=\frac{2x^3+4x^2y(x)+2xy(x)^2+2x+e^{\frac{1}{x}}-\log(x)}{\log(x)-e^{\frac{1}{x}}}$$

✓ **Mathematica** : cpu = 1.32028 (sec), leaf count = 38

$$\left\{\left\{y(x)\rightarrow -x+\tan\left(\int_1^x-\frac{2K[5]}{e^{\frac{1}{K[5]}}-\log(K[5])}dK[5]+c_1\right)\right\}\right\}$$

✓ **Maple** : cpu = 3.191 (sec), leaf count = 31

$$\left\{y(x)=-x+\tan\left(2c_1-2\left(\int-\frac{x}{-e^{\frac{1}{x}}+\ln(x)}dx\right)\right)\right\}$$

2.711 ODE No. 711

$$y'(x)=\frac{y(x)(-x\log(y(x))-\log(y(x))+1)}{x+1}$$

✓ **Mathematica** : cpu = 0.263492 (sec), leaf count = 28

$$\left\{\left\{y(x)\rightarrow e^{e^{-x-1}\operatorname{Ei}(x+1)+c_1e^{-x}}\right\}\right\}$$

✓ **Maple** : cpu = 0.266 (sec), leaf count = 31

$$\{y(x)=e^{c_1e^{-x}}e^{-\operatorname{Ei}(1,-x-1)e^{-x-1}}\}$$

2.712 ODE No. 712

$$y'(x)=\frac{\frac{x^2}{2}+x^3\sqrt{x^2-4y(x)+2x+1}+x+\frac{1}{2}}{x+1}$$

✓ **Mathematica** : cpu = 0.514569 (sec), leaf count = 105

$$\left\{\left\{y(x)\rightarrow \frac{1}{36}\Big(-4x^6+12x^5-33x^4+36x^3+24x^3\log(x+1)+24c_1x^3-27x^2-36x^2\log(x+1)-36c_1x^2+18x-36\log(x+1)\Big)\right\}\right\}$$

✓ **Maple** : cpu = 0.493 (sec), leaf count = 38

$$\left\{-\frac{2x^3}{3}+x^2+c_1-2x+2\ln(x+1)-\sqrt{x^2+2x-4y(x)+1}=0\right\}$$

2.713 ODE No. 713

$$y'(x)=\frac{-a^2-aby(x)-ab\sqrt{x}+ab+b^2x+b^2}{a\big(a(-y(x))-a\sqrt{x}+a+bx+b\big)}$$

✓ **Mathematica** : cpu = 0.143018 (sec), leaf count = 649

$$\left\{\left\{y(x)\rightarrow -\frac{a\sqrt{x}-a-bx-b}{a}+\frac{1}{a^2\operatorname{Root}\left[\#1^6\left(16x^3+16e^{12c_1}\right)-\frac{24\#1^4x^2}{a^4}+\frac{8\#1^3x^{3/2}}{a^6}+\frac{9\#1^2x}{a^8}-\frac{6\#1\sqrt{x}}{a^{10}}+\frac{1}{a^{12}}\&,1\right]}\right\}\right\},\left\{y(x)\rightarrow \right.$$

✓ **Maple** : cpu = 0.385 (sec), leaf count = 86

$$\left\{y(x)=\frac{3a\sqrt{x}\tanh\left(\text{RootOf}\left(729a^6x^3\left(\tanh^6(\_Z)\right)-2187a^6x^3\left(\tanh^4(\_Z)\right)+2187a^6x^3\left(\tanh^2(\_Z)\right)-729a^6x^3+64c_1e^2\right)\right)}{2a}\right.$$

2.714 ODE No. 714

$$y'(x)=-\frac{y(x)\left(x^3y(x)+x^2y(x)\log(x)-x^2+e^x-x\log(x)-\log\left(\frac{1}{x}\right)\right)}{x\left(e^x-\log\left(\frac{1}{x}\right)\right)}$$

✓ **Mathematica** : cpu = 1.58538 (sec), leaf count = 162

$$\left(\left(y(x)\rightarrow\frac{\exp\left(\int_1^x-\frac{-K[1]^2-\log(K[1])K[1]+e^{K[1]}-\log\left(\frac{1}{K[1]}\right)}{K[1]\left(e^{K[1]}-\log\left(\frac{1}{K[1]}\right)\right)}dK[1]\right)}{-\int_1^x-\frac{\exp\left(\int_1^{K[2]}-\frac{-K[1]^2-\log(K[1])K[1]+e^{K[1]}-\log\left(\frac{1}{K[1]}\right)}{K[1]\left(e^{K[1]}-\log\left(\frac{1}{K[1]}\right)\right)}dK[1]\right)\left(K[2]^3+\log(K[2])K[2]^2\right)}{K[2]\left(e^{K[2]}-\log\left(\frac{1}{K[2]}\right)\right)}dK[2]+c_1}\right)\right)$$

✓ **Maple** : cpu = 0.276 (sec), leaf count = 96

$$\left\{y(x)=\frac{e^{\int\frac{x^2+x\ln(x)-e^x+\ln\left(\frac{1}{x}\right)}{\left(e^x-\ln\left(\frac{1}{x}\right)\right)x}dx}}{c_1+\int\frac{(x+\ln(x))xe^{\int\frac{x^2+x\ln(x)-e^x+\ln\left(\frac{1}{x}\right)}{\left(e^x-\ln\left(\frac{1}{x}\right)\right)x}dx}}{e^x-\ln\left(\frac{1}{x}\right)}dx}\right\}$$

2.715 ODE No. 715

$$y'(x)=\frac{-\frac{x^2}{2}+x^3\sqrt{x^2+4y(x)-4x}+\frac{x}{2}+1}{x+1}$$

✓ **Mathematica** : cpu = 0.608698 (sec), leaf count = 104

$$\left\{\left\{y(x)\rightarrow\frac{1}{36}\left(4x^6-12x^5+33x^4-36x^3-24x^3\log(x+1)-24c_1x^3+27x^2+36x^2\log(x+1)+36c_1x^2+36x+36\log^2\right)\right.\right.$$

✓ **Maple** : cpu = 0.449 (sec), leaf count = 39

$$\left\{\frac{2x^3}{3}-x^2+c_1+2x-2\ln(x+1)-\sqrt{x^2-4x+4y(x)}=0\right\}$$

2.716 ODE No. 716

$$y'(x)=\frac{\sqrt{9x^4-4y(x)^3}+3x^4+3x^3}{(x+1)y(x)^2}$$

✓ **Mathematica** : cpu = 3.30506 (sec), leaf count = 133

$$\left\{\left\{y(x)\rightarrow\left(-\frac{3}{2}\right)^{2/3}\sqrt[3]{x^4-4\log^2(x+1)+8c_1\log(x+1)-4c_1^2}\right\},\left\{y(x)\rightarrow\left(\frac{3}{2}\right)^{2/3}\sqrt[3]{x^4-4\log^2(x+1)+8c_1\log(x+1)-\right.\right.$$

✓ **Maple** : cpu = 0.294 (sec), leaf count = 37

$$\left\{-c_1+\int_{\_b}^{y(x)}\frac{-a^2}{\sqrt{9x^4-4\_a^3}}d\_a-\ln(x+1)=0\right\}$$

2.717 ODE No. 717

$$y'(x) = \frac{\sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{ax}{2} - \frac{a}{2} - \frac{x^2}{2} - \frac{x}{2}}{x + 1}$$

✓ **Mathematica** : cpu = 0.7479 (sec), leaf count = 106

$$\text{Solve}\left[-\frac{1}{2}\sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{1}{2}a \log\left(\sqrt{a^2 + 2ax + x^2 + 4y(x)} + a + x\right) + \frac{1}{2}a \tanh^{-1}\left(\frac{2a + 2x}{2\sqrt{a^2 + 2ax + x^2 + 4y(x)}}\right)\right]$$

✓ **Maple** : cpu = 0.513 (sec), leaf count = 33

$$\left\{c_1 + \frac{a}{2} + 2\ln(x + 1) - \sqrt{a^2 + 2ax + x^2 + 4y(x)} = 0\right\}$$

2.718 ODE No. 718

$$y'(x) = e^{-x^2}x\left(e^{3x^2}y(x)^3 + e^{2x^2}y(x)^2 + 1\right)$$

✓ **Mathematica** : cpu = 0.383342 (sec), leaf count = 127

$$\text{Solve}\left[\frac{11}{3}\text{RootSum}\left[11\#1^3 + 15\sqrt[3]{11}\#1 + 11\&, \frac{\log\left(\frac{3e^{2x^2}xy(x)+e^{x^2}x}{\sqrt[3]{11}\sqrt[3]{e^{3x^2}x^3}} - \#1\right)}{11\#1^2 + 5\sqrt[3]{11}}\&\right] = \frac{11^{2/3}e^{x^2}x^3}{18\sqrt[3]{e^{3x^2}x^3}} + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 44

$$\left\{y(x) = \frac{\left(-11 \text{RootOf}\left(-5x^2 + 6c_1 + 20250\left(\int^{-Z} \frac{1}{121\_a^3+3375\_a-3375}d\_a\right)\right) - 15\right)e^{-x^2}}{45}\right\}$$

2.719 ODE No. 719

$$y'(x) = \frac{e^{-x}y(x)\left(x^2y(x)\log(2x) - e^x - x\log(2x)\right)}{x}$$

✓ **Mathematica** : cpu = 0.500507 (sec), leaf count = 49

$$\left\{\left\{y(x) \rightarrow \frac{2^{e^{-x}}x^{e^{-x}-1}}{2^{e^{-x}}x^{e^{-x}} + c_1e^{\text{Ei}(-x)}}\right\}\right\}$$

✓ **Maple** : cpu = 0.147 (sec), leaf count = 34

$$\left\{y(x) = \frac{1}{c_12^{-e^{-x}}x^{-e^{-x}+1}e^{-\text{Ei}(1,x)} + x}\right\}$$

2.720 ODE No. 720

$$y'(x) = \frac{x^3\left(\sqrt{9x^4 - 4y(x)^3} + 3x + 3\right)}{(x + 1)y(x)^2}$$

✓ **Mathematica** : cpu = 3.74297 (sec), leaf count = 314

$$\left\{\left\{y(x) \rightarrow \sqrt[3]{-x^6 + 3x^5 - 6x^4 + 9x^3 + 6x^3\log(x + 1) + 6c_1x^3 - 9x^2 - 9x^2\log(x + 1) - 9c_1x^2 - 9\log^2(x + 1) + 18x\log(x)}\right\}\right\}$$

✓ **Maple** : cpu = 0.2 (sec), leaf count = 48

$$\left\{-\frac{x^3}{3} + \frac{x^2}{2} - c_1 - x + \int_{-b}^{y(x)} \frac{-a^2}{\sqrt{9x^4 - 4a^3}}d\_a + \ln(x + 1) = 0\right\}$$

2.721 ODE No. 721

$$y'(x) = \frac{1}{36}\sqrt{x}\left(18x^{3/2} + x^6 - 12x^3y(x) + 36y(x)^2\right)$$

✓ **Mathematica** : cpu = 0.133799 (sec), leaf count = 27

$$\left\{\left\{y(x) \rightarrow \frac{x^3}{6} + \frac{1}{-\frac{2x^{3/2}}{3} + c_1}\right\}\right\}$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 19

$$\left\{y(x) = \frac{x^3}{6} + \frac{1}{-\frac{2x^{\frac{3}{2}}}{3} + c_1}\right\}$$

2.722 ODE No. 722

$$y'(x) = -\frac{y(x)^3}{x(-y(x) + 2y(x)\log(x) - 1)}$$

✓ **Mathematica** : cpu = 32.2393 (sec), leaf count = 490

Solve

$$\left[-\sqrt[3]{-2}\left((-2)^{2/3} - \frac{(1-2\log(x))^2\left(-\frac{1}{(2\log(x)-1)^3}\right)^{2/3}(y(x)(5-4\log(x))+2)}{2\sqrt[3]{2}(y(x)(2\log(x)-1)-1)}\right)\left(\frac{y(x)(4\log(x)-5)-2}{\sqrt[3]{2}\sqrt[3]{-\frac{1}{(2\log(x)-1)^3}}(2\log(x)-1)(y(x)(2\log(x)-1)-1)} + (-2)^{2/3}\right)\log(x) \right]$$

✓ **Maple** : cpu = 0.493 (sec), leaf count = 70

$$\left\{y(x) = \frac{e^{\text{RootOf}\left(3c_1e^{-Z}+_Ze^{-Z}-e^{-Z}\ln\left(\frac{e^{-Z}+2}{2x^4}\right)+2\right)}}{(2\ln(x)-1)e^{\text{RootOf}\left(3c_1e^{-Z}+_Ze^{-Z}-e^{-Z}\ln\left(\frac{e^{-Z}+2}{2x^4}\right)+2\right)}+1}\right\}$$

2.723 ODE No. 723

$$y'(x) = \frac{2a}{32a^3x^2 - 16a^2xy(x)^2 + 2ay(x)^4 + y(x)}$$

✓ **Mathematica** : cpu = 0.254167 (sec), leaf count = 663

$$\left\{\left\{y(x) \rightarrow -\frac{\sqrt[3]{-1024a^6c_1^3 + 9216a^5c_1x - 432a^2 + \sqrt{4(-192a^3x - 64a^4c_1^2)^3 + (-1024a^6c_1^3 + 9216a^5c_1x - 432a^2)^2}}}{12\sqrt[3]{2}a} + \dots\right\}\right\}$$

✓ **Maple** : cpu = 0.122 (sec), leaf count = 856

$$\left\{y(x) = \frac{-16c_1^2a^4 - 48a^3x + 8c_1\left(\left(64c_1^3a^4 - 576c_1a^3x + 3\sqrt{-12288c_1^4a^7x + 24576c_1^2a^6x^2 - 12288a^5x^3 + 384c_1^3a^4 - 3456c_1}\right)\right)}{\dots}\right\}$$

2.724 ODE No. 724

$$y'(x) = -\frac{y(x)^3}{x(-y(x) + y(x)\log(x) - 1)}$$

✓ **Mathematica** : cpu = 20.114 (sec), leaf count = 422

Solve

$$\left[\sqrt[3]{-2} \left(\frac{1-y(x)(\log(x)-4)}{\sqrt[3]{2} \sqrt[3]{-\frac{1}{(\log(x)-1)^3}} (\log(x)-1)(y(x)(\log(x)-1)-1)} + (-2)^{2/3} \right) \left(\frac{2^{2/3}(y(x)(\log(x)-4)-1)}{\sqrt[3]{-\frac{1}{(\log(x)-1)^3}} (\log(x)-1)(y(x)(\log(x)-1)-1)} + (-2)^{2/3} \right) \log \left(\frac{\sqrt[3]{-2}}{\sqrt[3]{-\frac{1}{(\log(x)-1)^3}}} \right) \right]$$

✓ **Maple** : cpu = 0.065 (sec), leaf count = 18

$$\left\{ y(x) = \frac{1}{-\text{LambertW}\left(c_1 e^{-2x}\right) + \ln(x) - 2} \right\}$$

2.725 ODE No. 725

$$y'(x) = \frac{x^2 \log(2x) + 2xy(x) \log(2x) + y(x)^2 \log(2x) - \log(x) + \log(2x)}{\log(x)}$$

✓ **Mathematica** : cpu = 0.360593 (sec), leaf count = 19

$$\{ \{ y(x) \rightarrow -x + \tan(\log(2)\text{li}(x) + x + c_1) \} \}$$

✓ **Maple** : cpu = 0.942 (sec), leaf count = 25

$$\{ y(x) = -x - \tan(c_1 - x + \ln(2) \text{Ei}(1, -\ln(x))) \}$$

2.726 ODE No. 726

$$y'(x) = \frac{a^2 - aby(x) - ab\sqrt{x} - b^2x + bc}{a\left(ay(x) + a\sqrt{x} + bx - c \right)}$$

✓ **Mathematica** : cpu = 0.135344 (sec), leaf count = 625

$$\left\{ \left\{ y(x) \rightarrow -\frac{a\sqrt{x} + bx - c}{a} + \frac{1}{a^2 \text{Root}\left[\#1^6 \left(16x^3 + 16e^{12c_1} \right) - \frac{24\#1^4 x^2}{a^4} + \frac{8\#1^3 x^{3/2}}{a^6} + \frac{9\#1^2 x}{a^8} - \frac{6\#1\sqrt{x}}{a^{10}} + \frac{1}{a^{12}} \&, 1 \right]} \right\} \right\}, \left\{ y(x) \rightarrow -\frac{a\sqrt{x} + bx - c}{a} + \frac{1}{a^2 \text{Root}\left[\#1^6 \left(16x^3 + 16e^{12c_1} \right) - \frac{24\#1^4 x^2}{a^4} + \frac{8\#1^3 x^{3/2}}{a^6} + \frac{9\#1^2 x}{a^8} - \frac{6\#1\sqrt{x}}{a^{10}} + \frac{1}{a^{12}} \&, 1 \right]} \right\}$$

✓ **Maple** : cpu = 0.296 (sec), leaf count = 83

$$\left\{ y(x) = \frac{3a\sqrt{x} \tanh\left(\text{RootOf}\left(-729a^6x^3\left(\tanh^6(\_Z)\right) + 2187a^6x^3\left(\tanh^4(\_Z)\right) - 2187a^6x^3\left(\tanh^2(\_Z)\right) + 729a^6x^3 + 64c_1\right)\right)}{2a} \right\}$$

2.727 ODE No. 727

$$y'(x) = \frac{y(x)(y(x) + 2x + 2)}{(x + 1)(\log(y(x)) + 2x - 1)}$$

✓ **Mathematica** : cpu = 0.570013 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow \frac{W\left(e^{-2x}(\log(x + 1) + c_1)\right)}{\log(x + 1) + c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.282 (sec), leaf count = 25

$$\{ y(x) = e^{-2x - \text{LambertW}\left((-c_1 + \ln(x + 1))e^{-2x}\right)} \}$$

2.728 ODE No. 728

$$y'(x) = \frac{y(x)(x^3 + 3y(x)^2)}{x(6y(x)^2 + x)}$$

✓ **Mathematica** : cpu = 0.364065 (sec), leaf count = 72

$$\left\{ \left(y(x) \rightarrow -\frac{\sqrt{x} \sqrt{W\left(\frac{6e^{x^2+2c_1}}{x}\right)}}{\sqrt{6}} \right), \left(y(x) \rightarrow \frac{\sqrt{x} \sqrt{W\left(\frac{6e^{x^2+2c_1}}{x}\right)}}{\sqrt{6}} \right) \right\}$$

✓ **Maple** : cpu = 0.388 (sec), leaf count = 50

$$\left\{ \frac{1}{\frac{6}{x} + \frac{1}{y(x)^2}} = \frac{\left(e^{\text{RootOf}\left(x^2 e^{-Z} + 3c_1 e^{-Z} + \_Z e^{-Z} - e^{-Z} \ln\left(\frac{(e^{-Z}+9)x}{2}\right) + 9\right)} + 9 \right) x}{54} \right\}$$

2.729 ODE No. 729

$$y'(x) = \frac{(x - y(x))y(x)}{x(x - y(x)^3)}$$

✓ **Mathematica** : cpu = 0.317317 (sec), leaf count = 327

$$\left\{ \left(y(x) \rightarrow \frac{\sqrt[3]{2}(-6\log(x) + 6c_1)}{3\sqrt[3]{54x + \sqrt{2916x^2 + 4(-6\log(x) + 6c_1)^3}}} - \frac{\sqrt[3]{54x + \sqrt{2916x^2 + 4(-6\log(x) + 6c_1)^3}}}{3\sqrt[3]{2}} \right), \left(y(x) \rightarrow \frac{(1 + i\sqrt{3})}{\sqrt[3]{2}} \right) \right\}$$

✓ **Maple** : cpu = 0.121 (sec), leaf count = 404

$$\left\{ y(x) = -\frac{-6c_1 + 6\ln(x) + \left(6ic_1 - 6i\ln(x) + i\left(-27x + 3\sqrt{24c_1^3 - 72c_1^2\ln(x) + 72c_1\ln(x)^2 - 24\ln(x)^3 + 81x^2} \right)^{\frac{2}{3}} \right) \sqrt{3} + \left(-27x + 3\sqrt{24c_1^3 - 72c_1^2\ln(x) + 72c_1\ln(x)^2 - 24\ln(x)^3 + 81x^2} \right)^{\frac{2}{3}}}{6\left(-27x + 3\sqrt{24c_1^3 - 72c_1^2\ln(x) + 72c_1\ln(x)^2 - 24\ln(x)^3 + 81x^2} \right)^{\frac{2}{3}}} \right\}$$

2.730 ODE No. 730

$$y'(x) = \frac{e^x(2y(x)^{3/2} - 3e^x)^3}{4\sqrt{y(x)}(2y(x)^{3/2} - 3e^x + 2)}$$

✓ **Mathematica** : cpu = 0.338759 (sec), leaf count = 83

$$\text{Solve}\left[-\frac{2}{3}\text{RootSum}\left[\#1^3 - \#1 - 1\&, \frac{\#1 \log\left(-\#1 + y(x)^{3/2} - \frac{3e^x}{2}\right) + \log\left(-\#1 + y(x)^{3/2} - \frac{3e^x}{2}\right)}{3\#1^2 - 1}\&\right] + e^x - c_1 = 0, y(x)\right]$$

✓ **Maple** : cpu = 1.445 (sec), leaf count = 41

$$\left\{ -c_1 - \left(\int^{y(x)^{\frac{3}{2}} - \frac{3e^x}{2}} \frac{2\_a + 2}{3\_a^3 - 3\_a - 3} d\_a \right) + e^x = 0 \right\}$$

2.731 ODE No. 731

$$y'(x) = \frac{2y(x) + 1}{x(2xy(x)^3 + xy(x)^2 - 2)}$$

✓ **Mathematica** : cpu = 0.541831 (sec), leaf count = 47

$$\text{Solve}\left[\frac{1}{64}\left(-4y(x)^2 + 4y(x) - 2\log(8y(x) + 4) + 3\right) - \frac{1}{4x(2y(x) + 1)} = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.255 (sec), leaf count = 42

$$\left\{y(x) = \frac{e^{\text{RootOf}\left(8c_1xe^{-Z} + 2_Zxe^{-Z} + 3xe^{-Z} - 4xe^{2_Z} + xe^{3_Z} + 16\right)}}{2} - \frac{1}{2}\right\}$$

2.732 ODE No. 732

$$y'(x) = \frac{x^3\sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{ax}{2} - \frac{a}{2} - \frac{x^2}{2} - \frac{x}{2}}{x + 1}$$

✓ **Mathematica** : cpu = 0.884886 (sec), leaf count = 116

$$\text{Solve}\left[-\frac{1}{2}\sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{1}{2}a\log\left(\sqrt{a^2 + 2ax + x^2 + 4y(x)} + a + x\right) + \frac{1}{2}a\tanh^{-1}\left(\frac{a + x}{\sqrt{a^2 + 2ax + x^2 + 4y(x)}}\right) + \right.$$

✓ **Maple** : cpu = 0.551 (sec), leaf count = 43

$$\left\{\frac{2x^3}{3} - x^2 + c_1 + 2x - 2\ln(x + 1) - \sqrt{a^2 + 2ax + x^2 + 4y(x)} = 0\right\}$$

2.733 ODE No. 733

$$y'(x) = \csc(x)\left(x^4\log(2x) - 2x^2y(x)\log(2x) + y(x)^2\log(2x) - \log(2x) + 2x\sin(x)\right)$$

✓ **Mathematica** : cpu = 13.3368 (sec), leaf count = 73

$$\left\{\left\{y(x) \rightarrow \frac{\exp\left(\int_1^x 2\csc(K[5])\log(2K[5])dK[5]\right)}{-\int_1^x \exp\left(\int_1^{K[6]} 2\csc(K[5])\log(2K[5])dK[5]\right)\csc(K[6])\log(2K[6])dK[6] + c_1} + x^2 + 1\right\}\right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

$$\text{dsolve}(\text{diff}(y(x), x) = (2*x*\sin(x) - \ln(2*x) + \ln(2*x)*x^4 - 2*\ln(2*x)*x^2*y(x) + \ln(2*x)*y(x)^2)/\sin(x), y(x))$$

2.734 ODE No. 734

$$y'(x) = \frac{y(x)\left(x^3 - x\log(y(x)) - \log(y(x))\right)}{x + 1}$$

✓ **Mathematica** : cpu = 0.282453 (sec), leaf count = 37

$$\left\{\left\{y(x) \rightarrow \exp\left(-e^{-x-1}\text{Ei}(x + 1) + x^2 - 3x - c_1e^{-x} + 4\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.481 (sec), leaf count = 39

$$\{y(x) = e^4e^{x^2}e^{c_1e^{-x}}e^{e^{-1}\text{Ei}(1, -x-1)e^{-x}}e^{-3x}\}$$

2.735 ODE No. 735

$$y'(x) = \frac{(2y(x)\log(x) - 1)^3}{x(-y(x) + 2y(x)\log(x) - 1)}$$

✓ **Mathematica** : cpu = 1.60855 (sec), leaf count = 573

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{2(2\log(x)K[1]-K[1]-1)}{8\log^3(x)K[1]^3+4\log(x)K[1]^3-2K[1]^3-12\log^2(x)K[1]^2-2K[1]^2+6\log(x)K[1]-1}+2\text{RootSum}\left[2K[1]^2-4\log(x)K[1]+2\right],\sqrt{-2K[1]^2+4\log(x)K[1]-2}\right]dx\right]$$

✓ **Maple** : cpu = 0.119 (sec), leaf count = 78

$$\left\{y(x)=\frac{71\text{RootOf}\left(3c_1-82944\left(\int^{-Z}\frac{1}{5041\_a^3-27648\_a+27648}d\_a\right)-16\ln(x)\right)-120}{(142\ln(x)-71)\text{RootOf}\left(3c_1-82944\left(\int^{-Z}\frac{1}{5041\_a^3-27648\_a+27648}d\_a\right)-16\ln(x)\right)-240\ln(x)+48}\right\}$$

2.736 ODE No. 736

$$y'(x) = \frac{x^4 - 2x^2y(x) + 2x^2 + y(x)^2 + 2x - 1}{x + 1}$$

✓ **Mathematica** : cpu = 0.235856 (sec), leaf count = 31

$$\left\{\left\{y(x)\rightarrow x^2+\frac{(x+1)^2}{-\frac{x^2}{2}-x+c_1}+1\right\}\right\}$$

✓ **Maple** : cpu = 0.256 (sec), leaf count = 43

$$\left\{y(x)=\frac{x^2+c_1\left(x^4+2x^3-x^2-2x-2\right)+1}{c_1\left(x^2+2x\right)+1}\right\}$$

2.737 ODE No. 737

$$y'(x) = \frac{x(2x^3 - 2xy(x) + x - 1)}{x^2 - y(x)}$$

✓ **Mathematica** : cpu = 0.0410636 (sec), leaf count = 36

$$\left\{\left\{\left\{y(x)\rightarrow x^2+\frac{1}{2}\left(1+W\left(-e^{\frac{4x^3}{3}-2x^2-1+c_1}\right)\right)\right\}\right\}\right\}$$

✓ **Maple** : cpu = 0.313 (sec), leaf count = 29

$$\left\{y(x)=x^2+\frac{\text{LambertW}\left(-2c_1e^{-1}e^{-2x^2}e^{\frac{4x^3}{3}}\right)}{2}+\frac{1}{2}\right\}$$

2.738 ODE No. 738

$$y'(x) = \frac{2a}{32a^3 - 16a^2xy(x)^2 + 2ax^2y(x)^4 - x^2y(x)}$$

✓ **Mathematica** : cpu = 0.530228 (sec), leaf count = 1347

$$\left\{\left\{y(x)\rightarrow -\frac{4a+e^{c_1}}{12a}+\frac{\sqrt[3]{4608x^2a^4-128x^3a^3+1152e^{c_1}x^2a^3-96e^{c_1}x^3a^2-432x^3a^2-24e^{2c_1}x^3a-2e^{3c_1}x^3+\sqrt{4(-192xa^4+128x^2a^3+1152e^{c_1}xa^3-96e^{c_1}x^3a^2-432x^3a^2-24e^{2c_1}x^3a-2e^{3c_1}x^3)}}{12a}}\right\}\right\}$$

✓ **Maple** : cpu = 1.059 (sec), leaf count = 1054

$$\left\{y(x)=\frac{-192c_1^2a^3x-x^2-2\left(-216c_1^3a^2x^3+576c_1^2a^3x^2+12c_1\sqrt{\frac{-49152c_1^4a^7+1536c_1^2a^4x+(324a^2c_1^4+3c_1)x^3+(-1728a^3c_1^3-12a)x^2}{x}}ax^2-x\right)}{\right.$$

2.739 ODE No. 739

$$y'(x)=\frac{2y(x)+1}{x\left(2xy(x)^2+xy(x)-2\right)}$$

✓ **Mathematica** : cpu = 0.447879 (sec), leaf count = 39

$$\text{Solve}\left[\frac{1}{8}(-2y(x)+\log(4y(x)+2))-1-\frac{1}{2x(2y(x)+1)}=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.25 (sec), leaf count = 35

$$\left\{y(x)=\frac{\mathrm{e}^{\mathrm{RootOf}\left(2c_1xe^{-Z}-Zxe^{-Z}-xe^{-Z}+xe^{2-Z}+4\right)}}{2}-\frac{1}{2}\right\}$$

2.740 ODE No. 740

$$y'(x)=\frac{x^4-2x^2y(x)^2+y(x)^4+x}{y(x)}$$

✓ **Mathematica** : cpu = 0.125116 (sec), leaf count = 74

$$\left\{\left\{y(x)\rightarrow-\frac{\sqrt{2x^3+2c_1x^2-1}}{\sqrt{2}\sqrt{x+c_1}}\right\},\left\{y(x)\rightarrow\frac{\sqrt{2x^3+2c_1x^2-1}}{\sqrt{2}\sqrt{x+c_1}}\right\}\right\}$$

✓ **Maple** : cpu = 0.096 (sec), leaf count = 72

$$\left\{y(x)=\frac{\sqrt{2}\sqrt{\left(c_1+x\right)\left(2x^3+2c_1x^2-1\right)}}{2c_1+2x},y(x)=-\frac{\sqrt{2}\sqrt{\left(c_1+x\right)\left(2x^3+2c_1x^2-1\right)}}{2c_1+2x}\right\}$$

2.741 ODE No. 741

$$y'(x)=\frac{x\left(ay(x)^2+bx^2\right)^3}{a^{5/2}y(x)\left(ay(x)^2+a+bx^2\right)}$$

✓ **Mathematica** : cpu = 2.05997 (sec), leaf count = 175

$$\text{Solve}\left[\frac{1}{2}\left(x^2-a^{3/2}\text{RootSum}\left[\#1^3b^3+3\#1^2ab^2y(x)^2+\#1a^{3/2}b^2+3\#1a^2by(x)^4+a^{5/2}by(x)^2+a^{5/2}b+a^3y(x)^6\&, \frac{ay(x)^2\log}{3}\right.\right.$$

✓ **Maple** : cpu = 1.13 (sec), leaf count = 246

$$\left\{c_1+\int_{-b}^x\frac{\left(-a^2b+ay(x)^2\right)^3_a}{\left(-a^2a^{\frac{3}{2}}b^2+\left(y(x)^2+1\right)a^{\frac{5}{2}}b+\left(-a^2b+ay(x)^2\right)^3\right)a^3}d_a+\int^{y(x)}\frac{\left(-a^{\frac{3}{2}}b^2x^2+\left(-f^2-1\right)a^{\frac{5}{2}}b-\left(-f^2a+bx^2\right)^3\right)}{a^{\frac{3}{2}}b^2x^2+\left(-f^2+\right.}\right.$$

2.742 ODE No. 742

$$y'(x) = -\frac{(-\cos(y(x)) + x + 1) \cos(y(x))}{(x + 1)(x \sin(y(x)) - 1)}$$

✓ **Mathematica** : cpu = 3.77682 (sec), leaf count = 3913

$$\left\{\left\{y(x) \rightarrow -\sec^{-1}\left(\frac{c_1 x^3}{x^2-1} + \frac{\log(x+1) x^3}{x^2-1} - \frac{c_1^3 x^3}{(x^2-1)\left(c_1^2+2 \log(x+1) c_1+\log ^2(x+1)+1\right)} - \frac{\log ^3(x+1)}{(x^2-1)\left(c_1^2+2 \log(x+1) c_1+\log ^2(x+1)+1\right)}\right.\right.$$

✓ **Maple** : cpu = 1.543 (sec), leaf count = 239

$$\left\{y(x)=\arctan \left(\frac{x+(-c_1+\ln (x+1)) \sqrt{c_1^2-2 c_1 \ln (x+1)-x^2+\ln (x+1)^2+1}}{c_1^2-2 c_1 \ln (x+1)+\ln (x+1)^2+1}, \frac{-c_1 x+x \ln (x+1)-\sqrt{c_1^2-2 c_1 \ln (x+1)-x^2+\ln (x+1)^2+1}}{c_1^2-2 c_1 \ln (x+1)+\ln (x+1)^2+1}\right)\right.$$

2.743 ODE No. 743

$$y'(x) = -\frac{i\left(x^4+8 x^2 y(x)^2+16 y(x)^4+8 i x\right)}{32 y(x)}$$

✓ **Mathematica** : cpu = 0.107878 (sec), leaf count = 406

$$\left\{\left\{y(x) \rightarrow -\frac{\sqrt{2} \sqrt{\left(\operatorname{Bi}\left(\frac{1}{2}(i-\sqrt{3}) x\right)+c_1 \operatorname{Ai}\left(\frac{1}{2}(i-\sqrt{3}) x\right)\right)\left(-\frac{1}{2} x^2\left(\operatorname{Bi}\left(\frac{1}{2}(i-\sqrt{3}) x\right)+c_1 \operatorname{Ai}\left(\frac{1}{2}(i-\sqrt{3}) x\right)\right)\right)+(1+i \sqrt{3}) \operatorname{Bi}'\left(\frac{1}{2}(i-\sqrt{3}) x\right)}{2 \operatorname{Bi}\left(\frac{1}{2}(i-\sqrt{3}) x\right)+2 c_1 \operatorname{Ai}\left(\frac{1}{2}(i-\sqrt{3}) x\right)}\right.\right.$$

✓ **Maple** : cpu = 0.521 (sec), leaf count = 296

$$\left\{y(x)=-\frac{\sqrt{2} \sqrt{\left(c_1 \operatorname{AiryAi}\left(\frac{(i-\sqrt{3}) x}{2}\right)+\operatorname{AiryBi}\left(\frac{(i-\sqrt{3}) x}{2}\right)\right)\left(c_1(1+i \sqrt{3}) \operatorname{AiryAi}\left(1, \frac{(i-\sqrt{3}) x}{2}\right)-\frac{\left(c_1 \operatorname{AiryAi}\left(\frac{(i-\sqrt{3}) x}{2}\right)+\operatorname{AiryBi}\left(\frac{(i-\sqrt{3}) x}{2}\right)\right)}{2}\right)}}{2 c_1 \operatorname{AiryAi}\left(\frac{(i-\sqrt{3}) x}{2}\right)+2 \operatorname{AiryBi}\left(\frac{(i-\sqrt{3}) x}{2}\right)}\right.$$

2.744 ODE No. 744

$$y'(x) = \frac{x}{x^4+2 x^2 y(x)^2+y(x)^4-y(x)}$$

✓ **Mathematica** : cpu = 0.175062 (sec), leaf count = 510

$$\left\{\left\{y(x) \rightarrow \frac{\sqrt[3]{144 c_1 x^2+\sqrt{4\left(12 x^2-4 c_1^2\right)^3+\left(144 c_1 x^2-108+16 c_1^3\right)^2}}-108+16 c_1^3}{6 \sqrt[3]{2}}-\frac{\sqrt[3]{144 c_1 x^2+\sqrt{4\left(12 x^2-4 c_1^2\right)^3+\left(144 c_1 x^2-108+16 c_1^3\right)^2}}}{3 \cdot 2^{2 / 3} \sqrt[3]{144 c_1 x^2+\sqrt{4\left(12 x^2-4 c_1^2\right)^3+\left(144 c_1 x^2-108+16 c_1^3\right)^2}}}\right.\right.$$

✓ **Maple** : cpu = 0.18 (sec), leaf count = 621

$$\left\{y(x)=-\frac{c_1^2-12 x^2+2 c_1\left(-c_1^3-36 c_1 x^2-54+6 \sqrt{24 c_1^2 x^4+48 x^6+3 c_1^3+\left(3 c_1^4+108 c_1\right) x^2+81}\right)^{\frac{1}{3}}+\left(-i c_1^2+12 i x^2+i\left(-c_1^3-36 c_1 x^2-54+6 \sqrt{24 c_1^2 x^4+48 x^6+3 c_1^3+\left(3 c_1^4+108 c_1\right) x^2+81}\right)^{\frac{1}{3}}\right)}{12\left(-c_1^3-36 c_1 x^2-54+6 \sqrt{24 c_1^2 x^4+48 x^6+3 c_1^3+\left(3 c_1^4+108 c_1\right) x^2+81}\right)^{\frac{1}{3}}+\left(-i c_1^2+12 i x^2+i\left(-c_1^3-36 c_1 x^2-54+6 \sqrt{24 c_1^2 x^4+48 x^6+3 c_1^3+\left(3 c_1^4+108 c_1\right) x^2+81}\right)^{\frac{1}{3}}\right)}$$

2.745 ODE No. 745

$$y'(x) = \frac{(y(x) \log(x) - 1)^3}{x(-y(x) + y(x) \log(x) - 1)}$$

✓ **Mathematica** : cpu = 1.495 (sec), leaf count = 546

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{\log(x)K[1]-K[1]-1}{\log^3(x)K[1]^3+\log(x)K[1]^3-K[1]^3-3\log^2(x)K[1]^2-K[1]^2+3\log(x)K[1]-1}+\text{RootSum}\left[K[1]^3-\#1K[1]^2+\#2K[1]-\#3\right]\right)dx,\right]$$

✓ **Maple** : cpu = 0.085 (sec), leaf count = 78

$$\left\{y(x)=\frac{47\text{RootOf}\left(3c_1-27783\left(\int^{-Z}\frac{1}{2209\_a^3-9261\_a+9261}d\_a\right)-7\ln(x)\right)-84}{(47\ln(x)-47)\text{RootOf}\left(3c_1-27783\left(\int^{-Z}\frac{1}{2209\_a^3-9261\_a+9261}d\_a\right)-7\ln(x)\right)-84\ln(x)+21}\right\}$$

2.746 ODE No. 746

$$y'(x) = -\frac{i\left(x^4 + 2x^2y(x)^2 + y(x)^4 + ix\right)}{y(x)}$$

✓ **Mathematica** : cpu = 0.0945512 (sec), leaf count = 274

$$\left\{\left\{y(x)\rightarrow-\frac{\sqrt{2}\sqrt{\left(\text{Bi}\left(2(-1)^{5/6}x\right)+c_1\text{Ai}\left(2(-1)^{5/6}x\right)\right)\left(-2x^2\left(\text{Bi}\left(2(-1)^{5/6}x\right)+c_1\text{Ai}\left(2(-1)^{5/6}x\right)\right)+\left(1+i\sqrt{3}\right)\text{Bi}'\left(2(-1)^{5/6}x\right)\right)}}{2\text{Bi}\left(2(-1)^{5/6}x\right)+2c_1\text{Ai}\left(2(-1)^{5/6}x\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.469 (sec), leaf count = 232

$$\left\{y(x)=\frac{\sqrt{2}\sqrt{\left(c_1\text{AiryAi}\left(-(-8i)^{\frac{1}{3}}x\right)+\text{AiryBi}\left(-(-8i)^{\frac{1}{3}}x\right)\right)\left(c_1\left(1+i\sqrt{3}\right)\text{AiryAi}\left(1,-(-8i)^{\frac{1}{3}}x\right)-2\left(c_1\text{AiryAi}\left(-(-8i)^{\frac{1}{3}}x\right)+\text{AiryBi}\left(-(-8i)^{\frac{1}{3}}x\right)\right)\right)}}{2c_1\text{AiryAi}\left(-(-8i)^{\frac{1}{3}}x\right)+2\text{AiryBi}\left(-(-8i)^{\frac{1}{3}}x\right)}\right\}$$

2.747 ODE No. 747

$$y'(x) = -\frac{y(x) \cot(x)\left(x^2y(x)(-\log(2x)) + x \log(2x) + \tan(x)\right)}{x}$$

✓ **Mathematica** : cpu = 3.21904 (sec), leaf count = 88

$$\left\{\left\{y(x)\rightarrow\frac{\exp\left(\int_1^x\frac{-\cot(K[1])K[1]\log(2K[1])-1}{K[1]}dK[1]\right)}{-\int_1^x\exp\left(\int_1^{K[2]}\frac{-\cot(K[1])K[1]\log(2K[1])-1}{K[1]}dK[1]\right)\cot(K[2])K[2]\log(2K[2])dK[2]+c_1}\right\}\right\}$$

✓ **Maple** : cpu = 0.31 (sec), leaf count = 75

$$\left\{y(x)=\frac{e^{\int\frac{-x\ln(x)-\ln(2)x-\tan(x)}{x\tan(x)}dx}}{c_1+\int-\frac{(\ln(x)+\ln(2))x e^{\int\frac{-x\ln(x)-\ln(2)x-\tan(x)}{x\tan(x)}dx}}{\tan(x)}dx\right\}$$

2.748 ODE No. 748

$$y'(x) = \frac{y(x)(y(x) + x)}{x(y(x)^3 + x)}$$

✓ **Mathematica** : cpu = 0.347576 (sec), leaf count = 285

$$\left\{ \left\{ y(x) \rightarrow \frac{2\sqrt[3]{2}(\log(x) + c_1)}{\sqrt[3]{54x + \sqrt{2916x^2 - 864(\log(x) + c_1)^3}}} + \frac{\sqrt[3]{54x + \sqrt{2916x^2 - 864(\log(x) + c_1)^3}}}{3\sqrt[3]{2}} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt[3]{2}(1 + \sqrt[3]{54x + \sqrt{2916x^2 - 864(\log(x) + c_1)^3}})}{3\sqrt[3]{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.105 (sec), leaf count = 404

$$\left\{ y(x) = -\frac{6c_1 + 6\ln(x) + \left(-6ic_1 - 6i\ln(x) + i\left(27x + 3\sqrt{-24c_1^3 - 72c_1^2\ln(x) - 72c_1\ln(x)^2 - 24\ln(x)^3 + 81x^2} \right)^{\frac{2}{3}} \right)\sqrt{3} + \left(27x + 3\sqrt{-24c_1^3 - 72c_1^2\ln(x) - 72c_1\ln(x)^2 - 24\ln(x)^3 + 81x^2} \right)^{\frac{2}{3}}}{6\left(27x + 3\sqrt{-24c_1^3 - 72c_1^2\ln(x) - 72c_1\ln(x)^2 - 24\ln(x)^3 + 81x^2} \right)^{\frac{2}{3}}}$$

2.749 ODE No. 749

$$y'(x) = \frac{x(x - y(x))^2(y(x) + x)^2}{y(x)}$$

✓ **Mathematica** : cpu = 0.191214 (sec), leaf count = 126

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{x^2 + x^2e^{2x^2+4c_1} - e^{2x^2+4c_1} + 1}}{\sqrt{1 + e^{2x^2+4c_1}}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{x^2 + x^2e^{2x^2+4c_1} - e^{2x^2+4c_1} + 1}}{\sqrt{1 + e^{2x^2+4c_1}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.123 (sec), leaf count = 192

$$\left\{ y(x) = \frac{\sqrt{\left(c_1(x^2 - 1)e^{-\frac{(x^2-2)x^2}{2}} + (x^2 + 1)e^{-\frac{(x^2+2)x^2}{2}} \right) \left(c_1e^{-\frac{(x^2-2)x^2}{2}} + e^{-\frac{(x^2+2)x^2}{2}} \right)}}{c_1e^{-\frac{(x^2-2)x^2}{2}} + e^{-\frac{(x^2+2)x^2}{2}}}, y(x) = -\frac{\sqrt{\left(c_1(x^2 - 1)e^{-\frac{(x^2-2)x^2}{2}} + (x^2 + 1)e^{-\frac{(x^2+2)x^2}{2}} \right) \left(c_1e^{-\frac{(x^2-2)x^2}{2}} + e^{-\frac{(x^2+2)x^2}{2}} \right)}}{c_1e^{-\frac{(x^2-2)x^2}{2}} + e^{-\frac{(x^2+2)x^2}{2}}} \right\}$$

2.750 ODE No. 750

$$y'(x) = \frac{y(x)(x^2 + 3y(x)^2)}{x(6y(x)^2 + x)}$$

✓ **Mathematica** : cpu = 0.345993 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{x}\sqrt{W\left(\frac{6e^{2x+2c_1}}{x}\right)}}{\sqrt{6}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{x}\sqrt{W\left(\frac{6e^{2x+2c_1}}{x}\right)}}{\sqrt{6}} \right\} \right\}$$

✓ **Maple** : cpu = 0.335 (sec), leaf count = 49

$$\left\{ \frac{1}{\frac{6}{x} + \frac{1}{y(x)^2}} = \frac{\left(e^{\text{RootOf}\left(3c_1e^{-Z} + Ze^{-Z} + 2xe^{-Z} - e^{-Z}\ln\left(\frac{e^{-Z}+9}{2}\right)x \right) + 9} \right)x}{54} \right\}$$

2.751 ODE No. 751

$$y'(x) = \frac{y(x)(x^4 + x \log(y(x)) + \log(y(x)))}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.190876 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow (x+1)^x e^{\frac{x^3}{2} - x^2 + c_1 x} \right\} \right\}$$

✓ **Maple** : cpu = 0.162 (sec), leaf count = 26

$$\left\{ y(x) = (x+1)^x e^{c_1 x} e^{-x^2} e^{\frac{x^3}{2}} \right\}$$

2.752 ODE No. 752

$$y'(x) = \frac{\cos(y(x))(x^3 \cos(y(x)) - x - 1)}{(x+1)(x \sin(y(x)) - 1)}$$

✓ **Mathematica** : cpu = 0.242643 (sec), leaf count = 849

$$\left\{ \left\{ y(x) \rightarrow \tan^{-1} \left(\frac{6(2x^4 - 3x^3 + 6x^2 + 6c_1 x - 6 \log(x+1)x + \sqrt{4x^6 - 12x^5 + 33x^4 + 12(2c_1 - 3)x^3 - 36c_1 x^2 + 72c_1 x + 36})}{4x^6 - 12x^5 + 33x^4 + 12(2c_1 - 3)x^3 - 36(c_1 - 1)x^2 + 72c_1 x + 36 \log^2(x+1) + 36} \right) \right\} \right\}$$

✓ **Maple** : cpu = 1.611 (sec), leaf count = 723

$$\left\{ y(x) = \arctan \left(\frac{36x + (-2x^3 + 3x^2 - 6c_1 - 6x + 6 \ln(x+1)) \sqrt{4x^6 - 12x^5 + 33x^4 - 36c_1 x^2 + (24c_1 - 36)x^3 + 36c_1^2 + 72c_1 x + (-36c_1 + 36)x^2 + 36 \ln(x+1)^2}}{4x^6 - 12x^5 + 33x^4 + (24c_1 - 36)x^3 + 36c_1^2 + 72c_1 x + (-36c_1 + 36)x^2 + 36 \ln(x+1)^2} \right) \right\}$$

2.753 ODE No. 753

$$y'(x) = \frac{y(x) \log(y(x))(x^4 \log(y(x)) + x + 1)}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.187137 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow \exp \left(-\frac{12x}{3x^4 - 4x^3 + 6x^2 - 12x + 12 \log(x+1) - 12c_1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.258 (sec), leaf count = 38

$$\left\{ y(x) = e^{-\frac{12x}{3x^4 - 4x^3 + 6x^2 - 12c_1 - 12x + 12 \ln(x+1)}} \right\}$$

2.754 ODE No. 754

$$y'(x) = \frac{x^3 + xy(x)^2 + xy(x) + y(x)^3}{x^2}$$

✓ **Mathematica** : cpu = 0.139996 (sec), leaf count = 47

$$\text{Solve} \left[\text{RootSum} \left[\#1^3 + \#1^2 + 1 \&, \frac{\log\left(\frac{y(x)}{x} - \#1\right)}{3\#1^2 + 2\#1} \& \right] = x + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 26

$$\left\{ y(x) = x \text{RootOf} \left(c_1 + x - \left(\int^{-Z} \frac{1}{-a^3 + -a^2 + 1} d_{-a} \right) \right) \right\}$$

2.755 ODE No. 755

$$y'(x) = \frac{y(x)^{3/2}}{x^2 - 2xy(x) + y(x)^2 + y(x)^{3/2}}$$

✓ **Mathematica** : cpu = 0.363633 (sec), leaf count = 2633

$$\left\{ \left\{ y(x) \rightarrow \frac{2}{3} \left(x + e^{c_1} + 2e^{2c_1} \right) - \frac{1}{3} \sqrt[3]{x^3 + 3e^{c_1}x^2 - 12e^{2c_1}x^2 + 3e^{2c_1}x + 12e^{3c_1}x + 48e^{4c_1}x + e^{3c_1} - 30e^{4c_1} - 96e^{5c_1} - 64e^{6c_1} + \dots} \right. \right.$$

✓ **Maple** : cpu = 0.134 (sec), leaf count = 44

$$\left\{ -c_1 - \frac{2x}{(-x + y(x))\sqrt{y(x)}} + \frac{2\sqrt{y(x)}}{-x + y(x)} + \frac{1}{-x + y(x)} = 0 \right\}$$

2.756 ODE No. 756

$$y'(x) = \frac{x^6 + 2x^3y(x) + x^2y(x)^2 + y(x)^3}{x^4}$$

✓ **Mathematica** : cpu = 0.199371 (sec), leaf count = 95

$$\text{Solve} \left[-\frac{29}{3} \text{RootSum} \left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log \left(\frac{\frac{3y(x)}{x^4} + \frac{1}{x^2}}{\sqrt[3]{29}\sqrt[3]{\frac{1}{x^6}}} - \#1 \right)}{\sqrt[3]{29} - 29\#1^2} \& \right] = \frac{1}{9} 29^{2/3} \left(\frac{1}{x^6} \right)^{2/3} x^5 + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 37

$$\left\{ y(x) = \frac{\left(29 \text{RootOf} \left(3c_1 + x - 81 \left(\int^x \frac{1}{841_a^3 - 27_a + 27} d_a \right) \right) - 3 \right) x^2}{9} \right\}$$

2.757 ODE No. 757

$$y'(x) = \frac{x^3 + 2x^2 - 4xy(x) - 4x - 8}{2x^2 - 8y(x) + 4x - 8}$$

✓ **Mathematica** : cpu = 0.0428142 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} (x^2 + 2x - 4) + 2 \left(1 + W \left(-e^{-\frac{x}{4} - 1 + c_1} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.1 (sec), leaf count = 26

$$\left\{ y(x) = \frac{x^2}{4} + \frac{x}{2} + 2 \text{LambertW} \left(\frac{c_1 e^{-\frac{1}{2}} e^{-\frac{x}{4}}}{2} \right) + 1 \right\}$$

2.758 ODE No. 758

$$y'(x) = \frac{y(x)(x^3y(x) + 2x + 2)}{(x + 1)(\log(y(x)) + 2x - 1)}$$

✓ **Mathematica** : cpu = 0.913132 (sec), leaf count = 459

$$\left\{\left\{y(x) \rightarrow \frac{6W\left(-\frac{1}{6}\sqrt[6]{e^{-12x}\left(2x^3-3x^2+6x-6\log(x+1)+6c_1\right)^6}\right)}{2x^3-3x^2+6x-6\log(x+1)+6c_1}\right\},\left\{y(x) \rightarrow \frac{6W\left(\frac{1}{6}\sqrt[6]{e^{-12x}\left(2x^3-3x^2+6x-6\log(x+1)+6c_1\right)^6}\right)}{2x^3-3x^2+6x-6\log(x+1)+6c_1}\right\}\right\}$$

✓ **Maple** : cpu = 0.278 (sec), leaf count = 41

$$\left\{y(x)=e^{-2x-\text{LambertW}\left(-\frac{\left(-2x^3+3x^2+6c_1-6x+6\ln(x+1)\right)e^{-2x}}{6}\right)}\right\}$$

2.759 ODE No. 759

$$y'(x)=-\frac{ix\left(x^8+18x^4y(x)^2+54ix^2+81y(x)^4\right)}{243y(x)}$$

✓ **Mathematica** : cpu = 0.103573 (sec), leaf count = 498

$$\left\{\left\{y(x) \rightarrow -\frac{\sqrt{\left(Y_{\frac{1}{3}}\left(\left(\frac{2}{9}-\frac{2i}{9}\right)\sqrt{\frac{2}{3}}x^3\right)+c_1J_{\frac{1}{3}}\left(\left(\frac{2}{9}-\frac{2i}{9}\right)\sqrt{\frac{2}{3}}x^3\right)\right)\left((1+i)\sqrt{6}x^3\left(Y_{\frac{4}{3}}\left(\left(\frac{2}{9}-\frac{2i}{9}\right)\sqrt{\frac{2}{3}}x^3\right)+c_1J_{\frac{4}{3}}\left(\left(\frac{2}{9}-\frac{2i}{9}\right)\sqrt{\frac{2}{3}}x^3\right)\right)\right)}{\sqrt{3}x\left(Y_{\frac{1}{3}}\left(\left(\frac{2}{9}-\frac{2i}{9}\right)\sqrt{\frac{2}{3}}x^3\right)+c_1J_{\frac{1}{3}}\left(\left(\frac{2}{9}-\frac{2i}{9}\right)\sqrt{\frac{2}{3}}x^3\right)\right)}\right\},\left\{y(x) \rightarrow \frac{\sqrt{\left(Y_{\frac{1}{3}}\left(\left(\frac{2}{9}-\frac{2i}{9}\right)\sqrt{\frac{2}{3}}x^3\right)+c_1J_{\frac{1}{3}}\left(\left(\frac{2}{9}-\frac{2i}{9}\right)\sqrt{\frac{2}{3}}x^3\right)\right)\left((1+i)\sqrt{6}x^3\left(Y_{\frac{4}{3}}\left(\left(\frac{2}{9}-\frac{2i}{9}\right)\sqrt{\frac{2}{3}}x^3\right)+c_1J_{\frac{4}{3}}\left(\left(\frac{2}{9}-\frac{2i}{9}\right)\sqrt{\frac{2}{3}}x^3\right)\right)\right)}}{\sqrt{3}x\left(Y_{\frac{1}{3}}\left(\left(\frac{2}{9}-\frac{2i}{9}\right)\sqrt{\frac{2}{3}}x^3\right)+c_1J_{\frac{1}{3}}\left(\left(\frac{2}{9}-\frac{2i}{9}\right)\sqrt{\frac{2}{3}}x^3\right)\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.589 (sec), leaf count = 305

$$\left\{y(x)=-\frac{\sqrt{3}\sqrt{\left((1+i)\left(c_1\text{BesselJ}\left(\frac{4}{3},\left(\frac{2}{27}-\frac{2i}{27}\right)\sqrt{6}x^3\right)+\text{BesselY}\left(\frac{4}{3},\left(\frac{2}{27}-\frac{2i}{27}\right)\sqrt{6}x^3\right)\right)\sqrt{6}x^3-9c_1\left(\frac{x^6}{27}+i\right)\text{BesselJ}\left(\frac{1}{3},\left(\frac{2}{27}-\frac{2i}{27}\right)\sqrt{6}x^3\right)\right)}}{3\left(c_1\text{BesselJ}\left(\frac{1}{3},\left(\frac{2}{27}-\frac{2i}{27}\right)\sqrt{6}x^3\right)+\text{BesselY}\left(\frac{1}{3},\left(\frac{2}{27}-\frac{2i}{27}\right)\sqrt{6}x^3\right)\right)}$$

2.760 ODE No. 760

$$y'(x)=\frac{\left(xy(x)^2+1\right)^3}{x^4y(x)\left(xy(x)^2+x+1\right)}$$

✓ **Mathematica** : cpu = 1.04407 (sec), leaf count = 112

Solve $\left[2\left(\frac{1}{10}\log\left(2x^2y(x)^4+2x^2y(x)^2+x^2+4xy(x)^2+2x+2\right)-\frac{1}{5}\log\left(xy(x)^2-x+1\right)-\frac{1}{10}\tan^{-1}\left(2xy(x)^4+2xy(x)^2+x+1\right)\right),y(x)\right]$

✓ **Maple** : cpu = 2.026 (sec), leaf count = 137

$$\left\{\frac{\left(y(x)-1\right)\left(2y(x)^4+2y(x)^2+1\right)\left(10c_1x-x\arctan\left(2y(x)^2+1\right)+x\arctan\left(2xy(x)^4+(2x+2)y(x)^2+x+1\right)+2x\ln\left(xy(x)^2-x+1\right)\right)}{10\left(2y(x)^6-y(x)^2-1\right)x}\right\}$$

2.761 ODE No. 761

$$y'(x)=\frac{-x^3+4x^2-4xy(x)-4x+8}{2x^2+8y(x)-8x+8}$$

✓ **Mathematica** : cpu = 0.0490669 (sec), leaf count = 33

$$\left\{\left\{y(x) \rightarrow W\left(-e^{-x-1+c_1}\right)+\frac{1}{4}\left(-x^2+4x-4\right)+1\right\}\right\}$$

✓ **Maple** : cpu = 0.09 (sec), leaf count = 18

$$\left\{y(x)=-\frac{x^2}{4}+x+\text{LambertW}\left(c_1e^{-x}\right)\right\}$$

2.762 ODE No. 762

$$y'(x) = \frac{y(x)(x(-\log(y(x))) - \log(y(x)) + x)}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.224779 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow (x+1)^{-1/x} e^{1-\frac{c_1}{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.176 (sec), leaf count = 22

$$\left\{ y(x) = e(x+1)^{-\frac{1}{x}} e^{\frac{c_1}{x}} \right\}$$

2.763 ODE No. 763

$$y'(x) = \frac{y(x)(x \log(y(x)) + \log(y(x)) + x)}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.177535 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow x^x (x+1)^{-x} e^{c_1 x} \right\} \right\}$$

✓ **Maple** : cpu = 0.122 (sec), leaf count = 14

$$\left\{ y(x) = \left(\frac{c_1 x}{x+1} \right)^x \right\}$$

2.764 ODE No. 764

$$y'(x) = \frac{y(x)(x^4 - x \log(y(x)) - \log(y(x)))}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.201401 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow (x+1)^{\frac{1}{x}} e^{\frac{x^3}{4} - \frac{x^2}{3} + \frac{x}{2} - \frac{25}{12x} - \frac{c_1}{x} - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.218 (sec), leaf count = 36

$$\left\{ y(x) = e^{-1} (x+1)^{\frac{1}{x}} e^{\frac{c_1}{x}} e^{\frac{x}{2}} e^{-\frac{x^2}{3}} e^{\frac{x^3}{4}} \right\}$$

2.765 ODE No. 765

$$y'(x) = \frac{y(x) \left(xy(x) \log\left(\frac{(x-1)(x+1)}{x}\right) - \log\left(\frac{(x-1)(x+1)}{x}\right) - 1 \right)}{x}$$

✓ **Mathematica** : cpu = 0.420932 (sec), leaf count = 138

$$\left\{ \left(y(x) \rightarrow \frac{\exp\left(-\text{Li}_2(1-x) + \text{Li}_2(-x) - \frac{1}{2} \log^2(x) + \log(x+1) \log(x) - \log\left(x - \frac{1}{x}\right) \log(x)\right)}{x \left(-\int_1^x \frac{\exp\left(-\frac{1}{2} \log^2(K[1]) + \log(K[1]+1) \log(K[1]) - \log\left(K[1] - \frac{1}{K[1]}\right) \log(K[1]) - \text{Li}_2(1-K[1]) + \text{Li}_2(-K[1])\right) \log\left(\frac{(K[1]-1)(K[1]+1)}{K[1]}\right)}{K[1]} dK[1] + c_1 \right)} \right) \right\}$$

✓ **Maple** : cpu = 0.198 (sec), leaf count = 106

$$\left\{ y(x) = \frac{x^{-\ln\left(\frac{(x-1)(x+1)}{x}\right)} x^{\ln(x+1)} e^{-\frac{\ln(x)^2}{2}} e^{-\text{dilog}(x)} e^{\text{dilog}(x+1)}}{\left(c_1 + \int -\frac{x^{-\ln\left(\frac{(x-1)(x+1)}{x}\right)} x^{\ln(x+1)} e^{-\frac{\ln(x)^2}{2}} e^{-\text{dilog}(x)} e^{\text{dilog}(x+1)} \ln\left(\frac{(x-1)(x+1)}{x}\right)}{x} dx \right) x} \right\}$$

2.766 ODE No. 766

$$y'(x) = \frac{y(x) \left(x^2 y(x) \log\left(\frac{(x-1)(x+1)}{x}\right) - x \log\left(\frac{(x-1)(x+1)}{x}\right) - \log(x) \right)}{x \log(x)}$$

✓ **Mathematica** : cpu = 0.383213 (sec), leaf count = 129

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp\left(\int_1^x \frac{-\log(K[1]) - K[1] \log\left(\frac{(K[1]-1)(K[1]+1)}{K[1]}\right)}{K[1] \log(K[1])} dK[1]\right)}{-\int_1^x \frac{\exp\left(\int_1^{K[2]} \frac{-\log(K[1]) - K[1] \log\left(\frac{(K[1]-1)(K[1]+1)}{K[1]}\right)}{K[1] \log(K[1])} dK[1]\right) K[2] \log\left(\frac{(K[2]-1)(K[2]+1)}{K[2]}\right)}{\log(K[2])} dK[2] + c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.186 (sec), leaf count = 89

$$\left\{ y(x) = \frac{e^{\int \frac{-x \ln\left(\frac{(x-1)(x+1)}{x}\right) - \ln(x)}{x \ln(x)} dx}}{c_1 + \int -\frac{x e^{\int \frac{-x \ln\left(\frac{(x-1)(x+1)}{x}\right) - \ln(x)}{x \ln(x)} dx} \ln\left(\frac{(x-1)(x+1)}{x}\right)}{\ln(x)} dx} \right\}$$

2.767 ODE No. 767

$$y'(x) = \frac{-x^3 + 2x^2 - 8xy(x) - 8x + 32}{4x^2 + 32y(x) - 8x + 32}$$

✓ **Mathematica** : cpu = 0.0417176 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{8}(-x^2 + 2x - 8) + 4 \left(1 + W\left(-e^{-\frac{x}{16} - 1 + c_1}\right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.1 (sec), leaf count = 26

$$\left\{ y(x) = -\frac{x^2}{8} + \frac{x}{4} + 4 \operatorname{LambertW}\left(\frac{c_1 e^{-\frac{3}{4}} e^{-\frac{x}{16}}}{4}\right) + 3 \right\}$$

2.768 ODE No. 768

$$y'(x) = \frac{y(x)(y(x) + 1)}{x(xy(x) - y(x) - 1)}$$

✓ **Mathematica** : cpu = 1.79393 (sec), leaf count = 66

$$\operatorname{Solve}\left[\frac{2^{2/3}\left(xy(x)\left(-\log\left(\frac{xy(x)}{(x-1)y(x)-1}\right) + \log\left(\frac{y(x)+1}{-xy(x)+y(x)+1}\right) + \log(x) + 1\right) - 1\right)}{9xy(x)} = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.129 (sec), leaf count = 26

$$\left\{ y(x) = -\frac{1}{x \operatorname{LambertW}\left(\frac{e^{-\frac{1}{x}}}{c_1 x}\right) + 1} \right\}$$

2.769 ODE No. 769

$$y'(x) = -\frac{ix\left(x^8 + 8x^4y(x)^2 + 16ix^2 + 16y(x)^4\right)}{32y(x)}$$

✓ **Mathematica** : cpu = 0.100868 (sec), leaf count = 360

$$\left\{\left\{y(x) \rightarrow -\frac{\sqrt{\left(Y_{\frac{1}{3}}\left(\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)+c_1J_{\frac{1}{3}}\left(\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)\right)\left((1+i)x^3\left(Y_{\frac{4}{3}}\left(\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)+c_1J_{\frac{4}{3}}\left(\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)\right)-\frac{1}{4}(x^6+8i)Y_{\frac{1}{3}}\left(\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)\right)}{x\left(Y_{\frac{1}{3}}\left(\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)+c_1J_{\frac{1}{3}}\left(\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)\right)}\right.\right.$$

✓ **Maple** : cpu = 0.537 (sec), leaf count = 251

$$\left\{y(x) = -\frac{\sqrt{4}\sqrt{\left((1+i)\left(c_1\operatorname{BesselJ}\left(\frac{4}{3},\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)+\operatorname{BesselY}\left(\frac{4}{3},\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)\right)x^3-2c_1\left(\frac{x^6}{8}+i\right)\operatorname{BesselJ}\left(\frac{1}{3},\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)+\left(-\frac{1}{4}(x^6+8i)Y_{\frac{1}{3}}\left(\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)\right)}{2\left(c_1\operatorname{BesselJ}\left(\frac{1}{3},\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)+\operatorname{BesselY}\left(\frac{1}{3},\left(\frac{1}{3}-\frac{i}{3}\right)x^3\right)\right)}\right.$$

2.770 ODE No. 770

$$y'(x) = \frac{2y(x)^6}{32x^2y(x)^4 + y(x)^3 + 16xy(x)^2 + 2}$$

✓ **Mathematica** : cpu = 0.247344 (sec), leaf count = 705

$$\left\{\left\{y(x) \rightarrow \frac{\sqrt[3]{8192x^3 + 18432c_1^2x^2 + \sqrt{4(-256x^2 + 192c_1^2x - 12c_1)^3 + (8192x^3 + 18432c_1^2x^2 - 2880c_1x + 108)^2} - 2880c_1x}}{3\sqrt[3]{2}(1 - 16c_1x)}\right.\right.$$

✓ **Maple** : cpu = 0.141 (sec), leaf count = 1105

$$\left\{y(x) = \frac{-256c_1^2x^2 + 32c_1\left((4096x^3 + 54)c_1^3 + 1440c_1^2x + 9216c_1x^2 + 96\left(\frac{c_1}{16} + x\right)\sqrt{3}\sqrt{(4096x^3 + 27)c_1^4 + 576c_1^3x + 2048c_1^2}\right)}{32x^2y(x)^4 + y(x)^3 + 16xy(x)^2 + 2}\right.$$

2.771 ODE No. 771

$$y'(x) = \frac{-a^2x^3 - 2abx^2 - 4axy(x) - 4ax + 8}{2ax^2 + 4bx + 8y(x) + 8}$$

✓ **Mathematica** : cpu = 0.055505 (sec), leaf count = 46

$$\left\{\left\{y(x) \rightarrow \frac{1}{4}(-ax^2 - 2bx - 4) - \frac{2\left(1 + W\left(-e^{-\frac{b^2x}{4}-1+c_1}\right)\right)}{b}\right.\right.$$

✓ **Maple** : cpu = 0.255 (sec), leaf count = 84

$$\left\{y(x) = \frac{-abx^2 - 2b^2x - 4b + 4e^{\frac{-2c_1b^2 - 4a\operatorname{LambertW}\left(-\frac{e^{-1}e^{-\frac{b}{2}}e^{-\frac{b^2x}{4}}e^{-\frac{c_1b^2}{2a}}\right) + (-b^2x - 2b - 4)a}}{4a}} - 8}{4b}\right.$$

2.772 ODE No. 772

$$y'(x) = \frac{y(x) \log(y(x))(x \log(y(x)) + x + 1)}{x(x + 1)}$$

✓ **Mathematica** : cpu = 0.219708 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{x}{-x + \log(x+1) + c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.217 (sec), leaf count = 18

$$\left\{ y(x) = e^{\frac{x}{c_1 - x + \ln(x+1)}} \right\}$$

2.773 ODE No. 773

$$y'(x) = \frac{y(x)^2 + xy(x) + x}{(x - 1)(y(x) + x)}$$

✓ **Mathematica** : cpu = 0.220879 (sec), leaf count = 61

$$\text{Solve} \left[\frac{1}{2} \log \left(\frac{y(x)^2}{x^2} + \frac{y(x)}{x} + 1 \right) + \frac{\tan^{-1} \left(\frac{\frac{2y(x)}{x} + 1}{\sqrt{3}} \right)}{\sqrt{3}} = \log(1 - x) - \log(x) + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.239 (sec), leaf count = 48

$$\left\{ y(x) = \frac{\sqrt{3} x \tan \left(\text{RootOf} \left(2c_1 \sqrt{3} - 2\_Z - \sqrt{3} \ln \left(\frac{3(\tan^2(\_Z)+1)x^2}{4(x-1)^2} \right) \right) \right)}{2} - \frac{x}{2} \right\}$$

2.774 ODE No. 774

$$y'(x) = \frac{-2ax^2 - x^3 - 4xy(x) - 4x + 8}{4ax + 2x^2 + 8y(x) + 8}$$

✓ **Mathematica** : cpu = 0.0451379 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4}(-2ax - x^2 - 4) - \frac{2 \left(1 + W \left(-e^{-\frac{a^2 x}{4} - 1 + c_1} \right) \right)}{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.165 (sec), leaf count = 51

$$\left\{ y(x) = \frac{-2a^2 x - a x^2 - 4a - 8 \text{LambertW} \left(-\frac{e^{-1} e^{-\frac{a}{2}} e^{\frac{c_1 a^2}{4}} e^{-\frac{a^2 x}{4}}}{2} \right) - 8}{4a} \right\}$$

2.775 ODE No. 775

$$y'(x) = \frac{-y(x) + \sqrt{y(x)} + x}{-y(x) + \sqrt{y(x)} + x + 1}$$

✓ **Mathematica** : cpu = 0.185328 (sec), leaf count = 943

$$\left\{ \left\{ y(x) \rightarrow \text{Root} \left[x^6 - 2e^{3c_1} x^3 + e^{6c_1} + \#1^6 + (-6x - 6)\#1^5 + (15x^2 + 24x + 9)\#1^4 + (-20x^3 - 36x^2 - 18x + 2e^{3c_1} - 4)\#1^3 - \right. \right. \right.$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 44

$$\left\{-x^3+y(x)^3+(-3x-3)y(x)^2-2y(x)^{\frac{3}{2}}-c_1+(3x^2+3x)y(x)=0\right\}$$

2.776 ODE No. 776

$$y'(x)=\frac{y(x)\left(x^2y(x)\log\left(\frac{x^2+1}{x}\right)-x\log\left(\frac{x^2+1}{x}\right)-\log\left(\frac{1}{x}\right)\right)}{x\log\left(\frac{1}{x}\right)}$$

✓ **Mathematica** : cpu = 0.430169 (sec), leaf count = 133

$$\left\{\left\{y(x)\rightarrow\frac{\exp\left(\int_1^x\frac{-\log\left(\frac{1}{K[1]}\right)-K[1]\log\left(\frac{K[1]^2+1}{K[1]}\right)}{K[1]\log\left(\frac{1}{K[1]}\right)}dK[1]\right)}{\exp\left(\int_1^{K[2]}\frac{-\log\left(\frac{1}{K[1]}\right)-K[1]\log\left(\frac{K[1]^2+1}{K[1]}\right)}{K[1]\log\left(\frac{1}{K[1]}\right)}dK[1]\right)K[2]\log\left(\frac{K[2]^2+1}{K[2]}\right)}\right.\\ \left.-\int_1^x\frac{dK[2]+c_1}{\log\left(\frac{1}{K[2]}\right)}\right\}$$

✓ **Maple** : cpu = 0.245 (sec), leaf count = 96

$$\left\{y(x)=\frac{e^{\int\frac{-x\ln\left(\frac{x^2+1}{x}\right)-\ln\left(\frac{1}{x}\right)}{x\ln\left(\frac{1}{x}\right)}dx}}{c_1+\int-\frac{x\ln\left(\frac{x^2+1}{x}\right)-\ln\left(\frac{1}{x}\right)}{\ln\left(\frac{1}{x}\right)}dx}\right\}$$

2.777 ODE No. 777

$$y'(x)=\frac{y(x)(y(x)+1)}{x(xy(x)^4-y(x)-1)}$$

✓ **Mathematica** : cpu = 0.318293 (sec), leaf count = 39

$$\text{Solve}\left[-\frac{1}{2}(y(x)+1)^2+2(y(x)+1)-\frac{1}{xy(x)}-\log(y(x)+1)=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.263 (sec), leaf count = 51

$$\{y(x)=e^{\text{RootOf}\left(2c_1xe^{-Z}+2\_Zxe^{-Z}-2c_1x-2\_Zx+7xe^{-Z}-5xe^{2\_Z}+xe^{3\_Z}-3x+2\right)}-1\}$$

2.778 ODE No. 778

$$y'(x)=\frac{x^9y(x)^3+x^6y(x)^2-3x^2y(x)+1}{x^3}$$

✓ **Mathematica** : cpu = 0.179966 (sec), leaf count = 95

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3+3\sqrt[3]{29}\#1-29\&,\frac{\log\left(\frac{3x^6y(x)+x^3}{\sqrt[3]{29}\sqrt[3]{x^9}}-\#1\right)}{\sqrt[3]{29}-29\#1^2}\&\right]=\frac{29^{2/3}\left(x^9\right)^{2/3}}{9x^5}+c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 37

$$\left\{y(x)=\frac{29\operatorname{RootOf}\left(3c_1+x-81\left(\int^{\frac{-Z}{841a^3-27a+27}}\frac{1}{d-a}d_a\right)\right)-3}{9x^3}\right\}$$

2.779 ODE No. 779

$$y'(x)=\frac{x^3y(x)+x^3+xy(x)^2+y(x)^3}{(x-1)x^3}$$

✓ **Mathematica** : cpu = 0.182902 (sec), leaf count = 57

$$\operatorname{Solve}\left[-\frac{1}{4}\log\left(\frac{y(x)^2}{x^2}+1\right)+\frac{1}{2}\log\left(\frac{y(x)}{x}+1\right)+\frac{1}{2}\tan^{-1}\left(\frac{y(x)}{x}\right)=\log(1-x)-\log(x)+c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 50

$$\left\{-c_1+\frac{\arctan\left(\frac{y(x)}{x}\right)}{2}+\ln(x)-\frac{\ln\left(\frac{x^2+y(x)^2}{x^2}\right)}{4}+\frac{\ln\left(\frac{x+y(x)}{x}\right)}{2}-\ln(x-1)=0\right\}$$

2.780 ODE No. 780

$$y'(x)=\frac{x\sqrt{x^2+y(x)^2}+xy(x)+y(x)}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.200274 (sec), leaf count = 15

$$\{\{y(x)\rightarrow x\sinh(\log(x+1)+c_1)\}\}$$

✓ **Maple** : cpu = 0.615 (sec), leaf count = 27

$$\left\{c_1+\frac{y(x)+\sqrt{x^2+y(x)^2}}{(x+1)x}=0\right\}$$

2.781 ODE No. 781

$$y'(x)=\frac{y(x)\left(x^4+x^3+3y(x)^2+x\right)}{x\left(6y(x)^2+x\right)}$$

✓ **Mathematica** : cpu = 0.486974 (sec), leaf count = 82

$$\left\{\left\{y(x)\rightarrow-\frac{\sqrt{x}\sqrt{W\left(6xe^{\frac{2x^3}{3}+x^2+2c_1}\right)}}{\sqrt{6}}\right\},\left\{y(x)\rightarrow\frac{\sqrt{x}\sqrt{W\left(6xe^{\frac{2x^3}{3}+x^2+2c_1}\right)}}{\sqrt{6}}\right\}\right\}$$

✓ **Maple** : cpu = 0.481 (sec), leaf count = 61

$$\left\{\frac{1}{\frac{6}{x}+\frac{1}{y(x)^2}}=\frac{\left(e^{\operatorname{RootOf}\left(2x^3e^{-Z}+3x^2e^{-Z}+9c_1e^{-Z}+3Ze^{-Z}-3e^{-Z}\ln\left(\frac{e^{-Z}+9}{2x}\right)+27\right)+9}\right)x}{54}\right\}$$

2.782 ODE No. 782

$$y'(x) = \frac{y(x) \coth\left(\frac{1}{x}\right) \left(x^2 y(x) \log\left(\frac{x^2+1}{x}\right) - x \log\left(\frac{x^2+1}{x}\right) - \tanh\left(\frac{1}{x}\right)\right)}{x}$$

✓ **Mathematica** : cpu = 4.7425 (sec), leaf count = 115

$$\left\{ \left(y(x) \rightarrow \frac{\exp\left(\int_1^x \frac{-\coth\left(\frac{1}{K[1]}\right) K[1] \log\left(\frac{K[1]^2+1}{K[1]}\right) - 1}{K[1]} dK[1]\right)}{-\int_1^x \exp\left(\int_1^{K[2]} \frac{-\coth\left(\frac{1}{K[1]}\right) K[1] \log\left(\frac{K[1]^2+1}{K[1]}\right) - 1}{K[1]} dK[1]\right) \coth\left(\frac{1}{K[2]}\right) K[2] \log\left(\frac{K[2]^2+1}{K[2]}\right) dK[2] + c_1} \right) \right\}$$

✓ **Maple** : cpu = 2.313 (sec), leaf count = 96

$$\left\{ y(x) = \frac{e^{\int \frac{-x \ln\left(\frac{x^2+1}{x}\right) - \tanh\left(\frac{1}{x}\right)}{x \tanh\left(\frac{1}{x}\right)} dx}}{c_1 + \int -\frac{x e^{\int \frac{-x \ln\left(\frac{x^2+1}{x}\right) - \tanh\left(\frac{1}{x}\right)}{x \tanh\left(\frac{1}{x}\right)} dx} \ln\left(\frac{x^2+1}{x}\right)}{\tanh\left(\frac{1}{x}\right)} dx} \right\}$$

2.783 ODE No. 783

$$y'(x) = -\frac{y(x) \coth(x) \left(x^2 y(x) (-\log(2x)) + x \log(2x) + \tanh(x)\right)}{x}$$

✓ **Mathematica** : cpu = 4.52942 (sec), leaf count = 88

$$\left\{ \left(y(x) \rightarrow \frac{\exp\left(\int_1^x \frac{-\coth(K[1]) K[1] \log(2K[1]) - 1}{K[1]} dK[1]\right)}{-\int_1^x \exp\left(\int_1^{K[2]} \frac{-\coth(K[1]) K[1] \log(2K[1]) - 1}{K[1]} dK[1]\right) \coth(K[2]) K[2] \log(2K[2]) dK[2] + c_1} \right) \right\}$$

✓ **Maple** : cpu = 0.2 (sec), leaf count = 75

$$\left\{ y(x) = \frac{e^{\int \frac{-x \ln(x) - \ln(2)x - \tanh(x)}{x \tanh(x)} dx}}{c_1 + \int -\frac{(\ln(x) + \ln(2))x e^{\int \frac{-x \ln(x) - \ln(2)x - \tanh(x)}{x \tanh(x)} dx}}{\tanh(x)} dx} \right\}$$

2.784 ODE No. 784

$$y'(x) = \operatorname{csch}(x) \left(x^2 \log(x) + 2xy(x) \log(x) + y(x)^2 \log(x) + \log(x) - \sinh(x)\right)$$

✓ **Mathematica** : cpu = 21.9564 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow -x + \tan\left(\int_1^x \operatorname{csch}(K[5]) \log(K[5]) dK[5] + c_1\right) \right\} \right\}$$

✓ **Maple** : cpu = 10.085 (sec), leaf count = 24

$$\left\{ y(x) = -x - \tan\left(c_1 - \left(\int \frac{\ln(x)}{\sinh(x)} dx\right)\right) \right\}$$

2.785 ODE No. 785

$$y'(x) = \frac{x^2 \sinh(x) + 2xy(x) \sinh(x) + y(x)^2 \sinh(x) - \log(x) + \sinh(x)}{\log(x)}$$

✓ **Mathematica** : cpu = 9.23553 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow -x + \tan \left(\int_1^x \frac{\sinh(K[5])}{\log(K[5])} dK[5] + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 50.054 (sec), leaf count = 24

$$\left\{ y(x) = -x - \tan \left(c_1 - \left(\int \frac{\sinh(x)}{\ln(x)} dx \right) \right) \right\}$$

2.786 ODE No. 786

$$y'(x) = \frac{axy(x)^2 \cosh(x) + bx^3 \cosh(x) + y(x) \log(x)}{x \log(x)}$$

✓ **Mathematica** : cpu = 4.45865 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{b} x \tan \left(\sqrt{a} \sqrt{b} \int_1^x \frac{\cosh(K[1])K[1]}{\log(K[1])} dK[1] + \sqrt{a} \sqrt{b} c_1 \right)}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 33

$$\left\{ y(x) = \frac{\sqrt{ab} x \tan \left(\sqrt{ab} \left(c_1 + \int \frac{x \cosh(x)}{\ln(x)} dx \right) \right)}{a} \right\}$$

2.787 ODE No. 787

$$y'(x) = \frac{x(2x^4 - 2x^2y(x) + x^2 - x - 1)}{(x + 1)(x^2 - y(x))}$$

✓ **Mathematica** : cpu = 29.7369 (sec), leaf count = 488

Solve

$$\left[\left(2 - \frac{x(x^2-x-1)(2x^2-2y(x)+3)}{\sqrt[3]{x^3(x^2-x-1)^3(x^2-y(x))}} \right) \left(\frac{x(x^2-x-1)(2x^2-2y(x)+3)}{\sqrt[3]{x^3(x^2-x-1)^3(x^2-y(x))}} + 4 \right) \left(1 - \frac{x(x^2-x-1)(2x^2-2y(x)+3)}{2\sqrt[3]{x^3(x^2-x-1)^3(x^2-y(x))}} \right) \log \left(\frac{2 - \frac{x(x^2-x-1)(2x^2-2y(x)+3)}{\sqrt[3]{x^3(x^2-x-1)^3(x^2-y(x))}}}{\sqrt[3]{2}} \right) + \left(\frac{x(x^2-x-1)(2x^2-2y(x)+3)}{2\sqrt[3]{x^3(x^2-x-1)^3(x^2-y(x))}} \right) \right. \\ \left. 18\sqrt[3]{2} \left(-\frac{(2x^2-2y(x)+3)^3}{8(x^2-y(x))^3} + \frac{3x(x^2-x-1)(2x^2-2y(x)+3)}{2\sqrt[3]{x^3(x^2-x-1)^3(x^2-y(x))}} - 2 \right) \right]$$

✓ **Maple** : cpu = 0.899 (sec), leaf count = 191

$$\left\{ y(x) = \frac{4x^2 e^{\text{RootOf}\left(8x^3 e^{-Z} - 36x^3 - 24x^2 e^{-Z} + 18c_1 e^{-Z} - 6_Z e^{-Z} + 108x^2 + 24x e^{-Z} + 6 e^{-Z} \ln\left(\frac{2e^{-Z}-9}{(x+1)^4}\right) - 81c_1 + 27_Z - 108x - 27 \ln\left(\frac{2e^{-Z}-9}{(x+1)^4}\right) + 27\right)} - 18x^2 - 9}{4 e^{\text{RootOf}\left(8x^3 e^{-Z} - 36x^3 - 24x^2 e^{-Z} + 18c_1 e^{-Z} - 6_Z e^{-Z} + 108x^2 + 24x e^{-Z} + 6 e^{-Z} \ln\left(\frac{2e^{-Z}-9}{(x+1)^4}\right) - 81c_1 + 27_Z - 108x - 27 \ln\left(\frac{2e^{-Z}-9}{(x+1)^4}\right) + 27\right)} - 18}$$

2.788 ODE No. 788

$$y'(x) = -\frac{y(x)\big(x^2y(x)(-\coth(x+1)) + \log(x-1) + x\coth(x+1)\big)}{x\log(x-1)}$$

✓ **Mathematica** : cpu = 27.7885 (sec), leaf count = 348

$$\left\{\left\{y(x) \rightarrow \frac{\exp\left(\int_1^x \frac{-e^2 \cosh(K[1])K[1]-\cosh(K[1])K[1]-e^2 \sinh(K[1])K[1]+\sinh(K[1])K[1]-e^2 \cosh(K[1]) \log(K[1]-1)+K[1] \log(K[1]-1)(e^2 \cosh(K[1])-\cosh(K[1])+e^2 \sinh(K[1])+\sinh(K[1]))}{K[1] \log(K[1]-1)(e^2 \cosh(K[1])-\cosh(K[1])+e^2 \sinh(K[1])+\sinh(K[1]))}dx\right)}{-\int_1^x \frac{\exp\left(\int_1^{K[2]} \frac{-e^2 \cosh(K[1])K[1]-\cosh(K[1])K[1]-e^2 \sinh(K[1])K[1]+\sinh(K[1])K[1]-e^2 \cosh(K[1]) \log(K[1]-1)+\cosh(K[1]) \log(K[1]-1)-e^2 \log(K[1]-1) \sinh(K[1])-\cosh(K[1]) \log(K[1]-1)(e^2 \cosh(K[1])-\cosh(K[1])+e^2 \sinh(K[1])+\sinh(K[1]))}{K[1] \log(K[1]-1)(e^2 \cosh(K[1])-\cosh(K[1])+e^2 \sinh(K[1])+\sinh(K[1]))}dK[1]\right)}{K[2] \log(K[2]-1)(e^2 \cosh(K[2])-\cosh(K[2])+e^2 \sinh(K[2])+\sinh(K[2]))}dK[2]+c_1}\right\}\right\}$$

✓ **Maple** : cpu = 0.31 (sec), leaf count = 108

$$\left\{y(x) = \frac{e^{\int -\frac{x \cosh(x+1)+\ln(x-1) \sinh(x+1)}{x \ln(x-1) \sinh(x+1)}dx}}{c_1 + \int -\frac{x \cosh(x+1)e^{\int -\frac{x \cosh(x+1)+\ln(x-1) \sinh(x+1)}{x \ln(x-1) \sinh(x+1)}dx}}{\ln(x-1) \sinh(x+1)}dx\right\}$$

2.789 ODE No. 789

$$y'(x) = \frac{x^2 \coth(x+1) + 2xy(x) \coth(x+1) + y(x)^2 \coth(x+1) - \log(x-1) + \coth(x+1)}{\log(x-1)}$$

✓ **Mathematica** : cpu = 76.9654 (sec), leaf count = 120

$$\left\{\left\{y(x) \rightarrow -\frac{e^2 x \sinh(x) - x \sinh(x) + e^2 x \cosh(x) + x \cosh(x)}{e^2 \sinh(x) - \sinh(x) + e^2 \cosh(x) + \cosh(x)} + \tan\left(\int_1^x \frac{e^2 \cosh(K[5]) + \cosh(K[5]) + e^2 \sinh(K[5]) + \sinh(K[5])}{\log(K[5]-1)(e^2 \cosh(K[5]) - \cosh(K[5]) + e^2 \sinh(K[5]) + \sinh(K[5]))}dK[5]\right)\right\}\right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(diff(y(x),x) = -(ln(x-1)-coth(1+x)*x^2-2*coth(1+x)*x*y(x)-coth(1+x)-coth(1+x)*y(x)^2-1),y(x))

2.790 ODE No. 790

$$y'(x) = \frac{x^4 \coth\left(\frac{x+1}{x-1}\right) - 2x^2y(x) \coth\left(\frac{x+1}{x-1}\right) + y(x)^2 \coth\left(\frac{x+1}{x-1}\right) + 2x \log\left(\frac{1}{x-1}\right) - \coth\left(\frac{x+1}{x-1}\right)}{\log\left(\frac{1}{x-1}\right)}$$

✓ **Mathematica** : cpu = 92.8859 (sec), leaf count = 127

$$\left\{\left\{y(x) \rightarrow \frac{\exp\left(\int_1^x \frac{2 \coth\left(\frac{K[5]}{K[5]-1} + \frac{1}{K[5]-1}\right)}{\log\left(\frac{1}{K[5]-1}\right)}dK[5]\right)}{-\int_1^x \frac{\exp\left(\int_1^{K[6]} \frac{2 \coth\left(\frac{K[5]}{K[5]-1} + \frac{1}{K[5]-1}\right)}{\log\left(\frac{1}{K[5]-1}\right)}dK[5]\right) \coth\left(\frac{K[6]}{K[6]-1} + \frac{1}{K[6]-1}\right)}{\log\left(\frac{1}{K[6]-1}\right)}dK[6]+c_1}\right\}\right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(diff(y(x),x) = (2*x*ln(1/(x-1))-coth((1+x)/(x-1))+coth((1+x)/(x-1))*y(x)^2-2*coth((1+x)/(x-1))*x^2*y(x)+coth((1+x)/(x-1))*x^4)/ln(1/(x-1)),y(x))

2.791 ODE No. 791

$$y'(x) = \frac{\operatorname{sech}\left(\frac{1}{x-1}\right)\left(x^5 + x^4 - 2x^3y(x) - 2x^2y(x) + 2x^2 \cosh\left(\frac{1}{x-1}\right) + xy(x)^2 + y(x)^2 - x - 2x \cosh\left(\frac{1}{x-1}\right) - 1\right)}{x-1}$$

✓ **Mathematica** : cpu = 8.21559 (sec), leaf count = 110

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp\left(\int_1^x \frac{2(K[5]+1)\operatorname{sech}\left(\frac{1}{K[5]-1}\right)}{K[5]-1} dK[5]\right)}{-\int_1^x \frac{\exp\left(\int_1^{K[6]} \frac{2(K[5]+1)\operatorname{sech}\left(\frac{1}{K[5]-1}\right)}{K[5]-1} dK[5]\right)(K[6]+1)\operatorname{sech}\left(\frac{1}{K[6]-1}\right)}{K[6]-1} dK[6] + c_1} + \frac{x^3 + x^2}{x + 1} + 1 \right\} \right\}$$

✓ **Maple** : cpu = 6.811 (sec), leaf count = 306

$$\left\{ y(x) = \frac{\frac{\int \frac{4(x+1)e^{\frac{1}{x-1}}}{\left(\frac{2}{e^{x-1}+1}\right)(x-1)} dx}{e^{\frac{2}{e^{x-1}+1}}} \frac{\int \frac{4(x+1)e^{\frac{1}{x-1}}}{\left(\frac{2}{e^{x-1}+1}\right)(x-1)} dx}{e^{\frac{2}{e^{x-1}+1}}} e^{\frac{2}{e^{x-1}+1}} + (x^2 + 1)e^{\frac{4c_1}{2}} e^{\frac{4c_1 e^{\frac{2}{x-1}}}{2}}}{\frac{\int \frac{4(x+1)e^{\frac{1}{x-1}}}{\left(\frac{2}{e^{x-1}+1}\right)(x-1)} dx}{-e^{\frac{2}{e^{x-1}+1}}} \frac{\int \frac{4(x+1)e^{\frac{1}{x-1}}}{\left(\frac{2}{e^{x-1}+1}\right)(x-1)} dx}{e^{\frac{2}{e^{x-1}+1}}} e^{\frac{2}{e^{x-1}+1}} + e^{\frac{4c_1}{2}} e^{\frac{4c_1 e^{\frac{2}{x-1}}}{2}}}$$

2.792 ODE No. 792

$$y'(x) = \frac{y(x)\operatorname{sech}\left(\frac{1}{x+1}\right)\left(x^3y(x) + x^2y(x) - x^2 - x - x\cosh\left(\frac{1}{x+1}\right) + \cosh\left(\frac{1}{x+1}\right)\right)}{(x-1)x}$$

✓ **Mathematica** : cpu = 3.10096 (sec), leaf count = 157

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp\left(\int_1^x \frac{-\operatorname{sech}\left(\frac{1}{K[1]+1}\right)K[1]^2-\operatorname{sech}\left(\frac{1}{K[1]+1}\right)K[1]-K[1]+1}{(K[1]-1)K[1]} dK[1]\right)}{-\int_1^x \frac{\exp\left(\int_1^{K[2]} \frac{-\operatorname{sech}\left(\frac{1}{K[1]+1}\right)K[1]^2-\operatorname{sech}\left(\frac{1}{K[1]+1}\right)K[1]-K[1]+1}{(K[1]-1)K[1]} dK[1]\right)\left(\operatorname{sech}\left(\frac{1}{K[2]+1}\right)K[2]^3+\operatorname{sech}\left(\frac{1}{K[2]+1}\right)K[2]^2\right)}{(K[2]-1)K[2]} dK[2] + c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.437 (sec), leaf count = 112

$$\left\{ y(x) = \frac{e^{\int \frac{-x^2-x+(-x+1)\cosh\left(\frac{1}{x+1}\right)}{(x-1)x\cosh\left(\frac{1}{x+1}\right)} dx}}{c_1 + \int -\frac{(x+1)x e^{\int \frac{-x^2-x+(-x+1)\cosh\left(\frac{1}{x+1}\right)}{(x-1)x\cosh\left(\frac{1}{x+1}\right)} dx}}{(x-1)\cosh\left(\frac{1}{x+1}\right)} dx} \right\}$$

2.793 ODE No. 793

$$y'(x) = -\frac{y(x)(xy(x) + 1)}{x(xy(x) - y(x) + 1)}$$

✓ **Mathematica** : cpu = 14.8539 (sec), leaf count = 399

Solve

$$\left[\frac{\sqrt[3]{-2}\left(\frac{2^{2/3}((x-1)y(x)-2)}{\sqrt[3]{-\frac{1}{(x-1)^3}}(x-1)((x-1)y(x)+1)} + (-2)^{2/3}\left(\frac{-xy(x)+y(x)+2}{\sqrt[3]{2}\sqrt[3]{-\frac{1}{(x-1)^3}}(x-1)((x-1)y(x)+1)} + (-2)^{2/3}\left(\frac{\sqrt[3]{-1}(-xy(x)+y(x)+2)}{\sqrt[3]{-\frac{1}{(x-1)^3}}(x-1)((x-1)y(x)+1)} + 1\right)\right)}{9\left(\frac{((x-1)y(x)-2)^3}{((x-1)y(x)+1)^3} + \frac{1}{\left(-\frac{1}{(x-1)^3}\right)}\right)} \right]$$

✓ **Maple** : cpu = 2.147 (sec), leaf count = 32

$$\left\{y(x)=-\frac{2\mathrm{e}^{3c_1-\mathrm{LambertW}\left(-\frac{2(x-1)\mathrm{e}^{-1}\mathrm{e}^{3c_1}}{x}\right)-1}}{x}\right\}$$

2.794 ODE No. 794

$$y'(x)=\frac{y(x)}{x\left(x^3y(x)^4+x^2y(x)^3+y(x)-1\right)}$$

✓ **Mathematica** : cpu = 0.229709 (sec), leaf count = 67

$$\mathrm{Solve}\left[\mathrm{RootSum}\left[\#1^3y(x)^3+\#1^2y(x)^2+1\&,\frac{\#1y(x)\log(x-\#1)+\log(x-\#1)}{3\#1y(x)+2}\&\right]+y(x)-\log(x)=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.59 (sec), leaf count = 32

$$\left\{-c_1+\int^{\mathrm{x}y(x)}\frac{1}{\left(_a^3+_a^2+1\right)_a}d_a-y(x)=0\right\}$$

2.795 ODE No. 795

$$y'(x)=\frac{a^3+3a^2x+3ax^2+ay(x)^2+x^3+y(x)^3+xy(x)^2}{(a+x)^3}$$

✓ **Mathematica** : cpu = 0.504701 (sec), leaf count = 111

$$\mathrm{Solve}\left[-\frac{19}{3}\mathrm{RootSum}\left[-19\#1^3+6\sqrt[3]{38}\#1-19\&,\frac{\log\left(\frac{\frac{3y(x)}{(a+x)^3}+\frac{1}{(a+x)^2}}{\sqrt[3]{38}\sqrt[3]{\frac{1}{(a+x)^6}}}-\#1\right)}{2\sqrt[3]{38}-19\#1^2}\&\right]=\frac{1}{9}38^{2/3}\left(\frac{1}{(a+x)^6}\right)^{2/3}(a+x)^4\log(a+x)+c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 37

$$\left\{y(x)=-(a+x)\mathrm{RootOf}\left(c_1-\left(\int^{-Z}\frac{1}{_a^3-_a^2-_a-1}d_a\right)+\ln(a+x)\right)\right\}$$

2.796 ODE No. 796

$$y'(x)=\frac{e^{-\frac{3x^2}{2}}xy(x)^3}{3\left(e^{\frac{3x^2}{2}}y(x)+3e^{\frac{3x^2}{2}}+3y(x)\right)}$$

✓ **Mathematica** : cpu = 11.6067 (sec), leaf count = 102

$$\mathrm{Solve}\left[\frac{1}{62}\left(-31\log\left(9e^{\frac{3x^2}{2}}(y(x)+3)y(x)+3e^{3x^2}(y(x)+3)^2-y(x)^2\right)+6\sqrt{93}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{31}}\left(2e^{\frac{3x^2}{2}}(y(x)+3)+3y(x)\right)}{y(x)}\right)\right)+93,y(x)\right]$$

✓ **Maple** : cpu = 1.647 (sec), leaf count = 143

$$\left\{y(x)=\mathrm{RootOf}\left(\left(9\mathrm{e}^{3x^2}+27\mathrm{e}^{\frac{3x^2}{2}}+7\mathrm{e}^{3x^2+\mathrm{RootOf}\left(\left(217\mathrm{e}^{3x^2+Z}\left(\tanh^2\left(\frac{\left(c_1-5Z\right)\sqrt{93}}{90}\right)\right)+42\sqrt{93}\mathrm{e}^{3x^2+Z}\tanh\left(\frac{\left(c_1-5Z\right)\sqrt{93}}{90}\right)-93\left(\tanh^2\left(\frac{\left(c_1-5Z\right)\sqrt{93}}{90}\right)\right)\right)}\right)\right)\right)$$

2.797 ODE No. 797

$$y'(x) = \frac{y(x)\left(x^3y(x)\cosh\left(\frac{x+1}{x-1}\right) + x^2y(x)\cosh\left(\frac{x+1}{x-1}\right) - x^2\cosh\left(\frac{x+1}{x-1}\right) - x\cosh\left(\frac{x+1}{x-1}\right) - 1\right)}{x}$$

✓ **Mathematica** : cpu = 2.13721 (sec), leaf count = 349

$$\left\{\left\{\left\{y(x) \rightarrow \frac{\exp\left(\frac{(3e^2-1)\text{Chi}\left(\frac{2}{x-1}\right)}{e} + \frac{(1+3e^2)\text{Shi}\left(\frac{2}{x-1}\right)}{e} - \frac{1}{4}ex^2\sinh\left(\frac{2}{x-1}\right) + \frac{x^2\sinh\left(\frac{2}{x-1}\right)}{4e} - \frac{1}{4}ex^2\cosh\left(\frac{2}{x-1}\right) - \frac{x^2\cosh\left(\frac{2}{x-1}\right)}{4e} - ex\sinh\left(\frac{2}{x-1}\right) - ex\cosh\left(\frac{2}{x-1}\right) + x\left(\exp\left(\frac{(3e^2-1)\text{Chi}\left(\frac{2}{x-1}\right)}{e} + \frac{(1+3e^2)\text{Shi}\left(\frac{2}{x-1}\right)}{e}\right) + \frac{1}{4}ex^2\sinh\left(\frac{2}{x-1}\right) - \frac{x^2\sinh\left(\frac{2}{x-1}\right)}{4e} + \frac{1}{4}ex^2\cosh\left(\frac{2}{x-1}\right) - \frac{x^2\cosh\left(\frac{2}{x-1}\right)}{4e} + ex\sinh\left(\frac{2}{x-1}\right) + ex\cosh\left(\frac{2}{x-1}\right)\right)}{x}\right\}\right\}\right.$$

✓ **Maple** : cpu = 0.696 (sec), leaf count = 168

$$\left\{y(x) = \frac{e^{-3e\text{Ei}\left(1,-\frac{2}{x-1}\right)+e^{-1}\text{Ei}\left(1,\frac{2}{x-1}\right)-\frac{(x^2-1)e^{-\frac{x-1}{x-1}}}{4}-\frac{(x^2+4x-5)e^{\frac{x+1}{x-1}}}{4}}{\left(c_1 + \int -(x+1)\cosh\left(\frac{x+1}{x-1}\right)e^{-3e\text{Ei}\left(1,-\frac{2}{x-1}\right)+e^{-1}\text{Ei}\left(1,\frac{2}{x-1}\right)+\frac{(-x^2+1)e^{-\frac{x-1}{x-1}}}{4}+\frac{(-x^2-4x+5)e^{\frac{x+1}{x-1}}}{4}}dx\right)x}\right\}$$

2.798 ODE No. 798

$$y'(x) = \frac{y(x)(y(x) + x + 1)}{(x + 1)(2y(x)^3 + y(x) + x)}$$

✓ **Mathematica** : cpu = 0.62424 (sec), leaf count = 27

$$\text{Solve}\left[y(x)^2 - \frac{x}{y(x)} + \log(y(x)) - \log(x + 1) = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.208 (sec), leaf count = 30

$$\{y(x) = e^{\text{RootOf}\left(c_1e^{-Z} - Ze^{-Z} + e^{-Z}\ln(x+1) + x - e^{3Z}\right)}\}$$

2.799 ODE No. 799

$$y'(x) = \frac{y(x)\left(e^{\frac{x+1}{x-1}}x^3y(x) + e^{\frac{x+1}{x-1}}x^2y(x) - e^{\frac{x+1}{x-1}}x^2 - e^{\frac{x+1}{x-1}}x - 1\right)}{x}$$

✓ **Mathematica** : cpu = 1.29574 (sec), leaf count = 126

$$\left\{\left\{\left\{y(x) \rightarrow \frac{\exp\left(6e\text{Ei}\left(\frac{2}{x-1}\right) + \frac{1}{2}e^{\frac{x}{x-1}+\frac{1}{x-1}}\left(x^2 + 4x - 5\right) - e^{\frac{2}{x-1}}\left(\frac{1}{2}e(x-1)^2 + 3e(x-1)\right)\right)}{x\left(e^{6e\text{Ei}\left(\frac{2}{x-1}\right)} + c_1e^{\frac{1}{2}e^{\frac{x}{x-1}+\frac{1}{x-1}}\left(x^2+4x-5\right)}\right)}\right\}\right\}\right.$$

✓ **Maple** : cpu = 0.309 (sec), leaf count = 147

$$\left\{y(x) = \frac{e^{-2xe^{\frac{x+1}{x-1}}}e^{-\frac{x^2e^{\frac{x+1}{x-1}}}{2}}e^{-6e\text{Ei}\left(1,-\frac{2}{x-1}\right)}e^{\frac{5e^{\frac{x+1}{x-1}}}{2}}{\left(c_1 + \int -(x+1)e^{\frac{x+1}{x-1}}e^{-2xe^{\frac{x+1}{x-1}}}e^{-\frac{x^2e^{\frac{x+1}{x-1}}}{2}}e^{-6e\text{Ei}\left(1,-\frac{2}{x-1}\right)}e^{\frac{5e^{\frac{x+1}{x-1}}}{2}}dx\right)x}\right\}$$

2.800 ODE No. 800

$$y'(x) = \frac{-b^3 + 6b^2x - 12bx^2 - 4by(x)^2 + 8x^3 + 8y(x)^3 + 8xy(x)^2}{(2x - b)^3}$$

✓ **Mathematica** : cpu = 0.489692 (sec), leaf count = 128

$$\text{Solve}\left[-\frac{19}{3}\text{RootSum}\left[-19\#1^3 + 6\sqrt[3]{38}\#1 - 19\&, \frac{\log\left(\frac{\frac{4}{(b-2x)^2} - \frac{24y(x)}{(b-2x)^3}}{4\sqrt[3]{38}\sqrt[3]{\frac{1}{(b-2x)^6}}} - \#1\right)}{2\sqrt[3]{38} - 19\#1^2}\&\right] = \frac{1}{9}38^{2/3}\left(\frac{1}{(b-2x)^6}\right)^{2/3}(b-2x)^4\log(b-2x)\right]$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 41

$$\left\{y(x) = \frac{(b-2x)\text{RootOf}\left(c_1 - \left(\int^{-Z} \frac{1}{-a^3 - a^2 - a - 1} d_a\right) + \ln(b-2x)\right)}{2}\right\}$$

2.801 ODE No. 801

$$y'(x) = \frac{1}{2}e^{\frac{x^2}{4}}\left(2e^{-\frac{3x^2}{4}}y(x)^3 + 2e^{-\frac{x^2}{2}}y(x)^2 + e^{-\frac{x^2}{4}}xy(x) + 2\right)$$

✓ **Mathematica** : cpu = 0.4164 (sec), leaf count = 126

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{3e^{-\frac{x^2}{2}}y(x) + e^{-\frac{x^2}{4}}}{\sqrt[3]{29}\sqrt[3]{e^{-\frac{3x^2}{4}}}} - \#1\right)}{\sqrt[3]{29} - 29\#1^2}\&\right] = \frac{1}{9}29^{2/3}e^{\frac{x^2}{2}}\left(e^{-\frac{3x^2}{4}}\right)^{2/3}x + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.07 (sec), leaf count = 63

$$\left\{y(x) = \frac{\left(-3e^{-\frac{x^2}{4}}e^{\frac{x^2}{4}} + 29\text{RootOf}\left(3c_1 + x - 81\left(\int^{-Z} \frac{1}{841a^3 - 27a + 27} d_a\right)\right)\right)e^{-\frac{x^2}{4}}e^{\frac{x^2}{2}}}{9}\right\}$$

2.802 ODE No. 802

$$y'(x) = \frac{-F1\left(y(x) + \frac{1}{x}\right) + \frac{1}{x}}{x}$$

✓ **Mathematica** : cpu = 0.148624 (sec), leaf count = 101

$$\text{Solve}\left[\int_1^{y(x)} -\frac{F1\left(K[2] + \frac{1}{x}\right)\int_1^x -\frac{F1'\left(K[2] + \frac{1}{K[1]}\right)}{K[1]^2\left(F1\left(K[2] + \frac{1}{K[1]}\right)\right)^2} dK[1] + 1}{F1\left(K[2] + \frac{1}{x}\right)} dK[2] + \int_1^x \left(\frac{1}{K[1]} + \frac{1}{F1\left(y(x) + \frac{1}{K[1]}\right)K[1]^2}\right) dK[1] = c_1\right]$$

✓ **Maple** : cpu = 0.272 (sec), leaf count = 27

$$\left\{y(x) = \frac{x\text{RootOf}\left(c_1 + \int^{-Z} \frac{1}{F1(a)} d_a - \ln(x)\right) - 1}{x}\right\}$$

2.803 ODE No. 803

$$y'(x) = \frac{{}_2F1\left(y(x)^2 - 2 \log(x)\right)}{x \sqrt{y(x)^2}}$$

✓ **Mathematica** : cpu = 0.488373 (sec), leaf count = 637

Solve $\left[\int_1^{y(x)} \left(\frac{K[2]}{\left({}_2F1\left(K[2]^2 - 2 \log(x)\right) - 1\right) \left({}_2F1\left(K[2]^2 - 2 \log(x)\right) + 1\right)} - \int_1^x \left(\frac{2K[2]{}_2F1'\left(K[2]^2 - 2 \log(K[1])\right) \left({}_2F1\left(K[1]^2 - 2 \log(x)\right) - 1\right)}{K[1] \left({}_2F1\left(K[2]^2 - 2 \log(K[1])\right) - 1\right)^2 \left({}_2F1\left(K[2]^2 - 2 \log(x)\right) + 1\right)} dx \right) dy(x) \right]$

✓ **Maple** : cpu = 0.442 (sec), leaf count = 65

$$\left\{ y(x) = \sqrt{2 \operatorname{RootOf}\left(c_1 - \left(\int^{-Z} \frac{1}{{}_2F1(2\_a) - 1} d\_a\right) + \ln(x)\right)} + 2 \ln(x), y(x) = -\sqrt{2 \operatorname{RootOf}\left(c_1 - \left(\int^{-Z} \frac{1}{{}_2F1(2\_a) - 1} d\_a\right) + \ln(x)\right)} \right\}$$

2.804 ODE No. 804

$$y'(x) = \frac{\frac{1}{2}x^4 \cos(2y(x)) + \frac{x^4}{2} - \frac{1}{2}x \sin(2y(x)) - \frac{1}{2} \sin(2y(x))}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.676966 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow \tan^{-1}\left(\frac{3x^4 - 4x^3 + 6x^2 - 12x + 12 \log(x+1) - 25 - 12c_1}{12x}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.986 (sec), leaf count = 38

$$\left\{ y(x) = \arctan\left(\frac{3x^4 - 4x^3 + 6x^2 - 12c_1 - 12x + 12 \ln(x+1)}{12x}\right) \right\}$$

2.805 ODE No. 805

$$y'(x) = \frac{x^4 \sqrt{x^2 + y(x)^2} + xy(x) + y(x)}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.196832 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow x \sinh\left(\frac{1}{6}\left(2x^3 - 3x^2 + 6x - 6 \log(x+1) + 11 + 6c_1\right)\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.761 (sec), leaf count = 42

$$\left\{ -\frac{x^3}{3} + \frac{x^2}{2} - c_1 - x - \ln(x) + \ln(x+1) + \ln\left(y(x) + \sqrt{x^2 + y(x)^2}\right) = 0 \right\}$$

2.806 ODE No. 806

$$y'(x) = \frac{-\frac{1}{2}x \sin(2y(x)) - \frac{1}{2} \sin(2y(x)) + \frac{1}{2}x \cos(2y(x)) + \frac{x}{2}}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.479953 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow \tan^{-1}\left(\frac{x - \log(x+1) - c_1}{x}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.587 (sec), leaf count = 22

$$\left\{ y(x) = -\arctan\left(\frac{-c_1 - x + \ln(x+1)}{x}\right) \right\}$$

2.807 ODE No. 807

$$y'(x) = -\frac{1}{-e^{y(x)}y(x)\_F1(y(x) - \log(x)) - x}$$

✓ **Mathematica** : cpu = 0.321982 (sec), leaf count = 59

$$\text{Solve}\left[-\int_1^{y(x)-\log(x)}\frac{K[1]\_F1(K[1]) + e^{-K[1]}}{\_F1(K[1])}dK[1] - y(x)\log(x) + \frac{\log^2(x)}{2} = -c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.522 (sec), leaf count = 43

$$\left\{\frac{\ln(x)^2}{2} - \ln(x)y(x) + c_1 - \left(\int^{-\ln(x)+y(x)}\frac{\_a\_F1(\_a) + e^{-\_a}}{\_F1(\_a)}d\_a\right) = 0\right\}$$

2.808 ODE No. 808

$$y'(x) = \frac{(y(x) + 1)(2y(x) + 1)}{x(2xy(x) - 2y(x) + x - 2)}$$

✓ **Mathematica** : cpu = 2.34826 (sec), leaf count = 149

$$\text{Solve}\left[\frac{2^{2/3}\left(x\log\left(-\frac{6\ 2^{2/3}(y(x)+1)}{2(x-1)y(x)+x-2}\right) - x\log\left(\frac{3\ 2^{2/3}(2xy(x)+x)}{2(x-1)y(x)+x-2}\right) + 2xy(x)\left(\log\left(-\frac{6\ 2^{2/3}(y(x)+1)}{2(x-1)y(x)+x-2}\right) - \log\left(\frac{3\ 2^{2/3}(2xy(x)+x)}{2(x-1)y(x)+x-2}\right) + \log(x) + \log(2)\right)\right)}{9(2xy(x) + x)}\right]$$

✓ **Maple** : cpu = 0.299 (sec), leaf count = 45

$$\left\{y(x) = \frac{-x\text{LambertW}\left(\frac{e^{-\frac{1}{x}}}{c_1x}\right) - 2}{2x\text{LambertW}\left(\frac{e^{-\frac{1}{x}}}{c_1x}\right) + 2}\right\}$$

2.809 ODE No. 809

$$y'(x) = \frac{64x^3 - 240x^2 + 64xy(x)^2 + 64y(x)^3 - 80y(x)^2 + 300x - 125}{(4x - 5)^3}$$

✓ **Mathematica** : cpu = 0.392603 (sec), leaf count = 128

$$\text{Solve}\left[-\frac{19}{3}\text{RootSum}\left[-19\#1^3 + 6\sqrt[3]{38}\#1 - 19\&, \frac{\log\left(\frac{\frac{192y(x)}{(4x-5)^3} + \frac{16}{(4x-5)^2}}{16\sqrt[3]{38}\sqrt[3]{\frac{1}{(4x-5)^6}}} - \#1\right)}{2\sqrt[3]{38} - 19\#1^2}\&\right] = \frac{1}{9}38^{2/3}\left(\frac{1}{(5-4x)^6}\right)^{2/3}(5-4x)^4\log(5-4x)\right]$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 41

$$\left\{y(x) = -\frac{(4x - 5)\text{RootOf}\left(c_1 - \left(\int^{-Z}\frac{1}{\_a^3-\_a^2-\_a-1}d\_a\right) + \ln(4x - 5)\right)}{4}\right\}$$

2.810 ODE No. 810

$$y'(x) = \frac{x^2\log^2(x) + y(x)^2 + y(x) - 2xy(x)\log(x) + x}{x}$$

✓ **Mathematica** : cpu = 0.134703 (sec), leaf count = 40

$$\left\{\left\{y(x) \rightarrow -\frac{1}{2}x^2\left(\frac{1-2x\log(x)}{x^2}-\frac{1}{x^2}\right)+\frac{1}{-1+\frac{c_1}{x}}\right\}\right\}$$

✓ **Maple** : cpu = 0.065 (sec), leaf count = 16

$$\left\{y(x)=\left(\ln(x)+\frac{1}{c_1-x}\right)x\right\}$$

2.811 ODE No. 811

$$y'(x)=\frac{x^4+x^3e^{y(x)}+xy(x)+e^{y(x)}y(x)-x\log\left(e^{y(x)}+x\right)-e^{y(x)}\log\left(e^{y(x)}+x\right)+x}{x^2}$$

✓ **Mathematica** : cpu = 2.04327 (sec), leaf count = 33

$$\left\{\left\{y(x) \rightarrow -\log\left(-\frac{1}{x}+\frac{e^{-\frac{x^3}{2}-c_1x}}{x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 1.769 (sec), leaf count = 32

$$\left\{y(x)=\frac{x^3}{2}+c_1x+\ln\left(-\frac{x}{e^{c_1x}e^{\frac{x^3}{2}}-1}\right)\right\}$$

2.812 ODE No. 812

$$y'(x)=x^3\sqrt{x^3-6y(x)}+\sqrt{x^3-6y(x)}+\frac{x^2}{2}+x^2\sqrt{x^3-6y(x)}$$

✓ **Mathematica** : cpu = 0.440085 (sec), leaf count = 78

$$\left\{\left\{y(x) \rightarrow \frac{1}{96}\left(-9x^8-24x^7-16x^6-72x^5-102x^4+72c_1x^4+8x^3+96c_1x^3-144x^2-24x+288c_1x-1-144c_1^2+24c_1\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.389 (sec), leaf count = 30

$$\left\{-\frac{3x^4}{4}-x^3+c_1-3x-\sqrt{x^3-6y(x)}=0\right\}$$

2.813 ODE No. 813

$$y'(x)=\frac{1}{2}\sqrt{a}\left(2\sqrt{ax^4+8y(x)}-\sqrt{a}x^3+2x^3\sqrt{ax^4+8y(x)}+2x^2\sqrt{ax^4+8y(x)}\right)$$

✓ **Mathematica** : cpu = 0.659587 (sec), leaf count = 93

$$\left\{\left\{y(x) \rightarrow \frac{144ax^8+384ax^7+256ax^6+1152ax^5+1464ax^4-1152ac_1x^4+96ax^3-1536ac_1x^3+2304ax^2+288ax-46}{1152}\right\}\right\}$$

✓ **Maple** : cpu = 0.707 (sec), leaf count = 40

$$\left\{-c_1+\frac{\left(-3x^4-4x^3-12x\right)\sqrt{a}}{12}+\frac{\sqrt{ax^4+8y(x)}}{4}=0\right\}$$

2.814 ODE No. 814

$$y'(x) = \frac{y(x)(x^7y(x)^2 - 3x^3y(x) - 3)}{x(x^3y(x) + 1)}$$

✓ **Mathematica** : cpu = 0.151694 (sec), leaf count = 72

$$\left\{ \left(y(x) \rightarrow \frac{x}{-x^4 + \frac{\sqrt{x+x(-2x+c_1)}}{\sqrt{\frac{1}{x^7}}}} \right) \right\} \left\{ y(x) \rightarrow -\frac{x}{x^4 + \frac{\sqrt{x+x(-2x+c_1)}}{\sqrt{\frac{1}{x^7}}}} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 38

$$\left\{ y(x) = \frac{1}{(\sqrt{c_1 - 2x} - 1)x^3}, y(x) = -\frac{1}{(\sqrt{c_1 - 2x} + 1)x^3} \right\}$$

2.815 ODE No. 815

$$y'(x) = \frac{e^{3x^2}x(y(x) + 3)^3}{81\left(e^{\frac{3x^2}{2}}y(x) + 3e^{\frac{3x^2}{2}} + 3y(x)\right)}$$

✓ **Mathematica** : cpu = 12.8585 (sec), leaf count = 99

Solve $\left[\frac{1}{186}\left((31 + 3\sqrt{93})\log\left(9(9 + \sqrt{93})y(x) - 2e^{\frac{3x^2}{2}}(y(x) + 3)\right) + (31 - 3\sqrt{93})\log\left(2e^{\frac{3x^2}{2}}(y(x) + 3) + 9(\sqrt{93} - 9)y(x)\right)\right)\right]$

✓ **Maple** : cpu = 0.975 (sec), leaf count = 168

$$\left\{ 15x^2 - c_1 - \frac{30\sqrt{93} \operatorname{arctanh}\left(\frac{\left(29e^{\frac{3x^2}{2}}y(x) + 87e^{\frac{3x^2}{2}} + 81y(x)\right)\sqrt{93}}{(279y(x) + 837)e^{\frac{3x^2}{2}} + 837y(x)}\right)}{31} + 5\ln\left(\frac{-24300y(x)^2 + 100(y(x) + 3)^2e^{3x^2} + (-8100y(x)^2 - 189\left((y(x) + 3)e^{\frac{3x^2}{2}} + 3y(x)\right)^2)}{189\left((y(x) + 3)e^{\frac{3x^2}{2}} + 3y(x)\right)^2}\right) \right\}$$

2.816 ODE No. 816

$$y'(x) = \frac{x(x - y(x))^3(y(x) + x)^3}{y(x)(x^2 - y(x)^2 - 1)}$$

✓ **Mathematica** : cpu = 0.288363 (sec), leaf count = 74

Solve $\left[\frac{1}{2}\left(\operatorname{RootSum}\left[\#1^3 - \#1 + 1\&, \frac{\#1\log(-\#1 + x^2 - y(x)^2) - \log(-\#1 + x^2 - y(x)^2)}{3\#1^2 - 1}\&\right] + x^2\right) = c_1, y(x)\right]$

✓ **Maple** : cpu = 0.611 (sec), leaf count = 190

$$\left\{ c_1 + \int_{-b}^x \frac{(-a - y(x))^3(-a + y(x))^3_a}{_a^6 - 3_a^4y(x)^2 + 3_a^2y(x)^4 - y(x)^6 - _a^2 + y(x)^2 + 1}d_a + \int^{y(x)}\left(-\frac{(-f^2 + x^2 - 1)_f}{-_f^6 + 3_f^4x^2 - 3_f^2x^4 + x^6 + _f^2 - x^2 + \right) \right\}$$

2.817 ODE No. 817

$$y'(x) = \frac{\csc(y(x)) \left(\frac{1}{2} x^3 \log(x) \cos(2y(x)) + \frac{1}{2} x^3 \log(x) - \cos(y(x)) \right)}{x \log(x)}$$

✓ **Mathematica** : cpu = 0.677796 (sec), leaf count = 63

$$\left\{ \left\{ y(x) \rightarrow -\sec^{-1} \left(\frac{-x^3 + 3x^3 \log(x) - 9c_1}{9 \log(x)} \right) \right\}, \left\{ y(x) \rightarrow \sec^{-1} \left(\frac{-x^3 + 3x^3 \log(x) - 9c_1}{9 \log(x)} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.824 (sec), leaf count = 27

$$\left\{ y(x) = \arccos \left(\frac{9 \ln(x)}{3x^3 \ln(x) - x^3 + 9c_1} \right) \right\}$$

2.818 ODE No. 818

$$y'(x) = \frac{y(x)}{x(xy(x)^4 + xy(x)^3 + xy(x) - 1)}$$

✓ **Mathematica** : cpu = 0.265931 (sec), leaf count = 34

$$\text{Solve} \left[\frac{y(x)^3}{3} + \frac{y(x)^2}{2} + \frac{1}{xy(x)} + \log(y(x)) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.317 (sec), leaf count = 34

$$\{y(x) = e^{\text{RootOf}(6c_1 x e^{-Z} - 6_Z x e^{-Z} - 3x e^{3-Z} - 2x e^{4-Z} - 6)}\}$$

2.819 ODE No. 819

$$y'(x) = x^2 \sqrt{x^2 + 3y(x)} + \sqrt{x^2 + 3y(x)} + x^3 \sqrt{x^2 + 3y(x)} - \frac{2x}{3}$$

✓ **Mathematica** : cpu = 0.353813 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{192} (9x^8 + 24x^7 + 16x^6 + 72x^5 + 96x^4 - 72c_1 x^4 - 96c_1 x^3 + 80x^2 - 288c_1 x + 144c_1^2) \right\} \right\}$$

✓ **Maple** : cpu = 0.429 (sec), leaf count = 30

$$\left\{ \frac{3x^4}{8} + \frac{x^3}{2} + c_1 + \frac{3x}{2} - \sqrt{x^2 + 3y(x)} = 0 \right\}$$

2.820 ODE No. 820

$$y'(x) = \frac{\csc(y(x)) \left(\frac{1}{2} x^2 \log(x) \cos(2y(x)) + \frac{1}{2} x^2 \log(x) - \cos(y(x)) \right)}{x \log(x)}$$

✓ **Mathematica** : cpu = 0.573191 (sec), leaf count = 63

$$\left\{ \left\{ y(x) \rightarrow -\sec^{-1} \left(\frac{-x^2 + 2x^2 \log(x) - 4c_1}{4 \log(x)} \right) \right\}, \left\{ y(x) \rightarrow \sec^{-1} \left(\frac{-x^2 + 2x^2 \log(x) - 4c_1}{4 \log(x)} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.737 (sec), leaf count = 27

$$\left\{ y(x) = \arccos \left(\frac{4 \ln(x)}{2x^2 \ln(x) - x^2 + 4c_1} \right) \right\}$$

2.821 ODE No. 821

$$y'(x) = \frac{y(x)(xy(x) + 1)}{x(x^3y(x)^4 - xy(x) - 1)}$$

✓ **Mathematica** : cpu = 0.241739 (sec), leaf count = 2093

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{4} - \frac{1}{2} \sqrt{\frac{c_1^2}{4} + \frac{\sqrt[3]{1944c_1^2x^6 + 1458x^5} + \sqrt{(1944c_1^2x^6 + 1458x^5)^2 - 4(54c_1x^4 + 144x^3)^3}}{18\sqrt[3]{2}x^3}} + \frac{\sqrt[3]{1944c_1^2x^6 + 1458x^5}}{x^3\sqrt[3]{1944c_1^2x^6 + 1458x^5}} \right\} \right\}$$

✓ **Maple** : cpu = 0.177 (sec), leaf count = 27

$$\left\{ c_1 - y(x) - \frac{1}{2x^2y(x)^2} - \frac{1}{3x^3y(x)^3} = 0 \right\}$$

2.822 ODE No. 822

$$y'(x) = \frac{1}{4}x(-4e^{-x^2}x^2y(x) - 4e^{-x^2}x^2 + 4e^{-x^2} + e^{-2x^2}x^4 + 4y(x)^2)$$

✓ **Mathematica** : cpu = 0.364136 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}e^{-x^2}x^2 + \frac{1}{-\frac{x^2}{2} + c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.146 (sec), leaf count = 25

$$\left\{ y(x) = \frac{x^2e^{-x^2}}{2} + \frac{1}{-\frac{x^2}{2} + c_1} \right\}$$

2.823 ODE No. 823

$$y'(x) = \frac{y(x)(y(x) + x)}{x(y(x)^4 + y(x)^3 + y(x) + x)}$$

✓ **Mathematica** : cpu = 0.486311 (sec), leaf count = 39

$$\text{Solve}\left[\frac{y(x)^3}{3} + \frac{y(x)^2}{2} + \log(y(x)) - \frac{y(x)\log(x) + x}{y(x)} = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.234 (sec), leaf count = 38

$$\{y(x) = e^{\text{RootOf}(6c_1e^{-Z} - 6_Ze^{-Z} + 6e^{-Z}\ln(x) + 6x - 3e^{3-Z} - 2e^{4-Z})}\}$$

2.824 ODE No. 824

$$y'(x) = \frac{y(x)(x^3 + x^2y(x) + y(x)^2)}{(x - 1)x^2(y(x) + x)}$$

✓ **Mathematica** : cpu = 0.281078 (sec), leaf count = 68

$$\text{Solve}\left[-\frac{1}{2}\log\left(\frac{y(x)^2}{x^2} + \frac{y(x)}{x} + 1\right) + \log\left(\frac{y(x)}{x}\right) + \frac{\tan^{-1}\left(\frac{\frac{2y(x)}{x} + 1}{\sqrt{3}}\right)}{\sqrt{3}} = \log(1 - x) - \log(x) + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.392 (sec), leaf count = 61

$$\left\{-c_1+\frac{\sqrt{3}\arctan\left(\frac{\left(x+2y(x)\right)\sqrt{3}}{3x}\right)}{3}+\ln(x)-\frac{\ln\left(\frac{x^2+xy(x)+y(x)^2}{x^2}\right)}{2}+\ln\left(\frac{y(x)}{x}\right)-\ln(x-1)=0\right\}$$

2.825 ODE No. 825

$$y'(x)=\frac{x\left(x^2y(x)^3+\left(x^2+1\right)^{3/2}y(x)^2+x^2\left(x^2+1\right)^{3/2}+\left(x^2+1\right)^{3/2}+y(x)^3\right)}{\left(x^2+1\right)^3}$$

✓ **Mathematica** : cpu = 0.587979 (sec), leaf count = 148

$$\text{Solve}\left[-\frac{19}{3}\text{RootSum}\left[-19\#1^3+6\sqrt[3]{38}\#1-19\&,\frac{\log\left(\frac{\frac{3xy(x)}{\left(x^2+1\right)^2}+\frac{x}{\left(x^2+1\right)^{3/2}}}{\sqrt[3]{38}\sqrt[3]{\frac{x^3}{\left(x^2+1\right)^{9/2}}}}-\#1\right)}{2\sqrt[3]{38}-19\#1^2}\&\right]=\frac{19^{2/3}\left(\frac{x^3}{\left(x^2+1\right)^{9/2}}\right)^{2/3}\left(x^2+1\right)^3\log\left(x^2+1\right)}{9\sqrt[3]{2}x^2}\right]$$

✓ **Maple** : cpu = 0.126 (sec), leaf count = 48

$$\left\{y(x)=\frac{\left(19\text{RootOf}\left(3c_1-1296\left(\int^{-Z}\frac{1}{361_a^3-432_a+432}d_a\right)+2\ln\left(x^2+1\right)\right)-6\right)\sqrt{x^2+1}}{18}\right\}$$

2.826 ODE No. 826

$$y'(x)=\frac{y(x)\left(3xy(x)^2+3y(x)^2+x\right)}{x(x+1)\left(6y(x)^2+x\right)}$$

✓ **Mathematica** : cpu = 0.580116 (sec), leaf count = 70

$$\left\{\left\{y(x)\rightarrow-\frac{\sqrt{x}\sqrt{W\left(\frac{6e^{2c_1}x}{\left(x+1\right)^2}\right)}}{\sqrt{6}}\right\},\left\{y(x)\rightarrow\frac{\sqrt{x}\sqrt{W\left(\frac{6e^{2c_1}x}{\left(x+1\right)^2}\right)}}{\sqrt{6}}\right\}\right\}$$

✓ **Maple** : cpu = 0.426 (sec), leaf count = 51

$$\left\{\frac{1}{\frac{6}{x}+\frac{1}{y(x)^2}}=\frac{\left(\text{e}^{\text{RootOf}\left(3c_1\text{e}^{-Z}+_Ze^{-Z}-\text{e}^{-Z}\ln\left(\frac{\left(x+1\right)^2\left(\text{e}^{-Z}+9\right)}{2x}\right)\right)+9}x\right)}{54}\right\}$$

2.827 ODE No. 827

$$y'(x)=\frac{x^2y(x)\sqrt{x^2+y(x)^2}+x^3\left(-\sqrt{x^2+y(x)^2}\right)+y(x)}{x}$$

✓ **Mathematica** : cpu = 0.305578 (sec), leaf count = 221

$$\left\{ \left\{ y(x) \rightarrow \frac{x - 2\sqrt{x^2 \tanh^2\left(\frac{1}{3}(-\sqrt{2}x^3 - 3\sqrt{2}c_1)\right)} - x^2 \tanh^4\left(\frac{1}{3}(-\sqrt{2}x^3 - 3\sqrt{2}c_1)\right)}{-1 + 2 \tanh^2\left(\frac{1}{3}(-\sqrt{2}x^3 - 3\sqrt{2}c_1)\right)} \right\}, \left\{ y(x) \rightarrow \frac{x + 2\sqrt{x^2 \tanh^2\left(\frac{1}{3}(-\sqrt{2}x^3 - 3\sqrt{2}c_1)\right)}}{-1 + 2 \tanh^2\left(\frac{1}{3}(-\sqrt{2}x^3 - 3\sqrt{2}c_1)\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.341 (sec), leaf count = 49

$$\left\{ \frac{\sqrt{2} x^3}{3} - c_1 - \ln(x) + \ln\left(\frac{2(x + y(x) + \sqrt{2x^2 + 2y(x)^2})x}{-x + y(x)}\right) = 0 \right\}$$

2.828 ODE No. 828

$$y'(x) = \frac{(y(x) + 1)(2y(x) + 1)}{x(2xy(x)^4 + xy(x)^3 - 2y(x) - 2)}$$

✓ **Mathematica** : cpu = 0.687373 (sec), leaf count = 56

$$\text{Solve}\left[-\frac{1}{8}y(x)^2 + \frac{3y(x)}{8} - \frac{1}{2x(2y(x) + 1)} - \frac{1}{2}\log(y(x) + 1) + \frac{1}{16}\log(2y(x) + 1) = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.318 (sec), leaf count = 54

$$\left\{ y(x) = \frac{e^{\text{RootOf}\left(8c_1x e^{-Z} - 2_Zx e^{-Z} + 16x e^{-Z} \ln\left(\frac{e^{-Z}}{2} + \frac{1}{2}\right) + 7x e^{-Z} - 8x e^{2-Z} + x e^{3-Z} + 16\right)}}{2} - \frac{1}{2} \right\}$$

2.829 ODE No. 829

$$y'(x) = \frac{x^6\sqrt{4x^2y(x) + 1} + x^5\sqrt{4x^2y(x) + 1} + x^3\sqrt{4x^2y(x) + 1} + \frac{1}{2}}{x^3}$$

✓ **Mathematica** : cpu = 0.532115 (sec), leaf count = 74

$$\left\{ \left\{ y(x) \rightarrow \frac{16x^{12} + 40x^{11} + 25x^{10} + 80x^9 + 100x^8 - 160c_1x^7 + 100x^6 - 200c_1x^6 - 400c_1x^4 + 400c_1^2x^2 - 100}{400x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.431 (sec), leaf count = 34

$$\left\{ \frac{2x^5}{5} + \frac{x^4}{2} + x^2 + c_1 - \frac{\sqrt{4x^2y(x) + 1}}{x} = 0 \right\}$$

2.830 ODE No. 830

$$y'(x) = \frac{(x - y(x))y(x)}{x(-y(x)^4 - y(x)^3 - y(x) + x)}$$

✓ **Mathematica** : cpu = 0.513535 (sec), leaf count = 37

$$\text{Solve}\left[-\frac{1}{3}y(x)^3 - \frac{y(x)^2}{2} - \frac{x}{y(x)} - \log(y(x)) + \log(x) = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.23 (sec), leaf count = 38

$$\{y(x) = e^{\text{RootOf}\left(6c_1e^{-Z} + 6_Ze^{-Z} - 6e^{-Z} \ln(x) + 6x + 3e^{3-Z} + 2e^{4-Z}\right)}\}$$

2.831 ODE No. 831

$$y'(x) = \frac{x^3\sqrt{4ax - y(x)^2} + x^2\sqrt{4ax - y(x)^2} + \sqrt{4ax - y(x)^2} + 2a}{y(x)}$$

✓ **Mathematica** : cpu = 3.37839 (sec), leaf count = 145

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{12}\sqrt{576ax - 9x^8 - 24x^7 - 16x^6 - 72x^5 - 96x^4 - 72c_1x^4 - 96c_1x^3 - 144x^2 - 288c_1x - 144c_1^2} \right\}, \left\{ y(x) \rightarrow \frac{1}{12}\sqrt{576ax - 9x^8 - 24x^7 - 16x^6 - 72x^5 - 96x^4 - 72c_1x^4 - 96c_1x^3 - 144x^2 - 288c_1x - 144c_1^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.386 (sec), leaf count = 35

$$\left\{ -\frac{x^4}{4} - \frac{x^3}{3} - c_1 - x - \sqrt{4ax - y(x)^2} = 0 \right\}$$

2.832 ODE No. 832

$$y'(x) = \frac{y(x)(y(x) + x + 1)}{(x + 1)(y(x)^4 + y(x)^3 + y(x)^2 + x)}$$

✓ **Mathematica** : cpu = 2.37499 (sec), leaf count = 2497

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2}\sqrt{\frac{3\sqrt[3]{2}(-8x + 3c_1 + 3\log(x + 1) + 2)}{\sqrt[3]{1944(c_1 + \log(x + 1))^2 + 972(c_1 + \log(x + 1)) + 3726x} + \sqrt{(1944(c_1 + \log(x + 1))^2 + 972(c_1 + \log(x + 1)) + 3726x)}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.189 (sec), leaf count = 31

$$\left\{ -\frac{y(x)^3}{3} - \frac{y(x)^2}{2} + c_1 + \ln(x + 1) - y(x) + \frac{x}{y(x)} = 0 \right\}$$

2.833 ODE No. 833

$$y'(x) = \frac{x^4(-\sqrt{x^2 + y(x)^2}) + x^3y(x)\sqrt{x^2 + y(x)^2} + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.305299 (sec), leaf count = 221

$$\left\{ \left\{ y(x) \rightarrow \frac{x - 2\sqrt{x^2 \tanh^2\left(\frac{1}{4}(-\sqrt{2}x^4 - 4\sqrt{2}c_1)\right)} - x^2 \tanh^4\left(\frac{1}{4}(-\sqrt{2}x^4 - 4\sqrt{2}c_1)\right)}{-1 + 2 \tanh^2\left(\frac{1}{4}(-\sqrt{2}x^4 - 4\sqrt{2}c_1)\right)} \right\} \right\}, \left\{ y(x) \rightarrow \frac{x + 2\sqrt{x^2 \tanh^2\left(\frac{1}{4}(-\sqrt{2}x^4 - 4\sqrt{2}c_1)\right)} - x^2 \tanh^4\left(\frac{1}{4}(-\sqrt{2}x^4 - 4\sqrt{2}c_1)\right)}{-1 + 2 \tanh^2\left(\frac{1}{4}(-\sqrt{2}x^4 - 4\sqrt{2}c_1)\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.301 (sec), leaf count = 49

$$\left\{ \frac{\sqrt{2}x^4}{4} - c_1 - \ln(x) + \ln\left(\frac{2(x + y(x) + \sqrt{2x^2 + 2y(x)^2})x}{-x + y(x)}\right) = 0 \right\}$$

2.834 ODE No. 834

$$y'(x) = \frac{y(x)(x^4 + 3xy(x)^2 + 3y(x)^2)}{x(x + 1)(6y(x)^2 + x)}$$

✓ **Mathematica** : cpu = 0.687405 (sec), leaf count = 90

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{x} \sqrt{W\left(\frac{6(x+1)^2 e^{x^2-2x-3+2c_1}}{x}\right)}}{\sqrt{6}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{x} \sqrt{W\left(\frac{6(x+1)^2 e^{x^2-2x-3+2c_1}}{x}\right)}}{\sqrt{6}} \right\} \right\}$$

✓ **Maple** : cpu = 0.736 (sec), leaf count = 60

$$\left\{ \frac{1}{\frac{6}{x} + \frac{1}{y(x)^2}} = \frac{\left(e^{\text{RootOf}\left(x^2 e^{-Z} + 3c_1 e^{-Z} + \_Z e^{-Z} - 2x e^{-Z} - e^{-Z} \ln\left(\frac{e^{-Z} + 9}{2(x+1)^2}\right) + 9\right)} + 9 \right) x}{54} \right\}$$

2.835 ODE No. 835

$$y'(x) = -\frac{1}{x\left(-\sqrt[3]{y(x)^3}\right)_F1\left(y(x)^3-3\log(x)\right)-x\left(y(x)^3\right)^{2/3}}$$

✗ **Mathematica** : cpu = 1.85578 (sec), leaf count = 0 , could not solve

```
DSolve[Derivative[1][y][x] == -(-(x*(y[x]^3)^(2/3)) - x*(y[x]^3)^(1/3)*_F1[-3*Log[x] + y[x]^3], y[x], x]
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve(diff(y(x),x) = -1/(-(y(x)^3)^(2/3)*x-_F1(y(x)^3-3*ln(x))*(y(x)^3)^(1/3)*x),y(x))
```

2.836 ODE No. 836

$$y'(x) = \frac{(x-y(x))y(x)(y(x)+1)}{x(xy(x)-y(x)+x)}$$

✓ **Mathematica** : cpu = 14.7816 (sec), leaf count = 379

$$\text{Solve}\left[\frac{1}{9}2^{2/3}\left[\frac{\left(1-\frac{(x-1)^2\left(\frac{x^6}{(x-1)^3}\right)^{2/3}((x+2)y(x)+x)}{x^4((x-1)y(x)+x)}\right)\left(\frac{\left(\frac{x^6}{(x-1)^3}\right)^{2/3}(x-1)^2((x+2)y(x)+x)}{x^4((x-1)y(x)+x)}+2\right)\left(1-\frac{(x-1)^2\left(\frac{x^6}{(x-1)^3}\right)^{2/3}((x+2)y(x)+x)}{x^4((x-1)y(x)+x)}\right)\log\left(2^{2/3}\left(1-\frac{(x-1)^2\left(\frac{x^6}{(x-1)^3}\right)^{2/3}((x+2)y(x)+x)}{x^4((x-1)y(x)+x)}\right)\right)}{\frac{3(x-1)^2\left(\frac{x^6}{(x-1)^3}\right)^{2/3}((x+2)y(x)+x)}{x^4((x-1)y(x)+x)}}\right]$$

✓ **Maple** : cpu = 0.301 (sec), leaf count = 73

$$\left\{ y(x) = -\frac{x e^{\text{RootOf}\left(3c_1 e^{-Z} + \_Z e^{-Z} - x e^{-Z} - e^{-Z} \ln\left(\frac{e^{-Z}}{2} + \frac{9}{2}\right) + 9\right)}}{(x-1)e^{\text{RootOf}\left(3c_1 e^{-Z} + \_Z e^{-Z} - x e^{-Z} - e^{-Z} \ln\left(\frac{e^{-Z}}{2} + \frac{9}{2}\right) + 9\right)} - 9} \right\}$$

2.837 ODE No. 837

$$y'(x) = -\frac{1}{-\sqrt[3]{y(x)^3} \log(x)_F1\left(3\text{Ei}(-\log(x))+y(x)^3\right)-\left(y(x)^3\right)^{2/3} \log(x)}$$

✗ **Mathematica** : cpu = 2.20892 (sec), leaf count = 0 , could not solve

```
DSolve[Derivative[1][y][x] == -(-(Log[x]*(y[x]^3)^(2/3)) - Log[x]*(y[x]^3)^(1/3)*_F1[3*ExpInLog[x]] + y[x]^3))^(1/3), y[x], x]
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve(diff(y(x),x) = -1/(-ln(x)*(y(x)^3)^(2/3)-_F1(y(x)^3+3*Ei(1,-ln(x)))*ln(x)*(y(x)^3)^(1/3)),y(x))
```

2.838 ODE No. 838

$$y'(x) = \frac{\frac{8x^{7/2}}{5} + \frac{4x^6}{25} - \frac{4}{5}x^3y(x) + \frac{6x^3}{5} - 4\sqrt{x}y(x) + y(x)^2 + 4x + \sqrt{x}}{x}$$

✓ **Mathematica** : cpu = 0.197133 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow \frac{2}{5}\sqrt{x}(x^{5/2} + 5) + \frac{1}{-\log(x) + c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.118 (sec), leaf count = 25

$$\left\{ y(x) = \frac{2\left(x^2 + \frac{5}{\sqrt{x}}\right)x}{5} + \frac{1}{c_1 - \ln(x)} \right\}$$

2.839 ODE No. 839

$$y'(x) = \frac{e^{\frac{y(x)}{x}} \left(x^2 + xe^{-\frac{y(x)}{x}} + e^{-\frac{y(x)}{x}} y(x) \right)}{x}$$

✓ **Mathematica** : cpu = 0.346186 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow -x \log\left(-\frac{x}{2} + \frac{e^{2c_1}}{2x}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.076 (sec), leaf count = 19

$$\left\{ y(x) = x \ln\left(\frac{2x}{-x^2 + c_1}\right) \right\}$$

2.840 ODE No. 840

$$y'(x) = \frac{e^{\frac{y(x)}{x}} \left(x^3 + xe^{-\frac{y(x)}{x}} + e^{-\frac{y(x)}{x}} y(x) \right)}{x}$$

✓ **Mathematica** : cpu = 0.340677 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow -x \log\left(-\frac{x^2}{3} + \frac{e^{3c_1}}{3x}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 19

$$\left\{ y(x) = x \ln\left(\frac{3x}{-x^3 + c_1}\right) \right\}$$

2.841 ODE No. 841

$$y'(x) = \frac{-2a^{3/2}bx^2y(x)^2 + 2a^{3/2}cy(x)^2 + a^{5/2}y(x)^4 + \sqrt{a}b^2x^4 - 2\sqrt{a}bcx^2 + \sqrt{a}c^2 + bx^3}{ax^2y(x)}$$

✓ **Mathematica** : cpu = 1.31015 (sec), leaf count = 236

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{2a^{5/2}bx^2 - 2a^{5/2}c + 4a^3b^2x^3 - 4a^3bcx + a^2x + 4\sqrt{a}b^2c_1x^2 - 4\sqrt{a}bcc_1 + 2bc_1x}}{\sqrt{2}\sqrt{2a^{3/2}bc_1 + a^{7/2} + 2a^4bx}} \right\} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{2a^{5/2}bx^2 - 2a^{5/2}c + 4a^3b^2x^3 - 4a^3bcx + a^2x + 4\sqrt{a}b^2c_1x^2 - 4\sqrt{a}bcc_1 + 2bc_1x}}{\sqrt{2}\sqrt{2a^{3/2}bc_1 + a^{7/2} + 2a^4bx}} \right\}$$

✓ **Maple** : cpu = 0.295 (sec), leaf count = 97

$$\left\{ y(x) = \frac{\sqrt{(c_1x + 1)\left(\frac{x}{2} + (c_1x + 1)(bx^2 - c)\sqrt{a}\right)a^{\frac{3}{2}}}}{(c_1x + 1)a^{\frac{3}{2}}}, y(x) = -\frac{2\sqrt{(c_1x + 1)\left(\frac{x}{2} + (c_1x + 1)(bx^2 - c)\sqrt{a}\right)a^{\frac{3}{2}}}}{(2c_1x + 2)a^{\frac{3}{2}}} \right\}$$

2.842 ODE No. 842

$$y'(x) = \frac{2x^2y(x)\log^2(x) + x^2y(x)^2\log(x) + x^2\log^3(x) + y(x)}{x\log(x)}$$

✓ **Mathematica** : cpu = 0.265561 (sec), leaf count = 186

$$\left\{ \left\{ y(x) \rightarrow -\frac{\frac{1}{4}x^2e^{\frac{1}{4}x^2(2\log(x)-1)}(2\log(x)-1)\left(\frac{x}{2} + \frac{1}{2}x(2\log(x)-1)\right) + \frac{1}{2}xe^{\frac{1}{4}x^2(2\log(x)-1)} + \frac{1}{2}xe^{\frac{1}{4}x^2(2\log(x)-1)}(2\log(x)-1) + c_1}{x\left(\frac{1}{4}x^2e^{\frac{1}{4}x^2(2\log(x)-1)}(2\log(x)-1) + c_1e^{\frac{1}{4}x^2(2\log(x)-1)}\right)} \right\} \right.$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 43

$$\left\{ y(x) = -\frac{(2x^2\ln(x) - x^2 + 2c_1 + 4)\ln(x)}{2x^2\ln(x) - x^2 + 2c_1} \right\}$$

2.843 ODE No. 843

$$y'(x) = \frac{2x^3y(x)\log^2(x) + x^3y(x)^2\log(x) + x^3\log^3(x) + y(x)}{x\log(x)}$$

✓ **Mathematica** : cpu = 0.23954 (sec), leaf count = 198

$$\left\{ \left\{ y(x) \rightarrow -\frac{\frac{1}{9}x^3e^{\frac{1}{9}x^3(3\log(x)-1)}(3\log(x)-1)\left(\frac{x^2}{3} + \frac{1}{3}x^2(3\log(x)-1)\right) + \frac{1}{3}x^2e^{\frac{1}{9}x^3(3\log(x)-1)} + \frac{1}{3}x^2e^{\frac{1}{9}x^3(3\log(x)-1)}(3\log(x)-1)}{x^2\left(\frac{1}{9}x^3e^{\frac{1}{9}x^3(3\log(x)-1)}(3\log(x)-1) + c_1e^{\frac{1}{9}x^3(3\log(x)-1)}\right)} \right\} \right.$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 43

$$\left\{ y(x) = -\frac{(6x^3\ln(x) - 2x^3 + 9c_1 + 18)\ln(x)}{6x^3\ln(x) - 2x^3 + 9c_1} \right\}$$

2.844 ODE No. 844

$$y'(x) = \frac{y(x)(y(x) + 1)(y(x) + x)}{x(xy(x) + y(x) + x)}$$

✓ **Mathematica** : cpu = 15.8113 (sec), leaf count = 386

Solve

$$\left[\frac{2^{2/3} \left(1 - \frac{\left(\frac{x^6}{(x+1)^3} \right)^{2/3} (x+1)^2 ((x-2)y(x)+x)}{x^4 ((x+1)y(x)+x)} \right) \left(\frac{\left(\frac{x^6}{(x+1)^3} \right)^{2/3} (x+1)^2 ((x-2)y(x)+x)}{x^4 ((x+1)y(x)+x)} + 2 \right) \left(\left(1 - \frac{\left(\frac{x^6}{(x+1)^3} \right)^{2/3} (x+1)^2 ((x-2)y(x)+x)}{x^4 ((x+1)y(x)+x)} \right) \log \left(2^{2/3} \left(1 - \frac{\left(\frac{x^6}{(x+1)^3} \right)^{2/3} (x+1)^2 ((x-2)y(x)+x)}{x^4 ((x+1)y(x)+x)} \right) \right)}{9 \left(\frac{3 \left(\frac{x^6}{(x+1)^3} \right)^{2/3} (x+1)^2 ((x-2)y(x)+x)}{x^4 ((x+1)y(x)+x)} - \right.} \right.$$

✓ **Maple** : cpu = 0.304 (sec), leaf count = 97

$$\left\{ y(x) = -\frac{x e^{\text{RootOf}\left(3c_1 e^{-Z} + _Z e^{-Z} + x e^{-Z} - e^{-Z} \ln\left(\frac{e^{-Z}}{2} + \frac{9}{2}\right) + 9\right)}}{x e^{\text{RootOf}\left(3c_1 e^{-Z} + _Z e^{-Z} + x e^{-Z} - e^{-Z} \ln\left(\frac{e^{-Z}}{2} + \frac{9}{2}\right) + 9\right)} + e^{\text{RootOf}\left(3c_1 e^{-Z} + _Z e^{-Z} + x e^{-Z} - e^{-Z} \ln\left(\frac{e^{-Z}}{2} + \frac{9}{2}\right) + 9\right)} + 9} \right\}$$

2.845 ODE No. 845

$$y'(x) = \frac{\sqrt{4y(x)^3 - 9x^4} + 3x^3 + x^3\sqrt{4y(x)^3 - 9x^4} + x^2\sqrt{4y(x)^3 - 9x^4}}{y(x)^2}$$

✓ **Mathematica** : cpu = 4.34805 (sec), leaf count = 266

$$\left\{\left\{y(x) \rightarrow -\frac{1}{2} \sqrt[3]{-\frac{1}{2} \sqrt[3]{9x^8 + 24x^7 + 16x^6 + 72x^5 + 114x^4 + 72c_1x^4 - 24x^3 + 96c_1x^3 + 144x^2 - 72x + 288c_1x + 9 + 144c_1}}$$

✓ **Maple** : cpu = 0.234 (sec), leaf count = 44

$$\left\{-\frac{x^4}{4}-\frac{x^3}{3}-c_1-x+\int_{-b}^{y(x)}\frac{-a^2}{\sqrt{-9x^4+4-a^3}}d_{-a}=0\right\}$$

2.846 ODE No. 846

$$y'(x) = \frac{1}{-\left(x^2\left(\frac{1}{y(x)}+1\right)\text{F1}\left(x\left(\frac{1}{y(x)}+1\right)\right)\right)+x^2\text{F1}\left(x\left(\frac{1}{y(x)}+1\right)\right)+x\left(\frac{1}{y(x)}+1\right)-x}$$

✓ **Mathematica** : cpu = 0.926777 (sec), leaf count = 365

$$\text{Solve}\left[\int_1^{y(x)}\left(\frac{x_{\text{F1}}\left(x\left(1+\frac{1}{K[2]}\right)\right)-1}{x_{\text{F1}}\left(x\left(1+\frac{1}{K[2]}\right)\right)K[2]-K[2]+x_{\text{F1}}\left(x\left(1+\frac{1}{K[2]}\right)\right)}-\int_1^x\left(\frac{-\text{F1}\left(K[1]\left(1+\frac{1}{K[2]}\right)\right)-\frac{K[1]\text{F1}'\left(K[1]\left(1+\frac{1}{K[2]}\right)\right)}{K[2]}}{K[1]\left(K[2]\text{F1}\left(K[1]\left(1+\frac{1}{K[2]}\right)\right)+\text{F1}\left(K[1]\left(1+\frac{1}{K[2]}\right)\right)}\right)dx\right]=0\right]$$

✓ **Maple** : cpu = 0.306 (sec), leaf count = 40

$$\left\{y(x)=\text{e}^{\text{RootOf}\left(c_1-Z-\left(\int\frac{x\text{e}^{-Z}}{\text{e}^{-Z}-1}\frac{1}{(-a_{\text{F1}}(a)-1)_{-a}}d_{-a}\right)\right)}-1\right\}$$

2.847 ODE No. 847

$$y'(x) = x^2\sqrt{x^2 - 4y(x) + 2x + 1} + \sqrt{x^2 - 4y(x) + 2x + 1} + x^3\sqrt{x^2 - 4y(x) + 2x + 1} + \frac{x}{2} + \frac{1}{2}$$

✓ **Mathematica** : cpu = 1.44182 (sec), leaf count = 103

$$\text{Solve}\left[\frac{x^4}{4}+\frac{x^3}{3}+\frac{1}{2}\sqrt{x^2-4y(x)+2x+1}+\frac{1}{2}\log\left(\sqrt{x^2-4y(x)+2x+1}+x+1\right)-\frac{1}{2}\tanh^{-1}\left(\frac{2x+2}{2\sqrt{x^2-4y(x)+2x+1}}\right)=0\right]$$

✓ **Maple** : cpu = 0.384 (sec), leaf count = 34

$$\left\{-\frac{x^4}{2}-\frac{2x^3}{3}+c_1-2x-\sqrt{x^2+2x-4y(x)+1}=0\right\}$$

2.848 ODE No. 848

$$y'(x) = \text{F1}(y(x) - \log(\sinh(x))) + \coth(x)$$

✓ **Mathematica** : cpu = 0.268143 (sec), leaf count = 157

$$\text{Solve}\left[\int_1^{y(x)}-\frac{\text{F1}(K[2]-\log(\sinh(x)))\int_1^x\left(\frac{(\coth(K[1])+\text{F1}(K[2]-\log(\sinh(K[1])))\text{F1}'(K[2]-\log(\sinh(K[1])))}{(\text{F1}(K[2]-\log(\sinh(K[1])))^2}-\frac{\text{F1}'(K[2]-\log(\sinh(K[1]))}{\text{F1}(K[2]-\log(\sinh(K[1]))}}\right)dx}{\text{F1}(K[2]-\log(\sinh(x)))}=0\right]$$

✓ **Maple** : cpu = 0.545 (sec), leaf count = 27

$$\left\{-c_1-x+\int_{-b}^{y(x)}\frac{1}{\_F1(\_a-\ln(\sinh(x)))}d\_a=0\right\}$$

2.849 ODE No. 849

$$y'(x)=x^2\sqrt{x^2+4y(x)-4x}+\sqrt{x^2+4y(x)-4x}+x^3\sqrt{x^2+4y(x)-4x}-\frac{x}{2}+1$$

✓ **Mathematica** : cpu = 1.27731 (sec), leaf count = 102

Solve $\left[\frac{x^4}{4}+\frac{x^3}{3}-\frac{1}{2}\sqrt{x^2+4y(x)-4x}+\log\left(-\sqrt{x^2+4y(x)-4x}-x+2\right)-\tanh^{-1}\left(\frac{2x-4}{2\sqrt{x^2+4y(x)-4x}}\right)-\frac{1}{2}\log(2-2y(x))\right]$

✓ **Maple** : cpu = 0.388 (sec), leaf count = 33

$$\left\{\frac{x^4}{2}+\frac{2x^3}{3}+c_1+2x-\sqrt{x^2-4x+4y(x)}=0\right\}$$

2.850 ODE No. 850

$$y'(x)=\_F1(y(x)-\log(\sin(x))+\log(\cos(x)+1))+\csc(x)$$

✓ **Mathematica** : cpu = 0.405695 (sec), leaf count = 1485

Solve $\left[\int_1^x-\frac{\left(\cot^2(K[1])+\csc(K[1])\cot(K[1])+1\right)\sin(K[1])(\csc(K[1])+\_F1(\log(\cos(K[1])+1)-\log(\sin(K[1])))}{-\cot^2(K[1])+\_F1(\log(\cos(K[1])+1)-\log(\sin(K[1]))) + y(x))\cot(K[1]) + \csc^2(K[1]) + \csc(K[1])\_F1(\log(\cos(K[1])+1)-\log(\sin(K[1])))}dx\right]$

✓ **Maple** : cpu = 1.058 (sec), leaf count = 32

$$\left\{-c_1-x+\int_{-b}^{y(x)}\frac{1}{\_F1(\_a+\ln(\cos(x)+1)-\ln(\sin(x)))}d\_a=0\right\}$$

2.851 ODE No. 851

$$y'(x)=\frac{a^3x^3+3a^2bx^2y(x)+a^2bx^2+3ab^2xy(x)^2+2ab^2xy(x)+b^3y(x)^3+b^3y(x)^2+b^3}{b^3}$$

✓ **Mathematica** : cpu = 0.439095 (sec), leaf count = 145

Solve $\left[-\frac{1}{3}(27a+29b)^{2/3}\text{RootSum}\left[\#1^3(27a+29b)^{2/3}-3\#1b^{2/3}+(27a+29b)^{2/3}\&, \frac{\log\left(\frac{3ax+b}{b}+3y(x)\right)-\#1}{\sqrt[3]{\frac{27a+29b}{b}}}\&\right]=\frac{1}{9}x\left(\frac{27a+29b}{b}\right)^{2/3}\right]$

✓ **Maple** : cpu = 0.061 (sec), leaf count = 42

$$\left\{y(x)=\frac{-ax+b\text{RootOf}\left(b\left(\int^Z\frac{1}{\_a^3b+\_a^2b+a+b}d\_a\right)+c_1-x\right)}{b}\right\}$$

2.852 ODE No. 852

$$y'(x) = \frac{\alpha^3 y(x)^3 + \alpha^3 y(x)^2 + \alpha^3 + 3\alpha^2 \beta x y(x)^2 + 2\alpha^2 \beta x y(x) + 3\alpha \beta^2 x^2 y(x) + \alpha \beta^2 x^2 + \beta^3 x^3}{\alpha^3}$$

✓ **Mathematica** : cpu = 0.417481 (sec), leaf count = 145

$$\text{Solve}\left[-\frac{1}{3}(29\alpha + 27\beta)^{2/3}\text{RootSum}\left[\#1^3(29\alpha + 27\beta)^{2/3} - 3\#1\alpha^{2/3} + (29\alpha + 27\beta)^{2/3}\&, \frac{\log\left(\frac{\frac{\alpha+3\beta x}{\alpha}+3y(x)}{\sqrt[3]{\frac{29\alpha+27\beta}{\alpha}}}-\#1\right)}{\alpha^{2/3}-\#1^2(29\alpha+27\beta)^{2/3}}\&\right]=\frac{1}{9}x\right]$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 42

$$\left\{y(x) = \frac{\alpha \text{RootOf}\left(\alpha\left(\int^{-Z} \frac{1}{-a^3\alpha + a^2\alpha + a\beta} d_a\right) + c_1 - x\right) - \beta x}{\alpha}\right\}$$

2.853 ODE No. 853

$$y'(x) = \frac{x^3 y(x)^3 + 6x^2 y(x)^2 + 14xy(x) + 2x + 12}{x^2(xy(x) + x + 2)}$$

✓ **Mathematica** : cpu = 0.198384 (sec), leaf count = 76

$$\left\{\left\{y(x) \rightarrow -\frac{x+2}{x} + \frac{1}{x^3\left(\frac{1}{x^3} - \frac{1}{x^3\sqrt{-2x+c_1}}\right)}\right\}, \left\{y(x) \rightarrow -\frac{x+2}{x} + \frac{1}{x^3\left(\frac{1}{x^3} + \frac{1}{x^3\sqrt{-2x+c_1}}\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 63

$$\left\{y(x) = \frac{-x - 2\sqrt{c_1 - 2x} - 2}{(\sqrt{c_1 - 2x} + 1)x}, y(x) = \frac{x - 2\sqrt{c_1 - 2x} + 2}{(\sqrt{c_1 - 2x} - 1)x}\right\}$$

2.854 ODE No. 854

$$y'(x) = \frac{y(x)(x^2 \log^2(y(x)) + 2x^2 \log(x) \log(y(x)) + x^2 \log^2(x) + \log(y(x)) + \log(x) - 1)}{x}$$

✓ **Mathematica** : cpu = 0.168058 (sec), leaf count = 24

$$\left\{\left\{y(x) \rightarrow \frac{e^{-\frac{3x}{x^3+3c_1}}}{x}\right\}\right\}$$

✓ **Maple** : cpu = 0.272 (sec), leaf count = 51

$$\left\{y(x) = x^{-\frac{3c_1}{x^3+3c_1}} x^{-\frac{x^3}{x^3+3c_1}} e^{-\frac{3x}{x^3+3c_1}}\right\}$$

2.855 ODE No. 855

$$y'(x) = \frac{y(x)(x^3 \log^2(y(x)) + 2x^3 \log(x) \log(y(x)) + x^3 \log^2(x) + \log(y(x)) + \log(x) - 1)}{x}$$

✓ **Mathematica** : cpu = 0.160301 (sec), leaf count = 24

$$\left\{\left\{y(x) \rightarrow \frac{e^{-\frac{4x}{x^4+4c_1}}}{x}\right\}\right\}$$

✓ **Maple** : cpu = 0.244 (sec), leaf count = 51

$$\left\{y(x)=x^{-\frac{4c_1}{x^4+4c_1}}x^{-\frac{x^4}{x^4+4c_1}}e^{-\frac{4x}{x^4+4c_1}}\right\}$$

2.856 ODE No. 856

$$y'(x)=\frac{x\left(-\mathrm{F1}\left(y(x)^2-2x\right)+\frac{1}{x}\right)}{\sqrt{y(x)^2}}$$

✓ **Mathematica** : cpu = 0.634202 (sec), leaf count = 103

Solve $\left[\int_1^{y(x)}\left(\frac{\sqrt{K[2]^2}}{-\mathrm{F1}\left(K[2]^2-2x\right)}-\int_1^x\frac{2K[2]_-\mathrm{F1}'\left(K[2]^2-2K[1]\right)}{\left(-\mathrm{F1}\left(K[2]^2-2K[1]\right)\right)^2}dK[1]\right)dK[2]+\int_1^x\left(-K[1]-\frac{1}{-\mathrm{F1}\left(y(x)^2-2K[1]\right)}\right)dK[1]$

✓ **Maple** : cpu = 0.318 (sec), leaf count = 65

$$\left\{y(x)=\sqrt{2x+2\operatorname{RootOf}\left(x^2+4c_1-2\left(\int^{-Z}\frac{1}{-\mathrm{F1}(2\_a)}d\_a\right)\right)},y(x)=-\sqrt{2x+2\operatorname{RootOf}\left(x^2+4c_1-2\left(\int^{-Z}\frac{1}{-\mathrm{F1}(2\_a)}d\_a\right)\right)}\right\}$$

2.857 ODE No. 857

$$y'(x)=x^2\sqrt{x^2+8y(x)-2x+1}+\sqrt{x^2+8y(x)-2x+1}+x^3\sqrt{x^2+8y(x)-2x+1}-\frac{x}{4}+\frac{1}{4}$$

✓ **Mathematica** : cpu = 1.04202 (sec), leaf count = 107

Solve $\left[\frac{x^4}{4}+\frac{x^3}{3}-\frac{1}{4}\sqrt{x^2+8y(x)-2x+1}+\frac{1}{4}\log\left(-\sqrt{x^2+8y(x)-2x+1}-x+1\right)-\frac{1}{4}\tanh^{-1}\left(\frac{2x-2}{2\sqrt{x^2+8y(x)-2x+1}}\right)\right]$

✓ **Maple** : cpu = 0.362 (sec), leaf count = 32

$$\left\{x^4+\frac{4x^3}{3}+c_1+4x-\sqrt{x^2-2x+8y(x)+1}=0\right\}$$

2.858 ODE No. 858

$$y'(x)=\frac{a^3y(x)^3+a^3y(x)^2+a^3+3a^2bxy(x)^2+2a^2bxy(x)+3ab^2x^2y(x)+ab^2x^2+b^3x^3}{a^3}$$

✓ **Mathematica** : cpu = 0.442401 (sec), leaf count = 145

Solve $\left[-\frac{1}{3}(29a+27b)^{2/3}\operatorname{RootSum}\left[\#1^3(29a+27b)^{2/3}-3\#1a^{2/3}+(29a+27b)^{2/3}\&, \frac{\log\left(\frac{\frac{a+3bx}{a}+3y(x)}{\sqrt[3]{\frac{29a+27b}{a}}}-\#1\right)}{a^{2/3}-\#1^2(29a+27b)^{2/3}}\&\right]=\frac{1}{9}x\left(\frac{29}{3}\right)\right]$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 42

$$\left\{y(x)=\frac{a\operatorname{RootOf}\left(a\left(\int^{-Z}\frac{1}{-a^3a+_a^2a+_a+b}d\_a\right)+c_1-x\right)-bx}{a}\right\}$$

2.859 ODE No. 859

$$y'(x) = \frac{{}_F1\left(y(x)^2 - 2x\right) + x}{x\sqrt{y(x)^2}}$$

✓ **Mathematica** : cpu = 0.895476 (sec), leaf count = 105

Solve $\left[\int_1^{y(x)}\left(\frac{\sqrt{K[2]^2}}{{}_F1\left(K[2]^2 - 2x\right)} - \int_1^x\frac{2K[2]{}_F1'\left(K[2]^2 - 2K[1]\right)}{\left({}_F1\left(K[2]^2 - 2K[1]\right)\right)^2}dK[1]\right)dK[2] + \int_1^x\left(-\frac{1}{{}_F1\left(y(x)^2 - 2K[1]\right)} - \frac{1}{K[1]}\right)dK[1]$

✓ **Maple** : cpu = 0.265 (sec), leaf count = 63

$$\left\{y(x) = \sqrt{2x + 2\operatorname{RootOf}\left(2c_1 - \left(\int^{-Z}\frac{1}{{}_F1(2\_a)}d\_a\right) + \ln(x)\right)}, y(x) = -\sqrt{2x + 2\operatorname{RootOf}\left(2c_1 - \left(\int^{-Z}\frac{1}{{}_F1(2\_a)}d\_a\right) + \ln(x)\right)}\right\}$$

2.860 ODE No. 860

$$y'(x) = \frac{\frac{1}{2}x^4\cos(2y(x)) + \frac{x^4}{2} + \frac{1}{2}x^3\cos(2y(x)) + \frac{x^3}{2} - \frac{1}{2}\sin(2y(x)) + \frac{1}{2}x\cos(2y(x)) + \frac{x}{2}}{x}$$

✓ **Mathematica** : cpu = 0.403556 (sec), leaf count = 33

$$\left\{\left\{y(x) \rightarrow \tan^{-1}\left(\frac{4x^5 + 5x^4 + 10x^2 + 10c_1}{20x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 1.744 (sec), leaf count = 29

$$\left\{y(x) = \arctan\left(\frac{4x^5 + 5x^4 + 10x^2 + 40c_1}{20x}\right)\right\}$$

2.861 ODE No. 861

$$y'(x) = \frac{e^{-1/x}\left({}_F1\left(e^{\frac{1}{x}}y(x)\right) + \frac{e^{\frac{1}{x}}y(x)}{x}\right)}{x}$$

✓ **Mathematica** : cpu = 1.55982 (sec), leaf count = 158

Solve $\left[\int_1^{y(x)}-\frac{{}_F1\left(e^{\frac{1}{x}}K[2]\right)\int_1^x\left(\frac{e^{\frac{1}{K[1]}}}{K[1]^2{}_F1\left(e^{\frac{1}{K[1]}}K[2]\right)} - \frac{e^{\frac{2}{K[1]}K[2]}{}_F1'\left(e^{\frac{1}{K[1]}K[2]}\right)}{K[1]^2\left({}_F1\left(e^{\frac{1}{K[1]}K[2]}\right)\right)^2}\right)dK[1] + e^{\frac{1}{x}}}{{}_F1\left(e^{\frac{1}{x}}K[2]\right)}dK[2] + \int_1^x\left(\frac{e^{\frac{1}{K[1]}y(x)}}{K[1]^2{}_F1\left(e^{\frac{1}{K[1]}y(x)}\right)} + \frac{e^{\frac{1}{K[1]}y(x)}}{y(x)}\right)dy(x)$

✓ **Maple** : cpu = 0.158 (sec), leaf count = 26

$$\left\{y(x) = \operatorname{RootOf}\left(c_1 + \int^{-Z}\frac{1}{{}_F1(\_a)}d\_a - \ln(x)\right)e^{-\frac{1}{x}}\right\}$$

2.862 ODE No. 862

$$y'(x) = \log(y(x) - 1)\left({}_F1(x) - \frac{\operatorname{Ei}(-\log(y(x) - 1))}{x}\right)$$

✗ **Mathematica** : cpu = 0.805693 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x] == Log[-1 + y[x]]*(-(ExpIntegralEi[-Log[-1 + y[x]]])/x) + _F1[x]],`

✓ **Maple** : cpu = 0.207 (sec), leaf count = 27

$$\left\{y(x) = e^{\operatorname{RootOf}\left(c_1x+x\left(\int \frac{F1(x)}{x}dx\right)+\operatorname{Ei}(1,-Z)\right)} + 1\right\}$$

2.863 ODE No. 863

$$y'(x) = \frac{x\sqrt{x^2 + y(x)^2} + x^4\sqrt{x^2 + y(x)^2} + x^3\sqrt{x^2 + y(x)^2} + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.243957 (sec), leaf count = 30

$$\left\{\left\{y(x) \rightarrow x \sinh\left(\frac{1}{12}\left(3x^4 + 4x^3 + 12x + 12c_1\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 14.23 (sec), leaf count = 38

$$\left\{-\frac{x^4}{4} - \frac{x^3}{3} - c_1 - x - \ln(x) + \ln\left(y(x) + \sqrt{x^2 + y(x)^2}\right) = 0\right\}$$

2.864 ODE No. 864

$$y'(x) = \frac{e^{\frac{x^2}{4}}y(x)\left(2e^{-\frac{3x^2}{4}}y(x)^2 + e^{-\frac{x^2}{2}}xy(x) + e^{-\frac{x^2}{4}}x\right)}{2e^{-\frac{x^2}{4}}y(x) + 2}$$

✓ **Mathematica** : cpu = 0.391417 (sec), leaf count = 137

$$\left\{\left\{y(x) \rightarrow \frac{2e^{\frac{x^2}{2}}}{-2e^{\frac{x^2}{4}} + \sqrt{2}\sqrt{2e^{\frac{x^2}{2}} + 2e^{\frac{x^2}{2}}(-2x + c_1)}}\right\}, \left\{y(x) \rightarrow -\frac{2e^{\frac{x^2}{2}}}{2e^{\frac{x^2}{4}} + \sqrt{2}\sqrt{2e^{\frac{x^2}{2}} + 2e^{\frac{x^2}{2}}(-2x + c_1)}}\right\}\right\}$$

✓ **Maple** : cpu = 0.097 (sec), leaf count = 162

$$\left\{y(x) = \frac{\left((\sqrt{c_1 - 2x} - 1)e^{-\frac{x^2}{4}}e^{\frac{x^2}{2}} - \sqrt{c_1 - 2x}e^{\frac{x^2}{4}}\right)e^{\frac{x^2}{4}}}{e^{-\frac{x^2}{4}}e^{\frac{x^2}{2}} + \sqrt{c_1 - 2x}e^{\frac{x^2}{4}}}, y(x) = \frac{\left((\sqrt{c_1 - 2x} + 1)e^{-\frac{x^2}{4}}e^{\frac{x^2}{2}} - \sqrt{c_1 - 2x}e^{\frac{x^2}{4}}\right)e^{\frac{x^2}{4}}}{-e^{-\frac{x^2}{4}}e^{\frac{x^2}{2}} + \sqrt{c_1 - 2x}e^{\frac{x^2}{4}}}\right\}$$

2.865 ODE No. 865

$$y'(x) = (1 - y(x))\left(-f(x) + \frac{y(x)\log(y(x) - 1)}{x(1 - y(x))\log(x)} - \frac{\log(y(x) - 1)}{x(1 - y(x))\log(x)}\right)$$

✓ **Mathematica** : cpu = 0.399838 (sec), leaf count = 87

$$\text{Solve}\left[\int_1^x\left(-\frac{f(K[1])}{\log(K[1])} - \frac{\log(y(x) - 1)}{K[1]\log^2(K[1])}\right)dK[1] + \int_1^{y(x)}\left(\frac{1}{(K[2] - 1)\log(x)} - \int_1^x -\frac{1}{K[1](K[2] - 1)\log^2(K[1])}dK[1]\right)dK[2]\right]$$

✓ **Maple** : cpu = 0.2 (sec), leaf count = 23

$$\left\{y(x) = x^{c_1}e^{\left(\int \frac{f(x)}{\ln(x)}dx\right)\ln(x)} + 1\right\}$$

2.866 ODE No. 866

$$y'(x) = x^2 \sqrt{a^2 + 2ax + x^2 + 4y(x)} + \sqrt{a^2 + 2ax + x^2 + 4y(x)} + x^3 \sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{a}{2} - \frac{x}{2}$$

✓ **Mathematica** : cpu = 0.888697 (sec), leaf count = 117

$$\text{Solve} \left[-\frac{1}{2} \sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{1}{2} a \log \left(\sqrt{a^2 + 2ax + x^2 + 4y(x)} + a + x \right) + \frac{1}{2} a \tanh^{-1} \left(\frac{2a + 2x}{2\sqrt{a^2 + 2ax + x^2 + 4y(x)}} \right) \right]$$

✓ **Maple** : cpu = 0.405 (sec), leaf count = 37

$$\left\{ \frac{x^4}{2} + \frac{2x^3}{3} + c_1 + 2x - \sqrt{a^2 + 2ax + x^2 + 4y(x)} = 0 \right\}$$

2.867 ODE No. 867

$$y'(x) = \frac{x^6}{27} + \frac{1}{3} x^4 y(x) + \frac{x^4}{9} + x^2 y(x)^2 + \frac{2}{3} x^2 y(x) + y(x)^3 + y(x)^2 - \frac{2x}{3} + 1$$

✓ **Mathematica** : cpu = 0.227419 (sec), leaf count = 77

$$\text{Solve} \left[-\frac{29}{3} \text{RootSum} \left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log \left(\frac{x^2 + 3y(x) + 1}{\sqrt[3]{29}} - \#1 \right)}{\sqrt[3]{29} - 29\#1^2} \& \right] = \frac{1}{9} 29^{2/3} x + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 30

$$\left\{ y(x) = -\frac{x^2}{3} + \text{RootOf} \left(c_1 - x + \int^{-Z} \frac{1}{\_a^3 + \_a^2 + 1} d\_a \right) \right\}$$

2.868 ODE No. 868

$$y'(x) = -x^6 + 3x^4 y(x) + x^4 - 3x^2 y(x)^2 - 2x^2 y(x) + y(x)^3 + y(x)^2 + 2x + 1$$

✓ **Mathematica** : cpu = 0.197384 (sec), leaf count = 79

$$\text{Solve} \left[-\frac{29}{3} \text{RootSum} \left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log \left(\frac{-3x^2 + 3y(x) + 1}{\sqrt[3]{29}} - \#1 \right)}{\sqrt[3]{29} - 29\#1^2} \& \right] = \frac{1}{9} 29^{2/3} x + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 28

$$\left\{ y(x) = x^2 + \text{RootOf} \left(c_1 - x + \int^{-Z} \frac{1}{\_a^3 + \_a^2 + 1} d\_a \right) \right\}$$

2.869 ODE No. 869

$$y'(x) = \frac{2x^5 + 2x^4 - 2x^3 y(x) + x^3 - 2x^2 y(x) + 3x^2 - 2y(x) - x + 1}{x^2 - y(x)}$$

✓ **Mathematica** : cpu = 0.0525664 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow x^2 + \frac{1}{2} \left(1 + W \left(-e^{x^4 + \frac{4x^3}{3} - 2x^2 + 4x - 1 + c_1} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.334 (sec), leaf count = 37

$$\left\{ y(x) = x^2 + \frac{\text{LambertW} \left(-2c_1 e^{-1} e^{x^4} e^{4x} e^{-2x^2} e^{\frac{4x^3}{3}} \right)}{2} + \frac{1}{2} \right\}$$

2.870 ODE No. 870

$$y'(x) = \frac{e^{\frac{y(x)}{x}} \left(x^4 + x^3 + x e^{-\frac{y(x)}{x}} + e^{-\frac{y(x)}{x}} y(x) + x \right)}{x}$$

✓ **Mathematica** : cpu = 1.74947 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow -x \log \left(\frac{-\frac{x^4}{4} - \frac{x^3}{3} - x - c_1}{x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.708 (sec), leaf count = 30

$$\left\{ y(x) = -x \ln \left(-\frac{3x^4 + 4x^3 + 12x + 12c_1}{12x} \right) \right\}$$

2.871 ODE No. 871

$$y'(x) = \frac{2xy(x)^2 + y(x)^2 + 4xy(x) \log(2x+1) + 2y(x) \log(2x+1) + 2x \log^2(2x+1) + \log^2(2x+1) - 2}{2x+1}$$

✓ **Mathematica** : cpu = 0.34061 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow -\log(2x+1) + \frac{1}{-x+c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.092 (sec), leaf count = 26

$$\left\{ y(x) = \frac{(c_1 - x) \ln(2x+1) - 1}{-c_1 + x} \right\}$$

2.872 ODE No. 872

$$y'(x) = \frac{14x^{7/2} + \frac{12x^6}{5} - 6x^3 y(x) - 6x^3 - 5\sqrt{x} y(x) + 10x - 5\sqrt{x} - 5}{x(2x^3 - 5y(x) + 10\sqrt{x} - 5)}$$

✓ **Mathematica** : cpu = 0.0663165 (sec), leaf count = 215

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{5}(2x^3 + 10\sqrt{x} - 5) - \frac{\sqrt{-x(2x^3 + 10\sqrt{x} - 5)^2 - 50x \left(-\frac{4x^{7/2}}{5} - \frac{2x^6}{25} + \frac{2x^3}{5} - 2x + 2\sqrt{x} + \log(x) \right) - 25c_1 x}}{5\sqrt{-\frac{1}{x}} x} \right\} \right\}, y$$

✓ **Maple** : cpu = 0.063 (sec), leaf count = 49

$$\left\{ y(x) = \frac{2x^3}{5} + 2\sqrt{x} - \sqrt{c_1 + 2 \ln(x)} - 1, y(x) = \frac{2x^3}{5} + 2\sqrt{x} + \sqrt{c_1 + 2 \ln(x)} - 1 \right\}$$

2.873 ODE No. 873

$$y'(x) = \frac{2y(x) + 1}{x(2xy(x)^4 + 3xy(x)^3 + xy(x)^2 + 2xy(x) + x - 2)}$$

✓ **Mathematica** : cpu = 0.947725 (sec), leaf count = 53

$$\text{Solve} \left[\frac{1}{192} (-16y(x)^3 - 12y(x)^2 + 12y(x) - 54 \log(4y(x) + 2) + 7) - \frac{1}{2x(2y(x) + 1)} = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.361 (sec), leaf count = 50

$$\left\{ y(x) = \frac{e^{\text{RootOf}(48c_1 x e^{-Z} + 54_Z x e^{-Z} + 7x e^{-Z} - 6x e^{2-Z} - 3x e^{3-Z} + 2x e^{4-Z} + 96)}}{2} - \frac{1}{2} \right\}$$

2.874 ODE No. 874

$$y'(x) = \frac{1}{512}x\left(a^3x^{12} + 24a^2x^8y(x) + 8a^2x^8 + 192ax^4y(x)^2 + 128ax^4y(x) - 256ax^2 + 512y(x)^3 + 512y(x)^2 + 512\right)$$

✓ **Mathematica** : cpu = 0.270099 (sec), leaf count = 101

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{\frac{1}{8}(3ax^5+8x)+3xy(x)}{\sqrt[3]{29}\sqrt[3]{x^3}} - \#1\right)}{\sqrt[3]{29} - 29\#1^2}\&\right] = \frac{1}{18}29^{2/3}\left(x^3\right)^{2/3} + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.057 (sec), leaf count = 40

$$\left\{y(x) = -\frac{ax^4}{8} + \frac{29\text{RootOf}\left(x^2 + 6c_1 - 162\left(\int^{-Z}\frac{1}{841\_a^3-27\_a+27}d\_a\right)\right)}{9} - \frac{1}{3}\right\}$$

2.875 ODE No. 875

$$y'(x) = \frac{x^5\left(-\sqrt{x^2 + y(x)^2}\right) + x^4y(x)\sqrt{x^2 + y(x)^2} + xy(x) + y(x)}{x(x + 1)}$$

✓ **Mathematica** : cpu = 0.300839 (sec), leaf count = 497

$$\left\{\left\{y(x) \rightarrow \frac{x - 2\sqrt{x^2 \tanh^2\left(\frac{1}{12}\left(-3\sqrt{2}x^4 + 4\sqrt{2}x^3 - 6\sqrt{2}x^2 + 12\sqrt{2}x - 12\sqrt{2}\log(x + 1) + 25\sqrt{2} - 12\sqrt{2}c_1\right)\right)} - x^2 \tanh^2\left(\frac{1}{12}\left(-3\sqrt{2}x^4 + 4\sqrt{2}x^3 - 6\sqrt{2}x^2 + 12\sqrt{2}x - 12\sqrt{2}\log(x + 1) + 25\sqrt{2} - 12\sqrt{2}c_1\right)\right)}{-1 + 2 \tanh^2\left(\frac{1}{12}\left(-3\sqrt{2}x^4 + 4\sqrt{2}x^3 - 6\sqrt{2}x^2 + 12\sqrt{2}x - 12\sqrt{2}\log(x + 1) + 25\sqrt{2} - 12\sqrt{2}c_1\right)\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.238 (sec), leaf count = 73

$$\left\{-c_1 - \ln(x) + \ln\left(\frac{2\left(x + y(x) + \sqrt{2x^2 + 2y(x)^2}\right)x}{-x + y(x)}\right) + \sqrt{2}\ln(x + 1) + \frac{\left(3x^4 - 4x^3 + 6x^2 - 12x\right)\sqrt{2}}{12} = 0\right\}$$

2.876 ODE No. 876

$$y'(x) = -\frac{y(x)^2\left(x^2y(x) - 2xy(x) + y(x) - 2x\right)}{2x(xy(x) - 2y(x) - 2)}$$

✓ **Mathematica** : cpu = 0.203807 (sec), leaf count = 135

$$\left\{\left\{y(x) \rightarrow -\frac{4x}{-2(x-2)x + \frac{2\sqrt{-x(x-2)^2-4x\left(-2\left(\frac{x^2}{8}-\frac{x}{2}+\frac{\log(x)}{4}\right)+c_1}\right)}}{\sqrt{-\frac{1}{x}}}\right\}, \left\{y(x) \rightarrow -\frac{4x}{2(x-2)x + \frac{2\sqrt{-x(x-2)^2-4x\left(-2\left(\frac{x^2}{8}-\frac{x}{2}+\frac{\log(x)}{4}\right)+c_1}\right)}}{\sqrt{-\frac{1}{x}}}\right\}\right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 41

$$\left\{y(x) = -\frac{4}{-2x + \sqrt{c_1 - 8\ln(x)} + 4}, y(x) = \frac{4}{2x + \sqrt{c_1 - 8\ln(x)} - 4}\right\}$$

2.877 ODE No. 877

$$y'(x) = \frac{x^6 - 3x^4y(x) + 2x^3 + 3x^2y(x)^2 - 2xy(x) - y(x)^3 - 2x}{x^2 - y(x) - 1}$$

✓ **Mathematica** : cpu = 0.207238 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow x^2 + \frac{1}{1 - \frac{1}{\sqrt{-2x+c_1}}} - 1 \right\}, \left\{ y(x) \rightarrow x^2 + \frac{1}{1 + \frac{1}{\sqrt{-2x+c_1}}} - 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 73

$$\left\{ y(x) = \frac{-2c_1x^2 + 2x^3 + \sqrt{2c_1 - 2x + 1} - 1}{-2c_1 + 2x}, y(x) = \frac{2c_1x^2 - 2x^3 + \sqrt{2c_1 - 2x + 1} + 1}{2c_1 - 2x} \right\}$$

2.878 ODE No. 878

$$y'(x) = \frac{-64a^3x^3 + 48a^2x^2y(x)^2 + 16a^2x^2 - 12axy(x)^4 - 8axy(x)^2 + y(x)^6 + y(x)^4 + 1}{y(x)}$$

✓ **Mathematica** : cpu = 0.475851 (sec), leaf count = 130

Solve $\left[2a\left(x - \frac{1}{2}\text{RootSum}\left[64\#1^3a^3 - 48\#1^2a^2y(x)^2 - 16\#1^2a^2 + 12\#1ay(x)^4 + 8\#1ay(x)^2 + 2a - y(x)^6 - y(x)^4 - 1\&, \frac{\log(2a^3x^3 - 4a^2x^2y(x)^2 - 4a^2x^2 + 12axy(x)^4 + 8axy(x)^2 + y(x)^6 + y(x)^4 + 1)}{48\#1^2a^2 - 24\#1ay(x)^2}\right)\right]$

✓ **Maple** : cpu = 39.088 (sec), leaf count = 75

$$\left\{ -c_1 + x + \int_{-b}^{y(x)} \frac{-a}{-a^6 + 12a^4ax - 48a^2a^2x^2 + 64a^3x^3 - a^4 + 8a^2ax - 16a^2x^2 + 2a - 1} d_{-a} = 0 \right\}$$

2.879 ODE No. 879

$$y'(x) = \frac{x^2(-\sqrt{x^2 + y(x)^2}) + xy(x)\sqrt{x^2 + y(x)^2} + xy(x) + y(x)}{x(x + 1)}$$

✓ **Mathematica** : cpu = 0.38423 (sec), leaf count = 239

$$\left\{ \left\{ y(x) \rightarrow \frac{x - 2\sqrt{x^2 \tanh^2(\sqrt{2}x - \sqrt{2} \log(x + 1) + \sqrt{2}c_1)} - x^2 \tanh^4(\sqrt{2}x - \sqrt{2} \log(x + 1) + \sqrt{2}c_1)}{-1 + 2 \tanh^2(\sqrt{2}x - \sqrt{2} \log(x + 1) + \sqrt{2}c_1)} \right\}, \left\{ y(x) \rightarrow \frac{x + 2\sqrt{x^2 \tanh^2(\sqrt{2}x - \sqrt{2} \log(x + 1) + \sqrt{2}c_1)} - x^2 \tanh^4(\sqrt{2}x - \sqrt{2} \log(x + 1) + \sqrt{2}c_1)}{-1 + 2 \tanh^2(\sqrt{2}x - \sqrt{2} \log(x + 1) + \sqrt{2}c_1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.275 (sec), leaf count = 55

$$\left\{ -c_1 + \sqrt{2}x - \ln(x) + \ln\left(\frac{2(x + y(x) + \sqrt{2x^2 + 2y(x)^2})x}{-x + y(x)}\right) - \sqrt{2} \ln(x + 1) = 0 \right\}$$

2.880 ODE No. 880

$$y'(x) = -\frac{2a}{128a^4x^3 - 96a^3x^2y(x)^2 - 32a^3x^2 + 24a^2xy(x)^4 + 16a^2xy(x)^2 - 2ay(x)^6 - 2ay(x)^4 - 2a - y(x)}$$

✓ **Mathematica** : cpu = 0.525248 (sec), leaf count = 131

Solve $\left[\frac{\text{RootSum}\left[-64\#1^3a^3 + 48\#1^2a^2y(x)^2 + 16\#1^2a^2 - 12\#1ay(x)^4 - 8\#1ay(x)^2 + y(x)^6 + y(x)^4 + 1\&, \frac{\log(2a^3x^3 - 4a^2x^2y(x)^2 - 4a^2x^2 + 12axy(x)^4 + 8axy(x)^2 + y(x)^6 + y(x)^4 + 1)}{48\#1^2a^2 - 24\#1ay(x)^2}\right]}{8a^2}\right]$

✓ **Maple** : cpu = 0.176 (sec), leaf count = 41

$$\left\{ -c_1 + \frac{y(x)}{2a} + \frac{\int^{-4ax+y(x)^2} \frac{1}{-a^3 + a^2 + 1} d_{-a}}{8a^2} = 0 \right\}$$

2.881 ODE No. 881

$$y'(x) = \frac{x^6 + 9x^4y(x) - 6x^3 + 27x^2y(x)^2 - 18xy(x) + 27y(x)^3 - 18x}{9x^2 + 27y(x) + 27}$$

✓ **Mathematica** : cpu = 0.1963 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{3}(-x^2 - 3) + \frac{1}{27\left(\frac{1}{27} - \frac{1}{\sqrt{-1458x+c_1}}\right)} \right\}, \left\{ y(x) \rightarrow \frac{1}{3}(-x^2 - 3) + \frac{1}{27\left(\frac{1}{27} + \frac{1}{\sqrt{-1458x+c_1}}\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 77

$$\left\{ y(x) = \frac{-2c_1x^2 + 2x^3 - 3\sqrt{2c_1 - 2x + 1} + 3}{6c_1 - 6x}, y(x) = \frac{-2c_1x^2 + 2x^3 + 3\sqrt{2c_1 - 2x + 1} + 3}{6c_1 - 6x} \right\}$$

2.882 ODE No. 882

$$y'(x) = -\frac{1}{216}\sqrt{x}\left(-108x^{3/2} + x^9 - 18x^6y(x) - 6x^6 + 108x^3y(x)^2 + 72x^3y(x) - 216y(x)^3 - 216y(x)^2 - 216\right)$$

✓ **Mathematica** : cpu = 0.290022 (sec), leaf count = 119

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{\frac{1}{2}(2\sqrt{x}-x^{7/2})+3\sqrt{x}y(x)}{\sqrt[3]{29}\sqrt[3]{x^{3/2}}}-\#1\right)}{\sqrt[3]{29}-29\#1^2}\&\right] = \frac{2}{27}29^{2/3}\sqrt{x}\left(x^{3/2}\right)^{2/3} + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.065 (sec), leaf count = 41

$$\left\{ y(x) = \frac{x^3}{6} + \frac{29\text{RootOf}\left(2x^{\frac{3}{2}} + 9c_1 - 243\left(\int^{\frac{-Z}{841\_a^3-27\_a+27}}\frac{1}{\sqrt[3]{29}\sqrt[3]{x^{3/2}}}d\_a\right)\right)}{9} - \frac{1}{3} \right\}$$

2.883 ODE No. 883

$$y'(x) = \frac{x\left(a^3y(x)^6 + a^3y(x)^4 + a^3 + 3a^2bx^2y(x)^4 + 2a^2bx^2y(x)^2 + 3ab^2x^4y(x)^2 + ab^2x^4 + b^3x^6\right)}{a^{7/2}y(x)}$$

✓ **Mathematica** : cpu = 1.37261 (sec), leaf count = 164

$$\text{Solve}\left[\frac{x^2}{2} - \frac{1}{2}a^{5/2}\text{RootSum}\left[\#1^3b^3 + 3\#1^2ab^2y(x)^2 + \#1^2ab^2 + 3\#1a^2by(x)^4 + 2\#1a^2by(x)^2 + a^{5/2}b + a^3y(x)^6 + a^3y(x)^4 + \right]$$

✓ **Maple** : cpu = 0.68 (sec), leaf count = 352

$$\left\{ c_1 + \int_{-b}^x \frac{\left(-a^6b^3 + 3\_a^4a\,b^2y(x)^2 + 3\_a^2a^2by(x)^4 + a^3y(x)^6 + \_a^4a\,b^2 + 2\_a^2a^2by(x)^2 + a^3y(x)^4 + a^3\right)\_a}{\left(-a^6b^3 + 3\_a^4a\,b^2y(x)^2 + 3\_a^2a^2by(x)^4 + a^3y(x)^6 + \_a^4a\,b^2 + 2\_a^2a^2by(x)^2 + a^3y(x)^4 + a^{\frac{5}{2}}b + a^3\right)a^{\frac{7}{2}}}d\_a + \int^y$$

2.884 ODE No. 884

$$y'(x) = \frac{x\left(-x^6 + 3x^4y(x)^2 + x^4 - 3x^2y(x)^4 - 2x^2y(x)^2 + y(x)^6 + y(x)^4 + 1\right)}{y(x)}$$

✓ **Mathematica** : cpu = 0.871618 (sec), leaf count = 71

$\text{Solve}\left[\frac{1}{4}\left(2\log(-x^2+y(x)^2+1)-2x^2-\frac{1}{y(x)(y(x)+x)}+\frac{1}{xy(x)-y(x)^2}-2\log(x-y(x))-2\log(y(x)+x)\right)=c_1,y(x)\right]$

✓ **Maple** : cpu = 0.451 (sec), leaf count = 107

$\left\{y(x)=-x+e^{\text{RootOf}\left(6x^3e^{-Z}+4c_1xe^{-Z}+6_Zxe^{-Z}-3x^2e^{2-Z}-6xe^{-Z}\ln\left(\frac{-2xe^{-Z}+e^{2-Z}+1}{-2x+e^{-Z}}\right)-2c_1e^{2-Z}-3_Ze^{2-Z}+3e^{2-Z}\ln\left(\frac{-2xe^{-Z}+e^{2-Z}+1}{-2x+e^{-Z}}\right)-3\right)}\right\}$

2.885 ODE No. 885

$$y'(x)=-\frac{i\left(x^6+12x^4y(x)^2+4x^4+48x^2y(x)^4+32x^2y(x)^2+64y(x)^6+64y(x)^4+32ix+64\right)}{128y(x)}$$

✗ **Mathematica** : cpu = 40.8199 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x]==((-1/128*I)*(64+(32*I)*x+4*x^4+x^6+32*x^2*y[x]^2+12*`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x),x)=-1/128*I*(32*I*x+64+64*y(x)^4+32*x^2*y(x)^2+4*x^4+64*y(x)^6+48*x^2*y(x)`

2.886 ODE No. 886

$$y'(x)=\frac{x^6y(x)^3-3x^5y(x)^2+x^4y(x)^2+3x^4y(x)-4x^3y(x)-x^3+2x^2+1}{x^4}$$

✓ **Mathematica** : cpu = 0.234412 (sec), leaf count = 82

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3+3\sqrt[3]{29}\#1-29\&,\frac{\log\left(\frac{3x^2y(x)-3x+1}{\sqrt[3]{29}}-\#1\right)}{\sqrt[3]{29}-29\#1^2}\&\right]=-\frac{29^{2/3}}{9x}+c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.028 (sec), leaf count = 42

$$\left\{y(x)=\frac{9x+29\text{RootOf}\left(3c_1x-81x\left(\int^{-Z}\frac{1}{841a^3-27a+27}d_a\right)-1\right)-3}{9x^2}\right\}$$

2.887 ODE No. 887

$$y'(x)=\frac{a^3x^3y(x)^3+3a^2x^2y(x)^2+a^2xy(x)+a^2x+3axy(x)+a+1}{a^2x^2(axy(x)+ax+1)}$$

✓ **Mathematica** : cpu = 0.283454 (sec), leaf count = 106

$$\left\{\left\{y(x)\rightarrow-\frac{ax+1}{ax}+\frac{1}{a^3x^3\left(\frac{1}{a^3x^3}-\frac{1}{x^3\sqrt{-2a^6x+c_1}}\right)}\right\},\left\{y(x)\rightarrow-\frac{ax+1}{ax}+\frac{1}{a^3x^3\left(\frac{1}{a^3x^3}+\frac{1}{x^3\sqrt{-2a^6x+c_1}}\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.051 (sec), leaf count = 72

$$\left\{y(x)=\frac{-ax-\sqrt{c_1-2x}-1}{\left(\sqrt{c_1-2x}+1\right)ax},y(x)=\frac{ax-\sqrt{c_1-2x}+1}{\left(\sqrt{c_1-2x}-1\right)ax}\right\}$$

2.888 ODE No. 888

$$y'(x) = \frac{x^4 y(x)^3 - 5x^3 y(x)^2 + 6x^2 y(x) - 2x y(x) - 2x + 1}{x^2(x^2 y(x) - x + 1)}$$

✓ **Mathematica** : cpu = 0.196148 (sec), leaf count = 78

$$\left\{ \left\{ y(x) \rightarrow \frac{x-1}{x^2} + \frac{1}{x^4 \left(\frac{1}{x^2} - \frac{1}{x^2 \sqrt{\frac{2}{x} + c_1}} \right)} \right\}, \left\{ y(x) \rightarrow \frac{x-1}{x^2} + \frac{1}{x^4 \left(\frac{1}{x^2} + \frac{1}{x^2 \sqrt{\frac{2}{x} + c_1}} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 79

$$\left\{ y(x) = \frac{\sqrt{\frac{c_1 x + 2}{x}} x - x + 1}{\left(\sqrt{\frac{c_1 x + 2}{x}} - 1 \right) x^2}, y(x) = \frac{\sqrt{\frac{c_1 x + 2}{x}} x + x - 1}{\left(\sqrt{\frac{c_1 x + 2}{x}} + 1 \right) x^2} \right\}$$

2.889 ODE No. 889

$$y'(x) = -\frac{e^x(-8y(x)^{9/2} + 36e^x y(x)^3 - 8y(x)^3 + 24e^x y(x)^{3/2} - 54e^{2x} y(x)^{3/2} - 18e^{2x} + 27e^{3x} - 8)}{8\sqrt{y(x)}}$$

✓ **Mathematica** : cpu = 1.82973 (sec), leaf count = 68

$$\text{Solve} \left[\frac{2}{3} \log \left(y(x)^{3/2} - \frac{3e^x}{2} \right) + e^x = \frac{4}{9e^x - 6y(x)^{3/2}} + \frac{2}{3} \log \left(y(x)^{3/2} - \frac{3e^x}{2} + 1 \right) + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.911 (sec), leaf count = 49

$$\left\{ -c_1 + e^x + \frac{2 \ln \left(y(x)^{\frac{3}{2}} - \frac{3e^x}{2} \right)}{3} - \frac{2 \ln \left(y(x)^{\frac{3}{2}} - \frac{3e^x}{2} + 1 \right)}{3} - \frac{4}{-6y(x)^{\frac{3}{2}} + 9e^x} = 0 \right\}$$

2.890 ODE No. 890

$$y'(x) = \frac{x}{x^6 + 3x^4 y(x)^2 + x^4 + 3x^2 y(x)^4 + 2x^2 y(x)^2 + y(x)^6 + y(x)^4 - y(x) + 1}$$

✓ **Mathematica** : cpu = 0.213356 (sec), leaf count = 103

$$\text{Solve} \left[y(x) - \frac{1}{2} \text{RootSum} \left[\#1^3 + 3\#1^2 y(x)^2 + \#1^2 + 3\#1 y(x)^4 + 2\#1 y(x)^2 + y(x)^6 + y(x)^4 + 1 \&, \frac{\log(x^2 - \#)}{3\#1^2 + 6\#1 y(x)^2 + 2\#1 + 1} \right], y(x) \right]$$

✓ **Maple** : cpu = 0.655 (sec), leaf count = 34

$$\left\{ -c_1 + \frac{\left(\int^{x^2 + y(x)^2} \frac{1}{-a^3 + -a^2 + 1} d-a \right)}{2} - y(x) = 0 \right\}$$

2.891 ODE No. 891

$$y'(x) = \frac{y(x)^2(x^4 y(x) + 2x^2 y(x) + 2x^2 - 2y(x))}{x^3(x^2 y(x) + x^2 - y(x))}$$

✓ **Mathematica** : cpu = 0.21048 (sec), leaf count = 135

$$\left\{\left\{y(x) \rightarrow \frac{x^5}{-x^3(x^2-1) + \frac{\sqrt{(x^2-1)^2 x + x^5 \left(-2\left(\frac{1}{2x^4} - \frac{1}{x^2} + \log(x)\right) + c_1}\right)}}{\sqrt{\frac{1}{x^5}}}\right\}, \left\{y(x) \rightarrow -\frac{x^5}{(x^2-1)x^3 + \frac{\sqrt{(x^2-1)^2 x + x^5 \left(-2\left(\frac{1}{2x^4} - \frac{1}{x^2} + \log(x)\right) + c_1}\right)}}{\sqrt{\frac{1}{x^5}}}\right\}\right\}$$

✓ **Maple** : cpu = 0.065 (sec), leaf count = 56

$$\left\{y(x) = \frac{x^2}{\sqrt{c_1 - 2 \ln(x)} x^2 - x^2 + 1}, y(x) = -\frac{x^2}{\sqrt{c_1 - 2 \ln(x)} x^2 + x^2 - 1}\right\}$$

2.892 ODE No. 892

$$y'(x) = \frac{e^{-\frac{2}{x^2-y(x)^2-1}} + x^2 + 2xy(x) + y(x)^2}{-e^{-\frac{2}{x^2-y(x)^2-1}} + x^2 + 2xy(x) + y(x)^2}$$

✓ **Mathematica** : cpu = 2.49072 (sec), leaf count = 1283

$$\text{Solve}\left[\int_1^x -e^{\int_1^{(K[2]-y(x))(K[2]+y(x))} \frac{2((K[1]-3)K[1]+1)}{\left(e^{-\frac{2}{K[1]-1}-K[1]}\right)(K[1]-1)^2} dK[1] + \frac{2}{K[2]^2-y(x)^2-1}} \frac{2((K[1]-3)K[1]+1)}{\left(e^{-\frac{2}{K[1]-1}-K[1]}\right)(K[1]-1)^2} dK[1] + \frac{2}{K[2]^2-y(x)^2-1}} K[2]^2 - 2e\right]$$

✓ **Maple** : cpu = 0.635 (sec), leaf count = 40

$$\left\{y(x) = -x + e^{\text{RootOf}\left(c_1 - \int^{-2xe^{-Z} + e^{2Z}} \frac{1}{\frac{2}{a+e^{-a+1}}} d_a\right)}\right\}$$

2.893 ODE No. 893

$$y'(x) = \frac{x^3y(x)^3 + x^3y(x)^2 + x^3 + 6x^2y(x)^2 + 4x^2y(x) + 12xy(x) + 6x + 8}{x^3}$$

✓ **Mathematica** : cpu = 0.211733 (sec), leaf count = 80

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{3y(x) + \frac{x+6}{x}}{\sqrt[3]{29}} - \#1\right)}{\sqrt[3]{29} - 29\#1^2}\&\right] = \frac{1}{9}29^{2/3}x + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 41

$$\left\{y(x) = \frac{29x \text{RootOf}\left(3c_1 + x - 81\left(\int^{-Z} \frac{1}{841a^3-27a+27} d_a\right)\right) - 3x - 18}{9x}\right\}$$

2.894 ODE No. 894

$$y'(x) = -\frac{i(x^6 + 3x^4y(x)^2 + x^4 + 3x^2y(x)^4 + 2x^2y(x)^2 + y(x)^6 + y(x)^4 + ix + 1)}{y(x)}$$

✗ **Mathematica** : cpu = 40.725 (sec), leaf count = 0 , could not solve

```
DSolve[Derivative[1][y][x] == ((-I)*(1 + I*x + x^4 + x^6 + 2*x^2*y[x]^2 + 3*x^4*y[x]^2 + y[x]^6), x] == (-I*(I*x+1+x^4+2*x^2*y(x)^2+y(x)^4+x^6+3*x^4*y(x)^2+3*x^2*y(x)^4+y(x)^6
```

2.895 ODE No. 895

$$y'(x) = \frac{x(a^3 x^{12} + 24 a^2 x^8 y(x) - 32 a^2 x^6 + 192 a x^4 y(x)^2 - 256 a x^2 y(x) - 256 a x^2 + 512 y(x)^3)}{64 a x^4 + 512 y(x) + 512}$$

✓ Mathematica : cpu = 0.315249 (sec), leaf count = 81

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{8}(-ax^4 - 8) + \frac{1}{512\left(\frac{1}{512} - \frac{1}{\sqrt{-262144x^2 + c_1}}\right)} \right\}, \left\{ y(x) \rightarrow \frac{1}{8}(-ax^4 - 8) + \frac{1}{512\left(\frac{1}{512} + \frac{1}{\sqrt{-262144x^2 + c_1}}\right)} \right\} \right\}$$

✓ Maple : cpu = 0.062 (sec), leaf count = 79

$$\left\{ y(x) = \frac{(-\sqrt{-x^2 + c_1} - 1)a x^4 - 8}{8 + 8\sqrt{-x^2 + c_1}}, y(x) = \frac{(-\sqrt{-x^2 + c_1} + 1)a x^4 + 8}{-8 + 8\sqrt{-x^2 + c_1}} \right\}$$

2.896 ODE No. 896

$$y'(x) = \frac{-x^6 + 3x^4 y(x)^2 + x^4 - 3x^2 y(x)^4 - 2x^2 y(x)^2 + y(x)^6 + y(x)^4 + x + 1}{y(x)}$$

✓ Mathematica : cpu = 0.349855 (sec), leaf count = 106

$$\text{Solve}\left[\frac{1}{2}\text{RootSum}\left[-\#1^3 + 3\#1^2 y(x)^2 + \#1^2 - 3\#1 y(x)^4 - 2\#1 y(x)^2 + y(x)^6 + y(x)^4 + 1\&, \frac{\log(x^2 - \#1)}{3\#1^2 - 6\#1 y(x)^2 - 2\#1 + 3y(x)}\right], y(x)\right]$$

✓ Maple : cpu = 0.511 (sec), leaf count = 63

$$\left\{ -c_1 + x + \int_{-b}^{y(x)} \frac{-a}{-a^6 + 3a^4 x^2 - 3a^2 x^4 + x^6 - a^4 + 2a^2 x^2 - x^4 - 1} d_a = 0 \right\}$$

2.897 ODE No. 897

$$y'(x) = \frac{\sqrt{x}(-108x^{3/2}y(x) + 18x^{9/2} - 108x^{3/2} + x^9 - 18x^6y(x) + 108x^3y(x)^2 - 216y(x)^3)}{36x^3 - 216y(x) - 216}$$

✓ Mathematica : cpu = 0.256293 (sec), leaf count = 79

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{6}(x^3 - 6) - \frac{1}{216\left(-\frac{1}{216} - \frac{1}{\sqrt{-62208x^{3/2} + c_1}}\right)} \right\}, \left\{ y(x) \rightarrow \frac{1}{6}(x^3 - 6) - \frac{1}{216\left(-\frac{1}{216} + \frac{1}{\sqrt{-62208x^{3/2} + c_1}}\right)} \right\} \right\}$$

✓ Maple : cpu = 0.086 (sec), leaf count = 87

$$\left\{ y(x) = \frac{\sqrt{-12x^{\frac{3}{2}} + 9c_1} x^3 - 3x^3 + 18}{6\sqrt{-12x^{\frac{3}{2}} + 9c_1} - 18}, y(x) = \frac{\sqrt{-12x^{\frac{3}{2}} + 9c_1} x^3 + 3x^3 - 18}{6\sqrt{-12x^{\frac{3}{2}} + 9c_1} + 18} \right\}$$

2.898 ODE No. 898

$$y'(x) = \frac{4x^6y(x)^3 + 2x^5y(x) + 2x^5 + 3x^4y(x)^2 + \frac{x^3}{2} + \frac{3}{4}x^2y(x) + \frac{1}{16}}{x^6(4x^2y(x) + 4x^2 + 1)}$$

✓ **Mathematica** : cpu = 0.236031 (sec), leaf count = 106

$$\left\{ \left\{ y(x) \rightarrow -\frac{4x^2 + 1}{4x^2} + \frac{1}{64x^8 \left(\frac{1}{64x^8} - \frac{1}{x^8 \sqrt{\frac{8192}{x} + c_1}} \right)} \right\}, \left\{ y(x) \rightarrow -\frac{4x^2 + 1}{4x^2} + \frac{1}{64x^8 \left(\frac{1}{64x^8} + \frac{1}{x^8 \sqrt{\frac{8192}{x} + c_1}} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 87

$$\left\{ y(x) = \frac{-4x^2 - \sqrt{\frac{c_1x+2}{x}} - 1}{4 \left(\sqrt{\frac{c_1x+2}{x}} + 1 \right) x^2}, y(x) = \frac{4x^2 - \sqrt{\frac{c_1x+2}{x}} + 1}{4 \left(\sqrt{\frac{c_1x+2}{x}} - 1 \right) x^2} \right\}$$

2.899 ODE No. 899

$$y'(x) = \frac{x^6y(x)^3 + x^6y(x)^2 + x^6 + \frac{x^5}{2} + \frac{3}{4}x^4y(x)^2 + \frac{1}{2}x^4y(x) + \frac{3}{16}x^2y(x) + \frac{x^2}{16} + \frac{1}{64}}{x^8}$$

✓ **Mathematica** : cpu = 0.265867 (sec), leaf count = 106

$$\text{Solve} \left[-\frac{29}{3} \text{RootSum} \left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log \left(\frac{\frac{3y(x)}{x^2} + \frac{4x^2+3}{4x^4}}{\sqrt[3]{29}\sqrt[3]{\frac{1}{x^6}}} - \#1 \right)}{\sqrt[3]{29} - 29\#1^2} \& \right] = -\frac{1}{9} 29^{2/3} \left(\frac{1}{x^6} \right)^{2/3} x^3 + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 47

$$\left\{ y(x) = \frac{116x^2 \text{RootOf} \left(3c_1x - 81x \left(\int^{-Z} \frac{1}{841\_a^3 - 27\_a + 27} d\_a \right) - 1 \right) - 12x^2 - 9}{36x^2} \right\}$$

2.900 ODE No. 900

$$y'(x) = \frac{2a(4ax - y(x)^2 - 1)}{128a^4x^3 - 96a^3x^2y(x)^2 + 24a^2xy(x)^4 - 2ay(x)^6 + 4axy(x) - y(x)^3 - y(x)}$$

✓ **Mathematica** : cpu = 0.237031 (sec), leaf count = 381

$$\left\{ \left\{ y(x) \rightarrow \text{Root} \left[8\#1^5a - 16\#1^4a^2c_1 - 64\#1^3a^2x + \#1^2(-2 + 128a^3c_1x) + 128\#1a^3x^2 - 256a^4c_1x^2 + 8ax - 1\&, 1 \right] \right\}, \left\{ y(x) \rightarrow \dots \right\} \right\}$$

✓ **Maple** : cpu = 0.074 (sec), leaf count = 48

$$\left\{ -c_1 + \frac{y(x)}{2a} - \frac{1}{16(-4ax + y(x)^2)^2 a^2} + \frac{1}{32a^3x - 8a^2y(x)^2} = 0 \right\}$$

2.901 ODE No. 901

$$y'(x) = \frac{y(x)(-ax \log(y(x)) + x^2 + y(x))}{x(ax - y(x) - y(x) \log(x) - y(x) \log(y(x)))}$$

✓ **Mathematica** : cpu = 0.700215 (sec), leaf count = 33

$$\text{Solve}\left[ax \log(y(x)) - \frac{x^2}{2} - y(x) \log(x) - y(x) \log(y(x)) = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.547 (sec), leaf count = 30

$$\{y(x) = e^{\text{RootOf}(-2\_Zax+2c_1a+2\_Ze^{-Z}+x^2+2e^{-Z}\ln(x))}\}$$

2.902 ODE No. 902

$$y'(x) = \frac{x^6 - 3x^4y(x)^2 + x^3 + 3x^2y(x)^4 - xy(x)^2 - y(x)^6 - x}{y(x)(x^2 - y(x)^2 - 1)}$$

✓ **Mathematica** : cpu = 0.245378 (sec), leaf count = 295

$$\left\{\left\{y(x) \rightarrow -\frac{1}{2}\sqrt{\frac{4x^3}{x-c_1} - \frac{4c_1x^2}{x-c_1} - \frac{\sqrt{-4x+1+4c_1}}{x-c_1} - \frac{1}{x-c_1}}\right\}, \left\{y(x) \rightarrow \frac{1}{2}\sqrt{\frac{4x^3}{x-c_1} - \frac{4c_1x^2}{x-c_1} - \frac{\sqrt{-4x+1+4c_1}}{x-c_1} - \frac{1}{x-c_1}}\right\}\right.$$

✓ **Maple** : cpu = 0.224 (sec), leaf count = 183

$$\left\{y(x) = \frac{\sqrt{(c_1+3x)(4c_1x^2+12x^3-\sqrt{-12c_1-36x+9}-3)}}{2c_1+6x}, y(x) = \frac{\sqrt{(c_1+3x)(4c_1x^2+12x^3+\sqrt{-12c_1-36x+9}-3)}}{2c_1+6x}\right.$$

2.903 ODE No. 903

$$y'(x) = \frac{\sin\left(\frac{y(x)}{x}\right) \csc\left(\frac{y(x)}{2x}\right) \sec\left(\frac{y(x)}{2x}\right) \left(2x^2 \sin\left(\frac{y(x)}{2x}\right) \cos\left(\frac{y(x)}{2x}\right) + y(x)\right)}{2x}$$

✓ **Mathematica** : cpu = 0.139949 (sec), leaf count = 19

$$\left\{\left\{y(x) \rightarrow 2x \cot^{-1}(e^{-x-c_1})\right\}\right\}$$

✓ **Maple** : cpu = 0.168 (sec), leaf count = 48

$$\left\{y(x) = x \arctan\left(\frac{2c_1e^x}{c_1^2e^{2x}+1}, \frac{-c_1^2e^{2x}+1}{c_1^2e^{2x}+1}\right)\right\}$$

2.904 ODE No. 904

$$y'(x) = \frac{\sin\left(\frac{y(x)}{x}\right) \csc\left(\frac{y(x)}{2x}\right) \sec\left(\frac{y(x)}{2x}\right) \left(2x^3 \sin\left(\frac{y(x)}{2x}\right) \cos\left(\frac{y(x)}{2x}\right) + y(x)\right)}{2x}$$

✓ **Mathematica** : cpu = 0.123113 (sec), leaf count = 23

$$\left\{\left\{y(x) \rightarrow 2x \cot^{-1}\left(e^{-\frac{x^2}{2}-c_1}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.177 (sec), leaf count = 64

$$\left\{y(x) = x \arctan\left(\frac{2c_1e^{\frac{x^2}{2}}}{c_1^2e^{x^2}+1}, \frac{-c_1^2e^{x^2}+1}{c_1^2e^{x^2}+1}\right)\right\}$$

2.905 ODE No. 905

$$y'(x) = \frac{a^3 x^3 y(x)^3 + a^3 x^3 y(x)^2 + a^3 x^3 + 3 a^2 x^2 y(x)^2 + 2 a^2 x^2 y(x) + a^2 x + 3 a x y(x) + a x + 1}{a^3 x^3}$$

✓ **Mathematica** : cpu = 0.262629 (sec), leaf count = 85

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{\frac{ax+3}{ax}+3y(x)}{\sqrt[3]{29}} - \#1\right)}{\sqrt[3]{29} - 29\#1^2}\&\right] = \frac{1}{9}29^{2/3}x + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 46

$$\left\{y(x) = \frac{29ax \text{RootOf}\left(3c_1 + x - 81\left(\int^{-Z} \frac{1}{841\_a^3 - 27\_a + 27} d\_a\right)\right) - 3ax - 9}{9ax}\right\}$$

2.906 ODE No. 906

$$y'(x) = \frac{x(x^2 + y(x)^2 + 1)}{x^6 + 3x^4 y(x)^2 + 3x^2 y(x)^4 - x^2 y(x) + y(x)^6 - y(x)^3 - y(x)}$$

✓ **Mathematica** : cpu = 0.160783 (sec), leaf count = 326

$$\left\{\left\{y(x) \rightarrow \text{Root}\left[4\#1^5 - 4\#1^4 c_1 + 8\#1^3 x^2 + \#1^2(2 - 8c_1 x^2) + 4\#1 x^4 - 4c_1 x^4 + 2x^2 + 1\&, 1\right]\right\}, \left\{y(x) \rightarrow \text{Root}\left[4\#1^5 - 4\#1^4 c_1\right]\right\}\right\}$$

✓ **Maple** : cpu = 0.298 (sec), leaf count = 37

$$\left\{c_1 - y(x) - \frac{1}{2x^2 + 2y(x)^2} - \frac{1}{4(x^2 + y(x)^2)^2} = 0\right\}$$

2.907 ODE No. 907

$$y'(x) = \frac{\frac{3x^2}{2} + x^2 \sin(x) - 2x^2 \cos(x) + \frac{1}{2}x^2 \cos(2x) - 2xy(x) + y(x)^2 + 2xy(x) \cos(x) + x - x \cos(x)}{x}$$

✓ **Mathematica** : cpu = 0.286452 (sec), leaf count = 22

$$\left\{\left\{y(x) \rightarrow -x(\cos(x) - 1) + \frac{1}{-\log(x) + c_1}\right\}\right\}$$

✓ **Maple** : cpu = 0.168 (sec), leaf count = 20

$$\left\{y(x) = -(\cos(x) - 1)x + \frac{1}{c_1 - \ln(x)}\right\}$$

2.908 ODE No. 908

$$y'(x) = \frac{4(a-1)(a+1)x}{a^6 x^4 - 3a^4 x^4 - 2a^4 x^2 y(x)^2 + 3a^2 x^4 + 4a^2 x^2 y(x)^2 + a^2 y(x)^4 - x^4 - 2x^2 y(x)^2 - y(x)^4 + 4y(x)}$$

✓ **Mathematica** : cpu = 1.14253 (sec), leaf count = 1269

$$\left\{\left\{y(x) \rightarrow \frac{c_1}{3(a^2 - 1)} + \frac{\sqrt[3]{-18x^2 c_1 a^6 + 54x^2 c_1 a^4 + 54a^4 - 54x^2 c_1 a^2 - 108a^2 + 2c_1^3 + 18x^2 c_1 + \sqrt{4(-3x^2 a^6 + 9x^2 a^4 - 9x^2 a^2 + 4a^4)}}{3\sqrt[3]{2}}\right\}\right\}$$

✓ **Maple** : cpu = 0.343 (sec), leaf count = 1742

$$\left\{y(x) = \frac{9^{\frac{2}{3}} \left(3a^6x^2 + (-9x^2 + c_1^2)a^4 + c_1^2 + (9x^2 - 2c_1^2)a^2 - 3x^2 + (-c_1a^2 + c_1)9^{\frac{1}{3}} \left((a-1)^2(a+1)^2 \left(\left(-\frac{a^2}{9} + \frac{1}{9} \right) c_1^3 + c_1(a-1) \right) \right) \right)}{\dots}$$

2.909 ODE No. 909

$$y'(x) = \frac{x^3y(x)^6 + x^3y(x)^4 + x^3 + 3x^2y(x)^4 + 2x^2y(x)^2 + 3xy(x)^2 + x + 1}{x^5y(x)}$$

✓ **Mathematica** : cpu = 0.185983 (sec), leaf count = 64

$$\text{Solve}\left[\frac{1}{2}\text{RootSum}\left[2\#1^3 + 2\#1^2 + 1\&, \frac{\log\left(\frac{xy(x)^2+1}{x} - \#1\right)}{3\#1^2 + 2\#1}\&\right] + \frac{1}{x} + c_1 = 0, y(x)\right]$$

✓ **Maple** : cpu = 0.559 (sec), leaf count = 84

$$\left\{y(x) = \frac{\sqrt{\left(x \text{RootOf}\left(c_1x + x\left(\int^{-Z} \frac{1}{2\_a^3+2\_a^2+1}d\_a\right) + 1\right) - 1\right)x}}{x}, y(x) = -\frac{\sqrt{\left(x \text{RootOf}\left(c_1x + x\left(\int^{-Z} \frac{1}{2\_a^3+2\_a^2+1}d\_a\right) + 1\right) + 1\right)x}}{x}\right\}$$

2.910 ODE No. 910

$$y'(x) = \frac{x^6 + 3x^5y(x) + 3x^4y(x)^2 + x^4 + x^3y(x)^3 + 2x^3y(x) + x^2y(x)^2 - y(x) - 2x + 1}{x}$$

✓ **Mathematica** : cpu = 0.237156 (sec), leaf count = 98

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{3x^3+3x^2y(x)+x}{\sqrt[3]{29}\sqrt[3]{x^3}} - \#1\right)}{\sqrt[3]{29} - 29\#1^2}\&\right] = \frac{29^{2/3}(x^3)^{2/3}}{9x} + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 42

$$\left\{y(x) = \frac{-9x^2 + 29 \text{RootOf}\left(3c_1 + x - 81\left(\int^{-Z} \frac{1}{841\_a^3-27\_a+27}d\_a\right)\right) - 3}{9x}\right\}$$

2.911 ODE No. 911

$$y'(x) = y(x)\left(-\text{F1}(x) + \frac{\log(y(x))}{x} - \cot(x)\log(y(x))\right)$$

✓ **Mathematica** : cpu = 0.931995 (sec), leaf count = 106

$$\text{Solve}\left[\int_1^x\left(\frac{2\log(y(x))\sin(K[1])}{K[1]^2} - \frac{2(\cos(K[1])\log(y(x)) - \sin(K[1])\text{F1}(K[1]))}{K[1]}\right)dK[1] + \int_1^{y(x)}\left(-\frac{2\sin(x)}{xK[2]} - \int_1^x\left(\frac{2\sin(l)}{K[1]^2}\right)dx\right)dy(x) = 0, y(x)\right]$$

✓ **Maple** : cpu = 0.796 (sec), leaf count = 30

$$\left\{y(x) = e^{\frac{c_1x}{\sin(x)}} e^{\frac{x\left(\int \frac{\text{F1}(x)\sin(x)}{x}dx\right)}{\sin(x)}}\right\}$$

2.912 ODE No. 912

$$y'(x) = \frac{2ax}{-128a^4 + 96a^3xy(x)^2 + 32a^3x - 24a^2x^2y(x)^4 - 16a^2x^2y(x)^2 + 2ax^3y(x)^6 + 2ax^3y(x)^4 + 2ax^3 - x^3y(x)}$$

✓ **Mathematica** : cpu = 1.36998 (sec), leaf count = 201

$$\text{Solve}\left[-\text{RootSum}\left[-\#1^3y(x)^6 - \#1^3y(x)^4 - \#1^3 + 12\#1^2ay(x)^4 + 8\#1^2ay(x)^2 - 48\#1a^2y(x)^2 - 16\#1a^2 + 64a^3\&, \frac{\quad}{3\#1^2y(x)}\right], y(x)\right]$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

$$\text{dsolve}(\text{diff}(y(x),x) = \frac{2ax}{(-x^3y(x)+2ax^3+2ay(x)^4x^3-16y(x)^2a^2x^2+32a^3x+24ay(x)^4a^2x^2+96y(x)^2xa^3-128a^4)}, y(x))$$

2.913 ODE No. 913

$$y'(x) = \frac{y(x)^3 + y(x) + y(x)^3(-\log^3(x)) + y(x)^3\log^2(x) + 3y(x)^2\log^2(x) - 2y(x)^2\log(x) - 3y(x)\log(x) + 1}{xy(x)}$$

✓ **Mathematica** : cpu = 0.907636 (sec), leaf count = 716

$$\text{Solve}\left[\int_1^{y(x)}\left(2\text{RootSum}\left[\#1^3K[1]^3 - \#1^2K[1]^3 - 2K[1]^3 - 3\#1^2K[1]^2 + 2\#1K[1]^2 + 3\#1K[1] - K[1] - 1\&, \frac{\quad}{3\#1^2K[1]^2 - 2}\right], y(x)\right]$$

✓ **Maple** : cpu = 0.086 (sec), leaf count = 43

$$\left\{y(x) = \frac{9}{56\text{RootOf}\left(3c_1 - 81\left(\int^{-Z}\frac{1}{3136_a^3-27_a+27}d_a\right) - \ln(x)\right) + 9\ln(x) - 3}\right\}$$

2.914 ODE No. 914

$$y'(x) = \frac{2a(-4a + xy(x)^2 + x)}{-128a^4 + 96a^3xy(x)^2 - 24a^2x^2y(x)^4 + 2ax^3y(x)^6 + 4ax^2y(x) - x^3y(x)^3 - x^3y(x)}$$

✓ **Mathematica** : cpu = 1.301 (sec), leaf count = 401

$$\left\{\left\{y(x) \rightarrow \text{Root}\left[8\#1^5ax^2 - 8\#1^4ac_1x^2 - 64\#1^3a^2x + \#1^2\left(2x^2 + 64a^2c_1x\right) + 128\#1a^3 - 128a^3c_1 - 8ax + x^2\&, 1\right]\right\}, \left\{y(x) \rightarrow \right.$$

✓ **Maple** : cpu = 3.685 (sec), leaf count = 71

$$\left\{c_1 + \frac{xy(x)^4 + (-4a + x)y(x)^2 - 2a}{2(-xy(x)^2 + 4a)^2ay(x)^4} + \frac{8ay(x)^5 + 2y(x)^2 + 1}{16a^2y(x)^4} = 0\right\}$$

2.915 ODE No. 915

$$y'(x) = \frac{y(x)^3 + y(x) - 8y(x)^3\log^3(x) + 4y(x)^3\log^2(x) + 12y(x)^2\log^2(x) - 4y(x)^2\log(x) - 6y(x)\log(x) + 1}{xy(x)}$$

✓ **Mathematica** : cpu = 0.985306 (sec), leaf count = 724

$$\text{Solve}\left[\int_1^{y(x)}\left(4\text{RootSum}\left[8\#1^3K[1]^3 - 4\#1^2K[1]^3 - 3K[1]^3 - 12\#1^2K[1]^2 + 4\#1K[1]^2 + 6\#1K[1] - K[1] - 1\&, \frac{\quad}{12\#1^2K[1]^2}\right], y(x)\right]$$

✓ **Maple** : cpu = 0.083 (sec), leaf count = 43

$$\left\{y(x)=\frac{9}{83\operatorname{RootOf}\left(3c_1-81\left(\int^{-Z}\frac{1}{6889\_a^3-27\_a+27}d\_a\right)-\ln(x)\right)+18\ln(x)-3}\right\}$$

2.916 ODE No. 916

$$y'(x)=\frac{y(x)\left(x^4\log^2(y(x))+2x^4\log(x)\log(y(x))+x^4\log^2(x)+x\log(y(x))+\log(y(x))-x+x\log(x)+\log(x)-1\right)}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.251155 (sec), leaf count = 43

$$\left\{\left\{y(x)\rightarrow\frac{\exp\left(\frac{12x}{-3x^4+4x^3-6x^2+12x-12\log(x+1)+c_1}\right)}{x}\right\}\right\}$$

✓ **Maple** : cpu = 0.318 (sec), leaf count = 73

$$\left\{y(x)=e^{\frac{-12\ln(x)\ln(x+1)-12x+\left(-3x^4+4x^3-6x^2+12x+12c_1\right)\ln(x)}{3x^4-4x^3+6x^2-12c_1-12x+12\ln(x+1)}}\right\}$$

2.917 ODE No. 917

$$y'(x)=\frac{y(x)\left(x\log^2(y(x))+2x\log(x)\log(y(x))+x\log(y(x))+\log(y(x))-x+x\log^2(x)+x\log(x)+\log(x)-1\right)}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.250359 (sec), leaf count = 28

$$\left\{\left\{y(x)\rightarrow\frac{e^{-\frac{x}{x-\log(x+1)-c_1}}}{x}\right\}\right\}$$

✓ **Maple** : cpu = 0.218 (sec), leaf count = 38

$$\left\{y(x)=e^{\frac{\ln(x)\ln(x+1)-x+\left(c_1-x\right)\ln(x)}{-c_1+x-\ln(x+1)}}\right\}$$

2.918 ODE No. 918

$$y'(x)=\frac{2y(x)^8}{128x^3y(x)^6+32x^2y(x)^6+96x^2y(x)^4+2y(x)^6+y(x)^5+16xy(x)^4+24xy(x)^2+2y(x)^2+2}$$

✓ **Mathematica** : cpu = 0.591407 (sec), leaf count = 720

$$\operatorname{Solve}\left[\int_1^{y(x)}\left(\operatorname{RootSum}\left[64\#1^3K[1]^6+16\#1^2K[1]^6+K[1]^6+48\#1^2K[1]^4+8\#1K[1]^4+12\#1K[1]^2+K[1]^2+1\&, \frac{1}{48\#1^2K[1]^6}\right]\right)dy(x)=0\right]$$

✓ **Maple** : cpu = 1.016 (sec), leaf count = 41

$$\left\{x-\operatorname{RootOf}\left(c_1y(x)+\left(\int^{-Z}\frac{1}{64\_a^3+16\_a^2+1}d\_a\right)y(x)+1\right)+\frac{1}{4y(x)^2}=0\right\}$$

2.919 ODE No. 919

$$y'(x) = \frac{(-y(x) + \sqrt{y(x)} + x)y(x)^{3/2}}{x^3 - 3x^2y(x) + 3xy(x)^2 + xy(x)^{3/2} - y(x)^3 - y(x)^{5/2} + y(x)^2}$$

✓ **Mathematica** : cpu = 0.355592 (sec), leaf count = 251

$$\left\{ \left\{ y(x) \rightarrow \text{Root}\left[\#1^9 c_1^4 - 6\#1^8 c_1^4 x + \#1^7 \left(15c_1^4 x^2 - 6c_1^2\right) + \#1^6 \left(-20c_1^4 x^3 + 30c_1^2 x - 4 + 2c_1^2\right) + \#1^5 \left(15c_1^4 x^4 - 60c_1^2 x^2\right)\right], \right. \right.$$

✓ **Maple** : cpu = 0.167 (sec), leaf count = 61

$$\left\{ -\frac{c_1 \left(-x + y(x)\right)^{\frac{3}{2}} y(x)^{\frac{3}{4}} + \left(x - y(x) - \sqrt{y(x)}\right) \sqrt{-2x + 2y(x) - \sqrt{y(x)}}}{\left(-x + y(x)\right)^{\frac{3}{2}} y(x)^{\frac{3}{4}}} = 0 \right\}$$

2.920 ODE No. 920

$$y'(x) = \frac{2y(x)^6(4xy(x)^2 + y(x)^2 + 1)}{128x^3y(x)^6 + 96x^2y(x)^4 + 4xy(x)^5 + y(x)^5 + y(x)^3 + 24xy(x)^2 + 2}$$

✓ **Mathematica** : cpu = 0.467939 (sec), leaf count = 301

$$\left\{ \left\{ y(x) \rightarrow \text{Root}\left[\#1^5 \left(128c_1 x^2 - 8x - 1\right) + 128\#1^4 x^2 + \#1^3 \left(-2 + 64c_1 x\right) + 64\#1^2 x + 8\#1c_1 + 8\&, 1\right], \left\{ y(x) \rightarrow \text{Root}\left[\#1^5 \left(128c_1 x^2 - 8x - 1\right) + 128\#1^4 x^2 + \#1^3 \left(-2 + 64c_1 x\right) + 64\#1^2 x + 8\#1c_1 + 8\&, 1\right], \right. \right.$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

$$\text{dsolve}(\text{diff}(y(x), x) = 2*y(x)^6*(1+4*x*y(x)^2+y(x)^2)/(y(x)^3+4*y(x)^5*x+y(x)^5+2+24*x*y(x)^2), y(x))$$

2.921 ODE No. 921

$$y'(x) = y(x) \left(-F1(x) + \frac{\log(y(x))}{x} - \frac{\log(y(x))}{x \log(x)} \right)$$

✓ **Mathematica** : cpu = 0.295569 (sec), leaf count = 92

$$\text{Solve}\left[\int_1^x \left(\frac{\log(y(x)) - \log(K[1]) \log(y(x))}{K[1]^2} - \frac{\log(K[1])_F1(K[1])}{K[1]}\right) dK[1] + \int_1^{y(x)} \left(\frac{\log(x)}{xK[2]} - \int_1^x \frac{\frac{1}{K[2]} - \frac{\log(K[1])}{K[2]}}{K[1]^2} dK[1]\right) dK[2] = 0, y(x)\right]$$

✓ **Maple** : cpu = 0.196 (sec), leaf count = 30

$$\left\{ y(x) = e^{\frac{c_1 x}{\ln(x)}} e^{\frac{x \left(\int \frac{-F1(x) \ln(x)}{x} dx\right)}{\ln(x)}} \right\}$$

2.922 ODE No. 922

$$y'(x) = \frac{y(x)^2}{x^3 - 3x^2y(x) + x^2\sqrt{y(x)} + 3xy(x)^2 - 2xy(x)^{3/2} - y(x)^3 + y(x)^{5/2} + y(x)^2 + y(x)^{3/2}}$$

✓ **Mathematica** : cpu = 1.42762 (sec), leaf count = 882

$$\text{Solve}\left[\int_1^{y(x)} \left(\frac{-x - K[1]}{2(-2x^3 + 6K[1]x^2 - 2\sqrt{K[1]}x^2 - 6K[1]^2x + 4K[1]^{3/2}x + K[1]x + 2K[1]^3 - 2K[1]^{5/2} - K[1]^2 - 2K[1]^{3/2})} + \int_1^x \frac{1}{2a^3 + 2a^2 - a + 2} da\right) dy(x) = 0, y(x)\right]$$

✓ **Maple** : cpu = 0.18 (sec), leaf count = 47

$$\left\{ -c_1 - \left(\int \frac{\frac{x}{\sqrt{y(x)}} - \sqrt{y(x)}}{2a^3 + 2a^2 - a + 2} da \right) + \frac{\ln(y(x))}{2} = 0 \right\}$$

2.923 ODE No. 923

$$y'(x) = \frac{x^2 + 2xy(x) + e^{-2(x-y(x))(y(x)+x)} + y(x)^2}{x^2 + 2xy(x) - e^{-2(x-y(x))(y(x)+x)} + y(x)^2}$$

✓ **Mathematica** : cpu = 1.62935 (sec), leaf count = 432

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{2e^{2(x-K[2])(x+K[2])}K[2]}{-e^{2(x-K[2])(x+K[2])}x^2+e^{2(x-K[2])(x+K[2])}K[2]^2+1}-\int_1^x\left(-\frac{2e^{2(K[1]-K[2])(K[1]+K[2])}K[1](2(K[1]-K[2])-2(K[1]-K[2])K[2])}{e^{2(K[1]-K[2])(K[1]+K[2])}K[1]^2-e^{2(K[1]-K[2])(K[1]+K[2])}K[1]K[2]+e^{2(K[1]-K[2])(K[1]+K[2])}K[2]^2}\right)dK[1]\right)dK[2]=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.413 (sec), leaf count = 36

$$\left\{y(x)=-x+e^{\text{RootOf}\left(c_1-{}_Z+\int^{-2xe^{-Z}+e^{2Z}}\frac{1}{-a+e^{2a}}d_a\right)}\right\}$$

2.924 ODE No. 924

$$y'(x) = \frac{y(x)\left(-F1(x) + \frac{\log^2(y(x))}{2x}\right)}{\log(y(x))}$$

✓ **Mathematica** : cpu = 0.312793 (sec), leaf count = 80

$$\text{Solve}\left[\int_1^x\left(-\frac{\log^2(y(x))}{2K[1]^2}-\frac{-F1(K[1])}{K[1]}\right)dK[1]+\int_1^{y(x)}\left(\frac{\log(K[2])}{xK[2]}-\int_1^x-\frac{\log(K[2])}{K[1]^2K[2]}dK[1]\right)dK[2]=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.128 (sec), leaf count = 46

$$\left\{y(x)=e^{\sqrt{2c_1x+2x\left(\int\frac{-F1(x)}{x}dx\right)}},y(x)=e^{-\sqrt{2}\sqrt{\left(c_1+\int\frac{-F1(x)}{x}dx\right)x}}\right\}$$

2.925 ODE No. 925

$$y'(x) = \frac{x^2 + 2xy(x) + e^{2(x-y(x))^2(y(x)+x)^2} + y(x)^2}{x^2 + 2xy(x) - e^{2(x-y(x))^2(y(x)+x)^2} + y(x)^2}$$

✓ **Mathematica** : cpu = 2.25506 (sec), leaf count = 228

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{2K[2]}{-x^2+e^{2(x-K[2])^2(x+K[2])^2}+K[2]^2}-\int_1^x\left(\frac{2K[1](-2K[2]-e^{2(K[1]-K[2])^2(K[1]+K[2])^2}(4(K[1]-K[2])^2(K[1]+K[2])^2-K[1]K[2])}{(K[1]^2-e^{2(K[1]-K[2])^2(K[1]+K[2])^2}-K[1]K[2])}\right)dK[1]\right)dK[2]=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.312 (sec), leaf count = 38

$$\left\{y(x)=-x+e^{\text{RootOf}\left(c_1-{}_Z+\int^{-2xe^{-Z}+e^{2Z}}\frac{1}{-a+e^{2a^2}}d_a\right)}\right\}$$

2.926 ODE No. 926

$$y'(x) = \frac{\frac{1}{16}x^3y(x)^3 - \frac{1}{2}x^2y(x)^3 - \frac{3}{8}x^2y(x)^2 + xy(x)^3 + xy(x)^2 + \frac{3}{4}xy(x) - \frac{1}{2}}{x(xy(x) - 2y(x) - 2)}$$

✓ **Mathematica** : cpu = 0.234324 (sec), leaf count = 128

$$\left\{\left(y(x)\rightarrow\frac{2}{x-2}+\frac{1}{16x(x-2)\left(-\frac{1}{64}-\frac{e^{2\left(\frac{1}{2}\log(2-x)-\frac{\log(x)}{2}\right)}}{\sqrt{2048\log(x)+c_1}}\right)}\right),\left(y(x)\rightarrow\frac{2}{x-2}+\frac{1}{16x(x-2)\left(-\frac{1}{64}+\frac{e^{2\left(\frac{1}{2}\log(2-x)-\frac{\log(x)}{2}\right)}}{\sqrt{2048\log(x)+c_1}}\right)}\right)\right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 67

$$\left\{y(x)=\frac{2\sqrt{c_1+8\ln(x)}-8}{\sqrt{c_1+8\ln(x)}\,x-4x+8},y(x)=\frac{2\sqrt{c_1+8\ln(x)}+8}{\sqrt{c_1+8\ln(x)}\,x+4x-8}\right\}$$

2.927 ODE No. 927

$$y'(x)=-\frac{1}{8}x\Big(12e^{-x^2}x^2y(x)^2+8e^{-x^2}x^2y(x)+8e^{-x^2}x^2-8e^{-x^2}+e^{-3x^2}x^6-6e^{-2x^2}x^4y(x)-2e^{-2x^2}x^4-8y(x)^3-8y(x)^2-8\Big)$$

✓ **Mathematica** : cpu = 0.737036 (sec), leaf count = 112

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3+3\sqrt[3]{29}\#1-29\&, \frac{\log\left(\frac{\frac{1}{2}e^{-x^2}x\left(2e^{x^2}-3x^2\right)+3xy(x)}{\sqrt[3]{29}\sqrt[3]{x^3}}-\#1\right)}{\sqrt[3]{29}-29\#1^2}\&\right]=\frac{1}{18}29^{2/3}\left(x^3\right)^{2/3}+c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.114 (sec), leaf count = 68

$$\left\{y(x)=\frac{\left(58\text{RootOf}\left(x^2+6c_1-162\left(\int^{-Z}\frac{1}{841\_a^3-27\_a+27}d\_a\right)\right)+\left(9x^2-6\,e^{x^2}\right)e^{-x^2}\right)e^{x^2}e^{-x^2}}{18}\right\}$$

2.928 ODE No. 928

$$y'(x)=\frac{e^{\frac{y(x)}{x}}\left(x^2e^{-\frac{y(x)}{x}}+xe^{-\frac{y(x)}{x}}+xe^{-\frac{y(x)}{x}}y(x)+e^{-\frac{y(x)}{x}}y(x)+x\right)}{x(x+1)}$$

✓ **Mathematica** : cpu = 1.26893 (sec), leaf count = 23

$$\left\{\left\{y(x)\rightarrow -x\log\left(-\frac{\log(x+1)-c_1}{x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.294 (sec), leaf count = 20

$$\left\{y(x)=-x\ln\left(\frac{c_1-\ln(x+1)}{x}\right)\right\}$$

2.929 ODE No. 929

$$y'(x)=\frac{-\frac{1}{32}x^3y(x)^3+\frac{1}{16}x^2y(x)^3+\frac{3}{16}x^2y(x)^2-\frac{1}{2}xy(x)^3+\frac{y(x)^3}{4}-\frac{1}{4}xy(x)^2-\frac{3}{8}xy(x)+\frac{y(x)}{4}+\frac{1}{4}}{xy(x)}$$

✓ **Mathematica** : cpu = 0.940212 (sec), leaf count = 683

$$\text{Solve}\left[\int_1^{y(x)}\left(-32\text{RootSum}\left[\#1^3K[1]^3-2\#1^2K[1]^3-8K[1]^3-6\#1^2K[1]^2+8\#1K[1]^2+12\#1K[1]-8K[1]-8\&,\frac{1}{3\#1^2K[1]^3}\&\right]\right)dy(x)=0,y(x)\right]$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 42

$$\left\{y(x)=\frac{18}{9x+58\text{RootOf}\left(12c_1-324\left(\int^{-Z}\frac{1}{841\_a^3-27\_a+27}d\_a\right)-\ln(x)\right)-6}\right\}$$

2.930 ODE No. 930

$$y'(x) = \frac{e^{\frac{y(x)}{x}} \left(x^4 + x^2 e^{-\frac{y(x)}{x}} + x e^{-\frac{y(x)}{x}} + x e^{-\frac{y(x)}{x}} y(x) + e^{-\frac{y(x)}{x}} y(x) \right)}{x(x+1)}$$

✓ **Mathematica** : cpu = 1.59835 (sec), leaf count = 39

$$\left\{ \left(y(x) \rightarrow -x \log \left(\frac{-\frac{x^3}{3} + \frac{x^2}{2} - x + \log(x+1) - c_1}{x} \right) \right) \right\}$$

✓ **Maple** : cpu = 0.434 (sec), leaf count = 36

$$\left\{ y(x) = -x \ln \left(\frac{-2x^3 + 3x^2 - 6c_1 - 6x + 6 \ln(x+1)}{6x} \right) \right\}$$

2.931 ODE No. 931

$$y'(x) = \frac{x^6 + 3x^5 y(x) + 3x^4 y(x)^2 + x^3 y(x)^3 - 2x^3 - 3x^2 y(x) - x y(x)^2 - y(x) - 2x}{x(x^2 + x y(x) + 1)}$$

✓ **Mathematica** : cpu = 0.211762 (sec), leaf count = 80

$$\left\{ \left(y(x) \rightarrow -\frac{x^2+1}{x} + \frac{1}{x^2 \left(\frac{1}{x} - \frac{1}{x \sqrt{-2x+c_1}} \right)} \right), \left(y(x) \rightarrow -\frac{x^2+1}{x} + \frac{1}{x^2 \left(\frac{1}{x} + \frac{1}{x \sqrt{-2x+c_1}} \right)} \right) \right\}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 73

$$\left\{ y(x) = \frac{-\sqrt{c_1-2x} x^2 - x^2 - 1}{(\sqrt{c_1-2x} + 1)x}, y(x) = \frac{-\sqrt{c_1-2x} x^2 + x^2 + 1}{(\sqrt{c_1-2x} - 1)x} \right\}$$

2.932 ODE No. 932

$$y'(x) = \frac{e^{-\frac{3x^2}{2}} x \left(3e^{3x^2} y(x)^3 + e^{\frac{9x^2}{2}} y(x)^3 + 18e^{3x^2} y(x)^2 + 9e^{\frac{9x^2}{2}} y(x)^2 + 27e^{3x^2} y(x) + 27e^{\frac{9x^2}{2}} y(x) + 27e^{\frac{9x^2}{2}} + 27y(x)^3 \right)}{243y(x)}$$

✓ **Mathematica** : cpu = 5.82178 (sec), leaf count = 4323

$$\text{Solve} \left[\frac{x^2}{2} + \frac{27}{2} \text{RootSum} \left[y(x)^3 \#1^3 + 9y(x)^2 \#1^3 + 27y(x) \#1^3 + 27 \#1^3 + 3y(x)^3 \#1^2 + 18y(x)^2 \#1^2 + 27y(x) \#1^2 - 243y(x)^3 \right], \right]$$

✓ **Maple** : cpu = 0.133 (sec), leaf count = 54

$$\left\{ y(x) = -\frac{369 e^{\frac{3x^2}{2}}}{-136 \text{RootOf} \left(-41x^2 + 27c_1 - 50243409 \left(\int^{-Z} \frac{1}{9248_a^3 - 1860867_a + 1860867} d_a \right) \right) + 123 e^{\frac{3x^2}{2}} + 123} \right\}$$

2.933 ODE No. 933

$$y'(x) = \frac{x^3 + x^3(-\log^3(x)) + x^3 \log^2(x) + 3x^2 y(x) \log^2(x) - 2x^2 y(x) \log(x) + x^2 + xy(x)^2 + xy(x) + y(x)^3 - 3xy(x)^2 \log(x)}{x^2}$$

✓ **Mathematica** : cpu = 0.27893 (sec), leaf count = 99

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log\left(\frac{\frac{3y(x)}{x^2} + \frac{1-3\log(x)}{x}}{\sqrt[3]{29}\sqrt[3]{\frac{1}{x^3}}} - \#1\right)}{\sqrt[3]{29} - 29\#1^2}\&\right] = \frac{29^{2/3}}{9\sqrt[3]{\frac{1}{x^3}}} + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.061 (sec), leaf count = 39

$$\left\{y(x) = \frac{\left(29 \text{RootOf}\left(3c_1 + x - 81\left(\int^{-Z} \frac{1}{841\_a^3 - 27\_a + 27} d\_a\right)\right) + 9 \ln(x) - 3\right)x}{9}\right\}$$

2.934 ODE No. 934

$$y'(x) = -\frac{x^6}{64} - \frac{3x^5}{32} + \frac{3}{16}x^4y(x) - \frac{x^4}{8} + \frac{3}{4}x^3y(x) + \frac{x^3}{8} - \frac{3}{4}x^2y(x)^2 + \frac{1}{4}x^2y(x) + \frac{x^2}{4} - \frac{3}{2}xy(x)^2 - xy(x) + y(x)^3 + y(x)^2 + \frac{x}{2} + 1$$

✓ **Mathematica** : cpu = 0.323667 (sec), leaf count = 102

$$\text{Solve}\left[-\frac{31}{3}\text{RootSum}\left[-31\#1^3 + 3\,2^{2/3}\sqrt[3]{31}\#1 - 31\&, \frac{\log\left(\sqrt[3]{\frac{2}{31}}\left(\frac{1}{4}(-3x^2 - 6x + 4) + 3y(x)\right) - \#1\right)}{2^{2/3}\sqrt[3]{31} - 31\#1^2}\&\right] = \frac{1}{9}\left(\frac{31}{2}\right)^{2/3}x + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.068 (sec), leaf count = 39

$$\left\{y(x) = \frac{x^2}{4} + \frac{x}{2} + \text{RootOf}\left(c_1 - x + 2\left(\int^{-Z} \frac{1}{2\_a^3 + 2\_a^2 + 1} d\_a\right)\right)\right\}$$

2.935 ODE No. 935

$$y'(x) = \frac{x^6}{64} - \frac{3x^5}{16} + \frac{3}{16}x^4y(x) + \frac{13x^4}{16} - \frac{3}{2}x^3y(x) - \frac{3x^3}{2} + \frac{3}{4}x^2y(x)^2 + \frac{7}{2}x^2y(x) + x^2 - 3xy(x)^2 - 2xy(x) + y(x)^3 + y(x)^2 - \frac{x}{2} + 1$$

✓ **Mathematica** : cpu = 62.1944 (sec), leaf count = 248

$$\text{Solve}\left[\frac{\sqrt[3]{2}\left(\frac{\frac{1}{4}(3x^2 - 12x + 4) + 3y(x)}{\sqrt[3]{2}} + 2^{2/3}\right)\left(2^{2/3} - 2^{2/3}\left(\frac{1}{4}(3x^2 - 12x + 4) + 3y(x)\right)\right)\left(\left(\frac{1}{4}(-3x^2 + 12x - 4) - 3y(x) + 1\right)\log\left(2^{2/3} - 2^{2/3}\left(\frac{1}{4}(3x^2 - 12x + 4) + 3y(x)\right)\right) + 3\left(\frac{1}{4}(3x^2 - 12x + 4) + 3y(x)\right)^3 + 3\left(\frac{1}{4}(3x^2 - 12x + 4) + 3y(x)\right)^2 + 3\left(\frac{1}{4}(3x^2 - 12x + 4) + 3y(x)\right) + 3\right)}{9\left(-\left(\frac{1}{4}(3x^2 - 12x + 4) + 3y(x)\right)^3 + 3\left(\frac{1}{4}(3x^2 - 12x + 4) + 3y(x)\right)^2 + 3\left(\frac{1}{4}(3x^2 - 12x + 4) + 3y(x)\right) + 3\right)} = \frac{1}{9}\left(\frac{31}{2}\right)^{2/3}x + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.212 (sec), leaf count = 55

$$\left\{y(x) = -\frac{x^2}{4} + x + \frac{e^{\text{RootOf}(c_1 e^{-Z} - Z e^{-Z} + x e^{-Z} + e^{-Z} \ln(e^{-Z} - 4) - 4c_1 + 4_Z - 4x - 4 \ln(e^{-Z} - 4) + 4)}}{4} - 1\right\}$$

2.936 ODE No. 936

$$y'(x) = \frac{x^6}{512} - \frac{3x^5}{256} + \frac{3}{64}x^4y(x) + \frac{5x^4}{128} - \frac{3}{16}x^3y(x) - \frac{5x^3}{64} + \frac{3}{8}x^2y(x)^2 + \frac{7}{16}x^2y(x) + \frac{x^2}{16} - \frac{3}{4}xy(x)^2 - \frac{1}{2}xy(x) + y(x)^3 + y(x)^2 - \frac{x}{4} + 1$$

✓ **Mathematica** : cpu = 0.353172 (sec), leaf count = 99

$$\text{Solve}\left[-\frac{89}{3}\text{RootSum}\left[-89\#1^3+6\sqrt[3]{178}\#1-89\&, \frac{\log\left(\frac{2^{2/3}\left(\frac{1}{8}(3x^2-6x+8)+3y(x)\right)}{\sqrt[3]{89}}-\#1\right)}{2\sqrt[3]{178}-89\#1^2}\&\right]=\frac{89^{2/3}x}{18\sqrt[3]{2}}+c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.068 (sec), leaf count = 39

$$\left\{y(x)=-\frac{x^2}{8}+\frac{x}{4}+\text{RootOf}\left(c_1-x+4\left(\int^{-Z}\frac{1}{4\_a^3+4\_a^2+3}d\_a\right)\right)\right\}$$

2.937 ODE No. 937

$$y'(x)=\frac{2xy(x)^3+y(x)^3-2y(x)+6xy(x)\log^2(2x+1)+3y(x)\log^2(2x+1)+6xy(x)^2\log(2x+1)+3y(x)^2\log(2x+1)+(2x+1)(y(x)+\log(2x+1)+1)}{(2x+1)(y(x)+\log(2x+1)+1)}$$

✓ **Mathematica** : cpu = 0.330358 (sec), leaf count = 124

$$\left\{\left\{y(x)\rightarrow\frac{1}{(2x+1)\left(\frac{2x+1}{4x^2+4x+1}-\frac{1}{(2x+1)\sqrt{-2x+c_1}}\right)}-\log(2x+1)-1\right\},\left\{y(x)\rightarrow\frac{1}{(2x+1)\left(\frac{2x+1}{4x^2+4x+1}+\frac{1}{(2x+1)\sqrt{-2x+c_1}}\right)}-\log(2x+1)-1\right\}\right\}$$

✓ **Maple** : cpu = 0.092 (sec), leaf count = 79

$$\left\{y(x)=\frac{-\sqrt{c_1-2x}\ln(2x+1)-\ln(2x+1)-1}{\sqrt{c_1-2x}+1},y(x)=\frac{-\sqrt{c_1-2x}\ln(2x+1)+\ln(2x+1)+1}{\sqrt{c_1-2x}-1}\right\}$$

2.938 ODE No. 938

$$y'(x)=\frac{x^6-3x^5+3x^4y(x)+4x^4-6x^3y(x)-3x^3+3x^2y(x)^2+5x^2y(x)-x^2-3xy(x)^2-2xy(x)+y(x)^3+y(x)^2+x+1}{x}$$

✓ **Mathematica** : cpu = 0.27716 (sec), leaf count = 108

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3+3\sqrt[3]{29}\#1-29\&, \frac{\log\left(\frac{\frac{3x^2-3x+1}{x}+\frac{3y(x)}{x}}{\sqrt[3]{29}\sqrt[3]{\frac{1}{x^3}}}}-\#1\right)}{\sqrt[3]{29}-29\#1^2}\&\right]=\frac{1}{9}29^{2/3}\left(\frac{1}{x^3}\right)^{2/3}x^2\log(x)+c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 39

$$\left\{y(x)=-x^2+x+\frac{29\text{RootOf}\left(3c_1-81\left(\int^{-Z}\frac{1}{841\_a^3-27\_a+27}d\_a\right)+\ln(x)\right)}{9}-\frac{1}{3}\right\}$$

2.939 ODE No. 939

$$y'(x)=\frac{x^6+6x^5-12x^4y(x)+12x^4-48x^3y(x)+16x^3+48x^2y(x)^2-48x^2y(x)+16x^2+96xy(x)^2-32xy(x)-64y(x)^3-16x^2-64y(x)+32x-64}{16x^2-64y(x)+32x-64}$$

✓ **Mathematica** : cpu = 0.593567 (sec), leaf count = 136

$$\text{Solve}\left[\frac{2}{5}\text{RootSum}\left[\#1^4+4\#1^3-8\#1^2y(x)-16\#1y(x)-8\#1+16y(x)^2+16y(x)+8\&,\frac{\#1^2(-\log(x-\#1))+4y(x)\log(x)}{-\#1^2-2}\&\right]=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.085 (sec), leaf count = 70

$$\left\{-c_1+x-\frac{2\arctan\left(\frac{x^2}{2}+x-2y(x)-1\right)}{5}-\frac{4\ln\left(-\frac{x^2}{4}-\frac{x}{2}+y(x)-1\right)}{5}+\frac{2\ln\left(-\frac{x^2}{2}-x+2y(x)+2\left(-\frac{x^2}{4}-\frac{x}{2}+y(x)\right)^2+1\right)}{5}\right\}$$

2.940 ODE No. 940

$$y'(x)=\frac{x^3\log^3(x)-3x^2y(x)\log^2(x)-x^2+x^2\log(x)-y(x)^3-y(x)^2-2xy(x)+3xy(x)^2\log(x)+xy(x)\log(x)}{x(-y(x)-x+x\log(x))}$$

✓ **Mathematica** : cpu = 0.283742 (sec), leaf count = 80

$$\left\{\left\{y(x)\rightarrow-\frac{1}{x\left(-\frac{1}{x^2}-\frac{1}{x^2\sqrt{-2x+c_1}}\right)}-x+x\log(x)\right\},\left\{y(x)\rightarrow-\frac{1}{x\left(-\frac{1}{x^2}+\frac{1}{x^2\sqrt{-2x+c_1}}\right)}-x+x\log(x)\right\}\right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 63

$$\left\{y(x)=\frac{\left(\sqrt{c_1-2x}\ln(x)-\ln(x)+1\right)x}{\sqrt{c_1-2x}-1},y(x)=\frac{\left(\sqrt{c_1-2x}\ln(x)+\ln(x)-1\right)x}{\sqrt{c_1-2x}+1}\right\}$$

2.941 ODE No. 941

$$y'(x)=\frac{x^6-12x^5+12x^4y(x)+48x^4-96x^3y(x)-72x^3+48x^2y(x)^2+192x^2y(x)+32x^2-192xy(x)^2-32xy(x)+64y(x)}{16x^2+64y(x)-64x+64}$$

✓ **Mathematica** : cpu = 0.509399 (sec), leaf count = 53

$$\text{Solve}\left[x-8\text{RootSum}\left[11776\#1^3-40\#1-1\&,\#1\log\left(17664\#1^2-1472\#1+11x^2+44y(x)-44x-40\right)\&\right]=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.055 (sec), leaf count = 35

$$\left\{y(x)=-\frac{x^2}{4}+x+\text{RootOf}\left(c_1-x+\int^{-Z}\frac{\_a+1}{\_a^3-\_a-1}d\_a\right)\right\}$$

2.942 ODE No. 942

$$y'(x)=\frac{-\exp\left(\frac{2(x-y(x))^3(y(x)+x)^3}{x^2-y(x)^2-1}\right)-x^2-2xy(x)-y(x)^2}{\exp\left(\frac{2(x-y(x))^3(y(x)+x)^3}{x^2-y(x)^2-1}\right)-x^2-2xy(x)-y(x)^2}$$

✓ **Mathematica** : cpu = 4.93284 (sec), leaf count = 349

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{2K[2]}{-x^2+\exp\left(\frac{2(x-K[2])^3(x+K[2])^3}{x^2-K[2]^2-1}\right)}+K[2]^2\right)-\int_1^x\left(\frac{2K[1]\left(-2K[2]-\exp\left(\frac{2(K[1]-K[2])^3(K[1]+K[2])^3}{K[1]^2-K[2]^2-1}\right)\right)\left(\frac{6(K[1]+K[2])^2(K[1]-K[2])^3}{K[1]^2-K[2]^2-1}\right)}{\left(K[1]^2-\exp\left(\frac{2(K[1]-K[2])^3(K[1]+K[2])^3}{K[1]^2-K[2]^2-1}\right)\right)}\right)dy(x)=0\right]$$

✓ **Maple** : cpu = 0.71 (sec), leaf count = 43

$$\left\{y(x)=-x+e^{\text{RootOf}\left(c_1-\_Z+\int^{-2xe^{-Z}+e^{2-Z}}\frac{1}{2\_a^3}d\_a\right)}\right\}$$

2.943 ODE No. 943

$$y'(x) = \frac{x^6 - 6x^5 + 24x^4y(x) + 12x^4 - 96x^3y(x) - 24x^3 + 192x^2y(x)^2 + 96x^2y(x) + 32x^2 - 384xy(x)^2 - 128xy(x) + 512y(x)}{64x^2 + 512y(x) - 128x + 512}$$

✓ **Mathematica** : cpu = 0.594247 (sec), leaf count = 53

$$\text{Solve}\left[x - 16\text{RootSum}\left[6656\#1^3 - 23\#1 - 1\&, \#1 \log\left(79872\#1^2 - 18304\#1 + 181x^2 + 1448y(x) - 362x - 184\right)\&\right] = c_1, y\right]$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 40

$$\left\{y(x) = -\frac{x^2}{8} + \frac{x}{4} + \text{RootOf}\left(c_1 - x + \int^{\underline{Z}} \frac{4\underline{a} + 4}{4\underline{a}^3 - \underline{a} - 1} d\underline{a}\right)\right\}$$

2.944 ODE No. 944

$$y'(x) = \frac{a^3x^6 + 6a^2bx^5 + 12a^2x^4y(x) - 8a^2x^3 + 12ab^2x^4 + 48abx^3y(x) - 16abx^2 + 48ax^2y(x)^2 - 32axy(x) - 32ax + 8b^3x^3}{16ax^2 + 32bx + 64y(x) + 64}$$

✓ **Mathematica** : cpu = 1.92047 (sec), leaf count = 233

$$\text{Solve}\left[x - 4\text{RootSum}\left[\#1^6a^3 + 6\#1^5a^2b + 12\#1^4a^2y(x) + 12\#1^4ab^2 + 48\#1^3aby(x) + 8\#1^3b^3 + 8\#1^2ab + 48\#1^2ay(x)^2 + 4\right] = c_1, y\right]$$

✓ **Maple** : cpu = 0.08 (sec), leaf count = 47

$$\left\{y(x) = -\frac{ax^2}{4} - \frac{bx}{2} + \text{RootOf}\left(bx + 2c_1 + 2\left(\int^{\underline{Z}} -\frac{(\underline{a} + 1)b}{2\underline{a}^3 + \underline{a}b + b} d\underline{a}\right)\right)\right\}$$

2.945 ODE No. 945

$$y'(x) = \frac{8a^3x^3 + 12a^2x^4 + 48a^2x^2y(x) + 6ax^5 + 48ax^3y(x) - 16ax^2 + 96axy(x)^2 + x^6 + 12x^4y(x) - 8x^3 + 48x^2y(x)^2 - 32x}{32ax + 16x^2 + 64y(x) + 64}$$

✓ **Mathematica** : cpu = 1.45908 (sec), leaf count = 213

$$\text{Solve}\left[x - 4\text{RootSum}\left[\#1^6 + 6\#1^5a + 12\#1^4a^2 + 12\#1^4y(x) + 8\#1^3a^3 + 48\#1^3ay(x) + 48\#1^2a^2y(x) + 8\#1^2a + 48\#1^2y(x)^2 + 4\right] = c_1, y\right]$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 41

$$\left\{y(x) = -\frac{ax}{2} - \frac{x^2}{4} + \text{RootOf}\left(c_1 - x + \int^{\underline{Z}} \frac{2\underline{a} + 2}{2\underline{a}^3 + \underline{a}a + a} d\underline{a}\right)\right\}$$

2.946 ODE No. 946

$$y'(x) = \frac{x(12e^{-x^2}x^2y(x)^2 + 8e^{-x^2}x^2y(x) - 8e^{-x^2}y(x) + 4e^{-2x^2}x^2 + 8e^{-x^2}x^2 - 8e^{-x^2} + e^{-3x^2}x^6 - 6e^{-2x^2}x^4y(x) - 4e^{-2x^2}x^4 - 8y(x))}{4e^{-x^2}x^2 - 8y(x) - 8}$$

✓ **Mathematica** : cpu = 0.747656 (sec), leaf count = 150

$$\left\{\left(y(x) \rightarrow -\frac{1}{2}e^{-x^2}(2e^{x^2} - x^2) + \frac{e^{-3x^2}}{8\left(\frac{1}{8}e^{-3x^2} - \frac{e^{-3x^2}}{\sqrt{-64x^2+c_1}}\right)}\right), \left(y(x) \rightarrow -\frac{1}{2}e^{-x^2}(2e^{x^2} - x^2) + \frac{e^{-3x^2}}{8\left(\frac{1}{8}e^{-3x^2} + \frac{e^{-3x^2}}{\sqrt{-64x^2+c_1}}\right)}\right)\right\}$$

✓ **Maple** : cpu = 0.15 (sec), leaf count = 85

$$\left\{y(x) = \frac{(\sqrt{-x^2+c_1}-1)x^2e^{-x^2}+2}{2\sqrt{-x^2+c_1}-2}, y(x) = \frac{(\sqrt{-x^2+c_1}+1)x^2e^{-x^2}-2}{2\sqrt{-x^2+c_1}+2}\right\}$$

2.947 ODE No. 947

$$y'(x) = \frac{x^3 \sin(x) + x^2 y(x)^2 + 2x^2 y(x) \cos(x) + \frac{x^2}{2} + x^2 \cos(x) + \frac{1}{2} x^2 \cos(2x) + 2xy(x) - 2xy(x) \sin(x) + x - x \sin(x) - x}{x^3}$$

✓ **Mathematica** : cpu = 0.421973 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow -\frac{-\sin(x) + x \cos(x) + 1}{x} + \frac{1}{-\log(x) + c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.273 (sec), leaf count = 44

$$\left\{ y(x) = \frac{-c_1 x \cos(x) + c_1 \sin(x) - c_1 + x + (x \cos(x) - \sin(x) + 1) \ln(x)}{(c_1 - \ln(x))x} \right\}$$

2.948 ODE No. 948

$$y'(x) = -\frac{216y(x)}{36x^2 + 4y(x)^8 + 12y(x)^7 + 33y(x)^6 + 60y(x)^5 - 24xy(x)^4 - 216y(x)^4 - 36xy(x)^3 - 252y(x)^3 - 72xy(x)^2 - 396y(x)^2}$$

✓ **Mathematica** : cpu = 0.750967 (sec), leaf count = 39

$$\text{Solve}\left[\frac{36}{y(x)(2y(x)^3 + 3y(x)^2 + 6y(x) + 6) - 6x} + \log(y(x)) = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.323 (sec), leaf count = 68

$$\{y(x) = e^{\text{RootOf}(36c_1x - 36c_1e^{-Z} - 36c_1e^{2Z} - 18c_1e^{3Z} - 12c_1e^{4Z} + 6_Zx - 6_Ze^{-Z} - 6_Ze^{2Z} - 3_Ze^{3Z} - 2_Ze^{4Z} - 36)}\}$$

2.949 ODE No. 949

$$y'(x) = \frac{x^6 - 3x^5 + 3x^4y(x) + x^4 - 6x^3y(x) + 2x^3 + 3x^2y(x)^2 + x^2y(x) - 3x^2 - 3xy(x)^2 + xy(x) + y(x)^3 + x}{x(x^2 + y(x) - x + 1)}$$

✓ **Mathematica** : cpu = 0.235079 (sec), leaf count = 76

$$\left\{ \left\{ y(x) \rightarrow -x^2 + x + \frac{1}{x\left(\frac{1}{x} - \frac{1}{x\sqrt{-2\log(x)+c_1}}\right)} - 1 \right\}, \left\{ y(x) \rightarrow -x^2 + x + \frac{1}{x\left(\frac{1}{x} + \frac{1}{x\sqrt{-2\log(x)+c_1}}\right)} - 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.048 (sec), leaf count = 81

$$\left\{ y(x) = \frac{-x^2 + x + (-x^2 + x)\sqrt{c_1 - 2\ln(x)} - 1}{1 + \sqrt{c_1 - 2\ln(x)}}, y(x) = \frac{x^2 - x + (-x^2 + x)\sqrt{c_1 - 2\ln(x)} + 1}{-1 + \sqrt{c_1 - 2\ln(x)}} \right\}$$

2.950 ODE No. 950

$$y'(x) = \frac{a^3x^6}{64} + \frac{3}{32}a^2bx^5 + \frac{3}{16}a^2x^4y(x) + \frac{a^2x^4}{16} + \frac{3}{16}ab^2x^4 + \frac{3}{4}abx^3y(x) + \frac{1}{4}abx^3 + \frac{3}{4}ax^2y(x)^2 + \frac{1}{2}ax^2y(x) - \frac{ax}{2} + \frac{b^3x^3}{8} + \frac{3}{4}b^2x^2y(x)$$

✓ **Mathematica** : cpu = 0.560059 (sec), leaf count = 141

$$\text{Solve}\left[-\frac{1}{3}(27b + 58)^{2/3}\text{RootSum}\left[\#1^3(27b + 58)^{2/3} - 3\ 2^{2/3}\#1 + (27b + 58)^{2/3}\&, \frac{\log\left(\frac{\sqrt[3]{2}\left(\frac{1}{4}(3ax^2+6bx+4)+3y(x)\right)}{\sqrt[3]{27b+58}} - \#1\right)}{2^{2/3} - \#1^2(27b + 58)^{2/3}}\&\right] = 0\right]$$

✓ **Maple** : cpu = 0.085 (sec), leaf count = 42

$$\left\{ y(x) = -\frac{a}{4}x^2 - \frac{bx}{2} + \text{RootOf}\left(c_1 - x + 2\left(\int^{-Z} \frac{1}{2\_a^3 + 2\_a^2 + b + 2} d\_a\right)\right) \right\}$$

2.951 ODE No. 951

$$y'(x) = \frac{a^3x^3}{8} + \frac{3a^2x^4}{16} + \frac{3}{4}a^2x^2y(x) + \frac{a^2x^2}{4} + \frac{3ax^5}{32} + \frac{3}{4}ax^3y(x) + \frac{ax^3}{4} + \frac{3}{2}axy(x)^2 + axy(x) + \frac{x^6}{64} + \frac{3}{16}x^4y(x) + \frac{x^4}{16} + \frac{3}{4}x^2y(x)^2 + \frac{1}{2}x^2y(x)$$

✓ **Mathematica** : cpu = 0.459707 (sec), leaf count = 140

$$\text{Solve}\left[-\frac{1}{3}(27a+58)^{2/3}\text{RootSum}\left[\#1^3(27a+58)^{2/3}-3\,2^{2/3}\#1+(27a+58)^{2/3}\&, \frac{\log\left(\frac{\sqrt[3]{2}\left(\frac{1}{4}(6ax+3x^2+4)+3y(x)\right)}{\sqrt[3]{27a+58}}-\#1\right)}{2^{2/3}-\#1^2(27a+58)^{2/3}}\&\right]=0\right]$$

✓ **Maple** : cpu = 0.07 (sec), leaf count = 41

$$\left\{y(x)=-\frac{ax}{2}-\frac{x^2}{4}+\text{RootOf}\left(c_1-x+2\left(\int^{-Z}\frac{1}{2\_a^3+2\_a^2+a+2}d\_a\right)\right)\right\}$$

2.952 ODE No. 952

$$y'(x) = \frac{-x^2\sqrt{x^2+y(x)^2}+xy(x)\sqrt{x^2+y(x)^2}+x^5(-\sqrt{x^2+y(x)^2})+x^4y(x)\sqrt{x^2+y(x)^2}-x^4\sqrt{x^2+y(x)^2}+x^3y(x)\sqrt{x^2+y(x)^2}}{x}$$

✓ **Mathematica** : cpu = 0.418874 (sec), leaf count = 341

$$\left\{\left\{y(x)\rightarrow\frac{x-2\sqrt{x^2\tanh^2\left(\frac{1}{20}\left(-4\sqrt{2}x^5-5\sqrt{2}x^4-10\sqrt{2}x^2-20\sqrt{2}c_1\right)\right)-x^2\tanh^4\left(\frac{1}{20}\left(-4\sqrt{2}x^5-5\sqrt{2}x^4-10\sqrt{2}x^2-20\sqrt{2}c_1\right)\right)}{-1+2\tanh^2\left(\frac{1}{20}\left(-4\sqrt{2}x^5-5\sqrt{2}x^4-10\sqrt{2}x^2-20\sqrt{2}c_1\right)\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.263 (sec), leaf count = 62

$$\left\{-c_1-\ln(x)+\ln\left(\frac{2\left(x+y(x)+\sqrt{x^2+2y(x)^2}\right)x}{-x+y(x)}\right)+\frac{\left(4x^5+5x^4+10x^2\right)\sqrt{2}}{20}=0\right\}$$

2.953 ODE No. 953

$$y'(x) = \frac{y(x)\left(x^4\log^2(y(x))+2x^4\log(x)\log(y(x))+x^4\log^2(x)+x^3\log^2(y(x))+2x^3\log(x)\log(y(x))+x^3\log^2(x)+x\log^2(y(x))\right)}{x}$$

✓ **Mathematica** : cpu = 0.285284 (sec), leaf count = 36

$$\left\{\left\{y(x)\rightarrow\frac{e^{-\frac{20x}{4x^5+5x^4+10x^2+20c_1}}}{x}\right\}\right\}$$

✓ **Maple** : cpu = 0.41 (sec), leaf count = 145

$$\left\{y(x)=x^{-\frac{20c_1}{4x^5+5x^4+10x^2+20c_1}}x^{-\frac{10x^2}{4x^5+5x^4+10x^2+20c_1}}x^{-\frac{5x^4}{4x^5+5x^4+10x^2+20c_1}}x^{-\frac{4x^5}{4x^5+5x^4+10x^2+20c_1}}e^{-\frac{20x}{4x^5+5x^4+10x^2+20c_1}}\right\}$$

2.954 ODE No. 954

$$y'(x) = \frac{\frac{24}{5}x^{7/2}y(x)-\frac{24x^{13/2}}{25}+\frac{8x^{7/2}}{5}-8x^{3/2}-\frac{8x^9}{125}+\frac{12}{25}x^6y(x)+\frac{4x^6}{25}-\frac{24x^4}{5}-\frac{6}{5}x^3y(x)^2-\frac{4}{5}x^3y(x)+\frac{6x^3}{5}+12xy(x)-6\sqrt{x}y(x)}{x}$$

✓ **Mathematica** : cpu = 0.386415 (sec), leaf count = 115

Solve $\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3+3\sqrt[3]{29}\#1-29\&, \frac{\log\left(\frac{-6x^3-30\sqrt{x}+5+\frac{3y(x)}{x}}{5x}-\#1\right)}{\frac{\sqrt[3]{29}\sqrt[3]{\frac{1}{x^3}}}{\sqrt[3]{29}-29\#1^2}}\&\right]=\frac{1}{9}29^{2/3}\left(\frac{1}{x^3}\right)^{2/3}x^2\log(x)+c_1,y(x)\right]$

✓ **Maple** : cpu = 0.09 (sec), leaf count = 53

$$\left\{y(x)=\frac{18x^{\frac{7}{2}}+145\sqrt{x}\text{RootOf}\left(3c_1-81\left(\int^{-Z}\frac{1}{841a^3-27a+27}d_a\right)+\ln(x)\right)+90x-15\sqrt{x}}{45\sqrt{x}}\right\}$$

2.955

ODE No. 955

$$y'(x)=\frac{-24x^{7/2}y(x)+\frac{24x^{13/2}}{5}+14x^{7/2}+40x^{3/2}+\frac{8x^9}{25}-\frac{12}{5}x^6y(x)+\frac{12x^6}{5}+24x^4+6x^3y(x)^2-6x^3y(x)-6x^3-60xy(x)+3}{x\left(2x^3-5y(x)+10\sqrt{x}-5\right)}$$

✓ **Mathematica** : cpu = 0.365493 (sec), leaf count = 112

$$\left\{\left\{y(x)\rightarrow\frac{1}{5}\left(2x^3+10\sqrt{x}-5\right)-\frac{1}{125x\left(-\frac{1}{125x}-\frac{1}{x\sqrt{-31250\log(x)+c_1}}\right)}\right\},\left\{y(x)\rightarrow\frac{1}{5}\left(2x^3+10\sqrt{x}-5\right)-\frac{1}{125x\left(-\frac{1}{125x}+\frac{1}{x\sqrt{-31250\log(x)+c_1}}\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.109 (sec), leaf count = 101

$$\left\{y(x)=\frac{-2x^3-10\sqrt{x}+\left(2x^3+10\sqrt{x}\right)\sqrt{c_1-2\ln(x)}+5}{5\sqrt{c_1-2\ln(x)}-5},y(x)=\frac{2x^3+10\sqrt{x}+\left(2x^3+10\sqrt{x}\right)\sqrt{c_1-2\ln(x)}-5}{5\sqrt{c_1-2\ln(x)}+5}\right\}$$

2.956

ODE No. 956

$$y'(x)=\frac{y(x)\left(y(x)e^{\frac{2\log^2(x)}{\log(x)+1}}x^{\frac{2}{\log(x)+1}+2}+y(x)e^{\frac{2\log^2(x)}{\log(x)+1}}\log^2(x)x^{\frac{2}{\log(x)+1}+2}+2y(x)e^{\frac{2\log^2(x)}{\log(x)+1}}\log(x)x^{\frac{2}{\log(x)+1}+2}-e^{\frac{2\log^2(x)}{\log(x)+1}}x^{\frac{2}{\log(x)+1}+2}-e^{\frac{2\log^2(x)}{\log(x)+1}}\right)}{x(\log(x)+1)}$$

✓ **Mathematica** : cpu = 1.3383 (sec), leaf count = 28

$$\left\{\left\{y(x)\rightarrow\frac{1}{\left(1+c_1e^{\frac{x^4}{4}}\right)(\log(x)+1)}\right\}\right\}$$

✓ **Maple** : cpu = 0.181 (sec), leaf count = 79

$$\left\{y(x)=\frac{e^{-\frac{x^4}{4}}}{(\ln(x)+1)\left((\ln(x)+1)x^{-\frac{2\ln(x)}{\ln(x)+1}}e^{\frac{-x^4\ln(x)-x^4+8\ln(x)^2+(-4\ln(x)-4)\ln(\ln(x)+1)}{4\ln(x)+4}}+c_1\right)}\right\}$$

2.957

ODE No. 957

$$y'(x)=\frac{y(x)\left(y(x)e^{\frac{2\log^2(x)}{\log(x)+1}}x^{\frac{2}{\log(x)+1}+3}+y(x)e^{\frac{2\log^2(x)}{\log(x)+1}}\log^2(x)x^{\frac{2}{\log(x)+1}+3}+2y(x)e^{\frac{2\log^2(x)}{\log(x)+1}}\log(x)x^{\frac{2}{\log(x)+1}+3}-e^{\frac{2\log^2(x)}{\log(x)+1}}x^{\frac{2}{\log(x)+1}+3}-e^{\frac{2\log^2(x)}{\log(x)+1}}\right)}{x(\log(x)+1)}$$

✓ **Mathematica** : cpu = 1.32372 (sec), leaf count = 28

$$\left\{\left\{y(x) \rightarrow \frac{1}{\left(1+c_1 e^{\frac{x^5}{5}}\right)(\log (x)+1)}\right\}\right\}$$

✓ **Maple** : cpu = 0.112 (sec), leaf count = 79

$$\left\{y(x)=\frac{e^{-\frac{x^5}{5}}}{(\ln (x)+1)\left((\ln (x)+1) x^{-\frac{2 \ln (x)}{\ln (x)+1}} e^{\frac{-x^5 \ln (x)-x^5+10 \ln (x)^2+(-5 \ln (x)-5) \ln (\ln (x)+1)}{5 \ln (x)+5}}+c_1\right)}\right\}$$

2.958 ODE No. 958

$$y'(x)=\frac{2xy(x)^3+y(x)^3+2xy(x)^2+y(x)^2+6xy(x)\log^2(2x+1)+3y(x)\log^2(2x+1)+6xy(x)^2\log(2x+1)+3y(x)^2\log(2x+1)}{x}$$

✓ **Mathematica** : cpu = 0.481477 (sec), leaf count = 82

$$\text{Solve}\left[-\frac{29}{3}\text{RootSum}\left[-29\#1^3+3\sqrt[3]{29}\#1-29\&, \frac{\log\left(\frac{3y(x)+3\log(2x+1)+1}{\sqrt[3]{29}}-\#1\right)}{\sqrt[3]{29}-29\#1^2}\&\right]=\frac{1}{9}29^{2/3}x+c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.071 (sec), leaf count = 40

$$\left\{y(x)=\frac{29\text{RootOf}\left(3c_1+x-81\left(\int^{\frac{-Z}{841\_a^3-27\_a+27}}\frac{1}{\_a}\,d\_a\right)\right)}{9}-\ln(2x+1)-\frac{1}{3}\right\}$$

2.959 ODE No. 959

$$y'(x)=\frac{\csc\left(\frac{y(x)}{2x}\right)\sec\left(\frac{y(x)}{2x}\right)\sec\left(\frac{y(x)}{x}\right)\left(x^3\sin\left(\frac{y(x)}{2x}\right)\sin\left(\frac{y(x)}{x}\right)\cos\left(\frac{y(x)}{2x}\right)-\frac{1}{2}y(x)\sin\left(\frac{y(x)}{x}\right)+\frac{1}{2}y(x)\sin\left(\frac{y(x)}{2x}\right)\cos\left(\frac{y(x)}{2x}\right)+\frac{1}{2}y(x)\right)}{x}$$

✓ **Mathematica** : cpu = 0.25737 (sec), leaf count = 20

$$\left\{\left\{y(x) \rightarrow x \sin^{-1}\left(e^{\frac{x^2}{2}+c_1}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.154 (sec), leaf count = 15

$$\left\{y(x)=x \arcsin\left(c_1 e^{\frac{x^2}{2}}\right)\right\}$$

2.960 ODE No. 960

$$y'(x)=\frac{\csc\left(\frac{y(x)}{2x}\right)\sec\left(\frac{y(x)}{2x}\right)\sec\left(\frac{y(x)}{x}\right)\left(x^2\sin\left(\frac{y(x)}{2x}\right)\sin\left(\frac{y(x)}{x}\right)\cos\left(\frac{y(x)}{2x}\right)-\frac{1}{2}y(x)\sin\left(\frac{y(x)}{x}\right)+\frac{1}{2}y(x)\sin\left(\frac{y(x)}{2x}\right)\cos\left(\frac{y(x)}{2x}\right)+\frac{1}{2}y(x)\right)}{x}$$

✓ **Mathematica** : cpu = 0.197503 (sec), leaf count = 14

$$\left\{\left\{y(x) \rightarrow x \sin^{-1}\left(e^{x+c_1}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.152 (sec), leaf count = 11

$$\{y(x)=x \arcsin(c_1 e^x)\}$$

✓ **Maple** : cpu = 0.092 (sec), leaf count = 91

$$\left\{y(x)=\frac{12x^3+58x^2\operatorname{RootOf}\left(6c_1-162\left(\int^{-Z}\frac{1}{841_a^3-27_a+27}d_a\right)+\ln\left(x^2+1\right)\right)-6x^2+58\operatorname{RootOf}\left(6c_1-162\left(\int^{-Z}\frac{1}{841_a^3-27_a+27}d_a\right)+\ln\left(x^2+1\right)\right)}{18x^2+18}\right.$$

2.968 ODE No. 968

$$y'(x)=\frac{\csc\left(\frac{y(x)}{2x}\right)\sec\left(\frac{y(x)}{2x}\right)\sec\left(\frac{y(x)}{x}\right)\left(x^4\sin\left(\frac{y(x)}{2x}\right)\sin\left(\frac{y(x)}{x}\right)\cos\left(\frac{y(x)}{2x}\right)-\frac{1}{2}xy(x)\sin\left(\frac{y(x)}{x}\right)-\frac{1}{2}y(x)\sin\left(\frac{y(x)}{x}\right)+\frac{1}{2}xy(x)\sin\left(\frac{y(x)}{2x}\right)\right)}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.403314 (sec), leaf count = 30

$$\left\{\left\{y(x)\rightarrow x\sin^{-1}\left((x+1)e^{\frac{x^2}{2}-x-\frac{3}{2}+c_1}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.179 (sec), leaf count = 22

$$\left\{y(x)=x\arcsin\left(c_1(x+1)e^{-x}e^{\frac{x^2}{2}}\right)\right\}$$

2.969 ODE No. 969

$$y'(x)=\frac{\csc\left(\frac{y(x)}{2x}\right)\sec\left(\frac{y(x)}{2x}\right)\sec\left(\frac{y(x)}{x}\right)\left(-\frac{1}{2}xy(x)\sin\left(\frac{y(x)}{x}\right)-\frac{1}{2}y(x)\sin\left(\frac{y(x)}{x}\right)+x\sin\left(\frac{y(x)}{2x}\right)\sin\left(\frac{y(x)}{x}\right)\cos\left(\frac{y(x)}{2x}\right)+\frac{1}{2}xy(x)\sin\left(\frac{y(x)}{2x}\right)\right)}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.280139 (sec), leaf count = 19

$$\left\{\left\{y(x)\rightarrow x\sin^{-1}\left(\frac{e^{c_1}x}{x+1}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.175 (sec), leaf count = 15

$$\left\{y(x)=x\arcsin\left(\frac{c_1x}{x+1}\right)\right\}$$

2.970 ODE No. 970

$$y'(x)=-\frac{216x^3-216x^2y(x)^4-324x^2y(x)^3-648x^2y(x)^2-648x^2y(x)-8y(x)^{12}-36y(x)^{11}-126y(x)^{10}-315y(x)^9+72y(x)^8}{216x^3-216x^2y(x)^4-324x^2y(x)^3-648x^2y(x)^2-648x^2y(x)-8y(x)^{12}-36y(x)^{11}-126y(x)^{10}-315y(x)^9+72y(x)^8}$$

✓ **Mathematica** : cpu = 0.806748 (sec), leaf count = 66

$$\operatorname{Solve}\left[\frac{36\left(2y(x)^4+3y(x)^3+6y(x)^2+6y(x)-6x-3\right)}{\left(y(x)\left(2y(x)^3+3y(x)^2+6y(x)+6\right)-6x\right)^2}+\log(y(x))=c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.671 (sec), leaf count = 181

$$\left\{\frac{-2c_1y(x)^4-3c_1y(x)^3-6c_1y(x)^2+6c_1x-6c_1y(x)+\left(2y(x)^4+3y(x)^3+6y(x)^2-6x+6y(x)\right)\ln\left(y(x)\right)-6\sqrt{-3c_1+3\ln\left(y(x)\right)}}{6c_1-6\ln\left(y(x)\right)}\right.$$

2.971 ODE No. 971

$$y'(x)=\frac{(xy(x)+1)^3}{x^5}$$

✓ **Mathematica** : cpu = 0.32885 (sec), leaf count = 157

Solve
$$\left[\frac{1}{3} \log \left(\frac{\frac{3}{x^3} + \frac{3y(x)}{x^2}}{3 \sqrt[3]{-\frac{1}{x^6}}} + 1 \right) - \frac{1}{6} \log \left(\frac{\left(\frac{3}{x^3} + \frac{3y(x)}{x^2} \right)^2}{9 \left(-\frac{1}{x^6} \right)^{2/3}} - \frac{\frac{3}{x^3} + \frac{3y(x)}{x^2}}{3 \sqrt[3]{-\frac{1}{x^6}}} + 1 \right) + \frac{\tan^{-1} \left(\frac{\frac{2 \left(\frac{3}{x^3} + \frac{3y(x)}{x^2} \right)}{3 \sqrt[3]{-\frac{1}{x^6}}} - 1}{\sqrt{3}} \right)}{\sqrt{3}} = - \left(-\frac{1}{x^6} \right)^{2/3} x^3 + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.309 (sec), leaf count = 86

$$\left\{ y(x) = \frac{\left(3 \left(-\frac{1}{x^6} \right)^{\frac{1}{3}} x^3 \tan \left(\text{RootOf} \left(-18 \left(-\frac{1}{x^6} \right)^{\frac{2}{3}} x^3 + 18 c_1 - 6 \sqrt{3} \_Z - \ln \left(\frac{\left(\tan(\_Z) + \sqrt{3} \right)^6}{\left(\tan^2(\_Z) + 1 \right)^3} \right) \right) \right) + \sqrt{3} \left(\left(-\frac{1}{x^6} \right)^{\frac{1}{3}} x^3 - 2 \right) \right) \sqrt{3}}{6x} \right\}$$

2.972 ODE No. 972

$$y'(x) = \frac{x \left(-2x^4 + 2x^2 y(x) - x^2 + 1 \right)}{y(x) - x^2}$$

✓ **Mathematica** : cpu = 0.0387993 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow x^2 + \frac{1}{2} \left(1 + W \left(-e^{x^4 - 2x^2 - 1 + c_1} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.223 (sec), leaf count = 27

$$\left\{ y(x) = x^2 + \frac{\text{LambertW} \left(-2c_1 e^{-1} e^{x^4} e^{-2x^2} \right)}{2} + \frac{1}{2} \right\}$$

2.973 ODE No. 973

$$y'(x) = e^{-2bx} y(x) \left(e^{bx} y(x) + e^{2bx} + y(x)^2 \right)$$

✓ **Mathematica** : cpu = 0.463225 (sec), leaf count = 146

Solve
$$\left[-\frac{1}{3} (9b - 7)^{2/3} \text{RootSum} \left[\#1^3 (9b - 7)^{2/3} - 9\#1b + 6\#1 + (9b - 7)^{2/3} \&, \frac{\log \left(\frac{3e^{-2bx} y(x) + e^{-bx}}{\sqrt[3]{(9b - 7)e^{-3bx}}} - \#1 \right)}{\#1^2 \left(-(9b - 7)^{2/3} \right) + 3b - 2} \& \right] = \frac{1}{9} x e^{2bx} \left((9b - 7)^{2/3} - 3b + 2 \right) + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.414 (sec), leaf count = 134

$$\left\{ y(x) = -\frac{e^{bx}}{2} - \frac{\sqrt{-(4b + 3)e^{2bx}} \tan \left(\text{RootOf} \left(2\_Z e^{bx} + c_1 \sqrt{-(4b - 3)e^{2bx}} - 2 \sqrt{-(4b - 3)e^{2bx}} x - \sqrt{-(4b - 3)e^{2bx}} \ln \left(\frac{4}{\sqrt[3]{(9b - 7)e^{-3bx}}} - \#1 \right) \right) \right)}{2} \right\}$$

2.974 ODE No. 974

$$y'(x) = -x^6 + 3x^4y(x) - 3x^2y(x)^2 + y(x)^3 + 2x$$

✓ **Mathematica** : cpu = 0.102738 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow x^2 - \frac{1}{\sqrt{-2x + c_1}} \right\}, \left\{ y(x) \rightarrow x^2 + \frac{1}{\sqrt{-2x + c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 57

$$\left\{ y(x) = \frac{\sqrt{2c_1 - 2x} x^2 - 1}{\sqrt{2c_1 - 2x}}, y(x) = \frac{\sqrt{2c_1 - 2x} x^2 + 1}{\sqrt{2c_1 - 2x}} \right\}$$

2.975 ODE No. 975

$$y'(x) = \frac{x^6}{27} + \frac{1}{3}x^4y(x) + x^2y(x)^2 + y(x)^3 - \frac{2x}{3}$$

✓ **Mathematica** : cpu = 0.112429 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow -\frac{x^2}{3} - \frac{1}{\sqrt{-2x + c_1}} \right\}, \left\{ y(x) \rightarrow -\frac{x^2}{3} + \frac{1}{\sqrt{-2x + c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 59

$$\left\{ y(x) = -\frac{\sqrt{-54c_1 - 2x} x^2 - 3}{3\sqrt{-54c_1 - 2x}}, y(x) = -\frac{\sqrt{-54c_1 - 2x} x^2 + 3}{3\sqrt{-54c_1 - 2x}} \right\}$$

2.976 ODE No. 976

$$y'(x) = \frac{y(x)(x^7y(x)^2 + x^4y(x) + x - 3)}{x}$$

✓ **Mathematica** : cpu = 0.2118 (sec), leaf count = 101

$$\text{Solve} \left[-\frac{7}{3} \text{RootSum} \left[-7\#1^3 + 6\sqrt[3]{-7}\#1 - 7\&, \frac{\log \left(\frac{3x^6y(x)+x^3}{\sqrt[3]{7}\sqrt[3]{-x^9}} - \#1 \right)}{2\sqrt[3]{-7} - 7\#1^2} \& \right] = \frac{7^{2/3}(-x^9)^{2/3}}{9x^5} + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.251 (sec), leaf count = 57

$$\left\{ y(x) = \frac{\sqrt{3} \tan \left(\text{RootOf} \left(3c_1\sqrt{3} - 2\_Z - 2\sqrt{3}x - \sqrt{3} \ln \left(\frac{\frac{9(\tan^2(\_Z))}{7} + \frac{9}{7}}{(-3\tan(\_Z) + \sqrt{3})^2} \right) \right) \right) - 1}{2x^3} \right\}$$

2.977 ODE No. 977

$$y'(x) = e^{2x^2}xy(x)(e^{-x^2}y(x) + e^{-2x^2} + y(x)^2)$$

✓ **Mathematica** : cpu = 0.447476 (sec), leaf count = 139

$$\text{Solve} \left[-\frac{25}{3} \text{RootSum} \left[-25\#1^3 + 24\sqrt[3]{-1}5^{2/3}\#1 - 25\&, \frac{\log \left(\frac{3e^{2x^2}xy(x)+e^{x^2}x}{5^{2/3}\sqrt[3]{-e^{3x^2}x^3}} - \#1 \right)}{8\sqrt[3]{-1}5^{2/3} - 25\#1^2} \& \right] = -\frac{5\sqrt[3]{5}e^{x^2}x^3}{18\sqrt[3]{-e^{3x^2}x^3}} + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.228 (sec), leaf count = 122

$$\left\{y(x)=\frac{\left(\sqrt{11}\tan\left(\text{RootOf}\left(-4\sqrt{11}\,x^2+9c_1\sqrt{11}-8\_Z+8\sqrt{11}\ln\left(36\tan(\_Z)-\frac{36\sqrt{11}}{11}\right)-4\sqrt{11}\ln\left(\frac{14256\,e^{2x^2}\left(\tan^2(\_Z)\right)}{25}\right)\right)\right)}{2}\right.+\right.$$

2.978 ODE No. 978

$$y'(x)=\frac{y(x)\left(x^2+xy(x)+y(x)^2+x\right)}{x^2}$$

✓ **Mathematica** : cpu = 0.231291 (sec), leaf count = 60

$$\text{Solve}\left[-\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+\frac{y(x)}{x}+1\right)+\log\left(\frac{y(x)}{x}\right)-\frac{\tan^{-1}\left(\frac{\frac{2y(x)}{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}=x+c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.2 (sec), leaf count = 71

$$\left\{y(x)=\frac{\sqrt{3}\,x\tan\left(\text{RootOf}\left(2c_1\sqrt{3}+2\_Z+2\sqrt{3}\,x-\sqrt{3}\ln\left(\frac{4}{3\left(\tan^2(\_Z)+1\right)}\right)-2\sqrt{3}\ln\left(\frac{\tan(\_Z)}{2}-\frac{\sqrt{3}}{6}\right)-\sqrt{3}\ln(3)\right)\right)}{2}-\frac{x}{2}\right\}$$

2.979 ODE No. 979

$$y'(x)=\frac{-x^3+3x^2y(x)-3xy(x)^2+y(x)^3+x}{x}$$

✓ **Mathematica** : cpu = 0.15847 (sec), leaf count = 37

$$\left\{\left\{y(x)\rightarrow x-\frac{1}{\sqrt{-2\log(x)+c_1}}\right\},\left\{y(x)\rightarrow x+\frac{1}{\sqrt{-2\log(x)+c_1}}\right\}\right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 57

$$\left\{y(x)=\frac{\sqrt{2c_1-2\ln(x)}\,x-1}{\sqrt{2c_1-2\ln(x)}},y(x)=\frac{\sqrt{2c_1-2\ln(x)}\,x+1}{\sqrt{2c_1-2\ln(x)}}\right\}$$

2.980 ODE No. 980

$$y'(x)=\frac{x^3y(x)^3+6x^2y(x)^2+12xy(x)+2x+8}{x^3}$$

✓ **Mathematica** : cpu = 0.147835 (sec), leaf count = 43

$$\left\{\left\{y(x)\rightarrow-\frac{2}{x}-\frac{1}{\sqrt{-2x+c_1}}\right\},\left\{y(x)\rightarrow-\frac{2}{x}+\frac{1}{\sqrt{-2x+c_1}}\right\}\right\}$$

✓ **Maple** : cpu = 0.015 (sec), leaf count = 35

$$\left\{y(x)=-\frac{2}{x}-\frac{1}{\sqrt{c_1-2x}},y(x)=-\frac{2}{x}+\frac{1}{\sqrt{c_1-2x}}\right\}$$

2.981 ODE No. 981

$$y'(x) = \frac{a^3 x^3 y(x)^3 + 3a^2 x^2 y(x)^2 + a^2 x + 3axy(x) + 1}{a^3 x^3}$$

✓ **Mathematica** : cpu = 0.167589 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{ax} - \frac{1}{\sqrt{-2x + c_1}} \right\}, \left\{ y(x) \rightarrow -\frac{1}{ax} + \frac{1}{\sqrt{-2x + c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 41

$$\left\{ y(x) = -\frac{1}{ax} - \frac{1}{\sqrt{c_1 - 2x}}, y(x) = -\frac{1}{ax} + \frac{1}{\sqrt{c_1 - 2x}} \right\}$$

2.982 ODE No. 982

$$y'(x) = \frac{1}{2} e^{-\frac{x^2}{2}} y(x) \left(2e^{\frac{x^2}{4}} y(x) + 2e^{\frac{x^2}{2}} + e^{\frac{x^2}{2}} x + 2y(x)^2 \right)$$

✓ **Mathematica** : cpu = 0.465858 (sec), leaf count = 132

$$\text{Solve} \left[-\frac{7}{3} \text{RootSum} \left[-7\#1^3 + 6\sqrt[3]{-7}\#1 - 7\&, -\frac{\log \left(\frac{3e^{-\frac{x^2}{2}} y(x) + e^{-\frac{x^2}{4}}}{\sqrt[3]{7} \sqrt{-e^{-\frac{3x^2}{4}}}} \right) - \#1}{2\sqrt[3]{-7} - 7\#1^2} \& \right] = \frac{1}{9} 7^{2/3} e^{\frac{x^2}{2}} \left(-e^{-\frac{3x^2}{4}} \right)^{2/3} x + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.315 (sec), leaf count = 145

$$\left\{ -c_1 + \frac{2x}{3} + \frac{2\sqrt{3} \arctan \left(\frac{\left(6e^{-\frac{x^2}{2}} y(x) + 2e^{-\frac{x^2}{4}} \right) \sqrt{3} e^{\frac{x^2}{4}}}{9} + \frac{\sqrt{3}}{9} \right)}{9} - \frac{2 \ln \left(\left(18e^{-\frac{x^2}{2}} y(x) + 6e^{-\frac{x^2}{4}} \right) e^{\frac{x^2}{4}} - 6 \right)}{3} + \frac{\ln \left(\frac{324 \left(e^{-\frac{x^2}{2}} y(x) + \frac{e^{-\frac{x^2}{4}}}{3} \right)^2}{7} \right)}{7} \right\}$$

2.983 ODE No. 983

$$y'(x) = \frac{-x^3 + 3x^2 y(x) + x^2 - 3xy(x)^2 + y(x)^3}{(x-1)(x+1)}$$

✓ **Mathematica** : cpu = 0.881286 (sec), leaf count = 238

$$\text{Solve} \left[\frac{1}{3} \log \left(\frac{\frac{3y(x)}{x^2-1} - \frac{3x}{x^2-1}}{3\sqrt[3]{\frac{1}{(x-1)^3(x+1)^3}}} + 1 \right) - \frac{1}{6} \log \left(\frac{\left(\frac{3y(x)}{x^2-1} - \frac{3x}{x^2-1} \right)^2}{9\left(\frac{1}{(x-1)^3(x+1)^3} \right)^{2/3}} - \frac{\frac{3y(x)}{x^2-1} - \frac{3x}{x^2-1}}{3\sqrt[3]{\frac{1}{(x-1)^3(x+1)^3}}} + 1 \right) + \frac{\tan^{-1} \left(\frac{\frac{2\left(\frac{3y(x)}{x^2-1} - \frac{3x}{x^2-1} \right)}{3\sqrt[3]{\frac{1}{(x-1)^3(x+1)^3}}} - 1}{\sqrt{3}} \right)}{\sqrt{3}} = \frac{1}{2} \left(\frac{1}{x^2 - 1} \right) \right]$$

✓ **Maple** : cpu = 0.566 (sec), leaf count = 188

$$\left\{y(x)=\frac{\sqrt{3}\left(\frac{2\sqrt{3}x}{3}+\frac{\left(x^2-1\right)\left(3\tan\left(\text{RootOf}\left(9\left(\frac{1}{\left(x+1\right)^3\left(x-1\right)^3}\right)^{\frac{2}{3}}x^4\ln\left(\frac{x-1}{x+1}\right)-18\left(\frac{1}{\left(x+1\right)^3\left(x-1\right)^3}\right)^{\frac{2}{3}}x^2\ln\left(\frac{x-1}{x+1}\right)+18c_1-6\sqrt{3}-Z+2\ln\left(\frac{1}{\left(x+1\right)^3\left(x-1\right)^3}\right)+9\left(\frac{1}{\left(x+1\right)^3\left(x-1\right)^3}\right)\right)}{3}\right)}{2}\right.$$

2.984 ODE No. 984

$$y'(x)=\frac{e^{-2x}(x-1)y(x)\left(x^2y(x)^2+e^xx y(x)+e^{2x}\right)}{x}$$

✓ **Mathematica** : cpu = 14.0122 (sec), leaf count = 428

$$\text{Solve}\left[\frac{\sqrt[3]{2}\left(\frac{3e^{-2x}x(x-1)y(x)+e^{-x}(x-1)}{\sqrt[3]{2}\sqrt[3]{e^{-3x}(x-1)^3}}+2^{2/3}\right)\left(2^{2/3}-\frac{2^{2/3}\left(3e^{-2x}x(x-1)y(x)+e^{-x}(x-1)\right)}{\sqrt[3]{e^{-3x}(x-1)^3}}\right)\left(\left(1-\frac{3e^{-2x}x(x-1)y(x)+e^{-x}(x-1)}{\sqrt[3]{e^{-3x}(x-1)^3}}\right)\log\left(2^{2/3}-\frac{2^{2/3}\left(3e^{-2x}x(x-1)y(x)+e^{-x}(x-1)\right)}{\sqrt[3]{e^{-3x}(x-1)^3}}\right)\right)}{9\left(-\frac{e^{3x}\left(3e^{-2x}x(x-1)y(x)+e^{-x}(x-1)\right)^3}{(x-1)^3}+\frac{3\left(3e^{-2x}x(x-1)y(x)+e^{-x}(x-1)\right)}{\sqrt[3]{e^{-3x}(x-1)^3}}\right)}\right]$$

✓ **Maple** : cpu = 0.32 (sec), leaf count = 40

$$\left\{y(x)=\frac{e^{x+\text{RootOf}\left(3c_1e^{-Z}+_Ze^{-Z}+xe^{-Z}-e^{-Z}\ln\left(\frac{e^{-Z}+9}{2}\right)+9\right)}}{9x}\right\}$$

2.985 ODE No. 985

$$y'(x)=\frac{(xy(x)+1)\left(x^2y(x)^2+x^2y(x)+x^2+2xy(x)+x+1\right)}{x^5}$$

✓ **Mathematica** : cpu = 0.375672 (sec), leaf count = 103

$$\text{Solve}\left[-\frac{17}{3}\text{RootSum}\left[-17\#1^3+3\sqrt[3]{-34}\#1-17\&,-\frac{\log\left(\frac{\frac{x+3}{x^3}+\frac{3y(x)}{x^2}}{\sqrt[3]{34}\sqrt[3]{-\frac{1}{x^6}}}-\#1\right)}{\sqrt[3]{-34}-17\#1^2}\&\right]=- \frac{1}{9}34^{2/3}\left(-\frac{1}{x^6}\right)^{2/3}x^3+c_1,y(x)\right]$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 43

$$\left\{y(x)=\frac{17x\text{RootOf}\left(3c_1x+162x\left(\int^{-Z}\frac{1}{289\_a^3+54\_a-54}d\_a\right)+2\right)-3x-9}{9x}\right\}$$

2.986 ODE No. 986

$$y'(x)=\frac{-x^3\log^3(x)+3x^2y(x)\log^2(x)+x^2+y(x)^3+xy(x)-3xy(x)^2\log(x)}{x^2}$$

✓ **Mathematica** : cpu = 0.13045 (sec), leaf count = 44

$$\left\{\left\{y(x)\rightarrow x\log(x)-\frac{x}{\sqrt{-2x+c_1}}\right\},\left\{y(x)\rightarrow x\log(x)+\frac{x}{\sqrt{-2x+c_1}}\right\}\right\}$$

✓ **Maple** : cpu = 0.048 (sec), leaf count = 36

$$\left\{y(x) = x \ln(x) - \frac{x}{\sqrt{c_1 - 2x}}, y(x) = x \ln(x) + \frac{x}{\sqrt{c_1 - 2x}}\right\}$$

2.987 ODE No. 987

$$y'(x) = \frac{y(x)}{x} - F(x)(y(x)^2 - ax^2)$$

✓ **Mathematica** : cpu = 0.171068 (sec), leaf count = 41

$$\left\{\left\{y(x) \rightarrow \sqrt{a} x \tanh\left(\sqrt{a} \int_1^x F(K[1])K[1]dK[1] + \sqrt{a} c_1\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.055 (sec), leaf count = 22

$$\left\{y(x) = \sqrt{a} x \tanh\left(\left(c_1 + \int xF(x)dx\right)\sqrt{a}\right)\right\}$$

2.988 ODE No. 988

$$y'(x) = \frac{y(x)}{x} - F(x)(-x^2 - 2xy(x) + y(x)^2)$$

✓ **Mathematica** : cpu = 0.232251 (sec), leaf count = 107

$$\left\{\left\{y(x) \rightarrow -\frac{x\left(-\exp\left(2\sqrt{2}\left(\int_1^x -F(K[1])K[1]dK[1] + c_1\right)\right) + \sqrt{2} \exp\left(2\sqrt{2}\left(\int_1^x -F(K[1])K[1]dK[1] + c_1\right)\right) - 1 - \sqrt{2}\right)}{1 + \exp\left(2\sqrt{2}\left(\int_1^x -F(K[1])K[1]dK[1] + c_1\right)\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 29

$$\left\{y(x) = \frac{(2 \tanh((c_1 + \int xF(x)dx)\sqrt{2}) + \sqrt{2})\sqrt{2} x}{2}\right\}$$

2.989 ODE No. 989

$$y'(x) = \frac{y(x)}{x} - F(x)(-ay(x)^2 - bx^2)$$

✓ **Mathematica** : cpu = 0.263336 (sec), leaf count = 56

$$\left\{\left\{y(x) \rightarrow \frac{\sqrt{b} x \tan\left(\sqrt{a} \sqrt{b} \int_1^x F(K[1])K[1]dK[1] + \sqrt{a} \sqrt{b} c_1\right)}{\sqrt{a}}\right\}\right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 29

$$\left\{y(x) = \frac{\sqrt{ab} x \tan(\sqrt{ab} (c_1 + \int xF(x)dx))}{a}\right\}$$

2.990 ODE No. 990

$$y'(x) = 2x - F(x)(-x^4 + 2x^2y(x) - y(x)^2 + 1)$$

✓ **Mathematica** : cpu = 0.231306 (sec), leaf count = 58

$$\left\{\left\{y(x) \rightarrow \frac{\exp\left(\int_1^x 2F(K[5])dK[5]\right)}{-\int_1^x \exp\left(\int_1^{K[6]} 2F(K[5])dK[5]\right)F(K[6])dK[6] + c_1} + x^2 + 1\right\}\right\}$$

✓ **Maple** : cpu = 0.469 (sec), leaf count = 44

$$\left\{ y(x) = \frac{c_1 x^2 - x^2 e^{\int 2F(x)dx} + c_1 + e^{\int 2F(x)dx}}{c_1 - e^{\int 2F(x)dx}} \right\}$$

2.991 ODE No. 991

$$y'(x) = \frac{y(x)}{x} - F(x)(x^2 + 2xy(x) - y(x)^2)$$

✓ **Mathematica** : cpu = 0.206408 (sec), leaf count = 104

$$\left\{ \left\{ y(x) \rightarrow -\frac{x \left(-\exp \left(2\sqrt{2} \left(\int_1^x F(K[1])K[1]dK[1] + c_1 \right) \right) + \sqrt{2} \exp \left(2\sqrt{2} \left(\int_1^x F(K[1])K[1]dK[1] + c_1 \right) \right) - 1 - \sqrt{2} \right) \right\}}{1 + \exp \left(2\sqrt{2} \left(\int_1^x F(K[1])K[1]dK[1] + c_1 \right) \right)} \right\}$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 29

$$\left\{ y(x) = \frac{(-2 \tanh((c_1 + \int xF(x)dx)\sqrt{2}) + \sqrt{2})\sqrt{2} x}{2} \right\}$$

2.992 ODE No. 992

$$y'(x) = \frac{y(x)}{x} - F(x)(-x^3 - 7xy(x)^2)$$

✓ **Mathematica** : cpu = 0.141589 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow \frac{x \tan \left(\sqrt{7} \int_1^x F(K[1])K[1]^2 dK[1] + \sqrt{7} c_1 \right)}{\sqrt{7}} \right\} \right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 25

$$\left\{ y(x) = \frac{\sqrt{7} x \tan((c_1 + \int x^2 F(x)dx)\sqrt{7})}{7} \right\}$$

2.993 ODE No. 993

$$y'(x) = \frac{y(x)}{x \log(x)} - F(x)(-y(x)^2 - 2y(x) \log(x) - \log^2(x))$$

✓ **Mathematica** : cpu = 1.82952 (sec), leaf count = 73

$$\left\{ \left\{ y(x) \rightarrow \frac{\int_1^x \frac{F(K[5])}{\sqrt{\frac{1}{\log^2(K[5])}}} dK[5] - 1 + c_1}{\sqrt{\frac{1}{\log^2(x)}} \int_1^x \frac{F(K[5])}{\sqrt{\frac{1}{\log^2(K[5])}}} dK[5] + c_1 \sqrt{\frac{1}{\log^2(x)}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 35

$$\left\{ y(x) = -\frac{(-c_1 + \int -2F(x) \ln(x)dx - 2) \ln(x)}{-c_1 + \int -2F(x) \ln(x)dx} \right\}$$

2.994 ODE No. 994

$$y'(x) = \frac{y(x)}{x \log(x)} - x^3(-y(x)^2 - 2y(x) \log(x) - \log^2(x))$$

✓ **Mathematica** : cpu = 0.244527 (sec), leaf count = 198

$$\left\{ \left\{ y(x) \rightarrow -\frac{\frac{1}{16}x^4 e^{\frac{1}{16}x^4(4\log(x)-1)}(4\log(x)-1)\left(\frac{x^3}{4} + \frac{1}{4}x^3(4\log(x)-1)\right) + \frac{1}{4}x^3 e^{\frac{1}{16}x^4(4\log(x)-1)} + \frac{1}{4}x^3 e^{\frac{1}{16}x^4(4\log(x)-1)}(4\log(x)-1)}{x^3\left(\frac{1}{16}x^4 e^{\frac{1}{16}x^4(4\log(x)-1)}(4\log(x)-1) + c_1 e^{\frac{1}{16}x^4(4\log(x)-1)}\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 43

$$\left\{ y(x) = -\frac{(4x^4 \ln(x) - x^4 + 8c_1 + 16) \ln(x)}{4x^4 \ln(x) - x^4 + 8c_1} \right\}$$

2.995 ODE No. 995

$$y'(x) = (y(x) - e^x)^2 + e^x$$

✓ **Mathematica** : cpu = 0.23995 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow e^x + \frac{1}{-x + c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.144 (sec), leaf count = 14

$$\left\{ y(x) = e^x + \frac{1}{c_1 - x} \right\}$$

2.996 ODE No. 996

$$y'(x) = \frac{(y(x) - \text{Si}(x))^2 + \sin(x)}{x}$$

✓ **Mathematica** : cpu = 0.177101 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow \text{Si}(x) + \frac{1}{-\log(x) + c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.079 (sec), leaf count = 15

$$\left\{ y(x) = \text{Si}(x) + \frac{1}{c_1 - \ln(x)} \right\}$$

2.997 ODE No. 997

$$y'(x) = (y(x) + \cos(x))^2 + \sin(x)$$

✓ **Mathematica** : cpu = 0.13083 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow -\cos(x) + \frac{1}{-x + c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.068 (sec), leaf count = 16

$$\left\{ y(x) = -\cos(x) + \frac{1}{c_1 - x} \right\}$$

2.998 ODE No. 998

$$y'(x) = \frac{(-\text{Ci}(x) + y(x) - \log(x))^2 + \cos(x)}{x}$$

✓ **Mathematica** : cpu = 0.601298 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow \text{Ci}(x) + \frac{x^2}{-\frac{x^2}{2} + c_1} + \log(x) + 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.267 (sec), leaf count = 27

$$\left\{ y(x) = \text{Ci}(x) + \ln(x) + \frac{-c_1 x^2 + 1}{c_1 x^2 + 1} \right\}$$

2.999 ODE No. 999

$$y'(x) = \frac{(y(x) - x + \log(x + 1))^2 + x}{x + 1}$$

✓ **Mathematica** : cpu = 0.334976 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow x - \log(x + 1) + \frac{1}{-\log(x + 1) + c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.06 (sec), leaf count = 36

$$\left\{ y(x) = \frac{c_1 x - \ln(x + 1)^2 + (-c_1 + x) \ln(x + 1) - 1}{c_1 + \ln(x + 1)} \right\}$$

2.1000 ODE No. 1000

$$y'(x) = \frac{x^3 + 2x^2 y(x) - x y(x) - y(x)^2 + x y(x) \log(x)}{x^2(x + \log(x))}$$

✓ **Mathematica** : cpu = 1.19896 (sec), leaf count = 351

$$\left\{ \left\{ y(x) \rightarrow \frac{x^2(x + \log(x)) \left(-\frac{\sqrt{x}(x + \log(x) + 2) \log(x) \exp\left(-\frac{1}{2} \int_1^x \frac{K[5] + \log(K[5]) + 2}{K[5]^2 + \log(K[5])K[5]} dK[5]\right)}{2(x^2 + x \log(x))} + c_1 \left(\frac{\exp\left(-\frac{1}{2} \int_1^x \frac{K[5] + \log(K[5]) + 2}{K[5]^2 + \log(K[5])K[5]} dK[5]\right)}{2\sqrt{x}} - \frac{\sqrt{x}(x + \log(x))}{\sqrt{x} \log(x) \exp\left(-\frac{1}{2} \int_1^x \frac{K[5] + \log(K[5]) + 2}{K[5]^2 + \log(K[5])K[5]} dK[5]\right)} + \right. \right. \right.$$

✓ **Maple** : cpu = 0.135 (sec), leaf count = 19

$$\left\{ y(x) = \frac{(c_1 x - 1)x}{c_1 \ln(x) + 1} \right\}$$

2.1001 ODE No. 1001

$$y''(x) = 0$$

✓ **Mathematica** : cpu = 0.003507 (sec), leaf count = 12

$$\{ \{ y(x) \rightarrow c_2 x + c_1 \} \}$$

✓ **Maple** : cpu = 0.004 (sec), leaf count = 9

$$\{ y(x) = c_1 x + c_2 \}$$

HAND SOLUTION

$$y'' = 0$$

Integration twice gives

$$y(x) = c_1x + c_2$$

2.1002 ODE No. 1002

$$y''(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0048476 (sec), leaf count = 16

$$\{ \{ y(x) \rightarrow c_1 \cos(x) + c_2 \sin(x) \} \}$$

✓ **Maple** : cpu = 0.005 (sec), leaf count = 13

$$\{ y(x) = c_1 \sin(x) + c_2 \cos(x) \}$$

HAND SOLUTION

$$y'' + y = 0$$

Let $y = e^{\lambda x}$, substitution in above gives

$$\begin{aligned} \lambda^2 e^{\lambda x} + e^{\lambda x} &= 0 \\ \lambda^2 + 1 &= 0 \end{aligned}$$

Hence $\lambda = \pm i$, therefore the solution is

$$\begin{aligned} y &= Ae^{ix} + Be^{-ix} \\ &= A(\cos x + i \sin x) + B(\cos x - i \sin x) \\ &= \cos x(A + B) + \sin x(Ai - iB) \\ &= \cos x(A + B) + \sin x(i(A - B)) \end{aligned}$$

Let $A + B = c_1, i(A - B) = c_2$ hence

$$y = c_1 \cos x + c_2 \sin x$$

2.1003 ODE No. 1003

$$-\sin(nx) + y''(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.108472 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow \frac{\cos^2(x)(-\sin(nx)) - \sin^2(x) \sin(nx)}{n^2 - 1} + c_1 \cos(x) + c_2 \sin(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.07 (sec), leaf count = 26

$$\left\{ y(x) = c_1 \cos(x) + c_2 \sin(x) - \frac{\sin(nx)}{n^2 - 1} \right\}$$

HAND SOLUTION

$$y'' + y = \sin nx \quad (1)$$

We start by solving the homogeneous equation

$$y'' + y = 0$$

Let $y = e^{\lambda x}$, substitution in above gives

$$\begin{aligned} \lambda^2 e^{\lambda x} + e^{\lambda x} &= 0 \\ \lambda^2 + 1 &= 0 \end{aligned}$$

Hence $\lambda = \pm i$, therefore the solution is

$$\begin{aligned} y_h &= Ae^{ix} + Be^{-ix} \\ &= A(\cos x + i \sin x) + B(\cos x - i \sin x) \\ &= \cos x(A + B) + \sin x(Ai - iB) \\ &= \cos x(A + B) + \sin x(i(A - B)) \end{aligned}$$

Let $A + B = c_1$, $i(A - B) = c_2$ hence

$$y_h = c_1 \cos x + c_2 \sin x$$

Now we solve for the particular solution using variation of parameters. Let

$$\begin{aligned} y_p &= u_1(x) \cos x + u_2(x) \sin x \\ y'_p &= u'_1 \cos x - u_1 \sin x + u'_2 \sin x + u_2 \cos x \\ &= u_2 \cos x - u_1 \sin x + u'_1 \cos x + u'_2 \sin x \end{aligned}$$

Let first condition be

$$u'_1 \cos x + u'_2 \sin x = 0 \quad (2)$$

Hence

$$\begin{aligned} y'_p &= u_2 \cos x - u_1 \sin x \\ y''_p &= u'_2 \cos x - u_2 \sin x - u'_1 \sin x - u_1 \cos x \end{aligned}$$

Substituting in (1) gives

$$\begin{aligned} y''_p + y_p &= \sin nx \\ u'_2 \cos x - u_2 \sin x - u'_1 \sin x - u_1 \cos x + u_1 \cos x + u_2 \sin x &= \sin nx \\ u'_2 \cos x - u'_1 \sin x &= \sin nx \end{aligned} \quad (3)$$

So we have two equations (1)(2) to solve for u_1, u_2

$$\begin{aligned} u'_1 \cos x + u'_2 \sin x &= 0 \\ u'_2 \cos x - u'_1 \sin x &= \sin nx \end{aligned}$$

From the first equation

$$u'_1 = -u'_2 \frac{\sin x}{\cos x} \quad (4)$$

Substituting in the second equation

$$\begin{aligned} u'_2 \cos x - \left(-u'_2 \frac{\sin x}{\cos x}\right) \sin x &= \sin nx \\ u'_2 \left(\cos x + \frac{\sin x}{\cos x} \sin x\right) &= \sin nx \\ u'_2 \left(\frac{\cos^2 x + \sin^2 x}{\cos x}\right) &= \sin nx \\ u'_2 &= \cos x \sin nx \end{aligned}$$

Hence

$$\begin{aligned} u_2 &= \int \cos x \sin(nx) dx \\ &= \frac{-n \cos x \cos(nx) - \sin x \sin(nx)}{n^2 - 1} \end{aligned}$$

From (4)

$$\begin{aligned} u_1' &= -\cos x \sin nx \frac{\sin x}{\cos x} \\ u_1 &= -\int \sin(nx) \sin x dx \\ &= \frac{n \cos(nx) \sin x - \cos x \sin(nx)}{n^2 - 1} \end{aligned}$$

Since $y_p = u_1(x) \cos x + u_2(x) \sin x$ then

$$\begin{aligned} y_p &= \left(\frac{n \cos(nx) \sin x - \cos x \sin(nx)}{n^2 - 1} \right) \cos x + \left(\frac{-n \cos x \cos(nx) - \sin x \sin(nx)}{n^2 - 1} \right) \sin x \\ &= \frac{n \cos(nx) \cos x \sin x - \cos^2 x \sin(nx) - n \cos x \sin x \cos(nx) - \sin^2 x \sin(nx)}{n^2 - 1} \\ &= \frac{-\sin(nx)(\cos^2 x + \sin^2 x)}{n^2 - 1} \\ &= \frac{\sin(nx)}{1 - n^2} \end{aligned}$$

Therefore, the full solution is (for $n^2 \neq 1$)

$$\begin{aligned} y &= y_h + y_p \\ &= c_1 \cos x + c_2 \sin x + \frac{\sin(nx)}{1 - n^2} \end{aligned}$$

Solution using undetermined coefficients: Since RHS is $\sin nx$ we guess $y_p = A \cos(nx) + B \sin(nx)$, therefore

$$\begin{aligned} y_p' &= -An \sin(nx) + Bn \cos(nx) \\ y_p'' &= -An^2 \cos(nx) - Bn^2 \sin(nx) \end{aligned}$$

Plug into the ODE gives

$$\begin{aligned} y_p'' + y_p &= \sin nx \\ -An^2 \cos(nx) - Bn^2 \sin(nx) + A \cos(nx) + B \sin(nx) &= \sin nx \\ \cos(nx)(-An^2 + A) + \sin(nx)(-Bn^2 + B) &= \sin(nx) \end{aligned}$$

Hence $-Bn^2 - B = 1$ and $-An^2 + A = 0$. Therefore $A = 0$ and from the first equation

$$\begin{aligned} B(n^2 + 1) &= -1 \\ B &= \frac{-1}{n^2 + 1} \end{aligned}$$

Hence

$$\begin{aligned} y_p &= A \cos(nx) + B \sin(nx) \\ &= \frac{\sin(nx)}{1 - n^2} \end{aligned}$$

Which is the same as variation of parameters method.

Note: Full solution should also really consider the case for $n = 1$. Will update later.

```
restart;
ode:=diff(diff(y(x),x),x)+y(x)-sin(n*x)=0;
y0:=_C1*cos(x)+_C2*sin(x)+sin(n*x)/(1-n^2);
odetest(y(x)=y0,ode);
0
```

2.1004 ODE No. 1004

$$-a \cos(bx) + y''(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.115846 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow \frac{-a \cos^2(x) \cos(bx) - a \sin^2(x) \cos(bx)}{b^2 - 1} + c_1 \cos(x) + c_2 \sin(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.063 (sec), leaf count = 27

$$\left\{ y(x) = c_1 \cos(x) + c_2 \sin(x) - \frac{a \cos(bx)}{b^2 - 1} \right\}$$

HAND SOLUTION

$$y'' + y = a \cos bx \quad (1)$$

We start by solving the homogeneous equation

$$y'' + y = 0$$

Let $y = e^{\lambda x}$, substitution in above gives

$$\begin{aligned} \lambda^2 e^{\lambda x} + e^{\lambda x} &= 0 \\ \lambda^2 + 1 &= 0 \end{aligned}$$

Hence $\lambda = \pm i$, therefore the solution is

$$\begin{aligned} y_h &= Ae^{ix} + Be^{-ix} \\ &= A(\cos x + i \sin x) + B(\cos x - i \sin x) \\ &= \cos x(A + B) + \sin x(Ai - iB) \\ &= \cos x(A + B) + \sin x(i(A - B)) \end{aligned}$$

Let $A + B = c_1, i(A - B) = c_2$ hence

$$y_h = c_1 \cos x + c_2 \sin x$$

Now we solve for the particular solution using variation of parameters. Let

$$\begin{aligned} y_p &= u_1(x) \cos x + u_2(x) \sin x \\ y'_p &= u'_1 \cos x - u_1 \sin x + u'_2 \sin x + u_2 \cos x \\ &= u_2 \cos x - u_1 \sin x + u'_1 \cos x + u'_2 \sin x \end{aligned}$$

Let first condition be

$$u'_1 \cos x + u'_2 \sin x = 0 \quad (2)$$

Hence

$$\begin{aligned} y'_p &= u_2 \cos x - u_1 \sin x \\ y''_p &= u'_2 \cos x - u_2 \sin x - u'_1 \sin x - u_1 \cos x \end{aligned}$$

Substituting in (1) gives

$$\begin{aligned} y''_p + y_p &= a \cos bx \\ u'_2 \cos x - u_2 \sin x - u'_1 \sin x - u_1 \cos x + u_1 \cos x + u_2 \sin x &= a \cos bx \\ u'_2 \cos x - u'_1 \sin x &= a \cos bx \end{aligned} \quad (3)$$

So we have two equations (1)(2) to solve for u_1, u_2

$$\begin{aligned} u'_1 \cos x + u'_2 \sin x &= 0 \\ u'_2 \cos x - u'_1 \sin x &= a \cos bx \end{aligned}$$

From the first equation

$$u_1' = -u_2' \frac{\sin x}{\cos x} \tag{4}$$

Substituting in the second equation

$$\begin{aligned} u_2' \cos x - \left(-u_2' \frac{\sin x}{\cos x}\right) \sin x &= a \cos bx \\ u_2' \left(\cos x + \frac{\sin x}{\cos x} \sin x\right) &= a \cos bx \\ u_2' \left(\frac{\cos^2 x + \sin^2 x}{\cos x}\right) &= a \cos bx \\ u_2' &= a \cos x \cos bx \end{aligned}$$

Hence

$$\begin{aligned} u_2 &= a \int \cos x \cos(bx)dx \\ &= a \frac{-\cos(bx) \sin x + b \cos x \sin(bx)}{b^2 - 1} \end{aligned}$$

From (4)

$$\begin{aligned} u_1' &= -a \cos(bx) \sin x \\ u_1 &= -a \int \cos(bx) \sin x dx \\ &= -a \frac{\cos(bx) \cos x + b \sin x \sin(bx)}{b^2 - 1} \end{aligned}$$

Since $y_p = u_1(x) \cos x + u_2(x) \sin x$ then

$$\begin{aligned} y_p &= \left(-a \frac{\cos(bx) \cos x + b \sin x \sin(bx)}{b^2 - 1}\right) \cos x + \left(a \frac{-\cos(bx) \sin x + b \cos x \sin(bx)}{b^2 - 1}\right) \sin x \\ &= \frac{-a \cos(bx) \cos^2 x - ab \cos x \sin x \sin(bx) - a \cos(bx) \sin^2 x + ab \sin x \cos x \sin(bx)}{b^2 - 1} \\ &= \frac{-a \cos(bx) \cos^2 x - a \cos(bx) \sin^2 x}{b^2 - 1} \\ &= \frac{-a \cos(bx) (\cos^2 x + \sin^2 x)}{b^2 - 1} \\ &= \frac{-a \cos(bx)}{b^2 - 1} \\ &= \frac{a \cos(bx)}{1 - b^2} \end{aligned}$$

Therefore, the full solution is (for $b^2 \neq 1$)

$$\begin{aligned} y &= y_h + y_p \\ &= c_1 \cos x + c_2 \sin x + \frac{a \cos(bx)}{1 - b^2} \end{aligned}$$

```
restart;
ode:=diff(diff(y(x),x),x)+y(x)-a*cos(b*x)=0;
y0:=_C1*cos(x)+_C2*sin(x)+a*cos(b*x)/(1-b^2);
odetest(y(x)=y0,ode);
0
```

2.1005 ODE No. 1005

$$-\sin(ax) \sin(bx) + y''(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.549023 (sec), leaf count = 1163

$$\left\{ \left\{ y(x) \rightarrow c_1 \cos(x) + c_2 \sin(x) + \frac{-\cos(x) \cos((a-b-1)x)a^3 + \cos(x) \cos((a-b+1)x)a^3 + \cos(x) \cos((a+b-1)x)a^3}{\dots} \right. \right.$$

✓ **Maple** : cpu = 0.128 (sec), leaf count = 82

$$\left\{y(x)=c_1 \cos (x)+c_2 \sin (x)+\frac{-(a+b+1)(a+b-1) \cos ((a-b) x)+(a-b+1)(a-b-1) \cos ((a+b) x)}{2 a^4+2 b^4+(-4 b^2-4) a^2-4 b^2+2}\right\}$$

HAND SOLUTION

$$y''+y=\sin ax \sin bx \tag{1}$$

We start by solving the homogeneous equation

$$y''+y=0$$

Let $y=e^{\lambda x}$, substitution in above gives

$$\begin{aligned}\lambda^2 e^{\lambda x}+e^{\lambda x} &=0 \\ \lambda^2+1 &=0\end{aligned}$$

Hence $\lambda=\pm i$, therefore the solution is

$$\begin{aligned}y_h &=A e^{i x}+B e^{-i x} \\ &=A(\cos x+i \sin x)+B(\cos x-i \sin x) \\ &=\cos x(A+B)+\sin x(A i-i B) \\ &=\cos x(A+B)+\sin x(i(A-B))\end{aligned}$$

Let $A+B=c_1, i(A-B)=c_2$ hence

$$\begin{aligned}y_h &=c_1 \cos x+c_2 \sin x \\ &=y_h=c_1 y_1+c_2 y_2\end{aligned}$$

Now we solve for the particular solution using variation of parameters.

$$y_p=u_1 y_1+u_2 y_2$$

Wonskian is

$$W=\left|\begin{array}{cc} y_1 & y_2 \\ y_1' & y_2' \end{array}\right|=\left|\begin{array}{cc} \cos x & \sin x \\ -\sin x & \cos x \end{array}\right|=\cos ^2 x+\sin ^2 x=1$$

Hence, using $f=\sin ax \sin bx$, which is the RHS of the ODE, and noting that a_0 is the coefficient of y'' which is one here, then

$$\begin{aligned}u_1 &=\int \frac{-y_2}{W} \frac{f}{a_0} d x=-\int \sin x \sin (a x) \sin (b x) d x \\ &=-\frac{1}{4}\left(\frac{\cos ((a-b-1) x)}{a-b-1}-\frac{\cos ((a-b+1) x)}{a-b+1}-\frac{\cos ((a+b-1) x)}{a+b-1}+\frac{\cos ((a+b+1) x)}{a+b+1}\right)\end{aligned}$$

And

$$\begin{aligned}u_2 &=\int \frac{y_1}{W} \frac{f}{a_0} d x=-\int \cos x \sin (a x) \sin (b x) d x \\ &=\frac{1}{4}\left(\frac{\sin ((a-b-1) x)}{a-b-1}+\frac{\sin ((a-b+1) x)}{a-b+1}-\frac{\sin ((a+b-1) x)}{a+b-1}-\frac{\sin ((a+b+1) x)}{a+b+1}\right)\end{aligned}$$

Since $y_p = u_1(x) \cos x + u_2(x) \sin x$ then

$$\begin{aligned} y_p &= -\frac{1}{4} \left(\frac{\cos((a-b-1)x)}{a-b-1} - \frac{\cos((a-b+1)x)}{a-b+1} - \frac{\cos((a+b-1)x)}{a+b-1} + \frac{\cos((a+b+1)x)}{a+b+1} \right) \cos x \\ &\quad + \frac{1}{4} \left(\frac{\sin((a-b-1)x)}{a-b-1} + \frac{\sin((a-b+1)x)}{a-b+1} - \frac{\sin((a+b-1)x)}{a+b-1} - \frac{\sin((a+b+1)x)}{a+b+1} \right) \sin x \\ &= \frac{1}{4} \left(-\frac{\cos^2((a-b-1)x)}{a-b-1} + \frac{\sin^2((a-b-1)x)}{a-b-1} \right) + \frac{1}{4} \left(\frac{\cos^2((a-b+1)x)}{a-b+1} + \frac{\sin^2((a-b+1)x)}{a-b+1} \right) \\ &\quad + \frac{1}{4} \left(\frac{\cos^2((a+b-1)x)}{a+b-1} - \frac{\sin^2((a+b-1)x)}{a+b-1} \right) + \frac{1}{4} \left(-\frac{\cos^2((a+b+1)x)}{a+b+1} - \frac{\sin^2((a+b+1)x)}{a+b+1} \right) \\ &= \frac{1}{4} \left(-\frac{\cos^2((a-b-1)x)}{a-b-1} + \frac{\sin^2((a-b-1)x)}{a-b-1} \right) + \frac{1}{4} \left(\frac{1}{a-b+1} \right) \\ &\quad + \frac{1}{4} \left(\frac{\cos^2((a+b-1)x)}{a+b-1} - \frac{\sin^2((a+b-1)x)}{a+b-1} \right) - \frac{1}{4} \left(\frac{1}{a+b+1} \right) \end{aligned}$$

Let $a-b-1 = \alpha, a+b-1 = \beta$ then

$$\begin{aligned} y_p &= \frac{1}{4} \left(\frac{\sin^2(\alpha x) - \cos^2(\alpha x)}{\alpha} \right) + \frac{1}{4} \left(\frac{\cos^2(\beta x) - \sin^2(\beta x)}{\beta} \right) + \frac{1}{4} \left(\frac{1}{\alpha+2} \right) - \frac{1}{4} \left(\frac{1}{\beta+2} \right) \\ &= \frac{1}{4} \left(\frac{\sin^2(\alpha x) - \cos^2(\alpha x)}{\alpha} + \frac{\cos^2(\beta x) - \sin^2(\beta x)}{\beta} + \frac{1}{\alpha+2} - \frac{1}{\beta+2} \right) \end{aligned}$$

Therefore, the full solution is

$$\begin{aligned} y &= y_h + y_p \\ &= c_1 \cos x + c_2 \sin x + \frac{1}{4} \left(\frac{\sin^2(\alpha x) - \cos^2(\alpha x)}{\alpha} + \frac{\cos^2(\beta x) - \sin^2(\beta x)}{\beta} + \frac{1}{\alpha+2} - \frac{1}{\beta+2} \right) \end{aligned}$$

I made mistake. Need to go over it again. I do not see it now. Maple does not verify.

```
restart;
ode:=diff(diff(y(x),x),x)+y(x)-sin(a*x)*sin(b*x)=0;
alpha:=a-b-1;
beta:=a+b-1;
yp:=1/4*(1/alpha*(sin(alpha*x)^2-cos(alpha*x)^2)+1/beta*(cos(beta*x)^2-sin(beta*x)^2))+(1/4)*(1/(alpha+2)-1/(beta+2));
y0:=yp+_C1*sin(x)+_C2*cos(x);
not zero
```

2.1006 ODE No. 1006

$$y''(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0039138 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow c_1 e^x + c_2 e^{-x} \right\} \right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 15

$$\{y(x) = c_1 e^{-x} + c_2 e^x\}$$

HAND SOLUTION

$$y'' - y = 0 \tag{1}$$

Let $y = e^{\lambda x}$, substitution in above gives

$$\begin{aligned}\lambda^2 e^{\lambda x} - e^{\lambda x} &= 0 \\ \lambda^2 - 1 &= 0\end{aligned}$$

Hence $\lambda = \pm 1$, therefore the solution is

$$y_h = Ae^x + Be^{-x}$$

2.1007 ODE No. 1007

$$-4e^{x^2}x^2 + y''(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0989217 (sec), leaf count = 135

$$\left\{ \left(y(x) \rightarrow \frac{e^{-\sqrt{2}x} \left(-2e^{x(x+\sqrt{2})}x + 2e^{(x-\sqrt{2})x+2\sqrt{2}x}x + \sqrt{2}e^{x(x+\sqrt{2})} + \sqrt{2}e^{(x-\sqrt{2})x+2\sqrt{2}x} \right)}{2\sqrt{2}} + c_1e^{\sqrt{2}x} + c_2e^{-\sqrt{2}x} \right) \right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 26

$$\left\{ y(x) = c_1e^{-\sqrt{2}x} + c_2e^{\sqrt{2}x} + e^{x^2} \right\}$$

Hand Solution

$$y'' - 2y = 4x^2e^{x^2} \tag{1}$$

We start by solving the homogeneous equation

$$y'' - 2y = 0$$

Let $y = e^{\lambda x}$, substitution in above gives

$$\begin{aligned}\lambda^2 e^{\lambda x} - 2e^{\lambda x} &= 0 \\ \lambda^2 - 2 &= 0\end{aligned}$$

Hence $\lambda = \pm\sqrt{2}$, therefore the solution is

$$\begin{aligned}y_h &= Ae^{\sqrt{2}x} + Be^{-\sqrt{2}x} \\ &= Ay_1 + By_2\end{aligned}$$

Now we solve for the particular solution using variation of parameters.

$$y_p = u_1y_1 + u_2y_2$$

wronskian is

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{\sqrt{2}x} & e^{-\sqrt{2}x} \\ \sqrt{2}e^{\sqrt{2}x} & -\sqrt{2}e^{-\sqrt{2}x} \end{vmatrix} = -\sqrt{2} - \sqrt{2} = -2\sqrt{2}$$

Hence, using $f = 4x^2e^{x^2}$, which is the RHS of the ODE, and noting that a_0 is the coefficient of y'' which is one here, then

$$\begin{aligned}u_1 &= \int \frac{-y_2}{W} \frac{f}{a_0} dx = - \int \frac{e^{-\sqrt{2}x}}{-2\sqrt{2}} 4x^2e^{x^2} dx \\ &= \frac{2}{\sqrt{2}} \int x^2e^{x^2-\sqrt{2}x} dx \\ &= \frac{2}{\sqrt{2}} \left(\frac{1}{4}e^{x(x-\sqrt{2})}(\sqrt{2} + 2x) \right)\end{aligned}$$

And

$$\begin{aligned} u_2 &= \int \frac{y_1}{W} \frac{f}{a_0} dx = \int \frac{e^{\sqrt{2}x}}{-2\sqrt{2}} 4x^2 e^{x^2} dx \\ &= \frac{-2}{\sqrt{2}} \int x^2 e^{x^2 \sqrt{2}x} dx \\ &= \frac{-2}{\sqrt{2}} \left(-\frac{1}{4} e^{x(x+\sqrt{2})} (\sqrt{2} - 2x) \right) \end{aligned}$$

Since $y_p = u_1 e^{\sqrt{2}x} + u_2 e^{\sqrt{2}x}$ then

$$\begin{aligned} y_p &= \frac{2}{\sqrt{2}} \left(\frac{1}{4} e^{x(x-\sqrt{2})} (\sqrt{2} + 2x) \right) e^{\sqrt{2}x} - \frac{2}{\sqrt{2}} \left(-\frac{1}{4} e^{x(x+\sqrt{2})} (\sqrt{2} - 2x) \right) e^{\sqrt{2}x} \\ &= \frac{2}{\sqrt{2}} \frac{1}{4} e^{x^2} (\sqrt{2} + 2x) + \frac{2}{\sqrt{2}} \frac{1}{4} e^{x^2} (\sqrt{2} - 2x) \\ &= \frac{1}{2} e^{x^2} + \frac{1}{2} e^{x^2} \\ &= e^{x^2} \end{aligned}$$

Therefore, the full solution is

$$\begin{aligned} y &= y_h + y_p \\ &= Ae^{\sqrt{2}x} + Be^{\sqrt{2}x} + e^{x^2} \end{aligned}$$

```
restart;
ode:=diff(y(x),x$2)-2*y(x)=4*x^2*exp(x^2);
y0:=-_C1*exp(sqrt(2)*x)+_C2*exp(-sqrt(2)*x)+exp(x^2);
odetest(y(x)=y0,ode);
0
```

2.1008 ODE No. 1008

$$a^2y(x) - \cot(ax) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0428319 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sin(ax) \left(\log\left(\cos\left(\frac{ax}{2}\right)\right) - \log\left(\sin\left(\frac{ax}{2}\right)\right) \right)}{a^2} + c_1 \cos(ax) + c_2 \sin(ax) \right\} \right\}$$

✓ **Maple** : cpu = 0.121 (sec), leaf count = 41

$$\left\{ y(x) = c_1 \cos(ax) + c_2 \sin(ax) + \frac{\ln\left(\frac{-\cos(ax)+1}{\sin(ax)}\right) \sin(ax)}{a^2} \right\}$$

2.1009 ODE No. 1009

$$ly(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0057355 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow c_1 \cos\left(\sqrt{l} x\right) + c_2 \sin\left(\sqrt{l} x\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 21

$$\left\{ y(x) = c_1 \sin\left(\sqrt{l} x\right) + c_2 \cos\left(\sqrt{l} x\right) \right\}$$

2.1010 ODE No. 1010

$$y(x)(ax + b) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0065964 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow c_1 \text{Ai} \left(\frac{-b - ax}{(-a)^{2/3}} \right) + c_2 \text{Bi} \left(\frac{-b - ax}{(-a)^{2/3}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.132 (sec), leaf count = 31

$$\left\{ y(x) = c_1 \text{AiryAi} \left(-\frac{ax + b}{a^{2/3}} \right) + c_2 \text{AiryBi} \left(-\frac{ax + b}{a^{2/3}} \right) \right\}$$

HAND SOLUTION

$$y'' + (ax + b)y = 0 \tag{1}$$

For $a \neq 0$. Let $y = \eta(\xi)$, $\xi = ax + b$, hence

$$\begin{aligned} \frac{dy}{dx} &= \frac{d\eta}{d\xi} \frac{d\xi}{dx} \\ &= \frac{d\eta}{d\xi} a \end{aligned}$$

And

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{d}{dx} \left(\frac{d\eta}{d\xi} a \right) \\ &= a \frac{d}{dx} \left(\frac{d\eta}{d\xi} \right) \\ &= a \left(\frac{d^2\eta}{d\xi^2} \frac{d\xi}{dx} \right) \\ &= a^2 \frac{d^2\eta}{d\xi^2} \end{aligned}$$

Therefore (1) becomes

$$\begin{aligned} a^2 \frac{d^2\eta}{d\xi^2} + \xi \eta(\xi) &= 0 \\ a^2 \eta'' + \xi \eta &= 0 \end{aligned} \tag{2}$$

This is Airy ODE but with plus sign instead of the normal Airy $\eta'' - \xi \eta = 0$. Let

$$\begin{aligned} \eta &= \sum_{n=0}^{\infty} c_n \xi^n \\ \eta' &= \sum_{n=0}^{\infty} n c_n \xi^{n-1} = \sum_{n=1}^{\infty} n c_n \xi^{n-1} = \sum_{n=0}^{\infty} (n+1) c_{n+1} \xi^n \\ \eta'' &= \sum_{n=0}^{\infty} n(n+1) c_{n+1} \xi^{n-1} = \sum_{n=1}^{\infty} n(n+1) c_{n+1} \xi^{n-1} = \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} \xi^n \end{aligned}$$

Hence (2) becomes

$$\begin{aligned}
& a^2 \sum_{n=0}^{\infty} (n+1)(n+2)c_{n+2}\xi^n + \xi \sum_{n=0}^{\infty} c_n \xi^n = 0 \\
& \sum_{n=0}^{\infty} a^2(n+1)(n+2)c_{n+2}\xi^n + \sum_{n=0}^{\infty} c_n \xi^{n+1} = 0 \\
& \sum_{n=0}^{\infty} a^2(n+1)(n+2)c_{n+2}\xi^n + \sum_{n=1}^{\infty} c_{n-1}\xi^n = 0 \\
& 2a^2c_2 + \sum_{n=1}^{\infty} (a^2(n+1)(n+2)c_{n+2} + c_{n-1})\xi^n = 0
\end{aligned}$$

Hence

$$2a^2c_2 = 0 \quad (3)$$

$$a^2(n+1)(n+2)c_{n+2} + c_{n-1} = 0 \quad n \geq 1 \quad (4)$$

From (3) and since $a \neq 0$

$$c_2 = 0$$

From (4)

$$c_{n+2} = \frac{-c_{n-1}}{a^2(n+1)(n+2)}$$

Hence for $n = 3$, we see from the above recurrence equation and because $c_2 = 0$ that

$$c_5 = \frac{-c_2}{a^2(4)(6)} = 0$$

Similarly, for $n = 6$

$$c_8 = \frac{-c_5}{a^2(7)(8)} = 0$$

Similarly, for $n = 9$

$$c_{11} = \frac{-c_8}{a^2(10)(11)} = 0$$

And so on. Hence we found so far that for $n = 3, 6, 9, 12, \dots$ all terms generated which are $c_5, c_8, c_{11}, \dots$ are zero.

Now, for $n = 1$, the recurrence equation gives

$$c_3 = \frac{-c_0}{a^2} \frac{1}{2 \cdot 3}$$

For $n = 2$

$$c_4 = \frac{-c_1}{a^2} \frac{1}{3 \cdot 4}$$

For $n = 4$

$$c_6 = \frac{-c_3}{a^2(5)(6)} = c_0 \left(\frac{1}{a^2} \frac{1}{5 \cdot 6} \right) \left(\frac{1}{a^2} \frac{1}{2 \cdot 3} \right)$$

For $n = 5$

$$c_7 = \frac{-c_4}{a^2(6)(7)} = c_1 \left(\frac{1}{a^2} \frac{1}{6 \cdot 7} \right) \left(\frac{1}{a^2} \frac{1}{3 \cdot 4} \right)$$

For $n = 7$

$$c_9 = \frac{-c_6}{a^2(8)(9)} = -c_0 \left(\frac{1}{a^2} \frac{1}{8 \cdot 9} \right) \left(\frac{1}{a^2} \frac{1}{5 \cdot 6} \right) \left(\frac{1}{a^2} \frac{1}{2 \cdot 3} \right)$$

For $n = 8$

$$c_{10} = \frac{-c_7}{a^2(9)(10)} = -c_1 \left(\frac{1}{a^2} \frac{1}{9 \cdot 10} \right) \left(\frac{1}{a^2} \frac{1}{6 \cdot 7} \right) \left(\frac{1}{a^2} \frac{1}{3 \cdot 4} \right)$$

Therefore, in summary, this is what we have so far. For $n = 3, 6, 9, 12, \dots$ all terms are zero. For $n = 1, 4, 7, \dots$ all terms are expressed using c_0 and for $n = 2, 5, 8, \dots$ all terms are expressed using c_1 . So there are two arbitrary constants c_0, c_1 .

In other words, $c_2, c_5, c_8, c_{11}, \dots = 0$ and $c_3, c_6, c_9, c_{12}, \dots = f(c_0)$ and $c_4, c_7, c_{10}, c_{13}, \dots = f(c_1)$.

$$\begin{aligned} \eta &= \sum_{n=0}^{\infty} c_n \xi^n \\ &= c_0 + c_1 \xi^1 + c_2 \xi^2 + c_3 \xi^3 + \dots \\ &= c_0 + (c_1 \xi^1) + 0 - \left(c_0 \frac{\xi^3}{a^2} \frac{1}{2 \cdot 3} \right) - \left(c_1 \frac{\xi^4}{a^2} \frac{1}{3 \cdot 4} \right) + 0 + c_0 \frac{\xi^6}{a^4} \frac{1}{2 \cdot 3 \cdot 5 \cdot 6} + c_1 \frac{\xi^7}{a^4} \frac{1}{3 \cdot 4 \cdot 6 \cdot 7} + 0 \\ &= c_0 \left(1 - \frac{\xi^3}{a^2} \frac{1}{2 \cdot 3} + \frac{\xi^6}{a^4} \frac{1}{2 \cdot 3 \cdot 5 \cdot 6} - \dots \right) + c_1 \left(\xi - \frac{\xi^4}{a^2} \frac{1}{3 \cdot 4} + \frac{\xi^7}{a^4} \frac{1}{3 \cdot 4 \cdot 6 \cdot 7} - \dots \right) \\ &= c_0 \left(1 - \frac{1}{a^2} \frac{\xi^3}{3!} + \frac{1 \cdot 4}{a^4} \frac{\xi^6}{6!} - \frac{1 \cdot 4 \cdot 7}{a^6} \frac{\xi^9}{9!} + \dots \right) + c_1 \left(\xi - \frac{\xi^4}{a^2} \frac{2}{4!} + \frac{\xi^7}{a^4} \frac{2 \cdot 5}{7!} - \frac{\xi^{10}}{a^6} \frac{2 \cdot 5 \cdot 8}{10!} + \dots \right) \end{aligned}$$

Hence

$$\eta = c_0 \left(\sum_{n=0}^{\infty} 3^n \left(\frac{1}{3} \right)_n \frac{(-1)^n}{a^{2n}} \frac{\xi^{3n}}{(3n)!} \right) + c_1 \left(\sum_{n=0}^{\infty} 3^n \left(\frac{2}{3} \right)_n \frac{(-1)^n}{a^{2n}} \frac{\xi^{3n+1}}{(3n+1)!} \right) \quad (5)$$

Where

$$\begin{aligned} 3^n \left(\frac{1}{3} \right)_n &= (1) \cdot (4) \cdot (7) \dots (3n-2) \\ 3^n \left(\frac{2}{3} \right)_n &= (2) \cdot (5) \cdot (8) \dots (3n-1) \end{aligned}$$

And

$$\left(\frac{1}{3} \right)_0 = \left(\frac{2}{3} \right)_0 = 1$$

Equation (5) can be simplified more by moving $\frac{(-1)^n}{a^{2n}}$ into ξ as follows

$$\eta = c_0 \left(\sum_{n=0}^{\infty} 3^n \left(\frac{1}{3} \right)_n \frac{1}{(3n)!} \left(\frac{-\xi}{a^{\frac{2}{3}}} \right)^{3n} \right) + ac_1 \left(\sum_{n=0}^{\infty} 3^n \left(\frac{2}{3} \right)_n \frac{1}{(3n+1)!} \left(\frac{-\xi}{a^{\frac{2}{3}}} \right)^{3n+1} \right)$$

Let $\left(\frac{-\xi}{a^{\frac{2}{3}}} \right) = z$ then the above is

$$\eta = c_0 \left(\sum_{n=0}^{\infty} 3^n \left(\frac{1}{3} \right)_n \frac{z^{3n}}{(3n)!} \right) + ac_1 \left(\sum_{n=0}^{\infty} 3^n \left(\frac{2}{3} \right)_n \frac{z^{3n+1}}{(3n+1)!} \right)$$

Let

$$f(\xi) = \sum_{n=0}^{\infty} 3^n \left(\frac{1}{3}\right)_n \frac{\xi^{3n}}{(3n)!}$$

$$g(\xi) = \sum_{n=0}^{\infty} 3^n \left(\frac{2}{3}\right)_n \frac{\xi^{3n+1}}{(3n+1)!}$$

Now looking at definition of AiryAI(z) we see

$$\text{AiryAI}(z) = r_1 f(z) - r_2 g(z)$$

$$\text{AiryBI}(z) = \sqrt{3} \left(r_1 f(z) + r_2 g(z) \right)$$

These are Airy functions AiryAI and AiryBI with appropriate choice of c_0, c_1 . See definition of these special functions. Converting back to x using $\xi = ax + b$ should result in solution given by CAS. Need to write these final details to make sure. Will finish later.

2.1011 ODE No. 1011

$$(-x^2 - 1)y(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0064255 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow c_1 D_{-1}(\sqrt{2}x) + c_2 D_0(i\sqrt{2}x) \right\} \right\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 17

$$\left\{ y(x) = (c_2 \operatorname{erf}(x) + c_1) e^{\frac{x^2}{2}} \right\}$$

HAND SOLUTION

$$y'' - (x^2 + 1)y = 0 \tag{1}$$

Second order with varying coefficient. Using power series, let $y = \sum_{n=0}^{\infty} c_n x^n$, hence

$$y' = \sum_{n=0}^{\infty} n c_n x^{n-1} = \sum_{n=1}^{\infty} n c_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n$$

$$y'' = \sum_{n=0}^{\infty} n(n+1) c_{n+1} x^{n-1} = \sum_{n=1}^{\infty} n(n+1) c_{n+1} x^{n-1} = \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n$$

Substituting back in the original ODE gives

$$\sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n - (x^2 + 1) \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n - x^2 \sum_{n=0}^{\infty} c_n x^n - \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n - \sum_{n=0}^{\infty} c_n x^{n+2} - \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n - \sum_{n=2}^{\infty} c_{n-2} x^n - \sum_{n=0}^{\infty} c_n x^n = 0$$

For $n = 0$

$$(n+1)(n+2) c_{n+2} - c_n = 0$$

$$2c_2 - c_0 = 0$$

$$c_2 = \frac{c_0}{2}$$

For $n = 1$

$$\begin{aligned}(n+1)(n+2)c_{n+2} - c_n &= 0 \\ (2)(3)c_3 - c_1 &= 0 \\ c_3 &= \frac{c_1}{6}\end{aligned}$$

For $n \geq 2$

$$\begin{aligned}(n+1)(n+2)c_{n+2} - c_{n-2} - c_n &= 0 \\ c_{n+2} &= \frac{c_{n-2} + c_n}{(n+1)(n+2)}\end{aligned}$$

Hence for $n = 2$

$$c_4 = \frac{c_0 + c_2}{(3)(4)} = \frac{c_0 + \frac{c_0}{2}}{(3)(4)} = \frac{2c_0 + c_0}{(2)(3)(4)} = c_0 \frac{3}{(2)(3)(4)}$$

For $n = 3$

$$c_5 = \frac{c_1 + c_3}{(4)(5)} = \frac{c_1 + \frac{c_1}{6}}{(4)(5)} = \frac{6c_1 + c_1}{(4)(5)(6)} = c_1 \frac{7}{(4)(5)(6)}$$

For $n = 4$

$$c_6 = \frac{c_2 + c_4}{(5)(6)} = \frac{\frac{c_0}{2} + c_0 \frac{3}{(2)(3)(4)}}{(5)(6)} = \frac{c_0(3)(4) + 3c_0}{(2)(3)(4)(5)(6)} = c_0 \frac{15}{(2)(3)(4)(5)(6)}$$

For $n = 5$

$$\begin{aligned}c_7 &= \frac{c_3 + c_5}{(6)(7)} \\ &= \frac{\frac{c_1}{6} + c_1 \frac{7}{(4)(5)(6)}}{(6)(7)} = \frac{3}{560}c_1\end{aligned}$$

And so on. Hence the series is

$$\begin{aligned}y &= \sum_{n=0}^{\infty} c_n x^n \\ &= c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots \\ &= c_0 + c_1 x + \frac{c_0}{2} x^2 + \frac{c_1}{6} x^3 + c_0 \frac{3}{(2)(3)(4)} x^4 + c_1 \frac{7}{(4)(5)(6)} x^5 + c_0 \frac{15}{(2)(3)(4)(5)(6)} x^6 + c_1 \frac{3}{560} x^7 + \dots \\ &= c_0 + c_1 x + \frac{c_0}{2} x^2 + \frac{c_1}{6} x^3 + c_0 \frac{1}{8} x^4 + c_1 \frac{7}{120} x^5 + c_0 \frac{1}{48} x^6 + c_1 \frac{3}{560} x^7 + \dots \\ &= c_0 \left(1 + \frac{x^2}{2} + \frac{1}{8} x^4 + \frac{1}{48} x^6 + \dots \right) + c_1 \left(x + \frac{1}{6} x^3 + \frac{7}{120} x^5 + \frac{3}{560} x^7 + \dots \right)\end{aligned}$$

Now the power series for $e^{\frac{x^2}{2}} = 1 + \frac{x^2}{2} + \frac{x^4}{8} + \frac{x^6}{48} + \dots$, so we can convert the first term above (the expression for c_0) to be $e^{\frac{x^2}{2}}$. Hence

$$c_0 \left(1 + \frac{x^2}{2} + \frac{1}{8} x^4 + \frac{1}{48} x^6 + \dots \right) = c_0 e^{\frac{x^2}{2}}$$

So now we have to work on the second term (the expression for c_1)

$$c_1 \left(x + \frac{1}{6} x^3 + \frac{7}{120} x^5 + \frac{3}{560} x^7 + \dots \right) = ?$$

Recall that series for error function is

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \left(x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{48} + \dots \right)$$

Multiplying $e^{\frac{x^2}{2}}$ by $\operatorname{erf}(x)$ gives

$$\begin{aligned}e^{\frac{x^2}{2}} \operatorname{erf}(x) &= \frac{2}{\sqrt{\pi}} \left(1 + \frac{x^2}{2} + \frac{x^4}{(2)(4)} + \frac{x^6}{(2)(4)(6)} + \dots \right) \left(x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{48} + \dots \right) \\ &= \frac{2}{\sqrt{\pi}} \left(x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{48} + \dots + \frac{x^3}{2} - \frac{x^5}{6} + \frac{x^7}{20} - \frac{x^{15}}{96} + \dots \right) \\ &= \frac{2}{\sqrt{\pi}} \left(x + \frac{x^3}{6} + \frac{7x^5}{120} + \frac{3}{560} x^7 + \dots \right)\end{aligned}$$

Comparing the above to the term next to c_1 above, we see they are the same with a multiplier $\frac{2}{\sqrt{\pi}}$, which can be absorbed into the constant c_1 , Hence

$$\begin{aligned} y &= c_0\left(1 + \frac{x^2}{2} + \frac{1}{8}x^4 + \frac{1}{48}x^6 + \cdots\right) + c_1\left(x + \frac{1}{6}x^3 + \frac{7}{120}x^5 + \frac{3}{560}x^7 + \cdots\right) \\ &= c_0e^{\frac{x^2}{2}} + c_1\left(e^{\frac{x^2}{2}} \operatorname{erf}(x)\right) \end{aligned}$$

Hence final solution is

$$y = c_0e^{\frac{x^2}{2}} + c_2\left(e^{\frac{x^2}{2}} \operatorname{erf}(x)\right)$$

Verification

```
restart;
ode:=diff(diff(y(x),x),x)-(x^2+1)*y(x)=0;
y0:=_C1*exp(x^2/2)+_C2*exp(x^2/2)*erf(x);
odetest(y(x)=y0,ode);
0
```

2.1012 ODE No. 1012

$$(-a-x^2)y(x)+y''(x)=0$$

✓ **Mathematica** : cpu = 0.0080078 (sec), leaf count = 47

$$\left\{\left\{y(x) \rightarrow c_1 D_{\frac{1}{2}(-a-1)}\left(\sqrt{2} x\right)+c_2 D_{\frac{a-1}{2}}\left(i \sqrt{2} x\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.164 (sec), leaf count = 29

$$\left\{y(x)=\frac{c_1 \operatorname{WhittakerM}\left(-\frac{a}{4}, \frac{1}{4}, x^2\right)+c_2 \operatorname{WhittakerW}\left(-\frac{a}{4}, \frac{1}{4}, x^2\right)}{\sqrt{x}}\right\}$$

2.1013 ODE No. 1013

$$\left(-a^2 x^2-a\right) y(x)+y''(x)=0$$

✓ **Mathematica** : cpu = 0.0143334 (sec), leaf count = 43

$$\left\{\left\{y(x) \rightarrow c_1 D_{-1}\left(\sqrt{2} \sqrt{a} x\right)+c_2 D_0\left(i \sqrt{2} \sqrt{a} x\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 22

$$\left\{y(x)=\left(c_2 \operatorname{erf}\left(\sqrt{a} x\right)+c_1\right) e^{\frac{a x^2}{2}}\right\}$$

2.1014 ODE No. 1014

$$y''(x)-c x^a y(x)=0$$

✓ **Mathematica** : cpu = 0.0229783 (sec), leaf count = 170

$$\left\{\left\{y(x) \rightarrow (a+2)^{-\frac{1}{a+2}} c_1 c^{\frac{1}{2(a+2)}} x^{\frac{a+1}{a+2}} \Gamma\left(1-\frac{1}{a+2}\right) I_{-\frac{1}{a+2}}\left(\frac{2 \sqrt{c} x^{\frac{a+2}{2}}}{a+2}\right)+(-1)^{\frac{1}{a+2}}(a+2)^{-\frac{1}{a+2}} c_2 c^{\frac{1}{2(a+2)}} x^{1-\frac{a+1}{a+2}} \Gamma\left(1+\frac{1}{a+2}\right) I_{\frac{1}{a+2}}\left(\frac{2 \sqrt{c} x^{\frac{a+2}{2}}}{a+2}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.231 (sec), leaf count = 63

$$\left\{y(x)=\left(c_1 \operatorname{BesselJ}\left(\frac{1}{a+2}, \frac{2 \sqrt{-c} x^{\frac{a}{2}+1}}{a+2}\right)+c_2 \operatorname{BesselY}\left(\frac{1}{a+2}, \frac{2 \sqrt{-c} x^{\frac{a}{2}+1}}{a+2}\right)\right) \sqrt{x}\right\}$$

2.1015 ODE No. 1015

$$y(x)\big(1 - a^2x^{2n}\big) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.332561 (sec), leaf count = 0 , could not solve
DSolve[(1 - a^2*x^(2*n))*y[x] + Derivative[2][y][x] == 0, y[x], x]
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x) = DESol\left(\left\{\left(-a^2x^{2n} + 1\right)_-Y(x) + \frac{d^2}{dx^2}_-Y(x)\right\}, \{_-Y(x)\}\right)\}$$

2.1016 ODE No. 1016

$$y(x)\big(ax^{2c} + bx^{c-1}\big) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.12795 (sec), leaf count = 312

$$\left\{\left\{y(x) \rightarrow 2^{\frac{c}{2(c+1)}}c_1\big(x^{c+1}\big)^{\frac{c}{2(c+1)}}x^{-c/2}e^{-\frac{\sqrt{a}x^{c+1}}{\sqrt{-c^2-2c-1}}}U\left(\frac{\frac{\sqrt{a}cb}{\sqrt{-(c+1)^2}} + \frac{\sqrt{a}b}{\sqrt{-(c+1)^2}} + ac}{2(ca+a)}, \frac{c}{c+1}, \frac{2\sqrt{a}x^{c+1}}{\sqrt{-c^2-2c-1}}\right) + 2^{\frac{c}{2(c+1)}}c_2\big(x^{c+1}\big)^{\frac{c}{2(c+1)}}x^{-\frac{c}{2}}\right.\right\}$$

✓ **Maple** : cpu = 0.346 (sec), leaf count = 91

$$\left\{y(x) = \left(c_1\operatorname{WhittakerM}\left(-\frac{ib}{(2c+2)\sqrt{a}}, \frac{1}{2c+2}, \frac{2i\sqrt{a}x^{c+1}}{c+1}\right) + c_2\operatorname{WhittakerW}\left(-\frac{ib}{(2c+2)\sqrt{a}}, \frac{1}{2c+2}, \frac{2i\sqrt{a}x^{c+1}}{c+1}\right)\right)x^{-\frac{c}{2}}\right\}$$

2.1017 ODE No. 1017

$$\big(e^{2x} - v^2\big)y(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.023942 (sec), leaf count = 46

$$\left\{\left\{y(x) \rightarrow c_1\Gamma(1-v)J_{-v}\left(\sqrt{e^{2x}}\right) + c_2\Gamma(v+1)J_v\left(\sqrt{e^{2x}}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 17

$$\{y(x) = c_1\operatorname{BesselJ}(v, e^x) + c_2\operatorname{BesselY}(v, e^x)\}$$

2.1018 ODE No. 1018

$$ae^{bx}y(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0208097 (sec), leaf count = 55

$$\left\{\left\{y(x) \rightarrow c_1I_0\left(\frac{2\sqrt{a}\sqrt{e^{bx}}}{b}\right) + 2c_2Y_0\left(\frac{2\sqrt{a}\sqrt{e^{bx}}}{b}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.131 (sec), leaf count = 39

$$\left\{y(x) = c_1\operatorname{BesselJ}\left(0, \frac{2\sqrt{a}e^{\frac{bx}{2}}}{b}\right) + c_2\operatorname{BesselY}\left(0, \frac{2\sqrt{a}e^{\frac{bx}{2}}}{b}\right)\right\}$$

2.1019 ODE No. 1019

$$y(x)\big(1-4a^2b^2x^2e^{2bx^2}\big)+y''(x)=0$$

✗ **Mathematica** : cpu = 0.699897 (sec), leaf count = 0 , could not solve

`DSolve[(1 - 4*a^2*b^2*E^(2*b*x^2))*x^2*y[x] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x)=DESol\left(\left\{\left(-4a^2b^2x^2e^{2bx^2}+1\right)_-Y(x)+\frac{d^2}{dx^2}_-Y(x)\right\},\{_-Y(x)\}\right)\}$$

2.1020 ODE No. 1020

$$y(x)\big(ae^{2x}+be^x+c\big)+y''(x)=0$$

✓ **Mathematica** : cpu = 0.577643 (sec), leaf count = 180

$$\left\{\left\{y(x)\rightarrow c_1e^{i\left(\sqrt{c}\log(e^x)-\sqrt{a}e^x\right)}U\left(\frac{i\left(b-i\sqrt{a}+2\sqrt{a}\sqrt{c}\right)}{2\sqrt{a}},2i\sqrt{c}+1,2i\sqrt{a}e^x\right)+c_2e^{i\left(\sqrt{c}\log(e^x)-\sqrt{a}e^x\right)}L_{-\frac{i\left(b-i\sqrt{a}+2\sqrt{a}\sqrt{c}\right)}{2\sqrt{a}}}^{2i\sqrt{c}}\left(2i\sqrt{a}e^x\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.29 (sec), leaf count = 58

$$\left\{y(x)=\left(c_1\operatorname{WhittakerM}\left(-\frac{ib}{2\sqrt{a}},i\sqrt{c},2i\sqrt{a}e^x\right)+c_2\operatorname{WhittakerW}\left(-\frac{ib}{2\sqrt{a}},i\sqrt{c},2i\sqrt{a}e^x\right)\right)e^{-\frac{x}{2}}\right\}$$

2.1021 ODE No. 1021

$$y(x)\big(a\cos^2(x)+b\big)+y''(x)=0$$

✓ **Mathematica** : cpu = 0.0278065 (sec), leaf count = 44

$$\left\{\left\{y(x)\rightarrow c_1\operatorname{MathieuC}\left[\frac{1}{2}(a+2b),-\frac{a}{4},x\right]+c_2\operatorname{MathieuS}\left[\frac{1}{2}(a+2b),-\frac{a}{4},x\right]\right\}\right\}$$

✓ **Maple** : cpu = 0.295 (sec), leaf count = 39

$$\left\{y(x)=c_1\operatorname{MathieuC}\left(-\frac{a}{2}-b,\frac{a}{4},ix\right)+c_2\operatorname{MathieuS}\left(-\frac{a}{2}-b,\frac{a}{4},ix\right)\right\}$$

2.1022 ODE No. 1022

$$y(x)(a\cos(2x)+b)+y''(x)=0$$

✓ **Mathematica** : cpu = 0.0203225 (sec), leaf count = 28

$$\left\{\left\{y(x)\rightarrow c_1\operatorname{MathieuC}\left[b,-\frac{a}{2},x\right]+c_2\operatorname{MathieuS}\left[b,-\frac{a}{2},x\right]\right\}\right\}$$

✓ **Maple** : cpu = 0.302 (sec), leaf count = 21

$$\left\{y(x)=c_1\operatorname{MathieuC}\left(b,-\frac{a}{2},x\right)+c_2\operatorname{MathieuS}\left(b,-\frac{a}{2},x\right)\right\}$$

2.1023 ODE No. 1023

$$y(x)\big(a\cos^2(x)+b\big)+y''(x)=0$$

✓ **Mathematica** : cpu = 0.0120762 (sec), leaf count = 44

$$\left\{\left\{y(x)\rightarrow c_1\operatorname{MathieuC}\left[\frac{1}{2}(a+2b),-\frac{a}{4},x\right]+c_2\operatorname{MathieuS}\left[\frac{1}{2}(a+2b),-\frac{a}{4},x\right]\right\}\right\}$$

✓ **Maple** : cpu = 0.306 (sec), leaf count = 29

$$\left\{y(x)=c_1\operatorname{MathieuC}\left(\frac{a}{2}+b,-\frac{a}{4},x\right)+c_2\operatorname{MathieuS}\left(\frac{a}{2}+b,-\frac{a}{4},x\right)\right\}$$

2.1024 ODE No. 1024

$$y''(x) + y(x)(-2 \tan^2(x) - 1) = 0$$

✓ **Mathematica** : cpu = 0.158162 (sec), leaf count = 84

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 \sqrt[4]{1 - \cos^2(x)} \sec(x)}{\sqrt[4]{\cos^2(x) - 1}} - \frac{c_2 \sqrt[4]{1 - \cos^2(x)} \sec(x) (\cos(x) \sqrt{1 - \cos^2(x)} - \sin^{-1}(\cos(x)))}{2 \sqrt[4]{\cos^2(x) - 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.167 (sec), leaf count = 30

$$\left\{ y(x) = \frac{ic_2 \cos(x) \sin(x) + c_2 \ln(\cos(x) + i \sin(x)) + c_1}{\cos(x)} \right\}$$

2.1025 ODE No. 1025

$$y(x)(-a - (m - 1)m \sec^2(x) - ((n - 1)n \csc^2(x))) + y''(x) = 0$$

✓ **Mathematica** : cpu = 1.05856 (sec), leaf count = 615

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 (-1)^{\frac{1}{2}(-2m-1)+1} \cos^2(x)^{\frac{1}{4}(-2m-1)+1} (\cos^2(x) - 1)^{\frac{1}{2} \left(\frac{4am+4\sqrt{-a}n^2+4an-4\sqrt{-a}n+4(-a)^{3/2}+8\sqrt{-a}a+\sqrt{-a}+4mn^2-4mn+m+4n^3-4n^2+n}{8a+8n^2-8n+2} + \frac{1}{2}(-\sqrt{-a}) \right)}}{1} \right\} \right\}$$

✓ **Maple** : cpu = 0.257 (sec), leaf count = 102

$$\left\{ y(x) = \left(c_1 (\cos^m(x)) \operatorname{hypergeom} \left(\left[\frac{m}{2} + \frac{n}{2} + \frac{i\sqrt{a}}{2}, \frac{m}{2} + \frac{n}{2} - \frac{i\sqrt{a}}{2} \right], \left[m + \frac{1}{2} \right], \cos^2(x) \right) + c_2 (\cos^{-m+1}(x)) \operatorname{hypergeom} \left(\left[\frac{m}{2} + \frac{n}{2} + \frac{i\sqrt{a}}{2}, \frac{m}{2} + \frac{n}{2} - \frac{i\sqrt{a}}{2} \right], \left[m + \frac{1}{2} \right], \cos^2(x) \right) \right) \right\}$$

2.1026 ODE No. 1026

$$y''(x) - y(x)(B + n(n + 1)\wp(x; g2, g3)) = 0$$

✗ **Mathematica** : cpu = 0.200477 (sec), leaf count = 0 , could not solve

`DSolve[-((B + n*(1 + n)*WeierstrassP[x, {g2, g3}])*y[x]) + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x) = DESol \left(\left\{ \left(-(n + 1)n \operatorname{WeierstrassP}(x, g2, g3) - B \right) _Y(x) + \frac{d^2}{dx^2} _Y(x) \right\}, \{ _Y(x) \} \right) \}$$

2.1027 ODE No. 1027

$$y(x)(a \operatorname{sn}(x|k)^2 + b) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.991642 (sec), leaf count = 235

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt{k \operatorname{sn}(x|k)^2 - 1} \operatorname{HeunG} \left[\frac{1}{k}, -\frac{b - k}{4k}, \frac{\sqrt{k - 4a} + 3\sqrt{k}}{4\sqrt{k}}, \frac{\sqrt{k} \sqrt{k - 4a} + 2a + k}{2\sqrt{k} (\sqrt{k - 4a} + \sqrt{k})}, \frac{1}{2}, \frac{1}{2}, \operatorname{sn}(x|k)^2 \right] + c_2 \operatorname{sn}(x|k) \sqrt{k \operatorname{sn}(x|k)^2 - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.459 (sec), leaf count = 69

$$\left\{ y(x) = c_2 HG \left(\frac{1}{k^2}, \frac{k^2 + b + 1}{4k^2}, \frac{n}{2} + 1, -\frac{n}{2} + \frac{1}{2}, \frac{3}{2}, \frac{1}{2}, \operatorname{sn}(x|k)^2 \right) \operatorname{sn}(x|k) + c_1 HG \left(\frac{1}{k^2}, \frac{b}{4k^2}, -\frac{n}{2}, \frac{n}{2} + \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \operatorname{sn}(x|k)^2 \right) \right\}$$

2.1028 ODE No. 1028

$$y''(x) - y(x)\left(ap(x) + b + \frac{p^4(x)}{30} + \frac{7p''(x)}{3}\right) = 0$$

✗ **Mathematica** : cpu = 0.286493 (sec), leaf count = 0 , could not solve

`DSolve[-(y[x]*(b + a*p[x] + (p^4)[x]/30 + (7*Derivative[2][p][x])/3)) + Derivative[2][y][x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x) = DESol\left[\left\{\left(-ap(x) - b - \frac{\frac{d^4}{dx^4}p(x)}{30} - \frac{7\frac{d^2}{dx^2}p(x)}{3}\right)Y(x) + \frac{d^2}{dx^2}Y(x)\right\}, \{Y(x)\}\right]\right\}$$

2.1029 ODE No. 1029

$$y''(x) - y(x)(f'(x) + f(x)^2) = 0$$

✓ **Mathematica** : cpu = 0.0229259 (sec), leaf count = 58

$$\left\{\left\{y(x) \rightarrow c_1 \exp\left(\int_1^x f(K[1])dK[1]\right) + c_2 \exp\left(\int_1^x f(K[2])dK[2]\right) \int_1^x \exp\left(\int_1^{K[4]} -2f(K[3])dK[3]\right)dK[4]\right\}\right\}$$

✓ **Maple** : cpu = 0.136 (sec), leaf count = 22

$$\{y(x) = c_2\left(c_1 + \int e^{\int -2f(x)dx}dx\right)e^{\int f(x)dx}\}$$

2.1030 ODE No. 1030

$$y(x)(l + P(x)) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.197984 (sec), leaf count = 0 , could not solve

`DSolve[(1 + P[x])*y[x] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x) = DESol\left[\left\{(l + P(x))Y(x) + \frac{d^2}{dx^2}Y(x)\right\}, \{Y(x)\}\right]\}$$

2.1031 ODE No. 1031

$$y''(x) - f(x)y(x) = 0$$

✗ **Mathematica** : cpu = 0.133659 (sec), leaf count = 0 , could not solve

`DSolve[-(f[x]*y[x]) + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x) = DESol\left[\left\{-Y(x)f(x) + \frac{d^2}{dx^2}Y(x)\right\}, \{Y(x)\}\right]\}$$

2.1032 ODE No. 1032

$$y(x)\left(\frac{\left(\frac{1}{4} - v^2\right)g'(x)^2}{g(x)} + g'(x)^2 + \frac{g^3(x)}{2g'(x)} - \frac{3g''(x)^2}{4g'(x)^2}\right) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.434963 (sec), leaf count = 0 , could not solve

`DSolve[y[x]*((g^3)[x]/(2*Derivative[1][g][x]) + Derivative[1][g][x]^2 + ((1/4 - v^2)*Derivat`

✓ **Maple** : cpu = 0.132 (sec), leaf count = 48

$$\left\{y(x)=\frac{c_1\operatorname{WhittakerM}\left(\frac{1}{2}i\nu^2-\frac{1}{8}i,\frac{1}{2},2ig(x)\right)+c_2\operatorname{WhittakerW}\left(\frac{1}{2}i\nu^2-\frac{1}{8}i,\frac{1}{2},2ig(x)\right)}{\sqrt{\frac{d}{dx}g(x)}}\right\}$$

2.1033 ODE No. 1033

$$ae^{-2x}y(x)+y''(x)+y'(x)=0$$

✓ **Mathematica** : cpu = 0.0129288 (sec), leaf count = 37

$$\left\{\left\{y(x)\rightarrow c_1\cos\left(\sqrt{a}\,e^{-x}\right)-c_2\sin\left(\sqrt{a}\,e^{-x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 27

$$\left\{y(x)=c_1\sin\left(\sqrt{a}\,e^{-x}\right)+c_2\cos\left(\sqrt{a}\,e^{-x}\right)\right\}$$

HAND SOLUTION

$$y''+y'+ae^{-2x}y=0$$

Let $y(x)=\eta(\xi)$ where $\xi=e^{-x}$, hence

$$\begin{aligned}\frac{dy}{dx}&=\frac{d\eta}{d\xi}\frac{d\xi}{dx} \\ &=\frac{d\eta}{d\xi}(-e^{-x})\end{aligned}$$

And

$$\begin{aligned}\frac{d^2y}{dx^2}&=\frac{d}{dx}\left(\frac{d\eta}{d\xi}(-e^{-x})\right) \\ &=\frac{d^2\eta}{d\xi^2}\frac{d\xi}{dx}(-e^{-x})+\frac{d\eta}{d\xi}(e^{-x}) \\ &=\frac{d^2\eta}{d\xi^2}(-e^{-x})(-e^{-x})+\frac{d\eta}{d\xi}(e^{-x}) \\ &=\frac{d^2\eta}{d\xi^2}(e^{-2x})+\frac{d\eta}{d\xi}(e^{-x})\end{aligned}$$

Hence the original ODE becomes

$$\begin{aligned}\frac{d^2\eta}{d\xi^2}(e^{-2x})+\frac{d\eta}{d\xi}(e^{-x})+\frac{d\eta}{d\xi}(-e^{-x})+ae^{-2x}\eta(\xi)&=0 \\ \eta''+a\eta&=0\end{aligned}$$

This is standard second order with constant coefficients. The solution is

$$\eta=c_1\cos\left(\sqrt{a}\,\xi\right)+c_2\sin\left(\sqrt{a}\,\xi\right)$$

Substituting back

$$y(x)=c_1\cos\left(\sqrt{a}\,e^{-x}\right)+c_2\sin\left(\sqrt{a}\,e^{-x}\right)$$

Verification

```
restart;
ode:=diff(diff(y(x),x),x)+diff(y(x),x)+a*exp(-2*x)*y(x)=0;
ys:=_C1*cos(sqrt(a)*exp(-x))+_C2*sin(sqrt(a)*exp(-x));
odetest(y(x)=ys,ode);
0
```

2.1034 ODE No. 1034

$$y''(x) - y'(x) + e^{2x}y(x) = 0$$

✓ **Mathematica** : cpu = 0.0096618 (sec), leaf count = 20

$$\left\{\left\{y(x) \rightarrow c_1 \cos(e^x) + c_2 \sin(e^x)\right\}\right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 15

$$\{y(x) = c_1 \sin(e^x) + c_2 \cos(e^x)\}$$

HAND SOLUTION

$$y'' - y' + e^{2x}y = 0$$

Let $y(x) = \eta(\xi)$ where $\xi = e^x$, hence

$$\begin{aligned}\frac{dy}{dx} &= \frac{d\eta}{d\xi} \frac{d\xi}{dx} \\ &= \frac{d\eta}{d\xi} e^x\end{aligned}$$

And

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{d}{dx} \left(\frac{d\eta}{d\xi} e^x \right) \\ &= \frac{d^2\eta}{d\xi^2} \frac{d\xi}{dx} (e^x) + \frac{d\eta}{d\xi} (e^x) \\ &= \frac{d^2\eta}{d\xi^2} (e^x)(e^x) + \frac{d\eta}{d\xi} (e^x) \\ &= \frac{d^2\eta}{d\xi^2} (e^{2x}) + \frac{d\eta}{d\xi} (e^x)\end{aligned}$$

Hence the original ODE becomes

$$\begin{aligned}\frac{d^2\eta}{d\xi^2} (e^{2x}) + \frac{d\eta}{d\xi} (e^x) - \frac{d\eta}{d\xi} (e^x) + e^{2x}\eta &= 0 \\ \eta'' + \eta &= 0\end{aligned}$$

This is standard second order with constant coefficients. The solution is

$$\eta = c_1 \cos(\xi) + c_2 \sin(\xi)$$

Substituting back

$$y(x) = c_1 \cos(e^x) + c_2 \sin(e^x)$$

Verification

```
restart;
ode:=diff(diff(y(x),x),x)-diff(y(x),x)+exp(2*x)*y(x)=0;
ys:=_C1*cos(exp(x))+_C2*sin(exp(x));
odetest(y(x)=ys,ode);
0
```

2.1035 ODE No. 1035

$$ay'(x) + by(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0059992 (sec), leaf count = 58

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{2}x(-\sqrt{a^2-4b}-a)} + c_2 e^{\frac{1}{2}x(\sqrt{a^2-4b}-a)} \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 41

$$\left\{ y(x) = c_1 e^{-\frac{(a-\sqrt{a^2-4b})x}{2}} + c_2 e^{-\frac{(a+\sqrt{a^2-4b})x}{2}} \right\}$$

2.1036 ODE No. 1036

$$ay'(x) + by(x) - f(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.155964 (sec), leaf count = 209

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{1}{2}x(-\sqrt{a^2-4b}-a)} \int_1^x -\frac{e^{\frac{aK[1]+\frac{1}{2}(\sqrt{a^2-4b}-a)K[1]}{f(K[1])}}}{\sqrt{a^2-4b}} dK[1] + e^{\frac{1}{2}x(\sqrt{a^2-4b}-a)} \int_1^x \frac{e^{\frac{aK[2]+\frac{1}{2}(-a-\sqrt{a^2-4b})K[2]}{f(K[2])}}}{\sqrt{a^2-4b}} dK[2] \right\} \right\}$$

✓ **Maple** : cpu = 0.145 (sec), leaf count = 134

$$\left\{ y(x) = c_1 e^{-\frac{(a+\sqrt{a^2-4b})x}{2}} + c_2 e^{-\frac{(a-\sqrt{a^2-4b})x}{2}} + \frac{\left(\left(\int e^{-\frac{(-a+\sqrt{a^2-4b})x}{2}} f(x) dx \right) e^{\frac{(a+\sqrt{a^2-4b})x}{2}} - \left(\int e^{\frac{(a+\sqrt{a^2-4b})x}{2}} f(x) dx \right) e^{-\frac{(-a+\sqrt{a^2-4b})x}{2}} \right)}{\sqrt{a^2-4b}} \right\}$$

2.1037 ODE No. 1037

$$ay'(x) + y(x)(-b^2x^2 - c) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0215399 (sec), leaf count = 101

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\frac{ax}{2}-\frac{bx^2}{2}} H_{\frac{-a^2-4b-4c}{8b}}(\sqrt{b}x) + c_2 e^{-\frac{ax}{2}-\frac{bx^2}{2}} {}_1F_1\left(-\frac{-a^2-4b-4c}{16b}; \frac{1}{2}; bx^2\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.157 (sec), leaf count = 64

$$\left\{ y(x) = \left(c_1 \text{KummerM}\left(\frac{a^2+12b+4c}{16b}, \frac{3}{2}, bx^2\right) + c_2 \text{KummerU}\left(\frac{a^2+12b+4c}{16b}, \frac{3}{2}, bx^2\right) \right) x e^{-\frac{(bx+a)x}{2}} \right\}$$

2.1038 ODE No. 1038

$$2ay'(x) + f(x)y(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.372802 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*y[x] + 2*a*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x) = DESol\left(\left\{2a\left(\frac{d}{dx}Y(x)\right) + Y(x)f(x) + \frac{d^2}{dx^2}Y(x)\right\}, \{Y(x)\}\right)\}$$

2.1039 ODE No. 1039

$$y''(x) + xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0099841 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow \sqrt{\frac{\pi}{2}} c_1 e^{-\frac{x^2}{2}} \operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right) + c_2 e^{-\frac{x^2}{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 25

$$\left\{ y(x) = \left(c_1 \operatorname{erf}\left(\frac{i\sqrt{2}x}{2}\right) + c_2 \right) e^{-\frac{x^2}{2}} \right\}$$

2.1040 ODE No. 1040

$$y''(x) + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0494561 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow c_1 x - \frac{1}{2} c_2 e^{-\frac{x^2}{2}} \left(\sqrt{2\pi} e^{\frac{x^2}{2}} \operatorname{xerf}\left(\frac{x}{\sqrt{2}}\right) + 2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 34

$$\left\{ y(x) = -c_2 e^{-\frac{x^2}{2}} + \left(-\frac{c_2 \sqrt{\pi} \sqrt{2} \operatorname{erf}\left(\frac{\sqrt{2}x}{2}\right)}{2} + c_1 \right) x \right\}$$

2.1041 ODE No. 1041

$$(n+1)y(x) + y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0108278 (sec), leaf count = 55

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\frac{x^2}{2}} H_n\left(\frac{x}{\sqrt{2}}\right) + c_2 e^{-\frac{x^2}{2}} {}_1F_1\left(-\frac{n}{2}; \frac{1}{2}; \frac{x^2}{2}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.141 (sec), leaf count = 41

$$\left\{ y(x) = \left(c_1 \operatorname{KummerM}\left(-\frac{n}{2} + \frac{1}{2}, \frac{3}{2}, \frac{x^2}{2}\right) + c_2 \operatorname{KummerU}\left(-\frac{n}{2} + \frac{1}{2}, \frac{3}{2}, \frac{x^2}{2}\right) \right) x e^{-\frac{x^2}{2}} \right\}$$

2.1042 ODE No. 1042

$$-ny(x) + y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0087322 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\frac{x^2}{2}} H_{-n-1}\left(\frac{x}{\sqrt{2}}\right) + c_2 e^{-\frac{x^2}{2}} {}_1F_1\left(\frac{n+1}{2}; \frac{1}{2}; \frac{x^2}{2}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.147 (sec), leaf count = 41

$$\left\{ y(x) = \left(c_1 \operatorname{KummerM}\left(\frac{n}{2} + 1, \frac{3}{2}, \frac{x^2}{2}\right) + c_2 \operatorname{KummerU}\left(\frac{n}{2} + 1, \frac{3}{2}, \frac{x^2}{2}\right) \right) x e^{-\frac{x^2}{2}} \right\}$$

2.1043 ODE No. 1043

$$y''(x) - xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.116475 (sec), leaf count = 69

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} c_2 \left(\sqrt{2\pi} x^2 \operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right) - \sqrt{2\pi} \operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right) - 2e^{\frac{x^2}{2}} x \right) + c_1(x^2 - 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.293 (sec), leaf count = 39

$$\left\{ y(x) = -2c_1 x e^{\frac{x^2}{2}} + (x-1)(x+1) \left(c_1 \sqrt{2} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{2} x}{2}\right) + c_2 \right) \right\}$$

HAND SOLUTION

$$y'' - xy' + 2y = 0 \quad (1)$$

Second order with varying coefficient. Using power series, let $y = \sum_{n=0}^{\infty} c_n x^n$, hence

$$\begin{aligned} y' &= \sum_{n=0}^{\infty} n c_n x^{n-1} = \sum_{n=1}^{\infty} n c_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n \\ y'' &= \sum_{n=0}^{\infty} n(n+1) c_{n+1} x^{n-1} = \sum_{n=1}^{\infty} n(n+1) c_{n+1} x^{n-1} = \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n \end{aligned}$$

Substituting back in the original ODE gives

$$\begin{aligned} \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n - x \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n + 2 \sum_{n=0}^{\infty} c_n x^n &= 0 \\ \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n - \sum_{n=0}^{\infty} (n+1) c_{n+1} x^{n+1} + \sum_{n=0}^{\infty} 2c_n x^n &= 0 \\ \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n - \sum_{n=1}^{\infty} n c_n x^n + \sum_{n=0}^{\infty} 2c_n x^n &= 0 \end{aligned}$$

For $n = 0$

$$\begin{aligned} (n+1)(n+2) c_{n+2} + 2c_n &= 0 \\ (1)(2) c_2 + 2c_0 &= 0 \\ c_2 &= -c_0 \end{aligned}$$

For $n \geq 1$

$$\begin{aligned} (n+1)(n+2) c_{n+2} - n c_n + 2c_n &= 0 \\ c_{n+2} &= \frac{c_n(n-2)}{(n+1)(n+2)} \end{aligned}$$

Hence for $n = 1$

$$c_3 = \frac{-c_1}{(2)(3)}$$

For $n = 2$

$$c_4 = \frac{c_2(2-2)}{(3)(4)} = 0$$

For $n = 3$

$$c_5 = \frac{c_3}{(4)(5)} = \frac{-c_1}{(2)(3)(4)(5)}$$

For $n = 4$ and since $c_4 = 0$ then

$$c_6 = \frac{c_4(n-2)}{(n+1)(n+2)} = 0$$

For $n = 5$

$$c_7 = \frac{3c_5}{(6)(7)} = -\frac{3c_1}{(2)(3)(4)(5)(6)(7)}$$

For $n = 6$ and since $c_6 = 0$ then

$$c_8 = \frac{c_6(n-2)}{(n+1)(n+2)} = 0$$

For $n = 7$

$$c_9 = \frac{5c_7}{(8)(9)} = -\frac{(3)(5)c_1}{(2)(3)(4)(5)(6)(7)(8)(9)}$$

And so on. Hence

$$\begin{aligned} y &= \sum_{n=0}^{\infty} c_n x^n \\ &= c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots \\ &= c_0 + c_1 x - c_0 x^2 - \frac{c_1}{(2)(3)} x^3 - \frac{c_1}{(2)(3)(4)(5)} x^5 - \frac{3c_1}{(2)(3)(4)(5)(6)(7)} x^7 - \frac{(3)(5)c_1}{(2)(3)(4)(5)(6)(7)(8)(9)} x^9 - \dots \\ &= c_0(1 - x^2) + c_1 \left(x - \frac{1}{(2)(3)} x^3 - \frac{1}{(2)(3)(4)(5)} x^5 - \frac{3}{(2)(3)(4)(5)(6)(7)} x^7 - \frac{(3)(5)}{(2)(3)(4)(5)(6)(7)(8)(9)} x^9 \right) \\ &= c_0(1 - x^2) + c_1 \left(x - \frac{1}{3!} x^3 - \frac{1}{5!} x^5 - \frac{3}{7!} x^7 - \frac{15}{9!} x^9 - \dots \right) \end{aligned}$$

Hence

$$y(x) = c_0(1 - x^2) + c_1 \left(x - \frac{1}{6} x^3 - \frac{1}{120} x^5 - \frac{1}{1680} x^7 - \frac{1}{24192} x^9 - \dots \right)$$

Verification

```
restart;
Order:=10:
sol:=dsolve(ode,y(x),series):
subs({y(0)=c0,D(y)(0)=c1},rhs(sol)):
sol:=convert(%,polynom):

sol:=collect(sol,{c0,c1});
sol := (-x^2+1)*c0+(x-(1/6)*x^3-(1/120)*x^5-(1/1680)*x^7-(1/24192)*x^9)*c1
```

2.1044 ODE No. 1044

$$-ay(x) + y''(x) - xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0069817 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow c_1 H_{-a} \left(\frac{x}{\sqrt{2}} \right) + c_2 {}_1F_1 \left(\frac{a}{2}; \frac{1}{2}; \frac{x^2}{2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.152 (sec), leaf count = 35

$$\left\{ y(x) = \left(c_1 \text{KummerM} \left(\frac{a}{2} + \frac{1}{2}, \frac{3}{2}, \frac{x^2}{2} \right) + c_2 \text{KummerU} \left(\frac{a}{2} + \frac{1}{2}, \frac{3}{2}, \frac{x^2}{2} \right) \right) x \right\}$$

2.1045 ODE No. 1045

$$y''(x) - xy'(x) + (x - 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0465083 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow \sqrt{\frac{\pi}{2}} c_2 e^{x-2} \operatorname{erfi} \left(\frac{x-2}{\sqrt{2}} \right) + c_1 e^x \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 21

$$\left\{ y(x) = \left(c_1 \operatorname{erf} \left(\frac{i\sqrt{2} (x-2)}{2} \right) + c_2 \right) e^x \right\}$$

2.1046 ODE No. 1046

$$ay(x) + y''(x) - 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.007764 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow c_1 H_{\frac{a}{2}}(x) + c_2 {}_1F_1\left(-\frac{a}{4}; \frac{1}{2}; x^2\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.151 (sec), leaf count = 31

$$\left\{ y(x) = \left(c_1 \text{KummerM}\left(-\frac{a}{4} + \frac{1}{2}, \frac{3}{2}, x^2\right) + c_2 \text{KummerU}\left(-\frac{a}{4} + \frac{1}{2}, \frac{3}{2}, x^2\right) \right) x \right\}$$

2.1047 ODE No. 1047

$$(4x^2 + 2)y(x) + y''(x) + 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0141536 (sec), leaf count = 27

$$\{ \{ y(x) \rightarrow c_1 e^{-x^2} + c_2 e^{-x^2} x \} \}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 16

$$\{y(x) = (c_2 x + c_1) e^{-x^2}\}$$

HAND SOLUTION

$$y'' + 4xy' + (4x^2 + 2)y = 0 \quad (1)$$

Second order with varying coefficient. Using power series, let $y = \sum_{n=0}^{\infty} c_n x^n$, hence

$$\begin{aligned} y' &= \sum_{n=0}^{\infty} n c_n x^{n-1} = \sum_{n=1}^{\infty} n c_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n \\ y'' &= \sum_{n=0}^{\infty} n(n+1) c_{n+1} x^{n-1} = \sum_{n=1}^{\infty} n(n+1) c_{n+1} x^{n-1} = \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n \end{aligned}$$

Substituting back in the original ODE gives

$$\begin{aligned} \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n + 4x \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n + (4x^2 + 2) \sum_{n=0}^{\infty} c_n x^n &= 0 \\ \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n + \sum_{n=0}^{\infty} 4(n+1) c_{n+1} x^{n+1} + \sum_{n=0}^{\infty} 4c_n x^{n+2} + \sum_{n=0}^{\infty} 2c_n x^n &= 0 \\ \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n + \sum_{n=1}^{\infty} 4n c_n x^n + \sum_{n=2}^{\infty} 4c_{n-2} x^n + \sum_{n=0}^{\infty} 2c_n x^n &= 0 \end{aligned}$$

For $n = 0$

$$\begin{aligned} (n+1)(n+2) c_{n+2} + 2c_n &= 0 \\ (1)(2) c_2 + 2c_0 &= 0 \\ c_2 &= -c_0 \end{aligned}$$

For $n = 1$

$$\begin{aligned} (n+1)(n+2) c_{n+2} + 4n c_n + 2c_n &= 0 \\ (2)(3) c_3 + 4c_1 + 2c_1 &= 0 \\ c_3 &= -c_1 \end{aligned}$$

For $n \geq 2$

$$\begin{aligned} (n+1)(n+2) c_{n+2} + 4n c_n + 4c_{n-2} + 2c_n &= 0 \\ c_{n+2} &= \frac{(-4n-2)c_n - 4c_{n-2}}{(n+1)(n+2)} \end{aligned}$$

Hence for $n = 2$

$$c_4 = \frac{(-8-2)c_2 - 4c_0}{(3)(4)} = \frac{(-8-2)(-c_0) - 4c_0}{(3)(4)} = c_0 \frac{6}{(3)(4)}$$

For $n = 3$

$$c_5 = \frac{(-12-2)c_3 - 4c_1}{(4)(5)} = \frac{(-12-2)(-c_1) - 4c_1}{(4)(5)} = c_1 \frac{10}{(4)(5)}$$

For $n = 4$

$$c_6 = \frac{(-16-2)c_4 - 4c_2}{(5)(6)} = \frac{(-16-2)\left(c_0 \frac{6}{(3)(4)}\right) - 4(-c_0)}{(5)(6)} = c_0 \frac{-5}{(5)(6)}$$

For $n = 5$

$$c_7 = \frac{(-20-2)c_5 - 4c_3}{(6)(7)} = \frac{(-20-2)\left(c_1 \frac{10}{(4)(5)}\right) - 4(-c_1)}{(6)(7)} = c_1 \frac{-7}{(6)(7)}$$

For $n = 6$

$$c_8 = \frac{(-24-2)c_6 - 4c_4}{(7)(8)} = \frac{(-24-2)\left(c_0 \frac{-5}{(5)(6)}\right) - 4\left(c_0 \frac{6}{(3)(4)}\right)}{(7)(8)} = c_0 \frac{7}{3} \frac{1}{(7)(8)}$$

And so on. Hence

$$\begin{aligned} y &= \sum_{n=0}^{\infty} c_n x^n \\ &= c_0 + c_1 x + c_2 x^2 + \dots \\ &= c_0 + c_1 x - c_0 x^2 - c_1 x^3 + c_0 \frac{6}{(3)(4)} x^4 + c_1 \frac{10}{(4)(5)} x^5 - c_0 \frac{5}{(5)(7)} x^6 - c_1 \frac{7}{(6)(7)} x^7 + c_0 \frac{12}{7} \frac{1}{(7)(8)} x^8 + \dots \\ &= c_0 \left(1 - x^2 + \frac{6}{(3)(4)} x^4 - \frac{5}{(5)(6)} x^6 + \frac{7}{3} \frac{1}{(7)(8)} x^8 + \dots \right) + c_1 \left(x - x^3 + \frac{10}{(4)(5)} x^5 - \frac{7}{(6)(7)} x^7 + \dots \right) \\ &= c_0 \left(1 - x^2 + \frac{1}{2} x^4 - \frac{1}{6} x^6 + \frac{1}{24} x^8 + \dots \right) + c_1 \left(x - x^3 + \frac{1}{2} x^5 - \frac{1}{6} x^7 + \dots \right) \end{aligned}$$

But Taylor series for $e^{-x^2} = 1 - x^2 + \frac{1}{2} x^4 - \frac{x^6}{4} + \dots$, therefore the above becomes

$$y = c_0 e^{-x^2} + c_1 x e^{-x^2}$$

Verification

```
restart;
restart;
ode:=diff(diff(y(x),x),x)+4*x*diff(y(x),x)+(4*x^2+2)*y(x)=0;
y0:=-C0*exp(-x^2)+_C1*x*exp(-x^2);
odetest(y(x)=y0,ode);
0
```

2.1048 ODE No. 1048

$$(2n + 3x^2 - 1)y(x) + y''(x) - 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0095042 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{x^2}{2}} H_n(x) + c_2 e^{\frac{x^2}{2}} {}_1F_1\left(-\frac{n}{2}; \frac{1}{2}; x^2\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.129 (sec), leaf count = 37

$$\left\{ y(x) = \left(c_1 \text{KummerM}\left(-\frac{n}{2} + \frac{1}{2}, \frac{3}{2}, x^2\right) + c_2 \text{KummerU}\left(-\frac{n}{2} + \frac{1}{2}, \frac{3}{2}, x^2\right) \right) x e^{\frac{x^2}{2}} \right\}$$

2.1049 ODE No. 1049

$$(4x^2 - 1)y(x) + y''(x) - 4xy'(x) - e^x = 0$$

✓ **Mathematica** : cpu = 0.0597842 (sec), leaf count = 109

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} \sqrt{\pi} e^{x(x-i)-\frac{i}{2}} \left(e^{2ix} \operatorname{erfi} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - ix \right) - ie^i \operatorname{erf} \left(-x + \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right) \right\} + c_1 e^{x(x-i)} - \frac{1}{2} ic_2 e^{(x-i)x+2ix} \right\}$$

✓ **Maple** : cpu = 0.182 (sec), leaf count = 66

$$\left\{ y(x) = \frac{\left(4c_1 \sin(x) + 4c_2 \cos(x) + (i \cos(x) + \sin(x)) e^{\frac{i}{2}} \sqrt{\pi} \operatorname{erf} \left(x - \frac{1}{2} - \frac{i}{2} \right) - (i \cos(x) - \sin(x)) e^{-\frac{i}{2}} \sqrt{\pi} \operatorname{erf} \left(x - \frac{1}{2} + \frac{i}{2} \right) \right)}{4} \right\}$$

2.1050 ODE No. 1050

$$(4x^2 - 2)y(x) + y''(x) - 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0113013 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{x^2} + c_2 e^{x^2} x \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 14

$$\{y(x) = (c_2 x + c_1) e^{x^2}\}$$

2.1051 ODE No. 1051

$$(4x^2 - 3)y(x) - e^{x^2} + y''(x) - 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0231736 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow -e^{(x-1)x+x} + c_1 e^{(x-1)x} + \frac{1}{2} c_2 e^{(x-1)x+2x} \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 27

$$\{y(x) = c_1 e^{(x-1)x} + c_2 e^{(x+1)x} - e^{x^2}\}$$

2.1052 ODE No. 1052

$$axy'(x) + by(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0162794 (sec), leaf count = 78

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\frac{ax^2}{2}} H_{\frac{b-a}{a}} \left(\frac{\sqrt{a} x}{\sqrt{2}} \right) + c_2 e^{-\frac{ax^2}{2}} {}_1F_1 \left(-\frac{b-a}{2a}; \frac{1}{2}; \frac{ax^2}{2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.164 (sec), leaf count = 58

$$\left\{ y(x) = \left(c_1 \operatorname{KummerM} \left(\frac{2a-b}{2a}, \frac{3}{2}, \frac{a x^2}{2} \right) + c_2 \operatorname{KummerU} \left(\frac{2a-b}{2a}, \frac{3}{2}, \frac{a x^2}{2} \right) \right) x e^{-\frac{a x^2}{2}} \right\}$$

2.1053 ODE No. 1053

$$a^2x^2y(x) + 2axy'(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0255472 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\frac{ax^2}{2} - \sqrt{a}x} + \frac{c_2 e^{\sqrt{a}x - \frac{ax^2}{2}}}{2\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 35

$$\left\{ y(x) = c_1 e^{-\frac{(ax-2\sqrt{a})x}{2}} + c_2 e^{-\frac{(ax+2\sqrt{a})x}{2}} \right\}$$

2.1054 ODE No. 1054

$$(ax + b)y'(x) + y(x)(cx + d) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0452633 (sec), leaf count = 172

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{cx}{a} - \frac{ax^2}{2} - bx} H_{\frac{-a^3+da^2-bca+c^2}{a^3}} \left(\frac{ab-2c}{\sqrt{2}a^{3/2}} + \frac{\sqrt{a}x}{\sqrt{2}} \right) + c_2 e^{\frac{cx}{a} - \frac{ax^2}{2} - bx} {}_1F_1 \left(-\frac{-a^3+da^2-bca+c^2}{2a^3}; \frac{1}{2}; \left(\frac{ab-2c}{\sqrt{2}a^{3/2}} + \frac{\sqrt{a}x}{\sqrt{2}} \right)^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.122 (sec), leaf count = 98

$$\left\{ y(x) = \left(c_1 \text{KummerM} \left(\frac{da^2-abc+c^2}{2a^3}, \frac{1}{2}, -\frac{(a^2x+ab-2c)^2}{2a^3} \right) + c_2 \text{KummerU} \left(\frac{da^2-abc+c^2}{2a^3}, \frac{1}{2}, -\frac{(a^2x+ab-2c)^2}{2a^3} \right) \right) \right\}$$

2.1055 ODE No. 1055

$$(ax + b)y'(x) + y(x)(a_1x^2 + b_1x + c_1) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.140322 (sec), leaf count = 421

$$\left\{ \left\{ y(x) \rightarrow c_1 \exp \left(\frac{-bx\sqrt{a^2-4a_1} - \frac{1}{2}ax^2\sqrt{a^2-4a_1} - \frac{1}{2}a^2x^2 - abx + 2a_1x^2 + 2b_1x}{2\sqrt{a^2-4a_1}} \right) H_{\frac{-a^3+2c_1a^2-\sqrt{a^2-4a_1}a^2+4a_1a-2bb_1a+2a_1b^2+2b_1^2}{2(a^2-4a_1)^{3/2}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.289 (sec), leaf count = 262

$$\left\{ y(x) = \left(c_1 \text{hypergeom} \left(\left[\frac{a^3-2a^2c_1-2b_1^2+(2b_1b-4a_1)a+(-2b^2+8c_1)a_1+(a^2-4a_1)^{\frac{3}{2}}}{4(a^2-4a_1)^{\frac{3}{2}}} \right], \left[\frac{1}{2} \right], \frac{(a^2x+ab-4a_1x-2b_1x)^2}{2(a^2-4a_1)^{\frac{3}{2}}} \right) \right) \right\}$$

2.1056 ODE No. 1056

$$x^2(-y'(x)) + y''(x) + xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0516783 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow c_1 x - \frac{c_2 \sqrt[3]{-x^3} \Gamma \left(-\frac{1}{3}, -\frac{x^3}{3} \right)}{3\sqrt[3]{3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.101 (sec), leaf count = 52

$$\left\{ y(x) = c_1 x + c_2 \left(-6 \sqrt[3]{3} (-x^3)^{\frac{1}{3}} \Gamma \left(\frac{2}{3}, -\frac{x^3}{3} \right) + 18 e^{\frac{x^3}{3}} + 6 \sqrt[3]{3} (-x^3)^{\frac{1}{3}} \Gamma \left(\frac{2}{3} \right) \right) \right\}$$

2.1057 ODE No. 1057

$$x^2(-y'(x)) + y''(x) - (x+1)^2 y(x) = 0$$

✓ **Mathematica** : cpu = 0.0559956 (sec), leaf count = 56

$$\left\{ \left\{ y(x) \rightarrow c_2 e^{\frac{x^3}{3} + x} \int_1^x e^{-\frac{1}{3}K[1]^3 - 2K[1]} dK[1] + c_1 e^{\frac{x^3}{3} + x} \right\} \right\}$$

✓ **Maple** : cpu = 0.264 (sec), leaf count = 50

$$\left\{ y(x) = c_1 HT \left(0, -3, 2 \cdot 3^{\frac{1}{3}}, \frac{3^{\frac{2}{3}} x}{3} \right) e^{-x} + c_2 HT \left(0, 3, 2 \cdot 3^{\frac{1}{3}}, -\frac{3^{\frac{2}{3}} x}{3} \right) e^{\frac{(x^2+3)x}{3}} \right\}$$

2.1058 ODE No. 1058

$$(x^4 - 2)xy(x) - ((x+1)x^2 y'(x)) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0641929 (sec), leaf count = 56

$$\left\{ \left\{ y(x) \rightarrow c_2 e^{\frac{x^3}{3}} \int_1^x e^{\frac{K[1]^4}{4} - \frac{K[1]^3}{3}} dK[1] + c_1 e^{\frac{x^3}{3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.261 (sec), leaf count = 29

$$\left\{ y(x) = \left(c_2 \left(\int e^{\frac{1}{4}x^4 - \frac{1}{3}x^3} dx \right) + c_1 \right) e^{\frac{x^3}{3}} \right\}$$

2.1059 ODE No. 1059

$$x^4 y'(x) - x^3 y(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0877516 (sec), leaf count = 39

$$\left\{ \left(y(x) \rightarrow c_1 x - \frac{c_2 \sqrt[5]{x^5} \Gamma\left(-\frac{1}{5}, \frac{x^5}{5}\right)}{5 \sqrt[5]{5}} \right) \right\}$$

✓ **Maple** : cpu = 0.085 (sec), leaf count = 56

$$\left\{ y(x) = \frac{\left(c_2 x^2 \text{WhittakerM}\left(\frac{2}{5}, \frac{9}{10}, \frac{x^5}{5}\right) e^{-\frac{x^5}{10}} + c_1 \right) x^8 + 9c_2 (x^5 + 4) \text{WhittakerM}\left(\frac{7}{5}, \frac{9}{10}, \frac{x^5}{5}\right) e^{-\frac{x^5}{10}}}{x^7} \right\}$$

2.1060 ODE No. 1060

$$ax^{q-1}y'(x) + bx^{q-2}y(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0306881 (sec), leaf count = 83

$$\left\{ \left\{ y(x) \rightarrow c_2 q^{-1/q} a^{\frac{1}{q}} (x^q)^{\frac{1}{q}} {}_1F_1\left(\frac{b}{aq} + \frac{1}{q}; 1 + \frac{1}{q}; -\frac{ax^q}{q}\right) + c_1 {}_1F_1\left(\frac{b}{aq}; 1 - \frac{1}{q}; -\frac{ax^q}{q}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.293 (sec), leaf count = 81

$$\left\{ y(x) = \left(c_1 \text{KummerM}\left(\frac{aq-b}{aq}, \frac{q+1}{q}, \frac{ax^q}{q}\right) + c_2 \text{KummerU}\left(\frac{aq-b}{aq}, \frac{q+1}{q}, \frac{ax^q}{q}\right) \right) x e^{-\frac{ax^q}{q}} \right\}$$

2.1061 ODE No. 1061

$$-e^{-\frac{x^{3/2}}{3}}x + y''(x) + \sqrt{x}y'(x) + \left(\frac{x}{4} + \frac{1}{4\sqrt{x}} - 9\right)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0672842 (sec), leaf count = 70

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{9}e^{3x-\frac{1}{3}(\sqrt{x}+9)x}x + c_1e^{-\frac{1}{3}(\sqrt{x}+9)x} + \frac{1}{6}c_2e^{6x-\frac{1}{3}(\sqrt{x}+9)x} \right\} \right\}$$

✓ **Maple** : cpu = 0.104 (sec), leaf count = 28

$$\left\{ y(x) = -\frac{(-9c_1 \cosh(3x) - 9c_2 \sinh(3x) + x)e^{-\frac{x^{\frac{3}{2}}}{3}}}{9} \right\}$$

2.1062 ODE No. 1062

$$\frac{(x + \sqrt{x} - 8)y(x)}{4x^2} + y''(x) - \frac{y'(x)}{\sqrt{x}} = 0$$

✓ **Mathematica** : cpu = 0.0211938 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{3}c_2e^{\sqrt{x}}x^2 + \frac{c_1e^{\sqrt{x}}}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 19

$$\left\{ y(x) = \frac{(c_2x^3 + c_1)e^{\sqrt{x}}}{x} \right\}$$

2.1063 ODE No. 1063

$$y''(x) - (2e^x + 1)y'(x) + e^{2x}y(x) - e^{3x} = 0$$

✓ **Mathematica** : cpu = 0.0487571 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow e^x + c_1e^{e^x} + c_2e^{x+e^x} + 2 \right\} \right\}$$

✓ **Maple** : cpu = 0.31 (sec), leaf count = 34

$$\left\{ y(x) = c_1 \cosh\left(\frac{x}{2}\right)e^{\frac{x}{2}+e^x} + c_2e^{\frac{x}{2}+e^x} \sinh\left(\frac{x}{2}\right) + e^x + 2 \right\}$$

2.1064 ODE No. 1064

$$ay'(x) + by(x) + y''(x) + \tan(x) = 0$$

✓ **Mathematica** : cpu = 0.519272 (sec), leaf count = 1400

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{1}{2}(-a-\sqrt{a^2-4b})x}c_1 + e^{\frac{1}{2}(\sqrt{a^2-4b}-a)x}c_2 + \frac{8\left(2{}_2F_1\left(1, \frac{1}{4}i\left(\sqrt{a^2-4b}-a\right); \frac{1}{4}i\left(\sqrt{a^2-4b}-a\right) + 1; -e^{2ix}\right)a^2 - 2{}_2F_1\left(1, -\frac{1}{4}i\left(\sqrt{a^2-4b}-a\right); \frac{1}{4}i\left(\sqrt{a^2-4b}-a\right) + 1; -e^{2ix}\right)a^2\right)}{2\sqrt{a^2-4b}} \right\} \right\}$$

✓ **Maple** : cpu = 0.445 (sec), leaf count = 134

$$\left\{ y(x) = c_1e^{-\frac{(a+\sqrt{a^2-4b})x}{2}} + c_2e^{-\frac{(a-\sqrt{a^2-4b})x}{2}} + \frac{\left(-\int e^{-\frac{(-a+\sqrt{a^2-4b})x}{2}}\tan(x)dx\right)e^{\frac{(a+\sqrt{a^2-4b})x}{2}} + \left(\int e^{\frac{(a+\sqrt{a^2-4b})x}{2}}\tan(x)dx\right)e^{-\frac{(-a+\sqrt{a^2-4b})x}{2}}}{\sqrt{a^2-4b}} \right\}$$

2.1065 ODE No. 1065

$$(n^2 - a^2)y(x) + 2n \cot(x)y'(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.158271 (sec), leaf count = 114

$$\left\{ \left\{ y(x) \rightarrow c_1 (\cos^2(x) - 1)^{\frac{1}{4}(1-2n)} P_{\frac{1}{2}(2n-1)}^{\frac{1}{2}(2n-1)}(\cos(x)) + c_2 (\cos^2(x) - 1)^{\frac{1}{4}(1-2n)} Q_{\frac{1}{2}(2n-1)}^{\frac{1}{2}(2n-1)}(\cos(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.271 (sec), leaf count = 60

$$\left\{ y(x) = \left(c_1 \text{LegendreP}\left(-\frac{1}{2} + \sqrt{-a^2 + 2n^2}, n - \frac{1}{2}, \cos(x)\right) + c_2 \text{LegendreQ}\left(-\frac{1}{2} + \sqrt{-a^2 + 2n^2}, n - \frac{1}{2}, \cos(x)\right) \right) \left(\sin^{-n} \right) \right\}$$

2.1066 ODE No. 1066

$$y''(x) + \tan(x)y'(x) + y(x) \cos^2(x) = 0$$

✓ **Mathematica** : cpu = 0.0258846 (sec), leaf count = 18

$$\{ \{ y(x) \rightarrow c_2 \sin(\sin(x)) + c_1 \cos(\sin(x)) \} \}$$

✓ **Maple** : cpu = 0.151 (sec), leaf count = 15

$$\{ y(x) = c_1 \sin(\sin(x)) + c_2 \cos(\sin(x)) \}$$

2.1067 ODE No. 1067

$$y''(x) + \tan(x)y'(x) - y(x) \cos^2(x) = 0$$

✓ **Mathematica** : cpu = 0.0277072 (sec), leaf count = 21

$$\{ \{ y(x) \rightarrow c_1 \cosh(\sin(x)) + ic_2 \sinh(\sin(x)) \} \}$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 17

$$\{ y(x) = c_1 e^{\sin(x)} + c_2 e^{-\sin(x)} \}$$

2.1068 ODE No. 1068

$$v(v+1)y(x) + y''(x) + \cot(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.129742 (sec), leaf count = 20

$$\{ \{ y(x) \rightarrow c_1 P_v(\cos(x)) + c_2 Q_v(\cos(x)) \} \}$$

✓ **Maple** : cpu = 0.288 (sec), leaf count = 45

$$\left\{ y(x) = c_2 \text{hypergeom}\left(\left[\frac{v}{2} + 1, -\frac{v}{2} + \frac{1}{2}\right], \left[\frac{3}{2}\right], \cos^2(x)\right) \cos(x) + c_1 \text{hypergeom}\left(\left[-\frac{v}{2}, \frac{v}{2} + \frac{1}{2}\right], \left[\frac{1}{2}\right], \cos^2(x)\right) \right\}$$

2.1069 ODE No. 1069

$$y''(x) - \cot(x)y'(x) + y(x) \sin^2(x) = 0$$

✓ **Mathematica** : cpu = 0.0306345 (sec), leaf count = 19

$$\{ \{ y(x) \rightarrow c_1 \cos(\cos(x)) - c_2 \sin(\cos(x)) \} \}$$

✓ **Maple** : cpu = 0.151 (sec), leaf count = 15

$$\{ y(x) = c_1 \sin(\cos(x)) + c_2 \cos(\cos(x)) \}$$

2.1070 ODE No. 1070

$$a \tan(x)y'(x) + by(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.285034 (sec), leaf count = 143

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1\left(-\frac{a}{4} - \frac{1}{4}\sqrt{a^2 + 4b}, \frac{1}{4}\sqrt{a^2 + 4b} - \frac{a}{4}; \frac{1}{2} - \frac{a}{2}; \cos^2(x)\right) + i^{a+1}c_2 \cos^{a+1}(x) {}_2F_1\left(\frac{a}{4} - \frac{1}{4}\sqrt{a^2 + 4b} + \frac{1}{2}, \frac{a}{4} + \frac{1}{4}\sqrt{a^2 + 4b} - \frac{a}{4}; \cos^2(x)\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.231 (sec), leaf count = 60

$$\left\{ y(x) = \left(c_1 \operatorname{LegendreP}\left(\frac{\sqrt{a^2 + 4b}}{2} - \frac{1}{2}, \frac{a}{2} + \frac{1}{2}, \sin(x)\right) + c_2 \operatorname{LegendreQ}\left(\frac{\sqrt{a^2 + 4b}}{2} - \frac{1}{2}, \frac{a}{2} + \frac{1}{2}, \sin(x)\right) \right) \left(\cos^{\frac{a}{2} + \frac{1}{2}}(x) \right) \right\}$$

2.1071 ODE No. 1071

$$(b^2 - a^2)y(x) + 2a \cot(ax)y'(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.074339 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-ibx} \csc(ax) - \frac{ic_2 e^{ibx} \csc(ax)}{2b} \right\} \right\}$$

✓ **Maple** : cpu = 0.077 (sec), leaf count = 24

$$\left\{ y(x) = \frac{c_1 \sin(bx) + c_2 \cos(bx)}{\sin(ax)} \right\}$$

2.1072 ODE No. 1072

$$y(x)(-4anp(x)^2 + a + bp(x)) + ap''(x)y'(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.346867 (sec), leaf count = 0 , could not solve

`DSolve[(a + b*p[x] - 4*a*n*p[x]^2)*y[x] + a*Derivative[1][y][x]*Derivative[2][p][x] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x) = DESol\left(\left\{a\left(\frac{d}{dx}Y(x)\right)\left(\frac{d^2}{dx^2}p(x)\right) + (-4anp(x)^2 + bp(x) + a)Y(x) + \frac{d^2}{dx^2}Y(x)\right\}, \{Y(x)\}\right)\}$$

2.1073 ODE No. 1073

$$\frac{y'(x)(-\wp(x; a, b)\wp'(x; a, b) + \wp(x; a, b)^3 - 6\wp(x; a, b)^2 + \frac{a}{2})}{\wp'(x; a, b) - \wp(x; a, b)^2} + \frac{y(x)(\wp(x; a, b)^2(-\wp'(x; a, b)) - (6\wp(x; a, b)^2 - \frac{a}{2})\wp(x; a, b) + \wp(x; a, b)^3)}{\wp(x; a, b)^2 + \wp'(x; a, b)}$$

✗ **Mathematica** : cpu = 1.41828 (sec), leaf count = 0 , could not solve

`DSolve[(((-(WeierstrassP[x, {a, b}]*(-1/2*a + 6*WeierstrassP[x, {a, b}]^2)) - WeierstrassP[x, {a, b}]^3 + 6*WeierstrassP[x, {a, b}]^2 + WeierstrassPPrime[x, {a, b}])) + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol\left(\left\{\frac{(-WeierstrassP(x, a, b)^2 \mathcal{P}'(x; a, b) + \mathcal{P}'(x; a, b)^2 - (6WeierstrassP(x, a, b)^2 - \frac{a}{2})WeierstrassP(x, a, b) + \wp(x; a, b)^3)}{WeierstrassP(x, a, b)^2 + \mathcal{P}'(x; a, b)}\right\}, \{\mathcal{P}(x; a, b)\}\right)\right\}$$

2.1074 ODE No. 1074

$$\frac{k^2 \operatorname{cn}(x|k) \operatorname{sn}(x|k) y'(x)}{\operatorname{dn}(x|k)} + n^2 y(x) \operatorname{dn}(x|k)^2 + y''(x) = 0$$

✗ **Mathematica** : cpu = 5.06514 (sec), leaf count = 0 , could not solve

`DSolve[n^2*JacobiDN[x, k]^2*y[x] + (k^2*JacobiCN[x, k]*JacobiSN[x, k]*Derivative[1][y][x])/J`

✓ **Maple** : cpu = 0.037 (sec), leaf count = 21

$$\{y(x) = c_1 \sin(n \operatorname{am}(x|k)) + c_2 \cos(n \operatorname{am}(x|k))\}$$

2.1075 ODE No. 1075

$$f(x)y'(x) + g(x)y(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.241473 (sec), leaf count = 0 , could not solve

`DSolve[g[x]*y[x] + f[x]*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x) = \operatorname{DESol}\left(\left\{-Y(x)g(x) + \left(\frac{d}{dx}Y(x)\right)f(x) + \frac{d^2}{dx^2}Y(x)\right\}, \{-Y(x)\}\right)\}$$

2.1076 ODE No. 1076

$$y(x)\left(a + f'(x)\right) + f(x)y'(x) - g(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.230854 (sec), leaf count = 0 , could not solve

`DSolve[-g[x] + y[x]*(a + Derivative[1][f][x]) + f[x]*Derivative[1][y][x] + Derivative[2][y][x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x) = \operatorname{DESol}\left(\left\{\left(\frac{d}{dx}Y(x)\right)f(x) + \left(a + \frac{d}{dx}f(x)\right)Y(x) + \frac{d^2}{dx^2}Y(x) - g(x)\right\}, \{-Y(x)\}\right)\}$$

2.1077 ODE No. 1077

$$y'(x)(af(x) + b) + y(x)(cf(x) + d) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.427753 (sec), leaf count = 0 , could not solve

`DSolve[(d + c*f[x])*y[x] + (b + a*f[x])*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x) = \operatorname{DESol}\left(\left\{(cf(x) + d)Y(x) + (af(x) + b)\left(\frac{d}{dx}Y(x)\right) + \frac{d^2}{dx^2}Y(x)\right\}, \{-Y(x)\}\right)\}$$

2.1078 ODE No. 1078

$$y(x)\left(a + \frac{f'(x)}{2} + \frac{f(x)^2}{4}\right) + f(x)y'(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0432429 (sec), leaf count = 76

$$\left\{\left\{y(x) \rightarrow c_1 \exp\left(-\frac{1}{2} \int_1^x f(K[1])dK[1] - i\sqrt{a}x\right) - \frac{ic_2 \exp\left(-\frac{1}{2} \int_1^x f(K[1])dK[1] + i\sqrt{a}x\right)}{2\sqrt{a}}\right\}\right\}$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 33

$$\left\{y(x) = \left(c_1 \sinh\left(\sqrt{-a}x\right) + c_2 \cosh\left(\sqrt{-a}x\right)\right)e^{-\frac{\left(\int f(x)dx\right)}{2}}\right\}$$

2.1079 ODE No. 1079

$$by(x)f(x)^{2a}-\frac{af'(x)y'(x)}{f(x)}+y''(x)=0$$

✓ **Mathematica** : cpu = 0.229097 (sec), leaf count = 315

$$\left\{\left\{y(x)\rightarrow-\frac{\sqrt{c_1}\exp\left(-\int_1^x-i\sqrt{b}f(K[1])^adK[1]-c_2\right)\left(-1+\exp\left(2\int_1^x-i\sqrt{b}f(K[1])^adK[1]+2c_2\right)\right)}{\sqrt{2}}\right\},\left\{y(x)\rightarrow\frac{\sqrt{c_1}\exp\left(-\int_1^x-i\sqrt{b}f(K[1])^adK[1]-c_2\right)}{\sqrt{2}}\right\}\right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 37

$$\left\{y(x)=c_1e^{\int i\sqrt{b}f(x)^adx}+c_2e^{-\left(\int i\sqrt{b}f(x)^adx\right)}\right\}$$

2.1080 ODE No. 1080

$$y(x)\left(a^2+\frac{af'(x)}{f(x)}-b^2f(x)^2\right)-y'(x)\left(2a+\frac{f'(x)}{f(x)}\right)+y''(x)=0$$

✓ **Mathematica** : cpu = 0.0665177 (sec), leaf count = 49

$$\left\{\left\{y(x)\rightarrow c_1\exp\left(b\int_1^xf(K[1])dK[1]+ax\right)+c_2\exp\left(ax-b\int_1^xf(K[2])dK[2]\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.577 (sec), leaf count = 74

$$\left\{y(x)=c_2e^{\int-\frac{be^{2c_1b}\int-2bf(x)dx}{e^{2c_1b}\int-2bf(x)dx-1}f(x)-ae^{2c_1b}\int-2bf(x)dx+bf(x)+a}dx}\right\}$$

2.1081 ODE No. 1081

$$-\frac{a^2y(x)f'(x)^2}{b^2+f(x)^2}+\frac{f(x)f^3(x)y'(x)}{b^2+f(x)^2}+y''(x)=0$$

✗ **Mathematica** : cpu = 0.748464 (sec), leaf count = 0 , could not solve

`DSolve[-((a^2*y[x]*Derivative[1][f][x]^2)/(b^2+f[x]^2))+ (f[x]*(f^3)[x]*Derivative[1][y][x])/(b^2+f[x]^2),y[x],x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x)=DESol\left[\left\{-\frac{a^2Y(x)\left(\frac{d}{dx}f(x)\right)^2}{b^2+f(x)^2}+\frac{\left(\frac{d}{dx}Y(x)\right)\left(\frac{d^3}{dx^3}f(x)\right)f(x)}{b^2+f(x)^2}+\frac{d^2}{dx^2}Y(x)\right\},\{Y(x)\}\right]\right\}$$

2.1082 ODE No. 1082

$$y(x)\left(\frac{(m^2-v^2)g'(x)^2}{g(x)}+g'(x)^2\right)-y'(x)\left(\frac{(2m-1)g'(x)}{g(x)}+\frac{g''(x)}{g'(x)}\right)+y''(x)=0$$

✗ **Mathematica** : cpu = 0.614736 (sec), leaf count = 0 , could not solve

`DSolve[y[x]*(Derivative[1][g][x]^2+((m^2-v^2)*Derivative[1][g][x]^2)/g[x])-Derivative[1][y[x]]*(Derivative[1][g][x])/g[x]+Derivative[2][g][x]/Derivative[1][g][x]]+Derivative[2][y[x]]==0,y[x],x]`

✓ **Maple** : cpu = 0.201 (sec), leaf count = 74

$$\left\{y(x)=\left(c_1\operatorname{KummerM}\left(\frac{1}{2}im^2-\frac{1}{2}iv^2+m+\frac{1}{2},2m+1,2ig(x)\right)+c_2\operatorname{KummerU}\left(\frac{1}{2}im^2-\frac{1}{2}iv^2+m+\frac{1}{2},2m+1,2ig(x)\right)\right)\right\}$$

2.1083 ODE No. 1083

$$-\frac{f'(x)y'(x)}{f(x)} + y(x) \left(-\frac{f''(x)}{2f(x)} + \frac{3f'(x)^2}{4f(x)^2} + \frac{\left(\frac{1}{4} - v^2\right)g'(x)^2}{g(x)^2} + g'(x)^2 + \frac{g^3(x)}{2g'(x)} - \frac{3g''(x)^2}{4g'(x)^2} \right) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.752187 (sec), leaf count = 0 , could not solve

`DSolve[-((Derivative[1][f][x]*Derivative[1][y][x])/f[x]) + y[x]*((3*Derivative[1][f][x]^2)/(f[x]^3) + (1/4 - v^2)*(Derivative[1][y][x]^2)/(Derivative[1][f][x]^2) + Derivative[1][y][x]^2 + (Derivative[1][f][x]^3)/(Derivative[1][f][x]^2) - (3*Derivative[1][f][x]^2*Derivative[1][y][x])/Derivative[1][f][x]^3) + y''[x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.122 (sec), leaf count = 31

$$\left\{ y(x) = \sqrt{\frac{f(x)g(x)}{\frac{d}{dx}g(x)}} \left(c_1 \text{BesselJ}\left(v, g(x)\right) + c_2 \text{BesselY}\left(v, g(x)\right) \right) \right\}$$

2.1084 ODE No. 1084

$$-y'(x) \left(\frac{2f'(x)}{f(x)} - \frac{g'(x)}{g(x)} + \frac{g''(x)}{g'(x)} \right) + y(x) \left(-\frac{f''(x)}{f(x)} + \frac{f'(x) \left(\frac{2f'(x)}{f(x)} - \frac{g'(x)}{g(x)} + \frac{g''(x)}{g'(x)} \right)}{f(x)} - \frac{v^2 g'(x)^2}{g(x)^2} + g'(x)^2 \right) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.162916 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow c_1 f(x) J_{\sqrt{v^2}}(g(x)) + c_2 f(x) Y_{\sqrt{v^2}}(g(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.105 (sec), leaf count = 20

$$\{y(x) = (c_1 \text{BesselJ}(v, g(x)) + c_2 \text{BesselY}(v, g(x)))f(x)\}$$

2.1085 ODE No. 1085

$$-y'(x) \left(\frac{(2v-1)g'(x)}{g(x)} + \frac{g''(x)}{g'(x)} + \frac{2h'(x)}{h(x)} \right) + y(x) \left(g'(x)^2 + \frac{h'(x) \left(\frac{(2v-1)g'(x)}{g(x)} + \frac{g''(x)}{g'(x)} + \frac{2h'(x)}{h(x)} \right)}{h(x)} - \frac{h''(x)}{h(x)} \right) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.158404 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow c_1 h(x) g(x)^v J_v(g(x)) + c_2 h(x) g(x)^v Y_v(g(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.094 (sec), leaf count = 24

$$\{y(x) = (c_1 \text{BesselJ}(v, g(x)) + c_2 \text{BesselY}(v, g(x)))g(x)^v h(x)\}$$

2.1086 ODE No. 1086

$$4y''(x) + 9xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0054594 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow c_1 \text{Ai}\left(\sqrt[3]{-1} \left(\frac{3}{2}\right)^{2/3} x\right) + c_2 \text{Bi}\left(\sqrt[3]{-1} \left(\frac{3}{2}\right)^{2/3} x\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 29

$$\left\{ y(x) = c_1 \text{AiryAi}\left(-\frac{3^{\frac{2}{3}} 2^{\frac{1}{3}} x}{2}\right) + c_2 \text{AiryBi}\left(-\frac{3^{\frac{2}{3}} 2^{\frac{1}{3}} x}{2}\right) \right\}$$

2.1087 ODE No. 1087

$$(-a - x^2)y(x) + 4y''(x) = 0$$

✓ **Mathematica** : cpu = 0.007803 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow c_1 D_{\frac{1}{4}(-a-2)}(x) + c_2 D_{\frac{a-2}{4}}(ix) \right\} \right\}$$

✓ **Maple** : cpu = 0.131 (sec), leaf count = 33

$$\left\{ y(x) = \frac{c_1 \text{WhittakerM}\left(-\frac{a}{8}, \frac{1}{4}, \frac{x^2}{2}\right) + c_2 \text{WhittakerW}\left(-\frac{a}{8}, \frac{1}{4}, \frac{x^2}{2}\right)}{\sqrt{x}} \right\}$$

2.1088 ODE No. 1088

$$4y''(x) + 4 \tan(x)y'(x) + y(x)(-5 \tan^2(x) - 2) = 0$$

✓ **Mathematica** : cpu = 0.0756578 (sec), leaf count = 180

$$\left\{ \left\{ y(x) \rightarrow \frac{3(-1)^{5/8} c_2 \left(4 \sqrt[4]{-1} 2^{3/4} \sinh^{-1} \left(\frac{1}{2} \sqrt[4]{-\frac{1}{2}} \sqrt[4]{-8 \cos^2(2x) - 16 \cos(2x) - 8} \right) - i \sqrt[4]{-8 \cos^2(2x) - 16 \cos(2x) - 8} \sqrt{8 + \dots} \right)}{8 \sqrt[8]{2} \sqrt[8]{-8 \cos^2(2x) - 16 \cos(2x) - 8}} \right\} \right\}$$

✓ **Maple** : cpu = 0.184 (sec), leaf count = 31

$$\left\{ y(x) = \frac{ic_2 \cos(x) \sin(x) - c_2 \ln(i \cos(x) + \sin(x)) + c_1}{\sqrt{\cos(x)}} \right\}$$

2.1089 ODE No. 1089

$$-y'(x)(ab + c + x) + ay''(x) + y(x)(b(c + x) + d) = 0$$

✓ **Mathematica** : cpu = 0.0441027 (sec), leaf count = 99

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{bx} H_d \left(\frac{x}{\sqrt{2} \sqrt{a}} - \frac{ab - c}{\sqrt{2} \sqrt{a}} \right) + c_2 e^{bx} {}_1F_1 \left(-\frac{d}{2}; \frac{1}{2}; \left(\frac{x}{\sqrt{2} \sqrt{a}} - \frac{ab - c}{\sqrt{2} \sqrt{a}} \right)^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.153 (sec), leaf count = 58

$$\left\{ y(x) = \left(c_1 \text{KummerM} \left(-\frac{d}{2}, \frac{1}{2}, \frac{(ab - c - x)^2}{2a} \right) + c_2 \text{KummerU} \left(-\frac{d}{2}, \frac{1}{2}, \frac{(ab - c - x)^2}{2a} \right) \right) e^{bx} \right\}$$

2.1090 ODE No. 1090

$$a(a^2 - 2be^{-ax})y'(x) + a^2y''(x) + b^2e^{-2ax}y(x) = 0$$

✓ **Mathematica** : cpu = 0.0285192 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\frac{be^{-ax}}{a^2}} - \frac{bc_2 e^{-\frac{be^{-ax}}{a^2} - ax}}{a^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.047 (sec), leaf count = 40

$$\left\{ y(x) = \left(c_1 \sinh\left(\frac{ax}{2}\right) + c_2 \cosh\left(\frac{ax}{2}\right) \right) e^{-\frac{a^3x + 2be^{-ax}}{2a^2}} \right\}$$

2.1091 ODE No. 1091

$$x(y''(x) + y(x)) - \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.0202742 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}(\text{Ci}(2x) \sin(x) - \text{Si}(2x) \cos(x) + \log(x) \sin(x)) + c_1 \cos(x) + c_2 \sin(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 35

$$\left\{ y(x) = c_1 \cos(x) + \frac{\text{Ci}(2x) \sin(x)}{2} - \frac{\text{Si}(2x) \cos(x)}{2} + \frac{(2c_2 + \ln(x)) \sin(x)}{2} \right\}$$

2.1092 ODE No. 1092

$$(a + x)y(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0775898 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow c_2 e^{-ix} x {}_1F_1\left(1 - \frac{1}{4}i(-2(a-2)-4); 2; 2ix\right) + c_1 e^{-ix} x U\left(1 - \frac{1}{4}i(-2(a-2)-4), 2, 2ix\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.055 (sec), leaf count = 29

$$\left\{ y(x) = c_1 \text{WhittakerM}\left(-\frac{ia}{2}, \frac{1}{2}, 2ix\right) + c_2 \text{WhittakerW}\left(-\frac{ia}{2}, \frac{1}{2}, 2ix\right) \right\}$$

2.1093 ODE No. 1093

$$xy''(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0163188 (sec), leaf count = 13

$$\{ \{ y(x) \rightarrow c_1 \log(x) + c_2 \} \}$$

✓ **Maple** : cpu = 0.006 (sec), leaf count = 10

$$\{ y(x) = c_2 \ln(x) + c_1 \}$$

2.1094 ODE No. 1094

$$ay(x) + xy''(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0194233 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow c_1 J_0(2\sqrt{a} \sqrt{x}) + 2c_2 Y_0(2\sqrt{a} \sqrt{x}) \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 29

$$\{ y(x) = c_1 \text{BesselJ}\left(0, 2\sqrt{a} \sqrt{x}\right) + c_2 \text{BesselY}\left(0, 2\sqrt{a} \sqrt{x}\right) \}$$

2.1095 ODE No. 1095

$$lxy(x) + xy''(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0075629 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow c_1 J_0(\sqrt{l} x) + c_2 Y_0(\sqrt{l} x) \right\} \right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 23

$$\{ y(x) = c_1 \text{BesselJ}\left(0, \sqrt{l} x\right) + c_2 \text{BesselY}\left(0, \sqrt{l} x\right) \}$$

2.1096 ODE No. 1096

$$(a+x)y(x)+xy''(x)+y'(x)=0$$

✓ **Mathematica** : cpu = 0.0121195 (sec), leaf count = 61

$$\left\{\left\{y(x)\rightarrow c_1e^{-ix}U\left(\frac{1}{2}i(a-i),1,2ix\right)+c_2e^{-ix}L_{-\frac{1}{2}i(a-i)}(2ix)\right\}\right\}$$

✓ **Maple** : cpu = 0.148 (sec), leaf count = 39

$$\left\{y(x)=\left(c_1\text{KummerM}\left(\frac{ia}{2}+\frac{1}{2},1,2ix\right)+c_2\text{KummerU}\left(\frac{ia}{2}+\frac{1}{2},1,2ix\right)\right)e^{-ix}\right\}$$

2.1097 ODE No. 1097

$$ay(x)+xy''(x)-y'(x)=0$$

✓ **Mathematica** : cpu = 0.0212636 (sec), leaf count = 46

$$\left\{\left\{y(x)\rightarrow 2ac_1xJ_2\left(2\sqrt{a}\sqrt{x}\right)-2ac_2xY_2\left(2\sqrt{a}\sqrt{x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 31

$$\left\{y(x)=\left(c_1\text{BesselJ}\left(2,2\sqrt{a}\sqrt{x}\right)+c_2\text{BesselY}\left(2,2\sqrt{a}\sqrt{x}\right)\right)x\right\}$$

2.1098 ODE No. 1098

$$-ax^3y(x)+xy''(x)-y'(x)=0$$

✓ **Mathematica** : cpu = 0.0085315 (sec), leaf count = 41

$$\left\{\left\{y(x)\rightarrow c_1\cosh\left(\frac{\sqrt{a}x^2}{2}\right)+ic_2\sinh\left(\frac{\sqrt{a}x^2}{2}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 27

$$\left\{y(x)=c_1\sinh\left(\frac{\sqrt{a}x^2}{2}\right)+c_2\cosh\left(\frac{\sqrt{a}x^2}{2}\right)\right\}$$

2.1099 ODE No. 1099

$$x^3\left(e^{x^3}-v^2\right)y(x)+xy''(x)-y'(x)=0$$

✗ **Mathematica** : cpu = 0.990392 (sec), leaf count = 0 , could not solve

`DSolve[(E^x^3 - v^2)*x^3*y[x] - Derivative[1][y][x] + x*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.148 (sec), leaf count = 25

$$\left\{y(x)=c_1\text{BesselJ}\left(v,e^{\frac{x^2}{2}}\right)+c_2\text{BesselY}\left(v,e^{\frac{x^2}{2}}\right)\right\}$$

2.1100 ODE No. 1100

$$xy''(x)+2y'(x)-xy(x)-e^x=0$$

✓ **Mathematica** : cpu = 0.0162022 (sec), leaf count = 44

$$\left\{\left\{y(x)\rightarrow \frac{e^x(2x-1)}{4x}+\frac{c_1e^{-x}}{x}+\frac{c_2e^x}{2x}\right\}\right\}$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 23

$$\left\{y(x)=\frac{c_1\cosh(x)}{x}+\frac{c_2\sinh(x)}{x}+\frac{e^x}{2}\right\}$$

HAND SOLUTION

$$xy'' + 2y' - xy = e^x \quad (1)$$

First method, much shorter, using transformation. Let $y_h = \frac{u(x)}{x}$, hence (now we are solving only the homogeneous part).

$$\begin{aligned} y' &= \frac{u'}{x} - \frac{u}{x^2} \\ y'' &= \frac{u''}{x} - \frac{u'}{x^2} - \frac{u'}{x^2} + 2\frac{u}{x^3} \end{aligned}$$

And (1) becomes

$$\begin{aligned} x\left(\frac{u''}{x} - \frac{u'}{x^2} - \frac{u'}{x^2} + 2\frac{u}{x^3}\right) + 2\left(\frac{u'}{x} - \frac{u}{x^2}\right) - x\left(\frac{u}{x}\right) &= 0 \\ u'' - 2\frac{u'}{x} + 2\frac{u}{x^2} + \frac{2u'}{x} - \frac{2u}{x^2} - u &= 0 \\ u'' - u &= 0 \end{aligned}$$

Hence the roots of the characteristic equation are ± 1 and the solution is

$$u = Ae^x + Be^{-x}$$

Hence

$$y_h = \frac{1}{x}(Ae^x + Be^{-x})$$

The particular solution is found below, and given in the second method. The transformation method is much simpler.

The second method, which is much longer, using series method. This is used if a transformation is not known or can not be found. There is singularity at $x = 0$. We need to check if the singularity is regular or not. Writing in standard form $y'' + p(x)y' + q(x)y = 0$ gives (we are looking at the homogeneous part now only)

$$y'' + \frac{2}{x}y' - y = 0$$

Hence $\lim_{x \rightarrow 0} xp(x) = \lim_{x \rightarrow 0} x \frac{2}{x} = 2$ which is analytic at $x = 0$. And $\lim_{x \rightarrow 0} x^2q(x) = \lim_{x \rightarrow 0} -x^2 = 0$ which is analytic. Hence the singularity is regular (removable). Using Frobenius series, assume that

$$y = \sum_{n=-\infty}^{\infty} c_n x^{n+r}$$

Where $c_n = 0$ for $n < 0$. Hence

$$\begin{aligned} y' &= \sum (n+r)c_n x^{n+r-1} \\ y'' &= \sum (n+r)(n+r-1)c_n x^{n+r-2} \end{aligned}$$

Substituting back in the original ODE gives

$$\sum (n+r)(n+r-1)c_n x^{n+r-1} + \sum 2(n+r)c_n x^{n+r-1} - \sum c_n x^{n+r+1} = 0$$

Adjusting so that all have same power x^{n+r} gives

$$\sum (n+r+1)(n+r)c_{n+1} x^{n+r} + \sum 2(n+r+1)c_{n+1} x^{n+r} - \sum c_{n-1} x^{n+r} = 0$$

Hence

$$\begin{aligned} (n+r+1)(n+r)c_{n+1} + 2(n+r+1)c_{n+1} - c_{n-1} &= 0 \\ (n+r+1)(2+(n+r))c_{n+1} - c_{n-1} &= 0 \end{aligned} \quad (2)$$

We want equation with c_0 in it. Hence let $n = -1$

$$(-1 + r + 1)(2 + (-1 + r))c_0 - c_{-2} = 0$$

But $c_n = 0$ for all $n < 0$ hence

$$(-1 + r + 1)(2 + (-1 + r))c_0 = 0$$

But $c_0 \neq 0$, as this is the basis for this method. Therefore, we obtain the indicial equation for r

$$\begin{aligned} (-1 + r + 1)(2 + (-1 + r)) &= 0 \\ r(r + 1) &= 0 \end{aligned}$$

Hence $r = 0$ or $r = -1$ are the roots. Now for each r we find a solution. Using $r = 0$, we go back the recurrence equation (2)

$$\begin{aligned} (n + 1)(2 + n)c_{n+1} - c_{n-1} &= 0 \\ c_{n+1} &= \frac{c_{n-1}}{(n + 1)(2 + n)} \end{aligned}$$

For $n = 0$

$$c_1 = \frac{c_{-1}}{(n + 1)(2 + n)} = 0$$

For $n = 1$

$$c_2 = \frac{c_0}{(2)(3)}$$

For $n = 2$

$$c_3 = \frac{c_1}{(n + 1)(2 + n)} = 0$$

For $n = 3$

$$c_4 = \frac{c_2}{(4)(5)} = \frac{c_0}{(2)(3)(4)(5)}$$

And so on. Hence, for $r = 0$ we have

$$\begin{aligned} y_{r=0} &= \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x^1 + c_2 x^2 + c_3 x^3 + \dots \\ &= c_0 + \frac{c_0}{6} x^2 + \frac{c_0}{120} x^4 + \dots \\ &= A \left(1 + \frac{1}{6} x^2 + \frac{1}{120} x^4 + \dots \right) \end{aligned} \tag{3}$$

Where A is used as arbitrary constant instead of a_0 . Now we find the solution for $r = -1$. we go back the recurrence equation (2)

$$\begin{aligned} (n - 1 + 1)(2 + (n - 1))c_{n+1} - c_{n-1} &= 0 \\ n(1 + n)c_{n+1} - c_{n-1} &= 0 \\ c_{n+1} &= \frac{c_{n-1}}{n(1 + n)} \end{aligned}$$

For $n = 0$

$$c_1 = \frac{c_{-1}}{n(1 + n)} = 0$$

For $n = 1$

$$c_2 = \frac{c_0}{2}$$

For $n = 2$

$$c_3 = \frac{c_1}{n(1 + n)} = 0$$

For $n = 3$

$$c_4 = \frac{c_2}{(3)(4)} = \frac{c_0}{(2)(3)(4)}$$

For $n = 4$

$$c_5 = \frac{c_3}{n(1 + n)} = 0$$

For $n = 5$

$$c_6 = \frac{c_4}{(5)(6)} = \frac{c_0}{(2)(3)(4)(5)(6)}$$

And so on. Hence solution is

$$\begin{aligned} y_{r=-1} &= \frac{1}{x} \sum_{n=0}^{\infty} c_n x^n = \frac{1}{x} \left(c_0 + \frac{c_0}{2} x^2 + \frac{c_0}{(2)(3)(4)} x^4 + \frac{c_0}{(2)(3)(4)(5)(6)} x^6 + \dots \right) \\ &= \frac{B}{x} \left(1 + \frac{1}{2} x^2 + \frac{1}{24} x^4 + \frac{1}{720} x^6 + \dots \right) \end{aligned}$$

Where B is used as arbitrary constant instead of a_0 . Therefore, the homogeneous solution found is

$$\begin{aligned} y_h &= y_{r=0} + y_{r=-1} \\ &= A \left(1 + \frac{1}{6} x^2 + \frac{1}{120} x^4 + \dots \right) + \frac{B}{x} \left(1 + \frac{1}{2} x^2 + \frac{1}{24} x^4 + \frac{1}{720} x^6 + \dots \right) \end{aligned}$$

But

$$e^x = 1 + x + \frac{1}{2} x^2 + \frac{1}{6} x^3 + \frac{1}{24} x^4 + \frac{1}{120} x^5 + \dots \quad (3)$$

And

$$e^{-x} = 1 - x + \frac{1}{2} x^2 - \frac{1}{6} x^3 + \frac{1}{24} x^4 - \frac{1}{120} x^5 + \dots \quad (4)$$

Hence adding (3)+(4) gives

$$\begin{aligned} e^x + e^{-x} &= 2 + 2 \frac{1}{2} x^2 + 2 \frac{1}{24} x^4 + \dots \\ &= 2 \left(1 + \frac{1}{2} x^2 + \frac{1}{24} x^4 + \frac{1}{720} x^6 + \dots \right) \end{aligned}$$

But $y_{r=-1} = \frac{B}{x} \left(1 + \frac{1}{2} x^2 + \frac{1}{24} x^4 + \frac{1}{720} x^6 + \dots \right)$, therefore comparing the result we found above, we see that we can write $y_{r=-1}$ as

$$y_{r=-1} = \frac{B}{x} \left(\frac{e^x + e^{-x}}{2} \right)$$

Similarly, we obtain $y_{r=0}$ expression

$$\begin{aligned} \frac{1}{x} e^x &= \frac{1}{x} \left(1 + x + \frac{1}{2} x^2 + \frac{1}{6} x^3 + \frac{1}{24} x^4 + \frac{1}{120} x^5 + \dots \right) \\ &= \frac{1}{x} + 1 + \frac{1}{2} x + \frac{1}{6} x^2 + \frac{1}{24} x^3 + \frac{1}{120} x^4 + \dots \end{aligned} \quad (3A)$$

And

$$\begin{aligned} \frac{1}{x} e^{-x} &= \frac{1}{x} \left(1 - x + \frac{1}{2} x^2 - \frac{1}{6} x^3 + \frac{1}{24} x^4 - \frac{1}{120} x^5 + \dots \right) \\ &= \frac{1}{x} - 1 + \frac{1}{2} x - \frac{1}{6} x^2 + \frac{1}{24} x^3 - \frac{1}{120} x^4 + \dots \end{aligned} \quad (4A)$$

Now (3A)-(4A) gives

$$\begin{aligned} \frac{1}{x} e^x - \frac{1}{x} e^{-x} &= \left(\frac{1}{x} + 1 + \frac{1}{2} x + \frac{1}{6} x^2 + \frac{1}{24} x^3 + \frac{1}{120} x^4 + \dots \right) - \left(\frac{1}{x} - 1 + \frac{1}{2} x - \frac{1}{6} x^2 + \frac{1}{24} x^3 - \frac{1}{120} x^4 + \dots \right) \\ &= 2 + 2 \frac{1}{6} x^2 + 2 \frac{1}{120} x^4 + \dots \\ &= 2 \left(1 + \frac{1}{6} x^2 + \frac{1}{120} x^4 + \dots \right) \end{aligned}$$

Hence

$$\left(1 + \frac{1}{6} x^2 + \frac{1}{120} x^4 + \dots \right) = \frac{1}{2x} (e^x - e^{-x})$$

But $y_{r=0} = A \left(1 + \frac{1}{6} x^2 + \frac{1}{120} x^4 + \dots \right)$, therefore comparing the result we found above, we see that we can write $y_{r=0}$ as

$$\begin{aligned} y_{r=0} &= A \left(\frac{1}{2x} (e^x - e^{-x}) \right) \\ &= \frac{A}{2x} (e^x - e^{-x}) \end{aligned}$$

Therefore

$$\begin{aligned}
 y_h &= y_{r=0} + y_{r=-1} \\
 &= \frac{A}{2x}(e^x - e^{-x}) + \frac{B}{2x}(e^x + e^{-x}) \\
 &= \frac{1}{x} \left(\frac{A}{2}e^x - \frac{A}{2}e^{-x} + \frac{B}{2}e^x + \frac{B}{2}e^{-x} \right) \\
 &= \frac{1}{x} \left(e^x \left(\frac{A}{2} + \frac{B}{2} \right) + e^{-x} \left(-\frac{A}{2} + \frac{B}{2} \right) \right)
 \end{aligned}$$

Let $\frac{A+B}{2} = A_0$, $\frac{B-A}{2} = B_0$ hence the above becomes

$$y_h = \frac{1}{x}(A_0e^x + B_0e^{-x})$$

We see this is the same solution using the transformation method given above. We now need to find particular solution. Let $y_1 = \frac{e^x}{x}$, $y_2 = \frac{e^{-x}}{x}$, hence $y'_1 = \frac{e^x}{x} - \frac{e^x}{x^2}$ and $y'_2 = \frac{-e^{-x}}{x^2} - \frac{e^{-x}}{x}$, hence the Wronskian is

$$\begin{aligned}
 W &= \begin{vmatrix} y_1 & y_2 \\ y'_1 & y'_2 \end{vmatrix} \\
 &= \begin{vmatrix} \frac{e^x}{x} & \frac{e^{-x}}{x} \\ \frac{e^x}{x} - \frac{e^x}{x^2} & \frac{-e^{-x}}{x^2} - \frac{e^{-x}}{x} \end{vmatrix} \\
 &= \frac{e^x}{x} \left(\frac{-e^{-x}}{x^2} - \frac{e^{-x}}{x} \right) - \frac{e^{-x}}{x} \left(\frac{e^x}{x} - \frac{e^x}{x^2} \right) \\
 &= \left(\frac{-1}{x^3} - \frac{1}{x^2} \right) - \left(\frac{1}{x^2} - \frac{1}{x^3} \right) \\
 &= -\frac{2}{x^2}
 \end{aligned}$$

Therefore, let $y_p = u_1y_1 + u_2y_2$ and hence

$$u_1 = - \int \frac{y_2}{aW} e^x dx$$

Where $a = x$ since the original ODE is $xy'' + 2y' - xy = e^x$, and a is the coefficient of y'' always. Hence the above becomes

$$u_1 = - \int \frac{\frac{e^{-x}}{x}}{x \left(-\frac{2}{x^2} \right)} e^x dx = \int \frac{e^{-x}}{2} e^x dx = \int \frac{1}{2} dx = \frac{x}{2}$$

And

$$u_2 = \int \frac{\frac{e^x}{x}}{x \left(-\frac{2}{x^2} \right)} e^x dx = - \int \frac{\frac{e^x}{x}}{\frac{2}{x}} e^x dx = -\frac{1}{2} \int e^{2x} dx = -\frac{1}{2} \frac{e^{2x}}{2} = -\frac{1}{4} e^{2x}$$

Hence

$$\begin{aligned}
 y_p &= u_1y_1 + u_2y_2 \\
 &= \frac{x}{2} \frac{e^x}{x} - \frac{1}{4} e^{2x} \frac{e^{-x}}{x} \\
 &= \frac{1}{2} e^x - \frac{1}{4x} e^x
 \end{aligned}$$

Therefore

$$y_p = e^x \left(\frac{1}{2} - \frac{1}{4x} \right)$$

Hence the general solution is

$$\begin{aligned}
 y &= y_h + y_p \\
 &= \frac{1}{x}(A_0e^x + B_0e^{-x}) + e^x \left(\frac{1}{2} - \frac{1}{4x} \right)
 \end{aligned}$$

Verification

```
restart;
ode:=x*diff(diff(y(x),x),x)+2*diff(y(x),x)-x*y(x)=exp(x);
y0:=1/x*( _C1* exp(x)+ _C2*exp(-x))+ (1/2-1/(4*x))*exp(x);
odetest(y(x)=y0,ode);
0
```

2.1101 ODE No. 1101

$$axy(x) + xy''(x) + 2y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0216162 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-i\sqrt{a}x}}{x} - \frac{ic_2 e^{i\sqrt{a}x}}{2\sqrt{a}x} \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 29

$$\left\{ y(x) = \frac{c_1 \sinh(\sqrt{-a}x) + c_2 \cosh(\sqrt{-a}x)}{x} \right\}$$

HAND SOLUTION

$$xy'' + 2y' + axy = 0 \tag{1}$$

First method much shorter, using transformation suggested by Kamke. Let $y = \frac{u(x)}{x}$, hence

$$\begin{aligned} y' &= \frac{u'}{x} - \frac{u}{x^2} \\ y'' &= \frac{u''}{x} - \frac{u'}{x^2} - \frac{u'}{x^2} + 2\frac{u}{x^3} \end{aligned}$$

And (1) becomes

$$\begin{aligned} x\left(\frac{u''}{x} - \frac{u'}{x^2} - \frac{u'}{x^2} + 2\frac{u}{x^3}\right) + 2\left(\frac{u'}{x} - \frac{u}{x^2}\right) + ax\left(\frac{u}{x}\right) &= 0 \\ u'' - 2\frac{u'}{x} + 2\frac{u}{x^2} + \frac{2u'}{x} - \frac{2u}{x^2} + au &= 0 \\ u'' + au &= 0 \end{aligned}$$

Hence the roots of the characteristic equation are $\pm\sqrt{-a}$ and the solution is

$$u = A \cos(\sqrt{a}x) + B \sin(\sqrt{a}x)$$

Hence

$$y = \frac{1}{x}\left(A \cos(\sqrt{a}x) + B \sin(\sqrt{a}x)\right)$$

Second method Using series method.

There is singularity at $x = 0$. We need to check if it regular or not. Writing in standard form $y'' + p(x)y' + q(x)y = 0$ gives (we are looking at the homogeneous part now only)

$$y'' + \frac{2}{x}y' + ay = 0$$

Hence $\lim_{x \rightarrow 0} xp(x) = \lim_{x \rightarrow 0} x \frac{2}{x} = 2$ which is analytic at $x = 0$. And $\lim_{x \rightarrow 0} x^2 q(x) = \lim_{x \rightarrow 0} -ax^2 = 0$ which is analytic. Hence the singularity is regular (removable). Using Frobenius series, assume that

$$y = \sum_{n=-\infty}^{\infty} c_n x^{n+r}$$

Where $c_n = 0$ for $n < 0$. Hence

$$\begin{aligned} y' &= \sum (n+r) c_n x^{n+r-1} \\ y'' &= \sum (n+r)(n+r-1) c_n x^{n+r-2} \end{aligned}$$

Substituting back in the original ODE gives

$$\sum (n+r)(n+r-1) c_n x^{n+r-1} + \sum 2(n+r) c_n x^{n+r-1} + \sum a c_n x^{n+r+1} = 0$$

Adjusting so that all have same power x^{n+r} gives

$$\sum (n+r+1)(n+r) c_{n+1} x^{n+r} + \sum 2(n+r+1) c_{n+1} x^{n+r} + \sum a c_{n-1} x^{n+r} = 0$$

Hence

$$\begin{aligned} (n+r+1)(n+r) c_{n+1} + 2(n+r+1) c_{n+1} + a c_{n-1} &= 0 \\ (n+r+1)(2+(n+r)) c_{n+1} + a c_{n-1} &= 0 \end{aligned} \quad (2)$$

We want equation with c_0 in it. Hence let $n = -1$

$$(-1+r+1)(2+(-1+r)) c_0 + a c_{-2} = 0$$

But $c_n = 0$ for all $n < 0$ hence

$$(-1+r+1)(2+(-1+r)) c_0 = 0$$

But $c_0 \neq 0$, as this is the basis for this method. Therefore, we obtain the indicial equation for r

$$\begin{aligned} (-1+r+1)(2+(-1+r)) &= 0 \\ r(r+1) &= 0 \end{aligned}$$

Hence $r = 0$ or $r = -1$ are the roots. Now for each r we find a solution. Using $r = 0$, we go back the recurrence equation (2)

$$\begin{aligned} (n+1)(2+n) c_{n+1} + a c_{n-1} &= 0 \\ c_{n+1} &= \frac{-a c_{n-1}}{(n+1)(2+n)} \end{aligned}$$

For $n = 0$

$$c_1 = \frac{-a c_{-1}}{(n+1)(2+n)} = 0$$

For $n = 1$

$$c_2 = \frac{-a c_0}{(2)(3)}$$

For $n = 2$

$$c_3 = \frac{-a c_1}{(n+1)(2+n)} = 0$$

For $n = 3$

$$c_4 = \frac{-a c_2}{(4)(5)} = \frac{-a(-a c_0)}{(2)(3)(4)(5)} = \frac{a^2 c_0}{(2)(3)(4)(5)}$$

And so on. Hence, for $r = 0$ we have

$$\begin{aligned} y_{r=0} &= \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x^1 + c_2 x^2 + c_3 x^3 + \dots \\ &= c_0 - a \frac{c_0}{6} x^2 + a^2 \frac{c_0}{120} x^4 + \dots \\ &= A \left(1 - a \frac{1}{6} x^2 + a^2 \frac{1}{120} x^4 + \dots \right) \end{aligned} \quad (3)$$

Where A is used as arbitrary constant instead of a_0 . Now we find the solution for $r = -1$. we go back the recurrence equation (2)

$$\begin{aligned}(n-1+1)(2+(n-1))c_{n+1} + ac_{n-1} &= 0 \\ n(1+n)c_{n+1} + ac_{n-1} &= 0 \\ c_{n+1} &= \frac{-ac_{n-1}}{n(1+n)}\end{aligned}$$

For $n = 0$

$$c_1 = \frac{-ac_{-1}}{n(1+n)} = 0$$

For $n = 1$

$$c_2 = \frac{-ac_0}{2}$$

For $n = 2$

$$c_3 = \frac{-ac_1}{n(1+n)} = 0$$

For $n = 3$

$$c_4 = \frac{-ac_2}{(3)(4)} = \frac{-a(-ac_0)}{(2)(3)(4)} = \frac{a^2c_0}{(2)(3)(4)}$$

For $n = 4$

$$c_5 = \frac{-ac_3}{n(1+n)} = 0$$

For $n = 5$

$$c_6 = \frac{-ac_4}{(5)(6)} = \frac{-a(a^2c_0)}{(2)(3)(4)(5)(6)} = \frac{-a^3c_0}{(2)(3)(4)(5)(6)}$$

And so on. Hence solution is

$$\begin{aligned}y_{r=-1} &= \frac{1}{x} \sum_{n=0}^{\infty} c_n x^n = \frac{1}{x} \left(c_0 - \frac{ac_0}{2} x^2 + \frac{a^2c_0}{(2)(3)(4)} x^4 - \frac{a^3c_0}{(2)(3)(4)(5)(6)} x^6 + \dots \right) \\ &= \frac{B}{x} \left(1 - \frac{a}{2} x^2 + \frac{a^2}{24} x^4 - \frac{a^3}{720} x^6 + \dots \right)\end{aligned}$$

Where B is used as arbitrary constant instead of a_0 . Therefore, the homogeneous solution found is

$$\begin{aligned}y_h &= y_{r=0} + y_{r=-1} \\ &= A \left(1 - a \frac{1}{6} x^2 + a^2 \frac{1}{120} x^4 + \dots \right) + \frac{B}{x} \left(1 - \frac{a}{2} x^2 + \frac{a^2}{24} x^4 - \frac{a^3}{720} x^6 + \dots \right) \\ &= A \left(1 - \frac{1}{6} (\sqrt{a} x)^2 + \frac{1}{120} (\sqrt{a} x)^4 + \dots \right) + \frac{B}{x} \left(1 - \frac{1}{2} (\sqrt{a} x)^2 + \frac{1}{24} (\sqrt{a} x)^4 - \frac{1}{720} (\sqrt{a} x)^6 + \dots \right)\end{aligned}$$

But

$$\sin x = x - \frac{1}{6} x^3 + \frac{1}{120} x^5 - \dots \quad (3)$$

And

$$\cos x = 1 - \frac{1}{2} x^2 + \frac{1}{24} x^4 - \frac{1}{720} x^6 + \dots \quad (4)$$

Therefore

$$\begin{aligned}\sin(\sqrt{a} x) &= \sqrt{a} x - \frac{1}{6} (\sqrt{a} x)^3 + \frac{1}{120} (\sqrt{a} x)^5 - \dots \\ \cos(\sqrt{a} x) &= 1 - \frac{1}{2} (\sqrt{a} x)^2 + \frac{1}{24} (\sqrt{a} x)^4 - \frac{1}{720} (\sqrt{a} x)^6 + \dots\end{aligned}$$

Therefore, using the above, we can write y_h as

$$\begin{aligned}y_h &= \frac{A}{\sqrt{a} x} \sqrt{a} x \left(1 - \frac{1}{6} (\sqrt{a} x)^2 + \frac{1}{120} (\sqrt{a} x)^4 + \dots \right) + \frac{B}{x} \cos(\sqrt{a} x) \\ &= \frac{A}{\sqrt{a} x} \left(\sqrt{a} x - \frac{1}{6} (\sqrt{a} x)^3 + \frac{1}{120} (\sqrt{a} x)^5 + \dots \right) + \frac{B}{x} \cos(\sqrt{a} x) \\ &= \frac{A}{\sqrt{a} x} \sin(\sqrt{a} x) + \frac{B}{x} \cos(\sqrt{a} x)\end{aligned}$$

Let $A_0 = \frac{A}{\sqrt{a}}$, hence the above becomes the same solution found using the transformation method

$$y(x) = \frac{1}{x} \left(A_0 \sin(\sqrt{a} x) + B \cos(\sqrt{a} x) \right)$$

Clearly, the transformation method is much faster and better. But the trick is to see the correct transformation needed and this is not always easy.

Third method Using Laplace transform. Using property $\mathcal{L} x f(x) = -\frac{d}{ds} F(s)$ then the Laplace transform of the ODE becomes

$$\begin{aligned} \mathcal{L}(xy'' + 2y' + axy) &= 0 \\ -\frac{d}{ds}(\mathcal{L}y'') + 2\left(-\frac{d}{ds}(\mathcal{L}y')\right) + a\left(-\frac{d}{ds}Y\right) &= 0 \\ -\frac{d}{ds}(s^2Y - sA - B) + 2\left(-\frac{d}{ds}(sY - A)\right) + a(-Y') &= 0 \end{aligned}$$

Where $A = y(0), B = y'(0)$. Simplifying gives

$$\begin{aligned} -(2sY + s^2Y' - A) - 2(Y + sY') - aY' &= 0 \\ Y'(-s^2 - 2s - a) + Y(-2s - 2) + A &= 0 \\ Y' + Y \frac{2s + 2}{s^2 + 2s + a} &= \frac{A}{s^2 + 2s + a} \end{aligned}$$

This is first order ODE, which is solved easily using an integrating factor. Solving for $Y(s)$ gives

$$Y(s) = \frac{As + c_1}{s^2 + a + 2s}$$

Where c_1 is constant of integration. The inverse Laplace of the above is

$$y(x) = e^{-x} \left(A \cos(x\sqrt{1-a}) + \frac{(-A + c_1)}{\sqrt{1-a}} \sinh(x\sqrt{1-a}) \right)$$

I need to find why the above does not verify. May be I made a mistake somewhere. Verification of the result of the first two methods is below.

```
restart;
ode:=x*diff(diff(y(x),x),x)+2*diff(y(x),x)+a*x*y(x)=0;
y0:=1/x*( _C1* cos(sqrt(a)*x)+ _C2*sin(sqrt(a)*x));
odetest(y(x)=y0,ode);
0
```

2.1102 ODE No. 1102

$$ax^2y(x) + xy''(x) + 2y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0066601 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 \text{Ai}\left(-\frac{ax}{(-a)^{2/3}}\right)}{x} + \frac{c_2 \text{Bi}\left(-\frac{ax}{(-a)^{2/3}}\right)}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.118 (sec), leaf count = 33

$$\left\{ y(x) = \frac{c_1 \text{BesselJ}\left(\frac{1}{3}, \frac{2\sqrt{a} x^{\frac{3}{2}}}{3}\right) + c_2 \text{BesselY}\left(\frac{1}{3}, \frac{2\sqrt{a} x^{\frac{3}{2}}}{3}\right)}{\sqrt{x}} \right\}$$

HAND SOLUTION

$$xy'' + 2y' + ax^2y = 0 \quad (1)$$

Since there is a term $2y$, we can use $y = \frac{u(x)}{x}$, hence

$$y' = \frac{u'}{x} - \frac{u}{x^2}$$

$$y'' = \frac{u''}{x} - \frac{u'}{x^2} - \frac{u'}{x^2} + 2\frac{u}{x^3}$$

And (1) becomes

$$x\left(\frac{u''}{x} - \frac{u'}{x^2} - \frac{u'}{x^2} + 2\frac{u}{x^3}\right) + 2\left(\frac{u'}{x} - \frac{u}{x^2}\right) + ax^2\left(\frac{u}{x}\right) = 0$$

$$u'' - 2\frac{u'}{x} + 2\frac{u}{x^2} + \frac{2u'}{x} - \frac{2u}{x^2} + axu = 0$$

$$u'' + axu = 0 \quad (2)$$

This is Emdon-Fowler. (form is $u'' + x^n u = 0$) with $n = 1$. Assume that

$$u = \sum_{n=0}^{\infty} c_n x^n$$

Hence

$$u' = \sum_{n=0} n c_n x^{n-1} = \sum_{n=1} n c_n x^{n-1} = \sum_{n=0} (n+1) c_{n+1} x^n$$

$$u'' = \sum_{n=0} n(n+1) c_{n+1} x^{n-1} = \sum_{n=1} n(n+1) c_{n+1} x^{n-1} = \sum_{n=0} (n+1)(n+2) c_{n+2} x^n$$

Substituting back in (2) gives

$$\sum_{n=0} (n+1)(n+2) c_{n+2} x^n + \sum_{n=0} a c_n x^{n+1} = 0$$

$$\sum_{n=0} (n+1)(n+2) c_{n+2} x^n + \sum_{n=1} a c_{n-1} x^n = 0$$

For $n = 0$

$$(1)(2)c_2 = 0$$

Hence $c_2 = 0$. For $n \geq 1$

$$(n+1)(n+2)c_{n+2} + a c_{n-1} = 0$$

$$c_{n+2} = \frac{-a c_{n-1}}{(n+1)(n+2)} \quad (3)$$

For $n = 1$, from (3)

$$c_3 = \frac{-a c_0}{(2)(3)}$$

For $n = 2$, from (3)

$$c_4 = \frac{-a c_1}{(3)(4)}$$

For $n = 3$, from (3)

$$c_5 = \frac{-a c_2}{(4)(5)} = 0$$

For $n = 4$, from (3)

$$c_6 = \frac{-a c_3}{(5)(6)} = \frac{-a}{(5)(6)} \left(\frac{-a c_0}{(2)(3)} \right) = \frac{a^2 c_0}{(2)(3)(5)(6)}$$

For $n = 5$, from (3)

$$c_7 = \frac{-a c_4}{(6)(7)} = \frac{-a}{(6)(7)} \left(\frac{-a c_1}{(3)(4)} \right) = \frac{a^2 c_1}{(3)(4)(6)(7)}$$

For $n = 6$, from (3)

$$c_8 = \frac{-a c_5}{(7)(8)} = 0$$

For $n = 7$, from (3)

$$c_9 = \frac{-ac_6}{(8)(9)} = \frac{-a}{(8)(9)} \left(\frac{a^2 c_0}{(2)(3)(5)(6)} \right) = \frac{-a^3 c_0}{(2)(3)(5)(6)(8)(9)}$$

For $n = 8$, from (3)

$$c_{10} = \frac{-ac_7}{(9)(10)} = \frac{-a}{(9)(10)} \left(\frac{a^2 c_1}{(3)(4)(6)(7)} \right) = \frac{-a^3 c_1}{(3)(4)(6)(7)(9)(10)}$$

And so on. Hence,

$$\begin{aligned} u &= \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x^1 + c_2 x^2 + c_3 x^3 + \cdots \\ &= c_0 + c_1 x + 0 + c_3 x^3 + c_4 x^4 + 0 + c_6 x^6 + c_7 x^7 + 0 + c_9 x^9 + \cdots \\ &= c_0 + c_1 x - \frac{ac_0}{(2)(3)} x^3 - \frac{ac_1}{(3)(4)} x^4 + \frac{a^2 c_0}{(2)(3)(5)(6)} x^6 + \frac{a^2 c_1}{(3)(4)(6)(7)} x^7 - \frac{a^3 c_0}{(2)(3)(5)(6)(8)(9)} x^9 - \frac{a^3 c_1}{(3)(4)(6)(7)(9)(10)} x^{10} + \cdots \\ &= c_0 \left(1 - \frac{a}{6} x^3 + \frac{a^2}{180} x^6 - \frac{a^3}{12\,960} x^9 + \cdots \right) + xc_1 \left(1 - \frac{a}{12} x^3 + \frac{a^2}{504} x^6 - \frac{a^3}{45\,360} x^9 + \cdots \right) \\ &= c_0 \left(1 - \frac{1}{6} \left(a^{\frac{1}{3}} x \right)^3 + \frac{1}{180} \left(a^{\frac{1}{3}} x \right)^6 - \frac{1}{12\,960} \left(a^{\frac{1}{3}} x \right)^9 + \cdots \right) + xc_1 \left(1 - \frac{1}{12} \left(a^{\frac{1}{3}} x \right)^3 + \frac{1}{504} \left(a^{\frac{1}{3}} x \right)^6 - \frac{1}{45\,360} \left(a^{\frac{1}{3}} x \right)^9 + \cdots \right) \end{aligned}$$

Comparing the above to the series expansion of Airy functions, we see that

$$u = c_0 \operatorname{AiryAI}\left(-a^{\frac{1}{3}} x\right) + c_1 \operatorname{AiryBI}\left(-a^{\frac{1}{3}} x\right)$$

And since $y = \frac{u(x)}{x}$ then

$$y = \frac{1}{x} \left(c_0 \operatorname{AiryAI}\left(-a^{\frac{1}{3}} x\right) + c_1 \operatorname{AiryBI}\left(-a^{\frac{1}{3}} x\right) \right)$$

Verification

```
restart;
#for series solution of u''+axu=0, use this:
Order:=10;
sol:=dsolve(ode,u(x),series);
sol:=convert(sol,polynomial);
sol:=subs({u(0)=c0,D(u)(0)=c1},rhs(sol));
collect(sol,{c0,c1});
(1-(1/6)*a*x^3+(1/180)*a^2*x^6-(1/12960)*a^3*x^9)*c0+(x-(1/12)*a*x^4+(1/504)*a
^2*x^7)*c1
#to verify final solution use this:
ode:=x*diff(y(x),x$2)+2*diff(y(x),x)+a*x^2*y(x)=0;
y0:=(1/x)*(_C0*AiryAi(-a^(1/3)*x)+_C1*AiryBi(-a^(1/3)*x));
odetest(y(x)=y0,ode);
0
```

2.1103 ODE No. 1103

$$ay(x) + xy''(x) - 2y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0225669 (sec), leaf count = 64

$$\left\{ \left\{ y(x) \rightarrow 6a^{3/2} c_1 x^{3/2} J_3\left(2\sqrt{a} \sqrt{x}\right) - 2ia^{3/2} c_2 x^{3/2} Y_3\left(2\sqrt{a} \sqrt{x}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 33

$$\left\{ y(x) = \left(c_1 \operatorname{BesselJ}\left(3, 2\sqrt{a} \sqrt{x}\right) + c_2 \operatorname{BesselY}\left(3, 2\sqrt{a} \sqrt{x}\right) \right) x^{\frac{3}{2}} \right\}$$

2.1104 ODE No. 1104

$$ay(x) + vy'(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0350844 (sec), leaf count = 104

$$\left\{ \left\{ y(x) \rightarrow c_2 a^{\frac{v-1}{2}-v+1} x^{\frac{v-1}{2}-v+1} \Gamma(2-v) J_{1-v}(2\sqrt{a}\sqrt{x}) + c_1 a^{\frac{1-v}{2}} x^{\frac{1-v}{2}} \Gamma(v) J_{v-1}(2\sqrt{a}\sqrt{x}) \right\} \right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 41

$$\left\{ y(x) = \left(c_1 \operatorname{BesselJ}\left(v-1, 2\sqrt{a}\sqrt{x}\right) + c_2 \operatorname{BesselY}\left(v-1, 2\sqrt{a}\sqrt{x}\right) \right) x^{-\frac{v}{2}+\frac{1}{2}} \right\}$$

2.1105 ODE No. 1105

$$ay'(x) + bxy(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0153551 (sec), leaf count = 64

$$\left\{ \left\{ y(x) \rightarrow c_1 x^{\frac{1-a}{2}} J_{\frac{a-1}{2}}(\sqrt{b}x) + c_2 x^{\frac{1-a}{2}} Y_{\frac{a-1}{2}}(\sqrt{b}x) \right\} \right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 39

$$\left\{ y(x) = \left(c_1 \operatorname{BesselJ}\left(\frac{a}{2} - \frac{1}{2}, \sqrt{b}x\right) + c_2 \operatorname{BesselY}\left(\frac{a}{2} - \frac{1}{2}, \sqrt{b}x\right) \right) x^{-\frac{a}{2}+\frac{1}{2}} \right\}$$

2.1106 ODE No. 1106

$$ay'(x) + bx^{a1}y(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0403912 (sec), leaf count = 441

$$\left\{ \left\{ y(x) \rightarrow c_1 \left(\frac{1}{a1} + 1 \right)^{\frac{a}{\left(\frac{1}{a1}+1\right)a1} - \frac{1}{\left(\frac{1}{a1}+1\right)a1}} a1^{\frac{a}{\left(\frac{1}{a1}+1\right)a1} - \frac{1}{\left(\frac{1}{a1}+1\right)a1}} b^{\frac{1}{2} \left(\frac{1}{\left(\frac{1}{a1}+1\right)a1} - \frac{a}{\left(\frac{1}{a1}+1\right)a1} \right)} \left(x^{a1} \right)^{\frac{1}{2} \left(\frac{1}{a1} + 1 \right) \left(\frac{1}{\left(\frac{1}{a1}+1\right)a1} - \frac{a}{\left(\frac{1}{a1}+1\right)a1} \right)} \Gamma \left(\frac{a}{a1+1} + \frac{a1}{a1-1} \right) \right. \right. \\ \left. \left. + c_2 \left(\frac{1}{a1} + 1 \right)^{\frac{a}{\left(\frac{1}{a1}+1\right)a1} - \frac{1}{\left(\frac{1}{a1}+1\right)a1}} a1^{\frac{a}{\left(\frac{1}{a1}+1\right)a1} - \frac{1}{\left(\frac{1}{a1}+1\right)a1}} b^{\frac{1}{2} \left(\frac{1}{\left(\frac{1}{a1}+1\right)a1} - \frac{a}{\left(\frac{1}{a1}+1\right)a1} \right)} \left(x^{a1} \right)^{\frac{1}{2} \left(\frac{1}{a1} + 1 \right) \left(\frac{1}{\left(\frac{1}{a1}+1\right)a1} - \frac{a}{\left(\frac{1}{a1}+1\right)a1} \right)} \Gamma \left(\frac{a}{a1+1} + \frac{a1}{a1-1} \right) \right. \right. \right. \\ \left. \left. \left. \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.237 (sec), leaf count = 71

$$\left\{ y(x) = \left(c_1 \operatorname{BesselJ}\left(\frac{a-1}{a1+1}, \frac{2\sqrt{b}x^{\frac{a1}{2}+\frac{1}{2}}}{a1+1}\right) + c_2 \operatorname{BesselY}\left(\frac{a-1}{a1+1}, \frac{2\sqrt{b}x^{\frac{a1}{2}+\frac{1}{2}}}{a1+1}\right) \right) x^{-\frac{a}{2}+\frac{1}{2}} \right\}$$

2.1107 ODE No. 1107

$$ay(x) + (b+x)y'(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.023153 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-x} U(b-a, b, x) + c_2 e^{-x} L_{a-b}^{b-1}(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.204 (sec), leaf count = 30

$$\{y(x) = (c_1 \operatorname{KummerM}(-a+b, b, x) + c_2 \operatorname{KummerU}(-a+b, b, x))e^{-x}\}$$

2.1108 ODE No. 1108

$$(a+b+x)y'(x) + ay(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0264551 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-x} U(b, a+b, x) + c_2 e^{-x} L_{-b}^{a+b-1}(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.183 (sec), leaf count = 26

$$\{y(x) = (c_1 \operatorname{KummerM}(b, a+b, x) + c_2 \operatorname{KummerU}(b, a+b, x))e^{-x}\}$$

2.1109 ODE No. 1109

$$xy''(x) - xy'(x) - y(x) - e^x x(x+1) = 0$$

✓ **Mathematica** : cpu = 0.133269 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow c_2(-e^x x \text{Ei}(-x) - 1) + e^x(x^2 + x - x \log(-x) - 1) + c_1 e^x x \right\} \right\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 33

$$\{y(x) = (-c_1 x \text{Ei}(1, x) + c_1 e^{-x} + c_2 x + x^2 - x \ln(x) - 1)e^x\}$$

2.1110 ODE No. 1110

$$-ay(x) + xy''(x) - xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0349301 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow c_2 G_{1,2}^{2,0} \left(-x \left| \begin{matrix} 1-a \\ 0,1 \end{matrix} \right. \right) + c_1 x {}_1F_1(a+1; 2; x) \right\} \right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 23

$$\{y(x) = (c_1 \text{KummerM}(a+1, 2, x) + c_2 \text{KummerU}(a+1, 2, x))x\}$$

2.1111 ODE No. 1111

$$xy''(x) - (x+1)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0199932 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow c_1 e^x + c_2(-x-1) \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 13

$$\{y(x) = c_1 x + c_2 e^x + c_1\}$$

HAND SOLUTION

$$xy'' - (x+1)y' + y = 0 \tag{1}$$

Taking Laplace transform of each term and using property of $\mathcal{L}(xf(x)) = -\frac{d}{ds}F(s)$ where $F(s) = \mathcal{L}f(x)$, then

$$\mathcal{L}(xy'') = -\frac{d}{ds}(\mathcal{L}y'')$$

Let $\mathcal{L}y(x) = Y(s) \equiv Y$. Now $\mathcal{L}y'' = s^2Y - sy(0) - y'(0)$. Assuming $y(0) = A, y'(0) = B$ then

$$\begin{aligned} \mathcal{L}(xy'') &= -\frac{d}{ds}(s^2Y - As - B) \\ &= -(2sY + s^2Y' - A) \end{aligned}$$

And

$$\begin{aligned} \mathcal{L}((x+1)y') &= \mathcal{L}(xy' + y') \\ &= -\frac{d}{ds}(\mathcal{L}y') + \mathcal{L}y' \\ &= -\frac{d}{ds}(sY - y(0)) + (sY - y(0)) \\ &= -\frac{d}{ds}(sY - A) + (sY - y(0)) \\ &= -(Y + sY') + (sY - A) \\ &= -Y - sY' + sY - A \end{aligned}$$

Hence Laplace transform of the ODE becomes

$$\begin{aligned} -(2sY + s^2Y' - A) - (-Y - sY' + sY - A) + Y &= 0 \\ -2sY - s^2Y' + A + Y + sY' - sY + A + Y &= 0 \\ Y'(s - s^2) + Y(-2s + 1 - s + 1) &= -2A \\ Y'(s^2 - s) + Y(3s - 2) &= 2A \\ Y' + \frac{(3s - 2)}{s(s - 1)}Y &= \frac{2A}{s(s - 1)} \end{aligned}$$

The integrating factor is $\mu = e^{\int \frac{(3s-2)}{s(s-1)} ds} = e^{\ln(s-1)+2\ln(s)} = (s - 1)s^2$, hence

$$\begin{aligned} d((s - 1)s^2Y) &= (s - 1)s^2 \frac{2A}{s(s - 1)} \\ (s - 1)s^2Y &= 2A \int s ds + c_1 \\ (s - 1)s^2Y &= 2A \frac{s^2}{2} + c_1 \\ Y &= \frac{As^2 + c_1}{(s - 1)s^2} \end{aligned}$$

Inverse Laplace transform gives

$$\begin{aligned} y(x) &= -c_1 + (A + c_1)e^x - c_1x \\ &= -c_1(1 + x) + (A + c_1)e^x \end{aligned}$$

Let $-c_1 = A_0, A + c_1 = B_0$, hence

$$y(x) = A_0(1 + x) + B_0e^x$$

Verification

```
rrestart;
ode:=x*diff(diff(y(x),x),x)-(1+x)*diff(y(x),x)+y(x)=0;
y0:=-C0*(1+x)+_C1*exp(x);
odetest(y(x)=y0,ode);
0
```

2.1112 ODE No. 1112

$$xy''(x) - (x + 1)y'(x) - 2(x - 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0223318 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{2x} - \frac{1}{9} c_2 e^{-x} (3x + 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 22

$$\{y(x) = c_1 e^{2x} + c_2 (3x + 1) e^{-x}\}$$

2.1113 ODE No. 1113

$$-ay(x) + (b - x)y'(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0139356 (sec), leaf count = 24

$$\{ \{ y(x) \rightarrow c_1 U(a, b, x) + c_2 L_{-a}^{b-1}(x) \} \}$$

✓ **Maple** : cpu = 0.178 (sec), leaf count = 17

$$\{y(x) = c_1 \text{KummerM}(a, b, x) + c_2 \text{KummerU}(a, b, x)\}$$

2.1114 ODE No. 1114

$$xy''(x) - 2(x-1)y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0419303 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow c_2 G_{1,2}^{2,0} \left(-2x \left| \begin{array}{c} \frac{1}{2} \\ -1, 0 \end{array} \right. \right) + c_1 e^x (I_0(x) - I_1(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 34

$$\{y(x) = (-c_2 \text{BesselK}(0, -x) + c_2 \text{BesselK}(1, -x) + c_1 (\text{BesselI}(0, x) - \text{BesselI}(1, x)))e^x\}$$

2.1115 ODE No. 1115

$$xy''(x) - (3x-2)y'(x) + (3-2x)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0519023 (sec), leaf count = 76

$$\left\{ \left\{ y(x) \rightarrow c_2 e^{-\frac{1}{2}(\sqrt{17}-3)x} {}_1F_1\left(1 - \frac{6}{\sqrt{17}}; 2; \sqrt{17}x\right) + c_1 e^{-\frac{1}{2}(\sqrt{17}-3)x} U\left(1 - \frac{6}{\sqrt{17}}, 2, \sqrt{17}x\right) \right\} \right\}$$

✓ **Maple** : cpu = 1.483 (sec), leaf count = 47

$$\left\{ y(x) = \left(c_1 \text{KummerM}\left(1 - \frac{6\sqrt{17}}{17}, 2, \sqrt{17}x\right) + c_2 \text{KummerU}\left(1 - \frac{6\sqrt{17}}{17}, 2, \sqrt{17}x\right) \right) e^{-\frac{(-3+\sqrt{17})x}{2}} \right\}$$

2.1116 ODE No. 1116

$$y'(x)(ax+b+n) + any(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0486651 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-ax} U(b, b+n, ax) + c_2 e^{-ax} L_{-b}^{b+n-1}(ax) \right\} \right\}$$

✓ **Maple** : cpu = 0.204 (sec), leaf count = 31

$$\{y(x) = (c_1 \text{KummerM}(b, b+n, ax) + c_2 \text{KummerU}(b, b+n, ax))e^{-ax}\}$$

2.1117 ODE No. 1117

$$-(x+1)(a+b)y'(x) + abxy(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.071989 (sec), leaf count = 107

$$\left\{ \left\{ y(x) \rightarrow c_1 U\left(-\frac{-a^2-ba-a+b}{a-b}, a+b+2, (a-b)x\right) e^{(a+b+1)\log(x)+bx} + c_2 L_{\frac{-a^2-ba-a+b}{a-b}}^{a+b+1}((a-b)x) e^{(a+b+1)\log(x)+bx} \right\} \right\}$$

✓ **Maple** : cpu = 0.204 (sec), leaf count = 82

$$\left\{ y(x) = \left(c_1 \text{KummerM}\left(\frac{a^2+ab+a-b}{a-b}, a+b+2, (a-b)x\right) + c_2 \text{KummerU}\left(\frac{a^2+ab+a-b}{a-b}, a+b+2, (a-b)x\right) \right) x^{a+b} \right\}$$

2.1118 ODE No. 1118

$$y'(x)(x(a+b)+m+n) + y(x)(abx+an+bm) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0657251 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-ax} U(m, m+n, (a-b)x) + c_2 e^{-ax} L_{-m}^{m+n-1}((a-b)x) \right\} \right\}$$

✓ **Maple** : cpu = 0.207 (sec), leaf count = 39

$$\{y(x) = (c_1 \text{KummerM}(m, m+n, (a-b)x) + c_2 \text{KummerU}(m, m+n, (a-b)x))e^{-ax}\}$$

2.1119 ODE No. 1119

$$y(x)\left(a^2x+2ab\right)-2(ax+b)y'(x)+xy''(x)=0$$

✓ **Mathematica** : cpu = 0.127317 (sec), leaf count = 77

$$\left\{\left\{y(x)\rightarrow c_1e^{ax}x^{b-\frac{1}{2}\sqrt{(2b+1)^2+\frac{1}{2}}+\frac{c_2e^{ax}x^{b+\frac{1}{2}\sqrt{(2b+1)^2+\frac{1}{2}}}}{\sqrt{(2b+1)^2}}\right\}\right\}$$

✓ **Maple** : cpu = 0.051 (sec), leaf count = 20

$$\{y(x)=\left(c_2x^{2b+1}+c_1\right)e^{ax}\}$$

2.1120 ODE No. 1120

$$(ax+b)y'(x)+y(x)(cx+d)+xy''(x)=0$$

✓ **Mathematica** : cpu = 0.0430742 (sec), leaf count = 168

$$\left\{\left\{y(x)\rightarrow c_1e^{-\frac{1}{2}x\sqrt{a^2-4c}-\frac{ax}{2}}U\left(-\frac{-ab-\sqrt{a^2-4c}b+2d}{2\sqrt{a^2-4c}},b,\sqrt{a^2-4c}x\right)+c_2e^{-\frac{1}{2}x\sqrt{a^2-4c}-\frac{ax}{2}}L^{\frac{b-1}{\frac{-ab-\sqrt{a^2-4c}b+2d}{2\sqrt{a^2-4c}}}}\left(\sqrt{a^2-4c}x\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.284 (sec), leaf count = 109

$$\left\{y(x)=\left(c_1\operatorname{KummerM}\left(\frac{ab+\sqrt{a^2-4c}b-2d}{2\sqrt{a^2-4c}},b,\sqrt{a^2-4c}x\right)+c_2\operatorname{KummerU}\left(\frac{ab+\sqrt{a^2-4c}b-2d}{2\sqrt{a^2-4c}},b,\sqrt{a^2-4c}x\right)\right)e^{\frac{ax}{2}}$$

2.1121 ODE No. 1121

$$-\left(x^2-x\right)y'(x)+xy''(x)+(x-1)y(x)=0$$

✓ **Mathematica** : cpu = 0.227533 (sec), leaf count = 41

$$\left\{\left\{y(x)\rightarrow c_2x\int_1^xe^{\frac{K[1]^2}{2}-K[1]}dK[1]+c_1x\right\}\right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 23

$$\left\{y(x)=\left(c_1\left(\int\frac{e^{\frac{(x-2)x}{2}}}{x^2}dx\right)+c_2\right)x\right\}$$

2.1122 ODE No. 1122

$$-\left(x^2-x-2\right)y'(x)+xy''(x)-x(x+3)y(x)=0$$

✓ **Mathematica** : cpu = 0.234063 (sec), leaf count = 57

$$\left\{\left\{y(x)\rightarrow c_2e^{\frac{x^2}{2}}\int_1^xe^{-\frac{1}{2}K[1]^2-K[1]}dK[1]+c_1e^{\frac{x^2}{2}}\right\}\right\}$$

✓ **Maple** : cpu = 0.25 (sec), leaf count = 28

$$\left\{y(x)=\left(c_2\left(\int\frac{e^{-\frac{(x+2)x}{2}}}{x^2}dx\right)+c_1\right)e^{\frac{x^2}{2}}\right\}$$

2.1123 ODE No. 1123

$$-(2ax^2 + 1)y'(x) + bx^3y(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0106146 (sec), leaf count = 91

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{2}\sqrt{b}x^2 \left(\frac{a}{\sqrt{b}} - \frac{\sqrt{a^2-b}}{\sqrt{b}} \right)} + c_2 e^{\frac{1}{2}\sqrt{b}x^2 \left(\frac{\sqrt{a^2-b}}{\sqrt{b}} + \frac{a}{\sqrt{b}} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.073 (sec), leaf count = 45

$$\left\{ y(x) = c_1 e^{\frac{(a+\sqrt{a^2-b})x^2}{2}} + c_2 e^{-\frac{(-a+\sqrt{a^2-b})x^2}{2}} \right\}$$

2.1124 ODE No. 1124

$$-2(x^2 - a)y'(x) + 2nxy(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0683357 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_1F_1\left(-\frac{n}{2}; a + \frac{1}{2}; x^2\right) + i^{1-2a} c_2 x^{1-2a} {}_1F_1\left(-a - \frac{n}{2} + \frac{1}{2}; \frac{3}{2} - a; x^2\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.194 (sec), leaf count = 29

$$\left\{ y(x) = c_1 \text{KummerM}\left(-\frac{n}{2}, a + \frac{1}{2}, x^2\right) + c_2 \text{KummerU}\left(-\frac{n}{2}, a + \frac{1}{2}, x^2\right) \right\}$$

2.1125 ODE No. 1125

$$-4x^5 - 4x^3y(x) + (4x^2 - 1)y'(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.112457 (sec), leaf count = 84

$$\left\{ \left\{ y(x) \rightarrow -\frac{3\sqrt{2}x^2 + 4x^2 + 6\sqrt{2} + 8}{\sqrt{2}(3 + 2\sqrt{2})} + c_1 e^{-((1-\sqrt{2})x^2)} + c_2 e^{-((1+\sqrt{2})x^2)} \right\} \right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 36

$$\left\{ y(x) = c_1 e^{-(1+\sqrt{2})x^2} + c_2 e^{(\sqrt{2}-1)x^2} - x^2 - 2 \right\}$$

2.1126 ODE No. 1126

$$(a^2x^3 + a)y(x) + (2ax^3 - 1)y'(x) + xy''(x) = 0$$

✗ **Mathematica** : cpu = 1.00992 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow (x) \right\} \right\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 19

$$\left\{ y(x) = (c_2 x^2 + c_1) e^{-\frac{ax^3}{3}} \right\}$$

2.1127 ODE No. 1127

$$y(x)(a^2x \log^2(x) + a \log(x) + a) + (2ax \log(x) + 1)y'(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0287341 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{ax} x^{-ax} + c_2 e^{ax} x^{-ax} \log(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 21

$$\{y(x) = (c_2 \ln(x) + c_1)x^{-ax}e^{ax}\}$$

2.1128 ODE No. 1128

$$(xf(x) + 2)y'(x) + f(x)y(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0210337 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \int_1^x \exp\left(-\int_1^{K[2]} f(K[1])dK[1]\right)dK[2]}{x} + \frac{c_1}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.235 (sec), leaf count = 32

$$\left\{ y(x) = \frac{c_2 \left(\int x^2 e^{\int \frac{-xf(x)-2}{x} dx} dx \right) + c_1}{x} \right\}$$

2.1129 ODE No. 1129

$$(x-3)y''(x) - (4x-9)y'(x) + (3x-6)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0622467 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{8}c_2 e^{3x-9}(4x^3 - 42x^2 + 150x - 183) + c_1 e^{x-3} \right\} \right\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 30

$$\{y(x) = c_1 e^x + c_2(4x^3 - 42x^2 + 150x - 183)e^{3x}\}$$

2.1130 ODE No. 1130

$$ay(x) + 2xy''(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0119579 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow c_1 \cos(\sqrt{2} \sqrt{a} \sqrt{x}) + c_2 \sin(\sqrt{2} \sqrt{a} \sqrt{x}) \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 31

$$\{y(x) = c_1 \sin(\sqrt{2} \sqrt{a} \sqrt{x}) + c_2 \cos(\sqrt{2} \sqrt{a} \sqrt{x})\}$$

2.1131 ODE No. 1131

$$ay(x) + 2xy''(x) - (x-1)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0097759 (sec), leaf count = 58

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt{x} U\left(\frac{1}{2}(1-2a), \frac{3}{2}, \frac{x}{2}\right) + c_2 \sqrt{x} L_{\frac{1}{2}(2a-1)}\left(\frac{x}{2}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.176 (sec), leaf count = 33

$$\left\{ y(x) = \left(c_1 \text{KummerM}\left(-a + \frac{1}{2}, \frac{3}{2}, \frac{x}{2}\right) + c_2 \text{KummerU}\left(-a + \frac{1}{2}, \frac{3}{2}, \frac{x}{2}\right) \right) \sqrt{x} \right\}$$

2.1132 ODE No. 1132

$$ay(x) + 2xy''(x) - (2x - 1)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0098392 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt{x} U\left(\frac{1-a}{2}, \frac{3}{2}, x\right) + c_2 \sqrt{x} L_{\frac{a-1}{2}}^{\frac{1}{2}}(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.198 (sec), leaf count = 29

$$\left\{ y(x) = \left(c_1 \text{KummerM}\left(-\frac{a}{2} + \frac{1}{2}, \frac{3}{2}, x\right) + c_2 \text{KummerU}\left(-\frac{a}{2} + \frac{1}{2}, \frac{3}{2}, x\right) \right) \sqrt{x} \right\}$$

2.1133 ODE No. 1133

$$(2x - 1)y''(x) - (3x - 4)y'(x) + (x - 3)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0548494 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow 2^{5/8} c_1 e^{x-\frac{1}{2}} - \frac{c_2 e^{x-\frac{1}{2}} \Gamma\left(-\frac{1}{4}, \frac{1}{4}(2x-1)\right)}{4\sqrt[8]{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.192 (sec), leaf count = 37

$$\left\{ y(x) = \frac{\left(c_1 \text{KummerM}\left(1, \frac{3}{4}, \frac{x}{2} - \frac{1}{4}\right) + c_2 \text{KummerU}\left(1, \frac{3}{4}, \frac{x}{2} - \frac{1}{4}\right) \right) e^{\frac{x}{2}}}{(2x-1)^{\frac{1}{4}}} \right\}$$

2.1134 ODE No. 1134

$$(-a - x)y(x) + 4xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0808838 (sec), leaf count = 78

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} c_2 e^{-x/2} x {}_1F_1\left(\frac{1}{128}(-32(-a-4i)-128i)+1; 2; x\right) + \frac{1}{4} c_1 e^{-x/2} x U\left(\frac{1}{128}(-32(-a-4i)-128i)+1, 2, x\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.045 (sec), leaf count = 21

$$\left\{ y(x) = c_1 \text{WhittakerM}\left(-\frac{a}{4}, \frac{1}{2}, x\right) + c_2 \text{WhittakerW}\left(-\frac{a}{4}, \frac{1}{2}, x\right) \right\}$$

2.1135 ODE No. 1135

$$4xy''(x) + 2y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0081757 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow c_1 \cosh(\sqrt{x}) + ic_2 \sinh(\sqrt{x}) \right\} \right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 17

$$\{y(x) = c_1 \sinh(\sqrt{x}) + c_2 \cosh(\sqrt{x})\}$$

2.1136 ODE No. 1136

$$4xy''(x) + 4y'(x) + (-x - 2)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0201868 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow c_2 e^{x/2} \text{Ei}(-x) + c_1 e^{x/2} \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 16

$$\left\{ y(x) = (c_2 \text{Ei}(1, x) + c_1) e^{\frac{x}{2}} \right\}$$

2.1137 ODE No. 1137

$$ly(x) + 4xy''(x) - (x + 2)y(x) + 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.0801933 (sec), leaf count = 74

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} c_2 e^{-x/2} x {}_1F_1\left(\frac{1}{128}(-32(l + (2 - 4i)) - 128i) + 1; 2; x\right) + \frac{1}{4} c_1 e^{-x/2} x U\left(\frac{1}{128}(-32(l + (2 - 4i)) - 128i) + 1, 2, x\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 25

$$\left\{ y(x) = c_1 \text{WhittakerM}\left(\frac{l}{4} + \frac{1}{2}, \frac{1}{2}, x\right) + c_2 \text{WhittakerW}\left(\frac{l}{4} + \frac{1}{2}, \frac{1}{2}, x\right) \right\}$$

2.1138 ODE No. 1138

$$y(x)(2m + 4n - x) + 4my'(x) + 4xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0218624 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-x/2} U(-n, m, x) + c_2 e^{-x/2} L_n^{m-1}(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.177 (sec), leaf count = 26

$$\left\{ y(x) = (c_1 \text{KummerM}(-n, m, x) + c_2 \text{KummerU}(-n, m, x)) e^{-\frac{x}{2}} \right\}$$

2.1139 ODE No. 1139

$$(-a - x)y(x) + 16xy''(x) + 8y'(x) = 0$$

✓ **Mathematica** : cpu = 0.012738 (sec), leaf count = 74

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{4}(2\log(x)-x)} U\left(\frac{a+6}{8}, \frac{3}{2}, \frac{x}{2}\right) + c_2 e^{\frac{1}{4}(2\log(x)-x)} L_{\frac{1}{8}(-a-6)}^{\frac{1}{2}}\left(\frac{x}{2}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.187 (sec), leaf count = 37

$$\left\{ y(x) = \left(c_1 \text{KummerM}\left(\frac{a}{8} + \frac{3}{4}, \frac{3}{2}, \frac{x}{2}\right) + c_2 \text{KummerU}\left(\frac{a}{8} + \frac{3}{4}, \frac{3}{2}, \frac{x}{2}\right) \right) \sqrt{x} e^{-\frac{x}{4}} \right\}$$

2.1140 ODE No. 1140

$$axy''(x) + by'(x) + cy(x) = 0$$

✓ **Mathematica** : cpu = 0.0376252 (sec), leaf count = 190

$$\left\{ \left\{ y(x) \rightarrow c_1 a^{\frac{1}{2}\left(\frac{b}{a}-1\right)} c^{\frac{1}{2}\left(1-\frac{b}{a}\right)} x^{\frac{1}{2}\left(1-\frac{b}{a}\right)} \Gamma\left(\frac{b}{a}\right) J_{\frac{b}{a}-1}\left(\frac{2\sqrt{c}\sqrt{x}}{\sqrt{a}}\right) + c_2 a^{\frac{1}{2}\left(1-\frac{b}{a}\right)-\frac{a-b}{a}} c^{\frac{a-b}{a}+\frac{1}{2}\left(\frac{b}{a}-1\right)} x^{\frac{a-b}{a}+\frac{1}{2}\left(\frac{b}{a}-1\right)} \Gamma\left(2-\frac{b}{a}\right) J_{1-\frac{b}{a}}\left(\frac{2\sqrt{c}\sqrt{x}}{\sqrt{a}}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 66

$$\left\{ y(x) = \left(c_1 \text{BesselJ}\left(\frac{-a+b}{a}, 2\sqrt{\frac{c}{a}}\sqrt{x}\right) + c_2 \text{BesselY}\left(\frac{-a+b}{a}, 2\sqrt{\frac{c}{a}}\sqrt{x}\right) \right) x^{\frac{a-b}{2a}} \right\}$$

2.1141 ODE No. 1141

$$(3a + bx)y'(x) + axy''(x) + 3by(x) = 0$$

✓ **Mathematica** : cpu = 0.126842 (sec), leaf count = 79

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\frac{bx}{a}} - \frac{c_2 e^{-\frac{bx}{a}} \left(a^2 e^{\frac{bx}{a}} - b^2 x^2 \operatorname{Ei}\left(\frac{bx}{a}\right) + abxe^{\frac{bx}{a}} \right)}{2a^2 x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 55

$$\left\{ y(x) = \frac{c_2 b^2 x^2 \operatorname{Ei}\left(1, -\frac{bx}{a}\right) e^{-\frac{bx}{a}} + c_1 x^2 e^{-\frac{bx}{a}} + c_2 (bx + a)a}{x^2} \right\}$$

2.1142 ODE No. 1142

$$cy(x)\sqrt[5]{ax+b} + 5(ax+b)y''(x) + 8ay'(x) = 0$$

✓ **Mathematica** : cpu = 0.0399267 (sec), leaf count = 108

$$\left\{ \left\{ y(x) \rightarrow \frac{6ac_1 \cos\left(\frac{\sqrt{5}\sqrt{c}(ax+b)^{3/5}}{3a}\right)}{\sqrt{5}\sqrt{c}(ax+b)^{3/5}} + \frac{3ac_2 \sin\left(\frac{\sqrt{5}\sqrt{c}(ax+b)^{3/5}}{3a}\right)}{\sqrt{5}\sqrt{c}(ax+b)^{3/5}} \right\} \right\}$$

✓ **Maple** : cpu = 0.224 (sec), leaf count = 53

$$\left\{ y(x) = \frac{c_1 \sinh\left(\frac{(ax+b)^{3/5} \sqrt{-5c}}{3a}\right) + c_2 \cosh\left(\frac{(ax+b)^{3/5} \sqrt{-5c}}{3a}\right)}{(ax+b)^{3/5}} \right\}$$

2.1143 ODE No. 1143

$$(a + bx)y'(x) + 2axy''(x) + cy(x) = 0$$

✓ **Mathematica** : cpu = 0.0289427 (sec), leaf count = 93

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{a \log(x) - bx}{2a}} U\left(-\frac{c-b}{b}, \frac{3}{2}, \frac{bx}{2a}\right) + c_2 e^{\frac{a \log(x) - bx}{2a}} L_{\frac{c-b}{b}}^{\frac{1}{2}}\left(\frac{bx}{2a}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.231 (sec), leaf count = 57

$$\left\{ y(x) = \left(c_1 \operatorname{KummerM}\left(\frac{b-c}{b}, \frac{3}{2}, \frac{bx}{2a}\right) + c_2 \operatorname{KummerU}\left(\frac{b-c}{b}, \frac{3}{2}, \frac{bx}{2a}\right) \right) \sqrt{x} e^{-\frac{bx}{2a}} \right\}$$

2.1144 ODE No. 1144

$$(3a + bx)y'(x) + 2axy''(x) + cy(x) = 0$$

✓ **Mathematica** : cpu = 0.0261744 (sec), leaf count = 88

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\frac{bx}{2a}} U\left(-\frac{2c-3b}{2b}, \frac{3}{2}, \frac{bx}{2a}\right) + c_2 e^{-\frac{bx}{2a}} L_{\frac{2c-3b}{2b}}^{\frac{1}{2}}\left(\frac{bx}{2a}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.23 (sec), leaf count = 60

$$\left\{ y(x) = \left(c_1 \operatorname{KummerM}\left(\frac{3b-2c}{2b}, \frac{3}{2}, \frac{bx}{2a}\right) + c_2 \operatorname{KummerU}\left(\frac{3b-2c}{2b}, \frac{3}{2}, \frac{bx}{2a}\right) \right) e^{-\frac{bx}{2a}} \right\}$$

2.1145 ODE No. 1145

$$y(x)(a_0x + b_0) + (a_1x + b_1)y'(x) + (a_2x + b_2)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.256016 (sec), leaf count = 386

$$\left\{ \left\{ y(x) \rightarrow c_1 U \left(-\frac{b_2 a_1^2 - a_2 b_1 a_1 - \sqrt{a_1^2 - 4 a_0 a_2} b_2 a_1 + 2 a_2^2 b_0 + a_2 \sqrt{a_1^2 - 4 a_0 a_2} b_1 - 2 a_0 a_2 b_2 - 2 a_2^2 \sqrt{a_1^2 - 4 a_0 a_2}}{2 a_2^2 \sqrt{a_1^2 - 4 a_0 a_2}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.238 (sec), leaf count = 248

$$\left\{ y(x) = \left(c_1 \text{KummerM} \left(\frac{-a_1^2 b_2 - 2 a_2^2 b_0 + (2 a_0 b_2 + a_1 b_1) a_2 + (a_1 b_2 + 2 a_2^2 - a_2 b_1) \sqrt{-4 a_0 a_2 + a_1^2}}{2 \sqrt{-4 a_0 a_2 + a_1^2} a_2^2}, \frac{a_1 b_2 + 2 a_2^2 - a_2 b_1}{a_2^2} \right) \right) \right\}$$

2.1146 ODE No. 1146

$$x^2 y''(x) - 6 y(x) = 0$$

✓ **Mathematica** : cpu = 0.0057946 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow c_2 x^3 + \frac{c_1}{x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.008 (sec), leaf count = 15

$$\left\{ y(x) = \frac{c_1 x^5 + c_2}{x^2} \right\}$$

2.1147 ODE No. 1147

$$x^2 y''(x) - 12 y(x) = 0$$

✓ **Mathematica** : cpu = 0.0048163 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow c_2 x^4 + \frac{c_1}{x^3} \right\} \right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 15

$$\left\{ y(x) = \frac{c_2 x^7 + c_1}{x^3} \right\}$$

2.1148 ODE No. 1148

$$a y(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0110452 (sec), leaf count = 77

$$\left\{ \left\{ y(x) \rightarrow c_1 x^{\frac{1}{2} \left(\frac{1}{\sqrt{a}} - \frac{\sqrt{1-4a}}{\sqrt{a}} \right)} \sqrt{a} + c_2 x^{\frac{1}{2} \left(\frac{\sqrt{1-4a}}{\sqrt{a}} + \frac{1}{\sqrt{a}} \right)} \sqrt{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 35

$$\left\{ y(x) = c_1 x^{\frac{1}{2} + \frac{\sqrt{-4a+1}}{2}} + c_2 x^{\frac{1}{2} - \frac{\sqrt{-4a+1}}{2}} \right\}$$

2.1149 ODE No. 1149

$$y(x)(ax + b) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0513605 (sec), leaf count = 212

$$\left\{ \left\{ y(x) \rightarrow c_2 a^{\frac{1}{2}(\sqrt{1-4b}+1)-\frac{1}{2}\sqrt{1-4b}} x^{\frac{1}{2}(\sqrt{1-4b}+1)-\frac{1}{2}\sqrt{1-4b}} \Gamma\left(\sqrt{1-4b} + 1\right) J_{\sqrt{1-4b}}\left(2\sqrt{a}\sqrt{x}\right) + c_1 a^{\frac{1}{2}(1-\sqrt{1-4b})+\frac{1}{2}\sqrt{1-4b}} x^{\frac{1}{2}(1-\sqrt{1-4b})+\frac{1}{2}\sqrt{1-4b}} \right. \right.$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 45

$$\left\{ y(x) = \left(c_1 \text{BesselJ}\left(\sqrt{-4b+1}, 2\sqrt{a}\sqrt{x}\right) + c_2 \text{BesselY}\left(\sqrt{-4b+1}, 2\sqrt{a}\sqrt{x}\right) \right) \sqrt{x} \right\}$$

2.1150 ODE No. 1150

$$x^2 y''(x) + (x^2 - 2)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0081937 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow \sqrt{\frac{2}{\pi}} c_2 \left(-\sin(x) - \frac{\cos(x)}{x} \right) + \sqrt{\frac{2}{\pi}} c_1 \left(\frac{\sin(x)}{x} - \cos(x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 27

$$\left\{ y(x) = \frac{(c_1 x + c_2) \cos(x) + (c_2 x - c_1) \sin(x)}{x} \right\}$$

2.1151 ODE No. 1151

$$(-ax^2 - 2)y(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0166555 (sec), leaf count = 129

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{\frac{2}{\pi}} c_2 \sqrt{x} \left(i \sinh(\sqrt{a} x) - \frac{i \cosh(\sqrt{a} x)}{\sqrt{a} x} \right)}{\sqrt{-i\sqrt{a} x}} + \frac{\sqrt{\frac{2}{\pi}} c_1 \sqrt{x} \left(\frac{\sinh(\sqrt{a} x)}{\sqrt{a} x} - \cosh(\sqrt{a} x) \right)}{\sqrt{-i\sqrt{a} x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.114 (sec), leaf count = 43

$$\left\{ y(x) = \frac{-c_1(ax - \sqrt{a})e^{\sqrt{a}x} + c_2(ax + \sqrt{a})e^{-\sqrt{a}x}}{x} \right\}$$

2.1152 ODE No. 1152

$$(a^2 x^2 - 6)y(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0149217 (sec), leaf count = 114

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{\frac{2}{\pi}} c_1 \sqrt{x} \left(\frac{3 \sin(ax)}{a^2 x^2} - \sin(ax) - \frac{3 \cos(ax)}{ax} \right)}{\sqrt{ax}} + \frac{\sqrt{\frac{2}{\pi}} c_2 \sqrt{x} \left(-\frac{3 \cos(ax)}{a^2 x^2} - \frac{3 \sin(ax)}{ax} + \cos(ax) \right)}{\sqrt{ax}} \right\} \right\}$$

✓ **Maple** : cpu = 0.246 (sec), leaf count = 53

$$\left\{ y(x) = \frac{(c_1 a^2 x^2 + 3c_2 ax - 3c_1) \cos(ax) + (c_2 a^2 x^2 - 3c_1 ax - 3c_2) \sin(ax)}{x^2} \right\}$$

2.1153 ODE No. 1153

$$y(x)\big(ax^2 + (1 - v)v\big) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0285052 (sec), leaf count = 56

$$\left\{\left\{y(x) \rightarrow c_1 \sqrt{x} J_{\frac{1}{2}(2v-1)}(\sqrt{a} x) + c_2 \sqrt{x} Y_{\frac{1}{2}(2v-1)}(\sqrt{a} x)\right\}\right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 31

$$\left\{y(x) = \left(c_1 \operatorname{BesselJ}\left(v - \frac{1}{2}, \sqrt{a} x\right) + c_2 \operatorname{BesselY}\left(v - \frac{1}{2}, \sqrt{a} x\right)\right) \sqrt{x}\right\}$$

2.1154 ODE No. 1154

$$y(x)\big(ax^2 + bx + c\big) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0168315 (sec), leaf count = 88

$$\left\{\left\{y(x) \rightarrow c_1 M_{-\frac{ib}{2\sqrt{a}}, -\frac{1}{2}i\sqrt{4c-1}}(2i\sqrt{a} x) + c_2 W_{-\frac{ib}{2\sqrt{a}}, -\frac{1}{2}i\sqrt{4c-1}}(2i\sqrt{a} x)\right\}\right\}$$

✓ **Maple** : cpu = 0.243 (sec), leaf count = 57

$$\left\{y(x) = c_1 \operatorname{WhittakerM}\left(-\frac{ib}{2\sqrt{a}}, \frac{\sqrt{-4c+1}}{2}, 2i\sqrt{a} x\right) + c_2 \operatorname{WhittakerW}\left(-\frac{ib}{2\sqrt{a}}, \frac{\sqrt{-4c+1}}{2}, 2i\sqrt{a} x\right)\right\}$$

2.1155 ODE No. 1155

$$y(x)\big(ax^k + (1 - b)b\big) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0514049 (sec), leaf count = 225

$$\left\{\left\{y(x) \rightarrow c_1 k^{-\frac{2(1-b)}{k} - \frac{2b}{k} + \frac{1}{k}} a^{\frac{1-b}{k} + \frac{1}{2}\left(\frac{2b}{k} - \frac{1}{k}\right)} (x^k)^{\frac{1-b}{k} + \frac{1}{2}\left(\frac{2b}{k} - \frac{1}{k}\right)} \Gamma\left(-\frac{2b}{k} + \frac{1}{k} + 1\right) J_{\frac{1-2b}{k}}\left(\frac{2\sqrt{a} \sqrt{x^k}}{k}\right) + c_2 k^{-1/k} a^{\frac{b}{k} + \frac{1}{2}\left(\frac{1}{k} - \frac{2b}{k}\right)} (x^k)^{\frac{b}{k} + \frac{1}{2}\left(\frac{1}{k} - \frac{2b}{k}\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.17 (sec), leaf count = 67

$$\left\{y(x) = \left(c_1 \operatorname{BesselJ}\left(\frac{\sqrt{(2b-1)^2}}{k}, \frac{2\sqrt{a} x^{\frac{k}{2}}}{k}\right) + c_2 \operatorname{BesselY}\left(\frac{\sqrt{(2b-1)^2}}{k}, \frac{2\sqrt{a} x^{\frac{k}{2}}}{k}\right)\right) \sqrt{x}\right\}$$

2.1156 ODE No. 1156

$$x^2y''(x) + \frac{y(x)}{\log(x)} - e^x x(x \log(x) + 2) = 0$$

✓ **Mathematica** : cpu = 0.0916499 (sec), leaf count = 32

$$\left\{\left\{y(x) \rightarrow c_2 \log(x) \left(\operatorname{li}(x) - \frac{x}{\log(x)}\right) + e^x \log(x) + c_1 \log(x)\right\}\right\}$$

✓ **Maple** : cpu = 0.247 (sec), leaf count = 71

$$\left\{y(x) = c_2 \ln(x) - c_1 (\operatorname{Ei}(1, -\ln(x)) \ln(x) + x) - \left((\operatorname{Ei}(1, -\ln(x)) \ln(x) + x) e^x \ln(x) - \left(\int \frac{(\operatorname{Ei}(1, -\ln(x)) \ln(x) + x)(x \ln(x))}{x}\right)\right)\right\}$$

2.1157 ODE No. 1157

$$ay'(x) + x^2y''(x) - xy(x) = 0$$

✗ **Mathematica** : cpu = 0.569527 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{\left\{y(x) \rightarrow (x)\right\}\right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x) = DESol\left[\left[\frac{d^2}{dx^2}Y(x) + \frac{a\left(\frac{d}{dx}Y(x)\right)}{x^2} - \frac{Y(x)}{x}\right], \{Y(x)\}\right]\right\}$$

2.1158 ODE No. 1158

$$y(x)(-ab - b^2x^2) + ay'(x) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.18039 (sec), leaf count = 43

$$\left\{\left\{y(x) \rightarrow c_2e^{bx} \int_1^x e^{\frac{a}{K[1]} - 2bK[1]} dK[1] + c_1e^{bx}\right\}\right\}$$

✓ **Maple** : cpu = 0.289 (sec), leaf count = 178

$$\left\{y(x) = \left(c_1HD\left(4\sqrt{2} \sqrt{ab}, -1 - 4\sqrt{2} \sqrt{ab}, 8\sqrt{2} \sqrt{ab}, -4\sqrt{2} \sqrt{ab} + 1, \frac{-a + \sqrt{2} \sqrt{ab} x}{a + \sqrt{2} \sqrt{ab} x}\right)e^{\frac{-bx^2+a}{x}} + c_2HD\left(-4\sqrt{2} \sqrt{ab}, -1\right)\right\}$$

2.1159 ODE No. 1159

$$-ax^2 + x^2y''(x) + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0070615 (sec), leaf count = 24

$$\left\{\left\{y(x) \rightarrow \frac{ax^2}{3} + c_2x + \frac{c_1}{x}\right\}\right\}$$

✓ **Maple** : cpu = 0.015 (sec), leaf count = 19

$$\left\{y(x) = \frac{a x^2}{3} + c_2x + \frac{c_1}{x}\right\}$$

2.1160 ODE No. 1160

$$ay(x) + x^2y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0124501 (sec), leaf count = 30

$$\left\{\left\{y(x) \rightarrow c_1 \cos(\sqrt{a} \log(x)) + c_2 \sin(\sqrt{a} \log(x))\right\}\right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 23

$$\left\{y(x) = c_1 \sin(\sqrt{a} \ln(x)) + c_2 \cos(\sqrt{a} \ln(x))\right\}$$

2.1161 ODE No. 1161

$$(-a - x)y(x) + x^2y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0384322 (sec), leaf count = 78

$$\left\{\left\{y(x) \rightarrow (-1)^{-\sqrt{a}}c_1\Gamma(1 - 2\sqrt{a})I_{-2\sqrt{a}}(2\sqrt{x}) + (-1)^{\sqrt{a}}c_2\Gamma(2\sqrt{a} + 1)I_{2\sqrt{a}}(2\sqrt{x})\right\}\right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 31

$$\left\{y(x) = c_1 \operatorname{BesselI}(2\sqrt{a}, 2\sqrt{x}) + c_2 \operatorname{BesselK}(2\sqrt{a}, 2\sqrt{x})\right\}$$

2.1162 ODE No. 1162

$$\left(x^2 - v^2\right)y(x) + x^2y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0518565 (sec), leaf count = 18

$$\{\{y(x) \rightarrow c_1J_v(x) + c_2Y_v(x)\}\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 15

$$\{y(x) = c_1 \operatorname{BesselJ}(v, x) + c_2 \operatorname{BesselY}(v, x)\}$$

2.1163 ODE No. 1163

$$-f(x) + \left(x^2 - v^2\right)y(x) + x^2y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.127894 (sec), leaf count = 72

$$\left\{\left\{y(x) \rightarrow J_v(x) \int_1^x -\frac{\pi Y_v(K[1])f(K[1])}{2K[1]}dK[1] + Y_v(x) \int_1^x \frac{\pi J_v(K[2])f(K[2])}{2K[2]}dK[2] + c_1J_v(x) + c_2Y_v(x)\right\}\right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 49

$$\left\{y(x) = c_1 \operatorname{BesselY}(v, x) + c_2 \operatorname{BesselJ}(v, x) - \frac{\pi \operatorname{BesselJ}(v, x)\left(\int \frac{\operatorname{BesselY}(v, x)f(x)}{x}dx\right)}{2} + \frac{\pi \operatorname{BesselY}(v, x)\left(\int \frac{\operatorname{BesselJ}(v, x)f(x)}{x}dx\right)}{2}\right\}$$

2.1164 ODE No. 1164

$$y(x)\left(lx^2 - v^2\right) + x^2y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.020164 (sec), leaf count = 30

$$\left\{\left\{y(x) \rightarrow c_1J_v\left(\sqrt{l}x\right) + c_2Y_v\left(\sqrt{l}x\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 23

$$\left\{y(x) = c_1 \operatorname{BesselJ}\left(v, \sqrt{l}x\right) + c_2 \operatorname{BesselY}\left(v, \sqrt{l}x\right)\right\}$$

2.1165 ODE No. 1165

$$(a + x)y'(x) + x^2y''(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.121724 (sec), leaf count = 26

$$\left\{\left\{y(x) \rightarrow \frac{c_2(a + x)}{a^2} + c_1xe^{a/x}\right\}\right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 19

$$\left\{y(x) = c_2xe^{\frac{a}{x}} + c_1(a + x)\right\}$$

2.1166 ODE No. 1166

$$-3x^3 + x^2y''(x) - xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0108962 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow \frac{3x^3}{4} + c_1x + c_2x \log(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 21

$$\left\{ y(x) = \frac{(4c_1 \ln(x) + 3x^2 + 4c_2)x}{4} \right\}$$

2.1167 ODE No. 1167

$$y(x)(ax^m + b) + x^2y''(x) - xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0589438 (sec), leaf count = 326

$$\left\{ \left\{ y(x) \rightarrow c_1 m^{-\frac{2(m-i\sqrt{b-1}m)}{m^2} - \frac{2i\sqrt{b-1}}{m}} a^{\frac{m-i\sqrt{b-1}m}{m^2} + \frac{i\sqrt{b-1}}{m}} (x^m)^{\frac{m-i\sqrt{b-1}m}{m^2} + \frac{i\sqrt{b-1}}{m}} \Gamma\left(1 - \frac{2i\sqrt{b-1}}{m}\right) J_{-\frac{2i\sqrt{b-1}}{m}}\left(\frac{2\sqrt{a}\sqrt{x^m}}{m}\right) + c_2 m^{\frac{2i\sqrt{b-1}}{m} - \frac{2(m-i\sqrt{b-1}m)}{m^2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.177 (sec), leaf count = 63

$$\left\{ y(x) = \left(c_1 \operatorname{BesselJ}\left(\frac{2\sqrt{-b+1}}{m}, \frac{2\sqrt{a} x^{\frac{m}{2}}}{m}\right) + c_2 \operatorname{BesselY}\left(\frac{2\sqrt{-b+1}}{m}, \frac{2\sqrt{a} x^{\frac{m}{2}}}{m}\right) \right) x \right\}$$

2.1168 ODE No. 1168

$$x^2y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0220508 (sec), leaf count = 15

$$\left\{ \left\{ y(x) \rightarrow c_2 - \frac{c_1}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 11

$$\left\{ y(x) = c_1 + \frac{c_2}{x} \right\}$$

2.1169 ODE No. 1169

$$y(x)(ax - b^2) + x^2y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0612584 (sec), leaf count = 236

$$\left\{ \left\{ y(x) \rightarrow c_1 a^{\frac{1}{2}(-\sqrt{4b^2+1}-1) + \frac{1}{2}\sqrt{4b^2+1}} x^{\frac{1}{2}(-\sqrt{4b^2+1}-1) + \frac{1}{2}\sqrt{4b^2+1}} \Gamma\left(1 - \sqrt{4b^2+1}\right) J_{-\sqrt{4b^2+1}}(2\sqrt{a}\sqrt{x}) + c_2 a^{\frac{1}{2}(\sqrt{4b^2+1}-1) - \frac{1}{2}\sqrt{4b^2+1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.189 (sec), leaf count = 49

$$\left\{ y(x) = \frac{c_1 \operatorname{BesselJ}\left(\sqrt{4b^2+1}, 2\sqrt{a}\sqrt{x}\right) + c_2 \operatorname{BesselY}\left(\sqrt{4b^2+1}, 2\sqrt{a}\sqrt{x}\right)}{\sqrt{x}} \right\}$$

2.1170 ODE No. 1170

$$y(x)(ax^2 + b) + x^2 y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0173903 (sec), leaf count = 58

$$\left\{ \left\{ y(x) \rightarrow c_1 j_{\frac{1}{2}(\sqrt{1-4b}-1)}(\sqrt{a}x) + c_2 y_{\frac{1}{2}(\sqrt{1-4b}-1)}(\sqrt{a}x) \right\} \right\}$$

✓ **Maple** : cpu = 0.142 (sec), leaf count = 43

$$\left\{ y(x) = \frac{c_1 \text{BesselJ}\left(\frac{\sqrt{-4b+1}}{2}, \sqrt{a}x\right) + c_2 \text{BesselY}\left(\frac{\sqrt{-4b+1}}{2}, \sqrt{a}x\right)}{\sqrt{x}} \right\}$$

2.1171 ODE No. 1171

$$y(x)(ax + lx^2 - n(n+1)) + x^2 y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0400409 (sec), leaf count = 142

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{n \log(x) - i\sqrt{l}x} U\left(\frac{i(a - 2i\sqrt{l}n - 2i\sqrt{l})}{2\sqrt{l}}, 2n + 2, 2i\sqrt{l}x\right) + c_2 e^{n \log(x) - i\sqrt{l}x} L_{-\frac{i(a-2i\sqrt{l}n-2i\sqrt{l})}{2\sqrt{l}}}^{2n+1}(2i\sqrt{l}x) \right\} \right\}$$

✓ **Maple** : cpu = 0.221 (sec), leaf count = 49

$$\left\{ y(x) = \frac{c_1 \text{WhittakerM}\left(-\frac{ia}{2\sqrt{l}}, n + \frac{1}{2}, 2i\sqrt{l}x\right) + c_2 \text{WhittakerW}\left(-\frac{ia}{2\sqrt{l}}, n + \frac{1}{2}, 2i\sqrt{l}x\right)}{x} \right\}$$

2.1172 ODE No. 1172

$$ay(x) + x^2 y''(x) + 2(x-1)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0669312 (sec), leaf count = 158

$$\left\{ \left\{ y(x) \rightarrow 2^{\frac{1}{2}(1-\sqrt{1-4a})} c_1 \left(\frac{1}{x}\right)^{\frac{1}{2}(1-\sqrt{1-4a})} {}_1F_1\left(\frac{1}{2} - \frac{1}{2}\sqrt{1-4a}; 1 - \sqrt{1-4a}; -\frac{2}{x}\right) + 2^{\frac{1}{2}(\sqrt{1-4a}+1)} c_2 \left(\frac{1}{x}\right)^{\frac{1}{2}(\sqrt{1-4a}+1)} {}_1F_1\left(\frac{1}{2}\sqrt{1-4a} + 1; 1 + \sqrt{1-4a}; -\frac{2}{x}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.154 (sec), leaf count = 47

$$\left\{ y(x) = \sqrt{\frac{1}{x}} \left(c_1 \text{BesselI}\left(\frac{\sqrt{-4a+1}}{2}, \frac{1}{x}\right) + c_2 \text{BesselK}\left(\frac{\sqrt{-4a+1}}{2}, \frac{1}{x}\right) \right) e^{-\frac{1}{x}} \right\}$$

2.1173 ODE No. 1173

$$2(a+x)y'(x) + (1-b)by(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.130936 (sec), leaf count = 74

$$\left\{ \left\{ y(x) \rightarrow (-2)^{1-b} c_1 a^{1-b} \left(\frac{1}{x}\right)^{1-b} {}_1F_1\left(1-b; 2-2b; \frac{2a}{x}\right) + (-2)^b c_2 a^b \left(\frac{1}{x}\right)^b {}_1F_1\left(b; 2b; \frac{2a}{x}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.213 (sec), leaf count = 37

$$\left\{ y(x) = \frac{\left(c_1 \text{BesselI}\left(b - \frac{1}{2}, \frac{a}{x}\right) + c_2 \text{BesselK}\left(b - \frac{1}{2}, \frac{a}{x}\right) \right) e^{\frac{a}{x}}}{\sqrt{x}} \right\}$$

2.1174 ODE No. 1174

$$x^5(-\log(x)) + x^2y''(x) - 2xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0082954 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{144} (12x^5 \log(x) - 7x^5) + c_2x^2 + c_1x \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 25

$$\left\{ y(x) = \frac{x^5 \ln(x)}{12} - \frac{7x^5}{144} + c_2x^2 + c_1x \right\}$$

2.1175 ODE No. 1175

$$(-ax^2 - 12a - 4) \cos(x) + x^2y''(x) - 2xy'(x) - 4y(x) - x \sin(x) = 0$$

✓ **Mathematica** : cpu = 0.228381 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow \frac{-2a \sin(x) - ax \cos(x) - \sin(x)}{x} + c_2x^4 + \frac{c_1}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.087 (sec), leaf count = 29

$$\left\{ y(x) = \frac{c_2x^5 - ax \cos(x) + c_1 + (-2a - 1) \sin(x)}{x} \right\}$$

2.1176 ODE No. 1176

$$x^2y''(x) + (x^2 + 2)y(x) - 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0166349 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow c_1e^{-ix}x - \frac{1}{2}ic_2e^{ix}x \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 15

$$\{y(x) = (c_1 \sin(x) + c_2 \cos(x))x\}$$

2.1177 ODE No. 1177

$$x^2y''(x) + (x^2 + 2)y'(x) + x^2(-\sec(x)) - 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.830027 (sec), leaf count = 141

$$\left\{ \left\{ y(x) \rightarrow \int_1^x e^{\frac{2}{K[1]} - K[1]} K[1]^2 dK[1] \int_1^x \frac{e^{K[3] - \frac{2}{K[3]}} \sec(K[3])}{K[3]^2} dK[3] + \int_1^x -\frac{e^{K[2] - \frac{2}{K[2]}} \sec(K[2]) \int_1^{K[2]} e^{\frac{2}{K[1]} - K[1]} K[1]^2 dK[1]}{K[2]^2} dK[2] \right\} \right\}$$

✓ **Maple** : cpu = 0.123 (sec), leaf count = 34

$$\left\{ y(x) = \left(c_1 \cos(x) - \left(\int \frac{\sin(x)}{x \cos(x)} dx \right) \cos(x) + (c_2 + \ln(x)) \sin(x) \right) x \right\}$$

2.1178 ODE No. 1178

$$x^3(-\sec(x)) + x^2y''(x) + (x^2 + 2)y(x) - 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0372219 (sec), leaf count = 74

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}e^{-ix}x \left(e^{2ix} \log(1 + e^{-2ix}) + \log(1 + e^{2ix}) \right) + c_1e^{-ix}x - \frac{1}{2}ic_2e^{ix}x \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 23

$$\{y(x) = (c_1 \cos(x) + \cos(x) \ln(\cos(x)) + (c_2 + x) \sin(x))x\}$$

2.1179 ODE No. 1179

$$(a^2x^2 + 2)y(x) + x^2y''(x) - 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0178435 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow c_1xe^{-iax} - \frac{ic_2xe^{iax}}{2a} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 19

$$\{y(x) = (c_1 \sin(ax) + c_2 \cos(ax))x\}$$

2.1180 ODE No. 1180

$$-f(x) + (-v^2 + x^2 + 1)y(x) + x^2y''(x) + 3xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.065767 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow \frac{J_v(x) \int_1^x -\frac{1}{2}\pi Y_v(K[1])f(K[1])dK[1] + Y_v(x) \int_1^x \frac{1}{2}\pi J_v(K[2])f(K[2])dK[2]}{x} + \frac{c_1J_v(x)}{x} + \frac{c_2Y_v(x)}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.065 (sec), leaf count = 49

$$\left\{ y(x) = \frac{2c_1 \operatorname{BesselY}(v, x) + 2c_2 \operatorname{BesselJ}(v, x) - \pi \operatorname{BesselJ}(v, x) \left(\int \operatorname{BesselY}(v, x) f(x) dx \right) + \pi \operatorname{BesselY}(v, x) \left(\int \operatorname{BesselJ}(v, x) f(x) dx \right)}{2x} \right\}$$

2.1181 ODE No. 1181

$$x^2y''(x) + (3x - 1)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0467607 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1e^{-1/x}}{x} - \frac{c_2e^{-1/x}\operatorname{Ei}\left(\frac{1}{x}\right)}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 25

$$\left\{ y(x) = \frac{\left(c_1 \operatorname{Ei}\left(1, -\frac{1}{x}\right) + c_2 \right) e^{-\frac{1}{x}}}{x} \right\}$$

2.1182 ODE No. 1182

$$x^2 y''(x) - 3xy'(x) + 4y(x) - 5x = 0$$

✓ **Mathematica** : cpu = 0.0108615 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow c_1 x^2 + 2c_2 x^2 \log(x) + 5x \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 20

$$\{y(x) = c_1 x^2 \ln(x) + c_2 x^2 + 5x\}$$

2.1183 ODE No. 1183

$$x^2 y''(x) + x^2(-\log(x)) - 3xy'(x) - 5y(x) = 0$$

✓ **Mathematica** : cpu = 0.0096156 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow c_2 x^5 - \frac{1}{9} x^2 \log(x) + \frac{c_1}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 22

$$\left\{ y(x) = c_2 x^5 - \frac{x^2 \ln(x)}{9} + \frac{c_1}{x} \right\}$$

2.1184 ODE No. 1184

$$-x^4 + x^2 y''(x) + x^2 - 4xy'(x) + 6y(x) = 0$$

✓ **Mathematica** : cpu = 0.0079021 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow c_2 x^3 + c_1 x^2 + \frac{1}{2} (x^4 + 2x^2 + 2x^2 \log(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 25

$$\left\{ y(x) = \frac{(2c_2 x + x^2 + 2c_1 + 2 \ln(x) + 2)x^2}{2} \right\}$$

2.1185 ODE No. 1185

$$(4 - 2x^3)y(x) + x^2 y''(x) + 5xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0303128 (sec), leaf count = 67

$$\left\{ \left(y(x) \rightarrow \frac{3\sqrt[3]{6} c_2 K_0\left(\frac{2}{3}\sqrt{2} x^{3/2}\right)}{x^2} - \frac{3\sqrt[3]{-3} c_1 I_0\left(\frac{2}{3}\sqrt{2} x^{3/2}\right)}{2^{2/3} x^2} \right) \right\}$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 33

$$\left\{ y(x) = \frac{c_1 \text{BesselI}\left(0, \frac{2\sqrt{2} x^{3/2}}{3}\right) + c_2 \text{BesselK}\left(0, \frac{2\sqrt{2} x^{3/2}}{3}\right)}{x^2} \right\}$$

2.1186 ODE No. 1186

$$x^3(-\sin(x)) + x^2 y''(x) - 5x y'(x) + 8y(x) = 0$$

✓ **Mathematica** : cpu = 0.0156754 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left(x^4 \text{Ci}(x) - x^3 \sin(x) + x^2 \cos(x) \right) + c_2 x^4 + c_1 x^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 36

$$\left\{ y(x) = \frac{x^4 \text{Ci}(x)}{2} - \frac{x^3 \sin(x)}{2} + \frac{(2c_2 x^2 + 2c_1 + \cos(x))x^2}{2} \right\}$$

2.1187 ODE No. 1187

$$axy'(x) + by(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0099878 (sec), leaf count = 99

$$\left\{ \left\{ y(x) \rightarrow c_1 x^{\frac{1}{2} \sqrt{b} \left(-\frac{\sqrt{a^2 - 2a - 4b + 1}}{\sqrt{b}} - \frac{a-1}{\sqrt{b}} \right)} + c_2 x^{\frac{1}{2} \sqrt{b} \left(\frac{\sqrt{a^2 - 2a - 4b + 1}}{\sqrt{b}} - \frac{a-1}{\sqrt{b}} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 53

$$\left\{ y(x) = c_1 x^{-\frac{a}{2} + \frac{1}{2} + \frac{\sqrt{a^2 - 2a - 4b + 1}}{2}} + c_2 x^{-\frac{a}{2} + \frac{1}{2} - \frac{\sqrt{a^2 - 2a - 4b + 1}}{2}} \right\}$$

2.1188 ODE No. 1188

$$(ax + b)y'(x) + cy(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.169052 (sec), leaf count = 266

$$\left\{ \left\{ y(x) \rightarrow c_1 i^{-\sqrt{a^2 - 2a - 4c + 1} + a - 1} b^{\frac{1}{2}(-\sqrt{a^2 - 2a - 4c + 1} + a - 1)} \left(\frac{1}{x} \right)^{\frac{1}{2}(-\sqrt{a^2 - 2a - 4c + 1} + a - 1)} {}_1F_1\left(\frac{a}{2} - \frac{1}{2} \sqrt{a^2 - 2a - 4c + 1} - \frac{1}{2}; 1 - \sqrt{a^2 - 2a - 4c + 1} - \frac{1}{2}; -\frac{b}{x^2}\right) + c_2 i^{\sqrt{a^2 - 2a - 4c + 1} - a + 1} b^{\frac{1}{2}(\sqrt{a^2 - 2a - 4c + 1} - a + 1)} \left(\frac{1}{x} \right)^{\frac{1}{2}(\sqrt{a^2 - 2a - 4c + 1} - a + 1)} {}_1F_1\left(\frac{a}{2} - \frac{1}{2} \sqrt{a^2 - 2a - 4c + 1} + \frac{1}{2}; 1 - \sqrt{a^2 - 2a - 4c + 1} + \frac{1}{2}; -\frac{b}{x^2}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.273 (sec), leaf count = 114

$$\left\{ y(x) = \left(c_1 \text{KummerM}\left(\frac{a}{2} - \frac{1}{2} + \frac{\sqrt{a^2 - 2a - 4c + 1}}{2}, 1 + \sqrt{a^2 - 2a - 4c + 1}, \frac{b}{x}\right) + c_2 \text{KummerU}\left(\frac{a}{2} - \frac{1}{2} + \frac{\sqrt{a^2 - 2a - 4c + 1}}{2}, 1 + \sqrt{a^2 - 2a - 4c + 1}, \frac{b}{x}\right) \right) \right\}$$

2.1189 ODE No. 1189

$$axy'(x) + y(x)(bx^m + c) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0581196 (sec), leaf count = 445

$$\left\{ \left\{ y(x) \rightarrow c_1 m^{-\frac{-\sqrt{a^2 - 2a - 4c + 1} - a + 1}{m} - \frac{\sqrt{a^2 - 2a - 4c + 1}}{m}} b^{\frac{-\sqrt{a^2 - 2a - 4c + 1} - a + 1}{2m} + \frac{\sqrt{a^2 - 2a - 4c + 1}}{2m}} (x^m)^{\frac{-\sqrt{a^2 - 2a - 4c + 1} - a + 1}{2m} + \frac{\sqrt{a^2 - 2a - 4c + 1}}{2m}} \Gamma\left(1 - \frac{\sqrt{a^2 - 2a - 4c + 1}}{m}\right) + c_2 m^{-\frac{-\sqrt{a^2 - 2a - 4c + 1} + a - 1}{m} - \frac{\sqrt{a^2 - 2a - 4c + 1}}{m}} b^{\frac{-\sqrt{a^2 - 2a - 4c + 1} + a - 1}{2m} + \frac{\sqrt{a^2 - 2a - 4c + 1}}{2m}} (x^m)^{\frac{-\sqrt{a^2 - 2a - 4c + 1} + a - 1}{2m} + \frac{\sqrt{a^2 - 2a - 4c + 1}}{2m}} \Gamma\left(1 + \frac{\sqrt{a^2 - 2a - 4c + 1}}{m}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.164 (sec), leaf count = 79

$$\left\{ y(x) = \left(c_1 \text{BesselJ}\left(\frac{\sqrt{a^2 - 2a - 4c + 1}}{m}, \frac{2\sqrt{b} x^{\frac{m}{2}}}{m}\right) + c_2 \text{BesselY}\left(\frac{\sqrt{a^2 - 2a - 4c + 1}}{m}, \frac{2\sqrt{b} x^{\frac{m}{2}}}{m}\right) \right) x^{-\frac{a}{2} + \frac{1}{2}} \right\}$$

2.1190 ODE No. 1190

$$y(x)(ax + b) + x^2y''(x) + x^2y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0230653 (sec), leaf count = 122

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{2}((\sqrt{1-4b}+1)\log(x)-2x)} U\left(\frac{1}{2}\left(-2a + \sqrt{1-4b} + 1\right), \sqrt{1-4b} + 1, x\right) + c_2 e^{\frac{1}{2}((\sqrt{1-4b}+1)\log(x)-2x)} L_{\frac{1}{2}(2a-\sqrt{1-4b}-1)}^{\sqrt{1-4b}}(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.192 (sec), leaf count = 38

$$\left\{ y(x) = \left(c_1 \operatorname{WhittakerM}\left(a, \frac{\sqrt{-4b+1}}{2}, x\right) + c_2 \operatorname{WhittakerW}\left(a, \frac{\sqrt{-4b+1}}{2}, x\right) \right) e^{-\frac{x}{2}} \right\}$$

2.1191 ODE No. 1191

$$x^2y''(x) + x^2y'(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0083381 (sec), leaf count = 110

$$\left\{ \left\{ y(x) \rightarrow \frac{2c_2 e^{\frac{1}{2}(\log(x)-x)} \left(i \sinh\left(\frac{x}{2}\right) - \frac{2i \cosh\left(\frac{x}{2}\right)}{x} \right)}{\sqrt{\pi} \sqrt{-ix}} + \frac{2c_1 e^{\frac{1}{2}(\log(x)-x)} \left(\frac{2 \sinh\left(\frac{x}{2}\right)}{x} - \cosh\left(\frac{x}{2}\right) \right)}{\sqrt{\pi} \sqrt{-ix}} \right\} \right\}$$

✓ **Maple** : cpu = 0.015 (sec), leaf count = 23

$$\left\{ y(x) = \frac{c_2(x+2)e^{-x} + c_1(x-2)}{x} \right\}$$

2.1192 ODE No. 1192

$$x^2y''(x) + (x^2 - 1)y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0887329 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow c_2 e^{-x} \int_1^x e^{K[1] - \frac{1}{K[1]}} dK[1] + c_1 e^{-x} \right\} \right\}$$

✓ **Maple** : cpu = 0.22 (sec), leaf count = 51

$$\left\{ y(x) = \left(c_1 HD\left(4, 3, -8, 5, \frac{x-1}{x+1}\right) e^{-x} + c_2 HD\left(-4, 3, -8, 5, \frac{x-1}{x+1}\right) e^{-\frac{1}{x}} \right) \sqrt{x} \right\}$$

2.1193 ODE No. 1193

$$x^2y''(x) + (x+1)xy'(x) + (x-9)y(x) = 0$$

✓ **Mathematica** : cpu = 0.213288 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1((x-8)x+20)}{x^3} - \frac{c_2 e^{-x}(x^3+9x^2+36x+60)}{x^3} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 38

$$\left\{ y(x) = \frac{c_2(x^3+9x^2+36x+60)e^{-x} + c_1(x^2-8x+20)}{x^3} \right\}$$

2.1194 ODE No. 1194

$$x^2 y''(x) + (x+1)xy'(x) + (3x-1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0999675 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-x} (x-3)x - \frac{c_2 e^{-x} (x^3 (-\text{Ei}(x)) + 3x^2 \text{Ei}(x) + e^x x^2 - 2e^x x - e^x)}{6x} \right\} \right\}$$

✓ **Maple** : cpu = 0.048 (sec), leaf count = 48

$$\left\{ y(x) = \frac{c_2 (x-3)x^2 \text{Ei}(1, -x)e^{-x} + c_1 (x-3)x^2 e^{-x} + c_2 (x^2 - 2x - 1)}{x} \right\}$$

2.1195 ODE No. 1195

$$x^2 y''(x) + (x+3)xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0293922 (sec), leaf count = 80

$$\left\{ \left\{ y(x) \rightarrow c_1 U(2 + \sqrt{2}, 1 + 2\sqrt{2}, x) e^{(\sqrt{2}-1)\log(x)-x} + c_2 L_{-2-\sqrt{2}}^{2\sqrt{2}}(x) e^{(\sqrt{2}-1)\log(x)-x} \right\} \right\}$$

✓ **Maple** : cpu = 0.23 (sec), leaf count = 93

$$\left\{ y(x) = - \frac{\left(-c_1 (x + \sqrt{2} + 1) \text{BesselI}\left(-\frac{1}{2} + \sqrt{2}, \frac{x}{2}\right) - c_1 (x - \sqrt{2} + 1) \text{BesselI}\left(\frac{1}{2} + \sqrt{2}, \frac{x}{2}\right) + c_2 \left((-x - \sqrt{2} - 1) \text{BesselK}\left(-\frac{1}{2} + \sqrt{2}, \frac{x}{2}\right) - (-x + \sqrt{2} - 1) \text{BesselK}\left(\frac{1}{2} + \sqrt{2}, \frac{x}{2}\right) \right) \right)}{\sqrt{x}} \right\}$$

2.1196 ODE No. 1196

$$x^2 y''(x) - (x-1)xy'(x) + (x-1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0294775 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 (x^2 \text{Ei}(x) - e^x x - e^x)}{2x} + c_1 x \right\} \right\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 31

$$\left\{ y(x) = \frac{c_2 x^2 \text{Ei}(1, -x) + c_1 x^2 + c_2 (x+1)e^x}{x} \right\}$$

2.1197 ODE No. 1197

$$(-a-x)y(x) + x^2 y''(x) - (x^2 - 2x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0152671 (sec), leaf count = 78

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{2}(x-\log(x))} J_{\frac{1}{2}\sqrt{4a+1}}\left(-\frac{ix}{2}\right) + c_2 e^{\frac{1}{2}(x-\log(x))} Y_{\frac{1}{2}\sqrt{4a+1}}\left(-\frac{ix}{2}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.132 (sec), leaf count = 43

$$\left\{ y(x) = \frac{\left(c_1 \text{BesselI}\left(\frac{\sqrt{4a+1}}{2}, \frac{x}{2}\right) + c_2 \text{BesselK}\left(\frac{\sqrt{4a+1}}{2}, \frac{x}{2}\right) \right) e^{\frac{x}{2}}}{\sqrt{x}} \right\}$$

2.1198 ODE No. 1198

$$x^2 y''(x) - (x^2 - 2x)y'(x) + (-3x - 2)y(x) = 0$$

✓ **Mathematica** : cpu = 0.04673 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow c_1 e^x x - \frac{c_2 (e^x x^3 \text{Ei}(-x) + x^2 - x + 2)}{6x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 37

$$\left\{ y(x) = \frac{c_2 x^3 \text{Ei}(1, x) e^x + c_1 x^3 e^x - c_2 (x^2 - x + 2)}{x^2} \right\}$$

2.1199 ODE No. 1199

$$x^2 y''(x) - (x + 4)x y'(x) + 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.0147162 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow c_2 e^x x^4 - \frac{1}{6} c_1 x (e^x x^3 \text{Ei}(-x) + x^2 - x + 2) \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 35

$$\{y(x) = (c_2 x^3 \text{Ei}(1, x) e^x + c_1 x^3 e^x - c_2 (x^2 - x + 2))x\}$$

2.1200 ODE No. 1200

$$(1 - v) v y(x) + x^2 y''(x) + 2x^2 y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0164944 (sec), leaf count = 62

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-x} \sqrt{x} J_{\frac{1}{2}(2v-1)}(-ix) + c_2 e^{-x} \sqrt{x} Y_{\frac{1}{2}(2v-1)}(-ix) \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 27

$$\left\{ y(x) = \left(c_1 \text{BesselI}\left(v - \frac{1}{2}, x\right) + c_2 \text{BesselK}\left(v - \frac{1}{2}, x\right) \right) \sqrt{x} e^{-x} \right\}$$

2.1201 ODE No. 1201

$$x^2 y''(x) + (2x + 1)x y'(x) - 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.119682 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-2x} (2x + 3)}{2x^2} + \frac{c_2 (2x^2 - 4x + 3)}{4x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 34

$$\left\{ y(x) = \frac{c_2 (2x + 3) e^{-2x}}{x^2} + \frac{c_1 (2x^2 - 4x + 3)}{x^2} \right\}$$

2.1202 ODE No. 1202

$$x^2y''(x) - 2(x+1)xy'(x) + 2(x+1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0124109 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow c_1x + \frac{1}{2}c_2e^{2x}x \right\} \right\}$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 14

$$\{y(x) = (c_2e^{2x} + c_1)x\}$$

2.1203 ODE No. 1203

$$ax^2y'(x) + x^2y''(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0161188 (sec), leaf count = 124

$$\left\{ \left(\left(y(x) \rightarrow \frac{2c_2e^{\frac{1}{2}(\log(x)-ax)} \left(i \sinh\left(\frac{ax}{2}\right) - \frac{2i \cosh\left(\frac{ax}{2}\right)}{ax} \right)}{\sqrt{\pi} \sqrt{-iax}} + \frac{2c_1e^{\frac{1}{2}(\log(x)-ax)} \left(\frac{2 \sinh\left(\frac{ax}{2}\right)}{ax} - \cosh\left(\frac{ax}{2}\right) \right)}{\sqrt{\pi} \sqrt{-iax}} \right) \right) \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 28

$$\left\{ y(x) = \frac{c_2(ax+2)e^{-ax} + c_1(ax-2)}{x} \right\}$$

2.1204 ODE No. 1204

$$x^2(a+2b)y'(x) + y(x)(bx^2(a+b)-2) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0164735 (sec), leaf count = 132

$$\left\{ \left(\left(y(x) \rightarrow \frac{2c_2e^{\frac{1}{2}(-ax-2bx+\log(x))} \left(i \sinh\left(\frac{ax}{2}\right) - \frac{2i \cosh\left(\frac{ax}{2}\right)}{ax} \right)}{\sqrt{\pi} \sqrt{-iax}} + \frac{2c_1e^{\frac{1}{2}(-ax-2bx+\log(x))} \left(\frac{2 \sinh\left(\frac{ax}{2}\right)}{ax} - \cosh\left(\frac{ax}{2}\right) \right)}{\sqrt{\pi} \sqrt{-iax}} \right) \right) \right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 35

$$\left\{ y(x) = \frac{c_1(ax-2)e^{-bx} + c_2(ax+2)e^{-(a+b)x}}{x} \right\}$$

2.1205 ODE No. 1205

$$ax^2y'(x) + f(x)y(x) + x^2y''(x) = 0$$

✗ **Mathematica** : cpu = 0.26733 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*y[x] + a*x^2*Derivative[1][y][x] + x^2*Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol\left(\left\{ a\left(\frac{d}{dx}Y(x)\right) + \frac{d^2}{dx^2}Y(x) + \frac{-Y(x)f(x)}{x^2} \right\}, \{-Y(x)\} \right) \right\}$$

2.1206 ODE No. 1206

$$y(x)(abx + cx^2 + d) + x(2ax + b)y'(x) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0881391 (sec), leaf count = 120

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{2}(-2ax - (b-1)\log(x))} J_{\frac{1}{2}\sqrt{b^2 - 2b - 4d + 1}} \left(-i\sqrt{a^2 - c} x \right) + c_2 e^{\frac{1}{2}(-2ax - (b-1)\log(x))} Y_{\frac{1}{2}\sqrt{b^2 - 2b - 4d + 1}} \left(-i\sqrt{a^2 - c} x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.081 (sec), leaf count = 76

$$\left\{ y(x) = \left(c_1 \text{BesselJ} \left(\frac{\sqrt{b^2 - 2b - 4d + 1}}{2}, \sqrt{-a^2 + c} x \right) + c_2 \text{BesselY} \left(\frac{\sqrt{b^2 - 2b - 4d + 1}}{2}, \sqrt{-a^2 + c} x \right) \right) x^{-\frac{b}{2} + \frac{1}{2}} e^{-ax} \right\}$$

2.1207 ODE No. 1207

$$x(ax + b)y'(x) + y(x)(a1x^2 + b1x + c1) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.090494 (sec), leaf count = 294

$$\left\{ \left\{ y(x) \rightarrow c_1 U \left(-\frac{-ab + 2b1 - \sqrt{a^2 - 4a1} - \sqrt{a^2 - 4a1} \sqrt{b^2 - 2b - 4c1 + 1}}{2\sqrt{a^2 - 4a1}}, \sqrt{b^2 - 2b - 4c1 + 1} + 1, \sqrt{a^2 - 4a1} x \right) \exp \left(\frac{1}{2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.333 (sec), leaf count = 110

$$\left\{ y(x) = \left(c_1 \text{WhittakerM} \left(-\frac{ab - 2b1}{2\sqrt{a^2 - 4a1}}, \frac{\sqrt{b^2 - 2b - 4c1 + 1}}{2}, \sqrt{a^2 - 4a1} x \right) + c_2 \text{WhittakerW} \left(-\frac{ab - 2b1}{2\sqrt{a^2 - 4a1}}, \frac{\sqrt{b^2 - 2b - 4c1 + 1}}{2}, \sqrt{a^2 - 4a1} x \right) \right) \right\}$$

2.1208 ODE No. 1208

$$x^3y'(x) + x^2y''(x) + (x^2 - 2)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0470233 (sec), leaf count = 59

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 e^{-\frac{x^2}{2}} \left(\sqrt{2\pi} e^{\frac{x^2}{2}} \text{erf} \left(\frac{x}{\sqrt{2}} \right) - 2x \right)}{2x} + \frac{c_1}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 35

$$\left\{ y(x) = \frac{-2c_2 x e^{-\frac{x^2}{2}} + c_2 \sqrt{\pi} \sqrt{2} \text{erf} \left(\frac{\sqrt{2} x}{2} \right) + c_1}{x} \right\}$$

2.1209 ODE No. 1209

$$x^2y''(x) + (x^2 + 2)xy'(x) + (x^2 - 2)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0175744 (sec), leaf count = 67

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-\frac{x^2}{2}} \left(e^{\frac{x^2}{2}} x - \sqrt{\frac{\pi}{2}} \text{erfi} \left(\frac{x}{\sqrt{2}} \right) \right)}{x^2} + \frac{c_2 e^{-\frac{x^2}{2}}}{x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.073 (sec), leaf count = 41

$$\left\{ y(x) = \frac{ic_2 \sqrt{\pi} \sqrt{2} x + \left(-c_2 \pi \text{erf} \left(\frac{i\sqrt{2} x}{2} \right) + c_1 \right) e^{-\frac{x^2}{2}}}{x^2} \right\}$$

2.1210 ODE No. 1210

$$y(x)\big(a((-1)^n-1)+2nx^2\big)-2x\big(x^2-a\big)y'(x)+x^2y''(x)=0$$

✓ **Mathematica** : cpu = 0.22301 (sec), leaf count = 252

$$\left\{\left\{y(x)\rightarrow c_1(-1)^{\frac{1}{4}}\Big(-\sqrt{4a^2-4a(-1)^{n+1}-2a+1}\Big)^{\frac{1}{2}}\Big(-\sqrt{4a^2-4a(-1)^{n+1}-2a+1}\Big) {}_1F_1\left(-\frac{a}{2}-\frac{n}{2}-\frac{1}{4}\sqrt{4a^2-4(-1)^na+1}+\frac{1}{4};1-\frac{1}{2}\sqrt{4a^2-4(-1)^na+1}\right),x^2\right\}\right\}$$

✓ **Maple** : cpu = 0.281 (sec), leaf count = 81

$$\left\{y(x)=\left(c_1\operatorname{WhittakerM}\left(\frac{a}{2}+\frac{n}{2}+\frac{1}{4},\frac{\sqrt{4a^2-4a(-1)^n+1}}{4},x^2\right)+c_2\operatorname{WhittakerW}\left(\frac{a}{2}+\frac{n}{2}+\frac{1}{4},\frac{\sqrt{4a^2-4a(-1)^n+1}}{4},x^2\right)\right)\right\}$$

2.1211 ODE No. 1211

$$4x^3y'(x)+x^2y''(x)+\big(4x^4+2x^2+1\big)y(x)=0$$

✓ **Mathematica** : cpu = 0.0541553 (sec), leaf count = 68

$$\left\{\left\{y(x)\rightarrow c_1e^{-x^2}x^{\frac{1}{2}-\frac{i\sqrt{3}}{2}}-\frac{ic_2e^{-x^2}x^{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}{\sqrt{3}}\right\}\right\}$$

✓ **Maple** : cpu = 0.065 (sec), leaf count = 36

$$\left\{y(x)=\left(c_1x^{\frac{1}{2}+\frac{i\sqrt{3}}{2}}+c_2x^{\frac{1}{2}-\frac{i\sqrt{3}}{2}}\right)e^{-x^2}\right\}$$

2.1212 ODE No. 1212

$$x\big(ax^2+b\big)y'(x)+f(x)y(x)+x^2y''(x)=0$$

✗ **Mathematica** : cpu = 0.554137 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*y[x] + x*(b + a*x^2)*Derivative[1][y][x] + x^2*Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x)=DESol\left[\left\{\frac{d^2}{dx^2}Y(x)+\frac{\left(ax^2+b\right)\left(\frac{d}{dx}Y(x)\right)}{x}+\frac{-Y(x)f(x)}{x^2}\right\},\{-Y(x)\}\right]\right\}$$

2.1213 ODE No. 1213

$$\big(x^3+1\big)xy'(x)+x^2y''(x)-y(x)=0$$

✓ **Mathematica** : cpu = 0.0876336 (sec), leaf count = 54

$$\left\{\left\{y(x)\rightarrow \frac{\sqrt[3]{3}c_1{}_1F_1\left(-\frac{1}{3};\frac{1}{3};-\frac{x^3}{3}\right)}{x}+\frac{c_2x{}_1F_1\left(\frac{1}{3};\frac{5}{3};-\frac{x^3}{3}\right)}{\sqrt[3]{3}}\right\}\right\}$$

✓ **Maple** : cpu = 0.234 (sec), leaf count = 53

$$\left\{y(x)=\left(c_1\operatorname{BesselI}\left(-\frac{1}{6},\frac{x^3}{6}\right)+c_1\operatorname{BesselI}\left(\frac{5}{6},\frac{x^3}{6}\right)-c_2\left(\operatorname{BesselK}\left(\frac{1}{6},\frac{x^3}{6}\right)-\operatorname{BesselK}\left(\frac{5}{6},\frac{x^3}{6}\right)\right)\right)x^{\frac{3}{2}}e^{-\frac{x^3}{6}}\right\}$$

2.1214 ODE No. 1214

$$y(x)\big(-a^2+x^2(2a+2n+1)+a(-1)^n-x^4\big)+x^2y''(x)=0$$

✓ **Mathematica** : cpu = 0.226946 (sec), leaf count = 260

$$\left\{\left\{y(x)\rightarrow\frac{c_1e^{-\frac{x^2}{2}}2^{\frac{1}{4}}\left(\sqrt{4a^2-4a(-1)^n+1}+2\right)\left(x^2\right)^{\frac{1}{4}}\left(\sqrt{4a^2-4a(-1)^n+1}+2\right)U\left(\frac{1}{4}\left(-2a-2n+\sqrt{4a^2-4(-1)^na+1}+1\right),\frac{1}{2}\left(\sqrt{4a^2-4(-1)^n}\right)\right)}{\sqrt{x}}\right.\right.$$

✓ **Maple** : cpu = 0.26 (sec), leaf count = 71

$$\left\{y(x)=\frac{c_1\operatorname{WhittakerM}\left(\frac{a}{2}+\frac{n}{2}+\frac{1}{4},\frac{\sqrt{4a^2-4a(-1)^n+1}}{4},x^2\right)+c_2\operatorname{WhittakerW}\left(\frac{a}{2}+\frac{n}{2}+\frac{1}{4},\frac{\sqrt{4a^2-4a(-1)^n+1}}{4},x^2\right)}{\sqrt{x}}\right\}$$

2.1215 ODE No. 1215

$$xy'(x)(ax^n+b)+y(x)\big(a1x^{2n}+b1x^n+c1\big)+x^2y''(x)=0$$

✓ **Mathematica** : cpu = 0.12285 (sec), leaf count = 664

$$\left\{\left\{y(x)\rightarrow c_1x^{\frac{1-n}{2}}2^{\frac{\sqrt{b^2n^2-2bn^2-4c1n^2+n^2}+n^2}{2n^2}}(x^n)^{\frac{\sqrt{b^2n^2-2bn^2-4c1n^2+n^2}+n^2}{2n^2}}\exp\left(\frac{1}{2}\left(-\frac{ax^n}{n}-b\log(x)\right)-\frac{\sqrt{a^2-4a1}x^n}{2n}\right)U\left(\frac{\sqrt{(b^2-2b-4c1+1)}}{n^2}\right)}\right.\right.$$

✓ **Maple** : cpu = 0.318 (sec), leaf count = 148

$$\left\{y(x)=\left(c_1\operatorname{WhittakerM}\left(-\frac{(b+n-1)a-2b1}{2\sqrt{a^2-4a1}n},\frac{\sqrt{b^2-2b-4c1+1}}{2n},\frac{\sqrt{a^2-4a1}x^n}{n}\right)+c_2\operatorname{WhittakerW}\left(-\frac{(b+n-1)a-2b1}{2\sqrt{a^2-4a1}n},\frac{\sqrt{b^2-2b-4c1+1}}{2n},\frac{\sqrt{a^2-4a1}x^n}{n}\right)\right)$$

2.1216 ODE No. 1216

$$xy'(x)\big(ax^{a1}+b\big)+y(x)\big(Ax^{2a1}+Bx^{a1}+Cx^{b1}+DD\big)+x^2y''(x)=0$$

✗ **Mathematica** : cpu = 1.28436 (sec), leaf count = 0 , could not solve

DSolve[(DD + B*x^a1 + A*x^(2*a1) + C*x^b1)*y[x] + x*(b + a*x^a1)*Derivative[1][y][x] + x^2*D

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x)=DESol\left[\left\{\frac{d^2}{dx^2}Y(x)+\frac{\left(ax^{a1}+b\right)\left(\frac{d}{dx}Y(x)\right)}{x}+\frac{\left(Ax^{2a1}+Bx^{a1}+Cx^{b1}+DD\right)Y(x)}{x^2}\right\},\{Y(x)\}\right]\right\}$$

2.1217 ODE No. 1217

$$y(x)(-a-x\tan(x))+x^2y''(x)-\big(2x^2\tan(x)-x\big)y'(x)=0$$

✓ **Mathematica** : cpu = 0.12077 (sec), leaf count = 30

$$\left\{\left\{y(x)\rightarrow c_1\sec(x)J_{\sqrt{a}}(x)+c_2\sec(x)Y_{\sqrt{a}}(x)\right\}\right\}$$

✓ **Maple** : cpu = 0.045 (sec), leaf count = 24

$$\left\{y(x)=\frac{c_1\operatorname{BesselJ}\left(\sqrt{a},x\right)+c_2\operatorname{BesselY}\left(\sqrt{a},x\right)}{\cos(x)}\right\}$$

2.1218 ODE No. 1218

$$y(x)(a + x \cot(x)) + x^2 y''(x) + (2x^2 \cot(x) + x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.125385 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow c_1 \csc(x) J_{i\sqrt{a}}(x) + c_2 \csc(x) Y_{i\sqrt{a}}(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.047 (sec), leaf count = 30

$$\left\{ y(x) = \frac{c_1 \text{BesselJ}(i\sqrt{a}, x) + c_2 \text{BesselY}(i\sqrt{a}, x)}{\sin(x)} \right\}$$

2.1219 ODE No. 1219

$$y(x)(ax^2 + bx + c + xf'(x) + f(x)^2 - f(x)) + 2xf(x)y'(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.143317 (sec), leaf count = 218

$$\left\{ \left\{ y(x) \rightarrow c_1 U\left(-\frac{-ib - \sqrt{a} - \sqrt{a}\sqrt{1-4c}}{2\sqrt{a}}, \sqrt{1-4c} + 1, 2i\sqrt{a}x\right) \exp\left(\int_1^x \frac{-2f(K[1]) - 2i\sqrt{a}K[1] + \sqrt{1-4c} + 1}{2K[1]} dK[1]\right) + \right. \right.$$

✓ **Maple** : cpu = 0.231 (sec), leaf count = 69

$$\left\{ y(x) = \left(c_1 \text{WhittakerM}\left(-\frac{ib}{2\sqrt{a}}, \frac{\sqrt{-4c+1}}{2}, 2i\sqrt{a}x\right) + c_2 \text{WhittakerW}\left(-\frac{ib}{2\sqrt{a}}, \frac{\sqrt{-4c+1}}{2}, 2i\sqrt{a}x\right) \right) e^{-\left(\int \frac{f(x)}{x} dx\right)} \right\}$$

2.1220 ODE No. 1220

$$y(x)(x^2(a + f'(x) + f(x)^2) + (1-v)v) + 2x^2 f(x)y'(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0477669 (sec), leaf count = 98

$$\left\{ \left\{ y(x) \rightarrow c_1 J_{\frac{1}{2}(2v-1)}(\sqrt{a}x) \exp\left(\int_1^x \frac{1-2f(K[1])K[1]}{2K[1]} dK[1]\right) + c_2 Y_{\frac{1}{2}(2v-1)}(\sqrt{a}x) \exp\left(\int_1^x \frac{1-2f(K[1])K[1]}{2K[1]} dK[1]\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 40

$$\left\{ y(x) = \left(c_1 \text{BesselJ}\left(v - \frac{1}{2}, \sqrt{a}x\right) + c_2 \text{BesselY}\left(v - \frac{1}{2}, \sqrt{a}x\right) \right) \sqrt{x} e^{-\frac{\left(\int 2f(x)dx\right)}{2}} \right\}$$

2.1221 ODE No. 1221

$$y(x)(x^2(-f'(x) + f(x)^2 + 1) - xf(x) - v^2) + (x - 2x^2 f(x))y'(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0286243 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow c_1 J_v(x) \exp\left(\int_1^x f(K[1])dK[1]\right) + c_2 Y_v(x) \exp\left(\int_1^x f(K[1])dK[1]\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 35

$$\left\{ y(x) = (c_1 \text{BesselJ}(v, x) + c_2 \text{BesselY}(v, x)) \sqrt{x} e^{-\frac{\left(\int \frac{-2xf(x)+1}{x} dx\right)}{2}} \right\}$$

2.1222 ODE No. 1222

$$(x^2 + 1)y''(x) + xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0162585 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow c_1 \cos(\sqrt{2} \sinh^{-1}(x)) + c_2 \sin(\sqrt{2} \sinh^{-1}(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 23

$$\{y(x) = c_1 \sin(\sqrt{2} \operatorname{arcsinh}(x)) + c_2 \cos(\sqrt{2} \operatorname{arcsinh}(x))\}$$

2.1223 ODE No. 1223

$$(x^2 + 1)y''(x) + xy'(x) - 9y(x) = 0$$

✓ **Mathematica** : cpu = 0.0155568 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow ic_2 \sinh(3 \sinh^{-1}(x)) + c_1 \cosh(3 \sinh^{-1}(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 19

$$\{y(x) = c_1 \sinh(3 \operatorname{arcsinh}(x)) + c_2 \cosh(3 \operatorname{arcsinh}(x))\}$$

2.1224 ODE No. 1224

$$ay(x) + (x^2 + 1)y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0146912 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow c_1 \cos(\sqrt{a} \sinh^{-1}(x)) + c_2 \sin(\sqrt{a} \sinh^{-1}(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 23

$$\{y(x) = c_1 \sin(\sqrt{a} \operatorname{arcsinh}(x)) + c_2 \cos(\sqrt{a} \operatorname{arcsinh}(x))\}$$

2.1225 ODE No. 1225

$$(x^2 + 1)y''(x) - xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0220558 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow c_2 \left(x \sinh^{-1}(x) - \sqrt{x^2 + 1} \right) + c_1 x \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 23

$$\left\{ y(x) = -c_2 \sqrt{x^2 + 1} + (c_2 \operatorname{arcsinh}(x) + c_1)x \right\}$$

2.1226 ODE No. 1226

$$(1 - v)vy(x) + (x^2 + 1)y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0122555 (sec), leaf count = 30

$$\{y(x) \rightarrow c_1 P_{v-1}(ix) + c_2 Q_{v-1}(ix)\}$$

✓ **Maple** : cpu = 0.167 (sec), leaf count = 25

$$\{y(x) = c_1 \operatorname{LegendreP}(v - 1, ix) + c_2 \operatorname{LegendreQ}(v - 1, ix)\}$$

2.1227 ODE No. 1227

$$(x^2 + 1)y''(x) - 2xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0268058 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow c_2 x - c_1 (x - i)^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 16

$$\{y(x) = c_2 x^2 + c_1 x - c_2\}$$

2.1228 ODE No. 1228

$$ay(x) + (x^2 + 1)y''(x) + 3xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0135503 (sec), leaf count = 82

$$\left\{ \left(\left\{ y(x) \rightarrow \frac{c_1 P_{\frac{1}{2}}^{\frac{1}{2}}(2\sqrt{1-a}-1)(ix)}{\sqrt[4]{x^2+1}} + \frac{c_2 Q_{\frac{1}{2}}^{\frac{1}{2}}(2\sqrt{1-a}-1)(ix)}{\sqrt[4]{x^2+1}} \right\} \right) \right\}$$

✓ **Maple** : cpu = 0.253 (sec), leaf count = 53

$$\left\{ y(x) = \frac{c_1 \left(x + \sqrt{x^2 + 1} \right)^{\sqrt{-a+1}} + c_2 \left(x + \sqrt{x^2 + 1} \right)^{-\sqrt{-a+1}}}{\sqrt{x^2 + 1}} \right\}$$

2.1229 ODE No. 1229

$$(x^2 + 1)y''(x) + 4xy'(x) + 2y(x) + 2x - 2\cos(x) = 0$$

✓ **Mathematica** : cpu = 0.032205 (sec), leaf count = 48

$$\left\{ \left(\left\{ y(x) \rightarrow \frac{c_1}{x^2 + 1} + \frac{c_2 x}{x^2 + 1} + \frac{-x^3 - 6\cos(x)}{3(x^2 + 1)} \right\} \right) \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 31

$$\left\{ y(x) = \frac{-x^3 + 3c_1 x + 3c_2 - 6\cos(x)}{3x^2 + 3} \right\}$$

2.1230 ODE No. 1230

$$axy'(x) + (a - 2)y(x) + (x^2 + 1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0203666 (sec), leaf count = 82

$$\left\{ \left(\left\{ y(x) \rightarrow c_1 \left(x^2 + 1 \right)^{\frac{2-a}{4}} P_{\frac{a-2}{a-4}}^{\frac{a-2}{2}}(ix) + c_2 \left(x^2 + 1 \right)^{\frac{2-a}{4}} Q_{\frac{a-4}{2}}^{\frac{a-2}{2}}(ix) \right\} \right) \right\}$$

✓ **Maple** : cpu = 0.214 (sec), leaf count = 36

$$\left\{ y(x) = c_2 x \operatorname{hypergeom} \left(\left[1, \frac{a}{2} - \frac{1}{2} \right], \left[\frac{3}{2} \right], -x^2 \right) + c_1 \left(x^2 + 1 \right)^{-\frac{a}{2}+1} \right\}$$

2.1231 ODE No. 1231

$$(x^2 - 1)y''(x) - v(v + 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0674738 (sec), leaf count = 58

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1\left(-\frac{v}{2} - \frac{1}{2}, \frac{v}{2}; \frac{1}{2}; x^2\right) + ic_2 x {}_2F_1\left(\frac{v}{2} + \frac{1}{2}, -\frac{v}{2}; \frac{3}{2}; x^2\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.19 (sec), leaf count = 52

$$\left\{ y(x) = -(x - 1)(x + 1) \left(c_2 x \operatorname{hypergeom}\left(\left[-\frac{v}{2} + 1, \frac{v}{2} + \frac{3}{2}\right], \left[\frac{3}{2}\right], x^2\right) + c_1 \operatorname{hypergeom}\left(\left[\frac{v}{2} + 1, -\frac{v}{2} + \frac{1}{2}\right], \left[\frac{1}{2}\right], x^2\right) \right) \right\}$$

2.1232 ODE No. 1232

$$\frac{nxP_n(x) - nP_{n-1}(x)}{x^2 - 1} - n(n + 1)y(x) + (x^2 - 1)y''(x) = 0$$

✓ **Mathematica** : cpu = 8.03994 (sec), leaf count = 6628

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1\left(-\frac{n}{2} - \frac{1}{2}, \frac{n}{2}; \frac{1}{2}; x^2\right) + \int_1^x \frac{4 \left({}_2F_1\left(\frac{1}{2}(-n - 1), \frac{n}{2}; \frac{1}{2}; K[1]^2\right) {}_2F_1\left(1 - \frac{n}{2}, \frac{n+3}{2}; \frac{5}{2}; K[1]^2\right) n^2 + {}_2F_1\left(\frac{1}{2}(-n - 1), \frac{n}{2}; \frac{1}{2}; K[1]^2\right) \right)}{4 \left({}_2F_1\left(\frac{1}{2}(-n - 1), \frac{n}{2}; \frac{1}{2}; K[1]^2\right) {}_2F_1\left(1 - \frac{n}{2}, \frac{n+3}{2}; \frac{5}{2}; K[1]^2\right) n^2 + {}_2F_1\left(\frac{1}{2}(-n - 1), \frac{n}{2}; \frac{1}{2}; K[1]^2\right) \right)} dx \right\} \right\}$$

✓ **Maple** : cpu = 0.28 (sec), leaf count = 409

$$\left\{ y(x) = 3(x - 1)(x + 1) \left(-\frac{c_1 x \operatorname{hypergeom}\left(\left[-\frac{n}{2} + 1, \frac{n}{2} + \frac{3}{2}\right], \left[\frac{3}{2}\right], x^2\right)}{3} + (n + 1)x \operatorname{hypergeom}\left(\left[-\frac{n}{2} + 1, \frac{n}{2} + \frac{3}{2}\right], \left[\frac{3}{2}\right], x^2\right) \right) \right\}$$

2.1233 ODE No. 1233

$$\frac{nxQ_n(x) - nQ_{n-1}(x)}{x^2 - 1} - n(n + 1)y(x) + (x^2 - 1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.541844 (sec), leaf count = 6628

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1\left(-\frac{n}{2} - \frac{1}{2}, \frac{n}{2}; \frac{1}{2}; x^2\right) + \int_1^x \frac{4 \left({}_2F_1\left(\frac{1}{2}(-n - 1), \frac{n}{2}; \frac{1}{2}; K[1]^2\right) {}_2F_1\left(1 - \frac{n}{2}, \frac{n+3}{2}; \frac{5}{2}; K[1]^2\right) n^2 + {}_2F_1\left(\frac{1}{2}(-n - 1), \frac{n}{2}; \frac{1}{2}; K[1]^2\right) \right)}{4 \left({}_2F_1\left(\frac{1}{2}(-n - 1), \frac{n}{2}; \frac{1}{2}; K[1]^2\right) {}_2F_1\left(1 - \frac{n}{2}, \frac{n+3}{2}; \frac{5}{2}; K[1]^2\right) n^2 + {}_2F_1\left(\frac{1}{2}(-n - 1), \frac{n}{2}; \frac{1}{2}; K[1]^2\right) \right)} dx \right\} \right\}$$

✓ **Maple** : cpu = 0.255 (sec), leaf count = 409

$$\left\{ y(x) = 3(x - 1)(x + 1) \left(-\frac{c_1 x \operatorname{hypergeom}\left(\left[-\frac{n}{2} + 1, \frac{n}{2} + \frac{3}{2}\right], \left[\frac{3}{2}\right], x^2\right)}{3} + (n + 1)x \operatorname{hypergeom}\left(\left[-\frac{n}{2} + 1, \frac{n}{2} + \frac{3}{2}\right], \left[\frac{3}{2}\right], x^2\right) \right) \right\}$$

2.1234 ODE No. 1234

$$(x^2 - 1)y''(x) + xy'(x) + 2 = 0$$

✓ **Mathematica** : cpu = 0.0787783 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow c_2 - \frac{1}{4} \left(\log\left(1 - \frac{x}{\sqrt{x^2 - 1}}\right) - \log\left(\frac{x}{\sqrt{x^2 - 1}} + 1\right) + c_1 \right)^2 \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0, could not solve

`dsolve((x^2-1)*diff(diff(y(x),x),x)+x*diff(y(x),x)+2=0,y(x))`

2.1235 ODE No. 1235

$$ay(x) + (x^2 - 1)y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0414979 (sec), leaf count = 97

$$\left\{ \left\{ y(x) \rightarrow c_1 \cos \left(\frac{1}{2} \sqrt{a} \left(\log \left(1 - \frac{x}{\sqrt{x^2 - 1}} \right) - \log \left(\frac{x}{\sqrt{x^2 - 1}} + 1 \right) \right) \right) - c_2 \sin \left(\frac{1}{2} \sqrt{a} \left(\log \left(1 - \frac{x}{\sqrt{x^2 - 1}} \right) - \log \left(\frac{x}{\sqrt{x^2 - 1}} + 1 \right) \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 45

$$\left\{ y(x) = \left(c_1 \left(x + \sqrt{x^2 - 1} \right)^{2i\sqrt{a}} + c_2 \right) \left(x + \sqrt{x^2 - 1} \right)^{-i\sqrt{a}} \right\}$$

2.1236 ODE No. 1236

$$f(x)y(x) + (x^2 - 1)y''(x) + xy'(x) = 0$$

✗ **Mathematica** : cpu = 0.527588 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*y[x] + x*Derivative[1][y][x] + (-1 + x^2)*Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left[\left\{ \frac{x \left(\frac{d}{dx} Y(x) \right)}{x^2 - 1} + \frac{Y(x)f(x)}{x^2 - 1} + \frac{d^2}{dx^2} Y(x) \right\}, \{ -Y(x) \} \right] \right\}$$

2.1237 ODE No. 1237

$$(x^2 - 1)y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0250307 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow c_1 \left(\frac{1}{2} \log(1 - x) - \frac{1}{2} \log(x + 1) \right) + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 20

$$\left\{ y(x) = c_1 - \frac{c_2(\ln(x + 1) - \ln(x - 1))}{2} \right\}$$

2.1238 ODE No. 1238

$$-a + (x^2 - 1)y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0311523 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}(a + c_1) \log(1 - x) + \frac{1}{2}(a - c_1) \log(x + 1) + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 26

$$\left\{ y(x) = c_2 + \frac{(-c_1 + a) \ln(x + 1)}{2} + \frac{(c_1 + a) \ln(x - 1)}{2} \right\}$$

2.1239 ODE No. 1239

$$-ly(x) + (x^2 - 1)y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0112002 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow c_1 P_{\frac{1}{2}(\sqrt{4l+1}-1)}(x) + c_2 Q_{\frac{1}{2}(\sqrt{4l+1}-1)}(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.168 (sec), leaf count = 35

$$\left\{ y(x) = c_1 \text{LegendreP}\left(\frac{\sqrt{4l+1}}{2} - \frac{1}{2}, x\right) + c_2 \text{LegendreQ}\left(\frac{\sqrt{4l+1}}{2} - \frac{1}{2}, x\right) \right\}$$

2.1240 ODE No. 1240

$$-v(v+1)y(x) + (x^2 - 1)y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.014132 (sec), leaf count = 18

$$\{ \{ y(x) \rightarrow c_1 P_v(x) + c_2 Q_v(x) \} \}$$

✓ **Maple** : cpu = 0.15 (sec), leaf count = 15

$$\{ y(x) = c_1 \text{LegendreP}(v, x) + c_2 \text{LegendreQ}(v, x) \}$$

2.1241 ODE No. 1241

$$(1-v)(v+2)y(x) + (x^2 - 1)y''(x) - 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0129755 (sec), leaf count = 30

$$\{ \{ y(x) \rightarrow c_1 (x^2 - 1) P_v^2(x) + c_2 (x^2 - 1) Q_v^2(x) \} \}$$

✓ **Maple** : cpu = 0.16 (sec), leaf count = 24

$$\{ y(x) = (x-1)(x+1) (c_1 \text{LegendreP}(v, 2, x) + c_2 \text{LegendreQ}(v, 2, x)) \}$$

2.1242 ODE No. 1242

$$(x^2 - 1)y''(x) + (x - x^2)y(x) - (3x + 1)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.226209 (sec), leaf count = 68

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-x}(x+1)^2 - c_2 e^{-x-2} \left(x^2 (-\text{Ei}(2(x+1))) - 2x \text{Ei}(2(x+1)) - \text{Ei}(2(x+1)) + 2e^{2x+2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.074 (sec), leaf count = 41

$$\{ y(x) = c_2 (x+1)^2 \text{Ei}(1, -2x-2) e^{-x-2} + c_1 (x+1)^2 e^{-x} + 2c_2 e^x \}$$

2.1243 ODE No. 1243

$$(x^2 - 1)y''(x) + (x^2 + 1)y(x) + 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0366797 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-ix}}{x^2 - 1} - \frac{ic_2 e^{ix}}{2(x^2 - 1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 21

$$\left\{ y(x) = \frac{c_1 \sin(x) + c_2 \cos(x)}{x^2 - 1} \right\}$$

2.1244 ODE No. 1244

$$(n-v)(n+v+1)y(x) + 2(n+1)xy'(x) + (x^2-1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0238543 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow c_1 (x^2-1)^{-n/2} P_v^n(x) + c_2 (x^2-1)^{-n/2} Q_v^n(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.161 (sec), leaf count = 27

$$\left\{ y(x) = (c_1 \text{LegendreP}(v, n, x) + c_2 \text{LegendreQ}(v, n, x)) (x^2-1)^{-\frac{n}{2}} \right\}$$

2.1245 ODE No. 1245

$$(n-v-1)(n+v)y(x) - 2(n-1)xy'(x) + (x^2-1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0197263 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow c_1 (x^2-1)^{n/2} P_v^n(x) + c_2 (x^2-1)^{n/2} Q_v^n(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.166 (sec), leaf count = 27

$$\left\{ y(x) = (c_1 \text{LegendreP}(v, n, x) + c_2 \text{LegendreQ}(v, n, x)) (x^2-1)^{\frac{n}{2}} \right\}$$

2.1246 ODE No. 1246

$$-2(v-1)xy'(x) - 2vy(x) + (x^2-1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0153973 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow c_1 (x^2-1)^{v/2} P_v^v(x) + c_2 (x^2-1)^{v/2} Q_v^v(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.224 (sec), leaf count = 28

$$\left\{ y(x) = \left(c_2 x \text{hypergeom} \left(\left[\frac{1}{2}, v+1 \right], \left[\frac{3}{2} \right], x^2 \right) + c_1 \right) (x^2-1)^v \right\}$$

2.1247 ODE No. 1247

$$2axy'(x) + (a-1)ay(x) + (x^2-1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.18418 (sec), leaf count = 128

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \sqrt{1-x^2} (x+1)^{\sqrt{(a-1)^2}} (x^2-1)^{-a/2} (1-x)^{-\sqrt{(a-1)^2}} e^{-\sqrt{(a-1)^2} \tanh^{-1}(x)}}{2\sqrt{(a-1)^2}} + c_1 \sqrt{1-x^2} (x^2-1)^{-a/2} e^{-\sqrt{(a-1)^2} \tanh^{-1}(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 27

$$\{y(x) = c_1 (x-1)^{-a+1} + c_2 (x+1)^{-a+1}\}$$

2.1248 ODE No. 1248

$$axy'(x) + y(x)(bx^2 + cx + d) + (x^2 - 1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.29482 (sec), leaf count = 238

$$\left\{ \left\{ y(x) \rightarrow c_2 \left(\frac{x}{2} - \frac{1}{2} \right)^{a/4} (x^2 - 1)^{-a/4} \left(\frac{x}{2} + \frac{1}{2} \right)^{1-\frac{a}{4}} e^{\sqrt{-b}x} \text{HeunC} \left[\frac{1}{4} a \left(a - 4\sqrt{-b} - 2 \right) - b + 4\sqrt{-b} + c - d, 2 \left(2\sqrt{-b} + c \right), 2 - \frac{a}{2} \right] \right. \right.$$

✓ **Maple** : cpu = 0.293 (sec), leaf count = 134

$$\left\{ y(x) = \left(c_2 \left(\frac{x}{2} - \frac{1}{2} \right)^{\frac{a}{4}} \left(\frac{x}{2} + \frac{1}{2} \right)^{-\frac{a}{4}+1} \text{HeunC} \left(4\sqrt{-b}, -\frac{a}{2} + 1, \frac{a}{2} - 1, 2c, -\frac{a^2}{8} + b - c + d + \frac{1}{2}, \frac{x}{2} + \frac{1}{2} \right) + c_1((x-1)(x+1))^{\frac{a}{4}} \right.$$

2.1249 ODE No. 1249

$$(ax + b)y'(x) + cy(x) + (x^2 - 1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.141176 (sec), leaf count = 193

$$\left\{ \left\{ y(x) \rightarrow c_2 2^{\frac{1}{2}(a+b-2)} (x-1)^{\frac{1}{2}(-a-b+2)} {}_2F_1 \left(-\frac{b}{2} - \frac{1}{2} \sqrt{a^2 - 2a - 4c + 1} + \frac{1}{2}, -\frac{b}{2} + \frac{1}{2} \sqrt{a^2 - 2a - 4c + 1} + \frac{1}{2}; -\frac{a}{2} - \frac{b}{2} + 2; \frac{1}{2} \right) \right. \right.$$

✓ **Maple** : cpu = 0.18 (sec), leaf count = 134

$$\left\{ y(x) = c_2 \left(\frac{x}{2} + \frac{1}{2} \right)^{-\frac{a}{2} + \frac{b}{2} + 1} \text{hypergeom} \left(\left[\frac{b}{2} + \frac{1}{2} + \frac{\sqrt{a^2 - 2a - 4c + 1}}{2}, \frac{b}{2} + \frac{1}{2} - \frac{\sqrt{a^2 - 2a - 4c + 1}}{2} \right], \left[-\frac{a}{2} + \frac{b}{2} + 2 \right], \frac{x}{2} + \frac{1}{2} \right) \right.$$

2.1250 ODE No. 1250

$$(x^2 - a^2)y''(x) + 8xy'(x) + 12y(x) = 0$$

✓ **Mathematica** : cpu = 0.0415534 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2(a^2 + 3x^2)}{3(a-x)^3(a+x)^3} + \frac{c_1}{(a+x)^3} \right\} \right\}$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 41

$$\left\{ y(x) = \frac{3c_2a^2x + c_2x^3 + c_1a^2 + 3c_1x^2}{(a-x)^3(a+x)^3} \right\}$$

2.1251 ODE No. 1251

$$x(x+1)y''(x) - (x-1)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0377426 (sec), leaf count = 25

$$\{ \{ y(x) \rightarrow c_1(x-1) + c_2(x \log(x) - \log(x) - 4) \} \}$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 20

$$\{ y(x) = c_2(x-1) \ln(x) + c_1(x-1) - 4c_2 \}$$

2.1252 ODE No. 1252

$$(ax + b)y'(x) + cy(x) + x(x + 1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.130369 (sec), leaf count = 151

$$\left\{ \left\{ y(x) \rightarrow c_2 x^{1-b} {}_2F_1\left(\frac{a}{2} - b - \frac{1}{2}\sqrt{a^2 - 2a - 4c + 1} + \frac{1}{2}, \frac{a}{2} - b + \frac{1}{2}\sqrt{a^2 - 2a - 4c + 1} + \frac{1}{2}; 2 - b; -x\right) + c_1 {}_2F_1\left(\frac{a}{2} - \frac{1}{2}\sqrt{a^2 - 2a - 4c + 1} + \frac{1}{2}, \frac{a}{2} - b + \frac{1}{2}\sqrt{a^2 - 2a - 4c + 1} + \frac{1}{2}; 2 - b; -x\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.145 (sec), leaf count = 124

$$\left\{ y(x) = c_2 (x + 1)^{-a+b+1} \operatorname{hypergeom}\left(\left[-\frac{a}{2} + b + \frac{1}{2} + \frac{\sqrt{a^2 - 2a - 4c + 1}}{2}, -\frac{a}{2} + b + \frac{1}{2} - \frac{\sqrt{a^2 - 2a - 4c + 1}}{2}\right], [-a + b + 2, \dots]\right) \right\}$$

2.1253 ODE No. 1253

$$x(x + 1)y''(x) + (3x + 2)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0217195 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{2} c_1}{x} + \frac{c_2 \log(2x + 2)}{\sqrt{2} x} \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 16

$$\left\{ y(x) = \frac{c_1 \ln(x + 1) + c_2}{x} \right\}$$

2.1254 ODE No. 1254

$$(x^2 + x - 2)y''(x) + (x^2 - x)y'(x) + (-6x^2 - 7x)y(x) = 0$$

✓ **Mathematica** : cpu = 0.150906 (sec), leaf count = 69

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{5} c_2 e^{-3x-5} (195 e^{5x} x \operatorname{Ei}(5 - 5x) - 195 e^{5x} \operatorname{Ei}(5 - 5x) + e^{5x} x + 44 e^5) - c_1 e^{2x} (x - 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.074 (sec), leaf count = 43

$$\{y(x) = 195 c_2 (x - 1) \operatorname{Ei}(1, 5x - 5) e^{2x-5} + c_1 (x - 1) e^{2x} - c_2 (x + 44) e^{-3x}\}$$

2.1255 ODE No. 1255

$$ay'(x) + (x - 1)xy''(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.714749 (sec), leaf count = 360

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 x^a (a^2 + 2ax - a + 2x^2 - 2x) (1 - x)^{-a} \left(-\frac{{}_2F_1\left(1, -a; 1 - a; \frac{(-a + \sqrt{1 - a^2} + 1)(x - 1)}{(-a + \sqrt{1 - a^2} - 1)x}\right)}{(1 - a^2)^{3/2}} + \frac{{}_2F_1\left(1, -a; 1 - a; \frac{(a + \sqrt{1 - a^2} - 1)(x - 1)}{(a + \sqrt{1 - a^2} + 1)x}\right)}{(1 - a^2)^{3/2}} + \frac{(x - 1) \left(\sqrt{1 - a^2} \right)}{2a^2 (a^2 + 3a + 4)} \right)}{2a^2 (a^2 + 3a + 4)} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 42

$$\{y(x) = c_2 (x - 1) x x^a (x - 1)^{-a} + c_1 (a^2 + 2x^2 + (2x - 1)a - 2x)\}$$

2.1256 ODE No. 1256

$$-v(v+1)y(x) + (x-1)xy''(x) + (2x-1)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0184036 (sec), leaf count = 26

$$\{y(x) \rightarrow c_1 P_v(2x-1) + c_2 Q_v(2x-1)\}$$

✓ **Maple** : cpu = 0.258 (sec), leaf count = 51

$$\left\{y(x) = c_1 x^v \operatorname{hypergeom}\left([-v, -v], [-2v], \frac{1}{x}\right) + c_2 x^{-v-1} \operatorname{hypergeom}\left([v+1, v+1], [2v+2], \frac{1}{x}\right)\right\}$$

2.1257 ODE No. 1257

$$((a+1)x+b)y'(x) + (x-1)xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.043411 (sec), leaf count = 33

$$\left\{\left\{y(x) \rightarrow \frac{c_1 x^{b+1} {}_2F_1(b+1, a+b+1; b+2; x)}{b+1} + c_2\right\}\right\}$$

✓ **Maple** : cpu = 0.219 (sec), leaf count = 27

$$\{y(x) = c_2 x^{b+1} \operatorname{hypergeom}([b+1, a+b+1], [b+2], x) + c_1\}$$

2.1258 ODE No. 1258

$$(ax+b)y'(x) + cy(x) + (x-1)xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.134097 (sec), leaf count = 146

$$\left\{\left\{y(x) \rightarrow (-1)^{b+1} c_2 x^{b+1} {}_2F_1\left(\frac{a}{2} + b - \frac{1}{2} \sqrt{a^2 - 2a - 4c + 1} + \frac{1}{2}, \frac{a}{2} + b + \frac{1}{2} \sqrt{a^2 - 2a - 4c + 1} + \frac{1}{2}; b+2; x\right) + c_1 {}_2F_1\left(\frac{a}{2} - \frac{1}{2} \sqrt{a^2 - 2a - 4c + 1} + \frac{1}{2}, \frac{a}{2} + b + \frac{1}{2} \sqrt{a^2 - 2a - 4c + 1} + \frac{1}{2}; b+2; x\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.155 (sec), leaf count = 110

$$\left\{y(x) = c_2 x^{b+1} \operatorname{hypergeom}\left(\left[\frac{a}{2} + b + \frac{1}{2} + \frac{\sqrt{a^2 - 2a - 4c + 1}}{2}, \frac{a}{2} + b + \frac{1}{2} - \frac{\sqrt{a^2 - 2a - 4c + 1}}{2}\right], [b+2], x\right) + c_1 \operatorname{hypergeom}\left(\left[\frac{a}{2} + b + \frac{1}{2} - \frac{\sqrt{a^2 - 2a - 4c + 1}}{2}, \frac{a}{2} + b + \frac{1}{2} + \frac{\sqrt{a^2 - 2a - 4c + 1}}{2}\right], [b+2], x\right)\right\}$$

2.1259 ODE No. 1259

$$((a+1)x+b)y'(x) - ly(x) + (x-1)xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.116602 (sec), leaf count = 120

$$\left\{\left\{y(x) \rightarrow (-1)^{b+1} c_2 x^{b+1} {}_2F_1\left(\frac{a}{2} + b - \frac{1}{2} \sqrt{a^2 + 4l} + 1, \frac{a}{2} + b + \frac{1}{2} \sqrt{a^2 + 4l} + 1; b+2; x\right) + c_1 {}_2F_1\left(\frac{a}{2} - \frac{1}{2} \sqrt{a^2 + 4l} + 1, \frac{a}{2} + b + \frac{1}{2} \sqrt{a^2 + 4l} + 1; b+2; x\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.16 (sec), leaf count = 92

$$\left\{y(x) = c_2 x^{b+1} \operatorname{hypergeom}\left(\left[\frac{a}{2} + b - \frac{\sqrt{a^2 + 4l}}{2} + 1, \frac{a}{2} + b + \frac{\sqrt{a^2 + 4l}}{2} + 1\right], [b+2], x\right) + c_1 \operatorname{hypergeom}\left(\left[\frac{a}{2} - \frac{\sqrt{a^2 + 4l}}{2} + 1, \frac{a}{2} + b + \frac{\sqrt{a^2 + 4l}}{2} + 1\right], [b+2], x\right)\right\}$$

2.1260 ODE No. 1260

$$y'(x)(x(a1 + b1 + 1) - d1) + a1b1d1 + (x - 1)xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.246186 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow a1b1x\Gamma(d1 + 1) {}_3\tilde{F}_2(1, a1 + b1 + 1, 1; d1 + 1, 2; x) - \frac{c_1x^{1-d1} {}_2F_1(1 - d1, a1 + b1 - d1 + 1; 2 - d1; x)}{d1 - 1} + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.712 (sec), leaf count = 77

$$\{y(x) = c_2 + \int -\left(a1b1(-\text{signum}(x - 1))^{-a1-b1+d1}\text{signum}(x - 1)^{a1+b1-d1}\text{hypergeom}([d1, -a1 - b1 + d1], [d1 + 1], x) - c_1\right)dx\}$$

2.1261 ODE No. 1261

$$y(x)(2lx(-n + p - 1) + 2lp + m) + 2(x(-2l + n + 1) - lx^2 + n + 1)y'(x) + x(x + 2)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.476033 (sec), leaf count = 148

$$\left\{ \left\{ y(x) \rightarrow c_2\left(-\frac{x}{2} - 1\right)^{\frac{n}{2} + \frac{1}{2}}x^{-n}(x + 2)^{-\frac{n}{2} - \frac{1}{2}}\text{HeunC}\left[-4ln - 2lp - m + n^2 + n, -4l(p - 1), 1 - n, n + 1, 4l, -\frac{x}{2}\right] + c_1\left(-\frac{x}{2} - 1\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.302 (sec), leaf count = 105

$$\left\{ y(x) = \left(c_2x^{-n}\text{HeunC}\left(4l, -n, n, -4lp, -\frac{n^2}{2} + \frac{(4n + 4p + 4)l}{2} + m - n, -\frac{x}{2}\right) + c_1\text{HeunC}\left(4l, n, n, -4lp, -\frac{n^2}{2} + \frac{(4n + 4p + 4)l}{2} + m - n, -\frac{x}{2}\right) \right) \right\}$$

2.1262 ODE No. 1262

$$(x^2 + x - 1)y'(x) + (x + 1)^2y''(x) + (-x - 2)y(x) = 0$$

✓ **Mathematica** : cpu = 0.294084 (sec), leaf count = 88

$$\left\{ \left\{ y(x) \rightarrow c_2e^{-x} \int_1^x \exp\left(-\frac{K[1]^2}{K[1] + 1} - \frac{K[1]}{K[1] + 1} + 2K[1] - \frac{1}{K[1] + 1}\right)(K[1] + 1)^{\frac{K[1]}{K[1] + 1} + \frac{1}{K[1] + 1}}dK[1] + c_1e^{-x} \right\} \right\}$$

✓ **Maple** : cpu = 0.273 (sec), leaf count = 53

$$\left\{ y(x) = (x + 1)\left(c_1HD\left(4, 4, -8, 12, \frac{x}{x + 2}\right)e^{-x} + c_2HD\left(-4, 4, -8, 12, \frac{x}{x + 2}\right)e^{\frac{x-1}{2x+2}}\right) \right\}$$

2.1263 ODE No. 1263

$$(-20x - 30)(x^2 + 3x)^{7/3} + x(x + 3)y''(x) + (3x - 1)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.429478 (sec), leaf count = 276

$$\left\{ \left\{ y(x) \rightarrow \frac{4x^{4/3} \int_1^x \left(45\sqrt[3]{3} {}_2F_1\left(-\frac{4}{3}, -\frac{4}{3}; -\frac{1}{3}; -\frac{K[1]}{3}\right)\sqrt[3]{K[1](K[1] + 3)}K[1]^{11/3} + \frac{675}{2}\sqrt[3]{3} {}_2F_1\left(-\frac{4}{3}, -\frac{4}{3}; -\frac{1}{3}; -\frac{K[1]}{3}\right)\sqrt[3]{K[1](K[1] + 3)}\right)dx}{(x^2 + 3x)x^{4/3}} + c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.138 (sec), leaf count = 52

$$\left\{ y(x) = \frac{\left(c_2 + \int \frac{\left(c_1 + 3(x^2 + 3x)^{\frac{7}{3}}(x + 3)x\right)(x + 3)^{\frac{7}{3}}}{(x^2 + 3x)x^{\frac{4}{3}}}dx\right)x^{\frac{4}{3}}}{(x + 3)^{\frac{7}{3}}} \right\}$$

2.1264 ODE No. 1264

$$(x^2 + 3x + 4)y''(x) + (x^2 + x + 1)y'(x) + (-2x - 3)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0570627 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow c_2(x^2 + x + 3) + c_1 e^{-x} \right\} \right\}$$

✓ **Maple** : cpu = 0.041 (sec), leaf count = 19

$$\{y(x) = c_1 e^{-x} + c_2(x^2 + x + 3)\}$$

2.1265 ODE No. 1265

$$(x - 2)(x - 1)y''(x) - (2x - 3)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0360861 (sec), leaf count = 64

$$\left\{ \left\{ y(x) \rightarrow c_1(x^2 - 3x + 2)P_{\frac{1}{2}(-1+\sqrt{5})}^2(2x - 3) + c_2(x^2 - 3x + 2)Q_{\frac{1}{2}(-1+\sqrt{5})}^2(2x - 3) \right\} \right\}$$

✓ **Maple** : cpu = 1.132 (sec), leaf count = 93

$$\left\{ y(x) = (x - 2)^2 \left(c_1(x - 1)^{\frac{\sqrt{5}}{2} - \frac{1}{2}} \text{hypergeom} \left(\left[\frac{1}{2} - \frac{\sqrt{5}}{2}, \frac{5}{2} - \frac{\sqrt{5}}{2} \right], [-\sqrt{5} + 1], \frac{1}{x - 1} \right) + c_2(x - 1)^{-\frac{1}{2} - \frac{\sqrt{5}}{2}} \text{hypergeom} \left(\left[\frac{1}{2} \right. \right. \right.$$

2.1266 ODE No. 1266

$$(x - 2)^2 y''(x) - (x - 2)y'(x) - 3y(x) = 0$$

✓ **Mathematica** : cpu = 0.0233969 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow c_1(x - 2)^3 + \frac{c_2}{x - 2} \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 19

$$\left\{ y(x) = \frac{c_1(x - 2)^4 + c_2}{x - 2} \right\}$$

2.1267 ODE No. 1267

$$-(l + 2x^2 - 5x)y'(x) + 2x^2 y''(x) + (1 - 4x)y(x) = 0$$

✓ **Mathematica** : cpu = 0.413805 (sec), leaf count = 166

$$\left\{ \left(y(x) \rightarrow \frac{c_1 e^{x - \frac{l}{2x}}}{\sqrt{x}} - \frac{\sqrt{\frac{\pi}{2}} c_2 e^{-\frac{l}{2x} - \sqrt{2} \sqrt{-l} + x} \left(\text{erf} \left(\frac{\sqrt{-l}}{\sqrt{2} \sqrt{x}} - \sqrt{x} \right) + e^{2\sqrt{2} \sqrt{-l}} \text{erf} \left(\frac{\sqrt{-l}}{\sqrt{2} \sqrt{x}} + \sqrt{x} \right) - e^{2\sqrt{2} \sqrt{-l}} + 1 \right)}{\sqrt{-l} \sqrt{x}} \right) \right\}$$

✓ **Maple** : cpu = 0.134 (sec), leaf count = 41

$$\left(y(x) = \frac{\left(c_1 \left(\int \frac{e^{-x} e^{\frac{l}{2x}}}{2x^{\frac{3}{2}}} dx \right) + c_2 \right) e^x e^{-\frac{l}{2x}}}{\sqrt{x}} \right)$$

2.1268 ODE No. 1268

$$y(x)(ax + b) + 2(x - 1)xy''(x) + (2x - 1)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.112443 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow c_1 \text{MathieuC}\left[-a - 2b, \frac{a}{2}, \cos^{-1}(\sqrt{x})\right] + c_2 \text{MathieuS}\left[-a - 2b, \frac{a}{2}, \cos^{-1}(\sqrt{x})\right] \right\} \right\}$$

✓ **Maple** : cpu = 0.248 (sec), leaf count = 39

$$\left\{ y(x) = c_1 \text{MathieuC}\left(-a - 2b, \frac{a}{2}, \arccos(\sqrt{x})\right) + c_2 \text{MathieuS}\left(-a - 2b, \frac{a}{2}, \arccos(\sqrt{x})\right) \right\}$$

2.1269 ODE No. 1269

$$((2v + 5)x - 2v - 3)y'(x) + (v + 1)y(x) + 2(x - 1)xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0736393 (sec), leaf count = 60

$$\left\{ \left\{ y(x) \rightarrow c_2 i^{-2v-1} x^{\frac{1}{2}(-2v-1)} {}_2F_1\left(\frac{1}{2}, -v; \frac{1}{2} - v; x\right) + c_1 {}_2F_1\left(\frac{1}{2}, v + 1; v + \frac{3}{2}; x\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.221 (sec), leaf count = 40

$$\left\{ y(x) = c_2 x^{-v-\frac{1}{2}} \text{hypergeom}\left(\left[\frac{1}{2}, -v\right], \left[-v + \frac{1}{2}\right], x\right) + c_1 \text{hypergeom}\left(\left[\frac{1}{2}, v + 1\right], \left[v + \frac{3}{2}\right], x\right) \right\}$$

2.1270 ODE No. 1270

$$(2x^2 + 6x + 4)y''(x) + (10x^2 + 21x + 8)y'(x) + (12x^2 + 17x + 8)y(x) = 0$$

✓ **Mathematica** : cpu = 2.04565 (sec), leaf count = 58

$$\left\{ \left\{ y(x) \rightarrow c_2 e^{-3x} (x + 2)^4 \int_1^x \frac{e^{K[1]} (K[1] + 1)^{3/2}}{(K[1] + 2)^5} dK[1] + c_1 e^{-3x} (x + 2)^4 \right\} \right\}$$

✓ **Maple** : cpu = 0.243 (sec), leaf count = 46

$$\left\{ y(x) = (x + 2)^4 \left(c_1 \text{HeunC}\left(-1, -\frac{5}{2}, 4, -\frac{7}{4}, \frac{7}{2}, -x - 1\right) + c_2 (x + 1)^{\frac{5}{2}} \text{HeunC}\left(-1, \frac{5}{2}, 4, -\frac{7}{4}, \frac{7}{2}, -x - 1\right) \right) e^{-2x} \right\}$$

2.1271 ODE No. 1271

$$4x^2 y''(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0099623 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt{x} + \frac{1}{2} c_2 \sqrt{x} \log(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 14

$$\{y(x) = (c_2 \ln(x) + c_1) \sqrt{x}\}$$

2.1272 ODE No. 1272

$$(4a^2 x^2 + 1)y(x) + 4x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0109969 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt{x} J_0(ax) + c_2 \sqrt{x} Y_0(ax) \right\} \right\}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 23

$$\{y(x) = (c_1 \text{BesselJ}(0, ax) + c_2 \text{BesselY}(0, ax)) \sqrt{x}\}$$

2.1273 ODE No. 1273

$$y(x)\big(4kx - 4m^2 - x^2 + 1\big) + 4x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0158697 (sec), leaf count = 20

$$\{ \{y(x) \rightarrow c_1 M_{k,m}(x) + c_2 W_{k,m}(x) \} \}$$

✓ **Maple** : cpu = 0.151 (sec), leaf count = 17

$$\{y(x) = c_1 \operatorname{WhittakerM}(k, m, x) + c_2 \operatorname{WhittakerW}(k, m, x)\}$$

2.1274 ODE No. 1274

$$(x - v^2)y(x) + 4x^2y''(x) + 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0426757 (sec), leaf count = 38

$$\{ \{y(x) \rightarrow c_1 \Gamma(1 - v) J_{-v}(\sqrt{x}) + c_2 \Gamma(v + 1) J_v(\sqrt{x}) \} \}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 19

$$\{y(x) = c_1 \operatorname{BesselJ}(v, \sqrt{x}) + c_2 \operatorname{BesselY}(v, \sqrt{x})\}$$

2.1275 ODE No. 1275

$$y(x)\big(2x(2l - m + 1) - m^2 - x^2 + 1\big) + 4x^2y''(x) + 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0282949 (sec), leaf count = 120

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{2}(\sqrt{m^2-1} \log(x)-x)} U\left(\frac{1}{2}(-2l+m+\sqrt{m^2-1}), \sqrt{m^2-1}+1, x\right) + c_2 e^{\frac{1}{2}(\sqrt{m^2-1} \log(x)-x)} L_{\frac{1}{2}(2l-m-\sqrt{m^2-1})}^{\sqrt{m^2-1}}(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.262 (sec), leaf count = 53

$$\left\{ y(x) = \frac{c_1 \operatorname{WhittakerM}\left(l - \frac{m}{2} + \frac{1}{2}, \frac{\sqrt{m-1} \sqrt{m+1}}{2}, x\right) + c_2 \operatorname{WhittakerW}\left(l - \frac{m}{2} + \frac{1}{2}, \frac{\sqrt{m-1} \sqrt{m+1}}{2}, x\right)}{\sqrt{x}} \right\}$$

2.1276 ODE No. 1276

$$-4e^x \sqrt{x^3} + 4x^2y''(x) - (4x^2 + 1)y(x) + 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0333451 (sec), leaf count = 55

$$\left\{ \left\{ y(x) \rightarrow \frac{e^x \sqrt{x^3} (2x - 1)}{4x^2} + \frac{c_1 e^{-x}}{\sqrt{x}} + \frac{c_2 e^x}{2\sqrt{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.181 (sec), leaf count = 31

$$\left\{ y(x) = \frac{c_1 \cosh(x)}{\sqrt{x}} + \frac{c_2 \sinh(x)}{\sqrt{x}} + \frac{\sqrt{x^3} e^x}{2x} \right\}$$

2.1277 ODE No. 1277

$$(-ax^2 - 1)y(x) + 4x^2y''(x) + 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0235184 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-\frac{\sqrt{a}x}{2}}}{\sqrt{x}} + \frac{c_2 e^{\frac{\sqrt{a}x}{2}}}{\sqrt{a}\sqrt{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.168 (sec), leaf count = 27

$$\left\{ y(x) = \frac{c_1 \sinh\left(\frac{\sqrt{a}x}{2}\right) + c_2 \cosh\left(\frac{\sqrt{a}x}{2}\right)}{\sqrt{x}} \right\}$$

2.1278 ODE No. 1278

$$f(x)y(x) + 4x^2y''(x) + 4xy'(x) = 0$$

✗ **Mathematica** : cpu = 0.405725 (sec), leaf count = 0 , could not solve

DSolve[f[x]*y[x] + 4*x*Derivative[1][y][x] + 4*x^2*Derivative[2][y][x] == 0, y[x], x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left[\left\{ \frac{d^2}{dx^2} Y(x) + \frac{\frac{d}{dx} Y(x)}{x} + \frac{-Y(x)f(x)}{4x^2} \right\}, \{ -Y(x) \} \right] \right\}$$

2.1279 ODE No. 1279

$$4x^2y''(x) + 5xy'(x) - y(x) - \log(x) = 0$$

✓ **Mathematica** : cpu = 0.116402 (sec), leaf count = 74

$$\left\{ \left\{ y(x) \rightarrow c_2 x^{\frac{1}{2} \left(\frac{\sqrt{17}}{4} - \frac{1}{4} \right)} + c_1 x^{\frac{1}{2} \left(-\frac{1}{4} - \frac{\sqrt{17}}{4} \right)} - \frac{256(\log(x) + 1)}{(\sqrt{17} - 1)^2 (1 + \sqrt{17})^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.169 (sec), leaf count = 32

$$\left\{ y(x) = c_1 x^{-\frac{1}{8} - \frac{\sqrt{17}}{8}} + c_2 x^{-\frac{1}{8} + \frac{\sqrt{17}}{8}} - \ln(x) - 1 \right\}$$

2.1280 ODE No. 1280

$$4x^2y''(x) + (-4x^2 - 12x - 3)y(x) + 8xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0483529 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 e^{-x} (4e^{2x} x^2 \text{Ei}(-2x) + 2x - 1)}{2x^{3/2}} + c_1 e^x \sqrt{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.148 (sec), leaf count = 40

$$\left\{ y(x) = \frac{-4c_2 x^2 \text{Ei}(1, 2x) e^x + c_1 x^2 e^x + c_2 (2x - 1) e^{-x}}{x^{\frac{3}{2}}} \right\}$$

2.1281 ODE No. 1281

$$4x^2y''(x) + (4x^2 - 4x - 1)y(x) - 4(2x - 1)xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.015682 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^x}{\sqrt{x}} + c_2 e^x \sqrt{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 15

$$\left\{ y(x) = \frac{(c_2 x + c_1) e^x}{\sqrt{x}} \right\}$$

2.1282 ODE No. 1282

$$4x^3y'(x) + 4x^2y''(x) + (x^2 - 4)(x^2 + 6)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0182571 (sec), leaf count = 39

$$\left\{ \left(\left\{ y(x) \rightarrow \frac{c_1 e^{-\frac{x^2}{4}}}{x^2} + \frac{1}{5} c_2 e^{-\frac{x^2}{4}} x^3 \right\} \right) \right\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 21

$$\left\{ y(x) = \frac{(c_2 x^5 + c_1) e^{-\frac{x^2}{4}}}{x^2} \right\}$$

2.1283 ODE No. 1283

$$4x^2y''(x) + 4x^2 \log(x)y'(x) + y(x)(x^2 \log^2(x) + 2x - 8) - 4\sqrt{e^x x^{-x}} x^2 = 0$$

✓ **Mathematica** : cpu = 0.0687054 (sec), leaf count = 90

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{x/2} x^{-\frac{x}{2}-1} + \frac{1}{3} c_2 e^{x/2} x^{2-\frac{x}{2}} + \frac{1}{9} (3x^2 \sqrt{e^x x^{-x}} \log(x) - x^2 \sqrt{e^x x^{-x}}) \right\} \right\}$$

✓ **Maple** : cpu = 0.141 (sec), leaf count = 48

$$\left\{ y(x) = \frac{\left(\ln(x) - \frac{1}{3} \right) \sqrt{x^{-x}} e^x x^2}{3} + \left(c_1 x^{-\frac{x}{2}+2} + c_2 x^{-\frac{x}{2}-1} \right) e^{\frac{x}{2}} \right\}$$

2.1284 ODE No. 1284

$$(2x + 1)^2 y''(x) - 2(2x + 1)y'(x) - 12y(x) - 3x - 1 = 0$$

✓ **Mathematica** : cpu = 0.0320147 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow \frac{-72x^2 - 56x - 7}{192(2x + 1)} + c_1(2x + 1)^3 + \frac{c_2}{2x + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 41

$$\left\{ y(x) = \frac{c_1}{2x + 1} + c_2(2x + 1)^3 + \frac{-72x^2 - 56x - 7}{384x + 192} \right\}$$

2.1285 ODE No. 1285

$$((4a + 2)x - a)y'(x) + (a - 1)ay(x) + x(4x - 1)y''(x) = 0$$

✓ **Mathematica** : cpu = 1.23168 (sec), leaf count = 269

$$\left\{ \left\{ y(x) \rightarrow c_2 x^{-a/2} \left(\sqrt{4x-1} + i \right)^{\frac{1}{2} + \frac{1}{2} i \sqrt{-(a-1)^2}} \left(-\sqrt{4x-1} + i \right)^{\frac{1}{2} - \frac{1}{2} i \sqrt{-(a-1)^2}} \int_1^x -\frac{\left(i - \sqrt{4K[1]-1} \right)^{i \sqrt{-(a-1)^2}} \left(\sqrt{4K[1]-1} + i \right)}{\sqrt{1-4K[1]} \left(\sqrt{4K[1]-1} - i \right)} \right. \right.$$

✓ **Maple** : cpu = 0.11 (sec), leaf count = 52

$$\left\{ y(x) = c_2 x^{-a+1} \text{hypergeom} \left(\left[-\frac{a}{2} + 1, -\frac{a}{2} + \frac{1}{2} \right], [-a+2], 4x \right) + c_1 \text{hypergeom} \left(\left[\frac{a}{2}, \frac{a}{2} - \frac{1}{2} \right], [a], 4x \right) \right\}$$

2.1286 ODE No. 1286

$$(3x - 1)^2 y''(x) + 3(3x - 1)y'(x) - 9y(x) - \log^2(3x - 1) = 0$$

✓ **Mathematica** : cpu = 0.106933 (sec), leaf count = 101

$$\left\{ \left\{ y(x) \rightarrow \frac{-6x - 3x \log^2(3x - 1) + \log^2(3x - 1) + \log(3x - 1) - \log(1 - 3x) + 1}{9(3x - 1)} + \frac{c_1 \left((1 - 3x)^2 + 1 \right)}{2(1 - 3x)} + \frac{ic_2 \left((1 - 3x)^2 - 1 \right)}{2(1 - 3x)} \right. \right.$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 32

$$\left\{ y(x) = -\frac{\ln(3x - 1)^2}{9} + \frac{c_1}{3x - 1} + c_2(3x - 1) - \frac{2}{9} \right\}$$

2.1287 ODE No. 1287

$$9(x - 1)xy''(x) + 3(2x - 1)y'(x) - 20y(x) = 0$$

✓ **Mathematica** : cpu = 0.014701 (sec), leaf count = 83

$$\left\{ \left\{ y(x) \rightarrow c_2 \sqrt[3]{1-x} \sqrt[3]{x} Q_1^{\frac{2}{3}}(2x-1) - \frac{c_1(2-2x)^{2/3} \sqrt[3]{1-x} x^{2/3} (6x-5)}{3 \cdot 2^{2/3} (x-1) \Gamma\left(\frac{4}{3}\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 27

$$\left\{ y(x) = c_1(6x - 5)x^{\frac{2}{3}} + c_2(6x - 1)(x - 1)^{\frac{2}{3}} \right\}$$

2.1288 ODE No. 1288

$$16x^2 y''(x) + (4x + 3)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0283907 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{i \sqrt{x}} \sqrt[4]{x} + ic_2 e^{-i \sqrt{x}} \sqrt[4]{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 21

$$\left\{ y(x) = \left(c_1 \sin(\sqrt{x}) + c_2 \cos(\sqrt{x}) \right) x^{\frac{1}{4}} \right\}$$

2.1289 ODE No. 1289

$$16x^2y''(x) + 32xy'(x) + (-4x - 5)y(x) = 0$$

✓ **Mathematica** : cpu = 0.125415 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 e^{-\sqrt{x}} (\sqrt{x} + 1)}{x^{5/4}} - \frac{c_1 e^{\sqrt{x}} (\sqrt{x} - 1)}{x^{5/4}} \right\} \right\}$$

✓ **Maple** : cpu = 0.111 (sec), leaf count = 33

$$\left\{ y(x) = \frac{c_1 (\sqrt{x} - 1) e^{\sqrt{x}} + c_2 (\sqrt{x} + 1) e^{-\sqrt{x}}}{x^{\frac{5}{4}}} \right\}$$

2.1290 ODE No. 1290

$$(27x^2 + 4)y''(x) + 27xy'(x) - 3y(x) = 0$$

✓ **Mathematica** : cpu = 0.12496 (sec), leaf count = 103

$$\left\{ \left\{ y(x) \rightarrow c_1 \cosh \left(\frac{\sqrt{-27x^2 - 4} \tan^{-1} \left(\frac{3x}{\sqrt{-9x^2 - \frac{4}{3}}} \right)}{3\sqrt{27x^2 + 4}} \right) + ic_2 \sinh \left(\frac{\sqrt{-27x^2 - 4} \tan^{-1} \left(\frac{3x}{\sqrt{-9x^2 - \frac{4}{3}}} \right)}{3\sqrt{27x^2 + 4}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 29

$$\left\{ y(x) = c_1 \sinh \left(\frac{\operatorname{arcsinh} \left(\frac{3\sqrt{3}x}{2} \right)}{3} \right) + c_2 \cosh \left(\frac{\operatorname{arcsinh} \left(\frac{3\sqrt{3}x}{2} \right)}{3} \right) \right\}$$

2.1291 ODE No. 1291

$$48(x - 1)xy''(x) + (152x - 40)y'(x) + 53y(x) = 0$$

✓ **Mathematica** : cpu = 0.0700549 (sec), leaf count = 92

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1 \left(\frac{13}{12} - \frac{\sqrt{5}}{6}, \frac{13}{12} + \frac{\sqrt{5}}{6}; \frac{5}{6}; x \right) + \sqrt[6]{-1} c_2 \sqrt[6]{x} {}_2F_1 \left(\frac{5}{4} - \frac{\sqrt{5}}{6}, \frac{5}{4} + \frac{\sqrt{5}}{6}; \frac{7}{6}; x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.123 (sec), leaf count = 50

$$\left\{ y(x) = c_2 x^{\frac{1}{6}} \operatorname{hypergeom} \left(\left[\frac{5}{4} - \frac{\sqrt{10}}{12}, \frac{5}{4} + \frac{\sqrt{10}}{12} \right], \left[\frac{7}{6} \right], x \right) + c_1 \operatorname{hypergeom} \left(\left[\frac{13}{12} - \frac{\sqrt{10}}{12}, \frac{13}{12} + \frac{\sqrt{10}}{12} \right], \left[\frac{5}{6} \right], x \right) \right\}$$

2.1292 ODE No. 1292

$$50(x - 1)xy''(x) + 25(2x - 1)y'(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0365894 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin \left(\frac{2}{5} \sin^{-1} (\sqrt{1-x}) \right) + c_1 \cos \left(\frac{2}{5} \sin^{-1} (\sqrt{1-x}) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 31

$$\left\{ y(x) = \frac{c_1 (\sqrt{x} + \sqrt{x-1})^{\frac{4}{5}} + c_2}{(\sqrt{x} + \sqrt{x-1})^{\frac{2}{5}}} \right\}$$

2.1293 ODE No. 1293

$$144(x-1)xy''(x) + (120x-48)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.288875 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow (-1)^{2/3} c_2 x^{2/3} {}_2F_1\left(\frac{7}{12}, \frac{7}{12}; \frac{5}{3}; x\right) + c_1 {}_2F_1\left(-\frac{1}{12}, -\frac{1}{12}; \frac{1}{3}; x\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.108 (sec), leaf count = 33

$$\left\{ y(x) = \left(c_1 \text{LegendreP}\left(-\frac{1}{2}, \frac{2}{3}, \sqrt{-x+1}\right) + c_2 \text{LegendreQ}\left(-\frac{1}{2}, \frac{2}{3}, \sqrt{-x+1}\right) \right) x^{\frac{1}{3}} \right\}$$

2.1294 ODE No. 1294

$$144(x-1)xy''(x) + (168x-96)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0648836 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1\left(\frac{1}{12}, \frac{1}{12}; \frac{2}{3}; x\right) + \sqrt[3]{-1} c_2 \sqrt[3]{x} {}_2F_1\left(\frac{5}{12}, \frac{5}{12}; \frac{4}{3}; x\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.108 (sec), leaf count = 33

$$\left\{ y(x) = \left(c_1 \text{LegendreP}\left(-\frac{1}{2}, \frac{1}{3}, \sqrt{-x+1}\right) + c_2 \text{LegendreQ}\left(-\frac{1}{2}, \frac{1}{3}, \sqrt{-x+1}\right) \right) x^{\frac{1}{6}} \right\}$$

2.1295 ODE No. 1295

$$ax^2y''(x) + bxy'(x) + y(x)(cx^2 + dx + f) = 0$$

✓ **Mathematica** : cpu = 0.195363 (sec), leaf count = 310

$$\left\{ \left\{ y(x) \rightarrow c_1 U\left(-\frac{-\sqrt{c}a - id\sqrt{a} - \sqrt{c}\sqrt{a^2 - 2ba - 4fa + b^2}}{2a\sqrt{c}}, \frac{\sqrt{a^2 - 2ba - 4fa + b^2}}{a} + 1, \frac{2i\sqrt{c}x}{\sqrt{a}}\right) \exp\left(\frac{\log(x)(\sqrt{a^2 - 2ab} + \sqrt{a^2 - 2ba - 4fa + b^2})}{2a\sqrt{c}}\right) \right. \right.$$

✓ **Maple** : cpu = 0.279 (sec), leaf count = 106

$$\left\{ y(x) = \left(c_1 \text{WhittakerM}\left(-\frac{id}{2\sqrt{a}\sqrt{c}}, \frac{\sqrt{a^2 + b^2 + (-2b - 4f)a}}{2a}, \frac{2i\sqrt{c}x}{\sqrt{a}}\right) + c_2 \text{WhittakerW}\left(-\frac{id}{2\sqrt{a}\sqrt{c}}, \frac{\sqrt{a^2 + b^2 + (-2b - 4f)a}}{2a} \right) \right.$$

2.1296 ODE No. 1296

$$y(x)(a_0x^2 + b_0x + c_0) + (a_1x^2 + b_1x)y'(x) + a_2x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.371816 (sec), leaf count = 356

$$\left\{ \left\{ y(x) \rightarrow c_1 U\left(-\frac{2b_0a_2 - \sqrt{a_1^2 - 4a_0a_2}a_2 - a_1b_1 - \sqrt{a_1^2 - 4a_0a_2}\sqrt{a_2^2 - 2b_1a_2 - 4c_0a_2 + b_1^2}}{2a_2\sqrt{a_1^2 - 4a_0a_2}}, \frac{\sqrt{a_2^2 - 2b_1a_2 - 4c_0a_2 + b_1^2}}{a_2} \right. \right.$$

✓ **Maple** : cpu = 0.446 (sec), leaf count = 150

$$\left\{ y(x) = \left(c_1 \text{WhittakerM}\left(-\frac{a_1b_1 - 2a_2b_0}{2\sqrt{-4a_0a_2 + a_1^2}a_2}, \frac{\sqrt{a_2^2 + b_1^2 + (-2b_1 - 4c_0)a_2}}{2a_2}, \frac{\sqrt{-4a_0a_2 + a_1^2}x}{a_2} \right) + c_2 \text{WhittakerW}\left(-\frac{a_1b_1 - 2a_2b_0}{2\sqrt{-4a_0a_2 + a_1^2}a_2}, \frac{\sqrt{a_2^2 + b_1^2 + (-2b_1 - 4c_0)a_2}}{2a_2} \right) \right.$$

2.1297 ODE No. 1297

$$(ax^2 + 1)y''(x) + axy'(x) + by(x) = 0$$

✓ **Mathematica** : cpu = 0.028371 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow c_1 \cos\left(\frac{\sqrt{b} \sinh^{-1}(\sqrt{a} x)}{\sqrt{a}}\right) + c_2 \sin\left(\frac{\sqrt{b} \sinh^{-1}(\sqrt{a} x)}{\sqrt{a}}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 63

$$\left\{ y(x) = \left(c_1 \left(\sqrt{a} x + \sqrt{a x^2 + 1} \right)^{\frac{2i\sqrt{b}}{\sqrt{a}}} + c_2 \left(\sqrt{a} x + \sqrt{a x^2 + 1} \right)^{-\frac{i\sqrt{b}}{\sqrt{a}}} \right) \right\}$$

2.1298 ODE No. 1298

$$(ax^2 + 1)y''(x) + bxy'(x) + cy(x) = 0$$

✓ **Mathematica** : cpu = 0.0669474 (sec), leaf count = 162

$$\left\{ \left\{ y(x) \rightarrow c_1 (ax^2 + 1)^{\frac{2a-b}{4a}} P_{\frac{b-2a}{2a}}^{\frac{2a-b}{4a}} \left(i\sqrt{a} x \right) + c_2 (ax^2 + 1)^{\frac{2a-b}{4a}} Q_{\frac{b-2a}{2a}}^{\frac{2a-b}{4a}} \left(i\sqrt{a} x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.138 (sec), leaf count = 124

$$\left\{ y(x) = \left(c_1 \text{LegendreP}\left(\frac{-a + \sqrt{a^2 + b^2 + (-2b - 4c)a}}{2a}, \frac{2a - b}{2a}, \sqrt{-a} x\right) + c_2 \text{LegendreQ}\left(\frac{-a + \sqrt{a^2 + b^2 + (-2b - 4c)a}}{2a}, \frac{2a - b}{2a}, \sqrt{-a} x\right) \right) \right\}$$

2.1299 ODE No. 1299

$$(a^2x^2 - 1)y''(x) + 2a^2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0283265 (sec), leaf count = 19

$$\left\{ \left\{ y(x) \rightarrow c_2 - \frac{c_1 \tanh^{-1}(ax)}{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 27

$$\left\{ y(x) = c_1 - \frac{c_2(-\ln(ax - 1) + \ln(ax + 1))}{2a} \right\}$$

2.1300 ODE No. 1300

$$(a^2x^2 - 1)y''(x) + 2a^2xy'(x) - 2a^2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0166793 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow ac_1x + c_2 \left(ax \left(\frac{1}{2} \log(ax + 1) - \frac{1}{2} \log(1 - ax) \right) - 1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 31

$$\left\{ y(x) = \frac{c_2ax \ln(ax - 1)}{2} - \frac{c_2ax \ln(ax + 1)}{2} + c_1x + c_2 \right\}$$

2.1301 ODE No. 1301

$$(ax^2 + bx)y''(x) - 2ay(x) + 2by'(x) = 0$$

✓ **Mathematica** : cpu = 0.0258393 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2(ax + b)^3}{3ax} + \frac{c_1}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 19

$$\left\{ y(x) = \frac{c_1 + c_2(ax + b)^3}{x} \right\}$$

2.1302 ODE No. 1302

$$A_0y(x)(ax + b) + A_1(ax + b)y'(x) + A_2(ax + b)^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0654025 (sec), leaf count = 243

$$\left\{ \left\{ y(x) \rightarrow c_1 \left(\frac{2b}{a} + 2x \right)^{\frac{A_1}{2aA_2}} (2aA_2x + 2A_2b)^{-\frac{A_1}{2aA_2}} \left(-\frac{A_0 \left(\frac{b}{a} + x \right)}{aA_2} \right)^{\frac{1}{2} - \frac{A_1}{2aA_2}} I_{\frac{A_1}{aA_2} - 1} \left(2\sqrt{-\frac{A_0 \left(\frac{b}{a} + x \right)}{aA_2}} \right) + c_2 (-1)^{1 - \frac{A_1}{aA_2}} \left(\frac{2b}{a} + 2x \right)^{\frac{A_1}{2aA_2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.083 (sec), leaf count = 98

$$\left\{ y(x) = \left(c_1 \text{BesselJ} \left(\frac{aA_2 - A_1}{A_2a}, 2\sqrt{\frac{ax + b}{A_2a^2}} \sqrt{A_0} \right) + c_2 \text{BesselY} \left(\frac{aA_2 - A_1}{A_2a}, 2\sqrt{\frac{ax + b}{A_2a^2}} \sqrt{A_0} \right) \right) (ax + b)^{-\frac{-aA_2 + A_1}{2A_2a}} \right\}$$

2.1303 ODE No. 1303

$$y''(x)(ax^2 + bx + c) + (dx + f)y'(x) + gy(x) = 0$$

✓ **Mathematica** : cpu = 5.12741 (sec), leaf count = 498

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1 \left(-\frac{a - d + \sqrt{(a - d)^2 - 4ag}}{2a}, \frac{-a + d + \sqrt{(a - d)^2 - 4ag}}{2a}; \frac{(b + \sqrt{b^2 - 4ac})d - 2af}{2a\sqrt{b^2 - 4ac}}; \frac{b + 2ax + \sqrt{b^2 - 4ac}}{2\sqrt{b^2 - 4ac}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.215 (sec), leaf count = 501

$$\left\{ y(x) = c_2 \left(2\sqrt{\frac{-4ac + b^2}{a^2}} a^2x + \sqrt{\frac{-4ac + b^2}{a^2}} ab - 4ac + b^2 \right)^{\frac{af - \frac{bd}{2} + \left(a - \frac{d}{2}\right) \sqrt{\frac{-4ac + b^2}{a^2}} a}{\sqrt{\frac{-4ac + b^2}{a^2}} a^2} \text{hypergeom} \left(\left[\frac{2af - bd + \left(a + \sqrt{a^2 + d^2}\right)}{2\sqrt{-4ac + b^2}} \right] \right) \right\}$$

2.1304 ODE No. 1304

$$x^3y''(x) + xy'(x) + (-2x - 3)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0573433 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \left(e^{\frac{1}{x}} \text{Ei} \left(-\frac{1}{x} \right) + 2x^3 - x^2 + x \right)}{6x} + \frac{c_1 e^{\frac{1}{x}}}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 38

$$\left\{ y(x) = \frac{c_2 \operatorname{Ei}\left(1, \frac{1}{x}\right) e^{\frac{1}{x}} + c_1 e^{\frac{1}{x}} - 2c_2 \left(x^2 - \frac{1}{2}x + \frac{1}{2}\right)x}{x} \right\}$$

2.1305 ODE No. 1305

$$x^3 y''(x) + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0801454 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow c_2 G_{1,2}^{2,0} \left(-\frac{2}{x} \middle| \begin{array}{c} \frac{1}{2} \\ -1, 0 \end{array} \right) + c_1 e^{\frac{1}{x}} \left(I_0\left(\frac{1}{x}\right) - I_1\left(\frac{1}{x}\right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.09 (sec), leaf count = 44

$$\left\{ y(x) = \left(c_2 \operatorname{BesselK}\left(0, -\frac{1}{x}\right) - c_2 \operatorname{BesselK}\left(1, -\frac{1}{x}\right) + c_1 \left(\operatorname{BesselI}\left(0, \frac{1}{x}\right) - \operatorname{BesselI}\left(1, \frac{1}{x}\right) \right) \right) e^{\frac{1}{x}} \right\}$$

2.1306 ODE No. 1306

$$y(x)(ax^2 + a + bx) + x^3 y''(x) + x^2 y'(x) = 0$$

✗ **Mathematica** : cpu = 1.05699 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow (x) \right\} \right\}$$

✓ **Maple** : cpu = 0.223 (sec), leaf count = 69

$$\left\{ y(x) = \left(c_2 \left(\int \frac{1}{x \operatorname{HD}\left(0, 8a + 4b, 0, 8a - 4b, \frac{x+1}{x-1}\right)^2} dx \right) + c_1 \right) \operatorname{HD}\left(0, 8a + 4b, 0, 8a - 4b, \frac{x+1}{x-1}\right) \right\}$$

2.1307 ODE No. 1307

$$x^3 y''(x) + (x+1)xy'(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.116997 (sec), leaf count = 54

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{\frac{1}{x}}(x+1)}{x} - \frac{c_2 \left(e^{\frac{1}{x}} x \operatorname{Ei}\left(-\frac{1}{x}\right) + e^{\frac{1}{x}} \operatorname{Ei}\left(-\frac{1}{x}\right) + x \right)}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.055 (sec), leaf count = 36

$$\left\{ y(x) = \frac{c_2 (x+1) \operatorname{Ei}\left(1, \frac{1}{x}\right) e^{\frac{1}{x}} + c_1 (x+1) e^{\frac{1}{x}} - c_2 x}{x} \right\}$$

2.1308 ODE No. 1308

$$x^3 y''(x) - x^2 y'(x) + xy(x) - \log^3(x) = 0$$

✓ **Mathematica** : cpu = 0.0144222 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow \frac{2 \log^3(x) + 6 \log^2(x) + 9 \log(x) + 6}{8x} + c_1 x + c_2 x \log(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 40

$$\left\{y(x) = \frac{8c_2x^2 + 2\ln(x)^3 + 6\ln(x)^2 + (8c_1x^2 + 9)\ln(x) + 6}{8x}\right\}$$

2.1309 ODE No. 1309

$$x^3y''(x) - (x^2 - 1)y'(x) + xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0999429 (sec), leaf count = 84

$$\left\{\left\{y(x) \rightarrow c_2G_{1,2}^{2,0}\left(-\frac{1}{2x^2}\middle|\begin{matrix} 1 \\ -\frac{1}{2}, -\frac{1}{2} \end{matrix}\right) + \sqrt{2}c_1e^{\frac{1}{4x^2}}x\left(1 - \frac{1}{2x^2}\right)I_0\left(\frac{1}{4x^2}\right) + \frac{I_1\left(\frac{1}{4x^2}\right)}{2x^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.148 (sec), leaf count = 85

$$\left\{y(x) = \frac{c_1\left(2x^2\operatorname{BesselI}\left(0, \frac{1}{4x^2}\right) - \operatorname{BesselI}\left(0, \frac{1}{4x^2}\right) + \operatorname{BesselI}\left(1, \frac{1}{4x^2}\right)\right)e^{\frac{1}{4x^2}}}{x} + \frac{c_2\left(2x^2\operatorname{BesselK}\left(0, -\frac{1}{4x^2}\right) - \operatorname{BesselK}\left(0, -\frac{1}{4x^2}\right)\right)}{x}\right\}$$

2.1310 ODE No. 1310

$$x^3y''(x) + 3x^2y'(x) + xy(x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.0094741 (sec), leaf count = 31

$$\left\{\left\{y(x) \rightarrow \frac{\log^2(x)}{2x} + \frac{c_1}{x} + \frac{c_2\log(x)}{x}\right\}\right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 20

$$\left\{y(x) = \frac{c_1\ln(x) + \frac{\ln(x)^2}{2} + c_2}{x}\right\}$$

2.1311 ODE No. 1311

$$-v(v+1)xy(x) + x(x^2+1)y''(x) + (2x^2+1)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.109689 (sec), leaf count = 63

$$\left\{\left\{y(x) \rightarrow c_2G_{2,2}^{2,0}\left(-x^2\middle|\begin{matrix} \frac{1-v}{2}, \frac{v+2}{2} \\ 0, 0 \end{matrix}\right) + c_1{}_2F_1\left(\frac{v}{2} + \frac{1}{2}, -\frac{v}{2}; 1; -x^2\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.201 (sec), leaf count = 52

$$\left\{y(x) = c_1\operatorname{hypergeom}\left(\left[-\frac{v}{2}, \frac{v}{2} + \frac{1}{2}\right], \left[\frac{1}{2}\right], x^2 + 1\right) + c_2\sqrt{x^2+1}\operatorname{hypergeom}\left(\left[\frac{v}{2} + 1, -\frac{v}{2} + \frac{1}{2}\right], \left[\frac{3}{2}\right], x^2 + 1\right)\right\}$$

2.1312 ODE No. 1312

$$x(x^2+1)y''(x) + 2(x^2-1)y'(x) - 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0188203 (sec), leaf count = 32

$$\left\{\left\{y(x) \rightarrow \frac{c_1}{x^2+1} + \frac{c_2x^3}{3(x^2+1)}\right\}\right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 19

$$\left\{ y(x) = \frac{c_2 x^3 + c_1}{x^2 + 1} \right\}$$

2.1313 ODE No. 1313

$$x(n-v)(n+v+1)y(x) + (2(n+1)x^2 + 2n+1)y'(x) + x(x^2+1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.175435 (sec), leaf count = 87

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1\left(\frac{n}{2} - \frac{v}{2}, \frac{n}{2} + \frac{v}{2} + \frac{1}{2}; n+1; -x^2\right) + c_2 x^{-2n} {}_2F_1\left(-\frac{n}{2} - \frac{v}{2}, -\frac{n}{2} + \frac{v}{2} + \frac{1}{2}; 1-n; -x^2\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.135 (sec), leaf count = 35

$$\left\{ y(x) = \left(c_1 \text{LegendreP}\left(v, n, \sqrt{x^2+1}\right) + c_2 \text{LegendreQ}\left(v, n, \sqrt{x^2+1}\right) \right) x^{-n} \right\}$$

2.1314 ODE No. 1314

$$x(n-v-1)(n+v)y(x) - (2(n-1)x^2 + 2n-1)y'(x) + x(x^2+1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.163228 (sec), leaf count = 87

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1\left(-\frac{n}{2} - \frac{v}{2}, -\frac{n}{2} + \frac{v}{2} + \frac{1}{2}; 1-n; -x^2\right) + c_2 x^{2n} {}_2F_1\left(\frac{n}{2} - \frac{v}{2}, \frac{n}{2} + \frac{v}{2} + \frac{1}{2}; n+1; -x^2\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.105 (sec), leaf count = 33

$$\left\{ y(x) = \left(c_1 \text{LegendreP}\left(v, n, \sqrt{x^2+1}\right) + c_2 \text{LegendreQ}\left(v, n, \sqrt{x^2+1}\right) \right) x^n \right\}$$

2.1315 ODE No. 1315

$$ax^3y(x) + (x^2-1)xy''(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.025699 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow c_1 \cos\left(\sqrt{a} \sqrt{x^2-1}\right) + c_2 \sin\left(\sqrt{a} \sqrt{x^2-1}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 45

$$\left\{ y(x) = c_1 \sin\left(\frac{(x-1)(x+1)\sqrt{a}}{\sqrt{x^2-1}}\right) + c_2 \cos\left(\frac{(x-1)(x+1)\sqrt{a}}{\sqrt{x^2-1}}\right) \right\}$$

2.1316 ODE No. 1316

$$x(x^2-1)y''(x) + (x^2-1)y'(x) - xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0748948 (sec), leaf count = 38

$$\left\{ \left(\left\{ y(x) \rightarrow c_2 G_{2,2}^{2,0}\left(x^2 \middle| \begin{matrix} \frac{1}{2}, \frac{3}{2} \\ 0, 0 \end{matrix} \right) + \frac{2c_1 E(x^2)}{\pi} \right\} \right) \right\}$$

✓ **Maple** : cpu = 0.074 (sec), leaf count = 18

$$\{y(x) = c_1 \text{EllipticE}(x) + c_2 (\text{EllipticCE}(x) - \text{EllipticCK}(x))\}$$

2.1317 ODE No. 1317

$$x(x^2 - 1)y''(x) + (3x^2 - 1)y'(x) + xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0846937 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow c_2 G_{2,2}^{2,0} \left(x^2 \middle| \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, 0 \end{matrix} \right) + \frac{2c_1 K(x^2)}{\pi} \right\} \right\}$$

✓ **Maple** : cpu = 0.086 (sec), leaf count = 13

$$\{y(x) = c_1 \operatorname{EllipticK}(x) + c_2 \operatorname{EllipticCK}(x)\}$$

2.1318 ODE No. 1318

$$(ax^2 + b)y'(x) + cxy(x) + x(x^2 - 1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.229986 (sec), leaf count = 172

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1 \left(\frac{a}{4} - \frac{1}{4} \sqrt{a^2 - 2a - 4c + 1} - \frac{1}{4}, \frac{a}{4} + \frac{1}{4} \sqrt{a^2 - 2a - 4c + 1} - \frac{1}{4}; \frac{1}{2} - \frac{b}{2}; x^2 \right) + i^{b+1} c_2 x^{b+1} {}_2F_1 \left(\frac{a}{4} + \frac{b}{2} - \frac{1}{4} \sqrt{a^2 - 2a - 4c + 1} - \frac{1}{4}, \frac{a}{4} + \frac{1}{4} \sqrt{a^2 - 2a - 4c + 1} - \frac{1}{4}; \frac{1}{2} - \frac{b}{2}; x^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.182 (sec), leaf count = 122

$$\left\{ y(x) = c_2 x^{b+1} \operatorname{hypergeom} \left(\left[\frac{a}{4} + \frac{b}{2} + \frac{1}{4} + \frac{\sqrt{a^2 - 2a - 4c + 1}}{4}, \frac{a}{4} + \frac{b}{2} + \frac{1}{4} - \frac{\sqrt{a^2 - 2a - 4c + 1}}{4} \right], \left[\frac{b}{2} + \frac{3}{2} \right], x^2 \right) + c_1 \operatorname{hypergeom} \left(\left[\frac{a}{4} - \frac{1}{4} \sqrt{a^2 - 2a - 4c + 1} - \frac{1}{4}, \frac{a}{4} + \frac{1}{4} \sqrt{a^2 - 2a - 4c + 1} - \frac{1}{4} \right], \left[\frac{1}{2} - \frac{b}{2} \right], x^2 \right) \right\}$$

2.1319 ODE No. 1319

$$x(x^2 + 2)y''(x) - y'(x) - 6xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0624675 (sec), leaf count = 60

$$\left\{ \left\{ y(x) \rightarrow c_1 x^{3/2} (x^2 + 2)^{3/4} - \frac{c_2 (x^2 + 2)^{3/4} {}_2F_1 \left(-\frac{3}{4}, \frac{7}{4}; \frac{1}{4}; -\frac{x^2}{2} \right)}{3 \cdot 2^{3/4}} \right\} \right\}$$

✓ **Maple** : cpu = 0.131 (sec), leaf count = 31

$$\left\{ y(x) = (x^2 + 2)^{\frac{3}{4}} \left(c_1 x^{\frac{3}{2}} + c_2 \operatorname{hypergeom} \left(\left[-\frac{3}{4}, \frac{7}{4} \right], \left[\frac{1}{4} \right], -\frac{x^2}{2} \right) \right) \right\}$$

2.1320 ODE No. 1320

$$x(x^2 - 2)y''(x) + (x^2 + 4x + 2)y(x) - (x^3 + 3x^2 - 2x - 2)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0741298 (sec), leaf count = 21

$$\{ \{ y(x) \rightarrow c_1 e^x x^2 + c_2 (x - 1) \} \}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 17

$$\{y(x) = c_2 x^2 e^x + c_1 (x - 1)\}$$

2.1321 ODE No. 1321

$$(x+1)x^2y''(x) - (2x+1)xy'(x) + (2x+1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0207762 (sec), leaf count = 18

$$\{\{y(x) \rightarrow c_1x + c_2x(x + \log(x))\}\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 15

$$\{y(x) = (c_2x + c_2 \ln(x) + c_1)x\}$$

2.1322 ODE No. 1322

$$(x+1)x^2y''(x) + 2(3x+2)xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0522572 (sec), leaf count = 44

$$\left\{\left\{y(x) \rightarrow c_1\left(-\frac{1}{3x^3} + \frac{1}{x^2} - \frac{3}{x} - \frac{1}{x+1} - 4\log(x) + 4\log(x+1)\right) + c_2\right\}\right\}$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 44

$$\left\{y(x) = c_1 + c_2\left(-4\ln(x) + 4\ln(x+1) - \frac{12x^3 + 6x^2 - 2x + 1}{3(x+1)x^3}\right)\right\}$$

2.1323 ODE No. 1323

$$y''(x) = \frac{2(x+1)y(x)}{(x-1)x} - \frac{2(x-2)y'(x)}{(x-1)x}$$

✗ **Mathematica** : cpu = 0.826801 (sec), leaf count = 0 , DifferentialRoot result

$$\{\{y(x) \rightarrow (x)\}\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 17

$$\left\{y(x) = \frac{c_1 + c_2(x-1)^3}{x^2}\right\}$$

2.1324 ODE No. 1324

$$y''(x) = \frac{(5x-4)y'(x)}{(x-1)x} - \frac{(9x-6)y(x)}{(x-1)x^2}$$

✓ **Mathematica** : cpu = 0.0245233 (sec), leaf count = 25

$$\{\{y(x) \rightarrow c_1x^3 - c_2x^2(x \log(x) + 1)\}\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 18

$$\{y(x) = (c_2x \ln(x) + c_1x + c_2)x^2\}$$

2.1325 ODE No. 1325

$$y''(x) = -\frac{y(x)(abx - \alpha\beta)}{(x-1)x^2} - \frac{y'(x)(x(a+b+1) + \alpha + \beta - 1)}{(x-1)x}$$

✓ **Mathematica** : cpu = 0.211231 (sec), leaf count = 52

$$\left\{\left\{y(x) \rightarrow (-1)^\alpha c_1x^\alpha {}_2F_1(a+\alpha, \alpha+b; \alpha-\beta+1; x) + (-1)^\beta c_2x^\beta {}_2F_1(a+\beta, b+\beta; -\alpha+\beta+1; x)\right\}\right\}$$

✓ **Maple** : cpu = 0.139 (sec), leaf count = 86

$$\{y(x) = (c_1x^\alpha \operatorname{hypergeom}([-b-\beta+1, -a-\beta+1], [\alpha-\beta+1], x) + c_2x^\beta \operatorname{hypergeom}([-a-\alpha+1, -\alpha-b+1], [-\alpha+\beta+1], x))\}$$

2.1326 ODE No. 1326

$$y''(x) = -\frac{y'(x)}{x+1} - \frac{y(x)}{x(x+1)^2}$$

✓ **Mathematica** : cpu = 0.0200498 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 x}{x+1} + \frac{c_2 (x \log(x) - 1)}{x+1} \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 22

$$\left\{ y(x) = \frac{c_2 x \ln(x) + c_1 x - c_2}{x+1} \right\}$$

2.1327 ODE No. 1327

$$y''(x) = \frac{2y'(x)}{(x-2)x} - \frac{y(x)}{(x-2)x^2}$$

✓ **Mathematica** : cpu = 0.145842 (sec), leaf count = 104

$$\left\{ \left\{ y(x) \rightarrow \left(-\frac{1}{2} \right)^{-\frac{1}{\sqrt{2}}} c_1 x^{-\frac{1}{\sqrt{2}}} {}_2F_1 \left(-\frac{1}{\sqrt{2}}, -1 - \frac{1}{\sqrt{2}}; 1 - \sqrt{2}; \frac{x}{2} \right) + \left(-\frac{1}{2} \right)^{\frac{1}{\sqrt{2}}} c_2 x^{\frac{1}{\sqrt{2}}} {}_2F_1 \left(\frac{1}{\sqrt{2}}, -1 + \frac{1}{\sqrt{2}}; 1 + \sqrt{2}; \frac{x}{2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.811 (sec), leaf count = 81

$$\left\{ y(x) = (x-2)^2 \left(c_1 x^{-\frac{\sqrt{2}}{2}} \text{hypergeom} \left(\left[2 - \frac{\sqrt{2}}{2}, 1 - \frac{\sqrt{2}}{2} \right], [1 - \sqrt{2}], \frac{x}{2} \right) + c_2 x^{\frac{\sqrt{2}}{2}} \text{hypergeom} \left(\left[2 + \frac{\sqrt{2}}{2}, 1 + \frac{\sqrt{2}}{2} \right], [1 + \sqrt{2}], \frac{x}{2} \right) \right) \right\}$$

2.1328 ODE No. 1328

$$y''(x) = \frac{2y(x)}{(x-1)^2 x}$$

✓ **Mathematica** : cpu = 0.01756 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 (-x^2 + 2x \log(x) + 1)}{x-1} - \frac{c_1 x}{x-1} \right\} \right\}$$

✓ **Maple** : cpu = 0.025 (sec), leaf count = 27

$$\left\{ y(x) = \frac{-c_2 x^2 + 2c_2 x \ln(x) + c_1 x + c_2}{x-1} \right\}$$

2.1329 ODE No. 1329

$$y''(x) = -\frac{y'(x)(-x(a(\delta + \text{gamma1}) + \alpha + \beta - \delta + 1) + a\text{gamma1} + x^2(\alpha + \beta + 1))}{(x-1)x(x-a)} - \frac{y(x)(\alpha\beta x - q)}{(x-1)x(x-a)}$$

✓ **Mathematica** : cpu = 0.723138 (sec), leaf count = 67

$$\left\{ \left\{ y(x) \rightarrow c_2 x^{1-\text{gamma1}} \text{HeunG}[a, q - (\text{gamma1} - 1)((a-1)\delta + \alpha + \beta - \text{gamma1} + 1), \beta - \text{gamma1} + 1, \alpha - \text{gamma1} + 1, -\gamma1 + 2, \delta, x] + c_1 \text{HG}[a, q, \alpha, \beta, \gamma1] \right\} \right\}$$

✓ **Maple** : cpu = 0.328 (sec), leaf count = 64

$$\{y(x) = c_2 x^{-\gamma1+1} HG(a, q - (\gamma1 - 1)(\alpha + \beta + (a-1)\delta - \gamma1 + 1), \beta - \gamma1 + 1, \alpha - \gamma1 + 1, -\gamma1 + 2, \delta, x) + c_1 HG(a, q, \alpha, \beta, \gamma1)\}$$

2.1330 ODE No. 1330

$$y''(x) = -\frac{y'(x)(Ax^2+Bx+C)}{(x-a)(x-b)(x-c)} - \frac{(DDx+e)y(x)}{(x-a)(x-b)(x-c)}$$

✓ **Mathematica** : cpu = 4.17085 (sec), leaf count = 1176

$$\left\{\left\{y(x) \rightarrow c_2 \text{HeunG}\left[\frac{a-c}{a-b}, \frac{A^2ba^4+B^2a^3+A(b^2-ab+(a+b)B+2C)a^3+(a-b)^2(aDD+e)a^2-2(a-b)\left(a-\frac{b}{2}\right)Ca}{(a-c)(a-b)}\right]\right\}\right\}$$

✓ **Maple** : cpu = 0.917 (sec), leaf count = 1147

$$\left\{y(x) = c_2(-a+x)^{\frac{(-A+1)a^2+bc-C+(-B-b-c)a}{(a-c)(a-b)}} HG\left[\frac{a-c}{a-b}, \frac{A^2a^4b+B^2a^3+(-ab+b^2+(a+b)B+2C)Aa^3+(2a-b)C^2-2(a-b)C}{(a-c)(a-b)}\right]\right\}$$

2.1331 ODE No. 1331

$$y''(x) = \frac{(x-4)y'(x)}{2(x-2)x} - \frac{(x-3)y(x)}{2(x-2)x^2}$$

✓ **Mathematica** : cpu = 0.0334524 (sec), leaf count = 55

$$\left\{\left\{y(x) \rightarrow \frac{c_1\sqrt[4]{x-2}\sqrt{x}}{\sqrt[4]{2-x}} + \frac{2c_2(x-2)^{3/4}\sqrt{x}}{\sqrt[4]{2-x}}\right\}\right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 19

$$\{y(x) = c_1\sqrt{x} + c_2\sqrt{(x-2)x}\}$$

2.1332 ODE No. 1332

$$y''(x) = \frac{y'(x)}{x+1} - \frac{(3x+1)y(x)}{4x^2(x+1)}$$

✓ **Mathematica** : cpu = 0.0210591 (sec), leaf count = 26

$$\left\{\left\{y(x) \rightarrow c_1\sqrt{x} + c_2\sqrt{x}(x+\log(x))\right\}\right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 17

$$\{y(x) = (c_2x + c_2\ln(x) + c_1)\sqrt{x}\}$$

2.1333 ODE No. 1333

$$y''(x) = \frac{v(v+1)y(x)}{4x^2} - \frac{(3x-1)y'(x)}{2(x-1)x}$$

✓ **Mathematica** : cpu = 0.0939401 (sec), leaf count = 70

$$\left\{\left\{y(x) \rightarrow c_1i^{-v}x^{-v/2}{}_2F_1\left(\frac{1}{2}, -v; \frac{1}{2}-v; x\right) + c_2i^{v+1}x^{\frac{v+1}{2}}{}_2F_1\left(\frac{1}{2}, v+1; v+\frac{3}{2}; x\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.122 (sec), leaf count = 45

$$\left\{y(x) = c_1x^{-\frac{v}{2}} \text{hypergeom}\left(\left[\frac{1}{2}, -v\right], \left[-v + \frac{1}{2}\right], x\right) + c_2x^{\frac{v}{2}+\frac{1}{2}} \text{hypergeom}\left(\left[\frac{1}{2}, v+1\right], \left[v + \frac{3}{2}\right], x\right)\right\}$$

2.1334 ODE No. 1334

$$y''(x) = -\frac{y(x)(x(a^2 - b^2) + c^2)}{4(x - 1)x^2} - \frac{((a + 1)x - 1)y'(x)}{(x - 1)x}$$

✓ **Mathematica** : cpu = 0.16994 (sec), leaf count = 114

$$\left\{ \left\{ y(x) \rightarrow i^{-c} c_1 x^{-c/2} {}_2F_1\left(\frac{a}{2} - \frac{b}{2} - \frac{c}{2}, \frac{a}{2} + \frac{b}{2} - \frac{c}{2}; 1 - c; x\right) + i^c c_2 x^{c/2} {}_2F_1\left(\frac{a}{2} - \frac{b}{2} + \frac{c}{2}, \frac{a}{2} + \frac{b}{2} + \frac{c}{2}; c + 1; x\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.124 (sec), leaf count = 89

$$\left\{ y(x) = \left(c_1 x^{\frac{c}{2}} \text{hypergeom}\left(\left[-\frac{a}{2} + \frac{b}{2} + \frac{c}{2} + 1, -\frac{a}{2} - \frac{b}{2} + \frac{c}{2} + 1\right], [c + 1], x\right) + c_2 x^{-\frac{c}{2}} \text{hypergeom}\left(\left[-\frac{a}{2} - \frac{b}{2} - \frac{c}{2} + 1, -\frac{a}{2} + \frac{b}{2} - \frac{c}{2} + 1\right], [c + 1], x\right) \right) \right\}$$

2.1335 ODE No. 1335

$$y''(x) = -\frac{y(x)(ax + b)}{4(x - 1)^2 x} - \frac{(3x - 1)y'(x)}{2(x - 1)x}$$

✓ **Mathematica** : cpu = 0.259016 (sec), leaf count = 893

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{1}{4}(-2\log(1-x) - \log(x))} \sqrt[4]{x} c_1 {}_2F_1\left(\frac{1}{4}\left(\sqrt{-8a - 4b - 4\sqrt{4a^2 + 4ba - a - b}} + 1\right), \frac{(-8a - 4b - 4\sqrt{4a^2 + 4ba - a - b})}{4\sqrt{4a^2 + 4ba - a - b}}; 1, x\right) + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.102 (sec), leaf count = 57

$$\left\{ y(x) = c_1 \text{LegendreP}\left(\frac{\sqrt{-4a + 1}}{2} - \frac{1}{2}, \sqrt{-a - b}, \sqrt{x}\right) + c_2 \text{LegendreQ}\left(\frac{\sqrt{-4a + 1}}{2} - \frac{1}{2}, \sqrt{-a - b}, \sqrt{x}\right) \right\}$$

2.1336 ODE No. 1336

$$y''(x) = -\frac{(1 - 3x)y(x)}{(x - 1)(2x - 1)^2}$$

✓ **Mathematica** : cpu = 0.0428465 (sec), leaf count = 70

$$\left\{ \left\{ y(x) \rightarrow c_2 \sqrt{1 - 2x} (2x \log(2(x - 1) + 1) - 2 \log(2(x - 1) + 1) - 2x \log(x - 1) + 2 \log(x - 1) - 1) - c_1 \sqrt{1 - 2x} (x - 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 44

$$\left\{ y(x) = (c_1 x - 2c_2(x - 1) \ln(x - 1) + 2c_2(x - 1) \ln(2x - 1) - c_1 - c_2) \sqrt{2x - 1} \right\}$$

2.1337 ODE No. 1337

$$y''(x) = -\frac{(a + 2b + 3x)y'(x)}{2(a + x)(b + x)} - \frac{(a - b)y(x)}{4(a + x)^2(b + x)}$$

✓ **Mathematica** : cpu = 0.0617827 (sec), leaf count = 62

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{\sqrt{\frac{b+x}{a-b}} + 1} + \frac{c_2 \sqrt{b+x}}{\sqrt{a-b} \sqrt{\frac{b+x}{a-b}} + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 27

$$\left\{ y(x) = \frac{c_1 \sqrt{b+x} + c_2}{\sqrt{\frac{a+x}{a-b}}} \right\}$$

2.1338 ODE No. 1338

$$y''(x) = \frac{y(x)}{3(x-2)x^2} + \frac{(6x-1)y'(x)}{3(x-2)x}$$

✓ **Mathematica** : cpu = 0.0454459 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow \frac{3}{935} c_2 x (18x^2 - 102x + 187) + c_1 \sqrt[6]{x} (2-x)^{17/6} \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 27

$$\left\{ y(x) = 18c_1 \left(x^2 - \frac{17}{3}x + \frac{187}{18} \right) x + c_2 (x-2)^{\frac{17}{6}} x^{\frac{1}{6}} \right\}$$

2.1339 ODE No. 1339

$$y''(x) = -\frac{y'(x)(a(b+2)x^2 + x(c-d+1))}{x^2(ax+1)} - \frac{y(x)(abx-cd)}{x^2(ax+1)}$$

✓ **Mathematica** : cpu = 0.211073 (sec), leaf count = 66

$$\left\{ \left\{ y(x) \rightarrow c_1 a^{-c} x^{-c} {}_2F_1(1-c, b-c; -c-d+1; -ax) + c_2 a^d x^d {}_2F_1(d+1, b+d; c+d+1; -ax) \right\} \right\}$$

✓ **Maple** : cpu = 0.167 (sec), leaf count = 76

$$\{y(x) = (c_1 x^d \operatorname{hypergeom}([c, -b+c+1], [c+d+1], -ax) + c_2 x^{-c} \operatorname{hypergeom}([-d, -b-d+1], [-c-d+1], -ax))(ax - 1)^{-d-c}\}$$

2.1340 ODE No. 1340

$$y''(x) = \frac{2(ax+2b)y'(x)}{x(ax+b)} - \frac{y(x)(2ax+6b)}{x^2(ax+b)}$$

✓ **Mathematica** : cpu = 0.0288691 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 x^3}{ax+b} + \frac{c_1 x^2}{ax+b} \right\} \right\}$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 20

$$\left\{ y(x) = \frac{(c_2 x + c_1) x^2}{ax+b} \right\}$$

2.1341 ODE No. 1341

$$y''(x) = -\frac{y(x)(avx-b)}{x^2(ax+b)} - \frac{(2ax+b)y'(x)}{x(ax+b)} + Ax$$

✓ **Mathematica** : cpu = 61.9786 (sec), leaf count = 2924

$$\left(\left(\left(y(x) \rightarrow \frac{ax c_1 {}_2F_1\left(\frac{3}{2} - \frac{1}{2}\sqrt{1-4v}, \frac{1}{2}\sqrt{1-4v} + \frac{3}{2}; 3; -\frac{ax}{b}\right)}{b} + c_2 G_{2,2}^{2,0}\left(-\frac{ax}{b} \middle| \begin{matrix} \frac{1}{2}(1-\sqrt{1-4v}), \frac{1}{2}(\sqrt{1-4v}+1) \\ -1, 1 \end{matrix} \right) \right) + \frac{ax {}_2F_1\left(\frac{3}{2} - \frac{1}{2}\sqrt{1-4v}, \frac{1}{2}\sqrt{1-4v} + \frac{3}{2}; 3; -\frac{ax}{b}\right)}{b} \right) \right)$$

✓ **Maple** : cpu = 0.2 (sec), leaf count = 201

$$\left\{y(x)=\frac{c_1(v+6)(v+2)(v+12)a^2x^{\frac{1}{2}-\frac{\sqrt{-4v+1}}{2}}\operatorname{hypergeom}\left(\left[-\frac{1}{2}+\frac{\sqrt{-4v+1}}{2},\frac{3}{2}+\frac{\sqrt{-4v+1}}{2}\right],\left[1+\sqrt{-4v+1}\right],-\frac{b}{ax}\right)+c_2(v+}{\right.$$

2.1342 ODE No. 1342

$$y''(x)=-\frac{ay(x)}{x^4}$$

✓ **Mathematica** : cpu = 0.084269 (sec), leaf count = 52

$$\left\{\left\{y(x)\rightarrow c_1xe^{\frac{i\sqrt{a}}{x}}-\frac{ic_2xe^{-\frac{i\sqrt{a}}{x}}}{2\sqrt{a}}\right\}\right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 31

$$\left\{y(x)=\left(c_1\sinh\left(\frac{\sqrt{-a}}{x}\right)+c_2\cosh\left(\frac{\sqrt{-a}}{x}\right)\right)x\right\}$$

2.1343 ODE No. 1343

$$y''(x)=-\frac{y(x)\left((1-a)ax^2-b(b+x)\right)}{x^4}$$

✓ **Mathematica** : cpu = 0.188589 (sec), leaf count = 73

$$\left\{\left\{y(x)\rightarrow c_1\left(2\left(ax+\frac{b}{2}\right)I_a\left(\frac{b}{x}\right)+bI_{a+1}\left(\frac{b}{x}\right)\right)+c_2\left(2\left(ax+\frac{b}{2}\right)K_a\left(\frac{b}{x}\right)-bK_{a+1}\left(\frac{b}{x}\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.132 (sec), leaf count = 58

$$\left\{y(x)=c_1b\operatorname{BesselI}\left(a+1,\frac{b}{x}\right)-c_2b\operatorname{BesselK}\left(a+1,\frac{b}{x}\right)+2\left(c_1\operatorname{BesselI}\left(a,\frac{b}{x}\right)+c_2\operatorname{BesselK}\left(a,\frac{b}{x}\right)\right)\left(ax+\frac{b}{2}\right)\right\}$$

2.1344 ODE No. 1344

$$y''(x)=-\frac{\left(e^{2/x}-v^2\right)y(x)}{x^4}$$

✓ **Mathematica** : cpu = 0.509737 (sec), leaf count = 173

$$\left\{\left\{y(x)\rightarrow\frac{c_12^{v+\frac{v+1}{2}}\left(e^{2/x}\right)^{\frac{v+1}{2}-\frac{1}{2}}\left(-e^{2/x}\right)^{\frac{1}{2}(-v-1)+\frac{1}{2}}I_v\left(\sqrt{-e^{2/x}}\right)}{\log\left(e^{2/x}\right)}+\frac{c_2(-1)^{-v}2^{v+\frac{v+1}{2}}\left(e^{2/x}\right)^{\frac{v+1}{2}-\frac{1}{2}}\left(-e^{2/x}\right)^{\frac{1}{2}(-v-1)+\frac{1}{2}}K_v\left(\sqrt{-e^{2/x}}\right)}{\log\left(e^{2/x}\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 23

$$\left\{y(x)=\left(c_1\operatorname{BesselJ}\left(v,e^{\frac{1}{x}}\right)+c_2\operatorname{BesselY}\left(v,e^{\frac{1}{x}}\right)\right)x\right\}$$

2.1345 ODE No. 1345

$$y''(x) = \frac{2y(x)}{x^4} - \frac{y'(x)}{x^3}$$

✓ **Mathematica** : cpu = 0.0589396 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{2x^2}} x - \sqrt{\frac{\pi}{2}} c_2 e^{\frac{1}{2x^2}} x \operatorname{erf}\left(\frac{1}{\sqrt{2}x}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 25

$$\left\{ y(x) = \left(c_2 \operatorname{erf}\left(\frac{\sqrt{2}}{2x}\right) + c_1 \right) x e^{\frac{1}{2x^2}} \right\}$$

2.1346 ODE No. 1346

$$y''(x) = \frac{(a+b)y'(x)}{x^2} - \frac{y(x)(x(a+b)+ab)}{x^4}$$

✓ **Mathematica** : cpu = 0.0931848 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 x e^{-\frac{a}{x}}}{a-b} + c_1 x e^{-\frac{b}{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 25

$$\left\{ y(x) = \left(c_1 e^{-\frac{a}{x}} + c_2 e^{-\frac{b}{x}} \right) x \right\}$$

2.1347 ODE No. 1347

$$y''(x) = -\frac{y(x)}{x^4} - \frac{y'(x)}{x}$$

✓ **Mathematica** : cpu = 0.0886726 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow c_2 J_0\left(\frac{1}{x}\right) + \frac{c_1 K_0\left(\frac{i}{x}\right)}{\sqrt{\pi}} \right\} \right\}$$

✓ **Maple** : cpu = 0.071 (sec), leaf count = 19

$$\left\{ y(x) = c_1 \operatorname{BesselJ}\left(0, \frac{1}{x}\right) + c_2 \operatorname{BesselY}\left(0, \frac{1}{x}\right) \right\}$$

2.1348 ODE No. 1348

$$y''(x) = -\frac{y(x)(a(x^4+1)+bx^2)}{x^4} - \frac{y'(x)}{x}$$

✓ **Mathematica** : cpu = 0.380489 (sec), leaf count = 34

$$\{ \{ y(x) \rightarrow c_1 \operatorname{MathieuC}[-b, a, i \log(x)] + c_2 \operatorname{MathieuS}[-b, a, i \log(x)] \} \}$$

✓ **Maple** : cpu = 0.253 (sec), leaf count = 73

$$\left\{ y(x) = \left(c_2 \left(\int \frac{1}{x \operatorname{HD}\left(0, 2a+b, 0, 2a-b, \frac{x^2+1}{x^2-1}\right)^2} dx \right) + c_1 \right) \operatorname{HD}\left(0, 2a+b, 0, 2a-b, \frac{x^2+1}{x^2-1}\right) \right\}$$

2.1349 ODE No. 1349

$$y''(x) = -\frac{y(x)}{x^4} - \frac{(x^2 + 1)y'(x)}{x^3}$$

✓ **Mathematica** : cpu = 0.10466 (sec), leaf count = 76

$$\left\{ \left\{ y(x) \rightarrow c_2 G_{1,2}^{2,0} \left(-\frac{1}{2x^2} \middle| \begin{array}{c} \frac{3}{2} \\ 0,0 \end{array} \right) + c_1 e^{\frac{1}{4x^2}} \left(\left(1 - \frac{1}{2x^2} \right) I_0 \left(\frac{1}{4x^2} \right) + \frac{I_1 \left(\frac{1}{4x^2} \right)}{2x^2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.118 (sec), leaf count = 85

$$\left\{ y(x) = \frac{c_1 \left(2x^2 \operatorname{BesselI} \left(0, \frac{1}{4x^2} \right) - \operatorname{BesselI} \left(0, \frac{1}{4x^2} \right) + \operatorname{BesselI} \left(1, \frac{1}{4x^2} \right) \right) e^{\frac{1}{4x^2}}}{x^2} + \frac{c_2 \left(2x^2 \operatorname{BesselK} \left(0, -\frac{1}{4x^2} \right) - \operatorname{BesselK} \left(0, -\frac{1}{4x^2} \right) \right)}{x^2} \right\}$$

2.1350 ODE No. 1350

$$y''(x) = -\frac{a^2 y(x)}{x^4} - \frac{2y'(x)}{x}$$

✓ **Mathematica** : cpu = 0.0082499 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow c_1 \cos \left(\frac{a}{x} \right) - c_2 \sin \left(\frac{a}{x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 21

$$\left\{ y(x) = c_1 \sin \left(\frac{a}{x} \right) + c_2 \cos \left(\frac{a}{x} \right) \right\}$$

2.1351 ODE No. 1351

$$y''(x) = \frac{y(x)}{x^4} - \frac{(2x^2 + 1)y'(x)}{x^3}$$

✓ **Mathematica** : cpu = 0.0187305 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{2x^2}} - \sqrt{\frac{\pi}{2}} c_2 e^{\frac{1}{2x^2}} \operatorname{erf} \left(\frac{1}{\sqrt{2}x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 24

$$\left\{ y(x) = \left(c_2 \operatorname{erf} \left(\frac{\sqrt{2}}{2x} \right) + c_1 \right) e^{\frac{1}{2x^2}} \right\}$$

2.1352 ODE No. 1352

$$y''(x) = -\frac{2(a+x)y'(x)}{x^2} - \frac{by(x)}{x^4}$$

✓ **Mathematica** : cpu = 0.0123503 (sec), leaf count = 89

$$\left\{ \left(\left\{ y(x) \rightarrow c_1 e^{-\frac{\sqrt{b} \left(-\frac{\sqrt{a^2-b}}{\sqrt{b}} - \frac{a}{\sqrt{b}} \right)}{x}} + c_2 e^{-\frac{\sqrt{b} \left(\frac{\sqrt{a^2-b}}{\sqrt{b}} - \frac{a}{\sqrt{b}} \right)}{x}} \right\} \right) \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 43

$$\left\{ y(x) = c_1 e^{\frac{a - \sqrt{a^2-b}}{x}} + c_2 e^{\frac{a + \sqrt{a^2-b}}{x}} \right\}$$

2.1353 ODE No. 1353

$$y''(x) = \frac{(2x^2 - 1)y'(x)}{x^3} - \frac{y(x)}{x^4}$$

✓ **Mathematica** : cpu = 0.8391 (sec), leaf count = 119

$$\left\{ \left\{ y(x) \rightarrow c_1 \left(x^3 + 2x - \frac{1}{x} \right) - \frac{c_2 \left(\sqrt{2\pi} x^4 \operatorname{erfi} \left(\frac{1}{\sqrt{2}x} \right) + 2\sqrt{2\pi} x^2 \operatorname{erfi} \left(\frac{1}{\sqrt{2}x} \right) - \sqrt{2\pi} \operatorname{erfi} \left(\frac{1}{\sqrt{2}x} \right) + 2e^{\frac{1}{2x^2}} x - 2e^{\frac{1}{2x^2}} x^3 \right)}{16x} \right\} \right\}$$

✓ **Maple** : cpu = 0.249 (sec), leaf count = 66

$$\left\{ y(x) = \frac{c_1 \sqrt{2} \sqrt{\pi} (x^4 + 2x^2 - 1) \operatorname{erfi} \left(\frac{\sqrt{2}}{2x} \right) + c_2 (x^4 + 2x^2 - 1) + (-2c_1 x^3 + 2c_1 x) e^{\frac{1}{2x^2}}}{x} \right\}$$

2.1354 ODE No. 1354

$$y''(x) = \frac{(2x^2 - 1)y'(x)}{x^3} - \frac{2y(x)}{x^4}$$

✓ **Mathematica** : cpu = 0.24373 (sec), leaf count = 108

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \left(-5\sqrt{2\pi} x^2 \operatorname{erfi} \left(\frac{1}{\sqrt{2}x} \right) + \sqrt{2\pi} \operatorname{erfi} \left(\frac{1}{\sqrt{2}x} \right) - 2e^{\frac{1}{2x^2}} x + 4e^{\frac{1}{2x^2}} x^5 + 8e^{\frac{1}{2x^2}} x^3 \right)}{12x^2} + c_1 \left(1 - \frac{1}{5x^2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.298 (sec), leaf count = 33

$$\left\{ y(x) = \frac{c_2 x^5 \operatorname{hypergeom} \left(\left[-\frac{5}{2} \right], \left[-\frac{1}{2} \right], \frac{1}{2x^2} \right) + 5c_1 x^2 - c_1}{x^2} \right\}$$

2.1355 ODE No. 1355

$$y''(x) = \frac{xy(x)}{x^3 + 1} - \frac{(x^3 - 1)y'(x)}{x(x^3 + 1)}$$

✓ **Mathematica** : cpu = 0.289484 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} c_2 \sqrt[3]{x^3 + 1} x^2 {}_2F_1 \left(\frac{2}{3}, \frac{4}{3}; \frac{5}{3}; -x^3 \right) + c_1 \sqrt[3]{x^3 + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.13 (sec), leaf count = 30

$$\left\{ y(x) = (x^3 + 1)^{\frac{1}{3}} \left(c_1 x^2 \operatorname{hypergeom} \left(\left[\frac{2}{3}, \frac{4}{3} \right], \left[\frac{5}{3} \right], -x^3 \right) + c_2 \right) \right\}$$

2.1356 ODE No. 1356

$$y''(x) = -\frac{y(x)(-n^2 - v(v+1)x^2)}{x^2(x^2 + 1)} - \frac{(2x^2 + 1)y'(x)}{x(x^2 + 1)}$$

✓ **Mathematica** : cpu = 0.24644 (sec), leaf count = 90

$$\left\{ \left\{ y(x) \rightarrow c_1 x^{-n} {}_2F_1 \left(-\frac{n}{2} - \frac{v}{2}, -\frac{n}{2} + \frac{v}{2} + \frac{1}{2}; 1 - n; -x^2 \right) + c_2 x^n {}_2F_1 \left(\frac{n}{2} - \frac{v}{2}, \frac{n}{2} + \frac{v}{2} + \frac{1}{2}; n + 1; -x^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.116 (sec), leaf count = 29

$$\left\{y(x) = c_1 \operatorname{LegendreP}\left(v, n, \sqrt{x^2 + 1}\right) + c_2 \operatorname{LegendreQ}\left(v, n, \sqrt{x^2 + 1}\right)\right\}$$

2.1357 ODE No. 1357

$$y''(x) = -\frac{(ax^2 + a - 1)y'(x)}{x(x^2 + 1)} - \frac{y(x)(bx^2 + c)}{x^2(x^2 + 1)}$$

✓ **Mathematica** : cpu = 0.578118 (sec), leaf count = 288

$$\left\{\left\{y(x) \rightarrow c_1 x^{\frac{1}{2}\left(-\sqrt{a^2-4a-4c+4}-a+2\right)} {}_2F_1\left(-\frac{1}{4}\sqrt{a^2-2a-4b+1}-\frac{1}{4}\sqrt{a^2-4a-4c+4}+\frac{1}{4}, \frac{1}{4}\sqrt{a^2-2a-4b+1}-\frac{1}{4}\sqrt{a^2-4a-4c+4}, x\right) + c_2 x^{\frac{1}{2}\left(-\sqrt{a^2-4a-4c+4}-a+2\right)} {}_2F_1\left(-\frac{1}{4}\sqrt{a^2-2a-4b+1}-\frac{1}{4}\sqrt{a^2-4a-4c+4}+\frac{1}{4}, \frac{1}{4}\sqrt{a^2-2a-4b+1}-\frac{1}{4}\sqrt{a^2-4a-4c+4}, x\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.145 (sec), leaf count = 97

$$\left\{y(x) = \left(c_1 \operatorname{LegendreP}\left(-\frac{1}{2} + \frac{\sqrt{a^2-2a-4b+1}}{2}, \frac{\sqrt{a^2-4a-4c+4}}{2}, \sqrt{x^2+1}\right) + c_2 \operatorname{LegendreQ}\left(-\frac{1}{2} + \frac{\sqrt{a^2-2a-4b+1}}{2}, \frac{\sqrt{a^2-4a-4c+4}}{2}, \sqrt{x^2+1}\right)\right)\right\}$$

2.1358 ODE No. 1358

$$y''(x) = \frac{(x^2-2)y'(x)}{x(x^2-1)} - \frac{(x^2-2)y(x)}{x^2(x^2-1)}$$

✓ **Mathematica** : cpu = 0.0492311 (sec), leaf count = 89

$$\left\{\left\{y(x) \rightarrow \frac{c_1 x^{\frac{4}{3}\sqrt{x^2-1}}}{\sqrt[4]{1-x^2}} - \frac{c_2 x^{\frac{4}{3}\sqrt{x^2-1}}\left(\log\left(1-\frac{x}{\sqrt{x^2-1}}\right) - \log\left(\frac{x}{\sqrt{x^2-1}}+1\right)\right)}{2\sqrt[4]{1-x^2}}\right\}\right\}$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 20

$$\left\{y(x) = \left(c_2 \ln\left(x + \sqrt{x^2-1}\right) + c_1\right)x\right\}$$

2.1359 ODE No. 1359

$$y''(x) = -\frac{v(v+1)y(x)}{x^2(x^2-1)} - \frac{2xy'(x)}{x^2-1}$$

✓ **Mathematica** : cpu = 0.0883112 (sec), leaf count = 86

$$\left\{\left\{y(x) \rightarrow c_1 i^{-v} x^{-v} {}_2F_1\left(\frac{1}{2} - \frac{v}{2}, -\frac{v}{2}; \frac{1}{2} - v; x^2\right) + c_2 i^{v+1} x^{v+1} {}_2F_1\left(\frac{v}{2} + \frac{1}{2}, \frac{v}{2} + 1; v + \frac{3}{2}; x^2\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.148 (sec), leaf count = 57

$$\left\{y(x) = c_1 x^{-v} \operatorname{hypergeom}\left(\left[-\frac{v}{2}, -\frac{v}{2} + \frac{1}{2}\right], \left[-v + \frac{1}{2}\right], x^2\right) + c_2 x^{v+1} \operatorname{hypergeom}\left(\left[\frac{v}{2} + 1, \frac{v}{2} + \frac{1}{2}\right], \left[v + \frac{3}{2}\right], x^2\right)\right\}$$

2.1360 ODE No. 1360

$$y''(x) = \frac{v(v+1)y(x)}{x^2} - \frac{2xy'(x)}{x^2-1}$$

✓ **Mathematica** : cpu = 0.0791025 (sec), leaf count = 68

$$\left\{\left\{y(x) \rightarrow c_1 i^{-v} x^{-v} {}_2F_1\left(\frac{1}{2}, -v; \frac{1}{2} - v; x^2\right) + c_2 i^{v+1} x^{v+1} {}_2F_1\left(\frac{1}{2}, v+1; v + \frac{3}{2}; x^2\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.124 (sec), leaf count = 47

$$\left\{y(x)=c_1x^{-v}\operatorname{hypergeom}\left(\left[\frac{1}{2},-v\right],\left[-v+\frac{1}{2}\right],x^2\right)+c_2x^{v+1}\operatorname{hypergeom}\left(\left[\frac{1}{2},v+1\right],\left[v+\frac{3}{2}\right],x^2\right)\right\}$$

2.1361 ODE No. 1361

$$y''(x)=\frac{2xy'(x)}{x^2-1}-\frac{(a(a+1)-a(a+3)x^2)y(x)}{x^2(x^2-1)}$$

✓ **Mathematica** : cpu = 0.480315 (sec), leaf count = 38

$$\left\{\left\{y(x)\rightarrow c_1x^{-a}+c_2(-2ax^2+2a-x^2+3)x^{a+1}\right\}\right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 33

$$\{y(x)=c_1x^{-a}+c_2(2ax^2+x^2-2a-3)x^{a+1}\}$$

2.1362 ODE No. 1362

$$y''(x)=\frac{2xy'(x)}{x^2-1}-\frac{y(x)\left((x^2-1)x^2(a-n)(a+n+1)+2ax^2+n(n+1)(x^2-1)\right)}{x^2(x^2-1)}$$

✗ **Mathematica** : cpu = 10.2771 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{\left\{y(x)\rightarrow (x)\right\}\right\}$$

✓ **Maple** : cpu = 0.231 (sec), leaf count = 109

$$\left\{y(x)=c_1x^{-n}\operatorname{HeunC}\left(0,-n-\frac{1}{2},-2,-\frac{1}{4}a^2+\frac{1}{4}n^2-\frac{1}{4}a+\frac{1}{4}n,\frac{1}{4}a^2-\frac{1}{4}n^2-\frac{1}{4}a-\frac{1}{4}n+\frac{3}{4},x^2\right)+c_2x^{n+1}\operatorname{HeunC}\left(0,n+\frac{1}{2},2,\frac{1}{4}a^2-\frac{1}{4}n^2+\frac{1}{4}a-\frac{1}{4}n,\frac{1}{4}a^2-\frac{1}{4}n^2-\frac{1}{4}a+\frac{1}{4}n,-x^2\right)\right\}$$

2.1363 ODE No. 1363

$$y''(x)=-\frac{(ax^2+a-2)y'(x)}{x(x^2-1)}-\frac{by(x)}{x^2}$$

✓ **Mathematica** : cpu = 0.601191 (sec), leaf count = 236

$$\left\{\left\{y(x)\rightarrow c_1(-1)^{\frac{1}{4}\left(-\sqrt{a^2-2a-4b+1}+a-1\right)}x^{\frac{1}{2}\left(-\sqrt{a^2-2a-4b+1}+a-1\right)}{}_2F_1\left(\frac{a}{2}-\frac{1}{2},\frac{a}{2}-\frac{1}{2},\frac{a}{2}-\frac{1}{2},\sqrt{a^2-2a-4b+1}-\frac{1}{2};1-\frac{1}{2}\sqrt{a^2-2a-4b+1},x^2\right)+c_2x^{\frac{a}{2}}\right\}\right\}$$

✓ **Maple** : cpu = 0.171 (sec), leaf count = 161

$$\left\{y(x)=\left(c_1x^{\frac{a}{2}-\frac{1}{2}+\frac{\sqrt{a^2-2a-4b+1}}{2}}\operatorname{hypergeom}\left(\left[-\frac{a}{2}+\frac{3}{2},-\frac{a}{2}+\frac{3}{2}+\frac{\sqrt{a^2-2a-4b+1}}{2}\right],\left[1+\frac{\sqrt{a^2-2a-4b+1}}{2}\right],x^2\right)+c_2x^{\frac{a}{2}}\right\}$$

2.1364 ODE No. 1364

$$y''(x)=\frac{y'(x)(2(a-1)x^2-2a+2bc(x^2-1)x^c)}{x(x^2-1)}-\frac{y(x)(bc(2a-c-1)x^{c+2}-bc(2a-c+1)x^c+x^2((a-1)a-v(v+1))-a(v+1))}{x^2(x^2-1)}$$

✓ **Mathematica** : cpu = 0.123224 (sec), leaf count = 42

$$\left\{\left\{y(x)\rightarrow c_1P_v(x)e^{a\log(x)+bx^c}+c_2Q_v(x)e^{a\log(x)+bx^c}\right\}\right\}$$

✓ **Maple** : cpu = 0.115 (sec), leaf count = 25

$$\{y(x)=(c_1\operatorname{LegendreP}(v,x)+c_2\operatorname{LegendreQ}(v,x))x^ae^{bx^c}\}$$

2.1365 ODE No. 1365

$$y''(x) = -\frac{ay(x)}{(x^2 + 1)^2}$$

✓ **Mathematica** : cpu = 0.0808674 (sec), leaf count = 104

$$\left\{ \left\{ y(x) \rightarrow \frac{ic_2 \sqrt{x^2 + 1} (1 - ix)^{\sqrt{a+1}} (1 + ix)^{-\sqrt{a+1}} e^{i\sqrt{a+1} \tan^{-1}(x)}}{2\sqrt{a+1}} + c_1 \sqrt{x^2 + 1} e^{i\sqrt{a+1} \tan^{-1}(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 59

$$\left\{ y(x) = \sqrt{x^2 + 1} \left(c_1 \left(\frac{x+i}{-x+i} \right)^{\frac{\sqrt{a+1}}{2}} + c_2 \left(\frac{x+i}{-x+i} \right)^{-\frac{\sqrt{a+1}}{2}} \right) \right\}$$

2.1366 ODE No. 1366

$$y''(x) = -\frac{2xy'(x)}{x^2 + 1} - \frac{y(x)}{(x^2 + 1)^2}$$

✓ **Mathematica** : cpu = 0.018508 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{\sqrt{x^2 + 1}} + \frac{c_2 x}{\sqrt{x^2 + 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 17

$$\left\{ y(x) = \frac{c_1 x + c_2}{\sqrt{x^2 + 1}} \right\}$$

2.1367 ODE No. 1367

$$y''(x) = -\frac{y(x) \left(a^2 (x^2 + 1)^2 + m^2 - n(n+1)(x^2 + 1) \right)}{(x^2 + 1)^2} - \frac{2xy'(x)}{x^2 + 1}$$

✓ **Mathematica** : cpu = 0.235702 (sec), leaf count = 229

$$\left\{ \left\{ y(x) \rightarrow c_1 (x^2 + 1)^{\frac{\sqrt{m^2}}{2}} \text{HeunC} \left[\frac{1}{4} \left(-a^2 - \sqrt{m^2} (\sqrt{m^2} + 1) + \frac{1}{4} (\sqrt{(2n+1)^2} - 1)^2 + \frac{1}{2} (\sqrt{(2n+1)^2} - 1) \right), -\frac{a^2}{4}, \frac{1}{2}, \sqrt{m^2} \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.23 (sec), leaf count = 88

$$\left\{ y(x) = \left(c_2 x \text{HeunC} \left(0, \frac{1}{2}, m, -\frac{a^2}{4}, \frac{1}{4} a^2 + \frac{1}{4} m^2 - \frac{1}{4} n^2 - \frac{1}{4} n + \frac{1}{4}, -x^2 \right) + c_1 \text{HeunC} \left(0, -\frac{1}{2}, m, -\frac{a^2}{4}, \frac{1}{4} a^2 + \frac{1}{4} m^2 - \frac{1}{4} n^2 - \frac{1}{4} n + \frac{1}{4}, -x^2 \right) \right) \right\}$$

2.1368 ODE No. 1368

$$y''(x) = -\frac{axy'(x)}{x^2 + 1} - \frac{by(x)}{(x^2 + 1)^2}$$

✓ **Mathematica** : cpu = 0.0208805 (sec), leaf count = 106

$$\left\{ \left\{ y(x) \rightarrow c_1 (x^2 + 1)^{\frac{2-a}{4}} P_{\frac{a-2}{2}}^{\frac{1}{2} \sqrt{a^2 - 4a + 4b + 4}}(ix) + c_2 (x^2 + 1)^{\frac{2-a}{4}} Q_{\frac{a-2}{2}}^{\frac{1}{2} \sqrt{a^2 - 4a + 4b + 4}}(ix) \right\} \right\}$$

✓ **Maple** : cpu = 0.115 (sec), leaf count = 71

$$\left\{ y(x) = \left(c_1 \text{LegendreP} \left(\frac{a}{2} - 1, \frac{\sqrt{a^2 - 4a + 4b + 4}}{2}, ix \right) + c_2 \text{LegendreQ} \left(\frac{a}{2} - 1, \frac{\sqrt{a^2 - 4a + 4b + 4}}{2}, ix \right) \right) (x^2 + 1)^{-\frac{a}{4} + \frac{1}{2}} \right\}$$

2.1369 ODE No. 1369

$$y''(x) = -\frac{ay(x)}{(x^2 - 1)^2}$$

✓ **Mathematica** : cpu = 0.0937368 (sec), leaf count = 106

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \sqrt{1-x^2} (x+1)^{\sqrt{1-a}} (1-x)^{-\sqrt{1-a}} e^{-\sqrt{1-a} \tanh^{-1}(x)}}{2\sqrt{1-a}} + c_1 \sqrt{1-x^2} e^{-\sqrt{1-a} \tanh^{-1}(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 55

$$\left\{ y(x) = \sqrt{x^2 - 1} \left(c_1 \left(\frac{x-1}{x+1} \right)^{\frac{\sqrt{-a+1}}{2}} + c_2 \left(\frac{x-1}{x+1} \right)^{-\frac{\sqrt{-a+1}}{2}} \right) \right\}$$

2.1370 ODE No. 1370

$$y''(x) = \frac{a^2 y(x)}{(x^2 - 1)^2} - \frac{2xy'(x)}{x^2 - 1}$$

✓ **Mathematica** : cpu = 0.0212122 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow c_1 \cosh\left(\frac{1}{2}a(\log(1-x) - \log(x+1))\right) + ic_2 \sinh\left(\frac{1}{2}a(\log(1-x) - \log(x+1))\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 19

$$\{y(x) = c_1 \sinh(a \operatorname{arctanh}(x)) + c_2 \cosh(a \operatorname{arctanh}(x))\}$$

2.1371 ODE No. 1371

$$y''(x) = -\frac{y(x)(-a^2 - \lambda(x^2 - 1))}{(x^2 - 1)^2} - \frac{2xy'(x)}{x^2 - 1}$$

✓ **Mathematica** : cpu = 0.0234926 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow c_1 P_{\frac{1}{2}(\sqrt{4\lambda+1}-1)}^a(x) + c_2 Q_{\frac{1}{2}(\sqrt{4\lambda+1}-1)}^a(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.106 (sec), leaf count = 37

$$\left\{ y(x) = c_1 \operatorname{LegendreP}\left(\frac{\sqrt{4\lambda+1}}{2} - \frac{1}{2}, a, x\right) + c_2 \operatorname{LegendreQ}\left(\frac{\sqrt{4\lambda+1}}{2} - \frac{1}{2}, a, x\right) \right\}$$

2.1372 ODE No. 1372

$$y''(x) = -\frac{y(x)((x^2 - 1)(ax^2 + bx + c) - k^2)}{(x^2 - 1)^2} - \frac{2xy'(x)}{x^2 - 1}$$

✓ **Mathematica** : cpu = 0.305321 (sec), leaf count = 202

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\sqrt{-a}x} (x^2 - 1)^{k/2} \operatorname{HeunC}\left[(k+1)(2\sqrt{-a} - k) - a + b - c, 2(2\sqrt{-a}(k+1) + b), k+1, k+1, 4\sqrt{-a}, \frac{x}{2} + \frac{1}{2}\right] + c_2 e^{-\sqrt{-a}x} (x^2 - 1)^{k/2} \operatorname{HeunC}\left[(k+1)(2\sqrt{-a} + k) - a + b - c, 2(2\sqrt{-a}(k+1) - b), k+1, k+1, 4\sqrt{-a}, \frac{x}{2} + \frac{1}{2}\right] \right\} \right\}$$

✓ **Maple** : cpu = 0.286 (sec), leaf count = 110

$$\left\{ y(x) = \left(c_2 \sqrt{2x-2} (x+1)^{-\frac{k}{2}} (x-1)^{\frac{k}{2}-\frac{1}{2}} \operatorname{HeunC}\left(4\sqrt{-a}, -k, k, 2b, \frac{k^2}{2} + a - b + c, \frac{x}{2} + \frac{1}{2}\right) + c_1 (x^2 - 1)^{\frac{k}{2}} \operatorname{HeunC}\left(4\sqrt{-a}, \right. \right.$$

2.1373 ODE No. 1373

$$y''(x) = -\frac{y(x)\left(-a^2(x^2-1)^2 - m^2 - n(n+1)(x^2-1)\right)}{(x^2-1)^2} - \frac{2xy'(x)}{x^2-1}$$

✓ **Mathematica** : cpu = 0.23455 (sec), leaf count = 113

$$\left\{ \left\{ y(x) \rightarrow c_1(x^2-1)^{m/2} \text{HeunC}\left[\frac{1}{4}(-a^2 - m(m+1) + n^2 + n), -\frac{a^2}{4}, \frac{1}{2}, m+1, 0, x^2\right] + c_2x(x^2-1)^{m/2} \text{HeunC}\left[\frac{1}{4}(-a^2 - (n+1)(n+2) + m^2 + m), -\frac{a^2}{4}, \frac{1}{2}, m+1, 0, x^2\right] \right\} \right\}$$

✓ **Maple** : cpu = 0.253 (sec), leaf count = 84

$$\left\{ y(x) = \left(c_2x \text{HeunC}\left(0, \frac{1}{2}, m, -\frac{a^2}{4}, \frac{1}{4}a^2 + \frac{1}{4}m^2 - \frac{1}{4}n^2 - \frac{1}{4}n + \frac{1}{4}, x^2\right) + c_1 \text{HeunC}\left(0, -\frac{1}{2}, m, -\frac{a^2}{4}, \frac{1}{4}a^2 + \frac{1}{4}m^2 - \frac{1}{4}n^2 - \frac{1}{4}n + \frac{1}{4}, x^2\right) \right) \right\}$$

2.1374 ODE No. 1374

$$y''(x) = \frac{2(2a-1)xy'(x)}{x^2-1} - \frac{y(x)(x^2(2a(2a-1) - v(v+1)) + 2a + v(v+1))}{(x^2-1)^2}$$

✓ **Mathematica** : cpu = 0.0325278 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow c_1(x^2-1)^a P_v(x) + c_2(x^2-1)^a Q_v(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.099 (sec), leaf count = 23

$$\{y(x) = (c_1 \text{LegendreP}(v, x) + c_2 \text{LegendreQ}(v, x))(x^2-1)^a\}$$

2.1375 ODE No. 1375

$$y''(x) = -\frac{y(x)(4ax^2(a-n) - (x^2-1)(2a + (v-n)(n+v+1)))}{(x^2-1)^2} - \frac{2x(-2a + n+1)y'(x)}{x^2-1}$$

✓ **Mathematica** : cpu = 0.0378384 (sec), leaf count = 54

$$\left\{ \left\{ y(x) \rightarrow c_1(x^2-1)^{\frac{1}{2}(2a-n)} P_v^n(x) + c_2(x^2-1)^{\frac{1}{2}(2a-n)} Q_v^n(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.109 (sec), leaf count = 29

$$\left\{ y(x) = (c_1 \text{LegendreP}(v, n, x) + c_2 \text{LegendreQ}(v, n, x))(x^2-1)^{a-\frac{n}{2}} \right\}$$

2.1376 ODE No. 1376

$$y''(x) = -\frac{by(x)}{x^2(a+x^2)} - \frac{(a+2x^2)y'(x)}{x(a+x^2)}$$

✓ **Mathematica** : cpu = 0.0336135 (sec), leaf count = 69

$$\left\{ \left(\left\{ y(x) \rightarrow c_1 \cos\left(\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{a+x^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right) - c_2 \sin\left(\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{a+x^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right) \right\} \right) \right\}$$

✓ **Maple** : cpu = 0.06 (sec), leaf count = 73

$$\left\{ y(x) = \left(c_2 \left(\frac{2a + 2\sqrt{x^2+a} \sqrt{a}}{x} \right)^{\frac{2i\sqrt{b}}{\sqrt{a}}} + c_1 \right) \left(\frac{2a + 2\sqrt{x^2+a} \sqrt{a}}{x} \right)^{-\frac{i\sqrt{b}}{\sqrt{a}}} \right\}$$

2.1377 ODE No. 1377

$$y''(x) = -\frac{b^2 y(x)}{(a^2 + x^2)^2}$$

✓ **Mathematica** : cpu = 0.187499 (sec), leaf count = 163

$$\left\{ \left\{ y(x) \rightarrow \frac{ic_2 \sqrt{a^2 + x^2} \left(1 - \frac{ix}{a}\right)^{\sqrt{\frac{a^2+b^2}{a^2}}} \left(1 + \frac{ix}{a}\right)^{-\sqrt{\frac{a^2+b^2}{a^2}}} e^{i\sqrt{\frac{a^2+b^2}{a^2}} \tan^{-1}\left(\frac{x}{a}\right)} + c_1 \sqrt{a^2 + x^2} e^{i\sqrt{\frac{b^2}{a^2}+1} \tan^{-1}\left(\frac{x}{a}\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.128 (sec), leaf count = 83

$$\left\{ y(x) = \sqrt{a^2 + x^2} \left(c_1 \left(\frac{ix - a}{ix + a} \right)^{\frac{\sqrt{a^2+b^2}}{2a}} + c_2 \left(\frac{ix - a}{ix + a} \right)^{-\frac{\sqrt{a^2+b^2}}{2a}} \right) \right\}$$

2.1378 ODE No. 1378

$$y''(x) = -\frac{2(x^2 - 1)y'(x)}{(x - 1)^2 x} - \frac{(-2x^2 + 2x + 2)y(x)}{(x - 1)^2 x^2}$$

✓ **Mathematica** : cpu = 0.0393729 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 x^2}{1 - x} + \frac{c_2 x (2x^2 \log(1 - x) - 2x^2 \log(x) + 2x - 2x \log(1 - x) + 2x \log(x) - 1)}{(x - 1)^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 48

$$\left\{ y(x) = \frac{(c_1 x^2 + c_2 (x - 1)x \ln(x) - c_2 (x - 1)x \ln(x - 1) + \frac{c_2}{2} + (-c_1 - c_2)x)x}{(x - 1)^2} \right\}$$

2.1379 ODE No. 1379

$$y''(x) = \frac{12y(x)}{(x + 1)^2 (x^2 + 2x + 3)}$$

✓ **Mathematica** : cpu = 0.057164 (sec), leaf count = 99

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \left(2x^3 + 4x^2 - 3\sqrt{2} x^2 \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right) + 8x - 6\sqrt{2} x \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right) - 9\sqrt{2} \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right) + 2 \right)}{2(x + 1)^2} + c_1 \left(\frac{2}{(x + 1)^2} + 1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 60

$$\left\{ y(x) = \frac{3c_2 (x^2 + 2x + 3) \arctan\left(\frac{(x+1)\sqrt{2}}{2}\right) + c_1 (x^2 + 2x + 3) - c_2 (x^3 + 2x^2 + 4x + 1)\sqrt{2}}{(x + 1)^2} \right\}$$

2.1380 ODE No. 1380

$$y''(x) = -\frac{by(x)}{x^2(x - a)^2}$$

✓ **Mathematica** : cpu = 0.20333 (sec), leaf count = 132

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2(x-a)^{\frac{1}{2}} \sqrt{\frac{a^2-4b}{a^2}} + \frac{1}{2} x^{\frac{1}{2}-\frac{1}{2}} \sqrt{\frac{a^2-4b}{a^2}}}{a \sqrt{\frac{a^2-4b}{a^2}}} + c_1(x-a)^{\frac{1}{2}-\frac{1}{2}} \sqrt{1-\frac{4b}{a^2}} x^{\frac{1}{2}} \sqrt{1-\frac{4b}{a^2}} + \frac{1}{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 67

$$\left\{ y(x) = \sqrt{(a-x)x} \left(c_1 \left(\frac{a-x}{x} \right)^{\frac{\sqrt{a^2-4b}}{2a}} + c_2 \left(\frac{x}{a-x} \right)^{\frac{\sqrt{a^2-4b}}{2a}} \right) \right\}$$

2.1381 ODE No. 1381

$$y''(x) = c - \frac{by(x)}{x^2(x-a)^2}$$

✓ **Mathematica** : cpu = 0.252297 (sec), leaf count = 589

$$\left\{ \left\{ y(x) \rightarrow - \frac{2cx^2(a-x) \left(1 - \frac{x}{a}\right)^{-\frac{1}{2}} \sqrt{\frac{a^2-4b}{a^2}} \left(\sqrt{\frac{a^2-4b}{a^2}} \left(1 - \frac{x}{a}\right) \sqrt{\frac{a^2-4b}{a^2}} {}_2F_1\left(\frac{1}{2}, \sqrt{1-\frac{4b}{a^2}} - \frac{1}{2}, \frac{1}{2}, \sqrt{1-\frac{4b}{a^2}} + \frac{3}{2}; \frac{1}{2}, \sqrt{1-\frac{4b}{a^2}} + \frac{5}{2}; \frac{x}{a}\right) - 3 \right)}{2cx^2(a-x) \left(1 - \frac{x}{a}\right)^{-\frac{1}{2}} \sqrt{\frac{a^2-4b}{a^2}} \left(\sqrt{\frac{a^2-4b}{a^2}} \left(1 - \frac{x}{a}\right) \sqrt{\frac{a^2-4b}{a^2}} {}_2F_1\left(\frac{1}{2}, \sqrt{1-\frac{4b}{a^2}} - \frac{1}{2}, \frac{1}{2}, \sqrt{1-\frac{4b}{a^2}} + \frac{3}{2}; \frac{1}{2}, \sqrt{1-\frac{4b}{a^2}} + \frac{5}{2}; \frac{x}{a}\right) - 3 \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.192 (sec), leaf count = 175

$$\left\{ y(x) = \frac{\left(\left(-c \left(\int \sqrt{(a-x)x} \left(\frac{a-x}{x} \right)^{-\frac{\sqrt{a^2-4b}}{2a}} dx \right) + c_2 \sqrt{a^2-4b} \right) \left(\frac{a-x}{x} \right)^{\frac{\sqrt{a^2-4b}}{2a}} + \left(c \left(\int \sqrt{(a-x)x} \left(\frac{x}{a-x} \right)^{-\frac{\sqrt{a^2-4b}}{2a}} dx \right) + c_1 \sqrt{a^2-4b} \right) \right)}{\sqrt{a^2-4b}}$$

2.1382 ODE No. 1382

$$y''(x) = \frac{cy(x)}{(x-a)^2(x-b)^2}$$

✓ **Mathematica** : cpu = 0.467723 (sec), leaf count = 154

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1(x-a)^{\frac{1}{2}} \left(\sqrt{\frac{4c}{(a-b)^2}+1}+1 \right) (x-b)^{\frac{1}{2}} \left(1-\sqrt{\frac{4c}{(a-b)^2}+1} \right) - c_2(x-a)^{\frac{1}{2}-\frac{1}{2}} \sqrt{\frac{4c}{(a-b)^2}+1} (x-b)^{\frac{1}{2}} \sqrt{\frac{4c}{(a-b)^2}+1} + \frac{1}{2}}{(a-b) \sqrt{\frac{4c}{(a-b)^2}+1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.13 (sec), leaf count = 104

$$\left\{ y(x) = \sqrt{(a-x)(b-x)} \left(c_1 \left(\frac{a-x}{b-x} \right)^{\frac{\sqrt{a^2-2ab+b^2+4c}}{2a-2b}} + c_2 \left(\frac{a-x}{b-x} \right)^{-\frac{\sqrt{a^2-2ab+b^2+4c}}{2a-2b}} \right) \right\}$$

2.1383 ODE No. 1383

$$y''(x) = -\frac{y'(x)((x-a)^2(\alpha+\beta+1)(x-b) + (x-a)(-\alpha-\beta+1)(x-b)^2)}{(x-a)^2(x-b)^2} - \frac{\alpha\beta(a-b)^2y(x)}{(x-a)^2(x-b)^2}$$

✓ **Mathematica** : cpu = 0.119147 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\alpha(\log(x-a)-\log(x-b))} + c_2 e^{\beta(\log(x-a)-\log(x-b))} \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 39

$$\left\{ y(x) = c_1 \left(\frac{a-x}{b-x} \right)^{\beta} + c_2 \left(\frac{a-x}{b-x} \right)^{\alpha} \right\}$$

2.1384 ODE No. 1384

$$y''(x) = -\frac{y(x)(- (a^2 - 1)x^2 + 2(a + 3)bx - b^2)}{4x^2}$$

✓ **Mathematica** : cpu = 0.022379 (sec), leaf count = 110

$$\left\{ \left\{ y(x) \rightarrow c_1 M_{\frac{(a+3)b}{2\sqrt{a^2-1}}, \frac{\sqrt{b(b^2+1)}}{2\sqrt{b}}} \left(\sqrt{a^2-1} x \right) + c_2 W_{\frac{(a+3)b}{2\sqrt{a^2-1}}, \frac{\sqrt{b(b^2+1)}}{2\sqrt{b}}} \left(\sqrt{a^2-1} x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.26 (sec), leaf count = 73

$$\left\{ y(x) = c_1 \text{WhittakerM} \left(\frac{(a+3)b}{2\sqrt{a^2-1}}, \frac{\sqrt{b^2+1}}{2}, \sqrt{a^2-1} x \right) + c_2 \text{WhittakerW} \left(\frac{(a+3)b}{2\sqrt{a^2-1}}, \frac{\sqrt{b^2+1}}{2}, \sqrt{a^2-1} x \right) \right\}$$

2.1385 ODE No. 1385

$$y''(x) = -\frac{(ax^2 + a - 3)y(x)}{4(x^2 + 1)^2}$$

✓ **Mathematica** : cpu = 0.0140124 (sec), leaf count = 78

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt{x^2 + 1} P_{\frac{1}{2}(\sqrt{1-a}-1)}^{\frac{1}{2}}(ix) + c_2 \sqrt{x^2 + 1} Q_{\frac{1}{2}(\sqrt{1-a}-1)}^{\frac{1}{2}}(ix) \right\} \right\}$$

✓ **Maple** : cpu = 0.064 (sec), leaf count = 55

$$\left\{ y(x) = (x^2 + 1)^{\frac{1}{4}} \left(c_1 \left(x + \sqrt{x^2 + 1} \right)^{\frac{\sqrt{-a+1}}{2}} + c_2 \left(x + \sqrt{x^2 + 1} \right)^{-\frac{\sqrt{-a+1}}{2}} \right) \right\}$$

2.1386 ODE No. 1386

$$y''(x) = \frac{18y(x)}{(2x+1)^2(x^2+x+1)}$$

✓ **Mathematica** : cpu = 0.064793 (sec), leaf count = 108

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1(x^2+x+1)}{(2x+1)^2} + \frac{c_2 \left(16x^3 + 24x^2 - 12\sqrt{3}x^2 \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) + 30x - 12\sqrt{3}x \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) - 12\sqrt{3} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) \right)}{(2x+1)^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 58

$$\left\{ y(x) = \frac{-36c_2(x^2+x+1) \arctan \left(\frac{(2x+1)\sqrt{3}}{3} \right) + c_1(x^2+x+1) + 16c_2 \left(x^3 + x^2 + \frac{11}{8}x + \frac{3}{16} \right) \sqrt{3}}{(2x+1)^2} \right\}$$

2.1387 ODE No. 1387

$$y''(x) = \frac{3y(x)}{4(x^2+x+1)^2}$$

✓ **Mathematica** : cpu = 0.0290872 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt{x^2+x+1} + \frac{2c_2 \sqrt{x^2+x+1} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right)}{\sqrt{3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 28

$$\left\{y(x) = \sqrt{x^2 + x + 1} \left(c_2 \arctan\left(\frac{(2x + 1)\sqrt{3}}{3}\right) + c_1 \right) \right\}$$

2.1388 ODE No. 1388

$$y''(x) = -\frac{y(x)(v(v + 1)(x - 1) - a^2x)}{4(x - 1)^2x^2} - \frac{(3x - 1)y'(x)}{2(x - 1)x}$$

✓ **Mathematica** : cpu = 0.262985 (sec), leaf count = 235

$$\left\{ \left\{ y(x) \rightarrow c_2(-1)^{\frac{1}{2}(-2v-3)+1} x^{\frac{1}{4}(-2v-3)+1} e^{\frac{1}{4}(-2\log(1-x)-\log(x))} (x-1)^{\frac{1}{2}\left(\frac{1}{2}(a+v+1)+\frac{1}{2}(a+v+2)+\frac{1}{2}(-2v-3)+1\right)} {}_2F_1\left(\frac{1}{2}(-2v-3) + \frac{1}{2}(a+v+1), \frac{1}{2}(-2v-3)+1, \frac{1}{2}(a+v+2)+\frac{1}{2}(-2v-3)+1, x\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.112 (sec), leaf count = 76

$$\left\{y(x) = \left(c_1x^{-\frac{v}{2}} \operatorname{hypergeom}\left(\left[-\frac{a}{2}-\frac{v}{2}, -\frac{a}{2}-\frac{v}{2}+\frac{1}{2}\right], \left[-v+\frac{1}{2}\right], x\right) + c_2x^{\frac{v}{2}+\frac{1}{2}} \operatorname{hypergeom}\left(\left[-\frac{a}{2}+\frac{v}{2}+1, -\frac{a}{2}+\frac{v}{2}+\frac{1}{2}\right], \left[-v+\frac{1}{2}\right], x\right)\right)\right\}$$

2.1389 ODE No. 1389

$$y''(x) = -\frac{y(x)(-4n^2x - v(v + 1)(x - 1)^2)}{4(x - 1)^2x^2} - \frac{(3x - 1)y'(x)}{2(x - 1)x}$$

✓ **Mathematica** : cpu = 0.330181 (sec), leaf count = 217

$$\left\{ \left\{ y(x) \rightarrow c_2(-1)^{\frac{1}{2}(-2v-3)+1} x^{\frac{1}{4}(-2v-3)+1} e^{\frac{1}{4}(-2\log(1-x)-\log(x))} (x-1)^{\frac{1}{2}\left(n+\frac{1}{2}(2n+1)+\frac{1}{2}(-2v-3)+v+2\right)} {}_2F_1\left(\frac{1}{2}(2n+1) + \frac{1}{2}(-2v-3)+1, \frac{1}{2}(-2v-3)+1, \frac{1}{2}(2n+1)+\frac{1}{2}(-2v-3)+v+2, x\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.124 (sec), leaf count = 68

$$\left\{y(x) = \left(c_1x^{-\frac{v}{2}} \operatorname{hypergeom}\left(\left[-n-v, -n+\frac{1}{2}\right], \left[-v+\frac{1}{2}\right], x\right) + c_2x^{\frac{v}{2}+\frac{1}{2}} \operatorname{hypergeom}\left(\left[-n+v+1, -n+\frac{1}{2}\right], \left[v+\frac{3}{2}\right], x\right)\right)\right\}$$

2.1390 ODE No. 1390

$$y''(x) = -\frac{3y(x)}{16(x - 1)^2x^2}$$

✓ **Mathematica** : cpu = 0.025122 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow 2c_2\sqrt[4]{1-x} x^{3/4} + c_1(1-x)^{3/4}\sqrt[4]{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 25

$$\left\{y(x) = c_1(x - 1)^{\frac{1}{4}}x^{\frac{3}{4}} + c_2(x - 1)^{\frac{3}{4}}x^{\frac{1}{4}}\right\}$$

2.1391 ODE No. 1391

$$y''(x) = \frac{(7ax^2 + 5)y'(x)}{x(ax^2 + 1)} - \frac{(15ax^2 + 5)y(x)}{x^2(ax^2 + 1)}$$

✓ **Mathematica** : cpu = 0.0406693 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow c_1x^5 - \frac{1}{4}c_2x(2ax^2 + 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 20

$$\{y(x) = c_1x^5 + 2c_2ax^3 + c_2x\}$$

2.1392 ODE No. 1392

$$y''(x) = -\frac{bxy'(x)}{a(x^2-1)} - \frac{y(x)(cx^2+dx+e)}{a(x^2-1)^2}$$

✓ **Mathematica** : cpu = 64.8077 (sec), leaf count = 1763961

Too large to display

✓ **Maple** : cpu = 0.233 (sec), leaf count = 561

$$\left\{y(x) = c_1 \left(\frac{x}{2} + \frac{1}{2}\right)^{\frac{2a - \sqrt{4a^2 + b^2 + (-4b - 4c + 4d - 4e)a}}{4a}} \operatorname{hypergeom}\left(\left[\frac{2a + \sqrt{4a^2 + b^2 + (-4b - 4c - 4d - 4e)a} + 2\sqrt{a^2 + b^2 + (-2b - 4c + 4d - 4e)a}}{4a}\right], \dots\right)\right\}$$

2.1393 ODE No. 1393

$$y''(x) = -\frac{y(x)(bx^2+cx+d)}{a(x-1)^2x^2}$$

✓ **Mathematica** : cpu = 15.271 (sec), leaf count = 413606

Too large to display

✓ **Maple** : cpu = 0.162 (sec), leaf count = 299

$$\left\{y(x) = c_1 x^{\frac{\sqrt{a} + \sqrt{a-4d}}{2\sqrt{a}}} (x-1)^{\frac{\sqrt{a} - \sqrt{a-4b-4c-4d}}{2\sqrt{a}}} \operatorname{hypergeom}\left(\left[-\frac{-\sqrt{a} + \sqrt{a-4b-4c-4d} - \sqrt{a-4d} + \sqrt{a-4b}}{2\sqrt{a}}, \frac{\sqrt{a} - \sqrt{a-4b-4c-4d}}{2\sqrt{a}}\right], \dots\right)\right\}$$

2.1394 ODE No. 1394

$$y''(x) = -\frac{cy(x)}{x^2(ax+b)^2} - \frac{2y'(x)}{x}$$

✓ **Mathematica** : cpu = 0.0391042 (sec), leaf count = 115

$$\left\{\left\{y(x) \rightarrow c_1 \exp\left(\frac{\sqrt{c}\left(-\frac{\sqrt{b^2-4c}}{\sqrt{c}} - \frac{b}{\sqrt{c}}\right)(\log(x) - \log(ax+b))}{2b}\right) + c_2 \exp\left(\frac{\sqrt{c}\left(\frac{\sqrt{b^2-4c}}{\sqrt{c}} - \frac{b}{\sqrt{c}}\right)(\log(x) - \log(ax+b))}{2b}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.108 (sec), leaf count = 79

$$\left\{y(x) = \sqrt{\frac{ax+b}{x}} \left(c_1 \left(\frac{x}{ax+b}\right)^{\frac{\sqrt{\frac{b^2-4c}{a^2}} a}} + c_2 \left(\frac{x}{ax+b}\right)^{-\frac{\sqrt{\frac{b^2-4c}{a^2}} a}}\right)\right\}$$

2.1395 ODE No. 1395

$$y''(x) = -\frac{y(x)}{(ax+b)^4}$$

✓ **Mathematica** : cpu = 0.0721561 (sec), leaf count = 59

$$\left\{\left\{y(x) \rightarrow c_1 e^{\frac{i}{a(ax+b)}} (ax+b) - \frac{1}{2} i c_2 e^{-\frac{i}{a(ax+b)}} (ax+b)\right\}\right\}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 39

$$\left\{y(x) = (ax+b) \left(c_1 \sin\left(\frac{1}{(ax+b)a}\right) + c_2 \cos\left(\frac{1}{(ax+b)a}\right)\right)\right\}$$

2.1396 ODE No. 1396

$$y''(x) = -\frac{Ay(x)}{(ax^2 + bx + c)^2}$$

✓ **Mathematica** : cpu = 0.963092 (sec), leaf count = 211

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \sqrt{ax^2 + bx + c} \exp\left(-\frac{\sqrt{4ac-b^2} \sqrt{1-\frac{4A}{b^2-4ac}} \tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)\right)}{\sqrt{b^2-4ac} \sqrt{1-\frac{4A}{b^2-4ac}}} + c_1 \sqrt{x(ax+b)+c} \exp\left(\frac{\sqrt{4ac-b^2} \sqrt{1-\frac{4A}{b^2-4ac}} \tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{b^2-4ac}}\right) \right. \right.$$

✓ **Maple** : cpu = 0.218 (sec), leaf count = 178

$$\left\{ y(x) = \sqrt{ax^2 + bx + c} \left(c_1 \left(\frac{-2ax - b + i\sqrt{4ac - b^2}}{2ax + b + i\sqrt{4ac - b^2}} \right)^{\frac{\sqrt{\frac{-4ac+b^2-4A}{a^2}} a}{2\sqrt{-4ac+b^2}}} + c_2 \left(\frac{-2ax - b + i\sqrt{4ac - b^2}}{2ax + b + i\sqrt{4ac - b^2}} \right)^{-\frac{\sqrt{\frac{-4ac+b^2-4A}{a^2}} a}{2\sqrt{-4ac+b^2}}} \right) \right\}$$

2.1397 ODE No. 1397

$$y''(x) = \frac{y(x)}{x^5} - \frac{y'(x)}{x^4}$$

✓ **Mathematica** : cpu = 0.0519583 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \Gamma\left(\frac{1}{3}, -\frac{1}{3x^3}\right)}{3^{2/3} \sqrt[3]{-\frac{1}{x^3}}} + c_1 x \right\} \right\}$$

✓ **Maple** : cpu = 0.111 (sec), leaf count = 27

$$\left\{ y(x) = \left(-c_2 \sqrt{3} \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{1}{3}, -\frac{1}{3x^3}\right) + c_1 + 2c_2 \pi \right) x \right\}$$

2.1398 ODE No. 1398

$$y''(x) = -\frac{(-(2v+1)^2 + x^2 - 1)y(x)}{(x^2 - 1)^2} - \frac{(3x^2 - 1)y'(x)}{x(x^2 - 1)}$$

✓ **Mathematica** : cpu = 0.15119 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow c_1 (x^2 - 1)^{-v-\frac{1}{2}} {}_2F_1\left(-v, -v; -2v; 1 - x^2\right) + c_2 (x^2 - 1)^{v+\frac{1}{2}} {}_2F_1\left(v+1, v+1; 2v+2; 1 - x^2\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.217 (sec), leaf count = 69

$$\left\{ y(x) = c_1 (x^2 - 1)^{-v-\frac{1}{2}} \text{hypergeom}\left([-v, -v], [-2v], -x^2 + 1\right) + c_2 (x^2 - 1)^{v+\frac{1}{2}} \text{hypergeom}\left([v+1, v+1], [2v+2], -x^2 + 1\right) \right\}$$

2.1399 ODE No. 1399

$$y''(x) = \frac{(3x+1)y'(x)}{(x-1)(x+1)} - \frac{36(x+1)^2 y(x)}{(x-1)^2(3x+5)^2}$$

✓ **Mathematica** : cpu = 0.0358289 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{2}(3 \log(1-x) + \log(3x+5))} + \frac{1}{2} c_2 e^{\frac{1}{2}(3 \log(1-x) + \log(3x+5))} (3 \log(1-x) + \log(3x+5)) \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 34

$$\left\{ y(x) = \sqrt{3x+5} (x-1)^{\frac{3}{2}} (3c_2 \ln(x-1) + c_2 \ln(3x+5) + c_1) \right\}$$

2.1400 ODE No. 1400

$$y''(x) = \frac{y'(x)}{x} - \frac{ay(x)}{x^6}$$

✓ **Mathematica** : cpu = 0.0992482 (sec), leaf count = 60

$$\left\{ \left\{ y(x) \rightarrow c_1 x^2 e^{\frac{i\sqrt{a}}{2x^2}} - \frac{ic_2 x^2 e^{\frac{i\sqrt{a}}{2x^2}}}{2\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 35

$$\left\{ y(x) = \left(c_1 \sinh\left(\frac{\sqrt{-a}}{2x^2}\right) + c_2 \cosh\left(\frac{\sqrt{-a}}{2x^2}\right) \right) x^2 \right\}$$

2.1401 ODE No. 1401

$$y''(x) = -\frac{(a+3x^2)y'(x)}{x^3} - \frac{by(x)}{x^6}$$

✓ **Mathematica** : cpu = 0.0104377 (sec), leaf count = 93

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\frac{\sqrt{b}\left(-\frac{\sqrt{a^2-4b}}{\sqrt{b}} - \frac{a}{\sqrt{b}}\right)}{4x^2}} + c_2 e^{-\frac{\sqrt{b}\left(\frac{\sqrt{a^2-4b}}{\sqrt{b}} - \frac{a}{\sqrt{b}}\right)}{4x^2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.068 (sec), leaf count = 45

$$\left\{ y(x) = c_1 e^{-\frac{-a+\sqrt{a^2-4b}}{4x^2}} + c_2 e^{\frac{a+\sqrt{a^2-4b}}{4x^2}} \right\}$$

2.1402 ODE No. 1402

$$y''(x) = -\frac{y(x)\left(4a(a+1)x^4 - 2a(x^2-1)x^2 + (x^2-1)^2(x^2-v^2)\right)}{x^2(x^2-1)^2} - \frac{((1-4a)x^2-1)y'(x)}{x(x^2-1)}$$

✓ **Mathematica** : cpu = 0.955371 (sec), leaf count = 86

$$\left\{ \left\{ y(x) \rightarrow c_2 (x^2-1)^{a+1} x^{-v} \text{HeunC}\left[-\frac{a}{2}+v-\frac{3}{4}, \frac{1}{4}, 1-v, 2, 0, x^2\right] + c_1 (x^2-1)^{a+1} x^v \text{HeunC}\left[\frac{1}{4}(-2a-4v-3), \frac{1}{4}, v+1, 2\right] \right\} \right\}$$

✓ **Maple** : cpu = 0.319 (sec), leaf count = 58

$$\left\{ y(x) = (x^2-1) \left(c_1 x^v \text{HeunC}\left(0, v, 1, \frac{1}{4}, \frac{a}{2} + \frac{1}{4}, x^2\right) + c_2 x^{-v} \text{HeunC}\left(0, -v, 1, \frac{1}{4}, \frac{a}{2} + \frac{1}{4}, x^2\right) \right) (x^2-1)^a \right\}$$

2.1403 ODE No. 1403

$$y''(x) = -y'(x)\left(\frac{-a1 - b1 + 1}{x - c1} + \frac{-a2 - b2 + 1}{x - c2} + \frac{-a3 - b3 + 1}{x - c3}\right) - \frac{y(x)\left(\frac{a1b1(c1-c2)(c1-c3)}{x-c1} + \frac{a2b2(c2-c1)(c2-c3)}{x-c2} + \frac{a3b3(c3-c1)(c3-c2)}{x-c3}\right)}{(x - c1)(x - c2)(x - c3)}$$

✓ **Mathematica** : cpu = 18.4876 (sec), leaf count = 316

$$\left\{\left\{y(x) \rightarrow c_1(x - c1)^{a1}(x - c2)^{a2}(x - c3)^{b3}\text{HeunG}\left[\frac{c1 - c3}{c1 - c2}, \frac{a1(c1(-a3 - 2b1 - b2 + 2) + c2(a3 + b1 - 1) + c3(b1 + b2 - a3))}{c1 - c2}, \frac{c1(-a3 - 2b1 - b2 + 2)c1 + (a3 + b1 - 1)c2 + (b1 + b2 - 1)c3}{c1 - c2}, \frac{a1 - (b1 - 1)(a2 + b3)c1}{c1 - c2}\right]\right.\right.$$

✓ **Maple** : cpu = 0.892 (sec), leaf count = 298

$$\left\{y(x) = \left(c_1(-c1 + x)^{a1}HG\left(\frac{c1 - c3}{c1 - c2}, \frac{((-a3 - 2b1 - b2 + 2)c1 + (a3 + b1 - 1)c2 + (b1 + b2 - 1)c3)a1 - (b1 - 1)(a2 + b3)c1}{c1 - c2}, \frac{c1(-a3 - 2b1 - b2 + 2)c1 + (a3 + b1 - 1)c2 + (b1 + b2 - 1)c3}{c1 - c2}, \frac{a1 - (b1 - 1)(a2 + b3)c1}{c1 - c2}\right)\right.$$

2.1404 ODE No. 1404

$$y''(x) = -\frac{(1 - 2x^2)y(x)}{4x^6} - \frac{(2x^2 + 1)y'(x)}{x^3}$$

✓ **Mathematica** : cpu = 0.0196547 (sec), leaf count = 33

$$\left\{\left\{y(x) \rightarrow \frac{c_1 e^{\frac{1}{4x^2}}}{x} + c_2 e^{\frac{1}{4x^2}}\right\}\right\}$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 19

$$\left\{y(x) = \frac{(c_1 x + c_2) e^{\frac{1}{4x^2}}}{x}\right\}$$

2.1405 ODE No. 1405

$$y''(x) = \frac{(2x^2 + 1)y'(x)}{x^3} - \frac{(ax^4 + 10x^2 + 1)y(x)}{4x^6}$$

✓ **Mathematica** : cpu = 0.0541893 (sec), leaf count = 77

$$\left\{\left\{y(x) \rightarrow c_1 e^{-\frac{1}{4x^2} x^{\frac{3}{2} - \frac{\sqrt{9-a}}{2}}} + \frac{c_2 e^{-\frac{1}{4x^2} x^{\frac{\sqrt{9-a}}{2} + \frac{3}{2}}}}{\sqrt{9-a}}\right\}\right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 42

$$\left\{y(x) = \left(c_1 x^{\frac{3}{2} + \frac{\sqrt{-a+9}}{2}} + c_2 x^{\frac{3}{2} - \frac{\sqrt{-a+9}}{2}}\right) e^{-\frac{1}{4x^2}}\right\}$$

2.1406 ODE No. 1406

$$y''(x) = -\frac{27xy(x)}{16(x^3 - 1)^2}$$

✓ **Mathematica** : cpu = 1.68214 (sec), leaf count = 258

$$\left\{\left\{y(x) \rightarrow \frac{\sqrt{2} c_2 (1 - x)^{3/4} \sqrt[4]{x^2 + x + 1} \int_1^x \frac{\sqrt{\sqrt{3} K[1] + \sqrt{2K[1] - i\sqrt{3} + 1}} \sqrt{2K[1] + i\sqrt{3} + 1} + \sqrt{3}}{2(1 - K[1])^{3/2} \sqrt{K[1]^2 + K[1] + 1}} dK[1]}{\sqrt[4]{\sqrt{3} x + \sqrt{2x - i\sqrt{3} + 1}} \sqrt{2x + i\sqrt{3} + 1} + \sqrt{3}} + \frac{\sqrt{2} c_1 (1 - x)^{3/4} \sqrt[4]{x^2 + x + 1} \int_1^x \frac{\sqrt{\sqrt{3} K[1] + \sqrt{2K[1] - i\sqrt{3} + 1}} \sqrt{2K[1] + i\sqrt{3} + 1} + \sqrt{3}}{2(1 - K[1])^{3/2} \sqrt{K[1]^2 + K[1] + 1}} dK[1]}{\sqrt[4]{\sqrt{3} x + \sqrt{2x - i\sqrt{3} + 1}} \sqrt{2x + i\sqrt{3} + 1} + \sqrt{3}}\right.\right.$$

✓ **Maple** : cpu = 0.163 (sec), leaf count = 44

$$\left\{y(x)=\left(x^3-1\right)^{\frac{1}{4}}\left(c_1 \operatorname{LegendreP}\left(-\frac{1}{6}, \frac{1}{3}, \sqrt{-x^3+1}\right)+c_2 \operatorname{LegendreQ}\left(-\frac{1}{6}, \frac{1}{3}, \sqrt{-x^3+1}\right)\right) \sqrt{x}\right\}$$

2.1407 ODE No. 1407

$$y''(x)=-y'(x)\left(\frac{b1(-a1-b1+1)}{b1x-a1}+\frac{b2(-a2-b2+1)}{b2x-a2}+\frac{b3(-a3-b3+1)}{b3x-a3}\right)-\frac{y(x)\left(\frac{a1b1(a1b2-a2b1)(a3b1-a1b3)}{b1x-a1}+\frac{a2b2}{(b1x-a1)}\right)}{(b1x-a1)}$$

✓ **Mathematica** : cpu = 88.7641 (sec), leaf count = 2002

$$\left\{\left\{y(x) \rightarrow\left(x-\frac{a2}{b2}\right)^{\frac{1}{2}\left(a2+b2-\sqrt{a2^2+6 b 2 a 2+b 2^2}\right)}\left(x-\frac{a3}{b3}\right)^{\frac{1}{2}\left(a3+b3+\sqrt{a3^2+6 b 3 a 3+b 3^2}\right)} c_1 \operatorname{HeunG}\left[\frac{a1}{b1}-\frac{a3}{b3}, \frac{a1\left(\frac{1}{2}\left(-a1-b1-\sqrt{a1^2+6 b 1 a 1+b 1^2}\right)\right)}{\frac{a1}{b1}-\frac{a2}{b2}}, \frac{a1\left(\frac{1}{2}\left(-a1-b1-\sqrt{a1^2+6 b 1 a 1+b 1^2}\right)\right)}{\frac{a1}{b1}-\frac{a2}{b2}}\right]\right.\right\}$$

✓ **Maple** : cpu = 1.664 (sec), leaf count = 2607

Expression too large to display

2.1408 ODE No. 1408

$$y''(x)=-\frac{y(x)\left(A x^2+B\right)}{x\left(x^2-a 1\right)\left(x^2-a 2\right)\left(x^2-a 3\right)}-\frac{y'(x)\left(x^2\left(\left(x^2-a 1\right)\left(x^2-a 2\right)+\left(x^2-a 1\right)\left(x^2-a 3\right)+\left(x^2-a 2\right)\left(x^2-a 3\right)\right)+\left(a 1 x^2+a 2 x^2+a 3 x^2\right)\right)}{x\left(x^2-a 1\right)\left(x^2-a 2\right)\left(x^2-a 3\right)}$$

✗ **Mathematica** : cpu = 50.8363 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{\left\{y(x) \rightarrow(x)\right\}\right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x)=DESol\left(\left\{\frac{d^2}{dx^2}-Y(x)+\frac{\left(A x^2+B\right)_-Y(x)}{\left(x^2-a 1\right)\left(x^2-a 2\right)\left(x^2-a 3\right) x}+\frac{\left(\left(\left(x^2-a 1\right)\left(x^2-a 2\right)+\left(x^2-a 2\right)\left(x^2-a 3\right)+\left(x^2-a 3\right)\left(x^2-a 2\right)\right)\right)}{\left(x^2-a 1\right)\left(x^2-a 2\right)}\right.\right.\right\}$$

2.1409 ODE No. 1409

$$y''(x)=-b^2 x^{-2 a} y(x)-\frac{a y'(x)}{x}$$

✓ **Mathematica** : cpu = 0.0189317 (sec), leaf count = 43

$$\left\{\left\{y(x) \rightarrow c_1 \cos \left(\frac{b x^{1-a}}{a-1}\right)-c_2 \sin \left(\frac{b x^{1-a}}{a-1}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 39

$$\left\{y(x)=c_1 \sin \left(\frac{b x^{-a+1}}{a-1}\right)+c_2 \cos \left(\frac{b x^{-a+1}}{a-1}\right)\right\}$$

2.1410 ODE No. 1410

$$y''(x) = -\frac{y'(x)(apx^b + q)}{x(ax^b - 1)} - \frac{y(x)(arx^b + s)}{x^2(ax^b - 1)}$$

✓ **Mathematica** : cpu = 0.0934547 (sec), leaf count = 481

$$\left\{ \left\{ y(x) \rightarrow c_1 i^{\frac{-\sqrt{q^2+2q+4s+1}+q+1}{b}} a^{\frac{-\sqrt{q^2+2q+4s+1}+q+1}{2b}} (x^b)^{\frac{-\sqrt{q^2+2q+4s+1}+q+1}{2b}} {}_2F_1\left(\frac{p}{2b} + \frac{q}{2b} - \frac{\sqrt{p^2-2p-4r+1}}{2b} - \frac{\sqrt{q^2+2q+4s+1}}{2b} \right. \right. \right.$$

✓ **Maple** : cpu = 0.319 (sec), leaf count = 253

$$\left\{ y(x) = c_1 x^{\frac{q}{2} + \frac{\sqrt{q^2+2q+4s+1}}{2} + \frac{1}{2}} \text{hypergeom}\left(\left[\frac{p+q+\sqrt{q^2+2q+4s+1}+\sqrt{p^2-2p-4r+1}}{2b}, \frac{p+q+\sqrt{q^2+2q+4s+1}}{2b} \right. \right. \right.$$

2.1411 ODE No. 1411

$$y''(x) = \frac{y(x)}{e^x + 1}$$

✓ **Mathematica** : cpu = 0.312218 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow c_1 (e^{-x} + 1) + c_2 e^{-x} (e^x \log(e^x + 1) + \log(e^x + 1) + 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 27

$$\{y(x) = (c_1(e^x + 1) \ln(e^x + 1) + c_2 e^x + c_1 + c_2) e^{-x}\}$$

2.1412 ODE No. 1412

$$y''(x) = \frac{y'(x)}{x \log(x)} + y(x) \log^2(x)$$

✓ **Mathematica** : cpu = 0.0133224 (sec), leaf count = 29

$$\{\{y(x) \rightarrow c_1 \cosh(x(\log(x) - 1)) + i c_2 \sinh(x(\log(x) - 1))\}\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 23

$$\{y(x) = c_1 \sinh((\ln(x) - 1)x) + c_2 \cosh((\ln(x) - 1)x)\}$$

2.1413 ODE No. 1413

$$y''(x) = \frac{y'(x)}{x(\log(x) - 1)} - \frac{y(x)}{x^2(\log(x) - 1)}$$

✓ **Mathematica** : cpu = 0.0481655 (sec), leaf count = 16

$$\{\{y(x) \rightarrow c_1 x - c_2 \log(x)\}\}$$

✓ **Maple** : cpu = 0.099 (sec), leaf count = 12

$$\{y(x) = c_1 x + c_2 \ln(x)\}$$

2.1414 ODE No. 1414

$$y''(x) = y(x) \left(-\operatorname{csch}^2(x) \right) \left((1-n)n - a^2 \sinh^2(x) \right)$$

✓ **Mathematica** : cpu = 0.872822 (sec), leaf count = 231

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 (-1)^{\frac{1}{2}(-2n-1)+1} \tanh^2(x)^{\frac{1}{4}(-2n-1)+1} \left(\tanh^2(x) - 1 \right)^{\frac{1}{2} \left(\frac{a+n}{2} + \frac{1}{2}(a+n+1) + \frac{1}{2}(-2n-1)+1 \right) - \frac{1}{2}} {}_2F_1 \left(\frac{1}{2}(-2n-1) + \frac{a+n}{2} + 1, \frac{1}{2}(-2n-1) + \frac{a+n}{2} + 1, \frac{1}{2}(-2n-1) + \frac{a+n}{2} + 1, \frac{1}{2}(-2n-1) + \frac{a+n}{2} + 1 \right)}{\sqrt{\tanh(x)}} \right. \right.$$

✓ **Maple** : cpu = 0.28 (sec), leaf count = 97

$$\left\{ y(x) = c_1 \left(\sinh^n(x) \right) \operatorname{hypergeom} \left(\left[-\frac{a}{2} + \frac{n}{2}, \frac{a}{2} + \frac{n}{2} \right], \left[\frac{1}{2} \right], \frac{\cosh(2x)}{2} + \frac{1}{2} \right) + \frac{c_2 (2 \cosh(2x) + 2)^{\frac{3}{4}} (2 \cosh(2x) - 2)^{\frac{1}{4}} \left(\sinh^n(x) \right)}{\sqrt{\tanh(x)}} \right\}$$

2.1415 ODE No. 1415

$$y''(x) = (a^2 - n^2)y(x) - 2n \coth(x)y'(x)$$

✓ **Mathematica** : cpu = 0.758103 (sec), leaf count = 273

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 (-1)^{\frac{1}{2}(-2n-1)+1} \tanh^2(x)^{\frac{1}{4}(-2n-1)+1} \left(\tanh^2(x) - 1 \right)^{\frac{1}{2} \left(\frac{a+n}{2} + \frac{1}{2}(a+n+1) + \frac{1}{2}(-2n-1)+1 \right)} {}_2F_1 \left(\frac{1}{2}(-2n-1) + \frac{a+n}{2} + 1, \frac{1}{2}(-2n-1) + \frac{a+n}{2} + 1, \frac{1}{2}(-2n-1) + \frac{a+n}{2} + 1, \frac{1}{2}(-2n-1) + \frac{a+n}{2} + 1 \right)}{\sqrt{\tanh(x)}} \right. \right.$$

✓ **Maple** : cpu = 0.154 (sec), leaf count = 36

$$\left\{ y(x) = \left(c_1 \operatorname{LegendreP} \left(a - \frac{1}{2}, n - \frac{1}{2}, \cosh(x) \right) + c_2 \operatorname{LegendreQ} \left(a - \frac{1}{2}, n - \frac{1}{2}, \cosh(x) \right) \right) \left(\sinh^{-n+\frac{1}{2}}(x) \right) \right\}$$

2.1416 ODE No. 1416

$$y''(x) = (n-v)(n+v+1)y(x) - (2n+1) \cot(x)y'(x)$$

✓ **Mathematica** : cpu = 0.181214 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow c_1 \left(\cos^2(x) - 1 \right)^{-n/2} P_v^n(\cos(x)) + c_2 \left(\cos^2(x) - 1 \right)^{-n/2} Q_v^n(\cos(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.19 (sec), leaf count = 26

$$\{y(x) = (c_1 \operatorname{LegendreP}(v, n, \cos(x)) + c_2 \operatorname{LegendreQ}(v, n, \cos(x))) (\sin^{-n}(x))\}$$

2.1417 ODE No. 1417

$$y''(x) = -\csc(x)y'(x) (\sin^2(x) - \cos(x)) - y(x) \sin^2(x)$$

✓ **Mathematica** : cpu = 0.141609 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{\cos(x)}{2}} \cos \left(\frac{1}{2} \sqrt{3} \cos(x) \right) + c_2 e^{\frac{\cos(x)}{2}} \sin \left(\frac{1}{2} \sqrt{3} \cos(x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.153 (sec), leaf count = 31

$$\left\{ y(x) = \left(c_1 \sin \left(\frac{\sqrt{3} \cos(x)}{2} \right) + c_2 \cos \left(\frac{\sqrt{3} \cos(x)}{2} \right) \right) e^{\frac{\cos(x)}{2}} \right\}$$

2.1418 ODE No. 1418

$$y''(x) = \frac{y(x) \sin(x)}{x \cos(x) - \sin(x)} - \frac{x \sin(x) y'(x)}{x \cos(x) - \sin(x)}$$

✓ **Mathematica** : cpu = 0.129966 (sec), leaf count = 15

$$\{ \{ y(x) \rightarrow c_1 x + c_2 \sin(x) \} \}$$

✓ **Maple** : cpu = 3.614 (sec), leaf count = 59

$$\left\{ y(x) = \left(c_2 \left(\int \cos(x) e^{\int \frac{-2x(\cos^3(x)) + 3(\cos^2(x)) \sin(x) - \sin(x)}{(x \cos(x) - \sin(x)) \cos(x) \sin(x)} dx} dx \right) + c_1 \right) \sin(x) \right\}$$

2.1419 ODE No. 1419

$$y''(x) = -\frac{\sec(x)y'(x)(x^2 \sin(x) - 2x \cos(x))}{x^2} - \frac{y(x) \sec(x)(2x \cos(x) - x \sin(x))}{x^2}$$

✗ **Mathematica** : cpu = 1.21778 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[2][y][x] == -((Sec[x]*(2*x*Cos[x] - x*Sin[x])*y[x])/x^2) - (Sec[x]*(-2*x*Cos[x] + x^2*Sin[x])*Derivative[1][y][x])/x^2, y[x], x]`

✓ **Maple** : cpu = 0.227 (sec), leaf count = 12

$$\{ y(x) = (c_2 \sin(x) + c_1)x \}$$

2.1420 ODE No. 1420

$$y(x)(-a \cos^2(x) - (n-1)n) + \cos^2(x)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.411958 (sec), leaf count = 134

$$\left\{ \left\{ y(x) \rightarrow c_1 i^{1-n} \cos^{1-n}(x) {}_2F_1\left(-\frac{n}{2} - \frac{i\sqrt{a}}{2} + \frac{1}{2}, -\frac{n}{2} + \frac{i\sqrt{a}}{2} + \frac{1}{2}; \frac{3}{2} - n; \cos^2(x)\right) + c_2 i^n \cos^n(x) {}_2F_1\left(\frac{n}{2} - \frac{i\sqrt{a}}{2}, \frac{n}{2} + \frac{i\sqrt{a}}{2}; \right. \right. \right.$$

✓ **Maple** : cpu = 0.369 (sec), leaf count = 123

$$\left\{ y(x) = c_1 (\cos^{-n}(x)) \operatorname{hypergeom}\left(\left[-\frac{n}{2} + \frac{i\sqrt{a}}{2} + 1, -\frac{n}{2} - \frac{i\sqrt{a}}{2} + 1\right], \left[-n + \frac{3}{2}\right], \frac{\cos(2x)}{2} + \frac{1}{2}\right) \sin(2x) + \frac{c_2(-2 \cos(2x) + \dots}{\dots} \right.$$

2.1421 ODE No. 1421

$$y''(x) = -a^2 n y(x) \sec^2(ax) ((n-1) \sin^2(ax) + \cos^2(ax)) - a(n-1) \sin(2ax) \sec^2(ax) y'(x)$$

✓ **Mathematica** : cpu = 0.162152 (sec), leaf count = 81

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-iax} \cos^{n-1}(ax) - \frac{ic_2 e^{2iax} \left(\frac{1}{2} e^{-iax} + \frac{1}{2} e^{iax}\right)^n}{a(1 + e^{2iax})} \right\} \right\}$$

✓ **Maple** : cpu = 0.127 (sec), leaf count = 27

$$\{ y(x) = c_2 (\cos^{n-1}(ax)) \sin(ax) + c_1 (\cos^n(ax)) \}$$

2.1422 ODE No. 1422

$$y''(x) = 2y(x) \csc^2(x)$$

✓ **Mathematica** : cpu = 0.0805171 (sec), leaf count = 58

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 \cos(x)}{\sqrt{\cos^2(x) - 1}} + \frac{c_2 \left(\cos(x) \left(-\sin^{-1}(\cos(x)) \right) - \sqrt{1 - \cos^2(x)} \right)}{\sqrt{1 - \cos^2(x)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.242 (sec), leaf count = 50

$$\left\{ y(x) = \frac{-ic_2 \ln(\cos(2x) + i \sin(2x)) \sin(2x) + c_1 \sin(2x) + 2c_2(\cos(2x) - 1)}{\cos(2x) - 1} \right\}$$

2.1423 ODE No. 1423

$$y''(x) = -ay(x) \csc^2(x)$$

✓ **Mathematica** : cpu = 0.0624715 (sec), leaf count = 70

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt[4]{\cos^2(x) - 1} P_{-\frac{1}{2}}^{\frac{1}{2}\sqrt{1-4a}}(\cos(x)) + c_2 \sqrt[4]{\cos^2(x) - 1} Q_{-\frac{1}{2}}^{\frac{1}{2}\sqrt{1-4a}}(\cos(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.325 (sec), leaf count = 132

$$\left\{ y(x) = \frac{(2 \cos(2x) + 2)^{\frac{1}{4}} \sqrt{-2 \cos(2x) + 2} \left(c_1 \operatorname{hypergeom} \left(\left[\frac{\sqrt{-4a+1}}{4} + \frac{1}{4}, \frac{\sqrt{-4a+1}}{4} + \frac{1}{4} \right], \left[\frac{1}{2} \right], \frac{\cos(2x)}{2} + \frac{1}{2} \right) + c_2 \sqrt{2 \cos(2x) - 2} \right)}{\sqrt{\sin(2x)}}$$

2.1424 ODE No. 1424

$$y(x) \left(-a \sin^2(x) - (n-1)n \right) + \sin^2(x) y''(x) = 0$$

✓ **Mathematica** : cpu = 0.213276 (sec), leaf count = 90

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt[4]{\cos^2(x) - 1} P_{\frac{1}{2}i(2\sqrt{a}+i)}^{\frac{1}{2}(2n-1)}(\cos(x)) + c_2 \sqrt[4]{\cos^2(x) - 1} Q_{\frac{1}{2}i(2\sqrt{a}+i)}^{\frac{1}{2}(2n-1)}(\cos(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.322 (sec), leaf count = 120

$$\left\{ y(x) = \frac{\left(c_1 \operatorname{hypergeom} \left(\left[\frac{n}{2} + \frac{i\sqrt{a}}{2}, \frac{n}{2} - \frac{i\sqrt{a}}{2} \right], \left[\frac{1}{2} \right], \frac{\cos(2x)}{2} + \frac{1}{2} \right) \left(\sqrt{\sin(2x)} \right) + c_2 (2 \cos(2x) + 2)^{\frac{3}{4}} (-2 \cos(2x) + 2)^{\frac{1}{4}} \operatorname{hypergeom} \left(\left[\frac{n}{2} + \frac{i\sqrt{a}}{2}, \frac{n}{2} - \frac{i\sqrt{a}}{2} \right], \left[\frac{1}{2} \right], \frac{\cos(2x)}{2} + \frac{1}{2} \right) \right)}{\sqrt{\sin(2x)}}$$

2.1425 ODE No. 1425

$$y''(x) = y(x) \csc^2(x) \left(a^2 \cos^2(x) + (3-2a) \cos(x) - 3a + 3 \right)$$

✓ **Mathematica** : cpu = 0.899361 (sec), leaf count = 236

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \sqrt{1 - \cos(x)} \left(-\frac{(2a-1)(\cos(x)+1)}{-2a \cos(x) + \cos(x) + 2} \right)^{a+\frac{1}{2}} (-2a \cos(x) + \cos(x) + 2) \left(1 - \cos^2(x) \right)^{-a} \left(\frac{(2a-1)(\cos(x)-1)}{(2a-1) \cos(x) - 2} \right)^{a-\frac{1}{2}} F_1 \left(2a; a - \frac{1}{2}; \frac{\cos(x)}{2} + \frac{1}{2} \right)}{2(1-2a)^2 a \sqrt{\cos(x)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.484 (sec), leaf count = 91

$$\left\{ y(x) = \frac{\left(c_2 \sqrt{2 \cos(x) + 2} (\cos(x) - 1)^{\frac{a}{2} - \frac{1}{4}} (\cos(x) + 1)^{-\frac{a}{2} - \frac{1}{4}} \operatorname{hypergeom} \left(\left[a - \frac{1}{2}, -a - \frac{1}{2} \right], \left[-a + \frac{3}{2} \right], \frac{\cos(x)}{2} + \frac{1}{2} \right) + 2c_1 \left(\left(a - \frac{1}{2} \right) \sqrt{2 \cos(x) + 2} \right)^{\frac{1}{2}} \right)}{(-2 \cos(x) + 2)^{\frac{3}{4}}}$$

$$\left\{y(x)=\frac{\sqrt{-2\cos(2x)+2}\left(2\cos(2x)+2\right)^{\frac{1}{4}}\left(c_1\operatorname{hypergeom}\left(\left[\frac{\sqrt{-4a-4c+1}}{4}+\frac{\sqrt{-a+b}}{2}+\frac{1}{4},\frac{\sqrt{-4a-4c+1}}{4}-\frac{\sqrt{-a+b}}{2}+\frac{1}{4}\right],\left[\frac{1}{2}\right],\frac{\cos(2x)}{2}\right)+c_2\operatorname{hypergeom}\left(\left[\frac{\sqrt{-4a-4c+1}}{4}+\frac{\sqrt{-a+b}}{2}+\frac{1}{4},\frac{\sqrt{-4a-4c+1}}{4}-\frac{\sqrt{-a+b}}{2}+\frac{1}{4}\right],\left[\frac{1}{2}\right],\frac{\cos(2x)}{2}\right)\right)}{\sqrt{-2\cos(2x)+2}}$$

2.1429 ODE No. 1429

$$y''(x)=y(x)\csc^2(x)-\cot(x)y'(x)$$

✓ **Mathematica** : cpu = 0.0424076 (sec), leaf count = 51

$$\left\{\left\{y(x)\rightarrow c_1\cosh\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)-ic_2\sinh\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.048 (sec), leaf count = 25

$$\left\{y(x)=\frac{c_1\sin(x)}{\cos(x)-1}+\frac{c_2(\cos(x)-1)}{\sin(x)}\right\}$$

2.1430 ODE No. 1430

$$y''(x)=-\left(y(x)\csc^2(x)\left(v(v+1)\sin^2(x)-n^2\right)\right)-\cot(x)y'(x)$$

✓ **Mathematica** : cpu = 0.415428 (sec), leaf count = 22

$$\{\{y(x)\rightarrow c_1P_v^n(\cos(x))+c_2Q_v^n(\cos(x))\}\}$$

✓ **Maple** : cpu = 0.344 (sec), leaf count = 85

$$\left\{y(x)=\left(c_1\operatorname{hypergeom}\left(\left[\frac{n}{2}-\frac{v}{2},\frac{n}{2}+\frac{v}{2}+\frac{1}{2}\right],\left[\frac{1}{2}\right],\frac{\cos(2x)}{2}+\frac{1}{2}\right)+c_2\sqrt{\cos(2x)+1}\operatorname{hypergeom}\left(\left[\frac{n}{2}+\frac{v}{2}+1,\frac{n}{2}-\frac{v}{2}+\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{\cos(2x)}{2}+\frac{1}{2}\right)\right)\right\}$$

2.1431 ODE No. 1431

$$y''(x)=\cot(2x)y'(x)-2y(x)$$

✓ **Mathematica** : cpu = 0.162883 (sec), leaf count = 80

$$\left\{\left\{y(x)\rightarrow c_1\left(\cos^2(x)-\frac{1}{2}\right)-\frac{2}{3}c_2\cos^{\frac{3}{2}}(x)\left(2\cos^2(x){}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};\cos^2(x)\right)-{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};\cos^2(x)\right)+3\left(1-\cos^2(x)\right)^{3/4}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.22 (sec), leaf count = 30

$$\left\{y(x)=\left(c_1\operatorname{LegendreP}\left(\frac{1}{4},\frac{3}{4},\cos(2x)\right)+c_2\operatorname{LegendreQ}\left(\frac{1}{4},\frac{3}{4},\cos(2x)\right)\right)\left(\sin^{\frac{3}{4}}(2x)\right)\right\}$$

2.1432 ODE No. 1432

$$y''(x)=-\cot(x)y'(x)-\frac{1}{4}y(x)\left(-17\sin^2(x)-1\right)\csc^2(x)$$

✓ **Mathematica** : cpu = 0.0702225 (sec), leaf count = 37

$$\left\{\left\{y(x)\rightarrow\frac{c_1e^{-2x}}{\sqrt{\sin(x)}}+\frac{c_2e^{2x}}{4\sqrt{\sin(x)}}\right\}\right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 22

$$\left\{y(x)=\frac{c_1\sinh(2x)+c_2\cosh(2x)}{\sqrt{\sin(x)}}\right\}$$

2.1433 ODE No. 1433

$$y''(x) = -\frac{y(x) \sec^2(x) (2x^2 + x^2 \sin^2(x) - 24 \cos^2(x))}{4x^2} - \tan(x)y'(x) + \sqrt{\cos(x)}$$

✓ **Mathematica** : cpu = 0.16091 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{5} c_2 x^3 \sqrt{\cos(x)} - \frac{1}{4} x^2 \sqrt{\cos(x)} + \frac{c_1 \sqrt{\cos(x)}}{x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.109 (sec), leaf count = 28

$$\left\{ y(x) = \frac{(4c_1 x^5 - x^4 + 4c_2) (\sqrt{\cos(x)})}{4x^2} \right\}$$

2.1434 ODE No. 1434

$$y''(x) = -\frac{b \cot(x) y'(x)}{a} - \frac{y(x) \csc^2(x) (c \cos^2(x) + d \cos(x) + e)}{a}$$

✓ **Mathematica** : cpu = 71.9133 (sec), leaf count = 1596424

Too large to display

✓ **Maple** : cpu = 0.656 (sec), leaf count = 517

$$\left\{ y(x) = \left(c_1 (2 \cos(x) + 2)^{-\frac{-2a + \sqrt{a^2 + b^2 + (-2b - 4c + 4d - 4e)a}}{4a}} \operatorname{hypergeom} \left(\left[-\frac{-2a + 2i\sqrt{4ac - b^2} - \sqrt{a^2 + b^2 + (-2b - 4c - 4d - 4e)a}}{4a} \right] \right) \right)$$

2.1435 ODE No. 1435

$$y''(x) = -4y(x) \sin(3x) \csc^3(x)$$

✓ **Mathematica** : cpu = 0.151365 (sec), leaf count = 70

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt[4]{\cos^2(x) - 1} P_{-\frac{1}{2} + 4i}^{\frac{i\sqrt{47}}{2}}(\cos(x)) + c_2 \sqrt[4]{\cos^2(x) - 1} Q_{-\frac{1}{2} + 4i}^{\frac{i\sqrt{47}}{2}}(\cos(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.159 (sec), leaf count = 38

$$\left\{ y(x) = \left(c_1 \operatorname{LegendreP} \left(-\frac{1}{2} + 4i, \frac{i\sqrt{47}}{2}, \cos(x) \right) + c_2 \operatorname{LegendreQ} \left(-\frac{1}{2} + 4i, \frac{i\sqrt{47}}{2}, \cos(x) \right) \right) (\sqrt{\sin(x)}) \right\}$$

2.1436 ODE No. 1436

$$y''(x) = -\frac{1}{4} y(x) \csc^2(x) (-4n^2 + 4v(v + 1) \sin^2(x) - \cos^2(x) + 2)$$

✓ **Mathematica** : cpu = 0.58564 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt[4]{\cos^2(x) - 1} P_v^n(\cos(x)) + c_2 \sqrt[4]{\cos^2(x) - 1} Q_v^n(\cos(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.299 (sec), leaf count = 113

$$\left\{ y(x) = \frac{\sqrt{-2 \cos(2x) + 2} (2 \cos(2x) + 2)^{\frac{1}{4}} \left(c_1 \operatorname{hypergeom} \left(\left[\frac{n}{2} - \frac{v}{2}, \frac{n}{2} + \frac{v}{2} + \frac{1}{2} \right], \left[\frac{1}{2} \right], \frac{\cos(2x)}{2} + \frac{1}{2} \right) + c_2 \sqrt{2 \cos(2x) + 2} \operatorname{hypergeom} \left(\left[\frac{n}{2} - \frac{v}{2}, \frac{n}{2} + \frac{v}{2} + \frac{1}{2} \right], \left[\frac{1}{2} \right], \frac{\cos(2x)}{2} + \frac{1}{2} \right) \right)}{\sqrt{\sin(2x)}} \right\}$$

2.1437 ODE No. 1437

$$y''(x) = \left(3 \sin^2(x) + 1\right) \csc(x) \sec(x) y'(x) + y(x) \tan^2(x)$$

✓ **Mathematica** : cpu = 0.316576 (sec), leaf count = 42

$$\left\{\left\{y(x) \rightarrow c_2 \cos^{\frac{1}{2}(\sqrt{13}-3)}(x) + c_1 \cos^{\frac{1}{2}(-3-\sqrt{13})}(x)\right\}\right\}$$

✓ **Maple** : cpu = 0.272 (sec), leaf count = 29

$$\left\{y(x) = c_1 \left(\cos^{-\frac{3}{2} + \frac{\sqrt{13}}{2}}(x)\right) + c_2 \left(\cos^{-\frac{3}{2} - \frac{\sqrt{13}}{2}}(x)\right)\right\}$$

2.1438 ODE No. 1438

$$y''(x) = y(x) \left(-\csc^2(x)\right) \sec^2(x) \left(-a \sin^2(x) \cos^2(x) - (m-1)m \sin^2(x) + (1-n)n \cos^2(x)\right)$$

✓ **Mathematica** : cpu = 1.32953 (sec), leaf count = 615

$$\left\{\left\{y(x) \rightarrow \frac{c_2(-1)^{\frac{1}{2}(-2m-1)+1} \cos^2(x)^{\frac{1}{4}(-2m-1)+1} \left(\cos^2(x) - 1\right)^{\frac{1}{2} \left(\frac{4am+4\sqrt{-a}n^2+4an-4\sqrt{-a}n+4(-a)^{3/2}+8\sqrt{-a}a+\sqrt{-a}+4mn^2-4mn+m+4n^3-4n^2+n}{8a+8n^2-8n+2} + \frac{1}{2}(-\sqrt{-a})\right)}}{c_1(-1)^{\frac{1}{2}(-2m-1)+1} \cos^2(x)^{\frac{1}{4}(-2m-1)+1} \left(\cos^2(x) - 1\right)^{\frac{1}{2} \left(\frac{4am+4\sqrt{-a}n^2+4an-4\sqrt{-a}n+4(-a)^{3/2}+8\sqrt{-a}a+\sqrt{-a}+4mn^2-4mn+m+4n^3-4n^2+n}{8a+8n^2-8n+2} + \frac{1}{2}(-\sqrt{-a})\right)}}\right\}\right\}$$

✓ **Maple** : cpu = 0.23 (sec), leaf count = 102

$$\left\{y(x) = \left(c_1(\cos^m(x)) \operatorname{hypergeom}\left(\left[\frac{m}{2} + \frac{n}{2} + \frac{i\sqrt{a}}{2}, \frac{m}{2} + \frac{n}{2} - \frac{i\sqrt{a}}{2}\right], \left[m + \frac{1}{2}\right], \cos^2(x)\right) + c_2(\cos^{-m+1}(x)) \operatorname{hypergeom}\left(\left[\frac{m}{2} + \frac{n}{2} + \frac{i\sqrt{a}}{2}, \frac{m}{2} + \frac{n}{2} - \frac{i\sqrt{a}}{2}\right], \left[m + \frac{1}{2}\right], \cos^2(x)\right)\right\}$$

2.1439 ODE No. 1439

$$y''(x) = \frac{\phi'(x)y'(x)}{\phi(x) - \phi(a)} - \frac{y(x)(\phi''(a) - n(n+1)(\phi(x) - \phi(a))^2)}{\phi(x) - \phi(a)}$$

✗ **Mathematica** : cpu = 0.862216 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[2][y][x] == (Derivative[1][phi][x]*Derivative[1][y][x])/(-phi[a] + phi[x]) (n*(1 + n)*(-phi[a] + phi[x])^2) + Derivative[2][phi][a]), y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x) = DESol\left[\left\{-\frac{\left(\frac{d}{dx}Y(x)\right)\left(\frac{d}{dx}\phi(x)\right)}{-\phi(a) + \phi(x)} + \frac{\left(-(n+1)(-\phi(a) + \phi(x))^2n + \frac{d^2}{da^2}\phi(a)\right)Y(x)}{-\phi(a) + \phi(x)} + \frac{d^2}{dx^2}Y(x)\right\}, \{-Y(x)\}\right]\right\}$$

2.1440 ODE No. 1440

$$y''(x) = -\frac{y'(x)(-\phi''(x) - \phi(x)\phi'(x) + \phi(x^3))}{\phi'(x) + \phi(x)^2} - \frac{y(x)(-\phi(x)\phi''(x) + \phi(x)^2(-\phi'(x)) + \phi'(x)^2)}{\phi'(x) + \phi(x)^2}$$

✗ **Mathematica** : cpu = 0.827814 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[2][y][x] == -((Derivative[1][y][x]*(phi[x^3] - phi[x]*Derivative[1][phi][x]) (phi[x]^2*Derivative[1][phi][x]) + Derivative[1][phi][x]^2 - phi[x]*Derivative[2][phi][x])), y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x) = DESol\left[\left[\left[\left(-\left(\frac{d}{dx}\phi(x)\right)\phi(x)^2 - \left(\frac{d^2}{dx^2}\phi(x)\right)\phi(x) + \left(\frac{d}{dx}\phi(x)\right)^2\right)Y(x) - \left(-\left(\frac{d}{dx}\phi(x)\right)\phi(x) - \frac{d^2}{dx^2}\phi(x) + \phi(x^3)\right)\left(\frac{d}{dx}Y(x)\right)\right]}{\phi(x)^2 + \frac{d}{dx}\phi(x)} + \frac{\left(-\left(\frac{d}{dx}\phi(x)\right)\phi(x) - \frac{d^2}{dx^2}\phi(x) + \phi(x^3)\right)\left(\frac{d}{dx}Y(x)\right)}{\phi(x)^2 + \frac{d}{dx}\phi(x)}\right]\right\}$$

2.1441 ODE No. 1441

$$y''(x) = -\frac{y'(x)(-\operatorname{cn}(x|k)\operatorname{dn}(x|k) - 2\operatorname{sn}(x|k))}{\operatorname{sn}(x|k)^2 - \operatorname{sn}(a|k)^2} - \frac{y(x)(6k^2\operatorname{sn}(a|k)^4 - 4(k^2 + 1)\operatorname{sn}(a|k)^2 + 2)}{\operatorname{sn}(x|k)^2 - \operatorname{sn}(a|k)^2} - \frac{1}{\operatorname{sn}(x|k)^2 - \operatorname{sn}(a|k)^2}$$

✗ **Mathematica** : cpu = 1.76096 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[2][y][x] == -(-JacobiSN[a, k]^2 + JacobiSN[x, k]^2)^(-1) - ((2 - 4*(1 + k^2) JacobiSN[a, k]^2 + JacobiSN[x, k]^2) - ((-(JacobiCN[x, k]*JacobiDN[x, k]) - 2*JacobiSN[x, k] JacobiSN[a, k]^2 + JacobiSN[x, k]^2), y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x) = DESol\left[\left\{-\frac{2\left(\frac{d}{dx}Y(x)\right)\operatorname{cn}(x|k)\operatorname{dn}(x|k)\operatorname{sn}(x|k)}{\operatorname{sn}(x|k)^2 - \operatorname{sn}(a|k)} - \frac{(-6k^2\operatorname{sn}(a|k)^4 + 4(k^2 + 1)\operatorname{sn}(a|k)^2 - 2)Y(x)}{\operatorname{sn}(x|k)^2 - \operatorname{sn}(a|k)} + \frac{d^2}{dx^2}Y(x)\right\}, \{Y(x)\}\right.\right.$$

2.1442 ODE No. 1442

$$y''(x) = \frac{y(x)}{f(x)} - \frac{xy'(x)}{f(x)}$$

✓ **Mathematica** : cpu = 0.0775299 (sec), leaf count = 46

$$\left\{\left\{y(x) \rightarrow c_2x \int_1^x \frac{\exp\left(-\int_1^{K[2]} \frac{K[1]}{f(K[1])} dK[1]\right)}{K[2]^2} dK[2] + c_1x\right\}\right\}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 30

$$\left\{y(x) = \left(c_1\left(\int e^{\int \frac{-\frac{x^2}{f(x)}-2}{x} dx} dx\right) + c_2\right)x\right\}$$

2.1443 ODE No. 1443

$$y''(x) = -\frac{f'(x)y'(x)}{2f(x)} - \frac{g(x)y(x)}{f(x)}$$

✗ **Mathematica** : cpu = 0.385637 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[2][y][x] == -((g[x]*y[x])/f[x]) - (Derivative[1][f][x]*Derivative[1][y][x])`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x) = DESol\left[\left\{\frac{-Y(x)g(x)}{f(x)} + \frac{\left(\frac{d}{dx}Y(x)\right)\left(\frac{d}{dx}f(x)\right)}{2f(x)} + \frac{d^2}{dx^2}Y(x)\right\}, \{Y(x)\}\right]\right\}$$

2.1444 ODE No. 1444

$$y''(x) = -by(x)f(x)^{2a} - \frac{af'(x)y'(x)}{f(x)}$$

✗ **Mathematica** : cpu = 1.60517 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[2][y][x] == -(b*f[x]^(2*a)*y[x]) - (a*Derivative[1][f][x]*Derivative[1][y][x])`

✓ **Maple** : cpu = 0.021 (sec), leaf count = 37

$$\left\{y(x) = c_1e^{\int i\sqrt{b}f(x)^a dx} + c_2e^{-\left(\int i\sqrt{b}f(x)^a dx\right)}\right\}$$

2.1445 ODE No. 1445

$$y''(x) = -\frac{y'(x)\left(2f(x)g(x)g'(x)^2 - \left(g(x)^2 - 1\right)\left(2f'(x)g'(x) + f(x)g''(x)\right)\right)}{f(x)\left(g(x)^2 - 1\right)g'(x)} - \frac{y(x)\left(\left(g(x)^2 - 1\right)\left(f'(x)\left(2f'(x)g'(x) + f(x)g''(x)\right)\right)\right)}{f(x)\left(g(x)^2 - 1\right)g'(x)}$$

✓ **Mathematica** : cpu = 0.272891 (sec), leaf count = 24

$$\{\{y(x) \rightarrow c_1 f(x) P_v(g(x)) + c_2 f(x) Q_v(g(x))\}\}$$

✓ **Maple** : cpu = 0.221 (sec), leaf count = 20

$$\{y(x) = \left(c_1 \operatorname{LegendreP}(v, g(x)) + c_2 \operatorname{LegendreQ}(v, g(x))\right) f(x)\}$$

2.1446 ODE No. 1446

$$y''(x) = -\frac{(x-1)y(x)}{x^4} - \frac{y'(x)}{x}$$

✓ **Mathematica** : cpu = 0.049076 (sec), leaf count = 33

$$\left\{\left\{y(x) \rightarrow c_1 e^{-1/x} - c_2 e^{-1/x} \operatorname{Ei}\left(\frac{2}{x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 22

$$\left\{y(x) = \left(c_2 \operatorname{Ei}\left(1, -\frac{2}{x}\right) + c_1\right) e^{-\frac{1}{x}}\right\}$$

2.1447 ODE No. 1447

$$y''(x) = -\frac{(-x-1)y(x)}{x^4} - \frac{y'(x)}{x}$$

✓ **Mathematica** : cpu = 0.0451344 (sec), leaf count = 29

$$\left\{\left\{y(x) \rightarrow c_1 e^{\frac{1}{x}} - c_2 e^{\frac{1}{x}} \operatorname{Ei}\left(-\frac{2}{x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 20

$$\left\{y(x) = \left(c_2 \operatorname{Ei}\left(1, \frac{2}{x}\right) + c_1\right) e^{\frac{1}{x}}\right\}$$

2.1448 ODE No. 1448

$$y''(x) = -\frac{b^2 y(x)}{\left(x^2 - a^2\right)^2}$$

✓ **Mathematica** : cpu = 0.270402 (sec), leaf count = 149

$$\left\{\left\{y(x) \rightarrow c_1 (x-a)^{\frac{1}{2}} \sqrt{1-\frac{b^2}{a^2}} + \frac{1}{2} (a+x)^{\frac{1}{2}-\frac{1}{2}} \sqrt{1-\frac{b^2}{a^2}} - \frac{c_2 (x-a)^{\frac{1}{2}-\frac{1}{2}} \sqrt{\frac{a^2-b^2}{a^2}} (a+x)^{\frac{1}{2}} \sqrt{\frac{a^2-b^2}{a^2}} + \frac{1}{2}}{2a\sqrt{\frac{a^2-b^2}{a^2}}}\right\}\right\}$$

✓ **Maple** : cpu = 0.118 (sec), leaf count = 77

$$\left\{y(x) = \sqrt{(a-x)(a+x)} \left(c_1 \left(\frac{a-x}{a+x}\right)^{\frac{\sqrt{a^2-b^2}}{2a}} + c_2 \left(\frac{a-x}{a+x}\right)^{-\frac{\sqrt{a^2-b^2}}{2a}}\right)\right\}$$

2.1449 ODE No. 1449

$$y^{(3)}(x) - \lambda y(x) = 0$$

✓ **Mathematica** : cpu = 0.0414995 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{(-1)^{2/3} \sqrt[3]{\lambda} x} + c_2 e^{-\sqrt[3]{-1} \sqrt[3]{\lambda} x} + c_3 e^{\sqrt[3]{\lambda} x} \right\} \right\}$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 47

$$\left\{ y(x) = c_1 e^{-\frac{(1+i\sqrt{3})\lambda^{\frac{1}{3}}x}{2}} + c_2 e^{\frac{(i\sqrt{3}-1)\lambda^{\frac{1}{3}}x}{2}} + c_3 e^{\lambda^{\frac{1}{3}}x} \right\}$$

2.1450 ODE No. 1450

$$ax^3y(x) - bx + y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 7.12367 (sec), leaf count = 3595

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{6} \sqrt[3]{a} c_3 {}_0F_2\left(; \frac{7}{6}, \frac{4}{3}; -\frac{ax^6}{216} \right) x^2 + \frac{\sqrt[6]{a} c_2 {}_0F_2\left(; \frac{5}{6}, \frac{7}{6}; -\frac{ax^6}{216} \right) x}{\sqrt{6}} + c_1 {}_0F_2\left(; \frac{2}{3}, \frac{5}{6}; -\frac{ax^6}{216} \right) + \frac{1}{6} \left(\sqrt[3]{a} {}_0F_2\left(; \frac{7}{6}, \frac{4}{3}; -\frac{ax^6}{216} \right) \int_1^{\infty} \right. \right. \right.$$

✓ **Maple** : cpu = 0.255 (sec), leaf count = 1616

Expression too large to display

2.1451 ODE No. 1451

$$y^{(3)}(x) - ax^b y(x) = 0$$

✓ **Mathematica** : cpu = 0.0153159 (sec), leaf count = 168

$$\left\{ \left\{ y(x) \rightarrow (-1)^{\frac{1}{b+3}} (b+3)^{-\frac{3}{b+3}} c_2 x a^{\frac{1}{b+3}} {}_0F_2\left(; 1 - \frac{1}{b+3}, 1 + \frac{1}{b+3}; \frac{ax^{b+3}}{(b+3)^3} \right) + (-1)^{\frac{2}{b+3}} (b+3)^{-\frac{6}{b+3}} c_3 x^2 a^{\frac{2}{b+3}} {}_0F_2\left(; 1 + \frac{1}{b+3}, 1 + \frac{2}{b+3}; \frac{ax^{b+3}}{(b+3)^3} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.155 (sec), leaf count = 114

$$\left\{ y(x) = c_3 x^2 \operatorname{hypergeom}\left(\left[\right], \left[\frac{b+4}{b+3}, \frac{b+5}{b+3} \right], \frac{ax^{b+3}}{(b+3)^3} \right) + c_2 x \operatorname{hypergeom}\left(\left[\right], \left[\frac{b+2}{b+3}, \frac{b+4}{b+3} \right], \frac{ax^{b+3}}{(b+3)^3} \right) + c_1 \operatorname{hypergeom}\left(\left[\right], \left[\frac{b+1}{b+3}, \frac{b+4}{b+3} \right], \frac{ax^{b+3}}{(b+3)^3} \right) \right\}$$

2.1452 ODE No. 1452

$$y^{(3)}(x) + 3y'(x) - 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.0065108 (sec), leaf count = 54

$$\left\{ \left\{ y(x) \rightarrow c_3 e^x + c_2 e^{-x/2} \cos\left(\frac{\sqrt{15} x}{2} \right) + c_1 e^{-x/2} \sin\left(\frac{\sqrt{15} x}{2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 35

$$\left\{ y(x) = c_2 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{15} x}{2} \right) + c_3 \cos\left(\frac{\sqrt{15} x}{2} \right) e^{-\frac{x}{2}} + c_1 e^x \right\}$$

2.1453 ODE No. 1453

$$a^2\bigl(-y'(x)\bigr)-e^{2ax}\sin^2(x)+y^{(3)}(x)=0$$

✓ **Mathematica** : cpu = 1.57547 (sec), leaf count = 128

$$\left\{\left\{y(x)\rightarrow\frac{e^{-ax}\Bigl(-9\bigl(a^2-4\bigr)a^4e^{3ax}\cos(2x)-3\bigl(11a^2-4\bigr)a^3e^{3ax}\sin(2x)+\bigl(9a^6+49a^4+56a^2+16\bigr)\bigl(12a^2c_1e^{2ax}-12a^2c_2+\right.}{12a^3\bigl(9a^6+49a^4+56a^2+16\bigr)}$$

✓ **Maple** : cpu = 0.15 (sec), leaf count = 122

$$\left\{y(x)=\frac{108\bigl(a^2+4\bigr)\bigl(a^2+1\bigr)\bigl(-c_1e^{-ax}+c_2e^{ax}+c_3a\bigr)\Bigl(a^2+\bigl(\frac{4}{9}\bigr)a^2+\bigl(9a^6+49a^4+56a^2+\bigl(-9a^6+36a^4\bigr)\cos(2x)+\bigl(-33a^5\right.}{108a^9+588a^7+672a^5+192a^3}$$

2.1454 ODE No. 1454

$$2axy'(x)+ay(x)+y^{(3)}(x)=0$$

✓ **Mathematica** : cpu = 0.0102548 (sec), leaf count = 79

$$\left\{\left\{y(x)\rightarrow c_1\mathrm{Ai}\left(\sqrt[3]{-\frac{1}{2}}\sqrt[3]{a}x\right)^2+c_3\mathrm{Bi}\left(\sqrt[3]{-\frac{1}{2}}\sqrt[3]{a}x\right)^2+c_2\mathrm{Ai}\left(\sqrt[3]{-\frac{1}{2}}\sqrt[3]{a}x\right)\mathrm{Bi}\left(\sqrt[3]{-\frac{1}{2}}\sqrt[3]{a}x\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 55

$$\left\{y(x)=c_1\mathrm{AiryAi}\left(-\frac{2^{\frac{2}{3}}a^{\frac{1}{3}}x}{2}\right)^2+c_2\mathrm{AiryBi}\left(-\frac{2^{\frac{2}{3}}a^{\frac{1}{3}}x}{2}\right)^2+c_3\mathrm{AiryAi}\left(-\frac{2^{\frac{2}{3}}a^{\frac{1}{3}}x}{2}\right)\mathrm{AiryBi}\left(-\frac{2^{\frac{2}{3}}a^{\frac{1}{3}}x}{2}\right)\right\}$$

2.1455 ODE No. 1455

$$x(a+b-1)y'(x)-aby(x)+x^2\bigl(-y''(x)\bigr)+y^{(3)}(x)=0$$

✓ **Mathematica** : cpu = 0.0291283 (sec), leaf count = 127

$$\left\{\left\{y(x)\rightarrow\sqrt[3]{-\frac{1}{3}}c_2x\,{}_2F_2\left(\frac{1}{3}-\frac{a}{3},\frac{1}{3}-\frac{b}{3};\frac{2}{3},\frac{4}{3};\frac{x^3}{3}\right)+c_1{}_2F_2\left(-\frac{a}{3},-\frac{b}{3};\frac{1}{3},\frac{2}{3};\frac{x^3}{3}\right)+\left(-\frac{1}{3}\right)^{2/3}c_3x^2{}_2F_2\left(\frac{2}{3}-\frac{a}{3},\frac{2}{3}-\frac{b}{3};\frac{4}{3},\frac{5}{3};\frac{x^3}{3}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.148 (sec), leaf count = 71

$$\left\{y(x)=c_2x^2\mathrm{hypergeom}\left(\left[-\frac{b}{3}+\frac{2}{3},-\frac{a}{3}+\frac{2}{3}\right],\left[\frac{4}{3},\frac{5}{3}\right],\frac{x^3}{3}\right)+c_3x\mathrm{hypergeom}\left(\left[-\frac{a}{3}+\frac{1}{3},-\frac{b}{3}+\frac{1}{3}\right],\left[\frac{2}{3},\frac{4}{3}\right],\frac{x^3}{3}\right)+c_1\mathrm{hype}$$

2.1456 ODE No. 1456

$$x^{2c-2}y'(x)+(c-1)x^{2c-3}y(x)+y^{(3)}(x)=0$$

✓ **Mathematica** : cpu = 0.0304634 (sec), leaf count = 183

$$\left\{\left\{y(x)\rightarrow c_1{}_1F_2\left(\frac{1}{2}-\frac{1}{2c};1-\frac{1}{c},1-\frac{1}{2c};-\frac{x^{2c}}{4c^2}\right)+4^{-1/c}c^{-2/c}c_3\bigl(x^{2c}\bigr)^{\frac{1}{c}}{}_1F_2\left(\frac{1}{2}+\frac{1}{2c};1+\frac{1}{2c},1+\frac{1}{c};-\frac{x^{2c}}{4c^2}\right)+2^{-1/c}c^{-1/c}c_2\bigl(x^{2c}\bigr)\right\}\right\}$$

✓ **Maple** : cpu = 0.131 (sec), leaf count = 73

$$\left\{y(x)=\left(c_1\mathrm{BesselJ}\left(\frac{1}{2c},\frac{x^c}{2c}\right)^2+c_2\mathrm{BesselY}\left(\frac{1}{2c},\frac{x^c}{2c}\right)^2+c_3\mathrm{BesselJ}\left(\frac{1}{2c},\frac{x^c}{2c}\right)\mathrm{BesselY}\left(\frac{1}{2c},\frac{x^c}{2c}\right)\right)x\right\}$$

2.1457 ODE No. 1457

$$-3y'(x)(a + 2\wp(x; g2, g3)) + by(x) + y^{(3)}(x) = 0$$

✘ **Mathematica** : cpu = 0.0484378 (sec), leaf count = 0 , could not solve

DSolve[b*y[x] - 3*(a + 2*WeierstrassP[x, {g2, g3}])*Derivative[1][y][x] + Derivative[3][y][x]

✘ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x) = DESol\left(\left\{b\_Y(x) + (-3a - 6 \operatorname{WeierstrassP}(x, g2, g3))\left(\frac{d}{dx}Y(x)\right) + \frac{d^3}{dx^3}Y(x)\right\}, \{\_Y(x)\}\right)\}$$

2.1458 ODE No. 1458

$$\frac{1}{2}y(x)\left((1 - n^2)\wp'(x; g2, g3) - a\right) + (1 - n^2)y'(x)\wp(x; g2, g3) + y^{(3)}(x) = 0$$

✘ **Mathematica** : cpu = 0.0209227 (sec), leaf count = 0 , could not solve

DSolve[((-a + (1 - n^2)*WeierstrassPPrime[x, {g2, g3}])*y[x])/2 + (1 - n^2)*WeierstrassP[x,

✘ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x) = DESol\left(\left(\left(-\frac{n^2 \mathcal{P}'(x; g2, g3)}{2} - \frac{a}{2} + \frac{\mathcal{P}'(x; g2, g3)}{2}\right)Y(x) + (-n^2 \operatorname{WeierstrassP}(x, g2, g3) + \operatorname{WeierstrassP}(x, g2, g3))\frac{d^2}{dx^2}Y(x) + \frac{d^3}{dx^3}Y(x)\right), \{Y(x)\}\right)\right\}$$

2.1459 ODE No. 1459

$$-y'(x)(a + 4n(n + 1)\wp(x; g2, g3)) - 2n(n + 1)y(x)\wp'(x; g2, g3) + y^{(3)}(x) = 0$$

✘ **Mathematica** : cpu = 0.0175501 (sec), leaf count = 0 , could not solve

DSolve[-2*n*(1 + n)*WeierstrassPPrime[x, {g2, g3}]*y[x] - (a + 4*n*(1 + n)*WeierstrassP[x, {g2, g3}])*Derivative[1][y][x] + Derivative[3][y][x]

✘ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x) = DESol\left(\left(\left(-n^2 \operatorname{WeierstrassP}(x, g2, g3) - n \operatorname{WeierstrassP}(x, g2, g3) - \frac{a}{4}\right)Y(x) + \frac{d^2}{dx^2}Y(x)\right), \{Y(x)\}\right)^2\right\}$$

2.1460 ODE No. 1460

$$y'(x)(a + A\wp(x; g2, g3)) + By(x)\wp'(x; g2, g3) + y^{(3)}(x) = 0$$

✘ **Mathematica** : cpu = 0.0132418 (sec), leaf count = 0 , could not solve

DSolve[B*WeierstrassPPrime[x, {g2, g3}]*y[x] + (a + A*WeierstrassP[x, {g2, g3}])*Derivative[1][y][x] + Derivative[3][y][x]

✘ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x) = DESol\left(\left\{B\_Y(x)\mathcal{P}'(x; g2, g3) + (A \operatorname{WeierstrassP}(x, g2, g3) + a)\left(\frac{d}{dx}Y(x)\right) + \frac{d^3}{dx^3}Y(x)\right\}, \{Y(x)\}\right)\}$$

2.1461 ODE No. 1461

$$-y'(x)\left(a + 3k^2 \operatorname{sn}(z|x)^2\right) + y(x)\left(b + \operatorname{csn}(z|x)^2 - 3k^2 \operatorname{cn}(z|x) \operatorname{dn}(z|x) \operatorname{sn}(z|x)\right) + y^{(3)}(x) = 0$$

✘ **Mathematica** : cpu = 0.0390423 (sec), leaf count = 0 , could not solve

DSolve[(b - 3*k^2*JacobiCN[z, x]*JacobiDN[z, x]*JacobiSN[z, x] + c*JacobiSN[z, x]^2)*y[x] - (a + 3*k^2*sn(z|x)^2 - 3*k^2*cn(z|x)*dn(z|x)*sn(z|x))*Derivative[1][y][x] + Derivative[3][y][x]

✘ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x) = DESol\left(\left\{(-3k^2 \operatorname{cn}(z|x) \operatorname{dn}(z|x) \operatorname{sn}(z|x) + \operatorname{csn}(z|x)^2 + b)Y(x) + (-3k^2 \operatorname{sn}(z|x)^2 - a)\left(\frac{d}{dx}Y(x)\right) + \frac{d^3}{dx^3}Y(x)\right\}, \{Y(x)\}\right)\}$$

2.1462 ODE No. 1462

$$-y'(x)(a + 6k^2 \sin^2(x)) + by(x) + y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0483607 (sec), leaf count = 0 , could not solve

`DSolve[b*y[x] - (a + 6*k^2*Sin[x]^2)*Derivative[1][y][x] + Derivative[3][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x) = DESol\left(\left\{b\_Y(x) + (-6k^2(\sin^2(x)) - a)\left(\frac{d}{dx}Y(x)\right) + \frac{d^3}{dx^3}Y(x)\right\}, \{\_Y(x)\}\right)\}$$

2.1463 ODE No. 1463

$$y(x)f'(x) + 2f(x)y'(x) + y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0483734 (sec), leaf count = 0 , could not solve

`DSolve[y[x]*Derivative[1][f][x] + 2*f[x]*Derivative[1][y][x] + Derivative[3][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x) = DESol\left(\left\{\frac{-Y(x)f(x)}{2} + \frac{d^2}{dx^2}Y(x)\right\}, \{\_Y(x)\}\right)^2\right\}$$

2.1464 ODE No. 1464

$$y^{(3)}(x) - 2y''(x) - 3y'(x) + 10y(x) = 0$$

✓ **Mathematica** : cpu = 0.0074825 (sec), leaf count = 34

$$\left\{\left\{y(x) \rightarrow c_3 e^{-2x} + c_2 e^{2x} \cos(x) + c_1 e^{2x} \sin(x)\right\}\right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 27

$$\{y(x) = c_2 e^{2x} \sin(x) + c_3 \cos(x) e^{2x} + c_1 e^{-2x}\}$$

2.1465 ODE No. 1465

$$-a^2 y'(x) + 2a^2 y(x) + y^{(3)}(x) - 2y''(x) - \sinh(x) = 0$$

✓ **Mathematica** : cpu = 0.108254 (sec), leaf count = 95

$$\left\{\left\{y(x) \rightarrow \frac{e^{-x}(3a^2 e^{2x} - a^2 - 3e^{2x} - 12e^x \sinh(x) - 6e^x \cosh(x) + 1)}{6(a-2)(a+2)(a^2-1)} + c_1 e^{-ax} + c_3 e^{ax} + c_2 e^{2x}\right\}\right\}$$

✓ **Maple** : cpu = 0.112 (sec), leaf count = 113

$$\left\{y(x) = \frac{3a^2 e^x - 12 e^x + (6c_2 a^4 - 30c_2 a^2 + 24c_2) e^{ax} + 3 e^{-x} + 6(a-1)\left(c_1 a^2 - 4c_1 - \frac{\cosh(3x)}{6} + \frac{\sinh(3x)}{6}\right)(a+1) e^{2x} + (6c_3 a^4 - 30c_3 a^2 + 24c_3) e^{ax}}{6a^4 - 30a^2 + 24}\right\}$$

2.1466 ODE No. 1466

$$a^3(-y(x)) + 3a^2 y'(x) - 3a y''(x) - e^{ax} + y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.010086 (sec), leaf count = 46

$$\left\{\left\{y(x) \rightarrow \frac{1}{6} x^3 e^{ax} + c_3 x^2 e^{ax} + c_2 x e^{ax} + c_1 e^{ax}\right\}\right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 27

$$\left\{y(x)=\frac{\left(x^3+6c_3x^2+6c_2x+6c_1\right)e^{ax}}{6}\right\}$$

2.1467 ODE No. 1467

$$a_0y(x)+a_1y'(x)+a_2y''(x)+y^{(3)}(x)=0$$

✓ **Mathematica** : cpu = 0.0054836 (sec), leaf count = 84

$$\left\{\left\{y(x)\rightarrow c_1e^{x\text{Root}\left[\#1^3+\#1^2a_2+\#1a_1+a_0\&,1\right]}+c_2e^{x\text{Root}\left[\#1^3+\#1^2a_2+\#1a_1+a_0\&,2\right]}+c_3e^{x\text{Root}\left[\#1^3+\#1^2a_2+\#1a_1+a_0\&,3\right]}\right\}\right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 590

$$\left\{y(x)=c_1e^{-\frac{\left(\frac{\left(-8a^2^3+36a_1a_2-108a_0+12\sqrt{12a_0a^2^3-3a_1^2a_2^2-54a_1a_2a_0+12a_1^3+81a_0^2}\right)^{\frac{1}{3}}a_2+\left(\frac{i\sqrt{3}}{12}+\frac{1}{12}\right)\left(-8a^2^3+36a_1a_2-108a_0+12\sqrt{12a_0a^2^3-3a_1^2a_2^2-54a_1a_2a_0+12a_1^3+81a_0^2}\right)^{\frac{1}{3}}\right)}{\left(-8a^2^3+36a_1a_2-108a_0+12\sqrt{12a_0a^2^3-3a_1^2a_2^2-54a_1a_2a_0+12a_1^3+81a_0^2}\right)^{\frac{1}{3}}}}\right\}$$

2.1468 ODE No. 1468

$$2\left(2a+4x^2-1\right)y'(x)-8axy(x)+y^{(3)}(x)-6xy''(x)=0$$

✓ **Mathematica** : cpu = 0.0750119 (sec), leaf count = 57

$$\left\{\left\{y(x)\rightarrow c_2H_{\frac{a}{2}}(x){}_1F_1\left(-\frac{a}{4};\frac{1}{2};x^2\right)+c_1H_{\frac{a}{2}}(x)^2+c_3{}_1F_1\left(-\frac{a}{4};\frac{1}{2};x^2\right)^2\right\}\right\}$$

✓ **Maple** : cpu = 0.117 (sec), leaf count = 59

$$\left\{y(x)=\left(c_1\text{KummerM}\left(-\frac{a}{4}+\frac{1}{2},\frac{3}{2},x^2\right)^2+c_2\text{KummerU}\left(-\frac{a}{4}+\frac{1}{2},\frac{3}{2},x^2\right)^2+c_3\text{KummerM}\left(-\frac{a}{4}+\frac{1}{2},\frac{3}{2},x^2\right)\text{KummerU}\left(-\frac{a}{4}+\frac{1}{2},\frac{3}{2},x^2\right)\right\}$$

2.1469 ODE No. 1469

$$a^3x^3y(x)+3a^2x^2y'(x)+3axy''(x)+y^{(3)}(x)=0$$

✓ **Mathematica** : cpu = 0.0140404 (sec), leaf count = 72

$$\left\{\left\{y(x)\rightarrow c_1e^{-\frac{ax^2}{2}}+c_2e^{-\frac{ax^2}{2}-\sqrt{3}\sqrt{a}x}+c_3e^{\sqrt{3}\sqrt{a}x-\frac{ax^2}{2}}\right\}\right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 37

$$\left\{y(x)=\left(c_2e^{\sqrt{3}\sqrt{a}x}+c_3e^{-\sqrt{3}\sqrt{a}x}+c_1\right)e^{-\frac{ax^2}{2}}\right\}$$

2.1470 ODE No. 1470

$$y^{(3)}(x)-\sin(x)y''(x)-2\cos(x)y'(x)+y(x)\sin(x)-\log(x)=0$$

✓ **Mathematica** : cpu = 3.90851 (sec), leaf count = 64

$$\left\{\left\{y(x)\rightarrow e^{-\cos(x)}\int_1^x\frac{1}{4}e^{\cos(K[1])}\left(2\log(K[1])K[1]^2-3K[1]^2+4c_1K[1]+4c_2\right)dK[1]+c_3e^{-\cos(x)}\right\}\right\}$$

✓ **Maple** : cpu = 0.094 (sec), leaf count = 36

$$\left\{y(x)=\left(c_3+\int\left(\frac{x^2\ln(x)}{2}+2c_1x-\frac{3x^2}{4}+c_2\right)e^{\cos(x)}dx\right)e^{-\cos(x)}\right\}$$

2.1471 ODE No. 1471

$$f(x)y''(x)+f(x)y(x)+y^{(3)}(x)+y'(x)=0$$

✓ **Mathematica** : cpu = 0.112313 (sec), leaf count = 84

$$\left\{\left\{y(x)\rightarrow c_3e^{ix}\int_1^xe^{-2iK[3]}\int_1^{K[3]}\exp\left(\int_1^{K[2]}(i-f(K[1]))dK[1]\right)dK[2]dK[3]+c_1e^{ix}+\frac{1}{2}ic_2e^{-ix}\right\}\right\}$$

✓ **Maple** : cpu = 0.157 (sec), leaf count = 36

$$\{y(x)=\left(c_1+\int\left(c_2+\int c_3e^{\int(-f(x)+i)dx}dx\right)e^{-2ix}dx\right)e^{ix}\}$$

2.1472 ODE No. 1472

$$f(x)\left(x^2y''(x)-2xy'(x)+2y(x)\right)+y^{(3)}(x)=0$$

✓ **Mathematica** : cpu = 0.135757 (sec), leaf count = 88

$$\left\{\left\{y(x)\rightarrow c_3x\left(\int_1^x\frac{\exp\left(-\int_1^{K[2]}f(K[1])K[1]^2dK[1]\right)}{K[2]^2}dK[2]-x\int_1^x\frac{\exp\left(-\int_1^{K[3]}f(K[1])K[1]^2dK[1]\right)}{K[3]^3}dK[3]\right)+c_2x^2+c_1x\right\}\right\}$$

✓ **Maple** : cpu = 0.159 (sec), leaf count = 33

$$\left\{y(x)=\left(c_3+\int\left(c_2\left(\int e^{-\left(\int\left(x^2f(x)+\frac{3}{x}\right)dx\right)}dx\right)+c_1\right)dx\right)x\right\}$$

2.1473 ODE No. 1473

$$y(x)\left(f(x)g(x)+g'(x)\right)+f(x)y''(x)+g(x)y'(x)+y^{(3)}(x)=0$$

✗ **Mathematica** : cpu = 0.0117768 (sec), leaf count = 0 , could not solve

`DSolve[y[x]*(f[x]*g[x] + Derivative[1][g][x]) + g[x]*Derivative[1][y][x] + f[x]*Derivative[2][y][x] + y[x]^3 == 0,x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x)=DESol\left(\left\{\left(\frac{d}{dx}-Y(x)\right)g(x)+\left(\frac{d^2}{dx^2}-Y(x)\right)f(x)+\left(f(x)g(x)+\frac{d}{dx}g(x)\right)-Y(x)+\frac{d^3}{dx^3}-Y(x)\right\},\{-Y(x)\}\right)\}$$

2.1474 ODE No. 1474

$$y'(x)\left(f'(x)+2f(x)^2+4g(x)\right)+y(x)\left(4f(x)g(x)+2g'(x)\right)+3f(x)y''(x)+y^{(3)}(x)=0$$

✗ **Mathematica** : cpu = 0.0145226 (sec), leaf count = 0 , could not solve

`DSolve[y[x]*(4*f[x]^2*g[x] + 2*Derivative[1][g][x]) + (2*f[x]^2 + 4*g[x] + Derivative[1][f][x])y[x] + 3*f[x]*Derivative[2][y][x] + y[x]^3 == 0,x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x)=DESol\left(\left\{-Y(x)g(x)+\left(\frac{d}{dx}-Y(x)\right)f(x)+\frac{d^2}{dx^2}-Y(x)\right\},\{-Y(x)\}\right)^2\}$$

2.1475 ODE No. 1475

$$4y^{(3)}(x) - 8y''(x) - 11y'(x) - 3y(x) + 18e^x = 0$$

✓ **Mathematica** : cpu = 0.0708775 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow e^x + c_1 e^{-x/2} + c_2 e^{-x/2} x + c_3 e^{3x} \right\} \right\}$$

✓ **Maple** : cpu = 0.025 (sec), leaf count = 23

$$\left\{ y(x) = c_1 e^{3x} + e^x + (c_3 x + c_2) e^{-\frac{x}{2}} \right\}$$

2.1476 ODE No. 1476

$$-36n^2 y'(x) \wp(x; g2, g3) - 2(n+3)(4n-3)ny(x)\phi'(x) + 27y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.166206 (sec), leaf count = 0 , could not solve

`DSolve[-2*n*(3 + n)*(-3 + 4*n)*y[x]*Derivative[1][phi][x] - 36*n^2*WeierstrassP[x, {g2, g3}]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x) = DESol\left(\left\{-36n^2 \operatorname{WeierstrassP}\left(x, g2, g3\right)\left(\frac{d}{dx}Y(x)\right) + \left(-8n^3 \mathcal{P}'\left(x; g2, g3\right) - 18n^2 \mathcal{P}'\left(x; g2, g3\right) + 18n \mathcal{P}'\left(x; g2, g3\right)\right\}\right)\right\}$$

2.1477 ODE No. 1477

$$xy^{(3)}(x) + 3y''(x) + xy(x) = 0$$

✓ **Mathematica** : cpu = 0.182079 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-x}}{x} + \frac{c_2 e^{\sqrt[3]{-1}x}}{x} + \frac{c_3 e^{-(-1)^{2/3}x}}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 41

$$\left\{ y(x) = \frac{c_2 e^{\frac{x}{2}} \sin\left(\frac{\sqrt{3}x}{2}\right) + c_3 \cos\left(\frac{\sqrt{3}x}{2}\right) e^{\frac{x}{2}} + c_1 e^{-x}}{x} \right\}$$

2.1478 ODE No. 1478

$$-ax^2y(x) + xy^{(3)}(x) + 3y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0445613 (sec), leaf count = 104

$$\left\{ \left\{ y(x) \rightarrow -\frac{2(-1)^{3/4}\sqrt{2}c_1 {}_0F_2\left(\frac{1}{2}, \frac{3}{4}; \frac{ax^4}{64}\right)}{\sqrt[4]{a}x} + c_2 {}_0F_2\left(\frac{3}{4}, \frac{5}{4}; \frac{ax^4}{64}\right) + \frac{\sqrt[4]{-1}\sqrt[4]{a}c_3 x {}_0F_2\left(\frac{5}{4}, \frac{3}{2}; \frac{ax^4}{64}\right)}{2\sqrt{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.102 (sec), leaf count = 48

$$\left\{ y(x) = c_3 x \operatorname{hypergeom}\left(\left[\right], \left[\frac{5}{4}, \frac{3}{2}\right], \frac{ax^4}{64}\right) + c_1 \operatorname{hypergeom}\left(\left[\right], \left[\frac{3}{4}, \frac{5}{4}\right], \frac{ax^4}{64}\right) + \frac{c_2 \operatorname{hypergeom}\left(\left[\right], \left[\frac{1}{2}, \frac{3}{4}\right], \frac{ax^4}{64}\right)}{x} \right\}$$

2.1479 ODE No. 1479

$$(a + b)y''(x) - ay(x) + xy^{(3)}(x) - xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.150543 (sec), leaf count = 153

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} i c_2 x {}_1F_2\left(\frac{a}{2} + \frac{1}{2}; \frac{3}{2}, \frac{a}{2} + \frac{b}{2} + \frac{1}{2}; \frac{x^2}{4}\right) + c_1 {}_1F_2\left(\frac{a}{2}; \frac{1}{2}, \frac{a}{2} + \frac{b}{2}; \frac{x^2}{4}\right) + c_3 \left(\frac{i}{2}\right)^{-a-b+2} x^{-a-b+2} {}_1F_2\left(1 - \frac{b}{2}; -\frac{a}{2} - \frac{b}{2} + \frac{3}{2}; \frac{x^2}{4}\right) \right. \right.$$

✓ **Maple** : cpu = 0.201 (sec), leaf count = 92

$$\left\{ y(x) = c_2 x \operatorname{hypergeom}\left(\left[\frac{a}{2} + \frac{1}{2}\right], \left[\frac{3}{2}, \frac{a}{2} + \frac{b}{2} + \frac{1}{2}\right], \frac{x^2}{4}\right) + c_3 x^{-a-b+2} \operatorname{hypergeom}\left(\left[-\frac{b}{2} + 1\right], \left[-\frac{a}{2} - \frac{b}{2} + 2, -\frac{a}{2} - \frac{b}{2} + \frac{3}{2}\right], \frac{x^2}{4}\right) \right.$$

2.1480 ODE No. 1480

$$-(2v + x)y''(x) - (-2v + x - 1)y'(x) + xy^{(3)}(x) + (x - 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.27502 (sec), leaf count = 93

$$\left\{ \left\{ y(x) \rightarrow \frac{c_3 e^x x^{2v+2} \Gamma\left(v + \frac{3}{2}\right) {}_1\tilde{F}_1\left(v + \frac{3}{2}; 2v + 3; -2x\right)}{\Gamma\left(\frac{1}{2} - v\right)} + c_2 2^{-2v-2} e^x G_{2,3}^{2,1}\left(2x \left| \begin{matrix} 1, v + \frac{3}{2} \\ 1, 2(v + 1), 0 \end{matrix} \right. \right) + c_1 e^x \right\} \right.$$

✓ **Maple** : cpu = 0.217 (sec), leaf count = 35

$$\{y(x) = c_2 x^{v+1} \operatorname{BesselI}(-v - 1, x) + c_3 x^{v+1} \operatorname{BesselK}(v + 1, x) + c_1 e^x\}$$

2.1481 ODE No. 1481

$$-f(x) + (x^2 - 3)y''(x) + xy^{(3)}(x) + 4xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.548543 (sec), leaf count = 432

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{240} e^{-\frac{x^2}{2}} \left(-240 x^5 \int_1^x \left(-\frac{1}{240} \left(15 \operatorname{Ei}\left(\frac{K[1]^2}{2}\right) + 16 e^{\frac{K[1]^2}{2}} \right) f(K[1]) + \frac{1}{15} \sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\frac{K[1]}{\sqrt{2}}\right) K[1] f(K[1]) + \frac{7 e^{\frac{K[1]^2}{2}} f(K[1])}{120 K[1]} \right) dx \right. \right.$$

✓ **Maple** : cpu = 0.067 (sec), leaf count = 44

$$\left\{ y(x) = \left(c_3 + \int \frac{(2c_1 x + c_2 - (\int \int -f(x) dx dx)) e^{\frac{x^2}{2}}}{x^6} dx \right) x^5 e^{-\frac{x^2}{2}} \right\}$$

2.1482 ODE No. 1482

$$axy(x) - b + 2xy^{(3)}(x) + 3y''(x) = 0$$

✓ **Mathematica** : cpu = 12.0268 (sec), leaf count = 3629

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_0F_2\left(\frac{2}{3}, \frac{5}{6}; -\frac{ax^3}{54}\right) + \frac{\sqrt[6]{a} \sqrt{x} c_2 {}_0F_2\left(\frac{5}{6}, \frac{7}{6}; -\frac{ax^3}{54}\right)}{\sqrt[6]{2} \sqrt{3}} + \frac{\sqrt[3]{a} x c_3 {}_0F_2\left(\frac{7}{6}, \frac{4}{3}; -\frac{ax^3}{54}\right)}{3 \sqrt[3]{2}} + \frac{1}{6} \left({}_0F_2\left(\frac{2}{3}, \frac{5}{6}; -\frac{ax^3}{54}\right) \int_1^x \frac{1}{t^2} dt \right. \right.$$

✓ **Maple** : cpu = 0.298 (sec), leaf count = 1616

Expression too large to display

2.1483 ODE No. 1483

$$-4(\nu + x - 1)y''(x) + (6\nu + 2x - 5)y'(x) + (1 - 2\nu)y(x) + 2xy^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.14485 (sec), leaf count = 112

$$\left\{ \left\{ y(x) \rightarrow \frac{c_3 e^x x \Gamma\left(\frac{5}{2} - 3\nu\right) \left(\frac{{}_2F_1\left(\frac{3}{2} - 3\nu; 1 - 2\nu; -x\right)}{3(2\nu - 1)x} + \frac{2}{3x\Gamma(2 - 2\nu)} \right)}{\Gamma\left(\frac{3}{2} - \nu\right)} + c_2 e^x G_{2,3}^{2,1} \left[x \left| \begin{matrix} 1, 3\nu - \frac{1}{2} \\ 1, 2\nu, 0 \end{matrix} \right. \right] + c_1 e^x \right\} \right\}$$

✓ **Maple** : cpu = 0.233 (sec), leaf count = 37

$$\left\{ y(x) = c_2 x^\nu \operatorname{BesselI}\left(\nu, \frac{x}{2}\right) e^{\frac{x}{2}} + c_3 x^\nu \operatorname{BesselK}\left(\nu, \frac{x}{2}\right) e^{\frac{x}{2}} + c_1 e^x \right\}$$

2.1484 ODE No. 1484

$$6y'(x)(ak + bx) + 3(2ax + k)y''(x) + y(x)(3bk + 2cx) + 2xy^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 62.495 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow (x) \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x) = \operatorname{DESol}\left(\left\{2x\left(\frac{d^3}{dx^3}Y(x)\right) + (3bk + 2cx)Y(x) + (6ak + 6bx)\left(\frac{d}{dx}Y(x)\right) + (6ax + 3k)\left(\frac{d^2}{dx^2}Y(x)\right)\right\}, \{Y(x)\}\right)\}$$

2.1485 ODE No. 1485

$$(x - 2)xy^{(3)}(x) - (x - 2)xy''(x) - 2y'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.239767 (sec), leaf count = 64

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4}c_3 x^2 \left(-\frac{4e^{x-2}\operatorname{Ei}(2-x)}{x^2} + \frac{2}{x^2} + \frac{2}{x} + \log(2-x) - \log(x) \right) + c_1 x^2 + c_2 e^x \right\} \right\}$$

✓ **Maple** : cpu = 0.3 (sec), leaf count = 51

$$\left\{ y(x) = -\frac{c_3 x^2 \ln(x)}{4} + \frac{c_3 x^2 \ln(x-2)}{4} + c_1 x^2 + c_3 \operatorname{Ei}(1, x-2)e^{x-2} + c_2 e^x + \frac{c_3(2x+2)}{4} \right\}$$

2.1486 ODE No. 1486

$$(2x - 1)y^{(3)}(x) - 8xy'(x) + 8y(x) = 0$$

✓ **Mathematica** : cpu = 0.331237 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4}c_3 x \left(\frac{e^{2x-2}\operatorname{Ei}(2-4x)}{x} - \frac{2\operatorname{Ei}(1-2x)}{e} - \frac{e^{-2x}}{x} \right) + c_1 x - c_2 e^{2x} \right\} \right\}$$

✓ **Maple** : cpu = 0.208 (sec), leaf count = 51

$$\left\{ y(x) = c_1 x + c_2 e^{2x} - \frac{c_3 \left(2e^{-1}x \operatorname{Ei}(1, 2x-1) - \operatorname{Ei}(1, 4x-2)e^{2x-2} - e^{-2x} \right)}{4} \right\}$$

2.1487 ODE No. 1487

$$(2x-1)y^{(3)}(x) + (x+4)y''(x) + 2y'(x) = 0$$

✓ **Mathematica** : cpu = 0.974209 (sec), leaf count = 87

$$\left\{ \left\{ y(x) \rightarrow \int_1^x \left(\frac{e^{-\frac{K[1]}{2}} c_1 \left(1 + \frac{1}{4 \left(\frac{K[1]}{2} - \frac{1}{4} \right)} \right)} + e^{-\frac{K[1]}{2}} c_2 L_{-\frac{1}{4}}^{\frac{5}{4}} \left(\frac{K[1]}{2} - \frac{1}{4} \right) \right) dK[1] + c_3 \right\} \right\}$$

✓ **Maple** : cpu = 0.141 (sec), leaf count = 38

$$\left\{ y(x) = \frac{\left(c_3 + \int \frac{(2c_1x+c_2)e^{\frac{x}{2}}}{(2x-1)^{\frac{3}{4}}} dx \right) e^{-\frac{x}{2}}}{(2x-1)^{\frac{1}{4}}} \right\}$$

2.1488 ODE No. 1488

$$ax^2y(x) + x^2y^{(3)}(x) - 6y'(x) = 0$$

✓ **Mathematica** : cpu = 0.61823 (sec), leaf count = 102

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-\sqrt[3]{a}x} \left(\sqrt[3]{a}x + 2 \right)}{x} + \frac{c_2 e^{\sqrt[3]{-1} \sqrt[3]{a}x} \left(\sqrt[3]{a}x + 2(-1)^{2/3} \right)}{x} + \frac{c_3 e^{-(-1)^{2/3} \sqrt[3]{a}x} \left(\sqrt[3]{a}x - 2\sqrt[3]{-1} \right)}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.5 (sec), leaf count = 135

$$\left\{ y(x) = \frac{c_1 \left(a^3x + 2(-a^4)^{\frac{2}{3}} \right) e^{\frac{(-a^4)^{\frac{1}{3}}x}{a}} - c_2 \left(ia^3x + (-i + \sqrt{3})(-a^4)^{\frac{2}{3}} \right) e^{\frac{i(\sqrt{3}+i)(-a^4)^{\frac{1}{3}}x}{2a}} - c_3 \left(ia^3x + (-i - \sqrt{3})(-a^4)^{\frac{2}{3}} \right) e^{\frac{i(i-\sqrt{3})(-a^4)^{\frac{1}{3}}x}{2a}}}{x}$$

2.1489 ODE No. 1489

$$x^2y^{(3)}(x) + (x+1)y''(x) - y(x) = 0$$

✗ **Mathematica** : cpu = 0.831859 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow (x) \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x) = DESol\left(\left\{x^2\left(\frac{d^3}{dx^3}-Y(x)\right)-_Y(x)+(x+1)\left(\frac{d^2}{dx^2}-Y(x)\right)\right\},\{\_Y(x)\}\right)\}$$

2.1490 ODE No. 1490

$$x^2y^{(3)}(x) + (x^2+1)y'(x) - xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0519733 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} c_1 x^2 {}_0\tilde{F}_1\left(;2;-\frac{x^2}{4}\right) + c_2 x Y_1(x) + c_3 \right\} \right\}$$

✓ **Maple** : cpu = 0.074 (sec), leaf count = 18

$$\{y(x) = c_2 x \operatorname{BesselJ}(1,x) + c_3 x \operatorname{BesselY}(1,x) + c_1\}$$

2.1491 ODE No. 1491

$$\left(-4a^2v^2+4a^2x^{2a}+1\right)y'(x)+x^2y^{(3)}(x)+3xy''(x)=4a^3x^{2a-1}y(x)$$

✓ **Mathematica** : cpu = 0.03516 (sec), leaf count = 102

$$\left\{\left\{y(x)\rightarrow c_2\left(x^{2a}\right)^{-\nu}{}_1F_2\left(-\nu-\frac{1}{2};1-2\nu,1-\nu;-x^{2a}\right)+c_3\left(x^{2a}\right)^{\nu}{}_1F_2\left(\nu-\frac{1}{2};\nu+1,2\nu+1;-x^{2a}\right)+c_1{}_1F_2\left(-\frac{1}{2};1-\nu,\nu+1;\right)\right\}\right.$$

✓ **Maple** : cpu = 0.099 (sec), leaf count = 88

$$\left\{y(x)=c_2x^{-2av}\operatorname{hypergeom}\left(\left[-\nu-\frac{1}{2}\right],[-2\nu+1,-\nu+1],-x^{2a}\right)+c_3x^{2av}\operatorname{hypergeom}\left(\left[\nu-\frac{1}{2}\right],[\nu+1,2\nu+1],-x^{2a}\right)+c_1\right.$$

2.1492 ODE No. 1492

$$\left(4x(n-m)+m(2m-1)+2x^2\right)y'(x)-2n(-2m+2x+1)y(x)-3x(x-m)y''(x)+x^2y^{(3)}(x)=0$$

✓ **Mathematica** : cpu = 0.391762 (sec), leaf count = 43

$$\left\{\left\{y(x)\rightarrow c_2U(-n,m,x)L_n^{m-1}(x)+c_1U(-n,m,x)^2+c_3L_n^{m-1}(x)^2\right\}\right\}$$

✓ **Maple** : cpu = 0.141 (sec), leaf count = 39

$$\{y(x)=c_1\operatorname{KummerM}(-n,m,x)^2+c_2\operatorname{KummerU}(-n,m,x)^2+c_3\operatorname{KummerM}(-n,m,x)\operatorname{KummerU}(-n,m,x)\}$$

2.1493 ODE No. 1493

$$-f(x)+x^2y^{(3)}(x)+\left(x^2+2\right)y'(x)+4xy''(x)+3xy(x)=0$$

✓ **Mathematica** : cpu = 1.60243 (sec), leaf count = 2585

$$\left\{\left\{\left\{y(x)\rightarrow J_0(x)c_1+2Y_0(x)c_2+\frac{2c_3{}_1F_2\left(1;\frac{1}{2},\frac{1}{2};-\frac{x^2}{4}\right)}{x}+\frac{xJ_0(x)\int_1^x\left(\frac{-16J_1(K[1])Y_0(K[1])^2f(K[1]){}_1F_2\left(3;\frac{5}{2},\frac{5}{2};-\frac{1}{4}K[1]^2\right)K[1]^3+16J_0(K[1])Y_0(K[1])^2}{2J_0(K[1])}\right)}{x}\right\}\right\}\right.$$

✓ **Maple** : cpu = 0.326 (sec), leaf count = 1033

Expression too large to display

2.1494 ODE No. 1494

$$x^2y^{(3)}(x)+5xy''(x)+4y'(x)-\log(x)=0$$

✓ **Mathematica** : cpu = 0.0211999 (sec), leaf count = 43

$$\left\{\left\{y(x)\rightarrow -\frac{x}{2}+\frac{1}{4}x\log(x)-\frac{c_1}{x}-\frac{2c_2}{x}-\frac{2c_2\log(x)}{x}+c_3\right\}\right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 32

$$\left\{y(x)=\frac{4c_1x-2x^2+4c_3+\left(x^2+4c_2\right)\ln(x)}{4x}\right\}$$

2.1495 ODE No. 1495

$$x^2y^{(3)}(x) + 6xy''(x) + 6y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0074537 (sec), leaf count = 24

$$\left\{\left\{y(x) \rightarrow -\frac{c_1}{2x^2} - \frac{c_2}{x} + c_3\right\}\right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 16

$$\left\{y(x) = c_1 + \frac{c_2}{x} + \frac{c_3}{x^2}\right\}$$

2.1496 ODE No. 1496

$$ax^2y(x) + x^2y^{(3)}(x) + 6xy''(x) + 6y'(x) = 0$$

✓ **Mathematica** : cpu = 0.258026 (sec), leaf count = 63

$$\left\{\left\{y(x) \rightarrow \frac{c_1e^{-\sqrt[3]{a}x}}{x^2} + \frac{c_2e^{\sqrt[3]{-1}\sqrt[3]{a}x}}{x^2} + \frac{c_3e^{-(-1)^{2/3}\sqrt[3]{a}x}}{x^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 57

$$\left\{y(x) = \frac{c_1e^{\frac{(-a)^{\frac{1}{3}}(i\sqrt{3}-1)x}{2}} + c_2e^{\frac{(-a)^{\frac{1}{3}}(1+i\sqrt{3})x}{2}} + c_3e^{(-a)^{\frac{1}{3}}x}}{x^2}\right\}$$

2.1497 ODE No. 1497

$$-3x(p+q)y''(x) + 3p(3q+1)y'(x) + x^2y^{(3)}(x) + x^2(-y(x)) = 0$$

✓ **Mathematica** : cpu = 0.538492 (sec), leaf count = 135

$$\left\{\left\{y(x) \rightarrow c_1{}_0F_2\left(\left;\frac{2}{3}-p,\frac{1}{3}-q;\frac{x^3}{27}\right)+c_2(-1)^{\frac{1}{3}(3p+1)}3^{-3p-1}x^{3p+1}{}_0F_2\left(\left;p+\frac{4}{3},p-q+\frac{2}{3};\frac{x^3}{27}\right)+c_3(-1)^{\frac{1}{3}(3q+2)}3^{-3q-2}x^{3q+2}{}_0F_2\left(\left;p+\frac{4}{3},p-q+\frac{2}{3};\frac{x^3}{27}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.227 (sec), leaf count = 77

$$\left\{y(x) = c_2x^{3p+1}\text{hypergeom}\left(\left[\right],\left[p+\frac{4}{3},p-q+\frac{2}{3}\right],\frac{x^3}{27}\right) + c_3x^{3q+2}\text{hypergeom}\left(\left[\right],\left[q+\frac{5}{3},-p+q+\frac{4}{3}\right],\frac{x^3}{27}\right) + c_1\text{hypergeom}\left(\left[\right],\left[p+\frac{4}{3},p-q+\frac{2}{3}\right],\frac{x^3}{27}\right)\right\}$$

2.1498 ODE No. 1498

$$(ax^2 + 6n)y'(x) - 2axy(x) - 2(n+1)xy''(x) + x^2y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 13.6002 (sec), leaf count = 584

$$\left\{\left\{y(x) \rightarrow -\frac{\pi c_32^{-n-\frac{3}{2}}x(\sqrt{a}x)^{-n-\frac{1}{2}}\left(-a^{3/2}2^{2n}x^3\sec(\pi n)\Gamma\left(\frac{3}{2}-n\right)\Gamma\left(n+\frac{3}{2}\right)J_{\frac{1}{2}(2n+1)}(\sqrt{a}x){}_1\tilde{F}_2\left(\frac{3}{2}-n;\frac{1}{2}-n,\frac{5}{2}-n;-\frac{ax^2}{4}\right)-\pi c_22^{-n-\frac{3}{2}}x(\sqrt{a}x)^{-n-\frac{1}{2}}\left(-a^{3/2}2^{2n}x^3\sec(\pi n)\Gamma\left(\frac{3}{2}-n\right)\Gamma\left(n+\frac{3}{2}\right)J_{\frac{1}{2}(2n+1)}(\sqrt{a}x){}_1\tilde{F}_2\left(\frac{3}{2}-n;\frac{1}{2}-n,\frac{5}{2}-n;-\frac{ax^2}{4}\right)-\pi c_12^{-n-\frac{3}{2}}x(\sqrt{a}x)^{-n-\frac{1}{2}}\left(-a^{3/2}2^{2n}x^3\sec(\pi n)\Gamma\left(\frac{3}{2}-n\right)\Gamma\left(n+\frac{3}{2}\right)J_{\frac{1}{2}(2n+1)}(\sqrt{a}x){}_1\tilde{F}_2\left(\frac{3}{2}-n;\frac{1}{2}-n,\frac{5}{2}-n;-\frac{ax^2}{4}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.222 (sec), leaf count = 53

$$\left\{y(x) = c_1x^{n+\frac{1}{2}}\text{BesselJ}\left(-n-\frac{1}{2},\sqrt{a}x\right) + c_2x^{n+\frac{1}{2}}\text{BesselY}\left(-n-\frac{1}{2},\sqrt{a}x\right) + c_3(ax^2 + 4n - 2)\right\}$$

2.1499 ODE No. 1499

$$-\left(\nu^2+x^2-\frac{1}{4}\right)y'(x)+\left(\nu^2+x^2-2x-\frac{1}{4}\right)y(x)+x^2y^{(3)}(x)-(x^2-2x)y''(x)=0$$

✓ **Mathematica** : cpu = 0.316308 (sec), leaf count = 97

$$\left\{\left\{y(x)\rightarrow\frac{c_3e^xx^{\nu+\frac{1}{2}}\Gamma\left(\nu+\frac{1}{2}\right){}_1\tilde{F}_1\left(\nu+\frac{1}{2};2\nu+1;-2x\right)}{\Gamma\left(\frac{3}{2}-\nu\right)}+c_22^{-\nu-\frac{1}{2}}e^xG_{2,3}^{2,1}\left(2x\left|\begin{array}{c}1,0\\\frac{1}{2}-\nu,\nu+\frac{1}{2},0\end{array}\right.\right)+c_1e^x\right\}\right\}$$

✓ **Maple** : cpu = 0.218 (sec), leaf count = 25

$$\{y(x)=c_2\sqrt{x}\text{ BesselI}(\nu,x)+c_3\sqrt{x}\text{ BesselK}(\nu,x)+c_1e^x\}$$

2.1500 ODE No. 1500

$$\nu(2x+1)y'(x)-\nu(x+1)y(x)-x(v+x)y''(x)+x^2y^{(3)}(x)=0$$

✗ **Mathematica** : cpu = 45.9694 (sec), leaf count = 0 , DifferentialRoot result

$$\{\{y(x)\rightarrow(x)\}\}$$

✓ **Maple** : cpu = 0.189 (sec), leaf count = 55

$$\left\{y(x)=c_2x^{\frac{\nu}{2}+\frac{1}{2}}\text{ BesselJ}\left(-\nu-1,2\sqrt{\nu}\sqrt{x}\right)+c_3x^{\frac{\nu}{2}+\frac{1}{2}}\text{ BesselY}\left(-\nu-1,2\sqrt{\nu}\sqrt{x}\right)+c_1e^x\right\}$$

2.1501 ODE No. 1501

$$\left(-\nu^2+x^2-2x+\frac{1}{4}\right)y'(x)+\left(\nu^2-\frac{1}{4}\right)y(x)+x^2y^{(3)}(x)-2\left(x^2-x\right)y''(x)=0$$

✓ **Mathematica** : cpu = 0.200746 (sec), leaf count = 86

$$\left\{\left\{y(x)\rightarrow\frac{c_3e^xx^{\nu+\frac{1}{2}}\Gamma\left(\nu+\frac{1}{2}\right){}_1\tilde{F}_1\left(\nu+\frac{1}{2};2\nu+1;-x\right)}{\Gamma\left(\frac{3}{2}-\nu\right)}+c_2e^xG_{2,3}^{2,1}\left(x\left|\begin{array}{c}1,0\\\frac{1}{2}-\nu,\nu+\frac{1}{2},0\end{array}\right.\right)+c_1e^x\right\}\right\}$$

✓ **Maple** : cpu = 0.194 (sec), leaf count = 37

$$\left\{y(x)=c_2\sqrt{x}\text{ BesselI}\left(\nu,\frac{x}{2}\right)e^{\frac{x}{2}}+c_3\sqrt{x}\text{ BesselK}\left(\nu,\frac{x}{2}\right)e^{\frac{x}{2}}+c_1e^x\right\}$$

2.1502 ODE No. 1502

$$-\left(x^4-6x\right)y''(x)-\left(2x^3-6\right)y'(x)+x^2y^{(3)}(x)+2x^2y(x)=0$$

✓ **Mathematica** : cpu = 0.104637 (sec), leaf count = 98

$$\left\{\left\{y(x)\rightarrow\frac{c_2\Gamma\left(\frac{1}{3}\right){}_2F_2\left(-\frac{2}{3},\frac{1}{3};\frac{2}{3},\frac{4}{3};\frac{x^3}{3}\right)}{3x\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt[3]{-\frac{1}{3}}c_3\Gamma\left(\frac{2}{3}\right){}_2F_2\left(-\frac{1}{3},\frac{2}{3};\frac{4}{3},\frac{5}{3};\frac{x^3}{3}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{c_1}{x^2}\right\}\right\}$$

✓ **Maple** : cpu = 0.409 (sec), leaf count = 104

$$\left\{y(x)=\frac{c_2\left(\int\left(x^3\text{ BesselI}\left(-\frac{5}{6},-\frac{x^3}{6}\right)+x^3\text{ BesselI}\left(\frac{1}{6},-\frac{x^3}{6}\right)-2\text{ BesselI}\left(\frac{1}{6},-\frac{x^3}{6}\right)\right)\sqrt{x}\text{ e}^{\frac{x^3}{6}}dx\right)+c_3\left(\int-\left(x^3\text{ BesselK}\left(\frac{1}{6},-\frac{x^3}{6}\right)\right)}{x^2}\right.}$$

2.1503 ODE No. 1503

$$(x^2 + 1)y^{(3)}(x) + \frac{1}{x^2} + 8xy''(x) + 10y'(x) - 2\log(x) - 3 = 0$$

✓ **Mathematica** : cpu = 0.645988 (sec), leaf count = 258

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{225} \left(-\frac{51x}{x^2 + 1} - \frac{34x}{(x^2 + 1)^2} - \frac{225c_2x}{x^2 + 1} - \frac{150c_2x}{(x^2 + 1)^2} - \frac{225c_1}{4(x^2 + 1)^2} - 9x + \frac{47}{x - i} + \frac{47}{x + i} + 45x\log(x) + 60i\log(-x) \right) \right\} \right.$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 67

$$\left\{ y(x) = \frac{225c_1x^4 - 9x^5 + 450c_1x^2 + (225c_2 - 50)x^3 + 225c_3 + (675c_2 - 225)x + (45x^5 + 150x^3 + 225x)\ln(x)}{225(x^2 + 1)^2} \right\}$$

2.1504 ODE No. 1504

$$(x^2 + 2)y^{(3)}(x) + (x^2 + 2)y'(x) - 2xy''(x) - 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.138555 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1x^2}{2} + \frac{1}{2}ic_2e^{-ix} - \frac{1}{4}c_3e^{ix} \right\} \right\}$$

✓ **Maple** : cpu = 0.175 (sec), leaf count = 18

$$\{y(x) = c_1x^2 + c_2\cos(x) + c_3\sin(x)\}$$

2.1505 ODE No. 1505

$$(2ax + b)y'(x) + ay(x) + 2(x - 1)xy^{(3)}(x) + 3(2x - 1)y''(x) = 0$$

✓ **Mathematica** : cpu = 60.4134 (sec), leaf count = 115

$$\left\{ \left\{ y(x) \rightarrow c_3\text{MathieuC}\left[-\frac{a}{2} - \frac{b}{2} + 1, \frac{a}{4}, \cos^{-1}(\sqrt{x})\right]\text{MathieuS}\left[-\frac{a}{2} - \frac{b}{2} + 1, \frac{a}{4}, \cos^{-1}(\sqrt{x})\right] + c_1\text{MathieuC}\left[-\frac{a}{2} - \frac{b}{2} + 1, \frac{a}{4}, \cos^{-1}(\sqrt{x})\right]\text{MathieuS}\left[-\frac{a}{2} - \frac{b}{2} + 1, \frac{a}{4}, \cos^{-1}(\sqrt{x})\right] + c_2\text{MathieuC}\left[-\frac{a}{2} - \frac{b}{2} + 1, \frac{a}{4}, \cos^{-1}(\sqrt{x})\right]\text{MathieuS}\left[-\frac{a}{2} - \frac{b}{2} + 1, \frac{a}{4}, \cos^{-1}(\sqrt{x})\right] \right\} \right.$$

✓ **Maple** : cpu = 0.202 (sec), leaf count = 79

$$\left\{ y(x) = c_1\text{MathieuC}\left(-\frac{a}{2} - \frac{b}{2} + 1, \frac{a}{4}, \arccos(\sqrt{x})\right)^2 + c_2\text{MathieuS}\left(-\frac{a}{2} - \frac{b}{2} + 1, \frac{a}{4}, \arccos(\sqrt{x})\right)^2 + c_3\text{MathieuC}\left(-\frac{a}{2} - \frac{b}{2} + 1, \frac{a}{4}, \arccos(\sqrt{x})\right)^2 + c_4\text{MathieuS}\left(-\frac{a}{2} - \frac{b}{2} + 1, \frac{a}{4}, \arccos(\sqrt{x})\right)^2 \right\}$$

2.1506 ODE No. 1506

$$4x^2y^{(3)}(x) + (x^2 + 14x - 1)y''(x) + 4(x + 1)y'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.437132 (sec), leaf count = 150

$$\left\{ \left\{ y(x) \rightarrow c_2e^{\frac{1}{4}\left(-x - \frac{1}{x} + 2\log(x)\right)} \int_1^x e^{\frac{K[1]^2 - 10\log(K[1])K[1] + 1}{4K[1]}} dK[1] - \sqrt{\pi} c_3 \left(\text{erfi}\left(\frac{1 - x}{2\sqrt{x}}\right) + \text{erfi}\left(\frac{x + 1}{2\sqrt{x}}\right) - i(e - 1) \right) e^{\frac{1}{4}\left(-x - \frac{1}{x} + 2\log(x)\right)} \right\} \right.$$

✓ **Maple** : cpu = 0.187 (sec), leaf count = 43

$$\left\{ y(x) = \left(c_3 + \int \frac{(2c_1x + c_2)e^{\frac{x}{4}}e^{\frac{1}{4x}}}{4x^{\frac{5}{2}}} dx \right) \sqrt{x} e^{-\frac{x}{4}}e^{-\frac{1}{4x}} \right\}$$

2.1507 ODE No. 1507

$$xy^{(3)}(x)(ax + b) + (ax + \beta)y''(x) - f(x) + xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 5.8388 (sec), leaf count = 70099

Too large to display

✓ **Maple** : cpu = 0.747 (sec), leaf count = 1210

$$\left\{y(x) = \left(\left(b \int -\frac{1}{ax \operatorname{HeunC}\left(0, \frac{-2b+\beta}{b}, \frac{-ab+(2b+\beta)a}{ab}, -\frac{b}{a^2}, \frac{a\beta^2-ab\beta+(4a-\alpha)b^2}{2ab^2}, -\frac{ax}{b}\right) \operatorname{HeunCPrime}\left(0, \frac{2b-\beta}{b}, \frac{-ab+(2b+\beta)a}{ab}, -\frac{b}{a^2}, \frac{a\beta^2-ab\beta+(4a-\alpha)b^2}{2ab^2}, -\frac{ax}{b}\right)}\right.\right.\right.$$

2.1508 ODE No. 1508

$$y(x)\left(ax^3 + \nu^2 - 1\right) + \left(1 - \nu^2\right)xy'(x) + x^3y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.893541 (sec), leaf count = 143

$$\left\{\left\{y(x) \rightarrow c_2 3^{\nu-1} a^{\frac{1-\nu}{3}} x^{1-\nu} {}_0F_2\left(; 1 - \frac{2\nu}{3}, 1 - \frac{\nu}{3}; -\frac{ax^3}{27}\right) + c_3 3^{-\nu-1} a^{\frac{\nu+1}{3}} x^{\nu+1} {}_0F_2\left(; \frac{\nu}{3} + 1, \frac{2\nu}{3} + 1; -\frac{ax^3}{27}\right) + \frac{1}{3} \sqrt[3]{a} c_1 x {}_0F_2\left(; 1 - \frac{2\nu}{3}, 1 - \frac{\nu}{3}; -\frac{ax^3}{27}\right)\right.\right.$$

✓ **Maple** : cpu = 0.138 (sec), leaf count = 81

$$\left\{y(x) = c_1 x \operatorname{hypergeom}\left(\left[\right], \left[\frac{\nu}{3} + 1, -\frac{\nu}{3} + 1\right], -\frac{a x^3}{27}\right) + c_2 x^{-\nu+1} \operatorname{hypergeom}\left(\left[\right], \left[-\frac{2\nu}{3} + 1, -\frac{\nu}{3} + 1\right], -\frac{a x^3}{27}\right) + c_3 x^{\nu+1} \operatorname{hypergeom}\left(\left[\right], \left[\frac{\nu}{3} + 1, \frac{2\nu}{3} + 1\right], -\frac{a x^3}{27}\right)\right.$$

2.1509 ODE No. 1509

$$\left(\left(1 - 4\nu^2\right)x + 4x^3\right)y'(x) + \left(4\nu^2 - 1\right)y(x) + x^3y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.0198809 (sec), leaf count = 34

$$\left\{\left\{y(x) \rightarrow c_1 x J_\nu(x)^2 + c_3 x Y_\nu(x)^2 + c_2 x J_\nu(x) Y_\nu(x)\right\}\right.$$

✓ **Maple** : cpu = 0.063 (sec), leaf count = 29

$$\{y(x) = \left(c_1 \operatorname{BesselJ}(\nu, x)^2 + c_2 \operatorname{BesselY}(\nu, x)^2 + c_3 \operatorname{BesselJ}(\nu, x) \operatorname{BesselY}(\nu, x)\right)x\}$$

2.1510 ODE No. 1510

$$y(x)\left(a(\nu - 1)x^{2\nu} + bx^{3\nu} + \nu^2 - 1\right) + x\left(ax^{2\nu} - \nu^2 + 1\right)y'(x) + x^3y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.0498376 (sec), leaf count = 102

$$\left\{\left\{y(x) \rightarrow c_1 x^{1-\nu} e^{\frac{x^\nu \operatorname{Root}\left[\#1^3 + \#1a + b\&, 1\right]}{\nu}} + c_2 x^{1-\nu} e^{\frac{x^\nu \operatorname{Root}\left[\#1^3 + \#1a + b\&, 2\right]}{\nu}} + c_3 x^{1-\nu} e^{\frac{x^\nu \operatorname{Root}\left[\#1^3 + \#1a + b\&, 3\right]}{\nu}}\right\}\right.$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x) = \operatorname{DESol}\left(\left\{x^3\left(\frac{d^3}{dx^3}-Y(x)\right) + \left(av x^{2\nu} - a x^{2\nu} + b x^{3\nu} + \nu^2 - 1\right)_-Y(x) + \left(ax x^{2\nu} - \nu^2 x + x\right)\left(\frac{d}{dx}-Y(x)\right)\right\}, \{-Y(x)\}\right)\}$$

2.1511 ODE No. 1511

$$x^3y^{(3)}(x) + (x + 8)x^3 - 6(x - 1)x^3 \log(x) + 3x^2y''(x) - 2xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0572187 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{x^2} + \frac{1}{450} \left(-50x^4 + 50x^4 \log(x) - 18x^3 - 135x^3 \log(x) \right) + c_2x + c_3x \log(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 49

$$\left\{ y(x) = \frac{-50x^6 - 18x^5 + 450c_1x^3 + 450c_2 + (50x^6 - 135x^5 + 450c_3x^3) \ln(x)}{450x^2} \right\}$$

2.1512 ODE No. 1512

$$(1 - a^2)xy'(x) + x^3y^{(3)}(x) + 3x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0460289 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow -\frac{c_1x^{-a}}{a} + \frac{c_2x^a}{a} + c_3 \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 18

$$\{y(x) = c_2x^a + c_3x^{-a} + c_1\}$$

2.1513 ODE No. 1513

$$x^3y^{(3)}(x) - 4x^2y''(x) + (x^2 + 8)xy'(x) - 2(x^2 + 4)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0910696 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow c_1x^2 + c_3x \cos(x) - c_2x \sin(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.168 (sec), leaf count = 18

$$\{y(x) = (c_1x + c_2 \sin(x) + c_3 \cos(x))x\}$$

2.1514 ODE No. 1514

$$(ax^3 - 12)y(x) + x^3y^{(3)}(x) + 6x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.68781 (sec), leaf count = 102

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1e^{-\sqrt[3]{a}x}(\sqrt[3]{a}x + 2)}{x^3} + \frac{c_2e^{\sqrt[3]{-1}\sqrt[3]{a}x}(\sqrt[3]{a}x + 2(-1)^{2/3})}{x^3} + \frac{c_3e^{-(-1)^{2/3}\sqrt[3]{a}x}(\sqrt[3]{a}x - 2\sqrt[3]{-1})}{x^3} \right\} \right\}$$

✓ **Maple** : cpu = 0.478 (sec), leaf count = 135

$$\left\{ y(x) = \frac{c_1 \left(a^3x + 2(-a^4)^{\frac{2}{3}} \right) e^{\frac{(-a^4)^{\frac{1}{3}}x}{a}} - c_2 \left(ia^3x + (-i + \sqrt{3})(-a^4)^{\frac{2}{3}} \right) e^{\frac{i(\sqrt{3}+i)(-a^4)^{\frac{1}{3}}x}{2a}} - c_3 \left(ia^3x + (-i - \sqrt{3})(-a^4)^{\frac{2}{3}} \right) e^{\frac{i(i-\sqrt{3})(-a^4)^{\frac{1}{3}}x}{2a}}}{x^3} \right.$$

2.1515 ODE No. 1515

$$y(x)\left(a(4c^2v^2 - a^2) + 4b^2c^2(c - a)x^{2c}\right) + y'(x)\left(3(a - 1)ax + 4b^2c^2x^{2c+1} - 4c^2v^2 + 1\right) + 3(1 - a)x^2y''(x) + x^3y^{(3)}(x) = 0$$

X Mathematica : cpu = 0.191315 (sec), leaf count = 0 , could not solve

$$\text{DSolve}[(a(-a^2 + 4c^2\nu^2) + 4b^2c^2(-a + c)x^{(2c)})y[x] + (1 - 4c^2\nu^2 + 3(-1 + a)ax + 4b^2c^2x^{(1 + 2c)})\text{Derivative}[1][y][x] + 3(1 - a)x^2\text{Derivative}[2][y][x]$$

X Maple : cpu = 0. (sec), leaf count = 0 , exception

cannot mix numeric and boolean arguments to max

2.1516 ODE No. 1516

$$x^3y^{(3)}(x) + (x + 3)x^2y''(x) + 5(x - 6)xy'(x) + (4x + 30)y(x) = 0$$

✓ **Mathematica** : cpu = 260.122 (sec), leaf count = 15142

Too large to display

✓ **Maple** : $\text{cpu} = 0.405$ (sec), leaf count = 188

$$\left\{ y(x) = \frac{c_3 x^7 + 29c_3 x^6 + 480c_3 x^5 + (c_1 + 5612c_3)x^4 + c_3(x^8 + 28x^7 + 450x^6 + 5100x^5 + 42900x^4 + 267120x^3 + 1179360x^2 + 207360x + 12800)}{x^8 + 28x^7 + 450x^6 + 5100x^5 + 42900x^4 + 267120x^3 + 1179360x^2 + 207360x + 12800} \right.$$

2.1517 ODE No. 1517

$$x^3y^{(3)}(x) - 2x^3 + x^2y''(x) + 2xy'(x) - y(x) + \log(x) = 0$$

✓ **Mathematica** : cpu = 0.288283 (sec), leaf count = 30686

Too large to display

✓ **Maple** : $\text{cpu} = 0.454$ (sec), leaf count = 866

$$\left\{ y(x) = c_1 x^{\frac{(-11+3\sqrt{69})(44+12\sqrt{69})^{\frac{2}{3}}}{600} - \frac{(44+12\sqrt{69})^{\frac{1}{3}}}{6} + \frac{2}{3}} - x^{\frac{(-11+3\sqrt{69})(44+12\sqrt{69})^{\frac{2}{3}}}{600} - \frac{(44+12\sqrt{69})^{\frac{1}{3}}}{6} + \frac{2}{3}} \int - \frac{(2x^3 - \ln(x)) \left(3\sqrt{69} (44 + 12\sqrt{69}) \right)}{\dots} \right.$$

2.1518 ODE No. 1518

$$x(x^2 + 1)y^{(3)}(x) + 3(2x^2 + 1)y''(x) - 12y(x) = 0$$

✓ **Mathematica** : cpu = 0.272014 (sec), leaf count = 104

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{3}c_1(2x^2 + 1) + \frac{1}{3}c_2x\sqrt{x^2 + 1} - \frac{c_3(2x^2 + 1)\left(3(x^4 + x^2)\tanh^{-1}(\sqrt{x^2 + 1}) - \sqrt{x^2 + 1}(3x^2 + 1)\right)}{6\sqrt{x^2 + 1}(2x^3 + x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.372 (sec), leaf count = 60

$$\left\{ y(x) = \frac{3c_2 \sqrt{x^2 + 1} x^2 \operatorname{arctanh}\left(\frac{1}{\sqrt{x^2 + 1}}\right) + 2c_3 x^3 + c_1 \sqrt{x^2 + 1} x^2 - 3c_2 x^2 + c_3 x - c_2}{x} \right\}$$

2.1519 ODE No. 1519

$$(x+3)x^2y^{(3)}(x)-3(x+2)xy''(x)+6(x+1)y'(x)-6y(x)=0$$

✓ **Mathematica** : cpu = 0.0239866 (sec), leaf count = 65

$$\left\{\left\{y(x)\rightarrow\frac{1}{4}c_1\left(x^3-3x^2+3x+3\right)+\frac{1}{2}c_2\left(-x^3+3x^2-x-1\right)+\frac{1}{8}c_3\left(3x^3-5x^2+x+1\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.131 (sec), leaf count = 19

$$\{y(x)=c_2x^3+c_1x^2+c_3x+c_3\}$$

2.1520 ODE No. 1520

$$y''(x)\Big(-6x(a1+a2+a3)+3a1a2+3a1a3+3a2a3+9x^2\Big)+2(x-a1)(x-a2)(x-a3)y^{(3)}(x)-2\Big(b+(n^2+n-3)x\Big)y'(x)-n$$

✓ **Mathematica** : cpu = 4.92295 (sec), leaf count = 534

$$\left\{\left\{y(x)\rightarrow c_1\text{HeunG}\left[\frac{a3-a1}{a2-a1},\frac{\frac{1}{4}(a1+a2+a3-b)+\frac{1}{4}a1(-n^2-n)}{a1-a2},\frac{1}{2}\left(\frac{1}{2}-\sqrt{n^2+n+\frac{1}{4}}\right),\frac{1}{2}\left(\sqrt{n^2+n+\frac{1}{4}}+\frac{1}{2}\right),\frac{1}{2},\frac{1}{2}\right]\right\}\right\}$$

✓ **Maple** : cpu = 0.424 (sec), leaf count = 288

$$\left\{y(x)=c_1HG\left(\frac{a1-a3}{a1-a2},\frac{(-n^2-n+1)a1+a2+a3-b}{4a1-4a2},-\frac{n}{2},\frac{n}{2}+\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{a1-x}{a1-a2}\right)^2-c_2(-a1+x)HG\left(\frac{a1-a3}{a1-a2},\frac{(-n^2-n+1)a1+a2+a3-b}{4a1-4a2},-\frac{n}{2},\frac{n}{2}+\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{a1-x}{a1-a2}\right)\right\}$$

2.1521 ODE No. 1521

$$(x+1)x^3y^{(3)}(x)-(4x+2)x^2y''(x)+(10x+4)xy'(x)-4(3x+1)y(x)=0$$

✓ **Mathematica** : cpu = 0.0564617 (sec), leaf count = 35

$$\left\{\left\{y(x)\rightarrow c_1x^2+c_3x^2\left(x+\frac{1}{x}+\log^2(x)\right)+c_2x^2\log(x)\right\}\right\}$$

✓ **Maple** : cpu = 0.349 (sec), leaf count = 28

$$\{y(x)=\left(c_3x\ln(x)^2+c_2x\ln(x)+c_3x^2+c_1x+c_3\right)x\}$$

2.1522 ODE No. 1522

$$4x^4y^{(3)}(x)-4x^3y''(x)+4x^2y'(x)-1=0$$

✓ **Mathematica** : cpu = 0.0149679 (sec), leaf count = 44

$$\left\{\left\{y(x)\rightarrow\frac{c_1x^2}{2}-\frac{c_2x^2}{4}+\frac{1}{2}c_2x^2\log(x)-\frac{1}{36x}+c_3\right\}\right\}$$

✓ **Maple** : cpu = 0.06 (sec), leaf count = 34

$$\left\{y(x)=\frac{18c_1x^3\ln(x)+(-9c_1+18c_2)x^3+36c_3x-1}{36x}\right\}$$

2.1523 ODE No. 1523

$$-(4x^2 + 2)x^2y''(x) + (10x^2 + 4)xy'(x) - 4(3x^2 + 1)y(x) + (x^2 + 1)x^3y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.320586 (sec), leaf count = 74

$$\left\{ \left\{ y(x) \rightarrow c_1(-x^3 + 3x^2 - x) + \frac{1}{2}c_2(x^3 - 2x^2 + x) - \frac{c_3x(-x^3 + 3x^2 - x)(\log(x) + 1)}{2(x^2 - 3x + 1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.351 (sec), leaf count = 23

$$\{y(x) = (c_2x \ln(x) + c_3x^2 + c_3 + (c_1 + c_2)x)x\}$$

2.1524 ODE No. 1524

$$x^6y^{(3)}(x) + x^2y''(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.167444 (sec), leaf count = 96

$$\left\{ \left\{ y(x) \rightarrow -\frac{\left(-\frac{1}{3}\right)^{2/3} c_2x\Gamma\left(\frac{1}{3}\right) {}_2F_2\left(-\frac{2}{3}, \frac{1}{3}; \frac{2}{3}, \frac{4}{3}; \frac{1}{3x^3}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{c_3\Gamma\left(\frac{2}{3}\right) {}_2F_2\left(-\frac{1}{3}, \frac{2}{3}; \frac{4}{3}, \frac{5}{3}; \frac{1}{3x^3}\right)}{9\Gamma\left(\frac{5}{3}\right)} + c_1x^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.439 (sec), leaf count = 98

$$\left\{ y(x) = \left(c_2 \left(\int \frac{\left(2x^3 \operatorname{BesselK}\left(\frac{1}{6}, -\frac{1}{6x^3}\right) - \operatorname{BesselK}\left(\frac{1}{6}, -\frac{1}{6x^3}\right) + \operatorname{BesselK}\left(\frac{5}{6}, -\frac{1}{6x^3}\right) \right) e^{\frac{1}{6x^3}}}{x^{\frac{11}{2}}} dx \right) + c_3 \left(\int \frac{\left(2x^3 \operatorname{BesselI}\left(\frac{1}{6}, -\frac{1}{6x^3}\right) \right)}{x^{\frac{11}{2}}} dx \right) \right)$$

2.1525 ODE No. 1525

$$ay(x) + x^6y^{(3)}(x) + 6x^5y''(x) = 0$$

✓ **Mathematica** : cpu = 0.3804 (sec), leaf count = 102

$$\left\{ \left\{ y(x) \rightarrow c_1e^{\frac{\sqrt[3]{a}}{x}} \left(2x - \sqrt[3]{a} \right) + c_2e^{\frac{(-1)^{2/3}\sqrt[3]{a}}{x}} \left(x - \frac{1}{2}(-1)^{2/3}\sqrt[3]{a} \right) + c_3e^{-\frac{\sqrt[3]{-1}\sqrt[3]{a}}{x}} \left(\frac{1}{2}\sqrt[3]{-1}\sqrt[3]{a} + x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.633 (sec), leaf count = 291

$$\left\{ y(x) = \frac{c_1(-8x^3 + a)^4 e^{-\frac{(-a^4)^{\frac{1}{3}}}{ax}}}{\left(2ax + (-a^4)^{\frac{1}{3}} \right)^3 \left(4a^2x^2 - 2(-a^4)^{\frac{1}{3}}ax + (-a^4)^{\frac{2}{3}} \right)^4} + \frac{c_2(-8x^3 + a)^4 e^{-\frac{i(i-\sqrt{3})(-a^4)^{\frac{1}{3}}}{2ax}}}{\left(-4iax + i(-a^4)^{\frac{1}{3}} - (-a^4)^{\frac{1}{3}}\sqrt{3} \right)^3 \left(-4iax + (-a^4)^{\frac{1}{3}}\sqrt{3} + i(-a^4)^{\frac{1}{3}} \right)}$$

2.1526 ODE No. 1526

$$(x^4 + 2x^2 + 2x + 1)x^2y^{(3)}(x) - (2x^6 + 3x^4 - 6x^2 - 6x - 1)y''(x) + (x^6 - 6x^3 - 15x^2 - 12x - 2)y'(x) + (x^4 + 4x^3 + 8x^2 + 6x + 1)y(x) = 0$$

✓ **Mathematica** : cpu = 130.195 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow c_1e^x + c_2e^xx + c_3e^{\frac{1}{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.21 (sec), leaf count = 19

$$\left\{ y(x) = c_2e^{\frac{1}{x}} + (c_3x + c_1)e^x \right\}$$

2.1527 ODE No. 1527

$$(x-a)^3(x-b)^3y^{(3)}(x)-cy(x)=0$$

✓ **Mathematica** : cpu = 130.128 (sec), leaf count = 165

$$\left\{\left\{y(x)\rightarrow c_1(x-b)^2\left(\frac{x-a}{x-b}\right)^{\text{Root}\left[-\#1^3+3\#1^2-2\#1+\frac{c}{(a-b)^3}\&,1\right]}+c_2(x-b)^2\left(\frac{x-a}{x-b}\right)^{\text{Root}\left[-\#1^3+3\#1^2-2\#1+\frac{c}{(a-b)^3}\&,2\right]}+c_3(x-b)^2\left(\frac{x-a}{x-b}\right)^{\text{Root}\left[-\#1^3+3\#1^2-2\#1+\frac{c}{(a-b)^3}\&,3\right]}\right.\right.$$

✓ **Maple** : cpu = 0.611 (sec), leaf count = 437

$$\left\{y(x)=\left(c_1(a-x)^{\frac{\text{RootOf}\left(-Z^3-4a^2b-4ab^2+(-3a-3b)Z^2+(2a^2+8ab+2b^2)Z-c,\text{index}=1\right)}{a-b}}(b-x)^{\frac{\text{RootOf}\left(-Z^3-4a^2b-4ab^2+(-3a-3b)Z^2+(2a^2+8ab+2b^2)Z-c,\text{index}=1\right)}{a-b}}+c_2(a-x)^{\frac{\text{RootOf}\left(-Z^3-4a^2b-4ab^2+(-3a-3b)Z^2+(2a^2+8ab+2b^2)Z-c,\text{index}=2\right)}{a-b}}(b-x)^{\frac{\text{RootOf}\left(-Z^3-4a^2b-4ab^2+(-3a-3b)Z^2+(2a^2+8ab+2b^2)Z-c,\text{index}=2\right)}{a-b}}+c_3(a-x)^{\frac{\text{RootOf}\left(-Z^3-4a^2b-4ab^2+(-3a-3b)Z^2+(2a^2+8ab+2b^2)Z-c,\text{index}=3\right)}{a-b}}(b-x)^{\frac{\text{RootOf}\left(-Z^3-4a^2b-4ab^2+(-3a-3b)Z^2+(2a^2+8ab+2b^2)Z-c,\text{index}=3\right)}{a-b}}\right\}$$

2.1528 ODE No. 1528

$$y^{(3)}(x)\sin(x)+(2\cos(x)+1)y''(x)-\sin(x)y'(x)-\cos(x)=0$$

✓ **Mathematica** : cpu = 0.996563 (sec), leaf count = 72

$$\left\{\left\{y(x)\rightarrow \frac{\sin\left(\frac{x}{2}\right)\left(-2\cos\left(\frac{x}{2}\right)\sin^{-1}(\cos(x))+\sqrt{2}\left(c_2x\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)(c_2\log(2(\cos(x)+1))+2c_1)\right)\right)}{\cos(x)-1}+c_3\right\}\right\}$$

✓ **Maple** : cpu = 0.227 (sec), leaf count = 71

$$\left\{y(x)=\frac{c_1\ln\left(\frac{-\cos(x)+1}{\sin(x)}\right)(\sin^2(x))-c_1\ln(\sin(x))(\sin^2(x))+c_3(\sin^2(x))-x(\cos^2(x))+x+(\cos(x)-1)(c_1x+c_2+1)\sin(x)}{(\cos(x)-1)\sin(x)}\right\}$$

2.1529 ODE No. 1529

$$y^{(3)}(x)(x+\sin(x))+3(\cos(x)+1)y''(x)-3\sin(x)y'(x)-y(x)\cos(x)+\sin(x)=0$$

✓ **Mathematica** : cpu = 0.0941548 (sec), leaf count = 47

$$\left\{\left\{y(x)\rightarrow \frac{c_3x^2}{x+\sin(x)}-\frac{\cos(x)}{x+\sin(x)}+\frac{c_2x}{x+\sin(x)}+\frac{c_1}{x+\sin(x)}\right\}\right\}$$

✓ **Maple** : cpu = 0.106 (sec), leaf count = 25

$$\left\{y(x)=\frac{c_1x^2+c_2x+c_3-\cos(x)}{x+\sin(x)}\right\}$$

2.1530 ODE No. 1530

$$y'(x)\left(4\nu(\nu+1)\sin^2(x)+\cos(2x)\right)+2\nu(\nu+1)y(x)\sin(2x)+y^{(3)}(x)\sin^2(x)+3\sin(x)\cos(x)y''(x)=0$$

✓ **Mathematica** : cpu = 0.0876347 (sec), leaf count = 35

$$\left\{\left\{y(x)\rightarrow c_3P_\nu(\cos(x))Q_\nu(\cos(x))+c_1P_\nu(\cos(x))^2+c_2Q_\nu(\cos(x))^2\right\}\right\}$$

✓ **Maple** : cpu = 0.257 (sec), leaf count = 113

$$\left\{y(x)=c_1\text{hypergeom}\left(\left[-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{2}\right],\left[\frac{1}{2}\right],\frac{\cos(2x)}{2}+\frac{1}{2}\right)^2+c_2(\cos(2x)+1)\text{hypergeom}\left(\left[\frac{\nu}{2}+1,-\frac{\nu}{2}+\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{\cos(2x)}{2}\right)\right\}$$

2.1531 ODE No. 1531

$A(x)\Big(f(x)y''(x)+g(x)y'(x)+h(x)y(x)\Big)+f'(x)y''(x)+f(x)y^{(3)}(x)+g'(x)y'(x)+g(x)y''(x)+y(x)h'(x)+h(x)y'(x)=0$

✗ **Mathematica** : cpu = 0.0253404 (sec), leaf count = 0 , could not solve

`DSolve[y[x]*Derivative[1][h][x] + h[x]*Derivative[1][y][x] + Derivative[1][g][x]*Derivative[1][y][x], y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$\{y(x)=DESol\left(\left\{\left(\frac{d^3}{dx^3}-Y(x)\right)f(x)+\left(A(x)h(x)+\frac{d}{dx}h(x)\right)-Y(x)+\left(A(x)g(x)+\frac{d}{dx}g(x)+h(x)\right)\left(\frac{d}{dx}-Y(x)\right)+\left(A(x)f(x)+\frac{d}{dx}f(x)+h(x)\right)\right\}\right)$

2.1532 ODE No. 1532

$ny(x)+y^{(3)}(x)+xy'(x)=0$

✓ **Mathematica** : cpu = 0.0157829 (sec), leaf count = 103

$\left\{\left\{y(x)\rightarrow\frac{c_2x{}_1F_2\left(\frac{n}{3}+\frac{1}{3};\frac{2}{3},\frac{4}{3};-\frac{x^3}{9}\right)}{3^{2/3}}+c_1{}_1F_2\left(\frac{n}{3};\frac{1}{3},\frac{2}{3};-\frac{x^3}{9}\right)+\frac{c_3x^2{}_1F_2\left(\frac{n}{3}+\frac{2}{3};\frac{4}{3},\frac{5}{3};-\frac{x^3}{9}\right)}{3\sqrt[3]{3}}\right\}\right\}$

✓ **Maple** : cpu = 0.101 (sec), leaf count = 58

$\left\{y(x)=c_3x^2\text{hypergeom}\left(\left[\frac{n}{3}+\frac{2}{3}\right],\left[\frac{4}{3},\frac{5}{3}\right],-\frac{x^3}{9}\right)+c_2x\text{hypergeom}\left(\left[\frac{n}{3}+\frac{1}{3}\right],\left[\frac{2}{3},\frac{4}{3}\right],-\frac{x^3}{9}\right)+c_1\text{hypergeom}\left(\left[\frac{n}{3}\right],\left[\frac{1}{3},\frac{2}{3}\right],-\frac{x^3}{9}\right)\right\}$

2.1533 ODE No. 1533

$-ny(x)+y^{(3)}(x)-xy'(x)=0$

✓ **Mathematica** : cpu = 0.0158216 (sec), leaf count = 113

$\left\{\left\{y(x)\rightarrow\frac{\sqrt[3]{-1}c_2x{}_1F_2\left(\frac{n}{3}+\frac{1}{3};\frac{2}{3},\frac{4}{3};\frac{x^3}{9}\right)}{3^{2/3}}+c_1{}_1F_2\left(\frac{n}{3};\frac{1}{3},\frac{2}{3};\frac{x^3}{9}\right)+\frac{(-1)^{2/3}c_3x^2{}_1F_2\left(\frac{n}{3}+\frac{2}{3};\frac{4}{3},\frac{5}{3};\frac{x^3}{9}\right)}{3\sqrt[3]{3}}\right\}\right\}$

✓ **Maple** : cpu = 0.106 (sec), leaf count = 58

$\left\{y(x)=c_3x^2\text{hypergeom}\left(\left[\frac{n}{3}+\frac{2}{3}\right],\left[\frac{4}{3},\frac{5}{3}\right],\frac{x^3}{9}\right)+c_2x\text{hypergeom}\left(\left[\frac{n}{3}+\frac{1}{3}\right],\left[\frac{2}{3},\frac{4}{3}\right],\frac{x^3}{9}\right)+c_1\text{hypergeom}\left(\left[\frac{n}{3}\right],\left[\frac{1}{3},\frac{2}{3}\right],\frac{x^3}{9}\right)\right\}$

2.1534 ODE No. 1534

$y^{(4)}(x)=0$

✓ **Mathematica** : cpu = 0.003467 (sec), leaf count = 24

$\{\{y(x)\rightarrow c_4x^3+c_3x^2+c_2x+c_1\}\}$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 21

$\left\{y(x)=\frac{1}{6}c_1x^3+\frac{1}{2}c_2x^2+c_3x+c_4\right\}$

2.1535 ODE No. 1535

$$-f(x) + y^{(4)}(x) + 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.406166 (sec), leaf count = 223

$$\left\{ \left\{ y(x) \rightarrow e^{-x} \left(\cos(x) \int_1^x \frac{1}{8} e^{K[1]} f(K[1]) (\cos(K[1]) - \sin(K[1])) (\cos^2(K[1]) + \sin^2(K[1])) dK[1] + \sin(x) \int_1^x \frac{1}{8} e^{K[2]} f(K[2]) dK[2] \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 36

$$\left\{ y(x) = c_1 \cos(x) e^x + c_2 e^x \sin(x) + c_3 \cos(x) e^{-x} + c_4 e^{-x} \sin(x) + \frac{f}{4} \right\}$$

2.1536 ODE No. 1536

$$\lambda y(x) + y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 0.0046415 (sec), leaf count = 76

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{(-1)^{3/4} \sqrt[4]{\lambda} x} + c_2 e^{-\frac{4}{\sqrt{-1}} \sqrt[4]{\lambda} x} + c_3 e^{(-1)^{3/4} \sqrt[4]{\lambda} x} + c_4 e^{\frac{4}{\sqrt{-1}} \sqrt[4]{\lambda} x} \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 50

$$\left\{ y(x) = c_1 e^{-i(-\lambda)^{\frac{1}{4}} x} + c_2 e^{i(-\lambda)^{\frac{1}{4}} x} + c_3 e^{-(-\lambda)^{\frac{1}{4}} x} + c_4 e^{(-\lambda)^{\frac{1}{4}} x} \right\}$$

2.1537 ODE No. 1537

$$-16e^{x^2} x^4 + y^{(4)}(x) - 12y''(x) + 12y(x) = 0$$

✓ **Mathematica** : cpu = 1.04673 (sec), leaf count = 1722

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{12} e^{-\left(\left(\sqrt{2(3-\sqrt{6})} - x \right) x \right) - \sqrt{2(3+\sqrt{6})} x - \sqrt{2(3-\sqrt{6})} x} \left(-2\sqrt{3+\sqrt{6}} e^{\sqrt{2(3+\sqrt{6})} x + 2\sqrt{2(3-\sqrt{6})} x} x^3 + 2\sqrt{3+\sqrt{6}} e^{\left(\sqrt{2(3-\sqrt{6})} - x \right) x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.16 (sec), leaf count = 67

$$\left\{ y(x) = c_1 e^{\sqrt{6-2\sqrt{6}} x} + c_2 e^{\sqrt{6+2\sqrt{6}} x} + c_3 e^{-\sqrt{6-2\sqrt{6}} x} + c_4 e^{-\sqrt{6+2\sqrt{6}} x} + e^{x^2} \right\}$$

2.1538 ODE No. 1538

$$a^4 y(x) + 2a^2 y''(x) - \cosh(ax) + y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 0.181197 (sec), leaf count = 66

$$\left\{ \left\{ y(x) \rightarrow \frac{\cos^2(ax) \cosh(ax) + \sin^2(ax) \cosh(ax)}{4a^4} + c_1 \cos(ax) + c_2 x \cos(ax) + c_3 \sin(ax) + c_4 x \sin(ax) \right\} \right\}$$

✓ **Maple** : cpu = 0.725 (sec), leaf count = 49

$$\left\{ y(x) = \frac{8((c_3 x + c_1) \cos(ax) + (c_4 x + c_2) \sin(ax)) a^4 + (e^{2ax} + 1) e^{-ax}}{8a^4} \right\}$$

2.1539 ODE No. 1539

$$a^4 \lambda y(x) + a^2(\lambda + 1)y''(x) + y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 0.0070489 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow c_1 \cos(a\sqrt{\lambda} x) + c_2 \sin(a\sqrt{\lambda} x) + c_3 \cos(ax) + c_4 \sin(ax) \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 35

$$\{y(x) = c_1 \sin(ax) + c_2 \cos(ax) + c_3 \sin(a\sqrt{\lambda} x) + c_4 \cos(a\sqrt{\lambda} x)\}$$

2.1540 ODE No. 1540

$$a(bx - 1)y''(x) + aby'(x) + \lambda y(x) + y^{(4)}(x) = 0$$

✗ **Mathematica** : cpu = 0.33144 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow (x) \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x) = DESol\left(\left\{ab\left(\frac{d}{dx}-Y(x)\right) + (bx - 1)a\left(\frac{d^2}{dx^2}-Y(x)\right) + \lambda_-Y(x) + \frac{d^4}{dx^4}-Y(x)\right\}, \{-Y(x)\}\right)\}$$

2.1541 ODE No. 1541

$$y''(x)(ax^2 + b\lambda + c) + y(x)(ax^2 + \beta\lambda + \gamma) + y^{(4)}(x) = 0$$

✗ **Mathematica** : cpu = 80.1946 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow (x) \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x) = DESol\left(\left\{(a x^2 + \beta \lambda + \gamma)_-Y(x) + (a x^2 + b \lambda + c)\left(\frac{d^2}{dx^2}-Y(x)\right) + \frac{d^4}{dx^4}-Y(x)\right\}, \{-Y(x)\}\right)\}$$

2.1542 ODE No. 1542

$$ay''(x)\wp(x; g2, g3) + by'(x)\wp'(x; g2, g3) + y(x)\left(c\left(6\wp(x; g2, g3)^2 - \frac{g2}{2}\right) + d\right) + y^{(4)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0254998 (sec), leaf count = 0 , could not solve

DSolve[(d + c*(-1/2*g2 + 6*WeierstrassP[x, {g2, g3}]^2))*y[x] + b*WeierstrassPPrime[x, {g2,

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol\left(\left\{a \operatorname{WeierstrassP}(x, g2, g3)\left(\frac{d^2}{dx^2}-Y(x)\right) + b\left(\frac{d}{dx}-Y(x)\right)\wp'(x; g2, g3) + \left(6c \operatorname{WeierstrassP}(x, g2, g3)^2 - \frac{g2}{2}\right)_-Y(x) + \frac{d^4}{dx^4}-Y(x)\right\}, \{-Y(x)\}\right)\right\}$$

2.1543 ODE No. 1543

$$-y''(x)(a + 12k^2\operatorname{sn}(z|x)^2) + y(x)(a\operatorname{sn}(z|x)^2 + \beta) + by'(x) + y^{(4)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0814704 (sec), leaf count = 0 , could not solve

DSolve[(beta + alpha*JacobiSN[z, x]^2)*y[x] + b*Derivative[1][y][x] - (a + 12*k^2*JacobiSN[z,

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x) = DESol\left(\left\{b\left(\frac{d}{dx}-Y(x)\right) + (a\operatorname{sn}(z|x)^2 + \beta)_-Y(x) + (-12k^2\operatorname{sn}(z|x)^2 - a)\left(\frac{d^2}{dx^2}-Y(x)\right) + \frac{d^4}{dx^4}-Y(x)\right\}, \{-Y(x)\}\right)\}$$

2.1544 ODE No. 1544

$$y(x)\big(3f''(x)+3f(x)^2\big)+10f'(x)y'(x)+10f(x)y''(x)+y^{(4)}(x)=0$$

✗ **Mathematica** : cpu = 0.0169331 (sec), leaf count = 0 , could not solve

DSolve[10*Derivative[1][f][x]*Derivative[1][y][x] + y[x]*(3*f[x]^2 + 3*Derivative[2][f][x])

✓ **Maple** : cpu = 0.017 (sec), leaf count = 41

$$\{y(x)=\sum_{a=1}^4-C_{-a}e^{x\text{RootOf}\big(-Z^4+10f\_Z^2+10df\_Z+3f^2+3ddf,index=-a\big)}\}$$

2.1545 ODE No. 1545

$$y^{(4)}(x)+2y^{(3)}(x)-3y''(x)-4y'(x)+4y(x)-32\sin(2x)+24\cos(2x)=0$$

✓ **Mathematica** : cpu = 0.679589 (sec), leaf count = 40

$$\left\{\left\{y(x)\rightarrow\sin(2x)+c_1e^{-2x}+c_2e^{-2x}x+c_3e^x+c_4e^xx\right\}\right\}$$

✓ **Maple** : cpu = 0.124 (sec), leaf count = 27

$$\{y(x)=(c_3x+c_1)e^x+(c_4x+c_2)e^{-2x}+\sin(2x)\}$$

2.1546 ODE No. 1546

$$a^4x^4y(x)+4a^3x^3y'(x)+6a^2x^2y''(x)+4axy^{(3)}(x)+y^{(4)}(x)=0$$

✓ **Mathematica** : cpu = 0.543225 (sec), leaf count = 300

$$\left\{\left\{y(x)\rightarrow\frac{2\left(\sqrt{6}-3\right)\sqrt{-\left(\left(\sqrt{6}-3\right)a\right)}c_3\exp\left(-\frac{ax^2}{2}-\sqrt{-\left(\left(\sqrt{6}-3\right)a\right)}x-\frac{\left(-3+\sqrt{3}+\sqrt{6}\right)ax}{\sqrt{-\left(\left(\sqrt{6}-3\right)a\right)}}\right)}{\left(-3-\sqrt{3}+\sqrt{6}\right)\left(-3+\sqrt{3}+\sqrt{6}\right)a}-\frac{2\left(\sqrt{6}-3\right)\sqrt{-\left(\left(\sqrt{6}-3\right)a\right)}c_4}{\left(3+\sqrt{3}+\sqrt{6}\right)a}\right\}\right\}$$

✓ **Maple** : cpu = 0.063 (sec), leaf count = 73

$$\left\{y(x)=\left(c_1e^{-\sqrt{-(\sqrt{6}-3)a}x}+c_2e^{\sqrt{-(\sqrt{6}-3)a}x}+c_3e^{-\sqrt{(3+\sqrt{6})a}x}+c_4e^{\sqrt{(3+\sqrt{6})a}x}\right)e^{-\frac{ax^2}{2}}\right\}$$

2.1547 ODE No. 1547

$$3y(x)\big(2g(x)f'(x)+5f(x)g'(x)+6f(x)^2g(x)+g''(x)+3g(x)^2\big)+y''(x)\big(4f'(x)+11f(x)^2+10g(x)\big)+y'(x)\big(f''(x)+7f(x)f'(x)+g''(x)+7g(x)g'(x)\big)=0$$

✗ **Mathematica** : cpu = 0.27168 (sec), leaf count = 0 , could not solve

DSolve[Derivative[1][y][x]*(6*f[x]^3 + 30*f[x]*g[x] + 7*f[x]*Derivative[1][f][x] + 10*Derivative[2][f][x])

✓ **Maple** : cpu = 0.02 (sec), leaf count = 87

$$\{y(x)=\sum_{a=1}^4-C_{-a}e^{x\text{RootOf}\big(-Z^4+6\_Z^3f+18f^2g+(11f^2+4df+10g)\_Z^2+6dfg+15dgf+9g^2+(6f^3+7dff+30fg+ddf+10dg)\_Z+3ddg,index=-a\big)}\}$$

2.1548 ODE No. 1548

$$4y^{(4)}(x)-12y^{(3)}(x)+11y''(x)-3y'(x)-4\cos(x)=0$$

✓ **Mathematica** : cpu = 0.0811143 (sec), leaf count = 50

$$\left\{\left\{y(x)\rightarrow\frac{18\sin(x)}{65}-\frac{14\cos(x)}{65}+2c_1e^{x/2}+\frac{2}{3}c_2e^{3x/2}+c_3e^x+c_4\right\}\right\}$$

✓ **Maple** : cpu = 0.086 (sec), leaf count = 32

$$\left\{y(x)=c_1e^x+2c_2e^{\frac{x}{2}}+\frac{2c_3e^{\frac{3x}{2}}}{3}+c_4-\frac{14\cos(x)}{65}+\frac{18\sin(x)}{65}\right\}$$

2.1549 ODE No. 1549

$$xy^{(4)}(x)+5y^{(3)}(x)-24=0$$

✓ **Mathematica** : cpu = 0.0320473 (sec), leaf count = 34

$$\left\{\left\{y(x)\rightarrow\frac{4x^3}{5}+c_4x^2-\frac{c_1}{24x^2}+c_3x+c_2\right\}\right\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 26

$$\left\{y(x)=\frac{c_2x^2}{2}+\frac{4x^3}{5}+c_3x+c_4-\frac{c_1}{24x^2}\right\}$$

2.1550 ODE No. 1550

$$12x^3y''(x)-\left(6x^2+1\right)y^{(3)}(x)-\left(9x^2-7\right)x^2y'(x)+2\left(x^2-3\right)x^3y(x)+xy^{(4)}(x)=0$$

✓ **Mathematica** : cpu = 3.30725 (sec), leaf count = 270

$$\left(\left(\left\{y(x)\rightarrow c_3e^{\frac{x^2}{2}}\int_1^xe^{\frac{K[1]^2}{2}}\left(\int\frac{\exp\left(\frac{1}{4}\sqrt{5}K[1]^2+\frac{1}{2}\left(-\frac{1}{2}K[1]^2-2\log(K[1])\right)\right)U\left(-\frac{-9+\sqrt{5}}{4\sqrt{5}},-\frac{1}{2},-\frac{1}{2}\sqrt{5}K[1]^2\right)}{ \sqrt{K[1]}\sqrt[4]{K[1]^2}}dK[1]\right)K[1]}{ \sqrt[4]{2}}dK[1]+c_4e^{\frac{x^2}{2}}\int_1^xe^{\frac{K[2]^2}{2}}\right.\right.\right.$$

✓ **Maple** : cpu = 2.871 (sec), leaf count = 157

$$\left\{y(x)=-c_3\left(\int\frac{\text{WhittakerM}\left(\frac{9\sqrt{5}}{20},\frac{3}{4},\frac{\sqrt{5}x^2}{2}\right)e^{-\frac{x^2}{4}}}{x^{\frac{3}{2}}}dx\right)e^{x^2}+c_3\left(\int\frac{\text{WhittakerM}\left(\frac{9\sqrt{5}}{20},\frac{3}{4},\frac{\sqrt{5}x^2}{2}\right)e^{\frac{x^2}{4}}}{x^{\frac{3}{2}}}dx\right)e^{\frac{x^2}{2}}-c_4\left(\int\frac{\text{WhittakerM}\left(\frac{9\sqrt{5}}{20},\frac{3}{4},\frac{\sqrt{5}x^2}{2}\right)e^{-\frac{x^2}{4}}}{x^{\frac{3}{2}}}dx\right)e^{x^2}+c_4\left(\int\frac{\text{WhittakerM}\left(\frac{9\sqrt{5}}{20},\frac{3}{4},\frac{\sqrt{5}x^2}{2}\right)e^{\frac{x^2}{4}}}{x^{\frac{3}{2}}}dx\right)e^{\frac{x^2}{2}}\right\}$$

2.1551 ODE No. 1551

$$-2\left(\nu^2x^2+6\right)y''(x)+\nu^2\left(\nu^2x^2+4\right)y(x)+x^2y^{(4)}(x)=0$$

✓ **Mathematica** : cpu = 0.366973 (sec), leaf count = 110

$$\left\{\left\{y(x)\rightarrow\frac{c_3(1-x)e^{-\nu x}\left(\nu^2x^2+\nu^2x+\nu^2+6\nu x+6\nu+15\right)}{x}+\frac{c_4(1-x)e^{\nu x}\left(\nu^2x^2+\nu^2x+\nu^2-6\nu x-6\nu+15\right)}{x}+\frac{c_1e^{-\nu x}}{x}+\frac{c_2e^{\nu x}}{x}\right\}\right\}$$

✓ **Maple** : cpu = 0.231 (sec), leaf count = 62

$$\left\{y(x)=\frac{\left(c_3\nu^2x^3-6c_3\nu x^2+15c_3x+c_1\right)e^{\nu x}+\left(c_4\nu^2x^3+6c_4\nu x^2+15c_4x+c_2\right)e^{-\nu x}}{x}\right\}$$

2.1552 ODE No. 1552

$$ay(x) - bx^2 + x^2y^{(4)}(x) + 2xy^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 300.003 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.08 (sec), leaf count = 89

$$\left\{y(x) = c_1\sqrt{x} \operatorname{BesselJ}\left(1, 2(-a)^{\frac{1}{4}}\sqrt{x}\right) + c_2\sqrt{x} \operatorname{BesselY}\left(1, 2(-a)^{\frac{1}{4}}\sqrt{x}\right) + c_3\sqrt{x} \operatorname{BesselJ}\left(1, 2\sqrt{-\sqrt{-a}}\sqrt{x}\right) + c_4\sqrt{x} \operatorname{BesselY}\left(1, 2\sqrt{-\sqrt{-a}}\sqrt{x}\right)\right\}$$

2.1553 ODE No. 1553

$$x^2y^{(4)}(x) + 4xy^{(3)}(x) + 2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0092985 (sec), leaf count = 29

$$\{y(x) \rightarrow c_2(-x) + c_4x + c_2x \log(x) - c_1 \log(x) + c_3\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 17

$$\{y(x) = c_3x + c_1 + (c_4x + c_2) \ln(x)\}$$

2.1554 ODE No. 1554

$$x^2y^{(4)}(x) + 6xy^{(3)}(x) + 6y''(x) = 0$$

✓ **Mathematica** : cpu = 0.009573 (sec), leaf count = 29

$$\left\{\left\{y(x) \rightarrow c_4x + \frac{1}{2}\left(\frac{c_1}{x} - 2c_2 \log(x)\right) + c_3\right\}\right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 18

$$\left\{y(x) = c_2 \ln(x) + c_4x + c_1 + \frac{c_3}{x}\right\}$$

2.1555 ODE No. 1555

$$\lambda^2(-y(x)) + x^2y^{(4)}(x) + 6xy^{(3)}(x) + 6y''(x) = 0$$

✓ **Mathematica** : cpu = 0.052573 (sec), leaf count = 156

$$\left\{\left\{y(x) \rightarrow c_4G_{0,4}^{2,0}\left(\frac{\lambda^2x^2}{16}\middle|\ -\frac{1}{2}, \frac{1}{2}, 0, 0\right) + c_2G_{0,4}^{2,0}\left(\frac{\lambda^2x^2}{16}\middle|\ 0, 0, -\frac{1}{2}, \frac{1}{2}\right) + \frac{c_1\left(I_1\left(2\sqrt{\lambda}\sqrt{x}\right) + I_1\left(2\sqrt{\lambda}\sqrt{x}\right)\right)}{2\sqrt{\lambda}\sqrt{x}} - \frac{ic_3\left(I_1\left(2\sqrt{\lambda}\sqrt{x}\right)\right)}{4\sqrt{\lambda}\sqrt{x}}\right\}\right\}$$

✓ **Maple** : cpu = 0.168 (sec), leaf count = 61

$$\left\{y(x) = \frac{c_1 \operatorname{BesselJ}\left(1, 2\sqrt{\lambda}\sqrt{x}\right) + c_2 \operatorname{BesselY}\left(1, 2\sqrt{\lambda}\sqrt{x}\right) + c_3 \operatorname{BesselJ}\left(1, 2\sqrt{-\lambda}\sqrt{x}\right) + c_4 \operatorname{BesselY}\left(1, 2\sqrt{-\lambda}\sqrt{x}\right)}{\sqrt{x}}\right\}$$

2.1556 ODE No. 1556

$$x^2y^{(4)}(x) + 8xy^{(3)}(x) + 12y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0107328 (sec), leaf count = 30

$$\left\{\left\{y(x) \rightarrow \frac{1}{6}\left(\frac{c_1}{x^2} + \frac{3c_2}{x}\right) + c_4x + c_3\right\}\right\}$$

✓ **Maple** : cpu = 0.015 (sec), leaf count = 19

$$\left\{y(x) = c_4x + c_1 + \frac{c_2}{x} + \frac{c_3}{x^2}\right\}$$

2.1557 ODE No. 1557

$$\lambda^2(-y(x)) + x^2y^{(4)}(x) + 8xy^{(3)}(x) + 12y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0572299 (sec), leaf count = 146

$$\left\{ \left\{ y(x) \rightarrow c_4 G_{0,4}^{2,0} \left(\frac{\lambda^2 x^2}{16} \middle| -1, 0, -\frac{1}{2}, \frac{1}{2} \right) + c_2 G_{0,4}^{2,0} \left(\frac{\lambda^2 x^2}{16} \middle| -\frac{1}{2}, \frac{1}{2}, -1, 0 \right) - \frac{3ic_1 \left(I_2(2\sqrt{\lambda} \sqrt{x}) - I_2(2\sqrt{\lambda} \sqrt{x}) \right)}{4\lambda x} - \frac{c_3 \left(I_2(2\sqrt{\lambda} \sqrt{x}) \right)}{4\lambda x} \right\} \right\}$$

✓ **Maple** : cpu = 0.112 (sec), leaf count = 61

$$\left\{ y(x) = \frac{c_1 \text{BesselJ}\left(2, 2\sqrt{\lambda} \sqrt{x}\right) + c_2 \text{BesselY}\left(2, 2\sqrt{\lambda} \sqrt{x}\right) + c_3 \text{BesselJ}\left(2, 2\sqrt{-\lambda} \sqrt{x}\right) + c_4 \text{BesselY}\left(2, 2\sqrt{-\lambda} \sqrt{x}\right)}{x} \right\}$$

2.1558 ODE No. 1558

$$-\frac{1}{16}b^4y(x) + x(2n - 2v + 4)y^{(3)}(x) + (n - v + 1)(n - v + 2)y''(x) + x^2y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 0.123714 (sec), leaf count = 319

$$\left\{ \left\{ y(x) \rightarrow c_4 i^{-n+v+1} 2^{3n-3v-3} b^{2(-n+v+1)+n-v-2} x^{\frac{1}{2}(n-v-2)-n+v+1} \Gamma(-n+v+2) \left(I_{v-n}(b\sqrt{x}) - I_{v-n}(b\sqrt{x}) \right) + c_3 i^{v-n} 2^{3n-3v-1} b^{2(v-n)} x^{\frac{1}{2}(n-v-2)-n+v+1} \Gamma(-n+v+2) \left(I_{v-n}(b\sqrt{x}) - I_{v-n}(b\sqrt{x}) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.167 (sec), leaf count = 67

$$\left\{ y(x) = \left(c_1 \text{BesselI}\left(n - v, b\sqrt{x}\right) + c_2 \text{BesselJ}\left(n - v, b\sqrt{x}\right) + c_3 \text{BesselK}\left(n - v, b\sqrt{x}\right) + c_4 \text{BesselY}\left(n - v, b\sqrt{x}\right) \right) x^{-\frac{n}{2} + \frac{v}{2}} \right\}$$

2.1559 ODE No. 1559

$$a^4(-x^3)y(x) + x^3y^{(4)}(x) + 2x^2y^{(3)}(x) - xy''(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.224862 (sec), leaf count = 100

$$\left\{ \left\{ y(x) \rightarrow c_4 G_{0,4}^{2,0} \left(\frac{a^4 x^4}{256} \middle| 0, 0, \frac{1}{2}, \frac{1}{2} \right) + c_2 G_{0,4}^{2,0} \left(\frac{a^4 x^4}{256} \middle| \frac{1}{2}, \frac{1}{2}, 0, 0 \right) + \frac{1}{8} ic_1 (I_0(ax) - J_0(ax)) + \frac{1}{2} c_3 (J_0(ax) + I_0(ax)) \right\} \right\}$$

✓ **Maple** : cpu = 0.161 (sec), leaf count = 33

$$\{y(x) = c_1 \text{BesselI}(0, ax) + c_2 \text{BesselJ}(0, ax) + c_3 \text{BesselK}(0, ax) + c_4 \text{BesselY}(0, ax)\}$$

2.1560 ODE No. 1560

$$x^3y^{(4)}(x) + 6x^2y^{(3)}(x) + 6xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0079386 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow c_4 x + \frac{1}{2} \left(\frac{c_1}{x} - 2c_2 \log(x) \right) + c_3 \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 18

$$\left\{ y(x) = c_2 \ln(x) + c_4 x + c_1 + \frac{c_3}{x} \right\}$$

2.1561 ODE No. 1561

$$y(x)\left(ax^4 + (n-2)n(n+1)(n+3)\right) - 2n(n+1)x^2y''(x) + 4n(n+1)xy'(x) + x^4y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 3.21729 (sec), leaf count = 400

$$\left\{\left\{y(x) \rightarrow c_1\left(-2^{n-\frac{5}{2}}\right)\sqrt{x}a^{\frac{2-n}{4}+\frac{1}{4}\left(n-\frac{3}{2}\right)}\Gamma\left(\frac{3}{2}-n\right)\left(\cos\left(\frac{3}{4}\pi\left(\frac{3}{2}-n\right)\right)\text{ber}_{-n-\frac{1}{2}}\left(\sqrt[4]{a}x\right)+\sin\left(\frac{3}{4}\pi\left(\frac{3}{2}-n\right)\right)\text{bei}_{-n-\frac{1}{2}}\left(\sqrt[4]{a}x\right)\right)+c_2\right\}\right.$$

✓ **Maple** : cpu = 0.305 (sec), leaf count = 69

$$\left\{y(x)=\left(c_1\text{BesselJ}\left(n+\frac{1}{2},(-a)^{\frac{1}{4}}x\right)+c_2\text{BesselY}\left(n+\frac{1}{2},(-a)^{\frac{1}{4}}x\right)+c_3\text{BesselJ}\left(n+\frac{1}{2},\sqrt{-\sqrt{-a}}\ x\right)+c_4\text{BesselY}\left(n+\frac{1}{2},\sqrt{-\sqrt{-a}}\ x\right)\right)\right.$$

2.1562 ODE No. 1562

$$-\left(4n^2-1\right)x^2y''(x)+\left(4n^2-1\right)xy'(x)+x^4y^{(4)}(x)-4x^4y(x)+4x^3y^{(3)}(x)=0$$

✓ **Mathematica** : cpu = 0.920553 (sec), leaf count = 140

$$\left\{\left\{y(x) \rightarrow c_1{}_0F_3\left(; \frac{1}{2}, 1-\frac{n}{2}, \frac{n}{2}+1; \frac{x^4}{64}\right)+\frac{1}{8}ic_2x^2{}_0F_3\left(; \frac{3}{2}, \frac{3}{2}-\frac{n}{2}, \frac{n}{2}+\frac{3}{2}; \frac{x^4}{64}\right)+c_3\left(\frac{i}{2}\right)^{-n}\Gamma(1-n)^2\left(\text{ber}_{-n}(x)^2+\text{bei}_{-n}(x)^2\right)\right\}\right.$$

✓ **Maple** : cpu = 0.359 (sec), leaf count = 77

$$\left\{y(x)=\left(c_1\text{BesselJ}\left(n,\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{2}\ x\right)+c_3\text{BesselY}\left(n,\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{2}\ x\right)\right)\text{BesselJ}\left(n,\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{2}\ x\right)+\left(c_2\text{BesselJ}\left(n,\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{2}\ x\right)+c_4\text{BesselY}\left(n,\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{2}\ x\right)\right)\text{BesselY}\left(n,\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{2}\ x\right)\right\}$$

2.1563 ODE No. 1563

$$\left(4n^2-4x^4-1\right)y(x)-\left(4n^2-1\right)x^2y''(x)-\left(4n^2-1\right)xy'(x)+x^4y^{(4)}(x)+4x^3y^{(3)}(x)=0$$

✓ **Mathematica** : cpu = 1.6412 (sec), leaf count = 232

$$\left\{\left\{y(x) \rightarrow \frac{\sqrt[4]{-1}c_2x{}_0F_3\left(; \frac{3}{2}, 1-\frac{n}{2}, \frac{n}{2}+1; \frac{x^4}{64}\right)}{2\sqrt{2}}-\frac{2(-1)^{3/4}\sqrt{2}c_1{}_0F_3\left(; \frac{1}{2}, \frac{1}{2}-\frac{n}{2}, \frac{n}{2}+\frac{1}{2}; \frac{x^4}{64}\right)}{x}+c_3(-1)^{\frac{1}{4}(1-2n)}2^{2n+\frac{1}{2}(2n-1)-1}x^{1-2n}\right\}\right.$$

✓ **Maple** : cpu = 0.251 (sec), leaf count = 87

$$\left\{y(x)=\frac{c_4\text{hypergeom}\left(\left[\right],\left[\frac{1}{2},\frac{n}{2}+\frac{1}{2},-\frac{n}{2}+\frac{1}{2}\right],\frac{x^4}{64}\right)+\left(c_2\text{bei}_{-n}(x)^2+c_2\text{ber}_{-n}(x)^2+c_3\text{hypergeom}\left(\left[\right],\left[\frac{3}{2},-\frac{n}{2}+1,\frac{n}{2}+1\right],\frac{x^4}{64}\right)\right)}{x}\right\}$$

2.1564 ODE No. 1564

$$\left(-12n^2-4x^4+3\right)y(x)-\left(4n^2+3\right)x^2y''(x)+\left(12n^2-3\right)xy'(x)+x^4y^{(4)}(x)+4x^3y^{(3)}(x)=0$$

✓ **Mathematica** : cpu = 1.13023 (sec), leaf count = 230

$$\left\{\left\{y(x) \rightarrow \frac{\sqrt[4]{-1}c_1x{}_0F_3\left(; \frac{1}{2}, \frac{3}{2}-\frac{n}{2}, \frac{n}{2}+\frac{3}{2}; \frac{x^4}{64}\right)}{2\sqrt{2}}+c_3(-1)^{\frac{1}{4}(-2n-1)}2^{2n+\frac{1}{2}(2n+1)+1}x^{-2n-1}{}_0F_3\left(; 1-n, \frac{1}{2}-\frac{n}{2}, -\frac{n}{2}; \frac{x^4}{64}\right)+c_4(-1)^{\frac{1}{4}(-2n-1)}2^{2n+\frac{1}{2}(2n+1)+1}x^{-2n-1}{}_0F_3\left(; 1-n, \frac{1}{2}-\frac{n}{2}, -\frac{n}{2}; \frac{x^4}{64}\right)\right\}\right.$$

✓ **Maple** : cpu = 0.24 (sec), leaf count = 88

$$\left\{y(x)=\frac{c_3x^4\text{hypergeom}\left(\left[\right],\left[\frac{3}{2},\frac{n}{2}+2,-\frac{n}{2}+2\right],\frac{x^4}{64}\right)+c_4x^2\text{hypergeom}\left(\left[\right],\left[\frac{1}{2},-\frac{n}{2}+\frac{3}{2},\frac{n}{2}+\frac{3}{2}\right],\frac{x^4}{64}\right)+c_2\text{bei}_{-n}(x)^2+c_2\text{ber}_{-n}(x)^2}{x}\right\}$$

2.1565 ODE No. 1565

$(x(-\rho^2 - \sigma^2 + 1) + 16x^3)y'(x) + y(x)(\rho^2\sigma^2 + 8x^2) + (x^2(-\rho^2 - \sigma^2 + 7) + 4x^4)y''(x) + x^4y^{(4)}(x) + 6x^3y^{(3)}(x) = 0$

✓ **Mathematica** : cpu = 0.4196 (sec), leaf count = 242

$\left\{ \left\{ y(x) \rightarrow c_1 x^{-\rho} {}_2F_3\left(\frac{1}{2} - \frac{\rho}{2}, 1 - \frac{\rho}{2}; 1 - \rho, -\frac{\rho}{2} - \frac{\sigma}{2} + 1, -\frac{\rho}{2} + \frac{\sigma}{2} + 1; -x^2\right) + c_3 x^{-\sigma} {}_2F_3\left(\frac{1}{2} - \frac{\sigma}{2}, 1 - \frac{\sigma}{2}; 1 - \sigma, -\frac{\rho}{2} - \frac{\sigma}{2} + 1, \frac{\rho}{2} - \frac{\sigma}{2} + 1; -x^2\right) \right\} \right\}$

✓ **Maple** : cpu = 0.353 (sec), leaf count = 71

$\{y(x) = \left(c_1 \operatorname{BesselJ}\left(\frac{\rho}{2} - \frac{\sigma}{2}, x\right) + c_2 \operatorname{BesselY}\left(\frac{\rho}{2} - \frac{\sigma}{2}, x\right)\right) \operatorname{BesselJ}\left(\frac{\rho}{2} + \frac{\sigma}{2}, x\right) + \left(c_3 \operatorname{BesselJ}\left(\frac{\rho}{2} - \frac{\sigma}{2}, x\right) + c_4 \operatorname{BesselY}\left(\frac{\rho}{2} - \frac{\sigma}{2}, x\right)\right) \operatorname{BesselY}\left(\frac{\rho}{2} + \frac{\sigma}{2}, x\right)\}$

2.1566 ODE No. 1566

$(x(-2\mu^2 - 2\nu^2 + 1) + 16x^3)y'(x) + y(x)\left((\mu^2 - \nu^2)^2 + 8x^2\right) + (x^2(-2\mu^2 - 2\nu^2 + 7) + 4x^4)y''(x) + x^4y^{(4)}(x) + 6x^3y^{(3)}(x) = 0$

✓ **Mathematica** : cpu = 0.495727 (sec), leaf count = 238

$\left\{ \left\{ y(x) \rightarrow c_1 x^{-\mu-\nu} {}_2F_3\left(-\frac{\mu}{2} - \frac{\nu}{2} + \frac{1}{2}, -\frac{\mu}{2} - \frac{\nu}{2} + 1; 1 - \mu, 1 - \nu, -\mu - \nu + 1; -x^2\right) + c_2 x^{\mu-\nu} {}_2F_3\left(\frac{\mu}{2} - \frac{\nu}{2} + \frac{1}{2}, \frac{\mu}{2} - \frac{\nu}{2} + 1; \mu + \nu, \mu - \nu + 1, \mu + \nu + 1; -x^2\right) \right\} \right\}$

✓ **Maple** : cpu = 0.309 (sec), leaf count = 35

$\{y(x) = (c_1 \operatorname{BesselJ}(\mu, x) + c_2 \operatorname{BesselY}(\mu, x)) \operatorname{BesselJ}(\nu, x) + (c_3 \operatorname{BesselJ}(\mu, x) + c_4 \operatorname{BesselY}(\mu, x)) \operatorname{BesselY}(\nu, x)\}$

2.1567 ODE No. 1567

$x^4y^{(4)}(x) + 8x^3y^{(3)}(x) + 12x^2y''(x) = 0$

✓ **Mathematica** : cpu = 0.0084236 (sec), leaf count = 30

$\left\{ \left\{ y(x) \rightarrow \frac{1}{6} \left(\frac{c_1}{x^2} + \frac{3c_2}{x} \right) + c_4 x + c_3 \right\} \right\}$

✓ **Maple** : cpu = 0.015 (sec), leaf count = 19

$\left\{ y(x) = c_4 x + c_1 + \frac{c_2}{x} + \frac{c_3}{x^2} \right\}$

2.1568 ODE No. 1568

$ay(x) + x^4y^{(4)}(x) + 8x^3y^{(3)}(x) + 12x^2y''(x) = 0$

✓ **Mathematica** : cpu = 0.0100379 (sec), leaf count = 122

$\left\{ \left\{ y(x) \rightarrow c_1 x^{\frac{1}{2} \left(-\sqrt{5-4\sqrt{1-a}} - 1 \right)} + c_2 x^{\frac{1}{2} \left(\sqrt{5-4\sqrt{1-a}} - 1 \right)} + c_3 x^{\frac{1}{2} \left(-\sqrt{4\sqrt{1-a}+5} - 1 \right)} + c_4 x^{\frac{1}{2} \left(\sqrt{4\sqrt{1-a}+5} - 1 \right)} \right\} \right\}$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 89

$\left\{ y(x) = c_1 x^{-\frac{1}{2} - \frac{\sqrt{5-4\sqrt{-a+1}}}{2}} + c_2 x^{-\frac{1}{2} + \frac{\sqrt{5-4\sqrt{-a+1}}}{2}} + c_3 x^{-\frac{1}{2} - \frac{\sqrt{5+4\sqrt{-a+1}}}{2}} + c_4 x^{-\frac{1}{2} + \frac{\sqrt{5+4\sqrt{-a+1}}}{2}} \right\}$

2.1569 ODE No. 1569

$xy'(x)\big((2a-1)C0+4b^2B0c^2x^{2c}\big)+(6-4a)x^3y^{(3)}(x)+x^2y''(x)\big(A0+4b^2c^2x^{2c}\big)+y(x)\big(4b^2c^2D0x^{2c}+E0\big)+x^4y^{(4)}(x)=0$

✗ **Mathematica** : cpu = 300.004 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.423 (sec), leaf count = 63

$\{y(x)=\left((c_2\operatorname{BesselJ}\left(\mu,b\,x^c\right)+c_3\operatorname{BesselY}\left(\mu,b\,x^c\right)\right)\operatorname{BesselJ}\left(\nu,b\,x^c\right)+\left(c_1\operatorname{BesselJ}\left(\mu,b\,x^c\right)+c_4\operatorname{BesselY}\left(\mu,b\,x^c\right)\right)\operatorname{BesselY}\left(\nu,b\,x^c\right)\right\}$

2.1570 ODE No. 1570

$y(x)\big((a^2-c^2\nu^2)\big(a^2+4ac-c^2\nu^2+4c^2\big)-b^4c^4x^{4c}\big)+x^2\big(2a^2+4(a+c-1)^2+4(a-1)(c-1)-2c^2\nu^2-1\big)y''(x)+x(2a+2c-1)y'(x)=0$

✓ **Mathematica** : cpu = 0.110918 (sec), leaf count = 470

$\left\{\left\{y(x)\rightarrow c_1\Gamma(1-\nu)(-1)^{\frac{a-c\nu}{4c}}2^{-\frac{2(a-c\nu)}{c}-\nu-1}b^{\frac{a-c\nu}{c}+\nu}\left(x^{4c}\right)^{\frac{a-c\nu}{4c}+\frac{\nu}{4}}\left(J_{-\nu}\left(b\sqrt[4]{x^{4c}}\right)+I_{-\nu}\left(b\sqrt[4]{x^{4c}}\right)\right)+c_2\Gamma(2-\nu)(-1)^{\frac{a-c\nu+2c}{4c}}2^{-\frac{2(a-c\nu+2c)}{c}-\nu-1}b^{\frac{a-c\nu}{c}+\nu}\left(x^{4c}\right)^{\frac{a-c\nu}{4c}+\frac{\nu}{4}}\left(J_{-\nu}\left(b\sqrt[4]{x^{4c}}\right)+I_{-\nu}\left(b\sqrt[4]{x^{4c}}\right)\right)\right\}$

✓ **Maple** : cpu = 0.107 (sec), leaf count = 49

$\{y(x)=(c_1\operatorname{BesselJ}(\nu,b\,x^c)+c_2\operatorname{BesselY}(\nu,b\,x^c)+c_3\operatorname{BesselJ}(\nu,ib\,x^c)+c_4\operatorname{BesselY}(\nu,ib\,x^c))x^a\}$

2.1571 ODE No. 1571

$-\frac{1}{16}b^4x^{2/v}y(x)+v^4x^4y^{(4)}(x)+v^3(4v-2)x^3y^{(3)}(x)+(\nu-1)v^2(2\nu-1)x^2y''(x)=0$

✓ **Mathematica** : cpu = 0.0695408 (sec), leaf count = 390

$\left\{\left\{y(x)\rightarrow c_1{}_0F_3\left(;1-\frac{v}{2},1-\frac{v}{2\nu},-\frac{v}{2\nu}-\frac{v}{2}+1;\frac{b^4v^4x^{2/v}}{256v^4}\right)+c_2\left(\frac{i}{16}\right)^v v^{2v}b^{2v}v^{-2v}\left(x^{2/v}\right)^{v/2}{}_0F_3\left(;\frac{v}{2}+1,1-\frac{v}{2\nu},-\frac{v}{2\nu}+\frac{v}{2}+1;\frac{b^4v^4x^{2/v}}{256v^4}\right)\right\}\right\}$

✓ **Maple** : cpu = 0.312 (sec), leaf count = 143

$\left\{y(x)=\left(c_1\operatorname{BesselJ}\left(\frac{1}{\lfloor\frac{1}{\nu}\rfloor},\frac{\sqrt{\frac{b^2}{\nu^2}}x^{\frac{\lfloor\frac{1}{\nu}\rfloor}{2}}\right)+c_2\operatorname{BesselY}\left(\frac{1}{\lfloor\frac{1}{\nu}\rfloor},\frac{\sqrt{\frac{b^2}{\nu^2}}x^{\frac{\lfloor\frac{1}{\nu}\rfloor}{2}}\right)+c_3\operatorname{BesselJ}\left(\frac{1}{\lfloor\frac{1}{\nu}\rfloor},\frac{\sqrt{-\frac{b^2}{\nu^2}}x^{\frac{\lfloor\frac{1}{\nu}\rfloor}{2}}\right)+c_4\operatorname{BesselY}\left(\frac{1}{\lfloor\frac{1}{\nu}\rfloor},\frac{\sqrt{-\frac{b^2}{\nu^2}}x^{\frac{\lfloor\frac{1}{\nu}\rfloor}{2}}\right)\right)\right\}$

2.1572 ODE No. 1572

$\big(-2\big(x^2-1\big)\big(\mu\big(\mu+1\big)+\nu\big(\nu+1\big)\big)+24x^3-8\big)y''(x)-6x\big(\mu\big(\mu+1\big)+\nu\big(\nu+1\big)-2\big)y'(x)+\big(\mu\big(\mu+1\big)-\nu\big(\nu+1\big)\big)^2-2\mu\big(\mu+1\big)-2\nu\big(\nu+1\big)\big)y(x)=0$

✗ **Mathematica** : cpu = 87.5806 (sec), leaf count = 0 , DifferentialRoot result

$\{\{y(x)\rightarrow (x)\}\}$

✓ **Maple** : cpu = 0.396 (sec), leaf count = 35

$\{y(x)=\left(c_1\operatorname{LegendreP}\left(\mu,x\right)+c_2\operatorname{LegendreQ}\left(\mu,x\right)\right)\operatorname{LegendreP}\left(\nu,x\right)+\left(c_3\operatorname{LegendreP}\left(\mu,x\right)+c_4\operatorname{LegendreQ}\left(\mu,x\right)\right)\operatorname{LegendreQ}\left(\nu,x\right)\}$

2.1573 ODE No. 1573

$$-\frac{1}{x^5} + (2x + e^x)y^{(4)}(x) + 4(e^x + 2)y^{(3)}(x) + 6e^xy''(x) + 4e^xy'(x) + e^xy(x) = 0$$

✓ **Mathematica** : cpu = 0.173271 (sec), leaf count = 77

$$\left\{ \left\{ y(x) \rightarrow \frac{c_4x^3}{2x + e^x} + \frac{c_3x^2}{2x + e^x} + \frac{1}{24(2x + e^x)x} + \frac{c_2x}{2x + e^x} + \frac{c_1}{2x + e^x} \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 41

$$\left\{ y(x) = \frac{24c_1x^4 + 24c_2x^3 + 24c_3x^2 + 24c_4x + 1}{24(2x + e^x)x} \right\}$$

2.1574 ODE No. 1574

$$y(x)\big(a^4\sin^4(x)-3\big)+y^{(4)}(x)\sin^4(x)+2y^{(3)}(x)\sin^3(x)\cos(x)+\big(\sin^2(x)-3\big)\sin^2(x)y''(x)+\big(2\sin^2(x)+3\big)\sin(x)\cos(x)y'(x)+y(x)\sin^6(x)=0$$

✓ **Mathematica** : cpu = 0.20003 (sec), leaf count = 270

$$\left\{ \left\{ y(x) \rightarrow c_1\sin(x) {}_2F_1\left(\frac{1}{4}\left(3-\sqrt{5-4\sqrt{1-a^4}}\right),\frac{1}{4}\left(\sqrt{5-4\sqrt{1-a^4}}+3\right);\frac{1}{2};\cos^2(x)\right) + c_3\sin(x)\cos(x) {}_2F_1\left(\frac{1}{4}\left(5-\sqrt{5-4\sqrt{1-a^4}}\right),\frac{1}{4}\left(\sqrt{5-4\sqrt{1-a^4}}+3\right);\frac{1}{2};\cos^2(x)\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.812 (sec), leaf count = 252

$$\left\{ y(x) = \left[c_1 \operatorname{hypergeom}\left[\left[\frac{3}{4} - \frac{\sqrt{-4\sqrt{-(a-1)(a+1)(a^2+1)}+5}}{4}, \frac{3}{4} + \frac{\sqrt{-4\sqrt{-(a-1)(a+1)(a^2+1)}+5}}{4}\right], \left[\frac{1}{2}\right], \cos^2(x)\right) \right] \right\}$$

2.1575 ODE No. 1575

$$-f(x)+y^{(4)}(x)\sin^6(x)+4y^{(3)}(x)\sin^5(x)\cos(x)-6\sin^6(x)y''(x)-4\sin^5(x)\cos(x)y'(x)+y(x)\sin^6(x)=0$$

✓ **Mathematica** : cpu = 8.53073 (sec), leaf count = 138

$$\left\{ \left\{ y(x) \rightarrow x^3\csc(x)\int_1^x\frac{1}{6}\csc^5(K[4])f(K[4])dK[4]+x^2\csc(x)\int_1^x-\frac{1}{2}\csc^5(K[3])f(K[3])K[3]dK[3]+x\csc(x)\int_1^x\frac{1}{2}\csc^5(K[2])f(K[2])K[2]dK[2] \right\} \right\}$$

✓ **Maple** : cpu = 0.622 (sec), leaf count = 638

$$\left\{ y(x) = \frac{-2\left(\csc^2(x)+\frac{3}{2}\right)f\,x^3\cot(x)\csc(x)-12\left(e^{2ix}-\frac{3e^{4ix}}{2}+e^{6ix}-\frac{e^{8ix}}{4}-\frac{1}{4}\right)f\,x^3\ln(-\cot(x)+\csc(x))+48c_4x^3+48c_3x^2}{} \right\}$$

2.1576 ODE No. 1576

$$2f'(x)\big(y^{(3)}(x)-a^2y'(x)\big)+f(x)\big(a^4y(x)-2a^2y''(x)+y^{(4)}(x)\big)=0$$

✗ **Mathematica** : cpu = 0.192138 (sec), leaf count = 0 , could not solve

`DSolve[2*Derivative[1][f][x]*(-(a^2*Derivative[1][y][x]) + Derivative[3][y][x]) + f[x]*(a^4*y(x) - 2*a^2*Derivative[2][y][x] + Derivative[4][y][x]) == 0, {y, f}, x]`

✓ **Maple** : cpu = 0.022 (sec), leaf count = 67

$$\left\{ y(x) = c_1e^{-ax} + c_2e^{ax} + c_3e^{\frac{(-df+\sqrt{a^2f^2+df^2})x}{f}} + c_4e^{-\frac{(df+\sqrt{a^2f^2+df^2})x}{f}} \right\}$$

2.1577 ODE No. 1577

$$f''(x)y''(x) + 2y^{(3)}(x)f'(x) + f(x)y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 0.0774019 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow \int_1^x \int_1^{K[2]} \left(\frac{c_1}{f(K[1])} + \frac{c_2 K[1]}{f(K[1])} \right) dK[1] dK[2] + c_4 x + c_3 \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 21

$$\left\{ y(x) = \frac{1}{6} c_1 x^3 + \frac{1}{2} c_2 x^2 + c_3 x + c_4 \right\}$$

2.1578 ODE No. 1578

$$a^4 y(x) - \lambda(ax - b)(y''(x) - a^2 y(x)) - 2a^2 y''(x) + y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 48.8374 (sec), leaf count = 141

$$\left\{ \left\{ y(x) \rightarrow c_3 e^{-ax} \int_1^x 2a e^{2aK[1]} \int e^{-aK[1]} \text{Ai} \left(\frac{a^2 + \lambda K[1]a - b\lambda}{(a\lambda)^{2/3}} \right) dK[1] dK[1] + c_4 e^{-ax} \int_1^x 2a e^{2aK[2]} \int e^{-aK[2]} \text{Bi} \left(\frac{a^2 + \lambda K[2]a - b\lambda}{(a\lambda)^{2/3}} \right) dK[2] dK[2] \right\} \right\}$$

✓ **Maple** : cpu = 0.482 (sec), leaf count = 89

$$\left\{ y(x) = \left(c_1 + \int \left(c_2 + \int \left(c_3 \text{AiryAi} \left(-\frac{(-a\lambda)^{1/3} (a^2 + (ax - b)\lambda)}{a\lambda} \right) + c_4 \text{AiryBi} \left(-\frac{(-a\lambda)^{1/3} (a^2 + (ax - b)\lambda)}{a\lambda} \right) \right) e^{ax} dx \right) e^{-2ax} \right\}$$

2.1579 ODE No. 1579

$$-ax - b \sin(x) - c \cos(x) + y^{(n)}(x) + 2y^{(3)}(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.937588 (sec), leaf count = 80

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{16} (8ax^2 + \cos(x)(b(2x^2 - 9) - 2(5cx + 8(c_4x - c_2 + c_3))) + \sin(x)(-6bx + c(13 - 2x^2) + 16(c_2x + c_1 + c_4))) \right\} \right\}$$

✓ **Maple** : cpu = 0.467 (sec), leaf count = 69

$$\left\{ y(x) = \frac{a x^2}{2} + c_5 + \frac{(b x^2 - 8c_2 + 8c_3 - 6b + (-8c_4 - 4c)x) \cos(x)}{8} + \frac{(-c x^2 + 8c_1 + 8c_4 + 6c + (8c_3 - 4b)x) \sin(x)}{8} \right\}$$

2.1580 ODE No. 1580

$$y^{(6)}(x) + y(x) - \sin\left(\frac{x}{2}\right) \sin\left(\frac{3x}{2}\right) = 0$$

✓ **Mathematica** : cpu = 2.44291 (sec), leaf count = 234

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{504} \left(-42 \sin^2\left(\frac{x}{2}\right) - 42 \sin^2(x) + 42x \sin(x) + 42 \sin\left(\frac{x}{2}\right) \sin\left(\frac{3x}{2}\right) + 21 \sin(x) \sin(2x) - 24 \sin\left(\frac{x}{2}\right) \sin\left(\frac{5x}{2}\right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 1.247 (sec), leaf count = 79

$$\left\{ y(x) = \frac{(504c_1 + 105) \cos(x)}{504} + \frac{\cos(2x)}{126} + \frac{(504c_3 \cos\left(\frac{x}{2}\right) + 504c_4 \sin\left(\frac{x}{2}\right)) e^{-\frac{\sqrt{3}x}{2}}}{504} + \frac{(504c_5 \cos\left(\frac{x}{2}\right) + 504c_6 \sin\left(\frac{x}{2}\right)) e^{\frac{\sqrt{3}x}{2}}}{504} \right\}$$

2.1581 ODE No. 1581

$$-axy(x)-b+y^{(5)}(x)=0$$

✗ **Mathematica** : cpu = 0.181005 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{\left\{y(x)\rightarrow (x)\right\}\right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(diff(diff(diff(y(x),x),x),x),x),x)-a*x*y(x)-b=0,y(x))`

2.1582 ODE No. 1582

$$avx^{\nu-1}y(x)+ax^{\nu}y'(x)+y^{(5)}(x)=0$$

✓ **Mathematica** : cpu = 0.122735 (sec), leaf count = 787

$$\left\{\left\{y(x)\rightarrow c_5\left(\frac{4}{\nu}+1\right)^{-\frac{16}{\nu+4}}\nu^{-\frac{16}{\nu+4}}a^{\frac{4}{\nu+4}}(x^{\nu})^{\frac{4\left(\frac{4}{\nu}+1\right)}{\nu+4}}{}_1F_4\left(\frac{4}{\nu\left(1+\frac{4}{\nu}\right)}+\frac{1}{1+\frac{4}{\nu}};1+\frac{1}{\left(1+\frac{4}{\nu}\right)\nu},1+\frac{2}{\left(1+\frac{4}{\nu}\right)\nu},1+\frac{3}{\left(1+\frac{4}{\nu}\right)\nu},1+\frac{4}{\left(1+\frac{4}{\nu}\right)\nu}\right)\right\}\right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\{y(x)=DESol\left(\left\{av\,x^{\nu-1}\_Y(x)+a\,x^{\nu}\left(\frac{d}{dx}\_Y(x)\right)+\frac{d^5}{dx^5}\_Y(x)\right\},\{\_Y(x)\}\right)\}$$

2.1583 ODE No. 1583

$$ay^{(4)}(x)-f(x)+y^{(5)}(x)=0$$

✓ **Mathematica** : cpu = 0.120663 (sec), leaf count = 92

$$\left\{\left\{y(x)\rightarrow \int_1^x\int_1^{K[5]}\int_1^{K[4]}\int_1^{K[3]}\left(e^{-aK[2]}c_1+e^{-aK[2]}\int_1^{K[2]}e^{aK[1]}f(K[1])dK[1]\right)dK[2]dK[3]dK[4]dK[5]+c_5x^3+c_4x^2+c_3x+c_2+c_1\right\}\right\}$$

✓ **Maple** : cpu = 0.067 (sec), leaf count = 40

$$\left\{y(x)=\frac{c_2x^3}{6}+\frac{fx^4}{24a}+\frac{c_3x^2}{2}+c_4x+c_5+\frac{c_1e^{-ax}}{a^4}\right\}$$

2.1584 ODE No. 1584

$$axy(x)-5my^{(4)}(x)+xy^{(5)}(x)=0$$

✓ **Mathematica** : cpu = 2.47595 (sec), leaf count = 216

$$\left\{\left\{y(x)\rightarrow c_55^{-5m-4}a^{\frac{1}{5}(5m+4)}x^{5m+4}{}_0F_4\left(;m+\frac{6}{5},m+\frac{7}{5},m+\frac{8}{5},m+\frac{9}{5};-\frac{ax^5}{3125}\right)+\frac{1}{125}a^{3/5}c_4x^3{}_0F_4\left(\frac{6}{5},\frac{7}{5},\frac{8}{5},\frac{4}{5};-m;-\frac{ax^5}{3125}\right)+c_3x^2+c_2+c_1\right\}\right\}$$

✓ **Maple** : cpu = 0.246 (sec), leaf count = 118

$$\left\{y(x)=c_4x^3\operatorname{hypergeom}\left(\left[\right],\left[\frac{6}{5},\frac{7}{5},\frac{8}{5},-m+\frac{4}{5}\right],-\frac{ax^5}{3125}\right)+c_3x^2\operatorname{hypergeom}\left(\left[\right],\left[\frac{4}{5},\frac{6}{5},\frac{7}{5},-m+\frac{3}{5}\right],-\frac{ax^5}{3125}\right)+c_2x\operatorname{hypergeom}\left(\left[\right],\left[\frac{3}{5},\frac{4}{5},\frac{5}{5},-m+\frac{2}{5}\right],-\frac{ax^5}{3125}\right)+c_1\right\}$$

2.1585 ODE No. 1585

$$xy(x)\bigl(ay'(x)+by''(x)+cy^{(3)}(x)+ey^{(4)}(x)\bigr)=0$$

✓ **Mathematica** : cpu = 0.181056 (sec), leaf count = 214

$$\left\{\left\{y(x)\rightarrow 0\right\},\left\{y(x)\rightarrow \frac{c_1e^{\text{xRoot}\left[\#1^3+\frac{\#1^2c}{e}+\frac{\#1b}{e}+\frac{a}{e}\&,1\right]}}{\text{Root}\left[\#1^3+\frac{\#1^2c}{e}+\frac{\#1b}{e}+\frac{a}{e}\&,1\right]}+\frac{c_2e^{\text{xRoot}\left[\#1^3+\frac{\#1^2c}{e}+\frac{\#1b}{e}+\frac{a}{e}\&,2\right]}}{\text{Root}\left[\#1^3+\frac{\#1^2c}{e}+\frac{\#1b}{e}+\frac{a}{e}\&,2\right]}+\frac{c_3e^{\text{xRoot}\left[\#1^3+\frac{\#1^2c}{e}+\frac{\#1b}{e}+\frac{a}{e}\&,3\right]}}{\text{Root}\left[\#1^3+\frac{\#1^2c}{e}+\frac{\#1b}{e}+\frac{a}{e}\&,3\right]}\right\}\right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 679

$$\left\{\begin{array}{l}y(x)=0,y(x)=c_2e^{\frac{-12be+\left(12ibe-4ic^2+i\left(-108ae^2+36bce-8c^3+12\sqrt{3}\sqrt{27a^2e^2+4ac^3-b^2c^2+(-18abc+4b^3)e}\right)e^{\frac{2}{3}}\right)\sqrt{3}+\left(2c+\left(-108ae^2+36bce-8c^3+12\sqrt{3}\sqrt{27a^2e^2+4ac^3-b^2c^2+(-18abc+4b^3)e}\right)^{\frac{1}{3}}e\right)^{\frac{1}{3}}}{12\left(-108ae^2+36bce-8c^3+12\sqrt{3}\sqrt{27a^2e^2+4ac^3-b^2c^2+(-18abc+4b^3)e}\right)^{\frac{1}{3}}e}\end{array}\right\}$$

2.1586 ODE No. 1586

$$-y^{(4)}(x)(x(aA(5)-A(4))+A(5))-y^{(3)}(x)(x(aA(4)-A(3))+A(4))-(x(aA(3)-A(2))+A(3))y''(x)-(x(aA(2)-A(1))+A(2))y'(x)+A(1))y(x)=0$$

✗ **Mathematica** : cpu = 83.181 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{\left\{y(x)\rightarrow (x)\right\}\right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{y(x)=c_1+\int DESol\left[\left[\frac{d^4}{dx^4}Y(x)-\frac{(axA_2-xA_1+A_2)_Y(x)}{x}-\frac{(axA_3-xA_2+A_3)\left(\frac{d}{dx}Y(x)\right)}{x}-\frac{(axA_4-xA_3+A_4)\left(\frac{d^2}{dx^2}Y(x)\right)}{x}\right]\right]\right\}$$

2.1587 ODE No. 1587

$$x^5y^{(10)}(x)-ay(x)=0$$

✓ **Mathematica** : cpu = 0.324153 (sec), leaf count = 492

$$\left\{\left\{y(x)\rightarrow \frac{(-1)^{4/5}a^{9/5}c_1x^9{}_0F_9\left(; \frac{6}{5},\frac{7}{5},\frac{8}{5},\frac{9}{5},2,\frac{11}{5},\frac{12}{5},\frac{13}{5},\frac{14}{5};\frac{ax^5}{9765625}\right)}{3814697265625}+\frac{(-1)^{3/5}a^{8/5}c_3x^8{}_0F_9\left(; \frac{4}{5},\frac{6}{5},\frac{7}{5},\frac{8}{5},\frac{9}{5},2,\frac{11}{5},\frac{12}{5},\frac{13}{5};\frac{ax^5}{9765625}\right)}{152587890625}\right\}\right\}$$

✓ **Maple** : cpu = 0.467 (sec), leaf count = 154

$$\left\{y(x)=\left(c_1\text{BesselI}\left(5,2a^{\frac{1}{10}}\sqrt{x}\right)+c_{10}\text{BesselY}\left(5,2(-1)^{\frac{9}{10}}a^{\frac{1}{10}}\sqrt{x}\right)+c_2\text{BesselY}\left(5,2ia^{\frac{1}{10}}\sqrt{x}\right)+c_3\text{BesselI}\left(5,2(-1)^{\frac{1}{10}}a^{\frac{1}{10}}\sqrt{x}\right)\right)\right\}$$

2.1588 ODE No. 1588

$$x^{10}y^{(5)}(x)-ay(x)=0$$

✓ **Mathematica** : cpu = 12.1395 (sec), leaf count = 114

$$\left\{\left\{y(x)\rightarrow c_1x^4e^{-\frac{\sqrt[5]{a}}{x}}+c_2x^4e^{\frac{\sqrt[5]{-1}\sqrt[5]{a}}{x}}+c_3x^4e^{-\frac{(-1)^{2/5}\sqrt[5]{a}}{x}}+c_4x^4e^{\frac{(-1)^{3/5}\sqrt[5]{a}}{x}}+c_5x^4e^{-\frac{(-1)^{4/5}\sqrt[5]{a}}{x}}\right\}\right\}$$

✓ **Maple** : cpu = 0.147 (sec), leaf count = 90

$$\left\{y(x)=c_5x^4\text{hypergeom}\left(\left[\right],\left[\frac{1}{5},\frac{2}{5},\frac{3}{5},\frac{4}{5}\right],-\frac{a}{3125x^5}\right)+c_4x^3\text{hypergeom}\left(\left[\right],\left[\frac{2}{5},\frac{3}{5},\frac{4}{5},\frac{6}{5}\right],-\frac{a}{3125x^5}\right)+c_3x^2\text{hypergeom}\left(\left[\right],\left[\frac{3}{5},\frac{4}{5},\frac{6}{5},\frac{7}{5}\right],-\frac{a}{3125x^5}\right)+c_2x\text{hypergeom}\left(\left[\right],\left[\frac{4}{5},\frac{6}{5},\frac{7}{5},\frac{8}{5}\right],-\frac{a}{3125x^5}\right)+c_1\text{hypergeom}\left(\left[\right],\left[\frac{5}{5},\frac{7}{5},\frac{8}{5},\frac{9}{5}\right],-\frac{a}{3125x^5}\right)\right\}$$

2.1589 ODE No. 1589

$$x^{11/2}y^{(11)}(x) - ay(x) = 0$$

✓ **Mathematica** : cpu = 0.0395231 (sec), leaf count = 670

$$\left\{\left\{y(x) \rightarrow \frac{4}{121}(-1)^{2/11}a^{2/11}c_2x\,{}_0F_{10}\left(;-\frac{7}{11},-\frac{5}{11},-\frac{3}{11},-\frac{1}{11},\frac{1}{11},\frac{3}{11},\frac{5}{11},\frac{7}{11},\frac{9}{11},\frac{13}{11},\frac{2048ax^{11/2}}{285311670611}\right)-\frac{1048576(-1)^{9/11}a^{20/11}}{285311670611}\right\}\right\}$$

✓ **Maple** : cpu = 8.053 (sec), leaf count = 4018

Expression too large to display

2.1590 ODE No. 1590

$$(x-a)^5(x-b)^5y^{(5)}(x) - cy(x) = 0$$

✓ **Mathematica** : cpu = 271.105 (sec), leaf count = 331

$$\left\{\left\{y(x) \rightarrow c_1(x-b)^4\left(\frac{x-a}{x-b}\right)^{\text{Root}\left[-\#1^5+10\#1^4-35\#1^3+50\#1^2-24\#1+\frac{c}{(a-b)^5}\&,1\right]}+c_2(x-b)^4\left(\frac{x-a}{x-b}\right)^{\text{Root}\left[-\#1^5+10\#1^4-35\#1^3+50\#1^2-24\#1+\frac{c}{(a-b)^5}\&,1\right]}\right\}\right\}$$

✓ **Maple** : cpu = 1.732 (sec), leaf count = 553

$$\left\{y(x) = \text{ODESolStruc}\left(e^{c_2+\int-\frac{4\left(\frac{f}{4}+a+\left(-\frac{f}{4}-b\right)e^{\left(c_1+\int-g(f)d_f\right)(a-b)}\right)-g(f)}d_f}{e^{\left(c_1+\int-g(f)d_f\right)(a-b)}-1}}\right),\left\{\frac{-96\left(\left(\frac{f^5}{96}+a^4b+\frac{13a^3b^2}{3}+\frac{13a^2b^3}{3}+ab^4+\left(\frac{5a}{48}+\frac{5b}{48}\right)-\frac{f^5}{96}\right)}{e^{\left(c_1+\int-g(f)d_f\right)(a-b)}-1}}\right)\right\}\right\}$$

2.1591 ODE No. 1591

$$y''(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0491518 (sec), leaf count = 26

$$\left\{\left\{y(x) \rightarrow \sqrt[3]{6}\wp\left(\frac{x+c_1}{\sqrt[3]{6}};0,c_2\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.857 (sec), leaf count = 12

$$\{y(x) = 6\text{WeierstrassP}(c_1+x,0,c_2)\}$$

2.1592 ODE No. 1592

$$y''(x) - 6y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0975503 (sec), leaf count = 14

$$\{\{y(x) \rightarrow \wp(x+c_1;0,c_2)\}\}$$

✓ **Maple** : cpu = 0.852 (sec), leaf count = 10

$$\{y(x) = \text{WeierstrassP}(c_1+x,0,c_2)\}$$

2.1593 ODE No. 1593

$$y''(x) - 6y(x)^2 - x = 0$$

✗ **Mathematica** : cpu = 20.0785 (sec), leaf count = 0 , could not solve

`DSolve[-x - 6*y[x]^2 + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x), x), x)-6*y(x)^2-x=0, y(x))`

2.1594 ODE No. 1594

$$y''(x) - 6y(x)^2 + 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.505858 (sec), leaf count = 373

Solve

$$\left[\frac{4 \left(\text{Root}\left[4\#1^3 - 4\#1^2 + c_1 \&, 2\right] - \text{Root}\left[4\#1^3 - 4\#1^2 + c_1 \&, 3\right] \right) \left(y(x) - \text{Root}\left[4\#1^3 - 4\#1^2 + c_1 \&, 1\right] \right) \left(y(x) - \text{Root}\left[4\#1^3 - 4\#1^2 + c_1 \&, 3\right] \right)}{\left(4y(x)^3 - 4y(x)^2 + c_1 \right) \left(\text{Root}\left[4\#1^3 - 4\#1^2 + c_1 \&, 3\right] \right)}$$

✓ **Maple** : cpu = 0.941 (sec), leaf count = 59

$$\left\{ -c_2 - x + \int^{y(x)} \frac{1}{\sqrt{4\_a^3 - 4\_a^2 + c_1}} d\_a = 0, -c_2 - x + \int^{y(x)} -\frac{1}{\sqrt{4\_a^3 - 4\_a^2 + c_1}} d\_a = 0 \right\}$$

2.1595 ODE No. 1595

$$ay(x)^2 + bx + c + y''(x) = 0$$

✗ **Mathematica** : cpu = 20.142 (sec), leaf count = 0 , could not solve

`DSolve[c + b*x + a*y[x]^2 + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x), x), x)+a*y(x)^2+b*x+c=0, y(x))`

2.1596 ODE No. 1596

$$a + y''(x) - 2y(x)^3 - xy(x) = 0$$

✗ **Mathematica** : cpu = 20.5128 (sec), leaf count = 0 , could not solve

`DSolve[a - x*y[x] - 2*y[x]^3 + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x), x), x)-2*y(x)^3-x*y(x)+a=0, y(x))`

2.1597 ODE No. 1597

$$y''(x) - ay(x)^3 = 0$$

✓ **Mathematica** : cpu = 2.52144 (sec), leaf count = 242

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt[4]{2} \sqrt{c_1} \sqrt{\frac{i \sqrt{a}}{\sqrt{c_1}}} \operatorname{sn}\left(\frac{(-1)^{3/4} \sqrt{\sqrt{2} \sqrt{a} \sqrt{c_1} x^2 + 2 \sqrt{2} \sqrt{a} \sqrt{c_1} c_2 x + \sqrt{2} \sqrt{a} \sqrt{c_1} c_2^2}}{\sqrt{2}}\right) - 1}{\sqrt{a}} \right\} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt[4]{2} \sqrt{c_1} \sqrt{\frac{i \sqrt{a}}{\sqrt{c_1}}} \operatorname{sn}\left(\frac{(-1)^{3/4} \sqrt{\sqrt{2} \sqrt{a} \sqrt{c_1} x^2 + 2 \sqrt{2} \sqrt{a} \sqrt{c_1} c_2 x + \sqrt{2} \sqrt{a} \sqrt{c_1} c_2^2}}{\sqrt{2}}\right) - 1}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 1.019 (sec), leaf count = 21

$$\left\{ y(x) = c_2 \operatorname{sn}\left(c_2 \left(c_1 + \frac{\sqrt{-2a} x}{2}\right) \middle| i\right) \right\}$$

2.1598 ODE No. 1598

$$-2a^2y(x)^3 + 2abxy(x) - b + y''(x) = 0$$

✗ **Mathematica** : cpu = 22.5214 (sec), leaf count = 0 , could not solve

```
DSolve[-b + 2*a*b*x*y[x] - 2*a^2*y[x]^3 + Derivative[2][y][x] == 0, y[x], x]
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve(diff(diff(y(x),x),x)-2*a^2*y(x)^3+2*a*b*x*y(x)-b=0,y(x))
```

2.1599 ODE No. 1599

$$ay(x)^3 + bxy(x) + cy(x) + d + y''(x) = 0$$

✗ **Mathematica** : cpu = 20.7826 (sec), leaf count = 0 , could not solve

```
DSolve[d + c*y[x] + b*x*y[x] + a*y[x]^3 + Derivative[2][y][x] == 0, y[x], x]
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve(diff(diff(y(x),x),x)+d+b*x*y(x)+c*y(x)+a*y(x)^3=0,y(x))
```

2.1600 ODE No. 1600

$$ay(x)^3 + by(x)^2 + cy(x) + d + y''(x) = 0$$

✓ **Mathematica** : cpu = 2.37264 (sec), leaf count = 1017

Solve

$$\left[4F\left(\sin^{-1}\left(\sqrt{\frac{\left(\text{Root}\left[3a\#1^4+4b\#1^3+6c\#1^2+12d\#1-6c_1\&,2\right]-\text{Root}\left[3a\#1^4+4b\#1^3+6c\#1^2+12d\#1-6c_1\&,4\right]\right)\left(y(x)-\text{Root}\left[3a\#1^4+4b\#1^3+6c\#1^2+12d\#1-6c_1\&,1\right]-\text{Root}\left[3a\#1^4+4b\#1^3+6c\#1^2+12d\#1-6c_1\&,4\right]\right)\left(y(x)-\text{Root}\left[3a\#1^4+4b\#1^3+6c\#1^2+12d\#1-6c_1\&,1\right]-\text{Root}\left[3a\#1^4+4b\#1^3+6c\#1^2+12d\#1-6c_1\&,4\right]\right)}{\left(\text{Root}\left[3a\#1^4+4b\#1^3+6c\#1^2+12d\#1-6c_1\&,1\right]-\text{Root}\left[3a\#1^4+4b\#1^3+6c\#1^2+12d\#1-6c_1\&,4\right]\right)\left(y(x)-\text{Root}\left[3a\#1^4+4b\#1^3+6c\#1^2+12d\#1-6c_1\&,1\right]-\text{Root}\left[3a\#1^4+4b\#1^3+6c\#1^2+12d\#1-6c_1\&,4\right]\right)}\right)\right]$$

✓ **Maple** : cpu = 0.916 (sec), leaf count = 89

$$\left\{ -c_2 - x + \int^{y(x)} -\frac{6}{\sqrt{-18\_a^4a - 24b\_a^3 - 36c\_a^2 - 72d\_a + 36c_1}}d\_a = 0, -c_2 - x + \int^{y(x)} \frac{6}{\sqrt{-18\_a^4a - 24b\_a^3 - 36c\_a^2 - 72d\_a + 36c_1}}d\_a = 0 \right\}$$

2.1601 ODE No. 1601

$$ax^ry(x)^n + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.0407634 (sec), leaf count = 0 , could not solve

```
DSolve[a*x^r*y[x]^n + Derivative[2][y][x] == 0, y[x], x]
```

✓ **Maple** : cpu = 4.318 (sec), leaf count = 151

$$\left\{ y(x) = ODESolStruc\left(-_a e^{c_1+\int -b(_a)d\_a}, \left\{ \frac{d}{d\_a}b(_a) = \frac{\left((n-1)^2a\_a^n b(_a) + ((n+r+1)\_a b(_a) + n+2r+3)(r+2)\right)}{(r+2)^2} \right. \right. \right.$$

2.1602 ODE No. 1602

$$(n+1)a^{2n}y(x)^{2n+1} + y''(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.148189 (sec), leaf count = 47

Solve

$$\left[\int_1^{y(x)} \frac{1}{\sqrt{c_1 - K[1]^2(a^{2n}K[1]^{2n} - 1)}}dK[1]^2 = (x + c_2)^2, y(x) \right]$$

✓ **Maple** : cpu = 0.243 (sec), leaf count = 73

$$\left\{-c_2-x+\int^{y(x)}\frac{1}{\sqrt{-a^2-\_a^{2n+2}a^{2n}+c_1}}d\_a=0,-c_2-x+\int^{y(x)}-\frac{1}{\sqrt{-a^2-\_a^{2n+2}a^{2n}+c_1}}d\_a=0\right\}$$

2.1603 ODE No. 1603

$$y''(x)-\frac{1}{\left(ay(x)^2+bxy(x)+cx^2+dy(x)+ex+k\right)^{3/2}}=0$$

✗ **Mathematica** : cpu = 60.4184 (sec), leaf count = 0 , could not solve
DSolve[-(k + e*x + c*x^2 + d*y[x] + b*x*y[x] + a*y[x]^2)^(-3/2) + Derivative[2][y][x] == 0,
✓ **Maple** : cpu = 43.954 (sec), leaf count = 8413

Expression too large to display

2.1604 ODE No. 1604

$$y''(x)-e^{y(x)}=0$$

✓ **Mathematica** : cpu = 0.0427803 (sec), leaf count = 34
$$\left\{\left\{y(x)\rightarrow\log\left(\frac{1}{2}c_1\left(-1+\tanh^2\left(\frac{1}{2}\sqrt{c_1(x+c_2)^2}\right)\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.269 (sec), leaf count = 23

$$\left\{y(x)=\ln\left(\frac{\tan^2\left(\frac{c_2+x}{2c_1}\right)+1}{2c_1^2}\right)\right\}$$

2.1605 ODE No. 1605

$$ae^x\sqrt{y(x)}+y''(x)=0$$

✗ **Mathematica** : cpu = 20.1682 (sec), leaf count = 0 , could not solve
DSolve[a*E^x*Sqrt[y[x]] + Derivative[2][y][x] == 0, y[x], x]
✓ **Maple** : cpu = 1.266 (sec), leaf count = 104

$$\left\{y(x)=ODESolStruc\left(\_ae^{2c_1+\int 2\_b(\_a)d\_a},\left[\left\{\frac{d}{d\_a}\_b(\_a)=\left(\sqrt{\_a}\_a\_b(\_a)+4\_a\_b(\_a)+4\right)\_b(\_a)^2\right\},\left\{\_a=e^{-2x}y(x),\_b(\_a)=\frac{d}{d\_a}y(x)\right\}\right]\right)\right\}$$

2.1606 ODE No. 1606

$$y''(x)+e^x\sin(y(x))=0$$

✗ **Mathematica** : cpu = 20.6398 (sec), leaf count = 0 , could not solve
DSolve[E^x*Sin[y[x]] + Derivative[2][y][x] == 0, y[x], x]
✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve
dsolve(diff(diff(y(x),x),x)+exp(x)*sin(y(x))=0,y(x))

2.1607 ODE No. 1607

$$a \sin(y(x)) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0862574 (sec), leaf count = 79

$$\left\{ \left\{ y(x) \rightarrow -2\operatorname{am}\left(\frac{1}{2}\sqrt{(2a+c_1)(x+c_2)^2}\middle|\frac{4a}{2a+c_1}\right) \right\}, \left\{ y(x) \rightarrow 2\operatorname{am}\left(\frac{1}{2}\sqrt{(2a+c_1)(x+c_2)^2}\middle|\frac{4a}{2a+c_1}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.165 (sec), leaf count = 49

$$\left\{-c_2-x+\int^{y(x)}\frac{1}{\sqrt{2a\cos(\_a)+c_1}}d\_a=0,-c_2-x+\int^{y(x)}-\frac{1}{\sqrt{2a\cos(\_a)+c_1}}d\_a=0\right\}$$

2.1608 ODE No. 1608

$$a^2 \sin(y(x)) - b \sin(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.0406499 (sec), leaf count = 0 , could not solve

`DSolve[-(b*Sin[x]) + a^2*Sin[y[x]] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x),x),x)+a^2*sin(y(x))-b*sin(x)=0,y(x))`

2.1609 ODE No. 1609

$$a^2 \sin(y(x)) - b f(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.0306451 (sec), leaf count = 0 , could not solve

`DSolve[-(b*f[x]) + a^2*Sin[y[x]] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x),x),x)+a^2*sin(y(x))-b*f(x)=0,y(x))`

2.1610 ODE No. 1610

$$y''(x) - \frac{h\left(\frac{y(x)}{\sqrt{x}}\right)}{x^{3/2}} = 0$$

✓ **Mathematica** : cpu = 4.06073 (sec), leaf count = 754

$$\left\{\operatorname{Solve}\left[\int_1^{y(x)}\frac{2}{\sqrt{x}\sqrt{\frac{K[3]^2+4xc_1+8x\int_1^{\frac{K[3]}{\sqrt{x}}}h(K[2])dK[2]}{x}}}dK[3]-\int_1^x\frac{2\left(\frac{y(x)}{2\sqrt{K[4]}}-\frac{\sqrt{\frac{y(x)^2}{2K[4]}+2c_1+4\int_1^{\frac{y(x)}{\sqrt{K[4]}}}h(K[2])dK[2]}}{\sqrt{2}}}\right)}{K[4]\sqrt{\frac{y(x)^2+4c_1K[4]+8K[4]\int_1^{\frac{y(x)}{\sqrt{K[4]}}}h(K[2])dK[2]}{K[4]}}}dx+\int_1^{y(x)}-\frac{4c_1+8}{\sqrt{2}\sqrt{K[4]}}dK[4],y(x)\right]\right\}$$

✓ **Maple** : cpu = 0.255 (sec), leaf count = 92

$$\left\{y(x)=\sqrt{x}\operatorname{RootOf}\left(2c_2-2\left(\int^{-Z}\frac{1}{\sqrt{-g^2+c_1+8\left(\int h(-g)d\_g\right)}}d\_g\right)-\ln(x)\right),y(x)=\sqrt{x}\operatorname{RootOf}\left(2c_2+2\left(\int^{-Z}\frac{1}{\sqrt{-g^2+c_1+8\left(\int h(-g)d\_g\right)}}d\_g\right)+\ln(x)\right)\right\}$$

2.1611 ODE No. 1611

$$y''(x) - 3y'(x) - y(x)^2 - 2y(x) = 0$$

✗ **Mathematica** : cpu = 3.58753 (sec), leaf count = 0 , could not solve

`DSolve[-2*y[x] - y[x]^2 - 3*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 1.618 (sec), leaf count = 57

$$\left\{y(x) = ODESolStruc\left(\_a, \left[\left\{-\_a^2 + \_b(\_a)\left(\frac{d}{d\_a}\_b(\_a)\right) - 2\_a - 3\_b(\_a) = 0\right\}, \left\{\_a = y(x), \_b(\_a) = \frac{d}{dx}y(x)\right\}, \left\{x = c_1\right\}\right.\right.$$

2.1612 ODE No. 1612

$$y''(x) - 7y'(x) - y(x)^{3/2} + 12y(x) = 0$$

✗ **Mathematica** : cpu = 21.4189 (sec), leaf count = 0 , could not solve

`DSolve[12*y[x] - y[x]^(3/2) - 7*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 2.113 (sec), leaf count = 57

$$\left\{y(x) = ODESolStruc\left(\_a, \left[\left\{\_b(\_a)\left(\frac{d}{d\_a}\_b(\_a)\right) - \_a^{\frac{3}{2}} + 12\_a - 7\_b(\_a) = 0\right\}, \left\{\_a = y(x), \_b(\_a) = \frac{d}{dx}y(x)\right\}, \left\{x = c_1\right\}\right.\right.$$

2.1613 ODE No. 1613

$$6a^2y(x) + 5ay'(x) + y''(x) - 6y(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.34106 (sec), leaf count = 35

$$\left\{\left\{y(x) \rightarrow a^2c_1^2e^{-2ax}\wp(e^{-ax}c_1 + c_2; 0, -1)\right\}\right\}$$

✓ **Maple** : cpu = 1.03 (sec), leaf count = 27

$$\left\{y(x) = \text{WeierstrassP}\left(c_1 - \frac{e^{-ax}}{a}, 0, c_2\right)e^{-2ax}\right\}$$

2.1614 ODE No. 1614

$$2a^2y(x) + 3ay'(x) + y''(x) - 2y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.964542 (sec), leaf count = 32

$$\left\{\left\{y(x) \rightarrow -iac_1e^{-ax}\text{sn}(e^{-ax}c_1 + c_2| -1)\right\}\right\}$$

✓ **Maple** : cpu = 1.122 (sec), leaf count = 33

$$\left\{y(x) = c_2e^{-ax}\text{sn}\left(c_2\left(c_1 - \frac{\sqrt{-e^{-2ax}}}{a}\right)|i\right)\right\}$$

2.1615 ODE No. 1615

$$-\frac{2(n+1)(n+2)y(x)\left(y(x)^{\frac{n}{n+1}} - 1\right)}{n^2} - \frac{(3n+4)y'(x)}{n} + y''(x) = 0$$

✗ **Mathematica** : cpu = 60.1484 (sec), leaf count = 0 , could not solve

`DSolve[(-2*(1 + n)*(2 + n)*y[x]*(-1 + y[x]^(n/(1 + n))))/n^2 - ((4 + 3*n)*Derivative[1][y][x])`

✓ **Maple** : cpu = 4.202 (sec), leaf count = 91

$$\left\{y(x) = ODESolStruc\left(\_a, \left[\left[\frac{n^2\_b(\_a)\left(\frac{d}{d\_a}\_b(\_a)\right) - 2(n+2)(n+1)\_a\_a^{\frac{n}{n+1}} + 2(n+1)(n+2)\_a + (-3n^2 - 4n)\_b(\_a)}{n^2}\right.\right.\right.$$

2.1616 ODE No. 1616

$$\frac{1}{4}(a^2-1)y(x)+ay'(x)+by(x)^n+y''(x)=0$$

✗ **Mathematica** : cpu = 16.7411 (sec), leaf count = 0 , could not solve

`DSolve[((-1 + a^2)*y[x])/4 + b*y[x]^n + a*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 1.225 (sec), leaf count = 63

$$\left\{y(x)=ODESolStruc\left(\_a,\left[\left\{\frac{-a\,a^2}{4}+a\_b(\_a)+b\_a^n+\_b(\_a)\left(\frac{d}{d\_a}\_b(\_a)\right)-\frac{a}{4}=0\right\},\left\{\_a=y(x),\_b(\_a)=\frac{d}{dx}y(x)\right\},\left\{\right\}\right.\right.$$

2.1617 ODE No. 1617

$$ay'(x)+bx^ry(x)^n+y''(x)=0$$

✗ **Mathematica** : cpu = 0.0386178 (sec), leaf count = 0 , could not solve

`DSolve[b*x^r*y[x]^n + a*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x),x),x)+a*diff(y(x),x)+b*x^r*y(x)^n=0,y(x))`

2.1618 ODE No. 1618

$$ay'(x)-2a+be^{y(x)}+y''(x)=0$$

✗ **Mathematica** : cpu = 27.5652 (sec), leaf count = 0 , could not solve

`DSolve[-2*a + b*E^y[x] + a*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 1.568 (sec), leaf count = 56

$$\left\{y(x)=ODESolStruc\left(\_a,\left[\left\{a\_b(\_a)+b\,e^{-a}+\_b(\_a)\left(\frac{d}{d\_a}\_b(\_a)\right)-2a=0\right\},\left\{\_a=y(x),\_b(\_a)=\frac{d}{dx}y(x)\right\},\left\{x=c_1+\right\}\right.\right.$$

2.1619 ODE No. 1619

$$ay'(x)+f(x)\sin(y(x))+y''(x)=0$$

✗ **Mathematica** : cpu = 0.0513776 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*Sin[y[x]] + a*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x),x),x)+a*diff(y(x),x)+f(x)*sin(y(x))=0,y(x))`

2.1620 ODE No. 1620

$$y''(x)+y(x)y'(x)-y(x)^3=0$$

✓ **Mathematica** : cpu = 3.44741 (sec), leaf count = 492

$$\left\{\left\{y(x)\rightarrow\text{InverseFunction}\left[\int_1^{\#1}\frac{2}{\frac{e^{6c_1K[1]^4}}{\sqrt[3]{e^{18c_1K[1]^6-2e^{12c_1}+2\sqrt{e^{24c_1}-e^{30c_1}}}}}-K[1]^2+e^{-6c_1}\sqrt[3]{e^{18c_1K[1]^6-2e^{12c_1}+2\sqrt{e^{24c_1}-e^{30c_1}}}}}}}\right.\right.$$

✓ **Maple** : cpu = 1.297 (sec), leaf count = 291

2.1629 ODE No. 1629

$$(f(x) + 3y(x))y'(x) + f(x)y(x)^2 + y''(x) + y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.0685738 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow \frac{\int_1^x \exp\left(-\int_1^{K[2]} f(K[1])dK[1]\right)c_1dK[2] + c_2}{\int_1^x \int_1^{K[5]} \exp\left(-\int_1^{K[4]} f(K[3])dK[3]\right)c_1dK[4]dK[5] + c_2x + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 38

$$\left\{ y(x) = \frac{c_2 + \int c_1 e^{-\left(\int f(x)dx\right)}dx}{c_2x + \int \int c_1 e^{-\left(\int f(x)dx\right)}dx dx + 1} \right\}$$

2.1630 ODE No. 1630

$$-4a^2y(x) - 3ay(x)^2 - b + y''(x) - 3y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 7.71636 (sec), leaf count = 3227

$$\left\{ \left\{ y(x) \rightarrow -\frac{2\left((-1)^{\frac{a^{3/2}\sqrt{4a^3-3b}-2a^3}}{4a^3}+1\right)2^{-\frac{3\left(a^{3/2}\sqrt{4a^3-3b}-2a^3\right)}{4a^3}+\frac{3\sqrt{4a^6-3a^3b}}{4a^3}+1}3^{\frac{a^{3/2}\sqrt{4a^3-3b}-2a^3}{4a^3}-\frac{\sqrt{4a^6-3a^3b}}{4a^3}}a^{-\frac{a^{3/2}\sqrt{4a^3-3b}-2a^3}{2a^3}+\frac{\sqrt{4a^6-3a^3b}}{2a^3}+1}b^{\frac{a^{3/2}\sqrt{4a^3-3b}-2a^3}{2a^3}+\frac{\sqrt{4a^6-3a^3b}}{2a^3}+1}}{a^{\frac{a^{3/2}\sqrt{4a^3-3b}-2a^3}{2a^3}+\frac{\sqrt{4a^6-3a^3b}}{2a^3}+1}b^{\frac{a^{3/2}\sqrt{4a^3-3b}-2a^3}{2a^3}+\frac{\sqrt{4a^6-3a^3b}}{2a^3}+1}}}\right\} \right\}$$

✓ **Maple** : cpu = 1.75 (sec), leaf count = 783

$$\left\{ -c_2 - x + \int^{y(x)} \frac{-9\_a^2a^2 - 12\_a\,a^3 - 3ab + \text{RootOf}\left(3c_{1\_aa}\,\text{BesselK}\left(\frac{4a^3-3b}{2\sqrt{4a^4-3ab}}\,a, -\frac{Z}{2a^2}\right) + 2c_1a^2\,\text{BesselK}\left(\frac{4a^3-3b}{2\sqrt{4a^4-3ab}}\,a, -\frac{Z}{2a^2}\right)\right)}{dx} \right\}$$

2.1631 ODE No. 1631

$$-(f(x) + 3y(x))y'(x) + f(x)y(x)^2 + y''(x) + y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.0550825 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow \frac{-\int_1^x \exp\left(\int_1^{K[2]} f(K[1])dK[1]\right)c_1dK[2] - c_2}{\int_1^x \int_1^{K[5]} \exp\left(\int_1^{K[4]} f(K[3])dK[3]\right)c_1dK[4]dK[5] + c_2x + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 38

$$\left\{ y(x) = \frac{-c_2 - \left(\int c_1 e^{\int f(x)dx}dx\right)}{c_2x + \int \int c_1 e^{\int f(x)dx}dx dx + 1} \right\}$$

2.1632 ODE No. 1632

$$y''(x) - 2ay(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0661677 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{c_1}\,\tan\left(\sqrt{a}\,\sqrt{c_1}\,x + \sqrt{a}\,\sqrt{c_1}\,c_2\right)}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 1.243 (sec), leaf count = 23

$$\left\{y(x)=\frac{\sqrt{c_1a}\,\tan\big(\sqrt{c_1a}\,(c_2+x)\big)}{a}\right\}$$

2.1633 ODE No. 1633

$$ay(x)y'(x)+by(x)^3+y''(x)=0$$

✓ **Mathematica** : cpu = 34.0885 (sec), leaf count = 92

$$\text{Solve}\left[\int_1^{y(x)}\frac{1}{K[2]^2\text{InverseFunction}\left[\frac{1}{4}\left(\log(b+\#1(a+2\#1))-\frac{2a\tan^{-1}\left(\frac{a+4\#1}{\sqrt{8b-a^2}}\right)}{\sqrt{8b-a^2}}\right)\right]\&}\left[c_1-\log(K[2])\right]}dK[2]=x-c_2,y(x)\right]$$

✓ **Maple** : cpu = 1.438 (sec), leaf count = 97

$$\left\{-c_2-x+\int^{y(x)}\frac{1}{\text{RootOf}\left(-2\_a^2a\operatorname{arctanh}\left(\frac{-\_a^2a+4\_Z}{\sqrt{(a^2-8b)\_a^4}}\right)+c_1\sqrt{(a^2-8b)\_a^4}-\sqrt{(a^2-8b)\_a^4}\ln(-\_a^4b+\_Z\_a^2a+2\_Z^2)\right)}d\_Z}\right\}$$

2.1634 ODE No. 1634

$$y'(x)h(x,y(x))+j(x,y(x))+y''(x)=0$$

✗ **Mathematica** : cpu = 0.148457 (sec), leaf count = 0 , could not solve

$$\text{DSolve}[j[x,y[x]]+h[x,y[x]]*\text{Derivative}[1][y][x]+\text{Derivative}[2][y][x]==0,y[x],x]$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

$$\text{dsolve}(\text{diff}(\text{diff}(y(x),x),x)+h(x,y(x))*\text{diff}(y(x),x)+j(x,y(x))=0,y(x))$$

2.1635 ODE No. 1635

$$ay'(x)^2+by(x)+y''(x)=0$$

✓ **Mathematica** : cpu = 0.591465 (sec), leaf count = 104

$$\left\{\left\{y(x)\rightarrow\text{InverseFunction}\left[\int_1^{\#1}-\frac{\sqrt{2}a}{\sqrt{2e^{-2aK[1]}c_1a^2-2bK[1]a+b}}dK[1]\&\right][x+c_2]\right\},\left\{y(x)\rightarrow\text{InverseFunction}\left[\int_1^{\#1}\frac{1}{\sqrt{2e^{-2aK[1]}c_1a^2-2bK[1]a+b}}dK[1]\&\right][x+c_2]\right\}\right\}$$

✓ **Maple** : cpu = 1.332 (sec), leaf count = 79

$$\left\{-c_2-x+\int^{y(x)}-\frac{2a}{\sqrt{4c_1a^2e^{-2\_aa}-4\_aab+2b}}d\_a=0,-c_2-x+\int^{y(x)}\frac{2a}{\sqrt{4c_1a^2e^{-2\_aa}-4\_aab+2b}}d\_a=0\right\}$$

2.1636 ODE No. 1636

$$ay'(x)|y'(x)|+by'(x)+cy(x)+y''(x)=0$$

✗ **Mathematica** : cpu = 7.58288 (sec), leaf count = 0 , could not solve

$$\text{DSolve}[c*y[x]+b*\text{Derivative}[1][y][x]+a*\text{Abs}[\text{Derivative}[1][y][x]]*\text{Derivative}[1][y][x]+\text{Derivative}[2][y][x]=0,y[x],x]$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

numeric exception: division by zero

2.1641 ODE No. 1641

$$g(x)y'(x) + h(y(x))y'(x)^2 + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0576651 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[\int_1^{\#1} \exp \left(- \int_1^{K[4]} -h(K[1])dK[1] \right) dK[4] \& \left\| \int_1^x - \exp \left(- \int_1^{K[5]} g(K[2])dK[2] \right) c_1 dK[5] + c_2 \right. \right. \right. \right.$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 29

$$\{-c_1\left(\int e^{-\left(\int g(x)dx\right)}dx\right)-c_2+\int^{y(x)}e^{\int h(b)db}db=0\}$$

2.1642 ODE No. 1642

$$f(x)h(y(x)) + g(x)y'(x) - \frac{j(y(x))y'(x)^2}{h(y(x))} + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.486739 (sec), leaf count = 0 , could not solve

$$\text{DSolve}[f[x]*h[y[x]] + g[x]*\text{Derivative}[1][y][x] - (j[y[x]]*\text{Derivative}[1][y][x]^2)/h[y[x]] + \text{Derivative}[2][y][x], y[x], x]$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

$$\text{dsolve}(\text{diff}(\text{diff}(y(x), x), x) - j(y(x))/h(y(x))*\text{diff}(y(x), x)^2 + g(x)*\text{diff}(y(x), x) + f(x)*h(y(x)) = 0, y(x))$$

2.1643 ODE No. 1643

$$f(x)y'(x) + g(x)j(y(x)) + h(y(x))y'(x)^2 + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.18991 (sec), leaf count = 0 , could not solve

$$\text{DSolve}[g[x]*j[y[x]] + f[x]*\text{Derivative}[1][y][x] + h[y[x]]*\text{Derivative}[1][y][x]^2 + \text{Derivative}[2][y][x], y[x], x]$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

$$\text{dsolve}((1 - D(j)(y(x)))/j(y(x))*\text{diff}(y(x), x)^2 + \text{diff}(y(x), x)*f(x) + \text{diff}(\text{diff}(y(x), x), x) + g(x)*j(y(x)) = 0, y(x))$$

2.1644 ODE No. 1644

$$h(y(x))y'(x)^2 + j(y(x))y'(x) + k(y(x)) + y''(x) = 0$$

✗ **Mathematica** : cpu = 41.9911 (sec), leaf count = 0 , could not solve

$$\text{DSolve}[k[y[x]] + j[y[x]]*\text{Derivative}[1][y][x] + h[y[x]]*\text{Derivative}[1][y][x]^2 + \text{Derivative}[2][y][x], y[x], x]$$

✓ **Maple** : cpu = 0.787 (sec), leaf count = 56

$$\left\{ y(x) = \text{ODESolStruc} \left(\_a, \left[\left\{ \_b(\_a)^2 h(\_a) + \_b(\_a) \left(\frac{d}{d\_a} \_b(\_a) \right) + \_b(\_a) + k(\_a) = 0 \right\}, \left\{ \_a = y(x), \_b(\_a) = \frac{d}{dx} y(x) \right\} \right. \right. \right.$$

2.1645 ODE No. 1645

$$(y'(x)^2 + 1)(y'(x)h(x, y(x)) + j(x, y(x))) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.0873754 (sec), leaf count = 0 , could not solve

$$\text{DSolve}[(j[x, y[x]] + h[x, y[x]]*\text{Derivative}[1][y][x])*(1 + \text{Derivative}[1][y][x]^2) + \text{Derivative}[2][y][x], y[x], x]$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

$$\text{dsolve}(\text{diff}(\text{diff}(y(x), x), x) + (\text{diff}(y(x), x)^2 + 1)*(h(x, y(x))*\text{diff}(y(x), x) + j(x, y(x))) = 0, y(x))$$

2.1646 ODE No. 1646

$$ay(x)\left(y'(x)^2+1\right)^2+y''(x)=0$$

✓ **Mathematica** : cpu = 10.8422 (sec), leaf count = 262

$$\left\{\left\{y(x) \rightarrow \text{InverseFunction}\left[-\frac{\sqrt{\frac{\#1^2(-a)+1+2c_1}{1+2c_1}}\sqrt{2\#1^2a-4c_1}E\left(\sin^{-1}\left(\sqrt{\frac{a}{2c_1+1}}\#1\right)\left|1+\frac{1}{2c_1}\right.\right)}{\sqrt{\frac{a}{1+2c_1}}\sqrt{\#1^2(-a)+1+2c_1}\sqrt{2-\frac{\#1^2a}{c_1}}}\&\right][x+c_2]\right\},\left\{y(x) \rightarrow \text{InverseFunction}\left[-\frac{\sqrt{\frac{\#1^2(-a)+1+2c_1}{1+2c_1}}\sqrt{2\#1^2a-4c_1}E\left(\sin^{-1}\left(\sqrt{\frac{a}{2c_1+1}}\#1\right)\left|1+\frac{1}{2c_1}\right.\right)}{\sqrt{\frac{a}{1+2c_1}}\sqrt{\#1^2(-a)+1+2c_1}\sqrt{2-\frac{\#1^2a}{c_1}}}\&\right][x+c_2]\right\}\right\}$$

✓ **Maple** : cpu = 1.322 (sec), leaf count = 94

$$\left\{-c_2-x+\int^{y(x)}\frac{\left(-a^2+2c_1\right)a}{\sqrt{-\left(\left(-a^2+2c_1\right)a-1\right)\left(-a^2+2c_1\right)a}}d\_a=0,-c_2-x+\int^{y(x)}-\frac{\left(-a^2+2c_1\right)a}{\sqrt{-\left(\left(-a^2+2c_1\right)a-1\right)\left(-a^2+2c_1\right)a}}d\_a=0\right\}$$

2.1647 ODE No. 1647

$$y''(x)-a\left(xy'(x)-y(x)\right)^r=0$$

✓ **Mathematica** : cpu = 0.507856 (sec), leaf count = 91

$$\left\{\left\{y(x) \rightarrow x\left(c_2-x\left(2-\frac{a(r-1)x^2}{c_1}\right)^{\frac{1}{r-1}}\left(x^{2r-2}\left(-a(r-1)x^2+2c_1\right)\right)^{\frac{1}{1-r}}{}_2F_1\left(-\frac{1}{2},\frac{1}{r-1};\frac{1}{2};\frac{a(r-1)x^2}{2c_1}\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.69 (sec), leaf count = 60

$$\left\{y(x)=\left(c_2+\int-\frac{\left((r-1)a\,x^2-c_1\right)2^{\frac{r}{r-1}}\left(-\frac{1}{(r-1)a\,x^2-c_1}\right)^{\frac{r}{r-1}}}{2x^2}dx\right)x\right\}$$

2.1648 ODE No. 1648

$$y''(x)-kx^ay(x)^by'(x)^c=0$$

✗ **Mathematica** : cpu = 0.0735638 (sec), leaf count = 0 , could not solve

`DSolve[-(k*x^a*y[x]^b*Derivative[1][y][x]^c) + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 1.552 (sec), leaf count = 205

$$\left\{y(x)=\text{ODESolStruc}\left[_ae^{c_1+\int\_b(\_a)d\_a},\left[\left[\frac{d}{d\_a}\_b(\_a)=\frac{\left(-(b+c-1)^2k\_a^b\left(-\frac{(a-c+2)(\_a\_b(\_a)+1)}{(b+c-1)\_b(\_a)}\right)^c\_b(\_a)+(a-c+2)((a-c+2)^2\right)}{(a-c+2)^2}\right]\right]\right\}$$

2.1649 ODE No. 1649

$$h(x,y(x))\left(y'(x)-\frac{y(x)}{x}\right)^a+y''(x)=0$$

✗ **Mathematica** : cpu = 2.65477 (sec), leaf count = 0 , could not solve

`DSolve[h[x, y[x]]*(-(y[x]/x) + Derivative[1][y][x])^a + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x),x),x)+(diff(y(x),x)-1/x*y(x))^a*h(x,y(x))=0,y(x))`

2.1650 ODE No. 1650

$$y''(x) - a\sqrt{y'(x)^2 + 1} = 0$$

✓ **Mathematica** : cpu = 0.119207 (sec), leaf count = 30

$$\left\{\left\{y(x) \rightarrow \frac{\cosh(c_1) \cosh(ax)}{a} + \frac{\sinh(c_1) \sinh(ax)}{a} + c_2\right\}\right\}$$

✓ **Maple** : cpu = 2.766 (sec), leaf count = 16

$$\left\{y(x) = c_2 + \frac{\cosh((c_1 + x)a)}{a}\right\}$$

2.1651 ODE No. 1651

$$a\left(-\sqrt{y'(x)^2 + 1}\right) - b + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.369387 (sec), leaf count = 414

$$\left\{\left\{y(x) \rightarrow \frac{a \operatorname{InverseFunction}\left[\frac{b \tan^{-1}\left(\frac{\#1 b}{\sqrt{\#1^2 + 1} \sqrt{a^2 - b^2}}\right) - \frac{b \tan^{-1}\left(\frac{\#1 a}{\sqrt{a^2 - b^2}}\right) + \sinh^{-1}(\#1)}{\sqrt{a^2 - b^2}}}{a}\right] \& [x + c_1]^2 - b}{\sqrt{1 + \operatorname{InverseFunction}\left[\frac{b \tan^{-1}\left(\frac{\#1 b}{\sqrt{\#1^2 + 1} \sqrt{a^2 - b^2}}\right) - \frac{b \tan^{-1}\left(\frac{\#1 a}{\sqrt{a^2 - b^2}}\right) + \sinh^{-1}(\#1)}{\sqrt{a^2 - b^2}}\right]}}}\right\}\right\}$$

✓ **Maple** : cpu = 2.411 (sec), leaf count = 31

$$\left\{y(x) = c_2 + \int \operatorname{RootOf}\left(c_1 + x - \left(\int^{-Z} \frac{1}{\sqrt{-f^2 + 1} a + b} d_f\right)\right) dx\right\}$$

2.1652 ODE No. 1652

$$y''(x) - a\sqrt{by(x)^2 + y'(x)^2} = 0$$

✓ **Mathematica** : cpu = 0.504302 (sec), leaf count = 76

$$\operatorname{Solve}\left[\int_1^{y(x)} \frac{1}{\operatorname{InverseFunction}\left[\int \frac{\#1}{K[1] \left(\frac{\#1^2}{K[1]^2} - a \sqrt{\frac{\#1^2}{K[1]^2} + b}\right)} d \frac{\#1}{K[1]} \& [c_1 - \log(K[1])]\right]} dK[1] = x - c_2, y(x)\right]$$

✓ **Maple** : cpu = 2.596 (sec), leaf count = 36

$$\left\{y(x) = e^{c_2 + \int \operatorname{RootOf}\left(c_1 + x - \left(\int^{-Z} \frac{1}{-\sqrt{f^2 + b} a} d_f\right)\right) dx}\right\}$$

2.1653 ODE No. 1653

$$y''(x) - a(y'(x)^2 + 1)^{3/2} = 0$$

✓ **Mathematica** : cpu = 0.173018 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow c_2 - \frac{i\sqrt{a^2x^2 + 2ac_1x - 1 + c_1^2}}{a} \right\}, \left\{ y(x) \rightarrow \frac{i\sqrt{a^2x^2 + 2ac_1x - 1 + c_1^2}}{a} + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 2.365 (sec), leaf count = 41

$$\left\{ y(x) = \frac{c_2a + ((c_1 + x)^2a^2 - 1)\sqrt{-\frac{1}{(c_1+x)^2a^2-1}}}{a} \right\}$$

2.1654 ODE No. 1654

$$y''(x) - 2ax(y'(x)^2 + 1)^{3/2} = 0$$

✓ **Mathematica** : cpu = 0.548774 (sec), leaf count = 308

$$\left\{ \left\{ y(x) \rightarrow c_2 - \frac{\sqrt{\frac{ax^2-1+c_1}{-1+c_1}} \sqrt{\frac{ax^2+1+c_1}{1+c_1}} \left(F\left(i \sinh^{-1}\left(x \sqrt{\frac{a}{c_1+1}}\right) \middle| \frac{c_1+1}{c_1-1} \right) + (-1+c_1)E\left(i \sinh^{-1}\left(x \sqrt{\frac{a}{c_1+1}}\right) \middle| \frac{c_1+1}{c_1-1} \right) \right)}{\sqrt{\frac{a}{1+c_1}} \sqrt{a^2x^4 + 2ac_1x^2 - 1 + c_1^2}} \right\}, \left\{ y(x) \rightarrow \sqrt{\frac{a}{1+c_1}} \sqrt{a^2x^4 + 2ac_1x^2 - 1 + c_1^2} \right\} \right\}$$

✓ **Maple** : cpu = 2.41 (sec), leaf count = 38

$$\left\{ y(x) = c_2 + \int \sqrt{-\frac{1}{(x^2 + 2c_1)^2a^2 - 1}} (x^2 + 2c_1)adx \right\}$$

2.1655 ODE No. 1655

$$y''(x) - ay(x)(y'(x)^2 + 1)^{3/2} = 0$$

✓ **Mathematica** : cpu = 1.07784 (sec), leaf count = 350

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[-\frac{\sqrt{\frac{\#1^2a-2+2c_1}{-1+c_1}} \sqrt{\frac{\#1^2a+2+2c_1}{1+c_1}} \left(F\left(i \sinh^{-1}\left(\sqrt{\frac{a}{2c_1+2}} \#1\right) \middle| \frac{c_1+1}{c_1-1} \right) + (-1+c_1)E\left(i \sinh^{-1}\left(\sqrt{\frac{a}{2c_1+2}} \#1\right) \middle| \frac{c_1+1}{c_1-1} \right) \right)}{\sqrt{\frac{a}{2+2c_1}} \sqrt{\#1^4a^2 + 4\#1^2ac_1 - 4 + 4c_1^2}} \right] \right\} \right\}$$

✓ **Maple** : cpu = 2.516 (sec), leaf count = 84

$$\left\{ -c_2 - x + \int^{y(x)} \frac{(-a^2 + 2c_1)a}{\sqrt{-(-a^2 + 2c_1)^2a^2 + 4}} d\_a = 0, -c_2 - x + \int^{y(x)} -\frac{(-a^2 + 2c_1)a}{\sqrt{-(-a^2 + 2c_1)^2a^2 + 4}} d\_a = 0 \right\}$$

2.1656 ODE No. 1656

$$y''(x) - a(y'(x)^2 + 1)^{3/2}(bx + c + y(x)) = 0$$

✓ **Mathematica** : cpu = 43.6659 (sec), leaf count = 9706

$$\left\{ \text{Solve} \left[\int_1^x \left(\frac{1}{b^2 + 1} - \frac{b\sqrt{-a^2(c^2 + 2bK[1]c + 2y(x)c + b^2K[1]^2 + y(x)^2 - 2c_1 + 2bK[1]y(x))^2(a^2c^4 + 4a^2bK[1]c^3 + 4a^2y(x)c^3 + (b^2 + 1)(a^2c^4 + 4a^2bK[1]c^3 + 4a^2y(x)c^3 + 6a^2b^2K[1]^2c^2 + 6a^2y(x)c^3)}}{(b^2 + 1)(a^2c^4 + 4a^2bK[1]c^3 + 4a^2y(x)c^3 + 6a^2b^2K[1]^2c^2 + 6a^2y(x)c^3)}} \right) dx \right] \right\}$$

✓ **Maple** : cpu = 3.269 (sec), leaf count = 771

$$\left\{y(x) = -bx + \text{RootOf}\left(c_2 - x + \int^{-Z} \frac{{_f^4 a^2 b^2 + 4\_f^3 a^2 b^2 c + 4\_f^2 a^2 b^2 c^2 - 4c_1\_f^2 a^2 b^2 - 8c_1\_f a^2 b^2 c + 4c_1^2 a^2 b^2 - 4b^4 - \sqrt{-}}{ }\right.\right.$$

2.1657 ODE No. 1657

$$y''(x)+y(x)^3y'(x)-y(x)y'(x)\sqrt{4y'(x)+y(x)^4}=0$$

✓ **Mathematica** : cpu = 0.436865 (sec), leaf count = 192

$$\left\{\left\{y(x) \rightarrow -\frac{i(\cosh(c_1)+\sinh(c_1))(\cos(2(x+c_2)(\cosh(3c_1)+\sinh(3c_1))))+i\sin(2(x+c_2)(\cosh(3c_1)+\sinh(3c_1))))-1}{\cos(2(x+c_2)(\cosh(3c_1)+\sinh(3c_1))))+i\sin(2(x+c_2)(\cosh(3c_1)+\sinh(3c_1))))+1}\right.\right.$$

✓ **Maple** : cpu = 2.586 (sec), leaf count = 35

$$\left\{y(x)=\frac{\tan\left(\frac{c_2+x}{\left(c_1^2\right)^{\frac{3}{2}}}\right)}{c_1},y(x)=\frac{\tanh\left(\frac{c_2+x}{\left(c_1^2\right)^{\frac{3}{2}}}\right)}{c_1}\right\}$$

2.1658 ODE No. 1658

$$y''(x)-h\big(y'(x),ax+by(x)\big)=0$$

✗ **Mathematica** : cpu = 0.383215 (sec), leaf count = 0 , could not solve

```
DSolve[-h[Derivative[1][y][x], a*x + b*y[x]] + Derivative[2][y][x] == 0, y[x], x]
```

✓ **Maple** : cpu = 0.232 (sec), leaf count = 115

$$\left\{y(x)=ODESolStruc\left(-\frac{-_ab+\left(c_1+\int\_b(_a)d\_a\right)a}{b},\left\{\frac{d}{d\_a}b(_a)=-_b(_a)^3h\left(\frac{-_a\_b(_a)+b}{b\_b(_a)},\_ab\right)\right\},\left\{-a=\frac{ax+by(x)}{b}\right.\right.$$

2.1659 ODE No. 1659

$$y''(x)-y(x)h\left(x,\frac{y'(x)}{y(x)}\right)=0$$

✗ **Mathematica** : cpu = 7.27977 (sec), leaf count = 0 , could not solve

```
DSolve[-(h[x, Derivative[1][y][x]/y[x]]*y[x]) + Derivative[2][y][x] == 0, y[x], x]
```

✓ **Maple** : cpu = 0.165 (sec), leaf count = 60

$$\left\{y(x)=ODESolStruc\left(e^{c_1+\int\_b(_a)d\_a},\left\{\frac{d}{d\_a}b(_a)=-_b(_a)^2+h(_a,_b(_a))\right\},\left\{-a=x,_b(_a)=\frac{\frac{d}{dx}y(x)}{y(x)}\right\},\left\{x=_a,y(x)=\frac{y(x)}{b(_a)}\right.\right.$$

2.1660 ODE No. 1660

$$y''(x)-x^{n-2}h\big(x^{-n}y(x),x^{1-n}y'(x)\big)=0$$

✗ **Mathematica** : cpu = 3.13394 (sec), leaf count = 0 , could not solve

```
DSolve[-(x^(-2 + n)*h[y[x]/x^n, x^(1 - n)*Derivative[1][y][x]]) + Derivative[2][y][x] == 0, y[x], x]
```

✓ **Maple** : cpu = 0.821 (sec), leaf count = 125

$$\left\{y(x) = ODESolStruc\left(-a e^{(c_1 + \int -b(_a)d_a)^n}, \left[\left\{\frac{d}{d_a}b(_a) = \left((n-1)an_b(_a) - b(_a)h\left(-a, \frac{an_b(_a) + 1}{b(_a)}\right) + 2n - 1\right)b(_a)\right.\right.\right.\right.$$

2.1661 ODE No. 1661

$$8y''(x) + 9y'(x)^4 = 0$$

✓ **Mathematica** : cpu = 0.0835306 (sec), leaf count = 92

$$\left\{\left\{y(x) \rightarrow c_2 - \frac{1}{3} \sqrt[3]{-\frac{1}{3} (9x - 8c_1)^{2/3}}\right\}, \left\{y(x) \rightarrow \frac{(9x - 8c_1)^{2/3}}{3\sqrt[3]{3}} + c_2\right\}, \left\{y(x) \rightarrow \frac{(-1)^{2/3} (9x - 8c_1)^{2/3}}{3\sqrt[3]{3}} + c_2\right\}\right\}$$

✓ **Maple** : cpu = 1.227 (sec), leaf count = 51

$$\left\{y(x) = c_2 + (c_1 + x)^{\frac{2}{3}}, y(x) = c_2 - \frac{(c_1 + x)^{\frac{2}{3}}(1 + i\sqrt{3})}{2}, y(x) = c_2 + \frac{(c_1 + x)^{\frac{2}{3}}(i\sqrt{3} - 1)}{2}\right\}$$

2.1662 ODE No. 1662

$$ay''(x) + cy(x) + h(y'(x)) = 0$$

✗ **Mathematica** : cpu = 0.979917 (sec), leaf count = 0 , could not solve

DSolve[h[Derivative[1][y][x]] + c*y[x] + a*Derivative[2][y][x] == 0, y[x], x]

✓ **Maple** : cpu = 0.464 (sec), leaf count = 56

$$\left\{y(x) = ODESolStruc\left(-a, \left[\left\{-b(_a)\left(\frac{d}{d_a}b(_a)\right) + \frac{ac + h(b(_a))}{a} = 0\right\}, \left\{-a = y(x), -b(_a) = \frac{d}{dx}y(x)\right\}, \left\{x = c_1 + \int -\frac{1}{b(_a)}d_a\right\}\right.\right.\right.$$

2.1663 ODE No. 1663

$$-xy(x)^n + xy''(x) + 2y'(x) = 0$$

✗ **Mathematica** : cpu = 0.0306819 (sec), leaf count = 0 , could not solve

DSolve[-(x*y[x]^n) + 2*Derivative[1][y][x] + x*Derivative[2][y][x] == 0, y[x], x]

✓ **Maple** : cpu = 1.092 (sec), leaf count = 125

$$\left\{y(x) = ODESolStruc\left(-a e^{c_1 + \int -b(_a)d_a}, \left[\left\{\frac{d}{d_a}b(_a) = -\frac{(2(n-3)a_b(_a) + (n-1)^2a^n_b(_a) + 2n-10)b(_a)^2}{4}\right\}, \left\{\frac{d}{d_a}b(_a) = -\frac{(2(n-3)a_b(_a) + (n-1)^2a^n_b(_a) + 2n-10)b(_a)^2}{4}\right\}\right.\right.\right.$$

2.1664 ODE No. 1664

$$ax^m y(x)^n + xy''(x) + 2y'(x) = 0$$

✗ **Mathematica** : cpu = 0.202213 (sec), leaf count = 0 , could not solve

DSolve[a*x^m*y[x]^n + 2*Derivative[1][y][x] + x*Derivative[2][y][x] == 0, y[x], x]

✓ **Maple** : cpu = 3.128 (sec), leaf count = 155

$$\left\{y(x) = ODESolStruc\left(-a e^{c_1 + \int -b(_a)d_a}, \left[\left\{\frac{d}{d_a}b(_a) = \frac{((n-1)^2a_a^n_b(_a) + (m+1)((m-n+2)a_b(_a) + 2m-n+1)b(_a)^2)}{(m+1)^2}\right\}, \left\{\frac{d}{d_a}b(_a) = \frac{((n-1)^2a_a^n_b(_a) + (m+1)((m-n+2)a_b(_a) + 2m-n+1)b(_a)^2)}{(m+1)^2}\right\}\right.\right.\right.$$

2.1665 ODE No. 1665

$$xy''(x) + 2y'(x) + xe^{y(x)} = 0$$

✗ **Mathematica** : cpu = 0.119726 (sec), leaf count = 0 , could not solve

`DSolve[E^y[x]*x + 2*Derivative[1][y][x] + x*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.552 (sec), leaf count = 84

$$\left\{y(x) = ODESolStruc\left(-2c_1 + {}_a - 2\left(\int {}_b({}_a)d{}_a\right), \left[\left\{\frac{d}{{}_d{}_a}{}_b({}_a) = (e^{-a} - 2){}_b({}_a)^3 + {}_b({}_a)^2\right\}, \left\{{}_a = 2\ln(x) + y(x),\right.\right.\right.$$

2.1666 ODE No. 1666

$$ay'(x) + bxe^{y(x)} + xy''(x) = 0$$

✗ **Mathematica** : cpu = 0.291787 (sec), leaf count = 0 , could not solve

`DSolve[b*E^y[x]*x + a*Derivative[1][y][x] + x*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 1.038 (sec), leaf count = 93

$$\left\{y(x) = ODESolStruc\left(-2c_1 + {}_a - 2\left(\int {}_b({}_a)d{}_a\right), \left[\left\{\frac{d}{{}_d{}_a}{}_b({}_a) = (b\,e^{-a} - 2a + 2){}_b({}_a)^3 + (a - 1){}_b({}_a)^2\right\}, \left\{{}_a = 2\right.\right.\right.$$

2.1667 ODE No. 1667

$$bx^{5-2a}e^{y(x)} + ay'(x) + xy''(x) = 0$$

✗ **Mathematica** : cpu = 0.353484 (sec), leaf count = 0 , could not solve

`DSolve[b*E^y[x]*x^(5 - 2*a) + a*Derivative[1][y][x] + x*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 1.448 (sec), leaf count = 121

$$\left\{y(x) = ODESolStruc\left(2c_1a - 6c_1 + {}_a + (2a - 6)\left(\int {}_b({}_a)d{}_a\right), \left[\left\{\frac{d}{{}_d{}_a}{}_b({}_a) = \left(2a^2 + b\,e^{-a} - 8a + 6\right){}_b({}_a)^3 + (a - 1\right.\right.\right.$$

2.1668 ODE No. 1668

$$xy''(x) + (y(x) - 1)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0587082 (sec), leaf count = 60

$$\left\{\left\{y(x) \rightarrow 2 + \sqrt{2}\sqrt{2 + c_1}\tanh\left(\frac{1}{2}\left(\sqrt{2}\sqrt{2 + c_1}\log(x) - 2\sqrt{2}\sqrt{2 + c_1}c_2\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 1.286 (sec), leaf count = 24

$$\left\{y(x) = \frac{2c_1 + \tanh\left(\frac{-c_2 + \ln(x)}{2c_1}\right)}{c_1}\right\}$$

2.1669 ODE No. 1669

$$-x^2y'(x)^2 + xy''(x) + 2y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.501198 (sec), leaf count = 160

$$\text{Solve}\left[\int_1^{y(x)} -\frac{x}{e^{xK[1]}c_1 + 2xK[1] + 1}dK[1] - \int_1^x \left(\int_1^{y(x)} \left(\frac{e^{K[1]K[2]}c_1K[1] + 2K[1]}{(e^{K[1]K[2]}c_1 + 2K[1]K[2] + 1)^2} - \frac{1}{e^{K[1]K[2]}c_1 + 2K[1]K[2] + 1}\right)dK[2]\right. \right.$$

✓ **Maple** : cpu = 1.308 (sec), leaf count = 32

$$\left\{y(x) = \frac{\text{RootOf}\left(c_2 + \int^{-Z} -\frac{1}{c_1e^{\underline{f}-2\underline{f}-1}}d\underline{f} - \ln(x)\right)}{x}\right\}$$

2.1670 ODE No. 1670

$$a\left(xy'(x) - y(x)\right)^2 - b + xy''(x) = 0$$

✓ **Mathematica** : cpu = 7.64263 (sec), leaf count = 51

$$\left\{\left\{y(x) \rightarrow x\left(\int_1^x \frac{\sqrt{-\frac{b}{a}} \tan\left(c_1 - a\sqrt{-\frac{b}{a}} K[2]\right)}{K[2]^2}dK[2] + c_2\right)\right\}\right\}$$

✓ **Maple** : cpu = 1.733 (sec), leaf count = 35

$$\left\{y(x) = \left(c_2 + \int \frac{i\sqrt{b} \tan\left(-i\sqrt{a} \sqrt{b} x + c_1\right)}{\sqrt{a} x^2}dx\right)x\right\}$$

2.1671 ODE No. 1671

$$2xy''(x) + y'(x)^3 + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.116508 (sec), leaf count = 59

$$\left\{\left\{y(x) \rightarrow c_2 - 2ie^{c_1}\sqrt{-x + e^{2c_1}}\right\}, \left\{y(x) \rightarrow 2ie^{c_1}\sqrt{-x + e^{2c_1}} + c_2\right\}\right\}$$

✓ **Maple** : cpu = 1.225 (sec), leaf count = 35

$$\left\{y(x) = c_2 - \frac{2\sqrt{c_1x - 1}}{c_1}, y(x) = c_2 + \frac{2\sqrt{c_1x - 1}}{c_1}\right\}$$

2.1672 ODE No. 1672

$$x^2y''(x) - a\left(y(x)^n - y(x)\right) = 0$$

✗ **Mathematica** : cpu = 12.8657 (sec), leaf count = 0 , could not solve

`DSolve[-(a*(-y[x] + y[x]^n)) + x^2*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 1.155 (sec), leaf count = 65

$$\left\{y(x) = \text{ODESolStruc}\left[_a, \left[\left\{\frac{d}{d\_a}b(_a) = (_aa\_b(_a) - a\_a^n\_b(_a) - 1)_b(_a)^2\right\}, \left\{_a = y(x),\_b(_a) = \frac{1}{x\left(\frac{d}{dx}y(x)\right)}\right\}\right], \left\{x\right.\right.$$

2.1673 ODE No. 1673

$$a\left(e^{y(x)}-1\right)+x^2y''(x)=0$$

✗ **Mathematica** : cpu = 26.4981 (sec), leaf count = 0 , could not solve

`DSolve[a*(-1 + E^y[x]) + x^2*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.797 (sec), leaf count = 60

$$\left\{y(x)=ODESolStruc\left[_a,\left[\left\{\frac{d}{d\_a}b(_a)=((e^{-a}-1)a\_b(_a)-1)\_b(_a)^2\right\},\left\{_a=y(x),\_b(_a)=\frac{1}{x\left(\frac{d}{dx}y(x)\right)}\right\},\left\{x=e^{c_1+\int}\right\}\right.\right.\right.$$

2.1674 ODE No. 1674

$$y(x)\left(a(a+b)+b^2c^2x^{2b}\right)-x(2a+b-1)y'(x)+x^2y''(x)=0$$

✓ **Mathematica** : cpu = 0.0460053 (sec), leaf count = 106

$$\left\{\left\{y(x)\rightarrow c_12^{-\frac{a}{b}}c^{\frac{a}{b}}\left(x^{2b}\right)^{\frac{a}{2b}}\cos\left(c\sqrt{x^{2b}}\right)+c_22^{-\frac{a+b}{b}}c^{\frac{a+b}{b}-1}\left(x^{2b}\right)^{\frac{a+b}{2b}-\frac{1}{2}}\sin\left(c\sqrt{x^{2b}}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.146 (sec), leaf count = 25

$$\{y(x)=\left(c_1\sin\left(c\,x^b\right)+c_2\cos\left(c\,x^b\right)\right)x^a\}$$

2.1675 ODE No. 1675

$$x^k\left(-h\left(x^ky(x),ky(x)+xy'(x)\right)\right)+(a+1)xy'(x)+x^2y''(x)=0$$

✗ **Mathematica** : cpu = 2.35555 (sec), leaf count = 0 , could not solve

`DSolve[-(x^k*h[x^k*y[x], k*y[x] + x*Derivative[1][y][x]]) + (1 + a)*x*Derivative[1][y][x] +`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(x^2*diff(diff(y(x),x),x)+(a+1)*x*diff(y(x),x)-x^k*h(x^k*y(x),x*diff(y(x),x)+k*y(x))=0`

2.1676 ODE No. 1676

$$a\left(xy'(x)-y(x)\right)^2-bx^2+x^2y''(x)=0$$

✓ **Mathematica** : cpu = 0.363928 (sec), leaf count = 134

$$\left\{\left\{y(x)\rightarrow x\left(\int_1^x\frac{i\sqrt{a}\sqrt{b}Y_1\left(-i\sqrt{a}\sqrt{b}K[2]\right)-i\sqrt{a}\sqrt{b}J_1\left(i\sqrt{a}\sqrt{b}K[2]\right)c_1}{a\left(Y_0\left(-i\sqrt{a}\sqrt{b}K[2]\right)+J_0\left(i\sqrt{a}\sqrt{b}K[2]\right)c_1\right)K[2]}dK[2]+c_2\right)\right\}\right\}$$

✓ **Maple** : cpu = 1.323 (sec), leaf count = 72

$$\left\{y(x)=\left(c_2+\int-\frac{\sqrt{-ab}\left(c_1\operatorname{BesselY}\left(1,\sqrt{-ab}\,x\right)+\operatorname{BesselJ}\left(1,\sqrt{-ab}\,x\right)\right)}{\left(c_1\operatorname{BesselY}\left(0,\sqrt{-ab}\,x\right)+\operatorname{BesselJ}\left(0,\sqrt{-ab}\,x\right)\right)ax}dx\right)x\right\}$$

2.1677 ODE No. 1677

$$ay(x)y'(x)^2+bx+x^2y''(x)=0$$

✗ **Mathematica** : cpu = 39.2725 (sec), leaf count = 0 , could not solve

`DSolve[b*x + a*y[x]*Derivative[1][y][x]^2 + x^2*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 2.769 (sec), leaf count = 101

$$\left\{y(x)=ODESolStruc\left[_ae^{c_1+\int}b(_a)^{d\_a},\left[\left\{\frac{d}{d\_a}b(_a)=\_aa\_b(_a)+\left(_a^3a+b\right)\_b(_a)^3+\left(2\_a^2a+1\right)\_b(_a)^2\right\},\left\{_a=\frac{y}{x}\right\}\right.\right.\right.$$

2.1678 ODE No. 1678

$$x^2y''(x) - \sqrt{ax^2y'(x)^2 + by(x)^2} = 0$$

✗ **Mathematica** : cpu = 1.01481 (sec), leaf count = 0 , could not solve

`DSolve[-Sqrt[b*y[x]^2 + a*x^2*Derivative[1][y][x]^2] + x^2*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 2.408 (sec), leaf count = 60

$$\left\{ \begin{array}{l} c_2 + \int^{\ln(x)} \text{RootOf}\left(c_1 - b + \int^{-Z} -\frac{y(x)}{-a^2y(x) - ay(x) - \sqrt{(a^2a+b)y(x)^2}} d\_a\right) d\_b \\ -e \end{array} + y(x) = 0 \right\}$$

2.1679 ODE No. 1679

$$(x^2 + 1)y''(x) + y'(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 0.201155 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow -x \cot(c_1) + \csc^2(c_1) \log(-x \sin(c_1) - \cos(c_1)) + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 1.277 (sec), leaf count = 27

$$\left\{ y(x) = c_2 + \ln(c_1x - 1) + \frac{x}{c_1} + \frac{\ln(c_1x - 1)}{c_1^2} \right\}$$

2.1680 ODE No. 1680

$$x^4(-y'(x)^2) + 4x^2y''(x) + 4y(x) = 0$$

✗ **Mathematica** : cpu = 7.74229 (sec), leaf count = 0 , could not solve

`DSolve[4*y[x] - x^4*Derivative[1][y][x]^2 + 4*x^2*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 1.809 (sec), leaf count = 103

$$\left\{ y(x) = \text{ODESolStruc} \left(\_a e^{-2c_1 + \int -2\_b(\_a) d\_a}, \left[\left\{ \frac{d}{d\_a} \_b(\_a) = (-\_a^2 + 7\_a) \_b(\_a)^3 + (\_a - 5) \_b(\_a)^2 - \frac{b(\_a)}{4} \right\}, \left\{ \_a = x^2 \right\} \right] \right) \right\}$$

2.1681 ODE No. 1681

$$ay(x)^3 + 9x^2y''(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 2.76213 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow c_2 \sqrt[3]{x} \operatorname{sn} \left(\left(c_1 + \frac{\sqrt{ax^{20/3}}}{\sqrt{2}x^3} \right) c_2 \middle| -1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 1.154 (sec), leaf count = 31

$$\left\{ y(x) = c_2 x^{\frac{1}{3}} \operatorname{sn} \left(c_2 \left(c_1 + \frac{\sqrt{2} \sqrt{a} x^{\frac{20}{3}}}{2x^3} \right) \middle| i \right) \right\}$$

2.1682 ODE No. 1682

$$x^3\big(y''(x)+y(x)y'(x)-y(x)^3\big)+12xy(x)+24=0$$

✓ **Mathematica** : cpu = 22.2405 (sec), leaf count = 41

$$\left\{\left\{y(x)\rightarrow\frac{2+x^3\wp'(x+c_1;0,c_2)}{x(-1+x^2\wp(x+c_1;0,c_2))}\right\}\right\}$$

✓ **Maple** : cpu = 1.964 (sec), leaf count = 94

$$\left\{y(x)=ODESolStruc\left[_ae^{c_1+\int _b(_a)d_a},\left\{\frac{d}{d_a}b(_a)=-\left(_a+\left(_a^3+_a^2-14_a-24\right)b(_a)-3\right)b(_a)^2\right\},\left\{ _a=xy(\right.\right.$$

2.1683 ODE No. 1683

$$x^3y''(x)-a\big(xy'(x)-y(x)\big)^2=0$$

✓ **Mathematica** : cpu = 0.0888071 (sec), leaf count = 26

$$\left\{\left\{y(x)\rightarrow-\frac{x\log\left(a\left(-\frac{c_1}{x}-c_2\right)\right)}{a}\right\}\right\}$$

✓ **Maple** : cpu = 1.171 (sec), leaf count = 23

$$\left\{y(x)=-\frac{x\ln\left(\frac{(c_1x-c_2)a}{x}\right)}{a}\right\}$$

2.1684 ODE No. 1684

$$xy(x)\Big(a-2x^2y(x)^2+3xy(x)\Big)+b+2x^3y''(x)+x^2(2xy(x)+9)y'(x)=0$$

✗ **Mathematica** : cpu = 54.0549 (sec), leaf count = 0 , could not solve

`DSolve[b + x*y[x]*(a + 3*x*y[x] - 2*x^2*y[x]^2) + x^2*(9 + 2*x*y[x])*Derivative[1][y][x] + 2`

✓ **Maple** : cpu = 2.791 (sec), leaf count = 100

$$\left\{y(x)=ODESolStruc\left[_ae^{c_1+\int _b(_a)d_a},\left\{\frac{d}{d_a}b(_a)=\frac{\left(-2_a+\left(-2_a^3+_a^2+(a-5)_a+b\right)b(_a)-3\right)b(_a)^2}{2}\right\},\left\{ -\right.\right.$$

2.1685 ODE No. 1685

$$axy(x)+b-\big(kx^{k-1}-12x^2\big)\big(3y'(x)+y(x)^2\big)+2\big(4x^3-x^k\big)\big(y''(x)+y(x)y'(x)-y(x)^3\big)=0$$

✗ **Mathematica** : cpu = 2.68139 (sec), leaf count = 0 , could not solve

`DSolve[b + a*x*y[x] - (-12*x^2 + k*x^(-1 + k))*(y[x]^2 + 3*Derivative[1][y][x]) + 2*(4*x^3 - y[x]^3 + y[x]*Derivative[1][y][x] + Derivative[2][y][x]) == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(2*(-x^k+4*x^3)*(diff(diff(y(x),x),x)+y(x)*diff(y(x),x)-y(x)^3)-(k*x^(k-1)-12*x^2)*(3*diff(y(x),x)+y(x)^2)+a*x*y(x)+b=0,y(x))`

2.1686 ODE No. 1686

$$a^2y(x)^n + x^4y''(x) = 0$$

✗ **Mathematica** : cpu = 0.0320163 (sec), leaf count = 0 , could not solve

```
DSolve[a^2*y[x]^n + x^4*Derivative[2][y][x] == 0, y[x], x]
```

✓ **Maple** : cpu = 1.433 (sec), leaf count = 128

$$\left\{y(x) = ODESolStruc\left[_a e^{c_1 + \int _b(_a)d_a}, \left\{\frac{d}{d_a}b(_a) = \frac{\left((n-1)^2 a^2 _a^n _b(_a) - 2(n-3) _a _b(_a) - 2n + 10\right) _b(_a)^2}{4}\right\}\right\}, \left\{\right.$$

2.1687 ODE No. 1687

$$x^4y''(x) - x(x^2 + 2y(x))y'(x) + 4y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0793184 (sec), leaf count = 262

$$\left\{\left\{y(x) \rightarrow -\frac{x^3\left(i\left(\frac{i}{\sqrt{c_1}} - \frac{\sqrt{-1-c_1}}{\sqrt{c_1}}\right)\sqrt{c_1} c_2 x^{-1+i\left(\frac{i}{\sqrt{c_1}} - \frac{\sqrt{-1-c_1}}{\sqrt{c_1}}\right)\sqrt{c_1}} + i\left(\frac{\sqrt{-1-c_1}}{\sqrt{c_1}} + \frac{i}{\sqrt{c_1}}\right)\sqrt{c_1} x^{-1+i\left(\frac{\sqrt{-1-c_1}}{\sqrt{c_1}} + \frac{i}{\sqrt{c_1}}\right)\sqrt{c_1}}\right)}{c_2 x^{i\left(\frac{i}{\sqrt{c_1}} - \frac{\sqrt{-1-c_1}}{\sqrt{c_1}}\right)\sqrt{c_1}} + x^{i\left(\frac{\sqrt{-1-c_1}}{\sqrt{c_1}} + \frac{i}{\sqrt{c_1}}\right)\sqrt{c_1}}}\right\}\right\}$$

✓ **Maple** : cpu = 1.247 (sec), leaf count = 21

$$\{y(x) = (c_1 \tanh(c_1(c_2 - \ln(x)))) + 1)x^2\}$$

2.1688 ODE No. 1688

$$x^4y''(x) - x^2y'(x)(y'(x) + x) + 4y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.594244 (sec), leaf count = 189

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{-e^{\frac{K[1]}{x^2}} c_1 x^2 + 2x^2 + 4K[1]} dK[1] - \int_1^x \left(\frac{K[2]\left(e^{\frac{y(x)}{K[2]^2}} c_1 + 2\left(-\frac{y(x)}{K[2]^2} - 1\right)\right)}{-e^{\frac{y(x)}{K[2]^2}} c_1 K[2]^2 + 2K[2]^2 + 4y(x)} + \int_1^{y(x)} -\frac{\frac{K[1]}{2e^{\frac{K[2]^2}} c_1 K[1]} - 2e^{\frac{K[1]}{K[2]^2}} c_1 K[1]}{\left(-e^{\frac{K[1]}{K[2]^2}} c_1 K[2]^2 + 2K[2]^2\right)} dK[1]\right]$$

✓ **Maple** : cpu = 1.247 (sec), leaf count = 32

$$\left\{y(x) = x^2 \text{RootOf}\left(c_2 - \left(\int^{-Z} \frac{1}{c_1 e^f + 4_f + 2} d_f\right) - \ln(x)\right)\right\}$$

2.1689 ODE No. 1689

$$x^4y''(x) + (xy'(x) - y(x))^3 = 0$$

✓ **Mathematica** : cpu = 0.30904 (sec), leaf count = 104

$$\left\{\left\{y(x) \rightarrow -ix \log\left(\frac{\frac{e^{c_2}}{x} - \frac{\sqrt{e^{2c_2} - 8ic_1 x^2}}{x}}{4c_1}\right)\right\}, \left\{y(x) \rightarrow -ix \log\left(\frac{\frac{\sqrt{e^{2c_2} - 8ic_1 x^2}}{x} + \frac{e^{c_2}}{x}}{4c_1}\right)\right\}\right\}$$

✓ **Maple** : cpu = 1.296 (sec), leaf count = 37

$$\left\{y(x) = \left(c_2 - \arctan\left(\frac{1}{\sqrt{c_1 x^2 - 1}}\right)\right)x, y(x) = \left(c_2 + \arctan\left(\frac{1}{\sqrt{c_1 x^2 - 1}}\right)\right)x\right\}$$

2.1690 ODE No. 1690

$$\sqrt{x}y''(x)-y(x)^{3/2}=0$$

✗ **Mathematica** : cpu = 21.7667 (sec), leaf count = 0 , could not solve

```
DSolve[-y[x]^(3/2) + Sqrt[x]*Derivative[2][y][x] == 0, y[x], x]
```

✓ **Maple** : cpu = 2.916 (sec), leaf count = 99

$$\left\{y(x)=ODESolStruc\left(-a\,e^{-3c_1+\int -3\_b(_a)d\_a},\left[\left\{\frac{d}{d\_a}b(_a)=-\_a^{\frac{3}{2}}b(_a)^3+12\_a\_b(_a)^3-7\_b(_a)^2\right\},\left\{-a=x^3y(x),\_b(_a)=\frac{1}{x^3y(x)}\right\}\right]\right\}$$

2.1691 ODE No. 1691

$$y''(x)(ax^2+bx+c)^{3/2}-f\left(\frac{y(x)}{\sqrt{ax^2+bx+c}}\right)=0$$

✓ **Mathematica** : cpu = 61.0355 (sec), leaf count = 251

$$\left\{\text{Solve}\left[2a\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)+2\sqrt{4ac-b^2}\int_1^{\frac{y(x)}{\sqrt{c+x(b+ax)}}}\frac{a}{\sqrt{4c_1a^2+(b^2-4ac)K[2]^2+8\int_1^{K[2]}f(K[1])dK[1]}}dK[2]=c_2V\right]\right\}$$

✓ **Maple** : cpu = 0.948 (sec), leaf count = 254

$$\left\{y(x)=\sqrt{a\,x^2+bx+c}\,\text{RootOf}\left(-2a\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)+c_2\sqrt{4ac-b^2}-2\sqrt{4ac-b^2}\left(\int^{-Z}\frac{a}{\sqrt{-4\_g^2ac+_g^2b^2+4c_1}}d\_g\right)\right)\right\}$$

2.1692 ODE No. 1692

$$x^{\frac{n}{n+1}}y''(x)-y(x)^{\frac{2n+1}{n+1}}=0$$

✗ **Mathematica** : cpu = 0.0664395 (sec), leaf count = 0 , could not solve

```
DSolve[-y[x]^((1 + 2*n)/(1 + n)) + x^(n/(1 + n))*Derivative[2][y][x] == 0, y[x], x]
```

✓ **Maple** : cpu = 4.227 (sec), leaf count = 156

$$\left\{y(x)=ODESolStruc\left(-a\,e^{-\frac{(c_1+\int\_b(_a)d\_a)(n+2)}{n}},\left[\left\{\frac{d}{d\_a}b(_a)=\frac{2\left(-\frac{n^2\_a^{\frac{2n+1}{n+1}}\_b(_a)}{2}+(n+2)(n+1)\_a\_b(_a)-\frac{3n^2}{2}-2n\right)\_b(_a)}{n^2}\right\}\right]\right\}$$

2.1693 ODE No. 1693

$$-h\big(y(x),f(x)y'(x)\big)+f(x)f'(x)y'(x)+f(x)^2y''(x)=0$$

✗ **Mathematica** : cpu = 0.829396 (sec), leaf count = 0 , could not solve

```
DSolve[-h[y[x], f[x]*Derivative[1][y][x]] + f[x]*Derivative[1][f][x]*Derivative[1][y][x] + f[x]^2Derivative[2][y][x] == 0, y[x], x]
```

✓ **Maple** : cpu = 0.326 (sec), leaf count = 68

$$\left\{y(x)=ODESolStruc\left(-a,\left[\left\{\frac{d}{d\_a}b(_a)=-\_b(_a)^3h\left(-a,\frac{1}{\_b(_a)}\right)\right\},\left\{-a=y(x),\_b(_a)=\frac{1}{\left(\frac{d}{dx}y(x)\right)f(x)}\right\}\right]\right\},\left\{x=\text{RootOf}\left(\left(\frac{d}{dx}y(x)\right)f(x)\right)\right\}$$

2.1694 ODE No. 1694

$$y(x)y''(x) - a = 0$$

✓ **Mathematica** : cpu = 0.134934 (sec), leaf count = 115

$$\left\{ \left\{ y(x) \rightarrow \exp \left(\frac{-2a \operatorname{erf}^{-1} \left(-i \sqrt{\frac{2}{\pi}} \sqrt{a e^{\frac{c_1}{a}} (x + c_2)^2} \right)^2 - c_1}{2a} \right) \right\} \right\}, \left\{ \left\{ y(x) \rightarrow \exp \left(\frac{-2a \operatorname{erf}^{-1} \left(i \sqrt{\frac{2}{\pi}} \sqrt{a e^{\frac{c_1}{a}} (x + c_2)^2} \right)^2 - c_1}{2a} \right) \right\} \right\} \right\}$$

✓ **Maple** : cpu = 1.28 (sec), leaf count = 54

$$\left\{ -c_2 - x + \int^{y(x)} \frac{1}{\sqrt{-2c_1 a + 2a \ln(-a)}} d\_a = 0, -c_2 - x + \int^{y(x)} -\frac{1}{\sqrt{-2(c_1 - \ln(-a))a}} d\_a = 0 \right\}$$

2.1695 ODE No. 1695

$$y(x)y''(x) - ax = 0$$

✗ **Mathematica** : cpu = 20.8141 (sec), leaf count = 0 , could not solve

`DSolve[-(a*x) + y[x]*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 1.84 (sec), leaf count = 103

$$\left\{ y(x) = \operatorname{ODESolStruc} \left(\_a \left(e^{c_1 + \int \_b(\_a) d\_a} \right)^{\frac{3}{2}}, \left\{ \frac{d}{d\_a} \_b(\_a) = \frac{(3\_a^2 - 4a) \_b(\_a)^3}{4\_a} + 2\_b(\_a)^2 \right\}, \left\{ \_a = \frac{y(x)}{x^{\frac{3}{2}}}, \_b(\_a) = -\frac{1}{2} \right\} \right. \right.$$

2.1696 ODE No. 1696

$$y(x)y''(x) - ax^2 = 0$$

✗ **Mathematica** : cpu = 19.3692 (sec), leaf count = 0 , could not solve

`DSolve[-(a*x^2) + y[x]*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 1.707 (sec), leaf count = 100

$$\left\{ y(x) = \operatorname{ODESolStruc} \left(\_a e^{2c_1 + \int 2\_b(\_a) d\_a}, \left\{ \frac{d}{d\_a} \_b(\_a) = \frac{(2\_a^2 - a) \_b(\_a)^3}{\_a} + 3\_b(\_a)^2 \right\}, \left\{ \_a = \frac{y(x)}{x^2}, \_b(\_a) = \frac{1}{x \left(\frac{d}{dx} y(x) \right)} \right\} \right. \right.$$

2.1697 ODE No. 1697

$$-a + y(x)y''(x) + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.211297 (sec), leaf count = 94

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{a^2 x^2 + 2a^2 c_2 x + a^2 c_2^2 - e^{2c_1}}}{\sqrt{a}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{a^2 x^2 + 2a^2 c_2 x + a^2 c_2^2 - e^{2c_1}}}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 1.198 (sec), leaf count = 39

$$\left\{ y(x) = \sqrt{a x^2 - 2c_1 x + 2c_2}, y(x) = -\sqrt{a x^2 - 2c_1 x + 2c_2} \right\}$$

2.1698 ODE No. 1698

$$-ax - b + y(x)y''(x) + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0369312 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{ax^3 + 3bx^2 + 3c_2x + 6c_1}}{\sqrt{3}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{ax^3 + 3bx^2 + 3c_2x + 6c_1}}{\sqrt{3}} \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x),x),x)*y(x)+y(x)^2-a*x-b=0,y(x))`

2.1699 ODE No. 1699

$$y(x)y''(x) + y'(x)^2 - y'(x) = 0$$

✓ **Mathematica** : cpu = 0.146007 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow c_1 \left(-W \left(-\frac{e^{-\frac{x}{c_1} - 1 - \frac{c_2}{c_1}}}{c_1} \right) \right) - c_1 \right\} \right\}$$

✓ **Maple** : cpu = 1.378 (sec), leaf count = 33

$$\left\{ y(x) = -c_1 \left(\text{LambertW} \left(-\frac{e^{-1} e^{-\frac{c_2}{c_1}} e^{-\frac{x}{c_1}}}{c_1} \right) + 1 \right) \right\}$$

2.1700 ODE No. 1700

$$y(x)y''(x) - y'(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 0.200315 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow -e^{-c_1} \sinh(e^{c_1}(x + c_2)) \right\}, \left\{ y(x) \rightarrow e^{-c_1} \sinh(e^{c_1}(x + c_2)) \right\} \right\}$$

✓ **Maple** : cpu = 1.553 (sec), leaf count = 86

$$\left\{ y(x) = \frac{\left(-c_1 e^{\frac{2c_2}{c_1} \frac{2x}{c_1}} + c_1 \right) e^{-\frac{c_2}{c_1} \frac{x}{c_1}}}{2}, y(x) = \frac{\left(c_1 e^{\frac{2c_2}{c_1} \frac{2x}{c_1}} - c_1 \right) e^{-\frac{c_2}{c_1} \frac{x}{c_1}}}{2} \right\}$$

2.1701 ODE No. 1701

$$y(x)y''(x) - y'(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.230964 (sec), leaf count = 80

$$\left\{ \left\{ y(x) \rightarrow -\frac{e^{-c_1} \tanh(e^{c_1}(x + c_2))}{\sqrt{-1 + \tanh^2(e^{c_1}(x + c_2))}} \right\}, \left\{ y(x) \rightarrow \frac{e^{-c_1} \tanh(e^{c_1}(x + c_2))}{\sqrt{-1 + \tanh^2(e^{c_1}(x + c_2))}} \right\} \right\}$$

✓ **Maple** : cpu = 1.475 (sec), leaf count = 42

$$\left\{ y(x) = \frac{c_1 \left(e^{\frac{2c_2}{c_1} \frac{2x}{c_1}} + 1 \right) e^{-\frac{c_2}{c_1} \frac{x}{c_1}}}{2} \right\}$$

2.1702 ODE No. 1702

$$e^{2x}\big(ay(x)^4 + b\big) + e^xy(x)\big(cy(x)^2 + d\big) + y(x)y''(x) - y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 1.45946 (sec), leaf count = 0 , could not solve

`DSolve[E^x*y[x]*(d + c*y[x]^2) + E^(2*x)*(b + a*y[x]^4) - Derivative[1][y][x]^2 + y[x]*Derivative[2][y][x], y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x), x), x)*y(x)-diff(y(x), x)^2+exp(x)*y(x)*(c*y(x)^2+d)+exp(2*x)*(b+a*y(x)^4), y(x))`

2.1703 ODE No. 1703

$$y(x)y''(x) - y'(x)^2 + y(x)^2(-\log(y(x))) = 0$$

✓ **Mathematica** : cpu = 0.41091 (sec), leaf count = 77

$$\left\{\left\{y(x) \rightarrow \exp\left(-\frac{1}{2}\sqrt{c_1} e^{-x-c_2}\left(-1 + e^{2x+2c_2}\right)\right)\right\}, \left\{y(x) \rightarrow \exp\left(\frac{1}{2}\sqrt{c_1} e^{-x-c_2}\left(-1 + e^{2x+2c_2}\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.217 (sec), leaf count = 21

$$\left\{y(x) = e^{-\frac{c_2e^x}{2}} e^{\frac{c_1e^xe^{-2x}}{2}}\right\}$$

2.1704 ODE No. 1704

$$y(x)^2\left(\frac{f''(x)}{f(x)} - \frac{f'(x)^2}{f(x)^2}\right) + f(x)y(x)^3 + y(x)y''(x) - y'(x)^2 - y'(x) = 0$$

✗ **Mathematica** : cpu = 14.0572 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*y[x]^3 - Derivative[1][y][x] - Derivative[1][y][x]^2 + y[x]^2*(-(Derivative[1][f][x]))], y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x), x), x)*y(x)-diff(y(x), x)^2-diff(y(x), x)+f(x)*y(x)^3+y(x)^2*(diff(diff(f(x), x), x)^2/f(x)^2)=0, y(x))`

2.1705 ODE No. 1705

$$-y(x)f'(x) + f(x)y'(x) + y(x)y''(x) - y'(x)^2 - y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.318924 (sec), leaf count = 252

$$\left\{\left\{y(x) \rightarrow -\frac{\exp\left(c_2 - \int_1^x \frac{y(K[3])^3+c_1^2y(K[3])^2+\int_1^{K[3]} \frac{-y(K[1])^3-f'(K[1])y(K[1])+f(K[1])y'(K[1])}{y(K[1])^2}dK[1]^2y(K[3])^2+2c_1\int_1^{K[3]} \frac{-y(K[1])^3-f'(K[1])y(K[1])+f(K[1])y'(K[1])}{y(K[1])^2}dK[1]}{y(K[3])^2\left(c_1+\int_1^{K[3]} \frac{-y(K[1])^3-f'(K[1])y(K[1])+f(K[1])y'(K[1])}{y(K[1])^2}dK[1]\right)}\right\}, \left\{y(x) \rightarrow -\frac{\int_1^x \frac{-y(K[1])^3-f'(K[1])y(K[1])+f(K[1])y'(K[1])}{y(K[1])^2}dK[1] + c_1}{\int_1^x \frac{-y(K[1])^3-f'(K[1])y(K[1])+f(K[1])y'(K[1])}{y(K[1])^2}dK[1]}\right\}\right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x), x), x)*y(x)-diff(y(x), x)^2+diff(y(x), x)*f(x)-diff(f(x), x)*y(x)-y(x)^3=0, y(x))`

2.1706 ODE No. 1706

$$-y(x)f''(x) + f'(x)y'(x) + f(x)y(x)^3 + y(x)y''(x) - y'(x)^2 - y(x)^4 = 0$$

✓ **Mathematica** : cpu = 0.585038 (sec), leaf count = 308

$$\left\{\left\{\begin{array}{l} \exp\left(c_2 - \int_1^x \frac{y(K[3])^4 - f(K[3])y(K[3])^3 + c_1^2 y(K[3])^2 + \int_1^{K[3]} \frac{-y(K[1])^4 + f(K[1])y(K[1])^3 - f''(K[1])y(K[1]) + f'(K[1])y'(K[1])}{y(K[1])^2} dK[1] y(K[3])^2 + 2c_1 \int_1^{K[3]} \frac{-y(K[1])^4 + f(K[1])y(K[1])^3 - f''(K[1])y(K[1]) + f'(K[1])y'(K[1])}{y(K[1])^2} dK[1]}{y(K[3])^2 \left(c_1 + \int_1^{K[3]} \frac{-y(K[1])^4 + f(K[1])y(K[1])^3 - f''(K[1])y(K[1]) + f'(K[1])y'(K[1])}{y(K[1])^2} dK[1]\right)} \int_1^x \frac{-y(K[1])^4 + f(K[1])y(K[1])^3 - f''(K[1])y(K[1]) + f'(K[1])y'(K[1])}{y(K[1])^2} dK[1] \right. \\ \left. \right\} y(x) \rightarrow - \frac{\exp\left(c_2 - \int_1^x \frac{y(K[3])^4 - f(K[3])y(K[3])^3 + c_1^2 y(K[3])^2 + \int_1^{K[3]} \frac{-y(K[1])^4 + f(K[1])y(K[1])^3 - f''(K[1])y(K[1]) + f'(K[1])y'(K[1])}{y(K[1])^2} dK[1] y(K[3])^2 + 2c_1 \int_1^{K[3]} \frac{-y(K[1])^4 + f(K[1])y(K[1])^3 - f''(K[1])y(K[1]) + f'(K[1])y'(K[1])}{y(K[1])^2} dK[1]}{y(K[3])^2 \left(c_1 + \int_1^{K[3]} \frac{-y(K[1])^4 + f(K[1])y(K[1])^3 - f''(K[1])y(K[1]) + f'(K[1])y'(K[1])}{y(K[1])^2} dK[1]\right)} \int_1^x \frac{-y(K[1])^4 + f(K[1])y(K[1])^3 - f''(K[1])y(K[1]) + f'(K[1])y'(K[1])}{y(K[1])^2} dK[1] \right. \\ \left. \right\} \end{array}\right.$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+diff(f(x),x)*diff(y(x),x)-diff(diff(f(x),x),x),y(x)^4=0,y(x))`

2.1707

ODE No. 1707

$$ay(x)y'(x) + by(x)^2 + y(x)y''(x) - y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0761008 (sec), leaf count = 31

$$\left\{\left\{y(x) \rightarrow c_2 e^{-\frac{bx}{a} - \frac{c_1 e^{-ax}}{a}}\right\}\right\}$$

✓ **Maple** : cpu = 1.363 (sec), leaf count = 39

$$\left\{y(x) = e^{\frac{b}{a^2}} e^{\frac{c_1 e^{-ax}}{a}} e^{-\frac{c_2}{a}} e^{-\frac{bx}{a}}\right\}$$

2.1708

ODE No. 1708

$$ay(x)y'(x) - 2ay(x)^2 + by(x)^3 + y(x)y''(x) - y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 43.574 (sec), leaf count = 0 , could not solve

`DSolve[-2*a*y[x]^2 + b*y[x]^3 + a*y[x]*Derivative[1][y][x] - Derivative[1][y][x]^2 + y[x]*De`

✓ **Maple** : cpu = 2.422 (sec), leaf count = 73

$$\left\{y(x) = ODESolStruc\left(-a, \left[\left[-b(-a) \left(\frac{d}{d-a} b(-a)\right) - \frac{-_a^3 b + 2\_a^2 a - \_a a\_b(-a) + \_b(-a)^2}{\_a} = 0\right], \left\{-a = y(x), \_b(-a) = \frac{d}{dx}\right.\right.\right.$$

2.1709

ODE No. 1709

$$2a^2y(x)^2 - (ay(x) - 1)y'(x) + ay(x) - 2b^2y(x)^3 + y(x)y''(x) - y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 56.2842 (sec), leaf count = 543

$$\left\{\left\{y(x) \rightarrow -\frac{1}{2a} + e^{2ax} \left(\frac{e^{-2ax} \left(c_1 \left(\sqrt{a^3 + 2b^2} - a^{3/2} \right) \Gamma\left(1 - \frac{\sqrt{a^3 + 2b^2}}{2a^{3/2}}\right) J_{-\frac{\sqrt{a^3 + 2b^2}}{2a^{3/2}}} \left(\frac{\sqrt{ab^2 e^{2ax} c_2}}{a^{3/2}} \right) + 2c_1 \Gamma\left(1 - \frac{\sqrt{a^3 + 2b^2}}{2a^{3/2}}\right) \sqrt{ab^2 c_2} e^{2ax} J_{\frac{\sqrt{a^3 + 2b^2}}{2a^{3/2}}} \left(\frac{\sqrt{ab^2 e^{2ax} c_2}}{a^{3/2}} \right) \right)}{4a \left(bc_1 \Gamma\left(1 - \frac{\sqrt{a^3 + 2b^2}}{2a^{3/2}}\right) J_{-\frac{\sqrt{a^3 + 2b^2}}{2a^{3/2}}} \left(\frac{\sqrt{ab^2 e^{2ax} c_2}}{a^{3/2}} \right) + 2c_1 \Gamma\left(1 - \frac{\sqrt{a^3 + 2b^2}}{2a^{3/2}}\right) \sqrt{ab^2 c_2} e^{2ax} J_{\frac{\sqrt{a^3 + 2b^2}}{2a^{3/2}}} \left(\frac{\sqrt{ab^2 e^{2ax} c_2}}{a^{3/2}} \right) \right)} \right. \right.$$

✓ **Maple** : cpu = 3.061 (sec), leaf count = 84

$$\left\{y(x) = ODESolStruc\left(-a, \left[\left[-b(-a) \left(\frac{d}{d-a} b(-a)\right) - \frac{2\_a^3 b^2 - 2\_a^2 a^2 + \_a a\_b(-a) - \_a a + \_b(-a)^2 - \_b(-a)}{\_a} = 0\right], \left\{-a = \right.\right.\right.$$

2.1710 ODE No. 1710

-y(x)(y(x)+1)(b^2y(x)^2-a^2)+(ay(x)-1)y'(x)+y(x)y''(x)-y'(x)^2=0

✗ **Mathematica** : cpu = 62.9165 (sec), leaf count = 0 , could not solve
DSolve[-(y[x]*(1+y[x])*(-a^2+b^2*y[x]^2))+(-1+a*y[x])*Derivative[1][y][x]-Derivati
✓ **Maple** : cpu = 4.236 (sec), leaf count = 91

{y(x)=ODESolStruc(-a,[{-b(a)(d/d_a-b(a))-(-a^4b^2+a^3b^2-a^2a^2-a a^2-aa_b(a)+b(a)^2+b(a)}=0},

2.1711 ODE No. 1711

y(x)^2 log(y(x))(cos^2(x)-n^2 cot^2(x))+y(x)y''(x)-y'(x)^2+y(x)y'(x)(tan(x)+cot(x))=0

✓ **Mathematica** : cpu = 98.9686 (sec), leaf count = 916

{c2+int_1^x-((-1)^{1-n}2^{n+1}(1/2(-n-1)+1/2)K_n(sqrt(cos^2(K[3])-1))cos(K[3])(2cos^2(K[3])-2)^{n+1/2}sin(K[3])(cos^2(K[3])-1)^{1/2(-n-1)-1/2}(-1)^{1-n}4^{n+1/2-1/2}K_n(sqrt(cos^2(K[3])-1))c1)/sqrt(1-cos^2(K[3]))+...}y(x) -> e^e

✓ **Maple** : cpu = 0.709 (sec), leaf count = 81

{y(x)=e^{(c2 BesselY(n,sin(x)))/(BesselJ(n,sin(x)) BesselY(n+1,sin(x))-BesselJ(n+1,sin(x)) BesselY(n,sin(x))) sin(x)}e^{- (c1 BesselJ(n,sin(x)))/(BesselJ(n,sin(x)) BesselY(n+1,sin(x))-BesselJ(n+1,sin(x)) BesselY(n,sin(x))) sin(x)}}}

2.1712 ODE No. 1712

-f(x)y(x)y'(x)-g(x)y(x)^2+y(x)y''(x)-y'(x)^2=0

✓ **Mathematica** : cpu = 0.101042 (sec), leaf count = 75

{y(x) -> c2 exp(int_1^x(exp(int_1^{K[3]} f(K[1])dK[1])c1+exp(int_1^{K[3]} f(K[1])dK[1])int_1^{K[3]} exp(-int_1^{K[2]} f(K[1])dK[1])g(K[2])

✓ **Maple** : cpu = 0.266 (sec), leaf count = 61

{y(x)=c2e^{(int e^int -f(x)dx g(x)dx)(int e^int f(x)dx dx)}e^{int (int -e^int f(x)dx dx)e^int -f(x)dx g(x)dx}e^{int -c1e^int f(x)dx dx}}

2.1713 ODE No. 1713

-y(x)(g'(x)-y(x)^2f'(x))+y'(x)(f(x)y(x)^2+g(x))+y(x)y''(x)-y'(x)^2=0

✗ **Mathematica** : cpu = 21.1553 (sec), leaf count = 0 , could not solve
DSolve[-(y[x]*(-(y[x]^2*Derivative[1][f][x]))+Derivative[1][g][x]))+(g[x]+f[x]*y[x]^2)*
✓ **Maple** : cpu = 0.352 (sec), leaf count = 54

{y(x)=ODESolStruc(-b(a),[{-b(a)^2f(a)+c1_b(a)+d/d_a-b(a)-g(a)}=0}],{-a=x,-b(a)=y(x)}, {x=-a,y(

2.1714 ODE No. 1714

$$y(x)y''(x) + 3y(x)y'(x) - 3y'(x)^2 - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.196927 (sec), leaf count = 25

$$\left\{\left\{y(x) \rightarrow \frac{c_2 e^x}{\sqrt{1 - 2e^{x+c_1}}}\right\}\right\}$$

✓ **Maple** : cpu = 1.282 (sec), leaf count = 68

$$\left\{y(x) = \frac{\sqrt{2} \sqrt{(c_1 e^x - c_2) e^{2x}}}{2c_1 e^x - 2c_2}, y(x) = -\frac{\sqrt{2} \sqrt{(c_1 e^x - c_2) e^{2x}}}{2c_1 e^x - 2c_2}\right\}$$

2.1715 ODE No. 1715

$$y(x)y''(x) - ay'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.180024 (sec), leaf count = 26

$$\left\{\left\{y(x) \rightarrow c_2(-ax + x - c_1)^{\frac{1}{1-a}}\right\}\right\}$$

✓ **Maple** : cpu = 1.24 (sec), leaf count = 25

$$\left\{y(x) = \left(\frac{1}{(-a + 1)(c_1 x + c_2)}\right)^{\frac{1}{a-1}}\right\}$$

2.1716 ODE No. 1716

$$a(y'(x)^2 + 1) + y(x)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.716898 (sec), leaf count = 172

$$\left\{\left\{y(x) \rightarrow \text{InverseFunction}\left[-\frac{\#1 \sqrt{1 - e^{2c_1} \#1^{-2a}} {}_2F_1\left(\frac{1}{2}, -\frac{1}{2a}; 1 - \frac{1}{2a}; e^{2c_1} \#1^{-2a}\right)}{\sqrt{-1 + e^{2c_1} \#1^{-2a}}}\right] \& \left[x + c_2\right]\right\}, \left\{y(x) \rightarrow \text{InverseFunction}\left[\right.\right.\right\}$$

✓ **Maple** : cpu = 1.478 (sec), leaf count = 68

$$\left\{-c_2 - x + \int^{y(x)} \frac{-a^a}{\sqrt{c_1 - a^{2a}}} d_a = 0, -c_2 - x + \int^{y(x)} -\frac{-a^a}{\sqrt{c_1 - a^{2a}}} d_a = 0\right\}$$

2.1717 ODE No. 1717

$$ay'(x)^2 + by(x)^3 + y(x)y''(x) = 0$$

✓ **Mathematica** : cpu = 58.0614 (sec), leaf count = 277

$$\left\{\text{Solve}\left[\frac{y(x)\sqrt{(2a+3)y(x)^{2a}}\sqrt{1-\frac{2by(x)^{2a+3}}{2ac_1+3c_1}}{}_2F_1\left(\frac{1}{2},\frac{a+1}{2a+3};\frac{a+1}{2a+3}+1;\frac{2by(x)^{2a+3}}{2ac_1+3c_1}\right)}{(a+1)\sqrt{-2by(x)^{2a+3}+2ac_1+3c_1}}=-x+c_2,y(x)\right],\text{Solve}\left[\frac{y(x)\sqrt{(2a+3)y(x)^{2a}}}{(a+1)\sqrt{-2by(x)^{2a+3}+2ac_1+3c_1}}=-x+c_2,y(x)\right]\right\}$$

✓ **Maple** : cpu = 1.49 (sec), leaf count = 107

$$\left\{-c_2 - x + \int^{y(x)} \frac{(2a+3)_-a^{2a}}{\sqrt{-(2a+3)(2b_-a^{2a+3}-c_1)_-a^{2a}}} d_a = 0, -c_2 - x + \int^{y(x)} \frac{(-2a-3)_-a^{2a}}{\sqrt{-(2a+3)(2b_-a^{2a+3}-c_1)_-a^{2a}}} d_a = 0\right\}$$

2.1718 ODE No. 1718

$dy(x)^{1-a} + ay'(x)^2 + by(x)y'(x) + cy(x)^2 + y(x)y''(x) = 0$

✓ **Mathematica** : cpu = 2.31064 (sec), leaf count = 744

$$\left\{\left\{y(x) \rightarrow -\frac{ad \exp\left(\frac{1}{2}x\left(\sqrt{-4ac + b^2 - 4c} + b\right) - \frac{x\left(b\sqrt{-4ac + b^2 - 4c} - 4(a+1)c + b^2\right)}{\sqrt{-4ac + b^2 - 4c} + b} - \frac{2(a+1)cx}{\sqrt{-4ac + b^2 - 4c} + b}\right)}{(a+1)c} - \frac{d \exp\left(\frac{1}{2}x\left(\sqrt{-4ac + b^2 - 4c} + b\right)\right)}{(a+1)c}\right\}\right\}$$

✓ **Maple** : cpu = 0.394 (sec), leaf count = 133

$$\left\{y(x) = \left(\frac{b^2c^2 + (-4a - 4)c^3}{\left(\sqrt{b^2 + (-4a - 4)c} d e^{\frac{(b + \sqrt{b^2 + (-4a - 4)c})x}{2}} + \left(c_1 e^{\sqrt{b^2 + (-4a - 4)c} x} - c_2\right)(a+1)c\right)^2}\right)^{-\frac{1}{2a+2}} e^{-\frac{bx}{2a+2}} e^{-\frac{\sqrt{b^2 + (-4a - 4)c} x}{2a+2}}\right\}$$

2.1719 ODE No. 1719

$ay'(x)^2 + f(x)y(x)y'(x) + g(x)y(x)^2 + y(x)y''(x) = 0$

✗ **Mathematica** : cpu = 42.7746 (sec), leaf count = 0 , could not solve

`DSolve[g[x]*y[x]^2 + f[x]*y[x]*Derivative[1][y][x] + a*Derivative[1][y][x]^2 + y[x]*Derivative[2][y][x] == 0,x]`

✓ **Maple** : cpu = 0.628 (sec), leaf count = 70

$$\left\{y(x) = ODESolStruc\left(e^{\int_1^x b(\_a) d\_a}, \left\{\frac{d}{d\_a} b(\_a) = (-a - 1) b(\_a)^2 - b(\_a) f(\_a) - g(\_a)\right\}, \left\{\_a = x, b(\_a) = \frac{\frac{d}{dx} y(x)}{y(x)}\right\}\right)\right\}$$

2.1720 ODE No. 1720

$ay'(x)^2 + by(x)^2y'(x) + cy(x)^4 + y(x)y''(x) = 0$

✓ **Mathematica** : cpu = 88.0957 (sec), leaf count = 105

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{K[2]^2 \text{InverseFunction}\left[\frac{\log(c + \#1(b + (a+2)\#1)) - \frac{2b \tan^{-1}\left(\frac{b + 2(a+2)\#1}{\sqrt{4(a+2)c - b^2}}\right)}{\sqrt{4(a+2)c - b^2}}}{2(a+2)}\right] \& [c_1 - \log(K[2])]} dK[2] = x - c_2, y(x)\right]$$

✓ **Maple** : cpu = 1.628 (sec), leaf count = 173

$$\left\{-c_2 - x + \int_1^{y(x)} \frac{2a + 4}{-a^2b + \sqrt{-a^4b^2 + (4a + 8)_a^4c} \tan\left(\text{RootOf}\left(2\_Z\_a^2b - 2\sqrt{4\_a^4ac - \_a^4b^2 + 8c\_a^4} a \ln(\_a) + c_1\sqrt{\_a^4b^2 - 4\_a^4ac}\right)\right)} d\_a\right\}$$

2.1721 ODE No. 1721

$$-\frac{ay(x)^3f'(x)}{a+2}+\frac{af(x)^2y(x)^4}{(a+2)^2}-\frac{(a-1)y'(x)^2}{a}-f(x)y(x)^2y'(x)+y(x)y''(x)=0$$

✓ **Mathematica** : cpu = 21.3503 (sec), leaf count = 41

$$\left\{\left\{y(x)\rightarrow-\frac{(a+2)(x+c_1)^a}{a\int_1^xf(K[3])(c_1+K[3])^adK[3]+c_2}\right\}\right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x),x),x)*y(x)-(a-1)/a*diff(y(x),x)^2-f(x)*y(x)^2*diff(y(x),x)+a/(a+2)^2*f(x)*diff(y(x),x)^2-a/(a+2)*diff(f(x),x)*y(x)^3=0,y(x))`

2.1722 ODE No. 1722

$$-2ay(x)\big(y'(x)^2+1\big)^{3/2}+y(x)y''(x)-y'(x)^2-1=0$$

✓ **Mathematica** : cpu = 2.34071 (sec), leaf count = 697

$$\left\{\left\{y(x)\rightarrow\text{InverseFunction}\left[-\frac{\sqrt{1-\frac{2\#1^2a^2}{-2ac_1+\sqrt{1-4ac_1}+1}}\sqrt{1+\frac{2\#1^2a^2}{2ac_1+\sqrt{1-4ac_1}-1}}\left((-2ac_1+\sqrt{1-4ac_1}+1\right)E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{a^2}{2ac_1+\sqrt{1-4ac_1}-1}}\right)\right)}{2\sqrt{2}a\sqrt{\frac{a^2}{2ac_1+\sqrt{1-4ac_1}-1}}}\right]\right\}\right\}$$

✓ **Maple** : cpu = 2.931 (sec), leaf count = 98

$$\left\{-c_2-x+\int^{y(x)}\frac{-a^2a+c_1}{\sqrt{-a^4a^2-2c_1a^2a+_a^2-c_1^2}}d_a=0,-c_2-x+\int^{y(x)}-\frac{-a^2a+c_1}{\sqrt{-a^4a^2-2c_1a^2a+_a^2-c_1^2}}d_a=0\right\}$$

2.1723 ODE No. 1723

$$(y(x)+x)y''(x)+y'(x)^2-y'(x)=0$$

✓ **Mathematica** : cpu = 0.851372 (sec), leaf count = 130

$$\left\{\left\{y(x)\rightarrow\frac{1}{2}\left(2x-\sqrt{2}e^{-2c_1}\sqrt{4e^{3c_1}x+e^{2c_1}+4e^{3c_1}c_2}+e^{-c_1}+4c_2\right)\right\},\left\{y(x)\rightarrow\frac{1}{2}\left(2x+\sqrt{2}e^{-2c_1}\sqrt{4e^{3c_1}x+e^{2c_1}+4e^{3c_1}c_2}+e^{-c_1}+4c_2\right)\right\}\right\}$$

✓ **Maple** : cpu = 1.301 (sec), leaf count = 16

$$\left\{y(x)=c_1+c_2\sqrt{c_1+2x}+x\right\}$$

2.1724 ODE No. 1724

$$(x-y(x))y''(x)+2y'(x)\big(y'(x)+1\big)=0$$

✓ **Mathematica** : cpu = 0.500577 (sec), leaf count = 38

$$\left\{\left\{y(x)\rightarrow-\frac{e^{-c_1}\left(e^{c_1}c_2x+1+e^{c_1}c_2^2\right)}{x+c_2}\right\}\right\}$$

✓ **Maple** : cpu = 2.094 (sec), leaf count = 21

$$\left\{y(x)=\frac{-c_2x+c_2^2+c_1}{c_2-x}\right\}$$

2.1725 ODE No. 1725

$$(x-y(x))y''(x)+\left(-y'(x)-1\right)\left(y'(x)^2+1\right)=0$$

✓ **Mathematica** : cpu = 2.2049 (sec), leaf count = 75

$$\left\{\left\{y(x)\rightarrow -\sqrt{-x^2-2c_2x+e^{2c_1}-c_2^2}-c_2\right\},\left\{y(x)\rightarrow \sqrt{-x^2-2c_2x+e^{2c_1}-c_2^2}-c_2\right\}\right\}$$

✓ **Maple** : cpu = 1.856 (sec), leaf count = 105

$$\left\{y(x)=x+\text{RootOf}\left(c_2-x+\int^{\_Z}\frac{c_1^2\_f^2-1}{-c_1^2\_f^2+c_1\sqrt{-c_1^2\_f^2+2\_f+2}}d\_f\right),y(x)=x+\text{RootOf}\left(c_2-x+\int^{\_Z}-\frac{c_1^2}{c_1^2\_f^2+c_1\sqrt{\_f^2+2}}d\_f\right)\right\}$$

2.1726 ODE No. 1726

$$(x-y(x))y''(x)-h\big(y'(x)\big)=0$$

✓ **Mathematica** : cpu = 0.376224 (sec), leaf count = 82

$$\text{Solve}\left[\left\{x=\int\frac{\exp\left(-\int_1^{K[4]}\frac{K[3]-1}{h(K[3])}dK[3]-c_1\right)}{h(K[4])}dK[4]+c_2,y(x)=x-\exp\left(-\int_1^{K[4]}\frac{K[3]-1}{h(K[3])}dK[3]-c_1\right)\right\},\{y(x),K[4]\}\right]$$

✓ **Maple** : cpu = 0.232 (sec), leaf count = 39

$$\left\{y(x)=x+\text{RootOf}\left(c_2-x+\int^{\_Z}\frac{1}{\text{RootOf}\left(c_1+\int^{\_Z}\frac{\_a-1}{h(\_a)}d\_a+\ln(-\_g)\right)-1}d\_g\right)\right\}$$

2.1727 ODE No. 1727

$$2y(x)y''(x)+y'(x)^2+1=0$$

✓ **Mathematica** : cpu = 0.449197 (sec), leaf count = 166

$$\left\{\left\{y(x)\rightarrow \text{InverseFunction}\left[-\frac{\sqrt{\#1}\left(\#1-e^{2c_1}\right)+e^{3c_1}\sqrt{1-\#1}e^{-2c_1}\sin^{-1}\left(\sqrt{\#1}e^{-c_1}\right)}{\sqrt{-\#1+e^{2c_1}}}\&\right][x+c_2]\right\},\left\{y(x)\rightarrow \text{InverseFunction}\left[\frac{\sqrt{\#1}\left(\#1-e^{2c_1}\right)+e^{3c_1}\sqrt{1-\#1}e^{-2c_1}\sin^{-1}\left(\sqrt{\#1}e^{-c_1}\right)}{\sqrt{-\#1+e^{2c_1}}}\&\right][x+c_2]\right\}\right\}$$

✓ **Maple** : cpu = 1.708 (sec), leaf count = 823

$$\left\{y(x)=\frac{c_1}{2}+\frac{\left(-c_1\text{RootOf}\left(c_1^2\_Z^2\left(\tan^2(\_Z)\right)-4c_1c_2\_Z\left(\tan^2(\_Z)\right)-4c_1\_Zx\left(\tan^2(\_Z)\right)+c_1^2\_Z^2+4c_2^2\left(\tan^2(\_Z)\right)+8c_2\right)\right)}{\left(-c_1\text{RootOf}\left(c_1^2\_Z^2\left(\tan^2(\_Z)\right)-4c_1c_2\_Z\left(\tan^2(\_Z)\right)-4c_1\_Zx\left(\tan^2(\_Z)\right)+c_1^2\_Z^2+4c_2^2\left(\tan^2(\_Z)\right)+8c_2\right)\right)}\right\}$$

2.1728 ODE No. 1728

$$a+2y(x)y''(x)-y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.0053534 (sec), leaf count = 31

$$\left\{\left\{y(x)\rightarrow \frac{x^2\left(-a+c_1^2\right)}{4c_2}+c_1x+c_2\right\}\right\}$$

✓ **Maple** : cpu = 1.189 (sec), leaf count = 24

$$\left\{y(x)=c_1x+c_2+\frac{\left(c_1^2-a\right)x^2}{4c_2}\right\}$$

2.1729 ODE No. 1729

$$a + f(x)y(x)^2 + 2y(x)y''(x) - y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 0.0225485 (sec), leaf count = 0 , could not solve

`DSolve[a + f[x]*y[x]^2 - Derivative[1][y][x]^2 + 2*y[x]*Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+f(x)*y(x)^2+a=0,y(x))`

2.1730 ODE No. 1730

$$2y(x)y''(x) - y'(x)^2 - 8y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.9652 (sec), leaf count = 135

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[-\frac{2\sqrt{\#1} \sqrt{1 + \frac{4\#1^2}{c_1}} {}_2F_1 \left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{4\#1^2}{c_1} \right)}{\sqrt{4\#1^2 + c_1}} \& \right] [x + c_2] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[\frac{2\sqrt{\#1} \sqrt{1 + \frac{4\#1^2}{c_1}} {}_2F_1 \left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{4\#1^2}{c_1} \right)}{\sqrt{4\#1^2 + c_1}} \& \right] [x + c_2] \right\} \right\}$$

✓ **Maple** : cpu = 1.29 (sec), leaf count = 53

$$\left\{ -c_2 - x + \int^{y(x)} \frac{1}{\sqrt{4\_a^3 + c_1\_a}} d\_a = 0, -c_2 - x + \int^{y(x)} -\frac{1}{\sqrt{4\_a^3 + c_1\_a}} d\_a = 0 \right\}$$

2.1731 ODE No. 1731

$$2y(x)y''(x) - y'(x)^2 - 8y(x)^3 - 4y(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.30115 (sec), leaf count = 351

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[-\frac{i\#1 \sqrt{4 + \frac{2c_1}{\#1 - \#1 \sqrt{1 - c_1}}} \sqrt{2 + \frac{c_1}{\#1 + \#1 \sqrt{1 - c_1}}} F \left(i \sinh^{-1} \left(\frac{\sqrt{\frac{c_1}{2\sqrt{1 - c_1} + 2}}}{\sqrt{\#1}} \right) \middle| \frac{\sqrt{1 - c_1} + 1}{1 - \sqrt{1 - c_1}} \right)}{\sqrt{\frac{c_1}{1 + \sqrt{1 - c_1}}} \sqrt{4\#1^2 + 4\#1 + c_1}} \& \right] [x + c_2] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[\frac{i\#1 \sqrt{4 + \frac{2c_1}{\#1 - \#1 \sqrt{1 - c_1}}} \sqrt{2 + \frac{c_1}{\#1 + \#1 \sqrt{1 - c_1}}} F \left(i \sinh^{-1} \left(\frac{\sqrt{\frac{c_1}{2\sqrt{1 - c_1} + 2}}}{\sqrt{\#1}} \right) \middle| \frac{\sqrt{1 - c_1} + 1}{1 - \sqrt{1 - c_1}} \right)}{\sqrt{\frac{c_1}{1 + \sqrt{1 - c_1}}} \sqrt{4\#1^2 + 4\#1 + c_1}} \& \right] [x + c_2] \right\} \right\}$$

✓ **Maple** : cpu = 1.265 (sec), leaf count = 61

$$\left\{ -c_2 - x + \int^{y(x)} \frac{1}{\sqrt{4\_a^3 + 4\_a^2 + c_1\_a}} d\_a = 0, -c_2 - x + \int^{y(x)} -\frac{1}{\sqrt{(4\_a^2 + 4\_a + c_1)\_a}} d\_a = 0 \right\}$$

2.1732 ODE No. 1732

$$2y(x)y''(x) - y'(x)^2 - 4(2y(x) + x)y(x)^2 = 0$$

✗ **Mathematica** : cpu = 1.09004 (sec), leaf count = 0 , could not solve

`DSolve[-4*y[x]^2*(x + 2*y[x]) - Derivative[1][y][x]^2 + 2*y[x]*Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2-4*(x+2*y(x))*y(x)^2=0,y(x))`

2.1733 ODE No. 1733

$$y(x)^2(ay(x) + b) + 2y(x)y''(x) - y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 2.27502 (sec), leaf count = 437

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[-\frac{i\sqrt{2}\#1^{3/2}\sqrt{2+\frac{4c_1}{\#1(-b+\sqrt{b^2+2ac_1})}}\sqrt{1-\frac{2c_1}{\#1(b+\sqrt{b^2+2ac_1})}}F\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{c_1}{\sqrt{b^2+2ac_1}-b}}}{\sqrt{\#1}}\right)\right)\frac{b-\sqrt{b^2+2ac_1}}{b+\sqrt{b^2+2ac_1}}}{\sqrt{\frac{c_1}{-b+\sqrt{b^2+2ac_1}}}\sqrt{-\#1(\#1^2a+2\#1b-2c_1)}} \right] \right\} \right\}$$

✓ **Maple** : cpu = 1.249 (sec), leaf count = 71

$$\left\{-c_2-x+\int^{y(x)}-\frac{2}{\sqrt{-2\_a^3a-4\_a^2b+4c_{1\_a}}}d\_a=0,-c_2-x+\int^{y(x)}\frac{2}{\sqrt{-2\_a^3a-4\_a^2b+4c_{1\_a}}}d\_a=0\right\}$$

2.1734 ODE No. 1734

$$ay(x)^3 + 2y(x)y''(x) - y'(x)^2 + 2xy(x)^2 + 1 = 0$$

✗ **Mathematica** : cpu = 1.39787 (sec), leaf count = 0 , could not solve

```
DSolve[1 + 2*x*y[x]^2 + a*y[x]^3 - Derivative[1][y][x]^2 + 2*y[x]*Derivative[2][y][x] == 0,
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+1+2*x*y(x)^2+a*y(x)^3=0,y(x))
```

2.1735 ODE No. 1735

$$y(x)^2(ay(x) + bx) + 2y(x)y''(x) - y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 0.866066 (sec), leaf count = 0 , could not solve

```
DSolve[y[x]^2*(b*x + a*y[x]) - Derivative[1][y][x]^2 + 2*y[x]*Derivative[2][y][x] == 0, y[x]
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+(a*y(x)+b*x)*y(x)^2=0,y(x))
```

2.1736 ODE No. 1736

$$2y(x)y''(x) - y'(x)^2 - 3y(x)^4 = 0$$

✓ **Mathematica** : cpu = 8.55933 (sec), leaf count = 129

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[-\frac{2\sqrt{\#1}\sqrt{1+\frac{\#1^3}{c_1}}{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{\#1^3}{c_1}\right)}{\sqrt{\#1^3+c_1}}\&[x+c_2] \right] \right\} \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[\frac{2\sqrt{\#1}\sqrt{1+\frac{\#1^3}{c_1}}{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{\#1^3}{c_1}\right)}{\sqrt{\#1^3+c_1}} \right] \right\}$$

✓ **Maple** : cpu = 1.409 (sec), leaf count = 49

$$\left\{-c_2-x+\int^{y(x)}\frac{1}{\sqrt{-a^4+c_{1\_a}}}d\_a=0,-c_2-x+\int^{y(x)}-\frac{1}{\sqrt{-a^4+c_{1\_a}}}d\_a=0\right\}$$

2.1737 ODE No. 1737

$$-4(a + x^2)y(x)^2 + b + 2y(x)y''(x) - y'(x)^2 - 3y(x)^4 - 8xy(x)^3 = 0$$

✗ **Mathematica** : cpu = 20.8404 (sec), leaf count = 0 , could not solve

```
DSolve[b - 4*(a + x^2)*y[x]^2 - 8*x*y[x]^3 - 3*y[x]^4 - Derivative[1][y][x]^2 + 2*y[x]*Derivative[2][y][x], y[x], x]
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+b-4*(x^2+a)*y(x)^2-8*x*y(x)^3-3*y(x)^4=0,y(x))
```

2.1738 ODE No. 1738

$$2y(x)^2(f'(x) + f(x)^2) + 3f(x)y(x)y'(x) + 2y(x)y''(x) - y'(x)^2 - 8y(x)^3 = 0$$

✗ **Mathematica** : cpu = 0.537722 (sec), leaf count = 0 , could not solve

```
DSolve[-8*y[x]^3 + 2*y[x]^2*(f[x]^2 + Derivative[1][f][x]) + 3*f[x]*y[x]*Derivative[1][y][x] + 2*y[x]*Derivative[2][y][x], y[x], x]
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+3*f(x)*y(x)*diff(y(x),x)+2*(f(x)^2+diff(f(x),x))*y(x)^2-8*y(x)^3=0,y(x))
```

2.1739 ODE No. 1739

$$f(x)y(x)^2 + 2y(x)y''(x) + 4y(x)^2y'(x) - y'(x)^2 + y(x)^4 + 1 = 0$$

✗ **Mathematica** : cpu = 0.0566015 (sec), leaf count = 0 , could not solve

```
DSolve[1 + f[x]*y[x]^2 + y[x]^4 + 4*y[x]^2*Derivative[1][y][x] - Derivative[1][y][x]^2 + 2*y[x]*Derivative[2][y][x], y[x], x]
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+4*y(x)^2*diff(y(x),x)+1+f(x)*y(x)^2+y(x)^4=0,y(x))
```

2.1740 ODE No. 1740

$$2y(x)y''(x) - 3y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.118795 (sec), leaf count = 16

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2}{(x + 2c_1)^2} \right\} \right\}$$

✓ **Maple** : cpu = 1.301 (sec), leaf count = 13

$$\left\{ y(x) = \frac{4}{(c_1x + c_2)^2} \right\}$$

2.1741 ODE No. 1741

$$2y(x)y''(x) - 3y'(x)^2 - 4y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.140424 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow c_2 \sec^2(x + 2c_1) \right\} \right\}$$

✓ **Maple** : cpu = 1.244 (sec), leaf count = 34

$$\left\{ y(x) = \frac{4}{-2c_1c_2 \cos(x) \sin(x) + c_2^2 + (c_1^2 - c_2^2)(\sin^2(x))} \right\}$$

2.1746 ODE No. 1746

$$-ax^2 - bx - c + 3y(x)y''(x) - 2y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.139219 (sec), leaf count = 118

Solve $\left[\int \frac{y(x)^{2/3}}{(ax^2 + bx + c)\sqrt{-\frac{2(ax^2+bx+c)^3}{y(x)^2} + \frac{c_1(ax^2+bx+c)}{y(x)^{2/3}} + 9(b^2 - 4ac)}} d\frac{ax^2 + bx + c}{y(x)^{2/3}} = - \int \frac{1}{3(ax^2 + bx + c)} dx + c_2, y(x) \right]$

✓ **Maple** : cpu = 1.656 (sec), leaf count = 207

$$\left\{ y(x) = \left(a x^2 + b x + c \right)^{\frac{3}{2}} \text{RootOf} \left(-2b \arctan \left(\frac{2ax + b}{\sqrt{4ac - b^2}} \right) + c_2 \sqrt{4ac - b^2} - 2\sqrt{4ac - b^2} \left(\int^{-Z} \frac{b}{\sqrt{4c_1 \_f^3 b^2 - 36\_f^2 ac + 9}} \right) \right. \right.$$

2.1747 ODE No. 1747

$$3y(x)y''(x) - 5y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.118964 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2}{(2x + 3c_1)^{3/2}} \right\} \right\}$$

✓ **Maple** : cpu = 1.225 (sec), leaf count = 17

$$\left\{ -c_1 x - c_2 - \frac{3}{2y(x)^{\frac{2}{3}}} = 0 \right\}$$

2.1748 ODE No. 1748

$$4y(x)y''(x) - 3y'(x)^2 + 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.255972 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow \frac{(c_1^2 x^2 + 2c_2 c_1^2 x - 64 + c_2^2 c_1^2)^2}{256 c_1^2} \right\} \right\}$$

✓ **Maple** : cpu = 1.247 (sec), leaf count = 67

$$\left\{ -c_2 - x - \frac{4\sqrt{c_1 y(x)^{\frac{3}{2}} + 4y(x)}}{c_1 \sqrt{y(x)}} = 0, -c_2 - x + \frac{4\sqrt{c_1 y(x)^{\frac{3}{2}} + 4y(x)}}{c_1 \sqrt{y(x)}} = 0, y(x) = 0 \right\}$$

2.1749 ODE No. 1749

$$4y(x)y''(x) - 3y'(x)^2 - 12y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.525416 (sec), leaf count = 153

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[-\frac{4\#1\sqrt{1 + \frac{4\#1^{3/2}}{c_1}} {}_2F_1 \left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; -\frac{4\#1^{3/2}}{c_1} \right)}{\sqrt{\#1^{3/2} (4\#1^{3/2} + c_1)}} \& \right] [x + c_2] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[\frac{4\#1\sqrt{1 + \frac{4\#1^{3/2}}{c_1}}}{\sqrt{\#1^{3/2} (4\#1^{3/2} + c_1)}} \right] [x + c_2] \right\} \right\}$$

✓ **Maple** : cpu = 2.003 (sec), leaf count = 57

$$\left\{ -c_2 - x + \int^{y(x)} \frac{1}{\sqrt{4\_a^3 + c_1\_a^{\frac{3}{2}}}} d\_a = 0, -c_2 - x + \int^{y(x)} -\frac{1}{\sqrt{4\_a^3 + c_1\_a^{\frac{3}{2}}}} d\_a = 0 \right\}$$

2.1754 ODE No. 1754

$$ny(x)y''(x) + (1 - n)y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.151869 (sec), leaf count = 17

$$\left\{\left\{y(x) \rightarrow c_2(x - c_1n)^n\right\}\right\}$$

✓ **Maple** : cpu = 1.221 (sec), leaf count = 15

$$\left\{y(x) = \left(\frac{c_1x + c_2}{n}\right)^n\right\}$$

2.1755 ODE No. 1755

$$ay(x)y''(x) + by'(x)^2 + c0 + c1y(x) + c2y(x)^2 + c3y(x)^3 + c4y(x)^4 = 0$$

✓ **Mathematica** : cpu = 6.0897 (sec), leaf count = 716

$$\left\{\left\{y(x) \rightarrow \text{InverseFunction}\left[\int_1^{\#1} -\frac{1}{\sqrt{4b^5c_1K[1]^{-\frac{2b}{a}} + 20ab^4c_1K[1]^{-\frac{2b}{a}} + 35a^2b^3c_1K[1]^{-\frac{2b}{a}} + 25a^3b^2c_1K[1]^{-\frac{2b}{a}} + 6a^4bc_1K[1]^{-\frac{2b}{a}} + c_0}}\right]}\right\}\right\}$$

✓ **Maple** : cpu = 1.46 (sec), leaf count = 418

$$\left\{-c_2 - x + \int^{y(x)} \frac{(3a + 2b)(2a + b)(a + 2b)(a + b)\sqrt{-36\left(a + \frac{b}{2}\right)(a + b)\left(a + \frac{2b}{3}\right)\left(\frac{2\left(a + \frac{b}{2}\right)(a + b)(a + 2b)bc_3 - a^{\frac{3a + 2b}{a}}}{3} + \left(a + \frac{2b}{3}\right)\left(\left(a + \frac{b}{2}\right)(a + 2b)bc_2 - a^{\frac{2a + 2b}{a}} + (a + b)c_1\right)\right)}}{dy}\right\}$$

2.1756 ODE No. 1756

$$ay(x)y''(x) + by'(x)^2 - \frac{y(x)y'(x)}{\sqrt{c^2 + x^2}} = 0$$

✓ **Mathematica** : cpu = 0.717703 (sec), leaf count = 211

$$\left\{\left\{y(x) \rightarrow c_2 \exp\left[\int_1^x -\frac{\left(\frac{K[2]}{\sqrt{c^2 + K[2]^2}} + 1\right)^{\frac{1}{2}/a}}{\left(1 - \frac{K[2]}{\sqrt{c^2 + K[2]^2}}\right)^{\frac{1}{2}/a} \int_1^{K[2]} \frac{\exp\left[\frac{\frac{1}{2} \log\left(\frac{K[1]}{\sqrt{c^2 + K[1]^2}} + 1\right) - \frac{1}{2} \log\left(1 - \frac{K[1]}{\sqrt{c^2 + K[1]^2}}\right)}{a} \right] \left(-\sqrt{c^2 + K[1]^2}^a - b\sqrt{c^2 + K[1]^2}\right)}{a\sqrt{c^2 + K[1]^2}} - dK[1] - c_1\left(1 - \frac{K[1]}{\sqrt{c^2 + K[1]^2}}\right)^{\frac{1}{2}/a}}\right]}\right\}\right\}$$

✓ **Maple** : cpu = 2.506 (sec), leaf count = 75

$$\left\{y(x) = \left(\frac{a}{(a + b)\left(\frac{c_1a2^{\frac{1}{a}}x^{\frac{1}{a}+1}\text{hypergeom}\left(\left[-\frac{1}{2a}, -\frac{1}{2a} - \frac{1}{2}\right], \left[-\frac{1}{a} + 1\right], -\frac{c^2}{x^2}\right)\right) + c_2}\right)^{-\frac{a}{a+b}}\right\}$$

2.1757 ODE No. 1757

$$(a+2)f(x)y(x)^2y'(x)+ay(x)y''(x)+ay(x)^3y'(x)-(a-1)y'(x)^2+f(x)^2y(x)^4=0$$

✗ **Mathematica** : cpu = 10.3835 (sec), leaf count = 0 , could not solve

`DSolve[f[x]^2*y[x]^4 + (2 + a)*f[x]*y[x]^2*Derivative[1][y][x] + a*y[x]^3*Derivative[1][y][x] + 1 + a)*Derivative[1][y][x]^2 + a*y[x]*Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(a*y(x)*diff(diff(y(x),x),x)-(a-1)*diff(y(x),x)^2+(a+2)*f(x)*y(x)^2*diff(y(x),x)+f(x)^2*y(x)^4=0,y(x),x)`

2.1758 ODE No. 1758

$$y''(x)(ay(x)+b)+cy'(x)^2=0$$

✓ **Mathematica** : cpu = 0.376399 (sec), leaf count = 36

$$\left\{\left\{y(x)\rightarrow\frac{-b+(-(c_1(a+c)(-x-c_2)))^{\frac{a}{a+c}}}{a}\right\}\right\}$$

✓ **Maple** : cpu = 1.394 (sec), leaf count = 42

$$\left\{y(x)=\frac{-b+(c_1x+c_2)(a+c)\left(\frac{1}{(a+c)(c_1x+c_2)}\right)^{\frac{c}{a+c}}}{a}\right\}$$

2.1759 ODE No. 1759

$$xy(x)y''(x)+xy'(x)^2-y(x)y'(x)=0$$

✓ **Mathematica** : cpu = 0.161166 (sec), leaf count = 18

$$\left\{\left\{y(x)\rightarrow c_2\sqrt{x^2+c_1}\right\}\right\}$$

✓ **Maple** : cpu = 1.181 (sec), leaf count = 31

$$\left\{y(x)=\sqrt{c_1x^2+2c_2},y(x)=-\sqrt{c_1x^2+2c_2}\right\}$$

2.1760 ODE No. 1760

$$ay(x)y'(x)+f(x)+xy(x)y''(x)+xy'(x)^2=0$$

✓ **Mathematica** : cpu = 0.0930389 (sec), leaf count = 108

$$\left\{\left\{y(x)\rightarrow-\sqrt{2}\sqrt{\int_1^x-K[2]^{-a}\left(c_1+\int_1^{K[2]}f(K[1])K[1]^{a-1}dK[1]\right)dK[2]+c_2}\right\}\right\},\left\{y(x)\rightarrow\sqrt{2}\sqrt{\int_1^x-K[2]^{-a}\left(c_1+\int_1^{K[2]}f(K[1])K[1]^{a-1}dK[1]\right)dK[2]+c_2}\right\}$$

✓ **Maple** : cpu = 0.115 (sec), leaf count = 114

$$\left\{y(x)=\frac{\sqrt{2}\sqrt{(a-1)\left(c_1x^{-a+1}+x^{-a+1}\left(\int\frac{x^af(x)}{x}dx\right)-c_2-\left(\int f(x)dx\right)\right)}}{a-1},y(x)=-\frac{\sqrt{2}\sqrt{(a-1)\left(c_1x^{-a+1}+x^{-a+1}\left(\int\frac{x^af(x)}{x}dx\right)-c_2-\left(\int f(x)dx\right)\right)}}{a-1}\right\}$$

2.1761 ODE No. 1761

$$x\bigl(ay(x)^4 + d\bigr) + y(x)\bigl(by(x)^2 + c\bigr) + xy(x)y''(x) - xy'(x)^2 + y(x)y'(x) = 0$$

✗ **Mathematica** : cpu = 1.3561 (sec), leaf count = 0 , could not solve

`DSolve[y[x]*(c + b*y[x]^2) + x*(d + a*y[x]^4) + y[x]*Derivative[1][y][x] - x*Derivative[1][y][x]^2 + x*y[x]*Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(x*y(x)*diff(diff(y(x), x), x) - x*diff(y(x), x)^2 + y(x)*diff(y(x), x) + x*(d + a*y(x)^4) + y(x)*(c + b*y(x)^2), y(x))`

2.1762 ODE No. 1762

$$ay(x)y'(x) + bxy(x)^3 + xy(x)y''(x) - xy'(x)^2 = 0$$

✗ **Mathematica** : cpu = 50.1901 (sec), leaf count = 0 , could not solve

`DSolve[b*x*y[x]^3 + a*y[x]*Derivative[1][y][x] - x*Derivative[1][y][x]^2 + x*y[x]*Derivative[1][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 2.309 (sec), leaf count = 108

$$\left\{y(x) = ODESolStruc\left(-a e^{-2c_1 + \int -2 b(-a) d_a}, \left[\left\{\frac{d}{d_a} b(-a) = -\frac{2\left(\left(-\frac{ab}{2} + a - 1\right) a^2 b(-a)^2 - \frac{(a-1) a b(-a)}{2} + \frac{1}{2}\right) b(-a)}{-a}\right\}, \left\{-\frac{1}{2} \frac{d}{d_a} b(-a) = -\frac{2\left(\left(-\frac{ab}{2} + a - 1\right) a^2 b(-a)^2 - \frac{(a-1) a b(-a)}{2} + \frac{1}{2}\right) b(-a)}{-a}\right\}\right]\right\}$$

2.1763 ODE No. 1763

$$ay(x)y'(x) + xy(x)y''(x) + 2xy'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.254972 (sec), leaf count = 40

$$\left\{\left\{y(x) \rightarrow c_2 \left(3x^{1-a} - (a-1)c_1\right)^{-\frac{a-1}{3(1-a)}}\right\}\right\}$$

✓ **Maple** : cpu = 1.236 (sec), leaf count = 148

$$\left\{y(x) = \frac{3^{\frac{1}{3}} \left((a-1)^2 (-c_1 x + c_2 (a-1) x^a) x^{2a}\right)^{\frac{1}{3}} x^{-a}}{a-1}, y(x) = -\frac{3^{\frac{1}{3}} \left((a-1)^2 (-c_1 x + c_2 (a-1) x^a) x^{2a}\right)^{\frac{1}{3}} (1 + i\sqrt{3}) x^{-a}}{2(a-1)}, y(x) = \frac{3^{\frac{1}{3}} \left((a-1)^2 (-c_1 x + c_2 (a-1) x^a) x^{2a}\right)^{\frac{1}{3}} (1 - i\sqrt{3}) x^{-a}}{2(a-1)}\right\}$$

2.1764 ODE No. 1764

$$xy(x)y''(x) - 2xy'(x)^2 + (y(x) + 1)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0707471 (sec), leaf count = 52

$$\left\{\left\{y(x) \rightarrow \frac{\tan\left(\frac{1}{2}\left(\sqrt{2} \sqrt{c_1} \log(x) - \sqrt{2} \sqrt{c_1} c_2\right)\right)}{\sqrt{2} \sqrt{c_1}}\right\}\right\}$$

✓ **Maple** : cpu = 1.406 (sec), leaf count = 18

$$\left\{y(x) = c_1 \tanh\left(\frac{-c_2 + \ln(x)}{2c_1}\right)\right\}$$

2.1765 ODE No. 1765

$$ay(x)y'(x) + xy(x)y''(x) - 2xy'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.273819 (sec), leaf count = 34

$$\left\{\left\{y(x) \rightarrow c_2 \left(x^{1-a} + (a-1)c_1\right)^{\frac{a-1}{1-a}}\right\}\right\}$$

✓ **Maple** : cpu = 1.218 (sec), leaf count = 27

$$\left\{ y(x) = -\frac{(a-1)x^a}{-c_1x + c_2(a-1)x^a} \right\}$$

2.1766 ODE No. 1766

$$xy(x)y''(x) - 4xy'(x)^2 + 4y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.179491 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2x}{\sqrt[3]{1+c_1x^3}} \right\} \right\}$$

✓ **Maple** : cpu = 1.242 (sec), leaf count = 64

$$\left\{ y(x) = \frac{x}{(-3c_2x^3 + c_1)^{\frac{1}{3}}}, y(x) = -\frac{(1+i\sqrt{3})x}{2(-3c_2x^3 + c_1)^{\frac{1}{3}}}, y(x) = \frac{(i\sqrt{3}-1)x}{2(-3c_2x^3 + c_1)^{\frac{1}{3}}} \right\}$$

2.1767 ODE No. 1767

$$\left(\frac{ax}{\sqrt{b^2-x^2}} - x \right) y'(x)^2 + xy(x)y''(x) - y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.342816 (sec), leaf count = 55

$$\left\{ \left\{ y(x) \rightarrow c_2 \exp \left(\frac{\sqrt{b^2-x^2}}{a} + \frac{c_1 \log(a\sqrt{b^2-x^2} - c_1)}{a^2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 2.704 (sec), leaf count = 50

$$\left\{ y(x) = c_2 e^{\int -\frac{\sqrt{b^2-x^2}x}{c_1\sqrt{b^2-x^2}+(b^2-x^2)^a} dx} \right\}$$

2.1768 ODE No. 1768

$$x(y(x)+x)y''(x) + xy'(x)^2 + (x-y(x))y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.04952 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow -x - \sqrt{x^2 + 2c_2x^2 + c_1} \right\}, \left\{ y(x) \rightarrow -x + \sqrt{x^2 + 2c_2x^2 + c_1} \right\} \right\}$$

✓ **Maple** : cpu = 1.215 (sec), leaf count = 43

$$\left\{ y(x) = -x - \sqrt{(-c_2+1)x^2 + c_1}, y(x) = -x + \sqrt{(-c_2+1)x^2 + c_1} \right\}$$

2.1769 ODE No. 1769

$$2xy(x)y''(x) - xy'(x)^2 + y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.168227 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow c_2(\sqrt{x} + c_1)^2 \right\} \right\}$$

✓ **Maple** : cpu = 1.314 (sec), leaf count = 21

$$\left\{ y(x) = c_1^2x + c_1c_2\sqrt{x} + \frac{c_2^2}{4} \right\}$$

2.1770 ODE No. 1770

$$x^2(y(x) - 1)y''(x) - 2x^2y'(x)^2 - 2x(y(x) - 1)y'(x) - 2(y(x) - 1)^2y(x) = 0$$

✓ **Mathematica** : cpu = 0.9568 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow 1 + \frac{1}{x^2 \left(-\frac{1}{x^2} - \frac{c_1}{x} + c_2 \right)} \right\} \right\}$$

✓ **Maple** : cpu = 1.257 (sec), leaf count = 26

$$\left\{ y(x) = \frac{(c_1x - c_2)x}{c_1x^2 - c_2x - 1} \right\}$$

2.1771 ODE No. 1771

$$x^2(y(x) + x)y''(x) - (xy'(x) - y(x))^2 = 0$$

✓ **Mathematica** : cpu = 0.122441 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow -x + c_2xe^{\frac{c_1}{x}} \right\} \right\}$$

✓ **Maple** : cpu = 1.365 (sec), leaf count = 22

$$\left\{ y(x) = -\frac{(c_1 - e^{-1}e^{\frac{c_2}{x}})x}{c_1} \right\}$$

2.1772 ODE No. 1772

$$a(xy'(x) - y(x))^2 + x^2(x - y(x))y''(x) = 0$$

✓ **Mathematica** : cpu = 0.691695 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow x \left(1 + \left((a-1) \left(\frac{(-1)^{a+1}c_1}{x} - c_2 \right) \right)^{\frac{1}{1-a}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 1.298 (sec), leaf count = 42

$$\{y(x) = x - \text{RootOf}(-c_2ax\_Z^a + c_1a\_Z^a + c_2x\_Z^a - c_1\_Z^a -\_Zx^a)\}$$

2.1773 ODE No. 1773

$$2x^2y(x)y''(x) - (x^2(y'(x)^2 + 1)) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.441898 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{x(c_1^2 \log^2(x) - 2c_2c_1^2 \log(x) + 4 + c_2^2c_1^2)}{4c_1} \right\} \right\}$$

✓ **Maple** : cpu = 1.185 (sec), leaf count = 30

$$\left\{ y(x) = \frac{(4c_2^2 \ln(x)^2 + 4c_1c_2 \ln(x) + c_1^2 + 1)x}{4c_2} \right\}$$

2.1774 ODE No. 1774

$$ax^2y(x)y''(x) + bx^2y'(x)^2 + cxy(x)y'(x) + dy(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.35511 (sec), leaf count = 92

$$\left\{ \left\{ y(x) \rightarrow c_2 \exp \left[- \frac{\log(x) \left(a \left(\sqrt{\frac{a^2 - 2a(c+2d) - 4bd + c^2}{a^2}} - 1 \right) + c \right) - 2a \log \left(x \sqrt{\frac{a^2 - 2a(c+2d) - 4bd + c^2}{a^2}} + c_1 \right)}{2(a+b)} \right] \right\} \right\}$$

✓ **Maple** : cpu = 1.5 (sec), leaf count = 136

$$\left\{ y(x) = x^{\frac{a}{2a+2b}} x^{-\frac{c}{2a+2b}} x^{-\frac{\sqrt{(-4a-4b)d+(a-c)^2}}{2a+2b}} \left(\frac{a^2 - 4bd + c^2 + (-2c - 4d)a}{\left(c_1 x^{\frac{\sqrt{(-4a-4b)d+(a-c)^2}}{a}} - c_2 \right)^2 (a+b)^2} \right)^{-\frac{a}{2a+2b}} \right\}$$

2.1775 ODE No. 1775

$$-a(x+2)y(x)^2 + x(x+1)^2y(x)y''(x) - x(x+1)^2y'(x)^2 + 2(x+1)^2y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.185213 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow c_2 e^{a \log(x+1) + \frac{-a-c_1}{x}} \right\} \right\}$$

✓ **Maple** : cpu = 1.5 (sec), leaf count = 31

$$\left\{ y(x) = \frac{(x+1)^a e^{\frac{c_2}{x}} e^{-a} e^{-\frac{a}{x}}}{c_1} \right\}$$

2.1776 ODE No. 1776

$$8(1-x^3)y(x)y''(x) - 4(1-x^3)y'(x)^2 - 12x^2y(x)y'(x) + 3xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 7.59826 (sec), leaf count = 1743

$$\left\{ \left\{ y(x) \rightarrow e^{\int_1^x \left[\frac{3(1-K[2])^{3/4} \sqrt[4]{K[2]^2 + K[2] + 1} \int_1^{K[2]} \frac{\sqrt{\sqrt{3}K[1] + \sqrt{2K[1]-i\sqrt{3}+1}} \sqrt{2K[1]+i\sqrt{3}+1} + \sqrt{3}}{2(1-K[1])^{3/2} \sqrt{K[1]^2 + K[1] + 1}} dK[1] K[2]^2}{2\sqrt{2}(K[2]^3-1)^{5/4} \sqrt[4]{\sqrt{3}K[2] + \sqrt{2K[2]-i\sqrt{3}+1}} \sqrt{2K[2]+i\sqrt{3}+1} + \sqrt{3}}} + c_1 \right] \right\} \right\}$$

✓ **Maple** : cpu = 1.55 (sec), leaf count = 49

$$\left\{y(x)=\frac{\left(c_1\operatorname{LegendreQ}\left(-\frac{1}{6},\frac{1}{3},\sqrt{-(x-1)(x^2+x+1)}\right)+\frac{c_2\operatorname{LegendreP}\left(-\frac{1}{6},\frac{1}{3},\sqrt{-(x-1)(x^2+x+1)}\right)}{2}\right)^2x}{c_1}\right\}$$

2.1777 ODE No. 1777

$$f0(x)y(x)y''(x)+f1(x)y'(x)^2+f2(x)y(x)y'(x)+f3(x)y(x)^2=0$$

✗ **Mathematica** : cpu = 46.0268 (sec), leaf count = 0 , could not solve

`DSolve[f3[x]*y[x]^2 + f2[x]*y[x]*Derivative[1][y][x] + f1[x]*Derivative[1][y][x]^2 + f0[x]*y`

✓ **Maple** : cpu = 0.704 (sec), leaf count = 79

$$\left\{y(x)=ODESolStruc\left(e^{c_1+\int _b(_a)d_a},\left[\left\{\frac{d}{d_a}b(_a)=\frac{\left(-f0(_a)-f1(_a)\right)_b(_a)^2-_b(_a)f2(_a)-f3(_a)}{f0(_a)}\right\},\left\{-a=x,_b\right.\right.\right.\right.$$

2.1778 ODE No. 1778

$$y(x)^2y''(x)-a=0$$

✓ **Mathematica** : cpu = 0.266365 (sec), leaf count = 65

$$\text{Solve}\left[\left[\frac{y(x)\sqrt{-\frac{2a}{y(x)}+c_1}}{c_1}+\frac{2a\tanh^{-1}\left(\frac{\sqrt{-\frac{2a}{y(x)}+c_1}}{\sqrt{c_1}}\right)}{c_1^{3/2}}\right]^2=(x+c_2)^2,y(x)\right]$$

✓ **Maple** : cpu = 2.039 (sec), leaf count = 245

$$\left\{y(x)=\frac{c_1\left(c_1a+e^{\operatorname{RootOf}\left(c_1^4a^2\operatorname{csgn}\left(\frac{1}{c_1}\right)-2c_1^3Zae^{-Z}-c_1^2\operatorname{csgn}\left(\frac{1}{c_1}\right)e^{2Z}-2c_2\operatorname{csgn}\left(\frac{1}{c_1}\right)e^{-Z}-2x\operatorname{csgn}\left(\frac{1}{c_1}\right)e^{-Z}\right)}e^{-\operatorname{RootOf}\left(c_1^4a^2\operatorname{csgn}\left(\frac{1}{c_1}\right)-2c_1^3Zae^{-Z}-c_1^2\operatorname{csgn}\left(\frac{1}{c_1}\right)e^{2Z}-2c_2\operatorname{csgn}\left(\frac{1}{c_1}\right)e^{-Z}-2x\operatorname{csgn}\left(\frac{1}{c_1}\right)e^{-Z}\right)}\right)^2}{2}\right\}$$

2.1779 ODE No. 1779

$$ax+y(x)^2y''(x)+y(x)y'(x)^2=0$$

✗ **Mathematica** : cpu = 22.866 (sec), leaf count = 0 , could not solve

`DSolve[a*x + y[x]*Derivative[1][y][x]^2 + y[x]^2*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 2.007 (sec), leaf count = 117

$$\left\{ -c_2 - \frac{\sqrt{3} \int^{\frac{y(x)}{x}} \frac{3 \left(\frac{a}{-g^3} \right)^{\frac{1}{3}} - g^2 \tan \left(\text{RootOf} \left(6c_1 - 2\sqrt{3} \_Z + 6 \int \frac{\left(\frac{a}{-g^3} \right)^{\frac{2}{3}} - g^2}{-g^3 + a} d\_g + \ln \left(\frac{\tan^2(\_Z) + 1}{\tan^2(\_Z) + 2\sqrt{3} \tan(\_Z) + 3} \right) \right) + \sqrt{3} \left(\left(\frac{a}{-g^3} \right)^{\frac{1}{3}} - 2 \right) g^2}{-g^3 + a} d\_g}{6} \right\} + \ln(x) = 0$$

2.1780 ODE No. 1780

$$-ax - b + y(x)^2 y''(x) + y(x) y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 20.3733 (sec), leaf count = 0 , could not solve

`DSolve[-b - a*x + y[x]*Derivative[1][y][x]^2 + y[x]^2*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 2.125 (sec), leaf count = 156

$$\left\{ -c_2 + \frac{b \ln(ax + b)}{a} - \frac{\sqrt{3} \int^{\frac{y(x)}{ax+b}} \frac{2 \left(3 \left(-\frac{a}{-g^3 b^3} \right)^{\frac{1}{3}} b \tan \left(\text{RootOf} \left(6b^2 \int \frac{\left(-\frac{a}{-g^3 b^3} \right)^{\frac{2}{3}} - g^2}{-g^3 a^2 - 1} d\_g + 6c_1 - 2\sqrt{3} \_Z + \ln \left(\frac{\tan^2(\_Z) + 1}{\tan^2(\_Z) + 2\sqrt{3} \tan(\_Z) + 3} \right) \right) \right) + \sqrt{3} \left(a - \frac{\left(-\frac{a}{-g^3 b^3} \right)^{\frac{1}{3}}}{2} \right) g^2}{-g^3 a^2 - 1}}{6} \right\}$$

2.1781 ODE No. 1781

$$(y(x)^2 + 1) y''(x) + (1 - 2y(x)) y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.237546 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow \frac{i(1 - c_1^{2i}(x + c_2)^{2i})}{1 + c_1^{2i}(x + c_2)^{2i}} \right\} \right\}$$

✓ **Maple** : cpu = 1.253 (sec), leaf count = 11

$$\{y(x) = \tan(\ln(c_1x + c_2))\}$$

2.1782 ODE No. 1782

$$(y(x)^2 + 1)y''(x) - 3y(x)y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.205672 (sec), leaf count = 93

$$\left\{ \left\{ y(x) \rightarrow -\frac{ic_1(x + c_2)}{\sqrt{c_1^2x^2 + 2c_2c_1^2x - 1 + c_2^2c_1^2}} \right\}, \left\{ y(x) \rightarrow \frac{ic_1(x + c_2)}{\sqrt{c_1^2x^2 + 2c_2c_1^2x - 1 + c_2^2c_1^2}} \right\} \right\}$$

✓ **Maple** : cpu = 1.271 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt{-\frac{1}{c_1^2x^2 + 2xc_2c_1 + c_2^2 - 1}} (c_1x + c_2) \right\}$$

2.1783 ODE No. 1783

$$(y(x)^2 + x)y''(x) - 2(x - y(x)^2)y'(x)^3 + (4y(x)y'(x) + 1)y'(x) = 0$$

✓ **Mathematica** : cpu = 1.38911 (sec), leaf count = 26

$$\text{Solve}[x = -y(x)^2 + c_2e^{e^{-c_1}y(x)}, y(x)]$$

✓ **Maple** : cpu = 1.331 (sec), leaf count = 23

$$\left\{ \frac{-c_1y(x) + c_2 + \ln(y(x)^2 + x) + 2}{y(x)} = 0 \right\}$$

2.1784 ODE No. 1784

$$(x^2 + y(x)^2)y''(x) + (y(x) - xy'(x))(y'(x)^2 + 1) = 0$$

✓ **Mathematica** : cpu = 0.325658 (sec), leaf count = 74

$$\text{Solve}\left[\frac{1}{2}\left(\log\left(1 - \frac{iy(x)}{x}\right) + \log\left(1 + \frac{iy(x)}{x}\right) + i \cot(c_1)\left(\log\left(1 - \frac{iy(x)}{x}\right) - \log\left(1 + \frac{iy(x)}{x}\right)\right)\right) = -\log(x) + c_2, y(x)\right]$$

✓ **Maple** : cpu = 1.961 (sec), leaf count = 82

$$\left\{ y(x) = x \tan\left(\text{RootOf}\left(x^{\frac{2}{c_1-1}}(\cos^2(_Z))e^{\frac{2c_2}{c_1-1}} - x^{\frac{2c_1}{c_1-1}}e^{\frac{2i_Z}{c_1-1}}e^{\frac{2ic_1-Z}{c_1-1}}e^{\frac{2c_1c_2}{c_1-1}}\right)\right)\right\}$$

2.1785 ODE No. 1785

$$(x^2 + y(x)^2)y''(x) - 2(xy'(x) - y(x))(y'(x)^2 + 1) = 0$$

✓ **Mathematica** : cpu = 0.443759 (sec), leaf count = 95

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}\left(-\sqrt{4x(-x + e^{c_2}) + e^{2c_2} \cot^2(c_1)} - e^{c_2} \cot(c_1)\right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2}\left(\sqrt{4x(-x + e^{c_2}) + e^{2c_2} \cot^2(c_1)} - e^{c_2} \cot(c_1)\right) \right\} \right\}$$

✓ **Maple** : cpu = 1.649 (sec), leaf count = 83

$$\left\{ y(x) = \frac{c_1 + 1 - \sqrt{-4c_2^2x^2 + c_1^2 - 4ic_2x + c_1(4ic_2x + 2) + 1}}{2c_2}, y(x) = \frac{c_1 + 1 + \sqrt{-4c_2^2x^2 + c_1^2 - 4ic_2x + c_1(4ic_2x + 2) + 1}}{2c_2} \right\}$$

2.1786 ODE No. 1786

$$f(x)(1-y(x))y(x)y'(x)+2(1-y(x))y(x)y''(x)-\left((1-2y(x))y'(x)^2\right)=0$$

✓ **Mathematica** : cpu = 0.067692 (sec), leaf count = 53

$$\left\{\left\{y(x)\rightarrow 1-\sin^2\left(\frac{1}{2}\left(-\int_1^x-\exp\left(-\int_1^{K[3]}\frac{1}{2}f(K[1])dK[1]\right)c_1dK[3]-c_2\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.247 (sec), leaf count = 42

$$\left\{y(x)=\frac{\left(2c_2e^{c_1\left(\int e^{-\frac{\left(\int f(x)dx\right)}{2}}dx\right)}+1\right)^2e^{\int -c_1e^{\int -\frac{f(x)}{2}}dx}dx}{8c_2}\right\}$$

2.1787 ODE No. 1787

$$h(y(x))+2(1-y(x))y(x)y''(x)-\left((1-3y(x))y'(x)^2\right)=0$$

✓ **Mathematica** : cpu = 1.00351 (sec), leaf count = 170

$$\left\{\left\{y(x)\rightarrow \text{InverseFunction}\left[\int_1^{\#1}-\frac{1}{(K[2]-1)\sqrt{K[2]}\sqrt{c_1+2\int_1^{K[2]}\frac{e^{-2\left(\log(1-K[1])+\frac{1}{2}\log(K[1])\right)}h(K[1])}{2(K[1]-1)K[1]}dK[1]}dK[2]\&[x+c_2]}\right]\right\},\left\{\right\}\right\}$$

✓ **Maple** : cpu = 0.34 (sec), leaf count = 80

$$\left\{-c_2-x+\int^{y(x)}\frac{1}{\sqrt{\left(c_1+\int\frac{h(\_b)}{(\_b-1)^3\_b^2}d\_b\right)\_b(\_b-1)}}d\_b=0,-c_2-x+\int^{y(x)}-\frac{1}{\sqrt{\left(c_1+\int\frac{h(\_b)}{(\_b-1)^3\_b^2}d\_b\right)\_b(\_b-1)}}d\_b\right\}$$

2.1788 ODE No. 1788

$$-4(1-y(x))y(x)^2\left(-f'(x)-f(x)^2-g'(x)+g(x)^2\right)+4y(x)y'(x)(f(x)y(x)+g(x))-2(1-y(x))y(x)y''(x)+(1-3y(x))y'(x)^2=0$$

✗ **Mathematica** : cpu = 1.41703 (sec), leaf count = 0 , could not solve

DSolve[-4*(1 - y[x])*y[x]^2*(-f[x]^2 + g[x]^2 - Derivative[1][f][x] - Derivative[1][g][x]))+4*y[x]*y'[x]*(f[x]*y[x]+g[x])-2*(1-y[x])*y[x]*y''[x]+(1-3*y[x])*y'[x]^2==0,x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{\sqrt{y(x)}-\frac{2\left(\frac{d}{dx}DESol\left(\left\{-\frac{c_1^2Y(x)e^{-2\left(\int f(x)dx\right)+2\left(\int g(x)dx\right)}}{4}-2\left(\frac{d}{dx}Y(x)\right)g(x)+\frac{d^2}{dx^2}Y(x)\right\},\{Y(x)\}\right)\right)e^{\int (f(x)-g(x))dx}}{c_1DESol\left(\left\{-\frac{c_1^2Y(x)e^{-2\left(\int f(x)dx\right)+2\left(\int g(x)dx\right)}}{4}-2\left(\frac{d}{dx}Y(x)\right)g(x)+\frac{d^2}{dx^2}Y(x)\right\},\{Y(x)\}\right)}=0\right\}$$

2.1789 ODE No. 1789

$$4y(x)^2(1-y(x))\left(-f'(x)+f(x)^2-g'(x)-g(x)^2\right)-4y(x)y'(x)(f(x)y(x)+g(x))+(1-y(x))^3\left(f_0(x)^2y(x)^2-f_1(x)^2\right)-2y(x)(1-y(x))\left(f_0(x)^2y(x)^2-f_1(x)^2\right)=0$$

✗ **Mathematica** : cpu = 3.65125 (sec), leaf count = 0 , could not solve

DSolve[(1 - y[x])^3*(-f1[x]^2 + f0[x]^2*y[x]^2) + 4*(1 - y[x])*y[x]^2*(f[x]^2 - g[x]^2 - Derivative[1][f][x] - Derivative[1][g][x]))-4*y[x]*y'[x]*(f[x]*y[x]+g[x])+(1-y[x])^3*(f0[x]^2*y[x]^2-f1[x]^2)-2*y[x]*(1-y[x])*(f0[x]^2*y[x]^2-f1[x]^2)==0,x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(-2*y(x)*(1-y(x))*diff(diff(y(x),x),x)+(1-3*y(x))*diff(y(x),x)^2-4*y(x)*diff(y(x),x)*(f0(x)^2*y(x)^2-f1(x)^2)+4*y(x)^2*(1-y(x))*(f(x)^2-g(x)^2-diff(g(x),x)-diff(f(x),x)))=0,x)

$$\left\{-c_2-x+\int^{y(x)}\frac{a\_b^{-\frac{c}{a}}(\_b-1)^{-\frac{-b-c}{a}}}{\sqrt{\left(c_1a-2\left(\int\frac{\_b^{-\frac{2c}{a}}(\_b-1)^{\frac{2b}{a}}(\_b-1)^{\frac{2c}{a}}h(\_b)}{(\_b-1)\_b}d\_b\right)a}}d\_b=0,-c_2-x+\int^{y(x)}-\frac{a\_b^{-\frac{c}{a}}(\_b-1)^{-\frac{-b-c}{a}}}{\sqrt{\left(c_1a-2\left(\int\frac{\_b^{-\frac{2c}{a}}(\_b-1)^{\frac{2b}{a}}(\_b-1)^{\frac{2c}{a}}h(\_b)}{(\_b-1)\_b}d\_b\right)a}}\right.$$

2.1793 ODE No. 1793

$$a(y(x)-1)y(x)y''(x)-\left((a-1)(2y(x)-1)y'(x)^2\right)+f(x)(y(x)-1)y(x)y'(x)=0$$

✓ **Mathematica** : cpu = 0.140583 (sec), leaf count = 83

$$\left\{\left\{y(x)\rightarrow \text{InverseFunction}\left[a\#1^{-1/a}(-((\#1-1)\#1))^{\frac{1}{a}}{}_2F_1\left(\frac{1}{a},\frac{a-1}{a};1+\frac{1}{a};1-\#1\right)\&\left[\int_1^x\exp\left(-\int_1^{K[1]}\frac{f(K[1])}{a}dK[1]\right)c_1\right.\right.\right.$$

✓ **Maple** : cpu = 1.296 (sec), leaf count = 40

$$\left\{c_1\mathrm{e}^{-\frac{fx}{a}}-c_2+\int^{y(x)}\frac{((\_a-1)\_a)^{\frac{1}{a}}}{(\_a-1)\_a}d\_a=0\right\}$$

2.1794 ODE No. 1794

$$ab(y(x)-1)y(x)y''(x)-\left(y'(x)^2((2ab-a-b)y(x)+(1-a)b)\right)+f(x)(y(x)-1)y(x)y'(x)=0$$

✓ **Mathematica** : cpu = 0.157967 (sec), leaf count = 69

$$\left\{\left\{y(x)\rightarrow \text{InverseFunction}\left[-a\#1^{\frac{1}{a}}{}_2F_1\left(\frac{1}{a},1-\frac{1}{b};1+\frac{1}{a};\#1\right)\&\left[\int_1^x\exp\left(-\int_1^{K[1]}\frac{f(K[1])}{ab}dK[1]\right)c_1dK[3]+c_2\right]\right\}\right\}$$

✓ **Maple** : cpu = 1.322 (sec), leaf count = 46

$$\left\{c_1\mathrm{e}^{-\frac{fx}{ab}}-c_2+\int^{y(x)}\frac{\frac{1}{a}\frac{1}{a}(\_a-1)^{\frac{1}{b}}}{(\_a-1)\_a}d\_a=0\right\}$$

2.1795 ODE No. 1795

$$xy(x)^2y''(x)-a=0$$

✓ **Mathematica** : cpu = 0.351018 (sec), leaf count = 116

$$\text{Solve}\left[-\frac{\sqrt{-\frac{2ay(x)}{x}-\frac{2c_1y(x)^2}{x^2}}}{2c_1}-\frac{a\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c_1}\left(\frac{y(x)}{x}+\frac{a}{2c_1}\right)}{\sqrt{-\frac{2ay(x)}{x}-\frac{2c_1y(x)^2}{x^2}}}\right)}{2\sqrt{2}c_1^{3/2}}-\frac{1}{x}-c_2=0,y(x)\right]$$

✓ **Maple** : cpu = 3.092 (sec), leaf count = 529

$$\left\{y(x)=\frac{c_1\left(9c_1a+\mathrm{e}^{\text{RootOf}\left(243c_1^4a^2x\operatorname{csgn}\left(\frac{1}{c_1}\right)-54c_1^3Zax\mathrm{e}^{-Z}-3c_1^2x\operatorname{csgn}\left(\frac{1}{c_1}\right)\mathrm{e}^{2-Z}-6c_2x\operatorname{csgn}\left(\frac{1}{c_1}\right)\mathrm{e}^{-Z}-2\operatorname{csgn}\left(\frac{1}{c_1}\right)\mathrm{e}^{-Z}\right)^2}\right)x\mathrm{e}^{-\text{RootOf}\left(243c_1^4a^2x\operatorname{csgn}\left(\frac{1}{c_1}\right)-54c_1^3Zax\mathrm{e}^{-Z}-3c_1^2x\operatorname{csgn}\left(\frac{1}{c_1}\right)\mathrm{e}^{2-Z}-6c_2x\operatorname{csgn}\left(\frac{1}{c_1}\right)\mathrm{e}^{-Z}-2\operatorname{csgn}\left(\frac{1}{c_1}\right)\mathrm{e}^{-Z}\right)}}{2}\right.$$

2.1796 ODE No. 1796

$$(a^2 - x^2)(a^2 - y(x)^2)y''(x) + (a^2 - x^2)y(x)y'(x)^2 - x(a^2 - y(x)^2)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.309609 (sec), leaf count = 363

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2}e^{-c_2} \left(1 - \frac{x}{\sqrt{x^2 - a^2}} \right)^{-\frac{c_1}{2}} \left(\frac{x}{\sqrt{x^2 - a^2}} + 1 \right)^{-\frac{c_1}{2}} \sqrt{2a^2 e^{2c_2} \left(1 - \frac{x}{\sqrt{x^2 - a^2}} \right)^{c_1} \left(\frac{x}{\sqrt{x^2 - a^2}} + 1 \right)^{c_1} - a^2 \left(\frac{x}{\sqrt{x^2 - a^2}} + 1 \right)^{c_1}} \right. \right.$$

✓ **Maple** : cpu = 1.62 (sec), leaf count = 51

$$\left\{ y(x) = \frac{\left(c_2^2 \left(x + \sqrt{-a^2 + x^2} \right)^{2c_1} + a^2 \right) \left(x + \sqrt{-a^2 + x^2} \right)^{-c_1}}{2c_2} \right\}$$

2.1797 ODE No. 1797

$$(y(x)-1)^3(a y(x)^2+b)+c x y(x)^2(y(x)-1)+d x^2 y(x)^2(y(x)+1)+2 x^2 y(x)(y(x)-1) y''(x)-x^2(3 y(x)-1) y'(x)^2+2 x y(x)(y(x)-1) y'(x)=0$$

✗ **Mathematica** : cpu = 11.4967 (sec), leaf count = 0 , could not solve

`DSolve[c*x*(-1 + y[x])*y[x]^2 + d*x^2*y[x]^2*(1 + y[x]) + (-1 + y[x])^3*(b + a*y[x]^2) + 2*x^2*y[x]^2*(y[x]-1)+c*x*y[x]^2*(y[x]-1)+d*x^2*y[x]^2*(y[x]+1)+2*x^2*y[x]*(y[x]-1)*y''[x]-x^2*(3*y[x]-1)*y'[x]^2+2*x*y[x]*(y[x]-1)*y'[x]==0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(2*x^2*y(x)*(-1+y(x))*diff(diff(y(x),x),x)-x^2*(3*y(x)-1)*diff(y(x),x)^2+2*x*y(x)*(-1+y(x))*diff(y(x),x)+(a*y(x)^2+b)*(-1+y(x))^3+c*x*y(x)^2*(-1+y(x))+d*x^2*y(x)^2*(1+y(x))=0, y(x))`

2.1798 ODE No. 1798

$$x^3y(x)^2y''(x) + (y(x) + x)(xy'(x) - y(x))^3 = 0$$

✓ **Mathematica** : cpu = 36.6769 (sec), leaf count = 248

`Solve[-Integrate[x^(y(x)/x)*i*sqrt(3)*sqrt(K[2])*Ji*sqrt(3)*(2*sqrt(K[2]))+sqrt(K[2])*Ji*sqrt(3)*(2*sqrt(K[2]))-2Ji*sqrt(3)*(2*sqrt(K[2]))*K[2]-2Yi*sqrt(3)*(2*sqrt(K[2]))*c1*K[2]+(Ji*sqrt(3)*(2*sqrt(K[2]))+Yi*sqrt(3)*(2*sqrt(K[2]))*c1)*K[2]^3/2,x),y(x)]`

✓ **Maple** : cpu = 1.401 (sec), leaf count = 166

$$\left\{ y(x) = x \operatorname{RootOf} \left(2c_2 - \left(\int^{-Z} \frac{-2c_1 \sqrt{f} \operatorname{BesselY}(1 + i\sqrt{3}, 2\sqrt{f}) + ic_1 \sqrt{3} \sqrt{f} \operatorname{BesselY}(i\sqrt{3}, 2\sqrt{f}) + c_1 \sqrt{f} \operatorname{BesselY}(i\sqrt{3}, 2\sqrt{f})}{(c_1 \operatorname{BesselY}(i\sqrt{3}, 2\sqrt{f}) + c_1 \operatorname{BesselY}(i\sqrt{3}, 2\sqrt{f}))} \right) \right. \right.$$

2.1799 ODE No. 1799

$$y(x)^3y''(x) - a = 0$$

✓ **Mathematica** : cpu = 2.1481 (sec), leaf count = 88

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{a + c_1^2 x^2 + 2c_2 c_1^2 x + c_2^2 c_1^2}}{\sqrt{c_1}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{a + c_1^2 x^2 + 2c_2 c_1^2 x + c_2^2 c_1^2}}{\sqrt{c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 1.566 (sec), leaf count = 46

$$\left\{ y(x) = \frac{\sqrt{c_1((c_2 + x)^2 c_1^2 + a)}}{c_1}, y(x) = -\frac{\sqrt{c_1((c_2 + x)^2 c_1^2 + a)}}{c_1} \right\}$$

2.1800 ODE No. 1800

$$y(x)\big(y(x)^2+1\big)y''(x)+\big(1-3y(x)^2\big)y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.651118 (sec), leaf count = 84

$$\left\{\left\{y(x)\rightarrow-\frac{\sqrt{-2c_1x-1-2c_2c_1}}{\sqrt{2}\sqrt{c_1x+c_2c_1}}\right\},\left\{y(x)\rightarrow\frac{\sqrt{-2c_1x-1-2c_2c_1}}{\sqrt{2}\sqrt{c_1x+c_2c_1}}\right\}\right\}$$

✓ **Maple** : cpu = 1.413 (sec), leaf count = 60

$$\left\{y(x)=\frac{\sqrt{-4(c_1x+c_2)\left(c_1x+c_2+\frac{1}{2}\right)}}{2c_1x+2c_2},y(x)=-\frac{\sqrt{-4(c_1x+c_2)\left(c_1x+c_2+\frac{1}{2}\right)}}{2c_1x+2c_2}\right\}$$

2.1801 ODE No. 1801

$$-a^2xy(x)^2+2y(x)^3y''(x)+y(x)^4-1=0$$

✗ **Mathematica** : cpu = 7.19496 (sec), leaf count = 0 , could not solve

`DSolve[-1 - a^2*x*y[x]^2 + y[x]^4 + 2*y[x]^3*Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(2*y(x)^3*diff(diff(y(x),x),x)+y(x)^4-a^2*x*y(x)^2-1=0,y(x))`

2.1802 ODE No. 1802

$$-ax^2-bx-c+2y(x)^3y''(x)+y(x)^2y'(x)^2=0$$

✗ **Mathematica** : cpu = 0.309693 (sec), leaf count = 0 , could not solve

`DSolve[-c - b*x - a*x^2 + y[x]^2*Derivative[1][y][x]^2 + 2*y[x]^3*Derivative[2][y][x] == 0,`

Maple : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(2*y(x)^3*diff(diff(y(x),x),x)+y(x)^2*diff(y(x),x)^2-a*x^2-b*x-c=0,y(x))`

2.1803 ODE No. 1803

$$-a0(a-y(x))^2(b-y(x))^2(c-y(x))^2-a2(a-y(x))^2(c-y(x))^2-a3(a-y(x))^2(b-y(x))^2+2(a-y(x))(b-y(x))(c-y(x))y''(x)+y'(x)^2($$

✓ **Mathematica** : cpu = 23.6236 (sec), leaf count = 10387

$$\left\{\text{Solve}\left[-\frac{2F\left(\sin^{-1}\left(\sqrt{\frac{\text{Root}\left[a0\#1^4+(-aa0-ba0-ca0-c_1)\#1^3+(-a1-a2-a3+aa0b+aa0c+a0bc+ac_1+bc_1+cc_1)\#1^2+(aa2+ca2+aa3+a1b+a3b+a1c-aa0bc-}$$

✓ **Maple** : cpu = 3.684 (sec), leaf count = 115620

Too large to display

2.1804 ODE No. 1804

$$y''(x)\big(-ay(x)-b+4y(x)^3\big)+\left(\frac{a}{2}-6y(x)^2\right)y'(x)^2=0$$

✓ **Mathematica** : cpu = 3.37907 (sec), leaf count = 415

$$\text{Solve}\left[\frac{2\sqrt{\frac{y(x)-\text{Root}\left[4\#1^3-\#1a-b\&,1\right]}{\text{Root}\left[4\#1^3-\#1a-b\&,3\right]-\text{Root}\left[4\#1^3-\#1a-b\&,1\right]}}\sqrt{\frac{y(x)-\text{Root}\left[4\#1^3-\#1a-b\&,2\right]}{\text{Root}\left[4\#1^3-\#1a-b\&,3\right]-\text{Root}\left[4\#1^3-\#1a-b\&,2\right]}}\left(y(x)-\text{Root}\left[4\#1^3-\#1a-b\&,3\right]\right)}{c_1\sqrt{2ay(x)+2b-8y(x)^3}\sqrt{\frac{y(x)-\text{Root}\left[4\#1^3-\#1a-b\&,2\right]}{\text{Root}\left[4\#1^3-\#1a-b\&,3\right]-\text{Root}\left[4\#1^3-\#1a-b\&,2\right]}}}\right]$$

✓ **Maple** : cpu = 1.208 (sec), leaf count = 31

$$\left\{-c_1x-c_2+\int^{y(x)}\frac{1}{\sqrt{4\_a^3-\_aa-b}}d\_a=0\right\}$$

2.1805 ODE No. 1805

$$\left(-ay(x)-b+4y(x)^3\right)\left(f(x)y'(x)+y''(x)\right)+\left(\frac{a}{2}-6y(x)^2\right)y'(x)^2=0$$

✓ **Mathematica** : cpu = 0.461913 (sec), leaf count = 438

Solve

$$\left[2\sqrt{\frac{y(x)-\text{Root}\left[4\#1^3-\#1a-b\&,1\right]}{\text{Root}\left[4\#1^3-\#1a-b\&,3\right]-\text{Root}\left[4\#1^3-\#1a-b\&,1\right]}}\sqrt{\frac{y(x)-\text{Root}\left[4\#1^3-\#1a-b\&,2\right]}{\text{Root}\left[4\#1^3-\#1a-b\&,3\right]-\text{Root}\left[4\#1^3-\#1a-b\&,2\right]}}\left(y(x)-\text{Root}\left[4\#1^3-\#1a-b\&,3\right]\right)\right.\\ \left.\sqrt{ay(x)+b-4y(x)^3}\sqrt{\frac{y(x)-\text{Root}\left[4\#1^3-\#1a-b\&,2\right]}{\text{Root}\left[4\#1^3-\#1a-b\&,3\right]-\text{Root}\left[4\#1^3-\#1a-b\&,2\right]}}\right]$$

✓ **Maple** : cpu = 1.066 (sec), leaf count = 34

$$\left\{c_1e^{-fx}-c_2+\int^{y(x)}\frac{1}{\sqrt{4\_a^3-\_aa-b}}d\_a=0\right\}$$

2.1806 ODE No. 1806

$$-f(x)((y(x)-1)y(x)(y(x)-x))^{3/2}+2(1-y(x))(x^2-2xy(x)+y(x))y(x)y'(x)-2(1-x)x(1-y(x))(x-y(x))y(x)y''(x)+(1-x)x(3y(x)-x^2)y'(x)^2)$$

✗ **Mathematica** : cpu = 18.7043 (sec), leaf count = 0 , could not solve

DSolve[-((1-y[x])^2*y[x]^2)-f[x]*((-1+y[x])*y[x]*(-x+y[x]))^(3/2)+2*(1-y[x])*y[x]*(3y[x]-x^2)y'[x]^2)

✓ **Maple** : cpu = 6.389 (sec), leaf count = 733

Too large to display

2.1807 ODE No. 1807

$$a(1-y(x))^2(x-y(x))^2y(x)^2+bx(1-y(x))^2(x-y(x))^2-c(1-x)(x-y(x))^2y(x)^2-d(1-x)x(1-y(x))^2y(x)^2+2(1-x)^2x^2(1-y(x))(x-y(x))y'(x)^2$$

✗ **Mathematica** : cpu = 25.0158 (sec), leaf count = 0 , could not solve

DSolve[b*x*(1-y[x])^2*(x-y[x])^2-d*(1-x)*x*(1-y[x])^2*y[x]^2-c*(1-x)*(x-y[x])^2*y[x]^2+a*(1-x)^2*x^2*(1-y[x])*(x-y[x])y'[x]^2

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(2*x^2*y(x)*(-x+1)^2*(1-y(x))*(x-y(x))*diff(diff(y(x),x),x)-x^2*(-x+1)^2*(x-2*x*y(x)-2*y(x)+3*y(x)^2)*diff(y(x),x)^2-2*x*y(x)*(-x+1)*(1-y(x))*(x^2+y(x)-2*x*y(x))*diff(y(x),x)^2*(x-y(x))^2-c*(-x+1)*y(x)^2*(x-y(x))^2-d*x*y(x)^2*(-x+1)*(1-y(x))^2+a*y(x)^2*(x-y(x))^2*(1-y(x))^2=0,y(x))

2.1808 ODE No. 1808

$$b\sqrt{\left(1-y(x)^2\right)\left(1-a^2y(x)^2\right)}y'(x)^2+\left(y(x)^2-1\right)\left(a^2y(x)^2-1\right)y''(x)+y(x)\left(-2a^2y(x)^2+a^2+1\right)y'(x)^2=0$$

✓ **Mathematica** : cpu = 1.24606 (sec), leaf count = 124

$$\left\{\left\{y(x)\rightarrow\text{InverseFunction}\left[\int_1^{\#1}\frac{\exp\left(\frac{b\sqrt{1-K[1]^2}\sqrt{1-a^2K[1]^2}F\left(\sin^{-1}(K[1])|a^2\right)}{\sqrt{(K[1]^2-1)\left(a^2K[1]^2-1\right)}}+\frac{1}{2}(-\log(1-K[1])-\log(K[1]+1)-\log(1-a^2K[1]^2))\right)}{c_1}\right.\right.\right.$$

✓ **Maple** : cpu = 0.204 (sec), leaf count = 72

$$\left\{ -c_1x - c_2 + \int^{y(x)} e^{\int \frac{-2b^3a^2 + b^2a^2 + b + \sqrt{(b^2-1)(b^2a^2-1)}b}{(b^2-1)(b^2a^2-1)} d_b} d_b = 0 \right\}$$

2.1809 ODE No. 1809

$$y''(x)(ax^2 + 2bx + c + y(x)^2)^2 + dy(x) = 0$$

✓ **Mathematica** : cpu = 26.4941 (sec), leaf count = 260

$$\left\{ \text{Solve} \left[a \tan^{-1} \left(\frac{ax + b}{\sqrt{ac - b^2}} \right) + \sqrt{ac - b^2} \int_1^{\frac{y(x)}{\sqrt{c+x(2b+ax)}}} \frac{a(K[2]^2 + 1)}{\sqrt{(K[2]^2 + 1)(d + (K[2]^2 + 1)(c_1a^2 + (b^2 - ac)K[2]^2))}} dK[2] = c_2 \right] \right\}$$

✓ **Maple** : cpu = 0.65 (sec), leaf count = 336

$$\left\{ y(x) = \sqrt{ax^2 + 2bx + c} \text{ RootOf} \left(-a \arctan \left(\frac{ax + b}{\sqrt{ac - b^2}} \right) + c_2 \sqrt{ac - b^2} - \sqrt{ac - b^2} \left(\int^{-Z} \frac{\sqrt{(\_f^2 + 1)(-\_f^4 ac + \_f^4 b^2 + c_1 \_f^2 a)}}{-\_f^4 ac + \_f^4 b^2 + c_1 \_f^2 a} d\_f \right) \right) \right\}$$

2.1810 ODE No. 1810

$$\sqrt{y(x)} y''(x) - a = 0$$

✓ **Mathematica** : cpu = 0.0876275 (sec), leaf count = 1677

$$\left\{ \left\{ y(x) \rightarrow \frac{3c_1^2}{16a^2} + \sqrt[3]{-\frac{221184c_1^6}{a^6} + \frac{159252480x^2c_1^3}{a^2} + \frac{159252480c_2^2c_1^3}{a^2} + \frac{318504960xc_2c_1^3}{a^2} + 2293235712a^2x^4 + 2293235712a^2c_2^4} \right\} \right\}$$

✓ **Maple** : cpu = 0.167 (sec), leaf count = 91

$$\left\{ -c_2 - x + \frac{-3c_1\sqrt{4a\sqrt{y(x)} - c_1} - (4a\sqrt{y(x)} - c_1)^{\frac{3}{2}}}{12a^2} = 0, -c_2 - x + \frac{3c_1\sqrt{4a\sqrt{y(x)} - c_1} + (4a\sqrt{y(x)} - c_1)^{\frac{3}{2}}}{12a^2} = 0 \right\}$$

2.1811 ODE No. 1811

$$\sqrt{x^2 + y(x)^2} y''(x) - a(y'(x)^2 + 1)^{3/2} = 0$$

✗ **Mathematica** : cpu = 300.033 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve((x^2+y(x)^2)^(1/2)*diff(diff(y(x),x),x)-a*(diff(y(x),x)^2+1)^(3/2)=0,y(x))

2.1812 ODE No. 1812

$$y(x)y''(x)(1 - \log(y(x))) + y'(x)^2(\log(y(x)) + 1) = 0$$

✓ **Mathematica** : cpu = 0.342689 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{c_1x-1+c_2c_1}{c_1(x+c_2)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.181 (sec), leaf count = 19

$$\left\{y(x)=e^{\frac{c_1x+c_2-1}{c_1x+c_2}}\right\}$$

2.1813 ODE No. 1813

$$Ay(x)\big(a\sin^2(y(x))+c\big)+y''(x)\big(a\sin^2(y(x))+b\big)+ay'(x)^2\sin(y(x))\cos(y(x))=0$$

✓ **Mathematica** : cpu = 29.0284 (sec), leaf count = 176

$$\left\{\left\{y(x)\rightarrow\text{InverseFunction}\left[\int_1^{\#1}-\frac{\sqrt{2}\sqrt{\cos(2K[1])a-a-2b}}{\sqrt{2aAK[1]^2+4AcK[1]^2-2aA\sin(2K[1])K[1]+2c_1-aA\cos(2K[1])}}dK[1]\&\right][x+\right.$$

✓ **Maple** : cpu = 0.463 (sec), leaf count = 138

$$\left\{-c_2-x+\int^{y(x)}\frac{\sqrt{2}\left(a\left(\sin^2(\_a)\right)+b\right)}{\sqrt{-\left(-2A\_aa\cos(\_a)\sin(\_a)+Aa\left(\sin^2(\_a)\right)+(a+2c)A\_a^2-2c_1\right)\left(a\left(\sin^2(\_a)\right)+b\right)}}d\_a=0,-c_2-x\right.$$

2.1814 ODE No. 1814

$$ah(y(x))y'(x)^2+h(y(x))y''(x)+j(y(x))=0$$

✓ **Mathematica** : cpu = 0.413204 (sec), leaf count = 120

$$\left\{\left\{y(x)\rightarrow\text{InverseFunction}\left[\int_1^{\#1}-\frac{e^{aK[2]}}{\sqrt{c_1+2\int_1^{K[2]}-\frac{e^{2aK[1]j(K[1])}}{h(K[1])}}dK[1]\&\right][x+c_2]\right\},\left\{y(x)\rightarrow\text{InverseFunction}\left[\int_1^{\#}\right.\right.$$

✓ **Maple** : cpu = 0.209 (sec), leaf count = 87

$$\left\{-c_2-x+\int^{y(x)}\frac{h(\_b)^a}{\sqrt{c_1-2\left(\int\frac{h(\_b)^{2a}}{h(\_b)}d\_b\right)}}d\_b=0,-c_2-x+\int^{y(x)}-\frac{h(\_b)^a}{\sqrt{c_1-2\left(\int\frac{2h(\_b)^{2a}}{h(\_b)}d\_b\right)}}d\_b=0\right\}$$

2.1815 ODE No. 1815

$$h(y(x))^2\left(-j\left(x,\frac{y'(x)}{h(y(x))}\right)\right)+h(y(x))y''(x)-h(y(x))y'(x)^2=0$$

✗ **Mathematica** : cpu = 1.39196 (sec), leaf count = 0 , could not solve

DSolve[-(h[y[x]]^2*j[x, Derivative[1][y][x]/h[y[x]]]) - h[y[x]]*Derivative[1][y][x]^2 + h[y[x]]

✓ **Maple** : cpu = 0.951 (sec), leaf count = 71

$$\left\{y(x)=ODESolStruc\left(\text{RootOf}\left(c_1+\int\_b(\_a)d\_a-\left(\int^{-Z}\frac{1}{h(\_f)}d\_f\right)\right),\left[\left\{\frac{d}{d\_a}b(\_a)=1\right\},\left\{-a=x,\_b(\_a)=\frac{\frac{d}{dx}y(x)}{h(y(x))}\right\},\right.\right.$$

2.1816 ODE No. 1816

$$x^2(-y(x))y'(x) + y'(x)y''(x) - xy(x)^2 = 0$$

✗ **Mathematica** : cpu = 55.8202 (sec), leaf count = 0 , could not solve

```
DSolve[-(x*y[x]^2) - x^2*y[x]*Derivative[1][y][x] + Derivative[1][y][x]*Derivative[2][y][x]
```

✓ **Maple** : cpu = 1.562 (sec), leaf count = 46

$$\{y(x) = ODESolStruc\left(-b(_a), \left[\left[-_a^2 b(_a)^2 + \left(\frac{d}{d\_a} b(_a)\right)^2 + c_1 = 0\right], \left\{-_a = x, -b(_a) = y(x)\right\}, \left\{x = -_a, y(x) = -b(_a)\right\}\right.$$

2.1817 ODE No. 1817

$$4y'(x)^2 + (xy'(x) - y(x))y''(x) = 0$$

✓ **Mathematica** : cpu = 20.8358 (sec), leaf count = 41

$$\left\{\left\{y(x) \rightarrow \frac{1}{2}c_2e^{-2-W\left(\frac{2x}{e^2c_1}\right)}\left(2+W\left(\frac{2x}{e^2c_1}\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.349 (sec), leaf count = 40

$$\{y(x) = e^{c_2 + \int^{\ln(x)} \left(e^{\text{RootOf}(c_1 e^{-Z} - Z e^{-Z} - b e^{-Z} + e^{-Z} \ln(e^{-Z} - 1) + 2)} - 1\right) d\_b}\}$$

2.1818 ODE No. 1818

$$(xy'(x) - y(x))y''(x) - (y'(x)^2 + 1)^2 = 0$$

✗ **Mathematica** : cpu = 1.41835 (sec), leaf count = 0 , could not solve

```
DSolve[-(1 + Derivative[1][y][x]^2)^2 + (-y[x] + x*Derivative[1][y][x])*Derivative[2][y][x]
```

✓ **Maple** : cpu = 0.474 (sec), leaf count = 66

$$\left\{y(x) = x \text{RootOf}\left(c_2 + \int^{-Z} \frac{-f + \text{RootOf}\left(c_1 - Z f - c_1 - Z \tan\left(\frac{1}{-Z}\right) + c_1 f \tan\left(\frac{1}{-Z}\right) + -Z f \tan\left(\frac{1}{-Z}\right) + c_1 + -Z - f + \tan\right)}{f^2 + 1} d\_f\right.\right.$$

2.1819 ODE No. 1819

$$ax^3y'(x)y''(x) + by(x)^2 = 0$$

✗ **Mathematica** : cpu = 1.99553 (sec), leaf count = 0 , could not solve

```
DSolve[b*y[x]^2 + a*x^3*Derivative[1][y][x]*Derivative[2][y][x] == 0, y[x], x]
```

✓ **Maple** : cpu = 0.117 (sec), leaf count = 42

$$\left\{y(x) = e^{c_2 + \int^{\ln(x)} \text{RootOf}\left(c_1 - b - \left(\int^{-Z} \frac{-aa}{-a^3 a - a^2 a + b} d\_a\right)\right) d\_b}\right\}$$

2.1820 ODE No. 1820

$$y''(x)\left(f1(x)y'(x) + f2(x)y(x)\right) + f3(x)y'(x)^2 + f4(x)y(x)y'(x) + f5(x)y(x)^2 = 0$$

✗ **Mathematica** : cpu = 303.679 (sec), leaf count = 0 , timed out

```
$Aborted
```

✓ **Maple** : cpu = 1.169 (sec), leaf count = 88

$$\left\{y(x) = ODESolStruc\left(e^{c_1 + \int -b(_a) d\_a}, \left[\left[\frac{d}{d\_a} b(_a) = \frac{-f1\_b(_a)^3 + (-f2 - f3)_b(_a)^2 - b(_a)f4(_a) - f5(_a)}{f1\_b(_a) + f2}\right], \left\{-a = x, -b(_a) = y(x)\right\}, \left\{x = -a, y(x) = -b(_a)\right\}\right.\right.$$

2.1821 ODE No. 1821

$$\left(x^2 + 2y(x)^2y'(x)\right)y''(x) + 2y(x)y'(x)^3 + 3xy'(x) + y(x) = 0$$

✗ **Mathematica** : cpu = 42.3157 (sec), leaf count = 0 , could not solve
`DSolve[y[x] + 3*x*Derivative[1][y][x] + 2*y[x]*Derivative[1][y][x]^3 + (x^2 + 2*y[x]^2*Deriv`
✓ **Maple** : cpu = 3.185 (sec), leaf count = 54

$$\{y(x) = ODESolStruc\left(\_b(\_a), \left[\left\{-b(\_a)^2\left(\frac{d}{d\_a}\_b(\_a)\right)^2 + \_a^2\left(\frac{d}{d\_a}\_b(\_a)\right) + \_a\_b(\_a) + c_1 = 0\right\}, \{-a = x, \_b(\_a) = y(x)\}\right), \{-a = x, \_b(\_a) = y(x)\}\right\}$$

2.1822 ODE No. 1822

$$\left(y'(x)^2 + y(x)^2\right)y''(x) + y(x)^3 = 0$$

✓ **Mathematica** : cpu = 1.11182 (sec), leaf count = 371
$$\left\{\left\{y(x) \rightarrow c_2 \exp\left[\frac{1}{12} - 2\sqrt{3} \tan^{-1}\left[\frac{1 + 2\text{InverseFunction}\left[\frac{\left(\sqrt{3} - i\right) \tan^{-1}\left(\frac{\#1}{\sqrt{\frac{1}{2}(1 - i\sqrt{3})}\right)} + \frac{\left(\sqrt{3} + i\right) \tan^{-1}\left(\frac{\#1}{\sqrt{\frac{1}{2}(1 + i\sqrt{3})}\right)}\right]}{\sqrt{6(1 - i\sqrt{3})}} + \frac{\left(\sqrt{3} + i\right) \tan^{-1}\left(\frac{\#1}{\sqrt{\frac{1}{2}(1 + i\sqrt{3})}\right)}{\sqrt{6(1 + i\sqrt{3})}}\right]}{\sqrt{3}}\right] \right\}\right\}$$

✓ **Maple** : cpu = 1.948 (sec), leaf count = 293

$$\left\{y(x) = c_2\left(\tan^2\left(\sqrt{3} \ x\right) + 1\right)^{-\frac{1}{4c_1^2+4}}\left(\tan^2\left(\sqrt{3} \ x\right) + 1\right)^{-\frac{c_1^2}{4c_1^2+4}}\left(c_1 + \tan\left(\sqrt{3} \ x\right)\right)^{\frac{1}{2c_1^2+2}}\left(c_1 + \tan\left(\sqrt{3} \ x\right)\right)^{\frac{c_1^2}{2c_1^2+2}}e^{\int-\frac{\sqrt{3c_1^2\left(\tan^2\left(\sqrt{3} \ x\right)\right)+4c_1}}{2}\right)}$$

2.1823 ODE No. 1823

$$y''(x)\left(a\left(xy'(x) - y(x)\right) + y'(x)^2\right) - b = 0$$

✓ **Mathematica** : cpu = 0.401265 (sec), leaf count = 281
$$\left\{\text{Solve}\left[-\int\frac{a\left(\frac{ax^2}{4} + y(x)\right) + \sqrt{4b\left(\frac{ax^2}{4} + y(x)\right) - 2c_1}}{\sqrt{\left(a^2\left(\frac{ax^2}{4} + y(x)\right)^2 - 4b\left(\frac{ax^2}{4} + y(x)\right) + 2c_1\right)\left(a\left(\frac{ax^2}{4} + y(x)\right) + \sqrt{4b\left(\frac{ax^2}{4} + y(x)\right) - 2c_1}\right)}}d\left(\frac{ax^2}{4} + y(x)\right) = -x\right.\right.$$

✓ **Maple** : cpu = 0.55 (sec), leaf count = 289

$$\left\{y(x) = -\frac{a \ x^2}{4} + \text{RootOf}\left(c_2 - x - \left(\int^{-x}\frac{\sqrt{\left(\_f^2a^2 - 4\_fb + 2c_1\right)\left(\_fa + \sqrt{4\_fb - 2c_1}\right)}}{\_f^2a^2 - 4\_fb + 2c_1}d\_f\right)\right), y(x) = -\frac{a \ x^2}{4} + \text{RootOf}\left(c_2 - \right.$$

2.1824 ODE No. 1824

$$y''(x)\Big(a\sqrt{y'(x)^2+1}-xy'(x)\Big)-y'(x)^2-1=0$$

✓ **Mathematica** : cpu = 0.956735 (sec), leaf count = 347

$$\left\{\left\{y(x)\rightarrow\frac{-2\sqrt{x^2\left(a^2-x^2+c_1^2\right)}+c_1x\log\left(-c_1\left(\sqrt{x^2\left(a^2-x^2+c_1^2\right)}+c_1x\right)+a^2(-x)+ax^2\right)+c_1x\log\left(c_1\left(\sqrt{x^2\left(a^2-x^2+c_1^2\right)}+c_1x\right)+a^2(-x)+ax^2\right)}{2x}\right.\right.$$

✓ **Maple** : cpu = 1.288 (sec), leaf count = 96

$$\left\{y(x)=c_2+\int\frac{-c_1a^2-\sqrt{\left(a^2-x^2+c_1^2\right)a^2}x}{a^3-ax^2}dx,y(x)=c_2+\int\frac{-c_1a^2+\sqrt{\left(a^2-x^2+c_1^2\right)a^2}x}{a^3-ax^2}dx\right\}$$

2.1825 ODE No. 1825

$$f(x)+y''(x)h(y'(x))+j(y(x))y'(x)=0$$

✗ **Mathematica** : cpu = 0.0367427 (sec), leaf count = 0 , could not solve

`DSolve[f[x] + j[y[x]]*Derivative[1][y][x] + h[Derivative[1][y][x]]*Derivative[2][y][x] == 0,`

✓ **Maple** : cpu = 1.306 (sec), leaf count = 49

$$\{y(x)=ODESolStruc\left(\_f\_b,\left[\left\{-\_bf+c_1+\int^{\_f\_b}1d\_a+\int^{\frac{d}{d\_b}\_f\_b}h(\_a)d\_a=0\right\},\{-b=x,\_f\_b=y(x)\},\{x=\_b,y(x)=\_f\_b\}\right]\right)$$

2.1826 ODE No. 1826

$$-ay(x)-b+y''(x)^2=0$$

✓ **Mathematica** : cpu = 1.14372 (sec), leaf count = 201

$$\left\{\text{Solve}\left[\frac{(ay(x)+b)^2\left(1-\frac{4(ay(x)+b)^{3/2}}{3ac_1}\right){}_2F_1\left(\frac{1}{2},\frac{2}{3},\frac{5}{3};\frac{4(b+ay(x))^{3/2}}{3ac_1}\right)^2}{a^2\left(-\frac{4(ay(x)+b)^{3/2}}{3a}+c_1\right)}=(x+c_2)^2,y(x)\right],\text{Solve}\left[\frac{(ay(x)+b)^2\left(1+\frac{4(ay(x)+b)^{3/2}}{3ac_1}\right){}_2F_1\left(\frac{1}{2},\frac{2}{3},\frac{5}{3};\frac{4(b+ay(x))^{3/2}}{3ac_1}\right)^2}{a^2\left(\frac{4(ay(x)+b)^{3/2}}{3a}+c_1\right)}=(x+c_2)^2,y(x)\right]\right\}$$

✓ **Maple** : cpu = 0.799 (sec), leaf count = 173

$$\left\{-c_2-x+\int^{y(x)}\frac{\sqrt{3}a}{\sqrt{\left(4\sqrt{-aa+b}\_aa-c_1+4\sqrt{-aa+b}b\right)a}}d\_a=0,-c_2-x+\int^{y(x)}-\frac{3a}{\sqrt{-12\left(-\frac{c_1}{4}+(\_aa+b)^{\frac{3}{2}}\right)a}}d\_a=0\right\}$$

2.1827 ODE No. 1827

$$a^2y''(x)^2-2axy''(x)+y'(x)=0$$

✗ **Mathematica** : cpu = 1.48238 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x]^2-2*a*x*Derivative[2][y][x]+a^2*Derivative[2][y][x]^2==0,y[x],x]`

✓ **Maple** : cpu = 3.5 (sec), leaf count = 81

$$\left\{y(x)=c_2+\int\text{RootOf}\left(c_1-\left(\int_{-g}^{-Z}\frac{1}{2\_fa-x^2+\sqrt{x^2-\_f}x}d\_f\right)\right)dx,y(x)=c_2+\int\text{RootOf}\left(c_1-\left(\int_{-g}^{-Z}\frac{1}{-2\_fa+x^2+\sqrt{x^2-\_f}x}d\_f\right)\right)dx\right\}$$

2.1828 ODE No. 1828

$$2(x^2 + 1)y''(x)^2 + 2y'(x)(y'(x) + x) - x(4y'(x) + x)y''(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0104344 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2} \sqrt{c_2 - c_1^2} x^2 + c_1 x + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 1.3 (sec), leaf count = 59

$$\left\{ y(x) = \frac{1}{2}c_1x^2 + c_1^2 + c_2^2 + c_2x, y(x) = c_1^2 + \frac{c_1 \operatorname{arcsinh}(x)}{2} - \frac{3x^2}{16} + \frac{\operatorname{arcsinh}(x)^2}{16} + \frac{\left(c_1 + \frac{\operatorname{arcsinh}(x)}{4}\right)\sqrt{x^2+1} x}{2} \right\}$$

2.1829 ODE No. 1829

$$3x^2y''(x)^2 + 4y'(x)^2 - 2(3xy'(x) + y(x))y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0060491 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1^2 x^2}{c_2} + c_1 x + c_2 \right\} \right\}$$

✓ **Maple** : $\text{cpu} = 0.782$ (sec), leaf count = 36

$$\left\{y(x) = 0, y(x) = c_1 x x^{\frac{2\sqrt{3}}{3}}, y(x) = \frac{c_1^2 x^2}{c_2} + c_1 x + c_2\right\}$$

2.1830 ODE No. 1830

$$(2 - 9x)x^2y''(x)^2 + 6y(x)y''(x) - 36xy'(x)^2 - 6(1 - 6x)xy'(x)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0276236 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1^2 x^3}{c_2} + c_1 x + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.987 (sec), leaf count = 308

$$\left\{ \begin{aligned} y(x) = 0, y(x) = & \frac{c_1 \sqrt{5} \sqrt{4} \sqrt{4x-1} x \left((9x-1)\sqrt{9} + 9\sqrt{9x^2-2x} \right)^{\frac{2\sqrt{9}}{9}} \left((9x-1)\sqrt{9} + 9\sqrt{9x^2-2x} \right)^{\frac{5\sqrt{9}}{18}} e^{\frac{\sqrt{9x^2-2x} (\sqrt{16}-4)}{2}}}{135 \sqrt{\frac{\frac{4}{5} + \frac{\sqrt{16} \left(x - \frac{1}{5} \right)}{\sqrt{9x^2-2x}}}{\sqrt{-\frac{(4x-1)^2}{9x^2-2x}}}}}, y(x) = \end{aligned} \right.$$

2.1831 ODE No. 1831

$$y(x)(xF(0,2)+xF(2,0))y''(x)+xF(2,2)y''(x)^2+xF(1,1)y''(x)+y'(x)\big((xF(1,2)+xF(2,1))y''(x)+y(x)(xF(0,1)+xF(1,0))\big)+$$

X Mathematica : cpu = 78.6372 (sec), leaf count = 0 , could not solve

$$\text{DSolve}[x \cdot F[0, 0] \cdot y[x]^2 + x \cdot F[1, 1] \cdot \text{Derivative}[2][y][x] + (x \cdot F[0, 2] + x \cdot F[2, 0]) \cdot y[x] \cdot \text{Deriv}$$

✓ **Maple** : cpu = 1.721 (sec), leaf count = 163

$$\left\{ y(x) = ODESolStruc \left(e^{c_1 + \int \_b(\_a) d\_a}, \left[\left\{ \frac{d}{d\_a} \_b(\_a) = \frac{-2\_b(\_a)^2 F_{2,2}(\_a) + (-F_{1,2}(\_a) - F_{2,1}(\_a)) \_b(\_a) - F_{0,2}(\_a) - F_{2,0}(\_a)}{2\_b(\_a)^2 F_{2,2}(\_a) + (-F_{1,2}(\_a) - F_{2,1}(\_a)) \_b(\_a) - F_{0,2}(\_a) - F_{2,0}(\_a)} \right. \right. \right.$$

2.1832 ODE No. 1832

$$y(x)y''(x)^2 - ae^{2x} = 0$$

✗ **Mathematica** : cpu = 20.4935 (sec), leaf count = 0 , could not solve

```
DSolve[-(a*E^(2*x)) + y[x]*Derivative[2][y][x]^2 == 0, y[x], x]
```

✓ **Maple** : cpu = 1.201 (sec), leaf count = 117

$$\left\{y(x) = ODESolStruc\left(-a e^{\frac{2c_1}{3} + \int \frac{2b(\_a)}{3} d\_a}, \left[\left\{\frac{d}{d\_a} b(\_a) = -\frac{(-4\_a^2 + 9\sqrt{\_a a})\_b(\_a)^3}{9\_a} + \frac{4\_b(\_a)^2}{3}\right\}\right], \left\{-a = e^{-\frac{2x}{3}} y(x), \_b(\_a) = \frac{2}{3} \frac{d}{d\_a} y(x)\right\}\right.\right.$$

2.1833 ODE No. 1833

$$y''(x)^2(a^2y(x)^2 - b^2) + y'(x)^2(a^2y'(x)^2 - 1) - 2a^2y(x)y'(x)^2y''(x) = 0$$

✓ **Mathematica** : cpu = 121.475 (sec), leaf count = 81

$$\left\{\left\{y(x) \rightarrow \frac{b\left(e^{\frac{\sqrt{-1+a^2c_1^2}(x+c_2)}}{b}} - c_1\right)}{\sqrt{-1+a^2c_1^2}}\right\}, \left\{y(x) \rightarrow c_1e^{c_2x} - \frac{\sqrt{b^2 + \frac{1}{c_2^2}}}{a}\right\}\right\}$$

✓ **Maple** : cpu = 6.829 (sec), leaf count = 162

$$\left\{y(x) = c_1, y(x) = \frac{b}{a}, y(x) = \frac{\left(-c_1 + e^{\frac{\sqrt{c_1^2a^2-1}(c_2+x)}}{b}\right)b}{\sqrt{c_1^2a^2-1}}, y(x) = \frac{b \tan\left(\frac{\sqrt{a^2}(c_1-x)}{ab}\right)}{\sqrt{\tan^2\left(\frac{\sqrt{a^2}(c_1-x)}{ab}\right) + 1} a}, y(x) = -\frac{b}{a}, y(x) = -\frac{b \tan\left(\frac{\sqrt{a^2}}{a}\right)}{\sqrt{\tan^2\left(\frac{\sqrt{a^2}(c_1-x)}{ab}\right) + 1} a}\right\}$$

2.1834 ODE No. 1834

$$\left(x^2y(x)y''(x) + x^2(-y'(x)^2) + y(x)^2\right)^2 - 4xy(x)(xy'(x) - y(x))^3 = 0$$

✓ **Mathematica** : cpu = 22.6742 (sec), leaf count = 19

$$\left\{\left\{y(x) \rightarrow c_1xe^{\frac{1}{-x+c_2}}\right\}\right\}$$

✓ **Maple** : cpu = 0.652 (sec), leaf count = 92

$$\left\{y(x) = 0, y(x) = c_1x, y(x) = ODESolStruc\left(e^{c_1 + \int \_b(\_a) d\_a}, \left[\left\{\frac{d}{d\_a} b(\_a) = \frac{2\sqrt{(\_a\_b(\_a)-1)\_a\_a\_b(\_a)} - 2\sqrt{(\_a\_b(\_a)-1)\_a\_a\_b(\_a)}}{\_a^2}\right\}\right]\right.\right.$$

2.1835 ODE No. 1835

$$32y''(x)(xy''(x) - y'(x))^3 + (2y(x)y''(x) - y'(x)^2)^3 = 0$$

✓ **Mathematica** : cpu = 0.0844814 (sec), leaf count = 143

$$\left\{\left\{y(x) \rightarrow \frac{1}{4}\left(-\frac{8c_1^3}{\sqrt[3]{3}\sqrt[3]{\sqrt{27c_1^{10}c_2^{10}-64c_1^9c_2^9}-9c_1^5c_2^5}}} + \frac{c_1^2}{c_2} - \frac{2\sqrt[3]{\sqrt{3}\sqrt{27c_1^{10}c_2^{10}-64c_1^9c_2^9}-9c_1^5c_2^5}}{3^{2/3}c_2^3}\right)x^2 + c_1x\right\}\right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve((2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2)^3+32*diff(diff(y(x),x),x)*(x*diff(diff(y(x),x),x))^3=0,y(x))
```


2.1836 ODE No. 1836

$$\sqrt{a y''(x)^2 + b y'(x)^2} + c y(x) y''(x) + d y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 10.3769 (sec), leaf count = 0 , could not solve

`DSolve[d*Derivative[1][y][x]^2 + c*y[x]*Derivative[2][y][x] + Sqrt[b*Derivative[1][y][x]^2 +`

✓ **Maple** : cpu = 1.172 (sec), leaf count = 116

$$\left\{ y(x) = 0, y(x) = c_1, y(x) = c_1 x + c_2, y(x) = ODESolStruc \left[-a, \left[\left[\frac{\left(-acd\_b(-a) + (-c^2\_a^2 + a) \left(\frac{d}{d\_a} b(-a) \right) + \sqrt{a d^2\_b(-a)} \right)}{-c^2\_a^2 + a} \right] \right] \right. \right.$$

2.1837 ODE No. 1837

$$y^{(3)}(x) - a^2 \left(y'(x)^5 + 2y'(x)^3 + y'(x) \right) = 0$$

✓ **Mathematica** : cpu = 10.2173 (sec), leaf count = 145

$$\left\{ \left\{ y(x) \rightarrow \int_1^x \text{InverseFunction} \left[-3 \int \frac{1}{\sqrt{3(a^2)^2 \#1^6 + 9(a^2)^2 \#1^4 + 9(a^2)^2 \#1^2 + 9c_1}} d\#1 \& [c_2 - K[1]] dK[1] + c_3 \right] \right\}, \left\{ y(x) \right.$$

✓ **Maple** : cpu = 0.547 (sec), leaf count = 95

$$\left\{ y(x) = c_3 + \int \text{RootOf} \left(c_2 + x - 3 \left(\int^{-Z} \frac{1}{\sqrt{3a^2\_f^6 + 9\_f^4 a^2 + 9\_f^2 a^2 + 9c_1}} d\_f \right) \right) dx, y(x) = c_3 + \int \text{RootOf} \left(c_2 + x + 3 \left(\int \right. \right.$$

2.1838 ODE No. 1838

$$y^{(3)}(x) + y(x) y''(x) - y'(x)^2 + 1 = 0$$

✗ **Mathematica** : cpu = 0.034647 (sec), leaf count = 0 , could not solve

`DSolve[1 - Derivative[1][y][x]^2 + y[x]*Derivative[2][y][x] + Derivative[3][y][x] == 0, y[x]`

✓ **Maple** : cpu = 0.816 (sec), leaf count = 73

$$\left\{ y(x) = ODESolStruc \left[-a, \left[\left[-a\_b(-a) \left(\frac{d}{d\_a} b(-a) \right) + b(-a)^2 \left(\frac{d^2}{d\_a^2} b(-a) \right) + b(-a) \left(\frac{d}{d\_a} b(-a) \right)^2 - b(-a)^2 + 1 = 0 \right] \right] \right. \right.$$

2.1839 ODE No. 1839

$$y^{(3)}(x) - y(x) y''(x) + y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 0.0336891 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x]^2 - y[x]*Derivative[2][y][x] + Derivative[3][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 1.057 (sec), leaf count = 116

$$\left\{ y(x) = ODESolStruc \left(e^{c_2 + \int -g(f) d\_f}, \left[\left[\frac{d}{d\_f} -g(-f) = \frac{6 \left(\left(-f - \frac{1}{6} \right) -g(-f) + \frac{1}{6} \right) (-f -g(-f) + 1) -g(-f)}{-f} \right] \right] \right. \right.$$

2.1840 ODE No. 1840

$ay(x)y''(x)+y^{(3)}(x)=0$

✗ **Mathematica** : cpu = 0.037189 (sec), leaf count = 0 , could not solve

`DSolve[a*y[x]*Derivative[2][y][x] + Derivative[3][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 1.15 (sec), leaf count = 129

$$\left\{y(x)=ODESolStruc\left(e^{c_2+\int -g(\_f)d\_f},\left[\left\{\frac{d}{d\_f}-g(\_f)=\frac{\left(6\_f^2-g(\_f)^2+2\_fa\_g(\_f)^2+7\_f-g(\_f)+a\_g(\_f)+1\right)-g(\_f)}{\_f}\right\},\left\{-\right.\right.\right.\right.$$

2.1841 ODE No. 1841

$-f(x)+x^2y^{(3)}(x)+xy''(x)+(2xy(x)-1)y'(x)+y(x)^2=0$

✗ **Mathematica** : cpu = 0.128177 (sec), leaf count = 0 , could not solve

`DSolve[-f[x] + y[x]^2 + (-1 + 2*x*y[x])*Derivative[1][y][x] + x*Derivative[2][y][x] + x^2*De`

✓ **Maple** : cpu = 0.73 (sec), leaf count = 60

$$\{y(x)=ODESolStruc\left(-b(\_a),\left[\left\{-a^2\left(\frac{d^2}{d\_a^2}-b(\_a)\right)+\_a\_b(\_a)^2-\_a\left(\frac{d}{d\_a}-b(\_a)\right)+c_1-\left(\int f(\_a)d\_a\right)=0\right\},\left\{-a=x,\_b\right.\right.\right.$$

2.1842 ODE No. 1842

$x^2y^{(3)}(x)+x(y(x)-1)y''(x)+xy'(x)^2+(1-y(x))y'(x)=0$

✓ **Mathematica** : cpu = 0.208281 (sec), leaf count = 286

$$\left\{\left\{y(x)\rightarrow\frac{2x\left(c_3\left(J_{\frac{\sqrt{c_2+2}}{\sqrt{2}}}\left(-\frac{1}{2}ix\sqrt{c_1}\right)-\frac{1}{4}i\sqrt{c_1}x\left(J_{\frac{\sqrt{c_2+2}}{\sqrt{2}}-1}\left(-\frac{1}{2}ix\sqrt{c_1}\right)-J_{\frac{\sqrt{c_2+2}}{\sqrt{2}}+1}\left(-\frac{1}{2}ix\sqrt{c_1}\right)\right)\right)+Y_{\frac{\sqrt{c_2+2}}{\sqrt{2}}}\left(-\frac{1}{2}ix\sqrt{c_1}\right)-\frac{1}{4}i\sqrt{c_1}\right)}{c_3xJ_{\frac{\sqrt{c_2+2}}{\sqrt{2}}}\left(-\frac{1}{2}ix\sqrt{c_1}\right)+xY_{\frac{\sqrt{c_2+2}}{\sqrt{2}}}\left(-\frac{1}{2}ix\sqrt{c_1}\right)}\right.\right.$$

✓ **Maple** : cpu = 0.821 (sec), leaf count = 190

$$\left\{-c_3+2\left(\int^{y(x)}\frac{\_h^2+2\operatorname{RootOf}\left(2c_2\sqrt{2}\\_Z\operatorname{BesselY}\left(\frac{\sqrt{c_1+4}}{2}+1,\frac{\sqrt{2}\_Z}{2}\right)+2c_2\_h\operatorname{BesselY}\left(\frac{\sqrt{c_1+4}}{2},\frac{\sqrt{2}\_Z}{2}\right)-2c_2\sqrt{c_1+4}\operatorname{BesselY}\left(\frac{\sqrt{c_1+4}}{2},\frac{\sqrt{2}\_Z}{2}\right)\right)}{\_h^2+2\operatorname{RootOf}\left(2c_2\sqrt{2}\\_Z\operatorname{BesselY}\left(\frac{\sqrt{c_1+4}}{2}+1,\frac{\sqrt{2}\_Z}{2}\right)+2c_2\_h\operatorname{BesselY}\left(\frac{\sqrt{c_1+4}}{2},\frac{\sqrt{2}\_Z}{2}\right)-2c_2\sqrt{c_1+4}\operatorname{BesselY}\left(\frac{\sqrt{c_1+4}}{2},\frac{\sqrt{2}\_Z}{2}\right)\right)}d\_h\right)\right.$$

2.1843 ODE No. 1843

$y^{(3)}(x)y(x)+y(x)^3y'(x)-y'(x)y''(x)=0$

✓ **Mathematica** : cpu = 2.92628 (sec), leaf count = 409

$$\left\{\left\{y(x)\rightarrow\operatorname{InverseFunction}\left[-\frac{2i\sqrt{1+\frac{\#1^2}{2\left(\sqrt{c_2^2-c_1}-c_2\right)}}\sqrt{1-\frac{\#1^2}{2\left(c_2+\sqrt{c_2^2-c_1}\right)}}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{1}{\sqrt{c_2^2-c_1}-c_2}}\#1\right)}{\sqrt{2}}\right)\right|_{\frac{c_2-\sqrt{c_2^2-c_1}}{c_2+\sqrt{c_2^2-c_1}}}\right)}{\sqrt{\frac{1}{\sqrt{c_2^2-c_1}-c_2}}\sqrt{-\frac{\#1^4}{2}+2\#1^2c_2-2c_1}}\right]&[x+c_3]$$

✓ **Maple** : cpu = 0.518 (sec), leaf count = 77

$$\left\{ -c_3 - x + \int^{y(x)} -\frac{2}{\sqrt{-a^4 + 4c_2a^2 - 4c_2^2 + 4c_1}} d_a = 0, -c_3 - x + \int^{y(x)} \frac{2}{\sqrt{-a^4 + 4c_2a^2 - 4c_2^2 + 4c_1}} d_a = 0 \right\}$$

2.1844 ODE No. 1844

$$4y(x)^2y^{(3)}(x) + 15y'(x)^3 - 18y(x)y'(x)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.115466 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{(c_3x^2 + c_2x + c_1)^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.833 (sec), leaf count = 17

$$\left\{ y(x) = \frac{c_3}{(c_1(c_2 + x)^2 - 4)^2} \right\}$$

2.1845 ODE No. 1845

$$9y(x)^2y^{(3)}(x) + 40y'(x)^3 - 45y(x)y'(x)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.112587 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{(c_3x^2 + c_2x + c_1)^{3/2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.652 (sec), leaf count = 22

$$\left\{ y(x) = \frac{c_3}{(x^2 + 2c_2x + c_2^2 - 9c_1)^{\frac{3}{2}}} \right\}$$

2.1846 ODE No. 1846

$$2y^{(3)}(x)y'(x) - 3y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0363249 (sec), leaf count = 51

$$\left\{ \{y(x) \rightarrow c_1\}, \left\{ y(x) \rightarrow \sqrt{\frac{2}{3}} e^{-\sqrt{\frac{3}{2}}x} (c_1 e^{\sqrt{6}x} - c_2) + c_3 \right\} \right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 28

$$\left\{ y(x) = c_1, y(x) = c_2 e^{\frac{\sqrt{6}x}{2}} + c_3 e^{-\frac{\sqrt{6}x}{2}} + c_1 \right\}$$

2.1847 ODE No. 1847

$$y^{(3)}(x)(y'(x)^2 + 1) - 3y'(x)y''(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.246633 (sec), leaf count = 95

$$\left\{ \left\{ y(x) \rightarrow c_3 - \frac{i\sqrt{c_1^2x^2 + 2c_2c_1^2x - 1 + c_2^2c_1^2}}{c_1} \right\}, \left\{ y(x) \rightarrow \frac{i\sqrt{c_1^2x^2 + 2c_2c_1^2x - 1 + c_2^2c_1^2}}{c_1} + c_3 \right\} \right\}$$

✓ **Maple** : cpu = 0.424 (sec), leaf count = 49

$$\left\{ y(x) = c_3 - \sqrt{-x^2 - 2c_2x - c_2^2 + c_1}, y(x) = c_3 + \sqrt{-x^2 - 2c_2x - c_2^2 + c_1} \right\}$$

2.1848 ODE No. 1848

$$y''(x)^2(-a - 3y'(x)) + y^{(3)}(x)(y'(x)^2 + 1) = 0$$

✓ **Mathematica** : cpu = 0.492681 (sec), leaf count = 187

$$\left\{ \left\{ y(x) \rightarrow c_3 - \frac{\left(1 - i \text{InverseFunction} \left[\frac{(\#1 - a) e^{-a \tan^{-1}(\#1)}}{\sqrt{\#1^2 + 1} (a^2 + 1) c_1} \right] \& \right) [x + c_2] \right)^{-\frac{1}{2} - \frac{ia}{2}} \left(1 + i \text{InverseFunction} \left[\frac{(\#1 - a) e^{-a \tan^{-1}(\#1)}}{\sqrt{\#1^2 + 1} (a^2 + 1) c_1} \right] \& \right) [x + c_2] \right\} \right\} \frac{1}{(a^2 + 1) c_1}$$

✓ **Maple** : cpu = 1.487 (sec), leaf count = 789

$$\left\{ y(x) = c_3 + \int \frac{\sin(\text{RootOf}(c_1^2 c_2^2 a^4 e^{2-Z_a} + 2c_2 c_1^2 a^4 x e^{2-Z_a} + c_1^2 a^4 x^2 e^{2-Z_a} + 2c_1^2 c_2^2 a^2 e^{2-Z_a} + 4c_2 c_1^2 a^2 x e^{2-Z_a} + 2c_1^2 a^2 x^2 e^{2-Z_a}))}{\cos(\text{RootOf}(c_1^2 c_2^2 a^4 e^{2-Z_a} + 2c_2 c_1^2 a^4 x e^{2-Z_a} + c_1^2 a^4 x^2 e^{2-Z_a} + 2c_1^2 c_2^2 a^2 e^{2-Z_a} + 4c_2 c_1^2 a^2 x e^{2-Z_a} + 2c_1^2 a^2 x^2 e^{2-Z_a}))} dx \right.$$

2.1849 ODE No. 1849

$$y^{(3)}(x)y''(x) - a\sqrt{b^2y''(x)^2 + 1} = 0$$

✓ **Mathematica** : cpu = 0.645312 (sec), leaf count = 426

$$\left\{ \left\{ y(x) \rightarrow \frac{\frac{(a^2b^4x^2+2ab^4c_1x+b^4c_1^2-1)^{3/2}}{3ab^2} + \frac{\sqrt{a^2b^4x^2+2ab^4c_1x+b^4c_1^2-1}}{ab^2} - \frac{c_1 \log\left(\sqrt{a^2b^4x^2+2ab^4c_1x+b^4c_1^2-1}+ab^2x+b^2c_1\right)}{a} - x \log\left(b^2\left(\sqrt{a^2b^4x^2}\right.\right.\right. \right.$$

✓ **Maple** : $\text{cpu} = 0.388 \text{ (sec)}$, leaf count = 197

$$\left\{ \begin{aligned} y(x) = c_2 x + c_3 + \int \frac{-\frac{\ln\left(\frac{(c_1+x)a^2 b^4}{\sqrt{a^2 b^4}} + \sqrt{\left((c_1+x)a b^2 + 1\right)\left((c_1+x)a b^2 - 1\right)}\right)}{\sqrt{a^2 b^4}} + \sqrt{\left((c_1+x)a b^2 + 1\right)\left((c_1+x)a b^2 - 1\right)} (c_1+x)}{2b} dx, y(x) = \end{aligned} \right.$$

2.1850 ODE No. 1850

$$y^{(4)}(x)y'(x) - y^{(3)}(x)y''(x) + y^{(3)}(x)y'(x)^3 = 0$$

X Mathematica : cpu = 0.147269 (sec), leaf count = 0 , could not solve

$$\text{DSolve}[\text{Derivative}[1][y][x]^3 \cdot \text{Derivative}[3][y][x] - \text{Derivative}[2][y][x] \cdot \text{Derivative}[3][y][x] +$$

✓ **Maple** : cpu = 1.645 (sec), leaf count = 164

$$\left\{ y(x) = ODESolStruc \left(c_3 + \int \frac{j(\_h) e^{-c_2 + \int -j(\_h) d\_h}}{\_h} d\_h, \left[\left\{ \frac{d}{d\_h} j(\_h) = \frac{(12\_h^2 j(\_h)^2 + 3\_h j(\_h)^2 + 10\_h j(\_h) + j(\_h))}{\_h} \right. \right. \right. \right.$$

2.1851 ODE No. 1851

$$y'(x)^3(f'(x)y'(x) + f(x)y''(x)) - y''(x)(f''(x)y'(x) + 2f'(x)y''(x) + f(x)y^{(3)}(x)) + y'(x)(f^{(3)}(x)y'(x) + 3f''(x)y''(x) + 3y^{(3)}(x))$$

X Mathematica : cpu = 0.704677 (sec), leaf count = 0 , could not solve

$$\text{DSolve}[2*q[x]*\text{Sin}[y[x]]*\text{Derivative}[1][y][x]^2 + \text{Derivative}[1][y][x]^3*(\text{Derivative}[1][f][x]*D(\text{Derivative}[1][q][x]*\text{Derivative}[1][y][x]) + q[x]*\text{Derivative}[2][y][x]) - \text{Derivative}[2][y][x]*$$

x Maple : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve(diff(y(x),x)*(diff(diff(diff(f(x),x),x),x)*diff(y(x),x)+3*diff(diff(f(x),x),x)*diff(d
diff(diff(y(x),x),x)*f*diff(diff(diff(y(x),x),x),x)+diff(y(x),x)^3*(diff(f(x),x)*diff(y(x),x
diff(q(x),x)*diff(y(x),x))*cos(y(x))=0,y(x))
```

2.1852 ODE No. 1852

$$3y^{(4)}(x)y''(x)-5y^{(3)}(x)^2=0$$

✓ **Mathematica** : cpu = 0.130194 (sec), leaf count = 28

$$\left\{\left\{y(x)\rightarrow c_2\left(-\sqrt{2x+3c_1}\right)+c_4x+c_3\right\}\right\}$$

✓ **Maple** : cpu = 0.416 (sec), leaf count = 28

$$\left\{y(x)=c_3x+3c_1(c_2+x)\sqrt{6}\sqrt{-\frac{c_1}{c_2+x}}+c_4\right\}$$

2.1853 ODE No. 1853

$$40y^{(3)}(x)^3+9y^{(5)}(x)y''(x)^2-45y^{(4)}(x)y^{(3)}(x)y''(x)=0$$

✓ **Mathematica** : cpu = 0.141571 (sec), leaf count = 43

$$\left\{\left\{y(x)\rightarrow c_5x-\frac{4\sqrt{x(c_3x+c_2)+c_1}}{c_2^2-4c_1c_3}+c_4\right\}\right\}$$

✓ **Maple** : cpu = 1.053 (sec), leaf count = 110

$$\left\{y(x)=c_4x+c_5+\int\int\text{RootOf}\left(c_3+x-\left(\int^{-Z}\frac{\text{RootOf}\left(20c_2+\int^{-Z}\left(e^{\text{RootOf}\left(81_k^2e^{-Z}+162c_1e^{-Z}-20_Ze^{-Z}+2187_k^2+20e^{-Z}\ln\left(e^{-Z}\right)}\right)}\right)}{\right)}\right)\right.\right.$$

2.1854 ODE No. 1854

$$y^{(n)}(x)-f\left(\frac{\partial^{n-1}y(x)}{\partial x^{n-1}}\right)=0$$

✗ **Mathematica** : cpu = 0.0413331 (sec), leaf count = 0 , could not solve

`DSolve[-f[D[y[x], {x, -1 + n}]] + Derivative[n][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

unable to handle ODEs of undefined differential order

2.1855 ODE No. 1855

$$y^{(n)}(x)-f\left(\frac{\partial^{n-2}y(x)}{\partial x^{n-2}}\right)=0$$

✗ **Mathematica** : cpu = 0.0032559 (sec), leaf count = 0 , could not solve

`DSolve[-f[D[y[x], {x, -2 + n}]] + Derivative[n][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

unable to handle ODEs of undefined differential order

2.1856 ODE No. 1856

$$\left\{x'(t)=ax(t),y'(t)=b\right\}$$

✓ **Mathematica** : cpu = 0.0180445 (sec), leaf count = 22

$$\left\{\left\{x(t)\rightarrow c_1e^{at},y(t)\rightarrow bt+c_2\right\}\right\}$$

✓ **Maple** : cpu = 0.055 (sec), leaf count = 19

$$\{\{x(t)=c_1e^{at},y(t)=bt+c_2\}\}$$

2.1857 ODE No. 1857

$$\left\{x'(t) = ay(t), y'(t) = -ax(t)\right\}$$

✓ **Mathematica** : cpu = 0.0095632 (sec), leaf count = 39

$$\{ \{x(t) \rightarrow c_1 \cos(at) + c_2 \sin(at), y(t) \rightarrow c_2 \cos(at) - c_1 \sin(at)\} \}$$

✓ **Maple** : cpu = 0.067 (sec), leaf count = 35

$$\{ \{x(t) = c_1 \sin(at) + c_2 \cos(at), y(t) = c_1 \cos(at) - c_2 \sin(at)\} \}$$

2.1858 ODE No. 1858

$$\left\{x'(t) = ay(t), y'(t) = bx(t)\right\}$$

✓ **Mathematica** : cpu = 0.0093449 (sec), leaf count = 182

$$\left\{ \left\{ x(t) \rightarrow \frac{1}{2} c_1 e^{-\sqrt{a} \sqrt{b} t} \left(e^{2 \sqrt{a} \sqrt{b} t} + 1 \right) + \frac{\sqrt{a} c_2 e^{-\sqrt{a} \sqrt{b} t} \left(e^{2 \sqrt{a} \sqrt{b} t} - 1 \right)}{2 \sqrt{b}}, y(t) \rightarrow \frac{\sqrt{b} c_1 e^{-\sqrt{a} \sqrt{b} t} \left(e^{2 \sqrt{a} \sqrt{b} t} - 1 \right)}{2 \sqrt{a}} + \frac{1}{2} c_2 e^{-\sqrt{a} \sqrt{b} t} \left(e^{2 \sqrt{a} \sqrt{b} t} + 1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.073 (sec), leaf count = 64

$$\left\{ \left\{ x(t) = c_1 e^{\sqrt{a} \sqrt{b} t} + c_2 e^{-\sqrt{a} \sqrt{b} t}, y(t) = \frac{\left(c_1 e^{\sqrt{a} \sqrt{b} t} - c_2 e^{-\sqrt{a} \sqrt{b} t} \right) \sqrt{b}}{\sqrt{a}} \right\} \right\}$$

2.1859 ODE No. 1859

$$\left\{x'(t) = ax(t) - y(t), y'(t) = ay(t) + x(t)\right\}$$

✓ **Mathematica** : cpu = 0.0047282 (sec), leaf count = 51

$$\{ \{x(t) \rightarrow c_1 e^{at} \cos(t) - c_2 e^{at} \sin(t), y(t) \rightarrow c_2 e^{at} \cos(t) + c_1 e^{at} \sin(t)\} \}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 37

$$\{ \{x(t) = (c_1 \sin(t) + c_2 \cos(t)) e^{at}, y(t) = (-c_1 \cos(t) + c_2 \sin(t)) e^{at}\} \}$$

2.1860 ODE No. 1860

$$\left\{x'(t) = ax(t) + by(t), y'(t) = by(t) + cx(t)\right\}$$

✓ **Mathematica** : cpu = 0.0348163 (sec), leaf count = 696

$$\left\{ \left\{ x(t) \rightarrow \frac{c_1 \left(a \left(-e^{\frac{1}{2} t \left(-\sqrt{a^2 - 2ab + b^2 + 4bc} + a + b \right)} \right) + a e^{\frac{1}{2} t \left(\sqrt{a^2 - 2ab + b^2 + 4bc} + a + b \right)} + b e^{\frac{1}{2} t \left(-\sqrt{a^2 - 2ab + b^2 + 4bc} + a + b \right)} + \sqrt{a^2 - 2ab + b^2 + 4bc} \right)}{2 \sqrt{a^2 - 2ab + b^2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.102 (sec), leaf count = 177

$$\left\{ \left\{ x(t) = c_1 e^{\frac{\left(a + b + \sqrt{a^2 + b^2 + (-2a + 4c)b} \right) t}{2}} + c_2 e^{\frac{\left(a + b - \sqrt{a^2 + b^2 + (-2a + 4c)b} \right) t}{2}}, y(t) = \frac{-c_1 \left(a - b - \sqrt{a^2 + b^2 + (-2a + 4c)b} \right) e^{\frac{\left(a + b + \sqrt{a^2 + b^2 + (-2a + 4c)b} \right) t}{2}}}{2} \right\} \right\}$$

2.1861 ODE No. 1861

$$\{ax'(t) + by'(t) = \alpha x(t) + \beta y(t), bx'(t) - ay'(t) = \beta x(t) - \alpha y(t)\}$$

✓ **Mathematica** : cpu = 0.0105553 (sec), leaf count = 183

$$\left\{ \left\{ x(t) \rightarrow c_1 e^{\frac{t(a\alpha+b\beta)}{a^2+b^2}} \cos\left(\frac{t(a\beta-\alpha b)}{a^2+b^2}\right) + c_2 e^{\frac{t(a\alpha+b\beta)}{a^2+b^2}} \sin\left(\frac{t(a\beta-\alpha b)}{a^2+b^2}\right), y(t) \rightarrow c_2 e^{\frac{t(a\alpha+b\beta)}{a^2+b^2}} \cos\left(\frac{t(a\beta-\alpha b)}{a^2+b^2}\right) - c_1 e^{\frac{t(a\alpha+b\beta)}{a^2+b^2}} \sin\left(\frac{t(a\beta-\alpha b)}{a^2+b^2}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.113 (sec), leaf count = 152

$$\left\{ \left\{ x(t) = c_1 e^{\frac{((i\beta+\alpha)a-(i\alpha-\beta)b)t}{a^2+b^2}} + c_2 e^{-\frac{((i\beta-\alpha)a-(i\alpha+\beta)b)t}{a^2+b^2}}, y(t) = i \left(c_1 e^{\frac{((i\beta+\alpha)a-(i\alpha-\beta)b)t}{a^2+b^2}} - c_2 e^{-\frac{((i\beta-\alpha)a-(i\alpha+\beta)b)t}{a^2+b^2}} \right) \right\} \right\}$$

2.1862 ODE No. 1862

$$\{x'(t) = -y(t), y'(t) = 2x(t) + 2y(t)\}$$

✓ **Mathematica** : cpu = 0.0126009 (sec), leaf count = 52

$$\left\{ \left\{ x(t) \rightarrow c_1 e^t (\cos(t) - \sin(t)) - c_2 e^t \sin(t), y(t) \rightarrow 2c_1 e^t \sin(t) + c_2 e^t (\sin(t) + \cos(t)) \right\} \right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 39

$$\{x(t) = (c_1 \sin(t) + c_2 \cos(t))e^t, y(t) = -((c_1 + c_2) \cos(t) + (c_1 - c_2) \sin(t))e^t\}$$

2.1863 ODE No. 1863

$$\{x'(t) + 3x(t) + 4y(t) = 0, 2x(t) + y'(t) + 5y(t) = 0\}$$

✓ **Mathematica** : cpu = 0.0068441 (sec), leaf count = 84

$$\left\{ \left\{ x(t) \rightarrow \frac{1}{3} c_1 e^{-7t} (2e^{6t} + 1) - \frac{2}{3} c_2 e^{-7t} (e^{6t} - 1), y(t) \rightarrow \frac{1}{3} c_2 e^{-7t} (e^{6t} + 2) - \frac{1}{3} c_1 e^{-7t} (e^{6t} - 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 35

$$\left\{ \left\{ x(t) = c_1 e^{-t} + c_2 e^{-7t}, y(t) = -\frac{c_1 e^{-t}}{2} + c_2 e^{-7t} \right\} \right\}$$

2.1864 ODE No. 1864

$$\{x'(t) = -5x(t) - 2y(t), y'(t) = x(t) - 7y(t)\}$$

✓ **Mathematica** : cpu = 0.0102639 (sec), leaf count = 59

$$\left\{ \left\{ x(t) \rightarrow c_1 e^{-6t} (\sin(t) + \cos(t)) - 2c_2 e^{-6t} \sin(t), y(t) \rightarrow c_1 e^{-6t} \sin(t) + c_2 e^{-6t} (\cos(t) - \sin(t)) \right\} \right\}$$

✓ **Maple** : cpu = 0.051 (sec), leaf count = 44

$$\left\{ \left\{ x(t) = (c_1 \sin(t) + c_2 \cos(t))e^{-6t}, y(t) = -\frac{((c_1 - c_2) \cos(t) - (c_1 + c_2) \sin(t))e^{-6t}}{2} \right\} \right\}$$

2.1865 ODE No. 1865

$$\left\{x'(t) = a1x(t) + b1y(t) + c1, y'(t) = a2x(t) + b2y(t) + c2\right\}$$

✓ **Mathematica** : cpu = 0.837384 (sec), leaf count = 2062

$$\left\{\left\{x(t) \rightarrow -\frac{b1e^{-\frac{1}{2}\left(a1+b2+\sqrt{a1^2-2b2a1+b2^2+4a2b1}\right)t}\left(2\left(\left(a1-b2+\sqrt{a1^2-2b2a1+b2^2+4a2b1}\right)c2-2a2c1\right)e^{\sqrt{a1^2-2b2a1+b2^2+4a2b1}t}-\frac{2\left(2a2c1+\left(-a1+b2+\sqrt{a1^2-2b2a1+b2^2+4a2b1}\right)c2\right)}{a1+b2+\sqrt{a1^2-2b2a1+b2^2+4a2b1}}\right)}{2\left(a1^2-2b2a1+b2^2+4a2b1\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.144 (sec), leaf count = 224

$$\left\{\left\{x(t) = c_1e^{\frac{\left(a1+b2-\sqrt{a1^2-2a1b2+4a2b1+b2^2}\right)t}{2}} + c_2e^{\frac{\left(a1+b2+\sqrt{a1^2-2a1b2+4a2b1+b2^2}\right)t}{2}} + \frac{c2b1-b2c1}{a1b2-a2b1}, y(t) = \frac{-c1(a1b2-a2b1)\left(a1-b2+\sqrt{a1^2-2a1b2+4a2b1+b2^2}\right)}{2\left(a1^2-2b2a1+b2^2+4a2b1\right)}\right\}\right\}$$

2.1866 ODE No. 1866

$$\left\{x'(t) + 2y(t) = 3t, y'(t) - 2x(t) = 4\right\}$$

✓ **Mathematica** : cpu = 0.0415217 (sec), leaf count = 132

$$\left\{\left\{x(t) \rightarrow \cos(2t)\left(\frac{3}{2}t\sin(2t) - \frac{5}{4}\cos(2t)\right) - \sin(2t)\left(\frac{5}{4}\sin(2t) + \frac{3}{2}t\cos(2t)\right) + c_1\cos(2t) - c_2\sin(2t), y(t) \rightarrow \cos(2t)\left(\frac{5}{4}\sin(2t) + \frac{3}{2}t\cos(2t)\right) - \sin(2t)\left(\frac{3}{2}t\sin(2t) - \frac{5}{4}\cos(2t)\right) + c_1\sin(2t) - c_2\cos(2t)\right\}\right\}$$

✓ **Maple** : cpu = 0.047 (sec), leaf count = 39

$$\left\{\left\{x(t) = c_1\cos(2t) + c_2\sin(2t) - \frac{5}{4}, y(t) = c_1\sin(2t) - c_2\cos(2t) + \frac{3t}{2}\right\}\right\}$$

2.1867 ODE No. 1867

$$\left\{-t^2 + x'(t) + y(t) + 6t + 1 = 0, y'(t) - x(t) = -3t^2 + 3t + 1\right\}$$

✓ **Mathematica** : cpu = 0.0998978 (sec), leaf count = 124

$$\left\{\left\{x(t) \rightarrow \cos(t)\left(\left(3t^2-t-13\right)\cos(t) + \left(t-12\right)t\sin(t)\right) - \sin(t)\left(\left(-3t^2+t+13\right)\sin(t) + \left(t-12\right)t\cos(t)\right) + c_1\cos(t) - c_2\sin(t), y(t) \rightarrow \sin(t)\left(\left(3t^2-t-13\right)\sin(t) + \left(t-12\right)t\cos(t)\right) + \cos(t)\left(\left(-3t^2+t+13\right)\cos(t) + \left(t-12\right)t\sin(t)\right) + c_1\sin(t) - c_2\cos(t)\right\}\right\}$$

✓ **Maple** : cpu = 0.047 (sec), leaf count = 42

$$\{ \{ x(t) = c_1 \cos(t) + c_2 \sin(t) + 3t^2 - t - 13, y(t) = c_1 \sin(t) - c_2 \cos(t) + t^2 - 12t \} \}$$

2.1868 ODE No. 1868

$$\left\{x'(t) + 3x(t) - y(t) = e^{2t}, x(t) + y'(t) + 5y(t) = e^t\right\}$$

✓ **Mathematica** : cpu = 0.0900776 (sec), leaf count = 162

$$\left\{\left\{x(t) \rightarrow -e^t(t+1)\left(\frac{t}{5} + \frac{1}{36}e^t(6t-7) - \frac{1}{25}\right) + e^t t\left(\frac{t}{5} + \frac{1}{36}e^t(6t-1) + \frac{4}{25}\right) + c_1e^{-4t}(t+1) + c_2e^{-4t}t, y(t) \rightarrow e^t t\left(\frac{t}{5} + \frac{1}{36}e^t(6t-1) + \frac{4}{25}\right) - e^t(t+1)\left(\frac{t}{5} + \frac{1}{36}e^t(6t-7) - \frac{1}{25}\right) + c_1e^{-4t} - c_2e^{-4t}t\right\}\right\}$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 64

$$\left\{\left\{x(t) = c_1te^{-4t} + c_2e^{-4t} + \frac{e^t}{25} + \frac{7e^{2t}}{36}, y(t) = -c_1te^{-4t} + c_1e^{-4t} - c_2e^{-4t} + \frac{4e^t}{25} - \frac{e^{2t}}{36}\right\}\right\}$$

2.1869 ODE No. 1869

$$\{x'(t) + 2x(t) + y'(t) + y(t) = t + e^{2t}, x'(t) - x(t) + y'(t) + 3y(t) = e^t - 1\}$$

✓ **Mathematica** : cpu = 0.108001 (sec), leaf count = 118

$$\left\{ \left\{ x(t) \rightarrow \frac{1}{5}(t - e^t + e^{2t} + 1) + \frac{5}{72} \left(\frac{12(5712t + 833e^t + 2352e^{2t} - 5508)}{20825} + c_1 e^{-7t/5} \right), y(t) \rightarrow \frac{1}{5}(-t + e^t - e^{2t} - 1) + \frac{5}{48} \left(\frac{12(5712t + 833e^t + 2352e^{2t} - 5508)}{20825} + c_1 e^{-7t/5} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 51

$$\left\{ \left\{ x(t) = c_1 e^{-\frac{7t}{5}} + \frac{3t}{7} - \frac{e^t}{6} + \frac{5e^{2t}}{17} - \frac{1}{49}, y(t) = \frac{3c_1 e^{-\frac{7t}{5}}}{2} + \frac{t}{7} + \frac{e^t}{4} - \frac{e^{2t}}{17} - \frac{26}{49} \right\} \right\}$$

2.1870 ODE No. 1870

$$\{x'(t) + y'(t) - y(t) = e^t, 2x'(t) + y'(t) + 2y(t) = \cos(t)\}$$

✓ **Mathematica** : cpu = 0.155578 (sec), leaf count = 122

$$\left\{ \left\{ x(t) \rightarrow -\frac{3}{4}c_2(e^{4t} - 1) + \frac{1}{68}e^{-4t}(e^{4t} - 1)(34e^t + 3\sin(t) - 12\cos(t)) + \frac{1}{4}(2e^{-3t} + 2e^t + \frac{3}{17}e^{-4t}\sin(t) + \sin(t) - \frac{12}{17}e^{-4t}\cos(t)) \right\} \right\}$$

✓ **Maple** : cpu = 0.152 (sec), leaf count = 47

$$\left\{ \left\{ x(t) = \frac{c_1 e^{4t}}{4} + c_2 - \frac{3\cos(t)}{17} + e^t + \frac{5\sin(t)}{17}, y(t) = -\frac{c_1 e^{4t}}{3} + \frac{4\cos(t)}{17} - \frac{2e^t}{3} - \frac{\sin(t)}{17} \right\} \right\}$$

2.1871 ODE No. 1871

$$\{4x'(t) + 2x(t) + 9y'(t) + 31y(t) = e^t, 3x'(t) + x(t) + 7y'(t) + 24y(t) = 3\}$$

✓ **Mathematica** : cpu = 0.351832 (sec), leaf count = 180

$$\left\{ \left\{ x(t) \rightarrow \frac{1}{442}(3(153e^t - 754)\sin(t) + 31(17e^t - 78)\cos(t))(\cos(t) - \sin(t)) + \frac{1}{221}\sin(t)((493e^t - 2340)\sin(t) + (34e^t - 17)\cos(t)) \right\} \right\}$$

✓ **Maple** : cpu = 0.132 (sec), leaf count = 62

$$\left\{ \left\{ x(t) = c_1 \cos(t)e^{-4t} + c_2 e^{-4t} \sin(t) + \frac{31e^t}{26} - \frac{93}{17}, y(t) = -\frac{2e^t}{13} + \frac{((-221c_1 - 221c_2)\cos(t) + 221(c_1 - c_2)\sin(t))e^{-4t}}{221} + \frac{1}{221} \right\} \right\}$$

2.1872 ODE No. 1872

$$\{4x'(t) + 11x(t) + 9y'(t) + 31y(t) = e^t, 3x'(t) + 8x(t) + 7y'(t) + 24y(t) = e^{2t}\}$$

✓ **Mathematica** : cpu = 0.129799 (sec), leaf count = 162

$$\left\{ \left\{ x(t) \rightarrow -e^t t \left(-\frac{4t}{5} + \frac{1}{36}e^t(30t + 19) - \frac{11}{25} \right) - e^t(t - 1) \left(\frac{4t}{5} - \frac{1}{36}e^t(30t + 49) + \frac{31}{25} \right) - c_1 e^{-4t}(t - 1) - c_2 e^{-4t}t, y(t) \rightarrow e^t(t + 1) \left(-\frac{4t}{5} + \frac{1}{36}e^t(30t + 19) - \frac{11}{25} \right) - e^t(t - 1) \left(\frac{4t}{5} - \frac{1}{36}e^t(30t + 49) + \frac{31}{25} \right) - c_1 e^{-4t}(t - 1) - c_2 e^{-4t}t \right\} \right\}$$

✓ **Maple** : cpu = 0.102 (sec), leaf count = 65

$$\left\{ \left\{ x(t) = c_1 t e^{-4t} + c_2 e^{-4t} + \frac{31e^t}{25} - \frac{49e^{2t}}{36}, y(t) = -c_1 t e^{-4t} - c_1 e^{-4t} - c_2 e^{-4t} - \frac{11e^t}{25} + \frac{19e^{2t}}{36} \right\} \right\}$$

2.1873 ODE No. 1873

$$\left\{4x'(t) + 44x(t) + 9y'(t) + 49y(t) = t, 3x'(t) + 34x(t) + 7y'(t) + 38y(t) = e^t\right\}$$

✓ **Mathematica** : cpu = 0.104857 (sec), leaf count = 322

$$\left\{\left\{x(t) \rightarrow -\frac{1}{5}e^{-6t}\left(e^{5t}-1\right)\left(\frac{16}{5}e^{6t}\left(\frac{t}{6}-\frac{1}{36}\right)+4e^{2t}-\frac{4e^{7t}}{7}-\frac{31}{5}e^t(t-1)\right)+\frac{1}{25}e^{-6t}\left(4e^{5t}+1\right)\left(e^{6t}\left(\frac{2t}{3}-\frac{1}{9}\right)-20e^{2t}-\frac{5e^{7t}}{7}+e^{5t}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.079 (sec), leaf count = 52

$$\left\{\left\{x(t)=c_1e^{-6t}+c_2e^{-t}+\frac{19t}{3}-\frac{29e^t}{7}-\frac{56}{9},y(t)=4c_1e^{-6t}-c_2e^{-t}-\frac{17t}{3}+\frac{24e^t}{7}+\frac{55}{9}\right\}\right\}$$

2.1874 ODE No. 1874

$$\left\{x'(t)=f(t)x(t)+g(t)y(t),y'(t)=f(t)y(t)-g(t)x(t)\right\}$$

✓ **Mathematica** : cpu = 0.010728 (sec), leaf count = 115

$$\left\{\left\{x(t) \rightarrow c_1 \exp \left(\int_1^t f(K[2]) d K[2]\right) \cos \left(\int_1^t g(K[1]) d K[1]\right)+c_2 \exp \left(\int_1^t f(K[2]) d K[2]\right) \sin \left(\int_1^t g(K[1]) d K[1]\right), y(t) \rightarrow c_1 \exp \left(\int_1^t g(K[1]) d K[1]\right) \cos \left(\int_1^t f(K[2]) d K[2]\right)-c_2 \exp \left(\int_1^t g(K[1]) d K[1]\right) \sin \left(\int_1^t f(K[2]) d K[2]\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.556 (sec), leaf count = 57

$$\left\{\left\{x(t)=c_2e^{\int\left(g(t)\tan\left(c_1-\left(\int g(t)dt\right)\right)+f(t)\right)dt},y(t)=c_2e^{\int\left(g(t)\tan\left(c_1-\left(\int g(t)dt\right)\right)+f(t)\right)dt}\tan\left(c_1-\left(\int g(t)dt\right)\right)\right\}\right\}$$

2.1875 ODE No. 1875

$$\left\{f(t)(ax(t)+by(t))+x'(t)=g(t),f(t)(cx(t)+dy(t))+y'(t)=h(t)\right\}$$

✓ **Mathematica** : cpu = 1.8693 (sec), leaf count = 3181

$$\left\{\left\{x(t) \rightarrow \frac{\left(-a+d+\sqrt{a^2-2da+d^2+4bc}\right)e^{\int_1^t-\frac{1}{2}\left(a+d-\sqrt{a^2-2da+d^2+4bc}\right)f(K[1])dK[1]}c_1}{2\sqrt{a^2-2da+d^2+4bc}}+\frac{\left(a-d+\sqrt{a^2-2da+d^2+4bc}\right)e^{\int_1^t\frac{1}{2}\left(a+d-\sqrt{a^2-2da+d^2+4bc}\right)f(K[1])dK[1]}c_1}{2\sqrt{a^2-2da+d^2+4bc}}\right\}\right\}$$

✓ **Maple** : cpu = 3.296 (sec), leaf count = 2602

Expression too large to display

2.1876 ODE No. 1876

$$\left\{x'(t)=x(t)\cos(t),y'(t)=x(t)e^{-\sin(t)}\right\}$$

✓ **Mathematica** : cpu = 0.0234421 (sec), leaf count = 39

$$\left\{\left\{x(t) \rightarrow c_1e^{\sin(t)},y(t) \rightarrow c_1\int_1^te^{\sin(K[1])-\sin(K[1])}dK[1]+c_2\right\}\right\}$$

✓ **Maple** : cpu = 0.207 (sec), leaf count = 18

$$\left\{\left\{x(t)=c_2e^{\sin(t)},y(t)=c_2t+c_1\right\}\right\}$$

2.1877 ODE No. 1877

$$\{tx'(t) + y(t) = 0, x(t) + ty'(t) = 0\}$$

✓ **Mathematica** : cpu = 0.0048264 (sec), leaf count = 31

$$\left\{ \left\{ x(t) \rightarrow c_1 t + \frac{c_2}{t}, y(t) \rightarrow \frac{c_2}{t} - c_1 t \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 31

$$\left\{ \left\{ x(t) = \frac{c_2 t^2 + c_1}{t}, y(t) = \frac{-c_2 t^2 + c_1}{t} \right\} \right\}$$

2.1878 ODE No. 1878

$$\{tx'(t) + 2x(t) = t, -((t+2)x(t)) + ty'(t) - ty(t) = -t\}$$

✓ **Mathematica** : cpu = 0.0130872 (sec), leaf count = 39

$$\left\{ \left\{ x(t) \rightarrow \frac{t}{3} + \frac{c_1}{t^2}, y(t) \rightarrow -\frac{c_1}{t^2} - \frac{t}{3} + c_2 e^t \right\} \right\}$$

✓ **Maple** : cpu = 0.091 (sec), leaf count = 39

$$\left\{ \left\{ x(t) = \frac{t}{3} + \frac{c_2}{t^2}, y(t) = \frac{3c_1 t^2 e^t - t^3 - 3c_2}{3t^2} \right\} \right\}$$

2.1879 ODE No. 1879

$$\{tx'(t) + 2(x(t) - y(t)) = t, x(t) + ty'(t) + 5y(t) = t^2\}$$

✓ **Mathematica** : cpu = 0.0229742 (sec), leaf count = 58

$$\left\{ \left\{ x(t) \rightarrow \frac{c_1}{t^4} + \frac{c_2}{t^3} + \frac{1}{30}t(2t+9), y(t) \rightarrow -\frac{c_1}{t^4} - \frac{c_2}{2t^3} + \frac{1}{60}t(8t-3) \right\} \right\}$$

✓ **Maple** : cpu = 0.088 (sec), leaf count = 54

$$\left\{ \left\{ x(t) = \frac{2t^6 + 9t^5 + 30c_1 t + 30c_2}{30t^4}, y(t) = \frac{8t^6 - 3t^5 - 30c_1 t - 60c_2}{60t^4} \right\} \right\}$$

2.1880 ODE No. 1880

$$\{t^2(1 - \sin(t))x'(t) = t^2y(t) + tx(t)(1 - 2\sin(t)), t^2(1 - \sin(t))y'(t) = x(t)(t\cos(t) - \sin(t)) + ty(t)(1 - t\cos(t))\}$$

✓ **Mathematica** : cpu = 0.0214935 (sec), leaf count = 29

$$\left\{ \left\{ x(t) \rightarrow c_1 t^2 + c_2 t, y(t) \rightarrow c_1 t + c_2 \sin(t) \right\} \right\}$$

✓ **Maple** : cpu = 0.1 (sec), leaf count = 23

$$\{x(t) = (c_1 t + c_2)t, y(t) = c_1 t + c_2 \sin(t)\}$$

2.1881 ODE No. 1881

$$\{x'(t) + y'(t) + y(t) = f(t), x''(t) + x(t) + y''(t) + y'(t) + y(t) = g(t)\}$$

✓ **Mathematica** : cpu = 0.0129763 (sec), leaf count = 44

$$\left\{ \left\{ x(t) \rightarrow -f''(t) - f'(t) - f(t) + g'(t) + g(t), y(t) \rightarrow f''(t) + f(t) - g'(t) \right\} \right\}$$

✓ **Maple** : cpu = 0.057 (sec), leaf count = 48

$$\{x(t) = -\frac{d^2}{dt^2}f(t) - \frac{d}{dt}f(t) + \frac{d}{dt}g(t) - f(t) + g(t), y(t) = \frac{d^2}{dt^2}f(t) - \frac{d}{dt}g(t) + f(t)\}$$

2.1882 ODE No. 1882

$$\left\{2x'(t) - 3x(t) + y'(t) = 0, x''(t) + y'(t) - 2y(t) = e^{2t}\right\}$$

✓ **Mathematica** : cpu = 1.73687 (sec), leaf count = 928

$$\left\{\left\{x(t) \rightarrow \frac{1}{46}e^{t/2}c_1\left(23\cos\left(\frac{\sqrt{23}t}{2}\right) + 23e^{t/2} - 3\sqrt{23}\sin\left(\frac{\sqrt{23}t}{2}\right)\right) + \frac{e^{3t/2}\left(23e^{t/2}\cos\left(\frac{\sqrt{23}t}{2}\right) - 7\sqrt{23}e^{t/2}\sin\left(\frac{\sqrt{23}t}{2}\right) + 46\right)}{12696}\right\}\right\}$$

✓ **Maple** : cpu = 0.235 (sec), leaf count = 99

$$\left\{\left\{x(t) = c_2\cos\left(\frac{\sqrt{23}t}{2}\right)e^{\frac{t}{2}} + c_3e^{\frac{t}{2}}\sin\left(\frac{\sqrt{23}t}{2}\right) + c_1e^t + \frac{e^{2t}}{4}, y(t) = c_1e^t - \frac{7\left(c_2 + \frac{c_3\sqrt{23}}{7}\right)\cos\left(\frac{\sqrt{23}t}{2}\right)e^{\frac{t}{2}}}{4} + \frac{(c_2\sqrt{23} - 7c_3)}{4}\right\}\right\}$$

2.1883 ODE No. 1883

$$\left\{x'(t) + x(t) - y'(t) = 2t, x''(t) - 9x(t) + y'(t) + 3y(t) = \sin(2t)\right\}$$

✓ **Mathematica** : cpu = 1.05945 (sec), leaf count = 602

$$\left\{\left\{x(t) \rightarrow \frac{e^{-4t}\left(20e^{4t}t + 7e^{4t} + 9\right)\left(10400\left(t^2 + 2t + 2\right) + \left(260t - 225e^{4t} - 351\right)\sin(2t) + 2\left(260t + 75e^{4t} - 91\right)\cos(2t)\right)}{83200} - \frac{e^{-4t}\left(20e^{4t}t + 7e^{4t} + 9\right)}{83200}\right\}\right\}$$

✓ **Maple** : cpu = 0.135 (sec), leaf count = 80

$$\left\{\left\{x(t) = c_3te^t + c_1e^t + c_2e^{-3t} + 2t - \frac{2\cos(2t)}{325} - \frac{36\sin(2t)}{325} + 4, y(t) = 2c_3te^t + 2c_1e^t + \frac{2c_2e^{-3t}}{3} - c_3e^t + 6t + \frac{16\cos(2t)}{325}\right\}\right\}$$

2.1884 ODE No. 1884

$$\left\{x'(t) - x(t) + 2y(t) = 0, x''(t) - 2y'(t) = 2t - \cos(2t)\right\}$$

✓ **Mathematica** : cpu = 0.446788 (sec), leaf count = 224

$$\left\{\left\{x(t) \rightarrow 7\left(t^2 - \frac{1}{2}\sin(2t) + c_2\right) + 8\left(\frac{1}{136}e^{-t/2}\left(2e^{t/2}\cos(2t) - 4\left(34e^{t/2}t^2 + 17e^{t/2}(t + 2) - 15e^{t/2}\sin(2t)\right)\right) + c_1e^{t/2} + c_2\left(e^{t/2}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.185 (sec), leaf count = 69

$$\left\{\left\{x(t) = 2c_1e^{\frac{t}{2}} - t^2 + c_2 - 4t + \frac{2\cos(2t)}{17} + \frac{\sin(2t)}{34}, y(t) = \frac{c_1e^{\frac{t}{2}}}{2} - \frac{t^2}{2} + \frac{c_2}{2} - t + \frac{\cos(2t)}{34} + \frac{9\sin(2t)}{68} + 2\right\}\right\}$$

2.1885 ODE No. 1885

$$\left\{tx'(t) - ty'(t) - 2y(t) = 0, tx''(t) + 2x'(t) + tx(t) = 0\right\}$$

✓ **Mathematica** : cpu = 0.0195699 (sec), leaf count = 66

$$\left\{\left\{x(t) \rightarrow \frac{c_2\cos(t)}{t} + \frac{c_3\sin(t)}{t}, y(t) \rightarrow \frac{c_1}{t^2} + c_2\left(\frac{\cos(t)}{t} - \frac{2\sin(t)}{t^2}\right) + c_3\left(\frac{2\cos(t)}{t^2} + \frac{\sin(t)}{t}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.175 (sec), leaf count = 47

$$\left\{\left\{x(t) = \frac{c_2\sin(t) + c_3\cos(t)}{t}, y(t) = \frac{c_1 + (c_3t + 2c_2)\cos(t) + (c_2t - 2c_3)\sin(t)}{t^2}\right\}\right\}$$

2.1890 ODE No. 1890

$$\left\{x''(t) = c^2x(t)\big(3\cos^2(at+b)-1\big) + \frac{3}{2}c^2y(t)\sin(2abt), y''(t) = \frac{3}{2}c^2x(t)\sin(2abt) + c^2y(t)\big(3\sin^2(at+b)-1\big)\right\}$$

✗ **Mathematica** : cpu = 2.05366 (sec), leaf count = 0 , could not solve

```
DSolve[{Derivative[2][x][t] == c^2*(-1 + 3*Cos[b + a*t]^2)*x[t] + (3*c^2*Sin[2*a*b*t]*y[t])/
1 + 3*Sin[b + a*t]^2)*y[t]}, {x[t], y[t]}, t]
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{x(t) = DESol\left(\left(-36c^4\big(\cos^2(b)\big)\big(\cos^2(at)\big)\big(\sin^2(b)\big)\big(\sin^2(at)\big) - 18c^4\cos(b)\big(\cos^3(at)\big)\big(\sin^3(b)\big)\sin(at) + 18c^4\cos(b)\cos^3(at)\right)\right\}$$

2.1891 ODE No. 1891

$$\{x''(t) + 6x(t) + 7y(t) = 0, 3x(t) + y''(t) + 2y(t) = 2t\}$$

✓ **Mathematica** : cpu = 1.05973 (sec), leaf count = 742

$$\left\{x(t) \rightarrow -\frac{7}{200}e^{-t}\big(e^{2t} - 2e^t\cos(3t) + 1\big)\bigg(-7e^{-t}\big(e^{2t}(t-1) + t + 1\big) - \frac{2}{9}\sin(3t) + \frac{2}{3}t\cos(3t)\bigg) + \frac{7}{600}e^{-t}\big(3e^{2t} + 14e^t\cos(3t) - 5\big)\right\}$$

✓ **Maple** : cpu = 0.084 (sec), leaf count = 64

$$\left\{x(t) = c_1e^t + c_2\cos(3t) + c_3e^{-t} + c_4\sin(3t) + \frac{14t}{9}, y(t) = -c_1e^t + \frac{3c_2\cos(3t)}{7} - c_3e^{-t} + \frac{3c_4\sin(3t)}{7} - \frac{4t}{3}\right\}$$

2.1892 ODE No. 1892

$$\{-ay'(t) + bx(t) + x''(t) = 0, ax'(t) + by(t) + y''(t) = 0\}$$

✓ **Mathematica** : cpu = 0.279632 (sec), leaf count = 4815

$$\left\{x(t) \rightarrow \frac{e^{-\frac{\sqrt{-a^2-2b-\sqrt{a^2(a^2+4b)}}{\sqrt{2}}t} - \frac{\sqrt{-a^2-2b+\sqrt{a^2(a^2+4b)}}{\sqrt{2}}t \left(e^{\frac{\sqrt{-a^2-2b-\sqrt{a^2(a^2+4b)}}{\sqrt{2}}t} a^2 - e^{\frac{\sqrt{-a^2-2b+\sqrt{a^2(a^2+4b)}}{\sqrt{2}}t} a^2 - e^{\sqrt{2}\sqrt{-a^2-2b-\sqrt{a^2(a^2+4b)}}t} + e^{\sqrt{2}\sqrt{-a^2-2b+\sqrt{a^2(a^2+4b)}}t} \right)}{a^2 - e^{\frac{\sqrt{-a^2-2b-\sqrt{a^2(a^2+4b)}}{\sqrt{2}}t} a^2 - e^{\frac{\sqrt{-a^2-2b+\sqrt{a^2(a^2+4b)}}{\sqrt{2}}t} a^2 - e^{\sqrt{2}\sqrt{-a^2-2b-\sqrt{a^2(a^2+4b)}}t} + e^{\sqrt{2}\sqrt{-a^2-2b+\sqrt{a^2(a^2+4b)}}t}}\right\}$$

✓ **Maple** : cpu = 0.167 (sec), leaf count = 463

$$\left\{x(t) = c_1e^{-\frac{\sqrt{-2a^2-4b-2\sqrt{(a^2+4b)a^2}}{2}t} + c_2e^{\frac{\sqrt{-2a^2-4b-2\sqrt{(a^2+4b)a^2}}{2}t} + c_3e^{-\frac{\sqrt{-2a^2-4b+2\sqrt{(a^2+4b)a^2}}{2}t} + c_4e^{\frac{\sqrt{-2a^2-4b+2\sqrt{(a^2+4b)a^2}}{2}t}, y(t) = -\frac{4c_1}{a^2}e^{-\frac{\sqrt{-2a^2-4b-2\sqrt{(a^2+4b)a^2}}{2}t} - \frac{4c_2}{a^2}e^{\frac{\sqrt{-2a^2-4b-2\sqrt{(a^2+4b)a^2}}{2}t} - \frac{4c_3}{a^2}e^{-\frac{\sqrt{-2a^2-4b+2\sqrt{(a^2+4b)a^2}}{2}t} - \frac{4c_4}{a^2}e^{\frac{\sqrt{-2a^2-4b+2\sqrt{(a^2+4b)a^2}}{2}t}\right\}$$

2.1893 ODE No. 1893

$$\left\{-A0y'(t)+a1x''(t)+b1x'(t)+c1x(t)=B0e^{i\omega t},A0x'(t)+a2y''(t)+b2y'(t)+c2y(t)=0\right\}$$

✓ **Mathematica** : cpu = 0.484848 (sec), leaf count = 5546

$$\left\{\left\{x(t)\rightarrow -A0a1^2c2c3\text{RootSum}\left[\#1^4+a2b1\#1^3+a1b2\#1^3+A0^2a1a2\#1^2+a1a2b1b2\#1^2+a1a2^2c1\#1^2+a1^2a2c2\#1\right]\right\}\right\}$$

✓ **Maple** : cpu = 0.954 (sec), leaf count = 1579

Expression too large to display

2.1894 ODE No. 1894

$$\left\{a\big(x'(t)-y'(t)\big)+b1x(t)+x''(t)=c1e^{i\omega t},a\big(y'(t)-x'(t)\big)+b2y(t)+y''(t)=c2e^{i\omega t}\right\}$$

✓ **Mathematica** : cpu = 0.527298 (sec), leaf count = 3386

$$\left\{\left\{x(t)\rightarrow -ab2c3\text{RootSum}\left[\#1^4+2a\#1^3+b1\#1^2+b2\#1^2+ab1\#1+ab2\#1+b1b2\&, \frac{e^{t\#1}}{4\#1^3+6a\#1^2+2b1\#1+2b2\#1}\right]\right\}\right\}$$

✓ **Maple** : cpu = 0.916 (sec), leaf count = 1056

$$\left\{\left\{x(t)=c_1\mathrm{e}^{t\text{RootOf}\big(\_Z^4+2\_Z^3a+(b1+b2)\_Z^2+b1b2+(ab1+ab2)\_Z,index=1\big)}+c_2\mathrm{e}^{t\text{RootOf}\big(\_Z^4+2\_Z^3a+(b1+b2)\_Z^2+b1b2+(ab1+ab2)\_Z,index=2\big)}+\right.\right\}$$

2.1895 ODE No. 1895

$$\left\{a11x''(t)+a12y''(t)+b11x'(t)+b12y'(t)+c11x(t)+c12y(t)=0,a21x''(t)+a22y''(t)+b21x'(t)+b22y'(t)+c21x(t)+c22y(t)=0\right\}$$

✓ **Mathematica** : cpu = 0.248366 (sec), leaf count = 7517

✓ **Maple** : cpu = 0.227 (sec), leaf count = 1008

$$\left\{\left\{x(t)=\sum_{_a=1}^4\_C_{\_a}\mathrm{e}^{t\text{RootOf}\big((a22a11-a21a12)\_Z^4+(a11b22-b21a12-a21b12+a22b11)\_Z^3+(a11c22-c21a12-a21c12+a22c11+b22b11-b12b21)\_Z^2+c11b22-c21b22\big)}\right\}\right\}$$

2.1896 ODE No. 1896

$$\left\{x''(t)-2x'(t)-y'(t)+y(t)=0,2x'(t)-x(t)+y^{(3)}(t)-y''(t)=t\right\}$$

✓ **Mathematica** : cpu = 0.570529 (sec), leaf count = 1132

$$\left\{\left\{x(t)\rightarrow \frac{1}{64}e^{-t}\big(2e^{2t}t^2-6e^{2t}t+7e^{2t}+1\big)\big(e^t(1-t)+e^{-t}\big(-2t^3-8t^2-17t-17\big)\big)+\frac{1}{64}e^{-t}\big(2e^{2t}t^2+6e^{2t}t+e^{2t}-1\big)\big(e^t(t-1)+e^{-t}\big(-2t^3-8t^2-17t-17\big)\big)\right\}\right\}$$

✓ **Maple** : cpu = 0.084 (sec), leaf count = 67

$$\left\{\left\{x(t)=-\frac{2c_2\mathrm{e}^{-t}}{3}-t+\frac{\big(-9c_5t^2-6c_4t-3c_3-18c_5\big)\mathrm{e}^t}{3}-2,y(t)=c_2\mathrm{e}^{-t}+\big(c_5t^3+c_4t^2+c_3t+c_1\big)\mathrm{e}^t-2\right\}\right\}$$

2.1897 ODE No. 1897

$$\{x''(t) + y''(t) + y'(t) = \sinh(2t), 2x''(t) + y''(t) = 2t\}$$

✓ **Mathematica** : cpu = 0.240627 (sec), leaf count = 280

$$\left\{ \left\{ x(t) \rightarrow t \left(\frac{t^2}{2} + \frac{t}{2} - \frac{e^{4t}}{8} + e^{2t} \left(\frac{t}{2} - \frac{1}{4} \right) \right) + \frac{1}{48} \left(-4(4t^2 - 3t + 3)t - 12e^{2t}t - 6e^{-2t} + 3e^{4t} \right) + \frac{1}{4}e^{-2t} \left(-2e^{2t} \left(\frac{t}{2} - \frac{1}{4} \right) + \frac{e^{4t}}{4} - t \right) \right. \right.$$

✓ **Maple** : cpu = 0.329 (sec), leaf count = 90

$$\left\{ \left\{ x(t) = \frac{t^3}{6} + c_3t + \frac{t^2}{4} + c_4 + \frac{(6c_2 - 6t - 9) \cosh(2t)}{24} + \frac{(-6c_2 + 6t + 6) \sinh(2t)}{24}, y(t) = -\frac{t^2}{2} + c_1 + c_3 + \frac{t}{2} + \frac{(-2c_2 + 2t)}{2} \right. \right.$$

2.1898 ODE No. 1898

$$\{x''(t) - x'(t) + y'(t) = 0, x''(t) - x(t) + y''(t) = 0\}$$

✓ **Mathematica** : cpu = 0.0320885 (sec), leaf count = 420

$$\left\{ \left\{ x(t) \rightarrow -\frac{1}{5}c_1e^{\frac{t}{2}-\frac{\sqrt{5}t}{2}} \left(\sqrt{5}e^{\sqrt{5}t} - 5e^{\frac{\sqrt{5}t}{2}+\frac{t}{2}} - \sqrt{5} \right) + \frac{c_2e^{\frac{t}{2}-\frac{\sqrt{5}t}{2}} \left(e^{\sqrt{5}t} - 1 \right)}{\sqrt{5}} - \frac{1}{10}c_4e^{\frac{t}{2}-\frac{\sqrt{5}t}{2}} \left(5e^{\sqrt{5}t} + \sqrt{5}e^{\sqrt{5}t} - 10e^{\frac{\sqrt{5}t}{2}+\frac{t}{2}} + 5 \right) \right. \right.$$

✓ **Maple** : cpu = 0.092 (sec), leaf count = 71

$$\left\{ \left\{ x(t) = c_1e^t - \frac{c_3(\sqrt{5}+1)e^{\frac{(\sqrt{5}+1)t}{2}}}{2} + \frac{c_4(\sqrt{5}-1)e^{-\frac{(\sqrt{5}-1)t}{2}}}{2}, y(t) = c_3e^{\frac{(\sqrt{5}+1)t}{2}} + c_4e^{-\frac{(\sqrt{5}-1)t}{2}} + c_2 \right\} \right\}$$

2.1899 ODE No. 1899

$$\{x'(t) = 2x(t), y'(t) = 3x(t) - 2y(t), z'(t) = 2y(t) + 3z(t)\}$$

✓ **Mathematica** : cpu = 0.0096933 (sec), leaf count = 112

$$\left\{ \left\{ x(t) \rightarrow c_1e^{2t}, y(t) \rightarrow \frac{3}{4}c_1e^{-2t}(e^{4t} - 1) + c_2e^{-2t}, z(t) \rightarrow \frac{3}{10}c_1e^{-2t}(2e^t + 3e^{2t} + 4e^{3t} + 1)(e^t - 1)^2 + \frac{2}{5}c_2e^{-2t}(e^{5t} - 1) + c_3e^t \right. \right.$$

✓ **Maple** : cpu = 0.113 (sec), leaf count = 52

$$\left\{ \left\{ x(t) = c_3e^{2t}, y(t) = c_2e^{-2t} + \frac{3c_3e^{2t}}{4}, z(t) = c_1e^{3t} - \frac{2c_2e^{-2t}}{5} - \frac{3c_3e^{2t}}{2} \right\} \right\}$$

2.1900 ODE No. 1900

$$\{x'(t) = 4x(t), y'(t) = x(t) - 2y(t), z'(t) = x(t) - 4y(t) + z(t)\}$$

✓ **Mathematica** : cpu = 0.0088893 (sec), leaf count = 94

$$\left\{ \left\{ x(t) \rightarrow c_1e^{4t}, y(t) \rightarrow \frac{1}{6}c_1e^{-2t}(e^{6t} - 1) + c_2e^{-2t}, z(t) \rightarrow \frac{1}{9}c_1e^{-2t}(e^{3t} + e^{6t} - 2) - \frac{4}{3}c_2e^{-2t}(e^{3t} - 1) + c_3e^t \right\} \right\}$$

✓ **Maple** : cpu = 0.116 (sec), leaf count = 50

$$\left\{ \left\{ x(t) = c_3e^{4t}, y(t) = c_2e^{-2t} + \frac{c_3e^{4t}}{6}, z(t) = c_1e^t + \frac{4c_2e^{-2t}}{3} + \frac{c_3e^{4t}}{9} \right\} \right\}$$

2.1901 ODE No. 1901

$$\left\{x'(t) = y(t) - z(t), y'(t) = x(t) + y(t), z'(t) = x(t) + z(t)\right\}$$

✓ **Mathematica** : cpu = 0.0088684 (sec), leaf count = 105

$$\left\{\left\{x(t) \rightarrow c_2(e^t - 1) + c_3(1 - e^t) + c_1, y(t) \rightarrow c_1(e^t - 1) + c_2(e^t t + 1) + c_3(-e^t t + e^t - 1), z(t) \rightarrow c_1(e^t - 1) + c_2(e^t t - e^t + 1)\right\}\right.$$

✓ **Maple** : cpu = 0.092 (sec), leaf count = 43

$$\{\{x(t) = c_2 e^t + c_3, y(t) = -c_3 + (c_2 t + c_1) e^t, z(t) = -c_3 + (c_1 + c_2(t - 1)) e^t\}\}$$

2.1902 ODE No. 1902

$$\left\{x'(t) - y(t) + z(t) = 0, -x(t) + y'(t) - y(t) = t, -x(t) + z'(t) - z(t) = t\right\}$$

✓ **Mathematica** : cpu = 0.0199899 (sec), leaf count = 226

$$\left\{\left\{x(t) \rightarrow e^{-t}(1 - e^t)(-t - 1) + e^{-t}(e^t - 1)(-t - 1) + c_2(e^t - 1) + c_3(1 - e^t) + c_1, y(t) \rightarrow e^{-t}(-t - 1)(-e^t t + e^t - 1) + e^{-t}(-t - 1)\right\}\right.$$

✓ **Maple** : cpu = 0.099 (sec), leaf count = 51

$$\{\{x(t) = c_2 e^t + c_3, y(t) = -c_3 - t + (c_2 t + c_1) e^t - 1, z(t) = -c_3 - t + (c_1 + c_2(t - 1)) e^t - 1\}\}$$

2.1903 ODE No. 1903

$$\left\{ax'(t) = bc(y(t) - z(t)), by'(t) = ac(z(t) - x(t)), cz'(t) = ab(x(t) - y(t))\right\}$$

✓ **Mathematica** : cpu = 0.0882043 (sec), leaf count = 1304

$$\left\{\left\{x(t) \rightarrow \frac{e^{-i\sqrt{a^2+b^2+c^2}t}\left(2e^{i\sqrt{a^2+b^2+c^2}t}a^2 + b^2e^{2i\sqrt{a^2+b^2+c^2}t} + c^2e^{2i\sqrt{a^2+b^2+c^2}t} + b^2 + c^2\right)c_1}{2(a^2 + b^2 + c^2)} - \frac{be^{-i\sqrt{a^2+b^2+c^2}t}\left(-1 + e^{i\sqrt{a^2+b^2+c^2}t}\right)}{2(a^2 + b^2 + c^2)}\right\}\right.$$

✓ **Maple** : cpu = 0.17 (sec), leaf count = 299

$$\left\{\left\{x(t) = c_2 \sin\left(\sqrt{a^2 + b^2 + c^2} t\right) + c_3 \cos\left(\sqrt{a^2 + b^2 + c^2} t\right) + c_1, y(t) = \frac{c_1 b^3 - \left(-c_2 \cos\left(\sqrt{a^2 + b^2 + c^2} t\right) + c_3 \sin\left(\sqrt{a^2 + b^2 + c^2} t\right)\right)}{2(a^2 + b^2 + c^2)}\right\}\right.$$

2.1904 ODE No. 1904

$$\left\{x'(t) = cy(t) - bz(t), y'(t) = az(t) - cx(t), z'(t) = bx(t) - ay(t)\right\}$$

✓ **Mathematica** : cpu = 0.0510571 (sec), leaf count = 1445

$$\left\{\left\{x(t) \rightarrow \frac{e^{-\sqrt{-a^2-b^2-c^2}t}\left(2e^{\sqrt{-a^2-b^2-c^2}t}a^2 + b^2e^{2\sqrt{-a^2-b^2-c^2}t} + c^2e^{2\sqrt{-a^2-b^2-c^2}t} + b^2 + c^2\right)c_1}{2(a^2 + b^2 + c^2)} - \frac{e^{-\sqrt{-a^2-b^2-c^2}t}\left(-1 + e^{\sqrt{-a^2-b^2-c^2}t}\right)}{2(a^2 + b^2 + c^2)}\right\}\right.$$

✓ **Maple** : cpu = 0.11 (sec), leaf count = 257

$$\left\{\left\{x(t) = c_2 \sin\left(\sqrt{a^2 + b^2 + c^2} t\right) + c_3 \cos\left(\sqrt{a^2 + b^2 + c^2} t\right) + c_1, y(t) = \frac{c_1(b^2 + c^2)b + \left(-c_3a^2b + c_2\sqrt{a^2 + b^2 + c^2}ac\right)}{2(a^2 + b^2 + c^2)}\right\}\right.$$

2.1905 ODE No. 1905

$$\left\{x'(t) = h(t)y(t) - g(t)z(t), y'(t) = f(t)z(t) - h(t)x(t), z'(t) = g(t)x(t) - f(t)y(t)\right\}$$

✗ **Mathematica** : cpu = 0.151997 (sec), leaf count = 0 , could not solve

`DSolve[{Derivative[1][x][t] == h[t]*y[t] - g[t]*z[t], Derivative[1][y][t] == -(h[t]*x[t]) +`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

Expression too large to display

2.1906 ODE No. 1906

$$\left\{x'(t) = x(t) + y(t) - z(t), y'(t) = -x(t) + y(t) + z(t), z'(t) = x(t) - y(t) + z(t)\right\}$$

✓ **Mathematica** : cpu = 0.0447676 (sec), leaf count = 278

$$\left\{\left\{x(t) \rightarrow \frac{1}{3}c_1e^t\left(2\cos(\sqrt{3}t) + 1\right) - \frac{1}{3}c_2e^t\left(-\sqrt{3}\sin(\sqrt{3}t) + \cos(\sqrt{3}t) - 1\right) - \frac{1}{3}c_3e^t\left(\sqrt{3}\sin(\sqrt{3}t) + \cos(\sqrt{3}t) - 1\right), y(t)\right.\right.$$

✓ **Maple** : cpu = 0.098 (sec), leaf count = 120

$$\left.\left\{x(t) = \left(c_2\sin(\sqrt{3}t) + c_3\cos(\sqrt{3}t) + c_1\right)e^t, y(t) = c_1e^t + \frac{\left(c_2\sqrt{3} - c_3\right)\cos(\sqrt{3}t)e^t}{2} + \frac{\left(-c_2 - c_3\sqrt{3}\right)e^t\sin(\sqrt{3}t)}{2}, z(t)\right\}\right\}$$

2.1907 ODE No. 1907

$$\left\{x'(t) = -3x(t) + 48y(t) - 28z(t), y'(t) = -4x(t) + 40y(t) - 22z(t), z'(t) = -6x(t) + 57y(t) - 31z(t)\right\}$$

✓ **Mathematica** : cpu = 0.0111799 (sec), leaf count = 179

$$\left\{\left\{x(t) \rightarrow c_1\left(-e^t\right)\left(2e^{2t} - 3\right) + 6c_2e^t\left(2e^t + 3e^{2t} - 5\right) - 2c_3e^t\left(4e^t + 5e^{2t} - 9\right), y(t) \rightarrow -2c_1e^t\left(e^{2t} - 1\right) + c_2e^t\left(3e^t + 18e^{2t} - 20\right)\right.\right.$$

✓ **Maple** : cpu = 0.094 (sec), leaf count = 66

$$\left.\left\{x(t) = c_1e^{3t} + c_2e^{2t} + c_3e^t, y(t) = c_1e^{3t} + \frac{c_2e^{2t}}{4} + \frac{2c_3e^t}{3}, z(t) = \frac{3c_1e^{3t}}{2} + \frac{c_2e^{2t}}{4} + c_3e^t\right\}\right\}$$

2.1908 ODE No. 1908

$$\left\{x'(t) = 6x(t) - 72y(t) + 44z(t), y'(t) = 4x(t) - 4y(t) + 26z(t), z'(t) = 6x(t) - 63y(t) + 38z(t)\right\}$$

✓ **Mathematica** : cpu = 0.040415 (sec), leaf count = 551

$$\left\{\left\{x(t) \rightarrow -36c_2\text{RootSum}\left[\#1^3 - 40\#1^2 + 1714\#1 + 1404\&, \frac{2\#1e^{\#1t} + e^{\#1t}}{3\#1^2 - 80\#1 + 1714}\&\right] + 4c_3\text{RootSum}\left[\#1^3 - 40\#1^2 + 1714\right.\right.$$

✓ **Maple** : cpu = 0.388 (sec), leaf count = 1285

Expression too large to display

2.1909 ODE No. 1909

$$\left\{x'(t) = ax(t) + \beta z(t) + gy(t), y'(t) = az(t) + by(t) + gx(t), z'(t) = \alpha y(t) + \beta x(t) + cz(t)\right\}$$

✓ **Mathematica** : cpu = 0.0567227 (sec), leaf count = 1630

$$\left\{\left\{x(t) \rightarrow -c_2\text{RootSum}\left[\#1^3 - a\#1^2 - b\#1^2 - c\#1^2 - \alpha^2\#1 - \beta^2\#1 - g^2\#1 + ab\#1 + ac\#1 + bc\#1 + a\alpha^2 + b\beta^2 + cg^2 - abc\right.\right.$$

✓ **Maple** : cpu = 16.364 (sec), leaf count = 33085

Too large to display

2.1910 ODE No. 1910

$$\left\{tx'(t) = 2x(t) - t, t^3y'(t) = t^2y(t) - x(t) + t, t^4z'(t) = t^3z(t) - t^2y(t) - x(t) + t\right\}$$

✓ **Mathematica** : cpu = 0.0087237 (sec), leaf count = 39

$$\left\{\left\{x(t) \rightarrow t + c_3t^2, y(t) \rightarrow c_2t + c_3, z(t) \rightarrow c_1t + \frac{c_3}{t} + c_2\right\}\right\}$$

✓ **Maple** : cpu = 0.133 (sec), leaf count = 37

$$\left\{\left\{x(t) = c_3t^2 + t, y(t) = c_2t + c_3, z(t) = \frac{c_1t^2 + c_2t + c_3}{t}\right\}\right\}$$

2.1911 ODE No. 1911

$$\left\{atx'(t) = bc(y(t) - z(t)), bty'(t) = ac(z(t) - x(t)), ctz'(t) = ab(x(t) - y(t))\right\}$$

✓ **Mathematica** : cpu = 0.0306786 (sec), leaf count = 1148

$$\left\{\left\{x(t) \rightarrow \frac{\left(2a^2t^{i\sqrt{a^2+b^2+c^2}} + b^2\left(t^{2i\sqrt{a^2+b^2+c^2}} + 1\right) + c^2\left(t^{2i\sqrt{a^2+b^2+c^2}} + 1\right)\right)c_1t^{-i\sqrt{a^2+b^2+c^2}} - b\left(t^{i\sqrt{a^2+b^2+c^2}} - 1\right)\left(ab\left(t^{i\sqrt{a^2+b^2+c^2}} - 1\right) + c_1\right)}{2\left(a^2 + b^2 + c^2\right)} - \frac{b\left(t^{i\sqrt{a^2+b^2+c^2}} - 1\right)\left(ab\left(t^{i\sqrt{a^2+b^2+c^2}} - 1\right) + c_1\right)}{2\left(a^2 + b^2 + c^2\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.162 (sec), leaf count = 309

$$\left\{\left\{x(t) = c_2 \sin\left(\sqrt{a^2 + b^2 + c^2} \ln(t)\right) + c_3 \cos\left(\sqrt{a^2 + b^2 + c^2} \ln(t)\right) + c_1, y(t) = \frac{c_1b^3 - \sqrt{a^2 + b^2 + c^2} \left(-c_2 \cos\left(\sqrt{a^2 + b^2 + c^2} \ln(t)\right) + c_3 \sin\left(\sqrt{a^2 + b^2 + c^2} \ln(t)\right)\right)}{a^2 + b^2 + c^2}\right\}\right\}$$

2.1912 ODE No. 1912

$$\left\{x1'(t) = ax2(t) + bx3(t) \cos(ct) + bx4(t) \sin(ct), x2'(t) = -ax1(t) + bx3(t) \sin(ct) - bx4(t) \cos(ct), x3'(t) = ax4(t) - bx1(t)\right\}$$

✓ **Mathematica** : cpu = 0.0109934 (sec), leaf count = 798

$$\left\{\left\{x1(t) \rightarrow c_3 \cos\left(\left(\frac{c}{2} - \frac{1}{2}\sqrt{4b^2 + (2a + c)^2}\right)t\right) + c_1 \cos\left(\left(\frac{c}{2} + \frac{1}{2}\sqrt{4b^2 + (2a + c)^2}\right)t\right) + c_4 \sin\left(\left(\frac{c}{2} - \frac{1}{2}\sqrt{4b^2 + (2a + c)^2}\right)t\right) + c_2 \sin\left(\left(\frac{c}{2} + \frac{1}{2}\sqrt{4b^2 + (2a + c)^2}\right)t\right)\right\}\right\}$$

✓ **Maple** : cpu = 1.156 (sec), leaf count = 2788

$$\left\{\left\{x1(t) = c_3 \sin(ct) + c_4 \cos(ct) + c_2, x2(t) = -c_3 \cos(ct) + c_4 \sin(ct) + c_1, x3(t) = \frac{(c_1a \cos(ct) - c_2a \sin(ct) - c_3(a + c))b}{(a + c)a}\right\}\right\}$$

2.1913 ODE No. 1913

$$\left\{x'(t) = -x(t)(x(t) + y(t)), y'(t) = y(t)(x(t) + y(t))\right\}$$

✓ **Mathematica** : cpu = 0.0339279 (sec), leaf count = 64

$$\left\{\left\{y(t) \rightarrow -\sqrt{c_1} \cot\left(\sqrt{c_1}t - \sqrt{c_1}c_2\right), x(t) \rightarrow -\sqrt{c_1} \tan\left(\sqrt{c_1}t - \sqrt{c_1}c_2\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.185 (sec), leaf count = 57

$$\left\{\left[\left\{x(t) = 0\right\}, \left\{y(t) = \frac{1}{c_1 - t}\right\}\right], \left[\left\{x(t) = \frac{\tanh\left(\frac{c_2+t}{c_1}\right)}{c_1}\right\}, \left\{y(t) = \frac{-x(t)^2 - \frac{d}{dt}x(t)}{x(t)}\right\}\right]\right\}$$

2.1919 ODE No. 1919

$$\left\{x'(t) = -\left(x(t)\left(x(t)^2 + y(t)^2\right)\right) + x(t) + y(t), y'(t) = -y(t)\left(x(t)^2 + y(t)^2\right) - x(t) + y(t)\right\}$$

✗ **Mathematica** : cpu = 0.438796 (sec), leaf count = 0 , could not solve

```
DSolve[{Derivative[1][x][t] == x[t] + y[t] - x[t]*(x[t]^2 + y[t]^2), Derivative[1][y][t] == x[t] + y[t] - y[t]*(x[t]^2 + y[t]^2)}, {x[t], y[t]}, t]
```

✓ **Maple** : cpu = 4.266 (sec), leaf count = 200

$$\left\{[\{x(t) = 0\}, \{y(t) = 0\}], \left[\left\{x(t) = ODESolStruc\left(-a, \left[\left[-b(a)\left(\frac{d}{d_a}b(a)\right) - \frac{-4_a^4 - 4_a^3_b(a) + 6_a^2_b(a)^2 + 4_a^2 - 0}{0}\right]\right)\right\}\right]\right\}$$

2.1920 ODE No. 1920

$$\left\{x'(t) = x(t)\left(x(t)^2 + y(t)^2 - 1\right) - y(t), y'(t) = y(t)\left(x(t)^2 + y(t)^2 - 1\right) + x(t)\right\}$$

✗ **Mathematica** : cpu = 0.236862 (sec), leaf count = 0 , could not solve

```
DSolve[{Derivative[1][x][t] == -y[t] + x[t]*(-1 + x[t]^2 + y[t]^2), Derivative[1][y][t] == x[t] + y[t]*(-1 + x[t]^2 + y[t]^2)}, {x[t], y[t]}, t]
```

✓ **Maple** : cpu = 3.939 (sec), leaf count = 202

$$\left\{[\{x(t) = 0\}, \{y(t) = 0\}], \left[\left\{x(t) = ODESolStruc\left(-a, \left[\left[-b(a)\left(\frac{d}{d_a}b(a)\right) + \frac{4_a^4 - 4_a^3_b(a) - 6_a^2_b(a)^2 - 4_a^2 - 0}{0}\right]\right)\right\}\right]\right\}$$

2.1921 ODE No. 1921

$$\left\{x'(t) = -y(t)\left(x(t)^2 + y(t)^2\right), y'(t) = \left\{\begin{array}{ll} x(t)^2 + y(t)^2 & x(t)^2 + y(t)^2 \geq 2x(t) \\ \left(x(t)^2 + y(t)^2\right)\left(\frac{x(t)}{2} - \frac{y(t)^2}{2x(t)}\right) & \text{True} \end{array}\right.\right\}$$

✗ **Mathematica** : cpu = 2.23123 (sec), leaf count = 0 , could not solve

```
DSolve[{Derivative[1][x][t] == -(y[t]*(x[t]^2 + y[t]^2)), Derivative[1][y][t] == Piecewise[{x[t]^2 + y[t]^2 >= 2 x[t], (x[t]^2 + y[t]^2) (x[t]/2 - y[t]^2/(2 x[t])}]}], {x[t], y[t]}, t]
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve({diff(x(t),t) = -y(t)*(x(t)^2+y(t)^2), diff(y(t),t) = piecewise(2*x(t) <= x(t)^2+y(t)^2, 1/2*y(t)^2/x(t))*(x(t)^2+y(t)^2)})
```

2.1922 ODE No. 1922

$$\left\{x'(t) = \left\{\begin{array}{ll} \sin\left(\frac{1}{x(t)^2+y(t)^2}\right)x(t)\left(x(t)^2 + y(t)^2 - 1\right) & x(t)^2 + y(t)^2 \neq 1 \\ 0 & \text{True} \end{array}\right.\right\} - y(t), y'(t) = \left\{\begin{array}{ll} \sin\left(\frac{1}{x(t)^2+y(t)^2}\right)y(t)\left(x(t)^2 + y(t)^2 - 1\right) & x(t)^2 + y(t)^2 \neq 1 \\ 0 & \text{True} \end{array}\right.\right\}$$

✗ **Mathematica** : cpu = 9.85611 (sec), leaf count = 0 , could not solve

```
DSolve[{Derivative[1][x][t] == Piecewise[{{Sin[(x[t]^2 + y[t]^2)^(-1)]*x[t]*(-1 + x[t]^2 + y[t]^2 - 1)]*y[t]*(-1 + x[t]^2 + y[t]^2), x[t]^2 + y[t]^2 != 1}}, 0] + x[t]}, {x[t], y[t]}, t]
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve({diff(x(t),t) = -y(t)+piecewise(x(t)^2+y(t)^2 <> 1,x(t)*(x(t)^2+y(t)^2-1)*sin(1/(x(t)^2+y(t)^2))}, {diff(y(t),t) = x(t)+piecewise(x(t)^2+y(t)^2 <> 1,y(t)*(x(t)^2+y(t)^2-1)*sin(1/(x(t)^2+y(t)^2))})
```

2.1923 ODE No. 1923

$$\left\{\left(t^2+1\right) x'(t)=y(t)-t x(t),\left(t^2+1\right) y'(t)=-x(t)-t y(t)\right\}$$

✓ **Mathematica** : cpu = 0.0105972 (sec), leaf count = 53

$$\left\{\left\{x(t) \rightarrow \frac{c_1}{t^2+1}+\frac{c_2 t}{t^2+1}, y(t) \rightarrow \frac{c_2}{t^2+1}-\frac{c_1 t}{t^2+1}\right\}\right\}$$

✓ **Maple** : cpu = 0.055 (sec), leaf count = 35

$$\left\{\left\{x(t)=\frac{c_1 t+c_2}{t^2+1}, y(t)=\frac{-c_2 t+c_1}{t^2+1}\right\}\right\}$$

2.1924 ODE No. 1924

$$\left\{\left(-t^2+x(t)^2+y(t)^2\right) x'(t)=-2 t x(t),\left(-t^2+x(t)^2+y(t)^2\right) y'(t)=-2 t y(t)\right\}$$

✓ **Mathematica** : cpu = 0.0720984 (sec), leaf count = 191

$$\left\{\left\{y(t) \rightarrow \frac{c_1\left(e^{c_2}-\sqrt{-4 t^2-4 c_1^2 t^2+e^{2 c_2}}\right)}{2\left(1+c_1^2\right)}, x(t) \rightarrow \frac{e^{c_2}-\sqrt{-4 t^2-4 c_1^2 t^2+e^{2 c_2}}}{2\left(1+c_1^2\right)}\right\},\left\{y(t) \rightarrow \frac{c_1\left(\sqrt{-4 t^2-4 c_1^2 t^2+e^{2 c_2}}+e^{c_2}\right)}{2\left(1+c_1^2\right)}, x(t) \rightarrow \frac{e^{c_2}+\sqrt{-4 t^2-4 c_1^2 t^2+e^{2 c_2}}}{2\left(1+c_1^2\right)}\right\}\right\}$$

✓ **Maple** : cpu = 0.391 (sec), leaf count = 180

$$\left[\left[\left\{x(t)=0\right\},\left\{y(t)=\frac{1-\sqrt{-4 c_1^2 t^2+1}}{2 c_1}, y(t)=\frac{1+\sqrt{-4 c_1^2 t^2+1}}{2 c_1}\right\}\right],\left[\left\{x(t)=\frac{c_1-\sqrt{-2 c_2 t^2+c_1^2}}{2 c_2}, x(t)=\frac{c_1+\sqrt{-2 c_2 t^2+c_1^2}}{2 c_2}\right\}\right]\right]$$

2.1925 ODE No. 1925

$$\left\{a y'(t)+t x'(t)-x(t)+y'(t)^2=0, x'(t) y'(t)+t y'(t)-y(t)=0\right\}$$

✓ **Mathematica** : cpu = 0.0189121 (sec), leaf count = 31

$$\left\{\left\{x(t) \rightarrow a c_2+c_1 t+c_2^2, y(t) \rightarrow c_2 t+c_1 c_2\right\}\right\}$$

✓ **Maple** : cpu = 0.354 (sec), leaf count = 194

$$\left[\left[\left\{x(t)=-\frac{t^2}{3}\right\},\left\{y(t)=-\frac{t^3}{27 a}\right\}\right],\left[\left\{x(t)=c_1 t+c_2,\left\{y(t)=-\frac{\left(t+\frac{d}{d t} x(t)\right)\left(t\left(\frac{d}{d t} x(t)\right)+\left(\frac{d}{d t} x(t)\right)^2-x(t)\right)}{a}\right\}\right],\left[\left\{x(t)=-\frac{3 c_1^2}{2 a}\right\}\right]\right]$$

2.1926 ODE No. 1926

$$\left\{x(t)=f\left(x'(t), y'(t)\right)+t x'(t), y(t)=g\left(x'(t), y'(t)\right)+t y'(t)\right\}$$

✓ **Mathematica** : cpu = 0.0059445 (sec), leaf count = 28

$$\left\{\left\{x(t) \rightarrow f\left(c_1, c_2\right)+c_1 t, y(t) \rightarrow g\left(c_1, c_2\right)+c_2 t\right\}\right\}$$

✓ **Maple** : cpu = 0.273 (sec), leaf count = 96

$$\left\{\left\{c_1+\int \operatorname{RootOf}\left(t\left(\frac{d}{d t} y(t)\right)+g\left(-Z, \frac{d}{d t} y(t)\right)-y(t)\right) d t=t \operatorname{RootOf}\left(t\left(\frac{d}{d t} y(t)\right)+g\left(-Z, \frac{d}{d t} y(t)\right)-y(t)\right)+f\left(\operatorname{RootOf}\left(t\left(\frac{d}{d t} y(t)\right)+g\left(-Z, \frac{d}{d t} y(t)\right)-y(t)\right)\right)\right\}\right\}$$

2.1927 ODE No. 1927

$$\left\{x''(t) = ae^{2x(t)} + e^{-2x(t)} \cos^2(y(t)) - e^{-x(t)}, y''(t) = e^{-2x(t)} \sin(y(t)) \cos(y(t)) - \tan(y(t)) \sec^2(y(t))\right\}$$

✗ **Mathematica** : cpu = 0.0076564 (sec), leaf count = 0 , could not solve

`DSolve[{Derivative[2][x][t] == -E^(-x[t]) + a*E^(2*x[t]) + Cos[y[t]]^2/E^(2*x[t])}, Derivative[2][y][t] == (k*y[t])/(x[t]^2 + y[t]^2)^(3/2), x[t] == 0, y[t] == 0]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve({diff(diff(x(t),t),t) = a*exp(2*x(t))-exp(-x(t))+exp(-2*x(t))*cos(y(t))^2, diff(diff(y(t),t),t) = k*y(t)/(x(t)^2+y(t)^2)^(3/2)}, {x(t), y(t)})`

2.1928 ODE No. 1928

$$\left\{x''(t) = \frac{kx(t)}{(x(t)^2 + y(t)^2)^{3/2}}, y''(t) = \frac{ky(t)}{(x(t)^2 + y(t)^2)^{3/2}}\right\}$$

✗ **Mathematica** : cpu = 0.0065055 (sec), leaf count = 0 , could not solve

`DSolve[{Derivative[2][x][t] == (k*x[t])/(x[t]^2 + y[t]^2)^(3/2), Derivative[2][y][t] == (k*y[t])/(x[t]^2 + y[t]^2)^(3/2), x[t] == 0, y[t] == 0]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve({diff(diff(x(t),t),t) = k*x(t)/(x(t)^2+y(t)^2)^(3/2), diff(diff(y(t),t),t) = k*y(t)/(x(t)^2+y(t)^2)^(3/2)}, {x(t), y(t)})`

2.1929 ODE No. 1929

$$\left\{x''(t) = -\frac{cy(t)x'(t)f(\sqrt{x'(t)^2 + y'(t)^2})}{\sqrt{x'(t)^2 + y'(t)^2}}, y''(t) = -\frac{cy(t)y'(t)f(\sqrt{x'(t)^2 + y'(t)^2})}{\sqrt{x'(t)^2 + y'(t)^2}} - g\right\}$$

✗ **Mathematica** : cpu = 0.0072309 (sec), leaf count = 0 , could not solve

`DSolve[{Derivative[2][x][t] == -((c*f[Sqrt[Derivative[1][x][t]^2 + Derivative[1][y][t]^2])*y[t] + g - (c*f[Sqrt[Derivative[1][x][t]^2 + Derivative[1][y][t]^2])*y[t]*Derivative[1][y][t])/Sqrt[x'(t)^2 + y'(t)^2]), Derivative[2][y][t] == -((c*f[Sqrt[Derivative[1][x][t]^2 + Derivative[1][y][t]^2])*x[t] + g - (c*f[Sqrt[Derivative[1][x][t]^2 + Derivative[1][y][t]^2])*x[t]*Derivative[1][x][t])/Sqrt[x'(t)^2 + y'(t)^2])], x[t] == 0, y[t] == 0]`

✓ **Maple** : cpu = 4.415 (sec), leaf count = 116

$$\left\{\left\{y(t) = ODESolStruc\left(-a, \left[\left[-b(-a)\left(\frac{d}{d_a}b(-a)\right) + \frac{C(-a)_b(-a)f\left(\sqrt{b(-a)^2}\right) + \sqrt{b(-a)^2}g}{\sqrt{b(-a)^2}} = 0\right], \left\{-a = y(t), -b(-a) = y'(t)\right\}\right.\right.\right\}$$

2.1930 ODE No. 1930

$$\left\{x'(t) = y(t) - z(t), y'(t) = x(t)^2 + y(t), z'(t) = x(t)^2 + z(t)\right\}$$

✓ **Mathematica** : cpu = 0.0337943 (sec), leaf count = 308

$$\left\{\left\{x(t) \rightarrow e^{-c_3}\left(e^t + e^{c_3c_1}\right), y(t) \rightarrow c_2\left(e^{-c_3}\left(e^t + e^{c_3c_1}\right) - c_1\right) + \left(e^{-c_3}\left(e^t + e^{c_3c_1}\right) - c_1\right)\left(-\frac{c_1^2}{e^{-c_3}\left(e^t + e^{c_3c_1}\right) - c_1} + e^{-c_3}\left(e^t + e^{c_3c_1}\right)\right), z(t) \rightarrow c_3\left(e^{-c_3}\left(e^t + e^{c_3c_1}\right) - c_1\right) + \left(e^{-c_3}\left(e^t + e^{c_3c_1}\right) - c_1\right)\left(-\frac{c_1^2}{e^{-c_3}\left(e^t + e^{c_3c_1}\right) - c_1} + e^{-c_3}\left(e^t + e^{c_3c_1}\right)\right)\right.\right\}$$

✓ **Maple** : cpu = 0.092 (sec), leaf count = 45

$$\left\{\left\{x(t) = c_2e^t + c_3, y(t) = \left(c_1 + \int e^{-t}x(t)^2dt\right)e^t, z(t) = -\frac{d}{dt}x(t) + y(t)\right\}\right\}$$

2.1931 ODE No. 1931

$\{ax'(t) = (b - c)y(t)z(t), by'(t) = (c - a)x(t)z(t), cz'(t) = (a - b)x(t)y(t)\}$

✓ **Mathematica** : cpu = 4.13771 (sec), leaf count = 10101

$$\left\{ \left\{ \left(\frac{\sqrt{2} b^2 \sqrt{a(a-c)} c_1 \operatorname{sn} \left(\frac{\frac{\sqrt{2} \sqrt{a} \sqrt{a-c} c \sqrt{c_2} t}{\sqrt{b} \sqrt{b-c}} - \frac{\sqrt{2} \sqrt{a} \sqrt{b} \sqrt{a-c} \sqrt{c_2} t}{\sqrt{b-c}} - \frac{\sqrt{2} \sqrt{a} \sqrt{a-c} c \sqrt{c_2} c_3}{\sqrt{b} \sqrt{b-c}} + \frac{\sqrt{2} \sqrt{a} \sqrt{b} \sqrt{a-c} \sqrt{c_2} c_3}{\sqrt{b-c}} - \frac{(a-b) b c_1}{(a-c) c c_2} \right)}{(a-c) \sqrt{b(b-c) c_1}} \right) - \frac{\sqrt{2} b \sqrt{a(a-c)} c c_1 \operatorname{sn} \left(\frac{\sqrt{2} \sqrt{a} \sqrt{a-c} c \sqrt{c_2} t}{\sqrt{b} \sqrt{b-c}} - \frac{\sqrt{2} \sqrt{a} \sqrt{b} \sqrt{a-c} \sqrt{c_2} t}{\sqrt{b-c}} - \frac{\sqrt{2} \sqrt{a} \sqrt{a-c} c \sqrt{c_2} c_3}{\sqrt{b} \sqrt{b-c}} + \frac{\sqrt{2} \sqrt{a} \sqrt{b} \sqrt{a-c} \sqrt{c_2} c_3}{\sqrt{b-c}} - \frac{(a-b) b c_1}{(a-c) c c_2} \right)}{a} \right\} \right\} x(t) \rightarrow$$

✓ **Maple** : cpu = 1.4 (sec), leaf count = 1117

$$\left\{ \left[\{x(t) = 0\}, \{y(t) = 0\}, \{z(t) = c_1\} \right], \left[\{x(t) = 0\}, \{y(t) = c_1\}, \{z(t) = 0\} \right], \left[\{x(t) = c_1\}, \{y(t) = 0\}, \{z(t) = 0\} \right], \left\{ x(t) = \operatorname{RootOf} \left(\frac{a^2 x^2 - 2 a b x + b^2}{a^2} \right) \right\} \right\}$$

2.1932 ODE No. 1932

$\{x'(t) = x(t)(y(t) - z(t)), y'(t) = y(t)(z(t) - x(t)), z'(t) = z(t)(x(t) - y(t))\}$

✗ **Mathematica** : cpu = 2.74736 (sec), leaf count = 0 , could not solve

```
DSolve[{Derivative[1][x][t] == x[t]*(y[t] - z[t]), Derivative[1][y][t] == y[t]*(-x[t] + z[t]), Derivative[1][z][t] == (x[t] - y[t])*z[t]}, {x[t], y[t], z[t]}, t]
```

✓ **Maple** : cpu = 1.242 (sec), leaf count = 383

$$\left\{ \left[\{x(t) = 0\}, \{y(t) = 0\}, \{z(t) = c_1\} \right], \left[\{x(t) = 0\}, \left\{ y(t) = \frac{c_1 e^{c_1 c_2} e^{c_1 t}}{e^{c_1 c_2} e^{c_1 t} - 1} \right\}, \left\{ z(t) = \frac{\frac{d}{dt} y(t)}{y(t)} \right\} \right], \left[\{x(t) = c_1\}, \{y(t) = 0\}, \{z(t) = 0\} \right] \right\}$$

2.1933 ODE No. 1933

$\{x'(t) + y'(t) = x(t)y(t), y'(t) + z'(t) = y(t)z(t), x'(t) + z'(t) = x(t)z(t)\}$

✗ **Mathematica** : cpu = 125.725 (sec), leaf count = 0 , could not solve

```
DSolve[{Derivative[1][x][t] + Derivative[1][y][t] == x[t]*y[t], Derivative[1][y][t] + Derivative[1][z][t] == y[t]*z[t], Derivative[1][x][t] + Derivative[1][z][t] == x[t]*z[t]}, {x[t], y[t], z[t]}, t]
```

✓ **Maple** : cpu = 1.618 (sec), leaf count = 4258

Expression too large to display

2.1934 ODE No. 1934

$$\left\{x'(t) = \frac{x(t)^2}{2} - \frac{y(t)}{24}, y'(t) = 2x(t)y(t) - 3z(t), z'(t) = 3x(t)z(t) - \frac{y(t)^2}{6}\right\}$$

✗ **Mathematica** : cpu = 67.4084 (sec), leaf count = 0 , could not solve

`DSolve[{Derivative[1][x][t] == x[t]^2/2 - y[t]/24, Derivative[1][y][t] == 2*x[t]*y[t] - 3*z[t], Derivative[1][z][t] == 1/6*y[t]^2 + 3*x[t]*z[t]}, {x[t], y[t], z[t]}, t]`

✓ **Maple** : cpu = 1.512 (sec), leaf count = 377

$$\left\{\left[\left\{y(t) = 0\right\}, \left\{x(t) = -\frac{2}{-2c_1 + t}\right\}, \left\{z(t) = 0\right\}\right], \left[\left\{y(t) = \frac{256}{(c_1t + c_2)^4}\right\}, \left\{x(t) = \frac{-\sqrt{3}y(t)^{\frac{3}{2}} + 3\frac{d}{dt}y(t)}{6y(t)}, x(t) = \frac{\sqrt{3}y(t)^{\frac{3}{2}} + 3\frac{d}{dt}y(t)}{6y(t)}\right\}\right]\right\}$$

2.1935 ODE No. 1935

$$\{x'(t) = x(t)(y(t)^2 - z(t)^2), y'(t) = y(t)(z(t)^2 - x(t)^2), z'(t) = z(t)(x(t)^2 - y(t)^2)\}$$

✗ **Mathematica** : cpu = 0.0432276 (sec), leaf count = 0 , could not solve

`DSolve[{Derivative[1][x][t] == x[t]*(y[t]^2 - z[t]^2), Derivative[1][y][t] == y[t]*(x[t]^2 - z[t]^2), Derivative[1][z][t] == (x[t]^2 - y[t]^2)*z[t]}, {x[t], y[t], z[t]}, t]`

✓ **Maple** : cpu = 2.276 (sec), leaf count = 741

$$\left\{\left[\left\{x(t) = 0\right\}, \left\{y(t) = 0\right\}, \left\{z(t) = c_1\right\}\right], \left[\left\{x(t) = 0\right\}, \left\{y(t) = \frac{\sqrt{c_1(e^{2c_1c_2}e^{2c_1t} - 1)}e^{2c_1c_2}e^{2c_1t}}{e^{2c_1c_2}e^{2c_1t} - 1}, y(t) = -\frac{\sqrt{c_1(e^{2c_1c_2}e^{2c_1t} - 1)}e^{2c_1c_2}e^{2c_1t}}{e^{2c_1c_2}e^{2c_1t} - 1}\right\}\right]\right\}$$

2.1936 ODE No. 1936

$$\{x'(t) = x(t)(y(t)^2 - z(t)^2), y'(t) = -y(t)(x(t)^2 + z(t)^2), z'(t) = z(t)(x(t)^2 + y(t)^2)\}$$

✗ **Mathematica** : cpu = 0.0439705 (sec), leaf count = 0 , could not solve

`DSolve[{Derivative[1][x][t] == x[t]*(y[t]^2 - z[t]^2), Derivative[1][y][t] == -(y[t]*(x[t]^2 + z[t]^2)), Derivative[1][z][t] == z[t]*(x[t]^2 + y[t]^2)}, {x[t], y[t], z[t]}, t]`

✓ **Maple** : cpu = 1.076 (sec), leaf count = 704

$$\left\{\left[\left\{x(t) = 0\right\}, \left\{y(t) = 0\right\}, \left\{z(t) = c_1\right\}\right], \left[\left\{x(t) = 0\right\}, \left\{y(t) = \frac{\sqrt{c_1e^{2c_1c_2}e^{2c_1t} - c_1e^{4c_1c_2}e^{4c_1t}}}{e^{2c_1c_2}e^{2c_1t} - 1}, y(t) = -\frac{\sqrt{c_1e^{2c_1c_2}e^{2c_1t} - c_1e^{4c_1c_2}e^{4c_1t}}}{e^{2c_1c_2}e^{2c_1t} - 1}\right\}\right]\right\}$$

2.1937 ODE No. 1937

$$\{x'(t) = -x(t)y(t)^2 + x(t) + y(t), y'(t) = x(t)^2y(t) - x(t) - y(t), z'(t) = y(t)^2 - x(t)^2\}$$

✗ **Mathematica** : cpu = 0.757668 (sec), leaf count = 0 , could not solve

`DSolve[{Derivative[1][x][t] == x[t] + y[t] - x[t]*y[t]^2, Derivative[1][y][t] == -x[t]^2*y[t] - y[t] + x[t]^2*y[t], Derivative[1][z][t] == -x[t]^2 + y[t]^2}, {x[t], y[t], z[t]}, t]`

✓ **Maple** : cpu = 1.039 (sec), leaf count = 242

$$\left\{\left[\left\{x(t) = 0\right\}, \left\{y(t) = 0\right\}, \left\{z(t) = c_1\right\}\right], \left[\left\{x(t) = ODESolStruc\left(-a, \left[\left[\frac{4a^5 + 2a^2b(a)\left(\frac{d}{da}b(a)\right) - 3a^3 - 2ab(a)^2 - \dots}{\dots}\right]\right)\right\}\right]\right\}$$

2.1938 ODE No. 1938

$$\left\{x''(t) = \frac{x(t)f'(r)}{r}, y''(t) = \frac{y(t)f'(r)}{r}, z''(t) = \frac{z(t)f'(r)}{r}\right\}$$

✓ **Mathematica** : cpu = 0.0075697 (sec), leaf count = 137

$$\left\{\left\{x(t) \rightarrow c_1 e^{-\frac{t\sqrt{f'(r)}}{\sqrt{r}}} + c_2 e^{\frac{t\sqrt{f'(r)}}{\sqrt{r}}}, y(t) \rightarrow c_3 e^{-\frac{t\sqrt{f'(r)}}{\sqrt{r}}} + c_4 e^{\frac{t\sqrt{f'(r)}}{\sqrt{r}}}, z(t) \rightarrow c_5 e^{-\frac{t\sqrt{f'(r)}}{\sqrt{r}}} + c_6 e^{\frac{t\sqrt{f'(r)}}{\sqrt{r}}}\right\}\right\}$$

✓ **Maple** : cpu = 0.211 (sec), leaf count = 101

$$\left\{\left\{x(t) = c_5 e^{\frac{t\sqrt{\frac{d}{dr}F(r)}}{\sqrt{r}}} + c_6 e^{-\frac{t\sqrt{\frac{d}{dr}F(r)}}{\sqrt{r}}}, y(t) = c_3 e^{\frac{t\sqrt{\frac{d}{dr}F(r)}}{\sqrt{r}}} + c_4 e^{-\frac{t\sqrt{\frac{d}{dr}F(r)}}{\sqrt{r}}}, z(t) = c_1 e^{\frac{t\sqrt{\frac{d}{dr}F(r)}}{\sqrt{r}}} + c_2 e^{-\frac{t\sqrt{\frac{d}{dr}F(r)}}{\sqrt{r}}}\right\}\right\}$$

2.1939 ODE No. 1939

$$\{(x(t)-y(t))(x(t)-z(t))x'(t)=f(t), (y(t)-x(t))(y(t)-z(t))y'(t)=f(t), (z(t)-x(t))(z(t)-y(t))z'(t)=f(t)\}$$

✓ **Mathematica** : cpu = 0.290648 (sec), leaf count = 2168

$$\left\{\left\{x(t) \rightarrow \frac{c_1}{3} + \frac{\sqrt[3]{2c_1^3 - 9c_2c_1 + 27c_3 + 27\int_1^t f(K[1])dK[1]} + \sqrt{4(3c_2 - c_1^2)^3 + (2c_1^3 - 9c_2c_1 + 27c_3 + 27\int_1^t f(K[1])dK[1])^2}}{3\sqrt[3]{2}}\right\}\right\}$$

✓ **Maple** : cpu = 1.798 (sec), leaf count = 899

$$\left\{\left[\left\{x(t) = c_3 + \int -\frac{3\left(c_1\left(c_1^3 + 11664c_2^2 - 23328c_2\left(\int f(t)dt\right) + 11664\left(\int f(t)dt\right)^2\right)\left(1 + i\sqrt{3}\right) + \left(1 - i\sqrt{3}\right)\left(1 + 108\sqrt{\frac{1}{c_1^3 + 11664c_2^2 - 23328c_2\left(\int f(t)dt\right) + 11664\left(\int f(t)dt\right)^2}}\right)\right)}{\left(1 + 108\sqrt{\frac{\left(-c_2 + \int f(t)dt\right)^2}{c_1^3 + 11664c_2^2 - 23328c_2\left(\int f(t)dt\right) + 11664\left(\int f(t)dt\right)^2}}\right)\left(c_1^3 + 11664c_2^2 - 23328c_2\left(\int f(t)dt\right) + 11664\left(\int f(t)dt\right)^2}\right)}\right]\right\}$$

2.1940 ODE No. 1940

$$\{x1'(t)\sin(x2(t))=x4(t)\sin(x3(t))+x5(t)\cos(x3(t)), x2'(t)=x4(t)\cos(x3(t))-x5(t)\sin(x3(t)), x1'(t)\cos(x2(t))+x3(t)\sin(x2(t))=x4(t)\cos(x3(t))-x5(t)\sin(x3(t))\}$$

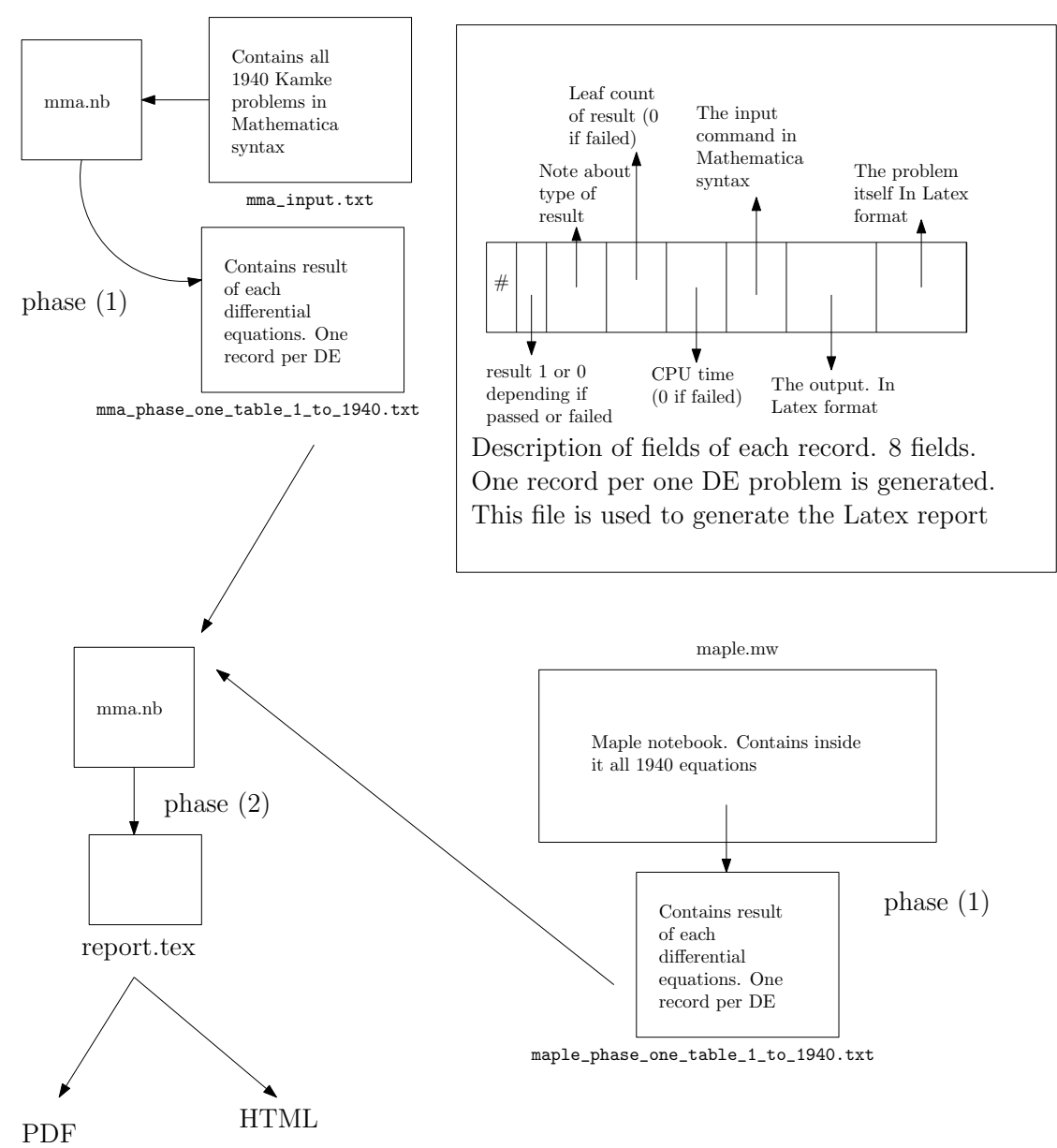
✗ **Mathematica** : cpu = 0.0080374 (sec), leaf count = 0 , could not solve

```
DSolve[{Sin[x2[t]]*Derivative[1][x1][t] == Sin[x3[t]]*x4[t] + Cos[x3[t]]*x5[t], Derivative[1][x2][t] == (a*(1 - lambda)*x5[t]) + Derivative[1][x4][t] == -(m*Cos[x3[t]]*Sin[x2[t]]), a*(1 - lambda)*x3[t] == m*x5[t]}, {x1[t], x2[t], x3[t], x4[t], x5[t]}, t]
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve({diff(x1(t),t)*sin(x2(t)) = x4(t)*sin(x3(t))+x5(t)*cos(x3(t)), diff(x3(t),t)+diff(x1(t),t)*cos(x2(t)) = x4(t)*cos(x3(t))-x5(t)*sin(x3(t)), diff(x2(t),t) = x4(t)*cos(x3(t))-x5(t)*sin(x3(t))}, {x1(t), x2(t), x3(t), x4(t), x5(t)}, t)
```

3 Appendix



Kamke Differential equations build process