

# HowTo, Matlab/Simulink for basic modeling

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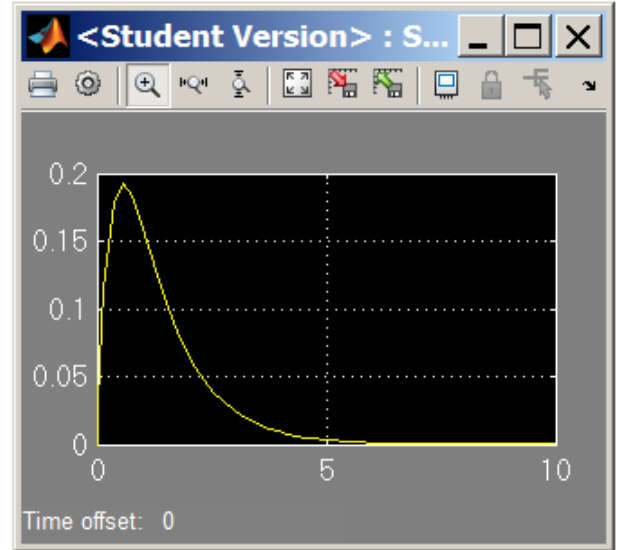
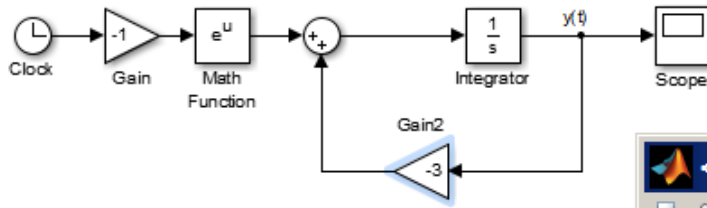
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## **1 How to solve first order ode with zero initial conditions?**

Solve  $y'(t) + 3y(t) = e^{-t}$  with  $y(0) = 0$

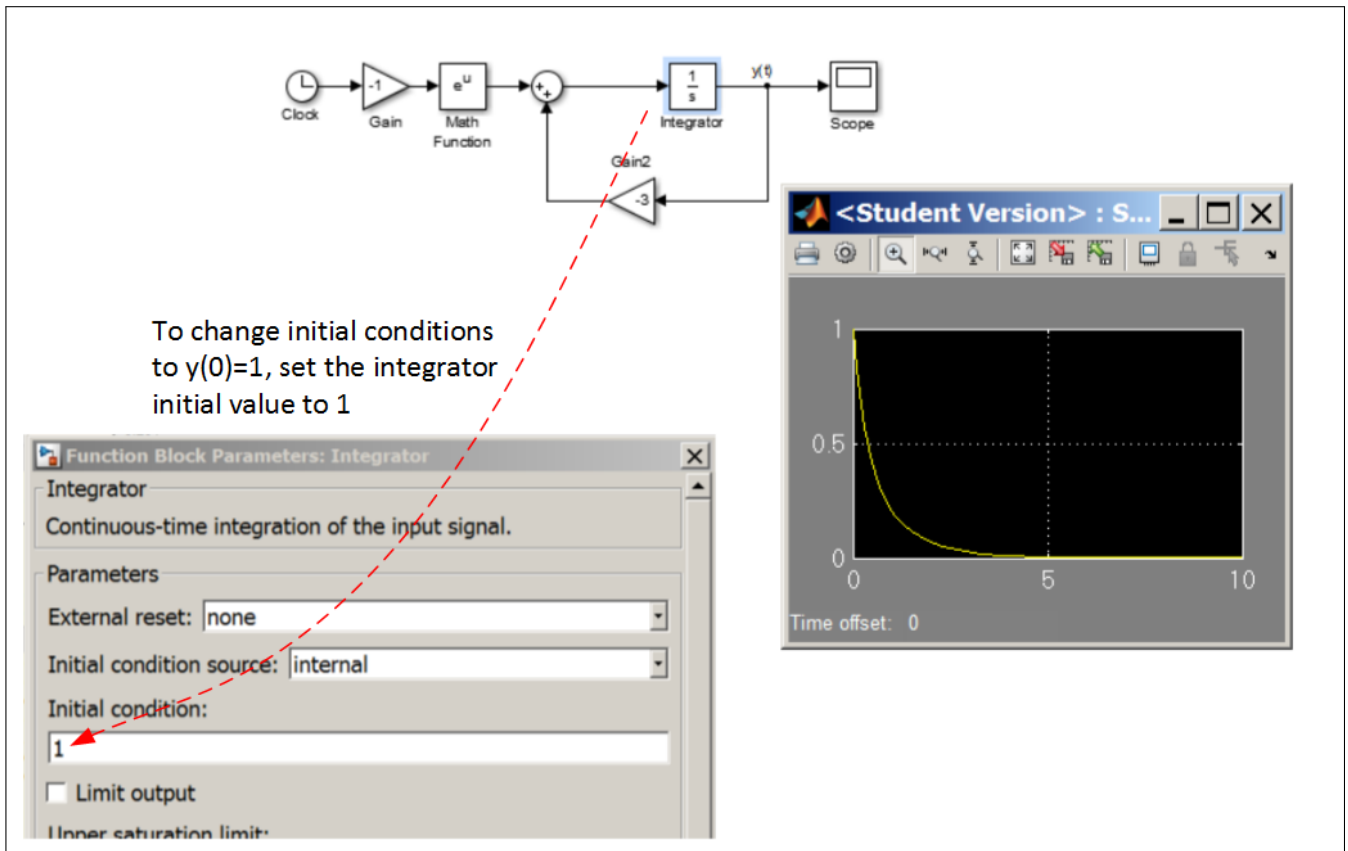
model\_1.slx



## 2 How to solve first order ode with non-zero initial conditions?

Solve  $y'(t) + 3y(t) = e^{-t}$  with  $y(0) = 1$

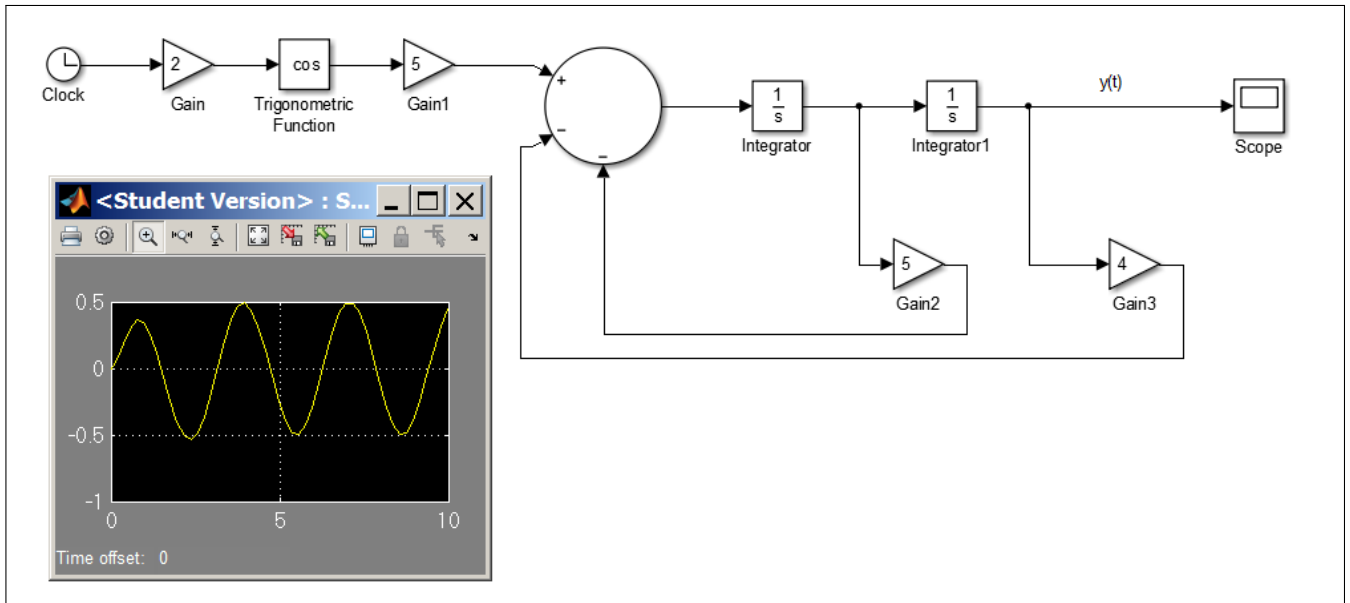
model\_2.slx



### 3 How to solve second order ode with zero initial conditions?

Solve  $y''(t) + 5y'(t) + 4y(t) = 5 \cos(2t)$  with  $y(0) = 0, y'(0) = 0$

model\_3.slx

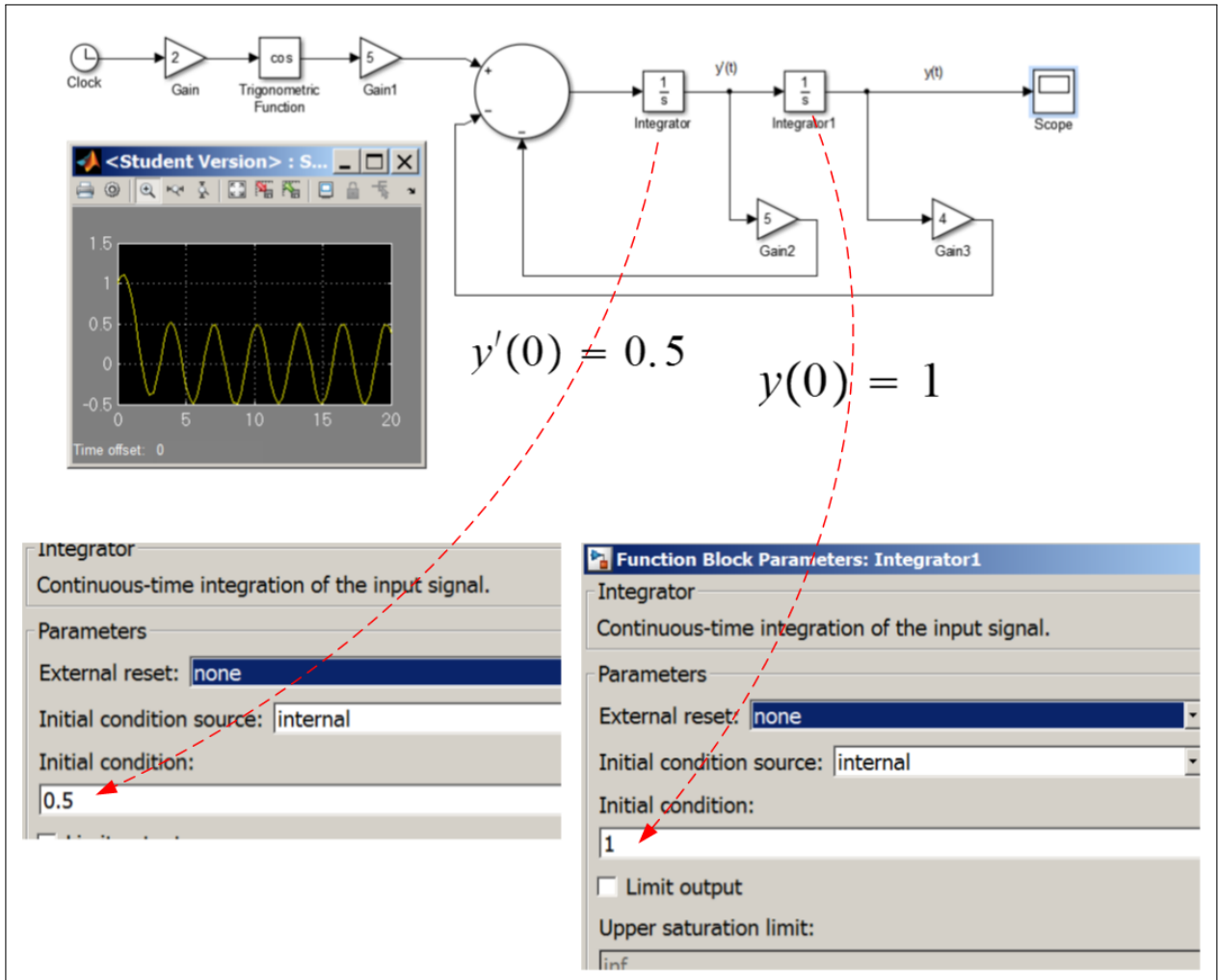


## 4 How to solve second order ode with non-zero initial conditions?

Solve  $y''(t) + 5y'(t) + 4y(t) = 5 \cos(2t)$  with  $y(0) = 1$ ,  $y'(0) = 0.5$

The initial conditions are set up by modifying the integrator  $\frac{1}{s}$  block as shown below.

model\_4.slx



## 5 How to solve mass/spring system with unit step input?

model\_e5.slx

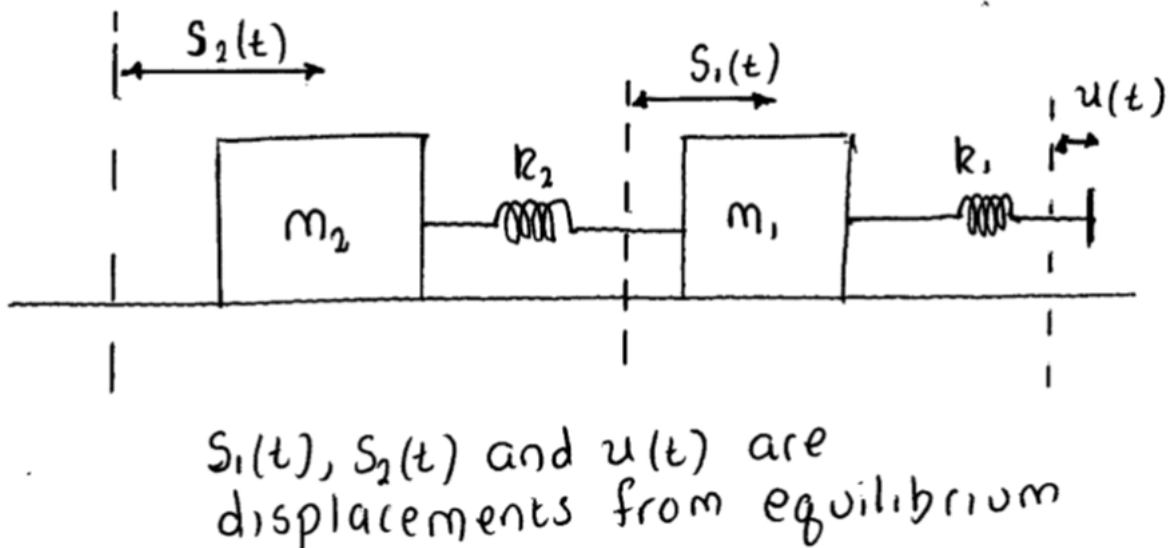
From HW problem I did for ECE 717, to be solved using Simulink

For the mass-spring system depicted on the next page, the input  $u(t)$  is taken to be the displacement of the supporting platform.

(a) Apply Newton's Laws to obtain the two governing differential equations of motion in  $s_1$ ,  $s_2$  and  $u$ .

(b) With states taken to be  $x_1 = s_1, x_2 = s_2, x_3 = \frac{ds_1}{dt}, x_4 = \frac{ds_2}{dt}$  and outputs  $y_1 = s_1, y_2 = s_2$ , obtain linear time-invariant state equations in the matrix form  $\dot{x} = Ax + Bu, y = Cx + Du$ .

(c) Use Simulink to obtain the unit step response for  $y_1$  and  $y_2$  using normalized parameter values  $k_1 = k_2 = 0.5, m_1 = 1, m_2 = 2$ . Assume the system is initially at rest; i.e.,  $x(0) = 0$ .



Starting with the assumption that the ground surface is smooth and there is no friction. Assuming that all parts are moving in the positive direction to the right. Taking a snap shot when  $s_2 > s_1$  so that the spring  $k_2$  is in compression. Spring  $k_1$  is in compression by also assuming that  $s_1 > u$  at this instance.

Any other assumptions will also lead to the same set of equations as long as they are used in consistent way when finding the forces in the springs.

Starting with drawing a free body diagram of each body showing all forces acting on them based on

the above assumption, and then using  $F = ma$  to find the equation of motion of each body  $m_1, m_2$ . The free body diagrams is shown below

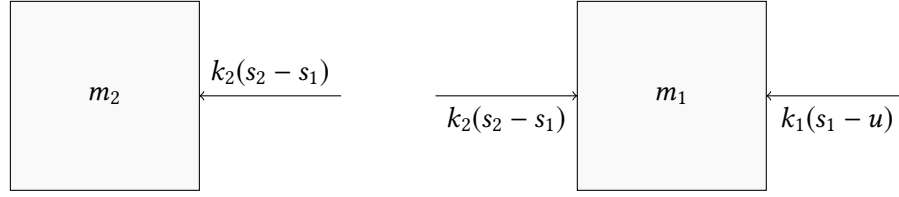


Figure 1: Free body diagram for problem 1

Now  $F = ma$  is applied to each body to obtain the equation of motions. For mass  $m_2$

$$m_2 s_2'' = -k_2 (s_2 - s_1)$$

$$s_2'' = -\frac{k_2}{m_2} (s_2 - s_1)$$

And for mass  $m_1$

$$m_1 s_1'' = k_2 (s_2 - s_1) - k_1 (s_1 - u)$$

$$s_1'' = \frac{k_2}{m_1} (s_2 - s_1) - \frac{k_1}{m_1} (s_1 - u)$$

Now the state space equations are found.

$$\begin{pmatrix} x_1 = s_1 \\ x_2 = s_2 \\ x_3 = s_1' \\ x_4 = s_2' \end{pmatrix} \xrightarrow{\frac{d}{dt}} \begin{pmatrix} x_1' = s_1' = x_3 \\ x_2' = s_2' = x_4 \\ x_3' = s_1'' = \frac{k_2}{m_1} (s_2 - s_1) - \frac{k_1}{m_1} (s_1 - u) = \frac{k_2}{m_1} (x_2 - x_1) - \frac{k_1}{m_1} (x_1 - u) \\ x_4' = s_2'' = -\frac{k_2}{m_2} (s_2 - s_1) = -\frac{k_2}{m_2} (x_2 - x_1) \end{pmatrix}$$

$$= \begin{pmatrix} x_3 \\ x_4 \\ x_1 \left( -\frac{k_2}{m_1} - \frac{k_1}{m_1} \right) + \frac{k_2}{m_1} x_2 + \frac{k_1}{m_1} u \\ \frac{k_2}{m_2} x_1 - \frac{k_2}{m_2} x_2 \end{pmatrix}$$

Hence

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{pmatrix} = \overbrace{\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\left(\frac{k_2}{m_1} + \frac{k_1}{m_1}\right) & \frac{k_2}{m_1} & 0 & 0 \\ \frac{k_2}{m_2} & -\frac{k_2}{m_2} & 0 & 0 \end{pmatrix}}^{A(n \times n)} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \overbrace{\begin{pmatrix} 0 \\ 0 \\ \frac{k_1}{m_1} \\ 0 \end{pmatrix}}^{B(n \times m)} u(t)$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \overbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}}^{C(r \times n)} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \overbrace{\begin{pmatrix} 0 \\ 0 \end{pmatrix}}^{D(r \times m)} u(t)$$

The above is in the form of  $x' = Ax + Bu$  and  $y = Cx + Du$  where  $r = 2$  is number of outputs,  $m = 1$  is the number of input and  $n = 4$  is the number of states.

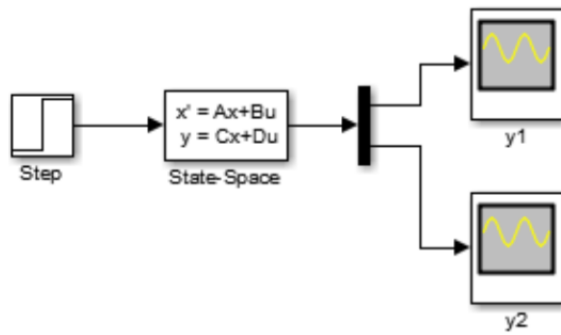
Using  $k_1 = k_2 = 0.5$ ,  $m_1 = 1$ ,  $m_2 = 2$  and  $x(0) = 0$  now the unit step response for  $y_1, y_2$  is found using Simulink. With the above values the system becomes

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0.5 & 0 & 0 \\ 0.25 & -0.25 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0.5 \\ 0 \end{pmatrix} u(t)$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} u(t)$$

Using simulink, state space block was used to implement the above. A step input source was used. Demux was used to send the  $y_1$  and  $y_2$  responses to two different time scopes. Simulation was set for 40 seconds to obtain long enough view of the response. The following figure shows the step response and the model used.





**Function Block Parameters: State-Space**

State Space

State-space model:  
 $dx/dt = Ax + Bu$   
 $y = Cx + Du$

Parameters

A:

B:

C:

D:

Initial conditions:

Absolute tolerance:

State Name: (e.g., 'position')

OK Cancel Help Apply

