Small note on finding Euler circuits in connected graphs using Fleury algorithm
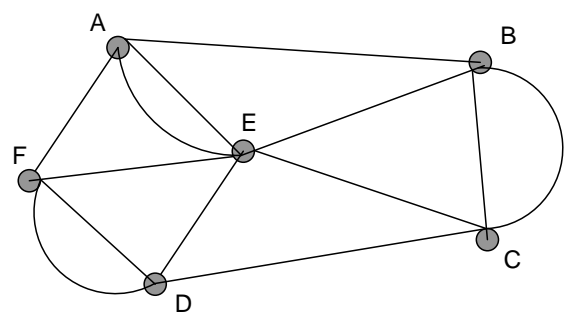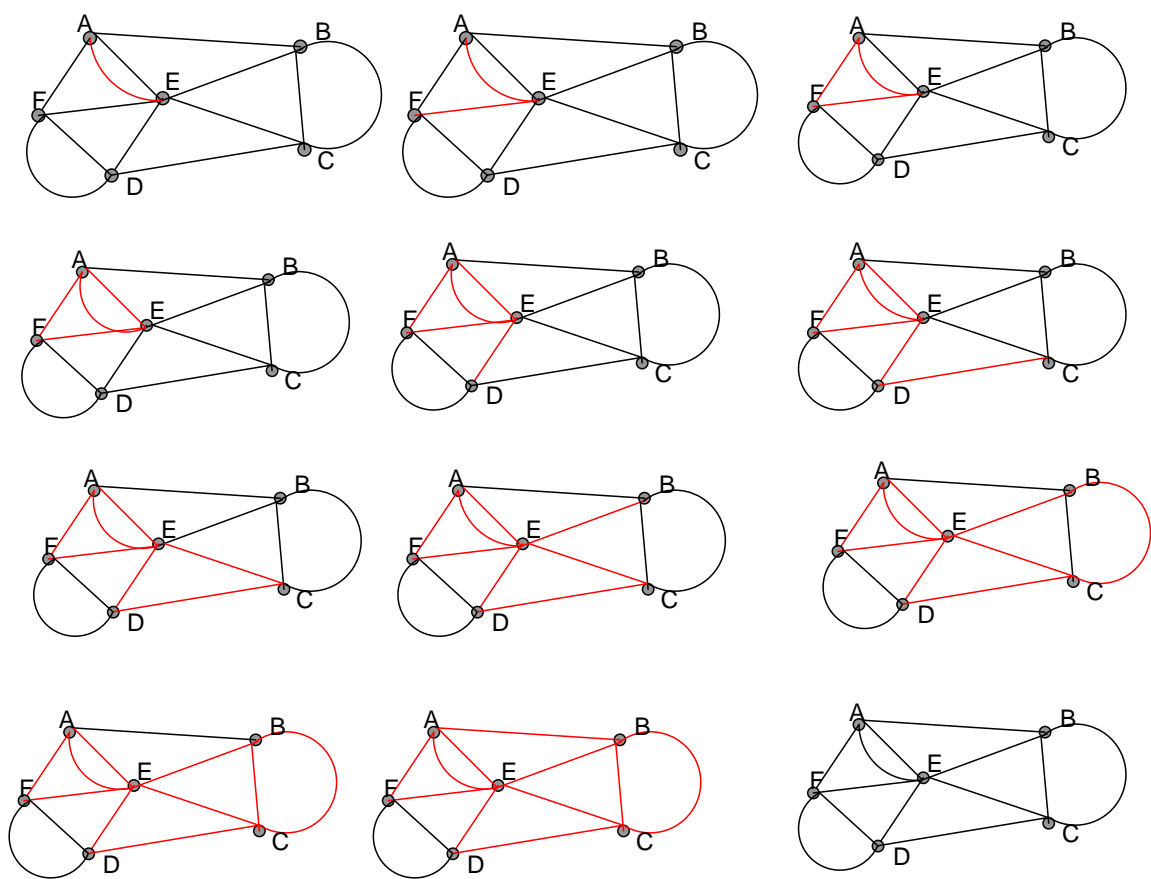
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Sometime in 2009

1   Example 1

Only connected graphs with has no vertices of odd degree can have an Euler circuit.
So the above graphs must have at least one Euler circuit. To find, use Fleury algorithm:
1. Start at any vertex.
2. Pick an edge to travel. If one of the edges is a bridge (Going along it, there is no way to come back to the vertex other than on it, then do not select it, unless it is the one edge left from that vertex).
2. Remove the edge traveled. If a vertex left with no edges leaving it, remove the vertex.
3. repeat step 2 until no more edges left.
4. The order of the edges traveled is the Euler circuit.

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Figure 1: Example 1

2 Example 2

Only connected graphs with has no vertices of odd degree can have an Euler circuit.
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3. repeat step 2 until no more edges left.
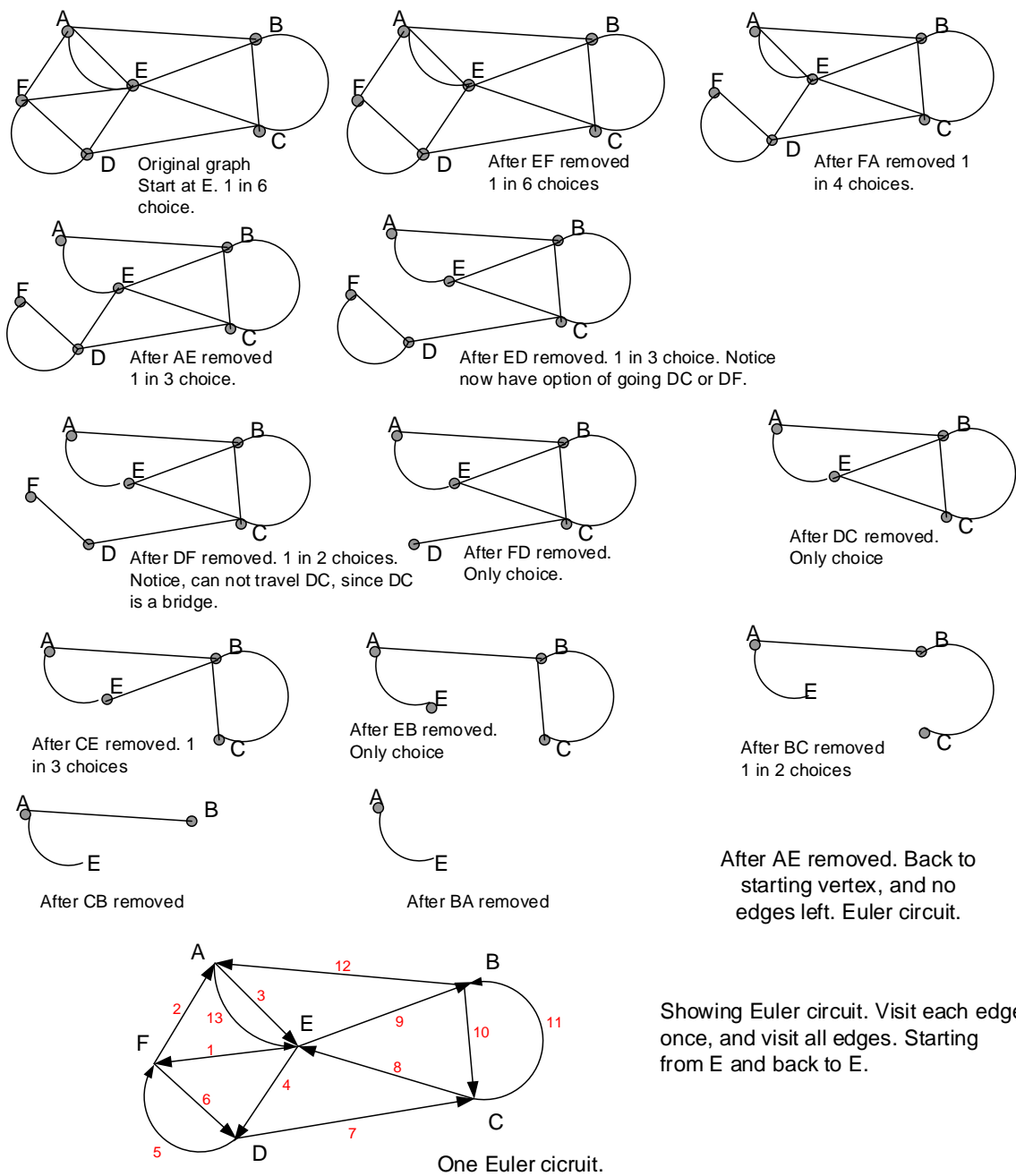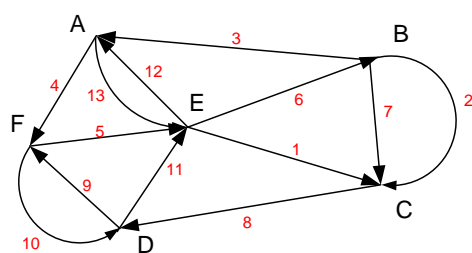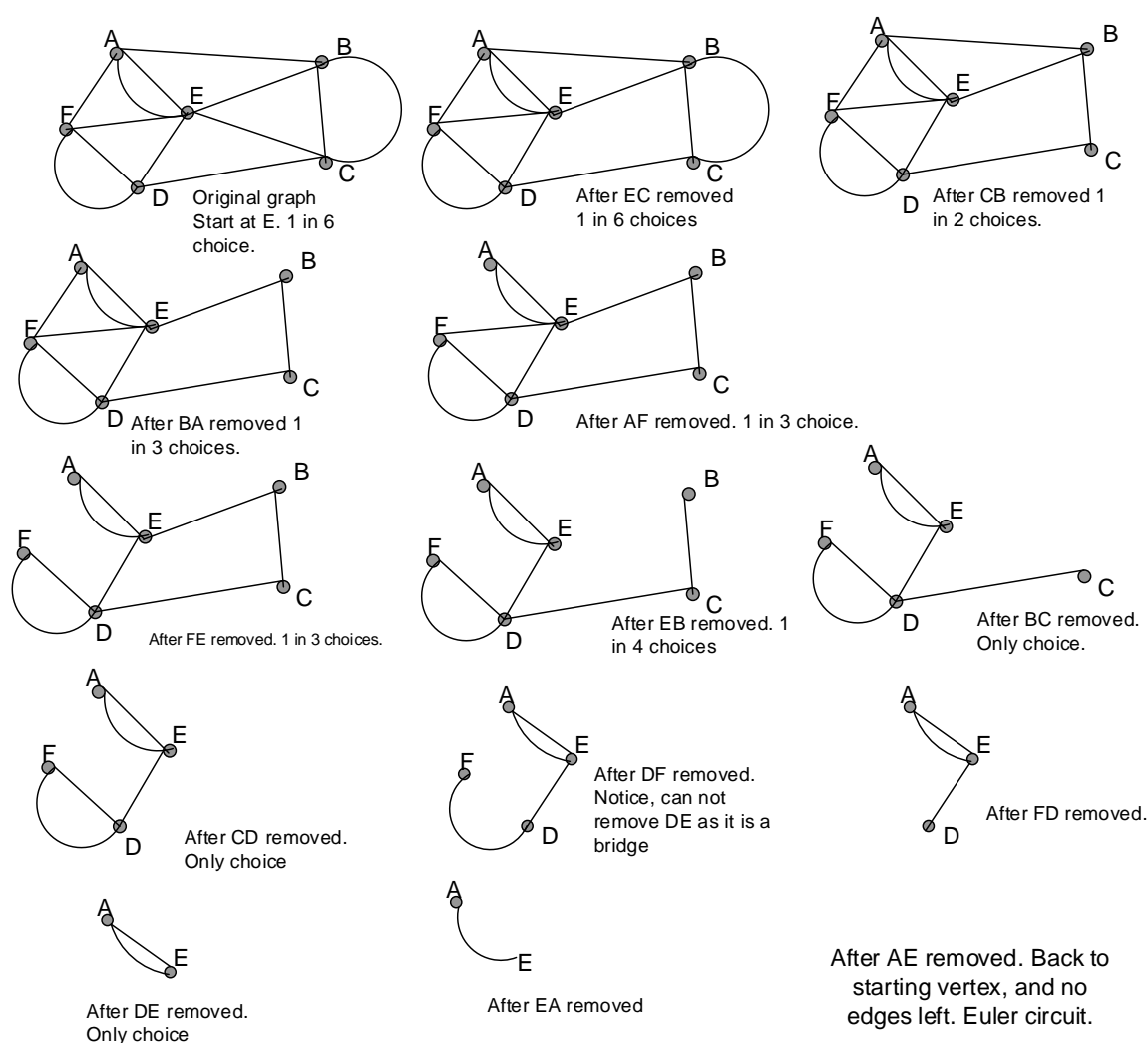4. The order of the edges traveled is the Euler circuit.

Original graph
Start at E. 1 in 6 choice.

After EF removed 1 in 6 choices

After FA removed 1 in 4 choices.

After AE removed 1 in 3 choice.

After ED removed. 1 in 3 choice. Notice now have option of going DC or DF.

After DF removed. 1 in 2 choices. Notice, can not travel DC, since DC is a bridge.

After FD removed. Only choice.

After DC removed. Only choice

After CE removed. 1 in 3 choices

After EB removed. Only choice

After BC removed 1 in 2 choices

After CB removed

After BA removed

After AE removed. Back to starting vertex, and no edges left. Euler circuit.

Showing Euler circuit. Visit each edge once, and visit all edges. Starting from E and back to E.

One Euler cicruit.

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Figure 2: Example 2

3   Example 3

A B E F D C Original graph Start at E. 1 in 6 choice.

A B E F D C After EC removed 1 in 6 choices

A B E F D C After CB removed 1 in 2 choices.

A B E F D C After BA removed 1 in 3 choices.

A B E F D C After AF removed. 1 in 3 choice.

A B E F D C After FE removed. 1 in 3 choices.

A B E F D C After EB removed. 1 in 4 choices

A E F D C After BC removed. Only choice.

A E F D After CD removed. Only choice

A E F D After DF removed. Notice, can not remove DE as it is a bridge

A E D After FD removed.

A E After DE removed. Only choice

A E After EA removed

After AE removed. Back to starting vertex, and no edges left. Euler circuit.

A B E F D C

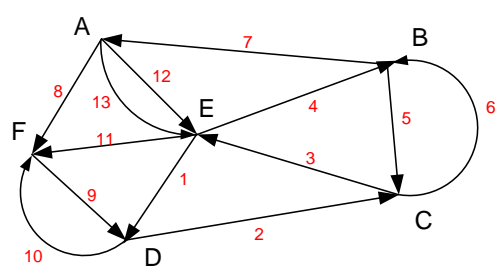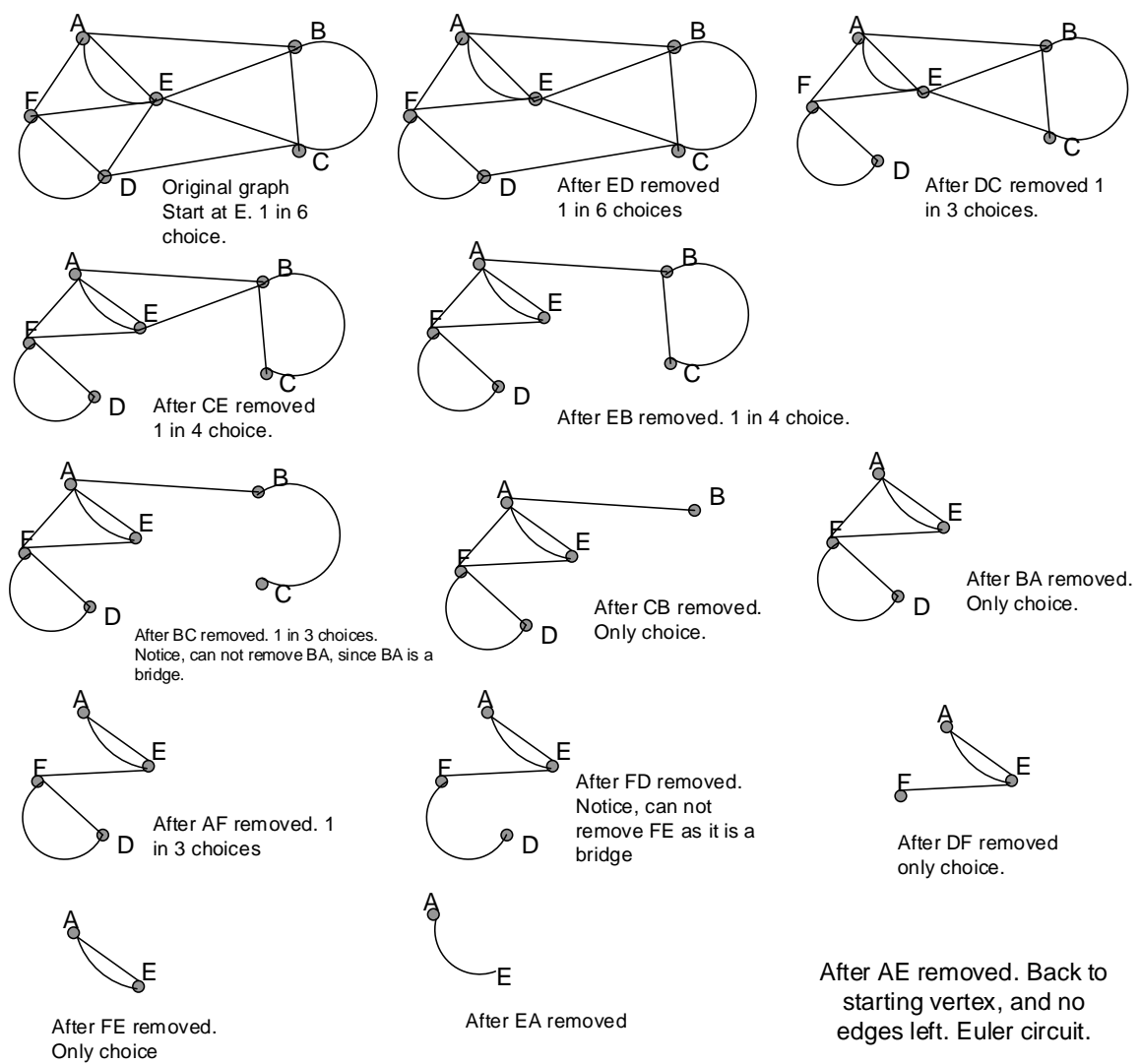Showing Euler circuit. Visit each edge once, and visit all edges. Starting from E and back to E.

Third Euler cicruit starting from E

Figure 3: Example 3

3

4   Example 4

A
B
E
F
D
C
Original graph
Start at E. 1 in 6
choice.

A
B
E
F
D
C
After ED removed
1 in 6 choices

A
B
F
E
C
D
After DC removed 1
in 3 choices.

A
B
F
E
D
C
After CE removed
1 in 4 choice.

A
B
F
E
D
C
After EB removed. 1 in 4 choice.

A
B
F
E
D
C
After BC removed. 1 in 3 choices.
Notice, can not remove BA, since BA is a
bridge.

A
B
F
E
D
After CB removed.
Only choice.

A
E
F
D
After BA removed.
Only choice.

A
E
F
D
After AF removed. 1
in 3 choices

A
E
F
D
After FD removed.
Notice, can not
remove FE as it is a
bridge

A
E
F
After DF removed
only choice.

A
E
After FE removed.
Only choice

A
E
After EA removed

After AE removed. Back to
starting vertex, and no
edges left. Euler circuit.

A
B
E
F
D
C
7
8
12
13
4
5
6
11
1
3
9
10
2

Showing Euler circuit. Visit each edge
once, and visit all edges. Starting
from E and back to E.

second Euler cicruit starting from E

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Figure 4: Example 4

Reference: **https://en.wikipedia.org/wiki/Eulerian_path** is reference on Euler circuits.