

# my Quantum Mechanics cheat sheet

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Table 1: QM cheat sheet

	Position Operator $X$	Momentum operator $P$	Hamiltonian operator $H$
Eigenvalue eigenvector relation	$X x\rangle = x x\rangle$ where $x$ is the eigenvalue (size) of the $ x\rangle$ which is the position vector associated with $x$ measured.	$P \phi_p\rangle = p \phi_p\rangle$ where $p$ is the momentum of the particle.	$H \Psi_{E_i}\rangle = E_i \Psi_{E_i}\rangle$ where $E_i$ is the energy level of the particle.
Normalization relation	$\int_{-\infty}^{\infty}  x\rangle \langle x  dx = 1$	$\int_{-\infty}^{\infty}  \phi_p\rangle \langle \phi_p  dp = 1$	$\int_{-\infty}^{\infty}  \Psi_{E_i}\rangle \langle \Psi_{E_i}  dE = 1$
orthogonality	$\langle x x'\rangle = \delta(x - x')$	$\langle \phi_p \phi_{p'}\rangle = \delta(p - p')$	$\langle \Psi_{E_i} \Psi_{E_j}\rangle = \delta(E_i - E_j)$
Matrix element of operator	$\langle x X x'\rangle = x'\delta(x - x')$ . Operator $X$ is diagonal matrix.	$\langle x P x'\rangle = -i\hbar\delta(x - x')\frac{d}{dx}$ where momentum operator $P$ is expressed in position operator $ x\rangle$ basis. Note that operator $P$ is not a diagonal matrix.	$\langle x H x'\rangle = ?$
Function form of the state function $ \Psi\rangle$	N/A ?	$P \phi_p\rangle = p \phi_p\rangle$ $\int P x'\rangle \langle x' \phi_p\rangle dx = p \int  x'\rangle \langle x' \phi_p\rangle dx$ $\int \langle x P x'\rangle \langle x' \phi_p\rangle dx = p \int \langle x x'\rangle \langle x' \phi_p\rangle dx$ $\int -i\hbar\delta(x - x')\frac{d}{dx'}\phi_p(x') dx = p \int \delta(x - x')\phi_p(x') dx$ $= \frac{2}{L} \frac{L}{2}$ $= 1$	$\langle \Psi \Psi\rangle = \int_{-\infty}^{\infty} \langle \Psi x\rangle \langle x \Psi\rangle dx$ $= \int_{-\infty}^{\infty} \langle x \Psi\rangle \langle x \Psi\rangle dx$ $= \int_{-\infty}^{\infty} \Psi^*(x)\Psi(x) dx$ $= \int_0^L \left( \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \right)^2 dx$ $= \frac{2}{L} \frac{L}{2}$ $= 1$
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	Position Operator $X$	Momentum operator $\Phi_p$	Hamiltonian operator
Vector form to function form	$\langle x   \Psi \rangle = \Psi(x)$	$\langle x   \phi_p \rangle = \phi_p(x)$	$\langle x   \Psi_E \rangle = \Psi_E(x)$
Expansion of state vector $ \Psi\rangle$	$ \Psi\rangle = \int_{-\infty}^{\infty}  x\rangle \langle x \Psi\rangle dx$	$ \Psi\rangle = \int_{-\infty}^{\infty}  \phi_p\rangle \langle \phi_p \Psi\rangle dp$	$ \Psi\rangle = \int_{-\infty}^{\infty}  E_i\rangle \langle E_i \Psi\rangle di$
State function $ \Psi\rangle$ For infinite potential deep well of width $x < 0 < L$	todo	todo	todo
Probability of measurement	<p>1. Probability of measuring <math>x</math> given system is in state <math> \Psi\rangle</math> is <math> \langle \Psi \Psi\rangle ^2</math>. For infinite potential deep well of width <math>x &lt; 0 &lt; L</math> this becomes</p> $\begin{aligned} \langle \Psi \Psi\rangle &= \int_{-\infty}^{\infty} \langle \Psi x\rangle \langle x \Psi\rangle dx \\ &= \int_{-\infty}^{\infty} \langle x \Psi\rangle \langle x \Psi\rangle dx \\ &= \int_{-\infty}^{\infty} \Psi^*(x)\Psi(x) dx \\ &= \int_0^L \left( \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \right)^2 dx \\ &= \frac{2}{L} \frac{L}{2} \\ &= 1 \end{aligned}$ <p>2. Probability of measuring <math>x</math> given system is in state <math>\phi_p</math></p>	$ \Psi\rangle = \int_{-\infty}^{\infty}  \phi_p\rangle \langle \phi_p \Psi\rangle dp$	$ \Psi\rangle = \int_{-\infty}^{\infty}  E_i\rangle \langle E_i \Psi\rangle di$