A simple method to do circular convolution

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This describes a simple method I found to do circular convolution, which I think is simpler than the method I saw in Digital Signal Processing, by Proakis, Manolakis.

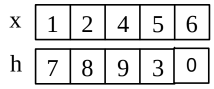
This is a method to compute the circular convolution for N points between two sequences, where N is the length of the longer of the two sequences (or the length of the sequences if they are of equal length).

Let the first sequence $x = \{\boxed{1}, 2, 4, 5, 6\}$ and the second sequence $h = \{7, \boxed{8}, 9, 3\}$, where the square around the number indicates the time $n = 0$.

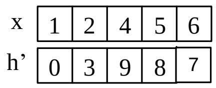
We want to find $y = x \circledast h$ where $\circledast$ is circular convolution.

The process requires as many steps as there are entries in the longer sequence x.

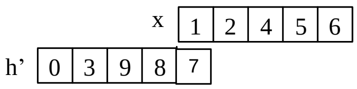
The process to to find $y[0]$ is illustrated using a diagram. The first step is to pad the smaller sequence by zeros so that it is the same length as the longer sequence. The method is explained in the diagrams



Step one: Write the 2 sequences under each others as given in the problem. Pad the smaller sequence with zeros.

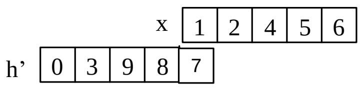


Step 2: Flip h, call result h'

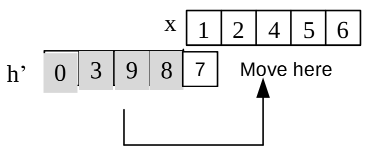


Slide h' to the left so that its last element is under the first element of x

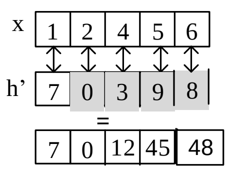
Step3: Slide the h' to the left so that its tail is under the head of x. This position is important, as we will return to this position again and again to compute new entries in y. Call this position the STARTING POSITION.



Step4: To calculate y[0] slide the START POSITION to the right by zero position. (so nothing to do).



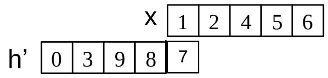
Step5: move the left most hanging part of h' to the right of the tail of h', this is the circular part of the convolution.



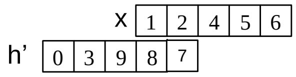
Sum = 112

Step6: Now do as with linear convolution, multiply the elements under each others and add the final vector. This will give the value of y[0]

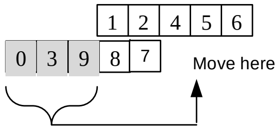
Now $y[1]$ is found using the same process as above, but h is moved to the right by 1 position instead of zero positions.



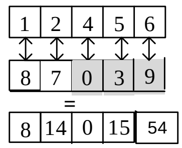
STARTING POSITION



To calculate y[1] slide the START POSITION to the right by 1 position.



move the left most hanging part of h to the right of the tail of h, this is the circular part of the convolution.

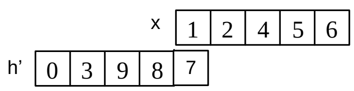


Sum = 91

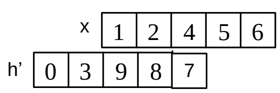
Now do as with linear convolution, multiply the elements under each others and add the final vector. This will give the value of y[1]

Notice that in the above step, we see that the origin (index $n = 0$) of sequence x happened to be aligned with the origin of the sequence h', this means that $y[1]$ is the origin of the y since this is the index for y being generated in this step.

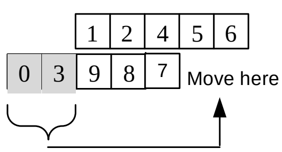
Now $y[2]$ is found using the same process as above, but h is moved to the right by 2 positions.



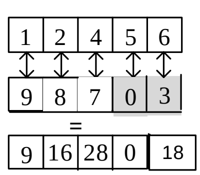
STARTING POSITION



To calculate y[2] slide the START POSITION to the right by 2 positions.



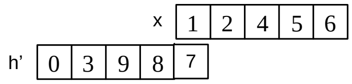
move the left most hanging part of h to the right of the tail of h, this is the circular part of the convolution.



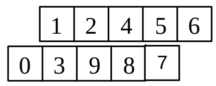
Sum = 71

Now do as with linear convolution, multiply the elements under each others and add the final vector. This will give the value of y[2]

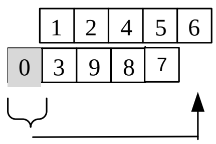
Now $y[3]$ is found using the same process as above, but h is moved to the right by 3 positions.



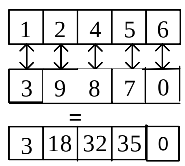
STARTING POSITION



To calculate y[3] slide the START POSITION to the right by 3 positions.



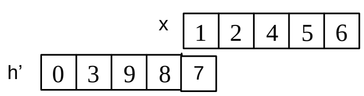
move the left most hanging part of h to the right of the tail of h, this is the circular part of the convolution.



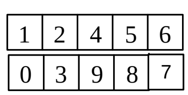
Sum = 88

Now do as with linear convolution, multiply the elements under each others and add the final vector. This will give the value of y[3]

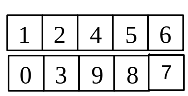
Now $y[4]$ is found using the same process as above, but h is moved to the right by 4 positions.



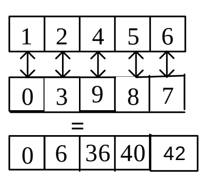
STARTING POSITION



To calculate y[4] slide the START POSITION to the right by 4 positions.



move the left most hanging part of h to the right of the tail of h. There is nothing to move now. Last step.



Sum = 124

Now do as with linear convolution, multiply the elements under each others and add the final vector. This will give the value of y[4]

Since now h' is completely under x, the process completes.

Hence $y = \{112, \boxed{91}, 71, 88, 124\}$. To verify

```
octave-3.2.4:39> x=[1 2 4 5 6];
octave-3.2.4:40> h=[7 8 9 3 0];
octave-3.2.4:41> X=fft(x); H=fft(h); y=ifft(X.*H)
y =

   112    91    71    88   124
```