my school cheat sheet for aerospace course

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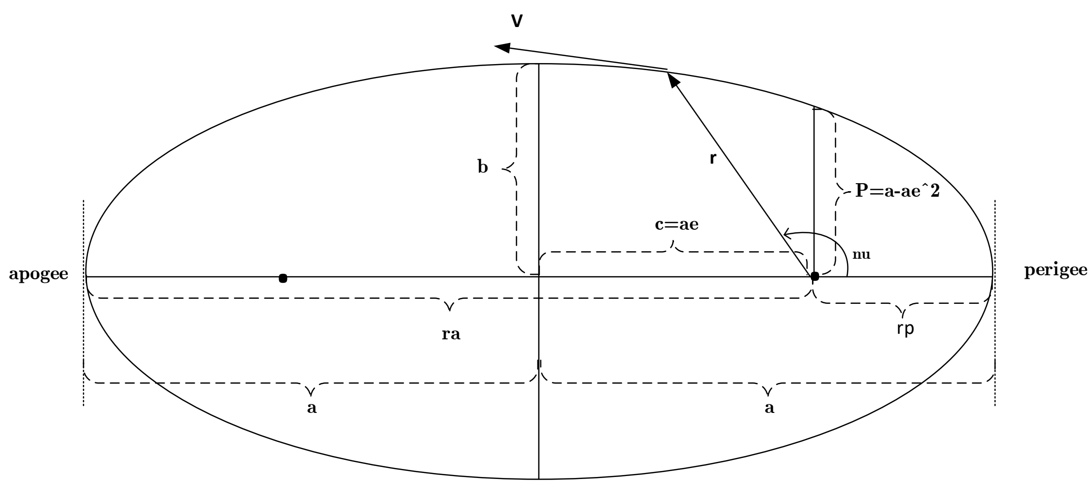

Figure 1: Eliptic orbit information

Equation of a conic section: $r = \frac{p}{1+e \ \cos \nu}$

Ellipse equation: $a^2 = b^2 + c^2$

relating energy to geometry

$$E = -\frac{\mu}{2a} \tag{1a}$$

relating energy to velocity and position:

$$E = \frac{V^2}{2} - \frac{\mu}{r} \tag{1b}$$

Velocity found from geometry and position: using equatings (1a) (1b), we get

$$\frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$
$$V = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)}$$

Velocity can be found knowing Energy and position: from (1b) we solve for V

$$V = \sqrt{2\left(\frac{\mu}{r} + E\right)}$$

Relating angular momentum to geometry:

$$p = \frac{h^2}{\mu}$$

Which is valid for any orbit.

$\mathbf{h} = \mathbf{r} \times \mathbf{V}$

$p = a - a\ e^2 \Longrightarrow p = a(1 - e^2)$ from geometry (1)

$r_p = a - a\ e \Longrightarrow r_p = a(1 - e)$ from geometry (2)

$r_a = a + a\ e \Longrightarrow r_a = a(1 + e)$ from geometry (3)

$c = a\ e$ by geometry definition.

e can also be found from physics as

$$e = \sqrt{1 + \frac{2Eh^2}{\mu^2}}$$

So, given the angular momentum and the mechanical energy, we can find e

$$e = \frac{r_a - r_p}{r_a + r_p}$$

$$\text{Period} = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$$r_p = \frac{P}{1 + e}$$

$$r_a = \frac{P}{1 - e}$$

These can be derived from (1) and (2)

$$\mathbf{e} = \frac{\mathbf{V} \times \mathbf{h}}{\mu} - \frac{\mathbf{r}}{r}$$

conic	eccentricity	Energy E	a
circle	0	$-ve$	$+ve$
ellipse	$0 \prec e \prec 1$	$-ve$	$+ve$
parabola	1	0	∞
hyperbola	$e \succ 0$	$+ve$	$-ve$

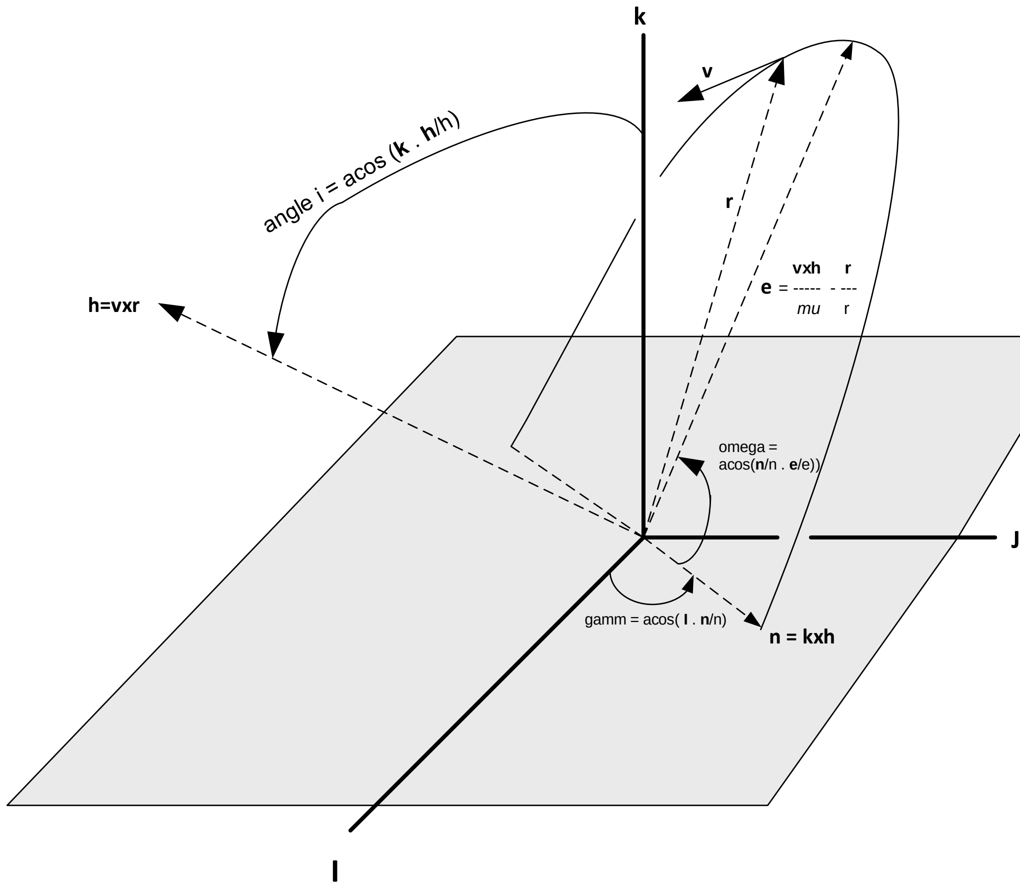

Figure 2: Elements

$$\tan\left(\frac{\nu}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)$$

$$M = E - e\sin E \tag{4}$$

where $M = \omega t$ where ω is the **average** angular velocity of the probe, and t is the time we wish to find the angle E at which the probe is located. hence M is the distance traveled (in radian angles) by the probe in the eccentric model. But $\omega = \frac{2\pi}{Period} = \frac{2\pi}{2\pi\sqrt{\frac{a^3}{\mu}}} = \sqrt{\frac{\mu}{a^3}}$(you see this have units as radians per unit time). so (4) is

$$\omega t = E - e\sin E$$

$$M = E - e\sin E$$

$$F(E) = E - e \sin E - M$$

Solve using Newton method. use

$$E_0 = M = \sqrt{\frac{\mu}{a^3}} t$$

$$\begin{aligned} E_1 &= E_0 - \frac{F(E_0)}{F'(E_0)} \\ &= E_0 - \frac{E_0 - e \sin E_0 - M}{1 - e \cos E_0} \end{aligned}$$

$$r = a(1 - e \cos E) = \frac{p}{1 + e \cos \nu}$$

1 Rocket Equations

effective I_{sp} officially is the total impulse per total mass expelled to generate this impulse.

i.e. effective $I_{sp} = \frac{I_{total}}{\text{total } m_p \text{ expelled}} = \frac{thrust*time}{total\ m_p}$

in units: $\frac{NT}{M} = \frac{M \frac{L}{T^2} T}{M} = \frac{L}{T}$ so it has the units of speed!

To convert it to I_{sp}, $I_{sp} = \frac{\text{effective } I_{sp}}{g\_c} = \frac{V_e}{g\_c}$ notice that effective I_{sp} is the same as V_e.

In units then $I_{sp} = \frac{\frac{L}{T}}{\left(\frac{L}{T^2}\right)} = T$ i.e. seconds, which is what we use

The above is the average specific impulse.

To find the instantaneous specific impulse $I_{spi} = \frac{\frac{d}{dt} I_{sp}}{\frac{d}{dt} m_p} = \frac{thrust}{\beta}$

From the net

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Specific impulse is defined as the number of seconds for which a
pound of propellant will produce a pound of thrust
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from the net

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Outside of the United States
specific impulse is in metres per second, and is identical to the effective
velocity of the exhaust gas from the rocket.
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from net

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Specific Impulse is a measure of the Thrust produced by an engine per
the mass flowrate of propellant and thus the correct SI unit is Ns/kg or
when the Newton is expanded and the units are cancelled down, m/s.
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from net

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The unit of seconds comes from some very silly cancelling when using old
units. If you measure thrust in lbf and you measure flowrate in lb/sec then
you get lbf.s/lb. Then if you cancel the two lb parts.... you get left with
seconds. This means that to make any use of the value it has to be multiplied
by g to put it into sensible units (s.m/s\symbol{94}2 = m/s again).
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impulse = change in momentum

Specific impulse is defined as dI/dm, in any system of units you care to name.

One definition I saw of specific impulse is

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how long you can thrust at a given force with a pound of fuel.
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from net

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specific impulse is a measure of how long
a given amount of fuel can provide a thrust equal to its own
weight.
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I_{sp} is the momentum gained per unit weight of propellant used during this momentum change. The momentum gained results from the loss of the m_p from the total mass.

Hence, assume we consume m_p propellant mass, then

$I_{sp} = \frac{(m_0 - m_f) V_e}{m_p \ g} = \frac{m_p V_e}{m_p \ g} = \frac{V_e}{g} = \frac{\frac{m}{\text{sec}}}{\frac{m}{\text{sec}^2}} = \text{sec}$

$m_o = m_L + m_p + m_s$

$m_f = m_L + m_s$

Final velocity $= V_e \ln Z$

Where $Z = \frac{m_0}{m_f} = \frac{m_L + m_s + m_p}{m_L + m_s}$

payload ratio (Prussing def) $\lambda = \frac{m_L}{m_s + m_p}$ this is class definition also

structural ratio $\epsilon = \frac{m_s}{m_p + m_s}$

payload ratio (Wisel def) $\pi = \frac{m_L}{m_L + m_s + m_p} = \frac{m_L}{m_0}$

$\frac{m_f}{m_0} = \epsilon + (1+\epsilon)\,\pi$

Notice that Z can also be written as $Z = \frac{1+\lambda}{\epsilon+\lambda}$

Methods: Given masses, if asked to find ΔV do

1. use similar stages. set up the $\lambda_1 = \lambda_2$ solve for m_{02}, m_{01}
2. set up $\epsilon_1 = \epsilon_2$, solve for m_{s1}, m_{s2}. Need to use result of step 1 (m_{02} or m_{01}) to help solve. Also, given that $m_{s1} + m_{s2} = m_s$ (which is given)
3. Now that we know the m_s and m_0 for the stages, we calculate λ_1 and ϵ_1 and find $Z = \frac{1+\lambda}{\epsilon+\lambda}$
4. $\Delta V = nV_e \ln Z$ where n is the number of stages.

β is the rate at which the propellant is consumed $= \frac{dm}{dt}$ assumed constant. Also called engine mass flow rate. in other words, it is the rate at which MASS is exiting the nozzle of the rocket engine. If we multiply this quantity by how fast this rate of mass is changing (i.e. V_e), we get an acceleration time mass, hence force, which is the engine thrust. This causes the rocket to go up.

For space shuttle, $V_e = 4500$ m/sec. $\beta = 222$ kg/sec, hence engine generates a thrust (force) of $4500 * 222 \simeq 1,000,000$ Newton

Impulse = thrust * time thrust applied.

i.e. total impulse $I = T\ t$

So, Thrust T, is the rate of change of impulse. The faster the impulse changes, the larger the thrust.

$$\text{thrust} = \beta\ V_e$$

$\beta = \frac{m_f\ -\ m_0}{t_{bo}} = \frac{-m_p}{t_{bo}}$ so, if I can find β given I_{sp}, then I can find the time it takes to reach burn out for some given m_p.

In other words, time it takes to burnout $t_{bo} = \frac{m_p}{\beta}$

Specific Impulse (ISP), or how much thrust you get from each pound of fuel is very important.

Generally, DELTA V = LN(MASS RATIO)* ISP*G That means that the Specific Impulse (ISP), or how much thrust you get from each pound of fuel is very important, and the Mass Ratio, or what percentage of your vehicle is propellant is less important. For each stage you can set an ISP to determine how much propellant you will use for the thrust you need. Then set a mass ratio to determine how much metal you wish to wrap around the propellant. The rule of Thumb is that higher stages get the better ISPs and Mass Ratios because they are smaller and they include the cost of the boosters. Boosters are the work horses, low ISP because of atmospheric back pressure, and heavy, but you can buy them by the pound cheap. Also, the ISP is set mostly by the propellant choice, the Mass Ratio on the other hand is determined by how much money you wish to spend on light weight materials. The lightest know material for construction is Unobtainium.

Solve for η :

$\Delta V_{total} - \sum_{i=1}^{N} V_{ei} \ln\left(\frac{\eta V e_i - 1}{\eta V_{ei} \varepsilon_i}\right) = 0$

Next find Z_i :

$Z_i = \frac{\eta V e_i - 1}{\eta V_{ei} \varepsilon_i}$