Application of Digital Signal Processing in Computed tomography (CT)

EE 518 project slides
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CT Overview
Uses X-ray to obtain multiple projections at different angles of the same cross section
Projection data processed using signal processing software to reconstruct 2D image of the cross section

DSP medical imaging software converts projection data to 2D section images. More projections leads to better images, but more x-ray exposure.
Simplified view of CT with parallel X-ray showing projection data capture

Components of parallel X-ray CT system. Many projections are obtained at different angles. Projections are combined in software using filtered backprojection to obtain an accurate 2D image of the section of the body shown above.
Illustrating the problem of image reconstruction on a simple 4 pixels image with 2 projections

A projection is an integral operation along the path of the ray. In other words, it sums the pixel values along its path, generating a vector of projection values.
The CT Inverse problem

Determine the original image from the projection data only.
The problem of image reconstruction from projections and possible solutions

The problem is the following:
Given a set of projections with corresponding angles that these projections obtained at, determined the original image

Method of solution

- Solving the CT inverse problem
  - Linear algebra approach
  - Frequency approach
    - Direct use of the Fourier Central Slice theorem and 2D Inverse Fourier Transform
    - Filtered Backprojection method (Used more in practice)
Solving the problem using linear algebra

\[ \begin{align*}
A + B &= 5 \\
C + D &= 8 \\
C + A &= 6 \\
D + B &= 7
\end{align*} \]

\[
\begin{pmatrix}
1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
A \\
B \\
C \\
D
\end{pmatrix} =
\begin{pmatrix}
5 \\
8 \\
6 \\
7
\end{pmatrix}
\]

Determinant of A is zero. No unique solution exist. Infinite number of solutions. Use least square approach.
Solving the problem using frequency domain with Fourier

Central Slice Theorem

2D Image

Projection #1

FFT

Projection #2

FFT

Align each vector into the 2D spectrum domain using the same angle used for the projection

2D spatial inverse Fourier transform (2D IFFT)

2D spectrum of image whose projections are slices which fills the matrix data

Using central slice theorem to solve the CT inverse problem
Solving the inverse CT problem using Filtered back projection

The more projections used, the better the approximation of filtered backprojection image to original (unknown) image becomes (See Matlab simulation)

Solving CT inverse problem using Filtered backprojection
Affect of Filtering using RAM-LAK on quality of back projection image, result found by simulation using Matlab radon/inverse radon functions.
Radon Transform

• Mathematically, a projection is performed using radon transform

\[
g(p, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos \theta + y \sin \theta - p) \, dx \, dy
\]
Sampling the projection data

Once a complete projection is obtained, it is sampled at frequency larger than its Nyquist spatial frequency

\[ g_\theta[p] \text{ sampled to sequence of samples} \]

\[ g_\theta(\frac{n}{2W}) \text{ where } f_s = 2W, \text{ where } W = \frac{1}{\tau} \]
Obtain Fourier transform of each sampled projection (using FFT)

\[ S(k, \theta_m) = \sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1} g \left( \frac{n}{2W}, \theta_m \right) e^{-j \frac{2\pi}{N} kn} \quad k = 0, 1, \ldots, N - 1 \]
First solution method
Apply the Fourier Central Slice Theorem

The Fourier transform of a projection taken at angle $\theta_m$ is equal to the values found along a slice in the 2D fourier transform of the original image itself, as long as this line goes through the origin of the 2D fourier transform plane and has the same angle $\theta_m$. 

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Second solution method: Filtered back projection

- Flow diagram shown earlier
- The mathematical expression of FBP derived from first principles in the project paper as follows

\[ h(x,y) = \frac{2W\pi}{M} \sum_{m=0}^{M-1} \left[ \frac{1}{N} \sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1} S(n \frac{2W}{N}, m \frac{\pi}{M}) \right] e^{\frac{4W\pi}{N} n \left( x \cos \left( \frac{m \pi}{M} \right) + y \sin \left( \frac{m \pi}{M} \right) \right)} \]

\[ N \] number of samples in projection
\[ M \] number of projections
\[ W \] largest spatial frequency in projection
Matlab simulation

- A Matlab application is written to simulate the CT reconstructions. Matlab radon and iradon used for the implementation.
Conclusions

1. CT image reconstruction is an inverse problem.
2. Hard problem to solve using linear algebra due to large number of equations to solve.
3. 2 methods based on frequency domain examined: Central slice theorem (SCT) and filtered back projection (FBP)
4. SCT requires gridding and interpolation of 2D spectrum to enable 2D IFFT.
5. FBP filters the projection spectrum before applying back projection. Back projection is an accumulative and averaging approach. Used more in practice than SCT.
6. Digital signal processing is critical to implement all the important current medical imaging methods such as CT, MRI, SPECT, PET and others
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