A Solution Manual For

An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906

AN ELEMENTARY TREATISE

ON

DIFFERENTIAL EQUATIONS

BY

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1.1 problem Ex 1

Internal problem ID [11132]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 8. Exact differential equations. Page 11
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{\frac{2yx+1}{y}+\frac{(y-x)\,y'}{y^2}=0}$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 18

```
dsolve((2*x*y(x)+1)/y(x)+ (y(x)-x)/y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\frac{x}{\operatorname{LambertW}\left(-\mathrm{e}^{x^2}c_1x\right)}$$

✓ Solution by Mathematica

Time used: 7.151 (sec). Leaf size: 29

```
DSolve[(2*x*y[x]+1)/y[x]+ (y[x]-x)/y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{x}{W\left(x\left(-e^{x^2-c_1}\right)\right)}$$
$$y(x)\to 0$$

1.2 problem Ex 2

Internal problem ID [11133]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 8. Exact differential equations. Page 11
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{\frac{y^2-2x^2}{y^2x-x^3}+\frac{(2y^2-x^2)\,y'}{y^3-yx^2}=0}$$

✓ Solution by Maple

Time used: 1.219 (sec). Leaf size: 223

```
dsolve((y(x)^2-2*x^2)/(x*y(x)^2-x^3)+ (2*y(x)^2-x^2)/(y(x)^3-x^2*y(x))*diff(y(x),x)=0,y(x),
```

$$y(x)=\frac{-xc_1-\frac{-2c_1^2x^2+\sqrt{2x^4c_1^4-2c_1\sqrt{c_1^6x^6+4}\,x}}{2xc_1}}{c_1}$$

$$y(x)=\frac{-xc_1+\frac{2c_1^2x^2+\sqrt{2x^4c_1^4-2c_1\sqrt{c_1^6x^6+4}\,x}}{2xc_1}}{c_1}$$

$$y(x)=\frac{-xc_1+\frac{2c_1^2x^2-\sqrt{2x^4c_1^4+2c_1\sqrt{c_1^6x^6+4}\,x}}{2xc_1}}{c_1}$$

$$y(x)=\frac{-xc_1+\frac{2c_1^2x^2+\sqrt{2x^4c_1^4+2c_1\sqrt{c_1^6x^6+4}\,x}}{2xc_1}}{c_1}$$

✓ Solution by Mathematica

Time used: 15.598 (sec). Leaf size: 277

```
DSolve[(y[x]^2-2*x^2)/(x*y[x]^2-x^3)+ (2*y[x]^2-x^2)/(y[x]^3-x^2*y[x])*y'[x]==0,y[x],x,Inclu
```

$$y(x) \to -\frac{\sqrt{x^2 - \frac{\sqrt{x^6 - 4e^{2c_1}}}{x}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{x^2 - \frac{\sqrt{x^6 - 4e^{2c_1}}}{x}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\frac{x^3 + \sqrt{x^6 - 4e^{2c_1}}}{x}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\frac{x^3 + \sqrt{x^6 - 4e^{2c_1}}}{x}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{x^2 - \frac{\sqrt{x^6}}{x}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{x^2 - \frac{\sqrt{x^6}}{x}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\frac{\sqrt{x^6} + x^3}{x}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\frac{\sqrt{x^6} + x^3}{x}}}{\sqrt{2}}$$

1.3 problem Ex 3

Internal problem ID [11134]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 8. Exact differential equations. Page 11
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{\frac{1}{\sqrt{y^2+x^2}}+\left(\frac{1}{y}-\frac{x}{y\sqrt{y^2+x^2}}\right)y'=0}$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 18

```
dsolve(1/sqrt(x^2+y(x)^2)+ ( 1/y(x)-(x/(y(x)*sqrt(x^2+y(x)^2))))*diff(y(x),x)=0,y(x), singso
```

$$-c_1+\sqrt{y(x)^2+x^2}+x=0$$

✓ Solution by Mathematica

Time used: 0.893 (sec). Leaf size: 62

```
DSolve[1/Sqrt[x^2+y[x]^2]+ ( 1/y[x]-(x/(y[x]*Sqrt[x^2+y[x]^2])))*y'[x]==0,y[x],x,IncludeSing
```

$$y(x)\to -e^{\frac{c_1}{2}}\sqrt{-2x+e^{c_1}}$$
$$y(x)\to e^{\frac{c_1}{2}}\sqrt{-2x+e^{c_1}}$$
$$y(x)\to 0$$
$$y(x)\to \text{ComplexInfinity}$$

1.4 problem Ex 4

Internal problem ID [11135]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 8. Exact differential equations. Page 11
Problem number: Ex 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + y = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve((y(x)+x)+ x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} + \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 17

```
DSolve[(y[x]+x)+ x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{2} + \frac{c_1}{x}$$

1.5 problem Ex 5

Internal problem ID [11136]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 8. Exact differential equations. Page 11
Problem number: Ex 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd ty`

$$\boxed{-2y + (2y - 2x - 3)\, y' = -6x - 1}$$

✓ Solution by Maple

Time used: 0.672 (sec). Leaf size: 36

```
dsolve((6*x-2*y(x)+1)+(2*y(x)-2*x-3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 2 - \frac{-(2x-1)\, c_1 + \sqrt{-2\,(2x-1)^2\, c_1^2 + 1}}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.208 (sec). Leaf size: 67

```
DSolve[(6*x-2*y[x]+1)+(2*y[x]-2*x-3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2} i \sqrt{8x^2 - 8x - 9 - 4c_1} + x + \frac{3}{2}$$
$$y(x) \to \frac{1}{2} i \sqrt{8x^2 - 8x - 9 - 4c_1} + x + \frac{3}{2}$$

2 Chapter 2, differential equations of the first order and the first degree. Article 9. Variables searated or separable. Page 13

2.1 problem Ex 1

Internal problem ID [11137]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 9. Variables searated or separable. Page 13
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\sec(x)\cos(y)^2 - \cos(x)\sin(y)\,y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve((sec(x)*cos(y(x))^2)-(cos(x)*sin(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{1}{\tan(x) + c_1}\right)$$

✓ Solution by Mathematica

Time used: 1.366 (sec). Leaf size: 45

```
DSolve[(Sec[x]*Cos[y[x]]^2)-(Cos[x]*Sin[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\sec^{-1}(\tan(x) + 2c_1)$$
$$y(x) \to \sec^{-1}(\tan(x) + 2c_1)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

2.2 problem Ex 2

Internal problem ID [11138]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 9. Variables searated or separable. Page 13
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+1)\,y^2 - x^3 y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve((1+x)*y(x)^2-x^3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2x^2}{2x^2c_1 + 2x + 1}$$

✓ Solution by Mathematica

Time used: 0.231 (sec). Leaf size: 29

```
DSolve[(1+x)*y[x]^2-x^3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^2}{-2c_1x^2 + 2x + 1}$$
$$y(x) \to 0$$

2.3 problem Ex 3

Internal problem ID [11139]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 9. Variables searated or separable. Page 13
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2(1-y^2)\,xy+(x^2+1)\,(y^2+1)\,y'=0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 75

```
dsolve(2*(1-y(x)^2)*x*y(x)+(1+x^2)*(1+y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\frac{x^2c_1}{2}+\frac{c_1}{2}-\frac{\sqrt{c_1^2x^4+2c_1^2x^2+c_1^2+4}}{2}$$
$$y(x)=\frac{x^2c_1}{2}+\frac{c_1}{2}+\frac{\sqrt{c_1^2x^4+2c_1^2x^2+c_1^2+4}}{2}$$

✓ Solution by Mathematica

Time used: 8.437 (sec). Leaf size: 98

```
DSolve[2*(1-y[x]^2)*x*y[x]+(1+x^2)*(1+y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \frac{1}{2}\left(-e^{c_1}\left(x^2+1\right)-\sqrt{4+e^{2c_1}\left(x^2+1\right)^2}\right)$$

$$y(x) \to \frac{1}{2}\left(\sqrt{4+e^{2c_1}\left(x^2+1\right)^2}-e^{c_1}\left(x^2+1\right)\right)$$

$$y(x) \to -1$$

$$y(x) \to 0$$

$$y(x) \to 1$$

2.4 problem Ex 4

Internal problem ID [11140]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 9. Variables searated or separable. Page 13
Problem number: Ex 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sin(x)\cos(y)^2 + \cos(x)^2 y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(sin(x)*cos(y(x))^2+cos(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\arctan(\sec(x) + c_1)$$

✓ Solution by Mathematica

Time used: 2.833 (sec). Leaf size: 31

```
DSolve[Sin[x]*Cos[y[x]]^2+Cos[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arctan(-\sec(x) + c_1)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

3 Chapter 2, differential equations of the first order and the first degree. Article 10. Homogeneous equations. Page 15

3.1 problem Ex 1

Internal problem ID [11141]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 10. Homogeneous equations. Page 15
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{x\,\mathrm{e}^{\frac{y}{x}}+y-y'x=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x*exp(y(x)/x)+y(x))-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\ln\left(-\frac{1}{\ln(x)+c_1}\right)x$$

✓ Solution by Mathematica

Time used: 0.527 (sec). Leaf size: 18

```
DSolve[(x*Exp[y[x]/x]+y[x])-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -x\log(-\log(x)-c_1)$$

3.2 problem Ex 2

Internal problem ID [11142]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 10. Homogeneous equations. Page 15
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{2yx^2 + 3y^3 - (x^3 + 2y^2x)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.984 (sec). Leaf size: 89

```
dsolve((2*x^2*y(x)+3*y(x)^3)-(x^3+2*x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2 - 2\sqrt{4x^2c_1 + 1}}\, x}{2}$$

$$y(x) = \frac{\sqrt{-2 - 2\sqrt{4x^2c_1 + 1}}\, x}{2}$$

$$y(x) = -\frac{\sqrt{-2 + 2\sqrt{4x^2c_1 + 1}}\, x}{2}$$

$$y(x) = \frac{\sqrt{-2 + 2\sqrt{4x^2c_1 + 1}}\, x}{2}$$

✓ Solution by Mathematica

Time used: 47.499 (sec). Leaf size: 277

```
DSolve[(2*x^2*y[x]+3*y[x]^3)-(x^3+2*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to -\frac{\sqrt{-x^2-\sqrt{x^4+4e^{2c_1}x^6}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-x^2-\sqrt{x^4+4e^{2c_1}x^6}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{-x^2+\sqrt{x^4+4e^{2c_1}x^6}}}{\sqrt{2}}$$

$$y(x) \to \sqrt{-\frac{x^2}{2}+\frac{1}{2}\sqrt{x^4+4e^{2c_1}x^6}}$$

$$y(x) \to -\frac{\sqrt{-\sqrt{x^4}-x^2}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-\sqrt{x^4}-x^2}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\sqrt{x^4}-x^2}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\sqrt{x^4}-x^2}}{\sqrt{2}}$$

3.3 problem Ex 3

Internal problem ID [11143]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 10. Homogeneous equations. Page 15
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y^2 - yx + y'x^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve((y(x)^2-x*y(x))+x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{\ln(x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.15 (sec). Leaf size: 19

```
DSolve[(y[x]^2-x*y[x])+x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{\log(x) + c_1}$$

$$y(x) \to 0$$

3.4 problem Ex 4

Internal problem ID [11144]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 10. Homogeneous equations. Page 15
Problem number: Ex 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{2yx^2 + y^3 - x^3y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(2*x^2*y(x)+y(x)^3-x^3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{\sqrt{-x^2 + c_1}}$$
$$y(x) = -\frac{x^2}{\sqrt{-x^2 + c_1}}$$

✓ Solution by Mathematica

Time used: 0.181 (sec). Leaf size: 47

```
DSolve[2*x^2*y[x]+y[x]^3-x^3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{\sqrt{-x^2 + c_1}}$$
$$y(x) \to \frac{x^2}{\sqrt{-x^2 + c_1}}$$
$$y(x) \to 0$$

3.5 problem Ex 5

Internal problem ID [11145]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 10. Homogeneous equations. Page 15
Problem number: Ex 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^3 + x^3 y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(y(x)^3+x^3*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{\sqrt{x^2 c_1 - 1}}$$
$$y(x) = -\frac{x}{\sqrt{x^2 c_1 - 1}}$$

✓ Solution by Mathematica

Time used: 0.356 (sec). Leaf size: 45

```
DSolve[y[x]^3+x^3*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{\sqrt{-1 - 2c_1 x^2}}$$
$$y(x) \to \frac{x}{\sqrt{-1 - 2c_1 x^2}}$$
$$y(x) \to 0$$

3.6 problem Ex 6

Internal problem ID [11146]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 10. Homogeneous equations. Page 15
Problem number: Ex 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y\cos\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)y' = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve((x+y(x)*cos(y(x)/x))-x*cos(y(x)/x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arcsin\left(\ln\left(x\right) + c_1\right)x$$

✓ Solution by Mathematica

Time used: 0.395 (sec). Leaf size: 13

```
DSolve[(x+y[x]*Cos[y[x]/x])-x*Cos[y[x]/x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\arcsin(\log(x) + c_1)$$

4 Chapter 2, differential equations of the first order and the first degree. Article 11. Equations in which M and N are linear but not homogeneous. Page 16

4.1 problem Ex 1

Internal problem ID [11147]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 11. Equations in which M and N are linear but not homogeneous. Page 16
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$3y + (x + y + 1)\, y' = -4x - 1$$

✓ Solution by Maple

Time used: 0.516 (sec). Leaf size: 29

```
dsolve((4*x+3*y(x)+1)+(x+y(x)+1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -3 - \frac{(x-2)\,(2\,\mathrm{LambertW}\,(c_1(x-2)) + 1)}{\mathrm{LambertW}\,(c_1\,(x-2))}$$

✓ Solution by Mathematica

Time used: 1.385 (sec). Leaf size: 159

```
DSolve[(4*x+3*y[x]+1)+(x+y[x]+1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{(-2)^{2/3}\left(-2x\log\left(\frac{3(-2)^{2/3}(y(x)+2x-1)}{y(x)+x+1}\right) + (2x-1)\log\left(-\frac{3(-2)^{2/3}(x-2)}{y(x)+x+1}\right) + \log\left(\frac{3(-2)^{2/3}(y(x)+2x-1)}{y(x)+x+1}\right) + \cdots\right.}{9(y(x)+2x-1)}\right.$$

4.2 problem Ex 2

Internal problem ID [11148]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 11. Equations in which M and N are linear but not homogeneous. Page 16
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{-y + (x + y + 3)\, y' = -4x - 2}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 33

```
dsolve((4*x-y(x)+2)+(x+y(x)+3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -2 - 2\tan\left(\operatorname{RootOf}\left(\ln\left(\frac{4}{\cos\left(\_Z\right)^2}\right) - \_Z + 2\ln\left(x+1\right) + 2c_1\right)\right)(x+1)$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 67

```
DSolve[(4*x-y[x]+2)+(x+y[x]+3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} &\text{Solve}\Bigg[2\arctan\left(\frac{1}{2} - \frac{5(x+1)}{2(y(x)+x+3)}\right) \\ &\quad + 2\log\left(\frac{4x^2 + y(x)^2 + 4y(x) + 8x + 8}{5(x+1)^2}\right) + 4\log(x+1) + 5c_1 = 0, y(x)\Bigg] \end{aligned}$$

4.3 problem Ex 3

Internal problem ID [11149]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 11. Equations in which M and N are linear but not homogeneous. Page 16
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y - (4x + 2y - 1)\, y' = -2x}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 35

```
dsolve((2*x+y(x))-(4*x+2*y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{-\operatorname{LambertW}\left(-2\,\mathrm{e}^{4}\mathrm{e}^{-25x}\mathrm{e}^{25c_1}\right)+4-25x+25c_1}}{5} + \frac{2}{5} - 2x$$

✓ Solution by Mathematica

Time used: 4.725 (sec). Leaf size: 39

```
DSolve[(2*x+y[x])-(4*x+2*y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{10} W\left(-e^{-25x-1+c_1}\right) - 2x + \frac{2}{5}$$
$$y(x) \to \frac{2}{5} - 2x$$

5 Chapter 2, differential equations of the first order and the first degree. Article 12. Equations of form $yf_1(xy) + xf_2(xy)y' = 0$. Page 18

5.1 problem Ex 1

Internal problem ID [11150]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 12. Equations of form $yf_1(xy) + xf_2(xy)y' = 0$. Page 18
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Riccati]`

$$\boxed{y + 2y^2x - y^3x^2 + 2y'yx^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve((y(x)+2*x*y(x)^2-x^2*y(x)^3)+(2*x^2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{\tanh\left(-\frac{\ln(x)}{2} + \frac{c_1}{2}\right)}{x}$$

✓ Solution by Mathematica

Time used: 1.44 (sec). Leaf size: 71

```
DSolve[(y[x]+2*x*y[x]^2-x^2*y[x]^3)+(2*x^2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to 0$$
$$y(x) \to \frac{i\tan\left(\frac{1}{2}i\log(x) + c_1\right)}{x}$$
$$y(x) \to 0$$
$$y(x) \to \frac{-x + e^{2i\text{Interval}[\{0,\pi\}]}}{x^2 + xe^{2i\text{Interval}[\{0,\pi\}]}}$$

5.2 problem Ex 2

Internal problem ID [11151]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 12. Equations of form $yf_1(xy) + xf_2(xy)y' = 0$. Page 18
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{2y + 3y^2x + (x + 2yx^2)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 43

```
dsolve((2*y(x)+3*x*y(x)^2)+(x+2*x^2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-x + \sqrt{4xc_1 + x^2}}{2x^2}$$
$$y(x) = -\frac{x + \sqrt{4xc_1 + x^2}}{2x^2}$$

✓ Solution by Mathematica

Time used: 0.888 (sec). Leaf size: 69

```
DSolve[(2*y[x]+3*x*y[x]^2)+(x+2*x^2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^{3/2} + \sqrt{x^2(x + 4c_1)}}{2x^{5/2}}$$
$$y(x) \to \frac{-x^{3/2} + \sqrt{x^2(x + 4c_1)}}{2x^{5/2}}$$

5.3 problem Ex 3

Internal problem ID [11152]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 12. Equations of form $yf_1(xy) + xf_2(xy)y' = 0$. Page 18
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y + y^2x + (x - yx^2)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 18

```
dsolve((y(x)+x*y(x)^2)+(x-x^2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{\text{LambertW}\left(-\frac{c_1}{x^2}\right) x}$$

✓ Solution by Mathematica

Time used: 8.358 (sec). Leaf size: 35

```
DSolve[(y[x]+x*y[x]^2)+(x-x^2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{xW\left(\frac{e^{-1+\frac{9c_1}{2^{2/3}}}}{x^2}\right)}$$

$$y(x) \to 0$$

6 Chapter 2, differential equations of the first order and the first degree. Article 13. Linear equations of first order. Page 19

6.1 problem Ex 1

Internal problem ID [11153]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 13. Linear equations of first order. Page 19
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \cot(x)\, y = \sec(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)+y(x)*cot(x)=sec(x),y(x), singsol=all)
```

$$y(x) = \frac{-\ln(\cos(x)) + c_1}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.081 (sec). Leaf size: 16

```
DSolve[y'[x]+y[x]*Cot[x]==Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \csc(x)(-\log(\cos(x)) + c_1)$$

6.2 problem Ex 2

Internal problem ID [11154]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 13. Linear equations of first order. Page 19
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + y(x+1) = e^x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x*diff(y(x),x)+(1+x)*y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{e^{2x}}{2} + c_1\right) e^{-x}}{x}$$

✓ Solution by Mathematica

Time used: 0.078 (sec). Leaf size: 25

```
DSolve[x*y'[x]+(1+x)*y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x + 2c_1 e^{-x}}{2x}$$

6.3 problem Ex 3

Internal problem ID [11155]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 13. Linear equations of first order. Page 19
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$
\boxed{y' - \frac{2y}{x+1} = (x+1)^3}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)-2*y(x)/(1+x)=(x+1)^3,y(x), singsol=all)
```

$$
y(x) = \left(\frac{1}{2}x^2 + x + c_1\right)(x+1)^2
$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 22

```
DSolve[y'[x]-2*y[x]/(1+x)==(x+1)^3,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \rightarrow (x+1)^2\left(\frac{x^2}{2} + x + c_1\right)
$$

6.4 problem Ex 4

Internal problem ID [11156]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 13. Linear equations of first order. Page 19
Problem number: Ex 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^3 + x)\, y' + 4yx^2 = 2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve((x+x^3)*diff(y(x),x)+4*x^2*y(x)=2,y(x), singsol=all)
```

$$y(x) = \frac{x^2 + 2\ln(x) + c_1}{(x^2 + 1)^2}$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 23

```
DSolve[(x+x^3)*y'[x]+4*x^2*y[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2 + 2\log(x) + c_1}{(x^2 + 1)^2}$$

6.5 problem Ex 5

Internal problem ID [11157]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 13. Linear equations of first order. Page 19
Problem number: Ex 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' x^2 + (-2x+1)\, y = x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)+(1-2*x)*y(x)=x^2,y(x), singsol=all)
```

$$y(x) = x^2 + \mathrm{e}^{\frac{1}{x}} c_1 x^2$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 19

```
DSolve[x^2*y'[x]+(1-2*x)*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2\left(1 + c_1 e^{\frac{1}{x}}\right)$$

7 Chapter 2, differential equations of the first order and the first degree. Article 14. Equations reducible to linear equations (Bernoulli). Page 21

7.1 problem Ex 1

Internal problem ID [11158]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 14. Equations reducible to linear equations (Bernoulli). Page 21
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{\left(-x^{2}+1\right) y'-2y(x+1)-y^{\frac{5}{2}}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve((1-x^2)*diff(y(x),x)-2*(1+x)*y(x)=y(x)^(5/2),y(x), singsol=all)
```

$$\frac{1}{y\left(x\right)^{\frac{3}{2}}}-\left(-\frac{1}{4\left(x-1\right)^{3}}+\frac{3}{16\left(x-1\right)^{2}}-\frac{3}{16\left(x-1\right)}-\frac{3\ln\left(x-1\right)}{32}+\frac{3\ln\left(x+1\right)}{32}+c_{1}\right)\left(x-1\right)^{3}=0$$

✓ Solution by Mathematica

Time used: 1.042 (sec). Leaf size: 76

```
DSolve[(1-x^2)*y'[x]-2*(1+x)*y[x]==y[x]^(5/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{8\sqrt[3]{2}}{\left(32c_1x^3-6x^2-96c_1x^2+18x-3(x-1)^3\log(x-1)+3(x-1)^3\log(x+1)+96c_1x-20-32c_1\right)^{2/3}}$$

$$y(x) \to 0$$

7.2 problem Ex 2

Internal problem ID [11159]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 14. Equations reducible to linear equations (Bernoulli). Page 21
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'y + y^2x = x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(y(x)*diff(y(x),x)+x*y(x)^2=x,y(x), singsol=all)
```

$$y(x) = \sqrt{e^{-x^2}c_1 + 1}$$
$$y(x) = -\sqrt{e^{-x^2}c_1 + 1}$$

✓ Solution by Mathematica

Time used: 2.1 (sec). Leaf size: 57

```
DSolve[y[x]*y'[x]+x*y[x]^2==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{1 + e^{-x^2+2c_1}}$$
$$y(x) \to \sqrt{1 + e^{-x^2+2c_1}}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

7.3 problem Ex 3

Internal problem ID [11160]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 14. Equations reducible to linear equations (Bernoulli). Page 21
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' \sin(y) + \sin(x)\cos(y) = \sin(x)}$$

✓ Solution by Maple

Time used: 0.234 (sec). Leaf size: 14

```
dsolve(sin(y(x))*diff(y(x),x)+sin(x)*cos(y(x))=sin(x),y(x), singsol=all)
```

$$y(x) = \arccos\left(\mathrm{e}^{-\cos(x)}c_1 + 1\right)$$

✓ Solution by Mathematica

Time used: 1.53 (sec). Leaf size: 81

```
DSolve[Sin[y[x]]*y'[x]+Sin[x]*Cos[y[x]]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

$$\text{Solve}\left[2\cos(x)\tan\left(\frac{y(x)}{2}\right)e^{\operatorname{arctanh}(\cos(y(x)))} - \sqrt{\sin^2(y(x))}\csc\left(\frac{y(x)}{2}\right)\sec\left(\frac{y(x)}{2}\right)\left(\log\left(\sec^2\left(\frac{y(x)}{2}\right)\right) - 2\log\left(\tan\left(\frac{y(x)}{2}\right)\right)\right) = c_1, y(x)\right]$$

$$y(x) \to 0$$

7.4 problem Ex 4

Internal problem ID [11161]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 14. Equations reducible to linear equations (Bernoulli). Page 21
Problem number: Ex 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{4y'x + 3y + e^{x} x^{4} y^{5} = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 75

```
dsolve(4*x*diff(y(x),x)+3*y(x)+exp(x)*x^4*y(x)^5=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{\sqrt{x\,e^{x} + x c_1}\,x}}$$
$$y(x) = \frac{1}{\sqrt{-\sqrt{x\,e^{x} + x c_1}\,x}}$$
$$y(x) = -\frac{1}{\sqrt{\sqrt{x\,e^{x} + x c_1}\,x}}$$
$$y(x) = -\frac{1}{\sqrt{-\sqrt{x\,e^{x} + x c_1}\,x}}$$

✓ Solution by Mathematica

Time used: 14.931 (sec). Leaf size: 88

```
DSolve[4*x*y'[x]+3*y[x]+Exp[x]*x^4*y[x]^5==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt[4]{x^3 (e^x + c_1)}}$$

$$y(x) \to -\frac{i}{\sqrt[4]{x^3 (e^x + c_1)}}$$

$$y(x) \to \frac{i}{\sqrt[4]{x^3 (e^x + c_1)}}$$

$$y(x) \to \frac{1}{\sqrt[4]{x^3 (e^x + c_1)}}$$

$$y(x) \to 0$$

7.5 problem Ex 5

Internal problem ID [11162]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 14. Equations reducible to linear equations (Bernoulli). Page 21
Problem number: Ex 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y' - \frac{y+1}{x+1} - \sqrt{y+1} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 160

```
dsolve(diff(y(x),x)- (y(x)+1)/(x+1)=sqrt(1+y(x)),y(x), singsol=all)
```

$$\frac{\sqrt{y(x)+1}\,x}{(-x^2-2x+y(x))\left(\sqrt{y(x)+1}-1-x\right)} + \frac{2x}{(-x^2-2x+y(x))\left(\sqrt{y(x)+1}-1-x\right)} + \frac{x^2}{(-x^2-2x+y(x))\left(\sqrt{y(x)+1}-1-x\right)} + \frac{\sqrt{y(x)+1}}{(-x^2-2x+y(x))\left(\sqrt{y(x)+1}-1-x\right)} + \frac{1}{(-x^2-2x+y(x))\left(\sqrt{y(x)+1}-1-x\right)} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.418 (sec). Leaf size: 60

```
DSolve[y'[x]- (y[x]+1)/(x+1)==Sqrt[1+y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2\sqrt{y(x)+1}\arctan\left(\frac{x+1}{\sqrt{-y(x)-1}}\right)}{\sqrt{-y(x)-1}}+\log\left(y(x)-(x+1)^2+1\right)-\log(x+1)=c_1,y(x)\right]$$

8 Chapter 2, differential equations of the first order and the first degree. Article 15. Page 22

8.1 problem Ex 1

Internal problem ID [11163]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 15. Page 22
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$x^4y(3y + 2y'x) + x^2(4y + 3y'x) = 0$$

✓ Solution by Maple

Time used: 2.375 (sec). Leaf size: 39

```
dsolve(x^4*y(x)*(3*y(x)+2*x*diff(y(x),x))+ x^2*(4*y(x)+3*x*diff(y(x),x))=0,y(x), singsol=all
```

$$y(x) = \frac{\mathrm{RootOf}\left(x^2\_Z^8 - 2c_1\_Z^2 - c_1\right)^6 x^2 - 2c_1}{x^2 c_1}$$

✓ Solution by Mathematica

Time used: 60.464 (sec). Leaf size: 1769

```
DSolve[x^4*y[x]*(3*y[x]+2*x*y'[x])+ x^2*(4*y[x]+3*x*y'[x])==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to -\frac{1}{2x^2} + \frac{\sqrt{\frac{3}{x^4} - \frac{2\,6^{2/3}e^{-2c_1}}{\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}} + \frac{\sqrt[3]{6}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right.}}{x^6}}}{2\sqrt{3}} - \frac{1}{2}\sqrt{\frac{2}{x^4} + \frac{2\,2^{2/3}e^{-2c_1}}{\sqrt[3]{3}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}} - \frac{\sqrt[3]{2}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e\right.}}{3^{2/3}x^6}}$$

$$y(x) \to -\frac{1}{2x^2} + \frac{\sqrt{\frac{3}{x^4} - \frac{2\,6^{2/3}e^{-2c_1}}{\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}} + \frac{\sqrt[3]{6}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right.}}{x^6}}}{2\sqrt{3}} + \frac{1}{2}\sqrt{\frac{2}{x^4} + \frac{2\,2^{2/3}e^{-2c_1}}{\sqrt[3]{3}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}} - \frac{\sqrt[3]{2}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e\right.}}{3^{2/3}x^6}}$$

$$y(x) \to -\frac{1}{2x^2} - \frac{\sqrt{\frac{3}{x^4} - \frac{2\,6^{2/3}e^{-2c_1}}{\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}} + \frac{\sqrt[3]{6}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right.}}{x^6}}}{2\sqrt{3}} - \frac{1}{2}\sqrt{\frac{2}{x^4} + \frac{2\,2^{2/3}e^{-2c_1}}{\ldots} - \frac{\sqrt[3]{2}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e\right.}}{\ldots}}$$

8.2 problem Ex 2

Internal problem ID [11164]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 15. Page 22
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2(3y - 6y'x) - x(y - 2y'x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(y(x)^2*(3*y(x)-6*x*diff(y(x),x))-x*(y(x)-2*x*diff(y(x),x))=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{3}\,\sqrt{x}}{3}$$
$$y(x) = \frac{\sqrt{3}\,\sqrt{x}}{3}$$
$$y(x) = c_1\sqrt{x}$$

✓ Solution by Mathematica

Time used: 6.194 (sec). Leaf size: 74

```
DSolve[y[x]^2*(3*y[x]-6*x*y'[x])- x*(4*y[x]-2*x*y'[x])==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{i\sqrt{x}\sqrt{W\left(-3e^{-3c_1}x^3\right)}}{\sqrt{3}}$$

$$y(x) \to \frac{i\sqrt{x}\sqrt{W\left(-3e^{-3c_1}x^3\right)}}{\sqrt{3}}$$

$$y(x) \to 0$$

8.3 problem Ex 3

Internal problem ID [11165]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 15. Page 22
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{2x^3y - y^2 - (2x^4 + yx)\, y' = 0}$$

✓ Solution by Maple

Time used: 1.093 (sec). Leaf size: 49

```
dsolve((2*x^3*y(x)-y(x)^2)-(2*x^4+x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\left(\sqrt{4x^4 + c_1^2} + c_1\right)}{2x}$$
$$y(x) = \frac{c_1\left(2c_1 - 2\sqrt{4x^4 + c_1^2}\right)}{4x}$$

✓ Solution by Mathematica

Time used: 1.279 (sec). Leaf size: 76

```
DSolve[(2*x^3*y[x]-y[x]^2)-(2*x^4+x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^4}{-x + \frac{\sqrt{1+4c_1x^4}}{\sqrt{\frac{1}{x^2}}}}$$

$$y(x) \to -\frac{2x^4}{x + \frac{\sqrt{1+4c_1x^4}}{\sqrt{\frac{1}{x^2}}}}$$

$$y(x) \to 0$$

9 Chapter 2, differential equations of the first order and the first degree. Article 16. Integrating factors by inspection. Page 23

9.1 problem Ex 1

Internal problem ID [11166]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 16. Integrating factors by inspection. Page 23
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y^2 - yx + y'x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve((y(x)^2-x*y(x))+x^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{\ln(x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.227 (sec). Leaf size: 19

```
DSolve[(y[x]^2-x*y[x])+x^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{\log(x) + c_1}$$

$$y(x) \to 0$$

9.2 problem Ex 2

Internal problem ID [11167]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 16. Integrating factors by inspection. Page 23
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{\frac{y'x-y}{\sqrt{x^2-y^2}}-y'x=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve((x*diff(y(x),x)-y(x))/sqrt(x^2-y(x)^2)=x*diff(y(x),x),y(x), singsol=all)
```

$$y(x)-\arctan\left(\frac{y(x)}{\sqrt{x^2-y(x)^2}}\right)-c_1=0$$

✓ Solution by Mathematica

Time used: 0.896 (sec). Leaf size: 29

```
DSolve[(x*y'[x]-y[x])/Sqrt[x^2-y[x]^2]==x*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{\sqrt{x^2-y(x)^2}}{y(x)}\right)+y(x)=c_1,y(x)\right]$$

9.3 problem Ex 3

Internal problem ID [11168]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 16. Integrating factors by inspection. Page 23
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y-(x-y)\,y'=-x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve((x+y(x))-(x-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\tan\left(\text{RootOf}\left(-2\_Z+\ln\left(\frac{1}{\cos\left(\_Z\right)^{2}}\right)+2\ln\left(x\right)+2c_1\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 36

```
DSolve[(x+y[x])-(x-y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right)-\arctan\left(\frac{y(x)}{x}\right)=-\log(x)+c_1,y(x)\right]$$

9.4 problem Ex 4

Internal problem ID [11169]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 16. Integrating factors by inspection. Page 23
Problem number: Ex 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y^2 - 2y'xy = -x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve((x^2+y(x)^2)-2*x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{xc_1 + x^2}$$
$$y(x) = -\sqrt{xc_1 + x^2}$$

✓ Solution by Mathematica

Time used: 0.304 (sec). Leaf size: 38

```
DSolve[(x^2+y[x]^2)-2*x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{x + c_1}$$
$$y(x) \to \sqrt{x}\sqrt{x + c_1}$$

9.5 problem Ex 5

Internal problem ID [11170]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 16. Integrating factors by inspection. Page 23
Problem number: Ex 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{-y^2 + 2y'xy = -x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve((x-y(x)^2)+2*x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{-\ln(x)\, x + xc_1}$$
$$y(x) = -\sqrt{-\ln(x)\, x + xc_1}$$

✓ Solution by Mathematica

Time used: 0.298 (sec). Leaf size: 44

```
DSolve[(x-y[x]^2)+2*x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{-\log(x) + c_1}$$
$$y(x) \to \sqrt{x}\sqrt{-\log(x) + c_1}$$

9.6 problem Ex 6

Internal problem ID [11171]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 16. Integrating factors by inspection. Page 23
Problem number: Ex 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$\boxed{y'x - y - y^2 = x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve(x*diff(y(x),x)-y(x)=x^2+y(x)^2,y(x), singsol=all)
```

$$y(x) = \tan(c_1 + x)\, x$$

✓ Solution by Mathematica

Time used: 0.277 (sec). Leaf size: 12

```
DSolve[x*y'[x]-y[x]==x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \tan(x + c_1)$$

10 Chapter 2, differential equations of the first order and the first degree. Article 17. Other forms which Integrating factors can be found. Page 25

10.1 problem Ex 1

Internal problem ID [11172]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 17. Other forms which Integrating factors can be found. Page 25
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl`

$$\boxed{6yx + 3y^2 + \left(2x^2 + 3yx\right) y' = -3x^2}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 63

```
dsolve((3*x^2+6*x*y(x)+3*y(x)^2)+(2*x^2+3*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-\frac{2x^2c_1}{3} - \frac{\sqrt{-2c_1^2x^4+6}}{6}}{c_1x}$$
$$y(x) = \frac{-\frac{2x^2c_1}{3} + \frac{\sqrt{-2c_1^2x^4+6}}{6}}{c_1x}$$

✓ Solution by Mathematica

Time used: 2.7 (sec). Leaf size: 135

```
DSolve[(3*x^2+6*x*y[x]+3*y[x]^2)+(2*x^2+3*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to -\frac{4x^2 + \sqrt{-2x^4 + 6e^{4c_1}}}{6x}$$

$$y(x) \to \frac{-4x^2 + \sqrt{-2x^4 + 6e^{4c_1}}}{6x}$$

$$y(x) \to -\frac{\sqrt{2}\sqrt{-x^4} + 4x^2}{6x}$$

$$y(x) \to \frac{\sqrt{2}\sqrt{-x^4} - 4x^2}{6x}$$

10.2 problem Ex 2

Internal problem ID [11173]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 17. Other forms which Integrating factors can be found. Page 25
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(x^2 + y^2 + 2y)\, y' = -2x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve((2*x)+(x^2+y(x)^2+2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$x^2 e^{y(x)} + e^{y(x)} y(x)^2 + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.245 (sec). Leaf size: 24

```
DSolve[(2*x)+(x^2+y[x]^2+2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x^2 e^{y(x)} + e^{y(x)} y(x)^2 = c_1, y(x)\right]$$

10.3 problem Ex 3

Internal problem ID [11174]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 17. Other forms which Integrating factors can be found. Page 25
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{y^4 + 2y + (xy^3 + 2y^4 - 4x)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve((y(x)^4+2*y(x))+(x*y(x)^3+2*y(x)^4-4*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$x - \frac{(-y(x)^2 + c_1)\, y(x)^2}{y(x)^3 + 2} = 0$$

✓ Solution by Mathematica

Time used: 60.318 (sec). Leaf size: 2021

```
DSolve[(y[x]^4+2*y[x])+(x*y[x]^3+2*y[x]^4-4*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\frac{1}{2}\sqrt{\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{3\sqrt[3]{2}}+\frac{\cdots}{3\sqrt[3]{54x^3+\sqrt{(54x^3+\cdots}}\cdots}}$$
$$-\frac{1}{2}\sqrt{-\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{3\sqrt[3]{2}}-\frac{\cdots}{3\sqrt[3]{54x^3+\sqrt{(54x^3\cdots}}\cdots}}$$
$$-\frac{x}{4}$$

$$y(x) \to -\frac{1}{2}\sqrt{\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{3\sqrt[3]{2}}+\frac{\cdots}{3\sqrt[3]{54x^3+\sqrt{(54x^3+\cdots}}\cdots}}$$
$$+\frac{1}{2}\sqrt{-\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{3\sqrt[3]{2}}-\frac{\cdots}{3\sqrt[3]{54x^3+\sqrt{(54x^3\cdots}}\cdots}}$$
$$-\frac{x}{4}$$

$$y(x)$$
$$\to \frac{1}{2}\sqrt{\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{3\sqrt[3]{2}}+\frac{\cdots}{3\sqrt[3]{54x^3+\sqrt{(54x^3+14\cdots}}\cdots}}$$
$$-\frac{1}{2}\sqrt{-\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{3\sqrt[3]{2}}-\frac{\cdots}{3\sqrt[3]{54x^3+\sqrt{(54x^3\cdots}}\cdots}}$$
$$-\frac{x}{4}$$

$$y(x)$$

$$\to \frac{1}{2}\sqrt{\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{\cdots}+\cdots}$$

10.4 problem Ex 4

Internal problem ID [11175]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 17. Other forms which Integrating factors can be found. Page 25
Problem number: Ex 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^3y - y^4 + (xy^3 - x^4)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve((x^3*y(x)-y(x)^4)+(y(x)^3*x-x^4)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = x\left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)$$
$$y(x) = x\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)$$
$$y(x) = x$$
$$y(x) = xc_1$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 99

```
DSolve[(x^3*y[x]-y[x]^4)+(y[x]^3*x-x^4)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x$$

$$y(x) \to -\frac{1}{2}i\left(\sqrt{3}-i\right)x$$

$$y(x) \to \frac{1}{2}i\left(\sqrt{3}+i\right)x$$

$$y(x) \to c_1x$$

$$y(x) \to x$$

$$y(x) \to -\frac{1}{2}i\left(\sqrt{3}-i\right)x$$

$$y(x) \to \frac{1}{2}i\left(\sqrt{3}+i\right)x$$

10.5 problem Ex 6

Internal problem ID [11176]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 17. Other forms which Integrating factors can be found. Page 25
Problem number: Ex 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$y^2 + 2ymx + \left(y^2 m - m\,x^2 - 2yx\right) y' = x^2$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 59

```
dsolve((y(x)^2-x^2+2*m*x*y(x))+(m*y(x)^2-m*x^2-2*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{-m + \sqrt{-4c_1^2x^2 - 4xc_1 + m^2}}{2c_1}$$

$$y(x) = \frac{m + \sqrt{-4c_1^2x^2 - 4xc_1 + m^2}}{2c_1}$$

✓ Solution by Mathematica

Time used: 3.604 (sec). Leaf size: 89

```
DSolve[(y[x]^2-x^2+2*m*x*y[x])+(m*y[x]^2-m*x^2-2*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolu
```

$$y(x) \to \frac{1}{2}\left(-\sqrt{e^{2c_1}m^2 - 4x^2 + 4e^{c_1}x} - e^{c_1}m\right)$$

$$y(x) \to \frac{1}{2}\left(\sqrt{e^{2c_1}m^2 - 4x^2 + 4e^{c_1}x} - e^{c_1}m\right)$$

11 Chapter 2, differential equations of the first order and the first degree. Article 18. Transformation of variables. Page 26

11.1 problem Ex 1

Internal problem ID [11177]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 18. Transformation of variables. Page 26
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - y + 2yx^2 = x^3}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x)-y(x)+2*x^2*y(x)-x^3=0,y(x), singsol=all)
```

$$y(x) = \frac{x}{2} + \mathrm{e}^{-x^2} c_1 x$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 21

```
DSolve[x*y'[x]-y[x]+2*x^2*y[x]-x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x\left(\frac{1}{2} + c_1 e^{-x^2}\right)$$

11.2 problem Ex 2

Internal problem ID [11178]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 18. Transformation of variables. Page 26
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], [_Abel, ‘2nd type‘, ‘class C‘], _d`

$$\boxed{y'(x+y)=1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve((x+y(x))*diff(y(x),x)-1=0,y(x), singsol=all)
```

$$y(x)=-\operatorname{LambertW}\left(-c_1 \mathrm{e}^{-x-1}\right)-x-1$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 24

```
DSolve[(x+y[x])*y'[x]-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\rightarrow -W\left(c_1\left(-e^{-x-1}\right)\right)-x-1$$

11.3 problem Ex 3

Internal problem ID [11179]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 18. Transformation of variables. Page 26
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y'y - y'x + y = -x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x+y(x)*diff(y(x),x)+y(x)-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \tan\left(\operatorname{RootOf}\left(-2\_Z + \ln\left(\frac{1}{\cos(\_Z)^2}\right) + 2\ln(x) + 2c_1\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 36

```
DSolve[x+y[x]*y'[x]+y[x]-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right) - \arctan\left(\frac{y(x)}{x}\right) = -\log(x) + c_1, y(x)\right]$$

11.4 problem Ex 4

Internal problem ID [11180]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 18. Transformation of variables. Page 26
Problem number: Ex 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - ay + by^2 = c\,x^{2a}}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 42

```
dsolve(x*diff(y(x),x)-a*y(x)+b*y(x)^2=c*x^(2*a),y(x), singsol=all)
```

$$y(x) = -\frac{i \tan\left(\frac{i x^a \sqrt{b}\,\sqrt{c} - c_1 a}{a}\right)\sqrt{c}\,x^a}{\sqrt{b}}$$

✓ Solution by Mathematica

Time used: 0.533 (sec). Leaf size: 153

```
DSolve[x*y'[x]-a*y[x]+b*y[x]^2==c*x^(2*a),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{c}x^a\left(-\cos\left(\frac{\sqrt{-b}\sqrt{c}x^a}{a}\right) + c_1 \sin\left(\frac{\sqrt{-b}\sqrt{c}x^a}{a}\right)\right)}{\sqrt{-b}\left(\sin\left(\frac{\sqrt{-b}\sqrt{c}x^a}{a}\right) + c_1 \cos\left(\frac{\sqrt{-b}\sqrt{c}x^a}{a}\right)\right)}$$

$$y(x) \to \frac{\sqrt{c}x^a \tan\left(\frac{\sqrt{-b}\sqrt{c}x^a}{a}\right)}{\sqrt{-b}}$$

12 Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29

12.1 problem Ex 1

Internal problem ID [11181]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x\sqrt{1-y^2}+y\sqrt{-x^2+1}\,y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(x*sqrt(1-y(x)^2)+y(x)*sqrt(1-x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$\frac{(x-1)(x+1)}{\sqrt{-x^2+1}}+\frac{(y(x)-1)(y(x)+1)}{\sqrt{1-y(x)^2}}+c_1=0$$

✓ Solution by Mathematica

Time used: 3.778 (sec). Leaf size: 77

```
DSolve[x*Sqrt[1-y[x]^2]+y[x]*Sqrt[1-x^2]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\sqrt{x^2-c_1\left(2\sqrt{1-x^2}+c_1\right)}$$
$$y(x)\to\sqrt{x^2-c_1\left(2\sqrt{1-x^2}+c_1\right)}$$
$$y(x)\to-1$$
$$y(x)\to 1$$

12.2 problem Ex 2

Internal problem ID [11182]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\sqrt{1-y^2}+\sqrt{-x^2+1}\,y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(sqrt(1-y(x)^2)+sqrt(1-x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\sin(\arcsin(x)+c_1)$$

✓ Solution by Mathematica

Time used: 0.496 (sec). Leaf size: 47

```
DSolve[Sqrt[1-y[x]^2]+Sqrt[1-x^2]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos\left(2\arctan\left(\frac{\sqrt{1-x^2}}{x+1}\right)+c_1\right)$$
$$y(x) \to -1$$
$$y(x) \to 1$$
$$y(x) \to \text{Interval}[\{-1,1\}]$$

12.3 problem Ex 3

Internal problem ID [11183]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - yx^2 = x^5}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)-x^2*y(x)=x^5,y(x), singsol=all)
```

$$y(x) = -x^3 - 3 + \mathrm{e}^{\frac{x^3}{3}} c_1$$

✓ Solution by Mathematica

Time used: 0.111 (sec). Leaf size: 24

```
DSolve[y'[x]-x^2*y[x]==x^5,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^3 + c_1 e^{\frac{x^3}{3}} - 3$$

12.4 problem Ex 4

Internal problem ID [11184]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$\boxed{(y-x)^2 y' = 1}$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 29

```
dsolve((y(x)-x)^2*diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x)+\frac{\ln(y(x)-x-1)}{2}-\frac{\ln(y(x)-x+1)}{2}-c_1=0$$

✓ Solution by Mathematica

Time used: 0.23 (sec). Leaf size: 33

```
DSolve[(y[x]-x)^2*y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[y(x)+\frac{1}{2}\log(-y(x)+x+1)-\frac{1}{2}\log(y(x)-x+1)=c_1,y(x)\right]$$

12.5 problem Ex 5

Internal problem ID [11185]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y'x + y + e^{x} x^{4} y^{4} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 80

```
dsolve(x*diff(y(x),x)+y(x)+x^4*y(x)^4*exp(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\left(3\,e^{x} + c_1\right)^{\frac{1}{3}} x}$$

$$y(x) = \frac{-\frac{1}{2(3\,e^{x}+c_1)^{\frac{1}{3}}} - \frac{i\sqrt{3}}{2(3\,e^{x}+c_1)^{\frac{1}{3}}}}{x}$$

$$y(x) = \frac{-\frac{1}{2(3\,e^{x}+c_1)^{\frac{1}{3}}} + \frac{i\sqrt{3}}{2(3\,e^{x}+c_1)^{\frac{1}{3}}}}{x}$$

✓ Solution by Mathematica

Time used: 11.276 (sec). Leaf size: 79

```
DSolve[x*y'[x]+y[x]+x^4*y[x]^4*Exp[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\sqrt[3]{x^3 (3e^x + c_1)}}$$
$$y(x) \to -\frac{\sqrt[3]{-1}}{\sqrt[3]{x^3 (3e^x + c_1)}}$$
$$y(x) \to \frac{(-1)^{2/3}}{\sqrt[3]{x^3 (3e^x + c_1)}}$$
$$y(x) \to 0$$

12.6 problem Ex 6

Internal problem ID [11186]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1-x)\,y+(1-y)\,xy'=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve((1-x)*y(x)+(1-y(x))*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\operatorname{LambertW}\left(-\frac{c_1 e^x}{x}\right)$$

✓ Solution by Mathematica

Time used: 4.764 (sec). Leaf size: 26

```
DSolve[(1-x)*y[x]+(1-y[x])*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -W\left(-\frac{e^{x-c_1}}{x}\right)$$
$$y(x)\to 0$$

12.7 problem Ex 7

Internal problem ID [11187]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{(y - x)\, y' + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve((y(x)-x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{LambertW}\left(-x\,\mathrm{e}^{-c_1}\right)+c_1}$$

✓ Solution by Mathematica

Time used: 5.289 (sec). Leaf size: 25

```
DSolve[(y[x]-x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{W\left(-e^{-c_1}x\right)}$$

$$y(x) \to 0$$

12.8 problem Ex 8

Internal problem ID [11188]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y'x - y - \sqrt{y^2 + x^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x)-y(x)=sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$\frac{\sqrt{y(x)^2 + x^2}}{x^2} + \frac{y(x)}{x^2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.582 (sec). Leaf size: 27

```
DSolve[x*y'[x]-y[x]==Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-c_1} \left(-1 + e^{2c_1} x^2\right)$$

12.9 problem Ex 10

Internal problem ID [11189]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y'x - y - \sqrt{x^2 - y^2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x)-y(x)=sqrt(x^2-y(x)^2),y(x), singsol=all)
```

$$-\arctan\left(\frac{y(x)}{\sqrt{x^2 - y(x)^2}}\right) + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.395 (sec). Leaf size: 18

```
DSolve[x*y'[x]-y[x]==Sqrt[x^2-y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\cosh(i\log(x) + c_1)$$

12.10 problem Ex 11

Internal problem ID [11190]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{x \sin\left(\frac{y}{x}\right) - y \cos\left(\frac{y}{x}\right) + x \cos\left(\frac{y}{x}\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve((x*sin(y(x)/x)-y(x)*cos(y(x)/x))+x*cos(y(x)/x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = x \arcsin\left(\frac{1}{x c_1}\right)$$

✓ Solution by Mathematica

Time used: 15.438 (sec). Leaf size: 21

```
DSolve[(x*Sin[y[x]/x]-y[x]*Cos[y[x]/x])+x*Cos[y[x]/x]*y'[x]==0,y[x],x,IncludeSingularSolutio
```

$$y(x) \to x \arcsin\left(\frac{e^{c_1}}{x}\right)$$
$$y(x) \to 0$$

12.11 problem Ex 12

Internal problem ID [11191]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$-2y + (2x - y + 4)\, y' = -x - 5$$

✓ Solution by Maple

Time used: 1.281 (sec). Leaf size: 182

```
dsolve((x-2*y(x)+5)+(2*x-y(x)+4)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 2 - \frac{(x+1)\left(c_1^2\left(-\frac{\left(3\sqrt{3}\sqrt{27c_1^2(x+1)^2-1}+27c_1(x+1)\right)^{\frac{1}{3}}}{6c_1(x+1)} - \frac{1}{2c_1(x+1)\left(3\sqrt{3}\sqrt{27c_1^2(x+1)^2-1}+27c_1(x+1)\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(3\sqrt{3}\sqrt{27c_1^2(\cdots}{\cdots}\right.}{\cdots}\right.\right.}{c_1^2}$$

✓ Solution by Mathematica

Time used: 60.282 (sec). Leaf size: 1601

```
DSolve[(x-2*y[x]+5)+(2*x-y[x]+4)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

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12.12 problem Ex 13

Internal problem ID [11192]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' + \frac{y}{(-x^2+1)^{\frac{3}{2}}} = \frac{x + \sqrt{-x^2+1}}{(-x^2+1)^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
dsolve(diff(y(x),x)+y(x)/(1-x^2)^(3/2)= (x+(1-x^2)^(1/2))/(1-x^2)^2,y(x), singsol=all)
```

$$y(x) = \left(\int \frac{e^{\frac{x}{\sqrt{-x^2+1}}}\left(x+\sqrt{-x^2+1}\right)}{(x-1)^2(x+1)^2}dx + c_1\right) e^{\frac{(x-1)(x+1)x}{(-x^2+1)^{\frac{3}{2}}}}$$

✓ Solution by Mathematica

Time used: 0.358 (sec). Leaf size: 38

```
DSolve[y'[x]+y[x]/(1-x^2)^(3/2)== (x+(1-x^2)^(1/2))/(1-x^2)^2,y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{x}{\sqrt{1-x^2}} + c_1 e^{-\frac{x}{\sqrt{1-x^2}}}$$

12.13 problem Ex 14

Internal problem ID [11193]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(-x^2+1)\, y' - yx - y^2 ax = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve((1-x^2)*diff(y(x),x)-x*y(x)=a*x*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{x-1}\,\sqrt{x+1}\,c_1 - a}$$

✓ Solution by Mathematica

Time used: 4.13 (sec). Leaf size: 47

```
DSolve[(1-x^2)*y'[x]-x*y[x]==a*x*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{c_1}}{-\sqrt{1-x^2} + ae^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{1}{a}$$

12.14 problem Ex 15

Internal problem ID [11194]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{xy^2(3y+y'x)-2y+y'x=0}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 45

```
dsolve((x*y(x)^2)*(3*y(x)+x*diff(y(x),x))-(2*y(x)-x*diff(y(x),x))=0,y(x), singsol=all)
```

$$y(x)=\frac{c_1+\sqrt{4x^5+c_1^2}}{2x^3}$$
$$y(x)=-\frac{-c_1+\sqrt{4x^5+c_1^2}}{2x^3}$$

✓ Solution by Mathematica

Time used: 1.836 (sec). Leaf size: 75

```
DSolve[(x*y[x]^2)*(3*y[x]+x*y'[x])-(2*y[x]-x*y'[x])==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x)\to -\frac{\sqrt{4x^5+e^{5c_1}}+e^{\frac{5c_1}{2}}}{2x^3}$$
$$y(x)\to \frac{\sqrt{4x^5+e^{5c_1}}-e^{\frac{5c_1}{2}}}{2x^3}$$

12.15 problem Ex 16

Internal problem ID [11195]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\,y'+y=\arctan(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1+x^2)*diff(y(x),x)+y(x)=arctan(x),y(x), singsol=all)
```

$$y(x)=\arctan(x)-1+\mathrm{e}^{-\arctan(x)}c_1$$

✓ Solution by Mathematica

Time used: 0.23 (sec). Leaf size: 18

```
DSolve[(1+x^2)*y'[x]+y[x]==ArcTan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\arctan(x)+c_1e^{-\arctan(x)}-1$$

12.16 problem Ex 17

Internal problem ID [11196]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$5yx - 3y^3 + (3x^2 - 7y^2x)\, y' = 0$$

✓ Solution by Maple

Time used: 2.516 (sec). Leaf size: 52

```
dsolve((5*x*y(x)-3*y(x)^3)+(3*x^2-7*x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{RootOf}\left(x\_Z^7 - \_Z^3 x^2 - \frac{c_1}{\sqrt{x}}\right)^2$$

$$y(x) = \mathrm{RootOf}\left(x\_Z^7 - \_Z^3 x^2 + \frac{c_1}{\sqrt{x}}\right)^2$$

✓ Solution by Mathematica

Time used: 7.756 (sec). Leaf size: 288

```
DSolve[(5*x*y[x]-3*y[x]^3)+(3*x^2-7*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - c_1{}^2\&, 1\right]$$
$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - c_1{}^2\&, 2\right]$$
$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - c_1{}^2\&, 3\right]$$
$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - c_1{}^2\&, 4\right]$$
$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - c_1{}^2\&, 5\right]$$
$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - c_1{}^2\&, 6\right]$$
$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - c_1{}^2\&, 7\right]$$

12.17 problem Ex 18

Internal problem ID [11197]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + y\cos(x) = \frac{\sin(2x)}{2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+y(x)*cos(x)=1/2*sin(2*x),y(x), singsol=all)
```

$$y(x) = \sin(x) - 1 + e^{-\sin(x)}c_1$$

✓ Solution by Mathematica

Time used: 0.089 (sec). Leaf size: 18

```
DSolve[y'[x]+y[x]*Cos[x]==1/2*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + c_1 e^{-\sin(x)} - 1$$

12.18 problem Ex 19

Internal problem ID [11198]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{y^2x + y - y'x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((x*y(x)^2+y(x))-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2x}{-x^2 + 2c_1}$$

✓ Solution by Mathematica

Time used: 0.207 (sec). Leaf size: 23

```
DSolve[(x*y[x]^2+y[x])-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2x}{x^2 - 2c_1}$$
$$y(x) \to 0$$

12.19 problem Ex 20

Internal problem ID [11199]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1-x)\,y-(y+1)\,xy'=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 14

```
dsolve((1-x)*y(x)-(1+y(x))*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{LambertW}\left(\frac{\mathrm{e}^{-x}x}{c_1}\right)$$

✓ Solution by Mathematica

Time used: 5.134 (sec). Leaf size: 21

```
DSolve[(1-x)*y[x]-(1+y[x])*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to W\left(xe^{-x+c_1}\right)$$
$$y(x) \to 0$$

12.20 problem Ex 21

Internal problem ID [11200]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{3yx^2 + (x^3 + x^3y^2)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(3*x^2*y(x)+(x^3+x^3*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{\frac{1}{\operatorname{LambertW}\left(\frac{c_1}{x^6}\right)}}}$$

✓ Solution by Mathematica

Time used: 6.245 (sec). Leaf size: 46

```
DSolve[3*x^2*y[x]+(x^3+x^3*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{W\left(\frac{e^{2c_1}}{x^6}\right)}$$

$$y(x) \to \sqrt{W\left(\frac{e^{2c_1}}{x^6}\right)}$$

$$y(x) \to 0$$

12.21 problem Ex 22

Internal problem ID [11201]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$(x^2+y^2)(x+yy')-(x^2+y^2+x)(y'x-y)=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
dsolve((x^2+y(x)^2)*(x+y(x)*diff(y(x),x))=(x^2+y(x)^2+x)*(x*diff(y(x),x)-y(x)),y(x), singsol
```

$$y(x) = -\frac{x}{\tan\left(\operatorname{RootOf}\left(-2\_Z+2\ln\left(\frac{x\left(2x\tan(\_Z)^2+\tan(\_Z)^2+2x+\tan(\_Z)\right)}{\tan(\_Z)^2}\right)-\ln\left(\frac{x^2\left(\tan(\_Z)^2+1\right)}{\tan(\_Z)^2}\right)+2c_1\right.\right.}$$

✓ Solution by Mathematica

Time used: 0.548 (sec). Leaf size: 53

```
DSolve[(x^2+y[x]^2)*(x+y[x]*y'[x])==(x^2+y[x]^2+x)*(x*y'[x]-y[x]),y[x],x,IncludeSingularSolu
```

$$\text{Solve}\left[\frac{1}{2}\arctan\left(\frac{x}{y(x)}\right)-\frac{1}{4}\log\left(x^2+y(x)^2\right)+\frac{1}{2}\log\left(2x^2+2y(x)^2-y(x)+x\right)=c_1,y(x)\right]$$

12.22 problem Ex 23

Internal problem ID [11202]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{3y + (2x + 3y - 5)\, y' = -2x + 1}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 21

```
dsolve((2*x+3*y(x)-1)+(2*x+3*y(x)-5)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{2x}{3} - 4\,\mathrm{LambertW}\left(-\frac{\mathrm{e}^{\frac{x}{12}} c_1 \mathrm{e}^{-\frac{7}{12}}}{12}\right) - \frac{7}{3}$$

✓ Solution by Mathematica

Time used: 5.457 (sec). Leaf size: 43

```
DSolve[(2*x+3*y[x]-1)+(2*x+3*y[x]-5)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -4W\left(-e^{\frac{x}{12}-1+c_1}\right) - \frac{2x}{3} - \frac{7}{3}$$
$$y(x) \to \frac{1}{3}(-2x - 7)$$

12.23 problem Ex 24

Internal problem ID [11203]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y^3 - 2yx^2 + (2y^2x - x^3)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 223

```
dsolve((y(x)^3-2*x^2*y(x))+(2*x*y(x)^2-x^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-xc_1 - \frac{-2c_1^2x^2 + \sqrt{2x^4c_1^4 - 2c_1\sqrt{c_1^6x^6 + 4x}\,x}}{2xc_1}}{c_1}$$

$$y(x) = \frac{-xc_1 + \frac{2c_1^2x^2 + \sqrt{2x^4c_1^4 - 2c_1\sqrt{c_1^6x^6 + 4x}\,x}}{2xc_1}}{c_1}$$

$$y(x) = \frac{-xc_1 + \frac{2c_1^2x^2 - \sqrt{2x^4c_1^4 + 2c_1\sqrt{c_1^6x^6 + 4x}\,x}}{2xc_1}}{c_1}$$

$$y(x) = \frac{-xc_1 + \frac{2c_1^2x^2 + \sqrt{2x^4c_1^4 + 2c_1\sqrt{c_1^6x^6 + 4x}\,x}}{2xc_1}}{c_1}$$

✓ Solution by Mathematica

Time used: 15.638 (sec). Leaf size: 277

```
DSolve[(y[x]^3-2*x^2*y[x])+(2*x*y[x]^2-x^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\frac{\sqrt{x^2 - \frac{\sqrt{x^6-4e^{2c_1}}}{x}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{x^2 - \frac{\sqrt{x^6-4e^{2c_1}}}{x}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\frac{x^3+\sqrt{x^6-4e^{2c_1}}}{x}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\frac{x^3+\sqrt{x^6-4e^{2c_1}}}{x}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{x^2 - \frac{\sqrt{x^6}}{x}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{x^2 - \frac{\sqrt{x^6}}{x}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\frac{\sqrt{x^6}+x^3}{x}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\frac{\sqrt{x^6}+x^3}{x}}}{\sqrt{2}}$$

12.24 problem Ex 25

Internal problem ID [11204]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{2x^3y^2 - y + \left(2y^3x^2 - x\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 522

```
dsolve((2*x^3*y(x)^2-y(x))+(2*x^2*y(x)^3-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}{6x} - \frac{6\left(\frac{x^2}{3}-\frac{c_1}{3}\right)x}{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}{12x} + \frac{3\left(\frac{x^2}{3}-\frac{c_1}{3}\right)x}{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}{6x} + \frac{6\left(\frac{x^2}{3}-\frac{c_1}{3}\right)x}{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}{12x} + \frac{3\left(\frac{x^2}{3}-\frac{c_1}{3}\right)x}{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}{6x} + \frac{6\left(\frac{x^2}{3}-\frac{c_1}{3}\right)x}{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 46.278 (sec). Leaf size: 358

```
DSolve[(2*x^3*y[x]^2-y[x])+(2*x^2*y[x]^3-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{\sqrt[3]{2}(-x^3+c_1x)}{\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3(x^3-c_1x)^3}}} + \frac{\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3(x^3-c_1x)^3}}}{3\sqrt[3]{2}x}$$

$$y(x) \to \frac{(1+i\sqrt{3})(x^3-c_1x)}{2^{2/3}\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3(x^3-c_1x)^3}}} - \frac{(1-i\sqrt{3})\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3(x^3-c_1x)^3}}}{6\sqrt[3]{2}x}$$

$$y(x) \to \frac{(1-i\sqrt{3})(x^3-c_1x)}{2^{2/3}\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3(x^3-c_1x)^3}}} - \frac{(1+i\sqrt{3})\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3(x^3-c_1x)^3}}}{6\sqrt[3]{2}x}$$

12.25 problem Ex 26

Internal problem ID [11205]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$(y^2+x^2)(x+y'y)+\sqrt{1+x^2+y^2}\,(-y'x+y)=0$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 27

```
dsolve((x^2+y(x)^2)*(x+y(x)*diff(y(x),x))+(1+x^2+y(x)^2)^(1/2)*(y(x)-x*diff(y(x),x))=0,y(x),
```

$$\arctan\left(\frac{y(x)}{x}\right)-\sqrt{x^2+y(x)^2+1}-c_1=0$$

✓ Solution by Mathematica

Time used: 0.454 (sec). Leaf size: 27

```
DSolve[(x^2+y[x]^2)*(x+y[x]*y'[x])+(1+x^2+y[x]^2)^(1/2)*(y[x]-x*y'[x])==0,y[x],x,IncludeSing
```

$$\text{Solve}\left[\arctan\left(\frac{x}{y(x)}\right)+\sqrt{x^2+y(x)^2+1}=c_1,y(x)\right]$$

12.26 problem Ex 27

Internal problem ID [11206]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\mathrm{e}^{\frac{y}{x}} + \mathrm{e}^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) y' = -1$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve((1+exp(y(x)/x))+exp(x/y(x))*(1-x/y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{RootOf}\left(\int^{\_Z} \frac{\mathrm{e}^{\frac{1}{\_a}}(\_a - 1)}{\_a\left(\_a\,\mathrm{e}^{\frac{1}{\_a}} - \mathrm{e}^{\frac{1}{\_a}} + \mathrm{e}^{\_a} + 1\right)} d\_a + \ln(x) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.879 (sec). Leaf size: 54

```
DSolve[(1+Exp[y[x]/x])+Exp(x/y[x])*(1-x/y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$\mathrm{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{K[1] - 1}{K[1]\left(K[1]\mathrm{Exp} - \mathrm{Exp} + e^{K[1]}K[1] + K[1]\right)} dK[1] = -\frac{\log(x)}{\mathrm{Exp}} + c_1, y(x)\right]$$

12.27 problem Ex 28

Internal problem ID [11207]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y'x + y - y^2 \ln(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x)+y(x)-y(x)^2*ln(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{1 + xc_1 + \ln(x)}$$

✓ Solution by Mathematica

Time used: 0.233 (sec). Leaf size: 20

```
DSolve[x*y'[x]+y[x]-y[x]^2*Log[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\log(x) + c_1x + 1}$$

$$y(x) \to 0$$

12.28 problem Ex 29

Internal problem ID [11208]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$
\boxed{x^3y^4 + y^3x^2 + y^2x + y + \left(y^3x^4 - x^3y^2 - x^3y + x\right) y' = 0}
$$

✗ Solution by Maple

```
dsolve((x^3*y(x)^4+x^2*y(x)^3+x*y(x)^2+y(x))+(x^4*y(x)^3-x^3*y(x)^2-x^3*y(x)+x)*diff(y(x),x)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x^3*y[x]^4+x^2*y[x]^3+x*y[x]^2+y[x])+(x^4*y[x]^3-x^3*y[x]^2-x^3*y[x]+x)*y'[x]==0,y[x
```

Not solved

12.29 problem Ex 30

Internal problem ID [11209]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter 2, differential equations of the first order and the first degree. Article 19. Summary. Page 29
Problem number: Ex 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{(2\sqrt{yx} - x)\, y' + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve((2*sqrt(x*y(x))-x)*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$\ln(y(x)) + \frac{x}{\sqrt{xy(x)}} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.376 (sec). Leaf size: 33

```
DSolve[(2*Sqrt[x*y[x]]-x)*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2}{\sqrt{\frac{y(x)}{x}}} + 2\log\left(\frac{y(x)}{x}\right) = -2\log(x) + c_1, y(x)\right]$$

13 Chapter IV, differential equations of the first order and higher degree than the first. Article 24. Equations solvable for p. Page 49

13.1 problem Ex 1

Internal problem ID [11210]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 24. Equations solvable for p. Page 49
Problem number: Ex 1.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 + y'(x + y) + yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)^2+(x+y(x))*diff(y(x),x)+x*y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{2} + c_1$$
$$y(x) = \mathrm{e}^{-x} c_1$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 32

```
DSolve[(y'[x])^2+(x+y[x])*y'[x]+x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x}$$
$$y(x) \to -\frac{x^2}{2} + c_1$$
$$y(x) \to 0$$

13.2 problem Ex 2

Internal problem ID [11211]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 24. Equations solvable for p. Page 49
Problem number: Ex 2.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{x{y'}^2 - 2y'y = x}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)-x=0,y(x), singsol=all)
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = -\frac{\left(-\frac{x^2}{c_1^2}+1\right)c_1}{2}$$

✓ Solution by Mathematica

Time used: 0.225 (sec). Leaf size: 71

```
DSolve[x*(y'[x])^2-2*y[x]*y'[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-c_1}\left(-x^2+e^{2c_1}\right)$$
$$y(x) \to \frac{1}{2}e^{-c_1}\left(-1+e^{2c_1}x^2\right)$$
$$y(x) \to -ix$$
$$y(x) \to ix$$

13.3 problem Ex 3

Internal problem ID [11212]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 24. Equations solvable for p. Page 49
Problem number: Ex 3.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type **[_quadrature]**

$$\boxed{y'^2 + y^2 = 1}$$

✓ Solution by Maple

Time used: 0.375 (sec). Leaf size: 29

```
dsolve(y(x)^2+diff(y(x),x)^2=1,y(x), singsol=all)
```

$$y(x) = -1$$
$$y(x) = 1$$
$$y(x) = -\sin(-x + c_1)$$
$$y(x) = \sin(-x + c_1)$$

✓ Solution by Mathematica

Time used: 0.211 (sec). Leaf size: 39

```
DSolve[y[x]^2+(y'[x])^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x + c_1)$$
$$y(x) \to \cos(x - c_1)$$
$$y(x) \to -1$$
$$y(x) \to 1$$
$$y(x) \to \text{Interval}[\{-1, 1\}]$$

13.4 problem Ex 4

Internal problem ID [11213]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 24. Equations solvable for *p*. Page 49
Problem number: Ex 4.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_linear]`

$$\boxed{(2y'x - y)^2 = 8x^3}$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 30

```
dsolve((2*x*diff(y(x),x)-y(x))^2=8*x^3,y(x), singsol=all)
```

$$y(x) = \left(-\sqrt{2}\,x + c_1\right)\sqrt{x}$$
$$y(x) = \left(\sqrt{2}\,x + c_1\right)\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.128 (sec). Leaf size: 42

```
DSolve[(2*x*y'[x]-y[x])^2==8*x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}\left(-\sqrt{2}x + c_1\right)$$
$$y(x) \to \sqrt{x}\left(\sqrt{2}x + c_1\right)$$

13.5 problem Ex 5

Internal problem ID [11214]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 24. Equations solvable for p. Page 49
Problem number: Ex 5.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(x^2+1)\,{y'}^2=1}$$

✓ Solution by Maple

Time used: 0.36 (sec). Leaf size: 17

```
dsolve((1+x^2)*diff(y(x),x)^2=1,y(x), singsol=all)
```

$$y(x)=\operatorname{arcsinh}(x)+c_1$$
$$y(x)=-\operatorname{arcsinh}(x)+c_1$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 45

```
DSolve[(1+x^2)*(y'[x])^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\log\left(\sqrt{x^2+1}-x\right)+c_1$$
$$y(x)\to \log\left(\sqrt{x^2+1}-x\right)+c_1$$

13.6 problem Ex 6

Internal problem ID [11215]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 24. Equations solvable for p. Page 49
Problem number: Ex 6.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 - (2x + y^2)\, y'^2 + (x^2 - y^2 + 2y^2 x)\, y' - (x^2 - y^2)\, y^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x)^3-(2*x+y(x)^2)*diff(y(x),x)^2+(x^2-y(x)^2+2*x*y(x)^2)*diff(y(x),x)-(x^2-
```

$$y(x) = \frac{1}{-x + c_1}$$
$$y(x) = -x - 1 + c_1 e^x$$
$$y(x) = x - 1 + e^{-x} c_1$$

✓ Solution by Mathematica

Time used: 0.276 (sec). Leaf size: 48

```
DSolve[(y'[x])^3-(2*x+y[x]^2)*(y'[x])^2+(x^2-y[x]^2+2*x*y[x]^2)*y'[x]-(x^2-y[x]^2)*y[x]^2==0
```

$$y(x) \to -\frac{1}{x + c_1}$$
$$y(x) \to x + c_1 e^{-x} - 1$$
$$y(x) \to -x + c_1 e^x - 1$$
$$y(x) \to 0$$

14 Chapter IV, differential equations of the first order and higher degree than the first. Article 25. Equations solvable for y. Page 52

14.1 problem Ex 1

Internal problem ID [11216]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 25. Equations solvable for y. Page 52
Problem number: Ex 1.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{2y'x - y + \ln(y') = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 61

```
dsolve(2*diff(y(x),x)*x-y(x)+ln(diff(y(x),x))=0,y(x), singsol=all)
```

$$y(x) = -1 + \sqrt{4xc_1 + 1} + \ln\left(\frac{-1 + \sqrt{4xc_1 + 1}}{2x}\right)$$
$$y(x) = -1 - \sqrt{4xc_1 + 1} + \ln\left(-\frac{1 + \sqrt{4xc_1 + 1}}{2x}\right)$$

✓ Solution by Mathematica

Time used: 0.157 (sec). Leaf size: 32

```
DSolve[2*y'[x]*x-y[x]+Log[y'[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[W\left(2xe^{y(x)}\right) - \log\left(W\left(2xe^{y(x)}\right) + 2\right) - y(x) = c_1, y(x)\right]$$

14.2 problem Ex 2

Internal problem ID [11217]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 25. Equations solvable for y. Page 52
Problem number: Ex 2.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{4xy'^2 + 2y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 51

```
dsolve(4*x*diff(y(x),x)^2+2*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{4}$$
$$y(x) = \left(\frac{4c_1}{x} + \frac{2\sqrt{xc_1}}{x}\right)x$$
$$y(x) = \left(\frac{4c_1}{x} - \frac{2\sqrt{xc_1}}{x}\right)x$$

✓ Solution by Mathematica

Time used: 0.196 (sec). Leaf size: 72

```
DSolve[4*x*(y'[x])^2+2*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{2c_1}\left(-2\sqrt{x} + e^{2c_1}\right)$$
$$y(x) \to \frac{1}{4}e^{-4c_1}\left(1 + 2e^{2c_1}\sqrt{x}\right)$$
$$y(x) \to 0$$
$$y(x) \to -\frac{x}{4}$$

14.3 problem Ex 3

Internal problem ID [11218]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 25. Equations solvable for y. Page 52
Problem number: Ex 3.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{x{y'}^2 - 2y'y = x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)-x=0,y(x), singsol=all)
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = -\frac{\left(-\frac{x^2}{c_1^2} + 1\right) c_1}{2}$$

✓ Solution by Mathematica

Time used: 0.186 (sec). Leaf size: 71

```
DSolve[x*(y'[x])^2-2*y[x]*y'[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-c_1}\left(-x^2 + e^{2c_1}\right)$$
$$y(x) \to \frac{1}{2}e^{-c_1}\left(-1 + e^{2c_1}x^2\right)$$
$$y(x) \to -ix$$
$$y(x) \to ix$$

14.4 problem Ex 4

Internal problem ID [11219]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 25. Equations solvable for y. Page 52
Problem number: Ex 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' + 2yx - y^2 = x^2}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 34

```
dsolve(diff(y(x),x)+2*x*y(x)=x^2+y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x\,\mathrm{e}^{2x}c_1 - \mathrm{e}^{2x}c_1 - x - 1}{-1 + \mathrm{e}^{2x}c_1}$$

✓ Solution by Mathematica

Time used: 0.208 (sec). Leaf size: 29

```
DSolve[y'[x]+2*x*y[x]==x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \frac{1}{\frac{1}{2} + c_1 e^{2x}} - 1$$
$$y(x) \to x - 1$$

14.5 problem Ex 5

Internal problem ID [11220]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 25. Equations solvable for y. Page 52
Problem number: Ex 5.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{y + y'x - x^4 y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 135

```
dsolve(y(x)=-x*diff(y(x),x)+x^4*diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{1}{4x^2}$$
$$y(x) = \frac{-c_1(2ix - c_1) - c_1^2 - 2x^2}{2c_1^2x^2}$$
$$y(x) = \frac{-c_1(-2ix - c_1) - c_1^2 - 2x^2}{2c_1^2x^2}$$
$$y(x) = \frac{c_1(2ix + c_1) - 2x^2 - c_1^2}{2c_1^2x^2}$$
$$y(x) = \frac{c_1(-2ix + c_1) - 2x^2 - c_1^2}{2c_1^2x^2}$$

✓ Solution by Mathematica

Time used: 0.809 (sec). Leaf size: 123

```
DSolve[y[x]==-x*y'[x]+x^4*(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x\sqrt{4x^2y(x)+1}\operatorname{arctanh}\left(\sqrt{4x^2y(x)+1}\right)}{\sqrt{4x^4y(x)+x^2}}-\frac{1}{2}\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{x\sqrt{4x^2y(x)+1}\operatorname{arctanh}\left(\sqrt{4x^2y(x)+1}\right)}{\sqrt{4x^4y(x)+x^2}}-\frac{1}{2}\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

14.6 problem Ex 6

Internal problem ID [11221]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 25. Equations solvable for y. Page 52
Problem number: Ex 6.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{{y'}^2 + 2y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 690

```
dsolve(diff(y(x),x)^2+2*x*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \left(\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{x^2}{2\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{x}{2}\right)^2 + 2x\left(\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{x^2}{2\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{x}{2}\right)$$

$$y(x) = \left(-\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x^2}{4\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{x}{2} - \frac{i\sqrt{3}\left(\frac{\left(6c_1-x^3+2\sqrt{-3c_1 \cdots}\right.}{2} \cdots\right.}{\cdots}\right.$$

$$+ 2x\left(-\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x^2}{4\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{x}{2} - \frac{i\sqrt{3}\left(\frac{\left(6c_1-x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{2} - \frac{x^2}{2\left(6c_1-x^3+2\sqrt{-3c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}\right)}{2}\right)$$

$$y(x) = \left(-\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x^2}{4\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{x}{2} + \frac{i\sqrt{3}\left(\frac{\left(6c_1-x^3+2\sqrt{-3c_1 \cdots}\right.}{2} \cdots\right.}{\cdots}\right.$$

✓ Solution by Mathematica

Time used: 60.154 (sec). Leaf size: 931

```
DSolve[(y'[x])^2+2*x*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-x^2 + \frac{x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}} + \sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2 - \frac{9i(\sqrt{3}-i)\,x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}} + 9i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2 + \frac{9i(\sqrt{3}+i)\,x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}} - 9\left(1+i\sqrt{3}\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{4}\left(-x^2 + \frac{x(x^3-8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}} + \sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2 + \frac{9(1+i\sqrt{3})\,x(-x^3+8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}} + 9i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2 + \frac{9i(\sqrt{3}+i)\,x(x^3-8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}} - 9\left(1+i\sqrt{3}\right)\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

15 Chapter IV, differential equations of the first order and higher degree than the first. Article 26. Equations solvable for x. Page 55

15.1 problem Ex 1

Internal problem ID [11222]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 26. Equations solvable for x. Page 55
Problem number: Ex 1.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y'y\left(2y'^2+3\right)=-x}$$

✓ Solution by Maple

Time used: 0.532 (sec). Leaf size: 776

```
dsolve(x+diff(y(x),x)*y(x)*(2*diff(y(x),x)^2+3)=0,y(x), singsol=all)
```

$$y(x) = -\frac{i\sqrt{2}\,x}{2}$$

$$y(x) = \frac{i\sqrt{2}\,x}{2}$$

$$y(x) = \operatorname{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{-2\left(\frac{\left(\_a^2-\sqrt{2\_a^2+1}+1\right)\_a}{(2\_a^2+1)^{\frac{3}{2}}}\right)^{\frac{2}{3}}\_a^2 + 2\left(\frac{\left(\_a^2-\sqrt{2\_a^2+1}+1\right)\_a}{(2\_a^2+1)^{\frac{3}{2}}}\right)^{\frac{1}{3}}\_a^3 - \left(\frac{\left(\_a^2-\sqrt{2\_a^2+1}+1\right)\_a}{(2\_a^2+1)^{\frac{3}{2}}}\right)^{\frac{2}{3}} + \_ \cdots}{\left(\frac{\left(\_a^2-\sqrt{2\_a^2+1}+1\right)\_a}{(2\_a^2+1)^{\frac{3}{2}}}\right)^{\frac{1}{3}}(2\_a^4+3\_a^2+1)} + c_1\right) x$$

$$y(x) = \operatorname{RootOf}\left(-2\ln(x) + \int^{\_Z} \frac{2i\left(\frac{\left(\_a^2-\sqrt{2\_a^2+1}+1\right)\_a}{(2\_a^2+1)^{\frac{3}{2}}}\right)^{\frac{2}{3}}\sqrt{3}\,\_a^2 + i\left(\frac{\left(\_a^2-\sqrt{2\_a^2+1}+1\right)\_a}{(2\_a^2+1)^{\frac{3}{2}}}\right)^{\frac{2}{3}}\sqrt{3} + i\sqrt{3}\,\_a^2 - 2\left(\frac{\left(\_a^2-\sqrt{2 \cdots}\right)}{(2\_ \cdots}\right. \cdots}{\left(\frac{\left(\_a^2-\sqrt{\cdots}\right)}{(2\_ \cdots}\right. \cdots} + 2c_1\right) x$$

$$y(x) = \operatorname{RootOf}\left(-2\ln(x) \cdots\right.$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x+y'[x]*y[x]*(2*(y'[x])^2+3)==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

15.2 problem Ex 2

Internal problem ID [11223]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 26. Equations solvable for x. Page 55
Problem number: Ex 2.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{a^2 y y'^2 - 2xy' + y = 0}$$

✓ Solution by Maple

Time used: 0.515 (sec). Leaf size: 65

```
dsolve(a^2*y(x)*diff(y(x),x)^2-2*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{a}$$
$$y(x) = \frac{x}{a}$$
$$y(x) = 0$$
$$y(x) = e^{\operatorname{RootOf}\left(\tanh(-\_Z+c_1-\ln(x))^2 e^{2\_Z} a^2 - \tanh(-\_Z+c_1-\ln(x))^2+1\right)} x$$

✓ Solution by Mathematica

Time used: 30.099 (sec). Leaf size: 244

```
DSolve[a^2*y[x]*(y'[x])^2-2*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\left(\cosh\left(\frac{a^2c_1}{2}\right)+\sinh\left(\frac{a^2c_1}{2}\right)\right)\sqrt{\cosh(a^2c_1)+\sinh(a^2c_1)-8ix}}{4a}$$
$$y(x) \to \frac{\left(\cosh\left(\frac{a^2c_1}{2}\right)+\sinh\left(\frac{a^2c_1}{2}\right)\right)\sqrt{\cosh(a^2c_1)+\sinh(a^2c_1)-8ix}}{4a}$$
$$y(x) \to -\frac{\left(\cosh\left(\frac{a^2c_1}{2}\right)+\sinh\left(\frac{a^2c_1}{2}\right)\right)\sqrt{\cosh(a^2c_1)+\sinh(a^2c_1)+8ix}}{4a}$$
$$y(x) \to \frac{\left(\cosh\left(\frac{a^2c_1}{2}\right)+\sinh\left(\frac{a^2c_1}{2}\right)\right)\sqrt{\cosh(a^2c_1)+\sinh(a^2c_1)+8ix}}{4a}$$
$$y(x) \to -\frac{x}{a}$$
$$y(x) \to \frac{x}{a}$$

15.3 problem Ex 3

Internal problem ID [11224]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 26. Equations solvable for x. Page 55
Problem number: Ex 3.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy'^2 - 2yy' = x}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)-x=0,y(x), singsol=all)
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = -\frac{\left(-\frac{x^2}{c_1^2}+1\right)c_1}{2}$$

✓ Solution by Mathematica

Time used: 0.213 (sec). Leaf size: 71

```
DSolve[x*(y'[x])^2-2*y[x]*y'[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-c_1}\left(-x^2 + e^{2c_1}\right)$$
$$y(x) \to \frac{1}{2}e^{-c_1}\left(-1 + e^{2c_1}x^2\right)$$
$$y(x) \to -ix$$
$$y(x) \to ix$$

15.4 problem Ex 4

Internal problem ID [11225]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 26. Equations solvable for x. Page 55
Problem number: Ex 4.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y'^3 - 4xyy' + 8y^2 = 0}$$

✓ Solution by Maple

Time used: 0.406 (sec). Leaf size: 36

```
dsolve(diff(y(x),x)^3-4*x*y(x)*diff(y(x),x)+8*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{4x^3}{27}$$
$$y(x) = 0$$
$$y(x) = \frac{x^2}{4c_1} - \frac{x}{8c_1^2} + \frac{1}{64c_1^3}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^3-4*x*y[x]*y'[x]+8*y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

16 Chapter IV, differential equations of the first order and higher degree than the first. Article 27. Clairaut equation. Page 56

16.1 problem Ex 1

Internal problem ID [11226]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 27. Clairaut equation. Page 56
Problem number: Ex 1.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$\boxed{(xy' - y)^2 - y'^2 = 1}$$

✓ Solution by Maple

Time used: 0.39 (sec). Leaf size: 57

```
dsolve((diff(y(x),x)*x-y(x))^2=diff(y(x),x)^2+1,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= \sqrt{-x^2 + 1} \\
y(x) &= -\sqrt{-x^2 + 1} \\
y(x) &= xc_1 - \sqrt{c_1^2 + 1} \\
y(x) &= xc_1 + \sqrt{c_1^2 + 1}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.192 (sec). Leaf size: 73

```
DSolve[(y'[x]*x-y[x])^2==(y'[x])^2+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \sqrt{1 + c_1^2}$$
$$y(x) \to c_1 x + \sqrt{1 + c_1^2}$$
$$y(x) \to -\sqrt{1 - x^2}$$
$$y(x) \to \sqrt{1 - x^2}$$

16.2 problem Ex 2

Internal problem ID [11227]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 27. Clairaut equation. Page 56
Problem number: Ex 2.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{4\,\mathrm{e}^{2y}{y'}^{2}+2y'x=1}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 21

```
dsolve(4*exp(2*y(x))*diff(y(x),x)^2+2*x*diff(y(x),x)-1=0,y(x), singsol=all)
```

$$y(x)=-\frac{\ln\left(\frac{1}{4\,\mathrm{e}^{2c_1}+2x}\right)}{2}+c_1$$

✓ Solution by Mathematica

Time used: 12.616 (sec). Leaf size: 119

```
DSolve[4*Exp[2*y[x]]*(y'[x])^2+2*x*y'[x]-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(-e^{\frac{c_1}{2}}\sqrt{-x+e^{c_1}}\right)$$

$$y(x) \to \log\left(e^{\frac{c_1}{2}}\sqrt{-x+e^{c_1}}\right)$$

$$y(x) \to \log\left(-e^{\frac{c_1}{2}}\sqrt{x+e^{c_1}}\right)$$

$$y(x) \to \log\left(e^{\frac{c_1}{2}}\sqrt{x+e^{c_1}}\right)$$

$$y(x) \to \frac{1}{2}\log\left(-\frac{x^2}{4}\right)$$

16.3 problem Ex 3

Internal problem ID [11228]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 27. Clairaut equation. Page 56
Problem number: Ex 3.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$4\,\mathrm{e}^{2y}{y'}^{2}+2\,\mathrm{e}^{2x}y'=\mathrm{e}^{2x}$$

✓ Solution by Maple

Time used: 2.141 (sec). Leaf size: 121

```
dsolve(4*exp(2*y(x))*diff(y(x),x)^2+2*exp(2*x)*diff(y(x),x)-exp(2*x)=0,y(x), singsol=all)
```

$$y(x)=\operatorname{arctanh}\left(\operatorname{RootOf}\left(-1+\left(\mathrm{e}^{4}+4\,\mathrm{e}^{\operatorname{RootOf}\left(\tanh\left(-\frac{\_Z}{2}+2+c_1-x\right)^{2}\mathrm{e}^{4}+4\tanh\left(-\frac{\_Z}{2}+2+c_1-x\right)^{2}\mathrm{e}^{-\_Z}-\mathrm{e}^{4}\right)}\right)\_Z^{2}\right)\mathrm{e}^{2}\right)+c_1$$

$$y(x)=-\operatorname{arctanh}\left(\operatorname{RootOf}\left(-1+\left(\mathrm{e}^{4}+4\,\mathrm{e}^{\operatorname{RootOf}\left(\tanh\left(-\frac{\_Z}{2}+2+c_1-x\right)^{2}\mathrm{e}^{4}+4\tanh\left(-\frac{\_Z}{2}+2+c_1-x\right)^{2}\mathrm{e}^{-\_Z}-\mathrm{e}^{4}\right)}\right)\_Z^{2}\right)\mathrm{e}^{2}\right)+c_1$$

✓ Solution by Mathematica

Time used: 2.772 (sec). Leaf size: 332

```
DSolve[4*Exp[2*y[x]]*(y'[x])^2+2*Exp[2*x]*y'[x]-Exp[2*x]==0,y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[-\frac{2e^{-x}\sqrt{4e^{2(y(x)+x)}+e^{4x}}\operatorname{arctanh}\left(\frac{-\sqrt{4e^{2y(x)}+e^{2x}}+e^x+1}{\sqrt{4e^{2y(x)}+e^{2x}}-e^x+1}\right)}{\sqrt{4e^{2y(x)}+e^{2x}}} - \frac{e^{-x}\sqrt{4e^{2(y(x)+x)}+e^{4x}}y(x)}{\sqrt{4e^{2y(x)}+e^{2x}}}+y(x)=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{2e^{-x}\sqrt{4e^{2(y(x)+x)}+e^{4x}}\operatorname{arctanh}\left(\frac{-\sqrt{4e^{2y(x)}+e^{2x}}+e^x+1}{\sqrt{4e^{2y(x)}+e^{2x}}-e^x+1}\right)}{\sqrt{4e^{2y(x)}+e^{2x}}} + \frac{e^{-x}\sqrt{4e^{2(y(x)+x)}+e^{4x}}y(x)}{\sqrt{4e^{2y(x)}+e^{2x}}}+y(x)=c_1,y(x)\right]$$

$$y(x)\to\frac{1}{2}\left(\log\left(-\frac{e^{4x}}{4}\right)-2x\right)$$

16.4 problem Ex 4

Internal problem ID [11229]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 27. Clairaut equation. Page 56
Problem number: Ex 4.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `` [`y=_G(x,y')`] ``

$$\boxed{e^{2y}{y'}^3 + \left(e^{2x} + e^{3x}\right) y' = e^{3x}}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 31

```
dsolve(exp(2*y(x))*diff(y(x),x)^3+(exp(2*x)+exp(3*x))*diff(y(x),x)-exp(3*x)=0,y(x), singsol=
```

$$y(x) = \frac{\ln\left(-(c_1+1)\left(e^{-2x}c_1^2 - 2\,e^{-x}c_1 + 1\right)\right)}{2} + x$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[Exp[2*y[x]]*(y'[x])^3+(Exp[2*x]+Exp[3*x])*y'[x]-Exp[3*x]==0,y[x],x,IncludeSingularSol
```

Timed out

16.5 problem Ex 5

Internal problem ID [11230]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 27. Clairaut equation. Page 56
Problem number: Ex 5.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{xy^2y'^2 - y^3y' = -x}$$

✓ Solution by Maple

Time used: 0.547 (sec). Leaf size: 141

```
dsolve(x*y(x)^2*diff(y(x),x)^2-y(x)^3*diff(y(x),x)+x=0,y(x), singsol=all)
```

$$y(x) = \sqrt{-2x}$$
$$y(x) = -\sqrt{-2x}$$
$$y(x) = \sqrt{x}\,\sqrt{2}$$
$$y(x) = -\sqrt{x}\,\sqrt{2}$$
$$y(x) = \mathrm{e}^{\frac{c_1}{2}+\frac{\mathrm{RootOf}\left(16x\,\mathrm{e}^{2c_1}\,\mathrm{e}^{2\_Z}+\mathrm{e}^{2\_Z}x^3-4\,\mathrm{e}^{2c_1}\,\mathrm{e}^{3\_Z}\right)}{2}-\frac{\ln(x)}{2}}$$
$$y(x) = \mathrm{e}^{-\frac{c_1}{2}+\frac{\mathrm{RootOf}\left(x^2\left(16\,\mathrm{e}^{-2c_1}\,\mathrm{e}^{2\_Z}x^2-4\,\mathrm{e}^{-2c_1}\,\mathrm{e}^{3\_Z}x+\mathrm{e}^{2\_Z}\right)\right)}{2}+\frac{\ln(x)}{2}}$$

✓ Solution by Mathematica

Time used: 6.367 (sec). Leaf size: 187

```
DSolve[x*y[x]^2*(y'[x])^2-y[x]^3*y'[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-2e^{-c_1}x^2 - \frac{e^{c_1}}{2}}$$
$$y(x) \to \sqrt{-2e^{-c_1}x^2 - \frac{e^{c_1}}{2}}$$
$$y(x) \to -\frac{\sqrt{4e^{-c_1}x^2 + e^{c_1}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{4e^{-c_1}x^2 + e^{c_1}}}{\sqrt{2}}$$
$$y(x) \to -\sqrt{2}\sqrt{x}$$
$$y(x) \to -i\sqrt{2}\sqrt{x}$$
$$y(x) \to i\sqrt{2}\sqrt{x}$$
$$y(x) \to \sqrt{2}\sqrt{x}$$

16.6 problem Ex 6

Internal problem ID [11231]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 27. Clairaut equation. Page 56
Problem number: Ex 6.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\left(y^2+x^2\right)\left(1+y'\right)^2-2(x+y)\left(1+y'\right)\left(x+yy'\right)+\left(x+yy'\right)^2=0$$

✓ Solution by Maple

Time used: 0.547 (sec). Leaf size: 106

```
dsolve((x^2+y(x)^2)*(1+diff(y(x),x))^2-2*(x+y(x))*(1+diff(y(x),x))*(x+y(x)*diff(y(x),x))+(x+
```

$$y(x)=0$$

$$y(x)=\operatorname{RootOf}\left(-2\ln(x)-\left(\int^{\_Z}\frac{2\_a^2+\sqrt{2\_a^3-4\_a^2+2\_a}}{\_a\left(\_a^2+1\right)}d\_a\right)+2c_1\right)x$$

$$y(x)=\operatorname{RootOf}\left(-2\ln(x)+\int^{\_Z}\frac{\sqrt{2}\sqrt{\_a\left(\_a-1\right)^2-2\_a^2}}{\_a\left(\_a^2+1\right)}d\_a+2c_1\right)x$$

✓ Solution by Mathematica

Time used: 7.379 (sec). Leaf size: 167

```
DSolve[(x^2+y[x]^2)*(1+y'[x])^2-2*(x+y[x])*(1+y'[x])*(x+y[x]*y'[x])+(x+y[x]*y'[x])^2==0,y[x]
```

$$y(x) \to -\sqrt{-x\left(x+2e^{\frac{c_1}{2}}\right)}-e^{\frac{c_1}{2}}$$

$$y(x) \to \sqrt{-x\left(x+2e^{\frac{c_1}{2}}\right)}-e^{\frac{c_1}{2}}$$

$$y(x) \to e^{\frac{c_1}{2}}-\sqrt{x\left(-x+2e^{\frac{c_1}{2}}\right)}$$

$$y(x) \to \sqrt{x\left(-x+2e^{\frac{c_1}{2}}\right)}+e^{\frac{c_1}{2}}$$

$$y(x) \to -\sqrt{-x^2}$$

$$y(x) \to \sqrt{-x^2}$$

16.7 problem Ex 7

Internal problem ID [11232]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 27. Clairaut equation. Page 56
Problem number: Ex 7.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y - 2xy' - y^2 {y'}^3 = 0$$

✓ Solution by Maple

Time used: 0.39 (sec). Leaf size: 107

```
dsolve(y(x)=2*diff(y(x),x)*x+y(x)^2*diff(y(x),x)^3,y(x), singsol=all)
```

$$y(x) = -\frac{2\,2^{\frac{1}{4}} 3^{\frac{1}{4}} (-x^3)^{\frac{1}{4}}}{3}$$

$$y(x) = \frac{2\,2^{\frac{1}{4}} 3^{\frac{1}{4}} (-x^3)^{\frac{1}{4}}}{3}$$

$$y(x) = -\frac{2i 2^{\frac{1}{4}} 3^{\frac{1}{4}} (-x^3)^{\frac{1}{4}}}{3}$$

$$y(x) = \frac{2i 2^{\frac{1}{4}} 3^{\frac{1}{4}} (-x^3)^{\frac{1}{4}}}{3}$$

$$y(x) = 0$$

$$y(x) = \sqrt{c_1^3 + 2xc_1}$$

$$y(x) = -\sqrt{c_1^3 + 2xc_1}$$

✓ Solution by Mathematica

Time used: 0.183 (sec). Leaf size: 119

```
DSolve[y[x]==2*y'[x]*x+y[x]^2*(y'[x])^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2c_1x + c_1{}^3}$$
$$y(x) \to \sqrt{2c_1x + c_1{}^3}$$
$$y(x) \to (-1-i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$
$$y(x) \to (1-i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$
$$y(x) \to (-1+i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$
$$y(x) \to (1+i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$

16.8 problem Ex 8

Internal problem ID [11233]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 27. Clairaut equation. Page 56
Problem number: Ex 8.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{a^2 y {y'}^2 - 2xy' + y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 65

```
dsolve(a^2*y(x)*diff(y(x),x)^2-2*x*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{a}$$
$$y(x) = \frac{x}{a}$$
$$y(x) = 0$$
$$y(x) = e^{\operatorname{RootOf}\left(\tanh(-\_Z + c_1 - \ln(x))^2 e^{2\_Z} a^2 - \tanh(-\_Z + c_1 - \ln(x))^2 + 1\right)} x$$

✓ Solution by Mathematica

Time used: 31.661 (sec). Leaf size: 244

```
DSolve[a^2*y[x]*(y'[x])^2-2*x*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\left(\cosh\left(\frac{a^2c_1}{2}\right)+\sinh\left(\frac{a^2c_1}{2}\right)\right)\sqrt{\cosh\left(a^2c_1\right)+\sinh\left(a^2c_1\right)-8ix}}{4a}$$
$$y(x) \to \frac{\left(\cosh\left(\frac{a^2c_1}{2}\right)+\sinh\left(\frac{a^2c_1}{2}\right)\right)\sqrt{\cosh\left(a^2c_1\right)+\sinh\left(a^2c_1\right)-8ix}}{4a}$$
$$y(x) \to -\frac{\left(\cosh\left(\frac{a^2c_1}{2}\right)+\sinh\left(\frac{a^2c_1}{2}\right)\right)\sqrt{\cosh\left(a^2c_1\right)+\sinh\left(a^2c_1\right)+8ix}}{4a}$$
$$y(x) \to \frac{\left(\cosh\left(\frac{a^2c_1}{2}\right)+\sinh\left(\frac{a^2c_1}{2}\right)\right)\sqrt{\cosh\left(a^2c_1\right)+\sinh\left(a^2c_1\right)+8ix}}{4a}$$
$$y(x) \to -\frac{x}{a}$$
$$y(x) \to \frac{x}{a}$$

16.9 problem Ex 9

Internal problem ID [11234]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 27. Clairaut equation. Page 56
Problem number: Ex 9.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$(x - y' - y)^2 - x^2(2yx - x^2y') = 0$$

✗ Solution by Maple

```
dsolve((x-diff(y(x),x)-y(x))^2=x^2*(2*x*y(x)-x^2*diff(y(x),x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x-y'[x]-y[x])^2==x^2*(2*x*y[x]-x^2*y'[x]),y[x],x,IncludeSingularSolutions -> True]
```

Not solved

17 Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59

17.1 problem Ex 1

Internal problem ID [11235]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 1.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y^2\left(y'^2+1\right)=a^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(y(x)^2*(1+diff(y(x),x)^2)=a^2,y(x), singsol=all)
```

$$y(x) = -a$$
$$y(x) = a$$
$$y(x) = \sqrt{a^2-c_1^2+2xc_1-x^2}$$
$$y(x) = -\sqrt{a^2-c_1^2+2xc_1-x^2}$$

✓ Solution by Mathematica

Time used: 0.344 (sec). Leaf size: 101

```
DSolve[y[x]^2*(1+(y'[x])^2)==a^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{a^2 - (x + c_1)^2}$$
$$y(x) \to \sqrt{a^2 - (x + c_1)^2}$$
$$y(x) \to -\sqrt{a^2 - (x - c_1)^2}$$
$$y(x) \to \sqrt{a^2 - (x - c_1)^2}$$
$$y(x) \to -a$$
$$y(x) \to a$$

17.2 problem Ex 2

Internal problem ID [11236]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 2.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{yy' - (x - b)\, y'^2 = a}$$

✓ Solution by Maple

Time used: 0.312 (sec). Leaf size: 50

```
dsolve(y(x)*diff(y(x),x)=(x-b)*diff(y(x),x)^2+a,y(x), singsol=all)
```

$$y(x) = -2\sqrt{-ba + ax}$$
$$y(x) = 2\sqrt{-ba + ax}$$
$$y(x) = xc_1 + \frac{-b\, c_1^2 + a}{c_1}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y(x)*y'[x]==(x-b)*(y'[x])^2+a,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

17.3 problem Ex 3

Internal problem ID [11237]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 3.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{x^3 {y'}^2 + x^2 y y' = -1}$$

✓ Solution by Maple

Time used: 0.422 (sec). Leaf size: 53

```
dsolve(x^3*diff(y(x),x)^2+x^2*y(x)*diff(y(x),x)+1=0,y(x), singsol=all)
```

$$y(x) = -\frac{2}{\sqrt{x}}$$
$$y(x) = \frac{2}{\sqrt{x}}$$
$$y(x) = \frac{c_1^2 x + 4}{2 x c_1}$$
$$y(x) = \frac{c_1^2 + 4x}{2 x c_1}$$

✓ Solution by Mathematica

Time used: 0.934 (sec). Leaf size: 77

```
DSolve[x^3*(y'[x])^2+x^2*y[x]*y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{-\frac{c_1}{2}}(x + 16e^{c_1})}{4x}$$
$$y(x) \to \frac{e^{-\frac{c_1}{2}}(x + 16e^{c_1})}{4x}$$
$$y(x) \to -\frac{2}{\sqrt{x}}$$
$$y(x) \to \frac{2}{\sqrt{x}}$$

17.4 problem Ex 4

Internal problem ID [11238]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 4.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$3xy'^2 - 6yy' + 2y = -x$$

✓ Solution by Maple

Time used: 0.265 (sec). Leaf size: 40

```
dsolve(3*x*diff(y(x),x)^2-6*y(x)*diff(y(x),x)+x+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = x$$
$$y(x) = -\frac{x}{3}$$
$$y(x) = \frac{\left(-\frac{(c_1+x)^2}{3c_1^2} - 1\right)x}{-\frac{2(c_1+x)}{c_1} + 2}$$

✓ Solution by Mathematica

Time used: 0.505 (sec). Leaf size: 67

```
DSolve[3*x*(y'[x])^2-6*y[x]*y'[x]+x+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{3}x\left(-1+2\cosh\left(-\log(x)+\sqrt{3}c_1\right)\right)$$

$$y(x) \to -\frac{1}{3}x\left(-1+2\cosh\left(\log(x)+\sqrt{3}c_1\right)\right)$$

$$y(x) \to -\frac{x}{3}$$

$$y(x) \to x$$

17.5 problem Ex 5

Internal problem ID [11239]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 5.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, _dAlembert]`

$$\boxed{y - y'^2(x+1) = 0}$$

✓ Solution by Maple

Time used: 0.36 (sec). Leaf size: 99

```
dsolve(y(x)=diff(y(x),x)^2*(x+1),y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \frac{x\left(x+1+\sqrt{xc_1+c_1+x+1}\right)^2}{(x+1)^2} + \frac{\left(x+1+\sqrt{xc_1+c_1+x+1}\right)^2}{(x+1)^2}$$

$$y(x) = \frac{x\left(-x-1+\sqrt{xc_1+c_1+x+1}\right)^2}{(x+1)^2} + \frac{\left(-x-1+\sqrt{xc_1+c_1+x+1}\right)^2}{(x+1)^2}$$

✓ Solution by Mathematica

Time used: 0.1 (sec). Leaf size: 57

```
DSolve[y[x]==(y'[x])^2*(x+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - c_1\sqrt{x+1} + 1 + \frac{c_1^2}{4}$$
$$y(x) \to x + c_1\sqrt{x+1} + 1 + \frac{c_1^2}{4}$$
$$y(x) \to 0$$

17.6 problem Ex 6

Internal problem ID [11240]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 6.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$\boxed{(xy' - y)(yy' + x) - a^2y' = 0}$$

✗ Solution by Maple

```
dsolve((diff(y(x),x)*x-y(x))*(diff(y(x),x)*y(x)+x)=a^2*diff(y(x),x),y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 0.6 (sec). Leaf size: 75

```
DSolve[(y'[x]*x-y[x])*(y'[x]*y[x]+x)==a^2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{c_1 \left(x^2 - \frac{a^2}{1 + c_1}\right)}$$
$$y(x) \to -i(a - x)$$
$$y(x) \to i(a - x)$$
$$y(x) \to -i(a + x)$$
$$y(x) \to i(a + x)$$

17.7 problem Ex 7

Internal problem ID [11241]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 7.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$y'^2 + 2y'y\cot(x) - y^2 = 0$$

✓ Solution by Maple

Time used: 0.234 (sec). Leaf size: 61

```
dsolve(diff(y(x),x)^2+2*diff(y(x),x)*y(x)*cot(x)=y(x)^2,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \frac{c_1(\tan(x)^2 + 1)\sqrt{\frac{\tan(x)^2}{\tan(x)^2+1}}}{\left(1 + \sqrt{\tan(x)^2 + 1}\right)\tan(x)}$$

$$y(x) = \frac{c_1 \mathrm{e}^{\operatorname{arctanh}\left(\frac{1}{\sqrt{\tan(x)^2+1}}\right)}\sqrt{\tan(x)^2 + 1}}{\tan(x)}$$

✓ Solution by Mathematica

Time used: 0.241 (sec). Leaf size: 36

```
DSolve[(y'[x])^2+2*y'[x]*y[x]*Cot[x]==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \csc^2\left(\frac{x}{2}\right)$$
$$y(x) \to c_1 \sec^2\left(\frac{x}{2}\right)$$
$$y(x) \to 0$$

17.8 problem Ex 8

Internal problem ID [11242]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 8.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$\boxed{(x^2+1)\,y'^2 - 2xyy' + y^2 = 1}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 57

```
dsolve((1+x^2)*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)+y(x)^2-1=0,y(x), singsol=all)
```

$$y(x) = \sqrt{x^2+1}$$
$$y(x) = -\sqrt{x^2+1}$$
$$y(x) = xc_1 - \sqrt{-c_1^2+1}$$
$$y(x) = xc_1 + \sqrt{-c_1^2+1}$$

✓ Solution by Mathematica

Time used: 0.168 (sec). Leaf size: 73

```
DSolve[(1+x^2)*(y'[x])^2-2*x*y[x]*y'[x]+y[x]^2-1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \sqrt{1 - c_1^2}$$
$$y(x) \to c_1 x + \sqrt{1 - c_1^2}$$
$$y(x) \to -\sqrt{x^2 + 1}$$
$$y(x) \to \sqrt{x^2 + 1}$$

17.9 problem Ex 9

Internal problem ID [11243]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 9.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'^2x^2 - 2(yx + 2y')\,y' + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)^2-2*(x*y(x)+2*diff(y(x),x))*diff(y(x),x)+y(x)^2=0,y(x), singsol=all)
```

$$y(x) = c_1(x - 2)$$
$$y(x) = c_1(x + 2)$$

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 26

```
DSolve[x^2*(y'[x])^2-2*(x*y[x]+2*y'[x])*y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to c_1(x - 2)$$
$$y(x) \to c_1(x + 2)$$
$$y(x) \to 0$$

17.10 problem Ex 10

Internal problem ID [11244]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 10.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y - xy' - \frac{y{y'}^2}{x^2} = 0$$

✓ Solution by Maple

Time used: 0.484 (sec). Leaf size: 91

```
dsolve(y(x)=x*diff(y(x),x)+y(x)*diff(y(x),x)^2/x^2,y(x), singsol=all)
```

$$y(x) = -\frac{i x^2}{2}$$
$$y(x) = \frac{i x^2}{2}$$
$$y(x) = 0$$
$$y(x) = -\frac{\sqrt{-4x^2 c_1 + c_1^2}}{4}$$
$$y(x) = \frac{\sqrt{-4x^2 c_1 + c_1^2}}{4}$$
$$y(x) = -\frac{2\sqrt{x^2 c_1 + 4}}{c_1}$$
$$y(x) = \frac{2\sqrt{x^2 c_1 + 4}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.986 (sec). Leaf size: 244

```
DSolve[y[x]==x*y'[x]+y[x]*(y'[x])^2/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt{x^6+4x^2y(x)^2}\log\left(\sqrt{x^4+4y(x)^2}+x^2\right)}{2x\sqrt{x^4+4y(x)^2}}+\frac{1}{2}\left(1-\frac{\sqrt{x^6+4x^2y(x)^2}}{x\sqrt{x^4+4y(x)^2}}\right)\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{2}\left(\frac{\sqrt{x^6+4x^2y(x)^2}}{x\sqrt{x^4+4y(x)^2}}+1\right)\log(y(x))-\frac{\sqrt{x^6+4x^2y(x)^2}\log\left(\sqrt{x^4+4y(x)^2}+x^2\right)}{2x\sqrt{x^4+4y(x)^2}}=c_1,y(x)\right]$$

$$y(x)\to-\frac{ix^2}{2}$$

$$y(x)\to\frac{ix^2}{2}$$

17.11 problem Ex 11

Internal problem ID [11245]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IV, differential equations of the first order and higher degree than the first. Article 28. Summary. Page 59
Problem number: Ex 11.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{x^2 {y'}^2 - 2xyy' + y^2 - y^2 x^2 = x^4}$$

✓ Solution by Maple

Time used: 0.656 (sec). Leaf size: 59

```
dsolve(x^2*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)+y(x)^2=x^2*y(x)^2+x^4,y(x), singsol=all)
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = -\frac{x\left(\frac{e^{2x}}{c_1^2} - 1\right) e^{-x} c_1}{2}$$
$$y(x) = \frac{x(e^{2x} c_1^2 - 1) e^{-x}}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.366 (sec). Leaf size: 60

```
DSolve[x^2*(y'[x])^2-2*x*y[x]*y'[x]+y[x]^2==x^2*y[x]^2+x^4,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{2} x e^{-x-c_1} \left(-1+e^{2(x+c_1)}\right)$$
$$y(x) \to \frac{1}{2} \left(x e^{-x+c_1} - x e^{x-c_1}\right)$$

18 Chapter V, Singular solutions. Article 30. Page 63

18.1 problem Ex 1

Internal problem ID [11246]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter V, Singular solutions. Article 30. Page 63
Problem number: Ex 1.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Clairaut]`

$$\boxed{y - xy' - \frac{1}{y'} = 0}$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 27

```
dsolve(y(x)=diff(y(x),x)*x+1/diff(y(x),x),y(x), singsol=all)
```

$$y(x) = -2\sqrt{x}$$
$$y(x) = 2\sqrt{x}$$
$$y(x) = xc_1 + \frac{1}{c_1}$$

✓ Solution by Mathematica

Time used: 0.016 (sec). Leaf size: 41

```
DSolve[y[x]==y'[x]*x+1/y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x + \frac{1}{c_1}$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -2\sqrt{x}$$
$$y(x) \to 2\sqrt{x}$$

18.2 problem Ex 2

Internal problem ID [11247]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter V, Singular solutions. Article 30. Page 63
Problem number: Ex 2.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy'^2 - 2yy' = x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)-x=0,y(x), singsol=all)
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = -\frac{\left(-\frac{x^2}{c_1^2} + 1\right) c_1}{2}$$

✓ Solution by Mathematica

Time used: 0.213 (sec). Leaf size: 71

```
DSolve[x*(y'[x])^2-2*y[x]*y'[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-c_1} \left(-x^2 + e^{2c_1}\right)$$
$$y(x) \to \frac{1}{2} e^{-c_1} \left(-1 + e^{2c_1} x^2\right)$$
$$y(x) \to -ix$$
$$y(x) \to ix$$

19 Chapter V, Singular solutions. Article 32. Page 69

19.1 problem Ex 5

Internal problem ID [11248]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter V, Singular solutions. Article 32. Page 69
Problem number: Ex 5.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _Clairaut]`

$$\boxed{{y'}^2 x^2 - 2(yx - 2)\, y' + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 35

```
dsolve(x^2*diff(y(x),x)^2-2*(x*y(x)-2)*diff(y(x),x)+y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{x}$$
$$y(x) = xc_1 - 2\sqrt{-c_1}$$
$$y(x) = xc_1 + 2\sqrt{-c_1}$$

✓ Solution by Mathematica

Time used: 0.416 (sec). Leaf size: 43

```
DSolve[x^2*(y'[x])^2-2*(x*y[x]-2)*y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4(-x + c_1)}{c_1^2}$$

$$y(x) \to -\frac{4(x + c_1)}{c_1^2}$$

$$y(x) \to 0$$

$$y(x) \to \frac{1}{x}$$

20 Chapter V, Singular solutions. Article 33. Page 73

20.1 problem Ex 1

Internal problem ID [11249]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter V, Singular solutions. Article 33. Page 73
Problem number: Ex 1.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 x^2 = (x-1)^2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x)^2-(x-1)^2=0,y(x), singsol=all)
```

$$y(x) = x - \ln(x) + c_1$$
$$y(x) = -x + \ln(x) + c_1$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 25

```
DSolve[x^2*(y'[x])^2-(x-1)^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - \log(x) + c_1$$
$$y(x) \to -x + \log(x) + c_1$$

20.2 problem Ex 2

Internal problem ID [11250]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter V, Singular solutions. Article 33. Page 73
Problem number: Ex 2.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$8(1+y')^3 - 27(x+y)(1-y')^3 = 0$$

✓ Solution by Maple

Time used: 0.719 (sec). Leaf size: 132

```
dsolve(8*(1+diff(y(x),x))^3=27*(x+y(x))*(1-diff(y(x),x))^3,y(x), singsol=all)
```

$$y(x) = -x$$

$$\frac{x}{2} - \frac{4\ln(27y(x)+27x+8)}{27} + \frac{4\ln\left(9(x+y(x))^{\frac{2}{3}} - 6(x+y(x))^{\frac{1}{3}} + 4\right)}{27} + \frac{4\ln\left(2+3(x+y(x))^{\frac{1}{3}}\right)}{27} - \frac{y(x)}{2} - \frac{(x+y(x))^{\frac{2}{3}}}{2} - c_1 = 0$$

$$\frac{x}{2} - \frac{y(x)}{2} - \frac{\left(i\sqrt{3}-1\right)(x+y(x))^{\frac{2}{3}}}{4} - c_1 = 0$$

$$\frac{x}{2} - \frac{y(x)}{2} + \frac{\left(1+i\sqrt{3}\right)(x+y(x))^{\frac{2}{3}}}{4} - c_1 = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[8*(1+y'[x])^3==27*(x+y[x])*(1-y'[x])^3,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

20.3 problem Ex 3

Internal problem ID [11251]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter V, Singular solutions. Article 33. Page 73
Problem number: Ex 3.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{4{y'}^2 = 9x}$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 19

```
dsolve(4*diff(y(x),x)^2=9*x,y(x), singsol=all)
```

$$y(x) = -x^{\frac{3}{2}} + c_1$$
$$y(x) = x^{\frac{3}{2}} + c_1$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 27

```
DSolve[4*y'[x]^2==9*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^{3/2} + c_1$$
$$y(x) \to x^{3/2} + c_1$$

20.4 problem Ex 4

Internal problem ID [11252]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter V, Singular solutions. Article 33. Page 73
Problem number: Ex 4.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y(3-4y)^2 {y'}^2+4y=4}$$

✓ Solution by Maple

Time used: 0.281 (sec). Leaf size: 58

```
dsolve(y(x)*(3-4*y(x))^2*diff(y(x),x)^2=4*(1-y(x)),y(x), singsol=all)
```

$$y(x)=1$$

$$x+\frac{y(x)^2\,(y(x)-1)}{\sqrt{-y\,(x)\,(y\,(x)-1)}}-c_1=0$$

$$x-\frac{y(x)^2\,(y(x)-1)}{\sqrt{-y\,(x)\,(y\,(x)-1)}}-c_1=0$$

✓ Solution by Mathematica

Time used: 60.436 (sec). Leaf size: 3751

```
DSolve[y[x]*(3-4*y[x])^2*y'[x]^2==4*(1-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

21 Chapter VII, Linear differential equations with constant coefficients. Article 43. Page 92

21.1 problem Ex 1

Internal problem ID [11253]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 43. Page 92
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 3y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2x} c_1 + c_2 \mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 18

```
DSolve[y''[x]-3*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x (c_2 e^x + c_1)$$

21.2 problem Ex 2

Internal problem ID [11254]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 43. Page 92
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{y'' - 6y' + 25y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-6*diff(y(x),x)+25*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{3x} \sin(4x) + c_2 e^{3x} \cos(4x)$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 26

```
DSolve[y''[x]-6*y'[x]+25*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{3x}(c_2 \cos(4x) + c_1 \sin(4x))$$

21.3 problem Ex 3

Internal problem ID [11255]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 43. Page 92
Problem number: Ex 3.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$3)-diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{-x} + c_3 e^{x}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 23

```
DSolve[y'''[x]-y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} - c_2 e^{-x} + c_3$$

21.4 problem Ex 4

Internal problem ID [11256]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 43. Page 92
Problem number: Ex 4.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 2y'' - y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)-diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}c_1 + c_2e^{2x} + c_3e^{x}$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 28

```
DSolve[y'''[x]-2*y''[x]-y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1e^{-x} + c_2e^{x} + c_3e^{2x}$$

22 Chapter VII, Linear differential equations with constant coefficients. Article 44. Roots of auxiliary equation repeated. Page 94

22.1 problem Ex 1

Internal problem ID [11257]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 44. Roots of auxiliary equation repeated. Page 94
Problem number: Ex 1.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{4y''' - 3y' + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(4*diff(y(x),x$3)-3*diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = e^{-x}c_1 + c_2e^{\frac{x}{2}} + c_3e^{\frac{x}{2}}x$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 29

```
DSolve[4*y'''[x]-3*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(e^{3x/2}(c_2x + c_1) + c_3\right)$$

22.2 problem Ex 2

Internal problem ID [11258]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 44. Roots of auxiliary equation repeated. Page 94
Problem number: Ex 2.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y'' - y' + y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)-diff(y(x),x)+y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x} c_1 + c_2 \mathrm{e}^{x} + c_3 x\, \mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 25

```
DSolve[y'''[x]-y''[x]-y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x} + e^{x}(c_3 x + c_2)$$

22.3 problem Ex 3

Internal problem ID [11259]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 44. Roots of auxiliary equation repeated. Page 94
Problem number: Ex 3.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 2y''' - 2y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$3)-2*diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^{-x} + c_3 e^{-x} x + c_4 e^{-x} x^2$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 32

```
DSolve[y''''[x]+2*y'''[x]-2*y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_3 x^2 + c_2 x + c_4 e^{2x} + c_1\right)$$

22.4 problem Ex 4

Internal problem ID [11260]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 44. Roots of auxiliary equation repeated. Page 94
Problem number: Ex 4.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 6y'' + 9y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$3)-6*diff(y(x),x$2)+9*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{3x} + c_3 e^{3x} x$$

✓ Solution by Mathematica

Time used: 0.078 (sec). Leaf size: 30

```
DSolve[y'''[x]-6*y''[x]+9*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9} e^{3x}(c_2(3x-1) + 3c_1) + c_3$$

23 Chapter VII, Linear differential equations with constant coefficients. Article 45. Roots of auxiliary equation complex. Page 95

23.1 problem Ex 2

Internal problem ID [11261]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 45. Roots of auxiliary equation complex. Page 95
Problem number: Ex 2.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 2y'' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$2)+y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(x) + c_2 \cos(x) + c_3 \sin(x)\, x + c_4 \cos(x)\, x$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 26

```
DSolve[y''''[x]+2*y''[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (c_2 x + c_1)\cos(x) + (c_4 x + c_3)\sin(x)$$

23.2 problem Ex 3

Internal problem ID [11262]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 45. Roots of auxiliary equation complex. Page 95
Problem number: Ex 3.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - y'' + y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2 e^{\frac{x}{2}} \sin\left(\frac{\sqrt{3}\,x}{2}\right) + c_3 e^{\frac{x}{2}} \cos\left(\frac{\sqrt{3}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.799 (sec). Leaf size: 75

```
DSolve[y'''[x]-y''[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(c_1 - \sqrt{3}c_2\right) e^{x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + \frac{1}{2}\left(\sqrt{3}c_1 + c_2\right) e^{x/2} \sin\left(\frac{\sqrt{3}x}{2}\right) + c_3$$

24 Chapter VII, Linear differential equations with constant coefficients. Article 47. Particular integral. Page 100

24.1 problem Ex 1

Internal problem ID [11263]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 47. Particular integral. Page 100
Problem number: Ex 1.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - y'' - 2y' = e^{-x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$3)-diff(y(x),x$2)-2*diff(y(x),x)=exp(-x),y(x), singsol=all)
```

$$y(x) = \frac{c_2 e^{2x}}{2} + \frac{e^{-x} x}{3} + \frac{e^{-x}}{3} - e^{-x} c_1 + c_3$$

✓ Solution by Mathematica

Time used: 0.166 (sec). Leaf size: 37

```
DSolve[y'''[x]-y''[x]-2*y'[x]==Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9} e^{-x}(3x + 4 - 9c_1) + \frac{1}{2} c_2 e^{2x} + c_3$$

24.2 problem Ex 2

Internal problem ID [11264]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 47. Particular integral. Page 100
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 3y' + 2y = \mathrm{e}^{\mathrm{e}^{x}}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=exp(exp(x)),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{e}^{x}-2x} - \mathrm{e}^{-2x}c_1 + c_2\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 25

```
DSolve[y''[x]+3*y'[x]+2*y[x]==Exp[Exp[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(e^{e^{x}} + c_2e^{x} + c_1\right)$$

24.3 problem Ex 3

Internal problem ID [11265]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 47. Particular integral. Page 100
Problem number: Ex 3.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' + 3y'' + 3y' + y = 2\,\mathrm{e}^{-x} - \mathrm{e}^{-x}x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve(diff(y(x),x$3)+3*diff(y(x),x$2)+3*diff(y(x),x)+y(x)=2*exp(-x)-x^2*exp(-x),y(x), sings
```

$$y(x) = \frac{x^3(x^2-20)\left(-x^2+2\right)\mathrm{e}^{-x}}{60x^2-120} + \mathrm{e}^{-x}c_1 + c_2x^2\mathrm{e}^{-x} + c_3\mathrm{e}^{-x}x$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 41

```
DSolve[y'''[x]+3*y''[x]+3*y'[x]+y[x]==2*Exp[-x]-x^2*Exp[-x],y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{60}e^{-x}\left(-x^5+20x^3+60c_3x^2+60c_2x+60c_1\right)$$

24.4 problem Ex 4

Internal problem ID [11266]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 47. Particular integral. Page 100
Problem number: Ex 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + y = \frac{e^x}{(1-x)^2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=exp(x)/(1-x)^2,y(x), singsol=all)
```

$$y(x) = c_2 e^x + e^x c_1 x + e^x(-1 - \ln(x-1))$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 23

```
DSolve[y''[x]-2*y'[x]+y[x]==Exp[x]/(1-x)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(-\log(x-1) + c_2 x - 1 + c_1)$$

25 Chapter VII, Linear differential equations with constant coefficients. Article 48. Page 103

25.1 problem Ex 1

Internal problem ID [11267]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 48. Page 103
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - 3y' + 2y = \mathrm{e}^{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = (-x + c_1\mathrm{e}^{x} + c_2)\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 22

```
DSolve[y''[x]-3*y'[x]+2*y[x]==Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(-x + c_2e^{x} - 1 + c_1)$$

25.2 problem Ex 2

Internal problem ID [11268]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 48. Page 103
Problem number: Ex 2.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$y''' - 3y'' - y' + 3y = x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x$3)-3*diff(y(x),x$2)-diff(y(x),x)+3*y(x)=x^2,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{3} + \frac{2x}{9} + \frac{20}{27} + c_1 e^x + c_2 e^{-x} + c_3 e^{3x}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 42

```
DSolve[y'''[x]-3*y''[x]-y'[x]+3*y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{27}(9x^2 + 6x + 20) + c_1 e^{-x} + c_2 e^x + c_3 e^{3x}$$

25.3 problem Ex 3

Internal problem ID [11269]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 48. Page 103
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+y(x)=sec(x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + c_1 \cos(x) + x \sin(x) - \ln(\sec(x)) \cos(x)$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 22

```
DSolve[y''[x]+y[x]==Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_2) \sin(x) + \cos(x)(\log(\cos(x)) + c_1)$$

25.4 problem Ex 4

Internal problem ID [11270]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 48. Page 103
Problem number: Ex 4.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - 4y'' + 5y' - 2y = x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x$2)+5*diff(y(x),x)-2*y(x)=x,y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} - \frac{5}{4} + c_1 e^x + c_2 e^{2x} + c_3 x\, e^x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 35

```
DSolve[y'''[x]-4*y''[x]+5*y'[x]-2*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x + x\left(-\frac{1}{2} + c_2 e^x\right) + c_3 e^{2x} - \frac{5}{4}$$

26 Chapter VII, Linear differential equations with constant coefficients. Article 49. Variation of parameters. Page 106

26.1 problem Ex 1

Internal problem ID [11271]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 49. Variation of parameters. Page 106
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \sec(x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(diff(y(x),x$2)+y(x)=sec(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + c_1 \cos(x) + x \sin(x) - \ln(\sec(x)) \cos(x)$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 22

```
DSolve[y''[x]+y[x]==Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_2) \sin(x) + \cos(x)(\log(\cos(x)) + c_1)$$

26.2 problem Ex 2

Internal problem ID [11272]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 49. Variation of parameters. Page 106
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \tan(x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)=tan(x),y(x), singsol=all)
```

$$y(x) = \sin(x) c_2 + c_1 \cos(x) - \cos(x) \ln(\sec(x) + \tan(x))$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 23

```
DSolve[y''[x]+y[x]==Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x)(-\text{arctanh}(\sin(x))) + c_1 \cos(x) + c_2 \sin(x)$$

27 Chapter VII, Linear differential equations with constant coefficients. Article 50. Method of undetermined coefficients. Page 107

27.1 problem Ex 1

Internal problem ID [11273]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 50. Method of undetermined coefficients. Page 107
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = x^2 + \cos(x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+4*y(x)=x^2+cos(x),y(x), singsol=all)
```

$$y(x) = \sin(2x)\, c_2 + \cos(2x)\, c_1 + \frac{x^2}{4} - \frac{1}{8} + \frac{\cos(x)}{3}$$

✓ Solution by Mathematica

Time used: 0.321 (sec). Leaf size: 36

```
DSolve[y''[x]+4*y[x]==x^2+Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{4} + \frac{\cos(x)}{3} + c_1 \cos(2x) + c_2 \sin(2x) - \frac{1}{8}$$

27.2 problem Ex 2

Internal problem ID [11274]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 50. Method of undetermined coefficients. Page 107
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' - 2y' + y = 2x\,\mathrm{e}^{2x} - \sin(x)^2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=2*x*exp(2*x)-sin(x)^2,y(x), singsol=all)
```

$$y(x) = c_2\mathrm{e}^{x} + \mathrm{e}^{x}c_1x - \frac{1}{2} + 2(x-2)\,\mathrm{e}^{2x} - \frac{3\cos(2x)}{50} - \frac{2\sin(2x)}{25}$$

✓ Solution by Mathematica

Time used: 1.17 (sec). Leaf size: 53

```
DSolve[y''[x]-2*y'[x]+y[x]==2*x*Exp[2*x]-Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2e^{2x}x - 4e^{2x} - \frac{2}{25}\sin(2x) - \frac{3}{50}\cos(2x) + c_2e^{x}x + c_1e^{x} - \frac{1}{2}$$

27.3 problem Ex 3

Internal problem ID [11275]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 50. Method of undetermined coefficients. Page 107
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = 2\,\mathrm{e}^{x} + x^{3} - x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+y(x)=2*exp(x)+x^3-x,y(x), singsol=all)
```

$$y(x) = \sin(x)\,c_2 + c_1\cos(x) + x^3 + \mathrm{e}^{x} - 7x$$

✓ Solution by Mathematica

Time used: 0.234 (sec). Leaf size: 25

```
DSolve[y''[x]+y[x]==2*Exp[x]+x^3-x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^3 - 7x + e^x + c_1\cos(x) + c_2\sin(x)$$

27.4 problem Ex 4

Internal problem ID [11276]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 50. Method of undetermined coefficients. Page 107
Problem number: Ex 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + y = 3\,e^{2x} - \cos(x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=3*exp(2*x)-cos(x),y(x), singsol=all)
```

$$y(x) = c_2 e^{-x} + e^{-x} x c_1 + \frac{e^{2x}}{3} - \frac{\sin(x)}{2}$$

✓ Solution by Mathematica

Time used: 0.454 (sec). Leaf size: 38

```
DSolve[y''[x]+2*y'[x]+y[x]==3*Exp[2*x]-Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6} e^{-x} \left(2e^{3x} - 3e^{x} \sin(x) + 6c_2 x + 6c_1\right)$$

27.5 problem Ex 5

Internal problem ID [11277]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 50. Method of undetermined coefficients. Page 107
Problem number: Ex 5.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{y''' - y = x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$3)-y(x)=x^2,y(x), singsol=all)
```

$$y(x) = -x^2 + c_1 e^x + c_2 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}\, x}{2}\right) + c_3 e^{-\frac{x}{2}} \sin\left(\frac{\sqrt{3}\, x}{2}\right)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 59

```
DSolve[y'''[x]-y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^2 + c_1 e^x + c_2 e^{-x/2} \cos\left(\frac{\sqrt{3}x}{2}\right) + c_3 e^{-x/2} \sin\left(\frac{\sqrt{3}x}{2}\right)$$

27.6 problem Ex 6

Internal problem ID [11278]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 50. Method of undetermined coefficients. Page 107
Problem number: Ex 6.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 2y'' - 3y' = 3x^2 + \sin(x)}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)-3*diff(y(x),x)=3*x^2+sin(x),y(x), singsol=all)
```

$$y(x) = -\frac{x^3}{3} + \frac{2x^2}{3} + \frac{e^{3x}c_1}{3} - c_2e^{-x} + \frac{\sin(x)}{10} + \frac{\cos(x)}{5} - \frac{14x}{9} + c_3$$

✓ Solution by Mathematica

Time used: 0.68 (sec). Leaf size: 58

```
DSolve[y'''[x]-2*y''[x]-3*y'[x]==3*x^2+Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{9}\left(-3x^3 + 6x^2 - 14x - 9c_1e^{-x} + 3c_2e^{3x} + 9c_3\right) + \frac{\sin(x)}{10} + \frac{\cos(x)}{5}$$

27.7 problem Ex 7

Internal problem ID [11279]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 50. Method of undetermined coefficients. Page 107
Problem number: Ex 7.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type **[[_high_order, _with_linear_symmetries]]**

$$\boxed{y'''' - 2y'' + y = \mathrm{e}^{x} + 4}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(x),x$4)-2*diff(y(x),x$2)+y(x)=exp(x)+4,y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{x} x^{2}}{8} - \frac{x\,\mathrm{e}^{x}}{4} + 4 + \frac{3\,\mathrm{e}^{x}}{16} + c_1 \mathrm{e}^{x} + c_2 \mathrm{e}^{-x} + c_3 x\,\mathrm{e}^{x} + c_4 \mathrm{e}^{-x} x$$

✓ Solution by Mathematica

Time used: 0.174 (sec). Leaf size: 47

```
DSolve[y''''[x]-2*y''[x]+y[x]==Exp[x]+4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}\left(\frac{x^{2}}{8} + \left(-\frac{1}{4} + c_4\right)x + \frac{3}{16} + c_3\right) + e^{-x}((2 + c_2)x + c_1) + 4$$

27.8 problem Ex 8

Internal problem ID [11280]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 50. Method of undetermined coefficients. Page 107
Problem number: Ex 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' - 2y' = e^{2x} + 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)=exp(2*x)+1,y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} + \frac{e^{2x}x}{2} - \frac{e^{2x}}{4} + \frac{e^{2x}c_1}{2} + c_2$$

✓ Solution by Mathematica

Time used: 0.126 (sec). Leaf size: 31

```
DSolve[y''[x]-2*y'[x]==Exp[2*x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{2} + \frac{1}{4}e^{2x}(2x - 1 + 2c_1) + c_2$$

27.9 problem Ex 9

Internal problem ID [11281]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 50. Method of undetermined coefficients. Page 107
Problem number: Ex 9.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' + 2y'' + y = \cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$2)+y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \left(-\frac{x^2}{8} + \frac{1}{4}\right)\cos(x) + \frac{x\sin(x)}{8} + c_1\cos(x) + \sin(x)\,c_2 + c_3\sin(x)\,x + c_4\cos(x)\,x$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 43

```
DSolve[y''''[x]+2*y''[x]+y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-\frac{x^2}{8} + c_2x + \frac{5}{16} + c_1\right)\cos(x) + \frac{1}{4}(x + 4c_4x + 4c_3)\sin(x)$$

28 Chapter VII, Linear differential equations with constant coefficients. Article 51. Cauchy linear equation. Page 114

28.1 problem Ex 1

Internal problem ID [11282]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 51. Cauchy linear equation. Page 114
Problem number: Ex 1.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{x^3 y''' + y'x - y = \ln(x)\, x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x^3*diff(y(x),x$3)+x*diff(y(x),x)-y(x)=x*ln(x),y(x), singsol=all)
```

$$y(x) = \frac{\ln(x)^4 x}{24} + x c_1 + c_2 x \ln(x)^2 + c_3 \ln(x)\, x$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 33

```
DSolve[x^3*y'''[x]+x*y'[x]-y[x]==x*Log[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{24} x \log^4(x) + c_1 x + c_3 x \log^2(x) + c_2 x \log(x)$$

28.2 problem Ex 2

Internal problem ID [11283]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 51. Cauchy linear equation. Page 114
Problem number: Ex 2.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _nonhomogeneous]]`

$$x^3 y''' + 2x^2 y'' + 2y = 10x + \frac{10}{x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 146

```
dsolve(x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)+2*y(x)=10*(x+1/x),y(x), singsol=all)
```

$$y(x) = \sin(\ln(x))\, x c_3 + \cos(\ln(x))\, x c_2 + \frac{(((10+20i)\ln(x) + 8 + 6i + (1+2i)\, c_1)\cos(\ln(x)) + \sin(\ln(x))\,((-20+10i)\ln(x) - 6 + 8i + (-2}{10} + \frac{(((10-20i)\ln(x) + 8 - 6i + (1-2i)\, c_1)\cos(\ln(x)) - \sin(\ln(x))\,((20+10i)\ln(x) + 6 + 8i + (2+i}{10} + \frac{5(i\sin(\ln(x)) + \cos(\ln(x)))\, x^{1-i}}{2} - \frac{5x^{1+i}(i\sin(\ln(x)) - \cos(\ln(x)))}{2}$$

✓ Solution by Mathematica

Time used: 0.187 (sec). Leaf size: 42

```
DSolve[x^3*y'''[x]+2*x^2*y''[x]+2*y[x]==10*(x+1/x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{25x^2 + 10\log(x) + 8 + 5c_3}{5x} + c_2 x \cos(\log(x)) + c_1 x \sin(\log(x))$$

28.3 problem Ex 3

Internal problem ID [11284]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 51. Cauchy linear equation. Page 114
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + 3xy' + y = \frac{1}{(1-x)^2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=1/(1-x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\ln(x)\,c_1}{x} + \frac{c_2}{x} - \frac{\ln(x-1) - \ln(x)}{x}$$

✓ Solution by Mathematica

Time used: 0.061 (sec). Leaf size: 27

```
DSolve[x^2*y''[x]+3*x*y'[x]+y[x]==1/(1-x)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-\log(1-x) + \log(x) + c_2\log(x) + c_1}{x}$$

28.4 problem Ex 4

Internal problem ID [11285]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 51. Cauchy linear equation. Page 114
Problem number: Ex 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$(x+1)^2 y'' - (x+1) y' + 6y = x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve((x+1)^2*diff(y(x),x$2)-(x+1)*diff(y(x),x)+6*y(x)=x,y(x), singsol=all)
```

$$y(x) = (x+1)\sin\left(\sqrt{5}\,\ln(x+1)\right)c_2 + (x+1)\cos\left(\sqrt{5}\,\ln(x+1)\right)c_1 + \frac{x}{5} + \frac{1}{30}$$

✓ Solution by Mathematica

Time used: 0.508 (sec). Leaf size: 49

```
DSolve[(x+1)^2*y''[x]-(x+1)*y'[x]+6*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{30}(6x+1) + c_2(x+1)\cos\left(\sqrt{5}\log(x+1)\right) + c_1(x+1)\sin\left(\sqrt{5}\log(x+1)\right)$$

29 Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117

29.1 problem Ex 1

Internal problem ID [11286]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 5y' + 6y = \cos(x) - e^{2x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$2)-5*diff(y(x),x)+6*y(x)=cos(x)-exp(2*x),y(x), singsol=all)
```

$$y(x) = c_2 e^{3x} + e^{2x} c_1 + e^{2x} x + e^{2x} - \frac{\sin(x)}{10} + \frac{\cos(x)}{10}$$

✓ Solution by Mathematica

Time used: 0.345 (sec). Leaf size: 34

```
DSolve[y''[x]-5*y'[x]+6*y[x]==Cos[x]-Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{10}\left(-\sin(x) + \cos(x) + 10e^{2x}(x + c_2 e^x + 1 + c_1)\right)$$

29.2 problem Ex 2

Internal problem ID [11287]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 2.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - y = \cos(x)\,e^{x}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$4)-y(x)=exp(x)*cos(x),y(x), singsol=all)
```

$$y(x) = -\frac{e^{x}\cos(x)}{5} + c_1\cos(x) + c_2 e^{x} + c_3\sin(x) + c_4 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 38

```
DSolve[y''''[x]-y[x]==Exp[x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x} + c_3 e^{-x} + \left(-\frac{e^{x}}{5} + c_2\right)\cos(x) + c_4\sin(x)$$

29.3 problem Ex 3

Internal problem ID [11288]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 2y' + y = 2x^3 - x\,\mathrm{e}^{3x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=2*x^3-x*exp(3*x),y(x), singsol=all)
```

$$y(x) = c_2\mathrm{e}^{-x} + \mathrm{e}^{-x}xc_1 + \frac{(-2x+1)\,\mathrm{e}^{3x}}{32} + 2x^3 - 12x^2 + 36x - 48$$

✓ Solution by Mathematica

Time used: 0.335 (sec). Leaf size: 48

```
DSolve[y''[x]+2*y'[x]+y[x]==2*x^3-x*Exp[3*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\left(x^3 - 6x^2 + 18x - 24\right) + \frac{1}{32}e^{3x}(1-2x) + e^{-x}(c_2x + c_1)$$

29.4 problem Ex 5

Internal problem ID [11289]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 5.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 4y' = x^2 - 3\,\mathrm{e}^{2x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x)=x^2-3*exp(2*x),y(x), singsol=all)
```

$$y(x) = -\frac{x^3}{12} - \frac{c_2 \mathrm{e}^{-2x}}{2} - \frac{3\,\mathrm{e}^{2x} x}{8} + \frac{9\,\mathrm{e}^{2x}}{32} + \frac{\mathrm{e}^{2x} c_1}{2} - \frac{x}{8} + c_3$$

✓ Solution by Mathematica

Time used: 0.311 (sec). Leaf size: 49

```
DSolve[y'''[x]-4*y'[x]==x^2-3*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^3}{12} - \frac{x}{8} + \frac{1}{32} e^{2x}(-12x + 9 + 16c_1) - \frac{1}{2} c_2 e^{-2x} + c_3$$

29.5 problem Ex 6

Internal problem ID [11290]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 6.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$\boxed{y'''' - 2y'' + y = \cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$4)-2*diff(y(x),x$2)+y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \frac{\cos(x)}{4} + c_1 e^x + c_2 e^{-x} + c_3 x\, e^x + c_4 e^{-x} x$$

✓ Solution by Mathematica

Time used: 0.131 (sec). Leaf size: 42

```
DSolve[y''''[x]-2*y''[x]+y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\cos(x)}{4} + e^{-x}\left(c_2 x + c_3 e^{2x} + c_4 e^{2x} x + c_1\right)$$

29.6 problem Ex 7

Internal problem ID [11291]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 7.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$x^4 y'''' + 6x^3 y''' + 9x^2 y'' + 3y'x + y = (1 + \ln(x))^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(x^4*diff(y(x),x$4)+6*x^3*diff(y(x),x$3)+9*x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=(1
```

$$y(x) = \ln(x)^2 + 2\ln(x) - 3 + c_1 \cos(\ln(x)) + c_2 \sin(\ln(x)) + c_3 \cos(\ln(x)) \ln(x) + c_4 \ln(x) \sin(\ln(x))$$

✓ Solution by Mathematica

Time used: 0.27 (sec). Leaf size: 39

```
DSolve[x^4*y''''[x]+6*x^3*y'''[x]+9*x^2*y''[x]+3*x*y'[x]+y[x]==(1+Log[x])^2,y[x],x,IncludeSi
```

$$y(x) \to \log^2(x) + 2\log(x) + (c_2 \log(x) + c_1)\cos(\log(x)) + (c_4 \log(x) + c_3)\sin(\log(x)) - 3$$

29.7 problem Ex 8

Internal problem ID [11292]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 8.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' + 2y'' + y' = x^2 - x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$3)+2*diff(y(x),x$2)+diff(y(x),x)=x^2-x,y(x), singsol=all)
```

$$y(x) = \frac{x^3}{3} - c_2 \mathrm{e}^{-x} + c_1\left(-\mathrm{e}^{-x}x - \mathrm{e}^{-x}\right) - \frac{5x^2}{2} + 8x + c_3$$

✓ Solution by Mathematica

Time used: 0.243 (sec). Leaf size: 39

```
DSolve[y'''[x]+2*y''[x]+y'[x]==x^2-x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}x\left(2x^2 - 15x + 48\right) - e^{-x}(c_2(x+1) + c_1) + c_3$$

29.8 problem Ex 9

Internal problem ID [11293]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \sin(x)^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$2)+4*y(x)=sin(x)^2,y(x), singsol=all)
```

$$y(x) = \sin(2x)\, c_2 + \cos(2x)\, c_1 - \frac{x \sin(2x)}{8} + \frac{1}{8} - \frac{\cos(2x)}{8}$$

✓ Solution by Mathematica

Time used: 0.16 (sec). Leaf size: 34

```
DSolve[y''[x]+4*y[x]==Sin[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}((-1 + 8c_1)\cos(2x) - (x - 8c_2)\sin(2x) + 1)$$

29.9 problem Ex 10

Internal problem ID [11294]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \sec(x)^2}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+4*y(x)=sec(x)^2,y(x), singsol=all)
```

$$y(x) = \sin(2x) c_2 + \cos(2x) c_1 + \left(-2\cos(x)^2 + 1\right)\ln(\sec(x)) + 2x\cos(x)\sin(x) - \sin(x)^2$$

✓ Solution by Mathematica

Time used: 0.168 (sec). Leaf size: 33

```
DSolve[y''[x]+4*y[x]==Sec[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \cos(2x)(\log(\cos(x)) + c_1) + \sin(x)(-\sin(x) + 2(x + c_2)\cos(x))$$

29.10 problem Ex 12

Internal problem ID [11295]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 12.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _with_linear_symmetries]]`

$$y'''' - y''' - 3y'' + 5y' - 2y = e^{3x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$4)-diff(y(x),x$3)-3*diff(y(x),x$2)+5*diff(y(x),x)-2*y(x)=exp(3*x),y(x), s
```

$$y(x) = \frac{e^{3x}}{40} + c_1 e^{x} + c_2 e^{-2x} + c_3 x\, e^{x} + c_4 e^{x} x^{2}$$

✓ Solution by Mathematica

Time used: 0.091 (sec). Leaf size: 39

```
DSolve[y''''[x]-y'''[x]-3*y''[x]+5*y'[x]-2*y[x]==Exp[3*x],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{e^{3x}}{40} + c_1 e^{-2x} + e^{x}(x(c_4 x + c_3) + c_2)$$

29.11 problem Ex 13

Internal problem ID [11296]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + y = \cos(x)\, x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x$2)+y(x)=x*cos(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_2 + c_1 \cos(x) + \frac{\cos(x)\, x}{4} + \frac{x^2 \sin(x)}{4} - \frac{\sin(x)}{4}$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 34

```
DSolve[y''[x]+y[x]==x*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}\left((2x^2 - 1 + 8c_2)\sin(x) + 2(x + 4c_1)\cos(x)\right)$$

29.12 problem Ex 14

Internal problem ID [11297]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 14.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^3y''' + 2x^2y'' - y'x + y = \frac{1}{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^3*diff(y(x),x$3)+2*x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=1/x,y(x), singsol=all)
```

$$y(x) = c_2 x \ln(x) + x c_3 + \frac{\ln(x) + 1 + c_1}{4x}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 33

```
DSolve[x^3*y'''[x]+2*x^2*y''[x]-x*y'[x]+y[x]==1/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log(x) + 1}{4x} + \frac{c_1}{x} + c_2 x + c_3 x \log(x)$$

29.13 problem Ex 15

Internal problem ID [11298]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VII, Linear differential equations with constant coefficients. Article 52. Summary. Page 117
Problem number: Ex 15.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$\boxed{y''' - y = x\,\mathrm{e}^{x} + \cos\left(x\right)^{2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 121

```
dsolve(diff(y(x),x$3)-y(x)=x*exp(x)+cos(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\cos\left(2x\right)}{10\left(5+2\sqrt{3}\right)\left(-5+2\sqrt{3}\right)} + \frac{4\sin\left(2x\right)}{5\left(5+2\sqrt{3}\right)\left(-5+2\sqrt{3}\right)} - \frac{13(3\,\mathrm{e}^{x}x^{2} - 6x\,\mathrm{e}^{x} + 4\,\mathrm{e}^{x} - 9)}{18\left(5+2\sqrt{3}\right)\left(-5+2\sqrt{3}\right)} + c_1\mathrm{e}^{x} + c_2\mathrm{e}^{-\frac{x}{2}}\cos\left(\frac{\sqrt{3}\,x}{2}\right) + c_3\mathrm{e}^{-\frac{x}{2}}\sin\left(\frac{\sqrt{3}\,x}{2}\right)$$

✓ Solution by Mathematica

Time used: 7.274 (sec). Leaf size: 98

```
DSolve[y'''[x]-y[x]==x*Exp[x]+Cos[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{x}x^{2}}{6} - \frac{e^{x}x}{3} + \frac{2e^{x}}{9} - \frac{4}{65}\sin(2x) - \frac{1}{130}\cos(2x) + c_1e^{x} + c_2e^{-x/2}\cos\left(\frac{\sqrt{3}x}{2}\right) + c_3e^{-x/2}\sin\left(\frac{\sqrt{3}x}{2}\right) - \frac{1}{2}$$

30 Chapter VIII, Linear differential equations of the second order. Article 53. Change of dependent variable. Page 125

30.1 problem Ex 1

Internal problem ID [11299]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 53. Change of dependent variable. Page 125
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' - x^2 y' + yx = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve(diff(y(x),x$2)-x^2*diff(y(x),x)+x*y(x)=x,y(x), singsol=all)
```

$$y(x) = c_2 x + \left(6\left(-x^3\right)^{\frac{1}{3}} 3^{\frac{2}{3}} \Gamma\left(\frac{2}{3}\right) - 6\left(-x^3\right)^{\frac{1}{3}} 3^{\frac{2}{3}} \Gamma\left(\frac{2}{3}, -\frac{x^3}{3}\right) + 18\,\mathrm{e}^{\frac{x^3}{3}}\right) c_1 + 1$$

✓ Solution by Mathematica

Time used: 0.286 (sec). Leaf size: 42

```
DSolve[y''[x]-x^2*y'[x]+x*y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{c_2 \sqrt[3]{-x^3}\,\Gamma\left(-\frac{1}{3}, -\frac{x^3}{3}\right)}{3\sqrt[3]{3}} + c_1 x + 1$$

30.2 problem Ex 2

Internal problem ID [11300]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 53. Change of dependent variable. Page 125
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (1+2x)\,y' + (x+1)\,y = x^2 - x - 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x$2)-(2*x+1)*diff(y(x),x)+(x+1)*y(x)=x^2-x-1,y(x), singsol=all)
```

$$y(x) = c_2 e^x + e^x x^2 c_1 + x$$

✓ Solution by Mathematica

Time used: 0.275 (sec). Leaf size: 25

```
DSolve[x*y''[x]-(2*x+1)*y'[x]+(x+1)*y[x]==x^2-x-1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} c_2 e^x x^2 + x + c_1 e^x$$

30.3 problem Ex 3

Internal problem ID [11301]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 53. Change of dependent variable. Page 125
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\, y''+2xy'-2y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve((1+x^2)*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2(\arctan(x)\, x + 1)$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 48

```
DSolve[(1+x^2)*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}i(2c_1x - c_2x\log(1-ix) + c_2x\log(1+ix) + 2ic_2)$$

30.4 problem Ex 4

Internal problem ID [11302]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 53. Change of dependent variable. Page 125
Problem number: Ex 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(1-x)\,y''+xy'-y=(1-x)^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((1-x)*diff(y(x),x$2)+x*diff(y(x),x)-y(x)=(1-x)^2,y(x), singsol=all)
```

$$y(x)=c_2x+c_1\mathrm{e}^x+x^2+1$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 22

```
DSolve[(1-x)*y''[x]+x*y'[x]-y[x]==(1-x)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x^2+x-c_2x+c_1e^x+1$$

30.5 problem Ex 5

Internal problem ID [11303]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 53. Change of dependent variable. Page 125
Problem number: Ex 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{\sin(x) y'' + 2y' \cos(x) + 3\sin(x) y = e^x}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(sin(x)*diff(y(x),x$2)+2*cos(x)*diff(y(x),x)+3*sin(x)*y(x)=exp(x),y(x), singsol=all)
```

$$y(x) = \csc(x)\sin(2x) c_2 + \csc(x)\cos(2x) c_1 + \frac{e^x \csc(x)}{5}$$

✓ Solution by Mathematica

Time used: 0.229 (sec). Leaf size: 56

```
DSolve[Sin[x]*y''[x]+2*Cos[x]*y'[x]+3*Sin[x]*y[x]==Exp[x],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{e^{-ix}\left(4ie^{(1+2i)x} + 5c_2e^{4ix} + 20ic_1\right)}{10\left(-1 + e^{2ix}\right)}$$

30.6 problem Ex 6

Internal problem ID [11304]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 53. Change of dependent variable. Page 125
Problem number: Ex 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$y'' - 2y' \tan(x) - (a^2 + 1)\, y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-2*tan(x)*diff(y(x),x)-(a^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sec(x) \sinh(ax) + c_2 \sec(x) \cosh(ax)$$

✓ Solution by Mathematica

Time used: 0.117 (sec). Leaf size: 32

```
DSolve[y''[x]-2*Tan[x]*y'[x]-(a^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sec(x) \left(c_1 e^{-ax} + \frac{c_2 e^{ax}}{2a} \right)$$

30.7 problem Ex 7

Internal problem ID [11305]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 53. Change of dependent variable. Page 125
Problem number: Ex 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{4x^2y'' + 4x^3y' + (x^2+1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.391 (sec). Leaf size: 43

```
dsolve(4*x^2*diff(y(x),x$2)+4*x^3*diff(y(x),x)+(x^2+1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \mathrm{e}^{-\frac{x^2}{4}} \operatorname{WhittakerM}\left(-\frac{1}{8}, 0, \frac{x^2}{2}\right)}{\sqrt{x}} + \frac{c_2 \mathrm{e}^{-\frac{x^2}{4}} \operatorname{WhittakerW}\left(-\frac{1}{8}, 0, \frac{x^2}{2}\right)}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.239 (sec). Leaf size: 60

```
DSolve[4*x^2*y''[x]+4*x^3*y'[x]+(x^2+1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 G_{1,2}^{2,0}\left(\frac{x^2}{16} \middle| \begin{array}{c} \frac{7}{8} \\ \frac{1}{4}, \frac{1}{4} \end{array}\right) + \frac{1}{2}\sqrt[4]{-1} c_1 \sqrt{x} \operatorname{Hypergeometric1F1}\left(\frac{3}{8}, 1, -\frac{x^2}{16}\right)$$

30.8 problem Ex 8

Internal problem ID [11306]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 53. Change of dependent variable. Page 125
Problem number: Ex 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{xy'' + 2y' - yx = 2\,\mathrm{e}^{x}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x$2)+2*diff(y(x),x)-x*y(x)=2*exp(x),y(x), singsol=all)
```

$$y(x) = \frac{\sinh(x)\,c_2}{x} + \frac{\cosh(x)\,c_1}{x} + \mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 35

```
DSolve[x*y''[x]+2*y'[x]-x*y[x]==2*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-x}(e^{2x}(2x - 1 + c_2) + 2c_1)}{2x}$$

31 Chapter VIII, Linear differential equations of the second order. Article 54. Change of independent variable. Page 127

31.1 problem Ex 1

Internal problem ID [11307]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 54. Change of independent variable. Page 127
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + (2\,e^{x} - 1)\,y' + e^{2x}y = e^{4x}}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
dsolve(diff(y(x),x$2)+(2*exp(x)-1)*diff(y(x),x)+exp(2*x)*y(x)=exp(4*x),y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{2}-e^{x}} \sinh\left(\frac{x}{2}\right) c_2 + e^{\frac{x}{2}-e^{x}} \cosh\left(\frac{x}{2}\right) c_1 - 4\,e^{x} + e^{2x} + 6$$

✓ Solution by Mathematica

Time used: 0.121 (sec). Leaf size: 39

```
DSolve[y''[x]+(2*Exp[x]-1)*y'[x]+Exp[2*x]*y[x]==Exp[4*x],y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -4e^{x} + e^{2x} + c_1 e^{-e^{x}} + c_2 e^{x-e^{x}} + 6$$

31.2 problem Ex 2

Internal problem ID [11308]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 54. Change of independent variable. Page 127
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']]`

$$\boxed{\left(-x^{2}+1\right) y''-x y'+4 y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)+4*y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{c_{1}}{\left(x+\sqrt{x^{2}-1}\right)^{2}}+c_{2}\left(x+\sqrt{x^{2}-1}\right)^{2}$$

✓ Solution by Mathematica

Time used: 0.316 (sec). Leaf size: 97

```
DSolve[(1-x^2)*y''[x]-x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_{1}\cosh\left(\frac{4\sqrt{1-x^{2}}\arctan\left(\frac{\sqrt{1-x^{2}}}{x+1}\right)}{\sqrt{x^{2}-1}}\right)-ic_{2}\sinh\left(\frac{4\sqrt{1-x^{2}}\arctan\left(\frac{\sqrt{1-x^{2}}}{x+1}\right)}{\sqrt{x^{2}-1}}\right)$$

31.3 problem Ex 3

Internal problem ID [11309]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 54. Change of independent variable. Page 127
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{y'' + y' \tan(x) + \cos(x)^2 y = 0}$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)+tan(x)*diff(y(x),x)+cos(x)^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(\sin(x)) + c_2 \cos(\sin(x))$$

✓ Solution by Mathematica

Time used: 0.078 (sec). Leaf size: 18

```
DSolve[y''[x]+Tan[x]*y'[x]+Cos[x]^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 \sin(\sin(x)) + c_1 \cos(\sin(x))$$

31.4 problem Ex 4

Internal problem ID [11310]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 54. Change of independent variable. Page 127
Problem number: Ex 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{x^6y'' + 3x^5y' + y = \frac{1}{x^2}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x^6*diff(y(x),x$2)+3*x^5*diff(y(x),x)+y(x)=1/x^2,y(x), singsol=all)
```

$$y(x) = \sin\left(\frac{1}{2x^2}\right)c_2 + \cos\left(\frac{1}{2x^2}\right)c_1 + \frac{1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.099 (sec). Leaf size: 32

```
DSolve[x^6*y''[x]+3*x^5*y'[x]+y[x]==1/x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x^2} + c_1\cos\left(\frac{1}{2x^2}\right) - c_2\sin\left(\frac{1}{2x^2}\right)$$

31.5 problem Ex 5

Internal problem ID [11311]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 54. Change of independent variable. Page 127
Problem number: Ex 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{xy'' - (2x^2 + 1)\, y' - 8yx^3 = 4x^3 e^{-x^2}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(x*diff(y(x),x$2)-(2*x^2+1)*diff(y(x),x)-8*x^3*y(x)=4*x^3*exp(-x^2),y(x), singsol=all)
```

$$y(x) = e^{2x^2} c_2 + e^{-x^2} c_1 - \frac{e^{-x^2} x^2}{3}$$

✓ Solution by Mathematica

Time used: 0.105 (sec). Leaf size: 38

```
DSolve[x*y''[x]-(2*x^2+1)*y'[x]-8*x^3*y[x]==4*x^3*Exp[-x^2],y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{9} e^{-x^2} \left(-3x^2 + 9c_1 e^{3x^2} - 1 + 9c_2\right)$$

32 Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129

32.1 problem Ex 1

Internal problem ID [11312]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[_Laguerre]`

$$\boxed{xy'' - (x+3)\,y' + 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x*diff(y(x),x$2)-(x+3)*diff(y(x),x)+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2(x^3 + 3x^2 + 6x + 6)$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 29

```
DSolve[x*y''[x]-(x+3)*y'[x]+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x - c_2(x^3 + 3x^2 + 6x + 6)$$

32.2 problem Ex 2

Internal problem ID [11313]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x-3)\,y''-(4x-9)\,y'+(3x-6)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve((x-3)*diff(y(x),x$2)-(4*x-9)*diff(y(x),x)+(3*x-6)*y(x)=0,y(x), singsol=all)
```

$$y(x)=c_1\mathrm{e}^{x}+c_2\mathrm{e}^{3x}\left(4x^3-42x^2+150x-183\right)$$

✓ Solution by Mathematica

Time used: 0.091 (sec). Leaf size: 42

```
DSolve[(x-3)*y''[x]-(4*x-9)*y'[x]+(3*x-6)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{8}c_2e^{3x-9}\left(4x^3-42x^2+150x-183\right)+c_1e^{x-3}$$

32.3 problem Ex 3

Internal problem ID [11314]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' + 4xy' + (-x^2 + 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)+4*x*diff(y(x),x)+(2-x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sinh(x)}{x^2} + \frac{c_2 \cosh(x)}{x^2}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 28

```
DSolve[x^2*y''[x]+4*x*y'[x]+(2-x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2c_1 e^{-x} + c_2 e^x}{2x^2}$$

32.4 problem Ex 4

Internal problem ID [11315]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129
Problem number: Ex 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(x^2+1)\,y''-2xy'+2y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+1)*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x)=xc_1+c_2(x^2-1)$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 21

```
DSolve[(x^2+1)*y''[x]-2*x*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2x-c_1(x-i)^2$$

32.5 problem Ex 5

Internal problem ID [11316]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129
Problem number: Ex 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{xy'' - (2x-1)\,y' + (x-1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x$2)-(2*x-1)*diff(y(x),x)+(x-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{x} + c_2 \ln(x)\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 17

```
DSolve[x*y''[x]-(2*x-1)*y'[x]+(x-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(c_2 \log(x) + c_1)$$

32.6 problem Ex 6

Internal problem ID [11317]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129
Problem number: Ex 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 4xy' + (x^2+6)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x$2)-4*x*diff(y(x),x)+(6+x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1x^2\sin(x) + c_2\cos(x)\,x^2$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 37

```
DSolve[x^2*y''[x]-4*x*y'[x]+(6+x^2)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}e^{-ix}x^2\left(2c_1 - ic_2e^{2ix}\right)$$

32.7 problem Ex 7

Internal problem ID [11318]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129
Problem number: Ex 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{(2x^3-1)\,y''-6x^2y'+6yx=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve((2*x^3-1)*diff(y(x),x$2)-6*x^2*diff(y(x),x)+6*x*y(x)=0,y(x), singsol=all)
```

$$y(x)=xc_1+c_2(x^3+1)$$

✓ Solution by Mathematica

Time used: 2.452 (sec). Leaf size: 19

```
DSolve[(2*x^3-1)*y''[x]-6*x^2*y'[x]+6*x*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1x-c_2(x^3+1)$$

32.8 problem Ex 8

Internal problem ID [11319]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129
Problem number: Ex 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2y'' - 2x(x+1)\,y' + 2(x+1)\,y = x^3$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)-2*x*(1+x)*diff(y(x),x)+2*(1+x)*y(x)=x^3,y(x), singsol=all)
```

$$y(x) = c_2 x + x\,\mathrm{e}^{2x} c_1 - \frac{x^2}{2}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 28

```
DSolve[x^2*y''[x]-2*x*(1+x)*y'[x]+2*(1+x)*y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4}x\left(2x - 2c_2e^{2x} + 1 - 4c_1\right)$$

32.9 problem Ex 9

Internal problem ID [11320]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129
Problem number: Ex 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{x^2y'' - 2nx(1+x)\,y' + \left(a^2x^2 + n^2 + n\right)y = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 95

```
dsolve(x^2*diff(y(x),x$2)-2*n*x*(1+x)*diff(y(x),x)+(n^2+n+a^2*x^2)*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \operatorname{WhittakerM}\left(\frac{i n^2}{\sqrt{a+n}\,\sqrt{a-n}}, \frac{1}{2}, 2i\sqrt{a+n}\,\sqrt{a-n}\,x\right) x^n \mathrm{e}^{nx} + c_2 \operatorname{WhittakerW}\left(\frac{i n^2}{\sqrt{a+n}\,\sqrt{a-n}}, \frac{1}{2}, 2i\sqrt{a+n}\,\sqrt{a-n}\,x\right) x^n \mathrm{e}^{nx}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*y''[x]-2*n*x*(1+x)*y'[x]+(n^2+n+a^2*x^2)*y[x]==0,y[x],x,IncludeSingularSolutions
```

Not solved

32.10 problem Ex 10

Internal problem ID [11321]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter VIII, Linear differential equations of the second order. Article 55. Summary. Page 129
Problem number: Ex 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^4 y'' + 2x^3(1+x)\,y' + y n^2 = 0$$

✓ Solution by Maple

Time used: 0.672 (sec). Leaf size: 299

```
dsolve(x^4*diff(y(x),x$2)+2*x^3*(1+x)*diff(y(x),x)+n^2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \operatorname{HeunD}\left(8(-n^2)^{\frac{1}{4}}, \frac{-8i(-n^2)^{\frac{3}{4}}-n+8\sqrt{-n^2}\,n}{n}, -\frac{16i(-n^2)^{\frac{3}{4}}}{n}, \frac{n-8i(-n^2)^{\frac{3}{4}}-8\sqrt{-n^2}\,n}{n}, \frac{i(-n^2)^{\frac{1}{4}}x+n}{i(-n^2)^{\frac{1}{4}}x-n}\right) e^{\frac{i\sqrt{-n^2}\,x^2+in^2-x^2n}{nx}}}{\sqrt{x}} + \frac{c_2 \operatorname{HeunD}\left(-8(-n^2)^{\frac{1}{4}}, \frac{-8i(-n^2)^{\frac{3}{4}}-n+8\sqrt{-n^2}\,n}{n}, -\frac{16i(-n^2)^{\frac{3}{4}}}{n}, \frac{n-8i(-n^2)^{\frac{3}{4}}-8\sqrt{-n^2}\,n}{n}, \frac{i(-n^2)^{\frac{1}{4}}x+n}{i(-n^2)^{\frac{1}{4}}x-n}\right) e^{\frac{-i\sqrt{-n^2}\,x^2-in^2}{nx}}}{\sqrt{x}}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^4*y''[x]+2*x^3*(1+x)*y'[x]+n^2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

33 Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 57. Dependent variable absent. Page 132

33.1 problem Ex 1

Internal problem ID [11322]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 57. Dependent variable absent. Page 132
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y], [_2nd_order, _reducible, _mu_y_y1]]`

$$\boxed{(x^2+1)\,y''+y'^2=-1}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 29

```
dsolve((1+x^2)*diff(y(x),x$2)+1+diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x)=\frac{x}{c_1}-\frac{(-c_1^2-1)\ln(xc_1-1)}{c_1^2}+c_2$$

✓ Solution by Mathematica

Time used: 12.052 (sec). Leaf size: 33

```
DSolve[(1+x^2)*y''[x]+1+(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -x\cot(c_1)+\csc^2(c_1)\log(-x\sin(c_1)-\cos(c_1))+c_2$$

33.2 problem Ex 2

Internal problem ID [11323]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 57. Dependent variable absent. Page 132
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y], [_3rd_order, _with_linear_symmetrie`

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 97

```
dsolve((x*diff(y(x),x$3)-diff(y(x),x$2))^2=diff(y(x),x$3)^2+1,y(x), singsol=all)
```

$$y(x) = -\frac{(-x^2+1)^{\frac{3}{2}}}{6} + \frac{x \arcsin(x)}{2} + \frac{\sqrt{-x^2+1}}{2} + xc_1 + c_2$$
$$y(x) = \frac{(-x^2+1)^{\frac{3}{2}}}{6} - \frac{x \arcsin(x)}{2} - \frac{\sqrt{-x^2+1}}{2} + xc_1 + c_2$$
$$y(x) = \frac{\sqrt{c_1^2-1}\, x^3}{6} + \frac{x^2 c_1}{2} + c_2 x + c_3$$

✓ Solution by Mathematica

Time used: 0.241 (sec). Leaf size: 75

```
DSolve[(x*y'''[x]-y''[x])^2==(y'''[x])^2+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 x^3}{6} - \frac{1}{2}\sqrt{1+c_1{}^2}x^2 + c_3 x + c_2$$
$$y(x) \to \frac{c_1 x^3}{6} + \frac{1}{2}\sqrt{1+c_1{}^2}x^2 + c_3 x + c_2$$

33.3 problem Ex 3

Internal problem ID [11324]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 57. Dependent variable absent. Page 132
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + xy' = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+x*diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sqrt{\pi}\, \sqrt{2}\, \operatorname{erf}\left(\frac{\sqrt{2}\, x}{2}\right)}{2} + x + c_2$$

✓ Solution by Mathematica

Time used: 0.137 (sec). Leaf size: 29

```
DSolve[y''[x]+x*y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{\frac{\pi}{2}} c_1 \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right) + x + c_2$$

33.4 problem Ex 4

Internal problem ID [11325]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 57. Dependent variable absent. Page 132
Problem number: Ex 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _quadrature]]`

$$\boxed{y'' = x\,\mathrm{e}^{x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x$2)=x*exp(x),y(x), singsol=all)
```

$$y(x) = (x-2)\,\mathrm{e}^{x} + xc_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 19

```
DSolve[y''[x]==x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x}(x-2) + c_2 x + c_1$$

33.5 problem Ex 5

Internal problem ID [11326]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 57. Dependent variable absent. Page 132
Problem number: Ex 5.
ODE order: 2.
ODE degree: 2.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(y' - xy'')^2 - y''^2 = 1}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 63

```
dsolve((diff(y(x),x)-x*diff(y(x),x$2))^2=1+diff(y(x),x$2)^2,y(x), singsol=all)
```

$$y(x) = \frac{x\sqrt{-x^2+1}}{2} + \frac{\arcsin(x)}{2} + c_1$$
$$y(x) = -\frac{x\sqrt{-x^2+1}}{2} - \frac{\arcsin(x)}{2} + c_1$$
$$y(x) = \frac{x^2\sqrt{c_1^2-1}}{2} + xc_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.215 (sec). Leaf size: 58

```
DSolve[(y'[x]-x*y''[x])^2==1+(y''[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1x^2}{2} - \sqrt{1+c_1{}^2}x + c_2$$
$$y(x) \to \frac{c_1x^2}{2} + \sqrt{1+c_1{}^2}x + c_2$$

34 Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 58. Independent variable absent. Page 135

34.1 problem Ex 1

Internal problem ID [11327]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 58. Independent variable absent. Page 135
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _with_potential_symmet`

$$\boxed{yy'' - y'^2 - y'y^2 = 0}$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 32

```
dsolve(y(x)*diff(y(x),x$2)-diff(y(x),x)^2-y(x)^2*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = -\frac{c_1 \mathrm{e}^{c_2 c_1} \mathrm{e}^{x c_1}}{-1 + \mathrm{e}^{c_2 c_1} \mathrm{e}^{x c_1}}$$

✓ Solution by Mathematica

Time used: 2.444 (sec). Leaf size: 43

```
DSolve[y[x]*y''[x]-y'[x]^2-y[x]^2*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{c_1 e^{c_1(x+c_2)}}{-1 + e^{c_1(x+c_2)}}$$
$$y(x) \to -\frac{1}{x + c_2}$$

34.2 problem Ex 2

Internal problem ID [11328]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 58. Independent variable absent. Page 135
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{yy'' - y'^2 = -1}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 79

```
dsolve(y(x)*diff(y(x),x$2)-diff(y(x),x)^2+1=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1\left(\mathrm{e}^{-\frac{2x}{c_1}}\,\mathrm{e}^{-\frac{2c_2}{c_1}} - 1\right)\mathrm{e}^{\frac{x}{c_1}}\,\mathrm{e}^{\frac{c_2}{c_1}}}{2}$$
$$y(x) = \frac{c_1\left(\mathrm{e}^{\frac{2x}{c_1}}\,\mathrm{e}^{\frac{2c_2}{c_1}} - 1\right)\mathrm{e}^{-\frac{x}{c_1}}\,\mathrm{e}^{-\frac{c_2}{c_1}}}{2}$$

✓ Solution by Mathematica

Time used: 60.222 (sec). Leaf size: 85

```
DSolve[y[x]*y''[x]-y'[x]^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ie^{-c_1}\tanh\left(e^{c_1}(x + c_2)\right)}{\sqrt{-\operatorname{sech}^2\left(e^{c_1}(x + c_2)\right)}}$$
$$y(x) \to \frac{ie^{-c_1}\tanh\left(e^{c_1}(x + c_2)\right)}{\sqrt{-\operatorname{sech}^2\left(e^{c_1}(x + c_2)\right)}}$$

34.3 problem Ex 3

Internal problem ID [11329]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 58. Independent variable absent. Page 135
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1]]`

$$\boxed{2y'' - e^y = 0}$$

✓ Solution by Maple

Time used: 0.609 (sec). Leaf size: 22

```
dsolve(2*diff(y(x),x$2)=exp(y(x)),y(x), singsol=all)
```

$$y(x) = \ln\left(\frac{\tan\left(\frac{x+c_2}{2c_1}\right)^2 + 1}{c_1^2}\right)$$

✓ Solution by Mathematica

Time used: 60.049 (sec). Leaf size: 30

```
DSolve[2*y''[x]==Exp[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(-c_1 \mathrm{sech}^2\left(\frac{1}{2}\sqrt{c_1(x+c_2)^2}\right)\right)$$

34.4 problem Ex 4

Internal problem ID [11330]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 58. Independent variable absent. Page 135
Problem number: Ex 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_x_y1],`

$$\boxed{yy'' + 2y' - y'^2 = 0}$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 23

```
dsolve(y(x)*diff(y(x),x$2)+2*diff(y(x),x)-diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{e^{c_2 c_1} e^{x c_1} - 2}{c_1}$$

✓ Solution by Mathematica

Time used: 2.726 (sec). Leaf size: 26

```
DSolve[y[x]*y''[x]+2*y'[x]-y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2 + e^{c_1(x+c_2)}}{c_1}$$
$$y(x) \to \text{Indeterminate}$$

35 Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 59. Linear equations with particular integral known. Page 136

35.1 problem Ex 1

Internal problem ID [11331]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 59. Linear equations with particular integral known. Page 136
Problem number: Ex 1.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{(x^2-2x+2)\, y''' - x^2 y'' + 2y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 17

```
dsolve((x^2-2*x+2)*diff(y(x),x$3)-x^2*diff(y(x),x$2)+2*x*diff(y(x),x)-2*y(x)=0,y(x), singsol
```

$$y(x) = xc_1 + c_2x^2 + c_3\mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 27

```
DSolve[(x^2-2*x+2)*y'''[x]-x^2*y''[x]+2*x*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{2}\left(c_2x^2 + 2c_1x + c_3e^x\right)$$

35.2 problem Ex 2

Internal problem ID [11332]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 59. Linear equations with particular integral known. Page 136
Problem number: Ex 2.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$\boxed{xy''' - y'' - y'x + y = -x^2 + 1}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 22

```
dsolve(x*diff(y(x),x$3)-diff(y(x),x$2)-x*diff(y(x),x)+y(x)=1-x^2,y(x), singsol=all)
```

$$y(x) = x^2 + 3 + xc_1 + c_2 e^x + c_3 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.242 (sec). Leaf size: 28

```
DSolve[x*y'''[x]-y''[x]-x*y'[x]+y[x]==1-x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + c_1 x - c_2 \cosh(x) + ic_3 \sinh(x) + 3$$

36 Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 60. Exact equation. Integrating factor. Page 139

36.1 problem Ex 1

Internal problem ID [11333]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 60. Exact equation. Integrating factor. Page 139
Problem number: Ex 1.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{(x+2)^2 y''' + (x+2) y'' + y' = 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
dsolve((x+2)^2*diff(y(x),x$3)+(x+2)*diff(y(x),x$2)+diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = c_1 \left(\frac{\cos(\ln(x+2))(x+2)}{2} + \frac{(x+2)\sin(\ln(x+2))}{2}\right) + c_2 \left(-\frac{\cos(\ln(x+2))(x+2)}{2} + \frac{(x+2)\sin(\ln(x+2))}{2}\right) + x + c_3$$

✓ Solution by Mathematica

Time used: 0.202 (sec). Leaf size: 45

```
DSolve[(x+2)^2*y'''[x]+(x+2)*y''[x]+y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \frac{1}{2}(c_1 - c_2)(x+2)\cos(\log(x+2)) + \frac{1}{2}(c_1 + c_2)(x+2)\sin(\log(x+2)) + c_3$$

36.2 problem Ex 2

Internal problem ID [11334]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 60. Exact equation. Integrating factor. Page 139
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{x^2y'' + 3xy' + y = x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=x,y(x), singsol=all)
```

$$y(x) = \frac{c_2}{x} + \frac{x}{4} + \frac{\ln(x)\, c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 26

```
DSolve[x^2*y''[x]+3*x*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2 + 4c_2 \log(x) + 4c_1}{4x}$$

36.3 problem Ex 3

Internal problem ID [11335]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 60. Exact equation. Integrating factor. Page 139
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$\boxed{(x-1)^2 y'' + 4(x-1)\, y' + 2y = \cos(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve((x-1)^2*diff(y(x),x$2)+4*(x-1)*diff(y(x),x)+2*y(x)=cos(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 x}{(x-1)^2} - \frac{\cos(x)}{(x-1)^2} + \frac{c_2}{(x-1)^2}$$

✓ Solution by Mathematica

Time used: 0.134 (sec). Leaf size: 24

```
DSolve[(x-1)^2*y''[x]+4*(x-1)*y'[x]+2*y[x]==Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-\cos(x) + c_1(x-1) + c_2}{(x-1)^2}$$

36.4 problem Ex 4

Internal problem ID [11336]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 60. Exact equation. Integrating factor. Page 139
Problem number: Ex 4.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _fully, _exact, _linear]]`

$$\boxed{(x^3-x)\,y''' + (8x^2-3)\,y'' + 14y'x + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve((x^3-x)*diff(y(x),x$3)+(8*x^2-3)*diff(y(x),x$2)+14*x*diff(y(x),x)+4*y(x)=0,y(x), sing
```

$$y(x) = \frac{c_3}{\sqrt{x-1}\,\sqrt{x+1}\,x} + \frac{c_1}{x} + \frac{c_2 \ln\left(x+\sqrt{x^2-1}\right)}{x\sqrt{x^2-1}}$$

✓ Solution by Mathematica

Time used: 0.135 (sec). Leaf size: 51

```
DSolve[(x^3-x)*y'''[x]+(8*x^2-3)*y''[x]+14*x*y'[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{-\frac{c_2}{\sqrt{x^2-1}} + \frac{c_3 \log\left(\sqrt{x^2-1}-x\right)}{\sqrt{x^2-1}} + c_1}{x}$$

36.5 problem Ex 5

Internal problem ID [11337]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 60. Exact equation. Integrating factor. Page 139
Problem number: Ex 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _exact, _nonlinear], [_3rd_order, _with_linear_s`

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 56

```
dsolve(2*x^3*y(x)*diff(y(x),x$3)+6*x^3*diff(y(x),x)*diff(y(x),x$2)+18*x^2*y(x)*diff(y(x),x$2
```

$$y(x) = 0$$
$$y(x) = \frac{\sqrt{-x\left(x^2c_1 + 2c_2x - 2c_3\right)}}{x^2}$$
$$y(x) = -\frac{\sqrt{-x\left(x^2c_1 + 2c_2x - 2c_3\right)}}{x^2}$$

✓ Solution by Mathematica

Time used: 0.389 (sec). Leaf size: 60

```
DSolve[2*x^3*y[x]*y'''[x]+6*x^3*y'[x]*y''[x]+18*x^2*y[x]*y''[x]+18*x^2*y'[x]^2+36*x*y[x]*y'[
```

$$y(x) \to -\frac{\sqrt{c_1x^2 + c_3x + 2c_2}}{x^{3/2}}$$
$$y(x) \to \frac{\sqrt{c_1x^2 + c_3x + 2c_2}}{x^{3/2}}$$

36.6 problem Ex 6

Internal problem ID [11338]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 60. Exact equation. Integrating factor. Page 139
Problem number: Ex 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^5 y'' + (2x^4 - x)\, y' - (2x^3 - 1)\, y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x^5*diff(y(x),x$2)+(2*x^4-x)*diff(y(x),x)-(2*x^3-1)*y(x)=0,y(x), singsol=all)
```

$$y(x) = xc_1 + c_2 x\, \mathrm{e}^{-\frac{1}{3x^3}}$$

✓ Solution by Mathematica

Time used: 0.152 (sec). Leaf size: 22

```
DSolve[x^5*y''[x]+(2*x^4-x)*y'[x]-(2*x^3-1)*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(c_2 e^{-\frac{1}{3x^3}} + c_1\right)$$

36.7 problem Ex 7

Internal problem ID [11339]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 60. Exact equation. Integrating factor. Page 139
Problem number: Ex 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2(-x^3+1)\,y''-y'x^3-2y=0$$

✗ Solution by Maple

```
dsolve(x^2*(1-x^3)*diff(y(x),x$2)-x^3*diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2*(1-x^3)*y''[x]-x^3*y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

36.8 problem Ex 8

Internal problem ID [11340]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 60. Exact equation. Integrating factor. Page 139
Problem number: Ex 8.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _with_linear_symmetries]]`

$$x^2y''' - 5xy'' + (4x^4+5)\,y' - 8yx^3 = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x$3)-5*x*diff(y(x),x$2)+(4*x^4+5)*diff(y(x),x)-8*x^3*y(x)=0,y(x), sings
```

$$y(x) = x^2c_1 + c_2\cos\left(x^2\right) + c_3\sin\left(x^2\right)$$

✓ Solution by Mathematica

Time used: 0.507 (sec). Leaf size: 44

```
DSolve[x^2*y'''[x]-5*x*y''[x]+(4*x^4+5)*y'[x]-8*x^3*y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to c_1x^2 + \frac{1}{2}ic_2e^{-ix^2} - \frac{1}{8}c_3e^{ix^2}$$

36.9 problem Ex 10

Internal problem ID [11341]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 60. Exact equation. Integrating factor. Page 139
Problem number: Ex 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + 2\cot(x)\, y' + 2\tan(x)\, {y'}^{2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)+2*cot(x)*diff(y(x),x)+2*tan(x)*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = -\frac{\mathrm{e}^{\frac{c_1}{2}}\, \mathrm{Ei}_1\left(\ln\left(\tan(x)\right) + \frac{c_1}{2}\right)}{2} + c_2$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y''[x]+2*Cot[x]*y'[x]+2*Tan[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

37 Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 61. Transformation of variables. Page 143

37.1 problem Ex 1

Internal problem ID [11342]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906

Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 61. Transformation of variables. Page 143

Problem number: Ex 1.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$x^2yy'' + (y'x - y)^2 = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 39

```
dsolve(x^2*y(x)*diff(y(x),x$2)+(x*diff(y(x),x)-y(x))^2=0,y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \sqrt{-2x^2c_1 + 2c_2x}$$
$$y(x) = -\sqrt{-2x^2c_1 + 2c_2x}$$

✓ Solution by Mathematica

Time used: 0.388 (sec). Leaf size: 23

```
DSolve[x^2*y[x]*y''[x]+(x*y'[x]-y[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2\sqrt{x}\sqrt{2x + c_1}$$

37.2 problem Ex 2

Internal problem ID [11343]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 61. Transformation of variables. Page 143
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$\boxed{x^3y'' - (y'x - y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 19

```
dsolve(x^3*diff(y(x),x$2)-(x*diff(y(x),x)-y(x))^2=0,y(x), singsol=all)
```

$$y(x) = -x \ln \left(\frac{xc_1 - c_2}{x}\right)$$

✓ Solution by Mathematica

Time used: 1.65 (sec). Leaf size: 21

```
DSolve[x^3*y''[x]-(x*y'[x]-y[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x \log \left(-\frac{c_2x + c_1}{x}\right)$$

37.3 problem Ex 3

Internal problem ID [11344]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 61. Transformation of variables. Page 143
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _reducible, _mu_xy]]`

$$\boxed{yy'' - y'^2 - y^2 \ln(y) + y^2x^2 = 0}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 27

```
dsolve(y(x)*diff(y(x),x$2)-diff(y(x),x)^2=y(x)^2*ln(y(x))-x^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{\mathrm{e}^{-2x} c_1 \mathrm{e}^{x}}{2}} \mathrm{e}^{-\frac{c_2 \mathrm{e}^{x}}{2}} \mathrm{e}^{x^2} \mathrm{e}^{2}$$

✓ Solution by Mathematica

Time used: 1.156 (sec). Leaf size: 30

```
DSolve[y[x]*y''[x]-y'[x]^2==y[x]^2*Log[y[x]]-x^2*y[x]^2,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to e^{x^2 - \frac{c_1 e^x}{2} - c_2 e^{-x} + 2}$$

37.4 problem Ex 4

Internal problem ID [11345]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 61. Transformation of variables. Page 143
Problem number: Ex 4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\boxed{\sin(x)^2 y'' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.437 (sec). Leaf size: 57

```
dsolve(sin(x)^2*diff(y(x),x$2)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{c_1 \sin(2x)}{-1+\cos(2x)} + \frac{c_2(-i \ln(\cos(2x) + i \sin(2x)) \sin(2x) + 2\cos(2x) - 2)}{-1+\cos(2x)}$$

✓ Solution by Mathematica

Time used: 0.339 (sec). Leaf size: 46

```
DSolve[Sin[x]^2*y''[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\cos(x)\left(c_1 - c_2 \log\left(\sqrt{-\sin^2(x)} - \cos(x)\right)\right)}{\sqrt{-\sin^2(x)}} - c_2$$

38 Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144

38.1 problem Ex 1

Internal problem ID [11346]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' - y'^2 = 1}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 20

```
dsolve(diff(y(x),x$2)=diff(y(x),x)^2+1,y(x), singsol=all)
```

$$y(x) = -\ln\left(\frac{c_1 \tan(x) - c_2}{\sec(x)}\right)$$

✓ Solution by Mathematica

Time used: 3.079 (sec). Leaf size: 16

```
DSolve[y''[x]==y'[x]^2+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_2 - \log(\cos(x + c_1))$$

38.2 problem Ex 2

Internal problem ID [11347]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(-x^2+1)\,y''-xy'=2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 57

```
dsolve((1-x^2)*diff(y(x),x$2)-x*diff(y(x),x)=2,y(x), singsol=all)
```

$$y(x)=\int \frac{-2\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)\sqrt{x-1}\,\sqrt{x+1}+x^2c_1-c_1}{(x-1)^{\frac{3}{2}}\,(x+1)^{\frac{3}{2}}}dx+c_2$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 48

```
DSolve[(1-x^2)*y''[x]-x*y'[x]==2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_2-\frac{1}{4}\left(\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)-\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)+c_1\right)^2$$

38.3 problem Ex 3

Internal problem ID [11348]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _exact, _nonlinear], _`

$$\boxed{y'' + yy' = 0}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$2)+y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\tanh\left(\frac{(x+c_2)\sqrt{2}}{2c_1}\right)\sqrt{2}}{c_1}$$

✓ Solution by Mathematica

Time used: 20.03 (sec). Leaf size: 34

```
DSolve[y''[x]+y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{2}\sqrt{c_1}\tanh\left(\frac{\sqrt{c_1}(x+c_2)}{\sqrt{2}}\right)$$

38.4 problem Ex 4

Internal problem ID [11349]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 4.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _fully, _exact, _linear]]`

$$\boxed{(x^3+1)\,y''' + 9x^2y'' + 18y'x + 6y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 60

```
dsolve((1+x^3)*diff(y(x),x$3)+9*x^2*diff(y(x),x$2)+18*x*diff(y(x),x)+6*y(x)=0,y(x), singsol=
```

$$y(x) = \frac{x^2c_1}{(x+1)\,(x^2-x+1)} + \frac{xc_2}{(x+1)\,(x^2-x+1)} + \frac{c_3}{(x+1)\,(x^2-x+1)}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 31

```
DSolve[(1+x^3)*y'''[x]+9*x^2*y''[x]+18*x*y'[x]+6*y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{c_3x^2 + 2c_2x + 2c_1}{2x^3 + 2}$$

38.5 problem Ex 5

Internal problem ID [11350]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 5.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{(x^2 - x)\, y'' + (4x + 2)\, y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve((x^2-x)*diff(y(x),x$2)+(4*x+2)*diff(y(x),x)+2*y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{(12x^3 \ln(x) - 3x^4 + 18x^2 - 6x + 1)\, c_1}{(x-1)^5} + \frac{x^3 c_2}{(x-1)^5}$$

✓ Solution by Mathematica

Time used: 0.086 (sec). Leaf size: 52

```
DSolve[(x^2-x)*y''[x]+(4*x+2)*y'[x]+2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-3c_2x^4 - 3c_1x^3 + 12c_2x^3 \log(x) + 18c_2x^2 - 6c_2x + c_2}{3(x-1)^5}$$

38.6 problem Ex 6

Internal problem ID [11351]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$y(1-\ln(y))\,y''+(1+\ln(y))\,y'^2=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(y(x)*(1-ln(y(x)))*diff(y(x),x$2)+(1+ln(y(x)))*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{\frac{xc_1+c_2-1}{xc_1+c_2}}$$

✓ Solution by Mathematica

Time used: 1.021 (sec). Leaf size: 34

```
DSolve[y[x]*(1-Log[y[x]])*y''[x]+(1+Log[y[x]])*y'[x]^2==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to e^{\frac{c_1x-1+c_2c_1}{c_1(x+c_2)}}$$
$$y(x)\to e$$

38.7 problem Ex 7

Internal problem ID [11352]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{y'' + \frac{y'}{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x$2)+1/x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_2 \ln(x) + c_1$$

✓ Solution by Mathematica

Time used: 0.018 (sec). Leaf size: 13

```
DSolve[y''[x]+1/x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \log(x) + c_2$$

38.8 problem Ex 8

Internal problem ID [11353]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _nonlinear], [_2nd_order, _reducible, _m`

$$x(x + 2y)\, y'' + 2xy'^2 + 4(x + y)\, y' + 2y = -x^2$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 75

```
dsolve(x*(x+2*y(x))*diff(y(x),x$2)+2*x*(diff(y(x),x))^2+4*(x+y(x))*diff(y(x),x)+2*y(x)+x^2=0
```

$$y(x) = \frac{-3x^2 + \sqrt{-3x^5 + 9x^4 - 36x^2c_1 + 36c_2x}}{6x}$$
$$y(x) = -\frac{3x^2 + \sqrt{-3x^5 + 9x^4 - 36x^2c_1 + 36c_2x}}{6x}$$

✓ Solution by Mathematica

Time used: 2.35 (sec). Leaf size: 104

```
DSolve[x*(x+2*y[x])*y''[x]+2*x*(y'[x])^2+4*(x+y[x])*y'[x]+2*y[x]+x^2==0,y[x],x,IncludeSingul
```

$$y(x) \to \frac{1}{6}\left(-3x - \sqrt{3}\sqrt{\frac{1}{x^2}}\sqrt{x\left(-x^4 + 3x^3 + 12c_2x + 12c_1\right)}\right)$$
$$y(x) \to \frac{1}{6}\left(-3x + \sqrt{3}\sqrt{\frac{1}{x^2}}\sqrt{x\left(-x^4 + 3x^3 + 12c_2x + 12c_1\right)}\right)$$

38.9 problem Ex 9

Internal problem ID [11354]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 9.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], [_2nd_order, _reducible, _mu_xy]]`

$$\boxed{y'' + y'^2 = -1}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$2)+diff(y(x),x)^2+1=0,y(x), singsol=all)
```

$$y(x) = \ln\left(-\frac{c_1 \tan(x) - c_2}{\sec(x)}\right)$$

✓ Solution by Mathematica

Time used: 3.113 (sec). Leaf size: 16

```
DSolve[y''[x]+y'[x]^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log(\cos(x - c_1)) + c_2$$

38.10 problem Ex 10

Internal problem ID [11355]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 10.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_y]]`

$$\boxed{(-x^2+1)\,y'' - \frac{y'}{x} = -x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve((1-x^2)*diff(y(x),x$2)-1/x*diff(y(x),x)+x^2=0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} + \sqrt{x-1}\,\sqrt{x+1}\,c_1 + c_2$$

✓ Solution by Mathematica

Time used: 0.084 (sec). Leaf size: 30

```
DSolve[(1-x^2)*y''[x]-1/x*y'[x]+x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} - c_1\sqrt{1-x^2} + c_2$$

38.11 problem Ex 11

Internal problem ID [11356]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 11.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{4x^2y''' + 8xy'' + y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(4*x^2*diff(y(x),x$3)+8*x*diff(y(x),x$2)+diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2\sqrt{x} + c_3\sqrt{x}\,\ln(x)$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 28

```
DSolve[4*x^2*y'''[x]+8*x*y''[x]+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}(c_2\log(x) + 2c_1 - 2c_2) + c_3$$

38.12 problem Ex 12

Internal problem ID [11357]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter IX, Miscellaneous methods for solving equations of higher order than first. Article 62. Summary. Page 144
Problem number: Ex 12.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$\boxed{\sin(x)\,y'' - y'\cos(x) + 2\sin(x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 36

```
dsolve(sin(x)*diff(y(x),x$2)-cos(x)*diff(y(x),x)+2*sin(x)*y(x)=0,y(x), singsol=all)
```

$$y(x) = \sin(x)^2 c_1 + c_2 \sin(x)^2 (\ln(\cos(x)+1) - \ln(\cos(x)-1) + 2\csc(x)\cot(x))$$

✓ Solution by Mathematica

Time used: 0.235 (sec). Leaf size: 45

```
DSolve[Sin[x]*y''[x]-Cos[x]*y'[x]+2*Sin[x]*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -c_1 \sin^2(x) - \frac{1}{4} c_2 (2\cos(x) + \sin^2(x)(\log(\cos(x)+1) - \log(1-\cos(x))))$$

39 Chapter X, System of simulataneous equations. Article 64. Systems of linear equations with constant coefficients. Page 150

39.1 problem Ex 1

Internal problem ID [11358]

Book: An elementary treatise on differential equations by Abraham Cohen. DC heath publishers. 1906
Section: Chapter X, System of simulataneous equations. Article 64. Systems of linear equations with constant coefficients. Page 150
Problem number: Ex 1.
ODE order: 1.
ODE degree: 1.

Solve

$$
\begin{aligned}
x'(t) &= -x(t) - \frac{2y(t)}{3} + \frac{e^t}{3} \\
y'(t) &= \frac{4x(t)}{3} + y(t) - t
\end{aligned}
$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 47

```
dsolve([3*diff(x(t),t)+3*x(t)+2*y(t)=exp(t),4*x(t)-3*diff(y(t),t)+3*y(t)=3*t],[x(t), y(t)],
```

$$
x(t) = -e^{-\frac{t}{3}}c_2 - \frac{e^{\frac{t}{3}}c_1}{2} - 6t
$$

$$
y(t) = e^{-\frac{t}{3}}c_2 + e^{\frac{t}{3}}c_1 + 9t + 9 + \frac{e^t}{2}
$$

✓ Solution by Mathematica

Time used: 1.125 (sec). Leaf size: 90

```
DSolve[{3*x'[t]+3*x[t]+2*y[t]==Exp[t],4*x[t]-3*y'[t]+3*y[t]==3*t},{x[t],y[t]},t,IncludeSingu
```

$$
\begin{aligned}
x(t) &\to e^{-t/3}\left(-6e^{t/3}t - (c_1 + c_2)e^{2t/3} + 2c_1 + c_2\right) \\
y(t) &\to 9(t+1) + \frac{e^t}{2} + 2(c_1 + c_2)e^{t/3} - (2c_1 + c_2)e^{-t/3}
\end{aligned}
$$