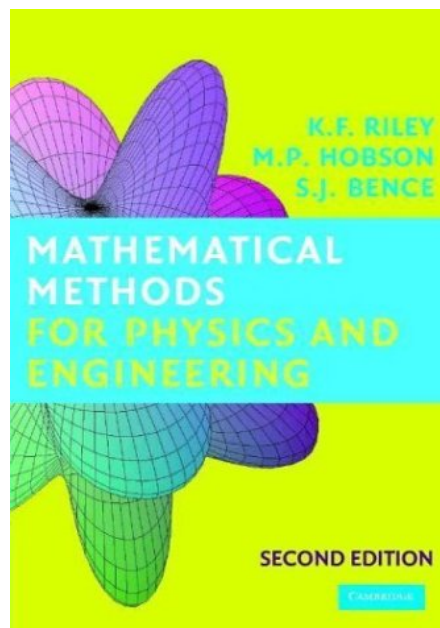**A Solution Manual For**

Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002

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March 3, 2024

Contents

1 Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490

1.1 problem Problem 14.2 (a)

Internal problem ID [2486]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.2 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$ y' - y^3 x = 0 $$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)-x*y(x)^3=0,y(x), singsol=all)
```

$$ y(x) = \frac{1}{\sqrt{-x^2 + c_1}} $$

$$ y(x) = -\frac{1}{\sqrt{-x^2 + c_1}} $$

✓ Solution by Mathematica

Time used: 0.17 (sec). Leaf size: 44

```
DSolve[y'[x]-x*y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to -\frac{1}{\sqrt{-x^2 - 2c_1}} $$

$$ y(x) \to \frac{1}{\sqrt{-x^2 - 2c_1}} $$

$$ y(x) \to 0 $$

3

1.2 problem Problem 14.2 (b)

Internal problem ID [2487]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002

Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490

Problem number: Problem 14.2 (b).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$\frac{y'}{\tan(x)} - \frac{y}{x^2+1} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)/tan(x)-y(x)/(1+x^2)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{\int \frac{\tan(x)}{x^2+1} dx}$$

✓ Solution by Mathematica

Time used: 9.987 (sec). Leaf size: 34

```
DSolve[y'[x]/Tan[x]-y[x]/(1+x^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \exp\left(\int_1^x \frac{\tan(K[1])}{K[1]^2+1} dK[1]\right)$$

$$y(x) \to 0$$

1.3 problem Problem 14.2 (c)

Internal problem ID [2488]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.2 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y'x^2 + xy^2 - 4y^2 = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)+x*y(x)^2=4*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x}{4 + x\ln(x) + c_1 x}$$

✓ Solution by Mathematica
Time used: 0.146 (sec). Leaf size: 25

```
DSolve[y'[x]+x*y[x]^2==4*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{x^2 - 8x - 2c_1}$$
$$y(x) \to 0$$

1.4 problem Problem 14.3 (a)

Internal problem ID [2489]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.3 (a).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_exact, _rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]

$$y\left(2y^2x^2 + 1\right)y' + x\left(y^4 + 1\right) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 119

```
dsolve(y(x)*(2*x^2*y(x)^2+1)*diff(y(x),x)+x*(y(x)^4+1)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2 - 2\sqrt{-4x^4 - 8c_1x^2 + 1}}}{2x}$$

$$y(x) = \frac{\sqrt{-2 - 2\sqrt{-4x^4 - 8c_1x^2 + 1}}}{2x}$$

$$y(x) = -\frac{\sqrt{2}\sqrt{-1 + \sqrt{-4x^4 - 8c_1x^2 + 1}}}{2x}$$

$$y(x) = \frac{\sqrt{2}\sqrt{-1 + \sqrt{-4x^4 - 8c_1x^2 + 1}}}{2x}$$

6

✓ Solution by Mathematica

Time used: 10.416 (sec). Leaf size: 197

```
DSolve[y[x]*(2*x^2*y[x]^2+1)*y'[x]+x*(y[x]^4+1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\sqrt{-\frac{1+\sqrt{-4x^4+8c_1x^2+1}}{x^2}}}{\sqrt{2}}$$

$$y(x) \rightarrow \frac{\sqrt{-\frac{1+\sqrt{-4x^4+8c_1x^2+1}}{x^2}}}{\sqrt{2}}$$

$$y(x) \rightarrow -\frac{\sqrt{\frac{-1+\sqrt{-4x^4+8c_1x^2+1}}{x^2}}}{\sqrt{2}}$$

$$y(x) \rightarrow \frac{\sqrt{\frac{-1+\sqrt{-4x^4+8c_1x^2+1}}{x^2}}}{\sqrt{2}}$$

$$y(x) \rightarrow -\sqrt[4]{-1}$$

$$y(x) \rightarrow \sqrt[4]{-1}$$

$$y(x) \rightarrow -(-1)^{3/4}$$

$$y(x) \rightarrow (-1)^{3/4}$$

1.5 problem Problem 14.3 (b)

Internal problem ID [2490]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.3 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$2xy' + y = -3x$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 13

```
dsolve(2*x*diff(y(x),x)+3*x+y(x)=0,y(x), singsol=all)
```

$$y(x) = -x + \frac{c_1}{\sqrt{x}}$$

✓ Solution by Mathematica
Time used: 0.027 (sec). Leaf size: 17

```
DSolve[2*x*y'[x]+3*x+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + \frac{c_1}{\sqrt{x}}$$

1.6 problem Problem 14.3 (c)

Internal problem ID [2491]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002

Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490

Problem number: Problem 14.3 (c).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [[_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Abel, '2nd ty

$$\left(\cos\left(x\right)^2 + y\sin\left(2x\right)\right) y' + y^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve((cos(x)^2+y(x)*sin(2*x))*diff(y(x),x)+y(x)^2=0,y(x), singsol=all)
```

$$c_1 + y(x)^2\tan\left(x\right) + y(x) = 0$$

✓ Solution by Mathematica

Time used: 23.536 (sec). Leaf size: 170

```
DSolve[(Cos[x]^2+y[x]*Sin[2*x])*y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\cot(x)}{2}$$
$$-\frac{\csc(2x)\sqrt{e^{-\operatorname{arctanh}(\cos(2x))}}\left(4c_1\sin(2x)e^{\operatorname{arctanh}(\cos(2x))} + \csc(2x) + (\cos(2x) + 2)\cot(2x)\right)}{2\sqrt{\csc(2x)e^{-\operatorname{arctanh}(\cos(2x))}}}$$

$$y(x) \to -\frac{\cot(x)}{2}$$
$$+\frac{\csc(2x)\sqrt{e^{-\operatorname{arctanh}(\cos(2x))}}\left(4c_1\sin(2x)e^{\operatorname{arctanh}(\cos(2x))} + \csc(2x) + (\cos(2x) + 2)\cot(2x)\right)}{2\sqrt{\csc(2x)e^{-\operatorname{arctanh}(\cos(2x))}}}$$

$$y(x) \to 0$$

1.7 problem Problem 14.5 (a)

Internal problem ID [2492]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002

Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490

Problem number: Problem 14.5 (a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_linear]

$$\left(1 - x^2\right) y' + 4yx = \left(1 - x^2\right)^{\frac{3}{2}}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve((1-x^2)*diff(y(x),x)+2*x*y(x)+2*x*y(x)=(1-x^2)^(3/2),y(x), singsol=all)
```

$$y(x) = \left(x^4 - 2x^2 + 1\right) c_1 - \sqrt{-x^2 + 1}\, x \left(x^2 - 1\right)$$

✓ Solution by Mathematica

Time used: 0.116 (sec). Leaf size: 29

```
DSolve[(1-x^2)*y'[x]+2*x*y[x]+2*x*y[x]==(1-x^2)^(3/2),y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \left(x^2 - 1\right)^2 \left(\frac{x}{\sqrt{1 - x^2}} + c_1\right)$$

10

1.8 problem Problem 14.5 (b)

Internal problem ID [2493]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002

Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490

Problem number: Problem 14.5 (b).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_linear]

$$y' - y \cot(x) = -\frac{1}{\sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)-y(x)*cot(x)+1/sin(x)=0,y(x), singsol=all)
```

$$y(x) = (\cot(x) + c_1) \sin(x)$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 13

```
DSolve[y'[x]-y[x]*Cot[x]+1/Sin[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \cos(x) + c_1 \sin(x)$$

1.9 problem Problem 14.5 (c)

Internal problem ID [2494]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.5 (c).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$\left(x + y^3\right) y' - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 260

```
dsolve((x+y(x)^3)*diff(y(x),x)=y(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}}{3} - \frac{2c_1}{\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}}{6} + \frac{c_1}{\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}}$$
$$- \frac{i\sqrt{3}\left(\frac{\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}}{3} + \frac{2c_1}{\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}}{6} + \frac{c_1}{\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}}$$
$$+ \frac{i\sqrt{3}\left(\frac{\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}}{3} + \frac{2c_1}{\left(27x + 3\sqrt{24c_1^3 + 81x^2}\right)^{\frac{1}{3}}}\right)}{2}$$

Time used: 1.757 (sec). Leaf size: 263

```
DSolve[(x+y[x]^3)*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2 \cdot 3^{2/3} c_1 - \sqrt[3]{3}\left(-9x + \sqrt{81x^2 + 24c_1{}^3}\right)^{2/3}}{3\sqrt[3]{-9x + \sqrt{81x^2 + 24c_1{}^3}}}$$

$$y(x) \to \frac{\sqrt[3]{3}\left(1 - i\sqrt{3}\right)\left(-9x + \sqrt{81x^2 + 24c_1{}^3}\right)^{2/3} - 2\sqrt[6]{3}\left(\sqrt{3} + 3i\right)c_1}{6\sqrt[3]{-9x + \sqrt{81x^2 + 24c_1{}^3}}}$$

$$y(x) \to \frac{\sqrt[3]{3}\left(1 + i\sqrt{3}\right)\left(-9x + \sqrt{81x^2 + 24c_1{}^3}\right)^{2/3} - 2\sqrt[6]{3}\left(\sqrt{3} - 3i\right)c_1}{6\sqrt[3]{-9x + \sqrt{81x^2 + 24c_1{}^3}}}$$

$$y(x) \to 0$$

1.10 problem Problem 14.6

Internal problem ID [2495]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002

Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490

Problem number: Problem 14.6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_rational, _Bernoulli]

$$y' + \frac{2x^2 + y^2 + x}{yx} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(diff(y(x),x) = - (2*x^2+y(x)^2+x)/(x*y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-9x^4 - 6x^3 + 9c_1}}{3x}$$

$$y(x) = \frac{\sqrt{-9x^4 - 6x^3 + 9c_1}}{3x}$$

✓ Solution by Mathematica

Time used: 0.251 (sec). Leaf size: 56

```
DSolve[y'[x] == - (2*x^2+y[x]^2+x)/(x*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^4 - \frac{2x^3}{3} + c_1}}{x}$$

$$y(x) \to \frac{\sqrt{-x^4 - \frac{2x^3}{3} + c_1}}{x}$$

1.11 problem Problem 14.11

Internal problem ID [2496]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cl

$$(y - x) y' + 3y = -2x$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 28

```
dsolve((y(x)-x)*diff(y(x),x)+2*x+3*y(x)=0,y(x), singsol=all)
```

$$y(x) = \tan\left(\text{RootOf}\left(-4\_Z + \ln\left(\frac{1}{\cos(\_Z)^2}\right) + 2\ln(x) + 2c_1\right)\right)x - x$$

✓ Solution by Mathematica
Time used: 0.037 (sec). Leaf size: 45

```
DSolve[(y[x]-x)*y'[x]+2*x+3*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{y(x)^2}{x^2} + \frac{2y(x)}{x} + 2\right) - 2\arctan\left(\frac{y(x)}{x} + 1\right) = -\log(x) + c_1, y(x)\right]$$

1.12 problem Problem 14.14

Internal problem ID [2497]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002

Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [[_homogeneous, 'class C'], [_Abel, '2nd type', 'class C'], _d

$$y' - \frac{1}{x + 2y + 1} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) = 1/(x+2*y(x)+1),y(x), singsol=all)
```

$$y(x) = -\,\mathrm{LambertW}\left(-\frac{c_1 \mathrm{e}^{-\frac{x}{2}-\frac{3}{2}}}{2}\right) - \frac{x}{2} - \frac{3}{2}$$

✓ Solution by Mathematica

Time used: 60.047 (sec). Leaf size: 34

```
DSolve[y'[x] == 1/(x+2*y[x]+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-2W\left(-\frac{1}{2}c_1 e^{-\frac{x}{2}-\frac{3}{2}}\right) - x - 3\right)$$

1.13 problem Problem 14.15

Internal problem ID [2498]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl

$$y' + \frac{y + x}{3x + 3y - 4} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) = - (x+y(x))/(3*x+3*y(x)-4),y(x), singsol=all)
```

$$y(x) = e^{-\operatorname{LambertW}\left(\frac{3\,e^x e^{-3} e^{-c_1}}{2}\right) + x - 3 - c_1} + 2 - x$$

✓ Solution by Mathematica

Time used: 3.788 (sec). Leaf size: 33

```
DSolve[y'[x] == - (x+y[x])/(3*x+3*y[x]-4),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{3}W\left(-e^{x-1+c_1}\right) - x + 2$$
$$y(x) \to 2 - x$$

1.14 problem Problem 14.16

Internal problem ID [2499]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - \tan(x)\cos(y)(\cos(y) + \sin(y)) = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 18

```
dsolve(diff(y(x),x) = tan(x)*cos(y(x))*( cos(y(x)) + sin(y(x)) ),y(x), singsol=all)
```

$$y(x) = -\arctan\left(\frac{\cos(x) - c_1}{\cos(x)}\right)$$

Solution by Mathematica

Time used: 60.547 (sec). Leaf size: 143

```
DSolve[y'[x]==Tan[x]*Cos[y[x]]*( Cos[y[x]] + Sin[y[x]] ),y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\arccos\left(-\frac{\cos(x)}{\sqrt{\cos(2x) - 2e^{\frac{c_1}{2}}\cos(x) + 1 + e^{c_1}}}\right)$$

$$y(x) \to \arccos\left(-\frac{\cos(x)}{\sqrt{\cos(2x) - 2e^{\frac{c_1}{2}}\cos(x) + 1 + e^{c_1}}}\right)$$

$$y(x) \to -\arccos\left(\frac{\cos(x)}{\sqrt{\cos(2x) - 2e^{\frac{c_1}{2}}\cos(x) + 1 + e^{c_1}}}\right)$$

$$y(x) \to \arccos\left(\frac{\cos(x)}{\sqrt{\cos(2x) - 2e^{\frac{c_1}{2}}\cos(x) + 1 + e^{c_1}}}\right)$$

1.15 problem Problem 14.17

Internal problem ID [2500]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002

Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490

Problem number: Problem 14.17.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [[_homogeneous, 'class G'], _exact, _rational, [_Abel, '2nd ty

$$x\left(1 - 2yx^2\right)y' + y - 3y^2x^2 = 0$$

With initial conditions

$$\left[y(1) = \frac{1}{2}\right]$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 35

```
dsolve([x*(1-2*x^2*y(x))*diff(y(x),x) +y(x) = 3*x^2*y(x)^2,y(1) = 1/2],y(x), singsol=all)
```

$$y(x) = \frac{1 - \sqrt{1 - x}}{2x^2}$$

$$y(x) = \frac{1 + \sqrt{1 - x}}{2x^2}$$

✓ Solution by Mathematica

Time used: 0.599 (sec). Leaf size: 53

```
DSolve[{x*(1-2*x^2*y[x])*y'[x] +y[x] == 3*x^2*y[x]^2,y[1]==1/2},y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{x - \sqrt{-((x-1)x^2)}}{2x^3}$$

$$y(x) \to \frac{\sqrt{-((x-1)x^2)} + x}{2x^3}$$

1.16 problem Problem 14.23 (a)

Internal problem ID [2501]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.23 (a) .
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$ y' + \frac{xy}{a^2 + x^2} = x $$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)+ (x*y(x))/(a^2+x^2)=x,y(x), singsol=all)
```

$$ y(x) = \frac{a^2}{3} + \frac{x^2}{3} + \frac{c_1}{\sqrt{a^2 + x^2}} $$

✓ Solution by Mathematica
Time used: 0.043 (sec). Leaf size: 31

```
DSolve[y'[x]+ (x*y[x])/(a^2+x^2)==x,y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to \frac{1}{3}\left(a^2 + x^2\right) + \frac{c_1}{\sqrt{a^2 + x^2}} $$

1.17 problem Problem 14.23 (b)

Internal problem ID [2502]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002

Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490

Problem number: Problem 14.23 (b) .

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_separable]

$$y' - \frac{4y^2}{x^2} + y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)= 4*y(x)^2/x^2 - y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x}{c_1 x + x^2 + 4}$$

✓ Solution by Mathematica

Time used: 0.15 (sec). Leaf size: 24

```
DSolve[y'[x]== 4*y[x]^2/x^2 - y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{x^2 - c_1 x + 4}$$

$$y(x) \to 0$$

1.18 problem Problem 14.24 (a)

Internal problem ID [2503]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.24 (a) .
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$\boxed{y' - \frac{y}{x} = 1}$$

With initial conditions

$$[y(1) = -1]$$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 10

```
dsolve([diff(y(x),x)-y(x)/x=1,y(1) = -1],y(x), singsol=all)
```

$$y(x) = (-1 + \ln(x)) x$$

✓ Solution by Mathematica
Time used: 0.025 (sec). Leaf size: 11

```
DSolve[{y'[x]-y[x]/x==1,y[1]==-1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(\log(x) - 1)$$

1.19 problem Problem 14.24 (b)

Internal problem ID [2504]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.24 (b) .
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$\boxed{y' - y\tan(x) = 1}$$

With initial conditions

$$\left[y\left(\frac{\pi}{4}\right) = 3 \right]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 13

```
dsolve([diff(y(x),x)-y(x)*tan(x)=1,y(1/4*Pi) = 3],y(x), singsol=all)
```

$$y(x) = \tan(x) + \sec(x)\sqrt{2}$$

✓ Solution by Mathematica
Time used: 0.049 (sec). Leaf size: 16

```
DSolve[{y'[x]-y[x]*Tan[x]==1,y[Pi/4]==3},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\sin(x) + \sqrt{2}\right)\sec(x)$$

1.20 problem Problem 14.24 (c)

Internal problem ID [2505]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002

Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.24 (c) .
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Riccati]`

$$y' - \frac{y^2}{x^2} = \frac{1}{4}$$

With initial conditions

$$[y(1) = 1]$$

✔ Solution by Maple

Time used: 0.047 (sec). Leaf size: 17

```
dsolve([diff(y(x),x)-y(x)^2/x^2=1/4,y(1) = 1],y(x), singsol=all)
```

$$y(x) = \frac{x(\ln(x) - 4)}{2\ln(x) - 4}$$

✔ Solution by Mathematica

Time used: 0.132 (sec). Leaf size: 20

```
DSolve[{y'[x]-y[x]^2/x^2==1/4,y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x(\log(x) - 4)}{2(\log(x) - 2)}$$

1.21 problem Problem 14.24 (d)

Internal problem ID [2506]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.24 (d) .
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [[_homogeneous, 'class A'], _rational, _Riccati]

$$ y' - \frac{y^2}{x^2} = \frac{1}{4} $$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)-y(x)^2/x^2=1/4,y(x), singsol=all)
```

$$ y(x) = \frac{x(\ln(x) + c_1 - 2)}{2\ln(x) + 2c_1} $$

✓ Solution by Mathematica

Time used: 0.096 (sec). Leaf size: 36

```
DSolve[y'[x]-y[x]^2/x^2==1/4,y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to \frac{x(\log(x) - 2 + 4c_1)}{2(\log(x) + 4c_1)} $$

$$ y(x) \to \frac{x}{2} $$

1.22 problem Problem 14.26

Internal problem ID [2507]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [_linear]

$$\sin(x)\, y' + 2y\cos(x) = 1$$

With initial conditions

$$\left[y\left(\frac{\pi}{2}\right) = 1\right]$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 10

```
dsolve([sin(x)*diff(y(x),x)+2*y(x)*cos(x)=1,y(1/2*Pi) = 1],y(x), singsol=all)
```

$$y(x) = \frac{1}{\cos(x) + 1}$$

✓ Solution by Mathematica
Time used: 0.046 (sec). Leaf size: 14

```
DSolve[{Sin[x]*y'[x]+2*y[x]*Cos[x]==1,y[Pi/2]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan\left(\frac{x}{2}\right)\csc(x)$$

1.23 problem Problem 14.28

Internal problem ID [2508]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, [_Abel, '2nd type', 'cl`

$$(5x + y - 7) y' - 3y = 3x + 3$$

✓ Solution by Maple
Time used: 0.563 (sec). Leaf size: 327

```
dsolve((5*x+y(x)-7)*diff(y(x),x)=3*(x+y(x)+1),y(x), singsol=all)
```

$$y(x) = -3$$

$$+ \frac{144(-2+x)\left(-\frac{\left(1-216(-2+x)^2 c_1 + 12\sqrt{324(-2+x)^4 c_1^2 - 3(-2+x)^2 c_1}\right)^{\frac{1}{3}}}{24} - \frac{1}{24\left(1-216(-2+x)^2 c_1 + 12\sqrt{324(-2-}}\right)}{-6\left(1 - 216\left(-2+x\right)^2 c_1 + 12\sqrt{324\left(-2+x\right)^4 c_1^2 - 3\left(-2+x\right)^2 c_1}\right)^{\frac{1}{3}} - \frac{6}{\left(1-216(-2+x)^2 c_1 + 12\sqrt{324(-2+x)^4}}}$$

✓ Solution by Mathematica
Time used: 60.172 (sec). Leaf size: 1626

```
DSolve[(5*x+y[x]-7)*y'[x]==3*(x+y[x]+1),y[x],x,IncludeSingularSolutions -> True]
```

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1.24 problem Problem 14.29

Internal problem ID [2509]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [[_homogeneous, 'class G'], _rational, _Bernoulli]

$$ xy' + y - \frac{y^2}{x^{\frac{3}{2}}} = 0 $$

With initial conditions

$$ [y(1) = 1] $$

✓ Solution by Maple
Time used: 0.062 (sec). Leaf size: 18

```
dsolve([x*diff(y(x),x)+y(x)-y(x)^2/x^(3/2)=0,y(1) = 1],y(x), singsol=all)
```

$$ y(x) = \frac{5x^{\frac{3}{2}}}{3x^{\frac{5}{2}} + 2} $$

✓ Solution by Mathematica
Time used: 0.162 (sec). Leaf size: 23

```
DSolve[{x*y'[x]+y[x]-y[x]^2/x^(3/2)==0,y[1]==1},y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to \frac{5x^{3/2}}{3x^{5/2} + 2} $$

1.25 problem Problem 14.30 (a)

Internal problem ID [2510]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002

Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490

Problem number: Problem 14.30 (a).

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$(2\sin(y) - x)\, y' - \tan(y) = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve([(2*sin(y(x))-x)*diff(y(x),x)=tan(y(x)),y(0) = 0],y(x), singsol=all)
```

$$y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 6

```
DSolve[{(2*Sin[y[x]]-x)*y'[x]==Tan[y[x]],y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 0$$

1.26   problem Problem 14.30 (b)

Internal problem ID [2511]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.30 (b).
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$(2\sin(y) - x)\, y' - \tan(y) = 0$$

With initial conditions

$$\left[y(0) = \frac{\pi}{2} \right]$$

✓   Solution by Maple

Time used: 6.969 (sec). Leaf size: 18

```
dsolve([(2*sin(y(x))-x)*diff(y(x),x)=tan(y(x)),y(0) = 1/2*Pi],y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{x}{2} + \frac{\sqrt{x^2 + 4}}{2} \right)$$

✓   Solution by Mathematica

Time used: 18.018 (sec). Leaf size: 67

```
DSolve[{(2*Sin[y[x]]-x)*y'[x]==Tan[y[x]],y[0]==Pi/2},y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \cot^{-1}\left(\sqrt{\frac{x^2}{2} - \frac{1}{2}\sqrt{x^4 + 4x^2}} \right)$$

$$y(x) \to \cot^{-1}\left(\frac{\sqrt{x^2 + \sqrt{x^2(x^2 + 4)}}}{\sqrt{2}} \right)$$

1.27 problem Problem 14.31

Internal problem ID [2512]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 14, First order ordinary differential equations. 14.4 Exercises, page 490
Problem number: Problem 14.31.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x], _Liouville, [_2nd_order, _reducible`

$$y'' + y'^2 + y' = 0$$

With initial conditions

$$[y(0) = 0]$$

✓ Solution by Maple
Time used: 0.078 (sec). Leaf size: 18

```
dsolve([diff(y(x),x$2)+ (diff(y(x),x))^2+diff(y(x),x)=0,y(0) = 0],y(x), singsol=all)
```

$$y(x) = \ln \left(e^x c_2 - c_2 + 1 \right) - x$$

✓ Solution by Mathematica
Time used: 0.395 (sec). Leaf size: 54

```
DSolve[{y''[x]+(y'[x])^2+y'[x]==0,y[0]==0},y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log \left(-e^x \right) - \log \left(e^x \right) - i\pi$$
$$y(x) \to -\log \left(e^x \right) + \log \left(-e^x + e^{c_1} \right) - \log \left(-1 + e^{c_1} \right)$$

2 Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523

2.1 problem Problem 15.1

Internal problem ID [2513]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$x'' + \omega_0^2 x = a \cos(\omega t)$$

With initial conditions

$$[x(0) = 0, x'(0) = 0]$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 28

```
dsolve([diff(x(t),t$2)+ (omega__0)^2*x(t)=a*cos(omega*t),x(0) = 0, D(x)(0) = 0],x(t), singso
```

$$x(t) = \frac{a(\cos(\omega_0 t) - \cos(\omega t))}{\omega^2 - \omega_0^2}$$

✓ Solution by Mathematica

Time used: 0.371 (sec). Leaf size: 33

```
DSolve[{x''[t]+(Subscript[\[Omega],0])^2*x[t]==a*Cos[\[Omega]*t],{x[0]==0,x'[0]==0}},x[t],t,
```

$$x(t) \to \frac{a(\cos(t\omega_0) - \cos(t\omega))}{\omega^2 - \omega_0^2}$$

2.2 problem Problem 15.2(a)

Internal problem ID [2514]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.2(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _missing_x]]`

$$\boxed{f'' + 2f' + 5f = 0}$$

With initial conditions

$$[f(0) = 1, f'(0) = 0]$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve([diff(f(t),t$2)+2*diff(f(t),t)+5*f(t)=0,f(0) = 1, D(f)(0) = 0],f(t), singsol=all)
```

$$f(t) = \frac{e^{-t}(\sin(2t) + 2\cos(2t))}{2}$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 25

```
DSolve[{f''[t]+2*f'[t]+5*f[t]==0,{f[0]==1,f'[0]==0}},f[t],t,IncludeSingularSolutions -> True
```

$$f(t) \rightarrow \frac{1}{2}e^{-t}(\sin(2t) + 2\cos(2t))$$

2.3 problem Problem 15.2(b)

Internal problem ID [2515]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.2(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$f'' + 2f' + 5f = e^{-t}\cos(3t)$$

With initial conditions

$$[f(0) = 0, f'(0) = 0]$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve([diff(f(t),t$2)+2*diff(f(t),t)+5*f(t)=exp(-t)*cos(3*t),f(0) = 0, D(f)(0) = 0],f(t), s
```

$$f(t) = -\frac{\left(4\cos(t)^3 - 2\cos(t)^2 - 3\cos(t) + 1\right)e^{-t}}{5}$$

✓ Solution by Mathematica

Time used: 0.118 (sec). Leaf size: 34

```
DSolve[{f''[t]+2*f'[t]+5*f[t]==Exp[-t]*Cos[3*t],{f[0]==0,f'[0]==0}},f[t],t,IncludeSingularSo
```

$$f(t) \rightarrow \frac{2}{5}e^{-t}\sin^2\left(\frac{t}{2}\right)(2\cos(t) + 2\cos(2t) + 1)$$

2.4 problem Problem 15.4

Internal problem ID [2516]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$f'' + 6f' + 9f = \mathrm{e}^{-t}$$

With initial conditions

$$[f(0) = 0, f'(0) = \lambda]$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 26

```
dsolve([diff(f(t),t$2)+6*diff(f(t),t)+9*f(t)=exp(-t),f(0) = 0, D(f)(0) = lambda],f(t), sings
```

$$f(t) = \frac{(-1 + (4\lambda - 2)\, t)\, \mathrm{e}^{-3t}}{4} + \frac{\mathrm{e}^{-t}}{4}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 28

```
DSolve[{f''[t]+6*f'[t]+9*f[t]==Exp[-t],{f[0]==0,f'[0]==\[Lambda]}},f[t],t,IncludeSingularSol
```

$$f(t) \to \frac{1}{4} e^{-3t} \left((4\lambda - 2)t + e^{2t} - 1 \right)$$

2.5 problem Problem 15.5(a)

Internal problem ID [2517]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.5(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$\boxed{f'' + 8f' + 12f = 12\,e^{-4t}}$$

With initial conditions

$$[f(0) = 0, f'(0) = 0]$$

✓ Solution by Maple
Time used: 0.015 (sec). Leaf size: 23

```
dsolve([diff(f(t),t$2)+8*diff(f(t),t)+12*f(t)=12*exp(-4*t),f(0) = 0, D(f)(0) = 0],f(t), sing
```

$$f(t) = \frac{3\,e^{-6t}}{2} + \frac{3\,e^{-2t}}{2} - 3\,e^{-4t}$$

✓ Solution by Mathematica
Time used: 0.022 (sec). Leaf size: 23

```
DSolve[{f''[t]+8*f'[t]+12*f[t]==12*Exp[-4*t],{f[0]==0,f'[0]==0}},f[t],t,IncludeSingularSolut
```

$$f(t) \to \frac{3}{2}e^{-6t}\left(e^{2t} - 1\right)^2$$

39

2.6 problem Problem 15.5(b)

Internal problem ID [2518]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.5(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$f'' + 8f' + 12f = 12\,e^{-4t}$$

With initial conditions

$$[f(0) = 0, f'(0) = -2]$$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 21

```
dsolve([diff(f(t),t$2)+8*diff(f(t),t)+12*f(t)=12*exp(-4*t),f(0) = 0, D(f)(0) = -2],f(t), sin
```

$$f(t) = 2\,e^{-6t} + e^{-2t} - 3\,e^{-4t}$$

✓ Solution by Mathematica
Time used: 0.019 (sec). Leaf size: 25

```
DSolve[{f''[t]+8*f'[t]+12*f[t]==12*Exp[-4*t],{f[0]==0,f'[0]==-2}},f[t],t,IncludeSingularSolu
```

$$f(t) \to e^{-6t}\left(-3e^{2t} + e^{4t} + 2\right)$$

2.7 problem Problem 15.7

Internal problem ID [2519]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.7.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$y'' + 2y' + y = 4\,e^{-x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+y(x)=4*exp(-x),y(x), singsol=all)
```

$$y(x) = e^{-x}c_2 + x\,e^{-x}c_1 + 2x^2 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 23

```
DSolve[y''[x]+2*y'[x]+y[x]==4*Exp[-x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(2x^2 + c_2 x + c_1\right)$$

2.8 problem Problem 15.9(a)

Internal problem ID [2520]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.9(a).
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type [[_3rd_order, _with_linear_symmetries]]

$$ y''' - 12y' + 16y = 32x - 8 $$

✓ Solution by Maple
Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x$3)-12*diff(y(x),x)+16*y(x)=32*x-8,y(x), singsol=all)
```

$$ y(x) = 1 + 2x + c_1 \mathrm{e}^{-4x} + c_2 \mathrm{e}^{2x} + c_3 \mathrm{e}^{2x} x $$

✓ Solution by Mathematica
Time used: 0.004 (sec). Leaf size: 35

```
DSolve[y'''[x]-12*y'[x]+16*y[x]==32*x-8,y[x],x,IncludeSingularSolutions -> True]
```

$$ y(x) \to c_1 e^{-4x} + c_2 e^{2x} + x\left(2 + c_3 e^{2x}\right) + 1 $$

2.9 problem Problem 15.9(b)

Internal problem ID [2521]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.9(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries], [_2nd_order, _reducibl`

$$
0 = \frac{y'^2}{y^2} - \frac{y''}{y} - \frac{2a \coth (2ax) \, y'}{y} + 2a^2
$$

Time used: 0.125 (sec). Leaf size: 407

```
dsolve(diff( 1/y(x)*diff(y(x),x),x)+(2*a*coth(2*a*x))*(1/y(x)*diff(y(x),x))=2*a^2,y(x), sing
```

$$y(x) = -\frac{2^{\frac{3}{4}} e^{-ax} e^{-\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\left(e^{4ax}+1\right)e^{-2ax}}{2}\right)}{c_1 a}} e^{-\frac{c_2}{c_1}} \left(\left(e^{8ax} - 2\,e^{4ax} + 1\right) e^{\frac{3\sqrt{2}\,\text{arctanh}\left(\frac{\left(e^{4ax}+1\right)e^{-2ax}}{2}\right)}{c_1 a}}\right)^{\frac{1}{4}}}{2}$$

$$y(x) = \frac{2^{\frac{3}{4}} e^{-ax} e^{-\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\left(e^{4ax}+1\right)e^{-2ax}}{2}\right)}{c_1 a}} e^{-\frac{c_2}{c_1}} \left(\left(e^{8ax} - 2\,e^{4ax} + 1\right) e^{\frac{3\sqrt{2}\,\text{arctanh}\left(\frac{\left(e^{4ax}+1\right)e^{-2ax}}{2}\right)}{c_1 a}}\right)^{\frac{1}{4}}}{2}$$

$$y(x) = -\frac{i\,2^{\frac{3}{4}} e^{-ax} e^{-\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\left(e^{4ax}+1\right)e^{-2ax}}{2}\right)}{c_1 a}} e^{-\frac{c_2}{c_1}} \left(\left(e^{8ax} - 2\,e^{4ax} + 1\right) e^{\frac{3\sqrt{2}\,\text{arctanh}\left(\frac{\left(e^{4ax}+1\right)e^{-2ax}}{2}\right)}{c_1 a}}\right)^{\frac{1}{4}}}{2}$$

$$y(x) = \frac{i\,2^{\frac{3}{4}} e^{-ax} e^{-\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\left(e^{4ax}+1\right)e^{-2ax}}{2}\right)}{c_1 a}} e^{-\frac{c_2}{c_1}} \left(\left(e^{8ax} - 2\,e^{4ax} + 1\right) e^{\frac{3\sqrt{2}\,\text{arctanh}\left(\frac{\left(e^{4ax}+1\right)e^{-2ax}}{2}\right)}{c_1 a}}\right)^{\frac{1}{4}}}{2}$$

Solution by Mathematica

Time used: 60.504 (sec). Leaf size: 287

```
DSolve[D[1/y[x]*y'[x],x]+(2*a*Coth[1/y[x]*y'[x]])==2*a^2,y[x],x,IncludeSingularSolutions ->
```

$y(x)$

$$\to c_2 \exp\left(\frac{-\text{PolyLog}\left(2, \dfrac{(a+1)\exp\left(-2\text{InverseFunction}\left[\dfrac{-((a+1)\log(1-\tanh(\#1)))+(a-1)\log(\tanh(\#1)+1)+2\log(1-a\tanh(\#1))}{2(a^2-1)}\right.\right.}{a-1}\right.}{}\right.$$

2.10 problem Problem 15.21

Internal problem ID [2522]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002

Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523

Problem number: Problem 15.21.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$x^2 y'' - xy' + y = x$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x$2)-x*diff(y(x),x)+y(x)=x,y(x), singsol=all)
```

$$y(x) = c_2 x + \ln(x) c_1 x + \frac{\ln(x)^2 x}{2}$$

✓ Solution by Mathematica

Time used: 0.019 (sec). Leaf size: 25

```
DSolve[x^2*y''[x]-x*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} x \left(\log^2(x) + 2c_2 \log(x) + 2c_1 \right)$$

2.11 problem Problem 15.22

Internal problem ID [2523]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002

Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523

Problem number: Problem 15.22.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _nonhomogeneous]]`

$$(x+1)^2 y'' + 3(x+1) y' + y = x^2$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve((x+1)^2*diff(y(x),x$2)+3*(x+1)*diff(y(x),x)+y(x)=x^2,y(x), singsol=all)
```

$$y(x) = \frac{\ln(x+1) c_1}{x+1} + \frac{c_2}{x+1} - \frac{-2x^3 + 3x^2 + 6\ln(x+1) - 6x}{18(x+1)}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 44

```
DSolve[(x+1)^2*y''[x]+3*(x+1)*y'[x]+y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^3 - 3x^2 + 6x + 6(-1 + 3c_2)\log(x+1) + 18c_1}{18(x+1)}$$

2.12 problem Problem 15.23

Internal problem ID [2524]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.23.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries], [_2nd_order, _linear,

$$(x - 2) y'' + 3y' + \frac{4y}{x^2} = 0$$

✓ Solution by Maple
Time used: 0.016 (sec). Leaf size: 30

```
dsolve((x-2)*diff(y(x),x$2)+3*diff(y(x),x)+4*y(x)/x^2=0,y(x), singsol=all)
```

$$y(x) = \frac{(3x - 4) c_1}{x (-2 + x)^2} + \frac{x^2 c_2}{(-2 + x)^2}$$

✓ Solution by Mathematica
Time used: 0.074 (sec). Leaf size: 45

```
DSolve[(x-2)*y''[x]+3*y'[x]+4*y[x]/x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{6c_1 x^3 + 3c_2 x - 4c_2}{6\sqrt{2 - x}(x - 2)^{3/2} x}$$

48

2.13 problem Problem 15.24(a)

Internal problem ID [2525]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.24(a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$y'' - y = x^n$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 81

```
dsolve(diff(y(x),x$2)-y(x)=x^n,y(x), singsol=all)
```

$$y(x) = e^{-x}c_2 + c_1 e^x$$
$$+ \frac{e^{-x}\left(-\left((n\Gamma(n,-x) - \Gamma(n+1))(-x)^{-n} + e^x\right)x^n(n+1) + e^{\frac{3x}{2}}x^{\frac{n}{2}}\text{WhittakerM}\left(\frac{n}{2},\frac{n}{2}+\frac{1}{2},x\right)\right)}{2n+2}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 58

```
DSolve[y''[x]-y[x]==x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}e^{-x}x^n(-x)^{-n}\Gamma(n+1,-x) - \frac{1}{2}e^x\Gamma(n+1,x) + c_1 e^x + c_2 e^{-x}$$

2.14 problem Problem 15.24(b)

Internal problem ID [2526]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.24(b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + y = 2x\,\mathrm{e}^x$$

✔ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+y(x)=2*x*exp(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^x c_2 + x\,\mathrm{e}^x c_1 + \frac{\mathrm{e}^x x^3}{3}$$

✔ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 25

```
DSolve[y''[x]-2*y'[x]+y[x]==2*x*Exp[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{3}\mathrm{e}^x \left(x^3 + 3c_2 x + 3c_1\right)$$

2.15 problem Problem 15.33

Internal problem ID [2527]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002

Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523

Problem number: Problem 15.33.

ODE order: 3.

ODE degree: 1.

CAS Maple gives this as type [[_3rd_order, _exact, _nonlinear]]

Solve

$$2yy''' + 2(y + 3y') y'' + 2(y')^2 - \sin(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 97

```
dsolve(2*y(x)*diff(y(x),x$3)+2*(y(x)+3*diff(y(x),x))*diff(y(x),x$2)+2*(diff(y(x),x))^2=sin(x
```

$$y(x) = -\frac{e^{-x}\sqrt{2}\sqrt{e^x\left(-4x\,e^x c_1 + \cos(x)\,e^x - \sin(x)\,e^x + 4c_1 e^x - 4c_3 e^x + 4c_2\right)}}{2}$$

$$y(x) = \frac{e^{-x}\sqrt{2}\sqrt{e^x\left(-4x\,e^x c_1 + \cos(x)\,e^x - \sin(x)\,e^x + 4c_1 e^x - 4c_3 e^x + 4c_2\right)}}{2}$$

✓ Solution by Mathematica

Time used: 0.473 (sec). Leaf size: 88

```
DSolve[2*y[x]*y'''[x]+2*(y[x]+3*y'[x])*y''[x]+2*(y'[x])^2==Sin[x],y[x],x,IncludeSingularSolu
```

$$y(x) \to -\frac{\sqrt{-\sin(x) + \cos(x) + 2c_1 x + 2c_3 e^{-x} - 2c_1 - 4c_2}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-\sin(x) + \cos(x) + 2c_1 x + 2c_3 e^{-x} - 2c_1 - 4c_2}}{\sqrt{2}}$$

2.16 problem Problem 15.34

Internal problem ID [2528]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.34.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$xy''' + 2y'' = Ax$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(x*diff(y(x),x$3)+2*diff(y(x),x$2)=A*x,y(x), singsol=all)
```

$$y(x) = \frac{A x^3}{18} - \ln(x) c_1 + c_2 x + c_3$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 26

```
DSolve[x*y'''[x]+2*y''[x]==A*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{Ax^3}{18} + c_3 x - c_1 \log(x) + c_2$$

2.17 problem Problem 15.35

Internal problem ID [2529]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 15, Higher order ordinary differential equations. 15.4 Exercises, page 523
Problem number: Problem 15.35.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _linear, _nonhomogeneous]]

$$ y'' + 4xy' + \left(4x^2 + 6\right) y = e^{-x^2} \sin\left(2x\right) $$

✓ Solution by Maple
Time used: 0.031 (sec). Leaf size: 42

```
dsolve(diff(y(x),x$2)+4*x*diff(y(x),x)+(4*x^2+6)*y(x)=exp(-x^2)*sin(2*x),y(x), singsol=all)
```

$$ y(x) = e^{-x^2} \cos\left(2x\right) c_2 + e^{-x^2} \sin\left(2x\right) c_1 - \frac{e^{-x^2} x \cos\left(2x\right)}{4} $$

✓ Solution by Mathematica
Time used: 0.13 (sec). Leaf size: 52

```
DSolve[y''[x]+4*x*y'[x]+(4*x^2+6)*y[x]==Exp[-x^2]*Sin[2*x],y[x],x,IncludeSingularSolutions -
```

$$ y(x) \rightarrow \frac{1}{32} e^{-x(x+2i)} \left(-4x - e^{4ix}\left(4x + i + 8ic_2\right) + i + 32c_1\right) $$

3 Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550

3.1 problem Problem 16.1

Internal problem ID [2530]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.1.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [_Gegenbauer]

$$\left(-z^2+1\right)y'' - 3zy' + \lambda y = 0$$

With the expansion point for the power series method at $z=0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
Order:=6;
dsolve((1-z^2)*diff(y(z),z$2)-3*z*diff(y(z),z)+lambda*y(z)=0,y(z),type='series',z=0);
```

$$y(z) = \left(1 - \frac{\lambda z^2}{2} + \frac{\lambda(\lambda-8)z^4}{24}\right)y(0)$$
$$+ \left(z - \frac{(\lambda-3)z^3}{6} + \frac{(\lambda-3)(\lambda-15)z^5}{120}\right)D(y)(0) + O\left(z^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 80

```
AsymptoticDSolveValue[(1-z^2)*y''[z]-3*z*y'[z]+\[Lambda]*y[z]==0,y[z],{z,0,5}]
```

$$y(z) \to c_2\left(\frac{\lambda^2 z^5}{120} - \frac{3\lambda z^5}{20} + \frac{3z^5}{8} - \frac{\lambda z^3}{6} + \frac{z^3}{2} + z\right) + c_1\left(\frac{\lambda^2 z^4}{24} - \frac{\lambda z^4}{3} - \frac{\lambda z^2}{2} + 1\right)$$

3.2 problem Problem 16.2

Internal problem ID [2531]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.2.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$4zy'' + 2(1-z)\,y' - y = 0$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 44

```
Order:=6;
dsolve(4*z*diff(y(z),z$2)+2*(1-z)*diff(y(z),z)-y(z)=0,y(z),type='series',z=0);
```

$$y(z) = c_1 \sqrt{z} \left(1 + \frac{1}{3}z + \frac{1}{15}z^2 + \frac{1}{105}z^3 + \frac{1}{945}z^4 + \frac{1}{10395}z^5 + \mathrm{O}\left(z^6\right) \right)$$
$$+ c_2 \left(1 + \frac{1}{2}z + \frac{1}{8}z^2 + \frac{1}{48}z^3 + \frac{1}{384}z^4 + \frac{1}{3840}z^5 + \mathrm{O}\left(z^6\right) \right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 85

```
AsymptoticDSolveValue[4*z*y''[z]+2*(1-z)*y'[z]-y[z]==0,y[z],{z,0,5}]
```

$$y(z) \to c_1 \sqrt{z} \left(\frac{z^5}{10395} + \frac{z^4}{945} + \frac{z^3}{105} + \frac{z^2}{15} + \frac{z}{3} + 1 \right) + c_2 \left(\frac{z^5}{3840} + \frac{z^4}{384} + \frac{z^3}{48} + \frac{z^2}{8} + \frac{z}{2} + 1 \right)$$

3.3 problem Problem 16.3

Internal problem ID [2532]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.3.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_Emden, _Fowler], [_2nd_order, _linear, '_with_symmetry_[0,F

$$zy'' - 2y' + 9z^5 y = 0$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
Order:=7;
dsolve(z*diff(y(z),z$2)-2*diff(y(z),z)+9*z^5*y(z)=0,y(z),type='series',z=0);
```

$$y(z) = c_1 z^3 \left(1 - \frac{1}{6} z^6 + O\left(z^7\right)\right) + c_2 \left(12 - 6z^6 + O\left(z^7\right)\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 12

```
AsymptoticDSolveValue[z*y''[z]-2*y'[z]+9*z^5*y[z]==0,y[z],{z,0,6}]
```

$$y(z) \rightarrow c_2 z^3 + c_1$$

3.4 problem Problem 16.4

Internal problem ID [2533]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002

Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.4.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$f'' + 2(z - 1) f' + 4f = 0$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
Order:=6;
dsolve(diff(f(z),z$2)+2*(z-1)*diff(f(z),z)+4*f(z)=0,f(z),type='series',z=0);
```

$$f(z) = \left(1 - 2z^2 - \frac{4}{3}z^3 + \frac{2}{3}z^4 + \frac{14}{15}z^5\right) f(0) + \left(z + z^2 - \frac{1}{3}z^3 - \frac{5}{6}z^4 - \frac{1}{6}z^5\right) D(f)(0) + O(z^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 127

```
AsymptoticDSolveValue[f''[z]+2*(z-a)*f'[z]+4*f[z]==0,f[z],{z,0,5}]
```

$$
\begin{aligned}
f(z) \to c_1 & \left(-\frac{4}{15}a^3 z^5 - \frac{2a^2 z^4}{3} + \frac{6az^5}{5} - \frac{4az^3}{3} + \frac{4z^4}{3} - 2z^2 + 1\right) \\
& + c_2 \left(\frac{2a^4 z^5}{15} + \frac{a^3 z^4}{3} - \frac{4a^2 z^5}{5} + \frac{2a^2 z^3}{3} - \frac{7az^4}{6} + az^2 + \frac{z^5}{2} - z^3 + z\right)
\end{aligned}
$$

3.5 problem Problem 16.6

Internal problem ID [2534]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.6.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$z^2 y'' - \frac{3zy'}{2} + (z+1)\, y = 0$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
Order:=6;
dsolve(z^2*diff(y(z),z$2)-3/2*z*diff(y(z),z)+(1+z)*y(z)=0,y(z),type='series',z=0);
```

$$y(z) = c_1 \sqrt{z}\left(1 + 2z - 2z^2 + \frac{4}{9}z^3 - \frac{2}{45}z^4 + \frac{4}{1575}z^5 + \mathrm{O}\left(z^6\right)\right)$$
$$+ c_2 z^2 \left(1 - \frac{2}{5}z + \frac{2}{35}z^2 - \frac{4}{945}z^3 + \frac{2}{10395}z^4 - \frac{4}{675675}z^5 + \mathrm{O}\left(z^6\right)\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 84

```
AsymptoticDSolveValue[z^2*y''[z]-3/2*z*y'[z]+(1+z)*y[z]==0,y[z],{z,0,5}]
```

$$y(z) \to c_1 \left(-\frac{4z^5}{675675} + \frac{2z^4}{10395} - \frac{4z^3}{945} + \frac{2z^2}{35} - \frac{2z}{5} + 1\right) z^2$$
$$+ c_2 \left(\frac{4z^5}{1575} - \frac{2z^4}{45} + \frac{4z^3}{9} - 2z^2 + 2z + 1\right) \sqrt{z}$$

3.6 problem Problem 16.8

Internal problem ID [2535]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.8.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [_Lienard]

$$zy'' - 2y' + zy = 0$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
Order:=6;
dsolve(z*diff(y(z),z$2)-2*diff(y(z),z)+z*y(z)=0,y(z),type='series',z=0);
```

$$y(z) = c_1 z^3 \left(1 - \frac{1}{10} z^2 + \frac{1}{280} z^4 + \mathrm{O}\left(z^6\right) \right) + c_2 \left(12 + 6z^2 - \frac{3}{2} z^4 + \mathrm{O}\left(z^6\right) \right)$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 44

```
AsymptoticDSolveValue[z*y''[z]-2*y'[z]+z*y[z]==0,y[z],{z,0,5}]
```

$$y(z) \to c_1 \left(-\frac{z^4}{8} + \frac{z^2}{2} + 1 \right) + c_2 \left(\frac{z^7}{280} - \frac{z^5}{10} + z^3 \right)$$

3.7 problem Problem 16.9

Internal problem ID [2536]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002

Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550

Problem number: Problem 16.9.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _exact, _linear, _homogeneous]]`

$$y'' - 2zy' - 2y = 0$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
Order:=6;
dsolve(diff(y(z),z$2)-2*z*diff(y(z),z)-2*y(z)=0,y(z),type='series',z=0);
```

$$y(z) = \left(1 + z^2 + \frac{1}{2}z^4\right) y(0) + \left(z + \frac{2}{3}z^3 + \frac{4}{15}z^5\right) D(y)(0) + O(z^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 38

```
AsymptoticDSolveValue[y''[z]-2*z*y'[z]-2*y[z]==0,y[z],{z,0,5}]
```

$$y(z) \to c_2 \left(\frac{4z^5}{15} + \frac{2z^3}{3} + z\right) + c_1 \left(\frac{z^4}{2} + z^2 + 1\right)$$

3.8 problem Problem 16.10

Internal problem ID [2537]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002

Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550

Problem number: Problem 16.10.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type [_Jacobi]

$$z(1-z)\, y'' + (1-z)\, y' + \lambda y = 0$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 261

```
Order:=6;
dsolve(z*(1-z)*diff(y(z),z$2)+(1-z)*diff(y(z),z)+lambda*y(z)=0,y(z),type='series',z=0);
```

$$
\begin{aligned}
y(z) = & \left(2\lambda z + \left(\frac{1}{4}\lambda - \frac{3}{4}\lambda^2\right)z^2 + \left(-\frac{37}{108}\lambda^2 + \frac{2}{27}\lambda + \frac{11}{108}\lambda^3\right)z^3 \right. \\
& + \left(\frac{139}{1728}\lambda^3 - \frac{649}{3456}\lambda^2 + \frac{1}{32}\lambda - \frac{25}{3456}\lambda^4\right)z^4 \\
& \left. + \left(-\frac{13}{1600}\lambda^4 + \frac{8467}{144000}\lambda^3 - \frac{2527}{21600}\lambda^2 + \frac{2}{125}\lambda + \frac{137}{432000}\lambda^5\right)z^5 + \mathrm{O}\left(z^6\right)\right)c_2 \\
& + \left(1 - \lambda z + \frac{1}{4}(-1+\lambda)\lambda z^2 - \frac{1}{36}\lambda(\lambda^2 - 5\lambda + 4)z^3 + \frac{1}{576}\lambda(\lambda^3 - 14\lambda^2 + 49\lambda - 36)z^4 \right. \\
& \left. - \frac{1}{14400}\lambda(-1+\lambda)(\lambda-4)(\lambda-16)(\lambda-9)z^5 + \mathrm{O}\left(z^6\right)\right)(c_2 \ln(z) + c_1)
\end{aligned}
$$

```
AsymptoticDSolveValue[z*(1-z)*y''[z]+(1-z)*y'[z]+\[Lambda]*y[z]==0,y[z],{z,0,5}]
```

$$y(z) \rightarrow \left(\frac{1}{25}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \lambda \right)\lambda \right. \right.$$

$$- \frac{1}{16}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \lambda \right)\lambda - \lambda \right)\lambda - \lambda \right)z^5$$

$$+ \frac{1}{16}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \lambda \right)\lambda - \lambda \right)z^4$$

$$+ \frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \lambda \right)z^3 + \frac{1}{4}(\lambda^2 - \lambda)z^2 - \lambda z + 1 \right)c_1$$

$$+ c_2 \left(-\frac{2}{125}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \lambda \right)\lambda \right. \right.$$

$$- \frac{1}{16}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \lambda \right)\lambda - \lambda \right)\lambda - \lambda \right)z^5 + \frac{1}{25}\left(\frac{\lambda^3}{2} \right.$$

$$- 2\lambda^2 + \frac{1}{4}(\lambda^2 - \lambda)\lambda + \frac{2}{27}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \lambda \right)\lambda - \frac{1}{9}\left(\frac{\lambda^3}{2} - 2\lambda^2 + \frac{1}{4}(\lambda^2 - \lambda)\lambda \right)\lambda$$

$$+ \frac{1}{32}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \lambda \right)\lambda - \lambda \right)\lambda$$

$$- \frac{1}{16}\left(\frac{\lambda^3}{2} - 2\lambda^2 + \frac{1}{4}(\lambda^2 - \lambda)\lambda + \frac{2}{27}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \lambda \right)\lambda - \frac{1}{9}\left(\frac{\lambda^3}{2} - 2\lambda^2 + \frac{1}{4}(\lambda^2 - \lambda)\lambda \right)\lambda \right)\lambda \right)z^5$$

$$- \frac{1}{32}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \lambda \right)\lambda - \lambda \right)z^4 + \frac{1}{16}\left(\frac{\lambda^3}{2} - 2\lambda^2 \right.$$

$$+ \frac{1}{4}(\lambda^2 - \lambda)\lambda + \frac{2}{27}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \lambda \right)\lambda - \frac{1}{9}\left(\frac{\lambda^3}{2} - 2\lambda^2 + \frac{1}{4}(\lambda^2 - \lambda)\lambda \right)\lambda \right)z^4$$

$$- \frac{2}{27}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \lambda \right)z^3 + \frac{1}{9}\left(\frac{\lambda^3}{2} - 2\lambda^2 + \frac{1}{4}(\lambda^2 - \lambda)\lambda \right)z^3 - \frac{\lambda^2 z^2}{2}$$

$$- \frac{1}{4}(\lambda^2 - \lambda)z^2 + 2\lambda z$$

$$+ \left(\frac{1}{25}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \lambda \right)\lambda - \frac{1}{16}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \lambda \right) \right. \right. \right.$$

$$+ \frac{1}{16}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \lambda \right)\lambda - \lambda \right)z^4$$

$$+ \frac{1}{9}\left(\lambda^2 - \frac{1}{4}(\lambda^2 - \lambda)\lambda - \lambda \right)z^3 + \frac{1}{4}(\lambda^2 - \lambda)z^2 - \lambda z + 1 \right)\log(z) \right)$$

3.9 problem Problem 16.11

Internal problem ID [2538]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.11.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_2nd_order, _with_linear_symmetries]]

$$zy'' + (2z - 3)\, y' + \frac{4y}{z} = 0$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
Order:=6;
dsolve(z*diff(y(z),z$2)+(2*z-3)*diff(y(z),z)+4/z*y(z)=0,y(z),type='series',z=0);
```

$$y(z) = z^2 \left((c_2 \ln(z) + c_1) \left(1 - 4z + 6z^2 - \frac{16}{3}z^3 + \frac{10}{3}z^4 - \frac{8}{5}z^5 + O\left(z^6\right) \right) \right.$$
$$\left. + \left(6z - 13z^2 + \frac{124}{9}z^3 - \frac{173}{18}z^4 + \frac{374}{75}z^5 + O\left(z^6\right) \right) c_2 \right)$$

64

Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 116

```
AsymptoticDSolveValue[z*y''[z]+(2*z-3)*y'[z]+4/z*y[z]==0,y[z],{z,0,5}]
```

$$y(z) \rightarrow c_1 \left(-\frac{8z^5}{5} + \frac{10z^4}{3} - \frac{16z^3}{3} + 6z^2 - 4z + 1\right) z^2$$
$$+ c_2 \left(\left(\frac{374z^5}{75} - \frac{173z^4}{18} + \frac{124z^3}{9} - 13z^2 + 6z\right) z^2\right.$$
$$\left. + \left(-\frac{8z^5}{5} + \frac{10z^4}{3} - \frac{16z^3}{3} + 6z^2 - 4z + 1\right) z^2 \log(z)\right)$$

3.10 problem Problem 16.12 (a)

Internal problem ID [2539]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002

Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.12 (a).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _with_linear_symmetries]]`

$$\left(z^2 + 5z + 6\right) y'' + 2y = 0$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
Order:=6;
dsolve((z^2+5*z+6)*diff(y(z),z$2)+2*y(z)=0,y(z),type='series',z=0);
```

$$y(z) = \left(1 - \frac{1}{6}z^2 + \frac{5}{108}z^3 - \frac{13}{1296}z^4 + \frac{5}{2592}z^5\right) y(0)$$
$$+ \left(z - \frac{1}{18}z^3 + \frac{5}{216}z^4 - \frac{17}{2160}z^5\right) D(y)(0) + O\left(z^6\right)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[(z^2+5*z+6)*y''[z]+2*y[z]==0,y[z],{z,0,5}]
```

$$y(z) \to c_2\left(-\frac{17z^5}{2160} + \frac{5z^4}{216} - \frac{z^3}{18} + z\right) + c_1\left(\frac{5z^5}{2592} - \frac{13z^4}{1296} + \frac{5z^3}{108} - \frac{z^2}{6} + 1\right)$$

3.11 problem Problem 16.12 (b)

Internal problem ID [2540]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.12 (b).
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [[_Emden, _Fowler]]

$$\left(z^2 + 5z + 7\right) y'' + 2y = 0$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
Order:=6;
dsolve((z^2+5*z+7)*diff(y(z),z$2)+2*y(z)=0,y(z),type='series',z=0);
```

$$y(z) = \left(1 - \frac{1}{7}z^2 + \frac{5}{147}z^3 - \frac{11}{2058}z^4 + \frac{5}{14406}z^5\right) y(0)$$
$$+ \left(z - \frac{1}{21}z^3 + \frac{5}{294}z^4 - \frac{47}{10290}z^5\right) D(y)(0) + O(z^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 63

```
AsymptoticDSolveValue[(z^2+5*z+7)*y''[z]+2*y[z]==0,y[z],{z,0,5}]
```

$$y(z) \to c_2\left(-\frac{47z^5}{10290} + \frac{5z^4}{294} - \frac{z^3}{21} + z\right) + c_1\left(\frac{5z^5}{14406} - \frac{11z^4}{2058} + \frac{5z^3}{147} - \frac{z^2}{7} + 1\right)$$

3.12 problem Problem 16.13

Internal problem ID [2541]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002

Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550

Problem number: Problem 16.13.

ODE order: 2.

ODE degree: 1.

CAS Maple gives this as type `[[_Emden, _Fowler]]`

$$y'' + \frac{y}{z^3} = 0$$

With the expansion point for the power series method at $z = 0$.

✗   Solution by Maple

```
Order:=6;
dsolve(diff(y(z),z$2)+1/z^3*y(z)=0,y(z),type='series',z=0);
```

No solution found

✓   Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 222

```
AsymptoticDSolveValue[y''[z]+1/z^3*y[z]==0,y[z],{z,0,5}]
```

$$y(z) \to c_1 e^{-\frac{2i}{\sqrt{z}}} z^{3/4} \left(-\frac{468131288625iz^{9/2}}{8796093022208} + \frac{66891825iz^{7/2}}{4294967296} - \frac{72765iz^{5/2}}{8388608} + \frac{105iz^{3/2}}{8192} \right.$$
$$+ \frac{33424574007825z^5}{281474976710656} - \frac{14783093325z^4}{549755813888} + \frac{2837835z^3}{268435456} - \frac{4725z^2}{524288} + \frac{15z}{512} - \frac{3i\sqrt{z}}{16}$$
$$\left. +1 \right) + c_2 e^{\frac{2i}{\sqrt{z}}} z^{3/4} \left(\frac{468131288625iz^{9/2}}{8796093022208} - \frac{66891825iz^{7/2}}{4294967296} + \frac{72765iz^{5/2}}{8388608} - \frac{105iz^{3/2}}{8192} + \frac{33424574007825z^5}{281474976710656} - \frac{1}{5} \right.$$

3.13 problem Problem 16.14

Internal problem ID [2542]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [_Laguerre]

$$zy'' + (1 - z)\,y' + \lambda y = 0$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 309

```
Order:=6;
dsolve(z*diff(y(z),z$2)+(1-z)*diff(y(z),z)+lambda*y(z)=0,y(z),type='series',z=0);
```

$$
\begin{aligned}
y(z) = &\left((2\lambda + 1)\,z + \left(\frac{1}{4}\lambda + \frac{1}{4} - \frac{3}{4}\lambda^2 \right) z^2 + \left(-\frac{2}{9}\lambda^2 + \frac{1}{27}\lambda + \frac{1}{18} + \frac{11}{108}\lambda^3 \right) z^3 \right. \\
&\quad + \left(\frac{7}{192}\lambda^3 - \frac{167}{3456}\lambda^2 + \frac{1}{192}\lambda + \frac{1}{96} - \frac{25}{3456}\lambda^4 \right) z^4 \\
&+ \left(\frac{1}{1500}\lambda - \frac{37}{4320}\lambda^2 + \frac{719}{86400}\lambda^3 - \frac{61}{21600}\lambda^4 + \frac{137}{432000}\lambda^5 + \frac{1}{600} \right) z^5 + \mathrm{O}\left(z^6\right) \Bigg)\, c_2 \\
&+ \left(1 - \lambda z + \frac{1}{4}(-1 + \lambda)\,\lambda z^2 - \frac{1}{36}(\lambda - 2)(-1 + \lambda)\,\lambda z^3 \right. \\
&+ \frac{1}{576}(\lambda - 3)(\lambda - 2)(-1 + \lambda)\,\lambda z^4 - \frac{1}{14400}(\lambda - 4)(\lambda - 3)(\lambda - 2)(-1 + \lambda)\,\lambda z^5 \\
&\left. + \mathrm{O}\left(z^6\right) \right) (c_2 \ln(z) + c_1)
\end{aligned}
$$

Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 415

```
AsymptoticDSolveValue[z*y''[z]+(1-z)*y'[z]+\[Lambda]*y[z]==0,y[z],{z,0,5}]
```

$$
\begin{aligned}
y(z) \to c_1 &\left(-\frac{(\lambda-4)(\lambda-3)(\lambda-2)(\lambda-1)\lambda z^5}{14400} + \frac{1}{576}(\lambda-3)(\lambda-2)(\lambda-1)\lambda z^4 \right. \\
&\left. -\frac{1}{36}(\lambda-2)(\lambda-1)\lambda z^3 + \frac{1}{4}(\lambda-1)\lambda z^2 - \lambda z + 1 \right) \\
+ c_2 &\left(\frac{(\lambda-4)(\lambda-3)(\lambda-2)(\lambda-1)z^5}{14400} + \frac{(\lambda-4)(\lambda-3)(\lambda-2)\lambda z^5}{14400} \right. \\
&+ \frac{(\lambda-4)(\lambda-3)(\lambda-1)\lambda z^5}{14400} + \frac{(\lambda-4)(\lambda-2)(\lambda-1)\lambda z^5}{14400} \\
&+ \frac{137(\lambda-4)(\lambda-3)(\lambda-2)(\lambda-1)\lambda z^5}{432000} + \frac{(\lambda-3)(\lambda-2)(\lambda-1)\lambda z^5}{14400} \\
&- \frac{1}{576}(\lambda-3)(\lambda-2)(\lambda-1)z^4 - \frac{1}{576}(\lambda-3)(\lambda-2)\lambda z^4 - \frac{1}{576}(\lambda-3)(\lambda-1)\lambda z^4 \\
&- \frac{25(\lambda-3)(\lambda-2)(\lambda-1)\lambda z^4}{3456} - \frac{1}{576}(\lambda-2)(\lambda-1)\lambda z^4 + \frac{1}{36}(\lambda-2)(\lambda-1)z^3 \\
&+ \frac{1}{36}(\lambda-2)\lambda z^3 + \frac{11}{108}(\lambda-2)(\lambda-1)\lambda z^3 + \frac{1}{36}(\lambda-1)\lambda z^3 - \frac{1}{4}(\lambda-1)z^2 - \frac{3}{4}(\lambda-1)\lambda z^2 \\
&- \frac{\lambda z^2}{4} + \left(-\frac{(\lambda-4)(\lambda-3)(\lambda-2)(\lambda-1)\lambda z^5}{14400} + \frac{1}{576}(\lambda-3)(\lambda-2)(\lambda-1)\lambda z^4 \right. \\
&\left.\left. - \frac{1}{36}(\lambda-2)(\lambda-1)\lambda z^3 + \frac{1}{4}(\lambda-1)\lambda z^2 - \lambda z + 1 \right) \log(z) + 2\lambda z + z \right)
\end{aligned}
$$

3.14 problem Problem 16.15

Internal problem ID [2543]

Book: Mathematical methods for physics and engineering, Riley, Hobson, Bence, second edition, 2002
Section: Chapter 16, Series solutions of ODEs. Section 16.6 Exercises, page 550
Problem number: Problem 16.15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type [_Gegenbauer, [_2nd_order, _linear, '_with_symmetry_[0,F(x)]']

$$\left(-z^2 + 1\right) y'' - zy' + m^2 y = 0$$

With the expansion point for the power series method at $z = 0$.

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 71

```
Order:=6;
dsolve((1-z^2)*diff(y(z),z$2)-z*diff(y(z),z)+m^2*y(z)=0,y(z),type='series',z=0);
```

$$y(z) = \left(1 - \frac{m^2 z^2}{2} + \frac{m^2 (m^2 - 4) z^4}{24}\right) y(0)$$
$$+ \left(z - \frac{(m^2 - 1) z^3}{6} + \frac{(m^4 - 10m^2 + 9) z^5}{120}\right) D(y)(0) + O(z^6)$$

✓ Solution by Mathematica

Time used: 0.001 (sec). Leaf size: 88

```
AsymptoticDSolveValue[(1-z^2)*y''[z]-z*y'[z]+m^2*y[z]==0,y[z],{z,0,5}]
```

$$y(z) \to c_2 \left(\frac{m^4 z^5}{120} - \frac{m^2 z^5}{12} - \frac{m^2 z^3}{6} + \frac{3z^5}{40} + \frac{z^3}{6} + z\right) + c_1 \left(\frac{m^4 z^4}{24} - \frac{m^2 z^4}{6} - \frac{m^2 z^2}{2} + 1\right)$$