

A Solution Manual For

First order enumerated odes

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1.1 problem 1

Internal problem ID [7317]

Book: First order enumerated odes

Section: section 1

Problem number: 1.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve(diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 7

```
DSolve[y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1$$

1.2 problem 2

Internal problem ID [7318]

Book: First order enumerated odes

Section: section 1

Problem number: 2.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' = a$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)=a,y(x), singsol=all)
```

$$y(x) = ax + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 11

```
DSolve[y'[x]==a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow ax + c_1$$

1.3 problem 3

Internal problem ID [7319]

Book: First order enumerated odes

Section: section 1

Problem number: 3.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' = x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)=x,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 15

```
DSolve[y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x^2}{2} + c_1$$

1.4 problem 4

Internal problem ID [7320]

Book: First order enumerated odes

Section: section 1

Problem number: 4.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' = 1$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 7

```
dsolve(diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = c_1 + x$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 9

```
DSolve[y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x + c_1$$

1.5 problem 5

Internal problem ID [7321]

Book: First order enumerated odes

Section: section 1

Problem number: 5.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' = ax$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=a*x,y(x), singsol=all)
```

$$y(x) = \frac{ax^2}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 16

```
DSolve[y'[x]==a*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{ax^2}{2} + c_1$$

1.6 problem 6

Internal problem ID [7322]

Book: First order enumerated odes

Section: section 1

Problem number: 6.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_separable`]

$$y' - axy = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)=a*x*y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{\frac{ax^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 23

```
DSolve[y'[x]==a*x*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{\frac{ax^2}{2}}$$

$$y(x) \rightarrow 0$$

1.7 problem 7

Internal problem ID [7323]

Book: First order enumerated odes

Section: section 1

Problem number: 7.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$y' - y = ax$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)=a*x+y(x),y(x), singsol=all)
```

$$y(x) = -ax - a + c_1e^x$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 18

```
DSolve[y'[x]==a*x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -a(x + 1) + c_1e^x$$

1.8 problem 8

Internal problem ID [7324]

Book: First order enumerated odes

Section: section 1

Problem number: 8.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$y' - by = ax$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)=a*x+b*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{ax}{b} - \frac{a}{b^2} + e^{xb}c_1$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 25

```
DSolve[y'[x]==a*x+b*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{abx + a}{b^2} + c_1e^{bx}$$

1.9 problem 9

Internal problem ID [7325]

Book: First order enumerated odes

Section: section 1

Problem number: 9.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x)=y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^x$$

✓ Solution by Mathematica

Time used: 0.022 (sec). Leaf size: 16

```
DSolve[y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^x$$

$$y(x) \rightarrow 0$$

1.10 problem 10

Internal problem ID [7326]

Book: First order enumerated odes

Section: section 1

Problem number: 10.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' - by = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=b*y(x),y(x), singsol=all)
```

$$y(x) = e^{xb} c_1$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 18

```
DSolve[y'[x]==b*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{bx}$$

$$y(x) \rightarrow 0$$

1.11 problem 11

Internal problem ID [7327]

Book: First order enumerated odes

Section: section 1

Problem number: 11.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$y' - by^2 = ax$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(diff(y(x),x)=a*x+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{(ba)^{\frac{1}{3}} \left(\text{AiryAi} \left(1, -(ba)^{\frac{1}{3}} x \right) c_1 + \text{AiryBi} \left(1, -(ba)^{\frac{1}{3}} x \right) \right)}{b \left(c_1 \text{AiryAi} \left(-(ba)^{\frac{1}{3}} x \right) + \text{AiryBi} \left(-(ba)^{\frac{1}{3}} x \right) \right)}$$

✓ Solution by Mathematica

Time used: 0.163 (sec). Leaf size: 331

```
DSolve[y'[x]==a*x+b*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\sqrt{a}\sqrt{bx}^{3/2} \left(-2 \text{BesselJ} \left(-\frac{2}{3}, \frac{2}{3} \sqrt{a}\sqrt{bx}^{3/2} \right) + c_1 \left(\text{BesselJ} \left(\frac{2}{3}, \frac{2}{3} \sqrt{a}\sqrt{bx}^{3/2} \right) - \text{BesselJ} \left(-\frac{4}{3}, \frac{2}{3} \sqrt{a}\sqrt{bx}^{3/2} \right) \right) \right)}{2bx \left(\text{BesselJ} \left(\frac{1}{3}, \frac{2}{3} \sqrt{a}\sqrt{bx}^{3/2} \right) + c_1 \text{BesselJ} \left(-\frac{1}{3}, \frac{2}{3} \sqrt{a}\sqrt{bx}^{3/2} \right) \right)}$$

$$y(x) \rightarrow \frac{\sqrt{a}\sqrt{bx}^{3/2} \text{BesselJ} \left(-\frac{4}{3}, \frac{2}{3} \sqrt{a}\sqrt{bx}^{3/2} \right) - \sqrt{a}\sqrt{bx}^{3/2} \text{BesselJ} \left(\frac{2}{3}, \frac{2}{3} \sqrt{a}\sqrt{bx}^{3/2} \right) + \text{BesselJ} \left(-\frac{1}{3}, \frac{2}{3} \sqrt{a}\sqrt{bx}^{3/2} \right)}{2bx \text{BesselJ} \left(-\frac{1}{3}, \frac{2}{3} \sqrt{a}\sqrt{bx}^{3/2} \right)}$$

1.12 problem 12

Internal problem ID [7328]

Book: First order enumerated odes

Section: section 1

Problem number: 12.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y'c = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve(c*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 7

```
DSolve[c*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1$$

1.13 problem 13

Internal problem ID [7329]

Book: First order enumerated odes

Section: section 1

Problem number: 13.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y'c = a$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(c*diff(y(x),x)=a,y(x), singsol=all)
```

$$y(x) = \frac{ax}{c} + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 14

```
DSolve[c*y'[x]==a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{ax}{c} + c_1$$

1.14 problem 14

Internal problem ID [7330]

Book: First order enumerated odes

Section: section 1

Problem number: 14.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y'c = ax$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(c*diff(y(x),x)=a*x,y(x), singsol=all)
```

$$y(x) = \frac{ax^2}{2c} + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 19

```
DSolve[c*y'[x]==a*x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{ax^2}{2c} + c_1$$

1.15 problem 15

Internal problem ID [7331]

Book: First order enumerated odes

Section: section 1

Problem number: 15.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y'c - y = ax$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(c*diff(y(x),x)=a*x+y(x),y(x), singsol=all)
```

$$y(x) = -ac - ax + e^{\frac{x}{c}} c_1$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 22

```
DSolve[c*y'[x]==a*x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -a(c + x) + c_1 e^{\frac{x}{c}}$$

1.16 problem 16

Internal problem ID [7332]

Book: First order enumerated odes

Section: section 1

Problem number: 16.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear, 'class A']`

$$y'c - by = ax$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(c*diff(y(x),x)=a*x+b*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{ax}{b} - \frac{ac}{b^2} + e^{\frac{xb}{c}} c_1$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 28

```
DSolve[c*y'[x]==a*x+b*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{a(bx + c)}{b^2} + c_1 e^{\frac{bx}{c}}$$

1.17 problem 17

Internal problem ID [7333]

Book: First order enumerated odes

Section: section 1

Problem number: 17.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y'c - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(c*diff(y(x),x)=y(x),y(x), singsol=all)
```

$$y(x) = e^{\frac{x}{c}} c_1$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 20

```
DSolve[c*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{\frac{x}{c}}$$

$$y(x) \rightarrow 0$$

1.18 problem 18

Internal problem ID [7334]

Book: First order enumerated odes

Section: section 1

Problem number: 18.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y'c - by = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(c*diff(y(x),x)=b*y(x),y(x), singsol=all)
```

$$y(x) = e^{\frac{xb}{c}} c_1$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 21

```
DSolve[c*y'[x]==b*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{\frac{bx}{c}}$$

$$y(x) \rightarrow 0$$

1.19 problem 19

Internal problem ID [7335]

Book: First order enumerated odes

Section: section 1

Problem number: 19.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$y'c - by^2 = ax$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 75

```
dsolve(c*diff(y(x),x)=a*x+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{ba}{c^2}\right)^{\frac{1}{3}} \left(\text{AiryAi}\left(1, -\left(\frac{ba}{c^2}\right)^{\frac{1}{3}} x\right) c_1 + \text{AiryBi}\left(1, -\left(\frac{ba}{c^2}\right)^{\frac{1}{3}} x\right) \right) c}{b \left(c_1 \text{AiryAi}\left(-\left(\frac{ba}{c^2}\right)^{\frac{1}{3}} x\right) + \text{AiryBi}\left(-\left(\frac{ba}{c^2}\right)^{\frac{1}{3}} x\right) \right)}$$

✓ Solution by Mathematica

Time used: 0.21 (sec). Leaf size: 628

`DSolve[c*y'[x]==a*x+b*y[x]^2,y[x],x,IncludeSingularSolutions -> True]`

$$y(x) \rightarrow \frac{c \left(x^{3/2} \sqrt{\frac{a}{c}} \sqrt{\frac{b}{c}} \left(-2 \text{BesselJ} \left(-\frac{2}{3}, \frac{2}{3} \sqrt{\frac{a}{c}} \sqrt{\frac{b}{c}} x^{3/2} \right) + c_1 \left(\text{BesselJ} \left(\frac{2}{3}, \frac{2}{3} \sqrt{\frac{a}{c}} \sqrt{\frac{b}{c}} x^{3/2} \right) - \text{BesselJ} \left(-\frac{4}{3}, \frac{2}{3} \sqrt{\frac{a}{c}} \sqrt{\frac{b}{c}} x^{3/2} \right) \right) \right)}{2bx \left(\text{BesselJ} \left(\frac{1}{3}, \frac{2}{3} \sqrt{\frac{a}{c}} \sqrt{\frac{b}{c}} x^{3/2} \right) + c_1 \text{BesselJ} \left(-\frac{1}{3}, \frac{2}{3} \sqrt{\frac{a}{c}} \sqrt{\frac{b}{c}} x^{3/2} \right) \right)}$$

$$y(x) \rightarrow \frac{c \left(x^{3/2} \sqrt{\frac{a}{c}} \sqrt{\frac{b}{c}} \text{BesselJ} \left(-\frac{4}{3}, \frac{2}{3} \sqrt{\frac{a}{c}} \sqrt{\frac{b}{c}} x^{3/2} \right) - x^{3/2} \sqrt{\frac{a}{c}} \sqrt{\frac{b}{c}} \text{BesselJ} \left(\frac{2}{3}, \frac{2}{3} \sqrt{\frac{a}{c}} \sqrt{\frac{b}{c}} x^{3/2} \right) + \text{BesselJ} \left(-\frac{1}{3}, \frac{2}{3} \sqrt{\frac{a}{c}} \sqrt{\frac{b}{c}} x^{3/2} \right) \right)}{2bx \text{BesselJ} \left(-\frac{1}{3}, \frac{2}{3} \sqrt{\frac{a}{c}} \sqrt{\frac{b}{c}} x^{3/2} \right)}$$

$$y(x) \rightarrow \frac{c \left(x^{3/2} \sqrt{\frac{a}{c}} \sqrt{\frac{b}{c}} \text{BesselJ} \left(-\frac{4}{3}, \frac{2}{3} \sqrt{\frac{a}{c}} \sqrt{\frac{b}{c}} x^{3/2} \right) - x^{3/2} \sqrt{\frac{a}{c}} \sqrt{\frac{b}{c}} \text{BesselJ} \left(\frac{2}{3}, \frac{2}{3} \sqrt{\frac{a}{c}} \sqrt{\frac{b}{c}} x^{3/2} \right) + \text{BesselJ} \left(-\frac{1}{3}, \frac{2}{3} \sqrt{\frac{a}{c}} \sqrt{\frac{b}{c}} x^{3/2} \right) \right)}{2bx \text{BesselJ} \left(-\frac{1}{3}, \frac{2}{3} \sqrt{\frac{a}{c}} \sqrt{\frac{b}{c}} x^{3/2} \right)}$$

1.20 problem 20

Internal problem ID [7336]

Book: First order enumerated odes

Section: section 1

Problem number: 20.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$y'c - \frac{ax + by^2}{r} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 91

```
dsolve(c*diff(y(x),x)=(a*x+b*y(x)^2)/r,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{ba}{r^2c^2}\right)^{\frac{1}{3}} \left(\text{AiryAi}\left(1, -\left(\frac{ba}{r^2c^2}\right)^{\frac{1}{3}}x\right) c_1 + \text{AiryBi}\left(1, -\left(\frac{ba}{r^2c^2}\right)^{\frac{1}{3}}x\right) \right) rc}{b \left(c_1 \text{AiryAi}\left(-\left(\frac{ba}{r^2c^2}\right)^{\frac{1}{3}}x\right) + \text{AiryBi}\left(-\left(\frac{ba}{r^2c^2}\right)^{\frac{1}{3}}x\right) \right)}$$

✓ Solution by Mathematica

Time used: 0.222 (sec). Leaf size: 517

```
DSolve[c*y'[x]==(a*x+b*y[x]^2)/r,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{cr \left(x^{3/2} \sqrt{\frac{a}{cr}} \sqrt{\frac{b}{cr}} \left(-2 \text{BesselJ}\left(-\frac{2}{3}, \frac{2}{3} \sqrt{\frac{a}{cr}} \sqrt{\frac{b}{cr}} x^{3/2}\right) + c_1 \left(\text{BesselJ}\left(\frac{2}{3}, \frac{2}{3} \sqrt{\frac{a}{cr}} \sqrt{\frac{b}{cr}} x^{3/2}\right) - \text{BesselJ}\left(-\frac{4}{3}, \frac{2}{3} \sqrt{\frac{a}{cr}} \sqrt{\frac{b}{cr}} x^{3/2}\right) \right) \right)}{2bx \left(\text{BesselJ}\left(\frac{1}{3}, \frac{2}{3} \sqrt{\frac{a}{cr}} \sqrt{\frac{b}{cr}} x^{3/2}\right) + c_1 \text{BesselJ}\left(-\frac{1}{3}, \frac{2}{3} \sqrt{\frac{a}{cr}} \sqrt{\frac{b}{cr}} x^{3/2}\right) \right)}$$

$$y(x) \rightarrow \frac{cr \left(x^{3/2} \sqrt{\frac{a}{cr}} \sqrt{\frac{b}{cr}} \text{BesselJ}\left(-\frac{4}{3}, \frac{2}{3} \sqrt{\frac{a}{cr}} \sqrt{\frac{b}{cr}} x^{3/2}\right) - x^{3/2} \sqrt{\frac{a}{cr}} \sqrt{\frac{b}{cr}} \text{BesselJ}\left(\frac{2}{3}, \frac{2}{3} \sqrt{\frac{a}{cr}} \sqrt{\frac{b}{cr}} x^{3/2}\right) + \text{BesselJ}\left(-\frac{1}{3}, \frac{2}{3} \sqrt{\frac{a}{cr}} \sqrt{\frac{b}{cr}} x^{3/2}\right) \right)}{2bx \text{BesselJ}\left(-\frac{1}{3}, \frac{2}{3} \sqrt{\frac{a}{cr}} \sqrt{\frac{b}{cr}} x^{3/2}\right)}$$

1.21 problem 21

Internal problem ID [7337]

Book: First order enumerated odes

Section: section 1

Problem number: 21.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [rational, Riccati]

$$y'c - \frac{ax + by^2}{rx} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 94

```
dsolve(c*diff(y(x),x)=(a*x+b*y(x)^2)/(r*x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{\frac{xba}{r^2c^2}} cr \left(\text{BesselY} \left(1, 2\sqrt{\frac{xba}{r^2c^2}} \right) c_1 + \text{BesselJ} \left(1, 2\sqrt{\frac{xba}{r^2c^2}} \right) \right)}{b \left(c_1 \text{BesselY} \left(0, 2\sqrt{\frac{xba}{r^2c^2}} \right) + \text{BesselJ} \left(0, 2\sqrt{\frac{xba}{r^2c^2}} \right) \right)}$$

✓ Solution by Mathematica

Time used: 0.295 (sec). Leaf size: 207

```
DSolve[c*y'[x]==(a*x+b*y[x]^2)/(r*x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\sqrt{a}\sqrt{x} \left(2 \text{BesselY} \left(1, \frac{2\sqrt{a}\sqrt{b}\sqrt{x}}{cr} \right) + c_1 \text{BesselJ} \left(1, \frac{2\sqrt{a}\sqrt{b}\sqrt{x}}{cr} \right) \right)}{\sqrt{b} \left(2 \text{BesselY} \left(0, \frac{2\sqrt{a}\sqrt{b}\sqrt{x}}{cr} \right) + c_1 \text{BesselJ} \left(0, \frac{2\sqrt{a}\sqrt{b}\sqrt{x}}{cr} \right) \right)}$$
$$y(x) \rightarrow \frac{\sqrt{a}\sqrt{x} \text{BesselJ} \left(1, \frac{2\sqrt{a}\sqrt{b}\sqrt{x}}{cr} \right)}{\sqrt{b} \text{BesselJ} \left(0, \frac{2\sqrt{a}\sqrt{b}\sqrt{x}}{cr} \right)}$$

1.22 problem 22

Internal problem ID [7338]

Book: First order enumerated odes

Section: section 1

Problem number: 22.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$y'c - \frac{ax + by^2}{rx^2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 106

```
dsolve(c*diff(y(x),x)=(a*x+b*y(x)^2)/(r*x^2),y(x), singsol=all)
```

$$y(x) = \frac{a \left(\text{BesselY} \left(0, 2\sqrt{\frac{ba}{c^2 r^2 x}} \right) c_1 + \text{BesselJ} \left(0, 2\sqrt{\frac{ba}{c^2 r^2 x}} \right) \right)}{cr \sqrt{\frac{ba}{c^2 r^2 x}} \left(c_1 \text{BesselY} \left(1, 2\sqrt{\frac{ba}{c^2 r^2 x}} \right) + \text{BesselJ} \left(1, 2\sqrt{\frac{ba}{c^2 r^2 x}} \right) \right)}$$

✓ Solution by Mathematica

Time used: 0.358 (sec). Leaf size: 492

`DSolve[c*y'[x]==(a*x+b*y[x]^2)/(r*x^2),y[x],x,IncludeSingularSolutions -> True]`

$y(x)$

$$\frac{2\sqrt{a}\sqrt{b} \operatorname{BesselY}\left(0, \frac{2\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}}}{cr}\right) + \frac{2cr \operatorname{BesselY}\left(1, \frac{2\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}}}{cr}\right)}{\sqrt{\frac{1}{x}}} - 2\sqrt{a}\sqrt{b} \operatorname{BesselY}\left(2, \frac{2\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}}}{cr}\right) - i\sqrt{a}\sqrt{b}c_1 \operatorname{BesselY}\left(1, \frac{2\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}}}{cr}\right)}{2b\sqrt{\frac{1}{x}} \left(2 \operatorname{BesselY}\left(1, \frac{2\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}}}{cr}\right) - ic_1 \operatorname{BesselY}\left(1, \frac{2\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}}}{cr}\right)\right)}$$

$y(x)$

$$\frac{x \left(\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}} \operatorname{BesselJ}\left(0, \frac{2\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}}}{cr}\right) + cr \operatorname{BesselJ}\left(1, \frac{2\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}}}{cr}\right) - \sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}} \operatorname{BesselJ}\left(2, \frac{2\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}}}{cr}\right) \right)}{2b \operatorname{BesselJ}\left(1, \frac{2\sqrt{a}\sqrt{b}\sqrt{\frac{1}{x}}}{cr}\right)}$$

1.23 problem 23

Internal problem ID [7339]

Book: First order enumerated odes

Section: section 1

Problem number: 23.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$y'c - \frac{ax + by^2}{y} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 69

```
dsolve(c*diff(y(x),x)=(a*x+b*y(x)^2)/y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{4e^{\frac{2xb}{c}}c_1b^2 - 4axb - 2ac}}{2b}$$

$$y(x) = \frac{\sqrt{4e^{\frac{2xb}{c}}c_1b^2 - 4axb - 2ac}}{2b}$$

✓ Solution by Mathematica

Time used: 5.371 (sec). Leaf size: 85

```
DSolve[c*y'[x]==(a*x+b*y[x]^2)/y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{i\sqrt{abx + \frac{ac}{2} + b^2c_1\left(-e^{\frac{2bx}{c}}\right)}}{b}$$

$$y(x) \rightarrow \frac{i\sqrt{abx + \frac{ac}{2} + b^2c_1\left(-e^{\frac{2bx}{c}}\right)}}{b}$$

1.24 problem 24

Internal problem ID [7340]

Book: First order enumerated odes

Section: section 1

Problem number: 24.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$a \sin(x) y x y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(a*sin(x)*y(x)*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 12

```
DSolve[a*Sin[x]*y[x]*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow c_1$$

1.25 problem 25

Internal problem ID [7341]

Book: First order enumerated odes

Section: section 1

Problem number: 25.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$f(x) \sin(x) y x y' \pi = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(f(x)*sin(x)*y(x)*x*diff(y(x),x)*Pi=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 12

```
DSolve[f(x)*Sin[x]*y[x]*x*y'[x]*Pi==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow c_1$$

1.26 problem 26

Internal problem ID [7342]

Book: First order enumerated odes

Section: section 1

Problem number: 26.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$y' - y = \sin(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)=sin(x)+y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x)}{2} - \frac{\sin(x)}{2} + c_1 e^x$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 24

```
DSolve[y'[x]==Sin[x]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{\sin(x)}{2} - \frac{\cos(x)}{2} + c_1 e^x$$

1.27 problem 27

Internal problem ID [7343]

Book: First order enumerated odes

Section: section 1

Problem number: 27.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [Riccati]

$$y' - y^2 = \sin(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x)=sin(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{c_1 \operatorname{MathieuSPrime}\left(0, -2, -\frac{\pi}{4} + \frac{x}{2}\right) + \operatorname{MathieuCPrime}\left(0, -2, -\frac{\pi}{4} + \frac{x}{2}\right)}{2\left(c_1 \operatorname{MathieuS}\left(0, -2, -\frac{\pi}{4} + \frac{x}{2}\right) + \operatorname{MathieuC}\left(0, -2, -\frac{\pi}{4} + \frac{x}{2}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.208 (sec). Leaf size: 105

```
DSolve[y'[x]==Sin[x]+y[x]^2,y[x],x,IncludeSingularSolutions->True]
```

$$y(x) \rightarrow \frac{-\operatorname{MathieuSPrime}\left[0, -2, \frac{1}{4}(\pi - 2x)\right] + c_1 \operatorname{MathieuCPrime}\left[0, -2, \frac{1}{4}(\pi - 2x)\right]}{2\left(\operatorname{MathieuS}\left[0, -2, \frac{1}{4}(2x - \pi)\right] + c_1 \operatorname{MathieuC}\left[0, -2, \frac{1}{4}(\pi - 2x)\right]\right)}$$

$$y(x) \rightarrow \frac{\operatorname{MathieuCPrime}\left[0, -2, \frac{1}{4}(\pi - 2x)\right]}{2\operatorname{MathieuC}\left[0, -2, \frac{1}{4}(\pi - 2x)\right]}$$

1.28 problem 28

Internal problem ID [7344]

Book: First order enumerated odes

Section: section 1

Problem number: 28.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' - \frac{y}{x} = \cos(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x)=cos(x)+y(x)/x,y(x), singsol=all)
```

$$y(x) = (\text{Ci}(x) + c_1) x$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 12

```
DSolve[y'[x]==Cos[x]+y[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x(\text{CosIntegral}(x) + c_1)$$

1.29 problem 29

Internal problem ID [7345]

Book: First order enumerated odes

Section: section 1

Problem number: 29.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [Riccati]

$$y' - \frac{y^2}{x} = \cos(x)$$

X Solution by Maple

```
dsolve(diff(y(x),x)=cos(x)+y(x)^2/x,y(x), singsol=all)
```

No solution found

X Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==Cos[x]+y[x]^2/x,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.30 problem 30

Internal problem ID [7346]

Book: First order enumerated odes

Section: section 1

Problem number: 30.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [Riccati]

$$y' - y - by^2 = x$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 105

```
dsolve(diff(y(x),x)=x+y(x)+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{2b^{\frac{1}{3}} \text{AiryAi}\left(1, -\frac{4xb-1}{4b^{\frac{2}{3}}}\right) c_1 + 2 \text{AiryBi}\left(1, -\frac{4xb-1}{4b^{\frac{2}{3}}}\right) b^{\frac{1}{3}} - \text{AiryAi}\left(-\frac{4xb-1}{4b^{\frac{2}{3}}}\right) c_1 - \text{AiryBi}\left(-\frac{4xb-1}{4b^{\frac{2}{3}}}\right)}{2b \left(\text{AiryAi}\left(-\frac{4xb-1}{4b^{\frac{2}{3}}}\right) c_1 + \text{AiryBi}\left(-\frac{4xb-1}{4b^{\frac{2}{3}}}\right) \right)}$$

✓ Solution by Mathematica

Time used: 0.222 (sec). Leaf size: 211

```
DSolve[y'[x]==x+y[x]+b*y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{-(-b)^{2/3} \text{AiryBi}\left(\frac{\frac{1}{4}-bx}{(-b)^{2/3}}\right) + 2b \text{AiryBiPrime}\left(\frac{\frac{1}{4}-bx}{(-b)^{2/3}}\right) + c_1 \left(2b \text{AiryAiPrime}\left(\frac{\frac{1}{4}-bx}{(-b)^{2/3}}\right) - (-b)^{2/3} \text{AiryBi}\left(\frac{\frac{1}{4}-bx}{(-b)^{2/3}}\right) \right)}{2(-b)^{5/3} \left(\text{AiryBi}\left(\frac{\frac{1}{4}-bx}{(-b)^{2/3}}\right) + c_1 \text{AiryAi}\left(\frac{\frac{1}{4}-bx}{(-b)^{2/3}}\right) \right)}$$

$$y(x) \rightarrow -\frac{\frac{2\sqrt[3]{-b} \text{AiryAiPrime}\left(\frac{\frac{1}{4}-bx}{(-b)^{2/3}}\right)}{\text{AiryAi}\left(\frac{\frac{1}{4}-bx}{(-b)^{2/3}}\right)} + 1}{2b}$$

1.31 problem 31

Internal problem ID [7347]

Book: First order enumerated odes

Section: section 1

Problem number: 31.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y'x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve(x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 7

```
DSolve[x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1$$

1.32 problem 32

Internal problem ID [7348]

Book: First order enumerated odes

Section: section 1

Problem number: 32.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$5y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve(5*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 7

```
DSolve[5*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1$$

1.33 problem 33

Internal problem ID [7349]

Book: First order enumerated odes

Section: section 1

Problem number: 33.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$ey' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve(exp(1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 7

```
DSolve[Exp[1]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1$$

1.34 problem 34

Internal problem ID [7350]

Book: First order enumerated odes

Section: section 1

Problem number: 34.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$\pi y' = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 5

```
dsolve(Pi*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 7

```
DSolve[Pi*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1$$

1.35 problem 35

Internal problem ID [7351]

Book: First order enumerated odes

Section: section 1

Problem number: 35.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y' \sin(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve(sin(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 7

```
DSolve[Sin[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1$$

1.36 problem 36

Internal problem ID [7352]

Book: First order enumerated odes

Section: section 1

Problem number: 36.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$f(x) y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve(f(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 7

```
DSolve[f[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1$$

1.37 problem 37

Internal problem ID [7353]

Book: First order enumerated odes

Section: section 1

Problem number: 37.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y'x = 1$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(x*diff(y(x),x)=1,y(x), singsol=all)
```

$$y(x) = \ln(x) + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 10

```
DSolve[x*y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \log(x) + c_1$$

1.38 problem 38

Internal problem ID [7354]

Book: First order enumerated odes

Section: section 1

Problem number: 38.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$y'x = \sin(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(x*diff(y(x),x)=sin(x),y(x), singsol=all)
```

$$y(x) = \text{Si}(x) + c_1$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 10

```
DSolve[x*y'[x]==Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \text{Si}(x) + c_1$$

1.39 problem 39

Internal problem ID [7355]

Book: First order enumerated odes

Section: section 1

Problem number: 39.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [`_quadrature`]

$$(x - 1)y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve((x-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 7

```
DSolve[(x-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1$$

1.40 problem 40

Internal problem ID [7356]

Book: First order enumerated odes

Section: section 1

Problem number: 40.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y'y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 11

```
dsolve(y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = -c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow c_1$$

1.41 problem 41

Internal problem ID [7357]

Book: First order enumerated odes

Section: section 1

Problem number: 41.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$y'xy = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow c_1$$

1.42 problem 42

Internal problem ID [7358]

Book: First order enumerated odes

Section: section 1

Problem number: 42.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$xy \sin(x) y' = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 9

```
dsolve(x*y(x)*sin(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[x*y[x]*Sin[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow c_1$$

1.43 problem 43

Internal problem ID [7359]

Book: First order enumerated odes

Section: section 1

Problem number: 43.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$\pi y \sin(x) y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(Pi*y(x)*sin(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[Pi*y[x]*Sin[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow c_1$$

1.44 problem 44

Internal problem ID [7360]

Book: First order enumerated odes

Section: section 1

Problem number: 44.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [_quadrature]

$$\sin(x) y' x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve(x*sin(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 7

```
DSolve[x*Sin[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1$$

1.45 problem 45

Internal problem ID [7361]

Book: First order enumerated odes

Section: section 1

Problem number: 45.

ODE order: 1.

ODE degree: 2.

CAS Maple gives this as type [_quadrature]

$$x \sin(x) y'^2 = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 5

```
dsolve(x*sin(x)*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 7

```
DSolve[x*Sine[x]*y'[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1$$

1.46 problem 46

Internal problem ID [7362]

Book: First order enumerated odes

Section: section 1

Problem number: 46.

ODE order: 1.

ODE degree: 2.

CAS Maple gives this as type [_quadrature]

$$yy'^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(y(x)*diff(y(x),x)^2=0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 12

```
DSolve[y[x]*(y'[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow c_1$$

1.47 problem 47

Internal problem ID [7363]

Book: First order enumerated odes

Section: section 1

Problem number: 47.

ODE order: 1.

ODE degree: 0.

CAS Maple gives this as type [_quadrature]

$$y'^n = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve(diff(y(x),x)^n=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 15

```
DSolve[(y'[x])^n==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 0^{\frac{1}{n}}x + c_1$$

1.48 problem 48

Internal problem ID [7364]

Book: First order enumerated odes

Section: section 1

Problem number: 48.

ODE order: 1.

ODE degree: 0.

CAS Maple gives this as type [_quadrature]

$$xy^n = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve(x*diff(y(x),x)^n=0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 15

```
DSolve[x*(y'[x])^n==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 0^{\frac{1}{n}}x + c_1$$

1.49 problem 49

Internal problem ID [7365]

Book: First order enumerated odes

Section: section 1

Problem number: 49.

ODE order: 1.

ODE degree: 2.

CAS Maple gives this as type [_quadrature]

$$y'^2 = x$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)^2=x,y(x), singsol=all)
```

$$y(x) = \frac{2x^{\frac{3}{2}}}{3} + c_1$$

$$y(x) = -\frac{2x^{\frac{3}{2}}}{3} + c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 33

```
DSolve[(y'[x])^2==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{2x^{3/2}}{3} + c_1$$

$$y(x) \rightarrow \frac{2x^{3/2}}{3} + c_1$$

1.50 problem 50

Internal problem ID [7366]

Book: First order enumerated odes

Section: section 1

Problem number: 50.

ODE order: 1.

ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$y'^2 - y = x$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)^2=x+y(x),y(x), singsol=all)
```

$$y(x) = \left(-\text{LambertW}\left(-c_1 e^{-\frac{x}{2}-1}\right) - 1\right)^2 - x$$

✓ Solution by Mathematica

Time used: 18.817 (sec). Leaf size: 100

```
DSolve[(y'[x])^2==x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow W\left(-e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right)^2 + 2W\left(-e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right) - x + 1$$

$$y(x) \rightarrow W\left(e^{\frac{1}{2}(-x-2+c_1)}\right)^2 + 2W\left(e^{\frac{1}{2}(-x-2+c_1)}\right) - x + 1$$

$$y(x) \rightarrow 1 - x$$

1.51 problem 51

Internal problem ID [7367]

Book: First order enumerated odes

Section: section 1

Problem number: 51.

ODE order: 1.

ODE degree: 2.

CAS Maple gives this as type `[_homogeneous, 'class A', _rational, _dAlembert]`

$$y'^2 - \frac{y}{x} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 39

```
dsolve(diff(y(x),x)^2=y(x)/x,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \frac{(x + \sqrt{xc_1})^2}{x}$$

$$y(x) = \frac{(-x + \sqrt{xc_1})^2}{x}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 46

```
DSolve[(y'[x])^2==y[x]/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4}(-2\sqrt{x} + c_1)^2$$

$$y(x) \rightarrow \frac{1}{4}(2\sqrt{x} + c_1)^2$$

$$y(x) \rightarrow 0$$

1.52 problem 52

Internal problem ID [7368]

Book: First order enumerated odes

Section: section 1

Problem number: 52.

ODE order: 1.

ODE degree: 2.

CAS Maple gives this as type [_separable]

$$y'^2 - \frac{y^2}{x} = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)^2=y(x)^2/x,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = c_1 e^{-2\sqrt{x}}$$

$$y(x) = c_1 e^{2\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 38

```
DSolve[(y'[x])^2==y[x]^2/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 e^{-2\sqrt{x}}$$

$$y(x) \rightarrow c_1 e^{2\sqrt{x}}$$

$$y(x) \rightarrow 0$$

1.53 problem 53

Internal problem ID [7369]

Book: First order enumerated odes

Section: section 1

Problem number: 53.

ODE order: 1.

ODE degree: 2.

CAS Maple gives this as type `[_homogeneous, 'class G']`

$$y'^2 - \frac{y^3}{x} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)^2=y(x)^3/x,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \text{WeierstrassP}\left(\frac{x2^{\frac{2}{3}}}{\sqrt{x2^{\frac{2}{3}}}} + c_1, 0, 0\right) 2^{\frac{2}{3}}$$

✓ Solution by Mathematica

Time used: 0.073 (sec). Leaf size: 42

```
DSolve[(y'[x])^2==y[x]^3/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{4}{(-2\sqrt{x} + c_1)^2}$$

$$y(x) \rightarrow \frac{4}{(2\sqrt{x} + c_1)^2}$$

$$y(x) \rightarrow 0$$

1.54 problem 54

Internal problem ID [7370]

Book: First order enumerated odes

Section: section 1

Problem number: 54.

ODE order: 1.

ODE degree: 3.

CAS Maple gives this as type `[_homogeneous, 'class G', _rational]`

$$y'^3 - \frac{y^2}{x} = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 453

```
dsolve(diff(y(x),x)^3=y(x)^2/x,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = -\frac{3x^{\frac{4}{3}}c_1}{8} + \frac{3x^{\frac{2}{3}}c_1^2}{8} - \frac{c_1^3}{8} + \frac{x^2}{8}$$

$$y(x) = -\frac{3\left(-\frac{x^{\frac{2}{3}}}{2} - \frac{i\sqrt{3}x^{\frac{2}{3}}}{2}\right)^2 c_1}{8} + \frac{3\left(-\frac{x^{\frac{2}{3}}}{2} - \frac{i\sqrt{3}x^{\frac{2}{3}}}{2}\right) c_1^2}{8} - \frac{c_1^3}{8} + \frac{x^2}{8}$$

$$y(x) = -\frac{3\left(-\frac{x^{\frac{2}{3}}}{2} + \frac{i\sqrt{3}x^{\frac{2}{3}}}{2}\right)^2 c_1}{8} + \frac{3\left(-\frac{x^{\frac{2}{3}}}{2} + \frac{i\sqrt{3}x^{\frac{2}{3}}}{2}\right) c_1^2}{8} - \frac{c_1^3}{8} + \frac{x^2}{8}$$

$$y(x) = \frac{3\left(2x^{\frac{2}{3}} + c_1\right)^2 c_1}{64} - \frac{3c_1^2\left(2x^{\frac{2}{3}} + c_1\right)}{64} + \frac{c_1^3}{64} + \frac{x^2}{8}$$

$$y(x) = \frac{3\left(-x^{\frac{2}{3}} + c_1 - i\sqrt{3}x^{\frac{2}{3}}\right)^2 c_1}{64} - \frac{3c_1^2\left(-x^{\frac{2}{3}} + c_1 - i\sqrt{3}x^{\frac{2}{3}}\right)}{64} + \frac{c_1^3}{64} + \frac{x^2}{8}$$

$$y(x) = \frac{3\left(-x^{\frac{2}{3}} + c_1 + i\sqrt{3}x^{\frac{2}{3}}\right)^2 c_1}{64} - \frac{3c_1^2\left(-x^{\frac{2}{3}} + c_1 + i\sqrt{3}x^{\frac{2}{3}}\right)}{64} + \frac{c_1^3}{64} + \frac{x^2}{8}$$

$$y(x) = -\frac{3\left(2x^{\frac{2}{3}} - c_1\right)^2 c_1}{64} - \frac{3c_1^2\left(2x^{\frac{2}{3}} - c_1\right)}{64} - \frac{c_1^3}{64} + \frac{x^2}{8}$$

$$y(x) = -\frac{3\left(-x^{\frac{2}{3}} - c_1 - i\sqrt{3}x^{\frac{2}{3}}\right)^2 c_1}{64} - \frac{3c_1^2\left(-x^{\frac{2}{3}} - c_1 - i\sqrt{3}x^{\frac{2}{3}}\right)}{64} - \frac{c_1^3}{64} + \frac{x^2}{8}$$

$$y(x) = -\frac{3\left(-x^{\frac{2}{3}} - c_1 + i\sqrt{3}x^{\frac{2}{3}}\right)^2 c_1}{64} - \frac{3c_1^2\left(-x^{\frac{2}{3}} - c_1 + i\sqrt{3}x^{\frac{2}{3}}\right)}{64} - \frac{c_1^3}{64} + \frac{x^2}{8}$$

✓ Solution by Mathematica

Time used: 0.083 (sec). Leaf size: 152

```
DSolve[(y'[x])^3==y[x]^2/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{216} (3x^{2/3} + 2c_1)^3$$

$$y(x) \rightarrow \frac{1}{216} \left(18i(\sqrt{3} + i) c_1^2 x^{2/3} - 27i(\sqrt{3} - i) c_1 x^{4/3} + 27x^2 + 8c_1^3 \right)$$

$$y(x) \rightarrow \frac{1}{216} \left(-18i(\sqrt{3} - i) c_1^2 x^{2/3} + 27i(\sqrt{3} + i) c_1 x^{4/3} + 27x^2 + 8c_1^3 \right)$$

$$y(x) \rightarrow 0$$

1.55 problem 55

Internal problem ID [7371]

Book: First order enumerated odes

Section: section 1

Problem number: 55.

ODE order: 1.

ODE degree: 2.

CAS Maple gives this as type `[_homogeneous, 'class G']`

$$y'^2 - \frac{1}{yx} = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 43

```
dsolve(diff(y(x),x)^2=1/(y(x)*x),y(x), singsol=all)
```

$$\frac{(xy(x))^{\frac{3}{2}}}{x^{\frac{3}{2}}} - 3\sqrt{x} - c_1 = 0$$

$$\frac{(xy(x))^{\frac{3}{2}}}{x^{\frac{3}{2}}} + 3\sqrt{x} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 3.748 (sec). Leaf size: 53

```
DSolve[(y'[x])^2==1/(y[x]*x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \left(\frac{3}{2}\right)^{2/3} (-2\sqrt{x} + c_1)^{2/3}$$

$$y(x) \rightarrow \left(\frac{3}{2}\right)^{2/3} (2\sqrt{x} + c_1)^{2/3}$$

1.56 problem 56

Internal problem ID [7372]

Book: First order enumerated odes

Section: section 1

Problem number: 56.

ODE order: 1.

ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$y'^2 - \frac{1}{xy^3} = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 43

```
dsolve(diff(y(x),x)^2=1/(x*y(x)^3),y(x), singsol=all)
```

$$\frac{(xy(x))^{\frac{5}{2}}}{x^{\frac{5}{2}}} - 5\sqrt{x} - c_1 = 0$$

$$\frac{(xy(x))^{\frac{5}{2}}}{x^{\frac{5}{2}}} + 5\sqrt{x} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.112 (sec). Leaf size: 53

```
DSolve[(y'[x])^2==1/(x*y[x]^3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \left(\frac{5}{2}\right)^{2/5} (-2\sqrt{x} + c_1)^{2/5}$$

$$y(x) \rightarrow \left(\frac{5}{2}\right)^{2/5} (2\sqrt{x} + c_1)^{2/5}$$

1.57 problem 57

Internal problem ID [7373]

Book: First order enumerated odes

Section: section 1

Problem number: 57.

ODE order: 1.

ODE degree: 2.

CAS Maple gives this as type [separable]

$$y'^2 - \frac{1}{x^2 y^3} = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)^2=1/(x^2*y(x)^3),y(x), singsol=all)
```

$$\ln(x) - \frac{2y(x)^{\frac{5}{2}}}{5} - c_1 = 0$$

$$\ln(x) + \frac{2y(x)^{\frac{5}{2}}}{5} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.139 (sec). Leaf size: 45

```
DSolve[(y'[x])^2==1/(x^2*y[x]^3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \left(\frac{5}{2}\right)^{2/5} (-\log(x) + c_1)^{2/5}$$

$$y(x) \rightarrow \left(\frac{5}{2}\right)^{2/5} (\log(x) + c_1)^{2/5}$$

1.58 problem 58

Internal problem ID [7374]

Book: First order enumerated odes

Section: section 1

Problem number: 58.

ODE order: 1.

ODE degree: 4.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational]`

$$y'^4 - \frac{1}{xy^3} = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 99

```
dsolve(diff(y(x),x)^4=1/(x*y(x)^3),y(x), singsol=all)
```

$$\frac{3(y(x) x^3)^{\frac{7}{4}}}{x^{\frac{21}{4}}} - 7x^{\frac{3}{4}} - c_1 = 0$$

$$\frac{3i(y(x) x^3)^{\frac{7}{4}}}{x^{\frac{21}{4}}} - 7x^{\frac{3}{4}} - c_1 = 0$$

$$\frac{3i(y(x) x^3)^{\frac{7}{4}}}{x^{\frac{21}{4}}} + 7x^{\frac{3}{4}} - c_1 = 0$$

$$\frac{3(y(x) x^3)^{\frac{7}{4}}}{x^{\frac{21}{4}}} + 7x^{\frac{3}{4}} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 7.225 (sec). Leaf size: 129

```
DSolve[(y'[x])^4==1/(x*y[x]^3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\left(-\frac{28x^{3/4}}{3} + 7c_1\right)^{4/7}}{2\sqrt[7]{2}}$$

$$y(x) \rightarrow \frac{\left(7c_1 - \frac{28}{3}ix^{3/4}\right)^{4/7}}{2\sqrt[7]{2}}$$

$$y(x) \rightarrow \frac{\left(\frac{28}{3}ix^{3/4} + 7c_1\right)^{4/7}}{2\sqrt[7]{2}}$$

$$y(x) \rightarrow \frac{\left(\frac{28x^{3/4}}{3} + 7c_1\right)^{4/7}}{2\sqrt[7]{2}}$$

1.59 problem 59

Internal problem ID [7375]

Book: First order enumerated odes

Section: section 1

Problem number: 59.

ODE order: 1.

ODE degree: 2.

CAS Maple gives this as type [separable]

$$y'^2 - \frac{1}{x^3 y^4} = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 185

```
dsolve(diff(y(x),x)^2=1/(x^3*y(x)^4),y(x), singsol=all)
```

$$y(x) = \left(\frac{c_1 \sqrt{x} - 6}{\sqrt{x}} \right)^{\frac{1}{3}}$$

$$y(x) = -\frac{\left(\frac{c_1 \sqrt{x} - 6}{\sqrt{x}} \right)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3} \left(\frac{c_1 \sqrt{x} - 6}{\sqrt{x}} \right)^{\frac{1}{3}}}{2}$$

$$y(x) = -\frac{\left(\frac{c_1 \sqrt{x} - 6}{\sqrt{x}} \right)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3} \left(\frac{c_1 \sqrt{x} - 6}{\sqrt{x}} \right)^{\frac{1}{3}}}{2}$$

$$y(x) = \left(\frac{c_1 \sqrt{x} + 6}{\sqrt{x}} \right)^{\frac{1}{3}}$$

$$y(x) = -\frac{\left(\frac{c_1 \sqrt{x} + 6}{\sqrt{x}} \right)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3} \left(\frac{c_1 \sqrt{x} + 6}{\sqrt{x}} \right)^{\frac{1}{3}}}{2}$$

$$y(x) = -\frac{\left(\frac{c_1 \sqrt{x} + 6}{\sqrt{x}} \right)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3} \left(\frac{c_1 \sqrt{x} + 6}{\sqrt{x}} \right)^{\frac{1}{3}}}{2}$$

✓ Solution by Mathematica

Time used: 3.775 (sec). Leaf size: 157

```
DSolve[(y'[x])^2==1/(x^3*y[x]^4),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sqrt[3]{-3} \sqrt[3]{-\frac{2}{\sqrt{x}} + c_1}$$

$$y(x) \rightarrow \sqrt[3]{3} \sqrt[3]{-\frac{2}{\sqrt{x}} + c_1}$$

$$y(x) \rightarrow (-1)^{2/3} \sqrt[3]{3} \sqrt[3]{-\frac{2}{\sqrt{x}} + c_1}$$

$$y(x) \rightarrow -\sqrt[3]{-3} \sqrt[3]{\frac{2}{\sqrt{x}} + c_1}$$

$$y(x) \rightarrow \sqrt[3]{3} \sqrt[3]{\frac{2}{\sqrt{x}} + c_1}$$

$$y(x) \rightarrow (-1)^{2/3} \sqrt[3]{3} \sqrt[3]{\frac{2}{\sqrt{x}} + c_1}$$

1.60 problem 60

Internal problem ID [7376]

Book: First order enumerated odes

Section: section 1

Problem number: 60.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$y' - \sqrt{1 + 6x + y} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 57

```
dsolve(diff(y(x),x)=(1+6*x+y(x))^(1/2),y(x), singsol=all)
```

$$\begin{aligned} x - 2\sqrt{1 + 6x + y(x)} - 6 \ln \left(-6 + \sqrt{1 + 6x + y(x)} \right) \\ + 6 \ln \left(6 + \sqrt{1 + 6x + y(x)} \right) + 6 \ln (-35 + y(x) + 6x) - c_1 = 0 \end{aligned}$$

✓ Solution by Mathematica

Time used: 13.35 (sec). Leaf size: 65

```
DSolve[y'[x]==(1+6*x+y[x])^(1/2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 36W \left(-\frac{1}{6} e^{\frac{1}{72}(-6x-73+6c_1)} \right)^2 + 72W \left(-\frac{1}{6} e^{\frac{1}{72}(-6x-73+6c_1)} \right) - 6x + 35$$

$$y(x) \rightarrow 35 - 6x$$

1.61 problem 61

Internal problem ID [7377]

Book: First order enumerated odes

Section: section 1

Problem number: 61.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$y' - (1 + 6x + y)^{\frac{1}{3}} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 79

```
dsolve(diff(y(x),x)=(1+6*x+y(x))^(1/3),y(x), singsol=all)
```

$$x - \frac{3(1 + 6x + y(x))^{\frac{2}{3}}}{2} + 36 \ln \left((1 + 6x + y(x))^{\frac{2}{3}} - 6(1 + 6x + y(x))^{\frac{1}{3}} + 36 \right) \\ - 72 \ln \left(6 + (1 + 6x + y(x))^{\frac{1}{3}} \right) - 36 \ln (217 + y(x) + 6x) + 18(1 + 6x + y(x))^{\frac{1}{3}} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.246 (sec). Leaf size: 66

```
DSolve[y'[x]==(1+6*x+y[x])^(1/3),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve} \left[\frac{1}{6} \left(y(x) - 9(y(x) + 6x + 1)^{2/3} + 108 \sqrt[3]{y(x) + 6x + 1} \right. \right. \\ \left. \left. - 648 \log \left(\sqrt[3]{y(x) + 6x + 1} + 6 \right) + 6x + 1 \right) - \frac{y(x)}{6} = c_1, y(x) \right]$$

1.62 problem 62

Internal problem ID [7378]

Book: First order enumerated odes

Section: section 1

Problem number: 62.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$y' - (1 + 6x + y)^{\frac{1}{4}} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 109

```
dsolve(diff(y(x),x)=(1+6*x+y(x))^(1/4),y(x), singsol=all)
```

$$\begin{aligned} & x + 216 \ln(1295 - y(x) - 6x) + 12\sqrt{1 + 6x + y(x)} \\ & - 216 \ln\left(\sqrt{1 + 6x + y(x)} + 36\right) + 216 \ln\left(\sqrt{1 + 6x + y(x)} - 36\right) \\ & - 144(1 + 6x + y(x))^{\frac{1}{4}} - 432 \ln\left((1 + 6x + y(x))^{\frac{1}{4}} - 6\right) \\ & + 432 \ln\left(6 + (1 + 6x + y(x))^{\frac{1}{4}}\right) - \frac{4(1 + 6x + y(x))^{\frac{3}{4}}}{3} - c_1 = 0 \end{aligned}$$

✓ Solution by Mathematica

Time used: 2.535 (sec). Leaf size: 79

```
DSolve[y'[x]==(1+6*x+y[x])^(1/4),y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} & \text{Solve}\left[\frac{1}{6}\left(y(x) - 8(y(x) + 6x + 1)^{3/4} + 72\sqrt{y(x) + 6x + 1} - 864\sqrt[4]{y(x) + 6x + 1}\right.\right. \\ & \left.\left.+ 5184 \log\left(\sqrt[4]{y(x) + 6x + 1} + 6\right) + 6x + 1\right) - \frac{y(x)}{6} = c_1, y(x)\right] \end{aligned}$$

1.63 problem 63

Internal problem ID [7379]

Book: First order enumerated odes

Section: section 1

Problem number: 63.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$y' - (a + xb + y)^4 = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 49

```
dsolve(diff(y(x),x)=(a+b*x+y(x))^4),y(x), singsol=all)
```

$$y(x) = -xb + \text{RootOf} \left(-x + \int \frac{1}{-a^4 + 4_a^3a + 6_a^2a^2 + 4_a a^3 + a^4 + b} d_a + c_1 \right)$$

✓ Solution by Mathematica

Time used: 0.429 (sec). Leaf size: 163

```
DSolve[y'[x]==(a+b*x+y[x])^4),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve} \left[\frac{2\sqrt{2} \arctan \left(1 - \frac{\sqrt{2}(a+bx+y(x))}{\sqrt[4]{b}} \right) - 2\sqrt{2} \arctan \left(\frac{\sqrt{2}(a+bx+y(x))}{\sqrt[4]{b}} + 1 \right) + \sqrt{2} \log \left((a + bx + y(x))^2 - \sqrt[4]{b} \right)}{8b^3} \right]$$

1.64 problem 64

Internal problem ID [7380]

Book: First order enumerated odes

Section: section 1

Problem number: 64.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$y' - (\pi + x + 7y)^{\frac{7}{2}} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 74

```
dsolve(diff(y(x),x)=(Pi+x+7*y(x))^(7/2),y(x), singsol=all)
```

$$y(x) = -\frac{x}{7} + \text{RootOf}\left(-x + 7\left(\int^{-Z} \frac{1}{7\pi^3\sqrt{\pi+7a} + 147\pi^2 a\sqrt{\pi+7a} + 1029\pi a^2\sqrt{\pi+7a} + 2401 a^3\sqrt{\pi+7a} + 1} d-a\right) + c_1\right)$$

✓ Solution by Mathematica

Time used: 30.556 (sec). Leaf size: 43

```
DSolve[y'[x]==(Pi+x+7*y[x])^(7/2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-(7y(x) + x + \pi) \left(\text{Hypergeometric2F1}\left(\frac{2}{7}, 1, \frac{9}{7}, -7(x + 7y(x) + \pi)^{7/2}\right) - 1\right) - 7y(x) = c_1, y(x)\right]$$

1.65 problem 65

Internal problem ID [7381]

Book: First order enumerated odes

Section: section 1

Problem number: 65.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$y' - (a + xb + yc)^6 = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 94

```
dsolve(diff(y(x),x)=(a+b*x+c*y(x))^6,y(x), singsol=all)
```

$$y(x) = \frac{\text{RootOf}\left(\left(\int^{-Z} \frac{1}{c^7 a^6 + 6 a^5 a c^6 + 15 a^4 a^2 c^5 + 20 a^3 a^3 c^4 + 15 a^2 a^4 c^3 + 6 a a^5 c^2 + a^6 c + b} d_a\right) c - x + c_1\right) c - xb}{c}$$

✓ Solution by Mathematica

Time used: 1.941 (sec). Leaf size: 274

```
DSolve[y'[x]==(a+b*x+c*y[x])^6,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{-4\sqrt[6]{b} \arctan\left(\frac{\sqrt[6]{c}(a+bx+cy(x))}{\sqrt[6]{b}}\right) + 2\sqrt[6]{b} \arctan\left(\sqrt{3} - \frac{2\sqrt[6]{c}(a+bx+cy(x))}{\sqrt[6]{b}}\right) - 2\sqrt[6]{b} \arctan\left(\frac{2\sqrt[6]{c}(a+bx+cy(x))}{\sqrt[6]{b}}\right)}{-\frac{cy(x)}{b} = c_1, y(x)}\right]$$

1.66 problem 66

Internal problem ID [7382]

Book: First order enumerated odes

Section: section 1

Problem number: 66.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type [separable]

$$y' - e^{x+y} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)=exp(x+y(x)),y(x), singsol=all)
```

$$y(x) = \ln\left(-\frac{1}{e^x + c_1}\right)$$

✓ Solution by Mathematica

Time used: 0.876 (sec). Leaf size: 18

```
DSolve[y'[x]==Exp[x+y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\log(-e^x - c_1)$$

1.67 problem 67

Internal problem ID [7383]

Book: First order enumerated odes

Section: section 1

Problem number: 67.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[_homogeneous, 'class C', _dAlembert]`

$$y' - e^{x+y} = 10$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)=10+exp(x+y(x)),y(x), singsol=all)
```

$$y(x) = -x + \ln\left(\frac{11}{e^{-11x}c_1 - 1}\right)$$

✓ Solution by Mathematica

Time used: 3.4 (sec). Leaf size: 42

```
DSolve[y'[x]==10+Exp[x+y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \log\left(-\frac{11e^{10x+11c_1}}{-1 + e^{11(x+c_1)}}\right)$$

$$y(x) \rightarrow \log(-11e^{-x})$$

1.68 problem 68

Internal problem ID [7384]

Book: First order enumerated odes

Section: section 1

Problem number: 68.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - 10e^{x+y} = x^2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x)=10*exp(x+y(x))+x^2,y(x), singsol=all)
```

$$y(x) = \frac{x^3}{3} - \ln\left(-c_1 - 10\left(\int e^x e^{\frac{x^3}{3}} dx\right)\right)$$

✓ Solution by Mathematica

Time used: 0.431 (sec). Leaf size: 115

```
DSolve[y'[x]==10*Exp[x+y[x]]+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} -\frac{1}{10}e^{-K[2]}\left(10e^{K[2]}\int_1^x -\frac{1}{10}e^{\frac{K[1]^3}{3}-K[2]}K[1]^2dK[1] + e^{\frac{x^3}{3}}\right)dK[2] + \int_1^x \left(\frac{1}{10}e^{\frac{K[1]^3}{3}-y(x)}K[1]^2 + e^{\frac{K[1]^3}{3}+K[1]}\right)dK[1] = c_1, y(x)\right]$$

1.69 problem 69

Internal problem ID [7385]

Book: First order enumerated odes

Section: section 1

Problem number: 69.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - e^{x+y}x = \sin(x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)=x*exp(x+y(x))+sin(x),y(x), singsol=all)
```

$$y(x) = -\cos(x) - \ln\left(-c_1 - \left(\int x e^x e^{-\cos(x)} dx\right)\right)$$

✓ Solution by Mathematica

Time used: 3.93 (sec). Leaf size: 100

```
DSolve[y'[x]==x*Exp[x+y[x]]+Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^x \left(-e^{K[1]-\cos(K[1])} K[1] - e^{-\cos(K[1])-y(x)} \sin(K[1])\right) dK[1] + \int_1^{y(x)} -e^{-\cos(x)-K[2]} \left(e^{\cos(x)+K[2]} \int_1^x e^{-\cos(K[1])-K[2]} \sin(K[1]) dK[1] - 1\right) dK[2] = c_1, y(x)\right]$$

1.70 problem 70

Internal problem ID [7386]

Book: First order enumerated odes

Section: section 1

Problem number: 70.

ODE order: 1.

ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$y' - 5e^{x^2+20y} = \sin(x)$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 30

```
dsolve(diff(y(x),x)=5*exp(x^2+20*y(x))+sin(x),y(x), singsol=all)
```

$$y(x) = -\cos(x) - \frac{\ln\left(-20c_1 - 100\left(\int e^{x^2} e^{-20\cos(x)} dx\right)\right)}{20}$$

✓ Solution by Mathematica

Time used: 10.354 (sec). Leaf size: 140

```
DSolve[y'[x]==5*Exp[x^2+20*y[x]]+Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^x -\frac{1}{100}e^{-20\cos(K[1])-20y(x)}\left(\sin(K[1]) + 5e^{K[1]^2+20y(x)}\right) dK[1] + \int_1^{y(x)} -\frac{1}{100}e^{-20\cos(x)-20K[2]}\left(100e^{20\cos(x)+20K[2]}\int_1^x\left(\frac{1}{5}e^{-20\cos(K[1])-20K[2]}\left(\sin(K[1]) + 5e^{K[1]^2+20K[2]}\right) - e^{K[1]^2-20\cos(x)}\right) dK[2] = c_1, y(x)\right]$$

2 section 2 (system of first order ode's)

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2.1 problem 1

Internal problem ID [7387]

Book: First order enumerated odes

Section: section 2 (system of first order ode's)

Problem number: 1.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) + y'(t) &= x(t) + y(t) + t \\x'(t) + y'(t) &= 2x(t) + 3y(t) + e^t\end{aligned}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 31

```
dsolve([diff(x(t),t)+diff(y(t),t)-x(t)=y(t)+t,diff(x(t),t)+diff(y(t),t)=2*x(t)+3*y(t)+exp(t))
```

$$x(t) = -3t - 2 - 2c_1e^t - e^t$$

$$y(t) = 2t + 1 + c_1e^t$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 37

```
DSolve[{x'[t]+y'[t]-x[t]==y[t]+t,x'[t]+y'[t]==2*x[t]+3*y[t]+Exp[t]},{x[t],y[t]},t,IncludeSin
```

$$x(t) \rightarrow -3t + (1 + 2c_1)e^t - 2$$

$$y(t) \rightarrow 2t - (1 + c_1)e^t + 1$$

2.2 problem 2

Internal problem ID [7388]

Book: First order enumerated odes

Section: section 2 (system of first order ode's)

Problem number: 2.

ODE order: 1.

ODE degree: 1.

Solve

$$\begin{aligned}x'(t) &= -x(t) - 2y(t) + t - e^t \\y'(t) &= 3x(t) + 5y(t) - t + 2e^t\end{aligned}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 95

```
dsolve([2*diff(x(t),t)+diff(y(t),t)-x(t)=y(t)+t,diff(x(t),t)+diff(y(t),t)=2*x(t)+3*y(t)+exp(t)
```

$$x(t) = \frac{e^{(2+\sqrt{3})t} c_2 \sqrt{3}}{3} - \frac{e^{-(2+\sqrt{3})t} c_1 \sqrt{3}}{3} - e^{(2+\sqrt{3})t} c_2 - e^{-(2+\sqrt{3})t} c_1 - 3t - 11$$

$$y(t) = e^{(2+\sqrt{3})t} c_2 + e^{-(2+\sqrt{3})t} c_1 - \frac{e^t}{2} + 2t + 7$$

✓ Solution by Mathematica

Time used: 10.209 (sec). Leaf size: 174

```
DSolve[{2*x'[t]+y'[t]-x[t]==y[t]+t,x'[t]+y'[t]==2*x[t]+3*y[t]+Exp[t]},{x[t],y[t]},t,IncludeS
```

$$x(t) \rightarrow \frac{1}{6} e^{-((\sqrt{3}-2)t)} \left(-6 e^{(\sqrt{3}-2)t} (3t + 11) + \left(-3(\sqrt{3} - 1) c_1 - 2\sqrt{3} c_2 \right) e^{2\sqrt{3}t} \right. \\ \left. + 3(1 + \sqrt{3}) c_1 + 2\sqrt{3} c_2 \right)$$
$$y(t) \rightarrow \frac{1}{2} \left(4t - e^t + \left(-\sqrt{3} c_1 - \sqrt{3} c_2 + c_2 \right) e^{-((\sqrt{3}-2)t)} + \left(\sqrt{3} c_1 + (1 + \sqrt{3}) c_2 \right) e^{(2+\sqrt{3})t} \right. \\ \left. + 14 \right)$$

2.3 problem 3

Internal problem ID [7389]

Book: First order enumerated odes

Section: section 2 (system of first order ode's)

Problem number: 3.

ODE order: 1.

ODE degree: 1.

Solve

$$x'(t) + y'(t) = x(t) + y(t) + t + \sin(t) + \cos(t)$$

$$x'(t) + y'(t) = 2x(t) + 3y(t) + e^t$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 45

```
dsolve([diff(x(t),t)+diff(y(t),t)-x(t)=y(t)+t+sin(t)+cos(t),diff(x(t),t)+diff(y(t),t)=2*x(t)
```

$$x(t) = -\sin(t) - 3\cos(t) - 2c_1e^t - 3t - 2 - e^t$$

$$y(t) = \sin(t) + 2\cos(t) + c_1e^t + 2t + 1$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 54

```
DSolve[{x'[t]+y'[t]-x[t]==y[t]+t+Sin[t]+Cos[t],x'[t]+y'[t]==2*x[t]+3*y[t]+Exp[t]},{x[t],y[t]
```

$$x(t) \rightarrow -3t + e^t - \sin(t) - 3\cos(t) + 2c_1e^t - 2$$

$$y(t) \rightarrow 2t - e^t + \sin(t) + 2\cos(t) - c_1e^t + 1$$