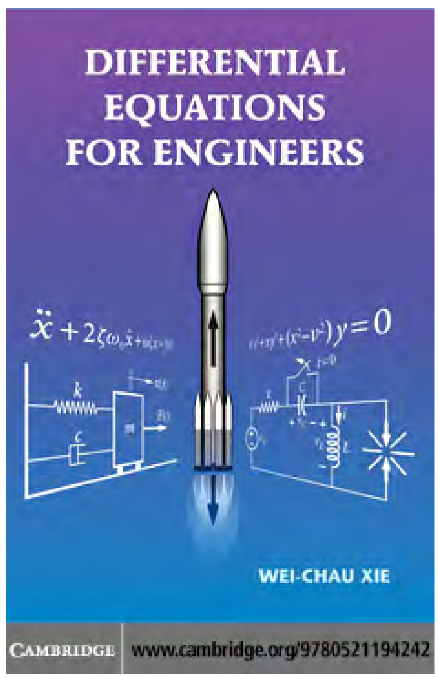
A Solution Manual For

Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010



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Contents

1 Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78

1.1 problem 1

Internal problem ID [3146]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{\cos(y)^2 + (1 + e^{-x}) \sin(y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(cos(y(x))^2+(1+exp(-x))*sin(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \pi - \arccos\left(\frac{1}{\ln(e^x + 1) + c_1}\right)$$

✓ Solution by Mathematica

Time used: 0.95 (sec). Leaf size: 57

```
DSolve[Cos[y[x]]^2+(1+Exp[-x])*Sin[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sec^{-1}(-\log(e^x + 1) + 2c_1)$$
$$y(x) \to \sec^{-1}(-\log(e^x + 1) + 2c_1)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

1.2 problem 2

Internal problem ID [3147]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{x^3 e^{x^2}}{\ln(y)\, y} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)=(x^3*exp(x^2))/(y(x)*ln(y(x))),y(x), singsol=all)
```

$$y(x) = e^{\frac{\operatorname{LambertW}\left(2\left(x^2 e^{x^2} - e^{x^2} + 2c_1\right)e^{-1}\right)}{2} + \frac{1}{2}}$$

✓ Solution by Mathematica

Time used: 60.191 (sec). Leaf size: 106

```
DSolve[y'[x]==(x^3*Exp[x^2])/(y[x]*Log[y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{2e^{x^2}(x^2-1)+4c_1}}{\sqrt{W\left(\frac{2e^{x^2}(x^2-1)+4c_1}{e}\right)}}$$

$$y(x) \to \frac{\sqrt{2e^{x^2}(x^2-1)+4c_1}}{\sqrt{W\left(\frac{2e^{x^2}(x^2-1)+4c_1}{e}\right)}}$$

1.3 problem 3

Internal problem ID [3148]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x\cos(y)^2 + e^x \tan(y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 69

```
dsolve(x*cos(y(x))^2+exp(x)*tan(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \operatorname{arccot}\left(\frac{\sqrt{-2\,(c_1 e^x - x - 1)\,e^x}}{2c_1 e^x - 2x - 2}\right)$$

$$y(x) = \pi - \operatorname{arccot}\left(\frac{\sqrt{-2\,(c_1 e^x - x - 1)\,e^x}}{2c_1 e^x - 2x - 2}\right)$$

✓ Solution by Mathematica

Time used: 15.741 (sec). Leaf size: 149

```
DSolve[x*Cos[y[x]]^2+Exp[x]*Tan[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sec^{-1}\left(-\sqrt{2}\sqrt{e^{-x}\left(x+4c_1e^x+1\right)}\right)$$
$$y(x) \to \sec^{-1}\left(-\sqrt{2}\sqrt{e^{-x}\left(x+4c_1e^x+1\right)}\right)$$
$$y(x) \to -\sec^{-1}\left(\sqrt{2}\sqrt{e^{-x}\left(x+4c_1e^x+1\right)}\right)$$
$$y(x) \to \sec^{-1}\left(\sqrt{2}\sqrt{e^{-x}\left(x+4c_1e^x+1\right)}\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

1.4 problem 4

Internal problem ID [3149]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(y^2+1)+(2y+1)\,\mathrm{e}^{-x}y'=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 28

```
dsolve(x*(y(x)^2+1)+(2*y(x)+1)*exp(-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=\tan\left(\mathrm{RootOf}\left(\mathrm{e}^{x}x+\ln\left(\frac{2}{1+\cos(2\_Z)}\right)+\_Z-\mathrm{e}^{x}+c_1\right)\right)$$

✓ Solution by Mathematica

Time used: 0.627 (sec). Leaf size: 43

```
DSolve[x*(y[x]^2+1)+(2*y[x]+1)*Exp[-x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\mathrm{InverseFunction}\left[\log\left(\#1^2+1\right)+\arctan(\#1)\&\right]\left[-e^x(x-1)+c_1\right]$$
$$y(x)\to -i$$
$$y(x)\to i$$

1.5 problem 5

Internal problem ID [3150]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^3x + y' \mathrm{e}^{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(x*y(x)^3+exp(x^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-\mathrm{e}^{-x^2} + c_1}}$$
$$y(x) = -\frac{1}{\sqrt{-\mathrm{e}^{-x^2} + c_1}}$$

✓ Solution by Mathematica

Time used: 7.124 (sec). Leaf size: 70

```
DSolve[x*y[x]^3+Exp[x^2]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ie^{\frac{x^2}{2}}}{\sqrt{1 + 2c_1e^{x^2}}}$$
$$y(x) \to \frac{ie^{\frac{x^2}{2}}}{\sqrt{1 + 2c_1e^{x^2}}}$$
$$y(x) \to 0$$

1.6 problem 6

Internal problem ID [3151]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x\cos(y)^2 + \tan(y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(x*cos(y(x))^2+tan(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \operatorname{arccot}\left(\frac{1}{\sqrt{-x^2 - 2c_1}}\right)$$
$$y(x) = \pi - \operatorname{arccot}\left(\frac{1}{\sqrt{-x^2 - 2c_1}}\right)$$

✓ Solution by Mathematica

Time used: 1.202 (sec). Leaf size: 103

```
DSolve[x*Cos[y[x]]^2+Tan[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sec^{-1}\left(-\sqrt{-x^2+8c_1}\right)$$
$$y(x) \to \sec^{-1}\left(-\sqrt{-x^2+8c_1}\right)$$
$$y(x) \to -\sec^{-1}\left(\sqrt{-x^2+8c_1}\right)$$
$$y(x) \to \sec^{-1}\left(\sqrt{-x^2+8c_1}\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

1.7 problem 7

Internal problem ID [3152]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^3x + (y+1)\,\mathrm{e}^{-x}y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 71

```
dsolve(x*y(x)^3+(y(x)+1)*exp(-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{-1+\sqrt{2\,\mathrm{e}^x x - 2\,\mathrm{e}^x + 2c_1 + 1}}{2\,(\mathrm{e}^x x - \mathrm{e}^x + c_1)}$$
$$y(x) = \frac{1+\sqrt{2\,\mathrm{e}^x x - 2\,\mathrm{e}^x + 2c_1 + 1}}{2\,\mathrm{e}^x x + 2c_1 - 2\,\mathrm{e}^x}$$

✓ Solution by Mathematica

Time used: 9.963 (sec). Leaf size: 88

```
DSolve[x*y[x]^3+(y[x]+1)*Exp[-x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1-\sqrt{2e^x(x-1)+1-2c_1}}{2e^x(x-1)-2c_1}$$
$$y(x) \to \frac{1+\sqrt{2e^x(x-1)+1-2c_1}}{2e^x(x-1)-2c_1}$$
$$y(x) \to 0$$

1.8 problem 8

Internal problem ID [3153]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y' + \frac{x}{y} = -2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)+x/y(x)+2=0,y(x), singsol=all)
```

$$y(x) = -\frac{x(\text{LambertW}(-c_1 x) + 1)}{\text{LambertW}(-c_1 x)}$$

✓ Solution by Mathematica

Time used: 0.125 (sec). Leaf size: 31

```
DSolve[y'[x]+x/y[x]+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{\frac{y(x)}{x} + 1} + \log\left(\frac{y(x)}{x} + 1\right) = -\log(x) + c_1, y(x)\right]$$

1.9 problem 9

Internal problem ID [3154]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$xy' - y - x\cot\left(\frac{y}{x}\right) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x)-y(x)=x*cot(y(x)/x),y(x), singsol=all)
```

$$y(x) = x\arccos\left(\frac{1}{c_1 x}\right)$$

✓ Solution by Mathematica

Time used: 25.917 (sec). Leaf size: 56

```
DSolve[x*y'[x]-y[x]==x*Cot[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\arccos\left(\frac{e^{-c_1}}{x}\right)$$
$$y(x) \to x\arccos\left(\frac{e^{-c_1}}{x}\right)$$
$$y(x) \to -\frac{\pi x}{2}$$
$$y(x) \to \frac{\pi x}{2}$$

1.10 problem 10

Internal problem ID [3155]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$x\cos\left(\frac{y}{x}\right)^2 - y + xy' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve((x*cos(y(x)/x)^2-y(x))+x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\arctan(\ln(x) + c_1)\, x$$

✓ Solution by Mathematica

Time used: 0.5 (sec). Leaf size: 37

```
DSolve[(x*Cos[y[x]/x]^2-y[x])+x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\arctan(-\log(x) + 2c_1)$$
$$y(x) \to -\frac{\pi x}{2}$$
$$y(x) \to \frac{\pi x}{2}$$

1.11 problem 11

Internal problem ID [3156]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xy' - y(1 + \ln(y) - \ln(x)) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(x*diff(y(x),x)=y(x)*(1+ln(y(x))-ln(x)),y(x), singsol=all)
```

$$y(x) = x\,\mathrm{e}^{c_1 x}$$

✓ Solution by Mathematica

Time used: 0.228 (sec). Leaf size: 20

```
DSolve[x*y'[x]==y[x]*(1+Log[y[x]]-Log[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to xe^{e^{c_1 x}}$$
$$y(x) \to x$$

1.12 problem 12

Internal problem ID [3157]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{yx + (y^2 + x^2)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.282 (sec). Leaf size: 223

```
dsolve(x*y(x)+(x^2+y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x^2 c_1 \left(c_1 x^2 + \sqrt{c_1^2 x^4 + 1}\right)}}{x \left(c_1 x^2 + \sqrt{c_1^2 x^4 + 1}\right) c_1}$$

$$y(x) = \frac{\sqrt{-x^2 c_1 \left(-c_1 x^2 + \sqrt{c_1^2 x^4 + 1}\right)}}{x \left(c_1 x^2 - \sqrt{c_1^2 x^4 + 1}\right) c_1}$$

$$y(x) = -\frac{\sqrt{x^2 c_1 \left(c_1 x^2 + \sqrt{c_1^2 x^4 + 1}\right)}}{x \left(c_1 x^2 + \sqrt{c_1^2 x^4 + 1}\right) c_1}$$

$$y(x) = -\frac{\sqrt{-x^2 c_1 \left(-c_1 x^2 + \sqrt{c_1^2 x^4 + 1}\right)}}{x \left(c_1 x^2 - \sqrt{c_1^2 x^4 + 1}\right) c_1}$$

✓ Solution by Mathematica

Time used: 9.087 (sec). Leaf size: 218

```
DSolve[x*y[x]+(x^2+y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 - \sqrt{x^4 + e^{4c_1}}}$$
$$y(x) \to \sqrt{-x^2 - \sqrt{x^4 + e^{4c_1}}}$$
$$y(x) \to -\sqrt{-x^2 + \sqrt{x^4 + e^{4c_1}}}$$
$$y(x) \to \sqrt{-x^2 + \sqrt{x^4 + e^{4c_1}}}$$
$$y(x) \to 0$$
$$y(x) \to -\sqrt{-\sqrt{x^4} - x^2}$$
$$y(x) \to \sqrt{-\sqrt{x^4} - x^2}$$
$$y(x) \to -\sqrt{\sqrt{x^4} - x^2}$$
$$y(x) \to \sqrt{\sqrt{x^4} - x^2}$$

1.13 problem 13

Internal problem ID [3158]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\left(1-\mathrm{e}^{-\frac{y}{x}}\right) y'-\frac{y}{x}=-1$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve((1-exp(- y(x)/x))*diff(y(x),x)+(1- y(x)/x)=0,y(x), singsol=all)
```

$$y(x)=-\frac{\mathrm{LambertW}\left(-\mathrm{e}^{-\frac{1}{c_1 x}}\right) c_1 x+1}{c_1}$$

✓ Solution by Mathematica

Time used: 60.202 (sec). Leaf size: 29

```
DSolve[(1-Exp[-y[x]/x])*y'[x]+(1-y[x]/x)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -xW\left(-e^{-\frac{e^{c_1}}{x}}\right)-e^{c_1}$$

1.14 problem 14

Internal problem ID [3159]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$-yx + y^2 - y'yx = -x^2$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 25

```
dsolve((x^2-x*y(x)+y(x)^2)-x*y(x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\operatorname{LambertW}\left(\frac{\mathrm{e}^{-c_1}\mathrm{e}^{-1}}{x}\right)-c_1-1} + x$$

✓ Solution by Mathematica

Time used: 3.69 (sec). Leaf size: 25

```
DSolve[(x^2-x*y[x]+y[x]^2)-x*y[x]*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(1 + W\left(\frac{e^{-1+c_1}}{x}\right)\right)$$

$$y(x) \to x$$

1.15 problem 15

Internal problem ID [3160]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$(3 + 2x + 4y)\, y' - 2y = x + 1$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve((3+2*x+4*y(x))*diff(y(x),x)=1+x+2*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} + \frac{\operatorname{LambertW}\left(e^{5} e^{8x} c_1\right)}{8} - \frac{5}{8}$$

✓ Solution by Mathematica

Time used: 4.849 (sec). Leaf size: 39

```
DSolve[(3+2*x+4*y[x])*y'[x]==1+x+2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}\left(W\left(-e^{8x-1+c_1}\right) - 4x - 5\right)$$

$$y(x) \to \frac{1}{8}(-4x - 5)$$

1.16 problem 16

Internal problem ID [3161]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y' - \frac{2x+y-1}{x-y-2} = 0$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 50

```
dsolve(diff(y(x),x)=(2*x+y(x)-1)/(x-y(x)-2),y(x), singsol=all)
```

$$y(x) = -1 - \tan\left(\operatorname{RootOf}\left(\sqrt{2}\,\ln\left(2\tan\left(\_Z\right)^{2}(x-1)^{2} + 2(x-1)^{2}\right) + 2\sqrt{2}\,c_1 + 2\_Z\right)\right)(x-1)\sqrt{2}$$

✓ Solution by Mathematica

Time used: 0.125 (sec). Leaf size: 75

```
DSolve[y'[x]==(2*x+y[x]-1)/(x-y[x]-2),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\sqrt{2}\arctan\left(\frac{y(x)+2x-1}{\sqrt{2}(-y(x)+x-2)}\right) + \log(9) = 2\log\left(\frac{2x^2+y(x)^2+2y(x)-4x+3}{(x-1)^2}\right) + 4\log(x-1) + 3c_1, y(x)\right]$$

1.17 problem 17

Internal problem ID [3162]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y - (2x + y - 4)\, y' = -2$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 49

```
dsolve(y(x)+2=(2*x+y(x)-4)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \frac{1 - 4c_1 + \sqrt{4c_1x - 12c_1 + 1}}{2c_1}$$
$$y(x) = -\frac{-1 + 4c_1 + \sqrt{4c_1x - 12c_1 + 1}}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.28 (sec). Leaf size: 82

```
DSolve[y[x]+2==(2*x+y[x]-4)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{1 + 4c_1(x - 3)} - 1 + 4c_1}{2c_1}$$
$$y(x) \to \frac{\sqrt{1 + 4c_1(x - 3)} + 1 - 4c_1}{2c_1}$$
$$y(x) \to -2$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to 1 - x$$

1.18 problem 18

Internal problem ID [3163]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - \sin(-y + x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(diff(y(x),x)=sin(x-y(x))^2,y(x), singsol=all)
```

$$y(x) = x + \arctan(c_1 - x)$$

✓ Solution by Mathematica

Time used: 0.199 (sec). Leaf size: 31

```
DSolve[y'[x]==Sin[x-y[x]]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}[2y(x) - 2(\tan(x - y(x)) - \arctan(\tan(x - y(x)))) = c_1, y(x)]$$

1.19 problem 19

Internal problem ID [3164]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _Riccati]`

$$y' - (4y+1)^2 - 8yx = (x+1)^2 + 1$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=(x+1)^2+(4*y(x)+1)^2+8*x*y(x)+1,y(x), singsol=all)
```

$$y(x) = -\frac{x}{4} - \frac{1}{4} - \frac{3\tan(-6x + 6c_1)}{8}$$

✓ Solution by Mathematica

Time used: 0.18 (sec). Leaf size: 49

```
DSolve[y'[x]==(x+1)^2+(4*y[x]+1)^2+8*x*y[x]+1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16}\left(-4x + \frac{1}{c_1 e^{12ix} - \frac{i}{12}} - (4+6i)\right)$$

$$y(x) \to \frac{1}{8}(-2x - (2+3i))$$

1.20 problem 20

Internal problem ID [3165]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{6xy^2 + (6yx^2 + 4y^3)\, y' = -3x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 125

```
dsolve((3*x^2+6*x*y(x)^2)+(6*x^2*y(x)+4*y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-6x^2 - 2\sqrt{9x^4 - 4x^3 - 4c_1}}}{2}$$
$$y(x) = \frac{\sqrt{-6x^2 - 2\sqrt{9x^4 - 4x^3 - 4c_1}}}{2}$$
$$y(x) = -\frac{\sqrt{-6x^2 + 2\sqrt{9x^4 - 4x^3 - 4c_1}}}{2}$$
$$y(x) = \frac{\sqrt{-6x^2 + 2\sqrt{9x^4 - 4x^3 - 4c_1}}}{2}$$

✓ Solution by Mathematica

Time used: 6.017 (sec). Leaf size: 163

```
DSolve[(3*x^2+6*x*y[x]^2)+(6*x^2*y[x]+4*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{\sqrt{-3x^2 - \sqrt{9x^4 - 4x^3 + 4c_1}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-3x^2 - \sqrt{9x^4 - 4x^3 + 4c_1}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{-3x^2 + \sqrt{9x^4 - 4x^3 + 4c_1}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-3x^2 + \sqrt{9x^4 - 4x^3 + 4c_1}}}{\sqrt{2}}$$

1.21 problem 21

Internal problem ID [3166]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$-xy^2 - 2y - (yx^2 + 2x)\, y' = -2x^2 - 3$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve((2*x^2-x*y(x)^2-2*y(x)+3)-(x^2*y(x)+2*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-2 - \frac{\sqrt{12x^3+18c_1+54x+36}}{3}}{x}$$
$$y(x) = \frac{-2 + \frac{\sqrt{12x^3+18c_1+54x+36}}{3}}{x}$$

✓ Solution by Mathematica

Time used: 0.646 (sec). Leaf size: 87

```
DSolve[(2*x^2-x*y[x]^2-2*y[x]+3)-(x^2*y[x]+2*x)*y'[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{6x + \sqrt{3}\sqrt{x^2\,(4x^3 + 18x + 12 + 3c_1)}}{3x^2}$$
$$y(x) \to \frac{-6x + \sqrt{3}\sqrt{x^2\,(4x^3 + 18x + 12 + 3c_1)}}{3x^2}$$

1.22 problem 22

Internal problem ID [3167]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$xy^2 - 2y + (yx^2 - 2x - 2y)\, y' = -x - 3$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 92

```
dsolve((x*y(x)^2+x-2*y(x)+3)+(x^2*y(x)-2*(x+y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2x + \sqrt{-x^4 - 2c_1x^2 - 6x^3 + 6x^2 + 4c_1 + 12x}}{x^2 - 2}$$
$$y(x) = -\frac{-2x + \sqrt{-x^4 - 2c_1x^2 - 6x^3 + 6x^2 + 4c_1 + 12x}}{x^2 - 2}$$

✓ Solution by Mathematica

Time used: 0.549 (sec). Leaf size: 95

```
DSolve[(x*y[x]^2+x-2*y[x]+3)+(x^2*y[x]-2*(x+y[x]))*y'[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{2x - \sqrt{-x^4 - 6x^3 + (6 + c_1)x^2 + 12x - 2c_1}}{x^2 - 2}$$
$$y(x) \to \frac{2x + \sqrt{-x^4 - 6x^3 + (6 + c_1)x^2 + 12x - 2c_1}}{x^2 - 2}$$

1.23 problem 23

Internal problem ID [3168]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]`

$$\boxed{3y(x^2-1)+(x^3+8y-3x)\,y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 67

```
dsolve((3*y(x)*(x^2-1))+(x^3+8*y(x)-3*x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\frac{x^3}{8}+\frac{3x}{8}-\frac{\sqrt{x^6-6x^4+9x^2-16c_1}}{8}$$
$$y(x)=-\frac{x^3}{8}+\frac{3x}{8}+\frac{\sqrt{x^6-6x^4+9x^2-16c_1}}{8}$$

✓ Solution by Mathematica

Time used: 0.17 (sec). Leaf size: 86

```
DSolve[(3*y[x]*(x^2-1))+(x^3+8*y[x]-3*x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{8}\left(-x^3-\sqrt{x^6-6x^4+9x^2+64c_1}+3x\right)$$
$$y(x)\to\frac{1}{8}\left(-x^3+\sqrt{x^6-6x^4+9x^2+64c_1}+3x\right)$$
$$y(x)\to 0$$

1.24 problem 24

Internal problem ID [3169]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_1st_order, ‘_with_symmetry_[F(x),G(x)*y+H(x)]‘]]`

$$\boxed{\ln(y) = -x^2 - \frac{xy'}{y}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((x^2+ln(y(x)))+(x/y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{x^2}{3}}\mathrm{e}^{-\frac{c_1}{x}}$$

✓ Solution by Mathematica

Time used: 0.247 (sec). Leaf size: 21

```
DSolve[(x^2+Log[y[x]])+(x/y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\frac{x^2}{3}+\frac{c_1}{x}}$$

1.25 problem 25

Internal problem ID [3170]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{2x\left(3x+y-y\,\mathrm{e}^{-x^2}\right)+\left(x^2+3y^2+\mathrm{e}^{-x^2}\right)y'=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 1085

```
dsolve((2*x*(3*x+y(x)-y(x)*exp(-x^2)))+(x^2+3*y(x)^2+exp(-x^2))*diff(y(x),x)=0,y(x), singsol
```

$$y(x) = \frac{e^{-x^2}\left(\left(-216x^3e^{x^2} + 12\sqrt{3}\,\sqrt{(112\,e^{3x^2}x^6 + 108\,e^{3x^2}c_1x^3 + 12\,e^{2x^2}x^4 + 27\,e^{3x^2}c_1^2 + 12x^2e^{x^2} + 4)\,e^{-x^2}} - 10\right.\right.}{6} - \frac{2\left(x^2e^{x^2} + 1\right)}{\left(\left(-216x^3e^{x^2} + 12\sqrt{3}\,\sqrt{(112\,e^{3x^2}x^6 + 108\,e^{3x^2}c_1x^3 + 12\,e^{2x^2}x^4 + 27\,e^{3x^2}c_1^2 + 12x^2e^{x^2} + 4)\,e^{-x^2}} - 108\,e\right.\right.}$$

$$y(x) = -\frac{e^{-x^2}\left(\left(-216x^3e^{x^2} + 12\sqrt{3}\,\sqrt{(112\,e^{3x^2}x^6 + 108\,e^{3x^2}c_1x^3 + 12\,e^{2x^2}x^4 + 27\,e^{3x^2}c_1^2 + 12x^2e^{x^2} + 4)\,e^{-x^2}} - \right.\right.}{12} + \frac{x^2e^{x^2} + 1}{\left(\left(-216x^3e^{x^2} + 12\sqrt{3}\,\sqrt{(112\,e^{3x^2}x^6 + 108\,e^{3x^2}c_1x^3 + 12\,e^{2x^2}x^4 + 27\,e^{3x^2}c_1^2 + 12x^2e^{x^2} + 4)\,e^{-x^2}} - 108\,e\right.\right.} - \frac{i\sqrt{3}\left(\frac{e^{-x^2}\left(\left(-216x^3e^{x^2}+12\sqrt{3}\sqrt{\left(112\,e^{3x^2}x^6+108\,e^{3x^2}c_1x^3+12\,e^{2x^2}x^4+27\,e^{3x^2}c_1^2+12x^2e^{x^2}+4\right)e^{-x^2}}-108\,e^{x^2}c_1\right)e^{2x^2}\right)^{\frac{1}{3}}}{6} + \frac{}{\left(\left(-\right.\right.}\right.}{2}$$

$$y(x) = -\frac{e^{-x^2}\left(\left(-216x^3e^{x^2} + 12\sqrt{3}\,\sqrt{(112\,e^{3x^2}x^6 + 108\,e^{3x^2}c_1x^3 + 12\,e^{2x^2}x^4 + 27\,e^{3x^2}c_1^2 + 12x^2e^{x^2} + 4)\,e^{-x^2}} - \right.\right.}{12} + \frac{x^2e^{x^2} + 1}{\left(\left(-216x^3e^{x^2} + 12\sqrt{3}\,\sqrt{(112\,e^{3x^2}x^6 + 108\,e^{3x^2}c_1x^3 + 12\,e^{2x^2}x^4 + 27\,e^{3x^2}c_1^2 + 12x^2e^{x^2} + 4)\,e^{-x^2}} - 108\,e\right.\right.} + \frac{i\sqrt{3}\left(\frac{e^{-x^2}\left(\left(-216x^3e^{x^2}+12\sqrt{3}\sqrt{\left(112\,e^{3x^2}x^6+108\,e^{3x^2}c_1x^3+12\,e^{2x^2}x^4+27\,e^{3x^2}c_1^2+12x^2e^{x^2}+4\right)e^{-x^2}}-108\,e^{x^2}c_1\right)e^{2x^2}\right)^{\frac{1}{3}}}{6} + \frac{}{\left(\left(-\right.\right.}\right.}{2}$$

✓ Solution by Mathematica

Time used: 37.566 (sec). Leaf size: 416

```
DSolve[(2*x*(3*x+y[x]-y[x]*Exp[-x^2]))+(x^2+3*y[x]^2+Exp[-x^2])*y'[x]==0,y[x],x,IncludeSingu
```

$$y(x) \to \frac{-6\sqrt[3]{2}\left(x^2+e^{-x^2}\right)+2^{2/3}\left(-54x^3+\sqrt{108\left(x^2+e^{-x^2}\right)^3+729\left(-2x^3+c_1\right)^2}+27c_1\right)^{2/3}}{6\sqrt[3]{-54x^3+\sqrt{108\left(x^2+e^{-x^2}\right)^3+729\left(-2x^3+c_1\right)^2}+27c_1}}$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)\left(x^2+e^{-x^2}\right)}{2^{2/3}\sqrt[3]{-54x^3+\sqrt{108\left(x^2+e^{-x^2}\right)^3+729\left(-2x^3+c_1\right)^2}+27c_1}} + \frac{\left(-1+i\sqrt{3}\right)\sqrt[3]{-54x^3+\sqrt{108\left(x^2+e^{-x^2}\right)^3+729\left(-2x^3+c_1\right)^2}+27c_1}}{6\sqrt[3]{2}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)\left(x^2+e^{-x^2}\right)}{2^{2/3}\sqrt[3]{-54x^3+\sqrt{108\left(x^2+e^{-x^2}\right)^3+729\left(-2x^3+c_1\right)^2}+27c_1}} - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-54x^3+\sqrt{108\left(x^2+e^{-x^2}\right)^3+729\left(-2x^3+c_1\right)^2}+27c_1}}{6\sqrt[3]{2}}$$

1.26 problem 26

Internal problem ID [3171]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, [_Abel, '2nd type', 'class B']]`

$$y+2y^2\sin(x)^2+(x+2yx-y\sin(2x))\,y'=-3$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 88

```
dsolve((3+y(x)+2*y(x)^2*sin(x)^2)+(x+2*x*y(x)-y(x)*sin(2*x))*diff(y(x),x)=0,y(x), singsol=al
```

$$y(x)=\frac{x+\sqrt{2c_1\sin(2x)+6x\sin(2x)-4c_1x-11x^2}}{\sin(2x)-2x}$$

$$y(x)=-\frac{-x+\sqrt{2c_1\sin(2x)+6x\sin(2x)-4c_1x-11x^2}}{\sin(2x)-2x}$$

✓ Solution by Mathematica

Time used: 1.378 (sec). Leaf size: 97

```
DSolve[(3+y[x]+2*y[x]^2*Sin[x]^2)+(x+2*x*y[x]-y[x]*Sin[2*x])*y'[x]==0,y[x],x,IncludeSingular
```

$$y(x)\to\frac{x-i\sqrt{x(11x+2c_1)-(6x+c_1)\sin(2x)}}{\sin(2x)-2x}$$

$$y(x)\to\frac{x+i\sqrt{x(11x+2c_1)-(6x+c_1)\sin(2x)}}{\sin(2x)-2x}$$

1.27 problem 27

Internal problem ID [3172]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{2yx + (x^2 + 2yx + y^2)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 56

```
dsolve((2*x*y(x))+(x^2+2*x*y(x)+y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -x + \sqrt{2}\, x \tan\left(\operatorname{RootOf}\left(2\sqrt{2}\, \ln\left(-x^3\left(\sqrt{2} - 2\tan(\_Z)\right)\left(\tan(\_Z)^2 + 1\right)\right) + \sqrt{2}\, \ln(2) + 6\sqrt{2}\, c_1 + 4\_Z\right)\right)$$

✓ Solution by Mathematica

Time used: 0.191 (sec). Leaf size: 62

```
DSolve[(2*x*y[x])+(x^2+2*x*y[x]+y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{3}\left(\sqrt{2}\arctan\left(\frac{\frac{y(x)}{x}+1}{\sqrt{2}}\right) + \log\left(\frac{y(x)^2}{x^2} + \frac{2y(x)}{x} + 3\right) + \log\left(\frac{y(x)}{x}\right)\right) = -\log(x) + c_1, y(x)\right]$$

1.28 problem 28

Internal problem ID [3173]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$-\sin(y)^2 + x\sin(2y)\,y' = -x^2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve((x^2-sin(y(x))^2)+(x*sin(2*y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arcsin\left(\sqrt{-c_1x - x^2}\right)$$
$$y(x) = -\arcsin\left(\sqrt{-c_1x - x^2}\right)$$

✓ Solution by Mathematica

Time used: 6.502 (sec). Leaf size: 39

```
DSolve[(x^2-Sin[y[x]]^2)+(x*Sin[2*y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arcsin\left(\sqrt{-x(x + 2c_1)}\right)$$
$$y(x) \to \arcsin\left(\sqrt{-x(x + 2c_1)}\right)$$

1.29 problem 29

Internal problem ID [3174]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$y(2x - y + 2) + 2(-y + x)\,y' = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 73

```
dsolve(y(x)*(2*x-y(x)+2)+2*(x-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(c_1 e^x x + \sqrt{e^{2x} c_1^2 x^2 + c_1 e^x}\right) e^{-x}}{c_1}$$

$$y(x) = -\frac{\left(-c_1 e^x x + \sqrt{e^{2x} c_1^2 x^2 + c_1 e^x}\right) e^{-x}}{c_1}$$

✓ Solution by Mathematica

Time used: 43.224 (sec). Leaf size: 125

```
DSolve[y[x]*(2*x-y[x]+2)+2*(x-y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - e^{-x}\sqrt{e^x\left(e^x x^2 - e^{2c_1}\right)}$$

$$y(x) \to x + e^{-x}\sqrt{e^x\left(e^x x^2 - e^{2c_1}\right)}$$

$$y(x) \to x - e^{-x}\sqrt{e^{2x} x^2}$$

$$y(x) \to e^{-x}\sqrt{e^{2x} x^2} + x$$

1.30 problem 30

Internal problem ID [3175]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{4yx + 3y^2 + x(x + 2y)\, y' = x}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 53

```
dsolve((4*x*y(x)+3*y(x)^2-x)+x*(x+2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{-x^3 + \sqrt{x^6 + x^5 - 4c_1 x}}{2x^2}$$
$$y(x) = -\frac{x^3 + \sqrt{x^6 + x^5 - 4c_1 x}}{2x^2}$$

✓ Solution by Mathematica

Time used: 0.621 (sec). Leaf size: 80

```
DSolve[(4*x*y[x]+3*y[x]^2-x)+x*(x+2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^4 + \sqrt{x^2}\sqrt{x^6 + x^5 + 4c_1 x}}{2x^3}$$
$$y(x) \to -\frac{x}{2} + \frac{\sqrt{x^2}\sqrt{x^6 + x^5 + 4c_1 x}}{2x^3}$$

1.31 problem 31

Internal problem ID [3176]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{y + x\left(y^2 + \ln(x)\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 275

```
dsolve((y(x))+x*(y(x)^2+ln(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-12c_1 + 4\sqrt{4\ln(x)^3 + 9c_1^2}\right)^{\frac{1}{3}}}{2} - \frac{2\ln(x)}{\left(-12c_1 + 4\sqrt{4\ln(x)^3 + 9c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(-12c_1 + 4\sqrt{4\ln(x)^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} + \frac{\ln(x)}{\left(-12c_1 + 4\sqrt{4\ln(x)^3 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(-12c_1+4\sqrt{4\ln(x)^3+9c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{2\ln(x)}{\left(-12c_1+4\sqrt{4\ln(x)^3+9c_1^2}\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(-12c_1 + 4\sqrt{4\ln(x)^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} + \frac{\ln(x)}{\left(-12c_1 + 4\sqrt{4\ln(x)^3 + 9c_1^2}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(-12c_1+4\sqrt{4\ln(x)^3+9c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{2\ln(x)}{\left(-12c_1+4\sqrt{4\ln(x)^3+9c_1^2}\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 1.211 (sec). Leaf size: 272

```
DSolve[(y[x])+x*(y[x]^2+Log[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{\sqrt{4\log^3(x)+9c_1{}^2}+3c_1}}{\sqrt[3]{2}} - \frac{\sqrt[3]{2}\log(x)}{\sqrt[3]{\sqrt{4\log^3(x)+9c_1{}^2}+3c_1}}$$

$$y(x) \to \frac{\sqrt[3]{2}\left(2+2i\sqrt{3}\right)\log(x)+i2^{2/3}\left(\sqrt{3}+i\right)\left(\sqrt{4\log^3(x)+9c_1{}^2}+3c_1\right)^{2/3}}{4\sqrt[3]{\sqrt{4\log^3(x)+9c_1{}^2}+3c_1}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)\log(x)}{2^{2/3}\sqrt[3]{\sqrt{4\log^3(x)+9c_1{}^2}+3c_1}} - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{\sqrt{4\log^3(x)+9c_1{}^2}+3c_1}}{2\sqrt[3]{2}}$$

$$y(x) \to 0$$

1.32 problem 32

Internal problem ID [3177]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], [_Abel`

$$y + (3yx^2 - x)\, y' = -x^2 - 2x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 65

```
dsolve((x^2+2*x+y(x))+(3*x^2*y(x)-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{-1 + \sqrt{-12\ln(x)\, x^2 - 6c_1x^2 - 6x^3 + 1}}{3x}$$
$$y(x) = \frac{1 + \sqrt{-12\ln(x)\, x^2 - 6c_1x^2 - 6x^3 + 1}}{3x}$$

✓ Solution by Mathematica

Time used: 0.543 (sec). Leaf size: 96

```
DSolve[(x^2+2*x+y[x])+(3*x^2*y[x]-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1 - \sqrt{\frac{1}{x^2}}x\sqrt{-6x^3 - 12x^2\log(x) + 9c_1x^2 + 1}}{3x}$$
$$y(x) \to \frac{1 + \sqrt{\frac{1}{x^2}}x\sqrt{-6x^3 - 12x^2\log(x) + 9c_1x^2 + 1}}{3x}$$

1.33 problem 33

Internal problem ID [3178]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$y^2 + (yx + y^2 - 1)\, y' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve((y(x)^2)+(x*y(x)+y(x)^2-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = e^{\mathrm{RootOf}\left(-e^{2\_Z}-2x\,e^{\_Z}+2c_1+2\_Z\right)}$$

✓ Solution by Mathematica

Time used: 0.15 (sec). Leaf size: 30

```
DSolve[(y[x]^2)+(x*y[x]+y[x]^2-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[x = \frac{\log(y(x)) - \frac{y(x)^2}{2}}{y(x)} + \frac{c_1}{y(x)}, y(x)\right]$$

1.34 problem 34

Internal problem ID [3179]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{3y^2 + x(x^2 + 3y^2 + 6y)\, y' = -3x^2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(3*(x^2+y(x)^2)+x*(x^2+3*y(x)^2+6*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$c_1 + e^{y(x)}\left(\frac{x^3}{3} + y(x)^2\, x\right) = 0$$

✓ Solution by Mathematica

Time used: 0.149 (sec). Leaf size: 26

```
DSolve[3*(x^2+y[x]^2)+x*(x^2+3*y[x]^2+6*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$\text{Solve}\left[x^3e^{y(x)} + 3xe^{y(x)}y(x)^2 = c_1, y(x)\right]$$

1.35 problem 35

Internal problem ID [3180]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$2y(x+y+2)+\left(y^2-x^2-4x-1\right)y'=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve(2*y(x)*(x+y(x)+2)+(y(x)^2-x^2-4*x-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-x-2+\frac{c_1}{2}-\frac{\sqrt{c_1^2-4c_1x-8c_1+12}}{2}$$
$$y(x)=-x-2+\frac{c_1}{2}+\frac{\sqrt{c_1^2-4c_1x-8c_1+12}}{2}$$

✓ Solution by Mathematica

Time used: 0.462 (sec). Leaf size: 74

```
DSolve[2*y[x]*(x+y[x]+2)+(y[x]^2-x^2-4*x-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x)\to\frac{1}{2}\left(-2x-\sqrt{4(-4+c_1)x-4+c_1{}^2}-c_1\right)$$
$$y(x)\to\frac{1}{2}\left(-2x+\sqrt{4(-4+c_1)x-4+c_1{}^2}-c_1\right)$$
$$y(x)\to 0$$

1.36 problem 36

Internal problem ID [3181]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{y^2 + 2y'y = -2x - 2}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve((2+y(x)^2+2*x)+(2*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 e^{-x} - 2x}$$
$$y(x) = -\sqrt{c_1 e^{-x} - 2x}$$

✓ Solution by Mathematica

Time used: 3.531 (sec). Leaf size: 43

```
DSolve[(2+y[x]^2+2*x)+(2*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-2x + c_1 e^{-x}}$$
$$y(x) \to \sqrt{-2x + c_1 e^{-x}}$$

1.37 problem 37

Internal problem ID [3182]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{2xy^2 - y + (y^2 + x + y)\,y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve((2*x*y(x)^2-y(x))+(y(x)^2+x+y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = e^{\mathrm{RootOf}\left(x^2 e^{\_Z} + e^{2\_Z} + e^{\_Z} c_1 + \_Z e^{\_Z} - x\right)}$$

✓ Solution by Mathematica

Time used: 0.18 (sec). Leaf size: 22

```
DSolve[(2*x*y[x]^2-y[x])+(y[x]^2+x+y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[x^2 - \frac{x}{y(x)} + y(x) + \log(y(x)) = c_1, y(x)\right]$$

1.38 problem 38

Internal problem ID [3183]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$y(y+x)+(x+2y-1)\,y'=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 93

```
dsolve(y(x)*(x+y(x))+(x+2*y(x)-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x)=-\frac{\left(\mathrm{e}^{x}x-\mathrm{e}^{x}-\sqrt{x^{2}\mathrm{e}^{2x}-2\,\mathrm{e}^{2x}x+\mathrm{e}^{2x}-4c_{1}\mathrm{e}^{x}}\right)\mathrm{e}^{-x}}{2}$$
$$y(x)=-\frac{\left(\mathrm{e}^{x}x-\mathrm{e}^{x}+\sqrt{x^{2}\mathrm{e}^{2x}-2\,\mathrm{e}^{2x}x+\mathrm{e}^{2x}-4c_{1}\mathrm{e}^{x}}\right)\mathrm{e}^{-x}}{2}$$

✓ Solution by Mathematica

Time used: 11.91 (sec). Leaf size: 80

```
DSolve[y[x]*(x+y[x])+(x+2*y[x]-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\left(-x-\frac{\sqrt{e^{x}(x-1)^{2}+4c_{1}}}{\sqrt{e^{x}}}+1\right)$$
$$y(x)\to\frac{1}{2}\left(-x+\frac{\sqrt{e^{x}(x-1)^{2}+4c_{1}}}{\sqrt{e^{x}}}+1\right)$$

1.39 problem 39

Internal problem ID [3184]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{2x(x^2 - \sin(y) + 1) + (x^2 + 1)\cos(y)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(2*x*(x^2-sin(y(x))+1)+(x^2+1)*cos(y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\arcsin\left(\ln\left(x^2+1\right)x^2 + c_1x^2 + \ln\left(x^2+1\right) + c_1\right)$$

✓ Solution by Mathematica

Time used: 7.478 (sec). Leaf size: 25

```
DSolve[2*x*(x^2-Sin[y[x]]+1)+(x^2+1)*Cos[y[x]]*y'[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\arcsin\left(\left(x^2+1\right)\left(\log\left(x^2+1\right)+8c_1\right)\right)$$

1.40 problem 41

Internal problem ID [3185]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$\boxed{y^2 + y - xy' = -x^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve((x^2+y(x)+y(x)^2)-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \tan(x + c_1)\, x$$

✓ Solution by Mathematica

Time used: 0.207 (sec). Leaf size: 12

```
DSolve[(x^2+y[x]+y[x]^2)-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \tan(x + c_1)$$

1.41 problem 42

Internal problem ID [3186]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _dAlembert]`

$$-\sqrt{y^2+x^2}+\left(y-\sqrt{y^2+x^2}\right)y'=-x$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 58

```
dsolve((x-sqrt(x^2+y(x)^2))+(y(x)-sqrt(x^2+y(x)^2))*diff(y(x),x)=0,y(x), singsol=all)
```

$$-c_1+\frac{\sqrt{x^2+y(x)^2}}{x^2y(x)}+\frac{1}{xy(x)}+\frac{1}{y(x)^2}+\frac{1}{x^2}+\frac{\sqrt{x^2+y(x)^2}}{xy(x)^2}=0$$

✓ Solution by Mathematica

Time used: 0.834 (sec). Leaf size: 34

```
DSolve[(x-Sqrt[x^2+y[x]^2])+(y[x]-Sqrt[x^2+y[x]^2])*y'[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x)\to -\frac{e^{c_1}(2x+e^{c_1})}{2(x+e^{c_1})}$$

$$y(x)\to 0$$

1.42 problem 43

Internal problem ID [3187]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$y\sqrt{y^2+1}+\left(x\sqrt{y^2+1}-y\right)y'=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve((y(x)*sqrt(1+y(x)^2))+(x*sqrt(1+y(x)^2)-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$x-\frac{\sqrt{y(x)^2+1}+c_1}{y(x)}=0$$

✓ Solution by Mathematica

Time used: 0.479 (sec). Leaf size: 82

```
DSolve[(y[x]*Sqrt[1+y[x]^2])+(x*Sqrt[1+y[x]^2]-y[x])*y'[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x)\to\frac{c_1x-\sqrt{x^2-1+c_1^2}}{x^2-1}$$

$$y(x)\to\frac{\sqrt{x^2-1+c_1^2}+c_1x}{x^2-1}$$

$$y(x)\to 0$$

$$y(x)\to -i$$

$$y(x)\to i$$

1.43 problem 44

Internal problem ID [3188]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y^2 - (yx + x^3)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
dsolve((y(x)^2)-(x*y(x)+x^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \left(-x - \sqrt{x^2 + c_1}\right) x$$
$$y(x) = \left(-x + \sqrt{x^2 + c_1}\right) x$$

✓ Solution by Mathematica

Time used: 0.551 (sec). Leaf size: 67

```
DSolve[(y[x]^2)-(x*y[x]+x^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^2 \left(1 + \sqrt{\frac{1}{x^3}} \sqrt{x\,(x^2 + c_1)}\right)$$
$$y(x) \to x^2 \left(-1 + \sqrt{\frac{1}{x^3}} \sqrt{x\,(x^2 + c_1)}\right)$$
$$y(x) \to 0$$

1.44 problem 45

Internal problem ID [3189]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘]]`

$$\boxed{y - 2x^3 \tan\left(\frac{y}{x}\right) - xy' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve(y(x)-2*x^3*tan(y(x)/x)-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arcsin\left(c_1 e^{-x^2}\right) x$$

✓ Solution by Mathematica

Time used: 59.679 (sec). Leaf size: 23

```
DSolve[y[x]-2*x^3*Tan[y[x]/x]-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \arcsin\left(e^{-x^2+c_1}\right)$$
$$y(x) \to 0$$

1.45 problem 46

Internal problem ID [3190]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$2y^2x^2 + y + (x^3y - x)\,y' = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve((2*x^2*y(x)^2+y(x))+(x^3*y(x)-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = x\,\mathrm{e}^{-\operatorname{LambertW}\left(-x^3\mathrm{e}^{-3c_1}\right)-3c_1}$$

✓ Solution by Mathematica

Time used: 2.365 (sec). Leaf size: 33

```
DSolve[(2*x^2*y[x]^2+y[x])+(x^3*y[x]-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{W\left(e^{-1+\frac{9c_1}{2^{2/3}}}x^3\right)}{x^2}$$
$$y(x) \to 0$$

1.46 problem 47

Internal problem ID [3191]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$y^2 + (yx + \tan(yx))\, y' = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 18

```
dsolve((y(x)^2)+(x*y(x)+tan(x*y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{RootOf}(\_Zc_1 \sin(\_Z) - x)}{x}$$

✓ Solution by Mathematica

Time used: 0.271 (sec). Leaf size: 14

```
DSolve[(y[x]^2)+(x*y[x]+Tan[x*y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}[y(x)\sin(xy(x)) = c_1, y(x)]$$

1.47 problem 48

Internal problem ID [3192]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{2y^4x - y + (4y^3x^3 - x)\,y' = 0}$$

✗ Solution by Maple

```
dsolve((2*x*y(x)^4-y(x))+(4*x^3*y(x)^3-x)*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(2*x*y[x]^4-y[x])+(4*x^3*y[x]^3-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.48 problem 49

Internal problem ID [3193]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$y^3 + y + \left(x^3 + y^2 - x\right) y' = -x^2$$

✗ Solution by Maple

```
dsolve((x^2+y(x)^3+y(x))+( x^3+y(x)^2-x )*diff(y(x),x)=0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(x^2+y[x]^3+y[x])+( x^3+y[x]^2-x )*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

1.49 problem 50

Internal problem ID [3194]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{y(y^2+1)+x(y^2-x+1)\,y'=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 153

```
dsolve((y(x)*(y(x)^2+1))+( x*(y(x)^2-x+1))*diff(y(x),x)=0,y(x), singsol=all)
```

$$c_1+\frac{-\sqrt{-\frac{2x^2}{(x-1)^2\left(\frac{1}{y(x)^2}-\frac{1}{x-1}\right)}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2x^2}{(x-1)^2\left(\frac{1}{y(x)^2}-\frac{1}{x-1}\right)}}\,(x-1)}{x\sqrt{\frac{2x+\frac{2}{\frac{1}{y(x)^2}-\frac{1}{x-1}}-2}{x-1}}}\right)+\sqrt{\frac{2x+\frac{2}{\frac{1}{y(x)^2}-\frac{1}{x-1}}-2}{x-1}}}{\sqrt{-\frac{2x^2}{(x-1)^2\left(\frac{1}{y(x)^2}-\frac{1}{x-1}\right)}}}=0$$

✓ Solution by Mathematica

Time used: 0.077 (sec). Leaf size: 34

```
DSolve[(y[x]*(y[x]^2+1))+( x*(y[x]^2-x+1))*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\operatorname{Solve}\left[\frac{1}{2}\left(-\arctan(y(x))-\frac{1}{y(x)}\right)+\frac{1}{2xy(x)}=c_1,y(x)\right]$$

1.50 problem 51

Internal problem ID [3195]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], [_Abel, ‘2nd type‘, ‘c`

$$\boxed{y^2 + (-y + \mathrm{e}^x)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 16

```
dsolve((y(x)^2)+( exp(x)-y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\mathrm{e}^x \operatorname{LambertW}\left(-c_1 \mathrm{e}^{-x}\right)$$

✓ Solution by Mathematica

Time used: 6.706 (sec). Leaf size: 306

```
DSolve[(y[x]^2)+( Exp[x]-y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{9}2^{2/3}\left(\frac{\left(\frac{e^x-\frac{3e^{2x}}{e^x-y(x)}}{\sqrt[3]{e^{3x}}}+2\right)\left(\frac{e^x(y(x)+2e^x)}{\sqrt[3]{e^{3x}}(e^x-y(x))}+1\right)\left(\left(\frac{e^x-\frac{3e^{2x}}{e^x-y(x)}}{\sqrt[3]{e^{3x}}}-1\right)\log\left(2^{2/3}\left(\frac{e^x-\frac{3e^{2x}}{e^x-y(x)}}{\sqrt[3]{e^{3x}}}+2\right)\right)+\left(\cdots\right.\right.}{\frac{(y(x)+2e^x)^3}{(e^x-y(x))^3}-\frac{3e^x(y(x)+2e^x)}{\sqrt[3]{e^{3x}}(e^x-y(x))}-2\cdots}\right.\right.$$

1.51 problem 52

Internal problem ID [3196]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$\boxed{y^2x^2 - 2y + \left(x^3y - x\right)y' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve((x^2*y(x)^2-2*y(x))+( x^3*y(x)-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{\text{LambertW}\left(-\frac{c_1}{x}\right)x^2}$$

✓ Solution by Mathematica

Time used: 6.74 (sec). Leaf size: 35

```
DSolve[(x^2*y[x]^2-2*y[x])+( x^3*y[x]-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{x^2 W\left(\frac{e^{-1+\frac{9c_1}{2^{2/3}}}}{x}\right)}$$

$$y(x) \to 0$$

1.52 problem 53

Internal problem ID [3197]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$2x^3y + y^3 - \left(x^4 + 2xy^2\right)y' = 0$$

✓ Solution by Maple

Time used: 0.5 (sec). Leaf size: 148

```
dsolve((2*x^3*y(x)+y(x)^3)-( x^4+2*x*y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = -\frac{x^{\frac{3}{2}} \operatorname{RootOf}\left(-16 + x^7 c_1\_Z^{12} - 4c_1 x^{\frac{11}{2}}\_Z^{10} + 6c_1 x^4\_Z^8 + \left(128x^{\frac{9}{2}} - 4c_1 x^{\frac{5}{2}}\right)\_Z^6 + \left(-192x^3 + c_1 x\right)\_}{2 \operatorname{RootOf}\left(-16 + x^7 c_1\_Z^{12} - 4c_1 x^{\frac{11}{2}}\_Z^{10} + 6c_1 x^4\_Z^8 + \left(128x^{\frac{9}{2}} - 4c_1 x^{\frac{5}{2}}\right)\_Z^6 + \left(-192x^3 + c_1 x\right)}$$

✓ Solution by Mathematica

Time used: 60.151 (sec). Leaf size: 2023

```
DSolve[(2*x^3*y[x]+y[x]^3)-( x^4+2*x*y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\sqrt{48x^3 + \frac{e^{4c_1}x^2}{\sqrt[3]{-3456e^{2c_1}x^7 + 144e^{4c_1}x^5 - e^{6c_1}x^3 + 192\sqrt{3}\sqrt{-e^{4c_1}x^{12}\left(-108x^2 + e^{2c_1}\right)}}} + \sqrt[3]{-3456e^{2c_1}x}$$

$$y(x) \to \sqrt{48x^3 + \frac{e^{4c_1}x^2}{\sqrt[3]{-3456e^{2c_1}x^7 + 144e^{4c_1}x^5 - e^{6c_1}x^3 + 192\sqrt{3}\sqrt{-e^{4c_1}x^{12}\left(-108x^2 + e^{2c_1}\right)}}} + \sqrt[3]{-3456e^{2c_1}x^7}$$

$$y(x) \to -\sqrt{\frac{i\left(\sqrt{3}+i\right)e^{4c_1}x^2+96x^3\sqrt[3]{-3456e^{2c_1}x^7 + 144e^{4c_1}x^5 - e^{6c_1}x^3 + 192\sqrt{3}\sqrt{-e^{4c_1}x^{12}\left(-108x^2 + e^{2c_1}\right)}}-2e^{2c_1}x\left(\right.}{\sqrt[3]{-3456e}}}$$

$$y(x) \to \sqrt{\frac{i\left(\sqrt{3}+i\right)e^{4c_1}x^2+96x^3\sqrt[3]{-3456e^{2c_1}x^7 + 144e^{4c_1}x^5 - e^{6c_1}x^3 + 192\sqrt{3}\sqrt{-e^{4c_1}x^{12}\left(-108x^2 + e^{2c_1}\right)}}-2e^{2c_1}x\left(48i\right.}{\sqrt[3]{-3456e^{2c}}}}$$

$$y(x) \to -\sqrt{\frac{-i\left(\sqrt{3}-i\right)e^{4c_1}x^2+96x^3\sqrt[3]{-3456e^{2c_1}x^7 + 144e^{4c_1}x^5 - e^{6c_1}x^3 + 192\sqrt{3}\sqrt{-e^{4c_1}x^{12}\left(-108x^2 + e^{2c_1}\right)}}+i\left(\sqrt{3}+\right.}{\sqrt[3]{-3456e}}}$$

$$y(x) \to \sqrt{\frac{-i\left(\sqrt{3}-i\right)e^{4c_1}x^2+96x^3\sqrt[3]{-3456e^{2c_1}x^7 + 144e^{4c_1}x^5 - e^{6c_1}x^3 + 192\sqrt{3}\sqrt{-e^{4c_1}x^{12}\left(-108x^2 + e^{2c_1}\right)}}+i\left(\sqrt{3}+i\right)\left(\right.}{\sqrt[3]{-3456e^{2c}}}}$$

1.53 problem 54

Internal problem ID [3198]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y\cos(x) - \sin(x)\, y' = -1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve((1+y(x)*cos(x))-( sin(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = (-\cot(x) + c_1)\sin(x)$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 15

```
DSolve[(1+y[x]*Cos[x])-( Sin[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\cos(x) + c_1 \sin(x)$$

1.54 problem 55

Internal problem ID [3199]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\left(\sin \left(y\right)^{2}+x \cot \left(y\right)\right) y^{\prime}=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 1223

```
dsolve((sin(y(x))^2+x*cot(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \arctan\left(-\frac{\sqrt{6\left(108x^2+12\sqrt{12x^6+81x^4}\right)^{\frac{1}{3}}-\frac{72x^2}{\left(108x^2+12\sqrt{12x^6+81x^4}\right)^{\frac{1}{3}}}}}{6}, \frac{\left(6\left(108x^2+12\sqrt{12x^6+81x^4}\right)^{\frac{1}{3}}-\right.}{216x}\right.$$

$$y(x) = \arctan\left(\frac{\sqrt{6\left(108x^2+12\sqrt{12x^6+81x^4}\right)^{\frac{1}{3}}-\frac{72x^2}{\left(108x^2+12\sqrt{12x^6+81x^4}\right)^{\frac{1}{3}}}}}{6}, -\frac{\left(6\left(108x^2+12\sqrt{12x^6+81x^4}\right)^{\frac{1}{3}}-\frac{72x^2}{\left(108x^2+12\sqrt{12x^6+81x^4}\right)^{\frac{1}{3}}}\right)^{\frac{3}{2}}}{216x}\right)$$

$$y(x) = \arctan\left(-\frac{\sqrt{-3\left(108x^2+12\sqrt{12x^6+81x^4}\right)^{\frac{1}{3}}+\frac{36x^2}{\left(108x^2+12\sqrt{12x^6+81x^4}\right)^{\frac{1}{3}}}-18i\sqrt{3}\left(\frac{\left(108x^2+12\sqrt{12x^6+81x^4}\right)^{\frac{1}{3}}}{6}\right.}}{6}\right.$$

$$y(x) = \arctan\left(\frac{\sqrt{-3\left(108x^2+12\sqrt{12x^6+81x^4}\right)^{\frac{1}{3}}+\frac{36x^2}{\left(108x^2+12\sqrt{12x^6+81x^4}\right)^{\frac{1}{3}}}-18i\sqrt{3}\left(\frac{\left(108x^2+12\sqrt{12x^6+81x^4}\right)^{\frac{1}{3}}}{6}+\right.}}{6}\right.$$

$$-\frac{\left(-3\left(108x^2+12\sqrt{12x^6+81x^4}\right)^{\frac{1}{3}}+\frac{36x^2}{\left(108x^2+12\sqrt{12x^6+81x^4}\right)^{\frac{1}{3}}}-18i\sqrt{3}\left(\frac{\left(108x^2+12\sqrt{12x^6+81x^4}\right)^{\frac{1}{3}}}{6}+\frac{}{\left(108x^2+1\right.}\right.\right.}{216x}$$

✓ Solution by Mathematica

Time used: 0.249 (sec). Leaf size: 1647

```
DSolve[(Sin[y[x]]^2+x*Cot[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(-\sqrt{-\frac{\sqrt[3]{\frac{2}{3}}x^2}{\sqrt[3]{\sqrt{3}\sqrt{x^4\left(4x^2+27\right)}-9x^2}}+\frac{\sqrt[3]{\sqrt{3}\sqrt{x^4\left(4x^2+27\right)}-9x^2}}{\sqrt[3]{2}3^{2/3}}+1}\right)$$

$$y(x) \to \arccos\left(-\sqrt{-\frac{\sqrt[3]{\frac{2}{3}}x^2}{\sqrt[3]{\sqrt{3}\sqrt{x^4\left(4x^2+27\right)}-9x^2}}+\frac{\sqrt[3]{\sqrt{3}\sqrt{x^4\left(4x^2+27\right)}-9x^2}}{\sqrt[3]{2}3^{2/3}}+1}\right)$$

$$y(x) \to -\arccos\left(\sqrt{-\frac{\sqrt[3]{\frac{2}{3}}x^2}{\sqrt[3]{\sqrt{3}\sqrt{x^4\left(4x^2+27\right)}-9x^2}}+\frac{\sqrt[3]{\sqrt{3}\sqrt{x^4\left(4x^2+27\right)}-9x^2}}{\sqrt[3]{2}3^{2/3}}+1}\right)$$

$$y(x) \to \arccos\left(\sqrt{-\frac{\sqrt[3]{\frac{2}{3}}x^2}{\sqrt[3]{\sqrt{3}\sqrt{x^4\left(4x^2+27\right)}-9x^2}}+\frac{\sqrt[3]{\sqrt{3}\sqrt{x^4\left(4x^2+27\right)}-9x^2}}{\sqrt[3]{2}3^{2/3}}+1}\right)$$

$$y(x) \to -\arccos\left(-\sqrt{\frac{\left(\sqrt{3}-3i\right)x^2}{2^{2/3}3^{5/6}\sqrt[3]{\sqrt{3}\sqrt{x^4\left(4x^2+27\right)}-9x^2}}+\frac{1}{12}\left(-i2^{2/3}3^{5/6}\sqrt[3]{\sqrt{3}\sqrt{x^4\left(4x^2+27\right)}-9x^2}-2^{2/}\cdots\right.}\right.$$

$$y(x) \to \arccos\left(-\sqrt{\frac{\left(\sqrt{3}-3i\right)x^2}{2^{2/3}3^{5/6}\sqrt[3]{\sqrt{3}\sqrt{x^4\left(4x^2+27\right)}-9x^2}}+\frac{1}{12}\left(-i2^{2/3}3^{5/6}\sqrt[3]{\sqrt{3}\sqrt{x^4\left(4x^2+27\right)}-9x^2}-2^{2/3}\sqrt[3]{\cdots}\right.}\right.$$

$$y(x) \to -\arccos\left(\sqrt{\frac{\left(\sqrt{3}-3i\right)x^2}{2^{2/3}3^{5/6}\sqrt[3]{\sqrt{3}\sqrt{x^4\left(4x^2+27\right)}-9x^2}}+\frac{1}{12}\left(-i2^{2/3}3^{5/6}\sqrt[3]{\sqrt{3}\sqrt{x^4\left(4x^2+27\right)}-9x^2}-2^{2/3}\cdots\right.}\right.$$

$$y(x) \to \arccos\left(\sqrt{\frac{\left(\sqrt{3}-3i\right)x^2}{2^{2/3}3^{5/6}\sqrt[3]{\sqrt{3}\sqrt{x^4\left(4x^2+27\right)}-9x^2}}+\frac{1}{12}\left(-i2^{2/3}3^{5/6}\sqrt[3]{\sqrt{3}\sqrt{x^4\left(4x^2+27\right)}-9x^2}-2^{2/3}\sqrt[3]{3\cdots}\right.}\right.$$

1.55 problem 56

Internal problem ID [3200]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{-(y - 2yx)\, y' = -1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 33

```
dsolve(1-(y(x)-2*x*y(x))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \sqrt{-\ln(-1+2x) + c_1}$$
$$y(x) = -\sqrt{-\ln(-1+2x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 45

```
DSolve[1-(y[x]-2*x*y[x])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-\log(1-2x) + 2c_1}$$
$$y(x) \to \sqrt{-\log(1-2x) + 2c_1}$$

1.56 problem 57

Internal problem ID [3201]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{-(1+2x\tan(y))\, y' = -1}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 41

```
dsolve(1-(1+2*x*tan(y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$\frac{c_1}{2\cos(2y(x))+2}+x-\frac{2y(x)+\sin(2y(x))}{2\cos(2y(x))+2}=0$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 36

```
DSolve[1-(1+2*x*Tan[y[x]])*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x=\left(\frac{y(x)}{2}+\frac{1}{4}\sin(2y(x))\right)\sec^2(y(x))+c_1\sec^2(y(x)),y(x)\right]$$

1.57 problem 58

Internal problem ID [3202]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\left(y^3 + \frac{x}{y}\right) y' = 1$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((y(x)^3+x/y(x))*diff(y(x),x)=1,y(x), singsol=all)
```

$$-c_1 y(x) + x - \frac{y(x)^4}{3} = 0$$

✓ Solution by Mathematica

Time used: 0.107 (sec). Leaf size: 997

```
DSolve[(y[x]^3+x/y[x])*y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{256x^3+81c_1{}^4}}}{\sqrt[3]{2}}-\frac{4\sqrt[3]{2}x}{\sqrt[3]{9c_1{}^2-\sqrt{256x^3+81c_1{}^4}}}}$$

$$-\frac{1}{2}\sqrt{\frac{4\sqrt[3]{2}x}{\sqrt[3]{9c_1{}^2-\sqrt{256x^3+81c_1{}^4}}}-\frac{\sqrt[3]{9c_1{}^2-\sqrt{256x^3+81c_1{}^4}}}{\sqrt[3]{2}}-\frac{6c_1}{\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{256x^3+81c_1{}^4}}}{\sqrt[3]{2}}-\frac{}{\sqrt[3]{9c_1}}}}}$$

$$y(x) \to \frac{1}{2}\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{256x^3+81c_1{}^4}}}{\sqrt[3]{2}}-\frac{4\sqrt[3]{2}x}{\sqrt[3]{9c_1{}^2-\sqrt{256x^3+81c_1{}^4}}}}$$

$$+\frac{1}{2}\sqrt{\frac{4\sqrt[3]{2}x}{\sqrt[3]{9c_1{}^2-\sqrt{256x^3+81c_1{}^4}}}-\frac{\sqrt[3]{9c_1{}^2-\sqrt{256x^3+81c_1{}^4}}}{\sqrt[3]{2}}-\frac{6c_1}{\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{256x^3+81c_1{}^4}}}{\sqrt[3]{2}}-\frac{}{\sqrt[3]{9c_1}}}}}$$

$$y(x) \to -\frac{1}{2}\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{256x^3+81c_1{}^4}}}{\sqrt[3]{2}}-\frac{4\sqrt[3]{2}x}{\sqrt[3]{9c_1{}^2-\sqrt{256x^3+81c_1{}^4}}}}$$

$$-\frac{1}{2}\sqrt{\frac{4\sqrt[3]{2}x}{\sqrt[3]{9c_1{}^2-\sqrt{256x^3+81c_1{}^4}}}-\frac{\sqrt[3]{9c_1{}^2-\sqrt{256x^3+81c_1{}^4}}}{\sqrt[3]{2}}+\frac{6c_1}{\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{256x^3+81c_1{}^4}}}{\sqrt[3]{2}}-\frac{}{\sqrt[3]{9c_1}}}}}$$

$$y(x)$$

$$\to \frac{1}{2}\sqrt{\frac{4\sqrt[3]{2}x}{\sqrt[3]{9c_1{}^2-\sqrt{256x^3+81c_1{}^4}}}-\frac{\sqrt[3]{9c_1{}^2-\sqrt{256x^3+81c_1{}^4}}}{\sqrt[3]{2}}+\frac{6c_1}{\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{256x^3+81c_1{}^4}}}{\sqrt[3]{2}}-\frac{}{\sqrt[3]{9c_1{}^2}}}}}$$

$$-\frac{1}{2}\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{256x^3+81c_1{}^4}}}{\sqrt[3]{2}}-\frac{4\sqrt[3]{2}x}{\sqrt[3]{9c_1{}^2-\sqrt{256x^3+81c_1{}^4}}}}$$

1.58 problem 59

Internal problem ID [3203]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_exponential_symmetries]]`

$$\boxed{(x - y^2)\, y' = -1}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 24

```
dsolve(1+(x-y(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$x - y(x)^2 + 2y(x) - 2 - \mathrm{e}^{-y(x)} c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.116 (sec). Leaf size: 24

```
DSolve[1+(x-y[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = y(x)^2 - 2y(x) + c_1 e^{-y(x)} + 2, y(x)\right]$$

1.59 problem 60

Internal problem ID [3204]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$\boxed{y^2 + \left(yx + y^2 - 1\right) y' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(y(x)^2+(x*y(x)+y(x)^2-1)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = e^{\mathrm{RootOf}\left(-e^{2\_Z}-2x\,e^{\_Z}+2c_1+2\_Z\right)}$$

✓ Solution by Mathematica

Time used: 0.135 (sec). Leaf size: 30

```
DSolve[y[x]^2+(x*y[x]+y[x]^2-1)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[x = \frac{\log(y(x)) - \frac{y(x)^2}{2}}{y(x)} + \frac{c_1}{y(x)}, y(x)\right]$$

1.60 problem 61

Internal problem ID [3205]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{y - (e^y + 2yx - 2x)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 62

```
dsolve(y(x)=(exp(y(x))+2*x*y(x)-2*x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(\_Z^2 x - c_1 + \_Z + e^{\operatorname{RootOf}\left(-x\, e^{2\_Z}\_Z^2 + \_Z e^{\_Z} + c_1 - e^{\_Z}\right)}\right) e^{-\operatorname{RootOf}\left(-x\, e^{2\_Z}\_Z^2 + \_Z e^{\_Z} + c_1 - e^{\_Z}\right)}$$

✓ Solution by Mathematica

Time used: 0.294 (sec). Leaf size: 34

```
DSolve[y[x]==(Exp[y[x]]+2*x*y[x]-2*x)*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = \frac{e^{y(x)}(-y(x) - 1)}{y(x)^2} + \frac{c_1 e^{2y(x)}}{y(x)^2}, y(x)\right]$$

1.61 problem 62

Internal problem ID [3206]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(2x+3)\,y'-y=\sqrt{2x+3}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve((2*x+3)*diff(y(x),x)=y(x)+sqrt(2*x+3),y(x), singsol=all)
```

$$y(x)=\left(\frac{\ln(2x+3)}{2}+c_1\right)\sqrt{2x+3}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 29

```
DSolve[(2*x+3)*y'[x]==y[x]+Sqrt[2*x+3],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\sqrt{2x+3}(\log(2x+3)+2c_1)$$

1.62 problem 63

Internal problem ID [3207]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y + (e^y y^2 - x)\, y' = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve(y(x)+(y(x)^2*exp(y(x))-x)*diff(y(x),x)=0,y(x), singsol=all)
```

$$x - \left(-e^{y(x)} + c_1\right) y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.195 (sec). Leaf size: 19

```
DSolve[y[x]+(y[x]^2*Exp[y[x]]-x)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = -e^{y(x)} y(x) + c_1 y(x), y(x)\right]$$

1.63 problem 64

Internal problem ID [3208]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 64.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - 3y\tan(x) = 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x)=1+3*y(x)*tan(x),y(x), singsol=all)
```

$$y(x) = \frac{9\sin(x) + \sin(3x) + 12c_1}{3\cos(3x) + 9\cos(x)}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 26

```
DSolve[y'[x]==1+3*y[x]*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}\sec^3(x)(9\sin(x) + \sin(3x) + 12c_1)$$

1.64 problem 65

Internal problem ID [3209]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 65.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(\cos(x)+1)\, y' - \sin(x)\,(\sin(x)+\sin(x)\cos(x)-y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve((1+cos(x))*diff(y(x),x)=sin(x)*( sin(x)+sin(x)*cos(x)-y(x) ),y(x), singsol=all)
```

$$y(x) = (-\sin(x)+x+c_1)(\cos(x)+1)$$

✓ Solution by Mathematica

Time used: 0.096 (sec). Leaf size: 24

```
DSolve[(1+Cos[x])*y'[x]==Sin[x]*( Sin[x]+Sin[x]*Cos[x]-y[x] ),y[x],x,IncludeSingularSolution
```

$$y(x) \to \cos^2\left(\frac{x}{2}\right)(2x - 2\sin(x) + c_1)$$

1.65 problem 66

Internal problem ID [3210]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 66.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' - \left(\sin\left(x\right)^{2} - y\right)\cos\left(x\right) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)=( sin(x)^2-y(x))*cos(x),y(x), singsol=all)
```

$$y(x) = \frac{5}{2} + \mathrm{e}^{-\sin(x)}c_1 - \frac{\cos\left(2x\right)}{2} - 2\sin\left(x\right)$$

✓ Solution by Mathematica

Time used: 0.147 (sec). Leaf size: 30

```
DSolve[y'[x]==( Sin[x]^2-y[x])*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\sin(x) - \frac{1}{2}\cos(2x) + c_1 e^{-\sin(x)} + \frac{5}{2}$$

1.66 problem 68

Internal problem ID [3211]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 68.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x+1)\,y'-y=x(x+1)^2}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1+x)*diff(y(x),x)-y(x)=x*(1+x)^2,y(x), singsol=all)
```

$$y(x)=\left(\frac{x^2}{2}+c_1\right)(x+1)$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 20

```
DSolve[(1+x)*y'[x]-y[x]==x*(1+x)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}(x+1)\left(x^2+2c_1\right)$$

1.67 problem 69

Internal problem ID [3212]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]`

$$y + \left(x - y(y+1)^2\right) y' = -1$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve((1+y(x))+(x-y(x)*(1+y(x))^2)* diff(y(x),x)=0,y(x), singsol=all)
```

$$x - \frac{\frac{y(x)^4}{4} + \frac{2y(x)^3}{3} + \frac{y(x)^2}{2} + c_1}{y(x) + 1} = 0$$

✓ Solution by Mathematica

Time used: 33.714 (sec). Leaf size: 1594

```
DSolve[(1+y[x])+(x-y[x]*(1+y[x])^2)* y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(-\sqrt{\frac{-24x+6+72c_1}{\sqrt[3]{27x^2-\frac{1}{432}\sqrt{186624\left(27x^2+1+12c_1\right)^2-4(-144x+36+432c_1)^3}+1+12c_1}}+6\sqrt[3]{27x^2 -}} \right.$$

$$-3\sqrt{-\frac{3\left(32x+\frac{64}{27}\right)}{4\sqrt{\frac{-24x+6+72c_1}{\sqrt[3]{27x^2-\frac{1}{432}\sqrt{186624\left(27x^2+1+12c_1\right)^2-4(-144x+36+432c_1)^3}+1+12c_1}}+6\sqrt[3]{27}}}}$$

$$\left. -4 \right)$$

$$y(x) \to \frac{1}{6}\left(-\sqrt{\frac{-24x+6+72c_1}{\sqrt[3]{27x^2-\frac{1}{432}\sqrt{186624\left(27x^2+1+12c_1\right)^2-4(-144x+36+432c_1)^3}+1+12c_1}}+6\sqrt[3]{27x^2 -}} \right.$$

$$+3\sqrt{-\frac{3\left(32x+\frac{64}{27}\right)}{4\sqrt{\frac{-24x+6+72c_1}{\sqrt[3]{27x^2-\frac{1}{432}\sqrt{186624\left(27x^2+1+12c_1\right)^2-4(-144x+36+432c_1)^3}+1+12c_1}}+6\sqrt[3]{27}}}}$$

$$\left. -4 \right)$$

1.68 problem 71.1

Internal problem ID [3213]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 71.1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + y^2 = x^2 + 1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)+y(x)^2=1+x^2,y(x), singsol=all)
```

$$y(x) = x - \frac{e^{-x^2}}{c_1 - \frac{\sqrt{\pi}\,\mathrm{erf}(x)}{2}}$$

✓ Solution by Mathematica

Time used: 0.136 (sec). Leaf size: 36

```
DSolve[y'[x]+y[x]^2==1+x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \frac{2e^{-x^2}}{\sqrt{\pi}\mathrm{erf}(x) + 2c_1}$$

$$y(x) \to x$$

1.69 problem 72

Internal problem ID [3214]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 72.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{3xy' - 3x y^4 \ln (x) - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 234

```
dsolve(3*x*diff(y(x),x)-3*x*y(x)^4*ln(x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-4x(6\ln (x)\, x^2 - 3x^2 - 4c_1)^2\right)^{\frac{1}{3}}}{6\ln (x)\, x^2 - 3x^2 - 4c_1}$$

$$y(x) = -\frac{\left(-4x(6\ln (x)\, x^2 - 3x^2 - 4c_1)^2\right)^{\frac{1}{3}}}{2\,(6\ln (x)\, x^2 - 3x^2 - 4c_1)} - \frac{i\sqrt{3}\left(-4x(6\ln (x)\, x^2 - 3x^2 - 4c_1)^2\right)^{\frac{1}{3}}}{2\,(6\ln (x)\, x^2 - 3x^2 - 4c_1)}$$

$$y(x) = -\frac{\left(-4x(6\ln (x)\, x^2 - 3x^2 - 4c_1)^2\right)^{\frac{1}{3}}}{2\,(6\ln (x)\, x^2 - 3x^2 - 4c_1)} + \frac{i\sqrt{3}\left(-4x(6\ln (x)\, x^2 - 3x^2 - 4c_1)^2\right)^{\frac{1}{3}}}{12\ln (x)\, x^2 - 6x^2 - 8c_1}$$

✓ Solution by Mathematica

Time used: 0.25 (sec). Leaf size: 120

```
DSolve[3*x*y'[x]-3*x*y[x]^4*Log[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(-2)^{2/3}\sqrt[3]{x}}{\sqrt[3]{3x^2 - 6x^2\log(x) + 4c_1}}$$

$$y(x) \to \frac{2^{2/3}\sqrt[3]{x}}{\sqrt[3]{3x^2 - 6x^2\log(x) + 4c_1}}$$

$$y(x) \to -\frac{\sqrt[3]{-1}2^{2/3}\sqrt[3]{x}}{\sqrt[3]{3x^2 - 6x^2\log(x) + 4c_1}}$$

$$y(x) \to 0$$

1.70 problem 73

Internal problem ID [3215]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 73.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, [_Abel, '2nd type', 'cl`

$$y' - \frac{4x^3y^2}{yx^4 + 2} = 0$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 45

```
dsolve(diff(y(x),x)=(4*x^3*y(x)^2)/(x^4*y(x)+2),y(x), singsol=all)
```

$$y(x) = \frac{x^4 - \sqrt{x^8 + 4c_1}}{2c_1}$$
$$y(x) = \frac{x^4 + \sqrt{x^8 + 4c_1}}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.409 (sec). Leaf size: 56

```
DSolve[y'[x]==(4*x^3*y[x]^2)/(x^4*y[x]+2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2}{-x^4 + \sqrt{x^8 + 4c_1}}$$
$$y(x) \to -\frac{2}{x^4 + \sqrt{x^8 + 4c_1}}$$
$$y(x) \to 0$$

1.71 problem 74

Internal problem ID [3216]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 74.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{y(6y^2 - x - 1) + 2xy' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(y(x)*(6*y(x)^2-x-1)+2*x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(c_1 e^{-x} + 6)\, x}}{c_1 e^{-x} + 6}$$
$$y(x) = -\frac{\sqrt{(c_1 e^{-x} + 6)\, x}}{c_1 e^{-x} + 6}$$

✓ Solution by Mathematica

Time used: 0.709 (sec). Leaf size: 65

```
DSolve[y[x]*(6*y[x]^2-x-1)+2*x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{x/2}\sqrt{x}}{\sqrt{6e^x + c_1}}$$
$$y(x) \to \frac{e^{x/2}\sqrt{x}}{\sqrt{6e^x + c_1}}$$
$$y(x) \to 0$$

1.72 problem 75

Internal problem ID [3217]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 75.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Bernoulli]`

$$\boxed{(x+1)\left(y'+y^2\right)-y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve((1+x)*(diff(y(x),x)+y(x)^2)-y(x)=0,y(x), singsol=all)
```

$$y(x)=\frac{2x+2}{x^2+2c_1+2x}$$

✓ Solution by Mathematica

Time used: 0.201 (sec). Leaf size: 28

```
DSolve[(1+x)*(y'[x]+y[x]^2)-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2(x+1)}{x^2+2x+2c_1}$$
$$y(x)\to 0$$

1.73 problem 76

Internal problem ID [3218]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 76.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y'yx + y^2 = \sin(x)}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 42

```
dsolve(x*y(x)*diff(y(x),x)+y(x)^2-sin(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{2\sin(x) - 2x\cos(x) + c_1}}{x}$$
$$y(x) = -\frac{\sqrt{2\sin(x) - 2x\cos(x) + c_1}}{x}$$

✓ Solution by Mathematica

Time used: 0.367 (sec). Leaf size: 50

```
DSolve[x*y[x]*y'[x]+y[x]^2-Sin[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{2\sin(x) - 2x\cos(x) + c_1}}{x}$$
$$y(x) \to \frac{\sqrt{2\sin(x) - 2x\cos(x) + c_1}}{x}$$

1.74 problem 77

Internal problem ID [3219]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 77.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$-y^4 + xy^3y' = -2x^3$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 73

```
dsolve((2*x^3-y(x)^4)+(x*y(x)^3)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \left(c_1x^4 + 8x^3\right)^{\frac{1}{4}}$$
$$y(x) = -\left(c_1x^4 + 8x^3\right)^{\frac{1}{4}}$$
$$y(x) = -i\left(c_1x^4 + 8x^3\right)^{\frac{1}{4}}$$
$$y(x) = i\left(c_1x^4 + 8x^3\right)^{\frac{1}{4}}$$

✓ Solution by Mathematica

Time used: 0.243 (sec). Leaf size: 88

```
DSolve[(2*x^3-y[x]^4)+(x*y[x]^3)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^{3/4}\sqrt[4]{8 + c_1x}$$
$$y(x) \to -ix^{3/4}\sqrt[4]{8 + c_1x}$$
$$y(x) \to ix^{3/4}\sqrt[4]{8 + c_1x}$$
$$y(x) \to x^{3/4}\sqrt[4]{8 + c_1x}$$

1.75 problem 78

Internal problem ID [3220]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 78.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' - y\tan(x) + \cos(x)\, y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)-y(x)*tan(x)+y(x)^2*cos(x)=0,y(x), singsol=all)
```

$$y(x) = \frac{1}{(x + c_1)\cos(x)}$$

✓ Solution by Mathematica

Time used: 0.22 (sec). Leaf size: 19

```
DSolve[y'[x]-y[x]*Tan[x]+y[x]^2*Cos[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sec(x)}{x + c_1}$$
$$y(x) \to 0$$

1.76 problem 79

Internal problem ID [3221]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cl`

$$6y^2 - x(2x^3 + y)\, y' = 0$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 227

```
dsolve(6*y(x)^2-(x*(2*x^3+y(x)))*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = x^3 \left(\frac{x^3 - \sqrt{x^6 + 8c_1x^3}}{2c_1} + 2\right)$$

$$y(x) = x^3 \left(\frac{x^3 + \sqrt{x^6 + 8c_1x^3}}{2c_1} + 2\right)$$

$$y(x) = x^3 \left(\frac{\left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)^3 \left(x^3 - \sqrt{x^6 + 8c_1x^3}\right)}{2c_1} + 2\right)$$

$$y(x) = x^3 \left(\frac{\left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)^3 \left(x^3 + \sqrt{x^6 + 8c_1x^3}\right)}{2c_1} + 2\right)$$

$$y(x) = x^3 \left(\frac{\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^3 \left(x^3 - \sqrt{x^6 + 8c_1x^3}\right)}{2c_1} + 2\right)$$

$$y(x) = x^3 \left(\frac{\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^3 \left(x^3 + \sqrt{x^6 + 8c_1x^3}\right)}{2c_1} + 2\right)$$

✓ Solution by Mathematica

Time used: 1.396 (sec). Leaf size: 123

```
DSolve[6*y[x]^2-(x*(2*x^3+y[x]))*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x^3 \left(-1 + \frac{2}{1 - \frac{4x^{3/2}}{\sqrt{16x^3+c_1}}}\right)$$

$$y(x) \to 2x^3 \left(-1 + \frac{2}{1 + \frac{4x^{3/2}}{\sqrt{16x^3+c_1}}}\right)$$

$$y(x) \to 0$$

$$y(x) \to 2x^3$$

$$y(x) \to \frac{2\left((x^3)^{3/2} - x^{9/2}\right)}{x^{3/2} + \sqrt{x^3}}$$

1.77 problem 80

Internal problem ID [3222]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 80.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{xy'^3 - yy'^2 = -1}$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 80

```
dsolve(x*(diff(y(x),x))^3-y(x)*(diff(y(x),x))^2+1=0,y(x), singsol=all)
```

$$y(x) = \frac{3\,2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{3\,2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{4} - \frac{3i\sqrt{3}\,2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{4}$$
$$y(x) = -\frac{3\,2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{4} + \frac{3i\sqrt{3}\,2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{4}$$
$$y(x) = c_1x + \frac{1}{c_1^2}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 69

```
DSolve[x*(y'[x])^3-y[x]*(y'[x])^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x + \frac{1}{c_1{}^2}$$

$$y(x) \to 3\left(-\frac{1}{2}\right)^{2/3} x^{2/3}$$

$$y(x) \to \frac{3x^{2/3}}{2^{2/3}}$$

$$y(x) \to -\frac{3\sqrt[3]{-1}x^{2/3}}{2^{2/3}}$$

1.78 problem 81

Internal problem ID [3223]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 81.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y - xy' - y'^3 = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 33

```
dsolve(y(x)=x*diff(y(x),x)+(diff(y(x),x))^3,y(x), singsol=all)
```

$$y(x) = -\frac{2\sqrt{-3x}\,x}{9}$$
$$y(x) = \frac{2\sqrt{-3x}\,x}{9}$$
$$y(x) = c_1^3 + c_1 x$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 54

```
DSolve[y[x]==x*y'[x]+(y'[x])^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\left(x + c_1{}^2\right)$$
$$y(x) \to -\frac{2ix^{3/2}}{3\sqrt{3}}$$
$$y(x) \to \frac{2ix^{3/2}}{3\sqrt{3}}$$

1.79 problem 82

Internal problem ID [3224]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 82.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x\left(y'^2-1\right)-2y'=0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 49

```
dsolve(x*( (diff(y(x),x))^2-1)=2*diff(y(x),x) ,y(x), singsol=all)
```

$$y(x)=\sqrt{x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)+\ln(x)+c_1$$
$$y(x)=-\sqrt{x^2+1}+\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)+\ln(x)+c_1$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 59

```
DSolve[x*( (y'[x])^2-1)==2*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\sqrt{x^2+1}+\log\left(\sqrt{x^2+1}-1\right)+c_1$$
$$y(x)\to-\sqrt{x^2+1}+\log\left(\sqrt{x^2+1}+1\right)+c_1$$

1.80 problem 83

Internal problem ID [3225]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 83.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{xy'(y'+2)-y=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 49

```
dsolve(x*diff(y(x),x)*(diff(y(x),x)+2)=y(x),y(x), singsol=all)
```

$$y(x)=-x$$
$$y(x)=\sqrt{c_1x}\left(\frac{\sqrt{c_1x}}{x}+2\right)$$
$$y(x)=-\sqrt{c_1x}\left(-\frac{\sqrt{c_1x}}{x}+2\right)$$

✓ Solution by Mathematica

Time used: 0.186 (sec). Leaf size: 63

```
DSolve[x*y'[x]*(y'[x]+2)==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{c_1}-2e^{\frac{c_1}{2}}\sqrt{x}$$
$$y(x)\to 2e^{-\frac{c_1}{2}}\sqrt{x}+e^{-c_1}$$
$$y(x)\to 0$$
$$y(x)\to -x$$

1.81 problem 84

Internal problem ID [3226]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 84.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[_quadrature]`

$$-y'\sqrt{y'^2+1} = -x$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 187

```
dsolve(x=diff(y(x),x)*sqrt( (diff(y(x),x))^2+1),y(x), singsol=all)
```

$$y(x) = \frac{i\sqrt{2}\left(-\frac{256\sqrt{\pi}\,\sqrt{2}\,x^3\cosh\left(\frac{3\,\mathrm{arcsinh}(2x)}{2}\right)}{3} - \frac{8\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{64}{3}x^4-\frac{8}{3}x^2+\frac{2}{3}\right)\sinh\left(\frac{3\,\mathrm{arcsinh}(2x)}{2}\right)}{\sqrt{4x^2+1}}\right)}{32\sqrt{\pi}} + c_1$$

$$y(x) = -\frac{i\sqrt{2}\left(-\frac{256\sqrt{\pi}\,\sqrt{2}\,x^3\cosh\left(\frac{3\,\mathrm{arcsinh}(2x)}{2}\right)}{3} - \frac{8\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{64}{3}x^4-\frac{8}{3}x^2+\frac{2}{3}\right)\sinh\left(\frac{3\,\mathrm{arcsinh}(2x)}{2}\right)}{\sqrt{4x^2+1}}\right)}{32\sqrt{\pi}} + c_1$$

$$y(x) = \int -\frac{\sqrt{-2+2\sqrt{4x^2+1}}}{2}dx + c_1$$

$$y(x) = \int \frac{\sqrt{-2+2\sqrt{4x^2+1}}}{2}dx + c_1$$

✓ Solution by Mathematica

Time used: 0.161 (sec). Leaf size: 207

```
DSolve[x==y'[x]*Sqrt[ (y'[x])^2+1],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{2}x\left(\sqrt{4x^2+1}-2\right)}{3\sqrt{\sqrt{4x^2+1}-1}}+c_1$$

$$y(x) \to \frac{\sqrt{2}x\left(\sqrt{4x^2+1}-2\right)}{3\sqrt{\sqrt{4x^2+1}-1}}+c_1$$

$$y(x) \to -\frac{\sqrt{2}x\left(4x^2+3\sqrt{4x^2+1}+3\right)}{3\left(-\sqrt{4x^2+1}-1\right)^{3/2}}+c_1$$

$$y(x) \to \frac{\sqrt{2}x\left(4x^2+3\sqrt{4x^2+1}+3\right)}{3\left(-\sqrt{4x^2+1}-1\right)^{3/2}}+c_1$$

1.82 problem 85

Internal problem ID [3227]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 85.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$2{y'}^2(-xy'+y)=1$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 57

```
dsolve(2*(diff(y(x),x))^2*(y(x)-x*diff(y(x),x))=1,y(x), singsol=all)
```

$$y(x)=\frac{3x^{\frac{2}{3}}}{2}$$
$$y(x)=-\frac{3x^{\frac{2}{3}}}{4}-\frac{3i\sqrt{3}\,x^{\frac{2}{3}}}{4}$$
$$y(x)=-\frac{3x^{\frac{2}{3}}}{4}+\frac{3i\sqrt{3}\,x^{\frac{2}{3}}}{4}$$
$$y(x)=c_1x+\frac{1}{2c_1^2}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 67

```
DSolve[2*(y'[x])^2*(y[x]-x*y'[x])==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 x + \frac{1}{2{c_1}^2}$$

$$y(x) \rightarrow \frac{3x^{2/3}}{2}$$

$$y(x) \rightarrow -\frac{3}{2}\sqrt[3]{-1}x^{2/3}$$

$$y(x) \rightarrow \frac{3}{2}(-1)^{2/3}x^{2/3}$$

1.83 problem 86

Internal problem ID [3228]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 86.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y - 2xy' - y^2 y'^3 = 0}$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 107

```
dsolve(y(x)=2*x*diff(y(x),x)+y(x)^2*(diff(y(x),x))^3,y(x), singsol=all)
```

$$y(x) = -\frac{2\, 2^{\frac{1}{4}} 3^{\frac{1}{4}} (-x^3)^{\frac{1}{4}}}{3}$$
$$y(x) = \frac{2\, 2^{\frac{1}{4}} 3^{\frac{1}{4}} (-x^3)^{\frac{1}{4}}}{3}$$
$$y(x) = -\frac{2i 2^{\frac{1}{4}} 3^{\frac{1}{4}} (-x^3)^{\frac{1}{4}}}{3}$$
$$y(x) = \frac{2i 2^{\frac{1}{4}} 3^{\frac{1}{4}} (-x^3)^{\frac{1}{4}}}{3}$$
$$y(x) = 0$$
$$y(x) = \sqrt{c_1^3 + 2c_1 x}$$
$$y(x) = -\sqrt{c_1^3 + 2c_1 x}$$

✓ Solution by Mathematica

Time used: 0.147 (sec). Leaf size: 119

```
DSolve[y[x]==2*x*y'[x]+y[x]^2*(y'[x])^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2c_1x + {c_1}^3}$$
$$y(x) \to \sqrt{2c_1x + {c_1}^3}$$
$$y(x) \to (-1-i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$
$$y(x) \to (1-i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$
$$y(x) \to (-1+i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$
$$y(x) \to (1+i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$

1.84 problem 87

Internal problem ID [3229]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 87.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y'^3 + y^2 - y'yx = 0$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 269

```
dsolve((diff(y(x),x))^3+y(x)^2=x*y(x)*diff(y(x),x),y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \frac{2x^4}{81\left(\frac{x}{3}-\frac{\sqrt{x^2+3c_1}}{3}\right)} - \frac{2x^3\sqrt{x^2+3c_1}}{81\left(\frac{x}{3}-\frac{\sqrt{x^2+3c_1}}{3}\right)} - \frac{c_1x^2}{27\left(\frac{x}{3}-\frac{\sqrt{x^2+3c_1}}{3}\right)} + \frac{2c_1x\sqrt{x^2+3c_1}}{27\left(\frac{x}{3}-\frac{\sqrt{x^2+3c_1}}{3}\right)} + \frac{c_1^2}{3x-3\sqrt{x^2+3c_1}}$$

$$y(x) = \frac{2x^4}{81\left(\frac{x}{3}+\frac{\sqrt{x^2+3c_1}}{3}\right)} + \frac{2x^3\sqrt{x^2+3c_1}}{81\left(\frac{x}{3}+\frac{\sqrt{x^2+3c_1}}{3}\right)} - \frac{c_1x^2}{27\left(\frac{x}{3}+\frac{\sqrt{x^2+3c_1}}{3}\right)} - \frac{2c_1x\sqrt{x^2+3c_1}}{27\left(\frac{x}{3}+\frac{\sqrt{x^2+3c_1}}{3}\right)} + \frac{c_1^2}{3x+3\sqrt{x^2+3c_1}}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^3+y[x]^2==x*y[x]*y'[x],y[x],x,IncludeSingularSolutions -> True]
```

Timed out

1.85 problem 88

Internal problem ID [3230]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 88.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{2xy' - y - y' \ln(y'y) = 0}$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 80

```
dsolve(2*x*diff(y(x),x)-y(x)=diff(y(x),x)*ln(y(x)*diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{1}{2}+x}$$
$$y(x) = -\mathrm{e}^{-\frac{1}{2}+x}$$
$$y(x) = \sqrt{-2\,\mathrm{e}^{-2x}\mathrm{e}^{2c_1}c_1 + 2\,\mathrm{e}^{-2x}\mathrm{e}^{2c_1}x}\,\mathrm{e}^{x}$$
$$y(x) = -\sqrt{-2\,\mathrm{e}^{-2x}\mathrm{e}^{2c_1}c_1 + 2\,\mathrm{e}^{-2x}\mathrm{e}^{2c_1}x}\,\mathrm{e}^{x}$$

✓ Solution by Mathematica

Time used: 0.35 (sec). Leaf size: 59

```
DSolve[2*x*y'[x]-y[x]==y'[x]*Log[y[x]*y'[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{c_1}\sqrt{-2x + i\pi + 2c_1}$$
$$y(x) \to e^{c_1}\sqrt{-2x + i\pi + 2c_1}$$
$$y(x) \to 0$$

1.86 problem 89

Internal problem ID [3231]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 89.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y - xy' + x^2 {y'}^3 = 0$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 123

```
dsolve(y(x)=x*diff(y(x),x)-x^2* (diff(y(x),x))^3,y(x), singsol=all)
```

$$y(x) = -x^2 \operatorname{RootOf}\left(4\_Z^4 c_1 x^2 + 8\_Z^2 c_1 x - \_Z + 4c_1\right)^3 + x \operatorname{RootOf}\left(4\_Z^4 c_1 x^2 + 8\_Z^2 c_1 x - \_Z + 4c_1\right)$$

$$y(x) = -x^2 \operatorname{RootOf}\left(4\_Z^4 c_1 x^2 - 16\_Z^2 c_1 x - \_Z + 16c_1\right)^3 + x \operatorname{RootOf}\left(4\_Z^4 c_1 x^2 - 16\_Z^2 c_1 x - \_Z + 16c_1\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]==x*y'[x]-x^2*(y'[x])^3,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

1.87 problem 90

Internal problem ID [3232]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 90.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$y(y-2xy')^3 - y'^2 = 0$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 577

```
dsolve(y(x)* (y(x)-2*x*diff(y(x),x))^3= (diff(y(x),x))^2 ,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{3}}{9x}$$

$$y(x) = \frac{\sqrt{3}}{9x}$$

$$y(x) = 0$$

$$y(x) = \frac{\operatorname{RootOf}\left(-\ln(x) + c_1 + 24\left(\int^{\_Z} \frac{\left(24\_a^3\sqrt{81\_a^2-3}-216\_a^4+36\cdots\right.}{36\left(24\_a^3\sqrt{81\_a^2-3}-216\_a^4+36\_a^2-1\right)^{\frac{1}{3}}\_a^2+\left(24\_a^3\sqrt{81\_a^2-3}-216\_a^4+36\cdots\right.}\cdots\right.\right.}{x}$$

$$y(x) = \frac{\operatorname{RootOf}\left(-\ln(x) + c_1 - 48\left(\int^{\_Z} \frac{\cdots}{i\left(24\_a^3\sqrt{81\_a^2-3}-216\_a^4+36\_a^2-1\right)^{\frac{2}{3}}\sqrt{3}+24i\sqrt{3}\,\_a^2-72\left(24\_a^3\sqrt{81\_a^2-3}-2\cdots\right.}\cdots\right.\right.}{\cdots}$$

$$y(x) = \frac{\operatorname{RootOf}\left(-\ln(x) + c_1 + 48\left(\int^{\_Z} \frac{\cdots}{i\left(24\_a^3\sqrt{81\_a^2-3}-216\_a^4+36\_a^2-1\right)^{\frac{2}{3}}\sqrt{3}+24i\sqrt{3}\,\_a^2+72\left(24\_a^3\sqrt{81\_a^2-3}-2\cdots\right.}\cdots\right.\right.}{\cdots}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]*(y[x]-2*x*y'[x])^3== (y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

1.88 problem 91

Internal problem ID [3233]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 91.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _dAlembert]`

$$xy' + y - 4\sqrt{y'} = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 63

```
dsolve(y(x)+x*diff(y(x),x) = 4*sqrt(diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = -\frac{4 \operatorname{LambertW}\left(-\frac{c_1 x}{2}\right)^2}{x} + 8\sqrt{\frac{\operatorname{LambertW}\left(-\frac{c_1 x}{2}\right)^2}{x^2}}$$
$$y(x) = -\frac{4 \operatorname{LambertW}\left(\frac{c_1 x}{2}\right)^2}{x} + 8\sqrt{\frac{\operatorname{LambertW}\left(\frac{c_1 x}{2}\right)^2}{x^2}}$$

✓ Solution by Mathematica

Time used: 1.157 (sec). Leaf size: 94

```
DSolve[y[x]+x*y'[x]==4*Sqrt[y'[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2e^{-\frac{1}{2}\sqrt{4-xy(x)}}\left(-2\sqrt{4-xy(x)}-4\right)}{y(x)}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{2e^{\frac{1}{2}\sqrt{4-xy(x)}}\left(2\sqrt{4-xy(x)}-4\right)}{y(x)}=c_1,y(x)\right]$$

$$y(x)\to 0$$

1.89 problem 92

Internal problem ID [3234]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 92.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{2xy' - y - \ln(y') = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 65

```
dsolve(2*x*diff(y(x),x) -y(x) = ln(diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = 1 + \sqrt{4c_1x + 1} - \ln\left(\frac{1 + \sqrt{4c_1x + 1}}{2x}\right)$$

$$y(x) = 1 - \sqrt{4c_1x + 1} - \ln\left(-\frac{-1 + \sqrt{4c_1x + 1}}{2x}\right)$$

✓ Solution by Mathematica

Time used: 0.106 (sec). Leaf size: 34

```
DSolve[2*x*y'[x] -y[x] == Log[y'[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[W\left(-2xe^{-y(x)}\right) - \log\left(W\left(-2xe^{-y(x)}\right) + 2\right) + y(x) = c_1, y(x)\right]$$

1.90 problem 111

Internal problem ID [3235]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 111.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{xy^2(xy' + y) = 1}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 96

```
dsolve(x*y(x)^2*(x*diff(y(x),x)+y(x) )=1,y(x), singsol=all)
```

$$y(x) = \frac{(12x^2 + 8c_1)^{\frac{1}{3}}}{2x}$$

$$y(x) = \frac{-\frac{(12x^2+8c_1)^{\frac{1}{3}}}{4} - \frac{i\sqrt{3}\,(12x^2+8c_1)^{\frac{1}{3}}}{4}}{x}$$

$$y(x) = \frac{-\frac{(12x^2+8c_1)^{\frac{1}{3}}}{4} + \frac{i\sqrt{3}\,(12x^2+8c_1)^{\frac{1}{3}}}{4}}{x}$$

✓ Solution by Mathematica

Time used: 0.233 (sec). Leaf size: 80

```
DSolve[x*y[x]^2*(x*y'[x]+y[x])==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{-\frac{1}{2}}\sqrt[3]{3x^2+2c_1}}{x}$$
$$y(x) \to \frac{\sqrt[3]{\frac{3x^2}{2}+c_1}}{x}$$
$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{\frac{3x^2}{2}+c_1}}{x}$$

1.91 problem 112

Internal problem ID [3236]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 112.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{5y + y'^2 - x(x + y') = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 105

```
dsolve(5*y(x)+(diff(y(x),x))^2=x*(x+diff(y(x),x)),y(x), singsol=all)
```

$$y(x) = \frac{x^2}{4}$$
$$y(x) = \frac{3x^2}{2} - \frac{x\left(5x - 2\sqrt{-5c_1}\right)}{2} + c_1$$
$$y(x) = \frac{3x^2}{2} - \frac{x\left(5x + 2\sqrt{-5c_1}\right)}{2} + c_1$$
$$y(x) = \frac{3x^2}{2} + \frac{x\left(-5x - 2\sqrt{-5c_1}\right)}{2} + c_1$$
$$y(x) = \frac{3x^2}{2} + \frac{x\left(-5x + 2\sqrt{-5c_1}\right)}{2} + c_1$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[5*y[x]+(y'[x])^2==x*(x+y'[x]),y[x],x,IncludeSingularSolutions -> True]
```

Timed out

1.92 problem 113

Internal problem ID [3237]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 113.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \frac{2+y}{x+1} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)= (y(x)+2)/(x+1),y(x), singsol=all)
```

$$y(x) = -2 + c_1(x+1)$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 18

```
DSolve[y'[x]== (y[x]+2)/(x+1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2 + c_1(x+1)$$
$$y(x) \to -2$$

1.93 problem 115

Internal problem ID [3238]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 115.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xy' - y + x\,\mathrm{e}^{\frac{y}{x}} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)= y(x)-x*exp(y(x)/x),y(x), singsol=all)
```

$$y(x) = -\ln\left(\ln\left(x\right) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.426 (sec). Leaf size: 16

```
DSolve[x*y'[x]== y[x]-x*Exp[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x \log(\log(x) - c_1)$$

1.94 problem 116

Internal problem ID [3239]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 116.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _Bernoulli]`

$$\boxed{y^2 \sin(2x) - 2y \cos(x)^2 y' = -1}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve((1+y(x)^2*sin(2*x))-(2*y(x)*cos(x)^2)*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x + c_1}}{\cos(x)}$$
$$y(x) = -\frac{\sqrt{x + c_1}}{\cos(x)}$$

✓ Solution by Mathematica

Time used: 0.321 (sec). Leaf size: 32

```
DSolve[(1+y[x]^2*Sin[2*x])-(2*y[x]*Cos[x]^2)*y'[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\sqrt{x + c_1} \sec(x)$$
$$y(x) \to \sqrt{x + c_1} \sec(x)$$

1.95 problem 117

Internal problem ID [3240]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 117.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{2\sqrt{yx} - y - xy' = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 58

```
dsolve((2*sqrt(x*y(x))-y(x))-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$\frac{\sqrt{y(x)\,x}}{(y(x)-x)\left(\sqrt{y(x)\,x}-x\right)x}+\frac{1}{(y(x)-x)\left(\sqrt{y(x)\,x}-x\right)}-c_1=0$$

✓ Solution by Mathematica

Time used: 0.209 (sec). Leaf size: 26

```
DSolve[(2*Sqrt[x*y[x]]-y[x])-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{\left(x+e^{\frac{c_1}{2}}\right)^2}{x}$$
$$y(x)\to x$$

1.96 problem 119

Internal problem ID [3241]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 2. First-Order and Simple Higher-Order Differential Equations. Page 78
Problem number: 119.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$y' = e^{\frac{xy'}{y}}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)=exp(x*diff(y(x),x)/y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{e^{-c_1 x}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.078 (sec). Leaf size: 21

```
DSolve[y'[x]==Exp[x*y'[x]/y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{c_1 - e^{-c_1} x}$$

2 Chapter 4. Linear Differential Equations. Page 183

2.1 problem 1

Internal problem ID [3242]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 1.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 2y'' + y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)+diff(y(x),x)-2*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} + c_2 \sin(x) + c_3 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 24

```
DSolve[y'''[x]-2*y''[x]+y'[x]-2*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 e^{2x} + c_1 \cos(x) + c_2 \sin(x)$$

2.2 problem 2

Internal problem ID [3243]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 2.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + y'' + 9y' + 9y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)+9*diff(y(x),x)+9*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{-x} + c_2 \sin(3x) + c_3 \cos(3x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 28

```
DSolve[y'''[x]+y''[x]+9*y'[x]+9*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_3 e^{-x} + c_1 \cos(3x) + c_2 \sin(3x)$$

2.3 problem 3

Internal problem ID [3244]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 3.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + y'' - y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$3)+diff(y(x),x$2)-diff(y(x),x)-y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x + c_2 e^{-x} + c_3 e^{-x} x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 26

```
DSolve[y'''[x]+y''[x]-y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} \left(c_2 x + c_3 e^{2x} + c_1\right)$$

2.4 problem 4

Internal problem ID [3245]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 4.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' + 8y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$3)+8*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-2x} c_1 + c_2 \mathrm{e}^{x} \sin\left(\sqrt{3}\, x\right) + c_3 \mathrm{e}^{x} \cos\left(\sqrt{3}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 42

```
DSolve[y'''[x]+8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-2x} + c_3 e^{x} \cos\left(\sqrt{3}x\right) + c_2 e^{x} \sin\left(\sqrt{3}x\right)$$

2.5 problem 5

Internal problem ID [3246]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 5.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_x]]`

$$\boxed{y''' - 8y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x$3)-8*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^{2x} + c_2 e^{-x} \sin\left(\sqrt{3}\, x\right) + c_3 e^{-x} \cos\left(\sqrt{3}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 42

```
DSolve[y'''[x]-8*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x}\left(c_1 e^{3x} + c_2 \cos\left(\sqrt{3}x\right) + c_3 \sin\left(\sqrt{3}x\right)\right)$$

2.6 problem 6

Internal problem ID [3247]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 6.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(diff(y(x),x$4)+4*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 e^x \sin(x) + c_2 e^x \cos(x) + c_3 e^{-x} \sin(x) + c_4 e^{-x} \cos(x)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 40

```
DSolve[y''''[x]+4*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-x} \left(\left(c_4 e^{2x} + c_1 \right) \cos(x) + \left(c_3 e^{2x} + c_2 \right) \sin(x) \right)$$

2.7 problem 7

Internal problem ID [3248]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 7.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' + 18y'' + 81y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$4)+18*diff(y(x),x$2)+81*y(x)=0,y(x), singsol=all)
```

$$y(x) = c_1 \sin(3x) + c_2 \cos(3x) + c_3 \sin(3x)\, x + c_4 \cos(3x)\, x$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 30

```
DSolve[y''''[x]+18*y''[x]+81*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (c_2x + c_1)\cos(3x) + (c_4x + c_3)\sin(3x)$$

2.8 problem 8

Internal problem ID [3249]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 8.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 4y'' + 16y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(y(x),x$4)-4*diff(y(x),x$2)+16*y(x)=0,y(x), singsol=all)
```

$$y(x) = -c_1 e^{\sqrt{3}\,x} \sin(x) + c_2 e^{-\sqrt{3}\,x} \sin(x) + c_3 e^{\sqrt{3}\,x} \cos(x) + c_4 e^{-\sqrt{3}\,x} \cos(x)$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 55

```
DSolve[y''''[x]-4*y''[x]+16*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-\sqrt{3}x}\left(\left(c_3 e^{2\sqrt{3}x} + c_2\right)\cos(x) + \left(c_1 e^{2\sqrt{3}x} + c_4\right)\sin(x)\right)$$

2.9 problem 9

Internal problem ID [3250]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 9.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y'''' - 2y''' + 2y'' - 2y' + y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 22

```
dsolve(diff(y(x),x$4)-2*diff(y(x),x$3)+2*diff(y(x),x$2)-2*diff(y(x),x)+y(x)=0,y(x), singsol=
```

$$y(x) = c_1 e^x + c_2 e^x x + c_3 \sin(x) + c_4 \cos(x)$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 27

```
DSolve[y''''[x]-2*y'''[x]+2*y''[x]-2*y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^x(c_4 x + c_3) + c_1 \cos(x) + c_2 \sin(x)$$

2.10 problem 10

Internal problem ID [3251]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 10.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$y'''' - 5y''' + 5y'' + 5y' - 6y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$4)-5*diff(y(x),x$3)+5*diff(y(x),x$2)+5*diff(y(x),x)-6*y(x)=0,y(x), singso
```

$$y(x) = c_1 e^x + c_2 e^{-x} + c_3 e^{2x} + c_4 e^{3x}$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 36

```
DSolve[y''''[x]-5*y'''[x]+5*y''[x]+5*y'[x]-6*y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to c_1 e^{-x} + c_2 e^x + c_3 e^{2x} + c_4 e^{3x}$$

2.11 problem 11

Internal problem ID [3252]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 11.
ODE order: 5.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(5)} - 6y'''' + 9y''' = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x$5)-6*diff(y(x),x$4)+9*diff(y(x),x$3)=0,y(x), singsol=all)
```

$$y(x) = c_1 + c_2x + c_3x^2 + c_4e^{3x} + c_5e^{3x}x$$

✓ Solution by Mathematica

Time used: 0.143 (sec). Leaf size: 35

```
DSolve[y'''''[x]-6*y''''[x]+9*y'''[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{27}e^{3x}(c_2(x-1)+c_1) + x(c_5x+c_4) + c_3$$

2.12 problem 12

Internal problem ID [3253]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 12.
ODE order: 6.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_x]]`

$$\boxed{y^{(6)} - 64y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
dsolve(diff(y(x),x$6)-64*y(x)=0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-2x} c_1 + c_2 \mathrm{e}^{2x} + c_3 \mathrm{e}^{x} \sin\left(\sqrt{3}\, x\right) + c_4 \mathrm{e}^{x} \cos\left(\sqrt{3}\, x\right) + c_5 \mathrm{e}^{-x} \sin\left(\sqrt{3}\, x\right) + c_6 \mathrm{e}^{-x} \cos\left(\sqrt{3}\, x\right)$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 68

```
DSolve[y''''''[x]-64*y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{-2x}\left(c_1 e^{4x} + e^{x}\left(c_2 e^{2x} + c_3\right) \cos\left(\sqrt{3}x\right) + e^{x}\left(c_6 e^{2x} + c_5\right) \sin\left(\sqrt{3}x\right) + c_4\right)$$

2.13 problem 13

Internal problem ID [3254]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 13.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' + 6y' + 10y = 3\,\mathrm{e}^{-3x}x - 2\cos(x)\,\mathrm{e}^{3x}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$2)+6*diff(y(x),x)+10*y(x)=3*x*exp(-3*x)-2*exp(3*x)*cos(x),y(x), singsol=a
```

$$y(x) = \mathrm{e}^{-3x}\sin(x)\,c_2 + \mathrm{e}^{-3x}\cos(x)\,c_1 + \frac{(-3\cos(x) - \sin(x))\,\mathrm{e}^{3x}}{60} + 3x\,\mathrm{e}^{-3x}$$

✓ Solution by Mathematica

Time used: 0.426 (sec). Leaf size: 46

```
DSolve[y''[x]+6*y'[x]+10*y[x]==3*x*Exp[-3*x]-2*Exp[3*x]*Cos[x],y[x],x,IncludeSingularSolutio
```

$$y(x) \to \frac{1}{60}e^{-3x}\left(180x - 3\left(e^{6x} - 20c_2\right)\cos(x) - \left(e^{6x} - 60c_1\right)\sin(x)\right)$$

2.14 problem 14

Internal problem ID [3255]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 14.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 8y' + 17y = e^{4x}(x^2 - 3\sin(x)\,x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
dsolve(diff(y(x),x$2)-8*diff(y(x),x)+17*y(x)=exp(4*x)*(x^2-3*x*sin(x)),y(x), singsol=all)
```

$$y(x) = e^{4x}\sin(x)\,c_2 + e^{4x}\cos(x)\,c_1 - \frac{e^{4x}(-3\cos(x)\,x^2 + 3x\sin(x) - 4x^2 + 8)}{4}$$

✓ Solution by Mathematica

Time used: 0.263 (sec). Leaf size: 47

```
DSolve[y''[x]-8*y'[x]+17*y[x]==Exp[4*x]*(x^2-3*x*Sin[x]),y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{1}{8}e^{4x}\left(8(x^2-2) + (6x^2 - 3 + 8c_2)\cos(x) + (-6x + 8c_1)\sin(x)\right)$$

2.15 problem 15

Internal problem ID [3256]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 15.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$y'' - 2y' + 2y = (x + e^x) \sin(x)$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$2)-2*diff(y(x),x)+2*y(x)=(x+exp(x))*sin(x),y(x), singsol=all)
```

$$y(x) = e^x \sin(x)\, c_2 + e^x \cos(x)\, c_1 + \frac{(-25\, e^x x + 20x + 28) \cos(x)}{50} + \frac{\sin(x)\,(5x + 2)}{25}$$

✓ Solution by Mathematica

Time used: 0.333 (sec). Leaf size: 48

```
DSolve[y''[x]-2*y'[x]+2*y[x]==(x+Exp[x])*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{50}((-5(5e^x - 4)\, x + 50c_2 e^x + 28) \cos(x) + 2(5x + 25c_1 e^x + 2) \sin(x))$$

2.16 problem 16

Internal problem ID [3257]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 16.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 4y = \sinh(x)\sin(2x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(diff(y(x),x$2)+4*y(x)=sinh(x)*sin(2*x),y(x), singsol=all)
```

$$y(x) = \sin(2x)\, c_2 + c_1 \cos(2x) + \frac{(-4\,\mathrm{e}^{x} - 4\,\mathrm{e}^{-x})\cos(2x)}{34} + \frac{\sin(2x)\,(\mathrm{e}^{x} - \mathrm{e}^{-x})}{34}$$

✓ Solution by Mathematica

Time used: 0.119 (sec). Leaf size: 46

```
DSolve[y''[x]+4*y[x]==Sinh[x]*Sin[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{34}(-(4-i)\cos((2+i)x) - (4+i)\cosh((1+2i)x) + 34c_1\cos(2x) + 34c_2\sin(2x))$$

2.17 problem 17

Internal problem ID [3258]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 17.
ODE order: 2.
ODE degree: 1.

CAS Maple gives this as type `[[_2nd_order, _linear, _nonhomogeneous]]`

$$\boxed{y'' + 2y' + 2y = \cosh(x)\sin(x)}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(diff(y(x),x$2)+2*diff(y(x),x)+2*y(x)=cosh(x)*sin(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-x}\sin(x)\, c_2 + \mathrm{e}^{-x}\cos(x)\, c_1 - \frac{\mathrm{e}^{-x}\cos(x)\, x}{4} - \frac{\mathrm{e}^{x}(\cos(x) - \sin(x))}{16}$$

✓ Solution by Mathematica

Time used: 0.199 (sec). Leaf size: 47

```
DSolve[y''[x]+2*y'[x]+2*y[x]==Cosh[x]*Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16}e^{-x}\left(\left(e^{2x} + 2 + 16c_1\right)\sin(x) - \left(e^{2x} + 4(x - 4c_2)\right)\cos(x)\right)$$

2.18 problem 18

Internal problem ID [3259]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 18.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$y''' + y' = \sin(x) + \cos(x) x$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x$3)+diff(y(x),x)=sin(x)+x*cos(x),y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x) x^2}{4} + \frac{\cos(x)}{2} + \frac{x \sin(x)}{4} - c_2 \cos(x) + \sin(x) c_1 + c_3$$

✓ Solution by Mathematica

Time used: 0.237 (sec). Leaf size: 36

```
DSolve[y'''[x]+y'[x]==Sin[x]+x*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{8}(2x^2 - 3 + 8c_2)\cos(x) + \left(\frac{x}{4} + c_1\right)\sin(x) + c_3$$

2.19 problem 19

Internal problem ID [3260]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 19.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _linear, _nonhomogeneous]]`

$$y''' - 2y'' + 4y' - 8y = e^{2x}\sin(2x) + 2x^2$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 96

```
dsolve(diff(y(x),x$3)-2*diff(y(x),x$2)+4*diff(y(x),x)-8*y(x)=exp(2*x)*sin(2*x)+2*x^2,y(x), s
```

$$y(x) = -\frac{e^{-2x}(2\,e^{4x} + 5\,e^{2x})\cos(2x)}{80} - \frac{e^{-2x}(4\,e^{4x} - 5\,e^{2x})\sin(2x)}{80} - \frac{e^{-2x}(4x^2 e^{2x} + 4\,e^{2x}x + e^{4x})}{16} + c_1\cos(2x) + c_2 e^{2x} + c_3\sin(2x)$$

✓ Solution by Mathematica

Time used: 0.241 (sec). Leaf size: 61

```
DSolve[y'''[x]-2*y''[x]+4*y'[x]-8*y[x]==Exp[2*x]*Sin[2*x]+2*x^2,y[x],x,IncludeSingularSoluti
```

$$y(x) \to \frac{1}{80}\left(-20x(x+1) + 5(-1+16c_3)e^{2x} - 2\left(e^{2x} - 40c_1\right)\cos(2x) - 4\left(e^{2x} - 20c_2\right)\sin(2x)\right)$$

2.20 problem 20

Internal problem ID [3261]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 20.
ODE order: 3.
ODE degree: 1.

CAS Maple gives this as type `[[_3rd_order, _missing_y]]`

$$\boxed{y''' - 4y'' + 3y' = x^2 + x\,\mathrm{e}^{2x}}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x$3)-4*diff(y(x),x$2)+3*diff(y(x),x)=x^2+x*exp(2*x),y(x), singsol=all)
```

$$y(x) = \frac{x^3}{9} + \frac{4x^2}{9} - \frac{\mathrm{e}^{2x}x}{2} + \frac{\mathrm{e}^{2x}}{4} + \mathrm{e}^{x}c_2 + \frac{\mathrm{e}^{3x}c_1}{3} + \frac{26x}{27} + c_3$$

✓ Solution by Mathematica

Time used: 0.239 (sec). Leaf size: 58

```
DSolve[y'''[x]-4*y''[x]+3*y'[x]==x^2+x*Exp[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{9} + \frac{4x^2}{9} + \frac{26x}{27} + \frac{1}{4}e^{2x}(1-2x) + c_1e^{x} + \frac{1}{3}c_2e^{3x} + c_3$$

2.21 problem 21

Internal problem ID [3262]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 21.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _missing_y]]`

$$\boxed{y'''' + 2y'' = 7x - 3\cos(x)}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve(diff(y(x),x$4)+2*diff(y(x),x$2)=7*x-3*cos(x),y(x), singsol=all)
```

$$y(x) = \frac{7x^3}{12} - \frac{\cos\left(\sqrt{2}\,x\right) c_1}{2} - \frac{\sin\left(\sqrt{2}\,x\right) c_2}{2} + 3\cos(x) + c_3x + c_4$$

✓ Solution by Mathematica

Time used: 0.603 (sec). Leaf size: 51

```
DSolve[y''''[x]+2*y''[x]==7*x-3*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{7x^3}{12} + 3\cos(x) + c_4x - \frac{1}{2}c_1\cos\left(\sqrt{2}x\right) - \frac{1}{2}c_2\sin\left(\sqrt{2}x\right) + c_3$$

2.22 problem 22

Internal problem ID [3263]

Book: Differential equations for engineers by Wei-Chau XIE, Cambridge Press 2010
Section: Chapter 4. Linear Differential Equations. Page 183
Problem number: 22.
ODE order: 4.
ODE degree: 1.

CAS Maple gives this as type `[[_high_order, _linear, _nonhomogeneous]]`

$$y'''' + 5y'' + 4y = \sin(x)\cos(2x)$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve(diff(y(x),x$4)+5*diff(y(x),x$2)+4*y(x)=sin(x)*cos(2*x),y(x), singsol=all)
```

$$y(x) = \frac{x\cos(x)}{12} + \frac{\sin(3x)}{80} - \frac{\sin(x)}{144} + \cos(x)c_1 + c_2\sin(x) + c_3\cos(2x) + c_4\sin(2x)$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 50

```
DSolve[y''''[x]+5*y''[x]+4*y[x]==Sin[x]*Cos[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(x)}{72} + \frac{1}{80}\sin(3x) + \left(\frac{x}{12} + c_3\right)\cos(x) + c_1\cos(2x) + c_4\sin(x) + c_2\sin(2x)$$