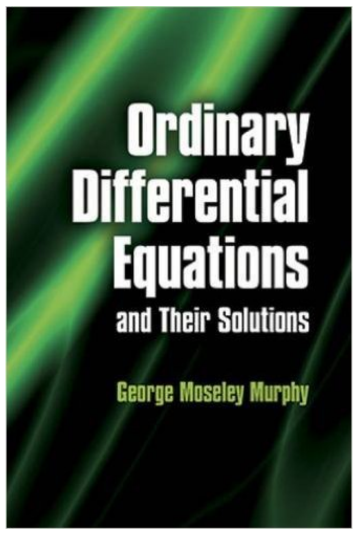
A Solution Manual For

Ordinary differential equations and their solutions. By George Moseley Murphy. 1960



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October 12, 2023

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1.1 problem 0

Internal problem ID [2755]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 0.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - af(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(diff(y(x),x) = a*f(x),y(x), singsol=all)
```

$$y(x) = \int af(x)\,dx + c_1$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 20

```
DSolve[y'[x]==a*f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \int_1^x af(K[1])dK[1] + c_1$$

1.2 problem 1

Internal problem ID [2756]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 1.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - x - \sin(x) - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) = x+sin(x)+y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\cos(x)}{2} - \frac{\sin(x)}{2} - x - 1 + c_1 e^x$$

✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 28

```
DSolve[y'[x]==x+Sin[x]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \frac{\sin(x)}{2} - \frac{\cos(x)}{2} + c_1 e^x - 1$$

1.3 problem 2

Internal problem ID [2757]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 2.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - x^2 - 3\cosh(x) - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 76

```
dsolve(diff(y(x),x) = x^2+3*cosh(x)+2*y(x),y(x), singsol=all)
```

$$y(x) = \frac{e^{2x}(-2x^2\cosh(2x) + 2x^2\sinh(2x) - 2x\cosh(2x) + 2x\sinh(2x) + 2\sinh(3x) - 2\cosh(3x) + 6\sinh(x)}{4}$$

✓ Solution by Mathematica

Time used: 0.084 (sec). Leaf size: 38

```
DSolve[y'[x]==x^2+3*Cosh[x]+2*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-2x(x+1) - 2e^{-x} - 6e^{x} + 4c_1e^{2x} - 1\right)$$

1.4 problem 3

Internal problem ID [2758]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 3.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, 'class A']]`

$$\boxed{y' - a - bx - cy = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x) = a+b*x+c*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{bx}{c} - \frac{a}{c} - \frac{b}{c^2} + \mathrm{e}^{cx} c_1$$

✓ Solution by Mathematica

Time used: 0.092 (sec). Leaf size: 28

```
DSolve[y'[x]==a+b*x+c*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ac + bcx + b}{c^2} + c_1 e^{cx}$$

1.5 problem 4

Internal problem ID [2759]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 4.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - a\cos(bx + c) - ky = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(x),x) = a*cos(b*x+c)+k*y(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{kx} c_1 + \frac{a(b\sin(xb + c) - \cos(xb + c)\, k)}{b^2 + k^2}$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 43

```
DSolve[y'[x]==a*Cos[b*x+c]+k*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a(b\sin(bx + c) - k\cos(bx + c))}{b^2 + k^2} + c_1 e^{kx}$$

1.6 problem 5

Internal problem ID [2760]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 5.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - a\sin(bx + c) - ky = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve(diff(y(x),x) = a*sin(b*x+c)+k*y(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{kx}c_1 - \frac{a(\sin(xb + c)\,k + b\cos(xb + c))}{b^2 + k^2}$$

✓ Solution by Mathematica

Time used: 0.121 (sec). Leaf size: 43

```
DSolve[y'[x]==a*Sin[b*x+c]+k*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a(k\sin(bx + c) + b\cos(bx + c))}{b^2 + k^2} + c_1 e^{kx}$$

1.7 problem 6

Internal problem ID [2761]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 6.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{y' - a - b\,\mathrm{e}^{kx} - cy = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x) = a+b*exp(k*x)+c*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{a}{c} + \frac{b\,\mathrm{e}^{-x(c-k)+cx}}{-c+k} + \mathrm{e}^{cx}c_1$$

✓ Solution by Mathematica

Time used: 0.156 (sec). Leaf size: 34

```
DSolve[y'[x]==a+b*Exp[k*x]+c*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a}{c} + \frac{be^{kx}}{k-c} + c_1e^{cx}$$

1.8 problem 7

Internal problem ID [2762]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 7.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - x(x^2 - y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x) = x*(x^2-y(x)),y(x), singsol=all)
```

$$y(x) = x^2 - 2 + \mathrm{e}^{-\frac{x^2}{2}} c_1$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 22

```
DSolve[y'[x]==x*(x^2-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x^2 + c_1 e^{-\frac{x^2}{2}} - 2$$

1.9 problem 8

Internal problem ID [2763]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 8.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' - x\left(e^{-x^2} + ay\right) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x) = x*(exp(-x^2)+a*y(x)),y(x), singsol=all)
```

$$y(x) = \left(-\frac{e^{-\frac{x^2(2+a)}{2}}}{2+a} + c_1\right) e^{\frac{a\,x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.114 (sec). Leaf size: 42

```
DSolve[y'[x]==x*(Exp[-x^2]+a*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{\frac{ax^2}{2}}\left(-e^{-\frac{1}{2}(a+2)x^2} + (a+2)c_1\right)}{a+2}$$

1.10 problem 9

Internal problem ID [2764]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 9.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' - x^2(a\,x^3 + yb) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) = x^2*(a*x^3+b*y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{x^3 a}{b} - \frac{3a}{b^2} + \mathrm{e}^{\frac{b\,x^3}{3}} c_1$$

✓ Solution by Mathematica

Time used: 0.08 (sec). Leaf size: 32

```
DSolve[y'[x]==x^2*(a*x^3+b*y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a(bx^3+3)}{b^2} + c_1 e^{\frac{bx^3}{3}}$$

1.11 problem 10

Internal problem ID [2765]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 10.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - a\,x^n y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x) = a*x^n*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{a\,x^{n+1}}{n+1}}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 27

```
DSolve[y'[x]==a*x^n*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{ax^{n+1}}{n+1}}$$
$$y(x) \to 0$$

1.12 problem 11

Internal problem ID [2766]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 11.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \cos(x)\sin(x) - \cos(x)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x) = cos(x)*sin(x)+y(x)*cos(x),y(x), singsol=all)
```

$$y(x) = -\sin(x) - 1 + \mathrm{e}^{\sin(x)} c_1$$

✓ Solution by Mathematica

Time used: 0.052 (sec). Leaf size: 18

```
DSolve[y'[x]==Cos[x]*Sin[x]+y[x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sin(x) + c_1 e^{\sin(x)} - 1$$

1.13 problem 12

Internal problem ID [2767]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 12.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - e^{\sin(x)} - \cos(x)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = exp(sin(x))+y(x)*cos(x),y(x), singsol=all)
```

$$y(x) = (x + c_1)\, e^{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.12 (sec). Leaf size: 14

```
DSolve[y'[x]==Exp[Sin[x]]+y[x]*Cos[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_1)e^{\sin(x)}$$

1.14 problem 13

Internal problem ID [2768]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 13.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \cot(x)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 8

```
dsolve(diff(y(x),x) = y(x)*cot(x),y(x), singsol=all)
```

$$y(x) = \sin(x)\, c_1$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 15

```
DSolve[y'[x]==y[x]*Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \sin(x)$$
$$y(x) \to 0$$

1.15 problem 14

Internal problem ID [2769]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 14.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - 1 + \cot(x) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x) = 1-y(x)*cot(x),y(x), singsol=all)
```

$$y(x) = \frac{-\cos(x) + c_1}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 15

```
DSolve[y'[x]==1-y[x]*Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\cot(x) + c_1 \csc(x)$$

1.16 problem 15

Internal problem ID [2770]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 15.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \csc(x)\, x + \cot(x)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x) = x*csc(x)-y(x)*cot(x),y(x), singsol=all)
```

$$y(x) = \frac{\frac{x^2}{2} + c_1}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 19

```
DSolve[y'[x]==x*Csc[x]-y[x]*Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(x^2 + 2c_1) \csc(x)$$

1.17 problem 16

Internal problem ID [2771]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 16.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - (2\csc(2x) + \cot(x))\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x) = (2*csc(2*x)+cot(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 \cot(x)(\cos(x) - \cos(3x))}{\sin(2x)\cot(x)^2 - \sin(2x) + 2\cot(x)}$$

✓ Solution by Mathematica

Time used: 0.106 (sec). Leaf size: 41

```
DSolve[y'[x]==(2*Csc[2*x]+Cot[x])*y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 \sin^{\frac{3}{2}}(x)\sqrt{\cos(2x)}\sqrt{\tan(2x)}}{\cos^{\frac{3}{2}}(x)}$$

$$y(x) \to 0$$

1.18 problem 17

Internal problem ID [2772]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 17.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \sec(x) + \cot(x)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x) = sec(x)-y(x)*cot(x),y(x), singsol=all)
```

$$y(x) = \frac{-\ln(\cos(x)) + c_1}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 16

```
DSolve[y'[x]==Sec[x]-y[x]*Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \csc(x)(-\log(\cos(x)) + c_1)$$

1.19 problem 18

Internal problem ID [2773]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 18.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - e^x \sin(x) - \cot(x)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = exp(x)*sin(x)+y(x)*cot(x),y(x), singsol=all)
```

$$y(x) = (e^x + c_1) \sin(x)$$

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 14

```
DSolve[y'[x]==Exp[x]*Sin[x]+y[x]*Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (e^x + c_1) \sin(x)$$

1.20 problem 19

Internal problem ID [2774]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 19.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \csc(x) + 2\cot(x)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)+csc(x)+2*y(x)*cot(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2\cos(x) + 2c_1}{-1 + \cos(2x)}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 15

```
DSolve[y'[x]+Csc[x]+2*y[x]*Cot[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \csc^2(x)(\cos(x) + c_1)$$

1.21 problem 20

Internal problem ID [2775]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 20.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' - 4\csc(x)\, x\sec(x)^2 + 2y\cot(2x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 81

```
dsolve(diff(y(x),x) = 4*csc(x)*x*sec(x)^2-2*y(x)*cot(2*x),y(x), singsol=all)
```

$$y(x) = \left(16 \operatorname{csgn}(\csc(2x))\left(-\frac{x\ln(1+ie^{ix})}{2} + \frac{x\ln(1-ie^{ix})}{2} + \frac{i\operatorname{dilog}(1+ie^{ix})}{2} - \frac{i\operatorname{dilog}(1-ie^{ix})}{2}\right) + c_1\right)\sqrt{\cot(2x)^2+1}$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 58

```
DSolve[y'[x]==2*Csc[x]*2*x*Sec[x]^2-2*y[x]*Cot[2*x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \csc(x)\sec(x)\left(-8ix\arctan\left(e^{ix}\right) - 4i\operatorname{PolyLog}\left(2, ie^{ix}\right) + 4i\operatorname{PolyLog}(2, \sin(x) - i\cos(x)) + c_1\right)$$

1.22 problem 21

Internal problem ID [2776]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 21.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - 2\cot(x)^2\cos(2x) + 2y\csc(2x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = 2*cot(x)^2*cos(2*x)-2*y(x)*csc(2*x),y(x), singsol=all)
```

$$y(x) = \left(2\ln(\sin(x)) + 2\cos(x)^2 + c_1\right)(\csc(2x) + \cot(2x))$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 25

```
DSolve[y'[x]==2*(Cot[x]^2*Cos[2*x]-y[x]*Csc[2*x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cot(x)(\cos(2x) + \log(\sin(x)) + \log(\tan(x)) + \log(\cos(x)) - 1 + c_1)$$

1.23 problem 22

Internal problem ID [2777]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 22.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' - 4\csc(x)\,x\left(\sin(x)^3 + y\right) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 116

```
dsolve(diff(y(x),x) = 4*csc(x)*x*(sin(x)^3+y(x)),y(x), singsol=all)
```

$$y(x) = \left(1+e^{ix}\right)^{-4x}\left(1-e^{ix}\right)^{4x} e^{4i(\operatorname{dilog}(1+e^{ix})-\operatorname{dilog}(1-e^{ix}))}\left(c_1 + \int -2x\left(1-e^{ix}\right)^{-4x}\left(1+e^{ix}\right)^{4x} e^{-4i(\operatorname{dilog}(1+e^{ix})-\operatorname{dilog}(1-e^{ix}))}(-1+\cos(2x))\,dx\right)$$

✓ Solution by Mathematica

Time used: 8.086 (sec). Leaf size: 109

```
DSolve[y'[x]==2*Csc[x]*2*x(Sin[x]^3+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(-8x\operatorname{arctanh}(e^{ix}) - 8i\operatorname{PolyLog}(2, e^{ix}) + 2i\operatorname{PolyLog}(2, e^{2ix})\right)\left(\int_1^x 4\exp\left(8\operatorname{arctanh}(e^{iK[1]})K[1] + 8i\operatorname{PolyLog}(2, e^{iK[1]}) - 2i\operatorname{PolyLog}(2, e^{2iK[1]})\right)K[1]\sin^2(K[1])dK[1] + c_1\right)$$

1.24 problem 23

Internal problem ID [2778]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 23.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' - 4\csc(x)\, x\left(1 - \tan(x)^2 + y\right) = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 118

```
dsolve(diff(y(x),x) = 4*csc(x)*x*(1-tan(x)^2+y(x)),y(x), singsol=all)
```

$$y(x) = \left(1+\mathrm{e}^{ix}\right)^{-4x}\left(1-\mathrm{e}^{ix}\right)^{4x}\left(\int -4\csc(x)\left(\sec(x)^2-2\right)x\left(1-\mathrm{e}^{ix}\right)^{-4x}\left(1+\mathrm{e}^{ix}\right)^{4x}\mathrm{e}^{-4i\left(\mathrm{dilog}\left(1+\mathrm{e}^{ix}\right)-\mathrm{dilog}\left(1-\mathrm{e}^{ix}\right)\right)}dx + c_1\right)\mathrm{e}^{4i\left(\mathrm{dilog}\left(1+\mathrm{e}^{ix}\right)-\mathrm{dilog}\left(1-\mathrm{e}^{ix}\right)\right)}$$

✓ Solution by Mathematica

Time used: 10.973 (sec). Leaf size: 117

```
DSolve[y'[x]==2*Csc[x]*2*x*(1-Tan[x]^2+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(-8x\,\mathrm{arctanh}\left(e^{ix}\right) - 8i\,\mathrm{PolyLog}\left(2, e^{ix}\right) + 2i\,\mathrm{PolyLog}\left(2, e^{2ix}\right)\right)\left(\int_1^x 4\exp\left(8\mathrm{arctanh}\left(e^{iK[1]}\right)K[1] + 8i\,\mathrm{PolyLog}\left(2, e^{iK[1]}\right) - 2i\,\mathrm{PolyLog}\left(2, e^{2iK[1]}\right)\right)\cos(2K[1])\csc(K[1])K[1]\sec^2(K[1])dK[1] + c_1\right)$$

1.25 problem 24

Internal problem ID [2779]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 24.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y\sec(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = y(x)*sec(x),y(x), singsol=all)
```

$$y(x) = c_1(\sec(x) + \tan(x))$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 24

```
DSolve[y'[x]==y[x]*Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{2\text{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)}$$
$$y(x) \to 0$$

1.26 problem 25

Internal problem ID [2780]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 1
Problem number: 25.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' + \tan(x) - (1 - y)\sec(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)+tan(x) = (1-y(x))*sec(x),y(x), singsol=all)
```

$$y(x) = \frac{x + c_1}{\sec(x) + \tan(x)}$$

✓ Solution by Mathematica

Time used: 0.784 (sec). Leaf size: 21

```
DSolve[y'[x]+Tan[x]==(1-y[x])*Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_1)e^{-2\operatorname{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)}$$

2 Various 2

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2.1 problem 26

Internal problem ID [2781]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 26.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - y\tan(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x) = y(x)*tan(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\cos(x)}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 15

```
DSolve[y'[x]==y[x]*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \sec(x)$$
$$y(x) \to 0$$

2.2 problem 27

Internal problem ID [2782]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 27.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' - \cos(x) - y\tan(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x) = cos(x)+y(x)*tan(x),y(x), singsol=all)
```

$$y(x) = \frac{\frac{x}{2} + \frac{\sin(2x)}{4} + c_1}{\cos(x)}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 21

```
DSolve[y'[x]==Cos[x]+y[x]*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}(\sin(x) + (x + 2c_1)\sec(x))$$

2.3 problem 28

Internal problem ID [2783]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 28.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \cos(x) + y\tan(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve(diff(y(x),x) = cos(x)-y(x)*tan(x),y(x), singsol=all)
```

$$y(x) = (x + c_1)\cos(x)$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 12

```
DSolve[y'[x]==Cos[x]-y[x]*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x + c_1)\cos(x)$$

2.4 problem 29

Internal problem ID [2784]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 29.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \sec(x) + y\tan(x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = sec(x)-y(x)*tan(x),y(x), singsol=all)
```

$$y(x) = (\tan(x) + c_1)\cos(x)$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 13

```
DSolve[y'[x]==Sec[x]-y[x]*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) + c_1\cos(x)$$

2.5 problem 30

Internal problem ID [2785]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 30.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \sin(2x) - y\tan(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) = sin(2*x)+y(x)*tan(x),y(x), singsol=all)
```

$$y(x) = \frac{-\frac{\cos(x)}{2} - \frac{\cos(3x)}{6} + c_1}{\cos(x)}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 19

```
DSolve[y'[x]==Sin[2*x]+y[x]*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2\cos^2(x)}{3} + c_1\sec(x)$$

2.6 problem 31

Internal problem ID [2786]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 31.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \sin(2x) + y\tan(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(diff(y(x),x) = sin(2*x)-y(x)*tan(x),y(x), singsol=all)
```

$$y(x) = (-2\cos(x) + c_1)\cos(x)$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 15

```
DSolve[y'[x]==Sin[2*x]-y[x]*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x)(-2\cos(x) + c_1)$$

2.7 problem 32

Internal problem ID [2787]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 32.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \sin(x) - 2y\tan(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) = sin(x)+2*y(x)*tan(x),y(x), singsol=all)
```

$$y(x) = \frac{-3\cos(x) - \cos(3x) + 12c_1}{6\cos(2x) + 6}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 19

```
DSolve[y'[x]==Sin[x]+2*y[x]*Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\cos(x)}{3} + c_1 \sec^2(x)$$

2.8 problem 33

Internal problem ID [2788]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 33.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - 2 - 2\sec(2x) - 2y\tan(2x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x) = 2+2*sec(2*x)+2*y(x)*tan(2*x),y(x), singsol=all)
```

$$y(x) = \left(2\,\mathrm{csgn}\left(\sec\left(2x\right)\right)\left(x + \frac{\sin\left(2x\right)}{2}\right) + c_1\right)\sqrt{1 + \tan\left(2x\right)^2}$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 20

```
DSolve[y'[x]==2*(1+Sec[2 x]+y[x] Tan[2 x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sec(2x)(2x + \sin(2x) + c_1)$$

2.9 problem 34

Internal problem ID [2789]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 34.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \csc(x) - 3y\tan(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x) = csc(x)+3*y(x)*tan(x),y(x), singsol=all)
```

$$y(x) = \frac{2\cos(x)^2 + 4\ln(\sin(x)) + 4c_1}{\cos(3x) + 3\cos(x)}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 24

```
DSolve[y'[x]==Csc[x]+3 y[x] Tan[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sec^3(x)\left(-\frac{1}{2}\sin^2(x) + \log(\sin(x)) + c_1\right)$$

2.10 problem 35

Internal problem ID [2790]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 35.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - (a + \cos(\ln(x)) + \sin(\ln(x)))\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x) = (a+cos(ln(x))+sin(ln(x)))*y(x),y(x), singsol=all)
```

$$y(x) = c_1 e^{x(\sin(\ln(x))+a)}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 22

```
DSolve[y'[x]==(a+Cos[Log[x]]+Sin[Log[x]]) y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x(a+\sin(\log(x)))}$$
$$y(x) \to 0$$

2.11 problem 36

Internal problem ID [2791]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 36.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - 6\,\mathrm{e}^{2x} + y\tanh(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = 6*exp(2*x)-y(x)*tanh(x),y(x), singsol=all)
```

$$y(x) = \frac{3\sinh(x) + \sinh(3x) + 3\cosh(x) + \cosh(3x) + c_1}{\cosh(x)}$$

✓ Solution by Mathematica

Time used: 0.144 (sec). Leaf size: 33

```
DSolve[y'[x]==6 Exp[2 x]- y[x] Tanh[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x(6e^x + 2e^{3x} + c_1)}{e^{2x} + 1}$$

2.12 problem 37

Internal problem ID [2792]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 37.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - f(x)\, f'(x) - f'(x)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x) = f(x)*diff(f(x),x)+diff(f(x),x)*y(x),y(x), singsol=all)
```

$$y(x) = -f(x) - 1 + \mathrm{e}^{f(x)} c_1$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 18

```
DSolve[y'[x]==f[x] f'[x] + f'[x] y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -f(x) + c_1 e^{f(x)} - 1$$

2.13 problem 38

Internal problem ID [2793]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 38.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - f(x) - g(x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x) = f(x)+g(x)*y(x),y(x), singsol=all)
```

$$y(x) = \left(\int f(x)\,\mathrm{e}^{-\left(\int g(x)dx\right)}dx + c_1\right)\mathrm{e}^{\int g(x)dx}$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 47

```
DSolve[y'[x]==f[x] + g[x] y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(\int_1^x g(K[1])dK[1]\right)\left(\int_1^x \exp\left(-\int_1^{K[2]} g(K[1])dK[1]\right) f(K[2])dK[2] + c_1\right)$$

2.14 problem 39

Internal problem ID [2794]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 39.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - x^2 + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 44

```
dsolve(diff(y(x),x) = x^2-y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x\left(\operatorname{BesselI}\left(-\frac{3}{4}, \frac{x^2}{2}\right) c_1 - \operatorname{BesselK}\left(\frac{3}{4}, \frac{x^2}{2}\right)\right)}{c_1 \operatorname{BesselI}\left(\frac{1}{4}, \frac{x^2}{2}\right) + \operatorname{BesselK}\left(\frac{1}{4}, \frac{x^2}{2}\right)}$$

✓ Solution by Mathematica

Time used: 0.128 (sec). Leaf size: 103

```
DSolve[y'[x]==x^2 - y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{ix\left(\operatorname{BesselJ}\left(-\frac{3}{4}, \frac{ix^2}{2}\right) - c_1 \operatorname{BesselJ}\left(\frac{3}{4}, \frac{ix^2}{2}\right)\right)}{\operatorname{BesselJ}\left(\frac{1}{4}, \frac{ix^2}{2}\right) + c_1 \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{ix^2}{2}\right)}$$

$$y(x) \to \frac{x \operatorname{BesselI}\left(\frac{3}{4}, \frac{x^2}{2}\right)}{\operatorname{BesselI}\left(-\frac{1}{4}, \frac{x^2}{2}\right)}$$

2.15 problem 40

Internal problem ID [2795]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 40.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + f(x)^2 - f'(x) - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(diff(y(x),x)+f(x)^2 = diff(f(x),x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = f(x) + \frac{e^{\int 2f(x)dx}}{c_1 - \left(\int e^{\int 2f(x)dx}dx\right)}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]+f[x]^2==f'[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

2.16 problem 41

Internal problem ID [2796]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 41.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' + 1 - x - y(x + y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(diff(y(x),x)+1-x = (x+y(x))*y(x),y(x), singsol=all)
```

$$y(x) = -1 + \frac{\mathrm{e}^{\frac{1}{2}x^2-2x}}{c_1 + \frac{i\sqrt{\pi}\,\mathrm{e}^{-2}\sqrt{2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,x}{2}-i\sqrt{2}\right)}{2}}$$

✓ Solution by Mathematica

Time used: 0.174 (sec). Leaf size: 54

```
DSolve[y'[x]+1-x==(x+y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 + \frac{2e^{\frac{1}{2}(x-2)^2}}{-\sqrt{2\pi}\mathrm{erfi}\left(\frac{x-2}{\sqrt{2}}\right) + 2e^2c_1}$$

$$y(x) \to -1$$

2.17 problem 42

Internal problem ID [2797]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 42.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _Riccati]`

$$\boxed{y' - (x+y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve(diff(y(x),x) = (x+y(x))^2,y(x), singsol=all)
```

$$y(x) = -x - \tan(c_1 - x)$$

✓ Solution by Mathematica

Time used: 0.478 (sec). Leaf size: 14

```
DSolve[y'[x]==(x+y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + \tan(x + c_1)$$

2.18 problem 43

Internal problem ID [2798]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 43.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' - (x - y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(y(x),x) = (x-y(x))^2,y(x), singsol=all)
```

$$y(x) = \frac{x\,\mathrm{e}^{-2x} c_1 + \mathrm{e}^{-2x} c_1 - x + 1}{\mathrm{e}^{-2x} c_1 - 1}$$

✓ Solution by Mathematica

Time used: 0.125 (sec). Leaf size: 29

```
DSolve[y'[x]==(x-y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \frac{1}{\frac{1}{2} + c_1 e^{2x}} - 1$$
$$y(x) \to x - 1$$

2.19 problem 44

Internal problem ID [2799]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 44.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _Riccati]`

$$\boxed{y' - 3 + 3x - 3y - (x - y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve(diff(y(x),x) = 3-3*x+3*y(x)+(x-y(x))^2,y(x), singsol=all)
```

$$y(x) = \frac{e^{-x} c_1 x - c_1 e^{-x} - x + 2}{c_1 e^{-x} - 1}$$

✓ Solution by Mathematica

Time used: 0.166 (sec). Leaf size: 25

```
DSolve[y'[x]==3(1-x+y[x])+(x-y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \frac{1}{1 + c_1 e^x} - 2$$
$$y(x) \to x - 2$$

2.20 problem 45

Internal problem ID [2800]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 45.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - 2x + y(x^2 + 1) - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve(diff(y(x),x) = 2*x-(x^2+1)*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = x^2 + 1 + \frac{e^{\frac{1}{3}x^3+x}}{c_1 - \left(\int e^{\frac{1}{3}x^3+x}dx\right)}$$

✓ Solution by Mathematica

Time used: 0.317 (sec). Leaf size: 58

```
DSolve[y'[x]==2 x-(1+x^2)y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{\frac{x^3}{3}+x}}{-\int_1^x e^{\frac{K[1]^3}{3}+K[1]}dK[1] + c_1} + x^2 + 1$$

$$y(x) \to x^2 + 1$$

2.21 problem 46

Internal problem ID [2801]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 46.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Riccati]`

$$\boxed{y' - x(x^3 + 2) + (2x^2 - y)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x) = x*(x^3+2)-(2*x^2-y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 x^2 + x^3 - 1}{x + c_1}$$

✓ Solution by Mathematica

Time used: 0.123 (sec). Leaf size: 24

```
DSolve[y'[x]==x(2+x^3)-(2 x^2-y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + \frac{1}{-x + c_1}$$
$$y(x) \to x^2$$

2.22 problem 47

Internal problem ID [2802]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 47.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Riccati]`

$$\boxed{y' - 1 - x(-x^3 + 2) - (2x^2 - y)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
dsolve(diff(y(x),x) = 1+x*(-x^3+2)+(2*x^2-y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{x^2 e^{-2x} c_1 - x^2 - e^{-2x} c_1 - 1}{e^{-2x} c_1 - 1}$$

✓ Solution by Mathematica

Time used: 0.13 (sec). Leaf size: 34

```
DSolve[y'[x]==1+x(2-x^3)+(2 x^2-y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 - \frac{2}{1 + 2c_1 e^{2x}} + 1$$

$$y(x) \to x^2 + 1$$

2.23 problem 48

Internal problem ID [2803]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 48.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - \cos(x) + (\sin(x) - y)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x) = cos(x)-(sin(x)-y(x))*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\mathrm{e}^{-\cos(x)}}{c_1 + \int \mathrm{e}^{-\cos(x)} dx} + \sin(x)$$

✓ Solution by Mathematica

Time used: 60.445 (sec). Leaf size: 39

```
DSolve[y'[x]==Cos[x]-(Sin[x]-y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sin(x) - \frac{c_1 e^{-\cos(x)}}{1 + c_1 \int_1^x e^{-\cos(K[1])} dK[1]}$$

2.24 problem 49

Internal problem ID [2804]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 49.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - \cos(2x) - (\sin(2x) + y)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 198

```
dsolve(diff(y(x),x) = cos(2*x)+(sin(2*x)+y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \left(\frac{2\,\mathrm{HeunCPrime}\left(1, \frac{1}{2}, -\frac{1}{2}, -1, \frac{7}{8}, \frac{\cos(2x)}{2} + \frac{1}{2}\right) c_1 \cos(2x)}{\sqrt{2\cos(2x)+2}\left(c_1\sqrt{2\cos(2x)+2}\,\mathrm{HeunC}\left(1, \frac{1}{2}, -\frac{1}{2}, -1, \frac{7}{8}, \frac{\cos(2x)}{2} + \frac{1}{2}\right) + \mathrm{HeunC}\left(1, -\frac{1}{2}, -\frac{1}{2}, -1, \frac{7}{8}, \right.\right.} + \frac{\mathrm{HeunCPrime}\left(1, -\frac{1}{2}, -\frac{1}{2}, -1, \frac{7}{8}, \frac{\cos(2x)}{2} + \frac{1}{2}\right)\sqrt{2\cos(2x)+2} + 2\,\mathrm{HeunC}\left(1, \frac{1}{2}, -\frac{1}{2}, -1, \frac{7}{8}, \frac{\cos(2x)}{2} + \frac{1}{2}\right) c}{\sqrt{2\cos(2x)+2}\left(c_1\sqrt{2\cos(2x)+2}\,\mathrm{HeunC}\left(1, \frac{1}{2}, -\frac{1}{2}, -1, \frac{7}{8}, \frac{\cos(2x)}{2} + \frac{1}{2}\right) + \mathrm{HeunC}\right.}$$

✓ Solution by Mathematica

Time used: 1.575 (sec). Leaf size: 73

```
DSolve[y'[x]==Cos[2 x]+(Sin[2 x]+y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(x) + \frac{e^{-\cos^2(x)}\tan(x)\sec(x)}{\sqrt{-\sin^2(x)}\left(\int_1^{\cos(x)} \frac{e^{-K[1]^2}}{K[1]^2\sqrt{K[1]^2-1}}dK[1] + c_1\right)}$$

$$y(x) \to \tan(x)$$

2.25 problem 50

Internal problem ID [2805]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 50.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - f(x) - x f(x)\, y - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(diff(y(x),x) = f(x)+x*f(x)*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{\int \frac{f(x)x^2-2}{x}dx}}{c_1 - \left(\int \mathrm{e}^{\int \frac{f(x)x^2-2}{x}dx}dx\right)} - \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 0.514 (sec). Leaf size: 76

```
DSolve[y'[x]==f[x]+x f[x] y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x + \frac{\exp\left(-\int_1^x -f(K[5])K[5]dK[5]\right)}{\int_1^x \frac{\exp\left(-\int_1^{K[6]} -f(K[5])K[5]dK[5]\right)}{K[6]^2}dK[6]+c_1}}{x^2}$$

$$y(x) \to -\frac{1}{x}$$

2.26 problem 51

Internal problem ID [2806]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 51.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' - (3 + x - 4y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 36

```
dsolve(diff(y(x),x) = (3+x-4*y(x))^2,y(x), singsol=all)
```

$$y(x) = \frac{2x\,e^{4x}c_1 + 5\,e^{4x}c_1 - 2x - 7}{-8 + 8\,e^{4x}c_1}$$

✓ Solution by Mathematica

Time used: 0.143 (sec). Leaf size: 41

```
DSolve[y'[x]==(3+x-4 y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{16}\left(4x + \frac{1}{\frac{1}{4} + c_1 e^{4x}} + 10\right)$$

$$y(x) \to \frac{1}{8}(2x + 5)$$

2.27 problem 52

Internal problem ID [2807]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 52.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _Riccati]`

$$\boxed{y' - (1 + 4x + 9y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x) = (1+4*x+9*y(x))^2,y(x), singsol=all)
```

$$y(x) = -\frac{4x}{9} - \frac{1}{9} - \frac{2\tan(-6x + 6c_1)}{27}$$

✓ Solution by Mathematica

Time used: 0.154 (sec). Leaf size: 49

```
DSolve[y'[x]==(1+4 x+9 y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{81}\left(-36x + \frac{1}{c_1 e^{12ix} - \frac{i}{12}} - (9 + 6i)\right)$$
$$y(x) \to \frac{1}{27}(-12x - (3 + 2i))$$

2.28 problem 53

Internal problem ID [2808]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 53.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - 3a - 3bx - 3by^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 80

```
dsolve(diff(y(x),x) = 3*a+3*b*x+3*b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(\text{AiryAi}\left(1, -\frac{3^{\frac{2}{3}}(xb+a)}{b^{\frac{1}{3}}}\right)c_1 + \text{AiryBi}\left(1, -\frac{3^{\frac{2}{3}}(xb+a)}{b^{\frac{1}{3}}}\right)\right)3^{\frac{2}{3}}}{b^{\frac{1}{3}}\left(3c_1\,\text{AiryAi}\left(-\frac{3^{\frac{2}{3}}(xb+a)}{b^{\frac{1}{3}}}\right) + 3\,\text{AiryBi}\left(-\frac{3^{\frac{2}{3}}(xb+a)}{b^{\frac{1}{3}}}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.19 (sec). Leaf size: 191

```
DSolve[y'[x]==3*(a+b*x+ b*y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{b\left(\text{AiryBiPrime}\left(-\frac{3^{2/3}b(a+bx)}{(-b^2)^{2/3}}\right) + c_1\,\text{AiryAiPrime}\left(-\frac{3^{2/3}b(a+bx)}{(-b^2)^{2/3}}\right)\right)}{\sqrt[3]{3}\,(-b^2)^{2/3}\left(\text{AiryBi}\left(-\frac{3^{2/3}b(a+bx)}{(-b^2)^{2/3}}\right) + c_1\,\text{AiryAi}\left(-\frac{3^{2/3}b(a+bx)}{(-b^2)^{2/3}}\right)\right)}$$

$$y(x) \to \frac{b\,\text{AiryAiPrime}\left(-\frac{3^{2/3}b(a+bx)}{(-b^2)^{2/3}}\right)}{\sqrt[3]{3}\,(-b^2)^{2/3}\,\text{AiryAi}\left(-\frac{3^{2/3}b(a+bx)}{(-b^2)^{2/3}}\right)}$$

2.29 problem 54

Internal problem ID [2809]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 2
Problem number: 54.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - a - by^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 23

```
dsolve(diff(y(x),x) = a+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\sqrt{ab}\,(x + c_1)\right)\sqrt{ab}}{b}$$

✓ Solution by Mathematica

Time used: 8.011 (sec). Leaf size: 68

```
DSolve[y'[x]==a+b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}\tan\left(\sqrt{a}\sqrt{b}(x + c_1)\right)}{\sqrt{b}}$$
$$y(x) \to -\frac{i\sqrt{a}}{\sqrt{b}}$$
$$y(x) \to \frac{i\sqrt{a}}{\sqrt{b}}$$

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3.1 problem 55

Internal problem ID [2810]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 55.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - ax - by^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(diff(y(x),x) = a*x+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{(ab)^{\frac{1}{3}} \left(\text{AiryAi}\left(1, -(ab)^{\frac{1}{3}} x\right) c_1 + \text{AiryBi}\left(1, -(ab)^{\frac{1}{3}} x\right)\right)}{b \left(c_1 \text{AiryAi}\left(-(ab)^{\frac{1}{3}} x\right) + \text{AiryBi}\left(-(ab)^{\frac{1}{3}} x\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.151 (sec). Leaf size: 165

```
DSolve[y'[x]==a x+b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}\sqrt{x}\left(-\text{BesselJ}\left(-\frac{2}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x^{3/2}\right) + c_1 \text{BesselJ}\left(\frac{2}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x^{3/2}\right)\right)}{\sqrt{b}\left(\text{BesselJ}\left(\frac{1}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x^{3/2}\right) + c_1 \text{BesselJ}\left(-\frac{1}{3}, \frac{2}{3}\sqrt{a}\sqrt{b}x^{3/2}\right)\right)}$$

$$y(x) \to \frac{ax^2 \, {}_0\tilde{F}_1\left(;\frac{5}{3}; -\frac{1}{9}abx^3\right)}{3 \, {}_0\tilde{F}_1\left(;\frac{2}{3}; -\frac{1}{9}abx^3\right)}$$

3.2 problem 56

Internal problem ID [2811]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 56.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a - bx - cy^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 85

```
dsolve(diff(y(x),x) = a+b*x+c*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{b}{\sqrt{c}}\right)^{\frac{1}{3}} \left(\text{AiryAi}\left(1, -\frac{xb+a}{\left(\frac{b}{\sqrt{c}}\right)^{\frac{2}{3}}}\right) c_1 + \text{AiryBi}\left(1, -\frac{xb+a}{\left(\frac{b}{\sqrt{c}}\right)^{\frac{2}{3}}}\right)\right)}{\sqrt{c}\left(c_1 \,\text{AiryAi}\left(-\frac{xb+a}{\left(\frac{b}{\sqrt{c}}\right)^{\frac{2}{3}}}\right) + \text{AiryBi}\left(-\frac{xb+a}{\left(\frac{b}{\sqrt{c}}\right)^{\frac{2}{3}}}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.181 (sec). Leaf size: 143

```
DSolve[y'[x]==a+b x+c y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{b\left(\text{AiryBiPrime}\left(-\frac{c(a+bx)}{(-bc)^{2/3}}\right) + c_1 \,\text{AiryAiPrime}\left(-\frac{c(a+bx)}{(-bc)^{2/3}}\right)\right)}{(-bc)^{2/3}\left(\text{AiryBi}\left(-\frac{c(a+bx)}{(-bc)^{2/3}}\right) + c_1 \,\text{AiryAi}\left(-\frac{c(a+bx)}{(-bc)^{2/3}}\right)\right)}$$

$$y(x) \to \frac{b \,\text{AiryAiPrime}\left(-\frac{c(a+bx)}{(-bc)^{2/3}}\right)}{(-bc)^{2/3} \,\text{AiryAi}\left(-\frac{c(a+bx)}{(-bc)^{2/3}}\right)}$$

3.3 problem 57

Internal problem ID [2812]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 57.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - a\,x^{n-1} - b\,x^{2n} - cy^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 499

```
dsolve(diff(y(x),x) = a*x^(n-1)+b*x^(2*n)+c*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\left(-2b^{\frac{3}{2}}c_1 n - 2b^{\frac{3}{2}}c_1\right)\mathrm{WhittakerW}\left(-\frac{i\sqrt{c}\,a-2\sqrt{b}\,n-2\sqrt{b}}{2\sqrt{b}\,(n+1)}, \frac{1}{2n+2}, \frac{2i\sqrt{c}\,\sqrt{b}\,x^{n+1}}{n+1}\right)}{2b^{\frac{3}{2}}\left(\mathrm{WhittakerW}\left(-\frac{i\sqrt{c}\,a}{2\sqrt{b}\,(n+1)}, \frac{1}{2n+2}, \frac{2i\sqrt{c}\,\sqrt{b}\,x^{n+1}}{n+1}\right)c_1 + \mathrm{WhittakerM}\left(-\frac{i\sqrt{c}\,a}{2\sqrt{b}\,(n+1)}, \frac{1}{2n+2}, \frac{2i\sqrt{c}\,\sqrt{b}\,x^{n+1}}{n+1}\right)\right)cx \cdots} - \frac{\left(2ix^{n+1}\sqrt{c}\,c_1 b^2 + i\sqrt{c}\,c_1 ab - b^{\frac{3}{2}}c_1 n\right)\mathrm{WhittakerW}\left(-\frac{i\sqrt{c}\,a}{2\sqrt{b}\,(n+1)}, \frac{1}{2n+2}, \frac{2i\sqrt{c}\,\sqrt{b}\,x^{n+1}}{n+1}\right) + \left(-i\sqrt{c}\,ab + b^{\frac{3}{2}}n + \cdots\right.}{2b^{\frac{3}{2}}\left(\mathrm{WhittakerW}\left(-\frac{i\sqrt{c}\,a}{2\sqrt{b}\,(n+1)}, \frac{1}{2n+2} \cdots\right.\right.}$$

✓ Solution by Mathematica

Time used: 0.923 (sec). Leaf size: 764

```
DSolve[y'[x]==a x^(n-1)+b x^(2 n)+c y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^n\left(\sqrt{b}c_1(n+1)\sqrt{-(n+1)^2}\,\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}a}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{b}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)+c_1\left(a\sqrt{c}(n\right.\right.}{\sqrt{c}(n+1)^2\left(\right.}$$

$$y(x) \to \frac{x^n\left(-\frac{\left(a\sqrt{c}(n+1)+\sqrt{b}\sqrt{-(n+1)^2}n\right)\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}a}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}+2\right),\frac{n}{n+1}+1,\frac{2\sqrt{b}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}{\text{HypergeometricU}\left(\frac{1}{2}\left(\frac{\sqrt{c}a}{\sqrt{b}\sqrt{-(n+1)^2}}+\frac{n}{n+1}\right),\frac{n}{n+1},\frac{2\sqrt{b}\sqrt{c}x^{n+1}}{\sqrt{-(n+1)^2}}\right)}-\sqrt{b}\sqrt{-(n+1)^2}(n+\right.}{\sqrt{c}(n+1)^2}$$

3.4 problem 58

Internal problem ID [2813]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 58.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Riccati, _special]]`

$$\boxed{y' - x^2 a - b y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 71

```
dsolve(diff(y(x),x) = a*x^2+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{ab}\, x \left(\operatorname{BesselJ}\left(-\frac{3}{4}, \frac{\sqrt{ab}\, x^2}{2}\right) c_1 + \operatorname{BesselY}\left(-\frac{3}{4}, \frac{\sqrt{ab}\, x^2}{2}\right)\right)}{b \left(c_1 \operatorname{BesselJ}\left(\frac{1}{4}, \frac{\sqrt{ab}\, x^2}{2}\right) + \operatorname{BesselY}\left(\frac{1}{4}, \frac{\sqrt{ab}\, x^2}{2}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 153

```
DSolve[y'[x]==a x^2+b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a} x \left(-\operatorname{BesselJ}\left(-\frac{3}{4}, \frac{1}{2}\sqrt{a}\sqrt{b}x^2\right) + c_1 \operatorname{BesselJ}\left(\frac{3}{4}, \frac{1}{2}\sqrt{a}\sqrt{b}x^2\right)\right)}{\sqrt{b}\left(\operatorname{BesselJ}\left(\frac{1}{4}, \frac{1}{2}\sqrt{a}\sqrt{b}x^2\right) + c_1 \operatorname{BesselJ}\left(-\frac{1}{4}, \frac{1}{2}\sqrt{a}\sqrt{b}x^2\right)\right)}$$

$$y(x) \to \frac{a x^3 \, {}_0\tilde{F}_1\left(;\frac{7}{4};-\frac{1}{16}abx^4\right)}{4 \, {}_0\tilde{F}_1\left(;\frac{3}{4};-\frac{1}{16}abx^4\right)}$$

3.5 problem 59

Internal problem ID [2814]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 59.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \mathrm{a0} - \mathrm{a1}\, y - \mathrm{a2}\, y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x) = a0+a1*y(x)+a2*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\mathrm{a1} - \tan\left(\frac{\sqrt{4\,\mathrm{a0}\,\mathrm{a2} - \mathrm{a1}^2}\,(x+c_1)}{2}\right)\sqrt{4\,\mathrm{a0}\,\mathrm{a2} - \mathrm{a1}^2}}{2\,\mathrm{a2}}$$

✓ Solution by Mathematica

Time used: 25.878 (sec). Leaf size: 106

```
DSolve[y'[x]==a0+a1 y[x]+ a2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-\mathrm{a1} + \sqrt{4\mathrm{a0a2} - \mathrm{a1}^2}\tan\left(\frac{1}{2}(x+c_1)\sqrt{4\mathrm{a0a2} - \mathrm{a1}^2}\right)}{2\mathrm{a2}}$$

$$y(x) \to \frac{\sqrt{\mathrm{a1}^2 - 4\mathrm{a0a2}} - \mathrm{a1}}{2\mathrm{a2}}$$

$$y(x) \to -\frac{\sqrt{\mathrm{a1}^2 - 4\mathrm{a0a2}} + \mathrm{a1}}{2\mathrm{a2}}$$

3.6 problem 60

Internal problem ID [2815]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 60.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - f(x) - ay - by^2 = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = f(x)+a*y(x)+b*y(x)^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]+a y[x]+b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

3.7 problem 61

Internal problem ID [2816]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 61.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - 1 - a(x - y)\, y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 71

```
dsolve(diff(y(x),x) = 1+a*(x-y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{2}\,\mathrm{erf}\left(\frac{\sqrt{2}\,\sqrt{a}\,x}{2}\right)\sqrt{\pi}\,ax + 2a^{\frac{3}{2}}c_1x + 2\sqrt{a}\,\mathrm{e}^{-\frac{a\,x^2}{2}}}{\sqrt{2}\,\mathrm{erf}\left(\frac{\sqrt{2}\,\sqrt{a}\,x}{2}\right)\sqrt{\pi}\,a + 2a^{\frac{3}{2}}c_1}$$

✓ Solution by Mathematica

Time used: 3.193 (sec). Leaf size: 59

```
DSolve[y'[x]==1+a(x-y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \frac{2c_1e^{-\frac{ax^2}{2}}}{\sqrt{a}\left(2\sqrt{a} + \sqrt{2\pi}c_1\mathrm{erf}\left(\frac{\sqrt{a}x}{\sqrt{2}}\right)\right)}$$

3.8 problem 62

Internal problem ID [2817]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 62.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - f(x) - g(x)\,y - a y^2 = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = f(x)+g(x)*y(x)+a*y(x)^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]+g[x] y[x]+a y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

3.9 problem 63

Internal problem ID [2818]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 63.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - xy(y + 3) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x) = x*y(x)*(3+y(x)),y(x), singsol=all)
```

$$y(x) = \frac{3}{-1 + 3\,\mathrm{e}^{-\frac{3x^2}{2}} c_1}$$

✓ Solution by Mathematica

Time used: 0.205 (sec). Leaf size: 35

```
DSolve[y'[x]==x*y[x](3+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3}{-1 + e^{-\frac{3}{2}(x^2+2c_1)}}$$

$$y(x) \to -3$$

$$y(x) \to 0$$

3.10 problem 64

Internal problem ID [2819]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 64.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' - 1 + x + x^3 - y(1 + 2x^2) + xy^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
dsolve(diff(y(x),x) = 1-x-x^3+(2*x^2+1)*y(x)-x*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{(c_1x^2 - c_1x + c_1)\,\mathrm{e}^{\frac{x(x^2+3)}{3}} + \mathrm{e}^{\frac{x^3}{3}}x}{(c_1x - c_1)\,\mathrm{e}^{\frac{x(x^2+3)}{3}} + \mathrm{e}^{\frac{x^3}{3}}}$$

✓ Solution by Mathematica

Time used: 0.196 (sec). Leaf size: 39

```
DSolve[y'[x]==1-x-x^3+(1+2 x^2)y[x]-x y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x((x-1)x+1) + c_1x}{e^x(x-1) + c_1}$$

$$y(x) \to x$$

3.11 problem 65

Internal problem ID [2820]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 65.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - x(2 + x^2 y - y^2) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 67

```
dsolve(diff(y(x),x) = x*(2+x^2*y(x)-y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{2c_1 e^{-\frac{x^4}{4}}}{\sqrt{\pi}\left(\operatorname{erf}\left(\frac{x^2}{2}\right) c_1 + 1\right)} + \frac{\operatorname{erf}\left(\frac{x^2}{2}\right)\sqrt{\pi}\, c_1 x^2 + x^2\sqrt{\pi}}{\sqrt{\pi}\left(\operatorname{erf}\left(\frac{x^2}{2}\right) c_1 + 1\right)}$$

✓ Solution by Mathematica

Time used: 0.259 (sec). Leaf size: 48

```
DSolve[y'[x]==x(2+x^2 y[x]-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + \frac{2e^{-\frac{x^4}{4}}}{\sqrt{\pi}\operatorname{erf}\left(\frac{x^2}{2}\right) + 2c_1}$$

$$y(x) \to x^2$$

3.12 problem 66

Internal problem ID [2821]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 66.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - x - (1 - 2x)\, y + (1 - x)\, y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x) = x+(1-2*x)*y(x)-(1-x)*y(x)^2,y(x), singsol=all)
```

$$y(x) = 1 - \frac{2\,\mathrm{e}^{-x}}{c_1 - 2x\,\mathrm{e}^{-x}}$$

✓ Solution by Mathematica

Time used: 0.155 (sec). Leaf size: 22

```
DSolve[y'[x]==x+(1-2 x)y[x]-(1-x)y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + \frac{1}{x + c_1 e^x}$$
$$y(x) \to 1$$

3.13 problem 67

Internal problem ID [2822]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 67.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - axy^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x) = a*x*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{2}{-a\,x^2 + 2c_1}$$

✓ Solution by Mathematica

Time used: 0.119 (sec). Leaf size: 24

```
DSolve[y'[x]==a x y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{ax^2 + 2c_1}$$

$$y(x) \to 0$$

3.14 problem 68

Internal problem ID [2823]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 68.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - x^n(a + by^2) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x) = x^n*(a+b*y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\frac{\sqrt{ab}\,(c_1 n + x^{n+1} + c_1)}{n+1}\right)\sqrt{ab}}{b}$$

✓ Solution by Mathematica

Time used: 0.291 (sec). Leaf size: 78

```
DSolve[y'[x]==x^n(a + b y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}\tan\left(\sqrt{a}\sqrt{b}\left(\frac{x^{n+1}}{n+1} + c_1\right)\right)}{\sqrt{b}}$$

$$y(x) \to -\frac{i\sqrt{a}}{\sqrt{b}}$$

$$y(x) \to \frac{i\sqrt{a}}{\sqrt{b}}$$

3.15 problem 69

Internal problem ID [2824]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 69.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - a\,x^m - x^n y^2 b = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 177

```
dsolve(diff(y(x),x) = a*x^m+b*x^n*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(\operatorname{BesselY}\left(\frac{1+m}{n+m+2}, \frac{2\sqrt{ab}\,x^{\frac{n}{2}+\frac{m}{2}+1}}{n+m+2}\right)c_1 + \operatorname{BesselJ}\left(\frac{1+m}{n+m+2}, \frac{2\sqrt{ab}\,x^{\frac{n}{2}+\frac{m}{2}+1}}{n+m+2}\right)\right)x^{\frac{n}{2}+\frac{m}{2}+1}\sqrt{ab}\,x^{-n}}{\left(\operatorname{BesselY}\left(-\frac{n+1}{n+m+2}, \frac{2\sqrt{ab}\,x^{\frac{n}{2}+\frac{m}{2}+1}}{n+m+2}\right)c_1 + \operatorname{BesselJ}\left(-\frac{n+1}{n+m+2}, \frac{2\sqrt{ab}\,x^{\frac{n}{2}+\frac{m}{2}+1}}{n+m+2}\right)\right)bx}$$

✓ Solution by Mathematica

Time used: 1.364 (sec). Leaf size: 992

```
DSolve[y'[x]==a x^m+ b x^n y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(n+1)x^{-n-1}\left((m+n+1)^{\frac{2(n+1)}{m+n+2}} \text{Gamma}\left(\frac{n+1}{m+n+2}\right)\left(-\sqrt{a}\sqrt{b}(m+n+1)\left(x^{m+n+1}\right)^{\frac{m+n+2}{2(m+n+1)}} \text{BesselJ}\left(-\frac{}{m}\right.\right.\right.}{}$$

$$y(x) \to \frac{x^{-n-1}\left(\frac{\frac{ab\left(x^{m+n+1}\right)^{\frac{1}{m+n+1}+1} {}_0F_1\left(;\frac{m+1}{m+n+2}+1;-\frac{ab\left(x^{m+n+1}\right)^{1+\frac{1}{m+n+1}}}{(m+n+2)^2}\right)}{m+1}+(n+1)\,{}_0F_1\left(;-\frac{n+1}{m+n+2};-\frac{ab\left(x^{m+n+1}\right)^{1+\frac{1}{m+n+1}}}{(m+n+2)^2}\right)}{{}_0F_1\left(;\frac{m+1}{m+n+2};-\frac{ab\left(x^{m+n+1}\right)^{1+\frac{1}{m+n+1}}}{(m+n+2)^2}\right)}-n-\right.}{2b}$$

$$y(x) \to \frac{x^{-n-1}\left(\frac{\frac{ab\left(x^{m+n+1}\right)^{\frac{1}{m+n+1}+1} {}_0F_1\left(;\frac{m+1}{m+n+2}+1;-\frac{ab\left(x^{m+n+1}\right)^{1+\frac{1}{m+n+1}}}{(m+n+2)^2}\right)}{m+1}+(n+1)\,{}_0F_1\left(;-\frac{n+1}{m+n+2};-\frac{ab\left(x^{m+n+1}\right)^{1+\frac{1}{m+n+1}}}{(m+n+2)^2}\right)}{{}_0F_1\left(;\frac{m+1}{m+n+2};-\frac{ab\left(x^{m+n+1}\right)^{1+\frac{1}{m+n+1}}}{(m+n+2)^2}\right)}-n-\right.}{2b}$$

3.16 problem 70

Internal problem ID [2825]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 70.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' - (a + by\cos(kx))\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(diff(y(x),x) = (a+b*y(x)*cos(k*x))*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{(a^2+k^2)\,\mathrm{e}^{ax}}{\mathrm{e}^{ax}k\sin(kx)\,b + a\,\mathrm{e}^{ax}\cos(kx)\,b - c_1a^2 - c_1k^2}$$

✓ Solution by Mathematica

Time used: 0.243 (sec). Leaf size: 47

```
DSolve[y'[x]==(a+b y[x] Cos[k x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{-\frac{b(a\cos(kx)+k\sin(kx))}{a^2+k^2} + c_1e^{-ax}}$$

$$y(x) \to 0$$

3.17 problem 71

Internal problem ID [2826]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 71.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \sin(x)\left(2\sec(x)^2 - y\right) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) = sin(x)*(2*sec(x)^2-y(x)),y(x), singsol=all)
```

$$y(x) = \left(2\sec(x)\,\mathrm{e}^{-\frac{1}{\sec(x)}} - 2\,\mathrm{Ei}_1\left(\frac{1}{\sec(x)}\right) + c_1\right)\mathrm{e}^{\cos(x)}$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 25

```
DSolve[y'[x]==Sin[x](2 Sec[x]^2-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\sec(x) + e^{\cos(x)}(2\,\mathrm{ExpIntegralEi}(-\cos(x)) + c_1)$$

3.18 problem 72

Internal problem ID [2827]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 72.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y' + 4\csc(x) - (3 - \cot(x))\, y - \sin(x)\, y^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 90

```
dsolve(diff(y(x),x)+4*csc(x) = (3-cot(x))*y(x)+y(x)^2*sin(x),y(x), singsol=all)
```

$$y(x) = \frac{-\frac{4c_1(\cos(x)+i\sin(x))^{-\frac{5i}{2}}}{c_1(\cos(x)+i\sin(x))^{-\frac{5i}{2}}+(\cos(x)+i\sin(x))^{\frac{5i}{2}}} + \frac{(\cos(x)+i\sin(x))^{\frac{5i}{2}}}{c_1(\cos(x)+i\sin(x))^{-\frac{5i}{2}}+(\cos(x)+i\sin(x))^{\frac{5i}{2}}}}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.269 (sec). Leaf size: 32

```
DSolve[y'[x]+4 Csc[x]==(3-Cot[x])y[x]+y[x]^2 Sin[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(-4 + \frac{1}{\frac{1}{5} + c_1 e^{5x}}\right)\csc(x)$$

$$y(x) \to -4\csc(x)$$

3.19 problem 73

Internal problem ID [2828]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 73.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y' - y\sec(x) - (\sin(x) - 1)^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x) = y(x)*sec(x)+(sin(x)-1)^2,y(x), singsol=all)
```

$$y(x) = \left(-3\sin(x) + 4\ln(\sec(x) + \tan(x)) + 4\ln(\cos(x)) - \frac{\cos(2x)}{4} + c_1\right)(\sec(x) + \tan(x))$$

✓ Solution by Mathematica

Time used: 12.197 (sec). Leaf size: 50

```
DSolve[y'[x]==y[x] Sec[x]+(Sin[x]-1)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4}e^{2\operatorname{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)}\left(\cos(2x) - 4\left(-3\sin(x) + 8\log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right) + c_1\right)\right)$$

3.20 problem 74

Internal problem ID [2829]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 74.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \tan(x)\left(1 - y^2\right) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)+tan(x)*(1-y(x)^2) = 0,y(x), singsol=all)
```

$$y(x) = -\tanh\left(-\ln\left(\cos\left(x\right)\right) + c_1\right)$$

✓ Solution by Mathematica

Time used: 0.548 (sec). Leaf size: 34

```
DSolve[y'[x]+Tan[x] (1-y[x]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 + \frac{2}{1 + e^{2c_1}\sec^2(x)}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

3.21 problem 75

Internal problem ID [2830]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 75.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y' - f(x) - g(x)\,y - h(x)\,y^2 = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = f(x)+g(x)*y(x)+h(x)*y(x)^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]+g[x] y[x]+h[x] y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

3.22 problem 76

Internal problem ID [2831]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 76.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - (a + yb + cy^2) f(x) = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 46

```
dsolve(diff(y(x),x) = (a+b*y(x)+c*y(x)^2)*f(x),y(x), singsol=all)
```

$$y(x) = -\frac{b - \tan\left(\frac{\sqrt{4ac-b^2}\left(\int f(x)dx+c_1\right)}{2}\right)\sqrt{4ac - b^2}}{2c}$$

✓ Solution by Mathematica

Time used: 0.253 (sec). Leaf size: 115

```
DSolve[y'[x]==(a+b y[x]+c y[x]^2)f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-b + \sqrt{4ac - b^2} \tan\left(\frac{1}{2}\sqrt{4ac - b^2}\left(\int_1^x f(K[1])dK[1] + c_1\right)\right)}{2c}$$

$$y(x) \to -\frac{\sqrt{b^2 - 4ac} + b}{2c}$$

$$y(x) \to \frac{\sqrt{b^2 - 4ac} - b}{2c}$$

3.23 problem 77

Internal problem ID [2832]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 77.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$\boxed{y' + (ax + y)\, y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 62

```
dsolve(diff(y(x),x)+(a*x+y(x))*y(x)^2=0,y(x), singsol=all)
```

$$y(x) = \frac{2a}{a^2x^2 + 2\,\mathrm{RootOf}\left(\mathrm{AiryBi}\left(\_Z\right)\left(-2a^2\right)^{\frac{1}{3}} c_1 x + \left(-2a^2\right)^{\frac{1}{3}} x\,\mathrm{AiryAi}\left(\_Z\right) + 2\,\mathrm{AiryBi}\left(1, \_Z\right) c_1 + 2\,\mathrm{AiryAi}\left(1\right.\right.}$$

✓ Solution by Mathematica

Time used: 0.241 (sec). Leaf size: 195

```
DSolve[y'[x]+(a x+y[x])y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\text{AiryAiPrime}\left(\frac{\sqrt[3]{-\frac{1}{2}}\sqrt[3]{a}}{y(x)}-\frac{1}{2}\sqrt[3]{-\frac{1}{2}}a^{4/3}x^2\right)-\left(-\frac{1}{2}\right)^{2/3}a^{2/3}x\,\text{AiryAi}\left(\frac{\sqrt[3]{-\frac{1}{2}}\sqrt[3]{a}}{y(x)}-\frac{1}{2}\sqrt[3]{-\frac{1}{2}}a^{4/3}x^2\right)}{\text{AiryBiPrime}\left(\frac{\sqrt[3]{-\frac{1}{2}}\sqrt[3]{a}}{y(x)}-\frac{1}{2}\sqrt[3]{-\frac{1}{2}}a^{4/3}x^2\right)-\left(-\frac{1}{2}\right)^{2/3}a^{2/3}x\,\text{AiryBi}\left(\frac{\sqrt[3]{-\frac{1}{2}}\sqrt[3]{a}}{y(x)}-\frac{1}{2}\sqrt[3]{-\frac{1}{2}}a^{4/3}x^2\right)}+c_1=0,y(x)\right]$$

3.24 problem 78

Internal problem ID [2833]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 78.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$\boxed{y' - (a\,e^{x} + y)\,y^{2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(diff(y(x),x) = (a*exp(x)+y(x))*y(x)^2,y(x), singsol=all)
```

$$c_1 + \frac{e^{-\frac{\left(a\,e^{x}+\frac{1}{y(x)}\right)^2}{2}}\,e^{-x}}{a} + \frac{\operatorname{erf}\left(\frac{\left(a\,e^{x}+\frac{1}{y(x)}\right)\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{\pi}}{2} = 0$$

✓ Solution by Mathematica

Time used: 0.68 (sec). Leaf size: 78

```
DSolve[y'[x]==(a Exp[x]+y[x])y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-iae^{x} = \frac{2e^{\frac{1}{2}\left(-iae^{x}-\frac{i}{y(x)}\right)^2}}{\sqrt{2\pi}\text{erfi}\left(\frac{-iae^{x}-\frac{i}{y(x)}}{\sqrt{2}}\right)+2c_1}, y(x)\right]$$

3.25 problem 79

Internal problem ID [2834]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 79.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$\boxed{y' + 3a(2x + y)\, y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(y(x),x)+3*a*(2*x+y(x))*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{3a\,x^2 + \operatorname{RootOf}\left((-3a)^{\frac{1}{3}} \operatorname{AiryBi}(\_Z)\, c_1 x + (-3a)^{\frac{1}{3}}\, x \operatorname{AiryAi}(\_Z) + \operatorname{AiryBi}(1, \_Z)\, c_1 + \operatorname{AiryAi}(1, \_Z)\right)}$$

✓ Solution by Mathematica

Time used: 0.299 (sec). Leaf size: 185

```
DSolve[y'[x]+3 a(2 x + y[x])y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt[3]{-3}\sqrt[3]{a}x \operatorname{AiryAi}\left((-3)^{2/3}a^{2/3}x^2 - \frac{(-1)^{2/3}}{\sqrt[3]{3}\sqrt[3]{a}y(x)}\right) + \operatorname{AiryAiPrime}\left((-3)^{2/3}a^{2/3}x^2 - \frac{(-1)^{2/3}}{\sqrt[3]{3}\sqrt[3]{a}y(x)}\right)}{\sqrt[3]{-3}\sqrt[3]{a}x \operatorname{AiryBi}\left((-3)^{2/3}a^{2/3}x^2 - \frac{(-1)^{2/3}}{\sqrt[3]{3}\sqrt[3]{a}y(x)}\right) + \operatorname{AiryBiPrime}\left((-3)^{2/3}a^{2/3}x^2 - \frac{(-1)^{2/3}}{\sqrt[3]{3}\sqrt[3]{a}y(x)}\right)} + c_1 = 0, y(x)\right]$$

3.26 problem 80

Internal problem ID [2835]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 80.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y(a + by^2) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 70

```
dsolve(diff(y(x),x) = y(x)*(a+b*y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(c_1 a \,\mathrm{e}^{-2ax} - b)\, a}}{c_1 a \,\mathrm{e}^{-2ax} - b}$$

$$y(x) = -\frac{\sqrt{(c_1 a \,\mathrm{e}^{-2ax} - b)\, a}}{c_1 a \,\mathrm{e}^{-2ax} - b}$$

✓ Solution by Mathematica

Time used: 1.739 (sec). Leaf size: 118

```
DSolve[y'[x]==y[x](a+b y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\sqrt{a}e^{a(x+c_1)}}{\sqrt{-1 + be^{2a(x+c_1)}}}$$

$$y(x) \to \frac{i\sqrt{a}e^{a(x+c_1)}}{\sqrt{-1 + be^{2a(x+c_1)}}}$$

$$y(x) \to 0$$

$$y(x) \to -\frac{i\sqrt{a}}{\sqrt{b}}$$

$$y(x) \to \frac{i\sqrt{a}}{\sqrt{b}}$$

3.27 problem 81

Internal problem ID [2836]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 81.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \mathrm{a0} - \mathrm{a1}\, y - \mathrm{a2}\, y^2 - \mathrm{a3}\, y^3 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x) = a0+a1*y(x)+a2*y(x)^2+a3*y(x)^3,y(x), singsol=all)
```

$$x - \left(\int^{y(x)} \frac{1}{\_a^3\, \mathrm{a3} + \_a^2\, \mathrm{a2} + \_a\, \mathrm{a1} + \mathrm{a0}} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.097 (sec). Leaf size: 54

```
DSolve[y'[x]==a0+a1 y[x]+a2 y[x]^2+ a3 y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\mathrm{RootSum}\left[\#1^3\mathrm{a3} + \#1^2\mathrm{a2} + \#1\mathrm{a1} + \mathrm{a0}\&, \frac{\log(y(x) - \#1)}{3\#1^2\mathrm{a3} + 2\#1\mathrm{a2} + \mathrm{a1}}\&\right] = x + c_1, y(x)\right]$$

3.28 problem 82

Internal problem ID [2837]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 82.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - xy^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = x*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-x^2 + c_1}}$$
$$y(x) = -\frac{1}{\sqrt{-x^2 + c_1}}$$

✓ Solution by Mathematica

Time used: 0.142 (sec). Leaf size: 44

```
DSolve[y'[x]==x y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{-x^2 - 2c_1}}$$
$$y(x) \to \frac{1}{\sqrt{-x^2 - 2c_1}}$$
$$y(x) \to 0$$

3.29 problem 83

Internal problem ID [2838]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 3
Problem number: 83.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + y(1 - xy^2) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(diff(y(x),x)+y(x)*(1-x*y(x)^2) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2}{\sqrt{2 + 4c_1 e^{2x} + 4x}}$$
$$y(x) = \frac{2}{\sqrt{2 + 4c_1 e^{2x} + 4x}}$$

✓ Solution by Mathematica

Time used: 2.605 (sec). Leaf size: 50

```
DSolve[y'[x]+y[x](1-x y[x]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{x + c_1 e^{2x} + \frac{1}{2}}}$$
$$y(x) \to \frac{1}{\sqrt{x + c_1 e^{2x} + \frac{1}{2}}}$$
$$y(x) \to 0$$

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4.1 problem 84

Internal problem ID [2839]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 84.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _Abel]`

$$y' - (a + bxy)\,y^2 = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 103

```
dsolve(diff(y(x),x) = (a+b*x*y(x))*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{e^{\mathrm{RootOf}\left(2\sqrt{a^2-4b}\,a\,\mathrm{arctanh}\left(\frac{2b\,e^{\_Z}+a}{\sqrt{a^2-4b}}\right)-\ln\left(x^2\left(b\,e^{2\_Z}+a\,e^{\_Z}+1\right)\right)a^2+2c_1a^2+2\_Z a^2+4\ln\left(x^2\left(b\,e^{2\_Z}+a\,e^{\_Z}+1\right)\right)b-8c_1b-8\_Zb\right)}}{x}$$

✓ Solution by Mathematica

Time used: 0.203 (sec). Leaf size: 94

```
DSolve[y'[x]==(a+b x y[x])y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{a^2\left(-\frac{2\arctan\left(\frac{a+2bxy(x)}{a\sqrt{\frac{4b}{a^2}-1}}\right)}{\sqrt{\frac{4b}{a^2}-1}}-\log\left(\frac{bxy(x)(a+bxy(x))+b}{b^2x^2y(x)^2}\right)\right)}{2b}=\frac{a^2\log(x)}{b}+c_1,y(x)\right]$$

4.2 problem 87

Internal problem ID [2840]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 87.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y' + 2xy(1 + axy^2) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 83

```
dsolve(diff(y(x),x)+2*x*y(x)*(1+a*x*y(x)^2) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2}{\sqrt{e^{2x^2} a\sqrt{\pi}\,\sqrt{2}\ \mathrm{erf}\left(\sqrt{2}\,x\right) + 4\,e^{2x^2} c_1 - 4ax}}$$

$$y(x) = \frac{2}{\sqrt{e^{2x^2} a\sqrt{\pi}\,\sqrt{2}\ \mathrm{erf}\left(\sqrt{2}\,x\right) + 4\,e^{2x^2} c_1 - 4ax}}$$

✓ Solution by Mathematica

Time used: 7.362 (sec). Leaf size: 96

```
DSolve[y'[x]+2 x y[x](1+ a x y[x]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{\sqrt{-4ax + e^{2x^2}\left(\sqrt{2\pi}a\mathrm{erf}\left(\sqrt{2}x\right) + 4c_1\right)}}$$

$$y(x) \to \frac{2}{\sqrt{-4ax + e^{2x^2}\left(\sqrt{2\pi}a\mathrm{erf}\left(\sqrt{2}x\right) + 4c_1\right)}}$$

$$y(x) \to 0$$

4.3 problem 90

Internal problem ID [2841]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 90.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + \left(\tan (x) + y^2 \sec (x)\right) y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(diff(y(x),x)+(tan(x)+y(x)^2*sec(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\cos (x)}{\sqrt{2 \sin (x) + c_1}}$$

$$y(x) = -\frac{\cos (x)}{\sqrt{2 \sin (x) + c_1}}$$

✓ Solution by Mathematica

Time used: 3.688 (sec). Leaf size: 48

```
DSolve[y'[x]+(Tan[x]+y[x]^2 Sec[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{\sec^2(x)(2\sin(x) + c_1)}}$$

$$y(x) \to \frac{1}{\sqrt{\sec^2(x)(2\sin(x) + c_1)}}$$

$$y(x) \to 0$$

4.4 problem 91

Internal problem ID [2842]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 91.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' + y^3 \sec(x) \tan(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(diff(y(x),x)+y(x)^3*sec(x)*tan(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(\cos(x) c_1 + 2) \cos(x)}}{\cos(x) c_1 + 2}$$
$$y(x) = -\frac{\sqrt{(\cos(x) c_1 + 2) \cos(x)}}{\cos(x) c_1 + 2}$$

✓ Solution by Mathematica

Time used: 0.361 (sec). Leaf size: 49

```
DSolve[y'[x]+y[x]^3 Sec[x] Tan[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{2}\sqrt{\sec(x) - c_1}}$$
$$y(x) \to \frac{1}{\sqrt{2}\sqrt{\sec(x) - c_1}}$$
$$y(x) \to 0$$

4.5 problem 92

Internal problem ID [2843]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 92.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Abel]`

$$y' - \text{f0}(x) - \text{f1}(x)\, y - \text{f2}(x)\, y^2 - \text{f3}(x)\, y^3 = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = f0(x)+f1(x)*y(x)+f2(x)*y(x)^2+f3(x)*y(x)^3,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f0[x]+f1[x]y[x]+f2[x] y[x]^2+f3[x]y[x]^3,y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

4.6 problem 94

Internal problem ID [2844]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 94.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _Chini]`

$$\boxed{y' - a\,x^{\frac{n}{-n+1}} - by^n = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 64

```
dsolve(diff(y(x),x) = a*x^(n/(1-n))+b*y(x)^n,y(x), singsol=all)
```

$$-\left(\int_{\_b}^{y(x)} \frac{x^{\frac{n}{n-1}}}{(bx(n-1)\_a^n + \_a)\,x^{\frac{n}{n-1}} + ax(n-1)} d\_a\right) + \frac{\ln(x)}{n-1} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.3 (sec). Leaf size: 117

```
DSolve[y'[x]==a*x^(n/(1-n))+b*y[x]^n,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{\left(\frac{bx^{-\frac{n}{1-n}}}{a}\right)^{\frac{1}{n}} y(x)} \frac{1}{K[1]^n - \left(\frac{(-1)^n a^{1-n}(n-1)^{-n}}{b}\right)^{\frac{1}{n}} K[1] + 1} dK[1] = \int_1^x aK[2]^{\frac{n}{1-n}} \left(\frac{bK[2]^{-\frac{n}{1-n}}}{a}\right)^{\frac{1}{n}} dK[2] + c_1, y(x)\right]$$

4.7 problem 95

Internal problem ID [2845]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 95.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' - f(x)\, y - g(x)\, y^k = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 81

```
dsolve(diff(y(x),x) = f(x)*y(x)+g(x)*y(x)^k,y(x), singsol=all)
```

$$y(x) = \left(\int \left(-k\, e^{\int (f(x)k-f(x))dx} g(x) + e^{\int (f(x)k-f(x))dx} g(x)\right) dx + c_1\right)^{-\frac{1}{k-1}} e^{\frac{(\int f(x)dx)k}{k-1}} e^{\int -\frac{f(x)}{k-1} dx}$$

✓ Solution by Mathematica

Time used: 10.26 (sec). Leaf size: 67

```
DSolve[y'[x]==f[x] y[x]+g[x]y[x]^k,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\exp\left(-\left((k-1)\int_1^x f(K[1])dK[1]\right)\right)\left(-(k-1)\int_1^x \exp\left((k-1)\int_1^{K[2]} f(K[1])dK[1]\right) g(K[2])dK[2] + c_1\right)\right)^{\frac{1}{1-k}}$$

4.8 problem 96

Internal problem ID [2846]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 96.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Chini]`

$$\boxed{y' - f(x) - g(x)\,y - h(x)\,y^n = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = f(x)+g(x)*y(x)+h(x)*y(x)^n,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]==f[x]+g[x]y[x]+h[x]y[x]^n,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

4.9 problem 98

Internal problem ID [2847]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 98.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{|y|} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(diff(y(x),x) = sqrt(abs(y(x))),y(x), singsol=all)
```

$$x - \left(\begin{cases} -2\sqrt{-y(x)} & y(x) \leq 0 \\ 2\sqrt{y(x)} & 0 < y(x) \end{cases}\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.159 (sec). Leaf size: 31

```
DSolve[y'[x]==Sqrt[Abs[y[x]]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{\sqrt{|K[1]|}} dK[1]\&\right][x + c_1]$$

$$y(x) \to 0$$

4.10 problem 99

Internal problem ID [2848]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 99.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$y' - a - yb - \sqrt{\mathrm{A0} + \mathrm{B0}\, y} = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve(diff(y(x),x) = a+b*y(x)+sqrt(A0+B0*y(x)),y(x), singsol=all)
```

$$x - \left(\int^{y(x)} \frac{1}{a + \_ab + \sqrt{\mathrm{B0}\,\_a + \mathrm{A0}}} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.672 (sec). Leaf size: 172

```
DSolve[y'[x]==a+b y[x]+Sqrt[A0+B0 y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{\log\left(-\mathrm{B0}\left(\sqrt{\#1\mathrm{B0}+\mathrm{A0}}+\#1b+a\right)\right) - \frac{2\mathrm{B0}\arctan\left(\frac{2b\sqrt{\#1\mathrm{B0}+\mathrm{A0}}+\mathrm{B0}}{\sqrt{\mathrm{B0}(4ab-\mathrm{B0})-4\mathrm{A0}b^2}}\right)}{\sqrt{\mathrm{B0}(4ab-\mathrm{B0})-4\mathrm{A0}b^2}}}{b}\&\right][x + c_1]$$

$$y(x) \to -\frac{\sqrt{-4ab\mathrm{B0} + 4\mathrm{A0}b^2 + \mathrm{B0}^2} + 2ab - \mathrm{B0}}{2b^2}$$

$$y(x) \to \frac{\sqrt{-4ab\mathrm{B0} + 4\mathrm{A0}b^2 + \mathrm{B0}^2} - 2ab + \mathrm{B0}}{2b^2}$$

4.11 problem 100

Internal problem ID [2849]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 100.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _Chini]`

$$\boxed{y' - ax - b\sqrt{y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 68

```
dsolve(diff(y(x),x) = a*x+b*sqrt(y(x)),y(x), singsol=all)
```

$$-\frac{\ln\left(\sqrt{y(x)}\, bx + a\, x^2 - 2y(x)\right)}{2} + \frac{b\sqrt{y(x)}\, \operatorname{arctanh}\left(\frac{b\sqrt{y(x)}+2ax}{\sqrt{y(x)(b^2+8a)}}\right)}{\sqrt{y(x)(b^2+8a)}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.266 (sec). Leaf size: 119

```
DSolve[y'[x]==a x+b Sqrt[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{b^2\left(-\frac{2b\,\text{arctanh}\left(\frac{b^2-4a\sqrt{\frac{b^2y(x)}{a^2x^2}}}{b\sqrt{8a+b^2}}\right)}{\sqrt{8a+b^2}}-\log\left(b^2\left(\sqrt{\frac{b^2y(x)}{a^2x^2}}+1\right)-\frac{2b^2y(x)}{ax^2}\right)\right)}{2a}=\frac{b^2\log(x)}{a}+c_1,y(x)\right]$$

4.12 problem 101

Internal problem ID [2850]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 101.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y' + x^3 - x\sqrt{x^4 + 4y} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 59

```
dsolve(diff(y(x),x)+x^3 = x*sqrt(x^4+4*y(x)),y(x), singsol=all)
```

$$-\frac{y(x)\, x^2}{x^2 + \sqrt{x^4 + 4y(x)}} + \frac{y(x)\sqrt{x^4 + 4y(x)}}{x^2 + \sqrt{x^4 + 4y(x)}} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.542 (sec). Leaf size: 30

```
DSolve[y'[x]+x^3==x Sqrt[x^4+4 y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2e^{2c_1}\left(x^2 + 2e^{2c_1}\right)$$
$$y(x) \to 0$$

4.13 problem 102

Internal problem ID [2851]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 102.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y' + 2y(1 - \sqrt{y}\, x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+2*y(x)*(1-x*sqrt(y(x))) = 0,y(x), singsol=all)
```

$$\frac{1}{\sqrt{y(x)}} - x - 1 - c_1 e^x = 0$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 21

```
DSolve[y'[x]+2 y[x] (1-x Sqrt[y[x]])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{(x + c_1 e^x + 1)^2}$$
$$y(x) \rightarrow 0$$

4.14 problem 103

Internal problem ID [2852]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 103.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{a + b y^2} = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(x),x) = sqrt(a+b*y(x)^2),y(x), singsol=all)
```

$$x - \frac{\ln\left(y(x)\sqrt{b} + \sqrt{a + b y(x)^2}\right)}{\sqrt{b}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.097 (sec). Leaf size: 82

```
DSolve[y'[x]==Sqrt[a+b y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{a}\tanh\left(\sqrt{b}(x + c_1)\right)}{\sqrt{b\operatorname{sech}^2\left(\sqrt{b}(x + c_1)\right)}}$$

$$y(x) \to \frac{\sqrt{a}\tanh\left(\sqrt{b}(x + c_1)\right)}{\sqrt{b\operatorname{sech}^2\left(\sqrt{b}(x + c_1)\right)}}$$

4.15 problem 104

Internal problem ID [2853]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 104.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - y\sqrt{a + yb} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(diff(y(x),x) = y(x)*sqrt(a+b*y(x)),y(x), singsol=all)
```

$$x + \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+by(x)}}{\sqrt{a}}\right)}{\sqrt{a}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 19.279 (sec). Leaf size: 42

```
DSolve[y'[x]==y[x] Sqrt[a+b y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a\operatorname{sech}^2\left(\frac{1}{2}\sqrt{a}(x + c_1)\right)}{b}$$

$$y(x) \to 0$$

$$y(x) \to -\frac{a}{b}$$

4.16 problem 105

Internal problem ID [2854]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 105.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' + (f(x) - y)\, g(x)\, \sqrt{(y-a)\,(y-b)} = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)+(f(x)-y(x))*g(x)*sqrt((y(x)-a)*(y(x)-b)) = 0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]+(f[x]-y[x])g[x] Sqrt[(y[x]-a)(y[x]-b)]==0,y[x],x,IncludeSingularSolutions -> Tru
```

Not solved

4.17 problem 106

Internal problem ID [2855]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 106.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{XY} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x) = sqrt(X*Y),y(x), singsol=all)
```

$$y(x) = \sqrt{XY}\, x + c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 17

```
DSolve[y'[x]==Sqrt[ X Y],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x\sqrt{XY} + c_1$$

4.18 problem 107

Internal problem ID [2856]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 107.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \cos(x)^2 \cos(y) = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 69

```
dsolve(diff(y(x),x) = cos(x)^2*cos(y(x)),y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{c_1^2 e^{x+\frac{\sin(2x)}{2}} - 1}{c_1^2 e^{x+\frac{\sin(2x)}{2}} + 1}, \frac{2c_1 e^{\frac{x}{2}+\frac{\sin(2x)}{4}}}{c_1^2 e^{x+\frac{\sin(2x)}{2}} + 1}\right)$$

✓ Solution by Mathematica

Time used: 0.954 (sec). Leaf size: 41

```
DSolve[y'[x]==Cos[x]^2 Cos[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\arctan\left(\tanh\left(\frac{1}{8}(2x + \sin(2x) + c_1)\right)\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

4.19 problem 108

Internal problem ID [2857]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 108.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \sec(x)^2 \cot(y) \cos(y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = sec(x)^2*cot(y(x))*cos(y(x)),y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{1}{\tan(x) + c_1}\right)$$

✓ Solution by Mathematica

Time used: 0.792 (sec). Leaf size: 45

```
DSolve[y'[x]==Sec[x]^2 Cot[y[x]] Cos[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sec^{-1}(\tan(x) + 2c_1)$$
$$y(x) \to \sec^{-1}(\tan(x) + 2c_1)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

4.20 problem 109

Internal problem ID [2858]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 109.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - a - b\cos(Ax + By) = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 96

```
dsolve(diff(y(x),x) = a+b*cos(A*x+B*y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{Ax + 2\arctan\left(\frac{\tan\left(\frac{c_1\sqrt{(aB+bB+A)(aB-bB+A)}}{2} - \frac{x\sqrt{(aB+bB+A)(aB-bB+A)}}{2}\right)\sqrt{(aB+bB+A)(aB-bB+A)}}{aB-bB+A}\right)}{B}$$

✓ Solution by Mathematica

Time used: 60.695 (sec). Leaf size: 80

```
DSolve[y'[x]==a+b Cos[A x+ B y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{Ax + 2\arctan\left(\frac{(B(a+b)+A)\tanh\left(\frac{1}{2}(-x+c_1)\sqrt{-((B(a-b)+A)(B(a+b)+A))}\right)}{\sqrt{-((B(a-b)+A)(B(a+b)+A))}}\right)}{B}$$

4.21 problem 110

Internal problem ID [2859]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 110.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' + f(x) + g(x)\sin(ay) + h(x)\cos(ay) = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)+f(x)+g(x)*sin(a*y(x))+h(x)*cos(a*y(x)) = 0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]+f[x]+g[x]Sin[a y[x]]+h[x] Cos[a y[x]]==0,y[x],x,IncludeSingularSolutions -> True
```

Not solved

4.22 problem 111

Internal problem ID [2860]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 111.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - a - b\cos(y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(diff(y(x),x) = a+b*cos(y(x)),y(x), singsol=all)
```

$$y(x) = 2\arctan\left(\frac{\tan\left(\frac{\sqrt{(a+b)(a-b)}\,(x+c_1)}{2}\right)\sqrt{(a+b)\,(a-b)}}{a-b}\right)$$

✓ Solution by Mathematica

Time used: 60.12 (sec). Leaf size: 47

```
DSolve[y'[x]==a+b Cos[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\arctan\left(\frac{(a+b)\tanh\left(\frac{1}{2}\sqrt{b^2-a^2}(x+c_1)\right)}{\sqrt{b^2-a^2}}\right)$$

4.23 problem 112

Internal problem ID [2861]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 112.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y' + x(\sin(2y) - x^2 \cos(y)^2) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)+x*(sin(2*y(x))-x^2*cos(y(x))^2) = 0,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{c_1 \mathrm{e}^{-x^2}}{2} + \frac{x^2}{2} - \frac{1}{2}\right)$$

✓ Solution by Mathematica

Time used: 19.387 (sec). Leaf size: 101

```
DSolve[y'[x]+x(Sin[2 y[x]]-x^2 Cos[y[x]]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arctan\left(\frac{1}{2}\left(x^2 - 8c_1 e^{-x^2} - 1\right)\right)$$
$$y(x) \to \arctan\left(\frac{1}{2}\left(x^2 - 8c_1 e^{-x^2} - 1\right)\right)$$
$$y(x) \to -\frac{1}{2}\pi e^{x^2}\sqrt{e^{-2x^2}}$$
$$y(x) \to \frac{1}{2}\pi e^{x^2}\sqrt{e^{-2x^2}}$$

4.24 problem 113

Internal problem ID [2862]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 113.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{y' + \tan(x)\sec(x)\cos(y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)+tan(x)*sec(x)*cos(y(x))^2 = 0,y(x), singsol=all)
```

$$y(x) = -\arctan(\sec(x) + c_1)$$

✓ Solution by Mathematica

Time used: 1.54 (sec). Leaf size: 31

```
DSolve[y'[x]+Tan[x] Sec[x] Cos[y[x]]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arctan(-\sec(x) + c_1)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

4.25 problem 114

Internal problem ID [2863]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 114.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \cot(x)\cot(y) = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 13

```
dsolve(diff(y(x),x) = cot(x)*cot(y(x)),y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{1}{\sin(x)\,c_1}\right)$$

✓ Solution by Mathematica

Time used: 5.655 (sec). Leaf size: 47

```
DSolve[y'[x]==Cot[x] Cot[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(-\frac{1}{2}c_1\csc(x)\right)$$
$$y(x) \to \arccos\left(-\frac{1}{2}c_1\csc(x)\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

4.26 problem 115

Internal problem ID [2864]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 4
Problem number: 115.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \cot(x)\cot(y) = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)+cot(x)*cot(y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \arccos(\sin(x) c_1)$$

✓ Solution by Mathematica

Time used: 5.735 (sec). Leaf size: 47

```
DSolve[y'[x]+Cot[x] Cot[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(-\frac{1}{2}c_1 \sin(x)\right)$$
$$y(x) \to \arccos\left(-\frac{1}{2}c_1 \sin(x)\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

5 Various 5

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5.1 problem 116

Internal problem ID [2865]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 116.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \sin(x)(\csc(y) - \cot(y)) = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 14

```
dsolve(diff(y(x),x) = sin(x)*(csc(y(x))-cot(y(x))),y(x), singsol=all)
```

$$y(x) = \arccos\left(\mathrm{e}^{-\cos(x)}c_1 + 1\right)$$

✓ Solution by Mathematica

Time used: 7.839 (sec). Leaf size: 26

```
DSolve[y'[x]==Sin[x](Csc[y[x]]-Cot[y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\arcsin\left(e^{\frac{1}{4}(-2\cos(x)+c_1)}\right)$$
$$y(x) \to 0$$

5.2 problem 117

Internal problem ID [2866]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 117.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \tan(x)\cot(y) = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 11

```
dsolve(diff(y(x),x) = tan(x)*cot(y(x)),y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{\cos(x)}{c_1}\right)$$

✓ Solution by Mathematica

Time used: 5.404 (sec). Leaf size: 47

```
DSolve[y'[x]==Tan[x] Cot[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(\frac{1}{2}c_1\cos(x)\right)$$

$$y(x) \to \arccos\left(\frac{1}{2}c_1\cos(x)\right)$$

$$y(x) \to -\frac{\pi}{2}$$

$$y(x) \to \frac{\pi}{2}$$

5.3 problem 118

Internal problem ID [2867]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 118.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \tan(x)\cot(y) = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 11

```
dsolve(diff(y(x),x)+tan(x)*cot(y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \arccos\left(\frac{c_1}{\cos(x)}\right)$$

✓ Solution by Mathematica

Time used: 5.978 (sec). Leaf size: 47

```
DSolve[y'[x]+Tan[x] Cot[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arccos\left(\frac{1}{2}c_1\sec(x)\right)$$
$$y(x) \to \arccos\left(\frac{1}{2}c_1\sec(x)\right)$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

5.4 problem 119

Internal problem ID [2868]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 119.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \sin(2x)\csc(2y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)+sin(2*x)*csc(2*y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\arccos(-\cos(2x) + 4c_1)}{2}$$

✓ Solution by Mathematica

Time used: 0.49 (sec). Leaf size: 41

```
DSolve[y'[x]+Sin[2 x]Csc[2 y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}\arccos(-\cos(2x) - 2c_1)$$
$$y(x) \to \frac{1}{2}\arccos(-\cos(2x) - 2c_1)$$

5.5 problem 120

Internal problem ID [2869]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 120.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - \tan(x)(\tan(y) + \sec(x)\sec(y)) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x) = tan(x)*(tan(y(x))+sec(x)*sec(y(x))),y(x), singsol=all)
```

$$y(x) = \arcsin(-\ln(\cos(x))\sec(x) + c_1\sec(x))$$

✓ Solution by Mathematica

Time used: 9.668 (sec). Leaf size: 20

```
DSolve[y'[x]==Tan[x] (Tan[y[x]]+ Sec[x] Sec[y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(\frac{1}{4}\sec(x)(-4\log(\cos(x)) + c_1)\right)$$

5.6 problem 121

Internal problem ID [2870]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 121.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \cos(x)\sec(y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 22

```
dsolve(diff(y(x),x) = cos(x)*sec(y(x))^2,y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{RootOf}\left(-\_Z + 4c_1 + 4\sin(x) - \sin(\_Z)\right)}{2}$$

✓ Solution by Mathematica

Time used: 0.313 (sec). Leaf size: 32

```
DSolve[y'[x]==Cos[x] Sec[y[x]]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \text{InverseFunction}\left[2\left(\frac{\#1}{2} + \frac{1}{4}\sin(2\#1)\right)\&\right][2\sin(x) + c_1]$$

5.7 problem 122

Internal problem ID [2871]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 122.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y' - \sec(x)^2 \sec(y)^3 = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 74

```
dsolve(diff(y(x),x) = sec(x)^2*sec(y(x))^3,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{3c_1 + 3\tan(x)}{\operatorname{RootOf}\left(\_Z^6 + 3\_Z^4 + 9c_1^2 + 18c_1\tan(x) + 9\tan(x)^2 - 4\right)^2 + 2}, \operatorname{RootOf}\left(\_Z^6 + 3\_Z^4 + 9c_1^2 + 18c_1\tan(x) + 9\tan(x)^2 - 4\right)\right)$$

✓ Solution by Mathematica

Time used: 23.315 (sec). Leaf size: 425

```
DSolve[y'[x]==Sec[x]^2 Sec[y[x]]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(\frac{\sqrt[3]{-3\tan(x)+\sqrt{9\tan^2(x)+18c_1\tan(x)-4+9{c_1}^2}-3c_1}}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}}{\sqrt[3]{-3\tan(x)+\sqrt{9\tan^2(x)+18c_1\tan(x)-4+9{c_1}^2}-3c_1}}\right)$$

$$y(x) \to -\arcsin\left(\frac{2\sqrt[3]{-2}-(-2)^{2/3}\left(-3\tan(x)+\sqrt{9\tan^2(x)+18c_1\tan(x)-4+9{c_1}^2}-3c_1\right)^{2/3}}{2\sqrt[3]{-3\tan(x)+\sqrt{9\tan^2(x)+18c_1\tan(x)-4+9{c_1}^2}-3c_1}}\right)$$

$$y(x) \to -\arcsin\left(\frac{\sqrt[3]{-2}\left(-3\tan(x)+\sqrt{9\tan^2(x)+18c_1\tan(x)-4+9{c_1}^2}-3c_1\right)^{2/3}-i\sqrt{3}+1}{2^{2/3}\sqrt[3]{-3\tan(x)+\sqrt{9\tan^2(x)+18c_1\tan(x)-4+9{c_1}^2}-3c_1}}\right)$$

$$y(x) \to \arcsin\left(\frac{\sqrt[3]{\sqrt{9\tan^2(x)-4}-3\tan(x)}}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}}{\sqrt[3]{\sqrt{9\tan^2(x)-4}-3\tan(x)}}\right)$$

$$y(x) \to -\arcsin\left(\frac{2\sqrt[3]{-2}-(-2)^{2/3}\left(\sqrt{9\tan^2(x)-4}-3\tan(x)\right)^{2/3}}{2\sqrt[3]{\sqrt{9\tan^2(x)-4}-3\tan(x)}}\right)$$

5.8 problem 123

Internal problem ID [2872]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 123.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - a - b\sin(y) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 43

```
dsolve(diff(y(x),x) = a+b*sin(y(x)),y(x), singsol=all)
```

$$y(x) = -2\arctan\left(\frac{b - \tan\left(\frac{\sqrt{a^2-b^2}\,(x+c_1)}{2}\right)\sqrt{a^2-b^2}}{a}\right)$$

✓ Solution by Mathematica

Time used: 60.182 (sec). Leaf size: 52

```
DSolve[y'[x]==a+b Sin[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\arctan\left(\frac{-b + \sqrt{(a-b)(a+b)}\tan\left(\frac{1}{2}\sqrt{(a-b)(a+b)}(x+c_1)\right)}{a}\right)$$

5.9 problem 125

Internal problem ID [2873]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 125.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' - (1 + \cos(x)\sin(y))\tan(y) = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x) = (1+cos(x)*sin(y(x)))*tan(y(x)),y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 1.935 (sec). Leaf size: 56

```
DSolve[y'[x]==(1+Cos[x] Sin[y[x]])Tan[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\csc^{-1}\left(\frac{1}{2}\left(\sin(x) + \cos(x) + 2c_1e^{-x}\right)\right)$$
$$y(x) \to -\csc^{-1}\left(\frac{1}{2}\left(\sin(x) + \cos(x) + 2c_1e^{-x}\right)\right)$$
$$y(x) \to 0$$

5.10 problem 126

Internal problem ID [2874]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 126.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + \csc(2x)\sin(2y) = 0}$$

✓ Solution by Maple

Time used: 0.234 (sec). Leaf size: 105

```
dsolve(diff(y(x),x)+csc(2*x)*sin(2*y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\arctan\left(-\frac{2c_1(\sin(4x)+2\sin(2x))}{c_1^2\cos(4x)-c_1^2-4\cos(2x)-\cos(4x)-3}, \frac{c_1^2\cos(4x)-c_1^2+4\cos(2x)+\cos(4x)+3}{c_1^2\cos(4x)-c_1^2-4\cos(2x)-\cos(4x)-3}\right)}{2}$$

✓ Solution by Mathematica

Time used: 6.234 (sec). Leaf size: 75

```
DSolve[y'[x]+Csc[2 x] Sin[2 y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arctan\left(e^{2c_1}\cot(x)\right)$$
$$y(x) \to 0$$
$$y(x) \to \frac{1}{2}\pi\tan(x)\sqrt{\cot^2(x)}$$
$$y(x) \to \frac{1}{2}\pi\left((-1)^{\left\lfloor \frac{\arg(\cot(x))}{\pi}+\frac{1}{2}\right\rfloor} - \sqrt{\tan^2(x)}\cot(x)\right)$$

5.11 problem 127

Internal problem ID [2875]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 127.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y' + f(x) + g(x)\tan(y) = 0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)+f(x)+g(x)*tan(y(x)) = 0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y'[x]+f[x]+g[x] Tan[ y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

5.12 problem 128

Internal problem ID [2876]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 128.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - \sqrt{a + b\cos(y)} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) = sqrt(a+b*cos(y(x))),y(x), singsol=all)
```

$$x - \left(\int^{y(x)} \frac{1}{\sqrt{a + b\cos(\_a)}} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.752 (sec). Leaf size: 55

```
DSolve[y'[x]==Sqrt[a+b Cos[ y[x]]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\,\text{JacobiAmplitude}\left(\frac{1}{2}\sqrt{a+b}(x + c_1), \frac{2b}{a+b}\right)$$
$$y(x) \to -\arccos\left(-\frac{a}{b}\right)$$
$$y(x) \to \arccos\left(-\frac{a}{b}\right)$$

5.13 problem 129

Internal problem ID [2877]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 129.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, `_with_symmetry_[F(x),G(x)]`]]`

$$\boxed{y' - e^y - x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x) = x+exp(y(x)),y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} - \ln\left(\frac{i\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\frac{i\sqrt{2}\,x}{2}\right)}{2} - c_1\right)$$

✓ Solution by Mathematica

Time used: 0.472 (sec). Leaf size: 40

```
DSolve[y'[x]==x+Exp[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(x^2 - 2\log\left(-\sqrt{\frac{\pi}{2}}\mathrm{erfi}\left(\frac{x}{\sqrt{2}}\right) - c_1\right)\right)$$

5.14 problem 130

Internal problem ID [2878]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 130.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - \mathrm{e}^{x+y} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

```
dsolve(diff(y(x),x) = exp(x+y(x)),y(x), singsol=all)
```

$$y(x) = \ln\left(-\frac{1}{\mathrm{e}^{x} + c_1}\right)$$

✓ Solution by Mathematica

Time used: 0.757 (sec). Leaf size: 18

```
DSolve[y'[x]==Exp[x+y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\log\left(-e^{x} - c_1\right)$$

5.15 problem 131

Internal problem ID [2879]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 131.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - e^{x}\left(a + b\,e^{-y}\right) = 0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 24

```
dsolve(diff(y(x),x) = exp(x)*(a+b*exp(-y(x))),y(x), singsol=all)
```

$$y(x) = -\ln\left(\frac{a}{e^{a\,e^{x}+c_1 a} - b}\right)$$

✓ Solution by Mathematica

Time used: 1.071 (sec). Leaf size: 24

```
DSolve[y'[x]==Exp[x](a+b Exp[-y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(\frac{-b + e^{a(e^{x}+c_1)}}{a}\right)$$

5.16 problem 132

Internal problem ID [2880]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 132.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' + y\ln(x)\ln(y) = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)+y(x)*ln(x)*ln(y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{x^{-x}\mathrm{e}^{x}}{c_1}}$$

✓ Solution by Mathematica

Time used: 0.193 (sec). Leaf size: 24

```
DSolve[y'[x]+y[x] Log[x] Log[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{x^{-x}e^{x+c_1}}$$
$$y(x) \to 1$$

5.17 problem 133

Internal problem ID [2881]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 133.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y' - x^{m-1}y^{-n+1}f(a\,x^m + by^n) = 0$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 174

```
dsolve(diff(y(x),x) = x^(m-1)*y(x)^(1-n)*f(a*x^m+b*y(x)^n),y(x), singsol=all)
```

$$y(x) = \left(-\frac{a\,x^m - \mathrm{RootOf}\left(\left(\left(\int^{\_Z} \frac{1}{\left(\left(\frac{\_a b - am}{b}\right)^{\frac{1}{n}}\right)^{-n}\left(m^{\frac{1}{m}}\right)^m f\left(a\left(m^{\frac{1}{m}}\right)^m + b\left(\left(\frac{\_a b - am}{b}\right)^{\frac{1}{n}}\right)^n\right) bn\_a - \left(\left(\frac{\_a b - am}{b}\right)^{\frac{1}{n}}\right)^{-n}\left(m^{\frac{1}{n}}\right.}\right.\right.\right.}{b}\right.$$

✓ Solution by Mathematica

Time used: 0.489 (sec). Leaf size: 242

```
DSolve[y'[x]==x^(m-1) y[x]^(1-n) f[a x^m + b y[x]^n],y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\int_1^{y(x)}\left(-\frac{amK[2]^{n-1}}{am + bnf\left(ax^m + bK[2]^n\right)}\right.\right.$$
$$-\int_1^x\left(\frac{abmnK[1]^{m-1}K[2]^{n-1}f'(aK[1]^m + bK[2]^n)}{am + bnf\left(aK[1]^m + bK[2]^n\right)} - \frac{ab^2mn^2f(aK[1]^m + bK[2]^n)\,K[1]^{m-1}K[2]^{n-1}f'(aK[1]}{\left(am + bnf\left(aK[1]^m + bK[2]^n\right)\right)^2}\right.$$
$$\left.+\int_1^x\frac{amf(aK[1]^m + by(x)^n)\,K[1]^{m-1}}{am + bnf\left(aK[1]^m + by(x)^n\right)}dK[1] = c_1, y(x)\right]$$

5.18 problem 134

Internal problem ID [2882]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 134.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' - af(y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x) = a*f(y(x)),y(x), singsol=all)
```

$$x - \left(\int^{y(x)} \frac{1}{af(\_a)} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.273 (sec). Leaf size: 35

```
DSolve[y'[x]==a f[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{f(K[1])} dK[1]\&\right][ax + c_1]$$
$$y(x) \to f^{(-1)}(0)$$

5.19 problem 135

Internal problem ID [2883]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 135.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y' - f(a+bx+cy) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 39

```
dsolve(diff(y(x),x) = f(a+b*x+c*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{RootOf}\left(\left(\int^{\_Z} \frac{1}{f(\_ac+a)c+b} d\_a\right) c - x + c_1\right) c - xb}{c}$$

✓ Solution by Mathematica

Time used: 0.209 (sec). Leaf size: 262

```
DSolve[y'[x]==f[a+b x +c y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} - \frac{f(a+bx+cK[2]) \int_1^x \left(\frac{c^2 f'(a+bK[1]+cK[2])}{b+cf(a+bK[1]+cK[2])} - \frac{c^3 f(a+bK[1]+cK[2]) f'(a+bK[1]+cK[2])}{(b+cf(a+bK[1]+cK[2]))^2}\right) dK[1]c + c + b\int_1^x \left(\frac{c^2 f'(a+bK[}{b+cf(a+bK[}\right.}{b+cf(a+bx+cK[2])} + \int_1^x \frac{cf(a+bK[1]+cy(x))}{b+cf(a+bK[1]+cy(x))} dK[1] = c_1, y(x)\right]$$

5.20 problem 136

Internal problem ID [2884]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 136.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y' - f(x)\,g(y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x) = f(x)*g(y(x)),y(x), singsol=all)
```

$$\int f(x)\,dx - \left(\int^{y(x)} \frac{1}{g(\_a)} d\_a\right) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.276 (sec). Leaf size: 42

```
DSolve[y'[x]==f[x] g[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{1}{g(K[1])} dK[1]\&\right]\left[\int_1^x f(K[2])dK[2] + c_1\right]$$
$$y(x) \to g^{(-1)}(0)$$

5.21 problem 137

Internal problem ID [2885]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 137.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' - \sec(x)^2 - y\sec(x)\operatorname{Csx}(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x) = sec(x)^2+y(x)*sec(x)*Csx(x),y(x), singsol=all)
```

$$y(x) = \left(\int \sec(x)^2 \mathrm{e}^{-\left(\int \operatorname{Csx}(x)\sec(x)dx\right)}dx + c_1\right)\mathrm{e}^{\int \operatorname{Csx}(x)\sec(x)dx}$$

✓ Solution by Mathematica

Time used: 0.127 (sec). Leaf size: 57

```
DSolve[y'[x]==Sec[x]^2+y[x] Sec[x]Csx[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \exp\left(\int_1^x \operatorname{Csx}(K[1])\sec(K[1])dK[1]\right)\left(\int_1^x \exp\left(-\int_1^{K[2]} \operatorname{Csx}(K[1])\sec(K[1])dK[1]\right)\sec^2(K[2])dK[2] + c_1\right)$$

5.22 problem 139

Internal problem ID [2886]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 139.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$2y' - 2\sin(y)^2 \tan(y) + x\sin(2y) = 0$$

✗ Solution by Maple

```
dsolve(2*diff(y(x),x) = 2*sin(y(x))^2*tan(y(x))-x*sin(2*y(x)),y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 60.356 (sec). Leaf size: 61

```
DSolve[2 y'[x]==2 Sin[y[x]]^2 Tan[y[x]]- x Sin[2 y[x]],y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\cot^{-1}\left(\sqrt{e^{x^2}\left(-\sqrt{\pi}\text{erf}(x) + 4c_1\right)}\right)$$
$$y(x) \to \cot^{-1}\left(\sqrt{e^{x^2}\left(-\sqrt{\pi}\text{erf}(x) + 4c_1\right)}\right)$$

5.23 problem 140

Internal problem ID [2887]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 140.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$2y' + ax - \sqrt{a^2x^2 - 4b\,x^2 - 4cy} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 269

```
dsolve(2*diff(y(x),x)+a*x = sqrt(a^2*x^2-4*b*x^2-4*c*y(x)),y(x), singsol=all)
```

$$\int_{\_b}^{x} -\frac{-a\_a + \sqrt{\_a^2a^2 - 4\_a^2b - 4cy(x)}}{-a\_a^2 + \_a\sqrt{\_a^2a^2 - 4\_a^2b - 4cy(x)} - 4y(x)}d\_a$$
$$+ \int^{y(x)} \left(\frac{2}{-a\,x^2 + x\sqrt{a^2x^2 - 4b\,x^2 - 4\_fc} - 4\_f} \right.$$
$$- \left(\int_{\_b}^{x} \left(\frac{2c}{\sqrt{\_a^2a^2 - 4\_a^2b - 4\_fc}\left(-a\_a^2 + \_a\sqrt{\_a^2a^2 - 4\_a^2b - 4\_fc} - 4\_f\right)} + \frac{\left(-a\_a + \sqrt{\_a^2a^2 - 4\right.}{\left(-a\_a^2 + \_\right.} \right.\right.$$
$$+ c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.564 (sec). Leaf size: 542

```
DSolve[2 y'[x]+a x==Sqrt[a^2 x^2-4 b x^2 -4 c y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\Bigg[\text{RootSum}\Bigg[\#1^4+2\#1^3c-2\#1^2a^2-4\#1^2ac+8\#1^2b+2\#1a^2c-8\#1bc+a^4-8a^2b+16b^2\&, \frac{\#1^3\log\left(\#1x-\sqrt{x^2\left(a^2-4b\right)-4cy(x)}+2\sqrt{-cy(x)}\right)+\#1^3(-\log(x))+\#1^2c\log\left(\#1x-\sqrt{x^2}\right.}{}$$

$$-\log\left(\sqrt{-cy(x)}\sqrt{x^2\left(a^2-4b\right)-4cy(x)}+2cy(x)\right)+\frac{1}{2}\log(y(x))+2\log(x)=c_1,y(x)\Bigg]$$

5.24 problem 141

Internal problem ID [2888]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 141.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$3y' - x - \sqrt{x^2 - 3y} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 234

```
dsolve(3*diff(y(x),x) = x+sqrt(x^2-3*y(x)),y(x), singsol=all)
```

$$\frac{2\sqrt{x^2-3y(x)}\,x^2}{(-x^2+4y(x))\,y(x)^2\left(x+\sqrt{x^2-3y(x)}\right)^2\left(-x+2\sqrt{x^2-3y(x)}\right)} - \frac{6\sqrt{x^2-3y(x)}}{(-x^2+4y(x))\,y(x)\left(x+\sqrt{x^2-3y(x)}\right)^2\left(-x+2\sqrt{x^2-3y(x)}\right)} - \frac{2x^3}{(-x^2+4y(x))\,y(x)^2\left(x+\sqrt{x^2-3y(x)}\right)^2\left(-x+2\sqrt{x^2-3y(x)}\right)} + \frac{9x}{(-x^2+4y(x))\,y(x)\left(x+\sqrt{x^2-3y(x)}\right)^2\left(-x+2\sqrt{x^2-3y(x)}\right)} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.157 (sec). Leaf size: 498

```
DSolve[3 y'[x]==x+Sqrt[x^2-3 y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}\left(x^2+\frac{x(x^3+216e^{3c_1})}{\sqrt[3]{x^6-540e^{3c_1}x^3+24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3}-5832e^{6c_1}}}+\sqrt[3]{x^6-540e^{3c_1}x^3+24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3}-5832e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{24}\left(2x^2+\frac{\left(-1-i\sqrt{3}\right)x(x^3+216e^{3c_1})}{\sqrt[3]{x^6-540e^{3c_1}x^3+24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3}-5832e^{6c_1}}}+i\left(\sqrt{3}+i\right)\sqrt[3]{x^6-540e^{3c_1}x^3+24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3}-5832e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{24}\left(2x^2+\frac{i\left(\sqrt{3}+i\right)x(x^3+216e^{3c_1})}{\sqrt[3]{x^6-540e^{3c_1}x^3+24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3}-5832e^{6c_1}}}-\left(1+i\sqrt{3}\right)\sqrt[3]{x^6-540e^{3c_1}x^3+24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3}-5832e^{6c_1}}\right)$$

5.25 problem 142

Internal problem ID [2889]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 142.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$y'x - \sqrt{a^2 - x^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 56

```
dsolve(x*diff(y(x),x) = sqrt(a^2-x^2),y(x), singsol=all)
```

$$y(x) = \sqrt{a^2 - x^2} - \frac{a^2 \ln\left(\frac{2a^2 + 2\sqrt{a^2}\,\sqrt{a^2 - x^2}}{x}\right)}{\sqrt{a^2}} + c_1$$

✓ Solution by Mathematica

Time used: 0.03 (sec). Leaf size: 40

```
DSolve[x y'[x]==Sqrt[a^2-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{a^2 - x^2} - a \coth^{-1}\left(\frac{a}{\sqrt{a^2 - x^2}}\right) + c_1$$

5.26 problem 143

Internal problem ID [2890]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 143.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x)+x+y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} + \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 17

```
DSolve[x y'[x]+x + y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{2} + \frac{c_1}{x}$$

5.27 problem 144

Internal problem ID [2891]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 144.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y'x + x^2 - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 11

```
dsolve(x*diff(y(x),x)+x^2-y(x) = 0,y(x), singsol=all)
```

$$y(x) = (c_1 - x)\,x$$

✓ Solution by Mathematica

Time used: 0.025 (sec). Leaf size: 13

```
DSolve[x y'[x]+x^2-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(-x + c_1)$$

5.28 problem 145

Internal problem ID [2892]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 145.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + y - x^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x) = x^3-y(x),y(x), singsol=all)
```

$$y(x) = \frac{\frac{x^4}{4} + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 19

```
DSolve[x y'[x]==x^3-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{4} + \frac{c_1}{x}$$

5.29 problem 146

Internal problem ID [2893]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 5
Problem number: 146.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - 1 - x^3 - y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x) = 1+x^3+y(x),y(x), singsol=all)
```

$$y(x) = \frac{1}{2}x^3 - 1 + c_1 x$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 18

```
DSolve[x y'[x]==1+x^3+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3}{2} + c_1 x - 1$$

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6.1 problem 147

Internal problem ID [2894]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 147.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - x^m - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x) = x^m+y(x),y(x), singsol=all)
```

$$y(x) = \frac{x^m}{m-1} + c_1 x$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 19

```
DSolve[x y'[x]==x^m+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^m}{m-1} + c_1 x$$

6.2 problem 148

Internal problem ID [2895]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 148.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - \sin(x)\, x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x) = x*sin(x)-y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sin(x) - x\cos(x) + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 19

```
DSolve[x y'[x]==x Sin[x]-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(x) - x\cos(x) + c_1}{x}$$

6.3 problem 149

Internal problem ID [2896]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 149.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - x^2 \sin(x) - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x) = x^2*sin(x)+y(x),y(x), singsol=all)
```

$$y(x) = (-\cos(x) + c_1)\, x$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 14

```
DSolve[x y'[x]==x^2 Sin[x]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x(-\cos(x) + c_1)$$

6.4 problem 150

Internal problem ID [2897]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 150.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - x^n \ln(x) + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(x*diff(y(x),x) = x^n*ln(x)-y(x),y(x), singsol=all)
```

$$y(x) = \frac{x^n \ln(x)}{n+1} - \frac{x^n}{n^2+2n+1} + \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.077 (sec). Leaf size: 29

```
DSolve[x y'[x]==x^n Log[x]-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^n((n+1)\log(x)-1)}{(n+1)^2} + \frac{c_1}{x}$$

6.5 problem 151

Internal problem ID [2898]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 151.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - \sin(x) + 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x) = sin(x)-2*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sin(x) - x\cos(x) + c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 19

```
DSolve[x y'[x]==Sin[x]-2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(x) - x\cos(x) + c_1}{x^2}$$

6.6 problem 152

Internal problem ID [2899]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 152.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - ay = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(x*diff(y(x),x) = a*y(x),y(x), singsol=all)
```

$$y(x) = c_1 x^a$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 16

```
DSolve[x y'[x]==a y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^a$$
$$y(x) \to 0$$

6.7 problem 153

Internal problem ID [2900]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 153.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - 1 - x - ay = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(x*diff(y(x),x) = 1+x+a*y(x),y(x), singsol=all)
```

$$y(x) = \left(-\frac{x^{-a}(ax + a - 1)}{a\,(a-1)} + c_1\right) x^a$$

✓ Solution by Mathematica

Time used: 0.079 (sec). Leaf size: 25

```
DSolve[x y'[x]==1+x+a y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^a - \frac{x}{a-1} - \frac{1}{a}$$

6.8 problem 154

Internal problem ID [2901]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 154.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - ax - yb = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*diff(y(x),x) = a*x+b*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{ax}{-1+b} + x^b c_1$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 22

```
DSolve[x y'[x]==a x + b y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{ax}{1-b} + c_1 x^b$$

6.9 problem 155

Internal problem ID [2902]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 155.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - x^2a - yb = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x) = a*x^2+b*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{a\,x^2}{b-2} + x^b c_1$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 24

```
DSolve[x y'[x]==a x^2+b y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{ax^2}{2-b} + c_1 x^b$$

6.10 problem 156

Internal problem ID [2903]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 156.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - a - x^n b - cy = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(x*diff(y(x),x) = a+b*x^n+c*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{a}{c} + \frac{b\,x^n}{n-c} + x^c c_1$$

✓ Solution by Mathematica

Time used: 0.109 (sec). Leaf size: 31

```
DSolve[x y'[x]==a+b x^n+c y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a}{c} - \frac{bx^n}{c-n} + c_1 x^c$$

6.11 problem 157

Internal problem ID [2904]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 157.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + 2 + (-x + 3)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x)+2+(3-x)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{2}{x} + \frac{4}{x^2} + \frac{4}{x^3} + \frac{e^x c_1}{x^3}$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 23

```
DSolve[x y'[x]+2+(3-x)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x(x+2) + c_1 e^x + 4}{x^3}$$

6.12 problem 158

Internal problem ID [2905]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 158.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + x + (ax + 2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(x*diff(y(x),x)+x+(a*x+2)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{a} + \frac{2}{a^2 x} - \frac{2}{a^3 x^2} + \frac{e^{-ax} c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 34

```
DSolve[x y'[x]+x+(2+a x)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\frac{ax(2-ax)-2}{a^3} + c_1 e^{-ax}}{x^2}$$

6.13 problem 159

Internal problem ID [2906]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 159.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x + y(bx + a) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)+(b*x+a)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-xb} x^{-a}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 24

```
DSolve[x y'[x]+(a+ b x)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^{-a} e^{-bx}$$
$$y(x) \to 0$$

6.14 problem 160

Internal problem ID [2907]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 160.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - x^3 - (-2x^2 + 1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x) = x^3+(-2*x^2+1)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{x}{2} + x\,\mathrm{e}^{-x^2} c_1$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 21

```
DSolve[x y'[x]==x^3+(1-2 x^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(\frac{1}{2} + c_1 e^{-x^2}\right)$$

6.15 problem 161

Internal problem ID [2908]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 161.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x - ax + \left(-b\,x^2 + 1\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x*diff(y(x),x) = a*x-(-b*x^2+1)*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{a}{xb} + \frac{e^{\frac{b\,x^2}{2}} c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 30

```
DSolve[x y'[x]==a x-(1-b x^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-a + bc_1 e^{\frac{bx^2}{2}}}{bx}$$

6.16 problem 162

Internal problem ID [2909]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 162.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x + x + \left(-x^2 a + 2\right) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve(x*diff(y(x),x)+x+(-a*x^2+2)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{\mathrm{e}^{-\frac{a x^2}{2}} x}{a} - \frac{\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\frac{\sqrt{2}\,\sqrt{a}\,x}{2}\right)}{2a^{\frac{3}{2}}} + c_1\right) \mathrm{e}^{\frac{a x^2}{2}}}{x^2}$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 69

```
DSolve[x y'[x]+x+(2-a x^2)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2\sqrt{a}x + e^{\frac{ax^2}{2}}\left(-\sqrt{2\pi}\mathrm{erf}\left(\frac{\sqrt{a}x}{\sqrt{2}}\right) + 2a^{3/2}c_1\right)}{2a^{3/2}x^2}$$

6.17 problem 163

Internal problem ID [2910]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 163.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x + x^2 + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve(x*diff(y(x),x)+x^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{c_1 x \operatorname{BesselY}(1, x)}{c_1 \operatorname{BesselY}(0, x) + \operatorname{BesselJ}(0, x)} - \frac{\operatorname{BesselJ}(1, x)\, x}{c_1 \operatorname{BesselY}(0, x) + \operatorname{BesselJ}(0, x)}$$

✓ Solution by Mathematica

Time used: 0.139 (sec). Leaf size: 45

```
DSolve[x y'[x]+x^2+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x(Y_1(x) + c_1 \operatorname{BesselJ}(1, x))}{Y_0(x) + c_1 \operatorname{BesselJ}(0, x)}$$

$$y(x) \to -\frac{x \operatorname{BesselJ}(1, x)}{\operatorname{BesselJ}(0, x)}$$

6.18 problem 164

Internal problem ID [2911]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 164.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$\boxed{y'x - x^2 - y(1+y) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 10

```
dsolve(x*diff(y(x),x) = x^2+y(x)*(1+y(x)),y(x), singsol=all)
```

$$y(x) = \tan(x + c_1)\, x$$

✓ Solution by Mathematica

Time used: 0.176 (sec). Leaf size: 12

```
DSolve[x y'[x]==x^2+y[x](1+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \tan(x + c_1)$$

6.19 problem 165

Internal problem ID [2912]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 165.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$y'x - y + y^2 - x^{\frac{2}{3}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 85

```
dsolve(x*diff(y(x),x)-y(x)+y(x)^2 = x^(2/3),y(x), singsol=all)
```

$$y(x) = -\frac{\left(\left(-|3x^{\frac{1}{3}} - 1|c_1 - \operatorname{abs}\left(1, 3x^{\frac{1}{3}} - 1\right)c_1\right)\mathrm{e}^{3x^{\frac{1}{3}}} + 3\,\mathrm{e}^{-3x^{\frac{1}{3}}}x^{\frac{1}{3}}\right)x^{\frac{1}{3}}}{c_1\mathrm{e}^{3x^{\frac{1}{3}}}|3x^{\frac{1}{3}} - 1| + \left(3x^{\frac{1}{3}} + 1\right)\mathrm{e}^{-3x^{\frac{1}{3}}}}$$

✓ Solution by Mathematica

Time used: 0.2 (sec). Leaf size: 103

```
DSolve[x y'[x]-y[x]+y[x]^2==x^(2/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3x^{2/3}}{3ic_1\sqrt[3]{x} + \frac{3(1+c_1{}^2)\sqrt[3]{x}\cosh\left(3\sqrt[3]{x}\right)}{\sinh\left(3\sqrt[3]{x}\right) + ic_1\cosh\left(3\sqrt[3]{x}\right)} - 1}$$

$$y(x) \to \frac{3x^{2/3}}{3\sqrt[3]{x}\tanh\left(3\sqrt[3]{x}\right) - 1}$$

6.20 problem 166

Internal problem ID [2913]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 166.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - a - by^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve(x*diff(y(x),x) = a+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\sqrt{ab}\,(\ln(x) + c_1)\right)\sqrt{ab}}{b}$$

✓ Solution by Mathematica

Time used: 9.445 (sec). Leaf size: 69

```
DSolve[x y'[x]==a+b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}\tan\left(\sqrt{a}\sqrt{b}(\log(x) + c_1)\right)}{\sqrt{b}}$$

$$y(x) \to -\frac{i\sqrt{a}}{\sqrt{b}}$$

$$y(x) \to \frac{i\sqrt{a}}{\sqrt{b}}$$

6.21 problem 167

Internal problem ID [2914]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 167.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$\boxed{y'x - x^2a - y - by^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(x*diff(y(x),x) = a*x^2+y(x)+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(c_1\sqrt{ab} + x\sqrt{ab}\right) x\sqrt{ab}}{b}$$

✓ Solution by Mathematica

Time used: 15.351 (sec). Leaf size: 33

```
DSolve[x y'[x]==a x^2+y[x]+b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}x\tan\left(\sqrt{a}\sqrt{b}(x + c_1)\right)}{\sqrt{b}}$$

6.22 problem 168

Internal problem ID [2915]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 168.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - a\,x^{2n} - (n + yb)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 38

```
dsolve(x*diff(y(x),x) = a*x^(2*n)+(n+b*y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\frac{x^n\sqrt{a}\,\sqrt{b}-c_1 n}{n}\right)\sqrt{a}\,x^n}{\sqrt{b}}$$

✓ Solution by Mathematica

Time used: 0.319 (sec). Leaf size: 139

```
DSolve[x y'[x]==a x^(2 n)+(n+b y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}x^n\left(-\cos\left(\frac{\sqrt{a}\sqrt{b}x^n}{n}\right)+c_1\sin\left(\frac{\sqrt{a}\sqrt{b}x^n}{n}\right)\right)}{\sqrt{b}\left(\sin\left(\frac{\sqrt{a}\sqrt{b}x^n}{n}\right)+c_1\cos\left(\frac{\sqrt{a}\sqrt{b}x^n}{n}\right)\right)}$$

$$y(x) \to \frac{\sqrt{a}x^n\tan\left(\frac{\sqrt{a}\sqrt{b}x^n}{n}\right)}{\sqrt{b}}$$

6.23 problem 169

Internal problem ID [2916]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 169.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x - x^n a - yb - cy^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 225

```
dsolve(x*diff(y(x),x) = a*x^n+b*y(x)+c*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x^{\frac{n}{2}}\sqrt{ac}\,c_1 \operatorname{BesselY}\left(\frac{b+n}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)}{c\left(\operatorname{BesselY}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)c_1 + \operatorname{BesselJ}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)\right)} + \frac{\operatorname{BesselJ}\left(\frac{b+n}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)\sqrt{ac}\,x^{\frac{n}{2}} - \operatorname{BesselY}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)c_1 b - b\operatorname{BesselJ}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)}{c\left(\operatorname{BesselY}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)c_1 + \operatorname{BesselJ}\left(\frac{b}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.298 (sec). Leaf size: 205

```
DSolve[x y'[x]==a x^n+b y[x]+c y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}x^{n/2}\left(-\operatorname{BesselJ}\left(\frac{b}{n}-1, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right) + c_1 \operatorname{BesselJ}\left(1-\frac{b}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right)\right)}{\sqrt{c}\left(\operatorname{BesselJ}\left(\frac{b}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right) + c_1 \operatorname{BesselJ}\left(-\frac{b}{n}, \frac{2\sqrt{a}\sqrt{c}x^{n/2}}{n}\right)\right)}$$

$$y(x) \to \frac{ax^n\, {}_0\tilde{F}_1\left(;2-\frac{b}{n};-\frac{acx^n}{n^2}\right)}{n\, {}_0\tilde{F}_1\left(;1-\frac{b}{n};-\frac{acx^n}{n^2}\right)}$$

6.24 problem 170

Internal problem ID [2917]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 170.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$y'x - k - x^n a - yb - cy^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 260

```
dsolve(x*diff(y(x),x) = k+a*x^n+b*y(x)+c*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(-\sqrt{b^2-4ck}\,c_1 - c_1 b\right) \text{BesselY}\left(\frac{\sqrt{b^2-4ck}}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right) + 2x^{\frac{n}{2}}\, \text{BesselY}\left(\frac{\sqrt{b^2-4ck}+n}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)\sqrt{ac}\,c_1 + \left(-\sqrt{b^2-\right.}{2c\left(\text{BesselY}\left(\frac{\sqrt{b^2-4ck}}{n}, \frac{2\sqrt{ac}\,x^{\frac{n}{2}}}{n}\right)c_1 + \text{BesselJ}\left(\right.\right.}$$

✓ Solution by Mathematica

Time used: 0.63 (sec). Leaf size: 602

```
DSolve[x y'[x]==k +a x^n+b y[x]+c y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-b\sqrt{x^n}\,\text{Gamma}\left(\frac{n+\sqrt{b^2-4ck}}{n}\right)\text{BesselJ}\left(\frac{\sqrt{b^2-4ck}}{n}, \frac{2\sqrt{a}\sqrt{c}\sqrt{x^n}}{n}\right) + \sqrt{a}\sqrt{c}x^n\left(\text{Gamma}\left(\frac{n+\sqrt{b^2-4ck}}{n}\right)\left(\text{BesselJ}\left(\right.\right.\right.}{2}$$

$$y(x) \to -\frac{\frac{2n\,{}_0\tilde{F}_1\left(;-\frac{\sqrt{b^2-4ck}}{n};-\frac{acx^n}{n^2}\right)}{{}_0\tilde{F}_1\left(;1-\frac{\sqrt{b^2-4ck}}{n};-\frac{acx^n}{n^2}\right)} + \sqrt{b^2-4ck} + b}{2c}$$

6.25 problem 171

Internal problem ID [2918]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 171.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Riccati, _special]]`

$$\boxed{y'x + a + xy^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(x*diff(y(x),x)+a+x*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{a}\left(\text{BesselJ}\left(0, 2\sqrt{a}\,\sqrt{x}\right) c_1 + \text{BesselY}\left(0, 2\sqrt{a}\,\sqrt{x}\right)\right)}{\sqrt{x}\,\left(c_1\,\text{BesselJ}\left(1, 2\sqrt{a}\,\sqrt{x}\right) + \text{BesselY}\left(1, 2\sqrt{a}\,\sqrt{x}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.21 (sec). Leaf size: 102

```
DSolve[x y'[x]+a+x y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}\left(c_1\,{}_0\tilde{F}_1(;1;-ax) + 2iY_0\left(2\sqrt{a}\sqrt{x}\right)\right)}{\sqrt{a}c_1x\,{}_0\tilde{F}_1(;2;-ax) + 2i\sqrt{x}Y_1\left(2\sqrt{a}\sqrt{x}\right)}$$

$$y(x) \to \frac{{}_0\tilde{F}_1(;1;-ax)}{x\,{}_0\tilde{F}_1(;2;-ax)}$$

6.26 problem 172

Internal problem ID [2919]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 172.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y'x + (-yx + 1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x)+(1-x*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{(\ln (x) - c_1)\, x}$$

✓ Solution by Mathematica

Time used: 0.126 (sec). Leaf size: 22

```
DSolve[x y'[x]+(1-x y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{-x \log(x) + c_1 x}$$

$$y(x) \to 0$$

6.27 problem 173

Internal problem ID [2920]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 173.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Bernoulli]`

$$\boxed{y'x - (-yx + 1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x) = (1-x*y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{2x}{x^2 + 2c_1}$$

✓ Solution by Mathematica

Time used: 0.136 (sec). Leaf size: 23

```
DSolve[x y'[x]==(1-x y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x}{x^2 + 2c_1}$$
$$y(x) \to 0$$

6.28 problem 174

Internal problem ID [2921]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 174.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{y'x - (1 + yx)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(x*diff(y(x),x) = (1+x*y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{2x}{-x^2 + 2c_1}$$

✓ Solution by Mathematica

Time used: 0.122 (sec). Leaf size: 23

```
DSolve[x y'[x]==(1+x y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2x}{x^2 - 2c_1}$$
$$y(x) \to 0$$

6.29 problem 175

Internal problem ID [2922]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 6
Problem number: 175.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y'x - a\,x^3(-yx+1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 126

```
dsolve(x*diff(y(x),x) = a*x^3*(1-x*y(x))*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{9\Gamma\left(\frac{2}{3}\right)\left(-a\,x^3\right)^{\frac{1}{3}}\left(-9a\,x^3\right)^{\frac{1}{3}}}{2\,\mathrm{e}^{-\frac{a\,x^3}{3}}3^{\frac{5}{6}}x\pi\left(-9a\,x^3\right)^{\frac{1}{3}} - 9\,\mathrm{e}^{-\frac{a\,x^3}{3}}c_1\Gamma\left(\frac{2}{3}\right)\left(-a\,x^3\right)^{\frac{1}{3}}\left(-9a\,x^3\right)^{\frac{1}{3}} - 9\,\mathrm{e}^{-\frac{a\,x^3}{3}}x\Gamma\left(\frac{1}{3}, -\frac{a\,x^3}{3}\right)\Gamma\left(\frac{2}{3}\right)\left(-a\,x^3\right)^{\frac{1}{3}} - \cdots}$$

✓ Solution by Mathematica

Time used: 0.224 (sec). Leaf size: 47

```
DSolve[x y'[x]==a x^3(1-x y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3e^{\frac{ax^3}{3}}}{-ax^4\,\mathrm{ExpIntegralE}\left(-\frac{1}{3}, -\frac{ax^3}{3}\right) + 3c_1}$$

$$y(x) \to 0$$

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7.1 problem 176

Internal problem ID [2923]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 176.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$\boxed{y'x - x^3 - y(1 + 2x^2) - xy^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x*diff(y(x),x) = x^3+(2*x^2+1)*y(x)+x*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{x(x^2 + 2c_1 + 2)}{x^2 + 2c_1}$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 28

```
DSolve[x y'[x]==x^3+(1+2 x^2)y[x]+x y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(-1 - \frac{2}{x^2 + 2c_1}\right)$$
$$y(x) \to -x$$

7.2 problem 177

Internal problem ID [2924]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 177.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{y'x - y(2yx + 1) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x) = y(x)*(1+2*x*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{x}{-x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.127 (sec). Leaf size: 22

```
DSolve[x y'[x]==y[x](1+2 x y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{-x^2 + c_1}$$
$$y(x) \to 0$$

7.3 problem 178

Internal problem ID [2925]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 178.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, ‘_with_symmetry_[F(x),G(x)]‘], _Riccat`

$$\boxed{y'x + bx + (2 + yax)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
dsolve(x*diff(y(x),x)+b*x+(2+a*x*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{-\frac{i\sqrt{a}\,\sqrt{b}\,x-1}{x} + \frac{e^{-2i\sqrt{a}\,\sqrt{b}\,x}}{c_1 - \frac{i e^{-2i\sqrt{a}\,\sqrt{b}\,x}}{2\sqrt{a}\,\sqrt{b}}}}{a}$$

✓ Solution by Mathematica

Time used: 2.953 (sec). Leaf size: 43

```
DSolve[x y'[x]+b x+(2+a x y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{ax} - \sqrt{\frac{b}{a}} \tan\left(ax\sqrt{\frac{b}{a}} - c_1\right)$$

7.4 problem 179

Internal problem ID [2926]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 179.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x + \mathrm{a0} + \mathrm{a1}\,x + (\mathrm{a2} + \mathrm{a3}\,xy)\,y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 848

```
dsolve(x*diff(y(x),x)+a0+a1*x+(a2+a3*x*y(x))*y(x) = 0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 0.429 (sec). Leaf size: 421

```
DSolve[x y'[x]+a0+a1 x+(a2+a3 x y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\left(\sqrt{\mathrm{a1}}c_1 \operatorname{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\mathrm{a3}}\mathrm{a0}}{\sqrt{\mathrm{a1}}} + \mathrm{a2}\right), \mathrm{a2}, 2i\sqrt{\mathrm{a1}}\sqrt{\mathrm{a3}}x\right) + c_1\left(\sqrt{\mathrm{a1}}\mathrm{a2} + i\mathrm{a0}\sqrt{\mathrm{a3}}\right) \operatorname{Hypergeomet}}{\sqrt{\mathrm{a3}}\left(c_1 \operatorname{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\mathrm{a3}}\mathrm{a0}}{\sqrt{\mathrm{a1}}}\right.\right.\right.}$$

$$y(x) \to \frac{\frac{\left(\mathrm{a0}\sqrt{\mathrm{a3}} - i\sqrt{\mathrm{a1}}\mathrm{a2}\right) \operatorname{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\mathrm{a3}}\mathrm{a0}}{\sqrt{\mathrm{a1}}} + \mathrm{a2} + 2\right), \mathrm{a2} + 1, 2i\sqrt{\mathrm{a1}}\sqrt{\mathrm{a3}}x\right)}{\operatorname{HypergeometricU}\left(\frac{1}{2}\left(\frac{i\sqrt{\mathrm{a3}}\mathrm{a0}}{\sqrt{\mathrm{a1}}} + \mathrm{a2}\right), \mathrm{a2}, 2i\sqrt{\mathrm{a1}}\sqrt{\mathrm{a3}}x\right)} - i\sqrt{\mathrm{a1}}}{\sqrt{\mathrm{a3}}}$$

7.5 problem 180

Internal problem ID [2927]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 180.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x + a\,x^2y^2 + 2y - b = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 104

```
dsolve(x*diff(y(x),x)+a*x^2*y(x)^2+2*y(x) = b,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-ab}\,c_1\,\text{BesselY}\left(1, \sqrt{-ab}\,x\right)}{ax\left(c_1\,\text{BesselY}\left(0, \sqrt{-ab}\,x\right) + \text{BesselJ}\left(0, \sqrt{-ab}\,x\right)\right)} - \frac{\text{BesselJ}\left(1, \sqrt{-ab}\,x\right)\sqrt{-ab}}{ax\left(c_1\,\text{BesselY}\left(0, \sqrt{-ab}\,x\right) + \text{BesselJ}\left(0, \sqrt{-ab}\,x\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.244 (sec). Leaf size: 125

```
DSolve[x y'[x]+a x^2 y[x]^2+2 y[x]==b,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{b}\left(iY_1\left(-i\sqrt{a}\sqrt{b}x\right) + c_1\,\text{BesselI}\left(1, \sqrt{a}\sqrt{b}x\right)\right)}{\sqrt{a}x\left(c_1\,{}_0\tilde{F}_1\left(;1;\frac{1}{4}abx^2\right) + Y_0\left(-i\sqrt{a}\sqrt{b}x\right)\right)}$$

$$y(x) \to \frac{b\,{}_0\tilde{F}_1\left(;2;\frac{1}{4}abx^2\right)}{2\,{}_0\tilde{F}_1\left(;1;\frac{1}{4}abx^2\right)}$$

7.6 problem 181

Internal problem ID [2928]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 181.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$y'x + x^m + \frac{(-m+n)\,y}{2} + x^n y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(x*diff(y(x),x)+x^m+1/2*(n-m)*y(x)+x^n*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\tan\left(\frac{c_1 m + c_1 n + 2x^{\frac{n}{2}+\frac{m}{2}}}{n+m}\right) x^{-\frac{n}{2}+\frac{m}{2}}$$

✓ Solution by Mathematica

Time used: 0.619 (sec). Leaf size: 40

```
DSolve[x y'[x]+x^m+((n-m)/2) y[x]+x^n y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^{\frac{m-n}{2}} \tan\left(\frac{2x^{\frac{m+n}{2}}}{m+n} - c_1\right)$$

7.7 problem 182

Internal problem ID [2929]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 182.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y'x + (a + b\,x^n y)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(x*diff(y(x),x)+(a+b*x^n*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{a-n}{x^a c_1 a - x^a c_1 n - b\,x^n}$$

✓ Solution by Mathematica

Time used: 0.294 (sec). Leaf size: 32

```
DSolve[x y'[x]+(a+b x^n y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{-\frac{bx^n}{a-n} + c_1 x^a}$$

$$y(x) \to 0$$

7.8 problem 183

Internal problem ID [2930]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 183.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$y'x - a\,x^m + yb + c\,x^n y^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 174

```
dsolve(x*diff(y(x),x) = a*x^m-b*y(x)-c*x^n*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\left(\text{BesselY}\left(\frac{b+m}{n+m}, \frac{2\sqrt{-ac}\,x^{\frac{n}{2}+\frac{m}{2}}}{n+m}\right)c_1 + \text{BesselJ}\left(\frac{b+m}{n+m}, \frac{2\sqrt{-ac}\,x^{\frac{n}{2}+\frac{m}{2}}}{n+m}\right)\right)x^{\frac{n}{2}+\frac{m}{2}}\sqrt{-ac}\,x^{-n+1}}{\left(\text{BesselY}\left(\frac{b-n}{n+m}, \frac{2\sqrt{-ac}\,x^{\frac{n}{2}+\frac{m}{2}}}{n+m}\right)c_1 + \text{BesselJ}\left(\frac{b-n}{n+m}, \frac{2\sqrt{-ac}\,x^{\frac{n}{2}+\frac{m}{2}}}{n+m}\right)\right)cx}$$

✓ Solution by Mathematica

Time used: 0.82 (sec). Leaf size: 433

```
DSolve[x y'[x]==a x^m-b y[x]-c x^n y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(m+n)x^{-n}\left((-1)^{\frac{n}{m+n}}(m+n)^{\frac{2n}{m+n}}\left((m+n)^2\right)^{\frac{b}{m+n}}\text{Gamma}\left(\frac{-b+m+2n}{m+n}\right){}_0\tilde{F}_1\left(;\frac{n-b}{m+n};\frac{acx^{m+n}}{(m+n)^2}\right)+\frac{c_1\left((m+n)^2\right)^{\frac{m}{\cdots}}}{\cdots}\right.}{c\left((-1)^{\frac{n}{m+n}}(m+n)^{\frac{2n}{m+n}}\left((m+n)^2\right)^{\frac{b}{m+n}}{}_0F_1\left(;\frac{n-b}{m+n}+1;\frac{acx^{m+n}}{(m+n)^2}\right)+c_1\left((m+n)^2\right)^{\frac{n}{m+n}}(-1)^{\frac{b}{m+n}}(m\cdots\right.}$$

$$y(x) \to \frac{ax^m\,{}_0\tilde{F}_1\left(;\frac{b+m}{m+n}+1;\frac{acx^{m+n}}{(m+n)^2}\right)}{(m+n)\,{}_0\tilde{F}_1\left(;\frac{b+m}{m+n};\frac{acx^{m+n}}{(m+n)^2}\right)}$$

7.9 problem 184

Internal problem ID [2931]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 184.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Riccati]`

$$\boxed{y'x-2x+y-a\,x^n(x-y)^2=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(x*diff(y(x),x) = 2*x-y(x)+a*x^n*(x-y(x))^2,y(x), singsol=all)
```

$$y(x)=\frac{x^n a x+c_1x^2-n+1}{a\,x^n+c_1x}$$

✓ Solution by Mathematica

Time used: 0.89 (sec). Leaf size: 164

```
DSolve[x y'[x]==2 x -y[x]+a x^n(x-y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x^{-n}\left(2ax^{n+\sqrt{(n-1)^2}+1}+2ac_1\sqrt{(n-1)^2}x^{n+1}-\left(n+\sqrt{(n-1)^2}-1\right)x^{\sqrt{(n-1)^2}}-c_1\left(-n+\sqrt{(n-1)^2}+1\right.\right.}{2a\left(x^{\sqrt{(n-1)^2}}+c_1\sqrt{(n-1)^2}\right)}$$

$$y(x)\to\frac{x^{-n}\left(2ax^{n+1}-n+\sqrt{(n-1)^2}+1\right)}{2a}$$

7.10 problem 185

Internal problem ID [2932]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 185.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{y'x + (1 - ay\ln(x))\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)+(1-a*y(x)*ln(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{a\ln(x) + c_1x + a}$$

✓ Solution by Mathematica

Time used: 0.162 (sec). Leaf size: 22

```
DSolve[x y'[x]+(1-a y[x] Log[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{a\log(x) + a + c_1x}$$

$$y(x) \to 0$$

7.11 problem 186

Internal problem ID [2933]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 186.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Riccati]`

$$\boxed{y'x - y - (x^2 - y^2) f(x) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve(x*diff(y(x),x) = y(x)+(x^2-y(x)^2)*f(x),y(x), singsol=all)
```

$$y(x) = \tanh\left(\int f(x)\,dx + c_1\right)x$$

✓ Solution by Mathematica

Time used: 0.369 (sec). Leaf size: 36

```
DSolve[x y'[x]==y[x]+(x^2-y[x]^2)f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\tanh\left(\int_1^x -f(K[1])dK[1] + c_1\right)$$
$$y(x) \to -x$$
$$y(x) \to x$$

7.12 problem 187

Internal problem ID [2934]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 187.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - y(1 + y^2) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(x*diff(y(x),x) = y(x)*(1+y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{x}{\sqrt{-x^2 + c_1}}$$
$$y(x) = -\frac{x}{\sqrt{-x^2 + c_1}}$$

✓ Solution by Mathematica

Time used: 0.641 (sec). Leaf size: 110

```
DSolve[x y'[x]==y[x](1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ie^{c_1}x}{\sqrt{-1+e^{2c_1}x^2}}$$

$$y(x) \to \frac{ie^{c_1}x}{\sqrt{-1+e^{2c_1}x^2}}$$

$$y(x) \to 0$$

$$y(x) \to -i$$

$$y(x) \to i$$

$$y(x) \to -\frac{ix}{\sqrt{x^2}}$$

$$y(x) \to \frac{ix}{\sqrt{x^2}}$$

7.13 problem 188

Internal problem ID [2935]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 188.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y'x + y(1 - xy^2) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(x*diff(y(x),x)+(1-x*y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{c_1 x^2 + 2x}}$$
$$y(x) = -\frac{1}{\sqrt{c_1 x^2 + 2x}}$$

✓ Solution by Mathematica

Time used: 0.373 (sec). Leaf size: 40

```
DSolve[x y'[x]+(1-x y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{x(2 + c_1 x)}}$$
$$y(x) \to \frac{1}{\sqrt{x(2 + c_1 x)}}$$
$$y(x) \to 0$$

7.14 problem 189

Internal problem ID [2936]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 189.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$y'x + y - a(x^2 + 1)\, y^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(x*diff(y(x),x)+y(x) = a*(x^2+1)*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-2x^2 \ln(x)\, a + c_1 x^2 + a}}$$

$$y(x) = -\frac{1}{\sqrt{-2x^2 \ln(x)\, a + c_1 x^2 + a}}$$

✓ Solution by Mathematica

Time used: 0.508 (sec). Leaf size: 56

```
DSolve[x y'[x]+y[x]==a(1+x^2)y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{-2ax^2 \log(x) + a + c_1 x^2}}$$

$$y(x) \to \frac{1}{\sqrt{-2ax^2 \log(x) + a + c_1 x^2}}$$

$$y(x) \to 0$$

7.15 problem 190

Internal problem ID [2937]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 190.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{y'x - ay - b(x^2+1)\,y^3 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 184

```
dsolve(x*diff(y(x),x) = a*y(x)+b*(x^2+1)*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-\left(ab\,x^{2+2a} + ab\,x^{2a} - c_1a^2 + b\,x^{2a} - c_1a\right) a\,x^{2a}\,(a+1)}}{ab\,x^{2+2a} + ab\,x^{2a} - c_1a^2 + b\,x^{2a} - c_1a}$$
$$y(x) = -\frac{\sqrt{-\left(ab\,x^{2+2a} + ab\,x^{2a} - c_1a^2 + b\,x^{2a} - c_1a\right) a\,x^{2a}\,(a+1)}}{ab\,x^{2+2a} + ab\,x^{2a} - c_1a^2 + b\,x^{2a} - c_1a}$$

✓ Solution by Mathematica

Time used: 3.844 (sec). Leaf size: 108

```
DSolve[x y'[x]==a y[x]+b(1+x^2)y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\sqrt{a}\sqrt{a+1}x^a}{\sqrt{bx^{2a}\left(ax^2+a+1\right) - a(a+1)c_1}}$$
$$y(x) \to \frac{i\sqrt{a}\sqrt{a+1}x^a}{\sqrt{bx^{2a}\left(ax^2+a+1\right) - a(a+1)c_1}}$$
$$y(x) \to 0$$

7.16 problem 191

Internal problem ID [2938]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 191.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y'x + 2y - a\,x^{2k} y^k = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve(x*diff(y(x),x)+2*y(x) = a*x^(2*k)*y(x)^k,y(x), singsol=all)
```

$$y(x) = 2^{\frac{1}{k-1}} \left(\frac{-ak\,x^2 + a\,x^2 + 2c_1}{x^2}\right)^{-\frac{1}{k-1}} x^{-\frac{2k}{k-1}}$$

✓ Solution by Mathematica

Time used: 15.267 (sec). Leaf size: 45

```
DSolve[x y'[x]+2 y[x]==a x^(2 k)y[x]^k,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{1}{2}ax^{2k} - \frac{1}{2}akx^{2k} + c_1x^{2k-2}\right)^{\frac{1}{1-k}}$$

7.17 problem 192

Internal problem ID [2939]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 192.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - 4y + 4\sqrt{y} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x) = 4*y(x)-4*sqrt(y(x)),y(x), singsol=all)
```

$$-c_1 x^2 + \sqrt{y(x)} - 1 = 0$$

✓ Solution by Mathematica

Time used: 0.176 (sec). Leaf size: 31

```
DSolve[x y'[x]==4(y[x]-Sqrt[y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(1 + e^{\frac{c_1}{2}} x^2\right)^2$$

$$y(x) \to 0$$

$$y(x) \to 1$$

7.18 problem 193

Internal problem ID [2940]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 193.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x + 2y - \sqrt{1 + y^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x)+2*y(x) = sqrt(1+y(x)^2),y(x), singsol=all)
```

$$\ln(x) + \int^{y(x)} -\frac{1}{-2\_a + \sqrt{\_a^2 + 1}} d\_a + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.208 (sec). Leaf size: 2509

```
DSolve[x y'[x]+2 y[x]==Sqrt[1+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

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7.19 problem 194

Internal problem ID [2941]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 194.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$y'x - y - \sqrt{x^2 + y^2} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x) = y(x)+sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$\frac{y(x)}{x^2} + \frac{\sqrt{x^2 + y(x)^2}}{x^2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.351 (sec). Leaf size: 27

```
DSolve[x y'[x]==y[x]+Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} e^{-c_1} \left(-1 + e^{2c_1} x^2\right)$$

7.20 problem 195

Internal problem ID [2942]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 195.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{y'x - y - \sqrt{x^2 - y^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(x*diff(y(x),x) = y(x)+sqrt(x^2-y(x)^2),y(x), singsol=all)
```

$$-\arctan\left(\frac{y(x)}{\sqrt{x^2 - y(x)^2}}\right) + \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.235 (sec). Leaf size: 18

```
DSolve[x y'[x]==y[x]+Sqrt[x^2-y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\cosh(i\log(x) + c_1)$$

7.21 problem 196

Internal problem ID [2943]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 196.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y'x - y - \sqrt{x^2 + y^2}\, x = 0}$$

✓ Solution by Maple

Time used: 0.437 (sec). Leaf size: 28

```
dsolve(x*diff(y(x),x) = y(x)+x*sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$\ln \left(y(x) + \sqrt{x^2 + y(x)^2}\right) - x - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.271 (sec). Leaf size: 12

```
DSolve[x y'[x]==y[x]+x Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \sinh(x + c_1)$$

7.22 problem 197

Internal problem ID [2944]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 197.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y'x - y + x(x-y)\sqrt{x^2+y^2} = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 49

```
dsolve(x*diff(y(x),x) = y(x)-x*(x-y(x))*sqrt(x^2+y(x)^2),y(x), singsol=all)
```

$$\ln\left(\frac{2x\left(\sqrt{2x^2+2y(x)^2}+y(x)+x\right)}{y(x)-x}\right)+\frac{\sqrt{2}\,x^2}{2}-\ln(x)-c_1=0$$

✓ Solution by Mathematica

Time used: 1.444 (sec). Leaf size: 71

```
DSolve[x y'[x]==y[x]-x(x-y[x])Sqrt[x^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}x\left(\sqrt{2}\tanh\left(\frac{x^2+2c_1}{2\sqrt{2}}\right)-\frac{1}{1+\sqrt{2}\tanh\left(\frac{x^2+2c_1}{2\sqrt{2}}\right)}+1\right)$$

$$y(x) \to x$$

7.23 problem 198

Internal problem ID [2945]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 198.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$y'x - y - a\sqrt{y^2 + b^2x^2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
dsolve(x*diff(y(x),x) = y(x)+a*sqrt(y(x)^2+b^2*x^2),y(x), singsol=all)
```

$$\frac{x^{-a}y(x)}{x} + \frac{x^{-a}\sqrt{y(x)^2 + b^2x^2}}{x} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.386 (sec). Leaf size: 73

```
DSolve[x y'[x]==y[x]+a Sqrt[y[x]^2+b^2 x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}be^{-c_1}\left(x^{1-a} - e^{2c_1}x^{a+1}\right)$$
$$y(x) \to \frac{1}{2}be^{-c_1}x^{1-a}\left(-1 + e^{2c_1}x^{2a}\right)$$

7.24 problem 199

Internal problem ID [2946]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 199.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y'x + (\sin(y) - 3x^2\cos(y))\cos(y) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)+(sin(y(x))-3*x^2*cos(y(x)))*cos(y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{x^3 + 2c_1}{x}\right)$$

✓ Solution by Mathematica

Time used: 1.74 (sec). Leaf size: 53

```
DSolve[x y'[x]+(Sin[y[x]]-3 x^2 Cos[y[x]]) Cos[y[x]]==0,y[x],x,IncludeSingularSolutions -> Tr
```

$$y(x) \to \arctan\left(x^2 + \frac{c_1}{2x}\right)$$
$$y(x) \to -\frac{1}{2}\pi\sqrt{\frac{1}{x^2}}x$$
$$y(x) \to \frac{1}{2}\pi\sqrt{\frac{1}{x^2}}x$$

7.25 problem 200

Internal problem ID [2947]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 200.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y'x + x - y + x\cos\left(\frac{y}{x}\right) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)+x-y(x)+x*cos(y(x)/x) = 0,y(x), singsol=all)
```

$$y(x) = -2\arctan\left(\ln\left(x\right) + c_1\right)x$$

✓ Solution by Mathematica

Time used: 0.371 (sec). Leaf size: 31

```
DSolve[x y'[x]+x -y[x]+x Cos[y[x]/x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\begin{aligned} y(x) &\to 2x\arctan(-\log(x) + c_1) \\ y(x) &\to -\pi x \\ y(x) &\to \pi x \end{aligned}$$

7.26 problem 201

Internal problem ID [2948]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 201.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$y'x + x\cos\left(\frac{y}{x}\right)^2 - y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x) = y(x)-x*cos(y(x)/x)^2,y(x), singsol=all)
```

$$y(x) = -\arctan(\ln(x) + c_1)\,x$$

✓ Solution by Mathematica

Time used: 0.421 (sec). Leaf size: 37

```
DSolve[x y'[x]==y[x]-x Cos[y[x]/x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\arctan(-\log(x) + 2c_1)$$
$$y(x) \to -\frac{\pi x}{2}$$
$$y(x) \to \frac{\pi x}{2}$$

7.27 problem 202

Internal problem ID [2949]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 202.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y'x - (-2x^2 + 1)\cot(y)^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(x*diff(y(x),x) = (-2*x^2+1)*cot(y(x))^2,y(x), singsol=all)
```

$$\frac{2x^2\cot(y(x)) + \pi\cot(y(x)) - 2\ln(x)\cot(y(x)) + 2c_1\cot(y(x)) - 2y(x)\cot(y(x)) + 2}{2\cot(y(x))} = 0$$

✓ Solution by Mathematica

Time used: 0.514 (sec). Leaf size: 55

```
DSolve[x y'[x]==(1-2 x^2)Cot[y[x]]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{1}{2}(\tan(\#1) - \arctan(\tan(\#1)))\&\right]\left[-\frac{x^2}{2} + \frac{\log(x)}{2} + c_1\right]$$
$$y(x) \to -\frac{\pi}{2}$$
$$y(x) \to \frac{\pi}{2}$$

7.28 problem 203

Internal problem ID [2950]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 203.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - y + \cot(y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x*diff(y(x),x) = y(x)-cot(y(x))^2,y(x), singsol=all)
```

$$\ln(x) + c_1 - \left(\int^{y(x)} -\frac{1}{\cot(\_a)^2 - \_a} d\_a\right) = 0$$

✓ Solution by Mathematica

Time used: 3.158 (sec). Leaf size: 49

```
DSolve[x y'[x]==y[x]-x Cot[y[x]]^2/x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\cos(2K[1]) - 1}{K[1]\cos(2K[1]) + \cos(2K[1]) - K[1] + 1} dK[1]\&\right][\log(x) + c_1]$$

7.29 problem 204

Internal problem ID [2951]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 204.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y'x + y + 2x\sec(yx) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)+y(x)+2*x*sec(x*y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\arcsin(-x^2 + c_1)}{x}$$

✓ Solution by Mathematica

Time used: 0.431 (sec). Leaf size: 19

```
DSolve[x y'[x]+y[x]+2 x Sec[x y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\arcsin(x^2 - c_1)}{x}$$

7.30 problem 205

Internal problem ID [2952]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 7
Problem number: 205.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$y'x - y + \sec\left(\frac{y}{x}\right)x = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)-y(x)+x*sec(y(x)/x) = 0,y(x), singsol=all)
```

$$y(x) = -\arcsin\left(\ln\left(x\right) + c_1\right)x$$

✓ Solution by Mathematica

Time used: 0.352 (sec). Leaf size: 15

```
DSolve[x y'[x]-y[x]+x Sec[y[x]/x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \arcsin(-\log(x) + c_1)$$

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8.1 problem 206

Internal problem ID [2953]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 206.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y'x - y - x\sec\left(\frac{y}{x}\right)^2 = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 35

```
dsolve(x*diff(y(x),x) = y(x)+x*sec(y(x)/x)^2,y(x), singsol=all)
```

$$\frac{\cos\left(\frac{y(x)}{x}\right)\sin\left(\frac{y(x)}{x}\right)x + y(x)}{2x} - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.252 (sec). Leaf size: 31

```
DSolve[x y'[x]==y[x]+x Sec[y[x]/x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{y(x)}{2x} + \frac{1}{4}\sin\left(\frac{2y(x)}{x}\right) = \log(x) + c_1, y(x)\right]$$

8.2 problem 207

Internal problem ID [2954]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 207.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [‘y=_G(x,y’)‘]

$$y'x - \sin(x - y) = 0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x) = sin(x-y(x)),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x y'[x]==Sin[x-y[x]],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

8.3 problem 208

Internal problem ID [2955]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 208.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y'x - y - x\sin\left(\frac{y}{x}\right) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 44

```
dsolve(x*diff(y(x),x) = y(x)+x*sin(y(x)/x),y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{2xc_1}{c_1^2x^2+1}, -\frac{c_1^2x^2-1}{c_1^2x^2+1}\right)x$$

✓ Solution by Mathematica

Time used: 2.702 (sec). Leaf size: 33

```
DSolve[x y'[x]==y[x]+x Sin[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x\arctan\left(e^{c_1}x\right)$$
$$y(x) \to 0$$
$$y(x) \to \pi\sqrt{x^2}$$

8.4 problem 209

Internal problem ID [2956]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 209.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x + \tan(y) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)+tan(y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{1}{c_1 x}\right)$$

✓ Solution by Mathematica

Time used: 12.792 (sec). Leaf size: 19

```
DSolve[x y'[x]+Tan[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \arcsin\left(\frac{e^{c_1}}{x}\right)$$
$$y(x) \to 0$$

8.5 problem 210

Internal problem ID [2957]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 210.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y'x + x + \tan(x + y) = 0}$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 117

```
dsolve(x*diff(y(x),x)+x+tan(x+y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{c_1}{x}, \frac{\sqrt{-c_1^2 + x^2}}{x}\right) - x$$
$$y(x) = \arctan\left(\frac{c_1}{x}, -\frac{\sqrt{-c_1^2 + x^2}}{x}\right) - x$$
$$y(x) = \arctan\left(-\frac{c_1}{x}, \frac{\sqrt{-c_1^2 + x^2}}{x}\right) - x$$
$$y(x) = \arctan\left(-\frac{c_1}{x}, -\frac{\sqrt{-c_1^2 + x^2}}{x}\right) - x$$

✓ Solution by Mathematica

Time used: 4.886 (sec). Leaf size: 16

```
DSolve[x y'[x]+x+Tan[x+y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -x + \arcsin\left(\frac{c_1}{x}\right)$$

8.6 problem 211

Internal problem ID [2958]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 211.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y'x - y + \tan\left(\frac{y}{x}\right)x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x*diff(y(x),x) = y(x)-x*tan(y(x)/x),y(x), singsol=all)
```

$$y(x) = x \arcsin\left(\frac{1}{c_1 x}\right)$$

✓ Solution by Mathematica

Time used: 12.901 (sec). Leaf size: 21

```
DSolve[x y'[x]==y[x]-x Tan[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \arcsin\left(\frac{e^{c_1}}{x}\right)$$

$$y(x) \to 0$$

8.7 problem 212

Internal problem ID [2959]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 212.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y'x - (1 + y^2)(x^2 + \arctan(y)) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x) = (1+y(x)^2)*(x^2+arctan(y(x))),y(x), singsol=all)
```

$$y(x) = \tan(c_1x + x^2)$$

✓ Solution by Mathematica

Time used: 0.292 (sec). Leaf size: 14

```
DSolve[x y'[x]==(1+y[x]^2)(x^2+ArcTan[y[x]]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(x(x + 2c_1))$$

8.8 problem 213

Internal problem ID [2960]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 213.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y'x - e^{\frac{y}{x}}x - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x) = y(x)+x*exp(y(x)/x),y(x), singsol=all)
```

$$y(x) = \ln\left(-\frac{1}{\ln(x) + c_1}\right)x$$

✓ Solution by Mathematica

Time used: 0.307 (sec). Leaf size: 18

```
DSolve[x y'[x]==y[x]+x Exp[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\log(-\log(x) - c_1)$$

8.9 problem 214

Internal problem ID [2961]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 214.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y'x - x - y - e^{\frac{y}{x}} x = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve(x*diff(y(x),x) = x+y(x)+x*exp(y(x)/x),y(x), singsol=all)
```

$$y(x) = \left(\ln\left(-\frac{x}{x\,e^{c_1} - 1}\right) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 4.524 (sec). Leaf size: 30

```
DSolve[x y'[x]==x+y[x]+x Exp[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x \log\left(-1 + \frac{1}{1 + e^{c_1} x}\right)$$
$$y(x) \to i\pi x$$

8.10 problem 215

Internal problem ID [2962]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 215.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x - y\ln(y) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 8

```
dsolve(x*diff(y(x),x) = y(x)*ln(y(x)),y(x), singsol=all)
```

$$y(x) = e^{c_1 x}$$

✓ Solution by Mathematica

Time used: 0.155 (sec). Leaf size: 18

```
DSolve[x y'[x]==y[x] Log[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{e^{c_1} x}$$
$$y(x) \to 1$$

8.11 problem 216

Internal problem ID [2963]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 216.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{y'x - (1 + \ln(x) - \ln(y))\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x) = (1+ln(x)-ln(y(x)))*y(x),y(x), singsol=all)
```

$$y(x) = x\,\mathrm{e}^{\frac{c_1}{x}}$$

✓ Solution by Mathematica

Time used: 0.189 (sec). Leaf size: 22

```
DSolve[x y'[x]==(1+Log[x]-Log[y[x]])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x e^{\frac{e^{c_1}}{x}}$$
$$y(x) \to x$$

8.12 problem 217

Internal problem ID [2964]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 217.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{y'x + (1 - \ln(x) - \ln(y))\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*diff(y(x),x)+(1-ln(x)-ln(y(x)))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{e^{c_1 x}}{x}$$

✓ Solution by Mathematica

Time used: 0.217 (sec). Leaf size: 26

```
DSolve[x y'[x]+(1-Log[x]-Log[y[x]])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{e^{-c_1} x}}{x}$$
$$y(x) \to \frac{1}{x}$$

8.13 problem 218

Internal problem ID [2965]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 218.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{y'x - y + 2\tanh\left(\frac{y}{x}\right)x = 0}$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 34

```
dsolve(x*diff(y(x),x) = y(x)-2*x*tanh(y(x)/x),y(x), singsol=all)
```

$$y(x) = \operatorname{arctanh}\left(\frac{1}{\sqrt{-c_1x^4+1}}\right)x$$
$$y(x) = -\operatorname{arctanh}\left(\frac{1}{\sqrt{-c_1x^4+1}}\right)x$$

✓ Solution by Mathematica

Time used: 11.023 (sec). Leaf size: 21

```
DSolve[x y'[x]==y[x]-2 x Tanh[y[x]/x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x\operatorname{arcsinh}\left(\frac{e^{c_1}}{x^2}\right)$$
$$y(x) \rightarrow 0$$

8.14 problem 219

Internal problem ID [2966]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 219.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y'x + ny - f(x)\, g(x^n y) = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 33

```
dsolve(x*diff(y(x),x)+n*y(x) = f(x)*g(x^n*y(x)),y(x), singsol=all)
```

$$y(x) = \mathrm{RootOf}\left(-\left(\int x^{n-1} f(x)\, dx\right) + \int^{\_Z} \frac{1}{g(\_a)} d\_a + c_1\right) x^{-n}$$

✓ Solution by Mathematica

Time used: 1.86 (sec). Leaf size: 41

```
DSolve[x y'[x]+ n y[x]==f[x] g[x^n y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\int_1^{x^n y(x)} \frac{1}{g(K[1])} dK[1] = \int_1^x f(K[2]) K[2]^{n-1} dK[2] + c_1, y(x)\right]$$

8.15 problem 220

Internal problem ID [2967]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 220.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{y'x - yf(x^m y^n) = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 39

```
dsolve(x*diff(y(x),x) = y(x)*f(x^m*y(x)^n),y(x), singsol=all)
```

$$\int_{\_b}^{y(x)} \frac{1}{(f(x^m\_a^n)\,n+m)\,\_a}d\_a - \frac{\ln(x)}{n} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.35 (sec). Leaf size: 186

```
DSolve[x y'[x]==y[x] f[x^m y[x]^n] ,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)}\left(-\frac{n}{(m+nf(x^mK[2]^n))\,K[2]}\right.\right.$$
$$\left.-\int_1^x\left(\frac{n^2K[1]^{m-1}K[2]^{n-1}f'(K[1]^mK[2]^n)}{m+nf(K[1]^mK[2]^n)} - \frac{n^3f(K[1]^mK[2]^n)\,K[1]^{m-1}K[2]^{n-1}f'(K[1]^mK[2]^n)}{(m+nf(K[1]^mK[2]^n))^2}\right)dK[1]\right)d$$
$$\left.+\int_1^x\frac{nf(K[1]^my(x)^n)}{(m+nf(K[1]^my(x)^n))\,K[1]}dK[1] = c_1, y(x)\right]$$

8.16 problem 221

Internal problem ID [2968]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 221.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x+1)\,y'-x^3(3x+4)-y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((1+x)*diff(y(x),x) = x^3*(4+3*x)+y(x),y(x), singsol=all)
```

$$y(x)=c_1(x+1)+x^4+x+1$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 18

```
DSolve[(1+x) y'[x]==x^3(4+3 x)+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x^4+(4+c_1)x+4+c_1$$

8.17 problem 222

Internal problem ID [2969]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 222.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x+1)\,y' - (x+1)^4 - 2y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((1+x)*diff(y(x),x) = (1+x)^4+2*y(x),y(x), singsol=all)
```

$$y(x) = \left(\frac{1}{2}x^2 + x + c_1\right)(x+1)^2$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 22

```
DSolve[(1+x) y'[x]==(1+x)^4+2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (x+1)^2\left(\frac{x^2}{2} + x + c_1\right)$$

8.18 problem 223

Internal problem ID [2970]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 223.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$(x+1)\,y' - \mathrm{e}^{x}(x+1)^{n+1} - ny = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve((1+x)*diff(y(x),x) = exp(x)*(1+x)^(n+1)+n*y(x),y(x), singsol=all)
```

$$y(x) = (\mathrm{e}^{x} + c_1)\,(x+1)^{n}$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 17

```
DSolve[(1+x) y'[x]==Exp[x](1+x)^(n+1)+n y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (e^{x} + c_1)\,(x+1)^{n}$$

8.19 problem 224

Internal problem ID [2971]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 224.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{(x+1)\,y' - ay - bxy^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve((1+x)*diff(y(x),x) = a*y(x)+b*x*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{(a+1)\,a}{(x+1)^{-a}\,c_1a^2 + (x+1)^{-a}\,c_1a - xba + b}$$

✓ Solution by Mathematica

Time used: 0.295 (sec). Leaf size: 38

```
DSolve[(1+x) y'[x]==a y[x]+b x y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\frac{b-abx}{a^2+a} + c_1(x+1)^{-a}}$$
$$y(x) \to 0$$

8.20 problem 225

Internal problem ID [2972]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 225.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Bernoulli]`

$$\boxed{(x+1)\,y'+y+(x+1)^4\,y^3=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 40

```
dsolve((1+x)*diff(y(x),x)+y(x)+(1+x)^4*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x)=\frac{1}{\sqrt{x^2+c_1+2x}\,(x+1)}$$
$$y(x)=-\frac{1}{\sqrt{x^2+c_1+2x}\,(x+1)}$$

✓ Solution by Mathematica

Time used: 0.472 (sec). Leaf size: 52

```
DSolve[(1+x) y'[x]+y[x]+(1+x)^4 y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{\sqrt{(x+1)^2(x(x+2)+c_1)}}$$
$$y(x)\to \frac{1}{\sqrt{(x+1)^2(x(x+2)+c_1)}}$$
$$y(x)\to 0$$

8.21 problem 226

Internal problem ID [2973]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 226.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{(x+1)\,y'-\left(1-x y^{3}\right) y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 275

```
dsolve((1+x)*diff(y(x),x) = (1-x*y(x)^3)*y(x),y(x), singsol=all)
```

$$y(x)=\frac{4^{\frac{1}{3}}\left(\left(3x^{4}+8x^{3}+6x^{2}+4c_{1}\right)^{2}\right)^{\frac{1}{3}}(x+1)}{3x^{4}+8x^{3}+6x^{2}+4c_{1}}$$

$$y(x)=\left(-\frac{4^{\frac{1}{3}}\left(\left(3x^{4}+8x^{3}+6x^{2}+4c_{1}\right)^{2}\right)^{\frac{1}{3}}}{2\left(3x^{4}+8x^{3}+6x^{2}+4c_{1}\right)}-\frac{i\sqrt{3}\,4^{\frac{1}{3}}\left(\left(3x^{4}+8x^{3}+6x^{2}+4c_{1}\right)^{2}\right)^{\frac{1}{3}}}{2\left(3x^{4}+8x^{3}+6x^{2}+4c_{1}\right)}\right)(x+1)$$

$$y(x)=\left(-\frac{4^{\frac{1}{3}}\left(\left(3x^{4}+8x^{3}+6x^{2}+4c_{1}\right)^{2}\right)^{\frac{1}{3}}}{2\left(3x^{4}+8x^{3}+6x^{2}+4c_{1}\right)}+\frac{i\sqrt{3}\,4^{\frac{1}{3}}\left(\left(3x^{4}+8x^{3}+6x^{2}+4c_{1}\right)^{2}\right)^{\frac{1}{3}}}{6x^{4}+16x^{3}+12x^{2}+8c_{1}}\right)(x+1)$$

✓ Solution by Mathematica

Time used: 0.274 (sec). Leaf size: 124

```
DSolve[(1+x) y'[x]==(1-x y[x]^3)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{(-2)^{2/3}(x+1)}{\sqrt[3]{-3x^4-8x^3-6x^2-4c_1}}$$

$$y(x) \to -\frac{2^{2/3}(x+1)}{\sqrt[3]{-3x^4-8x^3-6x^2-4c_1}}$$

$$y(x) \to \frac{\sqrt[3]{-1}2^{2/3}(x+1)}{\sqrt[3]{-3x^4-8x^3-6x^2-4c_1}}$$

$$y(x) \to 0$$

8.22 problem 227

Internal problem ID [2974]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 227.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$(x+1)\,y' - 1 - y - (x+1)\sqrt{1+y} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 160

```
dsolve((1+x)*diff(y(x),x) = 1+y(x)+(1+x)*sqrt(1+y(x)),y(x), singsol=all)
```

$$\frac{\sqrt{y(x)+1}\,x}{(-x^2-2x+y(x))\left(\sqrt{y(x)+1}-x-1\right)} + \frac{2x}{(-x^2-2x+y(x))\left(\sqrt{y(x)+1}-x-1\right)} + \frac{x^2}{(-x^2-2x+y(x))\left(\sqrt{y(x)+1}-x-1\right)} + \frac{\sqrt{y(x)+1}}{(-x^2-2x+y(x))\left(\sqrt{y(x)+1}-x-1\right)} + \frac{1}{(-x^2-2x+y(x))\left(\sqrt{y(x)+1}-x-1\right)} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.232 (sec). Leaf size: 60

```
DSolve[(1+x) y'[x]==(1+y[x])+(1+x)Sqrt[1+y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2\sqrt{y(x)+1}\arctan\left(\frac{x+1}{\sqrt{-y(x)-1}}\right)}{\sqrt{-y(x)-1}}+\log\left(y(x)-(x+1)^2+1\right)-\log(x+1)=c_1,y(x)\right]$$

8.23 problem 228

Internal problem ID [2975]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 228.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(x+a)\,y' - bx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve((a+x)*diff(y(x),x) = b*x,y(x), singsol=all)
```

$$y(x) = b(x - a\ln(x+a)) + c_1$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 19

```
DSolve[(a+x) y'[x]==b x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -ab\log(a+x) + bx + c_1$$

8.24 problem 229

Internal problem ID [2976]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 229.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x+a)\,y' - bx - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve((a+x)*diff(y(x),x) = b*x+y(x),y(x), singsol=all)
```

$$y(x) = \left(b\left(\ln\left(x+a\right) + \frac{a}{x+a}\right) + c_1\right)(x+a)$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 24

```
DSolve[(a+x) y'[x]==b x+ y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to b(a+x)\log(a+x) + ab + c_1(a+x)$$

8.25 problem 230

Internal problem ID [2977]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 230.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$(x+a)\,y'+b\,x^2+y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((a+x)*diff(y(x),x)+b*x^2+y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{-\frac{b\,x^3}{3}+c_1}{x+a}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 25

```
DSolve[(a+x) y'[x]+b x^2+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-bx^3+3c_1}{3(a+x)}$$

8.26 problem 231

Internal problem ID [2978]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 231.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x+a)\,y' - 2(x+a)^5 - 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((a+x)*diff(y(x),x) = 2*(a+x)^5+3*y(x),y(x), singsol=all)
```

$$y(x) = \left(2ax + x^2 + c_1\right)(x+a)^3$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 21

```
DSolve[(a+x) y'[x]==2(a+x)^5+3 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to (a+x)^3 \left(2ax + x^2 + c_1\right)$$

8.27 problem 232

Internal problem ID [2979]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 232.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+a)\,y'-b-cy=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((a+x)*diff(y(x),x) = b+c*y(x),y(x), singsol=all)
```

$$y(x)=-\frac{b}{c}+(x+a)^{c}\,c_1$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 30

```
DSolve[(a+x) y'[x]==(b+c y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{b}{c}+c_1(a+x)^c$$
$$y(x)\to -\frac{b}{c}$$

8.28 problem 233

Internal problem ID [2980]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 233.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x+a)\,y'-bx-cy=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve((a+x)*diff(y(x),x) = b*x+c*y(x),y(x), singsol=all)
```

$$y(x)=(x+a)^{c}\,c_1-\frac{b(cx+a)}{c\,(c-1)}$$

✓ Solution by Mathematica

Time used: 0.079 (sec). Leaf size: 32

```
DSolve[(a+x) y'[x]==b x+c y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{ab+bcx}{c-c^2}+c_1(a+x)^c$$

8.29 problem 234

Internal problem ID [2981]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 234.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+a)\,y'-y(1-ay)=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((a+x)*diff(y(x),x) = y(x)*(1-a*y(x)),y(x), singsol=all)
```

$$y(x)=\frac{x+a}{ax+c_1}$$

✓ Solution by Mathematica

Time used: 0.596 (sec). Leaf size: 31

```
DSolve[(a+x) y'[x]==y[x](1-a y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{a+\frac{e^{c_1}}{a+x}}$$

$$y(x)\to 0$$

$$y(x)\to\frac{1}{a}$$

8.30 problem 235

Internal problem ID [2982]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 235.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$(-x+a)\,y'-y-(cx+b)\,y^3=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 65

```
dsolve((a-x)*diff(y(x),x) = y(x)+(c*x+b)*y(x)^3,y(x), singsol=all)
```

$$y(x)=\frac{1}{\sqrt{c_1a^2-2ac_1x+c_1x^2+ac-2cx-b}}$$
$$y(x)=-\frac{1}{\sqrt{c_1a^2-2ac_1x+c_1x^2+ac-2cx-b}}$$

✓ Solution by Mathematica

Time used: 0.436 (sec). Leaf size: 66

```
DSolve[(a-x) y'[x]==y[x]+(b+c x)y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{\sqrt{c(a-2x)+c_1(a-x)^2-b}}$$
$$y(x)\to \frac{1}{\sqrt{c(a-2x)+c_1(a-x)^2-b}}$$
$$y(x)\to 0$$

8.31 problem 236

Internal problem ID [2983]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 236.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2y'x - 2x^3 + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(2*x*diff(y(x),x) = 2*x^3-y(x),y(x), singsol=all)
```

$$y(x) = \frac{2x^3}{7} + \frac{c_1}{\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 21

```
DSolve[2 x y'[x]==2 x^3-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^3}{7} + \frac{c_1}{\sqrt{x}}$$

8.32 problem 237

Internal problem ID [2984]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 237.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{2y'x + 1 - 4ixy - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 62

```
dsolve(2*x*diff(y(x),x)+1 = 4*I*x*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{i\,\mathrm{BesselJ}\,(1,x) + \mathrm{BesselK}\,(1,ix)\,c_1 - \mathrm{BesselK}\,(0,ix)\,c_1 + \mathrm{BesselJ}\,(0,x)}{i\,\mathrm{BesselJ}\,(1,x) + \mathrm{BesselK}\,(1,ix)\,c_1 + \mathrm{BesselK}\,(0,ix)\,c_1 - \mathrm{BesselJ}\,(0,x)}$$

✓ Solution by Mathematica

Time used: 0.364 (sec). Leaf size: 161

```
DSolve[2 x y'[x]+1==4 I x y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(1-i)c_1 e^{ix}\sqrt{x}((x-i)\,\mathrm{BesselJ}(0,x) - \mathrm{BesselJ}(1,x) + x\,\mathrm{BesselJ}(2,x)) - 4ixG_{1,2}^{2,0}\left(-2ix\,\middle|\,\begin{matrix}-1\\ -\frac{3}{2},-\frac{1}{2}\end{matrix}\right)}{G_{1,2}^{2,0}\left(-2ix\,\middle|\,\begin{matrix}1\\ -\frac{1}{2},\frac{1}{2}\end{matrix}\right) + (1+i)c_1 e^{ix}\sqrt{x}(\mathrm{BesselJ}(0,x) - i\,\mathrm{BesselJ}(1,x))}$$

$$y(x) \to -\frac{i((x-i)\,\mathrm{BesselJ}(0,x) - \mathrm{BesselJ}(1,x) + x\,\mathrm{BesselJ}(2,x))}{\mathrm{BesselJ}(0,x) - i\,\mathrm{BesselJ}(1,x)}$$

8.33 problem 238

Internal problem ID [2985]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 238.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2y'x - y(1 + y^2) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(2*x*diff(y(x),x) = y(x)*(1+y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(c_1 - x)\,x}}{c_1 - x}$$
$$y(x) = -\frac{\sqrt{(c_1 - x)\,x}}{c_1 - x}$$

✓ Solution by Mathematica

Time used: 0.511 (sec). Leaf size: 82

```
DSolve[2 x y'[x]==y[x](1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ie^{c_1}\sqrt{x}}{\sqrt{-1 + e^{2c_1}x}}$$
$$y(x) \to \frac{ie^{c_1}\sqrt{x}}{\sqrt{-1 + e^{2c_1}x}}$$
$$y(x) \to 0$$
$$y(x) \to -i$$
$$y(x) \to i$$

8.34 problem 239

Internal problem ID [2986]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 239.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2y'x + y(1 + y^2) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve(2*x*diff(y(x),x)+y(x)*(1+y(x)^2) = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{c_1 x - 1}}$$
$$y(x) = -\frac{1}{\sqrt{c_1 x - 1}}$$

✓ Solution by Mathematica

Time used: 0.398 (sec). Leaf size: 72

```
DSolve[2 x y'[x]+y[x](1+y[x]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ie^{c_1}}{\sqrt{-x + e^{2c_1}}}$$
$$y(x) \to \frac{ie^{c_1}}{\sqrt{-x + e^{2c_1}}}$$
$$y(x) \to 0$$
$$y(x) \to -i$$
$$y(x) \to i$$

8.35 problem 240

Internal problem ID [2987]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 8
Problem number: 240.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{2y'x - (1 + x - 6y^2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 54

```
dsolve(2*x*diff(y(x),x) = (1+x-6*y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(c_1 e^{-x} + 6)\, x}}{c_1 e^{-x} + 6}$$
$$y(x) = -\frac{\sqrt{(c_1 e^{-x} + 6)\, x}}{c_1 e^{-x} + 6}$$

✓ Solution by Mathematica

Time used: 0.638 (sec). Leaf size: 65

```
DSolve[2 x y'[x]==(1+x-6 y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{x/2}\sqrt{x}}{\sqrt{6e^x + c_1}}$$
$$y(x) \to \frac{e^{x/2}\sqrt{x}}{\sqrt{6e^x + c_1}}$$
$$y(x) \to 0$$

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9.1 problem 241

Internal problem ID [2988]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 241.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$2y'x + 4y + a + \sqrt{a^2 - 4b - 4cy} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(2*x*diff(y(x),x)+4*y(x)+a+sqrt(a^2-4*b-4*c*y(x)) = 0,y(x), singsol=all)
```

$$\ln(x) + \int^{y(x)} -\frac{1}{-2\_a - \frac{a}{2} - \frac{\sqrt{-4\_ac + a^2 - 4b}}{2}} d\_a + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.781 (sec). Leaf size: 177

```
DSolve[2 x y'[x]+4 y[x]+a +Sqrt[a^2-4 b- 4 c y[x]]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \text{InverseFunction}\left[\frac{1}{4}\left(\log\left(c\left(\sqrt{a^2 - 4(\#1c + b)} + 4\#1 + a\right)\right) - \frac{2c\arctan\left(\frac{c - 2\sqrt{a^2 - 4(\#1c+b)}}{\sqrt{-4a^2 - 4ac + 16b - c^2}}\right)}{\sqrt{-4a^2 - 4ac + 16b - c^2}}\right)\&\right]\left[-\frac{\log(x)}{2} + c_1\right]$$

$$y(x) \to \frac{1}{8}\left(-\sqrt{(2a + c)^2 - 16b} - 2a - c\right)$$

$$y(x) \to \frac{1}{8}\left(\sqrt{(2a + c)^2 - 16b} - 2a - c\right)$$

9.2 problem 242

Internal problem ID [2989]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 242.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(1-2x)\,y'-16-32x+6y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((1-2*x)*diff(y(x),x) = 16+32*x-6*y(x),y(x), singsol=all)
```

$$y(x)=\frac{4}{3}+8x+(-1+2x)^3\,c_1$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 22

```
DSolve[(1-2 x)y'[x]==2(8+16 x-3 y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 8x+c_1(2x-1)^3+\frac{4}{3}$$

9.3 problem 243

Internal problem ID [2990]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 243.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1+2x)\,y' - 4\,\mathrm{e}^{-y} + 2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve((1+2*x)*diff(y(x),x) = 4*exp(-y(x))-2,y(x), singsol=all)
```

$$y(x) = -\ln\left(\frac{1+2x}{-1+4x\,\mathrm{e}^{2c_1}+2\,\mathrm{e}^{2c_1}}\right) - 2c_1$$

✓ Solution by Mathematica

Time used: 0.654 (sec). Leaf size: 26

```
DSolve[(1+2 x)y'[x]==4 Exp[-y[x]]-2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \log\left(2+\frac{e^{c_1}}{2x+1}\right)$$
$$y(x) \to \log(2)$$

9.4 problem 244

Internal problem ID [2991]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 244.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$
\boxed{2(1-x)\, y' - 4x\sqrt{1-x} - y = 0}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(2*(1-x)*diff(y(x),x) = 4*x*sqrt(1-x)+y(x),y(x), singsol=all)
```

$$
y(x) = \frac{x^2}{\sqrt{1-x}} + \frac{c_1}{\sqrt{x-1}}
$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 27

```
DSolve[2(1-x)y'[x]==4 x Sqrt[1-x]+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to \frac{x^2 + \frac{c_1}{\sqrt{2}}}{\sqrt{1-x}}
$$

9.5 problem 245

Internal problem ID [2992]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 245.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Bernoulli]`

$$2(x+1)\,y'+2y+(x+1)^4\,y^3=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve(2*(1+x)*diff(y(x),x)+2*y(x)+(1+x)^4*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x)=-\frac{2}{\sqrt{2x^2+4c_1+4x}\,(x+1)}$$
$$y(x)=\frac{2}{\sqrt{2x^2+4c_1+4x}\,(x+1)}$$

✓ Solution by Mathematica

Time used: 0.517 (sec). Leaf size: 67

```
DSolve[2(1+x)y'[x]+2 y[x]+(1+x)^4 y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{\sqrt{2}}{\sqrt{(x+1)^2(x(x+2)+2c_1)}}$$
$$y(x)\to\frac{\sqrt{2}}{\sqrt{(x+1)^2(x(x+2)+2c_1)}}$$
$$y(x)\to 0$$

9.6 problem 246

Internal problem ID [2993]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 246.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{3y'x - 3x^{\frac{2}{3}} - (-3y + 1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(3*x*diff(y(x),x) = 3*x^(2/3)+(1-3*y(x))*y(x),y(x), singsol=all)
```

$$y(x) = i \tan\left(-3ix^{\frac{1}{3}} + c_1\right) x^{\frac{1}{3}}$$

✓ Solution by Mathematica

Time used: 0.176 (sec). Leaf size: 79

```
DSolve[3 x y'[x]==3 x^(2/3)+(1-3 y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{x}\left(i\cosh\left(3\sqrt[3]{x}\right) + c_1 \sinh\left(3\sqrt[3]{x}\right)\right)}{i\sinh\left(3\sqrt[3]{x}\right) + c_1 \cosh\left(3\sqrt[3]{x}\right)}$$
$$y(x) \to \sqrt[3]{x}\tanh\left(3\sqrt[3]{x}\right)$$

9.7 problem 247

Internal problem ID [2994]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 247.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$3y'x - (2 + xy^3)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 178

```
dsolve(3*x*diff(y(x),x) = (2+x*y(x)^3)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{3^{\frac{1}{3}} \left(x^2(-x^3 + 3c_1)^2\right)^{\frac{1}{3}}}{-x^3 + 3c_1}$$

$$y(x) = -\frac{3^{\frac{1}{3}} \left(x^2(-x^3 + 3c_1)^2\right)^{\frac{1}{3}}}{2\,(-x^3 + 3c_1)} - \frac{i3^{\frac{5}{6}} \left(x^2(-x^3 + 3c_1)^2\right)^{\frac{1}{3}}}{2\,(-x^3 + 3c_1)}$$

$$y(x) = -\frac{3^{\frac{1}{3}} \left(x^2(-x^3 + 3c_1)^2\right)^{\frac{1}{3}}}{2\,(-x^3 + 3c_1)} + \frac{i3^{\frac{5}{6}} \left(x^2(-x^3 + 3c_1)^2\right)^{\frac{1}{3}}}{-2x^3 + 6c_1}$$

✓ Solution by Mathematica

Time used: 0.209 (sec). Leaf size: 89

```
DSolve[3 x y'[x]==(2+x y[x]^3)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{-3}x^{2/3}}{\sqrt[3]{-x^3+3c_1}}$$

$$y(x) \to \frac{x^{2/3}}{\sqrt[3]{-\frac{x^3}{3}+c_1}}$$

$$y(x) \to \frac{(-1)^{2/3}x^{2/3}}{\sqrt[3]{-\frac{x^3}{3}+c_1}}$$

$$y(x) \to 0$$

9.8 problem 248

Internal problem ID [2995]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 248.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{3y'x - \left(1 + 3xy^3 \ln (x)\right) y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 234

```
dsolve(3*x*diff(y(x),x) = (1+3*x*y(x)^3*ln(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(-4x(6\ln (x)\, x^2 - 3x^2 - 4c_1)^2\right)^{\frac{1}{3}}}{6\ln (x)\, x^2 - 3x^2 - 4c_1}$$

$$y(x) = -\frac{\left(-4x(6\ln (x)\, x^2 - 3x^2 - 4c_1)^2\right)^{\frac{1}{3}}}{2\left(6\ln (x)\, x^2 - 3x^2 - 4c_1\right)} - \frac{i\sqrt{3}\left(-4x(6\ln (x)\, x^2 - 3x^2 - 4c_1)^2\right)^{\frac{1}{3}}}{2\left(6\ln (x)\, x^2 - 3x^2 - 4c_1\right)}$$

$$y(x) = -\frac{\left(-4x(6\ln (x)\, x^2 - 3x^2 - 4c_1)^2\right)^{\frac{1}{3}}}{2\left(6\ln (x)\, x^2 - 3x^2 - 4c_1\right)} + \frac{i\sqrt{3}\left(-4x(6\ln (x)\, x^2 - 3x^2 - 4c_1)^2\right)^{\frac{1}{3}}}{12\ln (x)\, x^2 - 6x^2 - 8c_1}$$

✓ Solution by Mathematica

Time used: 0.209 (sec). Leaf size: 120

```
DSolve[3 x y'[x]==(1+3 x y[x]^3 Log[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(-2)^{2/3}\sqrt[3]{x}}{\sqrt[3]{3x^2 - 6x^2\log(x) + 4c_1}}$$

$$y(x) \to \frac{2^{2/3}\sqrt[3]{x}}{\sqrt[3]{3x^2 - 6x^2\log(x) + 4c_1}}$$

$$y(x) \to -\frac{\sqrt[3]{-1}2^{2/3}\sqrt[3]{x}}{\sqrt[3]{3x^2 - 6x^2\log(x) + 4c_1}}$$

$$y(x) \to 0$$

9.9 problem 249

Internal problem ID [2996]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 249.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x^2 - a + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x^2*diff(y(x),x) = a-y(x),y(x), singsol=all)
```

$$y(x) = a + e^{\frac{1}{x}} c_1$$

✓ Solution by Mathematica

Time used: 0.029 (sec). Leaf size: 20

```
DSolve[x^2 y'[x]==a-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to a + c_1 e^{\frac{1}{x}}$$
$$y(x) \to a$$

9.10 problem 250

Internal problem ID [2997]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 250.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x^2 - a - bx - c\,x^2 - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x) = a+b*x+c*x^2+x*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{a}{2x} + xc\ln(x) - b + c_1x$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 26

```
DSolve[x^2 y'[x]==a+b x+c x^2+x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a}{2x} - b + cx\log(x) + c_1x$$

9.11 problem 251

Internal problem ID [2998]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 251.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' x^2 - a - bx - c\,x^2 + yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x) = a+b*x+c*x^2-x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{cx}{2} + b + \frac{\ln(x)\,a}{x} + \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 26

```
DSolve[x^2 y'[x]==a+b x+c x^2-x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a\log(x)}{x} + b + \frac{cx}{2} + \frac{c_1}{x}$$

9.12 problem 252

Internal problem ID [2999]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 252.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' x^2 + (1 - 2x) y - x^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)+(1-2*x)*y(x) = x^2,y(x), singsol=all)
```

$$y(x) = x^2 + e^{\frac{1}{x}} c_1 x^2$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 19

```
DSolve[x^2 y'[x]+(1-2 x)y[x]==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 \left(1 + c_1 e^{\frac{1}{x}}\right)$$

9.13 problem 253

Internal problem ID [3000]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 253.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x^2 - a - bxy = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x) = a+b*x*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{a}{(b+1)\,x} + x^b c_1$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 22

```
DSolve[x^2 y'[x]==a+b x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a}{bx+x} + c_1 x^b$$

9.14 problem 254

Internal problem ID [3001]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 254.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x^2 - y(bx + a) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x^2*diff(y(x),x) = (b*x+a)*y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{a}{x}} x^b$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 24

```
DSolve[x^2 y'[x]==(a+b x)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{a}{x}} x^b$$
$$y(x) \to 0$$

9.15 problem 255

Internal problem ID [3002]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 255.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x^2 + x(2+x)\,y - x\left(1 - \mathrm{e}^{-2x}\right) + 2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(x^2*diff(y(x),x)+x*(2+x)*y(x) = x*(1-exp(-2*x))-2,y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{-x} c_1}{x^2} + \frac{x\,\mathrm{e}^{-2x} + \mathrm{e}^{-2x} + x - 3}{x^2}$$

✓ Solution by Mathematica

Time used: 0.137 (sec). Leaf size: 31

```
DSolve[x^2 y'[x]+x(2+x)y[x]==x(1-Exp[-2 x])-2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-2x}(x + e^x(e^x(x-3) + c_1) + 1)}{x^2}$$

9.16 problem 256

Internal problem ID [3003]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 256.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$y'x^2 + 2x(1-x)\,y - \mathrm{e}^x(-1+2\,\mathrm{e}^x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^2*diff(y(x),x)+2*x*(1-x)*y(x) = exp(x)*(2*exp(x)-1),y(x), singsol=all)
```

$$y(x) = \frac{(2x + \mathrm{e}^{-x} + c_1)\,\mathrm{e}^{2x}}{x^2}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 24

```
DSolve[x^2 y'[x]+2 x(1-x)y[x]==Exp[x](2 Exp[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x(1 + e^x(2x + c_1))}{x^2}$$

9.17 problem 257

Internal problem ID [3004]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 257.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{y'x^2 + x^2 + yx + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x^2*diff(y(x),x)+x^2+x*y(x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x(\ln(x) + c_1 - 1)}{\ln(x) + c_1}$$

✓ Solution by Mathematica

Time used: 0.137 (sec). Leaf size: 25

```
DSolve[x^2 y'[x]+x^2+x y[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(-1 + \frac{1}{\log(x) - c_1}\right)$$
$$y(x) \to -x$$

9.18 problem 258

Internal problem ID [3005]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 258.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, _Riccati]`

$$\boxed{y'x^2 - (1 + 2x - y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.282 (sec). Leaf size: 24

```
dsolve(x^2*diff(y(x),x) = (1+2*x-y(x))^2,y(x), singsol=all)
```

$$y(x) = 1 + \frac{x(c_1x^3 - 4)}{c_1x^3 - 1}$$

✓ Solution by Mathematica

Time used: 0.233 (sec). Leaf size: 34

```
DSolve[x^2 y'[x]==(1+2 x-y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3x^4}{x^3 + 3c_1} + 4x + 1$$
$$y(x) \to 4x + 1$$

9.19 problem 259

Internal problem ID [3006]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 259.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'x^2 - a - by^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(x^2*diff(y(x),x) = a+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\frac{\sqrt{ab}\,(c_1x-1)}{x}\right)\sqrt{ab}}{b}$$

✓ Solution by Mathematica

Time used: 0.184 (sec). Leaf size: 75

```
DSolve[x^2 y'[x]==a + b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{a}\tan\left(\frac{\sqrt{a}\sqrt{b}(1-c_1x)}{x}\right)}{\sqrt{b}}$$

$$y(x) \to -\frac{i\sqrt{a}}{\sqrt{b}}$$

$$y(x) \to \frac{i\sqrt{a}}{\sqrt{b}}$$

9.20 problem 260

Internal problem ID [3007]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 260.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{y'x^2 - (x + ay)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x) = (x+a*y(x))*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{x}{a\ln(x) - c_1}$$

✓ Solution by Mathematica

Time used: 0.142 (sec). Leaf size: 22

```
DSolve[x^2 y'[x]==(x+a y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{-a\log(x) + c_1}$$
$$y(x) \to 0$$

9.21 problem 261

Internal problem ID [3008]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 261.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{y'x^2 - (ax + yb)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(x^2*diff(y(x),x) = (a*x+b*y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{x(a-1)}{c_1 x^{-a} x a - c_1 x^{-a} x - b}$$

✓ Solution by Mathematica

Time used: 0.171 (sec). Leaf size: 31

```
DSolve[x^2 y'[x]==(a x+b y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\frac{b}{x-ax} + c_1 x^{-a}}$$
$$y(x) \to 0$$

9.22 problem 262

Internal problem ID [3009]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 262.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$\boxed{y'x^2 + x^2a + bxy + cy^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 71

```
dsolve(x^2*diff(y(x),x)+a*x^2+b*x*y(x)+c*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x\left(\sqrt{4ac-b^2-2b-1}\,\tan\left(\frac{\ln(x)\sqrt{4ac-b^2-2b-1}}{2}+\frac{c_1\sqrt{4ac-b^2-2b-1}}{2}\right)+b+1\right)}{2c}$$

✓ Solution by Mathematica

Time used: 60.133 (sec). Leaf size: 62

```
DSolve[x^2 y'[x]+a x^2 +b x y[x]+c y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x\left(-\sqrt{4ac-(b+1)^2}\tan\left(\frac{1}{2}\sqrt{4ac-(b+1)^2}(-\log(x)+c_1)\right)+b+1\right)}{2c}$$

9.23 problem 263

Internal problem ID [3010]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 263.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$y'x^2 - a - x^n b - x^2 y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 275

```
dsolve(x^2*diff(y(x),x) = a+b*x^n+x^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x^{\frac{n}{2}}\sqrt{b}\, c_1 \operatorname{BesselY}\left(\frac{\sqrt{1-4a}+n}{n}, \frac{2\sqrt{b}\, x^{\frac{n}{2}}}{n}\right)}{x\left(\operatorname{BesselY}\left(\frac{\sqrt{1-4a}}{n}, \frac{2\sqrt{b}\, x^{\frac{n}{2}}}{n}\right) c_1 + \operatorname{BesselJ}\left(\frac{\sqrt{1-4a}}{n}, \frac{2\sqrt{b}\, x^{\frac{n}{2}}}{n}\right)\right)}$$
$$+ \frac{\left(-\sqrt{1-4a}\, c_1 - c_1\right) \operatorname{BesselY}\left(\frac{\sqrt{1-4a}}{n}, \frac{2\sqrt{b}\, x^{\frac{n}{2}}}{n}\right) + 2 \operatorname{BesselJ}\left(\frac{\sqrt{1-4a}+n}{n}, \frac{2\sqrt{b}\, x^{\frac{n}{2}}}{n}\right)\sqrt{b}\, x^{\frac{n}{2}} + \left(-\sqrt{1-4a} - 1\right)}{2x\left(\operatorname{BesselY}\left(\frac{\sqrt{1-4a}}{n}, \frac{2\sqrt{b}\, x^{\frac{n}{2}}}{n}\right) c_1 + \operatorname{BesselJ}\left(\frac{\sqrt{1-4a}}{n}, \frac{2\sqrt{b}\, x^{\frac{n}{2}}}{n}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.776 (sec). Leaf size: 807

```
DSolve[x^2 y'[x]==a+b x^n + x^2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{n^{\frac{2\sqrt{(1-4a)n^2}}{n^2}} b^{\frac{i\sqrt{4a-1}}{n}} (x^n)^{\frac{i\sqrt{4a-1}}{n}} \text{Gamma}\left(\frac{n+\sqrt{1-4a}}{n}\right) \left(2bx^n \left(\frac{\sqrt{b}\sqrt[3]{(x^n)^{3/2}}}{n}\right)^{\frac{\sqrt{(1-4a)n^2}}{n^2}} {}_0\tilde{F}_1\left(;\frac{\sqrt{(1-4a)n^2}}{n^2}+2;-\frac{bx}{n}\right.\right.}{2nx\left(n^{\frac{2\sqrt{}}{}}\right.}$$

$$y(x) \to \frac{\frac{2bx^n\, {}_0\tilde{F}_1\left(;2-\frac{\sqrt{(1-4a)n^2}}{n^2};-\frac{bx^n}{n^2}\right)}{n\, {}_0\tilde{F}_1\left(;1-\frac{\sqrt{(1-4a)n^2}}{n^2};-\frac{bx^n}{n^2}\right)} + i\sqrt{4a-1} - 1}{2x}$$

9.24 problem 264

Internal problem ID [3011]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 264.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{y'x^2 + 2 + xy(4 + yx) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^2*diff(y(x),x)+2+x*y(x)*(4+x*y(x)) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2c_1 - x}{x\,(c_1 - x)}$$

✓ Solution by Mathematica

Time used: 0.169 (sec). Leaf size: 26

```
DSolve[x^2 y'[x]+2 + x y[x](4+x y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{x} + \frac{1}{x + c_1}$$
$$y(x) \to -\frac{2}{x}$$

9.25 problem 265

Internal problem ID [3012]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 265.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{y'x^2 + 2 + ax(-yx + 1) - x^2y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 61

```
dsolve(x^2*diff(y(x),x)+2+a*x*(1-x*y(x))-x^2*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{(a^3x^3 - a^2x^2 + 2ax - 2)\,\mathrm{e}^{ax} - c_1}{x\left((a^2x^2 - 2ax + 2)\,\mathrm{e}^{ax} + c_1\right)}$$

✓ Solution by Mathematica

Time used: 0.317 (sec). Leaf size: 68

```
DSolve[x^2 y'[x]+2+a x(1-x y[x])-x^2 y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-e^{ax}(ax-1)\,(a^2x^2+2) + a^3c_1}{xe^{ax}(ax(ax-2)+2) + a^3c_1x}$$
$$y(x) \to \frac{1}{x}$$

9.26 problem 266

Internal problem ID [3013]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 9
Problem number: 266.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Riccati, _special]]`

$$\boxed{y'x^2 - a - b\,x^2y^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 40

```
dsolve(x^2*diff(y(x),x) = a+b*x^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{1 + \tan\left(\frac{\sqrt{4ab-1}\,(-\ln(x)+c_1)}{2}\right)\sqrt{4ab-1}}{2bx}$$

✓ Solution by Mathematica

Time used: 0.186 (sec). Leaf size: 77

```
DSolve[x^2 y'[x]==a+b x^2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-1 + \sqrt{1-4ab}\left(-1 + \frac{2c_1}{x^{\sqrt{1-4ab}}+c_1}\right)}{2bx}$$

$$y(x) \to \frac{\sqrt{1-4ab}-1}{2bx}$$

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10.1 problem 267

Internal problem ID [3014]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 267.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$y'x^2 - a - x^n b - c\,x^2y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 244

```
dsolve(x^2*diff(y(x),x) = a+b*x^n+c*x^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(-\sqrt{-4ac+1}\,c_1 - c_1\right) \mathrm{BesselY}\left(\frac{\sqrt{-4ac+1}}{n}, \frac{2\sqrt{cb}\,x^{\frac{n}{2}}}{n}\right) + 2x^{\frac{n}{2}}\,\mathrm{BesselY}\left(\frac{\sqrt{-4ac+1}+n}{n}, \frac{2\sqrt{cb}\,x^{\frac{n}{2}}}{n}\right)\sqrt{cb}\,c_1 + \left(-\sqrt{-4\right.}{2xc\left(\mathrm{BesselY}\left(\frac{\sqrt{-4ac+1}}{n}, \frac{2\sqrt{cb}\,x^{\frac{n}{2}}}{n}\right) c_1 + \mathrm{BesselJ}\right.}$$

✓ Solution by Mathematica

Time used: 0.917 (sec). Leaf size: 931

```
DSolve[x^2 y'[x]==a+b x^n+c x^2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{b^{\frac{i\sqrt{4ac-1}}{n}} c^{\frac{i\sqrt{4ac-1}}{n}} n^{\frac{2\sqrt{(1-4ac)n^2}}{n^2}} \text{Gamma}\left(\frac{n+\sqrt{1-4ac}}{n}\right) \left(2bcx^n \left(\frac{\sqrt{b}\sqrt{c}\sqrt[3]{(x^n)^{3/2}}}{n}\right)^{\frac{\sqrt{(1-4ac)n^2}}{n^2}} {}_0\tilde{F}_1\left(; \frac{\sqrt{(1-4ac)n^2}}{n^2} + 2; \right.\right.}{2cnx\sqrt{x^n}\left(b^{\frac{i\sqrt{4ac-}}{n}}\right.}$$

$$y(x) \to \frac{\frac{2bcx^n\, {}_0\tilde{F}_1\left(;2-\frac{\sqrt{(1-4ac)n^2}}{n^2};-\frac{bcx^n}{n^2}\right)}{n\, {}_0\tilde{F}_1\left(;1-\frac{\sqrt{(1-4ac)n^2}}{n^2};-\frac{bcx^n}{n^2}\right)} + i\sqrt{4ac-1} - 1}{2cx}$$

10.2 problem 268

Internal problem ID [3015]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 268.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{y'x^2 - a - bxy - c\,x^2y^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 57

```
dsolve(x^2*diff(y(x),x) = a+b*x*y(x)+c*x^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{b + 1 + \tan\left(\frac{\sqrt{4ac-b^2-2b-1}\,(-\ln(x)+c_1)}{2}\right)\sqrt{4ac - b^2 - 2b - 1}}{2cx}$$

✓ Solution by Mathematica

Time used: 0.28 (sec). Leaf size: 93

```
DSolve[x^2 y'[x]==a+b x y[x]+c x^2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{(b+1)^2 - 4ac}\left(1 - \frac{2c_1}{x^{\sqrt{(b+1)^2-4ac}} + c_1}\right) + b + 1}{2cx}$$

$$y(x) \to \frac{\sqrt{(b+1)^2 - 4ac} - b - 1}{2cx}$$

10.3 problem 269

Internal problem ID [3016]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 269.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$y'x^2 - a - bxy - c\,x^4y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 118

```
dsolve(x^2*diff(y(x),x) = a+b*x*y(x)+c*x^4*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{ac}\,c_1 \operatorname{BesselY}\left(-\frac{1}{2}-\frac{b}{2}, \sqrt{ac}\,x\right)}{x^2c\left(\operatorname{BesselY}\left(-\frac{3}{2}-\frac{b}{2}, \sqrt{ac}\,x\right)c_1 + \operatorname{BesselJ}\left(-\frac{3}{2}-\frac{b}{2}, \sqrt{ac}\,x\right)\right)} + \frac{\sqrt{ac}\operatorname{BesselJ}\left(-\frac{1}{2}-\frac{b}{2}, \sqrt{ac}\,x\right)}{x^2c\left(\operatorname{BesselY}\left(-\frac{3}{2}-\frac{b}{2}, \sqrt{ac}\,x\right)c_1 + \operatorname{BesselJ}\left(-\frac{3}{2}-\frac{b}{2}, \sqrt{ac}\,x\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.353 (sec). Leaf size: 225

```
DSolve[x^2 y'[x]==a+b x y[x]+c x^4 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}\left(\sec\left(\frac{\pi b}{2}\right)\operatorname{BesselJ}\left(\frac{1}{2}(-b-1), \sqrt{a}\sqrt{cx}\right) + \left(\tan\left(\frac{\pi b}{2}\right) - c_1\right)\operatorname{BesselJ}\left(\frac{b+1}{2}, \sqrt{a}\sqrt{cx}\right)\right)}{\sqrt{cx^2}\left(Y_{\frac{b+3}{2}}\left(\sqrt{a}\sqrt{cx}\right) + c_1\operatorname{BesselJ}\left(\frac{b+3}{2}, \sqrt{a}\sqrt{cx}\right)\right)}$$

$$y(x) \to -\frac{2\,{}_0\tilde{F}_1\left(;\frac{b+3}{2};-\frac{1}{4}acx^2\right)}{cx^3\,{}_0\tilde{F}_1\left(;\frac{b+5}{2};-\frac{1}{4}acx^2\right)}$$

$$y(x) \to -\frac{2\,{}_0\tilde{F}_1\left(;\frac{b+3}{2};-\frac{1}{4}acx^2\right)}{cx^3\,{}_0\tilde{F}_1\left(;\frac{b+5}{2};-\frac{1}{4}acx^2\right)}$$

10.4 problem 270

Internal problem ID [3017]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 270.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{y'x^2 + (x^2 + y^2 - x)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(x^2*diff(y(x),x)+(x^2+y(x)^2-x)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x}{\sqrt{c_1 e^{2x} - 1}}$$
$$y(x) = -\frac{x}{\sqrt{c_1 e^{2x} - 1}}$$

✓ Solution by Mathematica

Time used: 4.512 (sec). Leaf size: 47

```
DSolve[x^2 y'[x]+(x^2+y[x]^2-x)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{\sqrt{-1 + c_1 e^{2x}}}$$
$$y(x) \to \frac{x}{\sqrt{-1 + c_1 e^{2x}}}$$
$$y(x) \to 0$$

10.5 problem 271

Internal problem ID [3018]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 271.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y'x^2 - 2y(-y^2 + x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(x^2*diff(y(x),x) = 2*y(x)*(x-y(x)^2),y(x), singsol=all)
```

$$y(x) = -\frac{3x^2}{\sqrt{12x^3 + 9c_1}}$$
$$y(x) = \frac{3x^2}{\sqrt{12x^3 + 9c_1}}$$

✓ Solution by Mathematica

Time used: 0.212 (sec). Leaf size: 51

```
DSolve[x^2 y'[x]==2 y[x](x-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{\sqrt{\frac{4x^3}{3} + c_1}}$$
$$y(x) \to \frac{x^2}{\sqrt{\frac{4x^3}{3} + c_1}}$$
$$y(x) \to 0$$

10.6 problem 272

Internal problem ID [3019]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 272.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$y'x^2 - a\,x^2y^2 + ay^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 117

```
dsolve(x^2*diff(y(x),x) = a*x^2*y(x)^2-a*y(x)^3,y(x), singsol=all)
```

$$y(x) = -\frac{1}{ax + (-2a)^{\frac{2}{3}} \operatorname{RootOf}\left(\operatorname{AiryBi}\left(\frac{\_Z^2(-2a)^{\frac{1}{3}}x-1}{(-2a)^{\frac{1}{3}}x}\right)c_1\_Z + \_Z\operatorname{AiryAi}\left(\frac{\_Z^2(-2a)^{\frac{1}{3}}x-1}{(-2a)^{\frac{1}{3}}x}\right) + \operatorname{AiryBi}\left(1, \frac{\_Z^2}{(\cdot}\right.\right.}$$

✓ Solution by Mathematica

Time used: 0.429 (sec). Leaf size: 267

```
DSolve[x^2 y'[x]==a x^2 y[x]^2-a y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$\operatorname{Solve}\left[\frac{\left(-\frac{1}{2^{2/3}a^{2/3}y(x)} - \frac{\sqrt[3]{a}x}{2^{2/3}}\right)\operatorname{AiryAi}\left(\left(-\frac{\sqrt[3]{a}x}{2^{2/3}} - \frac{1}{2^{2/3}a^{2/3}y(x)}\right)^2 + \frac{1}{\sqrt[3]{2}\sqrt[3]{a}x}\right) + \operatorname{AiryAiPrime}\left(\left(-\frac{\sqrt[3]{a}x}{2^{2/3}} - \right.\right.}{\left(-\frac{1}{2^{2/3}a^{2/3}y(x)} - \frac{\sqrt[3]{a}x}{2^{2/3}}\right)\operatorname{AiryBi}\left(\left(-\frac{\sqrt[3]{a}x}{2^{2/3}} - \frac{1}{2^{2/3}a^{2/3}y(x)}\right)^2 + \frac{1}{\sqrt[3]{2}\sqrt[3]{a}x}\right) + \operatorname{AiryBiPrime}\left(\left(-\frac{\sqrt[3]{a}x}{2^{2/3}} - \right.\right.}\right.$$

$$\left. + c_1 = 0, y(x)\right]$$

10.7 problem 273

Internal problem ID [3020]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 273.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$y'x^2 + ay^2 + b\,x^2y^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 178

```
dsolve(x^2*diff(y(x),x)+a*y(x)^2+b*x^2*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2^{\frac{1}{3}}abx}{2^{\frac{1}{3}}a^2b - 2\left(a^2b^2\right)^{\frac{2}{3}} \operatorname{RootOf}\left(\operatorname{AiryBi}\left(-\frac{b2^{\frac{2}{3}}x - 2\_Z^2\left(a^2b^2\right)^{\frac{1}{3}}}{2\left(a^2b^2\right)^{\frac{1}{3}}}\right)c_1\_Z + \_Z\operatorname{AiryAi}\left(-\frac{b2^{\frac{2}{3}}x - 2\_Z^2\left(a^2b^2\right)^{\frac{1}{3}}}{2\left(a^2b^2\right)^{\frac{1}{3}}}\right) + A}$$

✓ Solution by Mathematica

Time used: 0.589 (sec). Leaf size: 343

```
DSolve[x^2 y'[x]+a y[x]^2+b x^2 y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\operatorname{Solve}\left[\frac{\left(\frac{a^{2/3}}{2^{2/3}\sqrt[3]{b}x} + \frac{1}{2^{2/3}\sqrt[3]{a}\sqrt[3]{b}y(x)}\right)\operatorname{AiryAi}\left(\left(\frac{a^{2/3}}{2^{2/3}\sqrt[3]{b}x} + \frac{1}{2^{2/3}\sqrt[3]{b}y(x)\sqrt[3]{a}}\right)^2 - \frac{\sqrt[3]{b}x}{\sqrt[3]{2}a^{2/3}}\right) + \operatorname{AiryAiPrime}\left(\left(\right.\right.}{\left(\frac{a^{2/3}}{2^{2/3}\sqrt[3]{b}x} + \frac{1}{2^{2/3}\sqrt[3]{a}\sqrt[3]{b}y(x)}\right)\operatorname{AiryBi}\left(\left(\frac{a^{2/3}}{2^{2/3}\sqrt[3]{b}x} + \frac{1}{2^{2/3}\sqrt[3]{b}y(x)\sqrt[3]{a}}\right)^2 - \frac{\sqrt[3]{b}x}{\sqrt[3]{2}a^{2/3}}\right) + \operatorname{AiryBiPrime}\left(\left(\right.\right.} + c_1 = 0, y(x)\right]$$

10.8 problem 274

Internal problem ID [3021]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 274.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y'x^2 - (ax + by^3)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 344

```
dsolve(x^2*diff(y(x),x) = (a*x+b*y(x)^3)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(x(3a-1)\left(3a\,x^{-3a+1}c_1 - x^{-3a+1}c_1 - 3b\right)^2\right)^{\frac{1}{3}}}{3a\,x^{-3a+1}c_1 - x^{-3a+1}c_1 - 3b}$$

$$y(x) = -\frac{\left(x(3a-1)\left(3a\,x^{-3a+1}c_1 - x^{-3a+1}c_1 - 3b\right)^2\right)^{\frac{1}{3}}}{2\left(3a\,x^{-3a+1}c_1 - x^{-3a+1}c_1 - 3b\right)} - \frac{i\sqrt{3}\left(x(3a-1)\left(3a\,x^{-3a+1}c_1 - x^{-3a+1}c_1 - 3b\right)^2\right)^{\frac{1}{3}}}{2\left(3a\,x^{-3a+1}c_1 - x^{-3a+1}c_1 - 3b\right)}$$

$$y(x) = -\frac{\left(x(3a-1)\left(3a\,x^{-3a+1}c_1 - x^{-3a+1}c_1 - 3b\right)^2\right)^{\frac{1}{3}}}{2\left(3a\,x^{-3a+1}c_1 - x^{-3a+1}c_1 - 3b\right)} + \frac{i\sqrt{3}\left(x(3a-1)\left(3a\,x^{-3a+1}c_1 - x^{-3a+1}c_1 - 3b\right)^2\right)^{\frac{1}{3}}}{6a\,x^{-3a+1}c_1 - 2x^{-3a+1}c_1 - 6b}$$

✓ Solution by Mathematica

Time used: 3.514 (sec). Leaf size: 149

```
DSolve[x^2 y'[x]==(a x+b y[x]^3)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{(1-3a)x^{3a+1}}}{\sqrt[3]{3bx^{3a}+(1-3a)c_1x}}$$

$$y(x) \to -\frac{\sqrt[3]{-1}\sqrt[3]{(1-3a)x^{3a+1}}}{\sqrt[3]{3bx^{3a}+(1-3a)c_1x}}$$

$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{(1-3a)x^{3a+1}}}{\sqrt[3]{3bx^{3a}+(1-3a)c_1x}}$$

$$y(x) \to 0$$

10.9 problem 275

Internal problem ID [3022]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 275.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{y'x^2 + yx + \sqrt{y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^2*diff(y(x),x)+x*y(x)+sqrt(y(x)) = 0,y(x), singsol=all)
```

$$\sqrt{y(x)} - \frac{1}{x} - \frac{c_1}{\sqrt{x}} = 0$$

✓ Solution by Mathematica

Time used: 0.149 (sec). Leaf size: 21

```
DSolve[x^2 y'[x]+x y[x]+Sqrt[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\left(1 + c_1\sqrt{x}\right)^2}{x^2}$$

10.10 problem 276

Internal problem ID [3023]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 276.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y'x^2 - \sec(y) - 3x\tan(y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x) = sec(y(x))+3*x*tan(y(x)),y(x), singsol=all)
```

$$y(x) = \arcsin\left(\frac{c_1 x^4 - 1}{4x}\right)$$

✓ Solution by Mathematica

Time used: 9.869 (sec). Leaf size: 23

```
DSolve[x^2 y'[x]==Sec[y[x]]+3 x Tan[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\arcsin\left(\frac{1}{4x} + 3c_1 x^3\right)$$

10.11 problem 277

Internal problem ID [3024]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 277.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(1 - x^2)\, y' - 1 + x^2 - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve((-x^2+1)*diff(y(x),x) = 1-x^2+y(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(\sqrt{-(x+1)^2 + 2x + 2} + \arcsin(x) + c_1\right)(x+1)}{\sqrt{-x^2 + 1}}$$

✓ Solution by Mathematica

Time used: 0.139 (sec). Leaf size: 54

```
DSolve[(1-x^2)y'[x]==1-x^2+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{x+1}\left(\sqrt{1-x^2} - 2\cot^{-1}\left(\frac{x-1}{\sqrt{1-x^2}}\right) + c_1\right)}{\sqrt{1-x}}$$

10.12 problem 278

Internal problem ID [3025]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 278.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$(1 - x^2)\, y' + 1 - yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve((-x^2+1)*diff(y(x),x)+1 = x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(x-1)(x+1)}\, \ln\left(x + \sqrt{x^2-1}\right)}{(x-1)(x+1)} + \frac{c_1}{\sqrt{x-1}\, \sqrt{x+1}}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 30

```
DSolve[(1-x^2)y'[x]+1==x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right) + c_1}{\sqrt{x^2-1}}$$

10.13 problem 279

Internal problem ID [3026]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 279.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$(1 - x^2) y' - 5 + yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve((-x^2+1)*diff(y(x),x) = 5-x*y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{x-1}\,\sqrt{x+1}\,c_1 + 5x$$

✓ Solution by Mathematica

Time used: 0.068 (sec). Leaf size: 21

```
DSolve[(1-x^2)y'[x]==5 -x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 5x + c_1\sqrt{x^2-1}$$

10.14 problem 280

Internal problem ID [3027]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 280.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$(x^2+1)\,y'+a+yx=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve((x^2+1)*diff(y(x),x)+a+x*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{-a\ \operatorname{arcsinh}(x)+c_1}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 23

```
DSolve[(1+x^2)y'[x]+a+x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-a\operatorname{arcsinh}(x)+c_1}{\sqrt{x^2+1}}$$

10.15 problem 281

Internal problem ID [3028]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 281.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\,y'+a-yx=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((x^2+1)*diff(y(x),x)+a-x*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\sqrt{x^2+1}\,c_1-ax$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 22

```
DSolve[(1+x^2)y'[x]+a-x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -ax+c_1\sqrt{x^2+1}$$

10.16 problem 282

Internal problem ID [3029]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 282.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$(1 - x^2)\, y' + a - yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve((-x^2+1)*diff(y(x),x)+a-x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{a\sqrt{(x-1)(x+1)}\, \ln\left(x+\sqrt{x^2-1}\right)}{(x-1)(x+1)} + \frac{c_1}{\sqrt{x-1}\,\sqrt{x+1}}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 32

```
DSolve[(1-x^2)y'[x]+a-x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a\text{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right) + c_1}{\sqrt{x^2-1}}$$

10.17 problem 283

Internal problem ID [3030]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 283.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1-x^2)\,y'-x+yx=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((-x^2+1)*diff(y(x),x)-x+x*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\sqrt{x-1}\,\sqrt{x+1}\,c_1+1$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 24

```
DSolve[(1-x^2)y'[x]-x +x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 1+c_1\sqrt{x^2-1}$$
$$y(x)\to 1$$

10.18 problem 284

Internal problem ID [3031]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 284.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$(1-x^2)\,y' - x^2 + yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
dsolve((-x^2+1)*diff(y(x),x)-x^2+x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\ln\left(x+\sqrt{x^2-1}\right)x^2}{\sqrt{x^2-1}} + x + \frac{\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{x^2-1}} + \sqrt{x-1}\,\sqrt{x+1}\,c_1$$

✓ Solution by Mathematica

Time used: 0.094 (sec). Leaf size: 43

```
DSolve[(1-x^2)y'[x]-x^2 +x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x^2-1}\log\left(\sqrt{x^2-1}-x\right) + c_1\sqrt{x^2-1} + x$$

10.19 problem 285

Internal problem ID [3032]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 285.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(1-x^2)\, y' + x^2 + yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 60

```
dsolve((-x^2+1)*diff(y(x),x)+x^2+x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\ln\left(x+\sqrt{x^2-1}\right)x^2}{\sqrt{x^2-1}} - x - \frac{\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{x^2-1}} + \sqrt{x-1}\,\sqrt{x+1}\,c_1$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 46

```
DSolve[(1-x^2)y'[x]+x^2 +x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x^2-1}\log\left(\sqrt{x^2-1}-x\right) + c_1\sqrt{x^2-1} - x$$

10.20 problem 286

Internal problem ID [3033]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 286.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\,y' - x(x^2+1) + yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve((x^2+1)*diff(y(x),x) = x*(x^2+1)-x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{x^2}{3} + \frac{1}{3} + \frac{c_1}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 27

```
DSolve[(1+x^2)y'[x]==x(1+x^2)-x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}(x^2+1) + \frac{c_1}{\sqrt{x^2+1}}$$

10.21 problem 287

Internal problem ID [3034]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 287.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\,y' - x(3x^2 - y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((x^2+1)*diff(y(x),x) = x*(3*x^2-y(x)),y(x), singsol=all)
```

$$y(x) = x^2 - 2 + \frac{c_1}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 22

```
DSolve[(1+x^2)y'[x]==x(3 x^2-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + \frac{c_1}{\sqrt{x^2+1}} - 2$$

10.22 problem 288

Internal problem ID [3035]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 288.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1 - x^2)\, y' + 2yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve((-x^2+1)*diff(y(x),x)+2*x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 x^2 - c_1$$

✓ Solution by Mathematica

Time used: 0.028 (sec). Leaf size: 18

```
DSolve[(1-x^2)y'[x]+2 x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 (x^2 - 1)$$
$$y(x) \to 0$$

10.23 problem 289

Internal problem ID [3036]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 289.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\,y' - 2x(x-y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((x^2+1)*diff(y(x),x) = 2*x*(x-y(x)),y(x), singsol=all)
```

$$y(x) = \frac{\frac{2x^3}{3} + c_1}{x^2+1}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 25

```
DSolve[(1+x^2)y'[x]==2 x(x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^3 + 3c_1}{3x^2+3}$$

10.24 problem 290

Internal problem ID [3037]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 290.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\,y'-2x(x^2+1)^2-2yx=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve((x^2+1)*diff(y(x),x) = 2*x*(x^2+1)^2+2*x*y(x),y(x), singsol=all)
```

$$y(x)=\left(x^2+c_1\right)\left(x^2+1\right)$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 17

```
DSolve[(1+x^2)y'[x]==2 x(1+x^2)^2+2 x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\left(x^2+1\right)\left(x^2+c_1\right)$$

10.25 problem 291

Internal problem ID [3038]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 10
Problem number: 291.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(1-x^2)\, y' + \cos(x) - 2yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((-x^2+1)*diff(y(x),x)+cos(x) = 2*x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 + \sin(x)}{(x-1)(x+1)}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 18

```
DSolve[(1-x^2)y'[x]+Cos[x]==2 x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sin(x) + c_1}{x^2 - 1}$$

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11.1 problem 292

Internal problem ID [3039]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 292.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x^2+1)\,y'-\tan(x)+2yx=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((x^2+1)*diff(y(x),x) = tan(x)-2*x*y(x),y(x), singsol=all)
```

$$y(x)=\frac{-\ln(\cos(x))+c_1}{x^2+1}$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 21

```
DSolve[(1+x^2)y'[x]==Tan[x]-2 x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-\log(\cos(x))+c_1}{x^2+1}$$

11.2 problem 293

Internal problem ID [3040]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 293.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(1-x^2)\, y' - a - 4yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve((-x^2+1)*diff(y(x),x) = a+4*x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{-a\left(\frac{1}{3}x^3 - x\right) + c_1}{(x-1)^2\,(x+1)^2}$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 30

```
DSolve[(1-x^2)y'[x]==a+4 x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-ax(x^2-3) + 3c_1}{3\,(x^2-1)^2}$$

11.3 problem 294

Internal problem ID [3041]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 294.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)\,y'-(2bx+a)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((x^2+1)*diff(y(x),x) = (2*b*x+a)*y(x),y(x), singsol=all)
```

$$y(x)=c_1\left(x^2+1\right)^b \mathrm{e}^{a\arctan(x)}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 26

```
DSolve[(1+x^2)y'[x]==(a+2 b x)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\left(x^2+1\right)^b e^{a\arctan(x)}$$
$$y(x)\to 0$$

11.4 problem 295

Internal problem ID [3042]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 295.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)\,y'-1-y^2=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve((x^2+1)*diff(y(x),x) = 1+y(x)^2,y(x), singsol=all)
```

$$y(x) = \tan(\arctan(x)+c_1)$$

✓ Solution by Mathematica

Time used: 0.237 (sec). Leaf size: 25

```
DSolve[(1+x^2)y'[x]==(1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \tan(\arctan(x)+c_1)$$
$$y(x) \to -i$$
$$y(x) \to i$$

11.5 problem 296

Internal problem ID [3043]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 296.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$(1-x^2)\, y'-1+y^2=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve((-x^2+1)*diff(y(x),x) = 1-y(x)^2,y(x), singsol=all)
```

$$y(x)=-\tanh\left(-\operatorname{arctanh}(x)+c_1\right)$$

✓ Solution by Mathematica

Time used: 0.524 (sec). Leaf size: 39

```
DSolve[(1-x^2)y'[x]==(1-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{x\cosh(c_1)-\sinh(c_1)}{\cosh(c_1)-x\sinh(c_1)}$$
$$y(x)\to -1$$
$$y(x)\to 1$$

11.6 problem 297

Internal problem ID [3044]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 297.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccat`

$$\boxed{(1-x^2)\,y'-1+(2x-y)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve((-x^2+1)*diff(y(x),x) = 1-(2*x-y(x))*y(x),y(x), singsol=all)
```

$$y(x)=x+\frac{1}{-\operatorname{arctanh}(x)+c_1}$$

✓ Solution by Mathematica

Time used: 0.222 (sec). Leaf size: 21

```
DSolve[(1-x^2)y'[x]==1-(2 x-y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x+\frac{1}{-\operatorname{arctanh}(x)+c_1}$$
$$y(x)\to x$$

11.7 problem 298

Internal problem ID [3045]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 298.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$(1-x^2)\,y'-n\left(y^2-2yx+1\right)=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 231

```
dsolve((-x^2+1)*diff(y(x),x) = n*(1-2*x*y(x)+y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{8(x+1)\,c_1\left(x\left(n-\frac{1}{2}\right)-\frac{n}{2}+\frac{1}{2}\right)\operatorname{HeunC}\left(0,-2n+1,0,0,n^2-n+\frac{1}{2},\frac{2}{x+1}\right)-n\left(-\frac{x}{2}-\frac{1}{2}\right)^{-2n+1}(x+1)\operatorname{Heu}}{4n\,(x+1)\left(\operatorname{HeunC}\left(0,-2n\right.\right.}$$

✓ Solution by Mathematica

Time used: 0.344 (sec). Leaf size: 47

```
DSolve[(1-x^2)y'[x]==n(1-2 x y[x]+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\operatorname{LegendreQ}(n,x)+c_1\operatorname{LegendreP}(n,x)}{\operatorname{LegendreQ}(n-1,x)+c_1\operatorname{LegendreP}(n-1,x)}$$

$$y(x) \to \frac{\operatorname{LegendreP}(n,x)}{\operatorname{LegendreP}(n-1,x)}$$

11.8 problem 299

Internal problem ID [3046]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 299.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)\,y'+xy(1-y)=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve((x^2+1)*diff(y(x),x)+x*y(x)*(1-y(x)) = 0,y(x), singsol=all)
```

$$y(x)=\frac{1}{1+\sqrt{x^2+1}\,c_1}$$

✓ Solution by Mathematica

Time used: 2.319 (sec). Leaf size: 33

```
DSolve[(1+x^2)y'[x]+x y[x](1-y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{1+e^{c_1}\sqrt{x^2+1}}$$
$$y(x)\to 0$$
$$y(x)\to 1$$

11.9 problem 300

Internal problem ID [3047]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 300.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1 - x^2)\, y' - xy(1 + ay) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve((-x^2+1)*diff(y(x),x) = x*y(x)*(1+a*y(x)),y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{x-1}\,\sqrt{x+1}\,c_1 - a}$$

✓ Solution by Mathematica

Time used: 4.095 (sec). Leaf size: 43

```
DSolve[(1-x^2)y'[x]==x y[x](1+a y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{-a + e^{-c_1}\sqrt{1-x^2}}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{1}{a}$$

11.10 problem 301

Internal problem ID [3048]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 301.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$(x^2+1)\,y'-1-y^2+2xy(1+y^2)=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 76

```
dsolve((x^2+1)*diff(y(x),x) = 1+y(x)^2-2*x*y(x)*(1+y(x)^2),y(x), singsol=all)
```

$$c_1+\frac{x}{\left(1+\left(\frac{1}{x}+\frac{x^2(x^2+1)}{x^4y(x)-x^3}\right)^2\right)^{\frac{1}{4}}}+\frac{(x+y(x))\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{5}{4}\right],\left[\frac{3}{2}\right],-\frac{(x+y(x))^2}{(y(x)x-1)^2}\right)}{2y(x)\,x-2}=0$$

✓ Solution by Mathematica

Time used: 0.402 (sec). Leaf size: 203

```
DSolve[(1+x^2)y'[x]==1+y[x]^2-2 x y[x](1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[c_1=\frac{\frac{1}{2}\left(\frac{1}{\frac{ix}{x^2+1}-\frac{ix^2y(x)}{x^2+1}}+\frac{i}{x}\right)\sqrt[4]{1-\left(\frac{1}{\frac{ix}{x^2+1}-\frac{ix^2y(x)}{x^2+1}}+\frac{i}{x}\right)^2}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{3}{2},\left(\frac{1}{\frac{ix}{x^2+1}-\frac{ix^2y(x)}{x^2+1}}\right.\right.}{\sqrt[4]{-1+\left(\frac{1}{\frac{ix}{x^2+1}-\frac{ix^2y(x)}{x^2+1}}+\frac{i}{x}\right)^2}}\right.$$

11.11 problem 302

Internal problem ID [3049]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 302.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$(x^2+1)\,y'+x\cos(y)\sin(y)-x(x^2+1)\cos(y)^2=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 191

```
dsolve((x^2+1)*diff(y(x),x)+x*sin(y(x))*cos(y(x)) = x*(x^2+1)*cos(y(x))^2,y(x), singsol=all)
```

$$y(x)=\frac{\arctan\left(\frac{6\sqrt{x^2+1}\,x^4+12\sqrt{x^2+1}\,x^2+18c_1x^2+6\sqrt{x^2+1}+18c_1}{\sqrt{x^2+1}\left(x^6+6\sqrt{x^2+1}\,c_1x^2+3x^4+6\sqrt{x^2+1}\,c_1+9c_1^2+12x^2+10\right)},-\frac{x^6+6\sqrt{x^2+1}\,c_1x^2+3x^4+6\sqrt{x^2+1}\,c_1+9c_1^2-6x^2-8}{x^6+6\sqrt{x^2+1}\,c_1x^2+3x^4+6\sqrt{x^2+1}\,c_1+9c_1^2+12x^2+10}\right)}{2}$$

✓ Solution by Mathematica

Time used: 8.738 (sec). Leaf size: 86

```
DSolve[(1+x^2)y'[x]+x Sin[y[x]] Cos[y[x]]==x(1+x^2) (Cos[y[x]])^2,y[x],x,IncludeSingularSolut
```

$$y(x)\to\arctan\left(\frac{1}{3}\left(x^2-\frac{6c_1}{\sqrt{x^2+1}}+1\right)\right)$$

$$y(x)\to-\frac{1}{2}\pi\sqrt{\frac{1}{x^2+1}}\sqrt{x^2+1}$$

$$y(x)\to\frac{1}{2}\pi\sqrt{\frac{1}{x^2+1}}\sqrt{x^2+1}$$

11.12 problem 303

Internal problem ID [3050]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 303.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$
(x^2+1)\,y'-1-x^2+y\,\operatorname{arccot}(x)=0
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve((x^2+1)*diff(y(x),x) = 1+x^2-y(x)*arccot(x),y(x), singsol=all)
```

$$
y(x)=\left(\int \mathrm{e}^{-\frac{\operatorname{arccot}(x)^2}{2}}dx+c_1\right)\mathrm{e}^{\frac{\operatorname{arccot}(x)^2}{2}}
$$

✓ Solution by Mathematica

Time used: 3.52 (sec). Leaf size: 37

```
DSolve[(1+x^2)y'[x]==(1+x^2)-y[x] ArcCot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x)\to e^{\frac{1}{2}\cot^{-1}(x)^2}\left(\int_1^x e^{-\frac{1}{2}\cot^{-1}(K[1])^2}dK[1]+c_1\right)
$$

11.13 problem 304

Internal problem ID [3051]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 304.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{(-x^2+4)\,y'+4y-(2+x)\,y^2=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve((-x^2+4)*diff(y(x),x)+4*y(x) = (2+x)*y(x)^2,y(x), singsol=all)
```

$$y(x)=\frac{x-2}{\ln(2+x)\,x+c_1x+2\ln(2+x)+2c_1}$$

✓ Solution by Mathematica

Time used: 0.211 (sec). Leaf size: 30

```
DSolve[(4-x^2)y'[x]+4 y[x]==(2+x)y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x-2}{(x+2)(\log(x+2)-c_1)}$$
$$y(x)\to 0$$

11.14 problem 305

Internal problem ID [3052]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 305.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(a^2 + x^2)\, y' - b - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve((a^2+x^2)*diff(y(x),x) = b+x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{xb}{a^2} + \sqrt{a^2 + x^2}\, c_1$$

✓ Solution by Mathematica

Time used: 0.082 (sec). Leaf size: 26

```
DSolve[(a^2+x^2)y'[x]==b+x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{bx}{a^2} + c_1\sqrt{a^2 + x^2}$$

11.15 problem 306

Internal problem ID [3053]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 306.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\left(a^{2}+x^{2}\right) y'-(b+y)\left(x+\sqrt{a^{2}+x^{2}}\right)=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 40

```
dsolve((a^2+x^2)*diff(y(x),x) = (b+y(x))*(x+sqrt(a^2+x^2)),y(x), singsol=all)
```

$$y(x)=\left(\frac{xb}{\sqrt{a^{2}+x^{2}}\,a^{2}}+c_{1}\right)\left(x\sqrt{a^{2}+x^{2}}+a^{2}+x^{2}\right)$$

✓ Solution by Mathematica

Time used: 0.277 (sec). Leaf size: 41

```
DSolve[(a^2+x^2)y'[x]==(b+y[x])(x+Sqrt[a^2+x^2]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{x\left(\sqrt{a^{2}+x^{2}}+x\right)(b-c_{1})}{a^{2}}-c_{1}$$

$$y(x)\to -b$$

11.16 problem 307

Internal problem ID [3054]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 307.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{(a^2 + x^2)\, y' + y(x - y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve((a^2+x^2)*diff(y(x),x)+(x-y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{a^2}{\sqrt{a^2 + x^2}\, c_1 a^2 - x}$$

✓ Solution by Mathematica

Time used: 0.264 (sec). Leaf size: 33

```
DSolve[(x^2+a^2)y'[x]+(x-y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{-\frac{x}{a^2} + c_1\sqrt{a^2 + x^2}}$$

$$y(x) \to 0$$

11.17 problem 308

Internal problem ID [3055]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 308.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{(a^2 + x^2) y' - a^2 - 3yx + 2y^2 = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 280

```
dsolve((a^2+x^2)*diff(y(x),x) = a^2+3*x*y(x)-2*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{a\left(-\frac{(ix-a)\sqrt{2}\,c_1\sqrt{\frac{ix-a}{a}}\ \mathrm{HeunC}\left(0,-\frac{1}{2},2,0,\frac{5}{4},\frac{ix+a}{ix-a}\right)}{2} + (ix-a)\sqrt{\frac{ix+a}{a}}\ \mathrm{HeunC}\left(0,\frac{1}{2},2,0,\frac{5}{4},\frac{ix+a}{ix-a}\right) + (ix+a)\left(-c_1 \cdots\right.\right.}{(ia+x)\left(c_1\sqrt{2}\sqrt{\frac{ix-a}{a}}\ \mathrm{HeunC}\left(0,-\frac{1}{2},2,0,\frac{5}{4},\frac{ix+a}{ix-a}\right) \cdots\right.}$$

✓ Solution by Mathematica

Time used: 1.081 (sec). Leaf size: 63

```
DSolve[(a^2+x^2)y'[x]==a^2+3 x y[x]-2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a^2 c_1(-x)\sqrt{a^2+x^2} + a^2 + 2x^2}{2x - a^2 c_1\sqrt{a^2+x^2}}$$

$$y(x) \to x$$

11.18 problem 309

Internal problem ID [3056]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 309.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(a^2 + x^2)\, y' + yx + bxy^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve((a^2+x^2)*diff(y(x),x)+x*y(x)+b*x*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{a^2 + x^2}\, c_1 - b}$$

✓ Solution by Mathematica

Time used: 3.916 (sec). Leaf size: 43

```
DSolve[(x^2+a^2)y'[x]+x y[x]+b x y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{-b + e^{-c_1}\sqrt{a^2 + x^2}}$$

$$y(x) \to 0$$

$$y(x) \to -\frac{1}{b}$$

11.19 problem 310

Internal problem ID [3057]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 310.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(1-x)\,y'-a-(x+1)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x*(1-x)*diff(y(x),x) = a+(1+x)*y(x),y(x), singsol=all)
```

$$y(x)=\frac{\left(-a\ln\left(x\right)-\frac{a}{x}+c_1\right)x}{\left(x-1\right)^2}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 24

```
DSolve[x(1-x)y'[x]==a+(1+x)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{ax\log(x)+a-c_1x}{(x-1)^2}$$

11.20 problem 311

Internal problem ID [3058]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 311.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(1-x)\,y'-2-2yx=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*(1-x)*diff(y(x),x) = 2+2*x*y(x),y(x), singsol=all)
```

$$y(x)=\frac{-2x+2\ln(x)+c_1}{(x-1)^2}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 21

```
DSolve[x(1-x)y'[x]==2(1+x y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{-2x+2\log(x)+c_1}{(x-1)^2}$$

11.21 problem 312

Internal problem ID [3059]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 312.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$x(1-x)\,y' - 2yx + 2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*(1-x)*diff(y(x),x) = 2*x*y(x)-2,y(x), singsol=all)
```

$$y(x) = \frac{2x - 2\ln(x) + c_1}{(x-1)^2}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 21

```
DSolve[x(1-x)y'[x]==2(x y[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x - 2\log(x) + c_1}{(x-1)^2}$$

11.22 problem 313

Internal problem ID [3060]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 313.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(x+1)\,y' - (1-2x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(x*(1+x)*diff(y(x),x) = (1-2*x)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 x}{(x+1)^3}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 19

```
DSolve[x(1+x)y'[x]==(1-2 x)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 x}{(x+1)^3}$$
$$y(x) \to 0$$

11.23 problem 314

Internal problem ID [3061]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 314.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(1-x)\,y'+(1+2x)\,y-a=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(x*(1-x)*diff(y(x),x)+(1+2*x)*y(x) = a,y(x), singsol=all)
```

$$y(x)=\frac{(x^3-3x^2+3x-1)\,c_1}{x}+\frac{a}{3x}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 23

```
DSolve[x(1-x)y'[x]+(1+2 x)y[x]==a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{a-3c_1(x-1)^3}{3x}$$

11.24 problem 315

Internal problem ID [3062]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 315.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$x(1-x)\,y'-a-2(2-x)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(x*(1-x)*diff(y(x),x) = a+2*(2-x)*y(x),y(x), singsol=all)
```

$$y(x)=\frac{\left(-a\left(\frac{1}{4x^4}-\frac{1}{3x^3}\right)+c_1\right)x^4}{(x-1)^2}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 29

```
DSolve[x(1-x)y'[x]==a+2(2-x)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{a(4x-3)+12c_1x^4}{12(x-1)^2}$$

11.25 problem 316

Internal problem ID [3063]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 316.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(1-x)\,y'+2-3yx+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*(1-x)*diff(y(x),x)+2-3*x*y(x)+y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{x^2+c_1-2x}{(x-1)^2\,x}$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 22

```
DSolve[x(1-x)y'[x]+(2-3 x y[x]+y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{(x-2)x+c_1}{(x-1)^2x}$$

11.26 problem 317

Internal problem ID [3064]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 317.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$x(x+1)\,y' - (x+1)\left(x^2-1\right) - \left(x^2+x-1\right)y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve(x*(1+x)*diff(y(x),x) = (1+x)*(x^2-1)+(x^2+x-1)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{e}^{x}(x+1)\,c_1}{x} - x - 1$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 22

```
DSolve[x(1+x)y'[x]==(x+1)(x^2-1)+(x^2+x-1)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{(x+1)\left(-x+c_1e^{x}\right)}{x}$$

11.27 problem 318

Internal problem ID [3065]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 318.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(-2+x)(x-3)\,y'+x^2-8y+3yx=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve((x-2)*(x-3)*diff(y(x),x)+x^2-8*y(x)+3*x*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{-\frac{1}{4}x^4+\frac{2}{3}x^3+c_1}{(x-2)^2\,(-3+x)}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 33

```
DSolve[(x-2)(x-3)y'[x]+x^2-8 y[x]+3 x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{(8-3x)x^3-12c_1}{12(x-3)(x-2)^2}$$

11.28 problem 319

Internal problem ID [3066]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 11
Problem number: 319.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$x(x+a)\,y' - (b+cy)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(x*(a+x)*diff(y(x),x) = (b+c*y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{b}{(x+a)^{\frac{b}{a}}\, x^{-\frac{b}{a}} c_1 b - c}$$

✓ Solution by Mathematica

Time used: 0.9 (sec). Leaf size: 53

```
DSolve[x(a+x)y'[x]==(b+c y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{b}{-c + e^{-bc_1} x^{-\frac{b}{a}} (a+x)^{\frac{b}{a}}}$$

$$y(x) \to 0$$

$$y(x) \to -\frac{b}{c}$$

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12.1 problem 320

Internal problem ID [3067]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 320.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+a)^2 y' - 2(x+a)(b+y) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve((a+x)^2*diff(y(x),x) = 2*(a+x)*(b+y(x)),y(x), singsol=all)
```

$$y(x) = -b + (x+a)^2 c_1$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 24

```
DSolve[(a+x)^2 y'[x]==2(a+x)(b+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -b + c_1(a+x)^2$$
$$y(x) \to -b$$

12.2 problem 321

Internal problem ID [3068]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 321.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, _Riccati]`

$$\boxed{(x-a)^2 y' + k(x+y-a)^2 + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 39

```
dsolve((x-a)^2*diff(y(x),x)+k*(x+y(x)-a)^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{(-x+a)\left(c_1 k(-x+a)-1\right)}{c_1 k\left(-x+a\right)+c_1\left(-x+a\right)-1}$$

✓ Solution by Mathematica

Time used: 0.203 (sec). Leaf size: 50

```
DSolve[(x-a)^2 y'[x]+k(x+y[x]-a)^2+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{k(a-x)}{k+1} + \frac{1}{\frac{k+1}{a-x} + c_1}$$
$$y(x) \to \frac{k(a-x)}{k+1}$$

12.3 problem 322

Internal problem ID [3069]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 322.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$(x-a)(x-b)y' + ky = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve((x-a)*(x-b)*diff(y(x),x)+k*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1(x-b)^{\frac{k}{a-b}}(x-a)^{-\frac{k}{a-b}}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 39

```
DSolve[(x-a)(x-b)y'[x]+k y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{k(\log(x-b)-\log(x-a))}{a-b}}$$
$$y(x) \to 0$$

12.4 problem 323

Internal problem ID [3070]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 323.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$(x-a)(x-b)y'-(x-a)(x-b)-(2x-a-b)y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 46

```
dsolve((x-a)*(x-b)*diff(y(x),x) = (x-a)*(x-b)+(2*x-a-b)*y(x),y(x), singsol=all)
```

$$y(x)=\left(-\frac{\ln(x-b)}{a-b}+\frac{\ln(x-a)}{a-b}+c_1\right)(x-a)(x-b)$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 42

```
DSolve[(x-a)(x-b)y'[x]==(x-a)(x-b)+(2 x-a-b)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to(x-a)(x-b)\left(\frac{\log(x-a)-\log(x-b)}{a-b}+c_1\right)$$

12.5 problem 324

Internal problem ID [3071]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 324.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x-a)(x-b)y'-cy^2=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 38

```
dsolve((x-a)*(x-b)*diff(y(x),x) = c*y(x)^2,y(x), singsol=all)
```

$$y(x)=-\frac{a-b}{c\ln(x-a)-c\ln(x-b)-c_1a+c_1b}$$

✓ Solution by Mathematica

Time used: 0.423 (sec). Leaf size: 44

```
DSolve[(x-a)(x-b)y'[x]==c y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{b-a}{c_1(a-b)+c\log(x-a)-c\log(x-b)}$$
$$y(x)\to 0$$

12.6 problem 325

Internal problem ID [3072]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 325.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$(x-a)(x-b)y'+k(y-a)(y-b)=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 131

```
dsolve((x-a)*(x-b)*diff(y(x),x)+k*(y(x)-a)*(y(x)-b) = 0,y(x), singsol=all)
```

$$y(x) = \frac{(x-b)^{-k}(x-a)^{k} a\,\mathrm{e}^{ac_1k-bc_1k}-(x-b)^{-k}(x-a)^{k} b\,\mathrm{e}^{ac_1k-bc_1k}+b\left(\frac{-x+b}{-x+a}\right)^{-k}\mathrm{e}^{ac_1k-bc_1k}-b}{-1+\left(\frac{-x+b}{-x+a}\right)^{-k}\mathrm{e}^{ac_1k-bc_1k}}$$

✓ Solution by Mathematica

Time used: 2.161 (sec). Leaf size: 68

```
DSolve[(x-a)(x-b)y'[x]+k(y[x]-a)(y[x]-b)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to a+\frac{(b-a)e^{ac_1}(x-b)^k}{e^{ac_1}(x-b)^k-e^{bc_1}(x-a)^k}$$

$$y(x)\to a$$

$$y(x)\to b$$

12.7 problem 326

Internal problem ID [3073]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 326.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccat`

$$(x-a)(x-b)y'+k(x+y-a)(x+y-b)+y^2=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 128

```
dsolve((x-a)*(x-b)*diff(y(x),x)+k*(x+y(x)-a)*(x+y(x)-b)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{k\left(\frac{bc_1(-x+b)^k}{c_1(-x+b)^k+(-x+a)^k}-\frac{xc_1(-x+b)^k}{c_1(-x+b)^k+(-x+a)^k}+\frac{a(-x+a)^k}{c_1(-x+b)^k+(-x+a)^k}-\frac{x(-x+a)^k}{c_1(-x+b)^k+(-x+a)^k}\right)}{k+1}$$

✓ Solution by Mathematica

Time used: 60.294 (sec). Leaf size: 99

```
DSolve[(x-a)(x-b)y'[x]+k(x+y[x]-a)(x+y[x]-b)+y[x]^2==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x)\to\frac{1}{2}\left(\frac{k(a+b-2x)}{k+1}+\sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}}\tan\left(\frac{(k+1)\sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}}(\log(x-b)-\log(x-a))}{2(a-b)}+c_1\right)\right)$$

12.8 problem 327

Internal problem ID [3074]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 327.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2y'x^2 - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

```
dsolve(2*x^2*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{-\frac{1}{2x}}$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 22

```
DSolve[2 x^2 y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{1}{2}/x}$$
$$y(x) \to 0$$

12.9 problem 328

Internal problem ID [3075]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 328.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$2y'x^2 + x\cot(x) - 1 + 2x^2y\cot(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(2*x^2*diff(y(x),x)+x*cot(x)-1+2*x^2*y(x)*cot(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2x} + \frac{c_1}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 18

```
DSolve[2 x^2 y'[x]+x Cot[x]-1+2 x^2 y[x] Cot[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2x} + c_1 \csc(x)$$

12.10 problem 329

Internal problem ID [3076]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 329.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{2y'x^2 + 1 + 2yx - x^2y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(2*x^2*diff(y(x),x)+1+2*x*y(x)-x^2*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\tanh\left(-\frac{\ln(x)}{2} + \frac{c_1}{2}\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.825 (sec). Leaf size: 61

```
DSolve[2 x^2 y'[x]+1+2 x y[x]- x^2 y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{i\tan\left(\frac{1}{2}i\log(x) + c_1\right)}{x}$$

$$y(x) \to \frac{-x + e^{2i\text{Interval}[\{0,\pi\}]}}{x^2 + xe^{2i\text{Interval}[\{0,\pi\}]}}$$

12.11 problem 330

Internal problem ID [3077]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 330.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Riccati]`

$$2y'x^2 - 2yx - (-x\cot(x) + 1)(x^2 - y^2) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(2*x^2*diff(y(x),x) = 2*x*y(x)+(1-x*cot(x))*(x^2-y(x)^2),y(x), singsol=all)
```

$$y(x) = -\tanh\left(\frac{\ln(\sin(x))}{2} - \frac{\ln(x)}{2} + \frac{c_1}{2}\right)x$$

✓ Solution by Mathematica

Time used: 0.969 (sec). Leaf size: 37

```
DSolve[2 x^2 y'[x]==2 x y[x]+(1-x Cot[x])(x^2-y[x]^2),y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to x\left(-1 + \frac{2x}{x + e^{2c_1}\sin(x)}\right)$$

$$y(x) \to -x$$

$$y(x) \to x$$

12.12 problem 331

Internal problem ID [3078]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 331.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{2(1-x^2)\,y'-\sqrt{1-x^2}-(x+1)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(2*(-x^2+1)*diff(y(x),x) = sqrt(-x^2+1)+(1+x)*y(x),y(x), singsol=all)
```

$$y(x)=\frac{c_1}{\sqrt{x-1}}+\frac{x+1}{\sqrt{-x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.311 (sec). Leaf size: 36

```
DSolve[2(1-x^2)y'[x]==Sqrt[1-x^2]+(1+x)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2\sqrt{x+1}+\sqrt{2}c_1}{2\sqrt{1-x}}$$

12.13 problem 332

Internal problem ID [3079]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 332.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(1-2x)\,y'+1+(1-4x)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*(1-2*x)*diff(y(x),x)+1+(1-4*x)*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{x+c_1}{(-1+2x)\,x}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 22

```
DSolve[x(1-2 x)y'[x]+1+(1-4 x)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{x-c_1}{x-2x^2}$$

12.14 problem 333

Internal problem ID [3080]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 333.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'], _Riccat`

$$x(1-2x)\,y' - 4x + (4x+1)\,y - y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*(1-2*x)*diff(y(x),x) = 4*x-(1+4*x)*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{-2x^2 + c_1}{c_1 - x}$$

✓ Solution by Mathematica

Time used: 0.194 (sec). Leaf size: 27

```
DSolve[x(1-2 x)y'[x]==4 x -(1+4 x)y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + \frac{x(2x-1)}{x - c_1}$$

$$y(x) \to 1$$

12.15 problem 334

Internal problem ID [3081]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 334.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$2x(1-x)\,y' + x + (1-2x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(2*x*(1-x)*diff(y(x),x)+x+(1-2*x)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{2} + \frac{\ln\left(-\frac{1}{2} + x + \sqrt{(x-1)\,x}\right)}{4\sqrt{(x-1)\,x}} + \frac{c_1}{\sqrt{(x-1)\,x}}$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 67

```
DSolve[2 x(1-x)y'[x]+x+(1-2 x)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-x^2 + x + \sqrt{x-1}\sqrt{x}\log\left(\sqrt{x-1} - \sqrt{x}\right) + 2c_1\sqrt{-((x-1)x)}}{2x - 2x^2}$$

12.16 problem 335

Internal problem ID [3082]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 335.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{2x(1-x)\,y'+x+(1-x)\,y^2=0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 97

```
dsolve(2*x*(1-x)*diff(y(x),x)+x+(1-x)*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{x\left(\mathrm{LegendreQ}\left(-\frac{1}{2},1,\frac{-x+2}{x}\right)c_1-\mathrm{LegendreQ}\left(\frac{1}{2},1,\frac{-x+2}{x}\right)c_1+\mathrm{LegendreP}\left(-\frac{1}{2},1,\frac{-x+2}{x}\right)-\mathrm{LegendreP}\left(\frac{1}{2},\right.\right.}{2\left(\mathrm{LegendreQ}\left(-\frac{1}{2},1,\frac{-x+2}{x}\right)c_1+\mathrm{LegendreP}\left(-\frac{1}{2},1,\frac{-x+2}{x}\right)\right)(x-1)}$$

✓ Solution by Mathematica

Time used: 0.334 (sec). Leaf size: 77

```
DSolve[2 x(1-x)y'[x]+x+(1-x)y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{2\left(\pi G_{2,2}^{2,0}\left(x\left|\begin{array}{c}\frac{1}{2},\frac{3}{2}\\ 0,1\end{array}\right.\right)+c_1(\mathrm{EllipticK}(x)-\mathrm{EllipticE}(x))\right)}{\pi G_{2,2}^{2,0}\left(x\left|\begin{array}{c}\frac{1}{2},\frac{3}{2}\\ 0,0\end{array}\right.\right)+2c_1\,\mathrm{EllipticE}(x)}$$

$$y(x)\to 1-\frac{\mathrm{EllipticK}(x)}{\mathrm{EllipticE}(x)}$$

12.17 problem 336

Internal problem ID [3083]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 336.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$2\left(x^{2}+x+1\right) y'-1-8x^{2}+(1+2x)\, y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(2*(x^2+x+1)*diff(y(x),x) = 1+8*x^2-(1+2*x)*y(x),y(x), singsol=all)
```

$$y(x)=2x-3+\frac{c_1}{\sqrt{x^2+x+1}}$$

✓ Solution by Mathematica

Time used: 0.197 (sec). Leaf size: 23

```
DSolve[2(1+x+x^2)y'[x]==1+8 x^2-(1+2 x)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_1}{\sqrt{x^2+x+1}}+2x-3$$

12.18 problem 337

Internal problem ID [3084]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 337.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{4(x^2+1)\,y'-4yx-x^2=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(4*(x^2+1)*diff(y(x),x)-4*x*y(x)-x^2 = 0,y(x), singsol=all)
```

$$y(x)=\left(-\frac{x}{4\sqrt{x^2+1}}+\frac{\operatorname{arcsinh}(x)}{4}+c_1\right)\sqrt{x^2+1}$$

✓ Solution by Mathematica

Time used: 0.087 (sec). Leaf size: 30

```
DSolve[4(1+x^2)y'[x]-4 x y[x]-x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{4}\left(-x+\sqrt{x^2+1}(\operatorname{arcsinh}(x)+4c_1)\right)$$

12.19 problem 338

Internal problem ID [3085]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 338.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Riccati]`

$$a\,x^2y' - x^2 - yax - y^2b^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(a*x^2*diff(y(x),x) = x^2+a*x*y(x)+b^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\tan\left(\frac{b(\ln(x)+c_1)}{a}\right)x}{b}$$

✓ Solution by Mathematica

Time used: 0.217 (sec). Leaf size: 22

```
DSolve[a x^2 y'[x]==x^2+a x y[x]+b^2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x\tan\left(b\left(\frac{\log(x)}{a}+c_1\right)\right)}{b}$$

12.20 problem 339

Internal problem ID [3086]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 339.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(b\,x^2+a)\,y'-A-By^2=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 42

```
dsolve((b*x^2+a)*diff(y(x),x) = A+B*y(x)^2,y(x), singsol=all)
```

$$y(x)=\frac{\tan\left(\frac{\sqrt{AB}\left(c_1\sqrt{ab}+\arctan\left(\frac{xb}{\sqrt{ab}}\right)\right)}{\sqrt{ab}}\right)\sqrt{AB}}{B}$$

✓ Solution by Mathematica

Time used: 27.024 (sec). Leaf size: 91

```
DSolve[(a+b x^2)y'[x]==(A+B y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{\sqrt{A}\tan\left(\sqrt{A}\sqrt{B}\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+c_1\right)\right)}{\sqrt{B}}$$

$$y(x)\to-\frac{i\sqrt{A}}{\sqrt{B}}$$

$$y(x)\to\frac{i\sqrt{A}}{\sqrt{B}}$$

12.21 problem 340

Internal problem ID [3087]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 340.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(b\,x^2+a)\,y'-cxy\ln(y)=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve((b*x^2+a)*diff(y(x),x) = c*x*y(x)*ln(y(x)),y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{\mathrm{e}^{c c_1}(b\,x^2+a)^{\frac{c}{2b}}}$$

✓ Solution by Mathematica

Time used: 0.353 (sec). Leaf size: 33

```
DSolve[(a+b x^2)y'[x]==c x y[x] Log[y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{e^{c_1}(a+bx^2)^{\frac{c}{2b}}}$$
$$y(x)\to 1$$

12.22 problem 341

Internal problem ID [3088]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 341.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(ax+1)\, y'+a-y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(x*(a*x+1)*diff(y(x),x)+a-y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{\left(\frac{a}{x}+c_1\right)x}{ax+1}$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 24

```
DSolve[x(1+a x)y'[x]+a-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{a+c_1x}{ax+1}$$
$$y(x)\to a$$

12.23 problem 342

Internal problem ID [3089]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 342.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$
\boxed{(bx+a)^2 y' + cy^2 + (bx+a)\, y^3 = 0}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 153

```
dsolve((b*x+a)^2*diff(y(x),x)+c*y(x)^2+(b*x+a)*y(x)^3 = 0,y(x), singsol=all)
```

$$
c_1 + \left(x + \frac{a}{b} + \frac{c\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(\frac{\sqrt{2}\,(b^2x+ab+cy(x))}{2\sqrt{b}\,y(x)(xb+a)}\right) e^{\frac{\left(b^2x+ab+cy(x)\right)^2}{2y(x)^2(xb+a)^2 b}}}{2b^{\frac{3}{2}}}\right) e^{-\frac{\left(b^2x+y(x)xb+ab+ay(x)+cy(x)\right)\left(b^2x-y(x)xb+ab-ay(x)+cy(x)\right)}{2y(x)^2(xb+a)^2 b}} = 0
$$

✓ Solution by Mathematica

Time used: 1.404 (sec). Leaf size: 149

```
DSolve[(a+b x)^2 y'[x]+c y[x]^2+(a+b x)y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{c}{\sqrt{-b(a+bx)^2}}=\frac{2\exp\left(\frac{1}{2}\left(-\frac{c}{\sqrt{-b(a+bx)^2}}-\frac{\left(-b(a+bx)^2\right)^{3/2}}{by(x)(a+bx)^3}\right)^2\right)}{\sqrt{2\pi}\,\text{erfi}\left(\frac{-\frac{c}{\sqrt{-b(a+bx)^2}}-\frac{\left(-b(a+bx)^2\right)^{3/2}}{by(x)(a+bx)^3}}{\sqrt{2}}\right)+2c_1},y(x)\right]$$

12.24 problem 343

Internal problem ID [3090]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 343.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x^3y' - yb\,x^2 - a = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x^3*diff(y(x),x) = a+b*x^2*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{a}{x^2\,(2+b)} + x^b c_1$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 23

```
DSolve[x^3 y'[x]==a + b x^2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a}{(b+2)x^2} + c_1x^b$$

12.25 problem 344

Internal problem ID [3091]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 344.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$x^3y' - 3 + x^2 - x^2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x^3*diff(y(x),x) = 3-x^2+x^2*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{1}{x^2} + 1 + c_1x$$

✓ Solution by Mathematica

Time used: 0.032 (sec). Leaf size: 16

```
DSolve[x^3 y'[x]==3 -x^2+x^2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{x^2} + c_1x + 1$$

12.26 problem 345

Internal problem ID [3092]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 345.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{x^3y' - x^4 - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x^3*diff(y(x),x) = x^4+y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x^2(\ln(x) - c_1 - 1)}{\ln(x) - c_1}$$

✓ Solution by Mathematica

Time used: 0.139 (sec). Leaf size: 27

```
DSolve[x^3 y'[x]==x^4+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2\left(1 - \frac{1}{\log(x) + c_1}\right)$$
$$y(x) \to x^2$$

12.27 problem 346

Internal problem ID [3093]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 346.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Bernoulli]`

$$\boxed{x^3y' - y(y + x^2) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^3*diff(y(x),x) = y(x)*(x^2+y(x)),y(x), singsol=all)
```

$$y(x) = \frac{x^2}{c_1x + 1}$$

✓ Solution by Mathematica

Time used: 0.127 (sec). Leaf size: 22

```
DSolve[x^3 y'[x]==y[x](x^2+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{1 + c_1x}$$

$$y(x) \to 0$$

12.28 problem 347

Internal problem ID [3094]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 347.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$\boxed{x^3y' - (y-1)\,x^2 - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 17

```
dsolve(x^3*diff(y(x),x) = x^2*(y(x)-1)+y(x)^2,y(x), singsol=all)
```

$$y(x) = -\tanh\left(\frac{c_1 x - 1}{x}\right) x$$

✓ Solution by Mathematica

Time used: 0.534 (sec). Leaf size: 28

```
DSolve[x^3 y'[x]==x^2(y[x]-1)+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\tanh\left(\frac{1}{x} - c_1\right)$$
$$y(x) \to -x$$
$$y(x) \to x$$

12.29 problem 348

Internal problem ID [3095]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 348.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$x^3y' - (x+1)\,y^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(x^3*diff(y(x),x) = (1+x)*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{2x^2}{2c_1x^2 + 2x + 1}$$

✓ Solution by Mathematica

Time used: 0.127 (sec). Leaf size: 29

```
DSolve[x^3 y'[x]==(1+x)y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^2}{-2c_1x^2 + 2x + 1}$$
$$y(x) \to 0$$

12.30 problem 349

Internal problem ID [3096]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 349.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Riccati]`

$$\boxed{x^3y' + 20 + x^2y(1 - x^2y) = 0}$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 26

```
dsolve(x^3*diff(y(x),x)+20+x^2*y(x)*(1-x^2*y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \frac{5x^9 + 4c_1}{x^2(-x^9 + c_1)}$$

✓ Solution by Mathematica

Time used: 0.15 (sec). Leaf size: 33

```
DSolve[x^3 y'[x]+20+x^2 y[x](1-x^2 y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4}{x^2} - \frac{9x^7}{x^9 + c_1}$$
$$y(x) \to \frac{4}{x^2}$$

12.31 problem 350

Internal problem ID [3097]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 350.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$x^3y' + 3 + (-2x + 3)\, x^2y - x^6y^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(x^3*diff(y(x),x)+3+(3-2*x)*x^2*y(x)-x^6*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{3(e^{4x}c_1 + 1)}{x^3\,(e^{4x}c_1 - 3)}$$

✓ Solution by Mathematica

Time used: 0.167 (sec). Leaf size: 34

```
DSolve[x^3 y'[x]+3+(3-2 x)x^2 y[x]-x^6 y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-3 + \frac{1}{\frac{1}{4}+c_1e^{4x}}}{x^3}$$

$$y(x) \to -\frac{3}{x^3}$$

12.32 problem 351

Internal problem ID [3098]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 351.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{x^3 y' - (2x^2 + y^2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 34

```
dsolve(x^3*diff(y(x),x) = (2*x^2+y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{x^2}{\sqrt{-x^2 + c_1}}$$
$$y(x) = -\frac{x^2}{\sqrt{-x^2 + c_1}}$$

✓ Solution by Mathematica

Time used: 0.164 (sec). Leaf size: 47

```
DSolve[x^3 y'[x]==(2 x^2+y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^2}{\sqrt{-x^2 + c_1}}$$
$$y(x) \to \frac{x^2}{\sqrt{-x^2 + c_1}}$$
$$y(x) \to 0$$

12.33 problem 352

Internal problem ID [3099]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 352.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{x^3y' - \cos(y)\left(\cos(y) - 2x^2\sin(y)\right) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^3*diff(y(x),x) = cos(y(x))*(cos(y(x))-2*x^2*sin(y(x))),y(x), singsol=all)
```

$$y(x) = \arctan\left(\frac{\ln(x) - c_1}{x^2}\right)$$

✓ Solution by Mathematica

Time used: 5.403 (sec). Leaf size: 55

```
DSolve[x^3 y'[x]==Cos[y[x]](Cos[y[x]]-2 x^2 Sin[y[x]]),y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \arctan\left(\frac{\log(x) + 4c_1}{x^2}\right)$$

$$y(x) \to -\frac{1}{2}\pi\sqrt{\frac{1}{x^4}}x^2$$

$$y(x) \to \frac{1}{2}\pi\sqrt{\frac{1}{x^4}}x^2$$

12.34 problem 353

Internal problem ID [3100]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 353.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(x^2+1)\,y' - x^2a - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*(x^2+1)*diff(y(x),x) = a*x^2+y(x),y(x), singsol=all)
```

$$y(x) = \frac{(a \operatorname{arcsinh}(x) + c_1)\,x}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 23

```
DSolve[x(1+x^2)y'[x]==a x^2+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x(a\text{arcsinh}(x) + c_1)}{\sqrt{x^2+1}}$$

12.35 problem 354

Internal problem ID [3101]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 12
Problem number: 354.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(1-x^2)\,y'-x^2a-y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve(x*(-x^2+1)*diff(y(x),x) = a*x^2+y(x),y(x), singsol=all)
```

$$y(x)=-\frac{xa\sqrt{(x-1)(x+1)}\,\ln\left(x+\sqrt{x^2-1}\right)}{(x-1)(x+1)}+\frac{xc_1}{\sqrt{x-1}\,\sqrt{x+1}}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 40

```
DSolve[x(1-x^2)y'[x]==a x^2+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x\left(-2a\cot^{-1}\left(\frac{x+1}{\sqrt{1-x^2}}\right)+c_1\right)}{\sqrt{1-x^2}}$$

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13.1 problem 355

Internal problem ID [3102]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 355.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(x^2+1)\,y'-a\,x^3-y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*(x^2+1)*diff(y(x),x) = a*x^3+y(x),y(x), singsol=all)
```

$$y(x)=ax+\frac{xc_1}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 21

```
DSolve[x(1+x^2)y'[x]==a x^3+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x\left(a+\frac{c_1}{\sqrt{x^2+1}}\right)$$

13.2 problem 356

Internal problem ID [3103]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 356.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$x(x^2+1)\,y' - a + x^2 y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(x*(x^2+1)*diff(y(x),x) = a-x^2*y(x),y(x), singsol=all)
```

$$y(x) = \frac{-a \operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right) + c_1}{\sqrt{x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.055 (sec). Leaf size: 31

```
DSolve[x(1+x^2)y'[x]==a-x^2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-a\operatorname{arctanh}\left(\sqrt{x^2+1}\right) + c_1}{\sqrt{x^2+1}}$$

13.3 problem 357

Internal problem ID [3104]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 357.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(x^2+1)\,y'-(1-x^2)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(x*(x^2+1)*diff(y(x),x) = (-x^2+1)*y(x),y(x), singsol=all)
```

$$y(x)=\frac{c_1x}{x^2+1}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 21

```
DSolve[x(1+x^2)y'[x]==(1-x^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_1x}{x^2+1}$$
$$y(x)\to 0$$

13.4 problem 358

Internal problem ID [3105]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 358.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$x(1-x^2)\,y' - (x^2-x+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*(-x^2+1)*diff(y(x),x) = (x^2-x+1)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{c_1 x}{\sqrt{x-1}\,(x+1)^{\frac{3}{2}}}$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 30

```
DSolve[x(1-x^2)y'[x]==(1-x+x^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1 x}{\sqrt{1-x}(x+1)^{3/2}}$$
$$y(x) \to 0$$

13.5 problem 359

Internal problem ID [3106]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 359.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(1-x^2)\, y' - a\, x^3 - (-2x^2+1)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*(-x^2+1)*diff(y(x),x) = a*x^3+(-2*x^2+1)*y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{x-1}\, x\sqrt{x+1}\, c_1 + ax$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 23

```
DSolve[x(1-x^2)y'[x]==a x^3+(1-2 x^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(a + c_1\sqrt{1-x^2}\right)$$

13.6 problem 360

Internal problem ID [3107]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 360.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$x(1-x^2)\,y' - x^3(1-x^2) - (-2x^2+1)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x*(-x^2+1)*diff(y(x),x) = x^3*(-x^2+1)+(-2*x^2+1)*y(x),y(x), singsol=all)
```

$$y(x) = x(x-1)\,(x+1) + \sqrt{x-1}\,x\sqrt{x+1}\,c_1$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 26

```
DSolve[x(1-x^2)y'[x]==x^3(1-x^2)+(1-2 x^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(x^2 + c_1\sqrt{1-x^2} - 1\right)$$

13.7 problem 361

Internal problem ID [3108]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 361.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(x^2+1)\, y' - 2 + 4x^2 y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*(x^2+1)*diff(y(x),x) = 2-4*x^2*y(x),y(x), singsol=all)
```

$$y(x) = \frac{x^2 + 2\ln(x) + c_1}{(x^2+1)^2}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 23

```
DSolve[x(1+x^2)y'[x]==2(1-2 x^2 y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2 + 2\log(x) + c_1}{(x^2+1)^2}$$

13.8 problem 362

Internal problem ID [3109]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 362.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(x^2+1)\,y' - x + (5x^2+3)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x*(x^2+1)*diff(y(x),x) = x-(5*x^2+3)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\frac{x^4}{4} + c_1}{x^3\,(x^2+1)}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 26

```
DSolve[x(1+x^2)y'[x]==x-(3+5 x^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^4 + 4c_1}{4\,(x^5 + x^3)}$$

13.9 problem 363

Internal problem ID [3110]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 363.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{x(1-x^2)\,y'+x^2+(1-x^2)\,y^2=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(x*(-x^2+1)*diff(y(x),x)+x^2+(-x^2+1)*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=-\frac{\text{EllipticK}(x)}{c_1\,\text{EllipticCE}(x)-c_1\,\text{EllipticCK}(x)+\text{EllipticE}(x)}+\frac{c_1\,\text{EllipticCE}(x)+\text{EllipticE}(x)}{c_1\,\text{EllipticCE}(x)-c_1\,\text{EllipticCK}(x)+\text{EllipticE}(x)}$$

✓ Solution by Mathematica

Time used: 0.466 (sec). Leaf size: 91

```
DSolve[x(1-x^2)y'[x]+x^2+(1-x^2)y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{2\left(\pi G_{2,2}^{2,0}\left(x^2\middle|\begin{array}{c}\frac{1}{2},\frac{3}{2}\\0,1\end{array}\right)+c_1(\text{EllipticK}(x^2)-\text{EllipticE}(x^2))\right)}{\pi G_{2,2}^{2,0}\left(x^2\middle|\begin{array}{c}\frac{1}{2},\frac{3}{2}\\0,0\end{array}\right)+2c_1\,\text{EllipticE}(x^2)}$$

$$y(x)\to1-\frac{\text{EllipticK}(x^2)}{\text{EllipticE}(x^2)}$$

13.10 problem 364

Internal problem ID [3111]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 364.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{x^2(1-x)\,y'-(2-x)\,xy+y^2=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(x^2*(1-x)*diff(y(x),x) = (2-x)*x*y(x)-y(x)^2,y(x), singsol=all)
```

$$y(x)=\frac{x^2}{c_1x-c_1+1}$$

✓ Solution by Mathematica

Time used: 0.187 (sec). Leaf size: 25

```
DSolve[x^2(1-x)y'[x]==(2-x)x y[x]-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x^2}{c_1(-x)+1+c_1}$$
$$y(x)\to 0$$

13.11 problem 365

Internal problem ID [3112]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 365.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{2x^3y' - (x^2 - y^2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(2*x^3*diff(y(x),x) = (x^2-y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{x}{\sqrt{c_1 x - 1}}$$
$$y(x) = -\frac{x}{\sqrt{c_1 x - 1}}$$

✓ Solution by Mathematica

Time used: 0.262 (sec). Leaf size: 39

```
DSolve[2 x^3 y'[x]==(x^2-y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{\sqrt{-1 + c_1 x}}$$
$$y(x) \to \frac{x}{\sqrt{-1 + c_1 x}}$$
$$y(x) \to 0$$

13.12 problem 366

Internal problem ID [3113]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 366.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{2x^3y' - (3x^2 + ay^2)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(2*x^3*diff(y(x),x) = (3*x^2+a*y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(-ax + c_1)\, x}\, x}{-ax + c_1}$$
$$y(x) = -\frac{\sqrt{(-ax + c_1)\, x}\, x}{-ax + c_1}$$

✓ Solution by Mathematica

Time used: 0.201 (sec). Leaf size: 49

```
DSolve[2 x^3 y'[x]==(3 x^2+a y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x^{3/2}}{\sqrt{-ax + c_1}}$$
$$y(x) \to \frac{x^{3/2}}{\sqrt{-ax + c_1}}$$
$$y(x) \to 0$$

13.13 problem 367

Internal problem ID [3114]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 367.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{6x^3y' - 4x^2y - (-3x+1)\,y^4 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 174

```
dsolve(6*x^3*diff(y(x),x) = 4*x^2*y(x)+(1-3*x)*y(x)^4,y(x), singsol=all)
```

$$y(x) = \frac{\left(-2x^2(-3x+\ln(x)-2c_1)^2\right)^{\frac{1}{3}}}{-3x+\ln(x)-2c_1}$$

$$y(x) = -\frac{\left(-2x^2(-3x+\ln(x)-2c_1)^2\right)^{\frac{1}{3}}}{2\left(-3x+\ln(x)-2c_1\right)} - \frac{i\sqrt{3}\left(-2x^2(-3x+\ln(x)-2c_1)^2\right)^{\frac{1}{3}}}{2\left(-3x+\ln(x)-2c_1\right)}$$

$$y(x) = -\frac{\left(-2x^2(-3x+\ln(x)-2c_1)^2\right)^{\frac{1}{3}}}{2\left(-3x+\ln(x)-2c_1\right)} + \frac{i\sqrt{3}\left(-2x^2(-3x+\ln(x)-2c_1)^2\right)^{\frac{1}{3}}}{-6x+2\ln(x)-4c_1}$$

✓ Solution by Mathematica

Time used: 0.187 (sec). Leaf size: 99

```
DSolve[6 x^3 y'[x]==4 x^2 y[x]+(1-3 x)y[x]^4,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{-2}x^{2/3}}{\sqrt[3]{3x - \log(x) + 2c_1}}$$

$$y(x) \to \frac{x^{2/3}}{\sqrt[3]{\frac{3x}{2} - \frac{\log(x)}{2} + c_1}}$$

$$y(x) \to \frac{(-1)^{2/3}x^{2/3}}{\sqrt[3]{\frac{3x}{2} - \frac{\log(x)}{2} + c_1}}$$

$$y(x) \to 0$$

13.14 problem 368

Internal problem ID [3115]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 368.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$\boxed{x(c x^2+bx+a)\, y'+x^2-(c x^2+bx+a)\, y-y^2=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 58

```
dsolve(x*(c*x^2+b*x+a)*diff(y(x),x)+x^2-(c*x^2+b*x+a)*y(x) = y(x)^2,y(x), singsol=all)
```

$$y(x) = -\tanh\left(\frac{c_1\sqrt{4ac-b^2}+2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}\right)x$$

✓ Solution by Mathematica

Time used: 1.125 (sec). Leaf size: 62

```
DSolve[x(a+b x +c x^2)y'[x]+x^2-(a+b x+c x^2)y[x]==y[x]^2,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -x\tanh\left(\frac{2\arctan\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+c_1\right)$$

$$y(x) \to -x$$

$$y(x) \to x$$

13.15 problem 369

Internal problem ID [3116]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 369.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{y' x^4 - (y + x^3)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x^4*diff(y(x),x) = (x^3+y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{2x^3}{2c_1x^2 + 1}$$

✓ Solution by Mathematica

Time used: 0.135 (sec). Leaf size: 26

```
DSolve[x^4 y'[x]==(x^3+y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{2x^3}{1 + 2c_1x^2}$$
$$y(x) \to 0$$

13.16 problem 370

Internal problem ID [3117]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 370.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Riccati, _special]]`

$$\boxed{y'x^4 + a^2 + y^2x^4 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(x^4*diff(y(x),x)+a^2+x^4*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{a^2}\,\tan\left(\frac{\sqrt{a^2}\,(c_1x-1)}{x}\right) - x}{x^2}$$

✓ Solution by Mathematica

Time used: 0.496 (sec). Leaf size: 54

```
DSolve[x^4 y'[x]+a^2+x^4 y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{x + a\left(\frac{1}{ac_1e^{\frac{2ia}{x}} - \frac{i}{2}} - i\right)}{x^2}$$

$$y(x) \rightarrow \frac{x - ia}{x^2}$$

13.17 problem 371

Internal problem ID [3118]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 371.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y'x^4 + x^3y + \csc(yx) = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 27

```
dsolve(x^4*diff(y(x),x)+x^3*y(x)+csc(x*y(x)) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{-\pi + \arccos\left(\frac{2c_1x^2+1}{2x^2}\right)}{x}$$

✓ Solution by Mathematica

Time used: 5.27 (sec). Leaf size: 40

```
DSolve[x^4 y'[x]+x^3 y[x]+ Csc[x y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\arccos\left(-\frac{1}{2x^2} + c_1\right)}{x}$$
$$y(x) \to \frac{\arccos\left(-\frac{1}{2x^2} + c_1\right)}{x}$$

13.18 problem 372

Internal problem ID [3119]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 372.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1-x^4)\,y'-2x(1-y^2)=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve((-x^4+1)*diff(y(x),x) = 2*x*(1-y(x)^2),y(x), singsol=all)
```

$$y(x)=-\tanh\left(\frac{\ln(x-1)}{2}-\frac{\ln(x^2+1)}{2}+\frac{\ln(x+1)}{2}+2c_1\right)$$

✓ Solution by Mathematica

Time used: 0.744 (sec). Leaf size: 43

```
DSolve[(1-x^4)y'[x]==2 x(1-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x^2\cosh(c_1)-\sinh(c_1)}{\cosh(c_1)-x^2\sinh(c_1)}$$
$$y(x)\to -1$$
$$y(x)\to 1$$

13.19 problem 373

Internal problem ID [3120]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 373.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{x(-x^3+1)\,y' - 2x + (-4x^3+1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x*(-x^3+1)*diff(y(x),x) = 2*x-(-4*x^3+1)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{-x^2 + c_1}{x\,(x^3 - 1)}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 21

```
DSolve[x(1-x^3)y'[x]==2 x-(1-4 x^3)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2 + c_1}{x - x^4}$$

13.20 problem 374

Internal problem ID [3121]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 374.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\boxed{x\left(-x^{3}+1\right)y'-x^{2}-\left(-2yx+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve(x*(-x^3+1)*diff(y(x),x) = x^2+(1-2*x*y(x))*y(x),y(x), singsol=all)
```

$$y(x)=\frac{x(x+c_1)}{c_1x^2+1}$$

✓ Solution by Mathematica

Time used: 0.328 (sec). Leaf size: 31

```
DSolve[x(1-x^3)y'[x]==x^2+(1-2 x y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x(1+2c_1x)}{x^2+2c_1}$$
$$y(x)\to x^2$$

13.21 problem 375

Internal problem ID [3122]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 375.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$x^2(1-x^2)\,y'-(x-3x^3y)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 72

```
dsolve(x^2*(-x^2+1)*diff(y(x),x) = (x-3*x^3*y(x))*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{x^2-1}\,x}{\sqrt{x-1}\,\sqrt{x+1}\,c_1\sqrt{x^2-1}-3\ln\left(x+\sqrt{x^2-1}\right)x^2+3\sqrt{x^2-1}\,x+3\ln\left(x+\sqrt{x^2-1}\right)}$$

✓ Solution by Mathematica

Time used: 0.291 (sec). Leaf size: 53

```
DSolve[x^2(1-x^2)y'[x]==(x-3 x^3 y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x}{3x+\sqrt{1-x^2}\left(-6\arctan\left(\frac{x}{\sqrt{1-x^2}-1}\right)+c_1\right)}$$

$$y(x)\to 0$$

13.22 problem 376

Internal problem ID [3123]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 376.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(-2x^3+1)\,y'-2(-x^3+1)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*(-2*x^3+1)*diff(y(x),x) = 2*(-x^3+1)*y(x),y(x), singsol=all)
```

$$y(x)=\frac{c_1x^2}{(2x^3-1)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 27

```
DSolve[x(1-2 x^3) y'[x]==2(1-x^3)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_1x^2}{\sqrt[3]{1-2x^3}}$$
$$y(x)\to 0$$

13.23 problem 377

Internal problem ID [3124]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 377.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$\left(c\,x^{2}+bx+a\right)^{2}\left(y'+y^{2}\right)+A=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 846

```
dsolve((c*x^2+b*x+a)^2*(diff(y(x),x)+y(x)^2)+A = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2\left(-i\sqrt{-\frac{4ac-b^{2}+4A}{c^{2}}}\,\sqrt{4ac-b^{2}}\left(\frac{i\sqrt{4ac-b^{2}}-2cx-b}{2cx+b+i\sqrt{4ac-b^{2}}}\right)^{-\frac{c\sqrt{-\frac{4ac-b^{2}+4A}{c^{2}}}}{2\sqrt{-4ac+b^{2}}}} c_{1}c+i\sqrt{-\frac{4ac-b^{2}+4A}{c^{2}}}\,\sqrt{4ac-b^{2}}\left(\frac{i\sqrt{4ac-b^{2}}-\cdots}{2cx+b+i\sqrt{4\cdots}}\right.\cdots}{\sqrt{-4ac\cdots}}$$

✓ Solution by Mathematica

Time used: 2.706 (sec). Leaf size: 312

```
DSolve[(a+b x+c x^2)^2 (y'[x]+y[x]^2)+A==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\frac{2(-4ac-4A+b^2)}{1+c_1\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}}\exp\left(\frac{2\sqrt{4ac-b^2}\sqrt{1-\frac{4A}{b^2-4ac}}\arctan\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{b^2-4ac}}\right)} + 2cx\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}} + b\sqrt{b^2-4ac}\sqrt{1-}}{2\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}}(a+x(b+cx))}$$

$$y(x) \to \frac{-\sqrt{b^2-4ac}\sqrt{1-\frac{4A}{b^2-4ac}}+b+2cx}{2(a+x(b+cx))}$$

13.24 problem 378

Internal problem ID [3125]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 378.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'x^5 - 1 + 3yx^4 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^5*diff(y(x),x) = 1-3*x^4*y(x),y(x), singsol=all)
```

$$y(x) = \frac{-\frac{1}{x} + c_1}{x^3}$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 15

```
DSolve[x^5 y'[x]==1-3 x^4 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-1 + c_1 x}{x^4}$$

13.25 problem 379

Internal problem ID [3126]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 13
Problem number: 379.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Riccati]`

$$x(1-x^4)\,y' - 2x(x^2-y^2) - (1-x^4)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(x*(-x^4+1)*diff(y(x),x) = 2*x*(x^2-y(x)^2)+(-x^4+1)*y(x),y(x), singsol=all)
```

$$y(x) = -\tanh\left(\frac{\ln(x-1)}{2} - \frac{\ln(x^2+1)}{2} + \frac{\ln(x+1)}{2} + 2c_1\right)x$$

✓ Solution by Mathematica

Time used: 0.321 (sec). Leaf size: 46

```
DSolve[x(1-x^4)y'[x]==2 x(x^2-y[x]^2)+(1-x^4) y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^3\cosh(c_1) - x\sinh(c_1)}{\cosh(c_1) - x^2\sinh(c_1)}$$
$$y(x) \to -x$$
$$y(x) \to x$$

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14.1 problem 380

Internal problem ID [3127]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 380.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Abel]`

$$\boxed{y'x^7 + 5x^3y^2 + 2(x^2+1)\,y^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
dsolve(x^7*diff(y(x),x)+5*x^3*y(x)^2+2*(x^2+1)*y(x)^3 = 0,y(x), singsol=all)
```

$$c_1 + \frac{x}{\left(\left(\frac{1}{x} + \frac{x^2}{y(x)}\right)^2 + 1\right)^{\frac{1}{4}}} + \frac{(x^3 + y(x))\,\mathrm{hypergeom}\left(\left[\frac{1}{2}, \frac{5}{4}\right], \left[\frac{3}{2}\right], -\frac{(x^3+y(x))^2}{x^2y(x)^2}\right)}{2y(x)\,x} = 0$$

✓ Solution by Mathematica

Time used: 0.298 (sec). Leaf size: 123

```
DSolve[x^7 y'[x]+5 x^3 y[x]^2+2(1+x^2)y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[c_1 = \frac{\frac{1}{2}\sqrt[4]{1 - \left(\frac{ix^2}{y(x)} + \frac{i}{x}\right)^2}\left(\frac{ix^2}{y(x)} + \frac{i}{x}\right)\mathrm{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{4}, \frac{3}{2}, \left(\frac{ix^2}{y(x)} + \frac{i}{x}\right)^2\right) + ix}{\sqrt[4]{-1 + \left(\frac{ix^2}{y(x)} + \frac{i}{x}\right)^2}}, y(x)\right]$$

14.2 problem 381

Internal problem ID [3128]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 381.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y' x^n - a - b\,x^{n-1} y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve(x^n*diff(y(x),x) = a+b*x^(n-1)*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{a\,x^{-n+1}}{n+b-1} + x^b c_1$$

✓ Solution by Mathematica

Time used: 0.102 (sec). Leaf size: 28

```
DSolve[x^n y'[x]==a+b x^(n-1) y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ax^{1-n}}{b+n-1} + c_1 x^b$$

14.3 problem 382

Internal problem ID [3129]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 382.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$\boxed{y'x^n - x^{-1+2n} + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
dsolve(x^n*diff(y(x),x) = x^(2*n-1)-y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(-\operatorname{BesselK}\left(n, 2\sqrt{x}\right) c_1 + \operatorname{BesselI}\left(n, 2\sqrt{x}\right)\right) x^n}{\sqrt{x}\ \left(\operatorname{BesselK}\left(n-1, 2\sqrt{x}\right) c_1 + \operatorname{BesselI}\left(n-1, 2\sqrt{x}\right)\right)}$$

✓ Solution by Mathematica

Time used: 0.312 (sec). Leaf size: 80

```
DSolve[x^n y'[x]==x^(2 n -1)-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^n\left((n-1)\,{}_0F_1(;1-n;x) + c_1(-1)^n x^n \operatorname{Gamma}(n)\,{}_0\tilde{F}_1(;n+1;x)\right)}{-x\,{}_0F_1(;2-n;x) + c_1(-1)^n x^n\,{}_0F_1(;n;x)}$$

$$y(x) \to \frac{x^n\,{}_0\tilde{F}_1(;n+1;x)}{{}_0\tilde{F}_1(;n;x)}$$

14.4 problem 384

Internal problem ID [3130]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 384.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Riccati]`

$$y'x^n + x^{-2+2n} + y^2 + (-n+1)\,x^{n-1} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 1097

```
dsolve(x^n*diff(y(x),x)+x^(2*n-2)+y(x)^2+(1-n)*x^(n-1) = 0,y(x), singsol=all)
```

Expression too large to display

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^n y'[x]+x^(2 n-2)+y[x]^2+(1-n)x^(n-1)==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

14.5 problem 385

Internal problem ID [3131]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 385.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _Riccati]`

$$\boxed{y'x^n - a^2x^{-2+2n} - y^2b^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 68

```
dsolve(x^n*diff(y(x),x) = a^2*x^(2*n-2)+b^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{x^{n-1}\left(n-1-\tan\left(\frac{\sqrt{4a^2b^2-n^2+2n-1}\,(-\ln(x)+c_1)}{2}\right)\sqrt{4a^2b^2-n^2+2n-1}\right)}{2b^2}$$

✓ Solution by Mathematica

Time used: 0.472 (sec). Leaf size: 115

```
DSolve[x^n y'[x]==a^2 x^(2 n-2)+b^2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^{n-1}\left(ab\sqrt{\frac{(n-1)^2}{a^2b^2}-4}\left(-1+\frac{2c_1}{x^{ab\sqrt{\frac{(n-1)^2}{a^2b^2}-4}}+c_1}\right)+n-1\right)}{2b^2}$$

$$y(x) \to \frac{x^{n-1}\left(ab\sqrt{\frac{(n-1)^2}{a^2b^2}-4}+n-1\right)}{2b^2}$$

14.6 problem 386

Internal problem ID [3132]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 386.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Riccati]`

$$y'x^n - x^{n-1}\left(a\,x^{2n} + ny - by^2\right) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
dsolve(x^n*diff(y(x),x) = x^(n-1)*(a*x^(2*n)+n*y(x)-b*y(x)^2),y(x), singsol=all)
```

$$y(x) = -\frac{i\tan\left(\frac{i x^n \sqrt{a}\,\sqrt{b} - c_1 n}{n}\right)\sqrt{a}\,x^n}{\sqrt{b}}$$

✓ Solution by Mathematica

Time used: 0.323 (sec). Leaf size: 145

```
DSolve[x^n y'[x]==x^(n-1)(a x^(2 n)+n y[x]-b y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}x^n\left(-\cosh\left(\frac{\sqrt{a}\sqrt{b}x^n}{n}\right) + c_1\sin\left(\frac{\sqrt{a}\sqrt{-b}x^n}{n}\right)\right)}{\sqrt{-b}\left(\sin\left(\frac{\sqrt{a}\sqrt{-b}x^n}{n}\right) + c_1\cosh\left(\frac{\sqrt{a}\sqrt{b}x^n}{n}\right)\right)}$$

$$y(x) \to \frac{\sqrt{a}x^n\tanh\left(\frac{\sqrt{a}\sqrt{b}x^n}{n}\right)}{\sqrt{b}}$$

14.7 problem 388

Internal problem ID [3133]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 388.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Chini]`

$$x^k y' - a\, x^m - b y^n = 0$$

✗ Solution by Maple

```
dsolve(x^k*diff(y(x),x) = a*x^m+b*y(x)^n,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^k y'[x]==a x^m + b y[x]^n,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

14.8 problem 389

Internal problem ID [3134]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 389.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{\sqrt{x^2+1}\,y' - 2x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x)*sqrt(x^2+1) = 2*x-y(x),y(x), singsol=all)
```

$$y(x) = \frac{x^2 + x\sqrt{x^2+1} - \operatorname{arcsinh}(x) + c_1}{x + \sqrt{x^2+1}}$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 39

```
DSolve[y'[x] Sqrt[1+x^2]==2 x -y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\sqrt{x^2+1} - x\right)\left(-\operatorname{arcsinh}(x) + x\left(\sqrt{x^2+1} + x\right) + c_1\right)$$

14.9 problem 390

Internal problem ID [3135]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 390.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'\sqrt{1-x^2}-1-y^2=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 9

```
dsolve(diff(y(x),x)*sqrt(-x^2+1) = 1+y(x)^2,y(x), singsol=all)
```

$$y(x)=\tan\left(\arcsin\left(x\right)+c_1\right)$$

✓ Solution by Mathematica

Time used: 0.256 (sec). Leaf size: 45

```
DSolve[y'[x] Sqrt[1-x^2]==1+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\rightarrow -\tan\left(2\cot^{-1}\left(\frac{x+1}{\sqrt{1-x^2}}\right)-c_1\right)$$
$$y(x)\rightarrow -i$$
$$y(x)\rightarrow i$$

14.10 problem 391

Internal problem ID [3136]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 391.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\left(-\sqrt{x^2+1}+x\right)y'-y-\sqrt{1+y^2}=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve((x-sqrt(x^2+1))*diff(y(x),x) = y(x)+sqrt(1+y(x)^2),y(x), singsol=all)
```

$$c_1+x^2+x\sqrt{x^2+1}+\operatorname{arcsinh}(x)+y(x)\sqrt{y(x)^2+1}+\operatorname{arcsinh}(y(x))-y(x)^2=0$$

✓ Solution by Mathematica

Time used: 0.885 (sec). Leaf size: 73

```
DSolve[(x-Sqrt[1+x^2])y'[x]==y[x]+Sqrt[1+ y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \text{InverseFunction}\left[\frac{1}{2}\left(\#1\left(\sqrt{\#1^2+1}-\#1\right)-\log\left(\sqrt{\#1^2+1}-\#1\right)\right)\&\right]\left[-\frac{\operatorname{arcsinh}(x)}{2}-\frac{1}{2}x\left(\sqrt{x^2+1}+x\right)+c_1\right]$$

14.11 problem 392

Internal problem ID [3137]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 392.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{y'\sqrt{a^2+x^2}+x+y-\sqrt{a^2+x^2}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x)*sqrt(a^2+x^2)+x+y(x) = sqrt(a^2+x^2),y(x), singsol=all)
```

$$y(x)=\frac{a^2\ln\left(x+\sqrt{a^2+x^2}\right)+c_1}{x+\sqrt{a^2+x^2}}$$

✓ Solution by Mathematica

Time used: 8.132 (sec). Leaf size: 78

```
DSolve[y'[x] Sqrt[a^2+x^2]+x+y[x]==Sqrt[a^2 + x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\left(\sqrt{a^2+x^2}-x\right)\operatorname{arctanh}\left(\frac{x}{\sqrt{a^2+x^2}}\right)+\frac{c_1\sqrt{1-\frac{x}{\sqrt{a^2+x^2}}}}{\sqrt{\frac{x}{\sqrt{a^2+x^2}}+1}}$$

14.12 problem 393

Internal problem ID [3138]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 393.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'\sqrt{b^2+x^2}-\sqrt{y^2+a^2}=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(y(x),x)*sqrt(b^2+x^2) = sqrt(y(x)^2+a^2),y(x), singsol=all)
```

$$\ln\left(x+\sqrt{b^2+x^2}\right)-\ln\left(y(x)+\sqrt{y(x)^2+a^2}\right)+c_1=0$$

✓ Solution by Mathematica

Time used: 10.997 (sec). Leaf size: 150

```
DSolve[y'[x] Sqrt[x^2+b^2]==Sqrt[y[x]^2+a^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{e^{-c_1}\sqrt{a^2e^{2c_1}\left(x\sqrt{b^2+x^2}\sinh(2c_1)+b^2\sinh^2(c_1)+x^2\cosh(2c_1)\right)}}{b}$$

$$y(x)\to \frac{e^{-c_1}\sqrt{a^2e^{2c_1}\left(x\sqrt{b^2+x^2}\sinh(2c_1)+b^2\sinh^2(c_1)+x^2\cosh(2c_1)\right)}}{b}$$

$$y(x)\to -ia$$

$$y(x)\to ia$$

14.13 problem 394

Internal problem ID [3139]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 394.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y'\sqrt{b^2-x^2}-\sqrt{a^2-y^2}=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(diff(y(x),x)*sqrt(b^2-x^2) = sqrt(a^2-y(x)^2),y(x), singsol=all)
```

$$\arctan\left(\frac{x}{\sqrt{b^2-x^2}}\right)-\arctan\left(\frac{y(x)}{\sqrt{a^2-y(x)^2}}\right)+c_1=0$$

✓ Solution by Mathematica

Time used: 4.819 (sec). Leaf size: 118

```
DSolve[y'[x] Sqrt[b^2-x^2]==Sqrt[a^2-y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{a\tan\left(\arctan\left(\frac{x}{\sqrt{b^2-x^2}}\right)+c_1\right)}{\sqrt{\sec^2\left(\arctan\left(\frac{x}{\sqrt{b^2-x^2}}\right)+c_1\right)}}$$

$$y(x)\to \frac{a\tan\left(\arctan\left(\frac{x}{\sqrt{b^2-x^2}}\right)+c_1\right)}{\sqrt{\sec^2\left(\arctan\left(\frac{x}{\sqrt{b^2-x^2}}\right)+c_1\right)}}$$

$$y(x)\to -a$$

$$y(x)\to a$$

14.14 problem 395

Internal problem ID [3140]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 395.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y'\sqrt{a^2+x^2}\,x - y\sqrt{b^2+y^2} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 74

```
dsolve(x*diff(y(x),x)*sqrt(a^2+x^2) = y(x)*sqrt(b^2+y(x)^2),y(x), singsol=all)
```

$$-\frac{\ln\left(\frac{2a^2+2\sqrt{a^2}\sqrt{a^2+x^2}}{x}\right)}{\sqrt{a^2}} + \frac{\ln\left(\frac{2b^2+2\sqrt{b^2}\sqrt{b^2+y(x)^2}}{y(x)}\right)}{\sqrt{b^2}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 25.279 (sec). Leaf size: 274

```
DSolve[x y'[x] Sqrt[a^2+x^2]==y[x] Sqrt[b^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2b^{3/2}e^{bc_1}\left(a\left(\sqrt{a^2+x^2}-a\right)\right)^{\frac{b}{2a}}\left(\sqrt{a^2+x^2}+a\right)^{\frac{b}{2a}}}{\sqrt{\left(-b\left(\sqrt{a^2+x^2}+a\right)^{\frac{b}{a}}+e^{2bc_1}\left(a\left(\sqrt{a^2+x^2}-a\right)\right)^{\frac{b}{a}}\right)^2}}$$

$$y(x) \to \frac{2b^{3/2}e^{bc_1}\left(a\left(\sqrt{a^2+x^2}-a\right)\right)^{\frac{b}{2a}}\left(\sqrt{a^2+x^2}+a\right)^{\frac{b}{2a}}}{\sqrt{\left(-b\left(\sqrt{a^2+x^2}+a\right)^{\frac{b}{a}}+e^{2bc_1}\left(a\left(\sqrt{a^2+x^2}-a\right)\right)^{\frac{b}{a}}\right)^2}}$$

$$y(x) \to 0$$

$$y(x) \to -ib$$

$$y(x) \to ib$$

14.15 problem 396

Internal problem ID [3141]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 396.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy'\sqrt{-a^2+x^2}-y\sqrt{y^2-b^2}=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 86

```
dsolve(x*diff(y(x),x)*sqrt(-a^2+x^2) = y(x)*sqrt(y(x)^2-b^2),y(x), singsol=all)
```

$$-\frac{\ln\left(\frac{-2a^2+2\sqrt{-a^2}\sqrt{-a^2+x^2}}{x}\right)}{\sqrt{-a^2}}+\frac{\ln\left(\frac{-2b^2+2\sqrt{-b^2}\sqrt{y(x)^2-b^2}}{y(x)}\right)}{\sqrt{-b^2}}+c_1=0$$

✓ Solution by Mathematica

Time used: 17.983 (sec). Leaf size: 95

```
DSolve[x y'[x] Sqrt[x^2-a^2]==y[x] Sqrt[y[x]^2-b^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -b\sqrt{\sec^2\left(b\left(\frac{\cot^{-1}\left(\frac{a}{\sqrt{x^2-a^2}}\right)}{a}+c_1\right)\right)}$$
$$y(x)\to b\sqrt{\sec^2\left(b\left(\frac{\cot^{-1}\left(\frac{a}{\sqrt{x^2-a^2}}\right)}{a}+c_1\right)\right)}$$
$$y(x)\to 0$$
$$y(x)\to -b$$
$$y(x)\to b$$

14.16 problem 397

Internal problem ID [3142]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 397.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'\sqrt{X}+\sqrt{Y}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)*sqrt(X)+sqrt(Y) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{Y}\,x}{\sqrt{X}} + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 21

```
DSolve[y'[x] Sqrt[X]+Sqrt[Y]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x\sqrt{Y}}{\sqrt{X}} + c_1$$

14.17 problem 398

Internal problem ID [3143]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 398.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'\sqrt{X} - \sqrt{Y} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)*sqrt(X) = sqrt(Y),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{Y}\,x}{\sqrt{X}} + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y'[x] Sqrt[X]==Sqrt[Y],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x\sqrt{Y}}{\sqrt{X}} + c_1$$

14.18 problem 399

Internal problem ID [3144]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 399.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Riccati, _special]]`

$$\boxed{x^{\frac{3}{2}}y' - a - b\,x^{\frac{3}{2}}y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 122

```
dsolve(x^(3/2)*diff(y(x),x) = a+b*x^(3/2)*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{2a\left(\text{BesselJ}\left(1,4\sqrt{a}\,\sqrt{b}\,x^{\frac{1}{4}}\right)c_1 + \text{BesselY}\left(1,4\sqrt{a}\,\sqrt{b}\,x^{\frac{1}{4}}\right)\right)}{\sqrt{x}\,\left(2\sqrt{a}\,\text{BesselJ}\left(0,4\sqrt{a}\,\sqrt{b}\,x^{\frac{1}{4}}\right)x^{\frac{1}{4}}\sqrt{b}\,c_1 + 2\,\text{BesselY}\left(0,4\sqrt{a}\,\sqrt{b}\,x^{\frac{1}{4}}\right)\sqrt{a}\,\sqrt{b}\,x^{\frac{1}{4}} - \text{BesselJ}\left(1,4\sqrt{a}\,\sqrt{b}\right.\right.}$$

✓ Solution by Mathematica

Time used: 0.213 (sec). Leaf size: 146

```
DSolve[x^(3/2) y'[x]==a+ b x^(3/2) y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{a}\left(-Y_1\left(4\sqrt{a}\sqrt{b}\sqrt[4]{x}\right) + c_1\,\text{BesselJ}\left(1,4\sqrt{a}\sqrt{b}\sqrt[4]{x}\right)\right)}{\sqrt{b}x^{3/4}\left(Y_2\left(4\sqrt{a}\sqrt{b}\sqrt[4]{x}\right) - c_1\,\text{BesselJ}\left(2,4\sqrt{a}\sqrt{b}\sqrt[4]{x}\right)\right)}$$

$$y(x) \to -\frac{{}_0\tilde{F}_1\left(;2;-4ab\sqrt{x}\right)}{bx\,{}_0F_1\left(;3;-4ab\sqrt{x}\right)}$$

14.19 problem 400

Internal problem ID [3145]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 400.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y'\sqrt{x^3+1}-\sqrt{y^3+1}=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve(diff(y(x),x)*sqrt(x^3+1) = sqrt(1+y(x)^3),y(x), singsol=all)
```

$$\int \frac{1}{\sqrt{x^3+1}}dx-\left(\int^{y(x)}\frac{1}{\sqrt{\_a^3+1}}d\_a\right)+c_1=0$$

✓ Solution by Mathematica

Time used: 40.442 (sec). Leaf size: 71

```
DSolve[y'[x] Sqrt[1+x^3]==Sqrt[1+y[x]^3],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \text{InverseFunction}\left[\#1\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\#1^3\right)\&\right]\left[x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)+c_1\right]$$

$$y(x)\to -1$$

$$y(x)\to \sqrt[3]{-1}$$

$$y(x)\to -(-1)^{2/3}$$

14.20 problem 401

Internal problem ID [3146]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 401.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'\sqrt{x(1-x)(-ax+1)}-\sqrt{y(1-y)(1-ay)}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x)*sqrt(x*(1-x)*(-a*x+1)) = sqrt(y(x)*(1-y(x))*(1-a*y(x))),y(x), singsol=all
```

$$\int \frac{1}{\sqrt{x(x-1)(ax-1)}}dx-\left(\int^{y(x)}\frac{1}{\sqrt{\_a(\_a-1)(a\_a-1)}}d\_a\right)+c_1=0$$

✓ Solution by Mathematica

Time used: 15.548 (sec). Leaf size: 72

```
DSolve[y'[x] Sqrt[x (1-x)(1-a x)]==Sqrt[y[x](1-y[x])(1-a y[x])],y[x],x,IncludeSingularSolutio
```

$$y(x)\to 1-\frac{1}{\operatorname{sn}\left(\frac{1}{2}i\sqrt{a}c_1-\operatorname{EllipticF}\left(i\operatorname{csch}^{-1}\left(\sqrt{x-1}\right),\frac{a-1}{a}\right)|\frac{a-1}{a}\right)^2}$$

$$y(x)\to 0$$

$$y(x)\to 1$$

$$y(x)\to \frac{1}{a}$$

14.21 problem 402

Internal problem ID [3147]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 402.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'\sqrt{1-x^4}-\sqrt{1-y^4}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)*sqrt(-x^4+1) = sqrt(1-y(x)^4),y(x), singsol=all)
```

$$\int \frac{1}{\sqrt{-x^4+1}}dx-\left(\int^{y(x)}\frac{1}{\sqrt{-\_a^4+1}}d\_a\right)+c_1=0$$

✓ Solution by Mathematica

Time used: 40.376 (sec). Leaf size: 38

```
DSolve[y'[x] Sqrt[1-x^4]==Sqrt[1-y[x]^4],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \operatorname{sn}(c_1+\operatorname{EllipticF}(\arcsin(x),-1)|-1)$$
$$y(x)\to -1$$
$$y(x)\to -i$$
$$y(x)\to i$$
$$y(x)\to 1$$

14.22 problem 403

Internal problem ID [3148]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 403.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'\sqrt{x^4+x^2+1}-\sqrt{1+y^2+y^4}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(diff(y(x),x)*sqrt(x^4+x^2+1) = sqrt(1+y(x)^2+y(x)^4),y(x), singsol=all)
```

$$\int \frac{1}{\sqrt{x^4+x^2+1}}dx-\left(\int^{y(x)}\frac{1}{\sqrt{\_a^4+\_a^2+1}}d\_a\right)+c_1=0$$

✓ Solution by Mathematica

Time used: 41.479 (sec). Leaf size: 189

```
DSolve[y'[x]Sqrt[1+x^2+x^4]==Sqrt[1+y[x]^2+y[x]^4],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{(-1)^{2/3}\sqrt{\sqrt[3]{-1}\#1^2+1}\sqrt{1-(-1)^{2/3}\#1^2}\,\text{EllipticF}\left(i\,\text{arcsinh}\left((-1)^{5/6}\#1\right),(-1)^{2/3}\right)}{\sqrt{\#1^4+\#1^2+1}}\&\right]$$

$$y(x) \to -\sqrt[3]{-1}$$

$$y(x) \to \sqrt[3]{-1}$$

$$y(x) \to -(-1)^{2/3}$$

$$y(x) \to (-1)^{2/3}$$

14.23 problem 404

Internal problem ID [3149]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 404.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'\sqrt{X} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 5

```
dsolve(diff(y(x),x)*sqrt(X) = 0,y(x), singsol=all)
```

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 7

```
DSolve[y'[x] Sqrt[X]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

14.24 problem 405

Internal problem ID [3150]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 405.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'\sqrt{X}+\sqrt{Y}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(diff(y(x),x)*sqrt(X)+sqrt(Y) = 0,y(x), singsol=all)
```

$$y(x)=-\frac{\sqrt{Y}\,x}{\sqrt{X}}+c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 21

```
DSolve[y'[x] Sqrt[X]+Sqrt[Y]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{x\sqrt{Y}}{\sqrt{X}}+c_1$$

14.25 problem 406

Internal problem ID [3151]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 406.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'\sqrt{X}-\sqrt{Y}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)*sqrt(X) = sqrt(Y),y(x), singsol=all)
```

$$y(x)=\frac{\sqrt{Y}\,x}{\sqrt{X}}+c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[y'[x] Sqrt[X]==Sqrt[Y],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x\sqrt{Y}}{\sqrt{X}}+c_1$$

14.26 problem 407

Internal problem ID [3152]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 407.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y'\left(x^{3}+1\right)^{\frac{2}{3}}+\left(y^{3}+1\right)^{\frac{2}{3}}=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(diff(y(x),x)*(x^3+1)^(2/3)+(1+y(x)^3)^(2/3) = 0,y(x), singsol=all)
```

$$x\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{2}{3}\right],\left[\frac{4}{3}\right],-x^{3}\right)+y(x)\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{2}{3}\right],\left[\frac{4}{3}\right],-y(x)^{3}\right)+c_{1}=0$$

✓ Solution by Mathematica

Time used: 2.834 (sec). Leaf size: 221

```
DSolve[y'[x](1+x^3)^(2/3)+(1+y[x]^3)^(2/3)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \text{InverseFunction}\left[\frac{3\sqrt[3]{\frac{\sqrt[3]{-1}-\#1}{1+\sqrt[3]{-1}}}(\#1+1)\left(\frac{\#1+(-1)^{2/3}}{(-1)^{2/3}-1}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{\sqrt[3]{-1}(\#1+1)}{\left(-1+\sqrt[3]{-1}\right)\#1\ldots}\right.}{\left(\#1^{3}+1\right)^{2/3}}\right.$$

$$y(x) \rightarrow -1$$

$$y(x) \rightarrow \sqrt[3]{-1}$$

$$y(x) \rightarrow -(-1)^{2/3}$$

14.27 problem 408

Internal problem ID [3153]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 14
Problem number: 408.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y'\left(4x^3 + \mathrm{a1}\,x + \mathrm{a0}\right)^{\frac{2}{3}} + \left(\mathrm{a0} + \mathrm{a1}\,y + 4y^3\right)^{\frac{2}{3}} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(diff(y(x),x)*(4*x^3+a1*x+a0)^(2/3)+(a0+a1*y(x)+4*y(x)^3)^(2/3) = 0,y(x), singsol=all)
```

$$\int \frac{1}{\left(4x^3 + \mathrm{a1}\,x + \mathrm{a0}\right)^{\frac{2}{3}}}dx + \int^{y(x)} \frac{1}{\left(4\_a^3 + \_a\,\mathrm{a1} + \mathrm{a0}\right)^{\frac{2}{3}}}d\_a + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.512 (sec). Leaf size: 558

```
DSolve[y'[x](a0+a1 x+4 x^3)^(2/3)+(a0+a1 y[x]+4 y[x]^3)^(2/3)==0,y[x],x,IncludeSingularSoluti
```

$$\text{Solve}\left[\frac{3(y(x)-\text{Root}[4\#1^3+\#1\text{a1}+\text{a0}\&,1])\left(\frac{y(x)-\text{Root}[4\#1^3+\#1\text{a1}+\text{a0}\&,2]}{\text{Root}[4\#1^3+\#1\text{a1}+\text{a0}\&,1]-\text{Root}[4\#1^3+\#1\text{a1}+\text{a0}\&,2]}\right)^{2/3}\sqrt[3]{\ldots}}{\ldots}\right.$$

$$-\frac{3(x-\text{Root}[4\#1^3+\#1\text{a1}+\text{a0}\&,1])\left(\frac{x-\text{Root}[4\#1^3+\#1\text{a1}+\text{a0}\&,2]}{\text{Root}[4\#1^3+\#1\text{a1}+\text{a0}\&,1]-\text{Root}[4\#1^3+\#1\text{a1}+\text{a0}\&,2]}\right)^{2/3}\sqrt[3]{\frac{\ldots}{\text{Root}[\ldots}}}{\ldots}$$

$$\left.+c_1,y(x)\right]$$

15 Various 15

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15.1 problem 409

Internal problem ID [3154]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 409.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{X^{\frac{2}{3}}y' - Y^{\frac{2}{3}} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(X^(2/3)*diff(y(x),x) = Y^(2/3),y(x), singsol=all)
```

$$y(x) = \frac{Y^{\frac{2}{3}}x}{X^{\frac{2}{3}}} + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 20

```
DSolve[X^(2/3) y'[x]== Y^(2/3),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{xY^{2/3}}{X^{2/3}} + c_1$$

15.2 problem 410

Internal problem ID [3155]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 410.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$y'\left(a+\cos\left(\frac{x}{2}\right)^2\right)-y\tan\left(\frac{x}{2}\right)\left(1+a+\cos\left(\frac{x}{2}\right)^2-y\right)=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 125

```
dsolve(diff(y(x),x)*(a+cos(1/2*x)^2) = y(x)*tan(1/2*x)*(1+a+cos(1/2*x)^2-y(x)),y(x), singsol=
```

$$y(x) = \frac{(2a+1+\cos(x))^{\frac{1}{a}}(\cos(x)+1)^{-\frac{1}{a}}}{\cos(x)\left(\int\frac{2(2a+1+\cos(x))^{\frac{1}{a}}(\cos(x)+1)^{-\frac{1}{a}}\tan\left(\frac{x}{2}\right)}{(\cos(x)+1)(2a+1+\cos(x))}dx\right)+\cos(x)c_1+\int\frac{2(2a+1+\cos(x))^{\frac{1}{a}}(\cos(x)+1)^{-\frac{1}{a}}\tan\left(\frac{x}{2}\right)}{(\cos(x)+1)(2a+1+\cos(x))}dx+c_1}$$

✓ Solution by Mathematica

Time used: 1.644 (sec). Leaf size: 60

```
DSolve[y'[x](a+Cos[x/2]^2)==y[x] Tan[x/2](1+a+Cos[x/2]^2-y[x]),y[x],x,IncludeSingularSolution
```

$$y(x)\to\frac{1}{\frac{\sin^2\left(\frac{x}{2}\right)}{a+1}+c_1\left(a+\cos^2\left(\frac{x}{2}\right)\right)^{-1/a}\cos^{\frac{2}{a}+2}\left(\frac{x}{2}\right)}$$

$$y(x)\to 0$$

15.3 problem 411

Internal problem ID [3156]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 411.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\left(-4\cos(x)^2+1\right)y'-\tan(x)\left(1+4\cos(x)^2\right)y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((1-4*cos(x)^2)*diff(y(x),x) = tan(x)*(1+4*cos(x)^2)*y(x),y(x), singsol=all)
```

$$y(x)=\frac{c_1(1+2\cos(2x))}{\cos(x)}$$

✓ Solution by Mathematica

Time used: 0.422 (sec). Leaf size: 23

```
DSolve[(1-4 Cos[x]^2)y'[x]==Tan[x](1+4 Cos[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1(2\cos(2x)+1)\sec(x)$$
$$y(x)\to 0$$

15.4 problem 412

Internal problem ID [3157]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 412.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1 - \sin(x))\, y' + \cos(x)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 10

```
dsolve((1-sin(x))*diff(y(x),x)+y(x)*cos(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1(\sin(x) - 1)$$

✓ Solution by Mathematica

Time used: 0.301 (sec). Leaf size: 18

```
DSolve[(1-Sin[x])y'[x]+y[x] Cos[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -c_1(\sin(x) - 1)$$
$$y(x) \to 0$$

15.5 problem 413

Internal problem ID [3158]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 413.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$(\cos(x) - \sin(x))\, y' + y(\sin(x) + \cos(x)) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve((cos(x)-sin(x))*diff(y(x),x)+y(x)*(cos(x)+sin(x)) = 0,y(x), singsol=all)
```

$$y(x) = c_1(\cos(x) - \sin(x))$$

✓ Solution by Mathematica

Time used: 0.114 (sec). Leaf size: 20

```
DSolve[(Cos[x]-Sin[x])y'[x]+y[x](Cos[x]+Sin[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(\cos(x) - \sin(x))$$
$$y(x) \to 0$$

15.6 problem 414

Internal problem ID [3159]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 414.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\left(\mathrm{a0}+\mathrm{a1}\sin\left(x\right)^{2}\right)y'+\mathrm{a2}\,x\left(\mathrm{a3}+\mathrm{a1}\sin\left(x\right)^{2}\right)+\mathrm{a1}\,y\sin\left(2x\right)=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 53

```
dsolve((a0+a1*sin(x)^2)*diff(y(x),x)+a2*x*(a3+a1*sin(x)^2)+a1*y(x)*sin(2*x) = 0,y(x), singsol
```

$$y(x)=\frac{\mathrm{a2}\left(\frac{\mathrm{a1}(\cos(2x)+2x\sin(2x))}{4}-\frac{\mathrm{a1}\;x^{2}}{2}-\mathrm{a3}\;x^{2}\right)+2c_{1}}{-\mathrm{a1}\cos\left(2x\right)+2\,\mathrm{a0}+\mathrm{a1}}$$

✓ Solution by Mathematica

Time used: 0.392 (sec). Leaf size: 54

```
DSolve[(a0+a1 Sin[x]^2)y'[x]+a2 x(a3+a1 Sin[x]^2)+a1 y[x] Sin[2 x]==0,y[x],x,IncludeSingularS
```

$$y(x)\to\frac{-2\mathrm{a2}x^{2}(\mathrm{a1}+2\mathrm{a3})+\mathrm{a1}\mathrm{a2}(2x\sin(2x)+\cos(2x))+4c_{1}}{4(2\mathrm{a0}-\mathrm{a1}\cos(2x)+\mathrm{a1})}$$

15.7 problem 415

Internal problem ID [3160]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 415.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$\boxed{(x - e^x) y' + e^x x + (1 - e^x) y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve((x-exp(x))*diff(y(x),x)+x*exp(x)+(1-exp(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{(x - 1) e^x + c_1}{-x + e^x}$$

✓ Solution by Mathematica

Time used: 0.089 (sec). Leaf size: 25

```
DSolve[(x-Exp[x])y'[x]+x Exp[x]+(1-Exp[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^x(x - 1) + c_1}{e^x - x}$$

15.8 problem 416

Internal problem ID [3161]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 416.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_linear]`

$$x\ln(x)\,y' - ax(\ln(x)+1) + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve(diff(y(x),x)*x*ln(x) = a*x*(1+ln(x))-y(x),y(x), singsol=all)
```

$$y(x) = ax + \frac{c_1}{\ln(x)}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 16

```
DSolve[y'[x] x Log[x]==a x(1+Log[x])-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to ax + \frac{c_1}{\log(x)}$$

15.9 problem 417

Internal problem ID [3162]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 417.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' + x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(y(x)*diff(y(x),x)+x = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 + c_1}$$
$$y(x) = -\sqrt{-x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.081 (sec). Leaf size: 39

```
DSolve[y[x] y'[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 + 2c_1}$$
$$y(x) \to \sqrt{-x^2 + 2c_1}$$

15.10 problem 418

Internal problem ID [3163]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 418.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' + x\,\mathrm{e}^{x^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve(y(x)*diff(y(x),x)+x*exp(x^2) = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{-\mathrm{e}^{x^2} + c_1}$$
$$y(x) = -\sqrt{-\mathrm{e}^{x^2} + c_1}$$

✓ Solution by Mathematica

Time used: 1.726 (sec). Leaf size: 43

```
DSolve[y[x] y'[x]+x Exp[x^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-e^{x^2} + 2c_1}$$
$$y(x) \to \sqrt{-e^{x^2} + 2c_1}$$

15.11 problem 419

Internal problem ID [3164]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 419.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\boxed{yy' + x^3 + y = 0}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+x^3+y(x) = 0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x] y'[x]+x^3+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

15.12 problem 420

Internal problem ID [3165]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 420.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{yy' + ax + yb = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 94

```
dsolve(y(x)*diff(y(x),x)+a*x+b*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(\_Z^2 - e^{\operatorname{RootOf}\left(x^2\left(-\tanh\left(\frac{\sqrt{b^2-4a}\,(2c_1+\_Z+2\ln(x))}{2b}\right)^2 b^2+4\tanh\left(\frac{\sqrt{b^2-4a}\,(2c_1+\_Z+2\ln(x))}{2b}\right)^2 a+b^2+4\,e^{-\_Z}-4a\right)\right)} + a + \_Zb\right) x$$

✓ Solution by Mathematica

Time used: 0.109 (sec). Leaf size: 74

```
DSolve[y[x] y'[x]+a x+b y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\operatorname{Solve}\left[\frac{1}{2}\log\left(a+\frac{by(x)}{x}+\frac{y(x)^2}{x^2}\right)-\frac{b\arctan\left(\frac{b+\frac{2y(x)}{x}}{\sqrt{4a-b^2}}\right)}{\sqrt{4a-b^2}}=-\log(x)+c_1,y(x)\right]$$

15.13 problem 421

Internal problem ID [3166]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 421.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy' + x\,\mathrm{e}^{-x}(1+y) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 46

```
dsolve(y(x)*diff(y(x),x)+x*exp(-x)*(1+y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\left(\mathrm{LambertW}\left(-\mathrm{e}^{c_1-1-x\,\mathrm{e}^{-x}-\mathrm{e}^{-x}}\right)\mathrm{e}^{x}-c_1\mathrm{e}^{x}+\mathrm{e}^{x}+x+1\right)\mathrm{e}^{-x}} - 1$$

✓ Solution by Mathematica

Time used: 4.754 (sec). Leaf size: 63

```
DSolve[y[x] y'[x]+x Exp[-x](1+y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -1 - W\left(-e^{-e^{-x}(x+(1+c_1)e^x+1)}\right)$$
$$y(x) \to -1$$
$$y(x) \to -W\left(-e^{-e^{-x}(x+e^x+1)}\right) - 1$$

15.14 problem 422

Internal problem ID [3167]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 422.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$\boxed{yy' + f(x) - g(x)\, y = 0}$$

✗ Solution by Maple

```
dsolve(y(x)*diff(y(x),x)+f(x) = g(x)*y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x] y'[x]+f[x]==g[x] y[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

15.15 problem 423

Internal problem ID [3168]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 423.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{yy' + 4x(x+1) + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 37

```
dsolve(y(x)*diff(y(x),x)+4*(1+x)*x+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{e^{-2x}c_1 - 4x^2}$$
$$y(x) = -\sqrt{e^{-2x}c_1 - 4x^2}$$

✓ Solution by Mathematica

Time used: 5.772 (sec). Leaf size: 47

```
DSolve[y[x] y'[x]+4(1+x)x+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sqrt{-4x^2 + c_1e^{-2x}}$$
$$y(x) \rightarrow \sqrt{-4x^2 + c_1e^{-2x}}$$

15.16 problem 424

Internal problem ID [3169]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 424.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$yy' - ax - by^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 61

```
dsolve(y(x)*diff(y(x),x) = a*x+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{4\,\mathrm{e}^{2xb}c_1b^2 - 4xba - 2a}}{2b}$$
$$y(x) = \frac{\sqrt{4\,\mathrm{e}^{2xb}c_1b^2 - 4xba - 2a}}{2b}$$

✓ Solution by Mathematica

Time used: 9.926 (sec). Leaf size: 77

```
DSolve[y[x] y'[x]==a x+b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\sqrt{a\left(bx + \frac{1}{2}\right) - b^2c_1e^{2bx}}}{b}$$
$$y(x) \to \frac{i\sqrt{a\left(bx + \frac{1}{2}\right) - b^2c_1e^{2bx}}}{b}$$

15.17 problem 425

Internal problem ID [3170]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 425.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{yy' - b\cos(x+c) - ay^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 116

```
dsolve(y(x)*diff(y(x),x) = b*cos(x+c)+a*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(4a^2+1)\left(4\,\mathrm{e}^{2ax}c_1a^2 - 4\cos(x+c)\,ab + \mathrm{e}^{2ax}c_1 + 2\sin(x+c)\,b\right)}}{4a^2+1}$$

$$y(x) = -\frac{\sqrt{(4a^2+1)\left(4\,\mathrm{e}^{2ax}c_1a^2 - 4\cos(x+c)\,ab + \mathrm{e}^{2ax}c_1 + 2\sin(x+c)\,b\right)}}{4a^2+1}$$

✓ Solution by Mathematica

Time used: 4.569 (sec). Leaf size: 106

```
DSolve[y[x] y'[x]== b Cos[x+c]+a y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{(4a^2+1)\,c_1e^{2ax} - 4ab\cos(c+x) + 2b\sin(c+x)}}{\sqrt{4a^2+1}}$$

$$y(x) \to \frac{\sqrt{(4a^2+1)\,c_1e^{2ax} - 4ab\cos(c+x) + 2b\sin(c+x)}}{\sqrt{4a^2+1}}$$

15.18 problem 426

Internal problem ID [3171]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 426.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$yy' - \mathrm{a0} - \mathrm{a1}\, y - \mathrm{a2}\, y^2 = 0$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 218

```
dsolve(y(x)*diff(y(x),x) = a0+a1*y(x)+a2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{4\,\mathrm{a0}\,\mathrm{a2}\tan\left(\mathrm{RootOf}\left(2c_1\,\mathrm{a2}\sqrt{4\,\mathrm{a0}\,\mathrm{a2}-\mathrm{a1}^2} + 2x\,\mathrm{a2}\sqrt{4\,\mathrm{a0}\,\mathrm{a2}-\mathrm{a1}^2} - \sqrt{4\,\mathrm{a0}\,\mathrm{a2}-\mathrm{a1}^2}\ln\left(\frac{4\,\mathrm{a0}\,\mathrm{a2}\tan(\_Z)^2 -}{}\right.\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 0.397 (sec). Leaf size: 123

```
DSolve[y[x] y'[x]==a0+a1 y[x]+a2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{\log(\#1(\#1\mathrm{a2}+\mathrm{a1})+\mathrm{a0}) - \frac{2\mathrm{a1}\arctan\left(\frac{2\#1\mathrm{a2}+\mathrm{a1}}{\sqrt{4\mathrm{a0}\mathrm{a2}-\mathrm{a1}^2}}\right)}{\sqrt{4\mathrm{a0}\mathrm{a2}-\mathrm{a1}^2}}}{2\mathrm{a2}}\&\right][x+c_1]$$

$$y(x) \to \frac{\sqrt{\mathrm{a1}^2-4\mathrm{a0}\mathrm{a2}}-\mathrm{a1}}{2\mathrm{a2}}$$

$$y(x) \to -\frac{\sqrt{\mathrm{a1}^2-4\mathrm{a0}\mathrm{a2}}+\mathrm{a1}}{2\mathrm{a2}}$$

15.19 problem 427

Internal problem ID [3172]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 427.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$yy' - ax - bxy^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve(y(x)*diff(y(x),x) = a*x+b*x*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{b\left(e^{b\,x^2}c_1 b - a\right)}}{b}$$

$$y(x) = -\frac{\sqrt{b\left(e^{b\,x^2}c_1 b - a\right)}}{b}$$

✓ Solution by Mathematica

Time used: 0.872 (sec). Leaf size: 98

```
DSolve[y[x] y'[x]==a x+b x y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-a + e^{b(x^2+2c_1)}}}{\sqrt{b}}$$

$$y(x) \to \frac{\sqrt{-a + e^{b(x^2+2c_1)}}}{\sqrt{b}}$$

$$y(x) \to -\frac{i\sqrt{a}}{\sqrt{b}}$$

$$y(x) \to \frac{i\sqrt{a}}{\sqrt{b}}$$

15.20 problem 428

Internal problem ID [3173]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 428.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$\boxed{yy' - \csc(x)^2 + y^2 \cot(x) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(y(x)*diff(y(x),x) = csc(x)^2-y(x)^2*cot(x),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{2x + c_1}}{\sin(x)}$$
$$y(x) = -\frac{\sqrt{2x + c_1}}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.468 (sec). Leaf size: 36

```
DSolve[y[x] y'[x]==Csc[x]^2- y[x]^2 Cot[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2x + c_1} \csc(x)$$
$$y(x) \to \sqrt{2x + c_1} \csc(x)$$

15.21 problem 429

Internal problem ID [3174]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 429.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{yy' - \sqrt{y^2 + a^2} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(y(x)*diff(y(x),x) = sqrt(y(x)^2+a^2),y(x), singsol=all)
```

$$x - \sqrt{y(x)^2 + a^2} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.217 (sec). Leaf size: 61

```
DSolve[y[x] y'[x]==Sqrt[a^2+y[x]^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-a^2 + (x + c_1)^2}$$
$$y(x) \to \sqrt{-a^2 + (x + c_1)^2}$$
$$y(x) \to -ia$$
$$y(x) \to ia$$

15.22 problem 430

Internal problem ID [3175]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 430.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{yy' - \sqrt{y^2 - a^2} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(y(x)*diff(y(x),x) = sqrt(y(x)^2-a^2),y(x), singsol=all)
```

$$x + \frac{(-y(x) + a)\,(y(x) + a)}{\sqrt{y(x)^2 - a^2}} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.215 (sec). Leaf size: 51

```
DSolve[y[x] y'[x]==Sqrt[y[x]^2-a^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{a^2 + (x + c_1)^2}$$
$$y(x) \to \sqrt{a^2 + (x + c_1)^2}$$
$$y(x) \to -a$$
$$y(x) \to a$$

15.23 problem 431

Internal problem ID [3176]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 431.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$\boxed{yy' + x + f\left(x^2 + y^2\right) g(x) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 30

```
dsolve(y(x)*diff(y(x),x)+x+f(x^2+y(x)^2)*g(x) = 0,y(x), singsol=all)
```

$$\int_{\_b}^{y(x)} \frac{\_a}{f\left(\_a^2 + x^2\right)} d\_a + \int g(x)\, dx - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.24 (sec). Leaf size: 95

```
DSolve[y[x] y'[x]+x+f[x^2+y[x]^2] g[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \left(\frac{K[2]}{f\left(x^2 + K[2]^2\right)} - \int_1^x -\frac{2K[1]K[2]f'(K[1]^2 + K[2]^2)}{f\left(K[1]^2 + K[2]^2\right)^2} dK[1]\right) dK[2] \right.$$
$$\left. + \int_1^x \left(g(K[1]) + \frac{K[1]}{f\left(K[1]^2 + y(x)^2\right)}\right) dK[1] = c_1, y(x)\right]$$

15.24 problem 432

Internal problem ID [3177]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 432.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(1+y)\,y'-x-y=0}$$

✓ Solution by Maple

Time used: 0.5 (sec). Leaf size: 73

```
dsolve((1+y(x))*diff(y(x),x) = x+y(x),y(x), singsol=all)
```

$$-\frac{\ln\left(-\frac{(x-1)^2-(x-1)(-y(x)-1)-(-y(x)-1)^2}{(x-1)^2}\right)}{2}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(x-2y(x)-3)\sqrt{5}}{5x-5}\right)}{5}-\ln(x-1)-c_1=0$$

✓ Solution by Mathematica

Time used: 0.099 (sec). Leaf size: 71

```
DSolve[(1+y[x])y'[x]==x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\log\left(\frac{x^2-y(x)^2+(x-3)y(x)-x-1}{(x-1)^2}\right)+\log(1-x)=\frac{\operatorname{arctanh}\left(\frac{y(x)+2x-1}{\sqrt{5}(y(x)+1)}\right)}{\sqrt{5}}+c_1,y(x)\right]$$

15.25 problem 433

Internal problem ID [3178]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 433.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1+y)\,y'-x^2(1-y)=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve((1+y(x))*diff(y(x),x) = x^2*(1-y(x)),y(x), singsol=all)
```

$$y(x)=2\,\mathrm{LambertW}\left(\frac{c_1\mathrm{e}^{-\frac{x^3}{6}-\frac{1}{2}}}{2}\right)+1$$

✓ Solution by Mathematica

Time used: 24.428 (sec). Leaf size: 66

```
DSolve[(1+y[x])y'[x]==x^2(1-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 1+2W\left(-\frac{1}{2}\sqrt{e^{-\frac{x^3}{3}-1+c_1}}\right)$$
$$y(x)\to 1+2W\left(\frac{1}{2}\sqrt{e^{-\frac{x^3}{3}-1+c_1}}\right)$$
$$y(x)\to 1$$

15.26 problem 434

Internal problem ID [3179]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 434.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd typ`

$$(x+y)\,y'+y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve((x+y(x))*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = -x-\sqrt{x^2+2c_1}$$
$$y(x) = -x+\sqrt{x^2+2c_1}$$

✓ Solution by Mathematica

Time used: 0.443 (sec). Leaf size: 84

```
DSolve[(x+y[x])y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x-\sqrt{x^2+e^{2c_1}}$$
$$y(x) \to -x+\sqrt{x^2+e^{2c_1}}$$
$$y(x) \to 0$$
$$y(x) \to -\sqrt{x^2}-x$$
$$y(x) \to \sqrt{x^2}-x$$

15.27 problem 435

Internal problem ID [3180]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 435.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(x-y)\,y'-y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 15

```
dsolve((x-y(x))*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{\mathrm{LambertW}\left(-x\,\mathrm{e}^{-c_1}\right)+c_1}$$

✓ Solution by Mathematica

Time used: 4.028 (sec). Leaf size: 25

```
DSolve[(x-y[x])y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to e^{W\left(-e^{-c_1}x\right)+c_1}$$
$$y(x)\to 0$$

15.28 problem 436

Internal problem ID [3181]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 436.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(x+y)\,y'+x-y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve((x+y(x))*diff(y(x),x)+x-y(x) = 0,y(x), singsol=all)
```

$$y(x)=\tan\left(\operatorname{RootOf}\left(2\_Z+\ln\left(\frac{1}{\cos\left(\_Z\right)^{2}}\right)+2\ln\left(x\right)+2c_{1}\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 34

```
DSolve[(x+y[x])y'[x]+(x-y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{y(x)}{x}\right)+\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right)=-\log(x)+c_1,y(x)\right]$$

15.29 problem 437

Internal problem ID [3182]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 437.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd typ`

$$(x+y)\,y'-x+y=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 51

```
dsolve((x+y(x))*diff(y(x),x) = x-y(x),y(x), singsol=all)
```

$$y(x)=\frac{-c_1x-\sqrt{2c_1^2x^2+1}}{c_1}$$
$$y(x)=\frac{-c_1x+\sqrt{2c_1^2x^2+1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.452 (sec). Leaf size: 94

```
DSolve[(x+y[x])y'[x]==x-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -x-\sqrt{2x^2+e^{2c_1}}$$
$$y(x)\to -x+\sqrt{2x^2+e^{2c_1}}$$
$$y(x)\to -\sqrt{2}\sqrt{x^2}-x$$
$$y(x)\to \sqrt{2}\sqrt{x^2}-x$$

15.30 problem 438

Internal problem ID [3183]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 438.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_linear, ‘class A‘]]`

$$\boxed{1 - y' - x - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(1-diff(y(x),x) = x+y(x),y(x), singsol=all)
```

$$y(x) = -x + 2 + c_1 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 18

```
DSolve[1-y'[x]==x+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x + c_1 e^{-x} + 2$$

15.31 problem 439

Internal problem ID [3184]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 439.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(x-y)\,y'-(2yx+1)\,y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve((x-y(x))*diff(y(x),x) = (1+2*x*y(x))*y(x),y(x), singsol=all)
```

$$y(x)=-\frac{x}{\mathrm{LambertW}\left(-\mathrm{e}^{x^{2}}c_{1}x\right)}$$

✓ Solution by Mathematica

Time used: 5.233 (sec). Leaf size: 29

```
DSolve[(x-y[x])y'[x]==(1+2 x y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{x}{W\left(x\left(-e^{x^{2}-c_{1}}\right)\right)}$$
$$y(x)\to 0$$

15.32 problem 440

Internal problem ID [3185]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 440.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(x+y)\,y'+\tan\left(y\right)=0}$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 26

```
dsolve((x+y(x))*diff(y(x),x)+tan(y(x)) = 0,y(x), singsol=all)
```

$$x-\frac{-\cos\left(y(x)\right)-y(x)\sin\left(y(x)\right)+c_1}{\sin\left(y\left(x\right)\right)}=0$$

✓ Solution by Mathematica

Time used: 0.192 (sec). Leaf size: 29

```
DSolve[(x+y[x])y'[x]+Tan[y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}[x=\csc(y(x))(-y(x)\sin(y(x))-\cos(y(x)))+c_1\csc(y(x)),y(x)]$$

15.33 problem 441

Internal problem ID [3186]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 441.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$(x-y)\,y'-\left(\mathrm{e}^{-\frac{x}{y}}+1\right)y=0$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 20

```
dsolve((x-y(x))*diff(y(x),x) = (exp(-x/y(x))+1)*y(x),y(x), singsol=all)
```

$$y(x)=-\frac{x}{\operatorname{LambertW}\left(\frac{c_1x}{c_1x-1}\right)}$$

✓ Solution by Mathematica

Time used: 1.197 (sec). Leaf size: 34

```
DSolve[(x-y[x])y'[x]==(Exp[-x/y[x]]+1)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{x}{W\left(\frac{x}{x-e^{c_1}}\right)}$$

$$y(x)\to -e^{W(1)}x$$

15.34 problem 442

Internal problem ID [3187]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 442.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(1+x+y)\,y'+1+4x+3y=0$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 29

```
dsolve((1+x+y(x))*diff(y(x),x)+1+4*x+3*y(x) = 0,y(x), singsol=all)
```

$$y(x)=-3-\frac{(x-2)\,(2\,\text{LambertW}\,(c_1(x-2))+1)}{\text{LambertW}\,(c_1\,(x-2))}$$

✓ Solution by Mathematica

Time used: 1.25 (sec). Leaf size: 159

```
DSolve[(1+x+y[x])y'[x]+1+4 x+3 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{(-2)^{2/3}\left(-2x\log\left(\frac{3(-2)^{2/3}(y(x)+2x-1)}{y(x)+x+1}\right)+(2x-1)\log\left(-\frac{3(-2)^{2/3}(x-2)}{y(x)+x+1}\right)+\log\left(\frac{3(-2)^{2/3}(y(x)+2x-1)}{y(x)+x+1}\right)+y(\right.}{9(y(x)+2x-1)}\right.$$

15.35 problem 443

Internal problem ID [3188]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 15
Problem number: 443.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd typ`

$$\boxed{(x+y+2)\,y'+y+x-1=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve((2+x+y(x))*diff(y(x),x) = 1-x-y(x),y(x), singsol=all)
```

$$y(x) = -2-x-\sqrt{-6c_1+6x+4}$$
$$y(x) = -2-x+\sqrt{-6c_1+6x+4}$$

✓ Solution by Mathematica

Time used: 0.114 (sec). Leaf size: 43

```
DSolve[(2+x+y[x])y'[x]==1-x-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -x-\sqrt{6x+4+c_1}-2$$
$$y(x)\to -x+\sqrt{6x+4+c_1}-2$$

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16.1 problem 444

Internal problem ID [3189]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 444.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(3-x-y)\,y'-1-x+3y=0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 28

```
dsolve((3-x-y(x))*diff(y(x),x) = 1+x-3*y(x),y(x), singsol=all)
```

$$y(x)=1+\frac{(x-2)\,(\mathrm{LambertW}\,(-2c_1(x-2))+2)}{\mathrm{LambertW}\,(-2c_1\,(x-2))}$$

✓ Solution by Mathematica

Time used: 1.011 (sec). Leaf size: 159

```
DSolve[(3-x-y[x])y'[x]==1+x-3 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2^{2/3}\left(x\left(-\log\left(-\frac{3\ 2^{2/3}(-y(x)+x-1)}{y(x)+x-3}\right)\right)+(x-1)\log\left(\frac{6\ 2^{2/3}(x-2)}{y(x)+x-3}\right)+\log\left(-\frac{3\ 2^{2/3}(-y(x)+x-1)}{y(x)+x-3}\right)+y(x)\left(-\right.\right.}{9(-y(x)+x-1)}\right.$$

16.2 problem 445

Internal problem ID [3190]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 445.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(3-x+y)\,y'-11+4x-3y=0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 30

```
dsolve((3-x+y(x))*diff(y(x),x) = 11-4*x+3*y(x),y(x), singsol=all)
```

$$y(x)=-1+\frac{(x-2)\left(2\,\mathrm{LambertW}\left(-c_1(x-2)\right)+1\right)}{\mathrm{LambertW}\left(-c_1\left(x-2\right)\right)}$$

✓ Solution by Mathematica

Time used: 1.347 (sec). Leaf size: 179

```
DSolve[(3-x+y[x])y'[x]==11-4 x+3 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{(-2)^{2/3}\left(-2x\log\left(\frac{3(-2)^{2/3}(-y(x)+2x-5)}{-y(x)+x-3}\right)+(2x-5)\log\left(-\frac{3(-2)^{2/3}(x-2)}{-y(x)+x-3}\right)+5\log\left(\frac{3(-2)^{2/3}(-y(x)+2x-5)}{-y(x)+x-3}\right)\right.}{9(-y(x)+2x-5)}\right.$$

16.3 problem 446

Internal problem ID [3191]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 446.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(2x + y) y' + x - 2y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve((2*x+y(x))*diff(y(x),x)+x-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \tan\left(\mathrm{RootOf}\left(4\_Z + \ln\left(\frac{1}{\cos(\_Z)^2}\right) + 2\ln(x) + 2c_1\right)\right) x$$

✓ Solution by Mathematica

Time used: 0.034 (sec). Leaf size: 36

```
DSolve[(2 x+y[x])y'[x]+(x-2 y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[2\arctan\left(\frac{y(x)}{x}\right) + \frac{1}{2}\log\left(\frac{y(x)^2}{x^2} + 1\right) = -\log(x) + c_1, y(x)\right]$$

16.4 problem 447

Internal problem ID [3192]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 447.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(2x - y + 2)\, y' + 3 + 6x - 3y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve((2+2*x-y(x))*diff(y(x),x)+3+6*x-3*y(x) = 0,y(x), singsol=all)
```

$$y(x) = 2x - \frac{3\,\mathrm{LambertW}\left(-\frac{\mathrm{e}^{\frac{25x}{3}}\,\mathrm{e}^{\frac{7}{3}}c_1}{3}\right)}{5} + \frac{7}{5}$$

✓ Solution by Mathematica

Time used: 3.602 (sec). Leaf size: 41

```
DSolve[(2+2 x-y[x])y'[x]+3(1+2 x- y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3}{5}W\left(-e^{\frac{25x}{3}-1+c_1}\right) + 2x + \frac{7}{5}$$

$$y(x) \to 2x + \frac{7}{5}$$

16.5 problem 448

Internal problem ID [3193]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 448.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], [_Abel, ‘2nd type‘, ‘class C‘], _dA`

$$\boxed{(2x - y + 3)\, y' + 2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve((3+2*x-y(x))*diff(y(x),x)+2 = 0,y(x), singsol=all)
```

$$y(x) = \operatorname{LambertW}\left(-2c_1 \mathrm{e}^{-2x-4}\right) + 2x + 4$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 22

```
DSolve[(3+2 x-y[x])y'[x]+2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to W\left(-2c_1 e^{-2(x+2)}\right) + 2x + 4$$

16.6 problem 449

Internal problem ID [3194]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 449.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(2x - y + 4)\, y' + 5 + x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 184

```
dsolve((4+2*x-y(x))*diff(y(x),x)+5+x-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = 2 + \frac{(x+1)\left(-c_1^2 - c_1^2\left(-\frac{\left(27c_1(x+1)+3\sqrt{3}\sqrt{27c_1^2(x+1)^2-1}\right)^{\frac{1}{3}}}{6c_1(x+1)} - \frac{1}{2c_1(x+1)\left(27c_1(x+1)+3\sqrt{3}\sqrt{27c_1^2(x+1)^2-1}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(27c_1 \right.}{}\right.}{}\right.\right.}{c_1^2}$$

✓ Solution by Mathematica

Time used: 60.166 (sec). Leaf size: 629

```
DSolve[(4+2 x-y[x])y'[x]+5+x-2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2(x+2) + \frac{3(x+1)}{\frac{1}{\sqrt[3]{-e^{\frac{3c_1}{4}}(x+1)^4 + 2e^{\frac{3c_1}{8}}(x+1)^2 + \sqrt{e^{\frac{3c_1}{8}}(x+1)^2\left(-1+e^{\frac{3c_1}{8}}(x+1)^2\right)^3} - 1}} - \frac{\sqrt[3]{-e^{\frac{3c_1}{4}}(x+1)^4 + 2e}}{}}$$

$$y(x) \to 2\left(x + \frac{3(x+1)}{\frac{-1-i\sqrt{3}}{\sqrt[3]{-e^{\frac{3c_1}{4}}(x+1)^4 + 2e^{\frac{3c_1}{8}}(x+1)^2 + \sqrt{e^{\frac{3c_1}{8}}(x+1)^2\left(-1+e^{\frac{3c_1}{8}}(x+1)^2\right)^3} - 1}} + \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{-e^{\frac{3c_1}{4}}(x+1}}{}} + 2\right)$$

$$y(x) \to 2\left(x\right.$$

16.7 problem 450

Internal problem ID [3195]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 450.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd typ`

$$(5-2x-y)\,y'+4-x-2y=0$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 32

```
dsolve((5-2*x-y(x))*diff(y(x),x)+4-x-2*y(x) = 0,y(x), singsol=all)
```

$$y(x)=1-\frac{2c_1(x-2)+\sqrt{3\,(x-2)^2\,c_1^2+1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 49

```
DSolve[(5-2 x-y[x])y'[x]+4-x-2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -2x-\sqrt{3(x-4)x+25+c_1}+5$$
$$y(x)\to -2x+\sqrt{3(x-4)x+25+c_1}+5$$

16.8 problem 451

Internal problem ID [3196]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 451.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(1 - 3x + y)\, y' - 2x + 2y = 0$$

✓ Solution by Maple

Time used: 0.469 (sec). Leaf size: 41

```
dsolve((1-3*x+y(x))*diff(y(x),x) = 2*x-2*y(x),y(x), singsol=all)
```

$$y(x) = \frac{1}{2} + \frac{(-1 + 2x)\left(-c_1 - c_1 \operatorname{RootOf}\left((-1 + 2x)^3 c_1\_Z^4 - \_Z - 3\right)\right)}{2c_1}$$

✓ Solution by Mathematica

Time used: 60.156 (sec). Leaf size: 4937

```
DSolve[(1-3 x+y[x])y'[x]==2(x-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

16.9 problem 452

Internal problem ID [3197]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 452.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd typ`

$$\boxed{(2-3x+y)\,y'+5-2x-3y=0}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 32

```
dsolve((2-3*x+y(x))*diff(y(x),x)+5-2*x-3*y(x) = 0,y(x), singsol=all)
```

$$y(x)=1-\frac{-3c_1(x-1)+\sqrt{11\,(x-1)^2\,c_1^2+1}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.118 (sec). Leaf size: 59

```
DSolve[(2-3 x+y[x])y'[x]+5-2 x-3 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 3x-i\sqrt{-11(x-2)x-4-c_1}-2$$
$$y(x)\to 3x+i\sqrt{-11(x-2)x-4-c_1}-2$$

16.10 problem 453

Internal problem ID [3198]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 453.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(4x - y) y' + 2x - 5y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
dsolve((4*x-y(x))*diff(y(x),x)+2*x-5*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{4c_1x - \sqrt{-12c_1x + 1} - 1}{2c_1}$$
$$y(x) = -\frac{4c_1x + \sqrt{-12c_1x + 1} - 1}{2c_1}$$

✓ Solution by Mathematica

Time used: 1.081 (sec). Leaf size: 80

```
DSolve[(4 x-y[x])y'[x]+2 x-5 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-4x - e^{\frac{c_1}{2}}\sqrt{12x + e^{c_1}} - e^{c_1}\right)$$
$$y(x) \to \frac{1}{2}\left(-4x + e^{\frac{c_1}{2}}\sqrt{12x + e^{c_1}} - e^{c_1}\right)$$

16.11 problem 454

Internal problem ID [3199]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 454.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(6-4x-y)\,y'-2x+y=0}$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 254

```
dsolve((6-4*x-y(x))*diff(y(x),x) = 2*x-y(x),y(x), singsol=all)
```

$$y(x) = 2+\frac{\left(12\sqrt{3}\,(x-1)\sqrt{\frac{(x-1)(27c_1(x-1)-4)}{c_1}}\,c_1^2+108(x-1)^2\,c_1^2-72c_1(x-1)+8\right)^{\frac{1}{3}}}{12c_1}$$
$$-\frac{6c_1(x-1)-1}{3c_1\left(12\sqrt{3}\,(x-1)\sqrt{\frac{(x-1)(27c_1(x-1)-4)}{c_1}}\,c_1^2+108\,(x-1)^2\,c_1^2-72c_1\,(x-1)+8\right)^{\frac{1}{3}}}$$
$$-\frac{3c_1(x-1)+1}{3c_1}$$
$$-\frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\,(x-1)\sqrt{\frac{(x-1)(27c_1(x-1)-4)}{c_1}}\,c_1^2+108(x-1)^2c_1^2-72c_1(x-1)+8\right)^{\frac{1}{3}}}{6c_1}+\frac{4c_1(x-1)-\frac{2}{3}}{c_1\left(12\sqrt{3}\,(x-1)\sqrt{\frac{(x-1)(27c_1(x-1)-4)}{c_1}}\,c_1^2+108(x-1)^2c_1^2\right.}\right.}{2}$$

✓ Solution by Mathematica

Time used: 60.095 (sec). Leaf size: 2581

```
DSolve[(6-4 x-y[x])y'[x]==2 x -y[x],y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

16.12 problem 455

Internal problem ID [3200]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 455.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(1+5x-y)\,y'+5+x-5y=0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 208

```
dsolve((1+5*x-y(x))*diff(y(x),x)+5+x-5*y(x) = 0,y(x), singsol=all)
```

$$y(x) = 1+\frac{\left(6\sqrt{3}\,x\sqrt{\frac{x(27c_1x+2)}{c_1}}\,c_1^2+54c_1^2x^2+18c_1x+1\right)^{\frac{1}{3}}}{6c_1} + \frac{12c_1x+1}{6c_1\left(6\sqrt{3}\,x\sqrt{\frac{x(27c_1x+2)}{c_1}}\,c_1^2+54c_1^2x^2+18c_1x+1\right)^{\frac{1}{3}}} - \frac{3c_1x+1}{3c_1} - \frac{i\sqrt{3}\left(\frac{\left(6\sqrt{3}\,x\sqrt{\frac{x(27c_1x+2)}{c_1}}\,c_1^2+54c_1^2x^2+18c_1x+1\right)^{\frac{1}{3}}}{3c_1}-\frac{12c_1x+1}{3c_1\left(6\sqrt{3}\,x\sqrt{\frac{x(27c_1x+2)}{c_1}}\,c_1^2+54c_1^2x^2+18c_1x+1\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 60.043 (sec). Leaf size: 925

```
DSolve[(1+5 x-y[x])y'[x]+5+x-5 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$y(x) \rightarrow$

$$-\frac{}{\text{Root}\left[\#1^6\left(186624x^4+186624e^{\frac{12c_1}{25}}x^6\right)+\#1^5\left(-186624x^3-186624e^{\frac{12c_1}{25}}x^5\right)+\#1^4\left(69984x^2+777\right.\right.}$$
$$+5x+1$$

$y(x) \rightarrow$

$$-\frac{}{\text{Root}\left[\#1^6\left(186624x^4+186624e^{\frac{12c_1}{25}}x^6\right)+\#1^5\left(-186624x^3-186624e^{\frac{12c_1}{25}}x^5\right)+\#1^4\left(69984x^2+777\right.\right.}$$
$$+5x+1$$

$y(x) \rightarrow$

$$-\frac{}{\text{Root}\left[\#1^6\left(186624x^4+186624e^{\frac{12c_1}{25}}x^6\right)+\#1^5\left(-186624x^3-186624e^{\frac{12c_1}{25}}x^5\right)+\#1^4\left(69984x^2+777\right.\right.}$$
$$+5x+1$$

$y(x) \rightarrow$

$$-\frac{}{\text{Root}\left[\#1^6\left(186624x^4+186624e^{\frac{12c_1}{25}}x^6\right)+\#1^5\left(-186624x^3-186624e^{\frac{12c_1}{25}}x^5\right)+\#1^4\left(69984x^2+777\right.\right.}$$
$$+5x+1$$

$y(x) \rightarrow$

$$-\frac{}{\text{Root}\left[\#1^6\left(186624x^4+186624e^{\frac{12c_1}{25}}x^6\right)+\#1^5\left(-186624x^3-186624e^{\frac{12c_1}{25}}x^5\right)+\#1^4\left(69984x^2+777\right.\right.}$$
$$+5x+1$$

$y(x) \rightarrow$

$$-\frac{}{\text{Root}\left[\#1^6\left(186624x^4+186624e^{\frac{12c_1}{25}}x^6\right)+\#1^5\left(-186624x^3-186624e^{\frac{12c_1}{25}}x^5\right)+\#1^4\left(69984x^2+777\right.\right.}$$
$$+5x+1$$

16.13 problem 456

Internal problem ID [3201]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 456.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(a+bx+y)\,y'+a-bx-y=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 91

```
dsolve((a+b*x+y(x))*diff(y(x),x)+a-b*x-y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{-b^{2}x+2\,\mathrm{LambertW}\left(\frac{\mathrm{e}^{-\frac{c_1 b^{2}}{2a}}\,\mathrm{e}^{\frac{b^{2}x}{2a}}\,\mathrm{e}^{-\frac{c_1 b}{a}}\,\mathrm{e}^{\frac{b}{2}}\,\mathrm{e}^{\frac{xb}{a}}\,\mathrm{e}^{-\frac{c_1}{2a}}\,\mathrm{e}^{-\frac{1}{2}}\,\mathrm{e}^{\frac{x}{2a}}}{2a}\right)a-ab-xb+a}{b+1}$$

✓ Solution by Mathematica

Time used: 5.219 (sec). Leaf size: 116

```
DSolve[(a+b x+y[x])y'[x]+a-b x-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2aW\left(-e^{\frac{(b+1)^2x}{2a}-1+c_1}\right)+a(-b)+a-b(b+1)x}{b+1}$$

$$y(x)\to\frac{a-ab}{b+1}-bx$$

$$y(x)\to\frac{2aW\left(-e^{\frac{(b+1)^2x}{2a}-1}\right)+a(-b)+a-b(b+1)x}{b+1}$$

16.14 problem 457

Internal problem ID [3202]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 457.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Abe`

$$\boxed{(x^2 - y)\,y' + x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve((x^2-y(x))*diff(y(x),x)+x = 0,y(x), singsol=all)
```

$$y(x) = x^2 + \frac{\text{LambertW}\left(4c_1 \mathrm{e}^{-2x^2-1}\right)}{2} + \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 5.228 (sec). Leaf size: 40

```
DSolve[(x^2-y[x])y'[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2 + \frac{1}{2}\left(1 + W\left(-e^{-2x^2-1+c_1}\right)\right)$$
$$y(x) \to x^2 + \frac{1}{2}$$

16.15 problem 458

Internal problem ID [3203]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 458.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$
\boxed{(x^2 - y)\, y' - 4yx = 0}
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 53

```
dsolve((x^2-y(x))*diff(y(x),x) = 4*x*y(x),y(x), singsol=all)
```

$$
y(x) = \frac{c_1\left(c_1 - \sqrt{c_1^2 - 4x^2}\right)}{2} - x^2
$$
$$
y(x) = \frac{c_1\left(c_1 + \sqrt{c_1^2 - 4x^2}\right)}{2} - x^2
$$

✓ Solution by Mathematica

Time used: 2.472 (sec). Leaf size: 206

```
DSolve[(x^2-y[x])y'[x]==4 x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x^2\left(1+\frac{2-2i}{\frac{i\sqrt{2}}{\sqrt{e^{\frac{2c_1}{9}}x^2-i}}-(1-i)}\right)$$

$$y(x)\to x^2\left(1+\frac{2-2i}{(-1+i)-\frac{i\sqrt{2}}{\sqrt{e^{\frac{2c_1}{9}}x^2-i}}}\right)$$

$$y(x)\to x^2\left(1+\frac{2-2i}{(-1+i)-\frac{\sqrt{2}}{\sqrt{e^{\frac{2c_1}{9}}x^2+i}}}\right)$$

$$y(x)\to x^2\left(1+\frac{2-2i}{\frac{\sqrt{2}}{\sqrt{e^{\frac{2c_1}{9}}x^2+i}}-(1-i)}\right)$$

$$y(x)\to 0$$

$$y(x)\to -x^2$$

16.16 problem 459

Internal problem ID [3204]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 16
Problem number: 459.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$(y - \csc(x)\cot(x))\, y' + \csc(x)\,(1 + \cos(x)\, y)\, y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 51

```
dsolve((y(x)-cot(x)*csc(x))*diff(y(x),x)+csc(x)*(1+y(x)*cos(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{-\csc(x)\cot(x)\sin(x) + \sqrt{c_1 + \cot(x)^2}}{\sin(x)}$$

$$y(x) = \frac{\csc(x)\cot(x)\sin(x) + \sqrt{c_1 + \cot(x)^2}}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 1.489 (sec). Leaf size: 85

```
DSolve[(y[x]-Cot[x] Csc[x])y'[x]+Csc[x](1+y[x] Cos[x])y[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \cot(x)\csc(x) - \frac{i\csc^2(x)\sqrt{(-1 + c_1)\cos(2x) - 1 - c_1}}{\sqrt{2}}$$

$$y(x) \to \cot(x)\csc(x) + \frac{i\csc^2(x)\sqrt{(-1 + c_1)\cos(2x) - 1 - c_1}}{\sqrt{2}}$$

17 Various 17

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17.1 problem 460

Internal problem ID [3205]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 460.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{2yy' + 2x + x^2 + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(2*y(x)*diff(y(x),x)+2*x+x^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 \mathrm{e}^{-x} - x^2}$$
$$y(x) = -\sqrt{c_1 \mathrm{e}^{-x} - x^2}$$

✓ Solution by Mathematica

Time used: 5.763 (sec). Leaf size: 47

```
DSolve[2 y[x] y'[x]+2 x+x^2+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 + c_1 e^{-x}}$$
$$y(x) \to \sqrt{-x^2 + c_1 e^{-x}}$$

17.2 problem 461

Internal problem ID [3206]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 461.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{2yy' - xy^2 - x^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 43

```
dsolve(2*y(x)*diff(y(x),x) = x*y(x)^2+x^3,y(x), singsol=all)
```

$$y(x) = \sqrt{e^{\frac{x^2}{2}} c_1 - x^2 - 2}$$
$$y(x) = -\sqrt{e^{\frac{x^2}{2}} c_1 - x^2 - 2}$$

✓ Solution by Mathematica

Time used: 7.212 (sec). Leaf size: 57

```
DSolve[2 y[x] y'[x]==x y[x]^2+x^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 + c_1 e^{\frac{x^2}{2}} - 2}$$
$$y(x) \to \sqrt{-x^2 + c_1 e^{\frac{x^2}{2}} - 2}$$

17.3 problem 462

Internal problem ID [3207]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 462.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(x-2y)\,y'-y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve((x-2*y(x))*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{\mathrm{LambertW}\left(-\frac{x\,\mathrm{e}^{-\frac{c_1}{2}}}{2}\right)+\frac{c_1}{2}}$$

✓ Solution by Mathematica

Time used: 4.826 (sec). Leaf size: 31

```
DSolve[(x-2 y[x])y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{x}{2W\left(-\frac{1}{2}e^{-\frac{c_1}{2}}x\right)}$$

$$y(x)\to 0$$

17.4 problem 463

Internal problem ID [3208]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 463.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(x + 2y)\, y' + 2x - y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve((x+2*y(x))*diff(y(x),x)+2*x-y(x) = 0,y(x), singsol=all)
```

$$y(x) = \tan\left(\mathrm{RootOf}\left(\ln\left(\frac{1}{\cos\left(\_Z\right)^2}\right) + \_Z + 2\ln\left(x\right) + 2c_1\right)\right) x$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 30

```
DSolve[(x+2 y[x])y'[x]+2 x -y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{y(x)}{x}\right) + \log\left(\frac{y(x)^2}{x^2} + 1\right) = -2\log(x) + c_1, y(x)\right]$$

17.5 problem 464

Internal problem ID [3209]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 464.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd typ`

$$\boxed{(x-2y)\,y'+2x+y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 53

```
dsolve((x-2*y(x))*diff(y(x),x)+2*x+y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{\frac{c_1x}{2}-\frac{\sqrt{5c_1^2x^2+4}}{2}}{c_1}$$
$$y(x)=\frac{\frac{c_1x}{2}+\frac{\sqrt{5c_1^2x^2+4}}{2}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.459 (sec). Leaf size: 102

```
DSolve[(x-2 y[x])y'[x]+2 x+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\left(x-\sqrt{5x^2-4e^{c_1}}\right)$$
$$y(x)\to\frac{1}{2}\left(x+\sqrt{5x^2-4e^{c_1}}\right)$$
$$y(x)\to\frac{1}{2}\left(x-\sqrt{5}\sqrt{x^2}\right)$$
$$y(x)\to\frac{1}{2}\left(\sqrt{5}\sqrt{x^2}+x\right)$$

17.6 problem 465

Internal problem ID [3210]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 465.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd typ`

$$\boxed{(1 + x - 2y) y' - 1 - 2x + y = 0}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 38

```
dsolve((1+x-2*y(x))*diff(y(x),x) = 1+2*x-y(x),y(x), singsol=all)
```

$$y(x) = \frac{1}{3} - \frac{-\frac{(3x+1)c_1}{2} + \frac{\sqrt{-3(3x+1)^2 c_1^2 + 4}}{2}}{3c_1}$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 65

```
DSolve[(1+x-2 y[x])y'[x]==1+2 x-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(x - i\sqrt{x(3x+2) - 1 - 4c_1} + 1\right)$$
$$y(x) \to \frac{1}{2}\left(x + i\sqrt{x(3x+2) - 1 - 4c_1} + 1\right)$$

17.7 problem 466

Internal problem ID [3211]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 466.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(x+2y+1)\,y'+1-x-2y=0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 21

```
dsolve((1+x+2*y(x))*diff(y(x),x)+1-x-2*y(x) = 0,y(x), singsol=all)
```

$$y(x)=-\frac{x}{2}+\frac{2\,\mathrm{LambertW}\left(\frac{\mathrm{e}^{\frac{9x}{4}}\,\mathrm{e}^{-\frac{1}{4}c_1}}{4}\right)}{3}+\frac{1}{6}$$

✓ Solution by Mathematica

Time used: 4.836 (sec). Leaf size: 43

```
DSolve[(1+x+2 y[x])y'[x]+1-x-2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{6}\left(4W\left(-e^{\frac{9x}{4}-1+c_1}\right)-3x+1\right)$$
$$y(x)\to\frac{1}{6}(1-3x)$$

17.8 problem 467

Internal problem ID [3212]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 467.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(x+2y+1)\,y'+7+x-4y=0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 254

```
dsolve((1+x+2*y(x))*diff(y(x),x)+7+x-4*y(x) = 0,y(x), singsol=all)
```

$$y(x)=1+\frac{\left(12\sqrt{3}\,(x+3)\sqrt{\frac{(x+3)(27(x+3)c_1-32)}{c_1}}\,c_1^2+108(x+3)^2c_1^2-576(x+3)\,c_1+512\right)^{\frac{1}{3}}}{12c_1}$$
$$-\frac{4(3(x+3)\,c_1-4)}{3c_1\left(12\sqrt{3}\,(x+3)\sqrt{\frac{(x+3)(27(x+3)c_1-32)}{c_1}}\,c_1^2+108\,(x+3)^2c_1^2-576\,(x+3)\,c_1+512\right)^{\frac{1}{3}}}$$
$$+\frac{3(x+3)\,c_1-4}{3c_1}$$
$$-\frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}\,(x+3)\sqrt{\frac{(x+3)(27(x+3)c_1-32)}{c_1}}\,c_1^2+108(x+3)^2c_1^2-576(x+3)c_1+512\right)^{\frac{1}{3}}}{6c_1}+\frac{8(x+3)c_1-\frac{32}{3}}{c_1\left(12\sqrt{3}\,(x+3)\sqrt{\frac{(x+3)(27(x+3)c_1-32)}{c_1}}\,c_1^2+108(x+\right.}\right.}{2}$$

✓ Solution by Mathematica

Time used: 60.098 (sec). Leaf size: 2617

```
DSolve[(1+x+2 y[x])y'[x]+7+x-4 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

17.9 problem 468

Internal problem ID [3213]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 468.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'],`

$$2(x+y)\,y' + x^2 + 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve(2*(x+y(x))*diff(y(x),x)+x^2+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -x - \frac{\sqrt{-3x^3+9x^2-9c_1}}{3}$$
$$y(x) = -x + \frac{\sqrt{-3x^3+9x^2-9c_1}}{3}$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 53

```
DSolve[2(x+y[x])y'[x]+x^2+2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \sqrt{-\frac{x^3}{3}+x^2+c_1}$$
$$y(x) \to -x + \sqrt{-\frac{x^3}{3}+x^2+c_1}$$

17.10 problem 469

Internal problem ID [3214]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 469.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd typ`

$$\boxed{(3+2x-2y)\,y'-1-6x+2y=0}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 36

```
dsolve((3+2*x-2*y(x))*diff(y(x),x) = 1+6*x-2*y(x),y(x), singsol=all)
```

$$y(x)=2-\frac{-(-1+2x)\,c_1+\sqrt{-2\,(-1+2x)^2\,c_1^2+1}}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.128 (sec). Leaf size: 63

```
DSolve[(3+2 x-2 y[x])y'[x]==1+6 x-2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x-\frac{1}{2}i\sqrt{8(x-1)x-9-4c_1}+\frac{3}{2}$$
$$y(x)\to x+\frac{1}{2}i\sqrt{8(x-1)x-9-4c_1}+\frac{3}{2}$$

17.11 problem 470

Internal problem ID [3215]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 470.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(1 - 4x - 2y)\, y' + 2x + y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve((1-4*x-2*y(x))*diff(y(x),x)+2*x+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{e^{-\operatorname{LambertW}\left(-2\, e^{4} e^{-25x} e^{25c_1}\right)+4-25x+25c_1}}{5} + \frac{2}{5} - 2x$$

✓ Solution by Mathematica

Time used: 3.56 (sec). Leaf size: 39

```
DSolve[(1-4 x-2 y[x])y'[x]+2 x+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{10} W\left(-e^{-25x-1+c_1}\right) - 2x + \frac{2}{5}$$

$$y(x) \to \frac{2}{5} - 2x$$

17.12 problem 471

Internal problem ID [3216]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 471.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(-2y+6x)\,y'-2-3x+y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
dsolve((6*x-2*y(x))*diff(y(x),x) = 2+3*x-y(x),y(x), singsol=all)
```

$$y(x)=\frac{\mathrm{e}^{-\operatorname{LambertW}\left(-\frac{\mathrm{e}^{\frac{25x}{4}}\mathrm{e}^{-1}\mathrm{e}^{-\frac{25c_1}{4}}}{2}\right)+\frac{25x}{4}-1-\frac{25c_1}{4}}}{5}+3x-\frac{2}{5}$$

✓ Solution by Mathematica

Time used: 3.845 (sec). Leaf size: 40

```
DSolve[(6 x-2 y[x])y'[x]==2+3 x-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 3x-\frac{2}{5}\left(1+W\left(-e^{\frac{25x}{4}-1+c_1}\right)\right)$$
$$y(x)\to 3x-\frac{2}{5}$$

17.13 problem 472

Internal problem ID [3217]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 472.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(19 + 9x + 2y)\, y' + 18 - 2x - 6y = 0$$

✓ Solution by Maple

Time used: 1.484 (sec). Leaf size: 29

```
dsolve((19+9*x+2*y(x))*diff(y(x),x)+18-2*x-6*y(x) = 0,y(x), singsol=all)
```

$$y(x) = 4 + \frac{4(x+3)\, c_1 + \sqrt{-40\,(x+3)\, c_1 + 1} - 1}{8c_1}$$

✓ Solution by Mathematica

Time used: 14.63 (sec). Leaf size: 236

```
DSolve[(19+9 x+2 y[x])y'[x]+18-2 x-6 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{9x}{2} + \frac{(5-5i)(x+3)}{\frac{i\sqrt{2}}{\sqrt{e^{\frac{2c_1}{9}}(x+3)-i}} + (1-i)} - \frac{19}{2}$$

$$y(x) \to -\frac{9x}{2} + \frac{(5-5i)(x+3)}{(1-i) - \frac{i\sqrt{2}}{\sqrt{e^{\frac{2c_1}{9}}(x+3)-i}}} - \frac{19}{2}$$

$$y(x) \to -\frac{9x}{2} + \frac{(5-5i)(x+3)}{(1-i) - \frac{\sqrt{2}}{\sqrt{e^{\frac{2c_1}{9}}(x+3)+i}}} - \frac{19}{2}$$

$$y(x) \to -\frac{9x}{2} + \frac{(5-5i)(x+3)}{\frac{\sqrt{2}}{\sqrt{e^{\frac{2c_1}{9}}(x+3)+i}} + (1-i)} - \frac{19}{2}$$

$$y(x) \to -2(x+1)$$

$$y(x) \to \frac{x+11}{2}$$

17.14 problem 473

Internal problem ID [3218]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 473.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]'],`

$$\boxed{(x^3 + 2y)\,y' - 3x(2 - yx) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 51

```
dsolve((x^3+2*y(x))*diff(y(x),x) = 3*x*(2-x*y(x)),y(x), singsol=all)
```

$$y(x) = -\frac{x^3}{2} - \frac{\sqrt{x^6 + 12x^2 - 4c_1}}{2}$$
$$y(x) = -\frac{x^3}{2} + \frac{\sqrt{x^6 + 12x^2 - 4c_1}}{2}$$

✓ Solution by Mathematica

Time used: 0.142 (sec). Leaf size: 65

```
DSolve[(x^3+2 y[x])y'[x]==3 x(2 - x y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(-x^3 - \sqrt{x^6 + 12x^2 + 4c_1}\right)$$
$$y(x) \to \frac{1}{2}\left(-x^3 + \sqrt{x^6 + 12x^2 + 4c_1}\right)$$

17.15 problem 474

Internal problem ID [3219]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 474.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class A']]`

$$(\tan(x)\sec(x) - 2y)\, y' + \sec(x)\,(1 + 2\sin(x)\, y) = 0$$

✗ Solution by Maple

```
dsolve((tan(x)*sec(x)-2*y(x))*diff(y(x),x)+sec(x)*(1+2*y(x)*sin(x)) = 0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(Tan[x] Sec[x]-2 y[x])y'[x]+Sec[x](1+2 y[x] Sin[x])==0,y[x],x,IncludeSingularSolutions
```

Not solved

17.16 problem 475

Internal problem ID [3220]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 475.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class B']]`

$$\left(x\,e^{-x}-2y\right)y'-2\,e^{-2x}x+\left(e^{-x}+x\,e^{-x}-2y\right)y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 88

```
dsolve((x*exp(-x)-2*y(x))*diff(y(x),x) = 2*x*exp(-2*x)-(exp(-x)+x*exp(-x)-2*y(x))*y(x),y(x),
```

$$y(x)=-\frac{\left(-e^{x}x+\sqrt{-3x^2e^{2x}+4c_1e^{2x}}\right)e^{-2x}}{2}$$
$$y(x)=\frac{e^{-2x}\left(e^{x}x+\sqrt{-3x^2e^{2x}+4c_1e^{2x}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 32.531 (sec). Leaf size: 81

```
DSolve[(x Exp[-x]-2 y[x])y'[x]==2 x Exp[-2 x]-(Exp[-x]+x Exp[-x]-2 y[x])y[x],y[x],x,IncludeSi
```

$$y(x)\to\frac{1}{2}e^{-2x}\left(e^{x}x-\sqrt{e^{2x}\left(-3x^2+4c_1\right)}\right)$$
$$y(x)\to\frac{1}{2}e^{-2x}\left(e^{x}x+\sqrt{e^{2x}\left(-3x^2+4c_1\right)}\right)$$

17.17 problem 476

Internal problem ID [3221]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 17
Problem number: 476.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$3yy' + 5\cot(x)\cot(y)\cos(y)^2 = 0$$

✓ Solution by Maple

Time used: 0.204 (sec). Leaf size: 61

```
dsolve(3*y(x)*diff(y(x),x)+5*cot(x)*cot(y(x))*cos(y(x))^2 = 0,y(x), singsol=all)
```

$$\frac{-3\tan(y(x))\cos(2y(x)) + 10\ln(\sin(x))\cos(2y(x)) + 10c_1\cos(2y(x)) - 3\tan(y(x)) + 10\ln(\sin(x)) + 10}{10\cos(2y(x)) + 10}$$
$$= 0$$

✓ Solution by Mathematica

Time used: 0.482 (sec). Leaf size: 30

```
DSolve[3 y[x] y'[x]+5 Cot[x] Cot[y[x]] Cos[y[x]]^2==0,y[x],x,IncludeSingularSolutions -> True
```

$$\text{Solve}\left[40\sin(x)e^{\frac{3}{10}(y(x)\sec^2(y(x))-\tan(y(x)))} = c_1, y(x)\right]$$

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18.1 problem 477

Internal problem ID [3222]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 477.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{3(-y+2)\,y'+yx=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(3*(2-y(x))*diff(y(x),x)+x*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-\operatorname{LambertW}\left(-\frac{\mathrm{e}^{-\frac{x^2}{12}-\frac{c_1}{6}}}{2}\right)-\frac{x^2}{12}-\frac{c_1}{6}}$$

✓ Solution by Mathematica

Time used: 19.587 (sec). Leaf size: 64

```
DSolve[3(2-y[x])y'[x]+x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -2W\left(-\frac{1}{2}\sqrt{e^{-\frac{x^2}{6}-c_1}}\right)$$
$$y(x)\to -2W\left(\frac{1}{2}\sqrt{e^{-\frac{x^2}{6}-c_1}}\right)$$
$$y(x)\to 0$$

18.2 problem 478

Internal problem ID [3223]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 478.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(x-3y)\,y'+4+3x-y=0}$$

✓ Solution by Maple

Time used: 0.875 (sec). Leaf size: 242

```
dsolve((x-3*y(x))*diff(y(x),x)+4+3*x-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2}$$

$$-\frac{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{6}\left(-144\left(108(2x+3)^{3}c_1+12\sqrt{-96(2x+3)^{9}c_1^{3}+81(2x+3)^{6}c_1^{2}}\right)^{\frac{1}{3}}-\frac{3456(2x+3)^{3}c_1}{\left(108(2x+3)^{3}c_1+12\sqrt{-96(2x+3)^{9}c_1^{3}+81(2x+3)^{6}c_1^{2}}\right)^{\frac{1}{3}}}+864i\sqrt{3}\left(\cdots\right.\right.}{2985984c_1}}{2\left(2x+3\right)^{2}}$$

✓ Solution by Mathematica

Time used: 60.041 (sec). Leaf size: 793

```
DSolve[(x-3 y[x])y'[x]+4+3 x-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{3}$$

$$-\frac{}{3\text{Root}\left[\#1^6\left(1024x^6+9216x^5+34560x^4+69120x^3+77760x^2+46656x+11664+16e^{12c_1}\right)+\#1^4\left(-\right.\right.}$$

$$y(x) \to \frac{x}{3}$$

$$-\frac{}{3\text{Root}\left[\#1^6\left(1024x^6+9216x^5+34560x^4+69120x^3+77760x^2+46656x+11664+16e^{12c_1}\right)+\#1^4\left(-\right.\right.}$$

$$y(x) \to \frac{x}{3}$$

$$-\frac{}{3\text{Root}\left[\#1^6\left(1024x^6+9216x^5+34560x^4+69120x^3+77760x^2+46656x+11664+16e^{12c_1}\right)+\#1^4\left(-\right.\right.}$$

$$y(x) \to \frac{x}{3}$$

$$-\frac{}{3\text{Root}\left[\#1^6\left(1024x^6+9216x^5+34560x^4+69120x^3+77760x^2+46656x+11664+16e^{12c_1}\right)+\#1^4\left(-\right.\right.}$$

$$y(x) \to \frac{x}{3}$$

$$-\frac{}{3\text{Root}\left[\#1^6\left(1024x^6+9216x^5+34560x^4+69120x^3+77760x^2+46656x+11664+16e^{12c_1}\right)+\#1^4\left(-\right.\right.}$$

$$y(x) \to \frac{x}{3}$$

$$-\frac{}{3\text{Root}\left[\#1^6\left(1024x^6+9216x^5+34560x^4+69120x^3+77760x^2+46656x+11664+16e^{12c_1}\right)+\#1^4\left(-\right.\right.}$$

18.3 problem 479

Internal problem ID [3224]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 479.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(4 - x - 3y)\, y' + 3 - x - 3y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve((4-x-3*y(x))*diff(y(x),x)+3-x-3*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{3} - \frac{\operatorname{LambertW}\left(-\frac{\mathrm{e}^{\frac{4x}{3}}\mathrm{e}^{\frac{5}{3}}c_1}{3}\right)}{2} + \frac{5}{6}$$

✓ Solution by Mathematica

Time used: 4.72 (sec). Leaf size: 43

```
DSolve[(4-x-3 y[x])y'[x]+3-x-3 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(-3W\left(-e^{\frac{4x}{3}-1+c_1}\right) - 2x + 5\right)$$
$$y(x) \to \frac{1}{6}(5 - 2x)$$

18.4 problem 480

Internal problem ID [3225]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 480.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(2 + 2x + 3y)\, y' - 1 + 2x + 3y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve((2+2*x+3*y(x))*diff(y(x),x) = 1-2*x-3*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{2x}{3} + 3\,\mathrm{LambertW}\left(\frac{\mathrm{e}^{-\frac{x}{9}} c_1 \mathrm{e}^{-\frac{7}{9}}}{9}\right) + \frac{7}{3}$$

✓ Solution by Mathematica

Time used: 4.809 (sec). Leaf size: 43

```
DSolve[(2+2 x+3 y[x])y'[x]==1-2 x-3 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(9W\left(-e^{-\frac{x}{9}-1+c_1}\right) - 2x + 7\right)$$
$$y(x) \to \frac{1}{3}(7 - 2x)$$

18.5 problem 481

Internal problem ID [3226]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 481.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(5-2x-3y)\,y'+1-2x-3y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 21

```
dsolve((5-2*x-3*y(x))*diff(y(x),x)+1-2*x-3*y(x) = 0,y(x), singsol=all)
```

$$y(x)=-\frac{2x}{3}-4\,\mathrm{LambertW}\left(-\frac{\mathrm{e}^{\frac{x}{12}}c_1\mathrm{e}^{-\frac{7}{12}}}{12}\right)-\frac{7}{3}$$

✓ Solution by Mathematica

Time used: 3.725 (sec). Leaf size: 43

```
DSolve[(5-2 x-3 y[x])y'[x]+1-2 x -3 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -4W\left(-e^{\frac{x}{12}-1+c_1}\right)-\frac{2x}{3}-\frac{7}{3}$$

$$y(x)\to \frac{1}{3}(-2x-7)$$

18.6 problem 482

Internal problem ID [3227]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 482.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(1+9x-3y)\,y'+2+3x-y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve((1+9*x-3*y(x))*diff(y(x),x)+2+3*x-y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{\mathrm{e}^{-\operatorname{LambertW}\left(3\,\mathrm{e}^{-20x}\mathrm{e}^{-3}\mathrm{e}^{20c_1}\right)-20x-3+20c_1}}{2}+3x+\frac{1}{2}$$

✓ Solution by Mathematica

Time used: 4.507 (sec). Leaf size: 37

```
DSolve[(1+9 x-3 y[x])y'[x]+2+3 x-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{6}\left(W\left(-e^{-20x-1+c_1}\right)+18x+3\right)$$
$$y(x)\to 3x+\frac{1}{2}$$

18.7 problem 483

Internal problem ID [3228]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 483.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(x+4y)\,y'+4x-y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 24

```
dsolve((x+4*y(x))*diff(y(x),x)+4*x-y(x) = 0,y(x), singsol=all)
```

$$y(x)=\tan\left(\mathrm{RootOf}\left(2\ln\left(\frac{1}{\cos\left(\_Z\right)^{2}}\right)+\_Z+4\ln\left(x\right)+4c_{1}\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.033 (sec). Leaf size: 32

```
DSolve[(x+4 y[x])y'[x]+4 x-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{y(x)}{x}\right)+2\log\left(\frac{y(x)^{2}}{x^{2}}+1\right)=-4\log(x)+c_{1},y(x)\right]$$

18.8 problem 484

Internal problem ID [3229]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 484.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(3 + 2x + 4y)\, y' - x - 2y - 1 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve((3+2*x+4*y(x))*diff(y(x),x) = 1+x+2*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{x}{2} + \frac{\text{LambertW}\left(e^{5} e^{8x} c_1\right)}{8} - \frac{5}{8}$$

✓ Solution by Mathematica

Time used: 4.682 (sec). Leaf size: 39

```
DSolve[(3+2 x+4 y[x])y'[x]==1+x+2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}\left(W\left(-e^{8x-1+c_1}\right) - 4x - 5\right)$$

$$y(x) \to \frac{1}{8}(-4x - 5)$$

18.9 problem 485

Internal problem ID [3230]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 485.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd typ`

$$(5 + 2x - 4y)\, y' - 3 - x + 2y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 41

```
dsolve((5+2*x-4*y(x))*diff(y(x),x) = 3+x-2*y(x),y(x), singsol=all)
```

$$y(x) = \frac{x}{2} + \frac{5}{4} - \frac{\sqrt{4c_1 - 4x + 25}}{4}$$
$$y(x) = \frac{x}{2} + \frac{5}{4} + \frac{\sqrt{4c_1 - 4x + 25}}{4}$$

✓ Solution by Mathematica

Time used: 0.104 (sec). Leaf size: 61

```
DSolve[(5+2 x-4 y[x])y'[x]==3+x-2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(2x - i\sqrt{4x - 25 - 16c_1} + 5\right)$$
$$y(x) \to \frac{1}{4}\left(2x + i\sqrt{4x - 25 - 16c_1} + 5\right)$$

18.10 problem 486

Internal problem ID [3231]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 486.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd typ`

$$(5+3x-4y)\,y'-2-7x+3y=0$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 38

```
dsolve((5+3*x-4*y(x))*diff(y(x),x) = 2+7*x-3*y(x),y(x), singsol=all)
```

$$y(x)=\frac{29}{19}-\frac{-\frac{3(-7+19x)c_1}{2}+\frac{\sqrt{-19(-7+19x)^2c_1^2+4}}{2}}{38c_1}$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 69

```
DSolve[(5+3 x-4 y[x])y'[x]==2+7 x-3 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{4}\left(3x-i\sqrt{x(19x-14)-25-16c_1}+5\right)$$
$$y(x)\to\frac{1}{4}\left(3x+i\sqrt{x(19x-14)-25-16c_1}+5\right)$$

18.11 problem 487

Internal problem ID [3232]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 487.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$4(-y-x+1)\,y'+2-x=0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 29

```
dsolve(4*(1-x-y(x))*diff(y(x),x)+2-x = 0,y(x), singsol=all)
```

$$y(x)=-1-\frac{(x-2)\left(-1+\mathrm{LambertW}\left(-c_1(x-2)\right)\right)}{2\,\mathrm{LambertW}\left(-c_1\left(x-2\right)\right)}$$

✓ Solution by Mathematica

Time used: 3.321 (sec). Leaf size: 109

```
DSolve[4(1-x-y[x])y'[x]+2-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2^{2/3}\left(x\log\left(\frac{x-2}{y(x)+x-1}\right)-x\log\left(\frac{2y(x)+x}{y(x)+x-1}\right)+2y(x)\left(\log\left(\frac{x-2}{y(x)+x-1}\right)-\log\left(\frac{2y(x)+x}{y(x)+x-1}\right)+1\right)+2x-2\right)}{9(2y(x)+x)}\right.$$

18.12 problem 488

Internal problem ID [3233]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 488.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(11 - 11x - 4y)\, y' - 62 + 8x + 25y = 0$$

✓ Solution by Maple

Time used: 0.281 (sec). Leaf size: 377

```
dsolve((11-11*x-4*y(x))*diff(y(x),x) = 62-8*x-25*y(x),y(x), singsol=all)
```

$$y(x) = \frac{22}{9} + \frac{36(-1+9x)\left(-\frac{\left(64-8748(-1+9x)^2c_1+108\sqrt{6561(-1+9x)^4c_1^2-96(-1+9x)^2c_1}\right)^{\frac{1}{3}}}{27} - \frac{16}{27\left(64-8748(-1+9x)^2c_1+108\sqrt{656\cdots}\right.}\cdots\right.}{-3\left(64-8748\left(-1+9x\right)^2c_1+108\sqrt{6561\left(-1+9x\right)^4c_1^2-96\left(-1+9x\right)^2c_1}\right)^{\frac{1}{3}} - \frac{\cdots}{\left(64-8748(-1+9x)^2c_1+108\cdots\right.}}$$

✓ Solution by Mathematica

Time used: 60.169 (sec). Leaf size: 1677

```
DSolve[(11-11 x-4 y[x])y'[x]==62-8x -25 y[x],y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

18.13 problem 489

Internal problem ID [3234]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 489.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(6+3x+5y)\, y' - 2 - x - 7y = 0$$

✓ Solution by Maple

Time used: 0.75 (sec). Leaf size: 32

```
dsolve((6+3*x+5*y(x))*diff(y(x),x) = 2+x+7*y(x),y(x), singsol=all)
```

$$y(x) = -\operatorname{RootOf}\left(\left(2+x\right)^{3} c_1\_Z^{12} - 5\_Z^{3} + 6\right)^{3}(2+x) + 2 + x$$

✓ Solution by Mathematica

Time used: 60.156 (sec). Leaf size: 4977

```
DSolve[(6+3 x+5 y[x])y'[x]==2 + x+7 y[x],y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

18.14 problem 490

Internal problem ID [3235]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 490.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(7x + 5y)\, y' + 10x + 8y = 0$$

✓ Solution by Maple

Time used: 0.281 (sec). Leaf size: 49

```
dsolve((7*x+5*y(x))*diff(y(x),x)+10*x+8*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x\left(-2c_1^2 + c_1^2 \operatorname{RootOf}\left(\_Z^{25}c_1x^5 - 2\_Z^{20}c_1x^5 + \_Z^{15}c_1x^5 - 1\right)^5\right)}{c_1^2}$$

✓ Solution by Mathematica

Time used: 2.16 (sec). Leaf size: 276

```
DSolve[(7 x+5 y[x])y'[x]+10 x+8 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \operatorname{Root}\left[\#1^5 + 8\#1^4x + 25\#1^3x^2 + 38\#1^2x^3 + 28\#1x^4 + 8x^5 - e^{c_1}\&, 1\right]$$
$$y(x) \to \operatorname{Root}\left[\#1^5 + 8\#1^4x + 25\#1^3x^2 + 38\#1^2x^3 + 28\#1x^4 + 8x^5 - e^{c_1}\&, 2\right]$$
$$y(x) \to \operatorname{Root}\left[\#1^5 + 8\#1^4x + 25\#1^3x^2 + 38\#1^2x^3 + 28\#1x^4 + 8x^5 - e^{c_1}\&, 3\right]$$
$$y(x) \to \operatorname{Root}\left[\#1^5 + 8\#1^4x + 25\#1^3x^2 + 38\#1^2x^3 + 28\#1x^4 + 8x^5 - e^{c_1}\&, 4\right]$$
$$y(x) \to \operatorname{Root}\left[\#1^5 + 8\#1^4x + 25\#1^3x^2 + 38\#1^2x^3 + 28\#1x^4 + 8x^5 - e^{c_1}\&, 5\right]$$

18.15 problem 491

Internal problem ID [3236]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 491.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class A']]`

$$\left(x + 4x^3 + 5y\right) y' + 7x^3 + 3x^2 y + 4y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 3020

```
dsolve((x+4*x^3+5*y(x))*diff(y(x),x)+7*x^3+3*x^2*y(x)+4*y(x) = 0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 60.417 (sec). Leaf size: 3641

```
DSolve[(x+4 x^3+5 y[x])y'[x]+7 x^3+3 x^2 y[x]+4 y[x]==0,y[x],x,IncludeSingularSolutions -> Tr
```

Too large to display

18.16 problem 492

Internal problem ID [3237]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 492.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(5 - x + 6y)\, y' - 3 + x - 4y = 0}$$

✓ Solution by Maple

Time used: 1.515 (sec). Leaf size: 29

```
dsolve((5-x+6*y(x))*diff(y(x),x) = 3-x+4*y(x),y(x), singsol=all)
```

$$y(x) = -1 + \frac{4c_1(x+1) + \sqrt{-8c_1\,(x+1)+9} - 3}{8c_1}$$

✓ Solution by Mathematica

Time used: 60.101 (sec). Leaf size: 377

```
DSolve[(5-x+6 y[x])y'[x]==3-x+4 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(x + \frac{2}{\frac{1}{x+1} + \sqrt{\frac{1}{(x+1)^2 + e^{\frac{4c_1}{9}}(x+1)^4} - \sqrt{-\frac{1}{2(x+1)^4 + e^{\frac{4c_1}{9}}(x+1)^6 + e^{-\frac{4c_1}{9}}(x+1)^2}}}} - 5\right)$$

$$y(x) \to \frac{1}{6}\left(x + \frac{2}{\frac{1}{x+1} - \sqrt{\frac{1}{(x+1)^2 + e^{\frac{4c_1}{9}}(x+1)^4} - \sqrt{-\frac{1}{2(x+1)^4 + e^{\frac{4c_1}{9}}(x+1)^6 + e^{-\frac{4c_1}{9}}(x+1)^2}}}} - 5\right)$$

$$y(x) \to \frac{1}{6}\left(x + \frac{2}{\frac{1}{x+1} + \sqrt{\sqrt{-\frac{1}{2(x+1)^4 + e^{\frac{4c_1}{9}}(x+1)^6 + e^{-\frac{4c_1}{9}}(x+1)^2}} + \frac{1}{(x+1)^2 + e^{\frac{4c_1}{9}}(x+1)^4}}} - 5\right)$$

$$y(x) \to \frac{1}{6}\left(x + \frac{2}{\frac{1}{x+1} - \sqrt{\sqrt{-\frac{1}{2(x+1)^4 + e^{\frac{4c_1}{9}}(x+1)^6 + e^{-\frac{4c_1}{9}}(x+1)^2}} + \frac{1}{(x+1)^2 + e^{\frac{4c_1}{9}}(x+1)^4}}} - 5\right)$$

18.17 problem 493

Internal problem ID [3238]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 493.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$3(x+2y)\,y'-1+x+2y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(3*(x+2*y(x))*diff(y(x),x) = 1-x-2*y(x),y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-\mathrm{LambertW}\left(-\mathrm{e}^{-\frac{x}{6}}\mathrm{e}^{-1}\mathrm{e}^{\frac{c_1}{6}}\right)-\frac{x}{6}-1+\frac{c_1}{6}}-1-\frac{x}{2}$$

✓ Solution by Mathematica

Time used: 3.815 (sec). Leaf size: 39

```
DSolve[3(x+2 y[x])y'[x]==1-x-2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -W\left(-e^{-\frac{x}{6}-1+c_1}\right)-\frac{x}{2}-1$$
$$y(x)\to -\frac{x}{2}-1$$

18.18 problem 494

Internal problem ID [3239]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 494.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{3y - 7x + 7 + (7y - 3x + 3) y' = 0}$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 706

```
dsolve((3-3*x+7*y(x))*diff(y(x),x)+7-7*x+3*y(x) = 0,y(x), singsol=all)
```

Expression too large to display

✓ Solution by Mathematica

Time used: 60.736 (sec). Leaf size: 7785

```
DSolve[(3-3 x+7 y[x])y'[x]+7-7 x+3 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

18.19 problem 495

Internal problem ID [3240]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 495.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(1 + x + 9y)\, y' + 1 + x + 5y = 0$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 29

```
dsolve((1+x+9*y(x))*diff(y(x),x)+1+x+5*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{(x+1)\left(2+3\,\mathrm{LambertW}\left(\frac{2c_1(x+1)}{3}\right)\right)}{9\,\mathrm{LambertW}\left(\frac{2c_1(x+1)}{3}\right)}$$

✓ Solution by Mathematica

Time used: 1.823 (sec). Leaf size: 145

```
DSolve[(1+x+9 y[x])y'[x]+1+x+5 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{(-2)^{2/3}\left((x+1)\left(3\log\left(-\frac{6(-2)^{2/3}(x+1)}{9y(x)+x+1}\right)-3\log\left(\frac{9(-2)^{2/3}(3y(x)+x+1)}{9y(x)+x+1}\right)+1\right)+9y(x)\left(\log\left(-\frac{6(-2)^{2/3}(x+}{9y(x)+x+1}\right.\right.\right.}{27(3y(x)+x+1)}\right.$$

18.20 problem 496

Internal problem ID [3241]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 496.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _exact, _rational, [_Abel, ‘2nd typ`

$$(8 + 5x - 12y)\, y' - 3 - 2x + 5y = 0$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 33

```
dsolve((8+5*x-12*y(x))*diff(y(x),x) = 3+2*x-5*y(x),y(x), singsol=all)
```

$$y(x) = -1 - \frac{-\frac{5(4+x)c_1}{12} + \frac{\sqrt{(4+x)^2 c_1^2+24}}{12}}{c_1}$$

✓ Solution by Mathematica

Time used: 0.125 (sec). Leaf size: 73

```
DSolve[(8+5 x-12 y[x])y'[x]==3+2 x-5 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}\left(5x - i\sqrt{-x(x+8) - 16(4+9c_1)} + 8\right)$$
$$y(x) \to \frac{1}{12}\left(5x + i\sqrt{-x(x+8) - 16(4+9c_1)} + 8\right)$$

18.21 problem 497

Internal problem ID [3242]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 497.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(140 + 7x - 16y)\, y' + 25 + 8x + y = 0$$

✓ Solution by Maple

Time used: 0.344 (sec). Leaf size: 334

```
dsolve((140+7*x-16*y(x))*diff(y(x),x)+25+8*x+y(x) = 0,y(x), singsol=all)
```

$$y(x) = 7$$
$$- \frac{-(4+x)^8 \operatorname{RootOf}\left((4+x)^8 c_1\_Z^{64} + 9(4+x)^8 c_1\_Z^{56} + 27(4+x)^8 c_1\_Z^{48} + 27(4+x)^8 c_1\_Z^{40} - 8\right)^5}{(4+x)^7 \operatorname{RootOf}\left((4+x)^8 c_1\_Z^{64} + 9\,(4+x)^8 c_1\_Z^{56} + 27\,(4+x)^8 c_1\_Z^{48} + 27\,(4+x)^8 c_1\_Z}$$

✓ Solution by Mathematica

Time used: 60.059 (sec). Leaf size: 1673

```
DSolve[(140+7 x-16 y[x])y'[x]+25+8 x+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

18.22 problem 498

Internal problem ID [3243]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 498.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(3 + 9x + 21y) y' - 45 - 7x + 5y = 0$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 62

```
dsolve((3+9*x+21*y(x))*diff(y(x),x) = 45+7*x-5*y(x),y(x), singsol=all)
```

$$y(x) = \frac{11}{3} - \frac{\text{RootOf}\left(\left(x+5\right)^{7} c_1\_Z^{49} - 12\left(x+5\right)^{7} c_1\_Z^{42} + 48\left(x+5\right)^{7} c_1\_Z^{35} - 64\left(x+5\right)^{7} c_1\_Z^{28} - 27\right)^{7} \left(x+5\right)}{3} + \frac{x}{3}$$

✓ Solution by Mathematica

Time used: 60.746 (sec). Leaf size: 7785

```
DSolve[(3+9 x+21 y[x])y'[x]==45 +7 x-5 y[x],y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

18.23 problem 499

Internal problem ID [3244]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 499.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(ax + yb)\, y' + x = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 98

```
dsolve((a*x+b*y(x))*diff(y(x),x)+x = 0,y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(b\_Z^2 - e^{\text{RootOf}\left(x^2\left(\tanh\left(\frac{\sqrt{a^2-4b}\,(2c_1+\_Z+2\ln(x))}{2a}\right)^2 a^2 - 4\tanh\left(\frac{\sqrt{a^2-4b}\,(2c_1+\_Z+2\ln(x))}{2a}\right)^2 b - 4b\,e^{-\_Z} - a^2 + 4b\right)\right)} + 1 + a\_Z\right) x$$

✓ Solution by Mathematica

Time used: 0.114 (sec). Leaf size: 75

```
DSolve[(a x+b y[x])y'[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{a \arctan\left(\frac{a+\frac{2by(x)}{x}}{\sqrt{4b-a^2}}\right)}{\sqrt{4b-a^2}} + \frac{1}{2}\log\left(\frac{ay(x)}{x} + \frac{by(x)^2}{x^2} + 1\right) = -\log(x) + c_1, y(x)\right]$$

18.24 problem 500

Internal problem ID [3245]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 500.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, [_Abel, '2nd type', 'cla`

$$\boxed{(ax + yb)\, y' + y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve((a*x+b*y(x))*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$x + \frac{by(x)}{a+1} - y(x)^{-a} c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 38

```
DSolve[(a x+b y[x])y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\log\left(a + \frac{by(x)}{x} + 1\right) + a\log\left(\frac{y(x)}{x}\right)}{a+1} = -\log(x) + c_1, y(x)\right]$$

18.25 problem 501

Internal problem ID [3246]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 501.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd typ`

$$\boxed{(ax + yb)\, y' + bx + ay = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 85

```
dsolve((a*x+b*y(x))*diff(y(x),x)+b*x+a*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{c_1 a x - \sqrt{a^2 c_1^2 x^2 - b^2 c_1^2 x^2 + b}}{b c_1}$$
$$y(x) = -\frac{c_1 a x + \sqrt{a^2 c_1^2 x^2 - b^2 c_1^2 x^2 + b}}{b c_1}$$

✓ Solution by Mathematica

Time used: 15.897 (sec). Leaf size: 135

```
DSolve[(a x+b y[x])y'[x]+b x+a y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{ax + \sqrt{x^2(a-b)(a+b) + be^{2c_1}}}{b}$$
$$y(x) \to \frac{-ax + \sqrt{x^2(a-b)(a+b) + be^{2c_1}}}{b}$$
$$y(x) \to -\frac{\sqrt{x^2(a-b)(a+b)} + ax}{b}$$
$$y(x) \to \frac{\sqrt{x^2(a-b)(a+b)} - ax}{b}$$

18.26 problem 502

Internal problem ID [3247]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 502.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(ax + yb)\, y' - bx - ay = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 64

```
dsolve((a*x+b*y(x))*diff(y(x),x) = b*x+a*y(x),y(x), singsol=all)
```

$$y(x) = x\, \mathrm{e}^{\mathrm{RootOf}\left(\mathrm{e}^{-\_Z} - \mathrm{e}^{\frac{2c_1 b}{a-b}}\, \mathrm{e}^{\frac{a\_Z}{a-b}}\, \mathrm{e}^{\frac{\_Zb}{a-b}}\, x^{\frac{2b}{a-b}} + 2\right)} + x$$

✓ Solution by Mathematica

Time used: 0.04 (sec). Leaf size: 48

```
DSolve[(a x+b y[x])y'[x]==b x+a y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}(a+b)\log\left(1 - \frac{y(x)}{x}\right) + \frac{1}{2}(b-a)\log\left(\frac{y(x)}{x} + 1\right) = -b\log(x) + c_1, y(x)\right]$$

18.27 problem 505

Internal problem ID [3248]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 505.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xyy' + 1 + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(x*y(x)*diff(y(x),x)+1+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-x^2 + c_1}}{x}$$
$$y(x) = -\frac{\sqrt{-x^2 + c_1}}{x}$$

✓ Solution by Mathematica

Time used: 0.323 (sec). Leaf size: 96

```
DSolve[x y[x] y'[x]+1+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^2 + e^{2c_1}}}{x}$$
$$y(x) \to \frac{\sqrt{-x^2 + e^{2c_1}}}{x}$$
$$y(x) \to -i$$
$$y(x) \to i$$
$$y(x) \to \frac{x}{\sqrt{-x^2}}$$
$$y(x) \to \frac{\sqrt{-x}}{\sqrt{x}}$$

18.28 problem 506

Internal problem ID [3249]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 506.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{xyy' - x - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(x*y(x)*diff(y(x),x) = x+y(x)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 x^2 - 2x}$$
$$y(x) = -\sqrt{c_1 x^2 - 2x}$$

✓ Solution by Mathematica

Time used: 0.272 (sec). Leaf size: 42

```
DSolve[x y[x] y'[x]==x+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{-2 + c_1 x}$$
$$y(x) \to \sqrt{x}\sqrt{-2 + c_1 x}$$

18.29 problem 507

Internal problem ID [3250]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 507.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{xyy' + x^2 + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 39

```
dsolve(x*y(x)*diff(y(x),x)+x^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2x^4 + 4c_1}}{2x}$$
$$y(x) = \frac{\sqrt{-2x^4 + 4c_1}}{2x}$$

✓ Solution by Mathematica

Time used: 0.2 (sec). Leaf size: 46

```
DSolve[x y[x] y'[x]+x^2+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-\frac{x^4}{2} + c_1}}{x}$$
$$y(x) \to \frac{\sqrt{-\frac{x^4}{2} + c_1}}{x}$$

18.30 problem 508

Internal problem ID [3251]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 508.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class D'], _rational, _Bernoulli]`

$$\boxed{xyy' + x^4 - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(x*y(x)*diff(y(x),x)+x^4-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 + c_1}\, x$$
$$y(x) = -\sqrt{-x^2 + c_1}\, x$$

✓ Solution by Mathematica

Time used: 0.381 (sec). Leaf size: 43

```
DSolve[x y[x] y'[x]+x^4-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^4 + c_1 x^2}$$
$$y(x) \to \sqrt{-x^4 + c_1 x^2}$$

18.31 problem 509

Internal problem ID [3252]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 509.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _Bernoulli]`

$$\boxed{xyy' - a\,x^3\cos(x) - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve(x*y(x)*diff(y(x),x) = a*x^3*cos(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{2\sin(x)\,a + c_1}\,x$$
$$y(x) = -\sqrt{2\sin(x)\,a + c_1}\,x$$

✓ Solution by Mathematica

Time used: 0.341 (sec). Leaf size: 38

```
DSolve[x y[x] y'[x]==a x^3 Cos[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x\sqrt{2a\sin(x) + c_1}$$
$$y(x) \to x\sqrt{2a\sin(x) + c_1}$$

18.32 problem 510

Internal problem ID [3253]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 510.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{xyy' - x^2 + yx - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve(x*y(x)*diff(y(x),x) = x^2-x*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\operatorname{LambertW}\left(\frac{\mathrm{e}^{-c_1}\mathrm{e}^{-1}}{x}\right)-c_1-1} + x$$

✓ Solution by Mathematica

Time used: 3.612 (sec). Leaf size: 25

```
DSolve[x y[x] y'[x]==x^2-x y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(1 + W\left(\frac{e^{-1+c_1}}{x}\right)\right)$$
$$y(x) \to x$$

18.33 problem 511

Internal problem ID [3254]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 511.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{xyy' + 2x^2 - 2yx - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
dsolve(x*y(x)*diff(y(x),x)+2*x^2-2*x*y(x)-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\operatorname{LambertW}\left(\mathrm{e}^{2c_1}\mathrm{e}^{-1}x^2\right)+2c_1-1}x^3 + x$$

✓ Solution by Mathematica

Time used: 3.713 (sec). Leaf size: 25

```
DSolve[x y[x] y'[x]+2 x^2-2 x y[x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x\left(1 + W\left(e^{-1+c_1}x^2\right)\right)$$
$$y(x) \to x$$

18.34 problem 512

Internal problem ID [3255]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 512.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xyy' - a - by^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 48

```
dsolve(x*y(x)*diff(y(x),x) = a+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{b\,(x^{2b}c_1 b - a)}}{b}$$
$$y(x) = -\frac{\sqrt{b\,(x^{2b}c_1 b - a)}}{b}$$

✓ Solution by Mathematica

Time used: 1.413 (sec). Leaf size: 94

```
DSolve[x y[x] y'[x]==a+b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-a + e^{2b(\log(x)+c_1)}}}{\sqrt{b}}$$
$$y(x) \to \frac{\sqrt{-a + e^{2b(\log(x)+c_1)}}}{\sqrt{b}}$$
$$y(x) \to -\frac{i\sqrt{a}}{\sqrt{b}}$$
$$y(x) \to \frac{i\sqrt{a}}{\sqrt{b}}$$

18.35 problem 513

Internal problem ID [3256]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 18
Problem number: 513.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Bernoulli]`

$$\boxed{xyy' - x^n a - by^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 98

```
dsolve(x*y(x)*diff(y(x),x) = a*x^n+b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-(2b-n)\left(-2x^{2b}c_1 b + x^{2b}c_1 n + 2a\,x^n\right)}}{2b-n}$$
$$y(x) = -\frac{\sqrt{-(2b-n)\left(-2x^{2b}c_1 b + x^{2b}c_1 n + 2a\,x^n\right)}}{2b-n}$$

✓ Solution by Mathematica

Time used: 4.261 (sec). Leaf size: 86

```
DSolve[x y[x] y'[x]==a x^n+b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-2ax^n + c_1(2b-n)x^{2b}}}{\sqrt{2b-n}}$$
$$y(x) \to \frac{\sqrt{-2ax^n + c_1(2b-n)x^{2b}}}{\sqrt{2b-n}}$$

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19.1 problem 514

Internal problem ID [3257]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 514.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$xyy' - (x^2 + 1)(1 - y^2) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 44

```
dsolve(x*y(x)*diff(y(x),x) = (x^2+1)*(1-y(x)^2),y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{e^{-x^2} c_1 + x^2}}{x}$$
$$y(x) = -\frac{\sqrt{e^{-x^2} c_1 + x^2}}{x}$$

✓ Solution by Mathematica

Time used: 5.394 (sec). Leaf size: 95

```
DSolve[x y[x] y'[x]==(1+x^2)(1-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{x^2 + e^{-x^2+2c_1}}}{x}$$
$$y(x) \to \frac{\sqrt{x^2 + e^{-x^2+2c_1}}}{x}$$
$$y(x) \to -1$$
$$y(x) \to 1$$
$$y(x) \to -\frac{x}{\sqrt{x^2}}$$
$$y(x) \to \frac{x}{\sqrt{x^2}}$$

19.2 problem 515

Internal problem ID [3258]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 515.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xyy' + x^2 \operatorname{arccot}\left(\frac{y}{x}\right) - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*y(x)*diff(y(x),x)+x^2*arccot(y(x)/x)-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(\int^{\_Z} \frac{\_a}{\operatorname{arccot}(\_a)} d\_a + \ln(x) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.557 (sec). Leaf size: 31

```
DSolve[x y[x] y'[x]+x^2 ArcCot[y[x]/x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\operatorname{Solve}\left[\int_1^{\frac{y(x)}{x}} \frac{K[1]}{\cot^{-1}(K[1])} dK[1] = -\log(x) + c_1, y(x)\right]$$

19.3 problem 516

Internal problem ID [3259]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 516.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{xyy' + x^2 e^{-\frac{2y}{x}} - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve(x*y(x)*diff(y(x),x)+x^2*exp(-2*y(x)/x)-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{(\text{LambertW}\left(-4(\ln(x) + c_1)\, e^{-1}\right) + 1)\, x}{2}$$

✓ Solution by Mathematica

Time used: 0.12 (sec). Leaf size: 25

```
DSolve[x y[x] y'[x]+x^2 Exp[(-2 y[x])/x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2} x \left(1 + W\left(\frac{4(-\log(x) + c_1)}{e}\right)\right)$$

19.4 problem 517

Internal problem ID [3260]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 517.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(1+yx)\,y'+y^2=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 14

```
dsolve((1+x*y(x))*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-\operatorname{LambertW}(x\,\mathrm{e}^{c_1})+c_1}$$

✓ Solution by Mathematica

Time used: 1.709 (sec). Leaf size: 21

```
DSolve[(1+x y[x])y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{W(e^{c_1}x)}{x}$$
$$y(x)\to 0$$

19.5 problem 518

Internal problem ID [3261]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 518.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(1+y)\,y' - (1-x)\,y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(x*(1+y(x))*diff(y(x),x)-(1-x)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \operatorname{LambertW}\left(\frac{\mathrm{e}^{-x}x}{c_1}\right)$$

✓ Solution by Mathematica

Time used: 3.355 (sec). Leaf size: 21

```
DSolve[x(1+y[x])y'[x]-(1-x)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to W\left(xe^{-x+c_1}\right)$$
$$y(x) \to 0$$

19.6 problem 519

Internal problem ID [3262]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 519.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$x(1-y)\,y'+(x+1)\,y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(x*(1-y(x))*diff(y(x),x)+(1+x)*y(x) = 0,y(x), singsol=all)
```

$$y(x)=-\operatorname{LambertW}\left(-\frac{\mathrm{e}^{-x}}{c_1 x}\right)$$

✓ Solution by Mathematica

Time used: 2.998 (sec). Leaf size: 28

```
DSolve[x(1-y[x])y'[x]+(1+x)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -W\left(-\frac{e^{-x-c_1}}{x}\right)$$
$$y(x)\to 0$$

19.7 problem 520

Internal problem ID [3263]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 520.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$x(1-y)\,y' + (1-x)\,y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*(1-y(x))*diff(y(x),x)+(1-x)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\operatorname{LambertW}\left(-\frac{\mathrm{e}^{x} c_1}{x}\right)$$

✓ Solution by Mathematica

Time used: 2.996 (sec). Leaf size: 26

```
DSolve[x(1-y[x])y'[x]+(1-x)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -W\left(-\frac{e^{x-c_1}}{x}\right)$$
$$y(x) \to 0$$

19.8 problem 521

Internal problem ID [3264]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 521.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x(y+2)\,y'+ax=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 37

```
dsolve(x*(2+y(x))*diff(y(x),x)+a*x = 0,y(x), singsol=all)
```

$$y(x) = -2-\sqrt{-2c_1a-2ax+4}$$
$$y(x) = -2+\sqrt{-2c_1a-2ax+4}$$

✓ Solution by Mathematica

Time used: 0.125 (sec). Leaf size: 50

```
DSolve[x(2+y[x])y'[x]+a x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2-\sqrt{2}\sqrt{-ax+2+c_1}$$
$$y(x) \to -2+\sqrt{2}\sqrt{-ax+2+c_1}$$

19.9 problem 522

Internal problem ID [3265]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 522.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]'], [_Abe`

$$(2 + 3x - yx) y' + y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
dsolve((2+3*x-x*y(x))*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$c_1 + \frac{e^{y(x)}}{xy(x)^3 - 2y(x)^2 - 4y(x) - 4} = 0$$

✓ Solution by Mathematica

Time used: 0.084 (sec). Leaf size: 35

```
DSolve[(2+3 x-x y[x])y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = -\frac{2(-y(x)^2 - 2y(x) - 2)}{y(x)^3} + \frac{c_1 e^{y(x)}}{y(x)^3}, y(x)\right]$$

19.10 problem 523

Internal problem ID [3266]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 523.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$x(4+y)\,y'-2x-2y-y^2=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 145

```
dsolve(x*(4+y(x))*diff(y(x),x) = 2*x+2*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x)=\frac{(4+x)^{\frac{3}{2}}\sqrt{\frac{c_1x+4c_1-4}{4+x}}\,x+4x^{\frac{3}{2}}+16\sqrt{x}}{(4+x)^{\frac{3}{2}}\sqrt{\frac{c_1x+4c_1-4}{4+x}}-x^{\frac{3}{2}}-4\sqrt{x}}$$

$$y(x)=\frac{(4+x)^{\frac{3}{2}}\sqrt{\frac{c_1x+4c_1-4}{4+x}}\,x-4x^{\frac{3}{2}}-16\sqrt{x}}{(4+x)^{\frac{3}{2}}\sqrt{\frac{c_1x+4c_1-4}{4+x}}+x^{\frac{3}{2}}+4\sqrt{x}}$$

✓ Solution by Mathematica

Time used: 1.038 (sec). Leaf size: 90

```
DSolve[x(4+y[x])y'[x]==2 x+2 y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{c_1x+\sqrt{x}\sqrt{x+4}\sqrt{-\frac{4}{x+4}+c_1}}{-1+c_1}$$

$$y(x)\to -4+\frac{1}{\frac{1}{x+4}+\frac{\sqrt{x}}{(x+4)^{3/2}\sqrt{-\frac{4}{x+4}+c_1}}}$$

$$y(x)\to x$$

19.11 problem 524

Internal problem ID [3267]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 524.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, ‘2nd type‘, ‘class B‘]]`

$$x(a+y)\,y' + bx + cy = 0$$

✗ Solution by Maple

```
dsolve(x*(a+y(x))*diff(y(x),x)+b*x+c*y(x) = 0,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x(a+y[x])y'[x]+b x+c y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Not solved

19.12 problem 525

Internal problem ID [3268]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 525.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(a+y)\,y' - y(Bx+A) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 46

```
dsolve(x*(a+y(x))*diff(y(x),x) = y(x)*(B*x+A),y(x), singsol=all)
```

$$y(x) = e^{\frac{A\ln(x)+Bx-a\,\mathrm{LambertW}\left(\frac{x^{\frac{A}{a}}\,e^{\frac{Bx}{a}+\frac{c_1}{a}}}{a}\right)+c_1}{a}}$$

✓ Solution by Mathematica

Time used: 1.029 (sec). Leaf size: 36

```
DSolve[x(a+y[x])y'[x]==y[x](A+B x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to aW\left(\frac{x^{\frac{A}{a}}e^{\frac{Bx+c_1}{a}}}{a}\right)$$

$$y(x) \to 0$$

19.13 problem 526

Internal problem ID [3269]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 526.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{x(x+y)\,y'+y^2=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 44

```
dsolve(x*(x+y(x))*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{1+\sqrt{c_1x^2+1}}{c_1x}$$
$$y(x)=-\frac{-1+\sqrt{c_1x^2+1}}{c_1x}$$

✓ Solution by Mathematica

Time used: 2.502 (sec). Leaf size: 80

```
DSolve[x(x+y[x])y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{e^{2c_1}-\sqrt{e^{2c_1}\left(x^2+e^{2c_1}\right)}}{x}$$
$$y(x)\to\frac{\sqrt{e^{2c_1}\left(x^2+e^{2c_1}\right)}+e^{2c_1}}{x}$$
$$y(x)\to 0$$

19.14 problem 527

Internal problem ID [3270]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 527.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$x(x-y)\,y'+y^2=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve(x*(x-y(x))*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-\operatorname{LambertW}\left(-\frac{\mathrm{e}^{-c_1}}{x}\right)-c_1}$$

✓ Solution by Mathematica

Time used: 2.213 (sec). Leaf size: 25

```
DSolve[x(x-y[x])y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\rightarrow -xW\left(-\frac{e^{-c_1}}{x}\right)$$
$$y(x)\rightarrow 0$$

19.15 problem 528

Internal problem ID [3271]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 528.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$x(x+y)\,y' - x^2 - y^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 32

```
dsolve(x*(x+y(x))*diff(y(x),x) = x^2+y(x)^2,y(x), singsol=all)
```

$$y(x) = x\,\mathrm{e}^{-\mathrm{LambertW}\left(\frac{\mathrm{e}^{-\frac{c_1}{2}}\mathrm{e}^{-\frac{1}{2}}}{2\sqrt{x}}\right)-\frac{c_1}{2}-\frac{1}{2}-\frac{\ln(x)}{2}} + x$$

✓ Solution by Mathematica

Time used: 7.08 (sec). Leaf size: 35

```
DSolve[x(x+y[x])y'[x]==x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + 2xW\left(\frac{e^{\frac{-1+c_1}{2}}}{2\sqrt{x}}\right)$$

$$y(x) \to x$$

19.16 problem 529

Internal problem ID [3272]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 529.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$x(x-y)\,y'+2x^2+3yx-y^2=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 59

```
dsolve(x*(x-y(x))*diff(y(x),x)+2*x^2+3*x*y(x)-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x^2-\sqrt{2c_1^2x^4+1}}{c_1x}$$
$$y(x)=\frac{c_1x^2+\sqrt{2c_1^2x^4+1}}{c_1x}$$

✓ Solution by Mathematica

Time used: 0.646 (sec). Leaf size: 99

```
DSolve[x(x-y[x])y'[x]+2 x^2+3 x y[x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x-\frac{\sqrt{2x^4+e^{2c_1}}}{x}$$
$$y(x)\to x+\frac{\sqrt{2x^4+e^{2c_1}}}{x}$$
$$y(x)\to x-\frac{\sqrt{2}\sqrt{x^4}}{x}$$
$$y(x)\to \frac{\sqrt{2}\sqrt{x^4}}{x}+x$$

19.17 problem 530

Internal problem ID [3273]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 530.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$x(x+y)\,y'-y(x+y)+x\sqrt{x^2-y^2}=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
dsolve(x*(x+y(x))*diff(y(x),x)-y(x)*(x+y(x))+x*sqrt(x^2-y(x)^2) = 0,y(x), singsol=all)
```

$$\arctan\left(\frac{y(x)}{\sqrt{x^2-y(x)^2}}\right)-\frac{\sqrt{x^2-y(x)^2}}{x}+\ln(x)-c_1=0$$

✓ Solution by Mathematica

Time used: 0.336 (sec). Leaf size: 109

```
DSolve[x(x+y[x])y'[x]-y[x](x+y[x])+x Sqrt[x^2-y[x]^2]==0,y[x],x,IncludeSingularSolutions -> T
```

$$\text{Solve}\left[\frac{2\sqrt{\frac{y(x)}{x}-1}\text{arctanh}\left(\frac{1}{\sqrt{\frac{\frac{y(x)}{x}-1}{\frac{y(x)}{x}+1}}}\right)+\left(\frac{y(x)}{x}-1\right)\sqrt{\frac{y(x)}{x}+1}}{\sqrt{\frac{\frac{y(x)}{x}-1}{\frac{y(x)}{x}+1}}\sqrt{\frac{y(x)}{x}+1}}=c_1-i\log(x),y(x)\right]$$

19.18 problem 531

Internal problem ID [3274]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 531.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$(a + x(x + y))\, y' - b(x + y)\, y = 0$$

✗ Solution by Maple

```
dsolve((a+x*(x+y(x)))*diff(y(x),x) = b*(x+y(x))*y(x),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a+x(x+y[x]))y'[x]==b(x+y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

Not solved

19.19 problem 532

Internal problem ID [3275]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 532.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$x(2x+y)\,y' - x^2 - yx + y^2 = 0$$

✓ Solution by Maple

Time used: 0.234 (sec). Leaf size: 63

```
dsolve(x*(2*x+y(x))*diff(y(x),x) = x^2+x*y(x)-y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{x\left(-\operatorname{RootOf}\left(3\_Z^{15}+\_Z^{9}-2c_1x^3\right)^9-c_1x^3\right)}{-\operatorname{RootOf}\left(3\_Z^{15}+\_Z^{9}-2c_1x^3\right)^9+2c_1x^3}$$

✓ Solution by Mathematica

Time used: 4.437 (sec). Leaf size: 431

```
DSolve[x(2 x+y[x])y'[x]==x^2+x y[x]-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[32\#1^5 - 80\#1^4x + 80\#1^3x^2 + \#1^2\left(-40x^3 + \frac{e^{6c_1}}{x^3}\right) + \#1\left(10x^4 + \frac{2e^{6c_1}}{x^2}\right) - x^5 + \frac{e^{6c_1}}{x}\&, 1\right]$$

$$y(x) \to \text{Root}\left[32\#1^5 - 80\#1^4x + 80\#1^3x^2 + \#1^2\left(-40x^3 + \frac{e^{6c_1}}{x^3}\right) + \#1\left(10x^4 + \frac{2e^{6c_1}}{x^2}\right) - x^5 + \frac{e^{6c_1}}{x}\&, 2\right]$$

$$y(x) \to \text{Root}\left[32\#1^5 - 80\#1^4x + 80\#1^3x^2 + \#1^2\left(-40x^3 + \frac{e^{6c_1}}{x^3}\right) + \#1\left(10x^4 + \frac{2e^{6c_1}}{x^2}\right) - x^5 + \frac{e^{6c_1}}{x}\&, 3\right]$$

$$y(x) \to \text{Root}\left[32\#1^5 - 80\#1^4x + 80\#1^3x^2 + \#1^2\left(-40x^3 + \frac{e^{6c_1}}{x^3}\right) + \#1\left(10x^4 + \frac{2e^{6c_1}}{x^2}\right) - x^5 + \frac{e^{6c_1}}{x}\&, 4\right]$$

$$y(x) \to \text{Root}\left[32\#1^5 - 80\#1^4x + 80\#1^3x^2 + \#1^2\left(-40x^3 + \frac{e^{6c_1}}{x^3}\right) + \#1\left(10x^4 + \frac{2e^{6c_1}}{x^2}\right) - x^5 + \frac{e^{6c_1}}{x}\&, 5\right]$$

19.20 problem 533

Internal problem ID [3276]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 533.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$x(4x - y)\, y' + 4x^2 - 6yx - y^2 = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 71

```
dsolve(x*(4*x-y(x))*diff(y(x),x)+4*x^2-6*x*y(x)-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2c_1x - \frac{1+\sqrt{-12c_1^2x^2+1}}{2xc_1}}{c_1}$$
$$y(x) = -\frac{2c_1x + \frac{-1+\sqrt{-12c_1^2x^2+1}}{2xc_1}}{c_1}$$

✓ Solution by Mathematica

Time used: 1.232 (sec). Leaf size: 90

```
DSolve[x(4 x -y[x])y'[x]+4 x^2-6 x y[x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{4x^2 + e^{\frac{c_1}{2}}\sqrt{12x^2 + e^{c_1}} + e^{c_1}}{2x}$$
$$y(x) \to -\frac{4x^2 - e^{\frac{c_1}{2}}\sqrt{12x^2 + e^{c_1}} + e^{c_1}}{2x}$$

19.21 problem 534

Internal problem ID [3277]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 534.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$x(y+x^3)\,y'-(-y+x^3)\,y=0$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 41

```
dsolve(x*(x^3+y(x))*diff(y(x),x) = (x^3-y(x))*y(x),y(x), singsol=all)
```

$$y(x)=\frac{c_1\left(c_1-\sqrt{x^4+c_1^2}\right)}{x}$$
$$y(x)=\frac{c_1\left(c_1+\sqrt{x^4+c_1^2}\right)}{x}$$

✓ Solution by Mathematica

Time used: 0.744 (sec). Leaf size: 72

```
DSolve[x(x^3+y[x])y'[x]==(x^3-y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{x^3}{-1+\sqrt{\frac{1}{x^2}}x\sqrt{1+c_1x^4}}$$
$$y(x)\to-\frac{x^4}{x+\frac{\sqrt{1+c_1x^4}}{\sqrt{\frac{1}{x^2}}}}$$
$$y(x)\to 0$$

19.22 problem 535

Internal problem ID [3278]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 535.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$x(2x^3+y)\,y'-(2x^3-y)\,y=0$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 47

```
dsolve(x*(2*x^3+y(x))*diff(y(x),x) = (2*x^3-y(x))*y(x),y(x), singsol=all)
```

$$y(x)=\frac{c_1\left(c_1-\sqrt{4x^4+c_1^2}\right)}{2x}$$
$$y(x)=\frac{c_1\left(c_1+\sqrt{4x^4+c_1^2}\right)}{2x}$$

✓ Solution by Mathematica

Time used: 0.719 (sec). Leaf size: 76

```
DSolve[x(2 x^3+y[x])y'[x]==(2 x^3-y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2x^4}{-x+\frac{\sqrt{1+4c_1x^4}}{\sqrt{\frac{1}{x^2}}}}$$
$$y(x)\to-\frac{2x^4}{x+\frac{\sqrt{1+4c_1x^4}}{\sqrt{\frac{1}{x^2}}}}$$
$$y(x)\to 0$$

19.23 problem 536

Internal problem ID [3279]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 536.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, [_Abel, '2nd type', 'cla`

$$x(2x^3+y)\,y'-6y^2=0$$

✓ Solution by Maple

Time used: 0.218 (sec). Leaf size: 227

```
dsolve(x*(2*x^3+y(x))*diff(y(x),x) = 6*y(x)^2,y(x), singsol=all)
```

$$y(x)=x^3\left(\frac{x^3-\sqrt{x^6+8c_1x^3}}{2c_1}+2\right)$$
$$y(x)=x^3\left(\frac{x^3+\sqrt{x^6+8c_1x^3}}{2c_1}+2\right)$$
$$y(x)=x^3\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)^3\left(x^3-\sqrt{x^6+8c_1x^3}\right)}{2c_1}+2\right)$$
$$y(x)=x^3\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)^3\left(x^3+\sqrt{x^6+8c_1x^3}\right)}{2c_1}+2\right)$$
$$y(x)=x^3\left(\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^3\left(x^3-\sqrt{x^6+8c_1x^3}\right)}{2c_1}+2\right)$$
$$y(x)=x^3\left(\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^3\left(x^3+\sqrt{x^6+8c_1x^3}\right)}{2c_1}+2\right)$$

✓ Solution by Mathematica

Time used: 1.306 (sec). Leaf size: 123

```
DSolve[x(2 x^3+y[x])y'[x]==6 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow 2x^3\left(-1+\frac{2}{1-\frac{4x^{3/2}}{\sqrt{16x^3+c_1}}}\right)$$

$$y(x) \rightarrow 2x^3\left(-1+\frac{2}{1+\frac{4x^{3/2}}{\sqrt{16x^3+c_1}}}\right)$$

$$y(x) \rightarrow 0$$

$$y(x) \rightarrow 2x^3$$

$$y(x) \rightarrow \frac{2\left((x^3)^{3/2}-x^{9/2}\right)}{x^{3/2}+\sqrt{x^3}}$$

19.24 problem 537

Internal problem ID [3280]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 537.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(1-x)\,y'y+x(1-y)=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(y(x)*(1-x)*diff(y(x),x)+x*(1-y(x)) = 0,y(x), singsol=all)
```

$$y(x)=\operatorname{LambertW}\left(\frac{e^{-x-1}}{c_1\,(x-1)}\right)+1$$

✓ Solution by Mathematica

Time used: 6.358 (sec). Leaf size: 28

```
DSolve[y[x](1-x)y'[x]+x(1-y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to 1+W\left(\frac{e^{-x-1+c_1}}{x-1}\right)$$
$$y(x)\to 1$$

19.25 problem 538

Internal problem ID [3281]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 538.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x+a)(x+b)y'-yx=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve((a+x)*(b+x)*diff(y(x),x) = x*y(x),y(x), singsol=all)
```

$$y(x)=c_1(x+a)^{\frac{a}{a-b}}(x+b)^{-\frac{b}{a-b}}$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 37

```
DSolve[(a+x)(b+x)y'[x]==x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1e^{\frac{a\log(a+x)-b\log(b+x)}{a-b}}$$
$$y(x)\to 0$$

19.26 problem 539

Internal problem ID [3282]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 539.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{2xyy' + 1 - 2x^3 - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(2*x*y(x)*diff(y(x),x)+1-2*x^3-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{x^3 + c_1 x + 1}$$
$$y(x) = -\sqrt{x^3 + c_1 x + 1}$$

✓ Solution by Mathematica

Time used: 0.274 (sec). Leaf size: 37

```
DSolve[2 x y[x] y'[x]+1-2 x^3-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x^3 + c_1 x + 1}$$
$$y(x) \to \sqrt{x^3 + c_1 x + 1}$$

19.27 problem 540

Internal problem ID [3283]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 540.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{2xyy' + a + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve(2*x*y(x)*diff(y(x),x)+a+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(-ax + c_1)\,x}}{x}$$
$$y(x) = -\frac{\sqrt{(-ax + c_1)\,x}}{x}$$

✓ Solution by Mathematica

Time used: 0.338 (sec). Leaf size: 115

```
DSolve[2 x y[x] y'[x]+a+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-ax + e^{2c_1}}}{\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{-ax + e^{2c_1}}}{\sqrt{x}}$$
$$y(x) \to -i\sqrt{a}$$
$$y(x) \to i\sqrt{a}$$
$$y(x) \to \frac{a\sqrt{x}}{\sqrt{-ax}}$$
$$y(x) \to \frac{\sqrt{-ax}}{\sqrt{x}}$$

19.28 problem 541

Internal problem ID [3284]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 541.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, `class G`], _rational, _Bernoulli]`

$$2xyy' - ax - y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(2*x*y(x)*diff(y(x),x) = a*x+y(x)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{ax \ln(x) + c_1 x}$$
$$y(x) = -\sqrt{ax \ln(x) + c_1 x}$$

✓ Solution by Mathematica

Time used: 0.211 (sec). Leaf size: 44

```
DSolve[2 x y[x] y'[x]==a x +y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{a \log(x) + c_1}$$
$$y(x) \to \sqrt{x}\sqrt{a \log(x) + c_1}$$

19.29 problem 542

Internal problem ID [3285]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 542.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _Bernoulli]`

$$\boxed{2xyy' + x^2 + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
dsolve(2*x*y(x)*diff(y(x),x)+x^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{3}\,\sqrt{x\,(-x^3+3c_1)}}{3x}$$
$$y(x) = \frac{\sqrt{3}\,\sqrt{x\,(-x^3+3c_1)}}{3x}$$

✓ Solution by Mathematica

Time used: 0.192 (sec). Leaf size: 60

```
DSolve[2 x y[x] y'[x]+x^2+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^3+3c_1}}{\sqrt{3}\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{-x^3+3c_1}}{\sqrt{3}\sqrt{x}}$$

19.30 problem 543

Internal problem ID [3286]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 543.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{2xyy' - x^2 - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(2*x*y(x)*diff(y(x),x) = x^2+y(x)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 x + x^2}$$
$$y(x) = -\sqrt{c_1 x + x^2}$$

✓ Solution by Mathematica

Time used: 0.165 (sec). Leaf size: 38

```
DSolve[2 x y[x] y'[x]==x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{x + c_1}$$
$$y(x) \to \sqrt{x}\sqrt{x + c_1}$$

19.31 problem 544

Internal problem ID [3287]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 544.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{2xyy' - 4x^2(1 + 2x) - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 41

```
dsolve(2*x*y(x)*diff(y(x),x) = 4*x^2*(1+2*x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{4x^3 + c_1x + 4x^2}$$
$$y(x) = -\sqrt{4x^3 + c_1x + 4x^2}$$

✓ Solution by Mathematica

Time used: 0.192 (sec). Leaf size: 48

```
DSolve[2 x y[x] y'[x]==4 x^2(1+2 x)+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{4x(x + 1) + c_1}$$
$$y(x) \to \sqrt{x}\sqrt{4x(x + 1) + c_1}$$

19.32 problem 545

Internal problem ID [3288]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 19
Problem number: 545.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{2xyy' + x^2(a\,x^3+1) - 6y^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
dsolve(2*x*y(x)*diff(y(x),x)+x^2*(a*x^3+1) = 6*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{4c_1x^4 + 4a\,x^3 + 1}\,x}{2}$$
$$y(x) = \frac{\sqrt{4c_1x^4 + 4a\,x^3 + 1}\,x}{2}$$

✓ Solution by Mathematica

Time used: 0.732 (sec). Leaf size: 59

```
DSolve[2 x y[x] y'[x]+x^2(1+a x^3)==6 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}\sqrt{4ax^5 + 4c_1x^6 + x^2}$$
$$y(x) \to \frac{1}{2}\sqrt{4ax^5 + 4c_1x^6 + x^2}$$

20 Various 20

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20.1 problem 546

Internal problem ID [3289]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 546.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$(3 - x + 2yx)\, y' + 3x^2 - y + y^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 63

```
dsolve((3-x+2*x*y(x))*diff(y(x),x)+3*x^2-y(x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{-3 + x + \sqrt{-4x^4 - 4c_1 x + x^2 - 6x + 9}}{2x}$$
$$y(x) = -\frac{3 - x + \sqrt{-4x^4 - 4c_1 x + x^2 - 6x + 9}}{2x}$$

✓ Solution by Mathematica

Time used: 0.492 (sec). Leaf size: 71

```
DSolve[(3-x+2 x y[x])y'[x]+3 x^2-y[x]+y[x]^2==0 ,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-\sqrt{9 + x\,(-4x^3 + x - 6 + 4c_1)} + x - 3}{2x}$$
$$y(x) \to \frac{\sqrt{9 + x\,(-4x^3 + x - 6 + 4c_1)} + x - 3}{2x}$$

20.2 problem 547

Internal problem ID [3290]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 547.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$x(x-2y)\,y'+y^2=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 55

```
dsolve(x*(x-2*y(x))*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{c_1x-\sqrt{c_1^2x^2+4c_1x}}{2c_1}$$
$$y(x)=\frac{c_1x+\sqrt{c_1^2x^2+4c_1x}}{2c_1}$$

✓ Solution by Mathematica

Time used: 4.261 (sec). Leaf size: 92

```
DSolve[x(x-2 y[x])y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\left(x-\sqrt{x\left(x-4e^{c_1}\right)}\right)$$
$$y(x)\to\frac{1}{2}\left(x+\sqrt{x\left(x-4e^{c_1}\right)}\right)$$
$$y(x)\to 0$$
$$y(x)\to\frac{1}{2}\left(x-\sqrt{x^2}\right)$$
$$y(x)\to\frac{1}{2}\left(\sqrt{x^2}+x\right)$$

20.3 problem 548

Internal problem ID [3291]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 548.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$x(x+2y)\,y'+(2x-y)\,y=0$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 33

```
dsolve(x*(x+2*y(x))*diff(y(x),x)+(2*x-y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{\operatorname{RootOf}\left(\_Z^{18}+3\_Z^{3}c_1x^3-c_1x^3\right)^{15}}{c_1x^2}$$

✓ Solution by Mathematica

Time used: 3.128 (sec). Leaf size: 385

```
DSolve[x(x+2 y[x])y'[x]+(2 x-y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \operatorname{Root}\left[\#1^6+15\#1^5x+90\#1^4x^2+270\#1^3x^3+405\#1^2x^4+243\#1x^5-e^{3c_1}x^3\&,1\right]$$
$$y(x)\to \operatorname{Root}\left[\#1^6+15\#1^5x+90\#1^4x^2+270\#1^3x^3+405\#1^2x^4+243\#1x^5-e^{3c_1}x^3\&,2\right]$$
$$y(x)\to \operatorname{Root}\left[\#1^6+15\#1^5x+90\#1^4x^2+270\#1^3x^3+405\#1^2x^4+243\#1x^5-e^{3c_1}x^3\&,3\right]$$
$$y(x)\to \operatorname{Root}\left[\#1^6+15\#1^5x+90\#1^4x^2+270\#1^3x^3+405\#1^2x^4+243\#1x^5-e^{3c_1}x^3\&,4\right]$$
$$y(x)\to \operatorname{Root}\left[\#1^6+15\#1^5x+90\#1^4x^2+270\#1^3x^3+405\#1^2x^4+243\#1x^5-e^{3c_1}x^3\&,5\right]$$
$$y(x)\to \operatorname{Root}\left[\#1^6+15\#1^5x+90\#1^4x^2+270\#1^3x^3+405\#1^2x^4+243\#1x^5-e^{3c_1}x^3\&,6\right]$$

20.4 problem 549

Internal problem ID [3292]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 549.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, [_Abel, ‘2nd typ`

$$x(x-2y)\,y'+(2x-y)\,y=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 68

```
dsolve(x*(x-2*y(x))*diff(y(x),x)+(2*x-y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{c_1^2x^2+\sqrt{c_1^4x^4+4c_1x}}{2x\,c_1^2}$$
$$y(x)=-\frac{-c_1^2x^2+\sqrt{c_1^4x^4+4c_1x}}{2x\,c_1^2}$$

✓ Solution by Mathematica

Time used: 0.659 (sec). Leaf size: 114

```
DSolve[x(x-2 y[x])y'[x]+(2 x - y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\left(x-\frac{\sqrt{x^3-4e^{c_1}}}{\sqrt{x}}\right)$$
$$y(x)\to\frac{1}{2}\left(x+\frac{\sqrt{x^3-4e^{c_1}}}{\sqrt{x}}\right)$$
$$y(x)\to\frac{x}{2}-\frac{\sqrt{x^3}}{2\sqrt{x}}$$
$$y(x)\to\frac{x^{3/2}+\sqrt{x^3}}{2\sqrt{x}}$$

20.5 problem 550

Internal problem ID [3293]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 550.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{x(1+x-2y)\,y'+(1-2x+y)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 499

```
dsolve(x*(1+x-2*y(x))*diff(y(x),x)+(1-2*x+y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{3\,5^{\frac{1}{3}}\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2+160c_1x+80c_1-x}{c_1}}-20x-20\right)c_1^2\right)^{\frac{1}{3}}}{40c_1} + \frac{3x5^{\frac{2}{3}}}{40\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2+160c_1x+80c_1-x}{c_1}}-20x-20\right)c_1^2\right)^{\frac{1}{3}}} - x - 1$$

$$y(x) = -\frac{3\,5^{\frac{1}{3}}\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2+160c_1x+80c_1-x}{c_1}}-20x-20\right)c_1^2\right)^{\frac{1}{3}}}{80c_1} - \frac{3x5^{\frac{2}{3}}}{80\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2+160c_1x+80c_1-x}{c_1}}-20x-20\right)c_1^2\right)^{\frac{1}{3}}} - x - 1 - \frac{i\sqrt{3}\left(\frac{3\,5^{\frac{1}{3}}\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2+160c_1x+80c_1-x}{c_1}}-20x-20\right)c_1^2\right)^{\frac{1}{3}}}{40c_1} - \frac{3x5^{\frac{2}{3}}}{40\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2+160c_1x+80c_1-x}{c_1}}-20x-20\right)c_1^2\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{3\,5^{\frac{1}{3}}\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2+160c_1x+80c_1-x}{c_1}}-20x-20\right)c_1^2\right)^{\frac{1}{3}}}{80c_1} - \frac{3x5^{\frac{2}{3}}}{80\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2+160c_1x+80c_1-x}{c_1}}-20x-20\right)c_1^2\right)^{\frac{1}{3}}} - x - 1 + \frac{i\sqrt{3}\left(\frac{3\,5^{\frac{1}{3}}\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2+160c_1x+80c_1-x}{c_1}}-20x-20\right)c_1^2\right)^{\frac{1}{3}}}{40c_1} - \frac{3x5^{\frac{2}{3}}}{40\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2+160c_1x+80c_1-x}{c_1}}-20x-20\right)c_1^2\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 43.309 (sec). Leaf size: 448

```
DSolve[x(1+x-2 y[x])y'[x]+(1-2 x+y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{\frac{2}{3}}x}{\sqrt[3]{\sqrt{3}\sqrt{c_1{}^3x^2\left(-4x+27c_1(x+1)^2\right)}+9c_1{}^2x(x+1)}} - \frac{\sqrt[3]{\sqrt{3}\sqrt{c_1{}^3x^2\left(-4x+27c_1(x+1)^2\right)}+9c_1{}^2x(x+1)}}{\sqrt[3]{2}3^{2/3}c_1} - x - 1$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{\sqrt{3}\sqrt{c_1{}^3x^2\left(-4x+27c_1(x+1)^2\right)}+9c_1{}^2x(x+1)}}{2\sqrt[3]{2}3^{2/3}c_1} + \frac{x+i\sqrt{3}x}{2^{2/3}\sqrt[3]{3}\sqrt[3]{\sqrt{3}\sqrt{c_1{}^3x^2\left(-4x+27c_1(x+1)^2\right)}+9c_1{}^2x(x+1)}} - x - 1$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{\sqrt{3}\sqrt{c_1{}^3x^2\left(-4x+27c_1(x+1)^2\right)}+9c_1{}^2x(x+1)}}{2\sqrt[3]{2}3^{2/3}c_1} + \frac{x-i\sqrt{3}x}{2^{2/3}\sqrt[3]{3}\sqrt[3]{\sqrt{3}\sqrt{c_1{}^3x^2\left(-4x+27c_1(x+1)^2\right)}+9c_1{}^2x(x+1)}} - x - 1$$

$$y(x) \to \text{Indeterminate}$$

$$y(x) \to -x - 1$$

20.6 problem 551

Internal problem ID [3294]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 551.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{x(1-x-2y)\,y'+(2x+y+1)\,y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 493

```
dsolve(x*(1-x-2*y(x))*diff(y(x),x)+(1+2*x+y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{3\,5^{\frac{1}{3}}\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2-160c_1x+80c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}}{40c_1} + \frac{3x5^{\frac{2}{3}}}{40\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2-160c_1x+80c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}} + x - 1$$

$$y(x) = -\frac{3\,5^{\frac{1}{3}}\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2-160c_1x+80c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}}{80c_1} - \frac{3x5^{\frac{2}{3}}}{80\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2-160c_1x+80c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}} + x - 1 - \frac{i\sqrt{3}\left(\frac{3\,5^{\frac{1}{3}}\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2-160c_1x+80c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}}{40c_1} - \frac{3x5^{\frac{2}{3}}}{40\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2-160c_1x+80c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{3\,5^{\frac{1}{3}}\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2-160c_1x+80c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}}{80c_1} - \frac{3x5^{\frac{2}{3}}}{80\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2-160c_1x+80c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}} + x - 1 + \frac{i\sqrt{3}\left(\frac{3\,5^{\frac{1}{3}}\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2-160c_1x+80c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}}{40c_1} - \frac{3x5^{\frac{2}{3}}}{40\left(x\left(\sqrt{5}\,\sqrt{\frac{80c_1x^2-160c_1x+80c_1-x}{c_1}}+20x-20\right)c_1^2\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 39.821 (sec). Leaf size: 463

```
DSolve[x(1-x-2 y[x])y'[x]+(1+2 x+y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{2}x}{\sqrt[3]{-27c_1^2x^2 + \sqrt{108c_1^3x^3 + (27c_1^2x - 27c_1^2x^2)^2} + 27c_1^2x}} + \frac{\sqrt[3]{-27c_1^2x^2 + \sqrt{108c_1^3x^3 + (27c_1^2x - 27c_1^2x^2)^2} + 27c_1^2x}}{3\sqrt[3]{2}c_1} + x - 1$$

$$y(x) \to \frac{(1 + i\sqrt{3})\,x}{2^{2/3}\sqrt[3]{-27c_1^2x^2 + \sqrt{108c_1^3x^3 + (27c_1^2x - 27c_1^2x^2)^2} + 27c_1^2x}} - \frac{(1 - i\sqrt{3})\sqrt[3]{-27c_1^2x^2 + \sqrt{108c_1^3x^3 + (27c_1^2x - 27c_1^2x^2)^2} + 27c_1^2x}}{6\sqrt[3]{2}c_1} + x - 1$$

$$y(x) \to \frac{(1 - i\sqrt{3})\,x}{2^{2/3}\sqrt[3]{-27c_1^2x^2 + \sqrt{108c_1^3x^3 + (27c_1^2x - 27c_1^2x^2)^2} + 27c_1^2x}} - \frac{(1 + i\sqrt{3})\sqrt[3]{-27c_1^2x^2 + \sqrt{108c_1^3x^3 + (27c_1^2x - 27c_1^2x^2)^2} + 27c_1^2x}}{6\sqrt[3]{2}c_1} + x - 1$$

$$y(x) \to \text{Indeterminate}$$

$$y(x) \to x - 1$$

20.7 problem 552

Internal problem ID [3295]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 552.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational, [_Abel, ‘2nd typ`

$$\boxed{2x(2x^2+y)\,y'+(12x^2+y)\,y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 50

```
dsolve(2*x*(2*x^2+y(x))*diff(y(x),x)+(12*x^2+y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{-2x^3+\sqrt{4x^6+c_1x}}{x}$$
$$y(x)=-\frac{2x^3+\sqrt{4x^6+c_1x}}{x}$$

✓ Solution by Mathematica

Time used: 0.443 (sec). Leaf size: 58

```
DSolve[2 x(2 x^2+y[x])y'[x]+(12 x^2+y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{2x^3+\sqrt{x\,(4x^5+c_1)}}{x}$$
$$y(x)\to\frac{-2x^3+\sqrt{x\,(4x^5+c_1)}}{x}$$

20.8 problem 553

Internal problem ID [3296]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 553.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, _Bernoulli]`

$$\boxed{2(x+1)\,y'y+2x-3x^2+y^2=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 52

```
dsolve(2*(1+x)*y(x)*diff(y(x),x)+2*x-3*x^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{\sqrt{(x+1)\,(x^3-x^2+c_1)}}{x+1}$$
$$y(x)=-\frac{\sqrt{(x+1)\,(x^3-x^2+c_1)}}{x+1}$$

✓ Solution by Mathematica

Time used: 0.333 (sec). Leaf size: 54

```
DSolve[2(1+x)y[x] y'[x]+2 x-3 x^2+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{\sqrt{(x-1)x^2+c_1}}{\sqrt{x+1}}$$
$$y(x)\to\frac{\sqrt{(x-1)x^2+c_1}}{\sqrt{x+1}}$$

20.9 problem 554

Internal problem ID [3297]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 554.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$x(2x+3y)\,y'-y^2=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 461

```
dsolve(x*(2*x+3*y(x))*diff(y(x),x) = y(x)^2,y(x), singsol=all)
```

$$y(x)=\frac{\frac{\left(108c_1x-8x^3c_1^3+12\sqrt{-12c_1^4x^4+81c_1^2x^2}\right)^{\frac{1}{3}}}{6}+\frac{2x^2c_1^2}{3\left(108c_1x-8x^3c_1^3+12\sqrt{-12c_1^4x^4+81c_1^2x^2}\right)^{\frac{1}{3}}}-\frac{c_1x}{3}}{c_1}$$

$$y(x)=\frac{-\frac{\left(108c_1x-8x^3c_1^3+12\sqrt{-12c_1^4x^4+81c_1^2x^2}\right)^{\frac{1}{3}}}{12}-\frac{x^2c_1^2}{3\left(108c_1x-8x^3c_1^3+12\sqrt{-12c_1^4x^4+81c_1^2x^2}\right)^{\frac{1}{3}}}-\frac{c_1x}{3}-\frac{i\sqrt{3}\left(\frac{\left(108c_1x-8x^3c_1^3+12\sqrt{-12c_1^4x^4\ldots}\right.}{6}\right.}{\ldots}}{c_1}$$

$$y(x)=\frac{-\frac{\left(108c_1x-8x^3c_1^3+12\sqrt{-12c_1^4x^4+81c_1^2x^2}\right)^{\frac{1}{3}}}{12}-\frac{x^2c_1^2}{3\left(108c_1x-8x^3c_1^3+12\sqrt{-12c_1^4x^4+81c_1^2x^2}\right)^{\frac{1}{3}}}-\frac{c_1x}{3}+\frac{i\sqrt{3}\left(\frac{\left(108c_1x-8x^3c_1^3+12\sqrt{-12c_1^4x^4\ldots}\right.}{6}\right.}{\ldots}}{c_1}$$

✓ Solution by Mathematica

Time used: 60.143 (sec). Leaf size: 413

```
DSolve[x(2 x+3 y[x])y'[x]==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(\frac{x^2}{\sqrt[3]{-x^3+\frac{3}{2}\sqrt{3}\sqrt{e^{c_1}x^2\left(-4x^2+27e^{c_1}\right)}+\frac{27e^{c_1}x}{2}}} + \sqrt[3]{-x^3+\frac{3}{2}\sqrt{3}\sqrt{e^{c_1}x^2\left(-4x^2+27e^{c_1}\right)}+\frac{27e^{c_1}x}{2}} - x\right)$$

$$y(x) \to \frac{1}{12}\left(-\frac{2\left(1+i\sqrt{3}\right)x^2}{\sqrt[3]{-x^3+\frac{3}{2}\sqrt{3}\sqrt{e^{c_1}x^2\left(-4x^2+27e^{c_1}\right)}+\frac{27e^{c_1}x}{2}}} + i2^{2/3}\left(\sqrt{3}+i\right)\sqrt[3]{-2x^3+3\sqrt{3}\sqrt{e^{c_1}x^2\left(-4x^2+27e^{c_1}\right)}+27e^{c_1}x} - 4x\right)$$

$$y(x) \to \frac{1}{12}\left(\frac{2i\left(\sqrt{3}+i\right)x^2}{\sqrt[3]{-x^3+\frac{3}{2}\sqrt{3}\sqrt{e^{c_1}x^2\left(-4x^2+27e^{c_1}\right)}+\frac{27e^{c_1}x}{2}}} - 2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{-2x^3+3\sqrt{3}\sqrt{e^{c_1}x^2\left(-4x^2+27e^{c_1}\right)}+27e^{c_1}x} - 4x\right)$$

20.10 problem 555

Internal problem ID [3298]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 555.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$x(2x+3y)\,y'+3(x+y)^2=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 63

```
dsolve(x*(2*x+3*y(x))*diff(y(x),x)+3*(x+y(x))^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{-\frac{2c_1x^2}{3}-\frac{\sqrt{-2c_1^2x^4+6}}{6}}{c_1x}$$
$$y(x)=\frac{-\frac{2c_1x^2}{3}+\frac{\sqrt{-2c_1^2x^4+6}}{6}}{c_1x}$$

✓ Solution by Mathematica

Time used: 1.714 (sec). Leaf size: 135

```
DSolve[x(2 x+3 y[x])y'[x]+3(x+y[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{4x^2+\sqrt{-2x^4+6e^{4c_1}}}{6x}$$
$$y(x)\to \frac{-4x^2+\sqrt{-2x^4+6e^{4c_1}}}{6x}$$
$$y(x)\to -\frac{\sqrt{2}\sqrt{-x^4}+4x^2}{6x}$$
$$y(x)\to \frac{\sqrt{2}\sqrt{-x^4}-4x^2}{6x}$$

20.11 problem 556

Internal problem ID [3299]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 556.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{(3 + 6yx + x^2)\, y' + 2x + 2yx + 3y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 71

```
dsolve((3+6*x*y(x)+x^2)*diff(y(x),x)+2*x+2*x*y(x)+3*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{-x^2 - 3 + \sqrt{x^4 - 12x^3 - 12c_1 x + 6x^2 + 9}}{6x}$$

$$y(x) = -\frac{x^2 + \sqrt{x^4 - 12x^3 - 12c_1 x + 6x^2 + 9} + 3}{6x}$$

✓ Solution by Mathematica

Time used: 0.499 (sec). Leaf size: 79

```
DSolve[(3+6 x y[x]+x^2)y'[x]+2 x+2 x y[x]+3 y[x]^2==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to -\frac{x^2 + \sqrt{9 + x(x((x-12)x + 6) + 36c_1)} + 3}{6x}$$

$$y(x) \to \frac{-x^2 + \sqrt{9 + x(x((x-12)x + 6) + 36c_1)} - 3}{6x}$$

20.12 problem 557

Internal problem ID [3300]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 557.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$3x(x+2y)\,y' + x^3 + 3y(2x+y) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

```
dsolve(3*x*(x+2*y(x))*diff(y(x),x)+x^3+3*y(x)*(2*x+y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \frac{-3x^2 + \sqrt{-3x^5 + 9x^4 - 12c_1x}}{6x}$$
$$y(x) = -\frac{3x^2 + \sqrt{-3x^5 + 9x^4 - 12c_1x}}{6x}$$

✓ Solution by Mathematica

Time used: 0.438 (sec). Leaf size: 71

```
DSolve[3 x(x+2 y[x])y'[x]+x^3+3 y[x](2 x+y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3x^2 + \sqrt{-3(x-3)x^4 + 36c_1x}}{6x}$$
$$y(x) \to \frac{-3x^2 + \sqrt{-3(x-3)x^4 + 36c_1x}}{6x}$$

20.13 problem 558

Internal problem ID [3301]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 558.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{axyy' - x^2 - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 80

```
dsolve(a*x*y(x)*diff(y(x),x) = x^2+y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(a-1)\left(x^{\frac{2}{a}}c_1a - x^{\frac{2}{a}}c_1 + x^2\right)}}{a-1}$$
$$y(x) = -\frac{\sqrt{(a-1)\left(x^{\frac{2}{a}}c_1a - x^{\frac{2}{a}}c_1 + x^2\right)}}{a-1}$$

✓ Solution by Mathematica

Time used: 4.031 (sec). Leaf size: 68

```
DSolve[a x y[x] y'[x]==x^2+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{x^2 + (a-1)c_1x^{2/a}}}{\sqrt{a-1}}$$
$$y(x) \to \frac{\sqrt{x^2 + (a-1)c_1x^{2/a}}}{\sqrt{a-1}}$$

20.14 problem 559

Internal problem ID [3302]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 559.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _Bernoulli]`

$$\boxed{axyy' + x^2 - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 84

```
dsolve(a*x*y(x)*diff(y(x),x)+x^2-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(a-1)\left(x^{\frac{2}{a}} c_1 a - x^{\frac{2}{a}} c_1 - x^2\right)}}{a-1}$$
$$y(x) = -\frac{\sqrt{(a-1)\left(x^{\frac{2}{a}} c_1 a - x^{\frac{2}{a}} c_1 - x^2\right)}}{a-1}$$

✓ Solution by Mathematica

Time used: 3.967 (sec). Leaf size: 72

```
DSolve[a x y[x] y'[x]+x^2-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^2 + (a-1)c_1 x^{2/a}}}{\sqrt{a-1}}$$
$$y(x) \to \frac{\sqrt{-x^2 + (a-1)c_1 x^{2/a}}}{\sqrt{a-1}}$$

20.15 problem 560

Internal problem ID [3303]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 560.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$x(a+yb)\,y'-cy=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
dsolve(x*(a+b*y(x))*diff(y(x),x) = c*y(x),y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-\frac{a\,\mathrm{LambertW}\left(\frac{b\,x^{\frac{c}{a}}\,\mathrm{e}^{\frac{c c_1}{a}}}{a}\right)-c\ln(x)-c c_1}{a}}$$

✓ Solution by Mathematica

Time used: 0.903 (sec). Leaf size: 36

```
DSolve[x(a+b y[x])y'[x]==c y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{aW\left(\frac{be^{\frac{c_1}{a}}x^{\frac{c}{a}}}{a}\right)}{b}$$

$$y(x)\to 0$$

20.16 problem 561

Internal problem ID [3304]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 561.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$x(x - ay)\, y' - y(y - ax) = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 95

```
dsolve(x*(x-a*y(x))*diff(y(x),x) = y(x)*(y(x)-a*x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-c_1 a - a\ln(x) - \mathrm{RootOf}\left(x\,\mathrm{e}^{c_1 a} x^a \mathrm{e}^{a\_Z} \mathrm{e}^{c_1} + \mathrm{e}^{c_1 a} x^a \mathrm{e}^{a\_Z} \mathrm{e}^{c_1} \mathrm{e}^{-\_Z} x - 1\right) a - c_1 + \mathrm{RootOf}\left(x\,\mathrm{e}^{c_1 a} x^a \mathrm{e}^{a\_Z} \mathrm{e}^{c_1} + \mathrm{e}^{c_1 a} x^a \mathrm{e}^{a\_Z} \mathrm{e}^{c_1} \mathrm{e}^{-\_Z} x - 1\right)}$$

✓ Solution by Mathematica

Time used: 0.159 (sec). Leaf size: 36

```
DSolve[x(x-a y[x])y'[x]==y[x](y[x]-a x),y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[(a-1)\log\left(1-\frac{y(x)}{x}\right)+\log\left(\frac{y(x)}{x}\right)=-(a+1)\log(x)+c_1, y(x)\right]$$

20.17 problem 564

Internal problem ID [3305]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 564.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, ‘_with_symmetry_[F(x)*G(y),0]‘], [_Abe`

$$x(x^n + ay)\, y' + (b + cy)\, y^2 = 0$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 107

```
dsolve(x*(x^n+a*y(x))*diff(y(x),x)+(b+c*y(x))*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{b}{\operatorname{RootOf}\left(-x^{-n}\_Z^{\frac{an}{b}} a^2bn - x^{-n}\_Z^{\frac{an}{b}} a\, b^2 + c_1a^2n^2 + \_Z^{\frac{an}{b}} acn - \_Z^{\frac{an+b}{b}} anb + c_1abn + \_Z^{\frac{an}{b}} bc\right) b - c}$$

✓ Solution by Mathematica

Time used: 1.375 (sec). Leaf size: 91

```
DSolve[x(x^n+a y[x])y'[x]+(b+c y[x])y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{y(x)^{-\frac{an+b}{b}}(cy(x) - an)(b + cy(x))^{\frac{an}{b}}}{a^2n^2(an + b)} - \frac{x^{-n}e^{-\frac{an(\log(y(x)) - \log(b+cy(x)))}{b}}}{an^2} = c_1, y(x)\right]$$

20.18 problem 565

Internal problem ID [3306]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 565.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$\left(1-x^{2} y\right) y'+1-x y^{2}=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 50

```
dsolve((1-x^2*y(x))*diff(y(x),x)+1-x*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{1+\sqrt{2c_{1}x^{2}+2x^{3}+1}}{x^{2}}$$
$$y(x)=-\frac{-1+\sqrt{2c_{1}x^{2}+2x^{3}+1}}{x^{2}}$$

✓ Solution by Mathematica

Time used: 0.527 (sec). Leaf size: 55

```
DSolve[(1-x^2 y[x])y'[x]+1-x y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{1-\sqrt{1+x^{2}(2x+c_{1})}}{x^{2}}$$
$$y(x)\to \frac{1+\sqrt{1+x^{2}(2x+c_{1})}}{x^{2}}$$

20.19 problem 566

Internal problem ID [3307]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 566.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{(1 - x^2 y)\, y' - 1 + x y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 1583

```
dsolve((1-x^2*y(x))*diff(y(x),x)-1+x*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{63x^3 - \frac{63x^2\left(c_1\left(-1+4\sqrt{-\frac{5\left(x^6-2x^3+1\right)}{c_1x^6-80x^6+160x^3-80}}\right)\left(c_1x^6-80x^6+160x^3-80\right)^2\right)^{\frac{1}{3}}}{c_1x^6-80x^6+160x^3-80} - \frac{63c_1x^4}{\left(c_1\left(-1+4\sqrt{-\frac{5\left(x^6-2x^3+1\right)}{c_1x^6-80x^6+160x^3-80}}\right)\left(c_1x^6 \cdots\right.\right.}}{4x^2\left(\frac{63x^2\left(c_1\left(-1+4\sqrt{-\frac{5\left(x^6-2x^3+1\right)}{c_1x^6-80x^6+160x^3-80}}\right)\left(c_1x^6-80x^6+160x^3-80\right)^2\right)^{\frac{1}{3}}}{4\left(c_1x^6-80x^6+160x^3-80\right)} + \frac{63c_1x^4}{4\left(c_1\left(-1+4\sqrt{-\frac{5\left(x^6-2x^3+1\right)}{c_1x^6-80x^6+160x^3-80}}\right)\left(c_1x^6-80x^6 \cdots\right.\right.}\right.}$$

$$y(x) = -\frac{63x^3 + \frac{63x^2\left(c_1\left(-1+4\sqrt{-\frac{5\left(x^6-2x^3+1\right)}{c_1x^6-80x^6+160x^3-80}}\right)\left(c_1x^6-80x^6+160x^3-80\right)^2\right)^{\frac{1}{3}}}{2\left(c_1x^6-80x^6+160x^3-80\right)} + \frac{63c_1x^4}{2\left(c_1\left(-1+4\sqrt{-\frac{5\left(x^6-2x^3+1\right)}{c_1x^6-80x^6+160x^3-80}}\right)\left(c_1x^6-80 \cdots\right.\right.}}{4x^2\left(-\frac{63x^2\left(c_1\left(-1+4\sqrt{-\frac{5\left(x^6-2x^3+1\right)}{c_1x^6-80x^6+160x^3-80}}\right)\left(c_1x^6-80x^6+160x^3-80\right)^2\right)^{\frac{1}{3}}}{8\left(c_1x^6-80x^6+160x^3-80\right)} - \frac{63c_1x^4}{8\left(c_1\left(-1+4\sqrt{-\frac{5\left(x^6-2x^3+1\right)}{c_1x^6-80x^6+160x^3-80}}\right)\left(c_1x^6 \cdots\right.\right.}\right.}$$

$$y(x) = -\frac{63x^3 + \frac{63x^2\left(c_1\left(-1+4\sqrt{-\frac{5\left(x^6-2x^3+1\right)}{c_1x^6-80x^6+160x^3-80}}\right)\left(c_1x^6-80x^6+160x^3-80\right)^2\right)^{\frac{1}{3}}}{2\left(c_1x^6-80x^6+160x^3-80\right)} + \frac{63c_1x^4}{2\left(c_1\left(-1+4\sqrt{-\frac{5\left(x^6-2x^3+1\right)}{c_1x^6-80x^6+160x^3-80}}\right)\left(c_1x^6-80 \cdots\right.\right.}}{4x^2\left(-\frac{63x^2\left(c_1\left(-1+4\sqrt{-\frac{5\left(x^6-2x^3+1\right)}{c_1x^6-80x^6+160x^3-80}}\right)\left(c_1x^6-80x^6+160x^3-80\right)^2\right)^{\frac{1}{3}}}{8\left(c_1x^6-80x^6+160x^3-80\right)} - \frac{63c_1x^4}{8\left(c_1\left(-1+4\sqrt{-\frac{5\left(x^6-2x^3+1\right)}{c_1x^6-80x^6+160x^3-80}}\right)\left(c_1x^6 \cdots\right.\right.}\right.}$$

✓ Solution by Mathematica

Time used: 35.673 (sec). Leaf size: 476

```
DSolve[(1-x^2 y[x])y'[x]-1+x y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{-(1-6c_1)^2x^3+\sqrt{(-1+6c_1)^3\left(2x^3+6c_1\left(x^3-1\right)^2-1\right)}+1+12c_1(-1+3c_1)}}{-1+6c_1} - \frac{x^2}{\sqrt[3]{-(1-6c_1)^2x^3+\sqrt{(-1+6c_1)^3\left(2x^3+6c_1\left(x^3-1\right)^2-1\right)}+1+12c_1(-1+3c_1)}} + x$$

$$y(x) \to \frac{i(\sqrt{3}+i)\sqrt[3]{-(1-6c_1)^2x^3+\sqrt{(-1+6c_1)^3\left(2x^3+6c_1\left(x^3-1\right)^2-1\right)}+1+12c_1(-1+3c_1)}}{-2+12c_1} + \frac{\left(1+i\sqrt{3}\right)x^2}{2\sqrt[3]{-(1-6c_1)^2x^3+\sqrt{(-1+6c_1)^3\left(2x^3+6c_1\left(x^3-1\right)^2-1\right)}+1+12c_1(-1+3c_1)}} + x$$

$$y(x) \to -\frac{i(\sqrt{3}-i)\sqrt[3]{-(1-6c_1)^2x^3+\sqrt{(-1+6c_1)^3\left(2x^3+6c_1\left(x^3-1\right)^2-1\right)}+1+12c_1(-1+3c_1)}}{-2+12c_1} + \frac{\left(1-i\sqrt{3}\right)x^2}{2\sqrt[3]{-(1-6c_1)^2x^3+\sqrt{(-1+6c_1)^3\left(2x^3+6c_1\left(x^3-1\right)^2-1\right)}+1+12c_1(-1+3c_1)}} + x$$

$$y(x) \to x$$

20.20 problem 567

Internal problem ID [3308]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 567.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$x(-yx+1)\,y'+(1+yx)\,y=0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 18

```
dsolve(x*(1-x*y(x))*diff(y(x),x)+(1+x*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x)=-\frac{1}{\operatorname{LambertW}\left(-\frac{c_1}{x^2}\right)x}$$

✓ Solution by Mathematica

Time used: 5.714 (sec). Leaf size: 35

```
DSolve[x(1-x y[x])y'[x]+(1+x y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{1}{xW\left(\frac{e^{-1+\frac{9c_1}{2^{2/3}}}}{x^2}\right)}$$

$$y(x)\to 0$$

20.21 problem 568

Internal problem ID [3309]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 568.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{x(2+yx)\,y'-3-2x^3+2y+xy^2=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(x*(2+x*y(x))*diff(y(x),x) = 3+2*x^3-2*y(x)-x*y(x)^2,y(x), singsol=all)
```

$$y(x)=\frac{-2-\sqrt{x^4-2c_1+6x+4}}{x}$$
$$y(x)=\frac{-2+\sqrt{x^4-2c_1+6x+4}}{x}$$

✓ Solution by Mathematica

Time used: 0.582 (sec). Leaf size: 62

```
DSolve[x(2+x y[x])y'[x]==3+2 x^3-2 y[x]-x y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{2x+\sqrt{x^2\left(x^4+6x+4+c_1\right)}}{x^2}$$
$$y(x)\to \frac{-2x+\sqrt{x^2\left(x^4+6x+4+c_1\right)}}{x^2}$$

20.22 problem 569

Internal problem ID [3310]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 569.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$x(2-yx)\,y'+2y-xy^2(1+yx)=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(x*(2-x*y(x))*diff(y(x),x)+2*y(x)-x*y(x)^2*(1+x*y(x)) = 0,y(x), singsol=all)
```

$$y(x)=-\frac{-1+\sqrt{1-4\ln(x)+4c_1}}{2(\ln(x)-c_1)\,x}$$

$$y(x)=\frac{1+\sqrt{1-4\ln(x)+4c_1}}{2(\ln(x)-c_1)\,x}$$

✓ Solution by Mathematica

Time used: 1.157 (sec). Leaf size: 85

```
DSolve[x(2-x y[x])y'[x]+2 y[x]-x y[x]^2(1+x y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2}{x+\sqrt{-\frac{1}{x^3}}x^2\sqrt{-x(-4\log(x)+1+4c_1)}}$$

$$y(x)\to\frac{2}{x+\left(-\frac{1}{x^3}\right)^{3/2}x^5\sqrt{x(4\log(x)-1-4c_1)}}$$

$$y(x)\to 0$$

20.23 problem 570

Internal problem ID [3311]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 570.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{x(3 - yx)\, y' - y(yx - 1) = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 74

```
dsolve(x*(3-x*y(x))*diff(y(x),x) = y(x)*(x*y(x)-1),y(x), singsol=all)
```

$$y(x) = -\frac{3\,\mathrm{LambertW}\left(\frac{2\left(-\frac{x^2}{8}\right)^{\frac{1}{3}} c_1}{3}\right)}{x}$$

$$y(x) = -\frac{3\,\mathrm{LambertW}\left(\frac{\left(-\frac{x^2}{8}\right)^{\frac{1}{3}} c_1\left(-1+i\sqrt{3}\right)}{3}\right)}{x}$$

$$y(x) = -\frac{3\,\mathrm{LambertW}\left(-\frac{\left(-\frac{x^2}{8}\right)^{\frac{1}{3}} c_1\left(1+i\sqrt{3}\right)}{3}\right)}{x}$$

✓ Solution by Mathematica

Time used: 15.002 (sec). Leaf size: 35

```
DSolve[x(3-x y[x])y'[x]==y[x](x y[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{3W\left(e^{-1+\frac{9c_1}{2^{2/3}}} x^{2/3}\right)}{x}$$

$$y(x) \to 0$$

20.24 problem 571

Internal problem ID [3312]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 571.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2(1-y)\,y'+(1-x)\,y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(x^2*(1-y(x))*diff(y(x),x)+(1-x)*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{\frac{x\ln(x)-\mathrm{LambertW}\left(-x\,\mathrm{e}^{c_1+\frac{1}{x}}\right)x+c_1x+1}{x}}$$

✓ Solution by Mathematica

Time used: 2.979 (sec). Leaf size: 26

```
DSolve[x^2(1-y[x])y'[x]+(1-x)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -W\left(x\left(-e^{\frac{1}{x}-c_1}\right)\right)$$
$$y(x)\to 0$$

20.25 problem 572

Internal problem ID [3313]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 572.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2(1-y)\,y' + y^2(x+1) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

```
dsolve(x^2*(1-y(x))*diff(y(x),x)+(1+x)*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = e^{\frac{x \ln(x)+\mathrm{LambertW}\left(-\frac{e^{-c_1+\frac{1}{x}}}{x}\right)x+c_1x-1}{x}}$$

✓ Solution by Mathematica

Time used: 5.178 (sec). Leaf size: 30

```
DSolve[x^2(1-y[x])y'[x]+(1+x)y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{W\left(-\frac{e^{\frac{1}{x}-c_1}}{x}\right)}$$

$$y(x) \to 0$$

20.26 problem 573

Internal problem ID [3314]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 573.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{(x^2+1)\,yy'+x(1-y^2)=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve((x^2+1)*y(x)*diff(y(x),x)+x*(1-y(x)^2) = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 x^2 + c_1 + 1}$$
$$y(x) = -\sqrt{c_1 x^2 + c_1 + 1}$$

✓ Solution by Mathematica

Time used: 0.797 (sec). Leaf size: 57

```
DSolve[(1+x^2)y[x] y'[x]+x(1-y[x]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{1 + e^{2c_1}(x^2+1)}$$
$$y(x) \to \sqrt{1 + e^{2c_1}(x^2+1)}$$
$$y(x) \to -1$$
$$y(x) \to 1$$

20.27 problem 574

Internal problem ID [3315]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 574.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$\boxed{(1-x^2)\, yy' + 2x^2 + xy^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 91

```
dsolve((-x^2+1)*y(x)*diff(y(x),x)+2*x^2+x*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{-\ln(x+1)\, x^2 + \ln(x-1)\, x^2 + c_1x^2 + \ln(x+1) - \ln(x-1) - c_1 - 2x}$$
$$y(x) = -\sqrt{-\ln(x+1)\, x^2 + \ln(x-1)\, x^2 + c_1x^2 + \ln(x+1) - \ln(x-1) - c_1 - 2x}$$

✓ Solution by Mathematica

Time used: 0.41 (sec). Leaf size: 61

```
DSolve[(1-x^2)y[x] y'[x]+2 x^2+x y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-2\left((x^2-1)\operatorname{arctanh}(x) + x\right) + c_1\left(x^2-1\right)}$$
$$y(x) \to \sqrt{-2\left((x^2-1)\operatorname{arctanh}(x) + x\right) + c_1\left(x^2-1\right)}$$

20.28 problem 575

Internal problem ID [3316]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 575.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational, _Bernoulli]`

$$\boxed{2y'x^2y - x^2(1+2x) + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve(2*x^2*y(x)*diff(y(x),x) = x^2*(1+2*x)-y(x)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{\mathrm{e}^{\frac{1}{x}} c_1 + x^2}$$
$$y(x) = -\sqrt{\mathrm{e}^{\frac{1}{x}} c_1 + x^2}$$

✓ Solution by Mathematica

Time used: 7.096 (sec). Leaf size: 43

```
DSolve[2 x^2 y[x] y'[x]==x^2(1+2 x)-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x^2 + c_1 e^{\frac{1}{x}}}$$
$$y(x) \to \sqrt{x^2 + c_1 e^{\frac{1}{x}}}$$

20.29 problem 576

Internal problem ID [3317]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 20
Problem number: 576.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$x(-2yx+1)\,y'+(2yx+1)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

```
dsolve(x*(1-2*x*y(x))*diff(y(x),x)+(1+2*x*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x)=-\frac{1}{2\,\mathrm{LambertW}\left(-\frac{c_1}{2x^2}\right)x}$$

✓ Solution by Mathematica

Time used: 5.6 (sec). Leaf size: 37

```
DSolve[x(1-2 x y[x])y'[x]+(1+2 x y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\rightarrow -\frac{1}{2xW\left(\frac{e^{-1+\frac{9c_1}{2^{2/3}}}}{x^2}\right)}$$

$$y(x)\rightarrow 0$$

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21.1 problem 577

Internal problem ID [3318]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 577.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{x(2yx+1)\,y'+(2+3yx)\,y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 43

```
dsolve(x*(1+2*x*y(x))*diff(y(x),x)+(2+3*x*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{-x+\sqrt{4c_1x+x^2}}{2x^2}$$
$$y(x)=-\frac{x+\sqrt{4c_1x+x^2}}{2x^2}$$

✓ Solution by Mathematica

Time used: 0.537 (sec). Leaf size: 69

```
DSolve[x(1+2 x y[x])y'[x]+(2+3 x y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{x^{3/2}+\sqrt{x^2(x+4c_1)}}{2x^{5/2}}$$
$$y(x)\to \frac{-x^{3/2}+\sqrt{x^2(x+4c_1)}}{2x^{5/2}}$$

21.2 problem 578

Internal problem ID [3319]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 578.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$x(2yx+1)\,y' + \left(1+2yx-x^2y^2\right)y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 59

```
dsolve(x*(1+2*x*y(x))*diff(y(x),x)+(1+2*x*y(x)-x^2*y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{-2+\sqrt{4-2\ln(x)+2c_1}}{2(\ln(x)-c_1)\,x}$$
$$y(x) = -\frac{2+\sqrt{4-2\ln(x)+2c_1}}{2(\ln(x)-c_1)\,x}$$

✓ Solution by Mathematica

Time used: 0.721 (sec). Leaf size: 78

```
DSolve[x(1+2 x y[x])y'[x]+(1+2 x y[x]-x^2 y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{1}{-2x+\sqrt{\frac{1}{x^3}}x^2\sqrt{x(-2\log(x)+4+c_1)}}$$
$$y(x) \to -\frac{1}{2x+\sqrt{\frac{1}{x^3}}x^2\sqrt{x(-2\log(x)+4+c_1)}}$$
$$y(x) \to 0$$

21.3 problem 579

Internal problem ID [3320]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 579.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$x^2(x-2y)\,y'-2x^3+4xy^2-y^3=0$$

✓ Solution by Maple

Time used: 0.234 (sec). Leaf size: 75

```
dsolve(x^2*(x-2*y(x))*diff(y(x),x) = 2*x^3-4*x*y(x)^2+y(x)^3,y(x), singsol=all)
```

$$y(x)=\frac{\left(3c_1x^2-\sqrt{3c_1x^2+1}-1\right)x}{c_1x^2-1}-x$$
$$y(x)=\frac{\left(3c_1x^2+\sqrt{3c_1x^2+1}-1\right)x}{c_1x^2-1}-x$$

✓ Solution by Mathematica

Time used: 13.974 (sec). Leaf size: 132

```
DSolve[x^2(x-2 y[x])y'[x]==2 x^3-4 x y[x]^2+y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{2x^3-\sqrt{e^{2c_1}x^2\left(-3x^2+e^{2c_1}\right)}}{x^2+e^{2c_1}}$$
$$y(x)\to\frac{2x^3+\sqrt{e^{2c_1}x^2\left(-3x^2+e^{2c_1}\right)}}{x^2+e^{2c_1}}$$
$$y(x)\to 2x$$
$$y(x)\to-\sqrt{x^2}$$
$$y(x)\to\sqrt{x^2}$$

21.4 problem 580

Internal problem ID [3321]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 580.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{2(x+1)\,xyy'-1-y^2=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 42

```
dsolve(2*(1+x)*x*y(x)*diff(y(x),x) = 1+y(x)^2,y(x), singsol=all)
```

$$y(x)=\frac{\sqrt{(x+1)\,(c_1x-1)}}{x+1}$$
$$y(x)=-\frac{\sqrt{(x+1)\,(c_1x-1)}}{x+1}$$

✓ Solution by Mathematica

Time used: 0.76 (sec). Leaf size: 114

```
DSolve[2(1+x)x y[x] y'[x]==1+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-1 + (-1 + e^{2c_1})\, x}}{\sqrt{x+1}}$$
$$y(x) \to \frac{\sqrt{-1 + (-1 + e^{2c_1})\, x}}{\sqrt{x+1}}$$
$$y(x) \to -i$$
$$y(x) \to i$$
$$y(x) \to \frac{\sqrt{x+1}}{\sqrt{-x-1}}$$
$$y(x) \to \frac{\sqrt{-x-1}}{\sqrt{x+1}}$$

21.5 problem 581

Internal problem ID [3322]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 581.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{3y'x^2y + 1 + 2xy^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 50

```
dsolve(3*x^2*y(x)*diff(y(x),x)+1+2*x*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-x^{\frac{7}{3}}\left(2x^{\frac{4}{3}} - c_1 x\right)}}{x^{\frac{7}{3}}}$$
$$y(x) = -\frac{\sqrt{-x^{\frac{7}{3}}\left(2x^{\frac{4}{3}} - c_1 x\right)}}{x^{\frac{7}{3}}}$$

✓ Solution by Mathematica

Time used: 3.737 (sec). Leaf size: 47

```
DSolve[3 x^2 y[x] y'[x]+1+2 x y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-\frac{2}{x} + \frac{c_1}{x^{4/3}}}$$
$$y(x) \to \sqrt{-\frac{2}{x} + \frac{c_1}{x^{4/3}}}$$

21.6 problem 582

Internal problem ID [3323]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 582.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$x^2(4x-3y)\,y' - \left(6x^2 - 3yx + 2y^2\right) y = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 44

```
dsolve(x^2*(4*x-3*y(x))*diff(y(x),x) = (6*x^2-3*x*y(x)+2*y(x)^2)*y(x),y(x), singsol=all)
```

$$2\ln\left(\frac{y(x)}{x}\right) - \ln\left(\frac{x^2+y(x)^2}{x^2}\right) - \frac{3\arctan\left(\frac{y(x)}{x}\right)}{2} - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.114 (sec). Leaf size: 43

```
DSolve[x^2(4 x-3 y[x])y'[x]==(6 x^2-3 x y[x]+2 y[x]^2)y[x],y[x],x,IncludeSingularSolutions ->
```

$$\text{Solve}\left[3\arctan\left(\frac{y(x)}{x}\right) + 2\log\left(\frac{y(x)^2}{x^2}+1\right) - 4\log\left(\frac{y(x)}{x}\right) = -2\log(x) + c_1, y(x)\right]$$

21.7 problem 583

Internal problem ID [3324]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 583.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(1 - x^3 y)\, y' - x^2 y^2 = 0}$$

✓ Solution by Maple

Time used: 0.36 (sec). Leaf size: 789

```
dsolve((1-x^3*y(x))*diff(y(x),x) = x^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(\frac{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} - \frac{c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}\right)^2 + 3}{2x^3}$$

$$y(x) = \frac{\left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)^6 \left(\frac{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} - \frac{c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}\right)^2 + 3}{2x^3}$$

$$y(x) = \frac{\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^6 \left(\frac{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} - \frac{c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}\right)^2 + 3}{2x^3}$$

$$y(x) = \frac{\frac{\left(-\frac{4\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{4c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}} - 4i\sqrt{3}\left(\frac{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}\right)\right)^2}{64} + 3}{2x^3}$$

$$y(x) = \frac{\frac{\left(-\frac{4\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{4c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}} + 4i\sqrt{3}\left(\frac{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}\right)\right)^2}{64} + 3}{2x^3}$$

$$y(x) = \frac{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)^6\left(-\frac{4\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{4c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}} - 4i\sqrt{3}\left(\frac{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}\right)\right)^2}{64} + 3}{2x^3}$$

$$y(x) = \frac{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)^6\left(-\frac{4\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{4c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}} + 4i\sqrt{3}\left(\frac{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}\right)\right)^2}{64} + 3}{2x^3}$$

$$y(x) = \frac{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^6\left(-\frac{4\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{4c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}} - 4i\sqrt{3}\left(\frac{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}\right)\right)^2}{64} + 3}{2x^3}$$

$$y(x) = \frac{\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^6\left(-\frac{4\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{4c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}} + 4i\sqrt{3}\left(\frac{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}{c_1} + \frac{c_1}{\left(x^3+\sqrt{c_1^6+x^6}\right)^{\frac{1}{3}}}\right)\right)^2}{64} + 3}{2x^3}$$

✓ Solution by Mathematica

Time used: 47.613 (sec). Leaf size: 331

```
DSolve[(1-x^3 y[x])y'[x]==x^2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{12c_1x^6+2\sqrt{6}\sqrt{c_1x^6\left(1+6c_1x^6\right)}+1}+\frac{1}{\sqrt[3]{12c_1x^6+2\sqrt{6}\sqrt{c_1x^6\left(1+6c_1x^6\right)}+1}}+1}{2x^3}$$

$$y(x) \to \frac{2i\left(\sqrt{3}+i\right)\sqrt[3]{12c_1x^6+2\sqrt{6}\sqrt{c_1x^6\left(1+6c_1x^6\right)}+1}-\frac{2\left(1+i\sqrt{3}\right)}{\sqrt[3]{12c_1x^6+2\sqrt{6}\sqrt{c_1x^6\left(1+6c_1x^6\right)}+1}}+4}{8x^3}$$

$$y(x) \to \frac{-2\left(1+i\sqrt{3}\right)\sqrt[3]{12c_1x^6+2\sqrt{6}\sqrt{c_1x^6\left(1+6c_1x^6\right)}+1}+\frac{2i\left(\sqrt{3}+i\right)}{\sqrt[3]{12c_1x^6+2\sqrt{6}\sqrt{c_1x^6\left(1+6c_1x^6\right)}+1}}+4}{8x^3}$$

$$y(x) \to 0$$

21.8 problem 584

Internal problem ID [3325]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 584.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational, _Bernoulli]`

$$\boxed{2y'yx^3 + a + 3x^2y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(2*x^3*y(x)*diff(y(x),x)+a+3*x^2*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{(-ax + c_1)\,x}}{x^2}$$
$$y(x) = -\frac{\sqrt{(-ax + c_1)\,x}}{x^2}$$

✓ Solution by Mathematica

Time used: 0.265 (sec). Leaf size: 44

```
DSolve[2 x^3 y[x] y'[x]+a+3 x^2 y[x]^2 ==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-ax + c_1}}{x^{3/2}}$$
$$y(x) \to \frac{\sqrt{-ax + c_1}}{x^{3/2}}$$

21.9 problem 585

Internal problem ID [3326]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 585.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_Abel, '2nd type', 'class B']]`

$$
x\left(3-2x^2y\right)y'-4x+3y-3x^2y^2=0
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 47

```
dsolve(x*(3-2*x^2*y(x))*diff(y(x),x) = 4*x-3*y(x)+3*x^2*y(x)^2,y(x), singsol=all)
```

$$
y(x)=\frac{3+\sqrt{-8x^3+4c_1x+9}}{2x^2}
$$
$$
y(x)=-\frac{-3+\sqrt{-8x^3+4c_1x+9}}{2x^2}
$$

✓ Solution by Mathematica

Time used: 0.64 (sec). Leaf size: 71

```
DSolve[x(3-2 x^2 y[x])y'[x]==4 x-3 y[x]+3 x^2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x)\to -\frac{-3x+\sqrt{x^2\left(-8x^3+4c_1x+9\right)}}{2x^3}
$$
$$
y(x)\to \frac{3x+\sqrt{x^2\left(-8x^3+4c_1x+9\right)}}{2x^3}
$$

21.10 problem 586

Internal problem ID [3327]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 586.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$x(3 + 2x^2y)\,y' + (4 + 3x^2y)\,y = 0$$

✓ Solution by Maple

Time used: 0.703 (sec). Leaf size: 39

```
dsolve(x*(3+2*x^2*y(x))*diff(y(x),x)+(4+3*x^2*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\operatorname{RootOf}\left(x^2\_Z^8 - 2\_Z^2c_1 - c_1\right)^6 x^2 - 2c_1}{x^2c_1}$$

✓ Solution by Mathematica

Time used: 60.283 (sec). Leaf size: 1769

```
DSolve[x(3+2 x^2 y[x])y'[x]+(4+3 x^2 y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2x^2} + \frac{\sqrt{\frac{3}{x^4} - \frac{2\ 6^{2/3}e^{-2c_1}}{\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}} + \frac{\sqrt[3]{6}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}}{x^6}}}{2\sqrt{3}}$$

$$-\frac{1}{2}\sqrt{\frac{2}{x^4} + \frac{2\ 2^{2/3}e^{-2c_1}}{\sqrt[3]{3}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}} - \frac{\sqrt[3]{2}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c}\right.}}{3^{2/3}x^6}}$$

$$y(x) \to -\frac{1}{2x^2} + \frac{\sqrt{\frac{3}{x^4} - \frac{2\ 6^{2/3}e^{-2c_1}}{\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}} + \frac{\sqrt[3]{6}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}}{x^6}}}{2\sqrt{3}}$$

$$+\frac{1}{2}\sqrt{\frac{2}{x^4} + \frac{2\ 2^{2/3}e^{-2c_1}}{\sqrt[3]{3}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}} - \frac{\sqrt[3]{2}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c}\right.}}{3^{2/3}x^6}}$$

$$y(x) \to -\frac{1}{2x^2} - \frac{\sqrt{\frac{3}{x^4} - \frac{2\ 6^{2/3}e^{-2c_1}}{\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}} + \frac{\sqrt[3]{6}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}}{x^6}}}{2\sqrt{3}}$$

$$-\frac{1}{2}\sqrt{\frac{2}{x^4} + \frac{2\ 2^{2/3}e^{-2c_1}}{\sqrt[3]{3}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c_1}x^8\right)}} - \frac{\sqrt[3]{2}\sqrt[3]{e^{-6c_1}\left(\sqrt{48e^{6c_1}x^{18}+81e^{8c_1}x^{16}}-9e^{4c}\right.}}{3^{2/3}x^6}}$$

21.11 problem 587

Internal problem ID [3328]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 587.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{8y'yx^3 + 3x^4 - 6x^2y^2 - y^4 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 54

```
dsolve(8*x^3*y(x)*diff(y(x),x)+3*x^4-6*x^2*y(x)^2-y(x)^4 = 0,y(x), singsol=all)
```

$$y(x) = \frac{x\sqrt{-(c_1x-1)(c_1x+3)}}{c_1x-1}$$
$$y(x) = -\frac{x\sqrt{-(c_1x-1)(c_1x+3)}}{c_1x-1}$$

✓ Solution by Mathematica

Time used: 4.822 (sec). Leaf size: 160

```
DSolve[8 x^3 y[x] y'[x]+3 x^4 -6 x^2 y[x]^2 -y[x]^4==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\frac{\sqrt{-x^2 \left(3+e^{8c_1}x\right)}}{\sqrt{-1+e^{8c_1}x}}$$

$$y(x) \to \frac{\sqrt{-x^2 \left(3+e^{8c_1}x\right)}}{\sqrt{-1+e^{8c_1}x}}$$

$$y(x) \to -i\sqrt{3}\sqrt{-x^2}$$

$$y(x) \to i\sqrt{3}\sqrt{-x^2}$$

$$y(x) \to \frac{x^{5/2}}{\sqrt{-x^3}}$$

$$y(x) \to \frac{\sqrt{-x^3}}{\sqrt{x}}$$

21.12 problem 588

Internal problem ID [3329]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 588.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{xy\left(b\,x^2+a\right)y'-A-By^2=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 82

```
dsolve(x*y(x)*(b*x^2+a)*diff(y(x),x) = A+B*y(x)^2,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-B\left(-x^{\frac{2B}{a}}\left(b\,x^2+a\right)^{-\frac{B}{a}}c_1B+A\right)}}{B}$$
$$y(x) = -\frac{\sqrt{-B\left(-x^{\frac{2B}{a}}\left(b\,x^2+a\right)^{-\frac{B}{a}}c_1B+A\right)}}{B}$$

✓ Solution by Mathematica

Time used: 1.812 (sec). Leaf size: 134

```
DSolve[x y[x] (a+b x^2)y'[x]==A+B y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-A + e^{2Bc_1} x^{\frac{2B}{a}} (a + bx^2)^{-\frac{B}{a}}}}{\sqrt{B}}$$

$$y(x) \to \frac{\sqrt{-A + e^{2Bc_1} x^{\frac{2B}{a}} (a + bx^2)^{-\frac{B}{a}}}}{\sqrt{B}}$$

$$y(x) \to -\frac{i\sqrt{A}}{\sqrt{B}}$$

$$y(x) \to \frac{i\sqrt{A}}{\sqrt{B}}$$

21.13 problem 589

Internal problem ID [3330]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 589.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _Bernoulli]`

$$\boxed{3y'x^4y-1+2x^3y^2=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 55

```
dsolve(3*x^4*y(x)*diff(y(x),x) = 1-2*x^3*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-5x^{\frac{13}{3}}\left(2x^{\frac{4}{3}}-5c_1x^3\right)}}{5x^{\frac{13}{3}}}$$
$$y(x) = \frac{\sqrt{-5x^{\frac{13}{3}}\left(2x^{\frac{4}{3}}-5c_1x^3\right)}}{5x^{\frac{13}{3}}}$$

✓ Solution by Mathematica

Time used: 3.685 (sec). Leaf size: 51

```
DSolve[3 x^4 y[x] y'[x]==1-2 x^3 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-\frac{2}{5x^3}+\frac{c_1}{x^{4/3}}}$$
$$y(x) \to \sqrt{-\frac{2}{5x^3}+\frac{c_1}{x^{4/3}}}$$

21.14 problem 590

Internal problem ID [3331]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 590.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_Abel, '2nd type', 'class B']]`

$$\boxed{x^7yy' - 2x^2 - 2 - 5x^3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 96

```
dsolve(x^7*y(x)*diff(y(x),x) = 2*x^2+2+5*x^3*y(x),y(x), singsol=all)
```

$$c_1 + \frac{-\frac{(y(x)x^3+1)\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{5}{4}\right],\left[\frac{3}{2}\right],-\frac{(y(x)x^3+1)^2}{x^2}\right)\left(\frac{x^6y(x)^2+2y(x)x^3+x^2+1}{x^2}\right)^{\frac{1}{4}}}{x} - 2x}{\left(\frac{x^6y(x)^2+2y(x)x^3+x^2+1}{x^2}\right)^{\frac{1}{4}}} = 0$$

✓ Solution by Mathematica

Time used: 0.36 (sec). Leaf size: 98

```
DSolve[x^7 y[x] y'[x]==2(1+x^2)+5 x^3 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[c_1 = \frac{\frac{i(x^3y(x)+1)\sqrt[4]{x^4y(x)^2+\frac{1}{x^2}+2xy(x)+1}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{3}{2},-\frac{(y(x)x^3+1)^2}{x^2}\right)}{2x} + ix}{\sqrt[4]{-\frac{(x^3y(x)+1)^2}{x^2}-1}}, y(x)\right]$$

21.15 problem 591

Internal problem ID [3332]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 591.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{yy'\sqrt{x^2+1}+x\sqrt{1+y^2}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(y(x)*diff(y(x),x)*sqrt(x^2+1)+x*sqrt(1+y(x)^2) = 0,y(x), singsol=all)
```

$$\sqrt{x^2+1}+\sqrt{1+y(x)^2}+c_1=0$$

✓ Solution by Mathematica

Time used: 0.272 (sec). Leaf size: 75

```
DSolve[y[x] y'[x]Sqrt[1+x^2]+x Sqrt[1+y[x]^2]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\sqrt{x^2+c_1\left(-2\sqrt{x^2+1}+c_1\right)}$$
$$y(x)\to \sqrt{x^2+c_1\left(-2\sqrt{x^2+1}+c_1\right)}$$
$$y(x)\to -i$$
$$y(x)\to i$$

21.16 problem 592

Internal problem ID [3333]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 592.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$(1+y)\,y'\sqrt{x^2+1}-y^3=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 49

```
dsolve((1+y(x))*diff(y(x),x)*sqrt(x^2+1) = y(x)^3,y(x), singsol=all)
```

$$y(x)=\frac{-1+\sqrt{1-2c_1-2\,\operatorname{arcsinh}(x)}}{2c_1+2\,\operatorname{arcsinh}(x)}$$
$$y(x)=-\frac{1+\sqrt{1-2c_1-2\,\operatorname{arcsinh}(x)}}{2\,(\operatorname{arcsinh}(x)+c_1)}$$

✓ Solution by Mathematica

Time used: 0.593 (sec). Leaf size: 52

```
DSolve[(1+y[x])y'[x]Sqrt[1+x^2]==y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{-1+\sqrt{-2\text{arcsinh}(x)+1-2c_1}}$$
$$y(x)\to-\frac{1}{1+\sqrt{-2\text{arcsinh}(x)+1-2c_1}}$$
$$y(x)\to 0$$

21.17 problem 593

Internal problem ID [3334]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 593.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_Abel, '2nd type', 'class C']]`

$$(\mathrm{g0}(x) + y\,\mathrm{g1}(x))\,y' - \mathrm{f0}(x) - \mathrm{f1}(x)\,y - \mathrm{f2}(x)\,y^2 - \mathrm{f3}(x)\,y^3 = 0$$

✗ Solution by Maple

```
dsolve((g0(x)+y(x)*g1(x))*diff(y(x),x) = f0(x)+f1(x)*y(x)+f2(x)*y(x)^2+f3(x)*y(x)^3,y(x), sin
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(g0[x]+y[x] g1[x])y'[x]==f0[x]+f1[x] y[x]+f2[x] y[x]^2+f3[x] y[x]^3,y[x],x,IncludeSing
```

Timed out

21.18 problem 594

Internal problem ID [3335]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 594.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2y' + x(-y+2) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 27

```
dsolve(y(x)^2*diff(y(x),x)+x*(2-y(x)) = 0,y(x), singsol=all)
```

$$\frac{x^2}{2} - \frac{y(x)^2}{2} - 2y(x) - 4\ln(y(x) - 2) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.318 (sec). Leaf size: 43

```
DSolve[y[x]^2 y'[x]+x(2-y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{\#1^2}{2} + 2\#1 + 4\log(\#1 - 2) - 6\&\right]\left[\frac{x^2}{2} + c_1\right]$$
$$y(x) \to 2$$

21.19 problem 595

Internal problem ID [3336]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 595.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2y' - x(1 + y^2) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 22

```
dsolve(y(x)^2*diff(y(x),x) = x*(1+y(x)^2),y(x), singsol=all)
```

$$y(x) = -\tan\left(\operatorname{RootOf}\left(x^2 + 2\tan(\_Z) + 2c_1 - 2\_Z\right)\right)$$

✓ Solution by Mathematica

Time used: 0.22 (sec). Leaf size: 39

```
DSolve[y[x]^2 y'[x]==x(1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}[\#1 - \arctan(\#1)\&]\left[\frac{x^2}{2} + c_1\right]$$

$$y(x) \to -i$$

$$y(x) \to i$$

21.20 problem 596

Internal problem ID [3337]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 596.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{(x + y^2)\, y' + y - bx - a = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 710

```
dsolve((x+y(x)^2)*diff(y(x),x)+y(x) = b*x+a,y(x), singsol=all)
```

$$y(x) = \frac{\left(6b\,x^2+12ax-12c_1+2\sqrt{9b^2x^4+36a\,x^3b+36a^2x^2-36bc_1x^2-72c_1ax+16x^3+36c_1^2}\right)^{\frac{1}{3}}}{2} - \frac{2x}{\left(6b\,x^2+12ax-12c_1+2\sqrt{9b^2x^4+36a\,x^3b+36a^2x^2-36bc_1x^2-72c_1ax+16x^3+36c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(6b\,x^2+12ax-12c_1+2\sqrt{9b^2x^4+36a\,x^3b+36a^2x^2-36bc_1x^2-72c_1ax+16x^3+36c_1^2}\right)^{\frac{1}{3}}}{4} + \frac{x}{\left(6b\,x^2+12ax-12c_1+2\sqrt{9b^2x^4+36a\,x^3b+36a^2x^2-36bc_1x^2-72c_1ax+16x^3+36c_1^2}\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(6b\,x^2+12ax-12c_1+2\sqrt{9b^2x^4+36a\,x^3b+36a^2x^2-36bc_1x^2-72c_1ax+16x^3+36c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{2\ldots}{\left(6b\,x^2+12ax-12c_1+2\sqrt{9b^2x^4+36a\,x^3b+3\ldots}\right.}\right.}{2}$$

$$y(x) = -\frac{\left(6b\,x^2+12ax-12c_1+2\sqrt{9b^2x^4+36a\,x^3b+36a^2x^2-36bc_1x^2-72c_1ax+16x^3+36c_1^2}\right)^{\frac{1}{3}}}{4} + \frac{x}{\left(6b\,x^2+12ax-12c_1+2\sqrt{9b^2x^4+36a\,x^3b+36a^2x^2-36bc_1x^2-72c_1ax+16x^3+36c_1^2}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(6b\,x^2+12ax-12c_1+2\sqrt{9b^2x^4+36a\,x^3b+36a^2x^2-36bc_1x^2-72c_1ax+16x^3+36c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{2\ldots}{\left(6b\,x^2+12ax-12c_1+2\sqrt{9b^2x^4+36a\,x^3b+3\ldots}\right.}\right.}{2}$$

✓ Solution by Mathematica

Time used: 5.341 (sec). Leaf size: 398

```
DSolve[(x+y[x]^2)y'[x]+y[x]==a+b x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2\ 2^{2/3}x + \sqrt[3]{2}\left(\sqrt{16x^3 + 9\left(2ax + bx^2 + 2c_1\right)^2} + 6ax + 3bx^2 + 6c_1\right)^{2/3}}{2\sqrt[3]{\sqrt{16x^3 + 9\left(2ax + bx^2 + 2c_1\right)^2} + 6ax + 3bx^2 + 6c_1}}$$

$$y(x) \to \frac{4\sqrt[3]{-2}x + i(\sqrt{3}+i)\left(\sqrt{x^2\left(9(2a+bx)^2 + 16x\right) + 36c_1x(2a+bx) + 36c_1{}^2} + 6ax + 3bx^2 + 6c_1\right)^{2/3}}{2\ 2^{2/3}\sqrt[3]{\sqrt{16x^3 + 9\left(2ax + bx^2 + 2c_1\right)^2} + 6ax + 3bx^2 + 6c_1}}$$

$$y(x) \to \frac{x - i\sqrt{3}x}{\sqrt[3]{2}\sqrt[3]{\sqrt{16x^3 + 9\left(2ax + bx^2 + 2c_1\right)^2} + 6ax + 3bx^2 + 6c_1}} - \frac{i(\sqrt{3}-i)\sqrt[3]{\sqrt{16x^3 + 9\left(2ax + bx^2 + 2c_1\right)^2} + 6ax + 3bx^2 + 6c_1}}{2\ 2^{2/3}}$$

21.21 problem 597

Internal problem ID [3338]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 597.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{\left(-y^{2}+x\right) y'-x^{2}+y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 402

```
dsolve((x-y(x)^2)*diff(y(x),x) = x^2-y(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(-4x^3+12c_1+4\sqrt{x^6-6c_1x^3-4x^3+9c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{2x}{\left(-4x^3+12c_1+4\sqrt{x^6-6c_1x^3-4x^3+9c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(-4x^3+12c_1+4\sqrt{x^6-6c_1x^3-4x^3+9c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x}{\left(-4x^3+12c_1+4\sqrt{x^6-6c_1x^3-4x^3+9c_1^2}\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(-4x^3+12c_1+4\sqrt{x^6-6c_1x^3-4x^3+9c_1^2}\right)^{\frac{1}{3}}}{2} - \frac{2x}{\left(-4x^3+12c_1+4\sqrt{x^6-6c_1x^3-4x^3+9c_1^2}\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(-4x^3+12c_1+4\sqrt{x^6-6c_1x^3-4x^3+9c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x}{\left(-4x^3+12c_1+4\sqrt{x^6-6c_1x^3-4x^3+9c_1^2}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(-4x^3+12c_1+4\sqrt{x^6-6c_1x^3-4x^3+9c_1^2}\right)^{\frac{1}{3}}}{2} - \frac{2x}{\left(-4x^3+12c_1+4\sqrt{x^6-6c_1x^3-4x^3+9c_1^2}\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 3.732 (sec). Leaf size: 326

```
DSolve[(x-y[x]^2)y'[x]==x^2-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2x + \sqrt[3]{2}\left(x^3 + \sqrt{x^6 + (-4 + 6c_1)x^3 + 9c_1{}^2} + 3c_1\right)^{2/3}}{2^{2/3}\sqrt[3]{x^3 + \sqrt{x^6 + (-4 + 6c_1)x^3 + 9c_1{}^2} + 3c_1}}$$

$$y(x) \to \frac{2^{2/3}(1 - i\sqrt{3})\left(x^3 + \sqrt{x^6 + (-4 + 6c_1)x^3 + 9c_1{}^2} + 3c_1\right)^{2/3} + \sqrt[3]{2}(2 + 2i\sqrt{3})\,x}{4\sqrt[3]{x^3 + \sqrt{x^6 + (-4 + 6c_1)x^3 + 9c_1{}^2} + 3c_1}}$$

$$y(x) \to \frac{2^{2/3}(1 + i\sqrt{3})\left(x^3 + \sqrt{x^6 + (-4 + 6c_1)x^3 + 9c_1{}^2} + 3c_1\right)^{2/3} + \sqrt[3]{2}(2 - 2i\sqrt{3})\,x}{4\sqrt[3]{x^3 + \sqrt{x^6 + (-4 + 6c_1)x^3 + 9c_1{}^2} + 3c_1}}$$

21.22 problem 598

Internal problem ID [3339]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 598.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(x^2+y^2)\,y'+yx=0}$$

✓ Solution by Maple

Time used: 0.36 (sec). Leaf size: 223

```
dsolve((x^2+y(x)^2)*diff(y(x),x)+x*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{\sqrt{x^2c_1\left(c_1x^2+\sqrt{c_1^2x^4+1}\right)}}{x\left(c_1x^2+\sqrt{c_1^2x^4+1}\right)c_1}$$

$$y(x)=\frac{\sqrt{-x^2c_1\left(-c_1x^2+\sqrt{c_1^2x^4+1}\right)}}{x\left(c_1x^2-\sqrt{c_1^2x^4+1}\right)c_1}$$

$$y(x)=-\frac{\sqrt{x^2c_1\left(c_1x^2+\sqrt{c_1^2x^4+1}\right)}}{x\left(c_1x^2+\sqrt{c_1^2x^4+1}\right)c_1}$$

$$y(x)=-\frac{\sqrt{-x^2c_1\left(-c_1x^2+\sqrt{c_1^2x^4+1}\right)}}{x\left(c_1x^2-\sqrt{c_1^2x^4+1}\right)c_1}$$

✓ Solution by Mathematica

Time used: 9.096 (sec). Leaf size: 218

```
DSolve[(x^2+y[x]^2)y'[x]+x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 - \sqrt{x^4 + e^{4c_1}}}$$
$$y(x) \to \sqrt{-x^2 - \sqrt{x^4 + e^{4c_1}}}$$
$$y(x) \to -\sqrt{-x^2 + \sqrt{x^4 + e^{4c_1}}}$$
$$y(x) \to \sqrt{-x^2 + \sqrt{x^4 + e^{4c_1}}}$$
$$y(x) \to 0$$
$$y(x) \to -\sqrt{-\sqrt{x^4} - x^2}$$
$$y(x) \to \sqrt{-\sqrt{x^4} - x^2}$$
$$y(x) \to -\sqrt{\sqrt{x^4} - x^2}$$
$$y(x) \to \sqrt{\sqrt{x^4} - x^2}$$

21.23 problem 599

Internal problem ID [3340]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 599.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(x^2+y^2)\, y'-yx=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 16

```
dsolve((x^2+y(x)^2)*diff(y(x),x) = x*y(x),y(x), singsol=all)
```

$$y(x)=\sqrt{\frac{1}{\operatorname{LambertW}(c_1x^2)}}\,x$$

✓ Solution by Mathematica

Time used: 7.196 (sec). Leaf size: 49

```
DSolve[(x^2+y[x]^2)y'[x]==x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{x}{\sqrt{W(e^{-2c_1}x^2)}}$$
$$y(x)\to \frac{x}{\sqrt{W(e^{-2c_1}x^2)}}$$
$$y(x)\to 0$$

21.24 problem 600

Internal problem ID [3341]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 600.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(x^2 - y^2)\, y' - 2yx = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 45

```
dsolve((x^2-y(x)^2)*diff(y(x),x) = 2*x*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{-1 + \sqrt{-4c_1^2x^2 + 1}}{2c_1}$$

$$y(x) = \frac{1 + \sqrt{-4c_1^2x^2 + 1}}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.981 (sec). Leaf size: 66

```
DSolve[(x^2-y[x]^2)y'[x]==2 x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(e^{c_1} - \sqrt{-4x^2 + e^{2c_1}}\right)$$

$$y(x) \to \frac{1}{2}\left(\sqrt{-4x^2 + e^{2c_1}} + e^{c_1}\right)$$

$$y(x) \to 0$$

21.25 problem 601

Internal problem ID [3342]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 601.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\left(x^2-y^2\right)y'+x(x+2y)=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 419

```
dsolve((x^2-y(x)^2)*diff(y(x),x)+x*(x+2*y(x)) = 0,y(x), singsol=all)
```

$$y(x)=\frac{\frac{\left(4+4x^3c_1^{\frac{3}{2}}+4\sqrt{-3x^6c_1^3+2x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}{2}+\frac{2x^2c_1}{\left(4+4x^3c_1^{\frac{3}{2}}+4\sqrt{-3x^6c_1^3+2x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}}{\sqrt{c_1}}$$

$$y(x)=\frac{-\frac{\left(4+4x^3c_1^{\frac{3}{2}}+4\sqrt{-3x^6c_1^3+2x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}{4}-\frac{x^2c_1}{\left(4+4x^3c_1^{\frac{3}{2}}+4\sqrt{-3x^6c_1^3+2x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}-\frac{i\sqrt{3}\left(\frac{\left(4+4x^3c_1^{\frac{3}{2}}+4\sqrt{-3x^6c_1^3+2x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}{2}-\frac{\cdots}{\left(4+4x^3c_1\cdots\right.}\cdots}{2}\cdots}{\sqrt{c_1}}$$

$$y(x)=\frac{-\frac{\left(4+4x^3c_1^{\frac{3}{2}}+4\sqrt{-3x^6c_1^3+2x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}{4}-\frac{x^2c_1}{\left(4+4x^3c_1^{\frac{3}{2}}+4\sqrt{-3x^6c_1^3+2x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}+\frac{i\sqrt{3}\left(\frac{\left(4+4x^3c_1^{\frac{3}{2}}+4\sqrt{-3x^6c_1^3+2x^3c_1^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}{2}-\frac{\cdots}{\left(4+4x^3c_1\cdots\right.}\cdots}{2}\cdots}{\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 60.216 (sec). Leaf size: 359

```
DSolve[(x^2-y[x]^2)y'[x]+x(x+2 y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{x^3+\sqrt{-3x^6+2e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}{\sqrt[3]{2}}+\frac{\sqrt[3]{2}x^2}{\sqrt[3]{x^3+\sqrt{-3x^6+2e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}$$

$$y(x) \to \frac{i\left(\sqrt[3]{2}\left(\sqrt{3}+i\right)\left(x^3+\sqrt{-3x^6+2e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}\right)^{2/3}-2\left(\sqrt{3}-i\right)x^2\right)}{2\ 2^{2/3}\sqrt[3]{x^3+\sqrt{-3x^6+2e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}$$

$$y(x) \to \frac{2i\sqrt[3]{2}\left(\sqrt{3}+i\right)x^2+2^{2/3}\left(-1-i\sqrt{3}\right)\left(x^3+\sqrt{-3x^6+2e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}\right)^{2/3}}{4\sqrt[3]{x^3+\sqrt{-3x^6+2e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}$$

21.26 problem 602

Internal problem ID [3343]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 602.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\left(x^{2}+y^{2}\right) y'+2x(2x+y)=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 417

```
dsolve((x^2+y(x)^2)*diff(y(x),x)+2*x*(2*x+y(x)) = 0,y(x), singsol=all)
```

$$y(x)=\frac{\frac{\left(4-16x^{3}c_{1}^{\frac{3}{2}}+4\sqrt{20x^{6}c_{1}^{3}-8x^{3}c_{1}^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}{2}-\frac{2x^{2}c_{1}}{\left(4-16x^{3}c_{1}^{\frac{3}{2}}+4\sqrt{20x^{6}c_{1}^{3}-8x^{3}c_{1}^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}}{\sqrt{c_{1}}}$$

$$y(x)=\frac{-\frac{\left(4-16x^{3}c_{1}^{\frac{3}{2}}+4\sqrt{20x^{6}c_{1}^{3}-8x^{3}c_{1}^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}{4}+\frac{x^{2}c_{1}}{\left(4-16x^{3}c_{1}^{\frac{3}{2}}+4\sqrt{20x^{6}c_{1}^{3}-8x^{3}c_{1}^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}-\frac{i\sqrt{3}\left(\frac{\left(4-16x^{3}c_{1}^{\frac{3}{2}}+4\sqrt{20x^{6}c_{1}^{3}-8x^{3}c_{1}^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}{2}+\cdots\right)}{2}\cdots}{\sqrt{c_{1}}}$$

$$y(x)=\frac{-\frac{\left(4-16x^{3}c_{1}^{\frac{3}{2}}+4\sqrt{20x^{6}c_{1}^{3}-8x^{3}c_{1}^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}{4}+\frac{x^{2}c_{1}}{\left(4-16x^{3}c_{1}^{\frac{3}{2}}+4\sqrt{20x^{6}c_{1}^{3}-8x^{3}c_{1}^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}+\frac{i\sqrt{3}\left(\frac{\left(4-16x^{3}c_{1}^{\frac{3}{2}}+4\sqrt{20x^{6}c_{1}^{3}-8x^{3}c_{1}^{\frac{3}{2}}+1}\right)^{\frac{1}{3}}}{2}+\cdots\right)}{2}\cdots}{\sqrt{c_{1}}}$$

✓ Solution by Mathematica

Time used: 42.663 (sec). Leaf size: 537

```
DSolve[(x^2+y[x]^2)y'[x]+2 x(2 x+y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}{\sqrt[3]{2}} - \frac{\sqrt[3]{2}x^2}{\sqrt[3]{-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}$$

$$y(x) \to \frac{2\sqrt[3]{-2}x^2+(-2)^{2/3}\left(-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}\right)^{2/3}}{2\sqrt[3]{-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}$$

$$y(x) \to -\frac{2(-1)^{2/3}x^2+\sqrt[3]{-2}\left(-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}\right)^{2/3}}{2^{2/3}\sqrt[3]{-4x^3+\sqrt{20x^6-8e^{3c_1}x^3+e^{6c_1}}+e^{3c_1}}}$$

$$y(x) \to \sqrt[3]{\sqrt{5}\sqrt{x^6}-2x^3} - \frac{x^2}{\sqrt[3]{\sqrt{5}\sqrt{x^6}-2x^3}}$$

$$y(x) \to \frac{\sqrt[3]{-1}\left(x^2+\sqrt[3]{-1}\left(\sqrt{5}\sqrt{x^6}-2x^3\right)^{2/3}\right)}{\sqrt[3]{\sqrt{5}\sqrt{x^6}-2x^3}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)x^2+\left(-1-i\sqrt{3}\right)\left(\sqrt{5}\sqrt{x^6}-2x^3\right)^{2/3}}{2\sqrt[3]{\sqrt{5}\sqrt{x^6}-2x^3}}$$

21.27 problem 603

Internal problem ID [3344]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 603.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$\boxed{(1 - x^2 + y^2)\, y' - 1 - x^2 + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve((1-x^2+y(x)^2)*diff(y(x),x) = 1+x^2-y(x)^2,y(x), singsol=all)
```

$$y(x)^2 + 2y(x)\, x + x^2 + 2\ln\left(y(x) - x\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.346 (sec). Leaf size: 25

```
DSolve[(1-x^2+y[x]^2)y'[x]==1+x^2-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[e^{\frac{1}{2}(y(x)+x)^2}(x - y(x)) = c_1, y(x)\right]$$

21.28 problem 604

Internal problem ID [3345]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 604.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]'`

$$\boxed{(a^2 + x^2 + y^2)\, y' + 2yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 500

```
dsolve((a^2+x^2+y(x)^2)*diff(y(x),x)+2*x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-12c_1 + 4\sqrt{4a^6 + 12a^4x^2 + 12a^2x^4 + 4x^6 + 9c_1^2}\right)^{\frac{1}{3}}}{2} - \frac{2(a^2 + x^2)}{\left(-12c_1 + 4\sqrt{4a^6 + 12a^4x^2 + 12a^2x^4 + 4x^6 + 9c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(-12c_1 + 4\sqrt{4a^6 + 12a^4x^2 + 12a^2x^4 + 4x^6 + 9c_1^2}\right)^{\frac{1}{3}}}{4} + \frac{a^2 + x^2}{\left(-12c_1 + 4\sqrt{4a^6 + 12a^4x^2 + 12a^2x^4 + 4x^6 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(-12c_1+4\sqrt{4a^6+12a^4x^2+12a^2x^4+4x^6+9c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{2a^2+2x^2}{\left(-12c_1+4\sqrt{4a^6+12a^4x^2+12a^2x^4+4x^6+9c_1^2}\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(-12c_1 + 4\sqrt{4a^6 + 12a^4x^2 + 12a^2x^4 + 4x^6 + 9c_1^2}\right)^{\frac{1}{3}}}{4} + \frac{a^2 + x^2}{\left(-12c_1 + 4\sqrt{4a^6 + 12a^4x^2 + 12a^2x^4 + 4x^6 + 9c_1^2}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(-12c_1+4\sqrt{4a^6+12a^4x^2+12a^2x^4+4x^6+9c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{2a^2+2x^2}{\left(-12c_1+4\sqrt{4a^6+12a^4x^2+12a^2x^4+4x^6+9c_1^2}\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 5.052 (sec). Leaf size: 317

```
DSolve[(a^2+x^2+y[x]^2)y'[x]+2 x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{2}\left(\sqrt{4\left(a^2+x^2\right)^3+9c_1{}^2}+3c_1\right)^{2/3}-2a^2-2x^2}{2^{2/3}\sqrt[3]{\sqrt{4\left(a^2+x^2\right)^3+9c_1{}^2}+3c_1}}$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)\left(a^2+x^2\right)}{2^{2/3}\sqrt[3]{\sqrt{4\left(a^2+x^2\right)^3+9c_1{}^2}+3c_1}}+\frac{i\left(\sqrt{3}+i\right)\sqrt[3]{\sqrt{4\left(a^2+x^2\right)^3+9c_1{}^2}+3c_1}}{2\sqrt[3]{2}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)\left(a^2+x^2\right)}{2^{2/3}\sqrt[3]{\sqrt{4\left(a^2+x^2\right)^3+9c_1{}^2}+3c_1}}-\frac{i\left(\sqrt{3}-i\right)\sqrt[3]{\sqrt{4\left(a^2+x^2\right)^3+9c_1{}^2}+3c_1}}{2\sqrt[3]{2}}$$

$$y(x) \to 0$$

21.29 problem 605

Internal problem ID [3346]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 605.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{(a^2 + x^2 + y^2)\, y' + b^2 + x^2 + 2yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 900

```
dsolve((a^2+x^2+y(x)^2)*diff(y(x),x)+b^2+x^2+2*x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-12b^2x - 4x^3 - 12c_1 + 4\sqrt{4a^6 + 12a^4x^2 + 12a^2x^4 + 9b^4x^2 + 6b^2x^4 + 5x^6 + 18b^2c_1x + 6c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{2} - \frac{2(a^2 + x^2)}{\left(-12b^2x - 4x^3 - 12c_1 + 4\sqrt{4a^6 + 12a^4x^2 + 12a^2x^4 + 9b^4x^2 + 6b^2x^4 + 5x^6 + 18b^2c_1x + 6c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(-12b^2x - 4x^3 - 12c_1 + 4\sqrt{4a^6 + 12a^4x^2 + 12a^2x^4 + 9b^4x^2 + 6b^2x^4 + 5x^6 + 18b^2c_1x + 6c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} + \frac{a^2 + x^2}{\left(-12b^2x - 4x^3 - 12c_1 + 4\sqrt{4a^6 + 12a^4x^2 + 12a^2x^4 + 9b^4x^2 + 6b^2x^4 + 5x^6 + 18b^2c_1x + 6c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(-12b^2x-4x^3-12c_1+4\sqrt{4a^6+12a^4x^2+12a^2x^4+9b^4x^2+6b^2x^4+5x^6+18b^2c_1x+6c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{\cdots}{\left(-12b^2x-4x^3-12c_1+4\sqrt{4a^6+\cdots}\right.}\right.}{2}$$

$$y(x) = -\frac{\left(-12b^2x - 4x^3 - 12c_1 + 4\sqrt{4a^6 + 12a^4x^2 + 12a^2x^4 + 9b^4x^2 + 6b^2x^4 + 5x^6 + 18b^2c_1x + 6c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} + \frac{a^2 + x^2}{\left(-12b^2x - 4x^3 - 12c_1 + 4\sqrt{4a^6 + 12a^4x^2 + 12a^2x^4 + 9b^4x^2 + 6b^2x^4 + 5x^6 + 18b^2c_1x + 6c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(-12b^2x-4x^3-12c_1+4\sqrt{4a^6+12a^4x^2+12a^2x^4+9b^4x^2+6b^2x^4+5x^6+18b^2c_1x+6c_1x^3+9c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{\cdots}{\left(-12b^2x-4x^3-12c_1+4\sqrt{4a^6+\cdots}\right.}\right.}{2}$$

✓ Solution by Mathematica

Time used: 7.051 (sec). Leaf size: 438

```
DSolve[(a^2+x^2+y[x]^2)y'[x]+b^2+x^2+2 x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{2}\left(\sqrt{4(a^2+x^2)^3+(3b^2x+x^3-3c_1)^2}-3b^2x-x^3+3c_1\right)^{2/3}-2a^2-2x^2}{2^{2/3}\sqrt[3]{\sqrt{4(a^2+x^2)^3+(3b^2x+x^3-3c_1)^2}-3b^2x-x^3+3c_1}}$$

$$y(x) \to \frac{(1+i\sqrt{3})(a^2+x^2)}{2^{2/3}\sqrt[3]{\sqrt{4(a^2+x^2)^3+(3b^2x+x^3-3c_1)^2}-3b^2x-x^3+3c_1}} + \frac{i(\sqrt{3}+i)\sqrt[3]{\sqrt{4(a^2+x^2)^3+(3b^2x+x^3-3c_1)^2}-3b^2x-x^3+3c_1}}{2\sqrt[3]{2}}$$

$$y(x) \to \frac{(1-i\sqrt{3})(a^2+x^2)}{2^{2/3}\sqrt[3]{\sqrt{4(a^2+x^2)^3+(3b^2x+x^3-3c_1)^2}-3b^2x-x^3+3c_1}} - \frac{i(\sqrt{3}-i)\sqrt[3]{\sqrt{4(a^2+x^2)^3+(3b^2x+x^3-3c_1)^2}-3b^2x-x^3+3c_1}}{2\sqrt[3]{2}}$$

21.30 problem 606

Internal problem ID [3347]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 21
Problem number: 606.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{(x + x^2 + y^2)\, y' - y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 30

```
dsolve((x+x^2+y(x)^2)*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$c_1 + \frac{e^{-2iy(x)}(ix + y(x))}{2iy(x) + 2x} = 0$$

✓ Solution by Mathematica

Time used: 0.106 (sec). Leaf size: 18

```
DSolve[(x+x^2+y[x]^2)y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[y(x) - \arctan\left(\frac{x}{y(x)}\right) = c_1, y(x)\right]$$

22 Various 22

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22.17 problem 625 695
22.18 problem 626 696
22.19 problem 627 699
22.20 problem 628 700
22.21 problem 629 703
22.22 problem 630 705

22.1 problem 607

Internal problem ID [3348]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 607.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(3x^2 - y^2)\, y' - 2yx = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 402

```
dsolve((3*x^2-y(x)^2)*diff(y(x),x) = 2*x*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(-108c_1^2x^2 + 12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1 + 8\right)^{\frac{1}{3}}}{6c_1} + \frac{2}{3c_1\left(-108c_1^2x^2 + 12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1 + 8\right)^{\frac{1}{3}}} + \frac{1}{3c_1}$$

$$y(x) = -\frac{\left(-108c_1^2x^2 + 12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1 + 8\right)^{\frac{1}{3}}}{12c_1} - \frac{1}{3c_1\left(-108c_1^2x^2 + 12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1 + 8\right)^{\frac{1}{3}}} + \frac{1}{3c_1} - \frac{i\sqrt{3}\left(\frac{\left(-108c_1^2x^2+12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1+8\right)^{\frac{1}{3}}}{6c_1} - \frac{2}{3c_1\left(-108c_1^2x^2+12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1+8\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(-108c_1^2x^2 + 12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1 + 8\right)^{\frac{1}{3}}}{12c_1} - \frac{1}{3c_1\left(-108c_1^2x^2 + 12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1 + 8\right)^{\frac{1}{3}}} + \frac{1}{3c_1} + \frac{i\sqrt{3}\left(\frac{\left(-108c_1^2x^2+12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1+8\right)^{\frac{1}{3}}}{6c_1} - \frac{2}{3c_1\left(-108c_1^2x^2+12\sqrt{3}\,x\sqrt{27c_1^2x^2-4}\,c_1+8\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 60.186 (sec). Leaf size: 458

```
DSolve[(3 x^2-y[x]^2)y'[x]==2 x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(\frac{\sqrt[3]{27e^{c_1}x^2 + 3\sqrt{81e^{2c_1}x^4 - 12e^{4c_1}x^2} - 2e^{3c_1}}}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}e^{2c_1}}{\sqrt[3]{27e^{c_1}x^2 + 3\sqrt{81e^{2c_1}x^4 - 12e^{4c_1}x^2} - 2e^{3c_1}}} - e^{c_1}\right)$$

$$y(x) \to \frac{i(\sqrt{3}+i)\sqrt[3]{27e^{c_1}x^2 + 3\sqrt{81e^{2c_1}x^4 - 12e^{4c_1}x^2} - 2e^{3c_1}}}{6\sqrt[3]{2}} - \frac{i(\sqrt{3}-i)e^{2c_1}}{3\ 2^{2/3}\sqrt[3]{27e^{c_1}x^2 + 3\sqrt{81e^{2c_1}x^4 - 12e^{4c_1}x^2} - 2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

$$y(x) \to -\frac{i(\sqrt{3}-i)\sqrt[3]{27e^{c_1}x^2 + 3\sqrt{81e^{2c_1}x^4 - 12e^{4c_1}x^2} - 2e^{3c_1}}}{6\sqrt[3]{2}} + \frac{i(\sqrt{3}+i)e^{2c_1}}{3\ 2^{2/3}\sqrt[3]{27e^{c_1}x^2 + 3\sqrt{81e^{2c_1}x^4 - 12e^{4c_1}x^2} - 2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

22.2 problem 608

Internal problem ID [3349]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 608.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$(x^4 + y^2)\, y' - 4x^3 y = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 67

```
dsolve((x^4+y(x)^2)*diff(y(x),x) = 4*x^3*y(x),y(x), singsol=all)
```

$$y(x) = \left(\frac{2x^2 + c_1 - \sqrt{4x^4 + c_1^2}}{2x^2} - 1\right) x^2$$
$$y(x) = \left(\frac{2x^2 + c_1 + \sqrt{4x^4 + c_1^2}}{2x^2} - 1\right) x^2$$

✓ Solution by Mathematica

Time used: 0.307 (sec). Leaf size: 58

```
DSolve[(x^4+y[x]^2)y'[x]==4 x^3 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(c_1 - \sqrt{4x^4 + c_1{}^2}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{4x^4 + c_1{}^2} + c_1\right)$$
$$y(x) \to 0$$

22.3 problem 609

Internal problem ID [3350]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 609.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y(1+y)\,y'-x(x+1)=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 720

```
dsolve(y(x)*(1+y(x))*diff(y(x),x) = x*(1+x),y(x), singsol=all)
```

$$y(x) = \frac{\left(-1+4x^3+6x^2+12c_1+2\sqrt{4x^6+12x^5+24c_1x^3+9x^4+36c_1x^2-2x^3+36c_1^2-3x^2-6c_1}\right)^{\frac{1}{3}}}{2} + \frac{1}{2\left(-1+4x^3+6x^2+12c_1+2\sqrt{4x^6+12x^5+24c_1x^3+9x^4+36c_1x^2-2x^3+36c_1^2-3x^2-6c_1}\right)^{\frac{1}{3}}} - \frac{1}{2}$$

$$y(x) = -\frac{\left(-1+4x^3+6x^2+12c_1+2\sqrt{4x^6+12x^5+24c_1x^3+9x^4+36c_1x^2-2x^3+36c_1^2-3x^2-6c_1}\right)^{\frac{1}{3}}}{4} - \frac{1}{4\left(-1+4x^3+6x^2+12c_1+2\sqrt{4x^6+12x^5+24c_1x^3+9x^4+36c_1x^2-2x^3+36c_1^2-3x^2-6c_1}\right)^{\frac{1}{3}}} - \frac{1}{2} - \frac{i\sqrt{3}\left(\frac{\left(-1+4x^3+6x^2+12c_1+2\sqrt{4x^6+12x^5+24c_1x^3+9x^4+36c_1x^2-2x^3+36c_1^2-3x^2-6c_1}\right)^{\frac{1}{3}}}{2} - \frac{\cdots}{2\left(-1+4x^3+6x^2+12c_1+2\sqrt{4x^6+12x^5+\cdots}\right.}\right.}{2}$$

$$y(x) = -\frac{\left(-1+4x^3+6x^2+12c_1+2\sqrt{4x^6+12x^5+24c_1x^3+9x^4+36c_1x^2-2x^3+36c_1^2-3x^2-6c_1}\right)^{\frac{1}{3}}}{4} - \frac{1}{4\left(-1+4x^3+6x^2+12c_1+2\sqrt{4x^6+12x^5+24c_1x^3+9x^4+36c_1x^2-2x^3+36c_1^2-3x^2-6c_1}\right)^{\frac{1}{3}}} - \frac{1}{2} + \frac{i\sqrt{3}\left(\frac{\left(-1+4x^3+6x^2+12c_1+2\sqrt{4x^6+12x^5+24c_1x^3+9x^4+36c_1x^2-2x^3+36c_1^2-3x^2-6c_1}\right)^{\frac{1}{3}}}{2} - \frac{\cdots}{2\left(-1+4x^3+6x^2+12c_1+2\sqrt{4x^6+12x^5+\cdots}\right.}\right.}{2}$$

✓ Solution by Mathematica

Time used: 4.229 (sec). Leaf size: 346

```
DSolve[y[x](1+y[x])y'[x]==x(1+x),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(\sqrt[3]{4x^3+6x^2+\sqrt{-1+(4x^3+6x^2-1+12c_1)^2}-1+12c_1} + \frac{1}{\sqrt[3]{4x^3+6x^2+\sqrt{-1+(4x^3+6x^2-1+12c_1)^2}-1+12c_1}} - 1\right)$$

$$y(x) \to \frac{1}{8}\left(2i\left(\sqrt{3}+i\right)\sqrt[3]{4x^3+6x^2+\sqrt{-1+(4x^3+6x^2-1+12c_1)^2}-1+12c_1} + \frac{-2-2i\sqrt{3}}{\sqrt[3]{4x^3+6x^2+\sqrt{-1+(4x^3+6x^2-1+12c_1)^2}-1+12c_1}} - 4\right)$$

$$y(x) \to \frac{1}{8}\left(-2\left(1+i\sqrt{3}\right)\sqrt[3]{4x^3+6x^2+\sqrt{-1+(4x^3+6x^2-1+12c_1)^2}-1+12c_1} + \frac{2i\left(\sqrt{3}+i\right)}{\sqrt[3]{4x^3+6x^2+\sqrt{-1+(4x^3+6x^2-1+12c_1)^2}-1+12c_1}} - 4\right)$$

22.4 problem 610

Internal problem ID [3351]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 610.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$(x+2y+y^2)\,y'+y(1+y)+(x+y)^2\,y^2=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 114

```
dsolve((x+2*y(x)+y(x)^2)*diff(y(x),x)+y(x)*(1+y(x))+(x+y(x))^2*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{-c_1x+x^2-1+\sqrt{c_1^2x^2-2c_1x^3+x^4+2c_1x-2x^2-4c_1+4x+1}}{2c_1-2x}$$

$$y(x)=-\frac{c_1x-x^2+\sqrt{c_1^2x^2-2c_1x^3+x^4+2c_1x-2x^2-4c_1+4x+1}+1}{2\,(c_1-x)}$$

✓ Solution by Mathematica

Time used: 2.268 (sec). Leaf size: 146

```
DSolve[(x+2 y[x]+y[x]^2)y'[x]+y[x](1+y[x])+(x+y[x])^2 y[x]^2==0,y[x],x,IncludeSingularSolutio
```

$$y(x)\to-\frac{x^2+\sqrt{(-x^2+c_1x+1)^2+4(x-c_1)}-c_1x-1}{2(x-c_1)}$$

$$y(x)\to\frac{-x^2+\sqrt{(-x^2+c_1x+1)^2+4(x-c_1)}+c_1x+1}{2(x-c_1)}$$

$$y(x)\to\frac{1}{2}\left(-\sqrt{x^2}-x\right)$$

$$y(x)\to\frac{1}{2}\left(\sqrt{x^2}-x\right)$$

22.5 problem 611

Internal problem ID [3352]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 611.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(x^2 + 2y + y^2)\, y' + 2x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve((x^2+2*y(x)+y(x)^2)*diff(y(x),x)+2*x = 0,y(x), singsol=all)
```

$$e^{y(x)}x^2 + e^{y(x)}y(x)^2 + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.151 (sec). Leaf size: 24

```
DSolve[(x^2+2 y[x]+y[x]^2)y'[x]+2 x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x^2e^{y(x)} + e^{y(x)}y(x)^2 = c_1, y(x)\right]$$

22.6 problem 612

Internal problem ID [3353]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 612.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]'`

$$\left(x^{3}+2y-y^{2}\right) y'+3x^{2}y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 493

```
dsolve((x^3+2*y(x)-y(x)^2)*diff(y(x),x)+3*x^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(12x^3+12c_1+8+4\sqrt{-4x^9-3x^6+18c_1x^3+9c_1^2+12c_1}\right)^{\frac{1}{3}}}{2} - \frac{2(-x^3-1)}{\left(12x^3+12c_1+8+4\sqrt{-4x^9-3x^6+18c_1x^3+9c_1^2+12c_1}\right)^{\frac{1}{3}}} + 1$$

$$y(x) = -\frac{\left(12x^3+12c_1+8+4\sqrt{-4x^9-3x^6+18c_1x^3+9c_1^2+12c_1}\right)^{\frac{1}{3}}}{4} + \frac{-x^3-1}{\left(12x^3+12c_1+8+4\sqrt{-4x^9-3x^6+18c_1x^3+9c_1^2+12c_1}\right)^{\frac{1}{3}}} + 1 - \frac{i\sqrt{3}\left(\frac{\left(12x^3+12c_1+8+4\sqrt{-4x^9-3x^6+18c_1x^3+9c_1^2+12c_1}\right)^{\frac{1}{3}}}{2} + \frac{-2x^3-2}{\left(12x^3+12c_1+8+4\sqrt{-4x^9-3x^6+18c_1x^3+9c_1^2+12c_1}\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(12x^3+12c_1+8+4\sqrt{-4x^9-3x^6+18c_1x^3+9c_1^2+12c_1}\right)^{\frac{1}{3}}}{4} + \frac{-x^3-1}{\left(12x^3+12c_1+8+4\sqrt{-4x^9-3x^6+18c_1x^3+9c_1^2+12c_1}\right)^{\frac{1}{3}}} + 1 + \frac{i\sqrt{3}\left(\frac{\left(12x^3+12c_1+8+4\sqrt{-4x^9-3x^6+18c_1x^3+9c_1^2+12c_1}\right)^{\frac{1}{3}}}{2} + \frac{-2x^3-2}{\left(12x^3+12c_1+8+4\sqrt{-4x^9-3x^6+18c_1x^3+9c_1^2+12c_1}\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 5.433 (sec). Leaf size: 409

```
DSolve[(x^3+2 y[x]-y[x]^2)y'[x]+3 x^2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{2}(x^3+1)}{\sqrt[3]{-3x^3+\sqrt{-4x^9-3x^6-18c_1x^3+3c_1(-4+3c_1)}-2+3c_1}} - \frac{\sqrt[3]{-3x^3+\sqrt{-4x^9-3x^6-18c_1x^3+3c_1(-4+3c_1)}-2+3c_1}}{\sqrt[3]{2}}+1$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)(x^3+1)}{2^{2/3}\sqrt[3]{-3x^3+\sqrt{-4x^9-3x^6-18c_1x^3+3c_1(-4+3c_1)}-2+3c_1}} + \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{-3x^3+\sqrt{-4x^9-3x^6-18c_1x^3+3c_1(-4+3c_1)}-2+3c_1}}{2\sqrt[3]{2}}+1$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)(x^3+1)}{2^{2/3}\sqrt[3]{-3x^3+\sqrt{-4x^9-3x^6-18c_1x^3+3c_1(-4+3c_1)}-2+3c_1}} + \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-3x^3+\sqrt{-4x^9-3x^6-18c_1x^3+3c_1(-4+3c_1)}-2+3c_1}}{2\sqrt[3]{2}}+1$$

$$y(x) \to 0$$

22.7 problem 613

Internal problem ID [3354]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 613.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$\boxed{(1 + y + yx + y^2) y' + 1 + y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 36

```
dsolve((1+y(x)+x*y(x)+y(x)^2)*diff(y(x),x)+1+y(x) = 0,y(x), singsol=all)
```

$$x - \left(-\frac{y(x)\,\mathrm{e}^{y(x)}}{y(x)+1} + c_1\right)\left(\mathrm{e}^{-y(x)} + y(x)\,\mathrm{e}^{-y(x)}\right) = 0$$

✓ Solution by Mathematica

Time used: 0.149 (sec). Leaf size: 23

```
DSolve[(1+y[x]+x y[x]+y[x]^2)y'[x]+1+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = -y(x) + c_1 e^{-y(x)}(y(x)+1), y(x)\right]$$

22.8 problem 614

Internal problem ID [3355]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 614.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$\boxed{(x+y)^2 y' - a^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve((x+y(x))^2*diff(y(x),x) = a^2,y(x), singsol=all)
```

$$y(x) = a\,\mathrm{RootOf}\left(\tan\left(\_Z\right)a - a\_Z + c_1 - x\right) - c_1$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 21

```
DSolve[(x+y[x])^2 y'[x]==a^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[y(x) - a\arctan\left(\frac{y(x)+x}{a}\right) = c_1, y(x)\right]$$

22.9 problem 615

Internal problem ID [3356]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 615.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{(x-y)^2 y'-a^2=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 36

```
dsolve((x-y(x))^2*diff(y(x),x) = a^2,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{\mathrm{RootOf}\left(a\ln\left(\mathrm{e}^{\_Z}+2a\right)-a\_Z-2\,\mathrm{e}^{\_Z}+2c_1-2a-2x\right)}+a+x$$

✓ Solution by Mathematica

Time used: 0.158 (sec). Leaf size: 49

```
DSolve[(x-y[x])^2 y'[x]==a^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\left(a^2\left(\frac{\log(a-y(x)+x)}{2a}-\frac{\log(-a-y(x)+x)}{2a}\right)\right)-y(x)=c_1,y(x)\right]$$

22.10 problem 616

Internal problem ID [3357]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 616.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(x^2 + 2yx - y^2)\, y' + x^2 - 2yx + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 37

```
dsolve((x^2+2*x*y(x)-y(x)^2)*diff(y(x),x)+x^2-2*x*y(x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(\int^{\_Z} \frac{\_a^2 - 2\_a - 1}{\_a^3 - 3\_a^2 + \_a - 1} d\_a + \ln(x) + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.13 (sec). Leaf size: 91

```
DSolve[(x^2+2 x y[x]-y[x]^2)y'[x]+x^2-2 x y[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> T
```

$$\text{Solve}\left[\text{RootSum}\left[\#1^3 - 3\#1^2 + \#1 - 1\&, \frac{\#1^2 \log\left(\frac{y(x)}{x} - \#1\right) - 2\#1 \log\left(\frac{y(x)}{x} - \#1\right) - \log\left(\frac{y(x)}{x} - \#1\right)}{3\#1^2 - 6\#1 + 1}\&\right] = -\log(x) + c_1, y(x)\right]$$

22.11 problem 619

Internal problem ID [3358]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 619.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(x+y)^2 y' - x^2 + 2yx - 5y^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 35

```
dsolve((x+y(x))^2*diff(y(x),x) = x^2-2*x*y(x)+5*y(x)^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(\mathrm{e}^{2\_Z}\ln(x)+\mathrm{e}^{2\_Z}c_1+\mathrm{e}^{2\_Z}\_Z-4\,\mathrm{e}^{\_Z}-2\right)}x + x$$

✓ Solution by Mathematica

Time used: 0.332 (sec). Leaf size: 41

```
DSolve[(x+y[x])^2 y'[x]==x^2-2 x y[x]+5 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2-\frac{4y(x)}{x}}{\left(\frac{y(x)}{x}-1\right)^2} + \log\left(\frac{y(x)}{x}-1\right) = -\log(x) + c_1, y(x)\right]$$

22.12 problem 620

Internal problem ID [3359]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 620.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational]`

$$\boxed{(a+b+x+y)^2 y' - 2(a+y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve((a+b+x+y(x))^2*diff(y(x),x) = 2*(a+y(x))^2,y(x), singsol=all)
```

$$y(x) = -a - \tan\left(\mathrm{RootOf}\left(-2\_Z + \ln\left(\tan\left(\_Z\right)\right) + \ln\left(x+b\right) + c_1\right)\right)(x+b)$$

✓ Solution by Mathematica

Time used: 0.146 (sec). Leaf size: 25

```
DSolve[(a+b+x+y[x])^2 y'[x]==2(a+y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\log(a+y(x)) - 2\arctan\left(\frac{b+x}{a+y(x)}\right) = c_1, y(x)\right]$$

22.13 problem 621

Internal problem ID [3360]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 621.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _dAlembert]`

$$\left(2x^2+4yx-y^2\right)y'-x^2+4yx+2y^2=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 441

```
dsolve((2*x^2+4*x*y(x)-y(x)^2)*diff(y(x),x) = x^2-4*x*y(x)-2*y(x)^2,y(x), singsol=all)
```

$$y(x)=\frac{\frac{\left(108x^3c_1^3+4+4\sqrt{-135c_1^6x^6+54x^3c_1^3+1}\right)^{\frac{1}{3}}}{2}+\frac{12x^2c_1^2}{\left(108x^3c_1^3+4+4\sqrt{-135c_1^6x^6+54x^3c_1^3+1}\right)^{\frac{1}{3}}}+2c_1x}{c_1}$$

$$y(x)=\frac{-\frac{\left(108x^3c_1^3+4+4\sqrt{-135c_1^6x^6+54x^3c_1^3+1}\right)^{\frac{1}{3}}}{4}-\frac{6x^2c_1^2}{\left(108x^3c_1^3+4+4\sqrt{-135c_1^6x^6+54x^3c_1^3+1}\right)^{\frac{1}{3}}}+2c_1x-\frac{i\sqrt{3}\left(\frac{\left(108x^3c_1^3+4+4\sqrt{-135c_1^6x^6+54\right.}{2}}{}}{c_1}$$

$$y(x)=\frac{-\frac{\left(108x^3c_1^3+4+4\sqrt{-135c_1^6x^6+54x^3c_1^3+1}\right)^{\frac{1}{3}}}{4}-\frac{6x^2c_1^2}{\left(108x^3c_1^3+4+4\sqrt{-135c_1^6x^6+54x^3c_1^3+1}\right)^{\frac{1}{3}}}+2c_1x+\frac{i\sqrt{3}\left(\frac{\left(108x^3c_1^3+4+4\sqrt{-135c_1^6x^6+54\right.}{2}}{}}{c_1}$$

✓ Solution by Mathematica

Time used: 31.148 (sec). Leaf size: 781

```
DSolve[(2 x^2+4 x y[x]-y[x]^2)y'[x]==x^2-4 x y[x]-2 y[x]^2,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \frac{\sqrt[3]{27x^3 + \sqrt{-135x^6 + 54e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}}{\sqrt[3]{2}} + \frac{6\sqrt[3]{2}x^2}{\sqrt[3]{27x^3 + \sqrt{-135x^6 + 54e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}} + 2x$$

$$y(x) \to -\frac{(1 - i\sqrt{3})\sqrt[3]{27x^3 + \sqrt{-135x^6 + 54e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}}{2\sqrt[3]{2}} - \frac{3\sqrt[3]{2}(1 + i\sqrt{3})\,x^2}{\sqrt[3]{27x^3 + \sqrt{-135x^6 + 54e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}} + 2x$$

$$y(x) \to -\frac{(1 + i\sqrt{3})\sqrt[3]{27x^3 + \sqrt{-135x^6 + 54e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}}{2\sqrt[3]{2}} - \frac{3\sqrt[3]{2}(1 - i\sqrt{3})\,x^2}{\sqrt[3]{27x^3 + \sqrt{-135x^6 + 54e^{3c_1}x^3 + e^{6c_1}} + e^{3c_1}}} + 2x$$

$$y(x) \to \frac{4\sqrt[3]{2}3^{2/3}x^2 + 4\sqrt[3]{\sqrt{15}\sqrt{-x^6} + 9x^3}x + 2^{2/3}\sqrt[3]{3}\left(\sqrt{15}\sqrt{-x^6} + 9x^3\right)^{2/3}}{2\sqrt[3]{\sqrt{15}\sqrt{-x^6} + 9x^3}}$$

$$y(x) \to \frac{-4\sqrt[3]{2}3^{2/3}x^2 + 12i\sqrt[3]{2}\sqrt[6]{3}x^2 + 8\sqrt[3]{\sqrt{15}\sqrt{-x^6} + 9x^3}x - i2^{2/3}3^{5/6}\left(\sqrt{15}\sqrt{-x^6} + 9x^3\right)^{2/3} - 2^{2/3}\sqrt[3]{3}\left(\sqrt{15}\sqrt{-\right.}{4\sqrt[3]{\sqrt{15}\sqrt{-x^6} + 9x^3}}$$

$$y(x) \to \frac{\sqrt[3]{3}\left(\sqrt{15}\sqrt{-x^6} + 9x^3\right)^{2/3}\text{Root}\left[2\#1^3 - 1\&, 3\right] - 2\sqrt[3]{-2}3^{2/3}x^2 + 2\sqrt[3]{\sqrt{15}\sqrt{-x^6} + 9x^3}x}{\sqrt[3]{\sqrt{15}\sqrt{-x^6} + 9x^3}}$$

22.14 problem 622

Internal problem ID [3361]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 622.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(3x+y)^2 y' - 4(3x+2y)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 47

```
dsolve((3*x+y(x))^2*diff(y(x),x) = 4*(3*x+2*y(x))*y(x),y(x), singsol=all)
```

$$3\ln\left(\frac{y(x)}{x}\right) - 3\ln\left(-\frac{-y(x)+3x}{x}\right) - \ln\left(\frac{x+y(x)}{x}\right) - \ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.187 (sec). Leaf size: 747

```
DSolve[(3 x+y[x])^2 y'[x]==4(3 x+2 y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-\sqrt{12\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}+16x^2-16e^{c_1}x+e^{2c_1}} -\sqrt{2}\sqrt{-6\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}-48x^2+\frac{\left(-8x+e^{c_1}\right)^3-72x^2\left(-8x+e^{c_1}\right)}{\sqrt{12\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}+16x^2-16e^{c_1}x+e^{2c_1}}}+(-8x} +8x-e^{c_1}\right)$$

$$y(x) \to \frac{1}{4}\left(-\sqrt{12\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}+16x^2-16e^{c_1}x+e^{2c_1}} +\sqrt{2}\sqrt{-6\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}-48x^2+\frac{\left(-8x+e^{c_1}\right)^3-72x^2\left(-8x+e^{c_1}\right)}{\sqrt{12\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}+16x^2-16e^{c_1}x+e^{2c_1}}}+(-8x} +8x-e^{c_1}\right)$$

$$y(x) \to \frac{1}{4}\left(\sqrt{12\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}+16x^2-16e^{c_1}x+e^{2c_1}} -\sqrt{2}\sqrt{-6\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}-48x^2+\frac{72x^2\left(-8x+e^{c_1}\right)-\left(-8x+e^{c_1}\right)^3}{\sqrt{12\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}+16x^2-16e^{c_1}x+e^{2c_1}}}+(-8x} +8x-e^{c_1}\right)$$

$$y(x) \to \frac{1}{4}\left(\sqrt{12\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}+16x^2-16e^{c_1}x+e^{2c_1}} +\sqrt{2}\sqrt{-6\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}-48x^2+\frac{72x^2\left(-8x+e^{c_1}\right)-\left(-8x+e^{c_1}\right)^3}{\sqrt{12\sqrt[3]{-e^{c_1}x^4\left(-16x+e^{c_1}\right)}+16x^2-16e^{c_1}x+e^{2c_1}}}+(-8x} +8x-e^{c_1}\right)$$

22.15 problem 623

Internal problem ID [3362]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 623.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational]`

$$\boxed{(1-3x-y)^2 y' - (-2y+1)(3-6x-4y) = 0}$$

✓ Solution by Maple

Time used: 0.328 (sec). Leaf size: 72

```
dsolve((1-3*x-y(x))^2*diff(y(x),x) = (1-2*y(x))*(3-6*x-4*y(x)),y(x), singsol=all)
```

$$-\ln\left(-\frac{6y(x)-4+6x}{6x-1}\right)+3\ln\left(\frac{-6y(x)+3}{6x-1}\right)-3\ln\left(\frac{-6y(x)+18x}{6x-1}\right)-\ln(6x-1)-c_1=0$$

✓ Solution by Mathematica

Time used: 60.206 (sec). Leaf size: 1069

```
DSolve[(1-3 x-y[x])^2 y'[x]==(1-2 y[x])(3-6 x-4 y[x]),y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{1}{6}\left(12x - \sqrt{(1-6x)^2 + 16e^{c_1}(6x-1) + 3\, 2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4\left(6x-1+e^{c_1}\right)} + 16e^{2c_1}} - \frac{1}{2}\sqrt{8\left(12x+1+4e^{c_1}\right)^2 - 96\left(3x(3x+1)+2e^{c_1}\right) - 12\, 2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4\left(6x-1+e^{c_1}\right)} + \frac{}{\sqrt{(1-6x}}} + 1 + 4e^{c_1}\right)$$

$$y(x) \to \frac{1}{6}\left(12x - \sqrt{(1-6x)^2 + 16e^{c_1}(6x-1) + 3\, 2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4\left(6x-1+e^{c_1}\right)} + 16e^{2c_1}} + \frac{1}{2}\sqrt{8\left(12x+1+4e^{c_1}\right)^2 - 96\left(3x(3x+1)+2e^{c_1}\right) - 12\, 2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4\left(6x-1+e^{c_1}\right)} + \frac{}{\sqrt{(1-6x}}} + 1 + 4e^{c_1}\right)$$

$$y(x) \to \frac{1}{6}\left(12x + \sqrt{(1-6x)^2 + 16e^{c_1}(6x-1) + 3\, 2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4\left(6x-1+e^{c_1}\right)} + 16e^{2c_1}} - \frac{1}{2}\sqrt{8\left(12x+1+4e^{c_1}\right)^2 - 96\left(3x(3x+1)+2e^{c_1}\right) - 12\, 2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4\left(6x-1+e^{c_1}\right)} + \frac{}{\sqrt{(1-6x}}} + 1 + 4e^{c_1}\right)$$

$$y(x) \to \frac{1}{6}\left(12x + \sqrt{(1-6x)^2 + 16e^{c_1}(6x-1) + 3\, 2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4\left(6x-1+e^{c_1}\right)} + 16e^{2c_1}} + \frac{1}{2}\sqrt{8\left(12x+1+4e^{c_1}\right)^2 - 96\left(3x(3x+1)+2e^{c_1}\right) - 12\, 2^{2/3}\sqrt[3]{-e^{c_1}(6x-1)^4\left(6x-1+e^{c_1}\right)} + \right.}$$

22.16 problem 624

Internal problem ID [3363]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 624.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\left(\cot(x) - 2y^2\right) y' - y^3 \csc(x) \sec(x) = 0$$

✗ Solution by Maple

```
dsolve((cot(x)-2*y(x)^2)*diff(y(x),x) = y(x)^3*csc(x)*sec(x),y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 5.38 (sec). Leaf size: 74

```
DSolve[(Cot[x]-2 y[x]^2)y'[x]==y[x]^3 Csc[x] Sec[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\sqrt{\cot(x)}\sqrt{W\left(-2e^{-8c_1}\tan(x)\right)}}{\sqrt{2}}$$
$$y(x) \to \frac{i\sqrt{\cot(x)}\sqrt{W\left(-2e^{-8c_1}\tan(x)\right)}}{\sqrt{2}}$$
$$y(x) \to 0$$

22.17 problem 625

Internal problem ID [3364]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 625.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, _Bernoulli]`

$$3y^2y' - 1 - x - ay^3 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 154

```
dsolve(3*y(x)^2*diff(y(x),x) = 1+x+a*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(\mathrm{e}^{ax} c_1 a^2 - ax - a - 1\right) a\right)^{\frac{1}{3}}}{a}$$

$$y(x) = -\frac{\left(\left(\mathrm{e}^{ax} c_1 a^2 - ax - a - 1\right) a\right)^{\frac{1}{3}}}{2a} - \frac{i\sqrt{3}\left(\left(\mathrm{e}^{ax} c_1 a^2 - ax - a - 1\right) a\right)^{\frac{1}{3}}}{2a}$$

$$y(x) = -\frac{\left(\left(\mathrm{e}^{ax} c_1 a^2 - ax - a - 1\right) a\right)^{\frac{1}{3}}}{2a} + \frac{i\sqrt{3}\left(\left(\mathrm{e}^{ax} c_1 a^2 - ax - a - 1\right) a\right)^{\frac{1}{3}}}{2a}$$

✓ Solution by Mathematica

Time used: 14.945 (sec). Leaf size: 111

```
DSolve[3 y[x]^2 y'[x]==1+x+a y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{\sqrt[3]{a^2 c_1 e^{ax} - a(x+1) - 1}}{a^{2/3}}$$

$$y(x) \rightarrow -\frac{\sqrt[3]{-1}\sqrt[3]{a^2 c_1 e^{ax} - a(x+1) - 1}}{a^{2/3}}$$

$$y(x) \rightarrow \frac{(-1)^{2/3}\sqrt[3]{a^2 c_1 e^{ax} - a(x+1) - 1}}{a^{2/3}}$$

22.18 problem 626

Internal problem ID [3365]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 626.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{(x^2 - 3y^2)\, y' + 1 + 2yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 392

```
dsolve((x^2-3*y(x)^2)*diff(y(x),x)+1+2*x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(108x+108c_1+12\sqrt{-12x^6+81c_1^2+162c_1x+81x^2}\right)^{\frac{1}{3}}}{6} + \frac{2x^2}{\left(108x+108c_1+12\sqrt{-12x^6+81c_1^2+162c_1x+81x^2}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(108x+108c_1+12\sqrt{-12x^6+81c_1^2+162c_1x+81x^2}\right)^{\frac{1}{3}}}{12} - \frac{x^2}{\left(108x+108c_1+12\sqrt{-12x^6+81c_1^2+162c_1x+81x^2}\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(108x+108c_1+12\sqrt{-12x^6+81c_1^2+162c_1x+81x^2}\right)^{\frac{1}{3}}}{6} - \frac{2x^2}{\left(108x+108c_1+12\sqrt{-12x^6+81c_1^2+162c_1x+81x^2}\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(108x+108c_1+12\sqrt{-12x^6+81c_1^2+162c_1x+81x^2}\right)^{\frac{1}{3}}}{12} - \frac{x^2}{\left(108x+108c_1+12\sqrt{-12x^6+81c_1^2+162c_1x+81x^2}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(108x+108c_1+12\sqrt{-12x^6+81c_1^2+162c_1x+81x^2}\right)^{\frac{1}{3}}}{6} - \frac{2x^2}{\left(108x+108c_1+12\sqrt{-12x^6+81c_1^2+162c_1x+81x^2}\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 4.669 (sec). Leaf size: 310

```
DSolve[(x^2-3 y[x]^2)y'[x]+1+2 x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{\sqrt{-108x^6+729(x-c_1)^2}-27x+27c_1}}{3\sqrt[3]{2}} - \frac{\sqrt[3]{2}x^2}{\sqrt[3]{\sqrt{-108x^6+729(x-c_1)^2}-27x+27c_1}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{\sqrt{-108x^6+729(x-c_1)^2}-27x+27c_1}}{6\sqrt[3]{2}} + \frac{\sqrt[3]{-\frac{2}{3}}x^2}{\sqrt[3]{\sqrt{3}\sqrt{-4x^6+27x^2-54c_1x+27c_1{}^2}-9x+9c_1}}$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{\sqrt{-108x^6+729(x-c_1)^2}-27x+27c_1}}{6\sqrt[3]{2}} + \frac{\left(1-i\sqrt{3}\right)x^2}{2^{2/3}\sqrt[3]{\sqrt{-108x^6+729(x-c_1)^2}-27x+27c_1}}$$

22.19 problem 627

Internal problem ID [3366]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 627.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{(2x^2 + 3y^2)\, y' + x(3x + y) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve((2*x^2+3*y(x)^2)*diff(y(x),x)+x*(3*x+y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \operatorname{RootOf}\left(\int^{\_Z} \frac{3\_a^2 + 2}{\_a^3 + \_a + 1} d\_a + 3\ln(x) + 3c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.138 (sec). Leaf size: 66

```
DSolve[(2 x^2+3 y[x]^2)y'[x]+x(3 x+y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\text{RootSum}\left[\#1^3 + \#1 + 1\&, \frac{3\#1^2 \log\left(\frac{y(x)}{x} - \#1\right) + 2\log\left(\frac{y(x)}{x} - \#1\right)}{3\#1^2 + 1}\&\right] = -3\log(x) + c_1, y(x)\right]$$

22.20 problem 628

Internal problem ID [3367]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 628.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{3(x^2 - y^2)\, y' + 3\,\mathrm{e}^x + 6(x+1)\, xy - 2y^3 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 622

```
dsolve(3*(x^2-y(x)^2)*diff(y(x),x)+3*exp(x)+6*x*y(x)*(1+x)-2*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x) = \frac{e^{-2x}\left(\left(4\,e^{3x}+4c_1+4\sqrt{-4x^6e^{4x}+e^{6x}+2\,e^{3x}c_1+c_1^2}\right)e^{4x}\right)^{\frac{1}{3}}}{2} + \frac{2x^2e^{2x}}{\left(\left(4\,e^{3x}+4c_1+4\sqrt{-4x^6e^{4x}+e^{6x}+2\,e^{3x}c_1+c_1^2}\right)e^{4x}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{e^{-2x}\left(\left(4\,e^{3x}+4c_1+4\sqrt{-4x^6e^{4x}+e^{6x}+2\,e^{3x}c_1+c_1^2}\right)e^{4x}\right)^{\frac{1}{3}}}{4} - \frac{x^2e^{2x}}{\left(\left(4\,e^{3x}+4c_1+4\sqrt{-4x^6e^{4x}+e^{6x}+2\,e^{3x}c_1+c_1^2}\right)e^{4x}\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{e^{-2x}\left(\left(4\,e^{3x}+4c_1+4\sqrt{-4x^6e^{4x}+e^{6x}+2\,e^{3x}c_1+c_1^2}\right)e^{4x}\right)^{\frac{1}{3}}}{2} - \frac{2x^2e^{2x}}{\left(\left(4\,e^{3x}+4c_1+4\sqrt{-4x^6e^{4x}+e^{6x}+2\,e^{3x}c_1+c_1^2}\right)e^{4x}\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{e^{-2x}\left(\left(4\,e^{3x}+4c_1+4\sqrt{-4x^6e^{4x}+e^{6x}+2\,e^{3x}c_1+c_1^2}\right)e^{4x}\right)^{\frac{1}{3}}}{4} - \frac{x^2e^{2x}}{\left(\left(4\,e^{3x}+4c_1+4\sqrt{-4x^6e^{4x}+e^{6x}+2\,e^{3x}c_1+c_1^2}\right)e^{4x}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{e^{-2x}\left(\left(4\,e^{3x}+4c_1+4\sqrt{-4x^6e^{4x}+e^{6x}+2\,e^{3x}c_1+c_1^2}\right)e^{4x}\right)^{\frac{1}{3}}}{2} - \frac{2x^2e^{2x}}{\left(\left(4\,e^{3x}+4c_1+4\sqrt{-4x^6e^{4x}+e^{6x}+2\,e^{3x}c_1+c_1^2}\right)e^{4x}\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 60.277 (sec). Leaf size: 497

```
DSolve[3(x^2-y[x]^2)y'[x]+3 Exp[x]+6 x y[x](1+x)-2 y[x]^3==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to -\frac{e^{-2x}\sqrt[3]{\sqrt{e^{8x}\left(-4e^{4x}x^6+e^{6x}-2c_1e^{3x}+c_1{}^2\right)}-e^{7x}+c_1e^{4x}}}{\sqrt[3]{2}} - \frac{\sqrt[3]{2}e^{2x}x^2}{\sqrt[3]{\sqrt{e^{8x}\left(-4e^{4x}x^6+e^{6x}-2c_1e^{3x}+c_1{}^2\right)}-e^{7x}+c_1e^{4x}}}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)e^{-2x}\sqrt[3]{\sqrt{e^{8x}\left(-4e^{4x}x^6+e^{6x}-2c_1e^{3x}+c_1{}^2\right)}-e^{7x}+c_1e^{4x}}}{2\sqrt[3]{2}} + \frac{\left(1+i\sqrt{3}\right)e^{2x}x^2}{2^{2/3}\sqrt[3]{\sqrt{e^{8x}\left(-4e^{4x}x^6+e^{6x}-2c_1e^{3x}+c_1{}^2\right)}-e^{7x}+c_1e^{4x}}}$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)e^{-2x}\sqrt[3]{\sqrt{e^{8x}\left(-4e^{4x}x^6+e^{6x}-2c_1e^{3x}+c_1{}^2\right)}-e^{7x}+c_1e^{4x}}}{2\sqrt[3]{2}} + \frac{\left(1-i\sqrt{3}\right)e^{2x}x^2}{2^{2/3}\sqrt[3]{\sqrt{e^{8x}\left(-4e^{4x}x^6+e^{6x}-2c_1e^{3x}+c_1{}^2\right)}-e^{7x}+c_1e^{4x}}}$$

22.21 problem 629

Internal problem ID [3368]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 629.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _dAlembert]`

$$\left(3x^2+2yx+4y^2\right)y'+2x^2+6yx+y^2=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 431

```
dsolve((3*x^2+2*x*y(x)+4*y(x)^2)*diff(y(x),x)+2*x^2+6*x*y(x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{\frac{\left(x^3c_1^3+8+2\sqrt{333c_1^6x^6+4x^3c_1^3+16}\right)^{\frac{1}{3}}}{4}-\frac{11x^2c_1^2}{4\left(x^3c_1^3+8+2\sqrt{333c_1^6x^6+4x^3c_1^3+16}\right)^{\frac{1}{3}}}-\frac{c_1x}{4}}{c_1}$$

$$y(x)=\frac{-\frac{\left(x^3c_1^3+8+2\sqrt{333c_1^6x^6+4x^3c_1^3+16}\right)^{\frac{1}{3}}}{8}+\frac{11x^2c_1^2}{8\left(x^3c_1^3+8+2\sqrt{333c_1^6x^6+4x^3c_1^3+16}\right)^{\frac{1}{3}}}-\frac{c_1x}{4}-\frac{i\sqrt{3}\left(\frac{\left(x^3c_1^3+8+2\sqrt{333c_1^6x^6+4x^3c_1^3+16}\right)^{\frac{1}{3}}}{4}+\cdots\right.}{2}\cdots}{c_1}$$

$$y(x)=\frac{-\frac{\left(x^3c_1^3+8+2\sqrt{333c_1^6x^6+4x^3c_1^3+16}\right)^{\frac{1}{3}}}{8}+\frac{11x^2c_1^2}{8\left(x^3c_1^3+8+2\sqrt{333c_1^6x^6+4x^3c_1^3+16}\right)^{\frac{1}{3}}}-\frac{c_1x}{4}+\frac{i\sqrt{3}\left(\frac{\left(x^3c_1^3+8+2\sqrt{333c_1^6x^6+4x^3c_1^3+16}\right)^{\frac{1}{3}}}{4}+\cdots\right.}{2}\cdots}{c_1}$$

✓ Solution by Mathematica

Time used: 53.355 (sec). Leaf size: 611

```
DSolve[(3 x^2+2 x y[x]+4 y[x]^2)y'[x]+2 x^2+6 x y[x]+y[x]^2==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{1}{4}\left(\sqrt[3]{x^3+2\sqrt{333x^6+4e^{3c_1}x^3+16e^{6c_1}}+8e^{3c_1}} - \frac{11x^2}{\sqrt[3]{x^3+2\sqrt{333x^6+4e^{3c_1}x^3+16e^{6c_1}}+8e^{3c_1}}} - x\right)$$

$$y(x) \to \frac{1}{16}\left(2i\left(\sqrt{3}+i\right)\sqrt[3]{x^3+2\sqrt{333x^6+4e^{3c_1}x^3+16e^{6c_1}}+8e^{3c_1}} + \frac{22(1+i\sqrt{3})x^2}{\sqrt[3]{x^3+2\sqrt{333x^6+4e^{3c_1}x^3+16e^{6c_1}}+8e^{3c_1}}} - 4x\right)$$

$$y(x) \to \frac{1}{16}\left(\left(-2-2i\sqrt{3}\right)\sqrt[3]{x^3+2\sqrt{333x^6+4e^{3c_1}x^3+16e^{6c_1}}+8e^{3c_1}} + \frac{22(1-i\sqrt{3})x^2}{\sqrt[3]{x^3+2\sqrt{333x^6+4e^{3c_1}x^3+16e^{6c_1}}+8e^{3c_1}}} - 4x\right)$$

$$y(x) \to \frac{1}{4}\left(\sqrt[3]{6\sqrt{37}\sqrt{x^6}+x^3} - \frac{11x^2}{\sqrt[3]{6\sqrt{37}\sqrt{x^6}+x^3}} - x\right)$$

$$y(x) \to \frac{1}{8}\left(\left(-1-i\sqrt{3}\right)\sqrt[3]{6\sqrt{37}\sqrt{x^6}+x^3} + \frac{11(1-i\sqrt{3})x^2}{\sqrt[3]{6\sqrt{37}\sqrt{x^6}+x^3}} - 2x\right)$$

$$y(x) \to \frac{1}{8}\left(i\left(\sqrt{3}+i\right)\sqrt[3]{6\sqrt{37}\sqrt{x^6}+x^3} + \frac{11(1+i\sqrt{3})x^2}{\sqrt[3]{6\sqrt{37}\sqrt{x^6}+x^3}} - 2x\right)$$

22.22 problem 630

Internal problem ID [3369]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 22
Problem number: 630.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational]`

$$(1-3x+2y)^2 y' - (4+2x-3y)^2 = 0$$

✓ Solution by Maple

Time used: 1.453 (sec). Leaf size: 309

```
dsolve((1-3*x+2*y(x))^2*diff(y(x),x) = (4+2*x-3*y(x))^2,y(x), singsol=all)
```

$$y(x) = \frac{14}{5} + \frac{(-11+5x)\left(\mathrm{RootOf}\left(59049(-11+5x)^9 c_1\_Z^{90} + \left(-295245(-11+5x)^9 c_1+1\right)\_Z^{81} + 459270(-11\right.\right.}{5\,\mathrm{RootOf}\left(59049\left(-11+5x\right)^9 c_1\_Z^{90} + \left(-295245\left(-11+5x\right)^9 c_1+1\right)\_Z^{81} + 459270\left(-11+5x\right.\right.}$$

✓ Solution by Mathematica

Time used: 60.2 (sec). Leaf size: 3501

```
DSolve[(1-3 x+2 y[x])^2 y'[x]==(4+2 x-3 y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

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23.1 problem 631

Internal problem ID [3370]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 631.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{(1 - 3x^2 y + 6y^2)\, y' + x^2 - 3x y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 797

```
dsolve((1-3*x^2*y(x)+6*y(x)^2)*diff(y(x),x)+x^2-3*x*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)$$

$$= \frac{\left(-108x^2 - 144x^3 - 432c_1 + 27x^6 + 12\sqrt{-54x^9 - 162c_1x^6 + 144x^6 + 216x^5 + 864c_1x^3 - 27x^4 + 648c_1x^2}\right.}{12}$$

$$- \frac{12\left(\frac{1}{6} - \frac{x^4}{16}\right)}{\left(-108x^2 - 144x^3 - 432c_1 + 27x^6 + 12\sqrt{-54x^9 - 162c_1x^6 + 144x^6 + 216x^5 + 864c_1x^3 - 27x^4 + 648c_1}\right.}$$

$$+ \frac{x^2}{4}$$

$$y(x) =$$

$$- \frac{\left(-108x^2 - 144x^3 - 432c_1 + 27x^6 + 12\sqrt{-54x^9 - 162c_1x^6 + 144x^6 + 216x^5 + 864c_1x^3 - 27x^4 + 648c_1}\right.}{24}$$

$$+ \frac{1 - \frac{3x^4}{8}}{\left(-108x^2 - 144x^3 - 432c_1 + 27x^6 + 12\sqrt{-54x^9 - 162c_1x^6 + 144x^6 + 216x^5 + 864c_1x^3 - 27x^4 + 648c_1}\right.}$$

$$+ \frac{x^2}{4}$$

$$- \frac{i\sqrt{3}\left(\frac{\left(-108x^2-144x^3-432c_1+27x^6+12\sqrt{-54x^9-162c_1x^6+144x^6+216x^5+864c_1x^3-27x^4+648c_1x^2+1296c_1^2+96}\right)^{\frac{1}{3}}}{12} + \frac{}{\left(-108x^2-14\right.}\right.}{2}$$

$$y(x) =$$

$$- \frac{\left(-108x^2 - 144x^3 - 432c_1 + 27x^6 + 12\sqrt{-54x^9 - 162c_1x^6 + 144x^6 + 216x^5 + 864c_1x^3 - 27x^4 + 648c_1}\right.}{24}$$

$$+ \frac{1 - \frac{3x^4}{8}}{\left(-108x^2 - 144x^3 - 432c_1 + 27x^6 + 12\sqrt{-54x^9 - 162c_1x^6 + 144x^6 + 216x^5 + 864c_1x^3 - 27x^4 + 648c_1}\right.}$$

$$+ \frac{x^2}{4}$$

$$+ \frac{i\sqrt{3}\left(\frac{\left(-108x^2-144x^3-432c_1+27x^6+12\sqrt{-54x^9-162c_1x^6+144x^6+216x^5+864c_1x^3-27x^4+648c_1x^2+1296c_1^2+96}\right)^{\frac{1}{3}}}{12} + \frac{}{\left(-108x^2-14\right.}\right.}{2}$$

✓ Solution by Mathematica

Time used: 7.46 (sec). Leaf size: 570

```
DSolve[(1-3 x^2 y[x]+6 y[x]^2)y'[x]+x^2-3 x y[x]^2==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{x^2}{4} - \frac{\sqrt[3]{-\frac{27x^6}{4}+36x^3+27x^2+\sqrt{4\left(6-\frac{9x^4}{4}\right)^3+\left(\frac{9}{4}\left(-3x^4+16x+12\right)x^2+108c_1\right)^2}+108c_1}}{6\sqrt[3]{2}} + \frac{6-\frac{9x^4}{4}}{3\,2^{2/3}\sqrt[3]{-\frac{27x^6}{4}+36x^3+27x^2+\sqrt{4\left(6-\frac{9x^4}{4}\right)^3+\left(\frac{9}{4}\left(-3x^4+16x+12\right)x^2+108c_1\right)^2}+108c_1}}$$

$$y(x) \to \frac{x^2}{4} + \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{-\frac{27x^6}{4}+36x^3+27x^2+\sqrt{4\left(6-\frac{9x^4}{4}\right)^3+\left(\frac{9}{4}\left(-3x^4+16x+12\right)x^2+108c_1\right)^2}+108c_1}}{12\sqrt[3]{2}} - \frac{\left(1+i\sqrt{3}\right)\left(6-\frac{9x^4}{4}\right)}{6\,2^{2/3}\sqrt[3]{-\frac{27x^6}{4}+36x^3+27x^2+\sqrt{4\left(6-\frac{9x^4}{4}\right)^3+\left(\frac{9}{4}\left(-3x^4+16x+12\right)x^2+108c_1\right)^2}+108c_1}}$$

$$y(x) \to \frac{x^2}{4} + \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-\frac{27x^6}{4}+36x^3+27x^2+\sqrt{4\left(6-\frac{9x^4}{4}\right)^3+\left(\frac{9}{4}\left(-3x^4+16x+12\right)x^2+108c_1\right)^2}+108c_1}}{12\sqrt[3]{2}} - \frac{\left(1-i\sqrt{3}\right)\left(6-\frac{9x^4}{4}\right)}{6\,2^{2/3}\sqrt[3]{-\frac{27x^6}{4}+36x^3+27x^2+\sqrt{4\left(6-\frac{9x^4}{4}\right)^3+\left(\frac{9}{4}\left(-3x^4+16x+12\right)x^2+108c_1\right)^2}+108c_1}}$$

23.2 problem 632

Internal problem ID [3371]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 632.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x),G(x)]']]`

$$(x-6y)^2 y' + a + 2yx - 6y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 115

```
dsolve((x-6*y(x))^2*diff(y(x),x)+a+2*x*y(x)-6*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{(-x^3-18ax-18c_1)^{\frac{1}{3}}}{6} + \frac{x}{6}$$
$$y(x) = -\frac{(-x^3-18ax-18c_1)^{\frac{1}{3}}}{12} - \frac{i\sqrt{3}\,(-x^3-18ax-18c_1)^{\frac{1}{3}}}{12} + \frac{x}{6}$$
$$y(x) = -\frac{(-x^3-18ax-18c_1)^{\frac{1}{3}}}{12} + \frac{i\sqrt{3}\,(-x^3-18ax-18c_1)^{\frac{1}{3}}}{12} + \frac{x}{6}$$

✓ Solution by Mathematica

Time used: 0.668 (sec). Leaf size: 115

```
DSolve[(x-6 y[x])^2 y'[x]+a+2 x y[x]-6 y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(x + \sqrt[3]{-18ax - x^3 + 18c_1}\right)$$
$$y(x) \to \frac{x}{6} + \frac{1}{12} i\left(\sqrt{3} + i\right) \sqrt[3]{-18ax - x^3 + 18c_1}$$
$$y(x) \to \frac{x}{6} - \frac{1}{12}\left(1 + i\sqrt{3}\right) \sqrt[3]{-18ax - x^3 + 18c_1}$$

23.3 problem 633

Internal problem ID [3372]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 633.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(x^2 + a y^2)\, y' - yx = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve((x^2+a*y(x)^2)*diff(y(x),x) = x*y(x),y(x), singsol=all)
```

$$y(x) = \sqrt{\frac{1}{a \operatorname{LambertW}\left(\frac{x^2 c_1}{a}\right)}}\, x$$

✓ Solution by Mathematica

Time used: 15.932 (sec). Leaf size: 71

```
DSolve[(x^2+a y[x]^2)y'[x]==x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x}{\sqrt{a}\sqrt{W\left(\frac{x^2 e^{-\frac{2c_1}{a}}}{a}\right)}}$$

$$y(x) \to \frac{x}{\sqrt{a}\sqrt{W\left(\frac{x^2 e^{-\frac{2c_1}{a}}}{a}\right)}}$$

$$y(x) \to 0$$

23.4 problem 634

Internal problem ID [3373]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 634.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(x^2 + yx + ay^2)\, y' - x^2 a - yx - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 216

```
dsolve((x^2+x*y(x)+a*y(x)^2)*diff(y(x),x) = a*x^2+x*y(x)+y(x)^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{3c_1 a+\mathrm{RootOf}\left(\mathrm{e}^{\_Z}-\mathrm{e}^{-\frac{6c_1 a}{2+a}}\mathrm{e}^{-\frac{2a\_Z}{2+a}}x^{-\frac{6a}{2+a}}\mathrm{e}^{\frac{2\_Z}{2+a}}-3\,\mathrm{e}^{-\frac{3c_1 a}{2+a}}\mathrm{e}^{-\frac{a\_Z}{2+a}}x^{-\frac{3a}{2+a}}\mathrm{e}^{\frac{\_Z}{2+a}}-3\right)a+3a\ln(x)-\mathrm{RootOf}\left(\mathrm{e}^{\_Z}-\mathrm{e}^{-\frac{6c_1 a}{2+a}}\mathrm{e}^{-\frac{2a\_Z}{2+a}}x^{-\frac{6a}{2+a}}\mathrm{e}^{\frac{2\_Z}{2+a}}-3\,\mathrm{e}^{-}\right.}{2+a}} + x$$

✓ Solution by Mathematica

Time used: 0.194 (sec). Leaf size: 54

```
DSolve[(x^2+x y[x]+a y[x]^2)y'[x]==a x^2+x y[x]+y[x]^2,y[x],x,IncludeSingularSolutions -> Tru
```

$$\mathrm{Solve}\left[\frac{1}{3}(a-1)\log\left(\frac{y(x)^2}{x^2}+\frac{y(x)}{x}+1\right)+\frac{1}{3}(a+2)\log\left(1-\frac{y(x)}{x}\right) = -a\log(x)+c_1, y(x)\right]$$

23.5 problem 635

Internal problem ID [3374]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 635.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$(x^2a + 2yx - ay^2)\, y' + x^2 - 2yax - y^2 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 59

```
dsolve((a*x^2+2*x*y(x)-a*y(x)^2)*diff(y(x),x)+x^2-2*a*x*y(x)-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{-a + \sqrt{-4c_1^2x^2 + a^2 - 4c_1x}}{2c_1}$$
$$y(x) = \frac{a + \sqrt{-4c_1^2x^2 + a^2 - 4c_1x}}{2c_1}$$

✓ Solution by Mathematica

Time used: 4.03 (sec). Leaf size: 87

```
DSolve[(a x^2+2 x y[x]-a y[x]^2)y'[x]+x^2-2 a x y[x]-y[x]^2==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{1}{2}\left(a(-e^{c_1}) - \sqrt{a^2e^{2c_1} + 4x\,(-x + e^{c_1})}\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{a^2e^{2c_1} + 4x\,(-x + e^{c_1})} - ae^{c_1}\right)$$

23.6 problem 637

Internal problem ID [3375]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 637.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{\left(x^2 a + 2bxy + cy^2\right) y' + k\,x^2 + 2yax + by^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 1666

```
dsolve((a*x^2+2*b*x*y(x)+c*y(x)^2)*diff(y(x),x)+k*x^2+2*a*x*y(x)+b*y(x)^2 = 0,y(x), singsol=a
```

$$y(x) = \frac{\frac{\left(12a\,x^3c_1^3bc-8x^3b^3c_1^3-4c_1^3c^2k\,x^3+4\sqrt{4a^3c\,c_1^6x^6-3a^2b^2c_1^6x^6-6abc\,c_1^6k\,x^6+4b^3c_1^6k\,x^6+c^2c_1^6k^2x^6+6a\,x^3c_1^3bc-4x^3b^3c_1^3-2c_1^3c^2k\,x^3+c^2}\,c+4c^2\right)^{\frac{1}{3}}}{2c}}{\quad}$$

$$y(x) = \frac{-\frac{\left(12a\,x^3c_1^3bc-8x^3b^3c_1^3-4c_1^3c^2k\,x^3+4\sqrt{4a^3c\,c_1^6x^6-3a^2b^2c_1^6x^6-6abc\,c_1^6k\,x^6+4b^3c_1^6k\,x^6+c^2c_1^6k^2x^6+6a\,x^3c_1^3bc-4x^3b^3c_1^3-2c_1^3c^2k\,x^3+c^2}\,c+4c^2\right)}{4c}}{\quad}$$

$$y(x) = \frac{-\frac{\left(12a\,x^3c_1^3bc-8x^3b^3c_1^3-4c_1^3c^2k\,x^3+4\sqrt{4a^3c\,c_1^6x^6-3a^2b^2c_1^6x^6-6abc\,c_1^6k\,x^6+4b^3c_1^6k\,x^6+c^2c_1^6k^2x^6+6a\,x^3c_1^3bc-4x^3b^3c_1^3-2c_1^3c^2k\,x^3+c^2}\,c+4c^2\right)}{4c}}{\quad}$$

✓ Solution by Mathematica

Time used: 60.36 (sec). Leaf size: 703

```
DSolve[(a x^2+2 b x y[x]+c y[x]^2)y'[x]+k x^2+2 a x y[x]+b y[x]^2==0,y[x],x,IncludeSingularSo
```

$y(x)$

$$\rightarrow \frac{2^{2/3}\sqrt[3]{\sqrt{-4x^6\left(b^2-ac\right)^3+\left(-x^3\left(-3abc+2b^3+c^2k\right)+c^2e^{3c_1}\right)^2}+3abcx^3-2b^3x^3+c^2\left(-kx^3+e^{3c_1}\right)+}}{2}$$

$y(x)$

$$\rightarrow \frac{9i2^{2/3}\left(\sqrt{3}+i\right)\sqrt[3]{\sqrt{-4x^6\left(b^2-ac\right)^3+\left(-x^3\left(-3abc+2b^3+c^2k\right)+c^2e^{3c_1}\right)^2}+3abcx^3-2b^3x^3+c^2\left(-kx^3\right.}}{}$$

$y(x)$

$$\rightarrow \frac{-9\ 2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{\sqrt{-4x^6\left(b^2-ac\right)^3+\left(-x^3\left(-3abc+2b^3+c^2k\right)+c^2e^{3c_1}\right)^2}+3abcx^3-2b^3x^3+c^2\left(-\right.}}{}$$

23.7 problem 638

Internal problem ID [3376]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 638.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(1-y^2)\,y' - y(x^2+1) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*(1-y(x)^2)*diff(y(x),x) = (x^2+1)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-\frac{1}{\operatorname{LambertW}\left(-e^{x^2} c_1 x^2\right)}}}$$

✓ Solution by Mathematica

Time used: 4.881 (sec). Leaf size: 62

```
DSolve[x(1-y[x]^2)y'[x]==(1+x^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i\sqrt{W\left(x^2\left(-e^{x^2-2c_1}\right)\right)}$$

$$y(x) \to i\sqrt{W\left(x^2\left(-e^{x^2-2c_1}\right)\right)}$$

$$y(x) \to 0$$

23.8 problem 639

Internal problem ID [3377]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 639.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{x\left(3x-y^{2}\right) y'+\left(5x-2y^{2}\right) y=0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 36

```
dsolve(x*(3*x-y(x)^2)*diff(y(x),x)+(5*x-2*y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$\ln \left(x \right)-c_{1}+\frac{6 \ln \left(\frac{y \left(x \right)}{\sqrt{x}}\right)}{13}-\frac{2 \ln \left(-\frac{-5 y \left(x \right)^{2}+13 x}{x}\right)}{65}=0$$

✓ Solution by Mathematica

Time used: 6.86 (sec). Leaf size: 661

```
DSolve[x(3 x-y[x]^2)y'[x]+(5 x-2 y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 1\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 2\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 3\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 4\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 5\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 6\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 7\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 8\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 9\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 10\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 11\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 12\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 13\right]$$

$$y(x) \to \text{Root}\left[-\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}}\&, 14\right]$$

23.9 problem 640

Internal problem ID [3378]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 640.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class D‘], _rational]`

$$x(x^2+y^2)\,y'-(x^2+x^4+y^2)\,y=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 26

```
dsolve(x*(x^2+y(x)^2)*diff(y(x),x) = (x^2+x^4+y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{-\frac{\mathrm{LambertW}\left(\mathrm{e}^{x^2}\mathrm{e}^{2c_1}\right)}{2}+\frac{x^2}{2}+c_1}x$$

✓ Solution by Mathematica

Time used: 5.008 (sec). Leaf size: 49

```
DSolve[x(x^2+y[x]^2)y'[x]==(x^2+x^4+y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -x\sqrt{W\left(e^{x^2+2c_1}\right)}$$
$$y(x)\to x\sqrt{W\left(e^{x^2+2c_1}\right)}$$
$$y(x)\to 0$$

23.10 problem 641

Internal problem ID [3379]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 641.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{x\left(1-x^{2}+y^{2}\right) y'+\left(1+x^{2}-y^{2}\right) y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 114

```
dsolve(x*(1-x^2+y(x)^2)*diff(y(x),x)+(1+x^2-y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$\frac{1}{\frac{1}{y(x)^{2}}-\frac{1}{x^{2}-1}}=-\frac{\sqrt{x-1}\, x\sqrt{x+1}}{\sqrt{c_{1}-\frac{2}{x+1}+\frac{2}{x-1}}}-\frac{(x-1)(x+1)}{2}$$
$$\frac{1}{\frac{1}{y(x)^{2}}-\frac{1}{x^{2}-1}}=\frac{\sqrt{x-1}\, x\sqrt{x+1}}{\sqrt{c_{1}-\frac{2}{x+1}+\frac{2}{x-1}}}-\frac{(x-1)(x+1)}{2}$$

✓ Solution by Mathematica

Time used: 1.391 (sec). Leaf size: 106

```
DSolve[x(1-x^2+y[x]^2)y'[x]+(1+x^2-y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\rightarrow \frac{-\sqrt{x^{2}-4c_{1}x^{2}+4c_{1}{}^{2}}+x-2c_{1}x}{2c_{1}}$$
$$y(x)\rightarrow \frac{\sqrt{x^{2}-4c_{1}x^{2}+4c_{1}{}^{2}}+x-2c_{1}x}{2c_{1}}$$
$$y(x)\rightarrow \text{Indeterminate}$$
$$y(x)\rightarrow -x-1$$
$$y(x)\rightarrow 1-x$$

23.11 problem 642

Internal problem ID [3380]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 642.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$x(a-x^2-y^2)\,y'+(a+x^2+y^2)\,y=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 112

```
dsolve(x*(a-x^2-y(x)^2)*diff(y(x),x)+(a+x^2+y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$\frac{1}{\frac{1}{y(x)^2}-\frac{1}{-x^2+a}}=-\frac{x\sqrt{x^2-a}}{\sqrt{c_1+\frac{4a}{x^2-a}}}+\frac{x^2}{2}-\frac{a}{2}$$

$$\frac{1}{\frac{1}{y(x)^2}-\frac{1}{-x^2+a}}=\frac{x\sqrt{x^2-a}}{\sqrt{c_1+\frac{4a}{x^2-a}}}+\frac{x^2}{2}-\frac{a}{2}$$

✓ Solution by Mathematica

Time used: 0.912 (sec). Leaf size: 65

```
DSolve[x(a-x^2-y[x]^2)y'[x]+(a+x^2+y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\left(c_1x-\sqrt{-4a+(4+c_1{}^2)\,x^2}\right)$$

$$y(x)\to\frac{1}{2}\left(\sqrt{-4a+(4+c_1{}^2)\,x^2}+c_1x\right)$$

23.12 problem 643

Internal problem ID [3381]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 643.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$x(2x^2 + y^2)\, y' - (2x^2 + 3y^2)\, y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 25

```
dsolve(x*(2*x^2+y(x)^2)*diff(y(x),x) = (2*x^2+3*y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\frac{\mathrm{LambertW}\left(\frac{2\,\mathrm{e}^{-4c_1}}{x^4}\right)}{2}+2c_1} x^3$$

✓ Solution by Mathematica

Time used: 8.72 (sec). Leaf size: 61

```
DSolve[x(2 x^2+y[x]^2)y'[x]==(2 x^2+3 y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{2}x}{\sqrt{W\left(\frac{2e^{-2c_1}}{x^4}\right)}}$$

$$y(x) \to \frac{\sqrt{2}x}{\sqrt{W\left(\frac{2e^{-2c_1}}{x^4}\right)}}$$

$$y(x) \to 0$$

23.13 problem 644

Internal problem ID [3382]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 644.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$
(x(a-x^2-y^2)+y)\, y'+x-(a-x^2-y^2)\, y=0
$$

✓ Solution by Maple

Time used: 0.204 (sec). Leaf size: 34

```
dsolve((x*(a-x^2-y(x)^2)+y(x))*diff(y(x),x)+x-(a-x^2-y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$
y(x)=\tan\left(\operatorname{RootOf}\left(2a\_Z+\ln\left(-\frac{x^2}{a\cos(\_Z)^2-x^2}\right)+c_1\right)\right)x
$$

✓ Solution by Mathematica

Time used: 0.169 (sec). Leaf size: 47

```
DSolve[(x(a-x^2-y[x]^2)+y[x])y'[x]+x-(a-x^2-y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$
\text{Solve}\left[\frac{-2a\arctan\left(\frac{y(x)}{x}\right)+\log\left(-a+x^2+y(x)^2\right)-\log\left(x^2+y(x)^2\right)}{2a}=c_1,y(x)\right]
$$

23.14 problem 645

Internal problem ID [3383]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 645.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x(a+y)^2 y' - b y^2 = 0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 33

```
dsolve(x*(a+y(x))^2*diff(y(x),x) = b*y(x)^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(\ln(x) b\,\mathrm{e}^{\_Z}+c_1 b\,\mathrm{e}^{\_Z}-2\_Z a\,\mathrm{e}^{\_Z}-\mathrm{e}^{2\_Z}+a^2\right)}$$

✓ Solution by Mathematica

Time used: 0.41 (sec). Leaf size: 37

```
DSolve[x(a+y[x])^2 y'[x]==b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{a^2}{\#1} + 2a\log(\#1) + \#1\&\right][b\log(x) + c_1]$$

$$y(x) \to 0$$

23.15 problem 646

Internal problem ID [3384]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 646.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$x(x^2 - yx + y^2)\, y' + (x^2 + yx + y^2)\, y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(x*(x^2-x*y(x)+y(x)^2)*diff(y(x),x)+(x^2+x*y(x)+y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \tan(\text{RootOf}(\ln(\tan(\_Z)) - \_Z + 2\ln(x) + 2c_1))\, x$$

✓ Solution by Mathematica

Time used: 0.135 (sec). Leaf size: 28

```
DSolve[x(x^2-x y[x]+y[x]^2)y'[x]+(x^2+x y[x]+y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$\text{Solve}\left[\log\left(\frac{y(x)}{x}\right) - \arctan\left(\frac{y(x)}{x}\right) = -2\log(x) + c_1, y(x)\right]$$

23.16 problem 647

Internal problem ID [3385]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 647.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$x(x^2 - yx - y^2)\, y' - (x^2 + yx - y^2)\, y = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 29

```
dsolve(x*(x^2-x*y(x)-y(x)^2)*diff(y(x),x) = (x^2+x*y(x)-y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(2\,\mathrm{e}^{\_Z} \ln(x)+\mathrm{e}^{2\_Z}+2\,\mathrm{e}^{\_Z} c_1+\_Z\,\mathrm{e}^{\_Z}+1\right)} x$$

✓ Solution by Mathematica

Time used: 0.175 (sec). Leaf size: 31

```
DSolve[x(x^2-x y[x]-y[x]^2)y'[x]==(x^2+x y[x]-y[x]^2)y[x],y[x],x,IncludeSingularSolutions ->
```

$$\text{Solve}\left[\frac{x}{y(x)} + \frac{y(x)}{x} + \log\left(\frac{y(x)}{x}\right) = -2\log(x) + c_1, y(x)\right]$$

23.17 problem 648

Internal problem ID [3386]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 648.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{x(x^2+yax+y^2)\,y'-(x^2+bxy+y^2)\,y=0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 43

```
dsolve(x*(x^2+a*x*y(x)+y(x)^2)*diff(y(x),x) = (x^2+b*x*y(x)+y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{\mathrm{RootOf}\left(\mathrm{e}^{\_Z}a\ln(x)-\ln(x)b\,\mathrm{e}^{\_Z}+\mathrm{e}^{\_Z}c_1a-c_1b\,\mathrm{e}^{\_Z}+\_Za\,\mathrm{e}^{\_Z}+\mathrm{e}^{2\_Z}-1\right)}x$$

✓ Solution by Mathematica

Time used: 0.249 (sec). Leaf size: 38

```
DSolve[x(x^2+a x y[x]+y[x]^2)y'[x]==(x^2+b x y[x]+y[x]^2)y[x],y[x],x,IncludeSingularSolutions
```

$$\mathrm{Solve}\left[a\log\left(\frac{y(x)}{x}\right)-\frac{x}{y(x)}+\frac{y(x)}{x}=(b-a)\log(x)+c_1,y(x)\right]$$

23.18 problem 649

Internal problem ID [3387]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 649.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{x\left(x^{2}-2y^{2}\right) y'-\left(2x^{2}-y^{2}\right) y=0}$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 807

```
dsolve(x*(x^2-2*y(x)^2)*diff(y(x),x) = (2*x^2-y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{216x^2c_1}{\left(6\left(8-108c_1^2x^2+12\sqrt{81c_1^4x^4-12c_1^2x^2}\right)^{\frac{1}{3}}+\frac{24}{\left(8-108c_1^2x^2+12\sqrt{81c_1^4x^4-12c_1^2x^2}\right)^{\frac{1}{3}}}+12\right)^{\frac{3}{2}}}$$

$$y(x) = \frac{216x^2c_1}{\left(6\left(8-108c_1^2x^2+12\sqrt{81c_1^4x^4-12c_1^2x^2}\right)^{\frac{1}{3}}+\frac{24}{\left(8-108c_1^2x^2+12\sqrt{81c_1^4x^4-12c_1^2x^2}\right)^{\frac{1}{3}}}+12\right)^{\frac{3}{2}}}$$

$$y(x) = -\frac{216x^2c_1}{\left(-3\left(8-108c_1^2x^2+12\sqrt{81c_1^4x^4-12c_1^2x^2}\right)^{\frac{1}{3}}-\frac{12}{\left(8-108c_1^2x^2+12\sqrt{81c_1^4x^4-12c_1^2x^2}\right)^{\frac{1}{3}}}+12-18i\sqrt{3}\left(\frac{\left(8-108c_1^2 \cdots\right.}{\cdots}\right.\right.}$$

$$y(x) = \frac{216x^2c_1}{\left(-3\left(8-108c_1^2x^2+12\sqrt{81c_1^4x^4-12c_1^2x^2}\right)^{\frac{1}{3}}-\frac{12}{\left(8-108c_1^2x^2+12\sqrt{81c_1^4x^4-12c_1^2x^2}\right)^{\frac{1}{3}}}+12-18i\sqrt{3}\left(\frac{\left(8-108c_1^2x^2+ \cdots\right.}{\cdots}\right.\right.}$$

$$y(x) = -\frac{216x^2c_1}{\left(-3\left(8-108c_1^2x^2+12\sqrt{81c_1^4x^4-12c_1^2x^2}\right)^{\frac{1}{3}}-\frac{12}{\left(8-108c_1^2x^2+12\sqrt{81c_1^4x^4-12c_1^2x^2}\right)^{\frac{1}{3}}}+12+18i\sqrt{3}\left(\frac{\left(8-108c_1^2 \cdots\right.}{\cdots}\right.\right.}$$

$$y(x) = \frac{216x^2c_1}{\left(-3\left(8-108c_1^2x^2+12\sqrt{81c_1^4x^4-12c_1^2x^2}\right)^{\frac{1}{3}}-\frac{12}{\left(8-108c_1^2x^2+12\sqrt{81c_1^4x^4-12c_1^2x^2}\right)^{\frac{1}{3}}}+12+18i\sqrt{3}\left(\frac{\left(8-108c_1^2x^2+ \cdots\right.}{\cdots}\right.\right.}$$

✓ Solution by Mathematica

Time used: 60.282 (sec). Leaf size: 831

```
DSolve[x(x^2-2 y[x]^2)y'[x]==(2 x^2-y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 + \frac{\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9e^{2c_1}x^4}}{\sqrt[3]{2}3^{2/3}} + \frac{\sqrt[3]{\frac{2}{3}}e^{2c_1}x^2}{\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9e^{2c_1}x^4}}}$$

$$y(x) \to \sqrt{-x^2 + \frac{\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9e^{2c_1}x^4}}{\sqrt[3]{2}3^{2/3}} + \frac{\sqrt[3]{\frac{2}{3}}e^{2c_1}x^2}{\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9e^{2c_1}x^4}}}$$

$$y(x) \to -\frac{1}{2}\sqrt{\left(\frac{2}{3}\right)^{2/3}\left(-1 - i\sqrt{3}\right)\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9e^{2c_1}x^4} + \frac{4}{3}x^2\left(-3 + \frac{(-3)^{2/3}\sqrt[3]{2}e^{2c_1}}{\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9}}\right.}$$

$$y(x) \to \frac{1}{2}\sqrt{\left(\frac{2}{3}\right)^{2/3}\left(-1 - i\sqrt{3}\right)\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9e^{2c_1}x^4} + \frac{4}{3}x^2\left(-3 + \frac{(-3)^{2/3}\sqrt[3]{2}e^{2c_1}}{\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9e}}\right.}$$

$$y(x) \to -\frac{1}{2}\sqrt{-4x^2 + i\left(\frac{2}{3}\right)^{2/3}\left(\sqrt{3} + i\right)\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9e^{2c_1}x^4} - \frac{4\sqrt[3]{-\frac{2}{3}}e^{2c_1}x^2}{\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9e^{2c_1}x}}}$$

$$y(x) \to \frac{1}{2}\sqrt{-4x^2 + i\left(\frac{2}{3}\right)^{2/3}\left(\sqrt{3} + i\right)\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9e^{2c_1}x^4} - \frac{4\sqrt[3]{-\frac{2}{3}}e^{2c_1}x^2}{\sqrt[3]{\sqrt{81e^{4c_1}x^8 - 12e^{6c_1}x^6} - 9e^{2c_1}x^4}}}$$

23.19 problem 650

Internal problem ID [3388]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 650.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{x(x^2+2y^2)\,y'-(2x^2+3y^2)\,y=0}$$

✓ Solution by Maple

Time used: 0.204 (sec). Leaf size: 89

```
dsolve(x*(x^2+2*y(x)^2)*diff(y(x),x) = (2*x^2+3*y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x)=-\frac{\sqrt{-2-2\sqrt{4c_1x^2+1}}\,x}{2}$$
$$y(x)=\frac{\sqrt{-2-2\sqrt{4c_1x^2+1}}\,x}{2}$$
$$y(x)=-\frac{\sqrt{-2+2\sqrt{4c_1x^2+1}}\,x}{2}$$
$$y(x)=\frac{\sqrt{-2+2\sqrt{4c_1x^2+1}}\,x}{2}$$

✓ Solution by Mathematica

Time used: 41.582 (sec). Leaf size: 277

```
DSolve[x(x^2+2 y[x]^2)y'[x]==(2 x^2+3 y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-x^2 - \sqrt{x^4 + 4e^{2c_1}x^6}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-x^2 - \sqrt{x^4 + 4e^{2c_1}x^6}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{-x^2 + \sqrt{x^4 + 4e^{2c_1}x^6}}}{\sqrt{2}}$$

$$y(x) \to \sqrt{-\frac{x^2}{2} + \frac{1}{2}\sqrt{x^4 + 4e^{2c_1}x^6}}$$

$$y(x) \to -\frac{\sqrt{-\sqrt{x^4} - x^2}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-\sqrt{x^4} - x^2}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\sqrt{x^4} - x^2}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\sqrt{x^4} - x^2}}{\sqrt{2}}$$

23.20 problem 651

Internal problem ID [3389]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 651.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$2x(5x^2+y^2)\,y'-x^2y+y^3=0$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 29

```
dsolve(2*x*(5*x^2+y(x)^2)*diff(y(x),x) = x^2*y(x)-y(x)^3,y(x), singsol=all)
```

$$y(x)=\operatorname{RootOf}\left(\_Z^{45}c_1x^9-\_Z^{18}-6\_Z^9-9\right)^{\frac{9}{2}}x$$

✓ Solution by Mathematica

Time used: 2.726 (sec). Leaf size: 216

```
DSolve[2 x(5 x^2+y[x]^2)y'[x]==x^2 y[x]-y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\operatorname{Root}\left[-\#1^5+\frac{\#1^2e^{3c_1}}{x^{3/2}}+3e^{3c_1}\sqrt{x}\&,1\right]$$

$$y(x)\to\operatorname{Root}\left[-\#1^5+\frac{\#1^2e^{3c_1}}{x^{3/2}}+3e^{3c_1}\sqrt{x}\&,2\right]$$

$$y(x)\to\operatorname{Root}\left[-\#1^5+\frac{\#1^2e^{3c_1}}{x^{3/2}}+3e^{3c_1}\sqrt{x}\&,3\right]$$

$$y(x)\to\operatorname{Root}\left[-\#1^5+\frac{\#1^2e^{3c_1}}{x^{3/2}}+3e^{3c_1}\sqrt{x}\&,4\right]$$

$$y(x)\to\operatorname{Root}\left[-\#1^5+\frac{\#1^2e^{3c_1}}{x^{3/2}}+3e^{3c_1}\sqrt{x}\&,5\right]$$

23.21 problem 652

Internal problem ID [3390]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 652.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$x(x^2 + yax + 2y^2)\, y' - (ax + 2y)\, y^2 = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve(x*(x^2+a*x*y(x)+2*y(x)^2)*diff(y(x),x) = (a*x+2*y(x))*y(x)^2,y(x), singsol=all)
```

$$y(x) = e^{\mathrm{RootOf}\left(e^{2\_Z}+a\,e^{\_Z}+c_1+\_Z+\ln(x)\right)}x$$

✓ Solution by Mathematica

Time used: 0.173 (sec). Leaf size: 34

```
DSolve[x(x^2+a x y[x]+2 y[x]^2)y'[x]==(a x+2 y[x])y[x]^2,y[x],x,IncludeSingularSolutions -> T
```

$$\mathrm{Solve}\left[\frac{ay(x)}{x} + \frac{y(x)^2}{x^2} + \log\left(\frac{y(x)}{x}\right) = -\log(x) + c_1, y(x)\right]$$

23.22 problem 653

Internal problem ID [3391]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 653.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _exact, _rational, _Bernoulli]`

$$\boxed{3y^2y'x - 2x + y^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 99

```
dsolve(3*x*y(x)^2*diff(y(x),x) = 2*x-y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(x^2 + c_1\right)x^2\right)^{\frac{1}{3}}}{x}$$

$$y(x) = -\frac{\left(\left(x^2 + c_1\right)x^2\right)^{\frac{1}{3}}}{2x} - \frac{i\sqrt{3}\left(\left(x^2 + c_1\right)x^2\right)^{\frac{1}{3}}}{2x}$$

$$y(x) = -\frac{\left(\left(x^2 + c_1\right)x^2\right)^{\frac{1}{3}}}{2x} + \frac{i\sqrt{3}\left(\left(x^2 + c_1\right)x^2\right)^{\frac{1}{3}}}{2x}$$

✓ Solution by Mathematica

Time used: 0.215 (sec). Leaf size: 72

```
DSolve[3 x y[x]^2 y'[x]==2 x-y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{x^2 + c_1}}{\sqrt[3]{x}}$$

$$y(x) \to -\frac{\sqrt[3]{-1}\sqrt[3]{x^2 + c_1}}{\sqrt[3]{x}}$$

$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{x^2 + c_1}}{\sqrt[3]{x}}$$

23.23 problem 654

Internal problem ID [3392]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 654.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$(1 - 4x + 3xy^2)\, y' - (2 - y^2)\, y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 24

```
dsolve((1-4*x+3*x*y(x)^2)*diff(y(x),x) = (2-y(x)^2)*y(x),y(x), singsol=all)
```

$$x + \frac{1}{y(x)^2} - \frac{c_1}{y(x)^2 \sqrt{y(x)^2 - 2}} = 0$$

✓ Solution by Mathematica

Time used: 60.161 (sec). Leaf size: 2348

```
DSolve[(1-4 x+3 x y[x]^2)y'[x]==(2-y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

23.24 problem 655

Internal problem ID [3393]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 655.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _exact, _rational]`

$$x(x-3y^2)\,y' + (2x-y^2)\,y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 327

```
dsolve(x*(x-3*y(x)^2)*diff(y(x),x)+(2*x-y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(12\sqrt{-12x^5+81c_1^2}+108c_1\right)x^2\right)^{\frac{1}{3}}}{6x} + \frac{2x^2}{\left(\left(12\sqrt{-12x^5+81c_1^2}+108c_1\right)x^2\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(\left(12\sqrt{-12x^5+81c_1^2}+108c_1\right)x^2\right)^{\frac{1}{3}}}{12x} - \frac{x^2}{\left(\left(12\sqrt{-12x^5+81c_1^2}+108c_1\right)x^2\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(\left(12\sqrt{-12x^5+81c_1^2}+108c_1\right)x^2\right)^{\frac{1}{3}}}{6x} - \frac{2x^2}{\left(\left(12\sqrt{-12x^5+81c_1^2}+108c_1\right)x^2\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(\left(12\sqrt{-12x^5+81c_1^2}+108c_1\right)x^2\right)^{\frac{1}{3}}}{12x} - \frac{x^2}{\left(\left(12\sqrt{-12x^5+81c_1^2}+108c_1\right)x^2\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(\left(12\sqrt{-12x^5+81c_1^2}+108c_1\right)x^2\right)^{\frac{1}{3}}}{6x} - \frac{2x^2}{\left(\left(12\sqrt{-12x^5+81c_1^2}+108c_1\right)x^2\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 35.053 (sec). Leaf size: 328

```
DSolve[x(x-3 y[x]^2)y'[x]+(2 x-y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2\sqrt[3]{3}x^3 + \sqrt[3]{2}\left(9c_1x^2 + \sqrt{-12x^9 + 81{c_1}^2x^4}\right)^{2/3}}{6^{2/3}x\sqrt[3]{9c_1x^2 + \sqrt{-12x^9 + 81{c_1}^2x^4}}}$$

$$y(x) \to \frac{2\sqrt[3]{2}\sqrt[6]{3}\left(\sqrt{3} + 3i\right)x^3 + \sqrt[3]{3}\left(1 - i\sqrt{3}\right)\left(18c_1x^2 + 2\sqrt{-12x^9 + 81{c_1}^2x^4}\right)^{2/3}}{12x\sqrt[3]{9c_1x^2 + \sqrt{-12x^9 + 81{c_1}^2x^4}}}$$

$$y(x) \to \frac{2\sqrt[3]{2}\sqrt[6]{3}\left(\sqrt{3} - 3i\right)x^3 + \sqrt[3]{3}\left(1 + i\sqrt{3}\right)\left(18c_1x^2 + 2\sqrt{-12x^9 + 81{c_1}^2x^4}\right)^{2/3}}{12x\sqrt[3]{9c_1x^2 + \sqrt{-12x^9 + 81{c_1}^2x^4}}}$$

23.25 problem 656

Internal problem ID [3394]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 656.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{3x\left(x+y^{2}\right)y'+x^{3}-3yx-2y^{3}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 450

```
dsolve(3*x*(x+y(x)^2)*diff(y(x),x)+x^3-3*x*y(x)-2*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-4c_1x^2 - 4x^3 + 4\sqrt{c_1^2x^4 + 2c_1x^5 + x^6 + 4x^3}\right)^{\frac{1}{3}}}{2} - \frac{2x}{\left(-4c_1x^2 - 4x^3 + 4\sqrt{c_1^2x^4 + 2c_1x^5 + x^6 + 4x^3}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(-4c_1x^2 - 4x^3 + 4\sqrt{c_1^2x^4 + 2c_1x^5 + x^6 + 4x^3}\right)^{\frac{1}{3}}}{4} + \frac{x}{\left(-4c_1x^2 - 4x^3 + 4\sqrt{c_1^2x^4 + 2c_1x^5 + x^6 + 4x^3}\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(-4c_1x^2-4x^3+4\sqrt{c_1^2x^4+2c_1x^5+x^6+4x^3}\right)^{\frac{1}{3}}}{2} + \frac{2x}{\left(-4c_1x^2-4x^3+4\sqrt{c_1^2x^4+2c_1x^5+x^6+4x^3}\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(-4c_1x^2 - 4x^3 + 4\sqrt{c_1^2x^4 + 2c_1x^5 + x^6 + 4x^3}\right)^{\frac{1}{3}}}{4} + \frac{x}{\left(-4c_1x^2 - 4x^3 + 4\sqrt{c_1^2x^4 + 2c_1x^5 + x^6 + 4x^3}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(-4c_1x^2-4x^3+4\sqrt{c_1^2x^4+2c_1x^5+x^6+4x^3}\right)^{\frac{1}{3}}}{2} + \frac{2x}{\left(-4c_1x^2-4x^3+4\sqrt{c_1^2x^4+2c_1x^5+x^6+4x^3}\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 36.017 (sec). Leaf size: 286

```
DSolve[3 x(x+y[x]^2)y'[x]+x^3-3 x y[x]-2 y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-2\sqrt[3]{2}x + 2^{2/3}\left(\sqrt{x^3\left(4 + x(x - c_1)^2\right)} + x^2(-x + c_1)\right)^{2/3}}{2\sqrt[3]{\sqrt{x^3\left(4 + x(x - c_1)^2\right)} + x^2(-x + c_1)}}$$

$$y(x) \to \frac{2\sqrt[3]{-2}x + (-2)^{2/3}\left(\sqrt{x^3\left(4 + x(x - c_1)^2\right)} + x^2(-x + c_1)\right)^{2/3}}{2\sqrt[3]{\sqrt{x^3\left(4 + x(x - c_1)^2\right)} + x^2(-x + c_1)}}$$

$$y(x) \to \frac{-\sqrt[3]{-2}\left(\sqrt{x^3\left(4 + x(x - c_1)^2\right)} + x^2(-x + c_1)\right)^{2/3} - i\sqrt{3}x + x}{2^{2/3}\sqrt[3]{\sqrt{x^3\left(4 + x(x - c_1)^2\right)} + x^2(-x + c_1)}}$$

23.26 problem 657

Internal problem ID [3395]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 657.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{x(x^3 - 3x^3y + 4y^2)\, y' - 6y^3 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve(x*(x^3-3*x^3*y(x)+4*y(x)^2)*diff(y(x),x) = 6*y(x)^3,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(-3\,\mathrm{e}^{\_Z} x^3 + 6c_1 x^3 + x^3\_Z + 2\,\mathrm{e}^{2\_Z}\right)}$$

✓ Solution by Mathematica

Time used: 0.161 (sec). Leaf size: 27

```
DSolve[x(x^3-3 x^3 y[x]+4 y[x]^2)y'[x]==6 y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{y(x)^2}{x^3} + \frac{1}{2}(\log(y(x)) - 3y(x)) = c_1, y(x)\right]$$

23.27 problem 658

Internal problem ID [3396]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 658.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _exact, _rational, _Bernoulli]`

$$\boxed{6y^2y'x + x + 2y^3 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 120

```
dsolve(6*x*y(x)^2*diff(y(x),x)+x+2*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(-2x^2 + 8c_1\right)x^2\right)^{\frac{1}{3}}}{2x}$$
$$y(x) = -\frac{\left(\left(-2x^2 + 8c_1\right)x^2\right)^{\frac{1}{3}}}{4x} - \frac{i\sqrt{3}\left(\left(-2x^2 + 8c_1\right)x^2\right)^{\frac{1}{3}}}{4x}$$
$$y(x) = -\frac{\left(\left(-2x^2 + 8c_1\right)x^2\right)^{\frac{1}{3}}}{4x} + \frac{i\sqrt{3}\left(\left(-2x^2 + 8c_1\right)x^2\right)^{\frac{1}{3}}}{4x}$$

✓ Solution by Mathematica

Time used: 0.224 (sec). Leaf size: 99

```
DSolve[6 x y[x]^2 y'[x]+x+2 y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{-x^2 + 4c_1}}{2^{2/3}\sqrt[3]{x}}$$
$$y(x) \to -\frac{\sqrt[3]{-1}\sqrt[3]{-x^2 + 4c_1}}{2^{2/3}\sqrt[3]{x}}$$
$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{-x^2 + 4c_1}}{2^{2/3}\sqrt[3]{x}}$$

23.28 problem 659

Internal problem ID [3397]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 659.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x(x+6y^2)\,y'+yx-3y^3=0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 25

```
dsolve(x*(x+6*y(x)^2)*diff(y(x),x)+x*y(x)-3*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x)=\frac{e^{-\frac{\operatorname{LambertW}\left(\frac{6\,e^{3c_1}}{x^3}\right)}{2}+\frac{3c_1}{2}}}{x}$$

✓ Solution by Mathematica

Time used: 4.246 (sec). Leaf size: 69

```
DSolve[x(x+6 y[x]^2)y'[x]+x y[x]-3 y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\frac{\sqrt{x}\sqrt{W\left(\frac{6e^{3c_1}}{x^3}\right)}}{\sqrt{6}}$$

$$y(x)\to\frac{\sqrt{x}\sqrt{W\left(\frac{6e^{3c_1}}{x^3}\right)}}{\sqrt{6}}$$

$$y(x)\to 0$$

23.29 problem 660

Internal problem ID [3398]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 660.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{x(x^2-6y^2)\,y'-4(x^2+3y^2)\,y=0}$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 47

```
dsolve(x*(x^2-6*y(x)^2)*diff(y(x),x) = 4*(x^2+3*y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{c_1\left(-1+\sqrt{-\frac{24x^6}{c_1^2}+1}\right)}{12x^2}$$
$$y(x) = \frac{c_1\left(1+\sqrt{-\frac{24x^6}{c_1^2}+1}\right)}{12x^2}$$

✓ Solution by Mathematica

Time used: 1.001 (sec). Leaf size: 67

```
DSolve[x(x^2-6 y[x]^2)y'[x]==4(x^2+3 y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{c_1}-\sqrt{-24x^6+e^{2c_1}}}{12x^2}$$
$$y(x) \to \frac{\sqrt{-24x^6+e^{2c_1}}+e^{c_1}}{12x^2}$$

23.30 problem 661

Internal problem ID [3399]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 661.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x(3x - 7y^2)\, y' + (5x - 3y^2)\, y = 0$$

✓ Solution by Maple

Time used: 0.5 (sec). Leaf size: 52

```
dsolve(x*(3*x-7*y(x)^2)*diff(y(x),x)+(5*x-3*y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(x\_Z^7 - x^2\_Z^3 - \frac{c_1}{\sqrt{x}}\right)^2$$
$$y(x) = \text{RootOf}\left(x\_Z^7 - x^2\_Z^3 + \frac{c_1}{\sqrt{x}}\right)^2$$

✓ Solution by Mathematica

Time used: 4.554 (sec). Leaf size: 288

```
DSolve[x(3 x-7 y[x]^2)y'[x]+(5  x-3 y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - c_1{}^2\&, 1\right]$$
$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - c_1{}^2\&, 2\right]$$
$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - c_1{}^2\&, 3\right]$$
$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - c_1{}^2\&, 4\right]$$
$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - c_1{}^2\&, 5\right]$$
$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - c_1{}^2\&, 6\right]$$
$$y(x) \to \text{Root}\left[4\#1^7x^3 - 8\#1^5x^4 + 4\#1^3x^5 - c_1{}^2\&, 7\right]$$

23.31 problem 662

Internal problem ID [3400]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 23
Problem number: 662.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'y^2x^2 + 1 - x + x^3 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 155

```
dsolve(x^2*y(x)^2*diff(y(x),x)+1-x+x^3 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(-12x^3 + 24x\ln\left(x\right) + 8c_1x + 24\right)x^2\right)^{\frac{1}{3}}}{2x}$$

$$y(x) = -\frac{\left(\left(-12x^3 + 24x\ln\left(x\right) + 8c_1x + 24\right)x^2\right)^{\frac{1}{3}}}{4x} - \frac{i\sqrt{3}\left(\left(-12x^3 + 24x\ln\left(x\right) + 8c_1x + 24\right)x^2\right)^{\frac{1}{3}}}{4x}$$

$$y(x) = -\frac{\left(\left(-12x^3 + 24x\ln\left(x\right) + 8c_1x + 24\right)x^2\right)^{\frac{1}{3}}}{4x} + \frac{i\sqrt{3}\left(\left(-12x^3 + 24x\ln\left(x\right) + 8c_1x + 24\right)x^2\right)^{\frac{1}{3}}}{4x}$$

✓ Solution by Mathematica

Time used: 0.313 (sec). Leaf size: 111

```
DSolve[x^2 y[x]^2 y'[x]+1-x+x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt[3]{-\frac{3}{2}}\sqrt[3]{-x^3+2x\log(x)+2c_1x+2}}{\sqrt[3]{x}}$$

$$y(x) \to \frac{\sqrt[3]{-\frac{3x^3}{2}+3x\log(x)+3c_1x+3}}{\sqrt[3]{x}}$$

$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{-\frac{3x^3}{2}+3x\log(x)+3c_1x+3}}{\sqrt[3]{x}}$$

24 Various 24

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24.1 problem 663

Internal problem ID [3401]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 663.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{(1 - x^2y^2)\, y' - xy^3 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 21

```
dsolve((1-x^2*y(x)^2)*diff(y(x),x) = x*y(x)^3,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\frac{\mathrm{LambertW}\left(-x^2\mathrm{e}^{-2c_1}\right)}{2}-c_1}$$

✓ Solution by Mathematica

Time used: 5.265 (sec). Leaf size: 60

```
DSolve[(1-x^2 y[x]^2)y'[x]==x y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{i\sqrt{W\left(-e^{-2c_1}x^2\right)}}{x}$$
$$y(x) \to \frac{i\sqrt{W\left(-e^{-2c_1}x^2\right)}}{x}$$
$$y(x) \to 0$$

24.2 problem 664

Internal problem ID [3402]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 664.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\boxed{(1 - x^2y^2)\, y' - (1 + yx)\, y^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve((1-x^2*y(x)^2)*diff(y(x),x) = (1+x*y(x))*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{1}{x}$$
$$y(x) = \mathrm{e}^{-\operatorname{LambertW}\left(-x\,\mathrm{e}^{-c_1}\right)-c_1}$$

✓ Solution by Mathematica

Time used: 2.124 (sec). Leaf size: 43

```
DSolve[(1-x^2 y[x]^2)y'[x]==(1+x y[x])y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{x}$$
$$y(x) \to -\frac{W(-e^{-c_1}x)}{x}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{1}{x}$$

24.3 problem 665

Internal problem ID [3403]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 665.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x(xy^2+1)\,y'+y=0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 137

```
dsolve(x*(1+x*y(x)^2)*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2xc_1\left(-2c_1-x+\sqrt{4c_1x+x^2}\right)}}{2xc_1}$$

$$y(x) = \frac{\sqrt{-2xc_1\left(-2c_1-x+\sqrt{4c_1x+x^2}\right)}}{2xc_1}$$

$$y(x) = -\frac{\sqrt{2}\sqrt{xc_1\left(2c_1+x+\sqrt{4c_1x+x^2}\right)}}{2xc_1}$$

$$y(x) = \frac{\sqrt{2}\sqrt{xc_1\left(2c_1+x+\sqrt{4c_1x+x^2}\right)}}{2xc_1}$$

✓ Solution by Mathematica

Time used: 0.297 (sec). Leaf size: 65

```
DSolve[x(1+x y[x]^2)y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left(c_1 - \frac{\sqrt{4 + c_1{}^2 x}}{\sqrt{x}}\right)$$
$$y(x) \to \frac{1}{2}\left(\frac{\sqrt{4 + c_1{}^2 x}}{\sqrt{x}} + c_1\right)$$
$$y(x) \to 0$$

24.4 problem 666

Internal problem ID [3404]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 666.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x(xy^2+1)\,y'-(2-3xy^2)\,y=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 45

```
dsolve(x*(1+x*y(x)^2)*diff(y(x),x) = (2-3*x*y(x)^2)*y(x),y(x), singsol=all)
```

$$y(x)=\frac{c_1+\sqrt{4x^5+c_1^2}}{2x^3}$$
$$y(x)=-\frac{-c_1+\sqrt{4x^5+c_1^2}}{2x^3}$$

✓ Solution by Mathematica

Time used: 1.075 (sec). Leaf size: 75

```
DSolve[x(1+x y[x]^2)y'[x]==(2-3 x y[x]^2)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\frac{\sqrt{4x^5+e^{5c_1}}+e^{\frac{5c_1}{2}}}{2x^3}$$
$$y(x)\to \frac{\sqrt{4x^5+e^{5c_1}}-e^{\frac{5c_1}{2}}}{2x^3}$$

24.5 problem 667

Internal problem ID [3405]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 667.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$x^2(a+y)^2 y' - (x^2+1)(y^2+a^2) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 92

```
dsolve(x^2*(a+y(x))^2*diff(y(x),x) = (x^2+1)*(y(x)^2+a^2),y(x), singsol=all)
```

$$y(x) = \frac{-ax\,\mathrm{RootOf}\left(\_Z^2a^2x^2 - 2c_1\_Za\,x^2 - 2\_Za\,x^3 + c_1^2x^2 + 2c_1x^3 + a^2x^2 + x^4 - x^2\mathrm{e}^{\_Z} + 2ax\_Z - 2c_1x - 2\right.}{x}$$

✓ Solution by Mathematica

Time used: 0.493 (sec). Leaf size: 48

```
DSolve[x^2 (a+y[x])^2 y'[x]==(1+x^2)(a^2+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \mathrm{InverseFunction}\left[a\log\left(\#1^2+a^2\right)+\#1\&\right]\left[x-\frac{1}{x}+c_1\right]$$
$$y(x) \to -ia$$
$$y(x) \to ia$$

24.6 problem 668

Internal problem ID [3406]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 668.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\left(x^{2}+1\right)\left(1+y^{2}\right) y'+2 x y\left(1-y^{2}\right)=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 75

```
dsolve((x^2+1)*(1+y(x)^2)*diff(y(x),x)+2*x*y(x)*(1-y(x)^2) = 0,y(x), singsol=all)
```

$$y(x)=\frac{c_{1} x^{2}}{2}+\frac{c_{1}}{2}-\frac{\sqrt{c_{1}^{2} x^{4}+2 c_{1}^{2} x^{2}+c_{1}^{2}+4}}{2}$$
$$y(x)=\frac{c_{1} x^{2}}{2}+\frac{c_{1}}{2}+\frac{\sqrt{c_{1}^{2} x^{4}+2 c_{1}^{2} x^{2}+c_{1}^{2}+4}}{2}$$

✓ Solution by Mathematica

Time used: 7.873 (sec). Leaf size: 98

```
DSolve[(1+x^2)(1+y[x]^2)y'[x]+2 x y[x](1-y[x]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{1}{2}\left(-e^{c_{1}}\left(x^{2}+1\right)-\sqrt{4+e^{2 c_{1}}\left(x^{2}+1\right)^{2}}\right)$$
$$y(x)\to \frac{1}{2}\left(\sqrt{4+e^{2 c_{1}}\left(x^{2}+1\right)^{2}}-e^{c_{1}}\left(x^{2}+1\right)\right)$$
$$y(x)\to -1$$
$$y(x)\to 0$$
$$y(x)\to 1$$

24.7 problem 669

Internal problem ID [3407]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 669.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$(x^2+1)(1+y^2)y'+2xy(1-y)^2=0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 40

```
dsolve((x^2+1)*(1+y(x)^2)*diff(y(x),x)+2*x*y(x)*(1-y(x))^2 = 0,y(x), singsol=all)
```

$$y(x)=\mathrm{e}^{\mathrm{RootOf}\left(\ln(x^2+1)\mathrm{e}^{\_Z}+2\,\mathrm{e}^{\_Z}c_1+\_Z\,\mathrm{e}^{\_Z}-\ln(x^2+1)-2c_1-\_Z-2\right)}$$

✓ Solution by Mathematica

Time used: 0.308 (sec). Leaf size: 40

```
DSolve[(1+x^2)(1+y[x]^2)y'[x]+2 x y[x](1-y[x])^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \text{InverseFunction}\left[\log(\#1)-\frac{2}{\#1-1}\&\right]\left[-\log\left(x^2+1\right)+c_1\right]$$

$$y(x)\to 0$$

$$y(x)\to 1$$

24.8 problem 670

Internal problem ID [3408]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 670.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{(1 - x^3 + 6x^2y^2)\, y' - (6 + 3yx - 4y^3)\, x = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 601

```
dsolve((1-x^3+6*x^2*y(x)^2)*diff(y(x),x) = (6+3*x*y(x)-4*y(x)^3)*x,y(x), singsol=all)
```

$$y(x) = \frac{\left(162x^3+6\sqrt{3}\,\sqrt{-2x^9+249x^6-162c_1x^4+27c_1^2x^2-6x^3+2}-54c_1x\right)^{\frac{1}{3}}}{6x} + \frac{x^3-1}{x\left(162x^3+6\sqrt{3}\,\sqrt{-2x^9+249x^6-162c_1x^4+27c_1^2x^2-6x^3+2}-54c_1x\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(162x^3+6\sqrt{3}\,\sqrt{-2x^9+249x^6-162c_1x^4+27c_1^2x^2-6x^3+2}-54c_1x\right)^{\frac{1}{3}}}{12x} - \frac{x^3-1}{2x\left(162x^3+6\sqrt{3}\,\sqrt{-2x^9+249x^6-162c_1x^4+27c_1^2x^2-6x^3+2}-54c_1x\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(162x^3+6\sqrt{3}\,\sqrt{-2x^9+249x^6-162c_1x^4+27c_1^2x^2-6x^3+2}-54c_1x\right)^{\frac{1}{3}}}{6x} - \frac{x^3-1}{x\left(162x^3+6\sqrt{3}\,\sqrt{-2x^9+249x^6-162c_1x^4+27c_1^2x^2-6x^3+2}\right.}\right.}{2}$$

$$y(x) = -\frac{\left(162x^3+6\sqrt{3}\,\sqrt{-2x^9+249x^6-162c_1x^4+27c_1^2x^2-6x^3+2}-54c_1x\right)^{\frac{1}{3}}}{12x} - \frac{x^3-1}{2x\left(162x^3+6\sqrt{3}\,\sqrt{-2x^9+249x^6-162c_1x^4+27c_1^2x^2-6x^3+2}-54c_1x\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(162x^3+6\sqrt{3}\,\sqrt{-2x^9+249x^6-162c_1x^4+27c_1^2x^2-6x^3+2}-54c_1x\right)^{\frac{1}{3}}}{6x} - \frac{x^3-1}{x\left(162x^3+6\sqrt{3}\,\sqrt{-2x^9+249x^6-162c_1x^4+27c_1^2x^2-6x^3+2}\right.}\right.}{2}$$

✓ Solution by Mathematica

Time used: 50.933 (sec). Leaf size: 424

```
DSolve[(1-x^3+6 x^2 y[x]^2)y'[x]==(6+3 x y[x]-4 y[x]^3)x,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\frac{\sqrt[3]{2}(x^3-1)}{\sqrt[3]{-324x^6+108c_1x^4+\sqrt{-864x^6(x^3-1)^3+(-324x^6+108c_1x^4)^2}}} - \frac{\sqrt[3]{-324x^6+108c_1x^4+\sqrt{-864x^6(x^3-1)^3+(-324x^6+108c_1x^4)^2}}}{6\sqrt[3]{2}x^2}$$

$$y(x) \to \frac{(1+i\sqrt{3})(x^3-1)}{2^{2/3}\sqrt[3]{-324x^6+108c_1x^4+\sqrt{-864x^6(x^3-1)^3+(-324x^6+108c_1x^4)^2}}} + \frac{(1-i\sqrt{3})\sqrt[3]{-324x^6+108c_1x^4+\sqrt{-864x^6(x^3-1)^3+(-324x^6+108c_1x^4)^2}}}{12\sqrt[3]{2}x^2}$$

$$y(x) \to \frac{(1-i\sqrt{3})(x^3-1)}{2^{2/3}\sqrt[3]{-324x^6+108c_1x^4+\sqrt{-864x^6(x^3-1)^3+(-324x^6+108c_1x^4)^2}}} + \frac{(1+i\sqrt{3})\sqrt[3]{-324x^6+108c_1x^4+\sqrt{-864x^6(x^3-1)^3+(-324x^6+108c_1x^4)^2}}}{12\sqrt[3]{2}x^2}$$

24.9 problem 671

Internal problem ID [3409]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 671.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$x\left(3+5x-12xy^2+4x^2y\right) y'+\left(3+10x-8xy^2+6x^2y\right) y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 931

```
dsolve(x*(3+5*x-12*x*y(x)^2+4*x^2*y(x))*diff(y(x),x)+(3+10*x-8*x*y(x)^2+6*x^2*y(x))*y(x) = 0,
```

$$y(x) = \frac{\left(\left(8x^5 + 180x^3 + 12\sqrt{3}\,\sqrt{8c_1x^5 - 25x^6 - 30x^5 + 180c_1x^3 - 509x^4 + 108c_1x^2 - 900x^3 + 108c_1^2 - 540x^2 -\right.\right.}{12x} + \frac{x^3 + 15x + 9}{3\left(\left(8x^5 + 180x^3 + 12\sqrt{3}\,\sqrt{8c_1x^5 - 25x^6 - 30x^5 + 180c_1x^3 - 509x^4 + 108c_1x^2 - 900x^3 + 108c_1^2 - 540x\right.\right.} + \frac{x}{6}$$

$$y(x) = -\frac{\left(\left(8x^5 + 180x^3 + 12\sqrt{3}\,\sqrt{8c_1x^5 - 25x^6 - 30x^5 + 180c_1x^3 - 509x^4 + 108c_1x^2 - 900x^3 + 108c_1^2 - 540x\right.\right.}{24x} - \frac{x^3 + 15x + 9}{6\left(\left(8x^5 + 180x^3 + 12\sqrt{3}\,\sqrt{8c_1x^5 - 25x^6 - 30x^5 + 180c_1x^3 - 509x^4 + 108c_1x^2 - 900x^3 + 108c_1^2 - 540x\right.\right.} + \frac{x}{6} - \frac{i\sqrt{3}\left(\frac{\left(\left(8x^5+180x^3+12\sqrt{3}\,\sqrt{8c_1x^5-25x^6-30x^5+180c_1x^3-509x^4+108c_1x^2-900x^3+108c_1^2-540x^2-108x+108x^2+216c_1}\right)x\right)^{\frac{1}{3}}}{12x} - \frac{}{3\left(\right.}\right.}{2}$$

$$y(x) = -\frac{\left(\left(8x^5 + 180x^3 + 12\sqrt{3}\,\sqrt{8c_1x^5 - 25x^6 - 30x^5 + 180c_1x^3 - 509x^4 + 108c_1x^2 - 900x^3 + 108c_1^2 - 540x\right.\right.}{24x} - \frac{x^3 + 15x + 9}{6\left(\left(8x^5 + 180x^3 + 12\sqrt{3}\,\sqrt{8c_1x^5 - 25x^6 - 30x^5 + 180c_1x^3 - 509x^4 + 108c_1x^2 - 900x^3 + 108c_1^2 - 540x\right.\right.} + \frac{x}{6} + \frac{i\sqrt{3}\left(\frac{\left(\left(8x^5+180x^3+12\sqrt{3}\,\sqrt{8c_1x^5-25x^6-30x^5+180c_1x^3-509x^4+108c_1x^2-900x^3+108c_1^2-540x^2-108x+108x^2+216c_1}\right)x\right)^{\frac{1}{3}}}{12x} - \frac{}{3\left(\right.}\right.}{2}$$

✓ Solution by Mathematica

Time used: 59.928 (sec). Leaf size: 621

```
DSolve[x(3+5 x-12 x y[x]^2+4 x^2 y[x])y'[x]+(3+10 x-8 x y[x]^2+6 x^2 y[x])y[x]==0,y[x],x,Incl
```

$$y(x) \to -\frac{(x^3+15x+9)\,x}{3\ 2^{2/3}\sqrt[3]{54c_1x^4-\left((2x^3+45x+27)\,x^6\right)+3\sqrt{3}\sqrt{x^8\left(-(5x+3)^2\,(x^3+20x+12)\,x-4c_1\,(2x^3+45x+\cdots\right.}}} - \frac{\sqrt[3]{-8x^9-180x^7-108x^6+216c_1x^4+4\sqrt{-4x^9\,(x^3+15x+9)^3+\left(x^6\,(2x^3+45x+27)-54c_1x^4\right)^2}}}{12x^2} + \frac{x}{6}$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)(x^3+15x+9)\,x}{6\ 2^{2/3}\sqrt[3]{54c_1x^4-\left((2x^3+45x+27)\,x^6\right)+3\sqrt{3}\sqrt{x^8\left(-(5x+3)^2\,(x^3+20x+12)\,x-4c_1\,(2x^3+45x+27\cdots\right.}}} + \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{-8x^9-180x^7-108x^6+216c_1x^4+4\sqrt{-4x^9\,(x^3+15x+9)^3+\left(x^6\,(2x^3+45x+27)-5\cdots\right.}}}{24x^2} + \frac{x}{6}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)(x^3+15x+9)\,x}{6\ 2^{2/3}\sqrt[3]{54c_1x^4-\left((2x^3+45x+27)\,x^6\right)+3\sqrt{3}\sqrt{x^8\left(-(5x+3)^2\,(x^3+20x+12)\,x-4c_1\,(2x^3+45x+27\cdots\right.}}} + \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-8x^9-180x^7-108x^6+216c_1x^4+4\sqrt{-4x^9\,(x^3+15x+9)^3+\left(x^6\,(2x^3+45x+27)-5\cdots\right.}}}{24x^2} + \frac{x}{6}$$

24.10 problem 672

Internal problem ID [3410]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 672.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^3(1+y^2)\,y'+3x^2y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 14

```
dsolve(x^3*(1+y(x)^2)*diff(y(x),x)+3*x^2*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{1}{\sqrt{\frac{1}{\operatorname{LambertW}\left(\frac{c_1}{x^6}\right)}}}$$

✓ Solution by Mathematica

Time used: 4.201 (sec). Leaf size: 46

```
DSolve[x^3(1+y[x]^2)y'[x]+3 x^2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to-\sqrt{W\left(\frac{e^{2c_1}}{x^6}\right)}$$
$$y(x)\to\sqrt{W\left(\frac{e^{2c_1}}{x^6}\right)}$$
$$y(x)\to 0$$

24.11 problem 673

Internal problem ID [3411]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 673.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x(-yx+1)^2 y' + (1+x^2y^2)\, y = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 34

```
dsolve(x*(1-x*y(x))^2*diff(y(x),x)+(1+x^2*y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{e^{\mathrm{RootOf}\left(-2\,e^{-\_Z}\ln(x)-e^{2\_Z}+2\,e^{-\_Z}c_1+2\_Z\,e^{-\_Z}+1\right)}}{x}$$

✓ Solution by Mathematica

Time used: 0.107 (sec). Leaf size: 25

```
DSolve[x(1-x y[x])^2 y'[x]+(1+x^2 y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[xy(x) - \frac{1}{xy(x)} - 2\log(y(x)) = c_1, y(x)\right]$$

24.12 problem 674

Internal problem ID [3412]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 674.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\left(1-y^{2} x^{4}\right) y'-x^{3} y^{3}=0$$

✓ Solution by Maple

Time used: 0.234 (sec). Leaf size: 191

```
dsolve((1-x^4*y(x)^2)*diff(y(x),x) = x^3*y(x)^3,y(x), singsol=all)
```

$$y(x)=\frac{\sqrt{-c_{1}-\sqrt{c_{1} x^{4}+c_{1}^{2}}}\left(\frac{-c_{1}-\sqrt{c_{1} x^{4}+c_{1}^{2}}}{c_{1}}+2\right)}{x^{4}}$$

$$y(x)=\frac{\sqrt{-c_{1}+\sqrt{c_{1} x^{4}+c_{1}^{2}}}\left(\frac{-c_{1}+\sqrt{c_{1} x^{4}+c_{1}^{2}}}{c_{1}}+2\right)}{x^{4}}$$

$$y(x)=-\frac{\sqrt{-c_{1}-\sqrt{c_{1} x^{4}+c_{1}^{2}}}\left(\frac{-c_{1}-\sqrt{c_{1} x^{4}+c_{1}^{2}}}{c_{1}}+2\right)}{x^{4}}$$

$$y(x)=-\frac{\sqrt{-c_{1}+\sqrt{c_{1} x^{4}+c_{1}^{2}}}\left(\frac{-c_{1}+\sqrt{c_{1} x^{4}+c_{1}^{2}}}{c_{1}}+2\right)}{x^{4}}$$

✓ Solution by Mathematica

Time used: 11.24 (sec). Leaf size: 122

```
DSolve[(1-x^4 y[x]^2)y'[x]==x^3 y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\frac{1-\sqrt{1+4c_1x^4}}{x^4}}$$
$$y(x) \to \sqrt{\frac{1-\sqrt{1+4c_1x^4}}{x^4}}$$
$$y(x) \to -\sqrt{\frac{1+\sqrt{1+4c_1x^4}}{x^4}}$$
$$y(x) \to \sqrt{\frac{1+\sqrt{1+4c_1x^4}}{x^4}}$$
$$y(x) \to 0$$

24.13 problem 675

Internal problem ID [3413]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 675.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{(3x - y^3)\, y' - x^2 + 3y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve((3*x-y(x)^3)*diff(y(x),x) = x^2-3*y(x),y(x), singsol=all)
```

$$-\frac{x^3}{3} + 3y(x)\, x - \frac{y(x)^4}{4} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.167 (sec). Leaf size: 1211

```
DSolve[(3 x-y[x]^3)y'[x]==x^2-3 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}}}}}{\sqrt{6}} - \frac{1}{2}\sqrt{-\frac{12\sqrt{6}x}{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}}}}} - \frac{2}{3}\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4 \cdots}}}$$

$$y(x) \to \frac{1}{2}\sqrt{-\frac{12\sqrt{6}x}{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}}}}} - \frac{2}{3}\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4 - \cdots}}} - \frac{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}}}}}{\sqrt{6}}$$

$$y(x) \to \frac{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}}}}}{\sqrt{6}} - \frac{1}{2}\sqrt{\frac{12\sqrt{6}x}{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}}}}} - \frac{2}{3}\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4 - \cdots}}}$$

$$y(x) \to \frac{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}}}}}{\sqrt{6}} + \frac{1}{2}\sqrt{\frac{12\sqrt{6}x}{\sqrt{\frac{4x^3+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}+12c_1}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}}}}} - \frac{2}{3}\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4 - \cdots}}}$$

24.14 problem 676

Internal problem ID [3414]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 676.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{\left(x^3-y^3\right)y'+x^2y=0}$$

✓ Solution by Maple

Time used: 0.375 (sec). Leaf size: 391

```
dsolve((x^3-y(x)^3)*diff(y(x),x)+x^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x}{\left(-x^3 c_1 \left(c_1 x^3 - \sqrt{c_1^2 x^6 + 1}\right)\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{x}{\left(-\left(c_1 x^3 + \sqrt{c_1^2 x^6 + 1}\right) x^3 c_1\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{x}{\left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)^2 \left(-x^3 c_1 \left(c_1 x^3 - \sqrt{c_1^2 x^6 + 1}\right)\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{x}{\left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)^2 \left(-\left(c_1 x^3 + \sqrt{c_1^2 x^6 + 1}\right) x^3 c_1\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{x}{\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^2 \left(-x^3 c_1 \left(c_1 x^3 - \sqrt{c_1^2 x^6 + 1}\right)\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{x}{\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^2 \left(-\left(c_1 x^3 + \sqrt{c_1^2 x^6 + 1}\right) x^3 c_1\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{x}{\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)^2 \left(-x^3 c_1 \left(c_1 x^3 - \sqrt{c_1^2 x^6 + 1}\right)\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{x}{\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)^2 \left(-\left(c_1 x^3 + \sqrt{c_1^2 x^6 + 1}\right) x^3 c_1\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{x}{\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^2 \left(-x^3 c_1 \left(c_1 x^3 - \sqrt{c_1^2 x^6 + 1}\right)\right)^{\frac{1}{3}}}$$

$$y(x) = \frac{x}{\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^2 \left(-\left(c_1 x^3 + \sqrt{c_1^2 x^6 + 1}\right) x^3 c_1\right)^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 6.352 (sec). Leaf size: 352

```
DSolve[(x^3-y[x]^3)y'[x]+x^2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt[3]{x^3 - \sqrt{x^6 - e^{6c_1}}}$$

$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{x^3 - \sqrt{x^6 - e^{6c_1}}}$$

$$y(x) \to (-1)^{2/3}\sqrt[3]{x^3 - \sqrt{x^6 - e^{6c_1}}}$$

$$y(x) \to \sqrt[3]{x^3 + \sqrt{x^6 - e^{6c_1}}}$$

$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{x^3 + \sqrt{x^6 - e^{6c_1}}}$$

$$y(x) \to (-1)^{2/3}\sqrt[3]{x^3 + \sqrt{x^6 - e^{6c_1}}}$$

$$y(x) \to 0$$

$$y(x) \to \sqrt[3]{x^3 - \sqrt{x^6}}$$

$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{x^3 - \sqrt{x^6}}$$

$$y(x) \to (-1)^{2/3}\sqrt[3]{x^3 - \sqrt{x^6}}$$

$$y(x) \to \sqrt[3]{\sqrt{x^6} + x^3}$$

$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{\sqrt{x^6} + x^3}$$

$$y(x) \to (-1)^{2/3}\sqrt[3]{\sqrt{x^6} + x^3}$$

24.15 problem 677

Internal problem ID [3415]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 677.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _exact, _rational, _dAlembert]`

$$\boxed{(x^3+y^3)\, y'+x^2(ax+3y)=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 29

```
dsolve((x^3+y(x)^3)*diff(y(x),x)+x^2*(a*x+3*y(x)) = 0,y(x), singsol=all)
```

$$y(x)=\frac{\operatorname{RootOf}\left(a\,x^4c_1^{\frac{4}{3}}+4x^3c_1\_Z+\_Z^4-1\right)}{c_1^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 60.16 (sec). Leaf size: 1402

```
DSolve[(x^3+y[x]^3)y'[x]+x^2(a x+3 y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt{\frac{\sqrt[3]{3}ax^4+\left(9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}\right)^{2/3}-\sqrt[3]{3}e^{4c_1}}{\sqrt[3]{9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}}}} - \sqrt{-\sqrt[3]{9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}} + \frac{}{\sqrt[3]{3x^6+}}}}{\sqrt{2}\sqrt[3]{3}}$$

$$y(x) \to \frac{\sqrt{\frac{\sqrt[3]{3}ax^4+\left(9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}\right)^{2/3}-\sqrt[3]{3}e^{4c_1}}{\sqrt[3]{9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}}}} + \sqrt{-\sqrt[3]{9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}} + \frac{}{\sqrt[3]{3x^6+}}}}{\sqrt{2}\sqrt[3]{3}}$$

$$y(x) \to -\frac{\sqrt{\frac{\sqrt[3]{3}ax^4+\left(9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}\right)^{2/3}-\sqrt[3]{3}e^{4c_1}}{\sqrt[3]{9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}}}} + \sqrt{-\sqrt[3]{9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}} + \frac{}{\sqrt[3]{3x^6}}}}{\sqrt{2}\sqrt[3]{3}}$$

$$y(x) \to \frac{\sqrt{-\sqrt[3]{9x^6+\sqrt{3}\sqrt{27x^{12}+(-ax^4+e^{4c_1})^3}} + \frac{-ax^4+e^{4c_1}}{\sqrt[3]{3x^6+\sqrt{9x^{12}+\frac{1}{3}(-ax^4+e^{4c_1})^3}}} + \frac{}{\sqrt{\frac{\sqrt[3]{3}ax^4+\left(9x^6+\sqrt{3}\sqrt{27x\ldots}\right.}{\sqrt[3]{9x^6+\sqrt{3}\sqrt{2\ldots}}}}}}}{\sqrt{2}\sqrt[3]{3}}$$

24.16 problem 678

Internal problem ID [3416]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 678.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$\boxed{(x - x^2 y - y^3)\, y' - x^3 + y - x y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 29

```
dsolve((x-x^2*y(x)-y(x)^3)*diff(y(x),x) = x^3-y(x)+x*y(x)^2,y(x), singsol=all)
```

$$-\frac{x^4}{4} - \frac{y(x)^2 x^2}{2} + y(x)\, x - \frac{y(x)^4}{4} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.19 (sec). Leaf size: 1807

```
DSolve[(x-x^2 y[x]-y[x]^3)y'[x]==x^3-y[x]+x y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-2x^2+\sqrt[3]{-8x^6+9(3+4c_1)x^2+3\sqrt{3}\sqrt{-16x^8+(27+8(9-2c_1)c_1)x^4+64c_1{}^3}}+\frac{\cdots}{\sqrt[3]{-8x^6+9(3+\cdots}}}}{\sqrt{6}}$$
$$-\frac{1}{2}\sqrt{-\frac{8x^2}{3}-\frac{4\sqrt{6}x}{\sqrt{-2x^2+\sqrt[3]{-8x^6+9(3+4c_1)x^2+3\sqrt{3}\sqrt{-16x^8+(27+8(9-2c_1)c_1)x^4+64c_1{}^3}}+\frac{\cdots}{\sqrt[3]{-\cdots}}}}\cdots}$$

$$y(x) \to \frac{1}{2}\sqrt{-\frac{8x^2}{3}-\frac{4\sqrt{6}x}{\sqrt{-2x^2+\sqrt[3]{-8x^6+9(3+4c_1)x^2+3\sqrt{3}\sqrt{-16x^8+(27+8(9-2c_1)c_1)x^4+64c_1{}^3}}+\frac{\cdots}{\sqrt[3]{-8\cdots}}}}\cdots}$$
$$-\frac{\sqrt{-2x^2+\sqrt[3]{-8x^6+9(3+4c_1)x^2+3\sqrt{3}\sqrt{-16x^8+(27+8(9-2c_1)c_1)x^4+64c_1{}^3}}+\frac{\cdots}{\sqrt[3]{-8x^6+9(3+\cdots}}}}{\sqrt{6}}$$

$$y(x) \to \frac{\sqrt{-2x^2+\sqrt[3]{-8x^6+9(3+4c_1)x^2+3\sqrt{3}\sqrt{-16x^8+(27+8(9-2c_1)c_1)x^4+64c_1{}^3}}+\frac{\cdots}{\sqrt[3]{-8x^6+9(3+4\cdots}}}}{\sqrt{6}}$$
$$-\frac{1}{2}\sqrt{-\frac{8x^2}{3}+\frac{4\sqrt{6}x}{\sqrt{-2x^2+\sqrt[3]{-8x^6+9(3+4c_1)x^2+3\sqrt{3}\sqrt{-16x^8+(27+8(9-2c_1)c_1)x^4+64c_1{}^3}}+\frac{\cdots}{\sqrt[3]{-\cdots}}}}\cdots}$$

$$y(x) \to \frac{\sqrt{-2x^2+\sqrt[3]{-8x^6+9(3+4c_1)x^2+3\sqrt{3}\sqrt{-16x^8+(27+8(9-2c_1)c_1)x^4+64c_1{}^3}}+\frac{\cdots}{\sqrt[3]{-8x^6+9(3+4\cdots}}}}{\sqrt{6}}$$
$$+\frac{1}{2}\sqrt{-\frac{8x^2}{3}+\frac{4\sqrt{6}x}{\sqrt{-2x^2+\sqrt[3]{-8x^6+9(3+4c_1)x^2+3\sqrt{3}\sqrt{-16x^8+(27+8(9-2c_1)c_1)x^4+64c_1{}^3}}+\frac{\cdots}{\sqrt[3]{-\cdots}}}}\cdots}$$

24.17 problem 679

Internal problem ID [3417]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 679.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\left(x\, a^{2}+y \left(x^{2}-y^{2}\right)\right) y^{\prime}+x \left(x^{2}-y^{2}\right)-y a^{2}=0$$

✗ Solution by Maple

```
dsolve((a^2*x+y(x)*(x^2-y(x)^2))*diff(y(x),x)+x*(x^2-y(x)^2) = a^2*y(x),y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 0.307 (sec). Leaf size: 48

```
DSolve[(a^2 x+y[x] (x^2-y[x]^2))y'[x]+x(x^2-y[x]^2)==a^2 y[x],y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[-\frac{1}{2}a^{2}\log(x-y(x))+\frac{1}{2}a^{2}\log(y(x)+x)+\frac{x^{2}}{2}+\frac{y(x)^{2}}{2}=c_{1},y(x)\right]$$

24.18 problem 680

Internal problem ID [3418]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 680.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational]`

$$(a + x^2 + y^2)\, yy' - x(a - x^2 - y^2) = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 113

```
dsolve((a+x^2+y(x)^2)*y(x)*diff(y(x),x) = x*(a-x^2-y(x)^2),y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 - a - 2\sqrt{a\,x^2 - c_1}}$$
$$y(x) = \sqrt{-x^2 - a + 2\sqrt{a\,x^2 - c_1}}$$
$$y(x) = -\sqrt{-x^2 - a - 2\sqrt{a\,x^2 - c_1}}$$
$$y(x) = -\sqrt{-x^2 - a + 2\sqrt{a\,x^2 - c_1}}$$

✓ Solution by Mathematica

Time used: 8.574 (sec). Leaf size: 149

```
DSolve[(a+x^2+y[x]^2)y[x] y'[x]==x(a-x^2-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-\sqrt{a^2 + 4ax^2 + 4c_1} - a - x^2}$$
$$y(x) \to \sqrt{-\sqrt{a^2 + 4ax^2 + 4c_1} - a - x^2}$$
$$y(x) \to -\sqrt{\sqrt{a^2 + 4ax^2 + 4c_1} - a - x^2}$$
$$y(x) \to \sqrt{\sqrt{a^2 + 4ax^2 + 4c_1} - a - x^2}$$

24.19 problem 681

Internal problem ID [3419]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 681.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _dAlembert]`

$$\boxed{(y^2+3x^2)\,yy'+x(x^2+3y^2)=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 119

```
dsolve((3*x^2+y(x)^2)*y(x)*diff(y(x),x)+x*(x^2+3*y(x)^2) = 0,y(x), singsol=all)
```

$$y(x)=\frac{\sqrt{-3c_1x^2-\sqrt{8c_1^2x^4+1}}}{\sqrt{c_1}}$$
$$y(x)=\frac{\sqrt{-3c_1x^2+\sqrt{8c_1^2x^4+1}}}{\sqrt{c_1}}$$
$$y(x)=-\frac{\sqrt{-3c_1x^2-\sqrt{8c_1^2x^4+1}}}{\sqrt{c_1}}$$
$$y(x)=-\frac{\sqrt{-3c_1x^2+\sqrt{8c_1^2x^4+1}}}{\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 8.621 (sec). Leaf size: 245

```
DSolve[(3 x^2+y[x]^2)y[x] y'[x]+x(x^2+3 y[x]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-3x^2 - \sqrt{8x^4 + e^{4c_1}}}$$
$$y(x) \to \sqrt{-3x^2 - \sqrt{8x^4 + e^{4c_1}}}$$
$$y(x) \to -\sqrt{-3x^2 + \sqrt{8x^4 + e^{4c_1}}}$$
$$y(x) \to \sqrt{-3x^2 + \sqrt{8x^4 + e^{4c_1}}}$$
$$y(x) \to -\sqrt{-2\sqrt{2}\sqrt{x^4} - 3x^2}$$
$$y(x) \to \sqrt{-2\sqrt{2}\sqrt{x^4} - 3x^2}$$
$$y(x) \to -\sqrt{2\sqrt{2}\sqrt{x^4} - 3x^2}$$
$$y(x) \to \sqrt{2\sqrt{2}\sqrt{x^4} - 3x^2}$$

24.20 problem 682

Internal problem ID [3420]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 682.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$(a - 3x^2 - y^2)\, yy' + x(y^2 - x^2 + a) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 122

```
dsolve((a-3*x^2-y(x)^2)*y(x)*diff(y(x),x)+x*(a-x^2+y(x)^2) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-\operatorname{LambertW}\left(-\left(-2x^2+a\right)c_1\mathrm{e}^2\right)\left(x^2 \operatorname{LambertW}\left(-\left(-2x^2+a\right)c_1\mathrm{e}^2\right) - 2x^2 + a\right)}}{\operatorname{LambertW}\left(-\left(-2x^2+a\right)c_1\mathrm{e}^2\right)}$$
$$y(x) = -\frac{\sqrt{-\operatorname{LambertW}\left(-\left(-2x^2+a\right)c_1\mathrm{e}^2\right)\left(x^2 \operatorname{LambertW}\left(-\left(-2x^2+a\right)c_1\mathrm{e}^2\right) - 2x^2 + a\right)}}{\operatorname{LambertW}\left(-\left(-2x^2+a\right)c_1\mathrm{e}^2\right)}$$

✓ Solution by Mathematica

Time used: 0.338 (sec). Leaf size: 39

```
DSolve[(a-3 x^2-y[x]^2)y[x] y'[x]+x(a-x^2+y[x]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\left(\frac{a+2y(x)^2}{x^2+y(x)^2} + \log\left(x^2+y(x)^2\right)\right) = c_1, y(x)\right]$$

24.21 problem 683

Internal problem ID [3421]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 683.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{2y'y^3 - x^3 + xy^2 = 0}$$

✓ Solution by Maple

Time used: 0.281 (sec). Leaf size: 711

```
dsolve(2*y(x)^3*diff(y(x),x) = x^3-x*y(x)^2,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}+\frac{2x^4c_1^2}{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}-2c_1x^2}}{2\sqrt{c_1}}$$

$$y(x) = \frac{\sqrt{2\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}+\frac{2x^4c_1^2}{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}-2c_1x^2}}{2\sqrt{c_1}}$$

$$y(x) = -\frac{\sqrt{-\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}-\frac{x^4c_1^2}{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}-2c_1x^2-2i\sqrt{3}\left(\frac{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}{2}-\frac{\cdots}{2\left(2+x^6\cdots\right.}\cdots}}{2\sqrt{c_1}}$$

$$y(x) = \frac{\sqrt{-\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}-\frac{x^4c_1^2}{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}-2c_1x^2-2i\sqrt{3}\left(\frac{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}{2}-\frac{\cdots}{2\left(2+x^6c_1^3+\cdots\right.}\cdots}}{2\sqrt{c_1}}$$

$$y(x) = -\frac{\sqrt{-\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}-\frac{x^4c_1^2}{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}-2c_1x^2+2i\sqrt{3}\left(\frac{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}{2}-\frac{\cdots}{2\left(2+x^6\cdots\right.}\cdots}}{2\sqrt{c_1}}$$

$$y(x) = \frac{\sqrt{-\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}-\frac{x^4c_1^2}{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}-2c_1x^2+2i\sqrt{3}\left(\frac{\left(2+x^6c_1^3+2\sqrt{x^6c_1^3+1}\right)^{\frac{1}{3}}}{2}-\frac{\cdots}{2\left(2+x^6c_1^3+\cdots\right.}\cdots}}{2\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 60.15 (sec). Leaf size: 714

```
DSolve[2 y[x]^3 y'[x]==x^3-x y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}-x^2+\frac{x^4}{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}-x^2+\frac{x^4}{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}}}}{\sqrt{2}}$$

$$y(x) \to -\frac{1}{2}\sqrt{\left(-1-i\sqrt{3}\right)\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}-2x^2+\frac{i\left(\sqrt{3}+i\right)x^4}{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}}}$$

$$y(x) \to \frac{1}{2}\sqrt{\left(-1-i\sqrt{3}\right)\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}-2x^2+\frac{i\left(\sqrt{3}+i\right)x^4}{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}}}$$

$$y(x) \to -\frac{1}{2}\sqrt{i\left(\sqrt{3}+i\right)\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}-2x^2+\frac{\left(-1-i\sqrt{3}\right)x^4}{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}}}$$

$$y(x) \to \frac{1}{2}\sqrt{i\left(\sqrt{3}+i\right)\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}-2x^2+\frac{\left(-1-i\sqrt{3}\right)x^4}{\sqrt[3]{x^6+2\sqrt{e^{24c_1}-e^{12c_1}x^6}-2e^{12c_1}}}}$$

24.22 problem 684

Internal problem ID [3422]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 684.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y(2y^2+1)\,y' - x(1+2x^2) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 113

```
dsolve(y(x)*(1+2*y(x)^2)*diff(y(x),x) = x*(2*x^2+1),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2-2\sqrt{4x^4+4x^2+8c_1+1}}}{2}$$

$$y(x) = \frac{\sqrt{-2-2\sqrt{4x^4+4x^2+8c_1+1}}}{2}$$

$$y(x) = -\frac{\sqrt{-2+2\sqrt{4x^4+4x^2+8c_1+1}}}{2}$$

$$y(x) = \frac{\sqrt{-2+2\sqrt{4x^4+4x^2+8c_1+1}}}{2}$$

✓ Solution by Mathematica

Time used: 2.326 (sec). Leaf size: 143

```
DSolve[y[x](1+2 y[x]^2)y'[x]==x(1+2 x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-1-\sqrt{(2x^2+1)^2+8c_1}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-1-\sqrt{(2x^2+1)^2+8c_1}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{-1+\sqrt{(2x^2+1)^2+8c_1}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-1+\sqrt{(2x^2+1)^2+8c_1}}}{\sqrt{2}}$$

24.23 problem 685

Internal problem ID [3423]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 685.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{(3x^2 + 2y^2)\, yy' + x^3 = 0}$$

✓ Solution by Maple

Time used: 0.219 (sec). Leaf size: 139

```
dsolve((3*x^2+2*y(x)^2)*y(x)*diff(y(x),x)+x^3 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-8c_1^2x^2 - 2\sqrt{8c_1^2x^2 + 1} + 2}}{4c_1}$$

$$y(x) = \frac{\sqrt{-8c_1^2x^2 - 2\sqrt{8c_1^2x^2 + 1} + 2}}{4c_1}$$

$$y(x) = -\frac{\sqrt{2}\,\sqrt{-4c_1^2x^2 + 1 + \sqrt{8c_1^2x^2 + 1}}}{4c_1}$$

$$y(x) = \frac{\sqrt{2}\,\sqrt{-4c_1^2x^2 + 1 + \sqrt{8c_1^2x^2 + 1}}}{4c_1}$$

✓ Solution by Mathematica

Time used: 21.513 (sec). Leaf size: 253

```
DSolve[(3 x^2+2 y[x]^2)y[x] y'[x]+x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-4x^2 - \sqrt{8e^{2c_1}x^2 + e^{4c_1}} + e^{2c_1}}}{2\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-4x^2 - \sqrt{8e^{2c_1}x^2 + e^{4c_1}} + e^{2c_1}}}{2\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{-4x^2 + \sqrt{8e^{2c_1}x^2 + e^{4c_1}} + e^{2c_1}}}{2\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-4x^2 + \sqrt{8e^{2c_1}x^2 + e^{4c_1}} + e^{2c_1}}}{2\sqrt{2}}$$

$$y(x) \to \text{Undefined}$$

$$y(x) \to -\frac{\sqrt{-x^2}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-x^2}}{\sqrt{2}}$$

24.24 problem 686

Internal problem ID [3424]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 686.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _dAlembert]`

$$\boxed{(5x^2 + 2y^2)\, yy' + x(x^2 + 5y^2) = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 125

```
dsolve((5*x^2+2*y(x)^2)*y(x)*diff(y(x),x)+x*(x^2+5*y(x)^2) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-10c_1x^2 - 2\sqrt{23c_1^2x^4 + 2}}}{2\sqrt{c_1}}$$

$$y(x) = \frac{\sqrt{-10c_1x^2 - 2\sqrt{23c_1^2x^4 + 2}}}{2\sqrt{c_1}}$$

$$y(x) = -\frac{\sqrt{-10c_1x^2 + 2\sqrt{23c_1^2x^4 + 2}}}{2\sqrt{c_1}}$$

$$y(x) = \frac{\sqrt{-10c_1x^2 + 2\sqrt{23c_1^2x^4 + 2}}}{2\sqrt{c_1}}$$

✓ Solution by Mathematica

Time used: 19.393 (sec). Leaf size: 295

```
DSolve[(5 x^2+2 y[x]^2)y[x] y'[x]+x(x^2+5 y[x]^2)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-5x^2 - \sqrt{23x^4 + 2e^{4c_1}}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-5x^2 - \sqrt{23x^4 + 2e^{4c_1}}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{-5x^2 + \sqrt{23x^4 + 2e^{4c_1}}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-5x^2 + \sqrt{23x^4 + 2e^{4c_1}}}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{-\sqrt{23}\sqrt{x^4} - 5x^2}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-\sqrt{23}\sqrt{x^4} - 5x^2}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\sqrt{23}\sqrt{x^4} - 5x^2}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\sqrt{23}\sqrt{x^4} - 5x^2}}{\sqrt{2}}$$

24.25 problem 687

Internal problem ID [3425]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 687.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{\left(x^{2}-x^{3}+3 x y^{2}+2 y^{3}\right) y'+2 x^{3}+3 x^{2} y+y^{2}-y^{3}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 662

```
dsolve((x^2-x^3+3*x*x*y(x)^2+2*y(x)^3)*diff(y(x),x)+2*x^3+3*x^2*y(x)+y(x)^2-y(x)^3 = 0,y(x), si
```

$$y(x) = \frac{\left(-108x^3 - 108c_1x + 12\sqrt{81x^6 + 162c_1x^4 + 81c_1^2x^2 + 12c_1^3 + 36c_1^2x + 36c_1x^2 + 12x^3}\right)^{\frac{1}{3}}}{6} - \frac{6\left(\frac{x}{3} + \frac{c_1}{3}\right)}{\left(-108x^3 - 108c_1x + 12\sqrt{81x^6 + 162c_1x^4 + 81c_1^2x^2 + 12c_1^3 + 36c_1^2x + 36c_1x^2 + 12x^3}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(-108x^3 - 108c_1x + 12\sqrt{81x^6 + 162c_1x^4 + 81c_1^2x^2 + 12c_1^3 + 36c_1^2x + 36c_1x^2 + 12x^3}\right)^{\frac{1}{3}}}{12} + \frac{x + c_1}{\left(-108x^3 - 108c_1x + 12\sqrt{81x^6 + 162c_1x^4 + 81c_1^2x^2 + 12c_1^3 + 36c_1^2x + 36c_1x^2 + 12x^3}\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(-108x^3-108c_1x+12\sqrt{81x^6+162c_1x^4+81c_1^2x^2+12c_1^3+36c_1^2x+36c_1x^2+12x^3}\right)^{\frac{1}{3}}}{6} + \frac{2c_1+2x}{\left(-108x^3-108c_1x+12\sqrt{81x^6+162c_1x^4+81c_1^2 \cdots}\right.}\right.}{2}$$

$$y(x) = -\frac{\left(-108x^3 - 108c_1x + 12\sqrt{81x^6 + 162c_1x^4 + 81c_1^2x^2 + 12c_1^3 + 36c_1^2x + 36c_1x^2 + 12x^3}\right)^{\frac{1}{3}}}{12} + \frac{x + c_1}{\left(-108x^3 - 108c_1x + 12\sqrt{81x^6 + 162c_1x^4 + 81c_1^2x^2 + 12c_1^3 + 36c_1^2x + 36c_1x^2 + 12x^3}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(-108x^3-108c_1x+12\sqrt{81x^6+162c_1x^4+81c_1^2x^2+12c_1^3+36c_1^2x+36c_1x^2+12x^3}\right)^{\frac{1}{3}}}{6} + \frac{2c_1+2x}{\left(-108x^3-108c_1x+12\sqrt{81x^6+162c_1x^4+81c_1^2 \cdots}\right.}\right.}{2}$$

✓ Solution by Mathematica

Time used: 8.305 (sec). Leaf size: 348

```
DSolve[(x^2-x^3+3 x y[x]^2+2 y[x]^3)y'[x]+2 x^3+3 x^2 y[x]+y[x]^2-y[x]^3==0,y[x],x,IncludeSin
```

$$y(x) \to \frac{\sqrt[3]{2}(x+c_1)}{\sqrt[3]{27x^3+\sqrt{729(x^3+c_1x)^2+108(x+c_1)^3}+27c_1x}} - \frac{\sqrt[3]{27x^3+\sqrt{729(x^3+c_1x)^2+108(x+c_1)^3}+27c_1x}}{3\sqrt[3]{2}}$$

$$y(x) \to \frac{(1-i\sqrt{3})\sqrt[3]{27x^3+\sqrt{729(x^3+c_1x)^2+108(x+c_1)^3}+27c_1x}}{6\sqrt[3]{2}} - \frac{\sqrt[3]{-2}(x+c_1)}{\sqrt[3]{27x^3+\sqrt{729(x^3+c_1x)^2+108(x+c_1)^3}+27c_1x}}$$

$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{2}(x+c_1)}{\sqrt[3]{27x^3+\sqrt{729(x^3+c_1x)^2+108(x+c_1)^3}+27c_1x}} + \frac{(1+i\sqrt{3})\sqrt[3]{27x^3+\sqrt{729(x^3+c_1x)^2+108(x+c_1)^3}+27c_1x}}{6\sqrt[3]{2}}$$

$$y(x) \to -x$$

24.26 problem 688

Internal problem ID [3426]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 688.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _exact, _rational, _dAlembert]`

$$\left(3x^3+6x^2y-3xy^2+20y^3\right)y'+4x^3+9x^2y+6xy^2-y^3=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 50

```
dsolve((3*x^3+6*x^2*y(x)-3*x*y(x)^2+20*y(x)^3)*diff(y(x),x)+4*x^3+9*x^2*y(x)+6*x*y(x)^2-y(x)^
```

$$y(x)=\frac{\operatorname{RootOf}\left(c_1^4x^4+3\_Zc_1^3x^3+3\_Z^2c_1^2x^2-\_Z^3c_1x+5\_Z^4-1\right)}{c_1}$$

✓ Solution by Mathematica

Time used: 60.176 (sec). Leaf size: 2201

```
DSolve[(3 x^3+6 x^2 y[x]-3 x y[x]^2+20 y[x]^3)y'[x]+4 x^3+9 x^2 y[x]+6 x y[x]^2-y[x]^3==0,y[x
```

$$y(x) \to \frac{1}{2}\sqrt{-\frac{39x^2}{100}+\frac{\sqrt[3]{99x^6+351e^{c_1}x^2+\sqrt{3}\sqrt{-67037x^{12}+185406e^{c_1}x^8-83733e^{2c_1}x^4+32000e^{3c_1}}}}{5\sqrt[3]{2}3^{2/3}}+\frac{\cdots}{5\sqrt[3]{99x\cdots}}\cdots}$$

$$-\frac{1}{2}\sqrt{-\frac{39x^2}{50}-\frac{\sqrt[3]{99x^6+351e^{c_1}x^2+\sqrt{3}\sqrt{-67037x^{12}+185406e^{c_1}x^8-83733e^{2c_1}x^4+32000e^{3c_1}}}}{5\sqrt[3]{2}3^{2/3}}+\frac{\cdots}{5\sqrt[3]{99\cdots}}\cdots}$$

$$+\frac{x}{20}$$

$$y(x) \to \frac{1}{2}\sqrt{-\frac{39x^2}{100}+\frac{\sqrt[3]{99x^6+351e^{c_1}x^2+\sqrt{3}\sqrt{-67037x^{12}+185406e^{c_1}x^8-83733e^{2c_1}x^4+32000e^{3c_1}}}}{5\sqrt[3]{2}3^{2/3}}+\frac{\cdots}{5\sqrt[3]{99x\cdots}}\cdots}$$

$$+\frac{1}{2}\sqrt{-\frac{39x^2}{50}-\frac{\sqrt[3]{99x^6+351e^{c_1}x^2+\sqrt{3}\sqrt{-67037x^{12}+185406e^{c_1}x^8-83733e^{2c_1}x^4+32000e^{3c_1}}}}{5\sqrt[3]{2}3^{2/3}}+\frac{\cdots}{5\sqrt[3]{99\cdots}}\cdots}$$

$$+\frac{x}{20}$$

$$y(x) \to$$

$$-\frac{1}{2}\sqrt{-\frac{39x^2}{100}+\frac{\sqrt[3]{99x^6+351e^{c_1}x^2+\sqrt{3}\sqrt{-67037x^{12}+185406e^{c_1}x^8-83733e^{2c_1}x^4+32000e^{3c_1}}}}{5\sqrt[3]{2}3^{2/3}}+\frac{\cdots}{5\sqrt[3]{99\cdots}}\cdots}$$

$$-\frac{1}{2}\sqrt{-\frac{39x^2}{50}-\frac{\sqrt[3]{99x^6+351e^{c_1}x^2+\sqrt{3}\sqrt{-67037x^{12}+185406e^{c_1}x^8-83733e^{2c_1}x^4+32000e^{3c_1}}}}{5\sqrt[3]{2}3^{2/3}}+\frac{\cdots}{5\sqrt[3]{99\cdots}}\cdots}$$

$$+\frac{x}{20}$$

24.27 problem 689

Internal problem ID [3427]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 689.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{\left(x^3 + a y^3\right) y' - x^2 y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 23

```
dsolve((x^3+a*y(x)^3)*diff(y(x),x) = x^2*y(x),y(x), singsol=all)
```

$$y(x) = \left(\frac{1}{a \operatorname{LambertW}\left(\frac{c_1 x^3}{a}\right)}\right)^{\frac{1}{3}} x$$

✓ Solution by Mathematica

Time used: 17.987 (sec). Leaf size: 113

```
DSolve[(x^3+a y[x]^3)y'[x]==x^2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{\sqrt[3]{a}\sqrt[3]{W\left(\frac{x^3 e^{-\frac{3c_1}{a}}}{a}\right)}}$$

$$y(x) \to -\frac{\sqrt[3]{-1}x}{\sqrt[3]{a}\sqrt[3]{W\left(\frac{x^3 e^{-\frac{3c_1}{a}}}{a}\right)}}$$

$$y(x) \to \frac{(-1)^{2/3}x}{\sqrt[3]{a}\sqrt[3]{W\left(\frac{x^3 e^{-\frac{3c_1}{a}}}{a}\right)}}$$

$$y(x) \to 0$$

24.28 problem 691

Internal problem ID [3428]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 691.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{x y^{3} y'-\left(1-x^{2}\right)\left(1+y^{2}\right)=0}$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 29

```
dsolve(x*y(x)^3*diff(y(x),x) = (-x^2+1)*(1+y(x)^2),y(x), singsol=all)
```

$$\frac{x^{2}}{2}-\ln \left(x\right)+\frac{y \left(x\right)^{2}}{2}-\frac{\ln \left(1+y \left(x\right)^{2}\right)}{2}+c_{1}=0$$

✓ Solution by Mathematica

Time used: 60.086 (sec). Leaf size: 61

```
DSolve[x y[x]^3 y'[x]==(1-x^2)(1+y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\sqrt{-1-W\left(-\frac{e^{x^2-1-2c_1}}{x^2}\right)}$$
$$y(x)\to \sqrt{-1-W\left(-\frac{e^{x^2-1-2c_1}}{x^2}\right)}$$

24.29 problem 692

Internal problem ID [3429]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 692.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{x(x-y^3)\,y'-(3x+y^3)\,y=0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 348

```
dsolve(x*(x-y(x)^3)*diff(y(x),x) = (3*x+y(x)^3)*y(x),y(x), singsol=all)
```

$$y(x) = \left(\frac{\left(-27x^4+3\sqrt{81x^8-3\,\mathrm{e}^{8c_1}}\right)^{\frac{1}{3}}}{3x^4}+\frac{\mathrm{e}^{\frac{8c_1}{3}}}{x^4\left(-27x^4+3\sqrt{81x^8-3\,\mathrm{e}^{8c_1}}\right)^{\frac{1}{3}}}\right)x^3$$

$$y(x) = \left(-\frac{\left(-27x^4+3\sqrt{81x^8-3\,\mathrm{e}^{8c_1}}\right)^{\frac{1}{3}}}{6x^4}-\frac{\mathrm{e}^{\frac{8c_1}{3}}}{2x^4\left(-27x^4+3\sqrt{81x^8-3\,\mathrm{e}^{8c_1}}\right)^{\frac{1}{3}}}-\frac{i\sqrt{3}\left(\frac{\left(-27x^4+3\sqrt{81x^8-3\,\mathrm{e}^{8c_1}}\right)^{\frac{1}{3}}}{3x^4}-\frac{\mathrm{e}^{\frac{8c_1}{3}}}{x^4\left(-27x^4+3\sqrt{81x^8-3\,\mathrm{e}^{8c_1}}\right)^{\frac{1}{3}}}\right)}{2}\right)x^3$$

$$y(x) = \left(-\frac{\left(-27x^4+3\sqrt{81x^8-3\,\mathrm{e}^{8c_1}}\right)^{\frac{1}{3}}}{6x^4}-\frac{\mathrm{e}^{\frac{8c_1}{3}}}{2x^4\left(-27x^4+3\sqrt{81x^8-3\,\mathrm{e}^{8c_1}}\right)^{\frac{1}{3}}}+\frac{i\sqrt{3}\left(\frac{\left(-27x^4+3\sqrt{81x^8-3\,\mathrm{e}^{8c_1}}\right)^{\frac{1}{3}}}{3x^4}-\frac{\mathrm{e}^{\frac{8c_1}{3}}}{x^4\left(-27x^4+3\sqrt{81x^8-3\,\mathrm{e}^{8c_1}}\right)^{\frac{1}{3}}}\right)}{2}\right)x^3$$

✓ Solution by Mathematica

Time used: 60.204 (sec). Leaf size: 356

```
DSolve[x(x-y[x]^3)y'[x]==(3 x+y[x]^3)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{\frac{8c_1}{3}}}{\sqrt[3]{-27x^7 + 3\sqrt{3}\sqrt{-x^6\left(-27x^8 + e^{8c_1}\right)}}} + \frac{\sqrt[3]{-9x^7 + \sqrt{3}\sqrt{-x^6\left(-27x^8 + e^{8c_1}\right)}}}{3^{2/3}x^2}$$

$$y(x) \to \frac{\frac{i\sqrt[6]{3}\left(\sqrt{3}+i\right)\left(-9x^7+\sqrt{3}\sqrt{-x^6\left(-27x^8+e^{8c_1}\right)}\right)^{2/3}}{x^2} - \left(\sqrt{3}+3i\right)e^{\frac{8c_1}{3}}}{2\ 3^{5/6}\sqrt[3]{-9x^7 + \sqrt{3}\sqrt{-x^6\left(-27x^8 + e^{8c_1}\right)}}}$$

$$y(x) \to \frac{\frac{\left(-1-i\sqrt{3}\right)\left(-9x^7+\sqrt{3}\sqrt{-x^6\left(-27x^8+e^{8c_1}\right)}\right)^{2/3}}{x^2} + i\sqrt[3]{3}\left(\sqrt{3}+i\right)e^{\frac{8c_1}{3}}}{2\ 3^{2/3}\sqrt[3]{-9x^7 + \sqrt{3}\sqrt{-x^6\left(-27x^8 + e^{8c_1}\right)}}}$$

24.30 problem 693

Internal problem ID [3430]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 693.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{x(2x^3+y^3)\,y' - (2x^3-x^2y+y^3)\,y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 443

```
dsolve(x*(2*x^3+y(x)^3)*diff(y(x),x) = (2*x^3-x^2*y(x)+y(x)^3)*y(x),y(x), singsol=all)
```

$$y(x) = \left(\frac{\left(54+6\sqrt{6c_1^3+18c_1^2\ln(x)+18c_1\ln(x)^2+6\ln(x)^3+81}\right)^{\frac{1}{3}}}{3} - \frac{3\left(\frac{2c_1}{3}+\frac{2\ln(x)}{3}\right)}{\left(54+6\sqrt{6c_1^3+18c_1^2\ln(x)+18c_1\ln(x)^2+6\ln(x)^3+81}\right)^{\frac{1}{3}}}\right)x$$

$$y(x) = \left(-\frac{\left(54+6\sqrt{6c_1^3+18c_1^2\ln(x)+18c_1\ln(x)^2+6\ln(x)^3+81}\right)^{\frac{1}{3}}}{6} + \frac{\ln(x)+c_1}{\left(54+6\sqrt{6c_1^3+18c_1^2\ln(x)+18c_1\ln(x)^2+6\ln(x)^3+81}\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(54+6\sqrt{6c_1^3+18c_1^2\ln(x)+18c_1\ln(x)^2+6\ln(x)^3+81}\right)^{\frac{1}{3}}}{3}+\frac{2c_1+2\ln(x)}{\left(54+6\sqrt{6c_1^3+18c_1^2\ln(x)+18c_1\ln(x)^2+6\ln(x)^3+81}\right)^{\frac{1}{3}}}\right)}{2}\right)x$$

$$y(x) = \left(-\frac{\left(54+6\sqrt{6c_1^3+18c_1^2\ln(x)+18c_1\ln(x)^2+6\ln(x)^3+81}\right)^{\frac{1}{3}}}{6} + \frac{\ln(x)+c_1}{\left(54+6\sqrt{6c_1^3+18c_1^2\ln(x)+18c_1\ln(x)^2+6\ln(x)^3+81}\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(54+6\sqrt{6c_1^3+18c_1^2\ln(x)+18c_1\ln(x)^2+6\ln(x)^3+81}\right)^{\frac{1}{3}}}{3}+\frac{2c_1+2\ln(x)}{\left(54+6\sqrt{6c_1^3+18c_1^2\ln(x)+18c_1\ln(x)^2+6\ln(x)^3+81}\right)^{\frac{1}{3}}}\right)}{2}\right)x$$

✓ Solution by Mathematica

Time used: 50.296 (sec). Leaf size: 336

```
DSolve[x(2 x^3+y[x]^3)y'[x]==(2 x^3-x^2 y[x]+y[x]^3)y[x],y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to \frac{6^{2/3}x^2(-\log(x)+c_1)+\sqrt[3]{6}\left(9x^3+\sqrt{3}\sqrt{x^6\left(27+2(\log(x)-c_1)^3\right)}\right)^{2/3}}{3\sqrt[3]{9x^3+\sqrt{3}\sqrt{x^6\left(27+2(\log(x)-c_1)^3\right)}}}$$

$$y(x) \to \frac{i(\sqrt{3}+i)\sqrt[3]{9x^3+\sqrt{3}\sqrt{x^6\left(27+2(\log(x)-c_1)^3\right)}}}{6^{2/3}} + \frac{\left(1+i\sqrt{3}\right)x^2(\log(x)-c_1)}{\sqrt[3]{6}\sqrt[3]{9x^3+\sqrt{3}\sqrt{x^6\left(27+2(\log(x)-c_1)^3\right)}}}$$

$$y(x) \to \frac{(-6)^{2/3}x^2(-\log(x)+c_1)-\sqrt[3]{-6}\left(9x^3+\sqrt{3}\sqrt{x^6\left(27+2(\log(x)-c_1)^3\right)}\right)^{2/3}}{3\sqrt[3]{9x^3+\sqrt{3}\sqrt{x^6\left(27+2(\log(x)-c_1)^3\right)}}}$$

24.31 problem 694

Internal problem ID [3431]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 694.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{x\left(2x^{3}-y^{3}\right)y'-\left(x^{3}-2y^{3}\right)y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 355

```
dsolve(x*(2*x^3-y(x)^3)*diff(y(x),x) = (x^3-2*y(x)^3)*y(x),y(x), singsol=all)
```

$$y(x) = \left(\frac{\left(-108+8x^3c_1^3+12\sqrt{-12x^3c_1^3+81}\right)^{\frac{1}{3}}}{6} + \frac{2x^2c_1^2}{3\left(-108+8x^3c_1^3+12\sqrt{-12x^3c_1^3+81}\right)^{\frac{1}{3}}} + \frac{c_1x}{3}\right)x$$

$$y(x) = \left(-\frac{\left(-108+8x^3c_1^3+12\sqrt{-12x^3c_1^3+81}\right)^{\frac{1}{3}}}{12} - \frac{x^2c_1^2}{3\left(-108+8x^3c_1^3+12\sqrt{-12x^3c_1^3+81}\right)^{\frac{1}{3}}} + \frac{c_1x}{3} - \frac{i\sqrt{3}\left(\frac{\left(-108+8x^3c_1^3+12\sqrt{-12x^3c_1^3+81}\right)^{\frac{1}{3}}}{6} - \frac{2x^2c_1^2}{3\left(-108+8x^3c_1^3+12\sqrt{-12x^3c_1^3+81}\right)^{\frac{1}{3}}}\right)}{2}\right)x$$

$$y(x) = \left(-\frac{\left(-108+8x^3c_1^3+12\sqrt{-12x^3c_1^3+81}\right)^{\frac{1}{3}}}{12} - \frac{x^2c_1^2}{3\left(-108+8x^3c_1^3+12\sqrt{-12x^3c_1^3+81}\right)^{\frac{1}{3}}} + \frac{c_1x}{3} + \frac{i\sqrt{3}\left(\frac{\left(-108+8x^3c_1^3+12\sqrt{-12x^3c_1^3+81}\right)^{\frac{1}{3}}}{6} - \frac{2x^2c_1^2}{3\left(-108+8x^3c_1^3+12\sqrt{-12x^3c_1^3+81}\right)^{\frac{1}{3}}}\right)}{2}\right)x$$

✓ Solution by Mathematica

Time used: 42.942 (sec). Leaf size: 542

```
DSolve[x(2 x^3-y[x]^3)y'[x]==(x^3-2 y[x]^3)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(e^{c_1}x^2 + \frac{\sqrt[3]{2e^{3c_1}x^6 - 27x^3 + 3\sqrt{81x^6 - 12e^{3c_1}x^9}}}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}e^{2c_1}x^4}{\sqrt[3]{2e^{3c_1}x^6 - 27x^3 + 3\sqrt{81x^6 - 12e^{3c_1}x^9}}}\right)$$

$$y(x) \to \frac{e^{c_1}x^2}{3} + \frac{i(\sqrt{3}+i)\sqrt[3]{2e^{3c_1}x^6 - 27x^3 + 3\sqrt{81x^6 - 12e^{3c_1}x^9}}}{6\sqrt[3]{2}} - \frac{i(\sqrt{3}-i)e^{2c_1}x^4}{3\ 2^{2/3}\sqrt[3]{2e^{3c_1}x^6 - 27x^3 + 3\sqrt{81x^6 - 12e^{3c_1}x^9}}}$$

$$y(x) \to \frac{e^{c_1}x^2}{3} - \frac{i(\sqrt{3}-i)\sqrt[3]{2e^{3c_1}x^6 - 27x^3 + 3\sqrt{81x^6 - 12e^{3c_1}x^9}}}{6\sqrt[3]{2}} + \frac{i(\sqrt{3}+i)e^{2c_1}x^4}{3\ 2^{2/3}\sqrt[3]{2e^{3c_1}x^6 - 27x^3 + 3\sqrt{81x^6 - 12e^{3c_1}x^9}}}$$

$$y(x) \to \frac{\sqrt[3]{\sqrt{x^6} - x^3}}{\sqrt[3]{2}}$$

$$y(x) \to -\frac{i(\sqrt{3}-i)\sqrt[3]{\sqrt{x^6} - x^3}}{2\sqrt[3]{2}}$$

$$y(x) \to \frac{i(\sqrt{3}+i)\sqrt[3]{\sqrt{x^6} - x^3}}{2\sqrt[3]{2}}$$

24.32 problem 695

Internal problem ID [3432]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 695.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$x(x^3 + 3x^2y + y^3)\, y' - (y^2 + 3x^2)\, y^2 = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve(x*(x^3+3*x^2*y(x)+y(x)^3)*diff(y(x),x) = (3*x^2+y(x)^2)*y(x)^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(\mathrm{e}^{3\_Z}+9\,\mathrm{e}^{\_Z}+3c_1+3\_Z+3\ln(x)\right)}x$$

✓ Solution by Mathematica

Time used: 0.157 (sec). Leaf size: 37

```
DSolve[x(x^3+3 x^2 y[x]+y[x]^3)y'[x]==(3 x^2+y[x]^2)y[x]^2,y[x],x,IncludeSingularSolutions ->
```

$$\mathrm{Solve}\left[\frac{y(x)^3}{3x^3} + \frac{3y(x)}{x} + \log\left(\frac{y(x)}{x}\right) = -\log(x) + c_1, y(x)\right]$$

24.33 problem 696

Internal problem ID [3433]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 696.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{x(x^3 - 2y^3)\, y' - (2x^3 - y^3)\, y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 437

```
dsolve(x*(x^3-2*y(x)^3)*diff(y(x),x) = (2*x^3-y(x)^3)*y(x),y(x), singsol=all)
```

$$y(x) = \frac{\left(-12x\left(9c_1x^2-\sqrt{3}\,\sqrt{\frac{x(27x^3c_1^3-4)}{c_1}}\right)c_1^2\right)^{\frac{1}{3}}}{6c_1} + \frac{2x}{\left(-12x\left(9c_1x^2-\sqrt{3}\,\sqrt{\frac{x(27x^3c_1^3-4)}{c_1}}\right)c_1^2\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(-12x\left(9c_1x^2-\sqrt{3}\,\sqrt{\frac{x(27x^3c_1^3-4)}{c_1}}\right)c_1^2\right)^{\frac{1}{3}}}{12c_1} - \frac{x}{\left(-12x\left(9c_1x^2-\sqrt{3}\,\sqrt{\frac{x(27x^3c_1^3-4)}{c_1}}\right)c_1^2\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(-12x\left(9c_1x^2-\sqrt{3}\,\sqrt{\frac{x(27x^3c_1^3-4)}{c_1}}\right)c_1^2\right)^{\frac{1}{3}}}{6c_1} - \frac{2x}{\left(-12x\left(9c_1x^2-\sqrt{3}\,\sqrt{\frac{x(27x^3c_1^3-4)}{c_1}}\right)c_1^2\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(-12x\left(9c_1x^2-\sqrt{3}\,\sqrt{\frac{x(27x^3c_1^3-4)}{c_1}}\right)c_1^2\right)^{\frac{1}{3}}}{12c_1} - \frac{x}{\left(-12x\left(9c_1x^2-\sqrt{3}\,\sqrt{\frac{x(27x^3c_1^3-4)}{c_1}}\right)c_1^2\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(-12x\left(9c_1x^2-\sqrt{3}\,\sqrt{\frac{x(27x^3c_1^3-4)}{c_1}}\right)c_1^2\right)^{\frac{1}{3}}}{6c_1} - \frac{2x}{\left(-12x\left(9c_1x^2-\sqrt{3}\,\sqrt{\frac{x(27x^3c_1^3-4)}{c_1}}\right)c_1^2\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 60.222 (sec). Leaf size: 294

```
DSolve[x(x^3-2 y[x]^3)y'[x]==(2 x^3-y[x]^3)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{2}\left(-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}\right)^{2/3}+2\sqrt[3]{3}e^{c_1}x}{6^{2/3}\sqrt[3]{-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}}}$$

$$y(x) \to \frac{(-1)^{2/3}\sqrt[3]{2}\left(-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}\right)^{2/3}-2\sqrt[3]{-3}e^{c_1}x}{6^{2/3}\sqrt[3]{-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}}}$$

$$y(x) \to \frac{-\sqrt[3]{-2}\sqrt[6]{3}\left(-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}\right)^{2/3}-\left(\left(\sqrt{3}-3i\right)e^{c_1}x\right)}{2^{2/3}3^{5/6}\sqrt[3]{-9x^3+\sqrt{81x^6-12e^{3c_1}x^3}}}$$

24.34 problem 697

Internal problem ID [3434]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 24
Problem number: 697.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x(x^4 - 2y^3)\, y' + (2x^4 + y^3)\, y = 0$$

✓ Solution by Maple

Time used: 0.563 (sec). Leaf size: 31

```
dsolve(x*(x^4-2*y(x)^3)*diff(y(x),x)+(2*x^4+y(x)^3)*y(x) = 0,y(x), singsol=all)
```

$$\ln(x) - c_1 + \frac{3\ln\left(-\frac{y(x)\left(2x^4 - y(x)^3\right)}{x^{\frac{16}{3}}}\right)}{10} = 0$$

✓ Solution by Mathematica

Time used: 60.147 (sec). Leaf size: 1139

```
DSolve[x(x^4-2 y[x]^3)y'[x]+(2 x^4+y[x]^3)y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(-\sqrt[6]{2}3^{2/3}\sqrt{\frac{4\sqrt[3]{6}c_1x^2+\left(9x^8-\sqrt{81x^{16}-384c_1{}^3x^6}\right)^{2/3}}{\sqrt[3]{9x^8-\sqrt{81x^{16}-384c_1{}^3x^6}}}}\right.$$

$$-3\sqrt{-\frac{\sqrt[3]{18x^8-2\sqrt{81x^{16}-384c_1{}^3x^6}}}{3^{2/3}}-\frac{4\,2^{2/3}c_1x^2}{\sqrt[3]{27x^8-3\sqrt{81x^{16}-384c_1{}^3x^6}}}-\frac{4\sqrt{3}x^4}{\sqrt{\frac{4\,6^{2/3}c_1x^2+\sqrt[3]{6}\left(9x^8-\sqrt{81x^{16}-\ldots\right.}}{\sqrt[3]{9x^8-\sqrt{81x^{16}-38\ldots}}}}}}$$

$$y(x) \to \frac{1}{6}\left(3\sqrt{-\frac{\sqrt[3]{18x^8-2\sqrt{81x^{16}-384c_1{}^3x^6}}}{3^{2/3}}-\frac{4\,2^{2/3}c_1x^2}{\sqrt[3]{27x^8-3\sqrt{81x^{16}-384c_1{}^3x^6}}}-\frac{4\sqrt{3}x^4}{\sqrt{\frac{4\,6^{2/3}c_1x^2+\sqrt[3]{6}\left(9x^8-\sqrt{81x^{16}\ldots\right.}}{\sqrt[3]{9x^8-\sqrt{81x^{16}-\ldots}}}}}}\right.$$

$$\left.-\sqrt[6]{2}3^{2/3}\sqrt{\frac{4\sqrt[3]{6}c_1x^2+\left(9x^8-\sqrt{81x^{16}-384c_1{}^3x^6}\right)^{2/3}}{\sqrt[3]{9x^8-\sqrt{81x^{16}-384c_1{}^3x^6}}}}\right)$$

$$y(x) \to \frac{1}{6}\left(\sqrt[6]{2}3^{2/3}\sqrt{\frac{4\sqrt[3]{6}c_1x^2+\left(9x^8-\sqrt{81x^{16}-384c_1{}^3x^6}\right)^{2/3}}{\sqrt[3]{9x^8-\sqrt{81x^{16}-384c_1{}^3x^6}}}}\right.$$

$$-3\sqrt{-\frac{\sqrt[3]{18x^8-2\sqrt{81x^{16}-384c_1{}^3x^6}}}{3^{2/3}}-\frac{4\,2^{2/3}c_1x^2}{\sqrt[3]{27x^8-3\sqrt{81x^{16}-384c_1{}^3x^6}}}+\frac{4\sqrt{3}x^4}{\sqrt{\frac{4\,6^{2/3}c_1x^2+\sqrt[3]{6}\left(9x^8-\sqrt{81x^{16}-\ldots\right.}}{\sqrt[3]{9x^8-\sqrt{81x^{16}-38\ldots}}}}}}$$

$$y(x) \to \frac{1}{6}\left(\ldots\sqrt{\frac{4\sqrt[3]{6}c_1x^2+\left(9x^8-\sqrt{81x^{16}-384c_1{}^3x^6}\right)^{2/3}}{\ldots}}\right.$$

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25.1 problem 698

Internal problem ID [3435]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 698.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{x(x + y + 2y^3)\, y' - y(x - y) = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 29

```
dsolve(x*(x+y(x)+2*y(x)^3)*diff(y(x),x) = (x-y(x))*y(x),y(x), singsol=all)
```

$$y(x) = e^{\mathrm{RootOf}\left(-e^{3\_Z} - e^{\_Z}\ln(x) + e^{\_Z}c_1 - \_Z e^{\_Z} + x\right)}$$

✓ Solution by Mathematica

Time used: 0.213 (sec). Leaf size: 23

```
DSolve[x(x+y[x]+2 y[x]^3)y'[x]==(x-y[x])y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[y(x)^2 - \frac{x}{y(x)} + \log(y(x)) + \log(x) = c_1, y(x)\right]$$

25.2 problem 699

Internal problem ID [3436]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 699.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(5x - y - 7xy^3)\, y' + 5y - y^4 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 33

```
dsolve((5*x-y(x)-7*x*y(x)^3)*diff(y(x),x)+5*y(x)-y(x)^4 = 0,y(x), singsol=all)
```

$$x - \frac{-\frac{y(x)^5}{5} + \frac{5y(x)^2}{2} + c_1}{y(x)\left(y(x)^3 - 5\right)^2} = 0$$

✓ Solution by Mathematica

Time used: 49.44 (sec). Leaf size: 342

```
DSolve[(5 x-y[x]-7 x y[x]^3)y'[x]+5 y[x]-y[x]^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 1\right]$$
$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 2\right]$$
$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 3\right]$$
$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 4\right]$$
$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 5\right]$$
$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 6\right]$$
$$y(x) \to \text{Root}\left[10\#1^7x + 2\#1^5 - 100\#1^4x - 25\#1^2 + 250\#1x - 10c_1\&, 7\right]$$
$$y(x) \to 0$$
$$y(x) \to -\sqrt[3]{-5}$$
$$y(x) \to \sqrt[3]{5}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{5}$$

25.3 problem 700

Internal problem ID [3437]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 700.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{x(1-2xy^3)\, y' + (1-2x^3y)\, y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 522

```
dsolve(x*(1-2*x*y(x)^3)*diff(y(x),x)+(1-2*x^3*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}{6x} - \frac{6\left(\frac{x^2}{3}-\frac{c_1}{3}\right)x}{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}{12x} + \frac{3\left(\frac{x^2}{3}-\frac{c_1}{3}\right)x}{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}{6x} + \frac{6\left(\frac{x^2}{3}-\frac{c_1}{3}\right)x}{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}{12x} + \frac{3\left(\frac{x^2}{3}-\frac{c_1}{3}\right)x}{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}{6x} + \frac{6\left(\frac{x^2}{3}-\frac{c_1}{3}\right)x}{\left(\left(-108+12\sqrt{12x^8-36c_1x^6+36c_1^2x^4-12c_1^3x^2+81}\right)x^2\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 44.293 (sec). Leaf size: 358

```
DSolve[x(1-2 x y[x]^3)y'[x]+(1-2 x^3 y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{2}(-x^3+c_1x)}{\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3\left(x^3-c_1x\right)^3}}}+\frac{\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3\left(x^3-c_1x\right)^3}}}{3\sqrt[3]{2}x}$$

$$y(x) \to \frac{\left(1+i\sqrt{3}\right)\left(x^3-c_1x\right)}{2^{2/3}\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3\left(x^3-c_1x\right)^3}}} - \frac{\left(1-i\sqrt{3}\right)\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3\left(x^3-c_1x\right)^3}}}{6\sqrt[3]{2}x}$$

$$y(x) \to \frac{\left(1-i\sqrt{3}\right)\left(x^3-c_1x\right)}{2^{2/3}\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3\left(x^3-c_1x\right)^3}}} - \frac{\left(1+i\sqrt{3}\right)\sqrt[3]{-27x^2+\sqrt{729x^4+108x^3\left(x^3-c_1x\right)^3}}}{6\sqrt[3]{2}x}$$

25.4 problem 701

Internal problem ID [3438]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 701.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$x(2 - xy^2 - 2xy^3)\, y' + 1 + 2y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 46

```
dsolve(x*(2-x*y(x)^2-2*x*y(x)^3)*diff(y(x),x)+1+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2}$$
$$y(x) = \frac{e^{\mathrm{RootOf}\left(x\,e^{3\_Z} - 4x\,e^{2\_Z} + 8c_1 x\,e^{\_Z} + 2\_Z\,e^{\_Z} x + 3x\,e^{\_Z} + 16\right)}}{2} - \frac{1}{2}$$

✓ Solution by Mathematica

Time used: 0.298 (sec). Leaf size: 47

```
DSolve[x(2-x y[x]^2-2 x y[x]^3)y'[x]+1+2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{1}{64}\left(-4y(x)^2 + 4y(x) - 2\log(8y(x) + 4) + 3\right) - \frac{1}{4x(2y(x) + 1)} = c_1, y(x)\right]$$

25.5 problem 702

Internal problem ID [3439]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 702.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact, _rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]'`

$$\boxed{(2 - 10x^2y^3 + 3y^2)\, y' - x(1 + 5y^4) = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 26

```
dsolve((2-10*x^2*y(x)^3+3*y(x)^2)*diff(y(x),x) = x*(1+5*y(x)^4),y(x), singsol=all)
```

$$\frac{(-5y(x)^4 - 1)\, x^2}{2} + y(x)^3 + 2y(x) + c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.223 (sec). Leaf size: 2097

```
DSolve[(2-10 x^2 y[x]^3+3 y[x]^2)y'[x]==x(1+5 y[x]^4),y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to$$

$$-\frac{\sqrt{3}x^2\sqrt{\frac{5\sqrt[3]{6}x^2\sqrt[3]{189x^2+\sqrt{3}\sqrt{27\left(21x^2-2c_1\right)^2-16\left(5x^4-10c_1x^2-2\right)^3}-18c_1}+\frac{\cdots}{\sqrt[3]{189x^2+\sqrt{3}\sqrt{27\cdots}}}}{x^4}}}{\cdots}$$

$$y(x) \to \frac{-\sqrt{3}x^2\sqrt{\frac{5\sqrt[3]{6}x^2\sqrt[3]{189x^2+\sqrt{3}\sqrt{27\left(21x^2-2c_1\right)^2-16\left(5x^4-10c_1x^2-2\right)^3}-18c_1}+\frac{\cdots}{\sqrt[3]{189x^2+\sqrt{3}\sqrt{27\cdots}}}}{x^4}}}{\cdots}$$

$$y(x) \to \frac{\sqrt{3}x^2\sqrt{\frac{5\sqrt[3]{6}x^2\sqrt[3]{189x^2+\sqrt{3}\sqrt{27\left(21x^2-2c_1\right)^2-16\left(5x^4-10c_1x^2-2\right)^3}-18c_1}+\frac{\cdots}{\sqrt[3]{189x^2+\sqrt{3}\sqrt{27(2\cdots}}}}{x^4}}}{\cdots}$$

$$y(x) \to \frac{\sqrt{3}x^2\sqrt{\frac{5\sqrt[3]{6}x^2\sqrt[3]{189x^2+\sqrt{3}\sqrt{27\left(21x^2-2c_1\right)^2-16\left(5x^4-10c_1x^2-2\right)^3}-18c_1}+\frac{\cdots}{\sqrt[3]{189x^2+\sqrt{3}\sqrt{27(2\cdots}}}}{x^4}}}{\cdots}$$

25.6 problem 703

Internal problem ID [3440]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 703.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{x\left(a+bx y^{3}\right) y^{\prime}+\left(a+c x^{3} y\right) y=0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 761

```
dsolve(x*(a+b*x*y(x)^3)*diff(y(x),x)+(a+c*x^3*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(27a+3\sqrt{-\frac{3(-c^3x^8+6c^2c_1x^6-12c\,c_1^2x^4+8c_1^3x^2-27a^2b)}{b}}\right)x^2b^2\right)^{\frac{1}{3}}}{3bx} + \frac{(-c\,x^2+2c_1)\,x}{\left(\left(27a+3\sqrt{-\frac{3(-c^3x^8+6c^2c_1x^6-12c\,c_1^2x^4+8c_1^3x^2-27a^2b)}{b}}\right)x^2b^2\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{\left(\left(27a+3\sqrt{-\frac{3(-c^3x^8+6c^2c_1x^6-12c\,c_1^2x^4+8c_1^3x^2-27a^2b)}{b}}\right)x^2b^2\right)^{\frac{1}{3}}}{6bx} - \frac{(-c\,x^2+2c_1)\,x}{2\left(\left(27a+3\sqrt{-\frac{3(-c^3x^8+6c^2c_1x^6-12c\,c_1^2x^4+8c_1^3x^2-27a^2b)}{b}}\right)x^2b^2\right)^{\frac{1}{3}}} - \frac{i\sqrt{3}\left(\frac{\left(\left(27a+3\sqrt{-\frac{3(-c^3x^8+6c^2c_1x^6-12c\,c_1^2x^4+8c_1^3x^2-27a^2b)}{b}}\right)x^2b^2\right)^{\frac{1}{3}}}{3bx} - \frac{(-c\,x^2+2c_1)x}{\left(\left(27a+3\sqrt{-\frac{3(-c^3x^8+6c^2c_1x^6-12c\,c_1^2x^4+8c_1^3x^2-27a^2b)}{b}}\right)x^2b^2\right)^{\frac{1}{3}}}\right)}{2}$$

$$y(x) = -\frac{\left(\left(27a+3\sqrt{-\frac{3(-c^3x^8+6c^2c_1x^6-12c\,c_1^2x^4+8c_1^3x^2-27a^2b)}{b}}\right)x^2b^2\right)^{\frac{1}{3}}}{6bx} - \frac{(-c\,x^2+2c_1)\,x}{2\left(\left(27a+3\sqrt{-\frac{3(-c^3x^8+6c^2c_1x^6-12c\,c_1^2x^4+8c_1^3x^2-27a^2b)}{b}}\right)x^2b^2\right)^{\frac{1}{3}}} + \frac{i\sqrt{3}\left(\frac{\left(\left(27a+3\sqrt{-\frac{3(-c^3x^8+6c^2c_1x^6-12c\,c_1^2x^4+8c_1^3x^2-27a^2b)}{b}}\right)x^2b^2\right)^{\frac{1}{3}}}{3bx} - \frac{(-c\,x^2+2c_1)x}{\left(\left(27a+3\sqrt{-\frac{3(-c^3x^8+6c^2c_1x^6-12c\,c_1^2x^4+8c_1^3x^2-27a^2b)}{b}}\right)x^2b^2\right)^{\frac{1}{3}}}\right)}{2}$$

✓ Solution by Mathematica

Time used: 60.15 (sec). Leaf size: 463

```
DSolve[x(a+b x y[x]^3)y'[x]+(a+c x^3 y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x(-cx^2+2c_1)}{\sqrt[3]{3}\sqrt[3]{9ab^2x^2+\sqrt{3}\sqrt{b^3x^4\left(27a^2b+x^2\left(cx^2-2c_1\right)^3\right)}}} + \frac{\sqrt[3]{9ab^2x^2+\sqrt{3}\sqrt{b^3x^4\left(27a^2b+x^2\left(cx^2-2c_1\right)^3\right)}}}{3^{2/3}bx}$$

$$y(x) \to \frac{\sqrt[3]{-\frac{1}{3}}x(cx^2-2c_1)}{\sqrt[3]{9ab^2x^2+\sqrt{3}\sqrt{b^3x^4\left(27a^2b+x^2\left(cx^2-2c_1\right)^3\right)}}} + \frac{i(\sqrt{3}+i)\sqrt[3]{9ab^2x^2+\sqrt{3}\sqrt{b^3x^4\left(27a^2b+x^2\left(cx^2-2c_1\right)^3\right)}}}{2\ 3^{2/3}bx}$$

$$y(x) \to \frac{\sqrt[3]{3}(-1-i\sqrt{3})\left(9ab^2x^2+\sqrt{3}\sqrt{b^3x^4\left(27a^2b+x^2\left(cx^2-2c_1\right)^3\right)}\right)^{2/3}-2(-3)^{2/3}bx^2(cx^2-2c_1)}{6bx\sqrt[3]{9ab^2x^2+\sqrt{3}\sqrt{b^3x^4\left(27a^2b+x^2\left(cx^2-2c_1\right)^3\right)}}}$$

25.7 problem 704

Internal problem ID [3441]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 704.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$x(1 - 2x^2y^3)\, y' + (1 - 2x^3y^2)\, y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 770

```
dsolve(x*(1-2*x^2*y(x)^3)*diff(y(x),x)+(1-2*x^3*y(x)^2)*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(\left(c_1^3x^2 - 6c_1^2x^3 + 12c_1x^4 - 8x^5 + 3\sqrt{-6c_1^3x^2 + 36c_1^2x^3 - 72c_1x^4 + 48x^5 + 81} - 27\right)x\right)^{\frac{1}{3}}}{6x} + \frac{\left(-2x + c_1\right)^2 x}{6\left(\left(c_1^3x^2 - 6c_1^2x^3 + 12c_1x^4 - 8x^5 + 3\sqrt{-6c_1^3x^2 + 36c_1^2x^3 - 72c_1x^4 + 48x^5 + 81} - 27\right)x\right)^{\frac{1}{3}}} - \frac{x}{3} + \frac{c_1}{6}$$

$$y(x) = -\frac{\left(\left(c_1^3x^2 - 6c_1^2x^3 + 12c_1x^4 - 8x^5 + 3\sqrt{-6c_1^3x^2 + 36c_1^2x^3 - 72c_1x^4 + 48x^5 + 81} - 27\right)x\right)^{\frac{1}{3}}}{12x} - \frac{\left(-2x + c_1\right)^2 x}{12\left(\left(c_1^3x^2 - 6c_1^2x^3 + 12c_1x^4 - 8x^5 + 3\sqrt{-6c_1^3x^2 + 36c_1^2x^3 - 72c_1x^4 + 48x^5 + 81} - 27\right)x\right)^{\frac{1}{3}}} - \frac{x}{3} + \frac{c_1}{6} - \frac{i\sqrt{3}\left(\frac{\left(\left(c_1^3x^2 - 6c_1^2x^3 + 12c_1x^4 - 8x^5 + 3\sqrt{-6c_1^3x^2 + 36c_1^2x^3 - 72c_1x^4 + 48x^5 + 81} - 27\right)x\right)^{\frac{1}{3}}}{6x} - \frac{(-2x+c_1)}{6\left(\left(c_1^3x^2 - 6c_1^2x^3 + 12c_1x^4 - 8x^5 + 3\sqrt{-6c_1^3x^2 -}\right.\right.}\right.}{2}$$

$$y(x) = -\frac{\left(\left(c_1^3x^2 - 6c_1^2x^3 + 12c_1x^4 - 8x^5 + 3\sqrt{-6c_1^3x^2 + 36c_1^2x^3 - 72c_1x^4 + 48x^5 + 81} - 27\right)x\right)^{\frac{1}{3}}}{12x} - \frac{\left(-2x + c_1\right)^2 x}{12\left(\left(c_1^3x^2 - 6c_1^2x^3 + 12c_1x^4 - 8x^5 + 3\sqrt{-6c_1^3x^2 + 36c_1^2x^3 - 72c_1x^4 + 48x^5 + 81} - 27\right)x\right)^{\frac{1}{3}}} - \frac{x}{3} + \frac{c_1}{6} + \frac{i\sqrt{3}\left(\frac{\left(\left(c_1^3x^2 - 6c_1^2x^3 + 12c_1x^4 - 8x^5 + 3\sqrt{-6c_1^3x^2 + 36c_1^2x^3 - 72c_1x^4 + 48x^5 + 81} - 27\right)x\right)^{\frac{1}{3}}}{6x} - \frac{(-2x+c_1)}{6\left(\left(c_1^3x^2 - 6c_1^2x^3 + 12c_1x^4 - 8x^5 + 3\sqrt{-6c_1^3x^2 -}\right.\right.}\right.}{2}$$

✓ Solution by Mathematica

Time used: 60.113 (sec). Leaf size: 582

```
DSolve[x(1-2 x^2 y[x]^3)y'[x]+(1-2 x^3 y[x]^2)y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{-2x^3 + c_1x^2 + \frac{x^4(-2x+c_1)^2}{\sqrt[3]{-8x^9 + 12c_1x^8 - 6c_1{}^2x^7 + c_1{}^3x^6 - 27x^4 + 3\sqrt{3}\sqrt{x^8\left(27 + 2x^2(2x-c_1)^3\right)}}} + \sqrt[3]{-8x^9 +}}{6x^2}$$

$$y(x) \to \frac{2x^2(-2x+c_1) - \frac{i\left(\sqrt{3}-i\right)x^4(-2x+c_1)^2}{\sqrt[3]{-8x^9 + 12c_1x^8 - 6c_1{}^2x^7 + c_1{}^3x^6 - 27x^4 + 3\sqrt{3}\sqrt{x^8\left(27 + 2x^2(2x-c_1)^3\right)}}} + i\left(\sqrt{3} +\right.}{12x^2}$$

$$y(x) \to \frac{2x^2(-2x+c_1) + \frac{i\left(\sqrt{3}+i\right)x^4(-2x+c_1)^2}{\sqrt[3]{-8x^9 + 12c_1x^8 - 6c_1{}^2x^7 + c_1{}^3x^6 - 27x^4 + 3\sqrt{3}\sqrt{x^8\left(27 + 2x^2(2x-c_1)^3\right)}}} - \left(1 + i\sqrt{\right.}{12x^2}$$

25.8 problem 705

Internal problem ID [3442]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 705.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x(-yx+1)\left(1-x^2y^2\right)y'+(1+yx)\left(1+x^2y^2\right)y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 42

```
dsolve(x*(1-x*y(x))*(1-x^2*y(x)^2)*diff(y(x),x)+(1+x*y(x))*(1+x^2*y(x)^2)*y(x) = 0,y(x), sing
```

$$y(x)=-\frac{1}{x}$$

$$y(x)=\frac{e^{\mathrm{RootOf}\left(-2\,e^{-\_Z}\ln(x)-e^{2\_Z}+2\,e^{-\_Z}c_1+2\_Z\,e^{-\_Z}+1\right)}}{x}$$

✓ Solution by Mathematica

Time used: 0.225 (sec). Leaf size: 35

```
DSolve[x(1-x y[x])(1-x^2 y[x]^2)y'[x]+(1+x y[x])(1+x^2 y[x]^2)y[x]==0,y[x],x,IncludeSingularS
```

$$y(x)\to-\frac{1}{x}$$

$$\text{Solve}\left[xy(x)-\frac{1}{xy(x)}-2\log(y(x))=c_1,y(x)\right]$$

25.9 problem 706

Internal problem ID [3443]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 706.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{(x^2 - y^4)\, y' - yx = 0}$$

✓ Solution by Maple

Time used: 0.079 (sec). Leaf size: 97

```
dsolve((x^2-y(x)^4)*diff(y(x),x) = x*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{2c_1 - 2\sqrt{c_1^2 - 4x^2}}}{2}$$

$$y(x) = \frac{\sqrt{2c_1 - 2\sqrt{c_1^2 - 4x^2}}}{2}$$

$$y(x) = -\frac{\sqrt{2c_1 + 2\sqrt{c_1^2 - 4x^2}}}{2}$$

$$y(x) = \frac{\sqrt{2c_1 + 2\sqrt{c_1^2 - 4x^2}}}{2}$$

✓ Solution by Mathematica

Time used: 2.15 (sec). Leaf size: 122

```
DSolve[(x^2-y[x]^4)y'[x]==x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-\sqrt{-x^2 + c_1{}^2} - c_1}$$
$$y(x) \to \sqrt{-\sqrt{-x^2 + c_1{}^2} - c_1}$$
$$y(x) \to -\sqrt{\sqrt{-x^2 + c_1{}^2} - c_1}$$
$$y(x) \to \sqrt{\sqrt{-x^2 + c_1{}^2} - c_1}$$
$$y(x) \to 0$$

25.10 problem 707

Internal problem ID [3444]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 707.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{(x^3 - y^4)\, y' - 3x^2 y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 25

```
dsolve((x^3-y(x)^4)*diff(y(x),x) = 3*x^2*y(x),y(x), singsol=all)
```

$$y(x) = \mathrm{RootOf}\left(x^9\_Z^4 + 3 - \mathrm{e}^{\frac{9c_1}{4}}\_Z\right) x^3$$

✓ Solution by Mathematica

Time used: 60.125 (sec). Leaf size: 1021

```
DSolve[(x^3-y[x]^4)y'[x]==3 x^2 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}+\frac{4\sqrt[3]{2}x^3}{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}}$$
$$-\frac{1}{2}\sqrt{-\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}-\frac{4\sqrt[3]{2}x^3}{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}-\frac{6c_1}{\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}+}}}$$

$$y(x) \to \frac{1}{2}\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}+\frac{4\sqrt[3]{2}x^3}{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}}$$
$$+\frac{1}{2}\sqrt{-\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}-\frac{4\sqrt[3]{2}x^3}{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}-\frac{6c_1}{\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}+}}}$$

$$y(x) \to -\frac{1}{2}\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}+\frac{4\sqrt[3]{2}x^3}{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}}$$
$$-\frac{1}{2}\sqrt{-\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}-\frac{4\sqrt[3]{2}x^3}{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}+\frac{6c_1}{\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}+}}}$$

$$y(x)$$
$$\to \frac{1}{2}\sqrt{-\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}-\frac{4\sqrt[3]{2}x^3}{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}+\frac{6c_1}{\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}+}}}$$
$$-\frac{1}{2}\sqrt{\frac{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}{\sqrt[3]{2}}+\frac{4\sqrt[3]{2}x^3}{\sqrt[3]{9c_1{}^2-\sqrt{-256x^9+81c_1{}^4}}}}$$

25.11 problem 708

Internal problem ID [3445]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 708.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\left(a^2x^2 + \left(x^2 + y^2\right)^2\right) y' - a^2xy = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 197

```
dsolve((a^2*x^2+(x^2+y(x)^2)^2)*diff(y(x),x) = a^2*x*y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2a^2 - 2x^2 - 2\sqrt{x^4 + (2a^2 - 2c_1)\, x^2 + (a^2 + c_1)^2} - 2c_1}}{2}$$

$$y(x) = \frac{\sqrt{-2a^2 - 2x^2 - 2\sqrt{x^4 + (2a^2 - 2c_1)\, x^2 + (a^2 + c_1)^2} - 2c_1}}{2}$$

$$y(x) = -\frac{\sqrt{-2a^2 - 2x^2 + 2\sqrt{x^4 + (2a^2 - 2c_1)\, x^2 + (a^2 + c_1)^2} - 2c_1}}{2}$$

$$y(x) = \frac{\sqrt{-2a^2 - 2x^2 + 2\sqrt{x^4 + (2a^2 - 2c_1)\, x^2 + (a^2 + c_1)^2} - 2c_1}}{2}$$

✓ Solution by Mathematica

Time used: 5.604 (sec). Leaf size: 272

```
DSolve[(a^2 x^2+(x^2+y[x]^2)^2)y'[x]==a^2 x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{-\sqrt{(a^2+x^2-c_1{}^2)^2+4c_1{}^2x^2}-a^2-x^2+c_1{}^2}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{-\sqrt{(a^2+x^2-c_1{}^2)^2+4c_1{}^2x^2}-a^2-x^2+c_1{}^2}}{\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\sqrt{(a^2+x^2-c_1{}^2)^2+4c_1{}^2x^2}-a^2-x^2+c_1{}^2}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\sqrt{(a^2+x^2-c_1{}^2)^2+4c_1{}^2x^2}-a^2-x^2+c_1{}^2}}{\sqrt{2}}$$

$$y(x) \to 0$$

$$y(x) \to -\sqrt{-x^2}$$

$$y(x) \to \sqrt{-x^2}$$

25.12 problem 709

Internal problem ID [3446]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 709.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{2(x-y^4)\,y'-y=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 89

```
dsolve(2*(x-y(x)^4)*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2\sqrt{c_1^2-4x}+2c_1}}{2}$$
$$y(x) = \frac{\sqrt{-2\sqrt{c_1^2-4x}+2c_1}}{2}$$
$$y(x) = -\frac{\sqrt{2\sqrt{c_1^2-4x}+2c_1}}{2}$$
$$y(x) = \frac{\sqrt{2\sqrt{c_1^2-4x}+2c_1}}{2}$$

✓ Solution by Mathematica

Time used: 2.134 (sec). Leaf size: 128

```
DSolve[2(x-y[x]^4)y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{c_1 - \sqrt{-4x + c_1{}^2}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{c_1 - \sqrt{-4x + c_1{}^2}}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{\sqrt{-4x + c_1{}^2} + c_1}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{\sqrt{-4x + c_1{}^2} + c_1}}{\sqrt{2}}$$
$$y(x) \to 0$$

25.13 problem 710

Internal problem ID [3447]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 710.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x)*G(y),0]']]`

$$\boxed{(4x - xy^3 - 2y^4)\, y' - (2 + y^3)\, y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 27

```
dsolve((4*x-x*y(x)^3-2*y(x)^4)*diff(y(x),x) = (2+y(x)^3)*y(x),y(x), singsol=all)
```

$$x - \frac{\left(-y(x)^2 + c_1\right) y(x)^2}{2 + y(x)^3} = 0$$

✓ Solution by Mathematica

Time used: 60.171 (sec). Leaf size: 2021

```
DSolve[(4 x-x y[x]^3-2 y[x]^4)y'[x]==(2+y[x]^3)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}\sqrt{\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{3\sqrt[3]{2}}+\frac{\cdots}{3\sqrt[3]{54x^3+\sqrt{(54x^3+14\cdots}}}}$$
$$-\frac{1}{2}\sqrt{-\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{3\sqrt[3]{2}}-\frac{\cdots}{3\sqrt[3]{54x^3+\sqrt{(54x^3+\cdots}}}}$$
$$-\frac{x}{4}$$

$$y(x) \to -\frac{1}{2}\sqrt{\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{3\sqrt[3]{2}}+\frac{\cdots}{3\sqrt[3]{54x^3+\sqrt{(54x^3+14\cdots}}}}$$
$$+\frac{1}{2}\sqrt{-\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{3\sqrt[3]{2}}-\frac{\cdots}{3\sqrt[3]{54x^3+\sqrt{(54x^3+\cdots}}}}$$
$$-\frac{x}{4}$$

$$y(x) \to \frac{1}{2}\sqrt{\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{3\sqrt[3]{2}}+\frac{\cdots}{3\sqrt[3]{54x^3+\sqrt{(54x^3+144\cdots}}}}$$
$$-\frac{1}{2}\sqrt{-\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{3\sqrt[3]{2}}-\frac{\cdots}{3\sqrt[3]{54x^3+\sqrt{(54x^3+\cdots}}}}$$
$$-\frac{x}{4}$$

$$y(x) \to \frac{1}{2}\sqrt{\frac{\sqrt[3]{54x^3+\sqrt{(54x^3+144c_1x-2c_1{}^3)^2-4(24x+c_1{}^2)^3}+144c_1x-2c_1{}^3}}{3\sqrt[3]{2}}+\frac{\cdots}{3\sqrt[3]{54x^3+\sqrt{(54x^3+144\cdots}}}}$$

25.14 problem 711

Internal problem ID [3448]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 711.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$(a\,x^3+(ax+yb)^3)\,yy'+x\big((ax+yb)^3+by^3\big)=0$$

✓ Solution by Maple

Time used: 0.281 (sec). Leaf size: 160

```
dsolve((a*x^3+(a*x+b*y(x))^3)*y(x)*diff(y(x),x)+x*((a*x+b*y(x))^3+b*y(x)^3) = 0,y(x), singsol
```

$$y(x) = \frac{x\left(c_1x-a\,\mathrm{RootOf}\left(a^2\_Z^4-2axc_1\_Z^3+(a^2c_1^2x^2+b^2c_1^2x^2+c_1^2x^2-b^2)\_Z^2-2a\,x^3c_1^3\_Z+c_1^4x^4\right)\right)}{b\,\mathrm{RootOf}\left(a^2\_Z^4-2axc_1\_Z^3+(a^2c_1^2x^2+b^2c_1^2x^2+c_1^2x^2-b^2)\_Z^2-2a\,x^3c_1^3\_Z+c_1^4x^4\right)}$$

✓ Solution by Mathematica

Time used: 61.439 (sec). Leaf size: 13289

```
DSolve[(a x^3+(a x+b y[x])^3)y[x] y'[x]+x((a x+b y[x])^3+b y[x]^3)==0,y[x],x,IncludeSingularS
```

Too large to display

25.15 problem 712

Internal problem ID [3449]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 712.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$\boxed{\left(x+2y+2x^2y^3+y^4x\right)y'+\left(y^4+1\right)y=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 579

```
dsolve((x+2*y(x)+2*x^2*y(x)^3+x*y(x)^4)*diff(y(x),x)+(1+y(x)^4)*y(x) = 0,y(x), singsol=all)
```

$$y(x)=\frac{\left(108c_1^3x^2+12\sqrt{3}\,\sqrt{27c_1^4x^2+4c_1x^4+18c_1^2x^2-x^2-4c_1}\,xc_1+36c_1x^2-8\right)^{\frac{1}{3}}}{6c_1x}-\frac{2(3c_1x^2-1)}{3c_1x\left(108c_1^3x^2+12\sqrt{3}\,\sqrt{27c_1^4x^2+4c_1x^4+18c_1^2x^2-x^2-4c_1}\,xc_1+36c_1x^2-8\right)^{\frac{1}{3}}}-\frac{1}{3c_1x}$$

$$y(x)=\frac{i\left(4-12c_1x^2-\left(108c_1^3x^2+12\sqrt{3}\,\sqrt{27c_1^4x^2+18c_1^2x^2+(4x^4-4)\,c_1-x^2}\,xc_1+36c_1x^2-8\right)^{\frac{2}{3}}\right)\sqrt{3}+12c}{12\left(108c_1^3x^2+12\sqrt{3}\,\sqrt{27c_1^4x^2+18c_1^2x^2+(4x^4-}\right.}$$

$$y(x)=\frac{i\left(-4+12c_1x^2+\left(108c_1^3x^2+12\sqrt{3}\,\sqrt{27c_1^4x^2+18c_1^2x^2+(4x^4-4)\,c_1-x^2}\,xc_1+36c_1x^2-8\right)^{\frac{2}{3}}\right)\sqrt{3}+12}{12\left(108c_1^3x^2+12\sqrt{3}\,\sqrt{27c_1^4x^2+18c_1^2x^2+(4x^4}\right.}$$

✓ Solution by Mathematica

Time used: 10.856 (sec). Leaf size: 631

```
DSolve[(x+2 y[x]+2 x^2 y[x]^3+x y[x]^4)y'[x]+(1+y[x]^4)y[x]==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to \frac{\frac{2c_1(3x^2+c_1)}{\sqrt[3]{\frac{9}{2}(3+c_1^2)x^2+\frac{3}{2}\sqrt{-12c_1^3x^6+(81-3c_1^4+54c_1^2)x^4+12c_1^3x^2}+c_1^3}}+2^{2/3}\sqrt[3]{9(3+c_1^2)x^2+3\sqrt{-}}}{6x}$$

$$y(x) \to \frac{-\frac{4\sqrt[3]{-2}c_1(3x^2+c_1)}{\sqrt[3]{9(3+c_1^2)x^2+3\sqrt{-12c_1^3x^6+(81-3c_1^4+54c_1^2)x^4+12c_1^3x^2}+2c_1^3}}+i2^{2/3}(\sqrt{3}+i)\sqrt[3]{9(3+c_1}}{12x}$$

$$y(x) \to \frac{\frac{4(-1)^{2/3}c_1(3x^2+c_1)}{\sqrt[3]{\frac{9}{2}(3+c_1^2)x^2+\frac{3}{2}\sqrt{-12c_1^3x^6+(81-3c_1^4+54c_1^2)x^4+12c_1^3x^2}+c_1^3}}-2^{2/3}(1+i\sqrt{3})\sqrt[3]{9(3+c_1^2)}}{12x}$$

$$y(x) \to 0$$

$$y(x) \to -\sqrt[4]{-1}$$

$$y(x) \to \sqrt[4]{-1}$$

$$y(x) \to -(-1)^{3/4}$$

$$y(x) \to (-1)^{3/4}$$

$$y(x) \to \frac{1}{2}x\left(-1+\frac{ix^2}{\sqrt{-x^4}}\right)$$

$$y(x) \to -\frac{x}{2}+\frac{i\sqrt{-x^4}}{2x}$$

25.16 problem 713

Internal problem ID [3450]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 713.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$2x(x^3+y^4)\,y'-(x^3+2y^4)\,y=0$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 293

```
dsolve(2*x*(x^3+y(x)^4)*diff(y(x),x) = (x^3+2*y(x)^4)*y(x),y(x), singsol=all)
```

$$y(x)=-\frac{\left(\left(16c_1+8x-8\sqrt{4c_1x+x^2}\right)x^3c_1^3\right)^{\frac{1}{4}}}{2c_1}$$

$$y(x)=\frac{\left(\left(16c_1+8x-8\sqrt{4c_1x+x^2}\right)x^3c_1^3\right)^{\frac{1}{4}}}{2c_1}$$

$$y(x)=-\frac{\left(\left(16c_1+8x+8\sqrt{4c_1x+x^2}\right)x^3c_1^3\right)^{\frac{1}{4}}}{2c_1}$$

$$y(x)=\frac{\left(\left(16c_1+8x+8\sqrt{4c_1x+x^2}\right)x^3c_1^3\right)^{\frac{1}{4}}}{2c_1}$$

$$y(x)=-\frac{i\left(\left(16c_1+8x-8\sqrt{4c_1x+x^2}\right)x^3c_1^3\right)^{\frac{1}{4}}}{2c_1}$$

$$y(x)=-\frac{i\left(\left(16c_1+8x+8\sqrt{4c_1x+x^2}\right)x^3c_1^3\right)^{\frac{1}{4}}}{2c_1}$$

$$y(x)=\frac{i\left(\left(16c_1+8x-8\sqrt{4c_1x+x^2}\right)x^3c_1^3\right)^{\frac{1}{4}}}{2c_1}$$

$$y(x)=\frac{i\left(\left(16c_1+8x+8\sqrt{4c_1x+x^2}\right)x^3c_1^3\right)^{\frac{1}{4}}}{2c_1}$$

✓ Solution by Mathematica

Time used: 3.81 (sec). Leaf size: 166

```
DSolve[2 x(x^3+y[x]^4)y'[x]==(x^3+2 y[x]^4)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{c_1x^2 - x^{3/2}\sqrt{4 + c_1{}^2x}}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{c_1x^2 - x^{3/2}\sqrt{4 + c_1{}^2x}}}{\sqrt{2}}$$
$$y(x) \to -\frac{\sqrt{x^{3/2}\sqrt{4 + c_1{}^2x} + c_1x^2}}{\sqrt{2}}$$
$$y(x) \to \frac{\sqrt{x^{3/2}\sqrt{4 + c_1{}^2x} + c_1x^2}}{\sqrt{2}}$$
$$y(x) \to 0$$

25.17 problem 714

Internal problem ID [3451]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 714.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x(1 - x^2 y^4)\, y' + y = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 129

```
dsolve(x*(1-x^2*y(x)^4)*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2xc_1\left(-x+\sqrt{-4c_1^2+x^2}\right)}}{2xc_1}$$

$$y(x) = \frac{\sqrt{-2xc_1\left(-x+\sqrt{-4c_1^2+x^2}\right)}}{2xc_1}$$

$$y(x) = -\frac{\sqrt{2}\,\sqrt{xc_1\left(x+\sqrt{-4c_1^2+x^2}\right)}}{2xc_1}$$

$$y(x) = \frac{\sqrt{2}\,\sqrt{xc_1\left(x+\sqrt{-4c_1^2+x^2}\right)}}{2xc_1}$$

✓ Solution by Mathematica

Time used: 11.773 (sec). Leaf size: 172

```
DSolve[x(1-x^2 y[x]^4)y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{c_1 - \frac{\sqrt{x^2 \left(-1 + c_1{}^2 x^2\right)}}{x^2}}$$

$$y(x) \to \sqrt{c_1 - \frac{\sqrt{x^2 \left(-1 + c_1{}^2 x^2\right)}}{x^2}}$$

$$y(x) \to -\sqrt{\frac{\sqrt{x^2 \left(-1 + c_1{}^2 x^2\right)}}{x^2} + c_1}$$

$$y(x) \to \sqrt{\frac{\sqrt{x^2 \left(-1 + c_1{}^2 x^2\right)}}{x^2} + c_1}$$

$$y(x) \to 0$$

$$y(x) \to -\frac{1}{\sqrt[4]{-x^2}}$$

$$y(x) \to \frac{1}{\sqrt[4]{-x^2}}$$

25.18 problem 715

Internal problem ID [3452]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 715.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{(x^2 - y^5)\, y' - 2yx = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 25

```
dsolve((x^2-y(x)^5)*diff(y(x),x) = 2*x*y(x),y(x), singsol=all)
```

$$y(x) = \text{RootOf}\left(x^8\_Z^5 + 4 - e^{\frac{8c_1}{5}}\_Z\right) x^2$$

✓ Solution by Mathematica

Time used: 1.923 (sec). Leaf size: 121

```
DSolve[(x^2-y[x]^5)y'[x]==2 x y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[\#1^5 + 4\#1c_1 + 4x^2\&, 1\right]$$
$$y(x) \to \text{Root}\left[\#1^5 + 4\#1c_1 + 4x^2\&, 2\right]$$
$$y(x) \to \text{Root}\left[\#1^5 + 4\#1c_1 + 4x^2\&, 3\right]$$
$$y(x) \to \text{Root}\left[\#1^5 + 4\#1c_1 + 4x^2\&, 4\right]$$
$$y(x) \to \text{Root}\left[\#1^5 + 4\#1c_1 + 4x^2\&, 5\right]$$
$$y(x) \to 0$$

25.19 problem 716

Internal problem ID [3453]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 716.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x(x^3+y^5)\,y'-(x^3-y^5)\,y=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve(x*(x^3+y(x)^5)*diff(y(x),x) = (x^3-y(x)^5)*y(x),y(x), singsol=all)
```

$$\ln(x)-c_1-\frac{5\ln\left(\frac{y(x)}{x^{\frac{3}{5}}}\right)}{2}+\frac{5\ln\left(-\frac{-4y(x)^5+x^3}{x^3}\right)}{8}=0$$

✓ Solution by Mathematica

Time used: 1.799 (sec). Leaf size: 141

```
DSolve[x(x^3+y[x]^5)y'[x]==(x^3-y[x]^5)y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \text{Root}\left[4\#1^5x-4\#1^4c_1-x^4\&,1\right]$$
$$y(x)\to \text{Root}\left[4\#1^5x-4\#1^4c_1-x^4\&,2\right]$$
$$y(x)\to \text{Root}\left[4\#1^5x-4\#1^4c_1-x^4\&,3\right]$$
$$y(x)\to \text{Root}\left[4\#1^5x-4\#1^4c_1-x^4\&,4\right]$$
$$y(x)\to \text{Root}\left[4\#1^5x-4\#1^4c_1-x^4\&,5\right]$$

25.20 problem 717

Internal problem ID [3454]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 717.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_rational]`

$$x^3\left(1+5x^3y^7\right)y'+\left(3x^5y^5-1\right)y^3=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(x^3*(1+5*x^3*y(x)^7)*diff(y(x),x)+(3*x^5*y(x)^5-1)*y(x)^3 = 0,y(x), singsol=all)
```

$$-x^3y(x)^5-\frac{1}{2x^2}+\frac{1}{2y\left(x\right)^2}+c_1=0$$

✓ Solution by Mathematica

Time used: 4.962 (sec). Leaf size: 253

```
DSolve[x^3(1+5 x^3 y[x]^7)y'[x]+(3 x^5 y[x]^5-1)y[x]^3==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to \text{Root}\left[2\#1^7x^5+\#1^2\left(1-2c_1x^2\right)-x^2\&,1\right]$$
$$y(x)\to \text{Root}\left[2\#1^7x^5+\#1^2\left(1-2c_1x^2\right)-x^2\&,2\right]$$
$$y(x)\to \text{Root}\left[2\#1^7x^5+\#1^2\left(1-2c_1x^2\right)-x^2\&,3\right]$$
$$y(x)\to \text{Root}\left[2\#1^7x^5+\#1^2\left(1-2c_1x^2\right)-x^2\&,4\right]$$
$$y(x)\to \text{Root}\left[2\#1^7x^5+\#1^2\left(1-2c_1x^2\right)-x^2\&,5\right]$$
$$y(x)\to \text{Root}\left[2\#1^7x^5+\#1^2\left(1-2c_1x^2\right)-x^2\&,6\right]$$
$$y(x)\to \text{Root}\left[2\#1^7x^5+\#1^2\left(1-2c_1x^2\right)-x^2\&,7\right]$$

25.21 problem 718

Internal problem ID [3455]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 718.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{(1+a(x+y))^n y'+a(x+y)^n=0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 45

```
dsolve((1+a*(x+y(x)))^n*diff(y(x),x)+a*(x+y(x))^n = 0,y(x), singsol=all)
```

$$y(x)=-x+\operatorname{RootOf}\left(-x-\left(\int^{\_Z}\frac{(a\_a+1)^n}{a\_a^n-(a\_a+1)^n}d\_a\right)+c_1\right)$$

✓ Solution by Mathematica

Time used: 6.435 (sec). Leaf size: 331

```
DSolve[(1+a (x+y[x]))^n y'[x]+a(x+y[x])^n==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^x\frac{a(K[1]+y(x))^n}{a(K[1]+y(x))^n-(a(K[1]+y(x))+1)^n}dK[1]+\int_1^{y(x)}\right.$$

$$-\frac{-a\int_1^x\left(\frac{an(K[1]+K[2])^{n-1}}{a(K[1]+K[2])^n-(a(K[1]+K[2])+1)^n}-\frac{a(K[1]+K[2])^n\left(an(K[1]+K[2])^{n-1}-an(a(K[1]+K[2])+1)^{n-1}\right)}{(a(K[1]+K[2])^n-(a(K[1]+K[2])+1)^n)^2}\right)dK[1](x+K[2])^{\cdots}}{(a\cdots}$$

25.22 problem 719

Internal problem ID [3456]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 719.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{x(a + x y^n)\, y' + y b = 0}$$

✓ Solution by Maple

Time used: 0.328 (sec). Leaf size: 39

```
dsolve(x*(a+x*y(x)^n)*diff(y(x),x)+b*y(x) = 0,y(x), singsol=all)
```

$$(y(x)^n)^{-a}\,(x y(x)^n - bn + a)^{bn}\, x^{-bn} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.27 (sec). Leaf size: 61

```
DSolve[x(a+x y[x]^n)y'[x]+b y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{an\log(ay(x) - bny(x))}{a - bn} - \frac{bn(\log(x) - \log\left(a - bn + xy(x)^n\right))}{a - bn} = c_1, y(x)\right]$$

25.23 problem 720

Internal problem ID [3457]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 720.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_Bernoulli]`

$$f(x)\,y^m y' + g(x)\,y^{m+1} + h(x)\,y^n = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 221

```
dsolve(f(x)*y(x)^m*diff(y(x),x)+g(x)*y(x)^(m+1)+h(x)*y(x)^n = 0,y(x), singsol=all)
```

$$y(x) = \left(-m\left(\int \frac{\mathrm{e}^{\int -\frac{g(x)n}{f(x)}dx}\mathrm{e}^{\left(\int \frac{g(x)}{f(x)}dx\right)m}\mathrm{e}^{\int \frac{g(x)}{f(x)}dx}h(x)}{f(x)}dx\right) + n\left(\int \frac{\mathrm{e}^{\int -\frac{g(x)n}{f(x)}dx}\mathrm{e}^{\left(\int \frac{g(x)}{f(x)}dx\right)m}\mathrm{e}^{\int \frac{g(x)}{f(x)}dx}h(x)}{f(x)}dx\right) + c_1 -$$

✓ Solution by Mathematica

Time used: 11.443 (sec). Leaf size: 185

```
DSolve[f[x] y[x]^m y'[x]+ g[x] y[x]^(m+1)+ h[x] y[x]^n==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \left(\exp\left((m-n+1)\int_1^x -\frac{g(K[1])}{f(K[1])}dK[1]\right)\left((m-n+1)\int_1^x -\frac{\exp\left((-m+n-1)\int_1^{K[2]} -\frac{g(K[1])}{f(K[1])}dK[1]\right)h(K[2])}{f(K[2])}dK[2]+c_1\right)\right)^{\frac{1}{m-n+1}}$$

$$y(x) \to \left((m-n+1)\exp\left((m-n+1)\int_1^x -\frac{g(K[1])}{f(K[1])}dK[1]\right)\int_1^x -\frac{\exp\left((-m+n-1)\int_1^{K[2]} -\frac{g(K[1])}{f(K[1])}dK[1]\right)h(K[2])}{f(K[2])}dK[2]\right)^{\frac{1}{m-n+1}}$$

25.24 problem 721

Internal problem ID [3458]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 721.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'\sqrt{b^2+y^2}-\sqrt{a^2+x^2}=0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 67

```
dsolve(diff(y(x),x)*sqrt(b^2+y(x)^2) = sqrt(a^2+x^2),y(x), singsol=all)
```

$$\frac{x\sqrt{a^2+x^2}}{2}+\frac{a^2\ln\left(x+\sqrt{a^2+x^2}\right)}{2}-\frac{y(x)\sqrt{b^2+y(x)^2}}{2}-\frac{b^2\ln\left(y(x)+\sqrt{b^2+y(x)^2}\right)}{2}+c_1=0$$

✓ Solution by Mathematica

Time used: 0.865 (sec). Leaf size: 93

```
DSolve[y'[x] Sqrt[y[x]^2+b^2]==Sqrt[x^2+a^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \text{InverseFunction}\left[\frac{1}{2}\#1\sqrt{\#1^2+b^2}-\frac{1}{2}b^2\log\left(\sqrt{\#1^2+b^2}-\#1\right)\&\right]\left[\frac{1}{2}x\sqrt{a^2+x^2}-\frac{1}{2}a^2\log\left(\sqrt{a^2+x^2}-x\right)+c_1\right]$$

25.25 problem 722

Internal problem ID [3459]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 722.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y'\sqrt{b^2-y^2}-\sqrt{a^2-x^2}=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 75

```
dsolve(diff(y(x),x)*sqrt(b^2-y(x)^2) = sqrt(a^2-x^2),y(x), singsol=all)
```

$$\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)}{2}-\frac{y(x)\sqrt{b^2-y(x)^2}}{2}-\frac{b^2\arctan\left(\frac{y(x)}{\sqrt{b^2-y(x)^2}}\right)}{2}+c_1=0$$

✓ Solution by Mathematica

Time used: 1.005 (sec). Leaf size: 97

```
DSolve[y'[x] Sqrt[b^2-y[x]^2]==Sqrt[a^2-x^2],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \text{InverseFunction}\left[\frac{1}{2}b^2\arctan\left(\frac{\#1}{\sqrt{b^2-\#1^2}}\right)+\frac{1}{2}\#1\sqrt{b^2-\#1^2}\&\right]\left[\frac{1}{2}\left(a^2\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)+x\sqrt{a^2-x^2}+2c_1\right)\right]$$

25.26 problem 723

Internal problem ID [3460]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 723.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type **[_separable]**

$$\boxed{y'\sqrt{y} - \sqrt{x} = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)*sqrt(y(x)) = sqrt(x),y(x), singsol=all)
```

$$y(x)^{\frac{3}{2}} - x^{\frac{3}{2}} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 20

```
DSolve[y'[x] Sqrt[Y]==Sqrt[X],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x\sqrt{X}}{\sqrt{Y}} + c_1$$

25.27 problem 724

Internal problem ID [3461]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 724.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{\left(\sqrt{x+y}+1\right)y'+1=0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve((1+sqrt(x+y(x)))*diff(y(x),x)+1 = 0,y(x), singsol=all)
```

$$-y(x)-2\sqrt{x+y(x)}-c_1=0$$

✓ Solution by Mathematica

Time used: 0.096 (sec). Leaf size: 39

```
DSolve[(1+Sqrt[x+y[x]])y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\rightarrow -2\sqrt{x+1+c_1}+2+c_1$$
$$y(x)\rightarrow 2\sqrt{x+1+c_1}+2+c_1$$

25.28 problem 725

Internal problem ID [3462]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 725.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{y'\sqrt{yx} + x - y - \sqrt{yx} = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 48

```
dsolve(diff(y(x),x)*sqrt(x*y(x))+x-y(x) = sqrt(x*y(x)),y(x), singsol=all)
```

$$\ln\left(\sqrt{y(x)\,x} + x\right) + 3\ln\left(\sqrt{y(x)\,x} - x\right) - \frac{2x}{\sqrt{y(x)\,x} - x} - 2\ln(x) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.151 (sec). Leaf size: 62

```
DSolve[y'[x] Sqrt[x y[x]]+x -y[x]==Sqrt[x y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{1 - \sqrt{\frac{y(x)}{x}}} + \frac{3}{2}\log\left(\sqrt{\frac{y(x)}{x}} - 1\right) + \frac{1}{2}\log\left(\sqrt{\frac{y(x)}{x}} + 1\right) = -\log(x) + c_1, y(x)\right]$$

25.29 problem 726

Internal problem ID [3463]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 726.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{(x - 2\sqrt{yx})\, y' - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((x-2*sqrt(x*y(x)))*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$\ln\left(y(x)\right) + \frac{x}{\sqrt{y\left(x\right)x}} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.236 (sec). Leaf size: 33

```
DSolve[(x-2 Sqrt[x y[x]])y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2}{\sqrt{\frac{y(x)}{x}}} + 2\log\left(\frac{y(x)}{x}\right) = -2\log(x) + c_1, y(x)\right]$$

25.30 problem 727

Internal problem ID [3464]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 727.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\left(y+\sqrt{1+y^2}\right)\left(x^2+1\right)^{\frac{3}{2}} y'-1-y^2=0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 28

```
dsolve((y(x)+sqrt(1+y(x)^2))*(x^2+1)^(3/2)*diff(y(x),x) = 1+y(x)^2,y(x), singsol=all)
```

$$\frac{x}{\sqrt{x^2+1}}-\operatorname{arcsinh}\left(y(x)\right)-\frac{\ln\left(1+y(x)^2\right)}{2}+c_1=0$$

✓ Solution by Mathematica

Time used: 15.075 (sec). Leaf size: 115

```
DSolve[(y[x]+Sqrt[1+y[x]^2])(1+x^2)^(3/2) y'[x]==1+y[x]^2,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to -\frac{i\left(1+e^{\frac{x}{\sqrt{x^2+1}}+c_1}\right)}{\sqrt{1+2e^{\frac{x}{\sqrt{x^2+1}}+c_1}}}$$

$$y(x)\to \frac{i\left(1+e^{\frac{x}{\sqrt{x^2+1}}+c_1}\right)}{\sqrt{1+2e^{\frac{x}{\sqrt{x^2+1}}+c_1}}}$$

$$y(x)\to -i$$

$$y(x)\to i$$

25.31 problem 728

Internal problem ID [3465]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 728.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$\left(y+\sqrt{1+y^2}\right)\left(x^2+1\right)^{\frac{3}{2}} y'-1-y^2=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 28

```
dsolve((y(x)+sqrt(1+y(x)^2))*(x^2+1)^(3/2)*diff(y(x),x) = 1+y(x)^2,y(x), singsol=all)
```

$$\frac{x}{\sqrt{x^2+1}}-\operatorname{arcsinh}\left(y(x)\right)-\frac{\ln\left(1+y(x)^2\right)}{2}+c_1=0$$

✓ Solution by Mathematica

Time used: 0.472 (sec). Leaf size: 115

```
DSolve[(1+x^2)^(3/2) (y[x]+Sqrt[1+y[x]^2])y'[x]==1+y[x]^2,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to -\frac{i\left(1+e^{\frac{x}{\sqrt{x^2+1}}+c_1}\right)}{\sqrt{1+2e^{\frac{x}{\sqrt{x^2+1}}+c_1}}}$$

$$y(x)\to \frac{i\left(1+e^{\frac{x}{\sqrt{x^2+1}}+c_1}\right)}{\sqrt{1+2e^{\frac{x}{\sqrt{x^2+1}}+c_1}}}$$

$$y(x)\to -i$$

$$y(x)\to i$$

25.32 problem 729

Internal problem ID [3466]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 729.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\left(x-\sqrt{x^2+y^2}\right)y'-y=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 18

```
dsolve((x-sqrt(x^2+y(x)^2))*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$-c_1+\sqrt{x^2+y(x)^2}+x=0$$

✓ Solution by Mathematica

Time used: 0.516 (sec). Leaf size: 57

```
DSolve[(x-Sqrt[x^2+y[x]^2])y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -e^{\frac{c_1}{2}}\sqrt{-2x+e^{c_1}}$$
$$y(x)\to e^{\frac{c_1}{2}}\sqrt{-2x+e^{c_1}}$$
$$y(x)\to 0$$

25.33 problem 730

Internal problem ID [3467]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 730.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$x\left(1-\sqrt{x^2-y^2}\right)y'-y=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve(x*(1-sqrt(x^2-y(x)^2))*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$y(x)-\arctan\left(\frac{y(x)}{\sqrt{x^2-y(x)^2}}\right)-c_1=0$$

✓ Solution by Mathematica

Time used: 0.505 (sec). Leaf size: 29

```
DSolve[x(1-Sqrt[x^2-y[x]^2])y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\arctan\left(\frac{\sqrt{x^2-y(x)^2}}{y(x)}\right)+y(x)=c_1,y(x)\right]$$

25.34 problem 731

Internal problem ID [3468]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 25
Problem number: 731.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _dAlembert]`

$$x\left(x+\sqrt{x^2+y^2}\right) y'+y\sqrt{x^2+y^2}=0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 139

```
dsolve(x*(x+sqrt(x^2+y(x)^2))*diff(y(x),x)+y(x)*sqrt(x^2+y(x)^2) = 0,y(x), singsol=all)
```

$$\int_{\_b}^{x} -\frac{\sqrt{\_a^2+y(x)^2}}{\_a\left(2\sqrt{\_a^2+y(x)^2}+\_a\right)}d\_a+\int^{y(x)}\left(-\frac{\sqrt{\_f^2+x^2}+x}{\_f\left(2\sqrt{\_f^2+x^2}+x\right)} - \left(\int_{\_b}^{x}\left(-\frac{\_f}{\sqrt{\_a^2+\_f^2}\,\_a\left(2\sqrt{\_a^2+\_f^2}+\_a\right)}+\frac{2\_f}{\_a\left(2\sqrt{\_a^2+\_f^2}+\_a\right)^2}\right)d\_a\right)\right)d\_f + c_1=0$$

✓ Solution by Mathematica

Time used: 60.226 (sec). Leaf size: 1435

```
DSolve[x(x+Sqrt[x^2+y[x]^2])y'[x] +y[x] Sqrt[x^2+y[x]^2]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to -\frac{1}{2}\sqrt{\frac{x^6 - x^4\sqrt[3]{-x^6+\frac{8e^{12c_1}}{x^6}+\frac{8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}}{x^6}+20e^{6c_1}} + x^2\left(-x^6+\frac{8e^{12c_1}}{x^6}+\frac{8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}}{x^6}+ \cdots\right.}{x^2\sqrt[3]{-x^6+\frac{8e^{12c_1}}{x^6}+\frac{8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}}{x^6}+20e^{6c_1}}}}$$

$$y(x) \to \frac{1}{2}\sqrt{\frac{x^6 - x^4\sqrt[3]{-x^6+\frac{8e^{12c_1}}{x^6}+\frac{8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}}{x^6}+20e^{6c_1}} + x^2\left(-x^6+\frac{8e^{12c_1}}{x^6}+\frac{8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}}{x^6}+20\cdots\right.}{x^2\sqrt[3]{-x^6+\frac{8e^{12c_1}}{x^6}+\frac{8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}}{x^6}+20e^{6c_1}}}}$$

$$y(x) \to -\frac{\sqrt{\left(-1-i\sqrt{3}\right)\sqrt[3]{-x^6+\frac{8e^{12c_1}}{x^6}+\frac{8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}}{x^6}+20e^{6c_1}} - 2x^2 + \frac{i\left(\sqrt{3}+i\right)x\cdots}{\sqrt[3]{-x^6+\frac{8e^{12c_1}}{x^6}+\frac{8\sqrt{e^{6c_1}\left(-\cdots\right.}}{\cdots}}}}}{2\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{\left(-1-i\sqrt{3}\right)\sqrt[3]{-x^6+\frac{8e^{12c_1}}{x^6}+\frac{8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}}{x^6}+20e^{6c_1}} - 2x^2 + \frac{i\left(\sqrt{3}+i\right)x^4}{\sqrt[3]{-x^6+\frac{8e^{12c_1}}{x^6}+\frac{8\sqrt{e^{6c_1}\left(-\cdots\right.}}{x^{\cdots}}}}}}{2\sqrt{2}}$$

$$y(x) \to -\frac{\sqrt{\frac{ix^2\left(x^2+\sqrt[3]{-x^6+\frac{8e^{12c_1}}{x^6}+\frac{8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}}{x^6}+20e^{6c_1}}\right)\left(\left(\sqrt{3}+i\right)\sqrt[3]{-x^6+\frac{8e^{12c_1}}{x^6}+\frac{8\sqrt{e^{6c_1}\left(-x^6+\cdots\right.}}{x^6}}\cdots\right.}{x^2\sqrt[3]{-x^6+\frac{8e^{12c_1}}{x^6}+\frac{8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}}{x^6}+20e^{6c_1}}}}}{2\sqrt{2}}$$

$$y(x) \to \sqrt{\frac{ix^2\left(x^2+\sqrt[3]{-x^6+\frac{8e^{12c_1}}{x^6}+\frac{8\sqrt{e^{6c_1}\left(-x^6+e^{6c_1}\right)^3}}{x^6}+20e^{6c_1}}\right)\left(\left(\sqrt{3}+i\right)\sqrt[3]{-x^6+\frac{8e^{12c_1}}{x^6}+\frac{8\sqrt{e^{6c_1}\left(-x^6+e\cdots\right.}}{x^6}}\cdots\right.}{\cdots}}$$

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26.1 problem 732

Internal problem ID [3469]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 732.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$xy\left(x+\sqrt{x^2-y^2}\right)y'-y^2x+\left(x^2-y^2\right)^{\frac{3}{2}}=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 36

```
dsolve(x*y(x)*(x+sqrt(x^2-y(x)^2))*diff(y(x),x) = x*y(x)^2-(x^2-y(x)^2)^(3/2),y(x), singsol=a
```

$$-\frac{2\sqrt{x^2-y(x)^2}}{x}+\frac{y(x)^2}{x^2}+2\ln(x)-c_1=0$$

✓ Solution by Mathematica

Time used: 21.91 (sec). Leaf size: 353

```
DSolve[x y[x](x+Sqrt[x^2-y[x]^2])y'[x]==x y[x]^2-(x^2-y[x]^2)^(3/2),y[x],x,IncludeSingularSol
```

$$y(x) \to -\sqrt{x^2(-2\log(x) - 1 + 2c_1) - 2\sqrt{x^4(2\log(x) + 1 - 2c_1)}}$$
$$y(x) \to \sqrt{x^2(-2\log(x) - 1 + 2c_1) - 2\sqrt{x^4(2\log(x) + 1 - 2c_1)}}$$
$$y(x) \to -\sqrt{2\sqrt{x^4(2\log(x) + 1 - 2c_1)} + x^2(-2\log(x) - 1 + 2c_1)}$$
$$y(x) \to \sqrt{2\sqrt{x^4(2\log(x) + 1 - 2c_1)} + x^2(-2\log(x) - 1 + 2c_1)}$$
$$y(x) \to -\sqrt{x^2(2\log(x) - 1 - 2c_1) - 2\sqrt{x^4(-2\log(x) + 1 + 2c_1)}}$$
$$y(x) \to \sqrt{x^2(2\log(x) - 1 - 2c_1) - 2\sqrt{x^4(-2\log(x) + 1 + 2c_1)}}$$
$$y(x) \to -\sqrt{2\sqrt{x^4(-2\log(x) + 1 + 2c_1)} + x^2(2\log(x) - 1 - 2c_1)}$$
$$y(x) \to \sqrt{2\sqrt{x^4(-2\log(x) + 1 + 2c_1)} + x^2(2\log(x) - 1 - 2c_1)}$$

26.2 problem 734

Internal problem ID [3470]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 734.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\left(x\sqrt{1+x^2+y^2}-y(x^2+y^2)\right)y'-x(x^2+y^2)-y\sqrt{1+x^2+y^2}=0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 27

```
dsolve((x*sqrt(1+x^2+y(x)^2)-y(x)*(x^2+y(x)^2))*diff(y(x),x) = x*(x^2+y(x)^2)+y(x)*sqrt(1+x^2
```

$$\arctan\left(\frac{y(x)}{x}\right)-\sqrt{1+x^2+y(x)^2}-c_1=0$$

✓ Solution by Mathematica

Time used: 0.255 (sec). Leaf size: 27

```
DSolve[(x Sqrt[1+x^2+y[x]^2]-y[x] (x^2+y[x]^2))y'[x]==x(x^2+y[x]^2)+y[x]Sqrt[1+x^2+y[x]^2],y[
```

$$\text{Solve}\left[\arctan\left(\frac{x}{y(x)}\right)+\sqrt{x^2+y(x)^2+1}=c_1,y(x)\right]$$

26.3 problem 736

Internal problem ID [3471]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 736.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `unknown`

$$y' \cos(y) (\cos(y) - \sin(A) \sin(x)) + \cos(x) (\cos(x) - \sin(A) \sin(y)) = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)*cos(y(x))*(cos(y(x))-sin(A)*sin(x))+cos(x)*(cos(x)-sin(A)*sin(y(x))) = 0,
```

No solution found

✓ Solution by Mathematica

Time used: 0.414 (sec). Leaf size: 43

```
DSolve[y'[x] Cos[y[x]](Cos[y[x]]- Sin[A] Sin[x])+Cos[x](Cos[x]-Sin[A]Sin[y[x]])==0,y[x],x,Inc
```

$$\text{Solve}\left[4\sin(A)\sin(x)\sin(y(x)) - 4\left(\frac{y(x)}{2} + \frac{1}{4}\sin(2y(x))\right) - 2x - \sin(2x) = c_1, y(x)\right]$$

26.4 problem 737

Internal problem ID [3472]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 737.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$(a\cos(bx+ay)-b\sin(ax+yb))\,y'+b\cos(bx+ay)-a\sin(ax+yb)=0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 40

```
dsolve((a*cos(b*x+a*y(x))-b*sin(a*x+b*y(x)))*diff(y(x),x)+b*cos(b*x+a*y(x))-a*sin(a*x+b*y(x))
```

$$y(x)=\frac{-xb+\operatorname{RootOf}\left(a^2x-b^2x+a\arccos\left(\sin\left(\_Z\right)+c_1\right)-a\pi+\_Zb\right)}{a}$$

✓ Solution by Mathematica

Time used: 0.719 (sec). Leaf size: 50

```
DSolve[(a Cos[b x+a y[x]]-b Sin[a x+ b y[x]])y'[x]+b Cos[b x+a y[x]]-a Sin[a x+b y[x]]==0,y[x
```

$$\begin{aligned}&\text{Solve}[\sin(ax)\sin(by(x))-\cos(ax)\cos(by(x))\\&\quad-\sin(bx)\cos(ay(x))-\cos(bx)\sin(ay(x))=c_1,y(x)]\end{aligned}$$

26.5 problem 739

Internal problem ID [3473]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 739.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[NONE]`

$$(x+\cos(x)\sec(y))\,y'+\tan(y)-y\sin(x)\sec(y)=0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 15

```
dsolve((x+cos(x)*sec(y(x)))*diff(y(x),x)+tan(y(x))-y(x)*sin(x)*sec(y(x)) = 0,y(x), singsol=al
```

$$y(x)\cos(x)+x\sin(y(x))+c_1=0$$

✓ Solution by Mathematica

Time used: 0.154 (sec). Leaf size: 17

```
DSolve[(x+Cos[x] Sec[y[x]])y'[x]+Tan[y[x]]-y[x] Sin[x] Sec[y[x]]==0,y[x],x,IncludeSingularSol
```

$$\text{Solve}[x\sin(y(x))+y(x)\cos(x)=c_1,y(x)]$$

26.6 problem 742

Internal problem ID [3474]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 742.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{(1 + (x + y)\tan(y))\, y' + 1 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 13

```
dsolve((1+(x+y(x))*tan(y(x)))*diff(y(x),x)+1 = 0,y(x), singsol=all)
```

$$x - c_1 \cos(y(x)) + y(x) = 0$$

✓ Solution by Mathematica

Time used: 0.289 (sec). Leaf size: 66

```
DSolve[(1+(x+y[x]) Tan[y[x]])y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[x = \cos(y(x))\left(-\operatorname{arctanh}(\sin(y(x))) - y(x)\sec(y(x)) - \log\left(\cos\left(\frac{y(x)}{2}\right) - \sin\left(\frac{y(x)}{2}\right)\right) + \log\left(\sin\left(\frac{y(x)}{2}\right) + \cos\left(\frac{y(x)}{2}\right)\right)\right) + c_1\cos(y(x)), y(x)\right]$$

26.7 problem 743

Internal problem ID [3475]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 743.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\boxed{x\left(x - y\tan\left(\frac{y}{x}\right)\right)y' + \left(x + y\tan\left(\frac{y}{x}\right)\right)y = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 24

```
dsolve(x*(x-y(x)*tan(y(x)/x))*diff(y(x),x)+(x+y(x)*tan(y(x)/x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{\cos\left(\operatorname{RootOf}\left(\_Zx^2\cos\left(\_Z\right) - c_1\right)\right)x}$$

✓ Solution by Mathematica

Time used: 0.351 (sec). Leaf size: 31

```
DSolve[x(x-y[x] Tan[y[x]/x])y'[x]+(x+y[x] Tan[y[x]/x])y[x]==0,y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[-\log\left(\frac{y(x)}{x}\right) - \log\left(\cos\left(\frac{y(x)}{x}\right)\right) = 2\log(x) + c_1, y(x)\right]$$

26.8 problem 744

Internal problem ID [3476]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 744.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$(\mathrm{e}^{x} + \mathrm{e}^{y} x)\, y' + \mathrm{e}^{x} y + \mathrm{e}^{y} = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve((exp(x)+x*exp(y(x)))*diff(y(x),x)+y(x)*exp(x)+exp(y(x)) = 0,y(x), singsol=all)
```

$$y(x) = -\left(\mathrm{LambertW}\left(x\,\mathrm{e}^{-x}\mathrm{e}^{-c_1\mathrm{e}^{-x}}\right)\mathrm{e}^{x} + c_1\right)\mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 2.246 (sec). Leaf size: 33

```
DSolve[(Exp[x]+x Exp[y[x]])y'[x]+y[x] Exp[x]+Exp[y[x]]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to c_1 e^{-x} - W\left(x e^{-x+c_1 e^{-x}}\right)$$

26.9 problem 745

Internal problem ID [3477]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 745.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{(1 - 2x - \ln(y))\, y' + 2y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 19

```
dsolve((1-2*x-ln(y(x)))*diff(y(x),x)+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{-\mathrm{LambertW}\left(-2\,\mathrm{e}^{-2x} c_1\right) - 2x}$$

✓ Solution by Mathematica

Time used: 60.142 (sec). Leaf size: 23

```
DSolve[(1-2 x -Log[y[x]])y'[x]+2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{W(-2c_1 e^{-2x})}{2c_1}$$

26.10 problem 746

Internal problem ID [3478]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 746.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_exact]`

$$\boxed{(\sinh(x) + x\cosh(y))\,y' + y\cosh(x) + \sinh(y) = 0}$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 179

```
dsolve((sinh(x)+x*cosh(y(x)))*diff(y(x),x)+y(x)*cosh(x)+sinh(y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\left(-x\,\mathrm{e}^{2\,\mathrm{RootOf}\left(\_Z\mathrm{e}^{\_Z+2x}-x\,\mathrm{e}^{\_Z+2x}+x\,\mathrm{e}^{2\_Z}+2c_1\mathrm{e}^{x+\_Z}-\mathrm{e}^{2x}x-\_Z\mathrm{e}^{\_Z}+x\,\mathrm{e}^{\_Z}\right)} - 2c_1\mathrm{e}^{\mathrm{RootOf}\left(\_Z\mathrm{e}^{\_Z+2x}-x\,\mathrm{e}^{\_Z+2x}+x\,\mathrm{e}^{2\_Z}+2c_1\mathrm{e}^{x+\_}\right.}}{\mathrm{e}^{2x}-1}$$

✓ Solution by Mathematica

Time used: 0.221 (sec). Leaf size: 17

```
DSolve[(Sinh[x]+x Cosh[y[x]])y'[x]+y[x] Cosh[x]+Sinh[y[x]]==0,y[x],x,IncludeSingularSolutions
```

$$\text{Solve}[x\sinh(y(x)) + y(x)\sinh(x) = c_1, y(x)]$$

26.11 problem 747

Internal problem ID [3479]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 747.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$y'(1+\sinh(x))\sinh(y)+\cosh(x)(\cosh(y)-1)=0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 113

```
dsolve(diff(y(x),x)*(1+sinh(x))*sinh(y(x))+cosh(x)*(cosh(y(x))-1) = 0,y(x), singsol=all)
```

$$y(x)=-2\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2(2c_1e^x+c_1e^{2x}-2e^x-c_1)e^x}{c_1^2}}}{-e^{2x}+\frac{2e^x}{c_1}-2e^x+1}\right)$$

$$y(x)=2\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2(2c_1e^x+c_1e^{2x}-2e^x-c_1)e^x}{c_1^2}}}{-e^{2x}+\frac{2e^x}{c_1}-2e^x+1}\right)$$

✓ Solution by Mathematica

Time used: 6.141 (sec). Leaf size: 32

```
DSolve[y'[x](1+Sinh[x])Sinh[y[x]]+Cosh[x](Cosh[y[x]]-1)==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x)\to 0$$

$$y(x)\to 2\operatorname{arcsinh}\left(\frac{c_1}{4\sqrt{\sinh(x)+1}}\right)$$

$$y(x)\to 0$$

26.12 problem 748

Internal problem ID [3480]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 748.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - x^n a = 0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 41

```
dsolve(diff(y(x),x)^2 = a*x^n,y(x), singsol=all)
```

$$y(x) = \frac{2x\sqrt{a\,x^n}}{n+2} + c_1$$
$$y(x) = -\frac{2x\sqrt{a\,x^n}}{n+2} + c_1$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 57

```
DSolve[(y'[x])^2 == a x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2\sqrt{a}x^{\frac{n}{2}+1}}{n+2} + c_1$$
$$y(x) \to \frac{2\sqrt{a}x^{\frac{n}{2}+1}}{n+2} + c_1$$

26.13 problem 749

Internal problem ID [3481]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 749.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)^2 = y(x),y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{1}{4}c_1^2 - \frac{1}{2}c_1 x + \frac{1}{4}x^2$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 36

```
DSolve[(y'[x])^2 == y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}(x - c_1)^2$$
$$y(x) \to \frac{1}{4}(x + c_1)^2$$
$$y(x) \to 0$$

26.14 problem 750

Internal problem ID [3482]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 750.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y'^2 - x + y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)^2 = x-y(x),y(x), singsol=all)
```

$$y(x) = -\left(\text{LambertW}\left(c_1 \mathrm{e}^{-\frac{x}{2}-1}\right) + 1\right)^2 + x$$

✓ Solution by Mathematica

Time used: 18.692 (sec). Leaf size: 94

```
DSolve[(y'[x])^2==x-y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -W\left(e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right)\left(2 + W\left(e^{-\frac{x}{2}-1-\frac{c_1}{2}}\right)\right) + x - 1$$
$$y(x) \to -W\left(-e^{\frac{1}{2}(-x-2+c_1)}\right)\left(2 + W\left(-e^{\frac{1}{2}(-x-2+c_1)}\right)\right) + x - 1$$
$$y(x) \to x - 1$$

26.15 problem 751

Internal problem ID [3483]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 751.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$\boxed{y'^2 - y - x^2 = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 287

```
dsolve(diff(y(x),x)^2 = x^2+y(x),y(x), singsol=all)
```

$$2\sqrt{17}\,\operatorname{arctanh}\left(\frac{\left(4\sqrt{x^2+y(x)}+x\right)\sqrt{17}}{17x}\right) + 2\sqrt{17}\,\operatorname{arctanh}\left(\frac{\left(-x+4\sqrt{x^2+y(x)}\right)\sqrt{17}}{17x}\right) + 2\sqrt{17}\,\operatorname{arctanh}\left(\frac{\left(-x^2+8y(x)\right)\sqrt{17}}{17x^2}\right) + 17\ln\left(x\sqrt{x^2+y(x)}+2y(x)\right) - 17\ln\left(-x\sqrt{x^2+y(x)}+2y(x)\right) - 17\ln\left(-x^4-y(x)\,x^2+4y(x)^2\right) - c_1 = 0$$

$$2\sqrt{17}\,\operatorname{arctanh}\left(\frac{\left(4\sqrt{x^2+y(x)}+x\right)\sqrt{17}}{17x}\right) + 2\sqrt{17}\,\operatorname{arctanh}\left(\frac{\left(-x+4\sqrt{x^2+y(x)}\right)\sqrt{17}}{17x}\right) - 2\sqrt{17}\,\operatorname{arctanh}\left(\frac{\left(-x^2+8y(x)\right)\sqrt{17}}{17x^2}\right) + 17\ln\left(x\sqrt{x^2+y(x)}+2y(x)\right) - 17\ln\left(-x\sqrt{x^2+y(x)}+2y(x)\right) + 17\ln\left(-x^4-y(x)\,x^2+4y(x)^2\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.959 (sec). Leaf size: 215

```
DSolve[(y'[x])^2==x^2+y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{34}\left(-34\log\left(\sqrt{x^2+y(x)}-x\right)\right.\right.$$
$$-\left(\sqrt{17}-17\right)\log\left(2x\sqrt{x^2+y(x)}-2x^2-\sqrt{17}y(x)+3y(x)\right)$$
$$\left.\left.+\left(17+\sqrt{17}\right)\log\left(2x\sqrt{x^2+y(x)}-2x^2+\left(3+\sqrt{17}\right)y(x)\right)\right)=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{34}\left(-34\log\left(\sqrt{x^2+y(x)}-x\right)\right.\right.$$
$$+\left(17+\sqrt{17}\right)\log\left(2x\sqrt{x^2+y(x)}-2x^2+\left(\sqrt{17}-5\right)y(x)\right)$$
$$\left.\left.-\left(\sqrt{17}-17\right)\log\left(2x\sqrt{x^2+y(x)}-2x^2-\left(5+\sqrt{17}\right)y(x)\right)\right)=c_1,y(x)\right]$$

26.16 problem 752

Internal problem ID [3484]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 752.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$y'^2 + x^2 - 4y = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 141

```
dsolve(diff(y(x),x)^2+x^2 = 4*y(x),y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} + \frac{e^{2\,\mathrm{LambertW}\left(\frac{x\sqrt{2}\,e^{\frac{c_1}{2}}}{2}\right)+\ln(2)-c_1}}{4} + \frac{e^{\mathrm{LambertW}\left(\frac{x\sqrt{2}\,e^{\frac{c_1}{2}}}{2}\right)+\frac{\ln(2)}{2}-\frac{c_1}{2}}\,x}{2}$$

$$y(x) = \frac{x^2\left(2\,\mathrm{LambertW}\left(-\frac{\sqrt{2}\,x c_1}{2}\right)^2 + 2\,\mathrm{LambertW}\left(-\frac{\sqrt{2}\,x c_1}{2}\right) + 1\right)}{4\,\mathrm{LambertW}\left(-\frac{\sqrt{2}\,x c_1}{2}\right)^2}$$

$$y(x) = \frac{x^2\left(2\,\mathrm{LambertW}\left(\frac{\sqrt{2}\,x c_1}{2}\right)^2 + 2\,\mathrm{LambertW}\left(\frac{\sqrt{2}\,x c_1}{2}\right) + 1\right)}{4\,\mathrm{LambertW}\left(\frac{\sqrt{2}\,x c_1}{2}\right)^2}$$

✓ Solution by Mathematica

Time used: 1.797 (sec). Leaf size: 162

```
DSolve[(y'[x])^2+x^2==4 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\operatorname{arctanh}\left(\frac{x}{\sqrt{4y(x)-x^2}}\right)+\frac{x\left(-\sqrt{4y(x)-x^2}\right)+(x^2-2y(x))\log\left(2y(x)-x^2\right)+2y(x)}{2\left(x^2-2y(x)\right)}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{x\sqrt{4y(x)-x^2}+(x^2-2y(x))\log\left(2y(x)-x^2\right)+2y(x)}{2\left(x^2-2y(x)\right)}-\operatorname{arctanh}\left(\frac{x}{\sqrt{4y(x)-x^2}}\right)=c_1,y(x)\right]$$

26.17 problem 753

Internal problem ID [3485]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 753.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{y'^2 + 3x^2 - 8y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 153

```
dsolve(diff(y(x),x)^2+3*x^2 = 8*y(x),y(x), singsol=all)
```

$$y(x) = \frac{3x^2}{8} + \frac{\text{RootOf}\left(\_Z^6 - 18x\_Z^5 + 135x^2\_Z^4 - 540x^3\_Z^3 + (1215x^4 - 16c_1)\_Z^2 + (-1458x^5 + 32c_1x)\_Z +}{8}$$

$$y(x) = \frac{3x^2}{8} + \frac{\text{RootOf}\left(\_Z^6 + 18x\_Z^5 + 135x^2\_Z^4 + 540x^3\_Z^3 + (1215x^4 - 16c_1)\_Z^2 + (1458x^5 - 32c_1x)\_Z + 72}{8}$$

✓ Solution by Mathematica

Time used: 60.625 (sec). Leaf size: 1865

```
DSolve[(y'[x])^2+3 x^2==8 y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{96}\Bigg(144x^2$$

$$-8\,2^{2/3}\sqrt[3]{-729x^4\cosh(2c_1) - 729x^4\sinh(2c_1) - 270x^2\cosh(4c_1) - 270x^2\sinh(4c_1) + 3\sqrt{3}\sqrt{x^2(\cosh(7c_1)}$$

$$-\frac{16\sqrt[3]{2}(54x^2\cosh(2c_1) + 54x^2\sinh(2c_1) + \mathrm{co}}{\sqrt[3]{-729x^4\cosh(2c_1) - 729x^4\sinh(2c_1) - 270x^2\cosh(4c_1) - 270x^2\sinh(4c_1) + 3\sqrt{3}\sqrt{x^2(\cosh(7c_1) + \mathrm{si}}}$$

$$+ 32\cosh(2c_1) + 32\sinh(2c_1)\Bigg)$$

$$y(x) \to \frac{1}{192}\Bigg(288x^2$$

$$+8\,2^{2/3}\left(1-i\sqrt{3}\right)\sqrt[3]{-729x^4\cosh(2c_1) - 729x^4\sinh(2c_1) - 270x^2\cosh(4c_1) - 270x^2\sinh(4c_1) + 3\sqrt{3}\sqrt{x}}$$

$$+\frac{16\sqrt[3]{2}(1 + i\sqrt{3})\,(54x^2\cosh(2c_1) + 54x^2\sinh(2c_1}{\sqrt[3]{-729x^4\cosh(2c_1) - 729x^4\sinh(2c_1) - 270x^2\cosh(4c_1) - 270x^2\sinh(4c_1) + 3\sqrt{3}\sqrt{x^2(\cosh(7c_1) + \mathrm{si}}}$$

$$+ 64\cosh(2c_1) + 64\sinh(2c_1)\Bigg)$$

$$y(x) \to \frac{1}{192}\Bigg(288x^2$$

$$+8\,2^{2/3}\left(1+i\sqrt{3}\right)\sqrt[3]{-729x^4\cosh(2c_1) - 729x^4\sinh(2c_1) - 270x^2\cosh(4c_1) - 270x^2\sinh(4c_1) + 3\sqrt{3}\sqrt{x}}$$

$$+\frac{16\sqrt[3]{2}(1 - i\sqrt{3})\,(54x^2\cosh(2c_1) + 54x^2\sinh(2c_1}{\sqrt[3]{-729x^4\cosh(2c_1) - 729x^4\sinh(2c_1) - 270x^2\cosh(4c_1) - 270x^2\sinh(4c_1) + 3\sqrt{3}\sqrt{x^2(\cosh(7c_1) + \mathrm{si}}}$$

$$+ 64\cosh(2c_1) + 64\sinh(2c_1)\Bigg)$$

$$y(x) \to \frac{1}{96}\Bigg(144x^2$$

$$-8\,2^{2/3}\sqrt[3]{729x^4\cosh(2c_1) + 729x^4\sinh(2c_1) - 270x^2\cosh(4c_1) - 270x^2\sinh(4c_1) + 3\sqrt{3}\sqrt{x^2(\cosh(7c_1)}$$

$$+\frac{16\sqrt[3]{2}(54x^2\cosh(2c_1) + 54x^2\sinh(2c_1) - \mathrm{co}}{\sqrt[3]{729x^4\cosh(2c_1) + 729x^4\sinh(2c_1) - 270x^2\cosh(4c_1) - 270x^2\sinh(4c_1) + 3\sqrt{3}\sqrt{x^2(\cosh(7c_1) + \mathrm{sinh}}}$$

26.18 problem 754

Internal problem ID [3486]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 754.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$\boxed{{y'}^{2}+x^{2}a+yb=0}$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)^2+a*x^2+b*y(x) = 0,y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 1.106 (sec). Leaf size: 581

```
DSolve[(y'[x])^2+a x^2+b y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\text{RootSum}\left[\#1^4-\#1^3b+2\#1^2a+\#1ab+a^2\&,\frac{2\#1^3\log\left(\#1x-\sqrt{-ax^2-by(x)}+\sqrt{-by(x)}\right)-2\#1^3\log(x)-\#1^2b\log\left(\#1x-\sqrt{-ax^2-by(x)}\right.}{}\right.\right.$$

$$\left.-\log\left(\sqrt{-by(x)}\sqrt{-ax^2-by(x)}+by(x)\right)+\frac{1}{2}\log(y(x))+2\log(x)=c_1,y(x)\right]$$

$$\text{Solve}\left[\text{RootSum}\left[\#1^4+\#1^3b+2\#1^2a-\#1ab+a^2\&,\frac{-2\#1^3\log\left(\#1x-\sqrt{-ax^2-by(x)}+\sqrt{-by(x)}\right)+2\#1^3\log(x)-\#1^2b\log\left(\#1x-\sqrt{-ax^2-by(x\right.}{}\right.\right.$$

$$\left.-\log\left(\sqrt{-by(x)}\sqrt{-ax^2-by(x)}+by(x)\right)+\frac{1}{2}\log(y(x))+2\log(x)=c_1,y(x)\right]$$

26.19 problem 755

Internal problem ID [3487]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 755.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - 1 - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)^2 = 1+y(x)^2,y(x), singsol=all)
```

$$y(x) = -i$$
$$y(x) = i$$
$$y(x) = -\sinh(c_1 - x)$$
$$y(x) = \sinh(c_1 - x)$$

✓ Solution by Mathematica

Time used: 0.189 (sec). Leaf size: 37

```
DSolve[(y'[x])^2==1+y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sinh(x - c_1)$$
$$y(x) \to \sinh(x + c_1)$$
$$y(x) \to -i$$
$$y(x) \to i$$

26.20 problem 756

Internal problem ID [3488]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 756.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - 1 + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)^2 = 1-y(x)^2,y(x), singsol=all)
```

$$y(x) = -1$$
$$y(x) = 1$$
$$y(x) = -\sin(c_1 - x)$$
$$y(x) = \sin(c_1 - x)$$

✓ Solution by Mathematica

Time used: 0.132 (sec). Leaf size: 39

```
DSolve[(y'[x])^2==1-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \cos(x + c_1)$$
$$y(x) \to \cos(x - c_1)$$
$$y(x) \to -1$$
$$y(x) \to 1$$
$$y(x) \to \text{Interval}[\{-1, 1\}]$$

26.21 problem 757

Internal problem ID [3489]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 757.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - a^2 + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 68

```
dsolve(diff(y(x),x)^2 = a^2-y(x)^2,y(x), singsol=all)
```

$$y(x) = -a$$
$$y(x) = a$$
$$y(x) = -\tan(c_1 - x)\sqrt{\frac{a^2}{\tan(c_1 - x)^2 + 1}}$$
$$y(x) = \tan(c_1 - x)\sqrt{\frac{a^2}{\tan(c_1 - x)^2 + 1}}$$

✓ Solution by Mathematica

Time used: 3.309 (sec). Leaf size: 111

```
DSolve[(y'[x])^2==a^2-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{a\tan(x-c_1)}{\sqrt{\sec^2(x-c_1)}}$$
$$y(x) \to \frac{a\tan(x-c_1)}{\sqrt{\sec^2(x-c_1)}}$$
$$y(x) \to -\frac{a\tan(x+c_1)}{\sqrt{\sec^2(x+c_1)}}$$
$$y(x) \to \frac{a\tan(x+c_1)}{\sqrt{\sec^2(x+c_1)}}$$
$$y(x) \to -a$$
$$y(x) \to a$$

26.22 problem 758

Internal problem ID [3490]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 758.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - y^2a^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)^2 = a^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{ax}$$
$$y(x) = c_1 \mathrm{e}^{-ax}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 31

```
DSolve[(y'[x])^2==a^2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-ax}$$
$$y(x) \to c_1 e^{ax}$$
$$y(x) \to 0$$

26.23 problem 759

Internal problem ID [3491]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 759.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$
{y'}^2 - a - b y^2 = 0
$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 114

```
dsolve(diff(y(x),x)^2 = a+b*y(x)^2,y(x), singsol=all)
```

$$
y(x) = \frac{\sqrt{-ab}}{b}
$$

$$
y(x) = -\frac{\sqrt{-ab}}{b}
$$

$$
y(x) = \frac{\left(e^{-2c_1\sqrt{b}} e^{2x\sqrt{b}} - a\right) e^{c_1\sqrt{b}} e^{-x\sqrt{b}}}{2\sqrt{b}}
$$

$$
y(x) = \frac{\left(e^{2c_1\sqrt{b}} e^{-2x\sqrt{b}} - a\right) e^{-c_1\sqrt{b}} e^{x\sqrt{b}}}{2\sqrt{b}}
$$

✓ Solution by Mathematica

Time used: 60.069 (sec). Leaf size: 171

```
DSolve[(y'[x])^2==a+b y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{a}\tanh\left(\sqrt{b}(x-c_1)\right)}{\sqrt{b\mathrm{sech}^2\left(\sqrt{b}(x-c_1)\right)}}$$

$$y(x) \to \frac{\sqrt{a}\tanh\left(\sqrt{b}(x-c_1)\right)}{\sqrt{b\mathrm{sech}^2\left(\sqrt{b}(x-c_1)\right)}}$$

$$y(x) \to -\frac{\sqrt{a}\tanh\left(\sqrt{b}(x+c_1)\right)}{\sqrt{b\mathrm{sech}^2\left(\sqrt{b}(x+c_1)\right)}}$$

$$y(x) \to \frac{\sqrt{a}\tanh\left(\sqrt{b}(x+c_1)\right)}{\sqrt{b\mathrm{sech}^2\left(\sqrt{b}(x+c_1)\right)}}$$

26.24 problem 760

Internal problem ID [3492]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 760.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{y'^2 - x^2y^2 = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)^2 = x^2*y(x)^2,y(x), singsol=all)
```

$$y(x) = e^{\frac{x^2}{2}} c_1$$
$$y(x) = c_1 e^{-\frac{x^2}{2}}$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 38

```
DSolve[(y'[x])^2==x^2 y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-\frac{x^2}{2}}$$
$$y(x) \to c_1 e^{\frac{x^2}{2}}$$
$$y(x) \to 0$$

26.25 problem 761

Internal problem ID [3493]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 761.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - (y-1)\,y^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 24

```
dsolve(diff(y(x),x)^2 = (y(x)-1)*y(x)^2,y(x), singsol=all)
```

$$y(x) = 1$$
$$y(x) = 0$$
$$y(x) = \tan\left(\frac{c_1}{2} - \frac{x}{2}\right)^2 + 1$$

✓ Solution by Mathematica

Time used: 1.086 (sec). Leaf size: 43

```
DSolve[(y'[x])^2==(y[x]-1)y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sec^2\left(\frac{x - c_1}{2}\right)$$
$$y(x) \to \sec^2\left(\frac{x + c_1}{2}\right)$$
$$y(x) \to 0$$
$$y(x) \to 1$$

26.26 problem 762

Internal problem ID [3494]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 762.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - (y-a)(y-b)(y-c) = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 79

```
dsolve(diff(y(x),x)^2 = (y(x)-a)*(y(x)-b)*(y(x)-c),y(x), singsol=all)
```

$$y(x) = a$$
$$y(x) = b$$
$$y(x) = c$$
$$x - \left(\int^{y(x)} \frac{1}{\sqrt{(\_a-a)(\_a-b)(\_a-c)}}d\_a\right) - c_1 = 0$$
$$x - \left(\int^{y(x)} -\frac{1}{\sqrt{(\_a-a)(\_a-b)(\_a-c)}}d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 31.466 (sec). Leaf size: 188

```
DSolve[(y'[x])^2==(y[x]-a)(y[x]-b)(y[x]-c),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \operatorname{ns}\left(\frac{1}{2}\sqrt{a-b}(c_1-ix)|\frac{a-c}{a-b}\right)^2\left(a\operatorname{sn}\left(\frac{1}{2}\sqrt{a-b}(c_1-ix)|\frac{a-c}{a-b}\right)^2-a+b\right)$$

$$y(x) \to \operatorname{ns}\left(\frac{1}{2}\sqrt{a-b}(ix+c_1)|\frac{a-c}{a-b}\right)^2\left(a\operatorname{sn}\left(\frac{1}{2}\sqrt{a-b}(ix+c_1)|\frac{a-c}{a-b}\right)^2-a+b\right)$$

$$y(x) \to a$$

$$y(x) \to b$$

$$y(x) \to c$$

26.27 problem 763

Internal problem ID [3495]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 763.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - a^2 y^n = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 83

```
dsolve(diff(y(x),x)^2 = a^2*y(x)^n,y(x), singsol=all)
```

$$y(x) = 2^{\frac{2}{-2+n}} \left(\frac{1}{a\left(c_1 n - nx - 2c_1 + 2x\right)}\right)^{\frac{2}{-2+n}}$$
$$y(x) = 2^{\frac{2}{-2+n}} \left(\frac{1}{a\left(-c_1 n + nx + 2c_1 - 2x\right)}\right)^{\frac{2}{-2+n}}$$

✓ Solution by Mathematica

Time used: 2.075 (sec). Leaf size: 77

```
DSolve[(y'[x])^2==a^2 y[x]^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2^{\frac{2}{n-2}}(-((n-2)(ax+c_1)))^{-\frac{2}{n-2}}$$
$$y(x) \to 2^{\frac{2}{n-2}}((n-2)(ax-c_1))^{-\frac{2}{n-2}}$$
$$y(x) \to 0^{\frac{1}{n}}$$

26.28 problem 764

Internal problem ID [3496]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 764.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - a^2\left(1 - \ln\left(y\right)^2\right) y^2 = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 47

```
dsolve(diff(y(x),x)^2 = a^2*(1-ln(y(x))^2)*y(x)^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{RootOf}\left(a^2 \mathrm{e}^{2\_Z}\left(\_Z^2-1\right)\right)}$$
$$y(x) = \mathrm{e}^{-\sin((c_1-x)a)}$$
$$y(x) = \mathrm{e}^{\sin(c_1 a - a x)}$$

✓ Solution by Mathematica

Time used: 12.426 (sec). Leaf size: 157

```
DSolve[(y'[x])^2==a^2(1-Log[y[x]]^2)y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{\cosh(\sin(ax + ic_1)) + \sinh\left(\sqrt{\sin^2(ax + ic_1)}\right)}$$

$$y(x) \to \cosh(\sin(ax + ic_1)) + \sinh\left(\sqrt{\sin^2(ax + ic_1)}\right)$$

$$y(x) \to \cosh(\sin(ax - ic_1)) - \sinh\left(\sqrt{\sin^2(ax - ic_1)}\right)$$

$$y(x) \to \cosh(\sin(ax - ic_1)) + \sinh\left(\sqrt{\sin^2(ax - ic_1)}\right)$$

$$y(x) \to \frac{1}{e}$$

$$y(x) \to e$$

26.29 problem 765

Internal problem ID [3497]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 26
Problem number: 765.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y'^2 + f(x)\,(y-a)\,(y-b) = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 212

```
dsolve(diff(y(x),x)^2+f(x)*(y(x)-a)*(y(x)-b) = 0,y(x), singsol=all)
```

$$\frac{\sqrt{(y(x)-a)\,(y(x)-b)}\,\ln\left(-\frac{b}{2}-\frac{a}{2}+y(x)+\sqrt{y(x)^2+(-b-a)\,y(x)+ab}\right)}{\sqrt{y(x)-a}\,\sqrt{y(x)-b}} + \int^x -\frac{\sqrt{-f(\_a)\,(-y(x)+a)\,(b-y(x))}}{\sqrt{y(x)-a}\,\sqrt{y(x)-b}}d\_a + c_1 = 0$$

$$\frac{\sqrt{(y(x)-a)\,(y(x)-b)}\,\ln\left(-\frac{b}{2}-\frac{a}{2}+y(x)+\sqrt{y(x)^2+(-b-a)\,y(x)+ab}\right)}{\sqrt{y(x)-a}\,\sqrt{y(x)-b}} + \int^x \frac{\sqrt{-f(\_a)\,(-y(x)+a)\,(b-y(x))}}{\sqrt{y(x)-a}\,\sqrt{y(x)-b}}d\_a + c_1 = 0$$

✓ Solution by Mathematica

Time used: 3.298 (sec). Leaf size: 89

```
DSolve[(y'[x])^2+ f[x](y[x]-a)(y[x]-b)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}\left((b-a)\cosh\left(\int_1^x -i\sqrt{f(K[2])}dK[2]+c_1\right)+a+b\right)$$
$$y(x) \to \frac{1}{2}\left((b-a)\cosh\left(\int_1^x i\sqrt{f(K[3])}dK[3]+c_1\right)+a+b\right)$$
$$y(x) \to a$$
$$y(x) \to b$$

27 Various 27

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27.1 problem 766

Internal problem ID [3498]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 766.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y'^2 + f(x)\,(y-a)^2\,(y-b) = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 110

```
dsolve(diff(y(x),x)^2+f(x)*(y(x)-a)^2*(y(x)-b) = 0,y(x), singsol=all)
```

$$\frac{2\arctan\left(\frac{\sqrt{y(x)-b}}{\sqrt{b-a}}\right)}{\sqrt{b-a}} + \int^{x} \frac{\sqrt{f(\_a)\,(b-y(x))}}{\sqrt{y(x)-b}} d\_a + c_1 = 0$$
$$\frac{2\arctan\left(\frac{\sqrt{y(x)-b}}{\sqrt{b-a}}\right)}{\sqrt{b-a}} + \int^{x} -\frac{\sqrt{f(\_a)\,(b-y(x))}}{\sqrt{y(x)-b}} d\_a + c_1 = 0$$

✓ Solution by Mathematica

Time used: 54.555 (sec). Leaf size: 103

```
DSolve[(y'[x])^2+f[x](y[x]-a)^2 (y[x]-b)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to b + (b-a)\tan^2\left(\frac{1}{2}\sqrt{a-b}\left(\int_1^x -\sqrt{f(K[1])}dK[1] + c_1\right)\right)$$
$$y(x) \to b + (b-a)\tan^2\left(\frac{1}{2}\sqrt{a-b}\left(\int_1^x \sqrt{f(K[2])}dK[2] + c_1\right)\right)$$
$$y(x) \to a$$
$$y(x) \to b$$

27.2 problem 767

Internal problem ID [3499]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 767.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y'^2 + f(x)\,(y-a)\,(y-b)\,(y-c) = 0$$

✓ Solution by Maple

Time used: 0.235 (sec). Leaf size: 158

```
dsolve(diff(y(x),x)^2+f(x)*(y(x)-a)*(y(x)-b)*(y(x)-c) = 0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\sqrt{-\left(-\_a+c\right)\left(-\_a+b\right)\left(-\_a+a\right)}}d\_a + \int^{x} -\frac{\sqrt{f\left(\_a\right)\left(c-y\left(x\right)\right)\left(b-y\left(x\right)\right)\left(-y\left(x\right)+a\right)}}{\sqrt{-\left(c-y\left(x\right)\right)\left(b-y\left(x\right)\right)\left(-y\left(x\right)+a\right)}}d\_a + c_1 = 0$$

$$\int^{y(x)} \frac{1}{\sqrt{-\left(-\_a+c\right)\left(-\_a+b\right)\left(-\_a+a\right)}}d\_a + \int^{x} \frac{\sqrt{f\left(\_a\right)\left(c-y\left(x\right)\right)\left(b-y\left(x\right)\right)\left(-y\left(x\right)+a\right)}}{\sqrt{-\left(c-y\left(x\right)\right)\left(b-y\left(x\right)\right)\left(-y\left(x\right)+a\right)}}d\_a + c_1 = 0$$

✓ Solution by Mathematica

Time used: 25.81 (sec). Leaf size: 228

```
DSolve[(y'[x])^2+f[x](y[x]-a)(y[x]-b)(y[x]-c)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{ns}\left(\frac{1}{2}\sqrt{a-b}\left(c_1+\int_1^x -\sqrt{f(K[1])}dK[1]\right)|\frac{a-c}{a-b}\right)^2\left(a\text{sn}\left(\frac{1}{2}\sqrt{a-b}\left(c_1+\int_1^x -\sqrt{f(K[1])}dK[1]\right)|\frac{a-c}{a-b}\right)^2-a+b\right)$$

$$y(x) \to \text{ns}\left(\frac{1}{2}\sqrt{a-b}\left(c_1+\int_1^x \sqrt{f(K[2])}dK[2]\right)|\frac{a-c}{a-b}\right)^2\left(a\text{sn}\left(\frac{1}{2}\sqrt{a-b}\left(c_1+\int_1^x \sqrt{f(K[2])}dK[2]\right)|\frac{a-c}{a-b}\right)^2-a+b\right)$$

$$y(x) \to a$$

$$y(x) \to b$$

$$y(x) \to c$$

27.3 problem 768

Internal problem ID [3500]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 768.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y'^2+f(x)(y-a)^2(y-b)(y-c)=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 378

```
dsolve(diff(y(x),x)^2+f(x)*(y(x)-a)^2*(y(x)-b)*(y(x)-c) = 0,y(x), singsol=all)
```

$$\frac{\ln\left(\frac{2\sqrt{a^2-ab-ac+cb}\,\sqrt{cb-cy(x)-by(x)+y(x)^2}+2ay(x)-by(x)-cy(x)-ab-ac+2cb}{y(x)-a}\right)\sqrt{a^2-ab-ac+cb}\,\sqrt{y(x)-b}\,\sqrt{y(x)-}}{(b-a)(a-c)\sqrt{cb-cy(x)-by(x)+y(x)^2}}$$
$$+\int^x\frac{\sqrt{-f(\_a)(c-y(x))(b-y(x))}}{\sqrt{y(x)-c}\,\sqrt{y(x)-b}}d\_a+c_1=0$$

$$\frac{\ln\left(\frac{2\sqrt{a^2-ab-ac+cb}\,\sqrt{cb-cy(x)-by(x)+y(x)^2}+2ay(x)-by(x)-cy(x)-ab-ac+2cb}{y(x)-a}\right)\sqrt{a^2-ab-ac+cb}\,\sqrt{y(x)-b}\,\sqrt{y(x)-}}{(b-a)(a-c)\sqrt{cb-cy(x)-by(x)+y(x)^2}}$$
$$+\int^x-\frac{\sqrt{-f(\_a)(c-y(x))(b-y(x))}}{\sqrt{y(x)-c}\,\sqrt{y(x)-b}}d\_a+c_1=0$$

✓ Solution by Mathematica

Time used: 60.203 (sec). Leaf size: 155

```
DSolve[(y'[x])^2+f[x](y[x]-a)^2 (y[x]-b) (y[x]-c)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to a - \frac{2(a-b)(a-c)}{(c-b)\cos\left(\sqrt{a-b}\sqrt{c-a}\left(\int_1^x -i\sqrt{f(K[1])}dK[1]+c_1\right)\right)+2a-b-c}$$

$$y(x) \to a - \frac{2(a-b)(a-c)}{(c-b)\cos\left(\sqrt{a-b}\sqrt{c-a}\left(\int_1^x i\sqrt{f(K[2])}dK[2]+c_1\right)\right)+2a-b-c}$$

27.4 problem 770

Internal problem ID [3501]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 770.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$y'^2 - f(x)^2 (y-a)(y-b)(y-c)^2 = 0$$

✓ Solution by Maple

Time used: 0.797 (sec). Leaf size: 821

```
dsolve(diff(y(x),x)^2 = f(x)^2*(y(x)-a)*(y(x)-b)*(y(x)-c)^2,y(x), singsol=all)
```

$$y(x) = \frac{a^2c - 2abc + 4ab\,e^{\left(\int f(x)dx\right)\sqrt{ab-ac-cb+c^2}+c_1\sqrt{ab-ac-cb+c^2}} - 2ac\,e^{\left(\int f(x)dx\right)\sqrt{ab-ac-cb+c^2}+c_1\sqrt{ab-ac-cb+c^2}} + b^2c - 2}{a^2 - 2ab + 2a\,e^{\left(\int f(x)dx\right)\sqrt{ab-ac-cb+c^2}+c_1\sqrt{ab-ac-cb+c^2}} + b^2 + 2b\,e^{\left(\int f(x)dx\right)\sqrt{ab-ac-cb+c^2}+c_1\sqrt{ab-ac-cb+c^2}} - 4\,e}$$

$$y(x) = \frac{a^2c - 2abc + 4ab\,e^{-\left(\int f(x)dx\right)\sqrt{ab-ac-cb+c^2}-c_1\sqrt{ab-ac-cb+c^2}} - 2ac\,e^{-\left(\int f(x)dx\right)\sqrt{ab-ac-cb+c^2}-c_1\sqrt{ab-ac-cb+c^2}} + b^2c}{a^2 - 2ab + 2a\,e^{-\left(\int f(x)dx\right)\sqrt{ab-ac-cb+c^2}-c_1\sqrt{ab-ac-cb+c^2}} + b^2 + 2b\,e^{-\left(\int f(x)dx\right)\sqrt{ab-ac-cb+c^2}-c_1\sqrt{ab-ac-cb+c^2}} -}$$

✓ Solution by Mathematica

Time used: 60.194 (sec). Leaf size: 133

```
DSolve[(y'[x])^2==f[x]^2 (y[x]-a)(y[x]-b)(y[x]-c)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c + \frac{2(a-c)(b-c)}{(a-b)\cos\left(\sqrt{c-a}\sqrt{b-c}\left(\int_1^x -f(K[1])dK[1] + c_1\right)\right) + a + b - 2c}$$

$$y(x) \to c + \frac{2(a-c)(b-c)}{(a-b)\cos\left(\sqrt{c-a}\sqrt{b-c}\left(\int_1^x f(K[2])dK[2] + c_1\right)\right) + a + b - 2c}$$

27.5 problem 771

Internal problem ID [3502]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 771.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type [`y=_G(x,y')`]

$$y'^2 - f(x)^2 (y - u(x))^2 (y - a)(y - b) = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)^2 = f(x)^2*(y(x)-u(x))^2*(y(x)-a)*(y(x)-b),y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^2==f[x]^2 (y[x]-u[x])^2 (y[x]-a)(y[x]-b),y[x],x,IncludeSingularSolutions -> Tr
```

Not solved

27.6 problem 772

Internal problem ID [3503]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 772.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 + 2y' + x = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 35

```
dsolve(diff(y(x),x)^2+2*diff(y(x),x)+x = 0,y(x), singsol=all)
```

$$y(x) = -x + \frac{2(1-x)^{\frac{3}{2}}}{3} + c_1$$
$$y(x) = -x - \frac{2(1-x)^{\frac{3}{2}}}{3} + c_1$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 47

```
DSolve[(y'[x])^2+2 y'[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{3}(1-x)^{3/2} - x + c_1$$
$$y(x) \to \frac{2}{3}(1-x)^{3/2} - x + c_1$$

27.7 problem 773

Internal problem ID [3504]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 773.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$y'^2 - 2y' + a(x - y) = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 42

```
dsolve(diff(y(x),x)^2-2*diff(y(x),x)+a*(x-y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \frac{ax - 1}{a}$$

$$y(x) = x + \frac{\frac{(c_1 - x)^2 a^2}{4} + (c_1 - x)\, a}{a}$$

✓ Solution by Mathematica

Time used: 0.296 (sec). Leaf size: 84

```
DSolve[(y'[x])^2-2 y'[x]+a(x-y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}a\left(x^2 - 2\sqrt{2}c_1 x + 2{c_1}^2\right) - \frac{1}{a} + x$$

$$y(x) \to \frac{1}{4}a\left(x^2 + 2\sqrt{2}c_1 x + 2{c_1}^2\right) - \frac{1}{a} + x$$

$$y(x) \to x - \frac{1}{a}$$

27.8 problem 774

Internal problem ID [3505]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 774.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - 2y' - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)^2-2*diff(y(x),x)-y(x)^2 = 0,y(x), singsol=all)
```

$$x - \frac{1}{y(x)} - \frac{\left(1+y(x)^2\right)^{\frac{3}{2}}}{y(x)} + y(x)\sqrt{1+y(x)^2} + \operatorname{arcsinh}(y(x)) - c_1 = 0$$

$$x + \frac{\left(1+y(x)^2\right)^{\frac{3}{2}}}{y(x)} - y(x)\sqrt{1+y(x)^2} - \operatorname{arcsinh}(y(x)) - \frac{1}{y(x)} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.912 (sec). Leaf size: 104

```
DSolve[(y'[x])^2-2 y'[x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{\sqrt{\#1^2+1} + \#1\log\left(\sqrt{\#1^2+1} - \#1\right) + 1}{\#1}\&\right][-x+c_1]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{\sqrt{\#1^2+1}}{\#1} - \log\left(\sqrt{\#1^2+1} - \#1\right) + \frac{1}{\#1}\&\right][x+c_1]$$

$$y(x) \to 0$$

27.9 problem 775

Internal problem ID [3506]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 775.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - 5y' + 6 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)^2-5*diff(y(x),x)+6 = 0,y(x), singsol=all)
```

$$y(x) = 3x + c_1$$
$$y(x) = 2x + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 21

```
DSolve[(y'[x])^2-5 y'[x]+6==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2x + c_1$$
$$y(x) \to 3x + c_1$$

27.10 problem 776

Internal problem ID [3507]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 776.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - 7y' + 12 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(diff(y(x),x)^2-7*diff(y(x),x)+12 = 0,y(x), singsol=all)
```

$$y(x) = 4x + c_1$$
$$y(x) = 3x + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 21

```
DSolve[(y'[x])^2-7 y'[x]+12==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 3x + c_1$$
$$y(x) \to 4x + c_1$$

27.11 problem 777

Internal problem ID [3508]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 777.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 + ay' + b = 0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 45

```
dsolve(diff(y(x),x)^2+a*diff(y(x),x)+b = 0,y(x), singsol=all)
```

$$y(x) = \left(-\frac{a}{2} - \frac{\sqrt{a^2-4b}}{2}\right)x + c_1$$
$$y(x) = \left(-\frac{a}{2} + \frac{\sqrt{a^2-4b}}{2}\right)x + c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 53

```
DSolve[(y'[x])^2+a y'[x]+b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}x\left(\sqrt{a^2-4b}+a\right) + c_1$$
$$y(x) \to \frac{1}{2}x\left(\sqrt{a^2-4b}-a\right) + c_1$$

27.12 problem 778

Internal problem ID [3509]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 778.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{{y'}^2 + ay' + bx = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 49

```
dsolve(diff(y(x),x)^2+a*diff(y(x),x)+b*x = 0,y(x), singsol=all)
```

$$y(x) = -\frac{ax}{2} + \frac{(a^2 - 4xb)^{\frac{3}{2}}}{12b} + c_1$$
$$y(x) = -\frac{(a^2 - 4xb)^{\frac{3}{2}}}{12b} - \frac{ax}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.017 (sec). Leaf size: 68

```
DSolve[(y'[x])^2+a y'[x]+b x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{(a^2 - 4bx)^{3/2} + 6abx}{12b} + c_1$$
$$y(x) \to \frac{1}{2}\left(\frac{(a^2 - 4bx)^{3/2}}{6b} - ax\right) + c_1$$

27.13 problem 779

Internal problem ID [3510]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 779.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 + ay' + yb = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 275

```
dsolve(diff(y(x),x)^2+a*diff(y(x),x)+b*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{a^2\left(\mathrm{LambertW}\left(-\frac{2\sqrt{-b}\,e^{\frac{c_1 b}{a}}e^{-\frac{xb}{a}}e^{-1}}{a}\right)+2\right)\mathrm{LambertW}\left(-\frac{2\sqrt{-b}\,e^{\frac{c_1 b}{a}}e^{-\frac{xb}{a}}e^{-1}}{a}\right)}{4b}$$

$$y(x) = -\frac{a^2\left(\mathrm{LambertW}\left(\frac{2\sqrt{-b}\,e^{\frac{c_1 b}{a}}e^{-\frac{xb}{a}}e^{-1}}{a}\right)+2\right)\mathrm{LambertW}\left(\frac{2\sqrt{-b}\,e^{\frac{c_1 b}{a}}e^{-\frac{xb}{a}}e^{-1}}{a}\right)}{4b}$$

$$y(x) = -\frac{e^{-\frac{a\ln\left(-\frac{1}{4b}\right)+2a\,\mathrm{LambertW}\left(\frac{2\,e^{\frac{c_1 b}{a}}e^{-\frac{xb}{a}}e^{-1}}{a\sqrt{-\frac{1}{b}}}\right)-2c_1 b+2xb+2a}{2a}}\left(e^{-\frac{a\ln\left(-\frac{1}{4b}\right)+2a\,\mathrm{LambertW}\left(\frac{2\,e^{\frac{c_1 b}{a}}e^{-\frac{xb}{a}}e^{-1}}{a\sqrt{-\frac{1}{b}}}\right)-2c_1 b+2xb+2a}{2a}}+2a\right)}{4b}$$

✓ Solution by Mathematica

Time used: 0.768 (sec). Leaf size: 119

```
DSolve[(y'[x])^2+a y'[x]+b y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{\sqrt{a^2-4\#1b}+a\log\left(b\left(a-\sqrt{a^2-4\#1b}\right)\right)}{2b}\&\right]\left[\frac{x}{2}+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{\sqrt{a^2-4\#1b}-a\log\left(b\left(\sqrt{a^2-4\#1b}+a\right)\right)}{2b}\&\right]\left[-\frac{x}{2}+c_1\right]$$

$$y(x) \to 0$$

27.14 problem 780

Internal problem ID [3511]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 780.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 + y'x + 1 = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 63

```
dsolve(diff(y(x),x)^2+x*diff(y(x),x)+1 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{4} - \frac{x\sqrt{x^2-4}}{4} + \ln\left(x + \sqrt{x^2-4}\right) + c_1$$
$$y(x) = \frac{x\sqrt{x^2-4}}{4} - \ln\left(x + \sqrt{x^2-4}\right) - \frac{x^2}{4} + c_1$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 79

```
DSolve[(y'[x])^2+x y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4}x\left(\sqrt{x^2-4} + x\right) - \log\left(\sqrt{x^2-4} - x\right) + c_1$$
$$y(x) \to \frac{1}{4}x\left(\sqrt{x^2-4} - x\right) + \log\left(\sqrt{x^2-4} - x\right) + c_1$$

27.15 problem 781

Internal problem ID [3512]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 781.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^2 + y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)^2+x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{4}$$
$$y(x) = c_1^2 + c_1 x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 23

```
DSolve[(y'[x])^2+x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x + c_1)$$
$$y(x) \to -\frac{x^2}{4}$$

27.16 problem 782

Internal problem ID [3513]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 782.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^2 - y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)^2-x*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{4}$$
$$y(x) = -c_1^2 + c_1 x$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 25

```
DSolve[(y'[x])^2-x y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x - c_1)$$
$$y(x) \to \frac{x^2}{4}$$

27.17 problem 783

Internal problem ID [3514]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 783.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^2 - y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 77

```
dsolve(diff(y(x),x)^2-x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$\frac{c_1}{\sqrt{2x + 2\sqrt{x^2 + 4y(x)}}} + \frac{2x}{3} - \frac{\sqrt{x^2 + 4y(x)}}{3} = 0$$

$$\frac{c_1}{\sqrt{-2\sqrt{x^2 + 4y(x)} + 2x}} + \frac{2x}{3} + \frac{\sqrt{x^2 + 4y(x)}}{3} = 0$$

✓ Solution by Mathematica

Time used: 60.171 (sec). Leaf size: 965

```
DSolve[(y'[x])^2-x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{\left(x^2+\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)^2+8e^{3c_1}x}{4\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}$$

$$y(x)\to \frac{1}{8}\left(4x^2+\frac{\left(-1-i\sqrt{3}\right)x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}\right.$$
$$\left.+i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x)\to \frac{1}{8}\left(4x^2+\frac{i\left(\sqrt{3}+i\right)x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}\right.$$
$$\left.-\left(1+i\sqrt{3}\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x)\to \frac{2\sqrt[3]{2}x^4+2^{2/3}\left(-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}\right)^{2/3}+4x^2\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}(4x^3}}}{8\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}}}$$

$$y(x)\to \frac{1}{16}\left(8x^2-\frac{4\sqrt[3]{-2}x(x^3-2e^{3c_1})}{\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}}}\right.$$
$$\left.+2(-2)^{2/3}\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}}\right)$$

$$y(x)\to \frac{x^2}{2}+\frac{(-1)^{2/3}x(x^3-2e^{3c_1})}{2\ 2^{2/3}\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}}}$$
$$-\frac{1}{4}\sqrt[3]{-\frac{1}{2}}\sqrt[3]{-2x^6-10e^{3c_1}x^3+\sqrt{e^{3c_1}\left(4x^3+e^{3c_1}\right)^3}+e^{6c_1}}$$

27.18 problem 784

Internal problem ID [3515]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 784.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$y'^2 + y'x + x - y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 39

```
dsolve(diff(y(x),x)^2+x*diff(y(x),x)+x-y(x) = 0,y(x), singsol=all)
```

$$y(x) = \left(\mathrm{LambertW}\left(\frac{c_1 e^{\frac{x}{2}-1}}{2}\right) - \frac{x}{2} + 2\right)x + \left(\mathrm{LambertW}\left(\frac{c_1 e^{\frac{x}{2}-1}}{2}\right) - \frac{x}{2} + 1\right)^2$$

✓ Solution by Mathematica

Time used: 2.318 (sec). Leaf size: 177

```
DSolve[(y'[x])^2+x y'[x]+x -y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[-\sqrt{x^2+4y(x)-4x} + 2\log\left(\sqrt{x^2+4y(x)-4x} - x + 2\right) - 2\log\left(-x\sqrt{x^2+4y(x)-4x} + x^2 + 4y(x) - 2x - 4\right) + x = c_1, y(x)\right]$$

$$\mathrm{Solve}\left[-4\,\mathrm{arctanh}\left(\frac{(x-5)\sqrt{x^2+4y(x)-4x} - x^2 - 4y(x) + 7x - 6}{(x-3)\sqrt{x^2+4y(x)-4x} - x^2 - 4y(x) + 5x - 2}\right) + \sqrt{x^2+4y(x)-4x} + x = c_1, y(x)\right]$$

27.19 problem 785

Internal problem ID [3516]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 785.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$y'^2 + (1 - x)\, y' + y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 29

```
dsolve(diff(y(x),x)^2+(1-x)*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{4}x^2 - \frac{1}{2}x + \frac{1}{4}$$
$$y(x) = -c_1^2 + c_1 x - c_1$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 28

```
DSolve[(y'[x])^2+(1-x)y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x - 1 - c_1)$$
$$y(x) \to \frac{1}{4}(x - 1)^2$$

27.20 problem 786

Internal problem ID [3517]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 786.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^2 - (x+1)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)^2-(1+x)*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{4}x^2 + \frac{1}{2}x + \frac{1}{4}$$
$$y(x) = -c_1^2 + c_1 x + c_1$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 28

```
DSolve[(y'[x])^2-(1+x)y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x+1-c_1)$$
$$y(x) \to \frac{1}{4}(x+1)^2$$

27.21 problem 787

Internal problem ID [3518]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 787.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$y'^2 - (2 - x)\, y' + 1 - y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)^2-(2-x)*diff(y(x),x)+1-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{4}x^2 + x$$
$$y(x) = c_1^2 + c_1 x - 2c_1 + 1$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 27

```
DSolve[(y'[x])^2-(2-x)y'[x]+1-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 1 + c_1(x - 2 + c_1)$$
$$y(x) \to -\frac{1}{4}(x - 4)x$$

27.22 problem 788

Internal problem ID [3519]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 788.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^2 + (a + x)\,y' - y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)^2+(a+x)*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{4}x^2 - \frac{1}{2}ax - \frac{1}{4}a^2$$
$$y(x) = ac_1 + c_1^2 + c_1x$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 26

```
DSolve[(y'[x])^2+(a+x)y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(a + x + c_1)$$
$$y(x) \to -\frac{1}{4}(a + x)^2$$

27.23 problem 789

Internal problem ID [3520]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 789.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 - 2y'x + 1 = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 65

```
dsolve(diff(y(x),x)^2-2*x*diff(y(x),x)+1 = 0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} - \frac{\sqrt{x^2-1}\,x}{2} + \frac{\ln\left(x+\sqrt{x^2-1}\right)}{2} + c_1$$
$$y(x) = \frac{x^2}{2} + \frac{\sqrt{x^2-1}\,x}{2} - \frac{\ln\left(x+\sqrt{x^2-1}\right)}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.05 (sec). Leaf size: 82

```
DSolve[(y'[x])^2-2 x y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}x\left(\sqrt{x^2-1}+x\right) - \coth^{-1}\left(\frac{x-1}{\sqrt{x^2-1}}\right) + c_1$$
$$y(x) \to \frac{x^2}{2} - \frac{1}{2}\sqrt{x^2-1}x + \coth^{-1}\left(\frac{x-1}{\sqrt{x^2-1}}\right) + c_1$$

27.24 problem 790

Internal problem ID [3521]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 790.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$
\boxed{y'^2 + 2y'x - 3x^2 = 0}
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)^2+2*x*diff(y(x),x)-3*x^2 = 0,y(x), singsol=all)
```

$$
y(x) = \frac{x^2}{2} + c_1
$$
$$
y(x) = -\frac{3x^2}{2} + c_1
$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 29

```
DSolve[(y'[x])^2+2 x y'[x]-3 x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to -\frac{3x^2}{2} + c_1
$$
$$
y(x) \to \frac{x^2}{2} + c_1
$$

27.25 problem 791

Internal problem ID [3522]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 791.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^2 + 2y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 690

```
dsolve(diff(y(x),x)^2+2*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = \left(\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{x^2}{2\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{x}{2}\right)^2 + 2x\left(\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{x^2}{2\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{x}{2}\right)$$

$$y(x) = \left(-\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x^2}{4\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{x}{2} - \frac{i\sqrt{3}\left(\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3}\right.}{2}\right.}{}\right.$$

$$+ 2x\left(-\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x^2}{4\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{x}{2} - \frac{i\sqrt{3}\left(\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{2} - \frac{x^2}{2\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}\right)}{2}\right)$$

$$y(x) = \left(-\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x^2}{4\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{x}{2} + \frac{i\sqrt{3}\left(\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3}\right.}{2}\right.}{}\right.$$

$$+ 2x\left(-\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x^2}{4\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{x}{2}\right.$$

✓ Solution by Mathematica

Time used: 60.18 (sec). Leaf size: 927

```
DSolve[(y'[x])^2+2 x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-x^2 + \frac{x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}} + \sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2 + \frac{\left(-9-9i\sqrt{3}\right)x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}} + 9i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2 + \frac{9i\left(\sqrt{3}+i\right)x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}} - 9\left(1+i\sqrt{3}\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{4}\left(-x^2 + \frac{x(x^3-8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}} + \sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2 + \frac{\left(-9-9i\sqrt{3}\right)x(x^3-8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}} + 9i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2 + \frac{9i\left(\sqrt{3}+i\right)x(x^3-8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}} - 9\left(1+i\sqrt{3}\right)\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

27.26 problem 792

Internal problem ID [3523]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 792.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{{y'}^2 + 2y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 690

```
dsolve(diff(y(x),x)^2+2*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = \left(\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{x^2}{2\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{x}{2}\right)^2 + 2x\left(\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{2} + \frac{x^2}{2\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{x}{2}\right)$$

$$y(x) = \left(-\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x^2}{4\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{x}{2} - \frac{i\sqrt{3}\left(\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3\right.}}{2}\right.}{}\right.$$

$$+ 2x\left(-\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x^2}{4\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{x}{2} - \frac{i\sqrt{3}\left(\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{2} - \frac{x^2}{2\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}\right)}{2}\right)$$

$$y(x) = \left(-\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x^2}{4\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{x}{2} + \frac{i\sqrt{3}\left(\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3\right.}}{2}\right.}{}\right.$$

$$+ 2x\left(-\frac{\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}}{4} - \frac{x^2}{4\left(6c_1 - x^3 + 2\sqrt{-3c_1x^3 + 9c_1^2}\right)^{\frac{1}{3}}} - \frac{x}{2}\right.$$

✓ Solution by Mathematica

Time used: 60.086 (sec). Leaf size: 927

```
DSolve[(y'[x])^2+2 x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}\left(-x^2+\frac{x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2+\frac{\left(-9-9i\sqrt{3}\right)x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+9i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2+\frac{9i\left(\sqrt{3}+i\right)x(x^3+8e^{3c_1})}{\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}-9\left(1+i\sqrt{3}\right)\sqrt[3]{-x^6+20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{4}\left(-x^2+\frac{x(x^3-8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2+\frac{\left(-9-9i\sqrt{3}\right)x(x^3-8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}+9i\left(\sqrt{3}+i\right)\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{72}\left(-18x^2+\frac{9i\left(\sqrt{3}+i\right)x(x^3-8e^{3c_1})}{\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}-9\left(1+i\sqrt{3}\right)\sqrt[3]{-x^6-20e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3}+8e^{6c_1}}\right)$$

27.27 problem 793

Internal problem ID [3524]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 793.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^2 - 2y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 21

```
dsolve(diff(y(x),x)^2-2*x*diff(y(x),x)+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2}$$
$$y(x) = -\frac{1}{2}c_1^2 + c_1 x$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 29

```
DSolve[(y'[x])^2-2 x y'[x]+2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \frac{{c_1}^2}{2}$$
$$y(x) \to \frac{x^2}{2}$$

27.28 problem 794

Internal problem ID [3525]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 794.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - (1+2x)\,y' - x(1-x) = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 45

```
dsolve(diff(y(x),x)^2-(1+2*x)*diff(y(x),x)-x*(1-x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} + \frac{x}{2} - \frac{(8x+1)^{\frac{3}{2}}}{24} + c_1$$
$$y(x) = \frac{x}{2} + \frac{(8x+1)^{\frac{3}{2}}}{24} + \frac{x^2}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 62

```
DSolve[(y'[x])^2-(1+2 x)y'[x]-x(1-x)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} + \frac{x}{2} - \frac{1}{24}(8x+1)^{3/2} + c_1$$
$$y(x) \to \frac{1}{2}\left(x^2 + x + \frac{1}{12}(8x+1)^{3/2}\right) + c_1$$

27.29 problem 795

Internal problem ID [3526]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 795.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$y'^2 + 2(1-x)\,y' - 2x + 2y = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 45

```
dsolve(diff(y(x),x)^2+2*(1-x)*diff(y(x),x)-2*x+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \left(\text{LambertW}\left(-c_1 e^{-x}\right) + x + 1\right) x - \frac{\left(\text{LambertW}\left(-c_1 e^{-x}\right) + x\right)^2}{2} - \text{LambertW}\left(-c_1 e^{-x}\right) - x$$

✓ Solution by Mathematica

Time used: 1.108 (sec). Leaf size: 171

```
DSolve[(y'[x])^2+2(1-x)y'[x]-2(x-y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[2\text{arctanh}\left(\frac{(x-2)\sqrt{x^2-2y(x)+1} - x^2 + 2y(x) + 2x - 1}{x\sqrt{x^2-2y(x)+1} - x^2 + 2y(x) - 1}\right) - \sqrt{x^2-2y(x)+1} + x = c_1, y(x)\right]$$

$$\text{Solve}\left[2\text{arctanh}\left(\frac{x\sqrt{x^2-2y(x)+1} - x^2 + 2y(x) - 1}{(x+2)\sqrt{x^2-2y(x)+1} - x^2 + 2y(x) - 2x - 1}\right) + \sqrt{x^2-2y(x)+1} + x = c_1, y(x)\right]$$

27.30 problem 796

Internal problem ID [3527]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 796.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{{y'}^{2}+3y'x-y=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 85

```
dsolve(diff(y(x),x)^2+3*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$\frac{c_{1}}{\left(-6x-2\sqrt{9x^{2}+4y\left(x\right)}\right)^{\frac{3}{2}}}+\frac{2x}{5}-\frac{\sqrt{9x^{2}+4y\left(x\right)}}{5}=0$$

$$\frac{c_{1}}{\left(-6x+2\sqrt{9x^{2}+4y\left(x\right)}\right)^{\frac{3}{2}}}+\frac{2x}{5}+\frac{\sqrt{9x^{2}+4y\left(x\right)}}{5}=0$$

✓ Solution by Mathematica

Time used: 13.797 (sec). Leaf size: 776

```
DSolve[(y'[x])^2+3 x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[16\#1^5 + 40\#1^4x^2 + 25\#1^3x^4 + 160\#1^2e^{5c_1}x + 360\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 64e^{10c_1}\&, 1\right]$$

$$y(x) \to \text{Root}\left[16\#1^5 + 40\#1^4x^2 + 25\#1^3x^4 + 160\#1^2e^{5c_1}x + 360\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 64e^{10c_1}\&, 2\right]$$

$$y(x) \to \text{Root}\left[16\#1^5 + 40\#1^4x^2 + 25\#1^3x^4 + 160\#1^2e^{5c_1}x + 360\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 64e^{10c_1}\&, 3\right]$$

$$y(x) \to \text{Root}\left[16\#1^5 + 40\#1^4x^2 + 25\#1^3x^4 + 160\#1^2e^{5c_1}x + 360\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 64e^{10c_1}\&, 4\right]$$

$$y(x) \to \text{Root}\left[16\#1^5 + 40\#1^4x^2 + 25\#1^3x^4 + 160\#1^2e^{5c_1}x + 360\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 64e^{10c_1}\&, 5\right]$$

$$y(x) \to \text{Root}\left[1024\#1^5 + 2560\#1^4x^2 + 1600\#1^3x^4 - 160\#1^2e^{5c_1}x - 360\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 1\right]$$

$$y(x) \to \text{Root}\left[1024\#1^5 + 2560\#1^4x^2 + 1600\#1^3x^4 - 160\#1^2e^{5c_1}x - 360\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 2\right]$$

$$y(x) \to \text{Root}\left[1024\#1^5 + 2560\#1^4x^2 + 1600\#1^3x^4 - 160\#1^2e^{5c_1}x - 360\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 3\right]$$

$$y(x) \to \text{Root}\left[1024\#1^5 + 2560\#1^4x^2 + 1600\#1^3x^4 - 160\#1^2e^{5c_1}x - 360\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 4\right]$$

$$y(x) \to \text{Root}\left[1024\#1^5 + 2560\#1^4x^2 + 1600\#1^3x^4 - 160\#1^2e^{5c_1}x - 360\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 5\right]$$

$$y(x) \to 0$$

27.31 problem 797

Internal problem ID [3528]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 27
Problem number: 797.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^2 - 4(x+1)\,y' + 4y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)^2-4*(1+x)*diff(y(x),x)+4*y(x) = 0,y(x), singsol=all)
```

$$y(x) = x^2 + 2x + 1$$
$$y(x) = c_1 x - \frac{1}{4}c_1^2 + c_1$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 27

```
DSolve[(y'[x])^2-4(1+x)y'[x]+4 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{4}c_1(-4x - 4 + c_1)$$
$$y(x) \to (x+1)^2$$

28 Various 28

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28.1 problem 798

Internal problem ID [3529]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 798.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 + axy' - bc\,x^2 = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 47

```
dsolve(diff(y(x),x)^2+a*x*diff(y(x),x) = b*c*x^2,y(x), singsol=all)
```

$$y(x) = \frac{\left(-a + \sqrt{a^2 + 4cb}\right) x^2}{4} + c_1$$
$$y(x) = -\frac{\left(a + \sqrt{a^2 + 4cb}\right) x^2}{4} + c_1$$

✓ Solution by Mathematica

Time used: 0.008 (sec). Leaf size: 59

```
DSolve[(y'[x])^2+a x y'[x]==b c x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}x^2\left(\sqrt{a^2 + 4bc} - a\right) + c_1$$
$$y(x) \to -\frac{1}{4}x^2\left(\sqrt{a^2 + 4bc} + a\right) + c_1$$

28.2 problem 799

Internal problem ID [3530]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 799.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^2 - y'ax + ya = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)^2-a*x*diff(y(x),x)+a*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{a\,x^2}{4}$$
$$y(x) = c_1 x - \frac{c_1^2}{a}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 29

```
DSolve[(y'[x])^2-a x y'[x]+a y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\left(x - \frac{c_1}{a}\right)$$
$$y(x) \to \frac{ax^2}{4}$$

28.3 problem 800

Internal problem ID [3531]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 800.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$y'^2 + axy' + b\,x^2 + cy = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)^2+a*x*diff(y(x),x)+b*x^2+c*y(x) = 0,y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 1.777 (sec). Leaf size: 1085

```
DSolve[(y'[x])^2+a x y'[x]+b x^2+c y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\text{RootSum}\left[\#1^4-2\#1^3c-2\#1^2a^2-4\#1^2ac+8\#1^2b-2\#1a^2c+8\#1bc+a^4-8a^2b\right.\right.$$
$$+16b^2\&,\frac{-\#1^3\log\left(\#1x-\sqrt{x^2\left(a^2-4b\right)-4cy(x)}+2\sqrt{-cy(x)}\right)+\#1^3\log(x)+\#1^2c\log\left(\#1x-\sqrt{x^2\left(\right.}\right.}{}$$
$$\left.-\log\left(\sqrt{-cy(x)}\sqrt{x^2\left(a^2-4b\right)-4cy(x)}+2cy(x)\right)+\frac{1}{2}\log(y(x))+2\log(x)=c_1,y(x)\right]$$

$$\text{Solve}\left[\text{RootSum}\left[\#1^4+2\#1^3c-2\#1^2a^2-4\#1^2ac+8\#1^2b+2\#1a^2c-8\#1bc+a^4-8a^2b\right.\right.$$
$$+16b^2\&,\frac{\#1^3\log\left(\#1x-\sqrt{x^2\left(a^2-4b\right)-4cy(x)}+2\sqrt{-cy(x)}\right)+\#1^3(-\log(x))+\#1^2c\log\left(\#1x-\sqrt{x^2}\right.}{}$$
$$\left.-\log\left(\sqrt{-cy(x)}\sqrt{x^2\left(a^2-4b\right)-4cy(x)}+2cy(x)\right)+\frac{1}{2}\log(y(x))+2\log(x)=c_1,y(x)\right]$$

28.4 problem 801

Internal problem ID [3532]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 801.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^2 + (bx + a)\, y' + c - by = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 50

```
dsolve(diff(y(x),x)^2+(b*x+a)*diff(y(x),x)+c = b*y(x),y(x), singsol=all)
```

$$y(x) = \frac{-b^2x^2 - 2xba - a^2 + 4c}{4b}$$

$$y(x) = c_1x + \frac{ac_1 + c_1^2 + c}{b}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 43

```
DSolve[(y'[x])^2+(a+b x)y'[x]+c==b y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c + c_1(a + bx + c_1)}{b}$$

$$y(x) \to -\frac{(a + bx)^2 - 4c}{4b}$$

28.5 problem 802

Internal problem ID [3533]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 802.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - 2y'x^2 + 2y'x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)^2-2*x^2*diff(y(x),x)+2*x*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{2}{3}x^3 - x^2 + c_1$$
$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 26

```
DSolve[(y'[x])^2-2 x^2 y'[x]+2 x y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1$$
$$y(x) \rightarrow \frac{2x^3}{3} - x^2 + c_1$$

28.6 problem 804

Internal problem ID [3534]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 804.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y'^2 + a\,x^3 y' - 2a\,x^2 y = 0$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)^2+a*x^3*diff(y(x),x)-2*a*x^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{a\,x^4}{8}$$
$$y(x) = c_1 x^2 + \frac{2c_1^2}{a}$$

✓ Solution by Mathematica

Time used: 0.658 (sec). Leaf size: 78

```
DSolve[(y'[x])^2+a x^3 y'[x]-2 a x^2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{8}e^{2c_1}\left(-2\sqrt{a}x^2 + e^{2c_1}\right)$$
$$y(x) \to 2\sqrt{a}e^{2c_1}x^2 + 8e^{4c_1}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{ax^4}{8}$$

28.7 problem 805

Internal problem ID [3535]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 805.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y'^2 - 2a\,x^3y' + 4a\,x^2y = 0}$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 27

```
dsolve(diff(y(x),x)^2-2*a*x^3*diff(y(x),x)+4*a*x^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{a\,x^4}{4}$$
$$y(x) = c_1x^2 - \frac{c_1^2}{a}$$

✓ Solution by Mathematica

Time used: 3.724 (sec). Leaf size: 262

```
DSolve[(y'[x])^2-2 a x^3 y'[x]+4 a x^2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{4}\left(\frac{\sqrt{a}x\sqrt{ax^4-4y(x)}}{\sqrt{ax^2\left(ax^4-4y(x)\right)}}+1\right)\log(y(x)) - \frac{\sqrt{a}x\sqrt{ax^4-4y(x)}\log\left(\sqrt{ax^4-4y(x)}+\sqrt{a}x^2\right)}{2\sqrt{ax^2\left(ax^4-4y(x)\right)}} = c_1, y(x)\right]$$

$$\text{Solve}\left[\frac{\sqrt{a}x\sqrt{ax^4-4y(x)}\log\left(\sqrt{ax^4-4y(x)}+\sqrt{a}x^2\right)}{2\sqrt{ax^2\left(ax^4-4y(x)\right)}} + \frac{1}{4}\left(1-\frac{\sqrt{a}x\sqrt{ax^4-4y(x)}}{\sqrt{ax^2\left(ax^4-4y(x)\right)}}\right)\log(y(x)) = c_1, y(x)\right]$$

$$y(x) \to \frac{ax^4}{4}$$

28.8 problem 806

Internal problem ID [3536]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 806.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y'^2 + 4x^5y' - 12x^4y = 0}$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)^2+4*x^5*diff(y(x),x)-12*x^4*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x^6}{3}$$
$$y(x) = c_1x^3 + \frac{3}{4}c_1^2$$

✓ Solution by Mathematica

Time used: 1.339 (sec). Leaf size: 217

```
DSolve[(y'[x])^2+4 x^5 y'[x]-12 x^4 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{6}\left(\log(y(x))-\frac{x^2\sqrt{x^6+3y(x)}\log(y(x))}{\sqrt{x^4(x^6+3y(x))}}\right)+\frac{x^2\sqrt{x^6+3y(x)}\log\left(\sqrt{x^6+3y(x)}+x^3\right)}{3\sqrt{x^4(x^6+3y(x))}}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{6}\left(\frac{x^2\sqrt{x^6+3y(x)}\log(y(x))}{\sqrt{x^4(x^6+3y(x))}}+\log(y(x))\right)-\frac{x^2\sqrt{x^6+3y(x)}\log\left(\sqrt{x^6+3y(x)}+x^3\right)}{3\sqrt{x^4(x^6+3y(x))}}=c_1,y(x)\right]$$

$$y(x)\to -\frac{x^6}{3}$$

28.9 problem 807

Internal problem ID [3537]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 807.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 - 2y' \cosh(x) + 1 = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)^2-2*diff(y(x),x)*cosh(x)+1 = 0,y(x), singsol=all)
```

$$y(x) = -e^{-x} + c_1$$
$$y(x) = e^{x} + c_1$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 24

```
DSolve[(y'[x])^2-2 y'[x] Cosh[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sinh(x) - \cosh(x) + c_1$$
$$y(x) \to e^{x} + c_1$$

28.10 problem 808

Internal problem ID [3538]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 808.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 + yy' - x(x + y) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)^2+y(x)*diff(y(x),x) = x*(x+y(x)),y(x), singsol=all)
```

$$y(x) = \frac{x^2}{2} + c_1$$
$$y(x) = -x + 1 + c_1 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 32

```
DSolve[(y'[x])^2+y[x] y'[x]==x(x+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} + c_1$$
$$y(x) \to -x + c_1 e^{-x} + 1$$

28.11 problem 809

Internal problem ID [3539]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 809.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y'^2 - yy' + e^x = 0}$$

✓ Solution by Maple

Time used: 0.282 (sec). Leaf size: 30

```
dsolve(diff(y(x),x)^2-y(x)*diff(y(x),x)+exp(x) = 0,y(x), singsol=all)
```

$$y(x) = -2\,e^{\frac{x}{2}}$$
$$y(x) = 2\,e^{\frac{x}{2}}$$
$$y(x) = \frac{1}{c_1} + c_1 e^x$$

✓ Solution by Mathematica

Time used: 60.215 (sec). Leaf size: 57

```
DSolve[(y'[x])^2-y[x] y'[x]+Exp[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\sqrt{-e^x \sinh^2\left(\frac{x-c_1}{2}\right)}$$
$$y(x) \to 2\sqrt{-e^x \sinh^2\left(\frac{x-c_1}{2}\right)}$$

28.12 problem 810

Internal problem ID [3540]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 810.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 + (x + y) y' + yx = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)^2+(x+y(x))*diff(y(x),x)+x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{2} + c_1$$
$$y(x) = c_1 \mathrm{e}^{-x}$$

✓ Solution by Mathematica

Time used: 0.035 (sec). Leaf size: 32

```
DSolve[(y'[x])^2+(x+y[x])y'[x]+x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-x}$$
$$y(x) \to -\frac{x^2}{2} + c_1$$
$$y(x) \to 0$$

28.13 problem 811

Internal problem ID [3541]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 811.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type **[_dAlembert]**

$$\boxed{y'^2 - 2yy' - 2x = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 217

```
dsolve(diff(y(x),x)^2-2*y(x)*diff(y(x),x)-2*x = 0,y(x), singsol=all)
```

$$\frac{\left(-2y(x)+2\sqrt{y(x)^2+2x}\right)c_1}{\sqrt{2y(x)^2+2x-2y(x)\sqrt{y(x)^2+2x}+1}}+x+\frac{\left(-y(x)+\sqrt{y(x)^2+2x}\right)\operatorname{arcsinh}\left(y(x)-\sqrt{y(x)^2+2x}\right)}{2\sqrt{2y(x)^2+2x-2y(x)\sqrt{y(x)^2+2x}+1}}=0$$

$$\frac{\left(2y(x)+2\sqrt{y(x)^2+2x}\right)c_1}{\sqrt{2y(x)^2+2x+2y(x)\sqrt{y(x)^2+2x}+1}}+x-\frac{\left(y(x)+\sqrt{y(x)^2+2x}\right)\operatorname{arcsinh}\left(y(x)+\sqrt{y(x)^2+2x}\right)}{2\sqrt{2y(x)^2+2x+2y(x)\sqrt{y(x)^2+2x}+1}}=0$$

✓ Solution by Mathematica

Time used: 0.688 (sec). Leaf size: 74

```
DSolve[(y'[x])^2-2 y[x] y'[x]-2 x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x=-\frac{K[1]\log\left(\sqrt{K[1]^2+1}-K[1]\right)}{2\sqrt{K[1]^2+1}}+\frac{c_1K[1]}{\sqrt{K[1]^2+1}},y(x)=\frac{K[1]}{2}-\frac{x}{K[1]}\right\},\{y(x),K[1]\}\right]$$

28.14 problem 812

Internal problem ID [3542]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 812.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 + (1 + 2y)\, y' + y(y - 1) = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 143

```
dsolve(diff(y(x),x)^2+(1+2*y(x))*diff(y(x),x)+y(x)*(y(x)-1) = 0,y(x), singsol=all)
```

$$x + \frac{3\ln\left(y(x)-1\right)}{2} - \frac{\ln\left(y(x)\right)}{2} + \frac{\ln\left(\sqrt{8y\left(x\right)+1}-1\right)}{2} - \frac{3\ln\left(\sqrt{8y\left(x\right)+1}-3\right)}{2} - \frac{\ln\left(\sqrt{8y\left(x\right)+1}+1\right)}{2} + \frac{3\ln\left(\sqrt{8y\left(x\right)+1}+3\right)}{2} - c_1 = 0$$

$$x + \frac{3\ln\left(y(x)-1\right)}{2} - \frac{\ln\left(y(x)\right)}{2} - \frac{\ln\left(\sqrt{8y\left(x\right)+1}-1\right)}{2} + \frac{3\ln\left(\sqrt{8y\left(x\right)+1}-3\right)}{2} + \frac{\ln\left(\sqrt{8y\left(x\right)+1}+1\right)}{2} - \frac{3\ln\left(\sqrt{8y\left(x\right)+1}+3\right)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 60.124 (sec). Leaf size: 1367

```
DSolve[(y'[x])^2+(1+2 y[x])y'[x]+y[x](y[x]-1)==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-2x}\left(128e^{x}(12e^{x}+e^{2c_1})+64\sqrt[3]{24\sqrt{3}\sqrt{-e^{7x+4c_1}\left(-27e^{x}+e^{2c_1}\right)^3}+540e^{4(x+c_1)}+5832e^{5x+2c_1}-e^{3x+6c_1}}\right.}{1536}$$

$$y(x) \to \frac{e^{-2x}\left(256e^{x}(12e^{x}+e^{2c_1})+64i\left(\sqrt{3}+i\right)\sqrt[3]{24\sqrt{3}\sqrt{-e^{7x+4c_1}\left(-27e^{x}+e^{2c_1}\right)^3}+540e^{4(x+c_1)}+5832e^{5x+2c_1}}\right.}{3072}$$

$$y(x) \to \frac{e^{-2x}\left(256e^{x}(12e^{x}+e^{2c_1})-64\left(1+i\sqrt{3}\right)\sqrt[3]{24\sqrt{3}\sqrt{-e^{7x+4c_1}\left(-27e^{x}+e^{2c_1}\right)^3}+540e^{4(x+c_1)}+5832e^{5x+2c_1}}\right.}{3072}$$

$$y(x) \to \frac{e^{-2(x+2c_1)}\left(128e^{x+2c_1}(1+12e^{x+2c_1})+64\sqrt[3]{e^{3x+6c_1}\left(-1+108e^{x+2c_1}\left(5+54e^{x+2c_1}\right)\right)+24\sqrt{3}\sqrt{e^{7(x+2c_1)}\left(-\right.}}\right.}{1536}$$

$$y(x) \to e^{-2(x+2c_1)}\left(256e^{x+2c_1}(1+12e^{x+2c_1})+64i\left(\sqrt{3}+i\right)\sqrt[3]{e^{3x+6c_1}\left(-1+108e^{x+2c_1}\left(5+54e^{x+2c_1}\right)\right)+24\sqrt{3}\sqrt{e}}\right.$$

$$y(x) \to e^{-2(x+2c_1)}\left(256e^{x+2c_1}(1+12e^{x+2c_1})-64\left(1+i\sqrt{3}\right)\sqrt[3]{e^{3x+6c_1}\left(-1+108e^{x+2c_1}\left(5+54e^{x+2c_1}\right)\right)+24\sqrt{3}\sqrt{e}}\right.$$

28.15 problem 813

Internal problem ID [3543]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 813.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 - 2(x - y)\, y' - 4yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(diff(y(x),x)^2-2*(x-y(x))*diff(y(x),x)-4*x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = x^2 + c_1$$
$$y(x) = \mathrm{e}^{-2x} c_1$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 28

```
DSolve[(y'[x])^2-2(x-y[x])y'[x]-4 x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-2x}$$
$$y(x) \to x^2 + c_1$$
$$y(x) \to 0$$

28.16 problem 814

Internal problem ID [3544]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 814.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - (4y+1)\,y' + (4y+1)\,y = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 193

```
dsolve(diff(y(x),x)^2-(1+4*y(x))*diff(y(x),x)+(1+4*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{4}$$

$$y(x) = \frac{\left(\frac{c_1 \mathrm{e}^{-2x}\left(\sqrt{-\mathrm{e}^{-2x} c_1}-2\right)}{\sqrt{-\mathrm{e}^{-2x} c_1}} - \mathrm{e}^{-2x} c_1 - 2\right)\mathrm{e}^{2x}}{2c_1}$$

$$y(x) = \frac{\left(\frac{c_1 \mathrm{e}^{-2x}\left(\sqrt{-\mathrm{e}^{-2x} c_1}+2\right)}{\sqrt{-\mathrm{e}^{-2x} c_1}} - \mathrm{e}^{-2x} c_1 - 2\right)\mathrm{e}^{2x}}{2c_1}$$

$$y(x) = -\frac{\left(-\frac{c_1 \mathrm{e}^{-2x}\left(\sqrt{-\mathrm{e}^{-2x} c_1}+2\right)}{\sqrt{-\mathrm{e}^{-2x} c_1}} + \mathrm{e}^{-2x} c_1 + 2\right)\mathrm{e}^{2x}}{2c_1}$$

$$y(x) = -\frac{\left(-\frac{c_1 \mathrm{e}^{-2x}\left(\sqrt{-\mathrm{e}^{-2x} c_1}-2\right)}{\sqrt{-\mathrm{e}^{-2x} c_1}} + \mathrm{e}^{-2x} c_1 + 2\right)\mathrm{e}^{2x}}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.206 (sec). Leaf size: 67

```
DSolve[(y'[x])^2-(1+4 y[x])y'[x]+(1+4 y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{x-4c_1}\left(e^x + 2e^{2c_1}\right)$$

$$y(x) \to \frac{1}{4}e^{x+2c_1}\left(-2 + e^{x+2c_1}\right)$$

$$y(x) \to -\frac{1}{4}$$

$$y(x) \to 0$$

28.17 problem 815

Internal problem ID [3545]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 815.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - 2(-3y+1)\,y' - (4-9y)\,y = 0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 123

```
dsolve(diff(y(x),x)^2-2*(1-3*y(x))*diff(y(x),x)-(4-9*y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{4}{9}$$

$$y(x) = \frac{\operatorname{RootOf}\left(\_Z^8 e^{24x} + 24\_Z^7 e^{24x} + 240\_Z^6 e^{24x} + 1280\_Z^5 e^{24x} + \left(3840\, e^{24x} - 1458\, e^{12x} c_1\right)\_Z^4 + \left(6144\, e^{24x}\right.\right.}{9} + \frac{4}{9}$$

✓ Solution by Mathematica

Time used: 60.286 (sec). Leaf size: 4769

```
DSolve[(y'[x])^2-2(1-3 y[x])y'[x]-(4-9 y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

28.18 problem 816

Internal problem ID [3546]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 816.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 + (a + 6y)\, y' + y(3a + b + 9y) = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 309

```
dsolve(diff(y(x),x)^2+(a+6*y(x))*diff(y(x),x)+y(x)*(3*a+b+9*y(x)) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\mathrm{RootOf}\left(3\ln\left(-\frac{4b}{(3\_Z-2b)^2}\right)a + 2\ln\left(-\frac{4b}{(3\_Z-2b)^2}\right)b - 3a\ln\left(-\frac{(\_Z+2a)^2}{4b}\right) + 18ac_1 + 6c_1b - 18ax\right.}{}$$

$$y(x) = -\frac{e^{\mathrm{RootOf}\left(-3a\ln\left(-\frac{1}{4b}\right)-3\ln\left(-\frac{\left(3\,e^{\_Z}+6a+2b\right)^2}{4b}\right)a-2\ln\left(-\frac{\left(3\,e^{\_Z}+6a+2b\right)^2}{4b}\right)b+18ac_1+6c_1b-6a\_Z-18ax-6xb\right)}\left(e^{\mathrm{RootOf}\left(-3a\ln\left(-\frac{1}{4}\right.\right.}\right.}{4b}$$

✓ Solution by Mathematica

Time used: 0.589 (sec). Leaf size: 175

```
DSolve[(y'[x])^2+(a+6 y[x])y'[x]+y[x](3 a+b+9 y[x])==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \text{InverseFunction}\left[\frac{(3a+2b)\log\left(-3\sqrt{a^2-4\#1b}+3a+2b\right)+3a\log\left(\sqrt{a^2-4\#1b}+a\right)}{6(3a+b)}\&\right]\left[-\frac{x}{2}+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3a\log\left(a-\sqrt{a^2-4\#1b}\right)+(3a+2b)\log\left(3\sqrt{a^2-4\#1b}+3a+2b\right)}{6(3a+b)}\&\right]\left[\frac{x}{2}+c_1\right]$$

$$y(x) \to 0$$

$$y(x) \to \frac{1}{9}(-3a-b)$$

28.19 problem 817

Internal problem ID [3547]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 817.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_dAlembert]`

$$\boxed{y'^2 + ayy' - ax = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 394

```
dsolve(diff(y(x),x)^2+a*y(x)*diff(y(x),x)-a*x = 0,y(x), singsol=all)
```

$$\frac{\left(-ay(x)+\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}\right)c_{1}}{\sqrt{-2ay\left(x\right)+2\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}-4}\sqrt{-2ay\left(x\right)+2\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}+4}}+x$$
$$+\frac{\left(-ay(x)+\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}\right)\ln\left(-\frac{ay(x)}{2}+\frac{\sqrt{a\left(ay(x)^{2}+4x\right)}}{2}+\frac{\sqrt{2a^{2}y(x)^{2}-2ay(x)\sqrt{a\left(ay(x)^{2}+4x\right)}+4ax-4}}{2}\right)}{a\sqrt{2a^{2}y\left(x\right)^{2}-2ay\left(x\right)\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}+4ax-4}}$$
$$=0$$

$$\frac{\left(ay(x)+\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}\right)c_{1}}{\sqrt{-2ay\left(x\right)-2\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}-4}\sqrt{-2ay\left(x\right)-2\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}+4}}+x$$
$$-\frac{\left(ay(x)+\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}\right)\ln\left(-\frac{ay(x)}{2}-\frac{\sqrt{a\left(ay(x)^{2}+4x\right)}}{2}+\frac{\sqrt{2a^{2}y(x)^{2}+2ay(x)\sqrt{a\left(ay(x)^{2}+4x\right)}+4ax-4}}{2}\right)}{a\sqrt{2a^{2}y\left(x\right)^{2}+2ay\left(x\right)\sqrt{a\left(ay\left(x\right)^{2}+4x\right)}+4ax-4}}$$
$$=0$$

✓ Solution by Mathematica

Time used: 0.627 (sec). Leaf size: 83

```
DSolve[(y'[x])^2+a y[x] y'[x]-a x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x=-\frac{2K[1]\arctan\left(\frac{\sqrt{1-K[1]^2}}{K[1]+1}\right)}{a\sqrt{1-K[1]^2}}+\frac{c_1K[1]}{\sqrt{1-K[1]^2}},y(x)=\frac{x}{K[1]}-\frac{K[1]}{a}\right\},\{y(x),K[1]\}\right]$$

28.20 problem 818

Internal problem ID [3548]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 818.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_dAlembert]`

$$\boxed{y'^2 - ayy' - ax = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 183

```
dsolve(diff(y(x),x)^2-a*y(x)*diff(y(x),x)-a*x = 0,y(x), singsol=all)
```

$$x + \frac{\left(-ay(x) + \sqrt{a\left(ay\left(x\right)^2 + 4x\right)}\right)\left(ac_1 + \operatorname{arcsinh}\left(\frac{ay(x)}{2} - \frac{\sqrt{a\left(ay(x)^2+4x\right)}}{2}\right)\right)}{\sqrt{2a^2y\left(x\right)^2 - 2ay\left(x\right)\sqrt{a\left(ay\left(x\right)^2 + 4x\right)} + 4ax + 4\,a}} = 0$$

$$x - \frac{\left(ay(x) + \sqrt{a\left(ay\left(x\right)^2 + 4x\right)}\right)\left(ac_1 + \operatorname{arcsinh}\left(\frac{ay(x)}{2} + \frac{\sqrt{a\left(ay(x)^2+4x\right)}}{2}\right)\right)}{\sqrt{2a^2y\left(x\right)^2 + 2ay\left(x\right)\sqrt{a\left(ay\left(x\right)^2 + 4x\right)} + 4ax + 4\,a}} = 0$$

✓ Solution by Mathematica

Time used: 0.897 (sec). Leaf size: 75

```
DSolve[(y'[x])^2-a y[x] y'[x]-a x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x=-\frac{K[1]\log\left(\sqrt{K[1]^2+1}-K[1]\right)}{a\sqrt{K[1]^2+1}}+\frac{c_1K[1]}{\sqrt{K[1]^2+1}},y(x)=\frac{K[1]}{a}-\frac{x}{K[1]}\right\},\{y(x),K[1]\}\right]$$

28.21 problem 819

Internal problem ID [3549]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 819.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 + (ax + yb)\, y' + yaxb = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

```
dsolve(diff(y(x),x)^2+(a*x+b*y(x))*diff(y(x),x)+a*b*x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{a\,x^2}{2} + c_1$$
$$y(x) = c_1 \mathrm{e}^{-xb}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 34

```
DSolve[(y'[x])^2+(a x+b y[x])y'[x]+a b x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{-bx}$$
$$y(x) \to -\frac{ax^2}{2} + c_1$$
$$y(x) \to 0$$

28.22 problem 820

Internal problem ID [3550]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 820.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y'^2 - xyy' + y^2 \ln (ay) = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 50

```
dsolve(diff(y(x),x)^2-x*diff(y(x),x)*y(x)+y(x)^2*ln(a*y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \frac{e^{\frac{x^2}{4}}}{a}$$
$$y(x) = \frac{e^{-c_1^2} e^{c_1 x}}{a}$$
$$y(x) = \frac{e^{-c_1^2} e^{-c_1 x}}{a}$$

✓ Solution by Mathematica

Time used: 0.308 (sec). Leaf size: 30

```
DSolve[(y'[x])^2-x y'[x] y[x]+y[x]^2 Log[a y[x]]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{e^{\frac{1}{4} c_1 (2x - c_1)}}{a}$$
$$y(x) \rightarrow 0$$

28.23 problem 821

Internal problem ID [3551]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 821.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y'^2 - (2yx + 1)\,y' + 2yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(diff(y(x),x)^2-(1+2*x*y(x))*diff(y(x),x)+2*x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 e^{x^2}$$
$$y(x) = x + c_1$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 21

```
DSolve[(y'[x])^2-(1+2 x y[x])y'[x]+2 x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{x^2}$$
$$y(x) \to x + c_1$$

28.24 problem 822

Internal problem ID [3552]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 28
Problem number: 822.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^2 - (4 + y^2)\, y' + 4 + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 83

```
dsolve(diff(y(x),x)^2-(4+y(x)^2)*diff(y(x),x)+4+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -2i$$
$$y(x) = 2i$$
$$x - \left(\int^{y(x)} \frac{1}{2 + \frac{\_a^2}{2} - \frac{\sqrt{\_a^2(\_a^2+4)}}{2}} d\_a\right) - c_1 = 0$$
$$x - \left(\int^{y(x)} \frac{1}{2 + \frac{\_a^2}{2} + \frac{\sqrt{\_a^2(\_a^2+4)}}{2}} d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.453 (sec). Leaf size: 55

```
DSolve[(y'[x])^2-(4+y[x]^2)y'[x]+4+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + \frac{1}{-x + 2c_1} - 2c_1$$
$$y(x) \to x - \frac{1}{x + 2c_1} + 2c_1$$
$$y(x) \to -2i$$
$$y(x) \to 2i$$

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29.1 problem 823

Internal problem ID [3553]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 823.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$y'^2 - (x - y)\, y y' - x y^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

```
dsolve(diff(y(x),x)^2-(x-y(x))*y(x)*diff(y(x),x)-x*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{x + c_1}$$
$$y(x) = e^{\frac{x^2}{2}} c_1$$

✓ Solution by Mathematica

Time used: 0.132 (sec). Leaf size: 34

```
DSolve[(y'[x])^2-(x-y[x])y[x] y'[x]-x y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{x - c_1}$$
$$y(x) \to c_1 e^{\frac{x^2}{2}}$$
$$y(x) \to 0$$

29.2 problem 824

Internal problem ID [3554]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 824.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$\boxed{y'^2 + y^2 y' x + y^3 = 0}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 127

```
dsolve(diff(y(x),x)^2+x*y(x)^2*diff(y(x),x)+y(x)^3 = 0,y(x), singsol=all)
```

$$y(x) = \frac{4}{x^2}$$
$$y(x) = 0$$
$$y(x) = \frac{2\sqrt{2}\, x c_1 - 2c_1^2}{c_1^2 \left(c_1^2 - 2x^2\right)}$$
$$y(x) = -\frac{2\left(\sqrt{2}\, x c_1 + c_1^2\right)}{c_1^2 \left(c_1^2 - 2x^2\right)}$$
$$y(x) = -\frac{\left(\sqrt{2}\, x c_1 - 2\right) c_1^2}{2 \left(c_1^2 x^2 - 2\right)}$$
$$y(x) = \frac{\left(\sqrt{2}\, x c_1 + 2\right) c_1^2}{2c_1^2 x^2 - 4}$$

✓ Solution by Mathematica

Time used: 0.754 (sec). Leaf size: 59

```
DSolve[(y'[x])^2+x y[x]^2 y'[x]+y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{e^{2c_1} - ie^{c_1}x}$$
$$y(x) \to \frac{1}{ie^{c_1}x + e^{2c_1}}$$
$$y(x) \to 0$$
$$y(x) \to \frac{4}{x^2}$$

29.3 problem 825

Internal problem ID [3555]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 825.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y'^2 - 2x^3y^2y' - 4y^3x^2 = 0}$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 135

```
dsolve(diff(y(x),x)^2-2*x^3*y(x)^2*diff(y(x),x)-4*x^2*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{4}{x^4}$$
$$y(x) = 0$$
$$y(x) = \frac{\left(\sqrt{2}\,x^2c_1 - 2\right)c_1^2}{2c_1^2x^4 - 4}$$
$$y(x) = -\frac{\left(\sqrt{2}\,x^2c_1 + 2\right)c_1^2}{2\left(c_1^2x^4 - 2\right)}$$
$$y(x) = -\frac{2\left(\sqrt{2}\,x^2c_1 - c_1^2\right)}{c_1^2\left(-2x^4 + c_1^2\right)}$$
$$y(x) = \frac{2\sqrt{2}\,x^2c_1 + 2c_1^2}{c_1^2\left(-2x^4 + c_1^2\right)}$$

✓ Solution by Mathematica

Time used: 1.393 (sec). Leaf size: 177

```
DSolve[(y'[x])^2-2 x^3 y[x]^2 y'[x]-4 x^2 y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x\sqrt{x^4y(x)+4}y(x)^{3/2}\log\left(\sqrt{x^4y(x)+4}+x^2\sqrt{y(x)}\right)}{2\sqrt{x^2y(x)^3\left(x^4y(x)+4\right)}}-\frac{1}{4}\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{xy(x)^{3/2}\sqrt{x^4y(x)+4}\log\left(\sqrt{x^4y(x)+4}+x^2\sqrt{y(x)}\right)}{2\sqrt{x^2y(x)^3\left(x^4y(x)+4\right)}}-\frac{1}{4}\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

$$y(x)\to -\frac{4}{x^4}$$

29.4 problem 826

Internal problem ID [3556]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 826.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$y'^2 - xy(x^2 + y^2)\, y' + x^4y^4 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 38

```
dsolve(diff(y(x),x)^2-x*y(x)*(x^2+y(x)^2)*diff(y(x),x)+x^4*y(x)^4 = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-x^2 + c_1}}$$

$$y(x) = -\frac{1}{\sqrt{-x^2 + c_1}}$$

$$y(x) = c_1 e^{\frac{x^4}{4}}$$

✓ Solution by Mathematica

Time used: 0.168 (sec). Leaf size: 60

```
DSolve[(y'[x])^2-x y[x](x^2+y[x]^2)y'[x]+x^4 y[x]^4==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\frac{1}{\sqrt{-x^2 - 2c_1}}$$

$$y(x) \to \frac{1}{\sqrt{-x^2 - 2c_1}}$$

$$y(x) \to c_1 e^{\frac{x^4}{4}}$$

$$y(x) \to 0$$

29.5 problem 827

Internal problem ID [3557]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 827.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$y'^2 + 2y'y^3x + y^4 = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 51

```
dsolve(diff(y(x),x)^2+2*x*y(x)^3*diff(y(x),x)+y(x)^4 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{x}$$
$$y(x) = \frac{1}{x}$$
$$y(x) = 0$$
$$y(x) = \frac{1}{\sqrt{-c_1^2 + 2c_1x}}$$
$$y(x) = -\frac{1}{\sqrt{-c_1^2 + 2c_1x}}$$

✓ Solution by Mathematica

Time used: 0.831 (sec). Leaf size: 161

```
DSolve[(y'[x])^2+2 x y[x]^3 y'[x]+y[x]^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{\sqrt{x^2y(x)^2-1}y(x)^2\text{arctanh}\left(\frac{xy(x)}{\sqrt{x^2y(x)^2-1}}\right)}{\sqrt{y(x)^4\left(x^2y(x)^2-1\right)}}-\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{y(x)^2\sqrt{x^2y(x)^2-1}\text{arctanh}\left(\frac{xy(x)}{\sqrt{x^2y(x)^2-1}}\right)}{\sqrt{y(x)^4\left(x^2y(x)^2-1\right)}}-\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

$$y(x)\to -\frac{1}{x}$$

$$y(x)\to \frac{1}{x}$$

29.6 problem 828

Internal problem ID [3558]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 828.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$y'^2 + 2yy' \cot(x) - y^2 = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 61

```
dsolve(diff(y(x),x)^2+2*y(x)*diff(y(x),x)*cot(x)-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \frac{c_1\left(1+\tan(x)^2\right)\sqrt{\frac{\tan(x)^2}{1+\tan(x)^2}}}{\left(1+\sqrt{1+\tan(x)^2}\right)\tan(x)}$$

$$y(x) = \frac{c_1 \mathrm{e}^{\operatorname{arctanh}\left(\frac{1}{\sqrt{1+\tan(x)^2}}\right)}\sqrt{1+\tan(x)^2}}{\tan(x)}$$

✓ Solution by Mathematica

Time used: 0.157 (sec). Leaf size: 36

```
DSolve[(y'[x])^2+2 y[x] y'[x] Cot[x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 \csc^2\left(\frac{x}{2}\right)$$

$$y(x) \to c_1 \sec^2\left(\frac{x}{2}\right)$$

$$y(x) \to 0$$

29.7 problem 829

Internal problem ID [3559]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 829.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y'^2 - 3xy^{\frac{2}{3}}y' + 9y^{\frac{5}{3}} = 0$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 141

```
dsolve(diff(y(x),x)^2-3*x*y(x)^(2/3)*diff(y(x),x)+9*y(x)^(5/3) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x^6}{64}$$
$$y(x) = 0$$
$$\ln(x) + \frac{\sqrt{-4\left(\frac{y(x)}{x^6}\right)^{\frac{5}{3}} + \left(\frac{y(x)}{x^6}\right)^{\frac{4}{3}}}\,\operatorname{arctanh}\left(\sqrt{-4\left(\frac{y(x)}{x^6}\right)^{\frac{1}{3}} + 1}\right)}{\left(\frac{y(x)}{x^6}\right)^{\frac{2}{3}}\sqrt{-4\left(\frac{y(x)}{x^6}\right)^{\frac{1}{3}} + 1}} + \frac{\ln\left(\frac{64y(x)}{x^6} - 1\right)}{6}$$
$$- \frac{\ln\left(4\left(\frac{y(x)}{x^6}\right)^{\frac{1}{3}} - 1\right)}{6} - \frac{\ln\left(16\left(\frac{y(x)}{x^6}\right)^{\frac{2}{3}} + 4\left(\frac{y(x)}{x^6}\right)^{\frac{1}{3}} + 1\right)}{6} + \frac{\ln\left(\frac{y(x)}{x^6}\right)}{6} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 17.048 (sec). Leaf size: 701

```
DSolve[(y'[x])^2-3 x y[x]^(2/3) y'[x]+9 y[x]^(5/3)==0,y[x],x,IncludeSingularSolutions -> True
```

$$\text{Solve}\left[\frac{8x^2\log(y(x)) - 6\sqrt{x^4}\log\left(x^2\sqrt{x^2-4\sqrt[3]{y(x)}}\right) - 3\sqrt{x^4}\log\left(4\sqrt[3]{y(x)}-x^2\right) + 6\left(\sqrt{x^4}-x^2\right)\log\left(16\right.}{}\right.$$

$$\left. - \frac{\sqrt{\left(x^2-4\sqrt[3]{y(x)}\right)y(x)^{4/3}}\log\left(\sqrt{x^2-4\sqrt[3]{y(x)}}-x\right)}{\sqrt{x^2-4\sqrt[3]{y(x)}}\,y(x)^{2/3}} = c_1, y(x)\right]$$

$$\text{Solve}\left[\frac{\sqrt{\left(x^2-4\sqrt[3]{y(x)}\right)y(x)^{4/3}}\log\left(\sqrt{x^2-4\sqrt[3]{y(x)}}-x\right)}{\sqrt{x^2-4\sqrt[3]{y(x)}}\,y(x)^{2/3}}\right.$$

$$+\frac{8x^2\log(y(x)) + 6\sqrt{x^4}\log\left(x^2\sqrt{x^2-4\sqrt[3]{y(x)}}\right) + 3\sqrt{x^4}\log\left(4\sqrt[3]{y(x)}-x^2\right) + 6\left(x^2-\sqrt{x^4}\right)\log\left(16x^2\sqrt{x\right.}{}$$

$$y(x) \to 0$$

29.8 problem 830

Internal problem ID [3560]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 830.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$y'^2 - e^{4x-2y}(y'-1) = 0$$

✓ Solution by Maple

Time used: 0.907 (sec). Leaf size: 259

```
dsolve(diff(y(x),x)^2 = exp(4*x-2*y(x))*(diff(y(x),x)-1),y(x), singsol=all)
```

$$x - \frac{\sqrt{-\left(4\,e^{-4x+2y(x)}-1\right)e^{-4y(x)+8x}}\,e^{-4x+2y(x)}\operatorname{arctanh}\left(\frac{1}{\sqrt{-4\,e^{-4x+2y(x)}+1}}\right)}{2\sqrt{-4\,e^{-4x+2y(x)}+1}} + \frac{\ln\left(e^{y(x)-2x}\right)}{2} - \frac{\ln\left(2\,e^{y(x)-2x}-1\right)}{4} - \frac{\ln\left(2\,e^{y(x)-2x}+1\right)}{4} + \frac{\ln\left(4\,e^{-4x+2y(x)}-1\right)}{4} - c_1 = 0$$

$$x + \frac{\ln\left(e^{y(x)-2x}\right)}{2} - \frac{\ln\left(2\,e^{y(x)-2x}-1\right)}{4} - \frac{\ln\left(2\,e^{y(x)-2x}+1\right)}{4} + \frac{\ln\left(4\,e^{-4x+2y(x)}-1\right)}{4} + \frac{\sqrt{-\left(4\,e^{-4x+2y(x)}-1\right)e^{-4y(x)+8x}}\,e^{-4x+2y(x)}\operatorname{arctanh}\left(\frac{1}{\sqrt{-4\,e^{-4x+2y(x)}+1}}\right)}{2\sqrt{-4\,e^{-4x+2y(x)}+1}} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 2.487 (sec). Leaf size: 383

```
DSolve[(y'[x])^2==Exp[4 x -2 y[x]] (y'[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{e^{-2x}\sqrt{e^{8x}-4e^{2y(x)+4x}}\operatorname{arctanh}\left(\frac{-\sqrt{e^{4x}-4e^{2y(x)}}+e^{2x}+1}{\sqrt{e^{4x}-4e^{2y(x)}}-e^{2x}+1}\right)}{\sqrt{e^{4x}-4e^{2y(x)}}} - \frac{e^{-2x}\sqrt{e^{8x}-4e^{2y(x)+4x}}y(x)}{2\sqrt{e^{4x}-4e^{2y(x)}}}+\frac{y(x)}{2}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{e^{-2x}\sqrt{e^{8x}-4e^{2y(x)+4x}}\operatorname{arctanh}\left(\frac{-\sqrt{e^{4x}-4e^{2y(x)}}+e^{2x}+1}{\sqrt{e^{4x}-4e^{2y(x)}}-e^{2x}+1}\right)}{\sqrt{e^{4x}-4e^{2y(x)}}} + \frac{\left(\sqrt{e^{4x}-4e^{2y(x)}}\sqrt{e^{8x}-4e^{2y(x)+4x}}-4e^{2(y(x)+x)}+e^{6x}\right)y(x)}{2e^{6x}-8e^{2(y(x)+x)}}=c_1,y(x)\right]$$

$$y(x)\to\frac{1}{2}\left(\log\left(\frac{e^{8x}}{4}\right)-4x\right)$$

29.9 problem 831

Internal problem ID [3561]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 831.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{2y'^2 + y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve(2*diff(y(x),x)^2+x*diff(y(x),x)-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = e^{2\,\mathrm{LambertW}\left(\frac{x\,e^{\frac{c_1}{4}}}{4}\right)-\frac{c_1}{2}} + \frac{x\,e^{\mathrm{LambertW}\left(\frac{x\,e^{\frac{c_1}{4}}}{4}\right)-\frac{c_1}{4}}}{2}$$

✓ Solution by Mathematica

Time used: 1.166 (sec). Leaf size: 130

```
DSolve[2 (y'[x])^2+x y'[x]-2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[-\frac{\frac{1}{2}x\sqrt{x^2+16y(x)} - 8y(x)\log\left(\sqrt{x^2+16y(x)}-x\right) + \frac{x^2}{2}}{8y(x)} = c_1, y(x)\right]$$

$$\mathrm{Solve}\left[\frac{\frac{1}{2}x\sqrt{x^2+16y(x)} - 8y(x)\log\left(\sqrt{x^2+16y(x)}-x\right) - \frac{x^2}{2}}{8y(x)} + \log(y(x)) = c_1, y(x)\right]$$

$$y(x) \to 0$$

29.10 problem 832

Internal problem ID [3562]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 832.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$2y'^2 - (1-x)\,y' - y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 29

```
dsolve(2*diff(y(x),x)^2-(1-x)*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{8}x^2 + \frac{1}{4}x - \frac{1}{8}$$
$$y(x) = 2c_1^2 + c_1x - c_1$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 28

```
DSolve[2 (y'[x])^2-(1-x)y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x - 1 + 2c_1)$$
$$y(x) \to -\frac{1}{8}(x-1)^2$$

29.11 problem 833

Internal problem ID [3563]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 833.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$2y'^2 - 2y'x^2 + 3yx = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 109

```
dsolve(2*diff(y(x),x)^2-2*x^2*diff(y(x),x)+3*x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x^3}{6}$$
$$y(x) = \frac{x^3}{3} - \frac{\left(x^2 - \sqrt{-6c_1x}\right)x}{3} + c_1$$
$$y(x) = \frac{x^3}{3} - \frac{\left(x^2 + \sqrt{-6c_1x}\right)x}{3} + c_1$$
$$y(x) = \frac{x^3}{3} + \frac{\left(-x^2 - \sqrt{-6c_1x}\right)x}{3} + c_1$$
$$y(x) = \frac{x^3}{3} + \frac{\left(-x^2 + \sqrt{-6c_1x}\right)x}{3} + c_1$$

✓ Solution by Mathematica

Time used: 2.563 (sec). Leaf size: 213

```
DSolve[2 (y'[x])^2-2 x^2 y'[x]+3 x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{3}\left(1-\frac{\sqrt{x^4-6xy(x)}}{\sqrt{x}\sqrt{x^3-6y(x)}}\right)\log(y(x))+\frac{2\sqrt{x^4-6xy(x)}\log\left(x^{3/2}+\sqrt{x^3-6y(x)}\right)}{3\sqrt{x}\sqrt{x^3-6y(x)}}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{3}\left(\frac{\sqrt{x^4-6xy(x)}}{\sqrt{x}\sqrt{x^3-6y(x)}}+1\right)\log(y(x))-\frac{2\sqrt{x^4-6xy(x)}\log\left(x^{3/2}+\sqrt{x^3-6y(x)}\right)}{3\sqrt{x}\sqrt{x^3-6y(x)}}=c_1,y(x)\right]$$

$$y(x)\to\frac{x^3}{6}$$

29.12 problem 834

Internal problem ID [3564]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 834.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$2y'^2 + 2(6y-1)\,y' + 3y(6y-1) = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 204

```
dsolve(2*diff(y(x),x)^2+2*(6*y(x)-1)*diff(y(x),x)+3*y(x)*(6*y(x)-1) = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{6}$$

$$y(x) = -\frac{e^{-3x}e^{3c_1}\left(\sqrt{6}\,e^{-\frac{3x}{2}}e^{\frac{3c_1}{2}} - 3\,e^{-3x}e^{3c_1}\right)}{3\,e^{-3x}e^{3c_1} - 2} - 2\,e^{-3x}e^{3c_1} + \frac{\frac{2\sqrt{6}\,e^{-\frac{3x}{2}}e^{\frac{3c_1}{2}}}{3} - 2\,e^{-3x}e^{3c_1}}{3\,e^{-3x}e^{3c_1} - 2}$$

$$y(x) = \frac{e^{-3x}e^{3c_1}\left(\sqrt{6}\,e^{-\frac{3x}{2}}e^{\frac{3c_1}{2}} + 3\,e^{-3x}e^{3c_1}\right)}{3\,e^{-3x}e^{3c_1} - 2} - 2\,e^{-3x}e^{3c_1} - \frac{2\left(\sqrt{6}\,e^{-\frac{3x}{2}}e^{\frac{3c_1}{2}} + 3\,e^{-3x}e^{3c_1}\right)}{3\left(3\,e^{-3x}e^{3c_1} - 2\right)}$$

✓ Solution by Mathematica

Time used: 0.259 (sec). Leaf size: 81

```
DSolve[2 (y'[x])^2+2(6 y[x]-1)y'[x]+3 y[x](6  y[x]-1)==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\frac{1}{6}e^{-3x+3c_1}\left(2e^{3x/2} + e^{3c_1}\right)$$

$$y(x) \to \frac{1}{6}e^{-3(x+2c_1)}\left(-1 + 2e^{\frac{3x}{2}+3c_1}\right)$$

$$y(x) \to 0$$

$$y(x) \to \frac{1}{6}$$

29.13 problem 835

Internal problem ID [3565]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 835.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{3{y'}^2 - 2y'x + y = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 656

```
dsolve(3*diff(y(x),x)^2-2*x*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = -3\left(\frac{\left(-54c_1+x^3+6\sqrt{-3c_1x^3+81c_1^2}\right)^{\frac{1}{3}}}{6}+\frac{x^2}{6\left(-54c_1+x^3+6\sqrt{-3c_1x^3+81c_1^2}\right)^{\frac{1}{3}}}+\frac{x}{6}\right)^2 + 2x\left(\frac{\left(-54c_1+x^3+6\sqrt{-3c_1x^3+81c_1^2}\right)^{\frac{1}{3}}}{6}+\frac{x^2}{6\left(-54c_1+x^3+6\sqrt{-3c_1x^3+81c_1^2}\right)^{\frac{1}{3}}}+\frac{x}{6}\right)$$

$$y(x) = -3\left(-\frac{\left(-54c_1+x^3+6\sqrt{-3c_1x^3+81c_1^2}\right)^{\frac{1}{3}}}{12}-\frac{x^2}{12\left(-54c_1+x^3+6\sqrt{-3c_1x^3+81c_1^2}\right)^{\frac{1}{3}}}+\frac{x}{6}-\frac{i\sqrt{3}\left(\left(-5\cdots\right.\right.}{\cdots}\right. + 2x\left(-\frac{\left(-54c_1+x^3+6\sqrt{-3c_1x^3+81c_1^2}\right)^{\frac{1}{3}}}{12}-\frac{x^2}{12\left(-54c_1+x^3+6\sqrt{-3c_1x^3+81c_1^2}\right)^{\frac{1}{3}}}+\frac{x}{6}-\frac{i\sqrt{3}\left(\frac{\left(-54c_1+x^3+6\sqrt{-3c_1x^3+81c_1^2}\right)^{\frac{1}{3}}}{6}-\frac{x^2}{6\left(-54c_1+x^3+6\sqrt{-3c_1x^3+81c_1^2}\right)^{\frac{1}{3}}}\right)}{2}\right)$$

$$y(x) = -3\left(-\frac{\left(-54c_1+x^3+6\sqrt{-3c_1x^3+81c_1^2}\right)^{\frac{1}{3}}}{12}-\frac{x^2}{12\left(-54c_1+x^3+6\sqrt{-3c_1x^3+81c_1^2}\right)^{\frac{1}{3}}}+\frac{x}{6}+\frac{i\sqrt{3}\left(\left(-5\cdots\right.\right.}{\cdots}\right.$$

✓ Solution by Mathematica

Time used: 60.138 (sec). Leaf size: 994

```
DSolve[3 (y'[x])^2-2 x y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}\left(x^2 + \frac{x(x^3+216e^{3c_1})}{\sqrt[3]{x^6 - 540e^{3c_1}x^3 + 24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3} - 5832e^{6c_1}}} + \sqrt[3]{x^6 - 540e^{3c_1}x^3 + 24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3} - 5832e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{24}\left(2x^2 + \frac{\left(-1-i\sqrt{3}\right)x(x^3+216e^{3c_1})}{\sqrt[3]{x^6 - 540e^{3c_1}x^3 + 24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3} - 5832e^{6c_1}}} + i\left(\sqrt{3}+i\right)\sqrt[3]{x^6 - 540e^{3c_1}x^3 + 24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3} - 5832e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{24}\left(2x^2 + \frac{i\left(\sqrt{3}+i\right)x(x^3+216e^{3c_1})}{\sqrt[3]{x^6 - 540e^{3c_1}x^3 + 24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3} - 5832e^{6c_1}}} - \left(1+i\sqrt{3}\right)\sqrt[3]{x^6 - 540e^{3c_1}x^3 + 24\sqrt{3}\sqrt{e^{3c_1}\left(-x^3+27e^{3c_1}\right)^3} - 5832e^{6c_1}}\right)$$

$$y(x) \to \frac{x^4 + \left(x^6 + 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3} - 8e^{6c_1}\right)^{2/3} + x^2\sqrt[3]{x^6 + 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3} - 8e^{6c_1}}\cdots}{12\sqrt[3]{x^6 + 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3} - 8e^{6c_1}}}$$

$$y(x) \to \frac{1}{24}\left(2x^2 + \frac{\left(1+i\sqrt{3}\right)x(-x^3+8e^{3c_1})}{\sqrt[3]{x^6 + 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3} - 8e^{6c_1}}} + i\left(\sqrt{3}+i\right)\sqrt[3]{x^6 + 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3} - 8e^{6c_1}}\right)$$

$$y(x) \to \frac{1}{24}\left(2x^2 + \frac{i\left(\sqrt{3}+i\right)x(x^3-8e^{3c_1})}{\sqrt[3]{x^6 + 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3} - 8e^{6c_1}}} - \left(1+i\sqrt{3}\right)\sqrt[3]{x^6 + 20e^{3c_1}x^3 + 8\sqrt{e^{3c_1}\left(x^3+e^{3c_1}\right)^3} - 8e^{6c_1}}\right)$$

29.14 problem 836

Internal problem ID [3566]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 836.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$3y'^2 + 4y'x + x^2 - y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 111

```
dsolve(3*diff(y(x),x)^2+4*x*diff(y(x),x)+x^2-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{3}$$
$$y(x) = -\frac{5x^2}{12} - \frac{x\left(-x - \sqrt{3}\, c_1\right)}{6} + \frac{c_1^2}{4}$$
$$y(x) = -\frac{5x^2}{12} - \frac{x\left(-x + \sqrt{3}\, c_1\right)}{6} + \frac{c_1^2}{4}$$
$$y(x) = -\frac{5x^2}{12} + \frac{x\left(x - \sqrt{3}\, c_1\right)}{6} + \frac{c_1^2}{4}$$
$$y(x) = -\frac{5x^2}{12} + \frac{x\left(x + \sqrt{3}\, c_1\right)}{6} + \frac{c_1^2}{4}$$

✓ Solution by Mathematica

Time used: 3.78 (sec). Leaf size: 79

```
DSolve[3 (y'[x])^2+4 x y'[x]+x^2-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{12}(-3x - 1 + e^{c_1})(x - 1 + e^{c_1})$$
$$y(x) \to \frac{1}{12}(-x + 1 + e^{c_1})(3x + 1 + e^{c_1})$$
$$y(x) \to -\frac{x^2}{3}$$
$$y(x) \to \frac{1}{12}((2 - 3x)x + 1)$$

29.15 problem 837

Internal problem ID [3567]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 837.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{4y'^2 - 9x = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 19

```
dsolve(4*diff(y(x),x)^2 = 9*x,y(x), singsol=all)
```

$$y(x) = -x^{\frac{3}{2}} + c_1$$
$$y(x) = x^{\frac{3}{2}} + c_1$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 27

```
DSolve[4 (y'[x])^2==9 x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x^{3/2} + c_1$$
$$y(x) \to x^{3/2} + c_1$$

29.16 problem 838

Internal problem ID [3568]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 838.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$4{y'}^2 + 2x\,\mathrm{e}^{-2y}y' - \mathrm{e}^{-2y} = 0$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 122

```
dsolve(4*diff(y(x),x)^2+2*x*exp(-2*y(x))*diff(y(x),x)-exp(-2*y(x)) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\ln\left(-\frac{4}{x^2}\right)}{2}$$

$$y(x) = c_1 - \operatorname{arctanh}\left(\frac{x}{\operatorname{RootOf}\left(\_Z^2 - x^2 - 4\,\mathrm{e}^{\operatorname{RootOf}\left(x^2\tanh\left(-\frac{\_Z}{2}+c_1\right)^2+4\,\mathrm{e}^{\_Z}\tanh\left(-\frac{\_Z}{2}+c_1\right)^2-x^2\right)}\right)}\right)$$

$$y(x) = c_1 + \operatorname{arctanh}\left(\frac{x}{\operatorname{RootOf}\left(\_Z^2 - x^2 - 4\,\mathrm{e}^{\operatorname{RootOf}\left(x^2\tanh\left(-\frac{\_Z}{2}+c_1\right)^2+4\,\mathrm{e}^{\_Z}\tanh\left(-\frac{\_Z}{2}+c_1\right)^2-x^2\right)}\right)}\right)$$

✓ Solution by Mathematica

Time used: 10.223 (sec). Leaf size: 119

```
DSolve[4 (y'[x])^2+2 x Exp[-2 y[x]] y'[x]-Exp[-2 y[x]]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \log\left(-e^{\frac{c_1}{2}}\sqrt{-x+e^{c_1}}\right)$$

$$y(x) \to \log\left(e^{\frac{c_1}{2}}\sqrt{-x+e^{c_1}}\right)$$

$$y(x) \to \log\left(-e^{\frac{c_1}{2}}\sqrt{x+e^{c_1}}\right)$$

$$y(x) \to \log\left(e^{\frac{c_1}{2}}\sqrt{x+e^{c_1}}\right)$$

$$y(x) \to \frac{1}{2}\log\left(-\frac{x^2}{4}\right)$$

29.17 problem 839

Internal problem ID [3569]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 839.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{4{y'}^2+2\,\mathrm{e}^{2x-2y}y'-\mathrm{e}^{2x-2y}=0}$$

✓ Solution by Maple

Time used: 1.109 (sec). Leaf size: 137

```
dsolve(4*diff(y(x),x)^2+2*exp(2*x-2*y(x))*diff(y(x),x)-exp(2*x-2*y(x)) = 0,y(x), singsol=all)
```

$$y(x)=c_1 -\operatorname{arctanh}\left(\frac{1}{\operatorname{RootOf}\left(\_Z^2-4\,\mathrm{e}^{\operatorname{RootOf}\left(16\tanh\left(-\frac{\_Z}{2}+c_1-x\right)^2\mathrm{e}^{2\_Z}+8\tanh\left(-\frac{\_Z}{2}+c_1-x\right)^2\mathrm{e}^{\_Z}+\tanh\left(-\frac{\_Z}{2}+c_1-x\right)^2-4\,\mathrm{e}^{\_Z}-\right.}\right.}\right.$$

$$y(x)=c_1 +\operatorname{arctanh}\left(\frac{1}{\operatorname{RootOf}\left(\_Z^2-4\,\mathrm{e}^{\operatorname{RootOf}\left(16\tanh\left(-\frac{\_Z}{2}+c_1-x\right)^2\mathrm{e}^{2\_Z}+8\tanh\left(-\frac{\_Z}{2}+c_1-x\right)^2\mathrm{e}^{\_Z}+\tanh\left(-\frac{\_Z}{2}+c_1-x\right)^2-4\,\mathrm{e}^{\_Z}-\right.}\right.}\right.$$

✓ Solution by Mathematica

Time used: 1.613 (sec). Leaf size: 332

```
DSolve[4 (y'[x])^2+2 Exp[2 x-2 y[x]] y'[x]-Exp[2 x-2 y[x]]==0,y[x],x,IncludeSingularSolutions
```

$$\text{Solve}\left[-\frac{2e^{-x}\sqrt{4e^{2(y(x)+x)}+e^{4x}}\,\text{arctanh}\left(\frac{-\sqrt{4e^{2y(x)}+e^{2x}}+e^{x}+1}{\sqrt{4e^{2y(x)}+e^{2x}}-e^{x}+1}\right)}{\sqrt{4e^{2y(x)}+e^{2x}}} - \frac{e^{-x}\sqrt{4e^{2(y(x)+x)}+e^{4x}}\,y(x)}{\sqrt{4e^{2y(x)}+e^{2x}}}+y(x)=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{2e^{-x}\sqrt{4e^{2(y(x)+x)}+e^{4x}}\,\text{arctanh}\left(\frac{-\sqrt{4e^{2y(x)}+e^{2x}}+e^{x}+1}{\sqrt{4e^{2y(x)}+e^{2x}}-e^{x}+1}\right)}{\sqrt{4e^{2y(x)}+e^{2x}}} + \frac{e^{-x}\sqrt{4e^{2(y(x)+x)}+e^{4x}}\,y(x)}{\sqrt{4e^{2y(x)}+e^{2x}}}+y(x)=c_1,y(x)\right]$$

$$y(x)\to\frac{1}{2}\left(\log\left(-\frac{e^{4x}}{4}\right)-2x\right)$$

29.18 problem 840

Internal problem ID [3570]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 840.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$5y'^2 + 3y'x - y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 85

```
dsolve(5*diff(y(x),x)^2+3*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$\frac{c_1}{\left(-30x - 10\sqrt{9x^2 + 20y(x)}\right)^{\frac{3}{2}}} + \frac{2x}{5} - \frac{\sqrt{9x^2 + 20y(x)}}{5} = 0$$
$$\frac{c_1}{\left(-30x + 10\sqrt{9x^2 + 20y(x)}\right)^{\frac{3}{2}}} + \frac{2x}{5} + \frac{\sqrt{9x^2 + 20y(x)}}{5} = 0$$

✓ Solution by Mathematica

Time used: 13.693 (sec). Leaf size: 771

```
DSolve[5 (y'[x])^2+3 x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[16\#1^5 + 8\#1^4x^2 + \#1^3x^4 + 4000\#1^2e^{5c_1}x + 1800\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 200000e^{10c_1}\&, 1\right]$$

$$y(x) \to \text{Root}\left[16\#1^5 + 8\#1^4x^2 + \#1^3x^4 + 4000\#1^2e^{5c_1}x + 1800\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 200000e^{10c_1}\&, 2\right]$$

$$y(x) \to \text{Root}\left[16\#1^5 + 8\#1^4x^2 + \#1^3x^4 + 4000\#1^2e^{5c_1}x + 1800\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 200000e^{10c_1}\&, 3\right]$$

$$y(x) \to \text{Root}\left[16\#1^5 + 8\#1^4x^2 + \#1^3x^4 + 4000\#1^2e^{5c_1}x + 1800\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 200000e^{10c_1}\&, 4\right]$$

$$y(x) \to \text{Root}\left[16\#1^5 + 8\#1^4x^2 + \#1^3x^4 + 4000\#1^2e^{5c_1}x + 1800\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 200000e^{10c_1}\&, 5\right]$$

$$y(x) \to \text{Root}\left[3200000\#1^5 + 1600000\#1^4x^2 + 200000\#1^3x^4 - 4000\#1^2e^{5c_1}x - 1800\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 1\right]$$

$$y(x) \to \text{Root}\left[3200000\#1^5 + 1600000\#1^4x^2 + 200000\#1^3x^4 - 4000\#1^2e^{5c_1}x - 1800\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 2\right]$$

$$y(x) \to \text{Root}\left[3200000\#1^5 + 1600000\#1^4x^2 + 200000\#1^3x^4 - 4000\#1^2e^{5c_1}x - 1800\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 3\right]$$

$$y(x) \to \text{Root}\left[3200000\#1^5 + 1600000\#1^4x^2 + 200000\#1^3x^4 - 4000\#1^2e^{5c_1}x - 1800\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 4\right]$$

$$y(x) \to \text{Root}\left[3200000\#1^5 + 1600000\#1^4x^2 + 200000\#1^3x^4 - 4000\#1^2e^{5c_1}x - 1800\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 5\right]$$

$$y(x) \to 0$$

29.19 problem 841

Internal problem ID [3571]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 841.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{5{y'}^2 + 6y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 85

```
dsolve(5*diff(y(x),x)^2+6*x*diff(y(x),x)-2*y(x) = 0,y(x), singsol=all)
```

$$\frac{c_1}{\left(-15x - 5\sqrt{9x^2 + 10y(x)}\right)^{\frac{3}{2}}} + \frac{2x}{5} - \frac{\sqrt{9x^2 + 10y(x)}}{5} = 0$$

$$\frac{c_1}{\left(-15x + 5\sqrt{9x^2 + 10y(x)}\right)^{\frac{3}{2}}} + \frac{2x}{5} + \frac{\sqrt{9x^2 + 10y(x)}}{5} = 0$$

✓ Solution by Mathematica

Time used: 13.858 (sec). Leaf size: 771

```
DSolve[5 (y'[x])^2+6 x y'[x]-2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{Root}\left[4\#1^5 + 4\#1^4x^2 + \#1^3x^4 + 1000\#1^2e^{5c_1}x + 900\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 25000e^{10c_1}\&, 1\right]$$

$$y(x) \to \text{Root}\left[4\#1^5 + 4\#1^4x^2 + \#1^3x^4 + 1000\#1^2e^{5c_1}x + 900\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 25000e^{10c_1}\&, 2\right]$$

$$y(x) \to \text{Root}\left[4\#1^5 + 4\#1^4x^2 + \#1^3x^4 + 1000\#1^2e^{5c_1}x + 900\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 25000e^{10c_1}\&, 3\right]$$

$$y(x) \to \text{Root}\left[4\#1^5 + 4\#1^4x^2 + \#1^3x^4 + 1000\#1^2e^{5c_1}x + 900\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 25000e^{10c_1}\&, 4\right]$$

$$y(x) \to \text{Root}\left[4\#1^5 + 4\#1^4x^2 + \#1^3x^4 + 1000\#1^2e^{5c_1}x + 900\#1e^{5c_1}x^3 + 216e^{5c_1}x^5 - 25000e^{10c_1}\&, 5\right]$$

$$y(x) \to \text{Root}\left[100000\#1^5 + 100000\#1^4x^2 + 25000\#1^3x^4 - 1000\#1^2e^{5c_1}x - 900\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 1\right]$$

$$y(x) \to \text{Root}\left[100000\#1^5 + 100000\#1^4x^2 + 25000\#1^3x^4 - 1000\#1^2e^{5c_1}x - 900\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 2\right]$$

$$y(x) \to \text{Root}\left[100000\#1^5 + 100000\#1^4x^2 + 25000\#1^3x^4 - 1000\#1^2e^{5c_1}x - 900\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 3\right]$$

$$y(x) \to \text{Root}\left[100000\#1^5 + 100000\#1^4x^2 + 25000\#1^3x^4 - 1000\#1^2e^{5c_1}x - 900\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 4\right]$$

$$y(x) \to \text{Root}\left[100000\#1^5 + 100000\#1^4x^2 + 25000\#1^3x^4 - 1000\#1^2e^{5c_1}x - 900\#1e^{5c_1}x^3 - 216e^{5c_1}x^5 - e^{10c_1}\&, 5\right]$$

$$y(x) \to 0$$

29.20 problem 842

Internal problem ID [3572]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 842.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{9{y'}^{2}+3x y^{4} y'+y^{5}=0}$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 109

```
dsolve(9*diff(y(x),x)^2+3*x*y(x)^4*diff(y(x),x)+y(x)^5 = 0,y(x), singsol=all)
```

$$y(x)=\frac{4^{\frac{1}{3}}}{x^{\frac{2}{3}}}$$
$$y(x)=-\frac{4^{\frac{1}{3}}}{2x^{\frac{2}{3}}}-\frac{i\sqrt{3}\,4^{\frac{1}{3}}}{2x^{\frac{2}{3}}}$$
$$y(x)=-\frac{4^{\frac{1}{3}}}{2x^{\frac{2}{3}}}+\frac{i\sqrt{3}\,4^{\frac{1}{3}}}{2x^{\frac{2}{3}}}$$
$$y(x)=0$$
$$y(x)=\frac{\operatorname{RootOf}\left(-\ln(x)+\int^{\_Z}\frac{\frac{3\_a^{3}}{2}+\frac{3\sqrt{\_a^{3}(\_a^{3}-4)}}{2}-6}{\_a(\_a^{3}-4)}d\_a+c_{1}\right)}{x^{\frac{2}{3}}}$$

✓ Solution by Mathematica

Time used: 1.01 (sec). Leaf size: 212

```
DSolve[9 (y'[x])^2+3 x y[x]^4 y'[x]+y[x]^5==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{\sqrt{x^2y(x)^3-4}\,y(x)^{5/2}\text{arctanh}\left(\frac{xy(x)^{3/2}}{\sqrt{x^2y(x)^3-4}}\right)}{\sqrt{y(x)^5\left(x^2y(x)^3-4\right)}}-\frac{3}{2}\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{y(x)^{5/2}\sqrt{x^2y(x)^3-4}\,\text{arctanh}\left(\frac{xy(x)^{3/2}}{\sqrt{x^2y(x)^3-4}}\right)}{\sqrt{y(x)^5\left(x^2y(x)^3-4\right)}}-\frac{3}{2}\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

$$y(x)\to \frac{(-2)^{2/3}}{x^{2/3}}$$

$$y(x)\to \frac{2^{2/3}}{x^{2/3}}$$

$$y(x)\to -\frac{\sqrt[3]{-1}2^{2/3}}{x^{2/3}}$$

29.21 problem 843

Internal problem ID [3573]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 843.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy'^2 - a = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 25

```
dsolve(x*diff(y(x),x)^2 = a,y(x), singsol=all)
```

$$y(x) = 2\sqrt{ax} + c_1$$
$$y(x) = -2\sqrt{ax} + c_1$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 39

```
DSolve[x (y'[x])^2==a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -2\sqrt{a}\sqrt{x} + c_1$$
$$y(x) \to 2\sqrt{a}\sqrt{x} + c_1$$

29.22 problem 844

Internal problem ID [3574]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 844.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$
x{y'}^2 + x^2 - a = 0
$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 46

```
dsolve(x*diff(y(x),x)^2 = -x^2+a,y(x), singsol=all)
```

$$
y(x) = \int \frac{\sqrt{x\,(-x^2+a)}}{x}dx + c_1
$$

$$
y(x) = \int -\frac{\sqrt{x\,(-x^2+a)}}{x}dx + c_1
$$

✓ Solution by Mathematica

Time used: 5.381 (sec). Leaf size: 93

```
DSolve[x (y'[x])^2==(a-x^2),y[x],x,IncludeSingularSolutions -> True]
```

$$
y(x) \to \frac{-2\sqrt{x}(a-x^2)^{3/2}\,\text{Hypergeometric2F1}\left(1,\frac{7}{4},\frac{5}{4},\frac{x^2}{a}\right) + ac_1}{a}
$$

$$
y(x) \to \frac{2\sqrt{x}(a-x^2)^{3/2}\,\text{Hypergeometric2F1}\left(1,\frac{7}{4},\frac{5}{4},\frac{x^2}{a}\right) + ac_1}{a}
$$

29.23 problem 845

Internal problem ID [3575]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 845.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy'^2 - y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 39

```
dsolve(x*diff(y(x),x)^2 = y(x),y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{\left(x + \sqrt{c_1 x}\right)^2}{x}$$
$$y(x) = \frac{\left(-x + \sqrt{c_1 x}\right)^2}{x}$$

✓ Solution by Mathematica

Time used: 0.047 (sec). Leaf size: 46

```
DSolve[x (y'[x])^2==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{1}{4}\left(-2\sqrt{x} + c_1\right)^2$$
$$y(x) \rightarrow \frac{1}{4}\left(2\sqrt{x} + c_1\right)^2$$
$$y(x) \rightarrow 0$$

29.24 problem 846

Internal problem ID [3576]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 846.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy'^2 + x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 73

```
dsolve(x*diff(y(x),x)^2+x-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \left(\frac{\left(\text{LambertW}\left(\frac{\sqrt{c_1 x}}{c_1}\right)+1\right)^2}{2\,\text{LambertW}\left(\frac{\sqrt{c_1 x}}{c_1}\right)^2}+\frac{1}{2}\right)x$$

$$y(x) = \left(\frac{\left(\text{LambertW}\left(-\frac{\sqrt{c_1 x}}{c_1}\right)+1\right)^2}{2\,\text{LambertW}\left(-\frac{\sqrt{c_1 x}}{c_1}\right)^2}+\frac{1}{2}\right)x$$

✓ Solution by Mathematica

Time used: 0.616 (sec). Leaf size: 97

```
DSolve[x (y'[x])^2+x-2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2}{\sqrt{\frac{2y(x)}{x}-1}-1}-2\log\left(\sqrt{\frac{2y(x)}{x}-1}-1\right)=\log(x)+c_1, y(x)\right]$$

$$\text{Solve}\left[\frac{2}{\sqrt{\frac{2y(x)}{x}-1}+1}+2\log\left(\sqrt{\frac{2y(x)}{x}-1}+1\right)=-\log(x)+c_1, y(x)\right]$$

29.25 problem 847

Internal problem ID [3577]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 847.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, _dAlembert]`

$$\boxed{xy'^2 + y' - y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 57

```
dsolve(x*diff(y(x),x)^2+diff(y(x),x) = y(x),y(x), singsol=all)
```

$$y(x) = e^{2\,\mathrm{RootOf}\left(-x\,e^{2\_Z}+2x\,e^{\_Z}+\_Z+c_1-x-e^{\_Z}\right)}x + e^{\mathrm{RootOf}\left(-x\,e^{2\_Z}+2x\,e^{\_Z}+\_Z+c_1-x-e^{\_Z}\right)}$$

✓ Solution by Mathematica

Time used: 0.887 (sec). Leaf size: 46

```
DSolve[x (y'[x])^2+y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x = \frac{\log(K[1]) - K[1]}{(K[1]-1)^2} + \frac{c_1}{(K[1]-1)^2}, y(x) = xK[1]^2 + K[1]\right\}, \{y(x), K[1]\}\right]$$

29.26 problem 848

Internal problem ID [3578]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 848.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, _dAlembert]`

$$\boxed{xy'^2 + 2y' - y = 0}$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 63

```
dsolve(x*diff(y(x),x)^2+2*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2\,\mathrm{RootOf}\left(-x\,\mathrm{e}^{2\_Z}+2x\,\mathrm{e}^{\_Z}-2\,\mathrm{e}^{\_Z}+c_1+2\_Z-x\right)}x + 2\,\mathrm{e}^{\mathrm{RootOf}\left(-x\,\mathrm{e}^{2\_Z}+2x\,\mathrm{e}^{\_Z}-2\,\mathrm{e}^{\_Z}+c_1+2\_Z-x\right)}$$

✓ Solution by Mathematica

Time used: 13.027 (sec). Leaf size: 50

```
DSolve[x (y'[x])^2+2 y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x = \frac{2\log(K[1]) - 2K[1]}{(K[1]-1)^2} + \frac{c_1}{(K[1]-1)^2}, y(x) = xK[1]^2 + 2K[1]\right\}, \{y(x), K[1]\}\right]$$

29.27 problem 849

Internal problem ID [3579]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 849.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, _dAlembert]`

$$\boxed{xy'^2 - 2y' - y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 63

```
dsolve(x*diff(y(x),x)^2-2*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{2\,\mathrm{RootOf}\left(-x\,\mathrm{e}^{2\_Z}+2x\,\mathrm{e}^{\_Z}+2\,\mathrm{e}^{\_Z}+c_1-2\_Z-x\right)}x - 2\,\mathrm{e}^{\mathrm{RootOf}\left(-x\,\mathrm{e}^{2\_Z}+2x\,\mathrm{e}^{\_Z}+2\,\mathrm{e}^{\_Z}+c_1-2\_Z-x\right)}$$

✓ Solution by Mathematica

Time used: 1.437 (sec). Leaf size: 50

```
DSolve[x (y'[x])^2-2 y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\left\{x = \frac{2K[1]-2\log(K[1])}{(K[1]-1)^2} + \frac{c_1}{(K[1]-1)^2}, y(x) = xK[1]^2 - 2K[1]\right\}, \{y(x), K[1]\}\right]$$

29.28 problem 850

Internal problem ID [3580]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 850.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, _dAlembert]`

$$\boxed{xy'^2 + 4y' - 2y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 64

```
dsolve(x*diff(y(x),x)^2+4*diff(y(x),x)-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{e^{2\,\mathrm{RootOf}\left(-x\,e^{2\_Z}+4x\,e^{\_Z}-4\,e^{\_Z}+c_1+8\_Z-4x\right)}x}{2} + 2\,e^{\mathrm{RootOf}\left(-x\,e^{2\_Z}+4x\,e^{\_Z}-4\,e^{\_Z}+c_1+8\_Z-4x\right)}$$

✓ Solution by Mathematica

Time used: 30.847 (sec). Leaf size: 90

```
DSolve[x (y'[x])^2+4 y'[x]-2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\left\{x = -\frac{2(2K[1]-y(K[1]))}{K[1]^2}, y(x) = 4\left(\frac{2}{K[1]}+\log(K[1])\right)\exp\left(-4\left(\frac{1}{2}\log(2-K[1])-\frac{1}{2}\log(K[1])\right)\right) + c_1\exp\left(-4\left(\frac{1}{2}\log(2-K[1])-\frac{1}{2}\log(K[1])\right)\right)\right\}, \{y(x), K[1]\}\right]$$

29.29 problem 851

Internal problem ID [3581]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 851.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy'^2 + y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 69

```
dsolve(x*diff(y(x),x)^2+x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = \left(\frac{1}{4 \operatorname{LambertW}\left(-\frac{1}{2\sqrt{\frac{c_1}{x}}}\right)^2} + \frac{1}{2 \operatorname{LambertW}\left(-\frac{1}{2\sqrt{\frac{c_1}{x}}}\right)}\right) x$$

$$y(x) = \left(\frac{1}{4 \operatorname{LambertW}\left(\frac{1}{2\sqrt{\frac{c_1}{x}}}\right)^2} + \frac{1}{2 \operatorname{LambertW}\left(\frac{1}{2\sqrt{\frac{c_1}{x}}}\right)}\right) x$$

✓ Solution by Mathematica

Time used: 0.563 (sec). Leaf size: 102

```
DSolve[x (y'[x])^2+x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{\sqrt{\frac{4y(x)}{x}+1}-1}-\log\left(\sqrt{\frac{4y(x)}{x}+1}-1\right)=\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{\sqrt{\frac{4y(x)}{x}+1}+1}+\log\left(\sqrt{\frac{4y(x)}{x}+1}+1\right)=-\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$y(x)\to 0$$

29.30 problem 852

Internal problem ID [3582]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 852.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$xy'^2 - (x^2 + 1)\, y' + x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve(x*diff(y(x),x)^2-(x^2+1)*diff(y(x),x)+x = 0,y(x), singsol=all)
```

$$y(x) = \ln(x) + c_1$$
$$y(x) = \frac{x^2}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 24

```
DSolve[x (y'[x])^2-(1+x^2)y'[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x^2}{2} + c_1$$
$$y(x) \to \log(x) + c_1$$

29.31 problem 853

Internal problem ID [3583]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 853.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _dAlembert]`

$$\boxed{xy'^2 + yy' + a = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 146

```
dsolve(x*diff(y(x),x)^2+y(x)*diff(y(x),x)+a = 0,y(x), singsol=all)
```

$$-\frac{c_1\left(\frac{-y(x)+\sqrt{-4ax+y(x)^2}}{x}\right)^{\frac{3}{2}}x^2}{\left(-y(x)+\sqrt{-4ax+y(x)^2}\right)^2}+x+\frac{4a\,x^2}{3\left(-y(x)+\sqrt{-4ax+y(x)^2}\right)^2}=0$$

$$\frac{\left(\frac{-2y(x)-2\sqrt{-4ax+y(x)^2}}{x}\right)^{\frac{3}{2}}x^2c_1}{\left(y(x)+\sqrt{-4ax+y(x)^2}\right)^2}+x+\frac{4a\,x^2}{3\left(y(x)+\sqrt{-4ax+y(x)^2}\right)^2}=0$$

✓ Solution by Mathematica

Time used: 60.288 (sec). Leaf size: 4845

```
DSolve[x (y'[x])^2+y[x] y'[x]+a==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

29.32 problem 854

Internal problem ID [3584]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 854.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Clairaut]`

$$\boxed{xy'^2 - yy' + a = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 33

```
dsolve(x*diff(y(x),x)^2-y(x)*diff(y(x),x)+a = 0,y(x), singsol=all)
```

$$y(x) = -2\sqrt{ax}$$
$$y(x) = 2\sqrt{ax}$$
$$y(x) = c_1x + \frac{a}{c_1}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 53

```
DSolve[x (y'[x])^2-y[x] y'[x]+a==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a}{c_1} + c_1x$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -2\sqrt{a}\sqrt{x}$$
$$y(x) \to 2\sqrt{a}\sqrt{x}$$

29.33 problem 855

Internal problem ID [3585]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 855.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy'^2 - yy' + ax = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 50

```
dsolve(x*diff(y(x),x)^2-y(x)*diff(y(x),x)+a*x = 0,y(x), singsol=all)
```

$$y(x) = \left(-a \,\mathrm{LambertW}\left(-\frac{x^2}{c_1^2 a}\right) + a\right) c_1 \sqrt{-\frac{x^2}{c_1^2 a \,\mathrm{LambertW}\left(-\frac{x^2}{c_1^2 a}\right)}}$$

✓ Solution by Mathematica

Time used: 0.906 (sec). Leaf size: 167

```
DSolve[x (y'[x])^2-y[x] y'[x]+a x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\frac{4a\arctan\left(\frac{y(x)}{x\sqrt{4a-\frac{y(x)^2}{x^2}}}\right) + \frac{y(x)\left(\sqrt{4a-\frac{y(x)^2}{x^2}} - \frac{iy(x)}{x}\right)}{x}}{8a} = \frac{1}{2} i \log(x) + c_1, y(x)\right]$$

$$\mathrm{Solve}\left[\frac{4a\arctan\left(\frac{y(x)}{x\sqrt{4a-\frac{y(x)^2}{x^2}}}\right) + \frac{y(x)\left(\sqrt{4a-\frac{y(x)^2}{x^2}} + \frac{iy(x)}{x}\right)}{x}}{8a} = c_1 - \frac{1}{2} i \log(x), y(x)\right]$$

29.34 problem 857

Internal problem ID [3586]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 857.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{xy'^2 + yy' + x^3 = 0}$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 337

```
dsolve(x*diff(y(x),x)^2+y(x)*diff(y(x),x)+x^3 = 0,y(x), singsol=all)
```

$$\int_{\_b}^{x} -\frac{-y(x)+\sqrt{-4\_a^4+y(x)^2}}{\left(-5y(x)+\sqrt{-4\_a^4+y(x)^2}\right)\_a}d\_a + \int^{y(x)} -\frac{2\left(8\left(\int_{\_b}^{x}\frac{\_a^3}{\left(-5\_f+\sqrt{-4\_a^4+\_f^2}\right)^2\sqrt{-4\_a^4+\_f^2}}d\_a\right)\sqrt{-4x^4+\_f^2} - 40\left(\int_{\_b}^{x}\frac{\_a^3}{\left(-5\_f+\sqrt{-4\_a^4+\_f^2}\right)^2\sqrt{-4\_a \cdots}} \cdots\right.\right.}{-5\_f+\sqrt{-4x^4+\_f^2} \cdots} + c_1 = 0$$

$$\int_{\_b}^{x} -\frac{y(x)+\sqrt{-4\_a^4+y(x)^2}}{\left(\sqrt{-4\_a^4+y(x)^2}+5y(x)\right)\_a}d\_a + \int^{y(x)} \frac{16\left(\int_{\_b}^{x}\frac{\_a^3}{\left(\sqrt{-4\_a^4+\_f^2}+5\_f\right)^2\sqrt{-4\_a^4+\_f^2}}d\_a\right)\sqrt{-4x^4+\_f^2} + 80\left(\int_{\_b}^{x}\frac{\_a^3}{\left(\sqrt{-4\_a^4+\_f^2}+5\_f\right)^2\sqrt{-4\_ \cdots}} \cdots\right.}{\sqrt{-4x^4+\_f^2}+5\_f \cdots} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.689 (sec). Leaf size: 107

```
DSolve[x (y'[x])^2+y[x] y'[x]+x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x^2\text{InverseFunction}\left[\int_1^{\#1} \frac{1}{5K[2]+\sqrt{K[2]^2-4}}dK[2]\&\right]\left[\int_1^x -\frac{1}{2K[3]}dK[3]+c_1\right]$$

$$y(x) \to x^2\text{InverseFunction}\left[\int_1^{\#1} \frac{1}{\sqrt{K[4]^2-4}-5K[4]}dK[4]\&\right]\left[\int_1^x \frac{1}{2K[5]}dK[5]+c_1\right]$$

29.35 problem 858

Internal problem ID [3587]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 29
Problem number: 858.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy'^2 - yy' + ay = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 55

```
dsolve(x*diff(y(x),x)^2-y(x)*diff(y(x),x)+a*y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = -\frac{\left(-\operatorname{LambertW}\left(-\frac{x\,\mathrm{e}}{c_1 a}\right)+1\right)^2 a^2 x}{-\left(-\operatorname{LambertW}\left(-\frac{x\,\mathrm{e}}{c_1 a}\right)+1\right)a+a}$$

✓ Solution by Mathematica

Time used: 2.805 (sec). Leaf size: 173

```
DSolve[x (y'[x])^2-y[x] y'[x]+a y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{-\sqrt{\frac{y(x)}{x}}\sqrt{\frac{y(x)}{x}-4a}-4a\log\left(\sqrt{\frac{y(x)}{x}-4a}-\sqrt{\frac{y(x)}{x}}\right)+\frac{y(x)}{x}}{4a} = -\frac{\log(x)}{2}+c_1, y(x)\right]$$

$$\text{Solve}\left[-\frac{\sqrt{\frac{y(x)}{x}}\sqrt{\frac{y(x)}{x}-4a}+4a\log\left(\sqrt{\frac{y(x)}{x}-4a}-\sqrt{\frac{y(x)}{x}}\right)+\frac{y(x)}{x}}{4a} = \frac{\log(x)}{2}+c_1, y(x)\right]$$

$$y(x) \to 0$$

30 Various 30

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30.1 problem 859

Internal problem ID [3588]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 859.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$xy'^2 + yy' - y^4 = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 99

```
dsolve(x*diff(y(x),x)^2+y(x)*diff(y(x),x)-y(x)^4 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{2\sqrt{-x}}$$

$$y(x) = \frac{1}{2\sqrt{-x}}$$

$$y(x) = 0$$

$$y(x) = -\frac{\sqrt{-x\left(\tanh\left(-\frac{\ln(x)}{2} + \frac{c_1}{2}\right)^2 - 1\right)}}{2x\tanh\left(-\frac{\ln(x)}{2} + \frac{c_1}{2}\right)}$$

$$y(x) = \frac{\sqrt{-x\left(\tanh\left(-\frac{\ln(x)}{2} + \frac{c_1}{2}\right)^2 - 1\right)}}{2x\tanh\left(-\frac{\ln(x)}{2} + \frac{c_1}{2}\right)}$$

✓ Solution by Mathematica

Time used: 0.526 (sec). Leaf size: 84

```
DSolve[x (y'[x])^2+y[x] y'[x]-y[x]^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2e^{\frac{c_1}{2}}}{-4x + e^{c_1}}$$

$$y(x) \to \frac{2e^{\frac{c_1}{2}}}{-4x + e^{c_1}}$$

$$y(x) \to 0$$

$$y(x) \to -\frac{i}{2\sqrt{x}}$$

$$y(x) \to \frac{i}{2\sqrt{x}}$$

30.2 problem 860

Internal problem ID [3589]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 860.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$\boxed{xy'^2 + (a - y)\, y' + b = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 41

```
dsolve(x*diff(y(x),x)^2+(a-y(x))*diff(y(x),x)+b = 0,y(x), singsol=all)
```

$$y(x) = a - 2\sqrt{xb}$$
$$y(x) = a + 2\sqrt{xb}$$
$$y(x) = c_1 x + \frac{a c_1 + b}{c_1}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 58

```
DSolve[x (y'[x])^2+(a-y[x])y'[x]+b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to a + \frac{b}{c_1} + c_1 x$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to a - 2\sqrt{b}\sqrt{x}$$
$$y(x) \to a + 2\sqrt{b}\sqrt{x}$$

30.3 problem 861

Internal problem ID [3590]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 861.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _dAlembert]`

$$xy'^2 + (x - y)\, y' + 1 - y = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 56

```
dsolve(x*diff(y(x),x)^2+(x-y(x))*diff(y(x),x)+1-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -x - 2\sqrt{x}$$
$$y(x) = -x + 2\sqrt{x}$$
$$y(x) = \frac{(-c_1^2 - c_1)\, x}{-c_1 - 1} - \frac{1}{-c_1 - 1}$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 46

```
DSolve[x (y'[x])^2+(x-y[x])y'[x]+1-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x + \frac{1}{1 + c_1}$$
$$y(x) \to -x - 2\sqrt{x}$$
$$y(x) \to 2\sqrt{x} - x$$

30.4 problem 862

Internal problem ID [3591]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 862.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _dAlembert]`

$$\boxed{xy'^2 + (a + x - y)\, y' - y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 63

```
dsolve(x*diff(y(x),x)^2+(a+x-y(x))*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = a - x - 2\sqrt{-ax}$$
$$y(x) = a - x + 2\sqrt{-ax}$$
$$y(x) = -\frac{(c_1^2 + c_1)\, x}{-c_1 - 1} - \frac{a c_1}{-c_1 - 1}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 60

```
DSolve[x (y'[x])^2+(a+x-y[x])y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1\left(x + \frac{a}{1 + c_1}\right)$$
$$y(x) \to \left(\sqrt{a} - i\sqrt{x}\right)^2$$
$$y(x) \to \left(\sqrt{a} + i\sqrt{x}\right)^2$$

30.5 problem 863

Internal problem ID [3592]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 863.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{xy'^2 - (-y + 3x)\,y' + y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 136

```
dsolve(x*diff(y(x),x)^2-(3*x-y(x))*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = x$$

$$-\frac{c_1\left(-y(x)+5x+\sqrt{9x^2-10y(x)\,x+y(x)^2}\right)}{x\left(\frac{-y(x)+3x+\sqrt{9x^2-10y(x)x+y(x)^2}}{x}\right)^{\frac{3}{2}}}+x=0$$

$$\frac{\left(y(x)-5x+\sqrt{9x^2-10y(x)\,x+y(x)^2}\right)c_1}{x\left(\frac{-2y(x)+6x-2\sqrt{9x^2-10y(x)x+y(x)^2}}{x}\right)^{\frac{3}{2}}}+x=0$$

✓ Solution by Mathematica

Time used: 60.311 (sec). Leaf size: 1225

```
DSolve[x (y'[x])^2-(3 x-y[x])y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{384}\left(\frac{\frac{4e^{8c_1}}{x^2}-6912e^{4c_1}}{\sqrt[3]{-\frac{-373248e^{4c_1}x^4+4320e^{8c_1}x^2-48\sqrt{6}x\sqrt{e^{8c_1}\left(216x^2+e^{4c_1}\right)^3}+e^{12c_1}}{x^3}}} + 4\sqrt[3]{\frac{373248e^{4c_1}x^4-4320e^{8c_1}x^2+48\sqrt{6}x\sqrt{e^{8c_1}\left(216x^2+e^{4c_1}\right)^3}-e^{12c_1}}{x^3}} - \frac{4e^{4c_1}}{x}\right)$$

$$y(x) \to \frac{1}{768}\left(\frac{\left(1+i\sqrt{3}\right)\left(6912e^{4c_1}-\frac{4e^{8c_1}}{x^2}\right)}{\sqrt[3]{-\frac{-373248e^{4c_1}x^4+4320e^{8c_1}x^2-48\sqrt{6}x\sqrt{e^{8c_1}\left(216x^2+e^{4c_1}\right)^3}+e^{12c_1}}{x^3}}} + 4i\left(\sqrt{3}+i\right)\sqrt[3]{-\frac{-373248e^{4c_1}x^4+4320e^{8c_1}x^2-48\sqrt{6}x\sqrt{e^{8c_1}\left(216x^2+e^{4c_1}\right)^3}+e^{12c_1}}{x^3}} - \frac{8e^{4c_1}}{x}\right)$$

$$y(x) \to \frac{1}{768}\left(\frac{\left(1-i\sqrt{3}\right)\left(6912e^{4c_1}-\frac{4e^{8c_1}}{x^2}\right)}{\sqrt[3]{-\frac{-373248e^{4c_1}x^4+4320e^{8c_1}x^2-48\sqrt{6}x\sqrt{e^{8c_1}\left(216x^2+e^{4c_1}\right)^3}+e^{12c_1}}{x^3}}} - 4\left(1+i\sqrt{3}\right)\sqrt[3]{-\frac{-373248e^{4c_1}x^4+4320e^{8c_1}x^2-48\sqrt{6}x\sqrt{e^{8c_1}\left(216x^2+e^{4c_1}\right)^3}+e^{12c_1}}{x^3}} - \frac{8e^{4c_1}}{x}\right)$$

$$y(x) \to \frac{1}{\ldots}\left(\frac{4-108e^{4c_1}x^2}{\ldots}\right.$$

30.6 problem 864

Internal problem ID [3593]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 864.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, _dAlembert]`

$$xy'^2 + a + bx - y - yb = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 59

```
dsolve(x*diff(y(x),x)^2+a+b*x-y(x)-b*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\left(\left(\operatorname{RootOf}\left(\_Z - \_Z^{\frac{1}{b}}\left(\frac{c_1}{x}\right)^{\frac{-1+b}{2b}} + 1 - b\right) + 1\right)^2 + b\right)x}{-b-1} - \frac{a}{-b-1}$$

✓ Solution by Mathematica

Time used: 90.22 (sec). Leaf size: 1197

```
DSolve[x (y'[x])^2+(a+b x-y[x])-b y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2(b+1)\left(-\log\left(\sqrt{-a+by(x)-bx+y(x)}+\sqrt{x}\right)+b\log\left(\sqrt{-a+by(x)-bx+y(x)}+b\sqrt{x}\right)+b\right.}{}\right.$$

$$-\frac{2(b+1)\left((b-1)\log\left(\sqrt{-a+by(x)+y(x)}\sqrt{-a+by(x)-bx+y(x)}+a-(b+1)y(x)\right)+\log\left(\sqrt{x}\sqrt{-\right.\right.}{}$$

$$\text{Solve}\left[\frac{2(b+1)\left(-\log\left(\sqrt{-a+by(x)-bx+y(x)}-\sqrt{x}\right)+b\log\left(\sqrt{-a+by(x)-bx+y(x)}-b\sqrt{x}\right)+b\right.}{}\right.$$

$$-\frac{2(b+1)\left((b-1)\log\left(\sqrt{-a+by(x)+y(x)}\sqrt{-a+by(x)-bx+y(x)}+a-(b+1)y(x)\right)+\log\left(-\sqrt{x}\sqrt{\right.\right.}{}$$

30.7 problem 865

Internal problem ID [3594]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 865.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _dAlembert]`

$$\boxed{xy'^2 - 2yy' + a = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 897

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)+a = 0,y(x), singsol=all)
```

$$y(x) = \frac{x\left(\frac{\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{1}{3}}}{6c_1}+\frac{2x^2}{3c_1\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{1}{3}}}+\frac{x}{3c_1}\right)}{2} + \frac{a}{\frac{\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{1}{3}}}{3c_1}+\frac{4x^2}{3c_1\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{1}{3}}}+\frac{2x}{3c_1}}$$

$$y(x) = \frac{x\left(-\frac{\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{1}{3}}}{12c_1}-\frac{x^2}{3c_1\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{1}{3}}}+\frac{x}{3c_1}-\frac{i\sqrt{3}\left(\frac{\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x\cdots\right.}\right.}{6c_1}\cdots\right.}{\cdots}\right.}{2} + \frac{a}{-\frac{\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{1}{3}}}{6c_1}-\frac{2x^2}{3c_1\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{1}{3}}}+\frac{2x}{3c_1}-i\sqrt{3}\left(\frac{\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a\cdots\right.}\right.}{6c_1}\cdots\right.}$$

$$y(x) = \frac{x\left(-\frac{\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{1}{3}}}{12c_1}-\frac{x^2}{3c_1\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{1}{3}}}+\frac{x}{3c_1}+\frac{i\sqrt{3}\left(\frac{\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x\cdots\right.}\right.}{6c_1}\cdots\right.}{\cdots}\right.}{2} + \frac{a}{-\frac{\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{1}{3}}}{6c_1}-\frac{2x^2}{3c_1\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a-4x^3\right)}\,c_1\right)^{\frac{1}{3}}}+\frac{2x}{3c_1}+i\sqrt{3}\left(\frac{\left(-36c_1^2a+8x^3+12\sqrt{a\left(9c_1^2a\cdots\right.}\right.}{6c_1}\cdots\right.}$$

✓ Solution by Mathematica

Time used: 60.162 (sec). Leaf size: 1550

```
DSolve[x (y'[x])^2-2 y[x] y'[x]+a==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-\frac{3c_1}{2}}\left(a^4x^4+\left(-a^6x^6+20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}+8e^{6c_1}\right)^{2/3}-a^2x^2\sqrt[3]{-a^6x^6+20a^3e^{3c_1}x^3\cdots}\right.}{4\sqrt[3]{-a^6x^6+20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}+8e^{6c_1}}}$$

$$y(x) \to \frac{ie^{-\frac{3c_1}{2}}\left(-\left(\left(\sqrt{3}-i\right)a^4x^4\right)+\left(\sqrt{3}+i\right)\left(-a^6x^6+20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}+8e^{6c_1}\right)^{2/3}+2ia\cdots\right.}{8\sqrt[3]{-a^6x^6+20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x\cdots\right.}}}$$

$$y(x) \to \frac{e^{-\frac{3c_1}{2}}\left(i\left(\sqrt{3}+i\right)a^4x^4+\left(-1-i\sqrt{3}\right)\left(-a^6x^6+20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}+8e^{6c_1}\right)^{2/3}-2a^2x\cdots\right.}{8\sqrt[3]{-a^6x^6+20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3\cdots\right.}}}$$

$$y(x) \to \frac{e^{-\frac{3c_1}{2}}\left(a^4x^4+\left(a^6x^6-20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}-8e^{6c_1}\right)^{2/3}+a^2x^2\sqrt[3]{a^6x^6-20a^3e^{3c_1}x^3+8\cdots}\right.}{4\sqrt[3]{a^6x^6-20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}-8e^{6c_1}}}$$

$$y(x) \to \frac{e^{-\frac{3c_1}{2}}\left(\left(-1-i\sqrt{3}\right)a^4x^4+i\left(\sqrt{3}+i\right)\left(a^6x^6-20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}-8e^{6c_1}\right)^{2/3}+2a^2x^2\cdots\right.}{8\sqrt[3]{a^6x^6-20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3\cdots\right.}}}$$

$$y(x) \to \frac{e^{-\frac{3c_1}{2}}\left(i\left(\sqrt{3}+i\right)a^4x^4+\left(-1-i\sqrt{3}\right)\left(a^6x^6-20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+e^{3c_1}\right)^3}-8e^{6c_1}\right)^{2/3}+2a^2x^2\cdots\right.}{8\sqrt[3]{a^6x^6-20a^3e^{3c_1}x^3+8\sqrt{e^{3c_1}\left(-a^3x^3+\cdots\right.}}}$$

30.8 problem 867

Internal problem ID [3595]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 867.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$xy'^2 - 2yy' + ax = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 33

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)+a*x = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{a}\, x$$
$$y(x) = -\sqrt{a}\, x$$
$$y(x) = \frac{\left(\frac{x^2}{c_1^2} + a\right) c_1}{2}$$

✓ Solution by Mathematica

Time used: 15.032 (sec). Leaf size: 400

```
DSolve[x (y'[x])^2-2 y[x] y'[x]+a x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{a}x\tan(c_1 - i\log(x))}{\sqrt{\sec^2(c_1 - i\log(x))}}$$

$$y(x) \to \frac{\sqrt{a}x\tan(c_1 - i\log(x))}{\sqrt{\sec^2(c_1 - i\log(x))}}$$

$$y(x) \to -\frac{\sqrt{a}x\tan(i\log(x) + c_1)}{\sqrt{\sec^2(i\log(x) + c_1)}}$$

$$y(x) \to \frac{\sqrt{a}x\tan(i\log(x) + c_1)}{\sqrt{\sec^2(i\log(x) + c_1)}}$$

$$y(x) \to -\sqrt{a}x$$

$$y(x) \to \sqrt{a}x$$

$$y(x) \to \frac{i\sqrt{a}e^{2i\text{Interval}[\{0,\pi\}]}\left(e^{2i\text{Interval}[\{0,\pi\}]}\sqrt{\frac{x^2e^{2i\text{Interval}[\{0,\pi\}]}}{\left(x^2+e^{2i\text{Interval}[\{0,\pi\}]}\right)^2}} - x^4\sqrt{\frac{x^2e^{2i\text{Interval}[\{0,\pi\}]}}{\left(x^2+e^{2i\text{Interval}[\{0,\pi\}]}\right)^2}}\right)}{2x}$$

$$y(x) \to \frac{i\sqrt{a}e^{2i\text{Interval}[\{0,\pi\}]}\left(x^4\sqrt{\frac{x^2e^{2i\text{Interval}[\{0,\pi\}]}}{\left(x^2+e^{2i\text{Interval}[\{0,\pi\}]}\right)^2}} - e^{2i\text{Interval}[\{0,\pi\}]}\sqrt{\frac{x^2e^{2i\text{Interval}[\{0,\pi\}]}}{\left(x^2+e^{2i\text{Interval}[\{0,\pi\}]}\right)^2}}\right)}{2x}$$

30.9 problem 868

Internal problem ID [3596]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 868.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy'^2 - 2yy' + x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 52

```
dsolve(x*diff(y(x),x)^2-2*y(x)*diff(y(x),x)+x+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \left(1 - \sqrt{2}\right) x$$
$$y(x) = \left(1 + \sqrt{2}\right) x$$
$$y(x) = -\frac{\left(\frac{(x+c_1)^2}{c_1^2} + 1\right) x}{-\frac{2(x+c_1)}{c_1} + 2}$$

✓ Solution by Mathematica

Time used: 0.234 (sec). Leaf size: 78

```
DSolve[x (y'[x])^2-2 y[x] y'[x]+x +2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2} e^{-c_1} x^2 + x - e^{c_1}$$
$$y(x) \to -e^{c_1} x^2 + x - \frac{e^{-c_1}}{2}$$
$$y(x) \to x - \sqrt{2} x$$
$$y(x) \to \left(1 + \sqrt{2}\right) x$$

30.10 problem 869

Internal problem ID [3597]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 869.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$xy'^2 - 3yy' + 9x^2 = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 51

```
dsolve(x*diff(y(x),x)^2-3*y(x)*diff(y(x),x)+9*x^2 = 0,y(x), singsol=all)
```

$$y(x) = -2x^{\frac{3}{2}}$$
$$y(x) = 2x^{\frac{3}{2}}$$
$$y(x) = \frac{4x^3 + c_1^2}{2c_1}$$
$$y(x) = \frac{c_1^2 x^3 + 4}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.29 (sec). Leaf size: 79

```
DSolve[x (y'[x])^2-3 y[x] y'[x]+9 x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}e^{-\frac{3c_1}{2}}\left(4x^3 + e^{3c_1}\right)$$
$$y(x) \to \frac{1}{2}e^{-\frac{3c_1}{2}}\left(4x^3 + e^{3c_1}\right)$$
$$y(x) \to -2x^{3/2}$$
$$y(x) \to 2x^{3/2}$$

30.11 problem 870

Internal problem ID [3598]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 870.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy'^2 - (2x + 3y)\,y' + 6y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x*diff(y(x),x)^2-(2*x+3*y(x))*diff(y(x),x)+6*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 x^3$$
$$y(x) = 2x + c_1$$

✓ Solution by Mathematica

Time used: 0.038 (sec). Leaf size: 26

```
DSolve[x (y'[x])^2-(2 x+3 y[x])y'[x]+6 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^3$$
$$y(x) \to 2x + c_1$$
$$y(x) \to 0$$

30.12 problem 871

Internal problem ID [3599]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 871.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _dAlembert]`

$$\boxed{xy'^2 - ayy' + b = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 646

```
dsolve(x*diff(y(x),x)^2-a*y(x)*diff(y(x),x)+b = 0,y(x), singsol=all)
```

$$\frac{c_1\left(2\left(\frac{ay(x)+\sqrt{a^2y(x)^2-4xb}}{2x}\right)^{\frac{1}{a-1}}a^3y(x)^2+2\left(\frac{ay(x)+\sqrt{a^2y(x)^2-4xb}}{2x}\right)^{\frac{1}{a-1}}\sqrt{a^2y\left(x\right)^2-4xb}\,a^2y(x)-\left(\frac{ay(x)+\sqrt{a^2y(x)^2-}}{2x}\right.\right.}{\left(ay\right.}$$

$$+x-\frac{4b\,x^2}{(2a-1)\left(ay\left(x\right)+\sqrt{a^2y\left(x\right)^2-4xb}\right)^2}=0$$

$$\frac{c_1\left(-2\left(-\frac{-ay(x)+\sqrt{a^2y(x)^2-4xb}}{2x}\right)^{\frac{1}{a-1}}a^3y(x)^2+2\left(-\frac{-ay(x)+\sqrt{a^2y(x)^2-4xb}}{2x}\right)^{\frac{1}{a-1}}\sqrt{a^2y\left(x\right)^2-4xb}\,a^2y(x)+\left(-\frac{-ay}{}\right.\right.}{}$$

$$+x-\frac{4b\,x^2}{(2a-1)\left(-ay\left(x\right)+\sqrt{a^2y\left(x\right)^2-4xb}\right)^2}=0$$

✓ Solution by Mathematica

Time used: 0.706 (sec). Leaf size: 143

```
DSolve[x (y'[x])^2-a y[x] y'[x]+b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2\left((a-1)\log\left(\sqrt{a^2y(x)^2-4bx}+(a-1)y(x)\right)+a\log\left(\sqrt{a^2y(x)^2-4bx}-ay(x)\right)\right)}{2a-1}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{2\left((a-1)\log\left(\sqrt{a^2y(x)^2-4bx}-ay(x)+y(x)\right)+a\log\left(\sqrt{a^2y(x)^2-4bx}+ay(x)\right)\right)}{2a-1}=c_1,y(x)\right]$$

30.13 problem 872

Internal problem ID [3600]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 872.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{xy'^2 + ayy' + bx = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 224

```
dsolve(x*diff(y(x),x)^2+a*y(x)*diff(y(x),x)+b*x = 0,y(x), singsol=all)
```

$$\frac{c_1 \left(\frac{a\left(a^2 y(x)^2+\sqrt{a^2 y(x)^2-4b\,x^2}\,a y(x)+a y(x)^2-2b\,x^2+\sqrt{a^2 y(x)^2-4b\,x^2}\,y(x)\right)}{2x^2}\right)^{-\frac{2+a}{2(a+1)}} \left(a y(x)+\sqrt{a^2 y\left(x\right)^2-4b\,x^2}\right)}{x} + x = 0$$

$$\frac{c_1 \left(-a y(x)+\sqrt{a^2 y\left(x\right)^2-4b\,x^2}\right) \left(-\frac{a\left(-a^2 y(x)^2+\sqrt{a^2 y(x)^2-4b\,x^2}\,a y(x)-a y(x)^2+2b\,x^2+\sqrt{a^2 y(x)^2-4b\,x^2}\,y(x)\right)}{2x^2}\right)^{-\frac{2+a}{2(a+1)}}}{x} + x = 0$$

✓ Solution by Mathematica

Time used: 2.046 (sec). Leaf size: 423

```
DSolve[x (y'[x])^2+a y[x] y'[x]+b x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{i\left(2\log\left(-i\sqrt{4b-\frac{a^2y(x)^2}{x^2}}+\frac{ay(x)}{x}+2i\sqrt{b}\right)+2(a+1)\log\left(i\sqrt{4b-\frac{a^2y(x)^2}{x^2}}+\frac{ay(x)}{x}-2i\sqrt{b}\right)-(a\right.}{4(a+1)}\right.$$

$$\left.-\frac{1}{2}i\log(x),y(x)\right]$$

$$\text{Solve}\left[\frac{i\left(2(a+1)\log\left(-i\sqrt{4b-\frac{a^2y(x)^2}{x^2}}+\frac{ay(x)}{x}+2i\sqrt{b}\right)+2\log\left(i\sqrt{4b-\frac{a^2y(x)^2}{x^2}}+\frac{ay(x)}{x}-2i\sqrt{b}\right)-(a+\right.}{4(a+1)}\right.$$

$$\left.+c_1,y(x)\right]$$

30.14 problem 873

Internal problem ID [3601]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 873.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy'^2 - (1 + yx)\, y' + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x*diff(y(x),x)^2-(1+x*y(x))*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \ln(x) + c_1$$
$$y(x) = c_1 e^x$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 20

```
DSolve[x (y'[x])^2-(1+x y[x])y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x$$
$$y(x) \to \log(x) + c_1$$

30.15 problem 874

Internal problem ID [3602]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 874.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy'^2 + (1-x)\,y'y - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 16

```
dsolve(x*diff(y(x),x)^2+(1-x)*y(x)*diff(y(x),x)-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x}$$
$$y(x) = c_1 \mathrm{e}^x$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 26

```
DSolve[x (y'[x])^2+(1-x)y[x] y'[x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x$$
$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to 0$$

30.16 problem 875

Internal problem ID [3603]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 875.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{xy'^2 + (1 - x^2 y)\, y' - yx = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

```
dsolve(x*diff(y(x),x)^2+(1-x^2*y(x))*diff(y(x),x)-x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\ln(x) + c_1$$
$$y(x) = \mathrm{e}^{\frac{x^2}{2}} c_1$$

✓ Solution by Mathematica

Time used: 0.011 (sec). Leaf size: 28

```
DSolve[x (y'[x])^2+(1-x^2 y[x])y'[x]-x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^{\frac{x^2}{2}}$$
$$y(x) \to -\log(x) + c_1$$

30.17 problem 876

Internal problem ID [3604]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 876.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, _dAlembert]`

$$(x+1)\,y'^2 - y = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 99

```
dsolve((1+x)*diff(y(x),x)^2 = y(x),y(x), singsol=all)
```

$$y(x) = 0$$
$$y(x) = \frac{\left(x+1+\sqrt{c_1x+c_1+x+1}\right)^2 x}{(x+1)^2} + \frac{\left(x+1+\sqrt{c_1x+c_1+x+1}\right)^2}{(x+1)^2}$$
$$y(x) = \frac{\left(-x-1+\sqrt{c_1x+c_1+x+1}\right)^2 x}{(x+1)^2} + \frac{\left(-x-1+\sqrt{c_1x+c_1+x+1}\right)^2}{(x+1)^2}$$

✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 57

```
DSolve[(1+x) (y'[x])^2==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x - c_1\sqrt{x+1} + 1 + \frac{c_1{}^2}{4}$$
$$y(x) \to x + c_1\sqrt{x+1} + 1 + \frac{c_1{}^2}{4}$$
$$y(x) \to 0$$

30.18 problem 877

Internal problem ID [3605]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 877.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _dAlembert]`

$$\boxed{(x+1)\,y'^2-(x+y)\,y'+y=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 59

```
dsolve((1+x)*diff(y(x),x)^2-(x+y(x))*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x)=x+2-2\sqrt{x+1}$$
$$y(x)=x+2+2\sqrt{x+1}$$
$$y(x)=\frac{(-c_1^2+c_1)\,x}{1-c_1}-\frac{c_1^2}{1-c_1}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 49

```
DSolve[(1+x) (y'[x])^2-(x+y[x])y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\left(x+1+\frac{1}{-1+c_1}\right)$$
$$y(x)\to x-2\sqrt{x+1}+2$$
$$y(x)\to x+2\sqrt{x+1}+2$$

30.19 problem 878

Internal problem ID [3606]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 878.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$(a-x)\,y'^2 + yy' - b = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 50

```
dsolve((a-x)*diff(y(x),x)^2+y(x)*diff(y(x),x)-b = 0,y(x), singsol=all)
```

$$y(x) = -2\sqrt{-ab+xb}$$
$$y(x) = 2\sqrt{-ab+xb}$$
$$y(x) = c_1x + \frac{-c_1^2a+b}{c_1}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 59

```
DSolve[(a-x) (y'[x])^2+y[x] y'[x]-b==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x-a) + \frac{b}{c_1}$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -2\sqrt{b(x-a)}$$
$$y(x) \to 2\sqrt{b(x-a)}$$

30.20 problem 880

Internal problem ID [3607]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 880.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, _dAlembert]`

$$2xy'^2 + (2x - y)\,y' + 1 - y = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 146

```
dsolve(2*x*diff(y(x),x)^2+(2*x-y(x))*diff(y(x),x)+1-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\left(-2\left(e^{\mathrm{RootOf}\left(-x\,e^{3\_Z}+2x\,e^{2\_Z}+e^{\_Z}c_1+\_Z\,e^{\_Z}-x\,e^{\_Z}+1\right)}-1\right)^2 - 2\,e^{\mathrm{RootOf}\left(-x\,e^{3\_Z}+2x\,e^{2\_Z}+e^{\_Z}c_1+\_Z\,e^{\_Z}-x\,e^{\_Z}+1\right)} + 2\right)e^{-\mathrm{RootOf}\left(-x\,e^{3\_Z}+2x\,e^{2\_Z}+e^{\_Z}c_1+\_Z\,e^{\_Z}-x\,e^{\_Z}+1\right)}x + e^{-\mathrm{RootOf}\left(-x\,e^{3\_Z}+2x\,e^{2\_Z}+e^{\_Z}c_1+\_Z\,e^{\_Z}-x\,e^{\_Z}+1\right)}$$

✓ Solution by Mathematica

Time used: 1.407 (sec). Leaf size: 49

```
DSolve[2 x (y'[x])^2+(2 x-y[x])y'[x]+1-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\left\{x = \frac{\frac{1}{K[1]+1} + \log(K[1]+1)}{K[1]^2} + \frac{c_1}{K[1]^2}, y(x) = 2xK[1] + \frac{1}{K[1]+1}\right\}, \{y(x), K[1]\}\right]$$

30.21 problem 881

Internal problem ID [3608]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 881.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$3xy'^2 - 6yy' + x + 2y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 40

```
dsolve(3*x*diff(y(x),x)^2-6*y(x)*diff(y(x),x)+x+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = x$$
$$y(x) = -\frac{x}{3}$$
$$y(x) = \frac{\left(-\frac{(x+c_1)^2}{3c_1^2} - 1\right)x}{-\frac{2(x+c_1)}{c_1} + 2}$$

✓ Solution by Mathematica

Time used: 0.31 (sec). Leaf size: 67

```
DSolve[3 x (y'[x])^2- 6 y[x] y'[x]+x +2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{3}\left(x - 2x\cosh\left(-\log(x) + \sqrt{3}c_1\right)\right)$$
$$y(x) \to \frac{1}{3}\left(x - 2x\cosh\left(\log(x) + \sqrt{3}c_1\right)\right)$$
$$y(x) \to -\frac{x}{3}$$
$$y(x) \to x$$

30.22 problem 882

Internal problem ID [3609]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 882.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$(1+3x)\,{y'}^2-3(2+y)\,y'+9=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 49

```
dsolve((1+3*x)*diff(y(x),x)^2-3*(2+y(x))*diff(y(x),x)+9 = 0,y(x), singsol=all)
```

$$y(x) = -2-2\sqrt{3x+1}$$
$$y(x) = -2+2\sqrt{3x+1}$$
$$y(x) = c_1x+\frac{c_1^2-6c_1+9}{3c_1}$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 60

```
DSolve[(1+3 x) (y'[x])^2-3(2+y[x])y'[x]+9==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to c_1\left(x+\frac{1}{3}\right)-2+\frac{3}{c_1}$$
$$y(x)\to \text{Indeterminate}$$
$$y(x)\to -2\left(\sqrt{3x+1}+1\right)$$
$$y(x)\to 2\left(\sqrt{3x+1}-1\right)$$

30.23 problem 883

Internal problem ID [3610]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 883.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, _dAlembert]`

$$(3x+5)\,y'^2-(3y+3)\,y'+y=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 745

```
dsolve((5+3*x)*diff(y(x),x)^2-(3+3*y(x))*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$\frac{\left(9\sqrt{-12y(x)\,x+9y(x)^2-2y(x)+9}-18x+27y(x)-3\right)\mathrm{e}^{-\frac{3\left(3+3y(x)+\sqrt{-12y(x)x+9y(x)^2-2y(x)+9}\right)}{2(5+3x)}}c_1}{5+3x}$$
$$+x$$
$$-\frac{\left(27\,\mathrm{Ei}_1\left(-\frac{3\left(3+3y(x)+\sqrt{-12y(x)x+9y(x)^2-2y(x)+9}\right)}{2(5+3x)}\right)\sqrt{-12y(x)\,x+9y(x)^2-2y(x)+9}-54\,\mathrm{Ei}_1\left(-\frac{3\left(3+3y\right.}{}\right.}{}$$
$$=0$$

$$\frac{\left(9\sqrt{-12y(x)\,x+9y(x)^2-2y(x)+9}+18x-27y(x)+3\right)\mathrm{e}^{\frac{-\frac{9}{2}-\frac{9y(x)}{2}+\frac{3\sqrt{-12y(x)x+9y(x)^2-2y(x)+9}}{2}}{5+3x}}c_1}{5+3x}$$
$$+x$$
$$+\frac{\left(27\,\mathrm{Ei}_1\left(\frac{-\frac{9}{2}-\frac{9y(x)}{2}+\frac{3\sqrt{-12y(x)x+9y(x)^2-2y(x)+9}}{2}}{5+3x}\right)\sqrt{-12y(x)\,x+9y(x)^2-2y(x)+9}+54\,\mathrm{Ei}_1\left(\frac{-\frac{9}{2}-\frac{9y(x)}{2}+\frac{3\sqrt{-12}}{}}{5}\right.}{}$$
$$=0$$

✓ Solution by Mathematica

Time used: 1.656 (sec). Leaf size: 106

```
DSolve[(5+3 x) (y'[x])^2-(3+3 y[x])y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x=\frac{e^{-3K[1]}(3K[1]-1)\left((9-27K[1])\,\text{ExpIntegralEi}(3K[1])+4e^{3K[1]}\right)}{9K[1]-3}+c_1e^{-3K[1]}(3K[1]-1),y(x)=\frac{3xK[1]^2}{3K[1]-1}+\frac{5K[1]^2-3K[1]}{3K[1]-1}\right\},\{y(x),K[1]\}\right]$$

30.24 problem 884

Internal problem ID [3611]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 884.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{4xy'^2 - (a - 3x)^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 30

```
dsolve(4*x*diff(y(x),x)^2 = (a-3*x)^2,y(x), singsol=all)
```

$$y(x) = -\sqrt{x}\,(-x + a) + c_1$$
$$y(x) = \sqrt{x}\,(-x + a) + c_1$$

✓ Solution by Mathematica

Time used: 0.021 (sec). Leaf size: 37

```
DSolve[4 x (y'[x])^2==(a-3 x)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}(a - x) + c_1$$
$$y(x) \to \sqrt{x}(x - a) + c_1$$

30.25 problem 885

Internal problem ID [3612]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 885.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$4xy'^2 + 2y'x - y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 51

```
dsolve(4*x*diff(y(x),x)^2+2*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x}{4}$$
$$y(x) = \left(\frac{4c_1}{x} + \frac{2\sqrt{c_1 x}}{x}\right)x$$
$$y(x) = \left(\frac{4c_1}{x} - \frac{2\sqrt{c_1 x}}{x}\right)x$$

✓ Solution by Mathematica

Time used: 0.12 (sec). Leaf size: 72

```
DSolve[4 x (y'[x])^2+2 x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{4}e^{2c_1}\left(-2\sqrt{x} + e^{2c_1}\right)$$
$$y(x) \to \frac{1}{4}e^{-4c_1}\left(1 + 2e^{2c_1}\sqrt{x}\right)$$
$$y(x) \to 0$$
$$y(x) \to -\frac{x}{4}$$

30.26 problem 886

Internal problem ID [3613]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 886.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G'], _rational, _dAlembert]`

$$\boxed{4x{y'}^2 - 3yy' + 3 = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 153

```
dsolve(4*x*diff(y(x),x)^2-3*y(x)*diff(y(x),x)+3 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2\sqrt{x\left(3+\sqrt{16c_1x+9}\right)}}{3} - \frac{2x}{\sqrt{x\left(3+\sqrt{16c_1x+9}\right)}}$$

$$y(x) = \frac{2\sqrt{x\left(3+\sqrt{16c_1x+9}\right)}}{3} + \frac{2x}{\sqrt{x\left(3+\sqrt{16c_1x+9}\right)}}$$

$$y(x) = -\frac{2\sqrt{-x\left(-3+\sqrt{16c_1x+9}\right)}}{3} - \frac{2x}{\sqrt{-x\left(-3+\sqrt{16c_1x+9}\right)}}$$

$$y(x) = \frac{2\sqrt{-x\left(-3+\sqrt{16c_1x+9}\right)}}{3} + \frac{2x}{\sqrt{-x\left(-3+\sqrt{16c_1x+9}\right)}}$$

✓ Solution by Mathematica

Time used: 27.438 (sec). Leaf size: 187

```
DSolve[4 x (y'[x])^2-3 y[x] y'[x]+3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{432x - e^{-\frac{c_1}{2}} \left(-144x + e^{c_1}\right)^{3/2} + e^{c_1}}}{6\sqrt{3}}$$

$$y(x) \to \frac{\sqrt{432x - e^{-\frac{c_1}{2}} \left(-144x + e^{c_1}\right)^{3/2} + e^{c_1}}}{6\sqrt{3}}$$

$$y(x) \to -\frac{\sqrt{432x + e^{-\frac{c_1}{2}} \left(-144x + e^{c_1}\right)^{3/2} + e^{c_1}}}{6\sqrt{3}}$$

$$y(x) \to \frac{\sqrt{432x + e^{-\frac{c_1}{2}} \left(-144x + e^{c_1}\right)^{3/2} + e^{c_1}}}{6\sqrt{3}}$$

30.27 problem 887

Internal problem ID [3614]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 887.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _dAlembert]`

$$\boxed{4xy'^2 + 4yy' - 1 = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 126

```
dsolve(4*x*diff(y(x),x)^2+4*y(x)*diff(y(x),x) = 1,y(x), singsol=all)
```

$$-\frac{c_1 \left(\frac{-y(x)+\sqrt{x+y(x)^2}}{x}\right)^{\frac{3}{2}} x^2}{\left(-y(x)+\sqrt{x+y(x)^2}\right)^2} + x - \frac{x^2}{3\left(-y(x)+\sqrt{x+y(x)^2}\right)^2} = 0$$

$$\frac{\left(\frac{-2y(x)-2\sqrt{x+y(x)^2}}{x}\right)^{\frac{3}{2}} x^2 c_1}{\left(y(x)+\sqrt{x+y(x)^2}\right)^2} + x - \frac{x^2}{3\left(y(x)+\sqrt{x+y(x)^2}\right)^2} = 0$$

✓ Solution by Mathematica

Time used: 60.259 (sec). Leaf size: 4057

```
DSolve[4 x (y'[x])^2+4 y[x] y'[x]==1,y[x],x,IncludeSingularSolutions -> True]
```

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30.28 problem 888

Internal problem ID [3615]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 888.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$4xy'^2 + 4yy' - y^4 = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 96

```
dsolve(4*x*diff(y(x),x)^2+4*y(x)*diff(y(x),x)-y(x)^4 = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{-x}}$$
$$y(x) = -\frac{1}{\sqrt{-x}}$$
$$y(x) = 0$$
$$y(x) = \frac{\sqrt{-x\left(\tanh\left(-\frac{\ln(x)}{2}+\frac{c_1}{2}\right)^2-1\right)}}{x\tanh\left(-\frac{\ln(x)}{2}+\frac{c_1}{2}\right)}$$
$$y(x) = -\frac{\sqrt{-x\left(\tanh\left(-\frac{\ln(x)}{2}+\frac{c_1}{2}\right)^2-1\right)}}{x\tanh\left(-\frac{\ln(x)}{2}+\frac{c_1}{2}\right)}$$

✓ Solution by Mathematica

Time used: 0.53 (sec). Leaf size: 80

```
DSolve[4 x (y'[x])^2+4 y[x] y'[x]-y[x]^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2e^{\frac{c_1}{2}}}{-x+e^{c_1}}$$
$$y(x) \to \frac{2e^{\frac{c_1}{2}}}{-x+e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{i}{\sqrt{x}}$$
$$y(x) \to \frac{i}{\sqrt{x}}$$

30.29 problem 889

Internal problem ID [3616]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 889.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$4(2-x)\,{y'}^{2}+1=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 23

```
dsolve(4*(2-x)*diff(y(x),x)^2+1 = 0,y(x), singsol=all)
```

$$y(x)=-\sqrt{x-2}+c_1$$
$$y(x)=\sqrt{x-2}+c_1$$

✓ Solution by Mathematica

Time used: 0.006 (sec). Leaf size: 31

```
DSolve[4(2-x) (y'[x])^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\sqrt{x-2}+c_1$$
$$y(x)\to \sqrt{x-2}+c_1$$

30.30 problem 890

Internal problem ID [3617]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 890.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$\boxed{16xy'^2 + 8yy' + y^6 = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 103

```
dsolve(16*x*diff(y(x),x)^2+8*y(x)*diff(y(x),x)+y(x)^6 = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{x^{\frac{1}{4}}}$$

$$y(x) = -\frac{1}{x^{\frac{1}{4}}}$$

$$y(x) = -\frac{i}{x^{\frac{1}{4}}}$$

$$y(x) = \frac{i}{x^{\frac{1}{4}}}$$

$$y(x) = 0$$

$$y(x) = \frac{\operatorname{RootOf}\left(-\ln\left(x\right) + c_1 + 4\left(\int^{\_Z} \frac{1}{\_a\sqrt{-\_a^4+1}}d\_a\right)\right)}{x^{\frac{1}{4}}}$$

$$y(x) = \frac{\operatorname{RootOf}\left(-\ln\left(x\right) + c_1 - 4\left(\int^{\_Z} \frac{1}{\_a\sqrt{-\_a^4+1}}d\_a\right)\right)}{x^{\frac{1}{4}}}$$

✓ Solution by Mathematica

Time used: 0.668 (sec). Leaf size: 171

```
DSolve[16 x(y'[x])^2+8 y[x] y'[x]+y[x]^6==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{2}e^{\frac{c_1}{4}}}{\sqrt{x+e^{c_1}}}$$
$$y(x) \to -\frac{i\sqrt{2}e^{\frac{c_1}{4}}}{\sqrt{x+e^{c_1}}}$$
$$y(x) \to \frac{i\sqrt{2}e^{\frac{c_1}{4}}}{\sqrt{x+e^{c_1}}}$$
$$y(x) \to \frac{\sqrt{2}e^{\frac{c_1}{4}}}{\sqrt{x+e^{c_1}}}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{1}{\sqrt[4]{x}}$$
$$y(x) \to -\frac{i}{\sqrt[4]{x}}$$
$$y(x) \to \frac{i}{\sqrt[4]{x}}$$
$$y(x) \to \frac{1}{\sqrt[4]{x}}$$

30.31 problem 891

Internal problem ID [3618]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 891.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x^2 y'^2 - a^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 20

```
dsolve(x^2*diff(y(x),x)^2 = a^2,y(x), singsol=all)
```

$$y(x) = a \ln(x) + c_1$$
$$y(x) = -a \ln(x) + c_1$$

✓ Solution by Mathematica

Time used: 0.003 (sec). Leaf size: 24

```
DSolve[x^2 (y'[x])^2==a^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -a \log(x) + c_1$$
$$y(x) \to a \log(x) + c_1$$

30.32 problem 892

Internal problem ID [3619]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 892.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{x^2 {y'}^2 - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x)^2 = y(x)^2,y(x), singsol=all)
```

$$y(x) = c_1 x$$
$$y(x) = \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.039 (sec). Leaf size: 24

```
DSolve[x^2 (y'[x])^2==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to c_1 x$$
$$y(x) \to 0$$

30.33 problem 893

Internal problem ID [3620]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 893.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$\boxed{x^2{y'}^2+x^2-y^2=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 44

```
dsolve(x^2*diff(y(x),x)^2+x^2-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{x(\text{LambertW}\left(-\mathrm{e}c_1x^4\right)-1)}{2\,\text{LambertW}\left(-\mathrm{e}c_1x^4\right)\sqrt{-\frac{1}{\text{LambertW}(-\mathrm{e}c_1x^4)}}}$$

✓ Solution by Mathematica

Time used: 2.678 (sec). Leaf size: 172

```
DSolve[x^2 (y'[x])^2+x^2-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{2}\left(-\frac{y(x)^2}{x^2}-\frac{\sqrt{\frac{y(x)}{x}-1}\sqrt{\frac{y(x)}{x}+1}y(x)}{x}-2\log\left(\sqrt{\frac{y(x)}{x}-1}-\sqrt{\frac{y(x)}{x}+1}\right)+1\right)=\log(x)+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{2}\left(\frac{y(x)^2}{x^2}-\frac{\sqrt{\frac{y(x)}{x}-1}\sqrt{\frac{y(x)}{x}+1}y(x)}{x}-2\log\left(\sqrt{\frac{y(x)}{x}-1}-\sqrt{\frac{y(x)}{x}+1}\right)-1\right)=-\log(x)+c_1,y(x)\right]$$

30.34 problem 894

Internal problem ID [3621]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 894.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_linear]`

$$\boxed{x^2 y'^2 - (x - y)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(x^2*diff(y(x),x)^2 = (x-y(x))^2,y(x), singsol=all)
```

$$y(x) = (-\ln(x) + c_1)\, x$$
$$y(x) = \frac{x}{2} + \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.07 (sec). Leaf size: 30

```
DSolve[x^2 (y'[x])^2==(x-y[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x}{2} + \frac{c_1}{x}$$
$$y(x) \to x(-\log(x) + c_1)$$

30.35 problem 895

Internal problem ID [3622]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 895.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$x^2 {y'}^2 + y^2 - y^4 = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 66

```
dsolve(x^2*diff(y(x),x)^2+y(x)^2-y(x)^4 = 0,y(x), singsol=all)
```

$$y(x) = -1$$
$$y(x) = 1$$
$$y(x) = 0$$
$$y(x) = \frac{\sqrt{\tan\left(-\ln\left(x\right)+c_1\right)^2+1}}{\tan\left(-\ln\left(x\right)+c_1\right)}$$
$$y(x) = -\frac{\sqrt{\tan\left(-\ln\left(x\right)+c_1\right)^2+1}}{\tan\left(-\ln\left(x\right)+c_1\right)}$$

✓ Solution by Mathematica

Time used: 1.547 (sec). Leaf size: 88

```
DSolve[x^2 (y'[x])^2+y[x]^2-y[x]^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{\sec^2(-\log(x) + c_1)}$$
$$y(x) \to \sqrt{\sec^2(-\log(x) + c_1)}$$
$$y(x) \to -\sqrt{\sec^2(\log(x) + c_1)}$$
$$y(x) \to \sqrt{\sec^2(\log(x) + c_1)}$$
$$y(x) \to -1$$
$$y(x) \to 0$$
$$y(x) \to 1$$

30.36 problem 896

Internal problem ID [3623]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 896.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$x^2{y'}^2 - y'x + y(1-y) = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)^2-x*diff(y(x),x)+y(x)*(1-y(x)) = 0,y(x), singsol=all)
```

$$y(x) = c_1 x$$
$$y(x) = \frac{x + c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.043 (sec). Leaf size: 31

```
DSolve[x^2 (y'[x])^2-x y'[x]+y[x](1-y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x$$
$$y(x) \to \frac{x + c_1}{x}$$
$$y(x) \to 0$$
$$y(x) \to 1$$

30.37 problem 897

Internal problem ID [3624]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 897.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$\boxed{x^2y'^2 + 2axy' + a^2 + x^2 - 2ay = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 86

```
dsolve(x^2*diff(y(x),x)^2+2*a*x*diff(y(x),x)+a^2+x^2-2*a*y(x) = 0,y(x), singsol=all)
```

$$y(x) - \mathrm{RootOf}\left(-a \operatorname{arcsinh}\left(\frac{\mathrm{RootOf}\left(-2ay(x) + a^2 + x^2 + 2a\_Z + \_Z^2\right)}{x}\right) - x\sqrt{-\frac{2a\,\mathrm{RootOf}\left(-2ay\left(x\right) + a^2 + x^2 + 2a\_Z + \_Z^2\right)}{x^2} - \frac{a^2}{x^2} + \frac{2a\_Z}{x^2} + c_1}\right) = 0$$

✓ Solution by Mathematica

Time used: 0.912 (sec). Leaf size: 82

```
DSolve[x^2 (y'[x])^2+2 a x y'[x]+a^2+x^2-2 a y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\left\{y(x) = \frac{2axK[1] + x^2K[1]^2 + a^2 + x^2}{2a}, x = \frac{a\log\left(\sqrt{K[1]^2+1} - K[1]\right)}{\sqrt{K[1]^2+1}} + \frac{c_1}{\sqrt{K[1]^2+1}}\right\}, \{y(x), K[1]\}\right]$$

30.38 problem 898

Internal problem ID [3625]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 898.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$x^2y'^2 - 2xy'y - x + y(1+y) = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 22

```
dsolve(x^2*diff(y(x),x)^2-2*x*diff(y(x),x)*y(x)-x+y(x)*(1+y(x)) = 0,y(x), singsol=all)
```

$$y(x) = x$$

$$y(x) = c_1\sqrt{x} - \frac{x\,c_1^2}{4} + x - 1$$

✓ Solution by Mathematica

Time used: 0.111 (sec). Leaf size: 55

```
DSolve[x^2 (y'[x])^2-2 x y[x] y'[x]-x+y[x](1+y[x])==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to x + \frac{{c_1}^2x}{4} - ic_1\sqrt{x} - 1$$

$$y(x) \to x + \frac{{c_1}^2x}{4} + ic_1\sqrt{x} - 1$$

30.39 problem 899

Internal problem ID [3626]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 30
Problem number: 899.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$x^2 {y'}^2 - 2xyy' - x^4 + (1 - x^2)\, y^2 = 0$$

✓ Solution by Maple

Time used: 0.234 (sec). Leaf size: 59

```
dsolve(x^2*diff(y(x),x)^2-2*x*diff(y(x),x)*y(x)-x^4+(-x^2+1)*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = -\frac{x\left(\frac{e^{2x}}{c_1^2} - 1\right) c_1 e^{-x}}{2}$$
$$y(x) = \frac{x(e^{2x} c_1^2 - 1)\, e^{-x}}{2c_1}$$

✓ Solution by Mathematica

Time used: 0.22 (sec). Leaf size: 26

```
DSolve[x^2 (y'[x])^2-2 x y[x] y'[x]-x^4+(1-x^2)y[x]^2==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to x \sinh(x + c_1)$$
$$y(x) \to -x \sinh(x - c_1)$$

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31.1 problem 900

Internal problem ID [3627]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 900.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$\boxed{x^2{y'}^2-(1+2yx)\,y'+1+y^2=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 42

```
dsolve(x^2*diff(y(x),x)^2-(1+2*x*y(x))*diff(y(x),x)+1+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x)=\frac{4x^2-1}{4x}$$
$$y(x)=c_1x-\sqrt{c_1-1}$$
$$y(x)=c_1x+\sqrt{c_1-1}$$

✓ Solution by Mathematica

Time used: 1.48 (sec). Leaf size: 62

```
DSolve[x^2 (y'[x])^2-(1+2 x y[x])y'[x]+1+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to x+e^{-2c_1}\left(x+e^{c_1}\right)$$
$$y(x)\to x+\frac{1}{4}e^{-2c_1}\left(x+2e^{c_1}\right)$$
$$y(x)\to x$$
$$y(x)\to x-\frac{1}{4x}$$

31.2 problem 901

Internal problem ID [3628]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 901.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational, _Clairaut]`

$$x^2y'^2 - (a + 2yx)\,y' + y^2 = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 36

```
dsolve(x^2*diff(y(x),x)^2-(a+2*x*y(x))*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{a}{4x}$$
$$y(x) = c_1x - \sqrt{ac_1}$$
$$y(x) = c_1x + \sqrt{ac_1}$$

✓ Solution by Mathematica

Time used: 0.347 (sec). Leaf size: 64

```
DSolve[x^2 (y'[x])^2-(a+2 x y[x])y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x - 2\sqrt{a}c_1}{4c_1{}^2}$$
$$y(x) \to \frac{x + 2\sqrt{a}c_1}{4c_1{}^2}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{a}{4x}$$

31.3 problem 902

Internal problem ID [3629]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 902.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{x^2 y'^2 - x(x-2y)\, y' + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 33

```
dsolve(x^2*diff(y(x),x)^2-x*(x-2*y(x))*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{x}{4}$$
$$y(x) = -\frac{c_1(c_1 - x)}{x}$$
$$y(x) = -\frac{c_1(x + c_1)}{x}$$

✓ Solution by Mathematica

Time used: 0.197 (sec). Leaf size: 64

```
DSolve[x^2 (y'[x])^2-x(x-2 y[x])y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{e^{-4c_1} - 2ie^{-2c_1}x}{4x}$$
$$y(x) \to \frac{2ie^{-2c_1}x + e^{-4c_1}}{4x}$$
$$y(x) \to 0$$

31.4 problem 903

Internal problem ID [3630]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 903.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$x^2y'^2 + 2x(2x + y)\,y' - 4a + y^2 = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 32

```
dsolve(x^2*diff(y(x),x)^2+2*x*(2*x+y(x))*diff(y(x),x)-4*a+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x^2 + a}{x}$$

$$y(x) = c_1 + \frac{\frac{c_1^2}{4} - a}{x}$$

✓ Solution by Mathematica

Time used: 1.242 (sec). Leaf size: 44

```
DSolve[x^2 (y'[x])^2+2 x(2 x+y[x])y'[x]-4 a+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{-a + c_1(-2x + c_1)}{x}$$

$$y(x) \to -2\sqrt{a}$$

$$y(x) \to 2\sqrt{a}$$

31.5 problem 904

Internal problem ID [3631]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 904.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x^2{y'}^2 + x\left(x^3 - 2y\right) y' - \left(2x^3 - y\right) y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 109

```
dsolve(x^2*diff(y(x),x)^2+x*(x^3-2*y(x))*diff(y(x),x)-(2*x^3-y(x))*y(x) = 0,y(x), singsol=all
```

$$y(x) = -\frac{x^3}{4}$$
$$y(x) = -\frac{x^3}{2} + x\,c_1^2 - \frac{(-x - 2c_1)\,x^2}{2}$$
$$y(x) = -\frac{x^3}{2} + x\,c_1^2 - \frac{(-x + 2c_1)\,x^2}{2}$$
$$y(x) = -\frac{x^3}{2} + x\,c_1^2 + \frac{(x - 2c_1)\,x^2}{2}$$
$$y(x) = -\frac{x^3}{2} + x\,c_1^2 + \frac{(x + 2c_1)\,x^2}{2}$$

✓ Solution by Mathematica

Time used: 1.743 (sec). Leaf size: 48

```
DSolve[x^2 (y'[x])^2+x(x^3-2 y[x])y'[x]-(2 x^3-y[x])y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to -e^{c_1}x(e^{c_1} - ix)$$
$$y(x) \to -e^{c_1}x(ix + e^{c_1})$$
$$y(x) \to 0$$

31.6 problem 905

Internal problem ID [3632]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 905.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$x^2 {y'}^2 + 3xyy' + 2y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)^2+3*x*diff(y(x),x)*y(x)+2*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x}$$
$$y(x) = \frac{c_1}{x^2}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 26

```
DSolve[x^2 (y'[x])^2+3 x y[x] y'[x]+2 y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^2}$$
$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to 0$$

31.7 problem 906

Internal problem ID [3633]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 906.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x^2{y'}^2 - 3xyy' + x^3 + 2y^2 = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 49

```
dsolve(x^2*diff(y(x),x)^2-3*x*diff(y(x),x)*y(x)+x^3+2*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -2x^{\frac{3}{2}}$$
$$y(x) = 2x^{\frac{3}{2}}$$
$$y(x) = \frac{x(c_1^2 + 4x)}{2c_1}$$
$$y(x) = \frac{x(x\,c_1^2 + 4)}{2c_1}$$

✓ Solution by Mathematica

Time used: 60.278 (sec). Leaf size: 961

```
DSolve[x^2 (y'[x])^2-3 x y[x] y'[x]+x^3+2 y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to \frac{e^{-\frac{3c_1}{2}}\left(2\sqrt[3]{2}e^{3c_1}x^3+\left(-4e^{3c_1}x^6-e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}\right)^{2/3}\right)}{2^{2/3}\sqrt[3]{-4e^{3c_1}x^6-e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}}}$$

$$y(x)\to \frac{ie^{-\frac{3c_1}{2}}\left(\left(\sqrt{3}+i\right)\left(-4e^{3c_1}x^6-e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}\right)^{2/3}-2\sqrt[3]{2}\left(\sqrt{3}-i\right)e^{3c_1}x^3\right)}{2\ 2^{2/3}\sqrt[3]{-4e^{3c_1}x^6-e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}}}$$

$$y(x)\to -\frac{ie^{-\frac{3c_1}{2}}\left(\left(\sqrt{3}-i\right)\left(-4e^{3c_1}x^6-e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}\right)^{2/3}-2\sqrt[3]{2}\left(\sqrt{3}+i\right)e^{3c_1}x^3\right)}{2\ 2^{2/3}\sqrt[3]{-4e^{3c_1}x^6-e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}}}$$

$$y(x)\to \frac{e^{-\frac{3c_1}{2}}\left(2\sqrt[3]{2}e^{3c_1}x^3+\left(4e^{3c_1}x^6+e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}\right)^{2/3}\right)}{2^{2/3}\sqrt[3]{4e^{3c_1}x^6+e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}}}$$

$$y(x)\to \frac{ie^{-\frac{3c_1}{2}}\left(\left(\sqrt{3}+i\right)\left(4e^{3c_1}x^6+e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}\right)^{2/3}-2\sqrt[3]{2}\left(\sqrt{3}-i\right)e^{3c_1}x^3\right)}{2\ 2^{2/3}\sqrt[3]{4e^{3c_1}x^6+e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}}}$$

$$y(x)\to -\frac{ie^{-\frac{3c_1}{2}}\left(\left(\sqrt{3}-i\right)\left(4e^{3c_1}x^6+e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}\right)^{2/3}-2\sqrt[3]{2}\left(\sqrt{3}+i\right)e^{3c_1}x^3\right)}{2\ 2^{2/3}\sqrt[3]{4e^{3c_1}x^6+e^{6c_1}x^3+\sqrt{e^{6c_1}x^6\left(-4x^3+e^{3c_1}\right)^2}}}$$

31.8 problem 907

Internal problem ID [3634]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 907.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$x^2{y'}^2 + 4xyy' - 5y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

```
dsolve(x^2*diff(y(x),x)^2+4*x*diff(y(x),x)*y(x)-5*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = c_1 x$$
$$y(x) = \frac{c_1}{x^5}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 24

```
DSolve[x^2 (y'[x])^2+4 x y[x] y'[x]-5 y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{x^5}$$
$$y(x) \to c_1 x$$
$$y(x) \to 0$$

31.9 problem 908

Internal problem ID [3635]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 908.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$x^2 {y'}^2 - 4x(y+2)\, y' + 4(y+2)\, y = 0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 122

```
dsolve(x^2*diff(y(x),x)^2-4*x*(2+y(x))*diff(y(x),x)+4*(2+y(x))*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -2$$

$$y(x) = \frac{\left(-\frac{2\sqrt{2}\,\sqrt{c_1 x^2}}{x^2} + 1\right) x^2}{c_1}$$

$$y(x) = \frac{\left(\frac{2\sqrt{2}\,\sqrt{c_1 x^2}}{x^2} + 1\right) x^2}{c_1}$$

$$y(x) = -\frac{-2c_1\left(-\sqrt{2}\,x + 4c_1\right) + 8c_1^2 - x^2}{c_1^2}$$

$$y(x) = -\frac{-2c_1\left(\sqrt{2}\,x + 4c_1\right) + 8c_1^2 - x^2}{c_1^2}$$

✓ Solution by Mathematica

Time used: 0.216 (sec). Leaf size: 69

```
DSolve[x^2 (y'[x])^2-4 x(2+y[x])y'[x]+4(2+y[x])y[x]==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to e^{-c_1} x \left(x - 2\sqrt{2} e^{\frac{c_1}{2}} \right)$$

$$y(x) \to e^{c_1} x^2 - 2\sqrt{2} e^{\frac{c_1}{2}} x$$

$$y(x) \to -2$$

$$y(x) \to 0$$

31.10 problem 909

Internal problem ID [3636]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 909.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$x^2 y'^2 - 5xyy' + 6y^2 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

```
dsolve(x^2*diff(y(x),x)^2-5*x*y(x)*diff(y(x),x)+6*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = c_1 x^3$$
$$y(x) = c_1 x^2$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 26

```
DSolve[x^2 (y'[x])^2-5 x y[x] y'[x]+6 y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x^2$$
$$y(x) \to c_1 x^3$$
$$y(x) \to 0$$

31.11 problem 910

Internal problem ID [3637]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 910.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$x^2 {y'}^2 + x(x^2 + yx - 2y)\, y' + (1 - x)(x^2 - y)\, y = 0$$

✗ Solution by Maple

```
dsolve(x^2*diff(y(x),x)^2+x*(x^2+x*y(x)-2*y(x))*diff(y(x),x)+(1-x)*(x^2-y(x))*y(x) = 0,y(x),
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2 (y'[x])^2+x(x^2+x y[x]-2 y[x])y'[x]+(1-x)(x^2-y[x])y[x]==0,y[x],x,IncludeSingularS
```

Not solved

31.12 problem 911

Internal problem ID [3638]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 911.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$x^2{y'}^2 + (2x + y)\, yy' + y^2 = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 125

```
dsolve(x^2*diff(y(x),x)^2+(2*x+y(x))*y(x)*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -4x$$
$$y(x) = 0$$
$$y(x) = \frac{2c_1^2(\sqrt{2}\, c_1 - x)}{2c_1^2 - x^2}$$
$$y(x) = -\frac{2c_1^2(\sqrt{2}\, c_1 + x)}{2c_1^2 - x^2}$$
$$y(x) = \frac{c_1^2(\sqrt{2}\, c_1 - 2x)}{2c_1^2 - 4x^2}$$
$$y(x) = -\frac{c_1^2(\sqrt{2}\, c_1 + 2x)}{2\,(c_1^2 - 2x^2)}$$

✓ Solution by Mathematica

Time used: 0.642 (sec). Leaf size: 63

```
DSolve[x^2 (y'[x])^2+(2 x+y[x])y[x] y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{4c_1}}{-x + e^{2c_1}}$$

$$y(x) \to \frac{e^{4c_1}}{4\left(4x + e^{2c_1}\right)}$$

$$y(x) \to 0$$

$$y(x) \to -4x$$

31.13 problem 912

Internal problem ID [3639]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 912.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$x^2y'^2 + (2x - y)\, yy' + y^2 = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 125

```
dsolve(x^2*diff(y(x),x)^2+(2*x-y(x))*y(x)*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = 4x$$
$$y(x) = 0$$
$$y(x) = -\frac{2c_1^2(\sqrt{2}\, c_1 - x)}{2c_1^2 - x^2}$$
$$y(x) = \frac{2c_1^2(\sqrt{2}\, c_1 + x)}{2c_1^2 - x^2}$$
$$y(x) = -\frac{c_1^2(\sqrt{2}\, c_1 - 2x)}{2\,(c_1^2 - 2x^2)}$$
$$y(x) = \frac{c_1^2(\sqrt{2}\, c_1 + 2x)}{2c_1^2 - 4x^2}$$

✓ Solution by Mathematica

Time used: 0.641 (sec). Leaf size: 61

```
DSolve[x^2 (y'[x])^2+(2 x-y[x])y[x] y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{4e^{-2c_1}}{2 + e^{2c_1}x}$$
$$y(x) \to \frac{1}{-4e^{4c_1}x - 2e^{2c_1}}$$
$$y(x) \to 0$$
$$y(x) \to 4x$$

31.14 problem 913

Internal problem ID [3640]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 913.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$x^2 {y'}^2 + (a + b\,x^2 y^3)\, y' + aby^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(x^2*diff(y(x),x)^2+(a+b*x^2*y(x)^3)*diff(y(x),x)+a*b*y(x)^3 = 0,y(x), singsol=all)
```

$$y(x) = \frac{a}{x} + c_1$$
$$y(x) = \frac{1}{\sqrt{2xb + c_1}}$$
$$y(x) = -\frac{1}{\sqrt{2xb + c_1}}$$

✓ Solution by Mathematica

Time used: 0.065 (sec). Leaf size: 49

```
DSolve[x^2 (y'[x])^2+(a+b x^2 y[x]^3)y'[x]+a b y[x]^3==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\frac{1}{\sqrt{2bx - 2c_1}}$$
$$y(x) \to \frac{1}{\sqrt{2bx - 2c_1}}$$
$$y(x) \to \frac{a}{x} + c_1$$

31.15 problem 914

Internal problem ID [3641]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 914.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{(1-x^2)\,y'^2-1+y^2=0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 166

```
dsolve((-x^2+1)*diff(y(x),x)^2 = 1-y(x)^2,y(x), singsol=all)
```

$$y(x)=-1$$
$$y(x)=1$$
$$\frac{\sqrt{(y(x)-1)(y(x)+1)}\,\ln\left(y(x)+\sqrt{y(x)^2-1}\right)}{\sqrt{y(x)-1}\,\sqrt{y(x)+1}}+\int^x-\frac{\sqrt{(\_a^2-1)\left(y(x)^2-1\right)}}{(\_a^2-1)\sqrt{y(x)-1}\,\sqrt{y(x)+1}}d\_a+c_1=0$$
$$\frac{\sqrt{(y(x)-1)(y(x)+1)}\,\ln\left(y(x)+\sqrt{y(x)^2-1}\right)}{\sqrt{y(x)-1}\,\sqrt{y(x)+1}}+\int^x\frac{\sqrt{(\_a^2-1)\left(y(x)^2-1\right)}}{(\_a^2-1)\sqrt{y(x)-1}\,\sqrt{y(x)+1}}d\_a+c_1=0$$

✓ Solution by Mathematica

Time used: 5.147 (sec). Leaf size: 218

```
DSolve[(1-x^2) (y'[x])^2==1-y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{-c_1}\sqrt{e^{2c_1}\left((2x^2-1)\cosh(2c_1)+2x\sqrt{x^2-1}\sinh(2c_1)+1\right)}}{\sqrt{2}}$$

$$y(x) \to \frac{e^{-c_1}\sqrt{e^{2c_1}\left((2x^2-1)\cosh(2c_1)+2x\sqrt{x^2-1}\sinh(2c_1)+1\right)}}{\sqrt{2}}$$

$$y(x) \to -\frac{1}{2}\sqrt{(4x^2-2)\cosh(2c_1)-4x\sqrt{x^2-1}\sinh(2c_1)+2}$$

$$y(x) \to \frac{1}{2}\sqrt{(4x^2-2)\cosh(2c_1)-4x\sqrt{x^2-1}\sinh(2c_1)+2}$$

$$y(x) \to -1$$

$$y(x) \to 1$$

31.16 problem 915

Internal problem ID [3642]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 915.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$(1-x^2)\, y'^2 + 2xyy' + 4x^2 = 0$$

✓ Solution by Maple

Time used: 0.282 (sec). Leaf size: 46

```
dsolve((-x^2+1)*diff(y(x),x)^2+2*x*y(x)*diff(y(x),x)+4*x^2 = 0,y(x), singsol=all)
```

$$y(x) = -2\sqrt{-x^2+1}$$
$$y(x) = 2\sqrt{-x^2+1}$$
$$y(x) = -c_1 + c_1 x^2 - \frac{1}{c_1}$$

✓ Solution by Mathematica

Time used: 0.348 (sec). Leaf size: 63

```
DSolve[(1-x^2) (y'[x])^2+2 x y[x] y'[x]+4 x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{-4x^2 + 4 + c_1^2}{2c_1}$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -2\sqrt{1-x^2}$$
$$y(x) \to 2\sqrt{1-x^2}$$

31.17 problem 916

Internal problem ID [3643]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 916.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$(a^2 + x^2)\, y'^2 - b^2 = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 40

```
dsolve((a^2+x^2)*diff(y(x),x)^2 = b^2,y(x), singsol=all)
```

$$y(x) = b \ln\left(x + \sqrt{a^2 + x^2}\right) + c_1$$
$$y(x) = -b \ln\left(x + \sqrt{a^2 + x^2}\right) + c_1$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 48

```
DSolve[(a^2+x^2) (y'[x])^2==b^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -b\operatorname{arctanh}\left(\frac{x}{\sqrt{a^2 + x^2}}\right) + c_1$$
$$y(x) \to b\operatorname{arctanh}\left(\frac{x}{\sqrt{a^2 + x^2}}\right) + c_1$$

31.18 problem 917

Internal problem ID [3644]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 917.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(a^2 - x^2)\, y'^2 + b^2 = 0}$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 44

```
dsolve((a^2-x^2)*diff(y(x),x)^2+b^2 = 0,y(x), singsol=all)
```

$$y(x) = b\ln\left(x + \sqrt{-a^2 + x^2}\right) + c_1$$

$$y(x) = -b\ln\left(x + \sqrt{-a^2 + x^2}\right) + c_1$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 52

```
DSolve[(a^2-x^2) (y'[x])^2+b^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -b\,\text{arctanh}\left(\frac{x}{\sqrt{x^2 - a^2}}\right) + c_1$$

$$y(x) \to b\,\text{arctanh}\left(\frac{x}{\sqrt{x^2 - a^2}}\right) + c_1$$

31.19 problem 918

Internal problem ID [3645]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 918.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(a^2 - x^2)\, y'^2 - b^2 = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 44

```
dsolve((a^2-x^2)*diff(y(x),x)^2 = b^2,y(x), singsol=all)
```

$$y(x) = b \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$
$$y(x) = -b \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 52

```
DSolve[(a^2-x^2) (y'[x])^2==b^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -b \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$
$$y(x) \to b \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$

31.20 problem 919

Internal problem ID [3646]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 919.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$(a^2 - x^2)\, y'^2 - x^2 = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 52

```
dsolve((a^2-x^2)*diff(y(x),x)^2 = x^2,y(x), singsol=all)
```

$$y(x) = -\frac{(-x+a)\,(x+a)}{\sqrt{(-x+a)\,(x+a)}} + c_1$$
$$y(x) = \frac{(-x+a)\,(x+a)}{\sqrt{(-x+a)\,(x+a)}} + c_1$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 43

```
DSolve[(a^2-x^2) (y'[x])^2==x^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{(a-x)(a+x)} + c_1$$
$$y(x) \to \sqrt{(a-x)(a+x)} + c_1$$

31.21 problem 920

Internal problem ID [3647]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 920.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{(a^2 - x^2)\, y'^2 + 2xyy' + x^2 = 0}$$

✓ Solution by Maple

Time used: 0.359 (sec). Leaf size: 51

```
dsolve((a^2-x^2)*diff(y(x),x)^2+2*x*y(x)*diff(y(x),x)+x^2 = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{a^2 - x^2}$$
$$y(x) = -\sqrt{a^2 - x^2}$$
$$y(x) = c_1 x^2 - c_1 a^2 - \frac{1}{4c_1}$$

✓ Solution by Mathematica

Time used: 0.415 (sec). Leaf size: 67

```
DSolve[(a^2-x^2) (y'[x])^2+2 x y[x] y'[x]+x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a^2 - x^2 + c_1{}^2}{2c_1}$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -\sqrt{(a-x)(a+x)}$$
$$y(x) \to \sqrt{(a-x)(a+x)}$$

31.22 problem 921

Internal problem ID [3648]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 921.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{(a^2 - x^2)\, y'^2 - 2xyy' - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve((a^2-x^2)*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{-x + a}$$
$$y(x) = \frac{c_1}{x + a}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 32

```
DSolve[(a^2-x^2) (y'[x])^2-2 x y[x] y'[x]-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1}{a - x}$$
$$y(x) \to \frac{c_1}{a + x}$$
$$y(x) \to 0$$

31.23 problem 922

Internal problem ID [3649]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 922.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$\boxed{(a^2+x^2)\, y'^2 - 2xy'y + b + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 90

```
dsolve((a^2+x^2)*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)+b+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{\sqrt{-a^2b - b\,x^2}}{a}$$
$$y(x) = -\frac{\sqrt{-a^2b - b\,x^2}}{a}$$
$$y(x) = c_1x - \sqrt{-a^2c_1^2 - b}$$
$$y(x) = c_1x + \sqrt{-a^2c_1^2 - b}$$

✓ Solution by Mathematica

Time used: 0.417 (sec). Leaf size: 100

```
DSolve[(a^2+x^2) (y'[x])^2-2 x y[x] y'[x]+b+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to c_1x - \sqrt{-b - a^2{c_1}^2}$$
$$y(x) \to \sqrt{-b - a^2{c_1}^2} + c_1x$$
$$y(x) \to -\frac{\sqrt{-b\,(a^2+x^2)}}{a}$$
$$y(x) \to \frac{\sqrt{-b\,(a^2+x^2)}}{a}$$

31.24 problem 924

Internal problem ID [3650]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 924.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_linear]`

$$4x^2{y'}^2 - 4xyy' - 8x^3 + y^2 = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 30

```
dsolve(4*x^2*diff(y(x),x)^2-4*x*y(x)*diff(y(x),x) = 8*x^3-y(x)^2,y(x), singsol=all)
```

$$y(x) = \left(-\sqrt{2}\,x + c_1\right)\sqrt{x}$$
$$y(x) = \left(\sqrt{2}\,x + c_1\right)\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.079 (sec). Leaf size: 42

```
DSolve[4 x^2 (y'[x])^2-4 x y[x] y'[x]==8 x^3 -y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{x}\left(-\sqrt{2}x + c_1\right)$$
$$y(x) \to \sqrt{x}\left(\sqrt{2}x + c_1\right)$$

31.25 problem 925

Internal problem ID [3651]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 925.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$a\,x^2 {y'}^2 - 2axyy' + a(-a+1)\,x^2 + y^2 = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 138

```
dsolve(a*x^2*diff(y(x),x)^2-2*a*x*y(x)*diff(y(x),x)+a*(1-a)*x^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = x\sqrt{-a}$$

$$y(x) = -x\sqrt{-a}$$

$$y(x) = \operatorname{RootOf}\left(-\ln(x) - \left(\int^{\_Z} \frac{\sqrt{(a\_a^2 - \_a^2 + a^2 - a)\,a}}{a\_a^2 - \_a^2 + a^2 - a} d\_a\right) + c_1\right) x$$

$$y(x) = \operatorname{RootOf}\left(-\ln(x) + \int^{\_Z} \frac{\sqrt{(a\_a^2 - \_a^2 + a^2 - a)\,a}}{a\_a^2 - \_a^2 + a^2 - a} d\_a + c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.615 (sec). Leaf size: 241

```
DSolve[a x^2 (y'[x])^2-2 a x y[x] y'[x]+a(1-a)x^2+y[x]^2==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(x^{2\sqrt{\frac{a-1}{a}}} - e^{2c_1}\right)$$

$$y(x) \to \frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(-x^{2\sqrt{\frac{a-1}{a}}} + e^{2c_1}\right)$$

$$y(x) \to -\frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(-1 + e^{2c_1}x^{2\sqrt{\frac{a-1}{a}}}\right)$$

$$y(x) \to \frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(-1 + e^{2c_1}x^{2\sqrt{\frac{a-1}{a}}}\right)$$

31.26 problem 926

Internal problem ID [3652]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 926.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\left(-a^2+1\right)x^2y'^2-2xyy'-a^2x^2+y^2=0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 229

```
dsolve((-a^2+1)*x^2*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)-a^2*x^2+y(x)^2 = 0,y(x), singsol=all
```

$$\ln(x)-\frac{\sqrt{-a^2}\,\arctan\left(\frac{a^2y(x)}{\sqrt{-a^2}\,\sqrt{-\frac{a^2x^2-x^2-y(x)^2}{x^2}}\,x}\right)}{a}+\frac{\ln\left(\frac{x^2+y(x)^2}{x^2}\right)}{2}+\frac{\ln\left(\frac{\sqrt{\frac{-a^2x^2+x^2+y(x)^2}{x^2}}\,x+y(x)}{x}\right)}{a}-c_1=0$$

$$\ln(x)+\frac{\sqrt{-a^2}\,\arctan\left(\frac{a^2y(x)}{\sqrt{-a^2}\,\sqrt{-\frac{a^2x^2-x^2-y(x)^2}{x^2}}\,x}\right)}{a}+\frac{\ln\left(\frac{x^2+y(x)^2}{x^2}\right)}{2}-\frac{\ln\left(\frac{\sqrt{\frac{-a^2x^2+x^2+y(x)^2}{x^2}}\,x+y(x)}{x}\right)}{a}-c_1=0$$

✓ Solution by Mathematica

Time used: 1.017 (sec). Leaf size: 223

```
DSolve[(1-a^2)x^2 (y'[x])^2-2 x y[x] y'[x]-a^2 x^2 + y[x]^2==0,y[x],x,IncludeSingularSolution
```

$$\text{Solve}\left[\frac{2i\arctan\left(\frac{y(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)-2ia\arctan\left(\frac{ay(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+a\log\left(\frac{y(x)^2}{x^2}+1\right)}{2a^2-2}=\frac{a\log\left(x-a^2x\right)}{1-a^2}\right.$$
$$\left.+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{-2i\arctan\left(\frac{y(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+2ia\arctan\left(\frac{ay(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+a\log\left(\frac{y(x)^2}{x^2}+1\right)}{2a^2-2}=\frac{a\log\left(x-a^2x\right)}{1-a^2}\right.$$
$$\left.+c_1,y(x)\right]$$

31.27 problem 927

Internal problem ID [3653]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 927.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x^3 {y'}^2 - a = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 31

```
dsolve(x^3*diff(y(x),x)^2 = a,y(x), singsol=all)
```

$$y(x) = -\frac{2\sqrt{ax}}{x} + c_1$$
$$y(x) = \frac{2\sqrt{ax}}{x} + c_1$$

✓ Solution by Mathematica

Time used: 0.004 (sec). Leaf size: 39

```
DSolve[x^3 (y'[x])^2==a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2\sqrt{a}}{\sqrt{x}} + c_1$$
$$y(x) \to \frac{2\sqrt{a}}{\sqrt{x}} + c_1$$

31.28 problem 928

Internal problem ID [3654]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 928.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x^3 {y'}^2 + y'x - y = 0$$

✗ Solution by Maple

```
dsolve(x^3*diff(y(x),x)^2+x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 105.518 (sec). Leaf size: 7052

```
DSolve[x^3 (y'[x])^2+x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

31.29 problem 929

Internal problem ID [3655]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 929.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x^3 {y'}^2 + y' x^2 y + a = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 66

```
dsolve(x^3*diff(y(x),x)^2+x^2*y(x)*diff(y(x),x)+a = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2\sqrt{ax}}{x}$$
$$y(x) = \frac{2\sqrt{ax}}{x}$$
$$y(x) = \frac{x\,c_1^2 + 4a}{2c_1 x}$$
$$y(x) = \frac{4ax + c_1^2}{2c_1 x}$$

✓ Solution by Mathematica

Time used: 0.836 (sec). Leaf size: 57

```
DSolve[x^3 (y'[x])^2+x^2 y[x] y'[x]+a==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{-\frac{c_1}{2}}(x + 4ae^{c_1})}{2x}$$
$$y(x) \to \frac{e^{-\frac{c_1}{2}}(x + 4ae^{c_1})}{2x}$$

31.30 problem 931

Internal problem ID [3656]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 931.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$x(1-x^2)\,y'^2-2(1-x^2)\,yy'+x(1-y^2)=0$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 33

```
dsolve(x*(-x^2+1)*diff(y(x),x)^2-2*(-x^2+1)*y(x)*diff(y(x),x)+x*(1-y(x)^2) = 0,y(x), singsol=
```

$$y(x)=-x$$
$$y(x)=x$$
$$y(x)=\sqrt{-c_1^2+1}+\sqrt{x^2-1}\,c_1$$

✓ Solution by Mathematica

Time used: 0.576 (sec). Leaf size: 75

```
DSolve[x(1-x^2) (y'[x])^2-2(1-x^2)y[x] y'[x]+x(1-y[x]^2)==0,y[x],x,IncludeSingularSolutions -
```

$$y(x)\to -x\cos\left(2\arctan\left(\sqrt{\frac{x-1}{x+1}}\right)+ic_1\right)$$
$$y(x)\to -x\cos\left(2\arctan\left(\sqrt{\frac{x-1}{x+1}}\right)-ic_1\right)$$
$$y(x)\to -x$$
$$y(x)\to x$$

31.31 problem 932

Internal problem ID [3657]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 932.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$4x(-x+a)\,(-x+b)\,y'^2-\left(ab-2(a+b)\,x+2x^2\right)^2=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 81

```
dsolve(4*x*(a-x)*(b-x)*diff(y(x),x)^2 = (a*b-2*x*(a+b)+2*x^2)^2,y(x), singsol=all)
```

$$y(x)=\int -\frac{ab-2ax-2xb+2x^2}{2\sqrt{x\,(-x+b)\,(-x+a)}}dx+c_1$$
$$y(x)=\int \frac{ab-2ax-2xb+2x^2}{2\sqrt{x\,(-x+b)\,(-x+a)}}dx+c_1$$

✓ Solution by Mathematica

Time used: 14.219 (sec). Leaf size: 299

```
DSolve[4 x(a-x)(b-x) (y'[x])^2==(a b-2 x(a+b)+2 x^2)^2,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to \frac{x\sqrt{1-\frac{a}{x}}\sqrt{\frac{x-b}{a-b}}\left(b(a+2b)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{x}{a}-1}\right),\frac{a}{a-b}\right)+2(a-b)(a+b)E\left(i\text{arcsinh}\left(\sqrt{\frac{x}{a}-1}\right)\middle|\frac{a}{a-b}\right.\right.}{3\sqrt{x(a-x)(x-b)}} + \frac{2}{3}i\sqrt{x(a-x)(x-b)}+c_1$$

$$y(x) \to -\frac{x\sqrt{1-\frac{a}{x}}\sqrt{\frac{x-b}{a-b}}\left(b(a+2b)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{x}{a}-1}\right),\frac{a}{a-b}\right)+2(a-b)(a+b)E\left(i\text{arcsinh}\left(\sqrt{\frac{x}{a}-1}\right)\middle|\frac{a}{a}\right.\right.}{3\sqrt{x(a-x)(x-b)}} - \frac{2}{3}i\sqrt{x(a-x)(x-b)}+c_1$$

31.32 problem 933

Internal problem ID [3658]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 933.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x^4 {y'}^2 - y'x - y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 135

```
dsolve(x^4*diff(y(x),x)^2-x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{4x^2}$$
$$y(x) = \frac{-c_1^2 - c_1(2ix - c_1) - 2x^2}{2x^2 c_1^2}$$
$$y(x) = \frac{-c_1^2 - c_1(-2ix - c_1) - 2x^2}{2x^2 c_1^2}$$
$$y(x) = \frac{c_1(2ix + c_1) - 2x^2 - c_1^2}{2c_1^2 x^2}$$
$$y(x) = \frac{c_1(-2ix + c_1) - 2x^2 - c_1^2}{2c_1^2 x^2}$$

✓ Solution by Mathematica

Time used: 0.531 (sec). Leaf size: 123

```
DSolve[x^4 (y'[x])^2-x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x\sqrt{4x^2y(x)+1}\text{arctanh}\left(\sqrt{4x^2y(x)+1}\right)}{\sqrt{4x^4y(x)+x^2}}-\frac{1}{2}\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{x\sqrt{4x^2y(x)+1}\text{arctanh}\left(\sqrt{4x^2y(x)+1}\right)}{\sqrt{4x^4y(x)+x^2}}-\frac{1}{2}\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

31.33 problem 934

Internal problem ID [3659]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 31
Problem number: 934.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$x^4{y'}^2 + 2x^3yy' - 4 = 0$$

✓ Solution by Maple

Time used: 0.046 (sec). Leaf size: 49

```
dsolve(x^4*diff(y(x),x)^2+2*x^3*y(x)*diff(y(x),x)-4 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2i}{x}$$
$$y(x) = \frac{2i}{x}$$
$$y(x) = \frac{2\sinh(-\ln(x) + c_1)}{x}$$
$$y(x) = -\frac{2\sinh(-\ln(x) + c_1)}{x}$$

✓ Solution by Mathematica

Time used: 0.678 (sec). Leaf size: 71

```
DSolve[x^4 (y'[x])^2+2 x^3 y[x] y'[x]-4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{4e^{c_1}}{x^2} - \frac{e^{-c_1}}{4}$$
$$y(x) \to \frac{e^{-c_1}}{4} - \frac{4e^{c_1}}{x^2}$$
$$y(x) \to -\frac{2i}{x}$$
$$y(x) \to \frac{2i}{x}$$

32 Various 32

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32.1 problem 935

Internal problem ID [3660]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 935.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$\boxed{x^4 {y'}^2 + y^2 y' x - y^3 = 0}$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 134

```
dsolve(x^4*diff(y(x),x)^2+x*y(x)^2*diff(y(x),x)-y(x)^3 = 0,y(x), singsol=all)
```

$$y(x) = -4x^2$$
$$y(x) = 0$$
$$y(x) = \frac{\left(\sqrt{2}\, c_1 - 2x\right) c_1^2 x}{2c_1^2 - 4x^2}$$
$$y(x) = -\frac{\left(\sqrt{2}\, c_1 + 2x\right) c_1^2 x}{2\left(c_1^2 - 2x^2\right)}$$
$$y(x) = -\frac{2\left(\sqrt{2}\, c_1 - x\, c_1^2\right) x}{c_1^2\left(c_1^2 x^2 - 2\right)}$$
$$y(x) = \frac{2\left(\sqrt{2}\, c_1 + x\, c_1^2\right) x}{c_1^2\left(c_1^2 x^2 - 2\right)}$$

✓ Solution by Mathematica

Time used: 0.725 (sec). Leaf size: 57

```
DSolve[x^4 (y'[x])^2+x y[x]^2 y'[x]-y[x]^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{2c_1}x}{x + ie^{c_1}}$$

$$y(x) \to \frac{e^{2c_1}x}{-x + ie^{c_1}}$$

$$y(x) \to 0$$

32.2 problem 936

Internal problem ID [3661]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 936.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{x^2(a^2 - x^2)\, y'^2 + 1 = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 90

```
dsolve(x^2*(a^2-x^2)*diff(y(x),x)^2+1 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\ln\left(\frac{-2a^2+2\sqrt{-a^2}\,\sqrt{-a^2+x^2}}{x}\right)}{\sqrt{-a^2}} + c_1$$

$$y(x) = \frac{\ln\left(\frac{-2a^2+2\sqrt{-a^2}\,\sqrt{-a^2+x^2}}{x}\right)}{\sqrt{-a^2}} + c_1$$

✓ Solution by Mathematica

Time used: 0.013 (sec). Leaf size: 116

```
DSolve[x^2(a^2-x^2) (y'[x])^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{x\sqrt{x^2-a^2}\cot^{-1}\left(\frac{a}{\sqrt{x^2-a^2}}\right)}{a\sqrt{x^4-a^2x^2}} + c_1$$

$$y(x) \to \frac{x\sqrt{x^2-a^2}\cot^{-1}\left(\frac{a}{\sqrt{x^2-a^2}}\right)}{a\sqrt{x^4-a^2x^2}} + c_1$$

32.3 problem 937

Internal problem ID [3662]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 937.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$3x^4{y'}^2 - yx - y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 223

```
dsolve(3*x^4*diff(y(x),x)^2-x*y(x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \frac{\left(-\operatorname{arctanh}\left(\frac{x}{\sqrt{x+1}} + \frac{1}{\sqrt{x+1}}\right)\sqrt{3}\,x - \frac{\sqrt{3}\,x}{\sqrt{x+1}} - \frac{\sqrt{3}}{\sqrt{x+1}} - 3c_1x\right)^2}{36x^2}$$

$$y(x) = \frac{\left(\operatorname{arctanh}\left(\frac{x}{\sqrt{x+1}} + \frac{1}{\sqrt{x+1}}\right)\sqrt{3}\,x + \frac{\sqrt{3}\,x}{\sqrt{x+1}} + \frac{\sqrt{3}}{\sqrt{x+1}} - 3c_1x\right)^2}{36x^2}$$

$$y(x) = \frac{\left(-\operatorname{arctanh}\left(\frac{x}{\sqrt{x+1}} + \frac{1}{\sqrt{x+1}}\right)\sqrt{3}\,x - \frac{\sqrt{3}\,x}{\sqrt{x+1}} - \frac{\sqrt{3}}{\sqrt{x+1}} + 3c_1x\right)^2}{36x^2}$$

$$y(x) = \frac{\left(\operatorname{arctanh}\left(\frac{x}{\sqrt{x+1}} + \frac{1}{\sqrt{x+1}}\right)\sqrt{3}\,x + \frac{\sqrt{3}\,x}{\sqrt{x+1}} + \frac{\sqrt{3}}{\sqrt{x+1}} + 3c_1x\right)^2}{36x^2}$$

✓ Solution by Mathematica

Time used: 0.156 (sec). Leaf size: 165

```
DSolve[3 x^4 (y'[x])^2-x y[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{x\left(x\operatorname{arctanh}\left(\sqrt{x+1}\right)^2+2\operatorname{arctanh}\left(\sqrt{x+1}\right)\left(\sqrt{x+1}-\sqrt{3}c_1x\right)+c_1\left(-2\sqrt{3}\sqrt{x+1}+3c_1x\right)\right)+x+1}{12x^2}$$

$$y(x) \to \frac{x\left(x\operatorname{arctanh}\left(\sqrt{x+1}\right)^2+2\operatorname{arctanh}\left(\sqrt{x+1}\right)\left(\sqrt{x+1}+\sqrt{3}c_1x\right)+c_1\left(2\sqrt{3}\sqrt{x+1}+3c_1x\right)\right)+x+1}{12x^2}$$

$$y(x) \to 0$$

32.4 problem 938

Internal problem ID [3663]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 938.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$4x^5{y'}^2 + 12y'x^4y + 9 = 0$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 53

```
dsolve(4*x^5*diff(y(x),x)^2+12*x^4*y(x)*diff(y(x),x)+9 = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{x^{\frac{3}{2}}}$$

$$y(x) = -\frac{1}{x^{\frac{3}{2}}}$$

$$y(x) = \frac{c_1^2x^3 + 1}{2c_1x^3}$$

$$y(x) = \frac{x^3 + c_1^2}{2c_1x^3}$$

✓ Solution by Mathematica

Time used: 6.91 (sec). Leaf size: 75

```
DSolve[4 x^5 (y'[x])^2+12 x^4 y[x] y'[x]+9==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{\sqrt{x^3 \operatorname{sech}^2\left(\frac{3}{2}(-\log(x) + c_1)\right)}}$$

$$y(x) \to \frac{1}{\sqrt{x^3 \operatorname{sech}^2\left(\frac{3}{2}(-\log(x) + c_1)\right)}}$$

$$y(x) \to -\frac{1}{x^{3/2}}$$

$$y(x) \to \frac{1}{x^{3/2}}$$

32.5 problem 939

Internal problem ID [3664]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 939.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{x^6 {y'}^2 - 2y'x - 4y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 143

```
dsolve(x^6*diff(y(x),x)^2-2*x*diff(y(x),x)-4*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{1}{4x^4}$$
$$y(x) = \frac{-2x^4 - c_1^2 - c_1(2ix^2 - c_1)}{2c_1^2x^4}$$
$$y(x) = \frac{-2x^4 - c_1^2 - c_1(-2ix^2 - c_1)}{2c_1^2x^4}$$
$$y(x) = \frac{-2x^4 + c_1(2ix^2 + c_1) - c_1^2}{2c_1^2x^4}$$
$$y(x) = \frac{-2x^4 + c_1(-2ix^2 + c_1) - c_1^2}{2c_1^2x^4}$$

✓ Solution by Mathematica

Time used: 0.542 (sec). Leaf size: 128

```
DSolve[x^6 (y'[x])^2-2 x y'[x]-4 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x\sqrt{4x^4y(x)+1}\text{arctanh}\left(\sqrt{4x^4y(x)+1}\right)}{2\sqrt{4x^6y(x)+x^2}}-\frac{1}{4}\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{x\sqrt{4x^4y(x)+1}\text{arctanh}\left(\sqrt{4x^4y(x)+1}\right)}{2\sqrt{4x^6y(x)+x^2}}-\frac{1}{4}\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

32.6 problem 940

Internal problem ID [3665]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 940.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$x^8 {y'}^2 + 3y'x + 9y = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 42

```
dsolve(x^8*diff(y(x),x)^2+3*x*diff(y(x),x)+9*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{1}{4x^6}$$
$$y(x) = \frac{-x^3 + c_1}{x^3 c_1^2}$$
$$y(x) = -\frac{x^3 + c_1}{x^3 c_1^2}$$

✓ Solution by Mathematica

Time used: 0.563 (sec). Leaf size: 130

```
DSolve[x^8 (y'[x])^2+3 x y'[x]+9 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{x\sqrt{4x^6y(x)-1}\arctan\left(\sqrt{4x^6y(x)-1}\right)}{3\sqrt{x^2-4x^8y(x)}} - \frac{1}{6}\log(y(x)) = c_1, y(x)\right]$$
$$\text{Solve}\left[\frac{\sqrt{x^2-4x^8y(x)}\arctan\left(\sqrt{4x^6y(x)-1}\right)}{3x\sqrt{4x^6y(x)-1}} - \frac{1}{6}\log(y(x)) = c_1, y(x)\right]$$
$$y(x) \to 0$$

32.7 problem 941

Internal problem ID [3666]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 941.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y{y'}^2 - a = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 239

```
dsolve(y(x)*diff(y(x),x)^2 = a,y(x), singsol=all)
```

$$y(x) = \frac{(-12c_1a^2 + 12a^2x)^{\frac{2}{3}}}{4a}$$

$$y(x) = \frac{\left(-\frac{(-12c_1a^2+12a^2x)^{\frac{1}{3}}}{4} - \frac{i\sqrt{3}\,(-12c_1a^2+12a^2x)^{\frac{1}{3}}}{4}\right)^2}{a}$$

$$y(x) = \frac{\left(-\frac{(-12c_1a^2+12a^2x)^{\frac{1}{3}}}{4} + \frac{i\sqrt{3}\,(-12c_1a^2+12a^2x)^{\frac{1}{3}}}{4}\right)^2}{a}$$

$$y(x) = \frac{(12c_1a^2 - 12a^2x)^{\frac{2}{3}}}{4a}$$

$$y(x) = \frac{\left(-\frac{(12c_1a^2-12a^2x)^{\frac{1}{3}}}{4} - \frac{i\sqrt{3}\,(12c_1a^2-12a^2x)^{\frac{1}{3}}}{4}\right)^2}{a}$$

$$y(x) = \frac{\left(-\frac{(12c_1a^2-12a^2x)^{\frac{1}{3}}}{4} + \frac{i\sqrt{3}\,(12c_1a^2-12a^2x)^{\frac{1}{3}}}{4}\right)^2}{a}$$

✓ Solution by Mathematica

Time used: 3.68 (sec). Leaf size: 54

```
DSolve[y[x] (y'[x])^2==a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{3}{2}\right)^{2/3} \left(-\sqrt{a}x + c_1\right)^{2/3}$$
$$y(x) \to \left(\frac{3}{2}\right)^{2/3} \left(\sqrt{a}x + c_1\right)^{2/3}$$

32.8 problem 942

Internal problem ID [3667]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 942.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$\boxed{yy'^2 - x\,a^2 = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 74

```
dsolve(y(x)*diff(y(x),x)^2 = a^2*x,y(x), singsol=all)
```

$$-\frac{c_1 x}{y(x)\left(\frac{a^2\left((y(x)x)^{\frac{3}{2}}a-y(x)^3\right)}{y(x)^3}\right)^{\frac{2}{3}}}+x=0$$
$$-\frac{c_1 x}{y(x)\left(-\frac{a^2\left((y(x)x)^{\frac{3}{2}}a+y(x)^3\right)}{y(x)^3}\right)^{\frac{2}{3}}}+x=0$$

✓ Solution by Mathematica

Time used: 3.55 (sec). Leaf size: 46

```
DSolve[y[x] (y'[x])^2==a^2 x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\left(-ax^{3/2}+\frac{3c_1}{2}\right)^{2/3}$$
$$y(x)\to\left(ax^{3/2}+\frac{3c_1}{2}\right)^{2/3}$$

32.9 problem 943

Internal problem ID [3668]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 943.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$yy'^2 - e^{2x} = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 50

```
dsolve(y(x)*diff(y(x),x)^2 = exp(2*x),y(x), singsol=all)
```

$$-\frac{\sqrt{y(x)\,e^{2x}}}{\sqrt{y(x)}} + \frac{2y(x)^{\frac{3}{2}}}{3} + c_1 = 0$$
$$\frac{\sqrt{y(x)\,e^{2x}}}{\sqrt{y(x)}} + \frac{2y(x)^{\frac{3}{2}}}{3} + c_1 = 0$$

✓ Solution by Mathematica

Time used: 2.148 (sec). Leaf size: 47

```
DSolve[y[x] (y'[x])^2==Exp[2 x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \left(\frac{3}{2}\right)^{2/3} (-e^x + c_1)^{2/3}$$
$$y(x) \to \left(\frac{3}{2}\right)^{2/3} (e^x + c_1)^{2/3}$$

32.10 problem 944

Internal problem ID [3669]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 944.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{yy'^2 + 2axy' - ay = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 115

```
dsolve(y(x)*diff(y(x),x)^2+2*a*x*diff(y(x),x)-a*y(x) = 0,y(x), singsol=all)
```

$$y(x) = x\sqrt{-a}$$
$$y(x) = -x\sqrt{-a}$$
$$y(x) = 0$$
$$y(x) = \operatorname{RootOf}\left(-\ln(x) - \left(\int^{\_Z} \frac{\_a^2 + \sqrt{a\_a^2 + a^2} + a}{\_a(\_a^2 + a)} d\_a\right) + c_1\right) x$$
$$y(x) = \operatorname{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{\_a^2 - \sqrt{a(\_a^2 + a)} + a}{\_a(\_a^2 + a)} d\_a + c_1\right) x$$

✓ Solution by Mathematica

Time used: 8.114 (sec). Leaf size: 88

```
DSolve[y[x] (y'[x])^2+2 a x y'[x]-a y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{e^{c_1}\left(-2\sqrt{a}x + e^{c_1}\right)}$$
$$y(x) \to \sqrt{e^{c_1}\left(-2\sqrt{a}x + e^{c_1}\right)}$$
$$y(x) \to 0$$
$$y(x) \to -i\sqrt{a}x$$
$$y(x) \to i\sqrt{a}x$$

32.11 problem 945

Internal problem ID [3670]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 945.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$yy'^2 - 4a^2xy' + ya^2 = 0$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 122

```
dsolve(y(x)*diff(y(x),x)^2-4*a^2*x*diff(y(x),x)+a^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \mathrm{RootOf}\left(-\ln(x) - \left(\int^{\_Z} \frac{\_a^2 - 2a^2 + \sqrt{-\_a^2a^2 + 4a^4}}{\_a\,(\_a^2 - 3a^2)} d\_a\right) + c_1\right) x$$

$$y(x) = \mathrm{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{\_a^2 - 2a^2 - \sqrt{-\_a^2a^2 + 4a^4}}{\_a\,(\_a^2 - 3a^2)} d\_a + c_1\right) x$$

✓ Solution by Mathematica

Time used: 8.588 (sec). Leaf size: 758

```
DSolve[y[x] (y'[x])^2-4 a^2 x y'[x]+a^2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{8\left(4a^2-\frac{y(x)^2}{x^2}\right)^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{\frac{y(x)}{x}-2a}}{2\sqrt{a}}\right)+\sqrt{a}\sqrt{\frac{y(x)}{ax}+2}\left(\sqrt{-\left(\frac{y(x)}{x}-2a\right)^2}\sqrt{2a+\frac{y(x)}{x}}\sqrt{4a^2-\frac{y(x)^2}{x^2}}\left(\right.\right.}{}\right.$$

$$\left.-\log(x)+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{\sqrt{a}\sqrt{\frac{y(x)}{ax}+2}\left(\sqrt{-\left(\frac{y(x)}{x}-2a\right)^2}\sqrt{2a+\frac{y(x)}{x}}\sqrt{4a^2-\frac{y(x)^2}{x^2}}\left(\log\left(3a^2-\frac{y(x)^2}{x^2}\right)+8\arctan\left(\frac{\sqrt{2a-\frac{y(x)}{x}}}{\sqrt{2a+\frac{y(x)}{x}}}\right)\right.\right.}{}\right.$$

$$\left.-\log(x)+c_1,y(x)\right]$$

$$y(x)\to 0$$

32.12 problem 946

Internal problem ID [3671]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 946.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{yy'^2 + axy' + yb = 0}$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 108

```
dsolve(y(x)*diff(y(x),x)^2+a*x*diff(y(x),x)+b*y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \text{RootOf}\left(-2\ln(x) - \left(\int^{\_Z} \frac{2\_a^2 + \sqrt{-4\_a^2 b + a^2} + a}{\_a(\_a^2 + a + b)} d\_a\right) + 2c_1\right) x$$

$$y(x) = \text{RootOf}\left(-2\ln(x) + \int^{\_Z} -\frac{2\_a^2 + a - \sqrt{-4\_a^2 b + a^2}}{\_a(\_a^2 + a + b)} d\_a + 2c_1\right) x$$

✓ Solution by Mathematica

Time used: 0.635 (sec). Leaf size: 162

```
DSolve[y[x] (y'[x])^2+a x y'[x]+b y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{a\log\left(\sqrt{a^2-\frac{4by(x)^2}{x^2}}+a\right)+(a+2b)\log\left(\sqrt{a^2-\frac{4by(x)^2}{x^2}}-a-2b\right)}{4(a+b)}=-\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{a\log\left(\sqrt{a^2-\frac{4by(x)^2}{x^2}}-a\right)+(a+2b)\log\left(\sqrt{a^2-\frac{4by(x)^2}{x^2}}+a+2b\right)}{4(a+b)}=-\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$y(x)\to 0$$

32.13 problem 947

Internal problem ID [3672]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 947.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, _dAlembert]`

$$yy'^2 - (-2bx + a)\, y' - yb = 0$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 201

```
dsolve(y(x)*diff(y(x),x)^2-(-2*b*x+a)*diff(y(x),x)-b*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{-2xb + a}{2\sqrt{-b}}$$

$$y(x) = \frac{-2xb + a}{2\sqrt{-b}}$$

$$y(x) = 0$$

$$y(x) = \sqrt{\frac{c_1 b + \sqrt{4b^3 c_1 x^2 - 4a\, b^2 c_1 x + a^2 b c_1}}{b}}$$

$$y(x) = \sqrt{\frac{c_1 b - \sqrt{4b^3 c_1 x^2 - 4a\, b^2 c_1 x + a^2 b c_1}}{b}}$$

$$y(x) = -\sqrt{\frac{c_1 b + \sqrt{4b^3 c_1 x^2 - 4a\, b^2 c_1 x + a^2 b c_1}}{b}}$$

$$y(x) = -\sqrt{\frac{c_1 b - \sqrt{4b^3 c_1 x^2 - 4a\, b^2 c_1 x + a^2 b c_1}}{b}}$$

✓ Solution by Mathematica

Time used: 1.077 (sec). Leaf size: 409

```
DSolve[y[x] (y'[x])^2-(a-2 b x)y'[x]-b y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\left(b-\sqrt{b^2}\right)\log(y(x))}{b}\right.$$

$$-\frac{-b\log\left(\sqrt{a^2-4abx+4b\left(bx^2+y(x)^2\right)}-a-2\sqrt{b^2}x\right)+\sqrt{b^2}\log\left(b\left(\sqrt{a^2-4abx+4b\left(bx^2+y(x)^2\right)}-a\right.\right.}{2\sqrt{b^2}}$$

$$\text{Solve}\left[\frac{-b\log\left(\sqrt{a^2-4abx+4b\left(bx^2+y(x)^2\right)}-a-2\sqrt{b^2}x\right)+\sqrt{b^2}\log\left(b\left(\sqrt{a^2-4abx+4b\left(bx^2+y(x)^2\right)}\right.\right.}{2\sqrt{b^2}}\right.$$

$$\left.+\frac{\left(\sqrt{b^2}+b\right)\log(y(x))}{b}=c_1,y(x)\right]$$

$$y(x)\to-\frac{i(2bx-a)}{2\sqrt{b}}$$

$$y(x)\to\frac{i(2bx-a)}{2\sqrt{b}}$$

32.14 problem 948

Internal problem ID [3673]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 948.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{yy'^2 + x^3y' - yx^2 = 0}$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 91

```
dsolve(y(x)*diff(y(x),x)^2+x^3*diff(y(x),x)-x^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{ix^2}{2}$$
$$y(x) = \frac{ix^2}{2}$$
$$y(x) = 0$$
$$y(x) = -\frac{\sqrt{-4c_1x^2 + c_1^2}}{4}$$
$$y(x) = \frac{\sqrt{-4c_1x^2 + c_1^2}}{4}$$
$$y(x) = -\frac{2\sqrt{c_1x^2 + 4}}{c_1}$$
$$y(x) = \frac{2\sqrt{c_1x^2 + 4}}{c_1}$$

✓ Solution by Mathematica

Time used: 1.282 (sec). Leaf size: 244

```
DSolve[y[x] (y'[x])^2+x^3 y'[x]-x^2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt{x^6+4x^2y(x)^2}\log\left(\sqrt{x^4+4y(x)^2}+x^2\right)}{2x\sqrt{x^4+4y(x)^2}}+\frac{1}{2}\left(1-\frac{\sqrt{x^6+4x^2y(x)^2}}{x\sqrt{x^4+4y(x)^2}}\right)\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{2}\left(\frac{\sqrt{x^6+4x^2y(x)^2}}{x\sqrt{x^4+4y(x)^2}}+1\right)\log(y(x))-\frac{\sqrt{x^6+4x^2y(x)^2}\log\left(\sqrt{x^4+4y(x)^2}+x^2\right)}{2x\sqrt{x^4+4y(x)^2}}=c_1,y(x)\right]$$

$$y(x)\to -\frac{ix^2}{2}$$

$$y(x)\to \frac{ix^2}{2}$$

32.15 problem 949

Internal problem ID [3674]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 949.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$yy'^2 + (x - y)\,y' - x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(y(x)*diff(y(x),x)^2+(x-y(x))*diff(y(x),x)-x = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 + c_1}$$
$$y(x) = -\sqrt{-x^2 + c_1}$$
$$y(x) = x + c_1$$

✓ Solution by Mathematica

Time used: 0.113 (sec). Leaf size: 47

```
DSolve[y[x] (y'[x])^2+(x-y[x])y'[x]-x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to x + c_1$$
$$y(x) \to -\sqrt{-x^2 + 2c_1}$$
$$y(x) \to \sqrt{-x^2 + 2c_1}$$

32.16 problem 950

Internal problem ID [3675]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 950.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{yy'^2-(x+y)\,y'+y=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 271

```
dsolve(y(x)*diff(y(x),x)^2-(x+y(x))*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x)=x$$
$$y(x)=0$$
$$\ln(x)-\frac{x\left(\frac{x^2+2y(x)x-3y(x)^2}{x^2}\right)^{\frac{3}{2}}}{2y(x)}-\operatorname{arctanh}\left(\frac{x+y(x)}{x\sqrt{\frac{x^2+2y(x)x-3y(x)^2}{x^2}}}\right)+\ln\left(\frac{y(x)}{x}\right)+\sqrt{\frac{x^2+2y(x)x-3y(x)^2}{x^2}}-\frac{3\sqrt{\frac{x^2+2y(x)x-3y(x)^2}{x^2}}\,y(x)}{2x}-\frac{x}{2y(x)}-c_1=0$$
$$\ln(x)+\frac{x\left(\frac{x^2+2y(x)x-3y(x)^2}{x^2}\right)^{\frac{3}{2}}}{2y(x)}+\operatorname{arctanh}\left(\frac{x+y(x)}{x\sqrt{\frac{x^2+2y(x)x-3y(x)^2}{x^2}}}\right)+\ln\left(\frac{y(x)}{x}\right)-\sqrt{\frac{x^2+2y(x)x-3y(x)^2}{x^2}}+\frac{3\sqrt{\frac{x^2+2y(x)x-3y(x)^2}{x^2}}\,y(x)}{2x}-\frac{x}{2y(x)}-c_1=0$$

✓ Solution by Mathematica

Time used: 4.171 (sec). Leaf size: 320

```
DSolve[y[x] (y'[x])^2-(x+y[x])y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[-\frac{x\left(-i\sqrt{\frac{y(x)}{x}-1}\sqrt{\frac{3y(x)}{x}+1}+\frac{4y(x)\log\left(\sqrt{\frac{3y(x)}{x}-3}-\sqrt{\frac{3y(x)}{x}+1}\right)}{x}-\frac{4y(x)\log\left(-i\left(\frac{3y(x)}{x}+1\right)+i\sqrt{\frac{3y(x)}{x}-3}\sqrt{\frac{3y(x)}{x}+1}+\sqrt{\cdots}\right.}{x}\right.}{4y(x)}\right.$$

$$\left.-\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$\text{Solve}\left[-\frac{x\left(i\sqrt{\frac{y(x)}{x}-1}\sqrt{\frac{3y(x)}{x}+1}+\frac{4y(x)\log\left(\sqrt{\frac{3y(x)}{x}-3}-\sqrt{\frac{3y(x)}{x}+1}\right)}{x}-\frac{4y(x)\log\left(i\left(\frac{3y(x)}{x}+1\right)-i\sqrt{\frac{3y(x)}{x}-3}\sqrt{\frac{3y(x)}{x}+1}+\sqrt{2-2\cdots}\right.}{x}\right.}{4y(x)}\right.$$

$$\left.-\frac{\log(x)}{2}+c_1,y(x)\right]$$

$$y(x)\to 0$$

32.17 problem 951

Internal problem ID [3676]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 951.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{yy'^2 - (1 + yx)\, y' + x = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(y(x)*diff(y(x),x)^2-(1+x*y(x))*diff(y(x),x)+x = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{2x + c_1}$$
$$y(x) = -\sqrt{2x + c_1}$$
$$y(x) = \frac{x^2}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.093 (sec). Leaf size: 52

```
DSolve[y[x] (y'[x])^2-(1+x y[x])y'[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{x + c_1}$$
$$y(x) \to \sqrt{2}\sqrt{x + c_1}$$
$$y(x) \to \frac{x^2}{2} + c_1$$

32.18 problem 952

Internal problem ID [3677]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 952.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y{y'}^2 + (-y^2 + x)\,y' - yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 34

```
dsolve(y(x)*diff(y(x),x)^2+(x-y(x)^2)*diff(y(x),x)-x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 + c_1}$$
$$y(x) = -\sqrt{-x^2 + c_1}$$
$$y(x) = c_1 e^x$$

✓ Solution by Mathematica

Time used: 0.096 (sec). Leaf size: 54

```
DSolve[y[x] (y'[x])^2+(x-y[x]^2)y'[x]-x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 e^x$$
$$y(x) \to -\sqrt{-x^2 + 2c_1}$$
$$y(x) \to \sqrt{-x^2 + 2c_1}$$
$$y(x) \to 0$$

32.19 problem 953

Internal problem ID [3678]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 953.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{yy'^2 + y - a = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 1583

```
dsolve(y(x)*diff(y(x),x)^2+y(x) = a,y(x), singsol=all)
```

$$y(x) = a$$

$$y(x) = \frac{\tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2 \_Z^2 + 4\tan(\_Z)^2 c_1 a \_Z - 4\tan(\_Z)^2 a x \_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c\right.\right.}{}$$

$$+ \tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2 \_Z^2 + 4\tan(\_Z)^2 c_1 a \_Z - 4\tan(\_Z)^2 a x \_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c_1 x + 4\tan(\_Z)^2 x^2 + a^2 \_Z^2 + 4c_1 \_Z a - 4 a x \_Z + 4c_1^2 - 8c_1 x - a^2 + 4x^2\right)\right) c_1$$
$$- \tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2 \_Z^2 + 4\tan(\_Z)^2 c_1 a \_Z - 4\tan(\_Z)^2 a x \_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c_1 x + 4\tan(\_Z)^2 x^2 + a^2 \_Z^2 + 4c_1 \_Z a - 4 a x \_Z + 4c_1^2 - 8c_1 x - a^2 + 4x^2\right)\right) x$$
$$+ \frac{a}{2}$$

$$y(x) = \frac{\tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2 \_Z^2 + 4\tan(\_Z)^2 c_1 a \_Z - 4\tan(\_Z)^2 a x \_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c\right.\right.}{}$$

$$+ \tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2 \_Z^2 + 4\tan(\_Z)^2 c_1 a \_Z - 4\tan(\_Z)^2 a x \_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c_1 x + 4\tan(\_Z)^2 x^2 + a^2 \_Z^2 + 4c_1 \_Z a - 4 a x \_Z + 4c_1^2 - 8c_1 x - a^2 + 4x^2\right)\right) c_1$$
$$- \tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2 \_Z^2 + 4\tan(\_Z)^2 c_1 a \_Z - 4\tan(\_Z)^2 a x \_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c_1 x + 4\tan(\_Z)^2 x^2 + a^2 \_Z^2 + 4c_1 \_Z a - 4 a x \_Z + 4c_1^2 - 8c_1 x - a^2 + 4x^2\right)\right) x$$
$$+ \frac{a}{2}$$

$$y(x) = \frac{\tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2 \_Z^2 - 4\tan(\_Z)^2 c_1 a \_Z + 4\tan(\_Z)^2 a x \_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c\right.\right.}{}$$

$$- \tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2 \_Z^2 - 4\tan(\_Z)^2 c_1 a \_Z + 4\tan(\_Z)^2 a x \_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c_1 x + 4\tan(\_Z)^2 x^2 + a^2 \_Z^2 - 4c_1 \_Z a + 4 a x \_Z + 4c_1^2 - 8c_1 x - a^2 + 4x^2\right)\right) c_1$$
$$+ \tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2 \_Z^2 - 4\tan(\_Z)^2 c_1 a \_Z + 4\tan(\_Z)^2 a x \_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c_1 x + 4\tan(\_Z)^2 x^2 + a^2 \_Z^2 - 4c_1 \_Z a + 4 a x \_Z + 4c_1^2 - 8c_1 x - a^2 + 4x^2\right)\right) x$$
$$+ \frac{a}{2}$$

$$y(x) = \frac{\tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2 \_Z^2 - 4\tan(\_Z)^2 c_1 a \_Z + 4\tan(\_Z)^2 a x \_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c\right.\right.}{}$$

$$- \tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2 \_Z^2 - 4\tan(\_Z)^2 c_1 a \_Z + 4\tan(\_Z)^2 a x \_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c_1 x + 4\tan(\_Z)^2 x^2 + a^2 \_Z^2 - 4c_1 \_Z a + 4 a x \_Z + 4c_1^2 - 8c_1 x - a^2 + 4x^2\right)\right) c_1$$
$$+ \tan\left(\operatorname{RootOf}\left(\tan(\_Z)^2 a^2 \_Z^2 - 4\tan(\_Z)^2 c_1 a \_Z + 4\tan(\_Z)^2 a x \_Z + 4\tan(\_Z)^2 c_1^2 - 8\tan(\_Z)^2 c_1 x + 4\tan(\_Z)^2 x^2 + a^2 \_Z^2 - 4c_1 \_Z a + 4 a x \_Z + 4c_1^2 - 8c_1 x - a^2 + 4x^2\right)\right) x$$
$$+ \frac{a}{2}$$

✓ Solution by Mathematica

Time used: 0.417 (sec). Leaf size: 106

```
DSolve[y[x] (y'[x])^2+y[x]==a,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[a\arctan\left(\frac{\sqrt{\#1}}{\sqrt{a-\#1}}\right)-\sqrt{\#1}\sqrt{a-\#1}\&\right][-x+c_1]$$

$$y(x) \to \text{InverseFunction}\left[a\arctan\left(\frac{\sqrt{\#1}}{\sqrt{a-\#1}}\right)-\sqrt{\#1}\sqrt{a-\#1}\&\right][x+c_1]$$

$$y(x) \to a$$

32.20 problem 954

Internal problem ID [3679]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 954.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$(x+y)\,y'^2 + 2y'x - y = 0$$

✓ Solution by Maple

Time used: 15.484 (sec). Leaf size: 119

```
dsolve((x+y(x))*diff(y(x),x)^2+2*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = \left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)x$$

$$y(x) = \left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)x$$

$$\ln(x) - \operatorname{arctanh}\left(\frac{2x + y(x)}{2x\sqrt{\frac{y(x)^2 + y(x)x + x^2}{x^2}}}\right) + \ln\left(\frac{y(x)}{x}\right) - c_1 = 0$$

$$\ln(x) + \operatorname{arctanh}\left(\frac{2x + y(x)}{2x\sqrt{\frac{y(x)^2 + y(x)x + x^2}{x^2}}}\right) + \ln\left(\frac{y(x)}{x}\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 4.308 (sec). Leaf size: 166

```
DSolve[(x+y[x]) (y'[x])^2+2 x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{2}{3}\sqrt{e^{c_1}\left(-3x+e^{c_1}\right)}-\frac{e^{c_1}}{3}$$
$$y(x) \to \frac{2}{3}\sqrt{e^{c_1}\left(-3x+e^{c_1}\right)}-\frac{e^{c_1}}{3}$$
$$y(x) \to e^{c_1}-2\sqrt{e^{c_1}\left(x+e^{c_1}\right)}$$
$$y(x) \to 2\sqrt{e^{c_1}\left(x+e^{c_1}\right)}+e^{c_1}$$
$$y(x) \to 0$$
$$y(x) \to -\frac{1}{2}i\left(\sqrt{3}-i\right)x$$
$$y(x) \to \frac{1}{2}i\left(\sqrt{3}+i\right)x$$

32.21 problem 955

Internal problem ID [3680]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 955.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$(2x - y)\, {y'}^2 - 2(1 - x)\, y' + 2 - y = 0$$

✓ Solution by Maple

Time used: 0.922 (sec). Leaf size: 78

```
dsolve((2*x-y(x))*diff(y(x),x)^2-2*(1-x)*diff(y(x),x)+2-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\sqrt{2}\, x + \sqrt{2} + x + 1$$
$$y(x) = \sqrt{2}\, x - \sqrt{2} + x + 1$$
$$y(x) = 2 + \frac{c_1}{2} - \frac{\sqrt{-c_1^2 + 4\,(x-1)\, c_1}}{2}$$
$$y(x) = 2 + c_1 - \sqrt{-c_1^2 + 2\,(x-1)\, c_1}$$

✓ Solution by Mathematica

Time used: 3.8 (sec). Leaf size: 187

```
DSolve[(2 x -y[x]) (y'[x])^2-2(1-x)y'[x]+2-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{1}{2}\sqrt{-e^{c_1}\left(4x-4+e^{c_1}\right)}+2-\frac{e^{c_1}}{2}$$

$$y(x) \to \frac{1}{2}\left(\sqrt{-e^{c_1}\left(4x-4+e^{c_1}\right)}+4-e^{c_1}\right)$$

$$y(x) \to -\sqrt{-e^{c_1}\left(2x-2+e^{c_1}\right)}+2-e^{c_1}$$

$$y(x) \to \sqrt{-e^{c_1}\left(2x-2+e^{c_1}\right)}+2-e^{c_1}$$

$$y(x) \to 2$$

$$y(x) \to x-\sqrt{2}\sqrt{(x-1)^2}+1$$

$$y(x) \to x+\sqrt{2}\sqrt{(x-1)^2}+1$$

32.22 problem 956

Internal problem ID [3681]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 956.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _rational, _dAlembert]`

$$2y{y'}^2 + (5-4x)\,y' + 2y = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 135

```
dsolve(2*y(x)*diff(y(x),x)^2+(5-4*x)*diff(y(x),x)+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = x - \frac{5}{4}$$
$$y(x) = -x + \frac{5}{4}$$
$$y(x) = 0$$
$$y(x) = \frac{\sqrt{4c_1 + 2\sqrt{-16c_1x^2 + 40c_1x - 25c_1}}}{2}$$
$$y(x) = -\frac{\sqrt{4c_1 + 2\sqrt{-16c_1x^2 + 40c_1x - 25c_1}}}{2}$$
$$y(x) = \frac{\sqrt{4c_1 - 2\sqrt{-16c_1x^2 + 40c_1x - 25c_1}}}{2}$$
$$y(x) = -\frac{\sqrt{4c_1 - 2\sqrt{-16c_1x^2 + 40c_1x - 25c_1}}}{2}$$

✓ Solution by Mathematica

Time used: 0.705 (sec). Leaf size: 160

```
DSolve[(2 y[x] (y'[x])^2)+(5-4 x)y'[x]+2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -i\sqrt{2}e^{\frac{c_1}{2}}\sqrt{4x - 5 + 8e^{c_1}}$$
$$y(x) \to i\sqrt{2}e^{\frac{c_1}{2}}\sqrt{4x - 5 + 8e^{c_1}}$$
$$y(x) \to -\frac{1}{4}ie^{\frac{c_1}{2}}\sqrt{8x - 10 + e^{c_1}}$$
$$y(x) \to \frac{1}{4}ie^{\frac{c_1}{2}}\sqrt{8x - 10 + e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to \frac{5}{4} - x$$
$$y(x) \to x - \frac{5}{4}$$

32.23 problem 957

Internal problem ID [3682]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 957.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$9y{y'}^2 + 4x^3y' - 4yx^2 = 0$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 91

```
dsolve(9*y(x)*diff(y(x),x)^2+4*x^3*diff(y(x),x)-4*x^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{ix^2}{3}$$
$$y(x) = \frac{ix^2}{3}$$
$$y(x) = 0$$
$$y(x) = -\frac{2\sqrt{c_1x^2 + 9}}{c_1}$$
$$y(x) = \frac{2\sqrt{c_1x^2 + 9}}{c_1}$$
$$y(x) = -\frac{\sqrt{-4c_1x^2 + c_1^2}}{6}$$
$$y(x) = \frac{\sqrt{-4c_1x^2 + c_1^2}}{6}$$

✓ Solution by Mathematica

Time used: 1.258 (sec). Leaf size: 244

```
DSolve[9 y[x] (y'[x])^2+4 x^3 y'[x]-4 x^2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt{x^6+9x^2y(x)^2}\log\left(\sqrt{x^4+9y(x)^2}+x^2\right)}{2x\sqrt{x^4+9y(x)^2}}+\frac{1}{2}\left(1-\frac{\sqrt{x^6+9x^2y(x)^2}}{x\sqrt{x^4+9y(x)^2}}\right)\log(y(x))=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{2}\left(\frac{\sqrt{x^6+9x^2y(x)^2}}{x\sqrt{x^4+9y(x)^2}}+1\right)\log(y(x))-\frac{\sqrt{x^6+9x^2y(x)^2}\log\left(\sqrt{x^4+9y(x)^2}+x^2\right)}{2x\sqrt{x^4+9y(x)^2}}=c_1,y(x)\right]$$

$$y(x)\to -\frac{ix^2}{3}$$

$$y(x)\to \frac{ix^2}{3}$$

32.24 problem 958

Internal problem ID [3683]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 958.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(1-ay)\,y'^2 - ay = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 815

```
dsolve((1-a*y(x))*diff(y(x),x)^2 = a*y(x),y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \frac{\operatorname{RootOf}\left(4c_1^2a^2 - 8a^2c_1x + 4a^2x^2 - 4\sqrt{a^2}\,c_1 \operatorname{RootOf}\left(4\tan\left(\_Z\right)^2 c_1^2a^2 - 8\tan\left(\_Z\right)^2 c_1a^2x + 4\tan\left(\_Z\right)^2\right.\right.}{}$$

$$y(x) = \frac{\operatorname{RootOf}\left(4c_1^2a^2 - 8a^2c_1x + 4a^2x^2 + 4\sqrt{a^2}\,c_1 \operatorname{RootOf}\left(4\tan\left(\_Z\right)^2 c_1^2a^2 - 8\tan\left(\_Z\right)^2 c_1a^2x + 4\tan\left(\_Z\right)^2\right.\right.}{}$$

✓ Solution by Mathematica

Time used: 0.547 (sec). Leaf size: 147

```
DSolve[(1-a y[x]) (y'[x])^2==a y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{2\arctan\left(\frac{\sqrt{\#1}\sqrt{a}}{\sqrt{1-\#1 a-1}}\right)}{\sqrt{a}} + \sqrt{\#1}\sqrt{1-\#1 a}\&\right]\left[-\sqrt{a}x + c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{2\arctan\left(\frac{\sqrt{\#1}\sqrt{a}}{\sqrt{1-\#1 a-1}}\right)}{\sqrt{a}} + \sqrt{\#1}\sqrt{1-\#1 a}\&\right]\left[\sqrt{a}x + c_1\right]$$

$$y(x) \to 0$$

32.25 problem 960

Internal problem ID [3684]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 960.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$(x^2 - ay)\, y'^2 - 2xyy' = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 28

```
dsolve((x^2-a*y(x))*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x^2}{a \operatorname{LambertW}\left(-\frac{c_1 x^2}{a}\right)}$$
$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 7.932 (sec). Leaf size: 310

```
DSolve[(x^2-a y[x]) (y'[x])^2-2 x y[x] y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1$$

$$\text{Solve}\left[\frac{\left(2-\frac{2(2axy(x)+x^3)}{\sqrt[3]{x^3}(x^2-ay(x))}\right)\left(\frac{\frac{6x^3}{x^2-ay(x)}-4x}{\sqrt[3]{x^3}}+4\right)\left(\left(1-\frac{x(2ay(x)+x^2)}{\sqrt[3]{x^3}(x^2-ay(x))}\right)\log\left(\frac{2-\frac{2\left(2axy(x)+x^3\right)}{\sqrt[3]{x^3}(x^2-ay(x))}}{\sqrt[3]{2}}\right)+\left(\frac{2axy(x}{\sqrt[3]{x^3}(x^2-}\right.\right.}{18\sqrt[3]{2}\left(-\frac{(2ay(x)+x^2)^3}{(x^2-ay(x))^3}+\frac{3(2axy(x)+x^3)}{\sqrt[3]{x^3}(x^2-ay(x))}-2\right)}\right.$$

$$\left.+c_1, y(x)\right]$$

$$y(x) \to 0$$

32.26 problem 961

Internal problem ID [3685]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 961.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$x y {y'}^2 + (x + y) y' + 1 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(x*y(x)*diff(y(x),x)^2+(x+y(x))*diff(y(x),x)+1 = 0,y(x), singsol=all)
```

$$y(x) = -\ln(x) + c_1$$
$$y(x) = \sqrt{-2x + c_1}$$
$$y(x) = -\sqrt{-2x + c_1}$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 53

```
DSolve[x y[x] (y'[x])^2+(x+y[x])y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{-x + c_1}$$
$$y(x) \to \sqrt{2}\sqrt{-x + c_1}$$
$$y(x) \to -\log(x) + c_1$$

32.27 problem 962

Internal problem ID [3686]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 32
Problem number: 962.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$xyy'^2 + (x^2 + y^2)\, y' + yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(x*y(x)*diff(y(x),x)^2+(x^2+y(x)^2)*diff(y(x),x)+x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{c_1}{x}$$
$$y(x) = \sqrt{-x^2 + c_1}$$
$$y(x) = -\sqrt{-x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.129 (sec). Leaf size: 54

```
DSolve[x y[x] (y'[x])^2+(x^2 + y[x]^2)y'[x]+x y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \frac{c_1}{x}$$
$$y(x) \to -\sqrt{-x^2 + 2c_1}$$
$$y(x) \to \sqrt{-x^2 + 2c_1}$$
$$y(x) \to 0$$

33 Various 33

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33.1 problem 963

Internal problem ID [3687]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 963.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$xyy'^2 + (x^2 - y^2)\, y' - yx = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(x*y(x)*diff(y(x),x)^2+(x^2-y(x)^2)*diff(y(x),x)-x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 x$$
$$y(x) = \sqrt{-x^2 + c_1}$$
$$y(x) = -\sqrt{-x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 65

```
DSolve[x y[x] (y'[x])^2+(x^2-y[x]^2)y'[x]-x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x$$
$$y(x) \to -\sqrt{-x^2 + 2c_1}$$
$$y(x) \to \sqrt{-x^2 + 2c_1}$$
$$y(x) \to -ix$$
$$y(x) \to ix$$

33.2 problem 964

Internal problem ID [3688]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 964.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$
x y {y'}^2 - \left(x^2 - y^2\right) y' - y x = 0
$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

```
dsolve(x*y(x)*diff(y(x),x)^2-(x^2-y(x)^2)*diff(y(x),x)-x*y(x) = 0,y(x), singsol=all)
```

$$
\begin{aligned}
y(x) &= \frac{c_1}{x} \\
y(x) &= \sqrt{x^2 + c_1} \\
y(x) &= -\sqrt{x^2 + c_1}
\end{aligned}
$$

✓ Solution by Mathematica

Time used: 0.122 (sec). Leaf size: 50

```
DSolve[x y[x] (y'[x])^2-(x^2-y[x]^2)y'[x]-x y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$
\begin{aligned}
y(x) &\to \frac{c_1}{x} \\
y(x) &\to -\sqrt{x^2 + 2c_1} \\
y(x) &\to \sqrt{x^2 + 2c_1} \\
y(x) &\to 0
\end{aligned}
$$

33.3 problem 965

Internal problem ID [3689]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 965.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$xyy'^2 + (a + x^2 - y^2)\, y' - xy = 0$$

✗ Solution by Maple

```
dsolve(x*y(x)*diff(y(x),x)^2+(a+x^2-y(x)^2)*diff(y(x),x)-x*y(x) = 0,y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 0.372 (sec). Leaf size: 112

```
DSolve[x y[x] (y'[x])^2+(a+x^2-y[x]^2)y'[x]-x y[x]==0,y[x],x,IncludeSingularSolutions -> True
```

$$y(x) \to \sqrt{c_1 \left(x^2 + \frac{a}{1 + c_1}\right)}$$

$$y(x) \to -\sqrt{\left(\sqrt{a} - ix\right)^2}$$

$$y(x) \to \sqrt{\left(\sqrt{a} - ix\right)^2}$$

$$y(x) \to -\sqrt{\left(\sqrt{a} + ix\right)^2}$$

$$y(x) \to \sqrt{\left(\sqrt{a} + ix\right)^2}$$

33.4 problem 966

Internal problem ID [3690]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 966.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$xyy'^2 - (a - b\,x^2 + y^2)\,y' - bxy = 0$$

✗ Solution by Maple

```
dsolve(x*y(x)*diff(y(x),x)^2-(a-b*x^2+y(x)^2)*diff(y(x),x)-b*x*y(x) = 0,y(x), singsol=all)
```

No solution found

✓ Solution by Mathematica

Time used: 1.511 (sec). Leaf size: 131

```
DSolve[x y[x] (y'[x])^2-(a-b x^2+y[x]^2)y'[x]-b x y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \sqrt{c_1\left(x^2 - \frac{a}{b + c_1}\right)}$$

$$y(x) \to -\sqrt{-\left(\sqrt{a} + \sqrt{b}x\right)^2}$$

$$y(x) \to \sqrt{-\left(\sqrt{a} + \sqrt{b}x\right)^2}$$

$$y(x) \to -\sqrt{-\left(\sqrt{a} - \sqrt{b}x\right)^2}$$

$$y(x) \to \sqrt{-\left(\sqrt{a} - \sqrt{b}x\right)^2}$$

33.5 problem 967

Internal problem ID [3691]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 967.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$x y {y'}^{2}+\left(3 x^{2}-2 y^{2}\right) y'-6 x y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(x*y(x)*diff(y(x),x)^2+(3*x^2-2*y(x)^2)*diff(y(x),x)-6*x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 x^2$$
$$y(x) = \sqrt{-3x^2+c_1}$$
$$y(x) = -\sqrt{-3x^2+c_1}$$

✓ Solution by Mathematica

Time used: 0.129 (sec). Leaf size: 54

```
DSolve[x y[x] (y'[x])^2+(3 x^2-2 y[x]^2)y'[x]-6 x y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to c_1 x^2$$
$$y(x) \to -\sqrt{-3x^2+2c_1}$$
$$y(x) \to \sqrt{-3x^2+2c_1}$$
$$y(x) \to 0$$

33.6 problem 968

Internal problem ID [3692]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 968.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _dAlembert]`

$$x(x-2y)\, y'^2 - 2xyy' - 2xy + y^2 = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 106

```
dsolve(x*(x-2*y(x))*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)-2*x*y(x)+y(x)^2 = 0,y(x), singsol=al
```

$$y(x) = 0$$

$$y(x) = \operatorname{RootOf}\left(-2\ln(x) - \left(\int^{\_Z} \frac{2\_a^2 + \sqrt{2\_a^3 - 4\_a^2 + 2\_a}}{\_a(\_a^2+1)} d\_a\right) + 2c_1\right) x$$

$$y(x) = \operatorname{RootOf}\left(-2\ln(x) + \int^{\_Z} \frac{\sqrt{2}\,\sqrt{\_a(\_a-1)^2 - 2\_a^2}}{\_a(\_a^2+1)} d\_a + 2c_1\right) x$$

✓ Solution by Mathematica

Time used: 4.484 (sec). Leaf size: 167

```
DSolve[x(x-2 y[x]) (y'[x])^2-2 x y[x] y'[x]-2 x y[x]+y[x]^2==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to -\sqrt{-x\left(x+2e^{\frac{c_1}{2}}\right)}-e^{\frac{c_1}{2}}$$

$$y(x) \to \sqrt{-x\left(x+2e^{\frac{c_1}{2}}\right)}-e^{\frac{c_1}{2}}$$

$$y(x) \to e^{\frac{c_1}{2}}-\sqrt{x\left(-x+2e^{\frac{c_1}{2}}\right)}$$

$$y(x) \to \sqrt{x\left(-x+2e^{\frac{c_1}{2}}\right)}+e^{\frac{c_1}{2}}$$

$$y(x) \to -\sqrt{-x^2}$$

$$y(x) \to \sqrt{-x^2}$$

33.7 problem 969

Internal problem ID [3693]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 969.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$x(x-2y)\,y'^2+6xyy'-2xy+y^2=0$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 118

```
dsolve(x*(x-2*y(x))*diff(y(x),x)^2+6*x*y(x)*diff(y(x),x)-2*x*y(x)+y(x)^2 = 0,y(x), singsol=al
```

$$y(x)=0$$

$$y(x)=\mathrm{RootOf}\left(-2\ln(x)-\left(\int^{\_Z}\frac{2\_a^2+\sqrt{2\_a^3+4\_a^2+2\_a}-4\_a}{\_a\left(\_a^2-4\_a+1\right)}d\_a\right)+2c_1\right)x$$

$$y(x)=\mathrm{RootOf}\left(-2\ln(x)+\int^{\_Z}\frac{\sqrt{2}\,\sqrt{\_a\left(\_a+1\right)^2}-2\_a^2+4\_a}{\_a\left(\_a^2-4\_a+1\right)}d\_a+2c_1\right)x$$

✓ Solution by Mathematica

Time used: 6.61 (sec). Leaf size: 196

```
DSolve[x(x-2 y[x]) (y'[x])^2+6 x y[x] y'[x]-2 x y[x]+y[x]^2==0,y[x],x,IncludeSingularSolution
```

$$y(x) \to 2x - \sqrt{x\left(3x - 2e^{\frac{c_1}{2}}\right)} - e^{\frac{c_1}{2}}$$

$$y(x) \to 2x + \sqrt{x\left(3x - 2e^{\frac{c_1}{2}}\right)} - e^{\frac{c_1}{2}}$$

$$y(x) \to 2x - \sqrt{x\left(3x + 2e^{\frac{c_1}{2}}\right)} + e^{\frac{c_1}{2}}$$

$$y(x) \to 2x + \sqrt{x\left(3x + 2e^{\frac{c_1}{2}}\right)} + e^{\frac{c_1}{2}}$$

$$y(x) \to 2x - \sqrt{3}\sqrt{x^2}$$

$$y(x) \to \sqrt{3}\sqrt{x^2} + 2x$$

33.8 problem 970

Internal problem ID [3694]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 970.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y^2 y'^2 - a^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 49

```
dsolve(y(x)^2*diff(y(x),x)^2 = a^2,y(x), singsol=all)
```

$$y(x) = \sqrt{2ax + c_1}$$
$$y(x) = -\sqrt{2ax + c_1}$$
$$y(x) = \sqrt{-2ax + c_1}$$
$$y(x) = -\sqrt{-2ax + c_1}$$

✓ Solution by Mathematica

Time used: 0.218 (sec). Leaf size: 85

```
DSolve[y[x]^2(y'[x])^2==a^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{-ax + c_1}$$
$$y(x) \to \sqrt{2}\sqrt{-ax + c_1}$$
$$y(x) \to -\sqrt{2}\sqrt{ax + c_1}$$
$$y(x) \to \sqrt{2}\sqrt{ax + c_1}$$

33.9 problem 971

Internal problem ID [3695]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 971.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y^2 y'^2 - a^2 + y^2 = 0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 59

```
dsolve(y(x)^2*diff(y(x),x)^2-a^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -a$$
$$y(x) = a$$
$$y(x) = \sqrt{a^2 - c_1^2 + 2c_1 x - x^2}$$
$$y(x) = -\sqrt{a^2 - c_1^2 + 2c_1 x - x^2}$$

✓ Solution by Mathematica

Time used: 0.215 (sec). Leaf size: 101

```
DSolve[y[x]^2 (y'[x])^2-a^2 +y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{a^2 - (x + c_1)^2}$$
$$y(x) \to \sqrt{a^2 - (x + c_1)^2}$$
$$y(x) \to -\sqrt{a^2 - (x - c_1)^2}$$
$$y(x) \to \sqrt{a^2 - (x - c_1)^2}$$
$$y(x) \to -a$$
$$y(x) \to a$$

33.10 problem 972

Internal problem ID [3696]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 972.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$y^2y'^2 - 3y'x + y = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 124

```
dsolve(y(x)^2*diff(y(x),x)^2-3*x*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{18^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{18^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{4} - \frac{i\sqrt{3}\,18^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{4}$$
$$y(x) = -\frac{18^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{4} + \frac{i\sqrt{3}\,18^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{4}$$
$$y(x) = 0$$
$$y(x) = \text{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{3\left(4\_a^3 - 3\sqrt{-4\_a^3+9} - 9\right)}{2\_a\left(4\_a^3-9\right)}d\_a + c_1\right)x^{\frac{2}{3}}$$

✓ Solution by Mathematica

Time used: 0.525 (sec). Leaf size: 247

```
DSolve[y[x]^2 (y'[x])^2-3 x y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{c_1}{3}} \sqrt[3]{e^{c_1} - 3ix}$$
$$y(x) \to -\sqrt[3]{-1} e^{\frac{c_1}{3}} \sqrt[3]{e^{c_1} - 3ix}$$
$$y(x) \to (-1)^{2/3} e^{\frac{c_1}{3}} \sqrt[3]{e^{c_1} - 3ix}$$
$$y(x) \to e^{\frac{c_1}{3}} \sqrt[3]{3ix + e^{c_1}}$$
$$y(x) \to -\sqrt[3]{-1} e^{\frac{c_1}{3}} \sqrt[3]{3ix + e^{c_1}}$$
$$y(x) \to (-1)^{2/3} e^{\frac{c_1}{3}} \sqrt[3]{3ix + e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to \left(-\frac{3}{2}\right)^{2/3} x^{2/3}$$
$$y(x) \to \left(\frac{3}{2}\right)^{2/3} x^{2/3}$$
$$y(x) \to -\sqrt[3]{-1} \left(\frac{3}{2}\right)^{2/3} x^{2/3}$$

33.11 problem 973

Internal problem ID [3697]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 973.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y^2 {y'}^2 - 6x^3 y' + 4yx^2 = 0$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 118

```
dsolve(y(x)^2*diff(y(x),x)^2-6*x^3*diff(y(x),x)+4*x^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{18^{\frac{1}{3}} x^{\frac{4}{3}}}{2}$$

$$y(x) = \left(-\frac{18^{\frac{1}{3}} x^{\frac{1}{3}}}{4} - \frac{i\sqrt{3}\, 18^{\frac{1}{3}} x^{\frac{1}{3}}}{4}\right) x$$

$$y(x) = \left(-\frac{18^{\frac{1}{3}} x^{\frac{1}{3}}}{4} + \frac{i\sqrt{3}\, 18^{\frac{1}{3}} x^{\frac{1}{3}}}{4}\right) x$$

$$y(x) = 0$$

$$y(x) = \operatorname{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{3\left(4\_a^3 - 3\sqrt{-4\_a^3 + 9} - 9\right)}{4\_a\left(4\_a^3 - 9\right)} d\_a + c_1\right) x^{\frac{4}{3}}$$

✓ Solution by Mathematica

Time used: 2.378 (sec). Leaf size: 304

```
DSolve[y[x]^2 (y'[x])^2-6 x^3 y'[x]+4 x^2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{\sqrt{9x^6-4x^2y(x)^3}\log\left(\sqrt{9x^4-4y(x)^3}+3x^2\right)}{2x\sqrt{9x^4-4y(x)^3}} - \frac{3}{4}\left(\frac{\sqrt{9x^6-4x^2y(x)^3}\log(y(x))}{x\sqrt{9x^4-4y(x)^3}}-\log(y(x))\right)=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{3}{4}\left(\frac{\sqrt{9x^6-4x^2y(x)^3}\log(y(x))}{x\sqrt{9x^4-4y(x)^3}}+\log(y(x))\right) - \frac{\sqrt{9x^6-4x^2y(x)^3}\log\left(\sqrt{9x^4-4y(x)^3}+3x^2\right)}{2x\sqrt{9x^4-4y(x)^3}}=c_1,y(x)\right]$$

$$y(x)\to\left(-\frac{3}{2}\right)^{2/3}x^{4/3}$$

$$y(x)\to\left(\frac{3}{2}\right)^{2/3}x^{4/3}$$

$$y(x)\to-\sqrt[3]{-1}\left(\frac{3}{2}\right)^{2/3}x^{4/3}$$

33.12 problem 974

Internal problem ID [3698]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 974.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y^2y'^2 - 4ayy' + 4a^2 - 4ax + y^2 = 0$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 71

```
dsolve(y(x)^2*diff(y(x),x)^2-4*a*y(x)*diff(y(x),x)+4*a^2-4*a*x+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -2\sqrt{ax}$$
$$y(x) = 2\sqrt{ax}$$
$$y(x) = \sqrt{4ax - c_1^2 + 2c_1x - x^2}$$
$$y(x) = -\sqrt{4ax - c_1^2 + 2c_1x - x^2}$$

✓ Solution by Mathematica

Time used: 0.648 (sec). Leaf size: 83

```
DSolve[y[x]^2 (y'[x])^2-4 a y[x] y'[x]+4 a^2-4 a x+y[x]^2==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to -\frac{\sqrt{4a^2x(4a-x) - 4ac_1x - c_1{}^2}}{2a}$$
$$y(x) \to \frac{\sqrt{4a^2x(4a-x) - 4ac_1x - c_1{}^2}}{2a}$$

33.13 problem 975

Internal problem ID [3699]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 975.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$y^2 y'^2 - (x+1)\, y y' + x = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 45

```
dsolve(y(x)^2*diff(y(x),x)^2-(1+x)*y(x)*diff(y(x),x)+x = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{2x + c_1}$$
$$y(x) = -\sqrt{2x + c_1}$$
$$y(x) = \sqrt{x^2 + c_1}$$
$$y(x) = -\sqrt{x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.123 (sec). Leaf size: 72

```
DSolve[y[x]^2 (y'[x])^2-(1+x)y[x] y'[x]+x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2}\sqrt{x + c_1}$$
$$y(x) \to \sqrt{2}\sqrt{x + c_1}$$
$$y(x) \to -\sqrt{x^2 + 2c_1}$$
$$y(x) \to \sqrt{x^2 + 2c_1}$$

33.14 problem 976

Internal problem ID [3700]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 976.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$\boxed{y^2 {y'}^2 + 2xyy' + x^2 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(y(x)^2*diff(y(x),x)^2+2*x*y(x)*diff(y(x),x)+x^2 = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 + 2c_1}$$
$$y(x) = -\sqrt{-x^2 + 2c_1}$$

✓ Solution by Mathematica

Time used: 0.049 (sec). Leaf size: 39

```
DSolve[y[x]^2 (y'[x])^2+2 x y[x] y'[x]+x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-x^2 + 2c_1}$$
$$y(x) \to \sqrt{-x^2 + 2c_1}$$

33.15 problem 977

Internal problem ID [3701]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 977.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{y^2 y'^2 + 2xyy' + a - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 57

```
dsolve(y(x)^2*diff(y(x),x)^2+2*x*y(x)*diff(y(x),x)+a-y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x^2 + a}$$
$$y(x) = -\sqrt{-x^2 + a}$$
$$y(x) = \sqrt{c_1^2 - 2c_1 x + a}$$
$$y(x) = -\sqrt{c_1^2 - 2c_1 x + a}$$

✓ Solution by Mathematica

Time used: 0.52 (sec). Leaf size: 61

```
DSolve[y[x]^2 (y'[x])^2+2 x y[x] y'[x]+a-y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{a + c_1(-2x + c_1)}$$
$$y(x) \to \sqrt{a + c_1(-2x + c_1)}$$
$$y(x) \to -\sqrt{a}$$
$$y(x) \to \sqrt{a}$$

33.16 problem 978

Internal problem ID [3702]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 978.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$y^2 {y'}^2 - 2xyy' - x^2 + 2y^2 = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 107

```
dsolve(y(x)^2*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)-x^2+2*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -x$$
$$y(x) = x$$
$$y(x) = \sqrt{-2\sqrt{2}\, x c_1 - c_1^2 - x^2}$$
$$y(x) = \sqrt{2\sqrt{2}\, x c_1 - c_1^2 - x^2}$$
$$y(x) = -\sqrt{-2\sqrt{2}\, x c_1 - c_1^2 - x^2}$$
$$y(x) = -\sqrt{2\sqrt{2}\, x c_1 - c_1^2 - x^2}$$

✓ Solution by Mathematica

Time used: 6.505 (sec). Leaf size: 169

```
DSolve[y[x]^2 (y'[x])^2-2 x y[x] y'[x]-x^2+2 y[x]^2==0,y[x],x,IncludeSingularSolutions -> Tru
```

$$y(x) \to -\sqrt{-x^2 - 4\sqrt{2}e^{c_1}x - 4e^{2c_1}}$$
$$y(x) \to \sqrt{-x^2 - 4\sqrt{2}e^{c_1}x - 4e^{2c_1}}$$
$$y(x) \to -\sqrt{-x^2 + 4\sqrt{2}e^{c_1}x - 4e^{2c_1}}$$
$$y(x) \to \sqrt{-x^2 + 4\sqrt{2}e^{c_1}x - 4e^{2c_1}}$$
$$y(x) \to -\sqrt{-x^2}$$
$$y(x) \to \sqrt{-x^2}$$

33.17 problem 979

Internal problem ID [3703]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 979.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{y^2{y'}^2 - 2xyy' + a - x^2 + 2y^2 = 0}$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 83

```
dsolve(y(x)^2*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)+a-x^2+2*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{4x^2-2a}}{2}$$
$$y(x) = \frac{\sqrt{4x^2-2a}}{2}$$
$$y(x) = -\frac{\sqrt{-8c_1^2+16c_1x-4x^2-2a}}{2}$$
$$y(x) = \frac{\sqrt{-8c_1^2+16c_1x-4x^2-2a}}{2}$$

✓ Solution by Mathematica

Time used: 0.667 (sec). Leaf size: 63

```
DSolve[y[x]^2 (y'[x])^2-2 x y[x] y'[x]+a -x^2+2 y[x]^2==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\sqrt{-\frac{a}{2}-x^2+4c_1x-2{c_1}^2}$$
$$y(x) \to \sqrt{-\frac{a}{2}-x^2+4c_1x-2{c_1}^2}$$

33.18 problem 980

Internal problem ID [3704]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 980.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$y^2 {y'}^2 + 2axyy' + (a-1)\,b + a\,x^2 + (1-a)\,y^2 = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 93

```
dsolve(y(x)^2*diff(y(x),x)^2+2*a*x*y(x)*diff(y(x),x)+(a-1)*b+a*x^2+(1-a)*y(x)^2 = 0,y(x), sin
```

$$y(x) = \sqrt{-a\,x^2 + b}$$
$$y(x) = -\sqrt{-a\,x^2 + b}$$
$$y(x) = \sqrt{c_1^2 a - 2ac_1 x - c_1^2 + 2c_1 x - x^2 + b}$$
$$y(x) = -\sqrt{c_1^2 a - 2ac_1 x - c_1^2 + 2c_1 x - x^2 + b}$$

✓ Solution by Mathematica

Time used: 1.072 (sec). Leaf size: 65

```
DSolve[y[x]^2 (y'[x])^2+2 a x y[x] y'[x]+(a-1)b+a x^2+(1-a)y[x]^2==0,y[x],x,IncludeSingularSo
```

$$y(x) \to -\sqrt{-2(a-1)c_1 x + (a-1)c_1{}^2 + b - x^2}$$
$$y(x) \to \sqrt{-2(a-1)c_1 x + (a-1)c_1{}^2 + b - x^2}$$

33.19 problem 981

Internal problem ID [3705]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 981.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(1-y^2)\, y'^2 - 1 = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 48

```
dsolve((1-y(x)^2)*diff(y(x),x)^2 = 1,y(x), singsol=all)
```

$$y(x) = \sin\left(\mathrm{RootOf}\left(\sin(\_Z)\sqrt{\frac{\cos(2\_Z)}{2}+\frac{1}{2}}+\_Z+2c_1-2x\right)\right)$$
$$y(x) = \sin\left(\mathrm{RootOf}\left(-\sin(\_Z)\sqrt{\frac{\cos(2\_Z)}{2}+\frac{1}{2}}-\_Z+2c_1-2x\right)\right)$$

✓ Solution by Mathematica

Time used: 0.058 (sec). Leaf size: 105

```
DSolve[(1-y[x]^2) (y'[x])^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \mathrm{InverseFunction}\left[\frac{1}{2}\#1\sqrt{1-\#1^2}-\arctan\left(\frac{\sqrt{1-\#1^2}}{\#1+1}\right)\&\right][-x+c_1]$$
$$y(x) \to \mathrm{InverseFunction}\left[\frac{1}{2}\#1\sqrt{1-\#1^2}-\arctan\left(\frac{\sqrt{1-\#1^2}}{\#1+1}\right)\&\right][x+c_1]$$

33.20 problem 982

Internal problem ID [3706]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 982.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(-y^2 + a^2)\, y'^2 - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 126

```
dsolve((a^2-y(x)^2)*diff(y(x),x)^2 = y(x)^2,y(x), singsol=all)
```

$$y(x) = 0$$
$$x - \sqrt{a^2 - y(x)^2} + \frac{a^2 \ln\left(\frac{2a^2+2\sqrt{a^2}\sqrt{a^2-y(x)^2}}{y(x)}\right)}{\sqrt{a^2}} - c_1 = 0$$
$$x + \sqrt{a^2 - y(x)^2} - \frac{a^2 \ln\left(\frac{2a^2+2\sqrt{a^2}\sqrt{a^2-y(x)^2}}{y(x)}\right)}{\sqrt{a^2}} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.325 (sec). Leaf size: 102

```
DSolve[(a^2-y[x]^2) (y'[x])^2==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\sqrt{a^2 - \#1^2} - a\text{arctanh}\left(\frac{\sqrt{a^2 - \#1^2}}{a}\right) \&\right][-x + c_1]$$
$$y(x) \to \text{InverseFunction}\left[\sqrt{a^2 - \#1^2} - a\text{arctanh}\left(\frac{\sqrt{a^2 - \#1^2}}{a}\right) \&\right][x + c_1]$$
$$y(x) \to 0$$

33.21 problem 983

Internal problem ID [3707]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 983.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{(a^2 - 2axy + y^2)\, y'^2 + 2ayy' + y^2 = 0}$$

✗ Solution by Maple

```
dsolve((a^2-2*a*x*y(x)+y(x)^2)*diff(y(x),x)^2+2*a*y(x)*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(a^2-2 a x y[x]+y[x]^2) (y'[x])^2+2 a y[x] y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolut
```

Not solved

33.22 problem 985

Internal problem ID [3708]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 985.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\boxed{((1-a)\,x^2+y^2)\,y'^2+2axyy'+x^2+(1-a)\,y^2=0}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 75

```
dsolve(((1-a)*x^2+y(x)^2)*diff(y(x),x)^2+2*a*x*y(x)*diff(y(x),x)+x^2+(1-a)*y(x)^2 = 0,y(x), s
```

$$y(x) = -ix$$
$$y(x) = ix$$
$$y(x) = \tan\left(\operatorname{RootOf}\left(-2\_Z\sqrt{a-1}-\ln\left(\frac{x^2}{\cos\left(\_Z\right)^2}\right)+2c_1\right)\right)x$$
$$y(x) = \tan\left(\operatorname{RootOf}\left(2\_Z\sqrt{a-1}-\ln\left(\frac{x^2}{\cos\left(\_Z\right)^2}\right)+2c_1\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.316 (sec). Leaf size: 101

```
DSolve[((1-a)x^2+y[x]^2)(y'[x])^2+2 a x y[x] y'[x]+x^2+(1-a)y[x]^2==0,y[x],x,IncludeSingularS
```

$$\text{Solve}\left[\sqrt{a-1}\arctan\left(\frac{y(x)}{x}\right)-\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right)=\log(x)+c_1,y(x)\right]$$
$$\text{Solve}\left[\sqrt{a-1}\arctan\left(\frac{y(x)}{x}\right)+\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right)=-\log(x)+c_1,y(x)\right]$$
$$y(x)\to -ix$$
$$y(x)\to ix$$

33.23 problem 986

Internal problem ID [3709]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 986.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$\left(\left(-4a^2+1\right)x^2+y^2\right)y'^2-8a^2xyy'+x^2+\left(-4a^2+1\right)y^2=0$$

✓ Solution by Maple

Time used: 0.188 (sec). Leaf size: 154

```
dsolve(((-4*a^2+1)*x^2+y(x)^2)*diff(y(x),x)^2-8*a^2*x*y(x)*diff(y(x),x)+x^2+(-4*a^2+1)*y(x)^2
```

$$y(x)=\operatorname{RootOf}\left(-\ln(x)+\int^{\_Z}-\frac{\_a^3-8\_a\,a^2-\sqrt{(4a^2-1)\left(\_a^2+1\right)^2}+\_a}{\_a^4-16\_a^2a^2+2\_a^2+1}d\_a+c_1\right)x$$

$$y(x)=\operatorname{RootOf}\left(-\ln(x)-\left(\int^{\_Z}\frac{\_a^3-8\_a\,a^2+\sqrt{4\_a^4a^2-\_a^4+8\_a^2a^2-2\_a^2+4a^2-1}+\_a}{\_a^4-16\_a^2a^2+2\_a^2+1}d\_a\right)+c_1\right)x$$

✓ Solution by Mathematica

Time used: 1.468 (sec). Leaf size: 328

```
DSolve[((1-4 a^2)x^2+y[x]^2) (y'[x])^2 - 8 a^2 x y[x] y'[x]+x^2+(1-4 a^2)y[x]^2==0,y[x],x,Inc
```

$$\text{Solve}\left[\frac{1}{4}\left(-\frac{2\sqrt{2a-1}\sqrt{2a+1}\arctan\left(\frac{\frac{y(x)}{x}-2a}{\sqrt{1-4a^2}}\right)}{\sqrt{1-4a^2}}-\frac{2\sqrt{2a-1}\sqrt{2a+1}\arctan\left(\frac{2a+\frac{y(x)}{x}}{\sqrt{1-4a^2}}\right)}{\sqrt{1-4a^2}}\right.\right.$$

$$\left.\left.+\log\left(-\frac{4ay(x)}{x}+\frac{y(x)^2}{x^2}+1\right)+\log\left(\frac{4ay(x)}{x}+\frac{y(x)^2}{x^2}+1\right)\right)=-\log(x)+c_1,y(x)\right]$$

$$\text{Solve}\left[-\frac{-2\sqrt{2a-1}\sqrt{2a+1}\arctan\left(\frac{\frac{y(x)}{x}-2a}{\sqrt{1-4a^2}}\right)-2\sqrt{2a-1}\sqrt{2a+1}\arctan\left(\frac{2a+\frac{y(x)}{x}}{\sqrt{1-4a^2}}\right)-\sqrt{1-4a^2}\left(\log\left(-\frac{4}{}\right.\right.}{4\sqrt{1-4a^2}}\right.$$

$$\left.-\log(x)+c_1,y(x)\right]$$

33.24 problem 987

Internal problem ID [3710]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 987.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\left(\left(-a^2+1\right)x^2+y^2\right)y'^2+2a^2xyy'+x^2+\left(-a^2+1\right)y^2=0$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 79

```
dsolve(((-a^2+1)*x^2+y(x)^2)*diff(y(x),x)^2+2*a^2*x*y(x)*diff(y(x),x)+x^2+(-a^2+1)*y(x)^2 = 0
```

$$y(x)=-ix$$
$$y(x)=ix$$
$$y(x)=\tan\left(\operatorname{RootOf}\left(-2\_Z\sqrt{a^2-1}-\ln\left(\frac{x^2}{\cos\left(\_Z\right)^2}\right)+2c_1\right)\right)x$$
$$y(x)=\tan\left(\operatorname{RootOf}\left(2\_Z\sqrt{a^2-1}-\ln\left(\frac{x^2}{\cos\left(\_Z\right)^2}\right)+2c_1\right)\right)x$$

✓ Solution by Mathematica

Time used: 0.372 (sec). Leaf size: 115

```
DSolve[((1-a^2)x^2+y[x]^2) (y'[x])^2 +2 a^2 x y[x] y'[x]+x^2+(1-a^2) y[x]^2==0,y[x],x,Include
```

$$\text{Solve}\left[\sqrt{a-1}\sqrt{a+1}\arctan\left(\frac{y(x)}{x}\right)-\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right)=\log(x)+c_1,y(x)\right]$$
$$\text{Solve}\left[\sqrt{a-1}\sqrt{a+1}\arctan\left(\frac{y(x)}{x}\right)+\frac{1}{2}\log\left(\frac{y(x)^2}{x^2}+1\right)=-\log(x)+c_1,y(x)\right]$$
$$y(x)\to -ix$$
$$y(x)\to ix$$

33.25 problem 988

Internal problem ID [3711]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 988.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$(y+x)^2 {y'}^2 - y^2 = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 48

```
dsolve((x+y(x))^2*diff(y(x),x)^2 = y(x)^2,y(x), singsol=all)
```

$$y(x) = \mathrm{e}^{\mathrm{LambertW}(x\,\mathrm{e}^{c_1})-c_1}$$
$$y(x) = -x - \sqrt{x^2+2c_1}$$
$$y(x) = -x + \sqrt{x^2+2c_1}$$

✓ Solution by Mathematica

Time used: 3.927 (sec). Leaf size: 101

```
DSolve[(x+y[x])^2 (y'[x])^2==y[x]^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -x - \sqrt{x^2+e^{2c_1}}$$
$$y(x) \to -x + \sqrt{x^2+e^{2c_1}}$$
$$y(x) \to \frac{x}{W\left(e^{-c_1}x\right)}$$
$$y(x) \to 0$$
$$y(x) \to -\sqrt{x^2} - x$$
$$y(x) \to \sqrt{x^2} - x$$

33.26 problem 989

Internal problem ID [3712]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 989.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$
(y+x)^2 y'^2 - \left(x^2 - xy - 2y^2\right) y' - (-y+x)\, y = 0
$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 85

```
dsolve((x+y(x))^2*diff(y(x),x)^2-(x^2-x*y(x)-2*y(x)^2)*diff(y(x),x)-(x-y(x))*y(x) = 0,y(x), s
```

$$
y(x) = -x - \sqrt{x^2 + 2c_1}
$$
$$
y(x) = -x + \sqrt{x^2 + 2c_1}
$$
$$
y(x) = \frac{-c_1 x - \sqrt{2c_1^2 x^2 + 1}}{c_1}
$$
$$
y(x) = \frac{-c_1 x + \sqrt{2c_1^2 x^2 + 1}}{c_1}
$$

✓ Solution by Mathematica

Time used: 0.502 (sec). Leaf size: 172

```
DSolve[(x+y[x])^2 (y'[x])^2 -(x^2-x y[x]-2 y[x]^2) y'[x]-(x-y[x])y[x]==0,y[x],x,IncludeSingul
```

$$y(x) \to -x - \sqrt{x^2 + e^{2c_1}}$$
$$y(x) \to -x + \sqrt{x^2 + e^{2c_1}}$$
$$y(x) \to -x - \sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to -x + \sqrt{2x^2 + e^{2c_1}}$$
$$y(x) \to -\sqrt{x^2} - x$$
$$y(x) \to \sqrt{x^2} - x$$
$$y(x) \to -\sqrt{2}\sqrt{x^2} - x$$
$$y(x) \to \sqrt{2}\sqrt{x^2} - x$$

33.27 problem 990

Internal problem ID [3713]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 990.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _dAlembert]`

$$\left(a^2-(-y+x)^2\right)y'^2+2a^2y'+a^2-(-y+x)^2=0$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 130

```
dsolve((a^2-(x-y(x))^2)*diff(y(x),x)^2+2*a^2*diff(y(x),x)+a^2-(x-y(x))^2 = 0,y(x), singsol=al
```

$$y(x)=x-\sqrt{2}\,a$$
$$y(x)=x+\sqrt{2}\,a$$
$$y(x)=x+\operatorname{RootOf}\left(-x+\int^{\_Z}-\frac{\_a^2-2a^2+\sqrt{-\_a^2\left(\_a^2-2a^2\right)}}{2\left(\_a^2-2a^2\right)}d\_a+c_1\right)$$
$$y(x)=x+\operatorname{RootOf}\left(-x+\int^{\_Z}-\frac{-2a^2+\_a^2-\sqrt{-\_a^2\left(\_a^2-2a^2\right)}}{2\left(\_a^2-2a^2\right)}d\_a+c_1\right)$$

✓ Solution by Mathematica

Time used: 50.988 (sec). Leaf size: 18407

```
DSolve[(a^2-(x-y[x])^2)(y'[x])^2+2 a^2 y'[x]+a^2-(x-y[x])^2==0,y[x],x,IncludeSingularSolution
```

Too large to display

33.28 problem 991

Internal problem ID [3714]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 991.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$2y^2y'^2 + 2xyy' - 1 + x^2 + y^2 = 0$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 103

```
dsolve(2*y(x)^2*diff(y(x),x)^2+2*x*y(x)*diff(y(x),x)-1+x^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2x^2+4}}{2}$$
$$y(x) = \frac{\sqrt{-2x^2+4}}{2}$$
$$y(x) = \sqrt{\text{RootOf}\left(-2\ln(x) + 2\,\text{arctanh}\left(\sqrt{-2\_Z-1}\right) - \ln(\_Z+1) + 2c_1\right)x^2 + 1}$$
$$y(x) = -\sqrt{\text{RootOf}\left(-2\ln(x) + 2\,\text{arctanh}\left(\sqrt{-2\_Z-1}\right) - \ln(\_Z+1) + 2c_1\right)x^2 + 1}$$

✓ Solution by Mathematica

Time used: 0.541 (sec). Leaf size: 57

```
DSolve[2 y[x]^2 (y'[x])^2 +2 x y[x] y'[x]-1+x^2+y[x]^2==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\sqrt{-x^2 + c_1x + 1 - \frac{c_1{}^2}{2}}$$
$$y(x) \to \sqrt{-x^2 + c_1x + 1 - \frac{c_1{}^2}{2}}$$

33.29 problem 992

Internal problem ID [3715]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 992.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$3y^2{y'}^2 - 2xyy' - x^2 + 4y^2 = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 203

```
dsolve(3*y(x)^2*diff(y(x),x)^2-2*x*y(x)*diff(y(x),x)-x^2+4*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{3}\,x}{3}$$

$$y(x) = \frac{\sqrt{3}\,x}{3}$$

$$\ln(x) - \frac{\sqrt{3}\sqrt{\frac{(\sqrt{3}\,x-3y(x))(\sqrt{3}\,x+3y(x))}{x^2}}}{6} + \frac{\sqrt{\frac{x^2-3y(x)^2}{x^2}}}{2} - \operatorname{arctanh}\left(\frac{\sqrt{\frac{x^2-3y(x)^2}{x^2}}}{2}\right) + \frac{\ln\left(\frac{x^2+y(x)^2}{x^2}\right)}{2} - c_1 = 0$$

$$\ln(x) + \frac{\sqrt{3}\sqrt{\frac{(\sqrt{3}\,x-3y(x))(\sqrt{3}\,x+3y(x))}{x^2}}}{6} - \frac{\sqrt{\frac{x^2-3y(x)^2}{x^2}}}{2} + \operatorname{arctanh}\left(\frac{\sqrt{\frac{x^2-3y(x)^2}{x^2}}}{2}\right) + \frac{\ln\left(\frac{x^2+y(x)^2}{x^2}\right)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.565 (sec). Leaf size: 179

```
DSolve[3  y[x]^2 (y'[x])^2 -2 x y[x] y'[x]-x^2+4 y[x]^2==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to -\frac{\sqrt{-3x^2 - 4ie^{3c_1}x + e^{6c_1}}}{\sqrt{3}}$$

$$y(x) \to \frac{\sqrt{-3x^2 - 4ie^{3c_1}x + e^{6c_1}}}{\sqrt{3}}$$

$$y(x) \to -\frac{\sqrt{-3x^2 + 4ie^{3c_1}x + e^{6c_1}}}{\sqrt{3}}$$

$$y(x) \to \frac{\sqrt{-3x^2 + 4ie^{3c_1}x + e^{6c_1}}}{\sqrt{3}}$$

$$y(x) \to -\sqrt{-x^2}$$

$$y(x) \to \sqrt{-x^2}$$

33.30 problem 993

Internal problem ID [3716]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 993.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_separable]`

$$4y^2{y'}^2 + 2(1+3x)\,xyy' + 3x^3 = 0$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 59

```
dsolve(4*y(x)^2*diff(y(x),x)^2+2*(1+3*x)*x*y(x)*diff(y(x),x)+3*x^3 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-2x^2+4c_1}}{2}$$
$$y(x) = \frac{\sqrt{-2x^2+4c_1}}{2}$$
$$y(x) = \sqrt{-x^3+c_1}$$
$$y(x) = -\sqrt{-x^3+c_1}$$

✓ Solution by Mathematica

Time used: 0.145 (sec). Leaf size: 81

```
DSolve[4 y[x]^2 (y'[x])^2 +2(1+3 x)x y[x] y'[x]+3 x^3==0,y[x],x,IncludeSingularSolutions -> T
```

$$y(x) \to -\sqrt{-x^3+2c_1}$$
$$y(x) \to \sqrt{-x^3+2c_1}$$
$$y(x) \to -\sqrt{-\frac{x^2}{2}+2c_1}$$
$$y(x) \to \sqrt{-\frac{x^2}{2}+2c_1}$$

33.31 problem 994

Internal problem ID [3717]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 994.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, [_Abel, ‘2nd type‘, ‘cla`

$$\left(x^2-4y^2\right)y'^2+6xyy'-4x^2+y^2=0$$

✓ Solution by Maple

Time used: 1.281 (sec). Leaf size: 92

```
dsolve((x^2-4*y(x)^2)*diff(y(x),x)^2+6*x*y(x)*diff(y(x),x)-4*x^2+y(x)^2 = 0,y(x), singsol=all
```

$$y(x)=-\frac{x\left(\operatorname{RootOf}\left(\_Z^{16}+2\_Z^4c_1x^4-c_1x^4\right)^4-1\right)}{\operatorname{RootOf}\left(\_Z^{16}+2\_Z^4c_1x^4-c_1x^4\right)^4}$$

$$y(x)=\frac{\frac{\operatorname{RootOf}\left(\_Z^{16}-2\_Z^4c_1x^4-c_1x^4\right)^{12}}{c_1}-x^4}{x^3}$$

✓ Solution by Mathematica

Time used: 60.112 (sec). Leaf size: 3017

```
DSolve[(x^2-4 y[x]^2) (y'[x])^2 +6 x y[x] y'[x]-4 x^2+y[x]^2==0,y[x],x,IncludeSingularSolutio
```

Too large to display

33.32 problem 995

Internal problem ID [3718]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 33
Problem number: 995.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$9y^2y'^2 - 3y'x + y = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 124

```
dsolve(9*y(x)^2*diff(y(x),x)^2-3*x*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{4} - \frac{i\sqrt{3}\,2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{4}$$
$$y(x) = -\frac{2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{4} + \frac{i\sqrt{3}\,2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{4}$$
$$y(x) = 0$$
$$y(x) = \operatorname{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{3\left(4\_a^3 - \sqrt{-4\_a^3+1} - 1\right)}{2\_a\,(4\_a^3-1)}d\_a + c_1\right) x^{\frac{2}{3}}$$

✓ Solution by Mathematica

Time used: 0.512 (sec). Leaf size: 243

```
DSolve[9 y[x]^2 (y'[x])^2 -3 x y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to e^{\frac{c_1}{3}} \sqrt[3]{e^{c_1} - ix}$$

$$y(x) \to -\sqrt[3]{-1} e^{\frac{c_1}{3}} \sqrt[3]{e^{c_1} - ix}$$

$$y(x) \to (-1)^{2/3} e^{\frac{c_1}{3}} \sqrt[3]{e^{c_1} - ix}$$

$$y(x) \to e^{\frac{c_1}{3}} \sqrt[3]{ix + e^{c_1}}$$

$$y(x) \to -\sqrt[3]{-1} e^{\frac{c_1}{3}} \sqrt[3]{ix + e^{c_1}}$$

$$y(x) \to (-1)^{2/3} e^{\frac{c_1}{3}} \sqrt[3]{ix + e^{c_1}}$$

$$y(x) \to 0$$

$$y(x) \to \left(-\frac{1}{2}\right)^{2/3} x^{2/3}$$

$$y(x) \to \frac{x^{2/3}}{2^{2/3}}$$

$$y(x) \to -\frac{\sqrt[3]{-1} x^{2/3}}{2^{2/3}}$$

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34.1 problem 996

Internal problem ID [3719]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 996.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{(2-3y)^2 y'^2 - 4 + 4y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 713

```
dsolve((2-3*y(x))^2*diff(y(x),x)^2 = 4-4*y(x),y(x), singsol=all)
```

$$y(x) = 1$$

$$y(x) = -\left(\frac{\left(108c_1 - 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{1}{3}}}{6} + \frac{2}{\left(108c_1 - 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2}\right.}\right. + 1$$

$$y(x) = -\left(-\frac{\left(108c_1 - 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{1}{3}}}{12} - \frac{1}{\left(108c_1 - 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x}\right.}\right. + 1$$

$$y(x) = -\left(-\frac{\left(108c_1 - 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{1}{3}}}{12} - \frac{1}{\left(108c_1 - 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x}\right.}\right. + 1$$

$$y(x) = -\left(\frac{\left(-108c_1 + 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{1}{3}}}{6} + \frac{2}{\left(-108c_1 + 108x + 12\sqrt{81c_1^2 - 162c_1x + 8}\right.}\right. + 1$$

$$y(x) = -\left(-\frac{\left(-108c_1 + 108x + 12\sqrt{81c_1^2 - 162c_1x + 81x^2 - 12}\right)^{\frac{1}{3}}}{12} - \frac{1}{\left(-108c_1 + 108x + 12\sqrt{81c_1^2 - 162c_1x +}\right.}\right. + 1$$

$$y(x) =$$

✓ Solution by Mathematica

Time used: 4.539 (sec). Leaf size: 746

```
DSolve[(2-3 y[x])^2 (y'[x])^2 ==4(1-y[x]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{6}\left(\sqrt[3]{-27(2x+c_1)^2+3\sqrt{3}\sqrt{(2x+c_1)^2\left(-16+27(2x+c_1)^2\right)}+8} + \frac{4}{\sqrt[3]{-27(2x+c_1)^2+3\sqrt{3}\sqrt{(2x+c_1)^2\left(-16+27(2x+c_1)^2\right)}+8}}+2\right)$$

$$y(x) \to \frac{1}{24}\left(2i\left(\sqrt{3}+i\right)\sqrt[3]{-27(2x+c_1)^2+3\sqrt{3}\sqrt{(2x+c_1)^2\left(-16+27(2x+c_1)^2\right)}+8} + \frac{-8-8i\sqrt{3}}{\sqrt[3]{-27(2x+c_1)^2+3\sqrt{3}\sqrt{(2x+c_1)^2\left(-16+27(2x+c_1)^2\right)}+8}}+8\right)$$

$$y(x) \to \frac{1}{24}\left(-2\left(1+i\sqrt{3}\right)\sqrt[3]{-27(2x+c_1)^2+3\sqrt{3}\sqrt{(2x+c_1)^2\left(-16+27(2x+c_1)^2\right)}+8} + \frac{-8+8i\sqrt{3}}{\sqrt[3]{-27(2x+c_1)^2+3\sqrt{3}\sqrt{(2x+c_1)^2\left(-16+27(2x+c_1)^2\right)}+8}}+8\right)$$

$$y(x) \to \frac{1}{6}\left(\sqrt[3]{-27(-2x+c_1)^2+3\sqrt{3}\sqrt{(-2x+c_1)^2\left(-16+27(-2x+c_1)^2\right)}+8} + \frac{4}{\sqrt[3]{-27(-2x+c_1)^2+3\sqrt{3}\sqrt{(-2x+c_1)^2\left(-16+27(-2x+c_1)^2\right)}+8}}+2\right)$$

$$y(x) \to \frac{1}{24}\left(2i\left(\sqrt{3}+i\right)\sqrt[3]{-27(-2x+c_1)^2+3\sqrt{3}\sqrt{(-2x+c_1)^2\left(-16+27(-2x+c_1)^2\right)}+8} + \frac{-8-8i\sqrt{3}}{\sqrt[3]{-27(-2x+c_1)^2+3\sqrt{3}\sqrt{(-2x+c_1)^2\left(-16+27(-2x+c_1)^2\right)}+8}}+8\right)$$

$$y(x) \to \frac{1}{24}\left(-2\left(1+i\sqrt{3}\right)\sqrt[3]{-27(-2x+c_1)^2+3\sqrt{3}\sqrt{(-2x+c_1)^2\left(-16+27(-2x+c_1)^2\right)}+8} + \frac{-8+8i\sqrt{3}}{\sqrt[3]{-27(-2x+c_1)^2+3\sqrt{3}\sqrt{(-2x+c_1)^2\left(-16+27(-2x+c_1)^2\right)}+8}}+8\right)$$

34.2 problem 997

Internal problem ID [3720]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 997.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$(-a^2+1)\,y^2y'^2 - 3a^2xyy' - a^2x^2 + y^2 = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 198

```
dsolve((-a^2+1)*y(x)^2*diff(y(x),x)^2-3*a^2*x*y(x)*diff(y(x),x)-a^2*x^2+y(x)^2 = 0,y(x), sing
```

$$y(x) = \operatorname{RootOf}\left(-2\ln(x) - \left(\int^{\_Z} \frac{\left(2\_a^2a^2 - 2\_a^2 + 3a^2 + \sqrt{4\_a^2a^2 + 5a^4 - 4\_a^2 + 4a^2}\right)\_a}{\_a^4a^2 - \_a^4 + 3\_a^2a^2 - \_a^2 + a^2}d\_a\right) + 2c_1\right)x$$

$$y(x) = \operatorname{RootOf}\left(-2\ln(x) + \int^{\_Z} -\frac{\left(2\_a^2a^2 - 2\_a^2 + 3a^2 - \sqrt{4\_a^2a^2 + 5a^4 - 4\_a^2 + 4a^2}\right)\_a}{\_a^4a^2 - \_a^4 + 3\_a^2a^2 - \_a^2 + a^2}d\_a + 2c_1\right)x$$

✓ Solution by Mathematica

Time used: 1.264 (sec). Leaf size: 342

```
DSolve[(1-a^2)y[x]^2 (y'[x])^2 -2 a^2 x y[x] y'[x]-a^2 x y[x] y'[x]-a^2 x^2+y[x]^2==0,y[x],x,
```

$$\text{Solve}\left[\frac{\log\left(-\left(a^2\left(\frac{2y(x)^2}{x^2}+3\right)\right)+\sqrt{5a^4+4a^2\left(\frac{y(x)^2}{x^2}+1\right)-\frac{4y(x)^2}{x^2}}+\frac{2y(x)^2}{x^2}\right)-\frac{2\arctan\left(\frac{1-\sqrt{5a^4+4a^2\left(\frac{y(x)^2}{x^2}+1\right)-4y\cdots}}{\sqrt{-5a^4+2a^2-1}}\right.}{\sqrt{-5a^4+2a^2-1}}}{4a^2-4}\right.$$

$$\left.+c_1, y(x)\right]$$

$$\text{Solve}\left[\frac{\log\left(a^2\left(\frac{2y(x)^2}{x^2}+3\right)+\sqrt{5a^4+4a^2\left(\frac{y(x)^2}{x^2}+1\right)-\frac{4y(x)^2}{x^2}}-\frac{2y(x)^2}{x^2}\right)-\frac{2\arctan\left(\frac{\sqrt{5a^4+4a^2\left(\frac{y(x)^2}{x^2}+1\right)-\frac{4y(x)^2}{x^2}+1}}{\sqrt{-5a^4+2a^2-1}}\right)}{\sqrt{-5a^4+2a^2-1}}}{4a^2-4}\right.$$

$$\left.+c_1, y(x)\right]$$

34.3 problem 998

Internal problem ID [3721]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 998.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational, [_1st_order, '_with_symmetry_[F(x),G(y)]']]`

$$\boxed{(a-b)\,y^2{y'}^2-2bxyy'-ab-b\,x^2+y^2a=0}$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 923

```
dsolve((a-b)*y(x)^2*diff(y(x),x)^2-2*b*x*y(x)*diff(y(x),x)-a*b-b*x^2+a*y(x)^2 = 0,y(x), sings
```

$$y(x) = \frac{\sqrt{b\,(x^2+a-b)\,(a-b)}}{a-b}$$

$$y(x) = -\frac{\sqrt{b\,(x^2+a-b)\,(a-b)}}{a-b}$$

$$\int_{\_b}^{x} -\frac{\_ab+\sqrt{a\,(\_a^2b-ay\,(x)^2+by\,(x)^2+ab-b^2)}}{-ay\,(x)^2+\_a^2b+by\,(x)^2+\sqrt{a\,(\_a^2b-ay\,(x)^2+by\,(x)^2+ab-b^2)}\,\_a+ab-b^2}d\_a + \int^{y(x)}\left(\frac{\_f(a-b)}{-a\_f^2+b\,x^2+b\_f^2+\sqrt{a\,(-a\_f^2+b\_f^2+b\,x^2+ab-b^2)}\,x+ab-b^2} - \left(\int_{\_b}^{x}\left(\frac{\left(\_ab+\sqrt{a\,(\_a^2b-a\_f^2+b\_f^2+ab-b^2)}\right)\left(-2a\_f+2\_fb+\frac{\_a a(-2a\_f+2\_fb)}{2\sqrt{a(\_a^2b-a\_f^2+b\_f^2+ab-b^2)}}\right)}{\left(-a\_f^2+\_a^2b+b\_f^2+\sqrt{a\,(\_a^2b-a\_f^2+b\_f^2+ab-b^2)}\,\_a+ab-b^2\right)^2}\right.\right.\right.$$

$$+ c_1 = 0$$

$$\int_{\_b}^{x} -\frac{-\_ab+\sqrt{a\,(\_a^2b-ay\,(x)^2+by\,(x)^2+ab-b^2)}}{ay\,(x)^2-\_a^2b-by\,(x)^2+\sqrt{a\,(\_a^2b-ay\,(x)^2+by\,(x)^2+ab-b^2)}\,\_a-ab+b^2}d\_a + \int^{y(x)}\left(-\frac{\_f(a-b)}{a\_f^2-b\,x^2-b\_f^2+\sqrt{a\,(-a\_f^2+b\_f^2+b\,x^2+ab-b^2)}\,x-ab+b^2} - \left(\int_{\_b}^{x}\left(\frac{\left(-\_ab+\sqrt{a\,(\_a^2b-a\_f^2+b\_f^2+ab-b^2)}\right)\left(2a\_f-2\_fb+\frac{\_a a(-2a\_f+2\_fb)}{2\sqrt{a(\_a^2b-a\_f^2+b\_f^2+ab-b^2)}}\right)}{\left(a\_f^2-\_a^2b-b\_f^2+\sqrt{a\,(\_a^2b-a\_f^2+b\_f^2+ab-b^2)}\,\_a-ab+b^2\right)^2}\right.\right.\right.$$

$$+ c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.384 (sec). Leaf size: 86

```
DSolve[(a-b) y[x]^2 (y'[x])^2 -2 b x y[x] y'[x]-a b -b x^2+a y[x]^2==0,y[x],x,IncludeSingular
```

$$y(x) \to -\frac{\sqrt{b\,(b-x^2)+a\,(-b+(x-c_1)^2)}}{\sqrt{b-a}}$$

$$y(x) \to \frac{\sqrt{b\,(b-x^2)+a\,(-b+(x-c_1)^2)}}{\sqrt{b-a}}$$

34.4 problem 999

Internal problem ID [3722]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 999.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$a^2\left(b^2-(cx-ya)^2\right)y'^2+2a\,b^2cy'+c^2\left(b^2-(cx-ya)^2\right)=0$$

✓ Solution by Maple

Time used: 0.328 (sec). Leaf size: 196

```
dsolve(a^2*(b^2-(c*x-a*y(x))^2)*diff(y(x),x)^2+2*a*b^2*c*diff(y(x),x)+c^2*(b^2-(c*x-a*y(x))^2
```

$$y(x)=\frac{cx-\sqrt{2}\,b}{a}$$
$$y(x)=\frac{cx+\sqrt{2}\,b}{a}$$
$$y(x)=\frac{\operatorname{RootOf}\left(-x+\int^{\_Z}-\frac{a\left(\_a^2a^2-2b^2+\sqrt{-a^2\_a^2\left(\_a^2a^2-2b^2\right)}\right)}{2\left(\_a^2a^2-2b^2\right)c}d\_a+c_1\right)a+cx}{a}$$
$$y(x)=\frac{\operatorname{RootOf}\left(-x+\int^{\_Z}-\frac{a\left(\_a^2a^2-2b^2-\sqrt{-a^2\_a^2\left(\_a^2a^2-2b^2\right)}\right)}{2\left(\_a^2a^2-2b^2\right)c}d\_a+c_1\right)a+cx}{a}$$

✓ Solution by Mathematica

Time used: 2.254 (sec). Leaf size: 71

```
DSolve[a^2 ( b^2 -(c x-a y[x])^2 ) (y'[x])^2 +2 a b^2 c y'[x]+c^2(b^2-(c x-a y[x])^2)==0,y[x]
```

$$y(x)\to\frac{cc_1-\sqrt{b^2-c^2(x-c_1)^2}}{a}$$
$$y(x)\to\frac{\sqrt{b^2-c^2(x-c_1)^2}+cc_1}{a}$$

34.5 problem 1000

Internal problem ID [3723]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1000.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{x y^2 {y'}^2 - y^3 y' + a^2 x = 0}$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 153

```
dsolve(x*y(x)^2*diff(y(x),x)^2-y(x)^3*diff(y(x),x)+a^2*x = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{-2ax}$$
$$y(x) = -\sqrt{-2ax}$$
$$y(x) = \sqrt{2}\,\sqrt{ax}$$
$$y(x) = -\sqrt{2}\,\sqrt{ax}$$
$$y(x) = \mathrm{e}^{\frac{c_1}{2}+\frac{\operatorname{RootOf}\left(16a^2 x\,\mathrm{e}^{2c_1}\,\mathrm{e}^{2\_Z}+x^3\mathrm{e}^{2\_Z}-4\,\mathrm{e}^{3\_Z}\mathrm{e}^{2c_1}\right)}{2}-\frac{\ln(x)}{2}}$$
$$y(x) = \mathrm{e}^{-\frac{c_1}{2}+\frac{\operatorname{RootOf}\left(x^2\left(16\,\mathrm{e}^{-2c_1}\,\mathrm{e}^{2\_Z}x^2a^2+\mathrm{e}^{2\_Z}-4\,\mathrm{e}^{3\_Z}\mathrm{e}^{-2c_1}\,x\right)\right)}{2}+\frac{\ln(x)}{2}}$$

✓ Solution by Mathematica

Time used: 22.657 (sec). Leaf size: 219

```
DSolve[x y[x]^2 (y'[x])^2 - y[x]^3 y'[x]+a^2 x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{-2a^2e^{-c_1}x^2 - \frac{e^{c_1}}{2}}$$

$$y(x) \to \sqrt{-2a^2e^{-c_1}x^2 - \frac{e^{c_1}}{2}}$$

$$y(x) \to -\frac{\sqrt{4a^2e^{-c_1}x^2 + e^{c_1}}}{\sqrt{2}}$$

$$y(x) \to \frac{\sqrt{4a^2e^{-c_1}x^2 + e^{c_1}}}{\sqrt{2}}$$

$$y(x) \to -\sqrt{2}\sqrt{a}\sqrt{x}$$

$$y(x) \to -i\sqrt{2}\sqrt{a}\sqrt{x}$$

$$y(x) \to i\sqrt{2}\sqrt{a}\sqrt{x}$$

$$y(x) \to \sqrt{2}\sqrt{a}\sqrt{x}$$

34.6 problem 1001

Internal problem ID [3724]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1001.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_rational]`

$$xy^2{y'}^2 + \left(a - x^3 - y^3\right) y' + x^2 y = 0$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 307

```
dsolve(x*y(x)^2*diff(y(x),x)^2+(a-x^3-y(x)^3)*diff(y(x),x)+x^2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \left(x^3 + a - 2x\sqrt{ax}\right)^{\frac{1}{3}}$$

$$y(x) = \left(x^3 + a + 2x\sqrt{ax}\right)^{\frac{1}{3}}$$

$$y(x) = -\frac{\left(x^3 + a - 2x\sqrt{ax}\right)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\left(x^3 + a - 2x\sqrt{ax}\right)^{\frac{1}{3}}}{2}$$

$$y(x) = -\frac{\left(x^3 + a - 2x\sqrt{ax}\right)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\left(x^3 + a - 2x\sqrt{ax}\right)^{\frac{1}{3}}}{2}$$

$$y(x) = -\frac{\left(x^3 + a + 2x\sqrt{ax}\right)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\left(x^3 + a + 2x\sqrt{ax}\right)^{\frac{1}{3}}}{2}$$

$$y(x) = -\frac{\left(x^3 + a + 2x\sqrt{ax}\right)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\left(x^3 + a + 2x\sqrt{ax}\right)^{\frac{1}{3}}}{2}$$

$$y(x) = 0$$

$$\int_{\_b}^{y(x)} \frac{\_a^2}{\sqrt{\_a^6 + \left(-2x^3 - 2a\right)\_a^3 + \left(-x^3 + a\right)^2}} d\_a + \frac{\ln(x)}{2} - c_1 = 0$$

$$\int_{\_b}^{y(x)} \frac{\_a^2}{\sqrt{\_a^6 + \left(-2x^3 - 2a\right)\_a^3 + \left(-x^3 + a\right)^2}} d\_a - \frac{\ln(x)}{2} - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.436 (sec). Leaf size: 194

```
DSolve[x y[x]^2 (y'[x])^2 +(a-x^3-y[x]^3) y'[x]+x^2 y[x]==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \frac{\sqrt[3]{a+(-1+c_1)x^3}}{\sqrt[3]{1-\frac{1}{c_1}}}$$
$$y(x) \to 0$$
$$y(x) \to \sqrt[3]{\left(\sqrt{a}-x^{3/2}\right)^2}$$
$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{\left(\sqrt{a}-x^{3/2}\right)^2}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{\left(\sqrt{a}-x^{3/2}\right)^2}$$
$$y(x) \to \sqrt[3]{\left(\sqrt{a}+x^{3/2}\right)^2}$$
$$y(x) \to -\sqrt[3]{-1}\sqrt[3]{\left(\sqrt{a}+x^{3/2}\right)^2}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{\left(\sqrt{a}+x^{3/2}\right)^2}$$

34.7 problem 1003

Internal problem ID [3725]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1003.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$2xy^2{y'}^2 - y^3y' - a = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 175

```
dsolve(2*x*y(x)^2*diff(y(x),x)^2-y(x)^3*diff(y(x),x)-a = 0,y(x), singsol=all)
```

$$y(x) = 2^{\frac{3}{4}}(-ax)^{\frac{1}{4}}$$
$$y(x) = -2^{\frac{3}{4}}(-ax)^{\frac{1}{4}}$$
$$y(x) = -i2^{\frac{3}{4}}(-ax)^{\frac{1}{4}}$$
$$y(x) = i2^{\frac{3}{4}}(-ax)^{\frac{1}{4}}$$
$$y(x) = \frac{2^{\frac{1}{4}}(a(c_1^2 - 2c_1x + x^2)\,c_1^3)^{\frac{1}{4}}}{c_1}$$
$$y(x) = -\frac{2^{\frac{1}{4}}(a(c_1^2 - 2c_1x + x^2)\,c_1^3)^{\frac{1}{4}}}{c_1}$$
$$y(x) = -\frac{i2^{\frac{1}{4}}(a(c_1^2 - 2c_1x + x^2)\,c_1^3)^{\frac{1}{4}}}{c_1}$$
$$y(x) = \frac{i2^{\frac{1}{4}}(a(c_1^2 - 2c_1x + x^2)\,c_1^3)^{\frac{1}{4}}}{c_1}$$

✓ Solution by Mathematica

Time used: 1.701 (sec). Leaf size: 151

```
DSolve[2 x y[x]^2 (y'[x])^2 -y[x]^3 y'[x] -a ==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{e^{-\frac{c_1}{4}}\sqrt{-8ax + e^{c_1}}}{\sqrt{2}}$$

$$y(x) \to \frac{e^{-\frac{c_1}{4}}\sqrt{-8ax + e^{c_1}}}{\sqrt{2}}$$

$$y(x) \to -(-2)^{3/4}\sqrt[4]{a}\sqrt[4]{x}$$

$$y(x) \to (-2)^{3/4}\sqrt[4]{a}\sqrt[4]{x}$$

$$y(x) \to (-1-i)\sqrt[4]{2}\sqrt[4]{a}\sqrt[4]{x}$$

$$y(x) \to (1+i)\sqrt[4]{2}\sqrt[4]{a}\sqrt[4]{x}$$

34.8 problem 1004

Internal problem ID [3726]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1004.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _Bernoulli]`

$$\boxed{4x^2y^2{y'}^2 - \left(y^2 + x^2\right)^2 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 75

```
dsolve(4*x^2*y(x)^2*diff(y(x),x)^2 = (x^2+y(x)^2)^2,y(x), singsol=all)
```

$$y(x) = \sqrt{c_1 x + x^2}$$
$$y(x) = -\sqrt{c_1 x + x^2}$$
$$y(x) = -\frac{\sqrt{3}\,\sqrt{x\left(-x^3 + 3c_1\right)}}{3x}$$
$$y(x) = \frac{\sqrt{3}\,\sqrt{x\left(-x^3 + 3c_1\right)}}{3x}$$

✓ Solution by Mathematica

Time used: 0.531 (sec). Leaf size: 97

```
DSolve[4 x^2 y[x]^2(y'[x])^2 ==(x^2+y[x]^2)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{x}\sqrt{x + c_1}$$
$$y(x) \to \sqrt{x}\sqrt{x + c_1}$$
$$y(x) \to -\frac{\sqrt{-x^3 + 3c_1}}{\sqrt{3}\sqrt{x}}$$
$$y(x) \to \frac{\sqrt{-x^3 + 3c_1}}{\sqrt{3}\sqrt{x}}$$

34.9 problem 1006

Internal problem ID [3727]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1006.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational]`

$$4y^3y'^2 - 4y'x + y = 0$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 83

```
dsolve(4*y(x)^3*diff(y(x),x)^2-4*x*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{-x}$$
$$y(x) = -\sqrt{-x}$$
$$y(x) = \sqrt{x}$$
$$y(x) = -\sqrt{x}$$
$$y(x) = 0$$
$$y(x) = \operatorname{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{2(\_a^4 - \sqrt{-\_a^4+1} - 1)}{\_a\,(\_a^4-1)}d\_a + c_1\right)\sqrt{x}$$

✓ Solution by Mathematica

Time used: 0.549 (sec). Leaf size: 282

```
DSolve[4 y[x]^3 (y'[x])^2 -4 x y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -e^{\frac{c_1}{4}}\sqrt[4]{e^{c_1}-2ix}$$
$$y(x) \to -ie^{\frac{c_1}{4}}\sqrt[4]{e^{c_1}-2ix}$$
$$y(x) \to ie^{\frac{c_1}{4}}\sqrt[4]{e^{c_1}-2ix}$$
$$y(x) \to e^{\frac{c_1}{4}}\sqrt[4]{e^{c_1}-2ix}$$
$$y(x) \to -e^{\frac{c_1}{4}}\sqrt[4]{2ix+e^{c_1}}$$
$$y(x) \to -ie^{\frac{c_1}{4}}\sqrt[4]{2ix+e^{c_1}}$$
$$y(x) \to ie^{\frac{c_1}{4}}\sqrt[4]{2ix+e^{c_1}}$$
$$y(x) \to e^{\frac{c_1}{4}}\sqrt[4]{2ix+e^{c_1}}$$
$$y(x) \to 0$$
$$y(x) \to -\sqrt{x}$$
$$y(x) \to -i\sqrt{x}$$
$$y(x) \to i\sqrt{x}$$
$$y(x) \to \sqrt{x}$$

34.10 problem 1012

Internal problem ID [3728]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1012.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{3xy^4{y'}^2 - y^5y' + 1 = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 295

```
dsolve(3*x*y(x)^4*diff(y(x),x)^2-y(x)^5*diff(y(x),x)+1 = 0,y(x), singsol=all)
```

$$y(x) = 12^{\frac{1}{6}} x^{\frac{1}{6}}$$

$$y(x) = -12^{\frac{1}{6}} x^{\frac{1}{6}}$$

$$y(x) = \left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) 12^{\frac{1}{6}} x^{\frac{1}{6}}$$

$$y(x) = \left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) 12^{\frac{1}{6}} x^{\frac{1}{6}}$$

$$y(x) = \left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) 12^{\frac{1}{6}} x^{\frac{1}{6}}$$

$$y(x) = \left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) 12^{\frac{1}{6}} x^{\frac{1}{6}}$$

$$y(x) = \frac{3^{\frac{1}{6}} \left(-\left(c_1^2 - 2c_1 x + x^2\right) c_1^5\right)^{\frac{1}{6}}}{c_1}$$

$$y(x) = -\frac{3^{\frac{1}{6}} \left(-\left(c_1^2 - 2c_1 x + x^2\right) c_1^5\right)^{\frac{1}{6}}}{c_1}$$

$$y(x) = \frac{\left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) 3^{\frac{1}{6}} \left(-\left(c_1^2 - 2c_1 x + x^2\right) c_1^5\right)^{\frac{1}{6}}}{c_1}$$

$$y(x) = \frac{\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) 3^{\frac{1}{6}} \left(-\left(c_1^2 - 2c_1 x + x^2\right) c_1^5\right)^{\frac{1}{6}}}{c_1}$$

$$y(x) = \frac{\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) 3^{\frac{1}{6}} \left(-\left(c_1^2 - 2c_1 x + x^2\right) c_1^5\right)^{\frac{1}{6}}}{c_1}$$

$$y(x) = \frac{\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) 3^{\frac{1}{6}} \left(-\left(c_1^2 - 2c_1 x + x^2\right) c_1^5\right)^{\frac{1}{6}}}{c_1}$$

✓ Solution by Mathematica

Time used: 1.352 (sec). Leaf size: 230

```
DSolve[3 x y[x]^4 (y'[x])^2 -y[x]^5 y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt[3]{-\frac{1}{2}}e^{-\frac{c_1}{6}}\sqrt[3]{12x+e^{c_1}}$$

$$y(x) \to e^{-\frac{c_1}{6}}\sqrt[3]{6x+\frac{e^{c_1}}{2}}$$

$$y(x) \to (-1)^{2/3}e^{-\frac{c_1}{6}}\sqrt[3]{6x+\frac{e^{c_1}}{2}}$$

$$y(x) \to -\sqrt[3]{-2}\sqrt[6]{3}\sqrt[6]{x}$$

$$y(x) \to \sqrt[3]{-2}\sqrt[6]{3}\sqrt[6]{x}$$

$$y(x) \to -\sqrt[3]{2}\sqrt[6]{3}\sqrt[6]{x}$$

$$y(x) \to \sqrt[3]{2}\sqrt[6]{3}\sqrt[6]{x}$$

$$y(x) \to -(-1)^{2/3}\sqrt[3]{2}\sqrt[6]{3}\sqrt[6]{x}$$

$$y(x) \to (-1)^{2/3}\sqrt[3]{2}\sqrt[6]{3}\sqrt[6]{x}$$

34.11 problem 1013

Internal problem ID [3729]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1013.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _rational]`

$$\boxed{9xy^4y'^2 - 3y^5y' - a = 0}$$

✓ Solution by Maple

Time used: 0.11 (sec). Leaf size: 295

```
dsolve(9*x*y(x)^4*diff(y(x),x)^2-3*y(x)^5*diff(y(x),x)-a = 0,y(x), singsol=all)
```

$$y(x) = 2^{\frac{1}{3}}(-ax)^{\frac{1}{6}}$$
$$y(x) = -2^{\frac{1}{3}}(-ax)^{\frac{1}{6}}$$
$$y(x) = \left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)2^{\frac{1}{3}}(-ax)^{\frac{1}{6}}$$
$$y(x) = \left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)2^{\frac{1}{3}}(-ax)^{\frac{1}{6}}$$
$$y(x) = \left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)2^{\frac{1}{3}}(-ax)^{\frac{1}{6}}$$
$$y(x) = \left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)2^{\frac{1}{3}}(-ax)^{\frac{1}{6}}$$
$$y(x) = \frac{\left(a\left(c_1^2-2c_1x+x^2\right)c_1^5\right)^{\frac{1}{6}}}{c_1}$$
$$y(x) = -\frac{\left(a\left(c_1^2-2c_1x+x^2\right)c_1^5\right)^{\frac{1}{6}}}{c_1}$$
$$y(x) = \frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(a\left(c_1^2-2c_1x+x^2\right)c_1^5\right)^{\frac{1}{6}}}{c_1}$$
$$y(x) = \frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(a\left(c_1^2-2c_1x+x^2\right)c_1^5\right)^{\frac{1}{6}}}{c_1}$$
$$y(x) = \frac{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(a\left(c_1^2-2c_1x+x^2\right)c_1^5\right)^{\frac{1}{6}}}{c_1}$$
$$y(x) = \frac{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(a\left(c_1^2-2c_1x+x^2\right)c_1^5\right)^{\frac{1}{6}}}{c_1}$$

✓ Solution by Mathematica

Time used: 10.173 (sec). Leaf size: 358

```
DSolve[9 x y[x]^4  (y'[x])^2 -3 y[x]^5 y'[x]-a==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt[3]{-\frac{1}{2}} e^{-\frac{c_1}{6}} \sqrt[3]{-4ax + e^{c_1}}$$
$$y(x) \to \frac{e^{-\frac{c_1}{6}} \sqrt[3]{-4ax + e^{c_1}}}{\sqrt[3]{2}}$$
$$y(x) \to \frac{(-1)^{2/3} e^{-\frac{c_1}{6}} \sqrt[3]{-4ax + e^{c_1}}}{\sqrt[3]{2}}$$
$$y(x) \to -\sqrt[3]{-\frac{1}{2}} \sqrt[3]{-e^{-\frac{c_1}{2}} (-4ax + e^{c_1})}$$
$$y(x) \to \frac{\sqrt[3]{e^{-\frac{c_1}{2}} (4ax - e^{c_1})}}{\sqrt[3]{2}}$$
$$y(x) \to \frac{(-1)^{2/3} \sqrt[3]{-e^{-\frac{c_1}{2}} (-4ax + e^{c_1})}}{\sqrt[3]{2}}$$
$$y(x) \to -i \sqrt[3]{2} \sqrt[6]{a} \sqrt[6]{x}$$
$$y(x) \to i \sqrt[3]{2} \sqrt[6]{a} \sqrt[6]{x}$$
$$y(x) \to -\sqrt[6]{-1} \sqrt[3]{2} \sqrt[6]{a} \sqrt[6]{x}$$
$$y(x) \to \sqrt[6]{-1} \sqrt[3]{2} \sqrt[6]{a} \sqrt[6]{x}$$
$$y(x) \to -(-1)^{5/6} \sqrt[3]{2} \sqrt[6]{a} \sqrt[6]{x}$$
$$y(x) \to (-1)^{5/6} \sqrt[3]{2} \sqrt[6]{a} \sqrt[6]{x}$$

34.12 problem 1014

Internal problem ID [3730]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1014.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$9(-x^2+1)\, y^4 {y'}^2 + 6x y^5 y' + 4x^2 = 0$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 245

```
dsolve(9*(-x^2+1)*y(x)^4*diff(y(x),x)^2+6*x*y(x)^5*diff(y(x),x)+4*x^2 = 0,y(x), singsol=all)
```

$$y(x) = (-4x^2+4)^{\frac{1}{6}}$$
$$y(x) = -(-4x^2+4)^{\frac{1}{6}}$$
$$y(x) = \left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(-4x^2+4)^{\frac{1}{6}}$$
$$y(x) = \left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-4x^2+4)^{\frac{1}{6}}$$
$$y(x) = \left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(-4x^2+4)^{\frac{1}{6}}$$
$$y(x) = \left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-4x^2+4)^{\frac{1}{6}}$$
$$y(x) = \frac{\left((-16c_1^2+4x^2-4)\, c_1^2\right)^{\frac{1}{3}}}{2c_1}$$
$$y(x) = -\frac{\left((-16c_1^2+4x^2-4)\, c_1^2\right)^{\frac{1}{3}}}{4c_1} - \frac{i\sqrt{3}\left((-16c_1^2+4x^2-4)\, c_1^2\right)^{\frac{1}{3}}}{4c_1}$$
$$y(x) = -\frac{\left((-16c_1^2+4x^2-4)\, c_1^2\right)^{\frac{1}{3}}}{4c_1} + \frac{i\sqrt{3}\left((-16c_1^2+4x^2-4)\, c_1^2\right)^{\frac{1}{3}}}{4c_1}$$

✓ Solution by Mathematica

Time used: 0.416 (sec). Leaf size: 199

```
DSolve[9(1-x^2) y[x]^4 (y'[x])^2 +6 x y[x]^5 y'[x]+4 x^2==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to -\frac{\sqrt[3]{-\frac{1}{2}}\sqrt[3]{-4x^2+4+c_1{}^2}}{\sqrt[3]{c_1}}$$
$$y(x) \to -1$$
$$y(x) \to 0$$
$$y(x) \to \sqrt[3]{-\frac{1}{2}}$$
$$y(x) \to \text{Indeterminate}$$
$$y(x) \to -\sqrt[3]{-2}\sqrt[6]{1-x^2}$$
$$y(x) \to \sqrt[3]{-2}\sqrt[6]{1-x^2}$$
$$y(x) \to -\sqrt[3]{2}\sqrt[6]{1-x^2}$$
$$y(x) \to \sqrt[3]{2}\sqrt[6]{1-x^2}$$
$$y(x) \to -(-1)^{2/3}\sqrt[3]{2}\sqrt[6]{1-x^2}$$
$$y(x) \to (-1)^{2/3}\sqrt[3]{2}\sqrt[6]{1-x^2}$$

34.13 problem 1015

Internal problem ID [3731]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1015.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^3 - bx - a = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 66

```
dsolve(diff(y(x),x)^3 = b*x+a,y(x), singsol=all)
```

$$y(x) = \frac{3(xb+a)^{\frac{4}{3}}}{4b} + c_1$$
$$y(x) = -\frac{3i(xb+a)^{\frac{4}{3}}\left(-i+\sqrt{3}\right)}{8b} + c_1$$
$$y(x) = \frac{3i(xb+a)^{\frac{4}{3}}\left(\sqrt{3}+i\right)}{8b} + c_1$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 80

```
DSolve[(y'[x])^3 ==a+b x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3(a+bx)^{4/3}}{4b} + c_1$$
$$y(x) \to -\frac{3\sqrt[3]{-1}(a+bx)^{4/3}}{4b} + c_1$$
$$y(x) \to \frac{3(-1)^{2/3}(a+bx)^{4/3}}{4b} + c_1$$

34.14 problem 1016

Internal problem ID [3732]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1016.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^3 - a\,x^n = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 77

```
dsolve(diff(y(x),x)^3 = a*x^n,y(x), singsol=all)
```

$$y(x) = \frac{3x(a\,x^n)^{\frac{1}{3}}}{n+3} + c_1$$
$$y(x) = \frac{3x\left(-1+i\sqrt{3}\right)(a\,x^n)^{\frac{1}{3}}}{2\,(n+3)} + c_1$$
$$y(x) = -\frac{3x\left(1+i\sqrt{3}\right)(a\,x^n)^{\frac{1}{3}}}{2\,(n+3)} + c_1$$

✓ Solution by Mathematica

Time used: 0.007 (sec). Leaf size: 95

```
DSolve[(y'[x])^3 ==a x^n,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{3\sqrt[3]{a}x^{\frac{n}{3}+1}}{n+3} + c_1$$
$$y(x) \to -\frac{3\sqrt[3]{-1}\sqrt[3]{a}x^{\frac{n}{3}+1}}{n+3} + c_1$$
$$y(x) \to \frac{3(-1)^{2/3}\sqrt[3]{a}x^{\frac{n}{3}+1}}{n+3} + c_1$$

34.15 problem 1017

Internal problem ID [3733]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1017.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y'^3 + x - y = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 209

```
dsolve(diff(y(x),x)^3+x-y(x) = 0,y(x), singsol=all)
```

$$x - \frac{3(y(x)-x)^{\frac{2}{3}}}{2} - 3(y(x)-x)^{\frac{1}{3}} - 3\ln\left((y(x)-x)^{\frac{1}{3}} - 1\right) - c_1 = 0$$

$$x + \frac{3(y(x)-x)^{\frac{2}{3}}}{4} - \frac{3i\sqrt{3}\,(y(x)-x)^{\frac{2}{3}}}{4} + \frac{3(y(x)-x)^{\frac{1}{3}}}{2} + \frac{3i\sqrt{3}\,(y(x)-x)^{\frac{1}{3}}}{2} - 3\ln\left(-\frac{(y(x)-x)^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\,(y(x)-x)^{\frac{1}{3}}}{2} - 1\right) - c_1 = 0$$

$$x + \frac{3(y(x)-x)^{\frac{2}{3}}}{4} + \frac{3i\sqrt{3}\,(y(x)-x)^{\frac{2}{3}}}{4} + \frac{3(y(x)-x)^{\frac{1}{3}}}{2} - \frac{3i\sqrt{3}\,(y(x)-x)^{\frac{1}{3}}}{2} - 3\ln\left(-\frac{(y(x)-x)^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\,(y(x)-x)^{\frac{1}{3}}}{2} - 1\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 11.046 (sec). Leaf size: 298

```
DSolve[(y'[x])^3 +x-y[x]==0 x,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{3}{2}(y(x)-x)^{2/3}+3\sqrt[3]{y(x)-x}+3\log\left(\sqrt[3]{y(x)-x}-1\right)-x=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{2}\left(\frac{1}{2}\sqrt[3]{y(x)-x}\left(4i(y(x)-x)^{2/3}+3\sqrt{3}\sqrt[3]{y(x)-x}-3i\sqrt[3]{y(x)-x}-6\sqrt{3}-6i\right)+6i\log\left(\sqrt{2-2i\sqrt{3}}-\right.\right.\right.$$

$$\left.-i(y(x)-x)=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{y(x)}{2}\right.$$

$$+\frac{1}{4}\left(-\frac{1}{2}\sqrt[3]{y(x)-x}\left(4(y(x)-x)^{2/3}+3i\sqrt{3}\sqrt[3]{y(x)-x}-3\sqrt[3]{y(x)-x}-6i\sqrt{3}-6\right)-6\log\left(2i\sqrt[3]{y(x)-x}+\sqrt{2}\right.\right.$$

34.16 problem 1018

Internal problem ID [3734]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1018.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y'^3 - (a + by + cy^2) f(x) = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 191

```
dsolve(diff(y(x),x)^3 = (a+b*y(x)+c*y(x)^2)*f(x),y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{(\_a^2c + \_ab + a)^{\frac{1}{3}}} d\_a + \int^{x} -\frac{\left(\left(a + by(x) + cy(x)^2\right) f(\_a)\right)^{\frac{1}{3}}}{\left(a + by(x) + cy(x)^2\right)^{\frac{1}{3}}} d\_a + c_1 = 0$$

$$\int^{y(x)} \frac{1}{(\_a^2c + \_ab + a)^{\frac{1}{3}}} d\_a + \int^{x} \frac{\left(\left(a + by(x) + cy(x)^2\right) f(\_a)\right)^{\frac{1}{3}} \left(1 + i\sqrt{3}\right)}{2\left(a + by(x) + cy(x)^2\right)^{\frac{1}{3}}} d\_a + c_1 = 0$$

$$\int^{y(x)} \frac{1}{(\_a^2c + \_ab + a)^{\frac{1}{3}}} d\_a + \int^{x} -\frac{\left(\left(a + by(x) + cy(x)^2\right) f(\_a)\right)^{\frac{1}{3}} \left(-1 + i\sqrt{3}\right)}{2\left(a + by(x) + cy(x)^2\right)^{\frac{1}{3}}} d\_a + c_1 = 0$$

✓ Solution by Mathematica

Time used: 21.144 (sec). Leaf size: 405

```
DSolve[(y'[x])^3 ==(a+b y[x]+c y[x]^2) f[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{(2\#1c+b)\sqrt[3]{\frac{c(\#1(\#1c+b)+a)}{4ac-b^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{(b+2c\#1)^2}{b^2-4ac}\right)}{\sqrt[3]{2}c\sqrt[3]{\#1(\#1c+b)+a}}\&\right]\left[\int_1^x \sqrt[3]{\ldots} + c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{(2\#1c+b)\sqrt[3]{\frac{c(\#1(\#1c+b)+a)}{4ac-b^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{(b+2c\#1)^2}{b^2-4ac}\right)}{\sqrt[3]{2}c\sqrt[3]{\#1(\#1c+b)+a}}\&\right]\left[\int_1^x -\sqrt[3]{-1}\sqrt[3]{f(K[2])}dK[2] + c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{(2\#1c+b)\sqrt[3]{\frac{c(\#1(\#1c+b)+a)}{4ac-b^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{(b+2c\#1)^2}{b^2-4ac}\right)}{\sqrt[3]{2}c\sqrt[3]{\#1(\#1c+b)+a}}\&\right]\left[\int_1^x (-\ldots + c_1\right]$$

$$y(x) \to -\frac{\sqrt{b^2-4ac}+b}{2c}$$

$$y(x) \to \frac{\sqrt{b^2-4ac}-b}{2c}$$

34.17 problem 1019

Internal problem ID [3735]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1019.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 - (y-a)^2 (y-b)^2 = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 126

```
dsolve(diff(y(x),x)^3 = (y(x)-a)^2*(y(x)-b)^2,y(x), singsol=all)
```

$$y(x) = a$$
$$y(x) = b$$
$$x - \left(\int^{y(x)} \frac{1}{\left((\_a-a)^2 (\_a-b)^2\right)^{\frac{1}{3}}} d\_a\right) - c_1 = 0$$
$$x - \left(\int^{y(x)} -\frac{2}{\left(1+i\sqrt{3}\right) \left((\_a-a)^2 (\_a-b)^2\right)^{\frac{1}{3}}} d\_a\right) - c_1 = 0$$
$$x - \left(\int^{y(x)} \frac{2}{\left(-1+i\sqrt{3}\right) \left((\_a-a)^2 (\_a-b)^2\right)^{\frac{1}{3}}} d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.226 (sec). Leaf size: 246

```
DSolve[(y'[x])^3 ==(y[x]-a)^2 (y[x]-b)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right][x+c_1]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[-\sqrt[3]{-1}x+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[(-1)^{2/3}x+c_1\right]$$

$$y(x) \to a$$

$$y(x) \to b$$

34.18 problem 1020

Internal problem ID [3736]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1020.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y'^3 + f(x)(y-a)^2(y-b)^2 = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 212

```
dsolve(diff(y(x),x)^3+f(x)*(y(x)-a)^2*(y(x)-b)^2 = 0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{((-\_a+a)(-\_a+b))^{\frac{2}{3}}} d\_a + \int^{x} -\frac{\left(-f(\_a)(-y(x)+a)^2(b-y(x))^2\right)^{\frac{1}{3}}}{((-y(x)+a)(b-y(x)))^{\frac{2}{3}}} d\_a + c_1 = 0$$

$$\int^{y(x)} \frac{1}{((-\_a+a)(-\_a+b))^{\frac{2}{3}}} d\_a + \int^{x} \frac{\left(-f(\_a)(-y(x)+a)^2(b-y(x))^2\right)^{\frac{1}{3}}(1+i\sqrt{3})}{2((-y(x)+a)(b-y(x)))^{\frac{2}{3}}} d\_a + c_1 = 0$$

$$\int^{y(x)} \frac{1}{((-\_a+a)(-\_a+b))^{\frac{2}{3}}} d\_a + \int^{x} -\frac{\left(-f(\_a)(-y(x)+a)^2(b-y(x))^2\right)^{\frac{1}{3}}(-1+i\sqrt{3})}{2((-y(x)+a)(b-y(x)))^{\frac{2}{3}}} d\_a + c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.987 (sec). Leaf size: 287

```
DSolve[(y'[x])^3 +f[x] (y[x]-a)^2 (y[x]-b)^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[\int_1^x -\sqrt[3]{f(K[1])}dK[\right.$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[\int_1^x \sqrt[3]{-1}\sqrt[3]{f(K[2])}d\right.$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[\int_1^x -(-1)^{2/3}\sqrt[3]{f(K\right.$$

$$y(x) \to a$$

$$y(x) \to b$$

34.19 problem 1021

Internal problem ID [3737]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1021.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y'^3 + f(x)(y-a)^2(y-b)^2(y-c)^2 = 0}$$

✓ Solution by Maple

Time used: 0.297 (sec). Leaf size: 275

```
dsolve(diff(y(x),x)^3+f(x)*(y(x)-a)^2*(y(x)-b)^2*(y(x)-c)^2 = 0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\left(-\left(-\_a+c\right)\left(-\_a+b\right)\left(-\_a+a\right)\right)^{\frac{2}{3}}} d\_a + \int^{x} -\frac{\left(-f(\_a)\left(c-y(x)\right)^{2}\left(b-y(x)\right)^{2}\left(-y(x)+a\right)^{2}\right)^{\frac{1}{3}}}{\left(-\left(c-y\left(x\right)\right)\left(b-y\left(x\right)\right)\left(-y\left(x\right)+a\right)\right)^{\frac{2}{3}}} d\_a + c_1 = 0$$

$$\int^{y(x)} \frac{1}{\left(-\left(-\_a+c\right)\left(-\_a+b\right)\left(-\_a+a\right)\right)^{\frac{2}{3}}} d\_a + \int^{x} \frac{\left(-f(\_a)\left(c-y(x)\right)^{2}\left(b-y(x)\right)^{2}\left(-y(x)+a\right)^{2}\right)^{\frac{1}{3}}\left(1+i\sqrt{3}\right)}{2\left(-\left(c-y\left(x\right)\right)\left(b-y\left(x\right)\right)\left(-y\left(x\right)+a\right)\right)^{\frac{2}{3}}} d\_a + c_1 = 0$$

$$\int^{y(x)} \frac{1}{\left(-\left(-\_a+c\right)\left(-\_a+b\right)\left(-\_a+a\right)\right)^{\frac{2}{3}}} d\_a + \int^{x} -\frac{\left(-f(\_a)\left(c-y(x)\right)^{2}\left(b-y(x)\right)^{2}\left(-y(x)+a\right)^{2}\right)^{\frac{1}{3}}\left(-1+i\sqrt{3}\right)}{2\left(-\left(c-y\left(x\right)\right)\left(b-y\left(x\right)\right)\left(-y\left(x\right)+a\right)\right)^{\frac{2}{3}}} d\_a + c_1 = 0$$

✓ Solution by Mathematica

Time used: 15.7 (sec). Leaf size: 421

```
DSolve[(y'[x])^3 +f[x](y[x]-a)^2 (y[x]-b)^2 (y[x]-c)^2==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \text{InverseFunction}\left[\frac{3\sqrt[3]{a-\#1}\sqrt[3]{c-\#1}\left(\frac{(b-\#1)(a-c)}{(c-\#1)(a-b)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{(c-b)(a-\#1)}{(a-b)(c-\#1)}\right)}{(b-\#1)^{2/3}(a-c)}\&\right]\left[\int\right.$$

$$y(x) \to \text{InverseFunction}\left[\frac{3\sqrt[3]{a-\#1}\sqrt[3]{c-\#1}\left(\frac{(b-\#1)(a-c)}{(c-\#1)(a-b)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{(c-b)(a-\#1)}{(a-b)(c-\#1)}\right)}{(b-\#1)^{2/3}(a-c)}\&\right]\left[\int\right.$$

$$y(x) \to \text{InverseFunction}\left[\frac{3\sqrt[3]{a-\#1}\sqrt[3]{c-\#1}\left(\frac{(b-\#1)(a-c)}{(c-\#1)(a-b)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{(c-b)(a-\#1)}{(a-b)(c-\#1)}\right)}{(b-\#1)^{2/3}(a-c)}\&\right]\left[\int\right.$$

$$y(x) \to a$$

$$y(x) \to b$$

$$y(x) \to c$$

34.20 problem 1022

Internal problem ID [3738]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1022.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^3 + y' + a - bx = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 334

```
dsolve(diff(y(x),x)^3+diff(y(x),x)+a-b*x = 0,y(x), singsol=all)
```

$$y(x) = \int -\frac{i\left(\sqrt{3}\left(108xb-108a+12\sqrt{81b^2x^2-162xba+81a^2+12}\right)^{\frac{2}{3}} - i\left(108xb-108a+12\sqrt{81b^2x^2-162xba\right.\right.}{12\left(108xb-108a+12\sqrt{81b^2x^2-162xba+81a^2+12}\right)^{\frac{1}{3}}} + c_1$$

$$y(x) = \int \frac{i\left(\sqrt{3}\left(108xb-108a+12\sqrt{81b^2x^2-162xba+81a^2+12}\right)^{\frac{2}{3}} + 12\sqrt{3} + i\left(108xb-108a+12\sqrt{81b^2x^2-\right.\right.}{12\left(108xb-108a+12\sqrt{81b^2x^2-162xba+81a^2+12}\right)^{\frac{1}{3}}} + c_1$$

$$y(x) = \int \frac{\left(108xb-108a+12\sqrt{81b^2x^2-162xba+81a^2+12}\right)^{\frac{2}{3}} - 12}{6\left(108xb-108a+12\sqrt{81b^2x^2-162xba+81a^2+12}\right)^{\frac{1}{3}}}dx + c_1$$

✓ Solution by Mathematica

Time used: 2.013 (sec). Leaf size: 667

```
DSolve[(y'[x])^3 +y'[x]+a-b x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{144}\left(\frac{2^{2/3}\sqrt[3]{3}\left(\sqrt{3}\sqrt{27(a-bx)^2+4}-9a+9bx\right)^{4/3}}{b} - \frac{4\sqrt[3]{2}3^{2/3}\left(\sqrt{3}\sqrt{27(a-bx)^2+4}-9a+9bx\right)^{2/3}}{b} - \frac{24\ 2^{2/3}\sqrt[3]{3}}{b\left(\sqrt{3}\sqrt{27(a-bx)^2+4}-9a+9bx\right)^{2/3}} + \frac{24\sqrt[3]{2}3^{2/3}}{b\left(\sqrt{3}\sqrt{27(a-bx)^2+4}-9a+9bx\right)^{4/3}} + 144c_1\right)$$

$$y(x) \to \frac{1}{288}\left(\frac{i2^{2/3}\sqrt[3]{3}(\sqrt{3}+i)\left(\sqrt{3}\sqrt{27(a-bx)^2+4}-9a+9bx\right)^{4/3}}{b} + \frac{4\sqrt[3]{2}\sqrt[6]{3}(\sqrt{3}+3i)\left(\sqrt{3}\sqrt{27(a-bx)^2+4}-9a+9bx\right)^{2/3}}{b} - \frac{48(-2)^{2/3}\sqrt[3]{3}}{b\left(\sqrt{3}\sqrt{27(a-bx)^2+4}-9a+9bx\right)^{2/3}} - \frac{48\sqrt[3]{-2}3^{2/3}}{b\left(\sqrt{3}\sqrt{27(a-bx)^2+4}-9a+9bx\right)^{4/3}} + 288c_1\right)$$

$$y(x) \to \frac{1}{288}\left(-\frac{i2^{2/3}\sqrt[3]{3}(\sqrt{3}-i)\left(\sqrt{3}\sqrt{27(a-bx)^2+4}-9a+9bx\right)^{4/3}}{b} + \frac{4\sqrt[3]{2}\sqrt[6]{3}(\sqrt{3}-3i)\left(\sqrt{3}\sqrt{27(a-bx)^2+4}-9a+9bx\right)^{2/3}}{b} + \frac{48\sqrt[3]{-3}2^{2/3}}{b\left(\sqrt{3}\sqrt{27(a-bx)^2+4}-9a+9bx\right)^{2/3}} + \frac{48(-3)^{2/3}\sqrt[3]{2}}{b\left(\sqrt{3}\sqrt{27(a-bx)^2+4}-9a+9bx\right)^{4/3}} + 288c_1\right)$$

34.21 problem 1023

Internal problem ID [3739]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1023.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 + y' - y = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 245

```
dsolve(diff(y(x),x)^3+diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$x - \left(\int^{y(x)} \frac{6\left(108\_a + 12\sqrt{81\_a^2 + 12}\right)^{\frac{1}{3}}}{\left(108\_a + 12\sqrt{81\_a^2 + 12}\right)^{\frac{2}{3}} - 12} d\_a\right) - c_1 = 0$$

$$x - \left(\int^{y(x)} \frac{12\left(108\_a + 12\sqrt{81\_a^2 + 12}\right)^{\frac{1}{3}}}{\left(1 + i\sqrt{3}\right)\left(\left(108\_a + 12\sqrt{81\_a^2 + 12}\right)^{\frac{1}{3}} + \sqrt{3} - 3i\right)\left(-\left(108\_a + 12\sqrt{81\_a^2 + 12}\right)^{\frac{1}{3}} - 3i + \sqrt{3}\right.}\right.$$
$$- c_1 = 0$$

$$x - \left(\int^{y(x)}\right.$$
$$- \frac{12\left(108\_a + 12\sqrt{81\_a^2 + 12}\right)^{\frac{1}{3}}}{\left(-1 + i\sqrt{3}\right)\left(\left(108\_a + 12\sqrt{81\_a^2 + 12}\right)^{\frac{1}{3}} + \sqrt{3} + 3i\right)\left(-\left(108\_a + 12\sqrt{81\_a^2 + 12}\right)^{\frac{1}{3}} + 3i + \sqrt{3}\right)} d\_a$$
$$- c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.345 (sec). Leaf size: 335

```
DSolve[(y'[x])^3 +y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int \frac{\sqrt[3]{\sqrt{729\#1^2+108}-27\#1}}{2^{2/3}\left(\sqrt{729\#1^2+108}-27\#1\right)^{2/3}-6\sqrt[3]{2}}d\#1\&\right]\left[-\frac{x}{6}+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[\int \frac{\sqrt[3]{\sqrt{729\#1^2+108}-27\#1}}{-i2^{2/3}\sqrt{3}\left(\sqrt{729\#1^2+108}-27\#1\right)^{2/3}+2^{2/3}\left(\sqrt{729\#1^2+108}-27\#1\right)^{2/3}-6i}\right.$$

$$y(x) \to \text{InverseFunction}\left[\int \frac{\sqrt[3]{\sqrt{729\#1^2+108}-27\#1}}{i2^{2/3}\sqrt{3}\left(\sqrt{729\#1^2+108}-27\#1\right)^{2/3}+2^{2/3}\left(\sqrt{729\#1^2+108}-27\#1\right)^{2/3}+6i\sqrt[3]{}}\right.$$

$$y(x) \to 0$$

34.22 problem 1024

Internal problem ID [3740]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1024.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^3 + y' - e^y = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 261

```
dsolve(diff(y(x),x)^3+diff(y(x),x) = exp(y(x)),y(x), singsol=all)
```

$$x - \left(\int^{y(x)} \frac{6\left(108\,e^{-}\_a + 12\sqrt{12 + 81\,e^{2}\_a}\right)^{\frac{1}{3}}}{\left(108\,e^{-}\_a + 12\sqrt{12 + 81\,e^{2}\_a}\right)^{\frac{2}{3}} - 12} d\_a\right) - c_1 = 0$$

$$x - \left(\int^{y(x)} \frac{12\left(108\,e^{-}\_a + 12\sqrt{12 + 81\,e^{2}\_a}\right)^{\frac{1}{3}}}{\left(1 + i\sqrt{3}\right)\left(\left(108\,e^{-}\_a + 12\sqrt{12 + 81\,e^{2}\_a}\right)^{\frac{1}{3}} + \sqrt{3} - 3i\right)\left(-\left(108\,e^{-}\_a + 12\sqrt{12 + 81\,e^{2}\_a}\right)^{\frac{1}{3}} - 3i +\right.}\right.$$
$$- c_1 = 0$$

$$x - \left(\int^{y(x)} \right.$$
$$-\frac{12\left(108\,e^{-}\_a + 12\sqrt{12 + 81\,e^{2}\_a}\right)^{\frac{1}{3}}}{\left(-1 + i\sqrt{3}\right)\left(\left(108\,e^{-}\_a + 12\sqrt{12 + 81\,e^{2}\_a}\right)^{\frac{1}{3}} + \sqrt{3} + 3i\right)\left(-\left(108\,e^{-}\_a + 12\sqrt{12 + 81\,e^{2}\_a}\right)^{\frac{1}{3}} + 3i + \sqrt{3}\right)} d\_$$
$$- c_1 = 0$$

✓ Solution by Mathematica

Time used: 172.268 (sec). Leaf size: 1244

```
DSolve[(y'[x])^3 +y'[x]==Exp[ y[x]],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{1}{36}\left(\frac{e^{-\#1}\left(2^{2/3}\sqrt[3]{\sqrt{81e^{2\#1}+12}-9e^{\#1}}\sqrt{81e^{2\#1}+12}-9\,2^{2/3}e^{\#1}\sqrt[3]{\sqrt{81e^{2\#1}+12}-9\right.}{\left(\sqrt{81e^{2\#1}+12}-9e^{\#1}\right)^{2/3}}\right.\right.$$

$$\left.\left.-12\sqrt[3]{6}\arctan\left(\frac{6^{2/3}\sqrt[3]{\sqrt{81e^{2\#1}+12}-9e^{\#1}}}{\sqrt[3]{2}\left(\sqrt{81e^{2\#1}+12}-9e^{\#1}\right)^{2/3}-2\sqrt[3]{3}}\right)\right)\right)+\frac{e^{-\#1}}{3\,6^{2/3}}\&\right]\left[-\frac{x}{6^{2/3}}+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{e^{-\#1}}{6\,2^{2/3}3^{5/6}}\right.$$

$$-\frac{1}{144}i\left(\frac{e^{-\#1}\left(-12i\sqrt[3]{2}\sqrt[6]{3}e^{\#1}\left(\sqrt{81e^{2\#1}+12}-9e^{\#1}\right)^{2/3}\arctan\left(\frac{6^{2/3}\sqrt[3]{\sqrt{81e^{2\#1}+12}-9e^{\#1}}}{\sqrt[3]{2}\left(\sqrt{81e^{2\#1}+12}-9e^{\#1}\right)^{2/3}-2\sqrt[3]{3}}\right)-3i\right.}{}\right.$$

$$y(x) \to \text{InverseFunction}\left[-\frac{e^{-\#1}}{6\,2^{2/3}3^{5/6}}\right.$$

$$+\frac{1}{144}i\left(\frac{e^{-\#1}\left(12i\sqrt[3]{2}\sqrt[6]{3}e^{\#1}\left(\sqrt{81e^{2\#1}+12}-9e^{\#1}\right)^{2/3}\arctan\left(\frac{6^{2/3}\sqrt[3]{\sqrt{81e^{2\#1}+12}-9e^{\#1}}}{\sqrt[3]{2}\left(\sqrt{81e^{2\#1}+12}-9e^{\#1}\right)^{2/3}-2\sqrt[3]{3}}\right)+3i2^{2\right.}{}\right.$$

34.23 problem 1025

Internal problem ID [3741]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1025.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^3 - 7y' + 6 = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(diff(y(x),x)^3-7*diff(y(x),x)+6 = 0,y(x), singsol=all)
```

$$y(x) = 2x + c_1$$
$$y(x) = x + c_1$$
$$y(x) = -3x + c_1$$

✓ Solution by Mathematica

Time used: 0.002 (sec). Leaf size: 29

```
DSolve[(y'[x])^3-7 y'[x]+6==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -3x + c_1$$
$$y(x) \to x + c_1$$
$$y(x) \to 2x + c_1$$

34.24 problem 1026

Internal problem ID [3742]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1026.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^3 - y'x + ya = 0}$$

✓ Solution by Maple

Time used: 0.063 (sec). Leaf size: 1223

```
dsolve(diff(y(x),x)^3-x*diff(y(x),x)+a*y(x) = 0,y(x), singsol=all)
```

$$c_1\left(48\left(\frac{i\sqrt{3}\left(-108ay(x)+12\sqrt{81a^2y(x)^2-12x^3}\right)^{\frac{2}{3}}-12i\sqrt{3}\,x-\left(-108ay(x)+12\sqrt{81a^2y(x)^2-12x^3}\right)^{\frac{2}{3}}-12x}{12\left(-108ay(x)+12\sqrt{81a^2y(x)^2-12x^3}\right)^{\frac{1}{3}}}\right)^{\frac{1}{a-1}}\left(-108ay(x)+12\sqrt{\cdots}\right.\right.$$

$$+x$$

$$-\frac{i\sqrt{3}\left(-108ay(x)+12\sqrt{81a^2y(x)^2-12x^3}\right)^{\frac{4}{3}}-144i\sqrt{3}\,x^2+\left(-108ay(x)+12\sqrt{81a^2y(x)^2-12x^3}\right)^{\frac{4}{3}}-\cdots}{24\left(-108ay(x)+12\sqrt{81a^2y(x)^2-12x^3}\right)^{\frac{2}{3}}(2a-\cdots}$$

$$=0$$

$$c_1\left(-24\left(-108ay(x)+12\sqrt{81a^2y(x)^2-12x^3}\right)^{\frac{2}{3}}\left(\frac{\left(-108ay(x)+12\sqrt{81a^2y(x)^2-12x^3}\right)^{\frac{2}{3}}+12x}{6\left(-108ay(x)+12\sqrt{81a^2y(x)^2-12x^3}\right)^{\frac{1}{3}}}\right)^{\frac{1}{a-1}}a+36\left(-108ay\cdots\right.\right.$$

$$\overline{\left(-108ay(x)+12\sqrt{81a^2y(x)^2-12\cdots}\right.}$$

$$+x$$

$$+\frac{\left(-108ay(x)+12\sqrt{81a^2y(x)^2-12x^3}\right)^{\frac{4}{3}}+24\left(-108ay(x)+12\sqrt{81a^2y(x)^2-12x^3}\right)^{\frac{2}{3}}x+144x^2}{12\left(-108ay(x)+12\sqrt{81a^2y(x)^2-12x^3}\right)^{\frac{2}{3}}(2a-3)}$$

$$=0$$

$$c_1\left(-48\left(-108ay(x)+12\sqrt{81a^2y(x)^2-12x^3}\right)^{\frac{2}{3}}\left(-\frac{i\sqrt{3}\left(-108ay(x)+12\sqrt{81a^2y(x)^2-12x^3}\right)^{\frac{2}{3}}-12i\sqrt{3}\,x+\left(-108ay(x)+12\cdots\right.}{12\left(-108ay(x)+12\sqrt{81a^2y(x)^2-12x^3}\right)^{\frac{1}{3}}}\right.\right.$$

$$+x$$

$$+\frac{i\sqrt{3}\left(-108ay(x)+12\sqrt{81a^2y(x)^2-12x^3}\right)^{\frac{4}{3}}-144i\sqrt{3}\,x^2-\left(-108ay(x)+12\sqrt{81a^2y(x)^2-12x^3}\right)^{\frac{4}{3}}+\cdots}{24\left(-108ay(x)+12\sqrt{81a^2y(x)^2-12x^3}\right)^{\frac{2}{3}}(2a-3\cdots}$$

$$=0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^3 -x y'[x]+a y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

34.25 problem 1027

Internal problem ID [3743]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1027.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^3 + 2y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 173

```
dsolve(diff(y(x),x)^3+2*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\left(-6\sqrt{x^2+3c_1}-6x\right)^{\frac{3}{2}}}{27} - \frac{2x\sqrt{-6\sqrt{x^2+3c_1}-6x}}{3}$$
$$y(x) = \frac{\left(-6\sqrt{x^2+3c_1}-6x\right)^{\frac{3}{2}}}{27} + \frac{2x\sqrt{-6\sqrt{x^2+3c_1}-6x}}{3}$$
$$y(x) = -\frac{\left(6\sqrt{x^2+3c_1}-6x\right)^{\frac{3}{2}}}{27} - \frac{2x\sqrt{6\sqrt{x^2+3c_1}-6x}}{3}$$
$$y(x) = \frac{\left(6\sqrt{x^2+3c_1}-6x\right)^{\frac{3}{2}}}{27} + \frac{2x\sqrt{6\sqrt{x^2+3c_1}-6x}}{3}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^3 +2 x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

34.26 problem 1028

Internal problem ID [3744]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1028.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{{y'}^{3} - 2y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 496

```
dsolve(diff(y(x),x)^3-2*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$-\frac{c_1}{\left(\frac{\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}+24x}{\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{1}{3}}}\right)^{\frac{2}{3}}}+x-\frac{\left(\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}+24x\right)^2}{96\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}}=0$$

$$-\frac{c_1}{\left(\frac{i\sqrt{3}\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}-24i\sqrt{3}\,x-\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}-24x}{\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{1}{3}}}\right)^{\frac{2}{3}}}+x-\frac{\left(i\sqrt{3}\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}-24i\sqrt{3}\,x-\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}-24x\right)^2}{384\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}}=0$$

$$-\frac{12^{\frac{2}{3}}c_1}{\left(\frac{-i\sqrt{3}\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}+24i\sqrt{3}\,x-\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}-24x}{\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{1}{3}}}\right)^{\frac{2}{3}}}+x-\frac{\left(i\sqrt{3}\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}-24i\sqrt{3}\,x+\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}+24x\right)^2}{384\left(108y(x)+12\sqrt{-96x^3+81y(x)^2}\right)^{\frac{2}{3}}}=0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^3 -2 x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

34.27 problem 1029

Internal problem ID [3745]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 34
Problem number: 1029.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 - axy' + x^3 = 0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 299

```
dsolve(diff(y(x),x)^3-a*x*diff(y(x),x)+x^3 = 0,y(x), singsol=all)
```

$$y(x) = \int -\frac{i\left(\sqrt{3}\left(-108x^3+12\sqrt{-3x^3\left(4a^3-27x^3\right)}\right)^{\frac{2}{3}} - 12\sqrt{3}\,ax - i\left(-108x^3+12\sqrt{-3x^3\left(4a^3-27x^3\right)}\right)^{\frac{2}{3}} - 12}{12\left(-108x^3+12\sqrt{-3x^3\left(4a^3-27x^3\right)}\right)^{\frac{1}{3}}} + c_1$$

$$y(x) = \int \frac{i\left(\sqrt{3}\left(-108x^3+12\sqrt{-3x^3\left(4a^3-27x^3\right)}\right)^{\frac{2}{3}} - 12\sqrt{3}\,ax + i\left(-108x^3+12\sqrt{-3x^3\left(4a^3-27x^3\right)}\right)^{\frac{2}{3}} + 12}{12\left(-108x^3+12\sqrt{-3x^3\left(4a^3-27x^3\right)}\right)^{\frac{1}{3}}} + c_1$$

$$y(x) = \int \frac{\left(-108x^3+12\sqrt{-3x^3\left(4a^3-27x^3\right)}\right)^{\frac{2}{3}} + 12ax}{6\left(-108x^3+12\sqrt{-3x^3\left(4a^3-27x^3\right)}\right)^{\frac{1}{3}}}dx + c_1$$

✓ Solution by Mathematica

Time used: 166.646 (sec). Leaf size: 309

```
DSolve[(y'[x])^3 -a x y'[x]+x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \int_1^x \frac{2\sqrt[3]{3}aK[1] + \sqrt[3]{2}\left(\sqrt{81K[1]^6 - 12a^3K[1]^3} - 9K[1]^3\right)^{2/3}}{6^{2/3}\sqrt[3]{\sqrt{81K[1]^6 - 12a^3K[1]^3} - 9K[1]^3}} dK[1] + c_1$$

$$y(x) \to \int_1^x \frac{\sqrt[3]{-1}\left(\sqrt[3]{-2}\left(\sqrt{81K[2]^6 - 12a^3K[2]^3} - 9K[2]^3\right)^{2/3} - 2\sqrt[3]{3}aK[2]\right)}{6^{2/3}\sqrt[3]{\sqrt{81K[2]^6 - 12a^3K[2]^3} - 9K[2]^3}} dK[2] + c_1$$

$$y(x) \to \int_1^x \frac{\sqrt[3]{-1}\left(2\sqrt[3]{-3}aK[3] - \sqrt[3]{2}\left(\sqrt{81K[3]^6 - 12a^3K[3]^3} - 9K[3]^3\right)^{2/3}\right)}{6^{2/3}\sqrt[3]{\sqrt{81K[3]^6 - 12a^3K[3]^3} - 9K[3]^3}} dK[3] + c_1$$

35 Various 35

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35.2 problem 1031 1277
35.3 problem 1034 1278
35.4 problem 1035 1280
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35.1 problem 1030

Internal problem ID [3746]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1030.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^3 + y'ax - ya = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 39

```
dsolve(diff(y(x),x)^3+a*x*diff(y(x),x)-a*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2\sqrt{-3ax}\,x}{9}$$
$$y(x) = \frac{2\sqrt{-3ax}\,x}{9}$$
$$y(x) = c_1x + \frac{c_1^3}{a}$$

✓ Solution by Mathematica

Time used: 0.01 (sec). Leaf size: 68

```
DSolve[(y'[x])^3 +a x y'[x]-a y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1{}^3}{a} + c_1x$$
$$y(x) \to -\frac{2i\sqrt{a}x^{3/2}}{3\sqrt{3}}$$
$$y(x) \to \frac{2i\sqrt{a}x^{3/2}}{3\sqrt{3}}$$

35.2 problem 1031

Internal problem ID [3747]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1031.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^3 - (bx + a)\, y' + by = 0}$$

✓ Solution by Maple

Time used: 0.218 (sec). Leaf size: 67

```
dsolve(diff(y(x),x)^3-(b*x+a)*diff(y(x),x)+b*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2\sqrt{3xb + 3a}\,(xb + a)}{9b}$$
$$y(x) = \frac{2\sqrt{3xb + 3a}\,(xb + a)}{9b}$$
$$y(x) = c_1 x + \frac{-c_1^3 + a c_1}{b}$$

✓ Solution by Mathematica

Time used: 0.012 (sec). Leaf size: 72

```
DSolve[(y'[x])^3 -(a+b x)y'[x]+b y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{c_1(a + bx - c_1{}^2)}{b}$$
$$y(x) \to -\frac{2(a + bx)^{3/2}}{3\sqrt{3}b}$$
$$y(x) \to \frac{2(a + bx)^{3/2}}{3\sqrt{3}b}$$

35.3 problem 1034

Internal problem ID [3748]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1034.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 - 2yy' + y^2 = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 243

```
dsolve(diff(y(x),x)^3-2*y(x)*diff(y(x),x)+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$x - \left(\int^{y(x)} \frac{6\left(-108\_a^2 + 12\sqrt{81\_a^4 - 96\_a^3}\right)^{\frac{1}{3}}}{\left(-108\_a^2 + 12\sqrt{81\_a^4 - 96\_a^3}\right)^{\frac{2}{3}} + 24\_a} d\_a\right) - c_1 = 0$$

$$x - \left(\int^{y(x)} -\frac{12\left(-108\_a^2 + 12\sqrt{81\_a^4 - 96\_a^3}\right)^{\frac{1}{3}}}{\left(1 + i\sqrt{3}\right)\left(-12i\sqrt{3}\,\_a + \left(-108\_a^2 + 12\sqrt{81\_a^4 - 96\_a^3}\right)^{\frac{2}{3}} - 12\_a\right)} d\_a\right) - c_1 = 0$$

$$x - \left(\int^{y(x)} \frac{12\left(-108\_a^2 + 12\sqrt{81\_a^4 - 96\_a^3}\right)^{\frac{1}{3}}}{\left(-1 + i\sqrt{3}\right)\left(12i\sqrt{3}\,\_a + \left(-108\_a^2 + 12\sqrt{81\_a^4 - 96\_a^3}\right)^{\frac{2}{3}} - 12\_a\right)} d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.405 (sec). Leaf size: 427

```
DSolve[(y'[x])^3 -2 y[x] y'[x]+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int \frac{\sqrt[3]{\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\#1^2}}{\sqrt[3]{2}\left(\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\#1^2\right)^{2/3}+4\sqrt[3]{3}\#1}d\#1\&\right]\left[\frac{x}{6^{2/3}}+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[\int \frac{\sqrt[3]{\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\#1^2}}{\sqrt[3]{2}3^{2/3}\left(\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\#1^2\right)^{2/3}-\sqrt[3]{2}\sqrt[6]{3}i\left(\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\right.}\right.$$

$$y(x) \to \text{InverseFunction}\left[\int \frac{\sqrt[3]{\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\#1^2}}{\sqrt[3]{2}3^{2/3}\left(\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\#1^2\right)^{2/3}+\sqrt[3]{2}\sqrt[6]{3}i\left(\sqrt{3}\sqrt{\#1^3(27\#1-32)}-9\right.}\right.$$

$$y(x) \to 0$$

35.4 problem 1035

Internal problem ID [3749]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1035.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y'^3 - axyy' + 2ay^2 = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 43

```
dsolve(diff(y(x),x)^3-a*x*y(x)*diff(y(x),x)+2*a*y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{a\,x^3}{27}$$
$$y(x) = 0$$
$$y(x) = \frac{x^2}{4c_1} - \frac{x}{2c_1^2 a} + \frac{1}{4c_1^3 a^2}$$

✓ Solution by Mathematica

Time used: 146.105 (sec). Leaf size: 13176

```
DSolve[(y'[x])^3 -a x y[x] y'[x]+2 a y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

35.5 problem 1037

Internal problem ID [3750]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1037.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y'^3 - xy^4y' - y^5 = 0$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 47

```
dsolve(diff(y(x),x)^3-x*y(x)^4*diff(y(x),x)-y(x)^5 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{3\sqrt{3}}{2x^{\frac{3}{2}}}$$
$$y(x) = \frac{3\sqrt{3}}{2x^{\frac{3}{2}}}$$
$$y(x) = 0$$
$$y(x) = c_1\sqrt{\frac{c_1^{10}}{(c_1^4x-1)^2}}$$

✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 64

```
DSolve[(y'[x])^3 -x y[x]^4 y'[x]- y[x]^5==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{c_1 x - c_1{}^3}$$

$$y(x) \to 0$$

$$y(x) \to \text{Indeterminate}$$

$$y(x) \to -\frac{3\sqrt{3}}{2x^{3/2}}$$

$$y(x) \to \frac{3\sqrt{3}}{2x^{3/2}}$$

35.6 problem 1038

Internal problem ID [3751]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1038.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y'^3 + \mathrm{e}^{3x-2y}(y'-1) = 0}$$

✓ Solution by Maple

Time used: 13.266 (sec). Leaf size: 540

```
dsolve(diff(y(x),x)^3+exp(3*x-2*y(x))*(diff(y(x),x)-1) = 0,y(x), singsol=all)
```

$$y(x) = \frac{3x}{2} + \operatorname{RootOf}\left(-x + 2\left(\int^{\_Z} -\frac{e^{2\_a}\left(18\sqrt{12\,e^{-6\_a}+81\,e^{-4\_a}}+162\,e^{-2\_a}\right)^{\frac{1}{3}}}{3\,e^{2\_a}\left(18\sqrt{12\,e^{-6\_a}+81\,e^{-4\_a}}+162\,e^{-2\_a}\right)^{\frac{1}{3}} - 2\left(\sqrt{12\,e^{-6\_a}+81\,e^{-4\_a}}+9\,e^{-2\_a}\right)^{\frac{2}{3}} e^{2\_a} + 2\,12^{\frac{1}{3}}} d\_a\right. \right.$$
$$\left. + c_1\right)$$

$$y(x) = \frac{3x}{2} + \operatorname{RootOf}\left(-x\right.$$
$$+12i\left(\int^{\_Z} \frac{e^{2\_a+3}\left(\sqrt{4\right.}}{2\,e^{2\_a+3}\left(6\,e^{-6\_a}+81\,e^{-4\_a}+9\,e^{-2\_a}\sqrt{12\,e^{-6\_a}+81\,e^{-4\_a}}\right)^{\frac{1}{3}} 3^{\frac{5}{6}} - 18i3^{\frac{1}{6}}\left(\sqrt{4\,e^{-6\_a}+27\,e^{-}\right.}\right.$$
$$\left. + c_1\right)$$

$$y(x) = \frac{3x}{2} + \operatorname{RootOf}\left(x\right.$$
$$+12i\left(\int^{\_Z} \frac{e^{2\_a+3}\left(\sqrt{4\right.}}{2\,e^{2\_a+3}\left(6\,e^{-6\_a}+81\,e^{-4\_a}+9\,e^{-2\_a}\sqrt{12\,e^{-6\_a}+81\,e^{-4\_a}}\right)^{\frac{1}{3}} 3^{\frac{5}{6}} + 18i3^{\frac{1}{6}}\left(\sqrt{4\,e^{-6\_a}+27\,e^{-}\right.}\right.$$
$$\left. - c_1\right)$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^3 +Exp[3 x -2 y[x]](y'[x]-1)==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

35.7 problem 1039

Internal problem ID [3752]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1039.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$y'^3 + e^{-2y}\left(e^{2x} + e^{3x}\right) y' - e^{3x-2y} = 0$$

✓ Solution by Maple

Time used: 0.406 (sec). Leaf size: 26

```
dsolve(diff(y(x),x)^3+exp(-2*y(x))*(exp(2*x)+exp(3*x))*diff(y(x),x)-exp(3*x-2*y(x)) = 0,y(x),
```

$$y(x) = x - \frac{\ln\left(-\frac{1}{(c_1+1)(c_1 e^{-x}-1)^2}\right)}{2}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^3 +Exp[-2 y[x]] (Exp[2 x]+Exp[3 x])(y'[x])-Exp[3 x-2 y[x]]==0,y[x],x,IncludeSi
```

Timed out

35.8 problem 1040

Internal problem ID [3753]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1040.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^3 + y'^2 - y = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 388

```
dsolve(diff(y(x),x)^3+diff(y(x),x)^2-y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$x - \left(\int^{y(x)} \frac{6\,6^{\frac{1}{3}}\left(-8+108\_a+12\sqrt{81\_a^2-12\_a}\right)^{\frac{1}{3}}}{6^{\frac{1}{3}}\left(-8+108\_a+12\sqrt{81\_a^2-12\_a}\right)^{\frac{2}{3}}+4\,6^{\frac{1}{3}}-4\left(\sqrt{3}\left(27\sqrt{3}\_a-2\sqrt{3}+9\sqrt{\_a(-4+27\_a}\right.\right.}\right. - c_1 = 0$$

$$x - \left(\int^{y(x)} -\frac{12\,6^{\frac{1}{3}}\left(-8+108\_a+12\sqrt{81\_a^2-12\_a}\right)^{\frac{1}{3}}}{i\sqrt{3}\,6^{\frac{1}{3}}\left(-8+108\_a+12\sqrt{81\_a^2-12\_a}\right)^{\frac{2}{3}}-4i\sqrt{3}\,6^{\frac{1}{3}}+6^{\frac{1}{3}}\left(-8+108\_a+12\sqrt{81\_a^2-12\_a}\right)^{\frac{2}{3}}+4\,6}\right. - c_1 = 0$$

$$x - \left(\int^{y(x)} \frac{12\,6^{\frac{1}{3}}\left(-8+108\_a+12\sqrt{81\_a^2-12}\right.}{i\sqrt{3}\,6^{\frac{1}{3}}\left(-8+108\_a+12\sqrt{81\_a^2-12\_a}\right)^{\frac{2}{3}}-4i\sqrt{3}\,6^{\frac{1}{3}}-6^{\frac{1}{3}}\left(-8+108\_a+12\sqrt{81\_a^2-12\_a}\right.}\right. - c_1 = 0$$

✓ Solution by Mathematica

Time used: 105.125 (sec). Leaf size: 515

```
DSolve[(y'[x])^3 + (y'[x])^2 -y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$\rightarrow \text{InverseFunction}\Bigg[\int_1^{\#1} \frac{\sqrt[3]{-27K[1]+3\sqrt{3}\sqrt{K[1](27K[1]-4)}+2}}{2^{2/3}\left(-27K[1]+3\sqrt{3}\sqrt{K[1](27K[1]-4)}+2\right)^{2/3}+2\sqrt[3]{-27K[1]+3\sqrt{3}\sqrt{K[1](\ldots}$

$y(x)$

$\rightarrow \text{InverseFunction}\Bigg[\int_1^{\#1} \frac{\sqrt[3]{-27K[2]+3\ldots}}{-i2^{2/3}\sqrt{3}\left(-27K[2]+3\sqrt{3}\sqrt{K[2](27K[2]-4)}+2\right)^{2/3}+2^{2/3}\left(-27K[2]+3\sqrt{3}\ldots\right.}$

$y(x)$

$\rightarrow \text{InverseFunction}\Bigg[\int_1^{\#1} \frac{\sqrt[3]{-27K[3]+3\ldots}}{i2^{2/3}\sqrt{3}\left(-27K[3]+3\sqrt{3}\sqrt{K[3](27K[3]-4)}+2\right)^{2/3}+2^{2/3}\left(-27K[3]+3\sqrt{3}\sqrt{\ldots}\right.}$

$y(x) \rightarrow 0$

35.9 problem 1041

Internal problem ID [3754]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1041.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^3 - y'^2 + y^2 = 0}$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 401

```
dsolve(diff(y(x),x)^3-diff(y(x),x)^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$x - \left(\int^{y(x)} \frac{6\left(8-108\_a^2+12\sqrt{81\_a^4-12\_a^2}\right)^{\frac{1}{3}}}{\left(8-108\_a^2+12\sqrt{81\_a^4-12\_a^2}\right)^{\frac{2}{3}}+2\left(-\frac{4\sqrt{3}\left(27\sqrt{3}\,\_a^2-2\sqrt{3}-9\sqrt{\_a^2\left(27\_a^2-4\right)}\right)}{3}\right)^{\frac{1}{3}}+4}d\_a\right) - c_1 = 0$$

$$x - \left(\int^{y(x)} \frac{12i\left(8-108\_a^2+12\sqrt{81\_a^4-12\_a^2}\right)^{\frac{1}{3}}}{\sqrt{3}\,\left(8-108\_a^2+12\sqrt{81\_a^4-12\_a^2}\right)^{\frac{2}{3}}-4\sqrt{3}-i\,\left(8-108\_a^2+12\sqrt{81\_a^4-12\_a^2}\right)^{\frac{2}{3}}+4i\left(\right.}\right. - c_1 = 0$$

$$x - \left(\int^{y(x)}\right.$$

$$-\frac{12i\left(8-108\_a^2+12\sqrt{81\_a^4-12\_a^2}\right)^{\frac{1}{3}}}{\sqrt{3}\,\left(8-108\_a^2+12\sqrt{81\_a^4-12\_a^2}\right)^{\frac{2}{3}}+i\,\left(8-108\_a^2+12\sqrt{81\_a^4-12\_a^2}\right)^{\frac{2}{3}}-4i\left(-\frac{4\sqrt{3}\left(27\sqrt{3}\,\_a^2\right.}{}\right.}$$

$$-c_1 = 0$$

✓ Solution by Mathematica

Time used: 47.57 (sec). Leaf size: 583

```
DSolve[(y'[x])^3 - (y'[x])^2 +y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt[3]{-27K[1]^2+3\sqrt{3}\sqrt{K[1]^2\left(27K[1]^2-4\right)}+2}}{2^{2/3}\left(-27K[1]^2+3\sqrt{3}\sqrt{K[1]^2\left(27K[1]^2-4\right)}+2\right)^{2/3}+2\sqrt[3]{-27K[1]^2+3\sqrt{3}\sqrt{K}}}\right.$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt[3]{-27K[2]^2}}{-i2^{2/3}\sqrt{3}\left(-27K[2]^2+3\sqrt{3}\sqrt{K[2]^2\left(27K[2]^2-4\right)}+2\right)^{2/3}-2^{2/3}\left(-27K[2]^2+3\right.}\right.$$

$$y(x) \to \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt[3]{-27K[3]^2}}{i2^{2/3}\sqrt{3}\left(-27K[3]^2+3\sqrt{3}\sqrt{K[3]^2\left(27K[3]^2-4\right)}+2\right)^{2/3}-2^{2/3}\left(-27K[3]^2+3\sqrt{}\right.}\right.$$

$$y(x) \to 0$$

35.10 problem 1042

Internal problem ID [3755]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1042.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{y'^3 - y'^2 + y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 70

```
dsolve(diff(y(x),x)^3-diff(y(x),x)^2+x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{x}{3} - \frac{2}{27} - \frac{2\sqrt{-27x^3 + 27x^2 - 9x + 1}}{27}$$
$$y(x) = \frac{x}{3} - \frac{2}{27} + \frac{2\sqrt{-27x^3 + 27x^2 - 9x + 1}}{27}$$
$$y(x) = c_1^3 - c_1^2 + c_1 x$$

✓ Solution by Mathematica

Time used: 0.024 (sec). Leaf size: 74

```
DSolve[(y'[x])^3 - (y'[x])^2 +x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x + (-1 + c_1)c_1)$$
$$y(x) \to \frac{1}{27}\left(9x - 2\left(\sqrt{-(3x-1)^3} + 1\right)\right)$$
$$y(x) \to \frac{1}{27}\left(9x + 2\sqrt{-(3x-1)^3} - 2\right)$$

35.11 problem 1043

Internal problem ID [3756]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1043.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{y'^3 - ay'^2 + by + abx = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 96

```
dsolve(diff(y(x),x)^3-a*diff(y(x),x)^2+b*y(x)+a*b*x = 0,y(x), singsol=all)
```

$$y(x) = -ax + \frac{\left(e^{\mathrm{RootOf}\left(-10\_Z a^2 - 3\,e^{2\_Z} + 16a\,e^{\_Z} + 2c_1 b - 13a^2 - 2xb\right)} - a\right)^2 a - \left(e^{\mathrm{RootOf}\left(-10\_Z a^2 - 3\,e^{2\_Z} + 16a\,e^{\_Z} + 2c_1 b - 13a^2 - 2xb\right)} - a\right)}{b}$$

✓ Solution by Mathematica

Time used: 0.61 (sec). Leaf size: 398

```
DSolve[(y'[x])^3 - a (y'[x])^2 +b y[x]+a b x==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x=\frac{5a\left(\frac{\sqrt[3]{2a^3+\sqrt{(2a^3-27abx-27by(x))^2-4a^6}-27abx-27by(x)}}{3\sqrt[3]{2}}+\frac{\cdots}{3\sqrt[3]{2a^3+\sqrt{(2a^3-27a\cdots}}}\right.}{\cdots}\right.\right.$$

$$\left.\left.+c_1\right\},y(x)\right]$$

35.12 problem 1044

Internal problem ID [3757]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1044.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^3 + \mathrm{a0}\, y'^2 + \mathrm{a1}\, y' + \mathrm{a2} + \mathrm{a3}\, y = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 1027

```
dsolve(diff(y(x),x)^3+a0*diff(y(x),x)^2+a1*diff(y(x),x)+a2+a3*y(x) = 0,y(x), singsol=all)
```

$$x - \left(\int^{y(x)} \frac{6\left(36\,\mathrm{a1}\,\mathrm{a}\right.}{\left(36\,\mathrm{a1}\,\mathrm{a0} - 108\,\mathrm{a3}\_a - 108\,\mathrm{a2} - 8\,\mathrm{a0}^3 + 12\sqrt{12\_a\;\mathrm{a0}^3\,\mathrm{a3} + 81\_a^2\,\mathrm{a3}^2 - 54\_a\,\mathrm{a0}\,\mathrm{a1}\,\mathrm{a3} + 12\,\mathrm{a2}\;\mathrm{a0}^3}\right.}\right. - c_1 = 0$$

$$x - \left(\int^{y(x)} \frac{}{\left(1 + i\sqrt{3}\right)\left(i\sqrt{3}\,\left(36\,\mathrm{a1}\,\mathrm{a0} - 108\,\mathrm{a3}\_a - 108\,\mathrm{a2} - 8\,\mathrm{a0}^3 + 12\sqrt{12\_a\;\mathrm{a0}^3\,\mathrm{a3} + 81\_a^2\,\mathrm{a3}^2 - 54\_a\,\mathrm{a0}}\right.\right.}\right. - c_1 = 0$$

$$x - \left(\int^{y(x)} \frac{}{\left(-1 + i\sqrt{3}\right)\left(i\sqrt{3}\,\left(36\,\mathrm{a1}\,\mathrm{a0} - 108\,\mathrm{a3}\_a - 108\,\mathrm{a2} - 8\,\mathrm{a0}^3 + 12\sqrt{12\_a\;\mathrm{a0}^3\,\mathrm{a3} + 81\_a^2\,\mathrm{a3}^2 - 54\_a\,\mathrm{a}}\right.\right.}\right. - c_1 = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^3 + a0 (y'[x])^2 +a1 y'[x]+a2 +a3 y[x]==0,y[x],x,IncludeSingularSolutions -> T
```

Timed out

35.13 problem 1046

Internal problem ID [3758]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1046.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 + (-3x + 1)\, y'^2 - x(-3x + 1)\, y' - 1 - x^3 = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 496

```
dsolve(diff(y(x),x)^3+(1-3*x)*diff(y(x),x)^2-x*(1-3*x)*diff(y(x),x)-1-x^3 = 0,y(x), singsol=a
```

$$y(x) = \int -\frac{i\left(\sqrt{3}\left(36x + 100 + 12\sqrt{3}\sqrt{(x+1)(4x^2 - 5x + 23)}\right)^{\frac{2}{3}} - i\left(36x + 100 + 12\sqrt{3}\sqrt{(x+1)(4x^2 - 5x + \cdots}\right.\right.}{12 \cdots} + c_1$$

$$y(x) = \int \frac{i\left(\sqrt{3}\left(36x + 100 + 12\sqrt{3}\sqrt{(x+1)(4x^2 - 5x + 23)}\right)^{\frac{2}{3}} + 12\sqrt{3}\,x - 4\sqrt{3} + i\left(36x + 100 + 12\sqrt{3}\sqrt{(x \cdots}\right.\right.}{12 \cdots} + c_1$$

$$y(x) = \int \frac{\left(36x + 100 + 12\sqrt{3}\sqrt{(x+1)(4x^2 - 5x + 23)}\right)^{\frac{2}{3}} + 6x\left(36x + 100 + 12\sqrt{3}\sqrt{(x+1)(4x^2 - 5x + 23)}\right) \cdots}{6\left(36x + 100 + 12\sqrt{3}\sqrt{(x+1)(4x^2 - 5x + \cdots}\right.} + c_1$$

✓ Solution by Mathematica

Time used: 110.186 (sec). Leaf size: 344

```
DSolve[(y'[x])^3+(1-3 x)(y'[x])^2-x(1-3 x)y'[x]-1 -x^3==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \to \int_1^x \frac{1}{6}\left(6K[1] - 2^{2/3}\sqrt[3]{-9K[1] + 3\sqrt{12K[1]^3 - 3K[1]^2 + 54K[1] + 69} - 25} + \frac{2\sqrt[3]{2}(3K[1]-1)}{\sqrt[3]{-9K[1] + 3\sqrt{12K[1]^3 - 3K[1]^2 + 54K[1] + 69} - 25}} - 2\right) dK[1] + c_1$$

$$y(x) \to \int_1^x \frac{1}{12}\left(\frac{4\sqrt[3]{-2}(1-3K[2])}{\sqrt[3]{-9K[2] + 3\sqrt{12K[2]^3 - 3K[2]^2 + 54K[2] + 69} - 25}} + 12K[2] - 2(-2)^{2/3}\sqrt[3]{-9K[2] + 3\sqrt{12K[2]^3 - 3K[2]^2 + 54K[2] + 69} - 25} - 4\right) dK[2] + c_1$$

$$y(x) \to \int_1^x \left(K[3] + \frac{1}{3}\sqrt[3]{-\frac{1}{2}}\sqrt[3]{-9K[3] + 3\sqrt{12K[3]^3 - 3K[3]^2 + 54K[3] + 69} - 25} + \frac{(-1)^{2/3}\sqrt[3]{2}(3K[3]-1)}{3\sqrt[3]{-9K[3] + 3\sqrt{12K[3]^3 - 3K[3]^2 + 54K[3] + 69} - 25}} - \frac{1}{3}\right) dK[3] + c_1$$

35.14 problem 1047

Internal problem ID [3759]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1047.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 - yy'^2 + y^2 = 0$$

✓ Solution by Maple

Time used: 0.156 (sec). Leaf size: 427

```
dsolve(diff(y(x),x)^3-y(x)*diff(y(x),x)^2+y(x)^2 = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$x - \left(\int^{y(x)} \frac{6\left(-108\_a^2 + 8\_a^3 + 12\sqrt{-12\_a^5 + 81\_a^4}\right)^{\frac{1}{3}}}{\left(-108\_a^2 + 8\_a^3 + 12\sqrt{-12\_a^5 + 81\_a^4}\right)^{\frac{2}{3}} + 2\_a\left(-108\_a^2 + 8\_a^3 + 12\sqrt{-3\_a^4\left(-27 + 4\_\right.}\right.}\right.$$

$$- c_1 = 0$$

$$x - \left(\int^{y(x)} \frac{12i\left(-108\_a^2 + 8\_a^3 + 12\sqrt{-12\right.}{\sqrt{3}\left(-108\_a^2 + 8\_a^3 + 12\sqrt{-12\_a^5 + 81\_a^4}\right)^{\frac{2}{3}} - 4\sqrt{3}\_a^2 + 4i\_a\left(-108\_a^2 + 8\_a^3 + 12\sqrt{-}\right.}\right.$$

$$- c_1 = 0$$

$$x - \left(\int^{y(x)} - \frac{12i\left(-108\_a^2 + 8\_a^3 + 12\sqrt{-12\_a^5 + 8\right.}{\sqrt{3}\left(-108\_a^2 + 8\_a^3 + 12\sqrt{-12\_a^5 + 81\_a^4}\right)^{\frac{2}{3}} - 4\sqrt{3}\_a^2 + i\left(-108\_a^2 + 8\_a^3 + 12\sqrt{-12\_a^5 + 81\right.}\right.$$

$$- c_1 = 0$$

✓ Solution by Mathematica

Time used: 56.309 (sec). Leaf size: 653

```
DSolve[(y'[x])^3 -y[x] (y'[x])^2+y[x]^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$y(x)$

$\rightarrow \text{InverseFunction}\left[\int_1^{\#1} \frac{\sqrt[3]{2K[1]^3 - 27K[1]^2 + 3\sqrt{3}\sqrt{-K[1]^4(4K}}}{2\sqrt[3]{2}K[1]^2 + 2\sqrt[3]{2K[1]^3 - 27K[1]^2 + 3\sqrt{3}\sqrt{-K[1]^4(4K[1]-27)}}K[1] + 2^{2/3}\left(2K\right.}\right.$

$y(x)$

$\rightarrow \text{InverseFunction}\left[\int_1^{\#1} \frac{}{2i\sqrt[3]{2}\sqrt{3}K[2]^2 - 2\sqrt[3]{2}K[2]^2 + 4\sqrt[3]{2K[2]^3 - 27K[2]^2 + 3\sqrt{3}\sqrt{-K[2]^4(4K[2]-27)}}}\right.$

$y(x)$

$\rightarrow \text{InverseFunction}\left[\int_1^{\#1} \frac{}{-2i\sqrt[3]{2}\sqrt{3}K[3]^2 - 2\sqrt[3]{2}K[3]^2 + 4\sqrt[3]{2K[3]^3 - 27K[3]^2 + 3\sqrt{3}\sqrt{-K[3]^4(4K[3]-2}}}\right.$

$y(x) \rightarrow 0$

35.15 problem 1048

Internal problem ID [3760]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1048.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 + (\cos(x)\cot(x) - y)\, y'^2 - (1 + y\cos(x)\cot(x))\, y' + y = 0$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)^3+(cos(x)*cot(x)-y(x))*diff(y(x),x)^2-(1+y(x)*cos(x)*cot(x))*diff(y(x),x)
```

$$y(x) = c_1 e^x$$
$$y(x) = -\ln(\csc(x) - \cot(x)) + c_1$$
$$y(x) = -\cos(x) + c_1$$

✓ Solution by Mathematica

Time used: 0.027 (sec). Leaf size: 45

```
DSolve[(y'[x])^3 +(Cos[x] Cot[x]-y[x])(y'[x])^2-(1+y[x] Cos[x] Cot[x])y'[x]+y[x]==0,y[x],x,In
```

$$y(x) \to c_1 e^x$$
$$y(x) \to -\cos(x) + c_1$$
$$y(x) \to -\log\left(\sin\left(\frac{x}{2}\right)\right) + \log\left(\cos\left(\frac{x}{2}\right)\right) + c_1$$

35.16 problem 1049

Internal problem ID [3761]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1049.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 + (2x - y^2)\, y'^2 - 2xy^2y' = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

```
dsolve(diff(y(x),x)^3+(2*x-y(x)^2)*diff(y(x),x)^2-2*x*y(x)^2*diff(y(x),x) = 0,y(x), singsol=a
```

$$y(x) = \frac{1}{c_1 - x}$$
$$y(x) = -x^2 + c_1$$
$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.057 (sec). Leaf size: 31

```
DSolve[(y'[x])^3 +(2 x-y[x]^2) (y'[x])^2 -2 x y[x]^2 y'[x]==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to -\frac{1}{x + c_1}$$
$$y(x) \to c_1$$
$$y(x) \to -x^2 + c_1$$

35.17 problem 1050

Internal problem ID [3762]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1050.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 - (2x + y^2)\,y'^2 + (x^2 - y^2 + 2xy^2)\,y' - (x^2 - y^2)\,y^2 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x)^3-(2*x+y(x)^2)*diff(y(x),x)^2+(x^2-y(x)^2+2*x*y(x)^2)*diff(y(x),x)-(x^2-y
```

$$y(x) = \frac{1}{c_1 - x}$$
$$y(x) = c_1 e^x - x - 1$$
$$y(x) = x - 1 + c_1 e^{-x}$$

✓ Solution by Mathematica

Time used: 0.15 (sec). Leaf size: 48

```
DSolve[(y'[x])^3 -(2 x+y[x]^2) (y'[x])^2 +(x^2 -y[x]^2+2 x y[x]^2) y'[x]-(x^2-y[x]^2)y[x]^2==
```

$$y(x) \to -\frac{1}{x + c_1}$$
$$y(x) \to x + c_1 e^{-x} - 1$$
$$y(x) \to -x + c_1 e^x - 1$$
$$y(x) \to 0$$

35.18 problem 1051

Internal problem ID [3763]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1051.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 - (x^2 + xy + y^2)\, y'^2 + xy(x^2 + xy + y^2)\, y' - x^3y^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)^3-(x^2+x*y(x)+y(x)^2)*diff(y(x),x)^2+x*y(x)*(x^2+x*y(x)+y(x)^2)*diff(y(x)
```

$$y(x) = \frac{x^3}{3} + c_1$$
$$y(x) = \frac{1}{c_1 - x}$$
$$y(x) = \mathrm{e}^{\frac{x^2}{2}} c_1$$

✓ Solution by Mathematica

Time used: 0.098 (sec). Leaf size: 48

```
DSolve[(y'[x])^3 -(x^2+x y[x]+ y[x]^2) (y'[x])^2 +x y[x](x^2 +x y[x]+ y[x]^2) y'[x]-x^3 y[x]^
```

$$y(x) \to -\frac{1}{x + c_1}$$
$$y(x) \to c_1 e^{\frac{x^2}{2}}$$
$$y(x) \to \frac{x^3}{3} + c_1$$
$$y(x) \to 0$$

35.19 problem 1052

Internal problem ID [3764]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1052.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$y'^3 - (x^2 + xy^2 + y^4)\, y'^2 + xy^2 (x^2 + xy^2 + y^4)\, y' - x^3 y^6 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 89

```
dsolve(diff(y(x),x)^3-(x^2+x*y(x)^2+y(x)^4)*diff(y(x),x)^2+x*y(x)^2*(x^2+x*y(x)^2+y(x)^4)*dif
```

$$y(x) = \frac{x^3}{3} + c_1$$
$$y(x) = \frac{1}{(-3x + c_1)^{\frac{1}{3}}}$$
$$y(x) = -\frac{1}{2(-3x + c_1)^{\frac{1}{3}}} - \frac{i\sqrt{3}}{2(-3x + c_1)^{\frac{1}{3}}}$$
$$y(x) = -\frac{1}{2(-3x + c_1)^{\frac{1}{3}}} + \frac{i\sqrt{3}}{2(-3x + c_1)^{\frac{1}{3}}}$$
$$y(x) = \frac{2}{-x^2 + 2c_1}$$

✓ Solution by Mathematica

Time used: 0.228 (sec). Leaf size: 110

```
DSolve[(y'[x])^3 -(x^2+x y[x]^2+ y[x]^4) (y'[x])^2 +x y[x]^2(x^2 +x y[x]^2+ y[x]^4) y'[x]-x^3
```

$$y(x) \to -\frac{\sqrt[3]{-\frac{1}{3}}}{\sqrt[3]{-x-c_1}}$$

$$y(x) \to \frac{1}{\sqrt[3]{3}\sqrt[3]{-x-c_1}}$$

$$y(x) \to \frac{(-1)^{2/3}}{\sqrt[3]{3}\sqrt[3]{-x-c_1}}$$

$$y(x) \to \frac{x^3}{3} + c_1$$

$$y(x) \to -\frac{2}{x^2+2c_1}$$

$$y(x) \to 0$$

35.20 problem 1053

Internal problem ID [3765]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1053.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{2{y'}^3 + y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 79

```
dsolve(2*diff(y(x),x)^3+x*diff(y(x),x)-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \left(-\frac{c_1}{12} - \frac{\sqrt{c_1^2 + 24x}}{12}\right)^3 + \frac{x\left(-\frac{c_1}{12} - \frac{\sqrt{c_1^2+24x}}{12}\right)}{2}$$

$$y(x) = \left(-\frac{c_1}{12} + \frac{\sqrt{c_1^2 + 24x}}{12}\right)^3 + \frac{x\left(-\frac{c_1}{12} + \frac{\sqrt{c_1^2+24x}}{12}\right)}{2}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2 (y'[x])^3 +x y'[x]-2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

35.21 problem 1054

Internal problem ID [3766]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1054.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$2y'^3 + y'^2 - y = 0$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 369

```
dsolve(2*diff(y(x),x)^3+diff(y(x),x)^2-y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$x - \left(\int^{y(x)} \frac{6\, 3^{\frac{1}{3}} \left(-1+54\_a+6\sqrt{81\_a^2-3\_a}\right)^{\frac{1}{3}}}{3^{\frac{1}{3}} \left(-1+54\_a+6\sqrt{81\_a^2-3\_a}\right)^{\frac{2}{3}} + 3^{\frac{1}{3}} - \left(\sqrt{3}\left(54\sqrt{3}\_a-\sqrt{3}+18\sqrt{\_a\,(-1+27\_a)}\right)\right)^{\frac{1}{3}}} d\right. - c_1 = 0$$

$$x - \left(\int^{y(x)} - \frac{12\, 3^{\frac{1}{3}} \left(-1+54\_a+6\sqrt{81\_a^2-3\_a}\right)^{\frac{1}{3}}}{i3^{\frac{5}{6}} \left(-1+54\_a+6\sqrt{81\_a^2-3\_a}\right)^{\frac{2}{3}} - i3^{\frac{5}{6}} + 3^{\frac{1}{3}} \left(-1+54\_a+6\sqrt{81\_a^2-3\_a}\right)^{\frac{2}{3}} + 3^{\frac{1}{3}} + 2\left(\sqrt{3}\left(54\sqrt{\right.\right.}\right. - c_1 = 0$$

$$x - \left(\int^{y(x)} - \frac{12\, 3^{\frac{1}{3}} \left(-1+54\_a+6\sqrt{81\_a^2-3\_a}\right)^{\frac{1}{3}}}{-i3^{\frac{5}{6}} \left(-1+54\_a+6\sqrt{81\_a^2-3\_a}\right)^{\frac{2}{3}} + i3^{\frac{5}{6}} + 3^{\frac{1}{3}} \left(-1+54\_a+6\sqrt{81\_a^2-3\_a}\right)^{\frac{2}{3}} + 3^{\frac{1}{3}} + 2\left(\sqrt{3}\left(54\right.\right.}\right. - c_1 = 0$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[2 (y'[x])^3 + (y'[x])^2 - y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

35.22 problem 1055

Internal problem ID [3767]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1055.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$3{y'}^3 - x^4 y' + 2yx^3 = 0$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 34

```
dsolve(3*diff(y(x),x)^3-x^4*diff(y(x),x)+2*x^3*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{x^3}{9}$$
$$y(x) = \frac{x^3}{9}$$
$$y(x) = \frac{x^2}{2c_1} - \frac{3}{2c_1^3}$$

✓ Solution by Mathematica

Time used: 90.585 (sec). Leaf size: 15992

```
DSolve[3 (y'[x])^3 - x^4 y'[x]+2 x^3 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

35.23 problem 1056

Internal problem ID [3768]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1056.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{4y'^3 + 4y' - x = 0}$$

✓ Solution by Maple

Time used: 0.157 (sec). Leaf size: 198

```
dsolve(4*diff(y(x),x)^3+4*diff(y(x),x) = x,y(x), singsol=all)
```

$$y(x) = \int -\frac{i\left(\sqrt{3}\left(27x+3\sqrt{81x^2+192}\right)^{\frac{2}{3}} - i\left(27x+3\sqrt{81x^2+192}\right)^{\frac{2}{3}} + 12\sqrt{3} + 12i\right)}{12\left(27x+3\sqrt{81x^2+192}\right)^{\frac{1}{3}}}dx + c_1$$

$$y(x) = \int \frac{i\left(\sqrt{3}\left(27x+3\sqrt{81x^2+192}\right)^{\frac{2}{3}} + 12\sqrt{3} + i\left(27x+3\sqrt{81x^2+192}\right)^{\frac{2}{3}} - 12i\right)}{12\left(27x+3\sqrt{81x^2+192}\right)^{\frac{1}{3}}}dx + c_1$$

$$y(x) = \int \frac{\left(27x+3\sqrt{81x^2+192}\right)^{\frac{2}{3}} - 12}{6\left(27x+3\sqrt{81x^2+192}\right)^{\frac{1}{3}}}dx + c_1$$

✓ Solution by Mathematica

Time used: 3.005 (sec). Leaf size: 360

```
DSolve[4 (y'[x])^3 +4 y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\sqrt[3]{-\frac{1}{3}}\left(27x^2 - 3\sqrt{81x^2+192}x + 16\right)}{\left(\sqrt{81x^2+192} - 9x\right)^{4/3}} + \frac{\left(1 - i\sqrt{3}\right)\left(3x\left(\sqrt{81x^2+192} - 9x\right) + 32\right)}{16\ 3^{2/3}\left(\sqrt{81x^2+192} - 9x\right)^{2/3}} + c_1$$

$$y(x) \to \frac{i\left(\sqrt{3} + i\right)\left(3x\left(\sqrt{81x^2+192} - 9x\right) - 16\right)}{2\sqrt[3]{3}\left(\sqrt{81x^2+192} - 9x\right)^{4/3}} + \frac{\left(1 + i\sqrt{3}\right)\left(3x\left(\sqrt{81x^2+192} - 9x\right) + 32\right)}{16\ 3^{2/3}\left(\sqrt{81x^2+192} - 9x\right)^{2/3}} + c_1$$

$$y(x) \to \frac{\left(\sqrt{81x^2+192} - 9x\right)^{4/3}}{48\ 3^{2/3}} - \frac{8}{3^{2/3}\left(\sqrt{81x^2+192} - 9x\right)^{2/3}} + \frac{9\sqrt[6]{3}x\left(\sqrt{27x^2+64} - 3\sqrt{3}x\right) - 16\ 3^{2/3}}{3\left(\sqrt{81x^2+192} - 9x\right)^{4/3}} + c_1$$

35.24 problem 1057

Internal problem ID [3769]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1057.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$8{y'}^{3}+12{y'}^{2}-27x-27y=0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 39

```
dsolve(8*diff(y(x),x)^3+12*diff(y(x),x)^2 = 27*x+27*y(x),y(x), singsol=all)
```

$$y(x)=-x+\frac{4}{27}$$
$$y(x)=-(-c_1+x)^{\frac{3}{2}}-c_1$$
$$y(x)=(-c_1+x)^{\frac{3}{2}}-c_1$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[8 (y'[x])^3 + 12 (y'[x])^2 ==27(x+y[x]),y[x],x,IncludeSingularSolutions -> True]
```

Timed out

35.25 problem 1058

Internal problem ID [3770]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1058.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{xy'^3 - yy'^2 + a = 0}$$

✓ Solution by Maple

Time used: 0.328 (sec). Leaf size: 92

```
dsolve(x*diff(y(x),x)^3-y(x)*diff(y(x),x)^2+a = 0,y(x), singsol=all)
```

$$y(x) = \frac{3\,2^{\frac{1}{3}}(a\,x^2)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{3\,2^{\frac{1}{3}}(a\,x^2)^{\frac{1}{3}}}{4} - \frac{3i\sqrt{3}\,2^{\frac{1}{3}}(a\,x^2)^{\frac{1}{3}}}{4}$$
$$y(x) = -\frac{3\,2^{\frac{1}{3}}(a\,x^2)^{\frac{1}{3}}}{4} + \frac{3i\sqrt{3}\,2^{\frac{1}{3}}(a\,x^2)^{\frac{1}{3}}}{4}$$
$$y(x) = c_1x + \frac{a}{c_1^2}$$

✓ Solution by Mathematica

Time used: 0.015 (sec). Leaf size: 89

```
DSolve[x (y'[x])^3 - y[x] (y'[x])^2 +a==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{a}{c_1{}^2} + c_1 x$$

$$y(x) \to \frac{3\sqrt[3]{a}x^{2/3}}{2^{2/3}}$$

$$y(x) \to -\frac{3\sqrt[3]{-1}\sqrt[3]{a}x^{2/3}}{2^{2/3}}$$

$$y(x) \to \frac{3(-1)^{2/3}\sqrt[3]{a}x^{2/3}}{2^{2/3}}$$

35.26 problem 1060

Internal problem ID [3771]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1060.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$x{y'}^3 - \left(x + x^2 + y\right){y'}^2 + \left(x^2 + y + xy\right)y' - xy = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(x*diff(y(x),x)^3-(x+x^2+y(x))*diff(y(x),x)^2+(x^2+y(x)+x*y(x))*diff(y(x),x)-x*y(x) = 0
```

$$y(x) = c_1 x$$
$$y(x) = x + c_1$$
$$y(x) = \frac{x^2}{2} + c_1$$

✓ Solution by Mathematica

Time used: 0.044 (sec). Leaf size: 36

```
DSolve[x (y'[x])^3 - (x+x^2+y[x])(y'[x])^2 + (x^2+y[x]+x y[x]) y'[x]-x y[x]==0,y[x],x,Include
```

$$y(x) \to c_1 x$$
$$y(x) \to x + c_1$$
$$y(x) \to \frac{x^2}{2} + c_1$$
$$y(x) \to 0$$

35.27 problem 1061

Internal problem ID [3772]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1061.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{xy'^3 - 2yy'^2 + 4x^2 = 0}$$

✓ Solution by Maple

Time used: 0.265 (sec). Leaf size: 831

```
dsolve(x*diff(y(x),x)^3-2*y(x)*diff(y(x),x)^2+4*x^2 = 0,y(x), singsol=all)
```

$$y(x) = \frac{3x^{\frac{4}{3}}}{2}$$

$$y(x) = \frac{3\left(-\frac{x^{\frac{1}{3}}}{2} - \frac{i\sqrt{3}\,x^{\frac{1}{3}}}{2}\right)x}{2}$$

$$y(x) = \frac{3\left(-\frac{x^{\frac{1}{3}}}{2} + \frac{i\sqrt{3}\,x^{\frac{1}{3}}}{2}\right)x}{2}$$

$$y(x) = -\frac{4x^2}{c_1} + \frac{c_1^2}{32}$$

$$y(x) = \frac{4x^2}{c_1} + \frac{c_1^2}{32}$$

$$y(x) = \frac{c_1\left(-1728x^2 + c_1^3 + 24\sqrt{-6c_1^3x^2 + 5184x^4}\right)^{\frac{1}{3}}}{96} + \frac{c_1^3}{96\left(-1728x^2 + c_1^3 + 24\sqrt{-6c_1^3x^2 + 5184x^4}\right)^{\frac{1}{3}}} + \frac{c_1^2}{96}$$

$$y(x) = \frac{c_1\left(1728x^2 + c_1^3 + 24\sqrt{6c_1^3x^2 + 5184x^4}\right)^{\frac{1}{3}}}{96} + \frac{c_1^3}{96\left(1728x^2 + c_1^3 + 24\sqrt{6c_1^3x^2 + 5184x^4}\right)^{\frac{1}{3}}} + \frac{c_1^2}{96}$$

$$y(x) = -\frac{c_1\left(-1728x^2 + c_1^3 + 24\sqrt{-6c_1^3x^2 + 5184x^4}\right)^{\frac{1}{3}}}{192} - \frac{c_1^3}{192\left(-1728x^2 + c_1^3 + 24\sqrt{-6c_1^3x^2 + 5184x^4}\right)^{\frac{1}{3}}} + \frac{c_1^2}{96} - \frac{ic_1\sqrt{3}\left(-1728x^2 + c_1^3 + 24\sqrt{-6c_1^3x^2 + 5184x^4}\right)^{\frac{1}{3}}}{192} + \frac{i\sqrt{3}\,c_1^3}{192\left(-1728x^2 + c_1^3 + 24\sqrt{-6c_1^3x^2 + 5184x^4}\right)^{\frac{1}{3}}}$$

$$y(x) = -\frac{c_1\left(-1728x^2 + c_1^3 + 24\sqrt{-6c_1^3x^2 + 5184x^4}\right)^{\frac{1}{3}}}{192} - \frac{c_1^3}{\quad} + \frac{c_1^2}{96}$$

✓ Solution by Mathematica

Time used: 169.172 (sec). Leaf size: 15120

```
DSolve[x (y'[x])^3 - 2 y[x](y'[x])^2 + 4 x^2==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

35.28 problem 1062

Internal problem ID [3773]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1062.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$2xy'^3 - 3yy'^2 - x = 0$$

✓ Solution by Maple

Time used: 0.281 (sec). Leaf size: 69

```
dsolve(2*x*diff(y(x),x)^3-3*y(x)*diff(y(x),x)^2-x = 0,y(x), singsol=all)
```

$$y(x) = \left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)x$$
$$y(x) = \left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)x$$
$$y(x) = -x$$
$$y(x) = -\frac{\left(-\frac{2(c_1x)^{\frac{3}{2}}}{c_1^3} + 1\right)c_1}{3}$$
$$y(x) = -\frac{\left(\frac{2(c_1x)^{\frac{3}{2}}}{c_1^3} + 1\right)c_1}{3}$$

✓ Solution by Mathematica

Time used: 28.358 (sec). Leaf size: 4317

```
DSolve[2 x (y'[x])^3 - 3 y[x] (y'[x])^2 -x==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

35.29 problem 1063

Internal problem ID [3774]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 35
Problem number: 1063.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$4xy'^3 - 6yy'^2 - x + 3y = 0$$

✓ Solution by Maple

Time used: 0.172 (sec). Leaf size: 102

```
dsolve(4*x*diff(y(x),x)^3-6*y(x)*diff(y(x),x)^2-x+3*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}\right) x$$

$$y(x) = \left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) x$$

$$y(x) = x$$

$$y(x) = \frac{\left(\frac{(x+c_1)\sqrt{2}\,\sqrt{c_1(x+c_1)}}{c_1^2} + 1\right) x}{-\frac{3(x+c_1)}{c_1} + 3}$$

$$y(x) = \frac{\left(-\frac{(x+c_1)\sqrt{2}\,\sqrt{c_1(x+c_1)}}{c_1^2} + 1\right) x}{-\frac{3(x+c_1)}{c_1} + 3}$$

✓ Solution by Mathematica

Time used: 1.202 (sec). Leaf size: 79

```
DSolve[4 x (y'[x])^3 -6 y[x] (y'[x])^2-x +3 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{\sqrt{2}\sqrt{c_1(x+c_1)^3}+c_1^2}{3c_1}$$

$$y(x) \to \frac{\sqrt{2}\sqrt{c_1(x+c_1)^3}}{3c_1}-\frac{c_1}{3}$$

$$y(x) \to \text{Indeterminate}$$

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36.1 problem 1064

Internal problem ID [3775]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1064.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{8xy'^3 - 12yy'^2 + 9y = 0}$$

✓ Solution by Maple

Time used: 0.234 (sec). Leaf size: 80

```
dsolve(8*x*diff(y(x),x)^3-12*y(x)*diff(y(x),x)^2+9*y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{3x}{2}$$
$$y(x) = \frac{3x}{2}$$
$$y(x) = 0$$
$$y(x) = \frac{(c_1(3c_1 + x))^{\frac{3}{2}}\, x}{c_1^3\left(-\frac{3(3c_1+x)}{c_1} + 9\right)}$$
$$y(x) = -\frac{(c_1(3c_1 + x))^{\frac{3}{2}}\, x}{c_1^3\left(-\frac{3(3c_1+x)}{c_1} + 9\right)}$$

✓ Solution by Mathematica

Time used: 0.356 (sec). Leaf size: 77

```
DSolve[8 x (y'[x])^3 -12 y[x] (y'[x])^2 + 9 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\frac{(x+3c_1)^{3/2}}{3\sqrt{c_1}}$$

$$y(x) \to \frac{(x+3c_1)^{3/2}}{3\sqrt{c_1}}$$

$$y(x) \to 0$$

$$y(x) \to \text{Indeterminate}$$

$$y(x) \to -\frac{3x}{2}$$

$$y(x) \to \frac{3x}{2}$$

36.2 problem 1065

Internal problem ID [3776]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1065.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$x^2y'^3 - 2xyy'^2 + y^2y' + 1 = 0$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 81

```
dsolve(x^2*diff(y(x),x)^3-2*x*y(x)*diff(y(x),x)^2+y(x)^2*diff(y(x),x)+1 = 0,y(x), singsol=all
```

$$y(x) = \frac{3(-2x)^{\frac{1}{3}}}{2}$$

$$y(x) = -\frac{3(-2x)^{\frac{1}{3}}}{4} - \frac{3i\sqrt{3}\,(-2x)^{\frac{1}{3}}}{4}$$

$$y(x) = -\frac{3(-2x)^{\frac{1}{3}}}{4} + \frac{3i\sqrt{3}\,(-2x)^{\frac{1}{3}}}{4}$$

$$y(x) = c_1x - \frac{1}{\sqrt{-c_1}}$$

$$y(x) = c_1x + \frac{1}{\sqrt{-c_1}}$$

✓ Solution by Mathematica

Time used: 65.518 (sec). Leaf size: 33909

```
DSolve[x^2 (y'[x])^3 -2 x y[x] (y'[x])^2 +  y[x]^2 y'[x]+1==0,y[x],x,IncludeSingularSolutions
```

Too large to display

36.3 problem 1066

Internal problem ID [3777]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1066.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$(a^2 - x^2)\, y'^3 + bx(a^2 - x^2)\, y'^2 - y' - bx = 0$$

✓ Solution by Maple

Time used: 0.187 (sec). Leaf size: 52

```
dsolve((a^2-x^2)*diff(y(x),x)^3+b*x*(a^2-x^2)*diff(y(x),x)^2-diff(y(x),x)-b*x = 0,y(x), sings
```

$$y(x) = -\frac{b\,x^2}{2} + c_1$$
$$y(x) = \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$
$$y(x) = -\arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$

✓ Solution by Mathematica

Time used: 0.005 (sec). Leaf size: 64

```
DSolve[(a^2-x^2) (y'[x])^3 +b x (a^2-x^2) (y'[x])^2 -y'[x] -b x==0,y[x],x,IncludeSingularSolu
```

$$y(x) \to -\frac{bx^2}{2} + c_1$$
$$y(x) \to -\arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$
$$y(x) \to \arctan\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c_1$$

36.4 problem 1067

Internal problem ID [3778]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1067.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$xy'^3 - 3x^2yy'^2 + x(x^5 + 3y^2)\,y' - 2x^5y - y^3 = 0$$

✗ Solution by Maple

```
dsolve(x*diff(y(x),x)^3-3*x^2*y(x)*diff(y(x),x)^2+x*(x^5+3*y(x)^2)*diff(y(x),x)-2*x^5*y(x)-y(
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x (y'[x])^3 -3 x^2 y[x] (y'[x])^2 +x(x^5+3 y[x]^2) y'[x]-2 x^5 y[x]- y[x]^3==0,y[x],x,
```

Timed out

36.5 problem 1068

Internal problem ID [3779]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1068.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_homogeneous, 'class G']]`

$$2x^3{y'}^3 + 6x^2y{y'}^2 - (1 - 6xy)\, yy' + 2y^3 = 0$$

✓ Solution by Maple

Time used: 0.75 (sec). Leaf size: 3159

```
dsolve(2*x^3*diff(y(x),x)^3+6*x^2*y(x)*diff(y(x),x)^2-(1-6*x*y(x))*y(x)*diff(y(x),x)+2*y(x)^3
```

$$y(x) = 0$$

Expression too large to display

Expression too large to display

Expression too large to display

✓ Solution by Mathematica

Time used: 62.067 (sec). Leaf size: 179

```
DSolve[2 x^3 (y'[x])^3 +6 x^2 y[x] (y'[x])^2 -(1-6 x y[x])y[x] y'[x]+2 y[x]^3==0,y[x],x,Inclu
```

$$y(x) \to \frac{\int_1^x \frac{\text{InverseFunction}\left[-\frac{2\sqrt{\#1^2-8\#1^3}\arctan\left(\sqrt{8\#1-1}\right)}{\#1\sqrt{8\#1-1}} - 14\log\left(\#1^2(8\#1-1)\right) + \log\left(\#1^{14}(8\#1-1)^{15/2}\left(\#1-\sqrt{\#1^2-8\#1^3}\right.\right.\right.}{K[1]}}{x}$$

36.6 problem 1070

Internal problem ID [3780]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1070.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{x^4 {y'}^3 - x^3 y {y'}^2 - x^2 y^2 y' + x y^3 - 1 = 0}$$

✓ Solution by Maple

Time used: 0.5 (sec). Leaf size: 464

```
dsolve(x^4*diff(y(x),x)^3-x^3*y(x)*diff(y(x),x)^2-x^2*y(x)^2*diff(y(x),x)+x*y(x)^3 = 1,y(x),
```

$$y(x) = \frac{3\,2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{4x}$$

$$y(x) = -\frac{3\,2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{8x} - \frac{3i\sqrt{3}\,2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{8x}$$

$$y(x) = -\frac{3\,2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{8x} + \frac{3i\sqrt{3}\,2^{\frac{1}{3}}(x^2)^{\frac{1}{3}}}{8x}$$

$$y(x) = \frac{\text{RootOf}\left(-\ln(x) + 3\left(\int^{\_Z} \frac{\left(-32\_a^3+6\sqrt{-96\_a^3+81}+54\right)^{\frac{1}{3}}}{4\,2^{\frac{2}{3}}\_a^2+2\_a\left(-32\_a^3+6\sqrt{-96\_a^3+81}+54\right)^{\frac{1}{3}}+\left(-16\_a^3+3\sqrt{-96\_a^3+81}+27\right)^{\frac{2}{3}}}d\_a\right)+\right.}{x^{\frac{1}{3}}}$$

$$y(x) = \frac{\text{RootOf}\left(-\ln(x) + \int^{\_Z} - \frac{3\left(-32\_a^3+6\sqrt{-96\_a^3+81}+54\right)^{\frac{1}{3}}\left(\sqrt{3}+i\right)}{2\left(2\sqrt{3}\,2^{\frac{2}{3}}\_a^2-2i2^{\frac{2}{3}}\_a^2-\_a\sqrt{3}\left(-32\_a^3+6\sqrt{-96\_a^3+81}+54\right)^{\frac{1}{3}}-i\_a\left(-32\_a^3+6\sqrt{-96\_a^3\right.\right.}\right.}{x^{\frac{1}{3}}}$$

$$y(x) = \frac{\text{RootOf}\left(-\ln(x) + \int^{\_Z} - \frac{3\left(-32\_a^3+6\sqrt{-96\_a^3+81}+54\right)^{\frac{1}{3}}\left(-i+\sqrt{3}\right)}{2\left(2\sqrt{3}\,2^{\frac{2}{3}}\_a^2+2i2^{\frac{2}{3}}\_a^2-\_a\sqrt{3}\left(-32\_a^3+6\sqrt{-96\_a^3+81}+54\right)^{\frac{1}{3}}+i\_a\left(-32\_a^3+6\sqrt{-96\_a^3\right.\right.}\right.}{x^{\frac{1}{3}}}$$

✓ Solution by Mathematica

Time used: 82.661 (sec). Leaf size: 67473

```
DSolve[x^4 (y'[x])^3 -x^3 y[x] (y'[x])^2 - x^2 y[x]^2 y'[x]+x y[x]^3==1,y[x],x,IncludeSingula
```

Too large to display

36.7 problem 1071

Internal problem ID [3781]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1071.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$x^6 {y'}^3 - y'x - y = 0$$

✓ Solution by Maple

Time used: 0.234 (sec). Leaf size: 36

```
dsolve(x^6*diff(y(x),x)^3-x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2\sqrt{3}}{9x^{\frac{3}{2}}}$$
$$y(x) = \frac{2\sqrt{3}}{9x^{\frac{3}{2}}}$$
$$y(x) = c_1^3 - \frac{c_1}{x}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^6 (y'[x])^3 -x y'[x] - y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

36.8 problem 1072

Internal problem ID [3782]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1072.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{yy'^3 - 3y'x + 3y = 0}$$

✓ Solution by Maple

Time used: 0.391 (sec). Leaf size: 607

```
dsolve(y(x)*diff(y(x),x)^3-3*x*diff(y(x),x)+3*y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \mathrm{RootOf}\left(-2\ln(x) + \int^{\_Z} \frac{-2\left(4\sqrt{\frac{9\_a^3-4}{\_a}}\,\_a^5 + 12\_a^6 - 24\_a^3 + 8\right)^{\frac{1}{3}}\_a^3 - 8\_a^3 + \left(4\sqrt{\frac{9\_a^3-4}{\_a}}\,\_a^5 + 12\_a^6 - 24\_a^3 + \cdots}{\_a^4\left(4\sqrt{\frac{9\_a^3-4}{\_a}}\,\_a^5 + 12\_a^6 - 24\_a^3 + 8\right)\cdots} + 2c_1\right)x$$

$$y(x) = \mathrm{RootOf}\left(-4\ln(x) + \int^{\_Z} \frac{8i\sqrt{3}\,\_a^3 + i\sqrt{3}\left(4\sqrt{\frac{9\_a^3-4}{\_a}}\,\_a^5 + 12\_a^6 - 24\_a^3 + 8\right)^{\frac{2}{3}} - 4\left(4\sqrt{\frac{9\_a^3-4}{\_a}}\,\_a^5 + 12\_a^6 - 24\_a\cdots}{\_a^4\left(4\sqrt{9\cdots}\right.} + 4c_1\right)x$$

$$y(x) = \mathrm{RootOf}\left(-4\ln(x) - \left(\int^{\_Z} \frac{8i\sqrt{3}\,\_a^3 + i\sqrt{3}\left(4\sqrt{\frac{9\_a^3-4}{\_a}}\,\_a^5 + 12\_a^6 - 24\_a^3 + 8\right)^{\frac{2}{3}} + 4\left(4\sqrt{\frac{9\_a^3-4}{\_a}}\,\_a^5 + 12\_a^6 - 24\cdots}{\_a^4\left(4\sqrt{\cdots}\right.} + 4c_1\right)x$$

✓ Solution by Mathematica

Time used: 151.081 (sec). Leaf size: 8706

```
DSolve[y[x] (y'[x])^3 -3 x y'[x] + 3 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

36.9 problem 1073

Internal problem ID [3783]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1073.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{2yy'^3 - 3y'x + 2y = 0}$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 740

```
dsolve(2*y(x)*diff(y(x),x)^3-3*x*diff(y(x),x)+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{2^{\frac{2}{3}} x}{2}$$

$$y(x) = \left(-\frac{2^{\frac{2}{3}}}{4} - \frac{i\sqrt{3}\, 2^{\frac{2}{3}}}{4}\right) x$$

$$y(x) = \left(-\frac{2^{\frac{2}{3}}}{4} + \frac{i\sqrt{3}\, 2^{\frac{2}{3}}}{4}\right) x$$

$$y(x) = 0$$

$$y(x) = \mathrm{RootOf}\left(-\ln(x) + \int^{\_Z} -\frac{2\left(\left(\sqrt{2}\sqrt{\frac{1}{\_a(2\_a^3-1)}}\,\_a^2+1\right)(2\_a^3-1)^2\right)^{\frac{1}{3}}\_a^3 + 2\_a^3 - \left(\left(\sqrt{2}\sqrt{\frac{1}{\_a(2\_a^3-1)}}\,\_a^2+1\right)(2\_a^3-\right.}{\_a(2\_a^3-1)\left(\left(\sqrt{2}\sqrt{\frac{1}{\_a(2\_a^3-1)}}\,\_a^2+1\right)(2\_a^3\right.} + c_1\right) x$$

$$y(x) = \mathrm{RootOf}\left(-2\ln(x) + \int^{\_Z} \frac{2i\sqrt{3}\,\_a^3 + i\sqrt{3}\left(\left(\sqrt{2}\sqrt{\frac{1}{\_a(2\_a^3-1)}}\,\_a^2+1\right)(2\_a^3-1)^2\right)^{\frac{2}{3}} - 4\left(\left(\sqrt{2}\sqrt{\frac{1}{\_a(2\_a^3-1)}}\,\_a^2+1\right)\right.}{\_a(2\_a^3-} + 2c_1\right) x$$

$$y(x) = \mathrm{RootOf}\left(-2\ln(x) - \left(\int^{\_Z} \frac{2i\sqrt{3}\,\_a^3 + i\sqrt{3}\left(\left(\sqrt{2}\sqrt{\frac{1}{\_a(2\_a^3-1)}}\,\_a^2+1\right)(2\_a^3-1)^2\right)^{\frac{2}{3}} + 4\left(\left(\sqrt{2}\sqrt{\frac{1}{\_a(2\_a^3-1)}}\,\_a^2+1\right.\right.}{\_a(2\_a}\right.\right.$$

✓ Solution by Mathematica

Time used: 172.119 (sec). Leaf size: 10331

```
DSolve[2 y[x] (y'[x])^3 -3 x y'[x]+2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

36.10 problem 1076

Internal problem ID [3784]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1076.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_quadrature]`

$$(x+2y)\,{y'}^3+3(y+x)\,{y'}^2+(y+2x)\,y'=0$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 65

```
dsolve((x+2*y(x))*diff(y(x),x)^3+3*(x+y(x))*diff(y(x),x)^2+(2*x+y(x))*diff(y(x),x) = 0,y(x),
```

$$y(x) = c_1 - x$$

$$y(x) = \frac{-\frac{c_1 x}{2} - \frac{\sqrt{-3c_1^2x^2+4}}{2}}{c_1}$$

$$y(x) = \frac{-\frac{c_1 x}{2} + \frac{\sqrt{-3c_1^2x^2+4}}{2}}{c_1}$$

$$y(x) = c_1$$

✓ Solution by Mathematica

Time used: 0.455 (sec). Leaf size: 130

```
DSolve[(x+2 y[x])(y'[x])^3+3 (x+y[x]) (y'[x])^2+ (2 x+y[x]) y'[x] ==0,y[x],x,IncludeSingularS
```

$$y(x) \to \frac{1}{2}\left(-x - \sqrt{-3x^2 + 4e^{c_1}}\right)$$
$$y(x) \to \frac{1}{2}\left(-x + \sqrt{-3x^2 + 4e^{c_1}}\right)$$
$$y(x) \to c_1$$
$$y(x) \to -x + c_1$$
$$y(x) \to \frac{1}{2}\left(-\sqrt{3}\sqrt{-x^2} - x\right)$$
$$y(x) \to \frac{1}{2}\left(\sqrt{3}\sqrt{-x^2} - x\right)$$

36.11 problem 1077

Internal problem ID [3785]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1077.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y^2 {y'}^3 - y'x + y = 0$$

✓ Solution by Maple

Time used: 0.329 (sec). Leaf size: 189

```
dsolve(y(x)^2*diff(y(x),x)^3-x*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = -\frac{2\sqrt{-24c_1^3 + 27c_1x - 3\sqrt{64c_1^6 - 144c_1^4x + 108c_1^2x^2 - 27x^3}}}{9}$$

$$y(x) = \frac{2\sqrt{-24c_1^3 + 27c_1x - 3\sqrt{64c_1^6 - 144c_1^4x + 108c_1^2x^2 - 27x^3}}}{9}$$

$$y(x) = -\frac{2\sqrt{-24c_1^3 + 27c_1x + 3\sqrt{64c_1^6 - 144c_1^4x + 108c_1^2x^2 - 27x^3}}}{9}$$

$$y(x) = \frac{2\sqrt{-24c_1^3 + 27c_1x + 3\sqrt{64c_1^6 - 144c_1^4x + 108c_1^2x^2 - 27x^3}}}{9}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]^2 (y'[x])^3- x y'[x] + y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

36.12 problem 1078

Internal problem ID [3786]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1078.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{y^2 {y'}^3 + 2y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.281 (sec). Leaf size: 107

```
dsolve(y(x)^2*diff(y(x),x)^3+2*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2\, 2^{\frac{1}{4}} 3^{\frac{1}{4}} (-x^3)^{\frac{1}{4}}}{3}$$
$$y(x) = \frac{2\, 2^{\frac{1}{4}} 3^{\frac{1}{4}} (-x^3)^{\frac{1}{4}}}{3}$$
$$y(x) = -\frac{2i 2^{\frac{1}{4}} 3^{\frac{1}{4}} (-x^3)^{\frac{1}{4}}}{3}$$
$$y(x) = \frac{2i 2^{\frac{1}{4}} 3^{\frac{1}{4}} (-x^3)^{\frac{1}{4}}}{3}$$
$$y(x) = 0$$
$$y(x) = \sqrt{c_1^3 + 2c_1 x}$$
$$y(x) = -\sqrt{c_1^3 + 2c_1 x}$$

✓ Solution by Mathematica

Time used: 0.118 (sec). Leaf size: 119

```
DSolve[y[x]^2 (y'[x])^3+2 x y'[x] -y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to -\sqrt{2c_1x + c_1{}^3}$$
$$y(x) \to \sqrt{2c_1x + c_1{}^3}$$
$$y(x) \to (-1 - i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$
$$y(x) \to (1 - i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$
$$y(x) \to (-1 + i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$
$$y(x) \to (1 + i)\left(\frac{2}{3}\right)^{3/4} x^{3/4}$$

36.13 problem 1079

Internal problem ID [3787]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1079.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$4y^2y'^3 - 2y'x + y = 0$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 95

```
dsolve(4*y(x)^2*diff(y(x),x)^3-2*x*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2^{\frac{3}{4}}3^{\frac{1}{4}}x^{\frac{3}{4}}}{3}$$
$$y(x) = \frac{2^{\frac{3}{4}}3^{\frac{1}{4}}x^{\frac{3}{4}}}{3}$$
$$y(x) = -\frac{i2^{\frac{3}{4}}3^{\frac{1}{4}}x^{\frac{3}{4}}}{3}$$
$$y(x) = \frac{i2^{\frac{3}{4}}3^{\frac{1}{4}}x^{\frac{3}{4}}}{3}$$
$$y(x) = 0$$
$$y(x) = \sqrt{-4c_1^3 + 2c_1x}$$
$$y(x) = -\sqrt{-4c_1^3 + 2c_1x}$$

✓ Solution by Mathematica

Time used: 83.382 (sec). Leaf size: 11250

```
DSolve[4 y[x]^2 (y'[x])^3 - 2 x y'[x] +y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

36.14 problem 1080

Internal problem ID [3788]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1080.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$16y^2{y'}^3 + 2y'x - y = 0$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 111

```
dsolve(16*y(x)^2*diff(y(x),x)^3+2*x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2^{\frac{1}{4}}3^{\frac{1}{4}}(-x^3)^{\frac{1}{4}}}{3}$$
$$y(x) = \frac{2^{\frac{1}{4}}3^{\frac{1}{4}}(-x^3)^{\frac{1}{4}}}{3}$$
$$y(x) = -\frac{i2^{\frac{1}{4}}3^{\frac{1}{4}}(-x^3)^{\frac{1}{4}}}{3}$$
$$y(x) = \frac{i2^{\frac{1}{4}}3^{\frac{1}{4}}(-x^3)^{\frac{1}{4}}}{3}$$
$$y(x) = 0$$
$$y(x) = \sqrt{16c_1^3 + 2c_1x}$$
$$y(x) = -\sqrt{16c_1^3 + 2c_1x}$$

✓ Solution by Mathematica

Time used: 0.115 (sec). Leaf size: 107

```
DSolve[16 y[x]^2 (y'[x])^3 +2 x y'[x] -y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \sqrt{c_1 \left(x + 2{c_1}^2\right)}$$

$$y(x) \to -\frac{\sqrt[4]{-2}x^{3/4}}{3^{3/4}}$$

$$y(x) \to \frac{(1-i)x^{3/4}}{\sqrt[4]{2}3^{3/4}}$$

$$y(x) \to \frac{i\sqrt[4]{-2}x^{3/4}}{3^{3/4}}$$

$$y(x) \to \frac{\sqrt[4]{-2}x^{3/4}}{3^{3/4}}$$

36.15 problem 1081

Internal problem ID [3789]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1081.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$x y^2 {y'}^3 - y^3 {y'}^2 + x(x^2+1) y' - x^2 y = 0$$

✗ Solution by Maple

```
dsolve(x*y(x)^2*diff(y(x),x)^3-y(x)^3*diff(y(x),x)^2+x*(x^2+1)*diff(y(x),x)-x^2*y(x) = 0,y(x)
```

No solution found

✓ Solution by Mathematica

Time used: 0.529 (sec). Leaf size: 399

```
DSolve[x y[x]^2 (y'[x])^3 -y[x]^3 (y'[x])^2 + x (1+x^2) y'[x] -x^2 y[x]==0,y[x],x,IncludeSing
```

$$y(x)\to -\sqrt{c_1\left(x^2+\frac{1}{1+{c_1}^2}\right)}$$

$$y(x)\to \sqrt{c_1\left(x^2+\frac{1}{1+{c_1}^2}\right)}$$

$$y(x)\to -\frac{\sqrt[4]{-8x^4+20x^2-\sqrt{-\left(8x^2-1\right)^3}+1}}{2^{3/4}}$$

$$y(x)\to -\frac{i\sqrt[4]{-8x^4+20x^2-\sqrt{-\left(8x^2-1\right)^3}+1}}{2^{3/4}}$$

$$y(x)\to \frac{i\sqrt[4]{-8x^4+20x^2-\sqrt{-\left(8x^2-1\right)^3}+1}}{2^{3/4}}$$

$$y(x)\to \frac{\sqrt[4]{-8x^4+20x^2-\sqrt{-\left(8x^2-1\right)^3}+1}}{2^{3/4}}$$

$$y(x)\to -\frac{\sqrt[4]{-8x^4+20x^2+\sqrt{-\left(8x^2-1\right)^3}+1}}{2^{3/4}}$$

$$y(x)\to -\frac{i\sqrt[4]{-8x^4+20x^2+\sqrt{-\left(8x^2-1\right)^3}+1}}{2^{3/4}}$$

$$y(x)\to \frac{i\sqrt[4]{-8x^4+20x^2+\sqrt{-\left(8x^2-1\right)^3}+1}}{2^{3/4}}$$

$$y(x)\to \frac{\sqrt[4]{-8x^4+20x^2+\sqrt{-\left(8x^2-1\right)^3}+1}}{2^{3/4}}$$

36.16 problem 1084

Internal problem ID [3790]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1084.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[‘y=_G(x,y’)‘]`

$$\boxed{y^3y'^3 - (-3x+1)\, y^2y'^2 + 3x^2yy' + x^3 - y^2 = 0}$$

✓ Solution by Maple

Time used: 0.422 (sec). Leaf size: 389

```
dsolve(y(x)^3*diff(y(x),x)^3-(1-3*x)*y(x)^2*diff(y(x),x)^2+3*x^2*y(x)*diff(y(x),x)+x^3-y(x)^2
```

$$y(x) = -\frac{\sqrt{-6-81x^2-6\sqrt{-216x^3+108x^2-18x+1}+54x}}{9}$$
$$y(x) = \frac{\sqrt{-6-81x^2-6\sqrt{-216x^3+108x^2-18x+1}+54x}}{9}$$
$$y(x) = -\frac{\sqrt{-6-81x^2+6\sqrt{-216x^3+108x^2-18x+1}+54x}}{9}$$
$$y(x) = \frac{\sqrt{-6-81x^2+6\sqrt{-216x^3+108x^2-18x+1}+54x}}{9}$$
$$y(x) = \sqrt{-\left(c_1^3\right)^{\frac{2}{3}}+2c_1x+c_1^3-x^2}$$
$$y(x) = -\sqrt{-\left(c_1^3\right)^{\frac{2}{3}}+2c_1x+c_1^3-x^2}$$
$$y(x) = -\frac{\sqrt{-2i\sqrt{3}\left(c_1^3\right)^{\frac{2}{3}}-4i\sqrt{3}\,c_1x+2\left(c_1^3\right)^{\frac{2}{3}}-4c_1x+4c_1^3-4x^2}}{2}$$
$$y(x) = \frac{\sqrt{-2i\sqrt{3}\left(c_1^3\right)^{\frac{2}{3}}-4i\sqrt{3}\,c_1x+2\left(c_1^3\right)^{\frac{2}{3}}-4c_1x+4c_1^3-4x^2}}{2}$$
$$y(x) = -\frac{\sqrt{2i\sqrt{3}\left(c_1^3\right)^{\frac{2}{3}}+4i\sqrt{3}\,c_1x+2\left(c_1^3\right)^{\frac{2}{3}}-4c_1x+4c_1^3-4x^2}}{2}$$
$$y(x) = \frac{\sqrt{2i\sqrt{3}\left(c_1^3\right)^{\frac{2}{3}}+4i\sqrt{3}\,c_1x+2\left(c_1^3\right)^{\frac{2}{3}}-4c_1x+4c_1^3-4x^2}}{2}$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[y[x]^3 (y'[x])^3 -(1-3 x) y[x]^2 (y'[x])^2 +3 x^2 y[x] y'[x]+x^3 - y[x]^2==0,y[x],x,In
```

Timed out

36.17 problem 1085

Internal problem ID [3791]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1085.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$y^4 {y'}^3 - 6y'x + 2y = 0$$

✓ Solution by Maple

Time used: 0.375 (sec). Leaf size: 183

```
dsolve(y(x)^4*diff(y(x),x)^3-6*x*diff(y(x),x)+2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \sqrt{-i\sqrt{3}\,x - x}$$
$$y(x) = \sqrt{i\sqrt{3}\,x - x}$$
$$y(x) = -\sqrt{-i\sqrt{3}\,x - x}$$
$$y(x) = -\sqrt{i\sqrt{3}\,x - x}$$
$$y(x) = \sqrt{2}\,\sqrt{x}$$
$$y(x) = -\sqrt{2}\,\sqrt{x}$$
$$y(x) = 0$$
$$y(x) = \frac{(-4c_1^3 + 24c_1 x)^{\frac{1}{3}}}{2}$$
$$y(x) = -\frac{(-4c_1^3 + 24c_1 x)^{\frac{1}{3}}}{4} - \frac{i\sqrt{3}\,(-4c_1^3 + 24c_1 x)^{\frac{1}{3}}}{4}$$
$$y(x) = -\frac{(-4c_1^3 + 24c_1 x)^{\frac{1}{3}}}{4} + \frac{i\sqrt{3}\,(-4c_1^3 + 24c_1 x)^{\frac{1}{3}}}{4}$$

✓ Solution by Mathematica

Time used: 69.598 (sec). Leaf size: 22649

```
DSolve[y[x]^4 (y'[x])^3 -6 x y'[x] +2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

36.18 problem 1086

Internal problem ID [3792]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1086.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^4 - (y-a)^3 (y-b)^2 = 0}$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 141

```
dsolve(diff(y(x),x)^4 = (y(x)-a)^3*(y(x)-b)^2,y(x), singsol=all)
```

$$y(x) = a$$
$$y(x) = b$$
$$x - \left(\int^{y(x)} \frac{1}{\left((\_a-a)^3 (\_a-b)^2\right)^{\frac{1}{4}}} d\_a\right) - c_1 = 0$$
$$x - \left(\int^{y(x)} \frac{i}{\left((\_a-a)^3 (\_a-b)^2\right)^{\frac{1}{4}}} d\_a\right) - c_1 = 0$$
$$x - \left(\int^{y(x)} -\frac{i}{\left((\_a-a)^3 (\_a-b)^2\right)^{\frac{1}{4}}} d\_a\right) - c_1 = 0$$
$$x - \left(\int^{y(x)} -\frac{1}{\left((\_a-a)^3 (\_a-b)^2\right)^{\frac{1}{4}}} d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.459 (sec). Leaf size: 333

```
DSolve[(y'[x])^4 == (y[x]-a)^3 (y[x]-b)^2 ,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[-\sqrt[4]{-1}x+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\sqrt[4]{-1}x+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[-(-1)^{3/4}x+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[(-1)^{3/4}x+c_1\right]$$

$$y(x) \to a$$

$$y(x) \to b$$

36.19 problem 1087

Internal problem ID [3793]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1087.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y'^4 + f(x) (y-a)^3 (y-b)^2 = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 270

```
dsolve(diff(y(x),x)^4+f(x)*(y(x)-a)^3*(y(x)-b)^2 = 0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\sqrt{\_a-b}\,(\_a-a)^{\frac{3}{4}}}d\_a + \int^{x} -\frac{\left(f(\_a)\,(-y(x)+a)^3\,(b-y(x))^2\right)^{\frac{1}{4}}}{\sqrt{y(x)-b}\,(y(x)-a)^{\frac{3}{4}}}d\_a + c_1 = 0$$

$$\int^{y(x)} \frac{1}{\sqrt{\_a-b}\,(\_a-a)^{\frac{3}{4}}}d\_a + \int^{x} \frac{i\left(f(\_a)\,(-y(x)+a)^3\,(b-y(x))^2\right)^{\frac{1}{4}}}{\sqrt{y(x)-b}\,(y(x)-a)^{\frac{3}{4}}}d\_a + c_1 = 0$$

$$\int^{y(x)} \frac{1}{\sqrt{\_a-b}\,(\_a-a)^{\frac{3}{4}}}d\_a + \int^{x} -\frac{i\left(f(\_a)\,(-y(x)+a)^3\,(b-y(x))^2\right)^{\frac{1}{4}}}{\sqrt{y(x)-b}\,(y(x)-a)^{\frac{3}{4}}}d\_a + c_1 = 0$$

$$\int^{y(x)} \frac{1}{\sqrt{\_a-b}\,(\_a-a)^{\frac{3}{4}}}d\_a + \int^{x} \frac{\left(f(\_a)\,(-y(x)+a)^3\,(b-y(x))^2\right)^{\frac{1}{4}}}{\sqrt{y(x)-b}\,(y(x)-a)^{\frac{3}{4}}}d\_a + c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.17 (sec). Leaf size: 369

```
DSolve[(y'[x])^4 +f[x] (y[x]-a)^3 (y[x]-b)^2==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x -\sqrt[4]{f(K[1])}dK[1]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x -i\sqrt[4]{f(K[2])}dK[2]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x i\sqrt[4]{f(K[3])}dK[3]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x \sqrt[4]{f(K[4])}dK[4]+c_1\right]$$

$$y(x) \to a$$

$$y(x) \to b$$

36.20 problem 1088

Internal problem ID [3794]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1088.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y'^4 + f(x)(y-a)^3 (y-b)^3 = 0}$$

✓ Solution by Maple

Time used: 0.359 (sec). Leaf size: 262

```
dsolve(diff(y(x),x)^4+f(x)*(y(x)-a)^3*(y(x)-b)^3 = 0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\left((-\_a+a)(-\_a+b)\right)^{\frac{3}{4}}} d\_a + \int^{x} -\frac{\left(-f(\_a)(-y(x)+a)^3 (b-y(x))^3\right)^{\frac{1}{4}}}{\left((-y(x)+a)(b-y(x))\right)^{\frac{3}{4}}} d\_a + c_1 = 0$$

$$\int^{y(x)} \frac{1}{\left((-\_a+a)(-\_a+b)\right)^{\frac{3}{4}}} d\_a + \int^{x} \frac{i\left(-f(\_a)(-y(x)+a)^3 (b-y(x))^3\right)^{\frac{1}{4}}}{\left((-y(x)+a)(b-y(x))\right)^{\frac{3}{4}}} d\_a + c_1 = 0$$

$$\int^{y(x)} \frac{1}{\left((-\_a+a)(-\_a+b)\right)^{\frac{3}{4}}} d\_a + \int^{x} -\frac{i\left(-f(\_a)(-y(x)+a)^3 (b-y(x))^3\right)^{\frac{1}{4}}}{\left((-y(x)+a)(b-y(x))\right)^{\frac{3}{4}}} d\_a + c_1 = 0$$

$$\int^{y(x)} \frac{1}{\left((-\_a+a)(-\_a+b)\right)^{\frac{3}{4}}} d\_a + \int^{x} \frac{\left(-f(\_a)(-y(x)+a)^3 (b-y(x))^3\right)^{\frac{1}{4}}}{\left((-y(x)+a)(b-y(x))\right)^{\frac{3}{4}}} d\_a + c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.598 (sec). Leaf size: 385

```
DSolve[(y'[x])^4 +f[x] (y[x]-a)^3 (y[x]-b)^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{3/4}}\&\right]\left[\int_1^x -\sqrt[4]{-1}\sqrt[4]{f(K[1]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{3/4}}\&\right]\left[\int_1^x \sqrt[4]{-1}\sqrt[4]{f(K[2])}d$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{3/4}}\&\right]\left[\int_1^x -(-1)^{3/4}\sqrt[4]{f(K$$

$$y(x) \to \text{InverseFunction}\left[-\frac{4\sqrt[4]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{3/4}}\&\right]\left[\int_1^x (-1)^{3/4}\sqrt[4]{f(K[4]$$

$$y(x) \to a$$

$$y(x) \to b$$

36.21 problem 1089

Internal problem ID [3795]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1089.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y'^4 + f(x)(y-a)^3(y-b)^3(y-c)^2 = 0$$

✓ Solution by Maple

Time used: 0.14 (sec). Leaf size: 92

```
dsolve(diff(y(x),x)^4+f(x)*(y(x)-a)^3*(y(x)-b)^3*(y(x)-c)^2 = 0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{(\_a-b)^{\frac{3}{4}}(\_a-a)^{\frac{3}{4}}\sqrt{\_a-c}}d\_a + \int^{x} -\frac{\left(-f(\_a)(c-y(x))^2(b-y(x))^3(-y(x)+a)^3\right)^{\frac{1}{4}}}{(y(x)-b)^{\frac{3}{4}}(y(x)-a)^{\frac{3}{4}}\sqrt{y(x)-c}}d\_a + c_1 = 0$$

✓ Solution by Mathematica

Time used: 19.784 (sec). Leaf size: 562

```
DSolve[(y'[x])^4 +f[x] (y[x]-a)^3 (y[x]-b)^3 (y[x]-c)^2 ==0,y[x],x,IncludeSingularSolutions -
```

$$y(x) \to \text{InverseFunction}\left[\frac{4\sqrt[4]{a-\#1}\sqrt{c-\#1}\left(\frac{(b-\#1)(a-c)}{(c-\#1)(a-b)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{(c-b)(a-\#1)}{(a-b)(c-\#1)}\right)}{(b-\#1)^{3/4}(a-c)}\&\right]\left[\int_{1}\right.$$

$$y(x) \to \text{InverseFunction}\left[\frac{4\sqrt[4]{a-\#1}\sqrt{c-\#1}\left(\frac{(b-\#1)(a-c)}{(c-\#1)(a-b)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{(c-b)(a-\#1)}{(a-b)(c-\#1)}\right)}{(b-\#1)^{3/4}(a-c)}\&\right]\left[\int_{1}\right.$$

$$y(x) \to \text{InverseFunction}\left[\frac{4\sqrt[4]{a-\#1}\sqrt{c-\#1}\left(\frac{(b-\#1)(a-c)}{(c-\#1)(a-b)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{(c-b)(a-\#1)}{(a-b)(c-\#1)}\right)}{(b-\#1)^{3/4}(a-c)}\&\right]\left[\int_{1}\right.$$

$$y(x) \to \text{InverseFunction}\left[\frac{4\sqrt[4]{a-\#1}\sqrt{c-\#1}\left(\frac{(b-\#1)(a-c)}{(c-\#1)(a-b)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{(c-b)(a-\#1)}{(a-b)(c-\#1)}\right)}{(b-\#1)^{3/4}(a-c)}\&\right]\left[\int_{1}\right.$$

$$y(x) \to a$$

$$y(x) \to b$$

$$y(x) \to c$$

36.22 problem 1090

Internal problem ID [3796]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1090.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{y'^4 + y'x - 3y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 37

```
dsolve(diff(y(x),x)^4+x*diff(y(x),x)-3*y(x) = 0,y(x), singsol=all)
```

$$\left[x(\_T) = \sqrt{\_T}\left(\frac{4\_T^{\frac{5}{2}}}{5} + c_1\right), y(\_T) = \frac{\_T^4}{3} + \frac{\_T^{\frac{3}{2}}\left(\frac{4\_T^{\frac{5}{2}}}{5} + c_1\right)}{3}\right]$$

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[(y'[x])^4 +x y'[x]-3 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

Timed out

36.23 problem 1092

Internal problem ID [3797]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1092.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘]]`

$$y'^4 - 4x^2yy'^2 + 16xy^2y' - 16y^3 = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 122

```
dsolve(diff(y(x),x)^4-4*x^2*y(x)*diff(y(x),x)^2+16*x*y(x)^2*diff(y(x),x)-16*y(x)^3 = 0,y(x),
```

$$y(x) = \frac{x^4}{16}$$

$$y(x) = 0$$

$$y(x)\left(\sqrt{x^2-4\sqrt{y(x)}}+x\right)^{\frac{2\sqrt{y(x)x^2-4y(x)^{\frac{3}{2}}}}{\sqrt{x^2-4\sqrt{y(x)}}\,\sqrt{y(x)}}}\left(\sqrt{x^2-4\sqrt{y(x)}}-x\right)^{-\frac{2\sqrt{y(x)x^2-4y(x)^{\frac{3}{2}}}}{\sqrt{x^2-4\sqrt{y(x)}}\,\sqrt{y(x)}}}-c_1 = 0$$

✓ Solution by Mathematica

Time used: 32.07 (sec). Leaf size: 519

```
DSolve[(y'[x])^4 -4 x^2 y[x] (y'[x])^2+16 x y[x]^2 y'[x]-16 y[x]^3==0,y[x],x,IncludeSingularS
```

$$\text{Solve}\left[\frac{\sqrt{\left(x^2+4\sqrt{y(x)}\right)y(x)}\log\left(\sqrt{x^2+4\sqrt{y(x)}}-x\right)}{\sqrt{x^2+4\sqrt{y(x)}}\sqrt{y(x)}}+\frac{1}{4}\left(\log(y(x))-\frac{\sqrt{x^2+4\sqrt{y(x)}}\sqrt{y(x)}\log(y(x))}{\sqrt{\left(x^2+4\sqrt{y(x)}\right)y(x)}}\right)=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{4}\left(\frac{\sqrt{x^2+4\sqrt{y(x)}}\sqrt{y(x)}\log(y(x))}{\sqrt{\left(x^2+4\sqrt{y(x)}\right)y(x)}}+\log(y(x))\right)-\frac{\sqrt{\left(x^2+4\sqrt{y(x)}\right)y(x)}\log\left(\sqrt{x^2+4\sqrt{y(x)}}-x\right)}{\sqrt{x^2+4\sqrt{y(x)}}\sqrt{y(x)}}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{1}{2}\left(\frac{\sqrt{x^2-4\sqrt{y(x)}}\sqrt{y(x)}\log(y(x))}{2\sqrt{x^2y(x)-4y(x)^{3/2}}}+\frac{1}{2}\log(y(x))\right)-\frac{\sqrt{\left(x^2-4\sqrt{y(x)}\right)y(x)}\log\left(\sqrt{x^2-4\sqrt{y(x)}}-x\right)}{\sqrt{x^2-4\sqrt{y(x)}}\sqrt{y(x)}}=c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{\sqrt{\left(x^2-4\sqrt{y(x)}\right)y(x)}\log\left(\sqrt{x^2-4\sqrt{y(x)}}-x\right)}{\sqrt{x^2-4\sqrt{y(x)}}\sqrt{y(x)}}+\left(\frac{1}{4}-\frac{\sqrt{x^2y(x)-4y(x)^{3/2}}}{4\sqrt{x^2-4\sqrt{y(x)}}\sqrt{y(x)}}\right)\log(y(x))=c_1,y(x)\right]$$

$$y(x)\to 0$$

$$y(x)\to\frac{x^4}{16}$$

36.24 problem 1093

Internal problem ID [3798]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1093.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[_quadrature]`

$$y'^4 + 4yy'^3 + 6y^2y'^2 - (1 - 4y^3)\,y' - (3 - y^3)\,y = 0$$

✓ Solution by Maple

Time used: 0.031 (sec). Leaf size: 441

```
dsolve(diff(y(x),x)^4+4*y(x)*diff(y(x),x)^3+6*y(x)^2*diff(y(x),x)^2-(1-4*y(x)^3)*diff(y(x),x)
```

$$x + \frac{\ln\left(-14640 \operatorname{RootOf}\left(\_Z^4 + 4y(x)\_Z^3 + 6y(x)^2\_Z^2 + \left(4y(x)^3 - 1\right)\_Z + y(x)^4 - 3y(x)\right)^3 y(x)^6 - 39648\right.}{}$$
$$- c_1 = 0$$

✓ Solution by Mathematica

Time used: 97.251 (sec). Leaf size: 2925

```
DSolve[(y'[x])^4 +4 y[x] (y'[x])^3+6 y[x]^2 (y'[x])^2-(1-4 y[x]^3) y'[x]- (3-y[x]^3) y[x]==0,
```

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36.25 problem 1094

Internal problem ID [3799]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1094.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[_quadrature]`

$$2y'^4 - yy' - 2 = 0$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 513

```
dsolve(2*diff(y(x),x)^4-y(x)*diff(y(x),x)-2 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{\sqrt{-6c_1^3 + 18c_1^2 x - 18c_1 x^2 + 6x^3 + 216c_1 - 216x - 6\sqrt{c_1^6 - 6c_1^5 x + 15c_1^4 x^2 - 20c_1^3 x^3 + 15c_1^2 x^4 - 6c_1 x^5 \cdots}}}{9}$$

$$y(x) = \frac{\sqrt{-6c_1^3 + 18c_1^2 x - 18c_1 x^2 + 6x^3 + 216c_1 - 216x - 6\sqrt{c_1^6 - 6c_1^5 x + 15c_1^4 x^2 - 20c_1^3 x^3 + 15c_1^2 x^4 - 6c_1 x^5 + x \cdots}}}{9}$$

$$y(x) = -\frac{\sqrt{-6c_1^3 + 18c_1^2 x - 18c_1 x^2 + 6x^3 + 216c_1 - 216x + 6\sqrt{c_1^6 - 6c_1^5 x + 15c_1^4 x^2 - 20c_1^3 x^3 + 15c_1^2 x^4 - 6c_1 x^5 \cdots}}}{9}$$

$$y(x) = \frac{\sqrt{-6c_1^3 + 18c_1^2 x - 18c_1 x^2 + 6x^3 + 216c_1 - 216x + 6\sqrt{c_1^6 - 6c_1^5 x + 15c_1^4 x^2 - 20c_1^3 x^3 + 15c_1^2 x^4 - 6c_1 x^5 + x \cdots}}}{9}$$

✓ Solution by Mathematica

Time used: 116.956 (sec). Leaf size: 12753

```
DSolve[2 (y'[x])^4 -y[x] y'[x]-2 ==0,y[x],x,IncludeSingularSolutions -> True]
```

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36.26 problem 1095

Internal problem ID [3800]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1095.
ODE order: 1.
ODE degree: 4.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries]]`

$$\boxed{xy'^4 - 2yy'^3 + 12x^3 = 0}$$

✓ Solution by Maple

Time used: 0.141 (sec). Leaf size: 62

```
dsolve(x*diff(y(x),x)^4-2*y(x)*diff(y(x),x)^3+12*x^3 = 0,y(x), singsol=all)
```

$$y(x) = -\frac{2\sqrt{-6x}\,x}{3}$$
$$y(x) = \frac{2\sqrt{-6x}\,x}{3}$$
$$y(x) = -\frac{2\sqrt{6}\,x^{\frac{3}{2}}}{3}$$
$$y(x) = \frac{2\sqrt{6}\,x^{\frac{3}{2}}}{3}$$
$$y(x) = 6c_1^3 + \frac{x^2}{2c_1}$$

✓ Solution by Mathematica

Time used: 39.939 (sec). Leaf size: 30947

```
DSolve[x (y'[x])^4 -2 y[x] (y'[x])^3+12 x^3==0,y[x],x,IncludeSingularSolutions -> True]
```

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36.27 problem 1098

Internal problem ID [3801]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1098.
ODE order: 1.
ODE degree: 5.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{3{y'}^{5} - yy' + 1 = 0}$$

✓ Solution by Maple

Time used: 0.032 (sec). Leaf size: 87

```
dsolve(3*diff(y(x),x)^5-y(x)*diff(y(x),x)+1 = 0,y(x), singsol=all)
```

$$y(x) = \frac{5\,\mathrm{RootOf}\left(1+8\_Z^{5}+(-2x+2c_1)\_Z^{2}\right)^{3}+2c_1-2x}{2\,\mathrm{RootOf}\left(1+8\_Z^{5}+(-2x+2c_1)\_Z^{2}\right)\left(4\,\mathrm{RootOf}\left(1+8\_Z^{5}+(-2x+2c_1)\_Z^{2}\right)^{3}+c_1-x\right)}$$

✓ Solution by Mathematica

Time used: 0.131 (sec). Leaf size: 176

```
DSolve[3 (y'[x])^5 -y[x] y'[x]+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{\text{Root}\left[3\#1^5 - K[1]\#1 + 1\&, 1\right]} dK[1] = x + c_1, y(x)\right]$$

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{\text{Root}\left[3\#1^5 - K[2]\#1 + 1\&, 2\right]} dK[2] = x + c_1, y(x)\right]$$

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{\text{Root}\left[3\#1^5 - K[3]\#1 + 1\&, 3\right]} dK[3] = x + c_1, y(x)\right]$$

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{\text{Root}\left[3\#1^5 - K[4]\#1 + 1\&, 4\right]} dK[4] = x + c_1, y(x)\right]$$

$$\text{Solve}\left[\int_1^{y(x)} \frac{1}{\text{Root}\left[3\#1^5 - K[5]\#1 + 1\&, 5\right]} dK[5] = x + c_1, y(x)\right]$$

36.28 problem 1099

Internal problem ID [3802]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1099.
ODE order: 1.
ODE degree: 6.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y'^6 - (y-a)^4 (y-b)^3 = 0}$$

✓ Solution by Maple

Time used: 0.062 (sec). Leaf size: 241

```
dsolve(diff(y(x),x)^6 = (y(x)-a)^4*(y(x)-b)^3,y(x), singsol=all)
```

$$y(x) = a$$
$$y(x) = b$$
$$x - \left(\int^{y(x)} \frac{1}{\left((\_a-a)^4 (\_a-b)^3\right)^{\frac{1}{6}}} d\_a\right) - c_1 = 0$$
$$x - \left(\int^{y(x)} \frac{2i}{\left(-i+\sqrt{3}\right)\left((\_a-a)^4 (\_a-b)^3\right)^{\frac{1}{6}}} d\_a\right) - c_1 = 0$$
$$x - \left(\int^{y(x)} -\frac{2i}{\left(\sqrt{3}+i\right)\left((\_a-a)^4 (\_a-b)^3\right)^{\frac{1}{6}}} d\_a\right) - c_1 = 0$$
$$x - \left(\int^{y(x)} \frac{2i}{\left(\sqrt{3}+i\right)\left((\_a-a)^4 (\_a-b)^3\right)^{\frac{1}{6}}} d\_a\right) - c_1 = 0$$
$$x - \left(\int^{y(x)} -\frac{2i}{\left(-i+\sqrt{3}\right)\left((\_a-a)^4 (\_a-b)^3\right)^{\frac{1}{6}}} d\_a\right) - c_1 = 0$$
$$x - \left(\int^{y(x)} -\frac{1}{\left((\_a-a)^4 (\_a-b)^3\right)^{\frac{1}{6}}} d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 2.148 (sec). Leaf size: 489

```
DSolve[(y'[x])^6 == (y[x]-a)^4 (y[x]-b)^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right][c_1-ix]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right][ix+c_1]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[-\sqrt[6]{-1}x+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\sqrt[6]{-1}x+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[-(-1)^{5/6}x+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[(-1)^{5/6}x+c_1\right]$$

$$y(x) \to a$$

$$y(x) \to b$$

36.29 problem 1100

Internal problem ID [3803]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1100.
ODE order: 1.
ODE degree: 6.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y'^6 + f(x)\,(y-a)^4\,(y-b)^3 = 0$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 68

```
dsolve(diff(y(x),x)^6+f(x)*(y(x)-a)^4*(y(x)-b)^3 = 0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\sqrt{\_a-b}\,(\_a-a)^{\frac{2}{3}}}d\_a + \int^{x} -\frac{\left(f(\_a)\,(-y(x)+a)^4\,(b-y(x))^3\right)^{\frac{1}{6}}}{\sqrt{y(x)-b}\,(y(x)-a)^{\frac{2}{3}}}d\_a + c_1 = 0$$

✓ Solution by Mathematica

Time used: 2.031 (sec). Leaf size: 561

```
DSolve[(y'[x])^6 +f[x] (y[x]-a)^4 (y[x]-b)^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x -\sqrt[6]{f(K[1])}dK[1]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x \sqrt[6]{f(K[2])}dK[2]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x -\sqrt[3]{-1}\sqrt[6]{f(K[3])}dK[3]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x \sqrt[3]{-1}\sqrt[6]{f(K[4])}dK[4]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x -(-1)^{2/3}\sqrt[6]{f(K[5])}dK[5]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{3\sqrt[3]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x (-1)^{2/3}\sqrt[6]{f(K[6])}dK[6]+c_1\right]$$

$$y(x) \to a$$

36.30 problem 1101

Internal problem ID [3804]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1101.
ODE order: 1.
ODE degree: 6.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$\boxed{y'^6 + f(x)(y-a)^5(y-b)^3 = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 69

```
dsolve(diff(y(x),x)^6+f(x)*(y(x)-a)^5*(y(x)-b)^3 = 0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{\sqrt{\_a-b}\,(\_a-a)^{\frac{5}{6}}} d\_a + \int^{x} -\frac{\left(-f(\_a)\,(b-y(x))^3\,(-y(x)+a)^5\right)^{\frac{1}{6}}}{\sqrt{y(x)-b}\,(y(x)-a)^{\frac{5}{6}}} d\_a + c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.995 (sec). Leaf size: 567

```
DSolve[(y'[x])^6 +f[x] (y[x]-a)^5 (y[x]-b)^3==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x -i\sqrt[6]{f(K[1])}dK[1]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x i\sqrt[6]{f(K[2])}dK[2]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x -\sqrt[6]{-1}\sqrt[6]{f(K[3])}dK[3]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x \sqrt[6]{-1}\sqrt[6]{f(K[4])}dK[4]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x -(-1)^{5/6}\sqrt[6]{f(K[5])}dK[5]+c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\sqrt{\frac{\#1-b}{a-b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{\sqrt{b-\#1}}\&\right]\left[\int_1^x (-1)^{5/6}\sqrt[6]{f(K[6])}dK[6]+c_1\right]$$

$$y(x) \to a$$

36.31 problem 1102

Internal problem ID [3805]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 36
Problem number: 1102.
ODE order: 1.
ODE degree: 6.

CAS Maple gives this as type `[[_1st_order, '_with_symmetry_[F(x),G(x)*y+H(x)]']]`

$$y'^6 + f(x)\,(y-a)^5\,(y-b)^4 = 0$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 68

```
dsolve(diff(y(x),x)^6+f(x)*(y(x)-a)^5*(y(x)-b)^4 = 0,y(x), singsol=all)
```

$$\int^{y(x)} \frac{1}{(\_a-b)^{\frac{2}{3}}\,(\_a-a)^{\frac{5}{6}}}d\_a + \int^{x} -\frac{\left(f(\_a)\,(b-y(x))^4\,(-y(x)+a)^5\right)^{\frac{1}{6}}}{(y\,(x)-b)^{\frac{2}{3}}\,(y\,(x)-a)^{\frac{5}{6}}}d\_a + c_1 = 0$$

✓ Solution by Mathematica

Time used: 1.942 (sec). Leaf size: 561

```
DSolve[(y'[x])^6 +f[x] (y[x]-a)^5 (y[x]-b)^4==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{2}{3},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[\int_1^x -\sqrt[6]{f(K[1])}dK[\right.$$

$$y(x) \rightarrow \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{2}{3},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[\int_1^x \sqrt[6]{f(K[2])}dK[2]\right.$$

$$y(x) \rightarrow \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{2}{3},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[\int_1^x -\sqrt[3]{-1}\sqrt[6]{f(K[3]}\right.$$

$$y(x) \rightarrow \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{2}{3},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[\int_1^x \sqrt[3]{-1}\sqrt[6]{f(K[4])}d\right.$$

$$y(x) \rightarrow \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{2}{3},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[\int_1^x -(-1)^{2/3}\sqrt[6]{f(K}\right.$$

$$y(x) \rightarrow \text{InverseFunction}\left[-\frac{6\sqrt[6]{a-\#1}\left(\frac{\#1-b}{a-b}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{2}{3},\frac{7}{6},\frac{a-\#1}{a-b}\right)}{(b-\#1)^{2/3}}\&\right]\left[\int_1^x (-1)^{2/3}\sqrt[6]{f(K[6]}\right.$$

$$y(x) \rightarrow a$$

$$y(x) \rightarrow b$$

37 Various 37

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37.1 problem 1104

Internal problem ID [3806]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1104.
ODE order: 1.
ODE degree: 6.

CAS Maple gives this as type `[_rational]`

$$x^2\left(y'^6+3y^4+3y^2+1\right)-a^2=0$$

✗ Solution by Maple

```
dsolve(x^2*(diff(y(x),x)^6+3*y(x)^4+3*y(x)^2+1) = a^2,y(x), singsol=all)
```

No solution found

✗ Solution by Mathematica

Time used: 0.0 (sec). Leaf size: 0

```
DSolve[x^2 ( (y'[x])^6 +3 (y[x])^4 +3 (y[x])^2 + 1)==a^2,y[x],x,IncludeSingularSolutions -> T
```

Not solved

37.2 problem 1115

Internal problem ID [3807]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1115.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class G‘], _Clairaut]`

$$\boxed{2\sqrt{ay'} + y'x - y = 0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 15

```
dsolve(2*sqrt(a*diff(y(x),x))+x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x) = 2\sqrt{c_1 a} + c_1 x$$

✓ Solution by Mathematica

Time used: 0.083 (sec). Leaf size: 25

```
DSolve[2 Sqrt[a y'[x]]+x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to 2\sqrt{ac_1} + c_1 x$$
$$y(x) \to 0$$

37.3 problem 1116

Internal problem ID [3808]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1116.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{(-y+x)\sqrt{y'}-a(1+y')=0}$$

✓ Solution by Maple

Time used: 0.094 (sec). Leaf size: 44

```
dsolve((x-y(x))*sqrt(diff(y(x),x)) = a*(1+diff(y(x),x)),y(x), singsol=all)
```

$$y(x)=x-2a$$
$$y(x)=x+\frac{-\frac{a^3}{(c_1-x)^2}-a}{\sqrt{\frac{a^2}{(c_1-x)^2}}}$$

✓ Solution by Mathematica

Time used: 67.98 (sec). Leaf size: 9683

```
DSolve[(x-y[x]) Sqrt[y'[x]]== a (1+y'[x]),y[x],x,IncludeSingularSolutions -> True]
```

Too large to display

37.4 problem 1117

Internal problem ID [3809]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1117.
ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type `[_separable]`

$$2(1+y)^{\frac{3}{2}}+3y'x-3y=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 33

```
dsolve(2*(1+y(x))^(3/2)+3*x*diff(y(x),x)-3*y(x) = 0,y(x), singsol=all)
```

$$\ln(x)+\int^{y(x)}-\frac{1}{-\frac{2\sqrt{\_a+1}\,\_a}{3}-\frac{2\sqrt{\_a+1}}{3}+\_a}d\_a+c_1=0$$

✓ Solution by Mathematica

Time used: 0.123 (sec). Leaf size: 55

```
DSolve[2 (1+y[x])^(3/2) + 3 x y'[x]-3 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{1}{3}\text{RootSum}\left[2\#1^3-3\#1^2+3\&,\frac{\log\left(\sqrt{y(x)+1}-\#1\right)}{\#1-1}\&\right]=-\frac{\log(x)}{3}+c_1,y(x)\right]$$

37.5 problem 1118

Internal problem ID [3810]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1118.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\sqrt{1+y'^2}+ay'-x=0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 118

```
dsolve(sqrt(1+diff(y(x),x)^2)+a*diff(y(x),x) = x,y(x), singsol=all)
```

$$y(x)=\frac{\frac{x\sqrt{a^2+x^2-1}}{2}+\frac{(4a^2-4)\ln\left(x+\sqrt{a^2+x^2-1}\right)}{8}+\frac{a\,x^2}{2}}{(a-1)\,(a+1)}+c_1$$

$$y(x)=-\frac{\frac{x\sqrt{a^2+x^2-1}}{2}+\frac{(4a^2-4)\ln\left(x+\sqrt{a^2+x^2-1}\right)}{8}-\frac{a\,x^2}{2}}{(a-1)\,(a+1)}+c_1$$

✓ Solution by Mathematica

Time used: 0.067 (sec). Leaf size: 113

```
DSolve[Sqrt[1+(y'[x])^2]+ a y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\frac{1}{2}\left(\frac{x\left(ax-\sqrt{a^2+x^2-1}\right)}{a^2-1}+\log\left(\sqrt{a^2+x^2-1}-x\right)\right)+c_1$$

$$y(x)\to\frac{1}{2}\left(\frac{x\left(\sqrt{a^2+x^2-1}+ax\right)}{a^2-1}-\log\left(\sqrt{a^2+x^2-1}-x\right)\right)+c_1$$

37.6 problem 1119

Internal problem ID [3811]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1119.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\sqrt{1+y'^2}+ay'-y=0$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 77

```
dsolve(sqrt(1+diff(y(x),x)^2)+a*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$x-\left(\int^{y(x)}\frac{(a-1)(a+1)}{a\_a+\sqrt{\_a^2+a^2-1}}d\_a\right)-c_1=0$$
$$x-\left(\int^{y(x)}-\frac{(a-1)(a+1)}{-a\_a+\sqrt{\_a^2+a^2-1}}d\_a\right)-c_1=0$$

✓ Solution by Mathematica

Time used: 0.808 (sec). Leaf size: 210

```
DSolve[Sqrt[1+(y'[x])^2]+ a y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \text{InverseFunction}\left[\frac{a\left(\log\left(\sqrt{\#1^2+a^2-1}-\#1-a+1\right)+\log\left(\sqrt{\#1^2+a^2-1}-\#1+a-1\right)\right)-(a+}{a^2-1} + c_1\right]$$

$$y(x) \to \text{InverseFunction}\left[\frac{a\left(\log\left(\sqrt{\#1^2+a^2-1}-\#1-a-1\right)+\log\left(\sqrt{\#1^2+a^2-1}-\#1+a+1\right)\right)-(a-}{a^2-1} + c_1\right]$$

$$y(x) \to 1$$

37.7 problem 1120

Internal problem ID [3812]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1120.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\sqrt{1+y'^2}-y'x=0}$$

✓ Solution by Maple

Time used: 0.078 (sec). Leaf size: 33

```
dsolve(sqrt(1+diff(y(x),x)^2) = x*diff(y(x),x),y(x), singsol=all)
```

$$y(x)=\ln\left(x+\sqrt{x^2-1}\right)+c_1$$
$$y(x)=-\ln\left(x+\sqrt{x^2-1}\right)+c_1$$

✓ Solution by Mathematica

Time used: 0.009 (sec). Leaf size: 41

```
DSolve[Sqrt[1+(y'[x])^2]==x y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to -\text{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)+c_1$$
$$y(x)\to \text{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)+c_1$$

37.8 problem 1123

Internal problem ID [3813]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1123.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$\boxed{\sqrt{a^2+b^2y'^2}+y'x-y=0}$$

✓ Solution by Maple

Time used: 0.343 (sec). Leaf size: 21

```
dsolve(sqrt(a^2+b^2*diff(y(x),x)^2)+x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x)=\sqrt{b^2c_1^2+a^2}+c_1x$$

✓ Solution by Mathematica

Time used: 0.398 (sec). Leaf size: 37

```
DSolve[Sqrt[a^2+b^2 (y'[x])^2] +x y'[x] -y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to\sqrt{a^2+b^2{c_1}^2}+c_1x$$
$$y(x)\to\sqrt{a^2}$$

37.9 problem 1125

Internal problem ID [3814]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1125.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _rational, _Clairaut]`

$$\boxed{a\sqrt{1+y'^2}+y'x-y=0}$$

✓ Solution by Maple

Time used: 0.25 (sec). Leaf size: 17

```
dsolve(a*sqrt(1+diff(y(x),x)^2)+x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x)=a\sqrt{c_1^2+1}+c_1x$$

✓ Solution by Mathematica

Time used: 0.06 (sec). Leaf size: 27

```
DSolve[a Sqrt[1+(y'[x])^2] + x y'[x] -y[x]==0 y'[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to a\sqrt{1+{c_1}^2}+c_1x$$
$$y(x)\to a$$

37.10 problem 1126

Internal problem ID [3815]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1126.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _dAlembert]`

$$a x \sqrt{1+y'^2}+y' x - y = 0$$

✓ Solution by Maple

Time used: 0.093 (sec). Leaf size: 223

```
dsolve(a*x*sqrt(1+diff(y(x),x)^2)+x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$x - \frac{e^{\frac{\operatorname{arcsinh}\left(\frac{\sqrt{-a^2x^2+x^2+y(x)^2}\,a+y(x)}{x(a^2-1)}\right)}{a}} c_1}{\sqrt{\frac{-a^2x^2+a^2y(x)^2+2\sqrt{-a^2x^2+x^2+y(x)^2}\,a y(x)+x^2+y(x)^2}{(a^2-1)^2 x^2}}} = 0$$

$$x - \frac{e^{-\frac{\operatorname{arcsinh}\left(\frac{\sqrt{-a^2x^2+x^2+y(x)^2}\,a-y(x)}{x(a^2-1)}\right)}{a}} c_1}{\sqrt{-\frac{a^2x^2-a^2y(x)^2+2\sqrt{-a^2x^2+x^2+y(x)^2}\,a y(x)-x^2-y(x)^2}{(a^2-1)^2 x^2}}} = 0$$

✓ Solution by Mathematica

Time used: 0.999 (sec). Leaf size: 223

```
DSolve[a x Sqrt[1+(y'[x])^2]+x y'[x] -y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\frac{2i\arctan\left(\frac{y(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)-2ia\arctan\left(\frac{ay(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+a\log\left(\frac{y(x)^2}{x^2}+1\right)}{2a^2-2}=\frac{a\log\left(x-a^2x\right)}{1-a^2}+c_1,y(x)\right]$$

$$\text{Solve}\left[\frac{-2i\arctan\left(\frac{y(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+2ia\arctan\left(\frac{ay(x)}{x\sqrt{a^2-\frac{y(x)^2}{x^2}-1}}\right)+a\log\left(\frac{y(x)^2}{x^2}+1\right)}{2a^2-2}=\frac{a\log\left(x-a^2x\right)}{1-a^2}+c_1,y(x)\right]$$

37.11 problem 1129

Internal problem ID [3816]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1129.
ODE order: 1.
ODE degree: 2.

CAS Maple gives this as type `[[_homogeneous, ‘class A‘], _rational, _dAlembert]`

$$\sqrt{(a\,x^2+y^2)\left(1+y'^2\right)}-yy'-ax=0$$

✓ Solution by Maple

Time used: 1.156 (sec). Leaf size: 180

```
dsolve(((a*x^2+y(x)^2)*(1+diff(y(x),x)^2))^(1/2)-y(x)*diff(y(x),x)-a*x = 0,y(x), singsol=all)
```

$$y(x)=\sqrt{-a}\,x$$
$$y(x)=-\sqrt{-a}\,x$$
$$y(x)=\frac{-x^{-\frac{-a+\sqrt{(a-1)a}}{a}}a^3+x^{\frac{a+\sqrt{(a-1)a}}{a}}c_1^2+x^{-\frac{-a+\sqrt{(a-1)a}}{a}}a^2}{2c_1\sqrt{(a-1)\,a}}$$
$$y(x)=-\frac{x^{\frac{a+\sqrt{(a-1)a}}{a}}a^3-x^{\frac{a+\sqrt{(a-1)a}}{a}}a^2-x^{-\frac{-a+\sqrt{(a-1)a}}{a}}c_1^2}{2\sqrt{(a-1)\,a}\,c_1}$$

✓ Solution by Mathematica

Time used: 0.702 (sec). Leaf size: 241

```
DSolve[((a x^2+y[x]^2)(1+(y'[x])^2))^(1/2) -y[x] y'[x]-a x==0,y[x],x,IncludeSingularSolutions
```

$$y(x) \to \frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(x^{2\sqrt{\frac{a-1}{a}}} - e^{2c_1}\right)$$

$$y(x) \to \frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(-x^{2\sqrt{\frac{a-1}{a}}} + e^{2c_1}\right)$$

$$y(x) \to -\frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(-1 + e^{2c_1}x^{2\sqrt{\frac{a-1}{a}}}\right)$$

$$y(x) \to \frac{1}{2}\sqrt{a}e^{-c_1}x^{1-\sqrt{\frac{a-1}{a}}}\left(-1 + e^{2c_1}x^{2\sqrt{\frac{a-1}{a}}}\right)$$

37.12 problem 1130

Internal problem ID [3817]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1130.
ODE order: 1.
ODE degree: 3.

CAS Maple gives this as type `[_Clairaut]`

$$\boxed{a\left(1+y'^3\right)^{\frac{1}{3}}+y'x-y=0}$$

✓ Solution by Maple

Time used: 0.547 (sec). Leaf size: 17

```
dsolve(a*(1+diff(y(x),x)^3)^(1/3)+x*diff(y(x),x)-y(x) = 0,y(x), singsol=all)
```

$$y(x)=a\left(c_1^3+1\right)^{\frac{1}{3}}+c_1x$$

✓ Solution by Mathematica

Time used: 0.178 (sec). Leaf size: 27

```
DSolve[a (1+ (y'[x])^3)^(1/3) +x y'[x]-y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x)\to a\sqrt[3]{1+{c_1}^3}+c_1x$$
$$y(x)\to a$$

37.13 problem 1132

Internal problem ID [3818]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1132.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_Clairaut]`

$$\cos(y') + y'x - y = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 27

```
dsolve(cos(diff(y(x),x))+x*diff(y(x),x) = y(x),y(x), singsol=all)
```

$$y(x) = \arcsin(x)\, x + \sqrt{-x^2 + 1}$$
$$y(x) = \cos(c_1) + c_1 x$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 18

```
DSolve[Cos[y'[x]]+x*y'[x]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x + \cos(c_1)$$
$$y(x) \to 1$$

37.14 problem 1133

Internal problem ID [3819]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1133.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{a\cos(y') + by' + x = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 18

```
dsolve(a*cos(diff(y(x),x))+b*diff(y(x),x)+x = 0,y(x), singsol=all)
```

$$y(x) = \int \operatorname{RootOf}\left(a\cos(\_Z) + \_Zb + x\right)dx + c_1$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 49

```
DSolve[a Cos[y'[x]] + b y'[x]+x ==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{y(x) = a\sin(K[1]) - aK[1]\cos(K[1]) - \frac{1}{2}bK[1]^2 + c_1, x = -a\cos(K[1]) - bK[1]\right\}, \{y(x), K[1]\}\right]$$

37.15 problem 1134

Internal problem ID [3820]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1134.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\sin(y') + y' - x = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 16

```
dsolve(sin(diff(y(x),x))+diff(y(x),x) = x,y(x), singsol=all)
```

$$y(x) = \int \operatorname{RootOf}(\sin(\_Z) + \_Z - x)\, dx + c_1$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 38

```
DSolve[Sin[y'[x]]+ y'[x]==x,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x = K[1] + \sin(K[1]), y(x) = \frac{K[1]^2}{2} + K[1]\sin(K[1]) + \cos(K[1]) + c_1\right\}, \{y(x), K[1]\}\right]$$

37.16 problem 1135

Internal problem ID [3821]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1135.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{y' \sin(y') + \cos(y') - y = 0}$$

✓ Solution by Maple

Time used: 0.266 (sec). Leaf size: 32

```
dsolve(diff(y(x),x)*sin(diff(y(x),x))+cos(diff(y(x),x)) = y(x),y(x), singsol=all)
```

$$y(x) = 1$$

$$x - \left(\int^{y(x)} \frac{1}{\mathrm{RootOf}\left(\_Z \sin\left(\_Z\right) + \cos\left(\_Z\right) - \_a\right)} d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 28

```
DSolve[y'[x] Sin[y'[x]]+ Cos[y'[x]]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}[\{x = \sin(K[1]) + c_1, y(x) = K[1]\sin(K[1]) + \cos(K[1])\}, \{y(x), K[1]\}]$$

37.17 problem 1137

Internal problem ID [3822]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1137.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_dAlembert]`

$$y'^2(x + \sin(y')) - y = 0$$

✓ Solution by Maple

Time used: 0.453 (sec). Leaf size: 68

```
dsolve(diff(y(x),x)^2*(x+sin(diff(y(x),x))) = y(x),y(x), singsol=all)
```

$$y(x) = 0$$

$$\left[x(\_T) = \frac{-\_T^2 \sin(\_T) - \cos(\_T) + \_T \sin(\_T) + c_1}{(\_T-1)^2}, y(\_T) = \frac{\_T^2(-\_T^2 \sin(\_T) - \cos(\_T) + \_T}{(\_T-1)^2}\right.$$

$$\left. + \_T^2 \sin(\_T)\right]$$

✓ Solution by Mathematica

Time used: 0.156 (sec). Leaf size: 61

```
DSolve[(y'[x])^2 (x+Sin[y'[x]])==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x = \frac{-(K[1]-1)K[1]\sin(K[1]) - \cos(K[1])}{(K[1]-1)^2} + \frac{c_1}{(K[1]-1)^2}, y(x) = xK[1]^2 + K[1]^2 \sin(K[1])\right\}, \{y(x), K[1]\}\right]$$

37.18 problem 1138

Internal problem ID [3823]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1138.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_Clairaut]`

$$\boxed{\left(1+y'^2\right)\sin\left(-y+y'x\right)^2-1=0}$$

✓ Solution by Maple

Time used: 0.203 (sec). Leaf size: 139

```
dsolve((1+diff(y(x),x)^2)*sin(y(x)-x*diff(y(x),x))^2 = 1,y(x), singsol=all)
```

$$y(x)=-x\sqrt{1-x}\,\sqrt{\frac{1}{x}}-\arcsin\left(\frac{1}{\sqrt{\frac{1}{x}}}\right)$$

$$y(x)=x\sqrt{1-x}\,\sqrt{\frac{1}{x}}+\arcsin\left(\frac{1}{\sqrt{\frac{1}{x}}}\right)$$

$$y(x)=-x\sqrt{x+1}\,\sqrt{-\frac{1}{x}}+\arcsin\left(\frac{1}{\sqrt{-\frac{1}{x}}}\right)$$

$$y(x)=x\sqrt{x+1}\,\sqrt{-\frac{1}{x}}-\arcsin\left(\frac{1}{\sqrt{-\frac{1}{x}}}\right)$$

$$y(x)=c_1x-\arcsin\left(\frac{1}{\sqrt{c_1^2+1}}\right)$$

$$y(x)=c_1x+\arcsin\left(\frac{1}{\sqrt{c_1^2+1}}\right)$$

✓ Solution by Mathematica

Time used: 0.354 (sec). Leaf size: 71

```
DSolve[(1+(y'[x])^2) (Sin[y[x]-x y'[x]])^2==1,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1 x - \frac{1}{2}\arccos\left(1 - \frac{2}{1 + c_1^2}\right)$$

$$y(x) \to \frac{1}{2}\arccos\left(1 - \frac{2}{1 + c_1^2}\right) + c_1 x$$

$$y(x) \to -\frac{\pi}{2}$$

$$y(x) \to \frac{\pi}{2}$$

37.19 problem 1140

Internal problem ID [3824]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1140.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\left(1+y'^2\right)\left(\arctan\left(y'\right)+ax\right)+y'=0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 30

```
dsolve((1+diff(y(x),x)^2)*(arctan(diff(y(x),x))+a*x)+diff(y(x),x) = 0,y(x), singsol=all)
```

$$y(x)=\int \tan\left(\mathrm{RootOf}\left(ax\tan\left(\_Z\right)^2+\tan\left(\_Z\right)^2\_Z+ax+\tan\left(\_Z\right)+\_Z\right)\right)dx+c_1$$

✓ Solution by Mathematica

Time used: 1.193 (sec). Leaf size: 58

```
DSolve[(1+(y'[x])^2)(ArcTan[y'[x]]+a x)+y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\mathrm{Solve}\left[\left\{y(x)=\frac{1}{a\left(K[1]^2+1\right)}+c_1,x=\frac{K[1]^2(-\arctan(K[1]))-\arctan(K[1])-K[1]}{a\left(K[1]^2+1\right)}\right\},\{y(x),K[1]\}\right]$$

37.20 problem 1141

Internal problem ID [3825]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1141.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{e^{y'-y} - y'^2 + 1 = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(exp(diff(y(x),x)-y(x))-diff(y(x),x)^2+1 = 0,y(x), singsol=all)
```

$$x - \left(\int^{y(x)} \frac{1}{\operatorname{RootOf}\left(-e^{\_Z-\_a} + \_Z^2 - 1\right)} d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.136 (sec). Leaf size: 44

```
DSolve[Exp[y'[x]-y[x]]-(y'[x])^2+1==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[\left\{x = -\log(1 - K[1]) + \log(K[1]) + \log(K[1] + 1) + c_1, y(x) = K[1] - \log\left(K[1]^2 - 1\right)\right\}, \{y(x), K[1]\}\right]$$

37.21 problem 1143

Internal problem ID [3826]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1143.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_quadrature]`

$$\boxed{\ln(y') + y'x + a = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 24

```
dsolve(ln(diff(y(x),x))+x*diff(y(x),x)+a = 0,y(x), singsol=all)
```

$$y(x) = \frac{\mathrm{LambertW}\left(x\,\mathrm{e}^{-a}\right)^2}{2} + \mathrm{LambertW}\left(x\,\mathrm{e}^{-a}\right) + c_1$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 30

```
DSolve[Log[y'[x]]+x y'[x]+ a ==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{1}{2}W\left(e^{-a}x\right)^2 + W\left(e^{-a}x\right) + c_1$$

37.22 problem 1144

Internal problem ID [3827]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1144.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{\ln(y') + y'x + a - y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

```
dsolve(ln(diff(y(x),x))+x*diff(y(x),x)+a = y(x),y(x), singsol=all)
```

$$y(x) = \ln\left(-\frac{1}{x}\right) + a - 1$$
$$y(x) = \ln(c_1) + c_1 x + a$$

✓ Solution by Mathematica

Time used: 0.046 (sec). Leaf size: 27

```
DSolve[Log[y'[x]]+x y'[x]+ a ==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to a + c_1 x + \log(c_1)$$
$$y(x) \to a + \log\left(-\frac{1}{x}\right) - 1$$

37.23 problem 1145

Internal problem ID [3828]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1145.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\ln(y') + y'x + a + by = 0$$

✓ Solution by Maple

Time used: 0.125 (sec). Leaf size: 66

```
dsolve(ln(diff(y(x),x))+x*diff(y(x),x)+a+b*y(x) = 0,y(x), singsol=all)
```

$$-\left(\mathrm{e}^{-by(x)-\mathrm{LambertW}\left(x\,\mathrm{e}^{-by(x)-a}\right)-a}\right)^{-\frac{1}{b+1}} c_1 + x - \frac{\mathrm{e}^{by(x)+\mathrm{LambertW}\left(x\,\mathrm{e}^{-by(x)-a}\right)+a}}{b} = 0$$

✓ Solution by Mathematica

Time used: 0.14 (sec). Leaf size: 59

```
DSolve[Log[y'[x]]+x y'[x]+ a +b y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[b\left(\frac{(b+1)\log\left(1-bW\left(xe^{-a-by(x)}\right)\right)}{b^2} + \frac{W\left(xe^{-a-by(x)}\right)}{b}\right) + by(x) = c_1, y(x)\right]$$

37.24 problem 1146

Internal problem ID [3829]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1146.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _dAlembert]`

$$\boxed{\ln(y') + 4y'x - 2y = 0}$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 67

```
dsolve(ln(diff(y(x),x))+4*x*diff(y(x),x)-2*y(x) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\ln\left(\frac{-1+\sqrt{16c_1x+1}}{4x}\right)}{2} - \frac{1}{2} + \frac{\sqrt{16c_1x+1}}{2}$$
$$y(x) = \frac{\ln\left(-\frac{1+\sqrt{16c_1x+1}}{4x}\right)}{2} - \frac{1}{2} - \frac{\sqrt{16c_1x+1}}{2}$$

✓ Solution by Mathematica

Time used: 0.092 (sec). Leaf size: 36

```
DSolve[Log[y'[x]]+4 x y'[x]-2 y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}\left[W\left(4xe^{2y(x)}\right) - \log\left(W\left(4xe^{2y(x)}\right) + 2\right) - 2y(x) = c_1, y(x)\right]$$

37.25 problem 1147

Internal problem ID [3830]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1147.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$\boxed{\ln(y') + a(y'x - y) = 0}$$

✓ Solution by Maple

Time used: 0.015 (sec). Leaf size: 36

```
dsolve(ln(diff(y(x),x))+a*(x*diff(y(x),x)-y(x)) = 0,y(x), singsol=all)
```

$$y(x) = \frac{\ln\left(-\frac{1}{ax}\right)}{a} - \frac{1}{a}$$
$$y(x) = c_1 x + \frac{\ln(c_1)}{a}$$

✓ Solution by Mathematica

Time used: 0.041 (sec). Leaf size: 36

```
DSolve[Log[y'[x]]+a( x y'[x]-y[x])==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to \frac{\log(c_1)}{a} + c_1 x$$
$$y(x) \to \frac{\log\left(-\frac{1}{ax}\right) - 1}{a}$$

37.26 problem 1148

Internal problem ID [3831]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1148.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_homogeneous, ‘class C‘], _dAlembert]`

$$\boxed{a(\ln(y') - y') - x + y = 0}$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 45

```
dsolve(a*(ln(diff(y(x),x))-diff(y(x),x))-x+y(x) = 0,y(x), singsol=all)
```

$$y(x) = x + a$$
$$y(x) = -a\left(\ln\left(\mathrm{e}^{-\frac{c_1}{a}+\frac{x}{a}}\right) - \mathrm{e}^{-\frac{c_1}{a}+\frac{x}{a}}\right) + x$$

✓ Solution by Mathematica

Time used: 0.357 (sec). Leaf size: 22

```
DSolve[a (Log[y'[x]]-y'[x])-x+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to ae^{\frac{x-c_1}{a}} + c_1$$

37.27 problem 1149

Internal problem ID [3832]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1149.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_separable]`

$$\boxed{y \ln (y') + y' - y \ln (y) - xy = 0}$$

✓ Solution by Maple

Time used: 0.109 (sec). Leaf size: 17

```
dsolve(y(x)*ln(diff(y(x),x))+diff(y(x),x)-y(x)*ln(y(x))-x*y(x) = 0,y(x), singsol=all)
```

$$y(x) = c_1 \mathrm{e}^{\frac{\mathrm{LambertW}(\mathrm{e}^x)(\mathrm{LambertW}(\mathrm{e}^x)+2)}{2}}$$

✓ Solution by Mathematica

Time used: 0.096 (sec). Leaf size: 24

```
DSolve[y[x] Log[y'[x]] + y'[x] -y[x] Log[y[x]] -x y[x]==0,y[x],x,IncludeSingularSolutions ->
```

$$y(x) \rightarrow c_1 e^{\frac{1}{2}W(e^x)(W(e^x)+2)}$$

37.28 problem 1150

Internal problem ID [3833]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1150.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[[_1st_order, _with_linear_symmetries], _Clairaut]`

$$y' \ln(y') - (x+1)\, y' + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)*ln(diff(y(x),x))-(1+x)*diff(y(x),x)+y(x) = 0,y(x), singsol=all)
```

$$y(x) = e^{x}$$
$$y(x) = c_1 x - c_1 \ln(c_1) + c_1$$

✓ Solution by Mathematica

Time used: 1.675 (sec). Leaf size: 21

```
DSolve[y'[x] Log[y'[x]] -(1+x) y'[x]+y[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \to c_1(x + 1 - \log(c_1))$$
$$y(x) \to 0$$

37.29 problem 1152

Internal problem ID [3834]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1152.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_Clairaut]`

$$y' \ln \left(y' + \sqrt{a + y'^2}\right) - \sqrt{1 + y'^2} - y'x + y = 0$$

✗ Solution by Maple

```
dsolve(diff(y(x),x)*ln(diff(y(x),x)+sqrt(a+diff(y(x),x)^2))-sqrt(1+diff(y(x),x)^2)-x*diff(y(x
```

No solution found

✓ Solution by Mathematica

Time used: 60.032 (sec). Leaf size: 37

```
DSolve[y'[x]*Log[y'[x]+Sqrt[a+(y'[x])^2]]-Sqrt[1+(y'[x])^2]-x*y'[x]+y[x]==0,y[x],x,IncludeSin
```

$$y(x) \rightarrow c_1 \left(x - \log \left(\sqrt{a + c_1{}^2} + c_1\right)\right) + \sqrt{1 + c_1{}^2}$$

37.30 problem 1153

Internal problem ID [3835]

Book: Ordinary differential equations and their solutions. By George Moseley Murphy. 1960
Section: Various 37
Problem number: 1153.
ODE order: 1.
ODE degree: 0.

CAS Maple gives this as type `[_dAlembert]`

$$\boxed{\ln(\cos(y')) + y' \tan(y') - y = 0}$$

✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 33

```
dsolve(ln(cos(diff(y(x),x)))+diff(y(x),x)*tan(diff(y(x),x)) = y(x),y(x), singsol=all)
```

$$y(x) = 0$$

$$x - \left(\int^{y(x)} \frac{1}{\operatorname{RootOf}(\ln(\cos(\_Z)) + \_Z\tan(\_Z) - \_a)} d\_a\right) - c_1 = 0$$

✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 29

```
DSolve[Log[Cos[y'[x]]]+y'[x] Tan[y'[x]]==y[x],y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve}[\{x = \tan(K[1]) + c_1, y(x) = K[1]\tan(K[1]) + \log(\cos(K[1]))\}, \{y(x), K[1]\}]$$