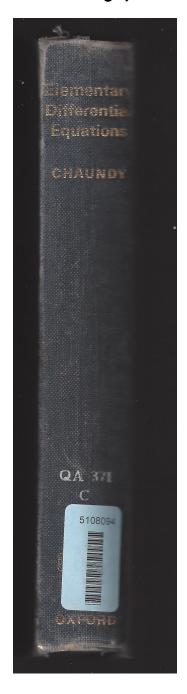
#### A Solution Manual For

# Elementary Differential equations, Chaundy, 1969



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October 12, 2023

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## 1 Exercises 3, page 60

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### 1.1 problem 1(a)

Internal problem ID [2520]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 1(a).

ODE order: 1. ODE degree: 1.

CAS Maple gives this as type [\_separable]

$$yy'-x=0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

dsolve(diff(y(x),x)\*y(x)=x,y(x), singsol=all)

$$y(x) = \sqrt{x^2 + c_1}$$

$$y(x) = -\sqrt{x^2 + c_1}$$

✓ Solution by Mathematica

Time used: 0.075 (sec). Leaf size: 35

DSolve[y'[x]\*y[x]==x,y[x],x,IncludeSingularSolutions -> True]

$$y(x) \to -\sqrt{x^2 + 2c_1}$$

$$y(x) o \sqrt{x^2 + 2c_1}$$

#### 1.2 problem 1(b)

Internal problem ID [2521]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 1(b).

ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [[\_linear, 'class A']]

$$y' - y - x^3 = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

 $dsolve(diff(y(x),x)-y(x)=x^3,y(x), singsol=all)$ 

$$y(x) = -x^3 - 3x^2 - 6x - 6 + c_1 e^x$$

✓ Solution by Mathematica

Time used: 0.037 (sec). Leaf size: 23

DSolve[y'[x]-y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]

$$y(x) \rightarrow -x(x(x+3)+6) + c_1e^x - 6$$

### 1.3 problem 1(c)

Internal problem ID [2522]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 1(c).

ODE order: 1. ODE degree: 1.

CAS Maple gives this as type [\_linear]

$$y' + \cot(x) y - x = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 18

dsolve(diff(y(x),x)+y(x)\*cot(x)=x,y(x), singsol=all)

$$y(x) = \frac{\sin(x) - x\cos(x) + c_1}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 17

DSolve[y'[x]+y[x]\*Cot[x]==x,y[x],x,IncludeSingularSolutions -> True]

$$y(x) \rightarrow -x \cot(x) + c_1 \csc(x) + 1$$

#### 1.4 problem 1(d)

Internal problem ID [2523]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 1(d).

ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [\_linear]

$$y' + \cot(x) y - \tan(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

dsolve(diff(y(x),x)+y(x)\*cot(x)=tan(x),y(x), singsol=all)

$$y(x) = \frac{-\sin(x) + \ln(\sec(x) + \tan(x)) + c_1}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.048 (sec). Leaf size: 16

DSolve[y'[x]+y[x]\*Cot[x]==Tan[x],y[x],x,IncludeSingularSolutions -> True]

$$y(x) \rightarrow -1 + \csc(x)(\operatorname{arctanh}(\sin(x)) + c_1)$$

#### 1.5 problem 1(e)

Internal problem ID [2524]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 1(e).

ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [linear]

$$y' + y \tan(x) - \cot(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

dsolve(diff(y(x),x)+y(x)\*tan(x)=cot(x),y(x), singsol=all)

$$y(x) = (-\ln(\csc(x) + \cot(x)) + c_1)\cos(x)$$

✓ Solution by Mathematica

Time used: 0.063 (sec). Leaf size: 27

DSolve[y'[x]+y[x]\*Tan[x]==Cot[x],y[x],x,IncludeSingularSolutions -> True]

$$y(x) \to \cos(x) \left( \log \left( \sin \left( \frac{x}{2} \right) \right) - \log \left( \cos \left( \frac{x}{2} \right) \right) + c_1 \right)$$

### 1.6 problem 1(f)

Internal problem ID [2525]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 1(f).

ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [\_linear]

$$y' + \ln(x) y - x^{-x} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

 $dsolve(diff(y(x),x)+y(x)*ln(x)=x^{-(-x)},y(x), singsol=all)$ 

$$y(x) = -x^{-x} + x^{-x} e^x c_1$$

✓ Solution by Mathematica

Time used: 0.071 (sec). Leaf size: 19

 $DSolve[y'[x]+y[x]*Log[x]==x^(-x),y[x],x,IncludeSingularSolutions \rightarrow True]$ 

$$y(x) \to x^{-x}(-1 + c_1 e^x)$$

### 1.7 problem 2(a)

Internal problem ID [2526]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 2(a).

ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [\_linear]

$$y'x + y - x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

dsolve(x\*diff(y(x),x)+y(x)=x,y(x), singsol=all)

$$y(x) = \frac{x}{2} + \frac{c_1}{x}$$

✓ Solution by Mathematica

Time used: 0.023 (sec). Leaf size: 17

DSolve[x\*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]

$$y(x) \rightarrow \frac{x}{2} + \frac{c_1}{x}$$

### 1.8 problem 2(b)

Internal problem ID [2527]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 2(b).

ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [\_linear]

$$y'x - y - x^3 = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 13

 $dsolve(x*diff(y(x),x)-y(x)=x^3,y(x), singsol=all)$ 

$$y(x) = \left(\frac{x^2}{2} + c_1\right)x$$

✓ Solution by Mathematica

Time used: 0.026 (sec). Leaf size: 17

DSolve[x\*y'[x]-y[x]==x^3,y[x],x,IncludeSingularSolutions -> True]

$$y(x) \to \frac{x^3}{2} + c_1 x$$

### 1.9 problem 2(c)

Internal problem ID [2528]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 2(c).

ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [\_linear]

$$y'x + ny - x^n = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 20

 $dsolve(x*diff(y(x),x)+n*y(x)=x^n,y(x), singsol=all)$ 

$$y(x) = \frac{x^n}{2n} + x^{-n}c_1$$

Solution by Mathematica

Time used: 0.045 (sec). Leaf size: 24

DSolve[x\*y'[x]+n\*y[x]==x^n,y[x],x,IncludeSingularSolutions -> True]

$$y(x) \to \frac{x^n}{2n} + c_1 x^{-n}$$

### 1.10 problem 2(d)

Internal problem ID [2529]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 2(d).

ODE order: 1. ODE degree: 1.

CAS Maple gives this as type [\_linear]

$$y'x - ny - x^n = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 12

 $dsolve(x*diff(y(x),x)-n*y(x)=x^n,y(x), singsol=all)$ 

$$y(x) = (\ln(x) + c_1) x^n$$

✓ Solution by Mathematica

Time used: 0.042 (sec). Leaf size: 14

DSolve[x\*y'[x]-n\*y[x]==x^n,y[x],x,IncludeSingularSolutions -> True]

$$y(x) \to x^n(\log(x) + c_1)$$

### 1.11 problem 2(e)

Internal problem ID [2530]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 2(e).

ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [\_linear]

$$(x^3 + x)y' + y - x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 22

 $dsolve((x^3+x)*diff(y(x),x)+y(x)=x,y(x), singsol=all)$ 

$$y(x) = \frac{\sqrt{x^2 + 1} c_1}{x} - \frac{1}{x}$$

✓ Solution by Mathematica

Time used: 0.036 (sec). Leaf size: 23

DSolve[(x^3+x)\*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]

$$y(x) \to \frac{-1 + c_1 \sqrt{x^2 + 1}}{x}$$

#### 1.12 problem 3(a)

Internal problem ID [2531]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 3(a).

ODE order: 1. ODE degree: 1.

CAS Maple gives this as type [linear]

$$\cot(x)y' + y - x = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

dsolve(cot(x)\*diff(y(x),x)+y(x)=x,y(x), singsol=all)

$$y(x) = \left(\frac{x}{\cos(x)} - \ln(\sec(x) + \tan(x)) + c_1\right) \cos(x)$$

✓ Solution by Mathematica

Time used: 0.072 (sec). Leaf size: 45

DSolve[Cot[x]\*y'[x]+y[x]==x,y[x],x,IncludeSingularSolutions -> True]

$$y(x) \to x + \cos(x) \left( \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right) \right) - \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right) \right) + c_1 \right)$$

#### 1.13 problem 3(b)

Internal problem ID [2532]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 3(b).

ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [linear]

$$\cot(x)y' + y - \tan(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 31

dsolve(cot(x)\*diff(y(x),x)+y(x)=tan(x),y(x), singsol=all)

$$y(x) = \left(\frac{\sin(x)^3}{2\cos(x)^2} + \frac{\sin(x)}{2} - \frac{\ln(\sec(x) + \tan(x))}{2} + c_1\right)\cos(x)$$

✓ Solution by Mathematica

Time used: 0.064 (sec). Leaf size: 25

DSolve[Cot[x]\*y'[x]+y[x]==Tan[x],y[x],x,IncludeSingularSolutions -> True]

$$y(x) \rightarrow \frac{1}{2}(\cos(x)(-\arctanh(\sin(x))) + \tan(x) + 2c_1\cos(x))$$

#### 1.14 problem 3(c)

Internal problem ID [2533]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 3(c).

ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [linear]

$$\tan(x)y' + y - \cot(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 21

dsolve(tan(x)\*diff(y(x),x)+y(x)=cot(x),y(x), singsol=all)

$$y(x) = \frac{\cos(x) + \ln(\csc(x) - \cot(x)) + c_1}{\sin(x)}$$

✓ Solution by Mathematica

Time used: 0.074 (sec). Leaf size: 29

DSolve[Tan[x]\*y'[x]+y[x]==Cot[x],y[x],x,IncludeSingularSolutions -> True]

$$y(x) \to \csc(x) \left(\cos(x) + \log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) + c_1\right)$$

#### 1.15 problem 3(a)

Internal problem ID [2534]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 3(a).

ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [\_linear]

$$\tan(x)y' - y + \cos(x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

dsolve(tan(x)\*diff(y(x),x)=y(x)-cos(x),y(x), singsol=all)

$$y(x) = \left(\cot\left(x\right) - \frac{\pi}{2} + x + c_1\right)\sin\left(x\right)$$

✓ Solution by Mathematica

Time used: 0.054 (sec). Leaf size: 17

 $DSolve[Tan[x]*y'[x] == y[x] - Cos[x], y[x], x, IncludeSingularSolutions \rightarrow True]$ 

$$y(x) \to \cos(x) + \sin(x)(\arctan(\tan(x)) + c_1)$$

### 1.16 problem 4(a)

Internal problem ID [2535]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 4(a).

ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [\_linear]

$$y' + \cos(x) y - \sin(2x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 17

 $\label{eq:diff} dsolve(diff(y(x),x)+y(x)*cos(x)=sin(2*x),y(x), singsol=all)$ 

$$y(x) = 2\sin(x) - 2 + e^{-\sin(x)}c_1$$

✓ Solution by Mathematica

Time used: 0.079 (sec). Leaf size: 20

DSolve[y'[x]+y[x]\*Cos[x]==Sin[2\*x],y[x],x,IncludeSingularSolutions -> True]

$$y(x) \to 2\sin(x) + c_1 e^{-\sin(x)} - 2$$

#### 1.17 problem 4(b)

Internal problem ID [2536]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 4(b).

ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [linear]

$$y'\cos(x) + y - \sin(2x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 25

dsolve(cos(x)\*diff(y(x),x)+y(x)=sin(2\*x),y(x), singsol=all)

$$y(x) = \frac{-2\sin(x) - 2\ln(\sin(x) - 1) + c_1}{\sec(x) + \tan(x)}$$

✓ Solution by Mathematica

Time used: 0.085 (sec). Leaf size: 42

DSolve[Cos[x]\*y'[x]+y[x]==Sin[2\*x],y[x],x,IncludeSingularSolutions -> True]

$$y(x) o e^{-2 \operatorname{arctanh}( an(rac{x}{2}))} \Big( -2 \sin(x) - 4 \log \Big( \cos \Big(rac{x}{2}\Big) - \sin \Big(rac{x}{2}\Big) \Big) + c_1 \Big)$$

### 1.18 problem 4(c)

Internal problem ID [2537]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 4(c).

ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [\_linear]

$$y' + \sin(x) y - \sin(2x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 15

 $\label{eq:diff} dsolve(diff(y(x),x)+y(x)*sin(x)=sin(2*x),y(x), singsol=all)$ 

$$y(x) = 2\cos(x) + 2 + e^{\cos(x)}c_1$$

✓ Solution by Mathematica

Time used: 0.069 (sec). Leaf size: 18

DSolve[y'[x]+y[x]\*Sin[x]==Sin[2\*x],y[x],x,IncludeSingularSolutions -> True]

$$y(x) \to 2\cos(x) + c_1 e^{\cos(x)} + 2$$

#### 1.19 problem 4(d)

Internal problem ID [2538]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 4(d).

ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [linear]

$$\sin(x) y' + y - \sin(2x) = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 23

dsolve(sin(x)\*diff(y(x),x)+y(x)=sin(2\*x),y(x), singsol=all)

$$y(x) = (-2\cos(x) + 2\ln(\cos(x) + 1) + c_1)(\csc(x) + \cot(x))$$

✓ Solution by Mathematica

Time used: 0.051 (sec). Leaf size: 29

DSolve[Sin[x]\*y'[x]+y[x]==Sin[2\*x],y[x],x,IncludeSingularSolutions -> True]

$$y(x) \to \cot\left(\frac{x}{2}\right)\left(-2\cos(x) + 4\log\left(\cos\left(\frac{x}{2}\right)\right) - 2 + c_1\right)$$

#### 1.20 problem 5(a)

Internal problem ID [2539]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 5(a).

ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [linear]

$$\sqrt{x^2 + 1} y' + y - 2x = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 34

 $dsolve(sqrt(1+x^2)*diff(y(x),x)+y(x)=2*x,y(x), singsol=all)$ 

$$y(x) = \frac{x^2 + x\sqrt{x^2 + 1} - \operatorname{arcsinh}(x) + c_1}{x + \sqrt{x^2 + 1}}$$

✓ Solution by Mathematica

Time used: 0.124 (sec). Leaf size: 39

DSolve[Sqrt[1+x^2]\*y'[x]+y[x]==2\*x,y[x],x,IncludeSingularSolutions -> True]

$$y(x) \to \left(\sqrt{x^2 + 1} - x\right) \left(-\operatorname{arcsinh}(x) + x\left(\sqrt{x^2 + 1} + x\right) + c_1\right)$$

#### 1.21 problem 5(b)

Internal problem ID [2540]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 5(b).

ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [linear]

$$\int \sqrt{x^2 + 1} \, y' - y - 2\sqrt{x^2 + 1} = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 32

 $dsolve(sqrt(1+x^2)*diff(y(x),x)-y(x)=2*sqrt(1+x^2),y(x), singsol=all)$ 

$$y(x) = (x\sqrt{x^2 + 1} + \operatorname{arcsinh}(x) - x^2 + c_1)(x + \sqrt{x^2 + 1})$$

✓ Solution by Mathematica

Time used: 0.073 (sec). Leaf size: 24

DSolve[Sqrt[1+x^2]\*y'[x]-y[x]==2\*Sqrt[1+x^2],y[x],x,IncludeSingularSolutions -> True]

$$y(x) \to x + \left(\sqrt{x^2 + 1} + x\right) \left(\operatorname{arcsinh}(x) + c_1\right)$$

#### 1.22 problem 5(c)

Internal problem ID [2541]

Book: Elementary Differential equations, Chaundy, 1969

Section: Exercises 3, page 60 Problem number: 5(c).

ODE order: 1.
ODE degree: 1.

CAS Maple gives this as type [linear]

$$\sqrt{(x+a)(x+b)}(2y'-3) + y = 0$$

✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 63

dsolve(sqrt((x+a)\*(x+b))\*(2\*diff(y(x),x)-3)+y(x)=0,y(x), singsol=all)

$$y(x) = \frac{2\left(\int \frac{3\sqrt{2a+2b+4x+4\sqrt{(x+a)(x+b)}}}{4}dx\right) + 2c_1}{\sqrt{2a+2b+4x+4\sqrt{x^2+(a+b)}x+ab}}$$

✓ Solution by Mathematica

Time used: 0.408 (sec). Leaf size: 115

 $DSolve[Sqrt[(x+a)*(x+b)]*(2*y'[x]-3)+y[x]==0,y[x],x,IncludeSingularSolutions \rightarrow True]$ 

$$y(x) \rightarrow \exp\left(-\frac{\sqrt{a+x}\sqrt{b+x}\operatorname{arctanh}\left(\frac{\sqrt{b+x}}{\sqrt{a+x}}\right)}{\sqrt{(a+x)(b+x)}}\right) \left(\int_{1}^{x} \frac{3}{2} \exp\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b+K[1]}}{\sqrt{a+K[1]}}\right)\sqrt{a+K[1]}\sqrt{b+K[1]}}{\sqrt{(a+K[1])(b+K[1])}}\right) dK[x] + c_{1}\right)$$

#### 1.23problem 5(d)

Internal problem ID [2542]

Book: Elementary Differential equations, Chaundy, 1969

**Section**: Exercises 3, page 60 Problem number: 5(d).

ODE order: 1. ODE degree: 1.

CAS Maple gives this as type [linear]

$$\sqrt{(x+a)(x+b)}y' + y - \sqrt{x+a} + \sqrt{x+b} = 0$$

Solution by Maple

Time used: 0.0 (sec). Leaf size: 103

dsolve(sqrt((x+a)\*(x+b))\*diff(y(x),x)+y(x)=sqrt(x+a)-sqrt(x+b),y(x), singsol=all)

$$y(x) = \frac{\frac{2(x+a)^{\frac{3}{2}}}{3} - \frac{2(x+b)^{\frac{3}{2}}}{3} + \frac{\sqrt{x+a}(x+b)(2x-b+3a)}{3\sqrt{(x+a)(x+b)}} - \frac{\sqrt{x+b}(x+a)(2x-a+3b)}{3\sqrt{(x+a)(x+b)}} + c_1}{\frac{a}{2} + \frac{b}{2} + x + \sqrt{x^2 + (a+b)x + ab}}$$

Solution by Mathematica

Time used: 2.353 (sec). Leaf size: 145

$$\int 2\sqrt{a+x}\sqrt{b+x} \operatorname{arctanh}\left(\frac{\sqrt{b+x}}{\sqrt{a+x}}\right) \sqrt{\left(\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b+K[1]}}{\sqrt{a+K[1]}}\right)\sqrt{a+K[1]}\sqrt{b+K[1]}}{\sqrt{(a+K[1])(b+K[1])}}\right) \left(\sqrt{a+K[1]}\right)} + \int \left(\sqrt{a+K[1]}\right) \left(\sqrt{a+K[1]}\right) \sqrt{a+K[1]} + \int \left(\sqrt{a+K[1]}\right) \left(\sqrt{a+K[1]}\right) \sqrt{a+K[1]} + \int \left(\sqrt{a+K[1]}\right) \sqrt{a+K[1]}\right) \left(\sqrt{a+K[1]}\right) \sqrt{a+K[1]} + \int \left(\sqrt{a+K[1]}\right) \sqrt{a+K[1]} + \int \left(\sqrt{a+K[1]}\right) \sqrt{a+K[1]}\right) \sqrt{a+K[1]} + \int \left(\sqrt{a+K[1]}\right) \sqrt{a+K[1]} + \int \left(\sqrt{a+K[1]}\right) \sqrt{a+K[1]}\right) \sqrt{a+K[1]} + \int \left(\sqrt{a+K[1]}\right) \sqrt{a+K[1]} + \int \left(\sqrt{a+K[1]}\right) \sqrt{a+K[1]}\right) \sqrt{a+K[1]} + \int \left(\sqrt{a+K[1]}\right) \sqrt{a+K[1]} + \int \left(\sqrt{a+K[1]}\right) \sqrt{a+K[1]} + \int \left(\sqrt{a+K[1]}\right) \sqrt{a+K[1]}\right) \sqrt{a+K[1]} + \int \left(\sqrt{a+K[1]}\right) \sqrt{a+K[1]} + \int \left(\sqrt{a+K[1]}\right) \sqrt{a+K[1]}\right) \sqrt{a+K[1]} + \int \left(\sqrt{a+K[1]}\right) \sqrt{a+K[1]} + \int \left(\sqrt{a+K[1]}$$

$$\rightarrow \exp\left(-\frac{2\sqrt{a+x}\sqrt{b+x}\mathrm{arctanh}\left(\frac{\sqrt{b+x}}{\sqrt{a+x}}\right)}{\sqrt{(a+x)(b+x)}}\right) \left(\int_{1}^{x} \frac{\exp\left(\frac{\mathrm{2arctanh}\left(\frac{\sqrt{b+K[1]}}{\sqrt{a+K[1]}}\right)\sqrt{a+K[1]}\sqrt{b+K[1]}}{\sqrt{(a+K[1])(b+K[1])}}\right) \left(\sqrt{a+K[1]}-\frac{\sqrt{a+K[1]}\sqrt{b+K[1]}}{\sqrt{a+K[1]}\sqrt{b+K[1]}}\right) \left(\sqrt{a+K[1]}-\frac{\sqrt{a+K[1]}\sqrt{$$

$$+ c_1$$