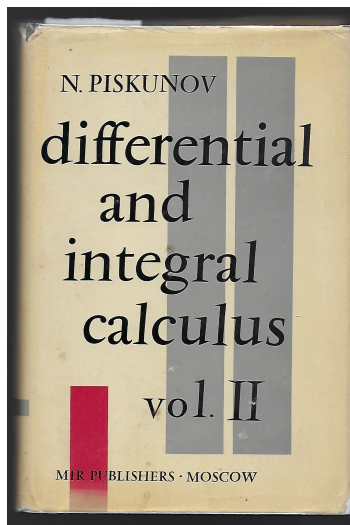


A Solution Manual For

**Differential and integral calculus,  
vol II By N. Piskunov. 1974**



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# Contents

1 Chapter 1

2

# 1 Chapter 1

1.1	problem Example, page 25 . . . . .	3
1.2	problem Example, page 27 . . . . .	4
1.3	problem Example, page 28 . . . . .	5
1.4	problem Example, page 30 . . . . .	6
1.5	problem Example, page 33 . . . . .	7
1.6	problem Example, page 36 . . . . .	8
1.7	problem Example, page 38 . . . . .	11
1.8	problem example page 46 . . . . .	12
1.9	problem example page 47 . . . . .	13
1.10	problem Example, page 49 . . . . .	14

## 1.1 problem Example, page 25

Internal problem ID [3836]

**Book:** Differential and integral calculus, vol II By N. Piskunov. 1974

**Section:** Chapter 1

**Problem number:** Example, page 25.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[[_homogeneous, 'class A'], _rational, _dAlembert]`

$$y' - \frac{xy}{x^2 - y^2} = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 19

```
dsolve(diff(y(x),x)=x*y(x)/(x^2-y(x)^2),y(x), singsol=all)
```

$$y(x) = \sqrt{-\frac{1}{\text{LambertW}(-c_1 x^2)}} x$$

### ✓ Solution by Mathematica

Time used: 8.043 (sec). Leaf size: 56

```
DSolve[y'[x]==x*y[x]/(x^2-y[x]^2),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{ix}{\sqrt{W(-e^{-2c_1 x^2})}}$$

$$y(x) \rightarrow \frac{ix}{\sqrt{W(-e^{-2c_1 x^2})}}$$

$$y(x) \rightarrow 0$$

## 1.2 problem Example, page 27

Internal problem ID [3837]

**Book:** Differential and integral calculus, vol II By N. Piskunov. 1974

**Section:** Chapter 1

**Problem number:** Example, page 27.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class C', _rational, [_Abel, '2nd type', 'cla`

$$y' - \frac{x + y - 3}{x - y - 1} = 0$$

### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 31

```
dsolve(diff(y(x),x)=(x+y(x)-3)/(x-y(x)-1),y(x), singsol=all)
```

$$y(x) = 1 - \tan \left( \text{RootOf} \left( 2\_Z + \ln \left( \frac{1}{\cos(\_Z)^2} \right) + 2 \ln(x - 2) + 2c_1 \right) \right) (x - 2)$$

### ✓ Solution by Mathematica

Time used: 0.056 (sec). Leaf size: 57

```
DSolve[y'[x]==(x+y[x]-3)/(x-y[x]-1),y[x],x,IncludeSingularSolutions -> True]
```

$$\text{Solve} \left[ 2 \arctan \left( \frac{y(x) + x - 3}{-y(x) + x - 1} \right) = \log \left( \frac{x^2 + y(x)^2 - 2y(x) - 4x + 5}{2(x - 2)^2} \right) + 2 \log(x - 2) + c_1, y(x) \right]$$

### 1.3 problem Example, page 28

Internal problem ID [3838]

**Book:** Differential and integral calculus, vol II By N. Piskunov. 1974

**Section:** Chapter 1

**Problem number:** Example, page 28.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class C', _rational, [_Abel, '2nd type', 'cla`

$$y' - \frac{2x + y - 1}{4x + 2y + 5} = 0$$

#### ✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 35

```
dsolve(diff(y(x),x)=(2*x+y(x)-1)/(4*x+2*y(x)+5),y(x), singsol=all)
```

$$y(x) = \frac{e^{-\text{LambertW}\left(\frac{18}{7} e^{\frac{25x}{7}} - \frac{25c_1}{7}\right) + \frac{18}{7} + \frac{25x}{7} - \frac{25c_1}{7}}}{5} - \frac{9}{5} - 2x$$

#### ✓ Solution by Mathematica

Time used: 3.947 (sec). Leaf size: 41

```
DSolve[y'[x]==(2*x+y[x]-1)/(4*x+2*y[x]+5),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow \frac{7}{10} W\left(-e^{\frac{25x}{7}-1+c_1}\right) - 2x - \frac{9}{5}$$

$$y(x) \rightarrow -2x - \frac{9}{5}$$

## 1.4 problem Example, page 30

Internal problem ID [3839]

**Book:** Differential and integral calculus, vol II By N. Piskunov. 1974

**Section:** Chapter 1

**Problem number:** Example, page 30.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [linear]

$$y' - \frac{2y}{x+1} - (x+1)^2 = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 13

```
dsolve(diff(y(x),x)-2*y(x)/(1+x)=(x+1)^2,y(x), singsol=all)
```

$$y(x) = (x + c_1)(x + 1)^2$$

### ✓ Solution by Mathematica

Time used: 0.031 (sec). Leaf size: 15

```
DSolve[y'[x]-2*y[x]/(1+x)==(x+1)^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow (x + 1)^2(x + c_1)$$

## 1.5 problem Example, page 33

Internal problem ID [3840]

**Book:** Differential and integral calculus, vol II By N. Piskunov. 1974

**Section:** Chapter 1

**Problem number:** Example, page 33.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type [\_Bernoulli]

$$y' + xy - x^3y^3 = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 35

```
dsolve(diff(y(x),x)+x*y(x)=x^3*y(x)^3,y(x), singsol=all)
```

$$y(x) = \frac{1}{\sqrt{e^{x^2}c_1 + x^2 + 1}}$$

$$y(x) = -\frac{1}{\sqrt{e^{x^2}c_1 + x^2 + 1}}$$

### ✓ Solution by Mathematica

Time used: 7.04 (sec). Leaf size: 50

```
DSolve[y'[x]+x*y[x]==x^3*y[x]^3,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{1}{\sqrt{x^2 + c_1e^{x^2} + 1}}$$

$$y(x) \rightarrow \frac{1}{\sqrt{x^2 + c_1e^{x^2} + 1}}$$

$$y(x) \rightarrow 0$$



## 1.6 problem Example, page 36

Internal problem ID [3841]

**Book:** Differential and integral calculus, vol II By N. Piskunov. 1974

**Section:** Chapter 1

**Problem number:** Example, page 36.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class A', _exact, _rational, _dAlembert]`

$$\frac{2x}{y^3} + \frac{(y^2 - 3x^2)y'}{y^4} = 0$$

✓ Solution by Maple

Time used: 0.016 (sec). Leaf size: 402

`dsolve(2*x/y(x)^3+ (y(x)^2-3*x^2)/(y(x)^4)*diff(y(x),x)=0,y(x), singsol=all)`

$$\begin{aligned}
 y(x) &= \frac{\left(12\sqrt{3}x\sqrt{27c_1^2x^2-4c_1-108c_1^2x^2+8}\right)^{\frac{1}{3}}}{6c_1} \\
 &+ \frac{2}{3c_1\left(12\sqrt{3}x\sqrt{27c_1^2x^2-4c_1-108c_1^2x^2+8}\right)^{\frac{1}{3}}} + \frac{1}{3c_1} \\
 y(x) &= -\frac{\left(12\sqrt{3}x\sqrt{27c_1^2x^2-4c_1-108c_1^2x^2+8}\right)^{\frac{1}{3}}}{12c_1} \\
 &- \frac{1}{3c_1\left(12\sqrt{3}x\sqrt{27c_1^2x^2-4c_1-108c_1^2x^2+8}\right)^{\frac{1}{3}}} + \frac{1}{3c_1} \\
 &- \frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}x\sqrt{27c_1^2x^2-4c_1-108c_1^2x^2+8}\right)^{\frac{1}{3}}}{6c_1} - \frac{2}{3c_1\left(12\sqrt{3}x\sqrt{27c_1^2x^2-4c_1-108c_1^2x^2+8}\right)^{\frac{1}{3}}}\right)}{2} \\
 y(x) &= -\frac{\left(12\sqrt{3}x\sqrt{27c_1^2x^2-4c_1-108c_1^2x^2+8}\right)^{\frac{1}{3}}}{12c_1} \\
 &- \frac{1}{3c_1\left(12\sqrt{3}x\sqrt{27c_1^2x^2-4c_1-108c_1^2x^2+8}\right)^{\frac{1}{3}}} + \frac{1}{3c_1} \\
 &+ \frac{i\sqrt{3}\left(\frac{\left(12\sqrt{3}x\sqrt{27c_1^2x^2-4c_1-108c_1^2x^2+8}\right)^{\frac{1}{3}}}{6c_1} - \frac{2}{3c_1\left(12\sqrt{3}x\sqrt{27c_1^2x^2-4c_1-108c_1^2x^2+8}\right)^{\frac{1}{3}}}\right)}{2}
 \end{aligned}$$

✓ Solution by Mathematica

Time used: 60.209 (sec). Leaf size: 458

`DSolve[2*x/y[x]^3+(y[x]^2-3*x^2)/(y[x]^4)*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]`

$$y(x) \rightarrow \frac{1}{3} \left( \frac{\sqrt[3]{27e^{c_1}x^2 + 3\sqrt{81e^{2c_1}x^4 - 12e^{4c_1}x^2} - 2e^{3c_1}}}{\sqrt[3]{2}} + \frac{\sqrt[3]{2}e^{2c_1}}{\sqrt[3]{27e^{c_1}x^2 + 3\sqrt{81e^{2c_1}x^4 - 12e^{4c_1}x^2} - 2e^{3c_1}}} - e^{c_1} \right)$$

$$y(x) \rightarrow \frac{i(\sqrt{3} + i) \sqrt[3]{27e^{c_1}x^2 + 3\sqrt{81e^{2c_1}x^4 - 12e^{4c_1}x^2} - 2e^{3c_1}}}{6\sqrt[3]{2}} - \frac{i(\sqrt{3} - i) e^{2c_1}}{3 \cdot 2^{2/3} \sqrt[3]{27e^{c_1}x^2 + 3\sqrt{81e^{2c_1}x^4 - 12e^{4c_1}x^2} - 2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

$$y(x) \rightarrow -\frac{i(\sqrt{3} - i) \sqrt[3]{27e^{c_1}x^2 + 3\sqrt{81e^{2c_1}x^4 - 12e^{4c_1}x^2} - 2e^{3c_1}}}{6\sqrt[3]{2}} + \frac{i(\sqrt{3} + i) e^{2c_1}}{3 \cdot 2^{2/3} \sqrt[3]{27e^{c_1}x^2 + 3\sqrt{81e^{2c_1}x^4 - 12e^{4c_1}x^2} - 2e^{3c_1}}} - \frac{e^{c_1}}{3}$$

## 1.7 problem Example, page 38

Internal problem ID [3842]

**Book:** Differential and integral calculus, vol II By N. Piskunov. 1974

**Section:** Chapter 1

**Problem number:** Example, page 38.

**ODE order:** 1.

**ODE degree:** 1.

CAS Maple gives this as type `[_homogeneous, 'class D', _rational, _Bernoulli]`

$$y + xy^2 - y'x = 0$$

### ✓ Solution by Maple

Time used: 0.0 (sec). Leaf size: 18

```
dsolve((y(x)+x*y(x)^2)-x*diff(y(x),x)=0,y(x), singsol=all)
```

$$y(x) = \frac{2x}{-x^2 + 2c_1}$$

### ✓ Solution by Mathematica

Time used: 0.152 (sec). Leaf size: 23

```
DSolve[(y[x]+x*y[x]^2)-x*y'[x]==0,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\frac{2x}{x^2 - 2c_1}$$

$$y(x) \rightarrow 0$$

## 1.8 problem example page 46

Internal problem ID [3843]

**Book:** Differential and integral calculus, vol II By N. Piskunov. 1974

**Section:** Chapter 1

**Problem number:** example page 46.

**ODE order:** 1.

**ODE degree:** 2.

CAS Maple gives this as type [\_quadrature]

$$y^2(1 + y'^2) - R^2 = 0$$

### ✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 59

```
dsolve(y(x)^2*(1+diff(y(x),x)^2)=R^2,y(x), singsol=all)
```

$$y(x) = -R$$

$$y(x) = R$$

$$y(x) = \sqrt{R^2 - c_1^2 + 2c_1x - x^2}$$

$$y(x) = -\sqrt{R^2 - c_1^2 + 2c_1x - x^2}$$

### ✓ Solution by Mathematica

Time used: 0.221 (sec). Leaf size: 101

```
DSolve[y[x]^2*(1+(y'[x])^2)=R^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow -\sqrt{R^2 - (x + c_1)^2}$$

$$y(x) \rightarrow \sqrt{R^2 - (x + c_1)^2}$$

$$y(x) \rightarrow -\sqrt{R^2 - (x - c_1)^2}$$

$$y(x) \rightarrow \sqrt{R^2 - (x - c_1)^2}$$

$$y(x) \rightarrow -R$$

$$y(x) \rightarrow R$$

## 1.9 problem example page 47

Internal problem ID [3844]

**Book:** Differential and integral calculus, vol II By N. Piskunov. 1974

**Section:** Chapter 1

**Problem number:** example page 47.

**ODE order:** 1.

**ODE degree:** 4.

CAS Maple gives this as type [Clairaut]

$$y - y'x - \frac{ay'}{\sqrt{1+y'^2}} = 0$$

### ✓ Solution by Maple

Time used: 1.5 (sec). Leaf size: 18

```
dsolve(y(x)=x*diff(y(x),x)+ a*diff(y(x),x)/(sqrt(1+diff(y(x),x)^2)),y(x), singsol=all)
```

$$y(x) = c_1x + \frac{ac_1}{\sqrt{c_1^2 + 1}}$$

### ✓ Solution by Mathematica

Time used: 35.248 (sec). Leaf size: 27

```
DSolve[y[x]==x*y'[x]+ a*y'[x]/(Sqrt[1+(y'[x])^2]),y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow c_1 \left( x + \frac{a}{\sqrt{1+c_1^2}} \right)$$

$$y(x) \rightarrow 0$$

## 1.10 problem Example, page 49

Internal problem ID [3845]

**Book:** Differential and integral calculus, vol II By N. Piskunov. 1974

**Section:** Chapter 1

**Problem number:** Example, page 49.

**ODE order:** 1.

**ODE degree:** 2.

CAS Maple gives this as type `[[_homogeneous, 'class C'], _rational, _dAlembert]`

$$y - xy'^2 - y'^2 = 0$$

### ✓ Solution by Maple

Time used: 0.047 (sec). Leaf size: 99

```
dsolve(y(x)=x*diff(y(x),x)^2+diff(y(x),x)^2,y(x), singsol=all)
```

$$y(x) = 0$$

$$y(x) = \frac{(x+1 + \sqrt{c_1x + c_1 + x + 1})^2 x}{(x+1)^2} + \frac{(x+1 + \sqrt{c_1x + c_1 + x + 1})^2}{(x+1)^2}$$

$$y(x) = \frac{(-x-1 + \sqrt{c_1x + c_1 + x + 1})^2 x}{(x+1)^2} + \frac{(-x-1 + \sqrt{c_1x + c_1 + x + 1})^2}{(x+1)^2}$$

### ✓ Solution by Mathematica

Time used: 0.062 (sec). Leaf size: 57

```
DSolve[y[x]==x*(y'[x])^2+(y'[x])^2,y[x],x,IncludeSingularSolutions -> True]
```

$$y(x) \rightarrow x - c_1\sqrt{x+1} + 1 + \frac{c_1^2}{4}$$

$$y(x) \rightarrow x + c_1\sqrt{x+1} + 1 + \frac{c_1^2}{4}$$

$$y(x) \rightarrow 0$$