Computer algebra independent integration tests

Summer 2022 edition

8-Special-functions/208-8.8-Polylogarithm-function

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [198]. This is test number [208].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (198)	0.00 (0)
Mathematica	98.99 (196)	1.01 (2)
Maple	74.75 (148)	25.25 (50)
Maxima	64.14 (127)	35.86 (71)
Fricas	52.53 (104)	47.47 (94)
Mupad	35.86 (71)	64.14 (127)
Sympy	24.24 (48)	75.76 (150)
Giac	8.08 (16)	91.92 (182)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

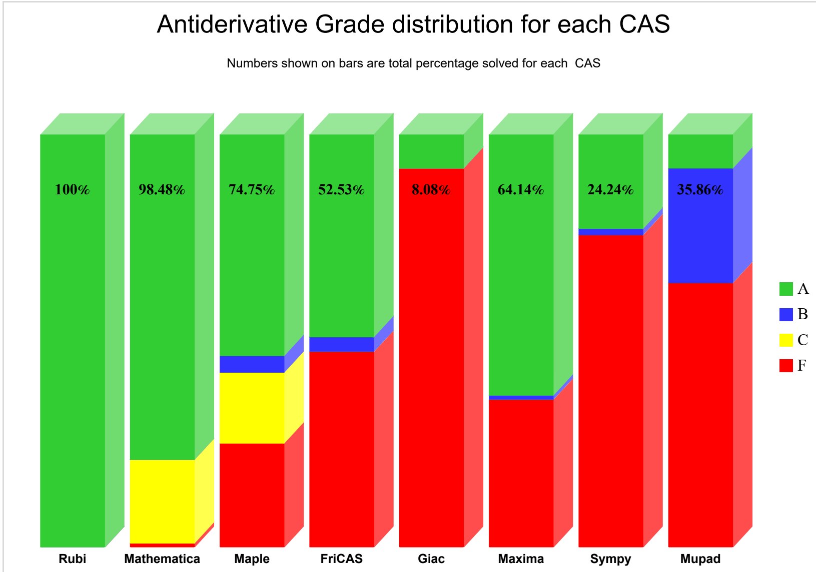
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

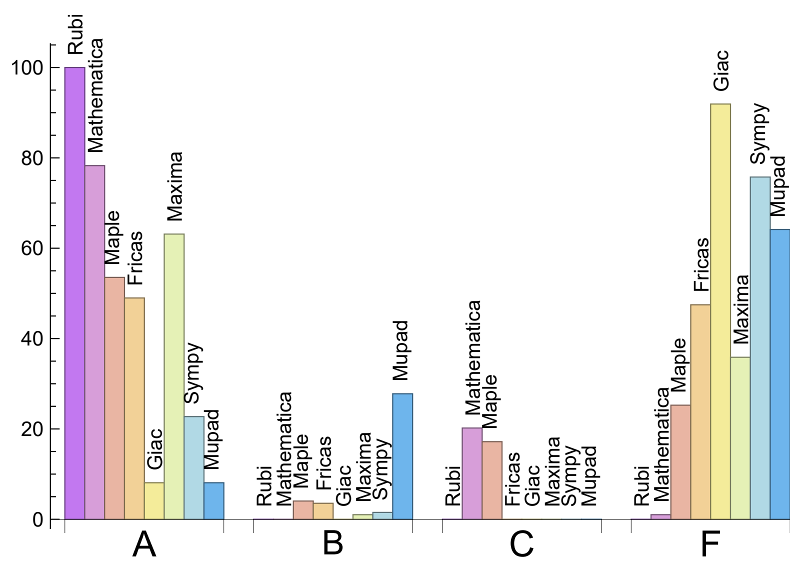
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	78.28	0.00	20.20	1.01
Maxima	63.13	1.01	0.00	35.86
Maple	53.54	4.04	17.17	25.25
Fricas	48.99	3.54	0.00	47.47
Sympy	22.73	1.52	0.00	75.76
Mupad	N/A	27.78	0.00	64.14
Giac	8.08	0.00	0.00	91.92

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	2	100.00 %	0.00 %	0.00 %
Maple	50	100.00 %	0.00 %	0.00 %
Fricas	94	100.00 %	0.00 %	0.00 %
Giac	182	100.00 %	0.00 %	0.00 %
Maxima	71	100.00 %	0.00 %	0.00 %
Sympy	150	82.00 %	17.33 %	0.67 %
Mupad	127	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

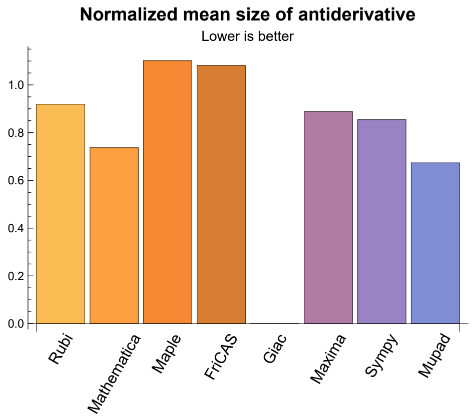
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

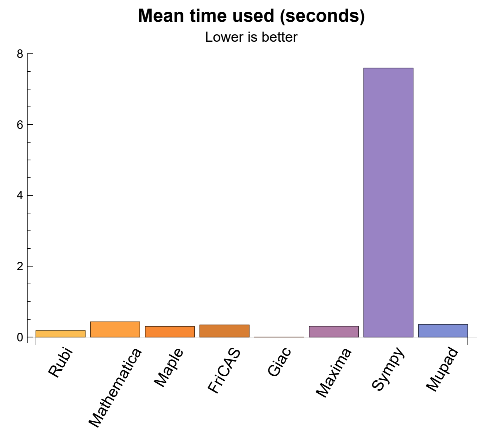
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.18	216.27	0.92	91.00	1.00
Mathematica	0.43	177.57	0.74	64.50	0.82
Maple	0.30	111.78	1.10	103.00	1.10
Maxima	0.31	131.57	0.89	78.00	0.94
Fricas	0.34	102.09	1.08	70.50	0.93
Sympy	7.60	87.44	0.85	41.50	0.75
Giac	0.00	0.00	0.00	0.00	0.00
Mupad	0.36	37.83	0.67	46.00	0.83

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0.0
0.2
0.4
0.6
0.8
1.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
2
4
6
8
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{96, 97, 98, 99, 100, 114, 115, 117, 118, 119, 120, 122, 123, 134, 160, 180\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{17, 18, 37, 38\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

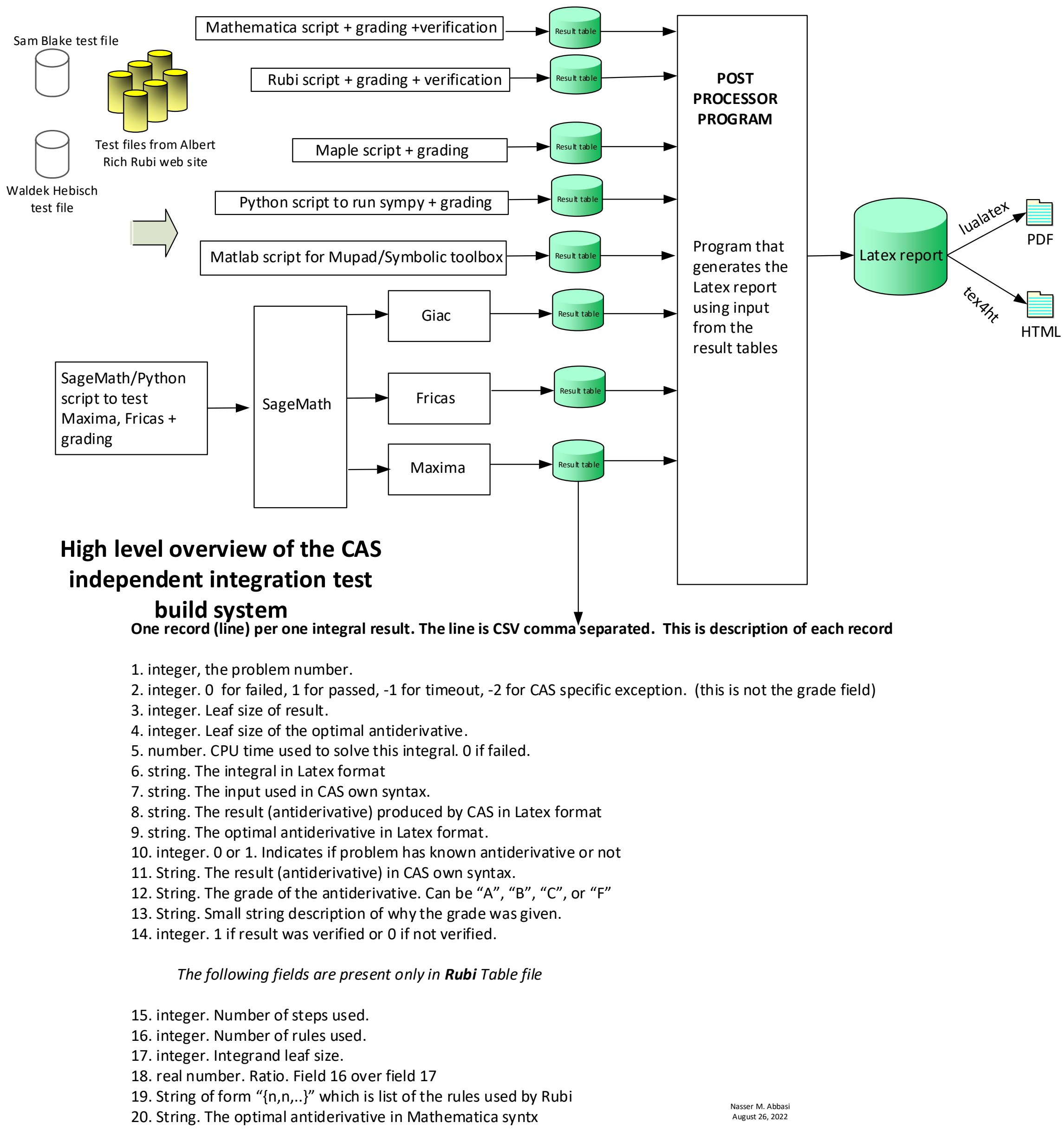
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 55, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 86, 87, 96, 97, 98, 99, 100, 102, 105, 108, 111, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 198 }

B grade: { }

C grade: { 17, 18, 30, 31, 37, 38, 52, 53, 54, 56, 57, 58, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 103, 104, 106, 107, 109, 110, 112, 113 }

F grade: { 101, 196 }

2.1.3 Maple

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 48, 55, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 96, 97, 98, 99, 100, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 134, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 153, 154, 155, 159, 160, 165, 173, 180 }

B grade: { 1, 39, 40, 41, 42, 43, 44, 137 }

C grade: { 45, 46, 47, 49, 50, 51, 52, 53, 54, 56, 57, 58, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F grade: { 101, 127, 131, 132, 133, 135, 136, 141, 149, 150, 151, 152, 156, 157, 158, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 96, 97, 98, 99, 100, 114, 115, 117, 118, 119, 120, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 137, 138, 139, 140, 142, 143, 145, 146, 147, 148, 153, 154, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 180, 184, 185, 186, 189, 190, 191, 192, 193, 197, 198 }

B grade: { 144, 155 }

C grade: { }

F grade: { 22, 35, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 121, 127, 135, 136, 141, 149, 150, 151, 152, 156, 157, 158, 159, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 187, 188, 194, 195, 196 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 48, 55, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 79, 80, 81, 82, 96, 97, 98, 99, 100, 114, 115, 117, 118, 119, 120, 122, 123, 124, 125, 126, 131, 132, 133, 134, 137, 138, 139, 140, 145, 146, 150, 151, 152, 153, 154, 155, 160, 165, 180 }

B grade: { 75, 76, 77, 78, 83, 84, 85 }

C grade: { }

F grade: { 6, 15, 22, 35, 45, 46, 47, 49, 50, 51, 52, 53, 54, 56, 57, 58, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 121, 127, 128, 129, 130, 135, 136, 141, 142, 143, 144, 147, 148, 149, 156, 157, 158, 159, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 19, 20, 21, 23, 24, 25, 26, 27, 28, 60, 96, 97, 98, 99, 100, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 134, 137, 138, 139, 140, 160, 180 }

B grade: { 29, 30, 31 }

C grade: { }

F grade: { 11, 12, 13, 14, 16, 17, 18, 22, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 121, 127, 128, 129, 130, 131, 132, 133, 135, 136, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

2.1.7 Giac

A grade: { 96, 97, 98, 99, 100, 114, 115, 117, 118, 119, 120, 122, 123, 134, 160, 180 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 121, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

2.1.8 Mupad

A grade: { 96, 97, 98, 99, 100, 114, 115, 117, 118, 119, 120, 122, 123, 134, 160, 180 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 101, 116, 126, 133, 140, 145, 146, 153, 154, 155, 165, 173 }

C grade: { }

F grade: { 15, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 121, 124, 125, 127, 128, 129, 130, 131, 132, 135, 136, 137, 138, 139, 141, 142, 143, 144, 147, 148, 149, 150, 151, 152, 156, 157, 158, 159, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	73	147	72	72	66	0	69
N.S.	1	1.00	0.85	1.71	0.84	0.84	0.77	0.00	0.80
time (sec)	N/A	0.036	0.032	0.356	0.270	0.370	3.188	0.000	0.309

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	65	120	64	64	58	0	61
N.S.	1	1.00	0.86	1.58	0.84	0.84	0.76	0.00	0.80
time (sec)	N/A	0.031	0.025	0.358	0.262	0.351	1.732	0.000	0.246

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	57	93	56	56	49	0	53
N.S.	1	1.00	0.86	1.41	0.85	0.85	0.74	0.00	0.80
time (sec)	N/A	0.028	0.021	0.375	0.282	0.367	0.892	0.000	0.226

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	48	66	48	48	41	0	46
N.S.	1	1.00	0.86	1.18	0.86	0.86	0.73	0.00	0.82
time (sec)	N/A	0.020	0.021	0.351	0.257	0.360	0.484	0.000	0.341

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	34	29	29	22	0	32
N.S.	1	1.00	0.90	1.17	1.00	1.00	0.76	0.00	1.10
time (sec)	N/A	0.006	0.011	0.288	0.258	0.352	0.251	0.000	0.257

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	0	3	0	5
N.S.	1	1.00	1.00	1.20	1.00	0.00	0.60	0.00	1.00
time (sec)	N/A	0.006	0.002	0.087	0.276	0.000	0.401	0.000	0.175

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	42	28	34	24	0	34
N.S.	1	1.00	1.00	1.17	0.78	0.94	0.67	0.00	0.94
time (sec)	N/A	0.015	0.010	0.386	0.267	0.349	0.365	0.000	0.202

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	50	61	40	47	42	0	51
N.S.	1	1.00	0.86	1.05	0.69	0.81	0.72	0.00	0.88
time (sec)	N/A	0.023	0.017	0.441	0.265	0.360	0.672	0.000	0.298

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	52	76	49	56	51	0	57
N.S.	1	1.00	0.76	1.12	0.72	0.82	0.75	0.00	0.84
time (sec)	N/A	0.026	0.023	0.453	0.269	0.385	1.229	0.000	0.319

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	60	95	58	65	60	0	60
N.S.	1	1.00	0.77	1.22	0.74	0.83	0.77	0.00	0.77
time (sec)	N/A	0.028	0.026	0.442	0.258	0.366	2.285	0.000	0.443

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	86	78	77	77	0	0	71
N.S.	1	1.00	0.98	0.89	0.88	0.88	0.00	0.00	0.81
time (sec)	N/A	0.039	0.009	0.129	0.255	0.358	0.000	0.000	0.819

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	69	69	69	0	0	63
N.S.	1	1.00	1.00	0.88	0.88	0.88	0.00	0.00	0.81
time (sec)	N/A	0.034	0.008	0.129	0.260	0.350	0.000	0.000	0.949

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	69	62	61	61	0	0	55
N.S.	1	1.00	1.01	0.91	0.90	0.90	0.00	0.00	0.81
time (sec)	N/A	0.024	0.007	0.139	0.260	0.385	0.000	0.000	0.904

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	39	41	39	39	0	0	37
N.S.	1	1.00	1.15	1.21	1.15	1.15	0.00	0.00	1.09
time (sec)	N/A	0.007	0.009	0.089	0.271	0.333	0.000	0.000	0.844

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	0	3	0	-1
N.S.	1	1.00	1.00	1.20	1.00	0.00	0.60	0.00	-0.20
time (sec)	N/A	0.006	0.002	0.128	0.252	0.000	0.194	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	57	33	39	0	0	36
N.S.	1	1.00	0.96	1.24	0.72	0.85	0.00	0.00	0.78
time (sec)	N/A	0.021	0.025	0.118	0.262	0.395	0.000	0.000	0.886

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	25	90	47	54	0	0	46
N.S.	1	1.00	0.36	1.29	0.67	0.77	0.00	0.00	0.66
time (sec)	N/A	0.030	0.008	0.208	0.254	0.364	0.000	0.000	1.273

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	25	106	56	63	0	0	62
N.S.	1	1.00	0.31	1.32	0.70	0.79	0.00	0.00	0.78
time (sec)	N/A	0.031	0.009	0.189	0.264	0.361	0.000	0.000	1.502

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	65	68	62	62	56	0	61
N.S.	1	1.00	0.88	0.92	0.84	0.84	0.76	0.00	0.82
time (sec)	N/A	0.042	0.018	0.056	0.261	0.366	4.648	0.000	0.194

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	60	54	54	48	0	53
N.S.	1	1.00	0.88	0.94	0.84	0.84	0.75	0.00	0.83
time (sec)	N/A	0.034	0.015	0.046	0.262	0.349	1.788	0.000	0.270

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	45	40	40	39	0	45
N.S.	1	1.00	0.93	0.98	0.87	0.87	0.85	0.00	0.98
time (sec)	N/A	0.018	0.008	0.294	0.250	0.349	0.675	0.000	0.206

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	0	0	0	0	9
N.S.	1	1.00	1.00	0.91	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.006	0.002	0.815	0.000	0.000	0.000	0.000	0.173

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	43	34	44	37	0	44
N.S.	1	1.00	1.00	0.88	0.69	0.90	0.76	0.00	0.90
time (sec)	N/A	0.027	0.012	0.077	0.265	0.359	0.805	0.000	0.210

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	51	52	46	55	49	0	53
N.S.	1	1.00	0.80	0.81	0.72	0.86	0.77	0.00	0.83
time (sec)	N/A	0.035	0.023	0.104	0.266	0.374	2.154	0.000	0.264

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	60	62	55	64	58	0	61
N.S.	1	1.00	0.81	0.84	0.74	0.86	0.78	0.00	0.82
time (sec)	N/A	0.037	0.027	0.115	0.255	0.419	5.199	0.000	0.278

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	65	63	80	159	94	0	60
N.S.	1	1.00	0.89	0.86	1.10	2.18	1.29	0.00	0.82
time (sec)	N/A	0.033	0.050	0.078	0.467	0.592	48.478	0.000	0.413

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	57	55	68	143	83	0	52
N.S.	1	1.00	0.90	0.87	1.08	2.27	1.32	0.00	0.83
time (sec)	N/A	0.026	0.040	0.070	0.466	0.535	13.127	0.000	0.280

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	39	43	49	107	60	0	39
N.S.	1	1.00	0.98	1.08	1.22	2.68	1.50	0.00	0.98
time (sec)	N/A	0.013	0.021	0.068	0.464	0.478	3.711	0.000	0.239

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	41	39	49	94	184	0	38
N.S.	1	1.00	0.98	0.93	1.17	2.24	4.38	0.00	0.90
time (sec)	N/A	0.019	0.014	0.069	0.457	0.605	12.264	0.000	0.264

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	47	47	57	114	275	0	47
N.S.	1	1.00	0.84	0.84	1.02	2.04	4.91	0.00	0.84
time (sec)	N/A	0.022	0.012	0.072	0.469	0.542	43.375	0.000	0.326

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	47	53	65	132	299	0	58
N.S.	1	1.00	0.71	0.80	0.98	2.00	4.53	0.00	0.88
time (sec)	N/A	0.025	0.013	0.076	0.460	0.516	140.026	0.000	0.336

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	80	77	77	0	0	73
N.S.	1	1.00	1.00	0.91	0.88	0.88	0.00	0.00	0.83
time (sec)	N/A	0.051	0.015	0.040	0.269	0.431	0.000	0.000	0.338

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	79	72	69	69	0	0	65
N.S.	1	1.00	1.01	0.92	0.88	0.88	0.00	0.00	0.83
time (sec)	N/A	0.045	0.013	0.043	0.254	0.452	0.000	0.000	0.305

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	56	53	53	0	0	57
N.S.	1	1.00	0.87	0.93	0.88	0.88	0.00	0.00	0.95
time (sec)	N/A	0.020	0.010	0.060	0.267	0.463	0.000	0.000	0.378

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	0	0	0	0	9
N.S.	1	1.00	1.00	0.91	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.006	0.003	0.037	0.000	0.000	0.000	0.000	0.202

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	68	41	51	0	0	54
N.S.	1	1.00	0.95	1.08	0.65	0.81	0.00	0.00	0.86
time (sec)	N/A	0.033	0.021	0.068	0.264	0.374	0.000	0.000	0.294

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	30	98	55	64	0	0	65
N.S.	1	1.00	0.38	1.26	0.71	0.82	0.00	0.00	0.83
time (sec)	N/A	0.042	0.010	0.102	0.253	0.359	0.000	0.000	0.728

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	30	115	64	73	0	0	73
N.S.	1	1.00	0.34	1.31	0.73	0.83	0.00	0.00	0.83
time (sec)	N/A	0.046	0.012	0.102	0.268	0.459	0.000	0.000	1.025

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	77	144	95	189	0	0	72
N.S.	1	1.00	0.89	1.66	1.09	2.17	0.00	0.00	0.83
time (sec)	N/A	0.036	0.112	0.114	0.477	0.687	0.000	0.000	0.552

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	69	136	81	173	0	0	64
N.S.	1	1.00	0.90	1.77	1.05	2.25	0.00	0.00	0.83
time (sec)	N/A	0.033	0.103	0.130	0.459	0.450	0.000	0.000	0.487

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	119	59	133	0	0	49
N.S.	1	1.00	1.00	2.38	1.18	2.66	0.00	0.00	0.98
time (sec)	N/A	0.016	0.064	0.115	0.480	0.466	0.000	0.000	0.364

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	112	58	112	0	0	53
N.S.	1	1.00	0.93	2.07	1.07	2.07	0.00	0.00	0.98
time (sec)	N/A	0.026	0.059	0.116	0.467	0.516	0.000	0.000	0.554

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	61	125	66	132	0	0	59
N.S.	1	1.00	0.87	1.79	0.94	1.89	0.00	0.00	0.84
time (sec)	N/A	0.028	0.063	0.118	0.467	0.365	0.000	0.000	0.771

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	69	138	74	150	0	0	70
N.S.	1	1.00	0.86	1.72	0.92	1.88	0.00	0.00	0.88
time (sec)	N/A	0.032	0.071	0.128	0.473	0.446	0.000	0.000	1.034

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	69	108	0	0	0	0	-1
N.S.	1	1.00	0.97	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.032	0.148	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	69	108	0	0	0	0	-1
N.S.	1	1.00	0.97	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.027	0.112	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	51	88	0	0	0	0	-1
N.S.	1	1.00	0.94	1.63	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.034	0.108	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	11	0	0	-1
N.S.	1	1.00	1.00	1.09	0.00	1.00	0.00	0.00	-0.09
time (sec)	N/A	0.007	0.003	0.234	0.000	0.404	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	60	106	0	0	0	0	-1
N.S.	1	1.00	0.87	1.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.039	0.122	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	61	108	0	0	0	0	-1
N.S.	1	1.00	0.78	1.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.040	0.110	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	61	108	0	0	0	0	-1
N.S.	1	1.00	0.80	1.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.041	0.116	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	41	132	0	0	0	0	-1
N.S.	1	1.00	0.47	1.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.009	0.200	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	41	132	0	0	0	0	-1
N.S.	1	1.00	0.47	1.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.007	0.199	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	39	105	0	0	0	0	-1
N.S.	1	1.00	0.57	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.006	0.194	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	11	0	0	-1
N.S.	1	1.00	1.00	1.09	0.00	1.00	0.00	0.00	-0.09
time (sec)	N/A	0.007	0.003	0.231	0.000	0.374	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	37	129	0	0	0	0	-1
N.S.	1	1.00	0.44	1.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.008	0.207	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	41	132	0	0	0	0	-1
N.S.	1	1.00	0.43	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.008	0.206	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	41	132	0	0	0	0	-1
N.S.	1	1.00	0.44	1.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.009	0.210	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	90	101	128	190	0	0	-1
N.S.	1	1.00	0.77	0.86	1.09	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.072	0.391	0.554	0.389	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	75	88	109	143	110	0	-1
N.S.	1	1.00	0.74	0.86	1.07	1.40	1.08	0.00	-0.01
time (sec)	N/A	0.035	0.056	0.368	0.467	0.395	30.884	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	63	74	83	135	0	0	-1
N.S.	1	1.00	0.79	0.92	1.04	1.69	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.058	0.421	0.465	0.410	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	51	59	71	132	0	0	-1
N.S.	1	1.00	0.75	0.87	1.04	1.94	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.052	0.377	0.466	0.365	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	57	75	89	150	0	0	-1
N.S.	1	1.00	0.64	0.84	1.00	1.69	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.055	0.353	0.466	0.399	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	65	88	108	170	0	0	-1
N.S.	1	1.00	0.61	0.83	1.02	1.60	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.064	0.374	0.480	0.390	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	98	149	156	279	0	0	-1
N.S.	1	1.00	0.64	0.97	1.02	1.82	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.178	0.143	0.481	0.392	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	88	141	143	229	0	0	-1
N.S.	1	1.00	0.65	1.04	1.05	1.68	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.150	0.070	0.484	0.392	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	73	133	122	173	0	0	-1
N.S.	1	1.00	0.60	1.10	1.01	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.146	0.066	0.493	0.381	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	57	127	94	161	0	0	-1
N.S.	1	1.00	0.59	1.31	0.97	1.66	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.104	0.081	0.486	0.381	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	58	111	78	156	0	0	-1
N.S.	1	1.00	0.68	1.31	0.92	1.84	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.078	0.066	0.470	0.392	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	64	122	97	175	0	0	-1
N.S.	1	1.00	0.59	1.13	0.90	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.070	0.069	0.468	0.401	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	72	135	118	195	0	0	-1
N.S.	1	1.00	0.58	1.08	0.94	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.098	0.086	0.483	0.426	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	101	145	160	194	0	0	-1
N.S.	1	1.00	0.72	1.04	1.14	1.39	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.073	0.424	0.472	0.425	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	91	134	139	172	0	0	-1
N.S.	1	1.00	0.73	1.07	1.11	1.38	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.049	0.391	0.472	0.389	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	57	128	128	156	0	0	-1
N.S.	1	1.00	0.50	1.11	1.11	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.055	0.411	0.482	0.396	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	62	114	123	170	0	0	-1
N.S.	1	1.00	0.60	1.11	1.19	1.65	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.051	0.413	0.468	0.404	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	62	116	125	196	0	0	-1
N.S.	1	1.00	0.56	1.05	1.13	1.77	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.049	0.389	0.469	0.453	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	70	131	151	212	0	0	-1
N.S.	1	1.00	0.56	1.04	1.20	1.68	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.055	0.412	0.480	0.437	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	89	155	178	237	0	0	-1
N.S.	1	1.00	0.55	0.96	1.11	1.47	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.075	0.158	0.475	0.399	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	89	155	175	213	0	0	-1
N.S.	1	1.00	0.55	0.96	1.09	1.32	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.074	0.141	0.474	0.407	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	68	147	153	187	0	0	-1
N.S.	1	1.00	0.47	1.01	1.05	1.28	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.069	0.142	0.469	0.392	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	68	147	141	169	0	0	-1
N.S.	1	1.00	0.51	1.10	1.05	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.066	0.160	0.476	0.384	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	71	131	132	184	0	0	-1
N.S.	1	1.00	0.58	1.07	1.08	1.51	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.063	0.141	0.473	0.437	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	71	131	134	211	0	0	-1
N.S.	1	1.00	0.54	0.99	1.02	1.60	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.076	0.144	0.475	0.423	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	79	142	163	226	0	0	-1
N.S.	1	1.00	0.54	0.97	1.11	1.54	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.068	0.151	0.474	0.400	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	84	142	168	223	0	0	-1
N.S.	1	1.00	0.57	0.97	1.14	1.52	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.074	0.157	0.482	0.405	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	82	121	0	0	0	0	-1
N.S.	1	1.00	0.81	1.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.088	0.492	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	82	121	0	0	0	0	-1
N.S.	1	1.00	0.82	1.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.078	0.522	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	48	109	0	0	0	0	-1
N.S.	1	1.00	0.52	1.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.016	0.517	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	48	121	0	0	0	0	-1
N.S.	1	1.00	0.49	1.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.020	0.520	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	48	121	0	0	0	0	-1
N.S.	1	1.00	0.46	1.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.020	0.510	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	50	145	0	0	0	0	-1
N.S.	1	1.00	0.40	1.16	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.023	0.250	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	50	145	0	0	0	0	-1
N.S.	1	1.00	0.40	1.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.020	0.254	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	50	133	0	0	0	0	-1
N.S.	1	1.00	0.43	1.16	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.015	0.257	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	50	145	0	0	0	0	-1
N.S.	1	1.00	0.42	1.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.019	0.244	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	50	145	0	0	0	0	-1
N.S.	1	1.00	0.39	1.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.020	0.242	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.005	0.006	0.014	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.004	0.006	0.014	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.10
time (sec)	N/A	0.002	0.005	0.012	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.004	0.006	0.015	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.005	0.006	0.016	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	0	0	0	0	0	0	7
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.78
time (sec)	N/A	0.005	0.006	0.020	0.000	0.000	0.000	0.000	0.370

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	53	144	0	0	0	0	-1
N.S.	1	1.00	0.68	1.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.032	0.115	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	88	173	0	0	0	0	-1
N.S.	1	1.00	0.86	1.70	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.043	0.221	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	119	198	0	0	0	0	-1
N.S.	1	1.00	0.98	1.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.055	0.449	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	72	177	0	0	0	0	-1
N.S.	1	1.00	0.77	1.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.037	0.125	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	126	218	0	0	0	0	-1
N.S.	1	1.00	1.07	1.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.058	0.226	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	166	259	0	0	0	0	-1
N.S.	1	1.00	1.17	1.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.074	0.479	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	72	177	0	0	0	0	-1
N.S.	1	1.00	0.77	1.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.038	0.165	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	126	218	0	0	0	0	-1
N.S.	1	1.00	1.07	1.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.057	0.240	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	166	259	0	0	0	0	-1
N.S.	1	1.00	1.17	1.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.066	0.492	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	80	148	0	0	0	0	-1
N.S.	1	1.00	0.79	1.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.046	0.127	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	50	180	0	0	0	0	-1
N.S.	1	1.00	0.38	1.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.017	0.276	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	52	217	0	0	0	0	-1
N.S.	1	1.00	0.34	1.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.017	0.994	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.10
time (sec)	N/A	0.004	0.020	0.027	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	8	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.12
time (sec)	N/A	0.002	0.002	0.028	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	0	0	5	0	7
N.S.	1	1.00	1.00	1.14	0.00	0.00	0.71	0.00	1.00
time (sec)	N/A	0.006	0.002	0.057	0.000	0.000	0.204	0.000	0.539

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.006	0.017	0.026	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.006	0.016	0.026	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.004	0.019	0.028	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.10
time (sec)	N/A	0.002	0.005	0.027	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	0	0	0	0	-1
N.S.	1	1.00	1.00	1.08	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.007	0.003	0.220	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.006	0.017	0.029	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.006	0.016	0.026	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	144	361	200	165	235	0	-1
N.S.	1	1.00	0.55	1.39	0.77	0.63	0.90	0.00	-0.00
time (sec)	N/A	0.217	0.140	0.582	0.265	0.350	6.862	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	96	166	145	110	153	0	-1
N.S.	1	1.00	0.63	1.09	0.95	0.72	1.01	0.00	-0.01
time (sec)	N/A	0.120	0.072	0.574	0.265	0.353	2.472	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	53	63	90	55	75	0	61
N.S.	1	1.00	0.88	1.05	1.50	0.92	1.25	0.00	1.02
time (sec)	N/A	0.033	0.013	0.425	0.257	0.351	0.929	0.000	0.593

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	422	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.244	0.093	0.468	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	73	94	114	0	0	0	-1
N.S.	1	1.00	0.87	1.12	1.36	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.038	1.306	0.281	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	131	163	193	0	0	0	-1
N.S.	1	1.00	0.76	0.94	1.12	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.113	1.349	0.266	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	210	272	302	0	0	0	-1
N.S.	1	1.00	0.76	0.99	1.09	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.197	0.208	1.338	0.282	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	296	0	264	219	0	0	-1
N.S.	1	1.00	0.85	0.00	0.76	0.63	0.00	0.00	-0.00
time (sec)	N/A	0.452	0.043	0.003	0.271	0.410	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	198	0	193	149	0	0	-1
N.S.	1	1.00	1.00	0.00	0.97	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.030	0.004	0.269	0.387	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	66	0	120	73	0	0	77
N.S.	1	1.00	0.79	0.00	1.43	0.87	0.00	0.00	0.92
time (sec)	N/A	0.047	0.017	0.003	0.259	0.382	0.000	0.000	2.181

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.025	0.025	0.004	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	486	486	477	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.377	0.521	0.004	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	629	629	573	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.468	1.330	0.003	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	605	605	485	1146	696	625	1030	0	-1
N.S.	1	1.00	0.80	1.89	1.15	1.03	1.70	0.00	-0.00
time (sec)	N/A	0.415	0.384	0.797	0.278	0.363	20.818	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	274	685	416	361	561	0	-1
N.S.	1	1.00	0.71	1.78	1.08	0.94	1.46	0.00	-0.00
time (sec)	N/A	0.241	0.116	0.800	0.272	0.352	10.209	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	161	249	219	174	252	0	-1
N.S.	1	1.00	0.77	1.19	1.04	0.83	1.20	0.00	-0.00
time (sec)	N/A	0.135	0.064	0.655	0.277	0.417	3.335	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	53	63	90	55	75	0	61
N.S.	1	1.00	0.88	1.05	1.50	0.92	1.25	0.00	1.02
time (sec)	N/A	0.033	0.012	0.438	0.261	0.356	0.938	0.000	0.002

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	591	591	622	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.347	0.173	0.823	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	108	213	172	0	0	0	-1
N.S.	1	1.00	0.78	1.54	1.25	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.097	2.107	0.272	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	190	346	482	0	0	0	-1
N.S.	1	1.00	0.68	1.24	1.73	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.187	0.236	2.116	0.273	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	313	648	1519	0	0	0	-1
N.S.	1	1.00	0.70	1.45	3.39	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.307	0.386	2.138	0.331	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	66	44	39	0	0	46
N.S.	1	1.00	1.00	1.43	0.96	0.85	0.00	0.00	1.00
time (sec)	N/A	0.046	0.033	0.829	0.253	0.361	0.000	0.000	0.044

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	66	44	39	0	0	46
N.S.	1	1.00	1.00	1.43	0.96	0.85	0.00	0.00	1.00
time (sec)	N/A	0.043	0.007	0.532	0.253	0.370	0.000	0.000	0.002

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	82	49	0	0	0	-1
N.S.	1	1.00	1.00	1.61	0.96	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.091	0.031	0.728	0.268	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	82	49	0	0	0	-1
N.S.	1	1.00	1.00	1.61	0.96	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.094	0.008	0.720	0.263	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.045	0.015	0.326	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	0	0	34	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	1.03	0.00	0.00	-0.03
time (sec)	N/A	0.039	0.006	0.325	0.000	0.416	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	0	0	34	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	1.03	0.00	0.00	-0.03
time (sec)	N/A	0.039	0.006	0.367	0.000	0.373	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	40	0	0	33	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	1.00	0.00	0.00	-0.03
time (sec)	N/A	0.038	1.218	0.322	0.000	0.367	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	37	61	36	0	0	33
N.S.	1	1.00	1.06	1.03	1.69	1.00	0.00	0.00	0.92
time (sec)	N/A	0.209	0.390	0.447	0.291	0.357	0.000	0.000	0.269

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	35	36	56	43	0	0	35
N.S.	1	1.00	0.97	1.00	1.56	1.19	0.00	0.00	0.97
time (sec)	N/A	0.242	0.357	0.428	0.275	0.384	0.000	0.000	0.195

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	53	207	88	0	0	81
N.S.	1	1.00	1.00	1.02	3.98	1.69	0.00	0.00	1.56
time (sec)	N/A	1.441	0.500	0.511	0.298	0.374	0.000	0.000	0.252

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	135	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.010	0.078	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.006	0.031	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.005	0.029	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	32	0	0	0	0	-1
N.S.	1	1.00	1.00	1.03	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.012	0.004	0.560	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.047	0.027	0.030	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	223	0	376	0	0	0	-1
N.S.	1	1.00	0.74	0.00	1.25	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.365	0.419	0.033	0.288	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	192	0	296	0	0	0	-1
N.S.	1	1.00	0.74	0.00	1.15	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.289	0.252	0.031	0.282	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	160	0	222	0	0	0	-1
N.S.	1	1.00	0.61	0.00	0.85	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.185	0.209	0.045	0.281	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	119	0	141	0	0	0	-1
N.S.	1	1.00	0.90	0.00	1.07	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.018	0.032	0.295	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	8	8	0	0	9
N.S.	1	1.00	1.00	0.91	0.73	0.73	0.00	0.00	0.82
time (sec)	N/A	0.019	0.008	0.064	0.252	0.356	0.000	0.000	0.240

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	115	0	113	0	0	0	-1
N.S.	1	1.00	1.04	0.00	1.02	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.096	0.042	0.306	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	185	0	162	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.85	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.193	0.032	0.511	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	246	0	188	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.77	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.253	0.174	0.033	0.507	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	277	0	214	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.75	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.313	0.163	0.040	0.504	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	423	252	0	0	0	0	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.306	0.335	0.166	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	211	0	0	0	0	0	-1
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.243	0.242	0.172	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	149	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.054	0.166	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	0	0	0	18
N.S.	1	1.00	1.00	0.95	0.00	0.00	0.00	0.00	0.90
time (sec)	N/A	0.041	0.013	0.149	0.000	0.000	0.000	0.000	0.248

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	150	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.181	0.123	0.203	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	238	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.258	0.177	0.236	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	301	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.351	0.190	0.237	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2995	2995	2610	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.104	9.808	0.105	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2252	2252	1996	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.038	6.625	0.108	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1653	1653	1546	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.141	3.130	0.098	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.016	1.766	0.102	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2498	2498	2247	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.775	7.163	0.110	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3119	3119	2673	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.227	13.832	0.128	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3733	3733	3331	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.866	18.154	0.115	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	661	661	425	0	415	0	0	0	-1
N.S.	1	1.00	0.64	0.00	0.63	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.645	0.527	0.008	0.268	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	546	546	362	0	345	0	0	0	-1
N.S.	1	1.00	0.66	0.00	0.63	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.464	0.414	0.006	0.280	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	285	0	258	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.66	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.289	0.340	0.004	0.271	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	137	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.220	0.146	0.007	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	135	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.503	0.007	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	285	0	213	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.64	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.347	0.817	0.010	0.310	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	389	0	287	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.62	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.443	0.997	0.006	0.317	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	584	584	505	0	341	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.58	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.553	1.062	0.007	0.310	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	900	900	583	0	518	0	0	0	-1
N.S.	1	1.00	0.65	0.00	0.58	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.754	0.854	0.037	0.272	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	645	645	472	0	412	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.64	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.539	0.682	0.045	0.268	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	298	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.390	0.236	0.037	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	280	0	0	0	0	0	-1
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.334	0.565	0.036	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.477	1.206	0.033	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	515	515	488	0	319	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.62	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.529	1.214	0.036	0.316	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	767	767	621	0	403	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.53	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.727	1.349	0.035	0.315	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [155] had the largest ratio of [76]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	9	0.333
2	A	4	3	1.00	9	0.333
3	A	4	3	1.00	9	0.333
4	A	4	3	1.00	7	0.429
5	A	3	3	1.00	5	0.600
6	A	1	1	1.00	9	0.111
7	A	5	5	1.00	9	0.556
8	A	4	3	1.00	9	0.333
9	A	4	3	1.00	9	0.333
10	A	4	3	1.00	9	0.333
11	A	5	3	1.00	9	0.333
12	A	5	3	1.00	9	0.333
13	A	5	3	1.00	7	0.429
14	A	4	3	1.00	5	0.600
15	A	1	1	1.00	9	0.111
16	A	6	5	1.00	9	0.556
17	A	5	3	1.00	9	0.333
18	A	5	3	1.00	9	0.333
19	A	5	4	1.00	11	0.364
20	A	5	4	1.00	11	0.364
21	A	4	4	1.00	9	0.444
22	A	1	1	1.00	11	0.091
23	A	6	6	1.00	11	0.546
24	A	5	4	1.00	11	0.364
25	A	5	4	1.00	11	0.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	5	4	1.00	11	0.364
27	A	5	4	1.00	11	0.364
28	A	4	4	1.00	7	0.571
29	A	3	3	1.00	11	0.273
30	A	4	4	1.00	11	0.364
31	A	5	4	1.00	11	0.364
32	A	6	4	1.00	11	0.364
33	A	6	4	1.00	11	0.364
34	A	5	4	1.00	9	0.444
35	A	1	1	1.00	11	0.091
36	A	7	6	1.00	11	0.546
37	A	6	4	1.00	11	0.364
38	A	6	4	1.00	11	0.364
39	A	6	4	1.00	11	0.364
40	A	6	4	1.00	11	0.364
41	A	5	4	1.00	7	0.571
42	A	4	3	1.00	11	0.273
43	A	5	4	1.00	11	0.364
44	A	6	4	1.00	11	0.364
45	A	3	3	1.00	11	0.273
46	A	3	3	1.00	9	0.333
47	A	3	3	1.00	7	0.429
48	A	1	1	1.00	11	0.091
49	A	3	3	1.00	11	0.273
50	A	3	3	1.00	11	0.273
51	A	3	3	1.00	11	0.273
52	A	4	3	1.00	11	0.273
53	A	4	3	1.00	9	0.333
54	A	4	3	1.00	7	0.429
55	A	1	1	1.00	11	0.091
56	A	4	3	1.00	11	0.273
57	A	4	3	1.00	11	0.273
58	A	4	3	1.00	11	0.273
59	A	7	5	1.00	13	0.385
60	A	6	5	1.00	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	5	5	1.00	13	0.385
62	A	4	4	1.00	13	0.308
63	A	5	5	1.00	13	0.385
64	A	6	5	1.00	13	0.385
65	A	9	5	1.00	13	0.385
66	A	8	5	1.00	13	0.385
67	A	7	5	1.00	13	0.385
68	A	6	5	1.00	13	0.385
69	A	5	4	1.00	13	0.308
70	A	6	5	1.00	13	0.385
71	A	7	5	1.00	13	0.385
72	A	9	8	1.00	15	0.533
73	A	8	8	1.00	15	0.533
74	A	8	8	1.00	15	0.533
75	A	7	7	1.00	15	0.467
76	A	7	7	1.00	15	0.467
77	A	8	8	1.00	15	0.533
78	A	10	8	1.00	15	0.533
79	A	10	8	1.00	15	0.533
80	A	9	8	1.00	15	0.533
81	A	9	8	1.00	15	0.533
82	A	8	7	1.00	15	0.467
83	A	8	7	1.00	15	0.467
84	A	9	8	1.00	15	0.533
85	A	9	8	1.00	15	0.533
86	A	4	4	1.00	15	0.267
87	A	4	4	1.00	15	0.267
88	A	4	4	1.00	15	0.267
89	A	4	4	1.00	15	0.267
90	A	4	4	1.00	15	0.267
91	A	5	4	1.00	15	0.267
92	A	5	4	1.00	15	0.267
93	A	5	4	1.00	15	0.267
94	A	5	4	1.00	15	0.267
95	A	5	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	0	0	0.00	0	0.000
97	A	0	0	0.00	0	0.000
98	A	0	0	0.00	0	0.000
99	A	0	0	0.00	0	0.000
100	A	0	0	0.00	0	0.000
101	A	2	1	1.00	15	0.067
102	A	3	3	1.00	11	0.273
103	A	4	3	1.00	11	0.273
104	A	5	3	1.00	11	0.273
105	A	4	4	1.00	13	0.308
106	A	5	4	1.00	13	0.308
107	A	6	4	1.00	13	0.308
108	A	4	4	1.00	13	0.308
109	A	5	4	1.00	13	0.308
110	A	6	4	1.00	13	0.308
111	A	4	4	1.00	13	0.308
112	A	5	4	1.00	13	0.308
113	A	6	4	1.00	13	0.308
114	A	0	0	0.00	0	0.000
115	A	0	0	0.00	0	0.000
116	A	1	1	1.00	9	0.111
117	A	0	0	0.00	0	0.000
118	A	0	0	0.00	0	0.000
119	A	0	0	0.00	0	0.000
120	A	0	0	0.00	0	0.000
121	A	1	1	1.00	11	0.091
122	A	0	0	0.00	0	0.000
123	A	0	0	0.00	0	0.000
124	A	13	8	1.00	13	0.615
125	A	10	8	1.00	11	0.727
126	A	7	7	1.00	9	0.778
127	A	3	3	1.00	13	0.231
128	A	7	9	1.00	13	0.692
129	A	11	11	1.00	13	0.846
130	A	14	11	1.00	13	0.846

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	33	13	1.00	13	1.000
132	A	19	12	1.00	11	1.091
133	A	9	8	1.00	9	0.889
134	A	0	0	0.00	0	0.000
135	A	6	5	1.00	13	0.385
136	A	12	13	1.00	13	1.000
137	A	16	8	1.00	17	0.471
138	A	13	8	1.00	17	0.471
139	A	10	8	1.00	15	0.533
140	A	7	7	1.00	9	0.778
141	A	3	3	1.00	17	0.176
142	A	8	5	1.00	17	0.294
143	A	12	8	1.00	17	0.471
144	A	15	9	1.00	17	0.529
145	A	5	5	1.00	9	0.556
146	A	5	5	1.00	12	0.417
147	A	8	6	1.00	12	0.500
148	A	8	6	1.00	15	0.400
149	A	1	1	1.00	34	0.029
150	A	1	1	1.00	34	0.029
151	A	1	1	1.00	34	0.029
152	A	1	1	1.00	37	0.027
153	A	2	2	1.00	53	0.038
154	A	2	2	1.00	53	0.038
155	A	4	4	1.00	76	0.053
156	A	5	3	1.00	19	0.158
157	A	4	3	1.00	19	0.158
158	A	3	3	1.00	17	0.176
159	A	2	2	1.00	15	0.133
160	A	0	0	0.00	0	0.000
161	A	38	16	1.00	16	1.000
162	A	31	16	1.00	16	1.000
163	A	22	17	1.00	14	1.214
164	A	15	12	1.00	13	0.923
165	A	1	2	1.00	16	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	10	13	1.00	16	0.812
167	A	23	17	1.00	16	1.062
168	A	30	17	1.00	16	1.062
169	A	37	17	1.00	16	1.062
170	A	25	18	1.00	20	0.900
171	A	21	18	1.00	18	1.000
172	A	18	14	1.00	17	0.824
173	A	3	3	1.00	20	0.150
174	A	12	10	1.00	20	0.500
175	A	20	17	1.00	20	0.850
176	A	28	19	1.00	20	0.950
177	A	108	20	1.00	27	0.741
178	A	67	20	1.00	25	0.800
179	A	42	17	1.00	24	0.708
180	A	0	0	0.00	0	0.000
181	A	22	9	1.00	27	0.333
182	A	44	16	1.00	27	0.593
183	A	78	18	1.00	27	0.667
184	A	52	17	1.00	21	0.810
185	A	40	21	1.00	19	1.105
186	A	26	20	1.00	18	1.111
187	A	18	15	1.00	21	0.714
188	A	13	17	1.00	21	0.810
189	A	30	20	1.00	21	0.952
190	A	41	19	1.00	21	0.905
191	A	51	19	1.00	21	0.905
192	A	60	22	1.00	24	0.917
193	A	43	21	1.00	23	0.913
194	A	29	24	1.00	26	0.923
195	A	19	21	1.00	26	0.808
196	A	32	22	1.00	26	0.846
197	A	43	20	1.00	26	0.769
198	A	61	19	1.00	26	0.731

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^4 \mathbf{PolyLog}(2, ax)\, dx$

Optimal. Leaf size=86

$$-\frac{x}{25a^4}-\frac{x^2}{50a^3}-\frac{x^3}{75a^2}-\frac{x^4}{100a}-\frac{x^5}{125}-\frac{\log(1-ax)}{25a^5}+\frac{1}{25}x^5\log(1-ax)+\frac{1}{5}x^5\text{PolyLog}(2,ax)$$

```
[Out] -1/25*x/a^4-1/50*x^2/a^3-1/75*x^3/a^2-1/100*x^4/a-1/125*x^5-1/25*ln(-a*x+1)
/a^5+1/25*x^5*ln(-a*x+1)+1/5*x^5*polylog(2,a*x)
```

Rubi [A]
time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6726, 2442, 45}

$$-\frac{\log(1-ax)}{25a^5}-\frac{x}{25a^4}-\frac{x^2}{50a^3}-\frac{x^3}{75a^2}+\frac{1}{5}x^5\text{Li}_2(ax)+\frac{1}{25}x^5\log(1-ax)-\frac{x^4}{100a}-\frac{x^5}{125}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*PolyLog[2, a*x],x]
```

```
[Out] -1/25*x/a^4 - x^2/(50*a^3) - x^3/(75*a^2) - x^4/(100*a) - x^5/125 - Log[1 -
 a*x]/(25*a^5) + (x^5*Log[1 - a*x])/25 + (x^5*PolyLog[2, a*x])/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\text{Li}_2(ax)\,dx &= \frac{1}{5}x^5\text{Li}_2(ax)+\frac{1}{5}\int x^4\log(1-ax)\,dx\\ &= \frac{1}{25}x^5\log(1-ax)+\frac{1}{5}x^5\text{Li}_2(ax)+\frac{1}{25}a\int\frac{x^5}{1-ax}\,dx\\ &= \frac{1}{25}x^5\log(1-ax)+\frac{1}{5}x^5\text{Li}_2(ax)+\frac{1}{25}a\int\left(-\frac{1}{a^5}-\frac{x}{a^4}-\frac{x^2}{a^3}-\frac{x^3}{a^2}-\frac{x^4}{a}-\frac{1}{a^5(-1+ax)}\right)\\ &= -\frac{x}{25a^4}-\frac{x^2}{50a^3}-\frac{x^3}{75a^2}-\frac{x^4}{100a}-\frac{x^5}{125}-\frac{\log(1-ax)}{25a^5}+\frac{1}{25}x^5\log(1-ax)+\frac{1}{5}x^5\text{Li}_2(ax)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 73, normalized size = 0.85

$$\frac{-ax(60+30ax+20a^2x^2+15a^3x^3+12a^4x^4)+60(-1+a^5x^5)\log(1-ax)+300a^5x^5\text{PolyLog}(2,ax)}{1500a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*PolyLog[2, a*x],x]
```

```
[Out] (-(a*x*(60 + 30*a*x + 20*a^2*x^2 + 15*a^3*x^3 + 12*a^4*x^4)) + 60*(-1 + a^5
*x^5)*Log[1 - a*x] + 300*a^5*x^5*PolyLog[2, a*x])/(1500*a^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(70) = 140.
time = 0.36, size = 147, normalized size = 1.71

method	result
meijerg	$\frac{-\frac{ax\left(12a^4x^4+15a^3x^3+20a^2x^2+30ax+60\right)}{1500}-\frac{\left(-6a^5x^5+6\right)\ln(-ax+1)}{150}+\frac{a^5x^5\,\text{polylog}(2,ax)}{5}}{a^5}$
derivativedivides	$\frac{\frac{a^5x^5\,\text{polylog}(2,ax)}{5}-\frac{(-ax+1)^5\ln(-ax+1)}{25}+\frac{(-ax+1)^5}{125}+\frac{\ln(-ax+1)(-ax+1)^4}{5}-\frac{(-ax+1)^4}{20}-\frac{2\ln(-ax+1)(-ax+1)^3}{5}+\frac{2(-ax+1)^3}{15}+}{a^5}$
default	$\frac{\frac{a^5x^5\,\text{polylog}(2,ax)}{5}-\frac{(-ax+1)^5\ln(-ax+1)}{25}+\frac{(-ax+1)^5}{125}+\frac{\ln(-ax+1)(-ax+1)^4}{5}-\frac{(-ax+1)^4}{20}-\frac{2\ln(-ax+1)(-ax+1)^3}{5}+\frac{2(-ax+1)^3}{15}+}{a^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*polylog(2,a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(1/5*a^5*x^5*polylog(2,a*x)-1/25*(-a*x+1)^5*ln(-a*x+1)+1/125*(-a*x+1)
^5+1/5*ln(-a*x+1)*(-a*x+1)^4-1/20*(-a*x+1)^4-2/5*ln(-a*x+1)*(-a*x+1)^3+2/15
*(-a*x+1)^3+2/5*ln(-a*x+1)*(-a*x+1)^2-1/5*(-a*x+1)^2-1/5*ln(-a*x+1)*(-a*x+1
)+1/5-1/5*a*x)
```

Maxima [A]
time = 0.27, size = 72, normalized size = 0.84

$$\frac{300\,a^5x^5\mathrm{Li}_2(ax) - 12\,a^5x^5 - 15\,a^4x^4 - 20\,a^3x^3 - 30\,a^2x^2 - 60\,ax + 60\,(a^5x^5 - 1)\log(-ax + 1)}{1500\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*polylog(2,a*x),x, algorithm="maxima")
```

```
[Out] 1/1500*(300*a^5*x^5*dilog(a*x) - 12*a^5*x^5 - 15*a^4*x^4 - 20*a^3*x^3 - 30*
a^2*x^2 - 60*a*x + 60*(a^5*x^5 - 1)*log(-a*x + 1))/a^5
```

Fricas [A]
time = 0.37, size = 72, normalized size = 0.84

$$\frac{300\,a^5x^5\mathrm{Li}_2(ax) - 12\,a^5x^5 - 15\,a^4x^4 - 20\,a^3x^3 - 30\,a^2x^2 - 60\,ax + 60\,(a^5x^5 - 1)\log(-ax + 1)}{1500\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*polylog(2,a*x),x, algorithm="fricas")
```

```
[Out] 1/1500*(300*a^5*x^5*dilog(a*x) - 12*a^5*x^5 - 15*a^4*x^4 - 20*a^3*x^3 - 30*
a^2*x^2 - 60*a*x + 60*(a^5*x^5 - 1)*log(-a*x + 1))/a^5
```

Sympy [A]
time = 3.19, size = 66, normalized size = 0.77

$$\begin{cases} -\frac{x^5\,\mathrm{Li}_1(ax)}{25} + \frac{x^5\,\mathrm{Li}_2(ax)}{5} - \frac{x^5}{125} - \frac{x^4}{100a} - \frac{x^3}{75a^2} - \frac{x^2}{50a^3} - \frac{x}{25a^4} + \frac{\mathrm{Li}_1(ax)}{25a^5} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*polylog(2,a*x),x)
```

```
[Out] Piecewise((-x**5*polylog(1, a*x)/25 + x**5*polylog(2, a*x)/5 - x**5/125 - x
**4/(100*a) - x**3/(75*a**2) - x**2/(50*a**3) - x/(25*a**4) + polylog(1, a*
x)/(25*a**5), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*polylog(2,a*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*dilog(a*x), x)
```

Mupad [B]
time = 0.31, size = 69, normalized size = 0.80

$$\frac{x^5\,\ln\left(1-a\,x\right)}{25}-\frac{\ln\left(a\,x-1\right)}{25\,a^5}-\frac{x}{25\,a^4}-\frac{x^5}{125}+\frac{x^5\,\mathrm{polylog}\left(2,a\,x\right)}{5}-\frac{x^4}{100\,a}-\frac{x^3}{75\,a^2}-\frac{x^2}{50\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*polylog(2, a*x),x)
```

```
[Out] (x^5*log(1 - a*x))/25 - log(a*x - 1)/(25*a^5) - x/(25*a^4) - x^5/125 + (x^5
*polylog(2, a*x))/5 - x^4/(100*a) - x^3/(75*a^2) - x^2/(50*a^3)
```

3.2 $\int x^3\mathbf{PolyLog}(2,ax)\,dx$

Optimal. Leaf size=76

$$-\frac{x}{16a^3}-\frac{x^2}{32a^2}-\frac{x^3}{48a}-\frac{x^4}{64}-\frac{\log(1-ax)}{16a^4}+\frac{1}{16}x^4\log(1-ax)+\frac{1}{4}x^4\text{PolyLog}(2,ax)$$

```
[Out] -1/16*x/a^3-1/32*x^2/a^2-1/48*x^3/a-1/64*x^4-1/16*ln(-a*x+1)/a^4+1/16*x^4*l
n(-a*x+1)+1/4*x^4*polylog(2,a*x)
```

Rubi [A]

time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {6726, 2442, 45}

$$-\frac{\log(1-ax)}{16a^4}-\frac{x}{16a^3}-\frac{x^2}{32a^2}+\frac{1}{4}x^4\text{Li}_2(ax)+\frac{1}{16}x^4\log(1-ax)-\frac{x^3}{48a}-\frac{x^4}{64}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*PolyLog[2, a*x],x]
```

```
[Out] -1/16*x/a^3 - x^2/(32*a^2) - x^3/(48*a) - x^4/64 - Log[1 - a*x]/(16*a^4) +
(x^4*Log[1 - a*x])/16 + (x^4*PolyLog[2, a*x])/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int x^3 \text{Li}_2(ax)\,dx &= \frac{1}{4}x^4\text{Li}_2(ax) + \frac{1}{4}\int x^3 \log(1-ax)\,dx \\ &= \frac{1}{16}x^4\log(1-ax) + \frac{1}{4}x^4\text{Li}_2(ax) + \frac{1}{16}a\int \frac{x^4}{1-ax}\,dx \\ &= \frac{1}{16}x^4\log(1-ax) + \frac{1}{4}x^4\text{Li}_2(ax) + \frac{1}{16}a\int \left(-\frac{1}{a^4} - \frac{x}{a^3} - \frac{x^2}{a^2} - \frac{x^3}{a} - \frac{1}{a^4(-1+ax)}\right)dx \\ &= -\frac{x}{16a^3} - \frac{x^2}{32a^2} - \frac{x^3}{48a} - \frac{x^4}{64} - \frac{\log(1-ax)}{16a^4} + \frac{1}{16}x^4\log(1-ax) + \frac{1}{4}x^4\text{Li}_2(ax)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 65, normalized size = 0.86

$$\frac{-ax(12+6ax+4a^2x^2+3a^3x^3)+12(-1+a^4x^4)\log(1-ax)+48a^4x^4\text{PolyLog}(2,ax)}{192a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*PolyLog[2, a*x],x]
```

```
[Out] (-(a*x*(12 + 6*a*x + 4*a^2*x^2 + 3*a^3*x^3)) + 12*(-1 + a^4*x^4)*Log[1 - a*
x] + 48*a^4*x^4*PolyLog[2, a*x])/(192*a^4)
```

Maple [A]
time = 0.36, size = 120, normalized size = 1.58

method	result
meijerg	$-\frac{\frac{ax\left(15a^3x^3+20a^2x^2+30ax+60\right)}{960}+\frac{\left(-5a^4x^4+5\right)\ln(-ax+1)}{80}-\frac{a^4x^4\,\text{polylog}(2,ax)}{4}}{a^4}$
derivativedivides	$\frac{\frac{a^4x^4\,\text{polylog}(2,ax)}{4}+\frac{\ln(-ax+1)(-ax+1)^4}{16}-\frac{(-ax+1)^4}{64}-\frac{\ln(-ax+1)(-ax+1)^3}{4}+\frac{(-ax+1)^3}{12}+\frac{3\ln(-ax+1)(-ax+1)^2}{8}-\frac{3(-ax+1)^2}{16}-}{a^4}$
default	$\frac{\frac{a^4x^4\,\text{polylog}(2,ax)}{4}+\frac{\ln(-ax+1)(-ax+1)^4}{16}-\frac{(-ax+1)^4}{64}-\frac{\ln(-ax+1)(-ax+1)^3}{4}+\frac{(-ax+1)^3}{12}+\frac{3\ln(-ax+1)(-ax+1)^2}{8}-\frac{3(-ax+1)^2}{16}-}{a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*polylog(2,a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*polylog(2,a*x)+1/16*ln(-a*x+1)*(-a*x+1)^4-1/64*(-a*x+1)^
4-1/4*ln(-a*x+1)*(-a*x+1)^3+1/12*(-a*x+1)^3+3/8*ln(-a*x+1)*(-a*x+1)^2-3/16*
(-a*x+1)^2-1/4*ln(-a*x+1)*(-a*x+1)+1/4-1/4*a*x)
```

Maxima [A]
time = 0.26, size = 64, normalized size = 0.84

$$\frac{48\,a^4x^4\text{Li}_2(ax) - 3\,a^4x^4 - 4\,a^3x^3 - 6\,a^2x^2 - 12\,ax + 12\,(a^4x^4-1)\log(-ax+1)}{192\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(2,a*x),x, algorithm="maxima")
```

```
[Out] 1/192*(48*a^4*x^4*dilog(a*x) - 3*a^4*x^4 - 4*a^3*x^3 - 6*a^2*x^2 - 12*a*x +
 12*(a^4*x^4 - 1)*log(-a*x + 1))/a^4
```

Fricas [A]
time = 0.35, size = 64, normalized size = 0.84

$$\frac{48\,a^4x^4\mathrm{Li}_2(ax) - 3\,a^4x^4 - 4\,a^3x^3 - 6\,a^2x^2 - 12\,ax + 12\,(a^4x^4 - 1)\log\left(-ax + 1\right)}{192\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(2,a*x),x, algorithm="fricas")
```

```
[Out] 1/192*(48*a^4*x^4*dilog(a*x) - 3*a^4*x^4 - 4*a^3*x^3 - 6*a^2*x^2 - 12*a*x +
 12*(a^4*x^4 - 1)*log(-a*x + 1))/a^4
```

Sympy [A]
time = 1.73, size = 58, normalized size = 0.76

$$\begin{cases} -\frac{x^4\,\mathrm{Li}_1(ax)}{16} + \frac{x^4\,\mathrm{Li}_2(ax)}{4} - \frac{x^4}{64} - \frac{x^3}{48a} - \frac{x^2}{32a^2} - \frac{x}{16a^3} + \frac{\mathrm{Li}_1(ax)}{16a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*polylog(2,a*x),x)
```

```
[Out] Piecewise((-x**4*polylog(1, a*x)/16 + x**4*polylog(2, a*x)/4 - x**4/64 - x*
*3/(48*a) - x**2/(32*a**2) - x/(16*a**3) + polylog(1, a*x)/(16*a**4), Ne(a,
 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(2,a*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*dilog(a*x), x)
```

Mupad [B]
time = 0.25, size = 61, normalized size = 0.80

$$\frac{x^4\,\ln\left(1-a\,x\right)}{16} - \frac{\ln\left(a\,x-1\right)}{16\,a^4} - \frac{x}{16\,a^3} - \frac{x^4}{64} + \frac{x^4\,\mathrm{polylog}(2,a\,x)}{4} - \frac{x^3}{48\,a} - \frac{x^2}{32\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*polylog(2, a*x),x)
```

```
[Out] (x^4*log(1 - a*x))/16 - log(a*x - 1)/(16*a^4) - x/(16*a^3) - x^4/64 + (x^4*
polylog(2, a*x))/4 - x^3/(48*a) - x^2/(32*a^2)
```

3.3 $\int x^2 \textbf{PolyLog}(2, ax)\, dx$

Optimal. Leaf size=66

$$-\frac{x}{9a^2}-\frac{x^2}{18a}-\frac{x^3}{27}-\frac{\log(1-ax)}{9a^3}+\frac{1}{9}x^3\log(1-ax)+\frac{1}{3}x^3\text{PolyLog}(2,ax)$$

```
[Out] -1/9*x/a^2-1/18*x^2/a-1/27*x^3-1/9*ln(-a*x+1)/a^3+1/9*x^3*ln(-a*x+1)+1/3*x^
3*polylog(2,a*x)
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6726, 2442, 45}

$$-\frac{\log(1-ax)}{9a^3}-\frac{x}{9a^2}+\frac{1}{3}x^3\text{Li}_2(ax)+\frac{1}{9}x^3\log(1-ax)-\frac{x^2}{18a}-\frac{x^3}{27}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*PolyLog[2, a*x],x]
```

```
[Out] -1/9*x/a^2 - x^2/(18*a) - x^3/27 - Log[1 - a*x]/(9*a^3) + (x^3*Log[1 - a*x]
)/9 + (x^3*PolyLog[2, a*x])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \text{Li}_2(ax)\,dx &= \frac{1}{3}x^3\text{Li}_2(ax) + \frac{1}{3}\int x^2\log(1-ax)\,dx \\
&= \frac{1}{9}x^3\log(1-ax) + \frac{1}{3}x^3\text{Li}_2(ax) + \frac{1}{9}a\int \frac{x^3}{1-ax}\,dx \\
&= \frac{1}{9}x^3\log(1-ax) + \frac{1}{3}x^3\text{Li}_2(ax) + \frac{1}{9}a\int \left(-\frac{1}{a^3} - \frac{x}{a^2} - \frac{x^2}{a} - \frac{1}{a^3(-1+ax)}\right)dx \\
&= -\frac{x}{9a^2} - \frac{x^2}{18a} - \frac{x^3}{27} - \frac{\log(1-ax)}{9a^3} + \frac{1}{9}x^3\log(1-ax) + \frac{1}{3}x^3\text{Li}_2(ax)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.86

$$\frac{-ax(6+3ax+2a^2x^2)+6(-1+a^3x^3)\log(1-ax)+18a^3x^3\text{PolyLog}(2,ax)}{54a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*PolyLog[2, a*x],x]
```

```
[Out] (-(a*x*(6 + 3*a*x + 2*a^2*x^2)) + 6*(-1 + a^3*x^3)*Log[1 - a*x] + 18*a^3*x^
3*PolyLog[2, a*x])/(54*a^3)
```

Maple [A]
time = 0.38, size = 93, normalized size = 1.41

method	result	size
meijerg	$\frac{-\frac{ax\left(4a^2x^2+6ax+12\right)}{108}-\frac{\left(-4a^3x^3+4\right)\ln(-ax+1)}{36}+\frac{a^3x^3\,\text{polylog}(2,ax)}{3}}{a^3}$	56
derivativedivides	$\frac{\frac{a^3x^3\,\text{polylog}(2,ax)}{3}-\frac{\ln(-ax+1)(-ax+1)^3}{9}+\frac{(-ax+1)^3}{27}+\frac{\ln(-ax+1)(-ax+1)^2}{3}-\frac{(-ax+1)^2}{6}-\frac{\ln(-ax+1)(-ax+1)}{3}+\frac{1}{3}-\frac{ax}{3}}{a^3}$	93
default	$\frac{\frac{a^3x^3\,\text{polylog}(2,ax)}{3}-\frac{\ln(-ax+1)(-ax+1)^3}{9}+\frac{(-ax+1)^3}{27}+\frac{\ln(-ax+1)(-ax+1)^2}{3}-\frac{(-ax+1)^2}{6}-\frac{\ln(-ax+1)(-ax+1)}{3}+\frac{1}{3}-\frac{ax}{3}}{a^3}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(2,a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*polylog(2,a*x)-1/9*ln(-a*x+1)*(-a*x+1)^3+1/27*(-a*x+1)^3
+1/3*ln(-a*x+1)*(-a*x+1)^2-1/6*(-a*x+1)^2-1/3*ln(-a*x+1)*(-a*x+1)+1/3-1/3*a
*x)
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.85

$$\frac{18\,a^3x^3\text{Li}_2(ax)-2\,a^3x^3-3\,a^2x^2-6\,ax+6\,(a^3x^3-1)\log(-ax+1)}{54\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,a*x),x, algorithm="maxima")
```

```
[Out] 1/54*(18*a^3*x^3*dilog(a*x) - 2*a^3*x^3 - 3*a^2*x^2 - 6*a*x + 6*(a^3*x^3 -
1)*log(-a*x + 1))/a^3
```

Fricas [A]
time = 0.37, size = 56, normalized size = 0.85

$$\frac{18\,a^3x^3\mathrm{Li}_2(ax)-2\,a^3x^3-3\,a^2x^2-6\,ax+6\,(a^3x^3-1)\log\left(-ax+1\right)}{54\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,a*x),x, algorithm="fricas")
```

```
[Out] 1/54*(18*a^3*x^3*dilog(a*x) - 2*a^3*x^3 - 3*a^2*x^2 - 6*a*x + 6*(a^3*x^3 -
1)*log(-a*x + 1))/a^3
```

Sympy [A]
time = 0.89, size = 49, normalized size = 0.74

$$\begin{cases} -\frac{x^3\,\mathrm{Li}_1(ax)}{9}+\frac{x^3\,\mathrm{Li}_2(ax)}{3}-\frac{x^3}{27}-\frac{x^2}{18a}-\frac{x}{9a^2}+\frac{\mathrm{Li}_1(ax)}{9a^3} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*polylog(2,a*x),x)
```

```
[Out] Piecewise((-x**3*polylog(1, a*x)/9 + x**3*polylog(2, a*x)/3 - x**3/27 - x**
2/(18*a) - x/(9*a**2) + polylog(1, a*x)/(9*a**3), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,a*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*dilog(a*x), x)
```

Mupad [B]
time = 0.23, size = 53, normalized size = 0.80

$$\frac{x^3\,\ln\left(1-a\,x\right)}{9}-\frac{\ln\left(a\,x-1\right)}{9\,a^3}-\frac{x}{9\,a^2}-\frac{x^3}{27}+\frac{x^3\,\mathrm{polylog}(2,a\,x)}{3}-\frac{x^2}{18\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(2, a*x),x)
```

```
[Out] (x^3*log(1 - a*x))/9 - log(a*x - 1)/(9*a^3) - x/(9*a^2) - x^3/27 + (x^3*pol
ylog(2, a*x))/3 - x^2/(18*a)
```

3.4 $\int x\textbf{PolyLog}(2, ax)\,dx$

Optimal. Leaf size=56

$$-\frac{x}{4a}-\frac{x^2}{8}-\frac{\log(1-ax)}{4a^2}+\frac{1}{4}x^2\log(1-ax)+\frac{1}{2}x^2\text{PolyLog}(2,ax)$$

```
[Out] -1/4*x/a-1/8*x^2-1/4*ln(-a*x+1)/a^2+1/4*x^2*ln(-a*x+1)+1/2*x^2*polylog(2,a*
x)
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6726, 2442, 45}

$$-\frac{\log(1-ax)}{4a^2}+\frac{1}{2}x^2\text{Li}_2(ax)+\frac{1}{4}x^2\log(1-ax)-\frac{x}{4a}-\frac{x^2}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x*PolyLog[2, a*x],x]
```

```
[Out] -1/4*x/a - x^2/8 - Log[1 - a*x]/(4*a^2) + (x^2*Log[1 - a*x])/4 + (x^2*PolyL
og[2, a*x])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int x\mathrm{Li}_2(ax)\,dx &= \frac{1}{2}x^2\mathrm{Li}_2(ax)+\frac{1}{2}\int x\log(1-ax)\,dx\\ &= \frac{1}{4}x^2\log(1-ax)+\frac{1}{2}x^2\mathrm{Li}_2(ax)+\frac{1}{4}a\int\frac{x^2}{1-ax}\,dx\\ &= \frac{1}{4}x^2\log(1-ax)+\frac{1}{2}x^2\mathrm{Li}_2(ax)+\frac{1}{4}a\int\left(-\frac{1}{a^2}-\frac{x}{a}-\frac{1}{a^2(-1+ax)}\right)dx\\ &= -\frac{x}{4a}-\frac{x^2}{8}-\frac{\log(1-ax)}{4a^2}+\frac{1}{4}x^2\log(1-ax)+\frac{1}{2}x^2\mathrm{Li}_2(ax)\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 48, normalized size = 0.86

$$\frac{-ax(2+ax)+2(-1+a^2x^2)\log(1-ax)+4a^2x^2\mathrm{PolyLog}(2,ax)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*PolyLog[2, a*x],x]
```

```
[Out] (-(a*x*(2 + a*x)) + 2*(-1 + a^2*x^2)*Log[1 - a*x] + 4*a^2*x^2*PolyLog[2, a*
x])/(8*a^2)
```

Maple [**A**]
time = 0.35, size = 66, normalized size = 1.18

method	result	size
meijerg	$-\frac{\frac{ax(3ax+6)}{24}+\frac{\left(-3a^2x^2+3\right)\ln(-ax+1)}{12}-\frac{a^2x^2\,\mathrm{polylog}(2,ax)}{2}}{a^2}$	49
derivativedivides	$\frac{\frac{a^2x^2\,\mathrm{polylog}(2,ax)}{2}+\frac{\ln(-ax+1)(-ax+1)^2}{4}-\frac{(-ax+1)^2}{8}-\frac{\ln(-ax+1)(-ax+1)}{2}+\frac{1}{2}-\frac{ax}{2}}{a^2}$	66
default	$\frac{\frac{a^2x^2\,\mathrm{polylog}(2,ax)}{2}+\frac{\ln(-ax+1)(-ax+1)^2}{4}-\frac{(-ax+1)^2}{8}-\frac{\ln(-ax+1)(-ax+1)}{2}+\frac{1}{2}-\frac{ax}{2}}{a^2}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(2,a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2*a^2*x^2*polylog(2,a*x)+1/4*ln(-a*x+1)*(-a*x+1)^2-1/8*(-a*x+1)^2-
1/2*ln(-a*x+1)*(-a*x+1)+1/2-1/2*a*x)
```

Maxima [**A**]
time = 0.26, size = 48, normalized size = 0.86

$$\frac{4\,a^2x^2\mathrm{Li}_2(ax)-a^2x^2-2\,ax+2\,(a^2x^2-1)\log(-ax+1)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x),x, algorithm="maxima")
```

```
[Out] 1/8*(4*a^2*x^2*dilog(a*x) - a^2*x^2 - 2*a*x + 2*(a^2*x^2 - 1)*log(-a*x + 1)
)/a^2
```

Fricas [A]
time = 0.36, size = 48, normalized size = 0.86

$$\frac{4\,a^2x^2\mathrm{Li}_2(ax) - a^2x^2 - 2\,ax + 2\,(a^2x^2 - 1)\log\left(-ax + 1\right)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x),x, algorithm="fricas")
```

```
[Out] 1/8*(4*a^2*x^2*dilog(a*x) - a^2*x^2 - 2*a*x + 2*(a^2*x^2 - 1)*log(-a*x + 1)
)/a^2
```

Sympy [A]
time = 0.48, size = 41, normalized size = 0.73

$$\begin{cases} -\frac{x^2\,\mathrm{Li}_1(ax)}{4} + \frac{x^2\,\mathrm{Li}_2(ax)}{2} - \frac{x^2}{8} - \frac{x}{4a} + \frac{\mathrm{Li}_1(ax)}{4a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x),x)
```

```
[Out] Piecewise((-x**2*polylog(1, a*x)/4 + x**2*polylog(2, a*x)/2 - x**2/8 - x/(4
*a) + polylog(1, a*x)/(4*a**2), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x),x, algorithm="giac")
```

```
[Out] integrate(x*dilog(a*x), x)
```

Mupad [B]
time = 0.34, size = 46, normalized size = 0.82

$$\frac{x^2\,\ln\left(1-a\,x\right)}{4} - \frac{\ln\left(1-a\,x\right)}{4\,a^2} - \frac{x}{4\,a} - \frac{x^2}{8} + \frac{x^2\,\mathrm{polylog}(2,a\,x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(2, a*x),x)
```

```
[Out] (x^2*log(1 - a*x))/4 - log(1 - a*x)/(4*a^2) - x/(4*a) - x^2/8 + (x^2*polylo
g(2, a*x))/2
```

3.5 $\int \textbf{PolyLog}(2, ax)\, dx$

Optimal. Leaf size=29

$$-x-\frac{(1-ax)\log(1-ax)}{a}+x\text{PolyLog}(2,ax)$$

```
[Out] -x-(-a*x+1)*ln(-a*x+1)/a+x*polylog(2,a*x)
```

Rubi [A]

time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6721, 2436, 2332}

$$x\text{Li}_2(ax)-\frac{(1-ax)\log(1-ax)}{a}-x$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x],x]
```

```
[Out] -x - ((1 - a*x)*Log[1 - a*x])/a + x*PolyLog[2, a*x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \text{Li}_2(ax)\,dx &= x\text{Li}_2(ax)+\int \log(1-ax)\,dx\\ &= x\text{Li}_2(ax)-\frac{\text{Subst}(\int \log(x)\,dx,x,1-ax)}{a}\\ &= -x-\frac{(1-ax)\log(1-ax)}{a}+x\text{Li}_2(ax)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.90

$$-x+\left(-\frac{1}{a}+x\right)\log(1-ax)+x\text{PolyLog}(2,ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x],x]
```

```
[Out] -x + (-a^(-1) + x)*Log[1 - a*x] + x*PolyLog[2, a*x]
```

Maple [A]
time = 0.29, size = 34, normalized size = 1.17

method	result	size
meijerg	$\frac{-ax-\frac{(-2ax+2)\ln(-ax+1)}{2}+ax\,\text{polylog}(2,ax)}{a}$	33
derivativedivides	$\frac{ax\,\text{polylog}(2,ax)-\ln(-ax+1)(-ax+1)+1-ax}{a}$	34
default	$\frac{ax\,\text{polylog}(2,ax)-\ln(-ax+1)(-ax+1)+1-ax}{a}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x*polylog(2,a*x)-ln(-a*x+1)*(-a*x+1)+1-a*x)
```

Maxima [A]
time = 0.26, size = 29, normalized size = 1.00

$$\frac{ax\text{Li}_2(ax)-ax+(ax-1)\log(-ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x),x, algorithm="maxima")
```

```
[Out] (a*x*dilog(a*x) - a*x + (a*x - 1)*log(-a*x + 1))/a
```

Fricas [A]
time = 0.35, size = 29, normalized size = 1.00

$$\frac{ax\text{Li}_2(ax)-ax+(ax-1)\log(-ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x),x, algorithm="fricas")
```

```
[Out] (a*x*dilog(a*x) - a*x + (a*x - 1)*log(-a*x + 1))/a
```

Sympy [A]
time = 0.25, size = 22, normalized size = 0.76

$$\begin{cases} -x\,\mathrm{Li}_1\,(ax) + x\,\mathrm{Li}_2\,(ax) - x + \frac{\mathrm{Li}_1(ax)}{a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x),x)
```

```
[Out] Piecewise((-x*polylog(1, a*x) + x*polylog(2, a*x) - x + polylog(1, a*x)/a,
Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x), x)
```

Mupad [B]
time = 0.26, size = 32, normalized size = 1.10

$$x\,\mathrm{polylog}(2, a\,x) - \frac{\ln\left(1 - a\,x\right)}{a} - x + x\,\ln\left(1 - a\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x),x)
```

```
[Out] x*polylog(2, a*x) - log(1 - a*x)/a - x + x*log(1 - a*x)
```

3.6 $\int \frac{\text{PolyLog}(2,ax)}{x}\,dx$

Optimal. Leaf size=5

$$\text{PolyLog}(3,ax)$$

```
[Out] polylog(3,a*x)
```

Rubi [A]
time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6724}

$$\text{Li}_3(ax)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x]/x,x]
```

```
[Out] PolyLog[3, a*x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\text{Li}_2(ax)}{x}\,dx = \text{Li}_3(ax)$$

Mathematica [A]
time = 0.00, size = 5, normalized size = 1.00

$$\text{PolyLog}(3,ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x]/x,x]
```

```
[Out] PolyLog[3, a*x]
```

Maple [A]
time = 0.09, size = 6, normalized size = 1.20

method	result	size
derivativedivides	$\text{polylog}(3, ax)$	6
default	$\text{polylog}(3, ax)$	6
meijerg	$\text{polylog}(3, ax)$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] polylog(3,a*x)
```

Maxima [A]
time = 0.28, size = 5, normalized size = 1.00

$$\text{Li}_3(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x,x, algorithm="maxima")
```

```
[Out] polylog(3, a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x,x, algorithm="fricas")
```

```
[Out] integral(dilog(a*x)/x, x)
```

Sympy [A]
time = 0.40, size = 3, normalized size = 0.60

$$\text{Li}_3(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x,x)
```

```
[Out] polylog(3, a*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x)/x, x)
```

Mupad [B]
time = 0.18, size = 5, normalized size = 1.00

$$\mathrm{polylog}(3, a\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x)/x,x)
```

```
[Out] polylog(3, a*x)
```

3.7 $\int \frac{\mathbf{PolyLog}(2,ax)}{x^2}\,dx$

Optimal. Leaf size=36

$$a\log(x) - a\log(1-ax) + \frac{\log(1-ax)}{x} - \frac{\mathrm{PolyLog}(2,ax)}{x}$$

```
[Out] a*ln(x)-a*ln(-a*x+1)+ln(-a*x+1)/x-polylog(2,a*x)/x
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6726, 2442, 36, 29, 31}

$$-\frac{\mathrm{Li}_2(ax)}{x} + a\log(x) - a\log(1-ax) + \frac{\log(1-ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x]/x^2,x]
```

```
[Out] a*Log[x] - a*Log[1 - a*x] + Log[1 - a*x]/x - PolyLog[2, a*x]/x
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_2(ax)}{x^2}\,dx &= -\frac{\text{Li}_2(ax)}{x}-\int \frac{\log(1-ax)}{x^2}\,dx\\ &= \frac{\log(1-ax)}{x}-\frac{\text{Li}_2(ax)}{x}+a\int \frac{1}{x(1-ax)}\,dx\\ &= \frac{\log(1-ax)}{x}-\frac{\text{Li}_2(ax)}{x}+a\int \frac{1}{x}\,dx+a^2\int \frac{1}{1-ax}\,dx\\ &= a\log(x)-a\log(1-ax)+\frac{\log(1-ax)}{x}-\frac{\text{Li}_2(ax)}{x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 1.00

$$a\log(x)-a\log(1-ax)+\frac{\log(1-ax)}{x}-\frac{\text{PolyLog}(2,ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x]/x^2,x]
[Out] a*Log[x] - a*Log[1 - a*x] + Log[1 - a*x]/x - PolyLog[2, a*x]/x
```

Maple [A]
time = 0.39, size = 42, normalized size = 1.17

method	result	size
derivativedivides	$a\left(-\frac{\text{polylog}(2,ax)}{ax}+\ln(-ax)+\frac{\ln(-ax+1)(-ax+1)}{ax}\right)$	42
default	$a\left(-\frac{\text{polylog}(2,ax)}{ax}+\ln(-ax)+\frac{\ln(-ax+1)(-ax+1)}{ax}\right)$	42
meijerg	$a\left(\frac{(-4ax+4)\ln(-ax+1)}{4ax}-\frac{\text{polylog}(2,ax)}{ax}+\ln(x)+\ln(-a)\right)$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x)/x^2,x,method=_RETURNVERBOSE)
[Out] a*(-1/a/x*polylog(2,a*x)+ln(-a*x)+ln(-a*x+1)*(-a*x+1)/a/x)
```

Maxima [A]
time = 0.27, size = 28, normalized size = 0.78

$$a\log(x) - \frac{(ax-1)\log(-ax+1) + \mathrm{Li}_2(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^2,x, algorithm="maxima")
[Out] a*log(x) - ((a*x - 1)*log(-a*x + 1) + dilog(a*x))/x
```

Fricas [A]
time = 0.35, size = 34, normalized size = 0.94

$$-\frac{ax\log(ax-1) - ax\log(x) + \mathrm{Li}_2(ax) - \log(-ax+1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^2,x, algorithm="fricas")
[Out] -(a*x*log(a*x - 1) - a*x*log(x) + dilog(a*x) - log(-a*x + 1))/x
```

Sympy [A]
time = 0.37, size = 24, normalized size = 0.67

$$a\log(x) + a\,\mathrm{Li}_1(ax) - \frac{\mathrm{Li}_1(ax)}{x} - \frac{\mathrm{Li}_2(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x**2,x)
[Out] a*log(x) + a*polylog(1, a*x) - polylog(1, a*x)/x - polylog(2, a*x)/x
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^2,x, algorithm="giac")
[Out] integrate(dilog(a*x)/x^2, x)
```

Mupad [B]
time = 0.20, size = 34, normalized size = 0.94

$$\frac{\ln(1-a\,x) - \mathrm{polylog}(2,a\,x)}{x} + a\,\ln(x) - a\,\ln(1-a\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x)/x^2,x)
[Out] (log(1 - a*x) - polylog(2, a*x))/x + a*log(x) - a*log(1 - a*x)
```

3.8 $\int \frac{\text{PolyLog}(2,ax)}{x^3} dx$

Optimal. Leaf size=58

$$-\frac{a}{4x}+\frac{1}{4}a^2\log(x)-\frac{1}{4}a^2\log(1-ax)+\frac{\log(1-ax)}{4x^2}-\frac{\text{PolyLog}(2,ax)}{2x^2}$$

```
[Out] -1/4*a/x+1/4*a^2*ln(x)-1/4*a^2*ln(-a*x+1)+1/4*ln(-a*x+1)/x^2-1/2*polylog(2,
a*x)/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6726, 2442, 46}

$$\frac{1}{4}a^2\log(x)-\frac{1}{4}a^2\log(1-ax)-\frac{\text{Li}_2(ax)}{2x^2}+\frac{\log(1-ax)}{4x^2}-\frac{a}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x]/x^3,x]
```

```
[Out] -1/4*a/x + (a^2*Log[x])/4 - (a^2*Log[1 - a*x])/4 + Log[1 - a*x]/(4*x^2) - P
olyLog[2, a*x]/(2*x^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{Li}_2(ax)}{x^3}\,dx &= -\frac{\text{Li}_2(ax)}{2x^2}-\frac{1}{2}\int \frac{\log(1-ax)}{x^3}\,dx\\
&= \frac{\log(1-ax)}{4x^2}-\frac{\text{Li}_2(ax)}{2x^2}+\frac{1}{4}a\int \frac{1}{x^2(1-ax)}\,dx\\
&= \frac{\log(1-ax)}{4x^2}-\frac{\text{Li}_2(ax)}{2x^2}+\frac{1}{4}a\int \left(\frac{1}{x^2}+\frac{a}{x}-\frac{a^2}{-1+ax}\right)dx\\
&= -\frac{a}{4x}+\frac{1}{4}a^2\log(x)-\frac{1}{4}a^2\log(1-ax)+\frac{\log(1-ax)}{4x^2}-\frac{\text{Li}_2(ax)}{2x^2}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 0.86

$$\frac{-ax+a^2x^2\log(x)+\log(1-ax)-a^2x^2\log(1-ax)-2\text{PolyLog}(2,ax)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x]/x^3,x]
```

```
[Out] (-(a*x) + a^2*x^2*Log[x] + Log[1 - a*x] - a^2*x^2*Log[1 - a*x] - 2*PolyLog[
2, a*x])/(4*x^2)
```

Maple [A]
time = 0.44, size = 61, normalized size = 1.05

method	result	size
derivativedivides	$a^2\left(-\frac{\text{polylog}(2,ax)}{2a^2x^2}-\frac{1}{4ax}+\frac{\ln(-ax)}{4}-\frac{\ln(-ax+1)(-ax+1)(-ax-1)}{4a^2x^2}\right)$	61
default	$a^2\left(-\frac{\text{polylog}(2,ax)}{2a^2x^2}-\frac{1}{4ax}+\frac{\ln(-ax)}{4}-\frac{\ln(-ax+1)(-ax+1)(-ax-1)}{4a^2x^2}\right)$	61
meijerg	$-a^2\left(-\frac{9ax+27}{36ax}-\frac{(-9a^2x^2+9)\ln(-ax+1)}{36a^2x^2}+\frac{\text{polylog}(2,ax)}{2a^2x^2}+\frac{1}{4}-\frac{\ln(x)}{4}-\frac{\ln(-a)}{4}+\frac{1}{ax}\right)$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2/a^2/x^2*polylog(2,a*x)-1/4/a/x+1/4*ln(-a*x)-1/4*ln(-a*x+1)*(-a*x+
1)*(-a*x-1)/a^2/x^2)
```

Maxima [A]
time = 0.26, size = 40, normalized size = 0.69

$$\frac{1}{4}\,a^2\log(x)-\frac{ax+(a^2x^2-1)\log(-ax+1)+2\,\text{Li}_2(ax)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*log(x) - 1/4*(a*x + (a^2*x^2 - 1)*log(-a*x + 1) + 2*dilog(a*x))/x^2
```

Fricas [A]
time = 0.36, size = 47, normalized size = 0.81

$$-\frac{a^2x^2\log(ax-1)-a^2x^2\log(x)+ax+2\operatorname{Li}_2(ax)-\log(-ax+1)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(a^2*x^2*log(a*x - 1) - a^2*x^2*log(x) + a*x + 2*dilog(a*x) - log(-a*x
 + 1))/x^2
```

Sympy [A]
time = 0.67, size = 42, normalized size = 0.72

$$\frac{a^2\log(x)}{4}+\frac{a^2\operatorname{Li}_1(ax)}{4}-\frac{a}{4x}-\frac{\operatorname{Li}_1(ax)}{4x^2}-\frac{\operatorname{Li}_2(ax)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x**3,x)
```

```
[Out] a**2*log(x)/4 + a**2*polylog(1, a*x)/4 - a/(4*x) - polylog(1, a*x)/(4*x**2)
 - polylog(2, a*x)/(2*x**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x)/x^3, x)
```

Mupad [B]
time = 0.30, size = 51, normalized size = 0.88

$$\frac{a^2\,\ln(x)}{2}+\frac{\ln(1-a\,x)}{4\,x^2}-\frac{a^2\,\ln(a\,x^2-x)}{4}-\frac{a}{4\,x}-\frac{\mathrm{polylog}(2,a\,x)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x)/x^3,x)
```

```
[Out] (a^2*log(x))/2 + log(1 - a*x)/(4*x^2) - (a^2*log(a*x^2 - x))/4 - a/(4*x) -
polylog(2, a*x)/(2*x^2)
```

3.9 $\int \frac{\text{PolyLog}(2,ax)}{x^4} dx$

Optimal. Leaf size=68

$$-\frac{a}{18x^2}-\frac{a^2}{9x}+\frac{1}{9}a^3\log(x)-\frac{1}{9}a^3\log(1-ax)+\frac{\log(1-ax)}{9x^3}-\frac{\text{PolyLog}(2,ax)}{3x^3}$$

```
[Out] -1/18*a/x^2-1/9*a^2/x+1/9*a^3*ln(x)-1/9*a^3*ln(-a*x+1)+1/9*ln(-a*x+1)/x^3-1
/3*polylog(2,a*x)/x^3
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6726, 2442, 46}

$$\frac{1}{9}a^3\log(x)-\frac{1}{9}a^3\log(1-ax)-\frac{a^2}{9x}-\frac{\text{Li}_2(ax)}{3x^3}+\frac{\log(1-ax)}{9x^3}-\frac{a}{18x^2}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x]/x^4,x]
```

```
[Out] -1/18*a/x^2 - a^2/(9*x) + (a^3*Log[x])/9 - (a^3*Log[1 - a*x])/9 + Log[1 - a
*x]/(9*x^3) - PolyLog[2, a*x]/(3*x^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\mathrm{Li}_2(ax)}{x^4}\,dx &= -\frac{\mathrm{Li}_2(ax)}{3x^3} - \frac{1}{3}\int \frac{\log(1-ax)}{x^4}\,dx \\
&= \frac{\log(1-ax)}{9x^3} - \frac{\mathrm{Li}_2(ax)}{3x^3} + \frac{1}{9}a\int \frac{1}{x^3(1-ax)}\,dx \\
&= \frac{\log(1-ax)}{9x^3} - \frac{\mathrm{Li}_2(ax)}{3x^3} + \frac{1}{9}a\int \left(\frac{1}{x^3} + \frac{a}{x^2} + \frac{a^2}{x} - \frac{a^3}{-1+ax}\right)dx \\
&= -\frac{a}{18x^2} - \frac{a^2}{9x} + \frac{1}{9}a^3\log(x) - \frac{1}{9}a^3\log(1-ax) + \frac{\log(1-ax)}{9x^3} - \frac{\mathrm{Li}_2(ax)}{3x^3}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 52, normalized size = 0.76

$$-\frac{ax(1+2ax) - 2a^3x^3\log(x) + 2(-1+a^3x^3)\log(1-ax) + 6\mathrm{PolyLog}(2,ax)}{18x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x]/x^4,x]
[Out] -1/18*(a*x*(1 + 2*a*x) - 2*a^3*x^3*Log[x] + 2*(-1 + a^3*x^3)*Log[1 - a*x] +
 6*PolyLog[2, a*x])/x^3
```

Maple [A]
time = 0.45, size = 76, normalized size = 1.12

method	result
derivativedivides	$a^3\left(-\frac{\mathrm{polylog}(2,ax)}{3a^3x^3} - \frac{1}{9ax} - \frac{1}{18a^2x^2} + \frac{\ln(-ax)}{9} + \frac{\ln(-ax+1)(-ax+1)\left((-ax+1)^2+3ax\right)}{9a^3x^3}\right)$
default	$a^3\left(-\frac{\mathrm{polylog}(2,ax)}{3a^3x^3} - \frac{1}{9ax} - \frac{1}{18a^2x^2} + \frac{\ln(-ax)}{9} + \frac{\ln(-ax+1)(-ax+1)\left((-ax+1)^2+3ax\right)}{9a^3x^3}\right)$
meijerg	$a^3\left(\frac{32a^2x^2+60ax+192}{432a^2x^2} + \frac{(-16a^3x^3+16)\ln(-ax+1)}{144a^3x^3} - \frac{\mathrm{polylog}(2,ax)}{3a^3x^3} - \frac{2}{27} + \frac{\ln(x)}{9} + \frac{\ln(-a)}{9} - \frac{1}{2a^2x^2} - \frac{1}{4a\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x)/x^4,x,method=_RETURNVERBOSE)
[Out] a^3*(-1/3/a^3/x^3*polylog(2,a*x)-1/9/a/x-1/18/a^2/x^2+1/9*ln(-a*x)+1/9*ln(-
a*x+1)*(-a*x+1)*((-a*x+1)^2+3*a*x)/a^3/x^3)
```

Maxima [A]
time = 0.27, size = 49, normalized size = 0.72

$$\frac{1}{9}\,a^3\log\left(x\right) - \frac{2\,a^2x^2 + ax + 2\left(a^3x^3-1\right)\log\left(-ax+1\right) + 6\,\mathrm{Li}_2(ax)}{18\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*a^3*log(x) - 1/18*(2*a^2*x^2 + a*x + 2*(a^3*x^3 - 1)*log(-a*x + 1) + 6*
dilog(a*x))/x^3
```

Fricas [A]
time = 0.39, size = 56, normalized size = 0.82

$$-\frac{2\,a^3x^3\log\left(ax-1\right)-2\,a^3x^3\log\left(x\right)+2\,a^2x^2+ax+6\,\mathrm{Li}_2(ax)-2\,\log\left(-ax+1\right)}{18\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/18*(2*a^3*x^3*log(a*x - 1) - 2*a^3*x^3*log(x) + 2*a^2*x^2 + a*x + 6*dilo
g(a*x) - 2*log(-a*x + 1))/x^3
```

Sympy [A]
time = 1.23, size = 51, normalized size = 0.75

$$\frac{a^3\log\left(x\right)}{9}+\frac{a^3\,\mathrm{Li}_1\left(ax\right)}{9}-\frac{a^2}{9x}-\frac{a}{18x^2}-\frac{\mathrm{Li}_1\left(ax\right)}{9x^3}-\frac{\mathrm{Li}_2\left(ax\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x**4,x)
```

```
[Out] a**3*log(x)/9 + a**3*polylog(1, a*x)/9 - a**2/(9*x) - a/(18*x**2) - polylog
(1, a*x)/(9*x**3) - polylog(2, a*x)/(3*x**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x)/x^4, x)
```

Mupad [B]
time = 0.32, size = 57, normalized size = 0.84

$$\frac{2\,a^3\,\ln\left(x\right)}{9}-\frac{\frac{a\,x}{18}-\frac{\ln(1-a\,x)}{9}+\frac{\mathrm{polylog}(2,a\,x)}{3}+\frac{a^2\,x^2}{9}}{x^3}-\frac{a^3\,\ln\left(a\,x^2-x\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x)/x^4,x)
```

```
[Out] (2*a^3*log(x))/9 - ((a*x)/18 - log(1 - a*x)/9 + polylog(2, a*x)/3 + (a^2*x^
2)/9)/x^3 - (a^3*log(a*x^2 - x))/9
```

3.10 $\int \frac{\text{PolyLog}(2,ax)}{x^5}\,dx$

Optimal. Leaf size=78

$$-\frac{a}{48x^3}-\frac{a^2}{32x^2}-\frac{a^3}{16x}+\frac{1}{16}a^4\log(x)-\frac{1}{16}a^4\log(1-ax)+\frac{\log(1-ax)}{16x^4}-\frac{\text{PolyLog}(2,ax)}{4x^4}$$

```
[Out] -1/48*a/x^3-1/32*a^2/x^2-1/16*a^3/x+1/16*a^4*ln(x)-1/16*a^4*ln(-a*x+1)+1/16
*ln(-a*x+1)/x^4-1/4*polylog(2,a*x)/x^4
```

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6726, 2442, 46}

$$\frac{1}{16}a^4\log(x)-\frac{1}{16}a^4\log(1-ax)-\frac{a^3}{16x}-\frac{a^2}{32x^2}-\frac{\text{Li}_2(ax)}{4x^4}+\frac{\log(1-ax)}{16x^4}-\frac{a}{48x^3}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x]/x^5,x]
```

```
[Out] -1/48*a/x^3 - a^2/(32*x^2) - a^3/(16*x) + (a^4*Log[x])/16 - (a^4*Log[1 - a*
x])/16 + Log[1 - a*x]/(16*x^4) - PolyLog[2, a*x]/(4*x^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax)}{x^5}\,dx &= -\frac{\text{Li}_2(ax)}{4x^4}-\frac{1}{4}\int \frac{\log(1-ax)}{x^5}\,dx\\
&= \frac{\log(1-ax)}{16x^4}-\frac{\text{Li}_2(ax)}{4x^4}+\frac{1}{16}a\int \frac{1}{x^4(1-ax)}\,dx\\
&= \frac{\log(1-ax)}{16x^4}-\frac{\text{Li}_2(ax)}{4x^4}+\frac{1}{16}a\int \left(\frac{1}{x^4}+\frac{a}{x^3}+\frac{a^2}{x^2}+\frac{a^3}{x}-\frac{a^4}{-1+ax}\right)dx\\
&= -\frac{a}{48x^3}-\frac{a^2}{32x^2}-\frac{a^3}{16x}+\frac{1}{16}a^4\log(x)-\frac{1}{16}a^4\log(1-ax)+\frac{\log(1-ax)}{16x^4}-\frac{\text{Li}_2(ax)}{4x^4}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 60, normalized size = 0.77

$$-\frac{ax(2+3ax+6a^2x^2)-6a^4x^4\log(x)+6(-1+a^4x^4)\log(1-ax)+24\text{PolyLog}(2,ax)}{96x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x]/x^5,x]
```

```
[Out] -1/96*(a*x*(2 + 3*a*x + 6*a^2*x^2) - 6*a^4*x^4*Log[x] + 6*(-1 + a^4*x^4)*Lo
g[1 - a*x] + 24*PolyLog[2, a*x])/x^4
```

Maple [A]
time = 0.44, size = 95, normalized size = 1.22

method	result
derivativedivides	$a^4\left(-\frac{\text{polylog}(2,ax)}{4a^4x^4}-\frac{1}{48a^3x^3}+\frac{\ln(-ax)}{16}-\frac{1}{16ax}-\frac{1}{32a^2x^2}-\frac{\ln(-ax+1)(-ax+1)\left((-ax+1)^3-4(-ax+1)^2\right.}{16a^4x^4}\right.$
default	$a^4\left(-\frac{\text{polylog}(2,ax)}{4a^4x^4}-\frac{1}{48a^3x^3}+\frac{\ln(-ax)}{16}-\frac{1}{16ax}-\frac{1}{32a^2x^2}-\frac{\ln(-ax+1)(-ax+1)\left((-ax+1)^3-4(-ax+1)^2\right.}{16a^4x^4}\right.$
meijerg	$-a^4\left(-\frac{225a^3x^3+350a^2x^2+675ax+2250}{7200a^3x^3}-\frac{(-25a^4x^4+25)\ln(-ax+1)}{400a^4x^4}+\frac{\text{polylog}(2,ax)}{4a^4x^4}+\frac{1}{32}-\frac{\ln(x)}{16}-\frac{\ln(-}{16}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*(-1/4/a^4/x^4*polylog(2,a*x)-1/48/a^3/x^3+1/16*ln(-a*x)-1/16/a/x-1/32/a
^2/x^2-1/16*ln(-a*x+1)*(-a*x+1)*((-a*x+1)^3-4*(-a*x+1)^2+2-6*a*x)/a^4/x^4)
```

Maxima [A]
time = 0.26, size = 58, normalized size = 0.74

$$\frac{1}{16}\,a^4\log\left(x\right)-\frac{6\,a^3x^3+3\,a^2x^2+2\,ax+6\,(a^4x^4-1)\log\left(-ax+1\right)+24\,\text{Li}_2(ax)}{96\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/16*a^4*log(x) - 1/96*(6*a^3*x^3 + 3*a^2*x^2 + 2*a*x + 6*(a^4*x^4 - 1)*log
(-a*x + 1) + 24*dilog(a*x))/x^4
```

Fricas [A]
time = 0.37, size = 65, normalized size = 0.83

$$-\frac{6\,a^4x^4\log\left(ax-1\right)-6\,a^4x^4\log\left(x\right)+6\,a^3x^3+3\,a^2x^2+2\,ax+24\,\mathrm{Li}_2(ax)-6\,\log\left(-ax+1\right)}{96\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^5,x, algorithm="fricas")
```

```
[Out] -1/96*(6*a^4*x^4*log(a*x - 1) - 6*a^4*x^4*log(x) + 6*a^3*x^3 + 3*a^2*x^2 +
2*a*x + 24*dilog(a*x) - 6*log(-a*x + 1))/x^4
```

Sympy [A]
time = 2.29, size = 60, normalized size = 0.77

$$\frac{a^4\log\left(x\right)}{16}+\frac{a^4\,\mathrm{Li}_1\left(ax\right)}{16}-\frac{a^3}{16x}-\frac{a^2}{32x^2}-\frac{a}{48x^3}-\frac{\mathrm{Li}_1\left(ax\right)}{16x^4}-\frac{\mathrm{Li}_2\left(ax\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x**5,x)
```

```
[Out] a**4*log(x)/16 + a**4*polylog(1, a*x)/16 - a**3/(16*x) - a**2/(32*x**2) - a
/(48*x**3) - polylog(1, a*x)/(16*x**4) - polylog(2, a*x)/(4*x**4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x)/x^5, x)
```

Mupad [B]
time = 0.44, size = 60, normalized size = 0.77

$$\frac{\ln\left(1-a\,x\right)}{16\,x^4}-\frac{\mathrm{polylog}(2,a\,x)}{4\,x^4}-\frac{a^3\,x^2+\frac{a^2\,x}{2}+\frac{a}{3}}{16\,x^3}-\frac{a^4\,\mathrm{atan}(a\,x\,2\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x)/x^5,x)
```

```
[Out] log(1 - a*x)/(16*x^4) - (a^4*atan(a*x*2i - 1i)*1i)/8 - polylog(2, a*x)/(4*x
^4) - (a/3 + (a^2*x)/2 + a^3*x^2)/(16*x^3)
```

3.11 $\int x^3\textbf{PolyLog}(3,ax)\,dx$

Optimal. Leaf size=88

$$\frac{x}{64a^3}+\frac{x^2}{128a^2}+\frac{x^3}{192a}+\frac{x^4}{256}+\frac{\log(1-ax)}{64a^4}-\frac{1}{64}x^4\log(1-ax)-\frac{1}{16}x^4\text{PolyLog}(2,ax)+\frac{1}{4}x^4\text{PolyLog}(3,ax)$$

```
[Out] 1/64*x/a^3+1/128*x^2/a^2+1/192*x^3/a+1/256*x^4+1/64*ln(-a*x+1)/a^4-1/64*x^4
*ln(-a*x+1)-1/16*x^4*polylog(2,a*x)+1/4*x^4*polylog(3,a*x)
```

Rubi [A]
time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6726, 2442, 45}

$$\frac{\log(1-ax)}{64a^4}+\frac{x}{64a^3}+\frac{x^2}{128a^2}-\frac{1}{16}x^4\text{Li}_2(ax)+\frac{1}{4}x^4\text{Li}_3(ax)-\frac{1}{64}x^4\log(1-ax)+\frac{x^3}{192a}+\frac{x^4}{256}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*PolyLog[3, a*x],x]
```

```
[Out] x/(64*a^3) + x^2/(128*a^2) + x^3/(192*a) + x^4/256 + Log[1 - a*x]/(64*a^4)
- (x^4*Log[1 - a*x])/64 - (x^4*PolyLog[2, a*x])/16 + (x^4*PolyLog[3, a*x])/
4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int x^3\text{Li}_3(ax)\,dx = \frac{1}{4}x^4\text{Li}_3(ax) - \frac{1}{4}\int x^3\text{Li}_2(ax)\,dx$$
$$= -\frac{1}{16}x^4\text{Li}_2(ax) + \frac{1}{4}x^4\text{Li}_3(ax) - \frac{1}{16}\int x^3\log(1-ax)\,dx$$
$$= -\frac{1}{64}x^4\log(1-ax) - \frac{1}{16}x^4\text{Li}_2(ax) + \frac{1}{4}x^4\text{Li}_3(ax) - \frac{1}{64}a\int \frac{x^4}{1-ax}\,dx$$
$$= -\frac{1}{64}x^4\log(1-ax) - \frac{1}{16}x^4\text{Li}_2(ax) + \frac{1}{4}x^4\text{Li}_3(ax) - \frac{1}{64}a\int \left(-\frac{1}{a^4} - \frac{x}{a^3} - \frac{x^2}{a^2} - \frac{x^3}{a} - \frac{\quad}{a^4(-$$
$$= \frac{x}{64a^3} + \frac{x^2}{128a^2} + \frac{x^3}{192a} + \frac{x^4}{256} + \frac{\log(1-ax)}{64a^4} - \frac{1}{64}x^4\log(1-ax) - \frac{1}{16}x^4\text{Li}_2(ax) + \frac{1}{4}x^4\text{Li}_3($$

Mathematica [A]
time = 0.01, size = 86, normalized size = 0.98

$$\frac{12ax + 6a^2x^2 + 4a^3x^3 + 3a^4x^4 + 12\log(1-ax) - 12a^4x^4\log(1-ax) - 48a^4x^4\text{PolyLog}(2,ax) + 192a^4x^4\text{PolyLog}(3,ax)}{768a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*PolyLog[3, a*x],x]
```

```
[Out] (12*a*x + 6*a^2*x^2 + 4*a^3*x^3 + 3*a^4*x^4 + 12*Log[1 - a*x] - 12*a^4*x^4*
Log[1 - a*x] - 48*a^4*x^4*PolyLog[2, a*x] + 192*a^4*x^4*PolyLog[3, a*x]])/(7
68*a^4)
```

Maple [A]
time = 0.13, size = 78, normalized size = 0.89

method	result	size
meijerg	$-\frac{-\frac{ax\left(15a^3x^3+20a^2x^2+30ax+60\right)}{3840}-\frac{\left(-5a^4x^4+5\right)\ln(-ax+1)}{320}+\frac{a^4x^4\,\text{polylog}(2,ax)}{16}-\frac{a^4x^4\,\text{polylog}(3,ax)}{4}}{a^4}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*polylog(3,a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^4*(-1/3840*a*x*(15*a^3*x^3+20*a^2*x^2+30*a*x+60)-1/320*(-5*a^4*x^4+5)*
ln(-a*x+1)+1/16*a^4*x^4*polylog(2,a*x)-1/4*a^4*x^4*polylog(3,a*x))
```

Maxima [A]
time = 0.26, size = 77, normalized size = 0.88

$$-\frac{48\,a^4x^4\text{Li}_2(ax) - 192\,a^4x^4\text{Li}_3(ax) - 3\,a^4x^4 - 4\,a^3x^3 - 6\,a^2x^2 - 12\,ax + 12\,(a^4x^4-1)\log(-ax+1)}{768\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(3,a*x),x, algorithm="maxima")
```

```
[Out] -1/768*(48*a^4*x^4*dilog(a*x) - 192*a^4*x^4*polylog(3, a*x) - 3*a^4*x^4 - 4
*a^3*x^3 - 6*a^2*x^2 - 12*a*x + 12*(a^4*x^4 - 1)*log(-a*x + 1))/a^4
```

Fricas [A]
time = 0.36, size = 77, normalized size = 0.88

$$-\frac{48\,a^4x^4\mathrm{Li}_2(ax)-192\,a^4x^4\mathrm{polylog}(3,ax)-3\,a^4x^4-4\,a^3x^3-6\,a^2x^2-12\,ax+12\,(a^4x^4-1)\log{(-ax+1)}}{768\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(3,a*x),x, algorithm="fricas")
```

```
[Out] -1/768*(48*a^4*x^4*dilog(a*x) - 192*a^4*x^4*polylog(3, a*x) - 3*a^4*x^4 - 4
*a^3*x^3 - 6*a^2*x^2 - 12*a*x + 12*(a^4*x^4 - 1)*log(-a*x + 1))/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\,\mathrm{Li}_3\,(ax)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*polylog(3,a*x),x)
```

```
[Out] Integral(x**3*polylog(3, a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(3,a*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*polylog(3, a*x), x)
```

Mupad [B]
time = 0.82, size = 71, normalized size = 0.81

$$\frac{\ln{(a\,x-1)}}{64\,a^4}-\frac{x^4\,\ln{(1-a\,x)}}{64}+\frac{x}{64\,a^3}+\frac{x^4}{256}-\frac{x^4\,\mathrm{polylog}(2,a\,x)}{16}+\frac{x^4\,\mathrm{polylog}(3,a\,x)}{4}+\frac{x^3}{192\,a}+\frac{x^2}{128\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*polylog(3, a*x),x)
```

```
[Out] log(a*x - 1)/(64*a^4) - (x^4*log(1 - a*x))/64 + x/(64*a^3) + x^4/256 - (x^4
*polylog(2, a*x))/16 + (x^4*polylog(3, a*x))/4 + x^3/(192*a) + x^2/(128*a^2
)
```

3.12 $\int x^2 \textbf{PolyLog}(3, ax)\, dx$

Optimal. Leaf size=78

$$\frac{x}{27a^2}+\frac{x^2}{54a}+\frac{x^3}{81}+\frac{\log(1-ax)}{27a^3}-\frac{1}{27}x^3\log(1-ax)-\frac{1}{9}x^3\text{PolyLog}(2,ax)+\frac{1}{3}x^3\text{PolyLog}(3,ax)$$

[Out] 1/27*x/a^2+1/54*x^2/a+1/81*x^3+1/27*ln(-a*x+1)/a^3-1/27*x^3*ln(-a*x+1)-1/9*x^3*polylog(2,a*x)+1/3*x^3*polylog(3,a*x)

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6726, 2442, 45}

$$\frac{\log(1-ax)}{27a^3}+\frac{x}{27a^2}-\frac{1}{9}x^3\text{Li}_2(ax)+\frac{1}{3}x^3\text{Li}_3(ax)-\frac{1}{27}x^3\log(1-ax)+\frac{x^2}{54a}+\frac{x^3}{81}$$

Antiderivative was successfully verified.

[In] Int[x^2*PolyLog[3, a*x],x]

[Out] x/(27*a^2) + x^2/(54*a) + x^3/81 + Log[1 - a*x]/(27*a^3) - (x^3*Log[1 - a*x])/27 - (x^3*PolyLog[2, a*x])/9 + (x^3*PolyLog[3, a*x])/3

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\text{Li}_3(ax)\,dx &= \frac{1}{3}x^3\text{Li}_3(ax)-\frac{1}{3}\int x^2\text{Li}_2(ax)\,dx\\ &= -\frac{1}{9}x^3\text{Li}_2(ax)+\frac{1}{3}x^3\text{Li}_3(ax)-\frac{1}{9}\int x^2\log(1-ax)\,dx\\ &= -\frac{1}{27}x^3\log(1-ax)-\frac{1}{9}x^3\text{Li}_2(ax)+\frac{1}{3}x^3\text{Li}_3(ax)-\frac{1}{27}a\int\frac{x^3}{1-ax}\,dx\\ &= -\frac{1}{27}x^3\log(1-ax)-\frac{1}{9}x^3\text{Li}_2(ax)+\frac{1}{3}x^3\text{Li}_3(ax)-\frac{1}{27}a\int\left(-\frac{1}{a^3}-\frac{x}{a^2}-\frac{x^2}{a}-\frac{1}{a^3(-1+a\right.\\ &= \frac{x}{27a^2}+\frac{x^2}{54a}+\frac{x^3}{81}+\frac{\log(1-ax)}{27a^3}-\frac{1}{27}x^3\log(1-ax)-\frac{1}{9}x^3\text{Li}_2(ax)+\frac{1}{3}x^3\text{Li}_3(ax)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 78, normalized size = 1.00

$$\frac{6ax+3a^2x^2+2a^3x^3+6\log(1-ax)-6a^3x^3\log(1-ax)-18a^3x^3\text{PolyLog}(2,ax)+54a^3x^3\text{PolyLog}(3,ax)}{162a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*PolyLog[3, a*x],x]
```

```
[Out] (6*a*x + 3*a^2*x^2 + 2*a^3*x^3 + 6*Log[1 - a*x] - 6*a^3*x^3*Log[1 - a*x] -
18*a^3*x^3*PolyLog[2, a*x] + 54*a^3*x^3*PolyLog[3, a*x])/(162*a^3)
```

Maple [A]
time = 0.13, size = 69, normalized size = 0.88

method	result	size
meijerg	$\frac{\frac{ax\left(4a^2x^2+6ax+12\right)}{324}+\frac{\left(-4a^3x^3+4\right)\ln(-ax+1)}{108}-\frac{a^3x^3\,\text{polylog}(2,ax)}{9}+\frac{a^3x^3\,\text{polylog}(3,ax)}{3}}{a^3}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(3,a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/324*a*x*(4*a^2*x^2+6*a*x+12)+1/108*(-4*a^3*x^3+4)*ln(-a*x+1)-1/9*a
^3*x^3*polylog(2,a*x)+1/3*a^3*x^3*polylog(3,a*x))
```

Maxima [A]
time = 0.26, size = 69, normalized size = 0.88

$$-\frac{18\,a^3x^3\text{Li}_2(ax)-54\,a^3x^3\text{Li}_3(ax)-2\,a^3x^3-3\,a^2x^2-6\,ax+6\,(a^3x^3-1)\log(-ax+1)}{162\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,a*x),x, algorithm="maxima")
```

```
[Out] -1/162*(18*a^3*x^3*dilog(a*x) - 54*a^3*x^3*polylog(3, a*x) - 2*a^3*x^3 - 3*
a^2*x^2 - 6*a*x + 6*(a^3*x^3 - 1)*log(-a*x + 1))/a^3
```

Fricas [A]
time = 0.35, size = 69, normalized size = 0.88

$$-\frac{18\,a^3x^3\mathrm{Li}_2(ax) - 54\,a^3x^3\mathrm{polylog}(3,ax) - 2\,a^3x^3 - 3\,a^2x^2 - 6\,ax + 6\,(a^3x^3 - 1)\log\left(-ax+1\right)}{162\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,a*x),x, algorithm="fricas")
```

```
[Out] -1/162*(18*a^3*x^3*dilog(a*x) - 54*a^3*x^3*polylog(3, a*x) - 2*a^3*x^3 - 3*
a^2*x^2 - 6*a*x + 6*(a^3*x^3 - 1)*log(-a*x + 1))/a^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\mathrm{Li}_3\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*polylog(3,a*x),x)
```

```
[Out] Integral(x**2*polylog(3, a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,a*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*polylog(3, a*x), x)
```

Mupad [B]
time = 0.95, size = 63, normalized size = 0.81

$$\frac{\ln\left(a\,x-1\right)}{27\,a^3}-\frac{x^3\,\ln\left(1-a\,x\right)}{27}+\frac{x}{27\,a^2}+\frac{x^3}{81}-\frac{x^3\,\mathrm{polylog}(2,a\,x)}{9}+\frac{x^3\,\mathrm{polylog}(3,a\,x)}{3}+\frac{x^2}{54\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(3, a*x),x)
```

```
[Out] log(a*x - 1)/(27*a^3) - (x^3*log(1 - a*x))/27 + x/(27*a^2) + x^3/81 - (x^3*
polylog(2, a*x))/9 + (x^3*polylog(3, a*x))/3 + x^2/(54*a)
```

3.13 $\int x\mathbf{PolyLog}(3, ax)\,dx$

Optimal. Leaf size=68

$$\frac{x}{8a}+\frac{x^2}{16}+\frac{\log(1-ax)}{8a^2}-\frac{1}{8}x^2\log(1-ax)-\frac{1}{4}x^2\text{PolyLog}(2,ax)+\frac{1}{2}x^2\text{PolyLog}(3,ax)$$

```
[Out] 1/8*x/a+1/16*x^2+1/8*ln(-a*x+1)/a^2-1/8*x^2*ln(-a*x+1)-1/4*x^2*polylog(2,a*
x)+1/2*x^2*polylog(3,a*x)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6726, 2442, 45}

$$\frac{\log(1-ax)}{8a^2}-\frac{1}{4}x^2\text{Li}_2(ax)+\frac{1}{2}x^2\text{Li}_3(ax)-\frac{1}{8}x^2\log(1-ax)+\frac{x}{8a}+\frac{x^2}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x*PolyLog[3, a*x],x]
```

```
[Out] x/(8*a) + x^2/16 + Log[1 - a*x]/(8*a^2) - (x^2*Log[1 - a*x])/8 - (x^2*PolyL
og[2, a*x])/4 + (x^2*PolyLog[3, a*x])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int x\text{Li}_3(ax)\,dx &= \frac{1}{2}x^2\text{Li}_3(ax) - \frac{1}{2}\int x\text{Li}_2(ax)\,dx \\ &= -\frac{1}{4}x^2\text{Li}_2(ax) + \frac{1}{2}x^2\text{Li}_3(ax) - \frac{1}{4}\int x\log(1-ax)\,dx \\ &= -\frac{1}{8}x^2\log(1-ax) - \frac{1}{4}x^2\text{Li}_2(ax) + \frac{1}{2}x^2\text{Li}_3(ax) - \frac{1}{8}a\int \frac{x^2}{1-ax}\,dx \\ &= -\frac{1}{8}x^2\log(1-ax) - \frac{1}{4}x^2\text{Li}_2(ax) + \frac{1}{2}x^2\text{Li}_3(ax) - \frac{1}{8}a\int \left(-\frac{1}{a^2} - \frac{x}{a} - \frac{1}{a^2(-1+ax)}\right)dx \\ &= \frac{x}{8a} + \frac{x^2}{16} + \frac{\log(1-ax)}{8a^2} - \frac{1}{8}x^2\log(1-ax) - \frac{1}{4}x^2\text{Li}_2(ax) + \frac{1}{2}x^2\text{Li}_3(ax)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 69, normalized size = 1.01

$$\frac{2ax + a^2x^2 + 2\log(1-ax) - 2a^2x^2\log(1-ax) - 4a^2x^2\text{PolyLog}(2,ax) + 8a^2x^2\text{PolyLog}(3,ax)}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*PolyLog[3, a*x],x]
```

```
[Out] (2*a*x + a^2*x^2 + 2*Log[1 - a*x] - 2*a^2*x^2*Log[1 - a*x] - 4*a^2*x^2*Poly
Log[2, a*x] + 8*a^2*x^2*PolyLog[3, a*x])/(16*a^2)
```

Maple [A]
time = 0.14, size = 62, normalized size = 0.91

method	result	size
meijerg	$-\frac{-\frac{ax(3ax+6)}{48}-\frac{\left(-3a^2x^2+3\right)\ln(-ax+1)}{24}+\frac{a^2x^2\,\text{polylog}(2,ax)}{4}-\frac{a^2x^2\,\text{polylog}(3,ax)}{2}}{a^2}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(3,a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*(-1/48*a*x*(3*a*x+6)-1/24*(-3*a^2*x^2+3)*ln(-a*x+1)+1/4*a^2*x^2*poly
log(2,a*x)-1/2*a^2*x^2*polylog(3,a*x))
```

Maxima [A]
time = 0.26, size = 61, normalized size = 0.90

$$-\frac{4\,a^2x^2\text{Li}_2(ax) - 8\,a^2x^2\text{Li}_3(ax) - a^2x^2 - 2\,ax + 2\,(a^2x^2-1)\log(-ax+1)}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x),x, algorithm="maxima")
```

```
[Out] -1/16*(4*a^2*x^2*dilog(a*x) - 8*a^2*x^2*polylog(3, a*x) - a^2*x^2 - 2*a*x +
 2*(a^2*x^2 - 1)*log(-a*x + 1))/a^2
```

Fricas [A]
time = 0.39, size = 61, normalized size = 0.90

$$-\frac{4\,a^2x^2\mathrm{Li}_2(ax)-8\,a^2x^2\mathrm{polylog}(3,ax)-a^2x^2-2\,ax+2\,(a^2x^2-1)\log\left(-ax+1\right)}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x),x, algorithm="fricas")
```

```
[Out] -1/16*(4*a^2*x^2*dilog(a*x) - 8*a^2*x^2*polylog(3, a*x) - a^2*x^2 - 2*a*x +
 2*(a^2*x^2 - 1)*log(-a*x + 1))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{Li}_3\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x),x)
```

```
[Out] Integral(x*polylog(3, a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x),x, algorithm="giac")
```

```
[Out] integrate(x*polylog(3, a*x), x)
```

Mupad [B]
time = 0.90, size = 55, normalized size = 0.81

$$\frac{\ln\left(a\,x-1\right)}{8\,a^2}-\frac{x^2\,\ln\left(1-a\,x\right)}{8}+\frac{x}{8\,a}+\frac{x^2}{16}-\frac{x^2\,\mathrm{polylog}(2,a\,x)}{4}+\frac{x^2\,\mathrm{polylog}(3,a\,x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(3, a*x),x)
```

```
[Out] log(a*x - 1)/(8*a^2) - (x^2*log(1 - a*x))/8 + x/(8*a) + x^2/16 - (x^2*polyl
og(2, a*x))/4 + (x^2*polylog(3, a*x))/2
```

3.14 $\int \mathbf{PolyLog}(3, ax)\,dx$

Optimal. Leaf size=34

$$x+\frac{(1-ax)\log(1-ax)}{a}-x\text{PolyLog}(2,ax)+x\text{PolyLog}(3,ax)$$

```
[Out] x+(-a*x+1)*ln(-a*x+1)/a-x*polylog(2,a*x)+x*polylog(3,a*x)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6721, 2436, 2332}

$$x(-\text{Li}_2(ax))+x\text{Li}_3(ax)+\frac{(1-ax)\log(1-ax)}{a}+x$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x],x]
```

```
[Out] x + ((1 - a*x)*Log[1 - a*x])/a - x*PolyLog[2, a*x] + x*PolyLog[3, a*x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \mathrm{Li}_3(ax)\,dx &= x\mathrm{Li}_3(ax) - \int \mathrm{Li}_2(ax)\,dx \\
&= -x\mathrm{Li}_2(ax) + x\mathrm{Li}_3(ax) - \int \log(1-ax)\,dx \\
&= -x\mathrm{Li}_2(ax) + x\mathrm{Li}_3(ax) + \frac{\mathrm{Subst}(\int \log(x)\,dx, x, 1-ax)}{a} \\
&= x + \frac{(1-ax)\log(1-ax)}{a} - x\mathrm{Li}_2(ax) + x\mathrm{Li}_3(ax)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.15

$$
x\left(1-\log(1-ax)+\frac{\log(1-ax)}{ax}-\mathrm{PolyLog}(2,ax)+\mathrm{PolyLog}(3,ax)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x],x]
```

```
[Out] x*(1 - Log[1 - a*x] + Log[1 - a*x]/(a*x) - PolyLog[2, a*x] + PolyLog[3, a*x
])
```

Maple [A]
time = 0.09, size = 41, normalized size = 1.21

method	result	size
meijerg	$\frac{ax+\frac{(-2ax+2)\ln(-ax+1)}{2}-ax\,\mathrm{polylog}(2,ax)+ax\,\mathrm{polylog}(3,ax)}{a}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x+1/2*(-2*a*x+2)*ln(-a*x+1)-a*x*polylog(2,a*x)+a*x*polylog(3,a*x))
```

Maxima [A]
time = 0.27, size = 39, normalized size = 1.15

$$
-\frac{ax\mathrm{Li}_2(ax) - ax\mathrm{Li}_3(ax) - ax + (ax-1)\log(-ax+1)}{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x),x, algorithm="maxima")
```

```
[Out] -(a*x*dilog(a*x) - a*x*polylog(3, a*x) - a*x + (a*x - 1)*log(-a*x + 1))/a
```

Fricas [A]
time = 0.33, size = 39, normalized size = 1.15

$$-\frac{ax\mathrm{Li}_2(ax) - ax\mathrm{polylog}(3, ax) - ax + (ax-1)\log(-ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x),x, algorithm="fricas")
[Out] -(a*x*dilog(a*x) - a*x*polylog(3, a*x) - a*x + (a*x - 1)*log(-a*x + 1))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Li}_3(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x),x)
[Out] Integral(polylog(3, a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x),x, algorithm="giac")
[Out] integrate(polylog(3, a*x), x)
```

Mupad [B]
time = 0.84, size = 37, normalized size = 1.09

$$x + \frac{\ln(a\,x-1)}{a} - x\,\mathrm{polylog}(2, a\,x) + x\,\mathrm{polylog}(3, a\,x) - x\,\ln(1-a\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x),x)
[Out] x + log(a*x - 1)/a - x*polylog(2, a*x) + x*polylog(3, a*x) - x*log(1 - a*x)
```

3.15 $\int \frac{\text{PolyLog}(3,ax)}{x} \, dx$

Optimal. Leaf size=5

$$\text{PolyLog}(4,ax)$$

[Out] polylog(4,a*x)

Rubi [A]
time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6724}

$$\text{Li}_4(ax)$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a*x]/x,x]

[Out] PolyLog[4, a*x]

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\text{Li}_3(ax)}{x} \, dx = \text{Li}_4(ax)$$

Mathematica [A]
time = 0.00, size = 5, normalized size = 1.00

$$\text{PolyLog}(4,ax)$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[3, a*x]/x,x]

[Out] PolyLog[4, a*x]

Maple [A]
time = 0.13, size = 6, normalized size = 1.20

method	result	size
derivativedivides	$\text{polylog}(4, ax)$	6
default	$\text{polylog}(4, ax)$	6
meijerg	$\text{polylog}(4, ax)$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] polylog(4,a*x)
```

Maxima [A]
time = 0.25, size = 5, normalized size = 1.00

$$\text{Li}_4(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x,x, algorithm="maxima")
```

```
[Out] polylog(4, a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x,x, algorithm="fricas")
```

```
[Out] integral(polylog(3, a*x)/x, x)
```

Sympy [A]
time = 0.19, size = 3, normalized size = 0.60

$$\text{Li}_4(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x,x)
```

```
[Out] polylog(4, a*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.20

$$\int \frac{\mathrm{polylog}(3, a\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x)/x,x)
```

```
[Out] int(polylog(3, a*x)/x, x)
```

3.16 $\int \frac{\text{PolyLog}(3,ax)}{x^2} \, dx$

Optimal. Leaf size=46

$$a\log(x) - a\log(1-ax) + \frac{\log(1-ax)}{x} - \frac{\text{PolyLog}(2,ax)}{x} - \frac{\text{PolyLog}(3,ax)}{x}$$

```
[Out] a*ln(x)-a*ln(-a*x+1)+ln(-a*x+1)/x-polylog(2,a*x)/x-polylog(3,a*x)/x
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6726, 2442, 36, 29, 31}

$$-\frac{\text{Li}_2(ax)}{x} - \frac{\text{Li}_3(ax)}{x} + a\log(x) - a\log(1-ax) + \frac{\log(1-ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x]/x^2,x]
```

```
[Out] a*Log[x] - a*Log[1 - a*x] + Log[1 - a*x]/x - PolyLog[2, a*x]/x - PolyLog[3,
 a*x]/x
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax)}{x^2}\,dx &= -\frac{\text{Li}_3(ax)}{x}+\int \frac{\text{Li}_2(ax)}{x^2}\,dx\\
&= -\frac{\text{Li}_2(ax)}{x}-\frac{\text{Li}_3(ax)}{x}-\int \frac{\log(1-ax)}{x^2}\,dx\\
&= \frac{\log(1-ax)}{x}-\frac{\text{Li}_2(ax)}{x}-\frac{\text{Li}_3(ax)}{x}+a\int \frac{1}{x(1-ax)}\,dx\\
&= \frac{\log(1-ax)}{x}-\frac{\text{Li}_2(ax)}{x}-\frac{\text{Li}_3(ax)}{x}+a\int \frac{1}{x}\,dx+a^2\int \frac{1}{1-ax}\,dx\\
&= a\log(x)-a\log(1-ax)+\frac{\log(1-ax)}{x}-\frac{\text{Li}_2(ax)}{x}-\frac{\text{Li}_3(ax)}{x}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 0.96

$$-\frac{-ax\log(-ax)-\log(1-ax)+ax\log(1-ax)+\text{PolyLog}(2,ax)+\text{PolyLog}(3,ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x]/x^2,x]
[Out] -((-(a*x*Log[-(a*x)]) - Log[1 - a*x] + a*x*Log[1 - a*x] + PolyLog[2, a*x] +
 PolyLog[3, a*x])/x)
```

Maple [A]
time = 0.12, size = 57, normalized size = 1.24

method	result	size
meijerg	$a\left(\frac{(-8ax+8)\ln(-ax+1)}{8ax}-\frac{\text{polylog}(2,ax)}{ax}-\frac{\text{polylog}(3,ax)}{ax}+\ln(x)+\ln(-a)\right)$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x)/x^2,x,method=_RETURNVERBOSE)
[Out] a*(1/8/a/x*(-8*a*x+8)*ln(-a*x+1)-1/a/x*polylog(2,a*x)-1/a/x*polylog(3,a*x)+
ln(x)+ln(-a))
```

Maxima [A]
time = 0.26, size = 33, normalized size = 0.72

$$a\log(x) - \frac{(ax-1)\log(-ax+1) + \mathrm{Li}_2(ax) + \mathrm{Li}_3(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x^2,x, algorithm="maxima")
[Out] a*log(x) - ((a*x - 1)*log(-a*x + 1) + dilog(a*x) + polylog(3, a*x))/x
```

Fricas [A]
time = 0.40, size = 39, normalized size = 0.85

$$-\frac{ax\log(ax-1) - ax\log(x) + \mathrm{Li}_2(ax) - \log(-ax+1) + \mathrm{polylog}(3, ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x^2,x, algorithm="fricas")
[Out] -(a*x*log(a*x - 1) - a*x*log(x) + dilog(a*x) - log(-a*x + 1) + polylog(3, a
*x))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_3(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x**2,x)
[Out] Integral(polylog(3, a*x)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x^2,x, algorithm="giac")
[Out] integrate(polylog(3, a*x)/x^2, x)
```

Mupad [B]
time = 0.89, size = 36, normalized size = 0.78

$$2\,a\,\mathrm{atanh}(2\,a\,x-1) - \frac{\mathrm{polylog}(2, a\,x) - \ln(1-a\,x) + \mathrm{polylog}(3, a\,x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x)/x^2,x)
[Out] 2*a*atanh(2*a*x - 1) - (polylog(2, a*x) - log(1 - a*x) + polylog(3, a*x))/x
```

3.17 $\int \frac{\text{PolyLog}(3,ax)}{x^3} dx$

Optimal. Leaf size=70

$$-\frac{a}{8x}+\frac{1}{8}a^2\log(x)-\frac{1}{8}a^2\log(1-ax)+\frac{\log(1-ax)}{8x^2}-\frac{\text{PolyLog}(2,ax)}{4x^2}-\frac{\text{PolyLog}(3,ax)}{2x^2}$$

```
[Out] -1/8*a/x+1/8*a^2*ln(x)-1/8*a^2*ln(-a*x+1)+1/8*ln(-a*x+1)/x^2-1/4*polylog(2,
a*x)/x^2-1/2*polylog(3,a*x)/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6726, 2442, 46}

$$\frac{1}{8}a^2\log(x)-\frac{1}{8}a^2\log(1-ax)-\frac{\text{Li}_2(ax)}{4x^2}-\frac{\text{Li}_3(ax)}{2x^2}+\frac{\log(1-ax)}{8x^2}-\frac{a}{8x}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x]/x^3,x]
```

```
[Out] -1/8*a/x + (a^2*Log[x])/8 - (a^2*Log[1 - a*x])/8 + Log[1 - a*x]/(8*x^2) - P
olyLog[2, a*x]/(4*x^2) - PolyLog[3, a*x]/(2*x^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{Li}_3(ax)}{x^3}\,dx &= -\frac{\text{Li}_3(ax)}{2x^2}+\frac{1}{2}\int \frac{\text{Li}_2(ax)}{x^3}\,dx \\
&= -\frac{\text{Li}_2(ax)}{4x^2}-\frac{\text{Li}_3(ax)}{2x^2}-\frac{1}{4}\int \frac{\log(1-ax)}{x^3}\,dx \\
&= \frac{\log(1-ax)}{8x^2}-\frac{\text{Li}_2(ax)}{4x^2}-\frac{\text{Li}_3(ax)}{2x^2}+\frac{1}{8}a\int \frac{1}{x^2(1-ax)}\,dx \\
&= \frac{\log(1-ax)}{8x^2}-\frac{\text{Li}_2(ax)}{4x^2}-\frac{\text{Li}_3(ax)}{2x^2}+\frac{1}{8}a\int \left(\frac{1}{x^2}+\frac{a}{x}-\frac{a^2}{-1+ax}\right)dx \\
&= -\frac{a}{8x}+\frac{1}{8}a^2\log(x)-\frac{1}{8}a^2\log(1-ax)+\frac{\log(1-ax)}{8x^2}-\frac{\text{Li}_2(ax)}{4x^2}-\frac{\text{Li}_3(ax)}{2x^2}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.01, size = 25, normalized size = 0.36

$$\frac{G_{5,5}^{2,4}\left(-ax\,\middle|\,\begin{array}{c}1,1,1,1,3\\1,2,0,0,0\end{array}\right)}{x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x]/x^3,x]
[Out] MeijerG[{{1, 1, 1, 1}, {3}}, {{1, 2}, {0, 0, 0}}, -(a*x)]/x^2
```

Maple [A]
time = 0.21, size = 90, normalized size = 1.29

method	result	siz
meijerg	$-a^2\left(-\frac{81ax+378}{432ax}-\frac{(-27a^2x^2+27)\ln(-ax+1)}{216a^2x^2}+\frac{\text{polylog}(2,ax)}{4a^2x^2}+\frac{\text{polylog}(3,ax)}{2a^2x^2}+\frac{3}{16}-\frac{\ln(x)}{8}-\frac{\ln(-a)}{8}+\frac{1}{ax}\right)$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x)/x^3,x,method=_RETURNVERBOSE)
[Out] -a^2*(-1/432/a/x*(81*a*x+378)-1/216/a^2/x^2*(-27*a^2*x^2+27)*ln(-a*x+1)+1/4
/a^2/x^2*polylog(2,a*x)+1/2/a^2/x^2*polylog(3,a*x)+3/16-1/8*ln(x)-1/8*ln(-a
)+1/a/x)
```

Maxima [A]
time = 0.25, size = 47, normalized size = 0.67

$$\frac{1}{8}\,a^2\log\left(x\right)-\frac{ax+\left(a^2x^2-1\right)\log\left(-ax+1\right)+2\,\text{Li}_2(ax)+4\,\text{Li}_3(ax)}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*log(x) - 1/8*(a*x + (a^2*x^2 - 1)*log(-a*x + 1) + 2*dilog(a*x) + 4*
polylog(3, a*x))/x^2
```

Fricas [A]
time = 0.36, size = 54, normalized size = 0.77

$$-\frac{a^2x^2\log\left(ax-1\right)-a^2x^2\log\left(x\right)+ax+2\operatorname{Li}_2(ax)-\log\left(-ax+1\right)+4\operatorname{polylog}(3,ax)}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(a^2*x^2*log(a*x - 1) - a^2*x^2*log(x) + a*x + 2*dilog(a*x) - log(-a*x
 + 1) + 4*polylog(3, a*x))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{Li}_3\left(ax\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x**3,x)
```

```
[Out] Integral(polylog(3, a*x)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x)/x^3, x)
```

Mupad [B]
time = 1.27, size = 46, normalized size = 0.66

$$\frac{a^2\,\mathrm{atanh}(2\,a\,x-1)}{4}-\frac{\frac{a\,x}{8}-\frac{\ln(1-a\,x)}{8}+\frac{\mathrm{polylog}(2,a\,x)}{4}+\frac{\mathrm{polylog}(3,a\,x)}{2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x)/x^3,x)
```

```
[Out] (a^2*atanh(2*a*x - 1))/4 - ((a*x)/8 - log(1 - a*x)/8 + polylog(2, a*x)/4 +
polylog(3, a*x)/2)/x^2
```

3.18 $\int \frac{\text{PolyLog}(3,ax)}{x^4}\,dx$

Optimal. Leaf size=80

$$-\frac{a}{54x^2}-\frac{a^2}{27x}+\frac{1}{27}a^3\log(x)-\frac{1}{27}a^3\log(1-ax)+\frac{\log(1-ax)}{27x^3}-\frac{\text{PolyLog}(2,ax)}{9x^3}-\frac{\text{PolyLog}(3,ax)}{3x^3}$$

```
[Out] -1/54*a/x^2-1/27*a^2/x+1/27*a^3*ln(x)-1/27*a^3*ln(-a*x+1)+1/27*ln(-a*x+1)/x
^3-1/9*polylog(2,a*x)/x^3-1/3*polylog(3,a*x)/x^3
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6726, 2442, 46}

$$\frac{1}{27}a^3\log(x)-\frac{1}{27}a^3\log(1-ax)-\frac{a^2}{27x}-\frac{\text{Li}_2(ax)}{9x^3}-\frac{\text{Li}_3(ax)}{3x^3}+\frac{\log(1-ax)}{27x^3}-\frac{a}{54x^2}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x]/x^4,x]
```

```
[Out] -1/54*a/x^2 - a^2/(27*x) + (a^3*Log[x])/27 - (a^3*Log[1 - a*x])/27 + Log[1
 - a*x]/(27*x^3) - PolyLog[2, a*x]/(9*x^3) - PolyLog[3, a*x]/(3*x^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\mathrm{Li}_3(ax)}{x^4}\,dx &= -\frac{\mathrm{Li}_3(ax)}{3x^3}+\frac{1}{3}\int \frac{\mathrm{Li}_2(ax)}{x^4}\,dx \\
&= -\frac{\mathrm{Li}_2(ax)}{9x^3}-\frac{\mathrm{Li}_3(ax)}{3x^3}-\frac{1}{9}\int \frac{\log(1-ax)}{x^4}\,dx \\
&= \frac{\log(1-ax)}{27x^3}-\frac{\mathrm{Li}_2(ax)}{9x^3}-\frac{\mathrm{Li}_3(ax)}{3x^3}+\frac{1}{27}a\int \frac{1}{x^3(1-ax)}\,dx \\
&= \frac{\log(1-ax)}{27x^3}-\frac{\mathrm{Li}_2(ax)}{9x^3}-\frac{\mathrm{Li}_3(ax)}{3x^3}+\frac{1}{27}a\int \left(\frac{1}{x^3}+\frac{a}{x^2}+\frac{a^2}{x}-\frac{a^3}{-1+ax}\right)dx \\
&= -\frac{a}{54x^2}-\frac{a^2}{27x}+\frac{1}{27}a^3\log(x)-\frac{1}{27}a^3\log(1-ax)+\frac{\log(1-ax)}{27x^3}-\frac{\mathrm{Li}_2(ax)}{9x^3}-\frac{\mathrm{Li}_3(ax)}{3x^3}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.01, size = 25, normalized size = 0.31

$$
\frac{G_{5,5}^{2,4}\left(-ax\,\middle|\,\begin{array}{c}1,1,1,1,4\\1,3,0,0,0\end{array}\right)}{x^3}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x]/x^4,x]
```

```
[Out] MeijerG[{{1, 1, 1, 1}, {4}}, {{1, 3}, {0, 0, 0}}, -(a*x)]/x^3
```

Maple [A]
time = 0.19, size = 106, normalized size = 1.32

method	result
meijerg	$a^3\left(\frac{64a^2x^2+152ax+832}{1728a^2x^2}+\frac{(-64a^3x^3+64)\ln(-ax+1)}{1728a^3x^3}-\frac{\mathrm{polylog}(2,ax)}{9a^3x^3}-\frac{\mathrm{polylog}(3,ax)}{3a^3x^3}-\frac{1}{27}+\frac{\ln(x)}{27}+\frac{\ln(-a)}{27}-\frac{1}{2a^2x\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(1/1728/a^2/x^2*(64*a^2*x^2+152*a*x+832)+1/1728/a^3/x^3*(-64*a^3*x^3+64
)*ln(-a*x+1)-1/9/a^3/x^3*polylog(2,a*x)-1/3/a^3/x^3*polylog(3,a*x)-1/27+1/2
7*ln(x)+1/27*ln(-a)-1/2/a^2/x^2-1/8/a/x)
```

Maxima [A]
time = 0.26, size = 56, normalized size = 0.70

$$
\frac{1}{27}\,a^3\log\left(x\right)-\frac{2\,a^2x^2+ax+2\,(a^3x^3-1)\log\left(-ax+1\right)+6\,\mathrm{Li}_2(ax)+18\,\mathrm{Li}_3(ax)}{54\,x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/27*a^3*log(x) - 1/54*(2*a^2*x^2 + a*x + 2*(a^3*x^3 - 1)*log(-a*x + 1) + 6
*dilog(a*x) + 18*polylog(3, a*x))/x^3
```

Fricas [A]
time = 0.36, size = 63, normalized size = 0.79

$$-\frac{2\,a^3x^3\log\left(ax-1\right)-2\,a^3x^3\log\left(x\right)+2\,a^2x^2+ax+6\,\mathrm{Li}_2(ax)-2\,\log\left(-ax+1\right)+18\,\mathrm{polylog}(3,ax)}{54\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/54*(2*a^3*x^3*log(a*x - 1) - 2*a^3*x^3*log(x) + 2*a^2*x^2 + a*x + 6*dilo
g(a*x) - 2*log(-a*x + 1) + 18*polylog(3, a*x))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{Li}_3\left(ax\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x**4,x)
```

```
[Out] Integral(polylog(3, a*x)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x)/x^4, x)
```

Mupad [B]
time = 1.50, size = 62, normalized size = 0.78

$$\frac{\ln\left(1-a\,x\right)}{27\,x^3}-\frac{\mathrm{polylog}(2,a\,x)}{9\,x^3}-\frac{\mathrm{polylog}(3,a\,x)}{3\,x^3}-\frac{x\,a^2+\frac{a}{2}}{27\,x^2}-\frac{a^3\,\mathrm{atan}(a\,x\,2\mathrm{i}-\mathrm{i})\;2\mathrm{i}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x)/x^4,x)
```

```
[Out] log(1 - a*x)/(27*x^3) - (a^3*atan(a*x*2i - 1i)*2i)/27 - polylog(2, a*x)/(9*
x^3) - polylog(3, a*x)/(3*x^3) - (a/2 + a^2*x)/(27*x^2)
```

3.19 $\int x^5\textbf{PolyLog}(2,ax^2)\,dx$

Optimal. Leaf size=74

$$-\frac{x^2}{18a^2}-\frac{x^4}{36a}-\frac{x^6}{54}-\frac{\log\left(1-ax^2\right)}{18a^3}+\frac{1}{18}x^6\log\left(1-ax^2\right)+\frac{1}{6}x^6\text{PolyLog}\left(2,ax^2\right)$$

```
[Out] -1/18*x^2/a^2-1/36*x^4/a-1/54*x^6-1/18*ln(-a*x^2+1)/a^3+1/18*x^6*ln(-a*x^2+
1)+1/6*x^6*polylog(2,a*x^2)
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.364$, Rules used = {6726, 2504, 2442, 45}

$$-\frac{\log\left(1-ax^2\right)}{18a^3}-\frac{x^2}{18a^2}+\frac{1}{6}x^6\text{Li}_2\left(ax^2\right)-\frac{x^4}{36a}+\frac{1}{18}x^6\log\left(1-ax^2\right)-\frac{x^6}{54}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*PolyLog[2, a*x^2],x]
```

```
[Out] -1/18*x^2/a^2 - x^4/(36*a) - x^6/54 - Log[1 - a*x^2]/(18*a^3) + (x^6*Log[1
- a*x^2])/18 + (x^6*PolyLog[2, a*x^2])/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int x^5\text{Li}_2\left(ax^2\right)\,dx &= \frac{1}{6}x^6\text{Li}_2\left(ax^2\right)+\frac{1}{3}\int x^5\log\left(1-ax^2\right)\,dx\\
&= \frac{1}{6}x^6\text{Li}_2\left(ax^2\right)+\frac{1}{6}\text{Subst}\left(\int x^2\log(1-ax)\,dx,x,x^2\right)\\
&= \frac{1}{18}x^6\log\left(1-ax^2\right)+\frac{1}{6}x^6\text{Li}_2\left(ax^2\right)+\frac{1}{18}a\text{Subst}\left(\int\frac{x^3}{1-ax}\,dx,x,x^2\right)\\
&= \frac{1}{18}x^6\log\left(1-ax^2\right)+\frac{1}{6}x^6\text{Li}_2\left(ax^2\right)+\frac{1}{18}a\text{Subst}\left(\int\left(-\frac{1}{a^3}-\frac{x}{a^2}-\frac{x^2}{a}-\frac{1}{a^3(-1+ax)}\right)\right.\\
&= -\frac{x^2}{18a^2}-\frac{x^4}{36a}-\frac{x^6}{54}-\frac{\log\left(1-ax^2\right)}{18a^3}+\frac{1}{18}x^6\log\left(1-ax^2\right)+\frac{1}{6}x^6\text{Li}_2\left(ax^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 65, normalized size = 0.88

$$\frac{-ax^2(6+3ax^2+2a^2x^4)+6(-1+a^3x^6)\log\left(1-ax^2\right)+18a^3x^6\text{PolyLog}(2,ax^2)}{108a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*PolyLog[2, a*x^2],x]
```

```
[Out] (-(a*x^2*(6 + 3*a*x^2 + 2*a^2*x^4)) + 6*(-1 + a^3*x^6)*Log[1 - a*x^2] + 18*
a^3*x^6*PolyLog[2, a*x^2])/(108*a^3)
```

Maple [A]
time = 0.06, size = 68, normalized size = 0.92

method	result	size
meijerg	$\frac{-\frac{x^2a\left(4a^2x^4+6a\,x^2+12\right)}{108}-\frac{\left(-4a^3x^6+4\right)\ln\left(-a\,x^2+1\right)}{36}+\frac{x^6a^3\,\mathrm{polylog}\left(2,a\,x^2\right)}{3}}{2a^3}$	65
default	$\frac{x^6\,\mathrm{polylog}(2,a\,x^2)}{6}+\frac{x^6\ln\left(-a\,x^2+1\right)}{18}+\frac{a\left(-\frac{\frac{1}{3}a^2x^6+\frac{1}{2}a\,x^4+x^2}{2a^3}-\frac{\ln\left(a\,x^2-1\right)}{2a^4}\right)}{9}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*polylog(2,a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^6*polylog(2,a*x^2)+1/18*x^6*ln(-a*x^2+1)+1/9*a*(-1/2/a^3*(1/3*a^2*x^6
+1/2*a*x^4+x^2)-1/2/a^4*ln(a*x^2-1))
```

Maxima [A]
time = 0.26, size = 62, normalized size = 0.84

$$\frac{18\,a^3x^6\mathrm{Li}_2(ax^2)-2\,a^3x^6-3\,a^2x^4-6\,ax^2+6\,(a^3x^6-1)\log\left(-ax^2+1\right)}{108\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*polylog(2,a*x^2),x, algorithm="maxima")
```

```
[Out] 1/108*(18*a^3*x^6*dilog(a*x^2) - 2*a^3*x^6 - 3*a^2*x^4 - 6*a*x^2 + 6*(a^3*x
^6 - 1)*log(-a*x^2 + 1))/a^3
```

Fricas [A]
time = 0.37, size = 62, normalized size = 0.84

$$\frac{18\,a^3x^6\mathrm{Li}_2(ax^2)-2\,a^3x^6-3\,a^2x^4-6\,ax^2+6\,(a^3x^6-1)\log\left(-ax^2+1\right)}{108\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*polylog(2,a*x^2),x, algorithm="fricas")
```

```
[Out] 1/108*(18*a^3*x^6*dilog(a*x^2) - 2*a^3*x^6 - 3*a^2*x^4 - 6*a*x^2 + 6*(a^3*x
^6 - 1)*log(-a*x^2 + 1))/a^3
```

Sympy [A]
time = 4.65, size = 56, normalized size = 0.76

$$\begin{cases} -\frac{x^6\,\mathrm{Li}_1\left(ax^2\right)}{18}+\frac{x^6\,\mathrm{Li}_2\left(ax^2\right)}{6}-\frac{x^6}{54}-\frac{x^4}{36a}-\frac{x^2}{18a^2}+\frac{\mathrm{Li}_1\left(ax^2\right)}{18a^3} & \text{for}\ a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*polylog(2,a*x**2),x)
```

```
[Out] Piecewise((-x**6*polylog(1, a*x**2)/18 + x**6*polylog(2, a*x**2)/6 - x**6/5
4 - x**4/(36*a) - x**2/(18*a**2) + polylog(1, a*x**2)/(18*a**3), Ne(a, 0)),
 (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*polylog(2,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^5*dilog(a*x^2), x)
```

Mupad [B]
time = 0.19, size = 61, normalized size = 0.82

$$\frac{x^6\,\mathrm{polylog}(2,a\,x^2)}{6}-\frac{\ln\left(a\,x^2-1\right)}{18\,a^3}+\frac{x^6\,\ln\left(1-a\,x^2\right)}{18}-\frac{x^6}{54}-\frac{x^2}{18\,a^2}-\frac{x^4}{36\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*polylog(2, a*x^2),x)
```

```
[Out] (x^6*polylog(2, a*x^2))/6 - log(a*x^2 - 1)/(18*a^3) + (x^6*log(1 - a*x^2))/
18 - x^6/54 - x^2/(18*a^2) - x^4/(36*a)
```

3.20 $\int x^3 \mathbf{PolyLog}(2, ax^2)\, dx$

Optimal. Leaf size=64

$$-\frac{x^2}{8a}-\frac{x^4}{16}-\frac{\log(1-ax^2)}{8a^2}+\frac{1}{8}x^4\log(1-ax^2)+\frac{1}{4}x^4\text{PolyLog}(2,ax^2)$$

```
[Out] -1/8*x^2/a-1/16*x^4-1/8*ln(-a*x^2+1)/a^2+1/8*x^4*ln(-a*x^2+1)+1/4*x^4*polyl
og(2,a*x^2)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6726, 2504, 2442, 45}

$$-\frac{\log(1-ax^2)}{8a^2}+\frac{1}{4}x^4\text{Li}_2(ax^2)-\frac{x^2}{8a}+\frac{1}{8}x^4\log(1-ax^2)-\frac{x^4}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*PolyLog[2, a*x^2],x]
```

```
[Out] -1/8*x^2/a - x^4/16 - Log[1 - a*x^2]/(8*a^2) + (x^4*Log[1 - a*x^2])/8 + (x^
4*PolyLog[2, a*x^2])/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3\text{Li}_2\left(ax^2\right)\,dx &= \frac{1}{4}x^4\text{Li}_2\left(ax^2\right)+\frac{1}{2}\int x^3\log\left(1-ax^2\right)\,dx\\
&= \frac{1}{4}x^4\text{Li}_2\left(ax^2\right)+\frac{1}{4}\text{Subst}\left(\int x\log(1-ax)\,dx,x,x^2\right)\\
&= \frac{1}{8}x^4\log\left(1-ax^2\right)+\frac{1}{4}x^4\text{Li}_2\left(ax^2\right)+\frac{1}{8}a\text{Subst}\left(\int\frac{x^2}{1-ax}\,dx,x,x^2\right)\\
&= \frac{1}{8}x^4\log\left(1-ax^2\right)+\frac{1}{4}x^4\text{Li}_2\left(ax^2\right)+\frac{1}{8}a\text{Subst}\left(\int\left(-\frac{1}{a^2}-\frac{x}{a}-\frac{1}{a^2(-1+ax)}\right)\,dx,x,x^2\right)\\
&= -\frac{x^2}{8a}-\frac{x^4}{16}-\frac{\log\left(1-ax^2\right)}{8a^2}+\frac{1}{8}x^4\log\left(1-ax^2\right)+\frac{1}{4}x^4\text{Li}_2\left(ax^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 0.88

$$\frac{-ax^2(2+ax^2)+2(-1+a^2x^4)\log\left(1-ax^2\right)+4a^2x^4\text{PolyLog}(2,ax^2)}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*PolyLog[2, a*x^2],x]
```

```
[Out] (-(a*x^2*(2 + a*x^2)) + 2*(-1 + a^2*x^4)*Log[1 - a*x^2] + 4*a^2*x^4*PolyLog
[2, a*x^2])/(16*a^2)
```

Maple [A]
time = 0.05, size = 60, normalized size = 0.94

method	result	size
meijerg	$-\frac{\frac{a\,x^2\left(3a\,x^2+6\right)}{24}+\frac{\left(-3a^2x^4+3\right)\ln\left(-a\,x^2+1\right)}{12}-\frac{a^2x^4\,\text{polylog}\left(2,a\,x^2\right)}{2}}{2a^2}$	57
default	$\frac{x^4\,\text{polylog}(2,a\,x^2)}{4}+\frac{x^4\ln(-a\,x^2+1)}{8}+\frac{a\left(-\frac{\frac{1}{2}a\,x^4+x^2}{2a^2}-\frac{\ln\left(a\,x^2-1\right)}{2a^3}\right)}{4}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*polylog(2,a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*polylog(2,a*x^2)+1/8*x^4*ln(-a*x^2+1)+1/4*a*(-1/2/a^2*(1/2*a*x^4+x^
2)-1/2/a^3*ln(a*x^2-1))
```

Maxima [A]
time = 0.26, size = 54, normalized size = 0.84

$$\frac{4\,a^2x^4\mathrm{Li}_2(ax^2) - a^2x^4 - 2\,ax^2 + 2\,(a^2x^4 - 1)\log\left(-ax^2 + 1\right)}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(2,a*x^2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*a^2*x^4*dilog(a*x^2) - a^2*x^4 - 2*a*x^2 + 2*(a^2*x^4 - 1)*log(-a*x
^2 + 1))/a^2
```

Fricas [A]
time = 0.35, size = 54, normalized size = 0.84

$$\frac{4\,a^2x^4\mathrm{Li}_2(ax^2) - a^2x^4 - 2\,ax^2 + 2\,(a^2x^4 - 1)\log\left(-ax^2 + 1\right)}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(2,a*x^2),x, algorithm="fricas")
```

```
[Out] 1/16*(4*a^2*x^4*dilog(a*x^2) - a^2*x^4 - 2*a*x^2 + 2*(a^2*x^4 - 1)*log(-a*x
^2 + 1))/a^2
```

Sympy [A]
time = 1.79, size = 48, normalized size = 0.75

$$\begin{cases} -\frac{x^4\,\mathrm{Li}_1\left(ax^2\right)}{8} + \frac{x^4\,\mathrm{Li}_2\left(ax^2\right)}{4} - \frac{x^4}{16} - \frac{x^2}{8a} + \frac{\mathrm{Li}_1\left(ax^2\right)}{8a^2} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*polylog(2,a*x**2),x)
```

```
[Out] Piecewise((-x**4*polylog(1, a*x**2)/8 + x**4*polylog(2, a*x**2)/4 - x**4/16
 - x**2/(8*a) + polylog(1, a*x**2)/(8*a**2), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(2,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^3*dilog(a*x^2), x)
```

Mupad [B]
time = 0.27, size = 53, normalized size = 0.83

$$\frac{x^4\,\mathrm{polylog}(2,a\,x^2)}{4}-\frac{\ln\left(a\,x^2-1\right)}{8\,a^2}+\frac{x^4\,\ln\left(1-a\,x^2\right)}{8}-\frac{x^4}{16}-\frac{x^2}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*polylog(2, a*x^2),x)
```

```
[Out] (x^4*polylog(2, a*x^2))/4 - log(a*x^2 - 1)/(8*a^2) + (x^4*log(1 - a*x^2))/8
 - x^4/16 - x^2/(8*a)
```

3.21 $\int x\mathbf{PolyLog}(2, ax^2)\, dx$

Optimal. Leaf size=46

$$-\frac{x^2}{2}-\frac{(1-ax^2)\log(1-ax^2)}{2a}+\frac{1}{2}x^2\text{PolyLog}(2,ax^2)$$

```
[Out] -1/2*x^2-1/2*(-a*x^2+1)*ln(-a*x^2+1)/a+1/2*x^2*polylog(2,a*x^2)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.444$, Rules used = {6726, 2504, 2436, 2332}

$$\frac{1}{2}x^2\text{Li}_2(ax^2)-\frac{(1-ax^2)\log(1-ax^2)}{2a}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*PolyLog[2, a*x^2],x]
```

```
[Out] -1/2*x^2 - ((1 - a*x^2)*Log[1 - a*x^2])/(2*a) + (x^2*PolyLog[2, a*x^2])/2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int x\mathrm{Li}_2\left(ax^2\right)\,dx &= \frac{1}{2}x^2\mathrm{Li}_2\left(ax^2\right)+\int x\log\left(1-ax^2\right)\,dx\\
&= \frac{1}{2}x^2\mathrm{Li}_2\left(ax^2\right)+\frac{1}{2}\mathrm{Subst}\left(\int\log(1-ax)\,dx,x,x^2\right)\\
&= \frac{1}{2}x^2\mathrm{Li}_2\left(ax^2\right)-\frac{\mathrm{Subst}\left(\int\log(x)\,dx,x,1-ax^2\right)}{2a}\\
&= -\frac{x^2}{2}-\frac{\left(1-ax^2\right)\log\left(1-ax^2\right)}{2a}+\frac{1}{2}x^2\mathrm{Li}_2\left(ax^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 0.93

$$\frac{-ax^2+\left(-1+ax^2\right)\log\left(1-ax^2\right)+ax^2\mathrm{PolyLog}(2,ax^2)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*PolyLog[2, a*x^2],x]
[Out] (-(a*x^2) + (-1 + a*x^2)*Log[1 - a*x^2] + a*x^2*PolyLog[2, a*x^2])/(2*a)
```

Maple [A]
time = 0.29, size = 45, normalized size = 0.98

method	result	size
meijerg	$\frac{-a\,x^2-\frac{\left(-2a\,x^2+2\right)\ln\left(-a\,x^2+1\right)}{2}+a\,x^2\,\mathrm{polylog}\left(2,a\,x^2\right)}{2a}$	44
derivativedivides	$\frac{a\,x^2\,\mathrm{polylog}\left(2,a\,x^2\right)-\ln\left(-a\,x^2+1\right)\left(-a\,x^2+1\right)+1-a\,x^2}{2a}$	45
default	$\frac{a\,x^2\,\mathrm{polylog}\left(2,a\,x^2\right)-\ln\left(-a\,x^2+1\right)\left(-a\,x^2+1\right)+1-a\,x^2}{2a}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(2,a*x^2),x,method=_RETURNVERBOSE)
[Out] 1/2/a*(a*x^2*polylog(2,a*x^2)-ln(-a*x^2+1)*(-a*x^2+1)+1-a*x^2)
```

Maxima [A]
time = 0.25, size = 40, normalized size = 0.87

$$\frac{ax^2\mathrm{Li}_2(ax^2)-ax^2+\left(ax^2-1\right)\log\left(-ax^2+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*(a*x^2*dilog(a*x^2) - a*x^2 + (a*x^2 - 1)*log(-a*x^2 + 1))/a
```

Fricas [A]
time = 0.35, size = 40, normalized size = 0.87

$$\frac{ax^2\mathrm{Li}_2(ax^2) - ax^2 + (ax^2-1)\log\left(-ax^2+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*x^2*dilog(a*x^2) - a*x^2 + (a*x^2 - 1)*log(-a*x^2 + 1))/a
```

Sympy [A]
time = 0.67, size = 39, normalized size = 0.85

$$\begin{cases} -\frac{x^2\,\mathrm{Li}_1\left(ax^2\right)}{2} + \frac{x^2\,\mathrm{Li}_2\left(ax^2\right)}{2} - \frac{x^2}{2} + \frac{\mathrm{Li}_1\left(ax^2\right)}{2a} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x**2),x)
```

```
[Out] Piecewise((-x**2*polylog(1, a*x**2)/2 + x**2*polylog(2, a*x**2)/2 - x**2/2
+ polylog(1, a*x**2)/(2*a), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x*dilog(a*x^2), x)
```

Mupad [B]
time = 0.21, size = 45, normalized size = 0.98

$$\frac{x^2\,\mathrm{polylog}(2,a\,x^2)}{2} - \frac{\ln\left(a\,x^2-1\right)}{2\,a} + \frac{x^2\,\ln\left(1-a\,x^2\right)}{2} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(2, a*x^2),x)
```

```
[Out] (x^2*polylog(2, a*x^2))/2 - log(a*x^2 - 1)/(2*a) + (x^2*log(1 - a*x^2))/2 -
 x^2/2
```

3.22 $\int \frac{\textbf{PolyLog}(2,ax^2)}{x}\,dx$

Optimal. Leaf size=11

$$\frac{1}{2}\text{PolyLog}\left(3,ax^2\right)$$

[Out] 1/2*polylog(3,a*x^2)

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6724}

$$\frac{\text{Li}_3(ax^2)}{2}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a*x^2]/x,x]

[Out] PolyLog[3, a*x^2]/2

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\text{Li}_2(ax^2)}{x}\,dx = \frac{\text{Li}_3(ax^2)}{2}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$\frac{1}{2}\text{PolyLog}\left(3,ax^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a*x^2]/x,x]

[Out] PolyLog[3, a*x^2]/2

Maple [A]
time = 0.82, size = 10, normalized size = 0.91

method	result	size
default	$\frac{\operatorname{polylog}(3,a\,x^2)}{2}$	10
meijerg	$\frac{\operatorname{polylog}(3,a\,x^2)}{2}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/x,x,method=_RETURNVERBOSE)
[Out] 1/2*polylog(3,a*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x,x, algorithm="maxima")
[Out] integrate(dilog(a*x^2)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x,x, algorithm="fricas")
[Out] integral(dilog(a*x^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_2\left(ax^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/x,x)
[Out] Integral(polylog(2, a*x**2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/x, x)
```

Mupad [B]
time = 0.17, size = 9, normalized size = 0.82

$$\frac{\mathrm{polylog}(3, a\,x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^2)/x,x)
```

```
[Out] polylog(3, a*x^2)/2
```

3.23 $\int \frac{\text{PolyLog}(2,ax^2)}{x^3} \, dx$

Optimal. Leaf size=49

$$a \log(x) - \frac{1}{2} a \log\left(1 - ax^2\right) + \frac{\log\left(1 - ax^2\right)}{2x^2} - \frac{\text{PolyLog}(2, ax^2)}{2x^2}$$

```
[Out] a*ln(x)-1/2*a*ln(-a*x^2+1)+1/2*ln(-a*x^2+1)/x^2-1/2*polylog(2,a*x^2)/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {6726, 2504, 2442, 36, 29, 31}

$$-\frac{\text{Li}_2(ax^2)}{2x^2} - \frac{1}{2} a \log\left(1 - ax^2\right) + \frac{\log\left(1 - ax^2\right)}{2x^2} + a \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/x^3,x]
```

```
[Out] a*Log[x] - (a*Log[1 - a*x^2])/2 + Log[1 - a*x^2]/(2*x^2) - PolyLog[2, a*x^2
]/(2*x^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax^2)}{x^3}\,dx &= -\frac{\text{Li}_2(ax^2)}{2x^2}-\int \frac{\log\left(1-ax^2\right)}{x^3}\,dx\\
&= -\frac{\text{Li}_2(ax^2)}{2x^2}-\frac{1}{2}\text{Subst}\left(\int \frac{\log(1-ax)}{x^2}\,dx,x,x^2\right)\\
&= \frac{\log\left(1-ax^2\right)}{2x^2}-\frac{\text{Li}_2(ax^2)}{2x^2}+\frac{1}{2}a\text{Subst}\left(\int \frac{1}{x(1-ax)}\,dx,x,x^2\right)\\
&= \frac{\log\left(1-ax^2\right)}{2x^2}-\frac{\text{Li}_2(ax^2)}{2x^2}+\frac{1}{2}a\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)+\frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{1-ax}\,dx,x,x^2\right)\\
&= a\log(x)-\frac{1}{2}a\log\left(1-ax^2\right)+\frac{\log\left(1-ax^2\right)}{2x^2}-\frac{\text{Li}_2(ax^2)}{2x^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 1.00

$$a\log(x)-\frac{1}{2}a\log\left(1-ax^2\right)+\frac{\log\left(1-ax^2\right)}{2x^2}-\frac{\text{PolyLog}(2,ax^2)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^2]/x^3,x]
```

```
[Out] a*Log[x] - (a*Log[1 - a*x^2])/2 + Log[1 - a*x^2]/(2*x^2) - PolyLog[2, a*x^2
]/(2*x^2)
```

Maple [A]
time = 0.08, size = 43, normalized size = 0.88

---

method	result	size
default	$-\frac{\operatorname{polylog}(2,a\,x^{2})}{2x^{2}}+\frac{\ln(-a\,x^{2}+1)}{2x^{2}}+a\left(-\frac{\ln(a\,x^{2}-1)}{2}+\ln(x)\right)$	43
meijerg	$\frac{a\left(\frac{(-4a\,x^{2}+4)\ln(-a\,x^{2}+1)}{4a\,x^{2}}-\frac{\operatorname{polylog}(2,a\,x^{2})}{a\,x^{2}}+2\ln(x)+\ln(-a)\right)}{2}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*polylog(2,a*x^2)/x^2+1/2*ln(-a*x^2+1)/x^2+a*(-1/2*ln(a*x^2-1)+ln(x))
```

Maxima [A]
time = 0.26, size = 34, normalized size = 0.69

$$a\log(x)-\frac{(ax^2-1)\log(-ax^2+1)+\mathrm{Li}_2(ax^2)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] a*log(x) - 1/2*((a*x^2 - 1)*log(-a*x^2 + 1) + dilog(a*x^2))/x^2
```

Fricas [A]
time = 0.36, size = 44, normalized size = 0.90

$$-\frac{ax^2\log(ax^2-1)-2\,ax^2\log(x)+\mathrm{Li}_2(ax^2)-\log(-ax^2+1)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*x^2*log(a*x^2 - 1) - 2*a*x^2*log(x) + dilog(a*x^2) - log(-a*x^2 + 1
))/x^2
```

Sympy [A]
time = 0.81, size = 37, normalized size = 0.76

$$a\log(x)+\frac{a\,\mathrm{Li}_1(ax^2)}{2}-\frac{\mathrm{Li}_1(ax^2)}{2x^2}-\frac{\mathrm{Li}_2(ax^2)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/x**3,x)
```

```
[Out] a*log(x) + a*polylog(1, a*x**2)/2 - polylog(1, a*x**2)/(2*x**2) - polylog(2
, a*x**2)/(2*x**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/x^3, x)
```

Mupad [B]
time = 0.21, size = 44, normalized size = 0.90

$$\frac{3\,a\,\ln\left(x\right)}{2}+\frac{\frac{\ln\left(1-a\,x^2\right)}{2}-\frac{\mathrm{polylog}\left(2,a\,x^2\right)}{2}}{x^2}-\frac{a\,\ln\left(a\,x^3-x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^2)/x^3,x)
```

```
[Out] (3*a*log(x))/2 + (log(1 - a*x^2)/2 - polylog(2, a*x^2)/2)/x^2 - (a*log(a*x^
3 - x))/2
```

3.24 $\int \frac{\text{PolyLog}(2,ax^2)}{x^5}\,dx$

Optimal. Leaf size=64

$$-\frac{a}{8x^2}+\frac{1}{4}a^2\log(x)-\frac{1}{8}a^2\log\left(1-ax^2\right)+\frac{\log\left(1-ax^2\right)}{8x^4}-\frac{\text{PolyLog}(2,ax^2)}{4x^4}$$

```
[Out] -1/8*a/x^2+1/4*a^2*ln(x)-1/8*a^2*ln(-a*x^2+1)+1/8*ln(-a*x^2+1)/x^4-1/4*poly
log(2,a*x^2)/x^4
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6726, 2504, 2442, 46}

$$-\frac{1}{8}a^2\log\left(1-ax^2\right)+\frac{1}{4}a^2\log(x)-\frac{\text{Li}_2(ax^2)}{4x^4}-\frac{a}{8x^2}+\frac{\log\left(1-ax^2\right)}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/x^5,x]
```

```
[Out] -1/8*a/x^2 + (a^2*Log[x])/4 - (a^2*Log[1 - a*x^2])/8 + Log[1 - a*x^2]/(8*x^
4) - PolyLog[2, a*x^2]/(4*x^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_2(ax^2)}{x^5}\,dx &= -\frac{\text{Li}_2(ax^2)}{4x^4}-\frac{1}{2}\int \frac{\log(1-ax^2)}{x^5}\,dx\\ &= -\frac{\text{Li}_2(ax^2)}{4x^4}-\frac{1}{4}\text{Subst}\left(\int \frac{\log(1-ax)}{x^3}\,dx,x,x^2\right)\\ &= \frac{\log(1-ax^2)}{8x^4}-\frac{\text{Li}_2(ax^2)}{4x^4}+\frac{1}{8}a\text{Subst}\left(\int \frac{1}{x^2(1-ax)}\,dx,x,x^2\right)\\ &= \frac{\log(1-ax^2)}{8x^4}-\frac{\text{Li}_2(ax^2)}{4x^4}+\frac{1}{8}a\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{a}{x}-\frac{a^2}{-1+ax}\right)dx,x,x^2\right)\\ &= -\frac{a}{8x^2}+\frac{1}{4}a^2\log(x)-\frac{1}{8}a^2\log(1-ax^2)+\frac{\log(1-ax^2)}{8x^4}-\frac{\text{Li}_2(ax^2)}{4x^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 51, normalized size = 0.80

$$-\frac{ax^2-2a^2x^4\log(x)+(-1+a^2x^4)\log(1-ax^2)+2\text{PolyLog}(2,ax^2)}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^2]/x^5,x]
[Out] -1/8*(a*x^2 - 2*a^2*x^4*Log[x] + (-1 + a^2*x^4)*Log[1 - a*x^2] + 2*PolyLog[
2, a*x^2])/x^4
```

Maple [A]
time = 0.10, size = 52, normalized size = 0.81

method	result	size
default	$-\frac{\text{polylog}(2,a x^2)}{4x^4}+\frac{\ln(-a x^2+1)}{8x^4}+\frac{a\left(-\frac{a\ln(a x^2-1)}{2}-\frac{1}{2x^2}+a\ln(x)\right)}{4}$	52
meijerg	$-\frac{a^2\left(-\frac{9a x^2+27}{36a x^2}-\frac{(-9a^2x^4+9)\ln(-a x^2+1)}{36a^2x^4}+\frac{\text{polylog}(2,a x^2)}{2a^2x^4}+\frac{1}{4}-\frac{\ln(x)}{2}-\frac{\ln(-a)}{4}+\frac{1}{a x^2}\right)}{2}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*polylog(2,a*x^2)/x^4+1/8*ln(-a*x^2+1)/x^4+1/4*a*(-1/2*a*ln(a*x^2-1)-1/
2/x^2+a*ln(x))
```

Maxima [A]
time = 0.27, size = 46, normalized size = 0.72

$$\frac{1}{4}\,a^2\log\left(x\right)-\frac{ax^2+\left(a^2x^4-1\right)\log\left(-ax^2+1\right)+2\,\mathrm{Li}_2(ax^2)}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*log(x) - 1/8*(a*x^2 + (a^2*x^4 - 1)*log(-a*x^2 + 1) + 2*dilog(a*x^2
))/x^4
```

Fricas [A]
time = 0.37, size = 55, normalized size = 0.86

$$-\frac{a^2x^4\log\left(ax^2-1\right)-2\,a^2x^4\log\left(x\right)+ax^2+2\,\mathrm{Li}_2(ax^2)-\log\left(-ax^2+1\right)}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(a^2*x^4*log(a*x^2 - 1) - 2*a^2*x^4*log(x) + a*x^2 + 2*dilog(a*x^2) -
log(-a*x^2 + 1))/x^4
```

Sympy [A]
time = 2.15, size = 49, normalized size = 0.77

$$\frac{a^2\log\left(x\right)}{4}+\frac{a^2\,\mathrm{Li}_1\left(ax^2\right)}{8}-\frac{a}{8x^2}-\frac{\mathrm{Li}_1\left(ax^2\right)}{8x^4}-\frac{\mathrm{Li}_2\left(ax^2\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/x**5,x)
```

```
[Out] a**2*log(x)/4 + a**2*polylog(1, a*x**2)/8 - a/(8*x**2) - polylog(1, a*x**2)
/(8*x**4) - polylog(2, a*x**2)/(4*x**4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/x^5, x)
```

Mupad [B]
time = 0.26, size = 53, normalized size = 0.83

$$\frac{a^2\,\ln\left(x\right)}{4}-\frac{\mathrm{polylog}(2,a\,x^2)}{4\,x^4}-\frac{a^2\,\ln\left(a\,x^2-1\right)}{8}-\frac{a}{8\,x^2}+\frac{\ln\left(1-a\,x^2\right)}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^2)/x^5,x)
```

```
[Out] (a^2*log(x))/4 - polylog(2, a*x^2)/(4*x^4) - (a^2*log(a*x^2 - 1))/8 - a/(8*
x^2) + log(1 - a*x^2)/(8*x^4)
```

3.25 $\int \frac{\text{PolyLog}(2,ax^2)}{x^7} dx$

Optimal. Leaf size=74

$$-\frac{a}{36x^4}-\frac{a^2}{18x^2}+\frac{1}{9}a^3\log(x)-\frac{1}{18}a^3\log\left(1-ax^2\right)+\frac{\log\left(1-ax^2\right)}{18x^6}-\frac{\text{PolyLog}(2,ax^2)}{6x^6}$$

```
[Out] -1/36*a/x^4-1/18*a^2/x^2+1/9*a^3*ln(x)-1/18*a^3*ln(-a*x^2+1)+1/18*ln(-a*x^2
+1)/x^6-1/6*polylog(2,a*x^2)/x^6
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6726, 2504, 2442, 46}

$$-\frac{1}{18}a^3\log\left(1-ax^2\right)+\frac{1}{9}a^3\log(x)-\frac{a^2}{18x^2}-\frac{\text{Li}_2(ax^2)}{6x^6}-\frac{a}{36x^4}+\frac{\log\left(1-ax^2\right)}{18x^6}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/x^7,x]
```

```
[Out] -1/36*a/x^4 - a^2/(18*x^2) + (a^3*Log[x])/9 - (a^3*Log[1 - a*x^2])/18 + Log
[1 - a*x^2]/(18*x^6) - PolyLog[2, a*x^2]/(6*x^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax^2)}{x^7}\,dx &= -\frac{\text{Li}_2(ax^2)}{6x^6} - \frac{1}{3}\int \frac{\log(1-ax^2)}{x^7}\,dx \\
&= -\frac{\text{Li}_2(ax^2)}{6x^6} - \frac{1}{6}\text{Subst}\left(\int \frac{\log(1-ax)}{x^4}\,dx, x, x^2\right) \\
&= \frac{\log(1-ax^2)}{18x^6} - \frac{\text{Li}_2(ax^2)}{6x^6} + \frac{1}{18}a\text{Subst}\left(\int \frac{1}{x^3(1-ax)}\,dx, x, x^2\right) \\
&= \frac{\log(1-ax^2)}{18x^6} - \frac{\text{Li}_2(ax^2)}{6x^6} + \frac{1}{18}a\text{Subst}\left(\int \left(\frac{1}{x^3} + \frac{a}{x^2} + \frac{a^2}{x} - \frac{a^3}{-1+ax}\right)dx, x, x^2\right) \\
&= -\frac{a}{36x^4} - \frac{a^2}{18x^2} + \frac{1}{9}a^3\log(x) - \frac{1}{18}a^3\log(1-ax^2) + \frac{\log(1-ax^2)}{18x^6} - \frac{\text{Li}_2(ax^2)}{6x^6}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 60, normalized size = 0.81

$$-\frac{ax^2(1+2ax^2) - 4a^3x^6\log(x) + 2(-1+a^3x^6)\log(1-ax^2) + 6\text{PolyLog}(2, ax^2)}{36x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^2]/x^7,x]
[Out] -1/36*(a*x^2*(1 + 2*a*x^2) - 4*a^3*x^6*Log[x] + 2*(-1 + a^3*x^6)*Log[1 - a*
x^2] + 6*PolyLog[2, a*x^2])/x^6
```

Maple [A]
time = 0.12, size = 62, normalized size = 0.84

method	result	size
default	$-\frac{\text{polylog}(2,a\,x^2)}{6x^6} + \frac{\ln(-a\,x^2+1)}{18x^6} + \frac{a\left(-\frac{a^2\ln(a\,x^2-1)}{2} - \frac{1}{4x^4} - \frac{a}{2x^2} + a^2\ln(x)\right)}{9}$	62
meijerg	$\frac{a^3\left(\frac{32a^2x^4+60a\,x^2+192}{432a^2x^4} + \frac{(-16a^3x^6+16)\ln(-a\,x^2+1)}{144a^3x^6} - \frac{\text{polylog}(2,a\,x^2)}{3a^3x^6} - \frac{2}{27} + \frac{2\ln(x)}{9} + \frac{\ln(-a)}{9} - \frac{1}{2a^2x^4} - \frac{1}{4a\,x^2}\right)}{2}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*polylog(2,a*x^2)/x^6+1/18*ln(-a*x^2+1)/x^6+1/9*a*(-1/2*a^2*ln(a*x^2-1)
-1/4/x^4-1/2*a/x^2+a^2*ln(x))
```

Maxima [A]
time = 0.25, size = 55, normalized size = 0.74

$$\frac{1}{9}\,a^3\log\left(x\right)-\frac{2\,a^2x^4+ax^2+2\left(a^3x^6-1\right)\log\left(-ax^2+1\right)+6\,\mathrm{Li}_2(ax^2)}{36\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/9*a^3*log(x) - 1/36*(2*a^2*x^4 + a*x^2 + 2*(a^3*x^6 - 1)*log(-a*x^2 + 1)
+ 6*dilog(a*x^2))/x^6
```

Fricas [A]
time = 0.42, size = 64, normalized size = 0.86

$$-\frac{2\,a^3x^6\log\left(ax^2-1\right)-4\,a^3x^6\log\left(x\right)+2\,a^2x^4+ax^2+6\,\mathrm{Li}_2(ax^2)-2\,\log\left(-ax^2+1\right)}{36\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/36*(2*a^3*x^6*log(a*x^2 - 1) - 4*a^3*x^6*log(x) + 2*a^2*x^4 + a*x^2 + 6*
dilog(a*x^2) - 2*log(-a*x^2 + 1))/x^6
```

Sympy [A]
time = 5.20, size = 58, normalized size = 0.78

$$\frac{a^3\log\left(x\right)}{9}+\frac{a^3\,\mathrm{Li}_1\left(ax^2\right)}{18}-\frac{a^2}{18x^2}-\frac{a}{36x^4}-\frac{\mathrm{Li}_1\left(ax^2\right)}{18x^6}-\frac{\mathrm{Li}_2\left(ax^2\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/x**7,x)
```

```
[Out] a**3*log(x)/9 + a**3*polylog(1, a*x**2)/18 - a**2/(18*x**2) - a/(36*x**4) -
 polylog(1, a*x**2)/(18*x**6) - polylog(2, a*x**2)/(6*x**6)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/x^7, x)
```

Mupad [B]
time = 0.28, size = 61, normalized size = 0.82

$$\frac{a^3\ln(x)}{9}-\frac{\operatorname{polylog}(2,a\,x^2)}{6\,x^6}-\frac{a^3\ln(a\,x^2-1)}{18}-\frac{a}{36\,x^4}+\frac{\ln(1-a\,x^2)}{18\,x^6}-\frac{a^2}{18\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^2)/x^7,x)
```

```
[Out] (a^3*log(x))/9 - polylog(2, a*x^2)/(6*x^6) - (a^3*log(a*x^2 - 1))/18 - a/(3
6*x^4) + log(1 - a*x^2)/(18*x^6) - a^2/(18*x^2)
```

3.26 $\int x^4\textbf{PolyLog}(2, ax^2)\,dx$

Optimal. Leaf size=73

$$-\frac{4x}{25a^2}-\frac{4x^3}{75a}-\frac{4x^5}{125}+\frac{4\tanh^{-1}\left(\sqrt{a}\,x\right)}{25a^{5/2}}+\frac{2}{25}x^5\log\left(1-ax^2\right)+\frac{1}{5}x^5\text{PolyLog}\left(2,ax^2\right)$$

```
[Out] -4/25*x/a^2-4/75*x^3/a-4/125*x^5+4/25*arctanh(x*a^(1/2))/a^(5/2)+2/25*x^5*l
n(-a*x^2+1)+1/5*x^5*polylog(2,a*x^2)
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6726, 2505, 308, 212}

$$\frac{4\tanh^{-1}\left(\sqrt{a}\,x\right)}{25a^{5/2}}-\frac{4x}{25a^2}+\frac{1}{5}x^5\text{Li}_2\left(ax^2\right)-\frac{4x^3}{75a}+\frac{2}{25}x^5\log\left(1-ax^2\right)-\frac{4x^5}{125}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*PolyLog[2, a*x^2],x]
```

```
[Out] (-4*x)/(25*a^2) - (4*x^3)/(75*a) - (4*x^5)/125 + (4*ArcTanh[Sqrt[a]*x])/(25
*a^(5/2)) + (2*x^5*Log[1 - a*x^2])/25 + (x^5*PolyLog[2, a*x^2])/5
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
```

```
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^4 \text{Li}_2\left(ax^2\right)\, dx &= \frac{1}{5}x^5 \text{Li}_2\left(ax^2\right) + \frac{2}{5}\int x^4 \log\left(1-ax^2\right)\, dx \\
&= \frac{2}{25}x^5 \log\left(1-ax^2\right) + \frac{1}{5}x^5 \text{Li}_2\left(ax^2\right) + \frac{1}{25}(4a)\int \frac{x^6}{1-ax^2}\, dx \\
&= \frac{2}{25}x^5 \log\left(1-ax^2\right) + \frac{1}{5}x^5 \text{Li}_2\left(ax^2\right) + \frac{1}{25}(4a)\int \left(-\frac{1}{a^3} - \frac{x^2}{a^2} - \frac{x^4}{a} + \frac{1}{a^3\left(1-ax^2\right)}\right) dx \\
&= -\frac{4x}{25a^2} - \frac{4x^3}{75a} - \frac{4x^5}{125} + \frac{2}{25}x^5 \log\left(1-ax^2\right) + \frac{1}{5}x^5 \text{Li}_2\left(ax^2\right) + \frac{4\int \frac{1}{1-ax^2}\, dx}{25a^2} \\
&= -\frac{4x}{25a^2} - \frac{4x^3}{75a} - \frac{4x^5}{125} + \frac{4\tanh^{-1}\left(\sqrt{a}\, x\right)}{25a^{5/2}} + \frac{2}{25}x^5 \log\left(1-ax^2\right) + \frac{1}{5}x^5 \text{Li}_2\left(ax^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 65, normalized size = 0.89

$$
\frac{1}{375}\left(-\frac{60x}{a^2} - \frac{20x^3}{a} - 12x^5 + \frac{60\tanh^{-1}\left(\sqrt{a}\, x\right)}{a^{5/2}} + 30x^5 \log\left(1-ax^2\right) + 75x^5 \text{PolyLog}\left(2, ax^2\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*PolyLog[2, a*x^2],x]
```

```
[Out] ((-60*x)/a^2 - (20*x^3)/a - 12*x^5 + (60*ArcTanh[Sqrt[a]*x])/a^(5/2) + 30*x
^5*Log[1 - a*x^2] + 75*x^5*PolyLog[2, a*x^2])/375
```

Maple [A]
time = 0.08, size = 63, normalized size = 0.86

method	result
default	$\frac{x^5 \operatorname{polylog}(2, a x^2)}{5} + \frac{2x^5 \ln(-a x^2+1)}{25} + \frac{4a\left(-\frac{\frac{1}{5}a^2 x^5 + \frac{1}{3}a x^3 + x}{a^3} + \frac{\operatorname{arctanh}\left(x\sqrt{a}\right)}{a^{\frac{7}{2}}}\right)}{25}$
meijerg	$-\frac{-\frac{2x(-a)^{\frac{7}{2}}\left(84a^2x^4+140a x^2+420\right)}{2625a^3} - \frac{4x(-a)^{\frac{7}{2}}\left(\ln\left(1-\sqrt{a x^2}\right)-\ln\left(1+\sqrt{a x^2}\right)\right)}{25a^3\sqrt{a x^2}} + \frac{4x^5(-a)^{\frac{7}{2}}\ln\left(-a x^2+1\right)}{25a} + \frac{2x^5(-a)^{\frac{7}{2}}\operatorname{polylog}\left(2, a x^2\right)}{5a}}{2a^2\sqrt{-a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*polylog(2,a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*polylog(2,a*x^2)+2/25*x^5*ln(-a*x^2+1)+4/25*a*(-1/a^3*(1/5*a^2*x^5+
1/3*a*x^3+x)+1/a^(7/2)*arctanh(x*a^(1/2)))
```

Maxima [A]
time = 0.47, size = 80, normalized size = 1.10

$$\frac{75\,a^2x^5\mathrm{Li}_2(ax^2)+30\,a^2x^5\log\left(-ax^2+1\right)-12\,a^2x^5-20\,ax^3-60\,x}{375\,a^2}-\frac{2\,\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right)}{25\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*polylog(2,a*x^2),x, algorithm="maxima")
```

```
[Out] 1/375*(75*a^2*x^5*dilog(a*x^2) + 30*a^2*x^5*log(-a*x^2 + 1) - 12*a^2*x^5 -
20*a*x^3 - 60*x)/a^2 - 2/25*log((a*x - sqrt(a))/(a*x + sqrt(a)))/a^(5/2)
```

Fricas [A]
time = 0.59, size = 159, normalized size = 2.18

$$\left[\frac{75\,a^3x^5\mathrm{Li}_2(ax^2)+30\,a^3x^5\log\left(-ax^2+1\right)-12\,a^3x^5-20\,a^2x^3-60\,ax+30\,\sqrt{a}\,\log\left(\frac{ax^2+2\,\sqrt{a}\,x+1}{ax^2-1}\right)}{375\,a^3},\frac{75\,a^3x^5\mathrm{Li}_2(ax^2)+30\,a^3x^5\log\left(-ax^2+1\right)-12\,a^3x^5-20\,a^2x^3-60\,ax-60\,\sqrt{-a}\,\arctan\left(\sqrt{-a}\,x\right)}{375\,a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*polylog(2,a*x^2),x, algorithm="fricas")
```

```
[Out] [1/375*(75*a^3*x^5*dilog(a*x^2) + 30*a^3*x^5*log(-a*x^2 + 1) - 12*a^3*x^5 -
 20*a^2*x^3 - 60*a*x + 30*sqrt(a)*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)
))/a^3, 1/375*(75*a^3*x^5*dilog(a*x^2) + 30*a^3*x^5*log(-a*x^2 + 1) - 12*a^
3*x^5 - 20*a^2*x^3 - 60*a*x - 60*sqrt(-a)*arctan(sqrt(-a)*x))/a^3]
```

Sympy [A]
time = 48.48, size = 94, normalized size = 1.29

$$\begin{cases} -\frac{2x^5\,\mathrm{Li}_1\left(ax^2\right)}{25}+\frac{x^5\,\mathrm{Li}_2\left(ax^2\right)}{5}-\frac{4x^5}{125}-\frac{4x^3}{75a}-\frac{4x}{25a^2}-\frac{4\log\left(x-\sqrt{\frac{1}{a}}\right)}{25a^3\sqrt{\frac{1}{a}}}-\frac{2\,\mathrm{Li}_1\left(ax^2\right)}{25a^3\sqrt{\frac{1}{a}}} & \text{for}\ a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*polylog(2,a*x**2),x)
```

```
[Out] Piecewise((-2*x**5*polylog(1, a*x**2)/25 + x**5*polylog(2, a*x**2)/5 - 4*x*
*5/125 - 4*x**3/(75*a) - 4*x/(25*a**2) - 4*log(x - sqrt(1/a))/(25*a**3*sqrt
(1/a)) - 2*polylog(1, a*x**2)/(25*a**3*sqrt(1/a)), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*polylog(2,a*x^2),x, algorithm="giac")
[Out] integrate(x^4*dilog(a*x^2), x)
```

Mupad [B]
time = 0.41, size = 60, normalized size = 0.82

$$\frac{x^5\,\mathrm{polylog}(2,a\,x^2)}{5}-\frac{4\,x}{25\,a^2}+\frac{2\,x^5\,\ln\left(1-a\,x^2\right)}{25}-\frac{4\,x^5}{125}-\frac{4\,x^3}{75\,a}-\frac{\mathrm{atan}\left(\sqrt{a}\,x\,1\mathrm{i}\right)\,4\mathrm{i}}{25\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*polylog(2, a*x^2),x)
[Out] (x^5*polylog(2, a*x^2))/5 - (atan(a^(1/2)*x*1i)*4i)/(25*a^(5/2)) - (4*x)/(2
5*a^2) + (2*x^5*log(1 - a*x^2))/25 - (4*x^5)/125 - (4*x^3)/(75*a)
```

3.27 $\int x^2\mathbf{PolyLog}(2, ax^2)\, dx$

Optimal. Leaf size=63

$$-\frac{4x}{9a}-\frac{4x^3}{27}+\frac{4\tanh^{-1}\left(\sqrt{a}\,x\right)}{9a^{3/2}}+\frac{2}{9}x^3\log\left(1-ax^2\right)+\frac{1}{3}x^3\text{PolyLog}(2,ax^2)$$

```
[Out] -4/9*x/a-4/27*x^3+4/9*arctanh(x*a^(1/2))/a^(3/2)+2/9*x^3*ln(-a*x^2+1)+1/3*x
^3*polylog(2,a*x^2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6726, 2505, 308, 212}

$$\frac{4\tanh^{-1}\left(\sqrt{a}\,x\right)}{9a^{3/2}}+\frac{1}{3}x^3\text{Li}_2\left(ax^2\right)+\frac{2}{9}x^3\log\left(1-ax^2\right)-\frac{4x}{9a}-\frac{4x^3}{27}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*PolyLog[2, a*x^2],x]
```

```
[Out] (-4*x)/(9*a) - (4*x^3)/27 + (4*ArcTanh[Sqrt[a]*x])/(9*a^(3/2)) + (2*x^3*Log
[1 - a*x^2])/9 + (x^3*PolyLog[2, a*x^2])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
```

```
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \text{Li}_2(ax^2)\, dx &= \frac{1}{3}x^3\text{Li}_2(ax^2) + \frac{2}{3}\int x^2 \log(1-ax^2)\, dx \\
&= \frac{2}{9}x^3 \log(1-ax^2) + \frac{1}{3}x^3\text{Li}_2(ax^2) + \frac{1}{9}(4a)\int \frac{x^4}{1-ax^2}\, dx \\
&= \frac{2}{9}x^3 \log(1-ax^2) + \frac{1}{3}x^3\text{Li}_2(ax^2) + \frac{1}{9}(4a)\int \left(-\frac{1}{a^2} - \frac{x^2}{a} + \frac{1}{a^2(1-ax^2)}\right) dx \\
&= -\frac{4x}{9a} - \frac{4x^3}{27} + \frac{2}{9}x^3 \log(1-ax^2) + \frac{1}{3}x^3\text{Li}_2(ax^2) + \frac{4\int \frac{1}{1-ax^2}\, dx}{9a} \\
&= -\frac{4x}{9a} - \frac{4x^3}{27} + \frac{4\tanh^{-1}(\sqrt{a}\,x)}{9a^{3/2}} + \frac{2}{9}x^3 \log(1-ax^2) + \frac{1}{3}x^3\text{Li}_2(ax^2)
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 0.90

$$\frac{1}{27}\left(-\frac{12x}{a} - 4x^3 + \frac{12\tanh^{-1}(\sqrt{a}\,x)}{a^{3/2}} + 6x^3\log(1-ax^2) + 9x^3\text{PolyLog}(2, ax^2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*PolyLog[2, a*x^2],x]
[Out] ((-12*x)/a - 4*x^3 + (12*ArcTanh[Sqrt[a]*x])/a^(3/2) + 6*x^3*Log[1 - a*x^2]
 + 9*x^3*PolyLog[2, a*x^2])/27
```

Maple [A]
time = 0.07, size = 55, normalized size = 0.87

method	result	size
default	$\frac{x^3 \operatorname{polylog}(2,a x^2)}{3} + \frac{2x^3 \ln(-a x^2+1)}{9} + \frac{4a\left(-\frac{\frac{1}{3}a x^3+x}{a^2} + \frac{\operatorname{arctanh}(x\sqrt{a})}{a^{\frac{5}{2}}}\right)}{9}$	55
meijerg	$\frac{-\frac{2x(-a)^{\frac{5}{2}}(20a x^2+60)}{135a^2} - \frac{4x(-a)^{\frac{5}{2}}\left(\ln\left(1-\sqrt{a x^2}\right)-\ln\left(1+\sqrt{a x^2}\right)\right)}{9a^2\sqrt{a x^2}} + \frac{4x^3(-a)^{\frac{5}{2}}\ln(-a x^2+1)}{9a} + \frac{2x^3(-a)^{\frac{5}{2}}\operatorname{polylog}(2,a x^2)}{3a}}{2a\sqrt{-a}}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(2,a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*polylog(2,a*x^2)+2/9*x^3*ln(-a*x^2+1)+4/9*a*(-1/a^2*(1/3*a*x^3+x)+a
rctanh(x*a^(1/2))/a^(5/2))
```

Maxima [A]
time = 0.47, size = 68, normalized size = 1.08

$$\frac{9\,ax^3\mathrm{Li}_2(ax^2)+6\,ax^3\log\left(-ax^2+1\right)-4\,ax^3-12\,x}{27\,a}-\frac{2\,\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right)}{9\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,a*x^2),x, algorithm="maxima")
```

```
[Out] 1/27*(9*a*x^3*dilog(a*x^2) + 6*a*x^3*log(-a*x^2 + 1) - 4*a*x^3 - 12*x)/a -
2/9*log((a*x - sqrt(a))/(a*x + sqrt(a)))/a^(3/2)
```

Fricas [A]
time = 0.54, size = 143, normalized size = 2.27

$$\left[\frac{9\,a^2x^3\mathrm{Li}_2(ax^2)+6\,a^2x^3\log\left(-ax^2+1\right)-4\,a^2x^3-12\,ax+6\,\sqrt{a}\,\log\left(\frac{ax^2+2\,\sqrt{a}\,x+1}{ax^2-1}\right)}{27\,a^2},\frac{9\,a^2x^3\mathrm{Li}_2(ax^2)+6\,a^2x^3\log\left(-ax^2+1\right)-4\,a^2x^3-12\,ax-12\,\sqrt{-a}\,\arctan\left(\sqrt{-a}\,x\right)}{27\,a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,a*x^2),x, algorithm="fricas")
```

```
[Out] [1/27*(9*a^2*x^3*dilog(a*x^2) + 6*a^2*x^3*log(-a*x^2 + 1) - 4*a^2*x^3 - 12*
a*x + 6*sqrt(a)*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)))/a^2, 1/27*(9*a^
2*x^3*dilog(a*x^2) + 6*a^2*x^3*log(-a*x^2 + 1) - 4*a^2*x^3 - 12*a*x - 12*sq
rt(-a)*arctan(sqrt(-a)*x))/a^2]
```

Sympy [A]
time = 13.13, size = 83, normalized size = 1.32

$$\begin{cases}-\frac{2x^3\operatorname{Li}_{1}\left(ax^2\right)}{9}+\frac{x^3\operatorname{Li}_{2}\left(ax^2\right)}{3}-\frac{4x^3}{27}-\frac{4x}{9a}-\frac{4\log\left(x-\sqrt{\frac{1}{a}}\right)}{9a^2\sqrt{\frac{1}{a}}}-\frac{2\operatorname{Li}_{1}\left(ax^2\right)}{9a^2\sqrt{\frac{1}{a}}} & \text{for}\: a\neq 0\\0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*polylog(2,a*x**2),x)
```

```
[Out] Piecewise((-2*x**3*polylog(1, a*x**2)/9 + x**3*polylog(2, a*x**2)/3 - 4*x**
3/27 - 4*x/(9*a) - 4*log(x - sqrt(1/a))/(9*a**2*sqrt(1/a)) - 2*polylog(1, a
*x**2)/(9*a**2*sqrt(1/a)), Ne(a, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*dilog(a*x^2), x)
```

Mupad [B]
time = 0.28, size = 52, normalized size = 0.83

$$\frac{x^3\,\mathrm{polylog}(2,a\,x^2)}{3}-\frac{4\,x}{9\,a}+\frac{2\,x^3\,\ln\left(1-a\,x^2\right)}{9}-\frac{4\,x^3}{27}-\frac{\mathrm{atan}\left(\sqrt{a}\,\,x\,1\mathrm{i}\right)\,4\mathrm{i}}{9\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(2, a*x^2),x)
```

```
[Out] (x^3*polylog(2, a*x^2))/3 - (atan(a^(1/2)*x*1i)*4i)/(9*a^(3/2)) - (4*x)/(9*
a) + (2*x^3*log(1 - a*x^2))/9 - (4*x^3)/27
```

3.28 $\int \textbf{PolyLog}(2, ax^2)\, dx$

Optimal. Leaf size=40

$$-4x + \frac{4\tanh^{-1}\left(\sqrt{a}\,x\right)}{\sqrt{a}} + 2x\log\left(1 - ax^2\right) + x\text{PolyLog}\left(2, ax^2\right)$$

```
[Out] -4*x+2*x*ln(-a*x^2+1)+x*polylog(2,a*x^2)+4*arctanh(x*a^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6721, 2498, 327, 212}

$$x\text{Li}_2\left(ax^2\right) + 2x\log\left(1 - ax^2\right) + \frac{4\tanh^{-1}\left(\sqrt{a}\,x\right)}{\sqrt{a}} - 4x$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2],x]
```

```
[Out] -4*x + (4*ArcTanh[Sqrt[a]*x])/Sqrt[a] + 2*x*Log[1 - a*x^2] + x*PolyLog[2, a
*x^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
```

```
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \text{Li}_2\left(ax^2\right)\,dx &= x\text{Li}_2\left(ax^2\right)+2\int \log\left(1-ax^2\right)\,dx\\
&= 2x\log\left(1-ax^2\right)+x\text{Li}_2\left(ax^2\right)+(4a)\int \frac{x^2}{1-ax^2}\,dx\\
&= -4x+2x\log\left(1-ax^2\right)+x\text{Li}_2\left(ax^2\right)+4\int \frac{1}{1-ax^2}\,dx\\
&= -4x+\frac{4\tanh^{-1}\left(\sqrt{a}\,x\right)}{\sqrt{a}}+2x\log\left(1-ax^2\right)+x\text{Li}_2\left(ax^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.98

$$\frac{4\tanh^{-1}\left(\sqrt{a}\,x\right)}{\sqrt{a}}+2x\left(-2+\log\left(1-ax^2\right)\right)+x\text{PolyLog}\left(2,ax^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^2],x]
[Out] (4*ArcTanh[Sqrt[a]*x])/Sqrt[a] + 2*x*(-2 + Log[1 - a*x^2]) + x*PolyLog[2, a
*x^2]
```

Maple [A]
time = 0.07, size = 43, normalized size = 1.08

method	result	size
default	$x\,\text{polylog}\left(2,a\,x^2\right)+2x\ln\left(-a\,x^2+1\right)+4a\left(-\frac{x}{a}+\frac{\text{arctanh}\left(x\sqrt{a}\right)}{a^{\frac{3}{2}}}\right)$	43
meijerg	$-\frac{-\frac{8x(-a)^{\frac{3}{2}}}{a}-\frac{4x(-a)^{\frac{3}{2}}\left(\ln\left(1-\sqrt{a\,x^2}\right)-\ln\left(1+\sqrt{a\,x^2}\right)\right)}{a\sqrt{a\,x^2}}+\frac{4x(-a)^{\frac{3}{2}}\ln\left(-a\,x^2+1\right)}{a}+\frac{2x(-a)^{\frac{3}{2}}\,\text{polylog}\left(2,a\,x^2\right)}{a}}{2\sqrt{-a}}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2),x,method=_RETURNVERBOSE)
[Out] x*polylog(2,a*x^2)+2*x*ln(-a*x^2+1)+4*a*(-x/a+arctanh(x*a^(1/2))/a^(3/2))
```

Maxima [A]
time = 0.46, size = 49, normalized size = 1.22

$$x\mathrm{Li}_2\left(ax^2\right)+2\,x\log\left(-ax^2+1\right)-4\,x-\frac{2\,\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2),x, algorithm="maxima")
```

```
[Out] x*dilog(a*x^2) + 2*x*log(-a*x^2 + 1) - 4*x - 2*log((a*x - sqrt(a))/(a*x + s
qrt(a)))/sqrt(a)
```

Fricas [A]
time = 0.48, size = 107, normalized size = 2.68

$$\left[\frac{ax\mathrm{Li}_2(ax^2)+2\,ax\log\left(-ax^2+1\right)-4\,ax+2\,\sqrt{a}\,\log\left(\frac{ax^2+2\,\sqrt{a}\,x+1}{ax^2-1}\right)}{a},\frac{ax\mathrm{Li}_2(ax^2)+2\,ax\log\left(-ax^2+1\right)-4\,ax-4\,\sqrt{-a}\,\arctan\left(\sqrt{-a}\,x\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2),x, algorithm="fricas")
```

```
[Out] [(a*x*dilog(a*x^2) + 2*a*x*log(-a*x^2 + 1) - 4*a*x + 2*sqrt(a)*log((a*x^2 +
 2*sqrt(a)*x + 1)/(a*x^2 - 1)))/a, (a*x*dilog(a*x^2) + 2*a*x*log(-a*x^2 + 1
) - 4*a*x - 4*sqrt(-a)*arctan(sqrt(-a)*x))/a]
```

Sympy [A]
time = 3.71, size = 60, normalized size = 1.50

$$\begin{cases} -2x\,\mathrm{Li}_1\left(ax^2\right)+x\,\mathrm{Li}_2\left(ax^2\right)-4x-\frac{4\log\left(x-\sqrt{\frac{1}{a}}\right)}{a\sqrt{\frac{1}{a}}}-\frac{2\,\mathrm{Li}_1\left(ax^2\right)}{a\sqrt{\frac{1}{a}}} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2),x)
```

```
[Out] Piecewise((-2*x*polylog(1, a*x**2) + x*polylog(2, a*x**2) - 4*x - 4*log(x -
 sqrt(1/a))/(a*sqrt(1/a)) - 2*polylog(1, a*x**2)/(a*sqrt(1/a)), Ne(a, 0)),
(0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2), x)
```

Mupad [B]
time = 0.24, size = 39, normalized size = 0.98

$$2\,x\,\ln\left(1-a\,x^2\right)-4\,x+x\,\mathrm{polylog}\left(2,a\,x^2\right)-\frac{\mathrm{atan}\left(\sqrt{a}\,\,x\,1\mathrm{i}\right)\,4\mathrm{i}}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^2),x)
```

```
[Out] 2*x*log(1 - a*x^2) - (atan(a^(1/2)*x*1i)*4i)/a^(1/2) - 4*x + x*polylog(2, a
*x^2)
```

3.29 $\int \frac{\text{PolyLog}(2,ax^2)}{x^2}\,dx$

Optimal. Leaf size=42

$$4\sqrt{a}\,\tanh^{-1}\left(\sqrt{a}\,x\right)+\frac{2\log\left(1-ax^2\right)}{x}-\frac{\text{PolyLog}(2,ax^2)}{x}$$

```
[Out] 2*ln(-a*x^2+1)/x-polylog(2,a*x^2)/x+4*arctanh(x*a^(1/2))*a^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6726, 2505, 212}

$$-\frac{\text{Li}_2(ax^2)}{x}+\frac{2\log\left(1-ax^2\right)}{x}+4\sqrt{a}\,\tanh^{-1}\left(\sqrt{a}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/x^2,x]
```

```
[Out] 4*Sqrt[a]*ArcTanh[Sqrt[a]*x] + (2*Log[1 - a*x^2])/x - PolyLog[2, a*x^2]/x
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_2(ax^2)}{x^2}\,dx &= -\frac{\text{Li}_2(ax^2)}{x} - 2\int \frac{\log(1-ax^2)}{x^2}\,dx \\ &= \frac{2\log(1-ax^2)}{x} - \frac{\text{Li}_2(ax^2)}{x} + (4a)\int \frac{1}{1-ax^2}\,dx \\ &= 4\sqrt{a}\,\tanh^{-1}\left(\sqrt{a}\,x\right) + \frac{2\log(1-ax^2)}{x} - \frac{\text{Li}_2(ax^2)}{x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 0.98

$$\frac{4\sqrt{a}\,x\tanh^{-1}\left(\sqrt{a}\,x\right) + 2\log\left(1-ax^2\right) - \text{PolyLog}(2,ax^2)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^2]/x^2,x]
[Out] (4*Sqrt[a]*x*ArcTanh[Sqrt[a]*x] + 2*Log[1 - a*x^2] - PolyLog[2, a*x^2])/x
```

Maple [A]
time = 0.07, size = 39, normalized size = 0.93

method	result	size
default	$\frac{2\ln(-a\,x^2+1)}{x}-\frac{\text{polylog}(2,a\,x^2)}{x}+4\,\text{arctanh}\left(x\sqrt{a}\right)\sqrt{a}$	39
meijerg	$\frac{a\left(-\frac{4x\sqrt{-a}\,\left(\ln\left(1-\sqrt{a\,x^2}\right)-\ln\left(1+\sqrt{a\,x^2}\right)\right)}{\sqrt{a\,x^2}}+\frac{4\sqrt{-a}\,\ln\left(-a\,x^2+1\right)}{xa}-\frac{2\sqrt{-a}\,\text{polylog}\left(2,a\,x^2\right)}{xa}\right)}{2\sqrt{-a}}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/x^2,x,method=_RETURNVERBOSE)
[Out] 2*ln(-a*x^2+1)/x-polylog(2,a*x^2)/x+4*arctanh(x*a^(1/2))*a^(1/2)
```

Maxima [A]
time = 0.46, size = 49, normalized size = 1.17

$$-2\sqrt{a}\,\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right) - \frac{\text{Li}_2(ax^2) - 2\log\left(-ax^2+1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] -2*sqrt(a)*log((a*x - sqrt(a))/(a*x + sqrt(a))) - (dilog(a*x^2) - 2*log(-a*
x^2 + 1))/x
```

Fricas [A]
time = 0.60, size = 94, normalized size = 2.24

$$\left[\frac{2\sqrt{a}\,x\log\left(\frac{ax^2+2\sqrt{a}\,x+1}{ax^2-1}\right)-\mathrm{Li}_2(ax^2)+2\log\left(-ax^2+1\right)}{x},-\frac{4\sqrt{-a}\,x\arctan\left(\sqrt{-a}\,x\right)+\mathrm{Li}_2(ax^2)-2\log\left(-ax^2+1\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] [(2*sqrt(a)*x*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)) - dilog(a*x^2) + 2
*log(-a*x^2 + 1))/x, -(4*sqrt(-a)*x*arctan(sqrt(-a)*x) + dilog(a*x^2) - 2*l
og(-a*x^2 + 1))/x]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(34) = 68$.
time = 12.26, size = 184, normalized size = 4.38

$$\begin{cases} -\frac{\pi^2}{6x} & \text{for}\ a=\frac{1}{x^2} \\ 0 & \text{for}\ a=0 \\ -\frac{4ax^3\sqrt{\frac{1}{a}}\log\left(x-\sqrt{\frac{1}{a}}\right)}{x^3-\frac{x}{a}}-\frac{2ax^3\sqrt{\frac{1}{a}}\,\mathrm{Li}_1\left(ax^2\right)}{x^3-\frac{x}{a}}-\frac{2x^2\,\mathrm{Li}_1\left(ax^2\right)}{x^3-\frac{x}{a}}-\frac{x^2\,\mathrm{Li}_2\left(ax^2\right)}{x^3-\frac{x}{a}}+\frac{4x\sqrt{\frac{1}{a}}\log\left(x-\sqrt{\frac{1}{a}}\right)}{x^3-\frac{x}{a}}+\frac{2x\sqrt{\frac{1}{a}}\,\mathrm{Li}_1\left(ax^2\right)}{x^3-\frac{x}{a}}+\frac{2\,\mathrm{Li}_1\left(ax^2\right)}{ax^3-x}+\frac{\mathrm{Li}_2\left(ax^2\right)}{ax^3-x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/x**2,x)
```

```
[Out] Piecewise((-pi**2/(6*x), Eq(a, x**(-2))), (0, Eq(a, 0)), (-4*a*x**3*sqrt(1/
a)*log(x - sqrt(1/a))/(x**3 - x/a) - 2*a*x**3*sqrt(1/a)*polylog(1, a*x**2)/
(x**3 - x/a) - 2*x**2*polylog(1, a*x**2)/(x**3 - x/a) - x**2*polylog(2, a*x
**2)/(x**3 - x/a) + 4*x*sqrt(1/a)*log(x - sqrt(1/a))/(x**3 - x/a) + 2*x*sqr
t(1/a)*polylog(1, a*x**2)/(x**3 - x/a) + 2*polylog(1, a*x**2)/(a*x**3 - x)
+ polylog(2, a*x**2)/(a*x**3 - x), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/x^2, x)
```

Mupad [B]
time = 0.26, size = 38, normalized size = 0.90

$$4\sqrt{a}\ \mathrm{atanh}\left(\sqrt{a}\ x\right)-\frac{\mathrm{polylog}(2,a\,x^2)}{x}+\frac{2\ \ln\left(1-a\,x^2\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^2)/x^2,x)
[Out] 4*a^(1/2)*atanh(a^(1/2)*x) - polylog(2, a*x^2)/x + (2*log(1 - a*x^2))/x
```

3.30 $\int \frac{\text{PolyLog}(2,ax^2)}{x^4} dx$

Optimal. Leaf size=56

$$-\frac{4a}{9x}+\frac{4}{9}a^{3/2}\tanh^{-1}\left(\sqrt{a}\,x\right)+\frac{2\log\left(1-ax^2\right)}{9x^3}-\frac{\text{PolyLog}(2,ax^2)}{3x^3}$$

```
[Out] -4/9*a/x+4/9*a^(3/2)*arctanh(x*a^(1/2))+2/9*ln(-a*x^2+1)/x^3-1/3*polylog(2,
a*x^2)/x^3
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6726, 2505, 331, 212}

$$\frac{4}{9}a^{3/2}\tanh^{-1}\left(\sqrt{a}\,x\right)-\frac{\text{Li}_2(ax^2)}{3x^3}+\frac{2\log\left(1-ax^2\right)}{9x^3}-\frac{4a}{9x}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/x^4,x]
```

```
[Out] (-4*a)/(9*x) + (4*a^(3/2)*ArcTanh[Sqrt[a]*x])/9 + (2*Log[1 - a*x^2])/(9*x^3
) - PolyLog[2, a*x^2]/(3*x^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax^2)}{x^4}\,dx &= -\frac{\text{Li}_2(ax^2)}{3x^3} - \frac{2}{3}\int \frac{\log\left(1-ax^2\right)}{x^4}\,dx \\
&= \frac{2\log\left(1-ax^2\right)}{9x^3} - \frac{\text{Li}_2(ax^2)}{3x^3} + \frac{1}{9}(4a)\int \frac{1}{x^2\left(1-ax^2\right)}\,dx \\
&= -\frac{4a}{9x} + \frac{2\log\left(1-ax^2\right)}{9x^3} - \frac{\text{Li}_2(ax^2)}{3x^3} + \frac{1}{9}\left(4a^2\right)\int \frac{1}{1-ax^2}\,dx \\
&= -\frac{4a}{9x} + \frac{4}{9}a^{3/2}\tanh^{-1}\left(\sqrt{a}\,x\right) + \frac{2\log\left(1-ax^2\right)}{9x^3} - \frac{\text{Li}_2(ax^2)}{3x^3}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.01, size = 47, normalized size = 0.84

$$-\frac{4ax^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};ax^2\right) - 2\log\left(1-ax^2\right) + 3\text{PolyLog}(2,ax^2)}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^2]/x^4,x]
```

```
[Out] -1/9*(4*a*x^2*Hypergeometric2F1[-1/2, 1, 1/2, a*x^2] - 2*Log[1 - a*x^2] + 3
*PolyLog[2, a*x^2])/x^3
```

Maple [A]
time = 0.07, size = 47, normalized size = 0.84

method	result	size
default	$-\frac{\text{polylog}(2,a\,x^2)}{3x^3}+\frac{2\ln(-a\,x^2+1)}{9x^3}+\frac{4a\left(\text{arctanh}\left(x\sqrt{a}\right)\sqrt{a}-\frac{1}{x}\right)}{9}$	47
meijerg	$-\frac{a^2\left(-\frac{8}{9x\sqrt{-a}}-\frac{4xa\left(\ln\left(1-\sqrt{a\,x^2}\right)-\ln\left(1+\sqrt{a\,x^2}\right)\right)}{9\sqrt{-a}\,\sqrt{a\,x^2}}+\frac{4\ln\left(-a\,x^2+1\right)}{9x^3\sqrt{-a}\,a}-\frac{2\,\text{polylog}\left(2,a\,x^2\right)}{3x^3\sqrt{-a}\,a}\right)}{2\sqrt{-a}}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*polylog(2,a*x^2)/x^3+2/9*ln(-a*x^2+1)/x^3+4/9*a*(arctanh(x*a^(1/2))*a^
(1/2)-1/x)
```

Maxima [A]
time = 0.47, size = 57, normalized size = 1.02

$$-\frac{2}{9}\,a^{\frac{3}{2}}\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right)-\frac{4\,ax^{2}+3\,\mathrm{Li}_{2}(ax^{2})-2\,\log\left(-ax^{2}+1\right)}{9\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] -2/9*a^(3/2)*log((a*x - sqrt(a))/(a*x + sqrt(a))) - 1/9*(4*a*x^2 + 3*dilog(
a*x^2) - 2*log(-a*x^2 + 1))/x^3
```

Fricas [A]
time = 0.54, size = 114, normalized size = 2.04

$$\left[\frac{2\,a^{\frac{3}{2}}x^{3}\log\left(\frac{ax^{2}+2\,\sqrt{a}\,x+1}{ax^{2}-1}\right)-4\,ax^{2}-3\,\mathrm{Li}_{2}(ax^{2})+2\,\log\left(-ax^{2}+1\right)}{9\,x^{3}},-\frac{4\,\sqrt{-a}\,ax^{3}\arctan\left(\sqrt{-a}\,x\right)+4\,ax^{2}+3\,\mathrm{Li}_{2}(ax^{2})-2\,\log\left(-ax^{2}+1\right)}{9\,x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/9*(2*a^(3/2)*x^3*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)) - 4*a*x^2 -
3*dilog(a*x^2) + 2*log(-a*x^2 + 1))/x^3, -1/9*(4*sqrt(-a)*a*x^3*arctan(sqrt
(-a)*x) + 4*a*x^2 + 3*dilog(a*x^2) - 2*log(-a*x^2 + 1))/x^3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(49) = 98$.
time = 43.38, size = 275, normalized size = 4.91

$$\begin{cases} -\frac{\pi^{2}}{18x^{3}} & \text{for}\: a = \frac{1}{x^{2}} \\ 0 & \text{for}\: a = 0 \\ -\frac{4a^{2}x^{5}\sqrt{\frac{1}{a}}\log\left(x-\sqrt{\frac{1}{a}}\right)}{9x^{5}-\frac{9x^{3}}{a}}-\frac{2a^{2}x^{5}\sqrt{\frac{1}{a}}\,\mathrm{Li}_{1}\left(ax^{2}\right)}{9x^{5}-\frac{9x^{3}}{a}}-\frac{4ax^{4}}{9x^{5}-\frac{9x^{3}}{a}}+\frac{4ax^{3}\sqrt{\frac{1}{a}}\log\left(x-\sqrt{\frac{1}{a}}\right)}{9x^{5}-\frac{9x^{3}}{a}}+\frac{2ax^{3}\sqrt{\frac{1}{a}}\,\mathrm{Li}_{1}\left(ax^{2}\right)}{9x^{5}-\frac{9x^{3}}{a}}-\frac{2x^{2}\,\mathrm{Li}_{1}\left(ax^{2}\right)}{9x^{5}-\frac{9x^{3}}{a}}-\frac{3x^{2}\,\mathrm{Li}_{2}\left(ax^{2}\right)}{9x^{5}-\frac{9x^{3}}{a}}+\frac{4x^{2}}{9x^{5}-\frac{9x^{3}}{a}}+\frac{2\,\mathrm{Li}_{1}\left(ax^{2}\right)}{9ax^{5}-9x^{3}}+\frac{3\,\mathrm{Li}_{2}\left(ax^{2}\right)}{9ax^{5}-9x^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/x**4,x)
```

```
[Out] Piecewise((-pi**2/(18*x**3), Eq(a, x**(-2))), (0, Eq(a, 0)), (-4*a**2*x**5*
sqrt(1/a)*log(x - sqrt(1/a))/(9*x**5 - 9*x**3/a) - 2*a**2*x**5*sqrt(1/a)*po
lylog(1, a*x**2)/(9*x**5 - 9*x**3/a) - 4*a*x**4/(9*x**5 - 9*x**3/a) + 4*a*x
**3*sqrt(1/a)*log(x - sqrt(1/a))/(9*x**5 - 9*x**3/a) + 2*a*x**3*sqrt(1/a)*p
olylog(1, a*x**2)/(9*x**5 - 9*x**3/a) - 2*x**2*polylog(1, a*x**2)/(9*x**5 -
 9*x**3/a) - 3*x**2*polylog(2, a*x**2)/(9*x**5 - 9*x**3/a) + 4*x**2/(9*x**5
```

```
 - 9*x**3/a) + 2*polylog(1, a*x**2)/(9*a*x**5 - 9*x**3) + 3*polylog(2, a*x*
*2)/(9*a*x**5 - 9*x**3), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/x^4, x)
```

Mupad [B]
time = 0.33, size = 47, normalized size = 0.84

$$\frac{2\,\ln\left(1-a\,x^2\right)}{9\,x^3}-\frac{4\,a}{9\,x}-\frac{\mathrm{polylog}(2,a\,x^2)}{3\,x^3}-\frac{a^{3/2}\,\mathrm{atan}\left(\sqrt{a}\,\,x\,1\mathrm{i}\right)\,4\mathrm{i}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^2)/x^4,x)
```

```
[Out] (2*log(1 - a*x^2))/(9*x^3) - polylog(2, a*x^2)/(3*x^3) - (4*a)/(9*x) - (a^(
3/2)*atan(a^(1/2)*x*1i)*4i)/9
```

3.31 $\int \frac{\text{PolyLog}(2,ax^2)}{x^6}\,dx$

Optimal. Leaf size=66

$$-\frac{4a}{75x^3}-\frac{4a^2}{25x}+\frac{4}{25}a^{5/2}\tanh^{-1}\left(\sqrt{a}\,x\right)+\frac{2\log\left(1-ax^2\right)}{25x^5}-\frac{\text{PolyLog}(2,ax^2)}{5x^5}$$

```
[Out] -4/75*a/x^3-4/25*a^2/x+4/25*a^(5/2)*arctanh(x*a^(1/2))+2/25*ln(-a*x^2+1)/x^
5-1/5*polylog(2,a*x^2)/x^5
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6726, 2505, 331, 212}

$$\frac{4}{25}a^{5/2}\tanh^{-1}\left(\sqrt{a}\,x\right)-\frac{4a^2}{25x}-\frac{\text{Li}_2(ax^2)}{5x^5}-\frac{4a}{75x^3}+\frac{2\log\left(1-ax^2\right)}{25x^5}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/x^6,x]
```

```
[Out] (-4*a)/(75*x^3) - (4*a^2)/(25*x) + (4*a^(5/2)*ArcTanh[Sqrt[a]*x])/25 + (2*L
og[1 - a*x^2])/(25*x^5) - PolyLog[2, a*x^2]/(5*x^5)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax^2)}{x^6}\,dx &= -\frac{\text{Li}_2(ax^2)}{5x^5}-\frac{2}{5}\int \frac{\log(1-ax^2)}{x^6}\,dx\\
&= \frac{2\log(1-ax^2)}{25x^5}-\frac{\text{Li}_2(ax^2)}{5x^5}+\frac{1}{25}(4a)\int \frac{1}{x^4(1-ax^2)}\,dx\\
&= -\frac{4a}{75x^3}+\frac{2\log(1-ax^2)}{25x^5}-\frac{\text{Li}_2(ax^2)}{5x^5}+\frac{1}{25}(4a^2)\int \frac{1}{x^2(1-ax^2)}\,dx\\
&= -\frac{4a}{75x^3}-\frac{4a^2}{25x}+\frac{2\log(1-ax^2)}{25x^5}-\frac{\text{Li}_2(ax^2)}{5x^5}+\frac{1}{25}(4a^3)\int \frac{1}{1-ax^2}\,dx\\
&= -\frac{4a}{75x^3}-\frac{4a^2}{25x}+\frac{4}{25}a^{5/2}\tanh^{-1}\left(\sqrt{a}\,x\right)+\frac{2\log(1-ax^2)}{25x^5}-\frac{\text{Li}_2(ax^2)}{5x^5}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.01, size = 47, normalized size = 0.71

$$-\frac{4ax^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};ax^2\right)-6\log(1-ax^2)+15\text{PolyLog}(2,ax^2)}{75x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^2]/x^6,x]
[Out] -1/75*(4*a*x^2*Hypergeometric2F1[-3/2, 1, -1/2, a*x^2] - 6*Log[1 - a*x^2] +
 15*PolyLog[2, a*x^2])/x^5
```

Maple [A]
time = 0.08, size = 53, normalized size = 0.80

method	result	size
default	$-\frac{\text{polylog}(2,a\,x^2)}{5x^5}+\frac{2\ln(-a\,x^2+1)}{25x^5}+\frac{4a\left(a^{\frac{3}{2}}\,\text{arctanh}\left(x\sqrt{a}\right)-\frac{1}{3x^3}-\frac{a}{x}\right)}{25}$	53
meijerg	$\frac{a^3\left(-\frac{8}{75x^3(-a)^{\frac{3}{2}}}-\frac{8a}{25x(-a)^{\frac{3}{2}}}-\frac{4x\,a^2\left(\ln\left(1-\sqrt{a\,x^2}\right)-\ln\left(1+\sqrt{a\,x^2}\right)\right)}{25(-a)^{\frac{3}{2}}\sqrt{a\,x^2}}+\frac{4\ln(-a\,x^2+1)}{25x^5(-a)^{\frac{3}{2}}a}-\frac{2\,\text{polylog}(2,a\,x^2)}{5x^5(-a)^{\frac{3}{2}}a}\right)}{2\sqrt{-a}}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*polylog(2,a*x^2)/x^5+2/25*ln(-a*x^2+1)/x^5+4/25*a*(a^(3/2)*arctanh(x*a
^(1/2))-1/3/x^3-a/x)
```

Maxima [A]
time = 0.46, size = 65, normalized size = 0.98

$$-\frac{2}{25}\,a^{\frac{5}{2}}\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right)-\frac{12\,a^2x^4+4\,ax^2+15\,\mathrm{Li}_2(ax^2)-6\,\log\left(-ax^2+1\right)}{75\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^6,x, algorithm="maxima")
```

```
[Out] -2/25*a^(5/2)*log((a*x - sqrt(a))/(a*x + sqrt(a))) - 1/75*(12*a^2*x^4 + 4*a
*x^2 + 15*dilog(a*x^2) - 6*log(-a*x^2 + 1))/x^5
```

Fricas [A]
time = 0.52, size = 132, normalized size = 2.00

$$\left[\frac{6\,a^{\frac{5}{2}}x^5\log\left(\frac{ax^2+2\,\sqrt{a}\,x+1}{ax^2-1}\right)-12\,a^2x^4-4\,ax^2-15\,\mathrm{Li}_2(ax^2)+6\,\log\left(-ax^2+1\right)}{75\,x^5},\ -\frac{12\,\sqrt{-a}\,a^2x^5\arctan\left(\sqrt{-a}\,x\right)+12\,a^2x^4+4\,ax^2+15\,\mathrm{Li}_2(ax^2)-6\,\log\left(-ax^2+1\right)}{75\,x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/75*(6*a^(5/2)*x^5*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)) - 12*a^2*x^
4 - 4*a*x^2 - 15*dilog(a*x^2) + 6*log(-a*x^2 + 1))/x^5, -1/75*(12*sqrt(-a)*
a^2*x^5*arctan(sqrt(-a)*x) + 12*a^2*x^4 + 4*a*x^2 + 15*dilog(a*x^2) - 6*log
(-a*x^2 + 1))/x^5]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(60) = 120$.
time = 140.03, size = 299, normalized size = 4.53

$$\begin{cases} -\frac{\pi^2}{30x^5} & \text{for}\: a = \frac{1}{x^2} \\ 0 & \text{for}\: a = 0 \\ -\frac{12a^3x^7\sqrt{\frac{1}{a}}\log\left(x-\sqrt{\frac{1}{a}}\right)}{75x^7-\frac{75x^5}{a}}-\frac{6a^3x^7\sqrt{\frac{1}{a}}\,\mathrm{Li}_1\left(ax^2\right)}{75x^7-\frac{75x^5}{a}}-\frac{12a^2x^6}{75x^7-\frac{75x^5}{a}}+\frac{12a^2x^5\sqrt{\frac{1}{a}}\log\left(x-\sqrt{\frac{1}{a}}\right)}{75x^7-\frac{75x^5}{a}}+\frac{6a^2x^5\sqrt{\frac{1}{a}}\,\mathrm{Li}_1\left(ax^2\right)}{75x^7-\frac{75x^5}{a}}+\frac{8ax^4}{75x^7-\frac{75x^5}{a}}-\frac{6x^2\,\mathrm{Li}_1\left(ax^2\right)}{75x^7-\frac{75x^5}{a}}-\frac{15x^2\,\mathrm{Li}_2\left(ax^2\right)}{75x^7-\frac{75x^5}{a}}+\frac{4x^2}{75x^7-\frac{75x^5}{a}}+\frac{6\,\mathrm{Li}_1\left(ax^2\right)}{75ax^7-75x^5}+\frac{15\,\mathrm{Li}_2\left(ax^2\right)}{75ax^7-75x^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/x**6,x)
```

```
[Out] Piecewise((-pi**2/(30*x**5), Eq(a, x**(-2))), (0, Eq(a, 0)), (-12*a**3*x**7
*sqrt(1/a)*log(x - sqrt(1/a))/(75*x**7 - 75*x**5/a) - 6*a**3*x**7*sqrt(1/a)
```

```
*polylog(1, a*x**2)/(75*x**7 - 75*x**5/a) - 12*a**2*x**6/(75*x**7 - 75*x**5
/a) + 12*a**2*x**5*sqrt(1/a)*log(x - sqrt(1/a))/(75*x**7 - 75*x**5/a) + 6*a
**2*x**5*sqrt(1/a)*polylog(1, a*x**2)/(75*x**7 - 75*x**5/a) + 8*a*x**4/(75*
x**7 - 75*x**5/a) - 6*x**2*polylog(1, a*x**2)/(75*x**7 - 75*x**5/a) - 15*x*
*2*polylog(2, a*x**2)/(75*x**7 - 75*x**5/a) + 4*x**2/(75*x**7 - 75*x**5/a)
+ 6*polylog(1, a*x**2)/(75*a*x**7 - 75*x**5) + 15*polylog(2, a*x**2)/(75*a*
x**7 - 75*x**5), True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^6,x, algorithm="giac")
[Out] integrate(dilog(a*x^2)/x^6, x)
```

Mupad [**B**]
time = 0.34, size = 58, normalized size = 0.88

$$\frac{2\,\ln\left(1-a\,x^2\right)}{25\,x^5}-\frac{4\,a^2\,x^2+\frac{4\,a}{3}}{25\,x^3}-\frac{\mathrm{polylog}(2,a\,x^2)}{5\,x^5}-\frac{a^{5/2}\,\mathrm{atan}\left(\sqrt{a}\ x\,1\mathrm{i}\right)\,4\mathrm{i}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^2)/x^6,x)
[Out] (2*log(1 - a*x^2))/(25*x^5) - polylog(2, a*x^2)/(5*x^5) - ((4*a)/3 + 4*a^2*
x^2)/(25*x^3) - (a^(5/2)*atan(a^(1/2)*x*1i)*4i)/25
```

3.32 $\int x^5 \textbf{PolyLog}(3, ax^2)\, dx$

Optimal. Leaf size=88

$$\frac{x^2}{54a^2}+\frac{x^4}{108a}+\frac{x^6}{162}+\frac{\log(1-ax^2)}{54a^3}-\frac{1}{54}x^6\log(1-ax^2)-\frac{1}{18}x^6\text{PolyLog}(2,ax^2)+\frac{1}{6}x^6\text{PolyLog}(3,ax^2)$$

```
[Out] 1/54*x^2/a^2+1/108*x^4/a+1/162*x^6+1/54*ln(-a*x^2+1)/a^3-1/54*x^6*ln(-a*x^2
+1)-1/18*x^6*polylog(2,a*x^2)+1/6*x^6*polylog(3,a*x^2)
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6726, 2504, 2442, 45}

$$\frac{\log(1-ax^2)}{54a^3}+\frac{x^2}{54a^2}-\frac{1}{18}x^6\text{Li}_2(ax^2)+\frac{1}{6}x^6\text{Li}_3(ax^2)+\frac{x^4}{108a}-\frac{1}{54}x^6\log(1-ax^2)+\frac{x^6}{162}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*PolyLog[3, a*x^2],x]
```

```
[Out] x^2/(54*a^2) + x^4/(108*a) + x^6/162 + Log[1 - a*x^2]/(54*a^3) - (x^6*Log[1
 - a*x^2])/54 - (x^6*PolyLog[2, a*x^2])/18 + (x^6*PolyLog[3, a*x^2])/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int x^5\text{Li}_3\left(ax^2\right)\,dx &= \frac{1}{6}x^6\text{Li}_3\left(ax^2\right)-\frac{1}{3}\int x^5\text{Li}_2\left(ax^2\right)\,dx\\ &= -\frac{1}{18}x^6\text{Li}_2\left(ax^2\right)+\frac{1}{6}x^6\text{Li}_3\left(ax^2\right)-\frac{1}{9}\int x^5\log\left(1-ax^2\right)\,dx\\ &= -\frac{1}{18}x^6\text{Li}_2\left(ax^2\right)+\frac{1}{6}x^6\text{Li}_3\left(ax^2\right)-\frac{1}{18}\text{Subst}\left(\int x^2\log(1-ax)\,dx,x,x^2\right)\\ &= -\frac{1}{54}x^6\log\left(1-ax^2\right)-\frac{1}{18}x^6\text{Li}_2\left(ax^2\right)+\frac{1}{6}x^6\text{Li}_3\left(ax^2\right)-\frac{1}{54}a\text{Subst}\left(\int\frac{x^3}{1-ax}\,dx,x,x^2\right)\\ &= -\frac{1}{54}x^6\log\left(1-ax^2\right)-\frac{1}{18}x^6\text{Li}_2\left(ax^2\right)+\frac{1}{6}x^6\text{Li}_3\left(ax^2\right)-\frac{1}{54}a\text{Subst}\left(\int\left(-\frac{1}{a^3}-\frac{x}{a^2}-\frac{x^2}{a}\right.\right.\\ &= \frac{x^2}{54a^2}+\frac{x^4}{108a}+\frac{x^6}{162}+\frac{\log\left(1-ax^2\right)}{54a^3}-\frac{1}{54}x^6\log\left(1-ax^2\right)-\frac{1}{18}x^6\text{Li}_2\left(ax^2\right)+\frac{1}{6}x^6\text{Li}_3\left(ax\right.\end{aligned}$$

Mathematica [A]
time = 0.01, size = 88, normalized size = 1.00

$$\frac{6ax^2+3a^2x^4+2a^3x^6+6\log\left(1-ax^2\right)-6a^3x^6\log\left(1-ax^2\right)-18a^3x^6\text{PolyLog}(2,ax^2)+54a^3x^6\text{PolyLog}(3,ax^2)}{324a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*PolyLog[3, a*x^2],x]
```

```
[Out] (6*a*x^2 + 3*a^2*x^4 + 2*a^3*x^6 + 6*Log[1 - a*x^2] - 6*a^3*x^6*Log[1 - a*x
^2] - 18*a^3*x^6*PolyLog[2, a*x^2] + 54*a^3*x^6*PolyLog[3, a*x^2])/(324*a^3
)
```

Maple [A]
time = 0.04, size = 80, normalized size = 0.91

method	result	size
meijerg	$\frac{\frac{x^2a\left(4a^2x^4+6a\,x^2+12\right)}{324}+\frac{\left(-4a^3x^6+4\right)\ln\left(-a\,x^2+1\right)}{108}-\frac{x^6a^3\,\text{polylog}\left(2,a\,x^2\right)}{9}+\frac{x^6a^3\,\text{polylog}\left(3,a\,x^2\right)}{3}}{2a^3}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*polylog(3,a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^3*(1/324*x^2*a*(4*a^2*x^4+6*a*x^2+12)+1/108*(-4*a^3*x^6+4)*ln(-a*x^2+
1)-1/9*x^6*a^3*polylog(2,a*x^2)+1/3*x^6*a^3*polylog(3,a*x^2))
```

Maxima [A]
time = 0.27, size = 77, normalized size = 0.88

$$-\frac{18\,a^3x^6\mathrm{Li}_2(ax^2)-54\,a^3x^6\mathrm{Li}_3(ax^2)-2\,a^3x^6-3\,a^2x^4-6\,ax^2+6\,(a^3x^6-1)\log\left(-ax^2+1\right)}{324\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*polylog(3,a*x^2),x, algorithm="maxima")
```

```
[Out] -1/324*(18*a^3*x^6*dilog(a*x^2) - 54*a^3*x^6*polylog(3, a*x^2) - 2*a^3*x^6
- 3*a^2*x^4 - 6*a*x^2 + 6*(a^3*x^6 - 1)*log(-a*x^2 + 1))/a^3
```

Fricas [A]
time = 0.43, size = 77, normalized size = 0.88

$$-\frac{18\,a^3x^6\mathrm{Li}_2(ax^2)-54\,a^3x^6\mathrm{polylog}(3,ax^2)-2\,a^3x^6-3\,a^2x^4-6\,ax^2+6\,(a^3x^6-1)\log\left(-ax^2+1\right)}{324\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*polylog(3,a*x^2),x, algorithm="fricas")
```

```
[Out] -1/324*(18*a^3*x^6*dilog(a*x^2) - 54*a^3*x^6*polylog(3, a*x^2) - 2*a^3*x^6
- 3*a^2*x^4 - 6*a*x^2 + 6*(a^3*x^6 - 1)*log(-a*x^2 + 1))/a^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\,\mathrm{Li}_3\left(ax^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*polylog(3,a*x**2),x)
```

```
[Out] Integral(x**5*polylog(3, a*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*polylog(3,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^5*polylog(3, a*x^2), x)
```

Mupad [B]
time = 0.34, size = 73, normalized size = 0.83

$$\frac{x^6\,\mathrm{polylog}(3,a\,x^2)}{6}-\frac{x^6\,\mathrm{polylog}(2,a\,x^2)}{18}+\frac{\ln\left(a\,x^2-1\right)}{54\,a^3}-\frac{x^6\,\ln\left(1-a\,x^2\right)}{54}+\frac{x^6}{162}+\frac{x^2}{54\,a^2}+\frac{x^4}{108\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*polylog(3, a*x^2),x)
```

```
[Out] (x^6*polylog(3, a*x^2))/6 - (x^6*polylog(2, a*x^2))/18 + log(a*x^2 - 1)/(54
*a^3) - (x^6*log(1 - a*x^2))/54 + x^6/162 + x^2/(54*a^2) + x^4/(108*a)
```

3.33 $\int x^3\mathbf{PolyLog}(3, ax^2)\,dx$

Optimal. Leaf size=78

$$\frac{x^2}{16a}+\frac{x^4}{32}+\frac{\log(1-ax^2)}{16a^2}-\frac{1}{16}x^4\log(1-ax^2)-\frac{1}{8}x^4\text{PolyLog}(2,ax^2)+\frac{1}{4}x^4\text{PolyLog}(3,ax^2)$$

```
[Out] 1/16*x^2/a+1/32*x^4+1/16*ln(-a*x^2+1)/a^2-1/16*x^4*ln(-a*x^2+1)-1/8*x^4*pol
ylog(2,a*x^2)+1/4*x^4*polylog(3,a*x^2)
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6726, 2504, 2442, 45}

$$\frac{\log(1-ax^2)}{16a^2}-\frac{1}{8}x^4\text{Li}_2(ax^2)+\frac{1}{4}x^4\text{Li}_3(ax^2)+\frac{x^2}{16a}-\frac{1}{16}x^4\log(1-ax^2)+\frac{x^4}{32}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*PolyLog[3, a*x^2],x]
```

```
[Out] x^2/(16*a) + x^4/32 + Log[1 - a*x^2]/(16*a^2) - (x^4*Log[1 - a*x^2])/16 - (
x^4*PolyLog[2, a*x^2])/8 + (x^4*PolyLog[3, a*x^2])/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int x^3\text{Li}_3(ax^2)\,dx &= \frac{1}{4}x^4\text{Li}_3(ax^2)-\frac{1}{2}\int x^3\text{Li}_2(ax^2)\,dx\\ &= -\frac{1}{8}x^4\text{Li}_2(ax^2)+\frac{1}{4}x^4\text{Li}_3(ax^2)-\frac{1}{4}\int x^3\log(1-ax^2)\,dx\\ &= -\frac{1}{8}x^4\text{Li}_2(ax^2)+\frac{1}{4}x^4\text{Li}_3(ax^2)-\frac{1}{8}\text{Subst}\left(\int x\log(1-ax)\,dx,x,x^2\right)\\ &= -\frac{1}{16}x^4\log(1-ax^2)-\frac{1}{8}x^4\text{Li}_2(ax^2)+\frac{1}{4}x^4\text{Li}_3(ax^2)-\frac{1}{16}a\text{Subst}\left(\int\frac{x^2}{1-ax}\,dx,x,x^2\right)\\ &= -\frac{1}{16}x^4\log(1-ax^2)-\frac{1}{8}x^4\text{Li}_2(ax^2)+\frac{1}{4}x^4\text{Li}_3(ax^2)-\frac{1}{16}a\text{Subst}\left(\int\left(-\frac{1}{a^2}-\frac{x}{a}-\frac{}{a^2(-}\right.\right.\\ &= \frac{x^2}{16a}+\frac{x^4}{32}+\frac{\log(1-ax^2)}{16a^2}-\frac{1}{16}x^4\log(1-ax^2)-\frac{1}{8}x^4\text{Li}_2(ax^2)+\frac{1}{4}x^4\text{Li}_3(ax^2)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 79, normalized size = 1.01

$$\frac{2ax^2+a^2x^4+2\log(1-ax^2)-2a^2x^4\log(1-ax^2)-4a^2x^4\text{PolyLog}(2,ax^2)+8a^2x^4\text{PolyLog}(3,ax^2)}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*PolyLog[3, a*x^2],x]
[Out] (2*a*x^2 + a^2*x^4 + 2*Log[1 - a*x^2] - 2*a^2*x^4*Log[1 - a*x^2] - 4*a^2*x^
4*PolyLog[2, a*x^2] + 8*a^2*x^4*PolyLog[3, a*x^2])/(32*a^2)
```

Maple [A]
time = 0.04, size = 72, normalized size = 0.92

method	result	size
meijerg	$-\frac{-\frac{a\,x^2(3a\,x^2+6)}{48}-\frac{(-3a^2x^4+3)\ln(-a\,x^2+1)}{24}+\frac{a^2x^4\,\text{polylog}(2,a\,x^2)}{4}-\frac{a^2x^4\,\text{polylog}(3,a\,x^2)}{2}}{2a^2}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*polylog(3,a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2*(-1/48*a*x^2*(3*a*x^2+6)-1/24*(-3*a^2*x^4+3)*ln(-a*x^2+1)+1/4*a^2*
x^4*polylog(2,a*x^2)-1/2*a^2*x^4*polylog(3,a*x^2))
```

Maxima [**A**]
time = 0.25, size = 69, normalized size = 0.88

$$-\frac{4\,a^2x^4\mathrm{Li}_2(ax^2)-8\,a^2x^4\mathrm{Li}_3(ax^2)-a^2x^4-2\,ax^2+2\,(a^2x^4-1)\log\left(-ax^2+1\right)}{32\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(3,a*x^2),x, algorithm="maxima")
```

```
[Out] -1/32*(4*a^2*x^4*dilog(a*x^2) - 8*a^2*x^4*polylog(3, a*x^2) - a^2*x^4 - 2*a
*x^2 + 2*(a^2*x^4 - 1)*log(-a*x^2 + 1))/a^2
```

Fricas [**A**]
time = 0.45, size = 69, normalized size = 0.88

$$-\frac{4\,a^2x^4\mathrm{Li}_2(ax^2)-8\,a^2x^4\mathrm{polylog}(3,ax^2)-a^2x^4-2\,ax^2+2\,(a^2x^4-1)\log\left(-ax^2+1\right)}{32\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(3,a*x^2),x, algorithm="fricas")
```

```
[Out] -1/32*(4*a^2*x^4*dilog(a*x^2) - 8*a^2*x^4*polylog(3, a*x^2) - a^2*x^4 - 2*a
*x^2 + 2*(a^2*x^4 - 1)*log(-a*x^2 + 1))/a^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\,\mathrm{Li}_3\left(ax^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*polylog(3,a*x**2),x)
```

```
[Out] Integral(x**3*polylog(3, a*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(3,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^3*polylog(3, a*x^2), x)
```

Mupad [B]
time = 0.30, size = 65, normalized size = 0.83

$$\frac{x^4\,\mathrm{polylog}(3,a\,x^2)}{4}-\frac{x^4\,\mathrm{polylog}(2,a\,x^2)}{8}+\frac{\ln\left(a\,x^2-1\right)}{16\,a^2}-\frac{x^4\,\ln\left(1-a\,x^2\right)}{16}+\frac{x^4}{32}+\frac{x^2}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*polylog(3, a*x^2),x)
```

```
[Out] (x^4*polylog(3, a*x^2))/4 - (x^4*polylog(2, a*x^2))/8 + log(a*x^2 - 1)/(16*
a^2) - (x^4*log(1 - a*x^2))/16 + x^4/32 + x^2/(16*a)
```

3.34 $\int x\mathbf{PolyLog}(3, ax^2)\, dx$

Optimal. Leaf size=60

$$\frac{x^2}{2} + \frac{(1-ax^2)\log(1-ax^2)}{2a} - \frac{1}{2}x^2\text{PolyLog}(2, ax^2) + \frac{1}{2}x^2\text{PolyLog}(3, ax^2)$$

```
[Out] 1/2*x^2+1/2*(-a*x^2+1)*ln(-a*x^2+1)/a-1/2*x^2*polylog(2,a*x^2)+1/2*x^2*poly
log(3,a*x^2)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6726, 2504, 2436, 2332}

$$-\frac{1}{2}x^2\text{Li}_2(ax^2) + \frac{1}{2}x^2\text{Li}_3(ax^2) + \frac{(1-ax^2)\log(1-ax^2)}{2a} + \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*PolyLog[3, a*x^2],x]
```

```
[Out] x^2/2 + ((1 - a*x^2)*Log[1 - a*x^2])/(2*a) - (x^2*PolyLog[2, a*x^2])/2 + (x
^2*PolyLog[3, a*x^2])/2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
```

```
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int x\mathrm{Li}_3(ax^2)\,dx &= \frac{1}{2}x^2\mathrm{Li}_3(ax^2) - \int x\mathrm{Li}_2(ax^2)\,dx \\
&= -\frac{1}{2}x^2\mathrm{Li}_2(ax^2) + \frac{1}{2}x^2\mathrm{Li}_3(ax^2) - \int x\log(1-ax^2)\,dx \\
&= -\frac{1}{2}x^2\mathrm{Li}_2(ax^2) + \frac{1}{2}x^2\mathrm{Li}_3(ax^2) - \frac{1}{2}\mathrm{Subst}\left(\int \log(1-ax)\,dx, x, x^2\right) \\
&= -\frac{1}{2}x^2\mathrm{Li}_2(ax^2) + \frac{1}{2}x^2\mathrm{Li}_3(ax^2) + \frac{\mathrm{Subst}\left(\int \log(x)\,dx, x, 1-ax^2\right)}{2a} \\
&= \frac{x^2}{2} + \frac{(1-ax^2)\log(1-ax^2)}{2a} - \frac{1}{2}x^2\mathrm{Li}_2(ax^2) + \frac{1}{2}x^2\mathrm{Li}_3(ax^2)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 0.87

$$\frac{1}{2}x^2\left(1-\log(1-ax^2)+\frac{\log(1-ax^2)}{ax^2}-\mathrm{PolyLog}(2,ax^2)+\mathrm{PolyLog}(3,ax^2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*PolyLog[3, a*x^2],x]
[Out] (x^2*(1 - Log[1 - a*x^2] + Log[1 - a*x^2]/(a*x^2) - PolyLog[2, a*x^2] + Pol
yLog[3, a*x^2]))/2
```

Maple [A]
time = 0.06, size = 56, normalized size = 0.93

method	result	size
meijerg	$\frac{a\,x^2+\frac{\left(-2a\,x^2+2\right)\ln\left(-a\,x^2+1\right)}{2}-a\,x^2\,\mathrm{polylog}(2,a\,x^2)+a\,x^2\,\mathrm{polylog}(3,a\,x^2)}{2a}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(3,a*x^2),x,method=_RETURNVERBOSE)
[Out] 1/2/a*(a*x^2+1/2*(-2*a*x^2+2)*ln(-a*x^2+1)-a*x^2*polylog(2,a*x^2)+a*x^2*pol
ylog(3,a*x^2))
```

Maxima [A]
time = 0.27, size = 53, normalized size = 0.88

$$-\frac{ax^2\mathrm{Li}_2(ax^2) - ax^2\mathrm{Li}_3(ax^2) - ax^2 + (ax^2-1)\log\left(-ax^2+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*x^2*dilog(a*x^2) - a*x^2*polylog(3, a*x^2) - a*x^2 + (a*x^2 - 1)*lo
g(-a*x^2 + 1))/a
```

Fricas [A]
time = 0.46, size = 53, normalized size = 0.88

$$-\frac{ax^2\mathrm{Li}_2(ax^2) - ax^2\mathrm{polylog}(3,ax^2) - ax^2 + (ax^2-1)\log\left(-ax^2+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(a*x^2*dilog(a*x^2) - a*x^2*polylog(3, a*x^2) - a*x^2 + (a*x^2 - 1)*lo
g(-a*x^2 + 1))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{Li}_3\left(ax^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x**2),x)
```

```
[Out] Integral(x*polylog(3, a*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x*polylog(3, a*x^2), x)
```

Mupad [B]
time = 0.38, size = 57, normalized size = 0.95

$$\frac{x^2\,\mathrm{polylog}(3,a\,x^2)}{2} - \frac{x^2\,\mathrm{polylog}(2,a\,x^2)}{2} + \frac{\ln\left(a\,x^2-1\right)}{2\,a} - \frac{x^2\,\ln\left(1-a\,x^2\right)}{2} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(3, a*x^2),x)
```

```
[Out] (x^2*polylog(3, a*x^2))/2 - (x^2*polylog(2, a*x^2))/2 + log(a*x^2 - 1)/(2*a
) - (x^2*log(1 - a*x^2))/2 + x^2/2
```

3.35 $\int \frac{\text{PolyLog}(3,ax^2)}{x}\,dx$

Optimal. Leaf size=11

$$\frac{1}{2}\text{PolyLog}\left(4,ax^2\right)$$

[Out] 1/2*polylog(4,a*x^2)

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6724}

$$\frac{\text{Li}_4(ax^2)}{2}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a*x^2]/x,x]

[Out] PolyLog[4, a*x^2]/2

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\text{Li}_3(ax^2)}{x}\,dx = \frac{\text{Li}_4(ax^2)}{2}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$\frac{1}{2}\text{PolyLog}\left(4,ax^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[3, a*x^2]/x,x]

[Out] PolyLog[4, a*x^2]/2

Maple [A]
time = 0.04, size = 10, normalized size = 0.91

method	result	size
meijerg	$\frac{\text{polylog}(4,a\,x^2)}{2}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*polylog(4,a*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(polylog(3, a*x^2)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(polylog(3, a*x^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Li}_3\left(ax^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/x,x)
```

```
[Out] Integral(polylog(3, a*x**2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/x, x)
```

Mupad [B]
time = 0.20, size = 9, normalized size = 0.82

$$\frac{\mathrm{polylog}(4, a\,x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^2)/x,x)
```

```
[Out] polylog(4, a*x^2)/2
```

3.36 $\int \frac{\text{PolyLog}(3,ax^2)}{x^3}\,dx$

Optimal. Leaf size=63

$$a\log(x)-\frac{1}{2}a\log\left(1-ax^2\right)+\frac{\log\left(1-ax^2\right)}{2x^2}-\frac{\text{PolyLog}(2,ax^2)}{2x^2}-\frac{\text{PolyLog}(3,ax^2)}{2x^2}$$

```
[Out] a*ln(x)-1/2*a*ln(-a*x^2+1)+1/2*ln(-a*x^2+1)/x^2-1/2*polylog(2,a*x^2)/x^2-1/
2*polylog(3,a*x^2)/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {6726, 2504, 2442, 36, 29, 31}

$$-\frac{\text{Li}_2(ax^2)}{2x^2}-\frac{\text{Li}_3(ax^2)}{2x^2}-\frac{1}{2}a\log\left(1-ax^2\right)+\frac{\log\left(1-ax^2\right)}{2x^2}+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/x^3,x]
```

```
[Out] a*Log[x] - (a*Log[1 - a*x^2])/2 + Log[1 - a*x^2]/(2*x^2) - PolyLog[2, a*x^2
]/(2*x^2) - PolyLog[3, a*x^2]/(2*x^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax^2)}{x^3}\,dx &= -\frac{\text{Li}_3(ax^2)}{2x^2}+\int \frac{\text{Li}_2(ax^2)}{x^3}\,dx \\
&= -\frac{\text{Li}_2(ax^2)}{2x^2}-\frac{\text{Li}_3(ax^2)}{2x^2}-\int \frac{\log\left(1-ax^2\right)}{x^3}\,dx \\
&= -\frac{\text{Li}_2(ax^2)}{2x^2}-\frac{\text{Li}_3(ax^2)}{2x^2}-\frac{1}{2}\text{Subst}\left(\int \frac{\log(1-ax)}{x^2}\,dx,x,x^2\right) \\
&= \frac{\log\left(1-ax^2\right)}{2x^2}-\frac{\text{Li}_2(ax^2)}{2x^2}-\frac{\text{Li}_3(ax^2)}{2x^2}+\frac{1}{2}a\text{Subst}\left(\int \frac{1}{x(1-ax)}\,dx,x,x^2\right) \\
&= \frac{\log\left(1-ax^2\right)}{2x^2}-\frac{\text{Li}_2(ax^2)}{2x^2}-\frac{\text{Li}_3(ax^2)}{2x^2}+\frac{1}{2}a\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)+\frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{1-ax} \right. \\
&= a\log(x)-\frac{1}{2}a\log\left(1-ax^2\right)+\frac{\log\left(1-ax^2\right)}{2x^2}-\frac{\text{Li}_2(ax^2)}{2x^2}-\frac{\text{Li}_3(ax^2)}{2x^2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 60, normalized size = 0.95

$$-\frac{-ax^2\log\left(-ax^2\right)-\log\left(1-ax^2\right)+ax^2\log\left(1-ax^2\right)+\text{PolyLog}(2,ax^2)+\text{PolyLog}(3,ax^2)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^2]/x^3,x]
```

```
[Out] -1/2*(-(a*x^2*Log[-(a*x^2)]) - Log[1 - a*x^2] + a*x^2*Log[1 - a*x^2] + Poly
Log[2, a*x^2] + PolyLog[3, a*x^2])/x^2
```

Maple [A]
time = 0.07, size = 68, normalized size = 1.08

method	result	size
meijerg	$\frac{a\left(\frac{\left(-8a\,x^{2}+8\right)\ln\left(-a\,x^{2}+1\right)}{8a\,x^{2}}-\frac{\mathrm{polylog}\left(2,a\,x^{2}\right)}{a\,x^{2}}-\frac{\mathrm{polylog}\left(3,a\,x^{2}\right)}{a\,x^{2}}+2\ln\left(x\right)+\ln\left(-a\right)\right)}{2}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*(1/8/a/x^2*(-8*a*x^2+8)*ln(-a*x^2+1)-1/a/x^2*polylog(2,a*x^2)-1/a/x^2
*polylog(3,a*x^2)+2*ln(x)+ln(-a))
```

Maxima [A]
time = 0.26, size = 41, normalized size = 0.65

$$a\log\left(x\right)-\frac{\left(ax^{2}-1\right)\log\left(-ax^{2}+1\right)+\mathrm{Li}_{2}(ax^{2})+\mathrm{Li}_{3}(ax^{2})}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] a*log(x) - 1/2*((a*x^2 - 1)*log(-a*x^2 + 1) + dilog(a*x^2) + polylog(3, a*x
^2))/x^2
```

Fricas [A]
time = 0.37, size = 51, normalized size = 0.81

$$-\frac{ax^{2}\log\left(ax^{2}-1\right)-2\,ax^{2}\log\left(x\right)+\mathrm{Li}_{2}(ax^{2})-\log\left(-ax^{2}+1\right)+\mathrm{polylog}(3,ax^{2})}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*x^2*log(a*x^2 - 1) - 2*a*x^2*log(x) + dilog(a*x^2) - log(-a*x^2 + 1
) + polylog(3, a*x^2))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{Li}_{3}\left(ax^{2}\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/x**3,x)
```

```
[Out] Integral(polylog(3, a*x**2)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^3,x, algorithm="giac")
[Out] integrate(polylog(3, a*x^2)/x^3, x)
```

Mupad [B]
time = 0.29, size = 54, normalized size = 0.86

$$-\frac{\mathrm{polylog}(2,a\,x^2)-\ln\left(1-a\,x^2\right)+\mathrm{polylog}(3,a\,x^2)-3\,a\,x^2\,\ln\left(x\right)+a\,x^2\,\ln\left(x\,\left(a\,x^2-1\right)\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^2)/x^3,x)
[Out] -(polylog(2, a*x^2) - log(1 - a*x^2) + polylog(3, a*x^2) - 3*a*x^2*log(x) +
 a*x^2*log(x*(a*x^2 - 1)))/(2*x^2)
```

3.37 $\int \frac{\text{PolyLog}(3,ax^2)}{x^5}\,dx$

Optimal. Leaf size=78

$$-\frac{a}{16x^2}+\frac{1}{8}a^2\log(x)-\frac{1}{16}a^2\log\left(1-ax^2\right)+\frac{\log\left(1-ax^2\right)}{16x^4}-\frac{\text{PolyLog}(2,ax^2)}{8x^4}-\frac{\text{PolyLog}(3,ax^2)}{4x^4}$$

```
[Out] -1/16*a/x^2+1/8*a^2*ln(x)-1/16*a^2*ln(-a*x^2+1)+1/16*ln(-a*x^2+1)/x^4-1/8*p
olylog(2,a*x^2)/x^4-1/4*polylog(3,a*x^2)/x^4
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6726, 2504, 2442, 46}

$$-\frac{1}{16}a^2\log\left(1-ax^2\right)+\frac{1}{8}a^2\log(x)-\frac{\text{Li}_2(ax^2)}{8x^4}-\frac{\text{Li}_3(ax^2)}{4x^4}-\frac{a}{16x^2}+\frac{\log\left(1-ax^2\right)}{16x^4}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/x^5,x]
```

```
[Out] -1/16*a/x^2 + (a^2*Log[x])/8 - (a^2*Log[1 - a*x^2])/16 + Log[1 - a*x^2]/(16
*x^4) - PolyLog[2, a*x^2]/(8*x^4) - PolyLog[3, a*x^2]/(4*x^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax^2)}{x^5}\,dx &= -\frac{\text{Li}_3(ax^2)}{4x^4}+\frac{1}{2}\int \frac{\text{Li}_2(ax^2)}{x^5}\,dx \\
&= -\frac{\text{Li}_2(ax^2)}{8x^4}-\frac{\text{Li}_3(ax^2)}{4x^4}-\frac{1}{4}\int \frac{\log(1-ax^2)}{x^5}\,dx \\
&= -\frac{\text{Li}_2(ax^2)}{8x^4}-\frac{\text{Li}_3(ax^2)}{4x^4}-\frac{1}{8}\text{Subst}\left(\int \frac{\log(1-ax)}{x^3}\,dx,x,x^2\right) \\
&= \frac{\log(1-ax^2)}{16x^4}-\frac{\text{Li}_2(ax^2)}{8x^4}-\frac{\text{Li}_3(ax^2)}{4x^4}+\frac{1}{16}a\text{Subst}\left(\int \frac{1}{x^2(1-ax)}\,dx,x,x^2\right) \\
&= \frac{\log(1-ax^2)}{16x^4}-\frac{\text{Li}_2(ax^2)}{8x^4}-\frac{\text{Li}_3(ax^2)}{4x^4}+\frac{1}{16}a\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{a}{x}-\frac{a^2}{-1+ax}\right)dx,x,x^2\right) \\
&= -\frac{a}{16x^2}+\frac{1}{8}a^2\log(x)-\frac{1}{16}a^2\log\left(1-ax^2\right)+\frac{\log(1-ax^2)}{16x^4}-\frac{\text{Li}_2(ax^2)}{8x^4}-\frac{\text{Li}_3(ax^2)}{4x^4}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.01, size = 30, normalized size = 0.38

$$\frac{G_{5,5}^{2,4}\left(-ax^2\middle|\begin{array}{c}1,1,1,1,3\\1,2,0,0,0\end{array}\right)}{2x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x^2]/x^5,x]
```

```
[Out] MeijerG[{{1, 1, 1, 1}, {3}}, {{1, 2}, {0, 0, 0}}, -(a*x^2)]/(2*x^4)
```

Maple [A]
time = 0.10, size = 98, normalized size = 1.26

method	result	size
meijerg	$-\frac{a^2\left(-\frac{81a\,x^2+378}{432a\,x^2}-\frac{\left(-27a^2x^4+27\right)\ln\left(-a\,x^2+1\right)}{216a^2x^4}+\frac{\text{polylog}\left(2,a\,x^2\right)}{4a^2x^4}+\frac{\text{polylog}\left(3,a\,x^2\right)}{2a^2x^4}+\frac{3}{16}-\frac{\ln(x)}{4}-\frac{\ln(-a)}{8}+\frac{1}{a\,x^2}\right)}{2}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2*(-1/432/a/x^2*(81*a*x^2+378)-1/216/a^2/x^4*(-27*a^2*x^4+27)*ln(-a*
x^2+1)+1/4/a^2/x^4*polylog(2,a*x^2)+1/2/a^2/x^4*polylog(3,a*x^2)+3/16-1/4*l
n(x)-1/8*ln(-a)+1/a/x^2)
```

Maxima [A]
time = 0.25, size = 55, normalized size = 0.71

$$\frac{1}{8}\,a^2\log\left(x\right)-\frac{ax^2+\left(a^2x^4-1\right)\log\left(-ax^2+1\right)+2\,\mathrm{Li}_2(ax^2)+4\,\mathrm{Li}_3(ax^2)}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*log(x) - 1/16*(a*x^2 + (a^2*x^4 - 1)*log(-a*x^2 + 1) + 2*dilog(a*x^
2) + 4*polylog(3, a*x^2))/x^4
```

Fricas [A]
time = 0.36, size = 64, normalized size = 0.82

$$-\frac{a^2x^4\log\left(ax^2-1\right)-2\,a^2x^4\log\left(x\right)+ax^2+2\,\mathrm{Li}_2(ax^2)-\log\left(-ax^2+1\right)+4\,\mathrm{polylog}(3,ax^2)}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/16*(a^2*x^4*log(a*x^2 - 1) - 2*a^2*x^4*log(x) + a*x^2 + 2*dilog(a*x^2) -
 log(-a*x^2 + 1) + 4*polylog(3, a*x^2))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{Li}_3\left(ax^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/x**5,x)
```

```
[Out] Integral(polylog(3, a*x**2)/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/x^5, x)
```

Mupad [B]
time = 0.73, size = 65, normalized size = 0.83

$$\frac{a^2\,\ln\left(x\right)}{8}-\frac{\mathrm{polylog}(2,a\,x^2)}{8\,x^4}-\frac{\mathrm{polylog}(3,a\,x^2)}{4\,x^4}-\frac{a^2\,\ln\left(a\,x^2-1\right)}{16}-\frac{a}{16\,x^2}+\frac{\ln\left(1-a\,x^2\right)}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^2)/x^5,x)
```

```
[Out] (a^2*log(x))/8 - polylog(2, a*x^2)/(8*x^4) - polylog(3, a*x^2)/(4*x^4) - (a
^2*log(a*x^2 - 1))/16 - a/(16*x^2) + log(1 - a*x^2)/(16*x^4)
```

3.38 $\int \frac{\text{PolyLog}(3,ax^2)}{x^7}\,dx$

Optimal. Leaf size=88

$$-\frac{a}{108x^4}-\frac{a^2}{54x^2}+\frac{1}{27}a^3\log(x)-\frac{1}{54}a^3\log\left(1-ax^2\right)+\frac{\log\left(1-ax^2\right)}{54x^6}-\frac{\text{PolyLog}(2,ax^2)}{18x^6}-\frac{\text{PolyLog}(3,ax^2)}{6x^6}$$

```
[Out] -1/108*a/x^4-1/54*a^2/x^2+1/27*a^3*ln(x)-1/54*a^3*ln(-a*x^2+1)+1/54*ln(-a*x
^2+1)/x^6-1/18*polylog(2,a*x^2)/x^6-1/6*polylog(3,a*x^2)/x^6
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6726, 2504, 2442, 46}

$$-\frac{1}{54}a^3\log\left(1-ax^2\right)+\frac{1}{27}a^3\log(x)-\frac{a^2}{54x^2}-\frac{\text{Li}_2(ax^2)}{18x^6}-\frac{\text{Li}_3(ax^2)}{6x^6}-\frac{a}{108x^4}+\frac{\log\left(1-ax^2\right)}{54x^6}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/x^7,x]
```

```
[Out] -1/108*a/x^4 - a^2/(54*x^2) + (a^3*Log[x])/27 - (a^3*Log[1 - a*x^2])/54 + L
og[1 - a*x^2]/(54*x^6) - PolyLog[2, a*x^2]/(18*x^6) - PolyLog[3, a*x^2]/(6*
x^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax^2)}{x^7}\,dx &= -\frac{\text{Li}_3(ax^2)}{6x^6} + \frac{1}{3}\int \frac{\text{Li}_2(ax^2)}{x^7}\,dx \\
&= -\frac{\text{Li}_2(ax^2)}{18x^6} - \frac{\text{Li}_3(ax^2)}{6x^6} - \frac{1}{9}\int \frac{\log\left(1-ax^2\right)}{x^7}\,dx \\
&= -\frac{\text{Li}_2(ax^2)}{18x^6} - \frac{\text{Li}_3(ax^2)}{6x^6} - \frac{1}{18}\text{Subst}\left(\int \frac{\log(1-ax)}{x^4}\,dx, x, x^2\right) \\
&= \frac{\log\left(1-ax^2\right)}{54x^6} - \frac{\text{Li}_2(ax^2)}{18x^6} - \frac{\text{Li}_3(ax^2)}{6x^6} + \frac{1}{54}a\text{Subst}\left(\int \frac{1}{x^3(1-ax)}\,dx, x, x^2\right) \\
&= \frac{\log\left(1-ax^2\right)}{54x^6} - \frac{\text{Li}_2(ax^2)}{18x^6} - \frac{\text{Li}_3(ax^2)}{6x^6} + \frac{1}{54}a\text{Subst}\left(\int \left(\frac{1}{x^3} + \frac{a}{x^2} + \frac{a^2}{x} - \frac{a^3}{-1+ax}\right) dx, x\right. \\
&= -\frac{a}{108x^4} - \frac{a^2}{54x^2} + \frac{1}{27}a^3\log(x) - \frac{1}{54}a^3\log\left(1-ax^2\right) + \frac{\log\left(1-ax^2\right)}{54x^6} - \frac{\text{Li}_2(ax^2)}{18x^6} - \frac{\text{Li}_3(a}{6x^6}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.01, size = 30, normalized size = 0.34

$$\frac{G_{5,5}^{2,4}\left(-ax^2\left|\begin{array}{c}1,1,1,1,4\\1,3,0,0,0\end{array}\right.\right)}{2x^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x^2]/x^7,x]
[Out] MeijerG[{{1, 1, 1, 1}, {4}}, {{1, 3}, {0, 0, 0}}, -(a*x^2)]/(2*x^6)
```

Maple [A]
time = 0.10, size = 115, normalized size = 1.31

method	result	si

meijerg	$\frac{a^3\left(\frac{64a^2x^4+152a\,x^2+832}{1728a^2x^4}+\frac{(-64a^3x^6+64)\ln(-a\,x^2+1)}{1728a^3x^6}-\frac{\mathrm{polylog}(2,a\,x^2)}{9a^3x^6}-\frac{\mathrm{polylog}(3,a\,x^2)}{3a^3x^6}-\frac{1}{27}+\frac{2\ln(x)}{27}+\frac{\ln(-a)}{27}-\frac{1}{2a^2x^4}-\frac{1}{8a\,x^2}\right)}{2}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^3*(1/1728/a^2/x^4*(64*a^2*x^4+152*a*x^2+832)+1/1728/a^3/x^6*(-64*a^3*
x^6+64)*ln(-a*x^2+1)-1/9/a^3/x^6*polylog(2,a*x^2)-1/3/a^3/x^6*polylog(3,a*x
^2)-1/27+2/27*ln(x)+1/27*ln(-a)-1/2/a^2/x^4-1/8/a/x^2)
```

Maxima [A]
time = 0.27, size = 64, normalized size = 0.73

$$\frac{1}{27}\,a^3\log\left(x\right)-\frac{2\,a^2x^4+ax^2+2\,(a^3x^6-1)\log\left(-ax^2+1\right)+6\,\mathrm{Li}_2(ax^2)+18\,\mathrm{Li}_3(ax^2)}{108\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/27*a^3*log(x) - 1/108*(2*a^2*x^4 + a*x^2 + 2*(a^3*x^6 - 1)*log(-a*x^2 + 1
) + 6*dilog(a*x^2) + 18*polylog(3, a*x^2))/x^6
```

Fricas [A]
time = 0.46, size = 73, normalized size = 0.83

$$-\frac{2\,a^3x^6\log\left(ax^2-1\right)-4\,a^3x^6\log\left(x\right)+2\,a^2x^4+ax^2+6\,\mathrm{Li}_2(ax^2)-2\,\log\left(-ax^2+1\right)+18\,\mathrm{polylog}(3,ax^2)}{108\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/108*(2*a^3*x^6*log(a*x^2 - 1) - 4*a^3*x^6*log(x) + 2*a^2*x^4 + a*x^2 + 6
*dilog(a*x^2) - 2*log(-a*x^2 + 1) + 18*polylog(3, a*x^2))/x^6
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{Li}_3\left(ax^2\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/x**7,x)
```

```
[Out] Integral(polylog(3, a*x**2)/x**7, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/x^7, x)
```

Mupad [B]
time = 1.02, size = 73, normalized size = 0.83

$$\frac{a^3\,\ln\left(x\right)}{27}-\frac{\mathrm{polylog}(2,a\,x^2)}{18\,x^6}-\frac{\mathrm{polylog}(3,a\,x^2)}{6\,x^6}-\frac{a^3\,\ln\left(a\,x^2-1\right)}{54}-\frac{a}{108\,x^4}+\frac{\ln\left(1-a\,x^2\right)}{54\,x^6}-\frac{a^2}{54\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^2)/x^7,x)
```

```
[Out] (a^3*log(x))/27 - polylog(2, a*x^2)/(18*x^6) - polylog(3, a*x^2)/(6*x^6) -
(a^3*log(a*x^2 - 1))/54 - a/(108*x^4) + log(1 - a*x^2)/(54*x^6) - a^2/(54*x
^2)
```

3.39 $\int x^4\textbf{PolyLog}(3, ax^2)\,dx$

Optimal. Leaf size=87

$$\frac{8x}{125a^2}+\frac{8x^3}{375a}+\frac{8x^5}{625}-\frac{8\tanh^{-1}\left(\sqrt{a}\,x\right)}{125a^{5/2}}-\frac{4}{125}x^5\log\left(1-ax^2\right)-\frac{2}{25}x^5\text{PolyLog}\left(2,ax^2\right)+\frac{1}{5}x^5\text{PolyLog}\left(3,ax^2\right)$$

```
[Out] 8/125*x/a^2+8/375*x^3/a+8/625*x^5-8/125*arctanh(x*a^(1/2))/a^(5/2)-4/125*x^
5*ln(-a*x^2+1)-2/25*x^5*polylog(2,a*x^2)+1/5*x^5*polylog(3,a*x^2)
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6726, 2505, 308, 212}

$$-\frac{8\tanh^{-1}\left(\sqrt{a}\,x\right)}{125a^{5/2}}+\frac{8x}{125a^2}-\frac{2}{25}x^5\text{Li}_2\left(ax^2\right)+\frac{1}{5}x^5\text{Li}_3\left(ax^2\right)+\frac{8x^3}{375a}-\frac{4}{125}x^5\log\left(1-ax^2\right)+\frac{8x^5}{625}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*PolyLog[3, a*x^2],x]
```

```
[Out] (8*x)/(125*a^2) + (8*x^3)/(375*a) + (8*x^5)/625 - (8*ArcTanh[Sqrt[a]*x])/(1
25*a^(5/2)) - (4*x^5*Log[1 - a*x^2])/125 - (2*x^5*PolyLog[2, a*x^2])/25 + (
x^5*PolyLog[3, a*x^2])/5
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
```

```
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\text{Li}_3\left(ax^2\right)\,dx &= \frac{1}{5}x^5\text{Li}_3\left(ax^2\right)-\frac{2}{5}\int x^4\text{Li}_2\left(ax^2\right)\,dx\\ &= -\frac{2}{25}x^5\text{Li}_2\left(ax^2\right)+\frac{1}{5}x^5\text{Li}_3\left(ax^2\right)-\frac{4}{25}\int x^4\log\left(1-ax^2\right)\,dx\\ &= -\frac{4}{125}x^5\log\left(1-ax^2\right)-\frac{2}{25}x^5\text{Li}_2\left(ax^2\right)+\frac{1}{5}x^5\text{Li}_3\left(ax^2\right)-\frac{1}{125}(8a)\int\frac{x^6}{1-ax^2}\,dx\\ &= -\frac{4}{125}x^5\log\left(1-ax^2\right)-\frac{2}{25}x^5\text{Li}_2\left(ax^2\right)+\frac{1}{5}x^5\text{Li}_3\left(ax^2\right)-\frac{1}{125}(8a)\int\left(-\frac{1}{a^3}-\frac{x^2}{a^2}-\frac{x^4}{a}\right.\\ &= \frac{8x}{125a^2}+\frac{8x^3}{375a}+\frac{8x^5}{625}-\frac{4}{125}x^5\log\left(1-ax^2\right)-\frac{2}{25}x^5\text{Li}_2\left(ax^2\right)+\frac{1}{5}x^5\text{Li}_3\left(ax^2\right)-\frac{8\int\frac{1}{1-a}}{125}\\ &= \frac{8x}{125a^2}+\frac{8x^3}{375a}+\frac{8x^5}{625}-\frac{8\tanh^{-1}\left(\sqrt{a}\,x\right)}{125a^{5/2}}-\frac{4}{125}x^5\log\left(1-ax^2\right)-\frac{2}{25}x^5\text{Li}_2\left(ax^2\right)+\frac{1}{5}x\end{aligned}$$

Mathematica [A]
time = 0.11, size = 77, normalized size = 0.89

$$\frac{\frac{120x}{a^2}+\frac{40x^3}{a}+24x^5-\frac{120\tanh^{-1}\left(\sqrt{a}\,x\right)}{a^{5/2}}-60x^5\log\left(1-ax^2\right)-150x^5\text{PolyLog}(2,ax^2)+375x^5\text{PolyLog}(3,ax^2)}{1875}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*PolyLog[3, a*x^2],x]
```

```
[Out] ((120*x)/a^2 + (40*x^3)/a + 24*x^5 - (120*ArcTanh[Sqrt[a]*x])/a^(5/2) - 60*
x^5*Log[1 - a*x^2] - 150*x^5*PolyLog[2, a*x^2] + 375*x^5*PolyLog[3, a*x^2])
/1875
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(69) = 138.
time = 0.11, size = 144, normalized size = 1.66

method	result
meijerg	$-\frac{\frac{2x(-a)^{\frac{7}{2}}\left(168a^2x^4+280a\,x^2+840\right)}{13125a^3}+\frac{8x(-a)^{\frac{7}{2}}\left(\ln\left(1-\sqrt{a\,x^2}\right)-\ln\left(1+\sqrt{a\,x^2}\right)\right)}{125a^3\sqrt{a\,x^2}}-\frac{8x^5(-a)^{\frac{7}{2}}\ln\left(-a\,x^2+1\right)}{125a}-\frac{4x^5(-a)^{\frac{7}{2}}\operatorname{polylog}\left(2,a\,x^2\right.}{25a}}{2a^2\sqrt{-a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*polylog(3,a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2/(-a)^(1/2)*(2/13125*x*(-a)^(7/2)*(168*a^2*x^4+280*a*x^2+840)/a^3+8
/125*x*(-a)^(7/2)/a^3/(a*x^2)^(1/2)*(ln(1-(a*x^2)^(1/2))-ln(1+(a*x^2)^(1/2)
))-8/125*x^5*(-a)^(7/2)/a*ln(-a*x^2+1)-4/25*x^5*(-a)^(7/2)/a*polylog(2,a*x^
2)+2/5*x^5*(-a)^(7/2)/a*polylog(3,a*x^2))
```

Maxima [A]
time = 0.48, size = 95, normalized size = 1.09

$$-\frac{150\,a^2x^5\mathrm{Li}_2(ax^2)+60\,a^2x^5\log\left(-ax^2+1\right)-375\,a^2x^5\mathrm{Li}_3(ax^2)-24\,a^2x^5-40\,ax^3-120\,x}{1875\,a^2}+\frac{4\,\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right)}{125\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*polylog(3,a*x^2),x, algorithm="maxima")
```

```
[Out] -1/1875*(150*a^2*x^5*dilog(a*x^2) + 60*a^2*x^5*log(-a*x^2 + 1) - 375*a^2*x^
5*polylog(3, a*x^2) - 24*a^2*x^5 - 40*a*x^3 - 120*x)/a^2 + 4/125*log((a*x -
 sqrt(a))/(a*x + sqrt(a)))/a^(5/2)
```

Fricas [A]
time = 0.69, size = 189, normalized size = 2.17

$$\left[-\frac{150\,a^3x^5\mathrm{Li}_2(ax^2)+60\,a^3x^5\log\left(-ax^2+1\right)-375\,a^3x^5\mathrm{polylog}(3,ax^2)-24\,a^3x^5-40\,a^2x^3-120\,ax-60\,\sqrt{a}\,\log\left(\frac{ax^2-2\,\sqrt{a}\,x+1}{ax^2-1}\right)}{1875\,a^3},\,-\frac{150\,a^3x^5\mathrm{Li}_2(ax^2)+60\,a^3x^5\log\left(-ax^2+1\right)-375\,a^3x^5\mathrm{polylog}(3,ax^2)-24\,a^3x^5-40\,a^2x^3-120\,ax-120\,\sqrt{-a}\,\arctan\left(\sqrt{-a}\,x\right)}{1875\,a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*polylog(3,a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/1875*(150*a^3*x^5*dilog(a*x^2) + 60*a^3*x^5*log(-a*x^2 + 1) - 375*a^3*x
^5*polylog(3, a*x^2) - 24*a^3*x^5 - 40*a^2*x^3 - 120*a*x - 60*sqrt(a)*log((
a*x^2 - 2*sqrt(a)*x + 1)/(a*x^2 - 1)))/a^3, -1/1875*(150*a^3*x^5*dilog(a*x^
2) + 60*a^3*x^5*log(-a*x^2 + 1) - 375*a^3*x^5*polylog(3, a*x^2) - 24*a^3*x^
5 - 40*a^2*x^3 - 120*a*x - 120*sqrt(-a)*arctan(sqrt(-a)*x))/a^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\,\mathrm{Li}_3\left(ax^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*polylog(3,a*x**2),x)
```

```
[Out] Integral(x**4*polylog(3, a*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*polylog(3,a*x^2),x, algorithm="giac")
[Out] integrate(x^4*polylog(3, a*x^2), x)
```

Mupad [B]
time = 0.55, size = 72, normalized size = 0.83

$$\frac{x^5\,\mathrm{polylog}(3,a\,x^2)}{5}-\frac{2\,x^5\,\mathrm{polylog}(2,a\,x^2)}{25}+\frac{8\,x}{125\,a^2}-\frac{4\,x^5\,\ln\left(1-a\,x^2\right)}{125}+\frac{8\,x^5}{625}+\frac{8\,x^3}{375\,a}+\frac{\mathrm{atan}\left(\sqrt{a}\,x\,1\mathrm{i}\right)\,8\mathrm{i}}{125\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*polylog(3, a*x^2),x)
[Out] (atan(a^(1/2)*x*1i)*8i)/(125*a^(5/2)) - (2*x^5*polylog(2, a*x^2))/25 + (x^5
*polylog(3, a*x^2))/5 + (8*x)/(125*a^2) - (4*x^5*log(1 - a*x^2))/125 + (8*x
^5)/625 + (8*x^3)/(375*a)
```

3.40 $\int x^2\textbf{PolyLog}(3, ax^2)\, dx$

Optimal. Leaf size=77

$$\frac{8x}{27a}+\frac{8x^3}{81}-\frac{8\tanh^{-1}\left(\sqrt{a}\,x\right)}{27a^{3/2}}-\frac{4}{27}x^3\log\left(1-ax^2\right)-\frac{2}{9}x^3\text{PolyLog}\left(2,ax^2\right)+\frac{1}{3}x^3\text{PolyLog}\left(3,ax^2\right)$$

```
[Out] 8/27*x/a+8/81*x^3-8/27*arctanh(x*a^(1/2))/a^(3/2)-4/27*x^3*ln(-a*x^2+1)-2/9
*x^3*polylog(2,a*x^2)+1/3*x^3*polylog(3,a*x^2)
```

Rubi [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6726, 2505, 308, 212}

$$-\frac{8\tanh^{-1}\left(\sqrt{a}\,x\right)}{27a^{3/2}}-\frac{2}{9}x^3\text{Li}_2\left(ax^2\right)+\frac{1}{3}x^3\text{Li}_3\left(ax^2\right)-\frac{4}{27}x^3\log\left(1-ax^2\right)+\frac{8x}{27a}+\frac{8x^3}{81}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*PolyLog[3, a*x^2],x]
```

```
[Out] (8*x)/(27*a) + (8*x^3)/81 - (8*ArcTanh[Sqrt[a]*x])/(27*a^(3/2)) - (4*x^3*Lo
g[1 - a*x^2])/27 - (2*x^3*PolyLog[2, a*x^2])/9 + (x^3*PolyLog[3, a*x^2])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
```

```
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2\text{Li}_3\left(ax^2\right)\,dx &= \frac{1}{3}x^3\text{Li}_3\left(ax^2\right)-\frac{2}{3}\int x^2\text{Li}_2\left(ax^2\right)\,dx\\
&= -\frac{2}{9}x^3\text{Li}_2\left(ax^2\right)+\frac{1}{3}x^3\text{Li}_3\left(ax^2\right)-\frac{4}{9}\int x^2\log\left(1-ax^2\right)\,dx\\
&= -\frac{4}{27}x^3\log\left(1-ax^2\right)-\frac{2}{9}x^3\text{Li}_2\left(ax^2\right)+\frac{1}{3}x^3\text{Li}_3\left(ax^2\right)-\frac{1}{27}(8a)\int\frac{x^4}{1-ax^2}\,dx\\
&= -\frac{4}{27}x^3\log\left(1-ax^2\right)-\frac{2}{9}x^3\text{Li}_2\left(ax^2\right)+\frac{1}{3}x^3\text{Li}_3\left(ax^2\right)-\frac{1}{27}(8a)\int\left(-\frac{1}{a^2}-\frac{x^2}{a}+\frac{1}{a^2\left(1-\right.}\right.\\
&= \frac{8x}{27a}+\frac{8x^3}{81}-\frac{4}{27}x^3\log\left(1-ax^2\right)-\frac{2}{9}x^3\text{Li}_2\left(ax^2\right)+\frac{1}{3}x^3\text{Li}_3\left(ax^2\right)-\frac{8\int\frac{1}{1-ax^2}\,dx}{27a}\\
&= \frac{8x}{27a}+\frac{8x^3}{81}-\frac{8\tanh^{-1}\left(\sqrt{a}\,x\right)}{27a^{3/2}}-\frac{4}{27}x^3\log\left(1-ax^2\right)-\frac{2}{9}x^3\text{Li}_2\left(ax^2\right)+\frac{1}{3}x^3\text{Li}_3\left(ax^2\right)
\end{aligned}
$$

Mathematica [**A**]
time = 0.10, size = 69, normalized size = 0.90

$$
\frac{1}{81}\left(\frac{24x}{a}+8x^3-\frac{24\tanh^{-1}\left(\sqrt{a}\,x\right)}{a^{3/2}}-12x^3\log\left(1-ax^2\right)-18x^3\text{PolyLog}\left(2,ax^2\right)+27x^3\text{PolyLog}\left(3,ax^2\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*PolyLog[3, a*x^2],x]
```

```
[Out] ((24*x)/a + 8*x^3 - (24*ArcTanh[Sqrt[a]*x])/a^(3/2) - 12*x^3*Log[1 - a*x^2]
 - 18*x^3*PolyLog[2, a*x^2] + 27*x^3*PolyLog[3, a*x^2])/81
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(61) = 122.
time = 0.13, size = 136, normalized size = 1.77

method	result
meijerg	$\frac{\frac{2x(-a)^{\frac{5}{2}}\left(40a\,x^2+120\right)}{405a^2}+\frac{8x(-a)^{\frac{5}{2}}\left(\ln\left(1-\sqrt{a\,x^2}\right)-\ln\left(1+\sqrt{a\,x^2}\right)\right)}{27a^2\sqrt{a\,x^2}}-\frac{8x^3(-a)^{\frac{5}{2}}\ln\left(-a\,x^2+1\right)}{27a}-\frac{4x^3(-a)^{\frac{5}{2}}\,\text{polylog}\left(2,a\,x^2\right)}{9a}+\frac{2x^3(-a)^{\frac{5}{2}}}{}}{2a\sqrt{-a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(3,a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a/(-a)^(1/2)*(2/405*x*(-a)^(5/2)*(40*a*x^2+120)/a^2+8/27*x*(-a)^(5/2)/a
^2/(a*x^2)^(1/2)*(ln(1-(a*x^2)^(1/2))-ln(1+(a*x^2)^(1/2)))-8/27*x^3*(-a)^(5
/2)/a*ln(-a*x^2+1)-4/9*x^3*(-a)^(5/2)/a*polylog(2,a*x^2)+2/3*x^3*(-a)^(5/2)
/a*polylog(3,a*x^2))
```

Maxima [A]
time = 0.46, size = 81, normalized size = 1.05

$$-\frac{18\,ax^3\mathrm{Li}_2(ax^2)+12\,ax^3\log\left(-ax^2+1\right)-27\,ax^3\mathrm{Li}_3(ax^2)-8\,ax^3-24\,x}{81\,a}+\frac{4\,\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right)}{27\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,a*x^2),x, algorithm="maxima")
```

```
[Out] -1/81*(18*a*x^3*dilog(a*x^2) + 12*a*x^3*log(-a*x^2 + 1) - 27*a*x^3*polylog(
3, a*x^2) - 8*a*x^3 - 24*x)/a + 4/27*log((a*x - sqrt(a))/(a*x + sqrt(a)))/a
^(3/2)
```

Fricas [A]
time = 0.45, size = 173, normalized size = 2.25

$$\left[-\frac{18\,a^2x^3\mathrm{Li}_2(ax^2)+12\,a^2x^3\log\left(-ax^2+1\right)-27\,a^2x^3\mathrm{polylog}(3,ax^2)-8\,a^2x^3-24\,ax-12\,\sqrt{a}\,\log\left(\frac{ax^2-2\,\sqrt{a}\,x+1}{ax^2-1}\right)}{81\,a^2},\,-\frac{18\,a^2x^3\mathrm{Li}_2(ax^2)+12\,a^2x^3\log\left(-ax^2+1\right)-27\,a^2x^3\mathrm{polylog}(3,ax^2)-8\,a^2x^3-24\,ax-24\,\sqrt{-a}\,\arctan\left(\sqrt{-a}\,x\right)}{81\,a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/81*(18*a^2*x^3*dilog(a*x^2) + 12*a^2*x^3*log(-a*x^2 + 1) - 27*a^2*x^3*p
olylog(3, a*x^2) - 8*a^2*x^3 - 24*a*x - 12*sqrt(a)*log((a*x^2 - 2*sqrt(a)*x
 + 1)/(a*x^2 - 1)))/a^2, -1/81*(18*a^2*x^3*dilog(a*x^2) + 12*a^2*x^3*log(-a
*x^2 + 1) - 27*a^2*x^3*polylog(3, a*x^2) - 8*a^2*x^3 - 24*a*x - 24*sqrt(-a)
*arctan(sqrt(-a)*x))/a^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\mathrm{Li}_3\left(ax^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*polylog(3,a*x**2),x)
```

```
[Out] Integral(x**2*polylog(3, a*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*polylog(3, a*x^2), x)
```

Mupad [B]
time = 0.49, size = 64, normalized size = 0.83

$$\frac{x^3\,\mathrm{polylog}(3,a\,x^2)}{3}-\frac{2\,x^3\,\mathrm{polylog}(2,a\,x^2)}{9}+\frac{8\,x}{27\,a}-\frac{4\,x^3\,\ln\left(1-a\,x^2\right)}{27}+\frac{8\,x^3}{81}+\frac{\mathrm{atan}\left(\sqrt{a}\,x\,1\mathrm{i}\right)\,8\mathrm{i}}{27\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(3, a*x^2),x)
```

```
[Out] (atan(a^(1/2)*x*1i)*8i)/(27*a^(3/2)) - (2*x^3*polylog(2, a*x^2))/9 + (x^3*p
olylog(3, a*x^2))/3 + (8*x)/(27*a) - (4*x^3*log(1 - a*x^2))/27 + (8*x^3)/81
```

3.41 $\int \textbf{PolyLog}(3, ax^2)\, dx$

Optimal. Leaf size=50

$$8x - \frac{8\tanh^{-1}\left(\sqrt{a}\,x\right)}{\sqrt{a}} - 4x\log\left(1-ax^2\right) - 2x\text{PolyLog}\left(2,ax^2\right) + x\text{PolyLog}\left(3,ax^2\right)$$

[Out] 8*x-4*x*ln(-a*x^2+1)-2*x*polylog(2,a*x^2)+x*polylog(3,a*x^2)-8*arctanh(x*a^(1/2))/a^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6721, 2498, 327, 212}

$$-2x\text{Li}_2\left(ax^2\right) + x\text{Li}_3\left(ax^2\right) - 4x\log\left(1-ax^2\right) - \frac{8\tanh^{-1}\left(\sqrt{a}\,x\right)}{\sqrt{a}} + 8x$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a*x^2],x]

[Out] 8*x - (8*ArcTanh[Sqrt[a]*x])/Sqrt[a] - 4*x*Log[1 - a*x^2] - 2*x*PolyLog[2, a*x^2] + x*PolyLog[3, a*x^2]

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \text{Li}_3(ax^2)\,dx &= x\text{Li}_3(ax^2) - 2\int \text{Li}_2(ax^2)\,dx \\
&= -2x\text{Li}_2(ax^2) + x\text{Li}_3(ax^2) - 4\int \log(1-ax^2)\,dx \\
&= -4x\log(1-ax^2) - 2x\text{Li}_2(ax^2) + x\text{Li}_3(ax^2) - (8a)\int \frac{x^2}{1-ax^2}\,dx \\
&= 8x - 4x\log(1-ax^2) - 2x\text{Li}_2(ax^2) + x\text{Li}_3(ax^2) - 8\int \frac{1}{1-ax^2}\,dx \\
&= 8x - \frac{8\tanh^{-1}(\sqrt{a}\,x)}{\sqrt{a}} - 4x\log(1-ax^2) - 2x\text{Li}_2(ax^2) + x\text{Li}_3(ax^2)
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 50, normalized size = 1.00

$$8x - \frac{8\tanh^{-1}(\sqrt{a}\,x)}{\sqrt{a}} - 4x\log(1-ax^2) - 2x\text{PolyLog}(2,ax^2) + x\text{PolyLog}(3,ax^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^2],x]
```

```
[Out] 8*x - (8*ArcTanh[Sqrt[a]*x])/Sqrt[a] - 4*x*Log[1 - a*x^2] - 2*x*PolyLog[2,
a*x^2] + x*PolyLog[3, a*x^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(46) = 92$.
time = 0.12, size = 119, normalized size = 2.38

method	result
meijerg	$-\frac{\frac{16x(-a)^{\frac{3}{2}}}{a}+\frac{8x(-a)^{\frac{3}{2}}\left(\ln\left(1-\sqrt{a\,x^2}\right)-\ln\left(1+\sqrt{a\,x^2}\right)\right)}{a\sqrt{a\,x^2}}-\frac{8x(-a)^{\frac{3}{2}}\ln\left(-a\,x^2+1\right)}{a}-\frac{4x(-a)^{\frac{3}{2}}\operatorname{polylog}\left(2,a\,x^2\right)}{a}+\frac{2x(-a)^{\frac{3}{2}}\operatorname{polylog}\left(3,a\,x^2\right)}{a}}{2\sqrt{-a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(-a)^(1/2)*(16*x*(-a)^(3/2)/a+8*x*(-a)^(3/2)/a/(a*x^2)^(1/2)*(ln(1-(a*
x^2)^(1/2))-ln(1+(a*x^2)^(1/2)))-8*x*(-a)^(3/2)/a*ln(-a*x^2+1)-4*x*(-a)^(3/
2)/a*polylog(2,a*x^2)+2*x*(-a)^(3/2)/a*polylog(3,a*x^2))
```

Maxima [A]
time = 0.48, size = 59, normalized size = 1.18

$$-2\,x\mathrm{Li}_2\left(ax^2\right)-4\,x\log\left(-ax^2+1\right)+x\mathrm{Li}_3(ax^2)+8\,x+\frac{4\,\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2),x, algorithm="maxima")
```

```
[Out] -2*x*dilog(a*x^2) - 4*x*log(-a*x^2 + 1) + x*polylog(3, a*x^2) + 8*x + 4*log
((a*x - sqrt(a))/(a*x + sqrt(a)))/sqrt(a)
```

Fricas [A]
time = 0.47, size = 133, normalized size = 2.66

$$\left[-\frac{2\,ax\mathrm{Li}_2(ax^2)+4\,ax\log\left(-ax^2+1\right)-ax\mathrm{polylog}(3,ax^2)-8\,ax-4\,\sqrt{a}\,\log\left(\frac{ax^2-2\,\sqrt{a}\,x+1}{ax^2-1}\right)}{a},-\frac{2\,ax\mathrm{Li}_2(ax^2)+4\,ax\log\left(-ax^2+1\right)-ax\mathrm{polylog}(3,ax^2)-8\,ax-8\,\sqrt{-a}\,\arctan\left(\sqrt{-a}\,x\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2),x, algorithm="fricas")
```

```
[Out] [-(2*a*x*dilog(a*x^2) + 4*a*x*log(-a*x^2 + 1) - a*x*polylog(3, a*x^2) - 8*a
*x - 4*sqrt(a)*log((a*x^2 - 2*sqrt(a)*x + 1)/(a*x^2 - 1)))/a, -(2*a*x*dilog
(a*x^2) + 4*a*x*log(-a*x^2 + 1) - a*x*polylog(3, a*x^2) - 8*a*x - 8*sqrt(-a
)*arctan(sqrt(-a)*x))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Li}_3\left(ax^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2),x)
```

```
[Out] Integral(polylog(3, a*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2), x)
```

Mupad [B]
time = 0.36, size = 49, normalized size = 0.98

$$8x - 4x\ \ln\left(1 - a\,x^2\right) - 2x\,\mathrm{polylog}\left(2, a\,x^2\right) + x\,\mathrm{polylog}\left(3, a\,x^2\right) + \frac{\mathrm{atan}\left(\sqrt{a}\ x\,1\mathrm{i}\right)\ 8\mathrm{i}}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^2),x)
```

```
[Out] 8*x + (atan(a^(1/2)*x*1i)*8i)/a^(1/2) - 4*x*log(1 - a*x^2) - 2*x*polylog(2,
 a*x^2) + x*polylog(3, a*x^2)
```

3.42 $\int \frac{\text{PolyLog}(3,ax^2)}{x^2} dx$

Optimal. Leaf size=54

$$8\sqrt{a}\tanh^{-1}\left(\sqrt{a}\,x\right)+\frac{4\log\left(1-ax^2\right)}{x}-\frac{2\text{PolyLog}(2,ax^2)}{x}-\frac{\text{PolyLog}(3,ax^2)}{x}$$

```
[Out] 4*ln(-a*x^2+1)/x-2*polylog(2,a*x^2)/x-polylog(3,a*x^2)/x+8*arctanh(x*a^(1/2
))*a^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6726, 2505, 212}

$$-\frac{2\text{Li}_2(ax^2)}{x}-\frac{\text{Li}_3(ax^2)}{x}+\frac{4\log\left(1-ax^2\right)}{x}+8\sqrt{a}\tanh^{-1}\left(\sqrt{a}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/x^2,x]
```

```
[Out] 8*Sqrt[a]*ArcTanh[Sqrt[a]*x] + (4*Log[1 - a*x^2])/x - (2*PolyLog[2, a*x^2])
/x - PolyLog[3, a*x^2]/x
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax^2)}{x^2}\,dx &= -\frac{\text{Li}_3(ax^2)}{x} + 2\int \frac{\text{Li}_2(ax^2)}{x^2}\,dx \\
&= -\frac{2\text{Li}_2(ax^2)}{x} - \frac{\text{Li}_3(ax^2)}{x} - 4\int \frac{\log\left(1-ax^2\right)}{x^2}\,dx \\
&= \frac{4\log\left(1-ax^2\right)}{x} - \frac{2\text{Li}_2(ax^2)}{x} - \frac{\text{Li}_3(ax^2)}{x} + (8a)\int \frac{1}{1-ax^2}\,dx \\
&= 8\sqrt{a}\,\tanh^{-1}\left(\sqrt{a}\,x\right) + \frac{4\log\left(1-ax^2\right)}{x} - \frac{2\text{Li}_2(ax^2)}{x} - \frac{\text{Li}_3(ax^2)}{x}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 50, normalized size = 0.93

$$\frac{8\sqrt{a}\,x\tanh^{-1}\left(\sqrt{a}\,x\right) + 4\log\left(1-ax^2\right) - 2\text{PolyLog}(2,ax^2) - \text{PolyLog}(3,ax^2)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^2]/x^2,x]
```

```
[Out] (8*Sqrt[a]*x*ArcTanh[Sqrt[a]*x] + 4*Log[1 - a*x^2] - 2*PolyLog[2, a*x^2] -
PolyLog[3, a*x^2])/x
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(50) = 100.
time = 0.12, size = 112, normalized size = 2.07

method	result
meijerg	$\frac{a\left(-\frac{8x\sqrt{-a}\,\left(\ln\left(1-\sqrt{a\,x^2}\right)-\ln\left(1+\sqrt{a\,x^2}\right)\right)}{\sqrt{a\,x^2}}+\frac{8\sqrt{-a}\,\ln\left(-a\,x^2+1\right)}{xa}-\frac{4\sqrt{-a}\,\text{polylog}\left(2,a\,x^2\right)}{xa}-\frac{2\sqrt{-a}\,\text{polylog}\left(3,a\,x^2\right)}{xa}\right.}{2\sqrt{-a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a/(-a)^(1/2)*(-8*x*(-a)^(1/2)/(a*x^2)^(1/2)*(ln(1-(a*x^2)^(1/2))-ln(1+(
a*x^2)^(1/2)))+8/x*(-a)^(1/2)/a*ln(-a*x^2+1)-4/x*(-a)^(1/2)/a*polylog(2,a*x
^2)-2/x*(-a)^(1/2)/a*polylog(3,a*x^2))
```

Maxima [A]
time = 0.47, size = 58, normalized size = 1.07

$$-4\sqrt{a}\,\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right) - \frac{2\,\text{Li}_2(ax^2) - 4\,\log\left(-ax^2+1\right) + \text{Li}_3(ax^2)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] -4*sqrt(a)*log((a*x - sqrt(a))/(a*x + sqrt(a))) - (2*dilog(a*x^2) - 4*log(-
a*x^2 + 1) + polylog(3, a*x^2))/x
```

Fricas [A]
time = 0.52, size = 112, normalized size = 2.07

$$\left[\frac{4\sqrt{a}\,x\log\left(\frac{ax^2+2\sqrt{a}\,x+1}{ax^2-1}\right)-2\,\mathrm{Li}_2(ax^2)+4\,\log\left(-ax^2+1\right)-\mathrm{polylog}(3,ax^2)}{x},-\frac{8\sqrt{-a}\,x\arctan\left(\sqrt{-a}\,x\right)+2\,\mathrm{Li}_2(ax^2)-4\,\log\left(-ax^2+1\right)+\mathrm{polylog}(3,ax^2)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] [(4*sqrt(a)*x*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)) - 2*dilog(a*x^2) +
 4*log(-a*x^2 + 1) - polylog(3, a*x^2))/x, -(8*sqrt(-a)*x*arctan(sqrt(-a)*x
) + 2*dilog(a*x^2) - 4*log(-a*x^2 + 1) + polylog(3, a*x^2))/x]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_3\left(ax^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/x**2,x)
```

```
[Out] Integral(polylog(3, a*x**2)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/x^2, x)
```

Mupad [B]
time = 0.55, size = 53, normalized size = 0.98

$$\frac{4\,\ln\left(1-a\,x^2\right)}{x}-\frac{\mathrm{polylog}(3,a\,x^2)}{x}-\frac{2\,\mathrm{polylog}(2,a\,x^2)}{x}-\sqrt{a}\,\mathrm{atan}\left(\sqrt{a}\,x\,1\mathrm{i}\right)\,8\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^2)/x^2,x)
```

```
[Out] (4*log(1 - a*x^2))/x - (2*polylog(2, a*x^2))/x - polylog(3, a*x^2)/x - a^(1
/2)*atan(a^(1/2)*x*1i)*8i
```

3.43 $\int \frac{\text{PolyLog}(3,ax^2)}{x^4} dx$

Optimal. Leaf size=70

$$-\frac{8a}{27x}+\frac{8}{27}a^{3/2}\tanh^{-1}\left(\sqrt{a}\,x\right)+\frac{4\log\left(1-ax^2\right)}{27x^3}-\frac{2\text{PolyLog}(2,ax^2)}{9x^3}-\frac{\text{PolyLog}(3,ax^2)}{3x^3}$$

```
[Out] -8/27*a/x+8/27*a^(3/2)*arctanh(x*a^(1/2))+4/27*ln(-a*x^2+1)/x^3-2/9*polylog
(2,a*x^2)/x^3-1/3*polylog(3,a*x^2)/x^3
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6726, 2505, 331, 212}

$$\frac{8}{27}a^{3/2}\tanh^{-1}\left(\sqrt{a}\,x\right)-\frac{2\text{Li}_2(ax^2)}{9x^3}-\frac{\text{Li}_3(ax^2)}{3x^3}+\frac{4\log\left(1-ax^2\right)}{27x^3}-\frac{8a}{27x}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/x^4,x]
```

```
[Out] (-8*a)/(27*x) + (8*a^(3/2)*ArcTanh[Sqrt[a]*x])/27 + (4*Log[1 - a*x^2])/(27*
x^3) - (2*PolyLog[2, a*x^2])/(9*x^3) - PolyLog[3, a*x^2]/(3*x^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_3(ax^2)}{x^4}\,dx &= -\frac{\text{Li}_3(ax^2)}{3x^3}+\frac{2}{3}\int \frac{\text{Li}_2(ax^2)}{x^4}\,dx\\ &= -\frac{2\text{Li}_2(ax^2)}{9x^3}-\frac{\text{Li}_3(ax^2)}{3x^3}-\frac{4}{9}\int \frac{\log(1-ax^2)}{x^4}\,dx\\ &= \frac{4\log(1-ax^2)}{27x^3}-\frac{2\text{Li}_2(ax^2)}{9x^3}-\frac{\text{Li}_3(ax^2)}{3x^3}+\frac{1}{27}(8a)\int \frac{1}{x^2(1-ax^2)}\,dx\\ &= -\frac{8a}{27x}+\frac{4\log(1-ax^2)}{27x^3}-\frac{2\text{Li}_2(ax^2)}{9x^3}-\frac{\text{Li}_3(ax^2)}{3x^3}+\frac{1}{27}(8a^2)\int \frac{1}{1-ax^2}\,dx\\ &= -\frac{8a}{27x}+\frac{8}{27}a^{3/2}\tanh^{-1}\left(\sqrt{a}\,x\right)+\frac{4\log(1-ax^2)}{27x^3}-\frac{2\text{Li}_2(ax^2)}{9x^3}-\frac{\text{Li}_3(ax^2)}{3x^3}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 61, normalized size = 0.87

$$-\frac{8ax^2-8a^{3/2}x^3\tanh^{-1}\left(\sqrt{a}\,x\right)-4\log(1-ax^2)+6\text{PolyLog}(2,ax^2)+9\text{PolyLog}(3,ax^2)}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^2]/x^4,x]
```

```
[Out] -1/27*(8*a*x^2 - 8*a^(3/2)*x^3*ArcTanh[Sqrt[a]*x] - 4*Log[1 - a*x^2] + 6*Po
lyLog[2, a*x^2] + 9*PolyLog[3, a*x^2])/x^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(56) = 112.
time = 0.12, size = 125, normalized size = 1.79

method	result	size
meijerg	$-\frac{a^2\left(-\frac{16}{27x\sqrt{-a}}-\frac{8xa\left(\ln\left(1-\sqrt{a\,x^2}\right)-\ln\left(1+\sqrt{a\,x^2}\right)\right)}{27\sqrt{-a}\,\sqrt{a\,x^2}}+\frac{8\ln(-a\,x^2+1)}{27x^3\sqrt{-a}\,a}-\frac{4\,\text{polylog}(2,a\,x^2)}{9x^3\sqrt{-a}\,a}-\frac{2\,\text{polylog}(3,a\,x^2)}{3x^3\sqrt{-a}\,a}\right)}{2\sqrt{-a}}$	125

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2/(-a)^(1/2)*(-16/27/x/(-a)^(1/2)-8/27*x/(-a)^(1/2)*a/(a*x^2)^(1/2)*
(ln(1-(a*x^2)^(1/2))-ln(1+(a*x^2)^(1/2)))+8/27/x^3/(-a)^(1/2)/a*ln(-a*x^2+1
)-4/9/x^3/(-a)^(1/2)/a*polylog(2,a*x^2)-2/3/x^3/(-a)^(1/2)/a*polylog(3,a*x^
2))
```

Maxima [A]
time = 0.47, size = 66, normalized size = 0.94

$$-\frac{4}{27}\,a^{\frac{3}{2}}\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right)-\frac{8\,ax^2+6\,\mathrm{Li}_2(ax^2)-4\,\log\left(-ax^2+1\right)+9\,\mathrm{Li}_3(ax^2)}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] -4/27*a^(3/2)*log((a*x - sqrt(a))/(a*x + sqrt(a))) - 1/27*(8*a*x^2 + 6*dilo
g(a*x^2) - 4*log(-a*x^2 + 1) + 9*polylog(3, a*x^2))/x^3
```

Fricas [A]
time = 0.37, size = 132, normalized size = 1.89

$$\left[\frac{4\,a^{\frac{3}{2}}x^3\log\left(\frac{ax^2+2\,\sqrt{a}\,x+1}{ax^2-1}\right)-8\,ax^2-6\,\mathrm{Li}_2(ax^2)+4\,\log\left(-ax^2+1\right)-9\,\mathrm{polylog}(3,ax^2)}{27\,x^3},-\frac{8\,\sqrt{-a}\,ax^3\arctan\left(\sqrt{-a}\,x\right)+8\,ax^2+6\,\mathrm{Li}_2(ax^2)-4\,\log\left(-ax^2+1\right)+9\,\mathrm{polylog}(3,ax^2)}{27\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/27*(4*a^(3/2)*x^3*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)) - 8*a*x^2 -
 6*dilog(a*x^2) + 4*log(-a*x^2 + 1) - 9*polylog(3, a*x^2))/x^3, -1/27*(8*sq
rt(-a)*a*x^3*arctan(sqrt(-a)*x) + 8*a*x^2 + 6*dilog(a*x^2) - 4*log(-a*x^2 +
 1) + 9*polylog(3, a*x^2))/x^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{Li}_3\left(ax^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/x**4,x)
```

```
[Out] Integral(polylog(3, a*x**2)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/x^4, x)
```

Mupad [B]
time = 0.77, size = 59, normalized size = 0.84

$$\frac{4\ln\left(1-a\,x^2\right)}{27\,x^3}-\frac{\mathrm{polylog}(3,a\,x^2)}{3\,x^3}-\frac{8\,a}{27\,x}-\frac{2\,\mathrm{polylog}(2,a\,x^2)}{9\,x^3}-\frac{a^{3/2}\,\mathrm{atan}\left(\sqrt{a}\;x\,1\mathrm{i}\right)\,8\mathrm{i}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^2)/x^4,x)
```

```
[Out] (4*log(1 - a*x^2))/(27*x^3) - (2*polylog(2, a*x^2))/(9*x^3) - polylog(3, a*
x^2)/(3*x^3) - (8*a)/(27*x) - (a^(3/2)*atan(a^(1/2)*x*1i)*8i)/27
```

3.44 $\int \frac{\text{PolyLog}(3,ax^2)}{x^6} dx$

Optimal. Leaf size=80

$$-\frac{8a}{375x^3}-\frac{8a^2}{125x}+\frac{8}{125}a^{5/2}\tanh^{-1}\left(\sqrt{a}\,x\right)+\frac{4\log\left(1-ax^2\right)}{125x^5}-\frac{2\text{PolyLog}(2,ax^2)}{25x^5}-\frac{\text{PolyLog}(3,ax^2)}{5x^5}$$

```
[Out] -8/375*a/x^3-8/125*a^2/x+8/125*a^(5/2)*arctanh(x*a^(1/2))+4/125*ln(-a*x^2+1
)/x^5-2/25*polylog(2,a*x^2)/x^5-1/5*polylog(3,a*x^2)/x^5
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6726, 2505, 331, 212}

$$\frac{8}{125}a^{5/2}\tanh^{-1}\left(\sqrt{a}\,x\right)-\frac{8a^2}{125x}-\frac{2\text{Li}_2(ax^2)}{25x^5}-\frac{\text{Li}_3(ax^2)}{5x^5}-\frac{8a}{375x^3}+\frac{4\log\left(1-ax^2\right)}{125x^5}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/x^6,x]
```

```
[Out] (-8*a)/(375*x^3) - (8*a^2)/(125*x) + (8*a^(5/2)*ArcTanh[Sqrt[a]*x])/125 + (
4*Log[1 - a*x^2])/(125*x^5) - (2*PolyLog[2, a*x^2])/(25*x^5) - PolyLog[3, a
*x^2]/(5*x^5)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{Li}_3(ax^2)}{x^6}\,dx &= -\frac{\text{Li}_3(ax^2)}{5x^5}+\frac{2}{5}\int \frac{\text{Li}_2(ax^2)}{x^6}\,dx\\
&= -\frac{2\text{Li}_2(ax^2)}{25x^5}-\frac{\text{Li}_3(ax^2)}{5x^5}-\frac{4}{25}\int \frac{\log(1-ax^2)}{x^6}\,dx\\
&= \frac{4\log(1-ax^2)}{125x^5}-\frac{2\text{Li}_2(ax^2)}{25x^5}-\frac{\text{Li}_3(ax^2)}{5x^5}+\frac{1}{125}(8a)\int \frac{1}{x^4(1-ax^2)}\,dx\\
&= -\frac{8a}{375x^3}+\frac{4\log(1-ax^2)}{125x^5}-\frac{2\text{Li}_2(ax^2)}{25x^5}-\frac{\text{Li}_3(ax^2)}{5x^5}+\frac{1}{125}(8a^2)\int \frac{1}{x^2(1-ax^2)}\,dx\\
&= -\frac{8a}{375x^3}-\frac{8a^2}{125x}+\frac{4\log(1-ax^2)}{125x^5}-\frac{2\text{Li}_2(ax^2)}{25x^5}-\frac{\text{Li}_3(ax^2)}{5x^5}+\frac{1}{125}(8a^3)\int \frac{1}{1-ax^2}\,dx\\
&= -\frac{8a}{375x^3}-\frac{8a^2}{125x}+\frac{8}{125}a^{5/2}\tanh^{-1}\left(\sqrt{a}\,x\right)+\frac{4\log(1-ax^2)}{125x^5}-\frac{2\text{Li}_2(ax^2)}{25x^5}-\frac{\text{Li}_3(ax^2)}{5x^5}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 69, normalized size = 0.86

$$
-\frac{8ax^2+24a^2x^4-24a^{5/2}x^5\tanh^{-1}\left(\sqrt{a}\,x\right)-12\log(1-ax^2)+30\text{PolyLog}(2,ax^2)+75\text{PolyLog}(3,ax^2)}{375x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^2]/x^6,x]
```

```
[Out] -1/375*(8*a*x^2 + 24*a^2*x^4 - 24*a^(5/2)*x^5*ArcTanh[Sqrt[a]*x] - 12*Log[1
 - a*x^2] + 30*PolyLog[2, a*x^2] + 75*PolyLog[3, a*x^2])/x^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(64) = 128$.
time = 0.13, size = 138, normalized size = 1.72

method	result

meijerg	$\frac{a^3\left(-\frac{16}{375x^3(-a)^{\frac{3}{2}}}-\frac{16a}{125x(-a)^{\frac{3}{2}}}-\frac{8x\,a^2\left(\ln\left(1-\sqrt{a\,x^2}\right)-\ln\left(1+\sqrt{a\,x^2}\right)\right)}{125(-a)^{\frac{3}{2}}\sqrt{a\,x^2}}+\frac{8\ln\left(-a\,x^2+1\right)}{125x^5(-a)^{\frac{3}{2}}a}-\frac{4\,\mathrm{polylog}\left(2,a\,x^2\right)}{25x^5(-a)^{\frac{3}{2}}a}-\frac{2\,\mathrm{polylog}\left(3,a\,x^2\right)}{5x^5(-a)^{\frac{3}{2}}a}\right)}{2\sqrt{-a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^3/(-a)^(1/2)*(-16/375/x^3/(-a)^(3/2)-16/125/x/(-a)^(3/2)*a-8/125*x/(-
a)^(3/2)*a^2/(a*x^2)^(1/2)*(ln(1-(a*x^2)^(1/2))-ln(1+(a*x^2)^(1/2)))+8/125/
x^5/(-a)^(3/2)/a*ln(-a*x^2+1)-4/25/x^5/(-a)^(3/2)/a*polylog(2,a*x^2)-2/5/x^
5/(-a)^(3/2)/a*polylog(3,a*x^2))
```

Maxima [A]
time = 0.47, size = 74, normalized size = 0.92

$$-\frac{4}{125}\,a^{\frac{5}{2}}\log\left(\frac{ax-\sqrt{a}}{ax+\sqrt{a}}\right)-\frac{24\,a^2x^4+8\,ax^2+30\,\mathrm{Li}_2(ax^2)-12\,\log\left(-ax^2+1\right)+75\,\mathrm{Li}_3(ax^2)}{375\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^6,x, algorithm="maxima")
```

```
[Out] -4/125*a^(5/2)*log((a*x - sqrt(a))/(a*x + sqrt(a))) - 1/375*(24*a^2*x^4 + 8
*a*x^2 + 30*dilog(a*x^2) - 12*log(-a*x^2 + 1) + 75*polylog(3, a*x^2))/x^5
```

Fricas [A]
time = 0.45, size = 150, normalized size = 1.88

$$\left[\frac{12\,a^{\frac{5}{2}}x^5\log\left(\frac{ax^2+2\,\sqrt{a}\,x+1}{ax^2-1}\right)-24\,a^2x^4-8\,ax^2-30\,\mathrm{Li}_2(ax^2)+12\,\log\left(-ax^2+1\right)-75\,\mathrm{polylog}(3,ax^2)}{375\,x^5},\,-\frac{24\,\sqrt{-a}\,a^2x^5\arctan\left(\sqrt{-a}\,x\right)+24\,a^2x^4+8\,ax^2+30\,\mathrm{Li}_2(ax^2)-12\,\log\left(-ax^2+1\right)+75\,\mathrm{polylog}(3,ax^2)}{375\,x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/375*(12*a^(5/2)*x^5*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)) - 24*a^2*
x^4 - 8*a*x^2 - 30*dilog(a*x^2) + 12*log(-a*x^2 + 1) - 75*polylog(3, a*x^2)
)/x^5, -1/375*(24*sqrt(-a)*a^2*x^5*arctan(sqrt(-a)*x) + 24*a^2*x^4 + 8*a*x^
2 + 30*dilog(a*x^2) - 12*log(-a*x^2 + 1) + 75*polylog(3, a*x^2))/x^5]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{Li}_3\left(ax^2\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/x**6,x)
```

```
[Out] Integral(polylog(3, a*x**2)/x**6, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/x^6, x)
```

Mupad [**B**]
time = 1.03, size = 70, normalized size = 0.88

$$\frac{4\ln\left(1-a\,x^2\right)}{125\,x^5}-\frac{\mathrm{polylog}(3,a\,x^2)}{5\,x^5}-\frac{8\,a^2\,x^2+\frac{8\,a}{3}}{125\,x^3}-\frac{2\,\mathrm{polylog}(2,a\,x^2)}{25\,x^5}-\frac{a^{5/2}\,\mathrm{atan}\left(\sqrt{a}\ x\,1\mathrm{i}\right)\ 8\mathrm{i}}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^2)/x^6,x)
```

```
[Out] (4*log(1 - a*x^2))/(125*x^5) - (2*polylog(2, a*x^2))/(25*x^5) - polylog(3,
a*x^2)/(5*x^5) - ((8*a)/3 + 8*a^2*x^2)/(125*x^3) - (a^(5/2)*atan(a^(1/2)*x*
1i)*8i)/125
```

3.45 $\int x^2\mathbf{PolyLog}(2, ax^q)\, dx$

Optimal. Leaf size=71

$$\frac{aq^2x^{3+q}\,{}_2F_1\left(1,\frac{3+q}{q};2+\frac{3}{q};ax^q\right)}{9(3+q)}+\frac{1}{9}qx^3\log\left(1-ax^q\right)+\frac{1}{3}x^3\text{PolyLog}(2,ax^q)$$

```
[Out] 1/9*a*q^2*x^(3+q)*hypergeom([1, (3+q)/q],[2+3/q],a*x^q)/(3+q)+1/9*q*x^3*ln(
1-a*x^q)+1/3*x^3*polylog(2,a*x^q)
```

Rubi [A]
time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6726, 2505, 371}

$$\frac{aq^2x^{q+3}\,{}_2F_1\left(1,\frac{q+3}{q};2+\frac{3}{q};ax^q\right)}{9(q+3)}+\frac{1}{3}x^3\text{Li}_2(ax^q)+\frac{1}{9}qx^3\log\left(1-ax^q\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*PolyLog[2, a*x^q],x]
```

```
[Out] (a*q^2*x^(3 + q)*Hypergeometric2F1[1, (3 + q)/q, 2 + 3/q, a*x^q])/(9*(3 + q
)) + (q*x^3*Log[1 - a*x^q])/9 + (x^3*PolyLog[2, a*x^q])/3
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int x^2 \text{Li}_2(ax^q)\,dx = \frac{1}{3}x^3\text{Li}_2(ax^q) + \frac{1}{3}q\int x^2\log(1-ax^q)\,dx$$
$$= \frac{1}{9}qx^3\log(1-ax^q) + \frac{1}{3}x^3\text{Li}_2(ax^q) + \frac{1}{9}(aq^2)\int \frac{x^{2+q}}{1-ax^q}\,dx$$
$$= \frac{aq^2x^{3+q}\,{}_2F_1\left(1,\frac{3+q}{q};2+\frac{3}{q};ax^q\right)}{9(3+q)} + \frac{1}{9}qx^3\log(1-ax^q) + \frac{1}{3}x^3\text{Li}_2(ax^q)$$

Mathematica [A]
time = 0.03, size = 69, normalized size = 0.97

$$\frac{qx^3\left(aqx^q\,{}_2F_1\left(1,\frac{3+q}{q};2+\frac{3}{q};ax^q\right)+(3+q)\log(1-ax^q)\right)}{9(3+q)} + \frac{1}{3}x^3\text{PolyLog}(2,ax^q)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*PolyLog[2, a*x^q],x]
[Out] (q*x^3*(a*q*x^q*Hypergeometric2F1[1, (3 + q)/q, 2 + 3/q, a*x^q] + (3 + q)*L
og[1 - a*x^q]))/(9*(3 + q)) + (x^3*PolyLog[2, a*x^q])/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.15, size = 108, normalized size = 1.52

method	result	size
meijerg	$-\frac{(-a)^{-\frac{3}{q}}\left(-\frac{q^2x^3(-a)^{\frac{3}{q}}\ln(1-a\,x^q)}{9}-\frac{q\,x^3(-a)^{\frac{3}{q}}\left(1+\frac{q}{3}\right)\text{polylog}(2,a\,x^q)}{3+q}-\frac{q^2x^{3+q}a(-a)^{\frac{3}{q}}\Phi\left(a\,x^q,1,\frac{3+q}{q}\right)}{9}\right)}{q}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(2,a*x^q),x,method=_RETURNVERBOSE)
[Out] -(-a)^(-3/q)/q*(-1/9*q^2*x^3*(-a)^(3/q)*ln(1-a*x^q)-q/(3+q)*x^3*(-a)^(3/q)*
(1+1/3*q)*polylog(2,a*x^q)-1/9*q^2*x^(3+q)*a*(-a)^(3/q)*LerchPhi(a*x^q,1,(3
+q)/q))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,a*x^q),x, algorithm="maxima")
```

```
[Out] -1/27*q^2*x^3 + 1/9*q*x^3*log(-a*x^q + 1) + 1/3*x^3*dilog(a*x^q) - q^2*inte
grate(1/9*x^2/(a*x^q - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(x^2*dilog(a*x^q), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{Li}_2(ax^q)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*polylog(2,a*x**q),x)
```

```
[Out] Integral(x**2*polylog(2, a*x**q), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,a*x^q),x, algorithm="giac")
```

```
[Out] integrate(x^2*dilog(a*x^q), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{polylog}(2, a\, x^q)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(2, a*x^q),x)
```

```
[Out] int(x^2*polylog(2, a*x^q), x)
```

3.46 $\int x\textbf{PolyLog}(2, ax^q)\, dx$

Optimal. Leaf size=71

$$\frac{aq^2x^{2+q}\,{}_2F_1\left(1,\frac{2+q}{q};2\left(1+\frac{1}{q}\right);ax^q\right)}{4(2+q)}+\frac{1}{4}qx^2\log\left(1-ax^q\right)+\frac{1}{2}x^2\text{PolyLog}(2,ax^q)$$

```
[Out] 1/4*a*q^2*x^(2+q)*hypergeom([1, (2+q)/q],[2+2/q],a*x^q)/(2+q)+1/4*q*x^2*ln(
1-a*x^q)+1/2*x^2*polylog(2,a*x^q)
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6726, 2505, 371}

$$\frac{aq^2x^{q+2}\,{}_2F_1\left(1,\frac{q+2}{q};2\left(1+\frac{1}{q}\right);ax^q\right)}{4(q+2)}+\frac{1}{2}x^2\text{Li}_2(ax^q)+\frac{1}{4}qx^2\log\left(1-ax^q\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*PolyLog[2, a*x^q],x]
```

```
[Out] (a*q^2*x^(2 + q)*Hypergeometric2F1[1, (2 + q)/q, 2*(1 + q^(-1)), a*x^q])/(4
*(2 + q)) + (q*x^2*Log[1 - a*x^q])/4 + (x^2*PolyLog[2, a*x^q])/2
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int x\text{Li}_2(ax^q)\,dx &= \frac{1}{2}x^2\text{Li}_2(ax^q)+\frac{1}{2}q\int x\log(1-ax^q)\,dx\\ &= \frac{1}{4}qx^2\log(1-ax^q)+\frac{1}{2}x^2\text{Li}_2(ax^q)+\frac{1}{4}(aq^2)\int\frac{x^{1+q}}{1-ax^q}\,dx\\ &= \frac{aq^2x^{2+q}\,{}_2F_1\left(1,\frac{2+q}{q};2\left(1+\frac{1}{q}\right);ax^q\right)}{4(2+q)}+\frac{1}{4}qx^2\log(1-ax^q)+\frac{1}{2}x^2\text{Li}_2(ax^q)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 69, normalized size = 0.97

$$\frac{qx^2\left(aqx^q\,{}_2F_1\left(1,\frac{2+q}{q};2+\frac{2}{q};ax^q\right)+(2+q)\log(1-ax^q)\right)}{4(2+q)}+\frac{1}{2}x^2\text{PolyLog}(2,ax^q)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*PolyLog[2, a*x^q],x]
```

```
[Out] (q*x^2*(a*q*x^q*Hypergeometric2F1[1, (2 + q)/q, 2 + 2/q, a*x^q] + (2 + q)*L
og[1 - a*x^q]))/(4*(2 + q)) + (x^2*PolyLog[2, a*x^q])/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.11, size = 108, normalized size = 1.52

method	result	size
meijerg	$-\frac{(-a)^{-\frac{2}{q}}\left(-\frac{q^2x^2(-a)^{\frac{2}{q}}\ln(1-a\,x^q)}{4}-\frac{q\,x^2(-a)^{\frac{2}{q}}\left(1+\frac{q}{2}\right)\text{polylog}(2,a\,x^q)}{2+q}-\frac{q^2x^{2+q}a(-a)^{\frac{2}{q}}\Phi\left(a\,x^q,1,\frac{2+q}{q}\right)}{4}\right)}{q}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(2,a*x^q),x,method=_RETURNVERBOSE)
```

```
[Out] -(-a)^(-2/q)/q*(-1/4*q^2*x^2*(-a)^(2/q)*ln(1-a*x^q)-q/(2+q)*x^2*(-a)^(2/q)*
(1+1/2*q)*polylog(2,a*x^q)-1/4*q^2*x^(2+q)*a*(-a)^(2/q)*LerchPhi(a*x^q,1,(2
+q)/q))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x^q),x, algorithm="maxima")
```

```
[Out] -1/8*q^2*x^2 + 1/4*q*x^2*log(-a*x^q + 1) + 1/2*x^2*dilog(a*x^q) - q^2*integ
rate(1/4*x/(a*x^q - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(x*dilog(a*x^q), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{Li}_2(ax^q)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x**q),x)
```

```
[Out] Integral(x*polylog(2, a*x**q), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x^q),x, algorithm="giac")
```

```
[Out] integrate(x*dilog(a*x^q), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{polylog}(2, a\,x^q)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(2, a*x^q),x)
```

```
[Out] int(x*polylog(2, a*x^q), x)
```

3.47 $\int \text{PolyLog}(2, ax^q)\, dx$

Optimal. Leaf size=54

$$\frac{aq^2x^{1+q}\,{}_2F_1\left(1,1+\frac{1}{q};2+\frac{1}{q};ax^q\right)}{1+q}+qx\log\left(1-ax^q\right)+x\text{PolyLog}(2,ax^q)$$

```
[Out] a*q^2*x^(1+q)*hypergeom([1, 1+1/q],[2+1/q],a*x^q)/(1+q)+q*x*ln(1-a*x^q)+x*p
olylog(2,a*x^q)
```

Rubi [A]

time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6721, 2498, 371}

$$\frac{aq^2x^{q+1}\,{}_2F_1\left(1,1+\frac{1}{q};2+\frac{1}{q};ax^q\right)}{q+1}+x\text{Li}_2(ax^q)+qx\log\left(1-ax^q\right)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^q],x]
```

```
[Out] (a*q^2*x^(1 + q)*Hypergeometric2F1[1, 1 + q^(-1), 2 + q^(-1), a*x^q])/(1 +
q) + q*x*Log[1 - a*x^q] + x*PolyLog[2, a*x^q]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \text{Li}_2(ax^q)\,dx &= x\text{Li}_2(ax^q) + q\int \log(1-ax^q)\,dx \\
&= qx\log(1-ax^q) + x\text{Li}_2(ax^q) + (aq^2)\int \frac{x^q}{1-ax^q}\,dx \\
&= \frac{aq^2x^{1+q}\,{}_2F_1\left(1,1+\frac{1}{q};2+\frac{1}{q};ax^q\right)}{1+q} + qx\log(1-ax^q) + x\text{Li}_2(ax^q)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 51, normalized size = 0.94

$$
qx\left(\frac{aqx^q\,{}_2F_1\left(1,1+\frac{1}{q};2+\frac{1}{q};ax^q\right)}{1+q} + \log(1-ax^q)\right) + x\text{PolyLog}(2,ax^q)
$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^q],x]
```

```
[Out] q*x*((a*q*x^q*Hypergeometric2F1[1, 1 + q^(-1), 2 + q^(-1), a*x^q])/(1 + q)
+ Log[1 - a*x^q]) + x*PolyLog[2, a*x^q]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.11, size = 88, normalized size = 1.63

method	result	size
meijerg	$-\frac{(-a)^{-\frac{1}{q}}\left(-q^2x(-a)^{\frac{1}{q}}\ln(1-a\,x^q)-qx(-a)^{\frac{1}{q}}\,\text{polylog}(2,a\,x^q)-q^2x^{1+q}a(-a)^{\frac{1}{q}}\,\Phi\left(a\,x^q,1,\frac{1+q}{q}\right)\right)}{q}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^q),x,method=_RETURNVERBOSE)
```

```
[Out] -1/q*(-a)^(-1/q)*(-q^2*x*(-a)^(1/q)*ln(1-a*x^q)-q*x*(-a)^(1/q)*polylog(2,a*
x^q)-q^2*x^(1+q)*a*(-a)^(1/q)*LerchPhi(a*x^q,1,(1+q)/q))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q),x, algorithm="maxima")
```

```
[Out] -q^2*x - q^2*integrate(1/(a*x^q - 1), x) + q*x*log(-a*x^q + 1) + x*dilog(a*
x^q)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q),x, algorithm="fricas")
[Out] integral(dilog(a*x^q), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Li}_2\left(ax^q\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**q),x)
[Out] Integral(polylog(2, a*x**q), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q),x, algorithm="giac")
[Out] integrate(dilog(a*x^q), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{polylog}(2,a\,x^q)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^q),x)
[Out] int(polylog(2, a*x^q), x)
```

3.48 $\int \frac{\text{PolyLog}(2,ax^q)}{x} dx$

Optimal. Leaf size=11

$$\frac{\text{PolyLog}(3, ax^q)}{q}$$

```
[Out] polylog(3,a*x^q)/q
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6724}

$$\frac{\text{Li}_3(ax^q)}{q}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^q]/x,x]
```

```
[Out] PolyLog[3, a*x^q]/q
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\text{Li}_2(ax^q)}{x} dx = \frac{\text{Li}_3(ax^q)}{q}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$\frac{\text{PolyLog}(3, ax^q)}{q}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^q]/x,x]
```

```
[Out] PolyLog[3, a*x^q]/q
```

Maple [A]
time = 0.23, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$\frac{\text{polylog}(3,a\,x^q)}{q}$	12
default	$\frac{\text{polylog}(3,a\,x^q)}{q}$	12
meijerg	$\frac{\text{polylog}(3,a\,x^q)}{q}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^q)/x,x,method=_RETURNVERBOSE)
```

```
[Out] polylog(3,a*x^q)/q
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x,x, algorithm="maxima")
```

```
[Out] -1/6*q^2*log(x)^3 + 1/2*q*log(-a*x^q + 1)*log(x)^2 - q^2*integrate(1/2*log(
x)^2/(a*x*x^q - x), x) + dilog(a*x^q)*log(x)
```

Fricas [A]
time = 0.40, size = 11, normalized size = 1.00

$$\frac{\text{polylog}(3, ax^q)}{q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x,x, algorithm="fricas")
```

```
[Out] polylog(3, a*x^q)/q
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Li}_2\left(ax^q\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**q)/x,x)
```

```
[Out] Integral(polylog(2, a*x**q)/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^q)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{\mathrm{polylog}(2, a\,x^q)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^q)/x,x)
```

```
[Out] int(polylog(2, a*x^q)/x, x)
```

3.49 $\int \frac{\text{PolyLog}(2,ax^q)}{x^2}\,dx$

Optimal. Leaf size=69

$$-\frac{aq^2x^{-1+q}\,{}_2F_1\left(1,-\frac{1-q}{q};2-\frac{1}{q};ax^q\right)}{1-q}+\frac{q\log\left(1-ax^q\right)}{x}-\frac{\text{PolyLog}(2,ax^q)}{x}$$

```
[Out] -a*q^2*x^(-1+q)*hypergeom([1, (-1+q)/q],[2-1/q],a*x^q)/(1-q)+q*ln(1-a*x^q)/
x-polylog(2,a*x^q)/x
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6726, 2505, 371}

$$-\frac{aq^2x^{q-1}\,{}_2F_1\left(1,-\frac{1-q}{q};2-\frac{1}{q};ax^q\right)}{1-q}-\frac{\text{Li}_2(ax^q)}{x}+\frac{q\log\left(1-ax^q\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^q]/x^2,x]
```

```
[Out] -((a*q^2*x^(-1 + q)*Hypergeometric2F1[1, -((1 - q)/q), 2 - q^(-1), a*x^q])/
(1 - q)) + (q*Log[1 - a*x^q])/x - PolyLog[2, a*x^q]/x
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_2(ax^q)}{x^2}\,dx &= -\frac{\text{Li}_2(ax^q)}{x} - q\int \frac{\log(1-ax^q)}{x^2}\,dx\\ &= \frac{q\log(1-ax^q)}{x} - \frac{\text{Li}_2(ax^q)}{x} + (aq^2)\int \frac{x^{-2+q}}{1-ax^q}\,dx\\ &= -\frac{aq^2x^{-1+q}\,{}_2F_1\left(1,-\frac{1-q}{q};2-\frac{1}{q};ax^q\right)}{1-q} + \frac{q\log(1-ax^q)}{x} - \frac{\text{Li}_2(ax^q)}{x}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 0.87

$$\frac{q\left(\frac{aqx^q\,{}_2F_1\left(1,\frac{-1+q}{q};2-\frac{1}{q};ax^q\right)}{-1+q}+\log(1-ax^q)\right)}{x} - \frac{\text{PolyLog}(2,ax^q)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^q]/x^2,x]
```

```
[Out] (q*((a*q*x^q*Hypergeometric2F1[1, (-1 + q)/q, 2 - q^(-1), a*x^q])/(-1 + q)
+ Log[1 - a*x^q]))/x - PolyLog[2, a*x^q]/x
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.12, size = 106, normalized size = 1.54

method	result	size
meijerg	$-\frac{(-a)^{\frac{1}{q}}\left(-\frac{q^2(-a)^{-\frac{1}{q}}\ln(1-a\,x^q)}{x}-\frac{q(-a)^{-\frac{1}{q}}(1-q)\,\text{polylog}(2,a\,x^q)}{(-1+q)x}-q^2x^{-1+q}a(-a)^{-\frac{1}{q}}\Phi\left(a\,x^q,1,\frac{-1+q}{q}\right)\right)}{q}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^q)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-a)^(1/q)/q*(-q^2/x*(-a)^(-1/q)*ln(1-a*x^q)-q/(-1+q)/x*(-a)^(-1/q)*(1-q)*
polylog(2,a*x^q)-q^2*x^(-1+q)*a*(-a)^(-1/q)*LerchPhi(a*x^q,1,(-1+q)/q))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x^2,x, algorithm="maxima")
```

```
[Out] -q^2*integrate(1/(a*x^2*x^q - x^2), x) + (q^2 + q*log(-a*x^q + 1) - dilog(a
*x^q))/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x^2,x, algorithm="fricas")
```

```
[Out] integral(dilog(a*x^q)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_2(ax^q)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**q)/x**2,x)
```

```
[Out] Integral(polylog(2, a*x**q)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x^2,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^q)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(2,a\,x^q)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^q)/x^2,x)
```

```
[Out] int(polylog(2, a*x^q)/x^2, x)
```

3.50 $\int \frac{\text{PolyLog}(2,ax^q)}{x^3}\,dx$

Optimal. Leaf size=78

$$-\frac{aq^2x^{-2+q}\,{}_2F_1\left(1,-\frac{2-q}{q};2\left(1-\frac{1}{q}\right);ax^q\right)}{4(2-q)}+\frac{q\log\left(1-ax^q\right)}{4x^2}-\frac{\text{PolyLog}(2,ax^q)}{2x^2}$$

```
[Out] -1/4*a*q^2*x^(-2+q)*hypergeom([1, (-2+q)/q],[2-2/q],a*x^q)/(2-q)+1/4*q*ln(1
-a*x^q)/x^2-1/2*polylog(2,a*x^q)/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6726, 2505, 371}

$$-\frac{aq^2x^{q-2}\,{}_2F_1\left(1,-\frac{2-q}{q};2\left(1-\frac{1}{q}\right);ax^q\right)}{4(2-q)}-\frac{\text{Li}_2(ax^q)}{2x^2}+\frac{q\log\left(1-ax^q\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^q]/x^3,x]
```

```
[Out] -1/4*(a*q^2*x^(-2 + q)*Hypergeometric2F1[1, -((2 - q)/q), 2*(1 - q^(-1)), a
*x^q])/(2 - q) + (q*Log[1 - a*x^q])/(4*x^2) - PolyLog[2, a*x^q]/(2*x^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\text{Li}_2(ax^q)}{x^3}\,dx = -\frac{\text{Li}_2(ax^q)}{2x^2} - \frac{1}{2}q\int \frac{\log(1-ax^q)}{x^3}\,dx$$
$$= \frac{q\log(1-ax^q)}{4x^2} - \frac{\text{Li}_2(ax^q)}{2x^2} + \frac{1}{4}(aq^2)\int \frac{x^{-3+q}}{1-ax^q}\,dx$$
$$= -\frac{aq^2x^{-2+q}\,{}_2F_1\left(1,-\frac{2-q}{q};2\left(1-\frac{1}{q}\right);ax^q\right)}{4(2-q)} + \frac{q\log(1-ax^q)}{4x^2} - \frac{\text{Li}_2(ax^q)}{2x^2}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.78

$$\frac{q\left(\frac{aqx^q\,{}_2F_1\left(1,\frac{-2+q}{q};2-\frac{2}{q};ax^q\right)}{-2+q}+\log(1-ax^q)\right)-2\text{PolyLog}(2,ax^q)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^q]/x^3,x]
```

```
[Out] (q*((a*q*x^q*Hypergeometric2F1[1, (-2 + q)/q, 2 - 2/q, a*x^q])/(-2 + q) + L
og[1 - a*x^q]) - 2*PolyLog[2, a*x^q])/(4*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.11, size = 108, normalized size = 1.38

method	result	size
meijerg	$-\frac{(-a)^{\frac{2}{q}}\left(-\frac{q^2(-a)^{-\frac{2}{q}}\ln(1-a\,x^q)}{4x^2}-\frac{q(-a)^{-\frac{2}{q}}\left(1-\frac{q}{2}\right)\text{polylog}(2,a\,x^q)}{(-2+q)x^2}-\frac{q^2x^{-2+q}a(-a)^{-\frac{2}{q}}\Phi\left(a\,x^q,1,\frac{-2+q}{q}\right)}{4}\right)}{q}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^q)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(-a)^(2/q)/q*(-1/4*q^2/x^2*(-a)^(-2/q)*ln(1-a*x^q)-q/(-2+q)/x^2*(-a)^(-2/q
)*(1-1/2*q)*polylog(2,a*x^q)-1/4*q^2*x^(-2+q)*a*(-a)^(-2/q)*LerchPhi(a*x^q,
1,(-2+q)/q))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x^3,x, algorithm="maxima")
```

```
[Out] -q^2*integrate(1/4/(a*x^3*x^q - x^3), x) + 1/8*(q^2 + 2*q*log(-a*x^q + 1) -
 4*dilog(a*x^q))/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x^3,x, algorithm="fricas")
```

```
[Out] integral(dilog(a*x^q)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_2\left(ax^q\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**q)/x**3,x)
```

```
[Out] Integral(polylog(2, a*x**q)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x^3,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^q)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(2,a\,x^q)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^q)/x^3,x)
```

```
[Out] int(polylog(2, a*x^q)/x^3, x)
```

3.51 $\int \frac{\text{PolyLog}(2,ax^q)}{x^4}\,dx$

Optimal. Leaf size=76

$$-\frac{aq^2x^{-3+q}\,{}_2F_1\left(1,-\frac{3-q}{q};2-\frac{3}{q};ax^q\right)}{9(3-q)}+\frac{q\log\left(1-ax^q\right)}{9x^3}-\frac{\text{PolyLog}(2,ax^q)}{3x^3}$$

```
[Out] -1/9*a*q^2*x^(-3+q)*hypergeom([1, (-3+q)/q],[2-3/q],a*x^q)/(3-q)+1/9*q*ln(1
-a*x^q)/x^3-1/3*polylog(2,a*x^q)/x^3
```

Rubi [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6726, 2505, 371}

$$-\frac{aq^2x^{q-3}\,{}_2F_1\left(1,-\frac{3-q}{q};2-\frac{3}{q};ax^q\right)}{9(3-q)}-\frac{\text{Li}_2(ax^q)}{3x^3}+\frac{q\log\left(1-ax^q\right)}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^q]/x^4,x]
```

```
[Out] -1/9*(a*q^2*x^(-3 + q)*Hypergeometric2F1[1, -((3 - q)/q), 2 - 3/q, a*x^q])/
(3 - q) + (q*Log[1 - a*x^q])/(9*x^3) - PolyLog[2, a*x^q]/(3*x^3)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax^q)}{x^4}\,dx &= -\frac{\text{Li}_2(ax^q)}{3x^3} - \frac{1}{3}q\int \frac{\log(1-ax^q)}{x^4}\,dx \\
&= \frac{q\log(1-ax^q)}{9x^3} - \frac{\text{Li}_2(ax^q)}{3x^3} + \frac{1}{9}(aq^2)\int \frac{x^{-4+q}}{1-ax^q}\,dx \\
&= -\frac{aq^2x^{-3+q}\,{}_2F_1\left(1,-\frac{3-q}{q};2-\frac{3}{q};ax^q\right)}{9(3-q)} + \frac{q\log(1-ax^q)}{9x^3} - \frac{\text{Li}_2(ax^q)}{3x^3}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.80

$$\frac{q\left(\frac{aqx^q\,{}_2F_1\left(1,\frac{-3+q}{q};2-\frac{3}{q};ax^q\right)}{-3+q} + \log(1-ax^q)\right) - 3\text{PolyLog}(2,ax^q)}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^q]/x^4,x]
```

```
[Out] (q*((a*q*x^q*Hypergeometric2F1[1, (-3 + q)/q, 2 - 3/q, a*x^q])/(-3 + q) + L
og[1 - a*x^q]) - 3*PolyLog[2, a*x^q])/(9*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.12, size = 108, normalized size = 1.42

method	result	size
meijerg	$-\frac{(-a)^{\frac{3}{q}}\left(-\frac{q^2(-a)^{-\frac{3}{q}}\ln(1-a\,x^q)}{9x^3}-\frac{q(-a)^{-\frac{3}{q}}\left(1-\frac{q}{3}\right)\text{polylog}(2,a\,x^q)}{(-3+q)x^3}-\frac{q^2x^{-3+q}a(-a)^{-\frac{3}{q}}\Phi\left(a\,x^q,1,\frac{-3+q}{q}\right)}{9}\right)}{q}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^q)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -(-a)^(3/q)/q*(-1/9*q^2/x^3*(-a)^(-3/q)*ln(1-a*x^q)-q/(-3+q)/x^3*(-a)^(-3/q
)*(1-1/3*q)*polylog(2,a*x^q)-1/9*q^2*x^(-3+q)*a*(-a)^(-3/q)*LerchPhi(a*x^q,
1,(-3+q)/q))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x^4,x, algorithm="maxima")
```

```
[Out] -q^2*integrate(1/9/(a*x^4*x^q - x^4), x) + 1/27*(q^2 + 3*q*log(-a*x^q + 1)
- 9*dilog(a*x^q))/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x^4,x, algorithm="fricas")
```

```
[Out] integral(dilog(a*x^q)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_2(ax^q)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**q)/x**4,x)
```

```
[Out] Integral(polylog(2, a*x**q)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x^4,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^q)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(2, a\,x^q)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^q)/x^4,x)
```

```
[Out] int(polylog(2, a*x^q)/x^4, x)
```

3.52 $\int x^2\text{PolyLog}(3, ax^q)\,dx$

Optimal. Leaf size=88

$$-\frac{aq^3x^{3+q}\,{}_2F_1\left(1,\frac{3+q}{q};2+\frac{3}{q};ax^q\right)}{27(3+q)}-\frac{1}{27}q^2x^3\log\left(1-ax^q\right)-\frac{1}{9}qx^3\text{PolyLog}(2,ax^q)+\frac{1}{3}x^3\text{PolyLog}(3,ax^q)$$

```
[Out] -1/27*a*q^3*x^(3+q)*hypergeom([1, (3+q)/q],[2+3/q],a*x^q)/(3+q)-1/27*q^2*x^
3*ln(1-a*x^q)-1/9*q*x^3*polylog(2,a*x^q)+1/3*x^3*polylog(3,a*x^q)
```

Rubi [A]
time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6726, 2505, 371}

$$-\frac{aq^3x^{q+3}\,{}_2F_1\left(1,\frac{q+3}{q};2+\frac{3}{q};ax^q\right)}{27(q+3)}-\frac{1}{9}qx^3\text{Li}_2(ax^q)+\frac{1}{3}x^3\text{Li}_3(ax^q)-\frac{1}{27}q^2x^3\log\left(1-ax^q\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*PolyLog[3, a*x^q],x]
```

```
[Out] -1/27*(a*q^3*x^(3 + q)*Hypergeometric2F1[1, (3 + q)/q, 2 + 3/q, a*x^q])/(3
+ q) - (q^2*x^3*Log[1 - a*x^q])/27 - (q*x^3*PolyLog[2, a*x^q])/9 + (x^3*Pol
yLog[3, a*x^q])/3
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\text{Li}_3(ax^q)\,dx &= \frac{1}{3}x^3\text{Li}_3(ax^q)-\frac{1}{3}q\int x^2\text{Li}_2(ax^q)\,dx\\ &= -\frac{1}{9}qx^3\text{Li}_2(ax^q)+\frac{1}{3}x^3\text{Li}_3(ax^q)-\frac{1}{9}q^2\int x^2\log(1-ax^q)\,dx\\ &= -\frac{1}{27}q^2x^3\log(1-ax^q)-\frac{1}{9}qx^3\text{Li}_2(ax^q)+\frac{1}{3}x^3\text{Li}_3(ax^q)-\frac{1}{27}(aq^3)\int\frac{x^{2+q}}{1-ax^q}\,dx\\ &= -\frac{aq^3x^{3+q}\,{}_2F_1\left(1,\frac{3+q}{q};2+\frac{3}{q};ax^q\right)}{27(3+q)}-\frac{1}{27}q^2x^3\log(1-ax^q)-\frac{1}{9}qx^3\text{Li}_2(ax^q)+\frac{1}{3}x^3\text{Li}_3(ax^q\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.01, size = 41, normalized size = 0.47

$$-\frac{x^3G_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,\frac{-3+q}{q}\\1,0,0,0,-\frac{3}{q}\end{array}\right)}{q}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*PolyLog[3, a*x^q],x]
```

```
[Out] -((x^3*MeijerG[{{1, 1, 1, 1, (-3 + q)/q}, {}}, {{1}, {0, 0, 0, -3/q}}, -(a*
x^q)])/q)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.20, size = 132, normalized size = 1.50

method	result	s
meijerg	$-\frac{(-a)^{-\frac{3}{q}}\left(\frac{q^3x^3(-a)^{\frac{3}{q}}\ln(1-a\,x^q)}{27}+\frac{q^2x^3(-a)^{\frac{3}{q}}\text{polylog}(2,a\,x^q)}{9}-\frac{q\,x^3(-a)^{\frac{3}{q}}\left(1+\frac{q}{3}\right)\text{polylog}(3,a\,x^q)}{3+q}+\frac{q^3x^{3+q}a(-a)^{\frac{3}{q}}\Phi\left(a\,x^q,1,\frac{3+q}{q}\right)}{27}\right)}{q}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(3,a*x^q),x,method=_RETURNVERBOSE)
```

```
[Out] -(-a)^(-3/q)/q*(1/27*q^3*x^3*(-a)^(3/q)*ln(1-a*x^q)+1/9*q^2*x^3*(-a)^(3/q)*
polylog(2,a*x^q)-q/(3+q)*x^3*(-a)^(3/q)*(1+1/3*q)*polylog(3,a*x^q)+1/27*q^3
*x^(3+q)*a*(-a)^(3/q)*LerchPhi(a*x^q,1,(3+q)/q))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,a*x^q),x, algorithm="maxima")
[Out] 1/81*q^3*x^3 - 1/27*q^2*x^3*log(-a*x^q + 1) - 1/9*q*x^3*dilog(a*x^q) + q^3*
integrate(1/27*x^2/(a*x^q - 1), x) + 1/3*x^3*polylog(3, a*x^q)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,a*x^q),x, algorithm="fricas")
[Out] integral(x^2*polylog(3, a*x^q), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{Li}_3(ax^q)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*polylog(3,a*x**q),x)
[Out] Integral(x**2*polylog(3, a*x**q), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,a*x^q),x, algorithm="giac")
[Out] integrate(x^2*polylog(3, a*x^q), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{polylog}(3, a\,x^q)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(3, a*x^q),x)
[Out] int(x^2*polylog(3, a*x^q), x)
```

3.53 $\int x\mathbf{PolyLog}(3, ax^q)\, dx$

Optimal. Leaf size=88

$$-\frac{aq^3x^{2+q}\,{}_2F_1\left(1,\frac{2+q}{q};2\left(1+\frac{1}{q}\right);ax^q\right)}{8(2+q)}-\frac{1}{8}q^2x^2\log\left(1-ax^q\right)-\frac{1}{4}qx^2\text{PolyLog}(2,ax^q)+\frac{1}{2}x^2\text{PolyLog}(3,ax^q)$$

```
[Out] -1/8*a*q^3*x^(2+q)*hypergeom([1, (2+q)/q],[2+2/q],a*x^q)/(2+q)-1/8*q^2*x^2*
ln(1-a*x^q)-1/4*q*x^2*polylog(2,a*x^q)+1/2*x^2*polylog(3,a*x^q)
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6726, 2505, 371}

$$-\frac{aq^3x^{q+2}\,{}_2F_1\left(1,\frac{q+2}{q};2\left(1+\frac{1}{q}\right);ax^q\right)}{8(q+2)}-\frac{1}{4}qx^2\text{Li}_2(ax^q)+\frac{1}{2}x^2\text{Li}_3(ax^q)-\frac{1}{8}q^2x^2\log\left(1-ax^q\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*PolyLog[3, a*x^q],x]
```

```
[Out] -1/8*(a*q^3*x^(2 + q)*Hypergeometric2F1[1, (2 + q)/q, 2*(1 + q^(-1)), a*x^q
])/(2 + q) - (q^2*x^2*Log[1 - a*x^q])/8 - (q*x^2*PolyLog[2, a*x^q])/4 + (x^
2*PolyLog[3, a*x^q])/2
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int x\text{Li}_3(ax^q)\,dx = \frac{1}{2}x^2\text{Li}_3(ax^q) - \frac{1}{2}q\int x\text{Li}_2(ax^q)\,dx$$
$$= -\frac{1}{4}qx^2\text{Li}_2(ax^q) + \frac{1}{2}x^2\text{Li}_3(ax^q) - \frac{1}{4}q^2\int x\log(1-ax^q)\,dx$$
$$= -\frac{1}{8}q^2x^2\log(1-ax^q) - \frac{1}{4}qx^2\text{Li}_2(ax^q) + \frac{1}{2}x^2\text{Li}_3(ax^q) - \frac{1}{8}(aq^3)\int \frac{x^{1+q}}{1-ax^q}\,dx$$
$$= -\frac{aq^3x^{2+q}\,{}_2F_1\left(1,\frac{2+q}{q};2\left(1+\frac{1}{q}\right);ax^q\right)}{8(2+q)} - \frac{1}{8}q^2x^2\log(1-ax^q) - \frac{1}{4}qx^2\text{Li}_2(ax^q) + \frac{1}{2}x^2\text{Li}_3(ax$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.01, size = 41, normalized size = 0.47

$$-\frac{x^2 G_{5,5}^{1,5}\left(-ax^q\middle|\begin{matrix}1,1,1,1,\frac{-2+q}{q}\\1,0,0,0,-\frac{2}{q}\end{matrix}\right)}{q}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*PolyLog[3, a*x^q],x]
```

```
[Out] -((x^2*MeijerG[{{1, 1, 1, 1, (-2 + q)/q}, {}}, {{1}, {0, 0, 0, -2/q}}, -(a*
x^q)])/q)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.20, size = 132, normalized size = 1.50

method	result	si
meijerg	$-\frac{(-a)^{-\frac{2}{q}}\left(\frac{q^3x^2(-a)^{\frac{2}{q}}\ln(1-a\,x^q)}{8}+\frac{q^2x^2(-a)^{\frac{2}{q}}\operatorname{polylog}(2,a\,x^q)}{4}-\frac{q\,x^2(-a)^{\frac{2}{q}}\left(1+\frac{q}{2}\right)\operatorname{polylog}(3,a\,x^q)}{2+q}+\frac{q^3x^{2+q}a(-a)^{\frac{2}{q}}\Phi\left(a\,x^q,1,\frac{2+q}{q}\right)}{8}\right)}{q}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(3,a*x^q),x,method=_RETURNVERBOSE)
```

```
[Out] -(-a)^(-2/q)/q*(1/8*q^3*x^2*(-a)^(2/q)*ln(1-a*x^q)+1/4*q^2*x^2*(-a)^(2/q)*p
olylog(2,a*x^q)-q/(2+q)*x^2*(-a)^(2/q)*(1+1/2*q)*polylog(3,a*x^q)+1/8*q^3*x
^(2+q)*a*(-a)^(2/q)*LerchPhi(a*x^q,1,(2+q)/q))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x^q),x, algorithm="maxima")
[Out] 1/16*q^3*x^2 - 1/8*q^2*x^2*log(-a*x^q + 1) - 1/4*q*x^2*dilog(a*x^q) + q^3*i
ntegrate(1/8*x/(a*x^q - 1), x) + 1/2*x^2*polylog(3, a*x^q)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x^q),x, algorithm="fricas")
[Out] integral(x*polylog(3, a*x^q), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{Li}_3(ax^q)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x**q),x)
[Out] Integral(x*polylog(3, a*x**q), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x^q),x, algorithm="giac")
[Out] integrate(x*polylog(3, a*x^q), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{polylog}(3, a\,x^q)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(3, a*x^q),x)
[Out] int(x*polylog(3, a*x^q), x)
```

3.54 $\int \text{PolyLog}(3, ax^q)\, dx$

Optimal. Leaf size=69

$$-\frac{aq^3x^{1+q}\,{}_2F_1\left(1,1+\frac{1}{q};2+\frac{1}{q};ax^q\right)}{1+q}-q^2x\log\left(1-ax^q\right)-qx\text{PolyLog}(2,ax^q)+x\text{PolyLog}(3,ax^q)$$

```
[Out] -a*q^3*x^(1+q)*hypergeom([1, 1+1/q],[2+1/q],a*x^q)/(1+q)-q^2*x*ln(1-a*x^q)-
q*x*polylog(2,a*x^q)+x*polylog(3,a*x^q)
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6721, 2498, 371}

$$-\frac{aq^3x^{q+1}\,{}_2F_1\left(1,1+\frac{1}{q};2+\frac{1}{q};ax^q\right)}{q+1}-qx\text{Li}_2(ax^q)+x\text{Li}_3(ax^q)-q^2x\log\left(1-ax^q\right)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^q],x]
```

```
[Out] -((a*q^3*x^(1 + q)*Hypergeometric2F1[1, 1 + q^(-1), 2 + q^(-1), a*x^q])/(1
+ q)) - q^2*x*Log[1 - a*x^q] - q*x*PolyLog[2, a*x^q] + x*PolyLog[3, a*x^q]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \mathrm{Li}_3(ax^q)\,dx &= x\mathrm{Li}_3(ax^q) - q\int \mathrm{Li}_2(ax^q)\,dx \\
&= -qx\mathrm{Li}_2(ax^q) + x\mathrm{Li}_3(ax^q) - q^2\int \log(1-ax^q)\,dx \\
&= -q^2x\log(1-ax^q) - qx\mathrm{Li}_2(ax^q) + x\mathrm{Li}_3(ax^q) - (aq^3)\int \frac{x^q}{1-ax^q}\,dx \\
&= -\frac{aq^3x^{1+q}\,{}_2F_1\left(1,1+\frac{1}{q};2+\frac{1}{q};ax^q\right)}{1+q} - q^2x\log(1-ax^q) - qx\mathrm{Li}_2(ax^q) + x\mathrm{Li}_3(ax^q)
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.01, size = 39, normalized size = 0.57

$$
-\frac{xG_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c} 1,1,1,1,\frac{-1+q}{q} \\ 1,0,0,0,-\frac{1}{q} \end{array}\right)}{q}
$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^q],x]
```

```
[Out] -((x*MeijerG[{{1, 1, 1, 1, (-1 + q)/q}, {}}, {{1}, {0, 0, 0, -q^(-1)}}, -(a
*x^q)])/q)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.19, size = 105, normalized size = 1.52

method	result	size
meijerg	$-\frac{(-a)^{-\frac{1}{q}}\left(q^3x(-a)^{\frac{1}{q}}\ln(1-a\,x^q)+q^2x(-a)^{\frac{1}{q}}\,\mathrm{polylog}(2,a\,x^q)-qx(-a)^{\frac{1}{q}}\,\mathrm{polylog}(3,a\,x^q)+q^3x^{1+q}a(-a)^{\frac{1}{q}}\,\Phi\left(a\,x^q,1,\frac{1+q}{q}\right)\right)}{q}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^q),x,method=_RETURNVERBOSE)
```

```
[Out] -1/q*(-a)^(-1/q)*(q^3*x*(-a)^(1/q)*ln(1-a*x^q)+q^2*x*(-a)^(1/q)*polylog(2,a
*x^q)-q*x*(-a)^(1/q)*polylog(3,a*x^q)+q^3*x^(1+q)*a*(-a)^(1/q)*LerchPhi(a*x
^q,1,(1+q)/q))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q),x, algorithm="maxima")
```

```
[Out] q^3*x + q^3*integrate(1/(a*x^q - 1), x) - q^2*x*log(-a*x^q + 1) - q*x*dilog
(a*x^q) + x*polylog(3, a*x^q)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(polylog(3, a*x^q), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Li}_3\left(ax^q\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**q),x)
```

```
[Out] Integral(polylog(3, a*x**q), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^q), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{polylog}(3,a\,x^q)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^q),x)
```

```
[Out] int(polylog(3, a*x^q), x)
```

3.55 $\int \frac{\text{PolyLog}(3,ax^q)}{x}\,dx$

Optimal. Leaf size=11

$$\frac{\text{PolyLog}(4,ax^q)}{q}$$

```
[Out] polylog(4,a*x^q)/q
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6724}

$$\frac{\text{Li}_4(ax^q)}{q}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^q]/x,x]
```

```
[Out] PolyLog[4, a*x^q]/q
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\text{Li}_3(ax^q)}{x}\,dx = \frac{\text{Li}_4(ax^q)}{q}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$\frac{\text{PolyLog}(4,ax^q)}{q}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^q]/x,x]
```

```
[Out] PolyLog[4, a*x^q]/q
```

Maple [A]
time = 0.23, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$\frac{\operatorname{polylog}(4,a\,x^{q})}{q}$	12
default	$\frac{\operatorname{polylog}(4,a\,x^{q})}{q}$	12
meijerg	$\frac{\operatorname{polylog}(4,a\,x^{q})}{q}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^q)/x,x,method=_RETURNVERBOSE)
```

```
[Out] polylog(4,a*x^q)/q
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x,x, algorithm="maxima")
```

```
[Out] 1/24*q^3*log(x)^4 - 1/6*q^2*log(-a*x^q + 1)*log(x)^3 + q^3*integrate(1/6*lo
g(x)^3/(a*x*x^q - x), x) - 1/2*q*dilog(a*x^q)*log(x)^2 + log(x)*polylog(3,
a*x^q)
```

Fricas [**A**]
time = 0.37, size = 11, normalized size = 1.00

$$\frac{\operatorname{polylog}(4, ax^q)}{q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x,x, algorithm="fricas")
```

```
[Out] polylog(4, a*x^q)/q
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_3(ax^q)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**q)/x,x)
```

```
[Out] Integral(polylog(3, a*x**q)/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x,x, algorithm="giac")
[Out] integrate(polylog(3, a*x^q)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{\mathrm{polylog}(3, a\,x^q)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^q)/x,x)
[Out] int(polylog(3, a*x^q)/x, x)
```

3.56 $\int \frac{\text{PolyLog}(3,ax^q)}{x^2} dx$

Optimal. Leaf size=84

$$-\frac{aq^3x^{-1+q}\,{}_2F_1\left(1,-\frac{1-q}{q};2-\frac{1}{q};ax^q\right)}{1-q}+\frac{q^2\log\left(1-ax^q\right)}{x}-\frac{q\text{PolyLog}(2,ax^q)}{x}-\frac{\text{PolyLog}(3,ax^q)}{x}$$

```
[Out] -a*q^3*x^(-1+q)*hypergeom([1, (-1+q)/q],[2-1/q],a*x^q)/(1-q)+q^2*ln(1-a*x^q
)/x-q*polylog(2,a*x^q)/x-polylog(3,a*x^q)/x
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6726, 2505, 371}

$$-\frac{aq^3x^{q-1}\,{}_2F_1\left(1,-\frac{1-q}{q};2-\frac{1}{q};ax^q\right)}{1-q}-\frac{q\text{Li}_2(ax^q)}{x}-\frac{\text{Li}_3(ax^q)}{x}+\frac{q^2\log\left(1-ax^q\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^q]/x^2,x]
```

```
[Out] -((a*q^3*x^(-1 + q)*Hypergeometric2F1[1, -((1 - q)/q), 2 - q^(-1), a*x^q])/
(1 - q)) + (q^2*Log[1 - a*x^q])/x - (q*PolyLog[2, a*x^q])/x - PolyLog[3, a*
x^q]/x
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax^q)}{x^2}\,dx &= -\frac{\text{Li}_3(ax^q)}{x} + q\int \frac{\text{Li}_2(ax^q)}{x^2}\,dx \\
&= -\frac{q\text{Li}_2(ax^q)}{x} - \frac{\text{Li}_3(ax^q)}{x} - q^2\int \frac{\log(1-ax^q)}{x^2}\,dx \\
&= \frac{q^2\log(1-ax^q)}{x} - \frac{q\text{Li}_2(ax^q)}{x} - \frac{\text{Li}_3(ax^q)}{x} + (aq^3)\int \frac{x^{-2+q}}{1-ax^q}\,dx \\
&= -\frac{aq^3x^{-1+q}\,{}_2F_1\left(1,-\frac{1-q}{q};2-\frac{1}{q};ax^q\right)}{1-q} + \frac{q^2\log(1-ax^q)}{x} - \frac{q\text{Li}_2(ax^q)}{x} - \frac{\text{Li}_3(ax^q)}{x}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.01, size = 37, normalized size = 0.44

$$-\frac{G_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,1+\frac{1}{q}\\1,0,0,0,\frac{1}{q}\end{array}\right)}{qx}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^q]/x^2,x]
```

```
[Out] -(MeijerG[{{1, 1, 1, 1, 1 + q^(-1)}, {}}, {{1}, {0, 0, 0, q^(-1)}}, -(a*x^q
)]/(q*x))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.21, size = 129, normalized size = 1.54

method	result
meijerg	$-\frac{(-a)^{\frac{1}{q}}\left(-\frac{q^3(-a)^{-\frac{1}{q}}\ln(1-a\,x^q)}{x}+\frac{q^2(-a)^{-\frac{1}{q}}\,\text{polylog}(2,a\,x^q)}{x}-\frac{q(-a)^{-\frac{1}{q}}(1-q)\,\text{polylog}(3,a\,x^q)}{(-1+q)x}-q^3x^{-1+q}a(-a)^{-\frac{1}{q}}\Phi\left(a\,x^q,1,\frac{-1+q}{q}\right)\right)}{q}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^q)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-a)^(1/q)/q*(-q^3/x*(-a)^(-1/q)*ln(1-a*x^q)+q^2/x*(-a)^(-1/q)*polylog(2,a
*x^q)-q/(-1+q)/x*(-a)^(-1/q)*(1-q)*polylog(3,a*x^q)-q^3*x^(-1+q)*a*(-a)^(-1
/q)*LerchPhi(a*x^q,1,(-1+q)/q))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x^2,x, algorithm="maxima")
[Out] -q^3*integrate(1/(a*x^2*x^q - x^2), x) + (q^3 + q^2*log(-a*x^q + 1) - q*dil
og(a*x^q) - polylog(3, a*x^q))/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x^2,x, algorithm="fricas")
[Out] integral(polylog(3, a*x^q)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_3(ax^q)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**q)/x**2,x)
[Out] Integral(polylog(3, a*x**q)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x^2,x, algorithm="giac")
[Out] integrate(polylog(3, a*x^q)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(3,a\,x^q)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^q)/x^2,x)
[Out] int(polylog(3, a*x^q)/x^2, x)
```

3.57 $\int \frac{\text{PolyLog}(3,ax^q)}{x^3}\,dx$

Optimal. Leaf size=95

$$-\frac{aq^3x^{-2+q}\,{}_2F_1\left(1,-\frac{2-q}{q};2\left(1-\frac{1}{q}\right);ax^q\right)}{8(2-q)}+\frac{q^2\log\left(1-ax^q\right)}{8x^2}-\frac{q\text{PolyLog}(2,ax^q)}{4x^2}-\frac{\text{PolyLog}(3,ax^q)}{2x^2}$$

```
[Out] -1/8*a*q^3*x^(-2+q)*hypergeom([1, (-2+q)/q],[2-2/q],a*x^q)/(2-q)+1/8*q^2*ln
(1-a*x^q)/x^2-1/4*q*polylog(2,a*x^q)/x^2-1/2*polylog(3,a*x^q)/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6726, 2505, 371}

$$-\frac{aq^3x^{q-2}\,{}_2F_1\left(1,-\frac{2-q}{q};2\left(1-\frac{1}{q}\right);ax^q\right)}{8(2-q)}-\frac{q\text{Li}_2(ax^q)}{4x^2}-\frac{\text{Li}_3(ax^q)}{2x^2}+\frac{q^2\log\left(1-ax^q\right)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^q]/x^3,x]
```

```
[Out] -1/8*(a*q^3*x^(-2 + q)*Hypergeometric2F1[1, -((2 - q)/q), 2*(1 - q^(-1)), a
*x^q])/(2 - q) + (q^2*Log[1 - a*x^q])/(8*x^2) - (q*PolyLog[2, a*x^q])/(4*x^
2) - PolyLog[3, a*x^q]/(2*x^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
```

```
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax^q)}{x^3}\,dx &= -\frac{\text{Li}_3(ax^q)}{2x^2} + \frac{1}{2}q\int \frac{\text{Li}_2(ax^q)}{x^3}\,dx \\
&= -\frac{q\text{Li}_2(ax^q)}{4x^2} - \frac{\text{Li}_3(ax^q)}{2x^2} - \frac{1}{4}q^2\int \frac{\log(1-ax^q)}{x^3}\,dx \\
&= \frac{q^2\log(1-ax^q)}{8x^2} - \frac{q\text{Li}_2(ax^q)}{4x^2} - \frac{\text{Li}_3(ax^q)}{2x^2} + \frac{1}{8}(aq^3)\int \frac{x^{-3+q}}{1-ax^q}\,dx \\
&= -\frac{aq^3x^{-2+q}\,{}_2F_1\left(1,-\frac{2-q}{q};2\left(1-\frac{1}{q}\right);ax^q\right)}{8(2-q)} + \frac{q^2\log(1-ax^q)}{8x^2} - \frac{q\text{Li}_2(ax^q)}{4x^2} - \frac{\text{Li}_3(ax^q)}{2x^2}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.01, size = 41, normalized size = 0.43

$$-\frac{G_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,\frac{2+q}{q}\\1,0,0,0,\frac{2}{q}\end{array}\right)}{qx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^q]/x^3,x]
[Out] -(MeijerG[{{1, 1, 1, 1, (2 + q)/q}, {}}, {{1}, {0, 0, 0, 2/q}}, -(a*x^q)]/(
q*x^2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.21, size = 132, normalized size = 1.39

method	result
meijerg	$-\frac{(-a)^{\frac{2}{q}}\left(-\frac{q^3(-a)^{-\frac{2}{q}}\ln(1-a\,x^q)}{8x^2}+\frac{q^2(-a)^{-\frac{2}{q}}\,\text{polylog}(2,a\,x^q)}{4x^2}-\frac{q(-a)^{-\frac{2}{q}}\left(1-\frac{q}{2}\right)\text{polylog}(3,a\,x^q)}{(-2+q)x^2}-\frac{q^3x^{-2+q}a(-a)^{-\frac{2}{q}}\Phi\left(a\,x^q,1,\frac{-2+q}{q}\right)}{8}\right)}{q}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^q)/x^3,x,method=_RETURNVERBOSE)
[Out] -(-a)^(2/q)/q*(-1/8*q^3/x^2*(-a)^(-2/q)*ln(1-a*x^q)+1/4*q^2/x^2*(-a)^(-2/q)
*polylog(2,a*x^q)-q/(-2+q)/x^2*(-a)^(-2/q)*(1-1/2*q)*polylog(3,a*x^q)-1/8*q
^3*x^(-2+q)*a*(-a)^(-2/q)*LerchPhi(a*x^q,1,(-2+q)/q))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x^3,x, algorithm="maxima")
[Out] -q^3*integrate(1/8/(a*x^3*x^q - x^3), x) + 1/16*(q^3 + 2*q^2*log(-a*x^q + 1
) - 4*q*dilog(a*x^q) - 8*polylog(3, a*x^q))/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x^3,x, algorithm="fricas")
[Out] integral(polylog(3, a*x^q)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_3(ax^q)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**q)/x**3,x)
[Out] Integral(polylog(3, a*x**q)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x^3,x, algorithm="giac")
[Out] integrate(polylog(3, a*x^q)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(3,a\,x^q)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^q)/x^3,x)
[Out] int(polylog(3, a*x^q)/x^3, x)
```

3.58 $\int \frac{\text{PolyLog}(3,ax^q)}{x^4}\,dx$

Optimal. Leaf size=93

$$-\frac{aq^3x^{-3+q}\,{}_2F_1\left(1,-\frac{3-q}{q};2-\frac{3}{q};ax^q\right)}{27(3-q)}+\frac{q^2\log\left(1-ax^q\right)}{27x^3}-\frac{q\text{PolyLog}(2,ax^q)}{9x^3}-\frac{\text{PolyLog}(3,ax^q)}{3x^3}$$

```
[Out] -1/27*a*q^3*x^(-3+q)*hypergeom([1, (-3+q)/q],[2-3/q],a*x^q)/(3-q)+1/27*q^2*
ln(1-a*x^q)/x^3-1/9*q*polylog(2,a*x^q)/x^3-1/3*polylog(3,a*x^q)/x^3
```

Rubi [A]
time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6726, 2505, 371}

$$-\frac{aq^3x^{q-3}\,{}_2F_1\left(1,-\frac{3-q}{q};2-\frac{3}{q};ax^q\right)}{27(3-q)}-\frac{q\text{Li}_2(ax^q)}{9x^3}-\frac{\text{Li}_3(ax^q)}{3x^3}+\frac{q^2\log\left(1-ax^q\right)}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^q]/x^4,x]
```

```
[Out] -1/27*(a*q^3*x^(-3 + q)*Hypergeometric2F1[1, -((3 - q)/q), 2 - 3/q, a*x^q])
/(3 - q) + (q^2*Log[1 - a*x^q])/(27*x^3) - (q*PolyLog[2, a*x^q])/(9*x^3) -
PolyLog[3, a*x^q]/(3*x^3)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
```

```
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax^q)}{x^4}\,dx &= -\frac{\text{Li}_3(ax^q)}{3x^3}+\frac{1}{3}q\int \frac{\text{Li}_2(ax^q)}{x^4}\,dx\\
&= -\frac{q\text{Li}_2(ax^q)}{9x^3}-\frac{\text{Li}_3(ax^q)}{3x^3}-\frac{1}{9}q^2\int \frac{\log\left(1-ax^q\right)}{x^4}\,dx\\
&= \frac{q^2\log\left(1-ax^q\right)}{27x^3}-\frac{q\text{Li}_2(ax^q)}{9x^3}-\frac{\text{Li}_3(ax^q)}{3x^3}+\frac{1}{27}\left(aq^3\right)\int \frac{x^{-4+q}}{1-ax^q}\,dx\\
&= -\frac{aq^3x^{-3+q}\,{}_2F_1\left(1,-\frac{3-q}{q};2-\frac{3}{q};ax^q\right)}{27(3-q)}+\frac{q^2\log\left(1-ax^q\right)}{27x^3}-\frac{q\text{Li}_2(ax^q)}{9x^3}-\frac{\text{Li}_3(ax^q)}{3x^3}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.01, size = 41, normalized size = 0.44

$$-\frac{G_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,\frac{3+q}{q}\\1,0,0,0,\frac{3}{q}\end{array}\right)}{qx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^q]/x^4,x]
[Out] -(MeijerG[{{1, 1, 1, 1, (3 + q)/q}, {}}, {{1}, {0, 0, 0, 3/q}}, -(a*x^q)]/(
q*x^3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.21, size = 132, normalized size = 1.42

method	result
meijerg	$-\frac{(-a)^{\frac{3}{q}}\left(-\frac{q^{3}(-a)^{-\frac{3}{q}}\ln(1-a\,x^{q})}{27x^{3}}+\frac{q^{2}(-a)^{-\frac{3}{q}}\operatorname{polylog}(2,a\,x^{q})}{9x^{3}}-\frac{q(-a)^{-\frac{3}{q}}\left(1-\frac{q}{3}\right)\operatorname{polylog}(3,a\,x^{q})}{(-3+q)x^{3}}-\frac{q^{3}x^{-3+q}a(-a)^{-\frac{3}{q}}\Phi\left(a\,x^{q},1,\frac{-3+q}{q}\right)}{27}\right)}{q}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^q)/x^4,x,method=_RETURNVERBOSE)
[Out] -(-a)^(3/q)/q*(-1/27*q^3/x^3*(-a)^(-3/q)*ln(1-a*x^q)+1/9*q^2/x^3*(-a)^(-3/q
)*polylog(2,a*x^q)-q/(-3+q)/x^3*(-a)^(-3/q)*(1-1/3*q)*polylog(3,a*x^q)-1/27
*q^3*x^(-3+q)*a*(-a)^(-3/q)*LerchPhi(a*x^q,1,(-3+q)/q))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x^4,x, algorithm="maxima")
[Out] -q^3*integrate(1/27/(a*x^4*x^q - x^4), x) + 1/81*(q^3 + 3*q^2*log(-a*x^q +
1) - 9*q*dilog(a*x^q) - 27*polylog(3, a*x^q))/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x^4,x, algorithm="fricas")
[Out] integral(polylog(3, a*x^q)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_3\left(ax^q\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**q)/x**4,x)
[Out] Integral(polylog(3, a*x**q)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x^4,x, algorithm="giac")
[Out] integrate(polylog(3, a*x^q)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(3,a\,x^q)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^q)/x^4,x)
[Out] int(polylog(3, a*x^q)/x^4, x)
```

3.59 $\int (dx)^{3/2}\text{PolyLog}(2,ax)\,dx$

Optimal. Leaf size=117

$$-\frac{8d\sqrt{dx}}{25a^2}-\frac{8(dx)^{3/2}}{75a}-\frac{8(dx)^{5/2}}{125d}+\frac{8d^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/2}}+\frac{4(dx)^{5/2}\log(1-ax)}{25d}+\frac{2(dx)^{5/2}\text{PolyLog}(2,ax}{5d}$$

```
[Out] -8/75*(d*x)^(3/2)/a-8/125*(d*x)^(5/2)/d+8/25*d^(3/2)*arctanh(a^(1/2)*(d*x)^
(1/2)/d^(1/2))/a^(5/2)+4/25*(d*x)^(5/2)*ln(-a*x+1)/d+2/5*(d*x)^(5/2)*polylo
g(2,a*x)/d-8/25*d*(d*x)^(1/2)/a^2
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6726, 2442, 52, 65, 212}

$$\frac{8d^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/2}}-\frac{8d\sqrt{dx}}{25a^2}+\frac{2(dx)^{5/2}\text{Li}_2(ax)}{5d}-\frac{8(dx)^{3/2}}{75a}+\frac{4(dx)^{5/2}\log(1-ax)}{25d}-\frac{8(dx)^{5/2}}{125d}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*PolyLog[2, a*x],x]
```

```
[Out] (-8*d*Sqrt[d*x])/(25*a^2) - (8*(d*x)^(3/2))/(75*a) - (8*(d*x)^(5/2))/(125*d
) + (8*d^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/(25*a^(5/2)) + (4*(d*x
)^(5/2)*Log[1 - a*x])/(25*d) + (2*(d*x)^(5/2)*PolyLog[2, a*x])/(5*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int (dx)^{3/2}\text{Li}_2(ax)\,dx &= \frac{2(dx)^{5/2}\text{Li}_2(ax)}{5d}+\frac{2}{5}\int (dx)^{3/2}\log(1-ax)\,dx\\
&= \frac{4(dx)^{5/2}\log(1-ax)}{25d}+\frac{2(dx)^{5/2}\text{Li}_2(ax)}{5d}+\frac{(4a)\int \frac{(dx)^{5/2}}{1-ax}\,dx}{25d}\\
&= -\frac{8(dx)^{5/2}}{125d}+\frac{4(dx)^{5/2}\log(1-ax)}{25d}+\frac{2(dx)^{5/2}\text{Li}_2(ax)}{5d}+\frac{4}{25}\int \frac{(dx)^{3/2}}{1-ax}\,dx\\
&= -\frac{8(dx)^{3/2}}{75a}-\frac{8(dx)^{5/2}}{125d}+\frac{4(dx)^{5/2}\log(1-ax)}{25d}+\frac{2(dx)^{5/2}\text{Li}_2(ax)}{5d}+\frac{(4d)\int \frac{\sqrt{dx}}{1-ax}\,dx}{25a}\\
&= -\frac{8d\sqrt{dx}}{25a^2}-\frac{8(dx)^{3/2}}{75a}-\frac{8(dx)^{5/2}}{125d}+\frac{4(dx)^{5/2}\log(1-ax)}{25d}+\frac{2(dx)^{5/2}\text{Li}_2(ax)}{5d}+\frac{(4d^2)\,\text{J}}{}\\
&= -\frac{8d\sqrt{dx}}{25a^2}-\frac{8(dx)^{3/2}}{75a}-\frac{8(dx)^{5/2}}{125d}+\frac{4(dx)^{5/2}\log(1-ax)}{25d}+\frac{2(dx)^{5/2}\text{Li}_2(ax)}{5d}+\frac{(8d)\text{Su}}{}\\
&= -\frac{8d\sqrt{dx}}{25a^2}-\frac{8(dx)^{3/2}}{75a}-\frac{8(dx)^{5/2}}{125d}+\frac{8d^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/2}}+\frac{4(dx)^{5/2}\log(1-ax}{25d}
\end{aligned}
$$

Mathematica [A]

time = 0.07, size = 90, normalized size = 0.77

$$\frac{2(dx)^{3/2}\left(\frac{4\tanh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{5a^{5/2}}+\frac{2}{75}\sqrt{x}\left(-\frac{2(15+5ax+3a^2x^2)}{a^2}+15x^2\log(1-ax)\right)+x^{5/2}\text{PolyLog}(2,ax)\right)}{5x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*PolyLog[2, a*x],x]
```

```
[Out] (2*(d*x)^(3/2)*((4*ArcTanh[Sqrt[a]*Sqrt[x]])/(5*a^(5/2)) + (2*Sqrt[x]*((-2*
(15 + 5*a*x + 3*a^2*x^2))/a^2 + 15*x^2*Log[1 - a*x]))/75 + x^(5/2)*PolyLog[
2, a*x]))/(5*x^(3/2))
```

Maple [A]

time = 0.39, size = 101, normalized size = 0.86

method	result
derivativedivides	$\frac{\frac{2(dx)^{\frac{5}{2}}\operatorname{polylog}(2,ax)}{5}+\frac{4(dx)^{\frac{5}{2}}\ln\left(\frac{-adx+d}{d}\right)}{25}+\frac{8a\left(-\frac{\frac{(dx)^{\frac{5}{2}}a^2}{5}+\frac{d(dx)^{\frac{3}{2}}a}{3}+d^2\sqrt{dx}}{a^3}+\frac{d^3\operatorname{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{a^3\sqrt{ad}}\right)}{25}}{d}$
default	$\frac{\frac{2(dx)^{\frac{5}{2}}\operatorname{polylog}(2,ax)}{5}+\frac{4(dx)^{\frac{5}{2}}\ln\left(\frac{-adx+d}{d}\right)}{25}+\frac{8a\left(-\frac{\frac{(dx)^{\frac{5}{2}}a^2}{5}+\frac{d(dx)^{\frac{3}{2}}a}{3}+d^2\sqrt{dx}}{a^3}+\frac{d^3\operatorname{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{a^3\sqrt{ad}}\right)}{25}}{d}$
meijerg	$\frac{(dx)^{\frac{3}{2}}\left(-\frac{2\sqrt{x}\,(-a)^{\frac{7}{2}}\left(84a^2x^2+140ax+420\right)}{2625a^3}-\frac{4\sqrt{x}\,(-a)^{\frac{7}{2}}\left(\ln\left(1-\sqrt{ax}\right)-\ln\left(1+\sqrt{ax}\right)\right)}{25a^3\sqrt{ax}}+\frac{4x^{\frac{5}{2}}(-a)^{\frac{7}{2}}\ln(-ax+1)}{25a}+\right.}{x^{\frac{3}{2}}(-a)^{\frac{3}{2}}a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*polylog(2,a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/5*(d*x)^(5/2)*polylog(2,a*x)+2/25*(d*x)^(5/2)*ln((-a*d*x+d)/d)+4/25*
a*(-1/a^3*(1/5*(d*x)^(5/2)*a^2+1/3*d*(d*x)^(3/2)*a+d^2*(d*x)^(1/2))+d^3/a^3
/(a*d)^(1/2)*arctanh(a*(d*x)^(1/2)/(a*d)^(1/2))))
```

Maxima [A]

time = 0.55, size = 128, normalized size = 1.09

$$-\frac{2\left(\frac{30\,d^3\log\left(\frac{\sqrt{dx}\,a-\sqrt{ad}}{\sqrt{dx}\,a+\sqrt{ad}}\right)}{\sqrt{ad}\,a^2}-\frac{75\,(dx)^{\frac{5}{2}}a^2\mathrm{Li}_2(ax)+30\,(dx)^{\frac{5}{2}}a^2\log(-adx+d)-6\left(5\,a^2\log(d)+2\,a^2\right)(dx)^{\frac{5}{2}}-20\,(dx)^{\frac{3}{2}}ad-60\,\sqrt{dx}\,d^2}{a^2}\right)}{375\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(2,a*x),x, algorithm="maxima")
```

```
[Out] -2/375*(30*d^3*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/(sq
rt(a*d)*a^2) - (75*(d*x)^(5/2)*a^2*dilog(a*x) + 30*(d*x)^(5/2)*a^2*log(-a*d
*x + d) - 6*(5*a^2*log(d) + 2*a^2)*(d*x)^(5/2) - 20*(d*x)^(3/2)*a*d - 60*sq
rt(d*x)*d^2)/a^2)/d
```

Fricas [A]
time = 0.39, size = 190, normalized size = 1.62

$$\left[\frac{2\left(30\,d\sqrt{\frac{d}{a}}\log\left(\frac{adx+2\,\sqrt{dx}\,a\sqrt{\frac{d}{a}}+d}{ax-1}\right)+\left(75\,a^2dx^2\mathrm{Li}_2(ax)+30\,a^2dx^2\log\left(-ax+1\right)-12\,a^2dx^2-20\,adx-60\,d\right)\sqrt{dx}\right)}{375\,a^2}, -\frac{2\left(60\,d\sqrt{-\frac{d}{a}}\arctan\left(\frac{\sqrt{dx}\,a\sqrt{-\frac{d}{a}}}{d}\right)-\left(75\,a^2dx^2\mathrm{Li}_2(ax)+30\,a^2dx^2\log\left(-ax+1\right)-12\,a^2dx^2-20\,adx-60\,d\right)\sqrt{dx}\right)}{375\,a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(2,a*x),x, algorithm="fricas")
```

```
[Out] [2/375*(30*d*sqrt(d/a)*log((a*d*x + 2*sqrt(d*x)*a*sqrt(d/a) + d)/(a*x - 1))
 + (75*a^2*d*x^2*dilog(a*x) + 30*a^2*d*x^2*log(-a*x + 1) - 12*a^2*d*x^2 - 2
0*a*d*x - 60*d)*sqrt(d*x))/a^2, -2/375*(60*d*sqrt(-d/a)*arctan(sqrt(d*x)*a*
sqrt(-d/a)/d) - (75*a^2*d*x^2*dilog(a*x) + 30*a^2*d*x^2*log(-a*x + 1) - 12*
a^2*d*x^2 - 20*a*d*x - 60*d)*sqrt(d*x))/a^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}}\,\mathrm{Li}_2\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*polylog(2,a*x),x)
```

```
[Out] Integral((d*x)**(3/2)*polylog(2, a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(2,a*x),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*dilog(a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{3/2}\,\mathrm{polylog}(2,a\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*polylog(2, a*x),x)
```

```
[Out] int((d*x)^(3/2)*polylog(2, a*x), x)
```

3.60 $\int \sqrt{dx}$ PolyLog$(2, ax)\, dx$

Optimal. Leaf size=102

$$-\frac{8\sqrt{dx}}{9a}-\frac{8(dx)^{3/2}}{27d}+\frac{8\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/2}}+\frac{4(dx)^{3/2}\log(1-ax)}{9d}+\frac{2(dx)^{3/2}\text{PolyLog}(2,ax)}{3d}$$

```
[Out] -8/27*(d*x)^(3/2)/d+4/9*(d*x)^(3/2)*ln(-a*x+1)/d+2/3*(d*x)^(3/2)*polylog(2,
a*x)/d+8/9*arctanh(a^(1/2)*(d*x)^(1/2)/d^(1/2))*d^(1/2)/a^(3/2)-8/9*(d*x)^(
1/2)/a
```

Rubi [A]
time = 0.03, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6726, 2442, 52, 65, 212}

$$\frac{8\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/2}}+\frac{2(dx)^{3/2}\text{Li}_2(ax)}{3d}-\frac{8\sqrt{dx}}{9a}+\frac{4(dx)^{3/2}\log(1-ax)}{9d}-\frac{8(dx)^{3/2}}{27d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*PolyLog[2, a*x],x]
```

```
[Out] (-8*Sqrt[d*x])/(9*a) - (8*(d*x)^(3/2))/(27*d) + (8*Sqrt[d]*ArcTanh[(Sqrt[a]
*Sqrt[d*x])/Sqrt[d]])/(9*a^(3/2)) + (4*(d*x)^(3/2)*Log[1 - a*x])/(9*d) + (2
*(d*x)^(3/2)*PolyLog[2, a*x])/(3*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{dx}\,\text{Li}_2(ax)\,dx &= \frac{2(dx)^{3/2}\text{Li}_2(ax)}{3d}+\frac{2}{3}\int \sqrt{dx}\,\log(1-ax)\,dx\\ &= \frac{4(dx)^{3/2}\log(1-ax)}{9d}+\frac{2(dx)^{3/2}\text{Li}_2(ax)}{3d}+\frac{(4a)\int \frac{(dx)^{3/2}}{1-ax}\,dx}{9d}\\ &= -\frac{8(dx)^{3/2}}{27d}+\frac{4(dx)^{3/2}\log(1-ax)}{9d}+\frac{2(dx)^{3/2}\text{Li}_2(ax)}{3d}+\frac{4}{9}\int \frac{\sqrt{dx}}{1-ax}\,dx\\ &= -\frac{8\sqrt{dx}}{9a}-\frac{8(dx)^{3/2}}{27d}+\frac{4(dx)^{3/2}\log(1-ax)}{9d}+\frac{2(dx)^{3/2}\text{Li}_2(ax)}{3d}+\frac{(4d)\int \frac{1}{\sqrt{dx}\,(1-ax)}\,d}{9a}\\ &= -\frac{8\sqrt{dx}}{9a}-\frac{8(dx)^{3/2}}{27d}+\frac{4(dx)^{3/2}\log(1-ax)}{9d}+\frac{2(dx)^{3/2}\text{Li}_2(ax)}{3d}+\frac{8\text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}}\,dx\right.}{9a}\\ &= -\frac{8\sqrt{dx}}{9a}-\frac{8(dx)^{3/2}}{27d}+\frac{8\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/2}}+\frac{4(dx)^{3/2}\log(1-ax)}{9d}+\frac{2(dx)^{3/2}}{3}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 75, normalized size = 0.74

$$\frac{2\sqrt{dx}\left(\frac{12\tanh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{a^{3/2}}+\frac{2\sqrt{x}\,(-6-2ax+3ax\log(1-ax))}{a}+9x^{3/2}\text{PolyLog}(2,ax)\right)}{27\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*PolyLog[2, a*x],x]
```

```
[Out] (2*Sqrt[d*x]*((12*ArcTanh[Sqrt[a]*Sqrt[x]])/a^(3/2) + (2*Sqrt[x]*(-6 - 2*a*
x + 3*a*x*Log[1 - a*x]))/a + 9*x^(3/2)*PolyLog[2, a*x]))/(27*Sqrt[x])
```

Maple [A]
time = 0.37, size = 88, normalized size = 0.86

method	result
derivativedivides	$\frac{\frac{2(dx)^{\frac{3}{2}}\operatorname{polylog}(2,ax)}{3}+\frac{4(dx)^{\frac{3}{2}}\ln\left(\frac{-adx+d}{d}\right)}{9}+\frac{8a\left(-\frac{\frac{a(dx)^{\frac{3}{2}}}{3}+\sqrt{dx}\,d}{a^{2}}+\frac{d^{2}\operatorname{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{a^{2}\sqrt{ad}}\right)}{9}}{d}$
default	$\frac{\frac{2(dx)^{\frac{3}{2}}\operatorname{polylog}(2,ax)}{3}+\frac{4(dx)^{\frac{3}{2}}\ln\left(\frac{-adx+d}{d}\right)}{9}+\frac{8a\left(-\frac{\frac{a(dx)^{\frac{3}{2}}}{3}+\sqrt{dx}\,d}{a^{2}}+\frac{d^{2}\operatorname{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{a^{2}\sqrt{ad}}\right)}{9}}{d}$
meijerg	$\frac{\sqrt{dx}\left(-\frac{2\sqrt{x}\,(-a)^{\frac{5}{2}}(20ax+60)}{135a^{2}}-\frac{4\sqrt{x}\,(-a)^{\frac{5}{2}}\left(\ln\left(1-\sqrt{ax}\right)-\ln\left(1+\sqrt{ax}\right)\right)}{9a^{2}\sqrt{ax}}+\frac{4x^{\frac{3}{2}}(-a)^{\frac{5}{2}}\ln(-ax+1)}{9a}+\frac{2x^{\frac{3}{2}}(-a)^{\frac{5}{2}}\cdots}{3\cdots}\cdots\right)}{\sqrt{x}\,\sqrt{-a}\,a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*polylog(2,a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/3*(d*x)^(3/2)*polylog(2,a*x)+2/9*(d*x)^(3/2)*ln((-a*d*x+d)/d)+4/9*a*
(-1/a^2*(1/3*a*(d*x)^(3/2)+(d*x)^(1/2)*d)+d^2/a^2/(a*d)^(1/2)*arctanh(a*(d*
x)^(1/2)/(a*d)^(1/2))))
```

Maxima [A]
time = 0.47, size = 109, normalized size = 1.07

$$-\frac{2\left(\frac{6\,d^{2}\log\left(\frac{\sqrt{dx}\,a-\sqrt{ad}}{\sqrt{dx}\,a+\sqrt{ad}}\right)}{\sqrt{ad}\,a}-\frac{9\,(dx)^{\frac{3}{2}}a\operatorname{Li}_2(ax)+6\,(dx)^{\frac{3}{2}}a\log(-adx+d)-2\,(dx)^{\frac{3}{2}}(3\,a\log(d)+2\,a)-12\,\sqrt{dx}\,d}{a}\right)}{27\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x),x, algorithm="maxima")
```

```
[Out] -2/27*(6*d^2*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/(sqrt
(a*d)*a) - (9*(d*x)^(3/2)*a*dilog(a*x) + 6*(d*x)^(3/2)*a*log(-a*d*x + d) -
2*(d*x)^(3/2)*(3*a*log(d) + 2*a) - 12*sqrt(d*x)*d)/a)/d
```

Fricas [A]
time = 0.39, size = 143, normalized size = 1.40

$$\left[\frac{2\left((9\,ax\mathrm{Li}_2(ax)+6\,ax\log\left(-ax+1\right)-4\,ax-12)\sqrt{dx}+6\,\sqrt{\frac{d}{a}}\log\left(\frac{adx+2\,\sqrt{dx}\,a\sqrt{\frac{d}{a}}+d}{ax-1}\right)\right)}{27\,a}, \frac{2\left((9\,ax\mathrm{Li}_2(ax)+6\,ax\log\left(-ax+1\right)-4\,ax-12)\sqrt{dx}-12\,\sqrt{-\frac{d}{a}}\arctan\left(\frac{\sqrt{dx}\,a\sqrt{-\frac{d}{a}}}{d}\right)\right)}{27\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x),x, algorithm="fricas")
```

```
[Out] [2/27*((9*a*x*dilog(a*x) + 6*a*x*log(-a*x + 1) - 4*a*x - 12)*sqrt(d*x) + 6*
sqrt(d/a)*log((a*d*x + 2*sqrt(d*x)*a*sqrt(d/a) + d)/(a*x - 1)))/a, 2/27*((9
*a*x*dilog(a*x) + 6*a*x*log(-a*x + 1) - 4*a*x - 12)*sqrt(d*x) - 12*sqrt(-d/
a)*arctan(sqrt(d*x)*a*sqrt(-d/a)/d))/a]
```

Sympy [A]
time = 30.88, size = 110, normalized size = 1.08

$$\frac{2\left(\begin{cases} -\frac{2(dx)^{\frac{3}{2}}\,\mathrm{Li}_1(ax)}{9}+\frac{(dx)^{\frac{3}{2}}\,\mathrm{Li}_2(ax)}{3}-\frac{4(dx)^{\frac{3}{2}}}{27}-\frac{4d\sqrt{dx}}{9a}-\frac{4d^2\log\left(-\sqrt{\frac{d}{a}}+\sqrt{dx}\right)}{9a^2\sqrt{\frac{d}{a}}}-\frac{2d^2\,\mathrm{Li}_1(ax)}{9a^2\sqrt{\frac{d}{a}}} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*polylog(2,a*x),x)
```

```
[Out] 2*Piecewise((-2*(d*x)**(3/2)*polylog(1, a*x)/9 + (d*x)**(3/2)*polylog(2, a*
x)/3 - 4*(d*x)**(3/2)/27 - 4*d*sqrt(d*x)/(9*a) - 4*d**2*log(-sqrt(d/a) + sq
rt(d*x))/(9*a**2*sqrt(d/a)) - 2*d**2*polylog(1, a*x)/(9*a**2*sqrt(d/a)), Ne
(a, 0)), (0, True))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*dilog(a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,x}\ \mathrm{polylog}(2,a\,x)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*polylog(2, a*x),x)
[Out] int((d*x)^(1/2)*polylog(2, a*x), x)
```

3.61 $\int \frac{\text{PolyLog}(2,ax)}{\sqrt{dx}} \, dx$

Optimal. Leaf size=80

$$-\frac{8\sqrt{dx}}{d}+\frac{8\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{a}\,\sqrt{d}}+\frac{4\sqrt{dx}\,\log(1-ax)}{d}+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax)}{d}$$

```
[Out] 8*arctanh(a^(1/2)*(d*x)^(1/2)/d^(1/2))/a^(1/2)/d^(1/2)-8*(d*x)^(1/2)/d+4*ln
(-a*x+1)*(d*x)^(1/2)/d+2*polylog(2,a*x)*(d*x)^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6726, 2442, 52, 65, 212}

$$\frac{2\sqrt{dx}\,\text{Li}_2(ax)}{d}+\frac{4\sqrt{dx}\,\log(1-ax)}{d}+\frac{8\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{a}\,\sqrt{d}}-\frac{8\sqrt{dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x]/Sqrt[d*x],x]
```

```
[Out] (-8*Sqrt[d*x])/d + (8*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[a]*Sqrt[d
]) + (4*Sqrt[d*x]*Log[1 - a*x])/d + (2*Sqrt[d*x]*PolyLog[2, a*x])/d
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax)}{\sqrt{dx}}\,dx &= \frac{2\sqrt{dx}\,\text{Li}_2(ax)}{d}+2\int \frac{\log(1-ax)}{\sqrt{dx}}\,dx\\
&= \frac{4\sqrt{dx}\,\log(1-ax)}{d}+\frac{2\sqrt{dx}\,\text{Li}_2(ax)}{d}+\frac{(4a)\int \frac{\sqrt{dx}}{1-ax}\,dx}{d}\\
&= -\frac{8\sqrt{dx}}{d}+\frac{4\sqrt{dx}\,\log(1-ax)}{d}+\frac{2\sqrt{dx}\,\text{Li}_2(ax)}{d}+4\int \frac{1}{\sqrt{dx}\,(1-ax)}\,dx\\
&= -\frac{8\sqrt{dx}}{d}+\frac{4\sqrt{dx}\,\log(1-ax)}{d}+\frac{2\sqrt{dx}\,\text{Li}_2(ax)}{d}+\frac{8\text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}}\,dx,x,\sqrt{dx}\right)}{d}\\
&= -\frac{8\sqrt{dx}}{d}+\frac{8\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{a}\,\sqrt{d}}+\frac{4\sqrt{dx}\,\log(1-ax)}{d}+\frac{2\sqrt{dx}\,\text{Li}_2(ax)}{d}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 63, normalized size = 0.79

$$\frac{8\sqrt{x}\,\tanh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)+4\sqrt{a}\,x(-2+\log(1-ax))+2\sqrt{a}\,x\text{PolyLog}(2,ax)}{\sqrt{a}\,\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x]/Sqrt[d*x],x]
```

```
[Out] (8*Sqrt[x]*ArcTanh[Sqrt[a]*Sqrt[x]] + 4*Sqrt[a]*x*(-2 + Log[1 - a*x]) + 2*S
qrt[a]*x*PolyLog[2, a*x])/(Sqrt[a]*Sqrt[d*x])
```

Maple [A]
time = 0.42, size = 74, normalized size = 0.92

method	result
derivativedivides	$\frac{2\sqrt{dx}\,\mathrm{polylog}(2,ax)+4\sqrt{dx}\,\ln\left(\frac{-adx+d}{d}\right)+8a\left(-\frac{\sqrt{dx}}{a}+\frac{d\,\mathrm{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{a\sqrt{ad}}\right)}{d}$
default	$\frac{2\sqrt{dx}\,\mathrm{polylog}(2,ax)+4\sqrt{dx}\,\ln\left(\frac{-adx+d}{d}\right)+8a\left(-\frac{\sqrt{dx}}{a}+\frac{d\,\mathrm{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{a\sqrt{ad}}\right)}{d}$
meijerg	$\frac{\sqrt{x}\,\sqrt{-a}\,\left(-\frac{8\sqrt{x}\,(-a)^{\frac{3}{2}}}{a}-\frac{4\sqrt{x}\,(-a)^{\frac{3}{2}}\left(\ln\left(1-\sqrt{ax}\right)-\ln\left(1+\sqrt{ax}\right)\right)}{a\sqrt{ax}}+\frac{4\sqrt{x}\,(-a)^{\frac{3}{2}}\ln(-ax+1)}{a}+\frac{2\sqrt{x}\,(\right.}{\sqrt{dx}\,a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*((d*x)^(1/2)*polylog(2,a*x)+2*(d*x)^(1/2)*ln((-a*d*x+d)/d)+4*a*(-(d*x)^
(1/2)/a+d/a/(a*d)^(1/2)*arctanh(a*(d*x)^(1/2)/(a*d)^(1/2))))
```

Maxima [A]
time = 0.46, size = 83, normalized size = 1.04

$$-\frac{2\left(2\,\sqrt{dx}\,(\log\left(d\right)+2)-\sqrt{dx}\,\mathrm{Li}_2(ax)-2\,\sqrt{dx}\,\log\left(-adx+d\right)+\frac{2\,d\log\left(\frac{\sqrt{dx}\,a-\sqrt{ad}}{\sqrt{dx}\,a+\sqrt{ad}}\right)}{\sqrt{ad}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(2*sqrt(d*x)*(log(d) + 2) - sqrt(d*x)*dilog(a*x) - 2*sqrt(d*x)*log(-a*d*
x + d) + 2*d*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/sqrt(
a*d))/d
```

Fricas [A]
time = 0.41, size = 135, normalized size = 1.69

$$\left[\frac{2\left(\sqrt{dx}\,(a\mathrm{Li}_2(ax)+2\,a\log\left(-ax+1\right)-4\,a)+2\,\sqrt{ad}\,\log\left(\frac{adx+2\,\sqrt{ad}\,\sqrt{dx}+d}{ax-1}\right)\right)}{ad},\ \frac{2\left(\sqrt{dx}\,(a\mathrm{Li}_2(ax)+2\,a\log\left(-ax+1\right)-4\,a)-4\,\sqrt{-ad}\,\arctan\left(\frac{\sqrt{-ad}\,\sqrt{dx}}{adx}\right)\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [2*(sqrt(d*x)*(a*dilog(a*x) + 2*a*log(-a*x + 1) - 4*a) + 2*sqrt(a*d)*log((a
*d*x + 2*sqrt(a*d)*sqrt(d*x) + d)/(a*x - 1)))/(a*d), 2*(sqrt(d*x)*(a*dilog(
a*x) + 2*a*log(-a*x + 1) - 4*a) - 4*sqrt(-a*d)*arctan(sqrt(-a*d)*sqrt(d*x)/
(a*d*x)))/(a*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_2(ax)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)**(1/2),x)
```

```
[Out] Integral(polylog(2, a*x)/sqrt(d*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x)/sqrt(d*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(2,a\,x)}{\sqrt{d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x)/(d*x)^(1/2),x)
```

```
[Out] int(polylog(2, a*x)/(d*x)^(1/2), x)
```

3.62 $\int \frac{\text{PolyLog}(2,ax)}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=68

$$\frac{8\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{4\log(1-ax)}{d\sqrt{dx}}-\frac{2\text{PolyLog}(2,ax)}{d\sqrt{dx}}$$

```
[Out] 8*arctanh(a^(1/2)*(d*x)^(1/2)/d^(1/2))*a^(1/2)/d^(3/2)+4*ln(-a*x+1)/d/(d*x)
^(1/2)-2*polylog(2,a*x)/d/(d*x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.308$, Rules used = {6726, 2442, 65, 212}

$$\frac{8\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\text{Li}_2(ax)}{d\sqrt{dx}}+\frac{4\log(1-ax)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x]/(d*x)^(3/2),x]
```

```
[Out] (8*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (4*Log[1 - a*x])
/(d*Sqrt[d*x]) - (2*PolyLog[2, a*x])/(d*Sqrt[d*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax)}{(dx)^{3/2}}\,dx &= -\frac{2\text{Li}_2(ax)}{d\sqrt{dx}} - 2\int \frac{\log(1-ax)}{(dx)^{3/2}}\,dx \\
&= \frac{4\log(1-ax)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax)}{d\sqrt{dx}} + \frac{(4a)\int \frac{1}{\sqrt{dx}\,(1-ax)}\,dx}{d} \\
&= \frac{4\log(1-ax)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax)}{d\sqrt{dx}} + \frac{(8a)\text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}}\,dx, x, \sqrt{dx}\right)}{d^2} \\
&= \frac{8\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{4\log(1-ax)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax)}{d\sqrt{dx}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 51, normalized size = 0.75

$$\frac{2x\left(4\sqrt{a}\,\sqrt{x}\,\tanh^{-1}\left(\sqrt{a}\,\sqrt{x}\right) + 2\log(1-ax) - \text{PolyLog}(2,ax)\right)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x]/(d*x)^(3/2),x]
```

```
[Out] (2*x*(4*Sqrt[a]*Sqrt[x]*ArcTanh[Sqrt[a]*Sqrt[x]] + 2*Log[1 - a*x] - PolyLog
[2, a*x]))/(d*x)^(3/2)
```

Maple [A]
time = 0.38, size = 59, normalized size = 0.87

method	result	size

derivativedivides	$\frac{-\frac{2\,\mathrm{polylog}(2,ax)}{\sqrt{dx}}+\frac{4\ln\left(\frac{-adx+d}{d}\right)}{\sqrt{dx}}+\frac{8a\,\mathrm{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{\sqrt{ad}}}{d}$	59
default	$\frac{-\frac{2\,\mathrm{polylog}(2,ax)}{\sqrt{dx}}+\frac{4\ln\left(\frac{-adx+d}{d}\right)}{\sqrt{dx}}+\frac{8a\,\mathrm{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{\sqrt{ad}}}{d}$	59
meijerg	$\frac{x^{\frac{3}{2}}(-a)^{\frac{3}{2}}\left(-\frac{4\sqrt{x}\,\sqrt{-a}\,\left(\ln\left(1-\sqrt{ax}\right)-\ln\left(1+\sqrt{ax}\right)\right)}{\sqrt{ax}}+\frac{4\sqrt{-a}\,\ln(-ax+1)}{\sqrt{x}\,a}-\frac{2\sqrt{-a}\,\mathrm{polylog}(2,ax)}{\sqrt{x}\,a}\right)}{(dx)^{\frac{3}{2}}a}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-polylog(2,a*x)/(d*x)^(1/2)+2/(d*x)^(1/2)*ln((-a*d*x+d)/d)+4*a/(a*d)^(
1/2)*arctanh(a*(d*x)^(1/2)/(a*d)^(1/2)))
```

Maxima [A]
time = 0.47, size = 71, normalized size = 1.04

$$-\frac{2\left(\frac{2\,a\log\left(\frac{\sqrt{dx}\,a-\sqrt{ad}}{\sqrt{dx}\,a+\sqrt{ad}}\right)}{\sqrt{ad}}+\frac{\mathrm{Li}_2(ax)-2\,\log(-adx+d)+2\,\log(d)}{\sqrt{dx}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*(2*a*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/sqrt(a*d)
+ (dilog(a*x) - 2*log(-a*d*x + d) + 2*log(d))/sqrt(d*x))/d
```

Fricas [A]
time = 0.37, size = 132, normalized size = 1.94

$$\left[\frac{2\left(2\,dx\sqrt{\frac{a}{d}}\,\log\left(\frac{ax+2\,\sqrt{dx}\,\sqrt{\frac{a}{d}}+1}{ax-1}\right)-\sqrt{dx}\,\left(\mathrm{Li}_2(ax)-2\,\log\left(-ax+1\right)\right)\right)}{d^2x},\,-\frac{2\left(4\,dx\sqrt{-\frac{a}{d}}\,\arctan\left(\frac{\sqrt{dx}\,\sqrt{-\frac{a}{d}}}{ax}\right)+\sqrt{dx}\,\left(\mathrm{Li}_2(ax)-2\,\log\left(-ax+1\right)\right)\right)}{d^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [2*(2*d*x*sqrt(a/d)*log((a*x + 2*sqrt(d*x)*sqrt(a/d) + 1)/(a*x - 1)) - sqrt
(d*x)*(dilog(a*x) - 2*log(-a*x + 1)))/(d^2*x), -2*(4*d*x*sqrt(-a/d)*arctan(
sqrt(d*x)*sqrt(-a/d)/(a*x)) + sqrt(d*x)*(dilog(a*x) - 2*log(-a*x + 1)))/(d^
2*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Li}_2\left(ax\right)}{\left(dx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)**(3/2),x)
```

```
[Out] Integral(polylog(2, a*x)/(d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x)/(d*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(2,a\,x)}{{\left(d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x)/(d*x)^(3/2),x)
```

```
[Out] int(polylog(2, a*x)/(d*x)^(3/2), x)
```

3.63 $\int \frac{\text{PolyLog}(2,ax)}{(dx)^{5/2}} dx$

Optimal. Leaf size=89

$$-\frac{8a}{9d^2\sqrt{dx}}+\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}}+\frac{4\log(1-ax)}{9d(dx)^{3/2}}-\frac{2\text{PolyLog}(2,ax)}{3d(dx)^{3/2}}$$

```
[Out] 8/9*a^(3/2)*arctanh(a^(1/2)*(d*x)^(1/2)/d^(1/2))/d^(5/2)+4/9*ln(-a*x+1)/d/(
d*x)^(3/2)-2/3*polylog(2,a*x)/d/(d*x)^(3/2)-8/9*a/d^2/(d*x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6726, 2442, 53, 65, 212}

$$\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}}-\frac{8a}{9d^2\sqrt{dx}}-\frac{2\text{Li}_2(ax)}{3d(dx)^{3/2}}+\frac{4\log(1-ax)}{9d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x]/(d*x)^(5/2),x]
```

```
[Out] (-8*a)/(9*d^2*Sqrt[d*x]) + (8*a^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])
/(9*d^(5/2)) + (4*Log[1 - a*x])/(9*d*(d*x)^(3/2)) - (2*PolyLog[2, a*x])/(3*
d*(d*x)^(3/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{Li}_2(ax)}{(dx)^{5/2}}\,dx &= -\frac{2\text{Li}_2(ax)}{3d(dx)^{3/2}} - \frac{2}{3}\int \frac{\log(1-ax)}{(dx)^{5/2}}\,dx \\
&= \frac{4\log(1-ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax)}{3d(dx)^{3/2}} + \frac{(4a)\int \frac{1}{(dx)^{3/2}(1-ax)}\,dx}{9d} \\
&= -\frac{8a}{9d^2\sqrt{dx}} + \frac{4\log(1-ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax)}{3d(dx)^{3/2}} + \frac{(4a^2)\int \frac{1}{\sqrt{dx}\,(1-ax)}\,dx}{9d^2} \\
&= -\frac{8a}{9d^2\sqrt{dx}} + \frac{4\log(1-ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax)}{3d(dx)^{3/2}} + \frac{(8a^2)\,\text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}}\,dx, x, \sqrt{dx}\right)}{9d^3} \\
&= -\frac{8a}{9d^2\sqrt{dx}} + \frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}} + \frac{4\log(1-ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax)}{3d(dx)^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 57, normalized size = 0.64

$$
-\frac{2x\left(4ax - 4a^{3/2}x^{3/2}\tanh^{-1}\left(\sqrt{a}\sqrt{x}\right) - 2\log(1-ax) + 3\text{PolyLog}(2,ax)\right)}{9(dx)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x]/(d*x)^(5/2),x]
```

```
[Out] (-2*x*(4*a*x - 4*a^(3/2)*x^(3/2)*ArcTanh[Sqrt[a]*Sqrt[x]] - 2*Log[1 - a*x]
+ 3*PolyLog[2, a*x]))/(9*(d*x)^(5/2))
```

Maple [A]
time = 0.35, size = 75, normalized size = 0.84

method	result	size
derivativedivides	$\frac{-\frac{2\,\mathrm{polylog}(2,ax)}{3(dx)^{\frac{3}{2}}}+\frac{4\ln\left(\frac{-adx+d}{d}\right)}{9(dx)^{\frac{3}{2}}}+\frac{8a\left(\frac{a\,\mathrm{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{d\sqrt{ad}}-\frac{1}{d\sqrt{dx}}\right)}{9}}{d}$	75
default	$\frac{-\frac{2\,\mathrm{polylog}(2,ax)}{3(dx)^{\frac{3}{2}}}+\frac{4\ln\left(\frac{-adx+d}{d}\right)}{9(dx)^{\frac{3}{2}}}+\frac{8a\left(\frac{a\,\mathrm{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{d\sqrt{ad}}-\frac{1}{d\sqrt{dx}}\right)}{9}}{d}$	75
meijerg	$\frac{x^{\frac{5}{2}}(-a)^{\frac{5}{2}}\left(-\frac{8}{9\sqrt{x}\,\sqrt{-a}}-\frac{4\sqrt{x}\,a\left(\ln\left(1-\sqrt{ax}\right)-\ln\left(1+\sqrt{ax}\right)\right)}{9\sqrt{-a}\,\sqrt{ax}}+\frac{4\ln(-ax+1)}{9x^{\frac{3}{2}}\sqrt{-a}\,a}-\frac{2\,\mathrm{polylog}(2,ax)}{3x^{\frac{3}{2}}\sqrt{-a}\,a}\right)}{(dx)^{\frac{5}{2}}a}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/3*polylog(2,a*x)/(d*x)^(3/2)+2/9/(d*x)^(3/2)*ln((-a*d*x+d)/d)+4/9*a
*(a/d/(a*d)^(1/2)*arctanh(a*(d*x)^(1/2)/(a*d)^(1/2))-1/d/(d*x)^(1/2)))
```

Maxima [A]
time = 0.47, size = 89, normalized size = 1.00

$$-\frac{2\left(\frac{2\,a^{2}\log\left(\frac{\sqrt{dx}\,a-\sqrt{ad}}{\sqrt{dx}\,a+\sqrt{ad}}\right)}{\sqrt{ad}\,d}+\frac{4\,adx+3\,d\mathrm{Li}_2(ax)-2\,d\log(-adx+d)+2\,d\log(d)}{(dx)^{\frac{3}{2}}d}\right)}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/9*(2*a^2*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/(sqrt(
a*d)*d) + (4*a*d*x + 3*d*dilog(a*x) - 2*d*log(-a*d*x + d) + 2*d*log(d))/((d
*x)^(3/2)*d))/d
```

Fricas [A]
time = 0.40, size = 150, normalized size = 1.69

$$\left[\frac{2\left(2\,adx^2\sqrt{\frac{a}{d}}\,\log\left(\frac{ax+2\sqrt{dx}\sqrt{\frac{a}{d}}+1}{ax-1}\right)-\left(4\,ax+3\,\mathrm{Li}_2(ax)-2\,\log\left(-ax+1\right)\right)\sqrt{dx}\right)}{9\,d^3x^2}, -\frac{2\left(4\,adx^2\sqrt{-\frac{a}{d}}\,\arctan\left(\frac{\sqrt{dx}\sqrt{-\frac{a}{d}}}{ax}\right)+\left(4\,ax+3\,\mathrm{Li}_2(ax)-2\,\log\left(-ax+1\right)\right)\sqrt{dx}\right)}{9\,d^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [2/9*(2*a*d*x^2*sqrt(a/d)*log((a*x + 2*sqrt(d*x)*sqrt(a/d) + 1)/(a*x - 1))
- (4*a*x + 3*dilog(a*x) - 2*log(-a*x + 1))*sqrt(d*x))/(d^3*x^2), -2/9*(4*a*
d*x^2*sqrt(-a/d)*arctan(sqrt(d*x)*sqrt(-a/d)/(a*x)) + (4*a*x + 3*dilog(a*x)
 - 2*log(-a*x + 1))*sqrt(d*x))/(d^3*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_2\left(ax\right)}{\left(dx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)**(5/2),x)
```

```
[Out] Integral(polylog(2, a*x)/(d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x)/(d*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(2,a\,x)}{{\left(d\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x)/(d*x)^(5/2),x)
```

```
[Out] int(polylog(2, a*x)/(d*x)^(5/2), x)
```

3.64 $\int \frac{\text{PolyLog}(2,ax)}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{8a}{75d^2(dx)^{3/2}}-\frac{8a^2}{25d^3\sqrt{dx}}+\frac{8a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}}+\frac{4\log(1-ax)}{25d(dx)^{5/2}}-\frac{2\text{PolyLog}(2,ax)}{5d(dx)^{5/2}}$$

```
[Out] -8/75*a/d^2/(d*x)^(3/2)+8/25*a^(5/2)*arctanh(a^(1/2)*(d*x)^(1/2)/d^(1/2))/d
^(7/2)+4/25*ln(-a*x+1)/d/(d*x)^(5/2)-2/5*polylog(2,a*x)/d/(d*x)^(5/2)-8/25*
a^2/d^3/(d*x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6726, 2442, 53, 65, 212}

$$\frac{8a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}}-\frac{8a^2}{25d^3\sqrt{dx}}-\frac{8a}{75d^2(dx)^{3/2}}-\frac{2\text{Li}_2(ax)}{5d(dx)^{5/2}}+\frac{4\log(1-ax)}{25d(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x]/(d*x)^(7/2),x]
```

```
[Out] (-8*a)/(75*d^2*(d*x)^(3/2)) - (8*a^2)/(25*d^3*Sqrt[d*x]) + (8*a^(5/2)*ArcTa
nh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/(25*d^(7/2)) + (4*Log[1 - a*x])/(25*d*(d*x
)^(5/2)) - (2*PolyLog[2, a*x])/(5*d*(d*x)^(5/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax)}{(dx)^{7/2}}\,dx &= -\frac{2\text{Li}_2(ax)}{5d(dx)^{5/2}} - \frac{2}{5}\int \frac{\log(1-ax)}{(dx)^{7/2}}\,dx \\
&= \frac{4\log(1-ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax)}{5d(dx)^{5/2}} + \frac{(4a)\int \frac{1}{(dx)^{5/2}(1-ax)}\,dx}{25d} \\
&= -\frac{8a}{75d^2(dx)^{3/2}} + \frac{4\log(1-ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax)}{5d(dx)^{5/2}} + \frac{(4a^2)\int \frac{1}{(dx)^{3/2}(1-ax)}\,dx}{25d^2} \\
&= -\frac{8a}{75d^2(dx)^{3/2}} - \frac{8a^2}{25d^3\sqrt{dx}} + \frac{4\log(1-ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax)}{5d(dx)^{5/2}} + \frac{(4a^3)\int \frac{1}{\sqrt{dx}\,(1-ax)}\,dx}{25d^3} \\
&= -\frac{8a}{75d^2(dx)^{3/2}} - \frac{8a^2}{25d^3\sqrt{dx}} + \frac{4\log(1-ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax)}{5d(dx)^{5/2}} + \frac{(8a^3)\,\text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}}\,dx, x, \sqrt{dx}\right)}{25d^4} \\
&= -\frac{8a}{75d^2(dx)^{3/2}} - \frac{8a^2}{25d^3\sqrt{dx}} + \frac{8a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}} + \frac{4\log(1-ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax)}{5d(dx)^{5/2}}
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 65, normalized size = 0.61

$$-\frac{2x\left(4ax+12a^2x^2-12a^{5/2}x^{5/2}\tanh^{-1}\left(\sqrt{a}\sqrt{x}\right)-6\log(1-ax)+15\text{PolyLog}(2,ax)\right)}{75(dx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x]/(d*x)^(7/2),x]
```

```
[Out] (-2*x*(4*a*x + 12*a^2*x^2 - 12*a^(5/2)*x^(5/2)*ArcTanh[Sqrt[a]*Sqrt[x]] - 6
*Log[1 - a*x] + 15*PolyLog[2, a*x]))/(75*(d*x)^(7/2))
```

Maple [A]
time = 0.37, size = 88, normalized size = 0.83

method	result
derivativedivides	$\frac{-\frac{2\,\mathrm{polylog}(2,ax)}{5(dx)^{\frac{5}{2}}}+\frac{4\ln\left(\frac{-adx+d}{d}\right)}{25(dx)^{\frac{5}{2}}}+\frac{8a\left(-\frac{1}{3d(dx)^{\frac{3}{2}}}-\frac{a}{d^2\sqrt{dx}}+\frac{a^2\,\mathrm{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{d^2\sqrt{ad}}\right)}{25}}{d}$
default	$\frac{-\frac{2\,\mathrm{polylog}(2,ax)}{5(dx)^{\frac{5}{2}}}+\frac{4\ln\left(\frac{-adx+d}{d}\right)}{25(dx)^{\frac{5}{2}}}+\frac{8a\left(-\frac{1}{3d(dx)^{\frac{3}{2}}}-\frac{a}{d^2\sqrt{dx}}+\frac{a^2\,\mathrm{arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)}{d^2\sqrt{ad}}\right)}{25}}{d}$
meijerg	$\frac{x^{\frac{7}{2}}(-a)^{\frac{7}{2}}\left(-\frac{8}{75x^{\frac{3}{2}}(-a)^{\frac{3}{2}}}-\frac{8a}{25\sqrt{x}\,(-a)^{\frac{3}{2}}}-\frac{4\sqrt{x}\,a^2\left(\ln\left(1-\sqrt{ax}\right)-\ln\left(1+\sqrt{ax}\right)\right)}{25(-a)^{\frac{3}{2}}\sqrt{ax}}+\frac{4\ln(-ax+1)}{25x^{\frac{5}{2}}(-a)^{\frac{3}{2}}a}-\frac{2\,\mathrm{polylog}(2,ax)}{5x^{\frac{5}{2}}(-a)^{\frac{3}{2}}a}\right.}{(dx)^{\frac{7}{2}}a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x)/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/5*polylog(2,a*x)/(d*x)^(5/2)+2/25/(d*x)^(5/2)*ln((-a*d*x+d)/d)+4/25
*a*(-1/3/d/(d*x)^(3/2)-a/d^2/(d*x)^(1/2)+a^2/d^2/(a*d)^(1/2)*arctanh(a*(d*x
)^(1/2)/(a*d)^(1/2))))
```

Maxima [A]
time = 0.48, size = 108, normalized size = 1.02

$$-\frac{2\left(\frac{6\,a^3\log\left(\frac{\sqrt{dx}\,a-\sqrt{ad}}{\sqrt{dx}\,a+\sqrt{ad}}\right)}{\sqrt{ad}\,d^2}+\frac{12\,a^2d^2x^2+4\,ad^2x+15\,d^2\mathrm{Li}_2(ax)-6\,d^2\log(-adx+d)+6\,d^2\log(d)}{(dx)^{\frac{5}{2}}d^2}\right)}{75\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/75*(6*a^3*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/(sqrt
(a*d)*d^2) + (12*a^2*d^2*x^2 + 4*a*d^2*x + 15*d^2*dilog(a*x) - 6*d^2*log(-a
*d*x + d) + 6*d^2*log(d))/((d*x)^(5/2)*d^2))/d
```

Fricas [A]
time = 0.39, size = 170, normalized size = 1.60

$$\left[\frac{2\left(6\,a^2dx^3\sqrt{\frac{a}{d}}\,\log\left(\frac{ax+2\,\sqrt{dx}\,\sqrt{\frac{a}{d}}+1}{ax-1}\right)-\left(12\,a^2x^2+4\,ax+15\,\mathrm{Li}_2(ax)-6\,\log\left(-ax+1\right)\right)\sqrt{dx}\right)}{75\,d^4x^3},-\frac{2\left(12\,a^2dx^3\sqrt{-\frac{a}{d}}\,\arctan\left(\frac{\sqrt{dx}\,\sqrt{-\frac{a}{d}}}{ax}\right)+\left(12\,a^2x^2+4\,ax+15\,\mathrm{Li}_2(ax)-6\,\log\left(-ax+1\right)\right)\sqrt{dx}\right)}{75\,d^4x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] [2/75*(6*a^2*d*x^3*sqrt(a/d)*log((a*x + 2*sqrt(d*x)*sqrt(a/d) + 1)/(a*x - 1
)) - (12*a^2*x^2 + 4*a*x + 15*dilog(a*x) - 6*log(-a*x + 1))*sqrt(d*x))/(d^4
*x^3), -2/75*(12*a^2*d*x^3*sqrt(-a/d)*arctan(sqrt(d*x)*sqrt(-a/d)/(a*x)) +
(12*a^2*x^2 + 4*a*x + 15*dilog(a*x) - 6*log(-a*x + 1))*sqrt(d*x))/(d^4*x^3)
]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x)/(d*x)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(2,a\,x)}{{(d\,x)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x)/(d*x)^(7/2),x)
[Out] int(polylog(2, a*x)/(d*x)^(7/2), x)
```

3.65 $\int (dx)^{5/2}\textbf{PolyLog}(3,ax)\,dx$

Optimal. Leaf size=153

$$\frac{16d^2\sqrt{dx}}{343a^3}+\frac{16d(dx)^{3/2}}{1029a^2}+\frac{16(dx)^{5/2}}{1715a}+\frac{16(dx)^{7/2}}{2401d}-\frac{16d^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/2}}-\frac{8(dx)^{7/2}\log(1-ax)}{343d}-\frac{4(dx)^{7/}}{}$$

```
[Out] 16/1029*d*(d*x)^(3/2)/a^2+16/1715*(d*x)^(5/2)/a+16/2401*(d*x)^(7/2)/d-16/34
3*d^(5/2)*arctanh(a^(1/2)*(d*x)^(1/2)/d^(1/2))/a^(7/2)-8/343*(d*x)^(7/2)*ln
(-a*x+1)/d-4/49*(d*x)^(7/2)*polylog(2,a*x)/d+2/7*(d*x)^(7/2)*polylog(3,a*x)
/d+16/343*d^2*(d*x)^(1/2)/a^3
```

Rubi [A]
time = 0.07, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6726, 2442, 52, 65, 212}

$$-\frac{16d^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/2}}+\frac{16d^2\sqrt{dx}}{343a^3}+\frac{16d(dx)^{3/2}}{1029a^2}-\frac{4(dx)^{7/2}\text{Li}_2(ax)}{49d}+\frac{2(dx)^{7/2}\text{Li}_3(ax)}{7d}+\frac{16(dx)^{5/2}}{1715a}-\frac{8(dx)^{7/2}\log(1-ax)}{343d}+\frac{16(dx)^{7/2}}{2401d}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*PolyLog[3, a*x],x]
```

```
[Out] (16*d^2*Sqrt[d*x])/(343*a^3) + (16*d*(d*x)^(3/2))/(1029*a^2) + (16*(d*x)^(5
/2))/(1715*a) + (16*(d*x)^(7/2))/(2401*d) - (16*d^(5/2)*ArcTanh[(Sqrt[a]*Sq
rt[d*x])/Sqrt[d]])/(343*a^(7/2)) - (8*(d*x)^(7/2)*Log[1 - a*x])/(343*d) - (
4*(d*x)^(7/2)*PolyLog[2, a*x])/(49*d) + (2*(d*x)^(7/2)*PolyLog[3, a*x])/(7*
d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int (dx)^{5/2}\mathrm{Li}_3(ax)\,dx = \frac{2(dx)^{7/2}\mathrm{Li}_3(ax)}{7d} - \frac{2}{7}\int (dx)^{5/2}\mathrm{Li}_2(ax)\,dx$$
$$= -\frac{4(dx)^{7/2}\mathrm{Li}_2(ax)}{49d} + \frac{2(dx)^{7/2}\mathrm{Li}_3(ax)}{7d} - \frac{4}{49}\int (dx)^{5/2}\log(1-ax)\,dx$$
$$= -\frac{8(dx)^{7/2}\log(1-ax)}{343d} - \frac{4(dx)^{7/2}\mathrm{Li}_2(ax)}{49d} + \frac{2(dx)^{7/2}\mathrm{Li}_3(ax)}{7d} - \frac{(8a)\int \frac{(dx)^{7/2}}{1-ax}\,dx}{343d}$$
$$= \frac{16(dx)^{7/2}}{2401d} - \frac{8(dx)^{7/2}\log(1-ax)}{343d} - \frac{4(dx)^{7/2}\mathrm{Li}_2(ax)}{49d} + \frac{2(dx)^{7/2}\mathrm{Li}_3(ax)}{7d} - \frac{8}{343}\int \frac{(dx}{1-}$$
$$= \frac{16(dx)^{5/2}}{1715a} + \frac{16(dx)^{7/2}}{2401d} - \frac{8(dx)^{7/2}\log(1-ax)}{343d} - \frac{4(dx)^{7/2}\mathrm{Li}_2(ax)}{49d} + \frac{2(dx)^{7/2}\mathrm{Li}_3(ax)}{7d}$$
$$= \frac{16d(dx)^{3/2}}{1029a^2} + \frac{16(dx)^{5/2}}{1715a} + \frac{16(dx)^{7/2}}{2401d} - \frac{8(dx)^{7/2}\log(1-ax)}{343d} - \frac{4(dx)^{7/2}\mathrm{Li}_2(ax)}{49d} + \frac{2(d}{}$$
$$= \frac{16d^2\sqrt{dx}}{343a^3} + \frac{16d(dx)^{3/2}}{1029a^2} + \frac{16(dx)^{5/2}}{1715a} + \frac{16(dx)^{7/2}}{2401d} - \frac{8(dx)^{7/2}\log(1-ax)}{343d} - \frac{4(dx)^{7/2}}{49}$$
$$= \frac{16d^2\sqrt{dx}}{343a^3} + \frac{16d(dx)^{3/2}}{1029a^2} + \frac{16(dx)^{5/2}}{1715a} + \frac{16(dx)^{7/2}}{2401d} - \frac{8(dx)^{7/2}\log(1-ax)}{343d} - \frac{4(dx)^{7/2}}{49}$$
$$= \frac{16d^2\sqrt{dx}}{343a^3} + \frac{16d(dx)^{3/2}}{1029a^2} + \frac{16(dx)^{5/2}}{1715a} + \frac{16(dx)^{7/2}}{2401d} - \frac{16d^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/2}} - \frac{8}{}$$

Mathematica [A]
time = 0.18, size = 98, normalized size = 0.64

$$\frac{2(dx)^{5/2}\left(\frac{8(105+35ax+21a^2x^2+15a^3x^3)}{a^3} - \frac{840\tanh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{7/2}\sqrt{x}} - 420x^3\log(1-ax) - 1470x^3\mathrm{PolyLog}(2,ax) + 5145x^3\mathrm{PolyLog}(3,ax)\right)}{36015x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*PolyLog[3, a*x],x]
```

```
[Out] (2*(d*x)^(5/2)*((8*(105 + 35*a*x + 21*a^2*x^2 + 15*a^3*x^3))/a^3 - (840*Arc
Tanh[Sqrt[a]*Sqrt[x]])/(a^(7/2)*Sqrt[x]) - 420*x^3*Log[1 - a*x] - 1470*x^3*
PolyLog[2, a*x] + 5145*x^3*PolyLog[3, a*x]))/(36015*x^2)
```

Maple [A]
time = 0.14, size = 149, normalized size = 0.97

method	result

meijerg	$\frac{(dx)^{\frac{5}{2}}\left(\frac{2\sqrt{x}\,(-a)^{\frac{9}{2}}(360a^3x^3+504a^2x^2+840ax+2520)}{108045a^4}+\frac{8\sqrt{x}\,(-a)^{\frac{9}{2}}\left(\ln\left(1-\sqrt{ax}\right)-\ln\left(1+\sqrt{ax}\right)\right)}{343a^4\sqrt{ax}}-\frac{8x^{\frac{7}{2}}(-a)^{\frac{9}{2}}\ln(-ax+1)}{343a}-\frac{4x^{\frac{7}{2}}}{}\right.}{x^{\frac{5}{2}}(-a)^{\frac{5}{2}}a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*polylog(3,a*x),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x)^(5/2)/x^(5/2)/(-a)^(5/2)/a*(2/108045*x^(1/2)*(-a)^(9/2)*(360*a^3*x^3+
504*a^2*x^2+840*a*x+2520)/a^4+8/343*x^(1/2)*(-a)^(9/2)/a^4/(a*x)^(1/2)*(ln(
1-(a*x)^(1/2))-ln(1+(a*x)^(1/2)))-8/343*x^(7/2)*(-a)^(9/2)/a*ln(-a*x+1)-4/4
9*x^(7/2)*(-a)^(9/2)/a*polylog(2,a*x)+2/7*x^(7/2)*(-a)^(9/2)/a*polylog(3,a*
x))
```

Maxima [A]
time = 0.48, size = 156, normalized size = 1.02

$$\frac{2\left(\frac{420\,d^4\log\left(\frac{\sqrt{dx}\,a-\sqrt{ad}}{\sqrt{dx}\,a+\sqrt{ad}}\right)}{\sqrt{ad}\,a^3}-\frac{1470\,(dx)^{\frac{7}{2}}a^3\mathrm{Li}_2(ax)+420\,(dx)^{\frac{7}{2}}a^3\log(-adx+d)-5145\,(dx)^{\frac{7}{2}}a^3\mathrm{Li}_3(ax)-168\,(dx)^{\frac{5}{2}}a^2d-60\,(7\,a^3\log(d)+2\,a^3)(dx)^{\frac{7}{2}}-280\,(dx)^{\frac{3}{2}}ad^2-840\,\sqrt{dx}\,d^3}{a^3}\right)}{36015\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*polylog(3,a*x),x, algorithm="maxima")
```

```
[Out] 2/36015*(420*d^4*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/(
sqrt(a*d)*a^3) - (1470*(d*x)^(7/2)*a^3*dilog(a*x) + 420*(d*x)^(7/2)*a^3*log
(-a*d*x + d) - 5145*(d*x)^(7/2)*a^3*polylog(3, a*x) - 168*(d*x)^(5/2)*a^2*d
 - 60*(7*a^3*log(d) + 2*a^3)*(d*x)^(7/2) - 280*(d*x)^(3/2)*a*d^2 - 840*sqrt
(d*x)*d^3)/a^3)/d
```

Fricas [A]
time = 0.39, size = 279, normalized size = 1.82

$$\left[\frac{2\left(5145\,\sqrt{dx}\,a^3d^2x^3\mathrm{polylog}(3,ax)+420\,d^2\sqrt{\frac{d}{a}}\log\left(\frac{adx-2\,\sqrt{dx}\,a\sqrt{\frac{d}{a}}+d}{ax-1}\right)-2\,(735\,a^3d^2x^3\mathrm{Li}_2(ax)+210\,a^3d^2x^3\log(-ax+1)-60\,a^3d^2x^3-84\,a^2d^2x^2-140\,ad^2x-420\,d^2)\sqrt{dx}\right)}{36015\,a^3},\ \frac{2\left(5145\,\sqrt{dx}\,a^3d^2x^3\mathrm{polylog}(3,ax)+840\,d^2\sqrt{-\frac{d}{a}}\arctan\left(\frac{\sqrt{dx}\,a\sqrt{-\frac{d}{a}}}{d}\right)-2\,(735\,a^3d^2x^3\mathrm{Li}_2(ax)+210\,a^3d^2x^3\log(-ax+1)-60\,a^3d^2x^3-84\,a^2d^2x^2-140\,ad^2x-420\,d^2)\sqrt{dx}\right)}{36015\,a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*polylog(3,a*x),x, algorithm="fricas")
```

```
[Out] [2/36015*(5145*sqrt(d*x)*a^3*d^2*x^3*polylog(3, a*x) + 420*d^2*sqrt(d/a)*lo
g((a*d*x - 2*sqrt(d*x)*a*sqrt(d/a) + d)/(a*x - 1)) - 2*(735*a^3*d^2*x^3*dil
og(a*x) + 210*a^3*d^2*x^3*log(-a*x + 1) - 60*a^3*d^2*x^3 - 84*a^2*d^2*x^2 -
 140*a*d^2*x - 420*d^2)*sqrt(d*x))/a^3, 2/36015*(5145*sqrt(d*x)*a^3*d^2*x^3
*polylog(3, a*x) + 840*d^2*sqrt(-d/a)*arctan(sqrt(d*x)*a*sqrt(-d/a)/d) - 2*
```

```
(735*a^3*d^2*x^3*dilog(a*x) + 210*a^3*d^2*x^3*log(-a*x + 1) - 60*a^3*d^2*x^
3 - 84*a^2*d^2*x^2 - 140*a*d^2*x - 420*d^2)*sqrt(d*x))/a^3]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{5}{2}} \operatorname{Li}_3(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*polylog(3,a*x),x)
[Out] Integral((d*x)**(5/2)*polylog(3, a*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*polylog(3,a*x),x, algorithm="giac")
[Out] integrate((d*x)^(5/2)*polylog(3, a*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{5/2}\,\mathrm{polylog}(3,a\,x)\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*polylog(3, a*x),x)
[Out] int((d*x)^(5/2)*polylog(3, a*x), x)
```

3.66 $\int (dx)^{3/2}\textbf{PolyLog}(3,ax)\,dx$

Optimal. Leaf size=136

$$\frac{16d\sqrt{dx}}{125a^2}+\frac{16(dx)^{3/2}}{375a}+\frac{16(dx)^{5/2}}{625d}-\frac{16d^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/2}}-\frac{8(dx)^{5/2}\log(1-ax)}{125d}-\frac{4(dx)^{5/2}\text{PolyLog}(2,$$

```
[Out] 16/375*(d*x)^(3/2)/a+16/625*(d*x)^(5/2)/d-16/125*d^(3/2)*arctanh(a^(1/2)*(d
*x)^(1/2)/d^(1/2))/a^(5/2)-8/125*(d*x)^(5/2)*ln(-a*x+1)/d-4/25*(d*x)^(5/2)*
polylog(2,a*x)/d+2/5*(d*x)^(5/2)*polylog(3,a*x)/d+16/125*d*(d*x)^(1/2)/a^2
```

Rubi [A]
time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6726, 2442, 52, 65, 212}

$$-\frac{16d^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/2}}+\frac{16d\sqrt{dx}}{125a^2}-\frac{4(dx)^{5/2}\text{Li}_2(ax)}{25d}+\frac{2(dx)^{5/2}\text{Li}_3(ax)}{5d}+\frac{16(dx)^{3/2}}{375a}-\frac{8(dx)^{5/2}\log(1-ax)}{125d}+\frac{16(dx)^{5/2}}{625d}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*PolyLog[3, a*x],x]
```

```
[Out] (16*d*Sqrt[d*x])/(125*a^2) + (16*(d*x)^(3/2))/(375*a) + (16*(d*x)^(5/2))/(6
25*d) - (16*d^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/(125*a^(5/2)) - (
8*(d*x)^(5/2)*Log[1 - a*x])/(125*d) - (4*(d*x)^(5/2)*PolyLog[2, a*x])/(25*d
) + (2*(d*x)^(5/2)*PolyLog[3, a*x])/(5*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int (dx)^{3/2}\text{Li}_3(ax)\,dx = \frac{2(dx)^{5/2}\text{Li}_3(ax)}{5d} - \frac{2}{5}\int (dx)^{3/2}\text{Li}_2(ax)\,dx$$

$$= -\frac{4(dx)^{5/2}\text{Li}_2(ax)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3(ax)}{5d} - \frac{4}{25}\int (dx)^{3/2}\log(1-ax)\,dx$$

$$= -\frac{8(dx)^{5/2}\log(1-ax)}{125d} - \frac{4(dx)^{5/2}\text{Li}_2(ax)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3(ax)}{5d} - \frac{(8a)\int \frac{(dx)^{5/2}}{1-ax}\,dx}{125d}$$

$$= \frac{16(dx)^{5/2}}{625d} - \frac{8(dx)^{5/2}\log(1-ax)}{125d} - \frac{4(dx)^{5/2}\text{Li}_2(ax)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3(ax)}{5d} - \frac{8}{125}\int \frac{(dx}{1-}$$

$$= \frac{16(dx)^{3/2}}{375a} + \frac{16(dx)^{5/2}}{625d} - \frac{8(dx)^{5/2}\log(1-ax)}{125d} - \frac{4(dx)^{5/2}\text{Li}_2(ax)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3(ax)}{5d}$$

$$= \frac{16d\sqrt{dx}}{125a^2} + \frac{16(dx)^{3/2}}{375a} + \frac{16(dx)^{5/2}}{625d} - \frac{8(dx)^{5/2}\log(1-ax)}{125d} - \frac{4(dx)^{5/2}\text{Li}_2(ax)}{25d} + \frac{2(dx}{}$$

$$= \frac{16d\sqrt{dx}}{125a^2} + \frac{16(dx)^{3/2}}{375a} + \frac{16(dx)^{5/2}}{625d} - \frac{8(dx)^{5/2}\log(1-ax)}{125d} - \frac{4(dx)^{5/2}\text{Li}_2(ax)}{25d} + \frac{2(dx}{}$$

$$= \frac{16d\sqrt{dx}}{125a^2} + \frac{16(dx)^{3/2}}{375a} + \frac{16(dx)^{5/2}}{625d} - \frac{16d^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/2}} - \frac{8(dx)^{5/2}\log(1-}{125d}$$

Mathematica [A]
time = 0.15, size = 88, normalized size = 0.65

$$\frac{2d\sqrt{dx}\left(4\left(\frac{30}{a^2}+\frac{10x}{a}+6x^2-\frac{30\tanh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{5/2}\sqrt{x}}-15x^2\log(1-ax)\right)-150x^2\text{PolyLog}(2,ax)+375x^2\text{PolyLog}(3,ax)\right)}{1875}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*PolyLog[3, a*x],x]
```

```
[Out] (2*d*Sqrt[d*x]*(4*(30/a^2 + (10*x)/a + 6*x^2 - (30*ArcTanh[Sqrt[a]*Sqrt[x]]
)/(a^(5/2)*Sqrt[x]) - 15*x^2*Log[1 - a*x]) - 150*x^2*PolyLog[2, a*x] + 375*
x^2*PolyLog[3, a*x]))/1875
```

Maple [A]
time = 0.07, size = 141, normalized size = 1.04

method	result
meijerg	$\frac{(dx)^{\frac{3}{2}}\left(\frac{2\sqrt{x}\,(-a)^{\frac{7}{2}}\left(168a^2x^2+280ax+840\right)}{13125a^3}+\frac{8\sqrt{x}\,(-a)^{\frac{7}{2}}\left(\ln\left(1-\sqrt{ax}\right)-\ln\left(1+\sqrt{ax}\right)\right)}{125a^3\sqrt{ax}}-\frac{8x^{\frac{5}{2}}(-a)^{\frac{7}{2}}\ln(-ax+1)}{125a}-\frac{4x^{\frac{5}{2}}(-a)^{\frac{7}{2}}\text{pol}}{25a}\right.}{x^{\frac{3}{2}}(-a)^{\frac{3}{2}}a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*polylog(3,a*x),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x)^(3/2)/x^(3/2)/(-a)^(3/2)/a*(2/13125*x^(1/2)*(-a)^(7/2)*(168*a^2*x^2+2
80*a*x+840)/a^3+8/125*x^(1/2)*(-a)^(7/2)/a^3/(a*x)^(1/2)*(ln(1-(a*x)^(1/2))
-ln(1+(a*x)^(1/2)))-8/125*x^(5/2)*(-a)^(7/2)/a*ln(-a*x+1)-4/25*x^(5/2)*(-a)
^(7/2)/a*polylog(2,a*x)+2/5*x^(5/2)*(-a)^(7/2)/a*polylog(3,a*x))
```

Maxima [A]
time = 0.48, size = 143, normalized size = 1.05

$$\frac{2\left(\frac{60\,d^3\log\left(\frac{\sqrt{dx}\,a-\sqrt{ad}}{\sqrt{dx}\,a+\sqrt{ad}}\right)}{\sqrt{ad}\,a^2}-\frac{150\,(dx)^{\frac{5}{2}}a^2\text{Li}_2(ax)+60\,(dx)^{\frac{5}{2}}a^2\log(-adx+d)-375\,(dx)^{\frac{5}{2}}a^2\text{Li}_3(ax)-12\,(5\,a^2\log(d)+2\,a^2)(dx)^{\frac{5}{2}}-40\,(dx)^{\frac{3}{2}}ad-120\,\sqrt{dx}\,d^2}{a^2}\right)}{1875\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x),x, algorithm="maxima")
```

```
[Out] 2/1875*(60*d^3*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/(sq
rt(a*d)*a^2) - (150*(d*x)^(5/2)*a^2*dilog(a*x) + 60*(d*x)^(5/2)*a^2*log(-a*
d*x + d) - 375*(d*x)^(5/2)*a^2*polylog(3, a*x) - 12*(5*a^2*log(d) + 2*a^2)*
(d*x)^(5/2) - 40*(d*x)^(3/2)*a*d - 120*sqrt(d*x)*d^2)/a^2)/d
```

Fricas [A]
time = 0.39, size = 229, normalized size = 1.68

$$\left[\frac{2\left(375\,\sqrt{dx}\,a^{2}dx^{2}\mathrm{polylog}(3,ax)+60\,d\sqrt{\frac{d}{a}}\,\log\left(\frac{adx-2\,\sqrt{dx}\,a\sqrt{\frac{d}{a}}+d}{ax-1}\right)-2\,(75\,a^{2}dx^{2}\mathrm{Li}_{2}(ax)+30\,a^{2}dx^{2}\log\left(-ax+1\right)-12\,a^{2}dx^{2}-20\,adx-60\,d)\sqrt{dx}\right)}{1875\,a^{2}},\ \frac{2\left(375\,\sqrt{dx}\,a^{2}dx^{2}\mathrm{polylog}(3,ax)+120\,d\sqrt{-\frac{d}{a}}\,\arctan\left(\frac{\sqrt{dx}\,a\sqrt{-\frac{d}{a}}}{d}\right)-2\,(75\,a^{2}dx^{2}\mathrm{Li}_{2}(ax)+30\,a^{2}dx^{2}\log\left(-ax+1\right)-12\,a^{2}dx^{2}-20\,adx-60\,d)\sqrt{dx}\right)}{1875\,a^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x),x, algorithm="fricas")
```

```
[Out] [2/1875*(375*sqrt(d*x)*a^2*d*x^2*polylog(3, a*x) + 60*d*sqrt(d/a)*log((a*d*
x - 2*sqrt(d*x)*a*sqrt(d/a) + d)/(a*x - 1)) - 2*(75*a^2*d*x^2*dilog(a*x) +
30*a^2*d*x^2*log(-a*x + 1) - 12*a^2*d*x^2 - 20*a*d*x - 60*d)*sqrt(d*x))/a^2
, 2/1875*(375*sqrt(d*x)*a^2*d*x^2*polylog(3, a*x) + 120*d*sqrt(-d/a)*arctan
(sqrt(d*x)*a*sqrt(-d/a)/d) - 2*(75*a^2*d*x^2*dilog(a*x) + 30*a^2*d*x^2*log(
-a*x + 1) - 12*a^2*d*x^2 - 20*a*d*x - 60*d)*sqrt(d*x))/a^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}}\,\mathrm{Li}_{3}\,(ax)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*polylog(3,a*x),x)
```

```
[Out] Integral((d*x)**(3/2)*polylog(3, a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*polylog(3, a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{3/2}\,\mathrm{polylog}(3,a\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*polylog(3, a*x),x)
```

```
[Out] int((d*x)^(3/2)*polylog(3, a*x), x)
```

3.67 $\int \sqrt{dx}\, \mathrm{PolyLog}(3, ax)\, dx$

Optimal. Leaf size=121

$$\frac{16\sqrt{dx}}{27a}+\frac{16(dx)^{3/2}}{81d}-\frac{16\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/2}}-\frac{8(dx)^{3/2}\log(1-ax)}{27d}-\frac{4(dx)^{3/2}\mathrm{PolyLog}(2,ax)}{9d}+\frac{2(dx)^{3}}{}$$

```
[Out] 16/81*(d*x)^(3/2)/d-8/27*(d*x)^(3/2)*ln(-a*x+1)/d-4/9*(d*x)^(3/2)*polylog(2
,a*x)/d+2/3*(d*x)^(3/2)*polylog(3,a*x)/d-16/27*arctanh(a^(1/2)*(d*x)^(1/2)/
d^(1/2))*d^(1/2)/a^(3/2)+16/27*(d*x)^(1/2)/a
```

Rubi [A]
time = 0.05, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6726, 2442, 52, 65, 212}

$$-\frac{16\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/2}}-\frac{4(dx)^{3/2}\mathrm{Li}_2(ax)}{9d}+\frac{2(dx)^{3/2}\mathrm{Li}_3(ax)}{3d}+\frac{16\sqrt{dx}}{27a}-\frac{8(dx)^{3/2}\log(1-ax)}{27d}+\frac{16(dx)^{3/2}}{81d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*PolyLog[3, a*x],x]
```

```
[Out] (16*Sqrt[d*x])/(27*a) + (16*(d*x)^(3/2))/(81*d) - (16*Sqrt[d]*ArcTanh[(Sqrt
[a]*Sqrt[d*x])/Sqrt[d]])/(27*a^(3/2)) - (8*(d*x)^(3/2)*Log[1 - a*x])/(27*d)
 - (4*(d*x)^(3/2)*PolyLog[2, a*x])/(9*d) + (2*(d*x)^(3/2)*PolyLog[3, a*x])/
(3*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{dx}\,\text{Li}_3(ax)\,dx &= \frac{2(dx)^{3/2}\text{Li}_3(ax)}{3d} - \frac{2}{3}\int \sqrt{dx}\,\text{Li}_2(ax)\,dx \\
&= -\frac{4(dx)^{3/2}\text{Li}_2(ax)}{9d} + \frac{2(dx)^{3/2}\text{Li}_3(ax)}{3d} - \frac{4}{9}\int \sqrt{dx}\,\log(1-ax)\,dx \\
&= -\frac{8(dx)^{3/2}\log(1-ax)}{27d} - \frac{4(dx)^{3/2}\text{Li}_2(ax)}{9d} + \frac{2(dx)^{3/2}\text{Li}_3(ax)}{3d} - \frac{(8a)\int \frac{(dx)^{3/2}}{1-ax}\,dx}{27d} \\
&= \frac{16(dx)^{3/2}}{81d} - \frac{8(dx)^{3/2}\log(1-ax)}{27d} - \frac{4(dx)^{3/2}\text{Li}_2(ax)}{9d} + \frac{2(dx)^{3/2}\text{Li}_3(ax)}{3d} - \frac{8}{27}\int \frac{\sqrt{dx}}{1-a
\end{aligned}$$

$$\begin{aligned}
&= \frac{16\sqrt{dx}}{27a} + \frac{16(dx)^{3/2}}{81d} - \frac{8(dx)^{3/2}\log(1-ax)}{27d} - \frac{4(dx)^{3/2}\text{Li}_2(ax)}{9d} + \frac{2(dx)^{3/2}\text{Li}_3(ax)}{3d} - \\
&= \frac{16\sqrt{dx}}{27a} + \frac{16(dx)^{3/2}}{81d} - \frac{8(dx)^{3/2}\log(1-ax)}{27d} - \frac{4(dx)^{3/2}\text{Li}_2(ax)}{9d} + \frac{2(dx)^{3/2}\text{Li}_3(ax)}{3d} - \\
&= \frac{16\sqrt{dx}}{27a} + \frac{16(dx)^{3/2}}{81d} - \frac{16\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/2}} - \frac{8(dx)^{3/2}\log(1-ax)}{27d} - \frac{4(dx)^{3/}}{9
\end{aligned}$$

Mathematica **[A]**

time = 0.15, size = 73, normalized size = 0.60

$$\frac{2}{81}\sqrt{dx}\left(4\left(\frac{6}{a}+2x-\frac{6\tanh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{a^{3/2}\sqrt{x}}-3x\log(1-ax)\right)-18x\text{PolyLog}(2,ax)+27x\text{PolyLog}(3,ax)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*PolyLog[3, a*x],x]
```

```
[Out] (2*Sqrt[d*x]*(4*(6/a + 2*x - (6*ArcTanh[Sqrt[a]*Sqrt[x]])/(a^(3/2)*Sqrt[x])
 - 3*x*Log[1 - a*x]) - 18*x*PolyLog[2, a*x] + 27*x*PolyLog[3, a*x]))/81
```

Maple [A]
time = 0.07, size = 133, normalized size = 1.10

method	result
meijerg	$\frac{\sqrt{dx}\left(\frac{2\sqrt{x}\,(-a)^{\frac{5}{2}}(40ax+120)}{405a^2}+\frac{8\sqrt{x}\,(-a)^{\frac{5}{2}}\left(\ln\left(1-\sqrt{ax}\right)-\ln\left(1+\sqrt{ax}\right)\right)}{27a^2\sqrt{ax}}-\frac{8x^{\frac{3}{2}}(-a)^{\frac{5}{2}}\ln(-ax+1)}{27a}-\frac{4x^{\frac{3}{2}}(-a)^{\frac{5}{2}}\text{polylog}(2,ax)}{9a}\right.}{\sqrt{x}\,\sqrt{-a}\,a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*polylog(3,a*x),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x)^(1/2)/x^(1/2)/(-a)^(1/2)/a*(2/405*x^(1/2)*(-a)^(5/2)*(40*a*x+120)/a^2
+8/27*x^(1/2)*(-a)^(5/2)/a^2/(a*x)^(1/2)*(ln(1-(a*x)^(1/2))-ln(1+(a*x)^(1/2
)))-8/27*x^(3/2)*(-a)^(5/2)/a*ln(-a*x+1)-4/9*x^(3/2)*(-a)^(5/2)/a*polylog(2
,a*x)+2/3*x^(3/2)*(-a)^(5/2)/a*polylog(3,a*x))
```

Maxima [A]
time = 0.49, size = 122, normalized size = 1.01

$$\frac{2\left(\frac{12\,d^2\log\left(\frac{\sqrt{dx}\,a-\sqrt{ad}}{\sqrt{dx}\,a+\sqrt{ad}}\right)}{\sqrt{ad}\,a}-\frac{18\,(dx)^{\frac{3}{2}}a\text{Li}_2(ax)+12\,(dx)^{\frac{3}{2}}a\log(-adx+d)-27\,(dx)^{\frac{3}{2}}a\text{Li}_3(ax)-4\,(dx)^{\frac{3}{2}}(3\,a\log(d)+2\,a)-24\,\sqrt{dx}\,d}{a}\right)}{81\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x),x, algorithm="maxima")
```

```
[Out] 2/81*(12*d^2*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/(sqrt
(a*d)*a) - (18*(d*x)^(3/2)*a*dilog(a*x) + 12*(d*x)^(3/2)*a*log(-a*d*x + d)
- 27*(d*x)^(3/2)*a*polylog(3, a*x) - 4*(d*x)^(3/2)*(3*a*log(d) + 2*a) - 24*
sqrt(d*x)*d)/a)/d
```

Fricas [A]
time = 0.38, size = 173, normalized size = 1.43

$$\left[\frac{2\left(27\sqrt{dx}\,ax\mathrm{polylog}(3,ax) - 2\,(9\,ax\mathrm{Li}_2(ax) + 6\,ax\log(-ax+1) - 4\,ax - 12)\sqrt{dx} + 12\sqrt{\frac{d}{a}}\log\left(\frac{adx - 2\sqrt{dx}\,a\sqrt{\frac{d}{a}} + d}{ax-1}\right)\right)}{81\,a}, \frac{2\left(27\sqrt{dx}\,ax\mathrm{polylog}(3,ax) - 2\,(9\,ax\mathrm{Li}_2(ax) + 6\,ax\log(-ax+1) - 4\,ax - 12)\sqrt{dx} + 24\sqrt{-\frac{d}{a}}\arctan\left(\frac{\sqrt{dx}\,a\sqrt{-\frac{d}{a}}}{d}\right)\right)}{81\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x),x, algorithm="fricas")
```

```
[Out] [2/81*(27*sqrt(d*x)*a*x*polylog(3, a*x) - 2*(9*a*x*dilog(a*x) + 6*a*x*log(-
a*x + 1) - 4*a*x - 12)*sqrt(d*x) + 12*sqrt(d/a)*log((a*d*x - 2*sqrt(d*x)*a*
sqrt(d/a) + d)/(a*x - 1)))/a, 2/81*(27*sqrt(d*x)*a*x*polylog(3, a*x) - 2*(9
*a*x*dilog(a*x) + 6*a*x*log(-a*x + 1) - 4*a*x - 12)*sqrt(d*x) + 24*sqrt(-d/
a)*arctan(sqrt(d*x)*a*sqrt(-d/a)/d))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\,\mathrm{Li}_3(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*polylog(3,a*x),x)
```

```
[Out] Integral(sqrt(d*x)*polylog(3, a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*polylog(3, a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,x}\,\mathrm{polylog}(3,a\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*polylog(3, a*x),x)
```

```
[Out] int((d*x)^(1/2)*polylog(3, a*x), x)
```

3.68 $\int \frac{\text{PolyLog}(3,ax)}{\sqrt{dx}} \, dx$

Optimal. Leaf size=97

$$\frac{16\sqrt{dx}}{d}-\frac{16\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{a}\sqrt{d}}-\frac{8\sqrt{dx}\log(1-ax)}{d}-\frac{4\sqrt{dx}\,\text{PolyLog}(2,ax)}{d}+\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax)}{d}$$

```
[Out] -16*arctanh(a^(1/2)*(d*x)^(1/2)/d^(1/2))/a^(1/2)/d^(1/2)+16*(d*x)^(1/2)/d-8
*ln(-a*x+1)*(d*x)^(1/2)/d-4*polylog(2,a*x)*(d*x)^(1/2)/d+2*polylog(3,a*x)*(
d*x)^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6726, 2442, 52, 65, 212}

$$-\frac{4\sqrt{dx}\,\text{Li}_2(ax)}{d}+\frac{2\sqrt{dx}\,\text{Li}_3(ax)}{d}-\frac{8\sqrt{dx}\log(1-ax)}{d}-\frac{16\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{a}\sqrt{d}}+\frac{16\sqrt{dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x]/Sqrt[d*x],x]
```

```
[Out] (16*Sqrt[d*x])/d - (16*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[a]*Sqrt[
d]) - (8*Sqrt[d*x]*Log[1 - a*x])/d - (4*Sqrt[d*x]*PolyLog[2, a*x])/d + (2*S
qrt[d*x]*PolyLog[3, a*x])/d
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{Li}_3(ax)}{\sqrt{dx}}\,dx &= \frac{2\sqrt{dx}\,\text{Li}_3(ax)}{d}-2\int \frac{\text{Li}_2(ax)}{\sqrt{dx}}\,dx\\
&= -\frac{4\sqrt{dx}\,\text{Li}_2(ax)}{d}+\frac{2\sqrt{dx}\,\text{Li}_3(ax)}{d}-4\int \frac{\log(1-ax)}{\sqrt{dx}}\,dx\\
&= -\frac{8\sqrt{dx}\,\log(1-ax)}{d}-\frac{4\sqrt{dx}\,\text{Li}_2(ax)}{d}+\frac{2\sqrt{dx}\,\text{Li}_3(ax)}{d}-\frac{(8a)\int \frac{\sqrt{dx}}{1-ax}\,dx}{d}\\
&= \frac{16\sqrt{dx}}{d}-\frac{8\sqrt{dx}\,\log(1-ax)}{d}-\frac{4\sqrt{dx}\,\text{Li}_2(ax)}{d}+\frac{2\sqrt{dx}\,\text{Li}_3(ax)}{d}-8\int \frac{1}{\sqrt{dx}\,(1-ax)}\,dx\\
&= \frac{16\sqrt{dx}}{d}-\frac{8\sqrt{dx}\,\log(1-ax)}{d}-\frac{4\sqrt{dx}\,\text{Li}_2(ax)}{d}+\frac{2\sqrt{dx}\,\text{Li}_3(ax)}{d}-\frac{16\text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}}\,dx,x
\end{aligned}
$$

$$
= \frac{16\sqrt{dx}}{d}-\frac{16\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{a}\,\sqrt{d}}-\frac{8\sqrt{dx}\,\log(1-ax)}{d}-\frac{4\sqrt{dx}\,\text{Li}_2(ax)}{d}+\frac{2\sqrt{dx}\,\text{Li}_3(ax
$$

Mathematica [A]

time = 0.10, size = 57, normalized size = 0.59

$$\frac{2x\left(8-\frac{8\tanh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{x}}-4\log(1-ax)-2\mathrm{PolyLog}(2,ax)+\mathrm{PolyLog}(3,ax)\right)}{\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x]/Sqrt[d*x],x]
```

```
[Out] (2*x*(8 - (8*ArcTanh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[x]) - 4*Log[1 - a*x] -
 2*PolyLog[2, a*x] + PolyLog[3, a*x]))/Sqrt[d*x]
```

Maple [A]
time = 0.08, size = 127, normalized size = 1.31

method	result
meijerg	$\frac{\sqrt{x}\,\sqrt{-a}\,\left(\frac{16\sqrt{x}\,(-a)^{\frac{3}{2}}}{a}+\frac{8\sqrt{x}\,(-a)^{\frac{3}{2}}\left(\ln\left(1-\sqrt{ax}\right)-\ln\left(1+\sqrt{ax}\right)\right)}{a\sqrt{ax}}-\frac{8\sqrt{x}\,(-a)^{\frac{3}{2}}\ln(-ax+1)}{a}-\frac{4\sqrt{x}\,(-a)^{\frac{3}{2}}\,\mathrm{polylog}(2}{a}\right.}{\sqrt{dx}\,a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(d*x)^(1/2)*x^(1/2)*(-a)^(1/2)/a*(16*x^(1/2)*(-a)^(3/2)/a+8*x^(1/2)*(-a)^
(3/2)/a/(a*x)^(1/2)*(ln(1-(a*x)^(1/2))-ln(1+(a*x)^(1/2)))-8*x^(1/2)*(-a)^(3
/2)/a*ln(-a*x+1)-4*x^(1/2)*(-a)^(3/2)/a*polylog(2,a*x)+2*x^(1/2)*(-a)^(3/2)
/a*polylog(3,a*x))
```

Maxima [A]
time = 0.49, size = 94, normalized size = 0.97

$$\frac{2\left(4\sqrt{dx}\left(\log(d)+2\right)-2\sqrt{dx}\,\mathrm{Li}_2(ax)-4\sqrt{dx}\,\log(-adx+d)+\frac{4\,d\log\left(\frac{\sqrt{dx}\,a-\sqrt{ad}}{\sqrt{dx}\,a+\sqrt{ad}}\right)}{\sqrt{ad}}+\sqrt{dx}\,\mathrm{Li}_3(ax)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(4*sqrt(d*x)*(log(d) + 2) - 2*sqrt(d*x)*dilog(a*x) - 4*sqrt(d*x)*log(-a*d
*x + d) + 4*d*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/sqrt
(a*d) + sqrt(d*x)*polylog(3, a*x))/d
```

Fricas [A]
time = 0.38, size = 161, normalized size = 1.66

$$\left[\frac{2\left(\sqrt{dx}\,a\mathrm{polylog}(3,ax)-2\sqrt{dx}\,(a\mathrm{Li}_2(ax)+2\,a\log\left(-ax+1\right)-4\,a)+4\sqrt{ad}\,\log\left(\frac{adx-2\sqrt{ad}\sqrt{dx}+d}{ax-1}\right)\right)}{ad}, \frac{2\left(\sqrt{dx}\,a\mathrm{polylog}(3,ax)-2\sqrt{dx}\,(a\mathrm{Li}_2(ax)+2\,a\log\left(-ax+1\right)-4\,a)+8\sqrt{-ad}\,\arctan\left(\frac{\sqrt{-ad}\sqrt{dx}}{adx}\right)\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [2*(sqrt(d*x)*a*polylog(3, a*x) - 2*sqrt(d*x)*(a*dilog(a*x) + 2*a*log(-a*x
+ 1) - 4*a) + 4*sqrt(a*d)*log((a*d*x - 2*sqrt(a*d)*sqrt(d*x) + d)/(a*x - 1)
))/(a*d), 2*(sqrt(d*x)*a*polylog(3, a*x) - 2*sqrt(d*x)*(a*dilog(a*x) + 2*a*
log(-a*x + 1) - 4*a) + 8*sqrt(-a*d)*arctan(sqrt(-a*d)*sqrt(d*x)/(a*d*x)))/(
a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_3\,(ax)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)**(1/2),x)
```

```
[Out] Integral(polylog(3, a*x)/sqrt(d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x)/sqrt(d*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(3,a\,x)}{\sqrt{d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x)/(d*x)^(1/2),x)
```

```
[Out] int(polylog(3, a*x)/(d*x)^(1/2), x)
```

3.69 $\int \frac{\text{PolyLog}(3,ax)}{(dx)^{3/2}} dx$

Optimal. Leaf size=85

$$\frac{16\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{8\log(1-ax)}{d\sqrt{dx}}-\frac{4\text{PolyLog}(2,ax)}{d\sqrt{dx}}-\frac{2\text{PolyLog}(3,ax)}{d\sqrt{dx}}$$

```
[Out] 16*arctanh(a^(1/2)*(d*x)^(1/2)/d^(1/2))*a^(1/2)/d^(3/2)+8*ln(-a*x+1)/d/(d*x
)^(1/2)-4*polylog(2,a*x)/d/(d*x)^(1/2)-2*polylog(3,a*x)/d/(d*x)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6726, 2442, 65, 212}

$$\frac{16\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{4\text{Li}_2(ax)}{d\sqrt{dx}}-\frac{2\text{Li}_3(ax)}{d\sqrt{dx}}+\frac{8\log(1-ax)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x]/(d*x)^(3/2),x]
```

```
[Out] (16*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (8*Log[1 - a*x]
)/(d*Sqrt[d*x]) - (4*PolyLog[2, a*x])/(d*Sqrt[d*x]) - (2*PolyLog[3, a*x])/(
d*Sqrt[d*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
```

```
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax)}{(dx)^{3/2}}\,dx &= -\frac{2\text{Li}_3(ax)}{d\sqrt{dx}} + 2\int \frac{\text{Li}_2(ax)}{(dx)^{3/2}}\,dx \\
&= -\frac{4\text{Li}_2(ax)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax)}{d\sqrt{dx}} - 4\int \frac{\log(1-ax)}{(dx)^{3/2}}\,dx \\
&= \frac{8\log(1-ax)}{d\sqrt{dx}} - \frac{4\text{Li}_2(ax)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax)}{d\sqrt{dx}} + \frac{(8a)\int \frac{1}{\sqrt{dx}\,(1-ax)}\,dx}{d} \\
&= \frac{8\log(1-ax)}{d\sqrt{dx}} - \frac{4\text{Li}_2(ax)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax)}{d\sqrt{dx}} + \frac{(16a)\text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}}\,dx, x, \sqrt{dx}\right)}{d^2} \\
&= \frac{16\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{8\log(1-ax)}{d\sqrt{dx}} - \frac{4\text{Li}_2(ax)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax)}{d\sqrt{dx}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 58, normalized size = 0.68

$$\frac{2x\left(8\sqrt{a}\,\sqrt{x}\,\tanh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)+4\log(1-ax)-2\text{PolyLog}(2,ax)-\text{PolyLog}(3,ax)\right)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x]/(d*x)^(3/2),x]
```

```
[Out] (2*x*(8*Sqrt[a]*Sqrt[x]*ArcTanh[Sqrt[a]*Sqrt[x]] + 4*Log[1 - a*x] - 2*PolyL
og[2, a*x] - PolyLog[3, a*x]))/(d*x)^(3/2)
```

Maple [A]
time = 0.07, size = 111, normalized size = 1.31

method	result
meijerg	$\frac{x^{\frac{3}{2}}(-a)^{\frac{3}{2}}\left(-\frac{8\sqrt{x}\,\sqrt{-a}\,\left(\ln\left(1-\sqrt{ax}\right)-\ln\left(1+\sqrt{ax}\right)\right)}{\sqrt{ax}}+\frac{8\sqrt{-a}\,\ln(-ax+1)}{\sqrt{x}\,a}-\frac{4\sqrt{-a}\,\operatorname{polylog}(2,ax)}{\sqrt{x}\,a}-\frac{2\sqrt{-a}\,\operatorname{polylog}(3,}{\sqrt{x}\,a}\right.}{(dx)^{\frac{3}{2}}a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(d*x)^(3/2)*x^(3/2)*(-a)^(3/2)/a*(-8*x^(1/2)*(-a)^(1/2)/(a*x)^(1/2)*(ln(1
-(a*x)^(1/2))-ln(1+(a*x)^(1/2)))+8/x^(1/2)*(-a)^(1/2)/a*ln(-a*x+1)-4/x^(1/2
)*(-a)^(1/2)/a*polylog(2,a*x)-2/x^(1/2)*(-a)^(1/2)/a*polylog(3,a*x))
```

Maxima [A]
time = 0.47, size = 78, normalized size = 0.92

$$-\frac{2\left(\frac{4\,a\log\left(\frac{\sqrt{dx}\,a-\sqrt{ad}}{\sqrt{dx}\,a+\sqrt{ad}}\right)}{\sqrt{ad}}+\frac{2\,\mathrm{Li}_2(ax)-4\,\log(-adx+d)+4\,\log(d)+\mathrm{Li}_3(ax)}{\sqrt{dx}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*(4*a*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/sqrt(a*d)
+ (2*dilog(a*x) - 4*log(-a*d*x + d) + 4*log(d) + polylog(3, a*x))/sqrt(d*x)
)/d
```

Fricas [A]
time = 0.39, size = 156, normalized size = 1.84

$$\left[\frac{2\left(4\,dx\sqrt{\frac{a}{d}}\,\log\left(\frac{ax+2\,\sqrt{dx}\,\sqrt{\frac{a}{d}}+1}{ax-1}\right)-2\,\sqrt{dx}\,\left(\mathrm{Li}_2(ax)-2\,\log\left(-ax+1\right)\right)-\sqrt{dx}\,\operatorname{polylog}(3,ax)\right)}{d^2x},-\frac{2\left(8\,dx\sqrt{-\frac{a}{d}}\,\arctan\left(\frac{\sqrt{dx}\,\sqrt{-\frac{a}{d}}}{ax}\right)+2\,\sqrt{dx}\,\left(\mathrm{Li}_2(ax)-2\,\log\left(-ax+1\right)\right)+\sqrt{dx}\,\operatorname{polylog}(3,ax)\right)}{d^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [2*(4*d*x*sqrt(a/d)*log((a*x + 2*sqrt(d*x)*sqrt(a/d) + 1)/(a*x - 1)) - 2*sq
rt(d*x)*(dilog(a*x) - 2*log(-a*x + 1)) - sqrt(d*x)*polylog(3, a*x))/(d^2*x)
, -2*(8*d*x*sqrt(-a/d)*arctan(sqrt(d*x)*sqrt(-a/d)/(a*x)) + 2*sqrt(d*x)*(di
log(a*x) - 2*log(-a*x + 1)) + sqrt(d*x)*polylog(3, a*x))/(d^2*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_3(ax)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)**(3/2),x)
```

```
[Out] Integral(polylog(3, a*x)/(d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x)/(d*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(3,a\,x)}{(d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x)/(d*x)^(3/2),x)
```

```
[Out] int(polylog(3, a*x)/(d*x)^(3/2), x)
```

3.70 $\int \frac{\text{PolyLog}(3,ax)}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=108

$$-\frac{16a}{27d^2\sqrt{dx}}+\frac{16a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}}+\frac{8\log(1-ax)}{27d(dx)^{3/2}}-\frac{4\text{PolyLog}(2,ax)}{9d(dx)^{3/2}}-\frac{2\text{PolyLog}(3,ax)}{3d(dx)^{3/2}}$$

```
[Out] 16/27*a^(3/2)*arctanh(a^(1/2)*(d*x)^(1/2)/d^(1/2))/d^(5/2)+8/27*ln(-a*x+1)/
d/(d*x)^(3/2)-4/9*polylog(2,a*x)/d/(d*x)^(3/2)-2/3*polylog(3,a*x)/d/(d*x)^(
3/2)-16/27*a/d^2/(d*x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6726, 2442, 53, 65, 212}

$$\frac{16a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}}-\frac{16a}{27d^2\sqrt{dx}}-\frac{4\text{Li}_2(ax)}{9d(dx)^{3/2}}-\frac{2\text{Li}_3(ax)}{3d(dx)^{3/2}}+\frac{8\log(1-ax)}{27d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x]/(d*x)^(5/2),x]
```

```
[Out] (-16*a)/(27*d^2*Sqrt[d*x]) + (16*a^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d
]])/(27*d^(5/2)) + (8*Log[1 - a*x])/(27*d*(d*x)^(3/2)) - (4*PolyLog[2, a*x]
)/(9*d*(d*x)^(3/2)) - (2*PolyLog[3, a*x])/(3*d*(d*x)^(3/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax)}{(dx)^{5/2}}\,dx &= -\frac{2\text{Li}_3(ax)}{3d(dx)^{3/2}}+\frac{2}{3}\int \frac{\text{Li}_2(ax)}{(dx)^{5/2}}\,dx\\
&= -\frac{4\text{Li}_2(ax)}{9d(dx)^{3/2}}-\frac{2\text{Li}_3(ax)}{3d(dx)^{3/2}}-\frac{4}{9}\int \frac{\log(1-ax)}{(dx)^{5/2}}\,dx\\
&= \frac{8\log(1-ax)}{27d(dx)^{3/2}}-\frac{4\text{Li}_2(ax)}{9d(dx)^{3/2}}-\frac{2\text{Li}_3(ax)}{3d(dx)^{3/2}}+\frac{(8a)\int \frac{1}{(dx)^{3/2}(1-ax)}\,dx}{27d}\\
&= -\frac{16a}{27d^2\sqrt{dx}}+\frac{8\log(1-ax)}{27d(dx)^{3/2}}-\frac{4\text{Li}_2(ax)}{9d(dx)^{3/2}}-\frac{2\text{Li}_3(ax)}{3d(dx)^{3/2}}+\frac{(8a^2)\int \frac{1}{\sqrt{dx}\,(1-ax)}\,dx}{27d^2}\\
&= -\frac{16a}{27d^2\sqrt{dx}}+\frac{8\log(1-ax)}{27d(dx)^{3/2}}-\frac{4\text{Li}_2(ax)}{9d(dx)^{3/2}}-\frac{2\text{Li}_3(ax)}{3d(dx)^{3/2}}+\frac{(16a^2)\,\text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}}\,dx,x,\sqrt{dx}\right)}{27d^3}\\
&= -\frac{16a}{27d^2\sqrt{dx}}+\frac{16a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}}+\frac{8\log(1-ax)}{27d(dx)^{3/2}}-\frac{4\text{Li}_2(ax)}{9d(dx)^{3/2}}-\frac{2\text{Li}_3(ax)}{3d(dx)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 64, normalized size = 0.59

$$-\frac{2x\left(8ax-8a^{3/2}x^{3/2}\tanh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)-4\log(1-ax)+6\text{PolyLog}(2,ax)+9\text{PolyLog}(3,ax)\right)}{27(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x]/(d*x)^(5/2),x]
```

```
[Out] (-2*x*(8*a*x - 8*a^(3/2)*x^(3/2)*ArcTanh[Sqrt[a]*Sqrt[x]] - 4*Log[1 - a*x]
+ 6*PolyLog[2, a*x] + 9*PolyLog[3, a*x]))/(27*(d*x)^(5/2))
```

Maple [A]
time = 0.07, size = 122, normalized size = 1.13

method	result	s
meijerg	$\frac{x^{\frac{5}{2}}(-a)^{\frac{5}{2}}\left(-\frac{16}{27\sqrt{x}\,\sqrt{-a}}-\frac{8\sqrt{x}\,a\left(\ln\left(1-\sqrt{ax}\right)-\ln\left(1+\sqrt{ax}\right)\right)}{27\sqrt{-a}\,\sqrt{ax}}+\frac{8\ln(-ax+1)}{27x^{\frac{3}{2}}\sqrt{-a}\,a}-\frac{4\operatorname{polylog}(2,ax)}{9x^{\frac{3}{2}}\sqrt{-a}\,a}-\frac{2\operatorname{polylog}(3,ax)}{3x^{\frac{3}{2}}\sqrt{-a}\,a}\right)}{(dx)^{\frac{5}{2}}a}$	

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(d*x)^(5/2)*x^(5/2)*(-a)^(5/2)/a*(-16/27/x^(1/2)/(-a)^(1/2)-8/27*x^(1/2)/
(-a)^(1/2)*a/(a*x)^(1/2)*(ln(1-(a*x)^(1/2))-ln(1+(a*x)^(1/2)))+8/27/x^(3/2)
/(-a)^(1/2)/a*ln(-a*x+1)-4/9/x^(3/2)/(-a)^(1/2)/a*polylog(2,a*x)-2/3/x^(3/2
)/(-a)^(1/2)/a*polylog(3,a*x))
```

Maxima [A]
time = 0.47, size = 97, normalized size = 0.90

$$-\frac{2\left(\frac{4\,a^2\log\left(\frac{\sqrt{dx}\,a-\sqrt{ad}}{\sqrt{dx}\,a+\sqrt{ad}}\right)}{\sqrt{ad}\,d}+\frac{8\,adx+6\,d\mathrm{Li}_2(ax)-4\,d\log(-adx+d)+4\,d\log(d)+9\,d\mathrm{Li}_3(ax)}{(dx)^{\frac{3}{2}}d}\right)}{27\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/27*(4*a^2*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/(sqrt
(a*d)*d) + (8*a*d*x + 6*d*dilog(a*x) - 4*d*log(-a*d*x + d) + 4*d*log(d) + 9
*d*polylog(3, a*x))/((d*x)^(3/2)*d))/d
```

Fricas [A]
time = 0.40, size = 175, normalized size = 1.62

$$\left[\frac{2\left(4\,adx^2\sqrt{\frac{a}{d}}\,\log\left(\frac{ax+2\sqrt{dx}\sqrt{\frac{a}{d}}+1}{ax-1}\right)-2\left(4\,ax+3\,\mathrm{Li}_2(ax)-2\,\log\left(-ax+1\right)\right)\sqrt{dx}-9\sqrt{dx}\,\operatorname{polylog}(3,ax)\right)}{27\,d^3x^2},\,-\frac{2\left(8\,adx^2\sqrt{-\frac{a}{d}}\,\arctan\left(\frac{\sqrt{dx}\sqrt{-\frac{a}{d}}}{ax}\right)+2\left(4\,ax+3\,\mathrm{Li}_2(ax)-2\,\log\left(-ax+1\right)\right)\sqrt{dx}+9\sqrt{dx}\,\operatorname{polylog}(3,ax)\right)}{27\,d^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [2/27*(4*a*d*x^2*sqrt(a/d)*log((a*x + 2*sqrt(d*x)*sqrt(a/d) + 1)/(a*x - 1))
 - 2*(4*a*x + 3*dilog(a*x) - 2*log(-a*x + 1))*sqrt(d*x) - 9*sqrt(d*x)*polyl
og(3, a*x))/(d^3*x^2), -2/27*(8*a*d*x^2*sqrt(-a/d)*arctan(sqrt(d*x)*sqrt(-a
/d)/(a*x)) + 2*(4*a*x + 3*dilog(a*x) - 2*log(-a*x + 1))*sqrt(d*x) + 9*sqrt(
d*x)*polylog(3, a*x))/(d^3*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_3(ax)}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)**(5/2),x)
```

```
[Out] Integral(polylog(3, a*x)/(d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x)/(d*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(3,a\,x)}{(d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x)/(d*x)^(5/2),x)
```

```
[Out] int(polylog(3, a*x)/(d*x)^(5/2), x)
```

3.71 $\int \frac{\text{PolyLog}(3,ax)}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{16a}{375d^2(dx)^{3/2}}-\frac{16a^2}{125d^3\sqrt{dx}}+\frac{16a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}}+\frac{8\log(1-ax)}{125d(dx)^{5/2}}-\frac{4\text{PolyLog}(2,ax)}{25d(dx)^{5/2}}-\frac{2\text{PolyLog}(3,ax)}{5d(dx)^{5/2}}$$

```
[Out] -16/375*a/d^2/(d*x)^(3/2)+16/125*a^(5/2)*arctanh(a^(1/2)*(d*x)^(1/2)/d^(1/2
))/d^(7/2)+8/125*ln(-a*x+1)/d/(d*x)^(5/2)-4/25*polylog(2,a*x)/d/(d*x)^(5/2)
-2/5*polylog(3,a*x)/d/(d*x)^(5/2)-16/125*a^2/d^3/(d*x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6726, 2442, 53, 65, 212}

$$\frac{16a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}}-\frac{16a^2}{125d^3\sqrt{dx}}-\frac{16a}{375d^2(dx)^{3/2}}-\frac{4\text{Li}_2(ax)}{25d(dx)^{5/2}}-\frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}}+\frac{8\log(1-ax)}{125d(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x]/(d*x)^(7/2),x]
```

```
[Out] (-16*a)/(375*d^2*(d*x)^(3/2)) - (16*a^2)/(125*d^3*Sqrt[d*x]) + (16*a^(5/2)*
ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/(125*d^(7/2)) + (8*Log[1 - a*x])/(125
*d*(d*x)^(5/2)) - (4*PolyLog[2, a*x])/(25*d*(d*x)^(5/2)) - (2*PolyLog[3, a*
x])/(5*d*(d*x)^(5/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\text{Li}_3(ax)}{(dx)^{7/2}}\,dx = -\frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}} + \frac{2}{5}\int \frac{\text{Li}_2(ax)}{(dx)^{7/2}}\,dx$$

$$= -\frac{4\text{Li}_2(ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}} - \frac{4}{25}\int \frac{\log(1-ax)}{(dx)^{7/2}}\,dx$$

$$= \frac{8\log(1-ax)}{125d(dx)^{5/2}} - \frac{4\text{Li}_2(ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}} + \frac{(8a)\int \frac{1}{(dx)^{5/2}(1-ax)}\,dx}{125d}$$

$$= -\frac{16a}{375d^2(dx)^{3/2}} + \frac{8\log(1-ax)}{125d(dx)^{5/2}} - \frac{4\text{Li}_2(ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}} + \frac{(8a^2)\int \frac{1}{(dx)^{3/2}(1-ax)}\,dx}{125d^2}$$

$$= -\frac{16a}{375d^2(dx)^{3/2}} - \frac{16a^2}{125d^3\sqrt{dx}} + \frac{8\log(1-ax)}{125d(dx)^{5/2}} - \frac{4\text{Li}_2(ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}} + \frac{(8a^3)\int \frac{1}{\sqrt{dx}\,(1-\cdots}}{125d^3}$$

$$= -\frac{16a}{375d^2(dx)^{3/2}} - \frac{16a^2}{125d^3\sqrt{dx}} + \frac{8\log(1-ax)}{125d(dx)^{5/2}} - \frac{4\text{Li}_2(ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}} + \frac{(16a^3)\,\text{Subst}\Big(\int \cdots}{\cdots}$$

$$= -\frac{16a}{375d^2(dx)^{3/2}} - \frac{16a^2}{125d^3\sqrt{dx}} + \frac{16a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}} + \frac{8\log(1-ax)}{125d(dx)^{5/2}} - \frac{4\text{Li}_2(ax)}{25d(dx)^{5/2}}$$

Mathematica [A]
time = 0.10, size = 72, normalized size = 0.58

$$-\frac{2x\left(8ax+24a^2x^2-24a^{5/2}x^{5/2}\tanh^{-1}\left(\sqrt{a}\sqrt{x}\right)-12\log(1-ax)+30\text{PolyLog}(2,ax)+75\text{PolyLog}(3,ax)\right)}{375(dx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x]/(d*x)^(7/2),x]
```

```
[Out] (-2*x*(8*a*x + 24*a^2*x^2 - 24*a^(5/2)*x^(5/2)*ArcTanh[Sqrt[a]*Sqrt[x]] - 1
2*Log[1 - a*x] + 30*PolyLog[2, a*x] + 75*PolyLog[3, a*x]))/(375*(d*x)^(7/2)
)
```

Maple [A]
time = 0.09, size = 135, normalized size = 1.08

method	result
meijerg	$\frac{x^{\frac{7}{2}}(-a)^{\frac{7}{2}}\left(-\frac{16}{375x^{\frac{3}{2}}(-a)^{\frac{3}{2}}}-\frac{16a}{125\sqrt{x}\,(-a)^{\frac{3}{2}}}-\frac{8\sqrt{x}\,a^2\left(\ln\left(1-\sqrt{ax}\right)-\ln\left(1+\sqrt{ax}\right)\right)}{125(-a)^{\frac{3}{2}}\sqrt{ax}}+\frac{8\ln(-ax+1)}{125x^{\frac{5}{2}}(-a)^{\frac{3}{2}}a}-\frac{4\,\text{polylog}(2,ax)}{25x^{\frac{5}{2}}(-a)^{\frac{3}{2}}a}-\frac{2\,\text{polylog}}{5x^{\frac{5}{2}}(-}\right.}{(dx)^{\frac{7}{2}}a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x)/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(d*x)^(7/2)*x^(7/2)*(-a)^(7/2)/a*(-16/375/x^(3/2)/(-a)^(3/2)-16/125/x^(1/
2)/(-a)^(3/2)*a-8/125*x^(1/2)/(-a)^(3/2)*a^2/(a*x)^(1/2)*(ln(1-(a*x)^(1/2))
-ln(1+(a*x)^(1/2)))+8/125/x^(5/2)/(-a)^(3/2)/a*ln(-a*x+1)-4/25/x^(5/2)/(-a)
^(3/2)/a*polylog(2,a*x)-2/5/x^(5/2)/(-a)^(3/2)/a*polylog(3,a*x))
```

Maxima [A]
time = 0.48, size = 118, normalized size = 0.94

$$-\frac{2\left(\frac{12\,a^3\log\left(\frac{\sqrt{dx}\,a-\sqrt{ad}}{\sqrt{dx}\,a+\sqrt{ad}}\right)}{\sqrt{ad}\,d^2}+\frac{24\,a^2d^2x^2+8\,ad^2x+30\,d^2\text{Li}_2(ax)-12\,d^2\log(-adx+d)+12\,d^2\log(d)+75\,d^2\text{Li}_3(ax)}{(dx)^{\frac{5}{2}}d^2}\right)}{375\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/375*(12*a^3*log((sqrt(d*x)*a - sqrt(a*d))/(sqrt(d*x)*a + sqrt(a*d)))/(sq
rt(a*d)*d^2) + (24*a^2*d^2*x^2 + 8*a*d^2*x + 30*d^2*dilog(a*x) - 12*d^2*log
(-a*d*x + d) + 12*d^2*log(d) + 75*d^2*polylog(3, a*x))/((d*x)^(5/2)*d^2))/d
```

Fricas [A]
time = 0.43, size = 195, normalized size = 1.56

$$\left[\frac{2\left(12\,a^{2}dx^{3}\sqrt{\frac{a}{d}}\log\left(\frac{ax+2\sqrt{dx}\sqrt{\frac{a}{d}}+1}{ax-1}\right)-2\left(12\,a^{2}x^{2}+4\,ax+15\,\mathrm{Li}_2(ax)-6\,\log\left(-ax+1\right)\right)\sqrt{dx}-75\,\sqrt{dx}\,\mathrm{polylog}(3,ax)\right)}{375\,d^{4}x^{3}},\ -\frac{2\left(24\,a^{2}dx^{3}\sqrt{-\frac{a}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{-\frac{a}{d}}}{ax}\right)+2\left(12\,a^{2}x^{2}+4\,ax+15\,\mathrm{Li}_2(ax)-6\,\log\left(-ax+1\right)\right)\sqrt{dx}+75\,\sqrt{dx}\,\mathrm{polylog}(3,ax)\right)}{375\,d^{4}x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(7/2),x, algorithm="fricas")
[Out] [2/375*(12*a^2*d*x^3*sqrt(a/d)*log((a*x + 2*sqrt(d*x)*sqrt(a/d) + 1)/(a*x -
 1)) - 2*(12*a^2*x^2 + 4*a*x + 15*dilog(a*x) - 6*log(-a*x + 1))*sqrt(d*x) -
 75*sqrt(d*x)*polylog(3, a*x))/(d^4*x^3), -2/375*(24*a^2*d*x^3*sqrt(-a/d)*a
rctan(sqrt(d*x)*sqrt(-a/d)/(a*x)) + 2*(12*a^2*x^2 + 4*a*x + 15*dilog(a*x) -
 6*log(-a*x + 1))*sqrt(d*x) + 75*sqrt(d*x)*polylog(3, a*x))/(d^4*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_3\left(ax\right)}{\left(dx\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)**(7/2),x)
[Out] Integral(polylog(3, a*x)/(d*x)**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(7/2),x, algorithm="giac")
[Out] integrate(polylog(3, a*x)/(d*x)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(3,a\,x)}{{\left(d\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x)/(d*x)^(7/2),x)
[Out] int(polylog(3, a*x)/(d*x)^(7/2), x)
```

3.72 $\int (dx)^{3/2}\text{PolyLog}(2,ax^2)\,dx$

Optimal. Leaf size=140

$$-\frac{32d\sqrt{dx}}{25a}-\frac{32(dx)^{5/2}}{125d}+\frac{16d^{3/2}\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/4}}+\frac{16d^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/4}}+\frac{8(dx)^{5/2}\log(1-ax^2)}{25d}$$

```
[Out] -32/125*(d*x)^(5/2)/d+16/25*d^(3/2)*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))/a^(
5/4)+16/25*d^(3/2)*arctanh(a^(1/4)*(d*x)^(1/2)/d^(1/2))/a^(5/4)+8/25*(d*x)^
(5/2)*ln(-a*x^2+1)/d+2/5*(d*x)^(5/2)*polylog(2,a*x^2)/d-32/25*d*(d*x)^(1/2)
/a
```

Rubi [A]
time = 0.07, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {6726, 2505, 16, 327, 335, 218, 214, 211}

$$\frac{16d^{3/2}\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/4}}+\frac{16d^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/4}}+\frac{2(dx)^{5/2}\text{Li}_2(ax^2)}{5d}+\frac{8(dx)^{5/2}\log(1-ax^2)}{25d}-\frac{32d\sqrt{dx}}{25a}-\frac{32(dx)^{5/2}}{125d}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*PolyLog[2, a*x^2],x]
```

```
[Out] (-32*d*Sqrt[d*x])/(25*a) - (32*(d*x)^(5/2))/(125*d) + (16*d^(3/2)*ArcTan[(a
^(1/4)*Sqrt[d*x])/Sqrt[d]])/(25*a^(5/4)) + (16*d^(3/2)*ArcTanh[(a^(1/4)*Sqr
t[d*x])/Sqrt[d]])/(25*a^(5/4)) + (8*(d*x)^(5/2)*Log[1 - a*x^2])/(25*d) + (2
*(d*x)^(5/2)*PolyLog[2, a*x^2])/(5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int (dx)^{3/2}\text{Li}_2\left(ax^2\right)\,dx = \frac{2(dx)^{5/2}\text{Li}_2(ax^2)}{5d} + \frac{4}{5}\int (dx)^{3/2}\log\left(1-ax^2\right)\,dx$$
$$= \frac{8(dx)^{5/2}\log\left(1-ax^2\right)}{25d} + \frac{2(dx)^{5/2}\text{Li}_2(ax^2)}{5d} + \frac{(16a)\int \frac{x(dx)^{5/2}}{1-ax^2}\,dx}{25d}$$
$$= \frac{8(dx)^{5/2}\log\left(1-ax^2\right)}{25d} + \frac{2(dx)^{5/2}\text{Li}_2(ax^2)}{5d} + \frac{(16a)\int \frac{(dx)^{7/2}}{1-ax^2}\,dx}{25d^2}$$
$$= -\frac{32(dx)^{5/2}}{125d} + \frac{8(dx)^{5/2}\log\left(1-ax^2\right)}{25d} + \frac{2(dx)^{5/2}\text{Li}_2(ax^2)}{5d} + \frac{16}{25}\int \frac{(dx)^{3/2}}{1-ax^2}\,dx$$
$$= -\frac{32d\sqrt{dx}}{25a} - \frac{32(dx)^{5/2}}{125d} + \frac{8(dx)^{5/2}\log\left(1-ax^2\right)}{25d} + \frac{2(dx)^{5/2}\text{Li}_2(ax^2)}{5d} + \frac{(16d^2)\int -$$
$$= -\frac{32d\sqrt{dx}}{25a} - \frac{32(dx)^{5/2}}{125d} + \frac{8(dx)^{5/2}\log\left(1-ax^2\right)}{25d} + \frac{2(dx)^{5/2}\text{Li}_2(ax^2)}{5d} + \frac{(32d)\text{Subs}$$
$$= -\frac{32d\sqrt{dx}}{25a} - \frac{32(dx)^{5/2}}{125d} + \frac{8(dx)^{5/2}\log\left(1-ax^2\right)}{25d} + \frac{2(dx)^{5/2}\text{Li}_2(ax^2)}{5d} + \frac{(16d^2)\text{Sub}$$
$$= -\frac{32d\sqrt{dx}}{25a} - \frac{32(dx)^{5/2}}{125d} + \frac{16d^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/4}} + \frac{16d^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right.}{25a^{5/4}}$$

Mathematica [A]
time = 0.07, size = 101, normalized size = 0.72

$$\frac{2(dx)^{3/2}\left(\frac{40\text{ArcTan}\left(\sqrt[4]{a}\sqrt{x}\right)+40\tanh^{-1}\left(\sqrt[4]{a}\sqrt{x}\right)+4\sqrt[4]{a}\sqrt{x}\left(-20-4ax^2+5ax^2\log\left(1-ax^2\right)\right)}{a^{5/4}}+25x^{5/2}\text{PolyLog}(2,ax^2)\right)}{125x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*PolyLog[2, a*x^2],x]
[Out] (2*(d*x)^(3/2)*((40*ArcTan[a^(1/4)*Sqrt[x]] + 40*ArcTanh[a^(1/4)*Sqrt[x]] +
 4*a^(1/4)*Sqrt[x]*(-20 - 4*a*x^2 + 5*a*x^2*Log[1 - a*x^2]))/a^(5/4) + 25*x
^(5/2)*PolyLog[2, a*x^2]))/(125*x^(3/2))
```

Maple [A]
time = 0.42, size = 145, normalized size = 1.04

method	result

meijerg	$-\frac{(dx)^{\frac{3}{2}}\left(-\frac{4\sqrt{x}\,(-a)^{\frac{9}{4}}(144a\,x^2+720)}{1125a^2}-\frac{16\sqrt{x}\,(-a)^{\frac{9}{4}}\left(\ln\left(1-(a\,x^2)^{\frac{1}{4}}\right)-\ln\left(1+(a\,x^2)^{\frac{1}{4}}\right)-2\arctan\left((a\,x^2)^{\frac{1}{4}}\right)\right)}{25a^2(a\,x^2)^{\frac{1}{4}}}+\frac{16x^{\frac{5}{2}}(-}{}\right.}{2x^{\frac{3}{2}}(-a)^{\frac{5}{4}}}$
derivativedivides	$\frac{\frac{2(dx)^{\frac{5}{2}}\operatorname{polylog}(2,a\,x^2)}{5}+\frac{8(dx)^{\frac{5}{2}}\ln\left(\frac{-a\,d^2x^2+d^2}{d^2}\right)}{25}+\frac{32a\left(-\frac{\frac{a(dx)^{\frac{5}{2}}}{5}+d^2\sqrt{dx}}{a^2}+\frac{d^2\left(\frac{d^2}{a}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{dx}+\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\right.\right.}{4a^2}\right.}{25}}{d}$
default	$\frac{\frac{2(dx)^{\frac{5}{2}}\operatorname{polylog}(2,a\,x^2)}{5}+\frac{8(dx)^{\frac{5}{2}}\ln\left(\frac{-a\,d^2x^2+d^2}{d^2}\right)}{25}+\frac{32a\left(-\frac{\frac{a(dx)^{\frac{5}{2}}}{5}+d^2\sqrt{dx}}{a^2}+\frac{d^2\left(\frac{d^2}{a}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{dx}+\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\right.\right.}{4a^2}\right.}{25}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*polylog(2,a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/5*(d*x)^(5/2)*polylog(2,a*x^2)+4/25*(d*x)^(5/2)*ln((-a*d^2*x^2+d^2)/
d^2)+16/25*a*(-1/a^2*(1/5*a*(d*x)^(5/2)+d^2*(d*x)^(1/2))+1/4*d^2/a^2*(d^2/a
)^(1/4)*(ln(((d*x)^(1/2)+(d^2/a)^(1/4))/((d*x)^(1/2)-(d^2/a)^(1/4)))+2*arct
an((d*x)^(1/2)/(d^2/a)^(1/4)))))
```

Maxima [A]

time = 0.47, size = 160, normalized size = 1.14

$$\frac{2\left(\frac{25\,(dx)^{\frac{5}{2}}a\mathrm{Li}_2(ax^2)+20\,(dx)^{\frac{5}{2}}a\log(-ad^2x^2+d^2)-8\,(dx)^{\frac{5}{2}}(5\,a\log(d)+2\,a)-80\sqrt{dx}\,d^2}{a}+\frac{20\left(\frac{2\,d^3\arctan\left(\frac{\sqrt{dx}\,\sqrt{a}}{\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}}-\frac{d^3\log\left(\frac{\sqrt{dx}\,\sqrt{a}-\sqrt{\sqrt{a}\,d}}{\sqrt{dx}\,\sqrt{a}+\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}}\right)}{a}\right)}{125\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(2,a*x^2),x, algorithm="maxima")
```

```
[Out] 2/125*((25*(d*x)^(5/2)*a*dilog(a*x^2) + 20*(d*x)^(5/2)*a*log(-a*d^2*x^2 + d
^2) - 8*(d*x)^(5/2)*(5*a*log(d) + 2*a) - 80*sqrt(d*x)*d^2)/a + 20*(2*d^3*ar
```

```
ctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/sqrt(sqrt(a)*d) - d^3*log((sqrt(d*x
)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a) + sqrt(sqrt(a)*d)))/sqrt(sq
rt(a)*d))/a)/d
```

Fricas [A]
time = 0.43, size = 194, normalized size = 1.39

$$-\frac{2\left(80\,a\left(\frac{d^6}{a^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^4d\left(\frac{d^6}{a^5}\right)^{\frac{3}{4}}-\sqrt{d^3x+a^2\sqrt{\frac{d^6}{a^5}}}\,a^4\left(\frac{d^6}{a^5}\right)^{\frac{3}{4}}}{d^6}\right)-20\,a\left(\frac{d^6}{a^5}\right)^{\frac{1}{4}}\log\left(8\sqrt{dx}\,d+8\,a\left(\frac{d^6}{a^5}\right)^{\frac{1}{4}}\right)+20\,a\left(\frac{d^6}{a^5}\right)^{\frac{1}{4}}\log\left(8\sqrt{dx}\,d-8\,a\left(\frac{d^6}{a^5}\right)^{\frac{1}{4}}\right)-\left(25\,adx^2\mathrm{Li}_2(ax^2)+20\,adx^2\log\left(-ax^2+1\right)-16\,adx^2-80\,d\right)\sqrt{dx}\right)}{125\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(2,a*x^2),x, algorithm="fricas")
[Out] -2/125*(80*a*(d^6/a^5)^(1/4)*arctan(-(sqrt(d*x)*a^4*d*(d^6/a^5)^(3/4) - sqr
t(d^3*x + a^2*sqrt(d^6/a^5))*a^4*(d^6/a^5)^(3/4))/d^6) - 20*a*(d^6/a^5)^(1/
4)*log(8*sqrt(d*x)*d + 8*a*(d^6/a^5)^(1/4)) + 20*a*(d^6/a^5)^(1/4)*log(8*sq
rt(d*x)*d - 8*a*(d^6/a^5)^(1/4)) - (25*a*d*x^2*dilog(a*x^2) + 20*a*d*x^2*lo
g(-a*x^2 + 1) - 16*a*d*x^2 - 80*d)*sqrt(d*x))/a
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*polylog(2,a*x**2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(2,a*x^2),x, algorithm="giac")
[Out] integrate((d*x)^(3/2)*dilog(a*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{polylog}\left(2,a\,x^2\right)\,{\left(d\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^2)*(d*x)^(3/2),x)
[Out] int(polylog(2, a*x^2)*(d*x)^(3/2), x)
```

3.73 $\int \sqrt{dx}$ PolyLog$(2, ax^2)\,dx$

Optimal. Leaf size=125

$$-\frac{32(dx)^{3/2}}{27d}-\frac{16\sqrt{d}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/4}}+\frac{16\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/4}}+\frac{8(dx)^{3/2}\log\left(1-ax^2\right)}{9d}+\frac{2(dx)^{3/2}\mathrm{Po}}{}$$

```
[Out] -32/27*(d*x)^(3/2)/d+8/9*(d*x)^(3/2)*ln(-a*x^2+1)/d+2/3*(d*x)^(3/2)*polylog
(2,a*x^2)/d-16/9*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))*d^(1/2)/a^(3/4)+16/9*a
rctanh(a^(1/4)*(d*x)^(1/2)/d^(1/2))*d^(1/2)/a^(3/4)
```

Rubi [A]
time = 0.06, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {6726, 2505, 16, 327, 335, 304, 211, 214}

$$-\frac{16\sqrt{d}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/4}}+\frac{16\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/4}}+\frac{2(dx)^{3/2}\mathrm{Li}_2\left(ax^2\right)}{3d}+\frac{8(dx)^{3/2}\log\left(1-ax^2\right)}{9d}-\frac{32(dx)^{3/2}}{27d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*PolyLog[2, a*x^2],x]
```

```
[Out] (-32*(d*x)^(3/2))/(27*d) - (16*Sqrt[d]*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])
/(9*a^(3/4)) + (16*Sqrt[d]*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(9*a^(3/4)
) + (8*(d*x)^(3/2)*Log[1 - a*x^2])/(9*d) + (2*(d*x)^(3/2)*PolyLog[2, a*x^2]
)/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \sqrt{dx}\,\text{Li}_2\left(ax^2\right)\,dx = \frac{2(dx)^{3/2}\text{Li}_2(ax^2)}{3d} + \frac{4}{3}\int \sqrt{dx}\,\log\left(1-ax^2\right)\,dx$$

$$= \frac{8(dx)^{3/2}\log\left(1-ax^2\right)}{9d} + \frac{2(dx)^{3/2}\text{Li}_2(ax^2)}{3d} + \frac{(16a)\int \frac{x(dx)^{3/2}}{1-ax^2}\,dx}{9d}$$

$$= \frac{8(dx)^{3/2}\log\left(1-ax^2\right)}{9d} + \frac{2(dx)^{3/2}\text{Li}_2(ax^2)}{3d} + \frac{(16a)\int \frac{(dx)^{5/2}}{1-ax^2}\,dx}{9d^2}$$

$$= -\frac{32(dx)^{3/2}}{27d} + \frac{8(dx)^{3/2}\log\left(1-ax^2\right)}{9d} + \frac{2(dx)^{3/2}\text{Li}_2(ax^2)}{3d} + \frac{16}{9}\int \frac{\sqrt{dx}}{1-ax^2}\,dx$$

$$= -\frac{32(dx)^{3/2}}{27d} + \frac{8(dx)^{3/2}\log\left(1-ax^2\right)}{9d} + \frac{2(dx)^{3/2}\text{Li}_2(ax^2)}{3d} + \frac{32\text{Subst}\left(\int \frac{x^2}{1-\frac{ax^4}{d^2}}\,dx, x, \sqrt{\right.}{9d}$$

$$= -\frac{32(dx)^{3/2}}{27d} + \frac{8(dx)^{3/2}\log\left(1-ax^2\right)}{9d} + \frac{2(dx)^{3/2}\text{Li}_2(ax^2)}{3d} + \frac{(16d)\text{Subst}\left(\int \frac{1}{d-\sqrt{a}\,x^2}\,d\right.}{9\sqrt{a}}$$

$$= -\frac{32(dx)^{3/2}}{27d} - \frac{16\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/4}} + \frac{16\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/4}} + \frac{8(dx)^{3/2}}{}$$

Mathematica [A]
time = 0.05, size = 91, normalized size = 0.73

$$\frac{2\sqrt{dx}\left(\frac{4\left(-6\text{ArcTan}\left(\sqrt[4]{a}\,\sqrt{x}\right)+6\tanh^{-1}\left(\sqrt[4]{a}\,\sqrt{x}\right)+a^{3/4}x^{3/2}\left(-4+3\log\left(1-ax^2\right)\right)\right)}{a^{3/4}} + 9x^{3/2}\text{PolyLog}(2,ax^2)\right)}{27\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*PolyLog[2, a*x^2],x]
```

```
[Out] (2*Sqrt[d*x]*((4*(-6*ArcTan[a^(1/4)*Sqrt[x]] + 6*ArcTanh[a^(1/4)*Sqrt[x]] +
 a^(3/4)*x^(3/2)*(-4 + 3*Log[1 - a*x^2])))/a^(3/4) + 9*x^(3/2)*PolyLog[2, a
*x^2]))/(27*Sqrt[x])
```

Maple [A]
time = 0.39, size = 134, normalized size = 1.07

method	result
meijerg	$-\frac{\sqrt{dx}\left(-\frac{64x^{\frac{3}{2}}(-a)^{\frac{7}{4}}}{27a} - \frac{16x^{\frac{3}{2}}(-a)^{\frac{7}{4}}\left(\ln\left(1-\left(a\,x^2\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^2\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(a\,x^2\right)^{\frac{1}{4}}\right)\right)}{9a\left(a\,x^2\right)^{\frac{3}{4}}} + \frac{16x^{\frac{3}{2}}(-a)^{\frac{7}{4}}\ln\left(-a\,x^2+1\right)}{9a}\right.}{2\sqrt{x}\,(-a)^{\frac{3}{4}}}$

derivativedivides	$\frac{\frac{2(dx)^{\frac{3}{2}}\operatorname{polylog}(2,a\,x^2)}{3}+\frac{8(dx)^{\frac{3}{2}}\ln\left(\frac{-a\,d^2x^2+d^2}{d^2}\right)}{9}+\frac{32a\left(-\frac{(dx)^{\frac{3}{2}}}{3a}-\frac{d^2\left(2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}\right)\right)}{4a^2\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}\right)}{9}}{d}$
default	$\frac{\frac{2(dx)^{\frac{3}{2}}\operatorname{polylog}(2,a\,x^2)}{3}+\frac{8(dx)^{\frac{3}{2}}\ln\left(\frac{-a\,d^2x^2+d^2}{d^2}\right)}{9}+\frac{32a\left(-\frac{(dx)^{\frac{3}{2}}}{3a}-\frac{d^2\left(2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}\right)\right)}{4a^2\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}\right)}{9}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*polylog(2,a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/3*(d*x)^(3/2)*polylog(2,a*x^2)+4/9*(d*x)^(3/2)*ln((-a*d^2*x^2+d^2)/d
^2)+16/9*a*(-1/3*(d*x)^(3/2)/a-1/4*d^2/a^2/(d^2/a)^(1/4)*(2*arctan((d*x)^(1
/2)/(d^2/a)^(1/4))-ln(((d*x)^(1/2)+(d^2/a)^(1/4))/((d*x)^(1/2)-(d^2/a)^(1/4
))))))
```

Maxima [A]
time = 0.47, size = 139, normalized size = 1.11

$$-\frac{2\left(12\,d^2\left(\frac{2\arctan\left(\frac{\sqrt{dx}\,\sqrt{a}}{\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}\,\sqrt{a}}+\frac{\log\left(\frac{\sqrt{dx}\,\sqrt{a}-\sqrt{\sqrt{a}\,d}}{\sqrt{dx}\,\sqrt{a}+\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}\,\sqrt{a}}\right)+8\,(dx)^{\frac{3}{2}}\,(3\,\log(d)+2)-9\,(dx)^{\frac{3}{2}}\,\mathrm{Li}_2(ax^2)-12\,(dx)^{\frac{3}{2}}\log\left(-ad^2x^2+d^2\right)\right)}{27\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x^2),x, algorithm="maxima")
```

```
[Out] -2/27*(12*d^2*(2*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/(sqrt(sqrt(a)*d)
*sqrt(a)) + log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a) +
sqrt(sqrt(a)*d)))/(sqrt(sqrt(a)*d)*sqrt(a))) + 8*(d*x)^(3/2)*(3*log(d) + 2)
 - 9*(d*x)^(3/2)*dilog(a*x^2) - 12*(d*x)^(3/2)*log(-a*d^2*x^2 + d^2))/d
```

Fricas [A]
time = 0.39, size = 172, normalized size = 1.38

$$\frac{2}{27}\sqrt{dx}\left(9\,x\mathrm{Li}_2(ax^2)+12\,x\log\left(-ax^2+1\right)-16\,x\right)+\frac{32}{9}\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,ad\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}-\sqrt{d^3x+ad^2\sqrt{\frac{d^2}{a^3}}}\,a\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}}{d^2}\right)+\frac{8}{9}\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}\log\left(512\,a^2\left(\frac{d^2}{a^3}\right)^{\frac{3}{4}}+512\,\sqrt{dx}\,d\right)-\frac{8}{9}\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}\log\left(-512\,a^2\left(\frac{d^2}{a^3}\right)^{\frac{3}{4}}+512\,\sqrt{dx}\,d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x^2),x, algorithm="fricas")
```

```
[Out] 2/27*sqrt(d*x)*(9*x*dilog(a*x^2) + 12*x*log(-a*x^2 + 1) - 16*x) + 32/9*(d^2
/a^3)^(1/4)*arctan(-(sqrt(d*x)*a*d*(d^2/a^3)^(1/4) - sqrt(d^3*x + a*d^2*sqr
t(d^2/a^3))*a*(d^2/a^3)^(1/4))/d^2) + 8/9*(d^2/a^3)^(1/4)*log(512*a^2*(d^2/
a^3)^(3/4) + 512*sqrt(d*x)*d) - 8/9*(d^2/a^3)^(1/4)*log(-512*a^2*(d^2/a^3)^
(3/4) + 512*sqrt(d*x)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx} \operatorname{Li}_2\left(ax^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*polylog(2,a*x**2),x)
```

```
[Out] Integral(sqrt(d*x)*polylog(2, a*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*dilog(a*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{polylog}\left(2,a\,x^2\right)\,\sqrt{d\,x} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^2)*(d*x)^(1/2),x)
```

```
[Out] int(polylog(2, a*x^2)*(d*x)^(1/2), x)
```

3.74 $\int \frac{\text{PolyLog}(2,ax^2)}{\sqrt{dx}}\,dx$

Optimal. Leaf size=115

$$-\frac{32\sqrt{dx}}{d}+\frac{16\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}}+\frac{16\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}}+\frac{8\sqrt{dx}\log(1-ax^2)}{d}+\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax^2)}{d}$$

```
[Out] 16*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))/a^(1/4)/d^(1/2)+16*arctanh(a^(1/4)*(
d*x)^(1/2)/d^(1/2))/a^(1/4)/d^(1/2)-32*(d*x)^(1/2)/d+8*ln(-a*x^2+1)*(d*x)^(
1/2)/d+2*polylog(2,a*x^2)*(d*x)^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {6726, 2505, 16, 327, 335, 218, 214, 211}

$$\frac{16\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}}+\frac{2\sqrt{dx}\,\text{Li}_2(ax^2)}{d}+\frac{8\sqrt{dx}\log(1-ax^2)}{d}+\frac{16\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}}-\frac{32\sqrt{dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/Sqrt[d*x],x]
```

```
[Out] (-32*Sqrt[d*x])/d + (16*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(a^(1/4)*Sqrt[
d]) + (16*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(a^(1/4)*Sqrt[d]) + (8*Sqrt
[d*x]*Log[1 - a*x^2])/d + (2*Sqrt[d*x]*PolyLog[2, a*x^2])/d
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{Li}_2(ax^2)}{\sqrt{dx}}\,dx &= \frac{2\sqrt{dx}\,\text{Li}_2(ax^2)}{d} + 4\int \frac{\log(1-ax^2)}{\sqrt{dx}}\,dx \\
&= \frac{8\sqrt{dx}\,\log(1-ax^2)}{d} + \frac{2\sqrt{dx}\,\text{Li}_2(ax^2)}{d} + \frac{(16a)\int \frac{x\sqrt{dx}}{1-ax^2}\,dx}{d} \\
&= \frac{8\sqrt{dx}\,\log(1-ax^2)}{d} + \frac{2\sqrt{dx}\,\text{Li}_2(ax^2)}{d} + \frac{(16a)\int \frac{(dx)^{3/2}}{1-ax^2}\,dx}{d^2} \\
&= -\frac{32\sqrt{dx}}{d} + \frac{8\sqrt{dx}\,\log(1-ax^2)}{d} + \frac{2\sqrt{dx}\,\text{Li}_2(ax^2)}{d} + 16\int \frac{1}{\sqrt{dx}\,(1-ax^2)}\,dx \\
&= -\frac{32\sqrt{dx}}{d} + \frac{8\sqrt{dx}\,\log(1-ax^2)}{d} + \frac{2\sqrt{dx}\,\text{Li}_2(ax^2)}{d} + \frac{32\text{Subst}\left(\int \frac{1}{1-\frac{ax^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{d} \\
&= -\frac{32\sqrt{dx}}{d} + \frac{8\sqrt{dx}\,\log(1-ax^2)}{d} + \frac{2\sqrt{dx}\,\text{Li}_2(ax^2)}{d} + 16\text{Subst}\left(\int \frac{1}{d-\sqrt{a}\,x^2}\,dx, x, \sqrt{dx}\right. \\
&= -\frac{32\sqrt{dx}}{d} + \frac{16\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}} + \frac{16\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}} + \frac{8\sqrt{dx}\,\log(1-ax^2)}{d} +
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.06, size = 57, normalized size = 0.50

$$
\frac{5x\Gamma\left(\frac{5}{4}\right)\left(-16+16\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};ax^2\right)+4\log(1-ax^2)+\text{PolyLog}(2,ax^2)\right)}{2\sqrt{dx}\,\Gamma\left(\frac{9}{4}\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^2]/Sqrt[d*x],x]
```

```
[Out] (5*x*Gamma[5/4]*(-16 + 16*Hypergeometric2F1[1/4, 1, 5/4, a*x^2] + 4*Log[1 -
 a*x^2] + PolyLog[2, a*x^2]))/(2*Sqrt[d*x]*Gamma[9/4])
```

Maple [A]
time = 0.41, size = 128, normalized size = 1.11

method	result
meijerg	$-\frac{\sqrt{x}\left(-\frac{64\sqrt{x}\,(-a)^{\frac{5}{4}}}{a}-\frac{16\sqrt{x}\,(-a)^{\frac{5}{4}}\left(\ln\left(1-\left(a\,x^2\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^2\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(a\,x^2\right)^{\frac{1}{4}}\right)\right)}{a\left(a\,x^2\right)^{\frac{1}{4}}}+\frac{16\sqrt{x}\,(-a)^{\frac{5}{4}}\ln(}{a}\right.}{2\sqrt{dx}\,(-a)^{\frac{1}{4}}}$

derivativedivides	$\frac{2\sqrt{dx}\,\mathrm{polylog}(2,a\,x^2)+8\sqrt{dx}\,\ln\left(\frac{-a\,d^2x^2+d^2}{d^2}\right)+32a\left(-\frac{\sqrt{dx}}{a}+\frac{\left(\frac{d^2}{a}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{dx}+\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^2}{a}\right)\cdots}\right.\right.}{4a}\right.}{d}$
default	$\frac{2\sqrt{dx}\,\mathrm{polylog}(2,a\,x^2)+8\sqrt{dx}\,\ln\left(\frac{-a\,d^2x^2+d^2}{d^2}\right)+32a\left(-\frac{\sqrt{dx}}{a}+\frac{\left(\frac{d^2}{a}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{dx}+\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^2}{a}\right)\cdots}\right.\right.}{4a}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*((d*x)^(1/2)*polylog(2,a*x^2)+4*(d*x)^(1/2)*ln((-a*d^2*x^2+d^2)/d^2)+16
*a*(-(d*x)^(1/2)/a+1/4/a*(d^2/a)^(1/4)*(ln(((d*x)^(1/2)+(d^2/a)^(1/4))/((d*
x)^(1/2)-(d^2/a)^(1/4)))+2*arctan((d*x)^(1/2)/(d^2/a)^(1/4)))))
```

Maxima [A]
time = 0.48, size = 128, normalized size = 1.11

$$-\frac{2\left(8\sqrt{dx}\left(\log(d)+2\right)-\frac{8\,d\arctan\left(\frac{\sqrt{dx}\sqrt{a}}{\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}}-\sqrt{dx}\,\mathrm{Li}_2(ax^2)-4\sqrt{dx}\,\log\left(-ad^2x^2+d^2\right)+\frac{4\,d\log\left(\frac{\sqrt{dx}\sqrt{a}-\sqrt{\sqrt{a}\,d}}{\sqrt{dx}\sqrt{a}+\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(8*sqrt(d*x)*(log(d) + 2) - 8*d*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d)
)/sqrt(sqrt(a)*d) - sqrt(d*x)*dilog(a*x^2) - 4*sqrt(d*x)*log(-a*d^2*x^2 + d
^2) + 4*d*log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a) + sq
rt(sqrt(a)*d)))/sqrt(sqrt(a)*d))/d
```

Fricas [A]
time = 0.40, size = 156, normalized size = 1.36

$$-\frac{2\left(16\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{d^2\sqrt{\frac{1}{ad^2}}+dx}\;ad\left(\frac{1}{ad^2}\right)^{\frac{3}{4}}-\sqrt{dx}\,ad\left(\frac{1}{ad^2}\right)^{\frac{3}{4}}\right)-4\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\log\left(d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)+4\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\log\left(-d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)-\sqrt{dx}\left(\mathrm{Li}_2(ax^2)+4\log\left(-ax^2+1\right)-16\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(16*d*(1/(a*d^2))^(1/4)*arctan(sqrt(d^2*sqrt(1/(a*d^2)) + d*x)*a*d*(1/(a
*d^2))^(3/4) - sqrt(d*x)*a*d*(1/(a*d^2))^(3/4)) - 4*d*(1/(a*d^2))^(1/4)*log
(d*(1/(a*d^2))^(1/4) + sqrt(d*x)) + 4*d*(1/(a*d^2))^(1/4)*log(-d*(1/(a*d^2)
)^(1/4) + sqrt(d*x)) - sqrt(d*x)*(dilog(a*x^2) + 4*log(-a*x^2 + 1) - 16))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_2\left(ax^2\right)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/(d*x)**(1/2),x)
```

```
[Out] Integral(polylog(2, a*x**2)/sqrt(d*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/sqrt(d*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(2,a\,x^2)}{\sqrt{d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^2)/(d*x)^(1/2),x)
```

```
[Out] int(polylog(2, a*x^2)/(d*x)^(1/2), x)
```

3.75 $\int \frac{\text{PolyLog}(2,ax^2)}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{16\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{16\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{8\log\left(1-ax^2\right)}{d\sqrt{dx}}-\frac{2\text{PolyLog}(2,ax^2)}{d\sqrt{dx}}$$

```
[Out] -16*a^(1/4)*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(3/2)+16*a^(1/4)*arctanh(
a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(3/2)+8*ln(-a*x^2+1)/d/(d*x)^(1/2)-2*polylog
(2,a*x^2)/d/(d*x)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {6726, 2505, 16, 335, 304, 211, 214}

$$-\frac{16\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{16\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\text{Li}_2(ax^2)}{d\sqrt{dx}}+\frac{8\log\left(1-ax^2\right)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/(d*x)^(3/2),x]
```

```
[Out] (-16*a^(1/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (16*a^(1/4)*Arc
Tanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (8*Log[1 - a*x^2])/(d*Sqrt[d*x
]) - (2*PolyLog[2, a*x^2])/(d*Sqrt[d*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\text{Li}_2(ax^2)}{(dx)^{3/2}}\,dx = -\frac{2\text{Li}_2(ax^2)}{d\sqrt{dx}} - 4\int \frac{\log(1-ax^2)}{(dx)^{3/2}}\,dx$$
$$= \frac{8\log(1-ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax^2)}{d\sqrt{dx}} + \frac{(16a)\int \frac{x}{\sqrt{dx}\,(1-ax^2)}\,dx}{d}$$
$$= \frac{8\log(1-ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax^2)}{d\sqrt{dx}} + \frac{(16a)\int \frac{\sqrt{dx}}{1-ax^2}\,dx}{d^2}$$
$$= \frac{8\log(1-ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax^2)}{d\sqrt{dx}} + \frac{(32a)\text{Subst}\left(\int \frac{x^2}{1-\frac{ax^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{d^3}$$
$$= \frac{8\log(1-ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax^2)}{d\sqrt{dx}} + \frac{(16\sqrt{a})\,\text{Subst}\left(\int \frac{1}{d-\sqrt{a}\,x^2}\,dx, x, \sqrt{dx}\right)}{d} - \frac{(16\sqrt{a})\,\text{Subst}}{}$$
$$= -\frac{16\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{16\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{8\log(1-ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax^2)}{d\sqrt{dx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.05, size = 62, normalized size = 0.60

$$\frac{x\Gamma\left(\frac{3}{4}\right)\left(16ax^2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};ax^2\right)+12\log\left(1-ax^2\right)-3\text{PolyLog}(2,ax^2)\right)}{2(dx)^{3/2}\Gamma\left(\frac{7}{4}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^2]/(d*x)^(3/2),x]
```

```
[Out] (x*Gamma[3/4]*(16*a*x^2*Hypergeometric2F1[3/4, 1, 7/4, a*x^2] + 12*Log[1 - a*x^2] - 3*PolyLog[2, a*x^2]))/(2*(d*x)^(3/2)*Gamma[7/4])
```

Maple [A]
time = 0.41, size = 114, normalized size = 1.11

method	result
meijerg	$-\frac{x^{\frac{3}{2}}(-a)^{\frac{1}{4}}\left(-\frac{16x^{\frac{3}{2}}(-a)^{\frac{3}{4}}\left(\ln\left(1-\left(a\,x^2\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^2\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(a\,x^2\right)^{\frac{1}{4}}\right)\right)}{\left(a\,x^2\right)^{\frac{3}{4}}}+\frac{16(-a)^{\frac{3}{4}}\ln\left(-a\,x^2+1\right)}{\sqrt{x}\,a}-\frac{4(-a)^{\frac{3}{4}}\,\text{polylo}}{\sqrt{x}}\right.}{2(dx)^{\frac{3}{2}}}$

derivativedivides	$\frac{-\frac{2\,\mathrm{polylog}\left(2,a\,x^{2}\right)}{\sqrt{dx}}+\frac{8\ln\left(\frac{-a\,d^{2}x^{2}+d^{2}}{d^{2}}\right)}{\sqrt{dx}}-\frac{8\left(2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)\right)}{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}}{d}$
default	$\frac{-\frac{2\,\mathrm{polylog}\left(2,a\,x^{2}\right)}{\sqrt{dx}}+\frac{8\ln\left(\frac{-a\,d^{2}x^{2}+d^{2}}{d^{2}}\right)}{\sqrt{dx}}-\frac{8\left(2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)\right)}{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-polylog(2,a*x^2)/(d*x)^(1/2)+4/(d*x)^(1/2)*ln((-a*d^2*x^2+d^2)/d^2)-4
/(d^2/a)^(1/4)*(2*arctan((d*x)^(1/2)/(d^2/a)^(1/4))-ln(((d*x)^(1/2)+(d^2/a)
^(1/4))/((d*x)^(1/2)-(d^2/a)^(1/4)))))
```

Maxima [A]
time = 0.47, size = 123, normalized size = 1.19

$$-\frac{2\left(4\,a\left(\frac{2\arctan\left(\frac{\sqrt{dx}\,\sqrt{a}}{\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}\,\sqrt{a}}+\frac{\log\left(\frac{\sqrt{dx}\,\sqrt{a}-\sqrt{\sqrt{a}\,d}}{\sqrt{dx}\,\sqrt{a}+\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}\,\sqrt{a}}\right)+\frac{\mathrm{Li}_2\left(ax^2\right)-4\,\log\left(-ad^2x^2+d^2\right)+8\,\log\left(d\right)}{\sqrt{dx}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*(4*a*(2*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/(sqrt(sqrt(a)*d)*sqrt(
a)) + log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a) + sqrt(s
qrt(a)*d)))/(sqrt(sqrt(a)*d)*sqrt(a))) + (dilog(a*x^2) - 4*log(-a*d^2*x^2 +
 d^2) + 8*log(d))/sqrt(d*x))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(78) = 156$.
time = 0.40, size = 170, normalized size = 1.65

$$\frac{2\left(16\,d^2x\left(\frac{a}{d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,ad\left(\frac{a}{d^6}\right)^{\frac{1}{4}}-\sqrt{ad^4\sqrt{\frac{a}{d^6}}+a^2dx}\,d\left(\frac{a}{d^6}\right)^{\frac{1}{4}}}{a}\right)+4\,d^2x\left(\frac{a}{d^6}\right)^{\frac{1}{4}}\log\left(512\,d^5\left(\frac{a}{d^6}\right)^{\frac{3}{4}}+512\,\sqrt{dx}\,a\right)-4\,d^2x\left(\frac{a}{d^6}\right)^{\frac{1}{4}}\log\left(-512\,d^5\left(\frac{a}{d^6}\right)^{\frac{3}{4}}+512\,\sqrt{dx}\,a\right)-\sqrt{dx}\,\left(\mathrm{Li}_2(ax^2)-4\,\log\left(-ax^2+1\right)\right)\right)}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(16*d^2*x*(a/d^6)^(1/4)*arctan(-(sqrt(d*x)*a*d*(a/d^6)^(1/4) - sqrt(a*d^4
*sqrt(a/d^6) + a^2*d*x)*d*(a/d^6)^(1/4))/a) + 4*d^2*x*(a/d^6)^(1/4)*log(512
*d^5*(a/d^6)^(3/4) + 512*sqrt(d*x)*a) - 4*d^2*x*(a/d^6)^(1/4)*log(-512*d^5*
(a/d^6)^(3/4) + 512*sqrt(d*x)*a) - sqrt(d*x)*(dilog(a*x^2) - 4*log(-a*x^2 +
 1)))/(d^2*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_2(ax^2)}{(dx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/(d*x)**(3/2),x)
```

```
[Out] Integral(polylog(2, a*x**2)/(d*x)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/(d*x)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(2, a\,x^2)}{(d\,x)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^2)/(d*x)^(3/2),x)
```

```
[Out] int(polylog(2, a*x^2)/(d*x)^(3/2), x)
```

3.76 $\int \frac{\text{PolyLog}(2,ax^2)}{(dx)^{5/2}} dx$

Optimal. Leaf size=111

$$\frac{16a^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}}+\frac{16a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}}+\frac{8\log(1-ax^2)}{9d(dx)^{3/2}}-\frac{2\text{PolyLog}(2,ax^2)}{3d(dx)^{3/2}}$$

```
[Out] 16/9*a^(3/4)*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(5/2)+16/9*a^(3/4)*arcta
nh(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(5/2)+8/9*ln(-a*x^2+1)/d/(d*x)^(3/2)-2/3*
polylog(2,a*x^2)/d/(d*x)^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {6726, 2505, 16, 335, 218, 214, 211}

$$\frac{16a^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}}+\frac{16a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}}-\frac{2\text{Li}_2(ax^2)}{3d(dx)^{3/2}}+\frac{8\log(1-ax^2)}{9d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/(d*x)^(5/2),x]
```

```
[Out] (16*a^(3/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(9*d^(5/2)) + (16*a^(3/4)*
ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(9*d^(5/2)) + (8*Log[1 - a*x^2])/(9*d
*(d*x)^(3/2)) - (2*PolyLog[2, a*x^2])/(3*d*(d*x)^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\text{Li}_2(ax^2)}{(dx)^{5/2}}\,dx = -\frac{2\text{Li}_2(ax^2)}{3d(dx)^{3/2}} - \frac{4}{3}\int \frac{\log(1-ax^2)}{(dx)^{5/2}}\,dx$$
$$= \frac{8\log(1-ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax^2)}{3d(dx)^{3/2}} + \frac{(16a)\int \frac{x}{(dx)^{3/2}(1-ax^2)}\,dx}{9d}$$
$$= \frac{8\log(1-ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax^2)}{3d(dx)^{3/2}} + \frac{(16a)\int \frac{1}{\sqrt{dx}\,(1-ax^2)}\,dx}{9d^2}$$
$$= \frac{8\log(1-ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax^2)}{3d(dx)^{3/2}} + \frac{(32a)\text{Subst}\left(\int \frac{1}{1-\frac{ax^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{9d^3}$$
$$= \frac{8\log(1-ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax^2)}{3d(dx)^{3/2}} + \frac{(16a)\text{Subst}\left(\int \frac{1}{d-\sqrt{a}\,x^2}\,dx, x, \sqrt{dx}\right)}{9d^2} + \frac{(16a)\text{Subst}\left(\int \frac{}{d+}\right.}{9}$$
$$= \frac{16a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}} + \frac{16a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}} + \frac{8\log(1-ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax^2)}{3d(dx)^{3/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.05, size = 62, normalized size = 0.56

$$\frac{x\Gamma\left(\frac{1}{4}\right)\left(16ax^2\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};ax^2\right)+4\log(1-ax^2)-3\text{PolyLog}(2,ax^2)\right)}{18(dx)^{5/2}\Gamma\left(\frac{5}{4}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^2]/(d*x)^(5/2),x]
```

```
[Out] (x*Gamma[1/4]*(16*a*x^2*Hypergeometric2F1[1/4, 1, 5/4, a*x^2] + 4*Log[1 - a
*x^2] - 3*PolyLog[2, a*x^2]))/(18*(d*x)^(5/2)*Gamma[5/4])
```

Maple [A]
time = 0.39, size = 116, normalized size = 1.05

method	result
meijerg	$-\frac{x^{\frac{5}{2}}(-a)^{\frac{3}{4}}\left(-\frac{16\sqrt{x}\,(-a)^{\frac{1}{4}}\left(\ln\left(1-\left(a\,x^2\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^2\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(a\,x^2\right)^{\frac{1}{4}}\right)\right)}{9\left(a\,x^2\right)^{\frac{1}{4}}}+\frac{16(-a)^{\frac{1}{4}}\ln\left(-a\,x^2+1\right)}{9x^{\frac{3}{2}}a}-\frac{4(-a)^{\frac{1}{4}}}{}\right.}{2(dx)^{\frac{5}{2}}}$

derivativedivides	$\frac{-\frac{2\,\mathrm{polylog}\left(2,a\,x^2\right)}{3(dx)^{\frac{3}{2}}}+\frac{8\ln\left(\frac{-a\,d^2x^2+d^2}{d^2}\right)}{9(dx)^{\frac{3}{2}}}+\frac{8a\left(\frac{d^2}{a}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{dx}+\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}\right)\right)}{9d^2}}{d}$
default	$\frac{-\frac{2\,\mathrm{polylog}\left(2,a\,x^2\right)}{3(dx)^{\frac{3}{2}}}+\frac{8\ln\left(\frac{-a\,d^2x^2+d^2}{d^2}\right)}{9(dx)^{\frac{3}{2}}}+\frac{8a\left(\frac{d^2}{a}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{dx}+\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^2}{a}\right)^{\frac{1}{4}}}\right)\right)}{9d^2}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/3*polylog(2,a*x^2)/(d*x)^(3/2)+4/9/(d*x)^(3/2)*ln((-a*d^2*x^2+d^2)/
d^2)+4/9*a*(d^2/a)^(1/4)/d^2*(ln(((d*x)^(1/2)+(d^2/a)^(1/4))/((d*x)^(1/2)-(
d^2/a)^(1/4)))+2*arctan((d*x)^(1/2)/(d^2/a)^(1/4))))
```

Maxima [A]
time = 0.47, size = 125, normalized size = 1.13

$$\frac{2\left(\frac{8\,a\arctan\left(\frac{\sqrt{dx}\,\sqrt{a}}{\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}\,d}-\frac{4\,a\log\left(\frac{\sqrt{dx}\,\sqrt{a}-\sqrt{\sqrt{a}\,d}}{\sqrt{dx}\,\sqrt{a}+\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}\,d}-\frac{3\,\mathrm{Li}_2\left(ax^2\right)-4\,\log\left(-ad^2x^2+d^2\right)+8\,\log\left(d\right)}{(dx)^{\frac{3}{2}}}\right)}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*(8*a*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/(sqrt(sqrt(a)*d)*d) - 4*
a*log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a) + sqrt(sqrt(
a)*d)))/(sqrt(sqrt(a)*d)*d) - (3*dilog(a*x^2) - 4*log(-a*d^2*x^2 + d^2) + 8
*log(d))/(d*x)^(3/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(78) = 156$.
time = 0.45, size = 196, normalized size = 1.77

$$-\frac{2\left(16\,d^3x^2\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,ad^7\left(\frac{a^3}{d^{10}}\right)^{\frac{3}{4}}-\sqrt{d^6\sqrt{\frac{a^3}{d^{10}}}+a^2dx}\,d^7\left(\frac{a^3}{d^{10}}\right)^{\frac{3}{4}}}{a^3}\right)-4\,d^3x^2\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}\log\left(8\,d^3\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}+8\,\sqrt{dx}\,a\right)+4\,d^3x^2\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}\log\left(-8\,d^3\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}+8\,\sqrt{dx}\,a\right)+\sqrt{dx}\,\left(3\,\mathrm{Li}_2\left(ax^2\right)-4\,\log\left(-ax^2+1\right)\right)\right)}{9\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/9*(16*d^3*x^2*(a^3/d^10)^(1/4)*arctan(-(sqrt(d*x)*a*d^7*(a^3/d^10)^(3/4)
 - sqrt(d^6*sqrt(a^3/d^10) + a^2*d*x)*d^7*(a^3/d^10)^(3/4))/a^3) - 4*d^3*x^
2*(a^3/d^10)^(1/4)*log(8*d^3*(a^3/d^10)^(1/4) + 8*sqrt(d*x)*a) + 4*d^3*x^2*
(a^3/d^10)^(1/4)*log(-8*d^3*(a^3/d^10)^(1/4) + 8*sqrt(d*x)*a) + sqrt(d*x)*(
3*dilog(a*x^2) - 4*log(-a*x^2 + 1)))/(d^3*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/(d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/(d*x)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(2, a\,x^2)}{(d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^2)/(d*x)^(5/2),x)
```

```
[Out] int(polylog(2, a*x^2)/(d*x)^(5/2), x)
```

3.77 $\int \frac{\text{PolyLog}(2,ax^2)}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{32a}{25d^3\sqrt{dx}}-\frac{16a^{5/4}\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}}+\frac{16a^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}}+\frac{8\log\left(1-ax^2\right)}{25d(dx)^{5/2}}-\frac{2\text{PolyLog}(2,ax^2)}{5d(dx)^{5/2}}$$

```
[Out] -16/25*a^(5/4)*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(7/2)+16/25*a^(5/4)*ar
ctanh(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(7/2)+8/25*ln(-a*x^2+1)/d/(d*x)^(5/2)-
2/5*polylog(2,a*x^2)/d/(d*x)^(5/2)-32/25*a/d^3/(d*x)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {6726, 2505, 16, 331, 335, 304, 211, 214}

$$-\frac{16a^{5/4}\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}}+\frac{16a^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}}-\frac{32a}{25d^3\sqrt{dx}}-\frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}}+\frac{8\log\left(1-ax^2\right)}{25d(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/(d*x)^(7/2),x]
```

```
[Out] (-32*a)/(25*d^3*Sqrt[d*x]) - (16*a^(5/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]
])/(25*d^(7/2)) + (16*a^(5/4)*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(25*d^(
7/2)) + (8*Log[1 - a*x^2])/(25*d*(d*x)^(5/2)) - (2*PolyLog[2, a*x^2])/(5*d*
(d*x)^(5/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\text{Li}_2(ax^2)}{(dx)^{7/2}}\,dx = -\frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}} - \frac{4}{5}\int \frac{\log(1-ax^2)}{(dx)^{7/2}}\,dx$$
$$= \frac{8\log(1-ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}} + \frac{(16a)\int \frac{x}{(dx)^{5/2}(1-ax^2)}\,dx}{25d}$$
$$= \frac{8\log(1-ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}} + \frac{(16a)\int \frac{1}{(dx)^{3/2}(1-ax^2)}\,dx}{25d^2}$$
$$= -\frac{32a}{25d^3\sqrt{dx}} + \frac{8\log(1-ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}} + \frac{(16a^2)\int \frac{\sqrt{dx}}{1-ax^2}\,dx}{25d^4}$$
$$= -\frac{32a}{25d^3\sqrt{dx}} + \frac{8\log(1-ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}} + \frac{(32a^2)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{ax^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{25d^5}$$
$$= -\frac{32a}{25d^3\sqrt{dx}} + \frac{8\log(1-ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}} + \frac{(16a^{3/2})\,\text{Subst}\left(\int \frac{1}{d-\sqrt{a}\,x^2}\,dx, x, \sqrt{dx}\right)}{25d^3} - $$
$$= -\frac{32a}{25d^3\sqrt{dx}} - \frac{16a^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}} + \frac{16a^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}} + \frac{8\log(1-ax^2)}{25d(dx)^{5/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.05, size = 70, normalized size = 0.56

$$-\frac{x\Gamma\left(-\frac{1}{4}\right)\left(-48ax^2+16a^2x^4\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};ax^2\right)+12\log(1-ax^2)-15\text{PolyLog}(2,ax^2)\right)}{150(dx)^{7/2}\Gamma\left(\frac{3}{4}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^2]/(d*x)^(7/2),x]
```

```
[Out] -1/150*(x*Gamma[-1/4]*(-48*a*x^2 + 16*a^2*x^4*Hypergeometric2F1[3/4, 1, 7/4
, a*x^2] + 12*Log[1 - a*x^2] - 15*PolyLog[2, a*x^2]))/((d*x)^(7/2)*Gamma[3/
4])
```

Maple [A]
time = 0.41, size = 131, normalized size = 1.04

method	result

meijerg	$-\frac{x^{\frac{7}{2}}(-a)^{\frac{5}{4}}\left(-\frac{64}{25\sqrt{x}\,(-a)^{\frac{1}{4}}}-\frac{16x^{\frac{3}{2}}a\left(\ln\left(1-\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^{2}\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(a\,x^{2}\right)^{\frac{1}{4}}\right)\right)}{25(-a)^{\frac{1}{4}}\left(a\,x^{2}\right)^{\frac{3}{4}}}+\frac{16\ln\left(-a\,x^{2}+1\right)}{25x^{\frac{5}{2}}(-a)^{\frac{1}{4}}a}-\frac{4\,\mathrm{poly}}{5x^{\frac{5}{2}}}\right.}{2(dx)^{\frac{7}{2}}}$
derivativedivides	$\frac{-\frac{2\,\mathrm{polylog}\left(2,a\,x^{2}\right)}{5(dx)^{\frac{5}{2}}}+\frac{8\ln\left(\frac{-a\,d^{2}x^{2}+d^{2}}{d^{2}}\right)}{25(dx)^{\frac{5}{2}}}+\frac{32a\left(-\frac{1}{d^{2}\sqrt{dx}}-\frac{2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)}{4d^{2}\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)}{25}}{d}$
default	$\frac{-\frac{2\,\mathrm{polylog}\left(2,a\,x^{2}\right)}{5(dx)^{\frac{5}{2}}}+\frac{8\ln\left(\frac{-a\,d^{2}x^{2}+d^{2}}{d^{2}}\right)}{25(dx)^{\frac{5}{2}}}+\frac{32a\left(-\frac{1}{d^{2}\sqrt{dx}}-\frac{2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)}{4d^{2}\left(\frac{d^{2}}{a}\right)^{\frac{1}{4}}}\right)}{25}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/5*polylog(2,a*x^2)/(d*x)^(5/2)+4/25/(d*x)^(5/2)*ln((-a*d^2*x^2+d^2)
/d^2)+16/25*a*(-1/d^2/(d*x)^(1/2)-1/4/d^2/(d^2/a)^(1/4)*(2*arctan((d*x)^(1/
2)/(d^2/a)^(1/4))-ln(((d*x)^(1/2)+(d^2/a)^(1/4))/((d*x)^(1/2)-(d^2/a)^(1/4)
)))))
```

Maxima [A]
time = 0.48, size = 151, normalized size = 1.20

$$-\frac{2\left(\frac{4\,a^{2}\left(\frac{2\arctan\left(\frac{\sqrt{dx}\,\sqrt{a}}{\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}\,\sqrt{a}}+\frac{\log\left(\frac{\sqrt{dx}\,\sqrt{a}-\sqrt{\sqrt{a}\,d}}{\sqrt{dx}\,\sqrt{a}+\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}\,\sqrt{a}}\right)}{d^{2}}+\frac{16\,ad^{2}x^{2}+5\,d^{2}\mathrm{Li}_{2}\left(ax^{2}\right)-4\,d^{2}\log\left(-ad^{2}x^{2}+d^{2}\right)+8\,d^{2}\log\left(d\right)}{(dx)^{\frac{5}{2}}d^{2}}\right)}{25\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/25*(4*a^2*(2*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/(sqrt(sqrt(a)*d)*
sqrt(a)) + log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a) + s
qrt(sqrt(a)*d)))/(sqrt(sqrt(a)*d)*sqrt(a)))/d^2 + (16*a*d^2*x^2 + 5*d^2*dil
og(a*x^2) - 4*d^2*log(-a*d^2*x^2 + d^2) + 8*d^2*log(d))/((d*x)^(5/2)*d^2))/
d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(89) = 178$.

time = 0.44, size = 212, normalized size = 1.68

$$\frac{2 \left(16 d^4 x^3 \left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{dx}\, a^4 d^3 \left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}} - \sqrt{a^5 d^8 \sqrt{\frac{a^5}{d^{14}}} + a^8 dx}\, d^3 \left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}}}{a^5}\right) + 4 d^4 x^3 \left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}} \log\left(512 d^{11} \left(\frac{a^5}{d^{14}}\right)^{\frac{3}{4}} + 512 \sqrt{dx}\, a^4\right) - 4 d^4 x^3 \left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}} \log\left(-512 d^{11} \left(\frac{a^5}{d^{14}}\right)^{\frac{3}{4}} + 512 \sqrt{dx}\, a^4\right) - \left(16 a x^2 + 5 \operatorname{Li}_2(a x^2) - 4 \log\left(-a x^2 + 1\right)\right) \sqrt{dx}\right)}{25 d^4 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/25*(16*d^4*x^3*(a^5/d^14)^(1/4)*arctan(-(sqrt(d*x)*a^4*d^3*(a^5/d^14)^(1/
4) - sqrt(a^5*d^8*sqrt(a^5/d^14) + a^8*d*x)*d^3*(a^5/d^14)^(1/4))/a^5) + 4*
d^4*x^3*(a^5/d^14)^(1/4)*log(512*d^11*(a^5/d^14)^(3/4) + 512*sqrt(d*x)*a^4)
 - 4*d^4*x^3*(a^5/d^14)^(1/4)*log(-512*d^11*(a^5/d^14)^(3/4) + 512*sqrt(d*x
)*a^4) - (16*a*x^2 + 5*dilog(a*x^2) - 4*log(-a*x^2 + 1))*sqrt(d*x))/(d^4*x^
3)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/(d*x)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/(d*x)^(7/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(2, a\,x^2)}{(d\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^2)/(d*x)^(7/2),x)
[Out] int(polylog(2, a*x^2)/(d*x)^(7/2), x)
```

3.78 $\int (dx)^{5/2}\text{PolyLog}(3, ax^2)\, dx$

Optimal. Leaf size=161

$$\frac{128d(dx)^{3/2}}{1029a}+\frac{128(dx)^{7/2}}{2401d}+\frac{64d^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/4}}-\frac{64d^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/4}}-\frac{32(dx)^{7/2}\log\left(1-ax^2\right.}{343d}$$

```
[Out] 128/1029*d*(d*x)^(3/2)/a+128/2401*(d*x)^(7/2)/d+64/343*d^(5/2)*arctan(a^(1/
4)*(d*x)^(1/2)/d^(1/2))/a^(7/4)-64/343*d^(5/2)*arctanh(a^(1/4)*(d*x)^(1/2)/
d^(1/2))/a^(7/4)-32/343*(d*x)^(7/2)*ln(-a*x^2+1)/d-8/49*(d*x)^(7/2)*polylog
(2,a*x^2)/d+2/7*(d*x)^(7/2)*polylog(3,a*x^2)/d
```

Rubi [A]
time = 0.09, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {6726, 2505, 16, 327, 335, 304, 211, 214}

$$\frac{64d^{5/2}\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/4}}-\frac{64d^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/4}}-\frac{8(dx)^{7/2}\text{Li}_2(ax^2)}{49d}+\frac{2(dx)^{7/2}\text{Li}_3(ax^2)}{7d}-\frac{32(dx)^{7/2}\log\left(1-ax^2\right)}{343d}+\frac{128d(dx)^{3/2}}{1029a}+\frac{128(dx)^{7/2}}{2401d}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*PolyLog[3, a*x^2],x]
```

```
[Out] (128*d*(d*x)^(3/2))/(1029*a) + (128*(d*x)^(7/2))/(2401*d) + (64*d^(5/2)*Arc
Tan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(343*a^(7/4)) - (64*d^(5/2)*ArcTanh[(a^(1
/4)*Sqrt[d*x])/Sqrt[d]])/(343*a^(7/4)) - (32*(d*x)^(7/2)*Log[1 - a*x^2])/(3
43*d) - (8*(d*x)^(7/2)*PolyLog[2, a*x^2])/(49*d) + (2*(d*x)^(7/2)*PolyLog[3
, a*x^2])/(7*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int (dx)^{5/2}\text{Li}_3\left(ax^2\right)\,dx = \frac{2(dx)^{7/2}\text{Li}_3(ax^2)}{7d} - \frac{4}{7}\int (dx)^{5/2}\text{Li}_2\left(ax^2\right)\,dx$$

$$= -\frac{8(dx)^{7/2}\text{Li}_2(ax^2)}{49d} + \frac{2(dx)^{7/2}\text{Li}_3(ax^2)}{7d} - \frac{16}{49}\int (dx)^{5/2}\log\left(1-ax^2\right)\,dx$$

$$= -\frac{32(dx)^{7/2}\log\left(1-ax^2\right)}{343d} - \frac{8(dx)^{7/2}\text{Li}_2(ax^2)}{49d} + \frac{2(dx)^{7/2}\text{Li}_3(ax^2)}{7d} - \frac{(64a)\int \frac{x(dx)^{7/2}}{1-ax^2}}{343d}$$

$$= -\frac{32(dx)^{7/2}\log\left(1-ax^2\right)}{343d} - \frac{8(dx)^{7/2}\text{Li}_2(ax^2)}{49d} + \frac{2(dx)^{7/2}\text{Li}_3(ax^2)}{7d} - \frac{(64a)\int \frac{(dx)^{9/2}}{1-ax^2}d}{343d^2}$$

$$= \frac{128(dx)^{7/2}}{2401d} - \frac{32(dx)^{7/2}\log\left(1-ax^2\right)}{343d} - \frac{8(dx)^{7/2}\text{Li}_2(ax^2)}{49d} + \frac{2(dx)^{7/2}\text{Li}_3(ax^2)}{7d} - \frac{64}{343}$$

$$= \frac{128d(dx)^{3/2}}{1029a} + \frac{128(dx)^{7/2}}{2401d} - \frac{32(dx)^{7/2}\log\left(1-ax^2\right)}{343d} - \frac{8(dx)^{7/2}\text{Li}_2(ax^2)}{49d} + \frac{2(dx)^{7/2}}{7}$$

$$= \frac{128d(dx)^{3/2}}{1029a} + \frac{128(dx)^{7/2}}{2401d} - \frac{32(dx)^{7/2}\log\left(1-ax^2\right)}{343d} - \frac{8(dx)^{7/2}\text{Li}_2(ax^2)}{49d} + \frac{2(dx)^{7/2}}{7}$$

$$= \frac{128d(dx)^{3/2}}{1029a} + \frac{128(dx)^{7/2}}{2401d} - \frac{32(dx)^{7/2}\log\left(1-ax^2\right)}{343d} - \frac{8(dx)^{7/2}\text{Li}_2(ax^2)}{49d} + \frac{2(dx)^{7/2}}{7}$$

$$= \frac{128d(dx)^{3/2}}{1029a} + \frac{128(dx)^{7/2}}{2401d} + \frac{64d^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/4}} - \frac{64d^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 89, normalized size = 0.55

$$-\frac{11d(dx)^{3/2}\Gamma\left(\frac{11}{4}\right)\left(-448-192ax^2+448\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};ax^2\right)+336ax^2\log\left(1-ax^2\right)+588ax^2\text{PolyLog}(2,ax^2)-1029ax^2\text{PolyLog}(3,ax^2)\right)}{14406a\Gamma\left(\frac{15}{4}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*PolyLog[3, a*x^2],x]
```

```
[Out] (-11*d*(d*x)^(3/2)*Gamma[11/4]*(-448 - 192*a*x^2 + 448*Hypergeometric2F1[3/
4, 1, 7/4, a*x^2] + 336*a*x^2*Log[1 - a*x^2] + 588*a*x^2*PolyLog[2, a*x^2]
- 1029*a*x^2*PolyLog[3, a*x^2]))/(14406*a*Gamma[15/4])
```

Maple [A]
time = 0.16, size = 155, normalized size = 0.96

method	result

meijerg	$-\frac{(dx)^{\frac{5}{2}}\left(\frac{4x^{\frac{3}{2}}(-a)^{\frac{11}{4}}(2112a\,x^2+4928)}{79233a^2}+\frac{64x^{\frac{3}{2}}(-a)^{\frac{11}{4}}\left(\ln\left(1-(a\,x^2)^{\frac{1}{4}}\right)-\ln\left(1+(a\,x^2)^{\frac{1}{4}}\right)+2\arctan\left((a\,x^2)^{\frac{1}{4}}\right)\right)}{343a^2(a\,x^2)^{\frac{3}{4}}}-\frac{64x^{\frac{7}{2}}(-a)^{\frac{11}{4}}\ln(-a\,x^2+}{343a}\right.}{2x^{\frac{5}{2}}(-a)^{\frac{7}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*polylog(3,a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(d*x)^(5/2)/x^(5/2)/(-a)^(7/4)*(4/79233*x^(3/2)*(-a)^(11/4)*(2112*a*x^
2+4928)/a^2+64/343*x^(3/2)*(-a)^(11/4)/a^2/(a*x^2)^(3/4)*(ln(1-(a*x^2)^(1/4
))-ln(1+(a*x^2)^(1/4))+2*arctan((a*x^2)^(1/4)))-64/343*x^(7/2)*(-a)^(11/4)/
a*ln(-a*x^2+1)-16/49*x^(7/2)*(-a)^(11/4)/a*polylog(2,a*x^2)+4/7*x^(7/2)*(-a
)^(11/4)/a*polylog(3,a*x^2))
```

Maxima [A]
time = 0.47, size = 178, normalized size = 1.11

$$\frac{2\left(\frac{336\,d^4\left(\frac{2\arctan\left(\frac{\sqrt{dx}\,\sqrt{a}}{\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}\,\sqrt{a}}+\frac{\log\left(\frac{\sqrt{dx}\,\sqrt{a}-\sqrt{\sqrt{a}\,d}}{\sqrt{dx}\,\sqrt{a}+\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}\,\sqrt{a}}\right)}{a}-\frac{588\,(dx)^{\frac{7}{2}}a\mathrm{Li}_2(ax^2)+336\,(dx)^{\frac{7}{2}}a\log(-ad^2x^2+d^2)-1029\,(dx)^{\frac{7}{2}}a\mathrm{Li}_3(ax^2)-96\,(dx)^{\frac{7}{2}}(7\,a\log(d)+2\,a)-448\,(dx)^{\frac{3}{2}}d^2}{a}\right)}{7203\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*polylog(3,a*x^2),x, algorithm="maxima")
```

```
[Out] 2/7203*(336*d^4*(2*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/(sqrt(sqrt(a)*
d)*sqrt(a)) + log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a)
+ sqrt(sqrt(a)*d)))/(sqrt(sqrt(a)*d)*sqrt(a)))/a - (588*(d*x)^(7/2)*a*dilog
(a*x^2) + 336*(d*x)^(7/2)*a*log(-a*d^2*x^2 + d^2) - 1029*(d*x)^(7/2)*a*poly
log(3, a*x^2) - 96*(d*x)^(7/2)*(7*a*log(d) + 2*a) - 448*(d*x)^(3/2)*d^2)/a)
/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(116) = 232$.
time = 0.40, size = 237, normalized size = 1.47

$$\frac{2\left(1029\sqrt{dx}\,ad^2x^3\mathrm{polylog}(3,ax^2)-1344\left(\frac{d^{10}}{a^7}\right)^{\frac{1}{4}}a\arctan\left(-\frac{\left(\frac{d^{10}}{a^7}\right)^{\frac{1}{4}}\sqrt{dx}\,a^2d^7-\sqrt{d^{15}x+\sqrt{\frac{d^{10}}{a^7}}\,a^3d^{10}}\left(\frac{d^{10}}{a^7}\right)^{\frac{1}{4}}a^2}{d^{10}}\right)-336\left(\frac{d^{10}}{a^7}\right)^{\frac{1}{4}}a\log\left(32768\sqrt{dx}\,d^7+32768\left(\frac{d^{10}}{a^7}\right)^{\frac{3}{4}}a^5\right)+336\left(\frac{d^{10}}{a^7}\right)^{\frac{1}{4}}a\log\left(32768\sqrt{dx}\,d^7-32768\left(\frac{d^{10}}{a^7}\right)^{\frac{3}{4}}a^5\right)-4\left(147\,ad^2x^3\mathrm{Li}_2(ax^2)+84\,ad^2x^3\log\left(-ax^2+1\right)-48\,ad^2x^3-112\,d^2x\right)\sqrt{dx}\right)}{7203\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*polylog(3,a*x^2),x, algorithm="fricas")
```

```
[Out] 2/7203*(1029*sqrt(d*x)*a*d^2*x^3*polylog(3, a*x^2) - 1344*(d^10/a^7)^(1/4)*
a*arctan(-((d^10/a^7)^(1/4)*sqrt(d*x)*a^2*d^7 - sqrt(d^15*x + sqrt(d^10/a^7
)*a^3*d^10)*(d^10/a^7)^(1/4)*a^2)/d^10) - 336*(d^10/a^7)^(1/4)*a*log(32768*
sqrt(d*x)*d^7 + 32768*(d^10/a^7)^(3/4)*a^5) + 336*(d^10/a^7)^(1/4)*a*log(32
768*sqrt(d*x)*d^7 - 32768*(d^10/a^7)^(3/4)*a^5) - 4*(147*a*d^2*x^3*dilog(a*
x^2) + 84*a*d^2*x^3*log(-a*x^2 + 1) - 48*a*d^2*x^3 - 112*d^2*x)*sqrt(d*x))/
a
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{5}{2}} \operatorname{Li}_3\left(ax^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*polylog(3,a*x**2),x)
```

```
[Out] Integral((d*x)**(5/2)*polylog(3, a*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*polylog(3,a*x^2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(5/2)*polylog(3, a*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{polylog}\left(3,a\,x^2\right)\,{\left(d\,x\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^2)*(d*x)^(5/2),x)
```

```
[Out] int(polylog(3, a*x^2)*(d*x)^(5/2), x)
```

3.79 $\int (dx)^{3/2}\textbf{PolyLog}(3, ax^2)\, dx$

Optimal. Leaf size=161

$$\frac{128d\sqrt{dx}}{125a}+\frac{128(dx)^{5/2}}{625d}-\frac{64d^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/4}}-\frac{64d^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/4}}-\frac{32(dx)^{5/2}\log\left(1-ax^2\right.}{125d}$$

```
[Out] 128/625*(d*x)^(5/2)/d-64/125*d^(3/2)*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))/a^
(5/4)-64/125*d^(3/2)*arctanh(a^(1/4)*(d*x)^(1/2)/d^(1/2))/a^(5/4)-32/125*(d
*x)^(5/2)*ln(-a*x^2+1)/d-8/25*(d*x)^(5/2)*polylog(2,a*x^2)/d+2/5*(d*x)^(5/2
)*polylog(3,a*x^2)/d+128/125*d*(d*x)^(1/2)/a
```

Rubi [A]
time = 0.08, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {6726, 2505, 16, 327, 335, 218, 214, 211}

$$-\frac{64d^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/4}}-\frac{64d^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/4}}-\frac{8(dx)^{5/2}\mathrm{Li}_2(ax^2)}{25d}+\frac{2(dx)^{5/2}\mathrm{Li}_3(ax^2)}{5d}-\frac{32(dx)^{5/2}\log\left(1-ax^2\right)}{125d}+\frac{128d\sqrt{dx}}{125a}+\frac{128(dx)^{5/2}}{625d}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*PolyLog[3, a*x^2],x]
```

```
[Out] (128*d*Sqrt[d*x])/(125*a) + (128*(d*x)^(5/2))/(625*d) - (64*d^(3/2)*ArcTan[
(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(125*a^(5/4)) - (64*d^(3/2)*ArcTanh[(a^(1/4)*
Sqrt[d*x])/Sqrt[d]])/(125*a^(5/4)) - (32*(d*x)^(5/2)*Log[1 - a*x^2])/(125*d
) - (8*(d*x)^(5/2)*PolyLog[2, a*x^2])/(25*d) + (2*(d*x)^(5/2)*PolyLog[3, a*
x^2])/(5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int (dx)^{3/2}\text{Li}_3(ax^2)\,dx = \frac{2(dx)^{5/2}\text{Li}_3(ax^2)}{5d} - \frac{4}{5}\int (dx)^{3/2}\text{Li}_2(ax^2)\,dx$$

$$= -\frac{8(dx)^{5/2}\text{Li}_2(ax^2)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3(ax^2)}{5d} - \frac{16}{25}\int (dx)^{3/2}\log(1-ax^2)\,dx$$

$$= -\frac{32(dx)^{5/2}\log(1-ax^2)}{125d} - \frac{8(dx)^{5/2}\text{Li}_2(ax^2)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3(ax^2)}{5d} - \frac{(64a)\int \frac{x(dx)^{5/2}}{1-ax^2}}{125d}$$

$$= -\frac{32(dx)^{5/2}\log(1-ax^2)}{125d} - \frac{8(dx)^{5/2}\text{Li}_2(ax^2)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3(ax^2)}{5d} - \frac{(64a)\int \frac{(dx)^{7/2}}{1-ax^2}}{125d^2}$$

$$= \frac{128(dx)^{5/2}}{625d} - \frac{32(dx)^{5/2}\log(1-ax^2)}{125d} - \frac{8(dx)^{5/2}\text{Li}_2(ax^2)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3(ax^2)}{5d} - \frac{6}{12}$$

$$= \frac{128d\sqrt{dx}}{125a} + \frac{128(dx)^{5/2}}{625d} - \frac{32(dx)^{5/2}\log(1-ax^2)}{125d} - \frac{8(dx)^{5/2}\text{Li}_2(ax^2)}{25d} + \frac{2(dx)^{5/2}}{5}$$

$$= \frac{128d\sqrt{dx}}{125a} + \frac{128(dx)^{5/2}}{625d} - \frac{32(dx)^{5/2}\log(1-ax^2)}{125d} - \frac{8(dx)^{5/2}\text{Li}_2(ax^2)}{25d} + \frac{2(dx)^{5/2}}{5}$$

$$= \frac{128d\sqrt{dx}}{125a} + \frac{128(dx)^{5/2}}{625d} - \frac{32(dx)^{5/2}\log(1-ax^2)}{125d} - \frac{8(dx)^{5/2}\text{Li}_2(ax^2)}{25d} + \frac{2(dx)^{5/2}}{5}$$

$$= \frac{128d\sqrt{dx}}{125a} + \frac{128(dx)^{5/2}}{625d} - \frac{64d^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/4}} - \frac{64d^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 89, normalized size = 0.55

$$-\frac{9d\sqrt{dx}\,\Gamma\left(\frac{9}{4}\right)\left(-320-64ax^2+320\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};ax^2\right)+80ax^2\log(1-ax^2)+100ax^2\text{PolyLog}(2,ax^2)-125ax^2\text{PolyLog}(3,ax^2)\right)}{1250a\Gamma\left(\frac{13}{4}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*PolyLog[3, a*x^2],x]
```

```
[Out] (-9*d*Sqrt[d*x]*Gamma[9/4]*(-320 - 64*a*x^2 + 320*Hypergeometric2F1[1/4, 1,
 5/4, a*x^2] + 80*a*x^2*Log[1 - a*x^2] + 100*a*x^2*PolyLog[2, a*x^2] - 125*
a*x^2*PolyLog[3, a*x^2]))/(1250*a*Gamma[13/4])
```

Maple [A]
time = 0.14, size = 155, normalized size = 0.96

method	result
meijerg	$-\frac{(dx)^{\frac{3}{2}}\left(\frac{4\sqrt{x}\,(-a)^{\frac{9}{4}}\left(576a\,x^{2}+2880\right)}{5625a^{2}}+\frac{64\sqrt{x}\,(-a)^{\frac{9}{4}}\left(\ln\left(1-\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(a\,x^{2}\right)^{\frac{1}{4}}\right)\right)}{125a^{2}\left(a\,x^{2}\right)^{\frac{1}{4}}}-\frac{64x^{\frac{5}{2}}(-a)^{\frac{9}{4}}\ln\left(-a\,x^{2}+\right.}{125a}}{2x^{\frac{3}{2}}(-a)^{\frac{5}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*polylog(3,a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(d*x)^(3/2)/x^(3/2)/(-a)^(5/4)*(4/5625*x^(1/2)*(-a)^(9/4)*(576*a*x^2+2
880)/a^2+64/125*x^(1/2)*(-a)^(9/4)/a^2/(a*x^2)^(1/4)*(ln(1-(a*x^2)^(1/4))-l
n(1+(a*x^2)^(1/4))-2*arctan((a*x^2)^(1/4)))-64/125*x^(5/2)*(-a)^(9/4)/a*ln(
-a*x^2+1)-16/25*x^(5/2)*(-a)^(9/4)/a*polylog(2,a*x^2)+4/5*x^(5/2)*(-a)^(9/4
)/a*polylog(3,a*x^2))
```

Maxima [A]

time = 0.47, size = 175, normalized size = 1.09

$$-\frac{2\left(\frac{100\,(dx)^{\frac{5}{2}}a\mathrm{Li}_2\left(ax^2\right)+80\,(dx)^{\frac{5}{2}}a\log\left(-ad^2x^2+d^2\right)-125\,(dx)^{\frac{5}{2}}a\mathrm{Li}_3\left(ax^2\right)-32\,(dx)^{\frac{5}{2}}(5\,a\log(d)+2\,a)-320\,\sqrt{dx}\,d^2}{a}+\frac{80\left(\frac{2\,d^3\arctan\left(\frac{\sqrt{dx}\,\sqrt{a}}{\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}}-\frac{d^3\log\left(\frac{\sqrt{dx}\,\sqrt{a}-\sqrt{\sqrt{a}\,d}}{\sqrt{dx}\,\sqrt{a}+\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}}\right)}{a}\right)}{625\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x^2),x, algorithm="maxima")
```

```
[Out] -2/625*((100*(d*x)^(5/2)*a*dilog(a*x^2) + 80*(d*x)^(5/2)*a*log(-a*d^2*x^2 +
 d^2) - 125*(d*x)^(5/2)*a*polylog(3, a*x^2) - 32*(d*x)^(5/2)*(5*a*log(d) +
2*a) - 320*sqrt(d*x)*d^2)/a + 80*(2*d^3*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(
a)*d))/sqrt(sqrt(a)*d) - d^3*log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqr
t(d*x)*sqrt(a) + sqrt(sqrt(a)*d)))/sqrt(sqrt(a)*d))/a)/d
```

Fricas [A]

time = 0.41, size = 213, normalized size = 1.32

$$\frac{2\left(125\,\sqrt{dx}\,adx^2\mathrm{polylog}(3,ax^2)+320\,a\left(\frac{d^6}{a^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^4d\left(\frac{d^6}{a^5}\right)^{\frac{3}{4}}-\sqrt{d^3x+a^2\sqrt{\frac{d^6}{a^5}}}\,a^4\left(\frac{d^6}{a^5}\right)^{\frac{3}{4}}}{d^6}\right)-80\,a\left(\frac{d^6}{a^5}\right)^{\frac{1}{4}}\log\left(32\,\sqrt{dx}\,d+32\,a\left(\frac{d^6}{a^5}\right)^{\frac{1}{4}}\right)+80\,a\left(\frac{d^6}{a^5}\right)^{\frac{1}{4}}\log\left(32\,\sqrt{dx}\,d-32\,a\left(\frac{d^6}{a^5}\right)^{\frac{1}{4}}\right)-4\left(25\,adx^2\mathrm{Li}_2(ax^2)+20\,adx^2\log\left(-ax^2+1\right)-16\,adx^2-80\,d\right)\sqrt{dx}\right)}{625\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x^2),x, algorithm="fricas")
```

```
[Out] 2/625*(125*sqrt(d*x)*a*d*x^2*polylog(3, a*x^2) + 320*a*(d^6/a^5)^(1/4)*arct
an(-(sqrt(d*x)*a^4*d*(d^6/a^5)^(3/4) - sqrt(d^3*x + a^2*sqrt(d^6/a^5))*a^4*
(d^6/a^5)^(3/4))/d^6) - 80*a*(d^6/a^5)^(1/4)*log(32*sqrt(d*x)*d + 32*a*(d^6
/a^5)^(1/4)) + 80*a*(d^6/a^5)^(1/4)*log(32*sqrt(d*x)*d - 32*a*(d^6/a^5)^(1/
4)) - 4*(25*a*d*x^2*dilog(a*x^2) + 20*a*d*x^2*log(-a*x^2 + 1) - 16*a*d*x^2
- 80*d)*sqrt(d*x))/a
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}} \operatorname{Li}_3\left(ax^2\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*polylog(3,a*x**2),x)
[Out] Integral((d*x)**(3/2)*polylog(3, a*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x^2),x, algorithm="giac")
[Out] integrate((d*x)^(3/2)*polylog(3, a*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{polylog}\left(3,a\,x^2\right)\,{\left(d\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^2)*(d*x)^(3/2),x)
[Out] int(polylog(3, a*x^2)*(d*x)^(3/2), x)
```

3.80 $\int \sqrt{dx}\,\text{PolyLog}(3,ax^2)\,dx$

Optimal. Leaf size=146

$$\frac{128(dx)^{3/2}}{81d}+\frac{64\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/4}}-\frac{64\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/4}}-\frac{32(dx)^{3/2}\log\left(1-ax^2\right)}{27d}-\frac{8(dx)^{3/2}\text{P}}{}$$

```
[Out] 128/81*(d*x)^(3/2)/d-32/27*(d*x)^(3/2)*ln(-a*x^2+1)/d-8/9*(d*x)^(3/2)*polyl
og(2,a*x^2)/d+2/3*(d*x)^(3/2)*polylog(3,a*x^2)/d+64/27*arctan(a^(1/4)*(d*x)
^(1/2)/d^(1/2))*d^(1/2)/a^(3/4)-64/27*arctanh(a^(1/4)*(d*x)^(1/2)/d^(1/2))*
d^(1/2)/a^(3/4)
```

Rubi [A]
time = 0.07, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {6726, 2505, 16, 327, 335, 304, 211, 214}

$$\frac{64\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/4}}-\frac{64\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/4}}-\frac{8(dx)^{3/2}\text{Li}_2(ax^2)}{9d}+\frac{2(dx)^{3/2}\text{Li}_3(ax^2)}{3d}-\frac{32(dx)^{3/2}\log\left(1-ax^2\right)}{27d}+\frac{128(dx)^{3/2}}{81d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*PolyLog[3, a*x^2],x]
```

```
[Out] (128*(d*x)^(3/2))/(81*d) + (64*Sqrt[d]*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])
/(27*a^(3/4)) - (64*Sqrt[d]*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(27*a^(3/
4)) - (32*(d*x)^(3/2)*Log[1 - a*x^2])/(27*d) - (8*(d*x)^(3/2)*PolyLog[2, a*
x^2])/(9*d) + (2*(d*x)^(3/2)*PolyLog[3, a*x^2])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \sqrt{dx}\,\mathrm{Li}_3(ax^2)\,dx = \frac{2(dx)^{3/2}\mathrm{Li}_3(ax^2)}{3d} - \frac{4}{3}\int \sqrt{dx}\,\mathrm{Li}_2(ax^2)\,dx$$
$$= -\frac{8(dx)^{3/2}\mathrm{Li}_2(ax^2)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_3(ax^2)}{3d} - \frac{16}{9}\int \sqrt{dx}\,\log(1-ax^2)\,dx$$
$$= -\frac{32(dx)^{3/2}\log(1-ax^2)}{27d} - \frac{8(dx)^{3/2}\mathrm{Li}_2(ax^2)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_3(ax^2)}{3d} - \frac{(64a)\int \frac{x(dx)^{3/2}}{1-ax^2}\,d}{27d}$$
$$= -\frac{32(dx)^{3/2}\log(1-ax^2)}{27d} - \frac{8(dx)^{3/2}\mathrm{Li}_2(ax^2)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_3(ax^2)}{3d} - \frac{(64a)\int \frac{(dx)^{5/2}}{1-ax^2}\,dx}{27d^2}$$
$$= \frac{128(dx)^{3/2}}{81d} - \frac{32(dx)^{3/2}\log(1-ax^2)}{27d} - \frac{8(dx)^{3/2}\mathrm{Li}_2(ax^2)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_3(ax^2)}{3d} - \frac{64}{27}\int$$
$$= \frac{128(dx)^{3/2}}{81d} - \frac{32(dx)^{3/2}\log(1-ax^2)}{27d} - \frac{8(dx)^{3/2}\mathrm{Li}_2(ax^2)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_3(ax^2)}{3d} - \frac{128S}{}$$
$$= \frac{128(dx)^{3/2}}{81d} - \frac{32(dx)^{3/2}\log(1-ax^2)}{27d} - \frac{8(dx)^{3/2}\mathrm{Li}_2(ax^2)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_3(ax^2)}{3d} - \frac{(64d}{}$$
$$= \frac{128(dx)^{3/2}}{81d} + \frac{64\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/4}} - \frac{64\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/4}} - \frac{32(dx)^{3/2}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 68, normalized size = 0.47

$$-\frac{7x\sqrt{dx}\,\Gamma\left(\frac{7}{4}\right)\left(-64+64\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};ax^2\right)+48\log(1-ax^2)+36\mathrm{PolyLog}(2,ax^2)-27\mathrm{PolyLog}(3,ax^2)\right)}{162\Gamma\left(\frac{11}{4}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*PolyLog[3, a*x^2],x]
```

```
[Out] (-7*x*Sqrt[d*x]*Gamma[7/4]*(-64 + 64*Hypergeometric2F1[3/4, 1, 7/4, a*x^2]
+ 48*Log[1 - a*x^2] + 36*PolyLog[2, a*x^2] - 27*PolyLog[3, a*x^2]))/(162*Ga
mma[11/4])
```

Maple [A]
time = 0.14, size = 147, normalized size = 1.01

method	result

meijerg	$-\frac{\sqrt{dx}\left(\frac{256x^{\frac{3}{2}}(-a)^{\frac{7}{4}}}{81a}+\frac{64x^{\frac{3}{2}}(-a)^{\frac{7}{4}}\left(\ln\left(1-\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^{2}\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(a\,x^{2}\right)^{\frac{1}{4}}\right)\right)}{27a\left(a\,x^{2}\right)^{\frac{3}{4}}}-\frac{64x^{\frac{3}{2}}(-a)^{\frac{7}{4}}\ln\left(-a\,x^{2}+1\right)}{27a}-\frac{16x^{\frac{3}{2}}(-a)^{\frac{7}{4}}}{}\right.}{2\sqrt{x}\,(-a)^{\frac{3}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*polylog(3,a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(d*x)^(1/2)/x^(1/2)/(-a)^(3/4)*(256/81*x^(3/2)*(-a)^(7/4)/a+64/27*x^(3
/2)*(-a)^(7/4)/a/(a*x^2)^(3/4)*(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))+2*a
rctan((a*x^2)^(1/4)))-64/27*x^(3/2)*(-a)^(7/4)/a*ln(-a*x^2+1)-16/9*x^(3/2)*
(-a)^(7/4)/a*polylog(2,a*x^2)+4/3*x^(3/2)*(-a)^(7/4)/a*polylog(3,a*x^2))
```

Maxima [A]
time = 0.47, size = 153, normalized size = 1.05

$$\frac{2\left(48\,d^{2}\left(\frac{2\arctan\left(\frac{\sqrt{dx}\,\sqrt{a}}{\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}\,\sqrt{a}}+\frac{\log\left(\frac{\sqrt{dx}\,\sqrt{a}-\sqrt{\sqrt{a}\,d}}{\sqrt{dx}\,\sqrt{a}+\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}\,\sqrt{a}}\right)+32\,(dx)^{\frac{3}{2}}\,(3\,\log(d)+2)-36\,(dx)^{\frac{3}{2}}\,\mathrm{Li}_2(ax^2)-48\,(dx)^{\frac{3}{2}}\log\left(-ad^2x^2+d^2\right)+27\,(dx)^{\frac{3}{2}}\,\mathrm{Li}_3(ax^2)\right)}{81\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x^2),x, algorithm="maxima")
```

```
[Out] 2/81*(48*d^2*(2*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/(sqrt(sqrt(a)*d)*
sqrt(a)) + log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a) + s
qrt(sqrt(a)*d)))/(sqrt(sqrt(a)*d)*sqrt(a))) + 32*(d*x)^(3/2)*(3*log(d) + 2)
 - 36*(d*x)^(3/2)*dilog(a*x^2) - 48*(d*x)^(3/2)*log(-a*d^2*x^2 + d^2) + 27*
(d*x)^(3/2)*polylog(3, a*x^2))/d
```

Fricas [A]
time = 0.39, size = 187, normalized size = 1.28

$$\frac{2}{3}\sqrt{dx}\,x\,\mathrm{polylog}(3,ax^2)-\frac{8}{81}\sqrt{dx}\left(9\,x\mathrm{Li}_2(ax^2)+12\,x\log\left(-ax^2+1\right)-16\,x\right)-\frac{128}{27}\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,ad\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}-\sqrt{d^3x+ad^2\sqrt{\frac{d^2}{a^3}}}\,a\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}}{d^2}\right)-\frac{32}{27}\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}\log\left(32768\,a^2\left(\frac{d^2}{a^3}\right)^{\frac{3}{4}}+32768\sqrt{dx}\,d\right)+\frac{32}{27}\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}\log\left(-32768\,a^2\left(\frac{d^2}{a^3}\right)^{\frac{3}{4}}+32768\sqrt{dx}\,d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x^2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(d*x)*x*polylog(3, a*x^2) - 8/81*sqrt(d*x)*(9*x*dilog(a*x^2) + 12*x
*log(-a*x^2 + 1) - 16*x) - 128/27*(d^2/a^3)^(1/4)*arctan(-(sqrt(d*x)*a*d*(d
^2/a^3)^(1/4) - sqrt(d^3*x + a*d^2*sqrt(d^2/a^3))*a*(d^2/a^3)^(1/4))/d^2) -
 32/27*(d^2/a^3)^(1/4)*log(32768*a^2*(d^2/a^3)^(3/4) + 32768*sqrt(d*x)*d) +
 32/27*(d^2/a^3)^(1/4)*log(-32768*a^2*(d^2/a^3)^(3/4) + 32768*sqrt(d*x)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\, \operatorname{Li}_3\left(ax^2\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*polylog(3,a*x**2),x)
```

```
[Out] Integral(sqrt(d*x)*polylog(3, a*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*polylog(3, a*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{polylog}\left(3,a\,x^2\right)\,\sqrt{d\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^2)*(d*x)^(1/2),x)
```

```
[Out] int(polylog(3, a*x^2)*(d*x)^(1/2), x)
```

3.81 $\int \frac{\text{PolyLog}(3,ax^2)}{\sqrt{dx}}\,dx$

Optimal. Leaf size=134

$$\frac{128\sqrt{dx}}{d}-\frac{64\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}}-\frac{64\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}}-\frac{32\sqrt{dx}\,\log(1-ax^2)}{d}-\frac{8\sqrt{dx}\,\text{PolyLog}(2,a$$

```
[Out] -64*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))/a^(1/4)/d^(1/2)-64*arctanh(a^(1/4)*
(d*x)^(1/2)/d^(1/2))/a^(1/4)/d^(1/2)+128*(d*x)^(1/2)/d-32*ln(-a*x^2+1)*(d*x
)^(1/2)/d-8*polylog(2,a*x^2)*(d*x)^(1/2)/d+2*polylog(3,a*x^2)*(d*x)^(1/2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {6726, 2505, 16, 327, 335, 218, 214, 211}

$$-\frac{64\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}}-\frac{8\sqrt{dx}\,\text{Li}_2(ax^2)}{d}+\frac{2\sqrt{dx}\,\text{Li}_3(ax^2)}{d}-\frac{32\sqrt{dx}\,\log(1-ax^2)}{d}-\frac{64\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}}+\frac{128\sqrt{dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/Sqrt[d*x],x]
```

```
[Out] (128*Sqrt[d*x])/d - (64*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(a^(1/4)*Sqrt[
d]) - (64*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(a^(1/4)*Sqrt[d]) - (32*Sqr
t[d*x]*Log[1 - a*x^2])/d - (8*Sqrt[d*x]*PolyLog[2, a*x^2])/d + (2*Sqrt[d*x]
*PolyLog[3, a*x^2])/d
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\text{Li}_3(ax^2)}{\sqrt{dx}}\,dx = \frac{2\sqrt{dx}\,\text{Li}_3(ax^2)}{d} - 4\int \frac{\text{Li}_2(ax^2)}{\sqrt{dx}}\,dx$$
$$= -\frac{8\sqrt{dx}\,\text{Li}_2(ax^2)}{d} + \frac{2\sqrt{dx}\,\text{Li}_3(ax^2)}{d} - 16\int \frac{\log(1-ax^2)}{\sqrt{dx}}\,dx$$
$$= -\frac{32\sqrt{dx}\,\log(1-ax^2)}{d} - \frac{8\sqrt{dx}\,\text{Li}_2(ax^2)}{d} + \frac{2\sqrt{dx}\,\text{Li}_3(ax^2)}{d} - \frac{(64a)\int \frac{x\sqrt{dx}}{1-ax^2}\,dx}{d}$$
$$= -\frac{32\sqrt{dx}\,\log(1-ax^2)}{d} - \frac{8\sqrt{dx}\,\text{Li}_2(ax^2)}{d} + \frac{2\sqrt{dx}\,\text{Li}_3(ax^2)}{d} - \frac{(64a)\int \frac{(dx)^{3/2}}{1-ax^2}\,dx}{d^2}$$
$$= \frac{128\sqrt{dx}}{d} - \frac{32\sqrt{dx}\,\log(1-ax^2)}{d} - \frac{8\sqrt{dx}\,\text{Li}_2(ax^2)}{d} + \frac{2\sqrt{dx}\,\text{Li}_3(ax^2)}{d} - 64\int \frac{1}{\sqrt{dx}\,(1-$$
$$= \frac{128\sqrt{dx}}{d} - \frac{32\sqrt{dx}\,\log(1-ax^2)}{d} - \frac{8\sqrt{dx}\,\text{Li}_2(ax^2)}{d} + \frac{2\sqrt{dx}\,\text{Li}_3(ax^2)}{d} - \frac{128\text{Subst}\left(\int \frac{}{1-}\right.}{}$$
$$= \frac{128\sqrt{dx}}{d} - \frac{32\sqrt{dx}\,\log(1-ax^2)}{d} - \frac{8\sqrt{dx}\,\text{Li}_2(ax^2)}{d} + \frac{2\sqrt{dx}\,\text{Li}_3(ax^2)}{d} - 64\text{Subst}\left(\int \frac{}{d}\right.$$
$$= \frac{128\sqrt{dx}}{d} - \frac{64\tan^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\,\sqrt{d}} - \frac{64\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\,\sqrt{d}} - \frac{32\sqrt{dx}\,\log(1-ax^2)}{d} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 68, normalized size = 0.51

$$-\frac{5x\Gamma\left(\frac{5}{4}\right)\left(-64+64\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};ax^2\right)+16\log\left(1-ax^2\right)+4\text{PolyLog}(2,ax^2)-\text{PolyLog}(3,ax^2)\right)}{2\sqrt{dx}\,\Gamma\left(\frac{9}{4}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^2]/Sqrt[d*x],x]
```

```
[Out] (-5*x*Gamma[5/4]*(-64 + 64*Hypergeometric2F1[1/4, 1, 5/4, a*x^2] + 16*Log[1
 - a*x^2] + 4*PolyLog[2, a*x^2] - PolyLog[3, a*x^2]))/(2*Sqrt[d*x]*Gamma[9/
4])
```

Maple [A]
time = 0.16, size = 147, normalized size = 1.10

method	result

meijerg	$-\frac{\sqrt{x}\left(\frac{256\sqrt{x}\,(-a)^{\frac{5}{4}}}{a}+\frac{64\sqrt{x}\,(-a)^{\frac{5}{4}}\left(\ln\left(1-\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(a\,x^{2}\right)^{\frac{1}{4}}\right)\right)}{a\left(a\,x^{2}\right)^{\frac{1}{4}}}-\frac{64\sqrt{x}\,(-a)^{\frac{5}{4}}\ln\left(-a\,x^{2}+1\right)}{a}-16\sqrt{\cdots}\right.}{2\sqrt{dx}\,(-a)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(d*x)^(1/2)*x^(1/2)/(-a)^(1/4)*(256*x^(1/2)*(-a)^(5/4)/a+64*x^(1/2)*(-
a)^(5/4)/a/(a*x^2)^(1/4)*(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))-2*arctan(
(a*x^2)^(1/4)))-64*x^(1/2)*(-a)^(5/4)/a*ln(-a*x^2+1)-16*x^(1/2)*(-a)^(5/4)/
a*polylog(2,a*x^2)+4*x^(1/2)*(-a)^(5/4)/a*polylog(3,a*x^2))
```

Maxima [A]
time = 0.48, size = 141, normalized size = 1.05

$$\frac{2\left(32\sqrt{dx}\left(\log\left(d\right)+2\right)-\frac{32\,d\arctan\left(\frac{\sqrt{dx}\sqrt{a}}{\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}}-4\sqrt{dx}\,\mathrm{Li}_2(ax^2)-16\sqrt{dx}\,\log\left(-ad^2x^2+d^2\right)+\frac{16\,d\log\left(\frac{\sqrt{dx}\sqrt{a}-\sqrt{\sqrt{a}\,d}}{\sqrt{dx}\sqrt{a}+\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}}+\sqrt{dx}\,\mathrm{Li}_3(ax^2)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(32*sqrt(d*x)*(log(d) + 2) - 32*d*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d
))/sqrt(sqrt(a)*d) - 4*sqrt(d*x)*dilog(a*x^2) - 16*sqrt(d*x)*log(-a*d^2*x^2
 + d^2) + 16*d*log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a)
 + sqrt(sqrt(a)*d)))/sqrt(sqrt(a)*d) + sqrt(d*x)*polylog(3, a*x^2))/d
```

Fricas [A]
time = 0.38, size = 169, normalized size = 1.26

$$\frac{2\left(64\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{d^2\sqrt{\frac{1}{ad^2}}+dx}\;ad\left(\frac{1}{ad^2}\right)^{\frac{3}{4}}-\sqrt{dx}\;ad\left(\frac{1}{ad^2}\right)^{\frac{3}{4}}\right)-16\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\log\left(d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)+16\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\log\left(-d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)-4\sqrt{dx}\left(\mathrm{Li}_2(ax^2)+4\log\left(-ax^2+1\right)-16\right)+\sqrt{dx}\,\mathrm{polylog}(3,ax^2)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(64*d*(1/(a*d^2))^(1/4)*arctan(sqrt(d^2*sqrt(1/(a*d^2)) + d*x)*a*d*(1/(a*
d^2))^(3/4) - sqrt(d*x)*a*d*(1/(a*d^2))^(3/4)) - 16*d*(1/(a*d^2))^(1/4)*log
(d*(1/(a*d^2))^(1/4) + sqrt(d*x)) + 16*d*(1/(a*d^2))^(1/4)*log(-d*(1/(a*d^2
))^(1/4) + sqrt(d*x)) - 4*sqrt(d*x)*(dilog(a*x^2) + 4*log(-a*x^2 + 1) - 16)
 + sqrt(d*x)*polylog(3, a*x^2))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_3\left(ax^2\right)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/(d*x)**(1/2),x)
```

```
[Out] Integral(polylog(3, a*x**2)/sqrt(d*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/sqrt(d*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(3,a\,x^2)}{\sqrt{d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^2)/(d*x)^(1/2),x)
```

```
[Out] int(polylog(3, a*x^2)/(d*x)^(1/2), x)
```

3.82 $\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{64\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{64\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{32\log(1-ax^2)}{d\sqrt{dx}}-\frac{8\text{PolyLog}(2,ax^2)}{d\sqrt{dx}}-\frac{2\text{PolyLo}}{d\sqrt{}}$$

```
[Out] -64*a^(1/4)*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(3/2)+64*a^(1/4)*arctanh(
a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(3/2)+32*ln(-a*x^2+1)/d/(d*x)^(1/2)-8*polylo
g(2,a*x^2)/d/(d*x)^(1/2)-2*polylog(3,a*x^2)/d/(d*x)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {6726, 2505, 16, 335, 304, 211, 214}

$$-\frac{64\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{64\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{8\text{Li}_2(ax^2)}{d\sqrt{dx}}-\frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}}+\frac{32\log(1-ax^2)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/(d*x)^(3/2),x]
```

```
[Out] (-64*a^(1/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (64*a^(1/4)*Arc
Tanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (32*Log[1 - a*x^2])/(d*Sqrt[d*
x]) - (8*PolyLog[2, a*x^2])/(d*Sqrt[d*x]) - (2*PolyLog[3, a*x^2])/(d*Sqrt[d
*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{Li}_3(ax^2)}{(dx)^{3/2}}\,dx &= -\frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}} + 4\int \frac{\text{Li}_2(ax^2)}{(dx)^{3/2}}\,dx \\
&= -\frac{8\text{Li}_2(ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}} - 16\int \frac{\log(1-ax^2)}{(dx)^{3/2}}\,dx \\
&= \frac{32\log(1-ax^2)}{d\sqrt{dx}} - \frac{8\text{Li}_2(ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}} + \frac{(64a)\int \frac{x}{\sqrt{dx}\,(1-ax^2)}\,dx}{d} \\
&= \frac{32\log(1-ax^2)}{d\sqrt{dx}} - \frac{8\text{Li}_2(ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}} + \frac{(64a)\int \frac{\sqrt{dx}}{1-ax^2}\,dx}{d^2} \\
&= \frac{32\log(1-ax^2)}{d\sqrt{dx}} - \frac{8\text{Li}_2(ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}} + \frac{(128a)\text{Subst}\left(\int \frac{x^2}{1-\frac{ax^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{d^3} \\
&= \frac{32\log(1-ax^2)}{d\sqrt{dx}} - \frac{8\text{Li}_2(ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}} + \frac{(64\sqrt{a})\,\text{Subst}\left(\int \frac{1}{d-\sqrt{a}\,x^2}\,dx, x, \sqrt{dx}\right)}{d} - \\
&= -\frac{64\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{64\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{32\log(1-ax^2)}{d\sqrt{dx}} - \frac{8\text{Li}_2(ax^2)}{d\sqrt{dx}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.06, size = 71, normalized size = 0.58

$$
\frac{x\Gamma\left(\frac{3}{4}\right)\left(64ax^2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};ax^2\right)+48\log\left(1-ax^2\right)-12\text{PolyLog}(2,ax^2)-3\text{PolyLog}(3,ax^2)\right)}{2(dx)^{3/2}\Gamma\left(\frac{7}{4}\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^2]/(d*x)^(3/2),x]
```

```
[Out] (x*Gamma[3/4]*(64*a*x^2*Hypergeometric2F1[3/4, 1, 7/4, a*x^2] + 48*Log[1 -
a*x^2] - 12*PolyLog[2, a*x^2] - 3*PolyLog[3, a*x^2]))/(2*(d*x)^(3/2)*Gamma[
7/4])
```

Maple [A]
time = 0.14, size = 131, normalized size = 1.07

method	result

meijerg	$-\frac{x^{\frac{3}{2}}(-a)^{\frac{1}{4}}\left(-\frac{64x^{\frac{3}{2}}(-a)^{\frac{3}{4}}\left(\ln\left(1-\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^{2}\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(a\,x^{2}\right)^{\frac{1}{4}}\right)\right)}{\left(a\,x^{2}\right)^{\frac{3}{4}}}+\frac{64(-a)^{\frac{3}{4}}\ln\left(-a\,x^{2}+1\right)}{\sqrt{x}\,a}-\frac{16(-a)^{\frac{3}{4}}\operatorname{polylog}\left(2,a\,x^{2}\right)}{\sqrt{x}\,a}-}{2(dx)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(d*x)^(3/2)*x^(3/2)*(-a)^(1/4)*(-64*x^(3/2)*(-a)^(3/4)/(a*x^2)^(3/4)*(
ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))+2*arctan((a*x^2)^(1/4)))+64/x^(1/2)
*(-a)^(3/4)/a*ln(-a*x^2+1)-16/x^(1/2)*(-a)^(3/4)/a*polylog(2,a*x^2)-4/x^(1/
2)*(-a)^(3/4)/a*polylog(3,a*x^2))
```

Maxima [A]
time = 0.47, size = 132, normalized size = 1.08

$$-\frac{2\left(16\,a\left(\frac{2\arctan\left(\frac{\sqrt{dx}\sqrt{a}}{\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}\,\sqrt{a}}+\frac{\log\left(\frac{\sqrt{dx}\sqrt{a}-\sqrt{\sqrt{a}\,d}}{\sqrt{dx}\sqrt{a}+\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}\,\sqrt{a}}\right)+\frac{4\,\mathrm{Li}_2(ax^2)-16\,\log(-ad^2x^2+d^2)+32\,\log(d)+\mathrm{Li}_3(ax^2)}{\sqrt{dx}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*(16*a*(2*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/(sqrt(sqrt(a)*d)*sqrt
(a)) + log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a) + sqrt(
sqrt(a)*d)))/(sqrt(sqrt(a)*d)*sqrt(a))) + (4*dilog(a*x^2) - 16*log(-a*d^2*x
^2 + d^2) + 32*log(d) + polylog(3, a*x^2))/sqrt(d*x))/d
```

Fricas [A]
time = 0.44, size = 184, normalized size = 1.51

$$\frac{2\left(64\,d^2x\left(\frac{a}{d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,ad\left(\frac{a}{d^6}\right)^{\frac{1}{4}}-\sqrt{ad^4\sqrt{\frac{a}{d^6}}+a^2dx}\,d\left(\frac{a}{d^6}\right)^{\frac{1}{4}}}{a}\right)+16\,d^2x\left(\frac{a}{d^6}\right)^{\frac{1}{4}}\log\left(32768\,d^5\left(\frac{a}{d^6}\right)^{\frac{3}{4}}+32768\sqrt{dx}\,a\right)-16\,d^2x\left(\frac{a}{d^6}\right)^{\frac{1}{4}}\log\left(-32768\,d^5\left(\frac{a}{d^6}\right)^{\frac{3}{4}}+32768\sqrt{dx}\,a\right)-4\sqrt{dx}\left(\mathrm{Li}_2(ax^2)-4\,\log\left(-ax^2+1\right)\right)-\sqrt{dx}\,\mathrm{polylog}(3,ax^2)\right)}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(64*d^2*x*(a/d^6)^(1/4)*arctan(-(sqrt(d*x)*a*d*(a/d^6)^(1/4) - sqrt(a*d^4
*sqrt(a/d^6) + a^2*d*x)*d*(a/d^6)^(1/4))/a) + 16*d^2*x*(a/d^6)^(1/4)*log(32
768*d^5*(a/d^6)^(3/4) + 32768*sqrt(d*x)*a) - 16*d^2*x*(a/d^6)^(1/4)*log(-32
```

```
768*d^5*(a/d^6)^(3/4) + 32768*sqrt(d*x)*a) - 4*sqrt(d*x)*(dilog(a*x^2) - 4*
log(-a*x^2 + 1)) - sqrt(d*x)*polylog(3, a*x^2))/(d^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_3(ax^2)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/(d*x)**(3/2),x)
[Out] Integral(polylog(3, a*x**2)/(d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(3/2),x, algorithm="giac")
[Out] integrate(polylog(3, a*x^2)/(d*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(3, a\,x^2)}{(d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^2)/(d*x)^(3/2),x)
[Out] int(polylog(3, a*x^2)/(d*x)^(3/2), x)
```

3.83 $\int \frac{\textbf{PolyLog}(3,ax^2)}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=132

$$\frac{64a^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}}+\frac{64a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}}+\frac{32\log\left(1-ax^2\right)}{27d(dx)^{3/2}}-\frac{8\text{PolyLog}(2,ax^2)}{9d(dx)^{3/2}}-\frac{2\text{PolyLog}}{3d(dx}$$

```
[Out] 64/27*a^(3/4)*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(5/2)+64/27*a^(3/4)*arc
tanh(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(5/2)+32/27*ln(-a*x^2+1)/d/(d*x)^(3/2)-
8/9*polylog(2,a*x^2)/d/(d*x)^(3/2)-2/3*polylog(3,a*x^2)/d/(d*x)^(3/2)
```

Rubi [A]

time = 0.07, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {6726, 2505, 16, 335, 218, 214, 211}

$$\frac{64a^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}}+\frac{64a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}}-\frac{8\text{Li}_2(ax^2)}{9d(dx)^{3/2}}-\frac{2\text{Li}_3(ax^2)}{3d(dx)^{3/2}}+\frac{32\log\left(1-ax^2\right)}{27d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/(d*x)^(5/2),x]
```

```
[Out] (64*a^(3/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(27*d^(5/2)) + (64*a^(3/4)
*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(27*d^(5/2)) + (32*Log[1 - a*x^2])/(
27*d*(d*x)^(3/2)) - (8*PolyLog[2, a*x^2])/(9*d*(d*x)^(3/2)) - (2*PolyLog[3,
 a*x^2])/(3*d*(d*x)^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\text{Li}_3(ax^2)}{(dx)^{5/2}}\,dx = -\frac{2\text{Li}_3(ax^2)}{3d(dx)^{3/2}} + \frac{4}{3}\int \frac{\text{Li}_2(ax^2)}{(dx)^{5/2}}\,dx$$
$$= -\frac{8\text{Li}_2(ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^2)}{3d(dx)^{3/2}} - \frac{16}{9}\int \frac{\log(1-ax^2)}{(dx)^{5/2}}\,dx$$
$$= \frac{32\log(1-ax^2)}{27d(dx)^{3/2}} - \frac{8\text{Li}_2(ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^2)}{3d(dx)^{3/2}} + \frac{(64a)\int \frac{x}{(dx)^{3/2}(1-ax^2)}\,dx}{27d}$$
$$= \frac{32\log(1-ax^2)}{27d(dx)^{3/2}} - \frac{8\text{Li}_2(ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^2)}{3d(dx)^{3/2}} + \frac{(64a)\int \frac{1}{\sqrt{dx}\,(1-ax^2)}\,dx}{27d^2}$$
$$= \frac{32\log(1-ax^2)}{27d(dx)^{3/2}} - \frac{8\text{Li}_2(ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^2)}{3d(dx)^{3/2}} + \frac{(128a)\text{Subst}\left(\int \frac{1}{1-\frac{ax^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{27d^3}$$
$$= \frac{32\log(1-ax^2)}{27d(dx)^{3/2}} - \frac{8\text{Li}_2(ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^2)}{3d(dx)^{3/2}} + \frac{(64a)\text{Subst}\left(\int \frac{1}{d-\sqrt{a}\,x^2}\,dx, x, \sqrt{dx}\right)}{27d^2} + \frac{(64}{}$$
$$= \frac{64a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}} + \frac{64a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}} + \frac{32\log(1-ax^2)}{27d(dx)^{3/2}} - \frac{8\text{Li}_2(ax^2)}{9d(dx)^{3/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 71, normalized size = 0.54

$$\frac{x\Gamma\left(\frac{1}{4}\right)\left(64ax^2\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};ax^2\right) + 16\log(1-ax^2) - 12\text{PolyLog}(2,ax^2) - 9\text{PolyLog}(3,ax^2)\right)}{54(dx)^{5/2}\Gamma\left(\frac{5}{4}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^2]/(d*x)^(5/2),x]
```

```
[Out] (x*Gamma[1/4]*(64*a*x^2*Hypergeometric2F1[1/4, 1, 5/4, a*x^2] + 16*Log[1 -
a*x^2] - 12*PolyLog[2, a*x^2] - 9*PolyLog[3, a*x^2]))/(54*(d*x)^(5/2)*Gamma
[5/4])
```

Maple [A]
time = 0.14, size = 131, normalized size = 0.99

method	result

meijerg	$-\frac{x^{\frac{5}{2}}(-a)^{\frac{3}{4}}\left(-\frac{64\sqrt{x}\,(-a)^{\frac{1}{4}}\left(\ln\left(1-\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(a\,x^{2}\right)^{\frac{1}{4}}\right)\right)}{27\left(a\,x^{2}\right)^{\frac{1}{4}}}+\frac{64(-a)^{\frac{1}{4}}\ln\left(-a\,x^{2}+1\right)}{27x^{\frac{3}{2}}a}-\frac{16(-a)^{\frac{1}{4}}\,\mathrm{polylog}\left(2,a\,x^{2}\right)}{9x^{\frac{3}{2}}a}\right.}{2(dx)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(d*x)^(5/2)*x^(5/2)*(-a)^(3/4)*(-64/27*x^(1/2)*(-a)^(1/4)/(a*x^2)^(1/4
)*(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))-2*arctan((a*x^2)^(1/4)))+64/27/x
^(3/2)*(-a)^(1/4)/a*ln(-a*x^2+1)-16/9/x^(3/2)*(-a)^(1/4)/a*polylog(2,a*x^2)
-4/3/x^(3/2)*(-a)^(1/4)/a*polylog(3,a*x^2))
```

Maxima [A]
time = 0.48, size = 134, normalized size = 1.02

$$\frac{2\left(\frac{32\,a\arctan\left(\frac{\sqrt{dx}\,\sqrt{a}}{\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}\,d}-\frac{16\,a\log\left(\frac{\sqrt{dx}\,\sqrt{a}-\sqrt{\sqrt{a}\,d}}{\sqrt{dx}\,\sqrt{a}+\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}\,d}-\frac{12\,\mathrm{Li}_2(ax^2)-16\,\log(-ad^2x^2+d^2)+32\,\log(d)+9\,\mathrm{Li}_3(ax^2)}{(dx)^{\frac{3}{2}}}\right)}{27\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/27*(32*a*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/(sqrt(sqrt(a)*d)*d) -
16*a*log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a) + sqrt(sq
rt(a)*d)))/(sqrt(sqrt(a)*d)*d) - (12*dilog(a*x^2) - 16*log(-a*d^2*x^2 + d^2
) + 32*log(d) + 9*polylog(3, a*x^2))/(d*x)^(3/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(95) = 190$.
time = 0.42, size = 211, normalized size = 1.60

$$-\frac{2\left(64\,d^3x^2\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,ad^7\left(\frac{a^3}{d^{10}}\right)^{\frac{3}{4}}-\sqrt{d^6\sqrt{\frac{a^3}{d^{10}}}+a^2dx}\,d^7\left(\frac{a^3}{d^{10}}\right)^{\frac{3}{4}}}{a^3}\right)-16\,d^3x^2\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}\log\left(32\,d^3\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}+32\,\sqrt{dx}\,a\right)+16\,d^3x^2\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}\log\left(-32\,d^3\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}+32\,\sqrt{dx}\,a\right)+4\,\sqrt{dx}\,(3\,\mathrm{Li}_2(ax^2)-4\,\log(-ax^2+1))+9\,\sqrt{dx}\,\mathrm{polylog}(3,ax^2)\right)}{27\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/27*(64*d^3*x^2*(a^3/d^10)^(1/4)*arctan(-(sqrt(d*x)*a*d^7*(a^3/d^10)^(3/4
) - sqrt(d^6*sqrt(a^3/d^10) + a^2*d*x)*d^7*(a^3/d^10)^(3/4))/a^3) - 16*d^3*
x^2*(a^3/d^10)^(1/4)*log(32*d^3*(a^3/d^10)^(1/4) + 32*sqrt(d*x)*a) + 16*d^3
```

```
*x^2*(a^3/d^10)^(1/4)*log(-32*d^3*(a^3/d^10)^(1/4) + 32*sqrt(d*x)*a) + 4*sq
rt(d*x)*(3*dilog(a*x^2) - 4*log(-a*x^2 + 1)) + 9*sqrt(d*x)*polylog(3, a*x^2
))/(d^3*x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_3\left(ax^2\right)}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/(d*x)**(5/2),x)
```

```
[Out] Integral(polylog(3, a*x**2)/(d*x)**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/(d*x)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(3,a\,x^2)}{{(d\,x)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^2)/(d*x)^(5/2),x)
```

```
[Out] int(polylog(3, a*x^2)/(d*x)^(5/2), x)
```

3.84 $\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{128a}{125d^3\sqrt{dx}}-\frac{64a^{5/4}\text{ArcTan}\left(\frac{\sqrt[4]{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}}+\frac{64a^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}}+\frac{32\log\left(1-ax^2\right)}{125d(dx)^{5/2}}-\frac{8\text{PolyLog}(2,ax}{25d(dx)^{5/2}}$$

```
[Out] -64/125*a^(5/4)*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(7/2)+64/125*a^(5/4)*
arctanh(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(7/2)+32/125*ln(-a*x^2+1)/d/(d*x)^(5
/2)-8/25*polylog(2,a*x^2)/d/(d*x)^(5/2)-2/5*polylog(3,a*x^2)/d/(d*x)^(5/2)-
128/125*a/d^3/(d*x)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {6726, 2505, 16, 331, 335, 304, 211, 214}

$$-\frac{64a^{5/4}\text{ArcTan}\left(\frac{\sqrt[4]{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}}+\frac{64a^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}}-\frac{128a}{125d^3\sqrt{dx}}-\frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}}-\frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}}+\frac{32\log\left(1-ax^2\right)}{125d(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/(d*x)^(7/2),x]
```

```
[Out] (-128*a)/(125*d^3*Sqrt[d*x]) - (64*a^(5/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[
d]])/(125*d^(7/2)) + (64*a^(5/4)*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(125
*d^(7/2)) + (32*Log[1 - a*x^2])/(125*d*(d*x)^(5/2)) - (8*PolyLog[2, a*x^2])
/(25*d*(d*x)^(5/2)) - (2*PolyLog[3, a*x^2])/(5*d*(d*x)^(5/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\text{Li}_3(ax^2)}{(dx)^{7/2}}\,dx = -\frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} + \frac{4}{5}\int \frac{\text{Li}_2(ax^2)}{(dx)^{7/2}}\,dx$$
$$= -\frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} - \frac{16}{25}\int \frac{\log(1-ax^2)}{(dx)^{7/2}}\,dx$$
$$= \frac{32\log(1-ax^2)}{125d(dx)^{5/2}} - \frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} + \frac{(64a)\int \frac{x}{(dx)^{5/2}(1-ax^2)}\,dx}{125d}$$
$$= \frac{32\log(1-ax^2)}{125d(dx)^{5/2}} - \frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} + \frac{(64a)\int \frac{1}{(dx)^{3/2}(1-ax^2)}\,dx}{125d^2}$$
$$= -\frac{128a}{125d^3\sqrt{dx}} + \frac{32\log(1-ax^2)}{125d(dx)^{5/2}} - \frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} + \frac{(64a^2)\int \frac{\sqrt{dx}}{1-ax^2}\,dx}{125d^4}$$
$$= -\frac{128a}{125d^3\sqrt{dx}} + \frac{32\log(1-ax^2)}{125d(dx)^{5/2}} - \frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} + \frac{(128a^2)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{ax^4}{d^2}}\,dx, x$$
$$125d^5$$
$$= -\frac{128a}{125d^3\sqrt{dx}} + \frac{32\log(1-ax^2)}{125d(dx)^{5/2}} - \frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} + \frac{(64a^{3/2})\,\text{Subst}\left(\int \frac{1}{d-\sqrt{a}\,x^2}\,c$$
$$125d^3$$
$$= -\frac{128a}{125d^3\sqrt{dx}} - \frac{64a^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}} + \frac{64a^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}} + \frac{32\log(1-ax}{125d(dx)^{5/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 79, normalized size = 0.54

$$-\frac{x\Gamma\left(-\frac{1}{4}\right)\left(-192ax^2+64a^2x^4\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};ax^2\right)+48\log(1-ax^2)-60\text{PolyLog}(2,ax^2)-75\text{PolyLog}(3,ax^2)\right)}{750(dx)^{7/2}\Gamma\left(\frac{3}{4}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^2]/(d*x)^(7/2),x]
```

```
[Out] -1/750*(x*Gamma[-1/4]*(-192*a*x^2 + 64*a^2*x^4*Hypergeometric2F1[3/4, 1, 7/
4, a*x^2] + 48*Log[1 - a*x^2] - 60*PolyLog[2, a*x^2] - 75*PolyLog[3, a*x^2]
))/((d*x)^(7/2)*Gamma[3/4])
```

Maple [A]
time = 0.15, size = 142, normalized size = 0.97

method	result

meijerg	$-\frac{x^{\frac{7}{2}}(-a)^{\frac{5}{4}}\left(-\frac{256}{125\sqrt{x}\,(-a)^{\frac{1}{4}}}-\frac{64x^{\frac{3}{2}}a\left(\ln\left(1-\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^{2}\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(a\,x^{2}\right)^{\frac{1}{4}}\right)\right)}{125(-a)^{\frac{1}{4}}\left(a\,x^{2}\right)^{\frac{3}{4}}}+\frac{64\ln\left(-a\,x^{2}+1\right)}{125x^{\frac{5}{2}}(-a)^{\frac{1}{4}}a}-\frac{16\,\mathrm{polylog}\left(2,a\,x^{2}\right)}{25x^{\frac{5}{2}}(-a)^{\frac{1}{4}}a}\right.}{2(dx)^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(d*x)^(7/2)*x^(7/2)*(-a)^(5/4)*(-256/125/x^(1/2)/(-a)^(1/4)-64/125*x^(
3/2)/(-a)^(1/4)*a/(a*x^2)^(3/4)*(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))+2*
arctan((a*x^2)^(1/4)))+64/125/x^(5/2)/(-a)^(1/4)/a*ln(-a*x^2+1)-16/25/x^(5/
2)/(-a)^(1/4)/a*polylog(2,a*x^2)-4/5/x^(5/2)/(-a)^(1/4)/a*polylog(3,a*x^2))
```

Maxima [A]
time = 0.47, size = 163, normalized size = 1.11

$$-\frac{2\left(\frac{16\,a^{2}\left(\frac{2\arctan\left(\frac{\sqrt{dx}\sqrt{a}}{\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}\,\sqrt{a}}+\frac{\log\left(\frac{\sqrt{dx}\sqrt{a}-\sqrt{\sqrt{a}\,d}}{\sqrt{dx}\sqrt{a}+\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}\,\sqrt{a}}\right)}{d^{2}}+\frac{64\,ad^{2}x^{2}+20\,d^{2}\mathrm{Li}_{2}\left(ax^{2}\right)-16\,d^{2}\log\left(-ad^{2}x^{2}+d^{2}\right)+32\,d^{2}\log\left(d\right)+25\,d^{2}\mathrm{Li}_{3}\left(ax^{2}\right)}{(dx)^{\frac{5}{2}}d^{2}}\right)}{125\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/125*(16*a^2*(2*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/(sqrt(sqrt(a)*d
)*sqrt(a)) + log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a) +
 sqrt(sqrt(a)*d)))/(sqrt(sqrt(a)*d)*sqrt(a)))/d^2 + (64*a*d^2*x^2 + 20*d^2*
dilog(a*x^2) - 16*d^2*log(-a*d^2*x^2 + d^2) + 32*d^2*log(d) + 25*d^2*polylo
g(3, a*x^2))/((d*x)^(5/2)*d^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(106) = 212$.
time = 0.40, size = 226, normalized size = 1.54

$$\frac{2\left(64\,d^{4}x^{3}\left(\frac{a^{5}}{d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^{4}d^{3}\left(\frac{a^{5}}{d^{14}}\right)^{\frac{1}{4}}-\sqrt{a^{5}d^{8}\sqrt{\frac{a^{5}}{d^{14}}}+a^{8}dx}\,d^{3}\left(\frac{a^{5}}{d^{14}}\right)^{\frac{1}{4}}}{a^{5}}\right)+16\,d^{4}x^{3}\left(\frac{a^{5}}{d^{14}}\right)^{\frac{1}{4}}\log\left(32768\,d^{11}\left(\frac{a^{5}}{d^{14}}\right)^{\frac{3}{4}}+32768\,\sqrt{dx}\,a^{4}\right)-16\,d^{4}x^{3}\left(\frac{a^{5}}{d^{14}}\right)^{\frac{1}{4}}\log\left(-32768\,d^{11}\left(\frac{a^{5}}{d^{14}}\right)^{\frac{3}{4}}+32768\,\sqrt{dx}\,a^{4}\right)-4\left(16\,ax^{2}+5\,\mathrm{Li}_{2}(ax^{2})-4\,\log\left(-ax^{2}+1\right)\right)\sqrt{dx}-25\,\sqrt{dx}\,\mathrm{polylog}(3,ax^{2})\right)}{125\,d^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/125*(64*d^4*x^3*(a^5/d^14)^(1/4)*arctan(-(sqrt(d*x)*a^4*d^3*(a^5/d^14)^(1
/4) - sqrt(a^5*d^8*sqrt(a^5/d^14) + a^8*d*x)*d^3*(a^5/d^14)^(1/4))/a^5) + 1
6*d^4*x^3*(a^5/d^14)^(1/4)*log(32768*d^11*(a^5/d^14)^(3/4) + 32768*sqrt(d*x
)*a^4) - 16*d^4*x^3*(a^5/d^14)^(1/4)*log(-32768*d^11*(a^5/d^14)^(3/4) + 327
68*sqrt(d*x)*a^4) - 4*(16*a*x^2 + 5*dilog(a*x^2) - 4*log(-a*x^2 + 1))*sqrt(
d*x) - 25*sqrt(d*x)*polylog(3, a*x^2))/(d^4*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_3\left(ax^2\right)}{(dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/(d*x)**(7/2),x)
```

```
[Out] Integral(polylog(3, a*x**2)/(d*x)**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/(d*x)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(3,a\,x^2)}{{(d\,x)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^2)/(d*x)^(7/2),x)
```

```
[Out] int(polylog(3, a*x^2)/(d*x)^(7/2), x)
```

3.85 $\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{9/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{128a}{1029d^3(dx)^{3/2}}+\frac{64a^{7/4}\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343d^{9/2}}+\frac{64a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343d^{9/2}}+\frac{32\log\left(1-ax^2\right)}{343d(dx)^{7/2}}-\frac{8\text{PolyLog}(2}{49d(dx)}$$

```
[Out] -128/1029*a/d^3/(d*x)^(3/2)+64/343*a^(7/4)*arctan(a^(1/4)*(d*x)^(1/2)/d^(1/
2))/d^(9/2)+64/343*a^(7/4)*arctanh(a^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(9/2)+32/
343*ln(-a*x^2+1)/d/(d*x)^(7/2)-8/49*polylog(2,a*x^2)/d/(d*x)^(7/2)-2/7*poly
log(3,a*x^2)/d/(d*x)^(7/2)
```

Rubi [A]
time = 0.07, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {6726, 2505, 16, 331, 335, 218, 214, 211}

$$\frac{64a^{7/4}\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343d^{9/2}}+\frac{64a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343d^{9/2}}-\frac{128a}{1029d^3(dx)^{3/2}}-\frac{8\text{Li}_2(ax^2)}{49d(dx)^{7/2}}-\frac{2\text{Li}_3(ax^2)}{7d(dx)^{7/2}}+\frac{32\log\left(1-ax^2\right)}{343d(dx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/(d*x)^(9/2),x]
```

```
[Out] (-128*a)/(1029*d^3*(d*x)^(3/2)) + (64*a^(7/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sq
rt[d]])/(343*d^(9/2)) + (64*a^(7/4)*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(
343*d^(9/2)) + (32*Log[1 - a*x^2])/(343*d*(d*x)^(7/2)) - (8*PolyLog[2, a*x^
2])/(49*d*(d*x)^(7/2)) - (2*PolyLog[3, a*x^2])/(7*d*(d*x)^(7/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\text{Li}_3(ax^2)}{(dx)^{9/2}}\,dx = -\frac{2\text{Li}_3(ax^2)}{7d(dx)^{7/2}} + \frac{4}{7}\int \frac{\text{Li}_2(ax^2)}{(dx)^{9/2}}\,dx$$
$$= -\frac{8\text{Li}_2(ax^2)}{49d(dx)^{7/2}} - \frac{2\text{Li}_3(ax^2)}{7d(dx)^{7/2}} - \frac{16}{49}\int \frac{\log(1-ax^2)}{(dx)^{9/2}}\,dx$$
$$= \frac{32\log(1-ax^2)}{343d(dx)^{7/2}} - \frac{8\text{Li}_2(ax^2)}{49d(dx)^{7/2}} - \frac{2\text{Li}_3(ax^2)}{7d(dx)^{7/2}} + \frac{(64a)\int \frac{x}{(dx)^{7/2}(1-ax^2)}\,dx}{343d}$$
$$= \frac{32\log(1-ax^2)}{343d(dx)^{7/2}} - \frac{8\text{Li}_2(ax^2)}{49d(dx)^{7/2}} - \frac{2\text{Li}_3(ax^2)}{7d(dx)^{7/2}} + \frac{(64a)\int \frac{1}{(dx)^{5/2}(1-ax^2)}\,dx}{343d^2}$$
$$= -\frac{128a}{1029d^3(dx)^{3/2}} + \frac{32\log(1-ax^2)}{343d(dx)^{7/2}} - \frac{8\text{Li}_2(ax^2)}{49d(dx)^{7/2}} - \frac{2\text{Li}_3(ax^2)}{7d(dx)^{7/2}} + \frac{(64a^2)\int \frac{1}{\sqrt{dx}\,(1-ax^2)}\,dx}{343d^4}$$
$$= -\frac{128a}{1029d^3(dx)^{3/2}} + \frac{32\log(1-ax^2)}{343d(dx)^{7/2}} - \frac{8\text{Li}_2(ax^2)}{49d(dx)^{7/2}} - \frac{2\text{Li}_3(ax^2)}{7d(dx)^{7/2}} + \frac{(128a^2)\,\text{Subst}\left(\int \frac{1}{1-\frac{ax^4}{d^2}}\right.}{343d^5}$$
$$= -\frac{128a}{1029d^3(dx)^{3/2}} + \frac{32\log(1-ax^2)}{343d(dx)^{7/2}} - \frac{8\text{Li}_2(ax^2)}{49d(dx)^{7/2}} - \frac{2\text{Li}_3(ax^2)}{7d(dx)^{7/2}} + \frac{(64a^2)\,\text{Subst}\left(\int \frac{1}{d-\sqrt{a}\,x}\right.}{343d^4}$$
$$= -\frac{128a}{1029d^3(dx)^{3/2}} + \frac{64a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343d^{9/2}} + \frac{64a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343d^{9/2}} + \frac{32\log(1-}{343d(d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 84, normalized size = 0.57

$$-\frac{\sqrt{dx}\,\Gamma\left(-\frac{3}{4}\right)\left(-64ax^2+192a^2x^4\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};ax^2\right)+48\log(1-ax^2)-84\text{PolyLog}(2,ax^2)-147\text{PolyLog}(3,ax^2)\right)}{686d^5x^4\Gamma\left(\frac{1}{4}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^2]/(d*x)^(9/2),x]
```

```
[Out] -1/686*(Sqrt[d*x]*Gamma[-3/4]*(-64*a*x^2 + 192*a^2*x^4*Hypergeometric2F1[1/
4, 1, 5/4, a*x^2] + 48*Log[1 - a*x^2] - 84*PolyLog[2, a*x^2] - 147*PolyLog[
3, a*x^2]))/(d^5*x^4*Gamma[1/4])
```

Maple [A]
time = 0.16, size = 142, normalized size = 0.97

method	result

meijerg	$-\frac{x^{\frac{9}{2}}(-a)^{\frac{7}{4}}\left(-\frac{256}{1029x^{\frac{3}{2}}(-a)^{\frac{3}{4}}}-\frac{64\sqrt{x}\,a\left(\ln\left(1-\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(a\,x^{2}\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(a\,x^{2}\right)^{\frac{1}{4}}\right)\right)}{343(-a)^{\frac{3}{4}}\left(a\,x^{2}\right)^{\frac{1}{4}}}+\frac{64\ln\left(-a\,x^{2}+1\right)}{343x^{\frac{7}{2}}(-a)^{\frac{3}{4}}a}-\frac{16\,\mathrm{polylog}\left(2,a\,x^{2}\right)}{49x^{\frac{7}{2}}(-a)^{\frac{3}{4}}a}-\right.}{2(dx)^{\frac{9}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/(d*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(d*x)^(9/2)*x^(9/2)*(-a)^(7/4)*(-256/1029/x^(3/2)/(-a)^(3/4)-64/343*x^
(1/2)/(-a)^(3/4)*a/(a*x^2)^(1/4)*(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))-2
*arctan((a*x^2)^(1/4)))+64/343/x^(7/2)/(-a)^(3/4)/a*ln(-a*x^2+1)-16/49/x^(7
/2)/(-a)^(3/4)/a*polylog(2,a*x^2)-4/7/x^(7/2)/(-a)^(3/4)/a*polylog(3,a*x^2)
)
```

Maxima [A]
time = 0.48, size = 168, normalized size = 1.14

$$\frac{2\left(\frac{48\left(\frac{2\,a^{2}\arctan\left(\frac{\sqrt{dx}\,\sqrt{a}}{\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}\,d}-\frac{a^{2}\log\left(\frac{\sqrt{dx}\,\sqrt{a}-\sqrt{\sqrt{a}\,d}}{\sqrt{dx}\,\sqrt{a}+\sqrt{\sqrt{a}\,d}}\right)}{\sqrt{\sqrt{a}\,d}\,d}\right)}{d^{2}}-\frac{64\,ad^{2}x^{2}+84\,d^{2}\mathrm{Li}_2\left(ax^{2}\right)-48\,d^{2}\log\left(-ad^{2}x^{2}+d^{2}\right)+96\,d^{2}\log\left(d\right)+147\,d^{2}\mathrm{Li}_3\left(ax^{2}\right)}{(dx)^{\frac{7}{2}}d^{2}}\right)}{1029\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/1029*(48*(2*a^2*arctan(sqrt(d*x)*sqrt(a)/sqrt(sqrt(a)*d))/(sqrt(sqrt(a)*d
)*d) - a^2*log((sqrt(d*x)*sqrt(a) - sqrt(sqrt(a)*d))/(sqrt(d*x)*sqrt(a) + s
qrt(sqrt(a)*d)))/(sqrt(sqrt(a)*d)*d))/d^2 - (64*a*d^2*x^2 + 84*d^2*dilog(a*
x^2) - 48*d^2*log(-a*d^2*x^2 + d^2) + 96*d^2*log(d) + 147*d^2*polylog(3, a*
x^2))/((d*x)^(7/2)*d^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(106) = 212.
time = 0.41, size = 223, normalized size = 1.52

$$-\frac{2\left(192\,d^{5}x^{4}\left(\frac{a^{7}}{d^{18}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^{2}d^{13}\left(\frac{a^{7}}{d^{18}}\right)^{\frac{3}{4}}-\sqrt{d^{10}\sqrt{\frac{a^{7}}{d^{18}}}+a^{4}dx}\,d^{13}\left(\frac{a^{7}}{d^{18}}\right)^{\frac{3}{4}}}{a^{7}}\right)-48\,d^{5}x^{4}\left(\frac{a^{7}}{d^{18}}\right)^{\frac{1}{4}}\log\left(32\,d^{5}\left(\frac{a^{7}}{d^{18}}\right)^{\frac{1}{4}}+32\,\sqrt{dx}\,a^{2}\right)+48\,d^{5}x^{4}\left(\frac{a^{7}}{d^{18}}\right)^{\frac{1}{4}}\log\left(-32\,d^{5}\left(\frac{a^{7}}{d^{18}}\right)^{\frac{1}{4}}+32\,\sqrt{dx}\,a^{2}\right)+4\left(16\,ax^{2}+21\,\mathrm{Li}_2(ax^{2})-12\,\log\left(-ax^{2}+1\right)\right)\sqrt{dx}+147\,\sqrt{dx}\,\mathrm{polylog}(3,ax^{2})\right)}{1029\,d^{5}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -2/1029*(192*d^5*x^4*(a^7/d^18)^(1/4)*arctan(-(sqrt(d*x)*a^2*d^13*(a^7/d^18
)^(3/4) - sqrt(d^10*sqrt(a^7/d^18) + a^4*d*x)*d^13*(a^7/d^18)^(3/4))/a^7) -
 48*d^5*x^4*(a^7/d^18)^(1/4)*log(32*d^5*(a^7/d^18)^(1/4) + 32*sqrt(d*x)*a^2
) + 48*d^5*x^4*(a^7/d^18)^(1/4)*log(-32*d^5*(a^7/d^18)^(1/4) + 32*sqrt(d*x)
*a^2) + 4*(16*a*x^2 + 21*dilog(a*x^2) - 12*log(-a*x^2 + 1))*sqrt(d*x) + 147
*sqrt(d*x)*polylog(3, a*x^2))/(d^5*x^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/(d*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/(d*x)^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(3, a\,x^2)}{{(d\,x)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^2)/(d*x)^(9/2),x)
```

```
[Out] int(polylog(3, a*x^2)/(d*x)^(9/2), x)
```

3.86 $\int (dx)^{3/2}\textbf{PolyLog}(2, ax^q)\, dx$

Optimal. Leaf size=101

$$\frac{8adq^2x^{2+q}\sqrt{dx}\ {}_2F_1\left(1, \frac{\frac{5}{2}+q}{q}; \frac{1}{2}\left(4+\frac{5}{q}\right); ax^q\right)}{25(5+2q)}+\frac{4q(dx)^{5/2}\log\left(1-ax^q\right)}{25d}+\frac{2(dx)^{5/2}\text{PolyLog}(2,ax^q)}{5d}$$

```
[Out] 4/25*q*(d*x)^(5/2)*ln(1-a*x^q)/d+2/5*(d*x)^(5/2)*polylog(2,a*x^q)/d+8/25*a*
d*q^2*x^(2+q)*hypergeom([1, (5/2+q)/q],[2+5/2/q],a*x^q)*(d*x)^(1/2)/(5+2*q)
```

Rubi [A]
time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6726, 2505, 20, 371}

$$\frac{8adq^2\sqrt{dx}\ x^{q+2}\ {}_2F_1\left(1, \frac{q+\frac{5}{2}}{q}; \frac{1}{2}\left(4+\frac{5}{q}\right); ax^q\right)}{25(2q+5)}+\frac{2(dx)^{5/2}\text{Li}_2(ax^q)}{5d}+\frac{4q(dx)^{5/2}\log\left(1-ax^q\right)}{25d}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*PolyLog[2, a*x^q],x]
```

```
[Out] (8*a*d*q^2*x^(2 + q)*Sqrt[d*x]*Hypergeometric2F1[1, (5/2 + q)/q, (4 + 5/q)/
2, a*x^q])/(25*(5 + 2*q)) + (4*q*(d*x)^(5/2)*Log[1 - a*x^q])/(25*d) + (2*(d
*x)^(5/2)*PolyLog[2, a*x^q])/(5*d)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (dx)^{3/2}\text{Li}_2(ax^q)\,dx &= \frac{2(dx)^{5/2}\text{Li}_2(ax^q)}{5d}+\frac{1}{5}(2q)\int (dx)^{3/2}\log(1-ax^q)\,dx\\ &= \frac{4q(dx)^{5/2}\log(1-ax^q)}{25d}+\frac{2(dx)^{5/2}\text{Li}_2(ax^q)}{5d}+\frac{(4aq^2)\int \frac{x^{-1+q}(dx)^{5/2}}{1-ax^q}\,dx}{25d}\\ &= \frac{4q(dx)^{5/2}\log(1-ax^q)}{25d}+\frac{2(dx)^{5/2}\text{Li}_2(ax^q)}{5d}+\frac{\left(4adq^2\sqrt{dx}\right)\int \frac{x^{\frac{3}{2}+q}}{1-ax^q}\,dx}{25\sqrt{x}}\\ &= \frac{8adq^2x^{2+q}\sqrt{dx}\,{}_2F_1\left(1,\frac{\frac{5}{2}+q}{q};\frac{1}{2}\left(4+\frac{5}{q}\right);ax^q\right)}{25(5+2q)}+\frac{4q(dx)^{5/2}\log(1-ax^q)}{25d}+\frac{2(dx)^{5/2}\text{I}}{5d}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 82, normalized size = 0.81

$$\frac{2x(dx)^{3/2}\left(4aq^2x^q\,{}_2F_1\left(1,\frac{\frac{5}{2}+q}{q};2+\frac{5}{2q};ax^q\right)+(5+2q)\left(2q\log(1-ax^q)+5\text{PolyLog}(2,ax^q)\right)\right)}{25(5+2q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*PolyLog[2, a*x^q],x]
```

```
[Out] (2*x*(d*x)^(3/2)*(4*a*q^2*x^q*Hypergeometric2F1[1, (5/2 + q)/q, 2 + 5/(2*q)
, a*x^q] + (5 + 2*q)*(2*q*Log[1 - a*x^q] + 5*PolyLog[2, a*x^q])))/(25*(5 +
2*q))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.49, size = 121, normalized size = 1.20

method	result	size
meijerg	$-\frac{(dx)^{\frac{3}{2}}(-a)^{-\frac{5}{2q}}\left(-\frac{4q^2x^{\frac{5}{2}}(-a)^{\frac{5}{2q}}\ln(1-a\,x^q)}{25}-\frac{2q\,x^{\frac{5}{2}}(-a)^{\frac{5}{2q}}\left(1+\frac{2q}{5}\right)\text{polylog}(2,a\,x^q)}{5+2q}-\frac{4q^2x^{\frac{5}{2}+q}a(-a)^{\frac{5}{2q}}\Phi\left(a\,x^q,1,\frac{5+2q}{2q}\right)}{25}\right)}{x^{\frac{3}{2}}q}$	121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*polylog(2,a*x^q),x,method=_RETURNVERBOSE)
```

```
[Out] -(d*x)^(3/2)/x^(3/2)*(-a)^(-5/2/q)/q*(-4/25*q^2*x^(5/2)*(-a)^(5/2/q)*ln(1-a
*x^q)-2*q/(5+2*q)*x^(5/2)*(-a)^(5/2/q)*(1+2/5*q)*polylog(2,a*x^q)-4/25*q^2*
x^(5/2+q)*a*(-a)^(5/2/q)*LerchPhi(a*x^q,1,1/2*(5+2*q)/q))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(2,a*x^q),x, algorithm="maxima")
```

```
[Out] 8*d^(3/2)*q^3*integrate(1/25*x^(3/2)/((2*a^2*q - 5*a^2)*x^(2*q) - 2*(2*a*q
- 5*a)*x^q + 2*q - 5), x) + 2/125*(25*((2*a*d^(3/2)*q - 5*a*d^(3/2))*x*x^q
- (2*d^(3/2)*q - 5*d^(3/2))*x)*x^(3/2)*dilog(a*x^q) + 10*((2*a*d^(3/2)*q^2
- 5*a*d^(3/2)*q)*x*x^q - (2*d^(3/2)*q^2 - 5*d^(3/2)*q)*x)*x^(3/2)*log(-a*x^
q + 1) + 4*(2*d^(3/2)*q^3*x - (2*a*d^(3/2)*q^3 - 5*a*d^(3/2)*q^2)*x*x^q)*x^
(3/2))/((2*a*q - 5*a)*x^q - 2*q + 5)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(2,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*d*x*dilog(a*x^q), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*polylog(2,a*x**q),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(2,a*x^q),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*dilog(a*x^q), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{3/2}\,\mathrm{polylog}(2,a\,x^q)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*polylog(2, a*x^q),x)
```

```
[Out] int((d*x)^(3/2)*polylog(2, a*x^q), x)
```

3.87 $\int \sqrt{dx}\, \textbf{PolyLog}(2, ax^q)\, dx$

Optimal. Leaf size=100

$$\frac{8aq^2x^{1+q}\sqrt{dx}\ {}_2F_1\left(1,\frac{\frac{3}{2}+q}{q};\frac{1}{2}\left(4+\frac{3}{q}\right);ax^q\right)}{9(3+2q)}+\frac{4q(dx)^{3/2}\log\left(1-ax^q\right)}{9d}+\frac{2(dx)^{3/2}\text{PolyLog}(2,ax^q)}{3d}$$

```
[Out] 4/9*q*(d*x)^(3/2)*ln(1-a*x^q)/d+2/3*(d*x)^(3/2)*polylog(2,a*x^q)/d+8/9*a*q^
2*x^(1+q)*hypergeom([1, (3/2+q)/q],[2+3/2/q],a*x^q)*(d*x)^(1/2)/(3+2*q)
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6726, 2505, 20, 371}

$$\frac{8aq^2\sqrt{dx}\ x^{q+1}\ {}_2F_1\left(1,\frac{q+\frac{3}{2}}{q};\frac{1}{2}\left(4+\frac{3}{q}\right);ax^q\right)}{9(2q+3)}+\frac{2(dx)^{3/2}\text{Li}_2(ax^q)}{3d}+\frac{4q(dx)^{3/2}\log\left(1-ax^q\right)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*PolyLog[2, a*x^q],x]
```

```
[Out] (8*a*q^2*x^(1 + q)*Sqrt[d*x]*Hypergeometric2F1[1, (3/2 + q)/q, (4 + 3/q)/2,
 a*x^q])/(9*(3 + 2*q)) + (4*q*(d*x)^(3/2)*Log[1 - a*x^q])/(9*d) + (2*(d*x)^
(3/2)*PolyLog[2, a*x^q])/(3*d)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{dx}\,\mathrm{Li}_2(ax^q)\,dx &= \frac{2(dx)^{3/2}\mathrm{Li}_2(ax^q)}{3d}+\frac{1}{3}(2q)\int \sqrt{dx}\,\log(1-ax^q)\,dx\\
&= \frac{4q(dx)^{3/2}\log(1-ax^q)}{9d}+\frac{2(dx)^{3/2}\mathrm{Li}_2(ax^q)}{3d}+\frac{(4aq^2)\int \frac{x^{-1+q}(dx)^{3/2}}{1-ax^q}\,dx}{9d}\\
&= \frac{4q(dx)^{3/2}\log(1-ax^q)}{9d}+\frac{2(dx)^{3/2}\mathrm{Li}_2(ax^q)}{3d}+\frac{\left(4aq^2\sqrt{dx}\right)\int \frac{x^{\frac{1}{2}+q}}{1-ax^q}\,dx}{9\sqrt{x}}\\
&= \frac{8aq^2x^{1+q}\sqrt{dx}\,{}_2F_1\left(1,\frac{\frac{3}{2}+q}{q};\frac{1}{2}\left(4+\frac{3}{q}\right);ax^q\right)}{9(3+2q)}+\frac{4q(dx)^{3/2}\log(1-ax^q)}{9d}+\frac{2(dx)^{3/2}\mathrm{Li}_2}{3d}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 82, normalized size = 0.82

$$\frac{2x\sqrt{dx}\left(4aq^2x^q\,{}_2F_1\left(1,\frac{\frac{3}{2}+q}{q};2+\frac{3}{2q};ax^q\right)+(3+2q)\left(2q\log(1-ax^q)+3\mathrm{PolyLog}(2,ax^q)\right)\right)}{9(3+2q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*PolyLog[2, a*x^q],x]
[Out] (2*x*Sqrt[d*x]*(4*a*q^2*x^q*Hypergeometric2F1[1, (3/2 + q)/q, 2 + 3/(2*q),
a*x^q] + (3 + 2*q)*(2*q*Log[1 - a*x^q] + 3*PolyLog[2, a*x^q])))/(9*(3 + 2*q
))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.52, size = 121, normalized size = 1.21

method	result	size
meijerg	$-\frac{\sqrt{dx}\,(-a)^{-\frac{3}{2q}}\left(-\frac{4q^2x^{\frac{3}{2}}(-a)^{\frac{3}{2q}}\ln(1-a\,x^q)}{9}-\frac{2q\,x^{\frac{3}{2}}(-a)^{\frac{3}{2q}}\left(1+\frac{2q}{3}\right)\mathrm{polylog}(2,a\,x^q)}{3+2q}-\frac{4q^2x^{\frac{3}{2}+q}a(-a)^{\frac{3}{2q}}\Phi\left(a\,x^q,1,\frac{3+2q}{2q}\right)}{9}\right)}{\sqrt{x}\,q}$	121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*polylog(2,a*x^q),x,method=_RETURNVERBOSE)
```

```
[Out] -(d*x)^(1/2)/x^(1/2)*(-a)^(-3/2/q)/q*(-4/9*q^2*x^(3/2)*(-a)^(3/2/q)*ln(1-a*
x^q)-2*q/(3+2*q)*x^(3/2)*(-a)^(3/2/q)*(1+2/3*q)*polylog(2,a*x^q)-4/9*q^2*x^
(3/2+q)*a*(-a)^(3/2/q)*LerchPhi(a*x^q,1,1/2*(3+2*q)/q))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x^q),x, algorithm="maxima")
```

```
[Out] 8*sqrt(d)*q^3*integrate(1/9*sqrt(x)/((2*a^2*q - 3*a^2)*x^(2*q) - 2*(2*a*q -
 3*a)*x^q + 2*q - 3), x) + 2/27*(9*((2*a*sqrt(d)*q - 3*a*sqrt(d))*x*x^q - (
2*sqrt(d)*q - 3*sqrt(d))*x)*sqrt(x)*dilog(a*x^q) + 6*((2*a*sqrt(d)*q^2 - 3*
a*sqrt(d)*q)*x*x^q - (2*sqrt(d)*q^2 - 3*sqrt(d)*q)*x)*sqrt(x)*log(-a*x^q +
1) + 4*(2*sqrt(d)*q^3*x - (2*a*sqrt(d)*q^3 - 3*a*sqrt(d)*q^2)*x*x^q)*sqrt(x
))/((2*a*q - 3*a)*x^q - 2*q + 3)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*dilog(a*x^q), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\, \mathrm{Li}_2\left(ax^q\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*polylog(2,a*x**q),x)
```

```
[Out] Integral(sqrt(d*x)*polylog(2, a*x**q), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x^q),x, algorithm="giac")
[Out] integrate(sqrt(d*x)*dilog(a*x^q), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,x}\ \mathrm{polylog}(2,a\,x^q)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*polylog(2, a*x^q),x)
[Out] int((d*x)^(1/2)*polylog(2, a*x^q), x)
```

3.88 $\int \frac{\text{PolyLog}(2,ax^q)}{\sqrt{dx}} \, dx$

Optimal. Leaf size=93

$$\frac{8aq^2x^q\sqrt{dx}\ {}_2F_1\left(1,\frac{\frac{1}{2}+q}{q};\frac{1}{2}\left(4+\frac{1}{q}\right);ax^q\right)}{d(1+2q)}+\frac{4q\sqrt{dx}\ \log\left(1-ax^q\right)}{d}+\frac{2\sqrt{dx}\ \text{PolyLog}(2,ax^q)}{d}$$

```
[Out] 8*a*q^2*x^q*hypergeom([1, (1/2+q)/q],[2+1/2/q],a*x^q)*(d*x)^(1/2)/d/(1+2*q)
+4*q*ln(1-a*x^q)*(d*x)^(1/2)/d+2*polylog(2,a*x^q)*(d*x)^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6726, 2505, 20, 371}

$$\frac{8aq^2\sqrt{dx}\ x^q\ {}_2F_1\left(1,\frac{q+\frac{1}{2}}{q};\frac{1}{2}\left(4+\frac{1}{q}\right);ax^q\right)}{d(2q+1)}+\frac{2\sqrt{dx}\ \text{Li}_2(ax^q)}{d}+\frac{4q\sqrt{dx}\ \log\left(1-ax^q\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^q]/Sqrt[d*x],x]
```

```
[Out] (8*a*q^2*x^q*Sqrt[d*x]*Hypergeometric2F1[1, (1/2 + q)/q, (4 + q^(-1))/2, a*
x^q])/(d*(1 + 2*q)) + (4*q*Sqrt[d*x]*Log[1 - a*x^q])/d + (2*Sqrt[d*x]*PolyL
og[2, a*x^q])/d
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
```

```
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_2(ax^q)}{\sqrt{dx}}\,dx &= \frac{2\sqrt{dx}\,\text{Li}_2(ax^q)}{d}+(2q)\int \frac{\log\left(1-ax^q\right)}{\sqrt{dx}}\,dx\\ &= \frac{4q\sqrt{dx}\,\log\left(1-ax^q\right)}{d}+\frac{2\sqrt{dx}\,\text{Li}_2(ax^q)}{d}+\frac{(4aq^2)\int \frac{x^{-1+q}\sqrt{dx}}{1-ax^q}\,dx}{d}\\ &= \frac{4q\sqrt{dx}\,\log\left(1-ax^q\right)}{d}+\frac{2\sqrt{dx}\,\text{Li}_2(ax^q)}{d}+\frac{\left(4aq^2\sqrt{dx}\right)\int \frac{x^{-\frac{1}{2}+q}}{1-ax^q}\,dx}{d\sqrt{x}}\\ &= \frac{8aq^2x^q\sqrt{dx}\,{}_2F_1\left(1,\frac{\frac{1}{2}+q}{q};\frac{1}{2}\left(4+\frac{1}{q}\right);ax^q\right)}{d(1+2q)}+\frac{4q\sqrt{dx}\,\log\left(1-ax^q\right)}{d}+\frac{2\sqrt{dx}\,\text{Li}_2(ax^q)}{d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.02, size = 48, normalized size = 0.52

$$-\frac{xG_{4,4}^{1,4}\left(-ax^q\middle|\begin{array}{c}1,1,1,1-\frac{1}{2q}\\ 1,0,0,-\frac{1}{2q}\end{array}\right)}{q\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^q]/Sqrt[d*x],x]
```

```
[Out] -((x*MeijerG[{{1, 1, 1, 1 - 1/(2*q)}, {}}, {{1}, {0, 0, -1/2*1/q}}, -(a*x^q
)])/(q*Sqrt[d*x]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.52, size = 109, normalized size = 1.17

method	result	size

meijerg	$-\frac{\sqrt{x}\,(-a)^{-\frac{1}{2q}}\left(-4q^2\sqrt{x}\,(-a)^{\frac{1}{2q}}\ln(1-a\,x^q)-2q\sqrt{x}\,(-a)^{\frac{1}{2q}}\operatorname{polylog}(2,a\,x^q)-4q^2x^{\frac{1}{2}+q}a(-a)^{\frac{1}{2q}}\Phi\left(a\,x^q,1,\frac{1+2q}{2q}\right)\right)}{\sqrt{dx}\,q}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^q)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(d*x)^(1/2)*x^(1/2)*(-a)^(-1/2/q)/q*(-4*q^2*x^(1/2)*(-a)^(1/2/q)*ln(1-a*
x^q)-2*q*x^(1/2)*(-a)^(1/2/q)*polylog(2,a*x^q)-4*q^2*x^(1/2+q)*a*(-a)^(1/2/
q)*LerchPhi(a*x^q,1,1/2*(1+2*q)/q))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 8*q^3*integrate(1/(((2*a^2*sqrt(d)*q - a^2*sqrt(d))*x^(2*q) - 2*(2*a*sqrt(d
)*q - a*sqrt(d))*x^q + 2*sqrt(d)*q - sqrt(d))*sqrt(x)), x) - 2*(((2*a*sqrt(
d)*q - a*sqrt(d))*x*x^q - (2*sqrt(d)*q - sqrt(d))*x)*dilog(a*x^q)/sqrt(x) +
 2*((2*a*sqrt(d)*q^2 - a*sqrt(d)*q)*x*x^q - (2*sqrt(d)*q^2 - sqrt(d)*q)*x)*
log(-a*x^q + 1)/sqrt(x) + 4*(2*sqrt(d)*q^3*x - (2*a*sqrt(d)*q^3 - a*sqrt(d)
*q^2)*x*x^q)/sqrt(x))/(2*d*q - (2*a*d*q - a*d)*x^q - d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*dilog(a*x^q)/(d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_2(ax^q)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**q)/(d*x)**(1/2),x)
```

```
[Out] Integral(polylog(2, a*x**q)/sqrt(d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(1/2),x, algorithm="giac")
[Out] integrate(dilog(a*x^q)/sqrt(d*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(2, a\,x^q)}{\sqrt{d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^q)/(d*x)^(1/2),x)
[Out] int(polylog(2, a*x^q)/(d*x)^(1/2), x)
```

3.89 $\int \frac{\text{PolyLog}(2,ax^q)}{(dx)^{3/2}} dx$

Optimal. Leaf size=97

$$-\frac{8aq^2x^q\,{}_2F_1\left(1,\frac{1}{2}\left(2-\frac{1}{q}\right);\frac{1}{2}\left(4-\frac{1}{q}\right);ax^q\right)}{d(1-2q)\sqrt{dx}}+\frac{4q\log(1-ax^q)}{d\sqrt{dx}}-\frac{2\text{PolyLog}(2,ax^q)}{d\sqrt{dx}}$$

```
[Out] -8*a*q^2*x^q*hypergeom([1, 1-1/2/q],[2-1/2/q],a*x^q)/d/(1-2*q)/(d*x)^(1/2)+
4*q*ln(1-a*x^q)/d/(d*x)^(1/2)-2*polylog(2,a*x^q)/d/(d*x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6726, 2505, 20, 371}

$$-\frac{8aq^2x^q\,{}_2F_1\left(1,\frac{1}{2}\left(2-\frac{1}{q}\right);\frac{1}{2}\left(4-\frac{1}{q}\right);ax^q\right)}{d(1-2q)\sqrt{dx}}-\frac{2\text{Li}_2(ax^q)}{d\sqrt{dx}}+\frac{4q\log(1-ax^q)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^q]/(d*x)^(3/2),x]
```

```
[Out] (-8*a*q^2*x^q*Hypergeometric2F1[1, (2 - q^(-1))/2, (4 - q^(-1))/2, a*x^q])/
(d*(1 - 2*q)*Sqrt[d*x]) + (4*q*Log[1 - a*x^q])/(d*Sqrt[d*x]) - (2*PolyLog[2
, a*x^q])/(d*Sqrt[d*x])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
```

```
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax^q)}{(dx)^{3/2}}\,dx &= -\frac{2\text{Li}_2(ax^q)}{d\sqrt{dx}} - (2q)\int \frac{\log\left(1-ax^q\right)}{(dx)^{3/2}}\,dx \\
&= \frac{4q\log\left(1-ax^q\right)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax^q)}{d\sqrt{dx}} + \frac{\left(4aq^2\right)\int \frac{x^{-1+q}}{\sqrt{dx}\,(1-ax^q)}\,dx}{d} \\
&= \frac{4q\log\left(1-ax^q\right)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax^q)}{d\sqrt{dx}} + \frac{\left(4aq^2\sqrt{x}\right)\int \frac{x^{-\frac{3}{2}+q}}{1-ax^q}\,dx}{d\sqrt{dx}} \\
&= -\frac{8aq^2x^q\,{}_2F_1\left(1,\frac{1}{2}\left(2-\frac{1}{q}\right);\frac{1}{2}\left(4-\frac{1}{q}\right);ax^q\right)}{d(1-2q)\sqrt{dx}} + \frac{4q\log\left(1-ax^q\right)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax^q)}{d\sqrt{dx}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.02, size = 48, normalized size = 0.49

$$-\frac{xG_{4,4}^{1,4}\left(-ax^q\middle|\begin{array}{c}1,1,1,1+\frac{1}{2q}\\ 1,0,0,\frac{1}{2q}\end{array}\right)}{q(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^q]/(d*x)^(3/2),x]
```

```
[Out] -((x*MeijerG[{{1, 1, 1, 1 + 1/(2*q)}, {}}, {{1}, {0, 0, 1/(2*q)}}, -(a*x^q)
])/(q*(d*x)^(3/2)))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.52, size = 121, normalized size = 1.25

method	result	size

meijerg	$-\frac{x^{\frac{3}{2}}(-a)^{\frac{1}{2q}}\left(-\frac{4q^2(-a)^{-\frac{1}{2q}}\ln(1-a\,x^q)}{\sqrt{x}}-\frac{2q(-a)^{-\frac{1}{2q}}(1-2q)\,\mathrm{polylog}(2,a\,x^q)}{(2q-1)\sqrt{x}}-4q^2x^{q-\frac{1}{2}}a(-a)^{-\frac{1}{2q}}\Phi\left(a\,x^q,1,\frac{2q-1}{2q}\right)\right)}{(dx)^{\frac{3}{2}}q}$	121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^q)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(d*x)^(3/2)*x^(3/2)*(-a)^(1/2/q)/q*(-4*q^2/x^(1/2)*(-a)^(-1/2/q)*ln(1-a*
x^q)-2*q/(2*q-1)/x^(1/2)*(-a)^(-1/2/q)*(1-2*q)*polylog(2,a*x^q)-4*q^2*x^(q-
1/2)*a*(-a)^(-1/2/q)*LerchPhi(a*x^q,1,1/2*(2*q-1)/q))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 8*q^3*integrate(1/((2*d^(3/2)*q + (2*a^2*d^(3/2)*q + a^2*d^(3/2))*x^(2*q) -
 2*(2*a*d^(3/2)*q + a*d^(3/2))*x^q + d^(3/2))*x^(3/2)), x) + 2*(((2*a*sqrt(
d)*q + a*sqrt(d))*x*x^q - (2*sqrt(d)*q + sqrt(d))*x)*dilog(a*x^q)/x^(3/2) -
 2*((2*a*sqrt(d)*q^2 + a*sqrt(d)*q)*x*x^q - (2*sqrt(d)*q^2 + sqrt(d)*q)*x)*
log(-a*x^q + 1)/x^(3/2) + 4*(2*sqrt(d)*q^3*x - (2*a*sqrt(d)*q^3 + a*sqrt(d)
*q^2)*x*x^q)/x^(3/2))/(2*d^2*q + d^2 - (2*a*d^2*q + a*d^2)*x^q)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*dilog(a*x^q)/(d^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_2\,(ax^q)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**q)/(d*x)**(3/2),x)
```

```
[Out] Integral(polylog(2, a*x**q)/(d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^q)/(d*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(2, a\,x^q)}{(d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^q)/(d*x)^(3/2),x)
```

```
[Out] int(polylog(2, a*x^q)/(d*x)^(3/2), x)
```

3.90 $\int \frac{\textbf{PolyLog}(2,ax^q)}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{8aq^2x^{-1+q}\,{}_2F_1\left(1,\frac{1}{2}\left(2-\frac{3}{q}\right);\frac{1}{2}\left(4-\frac{3}{q}\right);ax^q\right)}{9d^2(3-2q)\sqrt{dx}}+\frac{4q\log\left(1-ax^q\right)}{9d(dx)^{3/2}}-\frac{2\text{PolyLog}(2,ax^q)}{3d(dx)^{3/2}}$$

```
[Out] 4/9*q*ln(1-a*x^q)/d/(d*x)^(3/2)-2/3*polylog(2,a*x^q)/d/(d*x)^(3/2)-8/9*a*q^
2*x^(-1+q)*hypergeom([1, 1-3/2/q],[2-3/2/q],a*x^q)/d^2/(3-2*q)/(d*x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6726, 2505, 20, 371}

$$-\frac{8aq^2x^{q-1}\,{}_2F_1\left(1,\frac{1}{2}\left(2-\frac{3}{q}\right);\frac{1}{2}\left(4-\frac{3}{q}\right);ax^q\right)}{9d^2(3-2q)\sqrt{dx}}-\frac{2\text{Li}_2(ax^q)}{3d(dx)^{3/2}}+\frac{4q\log\left(1-ax^q\right)}{9d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^q]/(d*x)^(5/2),x]
```

```
[Out] (-8*a*q^2*x^(-1 + q)*Hypergeometric2F1[1, (2 - 3/q)/2, (4 - 3/q)/2, a*x^q])
/(9*d^2*(3 - 2*q)*Sqrt[d*x]) + (4*q*Log[1 - a*x^q])/(9*d*(d*x)^(3/2)) - (2*
PolyLog[2, a*x^q])/(3*d*(d*x)^(3/2))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
```

```
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_2(ax^q)}{(dx)^{5/2}}\,dx &= -\frac{2\text{Li}_2(ax^q)}{3d(dx)^{3/2}}-\frac{1}{3}(2q)\int \frac{\log(1-ax^q)}{(dx)^{5/2}}\,dx\\ &= \frac{4q\log(1-ax^q)}{9d(dx)^{3/2}}-\frac{2\text{Li}_2(ax^q)}{3d(dx)^{3/2}}+\frac{(4aq^2)\int \frac{x^{-1+q}}{(dx)^{3/2}(1-ax^q)}\,dx}{9d}\\ &= \frac{4q\log(1-ax^q)}{9d(dx)^{3/2}}-\frac{2\text{Li}_2(ax^q)}{3d(dx)^{3/2}}+\frac{\left(4aq^2\sqrt{x}\right)\int \frac{x^{-\frac{5}{2}+q}}{1-ax^q}\,dx}{9d^2\sqrt{dx}}\\ &= -\frac{8aq^2x^{-1+q}\,{}_2F_1\left(1,\frac{1}{2}\left(2-\frac{3}{q}\right);\frac{1}{2}\left(4-\frac{3}{q}\right);ax^q\right)}{9d^2(3-2q)\sqrt{dx}}+\frac{4q\log(1-ax^q)}{9d(dx)^{3/2}}-\frac{2\text{Li}_2(ax^q)}{3d(dx)^{3/2}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.02, size = 48, normalized size = 0.46

$$-\frac{xG_{4,4}^{1,4}\left(-ax^q\middle|\begin{array}{c}1,1,1,1+\frac{3}{2q}\\ 1,0,0,\frac{3}{2q}\end{array}\right)}{q(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^q]/(d*x)^(5/2),x]
```

```
[Out] -((x*MeijerG[{{1, 1, 1, 1 + 3/(2*q)}, {}}, {{1}, {0, 0, 3/(2*q)}}, -(a*x^q)
])/(q*(d*x)^(5/2)))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.51, size = 121, normalized size = 1.15

method	result	size

meijerg	$-\frac{x^{\frac{5}{2}}(-a)^{\frac{3}{2q}}\left(-\frac{4q^2(-a)^{-\frac{3}{2q}}\ln(1-a\,x^q)}{9x^{\frac{3}{2}}}-\frac{2q(-a)^{-\frac{3}{2q}}\left(1-\frac{2q}{3}\right)\mathrm{polylog}(2,a\,x^q)}{(-3+2q)x^{\frac{3}{2}}}-\frac{4q^2x^{q-\frac{3}{2}}a(-a)^{-\frac{3}{2q}}\Phi\left(a\,x^q,1,\frac{-3+2q}{2q}\right)}{9}\right)}{(dx)^{\frac{5}{2}}q}$	121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^q)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(d*x)^(5/2)*x^(5/2)*(-a)^(3/2/q)/q*(-4/9*q^2/x^(3/2)*(-a)^(-3/2/q)*ln(1-
a*x^q)-2*q/(-3+2*q)/x^(3/2)*(-a)^(-3/2/q)*(1-2/3*q)*polylog(2,a*x^q)-4/9*q^
2*x^(q-3/2)*a*(-a)^(-3/2/q)*LerchPhi(a*x^q,1,1/2*(-3+2*q)/q))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 8*q^3*integrate(1/9/((2*d^(5/2)*q + 3*d^(5/2) + (2*a^2*d^(5/2)*q + 3*a^2*d^
(5/2))*x^(2*q) - 2*(2*a*d^(5/2)*q + 3*a*d^(5/2))*x^q)*x^(5/2)), x) + 2/27*(
9*((2*a*sqrt(d)*q + 3*a*sqrt(d))*x*x^q - (2*sqrt(d)*q + 3*sqrt(d))*x)*dilog
(a*x^q)/x^(5/2) - 6*((2*a*sqrt(d)*q^2 + 3*a*sqrt(d)*q)*x*x^q - (2*sqrt(d)*q
^2 + 3*sqrt(d)*q)*x)*log(-a*x^q + 1)/x^(5/2) + 4*(2*sqrt(d)*q^3*x - (2*a*sq
rt(d)*q^3 + 3*a*sqrt(d)*q^2)*x*x^q)/x^(5/2))/(2*d^3*q + 3*d^3 - (2*a*d^3*q
+ 3*a*d^3)*x^q)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*dilog(a*x^q)/(d^3*x^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**q)/(d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(5/2),x, algorithm="giac")
[Out] integrate(dilog(a*x^q)/(d*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(2, a\,x^q)}{(d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^q)/(d*x)^(5/2),x)
[Out] int(polylog(2, a*x^q)/(d*x)^(5/2), x)
```

3.91 $\int (dx)^{3/2}\text{PolyLog}(3, ax^q)\, dx$

Optimal. Leaf size=125

$$-\frac{16adq^3x^{2+q}\sqrt{dx}\ {}_2F_1\left(1,\frac{\frac{5}{2}+q}{q};\frac{1}{2}\left(4+\frac{5}{q}\right);ax^q\right)}{125(5+2q)}-\frac{8q^2(dx)^{5/2}\log\left(1-ax^q\right)}{125d}-\frac{4q(dx)^{5/2}\text{PolyLog}(2,ax^q)}{25d}+\frac{2(dx)}{}$$

```
[Out] -8/125*q^2*(d*x)^(5/2)*ln(1-a*x^q)/d-4/25*q*(d*x)^(5/2)*polylog(2,a*x^q)/d+
2/5*(d*x)^(5/2)*polylog(3,a*x^q)/d-16/125*a*d*q^3*x^(2+q)*hypergeom([1, (5/
2+q)/q],[2+5/2/q],a*x^q)*(d*x)^(1/2)/(5+2*q)
```

Rubi [A]
time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6726, 2505, 20, 371}

$$-\frac{16adq^3\sqrt{dx}\ x^{q+2}\ {}_2F_1\left(1,\frac{q+\frac{5}{2}}{q};\frac{1}{2}\left(4+\frac{5}{q}\right);ax^q\right)}{125(2q+5)}-\frac{4q(dx)^{5/2}\text{Li}_2(ax^q)}{25d}+\frac{2(dx)^{5/2}\text{Li}_3(ax^q)}{5d}-\frac{8q^2(dx)^{5/2}\log\left(1-ax^q\right)}{125d}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*PolyLog[3, a*x^q],x]
```

```
[Out] (-16*a*d*q^3*x^(2 + q)*Sqrt[d*x]*Hypergeometric2F1[1, (5/2 + q)/q, (4 + 5/q
)/2, a*x^q])/(125*(5 + 2*q)) - (8*q^2*(d*x)^(5/2)*Log[1 - a*x^q])/(125*d) -
 (4*q*(d*x)^(5/2)*PolyLog[2, a*x^q])/(25*d) + (2*(d*x)^(5/2)*PolyLog[3, a*x
^q])/(5*d)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
```

```
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int (dx)^{3/2}\text{Li}_3(ax^q)\,dx &= \frac{2(dx)^{5/2}\text{Li}_3(ax^q)}{5d} - \frac{1}{5}(2q)\int (dx)^{3/2}\text{Li}_2(ax^q)\,dx \\
&= -\frac{4q(dx)^{5/2}\text{Li}_2(ax^q)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3(ax^q)}{5d} - \frac{1}{25}(4q^2)\int (dx)^{3/2}\log(1-ax^q)\,dx \\
&= -\frac{8q^2(dx)^{5/2}\log(1-ax^q)}{125d} - \frac{4q(dx)^{5/2}\text{Li}_2(ax^q)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3(ax^q)}{5d} - \frac{(8aq^3)\int \frac{x^{-1+}}{1}}{125} \\
&= -\frac{8q^2(dx)^{5/2}\log(1-ax^q)}{125d} - \frac{4q(dx)^{5/2}\text{Li}_2(ax^q)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3(ax^q)}{5d} - \frac{\left(8adq^3\sqrt{dx}\right.}{125} \\
&= -\frac{16adq^3x^{2+q}\sqrt{dx}\ {}_2F_1\left(1,\frac{\frac{5}{2}+q}{q};\frac{1}{2}\left(4+\frac{5}{q}\right);ax^q\right)}{125(5+2q)} - \frac{8q^2(dx)^{5/2}\log(1-ax^q)}{125d} - \frac{4q(d}{}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.02, size = 50, normalized size = 0.40

$$-\frac{x(dx)^{3/2}G_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,1-\frac{5}{2q}\\ 1,0,0,0,-\frac{5}{2q}\end{array}\right)}{q}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*PolyLog[3, a*x^q],x]
```

```
[Out] -((x*(d*x)^(3/2)*MeijerG[{{1, 1, 1, 1, 1 - 5/(2*q)}, {}}, {{1}, {0, 0, 0, -
5/(2*q)}}, -(a*x^q)])/q)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.25, size = 145, normalized size = 1.16

method	result
meijerg	$-\frac{(dx)^{\frac{3}{2}}(-a)^{-\frac{5}{2q}}\left(\frac{8q^3x^{\frac{5}{2}}(-a)^{\frac{5}{2q}}\ln(1-a\,x^q)}{125}+\frac{4q^2x^{\frac{5}{2}}(-a)^{\frac{5}{2q}}\,\mathrm{polylog}(2,a\,x^q)}{25}-\frac{2q\,x^{\frac{5}{2}}(-a)^{\frac{5}{2q}}\left(1+\frac{2q}{5}\right)\mathrm{polylog}(3,a\,x^q)}{5+2q}+\frac{8q^3x^{\frac{5}{2}+q}a(-a)^{\frac{5}{2q}}\,\Phi\left(a\right.}{125}\right.}{x^{\frac{3}{2}}q}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*polylog(3,a*x^q),x,method=_RETURNVERBOSE)
```

```
[Out] -(d*x)^(3/2)/x^(3/2)*(-a)^(-5/2/q)/q*(8/125*q^3*x^(5/2)*(-a)^(5/2/q)*ln(1-a
*x^q)+4/25*q^2*x^(5/2)*(-a)^(5/2/q)*polylog(2,a*x^q)-2*q/(5+2*q)*x^(5/2)*(-
a)^(5/2/q)*(1+2/5*q)*polylog(3,a*x^q)+8/125*q^3*x^(5/2+q)*a*(-a)^(5/2/q)*Le
rchPhi(a*x^q,1,1/2*(5+2*q)/q))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x^q),x, algorithm="maxima")
```

```
[Out] -16*d^(3/2)*q^4*integrate(1/125*x^(3/2)/(a^2*(2*q - 5)*x^(2*q) - 2*a*(2*q -
 5)*x^q + 2*q - 5), x) - 2/625*(50*((2*q^2 - 5*q)*a*d^(3/2)*x*x^q - (2*q^2
- 5*q)*d^(3/2)*x)*x^(3/2)*dilog(a*x^q) + 20*((2*q^3 - 5*q^2)*a*d^(3/2)*x*x^
q - (2*q^3 - 5*q^2)*d^(3/2)*x)*x^(3/2)*log(-a*x^q + 1) - 125*(a*d^(3/2)*(2*
q - 5)*x*x^q - d^(3/2)*(2*q - 5)*x)*x^(3/2)*polylog(3, a*x^q) + 8*(2*d^(3/2
)*q^4*x - (2*q^4 - 5*q^3)*a*d^(3/2)*x*x^q)*x^(3/2))/(a*(2*q - 5)*x^q - 2*q
+ 5)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*d*x*polylog(3, a*x^q), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}}\operatorname{Li}_3(ax^q)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*polylog(3,a*x**q),x)
```

```
[Out] Integral((d*x)**(3/2)*polylog(3, a*x**q), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x^q),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*polylog(3, a*x^q), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{3/2}\,\mathrm{polylog}(3,a\,x^q)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*polylog(3, a*x^q),x)
```

```
[Out] int((d*x)^(3/2)*polylog(3, a*x^q), x)
```

3.92 $\int \sqrt{dx}\, \textbf{PolyLog}(3, ax^q)\, dx$

Optimal. Leaf size=124

$$-\frac{16aq^3x^{1+q}\sqrt{dx}\ {}_2F_1\left(1,\frac{\frac{3}{2}+q}{q};\frac{1}{2}\left(4+\frac{3}{q}\right);ax^q\right)}{27(3+2q)}-\frac{8q^2(dx)^{3/2}\log\left(1-ax^q\right)}{27d}-\frac{4q(dx)^{3/2}\text{PolyLog}(2,ax^q)}{9d}+\frac{2(dx)^3}{}$$

```
[Out] -8/27*q^2*(d*x)^(3/2)*ln(1-a*x^q)/d-4/9*q*(d*x)^(3/2)*polylog(2,a*x^q)/d+2/
3*(d*x)^(3/2)*polylog(3,a*x^q)/d-16/27*a*q^3*x^(1+q)*hypergeom([1, (3/2+q)/
q],[2+3/2/q],a*x^q)*(d*x)^(1/2)/(3+2*q)
```

Rubi [A]
time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6726, 2505, 20, 371}

$$-\frac{16aq^3\sqrt{dx}\,x^{q+1}\,{}_2F_1\left(1,\frac{q+\frac{3}{2}}{q};\frac{1}{2}\left(4+\frac{3}{q}\right);ax^q\right)}{27(2q+3)}-\frac{4q(dx)^{3/2}\text{Li}_2(ax^q)}{9d}+\frac{2(dx)^{3/2}\text{Li}_3(ax^q)}{3d}-\frac{8q^2(dx)^{3/2}\log\left(1-ax^q\right)}{27d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*PolyLog[3, a*x^q],x]
```

```
[Out] (-16*a*q^3*x^(1 + q)*Sqrt[d*x]*Hypergeometric2F1[1, (3/2 + q)/q, (4 + 3/q)/
2, a*x^q])/(27*(3 + 2*q)) - (8*q^2*(d*x)^(3/2)*Log[1 - a*x^q])/(27*d) - (4*
q*(d*x)^(3/2)*PolyLog[2, a*x^q])/(9*d) + (2*(d*x)^(3/2)*PolyLog[3, a*x^q])/
(3*d)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
```

```
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{dx}\,\mathrm{Li}_3(ax^q)\,dx &= \frac{2(dx)^{3/2}\mathrm{Li}_3(ax^q)}{3d} - \frac{1}{3}(2q)\int \sqrt{dx}\,\mathrm{Li}_2(ax^q)\,dx \\
&= -\frac{4q(dx)^{3/2}\mathrm{Li}_2(ax^q)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_3(ax^q)}{3d} - \frac{1}{9}(4q^2)\int \sqrt{dx}\,\log(1-ax^q)\,dx \\
&= -\frac{8q^2(dx)^{3/2}\log(1-ax^q)}{27d} - \frac{4q(dx)^{3/2}\mathrm{Li}_2(ax^q)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_3(ax^q)}{3d} - \frac{(8aq^3)\int \frac{x^{-1+q}}{1-}}{27d} \\
&= -\frac{8q^2(dx)^{3/2}\log(1-ax^q)}{27d} - \frac{4q(dx)^{3/2}\mathrm{Li}_2(ax^q)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_3(ax^q)}{3d} - \frac{\left(8aq^3\sqrt{dx}\right)}{27\sqrt{x}} \\
&= -\frac{16aq^3x^{1+q}\sqrt{dx}\;{}_2F_1\left(1,\frac{\frac{3}{2}+q}{q};\frac{1}{2}\left(4+\frac{3}{q}\right);ax^q\right)}{27(3+2q)} - \frac{8q^2(dx)^{3/2}\log(1-ax^q)}{27d} - \frac{4q(dx)^{3}}{}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.02, size = 50, normalized size = 0.40

$$
-\frac{x\sqrt{dx}\,G_{5,5}^{1,5}\left(-ax^q\left|\begin{array}{c}1,1,1,1,1-\frac{3}{2q}\\1,0,0,0,-\frac{3}{2q}\end{array}\right.\right)}{q}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*PolyLog[3, a*x^q],x]
```

```
[Out] -((x*Sqrt[d*x]*MeijerG[{{1, 1, 1, 1, 1 - 3/(2*q)}, {}}, {{1}, {0, 0, 0, -3/
(2*q)}}, -(a*x^q)])/q)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.25, size = 145, normalized size = 1.17

method	result
meijerg	$-\frac{\sqrt{dx}\,(-a)^{-\frac{3}{2q}}\left(\frac{8q^3x^{\frac{3}{2}}(-a)^{\frac{3}{2q}}\ln(1-a\,x^q)}{27}+\frac{4q^2x^{\frac{3}{2}}(-a)^{\frac{3}{2q}}\operatorname{polylog}(2,a\,x^q)}{9}-\frac{2q\,x^{\frac{3}{2}}(-a)^{\frac{3}{2q}}\left(1+\frac{2q}{3}\right)\operatorname{polylog}(3,a\,x^q)}{3+2q}+\frac{8q^3x^{\frac{3}{2}+q}a(-a)^{\frac{3}{2q}}\Phi(}{27}\right.}{\sqrt{x}\,q}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*polylog(3,a*x^q),x,method=_RETURNVERBOSE)
```

```
[Out] -(d*x)^(1/2)/x^(1/2)*(-a)^(-3/2/q)/q*(8/27*q^3*x^(3/2)*(-a)^(3/2/q)*ln(1-a*
x^q)+4/9*q^2*x^(3/2)*(-a)^(3/2/q)*polylog(2,a*x^q)-2*q/(3+2*q)*x^(3/2)*(-a)
^(3/2/q)*(1+2/3*q)*polylog(3,a*x^q)+8/27*q^3*x^(3/2+q)*a*(-a)^(3/2/q)*Lerch
Phi(a*x^q,1,1/2*(3+2*q)/q))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x^q),x, algorithm="maxima")
```

```
[Out] -16*sqrt(d)*q^4*integrate(1/27*sqrt(x)/(a^2*(2*q - 3)*x^(2*q) - 2*a*(2*q -
3)*x^q + 2*q - 3), x) - 2/81*(18*((2*q^2 - 3*q)*a*sqrt(d)*x*x^q - (2*q^2 -
3*q)*sqrt(d)*x)*sqrt(x)*dilog(a*x^q) + 12*((2*q^3 - 3*q^2)*a*sqrt(d)*x*x^q
- (2*q^3 - 3*q^2)*sqrt(d)*x)*sqrt(x)*log(-a*x^q + 1) - 27*(a*sqrt(d)*(2*q -
 3)*x*x^q - sqrt(d)*(2*q - 3)*x)*sqrt(x)*polylog(3, a*x^q) + 8*(2*sqrt(d)*q
^4*x - (2*q^4 - 3*q^3)*a*sqrt(d)*x*x^q)*sqrt(x))/(a*(2*q - 3)*x^q - 2*q + 3
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*polylog(3, a*x^q), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\,\operatorname{Li}_3(ax^q)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*polylog(3,a*x**q),x)
```

```
[Out] Integral(sqrt(d*x)*polylog(3, a*x**q), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x^q),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*polylog(3, a*x^q), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,x}\,\mathrm{polylog}(3,a\,x^q)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*polylog(3, a*x^q),x)
```

```
[Out] int((d*x)^(1/2)*polylog(3, a*x^q), x)
```

3.93 $\int \frac{\text{PolyLog}(3,ax^q)}{\sqrt{dx}}\,dx$

Optimal. Leaf size=115

$$-\frac{16aq^3x^q\sqrt{dx}\;{}_2F_1\left(1,\frac{\frac{1}{2}+q}{q};\frac{1}{2}\left(4+\frac{1}{q}\right);ax^q\right)}{d(1+2q)}-\frac{8q^2\sqrt{dx}\,\log\left(1-ax^q\right)}{d}-\frac{4q\sqrt{dx}\,\text{PolyLog}(2,ax^q)}{d}+\frac{2\sqrt{dx}\,\text{Poly}}{d}$$

```
[Out] -16*a*q^3*x^q*hypergeom([1, (1/2+q)/q],[2+1/2/q],a*x^q)*(d*x)^(1/2)/d/(1+2*
q)-8*q^2*ln(1-a*x^q)*(d*x)^(1/2)/d-4*q*polylog(2,a*x^q)*(d*x)^(1/2)/d+2*pol
ylog(3,a*x^q)*(d*x)^(1/2)/d
```

Rubi [A]

time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6726, 2505, 20, 371}

$$-\frac{16aq^3\sqrt{dx}\,x^q\,{}_2F_1\left(1,\frac{q+\frac{1}{2}}{q};\frac{1}{2}\left(4+\frac{1}{q}\right);ax^q\right)}{d(2q+1)}-\frac{4q\sqrt{dx}\,\text{Li}_2(ax^q)}{d}+\frac{2\sqrt{dx}\,\text{Li}_3(ax^q)}{d}-\frac{8q^2\sqrt{dx}\,\log\left(1-ax^q\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^q]/Sqrt[d*x],x]
```

```
[Out] (-16*a*q^3*x^q*Sqrt[d*x]*Hypergeometric2F1[1, (1/2 + q)/q, (4 + q^(-1))/2,
a*x^q])/(d*(1 + 2*q)) - (8*q^2*Sqrt[d*x]*Log[1 - a*x^q])/d - (4*q*Sqrt[d*x]
*PolyLog[2, a*x^q])/d + (2*Sqrt[d*x]*PolyLog[3, a*x^q])/d
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
```

```
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_3(ax^q)}{\sqrt{dx}}\,dx &= \frac{2\sqrt{dx}\,\text{Li}_3(ax^q)}{d} - (2q)\int \frac{\text{Li}_2(ax^q)}{\sqrt{dx}}\,dx\\ &= -\frac{4q\sqrt{dx}\,\text{Li}_2(ax^q)}{d} + \frac{2\sqrt{dx}\,\text{Li}_3(ax^q)}{d} - (4q^2)\int \frac{\log(1-ax^q)}{\sqrt{dx}}\,dx\\ &= -\frac{8q^2\sqrt{dx}\,\log(1-ax^q)}{d} - \frac{4q\sqrt{dx}\,\text{Li}_2(ax^q)}{d} + \frac{2\sqrt{dx}\,\text{Li}_3(ax^q)}{d} - \frac{(8aq^3)\int \frac{x^{-1+q}\sqrt{dx}}{1-ax^q}\,dx}{d}\\ &= -\frac{8q^2\sqrt{dx}\,\log(1-ax^q)}{d} - \frac{4q\sqrt{dx}\,\text{Li}_2(ax^q)}{d} + \frac{2\sqrt{dx}\,\text{Li}_3(ax^q)}{d} - \frac{\left(8aq^3\sqrt{dx}\right)\int \frac{x^{-\frac{1}{2}+q}}{1-ax^q}\,dx}{d\sqrt{x}}\\ &= -\frac{16aq^3x^q\sqrt{dx}\;{}_2F_1\left(1,\frac{\frac{1}{2}+q}{q};\frac{1}{2}\left(4+\frac{1}{q}\right);ax^q\right)}{d(1+2q)} - \frac{8q^2\sqrt{dx}\,\log(1-ax^q)}{d} - \frac{4q\sqrt{dx}\,\text{Li}_2(ax^q)}{d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.02, size = 50, normalized size = 0.43

$$-\frac{xG_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,1-\frac{1}{2q}\\1,0,0,0,-\frac{1}{2q}\end{array}\right)}{q\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^q]/Sqrt[d*x],x]
```

```
[Out] -((x*MeijerG[{{1, 1, 1, 1, 1 - 1/(2*q)}, {}}, {{1}, {0, 0, 0, -1/2*1/q}}, -
(a*x^q)])/(q*Sqrt[d*x]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.26, size = 133, normalized size = 1.16

method	result
meijerg	$-\frac{\sqrt{x}\,(-a)^{-\frac{1}{2q}}\left(8q^3\sqrt{x}\,(-a)^{\frac{1}{2q}}\ln(1-a\,x^q)+4q^2\sqrt{x}\,(-a)^{\frac{1}{2q}}\operatorname{polylog}(2,a\,x^q)-2q\sqrt{x}\,(-a)^{\frac{1}{2q}}\operatorname{polylog}(3,a\,x^q)+8q^3x^{\frac{1}{2}+q}a(-a\right.}{\sqrt{dx}\,q}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^q)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(d*x)^(1/2)*x^(1/2)*(-a)^(-1/2/q)/q*(8*q^3*x^(1/2)*(-a)^(1/2/q)*ln(1-a*x
^q)+4*q^2*x^(1/2)*(-a)^(1/2/q)*polylog(2,a*x^q)-2*q*x^(1/2)*(-a)^(1/2/q)*po
lylog(3,a*x^q)+8*q^3*x^(1/2+q)*a*(-a)^(1/2/q)*LerchPhi(a*x^q,1,1/2*(1+2*q)/
q))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -16*q^4*integrate(1/((a^2*sqrt(d)*(2*q - 1)*x^(2*q) - 2*a*sqrt(d)*(2*q - 1)
*x^q + sqrt(d)*(2*q - 1))*sqrt(x)), x) - 2*(2*((2*q^2 - q)*a*x*x^q - (2*q^2
 - q)*x)*dilog(a*x^q)/sqrt(x) + 4*((2*q^3 - q^2)*a*x*x^q - (2*q^3 - q^2)*x)
*log(-a*x^q + 1)/sqrt(x) - (a*(2*q - 1)*x*x^q - (2*q - 1)*x)*polylog(3, a*x
^q)/sqrt(x) + 8*(2*q^4*x - (2*q^4 - q^3)*a*x*x^q)/sqrt(x))/(a*sqrt(d)*(2*q
- 1)*x^q - sqrt(d)*(2*q - 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*polylog(3, a*x^q)/(d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_3(ax^q)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**q)/(d*x)**(1/2),x)
```

```
[Out] Integral(polylog(3, a*x**q)/sqrt(d*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^q)/sqrt(d*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(3, a\, x^q)}{\sqrt{d\, x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^q)/(d*x)^(1/2),x)
```

```
[Out] int(polylog(3, a*x^q)/(d*x)^(1/2), x)
```

3.94 $\int \frac{\text{PolyLog}(3,ax^q)}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{16aq^3x^q\,{}_2F_1\left(1,\frac{1}{2}\left(2-\frac{1}{q}\right);\frac{1}{2}\left(4-\frac{1}{q}\right);ax^q\right)}{d(1-2q)\sqrt{dx}}+\frac{8q^2\log\left(1-ax^q\right)}{d\sqrt{dx}}-\frac{4q\text{PolyLog}(2,ax^q)}{d\sqrt{dx}}-\frac{2\text{PolyLog}(3,ax^q)}{d\sqrt{dx}}$$

```
[Out] -16*a*q^3*x^q*hypergeom([1, 1-1/2/q],[2-1/2/q],a*x^q)/d/(1-2*q)/(d*x)^(1/2)
+8*q^2*ln(1-a*x^q)/d/(d*x)^(1/2)-4*q*polylog(2,a*x^q)/d/(d*x)^(1/2)-2*polyl
og(3,a*x^q)/d/(d*x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6726, 2505, 20, 371}

$$-\frac{16aq^3x^q\,{}_2F_1\left(1,\frac{1}{2}\left(2-\frac{1}{q}\right);\frac{1}{2}\left(4-\frac{1}{q}\right);ax^q\right)}{d(1-2q)\sqrt{dx}}-\frac{4q\text{Li}_2(ax^q)}{d\sqrt{dx}}-\frac{2\text{Li}_3(ax^q)}{d\sqrt{dx}}+\frac{8q^2\log\left(1-ax^q\right)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^q]/(d*x)^(3/2),x]
```

```
[Out] (-16*a*q^3*x^q*Hypergeometric2F1[1, (2 - q^(-1))/2, (4 - q^(-1))/2, a*x^q])
/(d*(1 - 2*q)*Sqrt[d*x]) + (8*q^2*Log[1 - a*x^q])/(d*Sqrt[d*x]) - (4*q*Poly
Log[2, a*x^q])/(d*Sqrt[d*x]) - (2*PolyLog[3, a*x^q])/(d*Sqrt[d*x])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
```

```
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax^q)}{(dx)^{3/2}}\,dx &= -\frac{2\text{Li}_3(ax^q)}{d\sqrt{dx}} + (2q)\int \frac{\text{Li}_2(ax^q)}{(dx)^{3/2}}\,dx \\
&= -\frac{4q\text{Li}_2(ax^q)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^q)}{d\sqrt{dx}} - \left(4q^2\right)\int \frac{\log(1-ax^q)}{(dx)^{3/2}}\,dx \\
&= \frac{8q^2\log(1-ax^q)}{d\sqrt{dx}} - \frac{4q\text{Li}_2(ax^q)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^q)}{d\sqrt{dx}} + \frac{\left(8aq^3\right)\int \frac{x^{-1+q}}{\sqrt{dx}\,(1-ax^q)}\,dx}{d} \\
&= \frac{8q^2\log(1-ax^q)}{d\sqrt{dx}} - \frac{4q\text{Li}_2(ax^q)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^q)}{d\sqrt{dx}} + \frac{\left(8aq^3\sqrt{x}\right)\int \frac{x^{-\frac{3}{2}+q}}{1-ax^q}\,dx}{d\sqrt{dx}} \\
&= -\frac{16aq^3x^q\,{}_2F_1\left(1,\frac{1}{2}\left(2-\frac{1}{q}\right);\frac{1}{2}\left(4-\frac{1}{q}\right);ax^q\right)}{d(1-2q)\sqrt{dx}} + \frac{8q^2\log(1-ax^q)}{d\sqrt{dx}} - \frac{4q\text{Li}_2(ax^q)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^q)}{d\sqrt{dx}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.02, size = 50, normalized size = 0.42

$$-\frac{xG_{5,5}^{1,5}\left(-ax^q\left|\begin{array}{c}1,1,1,1,1+\frac{1}{2q}\\ 1,0,0,0,\frac{1}{2q}\end{array}\right.\right)}{q(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^q]/(d*x)^(3/2),x]
```

```
[Out] -((x*MeijerG[{{1, 1, 1, 1, 1 + 1/(2*q)}, {}}, {{1}, {0, 0, 0, 1/(2*q)}}, -(
a*x^q)])/(q*(d*x)^(3/2)))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.24, size = 145, normalized size = 1.22

method	result
meijerg	$-\frac{x^{\frac{3}{2}}(-a)^{\frac{1}{2q}}\left(-\frac{8q^3(-a)^{-\frac{1}{2q}}\ln(1-a\,x^q)}{\sqrt{x}}+\frac{4q^2(-a)^{-\frac{1}{2q}}\operatorname{polylog}(2,a\,x^q)}{\sqrt{x}}-\frac{2q(-a)^{-\frac{1}{2q}}(1-2q)\operatorname{polylog}(3,a\,x^q)}{(2q-1)\sqrt{x}}-8q^3x^{q-\frac{1}{2}}a(-a)^{-\frac{1}{2q}}\Phi\left(a\,x^q,1,\right.\right.}{(dx)^{\frac{3}{2}}q}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^q)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(d*x)^(3/2)*x^(3/2)*(-a)^(1/2/q)/q*(-8*q^3/x^(1/2)*(-a)^(-1/2/q)*ln(1-a*
x^q)+4*q^2/x^(1/2)*(-a)^(-1/2/q)*polylog(2,a*x^q)-2*q/(2*q-1)/x^(1/2)*(-a)^
(-1/2/q)*(1-2*q)*polylog(3,a*x^q)-8*q^3*x^(q-1/2)*a*(-a)^(-1/2/q)*LerchPhi(
a*x^q,1,1/2*(2*q-1)/q))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 16*q^4*integrate(1/((a^2*d^(3/2)*(2*q + 1)*x^(2*q) - 2*a*d^(3/2)*(2*q + 1)*
x^q + d^(3/2)*(2*q + 1))*x^(3/2)), x) - 2*(2*((2*q^2 + q)*a*x*x^q - (2*q^2
+ q)*x)*dilog(a*x^q)/x^(3/2) - 4*((2*q^3 + q^2)*a*x*x^q - (2*q^3 + q^2)*x)*
log(-a*x^q + 1)/x^(3/2) + (a*(2*q + 1)*x*x^q - (2*q + 1)*x)*polylog(3, a*x^
q)/x^(3/2) + 8*(2*q^4*x - (2*q^4 + q^3)*a*x*x^q)/x^(3/2))/(a*d^(3/2)*(2*q +
 1)*x^q - d^(3/2)*(2*q + 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*polylog(3, a*x^q)/(d^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_3(ax^q)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**q)/(d*x)**(3/2),x)
```

```
[Out] Integral(polylog(3, a*x**q)/(d*x)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^q)/(d*x)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(3, a\,x^q)}{{(d\,x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^q)/(d*x)^(3/2),x)
```

```
[Out] int(polylog(3, a*x^q)/(d*x)^(3/2), x)
```

3.95 $\int \frac{\text{PolyLog}(3,ax^q)}{(dx)^{5/2}} \, dx$

Optimal. Leaf size=129

$$-\frac{16aq^3x^{-1+q}\,{}_2F_1\left(1,\frac{1}{2}\left(2-\frac{3}{q}\right);\frac{1}{2}\left(4-\frac{3}{q}\right);ax^q\right)}{27d^2(3-2q)\sqrt{dx}}+\frac{8q^2\log\left(1-ax^q\right)}{27d(dx)^{3/2}}-\frac{4q\text{PolyLog}(2,ax^q)}{9d(dx)^{3/2}}-\frac{2\text{PolyLog}(3,ax^q)}{3d(dx)^{3/2}}$$

```
[Out] 8/27*q^2*ln(1-a*x^q)/d/(d*x)^(3/2)-4/9*q*polylog(2,a*x^q)/d/(d*x)^(3/2)-2/3
*polylog(3,a*x^q)/d/(d*x)^(3/2)-16/27*a*q^3*x^(-1+q)*hypergeom([1, 1-3/2/q]
,[2-3/2/q],a*x^q)/d^2/(3-2*q)/(d*x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6726, 2505, 20, 371}

$$-\frac{16aq^3x^{q-1}\,{}_2F_1\left(1,\frac{1}{2}\left(2-\frac{3}{q}\right);\frac{1}{2}\left(4-\frac{3}{q}\right);ax^q\right)}{27d^2(3-2q)\sqrt{dx}}-\frac{4q\text{Li}_2(ax^q)}{9d(dx)^{3/2}}-\frac{2\text{Li}_3(ax^q)}{3d(dx)^{3/2}}+\frac{8q^2\log\left(1-ax^q\right)}{27d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^q]/(d*x)^(5/2),x]
```

```
[Out] (-16*a*q^3*x^(-1 + q)*Hypergeometric2F1[1, (2 - 3/q)/2, (4 - 3/q)/2, a*x^q]
)/(27*d^2*(3 - 2*q)*Sqrt[d*x]) + (8*q^2*Log[1 - a*x^q])/(27*d*(d*x)^(3/2))
- (4*q*PolyLog[2, a*x^q])/(9*d*(d*x)^(3/2)) - (2*PolyLog[3, a*x^q])/(3*d*(d
*x)^(3/2))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
```

```
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_3(ax^q)}{(dx)^{5/2}}\,dx &= -\frac{2\text{Li}_3(ax^q)}{3d(dx)^{3/2}} + \frac{1}{3}(2q)\int \frac{\text{Li}_2(ax^q)}{(dx)^{5/2}}\,dx \\ &= -\frac{4q\text{Li}_2(ax^q)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^q)}{3d(dx)^{3/2}} - \frac{1}{9}(4q^2)\int \frac{\log(1-ax^q)}{(dx)^{5/2}}\,dx \\ &= \frac{8q^2\log(1-ax^q)}{27d(dx)^{3/2}} - \frac{4q\text{Li}_2(ax^q)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^q)}{3d(dx)^{3/2}} + \frac{(8aq^3)\int \frac{x^{-1+q}}{(dx)^{3/2}(1-ax^q)}\,dx}{27d} \\ &= \frac{8q^2\log(1-ax^q)}{27d(dx)^{3/2}} - \frac{4q\text{Li}_2(ax^q)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^q)}{3d(dx)^{3/2}} + \frac{\left(8aq^3\sqrt{x}\right)\int \frac{x^{-\frac{5}{2}+q}}{1-ax^q}\,dx}{27d^2\sqrt{dx}} \\ &= -\frac{16aq^3x^{-1+q}\,{}_2F_1\left(1,\frac{1}{2}\left(2-\frac{3}{q}\right);\frac{1}{2}\left(4-\frac{3}{q}\right);ax^q\right)}{27d^2(3-2q)\sqrt{dx}} + \frac{8q^2\log(1-ax^q)}{27d(dx)^{3/2}} - \frac{4q\text{Li}_2(ax^q)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^q)}{3d(dx)^{3/2}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.02, size = 50, normalized size = 0.39

$$-\frac{xG_{5,5}^{1,5}\left(-ax^q\left|\begin{array}{c}1,1,1,1,1+\frac{3}{2q}\\1,0,0,0,\frac{3}{2q}\end{array}\right.\right)}{q(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^q]/(d*x)^(5/2),x]
```

```
[Out] -((x*MeijerG[{{1, 1, 1, 1, 1 + 3/(2*q)}, {}}, {{1}, {0, 0, 0, 3/(2*q)}}, -(
a*x^q)])/(q*(d*x)^(5/2)))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.24, size = 145, normalized size = 1.12

method	result
meijerg	$-\frac{x^{\frac{5}{2}}(-a)^{\frac{3}{2q}}\left(-\frac{8q^3(-a)^{-\frac{3}{2q}}\ln(1-a\,x^q)}{27x^{\frac{3}{2}}}+\frac{4q^2(-a)^{-\frac{3}{2q}}\,\text{polylog}(2,a\,x^q)}{9x^{\frac{3}{2}}}-\frac{2q(-a)^{-\frac{3}{2q}}\left(1-\frac{2q}{3}\right)\text{polylog}(3,a\,x^q)}{(-3+2q)x^{\frac{3}{2}}}-\frac{8q^3x^{q-\frac{3}{2}}a(-a)^{-\frac{3}{2q}}\Phi\left(a\,x^q,1,\right.}{27}\right.}{(dx)^{\frac{5}{2}}q}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^q)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(d*x)^(5/2)*x^(5/2)*(-a)^(3/2/q)/q*(-8/27*q^3/x^(3/2)*(-a)^(-3/2/q)*ln(1
-a*x^q)+4/9*q^2/x^(3/2)*(-a)^(-3/2/q)*polylog(2,a*x^q)-2*q/(-3+2*q)/x^(3/2)
*(-a)^(-3/2/q)*(1-2/3*q)*polylog(3,a*x^q)-8/27*q^3*x^(q-3/2)*a*(-a)^(-3/2/q
)*LerchPhi(a*x^q,1,1/2*(-3+2*q)/q))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 16*q^4*integrate(1/27/((a^2*d^(5/2)*(2*q + 3)*x^(2*q) - 2*a*d^(5/2)*(2*q +
3)*x^q + d^(5/2)*(2*q + 3))*x^(5/2)), x) - 2/81*(18*((2*q^2 + 3*q)*a*x*x^q
- (2*q^2 + 3*q)*x)*dilog(a*x^q)/x^(5/2) - 12*((2*q^3 + 3*q^2)*a*x*x^q - (2*
q^3 + 3*q^2)*x)*log(-a*x^q + 1)/x^(5/2) + 27*(a*(2*q + 3)*x*x^q - (2*q + 3)
*x)*polylog(3, a*x^q)/x^(5/2) + 8*(2*q^4*x - (2*q^4 + 3*q^3)*a*x*x^q)/x^(5/
2))/(a*d^(5/2)*(2*q + 3)*x^q - d^(5/2)*(2*q + 3))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*polylog(3, a*x^q)/(d^3*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_3\left(ax^q\right)}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**q)/(d*x)**(5/2),x)
```

```
[Out] Integral(polylog(3, a*x**q)/(d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^q)/(d*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(3, a\,x^q)}{{(d\,x)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^q)/(d*x)^(5/2),x)
```

```
[Out] int(polylog(3, a*x^q)/(d*x)^(5/2), x)
```

3.96 $\int \text{PolyLog}\left(\frac{3}{2}, ax\right) dx$

Optimal. Leaf size=30

$$-x\text{PolyLog}\left(\frac{1}{2}, ax\right) + x\text{PolyLog}\left(\frac{3}{2}, ax\right) + \text{Int}\left(\text{PolyLog}\left(-\frac{1}{2}, ax\right), x\right)$$

```
[Out] -x*polylog(1/2,a*x)+x*polylog(3/2,a*x)+Unintegrable(polylog(-1/2,a*x),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \text{Li}_{\frac{3}{2}}(ax)\,dx$$

Verification is not applicable to the result.

```
[In] Int[PolyLog[3/2, a*x],x]
[Out] -(x*PolyLog[1/2, a*x]) + x*PolyLog[3/2, a*x] + Defer[Int][PolyLog[-1/2, a*x
], x]
```

Rubi steps

$$\begin{aligned}\int \text{Li}_{\frac{3}{2}}(ax)\,dx &= x\text{Li}_{\frac{3}{2}}(ax) - \int \text{Li}_{\frac{1}{2}}(ax)\,dx\\ &= -x\text{Li}_{\frac{1}{2}}(ax) + x\text{Li}_{\frac{3}{2}}(ax) + \int \text{Li}_{-\frac{1}{2}}(ax)\,dx\end{aligned}$$

Mathematica [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \text{PolyLog}\left(\frac{3}{2}, ax\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[PolyLog[3/2, a*x],x]
[Out] Integrate[PolyLog[3/2, a*x], x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \text{polylog}\left(\frac{3}{2}, ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3/2,a*x),x)
```

```
[Out] int(polylog(3/2,a*x),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3/2,a*x),x, algorithm="maxima")
```

```
[Out] integrate(polylog(3/2, a*x), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3/2,a*x),x, algorithm="fricas")
```

```
[Out] integral(polylog(3/2, a*x), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Li}_{\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3/2,a*x),x)
```

```
[Out] Integral(polylog(3/2, a*x), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3/2,a*x),x, algorithm="giac")
```

```
[Out] integrate(polylog(3/2, a*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \text{polylog}\left(\frac{3}{2}, a\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3/2, a*x),x)
[Out] int(polylog(3/2, a*x), x)
```

3.97 $\int \text{PolyLog}\left(\frac{1}{2}, ax\right)\,dx$

Optimal. Leaf size=22

$$x\text{PolyLog}\left(\frac{1}{2}, ax\right) - \text{Int}\left(\text{PolyLog}\left(-\frac{1}{2}, ax\right), x\right)$$

```
[Out] x*polylog(1/2,a*x)-Unintegrable(polylog(-1/2,a*x),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \text{Li}_{\frac{1}{2}}(ax)\,dx$$

Verification is not applicable to the result.

```
[In] Int[PolyLog[1/2, a*x],x]
[Out] x*PolyLog[1/2, a*x] - Defer[Int][PolyLog[-1/2, a*x], x]
```

Rubi steps

$$\int \text{Li}_{\frac{1}{2}}(ax)\,dx = x\text{Li}_{\frac{1}{2}}(ax) - \int \text{Li}_{-\frac{1}{2}}(ax)\,dx$$

Mathematica [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \text{PolyLog}\left(\frac{1}{2}, ax\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[PolyLog[1/2, a*x],x]
[Out] Integrate[PolyLog[1/2, a*x], x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \text{polylog}\left(\frac{1}{2}, ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(1/2,a*x),x)
```

```
[Out] int(polylog(1/2,a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(1/2,a*x),x, algorithm="maxima")
```

```
[Out] integrate(polylog(1/2, a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(1/2,a*x),x, algorithm="fricas")
```

```
[Out] integral(polylog(1/2, a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Li}_{\frac{1}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(1/2,a*x),x)
```

```
[Out] Integral(polylog(1/2, a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(1/2,a*x),x, algorithm="giac")
```

```
[Out] integrate(polylog(1/2, a*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \mathrm{polylog}\left(\frac{1}{2},a\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(1/2, a*x),x)
[Out] int(polylog(1/2, a*x), x)
```

3.98 $\int \text{PolyLog}\left(-\frac{1}{2}, ax\right) \, dx$

Optimal. Leaf size=10

$$\text{Int}\left(\text{PolyLog}\left(-\frac{1}{2}, ax\right), x\right)$$

[Out] Unintegrable(polylog(-1/2,a*x),x)

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \text{Li}_{-\frac{1}{2}}(ax) \, dx$$

Verification is not applicable to the result.

[In] Int[PolyLog[-1/2, a*x],x]

[Out] Defer[Int][PolyLog[-1/2, a*x], x]

Rubi steps

$$\int \text{Li}_{-\frac{1}{2}}(ax) \, dx = \int \text{Li}_{-\frac{1}{2}}(ax) \, dx$$

Mathematica [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \text{PolyLog}\left(-\frac{1}{2}, ax\right) \, dx$$

Verification is not applicable to the result.

[In] Integrate[PolyLog[-1/2, a*x],x]

[Out] Integrate[PolyLog[-1/2, a*x], x]

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \text{polylog}\left(-\frac{1}{2}, ax\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(-1/2,a*x),x)
```

```
[Out] int(polylog(-1/2,a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-1/2,a*x),x, algorithm="maxima")
```

```
[Out] integrate(polylog(-1/2, a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-1/2,a*x),x, algorithm="fricas")
```

```
[Out] integral(polylog(-1/2, a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Li}_{-\frac{1}{2}}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-1/2,a*x),x)
```

```
[Out] Integral(polylog(-1/2, a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-1/2,a*x),x, algorithm="giac")
```

```
[Out] integrate(polylog(-1/2, a*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.10

$$\int \mathrm{polylog}\left(-\frac{1}{2},a\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(-1/2, a*x),x)
```

```
[Out] int(polylog(-1/2, a*x), x)
```

3.99 $\int \text{PolyLog}\left(-\frac{3}{2}, ax\right) \, dx$

Optimal. Leaf size=22

$$x\text{PolyLog}\left(-\frac{1}{2}, ax\right) - \text{Int}\left(\text{PolyLog}\left(-\frac{1}{2}, ax\right), x\right)$$

```
[Out] x*polylog(-1/2,a*x)-Unintegrable(polylog(-1/2,a*x),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \text{Li}_{-\frac{3}{2}}(ax) \, dx$$

Verification is not applicable to the result.

```
[In] Int[PolyLog[-3/2, a*x],x]
[Out] x*PolyLog[-1/2, a*x] - Defer[Int][PolyLog[-1/2, a*x], x]
```

Rubi steps

$$\int \text{Li}_{-\frac{3}{2}}(ax) \, dx = x\text{Li}_{-\frac{1}{2}}(ax) - \int \text{Li}_{-\frac{1}{2}}(ax) \, dx$$

Mathematica [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \text{PolyLog}\left(-\frac{3}{2}, ax\right) \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[PolyLog[-3/2, a*x],x]
[Out] Integrate[PolyLog[-3/2, a*x], x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \text{polylog}\left(-\frac{3}{2}, ax\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(-3/2,a*x),x)
[Out] int(polylog(-3/2,a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-3/2,a*x),x, algorithm="maxima")
[Out] integrate(polylog(-3/2, a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-3/2,a*x),x, algorithm="fricas")
[Out] integral(polylog(-3/2, a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Li}_{-\frac{3}{2}}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-3/2,a*x),x)
[Out] Integral(polylog(-3/2, a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-3/2,a*x),x, algorithm="giac")
[Out] integrate(polylog(-3/2, a*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \mathrm{polylog}\left(-\frac{3}{2},a\,x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(-3/2, a*x),x)
[Out] int(polylog(-3/2, a*x), x)
```

3.100 $\int \mathrm{PolyLog}\left(-\frac{5}{2}, ax\right) dx$

Optimal. Leaf size=30

$$x\mathrm{PolyLog}\left(-\frac{3}{2}, ax\right) - x\mathrm{PolyLog}\left(-\frac{1}{2}, ax\right) + \mathrm{Int}\left(\mathrm{PolyLog}\left(-\frac{1}{2}, ax\right), x\right)$$

[Out] x*polylog(-3/2,a*x)-x*polylog(-1/2,a*x)+Unintegrable(polylog(-1/2,a*x),x)

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \mathrm{Li}_{-\frac{5}{2}}(ax)\,dx$$

Verification is not applicable to the result.

[In] Int[PolyLog[-5/2, a*x],x]

[Out] x*PolyLog[-3/2, a*x] - x*PolyLog[-1/2, a*x] + Defer[Int][PolyLog[-1/2, a*x]
, x]

Rubi steps

$$\begin{aligned}\int \mathrm{Li}_{-\frac{5}{2}}(ax)\,dx &= x\mathrm{Li}_{-\frac{3}{2}}(ax) - \int \mathrm{Li}_{-\frac{3}{2}}(ax)\,dx\\ &= x\mathrm{Li}_{-\frac{3}{2}}(ax) - x\mathrm{Li}_{-\frac{1}{2}}(ax) + \int \mathrm{Li}_{-\frac{1}{2}}(ax)\,dx\end{aligned}$$

Mathematica [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \mathrm{PolyLog}\left(-\frac{5}{2}, ax\right) dx$$

Verification is not applicable to the result.

[In] Integrate[PolyLog[-5/2, a*x],x]

[Out] Integrate[PolyLog[-5/2, a*x], x]

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \mathrm{polylog}\left(-\frac{5}{2}, ax\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(-5/2,a*x),x)
```

```
[Out] int(polylog(-5/2,a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-5/2,a*x),x, algorithm="maxima")
```

```
[Out] integrate(polylog(-5/2, a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-5/2,a*x),x, algorithm="fricas")
```

```
[Out] integral(polylog(-5/2, a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{Li}_{-\frac{5}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-5/2,a*x),x)
```

```
[Out] Integral(polylog(-5/2, a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-5/2,a*x),x, algorithm="giac")
```

```
[Out] integrate(polylog(-5/2, a*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{polylog}\left(-\frac{5}{2}, a\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(-5/2, a*x),x)
```

```
[Out] int(polylog(-5/2, a*x), x)
```

3.101 $\int \left(\text{PolyLog}\left(-\frac{3}{2},ax\right)+\text{PolyLog}\left(-\frac{1}{2},ax\right)\right)\,dx$

Optimal. Leaf size=9

$$x\text{PolyLog}\left(-\frac{1}{2},ax\right)$$

[Out] x*polylog(-1/2,a*x)

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.067$, Rules used = {6722}

$$x\text{Li}_{-\frac{1}{2}}(ax)$$

Antiderivative was successfully verified.

[In] Int[PolyLog[-3/2, a*x] + PolyLog[-1/2, a*x],x]

[Out] x*PolyLog[-1/2, a*x]

Rule 6722

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*(PolyL
og[n + 1, a*(b*x^p)^q]/(p*q)), x] - Dist[1/(p*q), Int[PolyLog[n + 1, a*(b*x
^p)^q], x], x] /; FreeQ[{a, b, p, q}, x] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \left(\text{Li}_{-\frac{3}{2}}(ax)+\text{Li}_{-\frac{1}{2}}(ax)\right)\,dx &= \int \text{Li}_{-\frac{3}{2}}(ax)\,dx+\int \text{Li}_{-\frac{1}{2}}(ax)\,dx\\ &= x\text{Li}_{-\frac{1}{2}}(ax)\end{aligned}$$

Mathematica [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(\text{PolyLog}\left(-\frac{3}{2},ax\right)+\text{PolyLog}\left(-\frac{1}{2},ax\right)\right)\,dx$$

Verification is not applicable to the result.

[In] Integrate[PolyLog[-3/2, a*x] + PolyLog[-1/2, a*x],x]

[Out] Integrate[PolyLog[-3/2, a*x] + PolyLog[-1/2, a*x], x]

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \operatorname{polylog}\left(-\frac{3}{2}, ax\right) + \operatorname{polylog}\left(-\frac{1}{2}, ax\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(-3/2,a*x)+polylog(-1/2,a*x),x)
[Out] int(polylog(-3/2,a*x)+polylog(-1/2,a*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-3/2,a*x)+polylog(-1/2,a*x),x, algorithm="maxima")
[Out] integrate(polylog(-1/2, a*x) + polylog(-3/2, a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-3/2,a*x)+polylog(-1/2,a*x),x, algorithm="fricas")
[Out] integral(polylog(-1/2, a*x) + polylog(-3/2, a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\operatorname{Li}_{-\frac{3}{2}}\left(ax\right) + \operatorname{Li}_{-\frac{1}{2}}\left(ax\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-3/2,a*x)+polylog(-1/2,a*x),x)
[Out] Integral(polylog(-3/2, a*x) + polylog(-1/2, a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-3/2,a*x)+polylog(-1/2,a*x),x, algorithm="giac")
```

```
[Out] integrate(polylog(-1/2, a*x) + polylog(-3/2, a*x), x)
```

Mupad [B]
time = 0.37, size = 7, normalized size = 0.78

$$x\,\mathrm{polylog}\left(-\frac{1}{2},a\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(-1/2, a*x) + polylog(-3/2, a*x),x)
```

```
[Out] x*polylog(-1/2, a*x)
```

3.102 $\int (dx)^m \textbf{PolyLog}(2, ax)\, dx$

Optimal. Leaf size=78

$$\frac{a(dx)^{2+m}\,{}_2F_1(1,2+m;3+m;ax)}{d^2(1+m)^2(2+m)}+\frac{(dx)^{1+m}\log(1-ax)}{d(1+m)^2}+\frac{(dx)^{1+m}\text{PolyLog}(2,ax)}{d(1+m)}$$

```
[Out] a*(d*x)^(2+m)*hypergeom([1, 2+m],[3+m],a*x)/d^2/(1+m)^2/(2+m)+(d*x)^(1+m)*l
n(-a*x+1)/d/(1+m)^2+(d*x)^(1+m)*polylog(2,a*x)/d/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6726, 2442, 66}

$$\frac{a(dx)^{m+2}\,{}_2F_1(1,m+2;m+3;ax)}{d^2(m+1)^2(m+2)}+\frac{\text{Li}_2(ax)(dx)^{m+1}}{d(m+1)}+\frac{\log(1-ax)(dx)^{m+1}}{d(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[2, a*x],x]
```

```
[Out] (a*(d*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, a*x])/(d^2*(1 + m)^2*(2
 + m)) + ((d*x)^(1 + m)*Log[1 - a*x])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLo
g[2, a*x])/(d*(1 + m))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int (dx)^m \text{Li}_2(ax)\,dx = \frac{(dx)^{1+m}\text{Li}_2(ax)}{d(1+m)} + \frac{\int (dx)^m \log(1-ax)\,dx}{1+m}$$
$$= \frac{(dx)^{1+m}\log(1-ax)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_2(ax)}{d(1+m)} + \frac{a\int \frac{(dx)^{1+m}}{1-ax}\,dx}{d(1+m)^2}$$
$$= \frac{a(dx)^{2+m}\,{}_2F_1(1,2+m;3+m;ax)}{d^2(1+m)^2(2+m)} + \frac{(dx)^{1+m}\log(1-ax)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_2(ax)}{d(1+m)}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.68

$$\frac{x(dx)^m(ax\,{}_2F_1(1,2+m;3+m;ax)+(2+m)(\log(1-ax)+(1+m)\text{PolyLog}(2,ax)))}{(1+m)^2(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*PolyLog[2, a*x],x]
```

```
[Out] (x*(d*x)^m*(a*x*Hypergeometric2F1[1, 2 + m, 3 + m, a*x] + (2 + m)*(Log[1 -
a*x] + (1 + m)*PolyLog[2, a*x])))/((1 + m)^2*(2 + m))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.12, size = 144, normalized size = 1.85

method	result
meijerg	$\frac{(dx)^m x^{-m}(-a)^{-m}\left(\frac{x^m(-a)^m\left(-a\,m^2x-2amx-m^2-3m-2\right)}{(2+m)(1+m)^3m}-\frac{x^{1+m}a(-a)^m(-m-2)\ln(-ax+1)}{(2+m)(1+m)^2}+\frac{x^{1+m}a(-a)^m\,\text{polylog}(2,ax)}{1+m}+\frac{x^m(-a)^m}{(1+}\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(2,a*x),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x)^m*x^(-m)*(-a)^(-m)/a*(1/(2+m)*x^m*(-a)^m*(-a*m^2*x-2*a*m*x-m^2-3*m-2)
/(1+m)^3/m-1/(2+m)*x^(1+m)*a*(-a)^m*(-m-2)/(1+m)^2*ln(-a*x+1)+x^(1+m)*a*(-a
)^m/(1+m)*polylog(2,a*x)+x^m*(-a)^m/(1+m)^2*LerchPhi(a*x,1,m))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x),x, algorithm="maxima")
```

```
[Out] -a*d^m*integrate(-x*x^m/(m^2 - (a*m^2 + 2*a*m + a)*x + 2*m + 1), x) + ((d^m
*m + d^m)*x*x^m*dilog(a*x) + d^m*x*x^m*log(-a*x + 1))/(m^2 + 2*m + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*dilog(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \operatorname{Li}_2(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(2,a*x),x)
```

```
[Out] Integral((d*x)**m*polylog(2, a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*dilog(a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\mathrm{polylog}(2,a\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(2, a*x),x)
```

```
[Out] int((d*x)^m*polylog(2, a*x), x)
```

3.103 $\int (dx)^m \textbf{PolyLog}(3, ax)\, dx$

Optimal. Leaf size=102

$$-\frac{a(dx)^{2+m}\,{}_2F_1(1,2+m;3+m;ax)}{d^2(1+m)^3(2+m)}-\frac{(dx)^{1+m}\log(1-ax)}{d(1+m)^3}-\frac{(dx)^{1+m}\text{PolyLog}(2,ax)}{d(1+m)^2}+\frac{(dx)^{1+m}\text{PolyLog}(3,ax)}{d(1+m)}$$

```
[Out] -a*(d*x)^(2+m)*hypergeom([1, 2+m],[3+m],a*x)/d^2/(1+m)^3/(2+m)-(d*x)^(1+m)*
ln(-a*x+1)/d/(1+m)^3-(d*x)^(1+m)*polylog(2,a*x)/d/(1+m)^2+(d*x)^(1+m)*polyl
og(3,a*x)/d/(1+m)
```

Rubi [A]
time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6726, 2442, 66}

$$-\frac{a(dx)^{m+2}\,{}_2F_1(1,m+2;m+3;ax)}{d^2(m+1)^3(m+2)}-\frac{\text{Li}_2(ax)(dx)^{m+1}}{d(m+1)^2}+\frac{\text{Li}_3(ax)(dx)^{m+1}}{d(m+1)}-\frac{\log(1-ax)(dx)^{m+1}}{d(m+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[3, a*x],x]
```

```
[Out] -((a*(d*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, a*x])/(d^2*(1 + m)^3*
(2 + m))) - ((d*x)^(1 + m)*Log[1 - a*x])/(d*(1 + m)^3) - ((d*x)^(1 + m)*Pol
yLog[2, a*x])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[3, a*x])/(d*(1 + m))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
```

```
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \text{Li}_3(ax)\, dx &= \frac{(dx)^{1+m}\text{Li}_3(ax)}{d(1+m)} - \frac{\int (dx)^m \text{Li}_2(ax)\, dx}{1+m}\\ &= -\frac{(dx)^{1+m}\text{Li}_2(ax)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_3(ax)}{d(1+m)} - \frac{\int (dx)^m \log(1-ax)\, dx}{(1+m)^2}\\ &= -\frac{(dx)^{1+m}\log(1-ax)}{d(1+m)^3} - \frac{(dx)^{1+m}\text{Li}_2(ax)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_3(ax)}{d(1+m)} - \frac{a\int \frac{(dx)^{1+m}}{1-ax}\, dx}{d(1+m)^3}\\ &= -\frac{a(dx)^{2+m}\,{}_2F_1(1,2+m;3+m;ax)}{d^2(1+m)^3(2+m)} - \frac{(dx)^{1+m}\log(1-ax)}{d(1+m)^3} - \frac{(dx)^{1+m}\text{Li}_2(ax)}{d(1+m)^2} + \frac{(dx)}{d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.04, size = 88, normalized size = 0.86

$$-\frac{x(dx)^m\Gamma(2+m)\left(a(1+m)x\Gamma(1+m)\,{}_2\tilde{F}_1(1,2+m;3+m;ax)+\log(1-ax)+(1+m)\text{PolyLog}(2,ax)-\text{PolyLog}(3,ax)-2m\text{PolyLog}(3,ax)-m^2\text{PolyLog}(3,ax)\right)}{(1+m)^4\Gamma(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*PolyLog[3, a*x],x]
```

```
[Out] -((x*(d*x)^m*Gamma[2 + m]*(a*(1 + m)*x*Gamma[1 + m]*HypergeometricPFQRegula
rized[{1, 2 + m}, {3 + m}, a*x] + Log[1 - a*x] + (1 + m)*PolyLog[2, a*x] -
PolyLog[3, a*x] - 2*m*PolyLog[3, a*x] - m^2*PolyLog[3, a*x]))/((1 + m)^4*Ga
mma[1 + m]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.22, size = 173, normalized size = 1.70

method	result
meijerg	$\frac{(dx)^m x^{-m}(-a)^{-m}\left(\frac{x^m(-a)^m\left(a\,m^2x+2amx+m^2+3m+2\right)}{(2+m)(1+m)^4 m}-\frac{x^{1+m}a(-a)^m\ln(-ax+1)}{(1+m)^3}+\frac{x^{1+m}a(-a)^m(-m-2)\operatorname{polylog}(2,ax)}{(2+m)(1+m)^2}+\frac{x^{1+m}a(-a)^m}{1+}\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(3,a*x),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x)^m*x^(-m)*(-a)^(-m)/a*(1/(2+m)*x^m*(-a)^m*(a*m^2*x+2*a*m*x+m^2+3*m+2)/
(1+m)^4/m-x^(1+m)*a*(-a)^m/(1+m)^3*ln(-a*x+1)+1/(2+m)*x^(1+m)*a*(-a)^m*(-m-
2)/(1+m)^2*polylog(2,a*x)+x^(1+m)*a*(-a)^m/(1+m)*polylog(3,a*x)+1/(2+m)*x^m
*(-a)^m*(-m-2)/(1+m)^3*LerchPhi(a*x,1,m))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x),x, algorithm="maxima")
[Out] a*d^m*integrate(-x*x^m/(m^3 - (m^3 + 3*m^2 + 3*m + 1)*a*x + 3*m^2 + 3*m + 1
), x) - (d^m*(m + 1)*x*x^m*dilog(a*x) - (m^2 + 2*m + 1)*d^m*x*x^m*polylog(3
, a*x) + d^m*x*x^m*log(-a*x + 1))/(m^3 + 3*m^2 + 3*m + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x),x, algorithm="fricas")
[Out] integral((d*x)^m*polylog(3, a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \operatorname{Li}_3(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(3,a*x),x)
[Out] Integral((d*x)**m*polylog(3, a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x),x, algorithm="giac")
[Out] integrate((d*x)^m*polylog(3, a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\mathrm{polylog}(3,a\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(3, a*x),x)
[Out] int((d*x)^m*polylog(3, a*x), x)
```

3.104 $\int (dx)^m \textbf{PolyLog}(4, ax)\, dx$

Optimal. Leaf size=121

$$\frac{a(dx)^{2+m}\,{}_2F_1(1,2+m;3+m;ax)}{d^2(1+m)^4(2+m)}+\frac{(dx)^{1+m}\log(1-ax)}{d(1+m)^4}+\frac{(dx)^{1+m}\text{PolyLog}(2,ax)}{d(1+m)^3}-\frac{(dx)^{1+m}\text{PolyLog}(3,ax)}{d(1+m)^2}$$

```
[Out] a*(d*x)^(2+m)*hypergeom([1, 2+m],[3+m],a*x)/d^2/(1+m)^4/(2+m)+(d*x)^(1+m)*l
n(-a*x+1)/d/(1+m)^4+(d*x)^(1+m)*polylog(2,a*x)/d/(1+m)^3-(d*x)^(1+m)*polylo
g(3,a*x)/d/(1+m)^2+(d*x)^(1+m)*polylog(4,a*x)/d/(1+m)
```

Rubi [A]
time = 0.06, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6726, 2442, 66}

$$\frac{a(dx)^{m+2}\,{}_2F_1(1,m+2;m+3;ax)}{d^2(m+1)^4(m+2)}+\frac{\text{Li}_2(ax)(dx)^{m+1}}{d(m+1)^3}-\frac{\text{Li}_3(ax)(dx)^{m+1}}{d(m+1)^2}+\frac{\text{Li}_4(ax)(dx)^{m+1}}{d(m+1)}+\frac{\log(1-ax)(dx)^{m+1}}{d(m+1)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[4, a*x],x]
```

```
[Out] (a*(d*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, a*x])/(d^2*(1 + m)^4*(2
 + m)) + ((d*x)^(1 + m)*Log[1 - a*x])/(d*(1 + m)^4) + ((d*x)^(1 + m)*PolyLo
g[2, a*x])/(d*(1 + m)^3) - ((d*x)^(1 + m)*PolyLog[3, a*x])/(d*(1 + m)^2) +
((d*x)^(1 + m)*PolyLog[4, a*x])/(d*(1 + m))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
```

```
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int (dx)^m \text{Li}_4(ax)\,dx &= \frac{(dx)^{1+m}\text{Li}_4(ax)}{d(1+m)} - \frac{\int (dx)^m \text{Li}_3(ax)\,dx}{1+m} \\
&= -\frac{(dx)^{1+m}\text{Li}_3(ax)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4(ax)}{d(1+m)} + \frac{\int (dx)^m \text{Li}_2(ax)\,dx}{(1+m)^2} \\
&= \frac{(dx)^{1+m}\text{Li}_2(ax)}{d(1+m)^3} - \frac{(dx)^{1+m}\text{Li}_3(ax)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4(ax)}{d(1+m)} + \frac{\int (dx)^m \log(1-ax)\,dx}{(1+m)^3} \\
&= \frac{(dx)^{1+m}\log(1-ax)}{d(1+m)^4} + \frac{(dx)^{1+m}\text{Li}_2(ax)}{d(1+m)^3} - \frac{(dx)^{1+m}\text{Li}_3(ax)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4(ax)}{d(1+m)} + \frac{a\int (}{d(} \\
&= \frac{a(dx)^{2+m}\,{}_2F_1(1,2+m;3+m;ax)}{d^2(1+m)^4(2+m)} + \frac{(dx)^{1+m}\log(1-ax)}{d(1+m)^4} + \frac{(dx)^{1+m}\text{Li}_2(ax)}{d(1+m)^3} - \frac{(dx)}{d(}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.05, size = 119, normalized size = 0.98

$$
\frac{x(dx)^m\Gamma(2+m)\left(a(1+m)x\Gamma(1+m)\,{}_2\tilde{F}_1(1,2+m;3+m;ax)+\log(1-ax)+(1+m)\text{PolyLog}(2,ax)-\text{PolyLog}(3,ax)-2m\text{PolyLog}(3,ax)-m^2\text{PolyLog}(3,ax)+\text{PolyLog}(4,ax)+3m\text{PolyLog}(4,ax)+3m^2\text{PolyLog}(4,ax)+m^3\text{PolyLog}(4,ax)\right)}{(1+m)^5\Gamma(1+m)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*PolyLog[4, a*x],x]
```

```
[Out] (x*(d*x)^m*Gamma[2 + m]*(a*(1 + m)*x*Gamma[1 + m]*HypergeometricPFQRegulari
zed[{1, 2 + m}, {3 + m}, a*x] + Log[1 - a*x] + (1 + m)*PolyLog[2, a*x] - Po
lyLog[3, a*x] - 2*m*PolyLog[3, a*x] - m^2*PolyLog[3, a*x] + PolyLog[4, a*x]
 + 3*m*PolyLog[4, a*x] + 3*m^2*PolyLog[4, a*x] + m^3*PolyLog[4, a*x]))/((1
+ m)^5*Gamma[1 + m])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.45, size = 198, normalized size = 1.64

method	result
meijerg	$\frac{(dx)^m x^{-m}(-a)^{-m}\left(\frac{x^m(-a)^m\left(-a\,m^2x-2amx-m^2-3m-2\right)}{(2+m)(1+m)^5 m}-\frac{x^{1+m}a(-a)^m(-m-2)\ln(-ax+1)}{(2+m)(1+m)^4}+\frac{x^{1+m}a(-a)^m\,\text{polylog}(2,ax)}{(1+m)^3}+\frac{x^{1+m}a(-}{}\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(4,a*x),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x)^m*x^(-m)*(-a)^(-m)/a*(1/(2+m)*x^m*(-a)^m*(-a*m^2*x-2*a*m*x-m^2-3*m-2)
/(1+m)^5/m-1/(2+m)*x^(1+m)*a*(-a)^m*(-m-2)/(1+m)^4*ln(-a*x+1)+x^(1+m)*a*(-a
)^m/(1+m)^3*polylog(2,a*x)+1/(2+m)*x^(1+m)*a*(-a)^m*(-m-2)/(1+m)^2*polylog(
3,a*x)+x^(1+m)*a*(-a)^m/(1+m)*polylog(4,a*x)+x^m*(-a)^m/(1+m)^4*LerchPhi(a*
x,1,m))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x),x, algorithm="maxima")
```

```
[Out] -a*d^m*integrate(-x*x^m/(m^4 + 4*m^3 + 6*m^2 - (a*m^4 + 4*a*m^3 + 6*a*m^2 +
 4*a*m + a)*x + 4*m + 1), x) + ((d^m*m + d^m)*x*x^m*dilog(a*x) + d^m*x*x^m*
log(-a*x + 1) + (d^m*m^3 + 3*d^m*m^2 + 3*d^m*m + d^m)*x*x^m*polylog(4, a*x)
 - (d^m*m^2 + 2*d^m*m + d^m)*x*x^m*polylog(3, a*x))/(m^4 + 4*m^3 + 6*m^2 +
4*m + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*polylog(4, a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \operatorname{Li}_4(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(4,a*x),x)
```

```
[Out] Integral((d*x)**m*polylog(4, a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x),x, algorithm="giac")
[Out] integrate((d*x)^m*polylog(4, a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\mathrm{polylog}(4,a\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(4, a*x),x)
[Out] int((d*x)^m*polylog(4, a*x), x)
```

3.105 $\int (dx)^m \mathbf{PolyLog}(2, ax^2)\, dx$

Optimal. Leaf size=94

$$\frac{4a(dx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};ax^2\right)}{d^3(1+m)^2(3+m)}+\frac{2(dx)^{1+m}\log\left(1-ax^2\right)}{d(1+m)^2}+\frac{(dx)^{1+m}\mathrm{PolyLog}(2,ax^2)}{d(1+m)}$$

```
[Out] 4*a*(d*x)^(3+m)*hypergeom([1, 3/2+1/2*m],[5/2+1/2*m],a*x^2)/d^3/(1+m)^2/(3+
m)+2*(d*x)^(1+m)*ln(-a*x^2+1)/d/(1+m)^2+(d*x)^(1+m)*polylog(2,a*x^2)/d/(1+m
)
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6726, 2505, 16, 371}

$$\frac{4a(dx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};ax^2\right)}{d^3(m+1)^2(m+3)}+\frac{\mathrm{Li}_2\left(ax^2\right)(dx)^{m+1}}{d(m+1)}+\frac{2\log\left(1-ax^2\right)(dx)^{m+1}}{d(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[2, a*x^2],x]
```

```
[Out] (4*a*(d*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, a*x^2])/(d^3*
(1 + m)^2*(3 + m)) + (2*(d*x)^(1 + m)*Log[1 - a*x^2])/(d*(1 + m)^2) + ((d*x
)^(1 + m)*PolyLog[2, a*x^2])/(d*(1 + m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int (dx)^m \mathrm{Li}_2(ax^2)\, dx &= \frac{(dx)^{1+m}\mathrm{Li}_2(ax^2)}{d(1+m)} + \frac{2\int (dx)^m \log(1-ax^2)\, dx}{1+m} \\
&= \frac{2(dx)^{1+m}\log(1-ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_2(ax^2)}{d(1+m)} + \frac{(4a)\int \frac{x(dx)^{1+m}}{1-ax^2}\, dx}{d(1+m)^2} \\
&= \frac{2(dx)^{1+m}\log(1-ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_2(ax^2)}{d(1+m)} + \frac{(4a)\int \frac{(dx)^{2+m}}{1-ax^2}\, dx}{d^2(1+m)^2} \\
&= \frac{4a(dx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};ax^2\right)}{d^3(1+m)^2(3+m)} + \frac{2(dx)^{1+m}\log(1-ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_2(ax^2)}{d(1+m)}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 72, normalized size = 0.77

$$\frac{x(dx)^m\left(4ax^2\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};ax^2\right)+(3+m)\left(2\log(1-ax^2)+(1+m)\mathrm{PolyLog}(2,ax^2)\right)\right)}{(1+m)^2(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*PolyLog[2, a*x^2],x]
```

```
[Out] (x*(d*x)^m*(4*a*x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, a*x^2] + (3
+ m)*(2*Log[1 - a*x^2] + (1 + m)*PolyLog[2, a*x^2])))/((1 + m)^2*(3 + m))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.12, size = 177, normalized size = 1.88

method	result
meijerg	$-\frac{(dx)^m x^{-m}(-a)^{-\frac{1}{2}-\frac{m}{2}}\left(\frac{2x^{1+m}(-a)^{\frac{3}{2}+\frac{m}{2}}(-12-4m)}{(3+m)(1+m)^3 a}-\frac{2x^{1+m}(-a)^{\frac{3}{2}+\frac{m}{2}}(-6-2m)\ln\left(-a\,x^2+1\right)}{(3+m)(1+m)^2 a}+\frac{2x^{1+m}(-a)^{\frac{3}{2}+\frac{m}{2}}\,\mathrm{polylog}\left(2,a\,x^2\right)}{(1+m)a}+\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(2,a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(d*x)^m*x^(-m)*(-a)^(-1/2-1/2*m)*(2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(-1
2-4*m)/(1+m)^3/a-2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(-6-2*m)/(1+m)^2/a*ln(-a*
```

```
x^2+1)+2*x^(1+m)*(-a)^(3/2+1/2*m)/(1+m)/a*polylog(2,a*x^2)+2/(3+m)*x^(1+m)*
(-a)^(3/2+1/2*m)*(6+2*m)/(1+m)^2/a*LerchPhi(a*x^2,1,1/2+1/2*m))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x^2),x, algorithm="maxima")
```

```
[Out] -4*a*d^m*integrate(x^2*x^m/((a*m^2 + 2*a*m + a)*x^2 - m^2 - 2*m - 1), x) +
((d^m*m + d^m)*x*x^m*dilog(a*x^2) + 2*d^m*x*x^m*log(-a*x^2 + 1))/(m^2 + 2*m
 + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x^2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*dilog(a*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \operatorname{Li}_2(ax^2)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(2,a*x**2),x)
```

```
[Out] Integral((d*x)**m*polylog(2, a*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x^2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*dilog(a*x^2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{polylog}\left(2,a\,x^2\right)\,{\left(d\,x\right)}^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^2)*(d*x)^m,x)
```

```
[Out] int(polylog(2, a*x^2)*(d*x)^m, x)
```

3.106 $\int (dx)^m \text{PolyLog}(3, ax^2)\, dx$

Optimal. Leaf size=118

$$-\frac{8a(dx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};ax^2\right)}{d^3(1+m)^3(3+m)}-\frac{4(dx)^{1+m}\log\left(1-ax^2\right)}{d(1+m)^3}-\frac{2(dx)^{1+m}\text{PolyLog}(2,ax^2)}{d(1+m)^2}+\frac{(dx)^{1+m}\text{PolyLog}(3,ax^2)}{d(1+m)}$$

```
[Out] -8*a*(d*x)^(3+m)*hypergeom([1, 3/2+1/2*m],[5/2+1/2*m],a*x^2)/d^3/(1+m)^3/(3
+m)-4*(d*x)^(1+m)*ln(-a*x^2+1)/d/(1+m)^3-2*(d*x)^(1+m)*polylog(2,a*x^2)/d/(
1+m)^2+(d*x)^(1+m)*polylog(3,a*x^2)/d/(1+m)
```

Rubi [A]
time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6726, 2505, 16, 371}

$$-\frac{8a(dx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};ax^2\right)}{d^3(m+1)^3(m+3)}-\frac{2\text{Li}_2\left(ax^2\right)(dx)^{m+1}}{d(m+1)^2}+\frac{\text{Li}_3\left(ax^2\right)(dx)^{m+1}}{d(m+1)}-\frac{4\log\left(1-ax^2\right)(dx)^{m+1}}{d(m+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[3, a*x^2],x]
```

```
[Out] (-8*a*(d*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, a*x^2])/(d^3
*(1 + m)^3*(3 + m)) - (4*(d*x)^(1 + m)*Log[1 - a*x^2])/(d*(1 + m)^3) - (2*(
d*x)^(1 + m)*PolyLog[2, a*x^2])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[3, a
*x^2])/(d*(1 + m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \text{Li}_3\left(ax^2\right)\,dx &= \frac{(dx)^{1+m}\text{Li}_3(ax^2)}{d(1+m)} - \frac{2\int (dx)^m \text{Li}_2(ax^2)\,dx}{1+m} \\
&= -\frac{2(dx)^{1+m}\text{Li}_2(ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_3(ax^2)}{d(1+m)} - \frac{4\int (dx)^m \log\left(1-ax^2\right)\,dx}{(1+m)^2} \\
&= -\frac{4(dx)^{1+m}\log\left(1-ax^2\right)}{d(1+m)^3} - \frac{2(dx)^{1+m}\text{Li}_2(ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_3(ax^2)}{d(1+m)} - \frac{(8a)\int \frac{x(dx)^{1+m}}{1-ax^2}}{d(1+m)^3} \\
&= -\frac{4(dx)^{1+m}\log\left(1-ax^2\right)}{d(1+m)^3} - \frac{2(dx)^{1+m}\text{Li}_2(ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_3(ax^2)}{d(1+m)} - \frac{(8a)\int \frac{(dx)^{2+m}}{1-ax^2}}{d^2(1+m)^3} \\
&= -\frac{8a(dx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};ax^2\right)}{d^3(1+m)^3(3+m)} - \frac{4(dx)^{1+m}\log\left(1-ax^2\right)}{d(1+m)^3} - \frac{2(dx)^{1+m}\text{Li}_2(ax^2)}{d(1+m)^2}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.06, size = 126, normalized size = 1.07

$$-\frac{2x(dx)^m\Gamma\left(\frac{3+m}{2}\right)\left(2a(1+m)x^2\Gamma\left(\frac{1+m}{2}\right)\,{}_2\tilde{F}_1\left(1,\frac{3+m}{2};\frac{5+m}{2};ax^2\right)+4\log\left(1-ax^2\right)+2(1+m)\text{PolyLog}(2,ax^2)-\text{PolyLog}(3,ax^2)-2m\text{PolyLog}(3,ax^2)-m^2\text{PolyLog}(3,ax^2)\right)}{(1+m)^4\Gamma\left(\frac{1+m}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*PolyLog[3, a*x^2],x]
```

```
[Out] (-2*x*(d*x)^m*Gamma[(3 + m)/2]*(2*a*(1 + m)*x^2*Gamma[(1 + m)/2]*Hypergeome
tricPFQRegularized[{1, (3 + m)/2}, {(5 + m)/2}, a*x^2] + 4*Log[1 - a*x^2] +
 2*(1 + m)*PolyLog[2, a*x^2] - PolyLog[3, a*x^2] - 2*m*PolyLog[3, a*x^2] -
m^2*PolyLog[3, a*x^2]))/((1 + m)^4*Gamma[(1 + m)/2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.23, size = 218, normalized size = 1.85

method	result

meijerg	$-\frac{(dx)^{m} x^{-m} (-a)^{-\frac{1}{2}-\frac{m}{2}} \left(\frac{2x^{1+m} (-a)^{\frac{3}{2}+\frac{m}{2}} (24+8m)}{(3+m)(1+m)^{4} a} - \frac{2x^{1+m} (-a)^{\frac{3}{2}+\frac{m}{2}} (12+4m) \ln\left(-a\,x^{2}+1\right)}{(3+m)(1+m)^{3} a} + \frac{2x^{1+m} (-a)^{\frac{3}{2}+\frac{m}{2}} (-6-2m) \operatorname{polylog}\left(2,a\,x^{2}\right.}{(3+m)(1+m)^{2} a}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(3,a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(d*x)^m*x^(-m)*(-a)^(-1/2-1/2*m)*(2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(24
+8*m)/(1+m)^4/a-2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(12+4*m)/(1+m)^3/a*ln(-a*x
^2+1)+2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(-6-2*m)/(1+m)^2/a*polylog(2,a*x^2)+
2*x^(1+m)*(-a)^(3/2+1/2*m)/(1+m)/a*polylog(3,a*x^2)+2/(3+m)*x^(1+m)*(-a)^(3
/2+1/2*m)*(-12-4*m)/(1+m)^3/a*LerchPhi(a*x^2,1,1/2+1/2*m))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x^2),x, algorithm="maxima")
```

```
[Out] 8*a*d^m*integrate(x^2*x^m/((m^3 + 3*m^2 + 3*m + 1)*a*x^2 - m^3 - 3*m^2 - 3*
m - 1), x) - (2*d^m*(m + 1)*x*x^m*dilog(a*x^2) - (m^2 + 2*m + 1)*d^m*x*x^m*
polylog(3, a*x^2) + 4*d^m*x*x^m*log(-a*x^2 + 1))/(m^3 + 3*m^2 + 3*m + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x^2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*polylog(3, a*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{m} \operatorname{Li}_{3}\left(ax^{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(3,a*x**2),x)
```

```
[Out] Integral((d*x)**m*polylog(3, a*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x^2),x, algorithm="giac")
[Out] integrate((d*x)^m*polylog(3, a*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{polylog}\left(3,a\,x^2\right)\,{\left(d\,x\right)}^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^2)*(d*x)^m,x)
[Out] int(polylog(3, a*x^2)*(d*x)^m, x)
```

3.107 $\int (dx)^m \textbf{PolyLog}(4, ax^2)\, dx$

Optimal. Leaf size=142

$$\frac{16a(dx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};ax^2\right)}{d^3(1+m)^4(3+m)}+\frac{8(dx)^{1+m}\log\left(1-ax^2\right)}{d(1+m)^4}+\frac{4(dx)^{1+m}\text{PolyLog}(2,ax^2)}{d(1+m)^3}-\frac{2(dx)^{1+m}\text{PolyLog}(3}{d(1+m)^2}$$

```
[Out] 16*a*(d*x)^(3+m)*hypergeom([1, 3/2+1/2*m],[5/2+1/2*m],a*x^2)/d^3/(1+m)^4/(3
+m)+8*(d*x)^(1+m)*ln(-a*x^2+1)/d/(1+m)^4+4*(d*x)^(1+m)*polylog(2,a*x^2)/d/(
1+m)^3-2*(d*x)^(1+m)*polylog(3,a*x^2)/d/(1+m)^2+(d*x)^(1+m)*polylog(4,a*x^2
)/d/(1+m)
```

Rubi [A]
time = 0.07, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6726, 2505, 16, 371}

$$\frac{16a(dx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};ax^2\right)}{d^3(m+1)^4(m+3)}+\frac{4\text{Li}_2(ax^2)\,(dx)^{m+1}}{d(m+1)^3}-\frac{2\text{Li}_3(ax^2)\,(dx)^{m+1}}{d(m+1)^2}+\frac{\text{Li}_4(ax^2)\,(dx)^{m+1}}{d(m+1)}+\frac{8\log\left(1-ax^2\right)(dx)^{m+1}}{d(m+1)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[4, a*x^2],x]
```

```
[Out] (16*a*(d*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, a*x^2])/(d^3
*(1 + m)^4*(3 + m)) + (8*(d*x)^(1 + m)*Log[1 - a*x^2])/(d*(1 + m)^4) + (4*(
d*x)^(1 + m)*PolyLog[2, a*x^2])/(d*(1 + m)^3) - (2*(d*x)^(1 + m)*PolyLog[3,
 a*x^2])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[4, a*x^2])/(d*(1 + m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \mathrm{Li}_4(ax^2)\,dx &= \frac{(dx)^{1+m}\mathrm{Li}_4(ax^2)}{d(1+m)} - \frac{2\int (dx)^m \mathrm{Li}_3(ax^2)\,dx}{1+m}\\
&= -\frac{2(dx)^{1+m}\mathrm{Li}_3(ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_4(ax^2)}{d(1+m)} + \frac{4\int (dx)^m \mathrm{Li}_2(ax^2)\,dx}{(1+m)^2}\\
&= \frac{4(dx)^{1+m}\mathrm{Li}_2(ax^2)}{d(1+m)^3} - \frac{2(dx)^{1+m}\mathrm{Li}_3(ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_4(ax^2)}{d(1+m)} + \frac{8\int (dx)^m \log(1-ax^2}{(1+m)^3}\\
&= \frac{8(dx)^{1+m}\log(1-ax^2)}{d(1+m)^4} + \frac{4(dx)^{1+m}\mathrm{Li}_2(ax^2)}{d(1+m)^3} - \frac{2(dx)^{1+m}\mathrm{Li}_3(ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_4(ax}{d(1+m)}\\
&= \frac{8(dx)^{1+m}\log(1-ax^2)}{d(1+m)^4} + \frac{4(dx)^{1+m}\mathrm{Li}_2(ax^2)}{d(1+m)^3} - \frac{2(dx)^{1+m}\mathrm{Li}_3(ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_4(ax}{d(1+m)}\\
&= \frac{16a(dx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};ax^2\right)}{d^3(1+m)^4(3+m)} + \frac{8(dx)^{1+m}\log(1-ax^2)}{d(1+m)^4} + \frac{4(dx)^{1+m}\mathrm{Li}_2(ax^2)}{d(1+m)^3} -\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.07, size = 166, normalized size = 1.17

$$\frac{2x(dx)^m\Gamma\left(\frac{3+m}{2}\right)\left(4a(1+m)x^2\Gamma\left(\frac{1+m}{2}\right)\,{}_2\tilde{F}_1\left(1,\frac{3+m}{2};\frac{5+m}{2};ax^2\right)+8\log(1-ax^2)+4(1+m)\mathrm{PolyLog}(2,ax^2)-2\mathrm{PolyLog}(3,ax^2)-4m\mathrm{PolyLog}(3,ax^2)-2m^2\mathrm{PolyLog}(3,ax^2)+\mathrm{PolyLog}(4,ax^2)+3m\mathrm{PolyLog}(4,ax^2)+3m^2\mathrm{PolyLog}(4,ax^2)+m^3\mathrm{PolyLog}(4,ax^2)\right)}{(1+m)^5\Gamma\left(\frac{1+m}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*PolyLog[4, a*x^2],x]
```

```
[Out] (2*x*(d*x)^m*Gamma[(3 + m)/2]*(4*a*(1 + m)*x^2*Gamma[(1 + m)/2]*Hypergeomet
ricPFQRegularized[{1, (3 + m)/2}, {(5 + m)/2}, a*x^2] + 8*Log[1 - a*x^2] +
4*(1 + m)*PolyLog[2, a*x^2] - 2*PolyLog[3, a*x^2] - 4*m*PolyLog[3, a*x^2] -
 2*m^2*PolyLog[3, a*x^2] + PolyLog[4, a*x^2] + 3*m*PolyLog[4, a*x^2] + 3*m^
2*PolyLog[4, a*x^2] + m^3*PolyLog[4, a*x^2]))/((1 + m)^5*Gamma[(1 + m)/2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.48, size = 259, normalized size = 1.82

method	result
meijerg	$-\frac{(dx)^m x^{-m}(-a)^{-\frac{1}{2}-\frac{m}{2}}\left(\frac{2x^{1+m}(-a)^{\frac{3}{2}+\frac{m}{2}}(-48-16m)}{(3+m)(1+m)^5 a}-\frac{2x^{1+m}(-a)^{\frac{3}{2}+\frac{m}{2}}(-24-8m)\ln\left(-a\,x^2+1\right)}{(3+m)(1+m)^4 a}+\frac{2x^{1+m}(-a)^{\frac{3}{2}+\frac{m}{2}}(12+4m)\operatorname{polylog}\left(2\right.}{(3+m)(1+m)^3 a}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(4,a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(d*x)^m*x^(-m)*(-a)^(-1/2-1/2*m)*(2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(-4
8-16*m)/(1+m)^5/a-2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(-24-8*m)/(1+m)^4/a*ln(-
a*x^2+1)+2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(12+4*m)/(1+m)^3/a*polylog(2,a*x^
2)+2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(-6-2*m)/(1+m)^2/a*polylog(3,a*x^2)+2*x
^(1+m)*(-a)^(3/2+1/2*m)/(1+m)/a*polylog(4,a*x^2)+2/(3+m)*x^(1+m)*(-a)^(3/2+
1/2*m)*(24+8*m)/(1+m)^4/a*LerchPhi(a*x^2,1,1/2+1/2*m))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x^2),x, algorithm="maxima")
```

```
[Out] -16*a*d^m*integrate(-x^2*x^m/(m^4 + 4*m^3 - (a*m^4 + 4*a*m^3 + 6*a*m^2 + 4*
a*m + a)*x^2 + 6*m^2 + 4*m + 1), x) + (4*(d^m*m + d^m)*x*x^m*dilog(a*x^2) +
 8*d^m*x*x^m*log(-a*x^2 + 1) + (d^m*m^3 + 3*d^m*m^2 + 3*d^m*m + d^m)*x*x^m*
polylog(4, a*x^2) - 2*(d^m*m^2 + 2*d^m*m + d^m)*x*x^m*polylog(3, a*x^2))/(m
^4 + 4*m^3 + 6*m^2 + 4*m + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x^2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*polylog(4, a*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \operatorname{Li}_4\left(ax^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(4,a*x**2),x)
```

```
[Out] Integral((d*x)**m*polylog(4, a*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x^2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*polylog(4, a*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{polylog}\left(4,a\,x^2\right)\,{\left(d\,x\right)}^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(4, a*x^2)*(d*x)^m,x)
```

```
[Out] int(polylog(4, a*x^2)*(d*x)^m, x)
```

3.108 $\int (dx)^m \textbf{PolyLog}(2, ax^3)\, dx$

Optimal. Leaf size=94

$$\frac{9a(dx)^{4+m}\,{}_2F_1\left(1,\frac{4+m}{3};\frac{7+m}{3};ax^3\right)}{d^4(1+m)^2(4+m)}+\frac{3(dx)^{1+m}\log\left(1-ax^3\right)}{d(1+m)^2}+\frac{(dx)^{1+m}\text{PolyLog}(2,ax^3)}{d(1+m)}$$

```
[Out] 9*a*(d*x)^(4+m)*hypergeom([1, 4/3+1/3*m],[7/3+1/3*m],a*x^3)/d^4/(1+m)^2/(4+
m)+3*(d*x)^(1+m)*ln(-a*x^3+1)/d/(1+m)^2+(d*x)^(1+m)*polylog(2,a*x^3)/d/(1+m
)
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6726, 2505, 16, 371}

$$\frac{9a(dx)^{m+4}\,{}_2F_1\left(1,\frac{m+4}{3};\frac{m+7}{3};ax^3\right)}{d^4(m+1)^2(m+4)}+\frac{\text{Li}_2\left(ax^3\right)(dx)^{m+1}}{d(m+1)}+\frac{3\log\left(1-ax^3\right)(dx)^{m+1}}{d(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[2, a*x^3],x]
```

```
[Out] (9*a*(d*x)^(4 + m)*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, a*x^3])/(d^4*
(1 + m)^2*(4 + m)) + (3*(d*x)^(1 + m)*Log[1 - a*x^3])/(d*(1 + m)^2) + ((d*x
)^(1 + m)*PolyLog[2, a*x^3])/(d*(1 + m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \text{Li}_2(ax^3)\,dx &= \frac{(dx)^{1+m}\text{Li}_2(ax^3)}{d(1+m)} + \frac{3\int (dx)^m \log(1-ax^3)\,dx}{1+m}\\ &= \frac{3(dx)^{1+m}\log(1-ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_2(ax^3)}{d(1+m)} + \frac{(9a)\int \frac{x^2(dx)^{1+m}}{1-ax^3}\,dx}{d(1+m)^2}\\ &= \frac{3(dx)^{1+m}\log(1-ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_2(ax^3)}{d(1+m)} + \frac{(9a)\int \frac{(dx)^{3+m}}{1-ax^3}\,dx}{d^3(1+m)^2}\\ &= \frac{9a(dx)^{4+m}\,{}_2F_1\left(1,\frac{4+m}{3};\frac{7+m}{3};ax^3\right)}{d^4(1+m)^2(4+m)} + \frac{3(dx)^{1+m}\log(1-ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_2(ax^3)}{d(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 72, normalized size = 0.77

$$\frac{x(dx)^m\left(9ax^3\,{}_2F_1\left(1,\frac{4+m}{3};\frac{7+m}{3};ax^3\right)+(4+m)\left(3\log\left(1-ax^3\right)+(1+m)\text{PolyLog}(2,ax^3)\right)\right)}{(1+m)^2(4+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*PolyLog[2, a*x^3],x]
```

```
[Out] (x*(d*x)^m*(9*a*x^3*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, a*x^3] + (4
+ m)*(3*Log[1 - a*x^3] + (1 + m)*PolyLog[2, a*x^3])))/((1 + m)^2*(4 + m))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.16, size = 177, normalized size = 1.88

method	result
meijerg	$-\frac{(dx)^m x^{-m}(-a)^{-\frac{1}{3}-\frac{m}{3}}\left(\frac{3x^{1+m}(-a)^{\frac{4}{3}+\frac{m}{3}}(-36-9m)}{(4+m)(1+m)^3 a}-\frac{3x^{1+m}(-a)^{\frac{4}{3}+\frac{m}{3}}(-12-3m)\ln\left(-a\,x^3+1\right)}{(4+m)(1+m)^2 a}+\frac{3x^{1+m}(-a)^{\frac{4}{3}+\frac{m}{3}}\operatorname{polylog}\left(2,a\,x^3\right)}{(1+m)a}+\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(2,a*x^3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(d*x)^m*x^(-m)*(-a)^(-1/3-1/3*m)*(3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(-3
6-9*m)/(1+m)^3/a-3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(-12-3*m)/(1+m)^2*ln(-a*x
```

```
^3+1)/a+3*x^(1+m)*(-a)^(4/3+1/3*m)/(1+m)/a*polylog(2,a*x^3)+3/(4+m)*x^(1+m)
*(-a)^(4/3+1/3*m)*(12+3*m)/(1+m)^2/a*LerchPhi(a*x^3,1,1/3*m+1/3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x^3),x, algorithm="maxima")
[Out] -9*a*d^m*integrate(x^3*x^m/((a*m^2 + 2*a*m + a)*x^3 - m^2 - 2*m - 1), x) +
((d^m*m + d^m)*x*x^m*dilog(a*x^3) + 3*d^m*x*x^m*log(-a*x^3 + 1))/(m^2 + 2*m
 + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x^3),x, algorithm="fricas")
[Out] integral((d*x)^m*dilog(a*x^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(2,a*x**3),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x^3),x, algorithm="giac")
[Out] integrate((d*x)^m*dilog(a*x^3), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{polylog}\left(2,a\,x^3\right)\,{\left(d\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, a*x^3)*(d*x)^m,x)
[Out] int(polylog(2, a*x^3)*(d*x)^m, x)
```

3.109 $\int (dx)^m \mathbf{PolyLog}(3, ax^3)\, dx$

Optimal. Leaf size=118

$$-\frac{27a(dx)^{4+m}\,{}_2F_1\left(1,\frac{4+m}{3};\frac{7+m}{3};ax^3\right)}{d^4(1+m)^3(4+m)}-\frac{9(dx)^{1+m}\log\left(1-ax^3\right)}{d(1+m)^3}-\frac{3(dx)^{1+m}\mathrm{PolyLog}(2,ax^3)}{d(1+m)^2}+\frac{(dx)^{1+m}\mathrm{PolyLog}(}{d(1+m)}$$

```
[Out] -27*a*(d*x)^(4+m)*hypergeom([1, 4/3+1/3*m],[7/3+1/3*m],a*x^3)/d^4/(1+m)^3/(
4+m)-9*(d*x)^(1+m)*ln(-a*x^3+1)/d/(1+m)^3-3*(d*x)^(1+m)*polylog(2,a*x^3)/d/
(1+m)^2+(d*x)^(1+m)*polylog(3,a*x^3)/d/(1+m)
```

Rubi [A]
time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6726, 2505, 16, 371}

$$-\frac{27a(dx)^{m+4}\,{}_2F_1\left(1,\frac{m+4}{3};\frac{m+7}{3};ax^3\right)}{d^4(m+1)^3(m+4)}-\frac{3\mathrm{Li}_2(ax^3)\,(dx)^{m+1}}{d(m+1)^2}+\frac{\mathrm{Li}_3(ax^3)\,(dx)^{m+1}}{d(m+1)}-\frac{9\log\left(1-ax^3\right)(dx)^{m+1}}{d(m+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[3, a*x^3],x]
```

```
[Out] (-27*a*(d*x)^(4 + m)*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, a*x^3])/(d^
4*(1 + m)^3*(4 + m)) - (9*(d*x)^(1 + m)*Log[1 - a*x^3])/(d*(1 + m)^3) - (3*
(d*x)^(1 + m)*PolyLog[2, a*x^3])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[3,
a*x^3])/(d*(1 + m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int (dx)^m \mathrm{Li}_3\left(ax^3\right)\,dx &= \frac{(dx)^{1+m}\mathrm{Li}_3(ax^3)}{d(1+m)} - \frac{3\int (dx)^m \mathrm{Li}_2(ax^3)\,dx}{1+m} \\
&= -\frac{3(dx)^{1+m}\mathrm{Li}_2(ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_3(ax^3)}{d(1+m)} - \frac{9\int (dx)^m \log\left(1-ax^3\right)\,dx}{(1+m)^2} \\
&= -\frac{9(dx)^{1+m}\log\left(1-ax^3\right)}{d(1+m)^3} - \frac{3(dx)^{1+m}\mathrm{Li}_2(ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_3(ax^3)}{d(1+m)} - \frac{(27a)\int \frac{x^2(dx)^{1}}{1-ax^3}}{d(1+m)^3} \\
&= -\frac{9(dx)^{1+m}\log\left(1-ax^3\right)}{d(1+m)^3} - \frac{3(dx)^{1+m}\mathrm{Li}_2(ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_3(ax^3)}{d(1+m)} - \frac{(27a)\int \frac{(dx)^{3+m}}{1-ax^3}}{d^3(1+m)^3} \\
&= -\frac{27a(dx)^{4+m}\,{}_2F_1\left(1,\frac{4+m}{3};\frac{7+m}{3};ax^3\right)}{d^4(1+m)^3(4+m)} - \frac{9(dx)^{1+m}\log\left(1-ax^3\right)}{d(1+m)^3} - \frac{3(dx)^{1+m}\mathrm{Li}_2(ax^3)}{d(1+m)^2}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.06, size = 126, normalized size = 1.07

$$
-\frac{3x(dx)^m\Gamma\left(\frac{4+m}{3}\right)\left(3a(1+m)x^3\Gamma\left(\frac{1+m}{3}\right)\,{}_2\tilde{F}_1\left(1,\frac{4+m}{3};\frac{7+m}{3};ax^3\right)+9\log\left(1-ax^3\right)+3(1+m)\mathrm{PolyLog}(2,ax^3)-\mathrm{PolyLog}(3,ax^3)-2m\mathrm{PolyLog}(3,ax^3)-m^2\mathrm{PolyLog}(3,ax^3)\right)}{(1+m)^4\Gamma\left(\frac{1+m}{3}\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*PolyLog[3, a*x^3],x]
```

```
[Out] (-3*x*(d*x)^m*Gamma[(4 + m)/3]*(3*a*(1 + m)*x^3*Gamma[(1 + m)/3]*Hypergeome
tricPFQRegularized[{1, (4 + m)/3}, {(7 + m)/3}, a*x^3] + 9*Log[1 - a*x^3] +
 3*(1 + m)*PolyLog[2, a*x^3] - PolyLog[3, a*x^3] - 2*m*PolyLog[3, a*x^3] -
m^2*PolyLog[3, a*x^3]))/((1 + m)^4*Gamma[(1 + m)/3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.24, size = 218, normalized size = 1.85

method	result
meijerg	$-\frac{(dx)^m x^{-m}(-a)^{-\frac{1}{3}-\frac{m}{3}}\left(\frac{3x^{1+m}(-a)^{\frac{4}{3}+\frac{m}{3}}(108+27m)}{(4+m)(1+m)^4 a}-\frac{3x^{1+m}(-a)^{\frac{4}{3}+\frac{m}{3}}(36+9m)\ln\left(-a\,x^3+1\right)}{(4+m)(1+m)^3 a}+\frac{3x^{1+m}(-a)^{\frac{4}{3}+\frac{m}{3}}(-12-3m)\,\mathrm{polylog}(}{(4+m)(1+m)^2 a}\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(3,a*x^3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(d*x)^m*x^(-m)*(-a)^(-1/3-1/3*m)*(3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(10
8+27*m)/(1+m)^4/a-3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(36+9*m)/(1+m)^3/a*ln(-a
*x^3+1)+3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(-12-3*m)/(1+m)^2*polylog(2,a*x^3)
/a+3*x^(1+m)*(-a)^(4/3+1/3*m)/(1+m)/a*polylog(3,a*x^3)+3/(4+m)*x^(1+m)*(-a)
^(4/3+1/3*m)*(-36-9*m)/(1+m)^3/a*LerchPhi(a*x^3,1,1/3*m+1/3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x^3),x, algorithm="maxima")
```

```
[Out] 27*a*d^m*integrate(x^3*x^m/((m^3 + 3*m^2 + 3*m + 1)*a*x^3 - m^3 - 3*m^2 - 3
*m - 1), x) - (3*d^m*(m + 1)*x*x^m*dilog(a*x^3) - (m^2 + 2*m + 1)*d^m*x*x^m
*polylog(3, a*x^3) + 9*d^m*x*x^m*log(-a*x^3 + 1))/(m^3 + 3*m^2 + 3*m + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x^3),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*polylog(3, a*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \operatorname{Li}_3\left(ax^3\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(3,a*x**3),x)
```

```
[Out] Integral((d*x)**m*polylog(3, a*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x^3),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*polylog(3, a*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{polylog}\left(3,a\,x^3\right)\,{\left(d\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, a*x^3)*(d*x)^m,x)
```

```
[Out] int(polylog(3, a*x^3)*(d*x)^m, x)
```

3.110 $\int (dx)^m \mathbf{PolyLog}(4, ax^3)\, dx$

Optimal. Leaf size=142

$$\frac{81a(dx)^{4+m}\,{}_2F_1\left(1,\frac{4+m}{3};\frac{7+m}{3};ax^3\right)}{d^4(1+m)^4(4+m)}+\frac{27(dx)^{1+m}\log\left(1-ax^3\right)}{d(1+m)^4}+\frac{9(dx)^{1+m}\text{PolyLog}(2,ax^3)}{d(1+m)^3}-\frac{3(dx)^{1+m}\text{PolyLog}(}{d(1+m)^2}$$

```
[Out] 81*a*(d*x)^(4+m)*hypergeom([1, 4/3+1/3*m],[7/3+1/3*m],a*x^3)/d^4/(1+m)^4/(4
+m)+27*(d*x)^(1+m)*ln(-a*x^3+1)/d/(1+m)^4+9*(d*x)^(1+m)*polylog(2,a*x^3)/d/
(1+m)^3-3*(d*x)^(1+m)*polylog(3,a*x^3)/d/(1+m)^2+(d*x)^(1+m)*polylog(4,a*x^
3)/d/(1+m)
```

Rubi [A]
time = 0.07, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6726, 2505, 16, 371}

$$\frac{81a(dx)^{m+4}\,{}_2F_1\left(1,\frac{m+4}{3};\frac{m+7}{3};ax^3\right)}{d^4(m+1)^4(m+4)}+\frac{9\text{Li}_2(ax^3)\,(dx)^{m+1}}{d(m+1)^3}-\frac{3\text{Li}_3(ax^3)\,(dx)^{m+1}}{d(m+1)^2}+\frac{\text{Li}_4(ax^3)\,(dx)^{m+1}}{d(m+1)}+\frac{27\log\left(1-ax^3\right)(dx)^{m+1}}{d(m+1)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[4, a*x^3],x]
```

```
[Out] (81*a*(d*x)^(4 + m)*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, a*x^3])/(d^4
*(1 + m)^4*(4 + m)) + (27*(d*x)^(1 + m)*Log[1 - a*x^3])/(d*(1 + m)^4) + (9*
(d*x)^(1 + m)*PolyLog[2, a*x^3])/(d*(1 + m)^3) - (3*(d*x)^(1 + m)*PolyLog[3
, a*x^3])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[4, a*x^3])/(d*(1 + m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \text{Li}_4\left(ax^3\right)\,dx &= \frac{(dx)^{1+m}\text{Li}_4(ax^3)}{d(1+m)} - \frac{3\int (dx)^m \text{Li}_3(ax^3)\,dx}{1+m} \\
&= -\frac{3(dx)^{1+m}\text{Li}_3(ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4(ax^3)}{d(1+m)} + \frac{9\int (dx)^m \text{Li}_2(ax^3)\,dx}{(1+m)^2} \\
&= \frac{9(dx)^{1+m}\text{Li}_2(ax^3)}{d(1+m)^3} - \frac{3(dx)^{1+m}\text{Li}_3(ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4(ax^3)}{d(1+m)} + \frac{27\int (dx)^m \log\left(1-a\right.}{(1+m)^3} \\
&= \frac{27(dx)^{1+m}\log\left(1-ax^3\right)}{d(1+m)^4} + \frac{9(dx)^{1+m}\text{Li}_2(ax^3)}{d(1+m)^3} - \frac{3(dx)^{1+m}\text{Li}_3(ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4(a}{d(1+m)} \\
&= \frac{27(dx)^{1+m}\log\left(1-ax^3\right)}{d(1+m)^4} + \frac{9(dx)^{1+m}\text{Li}_2(ax^3)}{d(1+m)^3} - \frac{3(dx)^{1+m}\text{Li}_3(ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4(a}{d(1+m)} \\
&= \frac{81a(dx)^{4+m}\,{}_2F_1\left(1,\frac{4+m}{3};\frac{7+m}{3};ax^3\right)}{d^4(1+m)^4(4+m)} + \frac{27(dx)^{1+m}\log\left(1-ax^3\right)}{d(1+m)^4} + \frac{9(dx)^{1+m}\text{Li}_2(ax^3)}{d(1+m)^3}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.07, size = 166, normalized size = 1.17

$$\frac{3x(dx)^m\Gamma\left(\frac{4+m}{3}\right)\left(9a(1+m)x^3\Gamma\left(\frac{1+m}{3}\right)\,{}_2\tilde{F}_1\left(1,\frac{4+m}{3};\frac{7+m}{3};ax^3\right)+27\log\left(1-ax^3\right)+9(1+m)\text{PolyLog}(2,ax^3)-3\text{PolyLog}(3,ax^3)-6m\text{PolyLog}(3,ax^3)-3m^2\text{PolyLog}(3,ax^3)+\text{PolyLog}(4,ax^3)+3m\text{PolyLog}(4,ax^3)+3m^2\text{PolyLog}(4,ax^3)+m^3\text{PolyLog}(4,ax^3)\right)}{(1+m)^5\Gamma\left(\frac{1+m}{3}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*PolyLog[4, a*x^3],x]
```

```
[Out] (3*x*(d*x)^m*Gamma[(4 + m)/3]*(9*a*(1 + m)*x^3*Gamma[(1 + m)/3]*Hypergeomet
ricPFQRegularized[{1, (4 + m)/3}, {(7 + m)/3}, a*x^3] + 27*Log[1 - a*x^3] +
 9*(1 + m)*PolyLog[2, a*x^3] - 3*PolyLog[3, a*x^3] - 6*m*PolyLog[3, a*x^3]
- 3*m^2*PolyLog[3, a*x^3] + PolyLog[4, a*x^3] + 3*m*PolyLog[4, a*x^3] + 3*m
^2*PolyLog[4, a*x^3] + m^3*PolyLog[4, a*x^3]))/((1 + m)^5*Gamma[(1 + m)/3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.49, size = 259, normalized size = 1.82

method	result
meijerg	$-\frac{(dx)^m x^{-m}(-a)^{-\frac{1}{3}-\frac{m}{3}}\left(\frac{3x^{1+m}(-a)^{\frac{4}{3}+\frac{m}{3}}(-324-81m)}{(4+m)(1+m)^5 a}-\frac{3x^{1+m}(-a)^{\frac{4}{3}+\frac{m}{3}}(-108-27m)\ln\left(-a\,x^3+1\right)}{(4+m)(1+m)^4 a}+\frac{3x^{1+m}(-a)^{\frac{4}{3}+\frac{m}{3}}(36+9m)\,\mathrm{polylo}}{(4+m)(1+m)^3 a}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(4,a*x^3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(d*x)^m*x^(-m)*(-a)^(-1/3-1/3*m)*(3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(-3
24-81*m)/(1+m)^5/a-3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(-108-27*m)/(1+m)^4/a*l
n(-a*x^3+1)+3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(36+9*m)/(1+m)^3/a*polylog(2,a
*x^3)+3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(-12-3*m)/(1+m)^2*polylog(3,a*x^3)/a
+3*x^(1+m)*(-a)^(4/3+1/3*m)/(1+m)/a*polylog(4,a*x^3)+3/(4+m)*x^(1+m)*(-a)^(
4/3+1/3*m)*(108+27*m)/(1+m)^4/a*LerchPhi(a*x^3,1,1/3*m+1/3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x^3),x, algorithm="maxima")
```

```
[Out] -81*a*d^m*integrate(-x^3*x^m/(m^4 - (a*m^4 + 4*a*m^3 + 6*a*m^2 + 4*a*m + a)
*x^3 + 4*m^3 + 6*m^2 + 4*m + 1), x) + (9*(d^m*m + d^m)*x*x^m*dilog(a*x^3) +
 27*d^m*x*x^m*log(-a*x^3 + 1) + (d^m*m^3 + 3*d^m*m^2 + 3*d^m*m + d^m)*x*x^m
*polylog(4, a*x^3) - 3*(d^m*m^2 + 2*d^m*m + d^m)*x*x^m*polylog(3, a*x^3))/(
m^4 + 4*m^3 + 6*m^2 + 4*m + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x^3),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*polylog(4, a*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \operatorname{Li}_4\left(ax^3\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(4,a*x**3),x)
[Out] Integral((d*x)**m*polylog(4, a*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x^3),x, algorithm="giac")
[Out] integrate((d*x)^m*polylog(4, a*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{polylog}\left(4,a\,x^3\right)\,{\left(d\,x\right)}^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(4, a*x^3)*(d*x)^m,x)
[Out] int(polylog(4, a*x^3)*(d*x)^m, x)
```

3.111 $\int (dx)^m \textbf{PolyLog}(2, ax^q)\,dx$

Optimal. Leaf size=101

$$\frac{aq^2x^{1+q}(dx)^m\,{}_2F_1\left(1,\frac{1+m+q}{q};\frac{1+m+2q}{q};ax^q\right)}{(1+m)^2(1+m+q)}+\frac{q(dx)^{1+m}\log(1-ax^q)}{d(1+m)^2}+\frac{(dx)^{1+m}\text{PolyLog}(2,ax^q)}{d(1+m)}$$

```
[Out] a*q^2*x^(1+q)*(d*x)^m*hypergeom([1, (1+m+q)/q],[(1+m+2*q)/q],a*x^q)/(1+m)^2
/(1+m+q)+q*(d*x)^(1+m)*ln(1-a*x^q)/d/(1+m)^2+(d*x)^(1+m)*polylog(2,a*x^q)/d
/(1+m)
```

Rubi [A]
time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6726, 2505, 20, 371}

$$\frac{aq^2x^{q+1}(dx)^m\,{}_2F_1\left(1,\frac{m+q+1}{q};\frac{m+2q+1}{q};ax^q\right)}{(m+1)^2(m+q+1)}+\frac{(dx)^{m+1}\text{Li}_2(ax^q)}{d(m+1)}+\frac{q(dx)^{m+1}\log(1-ax^q)}{d(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[2, a*x^q],x]
```

```
[Out] (a*q^2*x^(1 + q)*(d*x)^m*Hypergeometric2F1[1, (1 + m + q)/q, (1 + m + 2*q)/
q, a*x^q])/((1 + m)^2*(1 + m + q)) + (q*(d*x)^(1 + m)*Log[1 - a*x^q])/(d*(1
 + m)^2) + ((d*x)^(1 + m)*PolyLog[2, a*x^q])/(d*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
```

```
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \text{Li}_2(ax^q)\,dx &= \frac{(dx)^{1+m}\text{Li}_2(ax^q)}{d(1+m)} + \frac{q\int (dx)^m \log(1-ax^q)\,dx}{1+m}\\ &= \frac{q(dx)^{1+m}\log(1-ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_2(ax^q)}{d(1+m)} + \frac{(aq^2)\int \frac{x^{-1+q}(dx)^{1+m}}{1-ax^q}\,dx}{d(1+m)^2}\\ &= \frac{q(dx)^{1+m}\log(1-ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_2(ax^q)}{d(1+m)} + \frac{(aq^2x^{-m}(dx)^m)\int \frac{x^{m+q}}{1-ax^q}\,dx}{(1+m)^2}\\ &= \frac{aq^2x^{1+q}(dx)^m\,{}_2F_1\left(1,\frac{1+m+q}{q};\frac{1+m+2q}{q};ax^q\right)}{(1+m)^2(1+m+q)} + \frac{q(dx)^{1+m}\log(1-ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_2(}{d(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 80, normalized size = 0.79

$$\frac{x(dx)^m\left(aq^2x^q\,{}_2F_1\left(1,\frac{1+m+q}{q};\frac{1+m+2q}{q};ax^q\right)+(1+m+q)\left(q\log(1-ax^q)+(1+m)\text{PolyLog}(2,ax^q)\right)\right)}{(1+m)^2(1+m+q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*PolyLog[2, a*x^q],x]
```

```
[Out] (x*(d*x)^m*(a*q^2*x^q*Hypergeometric2F1[1, (1 + m + q)/q, (1 + m + 2*q)/q,
a*x^q] + (1 + m + q)*(q*Log[1 - a*x^q] + (1 + m)*PolyLog[2, a*x^q])))/((1 +
 m)^2*(1 + m + q))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.13, size = 148, normalized size = 1.47

method	result
meijerg	$-\frac{(dx)^m x^{-m}(-a)^{-\frac{m}{q}-\frac{1}{q}}\left(-\frac{q^2x^{1+m}(-a)^{\frac{m}{q}+\frac{1}{q}}\ln(1-a\,x^q)}{(1+m)^2}-\frac{q\,x^{1+m}(-a)^{\frac{m}{q}+\frac{1}{q}}\operatorname{polylog}(2,a\,x^q)}{1+m}-\frac{q^2x^{1+m+q}a(-a)^{\frac{m}{q}+\frac{1}{q}}\Phi\left(a\,x^q,1,\frac{1+m+}{q}\right.}{(1+m)^2}\right.}{q}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(2,a*x^q),x,method=_RETURNVERBOSE)
```

```
[Out] -(d*x)^m*x^(-m)*(-a)^(-m/q-1/q)/q*(-q^2*x^(1+m)*(-a)^(m/q+1/q)/(1+m)^2*ln(1
-a*x^q)-q*x^(1+m)*(-a)^(m/q+1/q)/(1+m)*polylog(2,a*x^q)-q^2*x^(1+m+q)*a*(-a
)^(m/q+1/q)/(1+m)^2*LerchPhi(a*x^q,1,(1+m+q)/q))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x^q),x, algorithm="maxima")
```

```
[Out] -d^m*q^2*integrate(-x^m/(m^2 - (a*m^2 + 2*a*m + a)*x^q + 2*m + 1), x) - (d^
m*q^2*x*x^m - (d^m*m + d^m)*q*x*x^m*log(-a*x^q + 1) - (d^m*m^2 + 2*d^m*m +
d^m)*x*x^m*dilog(a*x^q))/(m^3 + 3*m^2 + 3*m + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x^q),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*dilog(a*x^q), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \operatorname{Li}_2(ax^q)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(2,a*x**q),x)
```

```
[Out] Integral((d*x)**m*polylog(2, a*x**q), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x^q),x, algorithm="giac")
[Out] integrate((d*x)^m*dilog(a*x^q), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\mathrm{polylog}(2,a\,x^q)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(2, a*x^q),x)
[Out] int((d*x)^m*polylog(2, a*x^q), x)
```

3.112 $\int (dx)^m \mathbf{PolyLog}(3, ax^q)\, dx$

Optimal. Leaf size=130

$$-\frac{aq^3x^{1+q}(dx)^m\,{}_2F_1\left(1,\frac{1+m+q}{q};\frac{1+m+2q}{q};ax^q\right)}{(1+m)^3(1+m+q)}-\frac{q^2(dx)^{1+m}\log\left(1-ax^q\right)}{d(1+m)^3}-\frac{q(dx)^{1+m}\mathrm{PolyLog}(2,ax^q)}{d(1+m)^2}+\frac{(dx)^{1+m}\mathrm{P}}{d}$$

```
[Out] -a*q^3*x^(1+q)*(d*x)^m*hypergeom([1, (1+m+q)/q],[(1+m+2*q)/q],a*x^q)/(1+m)^
3/(1+m+q)-q^2*(d*x)^(1+m)*ln(1-a*x^q)/d/(1+m)^3-q*(d*x)^(1+m)*polylog(2,a*x
^q)/d/(1+m)^2+(d*x)^(1+m)*polylog(3,a*x^q)/d/(1+m)
```

Rubi [A]
time = 0.06, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6726, 2505, 20, 371}

$$-\frac{aq^3x^{q+1}(dx)^m\,{}_2F_1\left(1,\frac{m+q+1}{q};\frac{m+2q+1}{q};ax^q\right)}{(m+1)^3(m+q+1)}-\frac{q(dx)^{m+1}\mathrm{Li}_2(ax^q)}{d(m+1)^2}+\frac{(dx)^{m+1}\mathrm{Li}_3(ax^q)}{d(m+1)}-\frac{q^2(dx)^{m+1}\log\left(1-ax^q\right)}{d(m+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[3, a*x^q],x]
```

```
[Out] -((a*q^3*x^(1 + q)*(d*x)^m*Hypergeometric2F1[1, (1 + m + q)/q, (1 + m + 2*q
)/q, a*x^q])/((1 + m)^3*(1 + m + q))) - (q^2*(d*x)^(1 + m)*Log[1 - a*x^q])/
(d*(1 + m)^3) - (q*(d*x)^(1 + m)*PolyLog[2, a*x^q])/(d*(1 + m)^2) + ((d*x)^
(1 + m)*PolyLog[3, a*x^q])/(d*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
```

```
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int (dx)^m \mathrm{Li}_3(ax^q)\, dx &= \frac{(dx)^{1+m}\mathrm{Li}_3(ax^q)}{d(1+m)} - \frac{q\int (dx)^m \mathrm{Li}_2(ax^q)\, dx}{1+m} \\
&= -\frac{q(dx)^{1+m}\mathrm{Li}_2(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_3(ax^q)}{d(1+m)} - \frac{q^2\int (dx)^m \log(1-ax^q)\, dx}{(1+m)^2} \\
&= -\frac{q^2(dx)^{1+m}\log(1-ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m}\mathrm{Li}_2(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_3(ax^q)}{d(1+m)} - \frac{(aq^3)\int \frac{x^{-1+q}(d}{1-a}}{d(1+m} \\
&= -\frac{q^2(dx)^{1+m}\log(1-ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m}\mathrm{Li}_2(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_3(ax^q)}{d(1+m)} - \frac{(aq^3x^{-m}(dx)^m}{(1+} \\
&= -\frac{aq^3x^{1+q}(dx)^m\, {}_2F_1\left(1, \frac{1+m+q}{q}; \frac{1+m+2q}{q}; ax^q\right)}{(1+m)^3(1+m+q)} - \frac{q^2(dx)^{1+m}\log(1-ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m}}{d(1+}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.02, size = 50, normalized size = 0.38

$$
-\frac{x(dx)^m G_{5,5}^{1,5}\left(-ax^q \middle| \begin{array}{c} 1,1,1,1,1-\frac{1+m}{q} \\ 1,0,0,0,-\frac{1+m}{q} \end{array}\right)}{q}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*PolyLog[3, a*x^q],x]
```

```
[Out] -((x*(d*x)^m*MeijerG[{{1, 1, 1, 1, 1 - (1 + m)/q}, {}}, {{1}, {0, 0, 0, -((
1 + m)/q)}}, -(a*x^q)])/q)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.28, size = 180, normalized size = 1.38

method	result
meijerg	$-\frac{(dx)^m x^{-m}(-a)^{-\frac{m}{q}-\frac{1}{q}}\left(\frac{q^3 x^{1+m}(-a)^{\frac{m}{q}+\frac{1}{q}}\ln(1-a\,x^q)}{(1+m)^3}+\frac{q^2 x^{1+m}(-a)^{\frac{m}{q}+\frac{1}{q}}\,\mathrm{polylog}(2,a\,x^q)}{(1+m)^2}-\frac{q\,x^{1+m}(-a)^{\frac{m}{q}+\frac{1}{q}}\,\mathrm{polylog}(3,a\,x^q)}{1+m}+\frac{q^3 x^{1+}}{}\right.}{q}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(3,a*x^q),x,method=_RETURNVERBOSE)
```

```
[Out] -(d*x)^m*x^(-m)*(-a)^(-m/q-1/q)/q*(q^3*x^(1+m)*(-a)^(m/q+1/q)/(1+m)^3*ln(1-
a*x^q)+q^2*x^(1+m)*(-a)^(m/q+1/q)/(1+m)^2*polylog(2,a*x^q)-q*x^(1+m)*(-a)^(
m/q+1/q)/(1+m)*polylog(3,a*x^q)+q^3*x^(1+m+q)*a*(-a)^(m/q+1/q)/(1+m)^3*Lerc
hPhi(a*x^q,1,(1+m+q)/q))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x^q),x, algorithm="maxima")
```

```
[Out] d^m*q^3*integrate(-x^m/(m^3 - (m^3 + 3*m^2 + 3*m + 1)*a*x^q + 3*m^2 + 3*m +
 1), x) + (d^m*q^3*x*x^m - (m^2*q + 2*m*q + q)*d^m*x*x^m*dilog(a*x^q) - (m*
q^2 + q^2)*d^m*x*x^m*log(-a*x^q + 1) + (m^3 + 3*m^2 + 3*m + 1)*d^m*x*x^m*po
lylog(3, a*x^q))/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x^q),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*polylog(3, a*x^q), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \operatorname{Li}_3(ax^q)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(3,a*x**q),x)
```

```
[Out] Integral((d*x)**m*polylog(3, a*x**q), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x^q),x, algorithm="giac")
[Out] integrate((d*x)^m*polylog(3, a*x^q), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\mathrm{polylog}(3,a\,x^q)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(3, a*x^q),x)
[Out] int((d*x)^m*polylog(3, a*x^q), x)
```

3.113 $\int (dx)^m \textbf{PolyLog}(4, ax^q)\, dx$

Optimal. Leaf size=154

$$\frac{aq^4x^{1+q}(dx)^m\,{}_2F_1\left(1,\frac{1+m+q}{q};\frac{1+m+2q}{q};ax^q\right)}{(1+m)^4(1+m+q)}+\frac{q^3(dx)^{1+m}\log(1-ax^q)}{d(1+m)^4}+\frac{q^2(dx)^{1+m}\text{PolyLog}(2,ax^q)}{d(1+m)^3}-\frac{q(dx)^{1+m}\text{I}}{d(}$$

```
[Out] a*q^4*x^(1+q)*(d*x)^m*hypergeom([1, (1+m+q)/q],[(1+m+2*q)/q],a*x^q)/(1+m)^4
/(1+m+q)+q^3*(d*x)^(1+m)*ln(1-a*x^q)/d/(1+m)^4+q^2*(d*x)^(1+m)*polylog(2,a*
x^q)/d/(1+m)^3-q*(d*x)^(1+m)*polylog(3,a*x^q)/d/(1+m)^2+(d*x)^(1+m)*polylog
(4,a*x^q)/d/(1+m)
```

Rubi [A]
time = 0.07, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6726, 2505, 20, 371}

$$\frac{aq^4x^{q+1}(dx)^m\,{}_2F_1\left(1,\frac{m+q+1}{q};\frac{m+2q+1}{q};ax^q\right)}{(m+1)^4(m+q+1)}+\frac{q^2(dx)^{m+1}\text{Li}_2(ax^q)}{d(m+1)^3}-\frac{q(dx)^{m+1}\text{Li}_3(ax^q)}{d(m+1)^2}+\frac{(dx)^{m+1}\text{Li}_4(ax^q)}{d(m+1)}+\frac{q^3(dx)^{m+1}\log(1-ax^q)}{d(m+1)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[4, a*x^q],x]
```

```
[Out] (a*q^4*x^(1 + q)*(d*x)^m*Hypergeometric2F1[1, (1 + m + q)/q, (1 + m + 2*q)/
q, a*x^q])/((1 + m)^4*(1 + m + q)) + (q^3*(d*x)^(1 + m)*Log[1 - a*x^q])/(d*
(1 + m)^4) + (q^2*(d*x)^(1 + m)*PolyLog[2, a*x^q])/(d*(1 + m)^3) - (q*(d*x)
^(1 + m)*PolyLog[3, a*x^q])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[4, a*x^q
])/(d*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
```

```
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \text{Li}_4(ax^q)\,dx &= \frac{(dx)^{1+m}\text{Li}_4(ax^q)}{d(1+m)} - \frac{q\int (dx)^m \text{Li}_3(ax^q)\,dx}{1+m} \\
&= -\frac{q(dx)^{1+m}\text{Li}_3(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4(ax^q)}{d(1+m)} + \frac{q^2\int (dx)^m \text{Li}_2(ax^q)\,dx}{(1+m)^2} \\
&= \frac{q^2(dx)^{1+m}\text{Li}_2(ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m}\text{Li}_3(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4(ax^q)}{d(1+m)} + \frac{q^3\int (dx)^m \log(1-a\cdots}{(1+m)^3} \\
&= \frac{q^3(dx)^{1+m}\log(1-ax^q)}{d(1+m)^4} + \frac{q^2(dx)^{1+m}\text{Li}_2(ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m}\text{Li}_3(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4(a\cdots}{d(1+m)} \\
&= \frac{q^3(dx)^{1+m}\log(1-ax^q)}{d(1+m)^4} + \frac{q^2(dx)^{1+m}\text{Li}_2(ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m}\text{Li}_3(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4(a\cdots}{d(1+m)} \\
&= \frac{aq^4x^{1+q}(dx)^m\,{}_2F_1\left(1,\frac{1+m+q}{q};\frac{1+m+2q}{q};ax^q\right)}{(1+m)^4(1+m+q)} + \frac{q^3(dx)^{1+m}\log(1-ax^q)}{d(1+m)^4} + \frac{q^2(dx)^{1+m}\text{L}\cdots}{d(1+m\cdots}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.02, size = 52, normalized size = 0.34

$$-\frac{x(dx)^m G_{6,6}^{1,6}\left(-ax^q\left|\begin{array}{c}1,1,1,1,1,1-\frac{1+m}{q}\\1,0,0,0,0,-\frac{1+m}{q}\end{array}\right.\right)}{q}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*PolyLog[4, a*x^q],x]
```

```
[Out] -((x*(d*x)^m*MeijerG[{{1, 1, 1, 1, 1, 1 - (1 + m)/q}, {}}, {{1}, {0, 0, 0,
0, -((1 + m)/q)}}, -(a*x^q)])/q)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.99, size = 217, normalized size = 1.41

method	result
meijerg	$-\frac{(dx)^m x^{-m}(-a)^{-\frac{m}{q}-\frac{1}{q}}\left(-\frac{q^4 x^{1+m}(-a)^{\frac{m}{q}+\frac{1}{q}}\ln(1-a\,x^q)}{(1+m)^4}-\frac{q^3 x^{1+m}(-a)^{\frac{m}{q}+\frac{1}{q}}\operatorname{polylog}(2,a\,x^q)}{(1+m)^3}+\frac{q^2 x^{1+m}(-a)^{\frac{m}{q}+\frac{1}{q}}\operatorname{polylog}(3,a\,x^q)}{(1+m)^2}-\frac{q\,x^1}{}\right.}{q}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(4,a*x^q),x,method=_RETURNVERBOSE)
```

```
[Out] -(d*x)^m*x^(-m)*(-a)^(-m/q-1/q)/q*(-q^4*x^(1+m)*(-a)^(m/q+1/q)/(1+m)^4*ln(1
-a*x^q)-q^3*x^(1+m)*(-a)^(m/q+1/q)/(1+m)^3*polylog(2,a*x^q)+q^2*x^(1+m)*(-a
)^(m/q+1/q)/(1+m)^2*polylog(3,a*x^q)-q*x^(1+m)*(-a)^(m/q+1/q)/(1+m)*polylog
(4,a*x^q)-q^4*x^(1+m+q)*a*(-a)^(m/q+1/q)/(1+m)^4*LerchPhi(a*x^q,1,(1+m+q)/q
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x^q),x, algorithm="maxima")
```

```
[Out] -d^m*q^4*integrate(-x^m/(m^4 + 4*m^3 + 6*m^2 - (a*m^4 + 4*a*m^3 + 6*a*m^2 +
 4*a*m + a)*x^q + 4*m + 1), x) - (d^m*q^4*x*x^m - (d^m*m + d^m)*q^3*x*x^m*l
og(-a*x^q + 1) - (d^m*m^2 + 2*d^m*m + d^m)*q^2*x*x^m*dilog(a*x^q) + (d^m*m^
3 + 3*d^m*m^2 + 3*d^m*m + d^m)*q*x*x^m*polylog(3, a*x^q) - (d^m*m^4 + 4*d^m
*m^3 + 6*d^m*m^2 + 4*d^m*m + d^m)*x*x^m*polylog(4, a*x^q))/(m^5 + 5*m^4 + 1
0*m^3 + 10*m^2 + 5*m + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x^q),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*polylog(4, a*x^q), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \operatorname{Li}_4(ax^q)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(4,a*x**q),x)
```

```
[Out] Integral((d*x)**m*polylog(4, a*x**q), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x^q),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*polylog(4, a*x^q), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\mathrm{polylog}(4,a\,x^q)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(4, a*x^q),x)
```

```
[Out] int((d*x)^m*polylog(4, a*x^q), x)
```

3.114 $\int x\mathbf{PolyLog}(n,ax)\,dx$

Optimal. Leaf size=10

$$\mathrm{Int}(x\mathrm{PolyLog}(n,ax),x)$$

```
[Out] Unintegrable(x*polylog(n,a*x),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x\mathrm{Li}_n(ax)\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*PolyLog[n, a*x],x]
[Out] Defer[Int][x*PolyLog[n, a*x], x]
```

Rubi steps

$$\int x\mathrm{Li}_n(ax)\,dx = \int x\mathrm{Li}_n(ax)\,dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\mathrm{PolyLog}(n,ax)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*PolyLog[n, a*x],x]
[Out] Integrate[x*PolyLog[n, a*x], x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int x\,\mathrm{polylog}\left(n,ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(n,a*x),x)
```

```
[Out] int(x*polylog(n,a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,a*x),x, algorithm="maxima")
[Out] integrate(x*polylog(n, a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,a*x),x, algorithm="fricas")
[Out] integral(x*polylog(n, a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{Li}_n(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,a*x),x)
[Out] Integral(x*polylog(n, a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,a*x),x, algorithm="giac")
[Out] integrate(x*polylog(n, a*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.10

$$\int x \operatorname{polylog}(n, a\, x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(n, a*x),x)
[Out] int(x*polylog(n, a*x), x)
```

3.115 $\int \textbf{PolyLog}(n, ax)\,dx$

Optimal. Leaf size=8

$$\text{Int}(\text{PolyLog}(n, ax), x)$$

```
[Out] Unintegrable(polylog(n,a*x),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \text{Li}_n(ax)\,dx$$

Verification is not applicable to the result.

```
[In] Int[PolyLog[n, a*x],x]
[Out] Defer[Int][PolyLog[n, a*x], x]
```

Rubi steps

$$\int \text{Li}_n(ax)\,dx = \int \text{Li}_n(ax)\,dx$$

Mathematica [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \text{PolyLog}(n, ax)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[PolyLog[n, a*x],x]
[Out] Integrate[PolyLog[n, a*x], x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \text{polylog}(n, ax)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n,a*x),x)
```

```
[Out] int(polylog(n,a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x),x, algorithm="maxima")
[Out] integrate(polylog(n, a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x),x, algorithm="fricas")
[Out] integral(polylog(n, a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{Li}_{n}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x),x)
[Out] Integral(polylog(n, a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x),x, algorithm="giac")
[Out] integrate(polylog(n, a*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.12

$$\int \mathrm{polylog}(n,a\,x)\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n, a*x),x)
[Out] int(polylog(n, a*x), x)
```

3.116 $\int \frac{\text{PolyLog}(n,ax)}{x} \, dx$

Optimal. Leaf size=7

$$\text{PolyLog}(1+n,ax)$$

```
[Out] polylog(1+n,a*x)
```

Rubi [A]
time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6724}

$$\text{Li}_{n+1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[n, a*x]/x,x]
```

```
[Out] PolyLog[1 + n, a*x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\text{Li}_n(ax)}{x} \, dx = \text{Li}_{1+n}(ax)$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$\text{PolyLog}(1+n,ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[n, a*x]/x,x]
```

```
[Out] PolyLog[1 + n, a*x]
```

Maple [A]
time = 0.06, size = 8, normalized size = 1.14

method	result	size
derivativedivides	$\text{polylog}\,(1+n, ax)$	8
default	$\text{polylog}\,(1+n, ax)$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n,a*x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] polylog(1+n,a*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(polylog(n, a*x)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x)/x,x, algorithm="fricas")
```

```
[Out] integral(polylog(n, a*x)/x, x)
```

Sympy [A]
time = 0.20, size = 5, normalized size = 0.71

$$\text{Li}_{n+1}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x)/x,x)
```

```
[Out] polylog(n + 1, a*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x)/x,x, algorithm="giac")
```

```
[Out] integrate(polylog(n, a*x)/x, x)
```

Mupad [B]
time = 0.54, size = 7, normalized size = 1.00

$$\mathrm{polylog}(n+1, a\, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n, a*x)/x,x)
```

```
[Out] polylog(n + 1, a*x)
```

3.117 $\int \frac{\text{PolyLog}(n,ax)}{x^2}\,dx$

Optimal. Leaf size=12

$$\text{Int}\left(\frac{\text{PolyLog}(n,ax)}{x^2},x\right)$$

```
[Out] Unintegrable(polylog(n,a*x)/x^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{Li}_n(ax)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[PolyLog[n, a*x]/x^2,x]
[Out] Defer[Int][PolyLog[n, a*x]/x^2, x]
```

Rubi steps

$$\int \frac{\text{Li}_n(ax)}{x^2}\,dx = \int \frac{\text{Li}_n(ax)}{x^2}\,dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{PolyLog}(n,ax)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[PolyLog[n, a*x]/x^2,x]
[Out] Integrate[PolyLog[n, a*x]/x^2, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{polylog}\,(n,ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n,a*x)/x^2,x)
```

```
[Out] int(polylog(n,a*x)/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(polylog(n, a*x)/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral(polylog(n, a*x)/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_{n}(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x)/x**2,x)
```

```
[Out] Integral(polylog(n, a*x)/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(polylog(n, a*x)/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\mathrm{polylog}(n,a\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n, a*x)/x^2,x)
```

```
[Out] int(polylog(n, a*x)/x^2, x)
```

3.118 $\int \frac{\text{PolyLog}(n,ax)}{x^3}\,dx$

Optimal. Leaf size=12

$$\text{Int}\left(\frac{\text{PolyLog}(n,ax)}{x^3},x\right)$$

[Out] Unintegrable(polylog(n,a*x)/x^3,x)

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{Li}_n(ax)}{x^3}\,dx$$

Verification is not applicable to the result.

[In] Int[PolyLog[n, a*x]/x^3,x]

[Out] Defer[Int][PolyLog[n, a*x]/x^3, x]

Rubi steps

$$\int \frac{\text{Li}_n(ax)}{x^3}\,dx = \int \frac{\text{Li}_n(ax)}{x^3}\,dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{PolyLog}(n,ax)}{x^3}\,dx$$

Verification is not applicable to the result.

[In] Integrate[PolyLog[n, a*x]/x^3,x]

[Out] Integrate[PolyLog[n, a*x]/x^3, x]

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{polylog}\left(n,ax\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(polylog(n,a*x)/x^3,x)`

[Out] `int(polylog(n,a*x)/x^3,x)`

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(n,a*x)/x^3,x, algorithm="maxima")`

[Out] `integrate(polylog(n, a*x)/x^3, x)`

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(n,a*x)/x^3,x, algorithm="fricas")`

[Out] `integral(polylog(n, a*x)/x^3, x)`

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_n(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(n,a*x)/x**3,x)`

[Out] `Integral(polylog(n, a*x)/x**3, x)`

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(n,a*x)/x^3,x, algorithm="giac")`

[Out] `integrate(polylog(n, a*x)/x^3, x)`

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\mathrm{polylog}(n,a\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n, a*x)/x^3,x)
[Out] int(polylog(n, a*x)/x^3, x)
```

3.119 $\int x\mathrm{PolyLog}(n, ax^q)\, dx$

Optimal. Leaf size=12

$$\mathrm{Int}(x\mathrm{PolyLog}(n, ax^q)\,, x)$$

```
[Out] Unintegrable(x*polylog(n,a*x^q),x)
```

Rubi [A]

time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\mathrm{Li}_n(ax^q)\, dx$$

Verification is not applicable to the result.

```
[In] Int[x*PolyLog[n, a*x^q],x]
[Out] Defer[Int][x*PolyLog[n, a*x^q], x]
```

Rubi steps

$$\int x\mathrm{Li}_n(ax^q)\, dx = \int x\mathrm{Li}_n(ax^q)\, dx$$

Mathematica [A]

time = 0.02, size = 0, normalized size = 0.00

$$\int x\mathrm{PolyLog}(n, ax^q)\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*PolyLog[n, a*x^q],x]
[Out] Integrate[x*PolyLog[n, a*x^q], x]
```

Maple [A]

time = 0.03, size = 0, normalized size = 0.00

$$\int x\,\mathrm{polylog}\left(n, a\,x^q\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(n,a*x^q),x)
```

```
[Out] int(x*polylog(n,a*x^q),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,a*x^q),x, algorithm="maxima")
[Out] integrate(x*polylog(n, a*x^q), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,a*x^q),x, algorithm="fricas")
[Out] integral(x*polylog(n, a*x^q), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{Li}_n(ax^q)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,a*x**q),x)
[Out] Integral(x*polylog(n, a*x**q), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,a*x^q),x, algorithm="giac")
[Out] integrate(x*polylog(n, a*x^q), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int x\,\mathrm{polylog}(n,a\,x^q)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(n, a*x^q),x)
[Out] int(x*polylog(n, a*x^q), x)
```

3.120 $\int \mathbf{PolyLog}(n, ax^q)\, dx$

Optimal. Leaf size=10

$$\mathrm{Int}(\mathrm{PolyLog}(n, ax^q), x)$$

[Out] Unintegrable(polylog(n,a*x^q),x)

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \mathrm{Li}_n(ax^q)\, dx$$

Verification is not applicable to the result.

[In] Int[PolyLog[n, a*x^q],x]

[Out] Defer[Int][PolyLog[n, a*x^q], x]

Rubi steps

$$\int \mathrm{Li}_n(ax^q)\, dx = \int \mathrm{Li}_n(ax^q)\, dx$$

Mathematica [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{PolyLog}(n, ax^q)\, dx$$

Verification is not applicable to the result.

[In] Integrate[PolyLog[n, a*x^q],x]

[Out] Integrate[PolyLog[n, a*x^q], x]

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \mathrm{polylog}\left(n, a\,x^{q}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(n,a*x^q),x)

```
[Out] int(polylog(n,a*x^q),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q),x, algorithm="maxima")
[Out] integrate(polylog(n, a*x^q), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q),x, algorithm="fricas")
[Out] integral(polylog(n, a*x^q), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{Li}_n(ax^q)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x**q),x)
[Out] Integral(polylog(n, a*x**q), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q),x, algorithm="giac")
[Out] integrate(polylog(n, a*x^q), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.10

$$\int \operatorname{polylog}(n, a\, x^q)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n, a*x^q),x)
[Out] int(polylog(n, a*x^q), x)
```

3.121 $\int \frac{\text{PolyLog}(n,ax^q)}{x} \, dx$

Optimal. Leaf size=13

$$\frac{\text{PolyLog}(1+n,ax^q)}{q}$$

```
[Out] polylog(1+n,a*x^q)/q
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6724}

$$\frac{\text{Li}_{n+1}(ax^q)}{q}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[n, a*x^q]/x,x]
[Out] PolyLog[1 + n, a*x^q]/q
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\text{Li}_n(ax^q)}{x} \, dx = \frac{\text{Li}_{1+n}(ax^q)}{q}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{\text{PolyLog}(1+n,ax^q)}{q}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[n, a*x^q]/x,x]
[Out] PolyLog[1 + n, a*x^q]/q
```

Maple [A]
time = 0.22, size = 14, normalized size = 1.08

method	result	size
derivativedivides	$\frac{\mathrm{polylog}(1+n,a\,x^q)}{q}$	14
default	$\frac{\mathrm{polylog}(1+n,a\,x^q)}{q}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n,a*x^q)/x,x,method=_RETURNVERBOSE)
```

```
[Out] polylog(1+n,a*x^q)/q
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q)/x,x, algorithm="maxima")
```

```
[Out] integrate(polylog(n, a*x^q)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q)/x,x, algorithm="fricas")
```

```
[Out] integral(polylog(n, a*x^q)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_n\left(ax^q\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x**q)/x,x)
```

```
[Out] Integral(polylog(n, a*x**q)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q)/x,x, algorithm="giac")
```

```
[Out] integrate(polylog(n, a*x^q)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\mathrm{polylog}(n,a\,x^q)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n, a*x^q)/x,x)
```

```
[Out] int(polylog(n, a*x^q)/x, x)
```

3.122 $\int \frac{\text{PolyLog}(n,ax^q)}{x^2} \, dx$

Optimal. Leaf size=14

$$\text{Int}\left(\frac{\text{PolyLog}(n,ax^q)}{x^2},x\right)$$

```
[Out] Unintegrable(polylog(n,a*x^q)/x^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{Li}_n(ax^q)}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[PolyLog[n, a*x^q]/x^2,x]
```

```
[Out] Defer[Int][PolyLog[n, a*x^q]/x^2, x]
```

Rubi steps

$$\int \frac{\text{Li}_n(ax^q)}{x^2} \, dx = \int \frac{\text{Li}_n(ax^q)}{x^2} \, dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{PolyLog}(n,ax^q)}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[PolyLog[n, a*x^q]/x^2,x]
```

```
[Out] Integrate[PolyLog[n, a*x^q]/x^2, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{polylog}\left(n,a\,x^q\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n,a*x^q)/x^2,x)
[Out] int(polylog(n,a*x^q)/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q)/x^2,x, algorithm="maxima")
[Out] integrate(polylog(n, a*x^q)/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q)/x^2,x, algorithm="fricas")
[Out] integral(polylog(n, a*x^q)/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_{n}\left(ax^{q}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x**q)/x**2,x)
[Out] Integral(polylog(n, a*x**q)/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q)/x^2,x, algorithm="giac")
[Out] integrate(polylog(n, a*x^q)/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\mathrm{polylog}(n,a\,x^q)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n, a*x^q)/x^2,x)
[Out] int(polylog(n, a*x^q)/x^2, x)
```

3.123 $\int \frac{\text{PolyLog}(n,ax^q)}{x^3} \, dx$

Optimal. Leaf size=14

$$\text{Int}\left(\frac{\text{PolyLog}(n,ax^q)}{x^3},x\right)$$

```
[Out] Unintegrable(polylog(n,a*x^q)/x^3,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{Li}_n(ax^q)}{x^3} \, dx$$

Verification is not applicable to the result.

```
[In] Int[PolyLog[n, a*x^q]/x^3,x]
```

```
[Out] Defer[Int][PolyLog[n, a*x^q]/x^3, x]
```

Rubi steps

$$\int \frac{\text{Li}_n(ax^q)}{x^3} \, dx = \int \frac{\text{Li}_n(ax^q)}{x^3} \, dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{PolyLog}(n,ax^q)}{x^3} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[PolyLog[n, a*x^q]/x^3,x]
```

```
[Out] Integrate[PolyLog[n, a*x^q]/x^3, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{polylog}\left(n,a\,x^q\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n,a*x^q)/x^3,x)
[Out] int(polylog(n,a*x^q)/x^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q)/x^3,x, algorithm="maxima")
[Out] integrate(polylog(n, a*x^q)/x^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q)/x^3,x, algorithm="fricas")
[Out] integral(polylog(n, a*x^q)/x^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_n(ax^q)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x**q)/x**3,x)
[Out] Integral(polylog(n, a*x**q)/x**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q)/x^3,x, algorithm="giac")
[Out] integrate(polylog(n, a*x^q)/x^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\mathrm{polylog}(n,a\,x^q)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n, a*x^q)/x^3,x)
[Out] int(polylog(n, a*x^q)/x^3, x)
```

3.124 $\int x^2\mathbf{PolyLog}(2,c(a+bx))\,dx$

Optimal. Leaf size=260

$$-\frac{a^2x}{3b^2}+\frac{a(1-ac)x}{6b^2c}-\frac{(1-ac)^2x}{9b^2c^2}+\frac{ax^2}{12b}-\frac{(1-ac)x^2}{18bc}-\frac{x^3}{27}+\frac{a(1-ac)^2\log(1-ac-bcx)}{6b^3c^2}-\frac{(1-ac)^3\log(1-ac}{9b^3c^3}$$

```
[Out] -1/3*a^2*x/b^2+1/6*a*(-a*c+1)*x/b^2/c-1/9*(-a*c+1)^2*x/b^2/c^2+1/12*a*x^2/b
-1/18*(-a*c+1)*x^2/b/c-1/27*x^3+1/6*a*(-a*c+1)^2*ln(-b*c*x-a*c+1)/b^3/c^2-1
/9*(-a*c+1)^3*ln(-b*c*x-a*c+1)/b^3/c^3-1/6*a*x^2*ln(-b*c*x-a*c+1)/b+1/9*x^3
*ln(-b*c*x-a*c+1)-1/3*a^2*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b^3/c+1/3*a^3*pol
ylog(2,c*(b*x+a))/b^3+1/3*x^3*polylog(2,c*(b*x+a))
```

Rubi [A]
time = 0.22, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {6733, 45, 2463, 2436, 2332, 2442, 2440, 2438}

$$\frac{a^3\text{Li}_2(c(a+bx))}{3b^3}-\frac{a^2(-ac-bcx+1)\log(-ac-bcx+1)}{3b^3c}-\frac{a^2x}{3b^2}-\frac{(1-ac)^3\log(-ac-bcx+1)}{9b^3c^3}+\frac{a(1-ac)^2\log(-ac-bcx+1)}{6b^3c^2}-\frac{x(1-ac)^2}{9b^2c^2}+\frac{ax(1-ac)}{6b^2c}+\frac{1}{3}x^3\text{Li}_2(c(a+bx))+\frac{1}{9}x^3\log(-ac-bcx+1)-\frac{x^2(1-ac)}{18bc}-\frac{ax^2\log(-ac-bcx+1)}{6b}+\frac{ax^2}{12b}-\frac{x^3}{27}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] -1/3*(a^2*x)/b^2 + (a*(1 - a*c)*x)/(6*b^2*c) - ((1 - a*c)^2*x)/(9*b^2*c^2)
+ (a*x^2)/(12*b) - ((1 - a*c)*x^2)/(18*b*c) - x^3/27 + (a*(1 - a*c)^2*Log[1
 - a*c - b*c*x])/(6*b^3*c^2) - ((1 - a*c)^3*Log[1 - a*c - b*c*x])/(9*b^3*c^
3) - (a*x^2*Log[1 - a*c - b*c*x])/(6*b) + (x^3*Log[1 - a*c - b*c*x])/9 - (a
^2*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(3*b^3*c) + (a^3*PolyLog[2, c*(a
 + b*x)])/(3*b^3) + (x^3*PolyLog[2, c*(a + b*x)])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 6733

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Dist[b/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x))
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2 \mathrm{Li}_2(c(a+bx))\,dx = \frac{1}{3}x^3\mathrm{Li}_2(c(a+bx)) + \frac{1}{3}b\int \frac{x^3\log(1-ac-bcx)}{a+bx}\,dx$$
$$= \frac{1}{3}x^3\mathrm{Li}_2(c(a+bx)) + \frac{1}{3}b\int \left(\frac{a^2\log(1-ac-bcx)}{b^3} - \frac{ax\log(1-ac-bcx)}{b^2} + \frac{x^2\log(\right.$$
$$= \frac{1}{3}x^3\mathrm{Li}_2(c(a+bx)) + \frac{1}{3}\int x^2\log(1-ac-bcx)\,dx + \frac{a^2\int\log(1-ac-bcx)\,dx}{3b^2} - \frac{a^3}{}$$
$$= -\frac{ax^2\log(1-ac-bcx)}{6b} + \frac{1}{9}x^3\log(1-ac-bcx) + \frac{1}{3}x^3\mathrm{Li}_2(c(a+bx)) - \frac{a^3\mathrm{Subst}\Big(\int}{}$$
$$= -\frac{a^2x}{3b^2} - \frac{ax^2\log(1-ac-bcx)}{6b} + \frac{1}{9}x^3\log(1-ac-bcx) - \frac{a^2(1-ac-bcx)\log(1-}{3b^3c}$$
$$= -\frac{a^2x}{3b^2} + \frac{a(1-ac)x}{6b^2c} - \frac{(1-ac)^2x}{9b^2c^2} + \frac{ax^2}{12b} - \frac{(1-ac)x^2}{18bc} - \frac{x^3}{27} + \frac{a(1-ac)^2\log(1-a}{6b^3c^2}$$

Mathematica [A]
time = 0.14, size = 144, normalized size = 0.55

$$\frac{-bcx(12+66a^2c^2+6bcx+4b^2c^2x^2-3ac(14+5bcx))+6(-2+11a^3c^3+2b^3c^3x^3+6a^2c^2(-3+bcx)+a(9c-3b^2c^3x^2))\log(1-ac-bcx)+36c^3(a^3+b^3x^3)\,\mathrm{PolyLog}(2,c(a+bx))}{108b^3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] (-(b*c*x*(12 + 66*a^2*c^2 + 6*b*c*x + 4*b^2*c^2*x^2 - 3*a*c*(14 + 5*b*c*x))
) + 6*(-2 + 11*a^3*c^3 + 2*b^3*c^3*x^3 + 6*a^2*c^2*(-3 + b*c*x) + a*(9*c -
3*b^2*c^3*x^2))*Log[1 - a*c - b*c*x] + 36*c^3*(a^3 + b^3*x^3)*PolyLog[2, c*
(a + b*x)])/(108*b^3*c^3)
```

Maple [A]
time = 0.58, size = 361, normalized size = 1.39

method	result
derivativedivides	$\frac{-\frac{\mathrm{polylog}(2,xbc+ac)a^3c^3}{3}+\mathrm{polylog}(2,xbc+ac)a^2c^2(xbc+ac)-\mathrm{polylog}(2,xbc+ac)ac(xbc+ac)^2+\frac{\mathrm{polylog}(2,xbc+ac)(xbc+ac)^3}{3}-(}{}$
default	$\frac{-\frac{\mathrm{polylog}(2,xbc+ac)a^3c^3}{3}+\mathrm{polylog}(2,xbc+ac)a^2c^2(xbc+ac)-\mathrm{polylog}(2,xbc+ac)ac(xbc+ac)^2+\frac{\mathrm{polylog}(2,xbc+ac)(xbc+ac)^3}{3}-(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(2,c*(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3/b^3*(-1/3*polylog(2,b*c*x+a*c)*a^3*c^3+polylog(2,b*c*x+a*c)*a^2*c^2*(
b*c*x+a*c)-polylog(2,b*c*x+a*c)*a*c*(b*c*x+a*c)^2+1/3*polylog(2,b*c*x+a*c)*
```

```
(b*c*x+a*c)^3-((-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)-1+x*b*c+a*c)*a^2*c^2-(1/2*(-
b*c*x-a*c+1)^2*ln(-b*c*x-a*c+1)-1/4*(-b*c*x-a*c+1)^2)*a*c+((-b*c*x-a*c+1)*l
n(-b*c*x-a*c+1)-1+x*b*c+a*c)*a*c-1/9*(-b*c*x-a*c+1)^3*ln(-b*c*x-a*c+1)+1/27
*(-b*c*x-a*c+1)^3+1/3*(-b*c*x-a*c+1)^2*ln(-b*c*x-a*c+1)-1/6*(-b*c*x-a*c+1)^
2-1/3*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)+1/3-1/3*x*b*c-1/3*a*c+1/3*dilog(-b*c*
x-a*c+1)*a^3*c^3)
```

Maxima [A]
time = 0.26, size = 200, normalized size = 0.77

$$-\frac{\left(\log\left(bcx+ac\right)\log\left(-bcx-ac+1\right)+\mathrm{Li}_2(-bcx-ac+1)\right)a^3}{3\,b^3}+\frac{36\,b^3c^3x^3\mathrm{Li}_2(bcx+ac)-4\,b^3c^3x^3+3\,(5\,ab^2c^3-2\,b^2c^2)x^2-6\,(11\,a^2bc^3-7\,abc^2+2\,bc)x+6\,(2\,b^3c^3x^3-3\,ab^2c^3x^2+6\,a^2bc^3x+11\,a^3c^3-18\,a^2c^2+9\,ac-2)\log\left(-bcx-ac+1\right)}{108\,b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/3*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a^3
/b^3 + 1/108*(36*b^3*c^3*x^3*dilog(b*c*x + a*c) - 4*b^3*c^3*x^3 + 3*(5*a*b^
2*c^3 - 2*b^2*c^2)*x^2 - 6*(11*a^2*b*c^3 - 7*a*b*c^2 + 2*b*c)*x + 6*(2*b^3*
c^3*x^3 - 3*a*b^2*c^3*x^2 + 6*a^2*b*c^3*x + 11*a^3*c^3 - 18*a^2*c^2 + 9*a*c
 - 2)*log(-b*c*x - a*c + 1))/(b^3*c^3)
```

Fricas [A]
time = 0.35, size = 165, normalized size = 0.63

$$-\frac{4\,b^3c^3x^3-3\,(5\,ab^2c^3-2\,b^2c^2)x^2+6\,(11\,a^2bc^3-7\,abc^2+2\,bc)x-36\,(b^3c^3x^3+a^3c^3)\mathrm{Li}_2(bcx+ac)-6\,(2\,b^3c^3x^3-3\,ab^2c^3x^2+6\,a^2bc^3x+11\,a^3c^3-18\,a^2c^2+9\,ac-2)\log\left(-bcx-ac+1\right)}{108\,b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/108*(4*b^3*c^3*x^3 - 3*(5*a*b^2*c^3 - 2*b^2*c^2)*x^2 + 6*(11*a^2*b*c^3 -
 7*a*b*c^2 + 2*b*c)*x - 36*(b^3*c^3*x^3 + a^3*c^3)*dilog(b*c*x + a*c) - 6*(
2*b^3*c^3*x^3 - 3*a*b^2*c^3*x^2 + 6*a^2*b*c^3*x + 11*a^3*c^3 - 18*a^2*c^2 +
 9*a*c - 2)*log(-b*c*x - a*c + 1))/(b^3*c^3)
```

Sympy [A]
time = 6.86, size = 235, normalized size = 0.90

$$\begin{cases} 0 & \text{for}\: b = 0 \wedge c = 0 \\ \frac{x^{3}\,\mathrm{Li}_{2}\left(ac\right)}{3} & \text{for}\: b = 0 \\ 0 & \text{for}\: c = 0 \\ -\frac{11a^{3}\,\mathrm{Li}_{1}\left(ac+bcx\right)}{18b^{3}}+\frac{a^{3}\,\mathrm{Li}_{2}\left(ac+bcx\right)}{3b^{3}}-\frac{a^{2}x\,\mathrm{Li}_{1}\left(ac+bcx\right)}{3b^{2}}-\frac{11a^{2}x}{18b^{2}}+\frac{a^{2}\,\mathrm{Li}_{1}\left(ac+bcx\right)}{b^{3}c}+\frac{ax^{2}\,\mathrm{Li}_{1}\left(ac+bcx\right)}{6b}+\frac{5ax^{2}}{36b}+\frac{7ax}{18b^{2}c}-\frac{a\,\mathrm{Li}_{1}\left(ac+bcx\right)}{2b^{3}c^{2}}-\frac{x^{3}\,\mathrm{Li}_{1}\left(ac+bcx\right)}{9}+\frac{x^{3}\,\mathrm{Li}_{2}\left(ac+bcx\right)}{3}-\frac{x^{3}}{27}-\frac{x^{2}}{18bc}-\frac{x}{9b^{2}c^{2}}+\frac{\mathrm{Li}_{1}\left(ac+bcx\right)}{9b^{3}c^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*polylog(2,c*(b*x+a)),x)
```

```
[Out] Piecewise((0, Eq(b, 0) & Eq(c, 0)), (x**3*polylog(2, a*c)/3, Eq(b, 0)), (0,
 Eq(c, 0)), (-11*a**3*polylog(1, a*c + b*c*x)/(18*b**3) + a**3*polylog(2, a
```

```
*c + b*c*x)/(3*b**3) - a**2*x*polylog(1, a*c + b*c*x)/(3*b**2) - 11*a**2*x/
(18*b**2) + a**2*polylog(1, a*c + b*c*x)/(b**3*c) + a*x**2*polylog(1, a*c +
 b*c*x)/(6*b) + 5*a*x**2/(36*b) + 7*a*x/(18*b**2*c) - a*polylog(1, a*c + b*
c*x)/(2*b**3*c**2) - x**3*polylog(1, a*c + b*c*x)/9 + x**3*polylog(2, a*c +
 b*c*x)/3 - x**3/27 - x**2/(18*b*c) - x/(9*b**2*c**2) + polylog(1, a*c + b*
c*x)/(9*b**3*c**3), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,c*(b*x+a)),x, algorithm="giac")
[Out] integrate(x^2*dilog((b*x + a)*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{polylog}(2,c\,(a+b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(2, c*(a + b*x)),x)
[Out] int(x^2*polylog(2, c*(a + b*x)), x)
```

3.125 $\int x\mathbf{PolyLog}(2, c(a+bx))\, dx$

Optimal. Leaf size=152

$$\frac{ax}{2b}-\frac{(1-ac)x}{4bc}-\frac{x^2}{8}-\frac{(1-ac)^2\log(1-ac-bcx)}{4b^2c^2}+\frac{1}{4}x^2\log(1-ac-bcx)+\frac{a(1-ac-bcx)\log(1-ac-bcx)}{2b^2c}$$

[Out] 1/2*a*x/b-1/4*(-a*c+1)*x/b/c-1/8*x^2-1/4*(-a*c+1)^2*ln(-b*c*x-a*c+1)/b^2/c^2+1/4*x^2*ln(-b*c*x-a*c+1)+1/2*a*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b^2/c-1/2*a^2*polylog(2,c*(b*x+a))/b^2+1/2*x^2*polylog(2,c*(b*x+a))

Rubi [A]
time = 0.12, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {6733, 45, 2463, 2436, 2332, 2442, 2440, 2438}

$$-\frac{a^2\text{Li}_2(c(a+bx))}{2b^2}-\frac{(1-ac)^2\log(-ac-bcx+1)}{4b^2c^2}+\frac{a(-ac-bcx+1)\log(-ac-bcx+1)}{2b^2c}+\frac{1}{2}x^2\text{Li}_2(c(a+bx))+\frac{1}{4}x^2\log(-ac-bcx+1)-\frac{x(1-ac)}{4bc}+\frac{ax}{2b}-\frac{x^2}{8}$$

Antiderivative was successfully verified.

[In] Int[x*PolyLog[2, c*(a + b*x)],x]

[Out] (a*x)/(2*b) - ((1 - a*c)*x)/(4*b*c) - x^2/8 - ((1 - a*c)^2*Log[1 - a*c - b*c*x])/(4*b^2*c^2) + (x^2*Log[1 - a*c - b*c*x])/4 + (a*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(2*b^2*c) - (a^2*PolyLog[2, c*(a + b*x)])/(2*b^2) + (x^2*PolyLog[2, c*(a + b*x)])/2

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 6733

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Dist[b/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x))
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\text{Li}_2(c(a+bx))\,dx &= \frac{1}{2}x^2\text{Li}_2(c(a+bx))+\frac{1}{2}b\int\frac{x^2\log(1-ac-bcx)}{a+bx}\,dx\\ &= \frac{1}{2}x^2\text{Li}_2(c(a+bx))+\frac{1}{2}b\int\left(-\frac{a\log(1-ac-bcx)}{b^2}+\frac{x\log(1-ac-bcx)}{b}+\frac{a^2\log(}{b^2}\right.\\ &= \frac{1}{2}x^2\text{Li}_2(c(a+bx))+\frac{1}{2}\int x\log(1-ac-bcx)\,dx-\frac{a\int\log(1-ac-bcx)\,dx}{2b}+\frac{a^2\int}{}\\ &= \frac{1}{4}x^2\log(1-ac-bcx)+\frac{1}{2}x^2\text{Li}_2(c(a+bx))+\frac{a^2\text{Subst}\left(\int\frac{\log(1-cx)}{x}\,dx,x,a+bx\right)}{2b^2}+\\ &= \frac{ax}{2b}+\frac{1}{4}x^2\log(1-ac-bcx)+\frac{a(1-ac-bcx)\log(1-ac-bcx)}{2b^2c}-\frac{a^2\text{Li}_2(c(a+bx}{2b^2}\\ &= \frac{ax}{2b}-\frac{(1-ac)x}{4bc}-\frac{x^2}{8}-\frac{(1-ac)^2\log(1-ac-bcx)}{4b^2c^2}+\frac{1}{4}x^2\log(1-ac-bcx)+\frac{a(}{}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 96, normalized size = 0.63

$$\frac{-bcx(2-6ac+bcx)+(-2-6a^2c^2+2b^2c^2x^2-4ac(-2+bcx))\log(1-ac-bcx)-4c^2(a^2-b^2x^2)\,\text{PolyLog}(2,c(a+bx))}{8b^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*PolyLog[2, c*(a + b*x)],x]
[Out] (-(b*c*x*(2 - 6*a*c + b*c*x)) + (-2 - 6*a^2*c^2 + 2*b^2*c^2*x^2 - 4*a*c*(-2
 + b*c*x))*Log[1 - a*c - b*c*x] - 4*c^2*(a^2 - b^2*x^2)*PolyLog[2, c*(a + b
*x)])/(8*b^2*c^2)
```

Maple [A]
time = 0.57, size = 166, normalized size = 1.09

method	result
derivativedivides	$\frac{-\text{polylog}(2,xbc+ac)ac(xbc+ac)+\frac{\text{polylog}(2,xbc+ac)(xbc+ac)^2}{2}+((-xbc-ac+1)\ln(-xbc-ac+1)-1+xbc+ac)ac+\frac{(-xbc-}{}}{c^2b^2}$
default	$\frac{-\text{polylog}(2,xbc+ac)ac(xbc+ac)+\frac{\text{polylog}(2,xbc+ac)(xbc+ac)^2}{2}+((-xbc-ac+1)\ln(-xbc-ac+1)-1+xbc+ac)ac+\frac{(-xbc-}{}}{c^2b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(2,c*(b*x+a)),x,method=_RETURNVERBOSE)
[Out] 1/c^2/b^2*(-polylog(2,b*c*x+a*c)*a*c*(b*c*x+a*c)+1/2*polylog(2,b*c*x+a*c)*(
b*c*x+a*c)^2+((-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)-1+x*b*c+a*c)*a*c+1/4*(-b*c*x-
a*c+1)^2*ln(-b*c*x-a*c+1)-1/8*(-b*c*x-a*c+1)^2-1/2*(-b*c*x-a*c+1)*ln(-b*c*x
-a*c+1)+1/2-1/2*x*b*c-1/2*a*c)
```

Maxima [A]
time = 0.26, size = 145, normalized size = 0.95

$$\frac{(\log(bcx+ac)\log(-bcx-ac+1)+\mathrm{Li}_2(-bcx-ac+1))a^2}{2\,b^2}+\frac{4\,b^2c^2x^2\mathrm{Li}_2(bcx+ac)-b^2c^2x^2+2\,(3\,abc^2-bc)x+2\,(b^2c^2x^2-2\,abc^2x-3\,a^2c^2+4\,ac-1)\log(-bcx-ac+1)}{8\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a^2/
b^2 + 1/8*(4*b^2*c^2*x^2*dilog(b*c*x + a*c) - b^2*c^2*x^2 + 2*(3*a*b*c^2 -
b*c)*x + 2*(b^2*c^2*x^2 - 2*a*b*c^2*x - 3*a^2*c^2 + 4*a*c - 1)*log(-b*c*x -
 a*c + 1))/(b^2*c^2)
```

Fricas [A]
time = 0.35, size = 110, normalized size = 0.72

$$-\frac{b^2c^2x^2-2\,(3\,abc^2-bc)x-4\,(b^2c^2x^2-a^2c^2)\mathrm{Li}_2(bcx+ac)-2\,(b^2c^2x^2-2\,abc^2x-3\,a^2c^2+4\,ac-1)\log(-bcx-ac+1)}{8\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/8*(b^2*c^2*x^2 - 2*(3*a*b*c^2 - b*c)*x - 4*(b^2*c^2*x^2 - a^2*c^2)*dilog
(b*c*x + a*c) - 2*(b^2*c^2*x^2 - 2*a*b*c^2*x - 3*a^2*c^2 + 4*a*c - 1)*log(-
b*c*x - a*c + 1))/(b^2*c^2)
```

Sympy [A]
time = 2.47, size = 153, normalized size = 1.01

$$\begin{cases} 0 & \text{for}\: b = 0 \wedge c = 0 \\ \frac{x^2\,\mathrm{Li}_2(ac)}{2} & \text{for}\: b = 0 \\ 0 & \text{for}\: c = 0 \\ \frac{3a^2\,\mathrm{Li}_1(ac+bcx)}{4b^2} - \frac{a^2\,\mathrm{Li}_2(ac+bcx)}{2b^2} + \frac{ax\,\mathrm{Li}_1(ac+bcx)}{2b} + \frac{3ax}{4b} - \frac{a\,\mathrm{Li}_1(ac+bcx)}{b^2c} - \frac{x^2\,\mathrm{Li}_1(ac+bcx)}{4} + \frac{x^2\,\mathrm{Li}_2(ac+bcx)}{2} - \frac{x^2}{8} - \frac{x}{4bc} + \frac{\mathrm{Li}_1(ac+bcx)}{4b^2c^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,c*(b*x+a)),x)
```

```
[Out] Piecewise((0, Eq(b, 0) & Eq(c, 0)), (x**2*polylog(2, a*c)/2, Eq(b, 0)), (0,
 Eq(c, 0)), (3*a**2*polylog(1, a*c + b*c*x)/(4*b**2) - a**2*polylog(2, a*c
+ b*c*x)/(2*b**2) + a*x*polylog(1, a*c + b*c*x)/(2*b) + 3*a*x/(4*b) - a*pol
ylog(1, a*c + b*c*x)/(b**2*c) - x**2*polylog(1, a*c + b*c*x)/4 + x**2*polyl
og(2, a*c + b*c*x)/2 - x**2/8 - x/(4*b*c) + polylog(1, a*c + b*c*x)/(4*b**2
*c**2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*dilog((b*x + a)*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{polylog}(2,c\,(a+b\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(2, c*(a + b*x)),x)
```

```
[Out] int(x*polylog(2, c*(a + b*x)), x)
```

3.126 $\int \textbf{PolyLog}(2, c(a+bx))\,dx$

Optimal. Leaf size=60

$$-x-\frac{(1-ac-bcx)\log(1-ac-bcx)}{bc}+\frac{a\text{PolyLog}(2,c(a+bx))}{b}+x\text{PolyLog}(2,c(a+bx))$$

```
[Out] -x-(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b/c+a*polylog(2,c*(b*x+a))/b+x*polylog(2
,c*(b*x+a))
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {6730, 2494, 2436, 2332, 2468, 2440, 2438}

$$x\text{Li}_2(c(a+bx))+\frac{a\text{Li}_2(c(a+bx))}{b}-\frac{(-ac-bcx+1)\log(-ac-bcx+1)}{bc}-x$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)],x]
```

```
[Out] -x - ((1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c) + (a*PolyLog[2, c*(a +
b*x)])/b + x*PolyLog[2, c*(a + b*x)]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2468

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_)^(q_.), x_Symbol] :> In
t[ExpandToSum[u, x]^q*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a,
 b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] &&  !(BinomialMatch
Q[u, x] && LinearMatchQ[v, x])
```

Rule 2494

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rule 6730

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :> Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c*(a + b*x)^p], x]
, x] + Dist[a*p, Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /; Fr
eeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \text{Li}_2(c(a+bx))\,dx &= x\text{Li}_2(c(a+bx)) - a\int \frac{\log(1-c(a+bx))}{a+bx}\,dx + \int \log(1-c(a+bx))\,dx \\
&= x\text{Li}_2(c(a+bx)) - a\int \frac{\log(1-ac-bcx)}{a+bx}\,dx + \int \log(1-ac-bcx)\,dx \\
&= x\text{Li}_2(c(a+bx)) - \frac{a\text{Subst}\left(\int \frac{\log(1-cx)}{x}\,dx, x, a+bx\right)}{b} - \frac{\text{Subst}(\int \log(x)\,dx, x, 1-ac-}{bc} \\
&= -x - \frac{(1-ac-bcx)\log(1-ac-bcx)}{bc} + \frac{a\text{Li}_2(c(a+bx))}{b} + x\text{Li}_2(c(a+bx))
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.88

$$\frac{-c(a+bx)+(-1+c(a+bx))\log(1-c(a+bx))+c(a+bx)\text{PolyLog}(2,c(a+bx))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)],x]
```

```
[Out] (-(c*(a + b*x)) + (-1 + c*(a + b*x))*Log[1 - c*(a + b*x)] + c*(a + b*x)*Pol
yLog[2, c*(a + b*x)])/(b*c)
```

Maple [A]
time = 0.42, size = 63, normalized size = 1.05

method	result	size
derivativedivides	$\frac{(xbc+ac)\,\mathrm{polylog}(2,xbc+ac)-(-xbc-ac+1)\ln(-xbc-ac+1)+1-xbc-ac}{bc}$	63
default	$\frac{(xbc+ac)\,\mathrm{polylog}(2,xbc+ac)-(-xbc-ac+1)\ln(-xbc-ac+1)+1-xbc-ac}{bc}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,c*(b*x+a)),x,method=_RETURNVERBOSE)
[Out] 1/b/c*((b*c*x+a*c)*polylog(2,b*c*x+a*c)-(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)+1-x
*b*c-a*c)
```

Maxima [A]
time = 0.26, size = 90, normalized size = 1.50

$$-\frac{(\log(bcx+ac)\log(-bcx-ac+1)+\mathrm{Li}_2(-bcx-ac+1))a}{b}+\frac{bcx\mathrm{Li}_2(bcx+ac)-bcx+(bcx+ac-1)\log(-bcx-ac+1)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a)),x, algorithm="maxima")
[Out] -(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a/b + (
b*c*x*dilog(b*c*x + a*c) - b*c*x + (b*c*x + a*c - 1)*log(-b*c*x - a*c + 1))
/(b*c)
```

Fricas [A]
time = 0.35, size = 55, normalized size = 0.92

$$-\frac{bcx-(bcx+ac)\mathrm{Li}_2(bcx+ac)-(bcx+ac-1)\log(-bcx-ac+1)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a)),x, algorithm="fricas")
[Out] -(b*c*x - (b*c*x + a*c)*dilog(b*c*x + a*c) - (b*c*x + a*c - 1)*log(-b*c*x -
 a*c + 1))/(b*c)
```

Sympy [A]
time = 0.93, size = 75, normalized size = 1.25

$$\begin{cases} 0 & \text{for}\: c=0 \wedge (b=0 \vee c=0) \\ x\,\mathrm{Li}_2(ac) & \text{for}\: b=0 \\ -\frac{a\,\mathrm{Li}_1(ac+bcx)}{b}+\frac{a\,\mathrm{Li}_2(ac+bcx)}{b}-x\,\mathrm{Li}_1(ac+bcx)+x\,\mathrm{Li}_2(ac+bcx)-x+\frac{\mathrm{Li}_1(ac+bcx)}{bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a)),x)
```

```
[Out] Piecewise((0, Eq(c, 0) & (Eq(b, 0) | Eq(c, 0))), (x*polylog(2, a*c), Eq(b,
0)), (-a*polylog(1, a*c + b*c*x)/b + a*polylog(2, a*c + b*c*x)/b - x*polylo
g(1, a*c + b*c*x) + x*polylog(2, a*c + b*c*x) - x + polylog(1, a*c + b*c*x)
/(b*c), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(dilog((b*x + a)*c), x)
```

Mupad [B]
time = 0.59, size = 61, normalized size = 1.02

$$\frac{\mathrm{polylog}(2, c\,(a+b\,x))\;(a+b\,x)}{b}-x-\frac{\ln\left(1-c\,(a+b\,x)\right)}{b\,c}+\frac{\ln\left(1-c\,(a+b\,x)\right)\;(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, c*(a + b*x)),x)
```

```
[Out] (polylog(2, c*(a + b*x))*(a + b*x))/b - x - log(1 - c*(a + b*x))/(b*c) + (l
og(1 - c*(a + b*x))*(a + b*x))/b
```

3.127 $\int \frac{\text{PolyLog}(2,c(a+bx))}{x}\,dx$

Optimal. Leaf size=401

$$\log(x)\log\left(1+\frac{bx}{a}\right)\log(1-c(a+bx))+\frac{1}{2}\left(\log\left(1+\frac{bx}{a}\right)+\log\left(\frac{1-ac}{1-c(a+bx)}\right)-\log\left(\frac{(1-ac)(a+bx)}{a(1-c(a+bx))}\right)\right)$$

```
[Out] ln(x)*ln(1+b*x/a)*ln(1-c*(b*x+a))+1/2*(ln(1+b*x/a)+ln((-a*c+1)/(1-c*(b*x+a)
))-ln((-a*c+1)*(b*x+a)/a/(1-c*(b*x+a))))*ln(-a*(1-c*(b*x+a))/b/x)^2+1/2*(ln
(c*(b*x+a))-ln(1+b*x/a))*(ln(x)+ln(-a*(1-c*(b*x+a))/b/x))^2+(ln(1-c*(b*x+a)
)-ln(-a*(1-c*(b*x+a))/b/x))*polylog(2,-b*x/a)+ln(x)*polylog(2,c*(b*x+a))+ln
(-a*(1-c*(b*x+a))/b/x)*polylog(2,-b*x/a/(1-c*(b*x+a)))-ln(-a*(1-c*(b*x+a))/
b/x)*polylog(2,-b*c*x/(1-c*(b*x+a)))+(ln(x)+ln(-a*(1-c*(b*x+a))/b/x))*polyl
og(2,1-c*(b*x+a))-polylog(3,-b*x/a)+polylog(3,-b*x/a/(1-c*(b*x+a)))-polylog
(3,-b*c*x/(1-c*(b*x+a)))-polylog(3,1-c*(b*x+a))
```

Rubi [A]
time = 0.24, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6732, 2490, 2485}

$$\text{Li}_3\left(-\frac{bx}{a(1-c(a+bx))}\right)-\text{Li}_3\left(-\frac{bcx}{1-c(a+bx)}\right)-\text{Li}_3(1-c(a+bx))+\text{Li}_2\left(-\frac{bx}{a(1-c(a+bx))}\right)\log\left(-\frac{a(1-c(a+bx))}{bx}\right)-\text{Li}_2\left(-\frac{bcx}{1-c(a+bx)}\right)\log\left(-\frac{a(1-c(a+bx))}{bx}\right)+\text{Li}_2\left(-\frac{bx}{a}\right)\left(\log(1-c(a+bx))-\log\left(-\frac{a(1-c(a+bx))}{bx}\right)\right)+\log(x)\text{Li}_2(c(a+bx))+\text{Li}_2(1-c(a+bx))\left(\log\left(-\frac{a(1-c(a+bx))}{bx}\right)+\log(x)\right)+\frac{1}{2}\left(\log\left(\frac{1-ac}{1-c(a+bx)}\right)-\log\left(\frac{(1-ac)(a+bx)}{a(1-c(a+bx))}\right)+\log\left(\frac{bx}{a}+1\right)\right)\log^2\left(-\frac{a(1-c(a+bx))}{bx}\right)+\frac{1}{2}\left(\log(c(a+bx))-\log\left(\frac{bx}{a}+1\right)\right)\left(\log\left(-\frac{a(1-c(a+bx))}{bx}\right)+\log(x)\right)^2+\log(x)\log\left(\frac{bx}{a}+1\right)\log(1-c(a+bx))-\text{Li}_3\left(-\frac{bx}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/x,x]
```

```
[Out] Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*x)] + ((Log[1 + (b*x)/a] + Log[(1
- a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))
])*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/2 + ((Log[c*(a + b*x)] - Log[1 +
(b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/2 + (Log[1 - c*
(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*x)/a)] +
Log[x]*PolyLog[2, c*(a + b*x)] + Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLo
g[2, -((b*x)/(a*(1 - c*(a + b*x))))] - Log[-((a*(1 - c*(a + b*x)))/(b*x))]*
PolyLog[2, -((b*c*x)/(1 - c*(a + b*x)))] + (Log[x] + Log[-((a*(1 - c*(a + b
*x)))/(b*x))])*PolyLog[2, 1 - c*(a + b*x)] - PolyLog[3, -((b*x)/a)] + PolyL
og[3, -((b*x)/(a*(1 - c*(a + b*x))))] - PolyLog[3, -((b*c*x)/(1 - c*(a + b*
x)))] - PolyLog[3, 1 - c*(a + b*x)]
```

Rule 2485

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[(-b)*(x/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1/2)*(Log[(-b)*(x/a
)] - Log[(-(b*c - a*d))*(x/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))]
)*Log[a*((c + d*x)/(c*(a + b*x)))]^2, x] - Simp[(1/2)*(Log[(-b)*(x/a)] - Lo
g[(-d)*(x/c)])*(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])^2, x] + Si
mp[(Log[c + d*x] - Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1 + b*(x/a)
```

```
], x] + Simp[(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1
 + d*(x/c)], x] + Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2, c*((a +
b*x)/(a*(c + d*x)))], x] - Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2,
 d*((a + b*x)/(b*(c + d*x)))], x] - Simp[PolyLog[3, 1 + b*(x/a)], x] - Simp
[PolyLog[3, 1 + d*(x/c)], x] + Simp[PolyLog[3, c*((a + b*x)/(a*(c + d*x)))]
, x] - Simp[PolyLog[3, d*((a + b*x)/(b*(c + d*x)))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2490

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-(e*k - d*l)/l + e*(x/l))^n])*(f + g
*Log[h*(-(j*k - i*l)/l + j*(x/l))^m]), x], x, k + l*x], x] /; FreeQ[{a, b,
c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 6732

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[d + e*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, Int[Log[d
+ e*x]*(Log[1 - a*c - b*c*x]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_2(c(a+bx))}{x}\,dx &= \log(x)\text{Li}_2(c(a+bx)) + b\int \frac{\log(x)\log(1-ac-bcx)}{a+bx}\,dx\\ &= \log(x)\text{Li}_2(c(a+bx)) + \text{Subst}\left(\int \frac{\log\left(-\frac{a}{b}+\frac{x}{b}\right)\log\left(-\frac{-abc-b(1-ac)}{b}-cx\right)}{x}\,dx, x, a+\right.\\ &= \log(x)\log\left(1+\frac{bx}{a}\right)\log(1-c(a+bx)) + \frac{1}{2}\left(\log\left(1+\frac{bx}{a}\right)+\log\left(\frac{1-ac}{1-c(a+bx)}\right)\right.\end{aligned}$$

Mathematica [A]
time = 0.09, size = 422, normalized size = 1.05

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)]/x,x]
```

```
[Out] Log[x]*Log[1 + (b*x)/a]*Log[1 - a*c - b*c*x] + ((-Log[c*(a + b*x)] + Log[1
+ (b*x)/a])*Log[1 - a*c - b*c*x]*(-2*Log[x] + Log[1 - a*c - b*c*x]))/2 + (L
og[c*(a + b*x)] - Log[1 + (b*x)/a])*Log[1 - a*c - b*c*x]*Log[(a*(-1 + a*c +
 b*c*x))/(b*x)] + ((Log[(1 - a*c)/(b*c*x)] - Log[((1 - a*c)*(a + b*x))/(b*x
)] + Log[1 + (b*x)/a])*Log[(a*(-1 + a*c + b*c*x))/(b*x)]^2)/2 + (Log[1 - a*
c - b*c*x] - Log[(a*(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, -((b*x)/a)] + (L
og[x] + Log[(a*(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, 1 - a*c - b*c*x] + Lo
g[(a*(-1 + a*c + b*c*x))/(b*x)]*(-PolyLog[2, (a*(-1 + a*c + b*c*x))/(b*x)]
+ PolyLog[2, (-1 + a*c + b*c*x)/(b*c*x)]) + Log[x]*PolyLog[2, a*c + b*c*x]
- PolyLog[3, -((b*x)/a)] - PolyLog[3, 1 - a*c - b*c*x] + PolyLog[3, (a*(-1
+ a*c + b*c*x))/(b*x)] - PolyLog[3, (-1 + a*c + b*c*x)/(b*c*x)]
```

Maple [F]
time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{polylog}\left(2, c(bx+a)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,c*(b*x+a))/x,x)
```

```
[Out] int(polylog(2,c*(b*x+a))/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(dilog((b*x + a)*c)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(dilog(b*c*x + a*c)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_{2}\left(ac + bcx\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x,x)
```

```
[Out] Integral(polylog(2, a*c + b*c*x)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(dilog((b*x + a)*c)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{polylog}(2,c\,(a+b\,x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, c*(a + b*x))/x,x)
```

```
[Out] int(polylog(2, c*(a + b*x))/x, x)
```

3.128 $\int \frac{\text{PolyLog}(2,c(a+bx))}{x^2}\,dx$

Optimal. Leaf size=84

$$-\frac{b\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{a}-\frac{b\text{PolyLog}(2,c(a+bx))}{a}-\frac{\text{PolyLog}(2,c(a+bx))}{x}-\frac{b\text{PolyLog}\left(2,1-\frac{bcx}{1-ac}\right)}{a}$$

```
[Out] -b*ln(b*c*x/(-a*c+1))*ln(-b*c*x-a*c+1)/a-b*polylog(2,c*(b*x+a))/a-polylog(2
,c*(b*x+a))/x-b*polylog(2,1-b*c*x/(-a*c+1))/a
```

Rubi [A]
time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {6733, 36, 29, 31, 2463, 2441, 2352, 2440, 2438}

$$-\frac{b\text{Li}_2(c(a+bx))}{a}-\frac{\text{Li}_2(c(a+bx))}{x}-\frac{b\text{Li}_2\left(1-\frac{bcx}{1-ac}\right)}{a}-\frac{b\log\left(\frac{bcx}{1-ac}\right)\log(-ac-bcx+1)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/x^2,x]
```

```
[Out] -((b*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/a) - (b*PolyLog[2, c*(a +
 b*x)])/a - PolyLog[2, c*(a + b*x)]/x - (b*PolyLog[2, 1 - (b*c*x)/(1 - a*c)
])/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 6733

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Dist[b/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x))
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{Li}_2(c(a+bx))}{x^2}\,dx &= -\frac{\text{Li}_2(c(a+bx))}{x} - b\int \frac{\log(1-ac-bcx)}{x(a+bx)}\,dx \\
&= -\frac{\text{Li}_2(c(a+bx))}{x} - b\int \left(\frac{\log(1-ac-bcx)}{ax} - \frac{b\log(1-ac-bcx)}{a(a+bx)}\right)dx \\
&= -\frac{\text{Li}_2(c(a+bx))}{x} - \frac{b\int \frac{\log(1-ac-bcx)}{x}\,dx}{a} + \frac{b^2\int \frac{\log(1-ac-bcx)}{a+bx}\,dx}{a} \\
&= -\frac{b\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{a} - \frac{\text{Li}_2(c(a+bx))}{x} + \frac{b\text{Subst}\left(\int \frac{\log(1-cx)}{x}\,dx, x, a+bx\right.}{a} \\
&= -\frac{b\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{a} - \frac{b\text{Li}_2(c(a+bx))}{a} - \frac{\text{Li}_2(c(a+bx))}{x} - \frac{b\text{Li}_2\left(1-\frac{bcx}{1-a}\right.}{a}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 73, normalized size = 0.87

$$-\frac{(a+bx)\text{PolyLog}(2,c(a+bx))+bx\left(\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)+\text{PolyLog}\left(2,\frac{-1+ac+bcx}{-1+ac}\right)\right)}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)]/x^2,x]
```

```
[Out] -(((a + b*x)*PolyLog[2, c*(a + b*x)] + b*x*(Log[(b*c*x)/(1 - a*c)]*Log[1 -
a*c - b*c*x] + PolyLog[2, (-1 + a*c + b*c*x)/(-1 + a*c)]))/(a*x))
```

Maple [A]
time = 1.31, size = 94, normalized size = 1.12

method	result	size
derivativedivides	$cb\left(-\frac{\text{polylog}(2,xbc+ac)}{xbc}-\frac{\text{dilog}\left(-\frac{xbc}{ac-1}\right)+\ln(-xbc-ac+1)\ln\left(-\frac{xbc}{ac-1}\right)}{ac}-\frac{\text{dilog}(-xbc-ac+1)}{ac}\right)$	94
default	$cb\left(-\frac{\text{polylog}(2,xbc+ac)}{xbc}-\frac{\text{dilog}\left(-\frac{xbc}{ac-1}\right)+\ln(-xbc-ac+1)\ln\left(-\frac{xbc}{ac-1}\right)}{ac}-\frac{\text{dilog}(-xbc-ac+1)}{ac}\right)$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,c*(b*x+a))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*b*(-1/x/b/c*polylog(2,b*c*x+a*c)-(dilog(-x*b*c/(a*c-1))+ln(-b*c*x-a*c+1)*
ln(-x*b*c/(a*c-1)))/a/c-dilog(-b*c*x-a*c+1)/a/c)
```

Maxima [A]
time = 0.28, size = 114, normalized size = 1.36

$$\frac{(\log(bcx+ac)\log(-bcx-ac+1)+\text{Li}_2(-bcx-ac+1))b}{a}-\frac{\left(\log(-bcx-ac+1)\log\left(-\frac{bcx+ac-1}{ac-1}+1\right)+\text{Li}_2\left(\frac{bcx+ac-1}{ac-1}\right)\right)b}{a}-\frac{\text{Li}_2(bcx+ac)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x^2,x, algorithm="maxima")
```

```
[Out] (log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*b/a - (l
og(-b*c*x - a*c + 1)*log(-(b*c*x + a*c - 1)/(a*c - 1) + 1) + dilog((b*c*x +
 a*c - 1)/(a*c - 1)))*b/a - dilog(b*c*x + a*c)/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x^2,x, algorithm="fricas")
```

```
[Out] integral(dilog(b*c*x + a*c)/x^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_2(ac + bcx)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x**2,x)
```

```
[Out] Integral(polylog(2, a*c + b*c*x)/x**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x^2,x, algorithm="giac")
```

```
[Out] integrate(dilog((b*x + a)*c)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(2, c\,(a + b\,x))}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, c*(a + b*x))/x^2,x)
```

```
[Out] int(polylog(2, c*(a + b*x))/x^2, x)
```

3.129 $\int \frac{\text{PolyLog}(2,c(a+bx))}{x^3}\,dx$

Optimal. Leaf size=173

$$\frac{b^2c\log(x)}{2a(1-ac)}-\frac{b^2c\log(1-ac-bcx)}{2a(1-ac)}+\frac{b\log(1-ac-bcx)}{2ax}+\frac{b^2\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2}+\frac{b^2\text{PolyLog}(2,c(a+$$

```
[Out] 1/2*b^2*c*ln(x)/a/(-a*c+1)-1/2*b^2*c*ln(-b*c*x-a*c+1)/a/(-a*c+1)+1/2*b*ln(-
b*c*x-a*c+1)/a/x+1/2*b^2*ln(b*c*x/(-a*c+1))*ln(-b*c*x-a*c+1)/a^2+1/2*b^2*po
lylog(2,c*(b*x+a))/a^2-1/2*polylog(2,c*(b*x+a))/x^2+1/2*b^2*polylog(2,1-b*c
*x/(-a*c+1))/a^2
```

Rubi [A]
time = 0.13, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {6733, 46, 2463, 2442, 36, 29, 31, 2441, 2352, 2440, 2438}

$$\frac{b^2\text{Li}_2(c(a+bx))}{2a^2}+\frac{b^2\text{Li}_2\left(1-\frac{bcx}{1-ac}\right)}{2a^2}+\frac{b^2\log\left(\frac{bcx}{1-ac}\right)\log(-ac-bcx+1)}{2a^2}+\frac{b^2c\log(x)}{2a(1-ac)}-\frac{b^2c\log(-ac-bcx+1)}{2a(1-ac)}-\frac{\text{Li}_2(c(a+bx))}{2x^2}+\frac{b\log(-ac-bcx+1)}{2ax}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/x^3,x]
```

```
[Out] (b^2*c*Log[x])/(2*a*(1 - a*c)) - (b^2*c*Log[1 - a*c - b*c*x])/(2*a*(1 - a*c
)) + (b*Log[1 - a*c - b*c*x])/(2*a*x) + (b^2*Log[(b*c*x)/(1 - a*c)]*Log[1 -
 a*c - b*c*x])/(2*a^2) + (b^2*PolyLog[2, c*(a + b*x)])/(2*a^2) - PolyLog[2,
 c*(a + b*x)]/(2*x^2) + (b^2*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(2*a^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 6733

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
```

```
Dist[b/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x))
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\mathrm{Li}_2(c(a+bx))}{x^3}\,dx = -\frac{\mathrm{Li}_2(c(a+bx))}{2x^2} - \frac{1}{2}b\int \frac{\log(1-ac-bcx)}{x^2(a+bx)}\,dx$$

$$= -\frac{\mathrm{Li}_2(c(a+bx))}{2x^2} - \frac{1}{2}b\int \left(\frac{\log(1-ac-bcx)}{ax^2} - \frac{b\log(1-ac-bcx)}{a^2x} + \frac{b^2\log(1-ac-\ldots}{a^2(a+bx\ldots}\right.$$

$$= -\frac{\mathrm{Li}_2(c(a+bx))}{2x^2} - \frac{b\int \frac{\log(1-ac-bcx)}{x^2}\,dx}{2a} + \frac{b^2\int \frac{\log(1-ac-bcx)}{x}\,dx}{2a^2} - \frac{b^3\int \frac{\log(1-ac-bcx)}{a+bx}\,dx}{2a^2}$$

$$= \frac{b\log(1-ac-bcx)}{2ax} + \frac{b^2\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2} - \frac{\mathrm{Li}_2(c(a+bx))}{2x^2} - \frac{b^2\mathrm{Subst}\left(\int\ldots\right.}{\ldots}$$

$$= \frac{b\log(1-ac-bcx)}{2ax} + \frac{b^2\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2} + \frac{b^2\mathrm{Li}_2(c(a+bx))}{2a^2} - \frac{\mathrm{Li}_2(c(a+\ldots}{2x^2}$$

$$= \frac{b^2c\log(x)}{2a(1-ac)} - \frac{b^2c\log(1-ac-bcx)}{2a(1-ac)} + \frac{b\log(1-ac-bcx)}{2ax} + \frac{b^2\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-\ldots}{2a^2}$$

Mathematica [A]

time = 0.11, size = 131, normalized size = 0.76

$$\frac{-\left((-1+ac)\left(a^2-b^2x^2\right)\mathrm{PolyLog}(2,c(a+bx))\right)+bx\left(-abcx\log(x)+\left(a(-1+ac+bcx)+b(-1+ac)x\log\left(\frac{bcx}{1-ac}\right)\right)\log(1-ac-bcx)+b(-1+ac)x\mathrm{PolyLog}\left(2,\frac{-1+ac+bcx}{-1+ac}\right)\right)}{2a^2(-1+ac)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)]/x^3,x]
```

```
[Out] (-((-1 + a*c)*(a^2 - b^2*x^2)*PolyLog[2, c*(a + b*x)]) + b*x*(-(a*b*c*x*Log
[x]) + (a*(-1 + a*c + b*c*x) + b*(-1 + a*c)*x*Log[(b*c*x)/(1 - a*c)])*Log[1
 - a*c - b*c*x] + b*(-1 + a*c)*x*PolyLog[2, (-1 + a*c + b*c*x)/(-1 + a*c)])
)/(2*a^2*(-1 + a*c)*x^2)
```

Maple [A]

time = 1.35, size = 163, normalized size = 0.94

method	result
derivativedivides	$c^2b^2\left(-\frac{\mathrm{polylog}(2,xbc+ac)}{2x^2b^2c^2}+\frac{\mathrm{dilog}(-xbc-ac+1)}{2a^2c^2}+\frac{\mathrm{dilog}\left(-\frac{xbc}{ac-1}\right)+\ln(-xbc-ac+1)\ln\left(-\frac{xbc}{ac-1}\right)}{2a^2c^2}+\frac{-\frac{\ln(-xbc)}{ac-1}-\ldots}{\ldots}\right.$
default	$c^2b^2\left(-\frac{\mathrm{polylog}(2,xbc+ac)}{2x^2b^2c^2}+\frac{\mathrm{dilog}(-xbc-ac+1)}{2a^2c^2}+\frac{\mathrm{dilog}\left(-\frac{xbc}{ac-1}\right)+\ln(-xbc-ac+1)\ln\left(-\frac{xbc}{ac-1}\right)}{2a^2c^2}+\frac{-\frac{\ln(-xbc)}{ac-1}-\ldots}{\ldots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,c*(b*x+a))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*b^2*(-1/2/x^2/b^2/c^2*polylog(2,b*c*x+a*c)+1/2*dilog(-b*c*x-a*c+1)/a^2/
c^2+1/2*(dilog(-x*b*c/(a*c-1))+ln(-b*c*x-a*c+1)*ln(-x*b*c/(a*c-1)))/a^2/c^2
+1/2*(-1/(a*c-1)*ln(-x*b*c)-ln(-b*c*x-a*c+1)*(-b*c*x-a*c+1)/(a*c-1)/x/b/c)/
a/c)
```

Maxima [A]
time = 0.27, size = 193, normalized size = 1.12

$$-\frac{b^2 c \log (x)}{2 (a^2 c - a)} - \frac{(\log (bcx + ac) \log (-bcx - ac + 1) + \mathrm{Li}_2(-bcx - ac + 1))b^2}{2 a^2} + \frac{(\log (-bcx - ac + 1) \log \left(-\frac{bcx+ac-1}{ac-1} + 1\right) + \mathrm{Li}_2\left(\frac{bcx+ac-1}{ac-1}\right))b^2}{2 a^2} - \frac{(a^2c - a)\mathrm{Li}_2(bcx + ac) - (b^2cx^2 + (abc - b)x) \log (-bcx - ac + 1)}{2 (a^2c - a)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*c*log(x)/(a^2*c - a) - 1/2*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1)
 + dilog(-b*c*x - a*c + 1))*b^2/a^2 + 1/2*(log(-b*c*x - a*c + 1)*log(-(b*c*
x + a*c - 1)/(a*c - 1) + 1) + dilog((b*c*x + a*c - 1)/(a*c - 1)))*b^2/a^2 -
 1/2*((a^2*c - a)*dilog(b*c*x + a*c) - (b^2*c*x^2 + (a*b*c - b)*x)*log(-b*c
*x - a*c + 1))/((a^2*c - a)*x^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x^3,x, algorithm="fricas")
```

```
[Out] integral(dilog(b*c*x + a*c)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_2 \left(ac + bcx\right)}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x**3,x)
```

```
[Out] Integral(polylog(2, a*c + b*c*x)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x^3,x, algorithm="giac")
[Out] integrate(dilog((b*x + a)*c)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{polylog}(2, c\,(a+b\,x))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, c*(a + b*x))/x^3,x)
[Out] int(polylog(2, c*(a + b*x))/x^3, x)
```

3.130 $\int \frac{\text{PolyLog}(2,c(a+bx))}{x^4}\,dx$

Optimal. Leaf size=276

$$-\frac{b^2c}{6a(1-ac)x}+\frac{b^3c^2\log(x)}{6a(1-ac)^2}-\frac{b^3c\log(x)}{3a^2(1-ac)}-\frac{b^3c^2\log(1-ac-bcx)}{6a(1-ac)^2}+\frac{b^3c\log(1-ac-bcx)}{3a^2(1-ac)}+\frac{b\log(1-ac-b}{6ax^2}$$

```
[Out] -1/6*b^2*c/a/(-a*c+1)/x+1/6*b^3*c^2*ln(x)/a/(-a*c+1)^2-1/3*b^3*c*ln(x)/a^2/
(-a*c+1)-1/6*b^3*c^2*ln(-b*c*x-a*c+1)/a/(-a*c+1)^2+1/3*b^3*c*ln(-b*c*x-a*c+
1)/a^2/(-a*c+1)+1/6*b*ln(-b*c*x-a*c+1)/a/x^2-1/3*b^2*ln(-b*c*x-a*c+1)/a^2/x
-1/3*b^3*ln(b*c*x/(-a*c+1))*ln(-b*c*x-a*c+1)/a^3-1/3*b^3*polylog(2,c*(b*x+a
))/a^3-1/3*polylog(2,c*(b*x+a))/x^3-1/3*b^3*polylog(2,1-b*c*x/(-a*c+1))/a^3
```

Rubi [A]
time = 0.20, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {6733, 46, 2463, 2442, 36, 29, 31, 2441, 2352, 2440, 2438}

$$-\frac{b^3\text{Li}_2(c(a+bx))}{3a^3}-\frac{b^3\text{Li}_2\left(1-\frac{bcx}{1-ac}\right)}{3a^3}-\frac{b^3\log\left(\frac{bcx}{1-ac}\right)\log(-ac-bcx+1)}{3a^3}-\frac{b^3c\log(x)}{3a^2(1-ac)}+\frac{b^3c\log(-ac-bcx+1)}{3a^2(1-ac)}-\frac{b^2\log(-ac-bcx+1)}{3a^2x}+\frac{b^3c^2\log(x)}{6a(1-ac)^2}-\frac{b^3c^2\log(-ac-bcx+1)}{6a(1-ac)^2}-\frac{b^2c}{6ax(1-ac)}-\frac{\text{Li}_2(c(a+bx))}{3x^3}+\frac{b\log(-ac-bcx+1)}{6ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/x^4,x]
```

```
[Out] -1/6*(b^2*c)/(a*(1 - a*c)*x) + (b^3*c^2*Log[x])/(6*a*(1 - a*c)^2) - (b^3*c*
Log[x])/(3*a^2*(1 - a*c)) - (b^3*c^2*Log[1 - a*c - b*c*x])/(6*a*(1 - a*c)^2
) + (b^3*c*Log[1 - a*c - b*c*x])/(3*a^2*(1 - a*c)) + (b*Log[1 - a*c - b*c*x
])/(6*a*x^2) - (b^2*Log[1 - a*c - b*c*x])/(3*a^2*x) - (b^3*Log[(b*c*x)/(1 -
 a*c)]*Log[1 - a*c - b*c*x])/(3*a^3) - (b^3*PolyLog[2, c*(a + b*x)])/(3*a^3
) - PolyLog[2, c*(a + b*x)]/(3*x^3) - (b^3*PolyLog[2, 1 - (b*c*x)/(1 - a*c)
])/(3*a^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 6733

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Dist[b/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x))
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\text{Li}_2(c(a+bx))}{x^4}\,dx = -\frac{\text{Li}_2(c(a+bx))}{3x^3} - \frac{1}{3}b\int \frac{\log(1-ac-bcx)}{x^3(a+bx)}\,dx$$

$$= -\frac{\text{Li}_2(c(a+bx))}{3x^3} - \frac{1}{3}b\int \left(\frac{\log(1-ac-bcx)}{ax^3} - \frac{b\log(1-ac-bcx)}{a^2x^2} + \frac{b^2\log(1-ac}{a^3x}\right.$$

$$= -\frac{\text{Li}_2(c(a+bx))}{3x^3} - \frac{b\int \frac{\log(1-ac-bcx)}{x^3}\,dx}{3a} + \frac{b^2\int \frac{\log(1-ac-bcx)}{x^2}\,dx}{3a^2} - \frac{b^3\int \frac{\log(1-ac-bcx)}{x}\,dx}{3a^3}$$

$$= \frac{b\log(1-ac-bcx)}{6ax^2} - \frac{b^2\log(1-ac-bcx)}{3a^2x} - \frac{b^3\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{3a^3} - \text{Li}_2($$

$$= \frac{b\log(1-ac-bcx)}{6ax^2} - \frac{b^2\log(1-ac-bcx)}{3a^2x} - \frac{b^3\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{3a^3} - b^3\text{Li}$$

$$= -\frac{b^2c}{6a(1-ac)x} + \frac{b^3c^2\log(x)}{6a(1-ac)^2} - \frac{b^3c\log(x)}{3a^2(1-ac)} - \frac{b^3c^2\log(1-ac-bcx)}{6a(1-ac)^2} + \frac{b^3c\log(1-}{3a^2(1}$$

Mathematica [A]
time = 0.21, size = 210, normalized size = 0.76

$$-\frac{b\left(-\frac{2ab^2c(\log(x)-\log(1-ac-bcx))}{-1+ac} - \frac{a^2\log(1-ac-bcx)}{x^2} + \frac{2ab\log(1-ac-bcx)}{x} + 2b^2\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx) - \frac{a^2bc(-1+ac+bcx\log(x)-bcx\log(1-ac-bcx))}{(-1+ac)^2x} + 2b^2\text{PolyLog}(2,c(a+bx)) + 2b^2\text{PolyLog}\left(2,\frac{-1+ac+bcx}{-1+ac}\right)\right)}{6a^3} - \frac{\text{PolyLog}(2,ac+bcx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)]/x^4,x]
```

```
[Out] -1/6*(b*((-2*a*b^2*c*(Log[x] - Log[1 - a*c - b*c*x]))/(-1 + a*c) - (a^2*Log
[1 - a*c - b*c*x])/x^2 + (2*a*b*Log[1 - a*c - b*c*x])/x + 2*b^2*Log[(b*c*x)
/(1 - a*c)]*Log[1 - a*c - b*c*x] - (a^2*b*c*(-1 + a*c + b*c*x*Log[x] - b*c*
x*Log[1 - a*c - b*c*x]))/((-1 + a*c)^2*x) + 2*b^2*PolyLog[2, c*(a + b*x)] +
 2*b^2*PolyLog[2, (-1 + a*c + b*c*x)/(-1 + a*c)]))/a^3 - PolyLog[2, a*c + b
*c*x]/(3*x^3)
```

Maple [A]
time = 1.34, size = 272, normalized size = 0.99

method	result
derivativedivides	$c^3b^3\left(-\frac{\text{polylog}(2,xbc+ac)}{3x^3b^3c^3}-\frac{\text{dilog}\left(-\frac{xbc}{ac-1}\right)+\ln(-xbc-ac+1)\ln\left(-\frac{xbc}{ac-1}\right)}{3a^3c^3}-\frac{\text{dilog}(-xbc-ac+1)}{3a^3c^3}-\frac{-\frac{\ln(-xbc)}{2(ac-1)^2}-}{}\right.$
default	$c^3b^3\left(-\frac{\text{polylog}(2,xbc+ac)}{3x^3b^3c^3}-\frac{\text{dilog}\left(-\frac{xbc}{ac-1}\right)+\ln(-xbc-ac+1)\ln\left(-\frac{xbc}{ac-1}\right)}{3a^3c^3}-\frac{\text{dilog}(-xbc-ac+1)}{3a^3c^3}-\frac{-\frac{\ln(-xbc)}{2(ac-1)^2}-}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,c*(b*x+a))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*b^3*(-1/3/x^3/b^3/c^3*polylog(2,b*c*x+a*c)-1/3*(dilog(-x*b*c/(a*c-1))+l
n(-b*c*x-a*c+1)*ln(-x*b*c/(a*c-1)))/a^3/c^3-1/3*dilog(-b*c*x-a*c+1)/a^3/c^3
-1/3*(-1/2/(a*c-1)^2*ln(-x*b*c)-1/2/(a*c-1)^2/x/b*a+1/2/(a*c-1)^2/x/b/c+1/2
*ln(-b*c*x-a*c+1)*(-b*c*x+a*c-1)*(-b*c*x-a*c+1)/x^2/b^2/c^2/(a*c-1)^2)/a/c-
1/3*(-1/(a*c-1)*ln(-x*b*c)-ln(-b*c*x-a*c+1)*(-b*c*x-a*c+1)/(a*c-1)/x/b/c)/a
^2/c^2)
```

Maxima [A]
time = 0.28, size = 302, normalized size = 1.09

$$\frac{(\log(bcx+ac)\log(-bcx-ac+1)+\text{Li}_2(-bcx-ac+1))b^3}{3\,a^3}-\frac{\left(\log(-bcx-ac+1)\log\left(-\frac{bcx+ac-1}{ac-1}+1\right)+\text{Li}_2\left(\frac{bcx+ac-1}{ac-1}\right)\right)b^3}{3\,a^3}+\frac{(3\,ab^3c^2-2\,b^3c)\log(x)}{6\,(a^4c^2-2\,a^3c+a^2)}+\frac{(a^2b^2c^2-ab^2c)x^2-2\,(a^4c^2-2\,a^3c+a^2)\text{Li}_2(bcx+ac)-((3\,ab^3c^2-2\,b^3c)x^3+2\,(a^2b^2c^2-2\,ab^2c+b^2)x^2-(a^3bc^2-2\,a^2bc+ab)x)\log(-bcx-ac+1)}{6\,(a^4c^2-2\,a^3c+a^2)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*b^3/
a^3 - 1/3*(log(-b*c*x - a*c + 1)*log(-(b*c*x + a*c - 1)/(a*c - 1) + 1) + di
log((b*c*x + a*c - 1)/(a*c - 1)))*b^3/a^3 + 1/6*(3*a*b^3*c^2 - 2*b^3*c)*log
(x)/(a^4*c^2 - 2*a^3*c + a^2) + 1/6*((a^2*b^2*c^2 - a*b^2*c)*x^2 - 2*(a^4*c
^2 - 2*a^3*c + a^2)*dilog(b*c*x + a*c) - ((3*a*b^3*c^2 - 2*b^3*c)*x^3 + 2*(
a^2*b^2*c^2 - 2*a*b^2*c + b^2)*x^2 - (a^3*b*c^2 - 2*a^2*b*c + a*b)*x)*log(-
b*c*x - a*c + 1))/((a^4*c^2 - 2*a^3*c + a^2)*x^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x^4,x, algorithm="fricas")
```

```
[Out] integral(dilog(b*c*x + a*c)/x^4, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Li}_2\left(ac + bcx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x**4,x)
```

```
[Out] Integral(polylog(2, a*c + b*c*x)/x**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x^4,x, algorithm="giac")
```

```
[Out] integrate(dilog((b*x + a)*c)/x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{polylog}(2, c\,(a+b\,x))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, c*(a + b*x))/x^4,x)
```

```
[Out] int(polylog(2, c*(a + b*x))/x^4, x)
```

3.131 $\int x^2\mathrm{PolyLog}(3, c(a + bx))\, dx$

Optimal. Leaf size=347

$$\frac{11a^2x}{18b^2}-\frac{5a(1-ac)x}{36b^2c}+\frac{(1-ac)^2x}{27b^2c^2}-\frac{5ax^2}{72b}+\frac{(1-ac)x^2}{54bc}+\frac{x^3}{81}-\frac{5a(1-ac)^2\log(1-ac-bcx)}{36b^3c^2}+\frac{(1-ac)^3\log(1-}{27b^3c^3}$$

```
[Out] 11/18*a^2*x/b^2-5/36*a*(-a*c+1)*x/b^2/c+1/27*(-a*c+1)^2*x/b^2/c^2-5/72*a*x^
2/b+1/54*(-a*c+1)*x^2/b/c+1/81*x^3-5/36*a*(-a*c+1)^2*ln(-b*c*x-a*c+1)/b^3/c
^2+1/27*(-a*c+1)^3*ln(-b*c*x-a*c+1)/b^3/c^3+5/36*a*x^2*ln(-b*c*x-a*c+1)/b-1
/27*x^3*ln(-b*c*x-a*c+1)+11/18*a^2*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b^3/c-11
/18*a^3*polylog(2,c*(b*x+a))/b^3-1/3*a^2*x*polylog(2,c*(b*x+a))/b^2+1/6*a*x
^2*polylog(2,c*(b*x+a))/b-1/9*x^3*polylog(2,c*(b*x+a))+2/3*a^3*polylog(3,c*
(b*x+a))/b^3-1/3*(-b^3*x^3+a^3)*polylog(3,c*(b*x+a))/b^3
```

Rubi [A]
time = 0.45, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 13, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6734, 6730, 2494, 2436, 2332, 2468, 2440, 2438, 6733, 45, 2463, 2442, 6724}

$$-\frac{(a^3-b^3x^3)\,\mathrm{Li}_3(c(a+bx))}{3b^3}-\frac{11a^3\mathrm{Li}_2(c(a+bx))}{18b^3}+\frac{2a^3\mathrm{Li}_3(c(a+bx))}{3b^3}+\frac{11a^2(-ac-bcx+1)\log(-ac-bcx+1)}{18b^3c}-\frac{a^2x\mathrm{Li}_2(c(a+bx))}{3b^2}+\frac{11a^2x}{18b^2}+\frac{(1-ac)^3\log(-ac-bcx+1)}{27b^3c^3}-\frac{5a(1-ac)^2\log(-ac-bcx+1)}{36b^3c^2}+\frac{x(1-ac)^2}{27b^2c^2}-\frac{5ax(1-ac)}{36b^2c}-\frac{1}{9}x^3\mathrm{Li}_2(c(a+bx))+\frac{ax^2\mathrm{Li}_2(c(a+bx))}{6b}-\frac{1}{27}x^3\log(-ac-bcx+1)+\frac{x^2(1-ac)}{54bc}+\frac{5ax^2\log(-ac-bcx+1)}{36b}-\frac{5ax^2}{72b}+\frac{x^3}{81}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*PolyLog[3, c*(a + b*x)],x]
```

```
[Out] (11*a^2*x)/(18*b^2) - (5*a*(1 - a*c)*x)/(36*b^2*c) + ((1 - a*c)^2*x)/(27*b^
2*c^2) - (5*a*x^2)/(72*b) + ((1 - a*c)*x^2)/(54*b*c) + x^3/81 - (5*a*(1 - a
*c)^2*Log[1 - a*c - b*c*x])/(36*b^3*c^2) + ((1 - a*c)^3*Log[1 - a*c - b*c*x
])/(27*b^3*c^3) + (5*a*x^2*Log[1 - a*c - b*c*x])/(36*b) - (x^3*Log[1 - a*c
- b*c*x])/27 + (11*a^2*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(18*b^3*c) -
 (11*a^3*PolyLog[2, c*(a + b*x)])/(18*b^3) - (a^2*x*PolyLog[2, c*(a + b*x)]
)/(3*b^2) + (a*x^2*PolyLog[2, c*(a + b*x)])/(6*b) - (x^3*PolyLog[2, c*(a +
b*x)])/9 + (2*a^3*PolyLog[3, c*(a + b*x)])/(3*b^3) - ((a^3 - b^3*x^3)*PolyL
og[3, c*(a + b*x)])/(3*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2468

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_)^(q_.), x_Symbol] :> In
t[ExpandToSum[u, x]^q*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a,
 b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] &&  !(BinomialMatch
Q[u, x] && LinearMatchQ[v, x])
```

Rule 2494

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6730

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :> Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c*(a + b*x)^p], x]
, x] + Dist[a*p, Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /; Fr
eeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

Rule 6733

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Dist[b/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x))
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 6734

```
Int[(x_)^(m_.)*PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :>
Simp[(-(a^(m + 1) - b^(m + 1)*x^(m + 1)))*(PolyLog[n, c*(a + b*x)^p]/((m +
1)*b^(m + 1))), x] + Dist[p/((m + 1)*b^m), Int[ExpandIntegrand[PolyLog[n -
1, c*(a + b*x)^p], (a^(m + 1) - b^(m + 1)*x^(m + 1))/(a + b*x), x], x], x]
/; FreeQ[{a, b, c, p}, x] && GtQ[n, 0] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$\int x^2\mathrm{Li}_3(c(a+bx))\,dx = -\frac{(a^3-b^3x^3)\,\mathrm{Li}_3(c(a+bx))}{3b^3} + \frac{\int\left(-a^2\mathrm{Li}_2(c(a+bx)) + abx\mathrm{Li}_2(c(a+bx)) - b^2x^2\mathrm{I}\right.}{3b^2}$$

$$= -\frac{(a^3-b^3x^3)\,\mathrm{Li}_3(c(a+bx))}{3b^3} - \frac{1}{3}\int x^2\mathrm{Li}_2(c(a+bx))\,dx - \frac{a^2\int\mathrm{Li}_2(c(a+bx))\,dx}{3b^2}$$

$$= -\frac{a^2x\mathrm{Li}_2(c(a+bx))}{3b^2} + \frac{ax^2\mathrm{Li}_2(c(a+bx))}{6b} - \frac{1}{9}x^3\mathrm{Li}_2(c(a+bx)) + \frac{2a^3\mathrm{Li}_3(c(a+bx)}{3b^3}$$

$$= -\frac{a^2x\mathrm{Li}_2(c(a+bx))}{3b^2} + \frac{ax^2\mathrm{Li}_2(c(a+bx))}{6b} - \frac{1}{9}x^3\mathrm{Li}_2(c(a+bx)) + \frac{2a^3\mathrm{Li}_3(c(a+bx)}{3b^3}$$

$$= -\frac{a^2x\mathrm{Li}_2(c(a+bx))}{3b^2} + \frac{ax^2\mathrm{Li}_2(c(a+bx))}{6b} - \frac{1}{9}x^3\mathrm{Li}_2(c(a+bx)) + \frac{2a^3\mathrm{Li}_3(c(a+bx)}{3b^3}$$

$$= \frac{a^2x}{3b^2} + \frac{5ax^2\log(1-ac-bcx)}{36b} - \frac{1}{27}x^3\log(1-ac-bcx) + \frac{a^2(1-ac-bcx)\log(1}{3b^3c}$$

$$= \frac{11a^2x}{18b^2} + \frac{5ax^2\log(1-ac-bcx)}{36b} - \frac{1}{27}x^3\log(1-ac-bcx) + \frac{11a^2(1-ac-bcx)\,\mathrm{l}}{18b^3}$$

$$= \frac{11a^2x}{18b^2} - \frac{5a(1-ac)x}{36b^2c} + \frac{(1-ac)^2x}{27b^2c^2} - \frac{5ax^2}{72b} + \frac{(1-ac)x^2}{54bc} + \frac{x^3}{81} - \frac{5a(1-ac)^2\log}{36b}$$

Mathematica [A]
time = 0.04, size = 296, normalized size = 0.85

$$\frac{24ac - 150a^2c^2 + 575a^3c^3 + 24bcx - 138abc^2x + 510a^2bc^3x + 12b^2c^2x^2 - 57ab^2c^3x^2 + 8b^3c^3x^3 + 24\log(1-ac-bcx) - 162ac\log(1-ac-bcx) + 648a^2c^2\log(1-ac-bcx) - 510a^3c^3\log(1-ac-bcx) - 396a^2bc^3x\log(1-ac-bcx) + 90ab^2c^3x^2\log(1-ac-bcx) - 24b^3c^3x^3\log(1-ac-bcx) - 36c^3(11a^3 + 6a^2bx - 3ab^2x^2 + 2b^3x^3)\,\mathrm{PolyLog}(2, c(a+bx)) + 216c^3(a^3+b^3x^3)\,\mathrm{PolyLog}(3, c(a+bx))}{648b^3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*PolyLog[3, c*(a + b*x)],x]
```

```
[Out] (24*a*c - 150*a^2*c^2 + 575*a^3*c^3 + 24*b*c*x - 138*a*b*c^2*x + 510*a^2*b*
c^3*x + 12*b^2*c^2*x^2 - 57*a*b^2*c^3*x^2 + 8*b^3*c^3*x^3 + 24*Log[1 - a*c
- b*c*x] - 162*a*c*Log[1 - a*c - b*c*x] + 648*a^2*c^2*Log[1 - a*c - b*c*x]
- 510*a^3*c^3*Log[1 - a*c - b*c*x] - 396*a^2*b*c^3*x*Log[1 - a*c - b*c*x] +
 90*a*b^2*c^3*x^2*Log[1 - a*c - b*c*x] - 24*b^3*c^3*x^3*Log[1 - a*c - b*c*x
] - 36*c^3*(11*a^3 + 6*a^2*b*x - 3*a*b^2*x^2 + 2*b^3*x^3)*PolyLog[2, c*(a +
 b*x)] + 216*c^3*(a^3 + b^3*x^3)*PolyLog[3, c*(a + b*x)])/(648*b^3*c^3)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\mathrm{polylog}\left(3, c(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(3,c*(b*x+a)),x)
```

```
[Out] int(x^2*polylog(3,c*(b*x+a)),x)
```

Maxima [A]
time = 0.27, size = 264, normalized size = 0.76

$$\frac{11\,(\log\left(bcx+ac\right)\log\left(-bcx-ac+1\right)+\mathrm{Li}_2(-bcx-ac+1))a^3}{18\,b^3}+\frac{a^3\mathrm{Li}_3(bcx+ac)}{3\,b^3}+\frac{216\,b^3c^3x^3\mathrm{Li}_3(bcx+ac)+8\,b^3c^3x^3-3\,(19\,ab^2c^3-4\,b^2c^2)x^2+6\,(85\,a^2bc^3-23\,abc^2+4\,bc)x-36\,(2\,b^3c^3x^3-3\,ab^2c^3x^2+6\,a^2bc^3x)\mathrm{Li}_2(bcx+ac)-6\,(4\,b^3c^3x^3-15\,ab^2c^3x^2+66\,a^2bc^3x+85\,a^3c^3-108\,a^2c^2+27\,ac-4)\log\left(-bcx-ac+1\right)}{648\,b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] 11/18*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a^
3/b^3 + 1/3*a^3*polylog(3, b*c*x + a*c)/b^3 + 1/648*(216*b^3*c^3*x^3*polylo
g(3, b*c*x + a*c) + 8*b^3*c^3*x^3 - 3*(19*a*b^2*c^3 - 4*b^2*c^2)*x^2 + 6*(8
5*a^2*b*c^3 - 23*a*b*c^2 + 4*b*c)*x - 36*(2*b^3*c^3*x^3 - 3*a*b^2*c^3*x^2 +
 6*a^2*b*c^3*x)*dilog(b*c*x + a*c) - 6*(4*b^3*c^3*x^3 - 15*a*b^2*c^3*x^2 +
66*a^2*b*c^3*x + 85*a^3*c^3 - 108*a^2*c^2 + 27*a*c - 4)*log(-b*c*x - a*c +
1))/(b^3*c^3)
```

Fricas [A]
time = 0.41, size = 219, normalized size = 0.63

$$\frac{8\,b^3c^3x^3-3\,(19\,ab^2c^3-4\,b^2c^2)x^2+6\,(85\,a^2bc^3-23\,abc^2+4\,bc)x-36\,(2\,b^3c^3x^3-3\,ab^2c^3x^2+6\,a^2bc^3x+11\,a^3c^3)\mathrm{Li}_2(bcx+ac)-6\,(4\,b^3c^3x^3-15\,ab^2c^3x^2+66\,a^2bc^3x+85\,a^3c^3-108\,a^2c^2+27\,ac-4)\log\left(-bcx-ac+1\right)+216\,(b^3c^3x^3+a^3c^3)\mathrm{polylog}(3,bcx+ac)}{648\,b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/648*(8*b^3*c^3*x^3 - 3*(19*a*b^2*c^3 - 4*b^2*c^2)*x^2 + 6*(85*a^2*b*c^3 -
 23*a*b*c^2 + 4*b*c)*x - 36*(2*b^3*c^3*x^3 - 3*a*b^2*c^3*x^2 + 6*a^2*b*c^3*
x + 11*a^3*c^3)*dilog(b*c*x + a*c) - 6*(4*b^3*c^3*x^3 - 15*a*b^2*c^3*x^2 +
66*a^2*b*c^3*x + 85*a^3*c^3 - 108*a^2*c^2 + 27*a*c - 4)*log(-b*c*x - a*c +
1) + 216*(b^3*c^3*x^3 + a^3*c^3)*polylog(3, b*c*x + a*c))/(b^3*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\mathrm{Li}_3\left(ac+bcx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*polylog(3,c*(b*x+a)),x)
```

```
[Out] Integral(x**2*polylog(3, a*c + b*c*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,c*(b*x+a)),x, algorithm="giac")
[Out] integrate(x^2*polylog(3, (b*x + a)*c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{polylog}(3,c\,(a+b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(3, c*(a + b*x)),x)
[Out] int(x^2*polylog(3, c*(a + b*x)), x)
```

3.132 $\int x\textbf{PolyLog}(3,c(a+bx))\,dx$

Optimal. Leaf size=198

$$-\frac{3ax}{4b}+\frac{(1-ac)x}{8bc}+\frac{x^2}{16}+\frac{(1-ac)^2\log(1-ac-bcx)}{8b^2c^2}-\frac{1}{8}x^2\log(1-ac-bcx)-\frac{3a(1-ac-bcx)\log(1-ac-bc$$

```
[Out] -3/4*a*x/b+1/8*(-a*c+1)*x/b/c+1/16*x^2+1/8*(-a*c+1)^2*ln(-b*c*x-a*c+1)/b^2/
c^2-1/8*x^2*ln(-b*c*x-a*c+1)-3/4*a*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b^2/c+3/
4*a^2*polylog(2,c*(b*x+a))/b^2+1/2*a*x*polylog(2,c*(b*x+a))/b-1/4*x^2*polyl
og(2,c*(b*x+a))-1/2*(-b^2*x^2+a^2)*polylog(3,c*(b*x+a))/b^2
```

Rubi [A]

time = 0.19, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.091$, Rules used = {6734, 6730, 2494, 2436, 2332, 2468, 2440, 2438, 6733, 45, 2463, 2442}

$$-\frac{(a^2-b^2x^2)\operatorname{Li}_3(c(a+bx))}{2b^2}+\frac{3a^2\operatorname{Li}_2(c(a+bx))}{4b^2}+\frac{(1-ac)^2\log(-ac-bcx+1)}{8b^2c^2}-\frac{3a(-ac-bcx+1)\log(-ac-bcx+1)}{4b^2c}-\frac{1}{4}x^2\operatorname{Li}_2(c(a+bx))+\frac{ax\operatorname{Li}_2(c(a+bx))}{2b}-\frac{1}{8}x^2\log(-ac-bcx+1)+\frac{x(1-ac)}{8bc}-\frac{3ax}{4b}+\frac{x^2}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x*PolyLog[3, c*(a + b*x)],x]
```

```
[Out] (-3*a*x)/(4*b) + ((1 - a*c)*x)/(8*b*c) + x^2/16 + ((1 - a*c)^2*Log[1 - a*c
- b*c*x])/(8*b^2*c^2) - (x^2*Log[1 - a*c - b*c*x])/8 - (3*a*(1 - a*c - b*c*
x)*Log[1 - a*c - b*c*x])/(4*b^2*c) + (3*a^2*PolyLog[2, c*(a + b*x)])/(4*b^2
) + (a*x*PolyLog[2, c*(a + b*x)])/(2*b) - (x^2*PolyLog[2, c*(a + b*x)])/4 -
 ((a^2 - b^2*x^2)*PolyLog[3, c*(a + b*x)])/(2*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2468

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_)^(q_.), x_Symbol] :> In
t[ExpandToSum[u, x]^q*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a,
 b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] &&  !(BinomialMatch
Q[u, x] && LinearMatchQ[v, x])
```

Rule 2494

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rule 6730

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :> Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c*(a + b*x)^p], x]
, x] + Dist[a*p, Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /; Fr
eeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

Rule 6733

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Dist[b/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x))
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 6734

```
Int[(x_)^(m_.)*PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :>
Simp[(-(a^(m + 1) - b^(m + 1)*x^(m + 1)))*(PolyLog[n, c*(a + b*x)^p]/((m +
1)*b^(m + 1))), x] + Dist[p/((m + 1)*b^m), Int[ExpandIntegrand[PolyLog[n -
1, c*(a + b*x)^p], (a^(m + 1) - b^(m + 1)*x^(m + 1))/(a + b*x), x], x], x]
/; FreeQ[{a, b, c, p}, x] && GtQ[n, 0] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x\mathrm{Li}_3(c(a+bx))\,dx &= -\frac{(a^2-b^2x^2)\,\mathrm{Li}_3(c(a+bx))}{2b^2}+\frac{\int(a\mathrm{Li}_2(c(a+bx))-bx\mathrm{Li}_2(c(a+bx)))\,dx}{2b}\\
&= -\frac{(a^2-b^2x^2)\,\mathrm{Li}_3(c(a+bx))}{2b^2}-\frac{1}{2}\int x\mathrm{Li}_2(c(a+bx))\,dx+\frac{a\int\mathrm{Li}_2(c(a+bx))\,dx}{2b}\\
&= \frac{ax\mathrm{Li}_2(c(a+bx))}{2b}-\frac{1}{4}x^2\mathrm{Li}_2(c(a+bx))-\frac{(a^2-b^2x^2)\,\mathrm{Li}_3(c(a+bx))}{2b^2}+\frac{a\int\log(1-c(\cdots}{2b\cdots}\\
&= \frac{ax\mathrm{Li}_2(c(a+bx))}{2b}-\frac{1}{4}x^2\mathrm{Li}_2(c(a+bx))-\frac{(a^2-b^2x^2)\,\mathrm{Li}_3(c(a+bx))}{2b^2}+\frac{a\int\log(1-a\cdots}{2b\cdots}\\
&= \frac{ax\mathrm{Li}_2(c(a+bx))}{2b}-\frac{1}{4}x^2\mathrm{Li}_2(c(a+bx))-\frac{(a^2-b^2x^2)\,\mathrm{Li}_3(c(a+bx))}{2b^2}-\frac{1}{4}\int x\log(1-\cdots\\
&= -\frac{ax}{2b}-\frac{1}{8}x^2\log(1-ac-bcx)-\frac{a(1-ac-bcx)\log(1-ac-bcx)}{2b^2c}+\frac{a^2\mathrm{Li}_2(c(a+b\cdots}{2b^2}\\
&= -\frac{3ax}{4b}-\frac{1}{8}x^2\log(1-ac-bcx)-\frac{3a(1-ac-bcx)\log(1-ac-bcx)}{4b^2c}+\frac{3a^2\mathrm{Li}_2(c(a\cdots}{4b^2}\\
&= -\frac{3ax}{4b}+\frac{(1-ac)x}{8bc}+\frac{x^2}{16}+\frac{(1-ac)^2\log(1-ac-bcx)}{8b^2c^2}-\frac{1}{8}x^2\log(1-ac-bcx)-\cdots
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 198, normalized size = 1.00

$$\frac{2ac-15a^2c^2+2bcx-14abc^2x+b^2c^2x^2+2\log(1-ac-bcx)-16ac\log(1-ac-bcx)+14a^2c^2\log(1-ac-bcx)+12abc^2x\log(1-ac-bcx)-2b^2c^2x^2\log(1-ac-bcx)+4c^2(3a^2+2abx-b^2x^2)\,\mathrm{PolyLog}(2,c(a+bx))-8c^2(a^2-b^2x^2)\,\mathrm{PolyLog}(3,c(a+bx))}{16b^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*PolyLog[3, c*(a + b*x)],x]
```

```
[Out] (2*a*c - 15*a^2*c^2 + 2*b*c*x - 14*a*b*c^2*x + b^2*c^2*x^2 + 2*Log[1 - a*c
- b*c*x] - 16*a*c*Log[1 - a*c - b*c*x] + 14*a^2*c^2*Log[1 - a*c - b*c*x] +
12*a*b*c^2*x*Log[1 - a*c - b*c*x] - 2*b^2*c^2*x^2*Log[1 - a*c - b*c*x] + 4*
c^2*(3*a^2 + 2*a*b*x - b^2*x^2)*PolyLog[2, c*(a + b*x)] - 8*c^2*(a^2 - b^2*
x^2)*PolyLog[3, c*(a + b*x)])/(16*b^2*c^2)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{polylog}(3, c(bx + a))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(3,c*(b*x+a)),x)
```

```
[Out] int(x*polylog(3,c*(b*x+a)),x)
```

Maxima [A]
time = 0.27, size = 193, normalized size = 0.97

$$-\frac{3\left(\log(bcx + ac)\log(-bcx - ac + 1) + \operatorname{Li}_2(-bcx - ac + 1)\right)a^2}{4b^2} - \frac{a^2\operatorname{Li}_3(bcx + ac)}{2b^2} + \frac{8b^2c^2x^2\operatorname{Li}_3(bcx + ac) + b^2c^2x^2 - 2(7abc^2 - bc)x - 4(b^2c^2x^2 - 2abc^2x)\operatorname{Li}_2(bcx + ac) - 2(b^2c^2x^2 - 6abc^2x - 7a^2c^2 + 8ac - 1)\log(-bcx - ac + 1)}{16b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] -3/4*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a^2
/b^2 - 1/2*a^2*polylog(3, b*c*x + a*c)/b^2 + 1/16*(8*b^2*c^2*x^2*polylog(3,
 b*c*x + a*c) + b^2*c^2*x^2 - 2*(7*a*b*c^2 - b*c)*x - 4*(b^2*c^2*x^2 - 2*a*
b*c^2*x)*dilog(b*c*x + a*c) - 2*(b^2*c^2*x^2 - 6*a*b*c^2*x - 7*a^2*c^2 + 8*
a*c - 1)*log(-b*c*x - a*c + 1))/(b^2*c^2)
```

Fricas [A]
time = 0.39, size = 149, normalized size = 0.75

$$\frac{b^2c^2x^2 - 2(7abc^2 - bc)x - 4(b^2c^2x^2 - 2abc^2x - 3a^2c^2)\operatorname{Li}_2(bcx + ac) - 2(b^2c^2x^2 - 6abc^2x - 7a^2c^2 + 8ac - 1)\log(-bcx - ac + 1) + 8(b^2c^2x^2 - a^2c^2)\operatorname{polylog}(3, bcx + ac)}{16b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(b^2*c^2*x^2 - 2*(7*a*b*c^2 - b*c)*x - 4*(b^2*c^2*x^2 - 2*a*b*c^2*x -
3*a^2*c^2)*dilog(b*c*x + a*c) - 2*(b^2*c^2*x^2 - 6*a*b*c^2*x - 7*a^2*c^2 +
8*a*c - 1)*log(-b*c*x - a*c + 1) + 8*(b^2*c^2*x^2 - a^2*c^2)*polylog(3, b*c
*x + a*c))/(b^2*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{Li}_3(ac + bcx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,c*(b*x+a)),x)
[Out] Integral(x*polylog(3, a*c + b*c*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,c*(b*x+a)),x, algorithm="giac")
[Out] integrate(x*polylog(3, (b*x + a)*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{polylog}(3, c\,(a + b\,x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(3, c*(a + b*x)),x)
[Out] int(x*polylog(3, c*(a + b*x)), x)
```

3.133 $\int \textbf{PolyLog}(3, c(a+bx))\,dx$

Optimal. Leaf size=84

$$x+\frac{(1-ac-bcx)\log(1-ac-bcx)}{bc}-\frac{a\text{PolyLog}(2,c(a+bx))}{b}-x\text{PolyLog}(2,c(a+bx))+\frac{a\text{PolyLog}(3,c(a+b$$

```
[Out] x+(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b/c-a*polylog(2,c*(b*x+a))/b-x*polylog(2,
c*(b*x+a))+a*polylog(3,c*(b*x+a))/b+x*polylog(3,c*(b*x+a))
```

Rubi [A]
time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {6730, 2494, 2436, 2332, 2468, 2440, 2438, 6724}

$$x(-\text{Li}_2(c(a+bx)))+x\text{Li}_3(c(a+bx))-\frac{a\text{Li}_2(c(a+bx))}{b}+\frac{a\text{Li}_3(c(a+bx))}{b}+\frac{(-ac-bcx+1)\log(-ac-bcx+1)}{bc}+x$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, c*(a + b*x)],x]
```

```
[Out] x + ((1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c) - (a*PolyLog[2, c*(a + b
*x)])/b - x*PolyLog[2, c*(a + b*x)] + (a*PolyLog[3, c*(a + b*x)])/b + x*Pol
yLog[3, c*(a + b*x)]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2468

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_)^(q_.), x_Symbol] :> In
t[ExpandToSum[u, x]^q*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a,
 b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] &&  !(BinomialMatch
Q[u, x] && LinearMatchQ[v, x])
```

Rule 2494

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6730

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :> Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c*(a + b*x)^p], x]
, x] + Dist[a*p, Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /; Fr
eeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \mathrm{Li}_3(c(a+bx))\,dx &= x\mathrm{Li}_3(c(a+bx)) + a\int \frac{\mathrm{Li}_2(c(a+bx))}{a+bx}\,dx - \int \mathrm{Li}_2(c(a+bx))\,dx \\
&= -x\mathrm{Li}_2(c(a+bx)) + \frac{a\mathrm{Li}_3(c(a+bx))}{b} + x\mathrm{Li}_3(c(a+bx)) + a\int \frac{\log(1-c(a+bx))}{a+bx}\,dx - \\
&= -x\mathrm{Li}_2(c(a+bx)) + \frac{a\mathrm{Li}_3(c(a+bx))}{b} + x\mathrm{Li}_3(c(a+bx)) + a\int \frac{\log(1-ac-bcx)}{a+bx}\,dx - \\
&= -x\mathrm{Li}_2(c(a+bx)) + \frac{a\mathrm{Li}_3(c(a+bx))}{b} + x\mathrm{Li}_3(c(a+bx)) + \frac{a\mathrm{Subst}\left(\int \frac{\log(1-cx)}{x}\,dx, x, a+\right.}{b} \\
&= x + \frac{(1-ac-bcx)\log(1-ac-bcx)}{bc} - \frac{a\mathrm{Li}_2(c(a+bx))}{b} - x\mathrm{Li}_2(c(a+bx)) + \frac{a\mathrm{Li}_3(c(a}{b}
\end{aligned}
$$

Mathematica [A]

time = 0.02, size = 66, normalized size = 0.79

$$\frac{(a+bx)\left(1-\log(1-c(a+bx))+\frac{\log(1-c(a+bx))}{c(a+bx)}-\text{PolyLog}(2,c(a+bx))+\text{PolyLog}(3,c(a+bx))\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, c*(a + b*x)],x]
```

```
[Out] ((a + b*x)*(1 - Log[1 - c*(a + b*x)] + Log[1 - c*(a + b*x)]/(c*(a + b*x)) -
 PolyLog[2, c*(a + b*x)] + PolyLog[3, c*(a + b*x)]))/b
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \text{polylog}\left(3,c(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,c*(b*x+a)),x)
```

```
[Out] int(polylog(3,c*(b*x+a)),x)
```

Maxima [A]
time = 0.26, size = 120, normalized size = 1.43

$$\frac{(\log(bcx+ac)\log(-bcx-ac+1)+\text{Li}_2(-bcx-ac+1))a}{b}+\frac{a\text{Li}_3(bcx+ac)}{b}-\frac{bcx\text{Li}_2(bcx+ac)-bcx\text{Li}_3(bcx+ac)-bcx+(bcx+ac-1)\log(-bcx-ac+1)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] (log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a/b + a*
polylog(3, b*c*x + a*c)/b - (b*c*x*dilog(b*c*x + a*c) - b*c*x*polylog(3, b*
c*x + a*c) - b*c*x + (b*c*x + a*c - 1)*log(-b*c*x - a*c + 1))/(b*c)
```

Fricas [A]
time = 0.38, size = 73, normalized size = 0.87

$$\frac{bcx-(bcx+ac)\text{Li}_2(bcx+ac)-(bcx+ac-1)\log(-bcx-ac+1)+(bcx+ac)\text{polylog}(3,bcx+ac)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] (b*c*x - (b*c*x + a*c)*dilog(b*c*x + a*c) - (b*c*x + a*c - 1)*log(-b*c*x -
a*c + 1) + (b*c*x + a*c)*polylog(3, b*c*x + a*c))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{Li}_3\left(c(a+bx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a)),x)
```

```
[Out] Integral(polylog(3, c*(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, (b*x + a)*c), x)
```

Mupad [B]
time = 2.18, size = 77, normalized size = 0.92

$$x-\frac{\mathrm{polylog}(2,c\,(a+b\,x))\,\,(a+b\,x)}{b}+\frac{\mathrm{polylog}(3,c\,(a+b\,x))\,\,(a+b\,x)}{b}+\frac{\ln\left(c\,(a+b\,x)-1\right)}{b\,c}-\frac{\ln\left(1-c\,(a+b\,x)\right)\,\,(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, c*(a + b*x)),x)
```

```
[Out] x - (polylog(2, c*(a + b*x))*(a + b*x))/b + (polylog(3, c*(a + b*x))*(a + b
*x))/b + log(c*(a + b*x) - 1)/(b*c) - (log(1 - c*(a + b*x))*(a + b*x))/b
```

3.134 $\int \frac{\text{PolyLog}(3,c(a+bx))}{x}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{\text{PolyLog}(3,ac+bcx)}{x},x\right)$$

```
[Out] int(polylog(3,b*c*x+a*c)/x,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{Li}_3(c(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[PolyLog[3, c*(a + b*x)]/x,x]
```

```
[Out] Defer[Int][PolyLog[3, a*c + b*c*x]/x, x]
```

Rubi steps

$$\int \frac{\text{Li}_3(c(a+bx))}{x}\,dx=\int \frac{\text{Li}_3(ac+bcx)}{x}\,dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{PolyLog}(3,c(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[PolyLog[3, c*(a + b*x)]/x,x]
```

```
[Out] Integrate[PolyLog[3, c*(a + b*x)]/x, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{polylog}\left(3,c(bx+a)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,c*(b*x+a))/x,x)
```

```
[Out] int(polylog(3,c*(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(polylog(3, (b*x + a)*c)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(polylog(3, b*c*x + a*c)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_3(ac + bcx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x,x)
```

```
[Out] Integral(polylog(3, a*c + b*c*x)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, (b*x + a)*c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{polylog}(3, c\,(a + b\,x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, c*(a + b*x))/x,x)
[Out] int(polylog(3, c*(a + b*x))/x, x)
```

3.135 $\int \frac{\text{PolyLog}(3,c(a+bx))}{x^2}\,dx$

Optimal. Leaf size=486

$$\frac{b\log(x)\log\left(1+\frac{bx}{a}\right)\log(1-c(a+bx))}{a}+\frac{b\left(\log\left(1+\frac{bx}{a}\right)+\log\left(\frac{1-ac}{1-c(a+bx)}\right)-\log\left(\frac{(1-ac)(a+bx)}{a(1-c(a+bx))}\right)\right)\log^2\left(-\frac{a(1-c(\ldots}{b\ldots}\right.}{2a}$$

```
[Out] b*ln(x)*ln(1+b*x/a)*ln(1-c*(b*x+a))/a+1/2*b*(ln(1+b*x/a)+ln((-a*c+1)/(1-c*(
b*x+a)))-ln((-a*c+1)*(b*x+a)/a/(1-c*(b*x+a))))*ln(-a*(1-c*(b*x+a))/b/x)^2/a
+1/2*b*(ln(c*(b*x+a))-ln(1+b*x/a))*(ln(x)+ln(-a*(1-c*(b*x+a))/b/x))^2/a+b*(
ln(1-c*(b*x+a))-ln(-a*(1-c*(b*x+a))/b/x))*polylog(2,-b*x/a)/a+b*ln(x)*polyl
og(2,c*(b*x+a))/a+b*ln(-a*(1-c*(b*x+a))/b/x)*polylog(2,-b*x/a/(1-c*(b*x+a))
)/a-b*ln(-a*(1-c*(b*x+a))/b/x)*polylog(2,-b*c*x/(1-c*(b*x+a)))/a+b*(ln(x)+l
n(-a*(1-c*(b*x+a))/b/x))*polylog(2,1-c*(b*x+a))/a-b*polylog(3,-b*x/a)/a-2*b
*polylog(3,c*(b*x+a))/a+(b-a/x)*polylog(3,c*(b*x+a))/a+b*polylog(3,-b*x/a/(
1-c*(b*x+a)))/a-b*polylog(3,-b*c*x/(1-c*(b*x+a)))/a-b*polylog(3,1-c*(b*x+a)
)/a
```

Rubi [A]
time = 0.38, antiderivative size = 486, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6734, 6732, 2490, 2485, 6724}

$$-\frac{2b\,\text{Li}_3(c(a+bx))}{a}+\frac{\left(b-\frac{a}{x}\right)\text{Li}_3(c(a+bx))}{a}+\frac{b\,\text{Li}_3\left(-\frac{bx}{a(1-c(a+bx))}\right)}{a}-\frac{b\,\text{Li}_3\left(-\frac{bcx}{1-c(a+bx)}\right)}{a}-\frac{b\,\text{Li}_3(1-c(a+bx))}{a}+\frac{b\,\text{Li}_2\left(-\frac{bx}{a(1-c(a+bx))}\right)\log\left(-\frac{a(1-c(a+bx))}{bx}\right)}{a}-\frac{b\,\text{Li}_2\left(-\frac{bcx}{1-c(a+bx)}\right)\log\left(-\frac{a(1-c(a+bx))}{bx}\right)}{a}+\frac{b\,\text{Li}_2\left(-\frac{bx}{a}\right)\left(\log(1-c(a+bx))-\log\left(-\frac{a(1-c(a+bx))}{bx}\right)\right)}{a}+\frac{b\log(x)\text{Li}_2(c(a+bx))}{a}+\frac{b\,\text{Li}_2(1-c(a+bx))\left(\log\left(-\frac{a(1-c(a+bx))}{bx}\right)+\log(x)\right)}{a}+\frac{b\left(\log\left(\frac{1-ac}{1-c(a+bx)}\right)-\log\left(\frac{(1-ac)(a+bx)}{a(1-c(a+bx))}\right)+\log\left(\frac{bx}{a}+1\right)\right)\log^2\left(-\frac{a(1-c(a+bx))}{bx}\right)}{2a}+\frac{b\left(\log(c(a+bx))-\log\left(\frac{bx}{a}+1\right)\right)\left(\log\left(-\frac{a(1-c(a+bx))}{bx}\right)+\log(x)\right)^2}{2a}+\frac{b\log(x)\log\left(\frac{bx}{a}+1\right)\log(1-c(a+bx))}{a}-\frac{b\,\text{Li}_3\left(-\frac{bx}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, c*(a + b*x)]/x^2,x]
```

```
[Out] (b*Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*x)])/a + (b*(Log[1 + (b*x)/a] +
 Log[(1 - a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a
+ b*x)))])*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(2*a) + (b*(Log[c*(a + b*
x)] - Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/(
2*a) + (b*(Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*Poly
Log[2, -((b*x)/a)])/a + (b*Log[x]*PolyLog[2, c*(a + b*x)])/a + (b*Log[-((a*
(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))))])/a -
(b*Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*c*x)/(1 - c*(a + b*x
)))])/a + (b*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, 1 -
c*(a + b*x)])/a - (b*PolyLog[3, -((b*x)/a)])/a - (2*b*PolyLog[3, c*(a + b*x
)])/a + ((b - a/x)*PolyLog[3, c*(a + b*x)])/a + (b*PolyLog[3, -((b*x)/(a*(1
 - c*(a + b*x))))])/a - (b*PolyLog[3, -((b*c*x)/(1 - c*(a + b*x)))])/a - (b
*PolyLog[3, 1 - c*(a + b*x)])/a
```

Rule 2485

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[(-b)*(x/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1/2)*(Log[(-b)*(x/a
```

```
)] - Log[(-(b*c - a*d))*(x/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))]
)*Log[a*((c + d*x)/(c*(a + b*x)))]^2, x] - Simp[(1/2)*(Log[(-b)*(x/a)] - Lo
g[(-d)*(x/c)])*(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])^2, x] + Si
mp[(Log[c + d*x] - Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1 + b*(x/a)
], x] + Simp[(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1
 + d*(x/c)], x] + Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2, c*((a +
b*x)/(a*(c + d*x)))], x] - Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2,
 d*((a + b*x)/(b*(c + d*x)))], x] - Simp[PolyLog[3, 1 + b*(x/a)], x] - Simp
[PolyLog[3, 1 + d*(x/c)], x] + Simp[PolyLog[3, c*((a + b*x)/(a*(c + d*x)))]
, x] - Simp[PolyLog[3, d*((a + b*x)/(b*(c + d*x)))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2490

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-(e*k - d*l)/l + e*(x/l))^n])*(f + g
*Log[h*(-(j*k - i*l)/l + j*(x/l))^m]), x], x, k + l*x], x] /; FreeQ[{a, b,
c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6732

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[d + e*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, Int[Log[d
+ e*x]*(Log[1 - a*c - b*c*x]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rule 6734

```
Int[(x_)^(m_.)*PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :>
Simp[(-(a^(m + 1) - b^(m + 1)*x^(m + 1)))*(PolyLog[n, c*(a + b*x)^p]/((m +
1)*b^(m + 1))), x] + Dist[p/((m + 1)*b^m), Int[ExpandIntegrand[PolyLog[n -
1, c*(a + b*x)^p], (a^(m + 1) - b^(m + 1)*x^(m + 1))/(a + b*x), x], x], x]
/; FreeQ[{a, b, c, p}, x] && GtQ[n, 0] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\mathrm{Li}_3(c(a+bx))}{x^2}\,dx = \frac{\left(b-\frac{a}{x}\right)\mathrm{Li}_3(c(a+bx))}{a} - b^2\int\left(-\frac{\mathrm{Li}_2(c(a+bx))}{abx}+\frac{2\mathrm{Li}_2(c(a+bx))}{a(a+bx)}\right)dx$$

$$= \frac{\left(b-\frac{a}{x}\right)\mathrm{Li}_3(c(a+bx))}{a} + \frac{b\int\frac{\mathrm{Li}_2(c(a+bx))}{x}dx}{a} - \frac{(2b^2)\int\frac{\mathrm{Li}_2(c(a+bx))}{a+bx}dx}{a}$$

$$= \frac{b\log(x)\mathrm{Li}_2(c(a+bx))}{a} - \frac{2b\mathrm{Li}_3(c(a+bx))}{a} + \frac{\left(b-\frac{a}{x}\right)\mathrm{Li}_3(c(a+bx))}{a} + \frac{b^2\int\frac{\log(x)\log(1-}{a+ba}}{a}$$

$$= \frac{b\log(x)\mathrm{Li}_2(c(a+bx))}{a} - \frac{2b\mathrm{Li}_3(c(a+bx))}{a} + \frac{\left(b-\frac{a}{x}\right)\mathrm{Li}_3(c(a+bx))}{a} + \frac{b\mathrm{Subst}\left(\int\frac{\log(\cdot}{}\right.}{}$$

$$= \frac{b\log(x)\log\left(1+\frac{bx}{a}\right)\log(1-c(a+bx))}{a} + \frac{b\left(\log\left(1+\frac{bx}{a}\right)+\log\left(\frac{1-ac}{1-c(a+bx)}\right)-\log\left(\frac{(1-}{a(1}\right.\right.}{2a}$$

Mathematica [A]
time = 0.52, size = 477, normalized size = 0.98

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, c*(a + b*x)]/x^2,x]
```

```
[Out] -(PolyLog[3, c*(a + b*x)]/x) + (b*(Log[x]*Log[1 + (b*x)/a]*Log[1 - a*c - b*
c*x] + ((-Log[c*(a + b*x)] + Log[1 + (b*x)/a])*Log[1 - a*c - b*c*x]*(-2*Log
[x] + Log[1 - a*c - b*c*x]))/2 + (Log[c*(a + b*x)] - Log[1 + (b*x)/a])*Log[
1 - a*c - b*c*x]*Log[(a*(-1 + a*c + b*c*x))/(b*x)] + ((Log[(1 - a*c)/(b*c*x
)] - Log[((1 - a*c)*(a + b*x))/(b*x)] + Log[1 + (b*x)/a])*Log[(a*(-1 + a*c
+ b*c*x))/(b*x)]^2)/2 + (Log[1 - a*c - b*c*x] - Log[(a*(-1 + a*c + b*c*x))/
(b*x)])*PolyLog[2, -((b*x)/a)] + (Log[x] - Log[a + b*x])*PolyLog[2, c*(a +
b*x)] + Log[a + b*x]*PolyLog[2, c*(a + b*x)] + (Log[x] + Log[(a*(-1 + a*c +
 b*c*x))/(b*x)])*PolyLog[2, 1 - a*c - b*c*x] + Log[(a*(-1 + a*c + b*c*x))/(
b*x)]*(-PolyLog[2, (a*(-1 + a*c + b*c*x))/(b*x)] + PolyLog[2, (-1 + a*c + b
*c*x)/(b*c*x)]) - PolyLog[3, -((b*x)/a)] - PolyLog[3, c*(a + b*x)] - PolyLo
g[3, 1 - a*c - b*c*x] + PolyLog[3, (a*(-1 + a*c + b*c*x))/(b*x)] - PolyLog[
3, (-1 + a*c + b*c*x)/(b*c*x)]))/a
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{polylog}\left(3, c(bx+a)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,c*(b*x+a))/x^2,x)
[Out] int(polylog(3,c*(b*x+a))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x^2,x, algorithm="maxima")
[Out] b*integrate(dilog(b*c*x + a*c)/(b*x^2 + a*x), x) - polylog(3, b*c*x + a*c)/
x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x^2,x, algorithm="fricas")
[Out] integral(polylog(3, b*c*x + a*c)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_3\left(ac + bcx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x**2,x)
[Out] Integral(polylog(3, a*c + b*c*x)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x^2,x, algorithm="giac")
[Out] integrate(polylog(3, (b*x + a)*c)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{polylog}(3, c\,(a + b\,x))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, c*(a + b*x))/x^2,x)
[Out] int(polylog(3, c*(a + b*x))/x^2, x)
```

3.136 $\int \frac{\text{PolyLog}(3,c(a+bx))}{x^3}\,dx$

Optimal. Leaf size=629

$$-\frac{b^2\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2}-\frac{b^2\log(x)\log\left(1+\frac{bx}{a}\right)\log(1-c(a+bx))}{2a^2}-\frac{b^2\left(\log\left(1+\frac{bx}{a}\right)+\log\left(\frac{1-ac}{1-c(a+bx}\right.\right.}{}$$

```
[Out] -1/2*b^2*ln(b*c*x/(-a*c+1))*ln(-b*c*x-a*c+1)/a^2-1/2*b^2*ln(x)*ln(1+b*x/a)*
ln(1-c*(b*x+a))/a^2-1/4*b^2*(ln(1+b*x/a)+ln((-a*c+1)/(1-c*(b*x+a)))-ln((-a*
c+1)*(b*x+a)/a/(1-c*(b*x+a))))*ln(-a*(1-c*(b*x+a))/b/x)^2/a^2-1/4*b^2*(ln(c
*(b*x+a))-ln(1+b*x/a))*(ln(x)+ln(-a*(1-c*(b*x+a))/b/x))^2/a^2-1/2*b^2*(ln(1
-c*(b*x+a))-ln(-a*(1-c*(b*x+a))/b/x))*polylog(2,-b*x/a)/a^2-1/2*b^2*polylog
(2,c*(b*x+a))/a^2-1/2*b*polylog(2,c*(b*x+a))/a/x-1/2*b^2*ln(x)*polylog(2,c*
(b*x+a))/a^2-1/2*b^2*polylog(2,1-b*c*x/(-a*c+1))/a^2-1/2*b^2*ln(-a*(1-c*(b*
x+a))/b/x)*polylog(2,-b*x/a/(1-c*(b*x+a)))/a^2+1/2*b^2*ln(-a*(1-c*(b*x+a))/
b/x)*polylog(2,-b*c*x/(1-c*(b*x+a)))/a^2-1/2*b^2*(ln(x)+ln(-a*(1-c*(b*x+a))
/b/x))*polylog(2,1-c*(b*x+a))/a^2+1/2*b^2*polylog(3,-b*x/a)/a^2+1/2*(b^2-a^
2/x^2)*polylog(3,c*(b*x+a))/a^2-1/2*b^2*polylog(3,-b*x/a/(1-c*(b*x+a)))/a^2
+1/2*b^2*polylog(3,-b*c*x/(1-c*(b*x+a)))/a^2+1/2*b^2*polylog(3,1-c*(b*x+a))
/a^2
```

Rubi [A]
time = 0.47, antiderivative size = 629, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 13, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6734, 6733, 36, 29, 31, 2463, 2441, 2352, 2440, 2438, 6732, 2490, 2485}

$$\frac{\left(b^2-\frac{a^2}{x^2}\right)\text{Li}_3(c(a+bx))}{2a^2}-\frac{b^2\text{Li}_2(c(a+bx))}{2a^2}-\frac{b^2\text{Li}_2\left(1-\frac{bcx}{1-ac}\right)}{2a^2}-\frac{b^2\text{Li}_3\left(-\frac{bx}{a(1-c(a+bx))}\right)}{2a^2}+\frac{b^2\text{Li}_3\left(-\frac{bcx}{1-c(a+bx)}\right)}{2a^2}+\frac{b^2\text{Li}_3(1-c(a+bx))}{2a^2}-\frac{b^2\text{Li}_2\left(-\frac{bx}{a}\right)\left(\log(1-c(a+bx))-\log\left(-\frac{a(1-c(a+bx))}{bx}\right)\right)}{2a^2}-\frac{b^2\log(x)\text{Li}_2(c(a+bx))}{2a^2}-\frac{b^2\text{Li}_2\left(-\frac{bx}{a(1-c(a+bx))}\right)\log\left(-\frac{a(1-c(a+bx))}{bx}\right)}{2a^2}+\frac{b^2\text{Li}_2\left(-\frac{bcx}{1-c(a+bx)}\right)\log\left(-\frac{a(1-c(a+bx))}{bx}\right)}{2a^2}-\frac{b^2\text{Li}_2(1-c(a+bx))\left(\log\left(-\frac{a(1-c(a+bx))}{bx}\right)+\log(x)\right)}{2a^2}-\frac{b^2\left(\log\left(\frac{1-ac}{1-c(a+bx)}\right)-\log\left(\frac{(1-ac)(a+bx)}{a(1-c(a+bx))}\right)+\log\left(\frac{bx}{a}+1\right)\right)\log^2\left(-\frac{a(1-c(a+bx))}{bx}\right)}{4a^2}-\frac{b^2\left(\log(c(a+bx))-\log\left(\frac{bx}{a}+1\right)\right)\left(\log\left(-\frac{a(1-c(a+bx))}{bx}\right)+\log(x)\right)^2}{4a^2}-\frac{b^2\log\left(\frac{bcx}{1-ac}\right)\log(-ac-bcx+1)}{2a^2}-\frac{b^2\log(x)\log\left(\frac{bx}{a}+1\right)\log(1-c(a+bx))}{2a^2}+\frac{b^2\text{Li}_3\left(-\frac{bx}{a}\right)}{2a^2}-\frac{b\text{Li}_2(c(a+bx))}{2ax}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, c*(a + b*x)]/x^3,x]
```

```
[Out] -1/2*(b^2*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/a^2 - (b^2*Log[x]*Lo
g[1 + (b*x)/a]*Log[1 - c*(a + b*x)])/(2*a^2) - (b^2*(Log[1 + (b*x)/a] + Log
[(1 - a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*
x)))])*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(4*a^2) - (b^2*(Log[c*(a + b*
x)] - Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/(
4*a^2) - (b^2*(Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*
PolyLog[2, -((b*x)/a)])/(2*a^2) - (b^2*PolyLog[2, c*(a + b*x)])/(2*a^2) - (
b*PolyLog[2, c*(a + b*x)])/(2*a*x) - (b^2*Log[x]*PolyLog[2, c*(a + b*x)])/(
2*a^2) - (b^2*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(2*a^2) - (b^2*Log[-((a*(1
 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))))])/(2*a^2
) + (b^2*Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*c*x)/(1 - c*(a
 + b*x)))])/(2*a^2) - (b^2*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])*P
olyLog[2, 1 - c*(a + b*x)])/(2*a^2) + (b^2*PolyLog[3, -((b*x)/a)])/(2*a^2)
```

```
+ ((b^2 - a^2/x^2)*PolyLog[3, c*(a + b*x)])/(2*a^2) - (b^2*PolyLog[3, -((b*
x)/(a*(1 - c*(a + b*x))))])/(2*a^2) + (b^2*PolyLog[3, -((b*c*x)/(1 - c*(a +
 b*x)))])/(2*a^2) + (b^2*PolyLog[3, 1 - c*(a + b*x)])/(2*a^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2485

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[(-b)*(x/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1/2)*(Log[(-b)*(x/a
)] - Log[(-(b*c - a*d))*(x/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))]
)*Log[a*((c + d*x)/(c*(a + b*x)))]^2, x] - Simp[(1/2)*(Log[(-b)*(x/a)] - Lo
g[(-d)*(x/c)])*(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])^2, x] + Si
mp[(Log[c + d*x] - Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1 + b*(x/a)
], x] + Simp[(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1
 + d*(x/c)], x] + Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2, c*((a +
b*x)/(a*(c + d*x)))], x] - Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2,
 d*((a + b*x)/(b*(c + d*x)))], x] - Simp[PolyLog[3, 1 + b*(x/a)], x] - Simp
[PolyLog[3, 1 + d*(x/c)], x] + Simp[PolyLog[3, c*((a + b*x)/(a*(c + d*x)))]
, x] - Simp[PolyLog[3, d*((a + b*x)/(b*(c + d*x)))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2490

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-(e*k - d*l)/l + e*(x/l))^n])*(f + g
*Log[h*(-(j*k - i*l)/l + j*(x/l))^m]), x], x, k + l*x], x] /; FreeQ[{a, b,
c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 6732

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[d + e*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, Int[Log[d
+ e*x]*(Log[1 - a*c - b*c*x]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rule 6733

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Dist[b/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x))
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 6734

```
Int[(x_)^(m_.)*PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :>
Simp[(-(a^(m + 1) - b^(m + 1)*x^(m + 1)))*(PolyLog[n, c*(a + b*x)^p]/((m +
1)*b^(m + 1))), x] + Dist[p/((m + 1)*b^m), Int[ExpandIntegrand[PolyLog[n -
```

```
1, c*(a + b*x)^p], (a^(m + 1) - b^(m + 1)*x^(m + 1))/(a + b*x), x], x], x]
/; FreeQ[{a, b, c, p}, x] && GtQ[n, 0] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\text{Li}_3(c(a+bx))}{x^3}\,dx = \frac{\left(b^2-\frac{a^2}{x^2}\right)\text{Li}_3(c(a+bx))}{2a^2} - \frac{1}{2}b^3\int\left(-\frac{\text{Li}_2(c(a+bx))}{ab^2x^2}+\frac{\text{Li}_2(c(a+bx))}{a^2bx}\right)dx$$

$$= \frac{\left(b^2-\frac{a^2}{x^2}\right)\text{Li}_3(c(a+bx))}{2a^2} + \frac{b\int\frac{\text{Li}_2(c(a+bx))}{x^2}\,dx}{2a} - \frac{b^2\int\frac{\text{Li}_2(c(a+bx))}{x}\,dx}{2a^2}$$

$$= -\frac{b\text{Li}_2(c(a+bx))}{2ax} - \frac{b^2\log(x)\text{Li}_2(c(a+bx))}{2a^2} + \frac{\left(b^2-\frac{a^2}{x^2}\right)\text{Li}_3(c(a+bx))}{2a^2} - \frac{b^2\int\frac{\log(1-c}{x(a-}}{2c}$$

$$= -\frac{b\text{Li}_2(c(a+bx))}{2ax} - \frac{b^2\log(x)\text{Li}_2(c(a+bx))}{2a^2} + \frac{\left(b^2-\frac{a^2}{x^2}\right)\text{Li}_3(c(a+bx))}{2a^2} - \frac{b^2\text{Subst}\Big(}{}$$

$$= -\frac{b^2\log(x)\log\left(1+\frac{bx}{a}\right)\log(1-c(a+bx))}{2a^2} - \frac{b^2\Big(\log\left(1+\frac{bx}{a}\right)+\log\left(\frac{1-ac}{1-c(a+bx)}\right)-\log}{4a^2}$$

$$= -\frac{b^2\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2} - \frac{b^2\log(x)\log\left(1+\frac{bx}{a}\right)\log(1-c(a+bx))}{2a^2} - \frac{b^2\Big(\log}{}$$

$$= -\frac{b^2\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2} - \frac{b^2\log(x)\log\left(1+\frac{bx}{a}\right)\log(1-c(a+bx))}{2a^2} - \frac{b^2\Big(\log}{}$$

Mathematica [A]
time = 1.33, size = 573, normalized size = 0.91

$$-\text{PolyLog}(3,c(a+bx)) + \frac{bx\Big(-((a+bx\log(x)-bx\log(a+bx))\text{PolyLog}(2,c(a+bx)))+bx\Big(\log(c(a+bx))\log(1-ac-bcx)-\log(x)\log\left(1+\frac{bx}{a}\right)\log(1-ac-bcx)+\frac{1}{2}\left(\log(c(a+bx))-\log\left(1+\frac{bx}{a}\right)\right)\log(1-ac-bcx)(-2\log(x)+\log(1-ac-bcx))-\left(\log(c(a+bx))-\log\left(1+\frac{bx}{a}\right)\right)\log(1-ac-bcx)\log\left(\frac{a(-1+ac+bcx)}{bx}\right)-\frac{1}{2}\Big(\log\left(\frac{1-ac}{bcx}\right)-\log\left(\frac{(1-ac)(a+bx)}{bx}\right)+\log\Big(1+\frac{bx}{a}\Big) \ldots}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, c*(a + b*x)]/x^3,x]
```

```
[Out] (-PolyLog[3, c*(a + b*x)] + (b*x*(-((a + b*x*Log[x] - b*x*Log[a + b*x])*Pol
yLog[2, c*(a + b*x)]) + b*x*(Log[c*(a + b*x)]*Log[1 - a*c - b*c*x] - Log[x]
*Log[1 + (b*x)/a]*Log[1 - a*c - b*c*x] + ((Log[c*(a + b*x)] - Log[1 + (b*x)
/a])*Log[1 - a*c - b*c*x]*(-2*Log[x] + Log[1 - a*c - b*c*x]))/2 - (Log[c*(a
 + b*x)] - Log[1 + (b*x)/a])*Log[1 - a*c - b*c*x]*Log[(a*(-1 + a*c + b*c*x)
)/(b*x)] - ((Log[(1 - a*c)/(b*c*x)] - Log[((1 - a*c)*(a + b*x))/(b*x)] + Lo
g[1 + (b*x)/a])*Log[(a*(-1 + a*c + b*c*x))/(b*x)]^2)/2 - Log[x]*(Log[1 - a*
c - b*c*x] - Log[1 + (b*c*x)/(-1 + a*c)]) - (Log[1 - a*c - b*c*x] - Log[(a*
```

```
(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, -((b*x)/a)] + PolyLog[2, (b*c*x)/(1
- a*c)] - Log[a + b*x]*PolyLog[2, c*(a + b*x)] + PolyLog[2, 1 - a*c - b*c*x
] - (Log[x] + Log[(a*(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, 1 - a*c - b*c*x
] + Log[(a*(-1 + a*c + b*c*x))/(b*x)]*(PolyLog[2, (a*(-1 + a*c + b*c*x))/(b
*x)] - PolyLog[2, (-1 + a*c + b*c*x)/(b*c*x)]) + PolyLog[3, -((b*x)/a)] + P
olyLog[3, c*(a + b*x)] + PolyLog[3, 1 - a*c - b*c*x] - PolyLog[3, (a*(-1 +
a*c + b*c*x))/(b*x)] + PolyLog[3, (-1 + a*c + b*c*x)/(b*c*x)])))/a^2)/(2*x^
2)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{polylog}\left(3, c(bx+a)\right)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,c*(b*x+a))/x^3,x)
[Out] int(polylog(3,c*(b*x+a))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x^3,x, algorithm="maxima")
[Out] b*integrate(1/2*dilog(b*c*x + a*c)/(b*x^3 + a*x^2), x) - 1/2*polylog(3, b*c
*x + a*c)/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x^3,x, algorithm="fricas")
[Out] integral(polylog(3, b*c*x + a*c)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_{3}\left(ac + bcx\right)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x**3,x)
```

```
[Out] Integral(polylog(3, a*c + b*c*x)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x^3,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, (b*x + a)*c)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{polylog}(3, c\,(a+b\,x))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, c*(a + b*x))/x^3,x)
```

```
[Out] int(polylog(3, c*(a + b*x))/x^3, x)
```

3.137 $\int (d+ex)^3 \textbf{PolyLog}(2,c(a+bx))\,dx$

Optimal. Leaf size=605

$$-\frac{(bd-ae)^3x}{4b^3}-\frac{(bd-ae)^2(bcd+e-ace)x}{8b^3c}-\frac{(bd-ae)(bcd+e-ace)^2x}{12b^3c^2}-\frac{(bcd+e-ace)^3x}{16b^3c^3}-\frac{(bd-ae)^2(d}{16b^2e}$$

```
[Out] -1/4*(-a*e+b*d)^3*x/b^3-1/8*(-a*e+b*d)^2*(-a*c*e+b*c*d+e)*x/b^3/c-1/12*(-a*
e+b*d)*(-a*c*e+b*c*d+e)^2*x/b^3/c^2-1/16*(-a*c*e+b*c*d+e)^3*x/b^3/c^3-1/16*
(-a*e+b*d)^2*(e*x+d)^2/b^2/e-1/24*(-a*e+b*d)*(-a*c*e+b*c*d+e)*(e*x+d)^2/b^2
/c/e-1/32*(-a*c*e+b*c*d+e)^2*(e*x+d)^2/b^2/c^2/e-1/36*(-a*e+b*d)*(e*x+d)^3/
b/e-1/48*(-a*c*e+b*c*d+e)*(e*x+d)^3/b/c/e-1/64*(e*x+d)^4/e-1/8*(-a*e+b*d)^2
*(-a*c*e+b*c*d+e)^2*ln(-b*c*x-a*c+1)/b^4/c^2/e-1/12*(-a*e+b*d)*(-a*c*e+b*c*
d+e)^3*ln(-b*c*x-a*c+1)/b^4/c^3/e-1/16*(-a*c*e+b*c*d+e)^4*ln(-b*c*x-a*c+1)/
b^4/c^4/e-1/4*(-a*e+b*d)^3*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b^4/c+1/8*(-a*e+
b*d)^2*(e*x+d)^2*ln(-b*c*x-a*c+1)/b^2/e+1/12*(-a*e+b*d)*(e*x+d)^3*ln(-b*c*x
-a*c+1)/b/e+1/16*(e*x+d)^4*ln(-b*c*x-a*c+1)/e-1/4*(-a*e+b*d)^4*polylog(2,c*
(b*x+a))/b^4/e+1/4*(e*x+d)^4*polylog(2,c*(b*x+a))/e
```

Rubi [A]
time = 0.42, antiderivative size = 605, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {6733, 2465, 2436, 2332, 2440, 2438, 2442, 45}

$$-\frac{(-ace+bcd+e)^4\log(-ac-bcx+1)}{16b^4c^4e}-\frac{(bd-ae)(-ace+bcd+e)^3\log(-ac-bcx+1)}{12b^4c^3e}-\frac{(bd-ae)^2(-ace+bcd+e)^2\log(-ac-bcx+1)}{8b^4c^2e}-\frac{(bd-ae)^4\text{Li}_2(c(a+bx))}{4b^4e}-\frac{(-ac-bcx+1)(bd-ae)^3\log(-ac-bcx+1)}{4b^4c}-\frac{x(-ace+bcd+e)^3}{16b^3c^3}-\frac{x(bd-ae)(-ace+bcd+e)^2}{12b^3c^2}-\frac{x(bd-ae)^2(-ace+bcd+e)}{8b^3c}-\frac{x(bd-ae)^3}{4b^3}-\frac{(d+ex)^2(-ace+bcd+e)^2}{32b^2c^2e}-\frac{(d+ex)^2(bd-ae)(-ace+bcd+e)}{24b^2ce}+\frac{(d+ex)^2(bd-ae)^2\log(-ac-bcx+1)}{8b^2e}-\frac{(d+ex)^2(bd-ae)^2}{16b^2e}+\frac{(d+ex)^4\text{Li}_2(c(a+bx))}{4e}-\frac{(d+ex)^3(-ace+bcd+e)}{48bce}+\frac{(d+ex)^3(bd-ae)\log(-ac-bcx+1)}{12be}+\frac{(d+ex)^4\log(-ac-bcx+1)}{16e}-\frac{(d+ex)^3(bd-ae)}{36be}-\frac{(d+ex)^4}{64e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] -1/4*((b*d - a*e)^3*x)/b^3 - ((b*d - a*e)^2*(b*c*d + e - a*c*e)*x)/(8*b^3*c
) - ((b*d - a*e)*(b*c*d + e - a*c*e)^2*x)/(12*b^3*c^2) - ((b*c*d + e - a*c*
e)^3*x)/(16*b^3*c^3) - ((b*d - a*e)^2*(d + e*x)^2)/(16*b^2*e) - ((b*d - a*e
)*(b*c*d + e - a*c*e)*(d + e*x)^2)/(24*b^2*c*e) - ((b*c*d + e - a*c*e)^2*(d
 + e*x)^2)/(32*b^2*c^2*e) - ((b*d - a*e)*(d + e*x)^3)/(36*b*e) - ((b*c*d +
e - a*c*e)*(d + e*x)^3)/(48*b*c*e) - (d + e*x)^4/(64*e) - ((b*d - a*e)^2*(b
*c*d + e - a*c*e)^2*Log[1 - a*c - b*c*x])/(8*b^4*c^2*e) - ((b*d - a*e)*(b*c
*d + e - a*c*e)^3*Log[1 - a*c - b*c*x])/(12*b^4*c^3*e) - ((b*c*d + e - a*c*
e)^4*Log[1 - a*c - b*c*x])/(16*b^4*c^4*e) - ((b*d - a*e)^3*(1 - a*c - b*c*x
)*Log[1 - a*c - b*c*x])/(4*b^4*c) + ((b*d - a*e)^2*(d + e*x)^2*Log[1 - a*c
- b*c*x])/(8*b^2*e) + ((b*d - a*e)*(d + e*x)^3*Log[1 - a*c - b*c*x])/(12*b*
e) + ((d + e*x)^4*Log[1 - a*c - b*c*x])/(16*e) - ((b*d - a*e)^4*PolyLog[2,
c*(a + b*x)])/(4*b^4*e) + ((d + e*x)^4*PolyLog[2, c*(a + b*x)])/(4*e)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 6733

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Dist[b/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x))
```

```
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^3 \text{Li}_2(c(a+bx))\,dx = \frac{(d+ex)^4 \text{Li}_2(c(a+bx))}{4e} + \frac{b\int \frac{(d+ex)^4 \log(1-ac-bcx)}{a+bx}\,dx}{4e}$$

$$= \frac{(d+ex)^4 \text{Li}_2(c(a+bx))}{4e} + \frac{b\int \left(\frac{e(bd-ae)^3 \log(1-ac-bcx)}{b^4} + \frac{(bd-ae)^4 \log(1-ac-bcx)}{b^4(a+bx)} + \right.}{}$$

$$= \frac{(d+ex)^4 \text{Li}_2(c(a+bx))}{4e} + \frac{1}{4}\int (d+ex)^3 \log(1-ac-bcx)\,dx + \frac{(bd-ae)}{}$$

$$= \frac{(bd-ae)^2(d+ex)^2 \log(1-ac-bcx)}{8b^2e} + \frac{(bd-ae)(d+ex)^3 \log(1-ac-bc}{12be}$$

$$= -\frac{(bd-ae)^3 x}{4b^3} - \frac{(bd-ae)^3(1-ac-bcx)\log(1-ac-bcx)}{4b^4c} + \frac{(bd-ae)^2(d}{}$$

$$= -\frac{(bd-ae)^3 x}{4b^3} - \frac{(bd-ae)^2(bcd+e-ace)x}{8b^3c} - \frac{(bd-ae)(bcd+e-ace)^2 x}{12b^3c^2}$$

Mathematica [A]
time = 0.38, size = 485, normalized size = 0.80

$$\frac{12e(-1+ac+bcx)\left((3-13ac+23a^2c^2-25a^3c^3)e^2+bce(8(2-7ac+11a^2c^2)d+(3-10ac+13a^2c^2)ex)+b^3c^3x(36d^2+16dex+3e^2x^2)+b^2c^2(-36(-1+3ac)d^2-8(-2+5ac)dex+(3-7ac)e^2x^2)\right)\log(1-ac-bcx)+bc(300a^3c^3e^3x-6a^2c^2e^2x(46e+bc(176d+13ex))+4ac(39e^3x+3bce^2x(56d+5ex)+b^2c^2(-144d^3+324d^2ex+60de^2x^2+7e^3x^3))-x(36e^3+6bce^2(32d+3ex)+12b^2c^2e(36d^2+8dex+e^2x^2)+b^3c^3(576d^3+216d^2ex+64de^2x^2+9e^3x^3))+576b^2c^2d^3(-1+ac+bcx)\log(1-c(a+bx)))-144c^4(-4ab^3d^3+6a^2b^2d^2e-4a^3bde^2+a^4e^3-b^4x(4d^3+6d^2ex+4de^2x^2+e^3x^3))\text{PolyLog}(2,c(a+bx))}{576b^4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] (12*e*(-1 + a*c + b*c*x)*((3 - 13*a*c + 23*a^2*c^2 - 25*a^3*c^3)*e^2 + b*c*
e*(8*(2 - 7*a*c + 11*a^2*c^2)*d + (3 - 10*a*c + 13*a^2*c^2)*e*x) + b^3*c^3*
x*(36*d^2 + 16*d*e*x + 3*e^2*x^2) + b^2*c^2*(-36*(-1 + 3*a*c)*d^2 - 8*(-2 +
 5*a*c)*d*e*x + (3 - 7*a*c)*e^2*x^2))*Log[1 - a*c - b*c*x] + b*c*(300*a^3*c
^3*e^3*x - 6*a^2*c^2*e^2*x*(46*e + b*c*(176*d + 13*e*x)) + 4*a*c*(39*e^3*x
+ 3*b*c*e^2*x*(56*d + 5*e*x) + b^2*c^2*(-144*d^3 + 324*d^2*e*x + 60*d*e^2*x
^2 + 7*e^3*x^3)) - x*(36*e^3 + 6*b*c*e^2*(32*d + 3*e*x) + 12*b^2*c^2*e*(36*
d^2 + 8*d*e*x + e^2*x^2) + b^3*c^3*(576*d^3 + 216*d^2*e*x + 64*d*e^2*x^2 +
9*e^3*x^3)) + 576*b^2*c^2*d^3*(-1 + a*c + b*c*x)*Log[1 - c*(a + b*x)]) - 14
4*c^4*(-4*a*b^3*d^3 + 6*a^2*b^2*d^2*e - 4*a^3*b*d*e^2 + a^4*e^3 - b^4*x*(4*
d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3))*PolyLog[2, c*(a + b*x)])/(576*b^4
*c^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1145 vs. $2(567) = 1134$.
time = 0.80, size = 1146, normalized size = 1.89 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*polylog(2,c*(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/c*(1/4*c/b^3*e^3*polylog(2,b*c*x+a*c)*a^4-c/b^2*e^2*polylog(2,b*c*x+a*c
)*a^3*d+3/2*c/b*e*polylog(2,b*c*x+a*c)*a^2*d^2-c*polylog(2,b*c*x+a*c)*a*d^3
+1/4*c*b/e*polylog(2,b*c*x+a*c)*d^4-1/b^3*e^3*polylog(2,b*c*x+a*c)*a^3*(b*c
*x+a*c)+3/b^2*e^2*polylog(2,b*c*x+a*c)*a^2*d*(b*c*x+a*c)-3/b*e*polylog(2,b*
c*x+a*c)*a*d^2*(b*c*x+a*c)+polylog(2,b*c*x+a*c)*d^3*(b*c*x+a*c)+3/2/c/b^3*e
^3*polylog(2,b*c*x+a*c)*a^2*(b*c*x+a*c)^2-3/c/b^2*e^2*polylog(2,b*c*x+a*c)*
a*d*(b*c*x+a*c)^2+3/2/c/b*e*polylog(2,b*c*x+a*c)*d^2*(b*c*x+a*c)^2-1/c^2/b^
3*e^3*polylog(2,b*c*x+a*c)*a*(b*c*x+a*c)^3+1/c^2/b^2*e^2*polylog(2,b*c*x+a*
c)*d*(b*c*x+a*c)^3+1/4/c^3/b^3*e^3*polylog(2,b*c*x+a*c)*(b*c*x+a*c)^4+1/4/c
^3/b^3/e*((1/4*(-b*c*x-a*c+1)^4*ln(-b*c*x-a*c+1)-1/16*(-b*c*x-a*c+1)^4)*e^4
-(1/3*(-b*c*x-a*c+1)^3*ln(-b*c*x-a*c+1)-1/9*(-b*c*x-a*c+1)^3)*(-4*a*c*e^4+4
*b*c*d*e^3+3*e^4)-(1/2*(-b*c*x-a*c+1)^2*ln(-b*c*x-a*c+1)-1/4*(-b*c*x-a*c+1)
^2)*(-6*a^2*c^2*e^4+12*a*b*c^2*d*e^3-6*b^2*c^2*d^2*e^2+8*a*c*e^4-8*b*c*d*e^
3-3*e^4)+4*((-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)-1+x*b*c+a*c)*a^3*c^3*e^4-12*((-
b*c*x-a*c+1)*ln(-b*c*x-a*c+1)-1+x*b*c+a*c)*a^2*b*c^3*d*e^3+12*((-b*c*x-a*c+
1)*ln(-b*c*x-a*c+1)-1+x*b*c+a*c)*a*b^2*c^3*d^2*e^2-4*((-b*c*x-a*c+1)*ln(-b*
c*x-a*c+1)-1+x*b*c+a*c)*b^3*c^3*d^3*e-6*((-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)-1+
x*b*c+a*c)*a^2*c^2*e^4+12*((-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)-1+x*b*c+a*c)*a*b
*c^2*d*e^3-6*((-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)-1+x*b*c+a*c)*b^2*c^2*d^2*e^2+
4*((-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)-1+x*b*c+a*c)*a*c*e^4-4*((-b*c*x-a*c+1)*l
n(-b*c*x-a*c+1)-1+x*b*c+a*c)*b*c*d*e^3-e^4*((-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)
-1+x*b*c+a*c)-dilog(-b*c*x-a*c+1)*c^4*(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*
e^2-4*a*b^3*d^3*e+b^4*d^4)))
```

Maxima [A]
time = 0.28, size = 696, normalized size = 1.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*polylog(2,c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*a*b^3*d^3 - 6*a^2*b^2*d^2*e + 4*a^3*b*d*e^2 - a^4*e^3)*(log(b*c*x +
 a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))/b^4 - 1/576*(9*b^4*c
^4*x^4*e^3 + 4*(16*b^4*c^4*d*e^2 - 7*a*b^3*c^4*e^3 + 3*b^3*c^3*e^3)*x^3 + 6
*(36*b^4*c^4*d^2*e + 13*a^2*b^2*c^4*e^3 - 10*a*b^2*c^3*e^3 + 3*b^2*c^2*e^3
- 8*(5*a*b^3*c^4*e^2 - 2*b^3*c^3*e^2)*d)*x^2 + 12*(48*b^4*c^4*d^3 - 25*a^3*
b*c^4*e^3 + 23*a^2*b*c^3*e^3 - 13*a*b*c^2*e^3 - 36*(3*a*b^3*c^4*e - b^3*c^3
*e)*d^2 + 3*b*c*e^3 + 8*(11*a^2*b^2*c^4*e^2 - 7*a*b^2*c^3*e^2 + 2*b^2*c^2*e
^2)*d)*x - 144*(b^4*c^4*x^4*e^3 + 4*b^4*c^4*d*x^3*e^2 + 6*b^4*c^4*d^2*x^2*e
 + 4*b^4*c^4*d^3*x)*dilog(b*c*x + a*c) - 12*(3*b^4*c^4*x^4*e^3 - 25*a^4*c^4
*e^3 + 48*a^3*c^3*e^3 - 36*a^2*c^2*e^3 + 48*(a*b^3*c^4 - b^3*c^3)*d^3 + 4*(
```

```
4*b^4*c^4*d*e^2 - a*b^3*c^4*e^3)*x^3 - 36*(3*a^2*b^2*c^4*e - 4*a*b^2*c^3*e
+ b^2*c^2*e)*d^2 + 6*(6*b^4*c^4*d^2*e - 4*a*b^3*c^4*d*e^2 + a^2*b^2*c^4*e^3
)*x^2 + 16*a*c*e^3 + 8*(11*a^3*b*c^4*e^2 - 18*a^2*b*c^3*e^2 + 9*a*b*c^2*e^2
 - 2*b*c*e^2)*d + 12*(4*b^4*c^4*d^3 - 6*a*b^3*c^4*d^2*e + 4*a^2*b^2*c^4*d*e
^2 - a^3*b*c^4*e^3)*x - 3*e^3)*log(-b*c*x - a*c + 1))/(b^4*c^4)
```

Fricas [A]
time = 0.36, size = 625, normalized size = 1.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*polylog(2,c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/576*(576*b^4*c^4*d^3*x - 144*(4*b^4*c^4*d^3*x + 4*a*b^3*c^4*d^3 + (b^4*c
^4*x^4 - a^4*c^4)*e^3 + 4*(b^4*c^4*d*x^3 + a^3*b*c^4*d)*e^2 + 6*(b^4*c^4*d^
2*x^2 - a^2*b^2*c^4*d^2)*e)*dilog(b*c*x + a*c) + (9*b^4*c^4*x^4 - 4*(7*a*b^
3*c^4 - 3*b^3*c^3)*x^3 + 6*(13*a^2*b^2*c^4 - 10*a*b^2*c^3 + 3*b^2*c^2)*x^2
- 12*(25*a^3*b*c^4 - 23*a^2*b*c^3 + 13*a*b*c^2 - 3*b*c)*x)*e^3 + 16*(4*b^4*
c^4*d*x^3 - 3*(5*a*b^3*c^4 - 2*b^3*c^3)*d*x^2 + 6*(11*a^2*b^2*c^4 - 7*a*b^2
*c^3 + 2*b^2*c^2)*d*x)*e^2 + 216*(b^4*c^4*d^2*x^2 - 2*(3*a*b^3*c^4 - b^3*c^
3)*d^2*x)*e - 12*(48*b^4*c^4*d^3*x + 48*(a*b^3*c^4 - b^3*c^3)*d^3 + (3*b^4*
c^4*x^4 - 4*a*b^3*c^4*x^3 + 6*a^2*b^2*c^4*x^2 - 12*a^3*b*c^4*x - 25*a^4*c^4
 + 48*a^3*c^3 - 36*a^2*c^2 + 16*a*c - 3)*e^3 + 8*(2*b^4*c^4*d*x^3 - 3*a*b^3
*c^4*d*x^2 + 6*a^2*b^2*c^4*d*x + (11*a^3*b*c^4 - 18*a^2*b*c^3 + 9*a*b*c^2 -
 2*b*c)*d)*e^2 + 36*(b^4*c^4*d^2*x^2 - 2*a*b^3*c^4*d^2*x - (3*a^2*b^2*c^4 -
 4*a*b^2*c^3 + b^2*c^2)*d^2)*e)*log(-b*c*x - a*c + 1))/(b^4*c^4)
```

Sympy [A]
time = 20.82, size = 1030, normalized size = 1.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*polylog(2,c*(b*x+a)),x)
```

```
[Out] Piecewise((0, Eq(c, 0) & (Eq(b, 0) | Eq(c, 0))), ((d**3*x + 3*d**2*e*x**2/2
 + d*e**2*x**3 + e**3*x**4/4)*polylog(2, a*c), Eq(b, 0)), (25*a**4*e**3*pol
ylog(1, a*c + b*c*x)/(48*b**4) - a**4*e**3*polylog(2, a*c + b*c*x)/(4*b**4)
 - 11*a**3*d*e**2*polylog(1, a*c + b*c*x)/(6*b**3) + a**3*d*e**2*polylog(2,
 a*c + b*c*x)/b**3 + a**3*e**3*x*polylog(1, a*c + b*c*x)/(4*b**3) + 25*a**3
*e**3*x/(48*b**3) - a**3*e**3*polylog(1, a*c + b*c*x)/(b**4*c) + 9*a**2*d**
2*e*polylog(1, a*c + b*c*x)/(4*b**2) - 3*a**2*d**2*e*polylog(2, a*c + b*c*x
)/(2*b**2) - a**2*d*e**2*x*polylog(1, a*c + b*c*x)/b**2 - 11*a**2*d*e**2*x/
(6*b**2) - a**2*e**3*x**2*polylog(1, a*c + b*c*x)/(8*b**2) - 13*a**2*e**3*x
**2/(96*b**2) + 3*a**2*d*e**2*polylog(1, a*c + b*c*x)/(b**3*c) - 23*a**2*e*
```

```
*3*x/(48*b**3*c) + 3*a**2*e**3*polylog(1, a*c + b*c*x)/(4*b**4*c**2) - a*d*
*3*polylog(1, a*c + b*c*x)/b + a*d**3*polylog(2, a*c + b*c*x)/b + 3*a*d**2*
e*x*polylog(1, a*c + b*c*x)/(2*b) + 9*a*d**2*e*x/(4*b) + a*d*e**2*x**2*poly
log(1, a*c + b*c*x)/(2*b) + 5*a*d*e**2*x**2/(12*b) + a*e**3*x**3*polylog(1,
 a*c + b*c*x)/(12*b) + 7*a*e**3*x**3/(144*b) - 3*a*d**2*e*polylog(1, a*c +
b*c*x)/(b**2*c) + 7*a*d*e**2*x/(6*b**2*c) + 5*a*e**3*x**2/(48*b**2*c) - 3*a
*d*e**2*polylog(1, a*c + b*c*x)/(2*b**3*c**2) + 13*a*e**3*x/(48*b**3*c**2)
- a*e**3*polylog(1, a*c + b*c*x)/(3*b**4*c**3) - d**3*x*polylog(1, a*c + b*
c*x) + d**3*x*polylog(2, a*c + b*c*x) - d**3*x - 3*d**2*e*x**2*polylog(1, a
*c + b*c*x)/4 + 3*d**2*e*x**2*polylog(2, a*c + b*c*x)/2 - 3*d**2*e*x**2/8 -
 d*e**2*x**3*polylog(1, a*c + b*c*x)/3 + d*e**2*x**3*polylog(2, a*c + b*c*x
) - d*e**2*x**3/9 - e**3*x**4*polylog(1, a*c + b*c*x)/16 + e**3*x**4*polylo
g(2, a*c + b*c*x)/4 - e**3*x**4/64 + d**3*polylog(1, a*c + b*c*x)/(b*c) - 3
*d**2*e*x/(4*b*c) - d*e**2*x**2/(6*b*c) - e**3*x**3/(48*b*c) + 3*d**2*e*pol
ylog(1, a*c + b*c*x)/(4*b**2*c**2) - d*e**2*x/(3*b**2*c**2) - e**3*x**2/(32
*b**2*c**2) + d*e**2*polylog(1, a*c + b*c*x)/(3*b**3*c**3) - e**3*x/(16*b**
3*c**3) + e**3*polylog(1, a*c + b*c*x)/(16*b**4*c**4), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*polylog(2,c*(b*x+a)),x, algorithm="giac")
[Out] integrate((e*x + d)^3*dilog((b*x + a)*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{polylog}(2, c\,(a+b\,x))\ (d+e\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, c*(a + b*x))*(d + e*x)^3,x)
[Out] int(polylog(2, c*(a + b*x))*(d + e*x)^3, x)
```

3.138 $\int (d+ex)^2 \textbf{PolyLog}(2,c(a+bx))\,dx$

Optimal. Leaf size=385

$$-\frac{(bd-ae)^2x}{3b^2}-\frac{(bd-ae)(bcd+e-ace)x}{6b^2c}-\frac{(bcd+e-ace)^2x}{9b^2c^2}-\frac{(bd-ae)(d+ex)^2}{12be}-\frac{(bcd+e-ace)(d+e}{18bce}$$

```
[Out] -1/3*(-a*e+b*d)^2*x/b^2-1/6*(-a*e+b*d)*(-a*c*e+b*c*d+e)*x/b^2/c-1/9*(-a*c*e
+b*c*d+e)^2*x/b^2/c^2-1/12*(-a*e+b*d)*(e*x+d)^2/b/e-1/18*(-a*c*e+b*c*d+e)*(
e*x+d)^2/b/c/e-1/27*(e*x+d)^3/e-1/6*(-a*e+b*d)*(-a*c*e+b*c*d+e)^2*ln(-b*c*x
-a*c+1)/b^3/c^2/e-1/9*(-a*c*e+b*c*d+e)^3*ln(-b*c*x-a*c+1)/b^3/c^3/e-1/3*(-a
*e+b*d)^2*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b^3/c+1/6*(-a*e+b*d)*(e*x+d)^2*ln
(-b*c*x-a*c+1)/b/e+1/9*(e*x+d)^3*ln(-b*c*x-a*c+1)/e-1/3*(-a*e+b*d)^3*polylo
g(2,c*(b*x+a))/b^3/e+1/3*(e*x+d)^3*polylog(2,c*(b*x+a))/e
```

Rubi [A]
time = 0.24, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {6733, 2465, 2436, 2332, 2440, 2438, 2442, 45}

$$-\frac{(-ace+bcd+e)^3\log(-ac-bcx+1)}{9b^3c^3e}-\frac{(bd-ae)(-ace+bcd+e)^2\log(-ac-bcx+1)}{6b^3c^2e}-\frac{(bd-ae)^3\text{Li}_2(c(a+bx))}{3b^3e}-\frac{(-ac-bcx+1)(bd-ae)^2\log(-ac-bcx+1)}{3b^3c}-\frac{x(-ace+bcd+e)^2}{9b^2c^2}-\frac{x(bd-ae)(-ace+bcd+e)}{6b^2c}-\frac{x(bd-ae)^2}{3b^2}+\frac{(d+ex)^3\text{Li}_2(c(a+bx))}{3e}-\frac{(d+ex)^2(-ace+bcd+e)}{18bce}+\frac{(d+ex)^2(bd-ae)\log(-ac-bcx+1)}{6be}+\frac{(d+ex)^3\log(-ac-bcx+1)}{9e}-\frac{(d+ex)^2(bd-ae)}{12be}-\frac{(d+ex)^3}{27e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] -1/3*((b*d - a*e)^2*x)/b^2 - ((b*d - a*e)*(b*c*d + e - a*c*e)*x)/(6*b^2*c)
 - ((b*c*d + e - a*c*e)^2*x)/(9*b^2*c^2) - ((b*d - a*e)*(d + e*x)^2)/(12*b*e
) - ((b*c*d + e - a*c*e)*(d + e*x)^2)/(18*b*c*e) - (d + e*x)^3/(27*e) - ((b
*d - a*e)*(b*c*d + e - a*c*e)^2*Log[1 - a*c - b*c*x])/(6*b^3*c^2*e) - ((b*c
*d + e - a*c*e)^3*Log[1 - a*c - b*c*x])/(9*b^3*c^3*e) - ((b*d - a*e)^2*(1 -
 a*c - b*c*x)*Log[1 - a*c - b*c*x])/(3*b^3*c) + ((b*d - a*e)*(d + e*x)^2*Lo
g[1 - a*c - b*c*x])/(6*b*e) + ((d + e*x)^3*Log[1 - a*c - b*c*x])/(9*e) - ((
b*d - a*e)^3*PolyLog[2, c*(a + b*x)])/(3*b^3*e) + ((d + e*x)^3*PolyLog[2, c
*(a + b*x)])/(3*e)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 6733

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Dist[b/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x))
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^2 \mathrm{Li}_2(c(a+bx))\,dx = \frac{(d+ex)^3 \mathrm{Li}_2(c(a+bx))}{3e} + \frac{b\int \frac{(d+ex)^3 \log(1-ac-bcx)}{a+bx}\,dx}{3e}$$
$$= \frac{(d+ex)^3 \mathrm{Li}_2(c(a+bx))}{3e} + \frac{b\int \left(\frac{e(bd-ae)^2 \log(1-ac-bcx)}{b^3} + \frac{(bd-ae)^3 \log(1-ac-bcx)}{b^3(a+bx)} + \right.}{}$$
$$= \frac{(d+ex)^3 \mathrm{Li}_2(c(a+bx))}{3e} + \frac{1}{3}\int (d+ex)^2 \log(1-ac-bcx)\,dx + \frac{(bd-ae)}{}$$
$$= \frac{(bd-ae)(d+ex)^2 \log(1-ac-bcx)}{6be} + \frac{(d+ex)^3 \log(1-ac-bcx)}{9e} + \frac{(d+}{}$$
$$= -\frac{(bd-ae)^2 x}{3b^2} - \frac{(bd-ae)^2(1-ac-bcx)\log(1-ac-bcx)}{3b^3c} + \frac{(bd-ae)(d}{}$$
$$= -\frac{(bd-ae)^2 x}{3b^2} - \frac{(bd-ae)(bcd+e-ace)x}{6b^2c} - \frac{(bcd+e-ace)^2 x}{9b^2c^2} - \frac{(bd-a}{}$$

Mathematica [A]
time = 0.12, size = 274, normalized size = 0.71

$$\frac{6e(-1+ac+bcx)\left((2-7ac+11a^2c^2)e+b^2c^2x(9d+2ex)+bc((9-27ac)d+(2-5ac)ex)\right)\log(1-ac-bcx)+bc(-66a^2c^2e^2x-x(12e^2+6bce(9d+ex)+b^2c^2(108d^2+27dex+4e^2x^2))+3ac(14e^2x+bc(-36d^2+54dex+5e^2x^2))+108bcd^2(-1+ac+bcx)\log(1-c(a+bx)))+36c^3(3ab^2d^2-3a^2bde+a^3e^2+b^3x(3d^2+3dex+e^2x^2))\,\mathrm{PolyLog}(2,c(a+bx))}{108b^3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] (6*e*(-1 + a*c + b*c*x)*((2 - 7*a*c + 11*a^2*c^2)*e + b^2*c^2*x*(9*d + 2*e*
x) + b*c*((9 - 27*a*c)*d + (2 - 5*a*c)*e*x))*Log[1 - a*c - b*c*x] + b*c*(-6
6*a^2*c^2*e^2*x - x*(12*e^2 + 6*b*c*e*(9*d + e*x) + b^2*c^2*(108*d^2 + 27*d
*e*x + 4*e^2*x^2)) + 3*a*c*(14*e^2*x + b*c*(-36*d^2 + 54*d*e*x + 5*e^2*x^2)
) + 108*b*c*d^2*(-1 + a*c + b*c*x)*Log[1 - c*(a + b*x)]) + 36*c^3*(3*a*b^2*
d^2 - 3*a^2*b*d*e + a^3*e^2 + b^3*x*(3*d^2 + 3*d*e*x + e^2*x^2))*PolyLog[2,
 c*(a + b*x)])/(108*b^3*c^3)
```

Maple [A]
time = 0.80, size = 685, normalized size = 1.78

method	result
derivativedivides	$-\frac{c\,e^2 \operatorname{polylog}(2,xbc+ac)a^3}{3b^2} + \frac{ce \operatorname{polylog}(2,xbc+ac)a^2 d}{b} - c \operatorname{polylog}(2,xbc+ac)\,a\,d^2 + \frac{cb \operatorname{polylog}(2,xbc+ac)d^3}{3e} + \frac{e^2 \operatorname{polylog}(2,xbc+ac)}{b^2}$
default	$-\frac{c\,e^2 \operatorname{polylog}(2,xbc+ac)a^3}{3b^2} + \frac{ce \operatorname{polylog}(2,xbc+ac)a^2 d}{b} - c \operatorname{polylog}(2,xbc+ac)\,a\,d^2 + \frac{cb \operatorname{polylog}(2,xbc+ac)d^3}{3e} + \frac{e^2 \operatorname{polylog}(2,xbc+ac)}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*polylog(2,c*(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/c*(-1/3*c/b^2*e^2*polylog(2,b*c*x+a*c)*a^3+c/b*e*polylog(2,b*c*x+a*c)*a
^2*d-c*polylog(2,b*c*x+a*c)*a*d^2+1/3*c*b/e*polylog(2,b*c*x+a*c)*d^3+1/b^2*
e^2*polylog(2,b*c*x+a*c)*a^2*(b*c*x+a*c)-2/b*e*polylog(2,b*c*x+a*c)*a*d*(b*
c*x+a*c)+polylog(2,b*c*x+a*c)*d^2*(b*c*x+a*c)-1/c/b^2*e^2*polylog(2,b*c*x+a
*c)*a*(b*c*x+a*c)^2+1/c/b*e*polylog(2,b*c*x+a*c)*d*(b*c*x+a*c)^2+1/3/c^2/b^
2*e^2*polylog(2,b*c*x+a*c)*(b*c*x+a*c)^3-1/3/c^2/b^2/e*((1/3*(-b*c*x-a*c+1)
^3*ln(-b*c*x-a*c+1)-1/9*(-b*c*x-a*c+1)^3)*e^3-(1/2*(-b*c*x-a*c+1)^2*ln(-b*c
*x-a*c+1)-1/4*(-b*c*x-a*c+1)^2)*(-3*a*c*e^3+3*b*c*d*e^2+2*e^3)+3*((-b*c*x-a
*c+1)*ln(-b*c*x-a*c+1)-1+x*b*c+a*c)*a^2*c^2*e^3-6*((-b*c*x-a*c+1)*ln(-b*c*x
-a*c+1)-1+x*b*c+a*c)*a*b*c^2*d*e^2+3*((-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)-1+x*b
*c+a*c)*b^2*c^2*d^2*e-3*((-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)-1+x*b*c+a*c)*a*c*e
^3+3*((-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)-1+x*b*c+a*c)*b*c*d*e^2+((-b*c*x-a*c+1
)*ln(-b*c*x-a*c+1)-1+x*b*c+a*c)*e^3-dilog(-b*c*x-a*c+1)*c^3*(a^3*e^3-3*a^2*
b*d*e^2+3*a*b^2*d^2*e-b^3*d^3)))
```

Maxima [A]
time = 0.27, size = 416, normalized size = 1.08

$$-\frac{(3\,ab^2d^2-3\,a^2bde+a^3e^2)(\log(bcx+ac)\log(-bcx-ac+1)+\mathrm{Li}_2(-bcx-ac+1))}{3\,b^3}-\frac{4\,b^3c^3x^3e^2+3\,(9\,b^3c^3de-5\,ab^2c^3e^2+2\,b^2c^2e^2)x^2+6\,(18\,b^3c^3d^2+11\,a^2bc^3e^2-7\,abc^2e^2+2\,bce^2-9\,(3\,ab^2c^3e-b^2c^2e)d)x-36\,(b^3c^3x^3e^2+3\,b^3c^3dx^2e+3\,b^3c^3d^2x)\mathrm{Li}_2(bcx+ac)-6\,(2\,b^3c^3x^3e^2+11\,a^3c^3e^2-18\,a^2c^2e^2+18\,(ab^2c^3-b^2c^2)d^2+3\,(3\,b^3c^3de-ab^2c^3e^2)x^2+9\,ace^2-9\,(3\,a^2bc^3e-4\,abc^2e+bce)d+6\,(3\,b^3c^3d^2-3\,ab^2c^3de+a^2bc^3e^2)x-2\,e^2)\log(-bcx-ac+1)}{108\,b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*polylog(2,c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/3*(3*a*b^2*d^2 - 3*a^2*b*d*e + a^3*e^2)*(log(b*c*x + a*c)*log(-b*c*x - a
*c + 1) + dilog(-b*c*x - a*c + 1))/b^3 - 1/108*(4*b^3*c^3*x^3*e^2 + 3*(9*b^
3*c^3*d*e - 5*a*b^2*c^3*e^2 + 2*b^2*c^2*e^2)*x^2 + 6*(18*b^3*c^3*d^2 + 11*a
^2*b*c^3*e^2 - 7*a*b*c^2*e^2 + 2*b*c*e^2 - 9*(3*a*b^2*c^3*e - b^2*c^2*e)*d)
*x - 36*(b^3*c^3*x^3*e^2 + 3*b^3*c^3*d*x^2*e + 3*b^3*c^3*d^2*x)*dilog(b*c*x
 + a*c) - 6*(2*b^3*c^3*x^3*e^2 + 11*a^3*c^3*e^2 - 18*a^2*c^2*e^2 + 18*(a*b^
2*c^3 - b^2*c^2)*d^2 + 3*(3*b^3*c^3*d*e - a*b^2*c^3*e^2)*x^2 + 9*a*c*e^2 -
9*(3*a^2*b*c^3*e - 4*a*b*c^2*e + b*c*e)*d + 6*(3*b^3*c^3*d^2 - 3*a*b^2*c^3*
d*e + a^2*b*c^3*e^2)*x - 2*e^2)*log(-b*c*x - a*c + 1))/(b^3*c^3)
```

Fricas [A]
time = 0.35, size = 361, normalized size = 0.94

$$-\frac{108\,b^3c^3d^2x-36\,(3\,b^3c^3d^2x+3\,ab^2c^3d^2+(b^3c^3x^3+a^3c^3)e^2+3\,(b^3c^3dx^2-a^2bc^3d)e)\mathrm{Li}_2(bcx+ac)+(4\,b^3c^3x^3-3\,(5\,ab^2c^3-2\,b^2c^2)x^2+6\,(11\,a^2bc^3-7\,abc^2+2\,bc)x)e^2+27\,(b^3c^3dx^2-2\,(3\,ab^2c^3-b^2c^2)dx)e-6\,(18\,b^3c^3d^2x+18\,(ab^2c^3-b^2c^2)d^2+(2\,b^3c^3x^3-3\,ab^2c^3x^2+6\,a^2bc^3x+11\,a^3c^3-18\,a^2c^2+9\,ac-2)e^2+9\,(b^3c^3dx^2-2\,ab^2c^3dx-(3\,a^2bc^3-4\,abc^2+bc)d)e)\log(-bcx-ac+1)}{108\,b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*polylog(2,c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/108*(108*b^3*c^3*d^2*x - 36*(3*b^3*c^3*d^2*x + 3*a*b^2*c^3*d^2 + (b^3*c^
3*x^3 + a^3*c^3)*e^2 + 3*(b^3*c^3*d*x^2 - a^2*b*c^3*d)*e)*dilog(b*c*x + a*c
) + (4*b^3*c^3*x^3 - 3*(5*a*b^2*c^3 - 2*b^2*c^2)*x^2 + 6*(11*a^2*b*c^3 - 7*
a*b*c^2 + 2*b*c)*x)*e^2 + 27*(b^3*c^3*d*x^2 - 2*(3*a*b^2*c^3 - b^2*c^2)*d*x
)*e - 6*(18*b^3*c^3*d^2*x + 18*(a*b^2*c^3 - b^2*c^2)*d^2 + (2*b^3*c^3*x^3 -
 3*a*b^2*c^3*x^2 + 6*a^2*b*c^3*x + 11*a^3*c^3 - 18*a^2*c^2 + 9*a*c - 2)*e^2
 + 9*(b^3*c^3*d*x^2 - 2*a*b^2*c^3*d*x - (3*a^2*b*c^3 - 4*a*b*c^2 + b*c)*d)*
e)*log(-b*c*x - a*c + 1))/(b^3*c^3)
```

Sympy [A]
time = 10.21, size = 561, normalized size = 1.46

$$\begin{cases} 0 & \text{for}\: b = 0 \wedge c = 0 \\ \left(d^{2} x + d e x^{2} + \frac{e^{2} x^{3}}{3}\right) \operatorname{Li}_{2}\left(a c\right) & \text{for}\: b = 0 \\ 0 & \text{for}\: c = 0 \\ - \frac{11 a^{3} e^{2} \operatorname{Li}_{1}(a c + b c x)}{18 b^{3}} + \frac{a^{3} e^{2} \operatorname{Li}_{2}(a c + b c x)}{3 b^{3}} + \frac{3 a^{2} d e \operatorname{Li}_{1}(a c + b c x)}{2 b^{2}} - \frac{a^{2} d e \operatorname{Li}_{2}(a c + b c x)}{b^{2}} - \frac{a^{2} e^{2} x \operatorname{Li}_{1}(a c + b c x)}{3 b^{2}} - \frac{11 a^{2} e^{2} x}{18 b^{2}} + \frac{a^{2} e^{2} \operatorname{Li}_{1}(a c + b c x)}{b^{3} c} - \frac{a d^{2} \operatorname{Li}_{1}(a c + b c x)}{b} + \frac{a d^{2} \operatorname{Li}_{2}(a c + b c x)}{b} + \frac{a d e x \operatorname{Li}_{1}(a c + b c x)}{b} + \frac{3 a d e x}{2 b} + \frac{a e^{2} x^{2} \operatorname{Li}_{1}(a c + b c x)}{6 b} + \frac{5 a e^{2} x^{2}}{36 b} - \frac{2 a d e \operatorname{Li}_{1}(a c + b c x)}{b^{2} c} + \frac{7 a e^{2} x}{18 b^{2} c} - \frac{a e^{2} \operatorname{Li}_{1}(a c + b c x)}{2 b^{3} c^{2}} - d^{2} x \operatorname{Li}_{1}\left(a c + b c x\right) + d^{2} x \operatorname{Li}_{2}\left(a c + b c x\right) - d^{2} x - \frac{d e x^{2} \operatorname{Li}_{1}(a c + b c x)}{2} + d e x^{2} \operatorname{Li}_{2}\left(a c + b c x\right) - \frac{d e x^{2}}{4} - \frac{e^{2} x^{3} \operatorname{Li}_{1}(a c + b c x)}{9} + \frac{e^{2} x^{3} \operatorname{Li}_{2}(a c + b c x)}{3} - \frac{e^{2} x^{3}}{27} + \frac{d^{2} \operatorname{Li}_{1}(a c + b c x)}{b c} - \frac{d e x}{2 b c} - \frac{e^{2} x^{2}}{18 b c} + \frac{d e \operatorname{Li}_{1}(a c + b c x)}{2 b^{2} c^{2}} - \frac{e^{2} x}{9 b^{2} c^{2}} + \frac{e^{2} \operatorname{Li}_{1}(a c + b c x)}{9 b^{3} c^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*polylog(2,c*(b*x+a)),x)
```

```
[Out] Piecewise((0, Eq(b, 0) & Eq(c, 0)), ((d**2*x + d*e*x**2 + e**2*x**3/3)*poly
log(2, a*c), Eq(b, 0)), (0, Eq(c, 0)), (-11*a**3*e**2*polylog(1, a*c + b*c*
x)/(18*b**3) + a**3*e**2*polylog(2, a*c + b*c*x)/(3*b**3) + 3*a**2*d*e*poly
log(1, a*c + b*c*x)/(2*b**2) - a**2*d*e*polylog(2, a*c + b*c*x)/b**2 - a**2
*e**2*x*polylog(1, a*c + b*c*x)/(3*b**2) - 11*a**2*e**2*x/(18*b**2) + a**2*
e**2*polylog(1, a*c + b*c*x)/(b**3*c) - a*d**2*polylog(1, a*c + b*c*x)/b +
a*d**2*polylog(2, a*c + b*c*x)/b + a*d*e*x*polylog(1, a*c + b*c*x)/b + 3*a*
d*e*x/(2*b) + a*e**2*x**2*polylog(1, a*c + b*c*x)/(6*b) + 5*a*e**2*x**2/(36
*b) - 2*a*d*e*polylog(1, a*c + b*c*x)/(b**2*c) + 7*a*e**2*x/(18*b**2*c) - a
*e**2*polylog(1, a*c + b*c*x)/(2*b**3*c**2) - d**2*x*polylog(1, a*c + b*c*x
) + d**2*x*polylog(2, a*c + b*c*x) - d**2*x - d*e*x**2*polylog(1, a*c + b*c
*x)/2 + d*e*x**2*polylog(2, a*c + b*c*x) - d*e*x**2/4 - e**2*x**3*polylog(1
, a*c + b*c*x)/9 + e**2*x**3*polylog(2, a*c + b*c*x)/3 - e**2*x**3/27 + d**
2*polylog(1, a*c + b*c*x)/(b*c) - d*e*x/(2*b*c) - e**2*x**2/(18*b*c) + d*e*
polylog(1, a*c + b*c*x)/(2*b**2*c**2) - e**2*x/(9*b**2*c**2) + e**2*polylog
(1, a*c + b*c*x)/(9*b**3*c**3), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*polylog(2,c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*dilog((b*x + a)*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{polylog}(2, c\,(a+b\,x))\ {(d+e\,x)}^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, c*(a + b*x))*(d + e*x)^2,x)
[Out] int(polylog(2, c*(a + b*x))*(d + e*x)^2, x)
```

3.139 $\int (d+ex)\mathbf{PolyLog}(2,c(a+bx))\,dx$

Optimal. Leaf size=210

$$-\frac{(bd-ae)x}{2b}-\frac{(bcd+e-ace)x}{4bc}-\frac{(d+ex)^2}{8e}-\frac{(bcd+e-ace)^2\log(1-ac-bcx)}{4b^2c^2e}-\frac{(bd-ae)(1-ac-bcx)\,\mathrm{l}}{2b^2c}$$

```
[Out] -1/2*(-a*e+b*d)*x/b-1/4*(-a*c*e+b*c*d+e)*x/b/c-1/8*(e*x+d)^2/e-1/4*(-a*c*e+
b*c*d+e)^2*ln(-b*c*x-a*c+1)/b^2/c^2/e-1/2*(-a*e+b*d)*(-b*c*x-a*c+1)*ln(-b*c
*x-a*c+1)/b^2/c+1/4*(e*x+d)^2*ln(-b*c*x-a*c+1)/e-1/2*(-a*e+b*d)^2*polylog(2
,c*(b*x+a))/b^2/e+1/2*(e*x+d)^2*polylog(2,c*(b*x+a))/e
```

Rubi [A]
time = 0.14, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {6733, 2465, 2436, 2332, 2440, 2438, 2442, 45}

$$-\frac{(-ace+bcd+e)^2\log(-ac-bcx+1)}{4b^2c^2e}-\frac{(bd-ae)^2\mathrm{Li}_2(c(a+bx))}{2b^2e}-\frac{(-ac-bcx+1)(bd-ae)\log(-ac-bcx+1)}{2b^2c}+\frac{(d+ex)^2\mathrm{Li}_2(c(a+bx))}{2e}-\frac{x(-ace+bcd+e)}{4bc}+\frac{(d+ex)^2\log(-ac-bcx+1)}{4e}-\frac{x(bd-ae)}{2b}-\frac{(d+ex)^2}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] -1/2*((b*d - a*e)*x)/b - ((b*c*d + e - a*c*e)*x)/(4*b*c) - (d + e*x)^2/(8*e
) - ((b*c*d + e - a*c*e)^2*Log[1 - a*c - b*c*x])/(4*b^2*c^2*e) - ((b*d - a*
e)*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(2*b^2*c) + ((d + e*x)^2*Log[1 -
 a*c - b*c*x])/(4*e) - ((b*d - a*e)^2*PolyLog[2, c*(a + b*x)])/(2*b^2*e) +
((d + e*x)^2*PolyLog[2, c*(a + b*x)])/(2*e)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 6733

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Dist[b/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x))
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)\text{Li}_2(c(a+bx))\,dx = \frac{(d+ex)^2\text{Li}_2(c(a+bx))}{2e} + \frac{b\int \frac{(d+ex)^2\log(1-ac-bcx)}{a+bx}\,dx}{2e}$$

$$= \frac{(d+ex)^2\text{Li}_2(c(a+bx))}{2e} + \frac{b\int \left(\frac{e(bd-ae)\log(1-ac-bcx)}{b^2} + \frac{(bd-ae)^2\log(1-ac-bcx)}{b^2(a+bx)} + \frac{e}{}\right.}{2e}$$

$$= \frac{(d+ex)^2\text{Li}_2(c(a+bx))}{2e} + \frac{1}{2}\int (d+ex)\log(1-ac-bcx)\,dx + \frac{(bd-ae)\int 1}{}$$

$$= \frac{(d+ex)^2\log(1-ac-bcx)}{4e} + \frac{(d+ex)^2\text{Li}_2(c(a+bx))}{2e} + \frac{(bc)\int \frac{(d+ex)^2}{1-ac-bcx}\,dx}{4e}$$

$$= -\frac{(bd-ae)x}{2b} - \frac{(bd-ae)(1-ac-bcx)\log(1-ac-bcx)}{2b^2c} + \frac{(d+ex)^2\log(1}{4e}$$

$$= -\frac{(bd-ae)x}{2b} - \frac{(bcd+e-ace)x}{4bc} - \frac{(d+ex)^2}{8e} - \frac{(bcd+e-ace)^2\log(1-ac}{4b^2c^2e}$$

Mathematica [A]
time = 0.06, size = 161, normalized size = 0.77

$$\frac{e(-bcx(2-6ac+bcx)+(-2-6a^2c^2+2b^2c^2x^2-4ac(-2+bcx))\log(1-ac-bcx)-4a^2c^2\text{PolyLog}(2,c(a+bx)))}{8b^2c^2} + \frac{d(-c(a+bx)+(-1+c(a+bx))\log(1-c(a+bx))+c(a+bx)\text{PolyLog}(2,c(a+bx)))}{bc} + \frac{1}{2}ex^2\text{PolyLog}(2,ac+bcx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] (e*(-(b*c*x*(2 - 6*a*c + b*c*x)) + (-2 - 6*a^2*c^2 + 2*b^2*c^2*x^2 - 4*a*c*
(-2 + b*c*x))*Log[1 - a*c - b*c*x] - 4*a^2*c^2*PolyLog[2, c*(a + b*x)]))/(8
*b^2*c^2) + (d*(-(c*(a + b*x)) + (-1 + c*(a + b*x))*Log[1 - c*(a + b*x)] +
c*(a + b*x)*PolyLog[2, c*(a + b*x)]))/(b*c) + (e*x^2*PolyLog[2, a*c + b*c*x
])/2
```

Maple [A]
time = 0.66, size = 249, normalized size = 1.19

method	result
derivativedivides	$\frac{-\frac{\text{polylog}(2,xbc+ac)ae(xbc+ac)}{b}+\text{polylog}(2,xbc+ac)d(xbc+ac)+\frac{\text{polylog}(2,xbc+ac)e(xbc+ac)^2}{2cb}-\frac{-2ace((-xbc-ac+1)\ln(-xbc-}{}}{}$
default	$\frac{-\frac{\text{polylog}(2,xbc+ac)ae(xbc+ac)}{b}+\text{polylog}(2,xbc+ac)d(xbc+ac)+\frac{\text{polylog}(2,xbc+ac)e(xbc+ac)^2}{2cb}-\frac{-2ace((-xbc-ac+1)\ln(-xbc-}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*polylog(2,c*(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/c*(-1/b*polylog(2,b*c*x+a*c)*a*e*(b*c*x+a*c)+polylog(2,b*c*x+a*c)*d*(b*
c*x+a*c)+1/2/c/b*polylog(2,b*c*x+a*c)*e*(b*c*x+a*c)^2-1/2/b/c*(-2*a*c*e*((-
b*c*x-a*c+1)*ln(-b*c*x-a*c+1)-1+x*b*c+a*c)+2*d*c*b*((-b*c*x-a*c+1)*ln(-b*c*
x-a*c+1)-1+x*b*c+a*c)-e*(1/2*(-b*c*x-a*c+1)^2*ln(-b*c*x-a*c+1)-1/4*(-b*c*x-
a*c+1)^2)+e*((-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)-1+x*b*c+a*c)))
```

Maxima [A]
time = 0.28, size = 219, normalized size = 1.04

$$-\frac{(2\,abd-a^2e)(\log\left(bcx+ac\right)\log\left(-bcx-ac+1\right)+\mathrm{Li}_2(-bcx-ac+1))}{2\,b^2}-\frac{b^2c^2x^2e+2\,(4\,b^2c^2d-3\,abc^2e+bce)x-4\,(b^2c^2x^2e+2\,b^2c^2dx)\mathrm{Li}_2(bcx+ac)-2\,(b^2c^2x^2e-3\,a^2c^2e+4\,ace+4\,(abc^2-bc)d+2\,(2\,b^2c^2d-abc^2e)x-e)\log\left(-bcx-ac+1\right)}{8\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*polylog(2,c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*b*d - a^2*e)*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c
*x - a*c + 1))/b^2 - 1/8*(b^2*c^2*x^2*e + 2*(4*b^2*c^2*d - 3*a*b*c^2*e + b*
c*e)*x - 4*(b^2*c^2*x^2*e + 2*b^2*c^2*d*x)*dilog(b*c*x + a*c) - 2*(b^2*c^2*
x^2*e - 3*a^2*c^2*e + 4*a*c*e + 4*(a*b*c^2 - b*c)*d + 2*(2*b^2*c^2*d - a*b*
c^2*e)*x - e)*log(-b*c*x - a*c + 1))/(b^2*c^2)
```

Fricas [A]
time = 0.42, size = 174, normalized size = 0.83

$$-\frac{8\,b^2c^2dx-4\,(2\,b^2c^2dx+2\,abc^2d+(b^2c^2x^2-a^2c^2)e)\mathrm{Li}_2(bcx+ac)+(b^2c^2x^2-2\,(3\,abc^2-bc)x)e-2\,(4\,b^2c^2dx+4\,(abc^2-bc)d+(b^2c^2x^2-2\,abc^2x-3\,a^2c^2+4\,ac-1)e)\log\left(-bcx-ac+1\right)}{8\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*polylog(2,c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/8*(8*b^2*c^2*d*x - 4*(2*b^2*c^2*d*x + 2*a*b*c^2*d + (b^2*c^2*x^2 - a^2*c
^2)*e)*dilog(b*c*x + a*c) + (b^2*c^2*x^2 - 2*(3*a*b*c^2 - b*c)*x)*e - 2*(4*
b^2*c^2*d*x + 4*(a*b*c^2 - b*c)*d + (b^2*c^2*x^2 - 2*a*b*c^2*x - 3*a^2*c^2
+ 4*a*c - 1)*e)*log(-b*c*x - a*c + 1))/(b^2*c^2)
```

Sympy [A]
time = 3.33, size = 252, normalized size = 1.20

$$\begin{cases} 0 & \text{for}\: b = 0 \wedge c = 0 \\ \left(dx + \frac{ex^2}{2}\right) \mathrm{Li}_{2}\left(ac\right) & \text{for}\: b = 0 \\ 0 & \text{for}\: c = 0 \\ \frac{3a^2e\,\mathrm{Li}_{1}\left(ac+bcx\right)}{4b^2} - \frac{a^2e\,\mathrm{Li}_{2}\left(ac+bcx\right)}{2b^2} - \frac{ad\,\mathrm{Li}_{1}\left(ac+bcx\right)}{b} + \frac{ad\,\mathrm{Li}_{2}\left(ac+bcx\right)}{b} + \frac{aex\,\mathrm{Li}_{1}\left(ac+bcx\right)}{2b} + \frac{3aex}{4b} - \frac{ae\,\mathrm{Li}_{1}\left(ac+bcx\right)}{b^2c} - dx\,\mathrm{Li}_{1}\left(ac+bcx\right) + dx\,\mathrm{Li}_{2}\left(ac+bcx\right) - dx - \frac{ex^2\,\mathrm{Li}_{1}\left(ac+bcx\right)}{4} + \frac{ex^2\,\mathrm{Li}_{2}\left(ac+bcx\right)}{2} - \frac{ex^2}{8} + \frac{d\,\mathrm{Li}_{1}\left(ac+bcx\right)}{bc} - \frac{ex}{4bc} + \frac{e\,\mathrm{Li}_{1}\left(ac+bcx\right)}{4b^2c^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*polylog(2,c*(b*x+a)),x)
```

```
[Out] Piecewise((0, Eq(b, 0) & Eq(c, 0)), ((d*x + e*x**2/2)*polylog(2, a*c), Eq(b
, 0)), (0, Eq(c, 0)), (3*a**2*e*polylog(1, a*c + b*c*x)/(4*b**2) - a**2*e*p
olylog(2, a*c + b*c*x)/(2*b**2) - a*d*polylog(1, a*c + b*c*x)/b + a*d*polyl
og(2, a*c + b*c*x)/b + a*e*x*polylog(1, a*c + b*c*x)/(2*b) + 3*a*e*x/(4*b)
```

```
- a*e*polylog(1, a*c + b*c*x)/(b**2*c) - d*x*polylog(1, a*c + b*c*x) + d*x*
polylog(2, a*c + b*c*x) - d*x - e*x**2*polylog(1, a*c + b*c*x)/4 + e*x**2*p
olylog(2, a*c + b*c*x)/2 - e*x**2/8 + d*polylog(1, a*c + b*c*x)/(b*c) - e*x
/(4*b*c) + e*polylog(1, a*c + b*c*x)/(4*b**2*c**2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*polylog(2,c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*dilog((b*x + a)*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{polylog}(2,c\,(a+b\,x))\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, c*(a + b*x))*(d + e*x),x)
```

```
[Out] int(polylog(2, c*(a + b*x))*(d + e*x), x)
```

3.140 $\int \textbf{PolyLog}(2,c(a+bx))\,dx$

Optimal. Leaf size=60

$$-x-\frac{(1-ac-bcx)\log(1-ac-bcx)}{bc}+\frac{a\text{PolyLog}(2,c(a+bx))}{b}+x\text{PolyLog}(2,c(a+bx))$$

```
[Out] -x-(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b/c+a*polylog(2,c*(b*x+a))/b+x*polylog(2
,c*(b*x+a))
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6730, 2494, 2436, 2332, 2468, 2440, 2438}

$$x\text{Li}_2(c(a+bx))+\frac{a\text{Li}_2(c(a+bx))}{b}-\frac{(-ac-bcx+1)\log(-ac-bcx+1)}{bc}-x$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)],x]
```

```
[Out] -x - ((1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c) + (a*PolyLog[2, c*(a +
b*x)])/b + x*PolyLog[2, c*(a + b*x)]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2468

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_)^(q_.), x_Symbol] :> In
t[ExpandToSum[u, x]^q*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a,
 b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] &&  !(BinomialMatch
Q[u, x] && LinearMatchQ[v, x])
```

Rule 2494

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rule 6730

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :> Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c*(a + b*x)^p], x]
, x] + Dist[a*p, Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /; Fr
eeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \mathrm{Li}_2(c(a+bx))\,dx &= x\mathrm{Li}_2(c(a+bx)) - a\int \frac{\log(1-c(a+bx))}{a+bx}\,dx + \int \log(1-c(a+bx))\,dx\\ &= x\mathrm{Li}_2(c(a+bx)) - a\int \frac{\log(1-ac-bcx)}{a+bx}\,dx + \int \log(1-ac-bcx)\,dx\\ &= x\mathrm{Li}_2(c(a+bx)) - \frac{a\mathrm{Subst}\left(\int \frac{\log(1-cx)}{x}\,dx, x, a+bx\right)}{b} - \frac{\mathrm{Subst}(\int \log(x)\,dx, x, 1-ac-}{bc}\\ &= -x - \frac{(1-ac-bcx)\log(1-ac-bcx)}{bc} + \frac{a\mathrm{Li}_2(c(a+bx))}{b} + x\mathrm{Li}_2(c(a+bx))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.88

$$\frac{-c(a+bx)+(-1+c(a+bx))\log(1-c(a+bx))+c(a+bx)\mathrm{PolyLog}(2,c(a+bx))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)],x]
```

```
[Out] (-(c*(a + b*x)) + (-1 + c*(a + b*x))*Log[1 - c*(a + b*x)] + c*(a + b*x)*Pol
yLog[2, c*(a + b*x)])/(b*c)
```

Maple [A]
time = 0.44, size = 63, normalized size = 1.05

method	result	size
derivativedivides	$\frac{(xbc+ac)\,\mathrm{polylog}(2,xbc+ac)-(-xbc-ac+1)\ln(-xbc-ac+1)+1-xbc-ac}{bc}$	63
default	$\frac{(xbc+ac)\,\mathrm{polylog}(2,xbc+ac)-(-xbc-ac+1)\ln(-xbc-ac+1)+1-xbc-ac}{bc}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,c*(b*x+a)),x,method=_RETURNVERBOSE)
[Out] 1/b/c*((b*c*x+a*c)*polylog(2,b*c*x+a*c)-(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)+1-x
*b*c-a*c)
```

Maxima [A]
time = 0.26, size = 90, normalized size = 1.50

$$-\frac{(\log(bcx+ac)\log(-bcx-ac+1)+\mathrm{Li}_2(-bcx-ac+1))a}{b}+\frac{bcx\mathrm{Li}_2(bcx+ac)-bcx+(bcx+ac-1)\log(-bcx-ac+1)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a)),x, algorithm="maxima")
[Out] -(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a/b + (
b*c*x*dilog(b*c*x + a*c) - b*c*x + (b*c*x + a*c - 1)*log(-b*c*x - a*c + 1))
/(b*c)
```

Fricas [A]
time = 0.36, size = 55, normalized size = 0.92

$$-\frac{bcx-(bcx+ac)\mathrm{Li}_2(bcx+ac)-(bcx+ac-1)\log(-bcx-ac+1)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a)),x, algorithm="fricas")
[Out] -(b*c*x - (b*c*x + a*c)*dilog(b*c*x + a*c) - (b*c*x + a*c - 1)*log(-b*c*x -
 a*c + 1))/(b*c)
```

Sympy [A]
time = 0.94, size = 75, normalized size = 1.25

$$\begin{cases} 0 & \text{for}\: c=0 \wedge (b=0 \vee c=0) \\ x\,\mathrm{Li}_2(ac) & \text{for}\: b=0 \\ -\frac{a\,\mathrm{Li}_1(ac+bcx)}{b}+\frac{a\,\mathrm{Li}_2(ac+bcx)}{b}-x\,\mathrm{Li}_1(ac+bcx)+x\,\mathrm{Li}_2(ac+bcx)-x+\frac{\mathrm{Li}_1(ac+bcx)}{bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a)),x)
```

```
[Out] Piecewise((0, Eq(c, 0) & (Eq(b, 0) | Eq(c, 0))), (x*polylog(2, a*c), Eq(b,
0)), (-a*polylog(1, a*c + b*c*x)/b + a*polylog(2, a*c + b*c*x)/b - x*polylo
g(1, a*c + b*c*x) + x*polylog(2, a*c + b*c*x) - x + polylog(1, a*c + b*c*x)
/(b*c), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(dilog((b*x + a)*c), x)
```

Mupad [B]
time = 0.00, size = 61, normalized size = 1.02

$$\frac{\mathrm{polylog}(2,c\,(a+b\,x))\,\left(a+b\,x\right)}{b}-x-\frac{\ln\left(1-c\,(a+b\,x)\right)}{b\,c}+\frac{\ln\left(1-c\,(a+b\,x)\right)\,\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, c*(a + b*x)),x)
```

```
[Out] (polylog(2, c*(a + b*x))*(a + b*x))/b - x - log(1 - c*(a + b*x))/(b*c) + (l
og(1 - c*(a + b*x))*(a + b*x))/b
```

3.141 $\int \frac{\mathbf{PolyLog}(2,c(a+bx))}{d+ex}\,dx$

Optimal. Leaf size=591

$$\frac{\left(\log(c(a+bx))+\log\left(\frac{bcd+e-ace}{bc(d+ex)}\right)-\log\left(\frac{(bcd+e-ace)(a+bx)}{b(d+ex)}\right)\right)\log^2\left(\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right)}{2e}+\frac{\log(c(a+bx))\log(d+\ldots}{e}$$

```
[Out] 1/2*(ln(c*(b*x+a))+ln((-a*c*e+b*c*d+e)/b/c/(e*x+d))-ln((-a*c*e+b*c*d+e)*(b*
x+a)/b/(e*x+d)))*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))^2/e+ln(c*(b*x+a))*l
n(e*x+d)*ln(1-c*(b*x+a))/e-1/2*(ln(c*(b*x+a))-ln(-e*(b*x+a)/(-a*e+b*d)))*(l
n(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))^2/e+ln(e*x+d)*polylo
g(2,c*(b*x+a))/e+(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))*p
olylog(2,b*(e*x+d)/(-a*e+b*d))/e+(ln(e*x+d)-ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b
*x+a))))*polylog(2,1-c*(b*x+a))/e-ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))*po
lylog(2,-e*(1-c*(b*x+a))/b/c/(e*x+d))/e+ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a
)))*polylog(2,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/e-polylog(3,b*(e*x+d)/(-a
*e+b*d))/e-polylog(3,1-c*(b*x+a))/e-polylog(3,-e*(1-c*(b*x+a))/b/c/(e*x+d))
/e+polylog(3,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/e
```

Rubi [A]
time = 0.35, antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {6732, 2490, 2485}

$$-\frac{\text{Li}_3\left(-\frac{e(1-c(a+bx))}{bc(d+ex)}\right)}{e}+\frac{\text{Li}_3\left(\frac{(bd-ae)(1-c(a+bx))}{b(d+ex)}\right)}{e}-\frac{\text{Li}_2\left(-\frac{e(1-c(a+bx))}{bc(d+ex)}\right)\log\left(\frac{b(d+ex)}{(1-c(a+bx))(bd-ae)}\right)}{e}+\frac{\text{Li}_2\left(\frac{(bd-ae)(1-c(a+bx))}{b(d+ex)}\right)\log\left(\frac{b(d+ex)}{(1-c(a+bx))(bd-ae)}\right)}{e}+\frac{\log(d+ex)\text{Li}_2(c(a+bx))}{e}+\frac{\text{Li}_2\left(\frac{b(d+ex)}{bd-ae}\right)\left(\log\left(\frac{b(d+ex)}{(1-c(a+bx))(bd-ae)}\right)+\log(1-c(a+bx))\right)}{e}+\frac{\text{Li}_2(1-c(a+bx))\left(\log(d+ex)-\log\left(\frac{b(d+ex)}{(1-c(a+bx))(bd-ae)}\right)\right)}{e}+\frac{\left(\log\left(\frac{-ace+bcd+e}{bc(d+ex)}\right)-\log\left(\frac{(a+bx)(-ace+bcd+e)}{b(d+ex)}\right)+\log(c(a+bx))\right)\log^2\left(\frac{b(d+ex)}{(1-c(a+bx))(bd-ae)}\right)}{2e}-\frac{\left(\log(c(a+bx))-\log\left(-\frac{e(a+bx)}{bd-ae}\right)\right)\left(\log\left(\frac{b(d+ex)}{(1-c(a+bx))(bd-ae)}\right)+\log(1-c(a+bx))\right)^2}{2e}+\frac{\log(d+ex)\log(c(a+bx))\log(1-c(a+bx))}{e}-\frac{\text{Li}_3(1-c(a+bx))}{e}-\frac{\text{Li}_3\left(\frac{b(d+ex)}{bd-ae}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/(d + e*x),x]
```

```
[Out] ((Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d
 + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1
- c*(a + b*x)))]^2)/(2*e) + (Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b
*x)])/e - ((Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log[(b*(
d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])^2)/(2*e)
 + (Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e + ((Log[(b*(d + e*x))/((b*d - a
*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d + e*x))/(b
*d - a*e)])/e + ((Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a +
 b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/e - (Log[(b*(d + e*x))/((b*d - a*e)*
(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/e
 + (Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a
*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/e - PolyLog[3, (b*(d + e*x))/(b*d -
a*e)]/e - PolyLog[3, 1 - c*(a + b*x)]/e - PolyLog[3, -((e*(1 - c*(a + b*x))
)/(b*c*(d + e*x)))]/e + PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d +
e*x))]/e
```

Rule 2485

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[(-b)*(x/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1/2)*(Log[(-b)*(x/a
)] - Log[(-(b*c - a*d))*(x/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))]
)*Log[a*((c + d*x)/(c*(a + b*x)))]^2, x] - Simp[(1/2)*(Log[(-b)*(x/a)] - Lo
g[(-d)*(x/c)])*(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])^2, x] + Si
mp[(Log[c + d*x] - Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1 + b*(x/a)
], x] + Simp[(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1
 + d*(x/c)], x] + Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2, c*((a +
b*x)/(a*(c + d*x)))], x] - Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2,
 d*((a + b*x)/(b*(c + d*x)))], x] - Simp[PolyLog[3, 1 + b*(x/a)], x] - Simp
[PolyLog[3, 1 + d*(x/c)], x] + Simp[PolyLog[3, c*((a + b*x)/(a*(c + d*x)))]
, x] - Simp[PolyLog[3, d*((a + b*x)/(b*(c + d*x)))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2490

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-(e*k - d*l)/l + e*(x/l))^n])*(f + g
*Log[h*(-(j*k - i*l)/l + j*(x/l))^m]), x], x, k + l*x], x] /; FreeQ[{a, b,
c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 6732

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[d + e*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, Int[Log[d
+ e*x]*(Log[1 - a*c - b*c*x]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rubi steps

$$\int \frac{\text{Li}_2(c(a+bx))}{d+ex}\,dx = \frac{\log(d+ex)\text{Li}_2(c(a+bx))}{e} + \frac{b\int \frac{\log(1-ac-bcx)\log(d+ex)}{a+bx}\,dx}{e}$$

$$= \frac{\log(d+ex)\text{Li}_2(c(a+bx))}{e} + \frac{\text{Subst}\left(\int \frac{\log\left(-\frac{-abc-b(1-ac)}{b}-cx\right)\log\left(-\frac{-bd+ae}{b}+\frac{ex}{b}\right)}{x}\,dx, x, a+ \right.}{e}$$

$$= \frac{\left(\log(c(a+bx)) + \log\left(\frac{bcd+e-ace}{bc(d+ex)}\right) - \log\left(\frac{(bcd+e-ace)(a+bx)}{b(d+ex)}\right)\right)\log^2\left(\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right)}{2e}$$

Mathematica [A]

time = 0.17, size = 622, normalized size = 1.05

$$\frac{\log(c(a+bx))\log(1-ac-bcx)\log(d+ex)+\frac{1}{2}\left(\log(c(a+bx))-\log\left(\frac{e(a+bx)}{-bd+ae}\right)\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)\left(-2\log(1-ac-bcx)+\log\left(\frac{b(d+ex)}{bd-ae}\right)\right)+\left(-\log(c(a+bx))+\log\left(\frac{e(a+bx)}{-bd+ae}\right)\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)\log\left(-\frac{b(d+ex)}{(bd-ae)(-1+ac+bcx)}\right)+\frac{1}{2}\log^2\left(-\frac{b(d+ex)}{(bd-ae)(-1+ac+bcx)}\right)\left(\log(c(a+bx))-\log\left(\frac{(bcd+e-ace)(a+bx)}{(bd-ae)(-1+ac+bcx)}\right)+\log\left(\frac{bcd+e-ace}{e-ace-bcex}\right)\right)+\log(d+ex)\mathrm{PolyLog}(2,c(a+bx))+\left(\log(d+ex)-\log\left(-\frac{b(d+ex)}{(bd-ae)(-1+ac+bcx)}\right)\right)\mathrm{PolyLog}(2,1-ac-bcx)+\left(\log(1-ac-bcx)+\log\left(-\frac{b(d+ex)}{(bd-ae)(-1+ac+bcx)}\right)\right)\mathrm{PolyLog}\left(2,\frac{b(d+ex)}{bd-ae}\right)+\log\left(-\frac{b(d+ex)}{(bd-ae)(-1+ac+bcx)}\right)\left(\mathrm{PolyLog}\left(2,\frac{bc(d+ex)}{e(-1+ac+bcx)}\right)-\mathrm{PolyLog}\left(2,-\frac{b(d+ex)}{(bd-ae)(-1+ac+bcx)}\right)\right)-\mathrm{PolyLog}(3,1-ac-bcx)-\mathrm{PolyLog}\left(3,\frac{b(d+ex)}{bd-ae}\right)-\mathrm{PolyLog}\left(3,\frac{bc(d+ex)}{e(-1+ac+bcx)}\right)+\mathrm{PolyLog}\left(3,-\frac{b(d+ex)}{(bd-ae)(-1+ac+bcx)}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b*x)]*Log[1 - a*c - b*c*x]*Log[d + e*x] + ((Log[c*(a + b*x)] -
Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*(-2*Log[1
 - a*c - b*c*x] + Log[(b*(d + e*x))/(b*d - a*e)]))/2 + (-Log[c*(a + b*x)] +
 Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*Log[-((b
*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))] + (Log[-((b*(d + e*x))/((b*d
 - a*e)*(-1 + a*c + b*c*x)))]^2*(Log[c*(a + b*x)] - Log[((b*c*d + e - a*c*e
)*(a + b*x))/((b*d - a*e)*(-1 + a*c + b*c*x))] + Log[(b*c*d + e - a*c*e)/(e
 - a*c*e - b*c*e*x)]))/2 + Log[d + e*x]*PolyLog[2, c*(a + b*x)] + (Log[d +
e*x] - Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2, 1
 - a*c - b*c*x] + (Log[1 - a*c - b*c*x] + Log[-((b*(d + e*x))/((b*d - a*e)*
(-1 + a*c + b*c*x)))])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)] + Log[-((b*(d
+ e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]*(PolyLog[2, (b*c*(d + e*x))/(e*(
-1 + a*c + b*c*x))] - PolyLog[2, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b
*c*x)))]) - PolyLog[3, 1 - a*c - b*c*x] - PolyLog[3, (b*(d + e*x))/(b*d - a
*e)] - PolyLog[3, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))] + PolyLog[3, -((b
*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])/e
```

Maple [**F**]
time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{polylog}\left(2, c(bx+a)\right)}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,c*(b*x+a))/(e*x+d),x)
```

```
[Out] int(polylog(2,c*(b*x+a))/(e*x+d),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(dilog((b*x + a)*c)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d),x, algorithm="fricas")
[Out] integral(dilog(b*c*x + a*c)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_2(ac+bcx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d),x)
[Out] Integral(polylog(2, a*c + b*c*x)/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d),x, algorithm="giac")
[Out] integrate(dilog((b*x + a)*c)/(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{polylog}(2,c\,(a+b\,x))}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, c*(a + b*x))/(d + e*x),x)
[Out] int(polylog(2, c*(a + b*x))/(d + e*x), x)
```

3.142 $\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^2}\,dx$

Optimal. Leaf size=138

$$\frac{b\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{e(bd-ae)}+\frac{b\text{PolyLog}(2,c(a+bx))}{e(bd-ae)}-\frac{\text{PolyLog}(2,c(a+bx))}{e(d+ex)}+\frac{b\text{PolyLog}\left(2,\frac{e(1-ac-bcx}{bcd+e-ace}\right.}{e(bd-ae)}$$

```
[Out] b*ln(-b*c*x-a*c+1)*ln(b*c*(e*x+d)/(-a*c*e+b*c*d+e))/e/(-a*e+b*d)+b*polylog(
2,c*(b*x+a))/e/(-a*e+b*d)-polylog(2,c*(b*x+a))/e/(e*x+d)+b*polylog(2,e*(-b*
c*x-a*c+1)/(-a*c*e+b*c*d+e))/e/(-a*e+b*d)
```

Rubi [A]
time = 0.13, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6733, 2465, 2440, 2438, 2441}

$$\frac{b\text{Li}_2(c(a+bx))}{e(bd-ae)}-\frac{\text{Li}_2(c(a+bx))}{e(d+ex)}+\frac{b\text{Li}_2\left(\frac{e(-ac-bxc+1)}{bcd-ace+e}\right)}{e(bd-ae)}+\frac{b\log(-ac-bcx+1)\log\left(\frac{bc(d+ex)}{-ace+bcd+e}\right)}{e(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/(d + e*x)^2,x]
```

```
[Out] (b*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(e*(b*d -
 a*e)) + (b*PolyLog[2, c*(a + b*x)])/(e*(b*d - a*e)) - PolyLog[2, c*(a + b*
x)]/(e*(d + e*x)) + (b*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)
])/(e*(b*d - a*e))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 6733

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Dist[b/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x))
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(c(a+bx))}{(d+ex)^2}\,dx &= -\frac{\text{Li}_2(c(a+bx))}{e(d+ex)}-\frac{b\int\frac{\log(1-ac-bcx)}{(a+bx)(d+ex)}\,dx}{e}\\
&= -\frac{\text{Li}_2(c(a+bx))}{e(d+ex)}-\frac{b\int\left(\frac{b\log(1-ac-bcx)}{(bd-ae)(a+bx)}-\frac{e\log(1-ac-bcx)}{(bd-ae)(d+ex)}\right)dx}{e}\\
&= -\frac{\text{Li}_2(c(a+bx))}{e(d+ex)}+\frac{b\int\frac{\log(1-ac-bcx)}{d+ex}\,dx}{bd-ae}-\frac{b^2\int\frac{\log(1-ac-bcx)}{a+bx}\,dx}{e(bd-ae)}\\
&= \frac{b\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{e(bd-ae)}-\frac{\text{Li}_2(c(a+bx))}{e(d+ex)}-\frac{b\text{Subst}\left(\int\frac{\log(1-cx)}{x}\,dx,x,a+\right.}{e(bd-ae)}\\
&= \frac{b\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{e(bd-ae)}+\frac{b\text{Li}_2(c(a+bx))}{e(bd-ae)}-\frac{\text{Li}_2(c(a+bx))}{e(d+ex)}-\frac{b\text{Subst}\left(\int\right.}{}\\
&= \frac{b\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{e(bd-ae)}+\frac{b\text{Li}_2(c(a+bx))}{e(bd-ae)}-\frac{\text{Li}_2(c(a+bx))}{e(d+ex)}+\frac{b\text{Li}_2\left(\frac{e(1-}{bcd-}\right.}{e(bd-}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 108, normalized size = 0.78

$$\frac{-\frac{\text{PolyLog}(2,c(a+bx))}{d+ex}+\frac{b\left(\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)+\text{PolyLog}(2,c(a+bx))+\text{PolyLog}\left(2,\frac{e(-1+ac+bcx)}{-bcd-e+ace}\right)\right)}{bd-ae}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)]/(d + e*x)^2,x]
```

```
[Out] (-(PolyLog[2, c*(a + b*x)]/(d + e*x)) + (b*(Log[1 - a*c - b*c*x]*Log[(b*c*(
d + e*x))/(b*c*d + e - a*c*e)] + PolyLog[2, c*(a + b*x)] + PolyLog[2, (e*(-
1 + a*c + b*c*x))/(-(b*c*d) - e + a*c*e)]))/(b*d - a*e))/e
```

Maple [A]
time = 2.11, size = 213, normalized size = 1.54

method	result
derivativedivides	$\frac{\frac{c^2b^2 \operatorname{polylog}(2,xbc+ac)}{(aec-bcd-e(xbc+ac))e}+\frac{c^2b^2 \Big(-\frac{\operatorname{dilog}(-xbc-ac+1)}{c(ae-bd)}-\frac{\Big(\frac{\operatorname{dilog}\big(\frac{aec-bcd+e(-xbc-ac+1)-e}{aec-bcd-e}\big)}{e}+\frac{\ln(-xbc-ac+1)\ln\big(\frac{aec-bcd+e(-xbc}{aec-bcd}}{e}}{c(ae-bd)}}{e}}{bc}$
default	$\frac{\frac{c^2b^2 \operatorname{polylog}(2,xbc+ac)}{(aec-bcd-e(xbc+ac))e}+\frac{c^2b^2 \Big(-\frac{\operatorname{dilog}(-xbc-ac+1)}{c(ae-bd)}-\frac{\Big(\frac{\operatorname{dilog}\big(\frac{aec-bcd+e(-xbc-ac+1)-e}{aec-bcd-e}\big)}{e}+\frac{\ln(-xbc-ac+1)\ln\big(\frac{aec-bcd+e(-xbc}{aec-bcd}}{e}}{c(ae-bd)}}{e}}{bc}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,c*(b*x+a))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/c*(c^2*b^2/(a*e*c-b*c*d-e*(b*c*x+a*c))/e*polylog(2,b*c*x+a*c)+c^2*b^2/e
*(-dilog(-b*c*x-a*c+1)/c/(a*e-b*d)-(dilog((a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)/
(a*c*e-b*c*d-e))/e+ln(-b*c*x-a*c+1)*ln((a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)/(a*
c*e-b*c*d-e))/e)*e/c/(a*e-b*d)))
```

Maxima [A]
time = 0.27, size = 172, normalized size = 1.25

$$-\frac{(\log(bcx+ac)\log(-bcx-ac+1)+\operatorname{Li}_2(-bcx-ac+1))b}{bde-ae^2}+\frac{\left(\log(-bcx-ac+1)\log\left(\frac{bcxe+ace-e}{bcd-ace+e}+1\right)+\operatorname{Li}_2\left(-\frac{bcxe+ace-e}{bcd-ace+e}\right)\right)b}{bde-ae^2}-\frac{\operatorname{Li}_2(bcx+ac)}{xe^2+de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*b/(b*d*
e - a*e^2) + (log(-b*c*x - a*c + 1)*log((b*c*x*e + a*c*e - e)/(b*c*d - a*c*
e + e) + 1) + dilog(-(b*c*x*e + a*c*e - e)/(b*c*d - a*c*e + e)))*b/(b*d*e -
 a*e^2) - dilog(b*c*x + a*c)/(x*e^2 + d*e)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^2,x, algorithm="fricas")
[Out] integral(dilog(b*c*x + a*c)/(x^2*e^2 + 2*d*x*e + d^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_2\left(ac+bcx\right)}{\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)**2,x)
[Out] Integral(polylog(2, a*c + b*c*x)/(d + e*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^2,x, algorithm="giac")
[Out] integrate(dilog((b*x + a)*c)/(e*x + d)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{polylog}(2,c\,(a+b\,x))}{{(d+e\,x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, c*(a + b*x))/(d + e*x)^2,x)
[Out] int(polylog(2, c*(a + b*x))/(d + e*x)^2, x)
```

3.143 $\int \frac{\textbf{PolyLog}(2,c(a+bx))}{(d+ex)^3}\,dx$

Optimal. Leaf size=278

$$\frac{b^2c\log(1-ac-bcx)}{2e(bd-ae)(bcd+e-ace)}-\frac{b\log(1-ac-bcx)}{2e(bd-ae)(d+ex)}-\frac{b^2c\log(d+ex)}{2e(bd-ae)(bcd+e-ace)}+\frac{b^2\log(1-ac-bcx)\log\left(\frac{bc}{bcd}\right.}{2e(bd-ae)^2}$$

```
[Out] 1/2*b^2*c*ln(-b*c*x-a*c+1)/e/(-a*e+b*d)/(-a*c*e+b*c*d+e)-1/2*b*ln(-b*c*x-a*
c+1)/e/(-a*e+b*d)/(e*x+d)-1/2*b^2*c*ln(e*x+d)/e/(-a*e+b*d)/(-a*c*e+b*c*d+e)
+1/2*b^2*ln(-b*c*x-a*c+1)*ln(b*c*(e*x+d)/(-a*c*e+b*c*d+e))/e/(-a*e+b*d)^2+1
/2*b^2*polylog(2,c*(b*x+a))/e/(-a*e+b*d)^2-1/2*polylog(2,c*(b*x+a))/e/(e*x+
d)^2+1/2*b^2*polylog(2,e*(-b*c*x-a*c+1)/(-a*c*e+b*c*d+e))/e/(-a*e+b*d)^2
```

Rubi [A]
time = 0.19, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {6733, 2465, 2440, 2438, 2442, 36, 31, 2441}

$$\frac{b^2\text{Li}_2(c(a+bx))}{2e(bd-ae)^2}+\frac{b^2\text{Li}_2\left(\frac{e(-ac-bxc+1)}{bcd-ace+e}\right)}{2e(bd-ae)^2}+\frac{b^2c\log(-ac-bcx+1)}{2e(bd-ae)(-ace+bcd+e)}-\frac{b^2c\log(d+ex)}{2e(bd-ae)(-ace+bcd+e)}+\frac{b^2\log(-ac-bcx+1)\log\left(\frac{bc(d+ex)}{-ace+bcd+e}\right)}{2e(bd-ae)^2}-\frac{\text{Li}_2(c(a+bx))}{2e(d+ex)^2}-\frac{b\log(-ac-bcx+1)}{2e(d+ex)(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/(d + e*x)^3,x]
```

```
[Out] (b^2*c*Log[1 - a*c - b*c*x])/(2*e*(b*d - a*e)*(b*c*d + e - a*c*e)) - (b*Log
[1 - a*c - b*c*x])/(2*e*(b*d - a*e)*(d + e*x)) - (b^2*c*Log[d + e*x])/(2*e*
(b*d - a*e)*(b*c*d + e - a*c*e)) + (b^2*Log[1 - a*c - b*c*x]*Log[(b*c*(d +
e*x))/(b*c*d + e - a*c*e)])/(2*e*(b*d - a*e)^2) + (b^2*PolyLog[2, c*(a + b*
x)])/(2*e*(b*d - a*e)^2) - PolyLog[2, c*(a + b*x)]/(2*e*(d + e*x)^2) + (b^2
*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(2*e*(b*d - a*e)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 6733

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Dist[b/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x))
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\text{Li}_2(c(a+bx))}{(d+ex)^3}\,dx = -\frac{\text{Li}_2(c(a+bx))}{2e(d+ex)^2} - \frac{b\int \frac{\log(1-ac-bcx)}{(a+bx)(d+ex)^2}\,dx}{2e}$$

$$= -\frac{\text{Li}_2(c(a+bx))}{2e(d+ex)^2} - \frac{b\int \left(\frac{b^2\log(1-ac-bcx)}{(bd-ae)^2(a+bx)} - \frac{e\log(1-ac-bcx)}{(bd-ae)(d+ex)^2} - \frac{be\log(1-ac-bcx)}{(bd-ae)^2(d+ex)}\right)dx}{2e}$$

$$= -\frac{\text{Li}_2(c(a+bx))}{2e(d+ex)^2} + \frac{b^2\int \frac{\log(1-ac-bcx)}{d+ex}\,dx}{2(bd-ae)^2} - \frac{b^3\int \frac{\log(1-ac-bcx)}{a+bx}\,dx}{2e(bd-ae)^2} + \frac{b\int \frac{\log(1-ac-bcx)}{(d+ex)^2}\,dx}{2(bd-ae)}$$

$$= -\frac{b\log(1-ac-bcx)}{2e(bd-ae)(d+ex)} + \frac{b^2\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{2e(bd-ae)^2} - \frac{\text{Li}_2(c(a+bx))}{2e(d+ex)^2} - \frac{b^2\text{S}}{}$$

$$= -\frac{b\log(1-ac-bcx)}{2e(bd-ae)(d+ex)} + \frac{b^2\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{2e(bd-ae)^2} + \frac{b^2\text{Li}_2(c(a+bx))}{2e(bd-ae)^2} - \frac{\text{L}}{}$$

$$= \frac{b^2c\log(1-ac-bcx)}{2e(bd-ae)(bcd+e-ace)} - \frac{b\log(1-ac-bcx)}{2e(bd-ae)(d+ex)} - \frac{b^2c\log(d+ex)}{2e(bd-ae)(bcd+e-ace)} + \frac{b}{}$$

Mathematica [A]
time = 0.24, size = 190, normalized size = 0.68

$$\frac{-\frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^2} + \frac{b\left(-\frac{(bd-ae)\log(1-ac-bcx)}{d+ex} + \frac{bc(bd-ae)(\log(1-ac-bcx)-\log(d+ex))}{bcd+e-ace} + b\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right) + b\text{PolyLog}(2,c(a+bx)) + b\text{PolyLog}\left(2,\frac{e(-1+ac+bcx)}{-bcd+(-1+ac)e}\right)\right)}{(bd-ae)^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)]/(d + e*x)^3,x]
```

```
[Out] (-(PolyLog[2, c*(a + b*x)]/(d + e*x)^2) + (b*(-(((b*d - a*e)*Log[1 - a*c -
b*c*x])/(d + e*x)) + (b*c*(b*d - a*e)*(Log[1 - a*c - b*c*x] - Log[d + e*x])
)/(b*c*d + e - a*c*e) + b*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d +
 e - a*c*e)] + b*PolyLog[2, c*(a + b*x)] + b*PolyLog[2, (e*(-1 + a*c + b*c*
x))/(-(b*c*d) + (-1 + a*c)*e)]))/(b*d - a*e)^2)/(2*e)
```

Maple [A]
time = 2.12, size = 346, normalized size = 1.24

method	result
derivativedivides	$\frac{-\frac{c^3b^3\,\text{polylog}(2,xbc+ac)}{2(aec-bcd-e(xbc+ac))^2e} - \frac{c^3b^3\left(-\frac{\left(\frac{\text{dilog}\left(\frac{aec-bcd+e(-xbc-ac+1)-e}{aec-bcd-e}\right)}{e} + \frac{\ln(-xbc-ac+1)\ln\left(\frac{aec-bcd+e(-xbc-ac+1)-e}{aec-bcd-e}\right)}{e}\right)e}{c^2(ae-bd)^2}\right.}{}}{bc}$

default	$\frac{-\frac{c^{3}b^{3}\,\mathrm{polylog}(2,xbc+ac)}{2(aec-bcd-e(xbc+ac))^{2}e}-\frac{c^{3}b^{3}\left(-\frac{\left(\frac{\mathrm{dilog}\left(\frac{aec-bcd+e(-xbc-ac+1)-e}{aec-bcd-e}\right)}{e}+\frac{\ln(-xbc-ac+1)\ln\left(\frac{aec-bcd+e(-xbc-ac+1)-e}{aec-bcd-e}\right)}{e}\right)}{c^{2}(ae-bd)^{2}}\right.}{}}{bc}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,c*(b*x+a))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/c*(-1/2*c^3*b^3/(a*e*c-b*c*d-e*(b*c*x+a*c))^2/e*polylog(2,b*c*x+a*c)-1/
2*c^3*b^3/e*(-(dilog((a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)/(a*c*e-b*c*d-e))/e+ln
(-b*c*x-a*c+1)*ln((a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)/(a*c*e-b*c*d-e))/e)*e/c^
2/(a*e-b*d)^2-dilog(-b*c*x-a*c+1)/c^2/(a*e-b*d)^2-(-1/(a*c*e-b*c*d-e)*ln(a*
e*c-b*c*d+e*(-b*c*x-a*c+1)-e)/e+ln(-b*c*x-a*c+1)*(-b*c*x-a*c+1)/(a*c*e-b*c*
d-e)/(a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e))/c/(a*e-b*d)*e))
```

Maxima [A]
time = 0.27, size = 482, normalized size = 1.73

$$-\frac{b^{2}c\log(xe+d)}{2\left(b^{2}cd^{2}e+a^{2}ce^{3}-(2\,abce^{2}-be^{2})d-ae^{3}\right)}-\frac{\left(\log(bcx+ac)\log(-bcx-ac+1)+\mathrm{Li}_2(-bcx-ac+1)\right)b^{2}}{2\left(b^{2}d^{2}e-2\,abde^{2}+a^{2}e^{3}\right)}+\frac{\left(\log(-bcx-ac+1)\log\left(\frac{bcxe+ace-e}{bcd-ace+e}+1\right)+\mathrm{Li}_2\left(-\frac{bcxe+ace-e}{bcd-ace+e}\right)\right)b^{2}}{2\left(b^{2}d^{2}e-2\,abde^{2}+a^{2}e^{3}\right)}-\frac{(b^{2}cd^{2}+a^{2}ce^{2}-(2\,abce-be)d-ae^{2})\mathrm{Li}_2(bcx+ac)-(b^{2}cx^{2}e^{2}+(abce-be)d+(b^{2}cde+abce^{2}-be^{2})x)\log(-bcx-ac+1)}{2\left(b^{2}cd^{4}e-(2\,abce^{2}-be^{2})d^{3}+(a^{2}ce^{3}-ae^{3})d^{2}+(b^{2}cd^{2}e^{3}+a^{2}ce^{5}-(2\,abce^{4}-be^{4})d-ae^{5})x^{2}+2\left(b^{2}cd^{3}e^{2}-(2\,abce^{3}-be^{3})d^{2}+(a^{2}ce^{4}-ae^{4})d\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*c*log(x*e + d)/(b^2*c*d^2*e + a^2*c*e^3 - (2*a*b*c*e^2 - b*e^2)*d
- a*e^3) - 1/2*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c
 + 1))*b^2/(b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3) + 1/2*(log(-b*c*x - a*c + 1)
*log((b*c*x*e + a*c*e - e)/(b*c*d - a*c*e + e) + 1) + dilog(-(b*c*x*e + a*c
*e - e)/(b*c*d - a*c*e + e)))*b^2/(b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3) - 1/2
*((b^2*c*d^2 + a^2*c*e^2 - (2*a*b*c*e - b*e)*d - a*e^2)*dilog(b*c*x + a*c)
- (b^2*c*x^2*e^2 + (a*b*c*e - b*e)*d + (b^2*c*d*e + a*b*c*e^2 - b*e^2)*x)*l
og(-b*c*x - a*c + 1))/(b^2*c*d^4*e - (2*a*b*c*e^2 - b*e^2)*d^3 + (a^2*c*e^3
 - a*e^3)*d^2 + (b^2*c*d^2*e^3 + a^2*c*e^5 - (2*a*b*c*e^4 - b*e^4)*d - a*e^
5)*x^2 + 2*(b^2*c*d^3*e^2 - (2*a*b*c*e^3 - b*e^3)*d^2 + (a^2*c*e^4 - a*e^4)
*d)*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(dilog(b*c*x + a*c)/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(dilog((b*x + a)*c)/(e*x + d)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{polylog}(2, c\,(a+b\,x))}{(d+e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, c*(a + b*x))/(d + e*x)^3,x)
```

```
[Out] int(polylog(2, c*(a + b*x))/(d + e*x)^3, x)
```

3.144 $\int \frac{\textbf{PolyLog}(2,c(a+bx))}{(d+ex)^4}\,dx$

Optimal. Leaf size=448

$$\frac{b^2c}{6e(bd-ae)(bcd+e-ace)(d+ex)}+\frac{b^3c^2\log(1-ac-bcx)}{6e(bd-ae)(bcd+e-ace)^2}+\frac{b^3c\log(1-ac-bcx)}{3e(bd-ae)^2(bcd+e-ace)}-\frac{b\log(1-}{6e(bd-}$$

```
[Out] 1/6*b^2*c/e/(-a*e+b*d)/(-a*c*e+b*c*d+e)/(e*x+d)+1/6*b^3*c^2*ln(-b*c*x-a*c+1
)/e/(-a*e+b*d)/(-a*c*e+b*c*d+e)^2+1/3*b^3*c*ln(-b*c*x-a*c+1)/e/(-a*e+b*d)^2
/(-a*c*e+b*c*d+e)-1/6*b*ln(-b*c*x-a*c+1)/e/(-a*e+b*d)/(e*x+d)^2-1/3*b^2*ln(
-b*c*x-a*c+1)/e/(-a*e+b*d)^2/(e*x+d)-1/6*b^3*c^2*ln(e*x+d)/e/(-a*e+b*d)/(-a
*c*e+b*c*d+e)^2-1/3*b^3*c*ln(e*x+d)/e/(-a*e+b*d)^2/(-a*c*e+b*c*d+e)+1/3*b^3
*ln(-b*c*x-a*c+1)*ln(b*c*(e*x+d)/(-a*c*e+b*c*d+e))/e/(-a*e+b*d)^3+1/3*b^3*p
olylog(2,c*(b*x+a))/e/(-a*e+b*d)^3-1/3*polylog(2,c*(b*x+a))/e/(e*x+d)^3+1/3
*b^3*polylog(2,e*(-b*c*x-a*c+1)/(-a*c*e+b*c*d+e))/e/(-a*e+b*d)^3
```

Rubi [A]
time = 0.31, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {6733, 2465, 2440, 2438, 2442, 46, 36, 31, 2441}

$$\frac{b^3c^2\log(-ac-bcx+1)}{6e(bd-ae)(-ace+bcd+e)^2}-\frac{b^3c^2\log(d+ex)}{6e(bd-ae)(-ace+bcd+e)^2}+\frac{b^3\text{Li}_2(c(a+bx))}{3e(bd-ae)^3}+\frac{b^3\text{Li}_2\left(\frac{e(-ac-bxc+1)}{bcd-ace+e}\right)}{3e(bd-ae)^3}+\frac{b^3c\log(-ac-bcx+1)}{3e(bd-ae)^2(-ace+bcd+e)}-\frac{b^3c\log(d+ex)}{3e(bd-ae)^2(-ace+bcd+e)}+\frac{b^3\log(-ac-bcx+1)\log\left(\frac{bc(d+ex)}{-ace+bcd+e}\right)}{3e(bd-ae)^3}+\frac{b^2c}{6e(d+ex)(bd-ae)(-ace+bcd+e)}-\frac{b^2\log(-ac-bcx+1)}{3e(d+ex)(bd-ae)^2}-\frac{\text{Li}_2(c(a+bx))}{3e(d+ex)^3}-\frac{b\log(-ac-bcx+1)}{6e(d+ex)^2(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/(d + e*x)^4,x]
```

```
[Out] (b^2*c)/(6*e*(b*d - a*e)*(b*c*d + e - a*c*e)*(d + e*x)) + (b^3*c^2*Log[1 -
a*c - b*c*x])/(6*e*(b*d - a*e)*(b*c*d + e - a*c*e)^2) + (b^3*c*Log[1 - a*c
- b*c*x])/(3*e*(b*d - a*e)^2*(b*c*d + e - a*c*e)) - (b*Log[1 - a*c - b*c*x]
)/(6*e*(b*d - a*e)*(d + e*x)^2) - (b^2*Log[1 - a*c - b*c*x])/(3*e*(b*d - a*
e)^2*(d + e*x)) - (b^3*c^2*Log[d + e*x])/(6*e*(b*d - a*e)*(b*c*d + e - a*c*
e)^2) - (b^3*c*Log[d + e*x])/(3*e*(b*d - a*e)^2*(b*c*d + e - a*c*e)) + (b^3
*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(3*e*(b*d -
 a*e)^3) + (b^3*PolyLog[2, c*(a + b*x)])/(3*e*(b*d - a*e)^3) - PolyLog[2, c
*(a + b*x)]/(3*e*(d + e*x)^3) + (b^3*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*
d + e - a*c*e)])/(3*e*(b*d - a*e)^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 6733

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
```

```
Dist[b/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x))
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{Li}_2(c(a+bx))}{(d+ex)^4}\,dx &= -\frac{\operatorname{Li}_2(c(a+bx))}{3e(d+ex)^3} - \frac{b\int \frac{\log(1-ac-bcx)}{(a+bx)(d+ex)^3}\,dx}{3e} \\
&= -\frac{\operatorname{Li}_2(c(a+bx))}{3e(d+ex)^3} - \frac{b\int \left(\frac{b^3\log(1-ac-bcx)}{(bd-ae)^3(a+bx)} - \frac{e\log(1-ac-bcx)}{(bd-ae)(d+ex)^3} - \frac{be\log(1-ac-bcx)}{(bd-ae)^2(d+ex)^2} - \frac{b^2e\log(1-ac-}{(bd-ae)^3(d+}\right.}{3e} \\
&= -\frac{\operatorname{Li}_2(c(a+bx))}{3e(d+ex)^3} + \frac{b^3\int \frac{\log(1-ac-bcx)}{d+ex}\,dx}{3(bd-ae)^3} - \frac{b^4\int \frac{\log(1-ac-bcx)}{a+bx}\,dx}{3e(bd-ae)^3} + \frac{b^2\int \frac{\log(1-ac-bcx)}{(d+ex)^2}\,dx}{3(bd-ae)^2} \\
&= -\frac{b\log(1-ac-bcx)}{6e(bd-ae)(d+ex)^2} - \frac{b^2\log(1-ac-bcx)}{3e(bd-ae)^2(d+ex)} + \frac{b^3\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ac}\right.}{3e(bd-ae)^3} \\
&= -\frac{b\log(1-ac-bcx)}{6e(bd-ae)(d+ex)^2} - \frac{b^2\log(1-ac-bcx)}{3e(bd-ae)^2(d+ex)} + \frac{b^3\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ac}\right.}{3e(bd-ae)^3} \\
&= \frac{b^2c}{6e(bd-ae)(bcd+e-ace)(d+ex)} + \frac{b^3c^2\log(1-ac-bcx)}{6e(bd-ae)(bcd+e-ace)^2} + \frac{b^3c\log(1-}{3e(bd-ae)^2(}
\end{aligned}$$

Mathematica [A]
time = 0.39, size = 313, normalized size = 0.70

$$\frac{-\frac{2\operatorname{PolyLog}(2,c(a+bx))}{(d+ex)^3} + \frac{b\left(-\frac{(bd-ae)^2\log(1-ac-bcx)}{(d+ex)^2} - \frac{2b(bd-ae)\log(1-ac-bcx)}{d+ex} + \frac{2b^2c(bd-ae)(\log(1-ac-bcx)-\log(d+ex))}{bcd+e-ace} + \frac{bc(bd-ae)^2(bcd+e-ace+bc(d+ex)\log(1-ac-bcx)-bc(d+ex)\log(d+ex))}{(bcd+e-ace)^2(d+ex)} + 2b^2\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right) + 2b^2\operatorname{PolyLog}(2,c(a+bx)) + 2b^2\operatorname{PolyLog}\left(2,\frac{e(-1+ac+bcx)}{-bcd+(-1+ac)e}\right)\right)}{(bd-ae)^3}}{6e}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)]/(d + e*x)^4,x]
```

```
[Out] ((-2*PolyLog[2, c*(a + b*x)])/(d + e*x)^3 + (b*(-(((b*d - a*e)^2*Log[1 - a*
c - b*c*x])/(d + e*x)^2) - (2*b*(b*d - a*e)*Log[1 - a*c - b*c*x])/(d + e*x)
 + (2*b^2*c*(b*d - a*e)*(Log[1 - a*c - b*c*x] - Log[d + e*x]))/(b*c*d + e -
 a*c*e) + (b*c*(b*d - a*e)^2*(b*c*d + e - a*c*e + b*c*(d + e*x)*Log[1 - a*c
 - b*c*x] - b*c*(d + e*x)*Log[d + e*x]))/((b*c*d + e - a*c*e)^2*(d + e*x))
+ 2*b^2*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)] + 2*b
^2*PolyLog[2, c*(a + b*x)] + 2*b^2*PolyLog[2, (e*(-1 + a*c + b*c*x))/(-(b*c
*d) + (-1 + a*c)*e)]))/(b*d - a*e)^3)/(6*e)
```

Maple [A]
time = 2.14, size = 648, normalized size = 1.45

method	result
derivativedivides	$\frac{c^4b^4 \operatorname{polylog}(2,xbc+ac)}{3(aec-bcd-e(xbc+ac))^3e} + \frac{c^4b^4\left(-\left(\frac{ac}{2(aec-bcd-e)^2(aec-bcd+e(-xbc-ac+1)-e)} - \frac{dcb}{2(aec-bcd-e)^2e(aec-bcd+e(-xbc-ac+1)-e)} - \right.\right.}{}$
default	$\frac{c^4b^4 \operatorname{polylog}(2,xbc+ac)}{3(aec-bcd-e(xbc+ac))^3e} + \frac{c^4b^4\left(-\left(\frac{ac}{2(aec-bcd-e)^2(aec-bcd+e(-xbc-ac+1)-e)} - \frac{dcb}{2(aec-bcd-e)^2e(aec-bcd+e(-xbc-ac+1)-e)} - \right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,c*(b*x+a))/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/c*(1/3*c^4*b^4/(a*e*c-b*c*d-e*(b*c*x+a*c))^3/e*polylog(2,b*c*x+a*c)+1/3
*c^4*b^4/e*(-(1/2/(a*c*e-b*c*d-e)^2/(a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)*a*c-1/
2/(a*c*e-b*c*d-e)^2/e/(a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)*d*c*b-1/2/(a*c*e-b*c
*d-e)^2/(a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)-1/2/(a*c*e-b*c*d-e)^2/e*ln(a*e*c-b
*c*d+e*(-b*c*x-a*c+1)-e)+1/2*ln(-b*c*x-a*c+1)*(2*a*e*c-2*b*c*d+e*(-b*c*x-a*
c+1)-2*e)*(-b*c*x-a*c+1)/(a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)^2/(a*c*e-b*c*d-e)
^2)/c/(a*e-b*d)*e-(-1/(a*c*e-b*c*d-e)*ln(a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)/e+
ln(-b*c*x-a*c+1)*(-b*c*x-a*c+1)/(a*c*e-b*c*d-e)/(a*e*c-b*c*d+e*(-b*c*x-a*c+
1)-e))*e/c^2/(a*e-b*d)^2-(dilog((a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)/(a*c*e-b*c
*d-e))/e+ln(-b*c*x-a*c+1)*ln((a*e*c-b*c*d+e*(-b*c*x-a*c+1)-e)/(a*c*e-b*c*d-
e))/e)*e/c^3/(a*e-b*d)^3-dilog(-b*c*x-a*c+1)/c^3/(a*e-b*d)^3))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1519 vs. $2(443) = 886$.

time = 0.33, size = 1519, normalized size = 3.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*b^3
/(b^3*d^3*e - 3*a*b^2*d^2*e^2 + 3*a^2*b*d*e^3 - a^3*e^4) + 1/3*(log(-b*c*x
- a*c + 1)*log((b*c*x*e + a*c*e - e)/(b*c*d - a*c*e + e) + 1) + dilog(-(b*c
*x*e + a*c*e - e)/(b*c*d - a*c*e + e)))*b^3/(b^3*d^3*e - 3*a*b^2*d^2*e^2 +
3*a^2*b*d*e^3 - a^3*e^4) - 1/6*(3*b^4*c^2*d - 3*a*b^3*c^2*e + 2*b^3*c*e)*lo
g(x*e + d)/(b^4*c^2*d^4*e + a^4*c^2*e^5 - 2*a^3*c*e^5 - 2*(2*a*b^3*c^2*e^2
- b^3*c*e^2)*d^3 + (6*a^2*b^2*c^2*e^3 - 6*a*b^2*c*e^3 + b^2*e^3)*d^2 + a^2*
```

```
e^5 - 2*(2*a^3*b*c^2*e^4 - 3*a^2*b*c*e^4 + a*b*e^4)*d) + 1/6*(b^4*c^2*d^4 -
 (2*a*b^3*c^2*e - b^3*c*e)*d^3 + (a^2*b^2*c^2*e^2 - a*b^2*c*e^2)*d^2 + (b^4
*c^2*d^2*e^2 + a^2*b^2*c^2*e^4 - a*b^2*c*e^4 - (2*a*b^3*c^2*e^3 - b^3*c*e^3
)*d)*x^2 + 2*(b^4*c^2*d^3*e - (2*a*b^3*c^2*e^2 - b^3*c*e^2)*d^2 + (a^2*b^2*
c^2*e^3 - a*b^2*c*e^3)*d)*x - 2*(b^4*c^2*d^4 + a^4*c^2*e^4 - 2*a^3*c*e^4 -
2*(2*a*b^3*c^2*e - b^3*c*e)*d^3 + (6*a^2*b^2*c^2*e^2 - 6*a*b^2*c*e^2 + b^2*
e^2)*d^2 + a^2*e^4 - 2*(2*a^3*b*c^2*e^3 - 3*a^2*b*c*e^3 + a*b*e^3)*d)*dilog
(b*c*x + a*c) + (4*(a*b^3*c^2*e - b^3*c*e)*d^3 + (3*b^4*c^2*d*e^3 - 3*a*b^3
*c^2*e^4 + 2*b^3*c*e^4)*x^3 - (5*a^2*b^2*c^2*e^2 - 8*a*b^2*c*e^2 + 3*b^2*e^
2)*d^2 + (7*b^4*c^2*d^2*e^2 - 2*a^2*b^2*c^2*e^4 + 4*a*b^2*c*e^4 - 2*b^2*e^4
 - (5*a*b^3*c^2*e^3 - 2*b^3*c*e^3)*d)*x^2 + (a^3*b*c^2*e^3 - 2*a^2*b*c*e^3
+ a*b*e^3)*d + (4*b^4*c^2*d^3*e + a^3*b*c^2*e^4 - 2*a^2*b*c*e^4 + 2*(a*b^3*
c^2*e^2 - 2*b^3*c*e^2)*d^2 + a*b*e^4 - (7*a^2*b^2*c^2*e^3 - 12*a*b^2*c*e^3
+ 5*b^2*e^3)*d)*x)*log(-b*c*x - a*c + 1))/(b^4*c^2*d^7*e - 2*(2*a*b^3*c^2*e
^2 - b^3*c*e^2)*d^6 + (6*a^2*b^2*c^2*e^3 - 6*a*b^2*c*e^3 + b^2*e^3)*d^5 - 2
*(2*a^3*b*c^2*e^4 - 3*a^2*b*c*e^4 + a*b*e^4)*d^4 + (a^4*c^2*e^5 - 2*a^3*c*e
^5 + a^2*e^5)*d^3 + (b^4*c^2*d^4*e^4 + a^4*c^2*e^8 - 2*a^3*c*e^8 - 2*(2*a*b
^3*c^2*e^5 - b^3*c*e^5)*d^3 + (6*a^2*b^2*c^2*e^6 - 6*a*b^2*c*e^6 + b^2*e^6)
*d^2 + a^2*e^8 - 2*(2*a^3*b*c^2*e^7 - 3*a^2*b*c*e^7 + a*b*e^7)*d)*x^3 + 3*(
b^4*c^2*d^5*e^3 - 2*(2*a*b^3*c^2*e^4 - b^3*c*e^4)*d^4 + (6*a^2*b^2*c^2*e^5
- 6*a*b^2*c*e^5 + b^2*e^5)*d^3 - 2*(2*a^3*b*c^2*e^6 - 3*a^2*b*c*e^6 + a*b*e
^6)*d^2 + (a^4*c^2*e^7 - 2*a^3*c*e^7 + a^2*e^7)*d)*x^2 + 3*(b^4*c^2*d^6*e^2
 - 2*(2*a*b^3*c^2*e^3 - b^3*c*e^3)*d^5 + (6*a^2*b^2*c^2*e^4 - 6*a*b^2*c*e^4
 + b^2*e^4)*d^4 - 2*(2*a^3*b*c^2*e^5 - 3*a^2*b*c*e^5 + a*b*e^5)*d^3 + (a^4*
c^2*e^6 - 2*a^3*c*e^6 + a^2*e^6)*d^2)*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral(dilog(b*c*x + a*c)/(x^4*e^4 + 4*d*x^3*e^3 + 6*d^2*x^2*e^2 + 4*d^3*
x*e + d^4), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate(dilog((b*x + a)*c)/(e*x + d)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{polylog}(2, c\,(a+b\,x))}{(d+e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, c*(a + b*x))/(d + e*x)^4,x)
```

```
[Out] int(polylog(2, c*(a + b*x))/(d + e*x)^4, x)
```

3.145 $\int \frac{\text{PolyLog}(2,x)}{-1+x}\,dx$

Optimal. Leaf size=46

$$\log^2(1-x)\log(x)+2\log(1-x)\text{PolyLog}(2,1-x)+\log(1-x)\text{PolyLog}(2,x)-2\text{PolyLog}(3,1-x)$$

```
[Out] ln(1-x)^2*ln(x)+2*ln(1-x)*polylog(2,1-x)+ln(1-x)*polylog(2,x)-2*polylog(3,1
-x)
```

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6731, 2443, 2481, 2421, 6724}

$$-2\text{Li}_3(1-x)+2\text{Li}_2(1-x)\log(1-x)+\text{Li}_2(x)\log(1-x)+\log(x)\log^2(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, x]/(-1 + x),x]
```

```
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2
, x] - 2*PolyLog[3, 1 - x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\mathrm{Li}_2(x)}{-1+x}\,dx &= \log(1-x)\mathrm{Li}_2(x)+\int \frac{\log^2(1-x)}{x}\,dx\\ &= \log^2(1-x)\log(x)+\log(1-x)\mathrm{Li}_2(x)+2\int \frac{\log(1-x)\log(x)}{1-x}\,dx\\ &= \log^2(1-x)\log(x)+\log(1-x)\mathrm{Li}_2(x)-2\mathrm{Subst}\left(\int \frac{\log(1-x)\log(x)}{x}\,dx,x,1-x\right)\\ &= \log^2(1-x)\log(x)+2\log(1-x)\mathrm{Li}_2(1-x)+\log(1-x)\mathrm{Li}_2(x)-2\mathrm{Subst}\left(\int \frac{\mathrm{Li}_2(x)}{x}\,dx,x,1\right.\\ &= \log^2(1-x)\log(x)+2\log(1-x)\mathrm{Li}_2(1-x)+\log(1-x)\mathrm{Li}_2(x)-2\mathrm{Li}_3(1-x)\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 46, normalized size = 1.00

$$\log^2(1-x)\log(x)+2\log(1-x)\mathrm{PolyLog}(2,1-x)+\log(1-x)\mathrm{PolyLog}(2,x)-2\mathrm{PolyLog}(3,1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, x]/(-1 + x),x]
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2
, x] - 2*PolyLog[3, 1 - x]
```

Maple [**A**]
time = 0.83, size = 66, normalized size = 1.43

method	result
default	$\ln(-1+x)\,\mathrm{polylog}(2,x)+\ln(1-x)^2\ln(x)+2\ln(1-x)\,\mathrm{polylog}(2,1-x)-2\,\mathrm{polylog}(3,1-x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,x)/(-1+x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(-1+x)*polylog(2,x)+ln(1-x)^2*ln(x)+2*ln(1-x)*polylog(2,1-x)-2*polylog(3,
1-x)-(ln(-1+x)-ln(1-x))*dilog(1-x)
```

Maxima [A]
time = 0.25, size = 44, normalized size = 0.96

$$\log(x)\log(-x+1)^2 + \mathrm{Li}_2(x)\log(-x+1) + 2\,\mathrm{Li}_2(-x+1)\log(-x+1) - 2\,\mathrm{Li}_3(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,x)/(-1+x),x, algorithm="maxima")
```

```
[Out] log(x)*log(-x + 1)^2 + dilog(x)*log(-x + 1) + 2*dilog(-x + 1)*log(-x + 1) -
 2*polylog(3, -x + 1)
```

Fricas [A]
time = 0.36, size = 39, normalized size = 0.85

$$\log(x)\log(-x+1)^2 + (\mathrm{Li}_2(x) + 2\,\mathrm{Li}_2(-x+1))\log(-x+1) - 2\,\mathrm{polylog}(3,-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,x)/(-1+x),x, algorithm="fricas")
```

```
[Out] log(x)*log(-x + 1)^2 + (dilog(x) + 2*dilog(-x + 1))*log(-x + 1) - 2*polylog
(3, -x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_2(x)}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,x)/(-1+x),x)
```

```
[Out] Integral(polylog(2, x)/(x - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,x)/(-1+x),x, algorithm="giac")
```

```
[Out] integrate(dilog(x)/(x - 1), x)
```

Mupad [B]
time = 0.04, size = 46, normalized size = 1.00

$$\ln(1-x)^2\,\ln(x)-2\,\mathrm{polylog}(3,1-x)+2\,\ln(1-x)\,\mathrm{polylog}(2,1-x)+\ln(1-x)\,\mathrm{polylog}(2,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, x)/(x - 1),x)
```

```
[Out] log(1 - x)^2*log(x) - 2*polylog(3, 1 - x) + 2*log(1 - x)*polylog(2, 1 - x)
+ log(1 - x)*polylog(2, x)
```

3.146 $\int -\frac{\text{PolyLog}(2,x)}{1-x} \, dx$

Optimal. Leaf size=46

$$\log^2(1-x)\log(x)+2\log(1-x)\text{PolyLog}(2,1-x)+\log(1-x)\text{PolyLog}(2,x)-2\text{PolyLog}(3,1-x)$$

```
[Out] ln(1-x)^2*ln(x)+2*ln(1-x)*polylog(2,1-x)+ln(1-x)*polylog(2,x)-2*polylog(3,1
-x)
```

Rubi [**A**]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6731, 2443, 2481, 2421, 6724}

$$-2\text{Li}_3(1-x) + 2\text{Li}_2(1-x)\log(1-x) + \text{Li}_2(x)\log(1-x) + \log(x)\log^2(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[-(PolyLog[2, x]/(1 - x)),x]
```

```
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2
, x] - 2*PolyLog[3, 1 - x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rubi steps

$$\begin{aligned}\int -\frac{\text{Li}_2(x)}{1-x}\,dx &= \log(1-x)\text{Li}_2(x)+\int \frac{\log^2(1-x)}{x}\,dx\\ &= \log^2(1-x)\log(x)+\log(1-x)\text{Li}_2(x)+2\int \frac{\log(1-x)\log(x)}{1-x}\,dx\\ &= \log^2(1-x)\log(x)+\log(1-x)\text{Li}_2(x)-2\text{Subst}\left(\int \frac{\log(1-x)\log(x)}{x}\,dx,x,1-x\right)\\ &= \log^2(1-x)\log(x)+2\log(1-x)\text{Li}_2(1-x)+\log(1-x)\text{Li}_2(x)-2\text{Subst}\left(\int \frac{\text{Li}_2(x)}{x}\,dx,x,1\right.\\ &= \log^2(1-x)\log(x)+2\log(1-x)\text{Li}_2(1-x)+\log(1-x)\text{Li}_2(x)-2\text{Li}_3(1-x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 1.00

$\log^2(1-x)\log(x)+2\log(1-x)\text{PolyLog}(2,1-x)+\log(1-x)\text{PolyLog}(2,x)-2\text{PolyLog}(3,1-x)$

Antiderivative was successfully verified.

```
[In] Integrate[-(PolyLog[2, x]/(1 - x)),x]
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2
, x] - 2*PolyLog[3, 1 - x]
```

Maple [A]
time = 0.53, size = 66, normalized size = 1.43

method	result
default	$\ln(-1+x)\,\text{polylog}(2,x)+\ln(1-x)^2\ln(x)+2\ln(1-x)\,\text{polylog}(2,1-x)-2\,\text{polylog}(3,1-x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-polylog(2,x)/(1-x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(-1+x)*polylog(2,x)+ln(1-x)^2*ln(x)+2*ln(1-x)*polylog(2,1-x)-2*polylog(3,
1-x)-(ln(-1+x)-ln(1-x))*dilog(1-x)
```

Maxima [A]
time = 0.25, size = 44, normalized size = 0.96

$$\log(x)\log(-x+1)^{2}+\mathrm{Li}_2(x)\log(-x+1)+2\,\mathrm{Li}_2(-x+1)\log(-x+1)-2\,\mathrm{Li}_3(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-polylog(2,x)/(1-x),x, algorithm="maxima")
```

```
[Out] log(x)*log(-x + 1)^2 + dilog(x)*log(-x + 1) + 2*dilog(-x + 1)*log(-x + 1) -
 2*polylog(3, -x + 1)
```

Fricas [A]
time = 0.37, size = 39, normalized size = 0.85

$$\log(x)\log(-x+1)^{2}+\left(\mathrm{Li}_2(x)+2\,\mathrm{Li}_2(-x+1)\right)\log(-x+1)-2\,\mathrm{polylog}(3,-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-polylog(2,x)/(1-x),x, algorithm="fricas")
```

```
[Out] log(x)*log(-x + 1)^2 + (dilog(x) + 2*dilog(-x + 1))*log(-x + 1) - 2*polylog
(3, -x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{Li}_2(x)}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-polylog(2,x)/(1-x),x)
```

```
[Out] Integral(polylog(2, x)/(x - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-polylog(2,x)/(1-x),x, algorithm="giac")
```

```
[Out] integrate(dilog(x)/(x - 1), x)
```

Mupad [**B**]
time = 0.00, size = 46, normalized size = 1.00

$$\ln(1-x)^2 \ln(x) - 2\,\mathrm{polylog}(3, 1-x) + 2\,\ln(1-x)\,\mathrm{polylog}(2, 1-x) + \ln(1-x)\,\mathrm{polylog}(2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, x)/(x - 1),x)
```

```
[Out] log(1 - x)^2*log(x) - 2*polylog(3, 1 - x) + 2*log(1 - x)*polylog(2, 1 - x)
+ log(1 - x)*polylog(2, x)
```

3.147 $\int \frac{\text{PolyLog}(2,x)}{(-1+x)x}\,dx$

Optimal. Leaf size=51

$$\log^2(1-x)\log(x)+2\log(1-x)\text{PolyLog}(2,1-x)+\log(1-x)\text{PolyLog}(2,x)-2\text{PolyLog}(3,1-x)-\text{PolyLog}(3,x)$$

```
[Out] ln(1-x)^2*ln(x)+2*ln(1-x)*polylog(2,1-x)+ln(1-x)*polylog(2,x)-2*polylog(3,1
-x)-polylog(3,x)
```

Rubi [A]
time = 0.09, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6874, 6731, 2443, 2481, 2421, 6724}

$$-2\text{Li}_3(1-x)-\text{Li}_3(x)+2\text{Li}_2(1-x)\log(1-x)+\text{Li}_2(x)\log(1-x)+\log(x)\log^2(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, x]/((-1 + x)*x),x]
```

```
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2
, x] - 2*PolyLog[3, 1 - x] - PolyLog[3, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{\mathrm{Li}_2(x)}{(-1+x)x}\,dx &= \int \left(\frac{\mathrm{Li}_2(x)}{-1+x}-\frac{\mathrm{Li}_2(x)}{x}\right)dx\\ &= \int \frac{\mathrm{Li}_2(x)}{-1+x}\,dx-\int \frac{\mathrm{Li}_2(x)}{x}\,dx\\ &= \log(1-x)\mathrm{Li}_2(x)-\mathrm{Li}_3(x)+\int \frac{\log^2(1-x)}{x}\,dx\\ &= \log^2(1-x)\log(x)+\log(1-x)\mathrm{Li}_2(x)-\mathrm{Li}_3(x)+2\int \frac{\log(1-x)\log(x)}{1-x}\,dx\\ &= \log^2(1-x)\log(x)+\log(1-x)\mathrm{Li}_2(x)-\mathrm{Li}_3(x)-2\mathrm{Subst}\left(\int \frac{\log(1-x)\log(x)}{x}\,dx,x,1-\right.\\ &= \log^2(1-x)\log(x)+2\log(1-x)\mathrm{Li}_2(1-x)+\log(1-x)\mathrm{Li}_2(x)-\mathrm{Li}_3(x)-2\mathrm{Subst}\left(\int \frac{\mathrm{Li}}{}\right.\\ &= \log^2(1-x)\log(x)+2\log(1-x)\mathrm{Li}_2(1-x)+\log(1-x)\mathrm{Li}_2(x)-2\mathrm{Li}_3(1-x)-\mathrm{Li}_3(x)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 51, normalized size = 1.00

$$\log^2(1-x)\log(x)+2\log(1-x)\mathrm{PolyLog}(2,1-x)+\log(1-x)\mathrm{PolyLog}(2,x)-2\mathrm{PolyLog}(3,1-x)-\mathrm{PolyLog}(3,x)$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, x]/((-1 + x)*x),x]
```

```
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2
, x] - 2*PolyLog[3, 1 - x] - PolyLog[3, x]
```

Maple [A]
time = 0.73, size = 82, normalized size = 1.61

method	result
default	$\ln(-1+x)\operatorname{polylog}(2,x)+\ln(-1+x)^2\ln(x)+2\ln(-1+x)\operatorname{polylog}(2,1-x)-2\operatorname{polylog}(3,$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,x)/(-1+x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(-1+x)*polylog(2,x)+ln(-1+x)^2*ln(x)+2*ln(-1+x)*polylog(2,1-x)-2*polylog(
3,1-x)+ln(x)*ln(-1+x)*(ln(1-x)-ln(-1+x))+dilog(x)*(ln(1-x)-ln(-1+x))-polylo
g(3,x)
```

Maxima [A]
time = 0.27, size = 49, normalized size = 0.96

$$\log(x)\log(-x+1)^2+\mathrm{Li}_2(x)\log(-x+1)+2\,\mathrm{Li}_2(-x+1)\log(-x+1)-\mathrm{Li}_3(x)-2\,\mathrm{Li}_3(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,x)/(-1+x)/x,x, algorithm="maxima")
```

```
[Out] log(x)*log(-x + 1)^2 + dilog(x)*log(-x + 1) + 2*dilog(-x + 1)*log(-x + 1) -
 polylog(3, x) - 2*polylog(3, -x + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,x)/(-1+x)/x,x, algorithm="fricas")
```

```
[Out] integral(dilog(x)/(x^2 - x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{Li}_2(x)}{x(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,x)/(-1+x)/x,x)
```

```
[Out] Integral(polylog(2, x)/(x*(x - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,x)/(-1+x)/x,x, algorithm="giac")
[Out] integrate(dilog(x)/((x - 1)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{polylog}(2,x)}{x\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, x)/(x*(x - 1)),x)
[Out] int(polylog(2, x)/(x*(x - 1)), x)
```

3.148 $\int -\frac{\text{PolyLog}(2,x)}{(1-x)x}\,dx$

Optimal. Leaf size=51

$$\log^2(1-x)\log(x)+2\log(1-x)\text{PolyLog}(2,1-x)+\log(1-x)\text{PolyLog}(2,x)-2\text{PolyLog}(3,1-x)-\text{PolyLog}(3,x)$$

```
[Out] ln(1-x)^2*ln(x)+2*ln(1-x)*polylog(2,1-x)+ln(1-x)*polylog(2,x)-2*polylog(3,1
-x)-polylog(3,x)
```

Rubi [A]
time = 0.09, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6874, 6731, 2443, 2481, 2421, 6724}

$$-2\text{Li}_3(1-x)-\text{Li}_3(x)+2\text{Li}_2(1-x)\log(1-x)+\text{Li}_2(x)\log(1-x)+\log(x)\log^2(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[-(PolyLog[2, x]/((1 - x)*x)),x]
```

```
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2
, x] - 2*PolyLog[3, 1 - x] - PolyLog[3, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int -\frac{\mathrm{Li}_2(x)}{(1-x)x}\,dx &= -\int \left(-\frac{\mathrm{Li}_2(x)}{-1+x}+\frac{\mathrm{Li}_2(x)}{x}\right)dx \\
&= \int \frac{\mathrm{Li}_2(x)}{-1+x}\,dx-\int \frac{\mathrm{Li}_2(x)}{x}\,dx \\
&= \log(1-x)\mathrm{Li}_2(x)-\mathrm{Li}_3(x)+\int \frac{\log^2(1-x)}{x}\,dx \\
&= \log^2(1-x)\log(x)+\log(1-x)\mathrm{Li}_2(x)-\mathrm{Li}_3(x)+2\int \frac{\log(1-x)\log(x)}{1-x}\,dx \\
&= \log^2(1-x)\log(x)+\log(1-x)\mathrm{Li}_2(x)-\mathrm{Li}_3(x)-2\mathrm{Subst}\left(\int \frac{\log(1-x)\log(x)}{x}\,dx,x,1-\right. \\
&= \log^2(1-x)\log(x)+2\log(1-x)\mathrm{Li}_2(1-x)+\log(1-x)\mathrm{Li}_2(x)-\mathrm{Li}_3(x)-2\mathrm{Subst}\left(\int \frac{\mathrm{Li}}{}\right. \\
&= \log^2(1-x)\log(x)+2\log(1-x)\mathrm{Li}_2(1-x)+\log(1-x)\mathrm{Li}_2(x)-2\mathrm{Li}_3(1-x)-\mathrm{Li}_3(x)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 1.00

$$\log^2(1-x)\log(x)+2\log(1-x)\mathrm{PolyLog}(2,1-x)+\log(1-x)\mathrm{PolyLog}(2,x)-2\mathrm{PolyLog}(3,1-x)-\mathrm{PolyLog}(3,x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(PolyLog[2, x]/((1 - x)*x)),x]
```

```
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2
, x] - 2*PolyLog[3, 1 - x] - PolyLog[3, x]
```

Maple [A]
time = 0.72, size = 82, normalized size = 1.61

method	result
default	$\ln(-1+x)\,\mathrm{polylog}(2,x)+\ln(-1+x)^2\ln(x)+2\ln(-1+x)\,\mathrm{polylog}(2,1-x)-2\,\mathrm{polylog}(3,$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-polylog(2,x)/(1-x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(-1+x)*polylog(2,x)+ln(-1+x)^2*ln(x)+2*ln(-1+x)*polylog(2,1-x)-2*polylog(
3,1-x)+ln(x)*ln(-1+x)*(ln(1-x)-ln(-1+x))+dilog(x)*(ln(1-x)-ln(-1+x))-polylo
g(3,x)
```

Maxima [A]
time = 0.26, size = 49, normalized size = 0.96

$$\log(x)\log(-x+1)^2+\mathrm{Li}_2(x)\log(-x+1)+2\,\mathrm{Li}_2(-x+1)\log(-x+1)-\mathrm{Li}_3(x)-2\,\mathrm{Li}_3(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-polylog(2,x)/(1-x)/x,x, algorithm="maxima")
```

```
[Out] log(x)*log(-x + 1)^2 + dilog(x)*log(-x + 1) + 2*dilog(-x + 1)*log(-x + 1) -
 polylog(3, x) - 2*polylog(3, -x + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-polylog(2,x)/(1-x)/x,x, algorithm="fricas")
```

```
[Out] integral(dilog(x)/(x^2 - x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_2(x)}{x(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-polylog(2,x)/(1-x)/x,x)
```

```
[Out] Integral(polylog(2, x)/(x*(x - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-polylog(2,x)/(1-x)/x,x, algorithm="giac")
```

```
[Out] integrate(dilog(x)/((x - 1)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{polylog}(2,x)}{x\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, x)/(x*(x - 1)),x)
```

```
[Out] int(polylog(2, x)/(x*(x - 1)), x)
```

3.149 $\int \frac{\text{PolyLog}\left(n,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)} dx$

Optimal. Leaf size=35

$$\frac{\text{PolyLog}\left(1+n,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

```
[Out] polylog(1+n,e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/n
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = $\{6745\}$

$$\frac{\text{Li}_{n+1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[n, e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)),x]
```

```
[Out] PolyLog[1 + n, e*((a + b*x)/(c + d*x))^n]/((b*c - a*d)*n)
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\text{Li}_n\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)} dx = \frac{\text{Li}_{1+n}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 0.97

$$\frac{\text{PolyLog}\left(1+n,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bcn-adn}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[n, e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)),x]
```

```
[Out] PolyLog[1 + n, e*((a + b*x)/(c + d*x))^n]/(b*c*n - a*d*n)
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\text{polylog}\left(n, e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{\left(bx+a\right)\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(polylog(n,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="ma
xima")
```

```
[Out] integrate(polylog(n, ((b*x + a)/(d*x + c))^n*e)/((b*x + a)*(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="fr
icas")
```

```
[Out] integral(polylog(n, ((b*x + a)/(d*x + c))^n*e)/(b*d*x^2 + a*c + (b*c + a*d)
*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{Li}_{n}\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)}{\left(a+bx\right)\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,e*((b*x+a)/(d*x+c))**n)/(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(polylog(n, e*(a/(c + d*x) + b*x/(c + d*x))**n)/((a + b*x)*(c + d*x
)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="gi
ac")
```

```
[Out] integrate(polylog(n, e*((b*x + a)/(d*x + c))^n)/((b*x + a)*(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{polylog}\left(n,e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n, e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)),x)
```

```
[Out] int(polylog(n, e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)), x)
```

3.150 $\int \frac{\text{PolyLog}\left(3,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=33

$$\frac{\text{PolyLog}\left(4,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

```
[Out] polylog(4,e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/n
```

Rubi [A]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {6745}

$$\frac{\text{Li}_4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)),x]
```

```
[Out] PolyLog[4, e*((a + b*x)/(c + d*x))^n]/((b*c - a*d)*n)
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\text{Li}_3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \frac{\text{Li}_4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 0.97

$$\frac{\text{PolyLog}\left(4,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bcn-adn}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)),x]
```

```
[Out] PolyLog[4, e*((a + b*x)/(c + d*x))^n]/(b*c*n - a*d*n)
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{polylog}\left(3, e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{(bx+a)\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(polylog(3,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="ma
xima")
```

```
[Out] -1/6*(3*(n*log(b*x + a)^2 - 2*n*log(b*x + a)*log(d*x + c) + n*log(d*x + c)^
2)*dilog(e^(n*log(b*x + a) - n*log(d*x + c) + 1)) + (n^2*log(b*x + a)^3 - 3
*n^2*log(b*x + a)^2*log(d*x + c) + 3*n^2*log(b*x + a)*log(d*x + c)^2 - n^2*
log(d*x + c)^3)*log((d*x + c)^n - e^(n*log(b*x + a) + 1)) - (n^2*log(b*x +
a)^3 - 3*n^2*log(b*x + a)^2*log(d*x + c) + 3*n^2*log(b*x + a)*log(d*x + c)^
2 - n^2*log(d*x + c)^3)*log((d*x + c)^n) - 6*(log(b*x + a) - log(d*x + c))*
polylog(3, e^(n*log(b*x + a) - n*log(d*x + c) + 1)))/(b*c - a*d) + integrat
e(1/6*(n^3*e*log(b*x + a)^3 - 3*n^3*e*log(b*x + a)^2*log(d*x + c) + 3*n^3*e
*log(b*x + a)*log(d*x + c)^2 - n^3*e*log(d*x + c)^3)*(b*x + a)^n/((b*d*x^2*
e + a*c*e + (b*c + a*d)*x*e)*(b*x + a)^n - (b*d*x^2 + a*c + (b*c + a*d)*x)*
(d*x + c)^n), x)
```

Fricas [A]
time = 0.42, size = 34, normalized size = 1.03

$$\frac{\operatorname{polylog}\left(4, \left(\frac{bx+a}{dx+c}\right)^{n} e\right)}{(bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="fr
icas")
```

```
[Out] polylog(4, ((b*x + a)/(d*x + c))^n*e)/((b*c - a*d)*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{Li}_3\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,e*((b*x+a)/(d*x+c))**n)/(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(polylog(3, e*(a/(c + d*x) + b*x/(c + d*x))**n)/((a + b*x)*(c + d*x
)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="gi
ac")
```

```
[Out] integrate(polylog(3, e*((b*x + a)/(d*x + c))^n)/((b*x + a)*(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{polylog}\left(3,e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)),x)
```

```
[Out] int(polylog(3, e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)), x)
```

3.151 $\int \frac{\text{PolyLog}\left(2,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=33

$$\frac{\text{PolyLog}\left(3,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

```
[Out] polylog(3,e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/n
```

Rubi [A]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {6745}

$$\frac{\text{Li}_3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)),x]
[Out] PolyLog[3, e*((a + b*x)/(c + d*x))^n]/((b*c - a*d)*n)
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\text{Li}_2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \frac{\text{Li}_3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 0.97

$$\frac{\text{PolyLog}\left(3,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bcn-adn}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)),x]
[Out] PolyLog[3, e*((a + b*x)/(c + d*x))^n]/(b*c*n - a*d*n)
```

Maple [F]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{polylog}\left(2, e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{(bx+a)(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(polylog(2,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="ma
xima")
```

```
[Out] 1/2*(2*(log(b*x + a) - log(d*x + c))*dilog(e^(n*log(b*x + a) - n*log(d*x +
c) + 1)) + (n*log(b*x + a)^2 - 2*n*log(b*x + a)*log(d*x + c) + n*log(d*x +
c)^2)*log((d*x + c)^n - e^(n*log(b*x + a) + 1)) - (n*log(b*x + a)^2 - 2*n*l
og(b*x + a)*log(d*x + c) + n*log(d*x + c)^2)*log((d*x + c)^n))/(b*c - a*d)
+ integrate(-1/2*(n^2*e*log(b*x + a)^2 - 2*n^2*e*log(b*x + a)*log(d*x + c)
+ n^2*e*log(d*x + c)^2)*(b*x + a)^n/((b*d*x^2*e + a*c*e + (b*c*e + a*d*e)*x
)*(b*x + a)^n - (b*d*x^2 + a*c + (b*c + a*d)*x)*(d*x + c)^n), x)
```

Fricas [A]
time = 0.37, size = 34, normalized size = 1.03

$$\frac{\operatorname{polylog}\left(3, \left(\frac{bx+a}{dx+c}\right)^{n} e\right)}{(bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="fr
icas")
```

```
[Out] polylog(3, ((b*x + a)/(d*x + c))^n*e)/((b*c - a*d)*n)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,e*((b*x+a)/(d*x+c))**n)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="gi
ac")
```

```
[Out] integrate(dilog(e*((b*x + a)/(d*x + c))^n)/((b*x + a)*(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{polylog}\left(2,e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)),x)
```

```
[Out] int(polylog(2, e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)), x)
```

3.152 $\int -\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=33

$$\frac{\text{PolyLog}\left(2,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

```
[Out] polylog(2,e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/n
```

Rubi [**A**]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$, Rules used = $\{2598\}$

$$\frac{\text{Li}_2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[-(Log[1 - e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x))),x]
```

```
[Out] PolyLog[2, e*((a + b*x)/(c + d*x))^n]/((b*c - a*d)*n)
```

Rule 2598

```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Rubi steps

$$\int -\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \frac{\text{Li}_2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

Mathematica [**F**] Contains unresolved integral.
time = 1.22, size = 40, normalized size = 1.21

$$-\int \frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$$

Antiderivative was successfully verified.

```
[In] Integrate[-(Log[1 - e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x))),x]
```

```
[Out] -Integrate[Log[1 - e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)), x]
```

Maple [**F**]
time = 0.32, size = 0, normalized size = 0.00

$$\int -\frac{\ln \left(1-e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{(bx+a)\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-ln(1-e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(-ln(1-e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-log(1-e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="maxim
a")
```

```
[Out] -((log(b*x + a) - log(d*x + c))*log((d*x + c)^n - e^(n*log(b*x + a) + 1)) -
 (log(b*x + a) - log(d*x + c))*log((d*x + c)^n))/(b*c - a*d) + integrate((n
*e*log(b*x + a) - n*e*log(d*x + c))*(b*x + a)^n/((b*d*x^2*e + a*c*e + (b*c
+ a*d)*x*e)*(b*x + a)^n - (b*d*x^2 + a*c + (b*c + a*d)*x)*(d*x + c)^n), x)
```

Fricas [**A**]
time = 0.37, size = 33, normalized size = 1.00

$$\frac{\mathrm{Li}_2\left(\left(\frac{bx+a}{dx+c}\right)^{n} e\right)}{(bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-log(1-e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="frica
s")
```

```
[Out] dilog(((b*x + a)/(d*x + c))^n*e)/((b*c - a*d)*n)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-ln(1-e*((b*x+a)/(d*x+c))**n)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-log(1-e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="giac"
)
```

```
[Out] integrate(-log(-e*((b*x + a)/(d*x + c))^n + 1)/((b*x + a)*(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{\ln\left(1-e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-log(1 - e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)),x)
```

```
[Out] int(-log(1 - e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)), x)
```

3.153 $\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal. Leaf size=36

$$-\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

```
[Out] -ln(1-e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/n
```

Rubi [A]
time = 0.21, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 53, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {12, 6816}

$$-\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)*(1 - e*((a + b*x)/(c +
 d*x))^n)),x]
```

```
[Out] -(Log[1 - e*((a + b*x)/(c + d*x))^n]/((b*c - a*d)*n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6816

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rubi steps

$$\begin{aligned}\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx &= e\int \frac{\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx\\ &= -\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}\end{aligned}$$

Mathematica [A]
time = 0.39, size = 38, normalized size = 1.06

$$-\frac{e\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bcen-aden}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)*(1 - e*((a + b*x
)/(c + d*x))^n)),x]
```

```
[Out] -((e*Log[1 - e*((a + b*x)/(c + d*x))^n])/(b*c*e*n - a*d*e*n))
```

Maple [A]
time = 0.45, size = 37, normalized size = 1.03

method	result	size
norman	$\frac{\ln\left(-1+e\,{\mathrm e}^{n\ln\left(\frac{bx+a}{dx+c}\right)}\right)}{n(ad-cb)}$	37
risch	$-\frac{\ln(-dx-c)}{ad-cb}+\frac{\ln(bx+a)}{ad-cb}-\frac{\ln\left(\frac{bx+a}{dx+c}\right)}{ad-cb}+\frac{\ln\left(\left(\frac{bx+a}{dx+c}\right)^n-\frac{1}{e}\right)}{n(ad-cb)}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x,met
hod=_RETURNVERBOSE)
```

```
[Out] 1/n/(a*d-b*c)*ln(-1+e*exp(n*ln((b*x+a)/(d*x+c))))
```

Maxima [A]
time = 0.29, size = 61, normalized size = 1.69

$$\left(\frac{e^{(-1)}\log\left(dx+c\right)}{bc-ad}-\frac{e^{(-1)}\log\left(\left(dx+c\right)^n-e^{(n\log(bx+a)+1)}\right)}{bcn-adn}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c)
,x, algorithm="maxima")
```

```
[Out] (e^(-1)*log(d*x + c)/(b*c - a*d) - e^(-1)*log((d*x + c)^n - e^(n*log(b*x +
a) + 1))/(b*c*n - a*d*n))*e
```

Fricas [A]
time = 0.36, size = 36, normalized size = 1.00

$$-\frac{\log\left(\left(\frac{bx+a}{dx+c}\right)^n e-1\right)}{(bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c)
,x, algorithm="fricas")
```

```
[Out] -log(((b*x + a)/(d*x + c))^n*e - 1)/((b*c - a*d)*n)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-e*((b*x+a)/(d*x+c))**n/(-1+e*((b*x+a)/(d*x+c))**n)/(b*x+a)/(d*x+
c),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c)
,x, algorithm="giac")
```

```
[Out] integrate(-e*((b*x + a)/(d*x + c))^n/((b*x + a)*(d*x + c)*(e*((b*x + a)/(d*
x + c))^n - 1)), x)
```

Mupad [**B**]
time = 0.27, size = 33, normalized size = 0.92

$$\frac{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n-1\right)}{a\,d\,n-b\,c\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e*((a + b*x)/(c + d*x))^n)/((e*((a + b*x)/(c + d*x))^n - 1)*(a + b*x)
*(c + d*x)),x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n - 1)/(a*d*n - b*c*n)
```

3.154 $\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}\,dx$

Optimal. Leaf size=36

$$\frac{1}{(bc-ad)n\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}$$

```
[Out] 1/(-a*d+b*c)/n/(1-e*((b*x+a)/(d*x+c))^n)
```

Rubi [A]
time = 0.24, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 53, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {12, 6818}

$$\frac{1}{n(bc-ad)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)*(1 - e*((a + b*x)/(c +
 d*x))^n)^2),x]
```

```
[Out] 1/((b*c - a*d)*n*(1 - e*((a + b*x)/(c + d*x))^n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}\,dx &= e\int \frac{\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}\,dx\\ &= \frac{1}{(bc-ad)n\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 35, normalized size = 0.97

$$\frac{1}{(-bc+ad)n\left(-1+e\left(\frac{a+bx}{c+dx}\right)^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)*(1 - e*((a + b*x
)/(c + d*x))^n)^2),x]
```

```
[Out] 1/((-(b*c) + a*d)*n*(-1 + e*((a + b*x)/(c + d*x))^n))
```

Maple [A]
time = 0.43, size = 36, normalized size = 1.00

method	result	size
risch	$\frac{1}{n(ad-cb)\left(-1+e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}$	36
norman	$\frac{e\,{\mathrm e}^{n\ln\left(\frac{bx+a}{dx+c}\right)}}{n(ad-cb)\left(-1+e\,{\mathrm e}^{n\ln\left(\frac{bx+a}{dx+c}\right)}\right)}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)^2/(b*x+a)/(d*x+c),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 1/n/(a*d-b*c)/(-1+e*((b*x+a)/(d*x+c))^n)
```

Maxima [A]
time = 0.28, size = 56, normalized size = 1.56

$$\frac{e^{(n\log(bx+a)+1)}}{(bcn-adn)(dx+c)^n-(bcn-adn)e^{(n\log(bx+a)+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)^2/(b*x+a)/(d*x+c
),x, algorithm="maxima")
```

```
[Out] e^(n*log(b*x + a) + 1)/((b*c*n - a*d*n)*(d*x + c)^n - (b*c*n - a*d*n)*e^(n*
log(b*x + a) + 1))
```

Fricas [A]
time = 0.38, size = 43, normalized size = 1.19

$$-\frac{1}{(bc-ad)n\left(\frac{bx+a}{dx+c}\right)^n e-(bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)^2/(b*x+a)/(d*x+c
),x, algorithm="fricas")
```

```
[Out] -1/((b*c - a*d)*n*((b*x + a)/(d*x + c))^n*e - (b*c - a*d)*n)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e*((b*x+a)/(d*x+c))**n/(-1+e*((b*x+a)/(d*x+c))**n)**2/(b*x+a)/(d*
x+c),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)^2/(b*x+a)/(d*x+c
),x, algorithm="giac")
```

```
[Out] integrate(e*((b*x + a)/(d*x + c))^n/((b*x + a)*(d*x + c)*(e*((b*x + a)/(d*x
 + c))^n - 1)^2), x)
```

Mupad [B]
time = 0.19, size = 35, normalized size = 0.97

$$\frac{1}{n\,\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n-1\right)\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*((a + b*x)/(c + d*x))^n)/((e*((a + b*x)/(c + d*x))^n - 1)^2*(a + b*x
)*(c + d*x)),x)
```

```
[Out] 1/(n*(e*((a + b*x)/(c + d*x))^n - 1)*(a*d - b*c))
```

3.155 $\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n+e^2\left(\frac{a+bx}{c+dx}\right)^{2n}}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}\,dx$

Optimal. Leaf size=52

$$\frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(bc-ad)n\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}$$

```
[Out] e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)^2/(-a*d+b*c)/n
```

Rubi [A]
time = 1.44, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 76, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {6873, 12, 6824, 34}

$$\frac{e\left(\frac{a+bx}{c+dx}\right)^n}{n(bc-ad)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*((a + b*x)/(c + d*x))^n + e^2*((a + b*x)/(c + d*x))^(2*n))/((a + b*x
)*(c + d*x)*(1 - e*((a + b*x)/(c + d*x))^n)^3),x]
```

```
[Out] (e*((a + b*x)/(c + d*x))^n)/((b*c - a*d)*n*(1 - e*((a + b*x)/(c + d*x))^n)^
2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rule 6824

```
Int[(u_)*((c_.) + (d_.)*(v_))^(n_.)*((a_.) + (b_.)*(y_))^(m_.), x_Symbol] :
> With[{q = DerivativeDivides[y, u, x]}, Dist[q, Subst[Int[(a + b*x)^m*(c +
 d*x)^n, x], x, y], x] /;  !FalseQ[q]] /; FreeQ[{a, b, c, d, m, n}, x] && E
qQ[v, y]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\begin{aligned}
\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n+e^2\left(\frac{a+bx}{c+dx}\right)^{2n}}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}\,dx &= \int \frac{e\left(\frac{a+bx}{c+dx}\right)^n\left(1+e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}\,dx\\
&= e\int \frac{\left(\frac{a+bx}{c+dx}\right)^n\left(1+e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{1+x}{(1-x)^3}\,dx,x,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}\\
&= \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(bc-ad)n\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}
\end{aligned}$$

Mathematica [A]
time = 0.50, size = 52, normalized size = 1.00

$$-\frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(-bc+ad)n\left(-1+e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*((a + b*x)/(c + d*x))^n + e^2*((a + b*x)/(c + d*x))^(2*n))/((a
 + b*x)*(c + d*x)*(1 - e*((a + b*x)/(c + d*x))^n)^3),x]
```

```
[Out] -((e*((a + b*x)/(c + d*x))^n)/((-(b*c) + a*d)*n*(-1 + e*((a + b*x)/(c + d*x
))^n)^2))
```

Maple [A]
time = 0.51, size = 53, normalized size = 1.02

method	result	size
risch	$-\frac{e\left(\frac{bx+a}{dx+c}\right)^n}{n(ad-cb)\left(-1+e\left(\frac{bx+a}{dx+c}\right)^n\right)^2}$	53
norman	$-\frac{e\,\mathrm{e}^{n\ln\left(\frac{bx+a}{dx+c}\right)}}{n(ad-cb)\left(-1+e\,\mathrm{e}^{n\ln\left(\frac{bx+a}{dx+c}\right)}\right)^2}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1+e*((b*x+a)/(d*x+c))^n)*e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c)
)^n)^3/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -e/n/(a*d-b*c)*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(53) = 106$.
time = 0.30, size = 207, normalized size = 3.98

$$-\frac{1}{2}\left(\frac{e^{(2\,n\log(bx+a)+1)}-2\,e^{(n\log(bx+a)+n\log(dx+c))}}{(bcn-adn)(dx+c)^{2\,n}+(bcn-adn)e^{(2\,n\log(bx+a)+2)}-2\,(bcn-adn)e^{(n\log(bx+a)+n\log(dx+c)+1)}}-\frac{e^{(2\,n\log(bx+a)+1)}}{(bcn-adn)(dx+c)^{2\,n}+(bcn-adn)e^{(2\,n\log(bx+a)+2)}-2\,(bcn-adn)e^{(n\log(bx+a)+n\log(dx+c)+1)}}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(1+e*((b*x+a)/(d*x+c))^n)*e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(
d*x+c))^n)^3/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*((e^(2*n*log(b*x + a) + 1) - 2*e^(n*log(b*x + a) + n*log(d*x + c)))/((
b*c*n - a*d*n)*(d*x + c)^(2*n) + (b*c*n - a*d*n)*e^(2*n*log(b*x + a) + 2) -
 2*(b*c*n - a*d*n)*e^(n*log(b*x + a) + n*log(d*x + c) + 1)) - e^(2*n*log(b*
x + a) + 1)/((b*c*n - a*d*n)*(d*x + c)^(2*n) + (b*c*n - a*d*n)*e^(2*n*log(b
*x + a) + 2) - 2*(b*c*n - a*d*n)*e^(n*log(b*x + a) + n*log(d*x + c) + 1)))*
e
```

Fricas [A]
time = 0.37, size = 88, normalized size = 1.69

$$\frac{\left(\frac{bx+a}{dx+c}\right)^{n}e}{(bc-ad)n\left(\frac{bx+a}{dx+c}\right)^{2\,n}e^{2}-2\,(bc-ad)n\left(\frac{bx+a}{dx+c}\right)^{n}e+(bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(1+e*((b*x+a)/(d*x+c))^n)*e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(
d*x+c))^n)^3/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] ((b*x + a)/(d*x + c))^n*e/((b*c - a*d)*n*((b*x + a)/(d*x + c))^(2*n)*e^2 -
2*(b*c - a*d)*n*((b*x + a)/(d*x + c))^n*e + (b*c - a*d)*n)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(1+e*((b*x+a)/(d*x+c))**n)*e*((b*x+a)/(d*x+c))**n/(-1+e*((b*x+a)
/(d*x+c))**n)**3/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(1+e*((b*x+a)/(d*x+c))^n)*e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(
d*x+c))^n)^3/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(-(e*((b*x + a)/(d*x + c))^n + 1)*e*((b*x + a)/(d*x + c))^n/((b*x
+ a)*(d*x + c)*(e*((b*x + a)/(d*x + c))^n - 1)^3), x)
```

Mupad [B]
time = 0.25, size = 81, normalized size = 1.56

$$-\frac{e\left(\frac{a+b\,x}{c+d\,x}\right)^n}{n\,\left(a\,d-b\,c\right)\,\left(e^2\,{\left(\frac{a}{c+d\,x}+\frac{b\,x}{c+d\,x}\right)}^{2\,n}-2\,e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e*(e*((a + b*x)/(c + d*x))^n + 1)*((a + b*x)/(c + d*x))^n)/((e*((a +
b*x)/(c + d*x))^n - 1)^3*(a + b*x)*(c + d*x)),x)
```

```
[Out] -(e*((a + b*x)/(c + d*x))^n)/(n*(a*d - b*c)*(e^2*(a/(c + d*x) + (b*x)/(c +
d*x))^(2*n) - 2*e*((a + b*x)/(c + d*x))^n + 1))
```

3.156 $\int x^3 \mathbf{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx$

Optimal. Leaf size=135

$$\frac{x^3 \operatorname{PolyLog}\left(1+n, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{3x^2 \operatorname{PolyLog}\left(2+n, d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)} + \frac{6x \operatorname{PolyLog}\left(3+n, d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)} - 6\mathrm{Po}$$

```
[Out] x^3*polylog(1+n,d*(F^(c*(b*x+a)))^p)/b/c/p/ln(F)-3*x^2*polylog(2+n,d*(F^(c*
(b*x+a)))^p)/b^2/c^2/p^2/ln(F)^2+6*x*polylog(3+n,d*(F^(c*(b*x+a)))^p)/b^3/c
^3/p^3/ln(F)^3-6*polylog(4+n,d*(F^(c*(b*x+a)))^p)/b^4/c^4/p^4/ln(F)^4
```

Rubi [A]
time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {6744, 2320, 6724}

$$-\frac{6\mathrm{Li}_{n+4}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^4c^4p^4\log^4(F)} + \frac{6x\mathrm{Li}_{n+3}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)} - \frac{3x^2\mathrm{Li}_{n+2}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)} + \frac{x^3\mathrm{Li}_{n+1}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```

```
[Out] (x^3*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - (3*x^2*PolyLog
[2 + n, d*(F^(c*(a + b*x)))^p])/(b^2*c^2*p^2*Log[F]^2) + (6*x*PolyLog[3 + n
, d*(F^(c*(a + b*x)))^p])/(b^3*c^3*p^3*Log[F]^3) - (6*PolyLog[4 + n, d*(F^(
c*(a + b*x)))^p])/(b^4*c^4*p^4*Log[F]^4)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
```

```
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^3\text{Li}_n\left(d\left(F^{c(a+bx)}\right)^p\right)\,dx &= \frac{x^3\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}-\frac{3\int x^2\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)\,dx}{bcp\log(F)}\\ &= \frac{x^3\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}-\frac{3x^2\text{Li}_{2+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}+\frac{6\int x\text{Li}_{2+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}\\ &= \frac{x^3\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}-\frac{3x^2\text{Li}_{2+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}+\frac{6x\text{Li}_{3+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)}-\\ &= \frac{x^3\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}-\frac{3x^2\text{Li}_{2+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}+\frac{6x\text{Li}_{3+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)}-\\ &= \frac{x^3\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}-\frac{3x^2\text{Li}_{2+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}+\frac{6x\text{Li}_{3+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)}-\end{aligned}$$

Mathematica [A]
time = 0.01, size = 135, normalized size = 1.00

$$\frac{x^3\text{PolyLog}\left(1+n,d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}-\frac{3x^2\text{PolyLog}\left(2+n,d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}+\frac{6x\text{PolyLog}\left(3+n,d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)}-\frac{6\text{PolyLog}\left(4+n,d\left(F^{c(a+bx)}\right)^p\right)}{b^4c^4p^4\log^4(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```

```
[Out] (x^3*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - (3*x^2*PolyLog
[2 + n, d*(F^(c*(a + b*x)))^p])/(b^2*c^2*p^2*Log[F]^2) + (6*x*PolyLog[3 + n
, d*(F^(c*(a + b*x)))^p])/(b^3*c^3*p^3*Log[F]^3) - (6*PolyLog[4 + n, d*(F^(
c*(a + b*x)))^p])/(b^4*c^4*p^4*Log[F]^4)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int x^3\,\text{polylog}\left(n,d\left(F^{c(bx+a)}\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*polylog(n,d*(F^(c*(b*x+a)))^p),x)
```

```
[Out] int(x^3*polylog(n,d*(F^(c*(b*x+a)))^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="maxima")
[Out] integrate(x^3*polylog(n, F^((b*x + a)*c*p)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="fricas")
[Out] integral(x^3*polylog(n, (F^(b*c*x + a*c))^p*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{Li}_n\left(d\left(F^{ac}F^{bcx}\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*polylog(n,d*(F**(c*(b*x+a)))**p),x)
[Out] Integral(x**3*polylog(n, d*(F**(a*c)*F**(b*c*x))**p), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="giac")
[Out] integrate(x^3*polylog(n, (F^((b*x + a)*c))^p*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{polylog}\left(n,d\,{\left(F^{c\,(a+b\,x)}\right)}^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*polylog(n, d*(F^(c*(a + b*x)))^p),x)
[Out] int(x^3*polylog(n, d*(F^(c*(a + b*x)))^p), x)
```

3.157 $\int x^2 \textbf{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx$

Optimal. Leaf size=100

$$\frac{x^2 \text{PolyLog}\left(1+n, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}-\frac{2x\text{PolyLog}\left(2+n, d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}+\frac{2\text{PolyLog}\left(3+n, d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)}$$

```
[Out] x^2*polylog(1+n,d*(F^(c*(b*x+a)))^p)/b/c/p/ln(F)-2*x*polylog(2+n,d*(F^(c*(b
*x+a)))^p)/b^2/c^2/p^2/ln(F)^2+2*polylog(3+n,d*(F^(c*(b*x+a)))^p)/b^3/c^3/p
^3/ln(F)^3
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {6744, 2320, 6724}

$$\frac{2\text{Li}_{n+3}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)}-\frac{2x\text{Li}_{n+2}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}+\frac{x^2\text{Li}_{n+1}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```

```
[Out] (x^2*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - (2*x*PolyLog[2
 + n, d*(F^(c*(a + b*x)))^p])/(b^2*c^2*p^2*Log[F]^2) + (2*PolyLog[3 + n, d*
(F^(c*(a + b*x)))^p])/(b^3*c^3*p^3*Log[F]^3)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\text{Li}_n\left(d\left(F^{c(a+bx)}\right)^p\right)\,dx &= \frac{x^2\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}-\frac{2\int x\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)\,dx}{bcp\log(F)}\\ &= \frac{x^2\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}-\frac{2x\text{Li}_{2+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}+\frac{2\int \text{Li}_{2+n}\left(d\left(F^{c(a+bx)}\right)^p\right)\,d}{b^2c^2p^2\log^2(F)}\\ &= \frac{x^2\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}-\frac{2x\text{Li}_{2+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}+\frac{2\text{Subst}\left(\int \frac{\text{Li}_{2+n}(dx^p)}{x}\,dx,x\right.}{b^3c^3p^2\log^3(F)}\\ &= \frac{x^2\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}-\frac{2x\text{Li}_{2+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}+\frac{2\text{Li}_{3+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 100, normalized size = 1.00

$$\frac{x^2\text{PolyLog}\left(1+n,d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}-\frac{2x\text{PolyLog}\left(2+n,d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}+\frac{2\text{PolyLog}\left(3+n,d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
[Out] (x^2*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - (2*x*PolyLog[2
 + n, d*(F^(c*(a + b*x)))^p])/(b^2*c^2*p^2*Log[F]^2) + (2*PolyLog[3 + n, d*
(F^(c*(a + b*x)))^p])/(b^3*c^3*p^3*Log[F]^3)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^2\,\text{polylog}\left(n,d\left(F^{c(bx+a)}\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(n,d*(F^(c*(b*x+a)))^p),x)
[Out] int(x^2*polylog(n,d*(F^(c*(b*x+a)))^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="maxima")
```

```
[Out] integrate(x^2*polylog(n, F^((b*x + a)*c*p)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="fricas")
```

```
[Out] integral(x^2*polylog(n, (F^(b*c*x + a*c))^p*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{Li}_n\left(d\left(F^{ac}F^{bcx}\right)^p\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*polylog(n,d*(F**(c*(b*x+a)))**p),x)
```

```
[Out] Integral(x**2*polylog(n, d*(F**(a*c)*F**(b*c*x))**p), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="giac")
```

```
[Out] integrate(x^2*polylog(n, (F^((b*x + a)*c))^p*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{polylog}\left(n,d\,{\left(F^{c\,\left(a+b\,x\right)}\right)}^p\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(n, d*(F^(c*(a + b*x)))^p),x)
```

```
[Out] int(x^2*polylog(n, d*(F^(c*(a + b*x)))^p), x)
```

3.158 $\int x\textbf{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)\,dx$

Optimal. Leaf size=65

$$\frac{x\text{PolyLog}\left(1+n,d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}-\frac{\text{PolyLog}\left(2+n,d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}$$

```
[Out] x*polylog(1+n,d*(F^(c*(b*x+a)))^p)/b/c/p/ln(F)-polylog(2+n,d*(F^(c*(b*x+a))
)^p)/b^2/c^2/p^2/ln(F)^2
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.176$, Rules used = {6744, 2320, 6724}

$$\frac{x\text{Li}_{n+1}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}-\frac{\text{Li}_{n+2}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}$$

Antiderivative was successfully verified.

```
[In] Int[x*PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```

```
[Out] (x*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - PolyLog[2 + n, d
*(F^(c*(a + b*x)))^p]/(b^2*c^2*p^2*Log[F]^2)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\text{Li}_n\left(d\left(F^{c(a+bx)}\right)^p\right)\,dx &= \frac{x\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}-\frac{\int \text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)\,dx}{bcp\log(F)}\\ &= \frac{x\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}-\frac{\text{Subst}\left(\int \frac{\text{Li}_{1+n}(dx^p)}{x}\,dx,x,F^{c(a+bx)}\right)}{b^2c^2p\log^2(F)}\\ &= \frac{x\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}-\frac{\text{Li}_{2+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 65, normalized size = 1.00

$$\frac{x\text{PolyLog}\left(1+n,d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}-\frac{\text{PolyLog}\left(2+n,d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
[Out] (x*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - PolyLog[2 + n, d
*(F^(c*(a + b*x)))^p]/(b^2*c^2*p^2*Log[F]^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x\,\text{polylog}\left(n,d\left(F^{c(bx+a)}\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(n,d*(F^(c*(b*x+a)))^p),x)
[Out] int(x*polylog(n,d*(F^(c*(b*x+a)))^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="maxima")
[Out] integrate(x*polylog(n, F^((b*x + a)*c*p)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="fricas")
[Out] integral(x*polylog(n, (F^(b*c*x + a*c))^p*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{Li}_{n}\left(d\left(F^{ac}F^{bcx}\right)^{p}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,d*(F**(c*(b*x+a)))**p),x)
[Out] Integral(x*polylog(n, d*(F**(a*c)*F**(b*c*x))**p), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="giac")
[Out] integrate(x*polylog(n, (F^((b*x + a)*c))^p*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{polylog}\left(n,d\,{\left(F^{c\,\left(a+b\,x\right)}\right)}^p\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(n, d*(F^(c*(a + b*x)))^p),x)
[Out] int(x*polylog(n, d*(F^(c*(a + b*x)))^p), x)
```

3.159 $\int \text{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) \, dx$

Optimal. Leaf size=31

$$\frac{\text{PolyLog}\left(1+n, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}$$

```
[Out] polylog(1+n,d*(F^(c*(b*x+a)))^p)/b/c/p/ln(F)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2320, 6724}

$$\frac{\text{Li}_{n+1}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```

```
[Out] PolyLog[1 + n, d*(F^(c*(a + b*x)))^p]/(b*c*p*Log[F])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \text{Li}_n\left(d\left(F^{c(a+bx)}\right)^p\right) \, dx &= \frac{\text{Subst}\left(\int \frac{\text{Li}_n(dx^p)}{x} \, dx, x, F^{c(a+bx)}\right)}{bc\log(F)}\\ &= \frac{\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 1.00

$$\frac{\text{PolyLog}\left(1+n,d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
[Out] PolyLog[1 + n, d*(F^(c*(a + b*x)))^p]/(b*c*p*Log[F])
```

Maple [A]
time = 0.56, size = 32, normalized size = 1.03

method	result	size
derivativedivides	$\frac{\text{polylog}\left(1+n,d\left(F^{c(bx+a)}\right)^p\right)}{bcp\ln(F)}$	32
default	$\frac{\text{polylog}\left(1+n,d\left(F^{c(bx+a)}\right)^p\right)}{bcp\ln(F)}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n,d*(F^(c*(b*x+a)))^p),x,method=_RETURNVERBOSE)
[Out] polylog(1+n,d*(F^(c*(b*x+a)))^p)/b/c/p/ln(F)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="maxima")
[Out] integrate(polylog(n, F^((b*x + a)*c*p)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="fricas")
[Out] integral(polylog(n, (F^(b*c*x + a*c))^p*d), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{Li}_n\left(d\left(F^{c(a+bx)}\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,d*(F**(c*(b*x+a)))**p),x)
```

```
[Out] Integral(polylog(n, d*(F**(c*(a + b*x)))**p), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="giac")
```

```
[Out] integrate(polylog(n, (F^((b*x + a)*c))^p*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \operatorname{polylog}\left(n,d\,{\left(F^{c\,(a+b\,x)}\right)}^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n, d*(F^(c*(a + b*x)))^p),x)
```

```
[Out] int(polylog(n, d*(F^(c*(a + b*x)))^p), x)
```

3.160 $\int \frac{\text{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)}{x} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\text{PolyLog}\left(n,d\left(F^{ac+bcx}\right)^p\right)}{x},x\right)$$

```
[Out] CannotIntegrate(polylog(n,d*(F^(b*c*x+a*c))^p)/x,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{Li}_n\left(d\left(F^{c(a+bx)}\right)^p\right)}{x} dx$$

Verification is not applicable to the result.

```
[In] Int[PolyLog[n, d*(F^(c*(a + b*x)))^p]/x,x]
[Out] Defer[Int][PolyLog[n, d*(F^(a*c + b*c*x))^p]/x, x]
```

Rubi steps

$$\int \frac{\text{Li}_n\left(d\left(F^{c(a+bx)}\right)^p\right)}{x} dx = \int \frac{\text{Li}_n\left(d\left(F^{ac+bcx}\right)^p\right)}{x} dx$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)}{x} dx$$

Verification is not applicable to the result.

```
[In] Integrate[PolyLog[n, d*(F^(c*(a + b*x)))^p]/x,x]
[Out] Integrate[PolyLog[n, d*(F^(c*(a + b*x)))^p]/x, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{polylog}\left(n,d\left(F^{c(bx+a)}\right)^p\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n,d*(F^(c*(b*x+a)))^p)/x,x)
```

```
[Out] int(polylog(n,d*(F^(c*(b*x+a)))^p)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,d*(F^(c*(b*x+a)))^p)/x,x, algorithm="maxima")
```

```
[Out] integrate(polylog(n, F^((b*x + a)*c*p)*d)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,d*(F^(c*(b*x+a)))^p)/x,x, algorithm="fricas")
```

```
[Out] integral(polylog(n, (F^(b*c*x + a*c))^p*d)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{Li}_{n}\left(d\left(F^{ac}F^{bcx}\right)^{p}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,d*(F**(c*(b*x+a)))**p)/x,x)
```

```
[Out] Integral(polylog(n, d*(F**(a*c)*F**(b*c*x))**p)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,d*(F^(c*(b*x+a)))^p)/x,x, algorithm="giac")
```

```
[Out] integrate(polylog(n, (F^((b*x + a)*c))^p*d)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{polylog}\left(n, d\left(F^{c\,(a+b\,x)}\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n, d*(F^(c*(a + b*x)))^p)/x,x)
```

```
[Out] int(polylog(n, d*(F^(c*(a + b*x)))^p)/x, x)
```

3.161 $\int x^3 \log(1-cx)\textbf{PolyLog}(2,cx)\,dx$

Optimal. Leaf size=300

$$\frac{355x}{576c^3}+\frac{139x^2}{1152c^2}+\frac{67x^3}{1728c}+\frac{3x^4}{256}+\frac{139\log(1-cx)}{576c^4}-\frac{x^2\log(1-cx)}{8c^2}-\frac{5x^3\log(1-cx)}{72c}-\frac{3}{64}x^4\log(1-cx)+\frac{3(1-cx}{}$$

```
[Out] 355/576*x/c^3+139/1152*x^2/c^2+67/1728*x^3/c+3/256*x^4+139/576*ln(-c*x+1)/c
^4-1/8*x^2*ln(-c*x+1)/c^2-5/72*x^3*ln(-c*x+1)/c-3/64*x^4*ln(-c*x+1)+3/8*(-c
*x+1)*ln(-c*x+1)/c^4-1/16*ln(-c*x+1)^2/c^4+1/16*x^4*ln(-c*x+1)^2-1/4*ln(c*x
)*ln(-c*x+1)^2/c^4-1/4*x*polylog(2,c*x)/c^3-1/8*x^2*polylog(2,c*x)/c^2-1/12
*x^3*polylog(2,c*x)/c-1/16*x^4*polylog(2,c*x)-1/4*ln(-c*x+1)*polylog(2,c*x)
/c^4+1/4*x^4*ln(-c*x+1)*polylog(2,c*x)-1/2*ln(-c*x+1)*polylog(2,-c*x+1)/c^4
+1/2*polylog(3,-c*x+1)/c^4
```

Rubi [A]
time = 0.37, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 16, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6726, 2442, 45, 6738, 2445, 2457, 2436, 2332, 2437, 2338, 6721, 6731, 2443, 2481, 2421, 6724}

$$\frac{\text{Li}_3(1-cx)}{2c^4}-\frac{\text{Li}_2(cx)\log(1-cx)}{4c^4}-\frac{\text{Li}_2(1-cx)\log(1-cx)}{2c^4}-\frac{\log(cx)\log^2(1-cx)}{4c^4}-\frac{\log^2(1-cx)}{16c^4}+\frac{3(1-cx)\log(1-cx)}{8c^4}+\frac{139\log(1-cx)}{576c^4}-\frac{x\text{Li}_2(cx)}{4c^3}+\frac{355x}{576c^3}-\frac{x^2\text{Li}_2(cx)}{8c^2}+\frac{139x^2}{1152c^2}-\frac{x^2\log(1-cx)}{8c^2}-\frac{1}{16}x^4\text{Li}_2(cx)+\frac{1}{4}x^4\text{Li}_2(cx)\log(1-cx)-\frac{x^3\text{Li}_2(cx)}{12c}+\frac{1}{16}x^4\log^2(1-cx)-\frac{3}{64}x^4\log(1-cx)+\frac{67x^3}{1728c}-\frac{5x^3\log(1-cx)}{72c}+\frac{3x^4}{256}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[1 - c*x]*PolyLog[2, c*x],x]
```

```
[Out] (355*x)/(576*c^3) + (139*x^2)/(1152*c^2) + (67*x^3)/(1728*c) + (3*x^4)/256
+ (139*Log[1 - c*x])/(576*c^4) - (x^2*Log[1 - c*x])/(8*c^2) - (5*x^3*Log[1
- c*x])/(72*c) - (3*x^4*Log[1 - c*x])/64 + (3*(1 - c*x)*Log[1 - c*x])/(8*c^
4) - Log[1 - c*x]^2/(16*c^4) + (x^4*Log[1 - c*x]^2)/16 - (Log[c*x]*Log[1 -
c*x]^2)/(4*c^4) - (x*PolyLog[2, c*x])/(4*c^3) - (x^2*PolyLog[2, c*x])/(8*c^
2) - (x^3*PolyLog[2, c*x])/(12*c) - (x^4*PolyLog[2, c*x])/16 - (Log[1 - c*x
]*PolyLog[2, c*x])/(4*c^4) + (x^4*Log[1 - c*x]*PolyLog[2, c*x])/4 - (Log[1
- c*x]*PolyLog[2, 1 - c*x])/(2*c^4) + PolyLog[3, 1 - c*x]/(2*c^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
```

```
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2457

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6738

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[e*h*(n/(m + 1)), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\log(1-cx)\text{Li}_2(cx)\,dx &= \frac{1}{4}x^4\log(1-cx)\text{Li}_2(cx)+\frac{1}{4}\int x^3\log^2(1-cx)\,dx+\frac{1}{4}c\int\left(-\frac{\text{Li}_2(cx)}{c^4}-\frac{x\text{Li}_2(\right.\\ &= \frac{1}{16}x^4\log^2(1-cx)+\frac{1}{4}x^4\log(1-cx)\text{Li}_2(cx)-\frac{1}{4}\int x^3\text{Li}_2(cx)\,dx-\frac{\int\text{Li}_2(cx)}{4c^3}\\ &= \frac{1}{16}x^4\log^2(1-cx)-\frac{x\text{Li}_2(cx)}{4c^3}-\frac{x^2\text{Li}_2(cx)}{8c^2}-\frac{x^3\text{Li}_2(cx)}{12c}-\frac{1}{16}x^4\text{Li}_2(cx)-\frac{\log}{}\\ &= -\frac{x^2\log(1-cx)}{16c^2}-\frac{x^3\log(1-cx)}{36c}-\frac{1}{64}x^4\log(1-cx)+\frac{1}{16}x^4\log^2(1-cx)-\\ &= \frac{x}{4c^3}-\frac{x^2\log(1-cx)}{8c^2}-\frac{5x^3\log(1-cx)}{72c}-\frac{3}{64}x^4\log(1-cx)+\frac{(1-cx)\log(1}{4c^4}\\ &= \frac{277x}{576c^3}+\frac{61x^2}{1152c^2}+\frac{25x^3}{1728c}+\frac{x^4}{256}+\frac{61\log(1-cx)}{576c^4}-\frac{x^2\log(1-cx)}{8c^2}-\frac{5x^3\log}{7}\\ &= \frac{355x}{576c^3}+\frac{139x^2}{1152c^2}+\frac{67x^3}{1728c}+\frac{3x^4}{256}+\frac{139\log(1-cx)}{576c^4}-\frac{x^2\log(1-cx)}{8c^2}-\frac{5x^3\log}{}\end{aligned}$$

Mathematica [A]
time = 0.42, size = 223, normalized size = 0.74

$$\frac{4260cx+834c^2x^2+268c^3x^3+81c^4x^4+4260\log(1-cx)-2592cx\log(1-cx)-864c^2x^2\log(1-cx)-480c^3x^3\log(1-cx)-324c^4x^4\log(1-cx)-432\log^2(1-cx)+432c^4x^4\log^2(1-cx)-1728\log(cx)\log^2(1-cx)+144(-cx(12+6cx+4c^2x^2+3c^3x^3)+12(-1+c^4x^4)\log(1-cx))\text{PolyLog}(2,cx)-3456\log(1-cx)\text{PolyLog}(2,1-cx)+3456\text{PolyLog}(3,1-cx)}{6912c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[1 - c*x]*PolyLog[2, c*x],x]
```

```
[Out] (4260*c*x + 834*c^2*x^2 + 268*c^3*x^3 + 81*c^4*x^4 + 4260*Log[1 - c*x] - 25
92*c*x*Log[1 - c*x] - 864*c^2*x^2*Log[1 - c*x] - 480*c^3*x^3*Log[1 - c*x] -
 324*c^4*x^4*Log[1 - c*x] - 432*Log[1 - c*x]^2 + 432*c^4*x^4*Log[1 - c*x]^2
 - 1728*Log[c*x]*Log[1 - c*x]^2 + 144*(-(c*x*(12 + 6*c*x + 4*c^2*x^2 + 3*c^
3*x^3)) + 12*(-1 + c^4*x^4)*Log[1 - c*x])*PolyLog[2, c*x] - 3456*Log[1 - c*
x]*PolyLog[2, 1 - c*x] + 3456*PolyLog[3, 1 - c*x])/(6912*c^4)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^{3} \ln \left(-c x +1\right) \operatorname{polylog}\left(2, c x \right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] int(x^3*ln(-c*x+1)*polylog(2,c*x),x)
```

Maxima [A]
time = 0.29, size = 376, normalized size = 1.25

$$\frac{9\,c^{4}\left(\frac{3\,c^{3}x^{4}+4\,c^{2}x^{3}+6\,cx^{2}+12\,x}{c^{4}}+\frac{12\,\log\left(cx-1\right)}{c^{5}}\right)+24\,c^{3}\left(\frac{2\,c^{2}x^{3}+3\,cx^{2}+6\,x}{c^{3}}+\frac{6\,\log\left(cx-1\right)}{c^{4}}\right)+108\,c^{2}\left(\frac{cx^{2}+2\,x}{c^{2}}+\frac{2\,\log\left(cx-1\right)}{c^{3}}\right)+432\,c\left(\frac{x}{c}+\frac{\log\left(cx-1\right)}{c^{2}}\right)+\frac{2\,\left(27\,c^{4}x^{4}+92\,c^{3}x^{3}+300\,c^{2}x^{2}+1680\,cx-72\,\left(3\,c^{4}x^{4}+4\,c^{3}x^{3}+6\,c^{2}x^{2}+12\,cx+12\,\log\left(-cx+1\right)\right){\rm Li}_2\left(cx\right)-12\,\left(9\,c^{4}x^{4}+14\,c^{3}x^{3}+27\,c^{2}x^{2}+90\,cx-140\right)\log\left(-cx+1\right)\right)}{c}-\frac{1728\,\left(\log\left(cx\right)\log\left(-cx+1\right)^{2}+2\,{\rm Li}_2\left(-cx+1\right)\log\left(-cx+1\right)-2\,{\rm Li}_3\left(-cx+1\right)\right)}{c}}{6912\,c^{3}}+\frac{{\left(48\,c^{4}x^{4}{\rm Li}_2\left(cx\right)-3\,c^{4}x^{4}-4\,c^{3}x^{3}-6\,c^{2}x^{2}-12\,cx+12\,{\left(c^{4}x^{4}-1\right)}\log\left(-cx+1\right)\right)}\log\left(-cx+1\right)}{192\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
```

```
[Out] 1/6912*(9*c^4*((3*c^3*x^4 + 4*c^2*x^3 + 6*c*x^2 + 12*x)/c^4 + 12*log(c*x -
1)/c^5) + 24*c^3*((2*c^2*x^3 + 3*c*x^2 + 6*x)/c^3 + 6*log(c*x - 1)/c^4) + 1
08*c^2*((c*x^2 + 2*x)/c^2 + 2*log(c*x - 1)/c^3) + 432*c*(x/c + log(c*x - 1)
/c^2) + 2*(27*c^4*x^4 + 92*c^3*x^3 + 300*c^2*x^2 + 1680*c*x - 72*(3*c^4*x^4
 + 4*c^3*x^3 + 6*c^2*x^2 + 12*c*x + 12*log(-c*x + 1))*dilog(c*x) - 12*(9*c^
4*x^4 + 14*c^3*x^3 + 27*c^2*x^2 + 90*c*x - 140)*log(-c*x + 1))/c - 1728*(lo
g(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylog(3, -c*
x + 1))/c)/c^3 + 1/192*(48*c^4*x^4*dilog(c*x) - 3*c^4*x^4 - 4*c^3*x^3 - 6*c
^2*x^2 - 12*c*x + 12*(c^4*x^4 - 1)*log(-c*x + 1))*log(-c*x + 1)/c^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral(x^3*dilog(c*x)*log(-c*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3} \log{\left(- c x + 1 \right)} \operatorname{Li}_{2}\left(c x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] Integral(x**3*log(-c*x + 1)*polylog(2, c*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*dilog(c*x)*log(-c*x + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\ln\left(1-c\,x\right)\,\mathrm{polylog}(2,c\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(1 - c*x)*polylog(2, c*x),x)
```

```
[Out] int(x^3*log(1 - c*x)*polylog(2, c*x), x)
```

3.162 $\int x^2 \log(1-cx)\mathbf{PolyLog}(2,cx)\,dx$

Optimal. Leaf size=258

$$\frac{31x}{36c^2}+\frac{11x^2}{72c}+\frac{x^3}{27}+\frac{11\log(1-cx)}{36c^3}-\frac{7x^2\log(1-cx)}{36c}-\frac{1}{9}x^3\log(1-cx)+\frac{5(1-cx)\log(1-cx)}{9c^3}-\frac{\log^2(1-cx)}{9c^3}+\frac{1}{9}$$

```
[Out] 31/36*x/c^2+11/72*x^2/c+1/27*x^3+11/36*ln(-c*x+1)/c^3-7/36*x^2*ln(-c*x+1)/c
-1/9*x^3*ln(-c*x+1)+5/9*(-c*x+1)*ln(-c*x+1)/c^3-1/9*ln(-c*x+1)^2/c^3+1/9*x^
3*ln(-c*x+1)^2-1/3*ln(c*x)*ln(-c*x+1)^2/c^3-1/3*x*polylog(2,c*x)/c^2-1/6*x^
2*polylog(2,c*x)/c-1/9*x^3*polylog(2,c*x)-1/3*ln(-c*x+1)*polylog(2,c*x)/c^3
+1/3*x^3*ln(-c*x+1)*polylog(2,c*x)-2/3*ln(-c*x+1)*polylog(2,-c*x+1)/c^3+2/3
*polylog(3,-c*x+1)/c^3
```

Rubi [A]
time = 0.29, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 16, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6726, 2442, 45, 6738, 2445, 2457, 2436, 2332, 2437, 2338, 6721, 6731, 2443, 2481, 2421, 6724}

$$\frac{2\text{Li}_3(1-cx)}{3c^3}-\frac{\text{Li}_2(cx)\log(1-cx)}{3c^3}-\frac{2\text{Li}_2(1-cx)\log(1-cx)}{3c^3}-\frac{\log(cx)\log^2(1-cx)}{3c^3}-\frac{\log^2(1-cx)}{9c^3}+\frac{5(1-cx)\log(1-cx)}{9c^3}+\frac{11\log(1-cx)}{36c^3}-\frac{x\text{Li}_2(cx)}{3c^2}+\frac{31x}{36c^2}-\frac{1}{9}x^3\text{Li}_2(cx)+\frac{1}{3}x^3\text{Li}_2(cx)\log(1-cx)-\frac{x^2\text{Li}_2(cx)}{6c}+\frac{1}{9}x^3\log^2(1-cx)-\frac{1}{9}x^3\log(1-cx)+\frac{11x^2}{72c}-\frac{7x^2\log(1-cx)}{36c}+\frac{x^3}{27}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[1 - c*x]*PolyLog[2, c*x],x]
```

```
[Out] (31*x)/(36*c^2) + (11*x^2)/(72*c) + x^3/27 + (11*Log[1 - c*x])/(36*c^3) - (
7*x^2*Log[1 - c*x])/(36*c) - (x^3*Log[1 - c*x])/9 + (5*(1 - c*x)*Log[1 - c*
x])/(9*c^3) - Log[1 - c*x]^2/(9*c^3) + (x^3*Log[1 - c*x]^2)/9 - (Log[c*x]*L
og[1 - c*x]^2)/(3*c^3) - (x*PolyLog[2, c*x])/(3*c^2) - (x^2*PolyLog[2, c*x]
)/(6*c) - (x^3*PolyLog[2, c*x])/9 - (Log[1 - c*x]*PolyLog[2, c*x])/(3*c^3)
+ (x^3*Log[1 - c*x]*PolyLog[2, c*x])/3 - (2*Log[1 - c*x]*PolyLog[2, 1 - c*x
])/(3*c^3) + (2*PolyLog[3, 1 - c*x])/(3*c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2457

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6738

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
```

```
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[e*h*(n/(m + 1)), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$\int x^2 \log(1-cx)\text{Li}_2(cx)\,dx = \frac{1}{3}x^3\log(1-cx)\text{Li}_2(cx) + \frac{1}{3}\int x^2\log^2(1-cx)\,dx + \frac{1}{3}c\int\left(-\frac{\text{Li}_2(cx)}{c^3} - \frac{x\text{Li}_2(\right.$$

$$= \frac{1}{9}x^3\log^2(1-cx) + \frac{1}{3}x^3\log(1-cx)\text{Li}_2(cx) - \frac{1}{3}\int x^2\text{Li}_2(cx)\,dx - \frac{\int \text{Li}_2(cx)\,d}{3c^2}$$

$$= \frac{1}{9}x^3\log^2(1-cx) - \frac{x\text{Li}_2(cx)}{3c^2} - \frac{x^2\text{Li}_2(cx)}{6c} - \frac{1}{9}x^3\text{Li}_2(cx) - \frac{\log(1-cx)\text{Li}_2(cx}{3c^3}$$

$$= -\frac{x^2\log(1-cx)}{12c} - \frac{1}{27}x^3\log(1-cx) + \frac{1}{9}x^3\log^2(1-cx) - \frac{\log(cx)\log^2(1-c}{3c^3}$$

$$= \frac{x}{3c^2} - \frac{7x^2\log(1-cx)}{36c} - \frac{1}{9}x^3\log(1-cx) + \frac{(1-cx)\log(1-cx)}{3c^3} + \frac{1}{9}x^3\log^2($$

$$= \frac{73x}{108c^2} + \frac{13x^2}{216c} + \frac{x^3}{81} + \frac{13\log(1-cx)}{108c^3} - \frac{7x^2\log(1-cx)}{36c} - \frac{1}{9}x^3\log(1-cx) +$$

$$= \frac{31x}{36c^2} + \frac{11x^2}{72c} + \frac{x^3}{27} + \frac{11\log(1-cx)}{36c^3} - \frac{7x^2\log(1-cx)}{36c} - \frac{1}{9}x^3\log(1-cx) +$$

Mathematica [A]
time = 0.25, size = 192, normalized size = 0.74

$$\frac{186cx + 33c^2x^2 + 8c^3x^3 + 186\log(1-cx) - 120cx\log(1-cx) - 42c^2x^2\log(1-cx) - 24c^3x^3\log(1-cx) - 24\log^2(1-cx) + 24c^3x^3\log^2(1-cx) - 72\log(cx)\log^2(1-cx) + 12(-cx(6+3cx+2c^2x^2) + 6(-1+c^3x^3)\log(1-cx))\text{PolyLog}(2,cx) - 144\log(1-cx)\text{PolyLog}(2,1-cx) + 144\text{PolyLog}(3,1-cx)}{216c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[1 - c*x]*PolyLog[2, c*x],x]
```

```
[Out] (186*c*x + 33*c^2*x^2 + 8*c^3*x^3 + 186*Log[1 - c*x] - 120*c*x*Log[1 - c*x]
 - 42*c^2*x^2*Log[1 - c*x] - 24*c^3*x^3*Log[1 - c*x] - 24*Log[1 - c*x]^2 +
24*c^3*x^3*Log[1 - c*x]^2 - 72*Log[c*x]*Log[1 - c*x]^2 + 12*(-(c*x*(6 + 3*c
*x + 2*c^2*x^2)) + 6*(-1 + c^3*x^3)*Log[1 - c*x])*PolyLog[2, c*x] - 144*Log
[1 - c*x]*PolyLog[2, 1 - c*x] + 144*PolyLog[3, 1 - c*x])/(216*c^3)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^2 \ln(-cx+1)\,\text{polylog}(2, cx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] int(x^2*ln(-c*x+1)*polylog(2,c*x),x)
```

Maxima [A]
time = 0.28, size = 296, normalized size = 1.15

$$\frac{4\,c^3\left(\frac{2\,c^2x^3+3\,cx^2+6\,x}{c^3}+\frac{6\,\log(cx-1)}{c^4}\right)+18\,c^2\left(\frac{cx^2+2\,x}{c^2}+\frac{2\,\log(cx-1)}{c^3}\right)+72\,c\left(\frac{x}{c}+\frac{\log(cx-1)}{c^2}\right)+\frac{16\,c^3x^3+69\,c^2x^2+426\,cx-36\,(2\,c^3x^3+3\,c^2x^2+6\,cx+6\,\log(-cx+1))\mathrm{Li}_2(cx)-6\,(8\,c^3x^3+15\,c^2x^2+48\,cx-71)\log(-cx+1)}{c}-\frac{216\left(\log(cx)\log(-cx+1)^2+2\,\mathrm{Li}_2(-cx+1)\log(-cx+1)-2\,\mathrm{Li}_3(-cx+1)\right)}{c}}{648\,c^2}+\frac{(18\,c^3x^3\mathrm{Li}_2(cx)-2\,c^3x^3-3\,c^2x^2-6\,cx+6\,(c^3x^3-1)\log(-cx+1))\log(-cx+1)}{54\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
```

```
[Out] 1/648*(4*c^3*((2*c^2*x^3 + 3*c*x^2 + 6*x)/c^3 + 6*log(c*x - 1)/c^4) + 18*c^
2*((c*x^2 + 2*x)/c^2 + 2*log(c*x - 1)/c^3) + 72*c*(x/c + log(c*x - 1)/c^2)
+ (16*c^3*x^3 + 69*c^2*x^2 + 426*c*x - 36*(2*c^3*x^3 + 3*c^2*x^2 + 6*c*x +
6*log(-c*x + 1))*dilog(c*x) - 6*(8*c^3*x^3 + 15*c^2*x^2 + 48*c*x - 71)*log(
-c*x + 1))/c - 216*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x +
 1) - 2*polylog(3, -c*x + 1))/c)/c^2 + 1/54*(18*c^3*x^3*dilog(c*x) - 2*c^3*
x^3 - 3*c^2*x^2 - 6*c*x + 6*(c^3*x^3 - 1)*log(-c*x + 1))*log(-c*x + 1)/c^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral(x^2*dilog(c*x)*log(-c*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \log\left(-cx+1\right) \mathrm{Li}_2\left(cx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] Integral(x**2*log(-c*x + 1)*polylog(2, c*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*dilog(c*x)*log(-c*x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \ln(1-c x) \operatorname{polylog}(2, c x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(1 - c*x)*polylog(2, c*x),x)
```

```
[Out] int(x^2*log(1 - c*x)*polylog(2, c*x), x)
```

3.163 $\int x \log(1-cx)\textbf{PolyLog}(2,cx)\,dx$

Optimal. Leaf size=262

$$\frac{13x}{8c}+\frac{x^2}{16}+\frac{(1-cx)^2}{8c^2}+\frac{\log(1-cx)}{8c^2}-\frac{1}{8}x^2\log(1-cx)+\frac{3(1-cx)\log(1-cx)}{2c^2}-\frac{(1-cx)^2\log(1-cx)}{4c^2}-\frac{(1-cx)\log}{2}$$

```
[Out] 13/8*x/c+1/16*x^2+1/8*(-c*x+1)^2/c^2+1/8*ln(-c*x+1)/c^2-1/8*x^2*ln(-c*x+1)+
3/2*(-c*x+1)*ln(-c*x+1)/c^2-1/4*(-c*x+1)^2*ln(-c*x+1)/c^2-1/2*(-c*x+1)*ln(-
c*x+1)^2/c^2+1/4*(-c*x+1)^2*ln(-c*x+1)^2/c^2-1/2*ln(c*x)*ln(-c*x+1)^2/c^2-1
/2*x*polylog(2,c*x)/c-1/4*x^2*polylog(2,c*x)-1/2*ln(-c*x+1)*polylog(2,c*x)/
c^2+1/2*x^2*ln(-c*x+1)*polylog(2,c*x)-ln(-c*x+1)*polylog(2,-c*x+1)/c^2+poly
log(3,-c*x+1)/c^2
```

Rubi [A]
time = 0.19, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 17, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.214, Rules used = {6726, 2442, 45, 6738, 2448, 2436, 2333, 2332, 2437, 2342, 2341, 6721, 6731, 2443, 2481, 2421, 6724}

$$\frac{\text{Li}_3(1-cx)}{c^2}-\frac{\text{Li}_2(cx)\log(1-cx)}{2c^2}-\frac{\text{Li}_2(1-cx)\log(1-cx)}{c^2}+\frac{(1-cx)^2}{8c^2}+\frac{(1-cx)^2\log^2(1-cx)}{4c^2}-\frac{(1-cx)\log^2(1-cx)}{2c^2}-\frac{\log(cx)\log^2(1-cx)}{2c^2}-\frac{(1-cx)^2\log(1-cx)}{4c^2}+\frac{3(1-cx)\log(1-cx)}{2c^2}+\frac{\log(1-cx)}{8c^2}-\frac{1}{4}x^2\text{Li}_2(cx)+\frac{1}{2}x^2\text{Li}_2(cx)\log(1-cx)-\frac{x\text{Li}_2(cx)}{2c}-\frac{1}{8}x^2\log(1-cx)+\frac{13x}{8c}+\frac{x^2}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[1 - c*x]*PolyLog[2, c*x],x]
```

```
[Out] (13*x)/(8*c) + x^2/16 + (1 - c*x)^2/(8*c^2) + Log[1 - c*x]/(8*c^2) - (x^2*L
og[1 - c*x])/8 + (3*(1 - c*x)*Log[1 - c*x])/(2*c^2) - ((1 - c*x)^2*Log[1 -
c*x])/(4*c^2) - ((1 - c*x)*Log[1 - c*x]^2)/(2*c^2) + ((1 - c*x)^2*Log[1 - c
*x]^2)/(4*c^2) - (Log[c*x]*Log[1 - c*x]^2)/(2*c^2) - (x*PolyLog[2, c*x])/(2
*c) - (x^2*PolyLog[2, c*x])/4 - (Log[1 - c*x]*PolyLog[2, c*x])/(2*c^2) + (x
^2*Log[1 - c*x]*PolyLog[2, c*x])/2 - (Log[1 - c*x]*PolyLog[2, 1 - c*x])/c^2
 + PolyLog[3, 1 - c*x]/c^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6738

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[e*h*(n/(m + 1)), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$\int x\log(1-cx)\text{Li}_2(cx)\,dx = \frac{1}{2}x^2\log(1-cx)\text{Li}_2(cx) + \frac{1}{2}\int x\log^2(1-cx)\,dx + \frac{1}{2}c\int\left(-\frac{\text{Li}_2(cx)}{c^2} - \frac{x\text{Li}_2(c\cdots}{c}\right.$$

$$= \frac{1}{2}x^2\log(1-cx)\text{Li}_2(cx) + \frac{1}{2}\int\left(\frac{\log^2(1-cx)}{c} - \frac{(1-cx)\log^2(1-cx)}{c}\right)dx - \cdots$$

$$= -\frac{x\text{Li}_2(cx)}{2c} - \frac{1}{4}x^2\text{Li}_2(cx) - \frac{\log(1-cx)\text{Li}_2(cx)}{2c^2} + \frac{1}{2}x^2\log(1-cx)\text{Li}_2(cx) - \frac{1}{4}\cdots$$

$$= -\frac{1}{8}x^2\log(1-cx) - \frac{\log(cx)\log^2(1-cx)}{2c^2} - \frac{x\text{Li}_2(cx)}{2c} - \frac{1}{4}x^2\text{Li}_2(cx) - \frac{\log(1-\cdots}{}$$

$$= \frac{x}{2c} - \frac{1}{8}x^2\log(1-cx) + \frac{(1-cx)\log(1-cx)}{2c^2} - \frac{(1-cx)\log^2(1-cx)}{2c^2} + \frac{(1-\cdots}{}$$

$$= \frac{13x}{8c} + \frac{x^2}{16} + \frac{(1-cx)^2}{8c^2} + \frac{\log(1-cx)}{8c^2} - \frac{1}{8}x^2\log(1-cx) + \frac{3(1-cx)\log(1-c\cdots}{2c^2}$$

$$= \frac{13x}{8c} + \frac{x^2}{16} + \frac{(1-cx)^2}{8c^2} + \frac{\log(1-cx)}{8c^2} - \frac{1}{8}x^2\log(1-cx) + \frac{3(1-cx)\log(1-c\cdots}{2c^2}$$

Mathematica [A]
time = 0.21, size = 160, normalized size = 0.61

$$\frac{-14+22cx+3c^2x^2+22\log(1-cx)-16cx\log(1-cx)-6c^2x^2\log(1-cx)-4\log^2(1-cx)+4c^2x^2\log^2(1-cx)-8\log(cx)\log^2(1-cx)+\left(-4cx(2+cx)+8\left(-1+c^2x^2\right)\log(1-cx)\right)\text{PolyLog}(2,cx)-16\log(1-cx)\text{PolyLog}(2,1-cx)+16\text{PolyLog}(3,1-cx)}{16c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[1 - c*x]*PolyLog[2, c*x],x]
[Out] (-14 + 22*c*x + 3*c^2*x^2 + 22*Log[1 - c*x] - 16*c*x*Log[1 - c*x] - 6*c^2*x
^2*Log[1 - c*x] - 4*Log[1 - c*x]^2 + 4*c^2*x^2*Log[1 - c*x]^2 - 8*Log[c*x]*
```

```
Log[1 - c*x]^2 + (-4*c*x*(2 + c*x) + 8*(-1 + c^2*x^2)*Log[1 - c*x])*PolyLog
[2, c*x] - 16*Log[1 - c*x]*PolyLog[2, 1 - c*x] + 16*PolyLog[3, 1 - c*x])/(1
6*c^2)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x \ln \left(-cx+1\right) \operatorname{polylog}\left(2, cx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] int(x*ln(-c*x+1)*polylog(2,c*x),x)
```

Maxima [A]
time = 0.28, size = 222, normalized size = 0.85

$$\frac{c^2\left(\frac{cx^2+2x}{c^2}+\frac{2\log(cx-1)}{c^3}\right)+4c\left(\frac{x}{c}+\frac{\log(cx-1)}{c^2}\right)+\frac{2\left(c^2x^2+8cx-2\left(c^2x^2+2cx+2\log(-cx+1)\right)\mathrm{Li}_2(cx)-2\left(c^2x^2+3cx-4\right)\log(-cx+1)\right)}{c}-\frac{8\left(\log(cx)\log(-cx+1)^2+2\,\mathrm{Li}_2(-cx+1)\log(-cx+1)-2\,\mathrm{Li}_3(-cx+1)\right)}{c}}{16c}+\frac{\left(4c^2x^2\mathrm{Li}_2(cx)-c^2x^2-2cx+2\left(c^2x^2-1\right)\log\left(-cx+1\right)\right)\log\left(-cx+1\right)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
```

```
[Out] 1/16*(c^2*((c*x^2 + 2*x)/c^2 + 2*log(c*x - 1)/c^3) + 4*c*(x/c + log(c*x - 1
)/c^2) + 2*(c^2*x^2 + 8*c*x - 2*(c^2*x^2 + 2*c*x + 2*log(-c*x + 1))*dilog(c
*x) - 2*(c^2*x^2 + 3*c*x - 4)*log(-c*x + 1))/c - 8*(log(c*x)*log(-c*x + 1)^
2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylog(3, -c*x + 1))/c)/c + 1/8*(4
*c^2*x^2*dilog(c*x) - c^2*x^2 - 2*c*x + 2*(c^2*x^2 - 1)*log(-c*x + 1))*log(
-c*x + 1)/c^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral(x*dilog(c*x)*log(-c*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \log \left(-cx+1\right) \mathrm{Li}_2\left(cx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] Integral(x*log(-c*x + 1)*polylog(2, c*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate(x*dilog(c*x)*log(-c*x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(1-c\,x\right)\,\mathrm{polylog}\left(2,c\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(1 - c*x)*polylog(2, c*x),x)
```

```
[Out] int(x*log(1 - c*x)*polylog(2, c*x), x)
```

3.164 $\int \log(1-cx)\textbf{PolyLog}(2,cx)\,dx$

Optimal. Leaf size=132

$$3x+\frac{3(1-cx)\log(1-cx)}{c}-\frac{(1-cx)\log^2(1-cx)}{c}-\frac{\log(cx)\log^2(1-cx)}{c}-x\text{PolyLog}(2,cx)-\frac{\log(1-cx)\text{PolyI}}{c}$$

```
[Out] 3*x+3*(-c*x+1)*ln(-c*x+1)/c-(-c*x+1)*ln(-c*x+1)^2/c-ln(c*x)*ln(-c*x+1)^2/c-
x*polylog(2,c*x)-ln(-c*x+1)*polylog(2,c*x)/c+x*ln(-c*x+1)*polylog(2,c*x)-2*
ln(-c*x+1)*polylog(2,-c*x+1)/c+2*polylog(3,-c*x+1)/c
```

Rubi [A]
time = 0.14, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {6721, 2436, 2332, 6735, 2333, 6820, 6874, 6731, 2443, 2481, 2421, 6724}

$$-x\text{Li}_2(cx)+\frac{2\text{Li}_3(1-cx)}{c}+x\text{Li}_2(cx)\log(1-cx)-\frac{\text{Li}_2(cx)\log(1-cx)}{c}-\frac{2\text{Li}_2(1-cx)\log(1-cx)}{c}-\frac{(1-cx)\log^2(1-cx)}{c}-\frac{\log(cx)\log^2(1-cx)}{c}+\frac{3(1-cx)\log(1-cx)}{c}+3x$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 - c*x]*PolyLog[2, c*x],x]
```

```
[Out] 3*x + (3*(1 - c*x)*Log[1 - c*x])/c - ((1 - c*x)*Log[1 - c*x]^2)/c - (Log[c*
x]*Log[1 - c*x]^2)/c - x*PolyLog[2, c*x] - (Log[1 - c*x]*PolyLog[2, c*x])/c
 + x*Log[1 - c*x]*PolyLog[2, c*x] - (2*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c
+ (2*PolyLog[3, 1 - c*x])/c
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6735

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*PolyLog[2, (c_.)*
((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x*(g + h*Log[f*(d + e*x)^n])*PolyL
og[2, c*(a + b*x)], x] + (Dist[b, Int[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*
c - b*c*x]*ExpandIntegrand[x/(a + b*x), x], x], x] - Dist[e*h*n, Int[PolyLo
g[2, c*(a + b*x)]*ExpandIntegrand[x/(d + e*x), x], x], x]) /; FreeQ[{a, b,
```

```
c, d, e, f, g, h, n}, x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \log(1-cx)\mathrm{Li}_2(cx)\,dx &= x\log(1-cx)\mathrm{Li}_2(cx)+c\int\left(-\frac{1}{c}-\frac{1}{c(-1+cx)}\right)\mathrm{Li}_2(cx)\,dx+\int\log^2(1-cx)\,dx\\
&= x\log(1-cx)\mathrm{Li}_2(cx)-\frac{\mathrm{Subst}\left(\int\log^2(x)\,dx,x,1-cx\right)}{c}+c\int\frac{x\mathrm{Li}_2(cx)}{1-cx}\,dx\\
&= -\frac{(1-cx)\log^2(1-cx)}{c}+x\log(1-cx)\mathrm{Li}_2(cx)+\frac{2\mathrm{Subst}\left(\int\log(x)\,dx,x,1-cx\right)}{c}-\\
&= 2x+\frac{2(1-cx)\log(1-cx)}{c}-\frac{(1-cx)\log^2(1-cx)}{c}+x\log(1-cx)\mathrm{Li}_2(cx)-\int \mathrm{l}\\
&= 2x+\frac{2(1-cx)\log(1-cx)}{c}-\frac{(1-cx)\log^2(1-cx)}{c}-x\mathrm{Li}_2(cx)-\frac{\log(1-cx)\mathrm{Li}_2(}{c}\\
&= 2x+\frac{2(1-cx)\log(1-cx)}{c}-\frac{(1-cx)\log^2(1-cx)}{c}-\frac{\log(cx)\log^2(1-cx)}{c}-x\mathrm{Li}\\
&= 3x+\frac{3(1-cx)\log(1-cx)}{c}-\frac{(1-cx)\log^2(1-cx)}{c}-\frac{\log(cx)\log^2(1-cx)}{c}-x\mathrm{Li}\\
&= 3x+\frac{3(1-cx)\log(1-cx)}{c}-\frac{(1-cx)\log^2(1-cx)}{c}-\frac{\log(cx)\log^2(1-cx)}{c}-x\mathrm{Li}\\
&= 3x+\frac{3(1-cx)\log(1-cx)}{c}-\frac{(1-cx)\log^2(1-cx)}{c}-\frac{\log(cx)\log^2(1-cx)}{c}-x\mathrm{Li}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 119, normalized size = 0.90

$$\frac{-2+3cx+3\log(1-cx)-3cx\log(1-cx)-\log^2(1-cx)+cx\log^2(1-cx)-\log(cx)\log^2(1-cx)+(-cx+(-1+cx)\log(1-cx))\mathrm{PolyLog}(2,cx)-2\log(1-cx)\mathrm{PolyLog}(2,1-cx)+2\mathrm{PolyLog}(3,1-cx)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 - c*x]*PolyLog[2, c*x],x]
```

```
[Out] (-2 + 3*c*x + 3*Log[1 - c*x] - 3*c*x*Log[1 - c*x] - Log[1 - c*x]^2 + c*x*Lo
g[1 - c*x]^2 - Log[c*x]*Log[1 - c*x]^2 + (-(c*x) + (-1 + c*x)*Log[1 - c*x])
*PolyLog[2, c*x] - 2*Log[1 - c*x]*PolyLog[2, 1 - c*x] + 2*PolyLog[3, 1 - c*
x])/c
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \ln \left(-cx+1\right) \operatorname{polylog}\left(2, cx\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] int(ln(-c*x+1)*polylog(2,c*x),x)
```

Maxima [A]
time = 0.30, size = 141, normalized size = 1.07

$$c\left(\frac{x}{c}+\frac{\log\left(cx-1\right)}{c^2}\right)+\frac{\left(cx\mathrm{Li}_2(cx)-cx+\left(cx-1\right)\log\left(-cx+1\right)\right)\log\left(-cx+1\right)}{c}-\frac{\log\left(cx\right)\log\left(-cx+1\right)^2+2\,\mathrm{Li}_2(-cx+1)\log\left(-cx+1\right)-2\,\mathrm{Li}_3(-cx+1)}{c}+\frac{2\,cx-\left(cx+\log\left(-cx+1\right)\right)\mathrm{Li}_2(cx)-2\left(cx-1\right)\log\left(-cx+1\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
```

```
[Out] c*(x/c + log(c*x - 1)/c^2) + (c*x*dilog(c*x) - c*x + (c*x - 1)*log(-c*x + 1
))*log(-c*x + 1)/c - (log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x
 + 1) - 2*polylog(3, -c*x + 1))/c + (2*c*x - (c*x + log(-c*x + 1))*dilog(c*
x) - 2*(c*x - 1)*log(-c*x + 1))/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral(dilog(c*x)*log(-c*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log \left(-cx+1\right) \operatorname{Li}_2\left(cx\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-c*x+1)*polylog(2,c*x),x)
[Out] Integral(log(-c*x + 1)*polylog(2, c*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
[Out] integrate(dilog(c*x)*log(-c*x + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(1-c\,x\right)\,\mathrm{polylog}(2,c\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1 - c*x)*polylog(2, c*x),x)
[Out] int(log(1 - c*x)*polylog(2, c*x), x)
```

3.165 $\int \frac{\log(1-cx)\text{PolyLog}(2,cx)}{x}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{2}\text{PolyLog}(2,cx)^2$$

```
[Out] -1/2*polylog(2,c*x)^2
```

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6724, 6736}

$$-\frac{1}{2}\text{Li}_2(cx)^2$$

Antiderivative was successfully verified.

```
[In] Int[(Log[1 - c*x]*PolyLog[2, c*x])/x,x]
```

```
[Out] -1/2*PolyLog[2, c*x]^2
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6736

```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> Simp[-P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

Rubi steps

$$\int \frac{\log(1-cx)\text{Li}_2(cx)}{x}\,dx = -\frac{1}{2}\text{Li}_2(cx)^2$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$-\frac{1}{2}\text{PolyLog}(2,cx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[1 - c*x]*PolyLog[2, c*x])/x,x]
[Out] -1/2*PolyLog[2, c*x]^2
```

Maple [A]
time = 0.06, size = 10, normalized size = 0.91

method	result	size
derivativedivides	$-\frac{\operatorname{polylog}(2,cx)^2}{2}$	10
default	$-\frac{\operatorname{polylog}(2,cx)^2}{2}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-c*x+1)*polylog(2,c*x)/x,x,method=_RETURNVERBOSE)
[Out] -1/2*polylog(2,c*x)^2
```

Maxima [A]
time = 0.25, size = 8, normalized size = 0.73

$$-\frac{1}{2}\operatorname{Li}_2(cx)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="maxima")
[Out] -1/2*dilog(c*x)^2
```

Fricas [A]
time = 0.36, size = 8, normalized size = 0.73

$$-\frac{1}{2}\operatorname{Li}_2(cx)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="fricas")
[Out] -1/2*dilog(c*x)^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(-cx+1)\operatorname{Li}_2(cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-c*x+1)*polylog(2,c*x)/x,x)
```

```
[Out] Integral(log(-c*x + 1)*polylog(2, c*x)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="giac")
[Out] integrate(dilog(c*x)*log(-c*x + 1)/x, x)
```

Mupad [B]
time = 0.24, size = 9, normalized size = 0.82

$$-\frac{\operatorname{polylog}(2,c\,x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(1 - c*x)*polylog(2, c*x))/x,x)
[Out] -polylog(2, c*x)^2/2
```

3.166 $\int \frac{\log(1-cx)\text{PolyLog}(2,cx)}{x^2}\,dx$

Optimal. Leaf size=111

$$\frac{(1-cx)\log^2(1-cx)}{x}+c\log(cx)\log^2(1-cx)-2c\text{PolyLog}(2,cx)+c\log(1-cx)\text{PolyLog}(2,cx)-\frac{\log(1-cx)\text{Poly}}{x}$$

```
[Out] (-c*x+1)*ln(-c*x+1)^2/x+c*ln(c*x)*ln(-c*x+1)^2-2*c*polylog(2,c*x)+c*ln(-c*x
+1)*polylog(2,c*x)-ln(-c*x+1)*polylog(2,c*x)/x+2*c*ln(-c*x+1)*polylog(2,-c*
x+1)-c*polylog(3,c*x)-2*c*polylog(3,-c*x+1)
```

Rubi [A]
time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {6726, 2442, 36, 29, 31, 6738, 2444, 2438, 6724, 6731, 2443, 2481, 2421}

$$-2c\text{Li}_2(cx)-c\text{Li}_3(cx)-2c\text{Li}_3(1-cx)+c\text{Li}_2(cx)\log(1-cx)-\frac{\text{Li}_2(cx)\log(1-cx)}{x}+2c\text{Li}_2(1-cx)\log(1-cx)+\frac{(1-cx)\log^2(1-cx)}{x}+c\log(cx)\log^2(1-cx)$$

Antiderivative was successfully verified.

```
[In] Int[(Log[1 - c*x]*PolyLog[2, c*x])/x^2,x]
```

```
[Out] ((1 - c*x)*Log[1 - c*x]^2)/x + c*Log[c*x]*Log[1 - c*x]^2 - 2*c*PolyLog[2, c
*x] + c*Log[1 - c*x]*PolyLog[2, c*x] - (Log[1 - c*x]*PolyLog[2, c*x])/x + 2
*c*Log[1 - c*x]*PolyLog[2, 1 - c*x] - c*PolyLog[3, c*x] - 2*c*PolyLog[3, 1
- c*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
```

```
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6738

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[e*h*(n/(m + 1)), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(1-cx)\text{Li}_2(cx)}{x^2}\,dx &= -\frac{\log(1-cx)\text{Li}_2(cx)}{x}-c\int\left(\frac{\text{Li}_2(cx)}{x}-\frac{c\text{Li}_2(cx)}{-1+cx}\right)dx-\int\frac{\log^2(1-cx)}{x^2}\,dx\\ &= \frac{(1-cx)\log^2(1-cx)}{x}-\frac{\log(1-cx)\text{Li}_2(cx)}{x}-c\int\frac{\text{Li}_2(cx)}{x}\,dx+(2c)\int\frac{\log(1-c}{x}\\ &= \frac{(1-cx)\log^2(1-cx)}{x}-2c\text{Li}_2(cx)+c\log(1-cx)\text{Li}_2(cx)-\frac{\log(1-cx)\text{Li}_2(cx)}{x}-\\ &= \frac{(1-cx)\log^2(1-cx)}{x}+c\log(cx)\log^2(1-cx)-2c\text{Li}_2(cx)+c\log(1-cx)\text{Li}_2(cx\\ &= \frac{(1-cx)\log^2(1-cx)}{x}+c\log(cx)\log^2(1-cx)-2c\text{Li}_2(cx)+c\log(1-cx)\text{Li}_2(cx\\ &= \frac{(1-cx)\log^2(1-cx)}{x}+c\log(cx)\log^2(1-cx)-2c\text{Li}_2(cx)+c\log(1-cx)\text{Li}_2(cx\\ &= \frac{(1-cx)\log^2(1-cx)}{x}+c\log(cx)\log^2(1-cx)-2c\text{Li}_2(cx)+c\log(1-cx)\text{Li}_2(cx\end{aligned}$$

Mathematica [A]
time = 0.10, size = 115, normalized size = 1.04

$$2c\log(cx)\log(1-cx)-c\log^2(1-cx)+\frac{\log^2(1-cx)}{x}+c\log(cx)\log^2(1-cx)+\frac{(-1+cx)\log(1-cx)\text{PolyLog}(2,cx)}{x}+2c(1+\log(1-cx))\text{PolyLog}(2,1-cx)-c\text{PolyLog}(3,cx)-2c\text{PolyLog}(3,1-cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[1 - c*x]*PolyLog[2, c*x])/x^2,x]
```

```
[Out] 2*c*Log[c*x]*Log[1 - c*x] - c*Log[1 - c*x]^2 + Log[1 - c*x]^2/x + c*Log[c*x
]*Log[1 - c*x]^2 + ((-1 + c*x)*Log[1 - c*x]*PolyLog[2, c*x])/x + 2*c*(1 + L
og[1 - c*x])*PolyLog[2, 1 - c*x] - c*PolyLog[3, c*x] - 2*c*PolyLog[3, 1 - c
*x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\ln(-cx+1)\,\text{polylog}(2,cx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-c*x+1)*polylog(2,c*x)/x^2,x)
```

```
[Out] int(ln(-c*x+1)*polylog(2,c*x)/x^2,x)
```

Maxima [A]
time = 0.31, size = 113, normalized size = 1.02

$$(\log(cx)\log(-cx+1)^2+2\,\text{Li}_2(-cx+1)\log(-cx+1)-2\,\text{Li}_3(-cx+1))c+2\,(\log(cx)\log(-cx+1)+\text{Li}_2(-cx+1))c-c\text{Li}_3(cx)+\frac{(cx-1)\text{Li}_2(cx)\log(-cx+1)-(cx-1)\log(-cx+1)^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^2,x, algorithm="maxima")
```

```
[Out] (log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylog(3,
-c*x + 1))*c + 2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*c - c*polylog(3
, c*x) + ((c*x - 1)*dilog(c*x)*log(-c*x + 1) - (c*x - 1)*log(-c*x + 1)^2)/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral(dilog(c*x)*log(-c*x + 1)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(-cx+1\right)\mathrm{Li}_{2}\left(cx\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-c*x+1)*polylog(2,c*x)/x**2,x)
```

```
[Out] Integral(log(-c*x + 1)*polylog(2, c*x)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(dilog(c*x)*log(-c*x + 1)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(1-c\,x\right)\,\mathrm{polylog}(2,c\,x)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(1 - c*x)*polylog(2, c*x))/x^2,x)
```

```
[Out] int((log(1 - c*x)*polylog(2, c*x))/x^2, x)
```

3.167 $\int \frac{\log(1-cx)\textbf{PolyLog}(2,cx)}{x^3}\,dx$

Optimal. Leaf size=191

$$-c^2\log(x)+c^2\log(1-cx)-\frac{c\log(1-cx)}{x}-\frac{1}{4}c^2\log^2(1-cx)+\frac{\log^2(1-cx)}{4x^2}+\frac{1}{2}c^2\log(cx)\log^2(1-cx)-\frac{1}{2}c^2\text{PolyI}$$

```
[Out] -c^2*ln(x)+c^2*ln(-c*x+1)-c*ln(-c*x+1)/x-1/4*c^2*ln(-c*x+1)^2+1/4*ln(-c*x+1
)^2/x^2+1/2*c^2*ln(c*x)*ln(-c*x+1)^2-1/2*c^2*polylog(2,c*x)+1/2*c*polylog(2
,c*x)/x+1/2*c^2*ln(-c*x+1)*polylog(2,c*x)-1/2*ln(-c*x+1)*polylog(2,c*x)/x^2
+c^2*ln(-c*x+1)*polylog(2,-c*x+1)-1/2*c^2*polylog(3,c*x)-c^2*polylog(3,-c*x
+1)
```

Rubi [A]
time = 0.19, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 17, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.062$, Rules used = {6726, 2442, 46, 6738, 2445, 2457, 36, 29, 31, 2438, 2437, 2338, 6724, 6731, 2443, 2481, 2421}

$$-\frac{1}{2}c^2\text{Li}_2(cx)-\frac{1}{2}c^2\text{Li}_3(cx)-c^2\text{Li}_3(1-cx)+\frac{1}{2}c^2\text{Li}_2(cx)\log(1-cx)+c^2\text{Li}_2(1-cx)\log(1-cx)+\frac{1}{2}c^2\log(cx)\log^2(1-cx)-\frac{1}{4}c^2\log^2(1-cx)-c^2\log(x)+c^2\log(1-cx)-\frac{\text{Li}_2(cx)\log(1-cx)}{2x^2}+\frac{c\text{Li}_2(cx)}{2x}+\frac{\log^2(1-cx)}{4x^2}-\frac{c\log(1-cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[1 - c*x]*PolyLog[2, c*x])/x^3,x]
```

```
[Out] -(c^2*Log[x]) + c^2*Log[1 - c*x] - (c*Log[1 - c*x])/x - (c^2*Log[1 - c*x]^2
)/4 + Log[1 - c*x]^2/(4*x^2) + (c^2*Log[c*x]*Log[1 - c*x]^2)/2 - (c^2*PolyL
og[2, c*x])/2 + (c*PolyLog[2, c*x])/(2*x) + (c^2*Log[1 - c*x]*PolyLog[2, c*
x])/2 - (Log[1 - c*x]*PolyLog[2, c*x])/(2*x^2) + c^2*Log[1 - c*x]*PolyLog[2
, 1 - c*x] - (c^2*PolyLog[3, c*x])/2 - c^2*PolyLog[3, 1 - c*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2457

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6738

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
```

```
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[e*h*(n/(m + 1)), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log(1-cx)\text{Li}_2(cx)}{x^3}\,dx &= -\frac{\log(1-cx)\text{Li}_2(cx)}{2x^2} - \frac{1}{2}\int \frac{\log^2(1-cx)}{x^3}\,dx - \frac{1}{2}c\int \left(\frac{\text{Li}_2(cx)}{x^2} + \frac{c\text{Li}_2(cx)}{x} - \frac{c^2\text{L}}{-1}\right. \\
&= \frac{\log^2(1-cx)}{4x^2} - \frac{\log(1-cx)\text{Li}_2(cx)}{2x^2} + \frac{1}{2}c\int \frac{\log(1-cx)}{x^2(1-cx)}\,dx - \frac{1}{2}c\int \frac{\text{Li}_2(cx)}{x^2}\,dx - \\
&= \frac{\log^2(1-cx)}{4x^2} + \frac{c\text{Li}_2(cx)}{2x} + \frac{1}{2}c^2\log(1-cx)\text{Li}_2(cx) - \frac{\log(1-cx)\text{Li}_2(cx)}{2x^2} - \frac{1}{2}c^2\text{Li} \\
&= -\frac{c\log(1-cx)}{2x} + \frac{\log^2(1-cx)}{4x^2} + \frac{1}{2}c^2\log(cx)\log^2(1-cx) + \frac{c\text{Li}_2(cx)}{2x} + \frac{1}{2}c^2\log(\\
&= -\frac{c\log(1-cx)}{x} + \frac{\log^2(1-cx)}{4x^2} + \frac{1}{2}c^2\log(cx)\log^2(1-cx) - \frac{1}{2}c^2\text{Li}_2(cx) + \frac{c\text{Li}_2(c}{2x} \\
&= -\frac{1}{2}c^2\log(x) + \frac{1}{2}c^2\log(1-cx) - \frac{c\log(1-cx)}{x} - \frac{1}{4}c^2\log^2(1-cx) + \frac{\log^2(1-cx}{4x^2} \\
&= -c^2\log(x) + c^2\log(1-cx) - \frac{c\log(1-cx)}{x} - \frac{1}{4}c^2\log^2(1-cx) + \frac{\log^2(1-cx)}{4x^2} +
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 185, normalized size = 0.97

$$\frac{1}{4}\left(-2c^2\log(x) - 2c^2\log(cx) + 4c^2\log(1-cx) - \frac{4c\log(1-cx)}{x} + 2c^2\log(cx)\log(1-cx) - c^2\log^2(1-cx) + \frac{\log^2(1-cx)}{x^2} + 2c^2\log(cx)\log^2(1-cx) + \frac{2(cx+(-1+c^2x^2)\log(1-cx))\,\text{PolyLog}(2,cx)}{x^2} + 2c^2(1+2\log(1-cx))\text{PolyLog}(2,1-cx) - 2c^2\text{PolyLog}(3,cx) - 4c^2\text{PolyLog}(3,1-cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[1 - c*x]*PolyLog[2, c*x])/x^3,x]
```

```
[Out] (-2*c^2*Log[x] - 2*c^2*Log[c*x] + 4*c^2*Log[1 - c*x] - (4*c*Log[1 - c*x])/x
 + 2*c^2*Log[c*x]*Log[1 - c*x] - c^2*Log[1 - c*x]^2 + Log[1 - c*x]^2/x^2 +
2*c^2*Log[c*x]*Log[1 - c*x]^2 + (2*(c*x + (-1 + c^2*x^2)*Log[1 - c*x])*Poly
Log[2, c*x])/x^2 + 2*c^2*(1 + 2*Log[1 - c*x])*PolyLog[2, 1 - c*x] - 2*c^2*P
olyLog[3, c*x] - 4*c^2*PolyLog[3, 1 - c*x])/4
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\ln(-cx+1)\,\mathrm{polylog}(2,cx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-c*x+1)*polylog(2,c*x)/x^3,x)
```

```
[Out] int(ln(-c*x+1)*polylog(2,c*x)/x^3,x)
```

Maxima [A]
time = 0.51, size = 162, normalized size = 0.85

$$\frac{1}{2}\left(\log\left(cx\right)\log\left(-cx+1\right)^{2}+2\,\mathrm{Li}_{2}(-cx+1)\log\left(-cx+1\right)-2\,\mathrm{Li}_{3}(-cx+1)\right)c^{2}+\frac{1}{2}\left(\log\left(cx\right)\log\left(-cx+1\right)+\mathrm{Li}_{2}(-cx+1)\right)c^{2}-c^{2}\log\left(x\right)-\frac{1}{2}\,c^{2}\mathrm{Li}_{3}(cx)-\frac{\left(c^{2}x^{2}-1\right)\log\left(-cx+1\right)^{2}-2\left(cx+\left(c^{2}x^{2}-1\right)\log\left(-cx+1\right)\right)\mathrm{Li}_{2}(cx)-4\left(c^{2}x^{2}-cx\right)\log\left(-cx+1\right)}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylog
(3, -c*x + 1))*c^2 + 1/2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*c^2 - c
^2*log(x) - 1/2*c^2*polylog(3, c*x) - 1/4*((c^2*x^2 - 1)*log(-c*x + 1)^2 -
2*(c*x + (c^2*x^2 - 1)*log(-c*x + 1))*dilog(c*x) - 4*(c^2*x^2 - c*x)*log(-c
*x + 1))/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(dilog(c*x)*log(-c*x + 1)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(-cx+1\right)\mathrm{Li}_{2}\left(cx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-c*x+1)*polylog(2,c*x)/x**3,x)
```

```
[Out] Integral(log(-c*x + 1)*polylog(2, c*x)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(dilog(c*x)*log(-c*x + 1)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln(1-c\,x)\ \mathrm{polylog}(2,c\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(1 - c*x)*polylog(2, c*x))/x^3,x)
```

```
[Out] int((log(1 - c*x)*polylog(2, c*x))/x^3, x)
```

3.168 $\int \frac{\log(1-cx)\mathbf{PolyLog}(2,cx)}{x^4}\,dx$

Optimal. Leaf size=245

$$\frac{7c^2}{36x}-\frac{3}{4}c^3\log(x)+\frac{3}{4}c^3\log(1-cx)-\frac{7c\log(1-cx)}{36x^2}-\frac{5c^2\log(1-cx)}{9x}-\frac{1}{9}c^3\log^2(1-cx)+\frac{\log^2(1-cx)}{9x^3}+\frac{1}{3}c^3\log($$

```
[Out] 7/36*c^2/x-3/4*c^3*ln(x)+3/4*c^3*ln(-c*x+1)-7/36*c*ln(-c*x+1)/x^2-5/9*c^2*l
n(-c*x+1)/x-1/9*c^3*ln(-c*x+1)^2+1/9*ln(-c*x+1)^2/x^3+1/3*c^3*ln(c*x)*ln(-c
*x+1)^2-2/9*c^3*polylog(2,c*x)+1/6*c*polylog(2,c*x)/x^2+1/3*c^2*polylog(2,c
*x)/x+1/3*c^3*ln(-c*x+1)*polylog(2,c*x)-1/3*ln(-c*x+1)*polylog(2,c*x)/x^3+2
/3*c^3*ln(-c*x+1)*polylog(2,-c*x+1)-1/3*c^3*polylog(3,c*x)-2/3*c^3*polylog(
3,-c*x+1)
```

Rubi [A]

time = 0.25, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 17, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.062$, Rules used = {6726, 2442, 46, 6738, 2445, 2457, 36, 29, 31, 2438, 2437, 2338, 6724, 6731, 2443, 2481, 2421}

$$-\frac{2}{9}c^3\text{Li}_2(cx)-\frac{1}{3}c^3\text{Li}_3(cx)-\frac{2}{3}c^3\text{Li}_3(1-cx)+\frac{1}{3}c^3\text{Li}_2(cx)\log(1-cx)+\frac{2}{3}c^3\text{Li}_2(1-cx)\log(1-cx)+\frac{1}{3}c^3\log(cx)\log^2(1-cx)-\frac{1}{9}c^3\log^2(1-cx)-\frac{3}{4}c^3\log(x)+\frac{3}{4}c^3\log(1-cx)+\frac{c^2\text{Li}_2(cx)}{3x}+\frac{7c^2}{36x}-\frac{5c^2\log(1-cx)}{9x}-\frac{\text{Li}_2(cx)\log(1-cx)}{3x^3}+\frac{c\text{Li}_2(cx)}{6x^2}+\frac{\log^2(1-cx)}{9x^3}-\frac{7c\log(1-cx)}{36x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[1 - c*x]*PolyLog[2, c*x])/x^4,x]
```

```
[Out] (7*c^2)/(36*x) - (3*c^3*Log[x])/4 + (3*c^3*Log[1 - c*x])/4 - (7*c*Log[1 - c
*x])/(36*x^2) - (5*c^2*Log[1 - c*x])/(9*x) - (c^3*Log[1 - c*x]^2)/9 + Log[1
 - c*x]^2/(9*x^3) + (c^3*Log[c*x]*Log[1 - c*x]^2)/3 - (2*c^3*PolyLog[2, c*x
])/9 + (c*PolyLog[2, c*x])/(6*x^2) + (c^2*PolyLog[2, c*x])/(3*x) + (c^3*Log
[1 - c*x]*PolyLog[2, c*x])/3 - (Log[1 - c*x]*PolyLog[2, c*x])/(3*x^3) + (2*
c^3*Log[1 - c*x]*PolyLog[2, 1 - c*x])/3 - (c^3*PolyLog[3, c*x])/3 - (2*c^3*
PolyLog[3, 1 - c*x])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
```

```
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2457

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6738

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[e*h*(n/(m + 1)), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log(1-cx)\text{Li}_2(cx)}{x^4}\,dx = -\frac{\log(1-cx)\text{Li}_2(cx)}{3x^3} - \frac{1}{3}\int \frac{\log^2(1-cx)}{x^4}\,dx - \frac{1}{3}c\int \left(\frac{\text{Li}_2(cx)}{x^3} + \frac{c\text{Li}_2(cx)}{x^2} + \frac{c^2\text{L}}{}\right.$$

$$= \frac{\log^2(1-cx)}{9x^3} - \frac{\log(1-cx)\text{Li}_2(cx)}{3x^3} + \frac{1}{9}(2c)\int \frac{\log(1-cx)}{x^3(1-cx)}\,dx - \frac{1}{3}c\int \frac{\text{Li}_2(cx)}{x^3}\,dx$$

$$= \frac{\log^2(1-cx)}{9x^3} + \frac{c\text{Li}_2(cx)}{6x^2} + \frac{c^2\text{Li}_2(cx)}{3x} + \frac{1}{3}c^3\log(1-cx)\text{Li}_2(cx) - \frac{\log(1-cx)\text{Li}_2(}{3x^3}$$

$$= -\frac{c\log(1-cx)}{12x^2} - \frac{c^2\log(1-cx)}{3x} + \frac{\log^2(1-cx)}{9x^3} + \frac{1}{3}c^3\log(cx)\log^2(1-cx) + \frac{c\text{Li}}{}$$

$$= -\frac{7c\log(1-cx)}{36x^2} - \frac{5c^2\log(1-cx)}{9x} + \frac{\log^2(1-cx)}{9x^3} + \frac{1}{3}c^3\log(cx)\log^2(1-cx) -$$

$$= \frac{c^2}{12x} - \frac{5}{12}c^3\log(x) + \frac{5}{12}c^3\log(1-cx) - \frac{7c\log(1-cx)}{36x^2} - \frac{5c^2\log(1-cx)}{9x} - \frac{1}{9}c^3$$

$$= \frac{7c^2}{36x} - \frac{3}{4}c^3\log(x) + \frac{3}{4}c^3\log(1-cx) - \frac{7c\log(1-cx)}{36x^2} - \frac{5c^2\log(1-cx)}{9x} - \frac{1}{9}c^3\log$$

Mathematica [A]
time = 0.17, size = 246, normalized size = 1.00

$$\frac{7c^2x^2 - 4c^3x^3 - 15c^3x^3\log(x) - 12c^3x^3\log(cx) - 7cx\log(1-cx) - 20c^2x^2\log(1-cx) + 27c^3x^3\log(1-cx) + 8c^3x^3\log(cx)\log(1-cx) + 4\log^2(1-cx) - 4c^3x^3\log^2(1-cx) + 12c^3x^3\log(cx)\log^2(1-cx) + 6(cx(1+2cx) + 2(-1+c^3x^3)\log(1-cx))\,\text{PolyLog}(2,cx) + 8c^3x^3(1+3\log(1-cx))\text{PolyLog}(2,1-cx) - 12c^3x^3\text{PolyLog}(3,cx) - 24c^3x^3\text{PolyLog}(3,1-cx)}{36x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[1 - c*x]*PolyLog[2, c*x])/x^4,x]
```

```
[Out] (7*c^2*x^2 - 4*c^3*x^3 - 15*c^3*x^3*Log[x] - 12*c^3*x^3*Log[c*x] - 7*c*x*Lo
g[1 - c*x] - 20*c^2*x^2*Log[1 - c*x] + 27*c^3*x^3*Log[1 - c*x] + 8*c^3*x^3*
Log[c*x]*Log[1 - c*x] + 4*Log[1 - c*x]^2 - 4*c^3*x^3*Log[1 - c*x]^2 + 12*c^
3*x^3*Log[c*x]*Log[1 - c*x]^2 + 6*(c*x*(1 + 2*c*x) + 2*(-1 + c^3*x^3)*Log[1
 - c*x])*PolyLog[2, c*x] + 8*c^3*x^3*(1 + 3*Log[1 - c*x])*PolyLog[2, 1 - c*
x] - 12*c^3*x^3*PolyLog[3, c*x] - 24*c^3*x^3*PolyLog[3, 1 - c*x])/(36*x^3)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(-cx+1\right) \operatorname{polylog}\left(2,cx\right)}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-c*x+1)*polylog(2,c*x)/x^4,x)
```

```
[Out] int(ln(-c*x+1)*polylog(2,c*x)/x^4,x)
```

Maxima [A]
time = 0.51, size = 188, normalized size = 0.77

$$\frac{1}{3} \left(\log\left(cx\right) \log\left(-cx+1\right)^{2} + 2 \operatorname{Li}_2(-cx+1) \log\left(-cx+1\right) - 2 \operatorname{Li}_3(-cx+1)\right) c^{3} + \frac{2}{9} \left(\log\left(cx\right) \log\left(-cx+1\right) + \operatorname{Li}_2(-cx+1)\right) c^{3} - \frac{3}{4} c^{3} \log\left(x\right) - \frac{1}{3} c^{3} \operatorname{Li}_3(cx) + \frac{7 c^{2} x^{2} - 4 \left(c^{3} x^{3} - 1\right) \log\left(-cx+1\right)^{2} + 6 \left(2 c^{2} x^{2} + cx + 2 \left(c^{3} x^{3} - 1\right) \log\left(-cx+1\right)\right) \operatorname{Li}_2(cx) + \left(27 c^{3} x^{3} - 20 c^{2} x^{2} - 7 cx\right) \log\left(-cx+1\right)}{36 x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylog
(3, -c*x + 1))*c^3 + 2/9*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*c^3 - 3
/4*c^3*log(x) - 1/3*c^3*polylog(3, c*x) + 1/36*(7*c^2*x^2 - 4*(c^3*x^3 - 1)
*log(-c*x + 1)^2 + 6*(2*c^2*x^2 + c*x + 2*(c^3*x^3 - 1)*log(-c*x + 1))*dilo
g(c*x) + (27*c^3*x^3 - 20*c^2*x^2 - 7*c*x)*log(-c*x + 1))/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral(dilog(c*x)*log(-c*x + 1)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(-cx+1\right) \operatorname{Li}_{2}\left(cx\right)}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-c*x+1)*polylog(2,c*x)/x**4,x)
```

```
[Out] Integral(log(-c*x + 1)*polylog(2, c*x)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^4,x, algorithm="giac")
[Out] integrate(dilog(c*x)*log(-c*x + 1)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(1-c\,x\right)\,\mathrm{polylog}(2,c\,x)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(1 - c*x)*polylog(2, c*x))/x^4,x)
[Out] int((log(1 - c*x)*polylog(2, c*x))/x^4, x)
```

3.169 $\int \frac{\log(1-cx)\mathbf{PolyLog}(2,cx)}{x^5}\,dx$

Optimal. Leaf size=287

$$\frac{5c^2}{144x^2}+\frac{7c^3}{36x}-\frac{41}{72}c^4\log(x)+\frac{41}{72}c^4\log(1-cx)-\frac{5c\log(1-cx)}{72x^3}-\frac{c^2\log(1-cx)}{8x^2}-\frac{3c^3\log(1-cx)}{8x}-\frac{1}{16}c^4\log^2(1-$$

```
[Out] 5/144*c^2/x^2+7/36*c^3/x-41/72*c^4*ln(x)+41/72*c^4*ln(-c*x+1)-5/72*c*ln(-c*
x+1)/x^3-1/8*c^2*ln(-c*x+1)/x^2-3/8*c^3*ln(-c*x+1)/x-1/16*c^4*ln(-c*x+1)^2+
1/16*ln(-c*x+1)^2/x^4+1/4*c^4*ln(c*x)*ln(-c*x+1)^2-1/8*c^4*polylog(2,c*x)+1
/12*c*polylog(2,c*x)/x^3+1/8*c^2*polylog(2,c*x)/x^2+1/4*c^3*polylog(2,c*x)/
x+1/4*c^4*ln(-c*x+1)*polylog(2,c*x)-1/4*ln(-c*x+1)*polylog(2,c*x)/x^4+1/2*c
^4*ln(-c*x+1)*polylog(2,-c*x+1)-1/4*c^4*polylog(3,c*x)-1/2*c^4*polylog(3,-c
*x+1)
```

Rubi [A]
time = 0.31, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 17, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.062$, Rules used = {6726, 2442, 46, 6738, 2445, 2457, 36, 29, 31, 2438, 2437, 2338, 6724, 6731, 2443, 2481, 2421}

$$-\frac{1}{8}c^4\text{Li}_2(cx)-\frac{1}{4}c^4\text{Li}_3(cx)-\frac{1}{2}c^4\text{Li}_3(1-cx)+\frac{1}{4}c^4\text{Li}_2(cx)\log(1-cx)+\frac{1}{2}c^4\text{Li}_2(1-cx)\log(1-cx)+\frac{1}{4}c^4\log(cx)\log^2(1-cx)-\frac{1}{16}c^4\log^2(1-cx)-\frac{41}{72}c^4\log(x)+\frac{41}{72}c^4\log(1-cx)+\frac{c^3\text{Li}_2(cx)}{4x}+\frac{7c^3}{36x}-\frac{3c^3\log(1-cx)}{8x}+\frac{c^2\text{Li}_2(cx)}{8x^2}+\frac{5c^2}{144x^2}-\frac{c^2\log(1-cx)}{8x^2}-\frac{\text{Li}_2(cx)\log(1-cx)}{4x^4}+\frac{c\text{Li}_2(cx)}{12x^3}+\frac{\log^2(1-cx)}{16x^4}-\frac{5c\log(1-cx)}{72x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[1 - c*x]*PolyLog[2, c*x])/x^5,x]
```

```
[Out] (5*c^2)/(144*x^2) + (7*c^3)/(36*x) - (41*c^4*Log[x])/72 + (41*c^4*Log[1 - c
*x])/72 - (5*c*Log[1 - c*x])/(72*x^3) - (c^2*Log[1 - c*x])/(8*x^2) - (3*c^3
*Log[1 - c*x])/(8*x) - (c^4*Log[1 - c*x]^2)/16 + Log[1 - c*x]^2/(16*x^4) +
(c^4*Log[c*x]*Log[1 - c*x]^2)/4 - (c^4*PolyLog[2, c*x])/8 + (c*PolyLog[2, c
*x])/(12*x^3) + (c^2*PolyLog[2, c*x])/(8*x^2) + (c^3*PolyLog[2, c*x])/(4*x)
 + (c^4*Log[1 - c*x]*PolyLog[2, c*x])/4 - (Log[1 - c*x]*PolyLog[2, c*x])/(4
*x^4) + (c^4*Log[1 - c*x]*PolyLog[2, 1 - c*x])/2 - (c^4*PolyLog[3, c*x])/4
- (c^4*PolyLog[3, 1 - c*x])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2457

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
```

```
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6738

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[e*h*(n/(m + 1)), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(1-cx)\mathrm{Li}_2(cx)}{x^5}\,dx &= -\frac{\log(1-cx)\mathrm{Li}_2(cx)}{4x^4} - \frac{1}{4}\int \frac{\log^2(1-cx)}{x^5}\,dx - \frac{1}{4}c\int \left(\frac{\mathrm{Li}_2(cx)}{x^4} + \frac{c\mathrm{Li}_2(cx)}{x^3} + \frac{c^2\mathrm{L}}{} \right. \\
&= \frac{\log^2(1-cx)}{16x^4} - \frac{\log(1-cx)\mathrm{Li}_2(cx)}{4x^4} + \frac{1}{8}c\int \frac{\log(1-cx)}{x^4(1-cx)}\,dx - \frac{1}{4}c\int \frac{\mathrm{Li}_2(cx)}{x^4}\,dx - \\
&= \frac{\log^2(1-cx)}{16x^4} + \frac{c\mathrm{Li}_2(cx)}{12x^3} + \frac{c^2\mathrm{Li}_2(cx)}{8x^2} + \frac{c^3\mathrm{Li}_2(cx)}{4x} + \frac{1}{4}c^4\log(1-cx)\mathrm{Li}_2(cx) - \frac{\log}{} \\
&= -\frac{c\log(1-cx)}{36x^3} - \frac{c^2\log(1-cx)}{16x^2} - \frac{c^3\log(1-cx)}{4x} + \frac{\log^2(1-cx)}{16x^4} + \frac{1}{4}c^4\log(cx)\,\mathrm{lo} \\
&= -\frac{5c\log(1-cx)}{72x^3} - \frac{c^2\log(1-cx)}{8x^2} - \frac{3c^3\log(1-cx)}{8x} + \frac{\log^2(1-cx)}{16x^4} + \frac{1}{4}c^4\log(cx \\
&= \frac{c^2}{72x^2} + \frac{13c^3}{144x} - \frac{49}{144}c^4\log(x) + \frac{49}{144}c^4\log(1-cx) - \frac{5c\log(1-cx)}{72x^3} - \frac{c^2\log(1-}{8x^2} \\
&= \frac{5c^2}{144x^2} + \frac{7c^3}{36x} - \frac{41}{72}c^4\log(x) + \frac{41}{72}c^4\log(1-cx) - \frac{5c\log(1-cx)}{72x^3} - \frac{c^2\log(1-cx}{8x^2}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 277, normalized size = 0.97

$$\frac{5c^2x^2+28c^3x^3-18c^4x^4-49c^4x^4\log(x)-33c^4x^4\log(cx)-10cx\log(1-cx)-18c^2x^2\log(1-cx)-54c^3x^3\log(1-cx)+82c^4x^4\log(1-cx)+18c^4x^4\log(cx)\log(1-cx)+9\log^2(1-cx)-9c^4x^4\log^2(1-cx)+36c^4x^4\log(cx)\log^2(1-cx)+6(cx(2+3cx+6c^2x^2)+6(-1+c^4x^4)\log(1-cx))\mathrm{PolyLog}(2,cx)+18c^4x^4(1+4\log(1-cx))\mathrm{PolyLog}(2,1-cx)-36c^4x^4\mathrm{PolyLog}(3,cx)-72c^4x^4\mathrm{PolyLog}(3,1-cx)}{144x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[1 - c*x]*PolyLog[2, c*x])/x^5,x]
```

```
[Out] (5*c^2*x^2 + 28*c^3*x^3 - 18*c^4*x^4 - 49*c^4*x^4*Log[x] - 33*c^4*x^4*Log[c
*x] - 10*c*x*Log[1 - c*x] - 18*c^2*x^2*Log[1 - c*x] - 54*c^3*x^3*Log[1 - c*
```

```
x] + 82*c^4*x^4*Log[1 - c*x] + 18*c^4*x^4*Log[c*x]*Log[1 - c*x] + 9*Log[1 -
 c*x]^2 - 9*c^4*x^4*Log[1 - c*x]^2 + 36*c^4*x^4*Log[c*x]*Log[1 - c*x]^2 + 6
*(c*x*(2 + 3*c*x + 6*c^2*x^2) + 6*(-1 + c^4*x^4)*Log[1 - c*x])*PolyLog[2, c
*x] + 18*c^4*x^4*(1 + 4*Log[1 - c*x])*PolyLog[2, 1 - c*x] - 36*c^4*x^4*Poly
Log[3, c*x] - 72*c^4*x^4*PolyLog[3, 1 - c*x])/(144*x^4)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\ln(-cx+1)\,\mathrm{polylog}(2,cx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-c*x+1)*polylog(2,c*x)/x^5,x)
```

```
[Out] int(ln(-c*x+1)*polylog(2,c*x)/x^5,x)
```

Maxima [A]
time = 0.50, size = 214, normalized size = 0.75

$$\frac{1}{4}\left(\log(cx)\log(-cx+1)^2 + 2\,\mathrm{Li}_2(-cx+1)\log(-cx+1) - 2\,\mathrm{Li}_3(-cx+1)\right)c^4 + \frac{1}{8}\left(\log(cx)\log(-cx+1) + \mathrm{Li}_2(-cx+1)\right)c^4 - \frac{41}{72}c^4\log(x) - \frac{1}{4}c^4\mathrm{Li}_3(cx) + \frac{28c^3x^3 + 5c^2x^2 - 9(c^4x^4-1)\log(-cx+1)^2 + 6(6c^3x^3 + 3c^2x^2 + 2cx + 6(c^4x^4-1)\log(-cx+1))\mathrm{Li}_2(cx) + 2(41c^4x^4 - 27c^3x^3 - 9c^2x^2 - 5cx)\log(-cx+1)}{144x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylog
(3, -c*x + 1))*c^4 + 1/8*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*c^4 - 4
1/72*c^4*log(x) - 1/4*c^4*polylog(3, c*x) + 1/144*(28*c^3*x^3 + 5*c^2*x^2 -
 9*(c^4*x^4 - 1)*log(-c*x + 1)^2 + 6*(6*c^3*x^3 + 3*c^2*x^2 + 2*c*x + 6*(c^
4*x^4 - 1)*log(-c*x + 1))*dilog(c*x) + 2*(41*c^4*x^4 - 27*c^3*x^3 - 9*c^2*x
^2 - 5*c*x)*log(-c*x + 1))/x^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^5,x, algorithm="fricas")
```

```
[Out] integral(dilog(c*x)*log(-c*x + 1)/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(-cx+1)\,\mathrm{Li}_2(cx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-c*x+1)*polylog(2,c*x)/x**5,x)
```

```
[Out] Integral(log(-c*x + 1)*polylog(2, c*x)/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(dilog(c*x)*log(-c*x + 1)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(1-c\,x\right)\,\mathrm{polylog}(2,c\,x)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(1 - c*x)*polylog(2, c*x))/x^5,x)
```

```
[Out] int((log(1 - c*x)*polylog(2, c*x))/x^5, x)
```

3.170 $\int x^2(g+h\log(1-cx))\textbf{PolyLog}(2,cx)\,dx$

Optimal. Leaf size=423

$$\frac{121hx}{108c^2}+\frac{13hx^2}{216c}+\frac{hx^3}{81}+\frac{h(1-cx)^2}{6c^3}-\frac{2h(1-cx)^3}{81c^3}+\frac{13h\log(1-cx)}{108c^3}-\frac{hx^2\log(1-cx)}{12c}-\frac{1}{27}hx^3\log(1-cx)+\frac{h(}{}$$

```
[Out] 121/108*h*x/c^2+13/216*h*x^2/c+1/81*h*x^3+1/6*h*(-c*x+1)^2/c^3-2/81*h*(-c*x
+1)^3/c^3+13/108*h*ln(-c*x+1)/c^3-1/12*h*x^2*ln(-c*x+1)/c-1/27*h*x^3*ln(-c*
x+1)+1/3*h*(-c*x+1)*ln(-c*x+1)/c^3+1/9*h*ln(-c*x+1)^2/c^3-1/3*h*ln(c*x)*ln(
-c*x+1)^2/c^3+1/9*x^3*ln(-c*x+1)*(g+h*ln(-c*x+1))+1/3*(-c*x+1)*(g+2*h*ln(-c
*x+1))/c^3-1/6*(-c*x+1)^2*(g+2*h*ln(-c*x+1))/c^3+1/27*(-c*x+1)^3*(g+2*h*ln(
-c*x+1))/c^3-1/9*ln(-c*x+1)*(g+2*h*ln(-c*x+1))/c^3-1/3*h*x*polylog(2,c*x)/c
^2-1/6*h*x^2*polylog(2,c*x)/c-1/9*h*x^3*polylog(2,c*x)-1/3*h*ln(-c*x+1)*pol
ylog(2,c*x)/c^3+1/3*x^3*(g+h*ln(-c*x+1))*polylog(2,c*x)-2/3*h*ln(-c*x+1)*po
lylog(2,-c*x+1)/c^3+2/3*h*polylog(3,-c*x+1)/c^3
```

Rubi [A]
time = 0.31, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 18, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6738, 2483, 2458, 45, 2372, 12, 14, 2338, 6721, 2436, 2332, 6726, 2442, 6731, 2443, 2481, 2421, 6724}

$$\frac{(1-cx)^3(2h\log(1-cx)+g)}{27c^3}-\frac{(1-cx)^2(2h\log(1-cx)+g)}{6c^3}+\frac{(1-cx)(2h\log(1-cx)+g)}{3c^3}-\frac{\log(1-cx)(2h\log(1-cx)+g)}{9c^3}+\frac{2h\text{Li}_3(1-cx)}{3c^3}-\frac{h\text{Li}_2(cx)\log(1-cx)}{3c^3}-\frac{2h\text{Li}_2(1-cx)\log(1-cx)}{3c^3}-\frac{2h(1-cx)^3}{81c^3}+\frac{h(1-cx)^2}{6c^3}+\frac{h\log^2(1-cx)}{9c^3}-\frac{h\log(cx)\log^2(1-cx)}{3c^3}+\frac{13h\log(1-cx)}{108c^3}+\frac{h(1-cx)\log(1-cx)}{3c^3}-\frac{hx\text{Li}_2(cx)}{3c^2}+\frac{121hx}{108c^2}+\frac{1}{3}x^3\text{Li}_2(cx)(h\log(1-cx)+g)+\frac{1}{9}x^3\log(1-cx)(h\log(1-cx)+g)-\frac{1}{9}hx^3\text{Li}_2(cx)-\frac{hx^2\text{Li}_2(cx)}{6c}-\frac{1}{27}hx^3\log(1-cx)+\frac{13hx^2}{216c}-\frac{hx^2\log(1-cx)}{12c}+\frac{hx^3}{81}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(g + h*Log[1 - c*x])*PolyLog[2, c*x],x]
```

```
[Out] (121*h*x)/(108*c^2) + (13*h*x^2)/(216*c) + (h*x^3)/81 + (h*(1 - c*x)^2)/(6*
c^3) - (2*h*(1 - c*x)^3)/(81*c^3) + (13*h*Log[1 - c*x])/(108*c^3) - (h*x^2*
Log[1 - c*x])/(12*c) - (h*x^3*Log[1 - c*x])/27 + (h*(1 - c*x)*Log[1 - c*x])
/(3*c^3) + (h*Log[1 - c*x]^2)/(9*c^3) - (h*Log[c*x]*Log[1 - c*x]^2)/(3*c^3)
 + (x^3*Log[1 - c*x]*(g + h*Log[1 - c*x]))/9 + ((1 - c*x)*(g + 2*h*Log[1 -
c*x]))/(3*c^3) - ((1 - c*x)^2*(g + 2*h*Log[1 - c*x]))/(6*c^3) + ((1 - c*x)^
3*(g + 2*h*Log[1 - c*x]))/(27*c^3) - (Log[1 - c*x]*(g + 2*h*Log[1 - c*x]))/
(9*c^3) - (h*x*PolyLog[2, c*x])/(3*c^2) - (h*x^2*PolyLog[2, c*x])/(6*c) - (
h*x^3*PolyLog[2, c*x])/9 - (h*Log[1 - c*x]*PolyLog[2, c*x])/(3*c^3) + (x^3*
(g + h*Log[1 - c*x])*PolyLog[2, c*x])/3 - (2*h*Log[1 - c*x]*PolyLog[2, 1 -
c*x])/(3*c^3) + (2*h*PolyLog[3, 1 - c*x])/(3*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
```

```
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*
(a + b*Log[c*(d + e*x)^n])*((f + g*Log[c*(d + e*x)^n])/(m + 1)), x] - Dist[
e*(n/(m + 1)), Int[(x^(m + 1)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d +
e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, m}, x] && NeQ[m, -1]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6738

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[e*h*(n/(m + 1)), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$\int x^2(g+h\log(1-cx))\text{Li}_2(cx)\,dx = \frac{1}{3}x^3(g+h\log(1-cx))\text{Li}_2(cx)+\frac{1}{3}\int x^2\log(1-cx)(g+h\log(1-cx$$

$$=\frac{1}{9}x^3\log(1-cx)(g+h\log(1-cx))+\frac{1}{3}x^3(g+h\log(1-cx))\text{Li}_2(cx)+$$

$$=\frac{1}{9}x^3\log(1-cx)(g+h\log(1-cx))-\frac{hx\text{Li}_2(cx)}{3c^2}-\frac{hx^2\text{Li}_2(cx)}{6c}-\frac{1}{9}hx$$

$$=-\frac{hx^2\log(1-cx)}{12c}-\frac{1}{27}hx^3\log(1-cx)-\frac{h\log(cx)\log^2(1-cx)}{3c^3}+\frac{1}{9}x$$

$$=\frac{hx}{3c^2}-\frac{5hx^2\log(1-cx)}{36c}-\frac{2}{27}hx^3\log(1-cx)+\frac{h(1-cx)\log(1-cx)}{3c^3}$$

$$=\frac{61hx}{108c^2}+\frac{13hx^2}{216c}+\frac{hx^3}{81}+\frac{13h\log(1-cx)}{108c^3}-\frac{5hx^2\log(1-cx)}{36c}-\frac{2}{27}hx$$

$$=\frac{107hx}{108c^2}+\frac{23hx^2}{216c}+\frac{2hx^3}{81}+\frac{h(1-cx)^2}{12c^3}-\frac{h(1-cx)^3}{81c^3}+\frac{23h\log(1-ca}{108c^3}$$

$$=\frac{107hx}{108c^2}+\frac{23hx^2}{216c}+\frac{2hx^3}{81}+\frac{h(1-cx)^2}{12c^3}-\frac{h(1-cx)^3}{81c^3}+\frac{23h\log(1-ca}{108c^3}$$

Mathematica [A]
time = 0.33, size = 252, normalized size = 0.60

$$\frac{g(-cx(6+3cx+2c^2x^2)+6(-1+c^3x^3)\log(1-cx)+18c^3x^3\text{PolyLog}(2,cx))}{54c^3}+\frac{h(186cx+33c^2x^2+8c^3x^3+186\log(1-cx)-120cx\log(1-cx)-42c^2x^2\log(1-cx)-24c^3x^3\log(1-cx)-24\log^2(1-cx)+24c^3x^3\log^2(1-cx)-72\log(cx)\log^2(1-cx)+12(-cx(6+3cx+2c^2x^2)+6(-1+c^3x^3)\log(1-cx))\text{PolyLog}(2,cx)-144\log(1-cx)\text{PolyLog}(2,1-cx)+144\text{PolyLog}(3,1-cx))}{216c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(g + h*Log[1 - c*x])*PolyLog[2, c*x],x]
```

```
[Out] (g*(-(c*x*(6 + 3*c*x + 2*c^2*x^2)) + 6*(-1 + c^3*x^3)*Log[1 - c*x] + 18*c^3
*x^3*PolyLog[2, c*x]))/(54*c^3) + (h*(186*c*x + 33*c^2*x^2 + 8*c^3*x^3 + 18
6*Log[1 - c*x] - 120*c*x*Log[1 - c*x] - 42*c^2*x^2*Log[1 - c*x] - 24*c^3*x^
3*Log[1 - c*x] - 24*Log[1 - c*x]^2 + 24*c^3*x^3*Log[1 - c*x]^2 - 72*Log[c*x
]*Log[1 - c*x]^2 + 12*(-(c*x*(6 + 3*c*x + 2*c^2*x^2)) + 6*(-1 + c^3*x^3)*Lo
g[1 - c*x])*PolyLog[2, c*x] - 144*Log[1 - c*x]*PolyLog[2, 1 - c*x] + 144*Po
lyLog[3, 1 - c*x]))/(216*c^3)
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int x^2(g+h\ln(-cx+1))\,\text{polylog}(2,cx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

```
[Out] int(x^2*(g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="maxima")
```

```
[Out] -1/18*h*((2*c^3*x^3 + 3*c^2*x^2 + 6*c*x - 6*(c^3*x^3 - 1)*log(-c*x + 1))*di
log(c*x)/c^3 - integrate((6*(c^3*x^3 - 1)*log(-c*x + 1)^2 - (2*c^3*x^3 + 3*
c^2*x^2 + 6*c*x)*log(-c*x + 1))/x, x)/c^3) + 1/54*(18*c^3*x^3*dilog(c*x) -
2*c^3*x^3 - 3*c^2*x^2 - 6*c*x + 6*(c^3*x^3 - 1)*log(-c*x + 1))*g/c^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral(h*x^2*dilog(c*x)*log(-c*x + 1) + g*x^2*dilog(c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(g + h \log(-cx + 1)) \operatorname{Li}_2(cx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

```
[Out] Integral(x**2*(g + h*log(-c*x + 1))*polylog(2, c*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="giac")
```

[Out] integrate((h*log(-c*x + 1) + g)*x^2*dilog(c*x), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,(g+h\,\ln\left(1-c\,x\right))\;\mathrm{polylog}(2,c\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(g + h*log(1 - c*x))*polylog(2, c*x),x)

[Out] int(x^2*(g + h*log(1 - c*x))*polylog(2, c*x), x)

3.171 $\int x(g+h\log(1-cx))\textbf{PolyLog}(2,cx)\,dx$

Optimal. Leaf size=330

$$\frac{13hx}{8c}+\frac{hx^2}{16}+\frac{h(1-cx)^2}{8c^2}+\frac{h\log(1-cx)}{8c^2}-\frac{1}{8}hx^2\log(1-cx)+\frac{h(1-cx)\log(1-cx)}{2c^2}+\frac{h\log^2(1-cx)}{4c^2}-\frac{h\log(cx)}{}$$

```
[Out] 13/8*h*x/c+1/16*h*x^2+1/8*h*(-c*x+1)^2/c^2+1/8*h*ln(-c*x+1)/c^2-1/8*h*x^2*l
n(-c*x+1)+1/2*h*(-c*x+1)*ln(-c*x+1)/c^2+1/4*h*ln(-c*x+1)^2/c^2-1/2*h*ln(c*x
)*ln(-c*x+1)^2/c^2+1/4*x^2*ln(-c*x+1)*(g+h*ln(-c*x+1))+1/2*(-c*x+1)*(g+2*h*
ln(-c*x+1))/c^2-1/8*(-c*x+1)^2*(g+2*h*ln(-c*x+1))/c^2-1/4*ln(-c*x+1)*(g+2*h
*ln(-c*x+1))/c^2-1/2*h*x*polylog(2,c*x)/c-1/4*h*x^2*polylog(2,c*x)-1/2*h*ln
(-c*x+1)*polylog(2,c*x)/c^2+1/2*x^2*(g+h*ln(-c*x+1))*polylog(2,c*x)-h*ln(-c
*x+1)*polylog(2,-c*x+1)/c^2+h*polylog(3,-c*x+1)/c^2
```

Rubi [A]
time = 0.24, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 18, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6738, 2483, 2458, 45, 2372, 12, 14, 2338, 6721, 2436, 2332, 6726, 2442, 6731, 2443, 2481, 2421, 6724}

$$-\frac{(1-cx)^2(2h\log(1-cx)+g)}{8c^2}+\frac{(1-cx)(2h\log(1-cx)+g)}{2c^2}-\frac{\log(1-cx)(2h\log(1-cx)+g)}{4c^2}+\frac{h\text{Li}_3(1-cx)}{c^2}-\frac{h\text{Li}_2(cx)\log(1-cx)}{2c^2}-\frac{h\text{Li}_2(1-cx)\log(1-cx)}{c^2}+\frac{h(1-cx)^2}{8c^2}+\frac{h\log^2(1-cx)}{4c^2}-\frac{h\log(cx)\log^2(1-cx)}{2c^2}+\frac{h\log(1-cx)}{8c^2}+\frac{h(1-cx)\log(1-cx)}{2c^2}+\frac{1}{2}x^2\text{Li}_2(cx)(h\log(1-cx)+g)+\frac{1}{4}x^2\log(1-cx)(h\log(1-cx)+g)-\frac{1}{4}hx^2\text{Li}_2(cx)-\frac{hx\text{Li}_2(cx)}{2c}-\frac{1}{8}hx^2\log(1-cx)+\frac{13hx}{8c}+\frac{hx^2}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x*(g + h*Log[1 - c*x])*PolyLog[2, c*x],x]
```

```
[Out] (13*h*x)/(8*c) + (h*x^2)/16 + (h*(1 - c*x)^2)/(8*c^2) + (h*Log[1 - c*x])/(8
*c^2) - (h*x^2*Log[1 - c*x])/8 + (h*(1 - c*x)*Log[1 - c*x])/(2*c^2) + (h*Lo
g[1 - c*x]^2)/(4*c^2) - (h*Log[c*x]*Log[1 - c*x]^2)/(2*c^2) + (x^2*Log[1 -
c*x]*(g + h*Log[1 - c*x]))/4 + ((1 - c*x)*(g + 2*h*Log[1 - c*x]))/(2*c^2) -
 ((1 - c*x)^2*(g + 2*h*Log[1 - c*x]))/(8*c^2) - (Log[1 - c*x]*(g + 2*h*Log[
1 - c*x]))/(4*c^2) - (h*x*PolyLog[2, c*x])/(2*c) - (h*x^2*PolyLog[2, c*x])/
4 - (h*Log[1 - c*x]*PolyLog[2, c*x])/(2*c^2) + (x^2*(g + h*Log[1 - c*x])*Po
lyLog[2, c*x])/2 - (h*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c^2 + (h*PolyLog[3,
 1 - c*x])/c^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*
(a + b*Log[c*(d + e*x)^n])*((f + g*Log[c*(d + e*x)^n])/(m + 1)), x] - Dist[
e*(n/(m + 1)), Int[(x^(m + 1)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d +
e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, m}, x] && NeQ[m, -1]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6738

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[e*h*(n/(m + 1)), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$\int x(g+h\log(1-cx))\text{Li}_2(cx)\,dx = \frac{1}{2}x^2(g+h\log(1-cx))\text{Li}_2(cx)+\frac{1}{2}\int x\log(1-cx)(g+h\log(1-cx))\,dx$$

$$= \frac{1}{4}x^2\log(1-cx)(g+h\log(1-cx))+\frac{1}{2}x^2(g+h\log(1-cx))\text{Li}_2(cx)+\frac{1}{4}$$

$$= \frac{1}{4}x^2\log(1-cx)(g+h\log(1-cx))-\frac{hx\text{Li}_2(cx)}{2c}-\frac{1}{4}hx^2\text{Li}_2(cx)-\frac{h\log(}{}$$

$$= -\frac{1}{8}hx^2\log(1-cx)-\frac{h\log(cx)\log^2(1-cx)}{2c^2}+\frac{1}{4}x^2\log(1-cx)(g+h\log$$

$$= \frac{hx}{2c}-\frac{1}{4}hx^2\log(1-cx)+\frac{h(1-cx)\log(1-cx)}{2c^2}-\frac{h\log(cx)\log^2(1-cx)}{2c^2}$$

$$= \frac{7hx}{8c}+\frac{hx^2}{16}+\frac{h\log(1-cx)}{8c^2}-\frac{1}{4}hx^2\log(1-cx)+\frac{3h(1-cx)\log(1-cx}{4c^2}$$

$$= \frac{3hx}{2c}+\frac{hx^2}{8}+\frac{h(1-cx)^2}{16c^2}+\frac{h\log(1-cx)}{4c^2}-\frac{1}{4}hx^2\log(1-cx)+\frac{3h(1-}{}$$

$$= \frac{3hx}{2c}+\frac{hx^2}{8}+\frac{h(1-cx)^2}{16c^2}+\frac{h\log(1-cx)}{4c^2}-\frac{1}{4}hx^2\log(1-cx)+\frac{3h(1-}{}$$

Mathematica [A]
time = 0.24, size = 211, normalized size = 0.64

$$\frac{g(-cx(2+cx)+2(-1+c^2x^2)\log(1-cx)+4c^2x^2\text{PolyLog}(2,cx))}{8c^2}+\frac{h(-14+22cx+3c^2x^2+22\log(1-cx)-16cx\log(1-cx)-6c^2x^2\log(1-cx)-4\log^2(1-cx)+4c^2x^2\log^2(1-cx)-8\log(cx)\log^2(1-cx)+(-4cx(2+cx)+8(-1+c^2x^2)\log(1-cx))\text{PolyLog}(2,cx)-16\log(1-cx)\text{PolyLog}(2,1-cx)+16\text{PolyLog}(3,1-cx))}{16c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(g + h*Log[1 - c*x])*PolyLog[2, c*x],x]
```

```
[Out] (g*(-(c*x*(2 + c*x)) + 2*(-1 + c^2*x^2)*Log[1 - c*x] + 4*c^2*x^2*PolyLog[2,
 c*x]))/(8*c^2) + (h*(-14 + 22*c*x + 3*c^2*x^2 + 22*Log[1 - c*x] - 16*c*x*L
og[1 - c*x] - 6*c^2*x^2*Log[1 - c*x] - 4*Log[1 - c*x]^2 + 4*c^2*x^2*Log[1 -
 c*x]^2 - 8*Log[c*x]*Log[1 - c*x]^2 + (-4*c*x*(2 + c*x) + 8*(-1 + c^2*x^2)*
Log[1 - c*x])*PolyLog[2, c*x] - 16*Log[1 - c*x]*PolyLog[2, 1 - c*x] + 16*Po
lyLog[3, 1 - c*x]))/(16*c^2)
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int x(g+h\ln(-cx+1))\,\text{polylog}(2,cx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

```
[Out] int(x*(g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="maxima")
```

```
[Out] -1/4*h*((c^2*x^2 + 2*c*x - 2*(c^2*x^2 - 1)*log(-c*x + 1))*dilog(c*x)/c^2 -
integrate((2*(c^2*x^2 - 1)*log(-c*x + 1)^2 - (c^2*x^2 + 2*c*x)*log(-c*x + 1
))/x, x)/c^2) + 1/8*(4*c^2*x^2*dilog(c*x) - c^2*x^2 - 2*c*x + 2*(c^2*x^2 -
1)*log(-c*x + 1))*g/c^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral(h*x*dilog(c*x)*log(-c*x + 1) + g*x*dilog(c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(g + h\log(-cx + 1))\operatorname{Li}_2(cx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

```
[Out] Integral(x*(g + h*log(-c*x + 1))*polylog(2, c*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate((h*log(-c*x + 1) + g)*x*dilog(c*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(g+h\ln\left(1-c\,x\right)\right)\,\mathrm{polylog}(2,c\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g + h*log(1 - c*x))*polylog(2, c*x),x)
[Out] int(x*(g + h*log(1 - c*x))*polylog(2, c*x), x)
```

3.172 $\int (g+h\log(1-cx))\textbf{PolyLog}(2,cx)\,dx$

Optimal. Leaf size=167

$$-gx+3hx-\frac{g(1-cx)\log(1-cx)}{c}+\frac{3h(1-cx)\log(1-cx)}{c}-\frac{h(1-cx)\log^2(1-cx)}{c}-\frac{h\log(cx)\log^2(1-cx)}{c}$$

```
[Out] -g*x+3*h*x-g*(-c*x+1)*ln(-c*x+1)/c+3*h*(-c*x+1)*ln(-c*x+1)/c-h*(-c*x+1)*ln(
-c*x+1)^2/c-h*ln(c*x)*ln(-c*x+1)^2/c-h*x*polylog(2,c*x)-h*ln(-c*x+1)*polylo
g(2,c*x)/c+x*(g+h*ln(-c*x+1))*polylog(2,c*x)-2*h*ln(-c*x+1)*polylog(2,-c*x+
1)/c+2*h*polylog(3,-c*x+1)/c
```

Rubi [A]
time = 0.15, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {6735, 2411, 2407, 2332, 2333, 6820, 6874, 6721, 2436, 6731, 2443, 2481, 2421, 6724}

$$x\text{Li}_2(cx)(h\log(1-cx)+g)-\frac{g(1-cx)\log(1-cx)}{c}-hx\text{Li}_2(cx)+\frac{2h\text{Li}_3(1-cx)}{c}-\frac{h\text{Li}_2(cx)\log(1-cx)}{c}-\frac{2h\text{Li}_2(1-cx)\log(1-cx)}{c}-\frac{h(1-cx)\log^2(1-cx)}{c}-\frac{h\log(cx)\log^2(1-cx)}{c}+\frac{3h(1-cx)\log(1-cx)}{c}-gx+3hx$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*Log[1 - c*x])*PolyLog[2, c*x],x]
```

```
[Out] -(g*x) + 3*h*x - (g*(1 - c*x)*Log[1 - c*x])/c + (3*h*(1 - c*x)*Log[1 - c*x]
)/c - (h*(1 - c*x)*Log[1 - c*x]^2)/c - (h*Log[c*x]*Log[1 - c*x]^2)/c - h*x*
PolyLog[2, c*x] - (h*Log[1 - c*x]*PolyLog[2, c*x])/c + x*(g + h*Log[1 - c*x
])*PolyLog[2, c*x] - (2*h*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c + (2*h*PolyLo
g[3, 1 - c*x])/c
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2407

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(Log[(c_.)*(x_)^(n_.)]*(e_.
) + (d_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*x^n])^p*(d +
 e*Log[c*x^n])^q, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IntegerQ[p] &&
IntegerQ[q]
```

Rule 2411

```
Int[((a_.) + Log[v_]*(b_.))^(p_.)*((c_.) + Log[v_]*(d_.))^(q_.), x_Symbol]
:> Dist[1/Coeff[v, x, 1], Subst[Int[(a + b*Log[x])^p*(c + d*Log[x])^q, x],
x, v], x] /; FreeQ[{a, b, c, d, p, q}, x] && LinearQ[v, x] && NeQ[Coeff[v,
x, 0], 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6735

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*PolyLog[2, (c_.)*
((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x*(g + h*Log[f*(d + e*x)^n])*PolyL
og[2, c*(a + b*x)], x] + (Dist[b, Int[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*
c - b*c*x]*ExpandIntegrand[x/(a + b*x), x], x], x] - Dist[e*h*n, Int[PolyLo
g[2, c*(a + b*x)]*ExpandIntegrand[x/(d + e*x), x], x], x]) /; FreeQ[{a, b,
c, d, e, f, g, h, n}, x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int (g+h\log(1-cx))\text{Li}_2(cx)\,dx &= x(g+h\log(1-cx))\text{Li}_2(cx)+(ch)\int \left(-\frac{1}{c}-\frac{1}{c(-1+cx)}\right)\text{Li}_2(cx)\,dx+\\
&= x(g+h\log(1-cx))\text{Li}_2(cx)-\frac{\text{Subst}(\int \log(x)(g+h\log(x))\,dx,x,1-cx)}{c}\\
&= x(g+h\log(1-cx))\text{Li}_2(cx)-\frac{\text{Subst}\left(\int \left(g\log(x)+h\log^2(x)\right)\,dx,x,1-c\right.}{c}\\
&= x(g+h\log(1-cx))\text{Li}_2(cx)-\frac{g\text{Subst}(\int \log(x)\,dx,x,1-cx)}{c}-h\int \text{Li}_2(c\\
&= -gx-\frac{g(1-cx)\log(1-cx)}{c}-\frac{h(1-cx)\log^2(1-cx)}{c}-hx\text{Li}_2(cx)-\frac{h\,\text{lo}}{}\\
&= -gx+2hx-\frac{g(1-cx)\log(1-cx)}{c}+\frac{2h(1-cx)\log(1-cx)}{c}-\frac{h(1-cx}{}\\
&= -gx+3hx-\frac{g(1-cx)\log(1-cx)}{c}+\frac{3h(1-cx)\log(1-cx)}{c}-\frac{h(1-cx}{}\\
&= -gx+3hx-\frac{g(1-cx)\log(1-cx)}{c}+\frac{3h(1-cx)\log(1-cx)}{c}-\frac{h(1-cx}{}\\
&= -gx+3hx-\frac{g(1-cx)\log(1-cx)}{c}+\frac{3h(1-cx)\log(1-cx)}{c}-\frac{h(1-cx}{}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 149, normalized size = 0.89

$$
g\left(-x+\left(-\frac{1}{c}+x\right)\log(1-cx)+x\text{PolyLog}(2,cx)\right)+\frac{h\left(-2+3cx+3\log(1-cx)-3cx\log(1-cx)-\log^2(1-cx)+cx\log^2(1-cx)-\log(cx)\log^2(1-cx)+(-cx+(-1+cx)\log(1-cx))\text{PolyLog}(2,cx)-2\log(1-cx)\text{PolyLog}(2,1-cx)+2\text{PolyLog}(3,1-cx)\right)}{c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*Log[1 - c*x])*PolyLog[2, c*x],x]
```

```
[Out] g*(-x + (-c^(-1) + x)*Log[1 - c*x] + x*PolyLog[2, c*x]) + (h*(-2 + 3*c*x +
3*Log[1 - c*x] - 3*c*x*Log[1 - c*x] - Log[1 - c*x]^2 + c*x*Log[1 - c*x]^2 -
 Log[c*x]*Log[1 - c*x]^2 + (-(c*x) + (-1 + c*x)*Log[1 - c*x])*PolyLog[2, c*
x] - 2*Log[1 - c*x]*PolyLog[2, 1 - c*x] + 2*PolyLog[3, 1 - c*x]))/c
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$
\int \left(g+h\ln\left(-cx+1\right)\right)\text{polylog}\left(2,cx\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

```
[Out] int((g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="maxima")
```

```
[Out] -h*((c*x - (c*x - 1)*log(-c*x + 1))*dilog(c*x)/c - integrate(-(c*x*log(-c*x
 + 1) - (c*x - 1)*log(-c*x + 1)^2)/x, x)/c) + (c*x*dilog(c*x) - c*x + (c*x
- 1)*log(-c*x + 1))*g/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral(h*dilog(c*x)*log(-c*x + 1) + g*dilog(c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (g + h\log(-cx + 1))\operatorname{Li}_2(cx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

```
[Out] Integral((g + h*log(-c*x + 1))*polylog(2, c*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate((h*log(-c*x + 1) + g)*dilog(c*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g + h \ln(1 - cx)) \operatorname{polylog}(2, cx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*log(1 - c*x))*polylog(2, c*x),x)
[Out] int((g + h*log(1 - c*x))*polylog(2, c*x), x)
```

3.173 $\int \frac{(g+h\log(1-cx))\text{PolyLog}(2,cx)}{x}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{2}h\text{PolyLog}(2,cx)^2+g\text{PolyLog}(3,cx)$$

```
[Out] -1/2*h*polylog(2,c*x)^2+g*polylog(3,c*x)
```

Rubi [A]
time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6737, 6724, 6736}

$$g\text{Li}_3(cx)-\frac{1}{2}h\text{Li}_2(cx)^2$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x,x]
```

```
[Out] -1/2*(h*PolyLog[2, c*x]^2) + g*PolyLog[3, c*x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6736

```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> Simp[-P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

Rule 6737

```
Int[((Log[1 + (e_.)*(x_)]*(h_.) + (g_))*PolyLog[2, (c_.)*(x_)])/(x_), x_Sym
bol] :> Dist[g, Int[PolyLog[2, c*x]/x, x], x] + Dist[h, Int[(Log[1 + e*x]*P
olyLog[2, c*x])/x, x], x] /; FreeQ[{c, e, g, h}, x] && EqQ[c + e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(g+h\log(1-cx))\text{Li}_2(cx)}{x}\,dx &= g\int \frac{\text{Li}_2(cx)}{x}\,dx+h\int \frac{\log(1-cx)\text{Li}_2(cx)}{x}\,dx\\ &= -\frac{1}{2}h\text{Li}_2(cx)^2+g\text{Li}_3(cx)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$-\frac{1}{2}h\text{PolyLog}(2,cx)^2+g\text{PolyLog}(3,cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x,x]
[Out] -1/2*(h*PolyLog[2, c*x]^2) + g*PolyLog[3, c*x]
```

Maple [A]
time = 0.15, size = 19, normalized size = 0.95

method	result	size
default	$-\frac{h\,\text{polylog}(2,cx)^2}{2}+g\,\text{polylog}\left(3,cx\right)$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g+h*ln(-c*x+1))*polylog(2,c*x)/x,x,method=_RETURNVERBOSE)
[Out] -1/2*h*polylog(2,c*x)^2+g*polylog(3,c*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x,x, algorithm="maxima")
[Out] integrate((h*log(-c*x + 1) + g)*dilog(c*x)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x,x, algorithm="fricas")
[Out] integral((h*dilog(c*x)*log(-c*x + 1) + g*dilog(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(g+h\log{(-cx+1)})\operatorname{Li}_2(cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*ln(-c*x+1))*polylog(2,c*x)/x,x)
```

```
[Out] Integral((g + h*log(-c*x + 1))*polylog(2, c*x)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x,x, algorithm="giac")
```

```
[Out] integrate((h*log(-c*x + 1) + g)*dilog(c*x)/x, x)
```

Mupad [B]
time = 0.25, size = 18, normalized size = 0.90

$$g\,\mathrm{polylog}(3,c\,x)-\frac{h\,\mathrm{polylog}(2,c\,x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*log(1 - c*x))*polylog(2, c*x))/x,x)
```

```
[Out] g*polylog(3, c*x) - (h*polylog(2, c*x)^2)/2
```

3.174 $\int \frac{(g+h\log(1-cx))\textbf{PolyLog}(2,cx)}{x^2}\,dx$

Optimal. Leaf size=156

$$ch\log(cx)\log^2(1-cx)+\frac{\log(1-cx)(g+h\log(1-cx))}{x}+c(g+2h\log(1-cx))\log\left(1-\frac{1}{1-cx}\right)+ch\log(1-cx)$$

```
[Out] c*h*ln(c*x)*ln(-c*x+1)^2+ln(-c*x+1)*(g+h*ln(-c*x+1))/x+c*(g+2*h*ln(-c*x+1))
*ln(1-1/(-c*x+1))+c*h*ln(-c*x+1)*polylog(2,c*x)-(g+h*ln(-c*x+1))*polylog(2,
c*x)/x-2*c*h*polylog(2,1/(-c*x+1))+2*c*h*ln(-c*x+1)*polylog(2,-c*x+1)-c*h*p
olylog(3,c*x)-2*c*h*polylog(3,-c*x+1)
```

Rubi [A]
time = 0.18, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6738, 2483, 2458, 2379, 2438, 6724, 6731, 2443, 2481, 2421}

$$-\frac{\text{Li}_2(cx)(h\log(1-cx)+g)}{x}+\frac{\log(1-cx)(h\log(1-cx)+g)}{x}+c\log\left(1-\frac{1}{1-cx}\right)(2h\log(1-cx)+g)-2ch\text{Li}_2\left(\frac{1}{1-cx}\right)-ch\text{Li}_3(cx)-2ch\text{Li}_3(1-cx)+ch\text{Li}_2(cx)\log(1-cx)+2ch\text{Li}_2(1-cx)\log(1-cx)+ch\log(cx)\log^2(1-cx)$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^2,x]
```

```
[Out] c*h*Log[c*x]*Log[1 - c*x]^2 + (Log[1 - c*x]*(g + h*Log[1 - c*x]))/x + c*(g
+ 2*h*Log[1 - c*x])*Log[1 - (1 - c*x)^(-1)] + c*h*Log[1 - c*x]*PolyLog[2, c
*x] - ((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x - 2*c*h*PolyLog[2, (1 - c*x)
^(-1)] + 2*c*h*Log[1 - c*x]*PolyLog[2, 1 - c*x] - c*h*PolyLog[3, c*x] - 2*c
*h*PolyLog[3, 1 - c*x]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*
(a + b*Log[c*(d + e*x)^n])*((f + g*Log[c*(d + e*x)^n])/(m + 1)), x] - Dist[
e*(n/(m + 1)), Int[(x^(m + 1)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d +
e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, m}, x] && NeQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6738

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[e*h*(n/(m + 1)), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(g+h\log(1-cx))\mathrm{Li}_2(cx)}{x^2}\,dx &= -\frac{(g+h\log(1-cx))\mathrm{Li}_2(cx)}{x}-(ch)\int\left(\frac{\mathrm{Li}_2(cx)}{x}-\frac{c\mathrm{Li}_2(cx)}{-1+cx}\right)dx-\int \frac{\mathrm{lo}}{} \\ &= \frac{\log(1-cx)(g+h\log(1-cx))}{x}-\frac{(g+h\log(1-cx))\mathrm{Li}_2(cx)}{x}+c\int\frac{g+h}{x} \\ &= \frac{\log(1-cx)(g+h\log(1-cx))}{x}+ch\log(1-cx)\mathrm{Li}_2(cx)-\frac{(g+h\log(1-}{x} \\ &= ch\log(cx)\log^2(1-cx)+\frac{\log(1-cx)(g+h\log(1-cx))}{x}+ch\log(1-cx \\ &= cg\log(x)+ch\log(cx)\log^2(1-cx)+\frac{\log(1-cx)(g+h\log(1-cx))}{x}-\frac{c(}{} \\ &= cg\log(x)-\frac{1}{2}ch\log^2(1-cx)+ch\log(cx)\log^2(1-cx)+\frac{\log(1-cx)(g+}{x} \\ &= cg\log(x)-\frac{1}{2}ch\log^2(1-cx)+ch\log(cx)\log^2(1-cx)+\frac{\log(1-cx)(g+}{x}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 150, normalized size = 0.96

$$\frac{g(cx\log(x)+(1-cx)\log(1-cx)-\mathrm{PolyLog}(2,cx))}{x}+h\left(2c\log(cx)\log(1-cx)-c\log^2(1-cx)+\frac{\log^2(1-cx)}{x}+c\log(cx)\log^2(1-cx)+\frac{(-1+cx)\log(1-cx)\mathrm{PolyLog}(2,cx)}{x}+2c(1+\log(1-cx))\mathrm{PolyLog}(2,1-cx)-c\mathrm{PolyLog}(3,cx)-2c\mathrm{PolyLog}(3,1-cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^2,x]
```

```
[Out] (g*(c*x*Log[x] + (1 - c*x)*Log[1 - c*x] - PolyLog[2, c*x]))/x + h*(2*c*Log[
c*x]*Log[1 - c*x] - c*Log[1 - c*x]^2 + Log[1 - c*x]^2/x + c*Log[c*x]*Log[1
```

```
- c*x]^2 + ((-1 + c*x)*Log[1 - c*x]*PolyLog[2, c*x])/x + 2*c*(1 + Log[1 - c
*x])*PolyLog[2, 1 - c*x] - c*PolyLog[3, c*x] - 2*c*PolyLog[3, 1 - c*x])
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(g+h\ln(-cx+1))\,\mathrm{polylog}(2,cx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g+h*ln(-c*x+1))*polylog(2,c*x)/x^2,x)
[Out] int((g+h*ln(-c*x+1))*polylog(2,c*x)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^2,x, algorithm="maxima")
[Out] (c*log(x) - ((c*x - 1)*log(-c*x + 1) + dilog(c*x))/x)*g + h*integrate(dilog
(c*x)*log(-c*x + 1)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^2,x, algorithm="fricas")
[Out] integral((h*dilog(c*x)*log(-c*x + 1) + g*dilog(c*x))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g+h\log(-cx+1))\,\mathrm{Li}_2(cx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*ln(-c*x+1))*polylog(2,c*x)/x**2,x)
[Out] Integral((g + h*log(-c*x + 1))*polylog(2, c*x)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^2,x, algorithm="giac")
[Out] integrate((h*log(-c*x + 1) + g)*dilog(c*x)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g+h\,\ln(1-c\,x))\ \mathrm{polylog}(2,c\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*log(1 - c*x))*polylog(2, c*x))/x^2,x)
[Out] int(((g + h*log(1 - c*x))*polylog(2, c*x))/x^2, x)
```

3.175 $\int \frac{(g+h\log(1-cx))\text{PolyLog}(2,cx)}{x^3}\,dx$

Optimal. Leaf size=266

$$-c^2h\log(x)+\frac{1}{2}c^2h\log(1-cx)-\frac{ch\log(1-cx)}{2x}+\frac{1}{2}c^2h\log(cx)\log^2(1-cx)+\frac{\log(1-cx)(g+h\log(1-cx))}{4x^2}-$$

```
[Out] -c^2*h*ln(x)+1/2*c^2*h*ln(-c*x+1)-1/2*c*h*ln(-c*x+1)/x+1/2*c^2*h*ln(c*x)*ln
(-c*x+1)^2+1/4*ln(-c*x+1)*(g+h*ln(-c*x+1))/x^2-1/4*c*(-c*x+1)*(g+2*h*ln(-c*
x+1))/x+1/4*c^2*(g+2*h*ln(-c*x+1))*ln(1-1/(-c*x+1))+1/2*c*h*polylog(2,c*x)/
x+1/2*c^2*h*ln(-c*x+1)*polylog(2,c*x)-1/2*(g+h*ln(-c*x+1))*polylog(2,c*x)/x
^2-1/2*c^2*h*polylog(2,1/(-c*x+1))+c^2*h*ln(-c*x+1)*polylog(2,-c*x+1)-1/2*c
^2*h*polylog(3,c*x)-c^2*h*polylog(3,-c*x+1)
```

Rubi [A]
time = 0.26, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 17, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.850$, Rules used = {6738, 2483, 2458, 2389, 2379, 2438, 2351, 31, 6726, 2442, 36, 29, 6724, 6731, 2443, 2481, 2421}

$$\frac{1}{4}c^2\log\left(1-\frac{1}{1-cx}\right)(2h\log(1-cx)+g)-\frac{1}{2}c^2h\text{Li}_2\left(\frac{1}{1-cx}\right)-\frac{1}{2}c^2h\text{Li}_3(cx)-c^2h\text{Li}_3(1-cx)+\frac{1}{2}c^2h\text{Li}_2(cx)\log(1-cx)+c^2h\text{Li}_2(1-cx)\log(1-cx)+\frac{1}{2}c^2h\log(cx)\log^2(1-cx)-c^2h\log(x)+\frac{1}{2}c^2h\log(1-cx)-\frac{\text{Li}_2(cx)(h\log(1-cx)+g)}{2x^2}+\frac{\log(1-cx)(h\log(1-cx)+g)}{4x^2}-\frac{c(1-cx)(2h\log(1-cx)+g)}{4x}+\frac{ch\text{Li}_2(cx)}{2x}-\frac{ch\log(1-cx)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^3,x]
```

```
[Out] -(c^2*h*Log[x]) + (c^2*h*Log[1 - c*x])/2 - (c*h*Log[1 - c*x])/(2*x) + (c^2*
h*Log[c*x]*Log[1 - c*x]^2)/2 + (Log[1 - c*x]*(g + h*Log[1 - c*x]))/(4*x^2)
- (c*(1 - c*x)*(g + 2*h*Log[1 - c*x]))/(4*x) + (c^2*(g + 2*h*Log[1 - c*x])*
Log[1 - (1 - c*x)^(-1)])/4 + (c*h*PolyLog[2, c*x])/(2*x) + (c^2*h*Log[1 - c
*x]*PolyLog[2, c*x])/2 - ((g + h*Log[1 - c*x])*PolyLog[2, c*x])/(2*x^2) - (
c^2*h*PolyLog[2, (1 - c*x)^(-1)])/2 + c^2*h*Log[1 - c*x]*PolyLog[2, 1 - c*x
] - (c^2*h*PolyLog[3, c*x])/2 - c^2*h*PolyLog[3, 1 - c*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*
(a + b*Log[c*(d + e*x)^n])*((f + g*Log[c*(d + e*x)^n])/(m + 1)), x] - Dist[
e*(n/(m + 1)), Int[(x^(m + 1)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d +
e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, m}, x] && NeQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
```

```
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6738

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[e*h*(n/(m + 1)), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(g+h\log(1-cx))\text{Li}_2(cx)}{x^3}\,dx = -\frac{(g+h\log(1-cx))\text{Li}_2(cx)}{2x^2}-\frac{1}{2}\int \frac{\log(1-cx)(g+h\log(1-cx))}{x^3}\,dx -$$

$$= \frac{\log(1-cx)(g+h\log(1-cx))}{4x^2}-\frac{(g+h\log(1-cx))\text{Li}_2(cx)}{2x^2}+\frac{1}{4}c\int \frac{g+}{}$$

$$= \frac{\log(1-cx)(g+h\log(1-cx))}{4x^2}+\frac{ch\text{Li}_2(cx)}{2x}+\frac{1}{2}c^2h\log(1-cx)\text{Li}_2(cx) -$$

$$= -\frac{ch\log(1-cx)}{2x}+\frac{1}{2}c^2h\log(cx)\log^2(1-cx)+\frac{\log(1-cx)(g+h\log(1-}{4x^2}$$

$$= -\frac{3ch\log(1-cx)}{4x}+\frac{1}{2}c^2h\log(cx)\log^2(1-cx)-\frac{c(1-cx)(g+h\log(1-}{4x}$$

$$= \frac{1}{4}c^2g\log(x)-\frac{3}{4}c^2h\log(x)+\frac{1}{2}c^2h\log(1-cx)-\frac{3ch\log(1-cx)}{4x}-\frac{1}{8}c^2h\,l$$

$$= \frac{1}{4}c^2g\log(x)-c^2h\log(x)+\frac{3}{4}c^2h\log(1-cx)-\frac{3ch\log(1-cx)}{4x}-\frac{1}{8}c^2h\log$$

Mathematica [A]
time = 0.18, size = 238, normalized size = 0.89

$$\frac{g(-cx+c^2x^2\log(x)+\log(1-cx)-c^2x^2\log(1-cx)-2\text{PolyLog}(2,cx))}{4x^2}+\frac{1}{4}h\left(-2c^2\log(x)-2c^2\log(cx)+4c^2\log(1-cx)-\frac{4c\log(1-cx)}{x}+2c^2\log(cx)\log(1-cx)-c^2\log^2(1-cx)+\frac{\log^2(1-cx)}{x^2}+2c^2\log(cx)\log^2(1-cx)+\frac{2(cx+(-1+c^2x^2)\log(1-cx))\text{PolyLog}(2,cx)}{x^2}+2c^2(1+2\log(1-cx))\text{PolyLog}(2,1-cx)-2c^2\text{PolyLog}(3,cx)-4c^2\text{PolyLog}(3,1-cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^3,x]
```

```
[Out] (g*(-(c*x) + c^2*x^2*Log[x] + Log[1 - c*x] - c^2*x^2*Log[1 - c*x] - 2*PolyL
og[2, c*x]))/(4*x^2) + (h*(-2*c^2*Log[x] - 2*c^2*Log[c*x] + 4*c^2*Log[1 - c
```

```
*x] - (4*c*Log[1 - c*x])/x + 2*c^2*Log[c*x]*Log[1 - c*x] - c^2*Log[1 - c*x]
^2 + Log[1 - c*x]^2/x^2 + 2*c^2*Log[c*x]*Log[1 - c*x]^2 + (2*(c*x + (-1 + c
^2*x^2)*Log[1 - c*x])*PolyLog[2, c*x])/x^2 + 2*c^2*(1 + 2*Log[1 - c*x])*Pol
yLog[2, 1 - c*x] - 2*c^2*PolyLog[3, c*x] - 4*c^2*PolyLog[3, 1 - c*x]))/4
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(g + h \ln(-cx + 1)) \operatorname{polylog}(2, cx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g+h*ln(-c*x+1))*polylog(2,c*x)/x^3,x)
[Out] int((g+h*ln(-c*x+1))*polylog(2,c*x)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^3,x, algorithm="maxima")
[Out] 1/4*(c^2*log(x) - (c*x + (c^2*x^2 - 1)*log(-c*x + 1) + 2*dilog(c*x))/x^2)*g
 + h*integrate(dilog(c*x)*log(-c*x + 1)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^3,x, algorithm="fricas")
[Out] integral((h*dilog(c*x)*log(-c*x + 1) + g*dilog(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g + h \log(-cx + 1)) \operatorname{Li}_2(cx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*ln(-c*x+1))*polylog(2,c*x)/x**3,x)
[Out] Integral((g + h*log(-c*x + 1))*polylog(2, c*x)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^3,x, algorithm="giac")
[Out] integrate((h*log(-c*x + 1) + g)*dilog(c*x)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g+h\,\ln(1-c\,x))\,\,\mathrm{polylog}(2,c\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*log(1 - c*x))*polylog(2, c*x))/x^3,x)
[Out] int(((g + h*log(1 - c*x))*polylog(2, c*x))/x^3, x)
```

3.176 $\int \frac{(g+h\log(1-cx))\text{PolyLog}(2,cx)}{x^4}\,dx$

Optimal. Leaf size=340

$$\frac{7c^2h}{36x}-\frac{3}{4}c^3h\log(x)+\frac{19}{36}c^3h\log(1-cx)-\frac{ch\log(1-cx)}{12x^2}-\frac{c^2h\log(1-cx)}{3x}+\frac{1}{3}c^3h\log(cx)\log^2(1-cx)+\frac{\log(1-$$

```
[Out] 7/36*c^2*h/x-3/4*c^3*h*ln(x)+19/36*c^3*h*ln(-c*x+1)-1/12*c*h*ln(-c*x+1)/x^2
-1/3*c^2*h*ln(-c*x+1)/x+1/3*c^3*h*ln(c*x)*ln(-c*x+1)^2+1/9*ln(-c*x+1)*(g+h*
ln(-c*x+1))/x^3-1/18*c*(g+2*h*ln(-c*x+1))/x^2-1/9*c^2*(-c*x+1)*(g+2*h*ln(-c
*x+1))/x+1/9*c^3*(g+2*h*ln(-c*x+1))*ln(1-1/(-c*x+1))+1/6*c*h*polylog(2,c*x)
/x^2+1/3*c^2*h*polylog(2,c*x)/x+1/3*c^3*h*ln(-c*x+1)*polylog(2,c*x)-1/3*(g+
h*ln(-c*x+1))*polylog(2,c*x)/x^3-2/9*c^3*h*polylog(2,1/(-c*x+1))+2/3*c^3*h*
ln(-c*x+1)*polylog(2,-c*x+1)-1/3*c^3*h*polylog(3,c*x)-2/3*c^3*h*polylog(3,-
c*x+1)
```

Rubi [A]
time = 0.35, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 19, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.950$, Rules used = {6738, 2483, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46, 6726, 2442, 36, 29, 6724, 6731, 2443, 2481, 2421}

$$\frac{1}{9}c^3\log\left(1-\frac{1}{1-cx}\right)(2h\log(1-cx)+g)-\frac{2}{9}c^3h\text{Li}_2\left(\frac{1}{1-cx}\right)-\frac{1}{3}c^3h\text{Li}_3(cx)-\frac{2}{3}c^3h\text{Li}_3(1-cx)+\frac{1}{3}c^3h\text{Li}_2(cx)\log(1-cx)+\frac{2}{3}c^3h\text{Li}_2(1-cx)\log(1-cx)+\frac{1}{3}c^3h\log(cx)\log^2(1-cx)-\frac{3}{4}c^3h\log(x)+\frac{19}{36}c^3h\log(1-cx)-\frac{c^2(1-cx)(2h\log(1-cx)+g)}{9x}+\frac{c^2h\text{Li}_2(cx)}{3x}+\frac{7c^2h}{36x}-\frac{c^2h\log(1-cx)}{3x}-\frac{\text{Li}_2(cx)(h\log(1-cx)+g)}{3x^3}+\frac{\log(1-cx)(h\log(1-cx)+g)}{9x^3}-\frac{c(2h\log(1-cx)+g)}{18x^2}+\frac{ch\text{Li}_2(cx)}{6x^2}-\frac{ch\log(1-cx)}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^4,x]
```

```
[Out] (7*c^2*h)/(36*x) - (3*c^3*h*Log[x])/4 + (19*c^3*h*Log[1 - c*x])/36 - (c*h*L
og[1 - c*x])/(12*x^2) - (c^2*h*Log[1 - c*x])/(3*x) + (c^3*h*Log[c*x]*Log[1
- c*x]^2)/3 + (Log[1 - c*x]*(g + h*Log[1 - c*x]))/(9*x^3) - (c*(g + 2*h*Log
[1 - c*x]))/(18*x^2) - (c^2*(1 - c*x)*(g + 2*h*Log[1 - c*x]))/(9*x) + (c^3*
(g + 2*h*Log[1 - c*x])*Log[1 - (1 - c*x)^(-1)])/9 + (c*h*PolyLog[2, c*x])/(
6*x^2) + (c^2*h*PolyLog[2, c*x])/(3*x) + (c^3*h*Log[1 - c*x]*PolyLog[2, c*x
])/3 - ((g + h*Log[1 - c*x])*PolyLog[2, c*x])/(3*x^3) - (2*c^3*h*PolyLog[2,
 (1 - c*x)^(-1)])/9 + (2*c^3*h*Log[1 - c*x]*PolyLog[2, 1 - c*x])/3 - (c^3*h
*PolyLog[3, c*x])/3 - (2*c^3*h*PolyLog[3, 1 - c*x])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
```

```
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2483

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(c_.)
*((d_) + (e_.)*(x_))^(n_.)]*(g_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*
(a + b*Log[c*(d + e*x)^n])*((f + g*Log[c*(d + e*x)^n])/(m + 1)), x] - Dist[
```

```
e*(n/(m + 1)), Int[(x^(m + 1)*(b*f + a*g + 2*b*g*Log[c*(d + e*x)^n]))/(d +
e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, m}, x] && NeQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6738

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[e*h*(n/(m + 1)), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(g+h\log(1-cx))\text{Li}_2(cx)}{x^4}\,dx = -\frac{(g+h\log(1-cx))\text{Li}_2(cx)}{3x^3} - \frac{1}{3}\int \frac{\log(1-cx)(g+h\log(1-cx))}{x^4}\,dx$$

$$= \frac{\log(1-cx)(g+h\log(1-cx))}{9x^3} - \frac{(g+h\log(1-cx))\text{Li}_2(cx)}{3x^3} + \frac{1}{9}c\int g$$

$$= \frac{\log(1-cx)(g+h\log(1-cx))}{9x^3} + \frac{ch\text{Li}_2(cx)}{6x^2} + \frac{c^2h\text{Li}_2(cx)}{3x} + \frac{1}{3}c^3h\log(1$$

$$= -\frac{ch\log(1-cx)}{12x^2} - \frac{c^2h\log(1-cx)}{3x} + \frac{1}{3}c^3h\log(cx)\log^2(1-cx) + \frac{\log(1}{}$$

$$= -\frac{5ch\log(1-cx)}{36x^2} - \frac{4c^2h\log(1-cx)}{9x} + \frac{1}{3}c^3h\log(cx)\log^2(1-cx) - \frac{c(g}{}$$

$$= \frac{c^2h}{12x} - \frac{5}{12}c^3h\log(x) + \frac{5}{12}c^3h\log(1-cx) - \frac{5ch\log(1-cx)}{36x^2} - \frac{4c^2h\log(}{9x}$$

$$= \frac{7c^2h}{36x} + \frac{1}{9}c^3g\log(x) - \frac{3}{4}c^3h\log(x) + \frac{23}{36}c^3h\log(1-cx) - \frac{5ch\log(1-cx}{36x^2}$$

$$= \frac{7c^2h}{36x} + \frac{1}{9}c^3g\log(x) - \frac{3}{4}c^3h\log(x) + \frac{23}{36}c^3h\log(1-cx) - \frac{5ch\log(1-cx}{36x^2}$$

Mathematica [A]
time = 0.19, size = 301, normalized size = 0.89

$$-\frac{g(cx(1+2cx)-2c^3x^3\log(x)+2(-1+c^3x^3)\log(1-cx)+6\text{PolyLog}(2,cx))}{18x^3}+\frac{h(7c^2x^2-4c^3x^3-15c^3x^3\log(x)-12c^3x^3\log(cx)-7cx\log(1-cx)-20c^2x^2\log(1-cx)+27c^3x^3\log(1-cx)+8c^3x^3\log(cx)\log(1-cx)+4\log^2(1-cx)-4c^3x^3\log^2(1-cx)+12c^3x^3\log(cx)\log^2(1-cx)+6(cx(1+2cx)+2(-1+c^3x^3)\log(1-cx))\text{PolyLog}(2,cx)+8c^3x^3(1+3\log(1-cx))\text{PolyLog}(2,1-cx)-12c^3x^3\text{PolyLog}(3,cx)-24c^3x^3\text{PolyLog}(3,1-cx))}{36x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^4,x]
```

```
[Out] -1/18*(g*(c*x*(1 + 2*c*x) - 2*c^3*x^3*Log[x] + 2*(-1 + c^3*x^3)*Log[1 - c*x
] + 6*PolyLog[2, c*x]))/x^3 + (h*(7*c^2*x^2 - 4*c^3*x^3 - 15*c^3*x^3*Log[x]
 - 12*c^3*x^3*Log[c*x] - 7*c*x*Log[1 - c*x] - 20*c^2*x^2*Log[1 - c*x] + 27*
c^3*x^3*Log[1 - c*x] + 8*c^3*x^3*Log[c*x]*Log[1 - c*x] + 4*Log[1 - c*x]^2 -
 4*c^3*x^3*Log[1 - c*x]^2 + 12*c^3*x^3*Log[c*x]*Log[1 - c*x]^2 + 6*(c*x*(1
+ 2*c*x) + 2*(-1 + c^3*x^3)*Log[1 - c*x])*PolyLog[2, c*x] + 8*c^3*x^3*(1 +
3*Log[1 - c*x])*PolyLog[2, 1 - c*x] - 12*c^3*x^3*PolyLog[3, c*x] - 24*c^3*x
^3*PolyLog[3, 1 - c*x]))/(36*x^3)
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(g+h\ln(-cx+1))\,\text{polylog}(2,cx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g+h*ln(-c*x+1))*polylog(2,c*x)/x^4,x)
```

```
[Out] int((g+h*ln(-c*x+1))*polylog(2,c*x)/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/18*(2*c^3*log(x) - (2*c^2*x^2 + c*x + 2*(c^3*x^3 - 1)*log(-c*x + 1) + 6*d
ilog(c*x))/x^3)*g + h*integrate(dilog(c*x)*log(-c*x + 1)/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral((h*dilog(c*x)*log(-c*x + 1) + g*dilog(c*x))/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g + h\log(-cx + 1))\operatorname{Li}_2(cx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*ln(-c*x+1))*polylog(2,c*x)/x**4,x)
```

```
[Out] Integral((g + h*log(-c*x + 1))*polylog(2, c*x)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate((h*log(-c*x + 1) + g)*dilog(c*x)/x^4, x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g+h\,\ln\left(1-c\,x\right))\,\mathrm{polylog}(2,c\,x)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*log(1 - c*x))*polylog(2, c*x))/x^4,x)
[Out] int(((g + h*log(1 - c*x))*polylog(2, c*x))/x^4, x)
```

3.177 $\int x^2(g+h\log(f(d+ex)^n))\,\mathbf{PolyLog}(2,c(a+bx))\,dx$

Optimal. Leaf size=2995

result too large to display

```
[Out] -1/3*a^2*g*x/b^2+1/3*a^3*g*polylog(2,c*(b*x+a))/b^3-1/9*h*n*x^3*polylog(2,c
*(b*x+a))+1/6*a*(-a*c+1)*h*(e*x+d)*ln(f*(e*x+d)^n)/b^2/c/e-5/36*(-a*c+1)^2*
d*h*n*ln(-b*c*x-a*c+1)/b^2/c^2/e+4/9*d^2*h*n*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1
)/b/c/e^2+1/18*(-a*c+1)*d^2*h*n*ln(e*x+d)/b/c/e^2+1/6*a*d^2*h*n*ln(-b*c*x-a
*c+1)*ln(b*c*(e*x+d)/(-a*c*e+b*c*d+e))/b/e^2+1/3*a^2*d*h*n*ln(-b*c*x-a*c+1)
*ln(b*c*(e*x+d)/(-a*c*e+b*c*d+e))/b^2/e-2/27*h*n*x^3*ln(-b*c*x-a*c+1)+1/12*
a*x^2*(g+h*ln(f*(e*x+d)^n))/b+1/27*h*n*x^3+1/9*x^3*ln(-b*c*x-a*c+1)*(g+h*ln
(f*(e*x+d)^n))+1/3*x^3*(g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))-1/9*a^3*h
*n*polylog(2,c*(b*x+a))/b^3+1/9*d^3*h*n*polylog(2,e*(-b*c*x-a*c+1)/(-a*c*e+
b*c*d+e))/e^3+1/3*a^3*h*n*polylog(3,b*(e*x+d)/(-a*e+b*d))/b^3-1/3*d^3*h*n*p
olylog(3,b*(e*x+d)/(-a*e+b*d))/e^3+1/3*a^3*h*n*polylog(3,1-c*(b*x+a))/b^3-1
/3*d^3*h*n*polylog(3,1-c*(b*x+a))/e^3+1/3*a^3*h*n*polylog(3,-e*(1-c*(b*x+a)
)/b/c/(e*x+d))/b^3-1/3*d^3*h*n*polylog(3,-e*(1-c*(b*x+a))/b/c/(e*x+d))/e^3-
1/3*a^3*h*n*polylog(3,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/b^3+1/3*d^3*h*n*p
olylog(3,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/e^3-1/9*(-a*c+1)^2*g*x/b^2/c^2
+7/9*a^2*h*n*x/b^2+13/27*d^2*h*n*x/e^2-1/9*a*h*n*x^2/b-19/216*d*h*n*x^2/e+1
/3*a*d*h*n*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b^2/c/e-1/27*x^3*(g+h*ln(f*(e*x+
d)^n))-1/3*a^3*h*n*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))*polylog(2,(-a*e+b
*d)*(1-c*(b*x+a))/b/(e*x+d))/b^3+1/3*d^3*h*n*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(
b*x+a)))*polylog(2,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/e^3+2/27*(-a*c+1)^3*
h*n*ln(-b*c*x-a*c+1)/b^3/c^3+5/36*a*h*n*x^2*ln(-b*c*x-a*c+1)/b+5/36*d*h*n*x
^2*ln(-b*c*x-a*c+1)/e+1/9*d^3*h*n*ln(-b*c*x-a*c+1)*ln(b*c*(e*x+d)/(-a*c*e+b
*c*d+e))/e^3-1/3*a^2*h*(e*x+d)*ln(f*(e*x+d)^n)/b^2/e-1/3*a^2*(-a*c+1)*ln(e*
(-b*c*x-a*c+1)/(-a*c*e+b*c*d+e))*(g+h*ln(f*(e*x+d)^n))/b^3/c+1/6*a*(-a*c+1)
^2*ln(e*(-b*c*x-a*c+1)/(-a*c*e+b*c*d+e))*(g+h*ln(f*(e*x+d)^n))/b^3/c^2-1/6*
a^3*h*n*(ln(c*(b*x+a))+ln((-a*c*e+b*c*d+e)/b/c/(e*x+d))-ln((-a*c*e+b*c*d+e)
*(b*x+a)/b/(e*x+d)))*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))^2/b^3+1/6*d^3*h
*n*(ln(c*(b*x+a))+ln((-a*c*e+b*c*d+e)/b/c/(e*x+d))-ln((-a*c*e+b*c*d+e)*(b*x
+a)/b/(e*x+d)))*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))^2/e^3+1/6*a^3*h*n*(l
n(c*(b*x+a))-ln(-e*(b*x+a)/(-a*e+b*d)))*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+
a)))+ln(1-c*(b*x+a)))^2/b^3-1/6*d^3*h*n*(ln(c*(b*x+a))-ln(-e*(b*x+a)/(-a*e+
b*d)))*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))^2/e^3+1/3*d
^3*h*n*ln(e*x+d)*polylog(2,c*(b*x+a))/e^3-1/3*a^3*h*n*(ln(b*(e*x+d)/(-a*e+b
*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))*polylog(2,b*(e*x+d)/(-a*e+b*d))/b^3+1/3
*d^3*h*n*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))*polylog(2
,b*(e*x+d)/(-a*e+b*d))/e^3-1/3*a^3*h*n*(ln(e*x+d)-ln(b*(e*x+d)/(-a*e+b*d)/(
1-c*(b*x+a))))*polylog(2,1-c*(b*x+a))/b^3+1/3*d^3*h*n*(ln(e*x+d)-ln(b*(e*x+
d)/(-a*e+b*d)/(1-c*(b*x+a))))*polylog(2,1-c*(b*x+a))/e^3+1/3*a^3*h*n*ln(b*(
```

```
e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))*polylog(2,-e*(1-c*(b*x+a))/b/c/(e*x+d))/b^
3-1/3*d^3*h*n*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))*polylog(2,-e*(1-c*(b*x
+a))/b/c/(e*x+d))/e^3-1/3*a^3*h*(n*ln(e*x+d)-ln(f*(e*x+d)^n))*polylog(2,c*(
b*x+a))/b^3-1/27*d^3*h*n*ln(e*x+d)/e^3-1/18*(-a*c+1)*x^2*(g+h*ln(f*(e*x+d)^
n))/b/c+1/3*a^2*x*ln(-b*c*x-a*c+1)*(g+h*ln(f*(e*x+d)^n))/b^2-1/6*a*x^2*ln(-
b*c*x-a*c+1)*(g+h*ln(f*(e*x+d)^n))/b-1/9*(-a*c+1)^3*ln(e*(-b*c*x-a*c+1)/(-a
*c*e+b*c*d+e))*(g+h*ln(f*(e*x+d)^n))/b^3/c^3-7/36*(-a*c+1)*d*h*n*x/b/c/e+5/
12*a*d*h*n*x/b/e-1/3*a*d^2*h*n*polylog(2,c*(b*x+a))/b/e^2-1/6*a^2*d*h*n*pol
ylog(2,c*(b*x+a))/b^2/e+1/6*a*d^2*h*n*polylog(2,e*(-b*c*x-a*c+1)/(-a*c*e+b*
c*d+e))/b/e^2+1/3*a^2*d*h*n*polylog(2,e*(-b*c*x-a*c+1)/(-a*c*e+b*c*d+e))/b^
2/e-1/3*a^2*(-a*c+1)*h*n*polylog(2,b*c*(e*x+d)/(-a*c*e+b*c*d+e))/b^3/c+1/6*
a*(-a*c+1)^2*h*n*polylog(2,b*c*(e*x+d)/(-a*c*e+b*c*d+e))/b^3/c^2+1/6*a*(-a*
c+1)*g*x/b^2/c+5/27*(-a*c+1)^2*h*n*x/b^2/c^2+7/108*(-a*c+1)*h*n*x^2/b/c-1/3
*d^2*h*n*x*polylog(2,c*(b*x+a))/e^2+1/6*d*h*n*x^2*polylog(2,c*(b*x+a))/e-1/
9*(-a*c+1)^3*h*n*polylog(2,b*c*(e*x+d)/(-a*c*e+b*c*d+e))/b^3/c^3-5/36*a*(-a
*c+1)^2*h*n*ln(-b*c*x-a*c+1)/b^3/c^2+4/9*a^2*h*n*(-b*c*x-a*c+1)*ln(-b*c*x-a
*c+1)/b^3/c-1/12*a*d^2*h*n*ln(e*x+d)/b/e^2-1/9*(-a*c+1)^2*h*(e*x+d)*ln(f*(e
*x+d)^n)/b^2/c^2/e-1/3*a^3*h*n*ln(c*(b*x+a))*ln(e*x+d)*ln(1-c*(b*x+a))/b^3+
1/3*d^3*h*n*ln(c*(b*x+a))*ln(e*x+d)*ln(1-c*(b*x+a))/e^3-11/36*a*(-a*c+1)*h*
n*x/b^2/c
```

Rubi [A]
time = 3.10, antiderivative size = 2995, normalized size of antiderivative = 1.00, number of steps used = 108, number of rules used = 20, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.741$, Rules used = {6738, 2479, 45, 2463, 2436, 2332, 2441, 2440, 2438, 2489, 2442, 2490, 2488, 2487, 2485, 6730, 2494, 2468, 6733, 6732}

Too large to display

Antiderivative was successfully verified.

```
[In] Int[x^2*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] -1/3*(a^2*g*x)/b^2 + (a*(1 - a*c)*g*x)/(6*b^2*c) - ((1 - a*c)^2*g*x)/(9*b^2
*c^2) + (7*a^2*h*n*x)/(9*b^2) - (11*a*(1 - a*c)*h*n*x)/(36*b^2*c) + (5*(1 -
 a*c)^2*h*n*x)/(27*b^2*c^2) + (13*d^2*h*n*x)/(27*e^2) + (5*a*d*h*n*x)/(12*b
*e) - (7*(1 - a*c)*d*h*n*x)/(36*b*c*e) - (a*h*n*x^2)/(9*b) + (7*(1 - a*c)*h
*n*x^2)/(108*b*c) - (19*d*h*n*x^2)/(216*e) + (h*n*x^3)/27 - (5*a*(1 - a*c)^
2*h*n*Log[1 - a*c - b*c*x])/(36*b^3*c^2) + (2*(1 - a*c)^3*h*n*Log[1 - a*c -
 b*c*x])/(27*b^3*c^3) - (5*(1 - a*c)^2*d*h*n*Log[1 - a*c - b*c*x])/(36*b^2*
c^2*e) + (5*a*h*n*x^2*Log[1 - a*c - b*c*x])/(36*b) + (5*d*h*n*x^2*Log[1 - a
*c - b*c*x])/(36*e) - (2*h*n*x^3*Log[1 - a*c - b*c*x])/27 + (4*a^2*h*n*(1 -
 a*c - b*c*x)*Log[1 - a*c - b*c*x])/(9*b^3*c) + (4*d^2*h*n*(1 - a*c - b*c*x
)*Log[1 - a*c - b*c*x])/(9*b*c*e^2) + (a*d*h*n*(1 - a*c - b*c*x)*Log[1 - a*
c - b*c*x])/(3*b^2*c*e) - (d^3*h*n*Log[d + e*x])/(27*e^3) - (a*d^2*h*n*Log[
d + e*x])/(12*b*e^2) + ((1 - a*c)*d^2*h*n*Log[d + e*x])/(18*b*c*e^2) + (d^3
```

*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(9*e^3)
 + (a*d^2*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)]
)/(6*b*e^2) + (a^2*d*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d +
e - a*c*e)])/(3*b^2*e) - (a^2*h*(d + e*x)*Log[f*(d + e*x)^n])/(3*b^2*e) + (
a*(1 - a*c)*h*(d + e*x)*Log[f*(d + e*x)^n])/(6*b^2*c*e) - ((1 - a*c)^2*h*(d
 + e*x)*Log[f*(d + e*x)^n])/(9*b^2*c^2*e) + (a*x^2*(g + h*Log[f*(d + e*x)^n
]))/(12*b) - ((1 - a*c)*x^2*(g + h*Log[f*(d + e*x)^n]))/(18*b*c) - (x^3*(g
+ h*Log[f*(d + e*x)^n]))/27 + (a^2*x*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d +
 e*x)^n]))/(3*b^2) - (a*x^2*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + e*x)^n])
)/(6*b) + (x^3*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + e*x)^n]))/9 - (a^2*(1
 - a*c)*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*
x)^n]))/(3*b^3*c) + (a*(1 - a*c)^2*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a
*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(6*b^3*c^2) - ((1 - a*c)^3*Log[(e*(1 - a
*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(9*b^3*c^3) -
 (a^3*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Lo
g[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d -
 a*e)*(1 - c*(a + b*x)))]^2)/(6*b^3) + (d^3*h*n*(Log[c*(a + b*x)] + Log[(b*
c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*
(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(6*e^3)
- (a^3*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/(3*b^3) + (d
^3*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/(3*e^3) + (a^3*h
n(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e*x)
)/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])^2)/(6*b^3) - (d^
3*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e
*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])^2)/(6*e^3) +
(a^3*g*PolyLog[2, c*(a + b*x)])/(3*b^3) - (a^3*h*n*PolyLog[2, c*(a + b*x)])
/(9*b^3) - (a*d^2*h*n*PolyLog[2, c*(a + b*x)])/(3*b*e^2) - (a^2*d*h*n*PolyL
og[2, c*(a + b*x)])/(6*b^2*e) - (d^2*h*n*x*PolyLog[2, c*(a + b*x)])/(3*e^2)
 + (d*h*n*x^2*PolyLog[2, c*(a + b*x)])/(6*e) - (h*n*x^3*PolyLog[2, c*(a + b
*x)])/9 + (d^3*h*n*Log[d + e*x]*PolyLog[2, c*(a + b*x)])/(3*e^3) - (a^3*h*(
n*Log[d + e*x] - Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(3*b^3) + (x^
3*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/3 + (d^3*h*n*PolyLog[
2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(9*e^3) + (a*d^2*h*n*PolyLog
[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(6*b*e^2) + (a^2*d*h*n*Poly
Log[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(3*b^2*e) - (a^3*h*n*(Lo
g[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*Po
lyLog[2, (b*(d + e*x))/(b*d - a*e)])/(3*b^3) + (d^3*h*n*(Log[(b*(d + e*x))/
((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d +
 e*x))/(b*d - a*e)])/(3*e^3) - (a^2*(1 - a*c)*h*n*PolyLog[2, (b*c*(d + e*x)
)/(b*c*d + e - a*c*e)])/(3*b^3*c) + (a*(1 - a*c)^2*h*n*PolyLog[2, (b*c*(d +
 e*x))/(b*c*d + e - a*c*e)])/(6*b^3*c^2) - ((1 - a*c)^3*h*n*PolyLog[2, (b*c
*(d + e*x))/(b*c*d + e - a*c*e)])/(9*b^3*c^3) - (a^3*h*n*(Log[d + e*x] - Lo
g[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x
)])/(3*b^3) + (d^3*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 -
c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/(3*e^3) + (a^3*h*n*Log[(b*(d +

```
 e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/
(b*c*(d + e*x)))])/(3*b^3) - (d^3*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c
*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(3*e^3)
 - (a^3*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, (
(b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/(3*b^3) + (d^3*h*n*Log[(b*(d
 + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a
 + b*x)))/(b*(d + e*x))])/(3*e^3) + (a^3*h*n*PolyLog[3, (b*(d + e*x))/(b*d
- a*e)])/(3*b^3) - (d^3*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])/(3*e^3)
+ (a^3*h*n*PolyLog[3, 1 - c*(a + b*x)])/(3*b^3)...
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2468

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_)^(q_.), x_Symbol] :> In
t[ExpandToSum[u, x]^q*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a,
 b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] &&  !(BinomialMatch
Q[u, x] && LinearMatchQ[v, x])
```

Rule 2479

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[x*((a +
 b*Log[c*(d + e*x)^n])^p/(i + j*x)), x], x] - Dist[b*e*n*p, Int[x*(a + b*Lo
g[c*(d + e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2485

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[(-b)*(x/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1/2)*(Log[(-b)*(x/a
)] - Log[(-(b*c - a*d))*(x/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))]
)*Log[a*((c + d*x)/(c*(a + b*x)))]^2, x] - Simp[(1/2)*(Log[(-b)*(x/a)] - Lo
g[(-d)*(x/c)])*(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])^2, x] + Si
mp[(Log[c + d*x] - Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1 + b*(x/a)
], x] + Simp[(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1
 + d*(x/c)], x] + Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2, c*((a +
b*x)/(a*(c + d*x)))], x] - Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2,
 d*((a + b*x)/(b*(c + d*x)))], x] - Simp[PolyLog[3, 1 + b*(x/a)], x] - Simp
[PolyLog[3, 1 + d*(x/c)], x] + Simp[PolyLog[3, c*((a + b*x)/(a*(c + d*x)))]
, x] - Simp[PolyLog[3, d*((a + b*x)/(b*(c + d*x)))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2487

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Dist[m, Int[Log[i + j*x]*(Log[c*(d + e*x)^n]/x), x
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
 x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
 + j*x, h*(i + j*x)^m]
```

Rule 2488

```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(Log[(h_.)*((i_.)
 + (j_.)*(x_))^(m_.)]*(g_.) + (f_)))/(x_), x_Symbol] :> Dist[f, Int[(a + b*
Log[c*(d + e*x)^n])/x, x], x] + Dist[g, Int[Log[h*(i + j*x)^m]*((a + b*Log[
c*(d + e*x)^n])/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x
] && NeQ[e*i - d*j, 0]
```

Rule 2489

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Dist[g*j*(m/(r + 1)), Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
+ j*x)), x], x] - Dist[b*e*n*(p/(r + 1)), Int[x^(r + 1)*(a + b*Log[c*(d + e
*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2490

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-(e*k - d*l)/l + e*(x/l))^n])*(f + g
*Log[h*(-(j*k - i*l)/l + j*(x/l))^m]), x], x, k + l*x], x] /; FreeQ[{a, b,
c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2494

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rule 6730

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :> Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c*(a + b*x)^p], x]
```

```
, x] + Dist[a*p, Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /; Fr
eeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

Rule 6732

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[d + e*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, Int[Log[d
+ e*x]*(Log[1 - a*c - b*c*x]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rule 6733

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Dist[b/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x))
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 6738

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[e*h*(n/(m + 1)), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$\int x^2(g+h\log(f(d+ex)^n))\,\mathrm{Li}_2(c(a+bx))\,dx = \frac{1}{3}x^3(g+h\log(f(d+ex)^n))\,\mathrm{Li}_2(c(a+bx)) + \frac{1}{3}b\int\Big(\frac{a^2\,\mathrm{lo}}{}$$

$$= \frac{1}{3}x^3(g+h\log(f(d+ex)^n))\,\mathrm{Li}_2(c(a+bx)) + \frac{1}{3}\int x^2\log($$

$$= \frac{a^2x\log(1-ac-bcx)\,(g+h\log(f(d+ex)^n))}{3b^2} - \frac{ax^2\log(}{}$$

$$= \frac{a^2x\log(1-ac-bcx)\,(g+h\log(f(d+ex)^n))}{3b^2} - \frac{ax^2\log(}{}$$

$$= \frac{a^2x\log(1-ac-bcx)\,(g+h\log(f(d+ex)^n))}{3b^2} - \frac{ax^2\log(}{}$$

$$= -\frac{a^2gx}{3b^2} + \frac{a(1-ac)gx}{6b^2c} - \frac{(1-ac)^2gx}{9b^2c^2} + \frac{d^2hnx}{3e^2} + \frac{5ahnx^2}{}$$

$$= -\frac{a^2gx}{3b^2} + \frac{a(1-ac)gx}{6b^2c} - \frac{(1-ac)^2gx}{9b^2c^2} + \frac{4a^2hnx}{9b^2} + \frac{4d^2hn}{9e^2}$$

$$= -\frac{a^2gx}{3b^2} + \frac{a(1-ac)gx}{6b^2c} - \frac{(1-ac)^2gx}{9b^2c^2} + \frac{7a^2hnx}{9b^2} - \frac{11a(1}{3}$$

Mathematica [A]
time = 9.81, size = 2610, normalized size = 0.87

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] ((g - h*n*Log[d + e*x] + h*Log[f*(d + e*x)^n])*(-(b*c*x*(12 + 66*a^2*c^2 +
6*b*c*x + 4*b^2*c^2*x^2 - 3*a*c*(14 + 5*b*c*x))) + 6*(-2 + 11*a^3*c^3 + 2*b
^3*c^3*x^3 + 6*a^2*c^2*(-3 + b*c*x) + a*(9*c - 3*b^2*c^3*x^2))*Log[1 - a*c
- b*c*x] + 36*c^3*(a^3 + b^3*x^3)*PolyLog[2, c*(a + b*x)]))/(108*b^3*c^3) +
 (h*n*(36*b^3*c^3*(e*x*(-6*d^2 + 3*d*e*x - 2*e^2*x^2) + 6*(d^3 + e^3*x^3)*L
og[d + e*x])*PolyLog[2, c*(a + b*x)] - 216*b^2*c^2*d^2*e*(1 - a*c - b*c*x +
 (-1 + a*c + b*c*x - a*c*Log[c*(a + b*x)])*Log[1 - a*c - b*c*x] - a*c*PolyL
og[2, 1 - a*c - b*c*x]) - 27*b*c*d*e^2*(c*(-4*a^2*c + a*(4 - 6*b*c*x) + b*x
```

*(2 + b*c*x)) + (2 + 6*a^2*c^2 - 2*b^2*c^2*x^2 + 4*a*c*(-2 + b*c*x) - 4*a^2
*c^2*Log[c*(a + b*x)])*Log[1 - a*c - b*c*x] - 4*a^2*c^2*PolyLog[2, 1 - a*c
- b*c*x]) - 2*e^3*(-(c*(36*a^3*c^2 - 3*a*b*c*x*(14 + 5*b*c*x) + 6*a^2*c*(-6
 + 11*b*c*x) + 2*b*x*(6 + 3*b*c*x + 2*b^2*c^2*x^2))) - 6*(2 - 11*a^3*c^3 -
2*b^3*c^3*x^3 - 6*a^2*c^2*(-3 + b*c*x) + 3*a*c*(-3 + b^2*c^2*x^2) + 6*a^3*c
^3*Log[c*(a + b*x)])*Log[1 - a*c - b*c*x] - 36*a^3*c^3*PolyLog[2, 1 - a*c -
 b*c*x]) + 216*b^3*c^3*d^3*(Log[c*(a + b*x)]*Log[1 - a*c - b*c*x]*Log[d + e
x] + ((Log[c(a + b*x)] - Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e
*x))/(b*d - a*e)]*(-2*Log[1 - a*c - b*c*x] + Log[(b*(d + e*x))/(b*d - a*e)]
))/2 + (-Log[c*(a + b*x)] + Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d +
e*x))/(b*d - a*e)]*Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))] +
 (Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]^2*(Log[c*(a + b*x)
] - Log[((b*c*d + e - a*c*e)*(a + b*x))/((b*d - a*e)*(-1 + a*c + b*c*x))] +
 Log[(b*c*d + e - a*c*e)/(e - a*c*e - b*c*e*x)]))/2 + (Log[d + e*x] - Log[-
((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2, 1 - a*c - b*c
*x] + (Log[1 - a*c - b*c*x] + Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c +
b*c*x)))])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)] + Log[-((b*(d + e*x))/((b*
d - a*e)*(-1 + a*c + b*c*x)))]*(PolyLog[2, (b*c*(d + e*x))/(e*(-1 + a*c + b
*c*x))] - PolyLog[2, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]) -
PolyLog[3, 1 - a*c - b*c*x] - PolyLog[3, (b*(d + e*x))/(b*d - a*e)] - PolyL
og[3, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))] + PolyLog[3, -((b*(d + e*x))/
((b*d - a*e)*(-1 + a*c + b*c*x)))]) + 2*(b*c*(e*(48*(-1 + a*c)^2*e^2*x + 3*
b*c*(-1 + a*c)*(12*d^2 + 12*d*e*x - 5*e^2*x^2) + b^2*c^2*x*(48*d^2 - 15*d*e
*x + 8*e^2*x^2)) - 6*(d + e*x)*(6*(-1 + a*c)^2*e^2 + 3*b*c*(-1 + a*c)*e*(d
- e*x) + 2*b^2*c^2*(d^2 - d*e*x + e^2*x^2))*Log[d + e*x]) + 6*Log[1 - a*c -
 b*c*x]*(-(e*(-1 + a*c + b*c*x)*(2*(-1 + a*c)^2*e^2 + b*c*(-1 + a*c)*e*(3*d
 - 2*e*x) + b^2*c^2*(6*d^2 - 3*d*e*x + 2*e^2*x^2))) + 6*e^3*(-1 + 3*a*c - 3
*a^2*c^2 + a^3*c^3 + b^3*c^3*x^3)*Log[d + e*x] + 6*(b^3*c^3*d^3 - (-1 + a*c
)^3*e^3)*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)]) + 36*(b^3*c^3*d^3 - (-1
+ a*c)^3*e^3)*PolyLog[2, (e*(-1 + a*c + b*c*x))/(-(b*c*d) + (-1 + a*c)*e)]
- 108*a^2*c^2*e^2*(e - a*c*e - 2*b*c*e*x + b*c*d*Log[d + e*x] + b*c*e*x*Log
[d + e*x] - Log[1 - a*c - b*c*x]*(-(e*(-1 + a*c + b*c*x)) + e*(-1 + a*c + b
*c*x)*Log[d + e*x] + (b*c*d + e - a*c*e)*Log[(b*c*(d + e*x))/(b*c*d + e - a
*c*e)]) - (b*c*d + e - a*c*e)*PolyLog[2, (e*(-1 + a*c + b*c*x))/(-(b*c*d) +
 (-1 + a*c)*e)]) - 27*a*c*e*(b*c*(e*(d*(2 - 2*a*c - 3*b*c*x) + e*x*(3 - 3*a
*c + b*c*x)) + (d + e*x)*(2*(-1 + a*c)*e + b*c*(d - e*x))*Log[d + e*x]) + L
og[1 - a*c - b*c*x]*(e*(-1 + a*c + b*c*x)*((-1 + a*c)*e + b*c*(2*d - e*x))
- 2*e^2*(1 - 2*a*c + a^2*c^2 - b^2*c^2*x^2)*Log[d + e*x] + 2*(-(b^2*c^2*d^2
) + (-1 + a*c)^2*e^2)*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)]) + 2*(-(b^2*
c^2*d^2) + (-1 + a*c)^2*e^2)*PolyLog[2, (e*(-1 + a*c + b*c*x))/(-(b*c*d) +
(-1 + a*c)*e)]) - 108*a^3*c^3*e^3*(Log[c*(a + b*x)]*Log[1 - a*c - b*c*x]*Lo
g[d + e*x] + ((Log[c*(a + b*x)] - Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b
*(d + e*x))/(b*d - a*e)]*(-2*Log[1 - a*c - b*c*x] + Log[(b*(d + e*x))/(b*d
- a*e)]))/2 + (-Log[c*(a + b*x)] + Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(
b*(d + e*x))/(b*d - a*e)]*Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*

```
x)))] + (Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]^2*(Log[c*(a
 + b*x)] - Log[((b*c*d + e - a*c*e)*(a + b*x))/((b*d - a*e)*(-1 + a*c + b*c
*x))] + Log[(b*c*d + e - a*c*e)/(e - a*c*e - b*c*e*x)]))/2 + (Log[d + e*x]
- Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2, 1 - a*
c - b*c*x] + (Log[1 - a*c - b*c*x] + Log[-((b*(d + e*x))/((b*d - a*e)*(-1 +
 a*c + b*c*x)))])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)] + Log[-((b*(d + e*x
))/((b*d - a*e)*(-1 + a*c + b*c*x)))]*(PolyLog[2, (b*c*(d + e*x))/(e*(-1 +
a*c + b*c*x))] - PolyLog[2, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)
))]) - PolyLog[3, 1 - a*c - b*c*x] - PolyLog[3, (b*(d + e*x))/(b*d - a*e)]
- PolyLog[3, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))] + PolyLog[3, -((b*(d +
 e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]))))/(648*b^3*c^3*e^3)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int x^2(g + h\ln(f(ex+d)^n))\,\mathrm{polylog}(2, c(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x)
[Out] int(x^2*(g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="max
ima")
[Out] 1/18*(3*d*h*n*x^2*e^2 - 6*d^2*h*n*x*e + 6*d^3*h*n*log(x*e + d) + 6*h*x^3*e^
3*log((x*e + d)^n) - 2*(h*n*e^3 - 3*h*e^3*log(f) - 3*g*e^3)*x^3)*dilog(b*c*
x + a*c)*e^(-3) + integrate(1/18*(6*b*h*x^3*e^3*log(-b*c*x - a*c + 1)*log((
x*e + d)^n) + (3*b*d*h*n*x^2*e^2 - 6*b*d^2*h*n*x*e + 6*b*d^3*h*n*log(x*e +
d) - 2*(b*h*n*e^3 - 3*b*h*e^3*log(f) - 3*b*g*e^3)*x^3)*log(-b*c*x - a*c + 1
))/(b*x*e^3 + a*e^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="fri
cas")
```

```
[Out] integral(h*x^2*dilog(b*c*x + a*c)*log((x*e + d)^n*f) + g*x^2*dilog(b*c*x +
a*c), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="gia
c")
```

```
[Out] integrate((h*log((e*x + d)^n*f) + g)*x^2*dilog((b*x + a)*c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{polylog}(2,c\,(a+b\,x))\,\left(g+h\,\ln\left(f\,{(d+e\,x)}^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)),x)
```

```
[Out] int(x^2*polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)), x)
```

3.178 $\int x(g+h\log(f(d+ex)^n))\,\textbf{PolyLog}(2,c(a+bx))\,dx$

Optimal. Leaf size=2252

result too large to display

```
[Out] 1/2*a*g*x/b-1/2*a^2*g*polylog(2,c*(b*x+a))/b^2-1/4*h*n*x^2*polylog(2,c*(b*x
+a))-3/4*d*h*n*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b/c/e-1/2*a*d*h*n*ln(-b*c*x-
a*c+1)*ln(b*c*(e*x+d)/(-a*c*e+b*c*d+e))/b/e-1/4*h*n*x^2*ln(-b*c*x-a*c+1)+3/
16*h*n*x^2+1/2*(-a*c+1)*h*n*x/b/c+1/2*x^2*(g+h*ln(f*(e*x+d)^n))*polylog(2,c
*(b*x+a))+1/4*x^2*ln(-b*c*x-a*c+1)*(g+h*ln(f*(e*x+d)^n))-1/4*(-a*c+1)*g*x/b
/c-5/4*a*h*n*x/b-7/8*d*h*n*x/e+1/4*a^2*h*n*polylog(2,c*(b*x+a))/b^2-1/4*d^2
*h*n*polylog(2,e*(-b*c*x-a*c+1)/(-a*c*e+b*c*d+e))/e^2-1/2*a^2*h*n*polylog(3
,b*(e*x+d)/(-a*e+b*d))/b^2+1/2*d^2*h*n*polylog(3,b*(e*x+d)/(-a*e+b*d))/e^2-
1/2*a^2*h*n*polylog(3,1-c*(b*x+a))/b^2+1/2*d^2*h*n*polylog(3,1-c*(b*x+a))/e
^2-1/2*a^2*h*n*polylog(3,-e*(1-c*(b*x+a))/b/c/(e*x+d))/b^2+1/2*d^2*h*n*poly
log(3,-e*(1-c*(b*x+a))/b/c/(e*x+d))/e^2+1/2*a^2*h*n*polylog(3,(-a*e+b*d)*(1
-c*(b*x+a))/b/(e*x+d))/b^2-1/2*d^2*h*n*polylog(3,(-a*e+b*d)*(1-c*(b*x+a))/b
/(e*x+d))/e^2+1/4*a^2*h*n*(ln(c*(b*x+a))+ln((-a*c*e+b*c*d+e)/b/c/(e*x+d))-l
n((-a*c*e+b*c*d+e)*(b*x+a)/b/(e*x+d)))*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)
))^2/b^2-1/4*d^2*h*n*(ln(c*(b*x+a))+ln((-a*c*e+b*c*d+e)/b/c/(e*x+d))-ln((-a
*c*e+b*c*d+e)*(b*x+a)/b/(e*x+d)))*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))^2/
e^2-1/4*a^2*h*n*(ln(c*(b*x+a))-ln(-e*(b*x+a)/(-a*e+b*d)))*(ln(b*(e*x+d)/(-a
*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))^2/b^2+1/4*d^2*h*n*(ln(c*(b*x+a))-ln
(-e*(b*x+a)/(-a*e+b*d)))*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*
x+a)))^2/e^2-1/2*d^2*h*n*ln(e*x+d)*polylog(2,c*(b*x+a))/e^2+1/2*a^2*h*n*(ln
(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))*polylog(2,b*(e*x+d)/(
-a*e+b*d))/b^2-1/2*d^2*h*n*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(
b*x+a)))*polylog(2,b*(e*x+d)/(-a*e+b*d))/e^2+1/2*a^2*h*n*(ln(e*x+d)-ln(b*(e
*x+d)/(-a*e+b*d)/(1-c*(b*x+a))))*polylog(2,1-c*(b*x+a))/b^2-1/2*d^2*h*n*(ln
(e*x+d)-ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a))))*polylog(2,1-c*(b*x+a))/e^2-
1/2*a^2*h*n*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))*polylog(2,-e*(1-c*(b*x+a
))/b/c/(e*x+d))/b^2+1/2*d^2*h*n*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))*poly
log(2,-e*(1-c*(b*x+a))/b/c/(e*x+d))/e^2+1/2*a^2*h*n*ln(b*(e*x+d)/(-a*e+b*d)
/(1-c*(b*x+a)))*polylog(2,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/b^2-1/2*d^2*h
*n*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))*polylog(2,(-a*e+b*d)*(1-c*(b*x+a)
)/b/(e*x+d))/e^2+1/4*(-a*c+1)^2*h*n*ln(-b*c*x-a*c+1)/b^2/c^2-1/4*d^2*h*n*ln
(-b*c*x-a*c+1)*ln(b*c*(e*x+d)/(-a*c*e+b*c*d+e))/e^2+1/2*a*h*(e*x+d)*ln(f*(e
*x+d)^n)/b/e+1/2*a*(-a*c+1)*ln(e*(-b*c*x-a*c+1)/(-a*c*e+b*c*d+e))*(g+h*ln(f
*(e*x+d)^n))/b^2/c+1/8*d^2*h*n*ln(e*x+d)/e^2-1/2*a*x*ln(-b*c*x-a*c+1)*(g+h*
ln(f*(e*x+d)^n))/b-1/4*(-a*c+1)^2*ln(e*(-b*c*x-a*c+1)/(-a*c*e+b*c*d+e))*(g+
h*ln(f*(e*x+d)^n))/b^2/c^2+1/2*a^2*h*(n*ln(e*x+d)-ln(f*(e*x+d)^n))*polylog(
2,c*(b*x+a))/b^2+1/2*a*d*h*n*polylog(2,c*(b*x+a))/b/e-1/2*a*d*h*n*polylog(2
,e*(-b*c*x-a*c+1)/(-a*c*e+b*c*d+e))/b/e+1/2*a*(-a*c+1)*h*n*polylog(2,b*c*(e
```

```
*x+d)/(-a*c*e+b*c*d+e))/b^2/c-1/8*x^2*(g+h*ln(f*(e*x+d)^n))+1/2*d*h*n*x*pol
ylog(2,c*(b*x+a))/e-1/4*(-a*c+1)^2*h*n*polylog(2,b*c*(e*x+d)/(-a*c*e+b*c*d+
e))/b^2/c^2-3/4*a*h*n*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b^2/c-1/4*(-a*c+1)*h*
(e*x+d)*ln(f*(e*x+d)^n)/b/c/e+1/2*a^2*h*n*ln(c*(b*x+a))*ln(e*x+d)*ln(1-c*(b
*x+a))/b^2-1/2*d^2*h*n*ln(c*(b*x+a))*ln(e*x+d)*ln(1-c*(b*x+a))/e^2
```

Rubi [A]

time = 2.04, antiderivative size = 2252, normalized size of antiderivative = 1.00, number of steps used = 67, number of rules used = 20, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6738, 2479, 45, 2463, 2436, 2332, 2441, 2440, 2438, 2489, 2442, 2490, 2488, 2487, 2485, 6730, 2494, 2468, 6733, 6732}

Antiderivative was successfully verified.

```
[In] Int[x*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)],x]

[Out] (a*g*x)/(2*b) - ((1 - a*c)*g*x)/(4*b*c) - (5*a*h*n*x)/(4*b) + ((1 - a*c)*h*
n*x)/(2*b*c) - (7*d*h*n*x)/(8*e) + (3*h*n*x^2)/16 + ((1 - a*c)^2*h*n*Log[1
- a*c - b*c*x])/(4*b^2*c^2) - (h*n*x^2*Log[1 - a*c - b*c*x])/4 - (3*a*h*n*(
1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(4*b^2*c) - (3*d*h*n*(1 - a*c - b*c*
x)*Log[1 - a*c - b*c*x])/(4*b*c*e) + (d^2*h*n*Log[d + e*x])/(8*e^2) - (d^2*
h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(4*e^2)
- (a*d*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(
2*b*e) + (a*h*(d + e*x)*Log[f*(d + e*x)^n])/(2*b*e) - ((1 - a*c)*h*(d + e*x
)*Log[f*(d + e*x)^n])/(4*b*c*e) - (x^2*(g + h*Log[f*(d + e*x)^n]))/8 - (a*x
*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + e*x)^n]))/(2*b) + (x^2*Log[1 - a*c
- b*c*x]*(g + h*Log[f*(d + e*x)^n]))/4 + (a*(1 - a*c)*Log[(e*(1 - a*c - b*c
*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(2*b^2*c) - ((1 - a*c
)^2*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n
]))/(4*b^2*c^2) + (a^2*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c
*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(
d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(4*b^2) - (d^2*h*n*(Log[c*(a
+ b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*
e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*
x)))]^2)/(4*e^2) + (a^2*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*
x)])/(2*b^2) - (d^2*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])
/(2*e^2) - (a^2*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*
(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])
^2)/(4*b^2) + (d^2*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))
])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x
)])^2)/(4*e^2) - (a^2*g*PolyLog[2, c*(a + b*x)])/(2*b^2) + (a^2*h*n*PolyLog
[2, c*(a + b*x)])/(4*b^2) + (a*d*h*n*PolyLog[2, c*(a + b*x)])/(2*b*e) + (d*
h*n*x*PolyLog[2, c*(a + b*x)])/(2*e) - (h*n*x^2*PolyLog[2, c*(a + b*x)])/4
- (d^2*h*n*Log[d + e*x]*PolyLog[2, c*(a + b*x)])/(2*e^2) + (a^2*h*(n*Log[d
```

```
+ e*x] - Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(2*b^2) + (x^2*(g + h
*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/2 - (d^2*h*n*PolyLog[2, (e*(1
 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(4*e^2) - (a*d*h*n*PolyLog[2, (e*(1
- a*c - b*c*x))/(b*c*d + e - a*c*e)])/(2*b*e) + (a^2*h*n*(Log[(b*(d + e*x))
/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d
+ e*x))/(b*d - a*e)])/(2*b^2) - (d^2*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1
 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a
*e)])/(2*e^2) + (a*(1 - a*c)*h*n*PolyLog[2, (b*c*(d + e*x))/(b*c*d + e - a*
c*e)])/(2*b^2*c) - ((1 - a*c)^2*h*n*PolyLog[2, (b*c*(d + e*x))/(b*c*d + e -
 a*c*e)])/(4*b^2*c^2) + (a^2*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d -
a*e)*(1 - c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/(2*b^2) - (d^2*h*n*(
Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))])*PolyLog[
2, 1 - c*(a + b*x)])/(2*e^2) - (a^2*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 -
 c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(2*b^
2) + (d^2*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2,
 -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(2*e^2) + (a^2*h*n*Log[(b*(d +
e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a +
b*x)))/(b*(d + e*x))])/(2*b^2) - (d^2*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1
 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))]
)/(2*e^2) - (a^2*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])/(2*b^2) + (d^2*
h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])/(2*e^2) - (a^2*h*n*PolyLog[3, 1
- c*(a + b*x)])/(2*b^2) + (d^2*h*n*PolyLog[3, 1 - c*(a + b*x)])/(2*e^2) - (
a^2*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(2*b^2) + (d^
2*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(2*e^2) + (a^2*
h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/(2*b^2) - (d
^2*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/(2*e^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2468

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_)^(q_.), x_Symbol] :> In
t[ExpandToSum[u, x]^q*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a,
 b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] &&  !(BinomialMatch
Q[u, x] && LinearMatchQ[v, x])
```

Rule 2479

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[x*((a +
 b*Log[c*(d + e*x)^n])^p/(i + j*x)), x], x] - Dist[b*e*n*p, Int[x*(a + b*Lo
g[c*(d + e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2485

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[(-b)*(x/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1/2)*(Log[(-b)*(x/a
)] - Log[(-(b*c - a*d))*(x/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))]
)*Log[a*((c + d*x)/(c*(a + b*x)))]^2, x] - Simp[(1/2)*(Log[(-b)*(x/a)] - Lo
g[(-d)*(x/c)])*(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])^2, x] + Si
mp[(Log[c + d*x] - Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1 + b*(x/a)
], x] + Simp[(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1
 + d*(x/c)], x] + Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2, c*((a +
b*x)/(a*(c + d*x)))], x] - Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2,
 d*((a + b*x)/(b*(c + d*x)))], x] - Simp[PolyLog[3, 1 + b*(x/a)], x] - Simp
[PolyLog[3, 1 + d*(x/c)], x] + Simp[PolyLog[3, c*((a + b*x)/(a*(c + d*x)))]
, x] - Simp[PolyLog[3, d*((a + b*x)/(b*(c + d*x)))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2487

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Dist[m, Int[Log[i + j*x]*(Log[c*(d + e*x)^n]/x), x
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
 x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
 + j*x, h*(i + j*x)^m]
```

Rule 2488

```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(Log[(h_.)*((i_.)
 + (j_.)*(x_))^(m_.)]*(g_.) + (f_)))/(x_), x_Symbol] :> Dist[f, Int[(a + b*
Log[c*(d + e*x)^n])/x, x], x] + Dist[g, Int[Log[h*(i + j*x)^m]*((a + b*Log[
c*(d + e*x)^n])/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x
] && NeQ[e*i - d*j, 0]
```

Rule 2489

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Dist[g*j*(m/(r + 1)), Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
+ j*x)), x], x] - Dist[b*e*n*(p/(r + 1)), Int[x^(r + 1)*(a + b*Log[c*(d + e
*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2490

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-(e*k - d*l)/l + e*(x/l))^n])*(f + g
*Log[h*(-(j*k - i*l)/l + j*(x/l))^m]), x], x, k + l*x], x] /; FreeQ[{a, b,
c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2494

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rule 6730

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :> Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c*(a + b*x)^p], x]
, x] + Dist[a*p, Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /; Fr
eeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

Rule 6732

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[d + e*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, Int[Log[d
+ e*x]*(Log[1 - a*c - b*c*x]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rule 6733

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Dist[b/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x))
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 6738

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[e*h*(n/(m + 1)), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$\int x(g+h\log(f(d+ex)^n))\,\mathrm{Li}_2(c(a+bx))\,dx = \frac{1}{2}x^2(g+h\log(f(d+ex)^n))\,\mathrm{Li}_2(c(a+bx)) + \frac{1}{2}b\int\Bigg(-\frac{a\,\mathrm{lo}}{}$$

$$= \frac{1}{2}x^2(g+h\log(f(d+ex)^n))\,\mathrm{Li}_2(c(a+bx)) + \frac{1}{2}\int x\log(1-$$

$$= -\frac{ax\log(1-ac-bcx)\,(g+h\log(f(d+ex)^n))}{2b} + \frac{1}{4}x^2\log($$

$$= -\frac{ax\log(1-ac-bcx)\,(g+h\log(f(d+ex)^n))}{2b} + \frac{1}{4}x^2\log($$

$$= -\frac{ax\log(1-ac-bcx)\,(g+h\log(f(d+ex)^n))}{2b} + \frac{1}{4}x^2\log($$

$$= \frac{agx}{2b} - \frac{(1-ac)gx}{4bc} - \frac{dhnx}{2e} - \frac{dhn(1-ac-bcx)\log(1-a}{2bce}$$

$$= \frac{agx}{2b} - \frac{(1-ac)gx}{4bc} - \frac{3ahnx}{4b} - \frac{3dhnx}{4e} - \frac{3ahn(1-ac-bc}{}$$

$$= \frac{agx}{2b} - \frac{(1-ac)gx}{4bc} - \frac{5ahnx}{4b} + \frac{(1-ac)hnx}{4bc} - \frac{7dhnx}{8e} + \frac{1}{1}$$

Mathematica [A]
time = 6.62, size = 1996, normalized size = 0.89

Antiderivative was successfully verified.

```
[In] Integrate[x*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] ((g - h*n*Log[d + e*x] + h*Log[f*(d + e*x)^n])*(-(b*c*x*(2 - 6*a*c + b*c*x)
) + (-2 - 6*a^2*c^2 + 2*b^2*c^2*x^2 - 4*a*c*(-2 + b*c*x))*Log[1 - a*c - b*c
*x] - 4*c^2*(a^2 - b^2*x^2)*PolyLog[2, c*(a + b*x)]))/(8*b^2*c^2) + (h*n*(4
*b^2*c^2*(e*x*(2*d - e*x) - 2*(d^2 - e^2*x^2)*Log[d + e*x])*PolyLog[2, c*(a
 + b*x)] + 8*b*c*d*e*(1 - a*c - b*c*x + (-1 + a*c + b*c*x - a*c*Log[c*(a +
b*x)])*Log[1 - a*c - b*c*x] - a*c*PolyLog[2, 1 - a*c - b*c*x]) + e^2*(c*(-4
*a^2*c + a*(4 - 6*b*c*x) + b*x*(2 + b*c*x)) + (2 + 6*a^2*c^2 - 2*b^2*c^2*x^
2 + 4*a*c*(-2 + b*c*x) - 4*a^2*c^2*Log[c*(a + b*x)])*Log[1 - a*c - b*c*x] -
```

```
4*a^2*c^2*PolyLog[2, 1 - a*c - b*c*x]) - 8*b^2*c^2*d^2*(Log[c*(a + b*x)]*L
og[1 - a*c - b*c*x]*Log[d + e*x] + ((Log[c*(a + b*x)] - Log[(e*(a + b*x))/(
-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*(-2*Log[1 - a*c - b*c*x] + L
og[(b*(d + e*x))/(b*d - a*e)]))/2 + (-Log[c*(a + b*x)] + Log[(e*(a + b*x))/
(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*Log[-((b*(d + e*x))/((b*d -
 a*e)*(-1 + a*c + b*c*x)))] + (Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c +
 b*c*x)))]^2*(Log[c*(a + b*x)] - Log[((b*c*d + e - a*c*e)*(a + b*x))/((b*d
- a*e)*(-1 + a*c + b*c*x))] + Log[(b*c*d + e - a*c*e)/(e - a*c*e - b*c*e*x)
]))/2 + (Log[d + e*x] - Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)
))])*PolyLog[2, 1 - a*c - b*c*x] + (Log[1 - a*c - b*c*x] + Log[-((b*(d + e*
x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2, (b*(d + e*x))/(b*d - a*e
)] + Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]*(PolyLog[2, (b*
c*(d + e*x))/(e*(-1 + a*c + b*c*x))] - PolyLog[2, -((b*(d + e*x))/((b*d - a
*e)*(-1 + a*c + b*c*x)))]) - PolyLog[3, 1 - a*c - b*c*x] - PolyLog[3, (b*(d
 + e*x))/(b*d - a*e)] - PolyLog[3, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))]
+ PolyLog[3, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]) + 2*(b*c*(
e*(d*(2 - 2*a*c - 3*b*c*x) + e*x*(3 - 3*a*c + b*c*x)) + (d + e*x)*(2*(-1 +
a*c)*e + b*c*(d - e*x))*Log[d + e*x]) + Log[1 - a*c - b*c*x]*(e*(-1 + a*c +
 b*c*x)*((-1 + a*c)*e + b*c*(2*d - e*x)) - 2*e^2*(1 - 2*a*c + a^2*c^2 - b^2
*c^2*x^2)*Log[d + e*x] + 2*(-(b^2*c^2*d^2) + (-1 + a*c)^2*e^2)*Log[(b*c*(d
+ e*x))/(b*c*d + e - a*c*e)]) + 2*(-(b^2*c^2*d^2) + (-1 + a*c)^2*e^2)*PolyL
og[2, (e*(-1 + a*c + b*c*x))/(-(b*c*d) + (-1 + a*c)*e)] + 4*a*c*e*(e - a*c*
e - 2*b*c*e*x + b*c*d*Log[d + e*x] + b*c*e*x*Log[d + e*x] - Log[1 - a*c - b
*c*x]*(-(e*(-1 + a*c + b*c*x)) + e*(-1 + a*c + b*c*x)*Log[d + e*x] + (b*c*d
 + e - a*c*e)*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)]) - (b*c*d + e - a*c*
e)*PolyLog[2, (e*(-1 + a*c + b*c*x))/(-(b*c*d) + (-1 + a*c)*e)]) + 4*a^2*c^
2*e^2*(Log[c*(a + b*x)]*Log[1 - a*c - b*c*x]*Log[d + e*x] + ((Log[c*(a + b*
x)] - Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*(-2
*Log[1 - a*c - b*c*x] + Log[(b*(d + e*x))/(b*d - a*e)]))/2 + (-Log[c*(a + b
*x)] + Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*Lo
g[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))] + (Log[-((b*(d + e*x))
/((b*d - a*e)*(-1 + a*c + b*c*x)))]^2*(Log[c*(a + b*x)] - Log[((b*c*d + e -
 a*c*e)*(a + b*x))/((b*d - a*e)*(-1 + a*c + b*c*x))] + Log[(b*c*d + e - a*c
*e)/(e - a*c*e - b*c*e*x)]))/2 + (Log[d + e*x] - Log[-((b*(d + e*x))/((b*d
- a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2, 1 - a*c - b*c*x] + (Log[1 - a*c -
b*c*x] + Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2,
 (b*(d + e*x))/(b*d - a*e)] + Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c +
b*c*x)))]*(PolyLog[2, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))] - PolyLog[2,
-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]) - PolyLog[3, 1 - a*c -
b*c*x] - PolyLog[3, (b*(d + e*x))/(b*d - a*e)] - PolyLog[3, (b*c*(d + e*x))
/(e*(-1 + a*c + b*c*x))] + PolyLog[3, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*
c + b*c*x)))]))))/(16*b^2*c^2*e^2)
```

Maple [F]

time = 0.11, size = 0, normalized size = 0.00

$$\int x(g + h\ln(f(ex + d)^n))\operatorname{polylog}(2, c(bx + a))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x)
```

```
[Out] int(x*(g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="maxim
a")
```

```
[Out] 1/4*(2*d*h*n*x*e - 2*d^2*h*n*log(x*e + d) + 2*h*x^2*e^2*log((x*e + d)^n) -
(h*n*e^2 - 2*h*e^2*log(f) - 2*g*e^2)*x^2)*dilog(b*c*x + a*c)*e^(-2) + integ
rate(1/4*(2*b*h*x^2*e^2*log(-b*c*x - a*c + 1)*log((x*e + d)^n) + (2*b*d*h*n
*x*e - 2*b*d^2*h*n*log(x*e + d) - (b*h*n*e^2 - 2*b*h*e^2*log(f) - 2*b*g*e^2
)*x^2)*log(-b*c*x - a*c + 1))/(b*x*e^2 + a*e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="frica
s")
```

```
[Out] integral(h*x*dilog(b*c*x + a*c)*log((x*e + d)^n*f) + g*x*dilog(b*c*x + a*c)
, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="giac"
)
```

```
[Out] integrate((h*log((e*x + d)^n*f) + g)*x*dilog((b*x + a)*c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{polylog}(2,c\,(a+b\,x))\;\left(g+h\,\ln\left(f\,{\left(d+e\,x\right)}^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)),x)
```

```
[Out] int(x*polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)), x)
```

3.179 $\int (g + h \log(f(d + ex)^n)) \textbf{PolyLog}(2, c(a + bx)) \, dx$

Optimal. Leaf size=1653

$$-gx+3hnx-\frac{g(1-ac-bcx)\log(1-ac-bcx)}{bc}+\frac{2hn(1-ac-bcx)\log(1-ac-bcx)}{bc}+\frac{dhn\log(c(a+bx))}{}$$

```
[Out] -h*(e*x+d)*ln(f*(e*x+d)^n)/e-h*n*x*polylog(2,b*c*x+a*c)+h*x*ln(-b*c*x-a*c+1
)*ln(f*(e*x+d)^n)+a*g*polylog(2,c*(b*x+a))/b+3*h*n*x-g*x+x*(g+h*ln(f*(e*x+d
)^n))*polylog(2,c*(b*x+a))+d*h*n*ln(-b*c*x-a*c+1)*ln(b*c*(e*x+d)/(-a*c*e+b*
c*d+e))/e+d*h*n*(ln(-e*x-d)-ln(b*(e*x+d)/(-a*e+b*d)/(-b*c*x-a*c+1)))*polylo
g(2,-b*c*x-a*c+1)/e+d*h*n*ln(-e*x-d)*polylog(2,b*c*x+a*c)/e-d*h*n*ln(b*(e*x
+d)/(-a*e+b*d)/(-b*c*x-a*c+1))*polylog(2,-e*(-b*c*x-a*c+1)/b/c/(e*x+d))/e+d
*h*n*ln(b*(e*x+d)/(-a*e+b*d)/(-b*c*x-a*c+1))*polylog(2,(-a*e+b*d)*(-b*c*x-a
*c+1)/b/(e*x+d))/e+d*h*n*(ln(-b*c*x-a*c+1)+ln(b*(e*x+d)/(-a*e+b*d)/(-b*c*x-
a*c+1)))*polylog(2,b*(e*x+d)/(-a*e+b*d))/e-a*h*n*(ln(b*(e*x+d)/(-a*e+b*d)/(
1-c*(b*x+a)))+ln(1-c*(b*x+a)))*polylog(2,b*(e*x+d)/(-a*e+b*d))/b-a*h*n*(ln(
e*x+d)-ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a))))*polylog(2,1-c*(b*x+a))/b+a*h
*n*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))*polylog(2,-e*(1-c*(b*x+a))/b/c/(e
*x+d))/b-a*h*n*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))*polylog(2,(-a*e+b*d)*
(1-c*(b*x+a))/b/(e*x+d))/b+2*h*n*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b/c+1/2*d*
h*n*(ln(c*(b*x+a))+ln((-a*c*e+b*c*d+e)/b/c/(e*x+d))-ln((-a*c*e+b*c*d+e)*(b*
x+a)/b/(e*x+d)))*ln(b*(e*x+d)/(-a*e+b*d)/(-b*c*x-a*c+1))^2/e-1/2*d*h*n*(ln(
c*(b*x+a))-ln(-e*(b*x+a)/(-a*e+b*d)))*(ln(-b*c*x-a*c+1)+ln(b*(e*x+d)/(-a*e+
b*d)/(-b*c*x-a*c+1)))^2/e-(-a*c+1)*h*ln(e*(-b*c*x-a*c+1)/(-a*c*e+b*c*d+e))*
ln(f*(e*x+d)^n)/b/c-1/2*a*h*n*(ln(c*(b*x+a))+ln((-a*c*e+b*c*d+e)/b/c/(e*x+d
))-ln((-a*c*e+b*c*d+e)*(b*x+a)/b/(e*x+d)))*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*
x+a)))^2/b+1/2*a*h*n*(ln(c*(b*x+a))-ln(-e*(b*x+a)/(-a*e+b*d)))*(ln(b*(e*x+d
)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))^2/b-(-a*c+1)*h*n*polylog(2,b*c
*(e*x+d)/(-a*c*e+b*c*d+e))/b/c-a*h*n*polylog(2,c*(b*x+a))/b-a*h*(n*ln(e*x+d
)-ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/b+d*h*n*polylog(2,e*(-b*c*x-a*c+1)/
(-a*c*e+b*c*d+e))/e-g*(-b*c*x-a*c+1)*ln(-b*c*x-a*c+1)/b/c-d*h*n*polylog(3,-
b*c*x-a*c+1)/e-d*h*n*polylog(3,-e*(-b*c*x-a*c+1)/b/c/(e*x+d))/e+d*h*n*polyl
og(3,(-a*e+b*d)*(-b*c*x-a*c+1)/b/(e*x+d))/e+a*h*n*polylog(3,b*(e*x+d)/(-a*e
+b*d))/b-d*h*n*polylog(3,b*(e*x+d)/(-a*e+b*d))/e+a*h*n*polylog(3,1-c*(b*x+a
))/b+a*h*n*polylog(3,-e*(1-c*(b*x+a))/b/c/(e*x+d))/b-a*h*n*polylog(3,(-a*e+
b*d)*(1-c*(b*x+a))/b/(e*x+d))/b+d*h*n*ln(c*(b*x+a))*ln(-b*c*x-a*c+1)*ln(-e*
x-d)/e-a*h*n*ln(c*(b*x+a))*ln(e*x+d)*ln(1-c*(b*x+a))/b
```

Rubi [A]
time = 2.14, antiderivative size = 1653, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 17, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {6735, 2465, 2436, 2332, 2441, 2440, 2438, 6820, 45, 2463, 6874, 2479, 2490,

2487, 2485, 6730, 6732}

Antiderivative was successfully verified.

```
[In] Int[(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] -(g*x) + 3*h*n*x - (g*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c) + (2*h*
n*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c) + (d*h*n*Log[c*(a + b*x)]*L
og[1 - a*c - b*c*x]*Log[-d - e*x])/e + (d*h*n*Log[1 - a*c - b*c*x]*Log[(b*c
*(d + e*x))/(b*c*d + e - a*c*e)])/e + (d*h*n*(Log[c*(a + b*x)] + Log[(b*c*d
 + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d
+ e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))]^2)/(2*e) - (d*
h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log[1 - a*c - b
*c*x] + Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))])^2)/(2*e) - (h*(
d + e*x)*Log[f*(d + e*x)^n])/e + h*x*Log[1 - a*c - b*c*x]*Log[f*(d + e*x)^n
] - ((1 - a*c)*h*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*Log[f*(d +
e*x)^n])/(b*c) - (a*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d
 + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d +
 e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(2*b) - (a*h*n*Log[c*(a + b*x)]*
Log[d + e*x]*Log[1 - c*(a + b*x)])/b + (a*h*n*(Log[c*(a + b*x)] - Log[-((e*
(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x))
)] + Log[1 - c*(a + b*x)])^2)/(2*b) + (a*g*PolyLog[2, c*(a + b*x)])/b - (a*
h*n*PolyLog[2, c*(a + b*x)])/b - (a*h*(n*Log[d + e*x] - Log[f*(d + e*x)^n])
*PolyLog[2, c*(a + b*x)])/b + x*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a
+ b*x)] + (d*h*n*(Log[-d - e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c -
 b*c*x))])*PolyLog[2, 1 - a*c - b*c*x])/e + (d*h*n*PolyLog[2, (e*(1 - a*c -
 b*c*x))/(b*c*d + e - a*c*e)])/e - h*n*x*PolyLog[2, a*c + b*c*x] + (d*h*n*L
og[-d - e*x]*PolyLog[2, a*c + b*c*x])/e - (d*h*n*Log[(b*(d + e*x))/((b*d -
a*e)*(1 - a*c - b*c*x))]*PolyLog[2, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x))
)])/e + (d*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))]*PolyLog[2
, ((b*d - a*e)*(1 - a*c - b*c*x))/(b*(d + e*x))])/e + (d*h*n*(Log[1 - a*c -
 b*c*x] + Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))])*PolyLog[2, (b
*(d + e*x))/(b*d - a*e)])/e - (a*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c
*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)]
)/b - ((1 - a*c)*h*n*PolyLog[2, (b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(b*c)
 - (a*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))
])*PolyLog[2, 1 - c*(a + b*x)])/b + (a*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(
1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/b
- (a*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*
d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/b - (d*h*n*PolyLog[3, 1 - a*c -
 b*c*x])/e - (d*h*n*PolyLog[3, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))])/e
 + (d*h*n*PolyLog[3, ((b*d - a*e)*(1 - a*c - b*c*x))/(b*(d + e*x))])/e + (a
*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])/b - (d*h*n*PolyLog[3, (b*(d + e
*x))/(b*d - a*e)])/e + (a*h*n*PolyLog[3, 1 - c*(a + b*x)])/b + (a*h*n*PolyL
```

```
og[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/b - (a*h*n*PolyLog[3, ((b*
d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/b
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2479

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[x*((a +
 b*Log[c*(d + e*x)^n])^p/(i + j*x)), x], x] - Dist[b*e*n*p, Int[x*(a + b*Lo
g[c*(d + e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2485

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[(-b)*(x/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1/2)*(Log[(-b)*(x/a
)] - Log[(-(b*c - a*d))*(x/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))]
)*Log[a*((c + d*x)/(c*(a + b*x)))]^2, x] - Simp[(1/2)*(Log[(-b)*(x/a)] - Lo
g[(-d)*(x/c)])*(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])^2, x] + Si
mp[(Log[c + d*x] - Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1 + b*(x/a)
], x] + Simp[(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1
 + d*(x/c)], x] + Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2, c*((a +
b*x)/(a*(c + d*x)))], x] - Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2,
 d*((a + b*x)/(b*(c + d*x)))], x] - Simp[PolyLog[3, 1 + b*(x/a)], x] - Simp
[PolyLog[3, 1 + d*(x/c)], x] + Simp[PolyLog[3, c*((a + b*x)/(a*(c + d*x)))]
, x] - Simp[PolyLog[3, d*((a + b*x)/(b*(c + d*x)))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2487

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Dist[m, Int[Log[i + j*x]*(Log[c*(d + e*x)^n]/x), x
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
 x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
 + j*x, h*(i + j*x)^m]
```

Rule 2490

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-(e*k - d*l)/l + e*(x/l))^n])*(f + g
*Log[h*(-(j*k - i*l)/l + j*(x/l))^m]), x], x, k + l*x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 6730

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :> Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c*(a + b*x)^p], x]
, x] + Dist[a*p, Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /; Fr
eeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

Rule 6732

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[d + e*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, Int[Log[d
+ e*x]*(Log[1 - a*c - b*c*x]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rule 6735

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*PolyLog[2, (c_.)*
((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x*(g + h*Log[f*(d + e*x)^n])*PolyL
og[2, c*(a + b*x)], x] + (Dist[b, Int[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*
c - b*c*x]*ExpandIntegrand[x/(a + b*x), x], x], x] - Dist[e*h*n, Int[PolyLo
g[2, c*(a + b*x)]*ExpandIntegrand[x/(d + e*x), x], x], x]) /; FreeQ[{a, b,
c, d, e, f, g, h, n}, x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (g+h\log(f(d+ex)^n))\operatorname{Li}_2(c(a+bx))\,dx = x(g+h\log(f(d+ex)^n))\operatorname{Li}_2(c(a+bx)) + b\int \left(\frac{1}{b} - \frac{a}{b(a+bx}\right.$$

$$= x(g+h\log(f(d+ex)^n))\operatorname{Li}_2(c(a+bx)) + b\int \frac{x\log(1-ac-}{}$$

$$= x(g+h\log(f(d+ex)^n))\operatorname{Li}_2(c(a+bx)) + b\int \left(\frac{gx\log(1-a}{a+b}\right.$$

$$= x(g+h\log(f(d+ex)^n))\operatorname{Li}_2(c(a+bx)) + (bg)\int \frac{x\log(1-a}{a+b}$$

$$= x(g+h\log(f(d+ex)^n))\operatorname{Li}_2(c(a+bx)) - hnx\operatorname{Li}_2(ac+bcx)$$

$$= x(g+h\log(f(d+ex)^n))\operatorname{Li}_2(c(a+bx)) - hnx\operatorname{Li}_2(ac+bcx)$$

$$= hnx + \frac{hn(1-ac-bcx)\log(1-ac-bcx)}{bc} + \frac{dhn\log(c(a+b}{}$$

$$= -gx + hnx - \frac{g(1-ac-bcx)\log(1-ac-bcx)}{bc} + \frac{hn(1-ac}{}$$

$$= -gx + hnx - \frac{g(1-ac-bcx)\log(1-ac-bcx)}{bc} + \frac{hn(1-ac}{}$$

$$= -gx + hnx - \frac{g(1-ac-bcx)\log(1-ac-bcx)}{bc} + \frac{hn(1-ac}{}$$

$$= -gx + 3hnx - \frac{g(1-ac-bcx)\log(1-ac-bcx)}{bc} + \frac{2hn(1-}{}$$

$$= -gx + 3hnx - \frac{g(1-ac-bcx)\log(1-ac-bcx)}{bc} + \frac{2hn(1-}{}$$

Mathematica [A]
time = 3.13, size = 1546, normalized size = 0.94

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] ((g - h*n*Log[d + e*x] + h*Log[f*(d + e*x)^n])*(-(b*c*x) + (-1 + a*c + b*c*
x)*Log[1 - a*c - b*c*x] + c*(a + b*x)*PolyLog[2, c*(a + b*x)]))/(b*c) + (h*
n*((-(e*x) + (d + e*x)*Log[d + e*x])*PolyLog[2, c*(a + b*x)] + (-e + a*c*e
+ 2*b*c*e*x - b*c*d*Log[d + e*x] - b*c*e*x*Log[d + e*x] + Log[1 - a*c - b*c
*x]*(-(e*(-1 + a*c + b*c*x)) + e*(-1 + a*c + b*c*x)*Log[d + e*x] + (b*c*d +
 e - a*c*e)*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)]) + e*(-1 + a*c + b*c*x
 + (1 - a*c - b*c*x + a*c*Log[c*(a + b*x)])*Log[1 - a*c - b*c*x] + a*c*Poly
Log[2, 1 - a*c - b*c*x]) + (b*c*d + e - a*c*e)*PolyLog[2, (e*(-1 + a*c + b*
c*x))/(-(b*c*d) + (-1 + a*c)*e)] + b*c*d*(Log[c*(a + b*x)]*Log[1 - a*c - b*
c*x]*Log[d + e*x] + ((Log[c*(a + b*x)] - Log[(e*(a + b*x))/(-(b*d) + a*e)])
*Log[(b*(d + e*x))/(b*d - a*e)]*(-2*Log[1 - a*c - b*c*x] + Log[(b*(d + e*x)
)/(b*d - a*e)]))/2 + (-Log[c*(a + b*x)] + Log[(e*(a + b*x))/(-(b*d) + a*e)]
)*Log[(b*(d + e*x))/(b*d - a*e)]*Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c
 + b*c*x)))] + (Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]^2*(L
og[c*(a + b*x)] - Log[((b*c*d + e - a*c*e)*(a + b*x))/((b*d - a*e)*(-1 + a*
c + b*c*x))] + Log[(b*c*d + e - a*c*e)/(e - a*c*e - b*c*e*x)]))/2 + (Log[d
+ e*x] - Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2,
 1 - a*c - b*c*x] + (Log[1 - a*c - b*c*x] + Log[-((b*(d + e*x))/((b*d - a*e
)*(-1 + a*c + b*c*x)))])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)] + Log[-((b*(
d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]*(PolyLog[2, (b*c*(d + e*x))/(e
*(-1 + a*c + b*c*x))] - PolyLog[2, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c +
 b*c*x)))]) - PolyLog[3, 1 - a*c - b*c*x] - PolyLog[3, (b*(d + e*x))/(b*d -
 a*e)] - PolyLog[3, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))] + PolyLog[3, -(
(b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]) - a*c*e*(Log[c*(a + b*x)]
*Log[1 - a*c - b*c*x]*Log[d + e*x] + ((Log[c*(a + b*x)] - Log[(e*(a + b*x))
/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*(-2*Log[1 - a*c - b*c*x] +
 Log[(b*(d + e*x))/(b*d - a*e)]))/2 + (-Log[c*(a + b*x)] + Log[(e*(a + b*x)
)/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*Log[-((b*(d + e*x))/((b*d
 - a*e)*(-1 + a*c + b*c*x)))] + (Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c
 + b*c*x)))]^2*(Log[c*(a + b*x)] - Log[((b*c*d + e - a*c*e)*(a + b*x))/((b*
d - a*e)*(-1 + a*c + b*c*x))] + Log[(b*c*d + e - a*c*e)/(e - a*c*e - b*c*e*
x)]))/2 + (Log[d + e*x] - Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*
x)))])*PolyLog[2, 1 - a*c - b*c*x] + (Log[1 - a*c - b*c*x] + Log[-((b*(d +
e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2, (b*(d + e*x))/(b*d - a
*e)] + Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]*(PolyLog[2, (
b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))] - PolyLog[2, -((b*(d + e*x))/((b*d -
 a*e)*(-1 + a*c + b*c*x)))]) - PolyLog[3, 1 - a*c - b*c*x] - PolyLog[3, (b*
(d + e*x))/(b*d - a*e)] - PolyLog[3, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))
] + PolyLog[3, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])))/(b*c)))
/e
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (g + h \ln (f(ex + d)^n)) \operatorname{polylog}(2, c(bx + a)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x)
```

```
[Out] int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="maxima"
)
```

```
[Out] (d*h*n*log(x*e + d) + h*x*e*log((x*e + d)^n) - (h*n*e - h*e*log(f) - g*e)*x
)*dilog(b*c*x + a*c)*e^(-1) + integrate((b*h*x*e*log(-b*c*x - a*c + 1)*log(
(x*e + d)^n) + (b*d*h*n*log(x*e + d) - (b*h*n*e - b*h*e*log(f) - b*g*e)*x)*
log(-b*c*x - a*c + 1))/(b*x*e + a*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="fricas"
)
```

```
[Out] integral(h*dilog(b*c*x + a*c)*log((x*e + d)^n*f) + g*dilog(b*c*x + a*c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="giac")
[Out] integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{polylog}(2, c\,(a+b\,x))\,\left(g+h\,\ln\left(f\,{(d+e\,x)}^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)),x)
[Out] int(polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)), x)
```

3.180 $\int \frac{(g+h\log(f(d+ex)^n))\text{PolyLog}(2,c(a+bx))}{x}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))}{x},x\right)$$

```
[Out] Unintegrable((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(g+h\log(f(d+ex)^n))\,\text{Li}_2(c(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x,x]
[Out] Defer[Int][((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x, x]
```

Rubi steps

$$\int \frac{(g+h\log(f(d+ex)^n))\,\text{Li}_2(c(a+bx))}{x}\,dx = \int \frac{(g+h\log(f(d+ex)^n))\,\text{Li}_2(c(a+bx))}{x}\,dx$$

Mathematica [A]
time = 1.77, size = 0, normalized size = 0.00

$$\int \frac{(g+h\log(f(d+ex)^n))\,\text{PolyLog}(2,c(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x,x]
[Out] Integrate[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x, x]
```

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(g+h\ln(f(ex+d)^n))\,\text{polylog}(2,c(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x,x)
```

```
[Out] int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x,x, algorithm="maxim
a")
```

```
[Out] integrate((h*log((x*e + d)^n*f) + g)*dilog((b*x + a)*c)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x,x, algorithm="frica
s")
```

```
[Out] integral((h*dilog(b*c*x + a*c)*log((x*e + d)^n*f) + g*dilog(b*c*x + a*c))/x
, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g + h\log{(f(d + ex)^{n})})\operatorname{Li}_{2}(ac + bcx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a))/x,x)
```

```
[Out] Integral((g + h*log(f*(d + e*x)**n))*polylog(2, a*c + b*c*x)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x,x, algorithm="giac"
)
```

```
[Out] integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{polylog}(2, c\,(a+b\,x))\ (g+h\ \ln\left(f\,(d+e\,x)^n\right))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)))/x,x)
```

```
[Out] int((polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)))/x, x)
```

3.181 $\int \frac{(g+h\log(f(d+ex)^n))\text{PolyLog}(2,c(a+bx))}{x^2}\,dx$

Optimal. Leaf size=2498

result too large to display

[Out] -b*g*polylog(2,c*(b*x+a))/a-b*g*polylog(2,1-b*c*x/(-a*c+1))/a-(g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x+b*h*ln(b*c*x/(-a*c+1))*ln(-b*c*x-a*c+1)*(n*ln(e*x+d)-ln(f*(e*x+d)^n))/a+e*h*n*(ln(1-c*(b*x+a))-ln(-a*(1-c*(b*x+a))/b/x))*polylog(2,-b*x/a)/d+e*h*n*ln(x)*polylog(2,c*(b*x+a))/d-e*h*n*ln(e*x+d)*polylog(2,c*(b*x+a))/d-b*h*n*(ln(e*x+d)-ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a*c+1)))*polylog(2,1-b*c*x/(-a*c+1))/a-b*h*n*ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a*c+1))*polylog(2,d*(-b*c*x-a*c+1)/(-a*c+1)/(e*x+d))/a+b*h*n*ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a*c+1))*polylog(2,-e*(-b*c*x-a*c+1)/b/c/(e*x+d))/a+b*h*n*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))*polylog(2,b*(e*x+d)/(-a*e+b*d))/a-e*h*n*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))*polylog(2,b*(e*x+d)/(-a*e+b*d))/d-b*h*n*(ln(-b*c*x-a*c+1)+ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a*c+1)))*polylog(2,1+e*x/d)/a+e*h*n*ln(-a*(1-c*(b*x+a))/b/x)*polylog(2,-b*x/a/(1-c*(b*x+a)))/d-e*h*n*ln(-a*(1-c*(b*x+a))/b/x)*polylog(2,-b*c*x/(1-c*(b*x+a)))/d+b*h*n*(ln(e*x+d)-ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a))))*polylog(2,1-c*(b*x+a))/a-e*h*n*(ln(e*x+d)-ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a))))*polylog(2,1-c*(b*x+a))/d+e*h*n*(ln(x)+ln(-a*(1-c*(b*x+a))/b/x))*polylog(2,1-c*(b*x+a))/d-b*h*n*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))*polylog(2,-e*(1-c*(b*x+a))/b/c/(e*x+d))/a+e*h*n*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))*polylog(2,-e*(1-c*(b*x+a))/b/c/(e*x+d))/d+b*h*n*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))*polylog(2,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/a-e*h*n*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))*polylog(2,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/d-1/2*b*h*n*(ln(b*c*x/(-a*c+1))+ln((-a*c*e+b*c*d+e)/b/c/(e*x+d))-ln((-a*c*e+b*c*d+e)*x/(-a*c+1)/(e*x+d)))*ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a*c+1))^2/a+1/2*b*h*n*(ln(b*c*x/(-a*c+1))-ln(-e*x/d))*(ln(-b*c*x-a*c+1)+ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a*c+1)))^2/a+1/2*b*h*n*(ln(c*(b*x+a))+ln((-a*c*e+b*c*d+e)/b/c/(e*x+d))-ln((-a*c*e+b*c*d+e)*(b*x+a)/b/(e*x+d)))*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))^2/a-1/2*e*h*n*(ln(c*(b*x+a))+ln((-a*c*e+b*c*d+e)/b/c/(e*x+d))-ln((-a*c*e+b*c*d+e)*(b*x+a)/b/(e*x+d)))*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))^2/d-1/2*b*h*n*(ln(c*(b*x+a))-ln(-e*(b*x+a)/(-a*e+b*d)))*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))^2/a+1/2*e*h*n*(ln(c*(b*x+a))-ln(-e*(b*x+a)/(-a*e+b*d)))*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))^2/d+1/2*e*h*n*(ln(1+b*x/a)+ln((-a*c+1)/(1-c*(b*x+a)))-ln((-a*c+1)*(b*x+a)/a/(1-c*(b*x+a))))*ln(-a*(1-c*(b*x+a))/b/x)^2/d+1/2*e*h*n*(ln(c*(b*x+a))-ln(1+b*x/a))*(ln(x)+ln(-a*(1-c*(b*x+a))/b/x))^2/d-b*h*n*polylog(3,1-c*(b*x+a))/a-b*h*n*polylog(3,-e*(1-c*(b*x+a))/b/c/(e*x+d))/a+e*h*n*polylog(3,-e*(1-c*(b*x+a))/b/c/(e*x+d))/d+b*h*n*polylog(3,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/a-e*h*n*polylog(3,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/d-b*g*ln(b*c*x/(-a*c+1))*ln(-b*c*x-a*c+1)/a+b*h*(n*ln(e*x+d)-ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/a+b*h*(n*ln(e*x+d)-ln(f*(e*x+d)^n))*polylog(2,1-b*c*x/(-a*c+1)

```
)/a+b*h*n*polylog(3,1-b*c*x/(-a*c+1))/a-b*h*n*polylog(3,d*(-b*c*x-a*c+1)/(-
a*c+1)/(e*x+d))/a+b*h*n*polylog(3,-e*(-b*c*x-a*c+1)/b/c/(e*x+d))/a-b*h*n*po
lylog(3,b*(e*x+d)/(-a*e+b*d))/a+e*h*n*polylog(3,b*(e*x+d)/(-a*e+b*d))/d+e*h
*n*polylog(3,-b*x/a/(1-c*(b*x+a)))/d-e*h*n*polylog(3,-b*c*x/(1-c*(b*x+a)))/
d-e*h*n*polylog(3,-b*x/a)/d+b*h*n*polylog(3,1+e*x/d)/a-b*h*n*ln(b*c*x/(-a*c
+1))*ln(-b*c*x-a*c+1)*ln(e*x+d)/a+e*h*n*ln(x)*ln(1+b*x/a)*ln(1-c*(b*x+a))/d
+b*h*n*ln(c*(b*x+a))*ln(e*x+d)*ln(1-c*(b*x+a))/a-e*h*n*ln(c*(b*x+a))*ln(e*x
+d)*ln(1-c*(b*x+a))/d
```

Rubi [A]

time = 1.78, antiderivative size = 2498, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6738, 2488, 2441, 2352, 2487, 2485, 2490, 2438, 6732}

Too large to display

Antiderivative was successfully verified.

```
[In] Int[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^2,x]
```

```
[Out] -((b*g*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/a) - (b*h*n*Log[(b*c*x)
/(1 - a*c)]*Log[1 - a*c - b*c*x]*Log[d + e*x])/a - (b*h*n*(Log[(b*c*x)/(1 -
 a*c)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e
)*x)/((1 - a*c)*(d + e*x))])*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x)
)]^2)/(2*a) + (b*h*n*(Log[(b*c*x)/(1 - a*c)] - Log[-((e*x)/d)])*(Log[1 - a*
c - b*c*x] + Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])^2)/(2*a) + (
b*h*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*(n*Log[d + e*x] - Log[f*(d
+ e*x)^n]))/a + (b*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d
+ e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d +
e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(2*a) - (e*h*n*(Log[c*(a + b*x)]
+ Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a +
b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)
/(2*d) + (e*h*n*Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*x)])/d + (b*h*n*Lo
g[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/a - (e*h*n*Log[c*(a + b*x
)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/d - (b*h*n*(Log[c*(a + b*x)] - Log[-(
(e*(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*
x)))] + Log[1 - c*(a + b*x)])^2)/(2*a) + (e*h*n*(Log[c*(a + b*x)] - Log[-((
e*(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x
)))] + Log[1 - c*(a + b*x)])^2)/(2*d) + (e*h*n*(Log[1 + (b*x)/a] + Log[(1 -
 a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))]
)*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(2*d) + (e*h*n*(Log[c*(a + b*x)] -
 Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/(2*d)
+ (e*h*n*(Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyL
og[2, -((b*x)/a)])/d - (b*g*PolyLog[2, c*(a + b*x)])/a + (e*h*n*Log[x]*Poly
Log[2, c*(a + b*x)])/d - (e*h*n*Log[d + e*x]*PolyLog[2, c*(a + b*x)])/d + (
b*h*(n*Log[d + e*x] - Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/a - ((g
```

```
+ h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x - (b*g*PolyLog[2, 1 - (b
*c*x)/(1 - a*c)])/a - (b*h*n*(Log[d + e*x] - Log[((1 - a*c)*(d + e*x))/(d*(
1 - a*c - b*c*x))])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/a + (b*h*(n*Log[d +
e*x] - Log[f*(d + e*x)^n])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/a - (b*h*n*Lo
g[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))]*PolyLog[2, (d*(1 - a*c - b*c
*x))/((1 - a*c)*(d + e*x))])/a + (b*h*n*Log[((1 - a*c)*(d + e*x))/(d*(1 - a
*c - b*c*x))]*PolyLog[2, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))])/a + (b*
h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*
x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)])/a - (e*h*n*(Log[(b*(d + e*x))/(
(b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d +
e*x))/(b*d - a*e)])/d - (b*h*n*(Log[1 - a*c - b*c*x] + Log[((1 - a*c)*(d +
e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, 1 + (e*x)/d])/a + (e*h*n*Log[-((a*
(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))))])/d -
(e*h*n*Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*c*x)/(1 - c*(a +
 b*x)))])/d + (b*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*
(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/a - (e*h*n*(Log[d + e*x] - Log[(
b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])
/d + (e*h*n*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, 1 - c
*(a + b*x)])/d - (b*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*
PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/a + (e*h*n*Log[(b*(d
+ e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))
/(b*c*(d + e*x)))])/d + (b*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b
*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/a - (e*h*
n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e
)*(1 - c*(a + b*x)))/(b*(d + e*x))])/d - (e*h*n*PolyLog[3, -((b*x)/a)])/d +
 (b*h*n*PolyLog[3, 1 - (b*c*x)/(1 - a*c)])/a - (b*h*n*PolyLog[3, (d*(1 - a*
c - b*c*x))/((1 - a*c)*(d + e*x))])/a + (b*h*n*PolyLog[3, -((e*(1 - a*c - b
*c*x))/(b*c*(d + e*x)))])/a - (b*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])
/a + (e*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])/d + (b*h*n*PolyLog[3, 1
+ (e*x)/d])/a + (e*h*n*PolyLog[3, -((b*x)/(a*(1 - c*(a + b*x))))])/d - (e*h
*n*PolyLog[3, -((b*c*x)/(1 - c*(a + b*x)))])/d - (b*h*n*PolyLog[3, 1 - c*(a
 + b*x)])/a - (b*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/
a + (e*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/d + (b*h*n
*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/a - (e*h*n*Poly
Log[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/d
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2485

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[(-b)*(x/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1/2)*(Log[(-b)*(x/a
)] - Log[(-(b*c - a*d))*(x/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))]
)*Log[a*((c + d*x)/(c*(a + b*x)))]^2, x] - Simp[(1/2)*(Log[(-b)*(x/a)] - Lo
g[(-d)*(x/c)])*(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])^2, x] + Si
mp[(Log[c + d*x] - Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1 + b*(x/a)
], x] + Simp[(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1
 + d*(x/c)], x] + Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2, c*((a +
b*x)/(a*(c + d*x)))], x] - Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2,
 d*((a + b*x)/(b*(c + d*x)))], x] - Simp[PolyLog[3, 1 + b*(x/a)], x] - Simp
[PolyLog[3, 1 + d*(x/c)], x] + Simp[PolyLog[3, c*((a + b*x)/(a*(c + d*x)))]
, x] - Simp[PolyLog[3, d*((a + b*x)/(b*(c + d*x)))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2487

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Dist[m, Int[Log[i + j*x]*(Log[c*(d + e*x)^n]/x), x
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
 x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
 + j*x, h*(i + j*x)^m]
```

Rule 2488

```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(Log[(h_.)*((i_.)
 + (j_.)*(x_))^(m_.)]*(g_.) + (f_)))/(x_), x_Symbol] :> Dist[f, Int[(a + b*
Log[c*(d + e*x)^n])/x, x], x] + Dist[g, Int[Log[h*(i + j*x)^m]*((a + b*Log[
c*(d + e*x)^n])/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x
] && NeQ[e*i - d*j, 0]
```

Rule 2490

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-(e*k - d*l)/l + e*(x/l))^n])*(f + g
*Log[h*(-(j*k - i*l)/l + j*(x/l))^m]), x], x, k + l*x], x] /; FreeQ[{a, b,
c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 6732

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[d + e*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, Int[Log[d
+ e*x]*(Log[1 - a*c - b*c*x]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rule 6738

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[e*h*(n/(m + 1)), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(g+h\log(f(d+ex)^n))\,\text{Li}_2(c(a+bx))}{x^2}\,dx = -\frac{(g+h\log(f(d+ex)^n))\,\text{Li}_2(c(a+bx))}{x} - b\int \Bigg(\frac{\log(1-a$$

$$= -\frac{(g+h\log(f(d+ex)^n))\,\text{Li}_2(c(a+bx))}{x} - \frac{b\int \frac{\log(1-ac-bcx)(}{}}{}$$

$$= \frac{ehn\log(x)\text{Li}_2(c(a+bx))}{d} - \frac{ehn\log(d+ex)\text{Li}_2(c(a+bx))}{d}$$

$$= -\frac{bg\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{a} + \frac{ehn\log(x)\text{Li}_2(c(a+bx)}{d}$$

$$= -\frac{bg\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{a} - \frac{bhn\log\left(\frac{bcx}{1-ac}\right)\log(1-a}{a}$$

$$= -\frac{bg\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{a} - \frac{bhn\log\left(\frac{bcx}{1-ac}\right)\log(1-a}{a}$$

Mathematica [A]
time = 7.16, size = 2247, normalized size = 0.90

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^2,x]

[Out] -(((g - h*n*Log[d + e*x] + h*Log[f*(d + e*x)^n])*((a + b*x)*PolyLog[2, c*(a
 + b*x)] + b*x*(Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x] + PolyLog[2, (-
1 + a*c + b*c*x)/(-1 + a*c)])))/(a*x)) + (h*n*(a*(e*x*Log[x] - (d + e*x)*Lo
g[d + e*x])*PolyLog[2, c*(a + b*x)] + x*(a*e*(Log[x]*Log[1 + (b*x)/a]*Log[1
 - a*c - b*c*x] + ((-Log[c*(a + b*x)] + Log[1 + (b*x)/a])*Log[1 - a*c - b*c
*x]*(-2*Log[x] + Log[1 - a*c - b*c*x]))/2 + (Log[c*(a + b*x)] - Log[1 + (b*
x)/a])*Log[1 - a*c - b*c*x]*Log[(a*(-1 + a*c + b*c*x))/(b*x)] + ((Log[(1 -
a*c)/(b*c*x)] - Log[((1 - a*c)*(a + b*x))/(b*x)] + Log[1 + (b*x)/a])*Log[(a
*(-1 + a*c + b*c*x))/(b*x)]^2)/2 + (Log[1 - a*c - b*c*x] - Log[(a*(-1 + a*c
 + b*c*x))/(b*x)])*PolyLog[2, -((b*x)/a)] + (Log[x] + Log[(a*(-1 + a*c + b*
c*x))/(b*x)])*PolyLog[2, 1 - a*c - b*c*x] + Log[(a*(-1 + a*c + b*c*x))/(b*x
)]*(-PolyLog[2, (a*(-1 + a*c + b*c*x))/(b*x)] + PolyLog[2, (-1 + a*c + b*c*
x)/(b*c*x)]) - PolyLog[3, -((b*x)/a)] - PolyLog[3, 1 - a*c - b*c*x] + PolyL
og[3, (a*(-1 + a*c + b*c*x))/(b*x)] - PolyLog[3, (-1 + a*c + b*c*x)/(b*c*x)
]) - a*e*(Log[c*(a + b*x)]*Log[1 - a*c - b*c*x]*Log[d + e*x] + ((Log[c*(a +
 b*x)] - Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*
(-2*Log[1 - a*c - b*c*x] + Log[(b*(d + e*x))/(b*d - a*e)]))/2 + (-Log[c*(a
+ b*x)] + Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]
*Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))] + (Log[-((b*(d + e*
x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]^2*(Log[c*(a + b*x)] - Log[((b*c*d +
e - a*c*e)*(a + b*x))/((b*d - a*e)*(-1 + a*c + b*c*x))] + Log[(b*c*d + e -
a*c*e)/(e - a*c*e - b*c*e*x)]))/2 + (Log[d + e*x] - Log[-((b*(d + e*x))/((b
*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2, 1 - a*c - b*c*x] + (Log[1 - a*c
 - b*c*x] + Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog
[2, (b*(d + e*x))/(b*d - a*e)] + Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c
 + b*c*x)))]*(PolyLog[2, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))] - PolyLog[
2, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]) - PolyLog[3, 1 - a*c
 - b*c*x] - PolyLog[3, (b*(d + e*x))/(b*d - a*e)] - PolyLog[3, (b*c*(d + e*
x))/(e*(-1 + a*c + b*c*x))] + PolyLog[3, -((b*(d + e*x))/((b*d - a*e)*(-1 +
 a*c + b*c*x)))]) - b*d*(Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*Log[d
+ e*x] - Log[c*(a + b*x)]*Log[1 - a*c - b*c*x]*Log[d + e*x] - ((Log[c*(a +
b*x)] - Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*(
-2*Log[1 - a*c - b*c*x] + Log[(b*(d + e*x))/(b*d - a*e)]))/2 + (Log[c*(a +
b*x)] - Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*L
og[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))] + (Log[((-1 + a*c)*(d
 + e*x))/(d*(-1 + a*c + b*c*x))]^2*(Log[(b*c*x)/(1 - a*c)] - Log[((b*c*d +
e - a*c*e)*x)/(d*(-1 + a*c + b*c*x))] + Log[(b*c*d + e - a*c*e)/(e - a*c*e
- b*c*e*x)]))/2 - (Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]^2
*(Log[c*(a + b*x)] - Log[((b*c*d + e - a*c*e)*(a + b*x))/((b*d - a*e)*(-1 +
 a*c + b*c*x))] + Log[(b*c*d + e - a*c*e)/(e - a*c*e - b*c*e*x)]))/2 + (-Lo
g[(b*c*x)/(1 - a*c)] + Log[-((e*x)/d)])*Log[((-1 + a*c)*(d + e*x))/(d*(-1 +
 a*c + b*c*x))]*Log[1 + (e*x)/d] + ((Log[(b*c*x)/(1 - a*c)] - Log[-((e*x)/d
)])*Log[1 + (e*x)/d]*(-2*Log[1 - a*c - b*c*x] + Log[1 + (e*x)/d]))/2 - (Log
```

```
[d + e*x] - Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog
[2, 1 - a*c - b*c*x] + (Log[d + e*x] - Log[((-1 + a*c)*(d + e*x))/(d*(-1 +
a*c + b*c*x))])*PolyLog[2, (-1 + a*c + b*c*x)/(-1 + a*c)] - (Log[1 - a*c -
b*c*x] + Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2,
 (b*(d + e*x))/(b*d - a*e)] + Log[((-1 + a*c)*(d + e*x))/(d*(-1 + a*c + b*c
*x))]*(-PolyLog[2, ((-1 + a*c)*(d + e*x))/(d*(-1 + a*c + b*c*x))] + PolyLog
[2, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))]) - Log[-((b*(d + e*x))/((b*d -
a*e)*(-1 + a*c + b*c*x)))]*(PolyLog[2, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x
))] - PolyLog[2, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]) + (Log
[1 - a*c - b*c*x] + Log[((-1 + a*c)*(d + e*x))/(d*(-1 + a*c + b*c*x))])*Pol
yLog[2, 1 + (e*x)/d] + PolyLog[3, 1 - a*c - b*c*x] - PolyLog[3, (-1 + a*c +
 b*c*x)/(-1 + a*c)] + PolyLog[3, (b*(d + e*x))/(b*d - a*e)] + PolyLog[3, ((
-1 + a*c)*(d + e*x))/(d*(-1 + a*c + b*c*x))] - PolyLog[3, -((b*(d + e*x))/(
(b*d - a*e)*(-1 + a*c + b*c*x)))] - PolyLog[3, 1 + (e*x)/d]))))/(a*d*x)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(g + h\ln(f(ex+d)^n))\,\mathrm{polylog}(2, c(bx+a))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^2,x)
```

```
[Out] int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^2,x, algorithm="max
ima")
```

```
[Out] integrate((h*log((x*e + d)^n*f) + g)*dilog((b*x + a)*c)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^2,x, algorithm="fri
cas")
```

```
[Out] integral((h*dilog(b*c*x + a*c)*log((x*e + d)^n*f) + g*dilog(b*c*x + a*c))/x
^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a))/x**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^2,x, algorithm="gia
c")
```

```
[Out] integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{polylog}(2,c\,(a+b\,x))\,\left(g+h\,\ln\left(f\,{\left(d+e\,x\right)}^n\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)))/x^2,x)
```

```
[Out] int((polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)))/x^2, x)
```

3.182 $\int \frac{(g+h\log(f(d+ex)^n))\mathbf{PolyLog}(2,c(a+bx))}{x^3}\,dx$

Optimal. Leaf size=3119

result too large to display

```
[Out] 1/2*b^2*g*polylog(2,c*(b*x+a))/a^2+1/2*b^2*g*polylog(2,1-b*c*x/(-a*c+1))/a^
2-1/2*b*e*h*n*polylog(2,c*(b*x+a))/a/d+1/2*b*e*h*n*polylog(2,e*(-b*c*x-a*c+
1)/(-a*c*e+b*c*d+e))/a/d-1/2*b^2*c*h*n*polylog(2,b*c*(e*x+d)/(-a*c*e+b*c*d+
e))/a/(-a*c+1)+1/2*b^2*c*h*n*polylog(2,1+e*x/d)/a/(-a*c+1)-b*e*h*n*ln(b*c*x
/(-a*c+1))*ln(-b*c*x-a*c+1)/a/d+1/2*b*e*h*n*ln(-b*c*x-a*c+1)*ln(b*c*(e*x+d)
/(-a*c*e+b*c*d+e))/a/d-1/2*(g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^2+1
/2*e^2*h*n*polylog(3,-b*x/a)/d^2-1/2*b^2*h*n*polylog(3,1-b*c*x/(-a*c+1))/a^
2+1/2*b^2*h*n*polylog(3,d*(-b*c*x-a*c+1)/(-a*c+1)/(e*x+d))/a^2-1/2*b^2*h*n*
polylog(3,-e*(-b*c*x-a*c+1)/b/c/(e*x+d))/a^2+1/2*b^2*h*n*polylog(3,b*(e*x+d
)/(-a*e+b*d))/a^2-1/2*e^2*h*n*polylog(3,b*(e*x+d)/(-a*e+b*d))/d^2-1/2*b^2*h
*n*polylog(3,1+e*x/d)/a^2-1/2*e^2*h*n*polylog(3,-b*x/a/(1-c*(b*x+a)))/d^2+1
/2*e^2*h*n*polylog(3,-b*c*x/(1-c*(b*x+a)))/d^2+1/2*b^2*h*n*polylog(3,1-c*(b
*x+a))/a^2+1/2*b^2*h*n*polylog(3,-e*(1-c*(b*x+a))/b/c/(e*x+d))/a^2-1/2*e^2*
h*n*polylog(3,-e*(1-c*(b*x+a))/b/c/(e*x+d))/d^2-1/2*b^2*h*n*polylog(3,(-a*e
+b*d)*(1-c*(b*x+a))/b/(e*x+d))/a^2+1/2*e^2*h*n*polylog(3,(-a*e+b*d)*(1-c*(b
*x+a))/b/(e*x+d))/d^2-1/4*e^2*h*n*(ln(c*(b*x+a))-ln(-e*(b*x+a)/(-a*e+b*d)))
*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))^2/d^2-1/4*e^2*h*n
*(ln(1+b*x/a)+ln((-a*c+1)/(1-c*(b*x+a)))-ln((-a*c+1)*(b*x+a)/a/(1-c*(b*x+a)
)))*ln(-a*(1-c*(b*x+a))/b/x)^2/d^2-1/4*e^2*h*n*(ln(c*(b*x+a))-ln(1+b*x/a))*
(ln(x)+ln(-a*(1-c*(b*x+a))/b/x))^2/d^2-1/2*e^2*h*n*(ln(1-c*(b*x+a))-ln(-a*(
1-c*(b*x+a))/b/x))*polylog(2,-b*x/a)/d^2-1/2*e^2*h*n*ln(x)*polylog(2,c*(b*x
+a))/d^2+1/2*e^2*h*n*ln(e*x+d)*polylog(2,c*(b*x+a))/d^2+1/2*b^2*h*n*(ln(e*x
+d)-ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a*c+1)))*polylog(2,1-b*c*x/(-a*c+1))/a^2+
1/2*b^2*h*n*ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a*c+1))*polylog(2,d*(-b*c*x-a*c+1
)/(-a*c+1)/(e*x+d))/a^2-1/2*b^2*h*n*ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a*c+1))*p
olylog(2,-e*(-b*c*x-a*c+1)/b/c/(e*x+d))/a^2-1/2*b^2*h*n*(ln(b*(e*x+d)/(-a*e
+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))*polylog(2,b*(e*x+d)/(-a*e+b*d))/a^2+1
/2*e^2*h*n*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))*polylog
(2,b*(e*x+d)/(-a*e+b*d))/d^2+1/2*b^2*h*n*(ln(-b*c*x-a*c+1)+ln((-a*c+1)*(e*x
+d)/d/(-b*c*x-a*c+1)))*polylog(2,1+e*x/d)/a^2-1/2*e^2*h*n*ln(-a*(1-c*(b*x+a
))/b/x)*polylog(2,-b*x/a/(1-c*(b*x+a)))/d^2+1/2*e^2*h*n*ln(-a*(1-c*(b*x+a))
/b/x)*polylog(2,-b*c*x/(1-c*(b*x+a)))/d^2-1/2*b^2*h*n*(ln(e*x+d)-ln(b*(e*x+
d)/(-a*e+b*d)/(1-c*(b*x+a))))*polylog(2,1-c*(b*x+a))/a^2+1/2*e^2*h*n*(ln(e*
x+d)-ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a))))*polylog(2,1-c*(b*x+a))/d^2-1/2
*e^2*h*n*(ln(x)+ln(-a*(1-c*(b*x+a))/b/x))*polylog(2,1-c*(b*x+a))/d^2+1/2*b^
2*h*n*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))*polylog(2,-e*(1-c*(b*x+a))/b/c
/(e*x+d))/a^2-1/2*e^2*h*n*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))*polylog(2,
-e*(1-c*(b*x+a))/b/c/(e*x+d))/d^2-1/2*b^2*h*n*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*
(b*x+a)))*polylog(2,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/a^2+1/2*e^2*h*n*ln(
```

```
b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))*polylog(2,(-a*e+b*d)*(1-c*(b*x+a))/b/(e
*x+d))/d^2+1/2*b^2*g*ln(b*c*x/(-a*c+1))*ln(-b*c*x-a*c+1)/a^2+1/2*b*ln(-b*c*
x-a*c+1)*(g+h*ln(f*(e*x+d)^n))/a/x-1/2*b^2*h*(n*ln(e*x+d)-ln(f*(e*x+d)^n))*
polylog(2,c*(b*x+a))/a^2-1/2*b^2*h*(n*ln(e*x+d)-ln(f*(e*x+d)^n))*polylog(2,
1-b*c*x/(-a*c+1))/a^2+1/4*b^2*h*n*(ln(b*c*x/(-a*c+1))+ln((-a*c*e+b*c*d+e)/b
/c/(e*x+d))-ln((-a*c*e+b*c*d+e)*x/(-a*c+1)/(e*x+d)))*ln((-a*c+1)*(e*x+d)/d/
(-b*c*x-a*c+1))^2/a^2-1/4*b^2*h*n*(ln(b*c*x/(-a*c+1))-ln(-e*x/d))*(ln(-b*c*
x-a*c+1)+ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a*c+1)))^2/a^2-1/2*b^2*h*ln(b*c*x/(-
a*c+1))*ln(-b*c*x-a*c+1)*(n*ln(e*x+d)-ln(f*(e*x+d)^n))/a^2+1/2*b^2*c*ln(-e*
x/d)*(g+h*ln(f*(e*x+d)^n))/a/(-a*c+1)-1/2*b^2*c*ln(e*(-b*c*x-a*c+1)/(-a*c*e
+b*c*d+e))*(g+h*ln(f*(e*x+d)^n))/a/(-a*c+1)-1/4*b^2*h*n*(ln(c*(b*x+a))+ln((
-a*c*e+b*c*d+e)/b/c/(e*x+d))-ln((-a*c*e+b*c*d+e)*(b*x+a)/b/(e*x+d)))*ln(b*(
e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))^2/a^2+1/4*e^2*h*n*(ln(c*(b*x+a))+ln((-a*c*
e+b*c*d+e)/b/c/(e*x+d))-ln((-a*c*e+b*c*d+e)*(b*x+a)/b/(e*x+d)))*ln(b*(e*x+d
)/(-a*e+b*d)/(1-c*(b*x+a)))^2/d^2+1/4*b^2*h*n*(ln(c*(b*x+a))-ln(-e*(b*x+a)/
(-a*e+b*d)))*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))^2/a^2
-1/2*e*h*n*polylog(2,c*(b*x+a))/d/x+1/2*b^2*h*n*ln(b*c*x/(-a*c+1))*ln(-b*c*
x-a*c+1)*ln(e*x+d)/a^2-1/2*e^2*h*n*ln(x)*ln(1+b*x/a)*ln(1-c*(b*x+a))/d^2-1/
2*b^2*h*n*ln(c*(b*x+a))*ln(e*x+d)*ln(1-c*(b*x+a))/a^2+1/2*e^2*h*n*ln(c*(b*x
+a))*ln(e*x+d)*ln(1-c*(b*x+a))/d^2-b*e*h*n*polylog(2,1-b*c*x/(-a*c+1))/a/d
```

Rubi [A]

time = 2.23, antiderivative size = 3119, normalized size of antiderivative = 1.00, number of steps used = 44, number of rules used = 16, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {6738, 2489, 36, 29, 31, 2463, 2441, 2352, 2440, 2438, 2488, 2487, 2485, 2490, 6733, 6732}

Too large to display

Antiderivative was successfully verified.

```
[In] Int[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^3,x]
```

```
[Out] (b^2*g*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/(2*a^2) - (b*e*h*n*Log[
(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/(a*d) + (b^2*h*n*Log[(b*c*x)/(1 -
a*c)]*Log[1 - a*c - b*c*x]*Log[d + e*x])/(2*a^2) + (b*e*h*n*Log[1 - a*c - b
*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(2*a*d) + (b^2*h*n*(Log[(b*
c*x)/(1 - a*c)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d +
e - a*c*e)*x)/((1 - a*c)*(d + e*x))])*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c
 - b*c*x))]^2)/(4*a^2) - (b^2*h*n*(Log[(b*c*x)/(1 - a*c)] - Log[-((e*x)/d)]
)*(Log[1 - a*c - b*c*x] + Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])
^2)/(4*a^2) - (b^2*h*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*(n*Log[d +
 e*x] - Log[f*(d + e*x)^n]))/(2*a^2) + (b^2*c*Log[-((e*x)/d)]*(g + h*Log[f*
(d + e*x)^n]))/(2*a*(1 - a*c)) + (b*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d +
e*x)^n]))/(2*a*x) - (b^2*c*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(
g + h*Log[f*(d + e*x)^n]))/(2*a*(1 - a*c)) - (b^2*h*n*(Log[c*(a + b*x)] + L
```

```
og[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x
))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(4
*a^2) + (e^2*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x)
)] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/
((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(4*d^2) - (e^2*h*n*Log[x]*Log[1 + (b*x)
/a]*Log[1 - c*(a + b*x)])/(2*d^2) - (b^2*h*n*Log[c*(a + b*x)]*Log[d + e*x]*
Log[1 - c*(a + b*x)])/(2*a^2) + (e^2*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[
1 - c*(a + b*x)])/(2*d^2) + (b^2*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x)
)/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[
1 - c*(a + b*x)])^2)/(4*a^2) - (e^2*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b
*x))/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + L
og[1 - c*(a + b*x)])^2)/(4*d^2) - (e^2*h*n*(Log[1 + (b*x)/a] + Log[(1 - a*c
)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))])*Lo
g[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(4*d^2) - (e^2*h*n*(Log[c*(a + b*x)] -
 Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/(4*d^2
) - (e^2*h*n*(Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*P
olyLog[2, -((b*x)/a)])/(2*d^2) + (b^2*g*PolyLog[2, c*(a + b*x)])/(2*a^2) -
(b*e*h*n*PolyLog[2, c*(a + b*x)])/(2*a*d) - (e*h*n*PolyLog[2, c*(a + b*x)])
/(2*d*x) - (e^2*h*n*Log[x]*PolyLog[2, c*(a + b*x)])/(2*d^2) + (e^2*h*n*Log[
d + e*x]*PolyLog[2, c*(a + b*x)])/(2*d^2) - (b^2*h*(n*Log[d + e*x] - Log[f*
(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(2*a^2) - ((g + h*Log[f*(d + e*x)^n]
)*PolyLog[2, c*(a + b*x)])/(2*x^2) + (b*e*h*n*PolyLog[2, (e*(1 - a*c - b*c*
x))/(b*c*d + e - a*c*e)])/(2*a*d) + (b^2*g*PolyLog[2, 1 - (b*c*x)/(1 - a*c)
])/(2*a^2) - (b*e*h*n*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(a*d) + (b^2*h*n*(
Log[d + e*x] - Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2,
 1 - (b*c*x)/(1 - a*c)])/(2*a^2) - (b^2*h*(n*Log[d + e*x] - Log[f*(d + e*x)
^n])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(2*a^2) + (b^2*h*n*Log[((1 - a*c)*(
d + e*x))/(d*(1 - a*c - b*c*x))]*PolyLog[2, (d*(1 - a*c - b*c*x))/((1 - a*c
)*(d + e*x))])/(2*a^2) - (b^2*h*n*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b
*c*x))]*PolyLog[2, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))])/(2*a^2) - (b^
2*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a +
b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)])/(2*a^2) + (e^2*h*n*(Log[(b*(d
 + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2
, (b*(d + e*x))/(b*d - a*e)])/(2*d^2) - (b^2*c*h*n*PolyLog[2, (b*c*(d + e*x
))/(b*c*d + e - a*c*e)])/(2*a*(1 - a*c)) + (b^2*c*h*n*PolyLog[2, 1 + (e*x)/
d])/(2*a*(1 - a*c)) + (b^2*h*n*(Log[1 - a*c - b*c*x] + Log[((1 - a*c)*(d +
e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, 1 + (e*x)/d])/(2*a^2) - (e^2*h*n*L
og[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x)))
)])/(2*d^2) + (e^2*h*n*Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*
c*x)/(1 - c*(a + b*x)))])/(2*d^2) - (b^2*h*n*(Log[d + e*x] - Log[(b*(d + e*
x))/((b*d - a*e)*(1 - c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/(2*a^2)
+ (e^2*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x))
)])*PolyLog[2, 1 - c*(a + b*x)])/(2*d^2) - (e^2*h*n*(Log[x] + Log[-((a*(1 -
 c*(a + b*x)))/(b*x))])*PolyLog[2, 1 - c*(a + b*x)])/(2*d^2) + (b^2*h*n*Log
[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a +
```

```
 b*x)))/(b*c*(d + e*x)))])/(2*a^2) - (e^2*h*n*Log[(b*(d + e*x))/((b*d - a*e
)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])
/(2*d^2) - (b^2*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*Poly
Log[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/(2*a^2) + (e^2*h*n*L
og[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(
1 - c*(a + b*x)))/(b*(d + e*x))])/(2*d^2) + (e^2*h*n*PolyLog[3, -((b*x)/a)]
)/(2*d^2) - (b^2*h*n*PolyLog[3, 1 - (b*c*x)/(1 - a*c)])/(2*a^2) + (b^2*h*n*
PolyLog[3, (d*(1 - a*c - b*c*x))/((1 - a*c)*(d + e*x))])/(2*a^2) - (b^2*h*n
*PolyLog[3, -((e*(1 - a*c - b*c*x))/(b*c*(d + e...
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2485

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[(-b)*(x/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1/2)*(Log[(-b)*(x/a
)] - Log[(-(b*c - a*d))*(x/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))]
)*Log[a*((c + d*x)/(c*(a + b*x)))]^2, x] - Simp[(1/2)*(Log[(-b)*(x/a)] - Lo
g[(-d)*(x/c)])*(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])^2, x] + Si
mp[(Log[c + d*x] - Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1 + b*(x/a)
], x] + Simp[(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1
 + d*(x/c)], x] + Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2, c*((a +
b*x)/(a*(c + d*x)))], x] - Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2,
 d*((a + b*x)/(b*(c + d*x)))], x] - Simp[PolyLog[3, 1 + b*(x/a)], x] - Simp
[PolyLog[3, 1 + d*(x/c)], x] + Simp[PolyLog[3, c*((a + b*x)/(a*(c + d*x)))]
, x] - Simp[PolyLog[3, d*((a + b*x)/(b*(c + d*x)))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2487

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Dist[m, Int[Log[i + j*x]*(Log[c*(d + e*x)^n]/x), x
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
 x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
 + j*x, h*(i + j*x)^m]
```

Rule 2488

```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(Log[(h_.)*((i_.)
 + (j_.)*(x_))^(m_.)]*(g_.) + (f_)))/(x_), x_Symbol] :> Dist[f, Int[(a + b*
Log[c*(d + e*x)^n])/x, x], x] + Dist[g, Int[Log[h*(i + j*x)^m]*((a + b*Log[
c*(d + e*x)^n])/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x
] && NeQ[e*i - d*j, 0]
```

Rule 2489

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
```

```
] + (-Dist[g*j*(m/(r + 1)), Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
+ j*x)), x], x] - Dist[b*e*n*(p/(r + 1)), Int[x^(r + 1)*(a + b*Log[c*(d + e
*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2490

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-(e*k - d*l)/l + e*(x/l))^n])*(f + g
*Log[h*(-(j*k - i*l)/l + j*(x/l))^m]), x], x, k + l*x], x] /; FreeQ[{a, b,
c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 6732

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[d + e*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, Int[Log[d
+ e*x]*(Log[1 - a*c - b*c*x]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rule 6733

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Dist[b/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x))
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 6738

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[e*h*(n/(m + 1)), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(g+h\log(f(d+ex)^n))\operatorname{Li}_2(c(a+bx))}{x^3}\,dx = -\frac{(g+h\log(f(d+ex)^n))\operatorname{Li}_2(c(a+bx))}{2x^2} - \frac{1}{2}b\int \Bigg(\frac{\log(1-$$

$$= -\frac{(g+h\log(f(d+ex)^n))\operatorname{Li}_2(c(a+bx))}{2x^2} - \frac{b\int \frac{\log(1-ac-bcx)(}{}}{}$$

$$= \frac{b\log(1-ac-bcx)(g+h\log(f(d+ex)^n))}{2ax} - \frac{ehn\operatorname{Li}_2(c(a-}{2dx}$$

$$= \frac{b^2g\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2} + \frac{b\log(1-ac-bcx)(g+}{2ax}$$

$$= \frac{b^2g\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2} + \frac{b^2hn\log\left(\frac{bcx}{1-ac}\right)\log(1-a}{2a^2}$$

$$= \frac{b^2g\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2} + \frac{b^2hn\log\left(\frac{bcx}{1-ac}\right)\log(1-a}{2a^2}$$

$$= \frac{b^2g\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2} + \frac{b^2hn\log\left(\frac{bcx}{1-ac}\right)\log(1-a}{2a^2}$$

$$= \frac{b^2g\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2} + \frac{b^2hn\log\left(\frac{bcx}{1-ac}\right)\log(1-a}{2a^2}$$

Mathematica [A]
time = 13.83, size = 2673, normalized size = 0.86

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^3,x]
```

```
[Out] ((g - h*n*Log[d + e*x] + h*Log[f*(d + e*x)^n])*(-((-1 + a*c)*(a^2 - b^2*x^2
)*PolyLog[2, c*(a + b*x)]) + b*x*(-(a*b*c*x*Log[x]) + (a*(-1 + a*c + b*c*x)
 + b*(-1 + a*c)*x*Log[(b*c*x)/(1 - a*c)])*Log[1 - a*c - b*c*x] + b*(-1 + a*
c)*x*PolyLog[2, (-1 + a*c + b*c*x)/(-1 + a*c)])))/(2*a^2*(-1 + a*c)*x^2) -
(h*n*(((d*e*x + e^2*x^2*Log[x] + (d^2 - e^2*x^2)*Log[d + e*x])*PolyLog[2, c
*(a + b*x)])/x^2 + (b*d*e*(Log[x]*Log[1 - a*c - b*c*x] - Log[c*(a + b*x)]*L
og[1 - a*c - b*c*x] - Log[x]*Log[1 + (b*c*x)/(-1 + a*c)] - PolyLog[2, (b*c*
x)/(1 - a*c)] - PolyLog[2, 1 - a*c - b*c*x]))/a + e^2*(Log[x]*Log[1 + (b*x)
```

```
/a]*Log[1 - a*c - b*c*x] + ((-Log[c*(a + b*x)] + Log[1 + (b*x)/a])*Log[1 -
a*c - b*c*x]*(-2*Log[x] + Log[1 - a*c - b*c*x]))/2 + (Log[c*(a + b*x)] - Lo
g[1 + (b*x)/a])*Log[1 - a*c - b*c*x]*Log[(a*(-1 + a*c + b*c*x))/(b*x)] + ((
Log[(1 - a*c)/(b*c*x)] - Log[((1 - a*c)*(a + b*x))/(b*x)] + Log[1 + (b*x)/a
])*Log[(a*(-1 + a*c + b*c*x))/(b*x)]^2)/2 + (Log[1 - a*c - b*c*x] - Log[(a*
(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, -((b*x)/a)] + (Log[x] + Log[(a*(-1 +
 a*c + b*c*x))/(b*x)])*PolyLog[2, 1 - a*c - b*c*x] + Log[(a*(-1 + a*c + b*c
*x))/(b*x)]*(-PolyLog[2, (a*(-1 + a*c + b*c*x))/(b*x)] + PolyLog[2, (-1 + a
*c + b*c*x)/(b*c*x)]) - PolyLog[3, -((b*x)/a)] - PolyLog[3, 1 - a*c - b*c*x
] + PolyLog[3, (a*(-1 + a*c + b*c*x))/(b*x)] - PolyLog[3, (-1 + a*c + b*c*x
)/(b*c*x)]) - e^2*(Log[c*(a + b*x)]*Log[1 - a*c - b*c*x]*Log[d + e*x] + ((L
og[c*(a + b*x)] - Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d
 - a*e)]*(-2*Log[1 - a*c - b*c*x] + Log[(b*(d + e*x))/(b*d - a*e)]))/2 + (-
Log[c*(a + b*x)] + Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*
d - a*e)]*Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))] + (Log[-((
b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]^2*(Log[c*(a + b*x)] - Log[(
(b*c*d + e - a*c*e)*(a + b*x))/((b*d - a*e)*(-1 + a*c + b*c*x))] + Log[(b*c
*d + e - a*c*e)/(e - a*c*e - b*c*e*x)]))/2 + (Log[d + e*x] - Log[-((b*(d +
e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2, 1 - a*c - b*c*x] + (Lo
g[1 - a*c - b*c*x] + Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]
)*PolyLog[2, (b*(d + e*x))/(b*d - a*e)] + Log[-((b*(d + e*x))/((b*d - a*e)*
(-1 + a*c + b*c*x)))]*(PolyLog[2, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))] -
 PolyLog[2, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]) - PolyLog[3
, 1 - a*c - b*c*x] - PolyLog[3, (b*(d + e*x))/(b*d - a*e)] - PolyLog[3, (b*
c*(d + e*x))/(e*(-1 + a*c + b*c*x))] + PolyLog[3, -((b*(d + e*x))/((b*d - a
*e)*(-1 + a*c + b*c*x)))]) + (b*d^2*(-((a*Log[1 - a*c - b*c*x]*Log[d + e*x]
)/x) - b*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*Log[d + e*x] + b*Log[c
*(a + b*x)]*Log[1 - a*c - b*c*x]*Log[d + e*x] + (b*(Log[c*(a + b*x)] - Log[
(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*(-2*Log[1 - a
*c - b*c*x] + Log[(b*(d + e*x))/(b*d - a*e)]))/2 + b*(-Log[c*(a + b*x)] + L
og[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*Log[-((b*(
d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))] - (b*Log[((-1 + a*c)*(d + e*x)
)/(d*(-1 + a*c + b*c*x))]^2*(Log[(b*c*x)/(1 - a*c)] - Log[((b*c*d + e - a*c
*e)*x)/(d*(-1 + a*c + b*c*x))] + Log[(b*c*d + e - a*c*e)/(e - a*c*e - b*c*e
*x)]))/2 + (b*Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]^2*(Log
[c*(a + b*x)] - Log[((b*c*d + e - a*c*e)*(a + b*x))/((b*d - a*e)*(-1 + a*c
+ b*c*x))] + Log[(b*c*d + e - a*c*e)/(e - a*c*e - b*c*e*x)]))/2 + b*(Log[(b
*c*x)/(1 - a*c)] - Log[-((e*x)/d)])*Log[((-1 + a*c)*(d + e*x))/(d*(-1 + a*c
 + b*c*x))]*Log[1 + (e*x)/d] - (b*(Log[(b*c*x)/(1 - a*c)] - Log[-((e*x)/d)]
)*Log[1 + (e*x)/d]*(-2*Log[1 - a*c - b*c*x] + Log[1 + (e*x)/d]))/2 + b*(Log
[d + e*x] - Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog
[2, 1 - a*c - b*c*x] - b*(Log[d + e*x] - Log[((-1 + a*c)*(d + e*x))/(d*(-1
+ a*c + b*c*x))])*PolyLog[2, (-1 + a*c + b*c*x)/(-1 + a*c)] + (a*e*(Log[x]*
Log[1 - a*c - b*c*x] - Log[x]*Log[1 + (b*c*x)/(-1 + a*c)] - Log[1 - a*c - b
*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)] - PolyLog[2, (b*c*x)/(1 - a*
```

```
c)] - PolyLog[2, (e*(-1 + a*c + b*c*x))/(-(b*c*d) - e + a*c*e)]))/d + b*(Lo
g[1 - a*c - b*c*x] + Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]
)*PolyLog[2, (b*(d + e*x))/(b*d - a*e)] + b*Log[((-1 + a*c)*(d + e*x))/(d*(
-1 + a*c + b*c*x))]*(PolyLog[2, ((-1 + a*c)*(d + e*x))/(d*(-1 + a*c + b*c*x
))] - PolyLog[2, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))]) + b*Log[-((b*(d +
 e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]*(PolyLog[2, (b*c*(d + e*x))/(e*(-
1 + a*c + b*c*x))] - PolyLog[2, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*
c*x)))]) - b*(Log[1 - a*c - b*c*x] + Log[((-1 + a*c)*(d + e*x))/(d*(-1 + a*
c + b*c*x))])*PolyLog[2, 1 + (e*x)/d] + (a*b*c*(Log[d + e*x]*(Log[-((e*x)/d
)] - Log[1 - (b*c*(d + e*x))/(b*c*d + e - a*c*e)]) - PolyLog[2, (b*c*(d + e
*x))/(b*c*d + e - a*c*e)] + PolyLog[2, 1 + (e*x)/d]))/(-1 + a*c) - b*PolyLo
g[3, 1 - a*c - b*c*x] + b*PolyLog[3, (-1 + a*c + b*c*x)/(-1 + a*c)] - b*Pol
yLog[3, (b*(d + e*x))/(b*d - a*e)] - b*PolyLog[3, ((-1 + a*c)*(d + e*x))/(d
*(-1 + a*c + b*c*x))] + b*PolyLog[3, -((b*(d + ...
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(g+h\ln\left(f(ex+d)^{n}\right)\right)\operatorname{polylog}\left(2,c(bx+a)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^3,x)
[Out] int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^3,x, algorithm="max
ima")
[Out] integrate((h*log((x*e + d)^n*f) + g)*dilog((b*x + a)*c)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^3,x, algorithm="fri
cas")
```

```
[Out] integral((h*dilog(b*c*x + a*c)*log((x*e + d)^n*f) + g*dilog(b*c*x + a*c))/x
^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a))/x**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^3,x, algorithm="gia
c")
```

```
[Out] integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{polylog}(2, c\,(a+b\,x))\,\left(g+h\,\ln\left(f\,{\left(d+e\,x\right)}^n\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)))/x^3,x)
```

```
[Out] int((polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)))/x^3, x)
```

3.183 $\int \frac{(g+h\log(f(d+ex)^n))\mathbf{PolyLog}(2,c(a+bx))}{x^4}\,dx$

Optimal. Leaf size=3733

result too large to display

```
[Out] -1/3*b^3*g*polylog(2,1-b*c*x/(-a*c+1))/a^3-1/3*b^3*g*polylog(2,c*(b*x+a))/a
^3+1/6*b^2*e*h*n*polylog(2,c*(b*x+a))/a^2/d+1/3*b*e^2*h*n*polylog(2,c*(b*x+
a))/a/d^2+1/3*b*e*h*n*ln(-b*c*x-a*c+1)/a/d/x+1/2*b^2*e*h*n*ln(b*c*x/(-a*c+1
))*ln(-b*c*x-a*c+1)/a^2/d+1/2*b*e^2*h*n*ln(b*c*x/(-a*c+1))*ln(-b*c*x-a*c+1)
/a/d^2-1/3*b^2*e*h*n*ln(-b*c*x-a*c+1)*ln(b*c*(e*x+d)/(-a*c*e+b*c*d+e))/a^2/
d-1/6*b*e^2*h*n*ln(-b*c*x-a*c+1)*ln(b*c*(e*x+d)/(-a*c*e+b*c*d+e))/a/d^2+1/3
*b^3*h*n*polylog(3,-e*(-b*c*x-a*c+1)/b/c/(e*x+d))/a^3-1/3*b^3*h*n*polylog(3
,b*(e*x+d)/(-a*e+b*d))/a^3+1/3*e^3*h*n*polylog(3,b*(e*x+d)/(-a*e+b*d))/d^3+
1/3*b^3*h*n*polylog(3,1+e*x/d)/a^3+1/3*e^3*h*n*polylog(3,-b*x/a/(1-c*(b*x+a
)))/d^3-1/3*e^3*h*n*polylog(3,-b*c*x/(1-c*(b*x+a)))/d^3-1/3*b^3*h*n*polylog
(3,1-c*(b*x+a))/a^3-1/3*b^3*h*n*polylog(3,-e*(1-c*(b*x+a))/b/c/(e*x+d))/a^3
+1/3*e^3*h*n*polylog(3,-e*(1-c*(b*x+a))/b/c/(e*x+d))/d^3+1/3*b^3*h*n*polylo
g(3,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/a^3-1/3*e^3*h*n*polylog(3,(-a*e+b*d
)*(1-c*(b*x+a))/b/(e*x+d))/d^3-1/3*e^3*h*n*polylog(3,-b*x/a)/d^3+1/3*b^3*h*
n*polylog(3,1-b*c*x/(-a*c+1))/a^3-1/3*b^3*h*n*polylog(3,d*(-b*c*x-a*c+1)/(-
a*c+1)/(e*x+d))/a^3+1/2*b^2*c*e*h*n*ln(x)/a/(-a*c+1)/d-1/3*b^2*c*e*h*n*ln(-
b*c*x-a*c+1)/a/(-a*c+1)/d-1/6*b^2*c*e*h*n*ln(e*x+d)/a/(-a*c+1)/d-1/3*(g+h*l
n(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^3-1/3*b^3*h*n*ln(b*c*x/(-a*c+1))*ln(
-b*c*x-a*c+1)*ln(e*x+d)/a^3+1/3*e^3*h*n*ln(x)*ln(1+b*x/a)*ln(1-c*(b*x+a))/d
^3+1/3*b^3*h*n*ln(c*(b*x+a))*ln(e*x+d)*ln(1-c*(b*x+a))/a^3-1/3*e^3*h*n*ln(c
*(b*x+a))*ln(e*x+d)*ln(1-c*(b*x+a))/d^3-1/3*b^3*g*ln(b*c*x/(-a*c+1))*ln(-b*
c*x-a*c+1)/a^3+1/6*b*ln(-b*c*x-a*c+1)*(g+h*ln(f*(e*x+d)^n))/a/x^2-1/3*b^2*l
n(-b*c*x-a*c+1)*(g+h*ln(f*(e*x+d)^n))/a^2/x+1/3*b^3*h*(n*ln(e*x+d)-ln(f*(e*
x+d)^n))*polylog(2,c*(b*x+a))/a^3+1/3*b^3*h*(n*ln(e*x+d)-ln(f*(e*x+d)^n))*p
olylog(2,1-b*c*x/(-a*c+1))/a^3-1/3*e^3*h*n*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*
x+a)))*polylog(2,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/d^3-1/6*b^3*h*n*(ln(b*
c*x/(-a*c+1))+ln((-a*c*e+b*c*d+e)/b/c/(e*x+d))-ln((-a*c*e+b*c*d+e)*x/(-a*c+
1)/(e*x+d)))*ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a*c+1))^2/a^3+1/6*b^3*h*n*(ln(b*
c*x/(-a*c+1))-ln(-e*x/d))*(ln(-b*c*x-a*c+1)+ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a
*c+1)))^2/a^3+1/3*b^3*h*ln(b*c*x/(-a*c+1))*ln(-b*c*x-a*c+1)*(n*ln(e*x+d)-ln
(f*(e*x+d)^n))/a^3-1/6*b^2*c*(g+h*ln(f*(e*x+d)^n))/a/(-a*c+1)/x+1/6*b^3*c^2
*ln(-e*x/d)*(g+h*ln(f*(e*x+d)^n))/a/(-a*c+1)^2-1/3*b^3*c*ln(-e*x/d)*(g+h*ln
(f*(e*x+d)^n))/a^2/(-a*c+1)-1/6*b^3*c^2*ln(e*(-b*c*x-a*c+1)/(-a*c*e+b*c*d+e
))*(g+h*ln(f*(e*x+d)^n))/a/(-a*c+1)^2+1/3*b^3*c*ln(e*(-b*c*x-a*c+1)/(-a*c*e
+b*c*d+e))*(g+h*ln(f*(e*x+d)^n))/a^2/(-a*c+1)+1/6*b^3*h*n*(ln(c*(b*x+a))+ln
((-a*c*e+b*c*d+e)/b/c/(e*x+d))-ln((-a*c*e+b*c*d+e)*(b*x+a)/b/(e*x+d)))*ln(b
*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))^2/a^3-1/6*e^3*h*n*(ln(c*(b*x+a))+ln((-a*
c*e+b*c*d+e)/b/c/(e*x+d))-ln((-a*c*e+b*c*d+e)*(b*x+a)/b/(e*x+d)))*ln(b*(e*x
+d)/(-a*e+b*d)/(1-c*(b*x+a)))^2/d^3-1/6*b^3*h*n*(ln(c*(b*x+a))-ln(-e*(b*x+a
```

```
)/(-a*e+b*d)))*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))^2/a
^3+1/6*e^3*h*n*(ln(c*(b*x+a))-ln(-e*(b*x+a)/(-a*e+b*d)))*(ln(b*(e*x+d)/(-a*
e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))^2/d^3+1/6*e^3*h*n*(ln(1+b*x/a)+ln((-
a*c+1)/(1-c*(b*x+a)))-ln((-a*c+1)*(b*x+a)/a/(1-c*(b*x+a))))*ln(-a*(1-c*(b*x
+a))/b/x)^2/d^3+1/6*e^3*h*n*(ln(c*(b*x+a))-ln(1+b*x/a))*(ln(x)+ln(-a*(1-c*(
b*x+a))/b/x))^2/d^3+1/3*e^3*h*n*(ln(1-c*(b*x+a))-ln(-a*(1-c*(b*x+a))/b/x))*
polylog(2,-b*x/a)/d^3+1/3*e^3*h*n*ln(x)*polylog(2,c*(b*x+a))/d^3-1/3*e^3*h*
n*ln(e*x+d)*polylog(2,c*(b*x+a))/d^3-1/3*b^3*h*n*(ln(e*x+d)-ln((-a*c+1)*(e*
x+d)/d/(-b*c*x-a*c+1)))*polylog(2,1-b*c*x/(-a*c+1))/a^3-1/3*b^3*h*n*ln((-a*
c+1)*(e*x+d)/d/(-b*c*x-a*c+1))*polylog(2,d*(-b*c*x-a*c+1)/(-a*c+1)/(e*x+d))
/a^3+1/3*b^3*h*n*ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a*c+1))*polylog(2,-e*(-b*c*x
-a*c+1)/b/c/(e*x+d))/a^3+1/3*b^3*h*n*(ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a))
)+ln(1-c*(b*x+a)))*polylog(2,b*(e*x+d)/(-a*e+b*d))/a^3-1/3*e^3*h*n*(ln(b*(e
*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))+ln(1-c*(b*x+a)))*polylog(2,b*(e*x+d)/(-a*e+
b*d))/d^3-1/3*b^3*h*n*(ln(-b*c*x-a*c+1)+ln((-a*c+1)*(e*x+d)/d/(-b*c*x-a*c+1
)))*polylog(2,1+e*x/d)/a^3+1/3*e^3*h*n*ln(-a*(1-c*(b*x+a))/b/x)*polylog(2,-
b*x/a/(1-c*(b*x+a)))/d^3-1/3*e^3*h*n*ln(-a*(1-c*(b*x+a))/b/x)*polylog(2,-b*
c*x/(1-c*(b*x+a)))/d^3+1/3*b^3*h*n*(ln(e*x+d)-ln(b*(e*x+d)/(-a*e+b*d)/(1-c*
(b*x+a))))*polylog(2,1-c*(b*x+a))/a^3-1/3*e^3*h*n*(ln(e*x+d)-ln(b*(e*x+d)/(
-a*e+b*d)/(1-c*(b*x+a))))*polylog(2,1-c*(b*x+a))/d^3+1/3*e^3*h*n*(ln(x)+ln(
-a*(1-c*(b*x+a))/b/x))*polylog(2,1-c*(b*x+a))/d^3-1/3*b^3*h*n*ln(b*(e*x+d)/
(-a*e+b*d)/(1-c*(b*x+a)))*polylog(2,-e*(1-c*(b*x+a))/b/c/(e*x+d))/a^3+1/3*e
^3*h*n*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))*polylog(2,-e*(1-c*(b*x+a))/b/
c/(e*x+d))/d^3+1/3*b^3*h*n*ln(b*(e*x+d)/(-a*e+b*d)/(1-c*(b*x+a)))*polylog(2
,(-a*e+b*d)*(1-c*(b*x+a))/b/(e*x+d))/a^3-1/3*b^2*e*h*n*polylog(2,e*(-b*c*x-
a*c+1)/(-a*c*e+b*c*d+e))/a^2/d-1/6*b*e^2*h*n*polylog(2,e*(-b*c*x-a*c+1)/(-a
*c*e+b*c*d+e))/a/d^2+1/2*b^2*e*h*n*polylog(2,1-b*c*x/(-a*c+1))/a^2/d+1/2*b*
e^2*h*n*polylog(2,1-b*c*x/(-a*c+1))/a/d^2-1/6*b^3*c^2*h*n*polylog(2,b*c*(e*
x+d)/(-a*c*e+b*c*d+e))/a/(-a*c+1)^2+1/3*b^3*c*h*n*polylog(2,b*c*(e*x+d)/(-a
*c*e+b*c*d+e))/a^2/(-a*c+1)+1/6*b^3*c^2*h*n*polylog(2,1+e*x/d)/a/(-a*c+1)^2
-1/3*b^3*c*h*n*polylog(2,1+e*x/d)/a^2/(-a*c+1)-1/6*e*h*n*polylog(2,c*(b*x+a
))/d/x^2+1/3*e^2*h*n*polylog(2,c*(b*x+a))/d^2/x
```

Rubi [A]

time = 2.87, antiderivative size = 3733, normalized size of antiderivative = 1.00, number of steps used = 78, number of rules used = 18, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6738, 2489, 46, 2463, 2442, 36, 29, 31, 2441, 2352, 2440, 2438, 2488, 2487, 2485, 2490, 6733, 6732}

Too large to display

Antiderivative was successfully verified.

```
[In] Int[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^4,x]
```

```
[Out] (b^2*c*e*h*n*Log[x])/(2*a*(1 - a*c)*d) - (b^2*c*e*h*n*Log[1 - a*c - b*c*x])
/(3*a*(1 - a*c)*d) + (b*e*h*n*Log[1 - a*c - b*c*x])/(3*a*d*x) - (b^3*g*Log[
```

(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/(3*a^3) + (b^2*e*h*n*Log[(b*c*x)/(
1 - a*c)]*Log[1 - a*c - b*c*x])/(2*a^2*d) + (b*e^2*h*n*Log[(b*c*x)/(1 - a*c
)]*Log[1 - a*c - b*c*x])/(2*a*d^2) - (b^2*c*e*h*n*Log[d + e*x])/(6*a*(1 - a
*c)*d) - (b^3*h*n*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*Log[d + e*x])
/(3*a^3) - (b^2*e*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e -
 a*c*e)])/(3*a^2*d) - (b*e^2*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(
b*c*d + e - a*c*e)])/(6*a*d^2) - (b^3*h*n*(Log[(b*c*x)/(1 - a*c)] + Log[(b*
c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*x)/((1 - a*c)*
(d + e*x))])*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))]^2)/(6*a^3) +
(b^3*h*n*(Log[(b*c*x)/(1 - a*c)] - Log[-((e*x)/d)])*(Log[1 - a*c - b*c*x] +
 Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])^2)/(6*a^3) + (b^3*h*Log[
(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*(n*Log[d + e*x] - Log[f*(d + e*x)^n
]))/(3*a^3) - (b^2*c*(g + h*Log[f*(d + e*x)^n]))/(6*a*(1 - a*c)*x) + (b^3*c
^2*Log[-((e*x)/d)]*(g + h*Log[f*(d + e*x)^n]))/(6*a*(1 - a*c)^2) - (b^3*c*L
og[-((e*x)/d)]*(g + h*Log[f*(d + e*x)^n]))/(3*a^2*(1 - a*c)) + (b*Log[1 - a
*c - b*c*x]*(g + h*Log[f*(d + e*x)^n]))/(6*a*x^2) - (b^2*Log[1 - a*c - b*c*
x]*(g + h*Log[f*(d + e*x)^n]))/(3*a^2*x) - (b^3*c^2*Log[(e*(1 - a*c - b*c*x
))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(6*a*(1 - a*c)^2) + (b^
3*c*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n
]))/(3*a^2*(1 - a*c)) + (b^3*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e
)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Lo
g[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(6*a^3) - (e^3*h*n*(Log
[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e
- a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(
a + b*x)))]^2)/(6*d^3) + (e^3*h*n*Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*
x)])/(3*d^3) + (b^3*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])
/(3*a^3) - (e^3*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/(3*
d^3) - (b^3*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log
[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])^2)/
(6*a^3) + (e^3*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(
Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])^
2)/(6*d^3) + (e^3*h*n*(Log[1 + (b*x)/a] + Log[(1 - a*c)/(1 - c*(a + b*x))]
- Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))])*Log[-((a*(1 - c*(a + b*
x)))/(b*x))]^2)/(6*d^3) + (e^3*h*n*(Log[c*(a + b*x)] - Log[1 + (b*x)/a])*(L
og[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/(6*d^3) + (e^3*h*n*(Log[1 -
 c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*x)/a)]
)/(3*d^3) - (b^3*g*PolyLog[2, c*(a + b*x)])/(3*a^3) + (b^2*e*h*n*PolyLog[2,
 c*(a + b*x)])/(6*a^2*d) + (b*e^2*h*n*PolyLog[2, c*(a + b*x)])/(3*a*d^2) -
(e*h*n*PolyLog[2, c*(a + b*x)])/(6*d*x^2) + (e^2*h*n*PolyLog[2, c*(a + b*x)
])/(3*d^2*x) + (e^3*h*n*Log[x]*PolyLog[2, c*(a + b*x)])/(3*d^3) - (e^3*h*n*
Log[d + e*x]*PolyLog[2, c*(a + b*x)])/(3*d^3) + (b^3*h*(n*Log[d + e*x] - Lo
g[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(3*a^3) - ((g + h*Log[f*(d + e*x
)^n])*PolyLog[2, c*(a + b*x)])/(3*x^3) - (b^2*e*h*n*PolyLog[2, (e*(1 - a*c
- b*c*x))/(b*c*d + e - a*c*e)])/(3*a^2*d) - (b*e^2*h*n*PolyLog[2, (e*(1 - a
*c - b*c*x))/(b*c*d + e - a*c*e)])/(6*a*d^2) - (b^3*g*PolyLog[2, 1 - (b*c*x

```
)/(1 - a*c)])/(3*a^3) + (b^2*e*h*n*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(2*a^
2*d) + (b*e^2*h*n*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(2*a*d^2) - (b^3*h*n*(
Log[d + e*x] - Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2,
 1 - (b*c*x)/(1 - a*c)])/(3*a^3) + (b^3*h*(n*Log[d + e*x] - Log[f*(d + e*x)
^n])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(3*a^3) - (b^3*h*n*Log[((1 - a*c)*(
d + e*x))/(d*(1 - a*c - b*c*x))]*PolyLog[2, (d*(1 - a*c - b*c*x))/((1 - a*c
)*(d + e*x))])/(3*a^3) + (b^3*h*n*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b
*c*x))]*PolyLog[2, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))])/(3*a^3) + (b^
3*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a +
b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)])/(3*a^3) - (e^3*h*n*(Log[(b*(d
 + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2
, (b*(d + e*x))/(b*d - a*e)])/(3*d^3) - (b^3*c^2*h*n*PolyLog[2, (b*c*(d + e
*x))/(b*c*d + e - a*c*e)])/(6*a*(1 - a*c)^2) + (b^3*c*h*n*PolyLog[2, (b*c*(
d + e*x))/(b*c*d + e - a*c*e)])/(3*a^2*(1 - a*c)) + (b^3*c^2*h*n*PolyLog[2,
 1 + (e*x)/d])/(6*a*(1 - a*c)^2) - (b^3*c*h*n*PolyLog[2, 1 + (e*x)/d])/(3*a
^2*(1 - a*c)) - (b^3*h*n*(Log[1 - a*c - b*c*x] + Log[((1 - a*c)*(d + e*x))/
(d*(1 - a*c - b*c*x))])*PolyLog[2, 1 + (e*x)/d])/(3*a^3) + (e^3*h*n*Log[-((
a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))))])/(3
*d^3) - (e^3*h*n*Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*c*x)/(
1 - c*(a + b*x)))])/(3*d^3) + (b^3*h*n*(Log[d +...
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2485

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[(-b)*(x/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1/2)*(Log[(-b)*(x/a
)] - Log[(-(b*c - a*d))*(x/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))]
)*Log[a*((c + d*x)/(c*(a + b*x)))]^2, x] - Simp[(1/2)*(Log[(-b)*(x/a)] - Lo
g[(-d)*(x/c)])*(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])^2, x] + Si
mp[(Log[c + d*x] - Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1 + b*(x/a)
], x] + Simp[(Log[a + b*x] + Log[a*((c + d*x)/(c*(a + b*x)))])*PolyLog[2, 1
 + d*(x/c)], x] + Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2, c*((a +
b*x)/(a*(c + d*x)))], x] - Simp[Log[a*((c + d*x)/(c*(a + b*x)))]*PolyLog[2,
 d*((a + b*x)/(b*(c + d*x)))], x] - Simp[PolyLog[3, 1 + b*(x/a)], x] - Simp
```

```
[PolyLog[3, 1 + d*(x/c)], x] + Simp[PolyLog[3, c*((a + b*x)/(a*(c + d*x)))]
, x] - Simp[PolyLog[3, d*((a + b*x)/(b*(c + d*x)))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2487

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Dist[m, Int[Log[i + j*x]*(Log[c*(d + e*x)^n]/x), x
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
 x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
 + j*x, h*(i + j*x)^m]
```

Rule 2488

```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(Log[(h_.)*((i_.)
 + (j_.)*(x_))^(m_.)]*(g_.) + (f_)))/(x_), x_Symbol] :> Dist[f, Int[(a + b*
Log[c*(d + e*x)^n])/x, x], x] + Dist[g, Int[Log[h*(i + j*x)^m]*((a + b*Log[
c*(d + e*x)^n])/x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x
] && NeQ[e*i - d*j, 0]
```

Rule 2489

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Dist[g*j*(m/(r + 1)), Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
+ j*x)), x], x] - Dist[b*e*n*(p/(r + 1)), Int[x^(r + 1)*(a + b*Log[c*(d + e
*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2490

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-(e*k - d*l)/l + e*(x/l))^n])*(f + g
*Log[h*(-(j*k - i*l)/l + j*(x/l))^m]), x], x, k + l*x], x] /; FreeQ[{a, b,
c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 6732

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[d + e*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, Int[Log[d
+ e*x]*(Log[1 - a*c - b*c*x]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rule 6733

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Dist[b/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x))
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 6738

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[e*h*(n/(m + 1)), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(g+h\log(f(d+ex)^n))\,\mathrm{Li}_2(c(a+bx))}{x^4}\,dx = -\frac{(g+h\log(f(d+ex)^n))\,\mathrm{Li}_2(c(a+bx))}{3x^3} - \frac{1}{3}b\int\left(\frac{\log(1-\right.$$

$$= -\frac{(g+h\log(f(d+ex)^n))\,\mathrm{Li}_2(c(a+bx))}{3x^3} - \frac{b\int\frac{\log(1-ac-bcx)(}{}}{}$$

$$= \frac{b\log(1-ac-bcx)\,(g+h\log(f(d+ex)^n))}{6ax^2} - \frac{b^2\log(1-ac}{}$$

$$= -\frac{b^3g\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{3a^3} + \frac{b\log(1-ac-bcx)\,(g}{6ax}$$

$= \text{too large to display}$

Mathematica [A]
time = 18.15, size = 3331, normalized size = 0.89

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^4,x]
```

```
[Out] (g + h*(-(n*Log[d + e*x]) + Log[f*(d + e*x)^n]))*(-1/6*(b*((-2*a*b^2*c*(Log
[x] - Log[1 - a*c - b*c*x]))/(-1 + a*c) - (a^2*Log[1 - a*c - b*c*x])/x^2 +
(2*a*b*Log[1 - a*c - b*c*x])/x + 2*b^2*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c -
 b*c*x] - (a^2*b*c*(-1 + a*c + b*c*x*Log[x] - b*c*x*Log[1 - a*c - b*c*x]))/
```

```
((-1 + a*c)^2*x) + 2*b^2*PolyLog[2, c*(a + b*x)] + 2*b^2*PolyLog[2, (-1 + a
*c + b*c*x)/(-1 + a*c)]))/a^3 - PolyLog[2, a*c + b*c*x]/(3*x^3)) + (h*n*((-
2*Log[d + e*x]*PolyLog[2, c*(a + b*x)])/x^3 + (e*(-(d*(d - 2*e*x)) + 2*e^2*
x^2*Log[x] - 2*e^2*x^2*Log[d + e*x])*PolyLog[2, c*(a + b*x)])/(d^3*x^2) + (
2*b^2*(-((Log[1 - a*c - b*c*x]*Log[d + e*x])/x) + (e*(Log[x]*Log[1 - a*c -
b*c*x] - Log[x]*Log[1 + (b*c*x)/(-1 + a*c)] - Log[1 - a*c - b*c*x]*Log[(b*c
*(d + e*x))/(b*c*d + e - a*c*e)] - PolyLog[2, (b*c*x)/(1 - a*c)] - PolyLog[
2, (e*(-1 + a*c + b*c*x))/(-(b*c*d) - e + a*c*e)]))/d + (b*c*(Log[d + e*x]*
(Log[-((e*x)/d)] - Log[1 - (b*c*(d + e*x))/(b*c*d + e - a*c*e)]) - PolyLog[
2, (b*c*(d + e*x))/(b*c*d + e - a*c*e)] + PolyLog[2, 1 + (e*x)/d]))/(-1 + a
*c)))/a^2 - (b*(-((Log[1 - a*c - b*c*x]*Log[d + e*x])/x^2) + (e*((d*(b*c*x*
Log[x] - (-1 + a*c + b*c*x)*Log[1 - a*c - b*c*x]))/((-1 + a*c)*x) + e*(Log[
x]*(-Log[1 - a*c - b*c*x] + Log[1 + (b*c*x)/(-1 + a*c)]) + PolyLog[2, (b*c*
x)/(1 - a*c)]) + e*(Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a
*c*e)] + PolyLog[2, (e*(-1 + a*c + b*c*x))/(-(b*c*d) + (-1 + a*c)*e)])))/d^
2 + (b*c*(-(e*x*Log[e*x]) + a*c*e*x*Log[e*x] + d*Log[d + e*x] - a*c*d*Log[d
 + e*x] + e*x*Log[d + e*x] - a*c*e*x*Log[d + e*x] - b*c*d*x*Log[-((e*x)/d)]
*Log[d + e*x] + b*c*d*x*Log[d + e*x]*Log[1 - (b*c*(d + e*x))/(b*c*d + e - a
*c*e)] + b*c*d*x*PolyLog[2, (b*c*(d + e*x))/(b*c*d + e - a*c*e)] - b*c*d*x*
PolyLog[2, 1 + (e*x)/d]))/((-1 + a*c)^2*d*x)))/a + (2*b^3*(Log[c*(a + b*x)]
*Log[1 - a*c - b*c*x]*Log[d + e*x] + ((Log[c*(a + b*x)] - Log[(e*(a + b*x))
/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*(-2*Log[1 - a*c - b*c*x] +
 Log[(b*(d + e*x))/(b*d - a*e)]))/2 + (-Log[c*(a + b*x)] + Log[(e*(a + b*x)
)/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*Log[-((b*(d + e*x))/((b*d
 - a*e)*(-1 + a*c + b*c*x)))] + (Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c
 + b*c*x)))]^2*(Log[c*(a + b*x)] - Log[((b*c*d + e - a*c*e)*(a + b*x))/((b*
d - a*e)*(-1 + a*c + b*c*x))] + Log[(b*c*d + e - a*c*e)/(e - a*c*e - b*c*e*
x)]))/2 + (Log[d + e*x] - Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*
x)))])*PolyLog[2, 1 - a*c - b*c*x] + (Log[1 - a*c - b*c*x] + Log[-((b*(d +
e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2, (b*(d + e*x))/(b*d - a
*e)] + Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]*(PolyLog[2, (
b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))] - PolyLog[2, -((b*(d + e*x))/((b*d -
 a*e)*(-1 + a*c + b*c*x)))]) - PolyLog[3, 1 - a*c - b*c*x] - PolyLog[3, (b*
(d + e*x))/(b*d - a*e)] - PolyLog[3, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))
] + PolyLog[3, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]))/a^3 + (
e*((2*b*d*e*(Log[x]*Log[1 - a*c - b*c*x] - Log[c*(a + b*x)]*Log[1 - a*c - b
*c*x] - Log[x]*Log[1 + (b*c*x)/(-1 + a*c)] - PolyLog[2, (b*c*x)/(1 - a*c)]
- PolyLog[2, 1 - a*c - b*c*x]))/a - (b*d^2*((a*(b*c*x*Log[x] - (-1 + a*c +
b*c*x)*Log[1 - a*c - b*c*x]))/((-1 + a*c)*x) + b*(Log[x]*(-Log[1 - a*c - b*
c*x] + Log[1 + (b*c*x)/(-1 + a*c)]) + PolyLog[2, (b*c*x)/(1 - a*c)]) + b*(L
og[c*(a + b*x)]*Log[1 - a*c - b*c*x] + PolyLog[2, 1 - a*c - b*c*x])))/a^2 +
 2*e^2*(Log[x]*Log[1 + (b*x)/a]*Log[1 - a*c - b*c*x] + ((-Log[c*(a + b*x)]
+ Log[1 + (b*x)/a])*Log[1 - a*c - b*c*x]*(-2*Log[x] + Log[1 - a*c - b*c*x])
)/2 + (Log[c*(a + b*x)] - Log[1 + (b*x)/a])*Log[1 - a*c - b*c*x]*Log[(a*(-1
 + a*c + b*c*x))/(b*x)] + ((Log[(1 - a*c)/(b*c*x)] - Log[((1 - a*c)*(a + b*
```

```
x))/(b*x)] + Log[1 + (b*x)/a])*Log[(a*(-1 + a*c + b*c*x))/(b*x)]^2)/2 + (Lo
g[1 - a*c - b*c*x] - Log[(a*(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, -((b*x)/
a)] + (Log[x] + Log[(a*(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, 1 - a*c - b*c
*x] + Log[(a*(-1 + a*c + b*c*x))/(b*x)]*(-PolyLog[2, (a*(-1 + a*c + b*c*x))
/(b*x)] + PolyLog[2, (-1 + a*c + b*c*x)/(b*c*x)]) - PolyLog[3, -((b*x)/a)]
- PolyLog[3, 1 - a*c - b*c*x] + PolyLog[3, (a*(-1 + a*c + b*c*x))/(b*x)] -
PolyLog[3, (-1 + a*c + b*c*x)/(b*c*x)]) - 2*e^2*(Log[c*(a + b*x)]*Log[1 - a
*c - b*c*x]*Log[d + e*x] + ((Log[c*(a + b*x)] - Log[(e*(a + b*x))/(-(b*d) +
 a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*(-2*Log[1 - a*c - b*c*x] + Log[(b*(d
 + e*x))/(b*d - a*e)]))/2 + (-Log[c*(a + b*x)] + Log[(e*(a + b*x))/(-(b*d)
+ a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*Log[-((b*(d + e*x))/((b*d - a*e)*(-
1 + a*c + b*c*x)))] + (Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x))
)]^2*(Log[c*(a + b*x)] - Log[((b*c*d + e - a*c*e)*(a + b*x))/((b*d - a*e)*(
-1 + a*c + b*c*x))] + Log[(b*c*d + e - a*c*e)/(e - a*c*e - b*c*e*x)]))/2 +
(Log[d + e*x] - Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*Pol
yLog[2, 1 - a*c - b*c*x] + (Log[1 - a*c - b*c*x] + Log[-((b*(d + e*x))/((b*
d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)] + Log
[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x...
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(g + h\ln(f(ex+d)^n))\,\mathrm{polylog}(2, c(bx+a))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^4,x)
[Out] int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^4,x, algorithm="max
ima")
[Out] integrate((h*log((x*e + d)^n*f) + g)*dilog((b*x + a)*c)/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^4,x, algorithm="fri
cas")
```

```
[Out] integral((h*dilog(b*c*x + a*c)*log((x*e + d)^n*f) + g*dilog(b*c*x + a*c))/x
^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a))/x**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^4,x, algorithm="gia
c")
```

```
[Out] integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c)/x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{polylog}(2, c\,(a+b\,x))\,\left(g+h\,\ln\left(f\,{(d+e\,x)}^n\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)))/x^4,x)
```

```
[Out] int((polylog(2, c*(a + b*x))*(g + h*log(f*(d + e*x)^n)))/x^4, x)
```

3.184 $\int x^2(a+bx)\log(1-cx)\mathbf{PolyLog}(2,cx)\,dx$

Optimal. Leaf size=661

$$\frac{53bx}{192c^3}+\frac{11ax}{27c^2}+\frac{49(3b+4ac)x}{432c^3}+\frac{29bx^2}{384c^2}+\frac{5ax^2}{54c}+\frac{13(3b+4ac)x^2}{864c^2}+\frac{2ax^3}{81}+\frac{17bx^3}{576c}+\frac{(3b+4ac)x^3}{324c}+\frac{3bx^4}{256}+\frac{29b\log}{19}$$

```
[Out] -1/12*(4*a*c+3*b)*x*polylog(2,c*x)/c^3-1/24*(4*a*c+3*b)*x^2*polylog(2,c*x)/
c^2-1/36*(4*a*c+3*b)*x^3*polylog(2,c*x)/c-1/48*(4*a*c+3*b)*x^2*ln(-c*x+1)/c
^2-1/24*b*x^3*ln(-c*x+1)/c-1/108*(4*a*c+3*b)*x^3*ln(-c*x+1)/c+1/8*b*(-c*x+1
)*ln(-c*x+1)/c^4+2/9*a*(-c*x+1)*ln(-c*x+1)/c^3+1/12*(4*a*c+3*b)*(-c*x+1)*ln
(-c*x+1)/c^4-1/12*(4*a*c+3*b)*ln(c*x)*ln(-c*x+1)^2/c^4-1/12*(4*a*c+3*b)*ln(
-c*x+1)*polylog(2,c*x)/c^4-1/6*(4*a*c+3*b)*ln(-c*x+1)*polylog(2,-c*x+1)/c^4
-1/16*b*x^2*ln(-c*x+1)/c^2-1/9*a*x^2*ln(-c*x+1)/c+53/192*b*x/c^3+11/27*a*x/
c^2+49/432*(4*a*c+3*b)*x/c^3+29/384*b*x^2/c^2+5/54*a*x^2/c+13/864*(4*a*c+3*
b)*x^2/c^2+17/576*b*x^3/c+1/324*(4*a*c+3*b)*x^3/c-1/16*b*x^4*polylog(2,c*x)
+1/6*(4*a*c+3*b)*polylog(3,-c*x+1)/c^4+29/192*b*ln(-c*x+1)/c^4+5/27*a*ln(-c
*x+1)/c^3+13/432*(4*a*c+3*b)*ln(-c*x+1)/c^4-2/27*a*x^3*ln(-c*x+1)-3/64*b*x^
4*ln(-c*x+1)-1/16*b*ln(-c*x+1)^2/c^4-1/9*a*ln(-c*x+1)^2/c^3+1/9*a*x^3*ln(-c
*x+1)^2+1/16*b*x^4*ln(-c*x+1)^2+1/12*(3*b*x^4+4*a*x^3)*ln(-c*x+1)*polylog(2
,c*x)+2/81*a*x^3+3/256*b*x^4
```

Rubi [A]
time = 0.64, antiderivative size = 661, normalized size of antiderivative = 1.00, number of steps used = 52, number of rules used = 17, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {6874, 6726, 2442, 45, 6739, 2445, 2457, 2436, 2332, 2437, 2338, 6721, 6731, 2443, 2481, 2421, 6724}

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x],x]
```

```
[Out] (53*b*x)/(192*c^3) + (11*a*x)/(27*c^2) + (49*(3*b + 4*a*c)*x)/(432*c^3) + (
29*b*x^2)/(384*c^2) + (5*a*x^2)/(54*c) + (13*(3*b + 4*a*c)*x^2)/(864*c^2) +
 (2*a*x^3)/81 + (17*b*x^3)/(576*c) + ((3*b + 4*a*c)*x^3)/(324*c) + (3*b*x^4
)/256 + (29*b*Log[1 - c*x])/(192*c^4) + (5*a*Log[1 - c*x])/(27*c^3) + (13*(
3*b + 4*a*c)*Log[1 - c*x])/(432*c^4) - (b*x^2*Log[1 - c*x])/(16*c^2) - (a*x
^2*Log[1 - c*x])/(9*c) - ((3*b + 4*a*c)*x^2*Log[1 - c*x])/(48*c^2) - (2*a*x
^3*Log[1 - c*x])/27 - (b*x^3*Log[1 - c*x])/(24*c) - ((3*b + 4*a*c)*x^3*Log[
1 - c*x])/(108*c) - (3*b*x^4*Log[1 - c*x])/64 + (b*(1 - c*x)*Log[1 - c*x])/
(8*c^4) + (2*a*(1 - c*x)*Log[1 - c*x])/(9*c^3) + ((3*b + 4*a*c)*(1 - c*x)*L
og[1 - c*x])/(12*c^4) - (b*Log[1 - c*x]^2)/(16*c^4) - (a*Log[1 - c*x]^2)/(9
*c^3) + (a*x^3*Log[1 - c*x]^2)/9 + (b*x^4*Log[1 - c*x]^2)/16 - ((3*b + 4*a*
c)*Log[c*x]*Log[1 - c*x]^2)/(12*c^4) - ((3*b + 4*a*c)*x*PolyLog[2, c*x])/(1
```

```
2*c^3) - ((3*b + 4*a*c)*x^2*PolyLog[2, c*x])/(24*c^2) - ((3*b + 4*a*c)*x^3*
PolyLog[2, c*x])/(36*c) - (b*x^4*PolyLog[2, c*x])/16 - ((3*b + 4*a*c)*Log[1
 - c*x]*PolyLog[2, c*x])/(12*c^4) + ((4*a*x^3 + 3*b*x^4)*Log[1 - c*x]*PolyL
og[2, c*x])/12 - ((3*b + 4*a*c)*Log[1 - c*x]*PolyLog[2, 1 - c*x])/(6*c^4) +
 ((3*b + 4*a*c)*PolyLog[3, 1 - c*x])/(6*c^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
```

```
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2457

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6739

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*PolyLog[2,
(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[Px, x]}, Simp[u
*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Dist[b, Int[Expa
ndIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x), x
], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d +
 e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px, x
]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2(a+bx)\log(1-cx)\text{Li}_2(cx)\,dx = \frac{1}{12}\left(4ax^3+3bx^4\right)\log(1-cx)\text{Li}_2(cx) + c\int\left(\frac{(-3b-4ac)\text{Li}_2(cx)}{12c^4} - \right.$$

$$= \frac{1}{12}\left(4ax^3+3bx^4\right)\log(1-cx)\text{Li}_2(cx) + \frac{1}{3}a\int x^2\log^2(1-cx)\,dx + \frac{1}{4}b$$

$$= \frac{1}{9}ax^3\log^2(1-cx) + \frac{1}{16}bx^4\log^2(1-cx) - \frac{(3b+4ac)x\text{Li}_2(cx)}{12c^3} - \frac{(3b}{}$$

$$= -\frac{(3b+4ac)x^2\log(1-cx)}{48c^2} - \frac{(3b+4ac)x^3\log(1-cx)}{108c} - \frac{1}{64}bx^4\log(1$$

$$= \frac{(3b+4ac)x}{12c^3} - \frac{(3b+4ac)x^2\log(1-cx)}{48c^2} - \frac{(3b+4ac)x^3\log(1-cx)}{108c}$$

$$= \frac{bx}{64c^3} + \frac{49(3b+4ac)x}{432c^3} + \frac{bx^2}{128c^2} + \frac{13(3b+4ac)x^2}{864c^2} + \frac{bx^3}{192c} + \frac{(3b+4a}{324c}$$

$$= \frac{9bx}{64c^3} + \frac{2ax}{9c^2} + \frac{49(3b+4ac)x}{432c^3} + \frac{bx^2}{128c^2} + \frac{13(3b+4ac)x^2}{864c^2} + \frac{bx^3}{192c} + $$

$$= \frac{53bx}{192c^3} + \frac{11ax}{27c^2} + \frac{49(3b+4ac)x}{432c^3} + \frac{29bx^2}{384c^2} + \frac{5ax^2}{54c} + \frac{13(3b+4ac)x^2}{864c^2} + $$

Mathematica [A]
time = 0.53, size = 425, normalized size = 0.64

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x],x]
```

```
[Out] (4260*b*c*x + 5952*a*c^2*x + 834*b*c^2*x^2 + 1056*a*c^3*x^2 + 268*b*c^3*x^3
 + 256*a*c^4*x^3 + 81*b*c^4*x^4 + 4260*b*Log[1 - c*x] + 5952*a*c*Log[1 - c*
x] - 2592*b*c*x*Log[1 - c*x] - 3840*a*c^2*x*Log[1 - c*x] - 864*b*c^2*x^2*Lo
g[1 - c*x] - 1344*a*c^3*x^2*Log[1 - c*x] - 480*b*c^3*x^3*Log[1 - c*x] - 768
*a*c^4*x^3*Log[1 - c*x] - 324*b*c^4*x^4*Log[1 - c*x] - 432*b*Log[1 - c*x]^2
 - 768*a*c*Log[1 - c*x]^2 + 768*a*c^4*x^3*Log[1 - c*x]^2 + 432*b*c^4*x^4*Lo
g[1 - c*x]^2 - 1728*b*Log[c*x]*Log[1 - c*x]^2 - 2304*a*c*Log[c*x]*Log[1 - c
*x]^2 + 48*(-(c*x*(8*a*c*(6 + 3*c*x + 2*c^2*x^2) + 3*b*(12 + 6*c*x + 4*c^2*
x^2 + 3*c^3*x^3))) + 12*(4*a*c*(-1 + c^3*x^3) + 3*b*(-1 + c^4*x^4))*Log[1 -
 c*x])*PolyLog[2, c*x] - 1152*(3*b + 4*a*c)*Log[1 - c*x]*PolyLog[2, 1 - c*x
] + 3456*b*PolyLog[3, 1 - c*x] + 4608*a*c*PolyLog[3, 1 - c*x])/(6912*c^4)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{2}(bx+a)\ln\left(-cx+1\right)\operatorname{polylog}\left(2,cx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] int(x^2*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

Maxima [A]
time = 0.27, size = 415, normalized size = 0.63

$$-\frac{1}{6912}\,c\left(\frac{576\left(\log\left(cx\right)\log\left(-cx+1\right)^{2}+2\,\mathrm{Li}_{2}(-cx+1)\log\left(-cx+1\right)-2\,\mathrm{Li}_{3}(-cx+1)\right)(4\,ac+3\,b)}{c^{5}}-\frac{81\,bc^{4}x^{4}+4\,(64\,ac^{4}+67\,bc^{3})x^{3}+6\,(176\,ac^{3}+139\,bc^{2})x^{2}+12\,(496\,ac^{2}+355\,bc)x-48\,(9\,bc^{4}x^{4}+4\,(4\,ac^{4}+3\,bc^{3})x^{3}+6\,(4\,ac^{3}+3\,bc^{2})x^{2}+12\,(4\,ac^{2}+3\,bc)x+12\,(4\,ac+3\,b)\log\left(-cx+1\right))\mathrm{Li}_{2}(cx)-4\,(54\,bc^{4}x^{4}+4\,(32\,ac^{4}+21\,bc^{3})x^{3}+6\,(40\,ac^{3}+27\,bc^{2})x^{2}-1488\,ac+12\,(64\,ac^{2}+45\,bc)x-1065\,b)\log\left(-cx+1\right)}{c^{5}}\right)+\frac{1}{1728}\left(\frac{32\,(18\,c^{3}x^{3}\mathrm{Li}_{2}(cx)-2\,c^{3}x^{3}-3\,c^{2}x^{2}-6\,cx+6\,(c^{3}x^{3}-1)\log\left(-cx+1\right))a}{c^{3}}+\frac{9\,(48\,c^{4}x^{4}\mathrm{Li}_{2}(cx)-3\,c^{4}x^{4}-4\,c^{3}x^{3}-6\,c^{2}x^{2}-12\,cx+12\,(c^{4}x^{4}-1)\log\left(-cx+1\right))b}{c^{4}}\right)\log\left(-cx+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
```

```
[Out] -1/6912*c*(576*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1)
- 2*polylog(3, -c*x + 1))*(4*a*c + 3*b)/c^5 - (81*b*c^4*x^4 + 4*(64*a*c^4 +
 67*b*c^3)*x^3 + 6*(176*a*c^3 + 139*b*c^2)*x^2 + 12*(496*a*c^2 + 355*b*c)*x
 - 48*(9*b*c^4*x^4 + 4*(4*a*c^4 + 3*b*c^3)*x^3 + 6*(4*a*c^3 + 3*b*c^2)*x^2
+ 12*(4*a*c^2 + 3*b*c)*x + 12*(4*a*c + 3*b)*log(-c*x + 1))*dilog(c*x) - 4*(
54*b*c^4*x^4 + 4*(32*a*c^4 + 21*b*c^3)*x^3 + 6*(40*a*c^3 + 27*b*c^2)*x^2 -
1488*a*c + 12*(64*a*c^2 + 45*b*c)*x - 1065*b)*log(-c*x + 1))/c^5) + 1/1728*
(32*(18*c^3*x^3*dilog(c*x) - 2*c^3*x^3 - 3*c^2*x^2 - 6*c*x + 6*(c^3*x^3 - 1
)*log(-c*x + 1))*a/c^3 + 9*(48*c^4*x^4*dilog(c*x) - 3*c^4*x^4 - 4*c^3*x^3 -
 6*c^2*x^2 - 12*c*x + 12*(c^4*x^4 - 1)*log(-c*x + 1))*b/c^4)*log(-c*x + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a*x^2)*dilog(c*x)*log(-c*x + 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*x^2*dilog(c*x)*log(-c*x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \ln(1 - c\,x)\,\mathrm{polylog}(2, c\,x)\,(a + b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(1 - c*x)*polylog(2, c*x)*(a + b*x),x)
```

```
[Out] int(x^2*log(1 - c*x)*polylog(2, c*x)*(a + b*x), x)
```

3.185 $\int x(a+bx)\log(1-cx)\mathbf{PolyLog}(2,cx)\,dx$

Optimal. Leaf size=546

$$\frac{4bx}{9c^2}+\frac{ax}{c}+\frac{5(2b+3ac)x}{24c^2}+\frac{bx^2}{9c}+\frac{(2b+3ac)x^2}{48c}+\frac{bx^3}{27}+\frac{a(1-cx)^2}{8c^2}+\frac{2b\log(1-cx)}{9c^3}+\frac{(2b+3ac)\log(1-cx)}{24c^3}-\frac{bx^2}{}$$

```
[Out] -1/6*(3*a*c+2*b)*x*polylog(2,c*x)/c^2-1/12*(3*a*c+2*b)*x^2*polylog(2,c*x)/c
-1/6*(3*a*c+2*b)*ln(c*x)*ln(-c*x+1)^2/c^3-1/6*(3*a*c+2*b)*ln(-c*x+1)*polylo
g(2,c*x)/c^3-1/3*(3*a*c+2*b)*ln(-c*x+1)*polylog(2,-c*x+1)/c^3+a*(-c*x+1)*ln
(-c*x+1)/c^2-1/9*b*x^2*ln(-c*x+1)/c-1/24*(3*a*c+2*b)*x^2*ln(-c*x+1)/c+2/9*b
*(-c*x+1)*ln(-c*x+1)/c^3+1/6*(3*a*c+2*b)*(-c*x+1)*ln(-c*x+1)/c^3-1/4*a*(-c*
x+1)^2*ln(-c*x+1)/c^2-1/2*a*(-c*x+1)*ln(-c*x+1)^2/c^2+1/4*a*(-c*x+1)^2*ln(-
c*x+1)^2/c^2+5/24*(3*a*c+2*b)*x/c^2+1/9*b*x^2/c+1/48*(3*a*c+2*b)*x^2/c+1/8*
a*(-c*x+1)^2/c^2-1/9*b*x^3*polylog(2,c*x)+1/3*(3*a*c+2*b)*polylog(3,-c*x+1)
/c^3+4/9*b*x/c^2+1/27*b*x^3+2/9*b*ln(-c*x+1)/c^3+1/24*(3*a*c+2*b)*ln(-c*x+1
)/c^3-1/9*b*x^3*ln(-c*x+1)-1/9*b*ln(-c*x+1)^2/c^3+1/9*b*x^3*ln(-c*x+1)^2+1/
6*(2*b*x^3+3*a*x^2)*ln(-c*x+1)*polylog(2,c*x)+a*x/c
```

Rubi [A]
time = 0.46, antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 21, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 1.105$, Rules used = {6874, 6726, 2442, 45, 6739, 2448, 2436, 2333, 2332, 2437, 2342, 2341, 2445, 2457, 2338, 6721, 6731, 2443, 2481, 2421, 6724}

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x],x]
```

```
[Out] (4*b*x)/(9*c^2) + (a*x)/c + (5*(2*b + 3*a*c)*x)/(24*c^2) + (b*x^2)/(9*c) +
((2*b + 3*a*c)*x^2)/(48*c) + (b*x^3)/27 + (a*(1 - c*x)^2)/(8*c^2) + (2*b*Lo
g[1 - c*x])/(9*c^3) + ((2*b + 3*a*c)*Log[1 - c*x])/(24*c^3) - (b*x^2*Log[1
- c*x])/(9*c) - ((2*b + 3*a*c)*x^2*Log[1 - c*x])/(24*c) - (b*x^3*Log[1 - c*
x])/9 + (2*b*(1 - c*x)*Log[1 - c*x])/(9*c^3) + (a*(1 - c*x)*Log[1 - c*x])/c
^2 + ((2*b + 3*a*c)*(1 - c*x)*Log[1 - c*x])/(6*c^3) - (a*(1 - c*x)^2*Log[1
- c*x])/(4*c^2) - (b*Log[1 - c*x]^2)/(9*c^3) + (b*x^3*Log[1 - c*x]^2)/9 - (
a*(1 - c*x)*Log[1 - c*x]^2)/(2*c^2) + (a*(1 - c*x)^2*Log[1 - c*x]^2)/(4*c^2
) - ((2*b + 3*a*c)*Log[c*x]*Log[1 - c*x]^2)/(6*c^3) - ((2*b + 3*a*c)*x*Poly
Log[2, c*x])/(6*c^2) - ((2*b + 3*a*c)*x^2*PolyLog[2, c*x])/(12*c) - (b*x^3*
PolyLog[2, c*x])/9 - ((2*b + 3*a*c)*Log[1 - c*x]*PolyLog[2, c*x])/(6*c^3) +
 ((3*a*x^2 + 2*b*x^3)*Log[1 - c*x]*PolyLog[2, c*x])/6 - ((2*b + 3*a*c)*Log[
1 - c*x]*PolyLog[2, 1 - c*x])/(3*c^3) + ((2*b + 3*a*c)*PolyLog[3, 1 - c*x])
/(3*c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2457

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6739

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*PolyLog[2,
(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[Px, x]}, Simp[u
*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Dist[b, Int[Expa
ndIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x), x
], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d +
 e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px, x
]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x(a+bx)\log(1-cx)\text{Li}_2(cx)\,dx = \frac{1}{6}\left(3ax^2+2bx^3\right)\log(1-cx)\text{Li}_2(cx)+c\int\left(\frac{(-2b-3ac)\text{Li}_2(cx)}{6c^3}-\frac{(2b\right.$$

$$=\frac{1}{6}\left(3ax^2+2bx^3\right)\log(1-cx)\text{Li}_2(cx)+\frac{1}{2}a\int x\log^2(1-cx)\,dx+\frac{1}{3}b\int x$$

$$=\frac{1}{9}bx^3\log^2(1-cx)-\frac{(2b+3ac)x\text{Li}_2(cx)}{6c^2}-\frac{(2b+3ac)x^2\text{Li}_2(cx)}{12c}-\frac{1}{9}bx^3$$

$$=-\frac{(2b+3ac)x^2\log(1-cx)}{24c}-\frac{1}{27}bx^3\log(1-cx)+\frac{1}{9}bx^3\log^2(1-cx)-$$

$$=\frac{(2b+3ac)x}{6c^2}-\frac{(2b+3ac)x^2\log(1-cx)}{24c}-\frac{1}{27}bx^3\log(1-cx)+\frac{(2b+3}{}$$

$$=\frac{bx}{27c^2}+\frac{5(2b+3ac)x}{24c^2}+\frac{bx^2}{54c}+\frac{(2b+3ac)x^2}{48c}+\frac{bx^3}{81}+\frac{b\log(1-cx)}{27c^3}+\frac{(2}{}$$

$$=\frac{7bx}{27c^2}+\frac{ax}{c}+\frac{5(2b+3ac)x}{24c^2}+\frac{bx^2}{54c}+\frac{(2b+3ac)x^2}{48c}+\frac{bx^3}{81}+\frac{a(1-cx)^2}{8c^2}$$

$$=\frac{4bx}{9c^2}+\frac{ax}{c}+\frac{5(2b+3ac)x}{24c^2}+\frac{bx^2}{9c}+\frac{(2b+3ac)x^2}{48c}+\frac{bx^3}{27}+\frac{a(1-cx)^2}{8c^2}+$$

Mathematica [A]
time = 0.41, size = 362, normalized size = 0.66

$$\frac{-378ac+372bcx+594ac^2x+66bc^2x^2+81ac^3x^2+16bc^3x^3+372b\log(1-cx)+594ac\log(1-cx)-240bcx\log(1-cx)-432ac^2x\log(1-cx)-84bc^2x^2\log(1-cx)-162ac^3x^2\log(1-cx)-48bc^3x^3\log(1-cx)-48b\log^2(1-cx)-108ac\log^2(1-cx)+108ac^3x^2\log^2(1-cx)+48bc^3x^3\log^2(1-cx)-144b\log(cx)\log^2(1-cx)-216ac\log(cx)\log^2(1-cx)+12\left(-cx\left(9ac(2+cx)+2b\left(6+3cx+2c^2x^2\right)\right)+6\left(3ac\left(-1+c^2x^2\right)+2b\left(-1+c^3x^3\right)\right)\log(1-cx)\right)\text{PolyLog}(2,cx)-144(2b+3ac)\log(1-cx)\text{PolyLog}(2,1-cx)+288b\text{PolyLog}(3,1-cx)+432ac\text{PolyLog}(3,1-cx)}{432c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x],x]
```

```
[Out] (-378*a*c + 372*b*c*x + 594*a*c^2*x + 66*b*c^2*x^2 + 81*a*c^3*x^2 + 16*b*c^
3*x^3 + 372*b*Log[1 - c*x] + 594*a*c*Log[1 - c*x] - 240*b*c*x*Log[1 - c*x]
- 432*a*c^2*x*Log[1 - c*x] - 84*b*c^2*x^2*Log[1 - c*x] - 162*a*c^3*x^2*Log[
1 - c*x] - 48*b*c^3*x^3*Log[1 - c*x] - 48*b*Log[1 - c*x]^2 - 108*a*c*Log[1
- c*x]^2 + 108*a*c^3*x^2*Log[1 - c*x]^2 + 48*b*c^3*x^3*Log[1 - c*x]^2 - 144
*b*Log[c*x]*Log[1 - c*x]^2 - 216*a*c*Log[c*x]*Log[1 - c*x]^2 + 12*(-(c*x*(9
*a*c*(2 + c*x) + 2*b*(6 + 3*c*x + 2*c^2*x^2))) + 6*(3*a*c*(-1 + c^2*x^2) +
2*b*(-1 + c^3*x^3))*Log[1 - c*x])*PolyLog[2, c*x] - 144*(2*b + 3*a*c)*Log[1
```

```
 - c*x]*PolyLog[2, 1 - c*x] + 288*b*PolyLog[3, 1 - c*x] + 432*a*c*PolyLog[3
, 1 - c*x])/(432*c^3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x(bx+a)\ln\left(-cx+1\right)\operatorname{polylog}\left(2,cx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
[Out] int(x*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

Maxima [A]
time = 0.28, size = 345, normalized size = 0.63

$$-\frac{1}{432}\,c\left(\frac{72\,\left(\log\left(cx\right)\log\left(-cx+1\right)^{2}+2\,\mathrm{Li}_2(-cx+1)\log\left(-cx+1\right)-2\,\mathrm{Li}_3(-cx+1)\right)(3\,ac+2\,b)}{c^{4}}-\frac{16\,bc^{3}x^{3}+3\,(27\,ac^{3}+22\,bc^{2})x^{2}+6\,(99\,ac^{2}+62\,bc)x-12\,(4\,bc^{3}x^{3}+3\,(3\,ac^{3}+2\,bc^{2})x^{2}+6\,(3\,ac^{2}+2\,bc)x+6\,(3\,ac+2\,b)\log\left(-cx+1\right))\mathrm{Li}_2(cx)-2\,(16\,bc^{3}x^{3}+6\,(9\,ac^{3}+5\,bc^{2})x^{2}-297\,ac+6\,(27\,ac^{2}+16\,bc)x-186\,b)\log\left(-cx+1\right)}{c^{4}}\right)+\frac{1}{216}\left(\frac{27\,(4\,c^{2}x^{2}\mathrm{Li}_2(cx)-c^{2}x^{2}-2\,cx+2\,(c^{2}x^{2}-1)\log\left(-cx+1\right))a}{c^{2}}+\frac{4\,(18\,c^{3}x^{3}\mathrm{Li}_2(cx)-2\,c^{3}x^{3}-3\,c^{2}x^{2}-6\,cx+6\,(c^{3}x^{3}-1)\log\left(-cx+1\right))b}{c^{3}}\right)\log\left(-cx+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
[Out] -1/432*c*(72*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) -
2*polylog(3, -c*x + 1))*(3*a*c + 2*b)/c^4 - (16*b*c^3*x^3 + 3*(27*a*c^3 + 2
2*b*c^2)*x^2 + 6*(99*a*c^2 + 62*b*c)*x - 12*(4*b*c^3*x^3 + 3*(3*a*c^3 + 2*b
*c^2)*x^2 + 6*(3*a*c^2 + 2*b*c)*x + 6*(3*a*c + 2*b)*log(-c*x + 1))*dilog(c*
x) - 2*(16*b*c^3*x^3 + 6*(9*a*c^3 + 5*b*c^2)*x^2 - 297*a*c + 6*(27*a*c^2 +
16*b*c)*x - 186*b)*log(-c*x + 1))/c^4) + 1/216*(27*(4*c^2*x^2*dilog(c*x) -
c^2*x^2 - 2*c*x + 2*(c^2*x^2 - 1)*log(-c*x + 1))*a/c^2 + 4*(18*c^3*x^3*dilo
g(c*x) - 2*c^3*x^3 - 3*c^2*x^2 - 6*c*x + 6*(c^3*x^3 - 1)*log(-c*x + 1))*b/c
^3)*log(-c*x + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
[Out] integral((b*x^2 + a*x)*dilog(c*x)*log(-c*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+bx)\log\left(-cx+1\right)\mathrm{Li}_2\left(cx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] Integral(x*(a + b*x)*log(-c*x + 1)*polylog(2, c*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*x*dilog(c*x)*log(-c*x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\ln\left(1-c\,x\right)\,\mathrm{polylog}(2,c\,x)\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(1 - c*x)*polylog(2, c*x)*(a + b*x),x)
```

```
[Out] int(x*log(1 - c*x)*polylog(2, c*x)*(a + b*x), x)
```

3.186 $\int (a+bx)\log(1-cx)\mathbf{PolyLog}(2,cx)\,dx$

Optimal. Leaf size=390

$$2ax+\frac{9bx}{8c}+\frac{(b+2ac)x}{2c}+\frac{bx^2}{16}+\frac{b(1-cx)^2}{8c^2}+\frac{b\log(1-cx)}{8c^2}-\frac{1}{8}bx^2\log(1-cx)+\frac{b(1-cx)\log(1-cx)}{c^2}+\frac{2a(1-c$$

```
[Out] 2*a*x+9/8*b*x/c+1/2*(2*a*c+b)*x/c+1/16*b*x^2+1/8*b*(-c*x+1)^2/c^2+1/8*b*ln(
-c*x+1)/c^2-1/8*b*x^2*ln(-c*x+1)+b*(-c*x+1)*ln(-c*x+1)/c^2+2*a*(-c*x+1)*ln(
-c*x+1)/c+1/2*(2*a*c+b)*(-c*x+1)*ln(-c*x+1)/c^2-1/4*b*(-c*x+1)^2*ln(-c*x+1)
/c^2-1/2*b*(-c*x+1)*ln(-c*x+1)^2/c^2-a*(-c*x+1)*ln(-c*x+1)^2/c+1/4*b*(-c*x+
1)^2*ln(-c*x+1)^2/c^2-1/2*(2*a*c+b)*ln(c*x)*ln(-c*x+1)^2/c^2-1/2*(2*a*c+b)*
x*polylog(2,c*x)/c-1/4*b*x^2*polylog(2,c*x)-1/2*(2*a*c+b)*ln(-c*x+1)*polylo
g(2,c*x)/c^2+1/2*(b*x^2+2*a*x)*ln(-c*x+1)*polylog(2,c*x)-(2*a*c+b)*ln(-c*x+
1)*polylog(2,-c*x+1)/c^2+(2*a*c+b)*polylog(3,-c*x+1)/c^2
```

Rubi [A]
time = 0.29, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 20, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.111$, Rules used = {6733, 45, 2463, 2436, 2332, 2438, 2442, 6739, 2333, 2448, 2437, 2342, 2341, 6721, 6726, 6731, 2443, 2481, 2421, 6724}

$$\frac{(2ac+b)\text{Li}_3(1-cx)}{c^2}-\frac{(2ac+b)\text{Li}_2(cx)\log(1-cx)}{2c^2}-\frac{(2ac+b)\text{Li}_2(1-cx)\log(1-cx)}{c^2}-\frac{(2ac+b)\log(cx)\log^2(1-cx)}{2c^2}+\frac{(1-cx)(2ac+b)\log(1-cx)}{2c^2}+\frac{1}{2}\left(2ax+bx^2\right)\text{Li}_2(cx)\log(1-cx)-\frac{x(2ac+b)\text{Li}_2(cx)}{2c}+\frac{x(2ac+b)}{2c}-\frac{a(1-cx)\log^2(1-cx)}{c}+\frac{2a(1-cx)\log(1-cx)}{c}+2ax+\frac{b(1-cx)^2}{8c^2}+\frac{b(1-cx)^2\log^2(1-cx)}{4c^2}-\frac{b(1-cx)\log^2(1-cx)}{2c^2}-\frac{b(1-cx)^2\log(1-cx)}{4c^2}+\frac{b(1-cx)\log(1-cx)}{c^2}+\frac{b\log(1-cx)}{8c^2}-\frac{1}{4}bx^2\text{Li}_2(cx)-\frac{1}{8}bx^2\log(1-cx)+\frac{9bx}{8c}+\frac{bx^2}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x],x]
```

```
[Out] 2*a*x + (9*b*x)/(8*c) + ((b + 2*a*c)*x)/(2*c) + (b*x^2)/16 + (b*(1 - c*x)^2
)/(8*c^2) + (b*Log[1 - c*x])/(8*c^2) - (b*x^2*Log[1 - c*x])/8 + (b*(1 - c*x
)*Log[1 - c*x])/c^2 + (2*a*(1 - c*x)*Log[1 - c*x])/c + ((b + 2*a*c)*(1 - c*
x)*Log[1 - c*x])/(2*c^2) - (b*(1 - c*x)^2*Log[1 - c*x])/(4*c^2) - (b*(1 - c
*x)*Log[1 - c*x]^2)/(2*c^2) - (a*(1 - c*x)*Log[1 - c*x]^2)/c + (b*(1 - c*x)
^2*Log[1 - c*x]^2)/(4*c^2) - ((b + 2*a*c)*Log[c*x]*Log[1 - c*x]^2)/(2*c^2)
- ((b + 2*a*c)*x*PolyLog[2, c*x])/(2*c) - (b*x^2*PolyLog[2, c*x])/4 - ((b +
 2*a*c)*Log[1 - c*x]*PolyLog[2, c*x])/(2*c^2) + ((2*a*x + b*x^2)*Log[1 - c*
x]*PolyLog[2, c*x])/2 - ((b + 2*a*c)*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c^2
+ ((b + 2*a*c)*PolyLog[3, 1 - c*x])/c^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6733

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Dist[b/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x))
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 6739

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*PolyLog[2,
(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[Px, x]}, Simp[u
*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Dist[b, Int[Expa
ndIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x), x
], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d +
 e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px, x
]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\log(1-cx)\text{Li}_2(cx)\,dx &= \frac{1}{2}\left(2ax+bx^2\right)\log(1-cx)\text{Li}_2(cx)+c\int \left(\frac{(-b-2ac)\text{Li}_2(cx)}{2c^2}-\frac{bx\text{Li}_2(c\right.\right.\\
&= \frac{1}{2}\left(2ax+bx^2\right)\log(1-cx)\text{Li}_2(cx)+a\int \log^2(1-cx)\,dx+\frac{1}{2}b\int x\log^2(1\\
&= -\frac{(b+2ac)x\text{Li}_2(cx)}{2c}-\frac{1}{4}bx^2\text{Li}_2(cx)-\frac{(b+2ac)\log(1-cx)\text{Li}_2(cx)}{2c^2}+\frac{1}{2}\\
&= -\frac{1}{8}bx^2\log(1-cx)-\frac{a(1-cx)\log^2(1-cx)}{c}-\frac{(b+2ac)\log(cx)\log^2(1}{2c^2}\\
&= 2ax+\frac{(b+2ac)x}{2c}-\frac{1}{8}bx^2\log(1-cx)+\frac{2a(1-cx)\log(1-cx)}{c}+\frac{(b+}{}\\
&= 2ax+\frac{bx}{8c}+\frac{(b+2ac)x}{2c}+\frac{bx^2}{16}+\frac{b\log(1-cx)}{8c^2}-\frac{1}{8}bx^2\log(1-cx)+\frac{2a}{}\\
&= 2ax+\frac{9bx}{8c}+\frac{(b+2ac)x}{2c}+\frac{bx^2}{16}+\frac{b(1-cx)^2}{8c^2}+\frac{b\log(1-cx)}{8c^2}-\frac{1}{8}bx^2\log\end{aligned}$$

Mathematica [A]
time = 0.34, size = 285, normalized size = 0.73

$$\frac{-14b-32ac+22bcx+48ac^2x+3bc^2x^2+22b\log(1-cx)+48ac\log(1-cx)-16bcx\log(1-cx)-48ac^2x\log(1-cx)-6bc^2x^2\log(1-cx)-4b\log^2(1-cx)-16ac\log^2(1-cx)+16ac^2x\log^2(1-cx)+4bc^2x^2\log^2(1-cx)-8b\log(cx)\log^2(1-cx)-16ac\log(cx)\log^2(1-cx)+4(-cx(2b+4ac+bcx)+2(-1+cx)(b+2ac+bcx)\log(1-cx))\text{PolyLog}(2,cx)-16(b+2ac)\log(1-cx)\text{PolyLog}(2,1-cx)+16b\text{PolyLog}(3,1-cx)+32ac\text{PolyLog}(3,1-cx)}{16c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x],x]
```

```
[Out] (-14*b - 32*a*c + 22*b*c*x + 48*a*c^2*x + 3*b*c^2*x^2 + 22*b*Log[1 - c*x] +
 48*a*c*Log[1 - c*x] - 16*b*c*x*Log[1 - c*x] - 48*a*c^2*x*Log[1 - c*x] - 6*
b*c^2*x^2*Log[1 - c*x] - 4*b*Log[1 - c*x]^2 - 16*a*c*Log[1 - c*x]^2 + 16*a*
c^2*x*Log[1 - c*x]^2 + 4*b*c^2*x^2*Log[1 - c*x]^2 - 8*b*Log[c*x]*Log[1 - c*
x]^2 - 16*a*c*Log[c*x]*Log[1 - c*x]^2 + 4*(-(c*x*(2*b + 4*a*c + b*c*x)) + 2
*(-1 + c*x)*(b + 2*a*c + b*c*x)*Log[1 - c*x])*PolyLog[2, c*x] - 16*(b + 2*a
*c)*Log[1 - c*x]*PolyLog[2, 1 - c*x] + 16*b*PolyLog[3, 1 - c*x] + 32*a*c*Po
lyLog[3, 1 - c*x])/(16*c^2)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx+a\right)\ln\left(-cx+1\right)\operatorname{polylog}\left(2,cx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

Maxima [A]
time = 0.27, size = 258, normalized size = 0.66

$$-\frac{1}{16}c\left(\frac{8\left(\log\left(cx\right)\log\left(-cx+1\right)^{2}+2\,\mathrm{Li}_{2}(-cx+1)\log\left(-cx+1\right)-2\,\mathrm{Li}_{3}(-cx+1)\right)(2\,ac+b)}{c^{3}}-\frac{3\,bc^{2}x^{2}+2\,(24\,ac^{2}+11\,bc)x-4\,(bc^{2}x^{2}+2\,(2\,ac^{2}+bc)x+2\,(2\,ac+b)\log\left(-cx+1\right))\mathrm{Li}_{2}(cx)-2\,(2\,bc^{2}x^{2}-24\,ac+2\,(8\,ac^{2}+3\,bc)x-11\,b)\log\left(-cx+1\right)}{c^{3}}\right)+\frac{1}{8}\left(\frac{8\,(cx\mathrm{Li}_{2}(cx)-cx+(cx-1)\log\left(-cx+1\right))a}{c}+\frac{(4\,c^{2}x^{2}\mathrm{Li}_{2}(cx)-c^{2}x^{2}-2\,cx+2\,(c^{2}x^{2}-1)\log\left(-cx+1\right))b}{c^{2}}\right)\log\left(-cx+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
```

```
[Out] -1/16*c*(8*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*
polylog(3, -c*x + 1))*(2*a*c + b)/c^3 - (3*b*c^2*x^2 + 2*(24*a*c^2 + 11*b*c
)*x - 4*(b*c^2*x^2 + 2*(2*a*c^2 + b*c)*x + 2*(2*a*c + b)*log(-c*x + 1))*dil
og(c*x) - 2*(2*b*c^2*x^2 - 24*a*c + 2*(8*a*c^2 + 3*b*c)*x - 11*b)*log(-c*x
+ 1))/c^3) + 1/8*(8*(c*x*dilog(c*x) - c*x + (c*x - 1)*log(-c*x + 1))*a/c +
(4*c^2*x^2*dilog(c*x) - c^2*x^2 - 2*c*x + 2*(c^2*x^2 - 1)*log(-c*x + 1))*b/
c^2)*log(-c*x + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)*dilog(c*x)*log(-c*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)\log\left(-cx+1\right)\mathrm{Li}_2\left(cx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] Integral((a + b*x)*log(-c*x + 1)*polylog(2, c*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*dilog(c*x)*log(-c*x + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(1-c\,x\right)\,\mathrm{polylog}(2,c\,x)\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1 - c*x)*polylog(2, c*x)*(a + b*x),x)
```

```
[Out] int(log(1 - c*x)*polylog(2, c*x)*(a + b*x), x)
```

3.187 $\int \frac{(a+bx)\log(1-cx)\text{PolyLog}(2,cx)}{x}\,dx$

Optimal. Leaf size=153

$$3bx+\frac{3b(1-cx)\log(1-cx)}{c}-\frac{b(1-cx)\log^2(1-cx)}{c}-\frac{b\log(cx)\log^2(1-cx)}{c}-bx\text{PolyLog}(2,cx)-\frac{b\log(1-cx)}{}$$

```
[Out] 3*b*x+3*b*(-c*x+1)*ln(-c*x+1)/c-b*(-c*x+1)*ln(-c*x+1)^2/c-b*ln(c*x)*ln(-c*x
+1)^2/c-b*x*polylog(2,c*x)-b*ln(-c*x+1)*polylog(2,c*x)/c+b*x*ln(-c*x+1)*pol
ylog(2,c*x)-1/2*a*polylog(2,c*x)^2-2*b*ln(-c*x+1)*polylog(2,-c*x+1)/c+2*b*p
olylog(3,-c*x+1)/c
```

Rubi [A]
time = 0.22, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6874, 6721, 2436, 2332, 6724, 6740, 6736, 12, 6735, 2333, 6820, 6731, 2443, 2481, 2421}

$$-\frac{1}{2}a\text{Li}_2(cx)^2-bx\text{Li}_2(cx)+\frac{2b\text{Li}_3(1-cx)}{c}+bx\text{Li}_2(cx)\log(1-cx)-\frac{b\text{Li}_2(cx)\log(1-cx)}{c}-\frac{2b\text{Li}_2(1-cx)\log(1-cx)}{c}-\frac{b(1-cx)\log^2(1-cx)}{c}-\frac{b\log(cx)\log^2(1-cx)}{c}+\frac{3b(1-cx)\log(1-cx)}{c}+3bx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x,x]
```

```
[Out] 3*b*x + (3*b*(1 - c*x)*Log[1 - c*x])/c - (b*(1 - c*x)*Log[1 - c*x]^2)/c - (
b*Log[c*x]*Log[1 - c*x]^2)/c - b*x*PolyLog[2, c*x] - (b*Log[1 - c*x]*PolyLo
g[2, c*x])/c + b*x*Log[1 - c*x]*PolyLog[2, c*x] - (a*PolyLog[2, c*x]^2)/2 -
 (2*b*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c + (2*b*PolyLog[3, 1 - c*x])/c
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6735

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*PolyLog[2, (c_.)*
((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x*(g + h*Log[f*(d + e*x)^n])*PolyL
og[2, c*(a + b*x)], x] + (Dist[b, Int[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*
c - b*c*x]*ExpandIntegrand[x/(a + b*x), x], x], x] - Dist[e*h*n, Int[PolyLo
g[2, c*(a + b*x)]*ExpandIntegrand[x/(d + e*x), x], x], x]) /; FreeQ[{a, b,
c, d, e, f, g, h, n}, x]
```

Rule 6736

```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> Simp[-P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

Rule 6740

```
Int[((g_.) + Log[1 + (e_.)*(x_)]*(h_.))*(Px_)*(x_)^(m_)*PolyLog[2, (c_.)*(x
_)], x_Symbol] :> Dist[Coeff[Px, x, -m - 1], Int[(g + h*Log[1 + e*x])*(Poly
Log[2, c*x]/x), x], x] + Int[x^m*(Px - Coeff[Px, x, -m - 1]*x^(-m - 1))*(g
+ h*Log[1 + e*x])*PolyLog[2, c*x], x] /; FreeQ[{c, e, g, h}, x] && PolyQ[Px
, x] && ILtQ[m, 0] && EqQ[c + e, 0] && NeQ[Coeff[Px, x, -m - 1], 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(a+bx)\log(1-cx)\text{Li}_2(cx)}{x}\,dx = a\int \frac{\log(1-cx)\text{Li}_2(cx)}{x}\,dx + \int b\log(1-cx)\text{Li}_2(cx)\,dx$$

$$= -\frac{1}{2}a\text{Li}_2(cx)^2 + b\int \log(1-cx)\text{Li}_2(cx)\,dx$$

$$= bx\log(1-cx)\text{Li}_2(cx) - \frac{1}{2}a\text{Li}_2(cx)^2 + b\int \log^2(1-cx)\,dx + (bc)\int \Big(-$$

$$= bx\log(1-cx)\text{Li}_2(cx) - \frac{1}{2}a\text{Li}_2(cx)^2 - \frac{b\text{Subst}\left(\int \log^2(x)\,dx, x, 1-cx\right)}{c}$$

$$= -\frac{b(1-cx)\log^2(1-cx)}{c} + bx\log(1-cx)\text{Li}_2(cx) - \frac{1}{2}a\text{Li}_2(cx)^2 + \frac{(2b)\text{S}}{}$$

$$= 2bx + \frac{2b(1-cx)\log(1-cx)}{c} - \frac{b(1-cx)\log^2(1-cx)}{c} + bx\log(1-cx$$

$$= 2bx + \frac{2b(1-cx)\log(1-cx)}{c} - \frac{b(1-cx)\log^2(1-cx)}{c} - bx\text{Li}_2(cx) -$$

$$= 2bx + \frac{2b(1-cx)\log(1-cx)}{c} - \frac{b(1-cx)\log^2(1-cx)}{c} - \frac{b\log(cx)\log^2}{c}$$

$$= 3bx + \frac{3b(1-cx)\log(1-cx)}{c} - \frac{b(1-cx)\log^2(1-cx)}{c} - \frac{b\log(cx)\log^2}{c}$$

$$= 3bx + \frac{3b(1-cx)\log(1-cx)}{c} - \frac{b(1-cx)\log^2(1-cx)}{c} - \frac{b\log(cx)\log^2}{c}$$

$$= 3bx + \frac{3b(1-cx)\log(1-cx)}{c} - \frac{b(1-cx)\log^2(1-cx)}{c} - \frac{b\log(cx)\log^2}{c}$$

Mathematica [A]
time = 0.15, size = 137, normalized size = 0.90

$$\frac{b(-cx+(-1+cx)\log(1-cx))\text{PolyLog}(2,cx)}{c} - \frac{1}{2}a\text{PolyLog}(2,cx)^2 + \frac{b(-2+3cx+3\log(1-cx)-3cx\log(1-cx)-\log^2(1-cx)+cx\log^2(1-cx)-\log(cx)\log^2(1-cx)-2\log(1-cx)\text{PolyLog}(2,1-cx)+2\text{PolyLog}(3,1-cx))}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x,x]
```

```
[Out] (b*(-(c*x) + (-1 + c*x)*Log[1 - c*x])*PolyLog[2, c*x])/c - (a*PolyLog[2, c*
x]^2)/2 + (b*(-2 + 3*c*x + 3*Log[1 - c*x] - 3*c*x*Log[1 - c*x] - Log[1 - c*
x]^2 + c*x*Log[1 - c*x]^2 - Log[c*x]*Log[1 - c*x]^2 - 2*Log[1 - c*x]*PolyLo
g[2, 1 - c*x] + 2*PolyLog[3, 1 - c*x]))/c
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)\ln(-cx+1)\,\text{polylog}(2,cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x,x)
```

```
[Out] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)*dilog(c*x)*log(-c*x + 1)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx) \log(-cx + 1) \operatorname{Li}_2(cx)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x,x)
```

```
[Out] Integral((a + b*x)*log(-c*x + 1)*polylog(2, c*x)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln (1 - c\,x)\ \mathrm{polylog}(2, c\,x)\ (a + b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(1 - c*x)*polylog(2, c*x)*(a + b*x))/x,x)
[Out] int((log(1 - c*x)*polylog(2, c*x)*(a + b*x))/x, x)
```

3.188 $\int \frac{(a+bx)\log(1-cx)\text{PolyLog}(2,cx)}{x^2}\,dx$

Optimal. Leaf size=131

$$\frac{a(1-cx)\log^2(1-cx)}{x}+ac\log(cx)\log^2(1-cx)-2ac\text{PolyLog}(2,cx)+ac\log(1-cx)\text{PolyLog}(2,cx)-\frac{a\log(1-c$$

```
[Out] a*(-c*x+1)*ln(-c*x+1)^2/x+a*c*ln(c*x)*ln(-c*x+1)^2-2*a*c*polylog(2,c*x)+a*c
*ln(-c*x+1)*polylog(2,c*x)-a*ln(-c*x+1)*polylog(2,c*x)/x-1/2*b*polylog(2,c*
x)^2+2*a*c*ln(-c*x+1)*polylog(2,-c*x+1)-a*c*polylog(3,c*x)-2*a*c*polylog(3,
-c*x+1)
```

Rubi [A]
time = 0.20, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 17, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {6874, 6726, 2442, 36, 29, 31, 6724, 6740, 6736, 12, 6738, 2444, 2438, 6731, 2443, 2481, 2421}

$$-2ac\text{Li}_2(cx)-ac\text{Li}_3(cx)-2ac\text{Li}_3(1-cx)+ac\text{Li}_2(cx)\log(1-cx)-\frac{a\text{Li}_2(cx)\log(1-cx)}{x}+2ac\text{Li}_2(1-cx)\log(1-cx)+\frac{a(1-cx)\log^2(1-cx)}{x}+ac\log(cx)\log^2(1-cx)-\frac{1}{2}b\text{Li}_2(cx)^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^2,x]
```

```
[Out] (a*(1 - c*x)*Log[1 - c*x]^2)/x + a*c*Log[c*x]*Log[1 - c*x]^2 - 2*a*c*PolyLo
g[2, c*x] + a*c*Log[1 - c*x]*PolyLog[2, c*x] - (a*Log[1 - c*x]*PolyLog[2, c
*x])/x - (b*PolyLog[2, c*x]^2)/2 + 2*a*c*Log[1 - c*x]*PolyLog[2, 1 - c*x] -
 a*c*PolyLog[3, c*x] - 2*a*c*PolyLog[3, 1 - c*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6736

```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> Simp[-P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

Rule 6738

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x^(m + 1)*(g + h*Log[f*
(d + e*x)^n])*(PolyLog[2, c*(a + b*x)]/(m + 1)), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[e*h*(n/(m + 1)), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6740

```
Int[((g_.) + Log[1 + (e_.)*(x_)]*(h_.))*(Px_)*(x_)^(m_)*PolyLog[2, (c_.)*(x
_)], x_Symbol] :> Dist[Coeff[Px, x, -m - 1], Int[(g + h*Log[1 + e*x])*(Poly
Log[2, c*x]/x), x], x] + Int[x^m*(Px - Coeff[Px, x, -m - 1]*x^(-m - 1))*(g
+ h*Log[1 + e*x])*PolyLog[2, c*x], x] /; FreeQ[{c, e, g, h}, x] && PolyQ[Px
, x] && ILtQ[m, 0] && EqQ[c + e, 0] && NeQ[Coeff[Px, x, -m - 1], 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)\log(1-cx)\text{Li}_2(cx)}{x^2}\,dx &= b\int \frac{\log(1-cx)\text{Li}_2(cx)}{x}\,dx + \int \frac{a\log(1-cx)\text{Li}_2(cx)}{x^2}\,dx\\
&= -\frac{1}{2}b\text{Li}_2(cx)^2 + a\int \frac{\log(1-cx)\text{Li}_2(cx)}{x^2}\,dx\\
&= -\frac{a\log(1-cx)\text{Li}_2(cx)}{x} - \frac{1}{2}b\text{Li}_2(cx)^2 - a\int \frac{\log^2(1-cx)}{x^2}\,dx - (ac)\int (\\
&= \frac{a(1-cx)\log^2(1-cx)}{x} - \frac{a\log(1-cx)\text{Li}_2(cx)}{x} - \frac{1}{2}b\text{Li}_2(cx)^2 - (ac)\int\\
&= \frac{a(1-cx)\log^2(1-cx)}{x} - 2ac\text{Li}_2(cx) + ac\log(1-cx)\text{Li}_2(cx) - \frac{a\log(1}{}\\
&= \frac{a(1-cx)\log^2(1-cx)}{x} + ac\log(cx)\log^2(1-cx) - 2ac\text{Li}_2(cx) + ac\log\\
&= \frac{a(1-cx)\log^2(1-cx)}{x} + ac\log(cx)\log^2(1-cx) - 2ac\text{Li}_2(cx) + ac\log\\
&= \frac{a(1-cx)\log^2(1-cx)}{x} + ac\log(cx)\log^2(1-cx) - 2ac\text{Li}_2(cx) + ac\log\\
&= \frac{a(1-cx)\log^2(1-cx)}{x} + ac\log(cx)\log^2(1-cx) - 2ac\text{Li}_2(cx) + ac\log\end{aligned}$$

Mathematica [**A**]
time = 0.50, size = 135, normalized size = 1.03

$$2ac\log(cx)\log(1-cx) - ac\log^2(1-cx) + \frac{a\log^2(1-cx)}{x} + ac\log(cx)\log^2(1-cx) + \frac{a(-1+cx)\log(1-cx)\text{PolyLog}(2,cx)}{x} - \frac{1}{2}b\text{PolyLog}(2,cx)^2 + 2ac(1+\log(1-cx))\text{PolyLog}(2,1-cx) - ac\text{PolyLog}(3,cx) - 2ac\text{PolyLog}(3,1-cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^2,x]
```

```
[Out] 2*a*c*Log[c*x]*Log[1 - c*x] - a*c*Log[1 - c*x]^2 + (a*Log[1 - c*x]^2)/x + a
*c*Log[c*x]*Log[1 - c*x]^2 + (a*(-1 + c*x)*Log[1 - c*x]*PolyLog[2, c*x])/x
- (b*PolyLog[2, c*x]^2)/2 + 2*a*c*(1 + Log[1 - c*x])*PolyLog[2, 1 - c*x] -
a*c*PolyLog[3, c*x] - 2*a*c*PolyLog[3, 1 - c*x]
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)\ln(-cx+1)\,\text{polylog}(2,cx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^2,x)
```

```
[Out] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)\log(-cx+1)\operatorname{Li}_2(cx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x**2,x)
```

```
[Out] Integral((a + b*x)*log(-c*x + 1)*polylog(2, c*x)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln(1-c\,x)\ \mathrm{polylog}(2,c\,x)\ (a+b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(1 - c*x)*polylog(2, c*x)*(a + b*x))/x^2,x)
[Out] int((log(1 - c*x)*polylog(2, c*x)*(a + b*x))/x^2, x)
```

3.189 $\int \frac{(a+bx)\log(1-cx)\textbf{PolyLog}(2,cx)}{x^3}\,dx$

Optimal. Leaf size=331

$$-ac^2\log(x)+ac^2\log(1-cx)-\frac{ac\log(1-cx)}{x}-\frac{1}{4}ac^2\log^2(1-cx)+\frac{a\log^2(1-cx)}{4x^2}+\frac{b(1-cx)\log^2(1-cx)}{x}-\frac{b^2\log\ldots}{}$$

```
[Out] -a*c^2*ln(x)+a*c^2*ln(-c*x+1)-a*c*ln(-c*x+1)/x-1/4*a*c^2*ln(-c*x+1)^2+1/4*a
*ln(-c*x+1)^2/x^2+b*(-c*x+1)*ln(-c*x+1)^2/x-1/2*b^2*ln(c*x)*ln(-c*x+1)^2/a+
1/2*(a*c+b)^2*ln(c*x)*ln(-c*x+1)^2/a-2*b*c*polylog(2,c*x)-1/2*a*c^2*polylog
(2,c*x)+1/2*a*c*polylog(2,c*x)/x+1/2*(a*c+b)^2*ln(-c*x+1)*polylog(2,c*x)/a-
1/2*(b*x+a)^2*ln(-c*x+1)*polylog(2,c*x)/a/x^2-b^2*ln(-c*x+1)*polylog(2,-c*x
+1)/a+(a*c+b)^2*ln(-c*x+1)*polylog(2,-c*x+1)/a-1/2*c*(a*c+2*b)*polylog(3,c*
x)+b^2*polylog(3,-c*x+1)/a-(a*c+b)^2*polylog(3,-c*x+1)/a
```

Rubi [A]
time = 0.35, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 20, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.952$, Rules used = {6874, 6726, 2442, 46, 36, 29, 31, 37, 6741, 2445, 2457, 2438, 2437, 2338, 2444, 2443, 2481, 2421, 6724, 6731}

$$\frac{b^2\text{Li}_3(1-cx)}{a}-\frac{b^2\text{Li}_2(1-cx)\log(1-cx)}{a}-\frac{b^2\log(cx)\log^2(1-cx)}{2a}-\frac{(a+bx)^2\text{Li}_2(cx)\log(1-cx)}{2ax^2}-\frac{1}{2}c(ac+2b)\text{Li}_3(cx)-\frac{(ac+b)^2\text{Li}_3(1-cx)}{a}+\frac{(ac+b)^2\text{Li}_2(cx)\log(1-cx)}{2a}+\frac{(ac+b)^2\text{Li}_2(1-cx)\log(1-cx)}{a}+\frac{(ac+b)^2\log(cx)\log^2(1-cx)}{2a}-\frac{1}{2}ac^2\text{Li}_2(cx)-\frac{1}{4}ac^2\log^2(1-cx)-ac^2\log(x)+ac^2\log(1-cx)+\frac{ac\text{Li}_2(cx)}{2x}+\frac{a\log^2(1-cx)}{4x^2}-\frac{ac\log(1-cx)}{x}-2bc\text{Li}_2(cx)+\frac{b(1-cx)\log^2(1-cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^3,x]
```

```
[Out] -(a*c^2*Log[x]) + a*c^2*Log[1 - c*x] - (a*c*Log[1 - c*x])/x - (a*c^2*Log[1
- c*x]^2)/4 + (a*Log[1 - c*x]^2)/(4*x^2) + (b*(1 - c*x)*Log[1 - c*x]^2)/x -
 (b^2*Log[c*x]*Log[1 - c*x]^2)/(2*a) + ((b + a*c)^2*Log[c*x]*Log[1 - c*x]^2
)/(2*a) - 2*b*c*PolyLog[2, c*x] - (a*c^2*PolyLog[2, c*x])/2 + (a*c*PolyLog[
2, c*x])/(2*x) + ((b + a*c)^2*Log[1 - c*x]*PolyLog[2, c*x])/(2*a) - ((a + b
*x)^2*Log[1 - c*x]*PolyLog[2, c*x])/(2*a*x^2) - (b^2*Log[1 - c*x]*PolyLog[2
, 1 - c*x])/a + ((b + a*c)^2*Log[1 - c*x]*PolyLog[2, 1 - c*x])/a - (c*(2*b
+ a*c)*PolyLog[3, c*x])/2 + (b^2*PolyLog[3, 1 - c*x])/a - ((b + a*c)^2*Poly
Log[3, 1 - c*x])/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
```

```
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2457

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6741

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (D
ist[b, Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x],
 u/(a + b*x), x], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a
+ b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x]
 && PolyQ[Px, x] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(a+bx)\log(1-cx)\text{Li}_2(cx)}{x^3}\,dx = -\frac{(a+bx)^2\log(1-cx)\text{Li}_2(cx)}{2ax^2} + c\int\left(-\frac{a\text{Li}_2(cx)}{2x^2} + \frac{(-2b-ac)\text{Li}_2(cx)}{2x}\right.$$

$$= -\frac{(a+bx)^2\log(1-cx)\text{Li}_2(cx)}{2ax^2} - \frac{1}{2}a\int\frac{\log^2(1-cx)}{x^3}\,dx - b\int\frac{\log^2(1-}{x^2}$$

$$= \frac{a\log^2(1-cx)}{4x^2} + \frac{b(1-cx)\log^2(1-cx)}{x} - \frac{b^2\log(cx)\log^2(1-cx)}{2a} + \frac{ac\text{L}}{}$$

$$= -\frac{ac\log(1-cx)}{2x} + \frac{a\log^2(1-cx)}{4x^2} + \frac{b(1-cx)\log^2(1-cx)}{x} - \frac{b^2\log(cx)}{}$$

$$= -\frac{ac\log(1-cx)}{2x} + \frac{a\log^2(1-cx)}{4x^2} + \frac{b(1-cx)\log^2(1-cx)}{x} - \frac{b^2\log(cx)}{}$$

$$= -\frac{1}{2}ac^2\log(x) + \frac{1}{2}ac^2\log(1-cx) - \frac{ac\log(1-cx)}{x} + \frac{a\log^2(1-cx)}{4x^2} + \frac{b}{}$$

$$= -\frac{1}{2}ac^2\log(x) + \frac{1}{2}ac^2\log(1-cx) - \frac{ac\log(1-cx)}{x} - \frac{1}{4}ac^2\log^2(1-cx)$$

$$= -ac^2\log(x) + ac^2\log(1-cx) - \frac{ac\log(1-cx)}{x} - \frac{1}{4}ac^2\log^2(1-cx) + \frac{a}{}$$

Mathematica [A]
time = 0.82, size = 285, normalized size = 0.86

$$\frac{1}{4}\left(-4ac^2\log(cx) + 4ac^2\log(1-cx) - \frac{4ac\log(1-cx)}{x} + 8bc\log(cx)\log(1-cx) + 2ac^2\log(cx)\log(1-cx) - 4bc\log^2(1-cx) - ac^2\log^2(1-cx) + \frac{a\log^2(1-cx)}{x^2} + \frac{4b\log^2(1-cx)}{x} + 4bc\log(cx)\log^2(1-cx) + 2ac^2\log(cx)\log^2(1-cx) + \frac{2(acx+(-1+cx)(a+2bx+acx)\log(1-cx))\text{PolyLog}(2,cx)}{x^2} + 2c(4b+ac+2(2b+ac)\log(1-cx))\text{PolyLog}(2,1-cx) - 4bc\text{PolyLog}(3,cx) - 2ac^2\text{PolyLog}(3,cx) - 8bc\text{PolyLog}(3,1-cx) - 4ac^2\text{PolyLog}(3,1-cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^3,x]
```

```
[Out] (-4*a*c^2*Log[c*x] + 4*a*c^2*Log[1 - c*x] - (4*a*c*Log[1 - c*x])/x + 8*b*c*
Log[c*x]*Log[1 - c*x] + 2*a*c^2*Log[c*x]*Log[1 - c*x] - 4*b*c*Log[1 - c*x]^
2 - a*c^2*Log[1 - c*x]^2 + (a*Log[1 - c*x]^2)/x^2 + (4*b*Log[1 - c*x]^2)/x
+ 4*b*c*Log[c*x]*Log[1 - c*x]^2 + 2*a*c^2*Log[c*x]*Log[1 - c*x]^2 + (2*(a*c
*x + (-1 + c*x)*(a + 2*b*x + a*c*x)*Log[1 - c*x])*PolyLog[2, c*x])/x^2 + 2*
c*(4*b + a*c + 2*(2*b + a*c)*Log[1 - c*x])*PolyLog[2, 1 - c*x] - 4*b*c*Poly
Log[3, c*x] - 2*a*c^2*PolyLog[3, c*x] - 8*b*c*PolyLog[3, 1 - c*x] - 4*a*c^2
*PolyLog[3, 1 - c*x])/4
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)\ln(-cx+1)\,\text{polylog}(2,cx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^3,x)
```

```
[Out] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^3,x)
```

Maxima [A]
time = 0.31, size = 213, normalized size = 0.64

$$-ac^2\log(x)+\frac{1}{2}\left(ac^2+2\,bc\right)\left(\log(cx)\log(-cx+1)^2+2\,\mathrm{Li}_2(-cx+1)\log(-cx+1)-2\,\mathrm{Li}_3(-cx+1)\right)+\frac{1}{2}\left(ac^2+4\,bc\right)\left(\log(cx)\log(-cx+1)+\mathrm{Li}_2(-cx+1)\right)-\frac{1}{2}\left(ac^2+2\,bc\right)\mathrm{Li}_3(cx)-\frac{\left((ac^2+4\,bc)x^2-4\,bx-a\right)\log(-cx+1)^2-2\left(acx+\left((ac^2+2\,bc)x^2-2\,bx-a\right)\log(-cx+1)\right)\mathrm{Li}_2(cx)-4\left(ac^2x^2-acx\right)\log(-cx+1)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^3,x, algorithm="maxima")
```

```
[Out] -a*c^2*log(x) + 1/2*(a*c^2 + 2*b*c)*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*
x + 1)*log(-c*x + 1) - 2*polylog(3, -c*x + 1)) + 1/2*(a*c^2 + 4*b*c)*(log(c
*x)*log(-c*x + 1) + dilog(-c*x + 1)) - 1/2*(a*c^2 + 2*b*c)*polylog(3, c*x)
- 1/4*(((a*c^2 + 4*b*c)*x^2 - 4*b*x - a)*log(-c*x + 1)^2 - 2*(a*c*x + ((a*c
^2 + 2*b*c)*x^2 - 2*b*x - a)*log(-c*x + 1))*dilog(c*x) - 4*(a*c^2*x^2 - a*c
*x)*log(-c*x + 1))/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+bx)\log(-cx+1)\,\mathrm{Li}_2(cx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x**3,x)
```

```
[Out] Integral((a + b*x)*log(-c*x + 1)*polylog(2, c*x)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^3, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(1-c\,x\right)\,\mathrm{polylog}(2,c\,x)\,\left(a+b\,x\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(1 - c*x)*polylog(2, c*x)*(a + b*x))/x^3,x)
```

```
[Out] int((log(1 - c*x)*polylog(2, c*x)*(a + b*x))/x^3, x)
```

3.190 $\int \frac{(a+bx)\log(1-cx)\textbf{PolyLog}(2,cx)}{x^4}\,dx$

Optimal. Leaf size=460

$$\frac{7ac^2}{36x}-\frac{1}{2}bc^2\log(x)-\frac{5}{12}ac^3\log(x)-\frac{1}{6}c^2(3b+2ac)\log(x)+\frac{1}{2}bc^2\log(1-cx)+\frac{5}{12}ac^3\log(1-cx)+\frac{1}{6}c^2(3b+2ac)\,\mathrm{log}$$

```
[Out] 1/6*a*c*polylog(2,c*x)/x^2+1/6*c*(2*a*c+3*b)*polylog(2,c*x)/x-7/36*a*c*ln(-
c*x+1)/x^2-1/2*b*c*ln(-c*x+1)/x-2/9*a*c^2*ln(-c*x+1)/x-1/6*c*(2*a*c+3*b)*ln
(-c*x+1)/x+1/6*c^2*(2*a*c+3*b)*ln(c*x)*ln(-c*x+1)^2+1/6*c^2*(2*a*c+3*b)*ln(
-c*x+1)*polylog(2,c*x)+1/3*c^2*(2*a*c+3*b)*ln(-c*x+1)*polylog(2,-c*x+1)+7/3
6*a*c^2/x-1/2*b*c^2*polylog(2,c*x)-2/9*a*c^3*polylog(2,c*x)-1/6*c^2*(2*a*c+
3*b)*polylog(3,c*x)-1/3*c^2*(2*a*c+3*b)*polylog(3,-c*x+1)-1/2*b*c^2*ln(x)-5
/12*a*c^3*ln(x)-1/6*c^2*(2*a*c+3*b)*ln(x)+1/2*b*c^2*ln(-c*x+1)+5/12*a*c^3*l
n(-c*x+1)+1/6*c^2*(2*a*c+3*b)*ln(-c*x+1)-1/4*b*c^2*ln(-c*x+1)^2-1/9*a*c^3*l
n(-c*x+1)^2+1/9*a*ln(-c*x+1)^2/x^3+1/4*b*ln(-c*x+1)^2/x^2-1/6*(2*a/x^3+3*b/
x^2)*ln(-c*x+1)*polylog(2,c*x)
```

Rubi [A]

time = 0.44, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 41, number of rules used = 19, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.905$, Rules used = {6874, 6726, 2442, 46, 45, 6741, 2445, 2457, 36, 29, 31, 2438, 2437, 2338, 6724, 6731, 2443, 2481, 2421}

$$-\frac{1}{6}c^2(2ac+3b)\text{Li}_3(cx)-\frac{1}{3}c^2(2ac+3b)\text{Li}_3(1-cx)+\frac{1}{6}c^2(2ac+3b)\text{Li}_2(cx)\log(1-cx)+\frac{1}{3}c^2(2ac+3b)\text{Li}_2(1-cx)\log(1-cx)+\frac{1}{6}c^2(2ac+3b)\log(cx)\log^2(1-cx)-\frac{1}{6}c^2\log(x)(2ac+3b)+\frac{1}{6}c^2(2ac+3b)\log(1-cx)-\frac{1}{6}\left(\frac{2a}{x^3}+\frac{3b}{x^2}\right)\text{Li}_2(cx)\log(1-cx)+\frac{c(2ac+3b)\text{Li}_2(cx)}{6x}-\frac{c(2ac+3b)\log(1-cx)}{6x}-\frac{2}{9}ac^3\text{Li}_2(cx)-\frac{1}{9}ac^3\log^2(1-cx)-\frac{5}{12}ac^3\log(x)+\frac{5}{12}ac^3\log(1-cx)+\frac{7ac^2}{36x}-\frac{2ac^2\log(1-cx)}{9x}+\frac{ac\text{Li}_2(cx)}{6x^2}+\frac{a\log^2(1-cx)}{9x^3}-\frac{7ac\log(1-cx)}{36x^2}-\frac{1}{2}bc^2\text{Li}_2(cx)-\frac{1}{4}bc^2\log^2(1-cx)-\frac{1}{2}bc^2\log(x)+\frac{1}{2}bc^2\log(1-cx)+\frac{b\log^2(1-cx)}{4x^2}-\frac{bc\log(1-cx)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^4,x]
```

```
[Out] (7*a*c^2)/(36*x) - (b*c^2*Log[x])/2 - (5*a*c^3*Log[x])/12 - (c^2*(3*b + 2*a
*c)*Log[x])/6 + (b*c^2*Log[1 - c*x])/2 + (5*a*c^3*Log[1 - c*x])/12 + (c^2*(
3*b + 2*a*c)*Log[1 - c*x])/6 - (7*a*c*Log[1 - c*x])/(36*x^2) - (b*c*Log[1 -
 c*x])/(2*x) - (2*a*c^2*Log[1 - c*x])/(9*x) - (c*(3*b + 2*a*c)*Log[1 - c*x]
)/(6*x) - (b*c^2*Log[1 - c*x]^2)/4 - (a*c^3*Log[1 - c*x]^2)/9 + (a*Log[1 -
c*x]^2)/(9*x^3) + (b*Log[1 - c*x]^2)/(4*x^2) + (c^2*(3*b + 2*a*c)*Log[c*x]*
Log[1 - c*x]^2)/6 - (b*c^2*PolyLog[2, c*x])/2 - (2*a*c^3*PolyLog[2, c*x])/9
 + (a*c*PolyLog[2, c*x])/(6*x^2) + (c*(3*b + 2*a*c)*PolyLog[2, c*x])/(6*x)
+ (c^2*(3*b + 2*a*c)*Log[1 - c*x]*PolyLog[2, c*x])/6 - (((2*a)/x^3 + (3*b)/
x^2)*Log[1 - c*x]*PolyLog[2, c*x])/6 + (c^2*(3*b + 2*a*c)*Log[1 - c*x]*Poly
Log[2, 1 - c*x])/3 - (c^2*(3*b + 2*a*c)*PolyLog[3, c*x])/6 - (c^2*(3*b + 2*
a*c)*PolyLog[3, 1 - c*x])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2457

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6741

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (D
ist[b, Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x],
 u/(a + b*x), x], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a
+ b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x]
 && PolyQ[Px, x] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(a+bx)\log(1-cx)\text{Li}_2(cx)}{x^4}\,dx = -\frac{1}{6}\left(\frac{2a}{x^3}+\frac{3b}{x^2}\right)\log(1-cx)\text{Li}_2(cx) + c\int\left(-\frac{a\text{Li}_2(cx)}{3x^3}+\frac{(-3b-2ac)\text{I}}{6x^2}\right.$$
$$= -\frac{1}{6}\left(\frac{2a}{x^3}+\frac{3b}{x^2}\right)\log(1-cx)\text{Li}_2(cx) - \frac{1}{3}a\int\frac{\log^2(1-cx)}{x^4}\,dx - \frac{1}{2}b\int\frac{\text{lo}}{}$$
$$= \frac{a\log^2(1-cx)}{9x^3} + \frac{b\log^2(1-cx)}{4x^2} + \frac{ac\text{Li}_2(cx)}{6x^2} + \frac{c(3b+2ac)\text{Li}_2(cx)}{6x} + \frac{1}{6}$$
$$= -\frac{ac\log(1-cx)}{12x^2} - \frac{c(3b+2ac)\log(1-cx)}{6x} + \frac{a\log^2(1-cx)}{9x^3} + \frac{b\log^2(1}{4x}$$
$$= -\frac{ac\log(1-cx)}{12x^2} - \frac{c(3b+2ac)\log(1-cx)}{6x} + \frac{a\log^2(1-cx)}{9x^3} + \frac{b\log^2(1}{4x}$$
$$= \frac{ac^2}{12x} - \frac{1}{12}ac^3\log(x) - \frac{1}{6}c^2(3b+2ac)\log(x) + \frac{1}{12}ac^3\log(1-cx) + \frac{1}{6}c^2$$
$$= \frac{ac^2}{12x} - \frac{1}{12}ac^3\log(x) - \frac{1}{6}c^2(3b+2ac)\log(x) + \frac{1}{12}ac^3\log(1-cx) + \frac{1}{6}c^2$$
$$= \frac{7ac^2}{36x} - \frac{1}{2}bc^2\log(x) - \frac{5}{12}ac^3\log(x) - \frac{1}{6}c^2(3b+2ac)\log(x) + \frac{1}{2}bc^2\log($$

Mathematica [**A**]
time = 1.00, size = 389, normalized size = 0.85

$$\frac{1}{36}\left(-7ac^3+\frac{7ac^2}{x}-36bc^2\log(cx)-27ac^3\log(cx)+36bc^2\log(1-cx)+27ac^3\log(1-cx)-\frac{7ac\log(1-cx)}{x^2}-\frac{36bc\log(1-cx)}{x}-\frac{20ac^2\log(1-cx)}{x}+18bc^2\log(cx)\log(1-cx)+8ac^3\log(cx)\log(1-cx)-9bc^2\log^2(1-cx)-4ac^3\log^2(1-cx)+\frac{4a\log^2(1-cx)}{x^3}+\frac{9b\log^2(1-cx)}{x^2}+18bc^2\log(cx)\log^2(1-cx)+12ac^3\log(cx)\log^2(1-cx)+\frac{6(cx(a+3bx+2acx)+(-2a-3bx+3bc^2x^3+2ac^3x^3)\log(1-cx))\,\text{PolyLog}(2,cx)}{x^3}+2c^2(9b+4ac+6(3b+2ac)\log(1-cx))\text{PolyLog}(2,1-cx)-18bc^2\text{PolyLog}(3,cx)-12ac^3\text{PolyLog}(3,cx)-36bc^2\text{PolyLog}(3,1-cx)-24ac^3\text{PolyLog}(3,1-cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^4,x]
```

```
[Out] (-7*a*c^3 + (7*a*c^2)/x - 36*b*c^2*Log[c*x] - 27*a*c^3*Log[c*x] + 36*b*c^2*
Log[1 - c*x] + 27*a*c^3*Log[1 - c*x] - (7*a*c*Log[1 - c*x])/x^2 - (36*b*c*L
og[1 - c*x])/x - (20*a*c^2*Log[1 - c*x])/x + 18*b*c^2*Log[c*x]*Log[1 - c*x]
 + 8*a*c^3*Log[c*x]*Log[1 - c*x] - 9*b*c^2*Log[1 - c*x]^2 - 4*a*c^3*Log[1 -
 c*x]^2 + (4*a*Log[1 - c*x]^2)/x^3 + (9*b*Log[1 - c*x]^2)/x^2 + 18*b*c^2*Lo
g[c*x]*Log[1 - c*x]^2 + 12*a*c^3*Log[c*x]*Log[1 - c*x]^2 + (6*(c*x*(a + 3*b
*x + 2*a*c*x) + (-2*a - 3*b*x + 3*b*c^2*x^3 + 2*a*c^3*x^3)*Log[1 - c*x])*Po
lyLog[2, c*x])/x^3 + 2*c^2*(9*b + 4*a*c + 6*(3*b + 2*a*c)*Log[1 - c*x])*Pol
yLog[2, 1 - c*x] - 18*b*c^2*PolyLog[3, c*x] - 12*a*c^3*PolyLog[3, c*x] - 36
*b*c^2*PolyLog[3, 1 - c*x] - 24*a*c^3*PolyLog[3, 1 - c*x])/36
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)\ln(-cx+1)\,\text{polylog}(2,cx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^4,x)
```

```
[Out] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^4,x)
```

Maxima [A]
time = 0.32, size = 287, normalized size = 0.62

$$\frac{1}{6}\left(2\,ac^3+3\,bc^2\right)\left(\log\left(cx\right)\log\left(-cx+1\right)^2+2\,\mathrm{Li}_2(-cx+1)\log\left(-cx+1\right)-2\,\mathrm{Li}_3(-cx+1)\right)+\frac{1}{18}\left(4\,ac^3+9\,bc^2\right)\left(\log\left(cx\right)\log\left(-cx+1\right)+\mathrm{Li}_2(-cx+1)\right)-\frac{1}{4}\left(3\,ac^3+4\,bc^2\right)\log\left(x\right)-\frac{1}{6}\left(2\,ac^3+3\,bc^2\right)\mathrm{Li}_3(cx)+\frac{7\,ac^2x^2-\left((4\,ac^3+9\,bc^2)x^3-9\,bx-4\,a\right)\log\left(-cx+1\right)^2+6\left(acx+(2\,ac^2+3\,bc)x^2+\left((2\,ac^3+3\,bc^2)x^3-3\,bx-2\,a\right)\log\left(-cx+1\right)\right)\mathrm{Li}_2(cx)+\left(9\left(3\,ac^3+4\,bc^2\right)x^3-7\,acx-4\left(5\,ac^2+9\,bc\right)x^2\right)\log\left(-cx+1\right)}{36\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a*c^3 + 3*b*c^2)*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-
c*x + 1) - 2*polylog(3, -c*x + 1)) + 1/18*(4*a*c^3 + 9*b*c^2)*(log(c*x)*log
(-c*x + 1) + dilog(-c*x + 1)) - 1/4*(3*a*c^3 + 4*b*c^2)*log(x) - 1/6*(2*a*c
^3 + 3*b*c^2)*polylog(3, c*x) + 1/36*(7*a*c^2*x^2 - ((4*a*c^3 + 9*b*c^2)*x^
3 - 9*b*x - 4*a)*log(-c*x + 1)^2 + 6*(a*c*x + (2*a*c^2 + 3*b*c)*x^2 + ((2*a
*c^3 + 3*b*c^2)*x^3 - 3*b*x - 2*a)*log(-c*x + 1))*dilog(c*x) + (9*(3*a*c^3
+ 4*b*c^2)*x^3 - 7*a*c*x - 4*(5*a*c^2 + 9*b*c)*x^2)*log(-c*x + 1))/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^4,x, algorithm="giac")
[Out] integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(1-c\,x\right)\,\mathrm{polylog}(2,c\,x)\,\left(a+b\,x\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(1 - c*x)*polylog(2, c*x)*(a + b*x))/x^4,x)
[Out] int((log(1 - c*x)*polylog(2, c*x)*(a + b*x))/x^4, x)
```

3.191 $\int \frac{(a+bx)\log(1-cx)\mathrm{PolyLog}(2,cx)}{x^5}\,dx$

Optimal. Leaf size=584

$$\frac{5ac^2}{144x^2}+\frac{bc^2}{9x}+\frac{19ac^3}{144x}+\frac{c^2(4b+3ac)}{48x}-\frac{1}{3}bc^3\log(x)-\frac{37}{144}ac^4\log(x)-\frac{5}{48}c^3(4b+3ac)\log(x)+\frac{1}{3}bc^3\log(1-cx)+\frac{37}{144}$$

```
[Out] 1/12*a*c*polylog(2,c*x)/x^3+1/24*c*(3*a*c+4*b)*polylog(2,c*x)/x^2+1/12*c^2*
(3*a*c+4*b)*polylog(2,c*x)/x-5/72*a*c*ln(-c*x+1)/x^3-1/9*b*c*ln(-c*x+1)/x^2
-1/16*a*c^2*ln(-c*x+1)/x^2-1/48*c*(3*a*c+4*b)*ln(-c*x+1)/x^2-2/9*b*c^2*ln(-
c*x+1)/x-1/8*a*c^3*ln(-c*x+1)/x-1/12*c^2*(3*a*c+4*b)*ln(-c*x+1)/x+1/12*c^3*
(3*a*c+4*b)*ln(c*x)*ln(-c*x+1)^2+1/12*c^3*(3*a*c+4*b)*ln(-c*x+1)*polylog(2,
c*x)+1/6*c^3*(3*a*c+4*b)*ln(-c*x+1)*polylog(2,-c*x+1)-2/9*b*c^3*polylog(2,c
*x)-1/8*a*c^4*polylog(2,c*x)-1/12*c^3*(3*a*c+4*b)*polylog(3,c*x)-1/6*c^3*(3
*a*c+4*b)*polylog(3,-c*x+1)+5/144*a*c^2/x^2+1/9*b*c^2/x+19/144*a*c^3/x+1/48
*c^2*(3*a*c+4*b)/x-1/9*b*c^3*ln(-c*x+1)^2-1/16*a*c^4*ln(-c*x+1)^2+1/16*a*ln
(-c*x+1)^2/x^4+1/9*b*ln(-c*x+1)^2/x^3-1/12*(3*a/x^4+4*b/x^3)*ln(-c*x+1)*pol
ylog(2,c*x)-1/3*b*c^3*ln(x)-37/144*a*c^4*ln(x)-5/48*c^3*(3*a*c+4*b)*ln(x)+1
/3*b*c^3*ln(-c*x+1)+37/144*a*c^4*ln(-c*x+1)+5/48*c^3*(3*a*c+4*b)*ln(-c*x+1)
```

Rubi [A]

time = 0.55, antiderivative size = 584, normalized size of antiderivative = 1.00, number of steps used = 51, number of rules used = 19, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.905$, Rules used = {6874, 6726, 2442, 46, 45, 6741, 2445, 2457, 36, 29, 31, 2438, 2437, 2338, 6724, 6731, 2443, 2481, 2421}

$$-\frac{1}{12}c^3(3ac+4b)\mathrm{Li}_3(cx)-\frac{1}{6}c^3(3ac+4b)\mathrm{Li}_3(1-cx)+\frac{1}{12}c^3(3ac+4b)\mathrm{Li}_2(cx)\log(1-cx)+\frac{1}{6}c^3(3ac+4b)\mathrm{Li}_2(1-cx)\log(1-cx)+\frac{1}{12}c^3(3ac+4b)\log(cx)\log^2(1-cx)-\frac{5}{48}c^3\log(x)(3ac+4b)+\frac{5}{48}c^3(3ac+4b)\log(1-cx)+\frac{c^2(3ac+4b)\mathrm{Li}_2(cx)}{12x}+\frac{c^2(3ac+4b)}{48x}-\frac{c^2(3ac+4b)\log(1-cx)}{12x}+\frac{c(3ac+4b)\mathrm{Li}_2(cx)}{24x^2}-\frac{1}{12}\left(\frac{3a}{x^4}+\frac{4b}{x^3}\right)\mathrm{Li}_2(cx)\log(1-cx)-\frac{c(3ac+4b)\log(1-cx)}{48x^2}-\frac{1}{8}ac^4\mathrm{Li}_2(cx)-\frac{1}{16}ac^4\log^2(1-cx)-\frac{37}{144}ac^4\log(x)+\frac{37}{144}ac^4\log(1-cx)+\frac{19ac^3}{144x}-\frac{ac^3\log(1-cx)}{8x}+\frac{5ac^2}{144x^2}-\frac{ac^2\log(1-cx)}{16x^2}+\frac{ac\mathrm{Li}_2(cx)}{12x^3}+\frac{a\log^2(1-cx)}{16x^4}-\frac{5ac\log(1-cx)}{72x^3}-\frac{2}{9}bc^3\mathrm{Li}_2(cx)-\frac{1}{9}bc^3\log^2(1-cx)-\frac{1}{3}bc^3\log(x)+\frac{1}{3}bc^3\log(1-cx)+\frac{bc^2}{9x}-\frac{2bc^2\log(1-cx)}{9x}+\frac{b\log^2(1-cx)}{9x^3}-\frac{bc\log(1-cx)}{9x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^5,x]
```

```
[Out] (5*a*c^2)/(144*x^2) + (b*c^2)/(9*x) + (19*a*c^3)/(144*x) + (c^2*(4*b + 3*a*
c))/(48*x) - (b*c^3*Log[x])/3 - (37*a*c^4*Log[x])/144 - (5*c^3*(4*b + 3*a*c
)*Log[x])/48 + (b*c^3*Log[1 - c*x])/3 + (37*a*c^4*Log[1 - c*x])/144 + (5*c^
3*(4*b + 3*a*c)*Log[1 - c*x])/48 - (5*a*c*Log[1 - c*x])/(72*x^3) - (b*c*Log
[1 - c*x])/(9*x^2) - (a*c^2*Log[1 - c*x])/(16*x^2) - (c*(4*b + 3*a*c)*Log[1
 - c*x])/(48*x^2) - (2*b*c^2*Log[1 - c*x])/(9*x) - (a*c^3*Log[1 - c*x])/(8*
x) - (c^2*(4*b + 3*a*c)*Log[1 - c*x])/(12*x) - (b*c^3*Log[1 - c*x]^2)/9 - (
a*c^4*Log[1 - c*x]^2)/16 + (a*Log[1 - c*x]^2)/(16*x^4) + (b*Log[1 - c*x]^2)
/(9*x^3) + (c^3*(4*b + 3*a*c)*Log[c*x]*Log[1 - c*x]^2)/12 - (2*b*c^3*PolyLo
g[2, c*x])/9 - (a*c^4*PolyLog[2, c*x])/8 + (a*c*PolyLog[2, c*x])/(12*x^3) +
 (c*(4*b + 3*a*c)*PolyLog[2, c*x])/(24*x^2) + (c^2*(4*b + 3*a*c)*PolyLog[2,
 c*x])/(12*x) + (c^3*(4*b + 3*a*c)*Log[1 - c*x]*PolyLog[2, c*x])/12 - (((3*
a)/x^4 + (4*b)/x^3)*Log[1 - c*x]*PolyLog[2, c*x])/12 + (c^3*(4*b + 3*a*c)*L
```

```
og[1 - c*x]*PolyLog[2, 1 - c*x])/6 - (c^3*(4*b + 3*a*c)*PolyLog[3, c*x])/12
 - (c^3*(4*b + 3*a*c)*PolyLog[3, 1 - c*x])/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2457

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
```

```
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6741

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (D
ist[b, Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x],
 u/(a + b*x), x], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a
+ b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x]
 && PolyQ[Px, x] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(a+bx)\log(1-cx)\text{Li}_2(cx)}{x^5}\,dx = -\frac{1}{12}\left(\frac{3a}{x^4}+\frac{4b}{x^3}\right)\log(1-cx)\text{Li}_2(cx) + c\int\left(-\frac{a\text{Li}_2(cx)}{4x^4}+\frac{(-4b-3ac)\text{L}}{12x^3}\right.$$

$$= -\frac{1}{12}\left(\frac{3a}{x^4}+\frac{4b}{x^3}\right)\log(1-cx)\text{Li}_2(cx) - \frac{1}{4}a\int\frac{\log^2(1-cx)}{x^5}\,dx - \frac{1}{3}b\int \text{log}$$

$$= \frac{a\log^2(1-cx)}{16x^4}+\frac{b\log^2(1-cx)}{9x^3}+\frac{ac\text{Li}_2(cx)}{12x^3}+\frac{c(4b+3ac)\text{Li}_2(cx)}{24x^2}+\frac{c^2(}{}$$

$$= -\frac{ac\log(1-cx)}{36x^3}-\frac{c(4b+3ac)\log(1-cx)}{48x^2}-\frac{c^2(4b+3ac)\log(1-cx)}{12x}+$$

$$= -\frac{ac\log(1-cx)}{36x^3}-\frac{c(4b+3ac)\log(1-cx)}{48x^2}-\frac{c^2(4b+3ac)\log(1-cx)}{12x}+$$

$$= \frac{ac^2}{72x^2}+\frac{ac^3}{36x}+\frac{c^2(4b+3ac)}{48x}-\frac{1}{36}ac^4\log(x)-\frac{5}{48}c^3(4b+3ac)\log(x)+$$

$$= \frac{ac^2}{72x^2}+\frac{ac^3}{36x}+\frac{c^2(4b+3ac)}{48x}-\frac{1}{36}ac^4\log(x)-\frac{5}{48}c^3(4b+3ac)\log(x)+$$

$$= \frac{5ac^2}{144x^2}+\frac{bc^2}{9x}+\frac{19ac^3}{144x}+\frac{c^2(4b+3ac)}{48x}-\frac{1}{3}bc^3\log(x)-\frac{37}{144}ac^4\log(x)-$$

Mathematica [A]
time = 1.06, size = 505, normalized size = 0.86

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^5,x]
```

```
[Out] -1/144*(-5*a*c^2*x^2 - 28*b*c^2*x^3 - 28*a*c^3*x^3 + 28*b*c^3*x^4 + 33*a*c^
4*x^4 + 108*b*c^3*x^4*Log[c*x] + 82*a*c^4*x^4*Log[c*x] + 10*a*c*x*Log[1 - c
*x] + 28*b*c*x^2*Log[1 - c*x] + 18*a*c^2*x^2*Log[1 - c*x] + 80*b*c^2*x^3*Lo
g[1 - c*x] + 54*a*c^3*x^3*Log[1 - c*x] - 108*b*c^3*x^4*Log[1 - c*x] - 82*a*
c^4*x^4*Log[1 - c*x] - 32*b*c^3*x^4*Log[c*x]*Log[1 - c*x] - 18*a*c^4*x^4*Lo
g[c*x]*Log[1 - c*x] - 9*a*Log[1 - c*x]^2 - 16*b*x*Log[1 - c*x]^2 + 16*b*c^3
*x^4*Log[1 - c*x]^2 + 9*a*c^4*x^4*Log[1 - c*x]^2 - 48*b*c^3*x^4*Log[c*x]*Lo
g[1 - c*x]^2 - 36*a*c^4*x^4*Log[c*x]*Log[1 - c*x]^2 - 6*(c*x*(4*b*x*(1 + 2*
c*x) + a*(2 + 3*c*x + 6*c^2*x^2)) + (8*b*x*(-1 + c^3*x^3) + 6*a*(-1 + c^4*x
^4))*Log[1 - c*x])*PolyLog[2, c*x] - 2*c^3*x^4*(16*b + 9*a*c + 12*(4*b + 3*
a*c)*Log[1 - c*x])*PolyLog[2, 1 - c*x] + 48*b*c^3*x^4*PolyLog[3, c*x] + 36*
a*c^4*x^4*PolyLog[3, c*x] + 96*b*c^3*x^4*PolyLog[3, 1 - c*x] + 72*a*c^4*x^4
*PolyLog[3, 1 - c*x])/x^4
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)\ln(-cx+1)\,\mathrm{polylog}(2,cx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^5,x)
```

```
[Out] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^5,x)
```

Maxima [A]
time = 0.31, size = 341, normalized size = 0.58

$$\frac{1}{12}\left(3\,ac^4+4\,bc^3\right)\left(\log(cx)\log(-cx+1)^2+2\,\mathrm{Li}_2(-cx+1)\log(-cx+1)-2\,\mathrm{Li}_3(-cx+1)\right)+\frac{1}{72}\left(9\,ac^4+16\,bc^3\right)\left(\log(cx)\log(-cx+1)+\mathrm{Li}_2(-cx+1)\right)-\frac{1}{72}\left(41\,ac^4+54\,bc^3\right)\log(x)-\frac{1}{12}\left(3\,ac^4+4\,bc^3\right)\mathrm{Li}_3(cx)+\frac{5\,ac^2x^2+28\,(ac^3+bc^2)x^3-((9\,ac^4+16\,bc^3)x^4-16\,bx-9\,a)\log(-cx+1)^2+6\,(2\,(3\,ac^3+4\,bc^2)x^3+2\,acx+(3\,ac^2+4\,bc)x^2+2\,((3\,ac^4+4\,bc^3)x^4-4\,bx-3\,a)\log(-cx+1))\mathrm{Li}_2(cx)+2\,((41\,ac^4+54\,bc^3)x^4-(27\,ac^3+40\,bc^2)x^3-5\,acx-(9\,ac^2+14\,bc)x^2)\log(-cx+1)}{144\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a*c^4 + 4*b*c^3)*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(
-c*x + 1) - 2*polylog(3, -c*x + 1)) + 1/72*(9*a*c^4 + 16*b*c^3)*(log(c*x)*l
og(-c*x + 1) + dilog(-c*x + 1)) - 1/72*(41*a*c^4 + 54*b*c^3)*log(x) - 1/12*
(3*a*c^4 + 4*b*c^3)*polylog(3, c*x) + 1/144*(5*a*c^2*x^2 + 28*(a*c^3 + b*c^
2)*x^3 - ((9*a*c^4 + 16*b*c^3)*x^4 - 16*b*x - 9*a)*log(-c*x + 1)^2 + 6*(2*(
3*a*c^3 + 4*b*c^2)*x^3 + 2*a*c*x + (3*a*c^2 + 4*b*c)*x^2 + 2*((3*a*c^4 + 4*
b*c^3)*x^4 - 4*b*x - 3*a)*log(-c*x + 1))*dilog(c*x) + 2*((41*a*c^4 + 54*b*c
^3)*x^4 - (27*a*c^3 + 40*b*c^2)*x^3 - 5*a*c*x - (9*a*c^2 + 14*b*c)*x^2)*log
(-c*x + 1))/x^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^5,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^5, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x**5,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln(1-c\,x)\ \mathrm{polylog}(2,c\,x)\ (a+b\,x)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(1 - c*x)*polylog(2, c*x)*(a + b*x))/x^5,x)
```

```
[Out] int((log(1 - c*x)*polylog(2, c*x)*(a + b*x))/x^5, x)
```

3.192 $\int x(a+bx+cx^2)\log(1-dx)\textbf{PolyLog}(2,dx)\,dx$

Optimal. Leaf size=900

$$\frac{53cx}{192d^3}+\frac{11bx}{27d^2}+\frac{ax}{d}+\frac{(3c+4bd)x}{108d^3}+\frac{5(3c+4bd+6ad^2)\,x}{48d^3}+\frac{29cx^2}{384d^2}+\frac{5bx^2}{54d}+\frac{(3c+4bd)x^2}{216d^2}+\frac{(3c+4bd+6ad^2)\,x^2}{96d^2}$$

```
[Out] -1/12*(6*a*d^2+4*b*d+3*c)*x*polylog(2,d*x)/d^3-1/24*(6*a*d^2+4*b*d+3*c)*x^2
*polylog(2,d*x)/d^2-1/36*(4*b*d+3*c)*x^3*polylog(2,d*x)/d-1/4*a*(-d*x+1)^2*
ln(-d*x+1)/d^2-1/2*a*(-d*x+1)*ln(-d*x+1)^2/d^2+1/4*a*(-d*x+1)^2*ln(-d*x+1)^
2/d^2-1/12*(6*a*d^2+4*b*d+3*c)*ln(d*x)*ln(-d*x+1)^2/d^4-1/12*(6*a*d^2+4*b*d
+3*c)*ln(-d*x+1)*polylog(2,d*x)/d^4-1/6*(6*a*d^2+4*b*d+3*c)*ln(-d*x+1)*poly
log(2,-d*x+1)/d^4-1/16*c*x^2*ln(-d*x+1)/d^2-1/9*b*x^2*ln(-d*x+1)/d-1/48*(6*
a*d^2+4*b*d+3*c)*x^2*ln(-d*x+1)/d^2-1/24*c*x^3*ln(-d*x+1)/d-1/108*(4*b*d+3*
c)*x^3*ln(-d*x+1)/d+1/8*c*(-d*x+1)*ln(-d*x+1)/d^4+2/9*b*(-d*x+1)*ln(-d*x+1)
/d^3+1/12*(6*a*d^2+4*b*d+3*c)*(-d*x+1)*ln(-d*x+1)/d^4+a*(-d*x+1)*ln(-d*x+1)
/d^2+53/192*c*x/d^3+11/27*b*x/d^2+1/108*(4*b*d+3*c)*x/d^3+5/48*(6*a*d^2+4*b
*d+3*c)*x/d^3+29/384*c*x^2/d^2+5/54*b*x^2/d+1/216*(4*b*d+3*c)*x^2/d^2+1/96*
(6*a*d^2+4*b*d+3*c)*x^2/d^2+17/576*c*x^3/d+1/324*(4*b*d+3*c)*x^3/d+1/8*a*(-
d*x+1)^2/d^2-1/16*c*x^4*polylog(2,d*x)+1/6*(6*a*d^2+4*b*d+3*c)*polylog(3,-d
*x+1)/d^4+3/256*c*x^4+2/81*b*x^3+29/192*c*ln(-d*x+1)/d^4+5/27*b*ln(-d*x+1)/
d^3+1/108*(4*b*d+3*c)*ln(-d*x+1)/d^4+1/48*(6*a*d^2+4*b*d+3*c)*ln(-d*x+1)/d^
4-2/27*b*x^3*ln(-d*x+1)-3/64*c*x^4*ln(-d*x+1)-1/16*c*ln(-d*x+1)^2/d^4-1/9*b
*ln(-d*x+1)^2/d^3+1/9*b*x^3*ln(-d*x+1)^2+1/16*c*x^4*ln(-d*x+1)^2+1/12*(3*c*
x^4+4*b*x^3+6*a*x^2)*ln(-d*x+1)*polylog(2,d*x)+a*x/d
```

Rubi [A]
time = 0.75, antiderivative size = 900, normalized size of antiderivative = 1.00, number of steps used = 60, number of rules used = 22, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {6874, 6726, 2442, 45, 14, 6739, 2448, 2436, 2333, 2332, 2437, 2342, 2341, 2445, 2457, 2338, 6721, 6731, 2443, 2481, 2421, 6724}

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x],x]
```

```
[Out] (53*c*x)/(192*d^3) + (11*b*x)/(27*d^2) + (a*x)/d + ((3*c + 4*b*d)*x)/(108*d
^3) + (5*(3*c + 4*b*d + 6*a*d^2)*x)/(48*d^3) + (29*c*x^2)/(384*d^2) + (5*b*
x^2)/(54*d) + ((3*c + 4*b*d)*x^2)/(216*d^2) + ((3*c + 4*b*d + 6*a*d^2)*x^2)
/(96*d^2) + (2*b*x^3)/81 + (17*c*x^3)/(576*d) + ((3*c + 4*b*d)*x^3)/(324*d)
 + (3*c*x^4)/256 + (a*(1 - d*x)^2)/(8*d^2) + (29*c*Log[1 - d*x])/(192*d^4)
+ (5*b*Log[1 - d*x])/(27*d^3) + ((3*c + 4*b*d)*Log[1 - d*x])/(108*d^4) + ((
3*c + 4*b*d + 6*a*d^2)*Log[1 - d*x])/(48*d^4) - (c*x^2*Log[1 - d*x])/(16*d^
2) - (b*x^2*Log[1 - d*x])/(9*d) - ((3*c + 4*b*d + 6*a*d^2)*x^2*Log[1 - d*x]
```

```
)/(48*d^2) - (2*b*x^3*Log[1 - d*x])/27 - (c*x^3*Log[1 - d*x])/(24*d) - ((3*
c + 4*b*d)*x^3*Log[1 - d*x])/(108*d) - (3*c*x^4*Log[1 - d*x])/64 + (c*(1 -
d*x)*Log[1 - d*x])/(8*d^4) + (2*b*(1 - d*x)*Log[1 - d*x])/(9*d^3) + (a*(1 -
 d*x)*Log[1 - d*x])/d^2 + ((3*c + 4*b*d + 6*a*d^2)*(1 - d*x)*Log[1 - d*x])/
(12*d^4) - (a*(1 - d*x)^2*Log[1 - d*x])/(4*d^2) - (c*Log[1 - d*x]^2)/(16*d^
4) - (b*Log[1 - d*x]^2)/(9*d^3) + (b*x^3*Log[1 - d*x]^2)/9 + (c*x^4*Log[1 -
 d*x]^2)/16 - (a*(1 - d*x)*Log[1 - d*x]^2)/(2*d^2) + (a*(1 - d*x)^2*Log[1 -
 d*x]^2)/(4*d^2) - ((3*c + 4*b*d + 6*a*d^2)*Log[d*x]*Log[1 - d*x]^2)/(12*d^
4) - ((3*c + 4*b*d + 6*a*d^2)*x*PolyLog[2, d*x])/(12*d^3) - ((3*c + 4*b*d +
 6*a*d^2)*x^2*PolyLog[2, d*x])/(24*d^2) - ((3*c + 4*b*d)*x^3*PolyLog[2, d*x
])/(36*d) - (c*x^4*PolyLog[2, d*x])/16 - ((3*c + 4*b*d + 6*a*d^2)*Log[1 - d
*x]*PolyLog[2, d*x])/(12*d^4) + ((6*a*x^2 + 4*b*x^3 + 3*c*x^4)*Log[1 - d*x]
*PolyLog[2, d*x])/12 - ((3*c + 4*b*d + 6*a*d^2)*Log[1 - d*x]*PolyLog[2, 1 -
 d*x])/(6*d^4) + ((3*c + 4*b*d + 6*a*d^2)*PolyLog[3, 1 - d*x])/(6*d^4)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2457

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
```

```
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6739

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*PolyLog[2,
(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[Px, x]}, Simp[u
*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Dist[b, Int[Expa
ndIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x), x
], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d +
 e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px, x
]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+bx+cx^2\right)\log(1-dx)\mathrm{Li}_2(dx)\,dx = \frac{1}{12}\left(6ax^2+4bx^3+3cx^4\right)\log(1-dx)\mathrm{Li}_2(dx)+d\int\left(\frac{(-3c-4bc}{}\right.$$

$$= \frac{1}{12}\left(6ax^2+4bx^3+3cx^4\right)\log(1-dx)\mathrm{Li}_2(dx)+\frac{1}{2}a\int x\log^2(1-$$

$$= \frac{1}{9}bx^3\log^2(1-dx)+\frac{1}{16}cx^4\log^2(1-dx)-\frac{\left(3c+4bd+6ad^2\right)x\mathrm{L}}{12d^3}$$

$$= -\frac{\left(3c+4bd+6ad^2\right)x^2\log(1-dx)}{48d^2}-\frac{(3c+4bd)x^3\log(1-dx)}{108d}$$

$$= \frac{\left(3c+4bd+6ad^2\right)x}{12d^3}-\frac{\left(3c+4bd+6ad^2\right)x^2\log(1-dx)}{48d^2}-\frac{(3c+}{}$$

$$= \frac{cx}{64d^3}+\frac{(3c+4bd)x}{108d^3}+\frac{5\left(3c+4bd+6ad^2\right)x}{48d^3}+\frac{cx^2}{128d^2}+\frac{(3c+4b}{216d}$$

$$= \frac{9cx}{64d^3}+\frac{2bx}{9d^2}+\frac{ax}{d}+\frac{(3c+4bd)x}{108d^3}+\frac{5\left(3c+4bd+6ad^2\right)x}{48d^3}+\frac{cx^2}{128d}$$

$$= \frac{53cx}{192d^3}+\frac{11bx}{27d^2}+\frac{ax}{d}+\frac{(3c+4bd)x}{108d^3}+\frac{5\left(3c+4bd+6ad^2\right)x}{48d^3}+\frac{29}{38}$$

Mathematica [A]
time = 0.85, size = 583, normalized size = 0.65

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x],x]
```

```
[Out] ((355*c*d*x)/4 + 124*b*d^2*x + 198*a*d^3*x + (139*c*d^2*x^2)/8 + 22*b*d^3*x
^2 + 27*a*d^4*x^2 + (67*c*d^3*x^3)/12 + (16*b*d^4*x^3)/3 + (27*c*d^4*x^4)/1
6 + (355*c*Log[1 - d*x])/4 + 124*b*d*Log[1 - d*x] + 198*a*d^2*Log[1 - d*x]
- 54*c*d*x*Log[1 - d*x] - 80*b*d^2*x*Log[1 - d*x] - 144*a*d^3*x*Log[1 - d*x
] - 18*c*d^2*x^2*Log[1 - d*x] - 28*b*d^3*x^2*Log[1 - d*x] - 54*a*d^4*x^2*Lo
g[1 - d*x] - 10*c*d^3*x^3*Log[1 - d*x] - 16*b*d^4*x^3*Log[1 - d*x] - (27*c*
d^4*x^4*Log[1 - d*x])/4 - 9*c*Log[1 - d*x]^2 - 16*b*d*Log[1 - d*x]^2 - 36*a
*d^2*Log[1 - d*x]^2 + 36*a*d^4*x^2*Log[1 - d*x]^2 + 16*b*d^4*x^3*Log[1 - d*
x]^2 + 9*c*d^4*x^4*Log[1 - d*x]^2 - 36*c*Log[d*x]*Log[1 - d*x]^2 - 48*b*d*L
og[d*x]*Log[1 - d*x]^2 - 72*a*d^2*Log[d*x]*Log[1 - d*x]^2 + (-(d*x*(3*c*(12
 + 6*d*x + 4*d^2*x^2 + 3*d^3*x^3) + 4*d*(9*a*d*(2 + d*x) + 2*b*(6 + 3*d*x +
 2*d^2*x^2))) + 12*(-4*b*d - 6*a*d^2 + 6*a*d^4*x^2 + 4*b*d^4*x^3 + 3*c*(-1
```

```
 + d^4*x^4))*Log[1 - d*x])*PolyLog[2, d*x] - 24*(3*c + 4*b*d + 6*a*d^2)*Log
[1 - d*x]*PolyLog[2, 1 - d*x] + 24*(3*c + 4*b*d + 6*a*d^2)*PolyLog[3, 1 - d
*x])/(144*d^4)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x(c\,x^2+bx+a)\ln(-dx+1)\,\mathrm{polylog}(2,dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)
```

```
[Out] int(x*(c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)
```

Maxima [A]
time = 0.27, size = 518, normalized size = 0.58

$$-\frac{1}{6912}\,d\left(\frac{576\,(6\,ad^2+4\,bd+3\,c)(\log(dx)\log(-dx+1)^2+2\,\mathrm{Li}_2(-dx+1)\log(-dx+1)-2\,\mathrm{Li}_3(-dx+1))}{d^5}-\frac{81\,cd^4x^4+4\,(64\,bd^4+67\,cd^3)x^3+6\,(216\,ad^4+176\,bd^3+139\,cd^2)x^2+12\,(792\,ad^3+496\,bd^2+355\,cd)x-48\,(9\,cd^4x^4+4\,(4\,bd^4+3\,cd^3)x^3+6\,(6\,ad^4+4\,bd^3+3\,cd^2)x^2+12\,(6\,ad^3+4\,bd^2+3\,cd)x+12\,(6\,ad^2+4\,bd+3\,c)\log(-dx+1))\mathrm{Li}_2(dx)-4\,(54\,cd^4x^4+4\,(32\,bd^4+21\,cd^3)x^3-2376\,ad^2+6\,(72\,ad^4+40\,bd^3+27\,cd^2)x^2-1488\,bd+12\,(108\,ad^3+64\,bd^2+45\,cd)x-1065\,c)\log(-dx+1)}{d^5}\right)+\frac{1}{1728}\left(\frac{216\,(4\,d^2x^2\mathrm{Li}_2(dx)-d^2x^2-2\,dx+2\,(d^2x^2-1)\log(-dx+1))a}{d^2}+\frac{32\,(18\,d^3x^3\mathrm{Li}_2(dx)-2\,d^3x^3-3\,d^2x^2-6\,dx+6\,(d^3x^3-1)\log(-dx+1))b}{d^3}+\frac{9\,(48\,d^4x^4\mathrm{Li}_2(dx)-3\,d^4x^4-4\,d^3x^3-6\,d^2x^2-12\,dx+12\,(d^4x^4-1)\log(-dx+1))c}{d^4}\right)\log(-dx+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="maxima")
```

```
[Out] -1/6912*d*(576*(6*a*d^2 + 4*b*d + 3*c)*(log(d*x)*log(-d*x + 1)^2 + 2*dilog(
-d*x + 1)*log(-d*x + 1) - 2*polylog(3, -d*x + 1))/d^5 - (81*c*d^4*x^4 + 4*(
64*b*d^4 + 67*c*d^3)*x^3 + 6*(216*a*d^4 + 176*b*d^3 + 139*c*d^2)*x^2 + 12*(
792*a*d^3 + 496*b*d^2 + 355*c*d)*x - 48*(9*c*d^4*x^4 + 4*(4*b*d^4 + 3*c*d^3
)*x^3 + 6*(6*a*d^4 + 4*b*d^3 + 3*c*d^2)*x^2 + 12*(6*a*d^3 + 4*b*d^2 + 3*c*d
)*x + 12*(6*a*d^2 + 4*b*d + 3*c)*log(-d*x + 1))*dilog(d*x) - 4*(54*c*d^4*x^
4 + 4*(32*b*d^4 + 21*c*d^3)*x^3 - 2376*a*d^2 + 6*(72*a*d^4 + 40*b*d^3 + 27*
c*d^2)*x^2 - 1488*b*d + 12*(108*a*d^3 + 64*b*d^2 + 45*c*d)*x - 1065*c)*log(
-d*x + 1))/d^5) + 1/1728*(216*(4*d^2*x^2*dilog(d*x) - d^2*x^2 - 2*d*x + 2*(
d^2*x^2 - 1)*log(-d*x + 1))*a/d^2 + 32*(18*d^3*x^3*dilog(d*x) - 2*d^3*x^3 -
 3*d^2*x^2 - 6*d*x + 6*(d^3*x^3 - 1)*log(-d*x + 1))*b/d^3 + 9*(48*d^4*x^4*d
ilog(d*x) - 3*d^4*x^4 - 4*d^3*x^3 - 6*d^2*x^2 - 12*d*x + 12*(d^4*x^4 - 1)*l
og(-d*x + 1))*c/d^4)*log(-d*x + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="fricas")
```

```
[Out] integral((c*x^3 + b*x^2 + a*x)*dilog(d*x)*log(-d*x + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)*x*dilog(d*x)*log(-d*x + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x \ln(1 - d\,x)\ \mathrm{polylog}(2, d\,x)\ \left(c\,x^2 + b\,x + a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2),x)
[Out] int(x*log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2), x)
```

3.193 $\int (a+bx+cx^2)\log(1-dx)\textbf{PolyLog}(2,dx)\,dx$

Optimal. Leaf size=645

$$2ax+\frac{4cx}{9d^2}+\frac{bx}{d}+\frac{(2c+3bd)x}{24d^2}+\frac{(2c+3d(b+2ad))x}{6d^2}+\frac{cx^2}{9d}+\frac{(2c+3bd)x^2}{48d}+\frac{cx^3}{27}+\frac{b(1-dx)^2}{8d^2}+\frac{2c\log(1-dx)}{9d^3}+$$

```
[Out] -1/6*(2*c+3*d*(2*a*d+b))*x*polylog(2,d*x)/d^2-1/12*(3*b*d+2*c)*x^2*polylog(
2,d*x)/d+1/9*x^2*c/d+2*a*x-1/9*c*x^2*ln(-d*x+1)/d-1/24*(3*b*d+2*c)*x^2*ln(-
d*x+1)/d+2/9*c*(-d*x+1)*ln(-d*x+1)/d^3+2*a*(-d*x+1)*ln(-d*x+1)/d+1/6*(2*c+3
*d*(2*a*d+b))*(-d*x+1)*ln(-d*x+1)/d^3-1/4*b*(-d*x+1)^2*ln(-d*x+1)/d^2-1/2*b
*(-d*x+1)*ln(-d*x+1)^2/d^2+1/4*b*(-d*x+1)^2*ln(-d*x+1)^2/d^2-1/6*(2*c+3*d*(
2*a*d+b))*ln(d*x)*ln(-d*x+1)^2/d^3-1/6*(2*c+3*d*(2*a*d+b))*ln(-d*x+1)*polyl
og(2,d*x)/d^3-1/3*(2*c+3*d*(2*a*d+b))*ln(-d*x+1)*polylog(2,-d*x+1)/d^3+b*(-
d*x+1)*ln(-d*x+1)/d^2-a*(-d*x+1)*ln(-d*x+1)^2/d+4/9*c*x/d^2+1/24*(3*b*d+2*c
)*x/d^2+1/6*(2*c+3*d*(2*a*d+b))*x/d^2+1/48*(3*b*d+2*c)*x^2/d+1/8*b*(-d*x+1)
^2/d^2-1/9*c*x^3*polylog(2,d*x)+1/3*(2*c+3*d*(2*a*d+b))*polylog(3,-d*x+1)/d
^3+2/9*c*ln(-d*x+1)/d^3+1/24*(3*b*d+2*c)*ln(-d*x+1)/d^3-1/9*c*x^3*ln(-d*x+1
)-1/9*c*ln(-d*x+1)^2/d^3+1/9*c*x^3*ln(-d*x+1)^2+1/6*(2*c*x^3+3*b*x^2+6*a*x)
*ln(-d*x+1)*polylog(2,d*x)+1/27*c*x^3+b*x/d
```

Rubi [A]

time = 0.54, antiderivative size = 645, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 21, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {6874, 6721, 2436, 2332, 6726, 2442, 45, 6739, 2333, 2448, 2437, 2342, 2341, 2445, 2457, 2338, 6731, 2443, 2481, 2421, 6724}

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x],x]
```

```
[Out] 2*a*x + (4*c*x)/(9*d^2) + (b*x)/d + ((2*c + 3*b*d)*x)/(24*d^2) + ((2*c + 3*
d*(b + 2*a*d))*x)/(6*d^2) + (c*x^2)/(9*d) + ((2*c + 3*b*d)*x^2)/(48*d) + (c
*x^3)/27 + (b*(1 - d*x)^2)/(8*d^2) + (2*c*Log[1 - d*x])/(9*d^3) + ((2*c + 3
*b*d)*Log[1 - d*x])/(24*d^3) - (c*x^2*Log[1 - d*x])/(9*d) - ((2*c + 3*b*d)*
x^2*Log[1 - d*x])/(24*d) - (c*x^3*Log[1 - d*x])/9 + (2*c*(1 - d*x)*Log[1 -
d*x])/(9*d^3) + (b*(1 - d*x)*Log[1 - d*x])/d^2 + (2*a*(1 - d*x)*Log[1 - d*x
])/d + ((2*c + 3*d*(b + 2*a*d))*(1 - d*x)*Log[1 - d*x])/(6*d^3) - (b*(1 - d
*x)^2*Log[1 - d*x])/(4*d^2) - (c*Log[1 - d*x]^2)/(9*d^3) + (c*x^3*Log[1 - d
*x]^2)/9 - (b*(1 - d*x)*Log[1 - d*x]^2)/(2*d^2) - (a*(1 - d*x)*Log[1 - d*x]
^2)/d + (b*(1 - d*x)^2*Log[1 - d*x]^2)/(4*d^2) - ((2*c + 3*d*(b + 2*a*d))*L
og[d*x]*Log[1 - d*x]^2)/(6*d^3) - ((2*c + 3*d*(b + 2*a*d))*x*PolyLog[2, d*x
])/(6*d^2) - ((2*c + 3*b*d)*x^2*PolyLog[2, d*x])/(12*d) - (c*x^3*PolyLog[2,
 d*x])/9 - ((2*c + 3*d*(b + 2*a*d))*Log[1 - d*x]*PolyLog[2, d*x])/(6*d^3) +
```

```
 ((6*a*x + 3*b*x^2 + 2*c*x^3)*Log[1 - d*x]*PolyLog[2, d*x])/6 - ((2*c + 3*d
*(b + 2*a*d))*Log[1 - d*x]*PolyLog[2, 1 - d*x])/(3*d^3) + ((2*c + 3*d*(b +
2*a*d))*PolyLog[3, 1 - d*x])/(3*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2457

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6739

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*PolyLog[2,
(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[Px, x]}, Simp[u
*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Dist[b, Int[Expa
ndIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x), x
], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d +
 e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px, x
```

```
]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (a+bx+cx^2)\log(1-dx)\mathrm{Li}_2(dx)\,dx = \frac{1}{6}(6ax+3bx^2+2cx^3)\log(1-dx)\mathrm{Li}_2(dx)+d\int\left(\frac{(-2c-3d(b\cdots}{6\cdots}\right.$$

$$=\frac{1}{6}(6ax+3bx^2+2cx^3)\log(1-dx)\mathrm{Li}_2(dx)+a\int\log^2(1-dx)\,dx\cdots$$

$$=\frac{1}{9}cx^3\log^2(1-dx)-\frac{(2c+3d(b+2ad))x\mathrm{Li}_2(dx)}{6d^2}-\frac{(2c+3bd)x^2\cdots}{12d\cdots}$$

$$=-\frac{(2c+3bd)x^2\log(1-dx)}{24d}-\frac{1}{27}cx^3\log(1-dx)+\frac{1}{9}cx^3\log^2(1-\cdots$$

$$=2ax+\frac{(2c+3d(b+2ad))x}{6d^2}-\frac{(2c+3bd)x^2\log(1-dx)}{24d}-\frac{1}{27}cx^3\cdots$$

$$=2ax+\frac{cx}{27d^2}+\frac{(2c+3bd)x}{24d^2}+\frac{(2c+3d(b+2ad))x}{6d^2}+\frac{cx^2}{54d}+\frac{(2c\cdots}{\cdots}$$

$$=2ax+\frac{7cx}{27d^2}+\frac{bx}{d}+\frac{(2c+3bd)x}{24d^2}+\frac{(2c+3d(b+2ad))x}{6d^2}+\frac{cx^2}{54d}\cdots$$

$$=2ax+\frac{4cx}{9d^2}+\frac{bx}{d}+\frac{(2c+3bd)x}{24d^2}+\frac{(2c+3d(b+2ad))x}{6d^2}+\frac{cx^2}{9d}+\cdots$$

Mathematica [A]
time = 0.68, size = 472, normalized size = 0.73

$$\frac{31cdx+\frac{99}{2}bd^2x+108ad^3x+\frac{11}{2}cd^2x^2+\frac{27}{4}bd^3x^2+\frac{4}{3}cd^3x^3+31c\log(1-dx)+\frac{99}{2}bd\log(1-dx)+108ad^2\log(1-dx)-20cdx\log(1-dx)-36bd^2x\log(1-dx)-108ad^3x\log(1-dx)-7cd^2x^2\log(1-dx)-\frac{27}{2}bd^3x^2\log(1-dx)-4cd^3x^3\log(1-dx)-4c\log^2(1-dx)-9bd\log^2(1-dx)-36ad^2\log^2(1-dx)+36ad^3x\log^2(1-dx)+9bd^3x^2\log^2(1-dx)+4cd^3x^3\log^2(1-dx)-12c\log(dx)\log^2(1-dx)-18bd\log(dx)\log^2(1-dx)-36ad^2\log(dx)\log^2(1-dx)+\left(-dx(9d(2b+4ad+bdx)+2c(6+3dx+2d^2x^2))+6(-1+dx)(3d(b+2ad+bdx)+2c(1+dx+d^2x^2))\log(1-dx)\right)\mathrm{PolyLog}(2,dx)-12(2c+3d(b+2ad))\log(1-dx)\mathrm{PolyLog}(2,1-dx)+12(2c+3d(b+2ad))\mathrm{PolyLog}(3,1-dx)}{36d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x],x]
```

```
[Out] (31*c*d*x + (99*b*d^2*x)/2 + 108*a*d^3*x + (11*c*d^2*x^2)/2 + (27*b*d^3*x^2
)/4 + (4*c*d^3*x^3)/3 + 31*c*Log[1 - d*x] + (99*b*d*Log[1 - d*x])/2 + 108*a
*d^2*Log[1 - d*x] - 20*c*d*x*Log[1 - d*x] - 36*b*d^2*x*Log[1 - d*x] - 108*a
*d^3*x*Log[1 - d*x] - 7*c*d^2*x^2*Log[1 - d*x] - (27*b*d^3*x^2*Log[1 - d*x]
```

```
)/2 - 4*c*d^3*x^3*Log[1 - d*x] - 4*c*Log[1 - d*x]^2 - 9*b*d*Log[1 - d*x]^2
- 36*a*d^2*Log[1 - d*x]^2 + 36*a*d^3*x*Log[1 - d*x]^2 + 9*b*d^3*x^2*Log[1 -
 d*x]^2 + 4*c*d^3*x^3*Log[1 - d*x]^2 - 12*c*Log[d*x]*Log[1 - d*x]^2 - 18*b*
d*Log[d*x]*Log[1 - d*x]^2 - 36*a*d^2*Log[d*x]*Log[1 - d*x]^2 + (-(d*x*(9*d*
(2*b + 4*a*d + b*d*x) + 2*c*(6 + 3*d*x + 2*d^2*x^2))) + 6*(-1 + d*x)*(3*d*(
b + 2*a*d + b*d*x) + 2*c*(1 + d*x + d^2*x^2))*Log[1 - d*x])*PolyLog[2, d*x]
 - 12*(2*c + 3*d*(b + 2*a*d))*Log[1 - d*x]*PolyLog[2, 1 - d*x] + 12*(2*c +
3*d*(b + 2*a*d))*PolyLog[3, 1 - d*x])/(36*d^3)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(c\,x^{2}+b x +a \right) \ln \left(-d x +1\right) \operatorname{polylog}\left(2, d x \right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)
```

```
[Out] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)
```

Maxima [A]
time = 0.27, size = 412, normalized size = 0.64

$$-\frac{1}{432}\,d\left(\frac{72\,(6\,ad^2+3\,bd+2\,c)(\log(dx)\log(-dx+1)^2+2\,\mathrm{Li}_2(-dx+1)\log(-dx+1)-2\,\mathrm{Li}_3(-dx+1))}{d^4}-\frac{16\,cd^3x^3+3\,(27\,bd^3+22\,cd^2)x^2+6\,(216\,ad^3+99\,bd^2+62\,cd)x-12\,(4\,cd^3x^3+3\,(3\,bd^3+2\,cd^2)x^2+6\,(6\,ad^3+3\,bd^2+2\,cd)x+6\,(6\,ad^2+3\,bd+2\,c)\log(-dx+1))\mathrm{Li}_2(dx)-2\,(16\,cd^3x^3-648\,ad^2+6\,(9\,bd^3+5\,cd^2)x^2-297\,bd+6\,(72\,ad^3+27\,bd^2+16\,cd)x-186\,c)\log(-dx+1)}{d^4}\right)+\frac{1}{216}\left(\frac{216\,(dx\mathrm{Li}_2(dx)-dx+(dx-1)\log(-dx+1))a}{d}+\frac{27\,(4\,d^2x^2\mathrm{Li}_2(dx)-d^2x^2-2\,dx+2\,(d^2x^2-1)\log(-dx+1))b}{d^2}+\frac{4\,(18\,d^3x^3\mathrm{Li}_2(dx)-2\,d^3x^3-3\,d^2x^2-6\,dx+6\,(d^3x^3-1)\log(-dx+1))c}{d^3}\right)\log(-dx+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="maxima")
```

```
[Out] -1/432*d*(72*(6*a*d^2 + 3*b*d + 2*c)*(log(d*x)*log(-d*x + 1)^2 + 2*dilog(-d
*x + 1)*log(-d*x + 1) - 2*polylog(3, -d*x + 1))/d^4 - (16*c*d^3*x^3 + 3*(27
*b*d^3 + 22*c*d^2)*x^2 + 6*(216*a*d^3 + 99*b*d^2 + 62*c*d)*x - 12*(4*c*d^3*
x^3 + 3*(3*b*d^3 + 2*c*d^2)*x^2 + 6*(6*a*d^3 + 3*b*d^2 + 2*c*d)*x + 6*(6*a*
d^2 + 3*b*d + 2*c)*log(-d*x + 1))*dilog(d*x) - 2*(16*c*d^3*x^3 - 648*a*d^2
+ 6*(9*b*d^3 + 5*c*d^2)*x^2 - 297*b*d + 6*(72*a*d^3 + 27*b*d^2 + 16*c*d)*x
- 186*c)*log(-d*x + 1))/d^4) + 1/216*(216*(d*x*dilog(d*x) - d*x + (d*x - 1)
*log(-d*x + 1))*a/d + 27*(4*d^2*x^2*dilog(d*x) - d^2*x^2 - 2*d*x + 2*(d^2*x
^2 - 1)*log(-d*x + 1))*b/d^2 + 4*(18*d^3*x^3*dilog(d*x) - 2*d^3*x^3 - 3*d^2
*x^2 - 6*d*x + 6*(d^3*x^3 - 1)*log(-d*x + 1))*c/d^3)*log(-d*x + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx + cx^2\right) \log\left(-dx + 1\right) \operatorname{Li}_2\left(dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)
[Out] Integral((a + b*x + c*x**2)*log(-d*x + 1)*polylog(2, d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(1 - d\,x\right)\,\mathrm{polylog}(2, d\,x)\,\left(c\,x^2 + b\,x + a\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2),x)
[Out] int(log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2), x)
```

3.194 $\int \frac{(a+bx+cx^2)\log(1-dx)\text{PolyLog}(2,dx)}{x}\,dx$

Optimal. Leaf size=402

$$2bx+\frac{9cx}{8d}+\frac{(c+2bd)x}{2d}+\frac{cx^2}{16}+\frac{c(1-dx)^2}{8d^2}+\frac{c\log(1-dx)}{8d^2}-\frac{1}{8}cx^2\log(1-dx)+\frac{c(1-dx)\log(1-dx)}{d^2}+\frac{2b(1-dx}{}$$

```
[Out] 2*b*x+9/8*c*x/d+1/2*(2*b*d+c)*x/d+1/16*c*x^2+1/8*c*(-d*x+1)^2/d^2+1/8*c*ln(
-d*x+1)/d^2-1/8*c*x^2*ln(-d*x+1)+c*(-d*x+1)*ln(-d*x+1)/d^2+2*b*(-d*x+1)*ln(
-d*x+1)/d+1/2*(2*b*d+c)*(-d*x+1)*ln(-d*x+1)/d^2-1/4*c*(-d*x+1)^2*ln(-d*x+1)
/d^2-1/2*c*(-d*x+1)*ln(-d*x+1)^2/d^2-b*(-d*x+1)*ln(-d*x+1)^2/d+1/4*c*(-d*x+
1)^2*ln(-d*x+1)^2/d^2-1/2*(2*b*d+c)*ln(d*x)*ln(-d*x+1)^2/d^2-1/2*(2*b*d+c)*
x*polylog(2,d*x)/d-1/4*c*x^2*polylog(2,d*x)-1/2*(2*b*d+c)*ln(-d*x+1)*polylo
g(2,d*x)/d^2+1/2*(c*x^2+2*b*x)*ln(-d*x+1)*polylog(2,d*x)-1/2*a*polylog(2,d*
x)^2-(2*b*d+c)*ln(-d*x+1)*polylog(2,-d*x+1)/d^2+(2*b*d+c)*polylog(3,-d*x+1)
/d^2
```

Rubi [A]
time = 0.39, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 24, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {6874, 6721, 2436, 2332, 6724, 6726, 2442, 45, 6740, 6736, 1598, 6733, 2463, 2438, 6739, 2333, 2448, 2437, 2342, 2341, 6731, 2443, 2481, 2421}

$$-\frac{1}{2}a\text{Li}_2(dx)^2+\frac{(2bd+c)\text{Li}_3(1-dx)}{d^2}-\frac{(2bd+c)\text{Li}_2(dx)\log(1-dx)}{2d^2}-\frac{(2bd+c)\text{Li}_2(1-dx)\log(1-dx)}{d^2}-\frac{(2bd+c)\log(dx)\log^2(1-dx)}{2d^2}+\frac{(1-dx)(2bd+c)\log(1-dx)}{2d^2}+\frac{1}{2}\left(2bx+cx^2\right)\text{Li}_2(dx)\log(1-dx)-\frac{x(2bd+c)\text{Li}_2(dx)}{2d}+\frac{x(2bd+c)}{2d}-\frac{b(1-dx)\log^2(1-dx)}{d}+\frac{2b(1-dx)\log(1-dx)}{d}+2bx+\frac{c(1-dx)^2}{8d^2}+\frac{c(1-dx)^2\log^2(1-dx)}{4d^2}-\frac{c(1-dx)\log^2(1-dx)}{2d^2}-\frac{c(1-dx)^2\log(1-dx)}{4d^2}+\frac{c(1-dx)\log(1-dx)}{d^2}+\frac{c\log(1-dx)}{8d^2}-\frac{1}{4}cx^2\text{Li}_2(dx)-\frac{1}{8}cx^2\log(1-dx)+\frac{9cx}{8d}+\frac{cx^2}{16}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x,x]
```

```
[Out] 2*b*x + (9*c*x)/(8*d) + ((c + 2*b*d)*x)/(2*d) + (c*x^2)/16 + (c*(1 - d*x)^2
)/(8*d^2) + (c*Log[1 - d*x])/(8*d^2) - (c*x^2*Log[1 - d*x])/8 + (c*(1 - d*x
)*Log[1 - d*x])/d^2 + (2*b*(1 - d*x)*Log[1 - d*x])/d + ((c + 2*b*d)*(1 - d*
x)*Log[1 - d*x])/(2*d^2) - (c*(1 - d*x)^2*Log[1 - d*x])/(4*d^2) - (c*(1 - d
*x)*Log[1 - d*x]^2)/(2*d^2) - (b*(1 - d*x)*Log[1 - d*x]^2)/d + (c*(1 - d*x)
^2*Log[1 - d*x]^2)/(4*d^2) - ((c + 2*b*d)*Log[d*x]*Log[1 - d*x]^2)/(2*d^2)
- ((c + 2*b*d)*x*PolyLog[2, d*x])/(2*d) - (c*x^2*PolyLog[2, d*x])/4 - ((c +
 2*b*d)*Log[1 - d*x]*PolyLog[2, d*x])/(2*d^2) + ((2*b*x + c*x^2)*Log[1 - d*
x]*PolyLog[2, d*x])/2 - (a*PolyLog[2, d*x]^2)/2 - ((c + 2*b*d)*Log[1 - d*x]
*PolyLog[2, 1 - d*x])/d^2 + ((c + 2*b*d)*PolyLog[3, 1 - d*x])/d^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6733

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[(d + e*x)^(m + 1)*(PolyLog[2, c*(a + b*x)]/(e*(m + 1))), x] +
 Dist[b/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(Log[1 - a*c - b*c*x]/(a + b*x))
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 6736

```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> Simp[-P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

Rule 6739

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*PolyLog[2,
(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[Px, x]}, Simp[u
*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Dist[b, Int[Expa
ndIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x), x
], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d +
 e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px, x
```

```
]
```

Rule 6740

```
Int[((g_.) + Log[1 + (e_.)*(x_)]*(h_.))*(Px_)*(x_)^(m_)*PolyLog[2, (c_.)*(x
_)], x_Symbol] :> Dist[Coeff[Px, x, -m - 1], Int[(g + h*Log[1 + e*x])*(Poly
Log[2, c*x]/x), x], x] + Int[x^m*(Px - Coeff[Px, x, -m - 1]*x^(-m - 1))*(g
+ h*Log[1 + e*x])*PolyLog[2, c*x], x] /; FreeQ[{c, e, g, h}, x] && PolyQ[Px
, x] && ILtQ[m, 0] && EqQ[c + e, 0] && NeQ[Coeff[Px, x, -m - 1], 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx+cx^2)\log(1-dx)\text{Li}_2(dx)}{x}\,dx &= a\int \frac{\log(1-dx)\text{Li}_2(dx)}{x}\,dx + \int \frac{(bx+cx^2)\log(1-dx)\text{Li}_2(dx)}{x}\,dx \\
&= -\frac{1}{2}a\text{Li}_2(dx)^2 + \int (b+cx)\log(1-dx)\text{Li}_2(dx)\,dx \\
&= \frac{1}{2}(2bx+cx^2)\log(1-dx)\text{Li}_2(dx) - \frac{1}{2}a\text{Li}_2(dx)^2 + d\int \left(\frac{(-c-2b}{2} \right. \\
&= \frac{1}{2}(2bx+cx^2)\log(1-dx)\text{Li}_2(dx) - \frac{1}{2}a\text{Li}_2(dx)^2 + b\int \log^2(1-dx \\
&= -\frac{(c+2bd)x\text{Li}_2(dx)}{2d} - \frac{1}{4}cx^2\text{Li}_2(dx) - \frac{(c+2bd)\log(1-dx)\text{Li}_2(d}{2d^2} \\
&= -\frac{1}{8}cx^2\log(1-dx) - \frac{b(1-dx)\log^2(1-dx)}{d} - \frac{(c+2bd)\log(dx)}{2d^2} \\
&= 2bx + \frac{(c+2bd)x}{2d} - \frac{1}{8}cx^2\log(1-dx) + \frac{2b(1-dx)\log(1-dx)}{d} + \\
&= 2bx + \frac{cx}{8d} + \frac{(c+2bd)x}{2d} + \frac{cx^2}{16} + \frac{c\log(1-dx)}{8d^2} - \frac{1}{8}cx^2\log(1-dx \\
&= 2bx + \frac{9cx}{8d} + \frac{(c+2bd)x}{2d} + \frac{cx^2}{16} + \frac{c(1-dx)^2}{8d^2} + \frac{c\log(1-dx)}{8d^2} - \frac{1}{8}
\end{aligned}
$$

Mathematica [A]
time = 0.24, size = 298, normalized size = 0.74

$$
\frac{-14c-32bd+22cdx+48bd^2x+3cd^2x^2+22c\log(1-dx)+48bd\log(1-dx)-16cdx\log(1-dx)-48bd^2x\log(1-dx)-6cd^2x^2\log(1-dx)-4c\log^2(1-dx)-16bd\log^2(1-dx)+16bd^2x\log^2(1-dx)+4cd^2x^2\log^2(1-dx)-8c\log(dx)\log^2(1-dx)-16bd\log(dx)\log^2(1-dx)+4(-dx(2c+4bd+cdx)+2(-1+dx)(c+2bd+cdx)\log(1-dx))\text{PolyLog}(2,dx)-8ad^2\text{PolyLog}(2,dx)^2-16(c+2bd)\log(1-dx)\text{PolyLog}(2,1-dx)+16c\text{PolyLog}(3,1-dx)+32bd\text{PolyLog}(3,1-dx)}{16d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x,x]
```

```
[Out] (-14*c - 32*b*d + 22*c*d*x + 48*b*d^2*x + 3*c*d^2*x^2 + 22*c*Log[1 - d*x] +
 48*b*d*Log[1 - d*x] - 16*c*d*x*Log[1 - d*x] - 48*b*d^2*x*Log[1 - d*x] - 6*
c*d^2*x^2*Log[1 - d*x] - 4*c*Log[1 - d*x]^2 - 16*b*d*Log[1 - d*x]^2 + 16*b*
d^2*x*Log[1 - d*x]^2 + 4*c*d^2*x^2*Log[1 - d*x]^2 - 8*c*Log[d*x]*Log[1 - d*
x]^2 - 16*b*d*Log[d*x]*Log[1 - d*x]^2 + 4*(-(d*x*(2*c + 4*b*d + c*d*x)) + 2
*(-1 + d*x)*(c + 2*b*d + c*d*x)*Log[1 - d*x])*PolyLog[2, d*x] - 8*a*d^2*Pol
yLog[2, d*x]^2 - 16*(c + 2*b*d)*Log[1 - d*x]*PolyLog[2, 1 - d*x] + 16*c*Pol
yLog[3, 1 - d*x] + 32*b*d*PolyLog[3, 1 - d*x])/(16*d^2)
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^{2}+bx+a)\ln(-dx+1)\,\mathrm{polylog}(2,dx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x,x)
```

```
[Out] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx+cx^{2})\log(-dx+1)\operatorname{Li}_{2}(dx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x,x)
```

```
[Out] Integral((a + b*x + c*x**2)*log(-d*x + 1)*polylog(2, d*x)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln(1-d\,x)\,\mathrm{polylog}(2,d\,x)\,\left(c\,x^2+b\,x+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2))/x,x)
```

```
[Out] int((log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2))/x, x)
```

3.195 $\int \frac{(a+bx+cx^2)\log(1-dx)\text{PolyLog}(2,dx)}{x^2}\,dx$

Optimal. Leaf size=218

$$3cx+\frac{3c(1-dx)\log(1-dx)}{d}-\frac{c(1-dx)\log^2(1-dx)}{d}+\frac{a(1-dx)\log^2(1-dx)}{x}+\left(a-\frac{c}{d^2}\right)d\log(dx)\log^2(1-$$

```
[Out] 3*c*x+3*c*(-d*x+1)*ln(-d*x+1)/d-c*(-d*x+1)*ln(-d*x+1)^2/d+a*(-d*x+1)*ln(-d*
x+1)^2/x+(a-c/d^2)*d*ln(d*x)*ln(-d*x+1)^2-2*a*d*polylog(2,d*x)-c*x*polylog(
2,d*x)+(a-c/d^2)*d*ln(-d*x+1)*polylog(2,d*x)-(a/x-c*x)*ln(-d*x+1)*polylog(2
,d*x)-1/2*b*polylog(2,d*x)^2+2*(a-c/d^2)*d*ln(-d*x+1)*polylog(2,-d*x+1)-a*d
*polylog(3,d*x)-2*(a-c/d^2)*d*polylog(3,-d*x+1)
```

Rubi [A]

time = 0.33, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 21, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.808$, Rules used = {6874, 6721, 2436, 2332, 6726, 2442, 36, 29, 31, 6724, 6740, 6736, 14, 6741, 2333, 2444, 2438, 6731, 2443, 2481, 2421}

$$-2d\left(a-\frac{c}{d^2}\right)\text{Li}_3(1-dx)+d\left(a-\frac{c}{d^2}\right)\text{Li}_2(dx)\log(1-dx)+2d\left(a-\frac{c}{d^2}\right)\text{Li}_2(1-dx)\log(1-dx)+d\left(a-\frac{c}{d^2}\right)\log(dx)\log^2(1-dx)-\left(\frac{a}{x}-cx\right)\text{Li}_2(dx)\log(1-dx)-2ad\text{Li}_2(dx)-ad\text{Li}_3(dx)+\frac{a(1-dx)\log^2(1-dx)}{x}-\frac{1}{2}b\text{Li}_2(dx)^2-cx\text{Li}_2(dx)-\frac{c(1-dx)\log^2(1-dx)}{d}+\frac{3c(1-dx)\log(1-dx)}{d}+3cx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^2,x]
```

```
[Out] 3*c*x + (3*c*(1 - d*x)*Log[1 - d*x])/d - (c*(1 - d*x)*Log[1 - d*x]^2)/d + (
a*(1 - d*x)*Log[1 - d*x]^2)/x + (a - c/d^2)*d*Log[d*x]*Log[1 - d*x]^2 - 2*a
*d*PolyLog[2, d*x] - c*x*PolyLog[2, d*x] + (a - c/d^2)*d*Log[1 - d*x]*PolyL
og[2, d*x] - (a/x - c*x)*Log[1 - d*x]*PolyLog[2, d*x] - (b*PolyLog[2, d*x]^
2)/2 + 2*(a - c/d^2)*d*Log[1 - d*x]*PolyLog[2, 1 - d*x] - a*d*PolyLog[3, d*
x] - 2*(a - c/d^2)*d*PolyLog[3, 1 - d*x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
```

```
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6721

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6736

```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> Simp[-P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

Rule 6740

```
Int[((g_.) + Log[1 + (e_.)*(x_)]*(h_.))*(Px_)*(x_)^(m_)*PolyLog[2, (c_.)*(x
_)], x_Symbol] :> Dist[Coeff[Px, x, -m - 1], Int[(g + h*Log[1 + e*x])*(Poly
Log[2, c*x]/x), x], x] + Int[x^m*(Px - Coeff[Px, x, -m - 1]*x^(-m - 1))*(g
+ h*Log[1 + e*x])*PolyLog[2, c*x], x] /; FreeQ[{c, e, g, h}, x] && PolyQ[Px
, x] && ILtQ[m, 0] && EqQ[c + e, 0] && NeQ[Coeff[Px, x, -m - 1], 0]
```

Rule 6741

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (D
ist[b, Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x],
 u/(a + b*x), x], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a
+ b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x]
 && PolyQ[Px, x] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)\log(1-dx)\text{Li}_2(dx)}{x^2}\,dx = b\int \frac{\log(1-dx)\text{Li}_2(dx)}{x}\,dx + \int \frac{(a+cx^2)\log(1-dx)\text{Li}_2(dx)}{x^2}\,dx$$

$$= -\left(\frac{a}{x}-cx\right)\log(1-dx)\text{Li}_2(dx) - \frac{1}{2}b\text{Li}_2(dx)^2 + d\int \left(-\frac{c\text{Li}_2(dx}{d}\right.$$

$$= -\left(\frac{a}{x}-cx\right)\log(1-dx)\text{Li}_2(dx) - \frac{1}{2}b\text{Li}_2(dx)^2 - a\int \frac{\log^2(1-d}{x^2}$$

$$= \frac{a(1-dx)\log^2(1-dx)}{x} - cx\text{Li}_2(dx) - \frac{(c-ad^2)\log(1-dx)\text{Li}_2(}{d}$$

$$= -\frac{c(1-dx)\log^2(1-dx)}{d} + \frac{a(1-dx)\log^2(1-dx)}{x} - \frac{(c-ad^2)}{}$$

$$= 3cx + \frac{3c(1-dx)\log(1-dx)}{d} - \frac{c(1-dx)\log^2(1-dx)}{d} + \frac{a(1-}{}$$

$$= 3cx + \frac{3c(1-dx)\log(1-dx)}{d} - \frac{c(1-dx)\log^2(1-dx)}{d} + \frac{a(1-}{}$$

$$= 3cx + \frac{3c(1-dx)\log(1-dx)}{d} - \frac{c(1-dx)\log^2(1-dx)}{d} + \frac{a(1-}{}$$

Mathematica [A]
time = 0.56, size = 280, normalized size = 1.28

$$\frac{2(-cdx^2+(ad+cx)(-1+dx)\log(1-dx))\,\text{PolyLog}(2,dx) - bdx\text{PolyLog}(2,dx)^2 + 2\left(-2cx+3cdx^2+3cx\log(1-dx)-3cdx^2\log(1-dx)+2ad^2x\log(dx)\log(1-dx)+ad\log^2(1-dx)-cx\log^2(1-dx)-ad^2x\log^2(1-dx)+cdx^2\log^2(1-dx)-cx\log(dx)\log^2(1-dx)+ad^2x\log(dx)\log^2(1-dx)+2x(ad^2+(-c+ad^2)\log(1-dx))\,\text{PolyLog}(2,1-dx)-ad^2x\text{PolyLog}(3,dx)+2cx\text{PolyLog}(3,1-dx)-2ad^2x\text{PolyLog}(3,1-dx)\right)}{2dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^2,x]
```

```
[Out] (2*(-(c*d*x^2) + (a*d + c*x)*(-1 + d*x)*Log[1 - d*x])*PolyLog[2, d*x] - b*d
*x*PolyLog[2, d*x]^2 + 2*(-2*c*x + 3*c*d*x^2 + 3*c*x*Log[1 - d*x] - 3*c*d*x
^2*Log[1 - d*x] + 2*a*d^2*x*Log[d*x]*Log[1 - d*x] + a*d*Log[1 - d*x]^2 - c*
x*Log[1 - d*x]^2 - a*d^2*x*Log[1 - d*x]^2 + c*d*x^2*Log[1 - d*x]^2 - c*x*Lo
g[d*x]*Log[1 - d*x]^2 + a*d^2*x*Log[d*x]*Log[1 - d*x]^2 + 2*x*(a*d^2 + (-c
+ a*d^2)*Log[1 - d*x])*PolyLog[2, 1 - d*x] - a*d^2*x*PolyLog[3, d*x] + 2*c*
x*PolyLog[3, 1 - d*x] - 2*a*d^2*x*PolyLog[3, 1 - d*x]))/(2*d*x)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^2+bx+a)\ln(-dx+1)\,\text{polylog}(2,dx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^2,x)
```

```
[Out] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^2,x, algorithm="maxima
")
```

```
[Out] integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^2,x, algorithm="fricas
")
```

```
[Out] integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx + cx^2)\log(-dx + 1)\operatorname{Li}_2(dx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x**2,x)
```

```
[Out] Integral((a + b*x + c*x**2)*log(-d*x + 1)*polylog(2, d*x)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln(1-dx)\ \mathrm{polylog}(2,dx)\ (cx^2+bx+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2))/x^2,x)
[Out] int((log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2))/x^2, x)
```

3.196 $\int \frac{(a+bx+cx^2)\log(1-dx)\textbf{PolyLog}(2,dx)}{x^3}\,dx$

Optimal. Leaf size=343

$$-ad^2\log(x)+ad^2\log(1-dx)-\frac{ad\log(1-dx)}{x}-\frac{1}{4}ad^2\log^2(1-dx)+\frac{a\log^2(1-dx)}{4x^2}+\frac{b(1-dx)\log^2(1-dx)}{x}-b^2$$

```
[Out] -a*d^2*ln(x)+a*d^2*ln(-d*x+1)-a*d*ln(-d*x+1)/x-1/4*a*d^2*ln(-d*x+1)^2+1/4*a
*ln(-d*x+1)^2/x^2+b*(-d*x+1)*ln(-d*x+1)^2/x-1/2*b^2*ln(d*x)*ln(-d*x+1)^2/a+
1/2*(a*d+b)^2*ln(d*x)*ln(-d*x+1)^2/a-2*b*d*polylog(2,d*x)-1/2*a*d^2*polylog
(2,d*x)+1/2*a*d*polylog(2,d*x)/x+1/2*(a*d+b)^2*ln(-d*x+1)*polylog(2,d*x)/a-
1/2*(b*x+a)^2*ln(-d*x+1)*polylog(2,d*x)/a/x^2-1/2*c*polylog(2,d*x)^2-b^2*ln
(-d*x+1)*polylog(2,-d*x+1)/a+(a*d+b)^2*ln(-d*x+1)*polylog(2,-d*x+1)/a-1/2*d
*(a*d+2*b)*polylog(3,d*x)+b^2*polylog(3,-d*x+1)/a-(a*d+b)^2*polylog(3,-d*x+
1)/a
```

Rubi [A]

time = 0.48, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 22, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {6874, 6726, 2442, 46, 36, 29, 31, 6724, 6740, 6736, 37, 6741, 2445, 2457, 2438, 2437, 2338, 2444, 2443, 2481, 2421, 6731}

$$\frac{b^2\text{Li}_3(1-dx)}{a}-\frac{b^2\text{Li}_2(1-dx)\log(1-dx)}{a}-\frac{b^2\log(dx)\log^2(1-dx)}{2a}-\frac{(a+bx)^2\text{Li}_2(dx)\log(1-dx)}{2ax^2}-\frac{1}{2}d(ad+2b)\text{Li}_3(dx)-\frac{(ad+b)^2\text{Li}_3(1-dx)}{a}+\frac{(ad+b)^2\text{Li}_2(dx)\log(1-dx)}{2a}+\frac{(ad+b)^2\text{Li}_2(1-dx)\log(1-dx)}{a}+\frac{(ad+b)^2\log(dx)\log^2(1-dx)}{2a}-\frac{1}{2}ad^2\text{Li}_2(dx)-\frac{1}{4}ad^2\log^2(1-dx)-ad^2\log(x)+ad^2\log(1-dx)+\frac{ad\text{Li}_2(dx)}{2x}+\frac{a\log^2(1-dx)}{4x^2}-\frac{ad\log(1-dx)}{x}-2bd\text{Li}_2(dx)+\frac{b(1-dx)\log^2(1-dx)}{x}-\frac{1}{2}c\text{Li}_2(dx)^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^3,x]
```

```
[Out] -(a*d^2*Log[x]) + a*d^2*Log[1 - d*x] - (a*d*Log[1 - d*x])/x - (a*d^2*Log[1
- d*x]^2)/4 + (a*Log[1 - d*x]^2)/(4*x^2) + (b*(1 - d*x)*Log[1 - d*x]^2)/x -
 (b^2*Log[d*x]*Log[1 - d*x]^2)/(2*a) + ((b + a*d)^2*Log[d*x]*Log[1 - d*x]^2
)/(2*a) - 2*b*d*PolyLog[2, d*x] - (a*d^2*PolyLog[2, d*x])/2 + (a*d*PolyLog[
2, d*x])/(2*x) + ((b + a*d)^2*Log[1 - d*x]*PolyLog[2, d*x])/(2*a) - ((a + b
*x)^2*Log[1 - d*x]*PolyLog[2, d*x])/(2*a*x^2) - (c*PolyLog[2, d*x]^2)/2 - (
b^2*Log[1 - d*x]*PolyLog[2, 1 - d*x])/a + ((b + a*d)^2*Log[1 - d*x]*PolyLog
[2, 1 - d*x])/a - (d*(2*b + a*d)*PolyLog[3, d*x])/2 + (b^2*PolyLog[3, 1 - d
*x])/a - ((b + a*d)^2*PolyLog[3, 1 - d*x])/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2457

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6736

```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> Simp[-P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

Rule 6740

```
Int[((g_.) + Log[1 + (e_.)*(x_)]*(h_.))*(Px_)*(x_)^(m_)*PolyLog[2, (c_.)*(x
_)], x_Symbol] :> Dist[Coeff[Px, x, -m - 1], Int[(g + h*Log[1 + e*x])*(Poly
Log[2, c*x]/x), x], x] + Int[x^m*(Px - Coeff[Px, x, -m - 1]*x^(-m - 1))*(g
+ h*Log[1 + e*x])*PolyLog[2, c*x], x] /; FreeQ[{c, e, g, h}, x] && PolyQ[Px
, x] && ILtQ[m, 0] && EqQ[c + e, 0] && NeQ[Coeff[Px, x, -m - 1], 0]
```

Rule 6741

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (D
ist[b, Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x],
 u/(a + b*x), x], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a
+ b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x]
 && PolyQ[Px, x] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)\log(1-dx)\mathrm{Li}_2(dx)}{x^3}\,dx = c\int \frac{\log(1-dx)\mathrm{Li}_2(dx)}{x}\,dx + \int \frac{(a+bx)\log(1-dx)\mathrm{Li}_2(dx)}{x^3}\,dx$$

$$= -\frac{(a+bx)^2\log(1-dx)\mathrm{Li}_2(dx)}{2ax^2} - \frac{1}{2}c\mathrm{Li}_2(dx)^2 + d\int\left(-\frac{a\mathrm{Li}_2(dx)}{2x^2}\right.$$

$$= -\frac{(a+bx)^2\log(1-dx)\mathrm{Li}_2(dx)}{2ax^2} - \frac{1}{2}c\mathrm{Li}_2(dx)^2 - \frac{1}{2}a\int \frac{\log^2(1-dx}{x^3}$$

$$= \frac{a\log^2(1-dx)}{4x^2} + \frac{b(1-dx)\log^2(1-dx)}{x} - \frac{b^2\log(dx)\log^2(1-dx}{2a}$$

$$= -\frac{ad\log(1-dx)}{2x} + \frac{a\log^2(1-dx)}{4x^2} + \frac{b(1-dx)\log^2(1-dx)}{x} - \frac{b^2}{}$$

$$= -\frac{ad\log(1-dx)}{2x} + \frac{a\log^2(1-dx)}{4x^2} + \frac{b(1-dx)\log^2(1-dx)}{x} - \frac{b^2}{}$$

$$= -\frac{1}{2}ad^2\log(x) + \frac{1}{2}ad^2\log(1-dx) - \frac{ad\log(1-dx)}{x} + \frac{a\log^2(1-}{4x^2}$$

$$= -\frac{1}{2}ad^2\log(x) + \frac{1}{2}ad^2\log(1-dx) - \frac{ad\log(1-dx)}{x} - \frac{1}{4}ad^2\log^2(1$$

$$= -ad^2\log(x) + ad^2\log(1-dx) - \frac{ad\log(1-dx)}{x} - \frac{1}{4}ad^2\log^2(1-$$

Mathematica [F]
time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{(a+bx+cx^2)\log(1-dx)\mathrm{PolyLog}(2,dx)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^3,x]
[Out] Integrate[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^3, x]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^2+bx+a)\ln(-dx+1)\,\mathrm{polylog}(2,dx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^3,x)
```

```
[Out] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^3,x, algorithm="maxima
")
```

```
[Out] integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^3,x, algorithm="fricas
")
```

```
[Out] integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx + cx^2) \log (-dx + 1) \operatorname{Li}_2 (dx)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x**3,x)
```

```
[Out] Integral((a + b*x + c*x**2)*log(-d*x + 1)*polylog(2, d*x)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(1-d\,x\right)\,\mathrm{polylog}(2,d\,x)\,\left(c\,x^2+b\,x+a\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2))/x^3,x)
[Out] int((log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2))/x^3, x)
```

3.197 $\int \frac{(a+bx+cx^2)\log(1-dx)\textbf{PolyLog}(2,dx)}{x^4}\,dx$

Optimal. Leaf size=515

$$\frac{7ad^2}{36x}-\frac{1}{2}bd^2\log(x)-\frac{5}{12}ad^3\log(x)-\frac{1}{6}d^2(3b+2ad)\log(x)+\frac{1}{2}bd^2\log(1-dx)+\frac{5}{12}ad^3\log(1-dx)+\frac{1}{6}d^2(3b+2ad)$$

```
[Out] 1/6*a*d*polylog(2,d*x)/x^2+1/6*d*(2*a*d+3*b)*polylog(2,d*x)/x-7/36*a*d*ln(-
d*x+1)/x^2-1/2*b*d*ln(-d*x+1)/x-2/9*a*d^2*ln(-d*x+1)/x-1/6*d*(2*a*d+3*b)*ln
(-d*x+1)/x+1/6*d*(6*c+d*(2*a*d+3*b))*ln(d*x)*ln(-d*x+1)^2+1/6*d*(6*c+d*(2*a
*d+3*b))*ln(-d*x+1)*polylog(2,d*x)+1/3*d*(6*c+d*(2*a*d+3*b))*ln(-d*x+1)*pol
ylog(2,-d*x+1)+c*(-d*x+1)*ln(-d*x+1)^2/x+7/36*a*d^2/x-2*c*d*polylog(2,d*x)-
1/2*b*d^2*polylog(2,d*x)-2/9*a*d^3*polylog(2,d*x)-1/6*d*(6*c+d*(2*a*d+3*b))
*polylog(3,d*x)-1/3*d*(6*c+d*(2*a*d+3*b))*polylog(3,-d*x+1)-1/2*b*d^2*ln(x)
-5/12*a*d^3*ln(x)-1/6*d^2*(2*a*d+3*b)*ln(x)+1/2*b*d^2*ln(-d*x+1)+5/12*a*d^3
*ln(-d*x+1)+1/6*d^2*(2*a*d+3*b)*ln(-d*x+1)-1/4*b*d^2*ln(-d*x+1)^2-1/9*a*d^3
*ln(-d*x+1)^2+1/9*a*ln(-d*x+1)^2/x^3+1/4*b*ln(-d*x+1)^2/x^2-1/6*(2*a/x^3+3*
b/x^2+6*c/x)*ln(-d*x+1)*polylog(2,d*x)
```

Rubi [A]

time = 0.53, antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 20, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {6874, 6726, 2442, 46, 36, 29, 31, 14, 6741, 2445, 2457, 2438, 2437, 2338, 2444, 6724, 6731, 2443, 2481, 2421}

$$-\frac{1}{6}\text{Li}_2(dx)\log(1-dx)\left(\frac{2a}{x^3}+\frac{3b}{x^2}+\frac{6c}{x}\right)-\frac{1}{6}d\text{Li}_3(dx)(d(2ad+3b)+6c)-\frac{1}{3}d\text{Li}_3(1-dx)(d(2ad+3b)+6c)+\frac{1}{6}d\text{Li}_2(dx)\log(1-dx)(d(2ad+3b)+6c)+\frac{1}{3}d\text{Li}_2(1-dx)\log(1-dx)(d(2ad+3b)+6c)+\frac{1}{6}d\log(dx)\log^2(1-dx)(d(2ad+3b)+6c)-\frac{1}{6}d^2\log(x)(2ad+3b)+\frac{1}{6}d^2(2ad+3b)\log(1-dx)+\frac{d(2ad+3b)\text{Li}_2(dx)}{6x}-\frac{d(2ad+3b)\log(1-dx)}{6x}-\frac{2}{9}ad^3\text{Li}_2(dx)-\frac{1}{9}ad^3\log^2(1-dx)-\frac{5}{12}ad^3\log(x)+\frac{5}{12}ad^3\log(1-dx)+\frac{7ad^2}{36x}-\frac{2ad^2\log(1-dx)}{9x}+\frac{ad\text{Li}_2(dx)}{6x^2}+\frac{a\log^2(1-dx)}{9x^3}-\frac{7ad\log(1-dx)}{36x^2}-\frac{1}{2}bd^2\text{Li}_2(dx)-\frac{1}{4}bd^2\log^2(1-dx)-\frac{1}{2}bd^2\log(x)+\frac{1}{2}bd^2\log(1-dx)+\frac{b\log^2(1-dx)}{4x^2}-\frac{bd\log(1-dx)}{2x}-2cd\text{Li}_2(dx)+\frac{c(1-dx)\log^2(1-dx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^4,x]
```

```
[Out] (7*a*d^2)/(36*x) - (b*d^2*Log[x])/2 - (5*a*d^3*Log[x])/12 - (d^2*(3*b + 2*a
*d)*Log[x])/6 + (b*d^2*Log[1 - d*x])/2 + (5*a*d^3*Log[1 - d*x])/12 + (d^2*(
3*b + 2*a*d)*Log[1 - d*x])/6 - (7*a*d*Log[1 - d*x])/(36*x^2) - (b*d*Log[1 -
 d*x])/(2*x) - (2*a*d^2*Log[1 - d*x])/(9*x) - (d*(3*b + 2*a*d)*Log[1 - d*x]
)/(6*x) - (b*d^2*Log[1 - d*x]^2)/4 - (a*d^3*Log[1 - d*x]^2)/9 + (a*Log[1 -
d*x]^2)/(9*x^3) + (b*Log[1 - d*x]^2)/(4*x^2) + (c*(1 - d*x)*Log[1 - d*x]^2)
/x + (d*(6*c + d*(3*b + 2*a*d))*Log[d*x]*Log[1 - d*x]^2)/6 - 2*c*d*PolyLog[
2, d*x] - (b*d^2*PolyLog[2, d*x])/2 - (2*a*d^3*PolyLog[2, d*x])/9 + (a*d*Po
lyLog[2, d*x])/(6*x^2) + (d*(3*b + 2*a*d)*PolyLog[2, d*x])/(6*x) + (d*(6*c
+ d*(3*b + 2*a*d))*Log[1 - d*x]*PolyLog[2, d*x])/6 - (((2*a)/x^3 + (3*b)/x^
2 + (6*c)/x)*Log[1 - d*x]*PolyLog[2, d*x])/6 + (d*(6*c + d*(3*b + 2*a*d))*L
og[1 - d*x]*PolyLog[2, 1 - d*x])/3 - (d*(6*c + d*(3*b + 2*a*d))*PolyLog[3,
d*x])/6 - (d*(6*c + d*(3*b + 2*a*d))*PolyLog[3, 1 - d*x])/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2457

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6741

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (D
ist[b, Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x],
 u/(a + b*x), x], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a
+ b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x]
 && PolyQ[Px, x] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)\log(1-dx)\text{Li}_2(dx)}{x^4}\,dx = -\frac{1}{6}\left(\frac{2a}{x^3}+\frac{3b}{x^2}+\frac{6c}{x}\right)\log(1-dx)\text{Li}_2(dx) + d\int\left(-\frac{a\text{Li}_2(dx)}{3x^3}+\right.$$

$$= -\frac{1}{6}\left(\frac{2a}{x^3}+\frac{3b}{x^2}+\frac{6c}{x}\right)\log(1-dx)\text{Li}_2(dx) - \frac{1}{3}a\int\frac{\log^2(1-dx)}{x^4}\,d$$

$$= \frac{a\log^2(1-dx)}{9x^3} + \frac{b\log^2(1-dx)}{4x^2} + \frac{c(1-dx)\log^2(1-dx)}{x} + \frac{adl}{}$$

$$= -\frac{ad\log(1-dx)}{12x^2} - \frac{d(3b+2ad)\log(1-dx)}{6x} + \frac{a\log^2(1-dx)}{9x^3} +$$

$$= -\frac{ad\log(1-dx)}{12x^2} - \frac{d(3b+2ad)\log(1-dx)}{6x} + \frac{a\log^2(1-dx)}{9x^3} +$$

$$= \frac{ad^2}{12x} - \frac{1}{12}ad^3\log(x) - \frac{1}{6}d^2(3b+2ad)\log(x) + \frac{1}{12}ad^3\log(1-dx$$

$$= \frac{ad^2}{12x} - \frac{1}{12}ad^3\log(x) - \frac{1}{6}d^2(3b+2ad)\log(x) + \frac{1}{12}ad^3\log(1-dx$$

$$= \frac{7ad^2}{36x} - \frac{1}{2}bd^2\log(x) - \frac{5}{12}ad^3\log(x) - \frac{1}{6}d^2(3b+2ad)\log(x) + \frac{1}{2}$$

Mathematica [A]
time = 1.21, size = 488, normalized size = 0.95

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^4,x]
```

```
[Out] (-7*a*d^3 + (7*a*d^2)/x - 36*b*d^2*Log[d*x] - 27*a*d^3*Log[d*x] + 36*b*d^2*
Log[1 - d*x] + 27*a*d^3*Log[1 - d*x] - (7*a*d*Log[1 - d*x])/x^2 - (36*b*d*L
og[1 - d*x])/x - (20*a*d^2*Log[1 - d*x])/x + 72*c*d*Log[d*x]*Log[1 - d*x] +
 18*b*d^2*Log[d*x]*Log[1 - d*x] + 8*a*d^3*Log[d*x]*Log[1 - d*x] - 36*c*d*Lo
g[1 - d*x]^2 - 9*b*d^2*Log[1 - d*x]^2 - 4*a*d^3*Log[1 - d*x]^2 + (4*a*Log[1
 - d*x]^2)/x^3 + (9*b*Log[1 - d*x]^2)/x^2 + (36*c*Log[1 - d*x]^2)/x + 36*c*
d*Log[d*x]*Log[1 - d*x]^2 + 18*b*d^2*Log[d*x]*Log[1 - d*x]^2 + 12*a*d^3*Log
[d*x]*Log[1 - d*x]^2 + (6*(d*x*(a + 3*b*x + 2*a*d*x) + (-1 + d*x)*(3*x*(b +
 2*c*x + b*d*x) + 2*a*(1 + d*x + d^2*x^2))*Log[1 - d*x])*PolyLog[2, d*x])/x
^3 + 2*d*(36*c + 9*b*d + 4*a*d^2 + 6*(6*c + 3*b*d + 2*a*d^2)*Log[1 - d*x])*
PolyLog[2, 1 - d*x] - 36*c*d*PolyLog[3, d*x] - 18*b*d^2*PolyLog[3, d*x] - 1
2*a*d^3*PolyLog[3, d*x] - 72*c*d*PolyLog[3, 1 - d*x] - 36*b*d^2*PolyLog[3,
1 - d*x] - 24*a*d^3*PolyLog[3, 1 - d*x])/36
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^{2}+b x +a \right) \ln \left(-d x +1\right) \operatorname{polylog}\left(2, d x \right)}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^4,x)
```

```
[Out] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^4,x)
```

Maxima [A]
time = 0.32, size = 319, normalized size = 0.62

$$\frac{1}{6}\left(2\,ad^{3}+3\,bd^{2}+6\,cd\right)\left(\log\left(dx\right)\log\left(-dx+1\right)^{2}+2\,\mathrm{Li}_{2}(-dx+1)\log\left(-dx+1\right)-2\,\mathrm{Li}_{3}(-dx+1)\right)+\frac{1}{18}\left(4\,ad^{3}+9\,bd^{2}+36\,cd\right)\left(\log\left(dx\right)\log\left(-dx+1\right)+\mathrm{Li}_{2}(-dx+1)\right)-\frac{1}{4}\left(3\,ad^{3}+4\,bd^{2}\right)\log\left(x\right)-\frac{1}{6}\left(2\,ad^{3}+3\,bd^{2}+6\,cd\right)\mathrm{Li}_{3}(dx)+\frac{7\,ad^{2}x^{2}-\left(\left(4\,ad^{3}+9\,bd^{2}+36\,cd\right)x^{3}-36\,cx^{2}-9\,bx-4\,a\right)\log\left(-dx+1\right)^{2}+6\left(adx+\left(2\,ad^{2}+3\,bd\right)x^{2}+\left(\left(2\,ad^{3}+3\,bd^{2}+6\,cd\right)x^{3}-6\,cx^{2}-3\,bx-2\,a\right)\log\left(-dx+1\right)\right)\mathrm{Li}_{2}(dx)+\left(9\left(3\,ad^{3}+4\,bd^{2}\right)x^{3}-7\,adx-4\left(5\,ad^{2}+9\,bd\right)x^{2}\right)\log\left(-dx+1\right)}{36\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^4,x, algorithm="maxima
")
```

```
[Out] 1/6*(2*a*d^3 + 3*b*d^2 + 6*c*d)*(log(d*x)*log(-d*x + 1)^2 + 2*dilog(-d*x +
1)*log(-d*x + 1) - 2*polylog(3, -d*x + 1)) + 1/18*(4*a*d^3 + 9*b*d^2 + 36*c
*d)*(log(d*x)*log(-d*x + 1) + dilog(-d*x + 1)) - 1/4*(3*a*d^3 + 4*b*d^2)*lo
g(x) - 1/6*(2*a*d^3 + 3*b*d^2 + 6*c*d)*polylog(3, d*x) + 1/36*(7*a*d^2*x^2
- ((4*a*d^3 + 9*b*d^2 + 36*c*d)*x^3 - 36*c*x^2 - 9*b*x - 4*a)*log(-d*x + 1)
^2 + 6*(a*d*x + (2*a*d^2 + 3*b*d)*x^2 + ((2*a*d^3 + 3*b*d^2 + 6*c*d)*x^3 -
6*c*x^2 - 3*b*x - 2*a)*log(-d*x + 1))*dilog(d*x) + (9*(3*a*d^3 + 4*b*d^2)*x
^3 - 7*a*d*x - 4*(5*a*d^2 + 9*b*d)*x^2)*log(-d*x + 1))/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^4,x, algorithm="fricas
")
```

```
[Out] integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x**4,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^4,x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(1-d\,x\right)\,\mathrm{polylog}(2,d\,x)\,\left(c\,x^2+b\,x+a\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2))/x^4,x)
[Out] int((log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2))/x^4, x)
```

3.198 $\int \frac{(a+bx+cx^2)\log(1-dx)\textbf{PolyLog}(2,dx)}{x^5}\,dx$

Optimal. Leaf size=767

$$\frac{5ad^2}{144x^2}+\frac{bd^2}{9x}+\frac{19ad^3}{144x}+\frac{d^2(4b+3ad)}{48x}-\frac{1}{2}cd^2\log(x)-\frac{1}{3}bd^3\log(x)-\frac{37}{144}ad^4\log(x)-\frac{1}{48}d^3(4b+3ad)\log(x)-\frac{1}{12}d^2($$

```
[Out] 1/12*a*d*polylog(2,d*x)/x^3+1/24*d*(3*a*d+4*b)*polylog(2,d*x)/x^2+1/12*d*(6
*c+d*(3*a*d+4*b))*polylog(2,d*x)/x-1/2*c*d^2*polylog(2,d*x)-2/9*b*d^3*polyl
og(2,d*x)-1/8*a*d^4*polylog(2,d*x)-1/12*d^2*(6*c+d*(3*a*d+4*b))*polylog(3,d
*x)-1/6*d^2*(6*c+d*(3*a*d+4*b))*polylog(3,-d*x+1)-5/72*a*d*ln(-d*x+1)/x^3-1
/9*b*d*ln(-d*x+1)/x^2-1/16*a*d^2*ln(-d*x+1)/x^2-1/48*d*(3*a*d+4*b)*ln(-d*x+
1)/x^2-1/2*c*d*ln(-d*x+1)/x-2/9*b*d^2*ln(-d*x+1)/x-1/8*a*d^3*ln(-d*x+1)/x-1
/12*d*(6*c+d*(3*a*d+4*b))*ln(-d*x+1)/x+1/12*d^2*(6*c+d*(3*a*d+4*b))*ln(d*x)
*ln(-d*x+1)^2+1/12*d^2*(6*c+d*(3*a*d+4*b))*ln(-d*x+1)*polylog(2,d*x)+1/6*d^
2*(6*c+d*(3*a*d+4*b))*ln(-d*x+1)*polylog(2,-d*x+1)+5/144*a*d^2/x^2+1/9*b*d^
2/x+19/144*a*d^3/x+1/48*d^2*(3*a*d+4*b)/x+1/3*b*d^3*ln(-d*x+1)+37/144*a*d^4
*ln(-d*x+1)+1/48*d^3*(3*a*d+4*b)*ln(-d*x+1)+1/12*d^2*(6*c+d*(3*a*d+4*b))*ln
(-d*x+1)-1/4*c*d^2*ln(-d*x+1)^2-1/9*b*d^3*ln(-d*x+1)^2-1/16*a*d^4*ln(-d*x+1
)^2+1/16*a*ln(-d*x+1)^2/x^4+1/9*b*ln(-d*x+1)^2/x^3+1/4*c*ln(-d*x+1)^2/x^2-1
/12*(3*a/x^4+4*b/x^3+6*c/x^2)*ln(-d*x+1)*polylog(2,d*x)-1/2*c*d^2*ln(x)-1/3
*b*d^3*ln(x)-37/144*a*d^4*ln(x)-1/48*d^3*(3*a*d+4*b)*ln(x)-1/12*d^2*(6*c+d*
(3*a*d+4*b))*ln(x)+1/2*c*d^2*ln(-d*x+1)
```

Rubi [A]
time = 0.73, antiderivative size = 767, normalized size of antiderivative = 1.00, number of steps used = 61, number of rules used = 19, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.731$, Rules used = {6874, 6726, 2442, 46, 14, 6741, 2445, 2457, 36, 29, 31, 2438, 2437, 2338, 6724, 6731, 2443, 2481, 2421}

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^5,x]
```

```
[Out] (5*a*d^2)/(144*x^2) + (b*d^2)/(9*x) + (19*a*d^3)/(144*x) + (d^2*(4*b + 3*a*
d))/(48*x) - (c*d^2*Log[x])/2 - (b*d^3*Log[x])/3 - (37*a*d^4*Log[x])/144 -
(d^3*(4*b + 3*a*d)*Log[x])/48 - (d^2*(6*c + d*(4*b + 3*a*d))*Log[x])/12 + (
c*d^2*Log[1 - d*x])/2 + (b*d^3*Log[1 - d*x])/3 + (37*a*d^4*Log[1 - d*x])/14
4 + (d^3*(4*b + 3*a*d)*Log[1 - d*x])/48 + (d^2*(6*c + d*(4*b + 3*a*d))*Log[
1 - d*x])/12 - (5*a*d*Log[1 - d*x])/(72*x^3) - (b*d*Log[1 - d*x])/(9*x^2) -
 (a*d^2*Log[1 - d*x])/(16*x^2) - (d*(4*b + 3*a*d)*Log[1 - d*x])/(48*x^2) -
(c*d*Log[1 - d*x])/(2*x) - (2*b*d^2*Log[1 - d*x])/(9*x) - (a*d^3*Log[1 - d*
x])/(8*x) - (d*(6*c + d*(4*b + 3*a*d))*Log[1 - d*x])/(12*x) - (c*d^2*Log[1
```

```
- d*x]^2)/4 - (b*d^3*Log[1 - d*x]^2)/9 - (a*d^4*Log[1 - d*x]^2)/16 + (a*Log
[1 - d*x]^2)/(16*x^4) + (b*Log[1 - d*x]^2)/(9*x^3) + (c*Log[1 - d*x]^2)/(4*
x^2) + (d^2*(6*c + d*(4*b + 3*a*d))*Log[d*x]*Log[1 - d*x]^2)/12 - (c*d^2*Po
lyLog[2, d*x])/2 - (2*b*d^3*PolyLog[2, d*x])/9 - (a*d^4*PolyLog[2, d*x])/8
+ (a*d*PolyLog[2, d*x])/(12*x^3) + (d*(4*b + 3*a*d)*PolyLog[2, d*x])/(24*x^
2) + (d*(6*c + d*(4*b + 3*a*d))*PolyLog[2, d*x])/(12*x) + (d^2*(6*c + d*(4*
b + 3*a*d))*Log[1 - d*x]*PolyLog[2, d*x])/12 - (((3*a)/x^4 + (4*b)/x^3 + (6
*c)/x^2)*Log[1 - d*x]*PolyLog[2, d*x])/12 + (d^2*(6*c + d*(4*b + 3*a*d))*Lo
g[1 - d*x]*PolyLog[2, 1 - d*x])/6 - (d^2*(6*c + d*(4*b + 3*a*d))*PolyLog[3,
 d*x])/12 - (d^2*(6*c + d*(4*b + 3*a*d))*PolyLog[3, 1 - d*x])/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
```

```
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2457

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6726

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[(d*x)^(m + 1)*(PolyLog[n, a*(b*x^p)^q]/(d*(m + 1))), x] - Dist[p
*(q/(m + 1)), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6731

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[Log[1 - a*c - b*c*x]*(PolyLog[2, c*(a + b*x)]/e), x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 6741

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (D
ist[b, Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x],
 u/(a + b*x), x], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a
+ b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x]
 && PolyQ[Px, x] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)\log(1-dx)\text{Li}_2(dx)}{x^5}\,dx = -\frac{1}{12}\left(\frac{3a}{x^4}+\frac{4b}{x^3}+\frac{6c}{x^2}\right)\log(1-dx)\text{Li}_2(dx) + d\int\left(-\frac{a\text{Li}_2(dx)}{4x^4}+\right.$$
$$= -\frac{1}{12}\left(\frac{3a}{x^4}+\frac{4b}{x^3}+\frac{6c}{x^2}\right)\log(1-dx)\text{Li}_2(dx) - \frac{1}{4}a\int\frac{\log^2(1-dx)}{x^5}\,d$$
$$= \frac{a\log^2(1-dx)}{16x^4} + \frac{b\log^2(1-dx)}{9x^3} + \frac{c\log^2(1-dx)}{4x^2} + \frac{ad\text{Li}_2(dx)}{12x^3} +$$
$$= -\frac{ad\log(1-dx)}{36x^3} - \frac{d(4b+3ad)\log(1-dx)}{48x^2} - \frac{d(6c+d(4b+3ad}{12x}$$
$$= -\frac{ad\log(1-dx)}{36x^3} - \frac{d(4b+3ad)\log(1-dx)}{48x^2} - \frac{d(6c+d(4b+3ad}{12x}$$
$$= \frac{ad^2}{72x^2} + \frac{ad^3}{36x} + \frac{d^2(4b+3ad)}{48x} - \frac{1}{36}ad^4\log(x) - \frac{1}{48}d^3(4b+3ad)\log$$
$$= \frac{ad^2}{72x^2} + \frac{ad^3}{36x} + \frac{d^2(4b+3ad)}{48x} - \frac{1}{36}ad^4\log(x) - \frac{1}{48}d^3(4b+3ad)\log$$
$$= \frac{5ad^2}{144x^2} + \frac{bd^2}{9x} + \frac{19ad^3}{144x} + \frac{d^2(4b+3ad)}{48x} - \frac{1}{2}cd^2\log(x) - \frac{1}{3}bd^3\log($$

Mathematica [A]
time = 1.35, size = 621, normalized size = 0.81

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^5,x]
```

```
[Out] (-28*b*d^3 - 33*a*d^4 + (5*a*d^2)/x^2 + (28*b*d^2)/x + (28*a*d^3)/x - 144*c
*d^2*Log[d*x] - 108*b*d^3*Log[d*x] - 82*a*d^4*Log[d*x] + 144*c*d^2*Log[1 -
d*x] + 108*b*d^3*Log[1 - d*x] + 82*a*d^4*Log[1 - d*x] - (10*a*d*Log[1 - d*x
])/x^3 - (28*b*d*Log[1 - d*x])/x^2 - (18*a*d^2*Log[1 - d*x])/x^2 - (144*c*d
*Log[1 - d*x])/x - (80*b*d^2*Log[1 - d*x])/x - (54*a*d^3*Log[1 - d*x])/x +
72*c*d^2*Log[d*x]*Log[1 - d*x] + 32*b*d^3*Log[d*x]*Log[1 - d*x] + 18*a*d^4*
Log[d*x]*Log[1 - d*x] - 36*c*d^2*Log[1 - d*x]^2 - 16*b*d^3*Log[1 - d*x]^2 -
 9*a*d^4*Log[1 - d*x]^2 + (9*a*Log[1 - d*x]^2)/x^4 + (16*b*Log[1 - d*x]^2)/
x^3 + (36*c*Log[1 - d*x]^2)/x^2 + 72*c*d^2*Log[d*x]*Log[1 - d*x]^2 + 48*b*d
^3*Log[d*x]*Log[1 - d*x]^2 + 36*a*d^4*Log[d*x]*Log[1 - d*x]^2 + (6*(d*x*(4*
x*(b + 3*c*x + 2*b*d*x) + a*(2 + 3*d*x + 6*d^2*x^2)) + 2*(-4*b*x - 6*c*x^2
+ 6*c*d^2*x^4 + 4*b*d^3*x^4 + 3*a*(-1 + d^4*x^4))*Log[1 - d*x])*PolyLog[2,
d*x])/x^4 + 2*d^2*(36*c + 16*b*d + 9*a*d^2 + 12*(6*c + 4*b*d + 3*a*d^2)*Log
[1 - d*x])*PolyLog[2, 1 - d*x] - 72*c*d^2*PolyLog[3, d*x] - 48*b*d^3*PolyLo
```

```
g[3, d*x] - 36*a*d^4*PolyLog[3, d*x] - 144*c*d^2*PolyLog[3, 1 - d*x] - 96*b
*d^3*PolyLog[3, 1 - d*x] - 72*a*d^4*PolyLog[3, 1 - d*x])/144
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^{2}+bx+a\right)\ln \left(-dx+1\right) \operatorname{polylog}\left(2,dx\right)}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^5,x)
```

```
[Out] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^5,x)
```

Maxima [A]
time = 0.31, size = 403, normalized size = 0.53

$$\frac{1}{12}\left(3\,ad^{4}+4\,bd^{3}+6\,cd^{2}\right)\left(\log\left(dx\right)\log\left(-dx+1\right)^{2}+2\,\mathrm{Li}_{2}(-dx+1)\log\left(-dx+1\right)-2\,\mathrm{Li}_{3}(-dx+1)\right)+\frac{1}{72}\left(9\,ad^{4}+16\,bd^{3}+36\,cd^{2}\right)\left(\log\left(dx\right)\log\left(-dx+1\right)+\mathrm{Li}_{2}(-dx+1)\right)-\frac{1}{72}\left(41\,ad^{4}+54\,bd^{3}+72\,cd^{2}\right)\log\left(x\right)-\frac{1}{12}\left(3\,ad^{4}+4\,bd^{3}+6\,cd^{2}\right)\mathrm{Li}_{3}(dx)+\frac{5\,ad^{2}x^{2}+28\,(ad^{3}+bd^{2})x^{3}-((9\,ad^{4}+16\,bd^{3}+36\,cd^{2})x^{4}-36\,cx^{2}-16\,bx-9\,a)\log\left(-dx+1\right)^{2}+6\,(2\,(3\,ad^{3}+4\,bd^{2}+6\,cd)x^{3}+2\,adx+(3\,ad^{2}+4\,bd)x^{2}+2\,((3\,ad^{4}+4\,bd^{3}+6\,cd^{2})x^{4}-6\,cx^{2}-4\,bx-3\,a)\log\left(-dx+1\right))\mathrm{Li}_{2}(dx)+2\,((41\,ad^{4}+54\,bd^{3}+72\,cd^{2})x^{4}-(27\,ad^{3}+40\,bd^{2}+72\,cd)x^{3}-5\,adx-(9\,ad^{2}+14\,bd)x^{2})\log\left(-dx+1\right)}{144\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^5,x, algorithm="maxima
")
```

```
[Out] 1/12*(3*a*d^4 + 4*b*d^3 + 6*c*d^2)*(log(d*x)*log(-d*x + 1)^2 + 2*dilog(-d*x
 + 1)*log(-d*x + 1) - 2*polylog(3, -d*x + 1)) + 1/72*(9*a*d^4 + 16*b*d^3 +
36*c*d^2)*(log(d*x)*log(-d*x + 1) + dilog(-d*x + 1)) - 1/72*(41*a*d^4 + 54*
b*d^3 + 72*c*d^2)*log(x) - 1/12*(3*a*d^4 + 4*b*d^3 + 6*c*d^2)*polylog(3, d*
x) + 1/144*(5*a*d^2*x^2 + 28*(a*d^3 + b*d^2)*x^3 - ((9*a*d^4 + 16*b*d^3 + 3
6*c*d^2)*x^4 - 36*c*x^2 - 16*b*x - 9*a)*log(-d*x + 1)^2 + 6*(2*(3*a*d^3 + 4
*b*d^2 + 6*c*d)*x^3 + 2*a*d*x + (3*a*d^2 + 4*b*d)*x^2 + 2*((3*a*d^4 + 4*b*d
^3 + 6*c*d^2)*x^4 - 6*c*x^2 - 4*b*x - 3*a)*log(-d*x + 1))*dilog(d*x) + 2*((
41*a*d^4 + 54*b*d^3 + 72*c*d^2)*x^4 - (27*a*d^3 + 40*b*d^2 + 72*c*d)*x^3 -
5*a*d*x - (9*a*d^2 + 14*b*d)*x^2)*log(-d*x + 1))/x^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^5,x, algorithm="fricas
")
```

```
[Out] integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^5, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x**5,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(1-d\,x\right)\,\mathrm{polylog}\left(2,d\,x\right)\,\left(c\,x^2+b\,x+a\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2))/x^5,x)
```

```
[Out] int((log(1 - d*x)*polylog(2, d*x)*(a + b*x + c*x^2))/x^5, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0])  #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:])  #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```