Computer algebra independent integration tests

Summer 2022 edition

8-Special-functions/205-8.2-Fresnel-integral-functions

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September 27, 2022 Compiled on September 27, 2022 at 2:35am

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [218]. This is test number [205].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (218)	0.00 (0)
Mathematica	87.16 (190)	12.84 (28)
Fricas	87.16 (190)	12.84 (28)
Maple	70.64 (154)	29.36 (64)
Maxima	54.13 (118)	45.87 (100)
Sympy	54.13 (118)	45.87 (100)
Mupad	27.52 (60)	72.48 (158)
Giac	27.52 (60)	72.48 (158)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

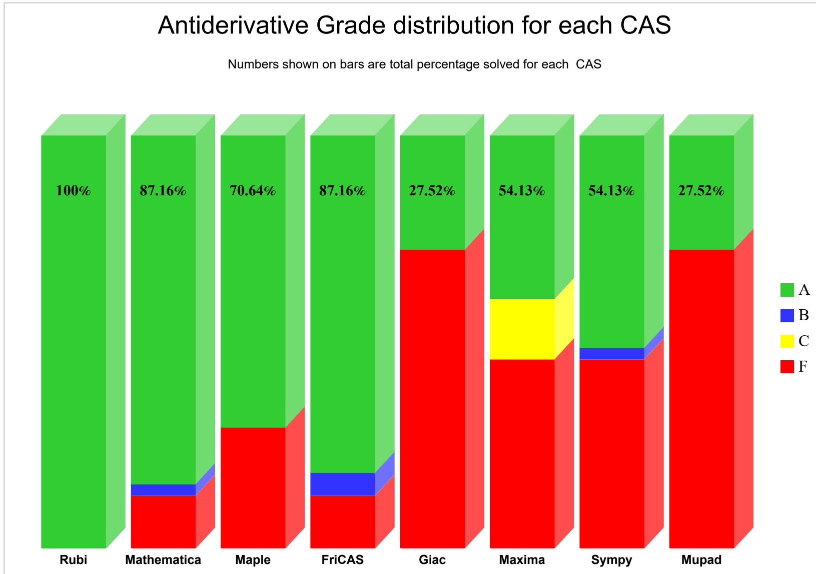
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

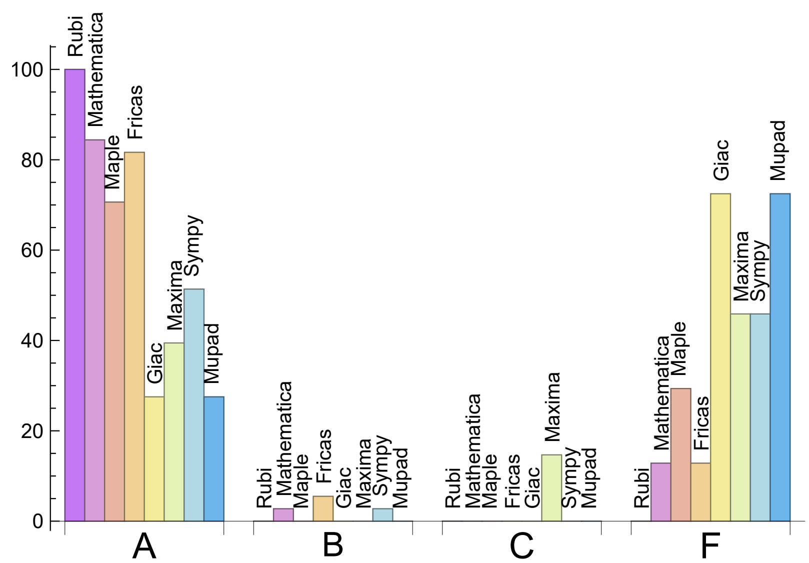
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	84.40	2.75	0.00	12.84
Fricas	81.65	5.50	0.00	12.84
Maple	70.64	0.00	0.00	29.36
Sympy	51.38	2.75	0.00	45.87
Maxima	39.45	0.00	14.68	45.87
Mupad	N/A	0.00	0.00	72.48
Giac	27.52	0.00	0.00	72.48

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	28	100.00 %	0.00 %	0.00 %
Maple	64	100.00 %	0.00 %	0.00 %
Fricas	28	100.00 %	0.00 %	0.00 %
Giac	158	100.00 %	0.00 %	0.00 %
Maxima	100	98.00 %	0.00 %	2.00 %
Sympy	100	100.00 %	0.00 %	0.00 %
Mupad	158	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

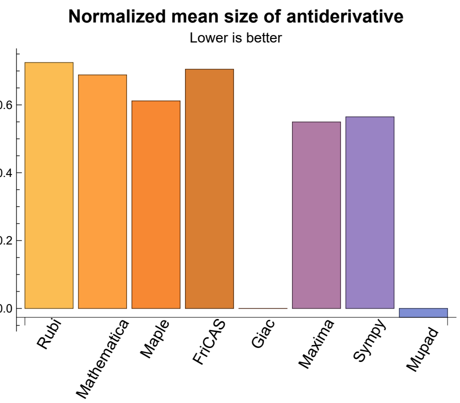
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

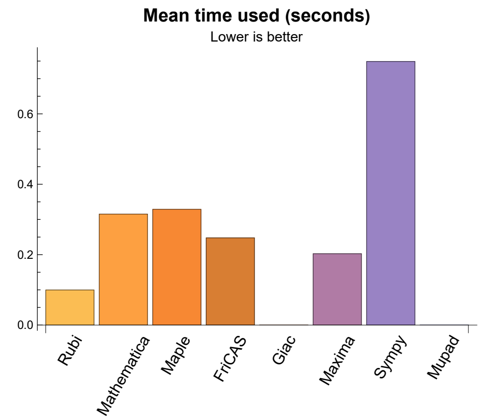
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.10	96.61	0.72	71.50	1.00
Mathematica	0.32	81.86	0.69	59.00	0.81
Maple	0.33	62.36	0.61	28.00	0.90
Maxima	0.20	43.09	0.55	0.00	0.00
Fricas	0.25	91.78	0.71	54.00	0.81
Sympy	0.75	41.35	0.56	0.00	0.00
Giac	0.00	0.00	0.00	0.00	0.00
Mupad	0.00	-1.00	-0.03	-1.00	-0.01

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0.0
0.2
0.4
0.6
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0.0
0.2
0.4
0.6
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 80, 81, 82, 84, 85, 86, 88, 89, 90, 100, 102, 103, 104, 106, 107, 108, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 189, 190, 191, 193, 194, 195, 197, 198, 199, 209, 211, 212, 213, 215, 216, 217\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```python
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

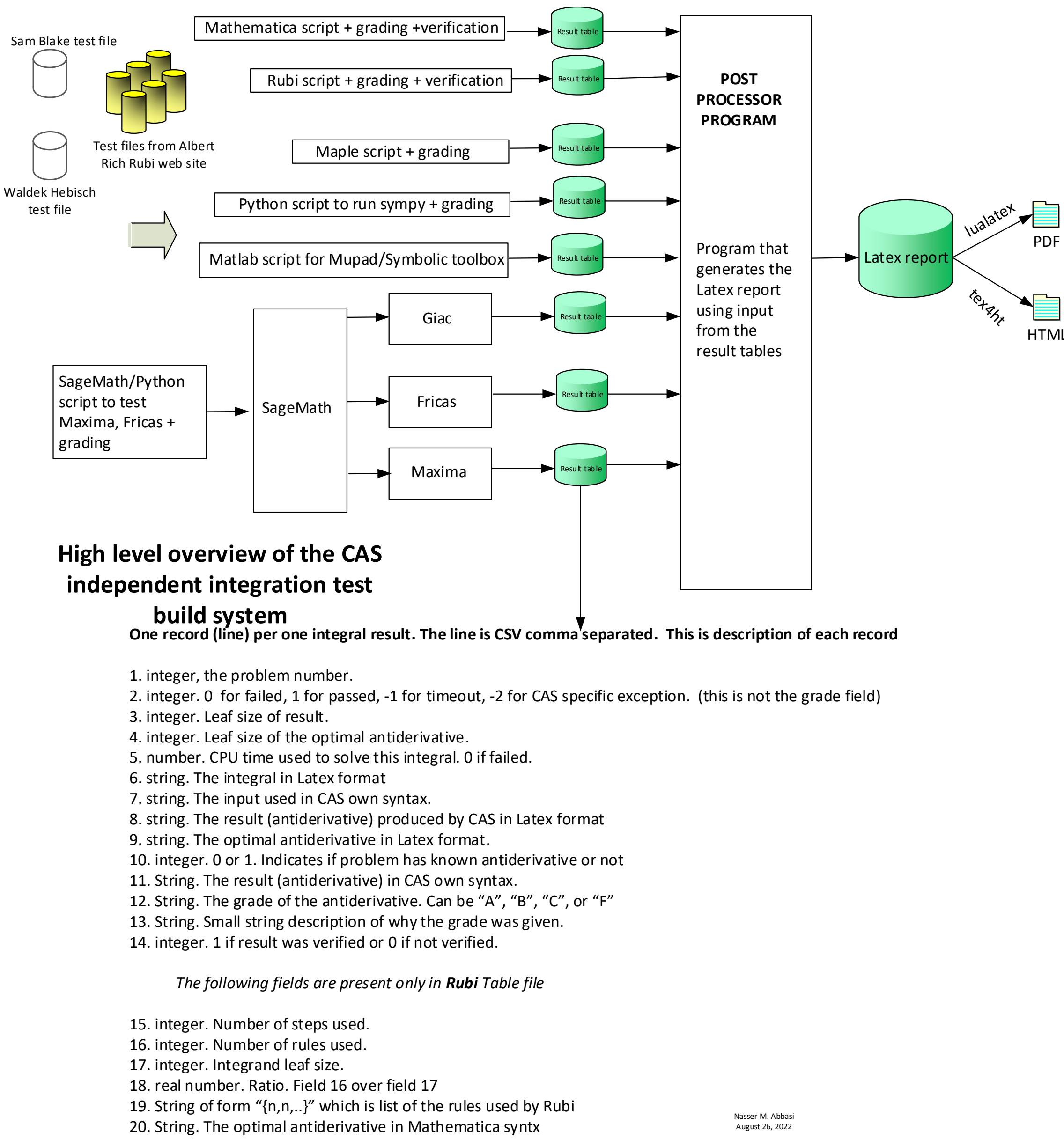
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 29, 30, 31, 32, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 138, 139, 140, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 167, 168, 169, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 201, 202, 203, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218 }

B grade: { 22, 28, 57, 131, 137, 166 }

C grade: { }

F grade: { 9, 33, 37, 49, 50, 61, 62, 63, 64, 73, 77, 91, 95, 99, 118, 142, 146, 158, 159, 170, 171, 172, 173, 182, 186, 200, 204, 208 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 36, 38, 39, 40, 41, 42, 44, 45, 46, 48, 51, 52, 53, 57, 65, 66, 67, 68, 69, 70, 72, 74, 76, 78, 79, 80, 81, 82, 84, 85, 86, 88, 89, 90, 92, 94, 96, 98, 100, 102, 103, 104, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 143, 145, 147, 148, 149, 150, 151, 153, 154, 155, 157, 160, 161, 162, 166, 174, 175, 176, 177, 178, 179, 181, 183, 185, 187, 188, 189, 190, 191, 193, 194, 195, 197, 198, 199, 201, 203, 205, 207, 209, 211, 212, 213, 215, 216, 217 }

B grade: { }

C grade: { }

F grade: { 31, 33, 35, 37, 43, 47, 49, 50, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 71, 73, 75, 77, 83, 87, 91, 93, 95, 97, 99, 101, 105, 109, 140, 142, 144, 146, 152, 156, 158, 159, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 180, 182, 184, 186, 192, 196, 200, 202, 204, 206, 208, 210, 214, 218 }

2.1.4 Maxima

A grade: { 2, 4, 6, 8, 22, 23, 24, 28, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 57, 65, 66, 67, 68, 69, 79, 80, 81, 82, 84, 85, 86, 88, 89, 90, 100, 102, 103, 104, 106, 107, 108, 111, 113, 115, 117, 131, 132, 133, 137, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 166, 174, 175, 176, 177, 178, 188, 189, 190, 191, 193, 194, 195, 197, 198, 199, 209, 211, 212, 213, 215, 216, 217 }

B grade: { }

C grade: { 1, 3, 5, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 25, 26, 27, 110, 112, 114, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 134, 135, 136 }

F grade: { 9, 19, 20, 21, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 70, 71, 72, 73, 74, 75, 76, 77, 78, 83, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 109, 118, 128, 129, 130, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 179, 180, 181, 182, 183, 184, 185, 186, 187, 192, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 214, 218 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 57, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 160, 161, 162, 166, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 201, 202, 203, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218 }

B grade: { 54, 55, 56, 58, 59, 60, 163, 164, 165, 167, 168, 169 }

C grade: { }

F grade: { 9, 33, 37, 49, 50, 61, 62, 63, 64, 73, 77, 91, 95, 99, 118, 142, 146, 158, 159, 170, 171, 172, 173, 182, 186, 200, 204, 208 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 11, 12, 13, 14, 15, 16, 17, 18, 23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 65, 66, 67, 68, 69, 71, 75, 79, 80, 81, 82, 84, 85, 86, 88, 89, 90, 93, 97, 100, 102, 103, 104, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 118, 120, 121, 122, 123, 124, 125, 126, 127, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 174, 175, 176, 177, 178, 180, 184, 188, 189, 190, 191, 193, 194, 195, 197, 198, 199, 202, 206, 209, 211, 212, 213, 215, 216, 217 }

B grade: { 8, 10, 70, 117, 119, 179 }

C grade: { }

F grade: { 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 72, 73, 74, 76, 77, 78, 83, 87, 91, 92, 94, 95, 96, 98, 99, 101, 105, 109, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 181, 182, 183, 185, 186, 187, 192, 196, 200, 201, 203, 204, 205, 207, 208, 210, 214, 218 }

2.1.7 Giac

A grade: { 23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 80, 81, 82, 84, 85, 86, 88, 89, 90, 100, 102, 103, 104, 106, 107, 108, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 189, 190, 191, 193, 194, 195, 197, 198, 199, 209, 211, 212, 213, 215, 216, 217 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 192, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 214, 218 }

2.1.8 Mupad

A grade: { 23, 24, 29, 30, 39, 40, 41, 42, 44, 45, 46, 48, 52, 53, 80, 81, 82, 84, 85, 86, 88, 89, 90, 100, 102, 103, 104, 106, 107, 108, 132, 133, 138, 139, 148, 149, 150, 151, 153, 154, 155, 157, 161, 162, 189, 190, 191, 193, 194, 195, 197, 198, 199, 209, 211, 212, 213, 215, 216, 217 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 43, 47, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 152, 156, 158, 159, 160, 163, 164, 165, 166, 167,

$168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 192, 196, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 214, 218\ \}$

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	88	123	126	84	184	0	-1
N.S.	1	1.00	0.71	0.99	1.02	0.68	1.48	0.00	-0.01
time (sec)	N/A	0.059	0.055	0.343	0.480	0.375	1.279	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	83	107	74	78	156	0	-1
N.S.	1	1.00	0.76	0.98	0.68	0.72	1.43	0.00	-0.01
time (sec)	N/A	0.074	0.039	0.355	0.270	0.356	0.945	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	79	96	110	85	53	0	-1
N.S.	1	1.00	0.80	0.97	1.11	0.86	0.54	0.00	-0.01
time (sec)	N/A	0.043	0.049	0.332	0.484	0.358	0.534	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	80	62	65	121	0	-1
N.S.	1	1.00	0.85	0.95	0.74	0.77	1.44	0.00	-0.01
time (sec)	N/A	0.054	0.033	0.333	0.281	0.340	0.895	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	70	94	58	112	0	-1
N.S.	1	1.00	1.00	0.95	1.27	0.78	1.51	0.00	-0.01
time (sec)	N/A	0.031	0.013	0.333	0.476	0.366	0.521	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	54	49	53	80	0	-1
N.S.	1	1.00	1.00	0.92	0.83	0.90	1.36	0.00	-0.02
time (sec)	N/A	0.035	0.010	0.346	0.262	0.386	0.461	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	70	51	53	0	-1
N.S.	1	1.00	1.00	0.90	1.43	1.04	1.08	0.00	-0.02
time (sec)	N/A	0.017	0.010	0.360	0.485	0.378	0.305	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	26	26	48	0	-1
N.S.	1	1.00	1.00	1.04	1.00	1.00	1.85	0.00	-0.04
time (sec)	N/A	0.003	0.003	0.425	0.268	0.334	0.473	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F	A	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	0	29	0	0	46	0	-1
N.S.	1	1.00	0.00	0.40	0.00	0.00	0.63	0.00	-0.01
time (sec)	N/A	0.030	0.012	0.328	0.000	0.000	0.298	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	38	25	42	0	-1
N.S.	1	1.00	1.00	1.04	1.41	0.93	1.56	0.00	-0.04
time (sec)	N/A	0.015	0.010	0.315	0.313	0.346	0.314	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	43	61	45	51	0	-1
N.S.	1	1.00	1.00	0.98	1.39	1.02	1.16	0.00	-0.02
time (sec)	N/A	0.020	0.011	0.327	0.530	0.352	0.338	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	49	42	62	56	0	-1
N.S.	1	1.00	1.00	0.94	0.81	1.19	1.08	0.00	-0.02
time (sec)	N/A	0.046	0.013	0.352	0.315	0.354	0.725	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	65	61	54	110	0	-1
N.S.	1	1.00	1.00	0.94	0.88	0.78	1.59	0.00	-0.01
time (sec)	N/A	0.029	0.012	0.332	0.516	0.340	0.610	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	71	48	65	46	0	-1
N.S.	1	1.00	1.00	0.92	0.62	0.84	0.60	0.00	-0.01
time (sec)	N/A	0.062	0.015	0.378	0.325	0.374	0.597	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	76	86	61	80	56	0	-1
N.S.	1	1.00	0.81	0.91	0.65	0.85	0.60	0.00	-0.01
time (sec)	N/A	0.041	0.044	0.336	0.530	0.363	0.807	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	85	93	46	98	68	0	-1
N.S.	1	1.00	0.83	0.91	0.45	0.96	0.67	0.00	-0.01
time (sec)	N/A	0.082	0.046	0.420	0.318	0.372	2.955	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	84	109	61	80	185	0	-1
N.S.	1	1.00	0.71	0.92	0.51	0.67	1.55	0.00	-0.01
time (sec)	N/A	0.054	0.044	0.322	0.518	0.350	1.819	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	96	115	48	91	48	0	-1
N.S.	1	1.00	0.76	0.91	0.38	0.72	0.38	0.00	-0.01
time (sec)	N/A	0.101	0.125	0.378	0.313	0.372	2.140	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	424	401	0	376	0	0	-1
N.S.	1	1.00	1.43	1.35	0.00	1.27	0.00	0.00	-0.00
time (sec)	N/A	0.278	0.557	0.522	0.000	0.364	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	236	251	0	248	0	0	-1
N.S.	1	1.00	1.22	1.30	0.00	1.28	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.305	0.513	0.000	0.369	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	61	109	0	132	0	0	-1
N.S.	1	1.00	0.50	0.90	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.167	0.406	0.000	0.345	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	89	33	43	45	0	0	-1
N.S.	1	1.00	2.47	0.92	1.19	1.25	0.00	0.00	-0.03
time (sec)	N/A	0.005	0.023	0.382	0.273	0.346	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.010	0.021	0.276	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.009	3.256	0.311	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	166	189	503	175	0	0	-1
N.S.	1	1.00	0.72	0.83	2.20	0.76	0.00	0.00	-0.00
time (sec)	N/A	0.127	0.241	0.400	1.161	0.369	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	115	121	424	147	0	0	-1
N.S.	1	1.00	0.78	0.82	2.88	1.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.170	0.402	0.911	0.354	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	51	80	307	104	0	0	-1
N.S.	1	1.00	0.53	0.83	3.20	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.123	0.412	0.839	0.371	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	89	33	43	45	0	0	-1
N.S.	1	1.00	2.47	0.92	1.19	1.25	0.00	0.00	-0.03
time (sec)	N/A	0.005	0.022	0.329	0.255	0.336	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.007	0.013	0.129	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.007	2.174	0.163	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	253	0	0	183	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.72	0.00	0.00	-0.00
time (sec)	N/A	0.281	0.010	0.105	0.000	0.353	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	171	324	0	184	0	0	-1
N.S.	1	1.00	0.72	1.36	0.00	0.77	0.00	0.00	-0.00
time (sec)	N/A	0.204	0.210	0.654	0.000	0.384	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.202	0.156	0.125	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	137	208	0	149	0	0	-1
N.S.	1	1.00	0.77	1.18	0.00	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.101	0.708	0.000	0.390	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	140	0	0	117	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.006	0.106	0.000	0.365	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	100	122	0	111	0	0	-1
N.S.	1	1.00	0.81	0.98	0.00	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.086	0.661	0.000	0.353	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.133	0.142	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	49	0	60	0	0	-1
N.S.	1	1.00	1.00	0.89	0.00	1.09	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.009	0.471	0.000	0.351	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.011	0.015	0.113	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.025	0.021	0.117	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.026	0.016	0.130	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.021	0.131	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	127	0	0	111	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.006	0.118	0.000	0.359	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.023	0.121	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.018	0.137	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	259	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.168	0.020	0.109	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	242	0	0	187	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.77	0.00	0.00	-0.00
time (sec)	N/A	0.267	0.010	0.107	0.000	0.372	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	286	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.244	0.021	0.115	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	497	497	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.277	0.444	0.283	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.144	0.398	0.143	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	67	60	0	89	0	0	-1
N.S.	1	1.00	0.96	0.86	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.010	0.521	0.000	0.386	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.015	0.026	0.240	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.015	0.060	0.284	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	319	0	0	448	0	0	-1
N.S.	1	1.00	1.38	0.00	0.00	1.94	0.00	0.00	-0.00
time (sec)	N/A	0.284	4.486	0.482	0.000	0.384	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	319	0	0	448	0	0	-1
N.S.	1	1.00	1.41	0.00	0.00	1.97	0.00	0.00	-0.00
time (sec)	N/A	0.252	4.379	0.428	0.000	0.371	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	316	0	0	445	0	0	-1
N.S.	1	1.00	1.48	0.00	0.00	2.08	0.00	0.00	-0.00
time (sec)	N/A	0.217	4.349	0.432	0.000	0.381	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	164	63	81	119	0	0	-1
N.S.	1	1.00	2.52	0.97	1.25	1.83	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.068	1.441	0.260	0.366	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	195	0	0	444	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	2.05	0.00	0.00	-0.00
time (sec)	N/A	0.257	2.580	0.418	0.000	0.374	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	200	0	0	460	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	2.02	0.00	0.00	-0.00
time (sec)	N/A	0.264	2.586	0.412	0.000	0.387	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	244	0	0	676	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	2.41	0.00	0.00	-0.00
time (sec)	N/A	0.403	3.541	0.204	0.000	0.410	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.023	0.151	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.023	0.140	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.034	0.260	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.035	0.462	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	0	-1
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.00	-0.08
time (sec)	N/A	0.010	0.005	0.061	0.268	0.367	0.200	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	0	-1
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.00	-0.08
time (sec)	N/A	0.008	0.004	0.057	0.260	0.363	0.101	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	8	0	-1
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.89	0.00	-0.11
time (sec)	N/A	0.010	0.014	0.115	0.252	0.332	0.103	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	10	0	-1
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.91	0.00	-0.09
time (sec)	N/A	0.010	0.004	0.059	0.259	0.342	0.264	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	14	0	-1
N.S.	1	1.00	1.00	0.92	0.85	0.85	1.08	0.00	-0.08
time (sec)	N/A	0.010	0.004	0.062	0.252	0.345	0.439	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	0	18	31	0	-1
N.S.	1	1.00	1.00	1.06	0.00	1.06	1.82	0.00	-0.06
time (sec)	N/A	0.012	0.006	0.069	0.000	0.357	0.647	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	232	0	0	169	301	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.73	1.30	0.00	-0.00
time (sec)	N/A	0.267	0.010	0.024	0.000	0.353	16.723	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	153	318	0	167	0	0	-1
N.S.	1	1.00	0.71	1.47	0.00	0.77	0.00	0.00	-0.00
time (sec)	N/A	0.183	0.200	0.589	0.000	0.358	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.194	0.337	0.026	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	120	202	0	132	0	0	-1
N.S.	1	1.00	0.76	1.28	0.00	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.124	0.585	0.000	0.351	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	105	151	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.88	1.26	0.00	-0.01
time (sec)	N/A	0.085	0.006	0.023	0.000	0.350	1.650	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	83	115	0	94	0	0	-1
N.S.	1	1.00	0.79	1.10	0.00	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.068	0.639	0.000	0.360	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.165	0.024	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	46	0	47	0	0	-1
N.S.	1	1.00	0.90	0.94	0.00	0.96	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.019	0.426	0.000	0.346	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	0	-1
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.00	-0.08
time (sec)	N/A	0.008	0.003	0.072	0.255	0.351	0.100	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.012	0.026	0.025	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.013	0.022	0.023	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.024	0.023	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	109	0	0	98	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.007	0.029	0.000	0.371	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.023	0.023	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.024	0.029	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	241	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.142	0.026	0.023	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	224	0	0	172	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.77	0.00	0.00	-0.00
time (sec)	N/A	0.247	0.011	0.029	0.000	0.374	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.214	0.024	0.028	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	263	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.326	0.022	0.033	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.010	0.052	0.226	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.298	0.035	0.219	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	163	321	0	169	0	0	-1
N.S.	1	1.00	0.75	1.48	0.00	0.78	0.00	0.00	-0.00
time (sec)	N/A	0.184	0.134	0.766	0.000	0.361	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	184	0	0	141	264	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.77	1.43	0.00	-0.01
time (sec)	N/A	0.169	0.008	0.191	0.000	0.343	5.462	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	126	212	0	139	0	0	-1
N.S.	1	1.00	0.76	1.28	0.00	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.127	0.824	0.000	0.369	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.030	0.206	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	90	119	0	97	0	0	-1
N.S.	1	1.00	0.83	1.10	0.00	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.061	0.856	0.000	0.351	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	58	114	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.79	1.56	0.00	-0.01
time (sec)	N/A	0.039	0.005	0.207	0.000	0.364	0.505	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	48	52	0	53	0	0	-1
N.S.	1	1.00	0.81	0.88	0.00	0.90	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.022	0.814	0.000	0.374	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.011	0.012	0.193	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.012	0.026	0.205	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	46	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.96	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.006	0.201	0.000	0.360	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.023	0.230	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.024	0.224	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	156	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.025	0.194	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	163	0	0	141	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.009	0.213	0.000	0.358	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	231	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.135	0.023	0.217	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	0.023	0.188	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	271	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.206	0.028	0.233	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	278	0	0	203	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.73	0.00	0.00	-0.00
time (sec)	N/A	0.337	0.013	0.239	0.000	0.365	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	89	123	126	85	184	0	-1
N.S.	1	1.00	0.72	0.99	1.02	0.69	1.48	0.00	-0.01
time (sec)	N/A	0.057	0.055	0.297	0.491	0.360	1.251	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	83	107	74	79	153	0	-1
N.S.	1	1.00	0.76	0.98	0.68	0.72	1.40	0.00	-0.01
time (sec)	N/A	0.080	0.041	0.312	0.268	0.370	1.184	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	80	97	110	86	49	0	-1
N.S.	1	1.00	0.81	0.98	1.11	0.87	0.49	0.00	-0.01
time (sec)	N/A	0.045	0.043	0.320	0.481	0.349	0.527	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	81	61	66	116	0	-1
N.S.	1	1.00	0.85	0.96	0.73	0.79	1.38	0.00	-0.01
time (sec)	N/A	0.056	0.030	0.328	0.282	0.350	0.621	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	70	94	59	112	0	-1
N.S.	1	1.00	1.00	0.95	1.27	0.80	1.51	0.00	-0.01
time (sec)	N/A	0.032	0.013	0.296	0.477	0.329	0.507	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	54	49	54	80	0	-1
N.S.	1	1.00	1.00	0.92	0.83	0.92	1.36	0.00	-0.02
time (sec)	N/A	0.036	0.010	0.296	0.265	0.350	0.543	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	70	51	49	0	-1
N.S.	1	1.00	1.00	0.90	1.43	1.04	1.00	0.00	-0.02
time (sec)	N/A	0.018	0.009	0.303	0.482	0.364	0.298	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	27	28	44	0	-1
N.S.	1	1.00	1.00	1.04	1.00	1.04	1.63	0.00	-0.04
time (sec)	N/A	0.004	0.003	0.386	0.265	0.373	0.365	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F	A	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	0	23	0	0	41	0	-1
N.S.	1	1.00	0.00	0.33	0.00	0.00	0.59	0.00	-0.01
time (sec)	N/A	0.033	0.012	0.379	0.000	0.000	0.288	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	34	38	53	0	-1
N.S.	1	1.00	1.00	1.04	1.26	1.41	1.96	0.00	-0.04
time (sec)	N/A	0.017	0.009	0.365	0.345	0.351	0.504	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	43	61	42	51	0	-1
N.S.	1	1.00	1.00	0.98	1.39	0.95	1.16	0.00	-0.02
time (sec)	N/A	0.020	0.009	0.309	0.532	0.377	0.358	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	49	44	44	42	0	-1
N.S.	1	1.00	1.00	0.94	0.85	0.85	0.81	0.00	-0.02
time (sec)	N/A	0.046	0.012	0.324	0.322	0.370	0.399	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	64	61	56	110	0	-1
N.S.	1	1.00	1.00	0.93	0.88	0.81	1.59	0.00	-0.01
time (sec)	N/A	0.031	0.012	0.297	0.529	0.357	0.605	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	71	46	85	65	0	-1
N.S.	1	1.00	1.00	0.92	0.60	1.10	0.84	0.00	-0.01
time (sec)	N/A	0.067	0.015	0.375	0.315	0.355	1.340	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	74	87	61	79	56	0	-1
N.S.	1	1.00	0.79	0.93	0.65	0.84	0.60	0.00	-0.01
time (sec)	N/A	0.042	0.076	0.318	0.525	0.363	0.806	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	84	93	48	78	44	0	-1
N.S.	1	1.00	0.82	0.91	0.47	0.76	0.43	0.00	-0.01
time (sec)	N/A	0.082	0.090	0.319	0.328	0.360	1.123	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	85	108	61	81	185	0	-1
N.S.	1	1.00	0.71	0.91	0.51	0.68	1.55	0.00	-0.01
time (sec)	N/A	0.053	0.047	0.315	0.535	0.362	1.843	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	96	115	46	111	76	0	-1
N.S.	1	1.00	0.76	0.91	0.36	0.87	0.60	0.00	-0.01
time (sec)	N/A	0.100	0.058	0.421	0.311	0.359	7.385	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	424	398	0	375	0	0	-1
N.S.	1	1.00	1.42	1.34	0.00	1.26	0.00	0.00	-0.00
time (sec)	N/A	0.256	0.531	0.494	0.000	0.357	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	237	251	0	249	0	0	-1
N.S.	1	1.00	1.22	1.29	0.00	1.28	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.306	0.487	0.000	0.358	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	74	108	0	132	0	0	-1
N.S.	1	1.00	0.61	0.89	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.168	0.435	0.000	0.372	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	90	34	44	47	0	0	-1
N.S.	1	1.00	2.43	0.92	1.19	1.27	0.00	0.00	-0.03
time (sec)	N/A	0.005	0.023	0.385	0.258	0.337	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.010	0.019	0.361	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.009	1.588	0.368	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	166	187	502	176	0	0	-1
N.S.	1	1.00	0.73	0.82	2.21	0.78	0.00	0.00	-0.00
time (sec)	N/A	0.122	0.206	0.395	1.184	0.391	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	116	122	423	148	0	0	-1
N.S.	1	1.00	0.78	0.82	2.86	1.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.204	0.379	0.903	0.374	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	59	79	311	104	0	0	-1
N.S.	1	1.00	0.62	0.83	3.27	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.101	0.381	0.823	0.364	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	90	34	44	47	0	0	-1
N.S.	1	1.00	2.43	0.92	1.19	1.27	0.00	0.00	-0.03
time (sec)	N/A	0.005	0.021	0.243	0.262	0.356	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.007	0.014	0.257	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.008	0.974	0.243	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	253	0	0	183	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.72	0.00	0.00	-0.00
time (sec)	N/A	0.293	0.013	0.141	0.000	0.356	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	170	324	0	184	0	0	-1
N.S.	1	1.00	0.71	1.36	0.00	0.77	0.00	0.00	-0.00
time (sec)	N/A	0.204	0.167	0.795	0.000	0.375	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.207	0.142	0.122	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	137	209	0	149	0	0	-1
N.S.	1	1.00	0.77	1.18	0.00	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.095	0.805	0.000	0.377	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	140	0	0	118	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.008	0.131	0.000	0.350	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	100	122	0	111	0	0	-1
N.S.	1	1.00	0.81	0.98	0.00	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.066	0.748	0.000	0.350	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.132	0.116	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	49	0	59	0	0	-1
N.S.	1	1.00	1.00	0.91	0.00	1.09	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.010	0.398	0.000	0.356	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.011	0.013	0.019	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.023	0.020	0.121	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.024	0.016	0.115	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.020	0.111	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	127	0	0	102	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.006	0.106	0.000	0.367	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.019	0.110	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.015	0.124	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	259	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.163	0.020	0.064	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	242	0	0	178	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.74	0.00	0.00	-0.00
time (sec)	N/A	0.266	0.015	0.138	0.000	0.382	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	286	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.240	0.021	0.107	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.275	0.439	0.237	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.136	0.392	0.127	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	60	0	88	0	0	-1
N.S.	1	1.00	0.96	0.87	0.00	1.28	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.010	0.477	0.000	0.355	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.015	0.028	0.171	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.015	0.058	0.300	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	318	0	0	448	0	0	-1
N.S.	1	1.00	1.38	0.00	0.00	1.94	0.00	0.00	-0.00
time (sec)	N/A	0.264	4.464	0.545	0.000	0.373	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	318	0	0	448	0	0	-1
N.S.	1	1.00	1.40	0.00	0.00	1.97	0.00	0.00	-0.00
time (sec)	N/A	0.235	4.335	0.470	0.000	0.395	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	315	0	0	445	0	0	-1
N.S.	1	1.00	1.47	0.00	0.00	2.08	0.00	0.00	-0.00
time (sec)	N/A	0.187	4.324	0.542	0.000	0.386	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	165	64	82	121	0	0	-1
N.S.	1	1.00	2.50	0.97	1.24	1.83	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.068	1.471	0.255	0.369	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	194	0	0	444	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	2.05	0.00	0.00	-0.00
time (sec)	N/A	0.259	2.583	0.468	0.000	0.377	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	199	0	0	460	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	2.02	0.00	0.00	-0.00
time (sec)	N/A	0.253	2.583	0.500	0.000	0.352	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	244	0	0	674	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	2.41	0.00	0.00	-0.00
time (sec)	N/A	0.370	3.496	0.129	0.000	0.388	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.023	0.037	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.024	0.039	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.032	0.186	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.033	0.390	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	0	-1
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.00	-0.08
time (sec)	N/A	0.011	0.007	0.172	0.260	0.351	0.199	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	0	-1
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.00	-0.08
time (sec)	N/A	0.008	0.006	0.188	0.261	0.368	0.099	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	10	0	-1
N.S.	1	1.00	1.00	1.11	1.00	1.00	1.11	0.00	-0.11
time (sec)	N/A	0.010	0.009	0.234	0.260	0.333	0.101	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	12	0	-1
N.S.	1	1.00	1.00	1.09	1.00	1.00	1.09	0.00	-0.09
time (sec)	N/A	0.010	0.006	0.159	0.259	0.324	0.263	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	15	0	-1
N.S.	1	1.00	1.00	0.92	0.85	0.85	1.15	0.00	-0.08
time (sec)	N/A	0.011	0.006	0.186	0.254	0.325	0.436	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	0	18	34	0	-1
N.S.	1	1.00	1.00	1.06	0.00	1.06	2.00	0.00	-0.06
time (sec)	N/A	0.013	0.009	0.190	0.000	0.344	0.610	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	231	0	0	169	301	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.73	1.30	0.00	-0.00
time (sec)	N/A	0.277	0.011	0.229	0.000	0.350	16.704	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	154	317	0	167	0	0	-1
N.S.	1	1.00	0.72	1.47	0.00	0.78	0.00	0.00	-0.00
time (sec)	N/A	0.185	0.170	0.902	0.000	0.366	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.180	0.263	0.279	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	120	202	0	132	0	0	-1
N.S.	1	1.00	0.76	1.29	0.00	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.092	0.947	0.000	0.373	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	105	151	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.88	1.26	0.00	-0.01
time (sec)	N/A	0.089	0.007	0.243	0.000	0.358	1.612	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	83	114	0	94	0	0	-1
N.S.	1	1.00	0.80	1.10	0.00	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.060	0.893	0.000	0.359	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.150	0.238	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	44	45	0	47	0	0	-1
N.S.	1	1.00	0.92	0.94	0.00	0.98	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.020	0.609	0.000	0.351	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	0	-1
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.00	-0.08
time (sec)	N/A	0.009	0.005	0.159	0.259	0.356	0.099	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.012	0.021	0.240	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.013	0.020	0.244	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.027	0.232	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	109	0	0	93	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.009	0.226	0.000	0.374	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.026	0.244	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.023	0.246	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	241	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.139	0.023	0.250	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	224	0	0	168	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.75	0.00	0.00	-0.00
time (sec)	N/A	0.257	0.010	0.224	0.000	0.388	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.213	0.023	0.234	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	263	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.341	0.025	0.229	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.009	0.050	0.038	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.307	0.031	0.027	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	163	322	0	169	0	0	-1
N.S.	1	1.00	0.75	1.48	0.00	0.78	0.00	0.00	-0.00
time (sec)	N/A	0.182	0.124	0.849	0.000	0.370	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	185	0	0	141	264	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.76	1.43	0.00	-0.01
time (sec)	N/A	0.173	0.008	0.027	0.000	0.365	5.641	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	126	212	0	139	0	0	-1
N.S.	1	1.00	0.75	1.27	0.00	0.83	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.109	0.816	0.000	0.360	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.026	0.025	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	90	120	0	97	0	0	-1
N.S.	1	1.00	0.83	1.10	0.00	0.89	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.063	0.796	0.000	0.369	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	0	0	67	114	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.91	1.54	0.00	-0.01
time (sec)	N/A	0.040	0.007	0.026	0.000	0.362	0.440	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	48	52	0	52	0	0	-1
N.S.	1	1.00	0.80	0.87	0.00	0.87	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.019	0.622	0.000	0.359	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.012	0.026	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.012	0.022	0.032	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	45	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.94	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.005	0.026	0.000	0.400	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.026	0.026	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.026	0.025	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	156	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.023	0.029	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	163	0	0	141	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.009	0.026	0.000	0.345	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	231	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.137	0.023	0.027	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.220	0.023	0.033	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	271	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.208	0.024	0.027	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	278	0	0	203	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.73	0.00	0.00	-0.00
time (sec)	N/A	0.344	0.013	0.026	0.000	0.372	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [61] had the largest ratio of [22]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.00	8	0.500
2	A	6	4	1.00	8	0.500
3	A	5	4	1.00	8	0.500
4	A	5	4	1.00	8	0.500
5	A	4	4	1.00	8	0.500
6	A	4	4	1.00	8	0.500
7	A	3	3	1.00	6	0.500
8	A	1	1	1.00	4	0.250
9	A	3	3	1.00	8	0.375
10	A	2	2	1.00	8	0.250
11	A	3	3	1.00	8	0.375
12	A	4	4	1.00	8	0.500
13	A	4	4	1.00	8	0.500
14	A	5	4	1.00	8	0.500
15	A	5	4	1.00	8	0.500
16	A	6	4	1.00	8	0.500
17	A	6	4	1.00	8	0.500
18	A	7	4	1.00	8	0.500
19	A	14	10	1.00	14	0.714
20	A	11	9	1.00	14	0.643
21	A	8	7	1.00	12	0.583
22	A	1	1	1.00	6	0.167
23	A	0	0	0.00	0	0.000
24	A	0	0	0.00	0	0.000
25	A	14	10	1.00	10	1.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	11	9	1.00	10	0.900
27	A	8	7	1.00	8	0.875
28	A	1	1	1.00	6	0.167
29	A	0	0	0.00	0	0.000
30	A	0	0	0.00	0	0.000
31	A	23	10	1.00	10	1.000
32	A	19	10	1.00	10	1.000
33	A	16	9	1.00	10	0.900
34	A	12	9	1.00	10	0.900
35	A	10	9	1.00	10	0.900
36	A	8	6	1.00	10	0.600
37	A	5	5	1.00	8	0.625
38	A	4	4	1.00	6	0.667
39	A	0	0	0.00	0	0.000
40	A	0	0	0.00	0	0.000
41	A	0	0	0.00	0	0.000
42	A	0	0	0.00	0	0.000
43	A	9	9	1.00	10	0.900
44	A	0	0	0.00	0	0.000
45	A	0	0	0.00	0	0.000
46	A	0	0	0.00	0	0.000
47	A	20	10	1.00	10	1.000
48	A	0	0	0.00	0	0.000
49	A	18	13	1.00	16	0.812
50	A	10	9	1.00	14	0.643
51	A	4	3	1.00	8	0.375
52	A	0	0	0.00	0	0.000
53	A	0	0	0.00	0	0.000
54	A	10	7	1.00	17	0.412
55	A	10	7	1.00	15	0.467
56	A	10	7	1.00	13	0.538
57	A	3	1	1.00	17	0.059
58	A	10	7	1.00	17	0.412
59	A	10	7	1.00	17	0.412
60	A	10	7	1.00	19	0.368

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	4	4	1.00	22	0.182
62	A	4	4	1.00	22	0.182
63	A	4	4	1.00	19	0.210
64	A	4	4	1.00	19	0.210
65	A	2	2	1.00	19	0.105
66	A	2	2	1.00	17	0.118
67	A	2	2	1.00	19	0.105
68	A	2	2	1.00	19	0.105
69	A	2	2	1.00	19	0.105
70	A	2	2	1.00	19	0.105
71	A	22	9	1.00	20	0.450
72	A	18	9	1.00	20	0.450
73	A	15	8	1.00	20	0.400
74	A	11	8	1.00	20	0.400
75	A	9	8	1.00	20	0.400
76	A	7	5	1.00	20	0.250
77	A	4	4	1.00	20	0.200
78	A	2	2	1.00	18	0.111
79	A	2	2	1.00	17	0.118
80	A	0	0	0.00	0	0.000
81	A	0	0	0.00	0	0.000
82	A	0	0	0.00	0	0.000
83	A	8	8	1.00	20	0.400
84	A	0	0	0.00	0	0.000
85	A	0	0	0.00	0	0.000
86	A	0	0	0.00	0	0.000
87	A	19	9	1.00	20	0.450
88	A	0	0	0.00	0	0.000
89	A	0	0	0.00	0	0.000
90	A	0	0	0.00	0	0.000
91	A	23	8	1.00	20	0.400
92	A	18	8	1.00	20	0.400
93	A	16	9	1.00	20	0.450
94	A	13	9	1.00	20	0.450
95	A	10	8	1.00	20	0.400

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	7	6	1.00	20	0.300
97	A	5	5	1.00	20	0.250
98	A	4	3	1.00	18	0.167
99	A	1	1	1.00	17	0.059
100	A	0	0	0.00	0	0.000
101	A	4	4	1.00	20	0.200
102	A	0	0	0.00	0	0.000
103	A	0	0	0.00	0	0.000
104	A	0	0	0.00	0	0.000
105	A	13	9	1.00	20	0.450
106	A	0	0	0.00	0	0.000
107	A	0	0	0.00	0	0.000
108	A	0	0	0.00	0	0.000
109	A	26	9	1.00	20	0.450
110	A	6	4	1.00	8	0.500
111	A	6	4	1.00	8	0.500
112	A	5	4	1.00	8	0.500
113	A	5	4	1.00	8	0.500
114	A	4	4	1.00	8	0.500
115	A	4	4	1.00	8	0.500
116	A	3	3	1.00	6	0.500
117	A	1	1	1.00	4	0.250
118	A	3	3	1.00	8	0.375
119	A	2	2	1.00	8	0.250
120	A	3	3	1.00	8	0.375
121	A	4	4	1.00	8	0.500
122	A	4	4	1.00	8	0.500
123	A	5	4	1.00	8	0.500
124	A	5	4	1.00	8	0.500
125	A	6	4	1.00	8	0.500
126	A	6	4	1.00	8	0.500
127	A	7	4	1.00	8	0.500
128	A	14	10	1.00	14	0.714
129	A	11	9	1.00	14	0.643
130	A	8	7	1.00	12	0.583

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	1	1	1.00	6	0.167
132	A	0	0	0.00	0	0.000
133	A	0	0	0.00	0	0.000
134	A	14	10	1.00	10	1.000
135	A	11	9	1.00	10	0.900
136	A	8	7	1.00	8	0.875
137	A	1	1	1.00	6	0.167
138	A	0	0	0.00	0	0.000
139	A	0	0	0.00	0	0.000
140	A	23	11	1.00	10	1.100
141	A	19	10	1.00	10	1.000
142	A	16	10	1.00	10	1.000
143	A	12	9	1.00	10	0.900
144	A	10	10	1.00	10	1.000
145	A	8	6	1.00	10	0.600
146	A	5	5	1.00	8	0.625
147	A	4	4	1.00	6	0.667
148	A	0	0	0.00	0	0.000
149	A	0	0	0.00	0	0.000
150	A	0	0	0.00	0	0.000
151	A	0	0	0.00	0	0.000
152	A	9	9	1.00	10	0.900
153	A	0	0	0.00	0	0.000
154	A	0	0	0.00	0	0.000
155	A	0	0	0.00	0	0.000
156	A	20	10	1.00	10	1.000
157	A	0	0	0.00	0	0.000
158	A	18	13	1.00	16	0.812
159	A	10	9	1.00	14	0.643
160	A	4	3	1.00	8	0.375
161	A	0	0	0.00	0	0.000
162	A	0	0	0.00	0	0.000
163	A	10	7	1.00	17	0.412
164	A	10	7	1.00	15	0.467
165	A	10	7	1.00	13	0.538

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	3	1	1.00	17	0.059
167	A	10	7	1.00	17	0.412
168	A	10	7	1.00	17	0.412
169	A	10	7	1.00	19	0.368
170	A	4	4	1.00	22	0.182
171	A	4	4	1.00	22	0.182
172	A	4	4	1.00	19	0.210
173	A	4	4	1.00	19	0.210
174	A	2	2	1.00	19	0.105
175	A	2	2	1.00	17	0.118
176	A	2	2	1.00	19	0.105
177	A	2	2	1.00	19	0.105
178	A	2	2	1.00	19	0.105
179	A	2	2	1.00	19	0.105
180	A	22	10	1.00	20	0.500
181	A	18	9	1.00	20	0.450
182	A	15	9	1.00	20	0.450
183	A	11	8	1.00	20	0.400
184	A	9	9	1.00	20	0.450
185	A	7	5	1.00	20	0.250
186	A	4	4	1.00	20	0.200
187	A	2	2	1.00	18	0.111
188	A	2	2	1.00	17	0.118
189	A	0	0	0.00	0	0.000
190	A	0	0	0.00	0	0.000
191	A	0	0	0.00	0	0.000
192	A	8	8	1.00	20	0.400
193	A	0	0	0.00	0	0.000
194	A	0	0	0.00	0	0.000
195	A	0	0	0.00	0	0.000
196	A	19	9	1.00	20	0.450
197	A	0	0	0.00	0	0.000
198	A	0	0	0.00	0	0.000
199	A	0	0	0.00	0	0.000
200	A	23	9	1.00	20	0.450

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	18	8	1.00	20	0.400
202	A	16	10	1.00	20	0.500
203	A	13	9	1.00	20	0.450
204	A	10	9	1.00	20	0.450
205	A	7	6	1.00	20	0.300
206	A	5	5	1.00	20	0.250
207	A	4	3	1.00	18	0.167
208	A	1	1	1.00	17	0.059
209	A	0	0	0.00	0	0.000
210	A	4	4	1.00	20	0.200
211	A	0	0	0.00	0	0.000
212	A	0	0	0.00	0	0.000
213	A	0	0	0.00	0	0.000
214	A	13	9	1.00	20	0.450
215	A	0	0	0.00	0	0.000
216	A	0	0	0.00	0	0.000
217	A	0	0	0.00	0	0.000
218	A	26	9	1.00	20	0.450

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^7 S(bx)\,dx$

Optimal. Leaf size=124

$$-\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3}+\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi}-\frac{105S(bx)}{8b^8\pi^4}+\frac{1}{8}x^8S(bx)+\frac{105x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4}-\frac{7x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2}$$

```
[Out] -35/8*x^3*cos(1/2*b^2*Pi*x^2)/b^5/Pi^3+1/8*x^7*cos(1/2*b^2*Pi*x^2)/b/Pi-105
/8*FresnelS(b*x)/b^8/Pi^4+1/8*x^8*FresnelS(b*x)+105/8*x*sin(1/2*b^2*Pi*x^2)
/b^7/Pi^4-7/8*x^5*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A]
time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6561, 3466, 3467, 3432}

$$-\frac{105S(bx)}{8\pi^4b^8}+\frac{x^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi b}+\frac{105x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^4b^7}-\frac{35x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^3b^5}-\frac{7x^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^2b^3}+\frac{1}{8}x^8S(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^7*FresnelS[b*x],x]
```

```
[Out] (-35*x^3*Cos[(b^2*Pi*x^2)/2])/(8*b^5*Pi^3) + (x^7*Cos[(b^2*Pi*x^2)/2])/(8*b
*Pi) - (105*FresnelS[b*x])/(8*b^8*Pi^4) + (x^8*FresnelS[b*x])/8 + (105*x*Si
n[(b^2*Pi*x^2)/2])/(8*b^7*Pi^4) - (7*x^5*Sin[(b^2*Pi*x^2)/2])/(8*b^3*Pi^2)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^7 S(bx)\,dx &= \frac{1}{8}x^8 S(bx) - \frac{1}{8}b\int x^8 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\
&= \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{1}{8}x^8 S(bx) - \frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{8b\pi}\\
&= \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{1}{8}x^8 S(bx) - \frac{7x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} + \frac{35\int x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{8b^3\pi^2}\\
&= -\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{1}{8}x^8 S(bx) - \frac{7x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} + \frac{105\int x^2\cos\left(\frac{1}{2}b^2\right.}{8b^5\pi^3}\\
&= -\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{1}{8}x^8 S(bx) + \frac{105x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{7x^5\sin\left(\frac{1}{2}b^2\pi x^2\right.}{8b^3\pi^2}\\
&= -\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} - \frac{105S(bx)}{8b^8\pi^4} + \frac{1}{8}x^8 S(bx) + \frac{105x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{7x^5
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 88, normalized size = 0.71

$$\frac{b^3\pi x^3(-35+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right) + (-105+b^8\pi^4x^8)\,S(bx) - 7bx(-15+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^8\pi^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*FresnelS[b*x],x]
```

```
[Out] (b^3*Pi*x^3*(-35 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8
)*FresnelS[b*x] - 7*b*x*(-15 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(8*b^8*Pi
^4)
```

Maple [A]
time = 0.34, size = 123, normalized size = 0.99

method	result	size
meijerg	$\frac{\pi b^3 x^{11}\,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{11}{4}\right],\left[\frac{3}{2},\frac{7}{4},\frac{15}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{66}$	29

derivativedivides	$\frac{\frac{S(bx)b^8x^8}{8}+\frac{b^7x^7\cos\left(\frac{b^2\pi x^2}{2}\right)}{8\pi}-\frac{7\left(\frac{b^5x^5\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{5\left(-\frac{b^3x^3\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{3bx\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{3\,S(bx)}{\pi}}{\pi}\right)}{\pi}\right)}{8\pi}}{b^8}$	123
default	$\frac{\frac{S(bx)b^8x^8}{8}+\frac{b^7x^7\cos\left(\frac{b^2\pi x^2}{2}\right)}{8\pi}-\frac{7\left(\frac{b^5x^5\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{5\left(-\frac{b^3x^3\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{3bx\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{3\,S(bx)}{\pi}}{\pi}\right)}{\pi}\right)}{8\pi}}{b^8}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^8*(1/8*FresnelS(b*x)*b^8*x^8+1/8/Pi*b^7*x^7*cos(1/2*b^2*Pi*x^2)-7/8/Pi*
(1/Pi*b^5*x^5*sin(1/2*b^2*Pi*x^2)-5/Pi*(-1/Pi*b^3*x^3*cos(1/2*b^2*Pi*x^2)+3
/Pi*(1/Pi*b*x*sin(1/2*b^2*Pi*x^2)-1/Pi*FresnelS(b*x)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.48, size = 126, normalized size = 1.02

$$\frac{1}{8}\,x^8\,S(bx)-\frac{\sqrt{\frac{1}{2}}\left((105i+105)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{\frac{1}{2}i\pi}\,bx\right)-(105i-105)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{-\frac{1}{2}i\pi}\,bx\right)-4\left(\sqrt{\frac{1}{2}}\,\pi^4b^7x^7-35\sqrt{\frac{1}{2}}\,\pi^2b^3x^3\right)\cos\left(\frac{1}{2}\pi b^2x^2\right)+28\left(\sqrt{\frac{1}{2}}\,\pi^3b^5x^5-15\sqrt{\frac{1}{2}}\,\pi bx\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)\right)}{16\,\pi^5b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] 1/8*x^8*fresnel_sin(b*x) - 1/16*sqrt(1/2)*((105*I + 105)*(1/4)^(1/4)*pi*erf
(sqrt(1/2*I*pi)*b*x) - (105*I - 105)*(1/4)^(1/4)*pi*erf(sqrt(-1/2*I*pi)*b*x
) - 4*(sqrt(1/2)*pi^4*b^7*x^7 - 35*sqrt(1/2)*pi^2*b^3*x^3)*cos(1/2*pi*b^2*x
^2) + 28*(sqrt(1/2)*pi^3*b^5*x^5 - 15*sqrt(1/2)*pi*b*x)*sin(1/2*pi*b^2*x^2)
)/(pi^5*b^8)
```

Fricas [A]
time = 0.37, size = 84, normalized size = 0.68

$$\frac{\left(\pi^3b^7x^7-35\,\pi b^3x^3\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)+\left(\pi^4b^8x^8-105\right)S\left(bx\right)-7\left(\pi^2b^5x^5-15\,bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{8\,\pi^4b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnel_sin(b*x),x, algorithm="fricas")
```

```
[Out] 1/8*((pi^3*b^7*x^7 - 35*pi*b^3*x^3)*cos(1/2*pi*b^2*x^2) + (pi^4*b^8*x^8 - 1
05)*fresnel_sin(b*x) - 7*(pi^2*b^5*x^5 - 15*b*x)*sin(1/2*pi*b^2*x^2))/(pi^4
*b^8)
```

Sympy [A]
time = 1.28, size = 184, normalized size = 1.48

$$\frac{231x^8S(bx)\,\Gamma\left(\frac{3}{4}\right)}{512\Gamma\left(\frac{15}{4}\right)}+\frac{231x^7\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{512\pi b\Gamma\left(\frac{15}{4}\right)}-\frac{1617x^5\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{512\pi^2b^3\Gamma\left(\frac{15}{4}\right)}-\frac{8085x^3\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{512\pi^3b^5\Gamma\left(\frac{15}{4}\right)}+\frac{24255x\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{512\pi^4b^7\Gamma\left(\frac{15}{4}\right)}-\frac{24255S(bx)\,\Gamma\left(\frac{3}{4}\right)}{512\pi^4b^8\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*fresnels(b*x),x)
```

```
[Out] 231*x**8*fresnels(b*x)*gamma(3/4)/(512*gamma(15/4)) + 231*x**7*cos(pi*b**2*
x**2/2)*gamma(3/4)/(512*pi*b*gamma(15/4)) - 1617*x**5*sin(pi*b**2*x**2/2)*g
amma(3/4)/(512*pi**2*b**3*gamma(15/4)) - 8085*x**3*cos(pi*b**2*x**2/2)*gamm
a(3/4)/(512*pi**3*b**5*gamma(15/4)) + 24255*x*sin(pi*b**2*x**2/2)*gamma(3/4
)/(512*pi**4*b**7*gamma(15/4)) - 24255*fresnels(b*x)*gamma(3/4)/(512*pi**4*
b**8*gamma(15/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnel_sin(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^7*fresnel_sin(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^7\,\mathrm{FresnelS}(b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelS(b*x),x)
```

```
[Out] int(x^7*FresnelS(b*x), x)
```

3.2 $\int x^6 S(bx)\,dx$

Optimal. Leaf size=109

$$-\frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3}+\frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi}+\frac{1}{7}x^7S(bx)+\frac{48\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4}-\frac{6x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2}$$

```
[Out] -24/7*x^2*cos(1/2*b^2*Pi*x^2)/b^5/Pi^3+1/7*x^6*cos(1/2*b^2*Pi*x^2)/b/Pi+1/7
*x^7*FresnelS(b*x)+48/7*sin(1/2*b^2*Pi*x^2)/b^7/Pi^4-6/7*x^4*sin(1/2*b^2*Pi
*x^2)/b^3/Pi^2
```

Rubi [A]
time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6561, 3460, 3377, 2717}

$$\frac{x^6\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b}+\frac{48\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7}-\frac{24x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5}-\frac{6x^4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3}+\frac{1}{7}x^7S(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^6*FresnelS[b*x],x]
```

```
[Out] (-24*x^2*Cos[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) + (x^6*Cos[(b^2*Pi*x^2)/2])/(7*b
*Pi) + (x^7*FresnelS[b*x])/7 + (48*Sin[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) - (6*x
^4*Sin[(b^2*Pi*x^2)/2])/(7*b^3*Pi^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^6 S(bx)\,dx &= \frac{1}{7}x^7 S(bx) - \frac{1}{7}b\int x^7 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{1}{7}x^7 S(bx) - \frac{1}{14}b\text{Subst}\left(\int x^3 \sin\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right) \\
&= \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{1}{7}x^7 S(bx) - \frac{3\text{Subst}\left(\int x^2\cos\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{7b\pi} \\
&= \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{1}{7}x^7 S(bx) - \frac{6x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{12\text{Subst}\left(\int x\sin\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{7b^3\pi^2} \\
&= -\frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} + \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{1}{7}x^7 S(bx) - \frac{6x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{24\text{Subst}\left(\int \cos\left(\frac{1}{2}\right.\right.}{7b^5} \\
&= -\frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} + \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{1}{7}x^7 S(bx) + \frac{48\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4} - \frac{6x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 83, normalized size = 0.76

$$\frac{x^2(-24+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} + \frac{1}{7}x^7 S(bx) - \frac{6(-8+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*FresnelS[b*x],x]

[Out] (x^2*(-24 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) + (x^7*FresnelS
[b*x])/7 - (6*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4)
```

Maple [A]
time = 0.36, size = 107, normalized size = 0.98

method	result	size
meijerg	$\frac{\pi b^3 x^{10}\,\text{hypergeom}\left(\left[\frac{3}{4},\frac{5}{2}\right],\left[\frac{3}{2},\frac{7}{4},\frac{7}{2}\right],-\frac{x^4\pi^2 b^4}{16}\right)}{60}$	29

derivativedivides	$\frac{\frac{S(bx)b^7x^7}{7}+\frac{b^6x^6\cos\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}-\frac{6\left(\frac{b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{4\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}\right)}{7\pi}}{b^7}$	107
default	$\frac{\frac{S(bx)b^7x^7}{7}+\frac{b^6x^6\cos\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}-\frac{6\left(\frac{b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{4\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}\right)}{7\pi}}{b^7}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^7*(1/7*FresnelS(b*x)*b^7*x^7+1/7/Pi*b^6*x^6*cos(1/2*b^2*Pi*x^2)-6/7/Pi*
(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2
/Pi^2*sin(1/2*b^2*Pi*x^2))))
```

Maxima [A]
time = 0.27, size = 74, normalized size = 0.68

$$\frac{1}{7}x^7\,\mathrm{S}\left(bx\right)+\frac{\left(\pi^3b^6x^6-24\,\pi b^2x^2\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)-6\left(\pi^2b^4x^4-8\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{7\,\pi^4b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] 1/7*x^7*fresnel_sin(b*x) + 1/7*((pi^3*b^6*x^6 - 24*pi*b^2*x^2)*cos(1/2*pi*b
^2*x^2) - 6*(pi^2*b^4*x^4 - 8)*sin(1/2*pi*b^2*x^2))/(pi^4*b^7)
```

Fricas [A]
time = 0.36, size = 78, normalized size = 0.72

$$\frac{\pi^4b^7x^7\,\mathrm{S}\left(bx\right)+\left(\pi^3b^6x^6-24\,\pi b^2x^2\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)-6\left(\pi^2b^4x^4-8\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{7\,\pi^4b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnel_sin(b*x),x, algorithm="fricas")
```

```
[Out] 1/7*(pi^4*b^7*x^7*fresnel_sin(b*x) + (pi^3*b^6*x^6 - 24*pi*b^2*x^2)*cos(1/2
*pi*b^2*x^2) - 6*(pi^2*b^4*x^4 - 8)*sin(1/2*pi*b^2*x^2))/(pi^4*b^7)
```

Sympy [A]
time = 0.95, size = 156, normalized size = 1.43

$$\frac{3x^7S(bx)\,\Gamma\left(\frac{3}{4}\right)}{28\Gamma\left(\frac{7}{4}\right)}+\frac{3x^6\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{28\pi b\Gamma\left(\frac{7}{4}\right)}-\frac{9x^4\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{14\pi^2b^3\Gamma\left(\frac{7}{4}\right)}-\frac{18x^2\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{7\pi^3b^5\Gamma\left(\frac{7}{4}\right)}+\frac{36\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{7\pi^4b^7\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnels(b*x),x)
```

```
[Out] 3*x**7*fresnels(b*x)*gamma(3/4)/(28*gamma(7/4)) + 3*x**6*cos(pi*b**2*x**2/2
)*gamma(3/4)/(28*pi*b*gamma(7/4)) - 9*x**4*sin(pi*b**2*x**2/2)*gamma(3/4)/(
14*pi**2*b**3*gamma(7/4)) - 18*x**2*cos(pi*b**2*x**2/2)*gamma(3/4)/(7*pi**3
*b**5*gamma(7/4)) + 36*sin(pi*b**2*x**2/2)*gamma(3/4)/(7*pi**4*b**7*gamma(7
/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnel_sin(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^6*fresnel_sin(b*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\,\mathrm{FresnelS}(b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelS(b*x),x)
```

```
[Out] int(x^6*FresnelS(b*x), x)
```

3.3 $\int x^5 S(bx)\,dx$

Optimal. Leaf size=99

$$-\frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3}+\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi}+\frac{5\text{FresnelC}(bx)}{2b^6\pi^3}+\frac{1}{6}x^6S(bx)-\frac{5x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2}$$

```
[Out] -5/2*x*cos(1/2*b^2*Pi*x^2)/b^5/Pi^3+1/6*x^5*cos(1/2*b^2*Pi*x^2)/b/Pi+5/2*Fr
esnelC(b*x)/b^6/Pi^3+1/6*x^6*FresnelS(b*x)-5/6*x^3*sin(1/2*b^2*Pi*x^2)/b^3/
Pi^2
```

Rubi [A]
time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6561, 3466, 3467, 3433}

$$\frac{5\text{FresnelC}(bx)}{2\pi^3b^6}+\frac{x^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi b}-\frac{5x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^3b^5}-\frac{5x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi^2b^3}+\frac{1}{6}x^6S(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*FresnelS[b*x],x]
```

```
[Out] (-5*x*Cos[(b^2*Pi*x^2)/2])/(2*b^5*Pi^3) + (x^5*Cos[(b^2*Pi*x^2)/2])/(6*b*Pi
) + (5*FresnelC[b*x])/(2*b^6*Pi^3) + (x^6*FresnelS[b*x])/6 - (5*x^3*Sin[(b^
2*Pi*x^2)/2])/(6*b^3*Pi^2)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^5 S(bx)\,dx &= \frac{1}{6}x^6 S(bx) - \frac{1}{6}b\int x^6 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{1}{6}x^6 S(bx) - \frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{6b\pi} \\
&= \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{1}{6}x^6 S(bx) - \frac{5x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{5\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b^3\pi^2} \\
&= -\frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{1}{6}x^6 S(bx) - \frac{5x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{5\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b^5\pi^3} \\
&= -\frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{5C(bx)}{2b^6\pi^3} + \frac{1}{6}x^6 S(bx) - \frac{5x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 79, normalized size = 0.80

$$\frac{bx(-15+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right) + 15\text{FresnelC}(bx) + b^6\pi^3x^6S(bx) - 5b^3\pi x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^6\pi^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*FresnelS[b*x],x]
[Out] (b*x*(-15 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + 15*FresnelC[b*x] + b^6*Pi^3
*x^6*FresnelS[b*x] - 5*b^3*Pi*x^3*Sin[(b^2*Pi*x^2)/2])/(6*b^6*Pi^3)
```

Maple [A]
time = 0.33, size = 96, normalized size = 0.97

method	result	size
meijerg	$\frac{\pi b^3x^9\,\text{hypergeom}\left(\left[\frac{3}{4},\frac{9}{4}\right],\left[\frac{3}{2},\frac{7}{4},\frac{13}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{54}$	29
derivativedivides	$\frac{\frac{\mathrm{S}(bx)b^6x^6}{6}+\frac{b^5x^5\cos\left(\frac{b^2\pi x^2}{2}\right)}{6\pi}-\frac{5\left(\frac{b^3x^3\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{3\left(-\frac{bx\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\text{FresnelC}(bx)}{\pi}\right)}{\pi}\right)}{6\pi}}{b^6}$	96

default	$\frac{\frac{S(bx)b^6x^6}{6}+\frac{b^5x^5\cos\left(\frac{b^2\pi x^2}{2}\right)}{6\pi}-\frac{5\left(\frac{b^3x^3\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{3\left(-\frac{bx\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\mathrm{FresnelC}(bx)}{\pi}\right)}{\pi}\right)}{6\pi}}{b^6}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^6*(1/6*FresnelS(b*x)*b^6*x^6+1/6/Pi*b^5*x^5*cos(1/2*b^2*Pi*x^2)-5/6/Pi*
(1/Pi*b^3*x^3*sin(1/2*b^2*Pi*x^2)-3/Pi*(-1/Pi*b*x*cos(1/2*b^2*Pi*x^2)+1/Pi*
FresnelC(b*x))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.48, size = 110, normalized size = 1.11

$$\frac{1}{6}\,x^6\,S\left(bx\right)-\frac{\sqrt{\frac{1}{2}}\left(20\,\sqrt{\frac{1}{2}}\,\pi^2b^3x^3\sin\left(\frac{1}{2}\,\pi b^2x^2\right)+\left(15i-15\right)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{\frac{1}{2}i\pi}\,bx\right)-\left(15i+15\right)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{-\frac{1}{2}i\pi}\,bx\right)-4\left(\sqrt{\frac{1}{2}}\,\pi^3b^5x^5-15\,\sqrt{\frac{1}{2}}\,\pi bx\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\right)}{12\,\pi^4b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] 1/6*x^6*fresnel_sin(b*x) - 1/12*sqrt(1/2)*(20*sqrt(1/2)*pi^2*b^3*x^3*sin(1/
2*pi*b^2*x^2) + (15*I - 15)*(1/4)^(1/4)*pi*erf(sqrt(1/2*I*pi)*b*x) - (15*I
+ 15)*(1/4)^(1/4)*pi*erf(sqrt(-1/2*I*pi)*b*x) - 4*(sqrt(1/2)*pi^3*b^5*x^5 -
 15*sqrt(1/2)*pi*b*x)*cos(1/2*pi*b^2*x^2))/(pi^4*b^6)
```

Fricas [A]
time = 0.36, size = 85, normalized size = 0.86

$$\frac{\pi^3b^7x^6\,S\left(bx\right)-5\,\pi b^4x^3\sin\left(\frac{1}{2}\,\pi b^2x^2\right)+\left(\pi^2b^6x^5-15\,b^2x\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)+15\,\sqrt{b^2}\,C\left(\sqrt{b^2}\,x\right)}{6\,\pi^3b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnel_sin(b*x),x, algorithm="fricas")
```

```
[Out] 1/6*(pi^3*b^7*x^6*fresnel_sin(b*x) - 5*pi*b^4*x^3*sin(1/2*pi*b^2*x^2) + (pi
^2*b^6*x^5 - 15*b^2*x)*cos(1/2*pi*b^2*x^2) + 15*sqrt(b^2)*fresnel_cos(sqrt(
b^2)*x))/(pi^3*b^7)
```

Sympy [A]
time = 0.53, size = 53, normalized size = 0.54

$$\frac{\pi b^3x^9\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{9}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{9}{4}\\\frac{3}{2},\frac{7}{4},\frac{13}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*fresnels(b*x),x)
```

```
[Out] pi*b**3*x**9*gamma(3/4)*gamma(9/4)*hyper((3/4, 9/4), (3/2, 7/4, 13/4), -pi*
*2*b**4*x**4/16)/(32*gamma(7/4)*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnel_sin(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^5*fresnel_sin(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \,\mathrm{FresnelS}(b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelS(b*x),x)
```

```
[Out] int(x^5*FresnelS(b*x), x)
```

3.4 $\int x^4 S(bx)\,dx$

Optimal. Leaf size=84

$$-\frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3}+\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi}+\frac{1}{5}x^5S(bx)-\frac{4x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2}$$

```
[Out] -8/5*cos(1/2*b^2*Pi*x^2)/b^5/Pi^3+1/5*x^4*cos(1/2*b^2*Pi*x^2)/b/Pi+1/5*x^5*
FresnelS(b*x)-4/5*x^2*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A]
time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6561, 3460, 3377, 2718}

$$\frac{x^4\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b}-\frac{8\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5}-\frac{4x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3}+\frac{1}{5}x^5S(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*FresnelS[b*x],x]
```

```
[Out] (-8*Cos[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) + (x^4*Cos[(b^2*Pi*x^2)/2])/(5*b*Pi)
+ (x^5*FresnelS[b*x])/5 - (4*x^2*Sin[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
```

```
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^4 S(bx)\,dx &= \frac{1}{5}x^5 S(bx) - \frac{1}{5}b\int x^5 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{1}{5}x^5 S(bx) - \frac{1}{10}b\text{Subst}\left(\int x^2 \sin\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right) \\
&= \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{1}{5}x^5 S(bx) - \frac{2\text{Subst}\left(\int x\cos\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{5b\pi} \\
&= \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{1}{5}x^5 S(bx) - \frac{4x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} + \frac{4\text{Subst}\left(\int \sin\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{5b^3\pi^2} \\
&= -\frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} + \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{1}{5}x^5 S(bx) - \frac{4x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 71, normalized size = 0.85

$$\frac{(-8+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} + \frac{1}{5}x^5 S(bx) - \frac{4x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*FresnelS[b*x],x]
```

```
[Out] ((-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) + (x^5*FresnelS[b*x]
)/5 - (4*x^2*Sin[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2)
```

Maple [A]
time = 0.33, size = 80, normalized size = 0.95

method	result	size
meijerg	$\frac{\pi b^3 x^8 \,\text{hypergeom}\left(\left[\frac{3}{4},2\right],\left[\frac{3}{2},\frac{7}{4},3\right],-\frac{x^4\pi^2 b^4}{16}\right)}{48}$	29
derivativedivides	$\frac{\frac{\mathrm{S}(bx)b^5x^5}{5}+\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi}-\frac{4\left(\frac{b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{5\pi}}{b^5}$	80
default	$\frac{\frac{\mathrm{S}(bx)b^5x^5}{5}+\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi}-\frac{4\left(\frac{b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{5\pi}}{b^5}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/5*FresnelS(b*x)*b^5*x^5+1/5/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)-4/5/Pi*
(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2)))
```

Maxima [A]
time = 0.28, size = 62, normalized size = 0.74

$$\frac{1}{5}\,x^5\,\mathrm{S}\left(bx\right)-\frac{4\,\pi b^2x^2\sin\left(\frac{1}{2}\,\pi b^2x^2\right)-\left(\pi^2b^4x^4-8\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)}{5\,\pi^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*fresnel_sin(b*x) - 1/5*(4*pi*b^2*x^2*sin(1/2*pi*b^2*x^2) - (pi^2*b^
4*x^4 - 8)*cos(1/2*pi*b^2*x^2))/(pi^3*b^5)
```

Fricas [A]
time = 0.34, size = 65, normalized size = 0.77

$$\frac{\pi^3b^5x^5\,\mathrm{S}\left(bx\right)-4\,\pi b^2x^2\sin\left(\frac{1}{2}\,\pi b^2x^2\right)+\left(\pi^2b^4x^4-8\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)}{5\,\pi^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnel_sin(b*x),x, algorithm="fricas")
```

```
[Out] 1/5*(pi^3*b^5*x^5*fresnel_sin(b*x) - 4*pi*b^2*x^2*sin(1/2*pi*b^2*x^2) + (pi
^2*b^4*x^4 - 8)*cos(1/2*pi*b^2*x^2))/(pi^3*b^5)
```

Sympy [A]
time = 0.89, size = 121, normalized size = 1.44

$$\frac{3x^5S(bx)\,\Gamma\left(\frac{3}{4}\right)}{20\Gamma\left(\frac{7}{4}\right)}+\frac{3x^4\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{20\pi b\Gamma\left(\frac{7}{4}\right)}-\frac{3x^2\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{5\pi^2b^3\Gamma\left(\frac{7}{4}\right)}-\frac{6\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{5\pi^3b^5\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnels(b*x),x)
```

```
[Out] 3*x**5*fresnels(b*x)*gamma(3/4)/(20*gamma(7/4)) + 3*x**4*cos(pi*b**2*x**2/2
)*gamma(3/4)/(20*pi*b*gamma(7/4)) - 3*x**2*sin(pi*b**2*x**2/2)*gamma(3/4)/(
5*pi**2*b**3*gamma(7/4)) - 6*cos(pi*b**2*x**2/2)*gamma(3/4)/(5*pi**3*b**5*g
amma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnel_sin(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnel_sin(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\mathrm{FresnelS}(b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelS(b*x),x)
```

```
[Out] int(x^4*FresnelS(b*x), x)
```

3.5 $\int x^3 S(bx)\, dx$

Optimal. Leaf size=74

$$\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi}+\frac{3S(bx)}{4b^4\pi^2}+\frac{1}{4}x^4S(bx)-\frac{3x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2}$$

```
[Out] 1/4*x^3*cos(1/2*b^2*Pi*x^2)/b/Pi+3/4*FresnelS(b*x)/b^4/Pi^2+1/4*x^4*Fresnel
S(b*x)-3/4*x*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6561, 3466, 3467, 3432}

$$\frac{3S(bx)}{4\pi^2b^4}+\frac{x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b}-\frac{3x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^2b^3}+\frac{1}{4}x^4S(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelS[b*x],x]
```

```
[Out] (x^3*Cos[(b^2*Pi*x^2)/2])/(4*b*Pi) + (3*FresnelS[b*x])/(4*b^4*Pi^2) + (x^4*
FresnelS[b*x])/4 - (3*x*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
```

```
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3 S(bx)\,dx &= \frac{1}{4}x^4 S(bx) - \frac{1}{4}b\int x^4 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\ &= \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{1}{4}x^4 S(bx) - \frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{4b\pi}\\ &= \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{1}{4}x^4 S(bx) - \frac{3x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3\int \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{4b^3\pi^2}\\ &= \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{3S(bx)}{4b^4\pi^2} + \frac{1}{4}x^4 S(bx) - \frac{3x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 74, normalized size = 1.00

$$\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{3S(bx)}{4b^4\pi^2} + \frac{1}{4}x^4 S(bx) - \frac{3x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelS[b*x],x]
```

```
[Out] (x^3*Cos[(b^2*Pi*x^2)/2])/(4*b*Pi) + (3*FresnelS[b*x])/(4*b^4*Pi^2) + (x^4*
FresnelS[b*x])/4 - (3*x*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2)
```

Maple [A]
time = 0.33, size = 70, normalized size = 0.95

method	result	size
meijerg	$\frac{\frac{\pi x^3 b^3\cos\left(\frac{b^2\pi x^2}{2}\right)}{4}-\frac{3\sin\left(\frac{b^2\pi x^2}{2}\right)bx}{4}+\frac{\left(21x^4\pi^2 b^4+63\right)\mathrm{S}(bx)}{84}}{\pi^2 b^4}$	62
derivativedivides	$\frac{\frac{\mathrm{S}(bx)b^4x^4}{4}+\frac{b^3x^3\cos\left(\frac{b^2\pi x^2}{2}\right)}{4\pi}-\frac{3\left(\frac{bx\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{\mathrm{S}(bx)}{\pi}\right)}{4\pi}}{b^4}$	70
default	$\frac{\frac{\mathrm{S}(bx)b^4x^4}{4}+\frac{b^3x^3\cos\left(\frac{b^2\pi x^2}{2}\right)}{4\pi}-\frac{3\left(\frac{bx\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{\mathrm{S}(bx)}{\pi}\right)}{4\pi}}{b^4}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/4*FresnelS(b*x)*b^4*x^4+1/4/Pi*b^3*x^3*cos(1/2*b^2*Pi*x^2)-3/4/Pi*
(1/Pi*b*x*sin(1/2*b^2*Pi*x^2)-1/Pi*FresnelS(b*x)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.48, size = 94, normalized size = 1.27

$$\frac{1}{4}x^4\,\mathrm{S}\left(bx\right)+\frac{\sqrt{\frac{1}{2}}\left(4\sqrt{\frac{1}{2}}\pi^2b^3x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)-12\sqrt{\frac{1}{2}}\pi bx\sin\left(\frac{1}{2}\pi b^2x^2\right)+(3i+3)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{\frac{1}{2}i\pi}\,bx\right)-(3i-3)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{-\frac{1}{2}i\pi}\,bx\right)\right)}{8\,\pi^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*fresnel_sin(b*x) + 1/8*sqrt(1/2)*(4*sqrt(1/2)*pi^2*b^3*x^3*cos(1/2*
pi*b^2*x^2) - 12*sqrt(1/2)*pi*b*x*sin(1/2*pi*b^2*x^2) + (3*I + 3)*(1/4)^(1/
4)*pi*erf(sqrt(1/2*I*pi)*b*x) - (3*I - 3)*(1/4)^(1/4)*pi*erf(sqrt(-1/2*I*pi
)*b*x))/(pi^3*b^4)
```

Fricas [A]
time = 0.37, size = 58, normalized size = 0.78

$$\frac{\pi b^3x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)-3\,bx\sin\left(\frac{1}{2}\pi b^2x^2\right)+\left(\pi^2b^4x^4+3\right)\mathrm{S}\left(bx\right)}{4\,\pi^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_sin(b*x),x, algorithm="fricas")
```

```
[Out] 1/4*(pi*b^3*x^3*cos(1/2*pi*b^2*x^2) - 3*b*x*sin(1/2*pi*b^2*x^2) + (pi^2*b^4
*x^4 + 3)*fresnel_sin(b*x))/(pi^2*b^4)
```

Sympy [A]
time = 0.52, size = 112, normalized size = 1.51

$$\frac{21x^4S(bx)\,\Gamma\left(\frac{3}{4}\right)}{64\Gamma\left(\frac{11}{4}\right)}+\frac{21x^3\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{64\pi b\Gamma\left(\frac{11}{4}\right)}-\frac{63x\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{64\pi^2b^3\Gamma\left(\frac{11}{4}\right)}+\frac{63S(bx)\,\Gamma\left(\frac{3}{4}\right)}{64\pi^2b^4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnels(b*x),x)
```

```
[Out] 21*x**4*fresnels(b*x)*gamma(3/4)/(64*gamma(11/4)) + 21*x**3*cos(pi*b**2*x**
2/2)*gamma(3/4)/(64*pi*b*gamma(11/4)) - 63*x*sin(pi*b**2*x**2/2)*gamma(3/4)
/(64*pi**2*b**3*gamma(11/4)) + 63*fresnels(b*x)*gamma(3/4)/(64*pi**2*b**4*g
amma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_sin(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnel_sin(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \,\mathrm{FresnelS}(b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(b*x),x)
```

```
[Out] int(x^3*FresnelS(b*x), x)
```

3.6 $\int x^2 S(bx)\,dx$

Optimal. Leaf size=59

$$\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}+\frac{1}{3}x^3S(bx)-\frac{2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2}$$

```
[Out] 1/3*x^2*cos(1/2*b^2*Pi*x^2)/b/Pi+1/3*x^3*FresnelS(b*x)-2/3*sin(1/2*b^2*Pi*x
^2)/b^3/Pi^2
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6561, 3460, 3377, 2717}

$$\frac{x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}-\frac{2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{1}{3}x^3S(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelS[b*x],x]
```

```
[Out] (x^2*Cos[(b^2*Pi*x^2)/2])/(3*b*Pi) + (x^3*FresnelS[b*x])/3 - (2*Sin[(b^2*Pi
*x^2)/2])/(3*b^3*Pi^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
```

```
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2 S(bx)\,dx &= \frac{1}{3}x^3 S(bx) - \frac{1}{3}b\int x^3 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{1}{3}x^3 S(bx) - \frac{1}{6}b\text{Subst}\left(\int x\sin\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right) \\
&= \frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{1}{3}x^3 S(bx) - \frac{\text{Subst}\left(\int \cos\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{3b\pi} \\
&= \frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{1}{3}x^3 S(bx) - \frac{2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2}
\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 59, normalized size = 1.00

$$\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{1}{3}x^3 S(bx) - \frac{2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelS[b*x],x]
[Out] (x^2*Cos[(b^2*Pi*x^2)/2])/(3*b*Pi) + (x^3*FresnelS[b*x])/3 - (2*Sin[(b^2*Pi
*x^2)/2])/(3*b^3*Pi^2)
```

Maple [**A**]
time = 0.35, size = 54, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\frac{\mathrm{S}(bx)b^3x^3}{3}+\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{3\pi}-\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\pi^2}}{b^3}$	54
default	$\frac{\frac{\mathrm{S}(bx)b^3x^3}{3}+\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{3\pi}-\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\pi^2}}{b^3}$	54
meijerg	$\frac{\frac{\sqrt{\pi}\,x^2b^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{3}-\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\sqrt{\pi}}+\frac{\pi^{\frac{3}{2}}x^3b^3\mathrm{S}(bx)}{3}}{\pi^{\frac{3}{2}}b^3}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/3*FresnelS(b*x)*b^3*x^3+1/3/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)-2/3/Pi^
2*sin(1/2*b^2*Pi*x^2))
```

Maxima [A]
time = 0.26, size = 49, normalized size = 0.83

$$\frac{1}{3}\,x^3\,\mathrm{S}\left(bx\right)+\frac{\pi b^2x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)-2\,\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{3\,\pi^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*fresnel_sin(b*x) + 1/3*(pi*b^2*x^2*cos(1/2*pi*b^2*x^2) - 2*sin(1/2*
pi*b^2*x^2))/(pi^2*b^3)
```

Fricas [A]
time = 0.39, size = 53, normalized size = 0.90

$$\frac{\pi^2b^3x^3\,\mathrm{S}\left(bx\right)+\pi b^2x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)-2\,\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{3\,\pi^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_sin(b*x),x, algorithm="fricas")
```

```
[Out] 1/3*(pi^2*b^3*x^3*fresnel_sin(b*x) + pi*b^2*x^2*cos(1/2*pi*b^2*x^2) - 2*sin
(1/2*pi*b^2*x^2))/(pi^2*b^3)
```

Sympy [A]
time = 0.46, size = 80, normalized size = 1.36

$$\frac{x^3S(bx)\,\Gamma\left(\frac{3}{4}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x^2\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{4\pi b\Gamma\left(\frac{7}{4}\right)}-\frac{\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{2\pi^2b^3\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnels(b*x),x)
```

```
[Out] x**3*fresnels(b*x)*gamma(3/4)/(4*gamma(7/4)) + x**2*cos(pi*b**2*x**2/2)*gam
ma(3/4)/(4*pi*b*gamma(7/4)) - sin(pi*b**2*x**2/2)*gamma(3/4)/(2*pi**2*b**3*
gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_sin(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnel_sin(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2 \operatorname{FresnelS}(b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(b*x),x)
```

```
[Out] int(x^2*FresnelS(b*x), x)
```

3.7 $\int xS(bx)\,dx$

Optimal. Leaf size=49

$$\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi}-\frac{\text{FresnelC}(bx)}{2b^2\pi}+\frac{1}{2}x^2S(bx)$$

[Out] 1/2*x*cos(1/2*b^2*Pi*x^2)/b/Pi-1/2*FresnelC(b*x)/b^2/Pi+1/2*x^2*FresnelS(b*x)

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6561, 3466, 3433}

$$-\frac{\text{FresnelC}(bx)}{2\pi b^2}+\frac{x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b}+\frac{1}{2}x^2S(bx)$$

Antiderivative was successfully verified.

[In] Int[x*FresnelS[b*x],x]

[Out] (x*Cos[(b^2*Pi*x^2)/2])/(2*b*Pi) - FresnelC[b*x]/(2*b^2*Pi) + (x^2*FresnelS[b*x])/2

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int xS(bx)\,dx = \frac{1}{2}x^2S(bx) - \frac{1}{2}b\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$
$$= \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{1}{2}x^2S(bx) - \frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b\pi}$$
$$= \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} - \frac{C(bx)}{2b^2\pi} + \frac{1}{2}x^2S(bx)$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 1.00

$$\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} - \frac{\text{FresnelC}(bx)}{2b^2\pi} + \frac{1}{2}x^2S(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*FresnelS[b*x],x]
```

```
[Out] (x*Cos[(b^2*Pi*x^2)/2])/(2*b*Pi) - FresnelC[b*x]/(2*b^2*Pi) + (x^2*FresnelS
[b*x])/2
```

Maple [A]
time = 0.36, size = 44, normalized size = 0.90

method	result	size
meijerg	$\frac{\pi b^3x^5 \text{ hypergeom}\left(\left[\frac{3}{4},\frac{5}{4}\right],\left[\frac{3}{2},\frac{7}{4},\frac{9}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{30}$	29
derivativedivides	$\frac{\frac{\text{S}(bx)b^2x^2}{2}+\frac{bx\cos\left(\frac{b^2\pi x^2}{2}\right)}{2\pi}-\frac{\text{FresnelC}(bx)}{2\pi}}{b^2}$	44
default	$\frac{\frac{\text{S}(bx)b^2x^2}{2}+\frac{bx\cos\left(\frac{b^2\pi x^2}{2}\right)}{2\pi}-\frac{\text{FresnelC}(bx)}{2\pi}}{b^2}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*FresnelS(b*x)*b^2*x^2+1/2/Pi*b*x*cos(1/2*b^2*Pi*x^2)-1/2/Pi*Fres
nelC(b*x))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.48, size = 70, normalized size = 1.43

$$\frac{1}{2}x^2\,\text{S}(bx) + \frac{\sqrt{\frac{1}{2}}\left(4\sqrt{\frac{1}{2}}\,\pi bx\cos\left(\frac{1}{2}\pi b^2x^2\right) + (i-1)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\text{erf}\left(\sqrt{\frac{1}{2}i\pi}\,bx\right) - (i+1)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\text{erf}\left(\sqrt{-\frac{1}{2}i\pi}\,bx\right)\right)}{4\pi^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*fresnel_sin(b*x) + 1/4*sqrt(1/2)*(4*sqrt(1/2)*pi*b*x*cos(1/2*pi*b^2
*x^2) + (I - 1)*(1/4)^(1/4)*pi*erf(sqrt(1/2*I*pi)*b*x) - (I + 1)*(1/4)^(1/4
)*pi*erf(sqrt(-1/2*I*pi)*b*x))/(pi^2*b^2)
```

Fricas [A]
time = 0.38, size = 51, normalized size = 1.04

$$\frac{\pi b^3 x^2\, \mathrm{S}\left(bx\right) + b^2 x \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - \sqrt{b^2}\ \mathrm{C}\left(\sqrt{b^2}\ x\right)}{2\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_sin(b*x),x, algorithm="fricas")
```

```
[Out] 1/2*(pi*b^3*x^2*fresnel_sin(b*x) + b^2*x*cos(1/2*pi*b^2*x^2) - sqrt(b^2)*fr
esnel_cos(sqrt(b^2)*x))/(pi*b^3)
```

Sympy [A]
time = 0.31, size = 53, normalized size = 1.08

$$\frac{\pi b^3 x^5 \Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right) {}_{2}F_{3}\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\right| -\frac{\pi^2 b^4 x^4}{16}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x),x)
```

```
[Out] pi*b**3*x**5*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4), -pi**
2*b**4*x**4/16)/(32*gamma(7/4)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_sin(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*fresnel_sin(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{FresnelS}(b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(b*x),x)
```

```
[Out] int(x*FresnelS(b*x), x)
```

3.8 $\int S(bx)\,dx$

Optimal. Leaf size=26

$$\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}+xS(bx)$$

[Out] cos(1/2*b^2*Pi*x^2)/b/Pi+x*FresnelS(b*x)

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6553}

$$\frac{\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+xS(bx)$$

Antiderivative was successfully verified.

[In] Int[FresnelS[b*x],x]

[Out] Cos[(b^2*Pi*x^2)/2]/(b*Pi) + x*FresnelS[b*x]

Rule 6553

```
Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelS[a +
 b*x]/b), x] + Simp[Cos[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int S(bx)\,dx=\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}+xS(bx)$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 1.00

$$\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}+xS(bx)$$

Antiderivative was successfully verified.

[In] Integrate[FresnelS[b*x],x]

[Out] Cos[(b^2*Pi*x^2)/2]/(b*Pi) + x*FresnelS[b*x]

Maple [A]
time = 0.42, size = 27, normalized size = 1.04

method	result	size
derivativedivides	$\frac{\mathrm{S}(bx)bx+\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}}{b}$	27
default	$\frac{\mathrm{S}(bx)bx+\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}}{b}$	27
meijerg	$\frac{\pi b^3x^4 \,\mathrm{hypergeom}\left(\left[\frac{3}{4},1\right],\left[\frac{3}{2},\frac{7}{4},2\right],-\frac{x^4\pi^2b^4}{16}\right)}{24}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(FresnelS(b*x)*b*x+1/Pi*cos(1/2*b^2*Pi*x^2))
```

Maxima [A]
time = 0.27, size = 26, normalized size = 1.00

$$\frac{bx\,\mathrm{S}\left(bx\right)+\frac{\cos\left(\frac{1}{2}\,\pi b^2x^2\right)}{\pi}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] (b*x*fresnel_sin(b*x) + cos(1/2*pi*b^2*x^2)/pi)/b
```

Fricas [A]
time = 0.33, size = 26, normalized size = 1.00

$$\frac{\pi bx\,\mathrm{S}\left(bx\right)+\cos\left(\frac{1}{2}\,\pi b^2x^2\right)}{\pi b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x),x, algorithm="fricas")
```

```
[Out] (pi*b*x*fresnel_sin(b*x) + cos(1/2*pi*b^2*x^2))/(pi*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.
time = 0.47, size = 48, normalized size = 1.85

$$\frac{3xS(bx)\,\Gamma\left(\frac{3}{4}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{3\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{4\pi b\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x),x)
```

```
[Out] 3*x*fresnels(b*x)*gamma(3/4)/(4*gamma(7/4)) + 3*cos(pi*b**2*x**2/2)*gamma(3
/4)/(4*pi*b*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x),x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \mathrm{FresnelS}(b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x),x)
```

```
[Out] int(FresnelS(b*x), x)
```

3.9 $\int \frac{S(bx)}{x}\,dx$

Optimal. Leaf size=73

$$\frac{1}{2}ibx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{2}ibx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)$$

```
[Out] 1/2*I*b*x*hypergeom([1/2, 1/2],[3/2, 3/2],-1/2*I*b^2*Pi*x^2)-1/2*I*b*x*hype
rgeom([1/2, 1/2],[3/2, 3/2],1/2*I*b^2*Pi*x^2)
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6559, 6493, 6495}

$$\frac{1}{2}ibx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{2}ibx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x,x]
```

```
[Out] (I/2)*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (-1/2*I)*b^2*Pi*x^2] -
(I/2)*b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (I/2)*b^2*Pi*x^2]
```

Rule 6493

```
Int[Erf[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[2*b*(x/Sqrt[Pi])*Hypergeometric
PFQ[{1/2, 1/2}, {3/2, 3/2}, (-b^2)*x^2], x] /; FreeQ[b, x]
```

Rule 6495

```
Int[Erfi[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[2*b*(x/Sqrt[Pi])*Hypergeometri
cPFQ[{1/2, 1/2}, {3/2, 3/2}, b^2*x^2], x] /; FreeQ[b, x]
```

Rule 6559

```
Int[FresnelS[(b_.)*(x_)]/(x_), x_Symbol] :> Dist[(1 + I)/4, Int[Erf[(Sqrt[P
i]/2)*(1 + I)*b*x]/x, x], x] + Dist[(1 - I)/4, Int[Erf[(Sqrt[Pi]/2)*(1 - I)
*b*x]/x, x], x] /; FreeQ[b, x]
```

Rubi steps

$$\begin{aligned}\int \frac{S(bx)}{x}\,dx &= \left(-\frac{1}{4}-\frac{i}{4}\right)\int \frac{\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)}{x}\,dx+\left(\frac{1}{4}+\frac{i}{4}\right)\int \frac{\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)}{x}\,dx\\ &= \frac{1}{2}ibx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{2}ibx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)\end{aligned}$$

Mathematica [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{S(bx)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelS[b*x]/x,x]
[Out] Integrate[FresnelS[b*x]/x, x]
```

Maple [A]
time = 0.33, size = 29, normalized size = 0.40

method	result	size
meijerg	$\frac{\pi\,x^3 b^3\,\mathrm{hypergeom}\left([\frac{3}{4},\frac{3}{4}],[\frac{3}{2},\frac{7}{4},\frac{7}{4}],-\frac{x^4\pi^2 b^4}{16}\right)}{18}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x,x,method=_RETURNVERBOSE)
[Out] 1/18*Pi*x^3*b^3*hypergeom([3/4,3/4],[3/2,7/4,7/4],-1/16*x^4*Pi^2*b^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)/x, x)
```

Sympy [A]
time = 0.30, size = 46, normalized size = 0.63

$$\frac{\pi b^3 x^3 \Gamma^2\left(\frac{3}{4}\right) {}_2F_3\left(\begin{matrix} \frac{3}{4}, \frac{3}{4} \\ \frac{3}{2}, \frac{7}{4}, \frac{7}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{32\Gamma^2\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x,x)
```

```
[Out] pi*b**3*x**3*gamma(3/4)**2*hyper((3/4, 3/4), (3/2, 7/4, 7/4), -pi**2*b**4*x
**4/16)/(32*gamma(7/4)**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}(b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x,x)
```

```
[Out] int(FresnelS(b*x)/x, x)
```

3.10 $\int \frac{S(bx)}{x^2}\,dx$

Optimal. Leaf size=27

$$-\frac{S(bx)}{x}+\frac{1}{2}b\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

```
[Out] -FresnelS(b*x)/x+1/2*b*Si(1/2*b^2*Pi*x^2)
```

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6561, 3456}

$$\frac{1}{2}b\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{S(bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^2,x]
```

```
[Out] -(FresnelS[b*x]/x) + (b*SinIntegral[(b^2*Pi*x^2)/2])/2
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{S(bx)}{x^2}\,dx &= -\frac{S(bx)}{x}+b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx\\ &= -\frac{S(bx)}{x}+\frac{1}{2}b\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.00

$$-\frac{S(bx)}{x}+\frac{1}{2}b\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^2,x]
[Out] -(FresnelS[b*x]/x) + (b*SinIntegral[(b^2*Pi*x^2)/2])/2
```

Maple [A]
time = 0.32, size = 28, normalized size = 1.04

method	result	size
derivativedivides	$b\left(-\frac{S(bx)}{bx}+\frac{\text{sinIntegral}\left(\frac{b^2\pi x^2}{2}\right)}{2}\right)$	28
default	$b\left(-\frac{S(bx)}{bx}+\frac{\text{sinIntegral}\left(\frac{b^2\pi x^2}{2}\right)}{2}\right)$	28
meijerg	$\frac{\pi b^3x^2\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2},\frac{3}{2},\frac{7}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{12}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^2,x,method=_RETURNVERBOSE)
[Out] b*(-FresnelS(b*x)/b/x+1/2*Si(1/2*b^2*Pi*x^2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.31, size = 38, normalized size = 1.41

$$-\frac{1}{4}\,b\left(i\,\text{Ei}\left(\frac{1}{2}i\,\pi b^2x^2\right)-i\,\text{Ei}\left(-\frac{1}{2}i\,\pi b^2x^2\right)\right)-\frac{\text{S}\,(bx)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^2,x, algorithm="maxima")
[Out] -1/4*b*(I*Ei(1/2*I*pi*b^2*x^2) - I*Ei(-1/2*I*pi*b^2*x^2)) - fresnel_sin(b*x
)/x
```

Fricas [A]
time = 0.35, size = 25, normalized size = 0.93

$$\frac{bx\,\text{Si}\left(\frac{1}{2}\,\pi b^2x^2\right)-2\,\text{S}\,(bx)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^2,x, algorithm="fricas")
[Out] 1/2*(b*x*sin_integral(1/2*pi*b^2*x^2) - 2*fresnel_sin(b*x))/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.
time = 0.31, size = 42, normalized size = 1.56

$$\frac{\pi b^3 x^2 \Gamma\left(\frac{3}{4}\right) {}_2F_3\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2}, \frac{3}{2}, \frac{7}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{16\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**2,x)
```

```
[Out] pi*b**3*x**2*gamma(3/4)*hyper((1/2, 3/4), (3/2, 3/2, 7/4), -pi**2*b**4*x**4
/16)/(16*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{FresnelS}(b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^2,x)
```

```
[Out] int(FresnelS(b*x)/x^2, x)
```

3.11 $\int \frac{S(bx)}{x^3}\,dx$

Optimal. Leaf size=44

$$\frac{1}{2}b^2\pi \text{FresnelC}(bx) - \frac{S(bx)}{2x^2} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x}$$

```
[Out] 1/2*b^2*Pi*FresnelC(b*x)-1/2*FresnelS(b*x)/x^2-1/2*b*sin(1/2*b^2*Pi*x^2)/x
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6561, 3468, 3433}

$$\frac{1}{2}\pi b^2 \text{FresnelC}(bx) - \frac{b\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{2x} - \frac{S(bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^3,x]
```

```
[Out] (b^2*Pi*FresnelC[b*x])/2 - FresnelS[b*x]/(2*x^2) - (b*Sin[(b^2*Pi*x^2)/2])/
(2*x)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{S(bx)}{x^3}\,dx &= -\frac{S(bx)}{2x^2} + \frac{1}{2}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx \\ &= -\frac{S(bx)}{2x^2} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x} + \frac{1}{2}(b^3\pi)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\ &= \frac{1}{2}b^2\pi C(bx) - \frac{S(bx)}{2x^2} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 1.00

$$\frac{1}{2}b^2\pi\text{FresnelC}(bx) - \frac{S(bx)}{2x^2} - \frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^3,x]
[Out] (b^2*Pi*FresnelC[b*x])/2 - FresnelS[b*x]/(2*x^2) - (b*Sin[(b^2*Pi*x^2)/2])/
(2*x)
```

Maple [A]
time = 0.33, size = 43, normalized size = 0.98

method	result	size
meijerg	$\frac{\pi b^3 x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{5}{4},\frac{3}{2},\frac{7}{4}\right],-\frac{x^4\pi^2 b^4}{16}\right)}{6}$	27
derivativedivides	$b^2\left(-\frac{\text{S}(bx)}{2b^2x^2} - \frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2bx} + \frac{\pi\,\text{FresnelC}(bx)}{2}\right)$	43
default	$b^2\left(-\frac{\text{S}(bx)}{2b^2x^2} - \frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2bx} + \frac{\pi\,\text{FresnelC}(bx)}{2}\right)$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^3,x,method=_RETURNVERBOSE)
[Out] b^2*(-1/2*FresnelS(b*x)/b^2/x^2-1/2/b/x*sin(1/2*b^2*Pi*x^2)+1/2*Pi*FresnelC
(b*x))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 61, normalized size = 1.39

$$-\frac{\sqrt{\frac{1}{2}}\,\sqrt{\pi x^2}\,\left((i-1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},\frac{1}{2}i\,\pi b^2x^2\right) - (i+1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},-\frac{1}{2}i\,\pi b^2x^2\right)\right)b^2}{16\,x} - \frac{\text{S}\,(bx)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(1/2)*sqrt(pi*x^2)*((I - 1)*sqrt(2)*gamma(-1/2, 1/2*I*pi*b^2*x^2)
 - (I + 1)*sqrt(2)*gamma(-1/2, -1/2*I*pi*b^2*x^2))*b^2/x - 1/2*fresnel_sin(
b*x)/x^2
```

Fricas [A]
time = 0.35, size = 45, normalized size = 1.02

$$\frac{\pi\sqrt{b^2}\,bx^2\,\mathrm{C}\left(\sqrt{b^2}\,x\right)-bx\sin\left(\frac{1}{2}\,\pi b^2x^2\right)-\mathrm{S}\left(bx\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(pi*sqrt(b^2)*b*x^2*fresnel_cos(sqrt(b^2)*x) - b*x*sin(1/2*pi*b^2*x^2)
- fresnel_sin(b*x))/x^2
```

Sympy [A]
time = 0.34, size = 51, normalized size = 1.16

$$\frac{\pi b^3x\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{5}{4},\frac{3}{2},\frac{7}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{32\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**3,x)
```

```
[Out] pi*b**3*x*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (5/4, 3/2, 7/4), -pi**2*b
**4*x**4/16)/(32*gamma(5/4)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\mathrm{FresnelS}(b\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^3,x)
```

```
[Out] int(FresnelS(b*x)/x^3, x)
```

3.12 $\int \frac{S(bx)}{x^4}\,dx$

Optimal. Leaf size=52

$$\frac{1}{12}b^3\pi\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{S(bx)}{3x^3}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2}$$

```
[Out] 1/12*b^3*Pi*Ci(1/2*b^2*Pi*x^2)-1/3*FresnelS(b*x)/x^3-1/6*b*sin(1/2*b^2*Pi*x
^2)/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6561, 3460, 3378, 3383}

$$-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^2}+\frac{1}{12}\pi b^3\text{CosIntegral}\left(\frac{1}{2}\pi b^2x^2\right)-\frac{S(bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^4,x]
```

```
[Out] (b^3*Pi*CosIntegral[(b^2*Pi*x^2)/2])/12 - FresnelS[b*x]/(3*x^3) - (b*Sin[(b
^2*Pi*x^2)/2])/(6*x^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)}{x^4}\,dx &= -\frac{S(bx)}{3x^3}+\frac{1}{3}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx\\
&= -\frac{S(bx)}{3x^3}+\frac{1}{6}b\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx,x,x^2\right)\\
&= -\frac{S(bx)}{3x^3}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2}+\frac{1}{12}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx,x,x^2\right)\\
&= \frac{1}{12}b^3\pi\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{S(bx)}{3x^3}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 1.00

$$\frac{1}{12}b^3\pi\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{S(bx)}{3x^3}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^4,x]
```

```
[Out] (b^3*Pi*CosIntegral[(b^2*Pi*x^2)/2])/12 - FresnelS[b*x]/(3*x^3) - (b*Sin[(b
^2*Pi*x^2)/2])/(6*x^2)
```

Maple [A]
time = 0.35, size = 49, normalized size = 0.94

method	result	size
derivativedivides	$b^3\left(-\frac{\mathrm{S}(bx)}{3b^3x^3}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{6b^2x^2}+\frac{\pi\,\text{cosineIntegral}\left(\frac{b^2\pi x^2}{2}\right)}{12}\right)$	49
default	$b^3\left(-\frac{\mathrm{S}(bx)}{3b^3x^3}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{6b^2x^2}+\frac{\pi\,\text{cosineIntegral}\left(\frac{b^2\pi x^2}{2}\right)}{12}\right)$	49
meijerg	$\frac{\pi^{\frac{3}{2}}b^3\left(-\frac{\pi^{\frac{3}{2}}x^4b^4\,\text{hypergeom}\left(\left[1,1,\frac{7}{4}\right],\left[2,2,\frac{5}{2},\frac{11}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{21}+\frac{\frac{16\gamma}{3}-\frac{16\ln(2)}{3}-\frac{80}{9}+\frac{32\ln(x)}{3}+\frac{16\ln(\pi)}{3}+\frac{32\ln(b)}{3}}{\sqrt{\pi}}\right)}{64}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*(-1/3*FresnelS(b*x)/b^3/x^3-1/6/b^2/x^2*sin(1/2*b^2*Pi*x^2)+1/12*Pi*Ci(
1/2*b^2*Pi*x^2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.32, size = 42, normalized size = 0.81

$$\frac{1}{24}\left(\pi\Gamma\left(-1, \frac{1}{2}i\,\pi b^2x^2\right) + \pi\Gamma\left(-1, -\frac{1}{2}i\,\pi b^2x^2\right)\right)b^3 - \frac{\mathrm{S}\,(bx)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/24*(pi*gamma(-1, 1/2*I*pi*b^2*x^2) + pi*gamma(-1, -1/2*I*pi*b^2*x^2))*b^3
 - 1/3*fresnel_sin(b*x)/x^3
```

Fricas [A]
time = 0.35, size = 62, normalized size = 1.19

$$\frac{\pi b^3x^3\,\mathrm{Ci}\left(\frac{1}{2}\,\pi b^2x^2\right) + \pi b^3x^3\,\mathrm{Ci}\left(-\frac{1}{2}\,\pi b^2x^2\right) - 4\,bx\sin\left(\frac{1}{2}\,\pi b^2x^2\right) - 8\,\mathrm{S}\,(bx)}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^4,x, algorithm="fricas")
```

```
[Out] 1/24*(pi*b^3*x^3*cos_integral(1/2*pi*b^2*x^2) + pi*b^3*x^3*cos_integral(-1/
2*pi*b^2*x^2) - 4*b*x*sin(1/2*pi*b^2*x^2) - 8*fresnel_sin(b*x))/x^3
```

Sympy [A]
time = 0.72, size = 56, normalized size = 1.08

$$-\frac{\pi^3b^7x^4\Gamma\left(\frac{7}{4}\right)\,{}_3F_4\left(\begin{matrix}1, 1, \frac{7}{4}\\ 2, 2, \frac{5}{2}, \frac{11}{4}\end{matrix}\middle| -\frac{\pi^2b^4x^4}{16}\right)}{768\Gamma\left(\frac{11}{4}\right)} + \frac{\pi b^3\log\left(b^4x^4\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**4,x)
```

```
[Out] -pi**3*b**7*x**4*gamma(7/4)*hyper((1, 1, 7/4), (2, 2, 5/2, 11/4), -pi**2*b*
*4*x**4/16)/(768*gamma(11/4)) + pi*b**3*log(b**4*x**4)/24
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)/x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{FresnelS}(b\,x)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^4,x)
```

```
[Out] int(FresnelS(b*x)/x^4, x)
```

3.13 $\int \frac{S(bx)}{x^5}\,dx$

Optimal. Leaf size=69

$$-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x}-\frac{1}{12}b^4\pi^2S(bx)-\frac{S(bx)}{4x^4}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}$$

```
[Out] -1/12*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x-1/12*b^4*Pi^2*FresnelS(b*x)-1/4*FresnelS
(b*x)/x^4-1/12*b*sin(1/2*b^2*Pi*x^2)/x^3
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6561, 3468, 3469, 3432}

$$-\frac{1}{12}\pi^2b^4S(bx)-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{12x^3}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{12x}-\frac{S(bx)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^5,x]
```

```
[Out] -1/12*(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x - (b^4*Pi^2*FresnelS[b*x])/12 - Fresne
lS[b*x]/(4*x^4) - (b*Sin[(b^2*Pi*x^2)/2])/(12*x^3)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
```

```
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)}{x^5}\,dx &= -\frac{S(bx)}{4x^4}+\frac{1}{4}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx\\
&= -\frac{S(bx)}{4x^4}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}+\frac{1}{12}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x}-\frac{S(bx)}{4x^4}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}-\frac{1}{12}\left(b^5\pi^2\right)\int \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x}-\frac{1}{12}b^4\pi^2S(bx)-\frac{S(bx)}{4x^4}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 69, normalized size = 1.00

$$-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x}-\frac{1}{12}b^4\pi^2S(bx)-\frac{S(bx)}{4x^4}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^5,x]
[Out] -1/12*(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x - (b^4*Pi^2*FresnelS[b*x])/12 - Fresne
lS[b*x]/(4*x^4) - (b*Sin[(b^2*Pi*x^2)/2])/(12*x^3)
```

Maple [A]
time = 0.33, size = 65, normalized size = 0.94

method	result	size
derivativedivides	$b^4\left(-\frac{\mathrm{S}(bx)}{4b^4x^4}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{12b^3x^3}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{bx}-\pi\,\mathrm{S}(bx)\right)}{12}\right)$	65
default	$b^4\left(-\frac{\mathrm{S}(bx)}{4b^4x^4}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{12b^3x^3}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{bx}-\pi\,\mathrm{S}(bx)\right)}{12}\right)$	65
meijerg	$\frac{\pi^2b^4\left(-\frac{32\cos\left(\frac{b^2\pi x^2}{2}\right)}{3\pi xb}-\frac{32\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\pi^2x^3b^3}-\frac{32\left(x^4\pi^2b^4+3\right)\mathrm{S}(bx)}{3\pi^2x^4b^4}\right)}{128}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] b^4*(-1/4*FresnelS(b*x)/b^4/x^4-1/12/b^3/x^3*sin(1/2*b^2*Pi*x^2)+1/12*Pi*(-
1/b/x*cos(1/2*b^2*Pi*x^2)-Pi*FresnelS(b*x)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 61, normalized size = 0.88

$$-\frac{\sqrt{\frac{1}{2}}\,(\pi x^2)^{\frac{3}{2}}\left(-(i+1)\,\sqrt{2}\,\Gamma\left(-\frac{3}{2},\frac{1}{2}i\,\pi b^2x^2\right)+(i-1)\,\sqrt{2}\,\Gamma\left(-\frac{3}{2},-\frac{1}{2}i\,\pi b^2x^2\right)\right)b^4}{64\,x^3}-\frac{\mathrm{S}\,(bx)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/64*sqrt(1/2)*(pi*x^2)^(3/2)*(-(I + 1)*sqrt(2)*gamma(-3/2, 1/2*I*pi*b^2*x
^2) + (I - 1)*sqrt(2)*gamma(-3/2, -1/2*I*pi*b^2*x^2))*b^4/x^3 - 1/4*fresnel
_sin(b*x)/x^4
```

Fricas [A]
time = 0.34, size = 54, normalized size = 0.78

$$-\frac{\pi b^3x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)+bx\sin\left(\frac{1}{2}\,\pi b^2x^2\right)+\left(\pi^2b^4x^4+3\right)\mathrm{S}\,(bx)}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(pi*b^3*x^3*cos(1/2*pi*b^2*x^2) + b*x*sin(1/2*pi*b^2*x^2) + (pi^2*b^4
*x^4 + 3)*fresnel_sin(b*x))/x^4
```

Sympy [A]
time = 0.61, size = 110, normalized size = 1.59

$$\frac{\pi^2b^4S(bx)\,\Gamma\left(-\frac{1}{4}\right)}{64\Gamma\left(\frac{7}{4}\right)}+\frac{\pi b^3\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{1}{4}\right)}{64x\Gamma\left(\frac{7}{4}\right)}+\frac{b\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{1}{4}\right)}{64x^3\Gamma\left(\frac{7}{4}\right)}+\frac{3S(bx)\,\Gamma\left(-\frac{1}{4}\right)}{64x^4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**5,x)
```

```
[Out] pi**2*b**4*fresnels(b*x)*gamma(-1/4)/(64*gamma(7/4)) + pi*b**3*cos(pi*b**2*
x**2/2)*gamma(-1/4)/(64*x*gamma(7/4)) + b*sin(pi*b**2*x**2/2)*gamma(-1/4)/(
64*x**3*gamma(7/4)) + 3*fresnels(b*x)*gamma(-1/4)/(64*x**4*gamma(7/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^5,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)/x^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}(b\,x)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^5,x)
[Out] int(FresnelS(b*x)/x^5, x)
```

3.14 $\int \frac{S(bx)}{x^6} \, dx$

Optimal. Leaf size=77

$$-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}-\frac{S(bx)}{5x^5}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4}-\frac{1}{80}b^5\pi^2\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

```
[Out] -1/40*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x^2-1/5*FresnelS(b*x)/x^5-1/80*b^5*Pi^2*Si
(1/2*b^2*Pi*x^2)-1/20*b*sin(1/2*b^2*Pi*x^2)/x^4
```

Rubi [A]
time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6561, 3460, 3378, 3380}

$$-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{20x^4}-\frac{1}{80}\pi^2b^5\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{40x^2}-\frac{S(bx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^6,x]
```

```
[Out] -1/40*(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^2 - FresnelS[b*x]/(5*x^5) - (b*Sin[(b^
2*Pi*x^2)/2])/(20*x^4) - (b^5*Pi^2*SinIntegral[(b^2*Pi*x^2)/2])/80
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{S(bx)}{x^6}\,dx &= -\frac{S(bx)}{5x^5}+\frac{1}{5}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx\\ &= -\frac{S(bx)}{5x^5}+\frac{1}{10}b\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx,x,x^2\right)\\ &= -\frac{S(bx)}{5x^5}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4}+\frac{1}{40}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx,x,x^2\right)\\ &= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}-\frac{S(bx)}{5x^5}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4}-\frac{1}{80}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx,x,x^2\right)\\ &= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}-\frac{S(bx)}{5x^5}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4}-\frac{1}{80}b^5\pi^2\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 77, normalized size = 1.00

$$-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}-\frac{S(bx)}{5x^5}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4}-\frac{1}{80}b^5\pi^2\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^6,x]
```

```
[Out] -1/40*(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^2 - FresnelS[b*x]/(5*x^5) - (b*Sin[(b^
2*Pi*x^2)/2])/(20*x^4) - (b^5*Pi^2*SinIntegral[(b^2*Pi*x^2)/2])/80
```

Maple [A]
time = 0.38, size = 71, normalized size = 0.92

method	result	size
meijerg	$-\frac{\pi b^3 \,\text{hypergeom}\left(\left[-\frac{1}{2},\frac{3}{4}\right],\left[\frac{1}{2},\frac{3}{2},\frac{7}{4}\right],-\frac{x^4\pi^2 b^4}{16}\right)}{12x^2}$	29
derivativedivides	$b^5\left(-\frac{\text{S}(bx)}{5b^5x^5}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{20b^4x^4}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}-\frac{\pi\,\text{sinIntegral}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{20}\right.$	71

default	$b^5\left(-\frac{S(bx)}{5b^5x^5}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{20b^4x^4}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}-\frac{\pi \operatorname{sinIntegral}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{20}\right)$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] b^5*(-1/5*FresnelS(b*x)/b^5/x^5-1/20/b^4/x^4*sin(1/2*b^2*Pi*x^2)+1/20*Pi*(-
1/2/b^2/x^2*cos(1/2*b^2*Pi*x^2)-1/4*Pi*Si(1/2*b^2*Pi*x^2)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.32, size = 48, normalized size = 0.62

$$-\frac{1}{80}\left(-i\,\pi^2\Gamma\left(-2,\frac{1}{2}i\,\pi b^2x^2\right)+i\,\pi^2\Gamma\left(-2,-\frac{1}{2}i\,\pi b^2x^2\right)\right)b^5-\frac{\mathrm{S}\left(bx\right)}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^6,x, algorithm="maxima")
```

```
[Out] -1/80*(-I*pi^2*gamma(-2, 1/2*I*pi*b^2*x^2) + I*pi^2*gamma(-2, -1/2*I*pi*b^2
*x^2))*b^5 - 1/5*fresnel_sin(b*x)/x^5
```

Fricas [A]
time = 0.37, size = 65, normalized size = 0.84

$$-\frac{\pi^2b^5x^5\,\mathrm{Si}\left(\frac{1}{2}\,\pi b^2x^2\right)+2\,\pi b^3x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)+4\,bx\sin\left(\frac{1}{2}\,\pi b^2x^2\right)+16\,\mathrm{S}\left(bx\right)}{80\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^6,x, algorithm="fricas")
```

```
[Out] -1/80*(pi^2*b^5*x^5*sin_integral(1/2*pi*b^2*x^2) + 2*pi*b^3*x^3*cos(1/2*pi*
b^2*x^2) + 4*b*x*sin(1/2*pi*b^2*x^2) + 16*fresnel_sin(b*x))/x^5
```

Sympy [A]
time = 0.60, size = 46, normalized size = 0.60

$$-\frac{\pi b^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_3\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{1}{2},\frac{3}{2},\frac{7}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{16x^2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**6,x)
```

```
[Out] -pi*b**3*gamma(3/4)*hyper((-1/2, 3/4), (1/2, 3/2, 7/4), -pi**2*b**4*x**4/16
)/(16*x**2*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^6,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}(b\,x)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^6,x)
```

```
[Out] int(FresnelS(b*x)/x^6, x)
```

3.15 $\int \frac{S(bx)}{x^7} \, dx$

Optimal. Leaf size=94

$$-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}-\frac{1}{90}b^6\pi^3\text{FresnelC}(bx)-\frac{S(bx)}{6x^6}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x}$$

```
[Out] -1/90*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x^3-1/90*b^6*Pi^3*FresnelC(b*x)-1/6*Fresne
lS(b*x)/x^6-1/30*b*sin(1/2*b^2*Pi*x^2)/x^5+1/90*b^5*Pi^2*sin(1/2*b^2*Pi*x^2
)/x
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6561, 3468, 3469, 3433}

$$-\frac{1}{90}\pi^3b^6\text{FresnelC}(bx)-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{30x^5}+\frac{\pi^2b^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{90x}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{90x^3}-\frac{S(bx)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^7,x]
```

```
[Out] -1/90*(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^3 - (b^6*Pi^3*FresnelC[b*x])/90 - Fres
nelS[b*x]/(6*x^6) - (b*Sin[(b^2*Pi*x^2)/2])/(30*x^5) + (b^5*Pi^2*Sin[(b^2*P
i*x^2)/2])/(90*x)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{S(bx)}{x^7}\,dx &= -\frac{S(bx)}{6x^6}+\frac{1}{6}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx\\ &= -\frac{S(bx)}{6x^6}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}+\frac{1}{30}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx\\ &= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}-\frac{S(bx)}{6x^6}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}-\frac{1}{90}\left(b^5\pi^2\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\ &= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}-\frac{S(bx)}{6x^6}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x}-\frac{1}{90}\left(b^7\pi^3\right)\int \cos\left(\frac{1}{2}b^2\pi\right.\\ &= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}-\frac{1}{90}b^6\pi^3C(bx)-\frac{S(bx)}{6x^6}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 76, normalized size = 0.81

$$\frac{1}{90}\left(-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}-b^6\pi^3\mathrm{FresnelC}(bx)-\frac{15S(bx)}{x^6}+\frac{b(-3+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^7,x]
```

```
[Out] (-((b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^3) - b^6*Pi^3*FresnelC[b*x] - (15*Fresnel
S[b*x])/x^6 + (b*(-3 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/x^5)/90
```

Maple [A]
time = 0.34, size = 86, normalized size = 0.91

method	result	size
meijerg	$-\frac{\pi b^3\,\mathrm{hypergeom}\left(\left[-\frac{3}{4},\frac{3}{4}\right],\left[\frac{1}{4},\frac{3}{2},\frac{7}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{18x^3}$	29

derivativedivides	$b^6\left(-\frac{\mathrm{S}(bx)}{6b^6x^6}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{30b^5x^5}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{3b^3x^3}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{bx}+\pi\,\mathrm{FresnelC}(bx)\right)}{3}\right)}{30}\right)$	86
default	$b^6\left(-\frac{\mathrm{S}(bx)}{6b^6x^6}-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{30b^5x^5}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{3b^3x^3}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{bx}+\pi\,\mathrm{FresnelC}(bx)\right)}{3}\right)}{30}\right)$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] b^6*(-1/6*FresnelS(b*x)/b^6/x^6-1/30/b^5/x^5*sin(1/2*b^2*Pi*x^2)+1/30*Pi*(-
1/3/b^3/x^3*cos(1/2*b^2*Pi*x^2)-1/3*Pi*(-1/b/x*sin(1/2*b^2*Pi*x^2)+Pi*Fresn
elC(b*x))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 61, normalized size = 0.65

$$-\frac{\sqrt{\frac{1}{2}}\left(\pi x^2\right)^{\frac{5}{2}}\left(-(i-1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},\frac{1}{2}i\pi b^2x^2\right)+(i+1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},-\frac{1}{2}i\pi b^2x^2\right)\right)b^6}{192\,x^5}-\frac{\mathrm{S}\left(bx\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^7,x, algorithm="maxima")
```

```
[Out] -1/192*sqrt(1/2)*(pi*x^2)^(5/2)*(-(I - 1)*sqrt(2)*gamma(-5/2, 1/2*I*pi*b^2*
x^2) + (I + 1)*sqrt(2)*gamma(-5/2, -1/2*I*pi*b^2*x^2))*b^6/x^5 - 1/6*fresne
l_sin(b*x)/x^6
```

Fricas [A]
time = 0.36, size = 80, normalized size = 0.85

$$-\frac{\pi^3\sqrt{b^2}\,b^5x^6\,\mathrm{C}\left(\sqrt{b^2}\,x\right)+\pi b^3x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)-\left(\pi^2b^5x^5-3\,bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)+15\,\mathrm{S}\left(bx\right)}{90\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^7,x, algorithm="fricas")
```

```
[Out] -1/90*(pi^3*sqrt(b^2)*b^5*x^6*fresnel_cos(sqrt(b^2)*x) + pi*b^3*x^3*cos(1/2
*pi*b^2*x^2) - (pi^2*b^5*x^5 - 3*b*x)*sin(1/2*pi*b^2*x^2) + 15*fresnel_sin(
b*x))/x^6
```

Sympy [A]
time = 0.81, size = 56, normalized size = 0.60

$$\frac{\pi b^3\Gamma\left(-\frac{3}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}-\frac{3}{4},\frac{3}{4}\\ \frac{1}{4},\frac{3}{2},\frac{7}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{32x^3\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**7,x)
```

```
[Out] pi*b**3*gamma(-3/4)*gamma(3/4)*hyper((-3/4, 3/4), (1/4, 3/2, 7/4), -pi**2*b
**4*x**4/16)/(32*x**3*gamma(1/4)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^7,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)/x^7, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}(b\,x)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^7,x)
```

```
[Out] int(FresnelS(b*x)/x^7, x)
```

3.16 $\int \frac{S(bx)}{x^8}\,dx$

Optimal. Leaf size=102

$$-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4}-\frac{1}{672}b^7\pi^3\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{S(bx)}{7x^7}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2}$$

```
[Out] -1/672*b^7*Pi^3*Ci(1/2*b^2*Pi*x^2)-1/168*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x^4-1/7
*FresnelS(b*x)/x^7-1/42*b*sin(1/2*b^2*Pi*x^2)/x^6+1/336*b^5*Pi^2*sin(1/2*b^
2*Pi*x^2)/x^2
```

Rubi [A]
time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6561, 3460, 3378, 3383}

$$-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{42x^6}-\frac{1}{672}\pi^3b^7\text{CosIntegral}\left(\frac{1}{2}\pi b^2x^2\right)+\frac{\pi^2b^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{336x^2}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{168x^4}-\frac{S(bx)}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^8,x]
```

```
[Out] -1/168*(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^4 - (b^7*Pi^3*CosIntegral[(b^2*Pi*x^2
)/2])/672 - FresnelS[b*x]/(7*x^7) - (b*Sin[(b^2*Pi*x^2)/2])/(42*x^6) + (b^5
*Pi^2*Sin[(b^2*Pi*x^2)/2])/(336*x^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)}{x^8}\,dx &= -\frac{S(bx)}{7x^7}+\frac{1}{7}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx \\
&= -\frac{S(bx)}{7x^7}+\frac{1}{14}b\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^4}\,dx,x,x^2\right) \\
&= -\frac{S(bx)}{7x^7}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6}+\frac{1}{84}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx,x,x^2\right) \\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4}-\frac{S(bx)}{7x^7}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6}-\frac{1}{336}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx,x,x^2\right) \\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4}-\frac{S(bx)}{7x^7}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2}-\frac{1}{672}\left(b^7\pi^3\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx,x,x^2\right) \\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4}-\frac{1}{672}b^7\pi^3\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{S(bx)}{7x^7}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 85, normalized size = 0.83

$$\frac{1}{672}\left(-\frac{4b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}-b^7\pi^3\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{96S(bx)}{x^7}+\frac{2b\left(-8+b^4\pi^2x^4\right)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^8,x]
```

```
[Out] ((-4*b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^4 - b^7*Pi^3*CosIntegral[(b^2*Pi*x^2)/2]
 - (96*FresnelS[b*x])/x^7 + (2*b*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/x
^6)/672
```

Maple [A]
time = 0.42, size = 93, normalized size = 0.91

method	result	s

meijerg	$\frac{\pi^{\frac{7}{2}} b^{7}\left(\frac{\pi^{\frac{3}{2}} x^{4} b^{4}\, \mathrm{hypergeom}\left(\left[1,1,\frac{11}{4}\right],\left[2,3,\frac{7}{2},\frac{15}{4}\right],-\frac{x^{4} \pi^{2} b^{4}}{16}\right)}{165}-\frac{16\left(-\frac{89}{21}+2\gamma-2\ln(2)+4\ln(x)+2\ln(\pi)+4\ln(b)\right)}{21\sqrt{\pi}}-\frac{128}{3\pi^{\frac{5}{2}} x^{4} b^{4}}\right)}{1024}$	79
derivativedivides	$b^{7}\left(-\frac{\mathrm{S}(bx)}{7b^{7}x^{7}}-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{42b^{6}x^{6}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{4b^{4}x^{4}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{2b^{2}x^{2}}+\frac{\pi\, \mathrm{cosineIntegral}\left(\frac{b^{2}\pi x^{2}}{2}\right)}{4}\right)}{4}\right)}{42}\right)$	93
default	$b^{7}\left(-\frac{\mathrm{S}(bx)}{7b^{7}x^{7}}-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{42b^{6}x^{6}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{4b^{4}x^{4}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{2b^{2}x^{2}}+\frac{\pi\, \mathrm{cosineIntegral}\left(\frac{b^{2}\pi x^{2}}{2}\right)}{4}\right)}{4}\right)}{42}\right)$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] b^7*(-1/7*FresnelS(b*x)/b^7/x^7-1/42/b^6/x^6*sin(1/2*b^2*Pi*x^2)+1/42*Pi*(-
1/4/b^4/x^4*cos(1/2*b^2*Pi*x^2)-1/4*Pi*(-1/2/b^2/x^2*sin(1/2*b^2*Pi*x^2)+1/
4*Pi*Ci(1/2*b^2*Pi*x^2))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.32, size = 46, normalized size = 0.45

$$-\frac{1}{224}\left(\pi^{3}\Gamma\left(-3,\frac{1}{2}i\,\pi b^{2}x^{2}\right)+\pi^{3}\Gamma\left(-3,-\frac{1}{2}i\,\pi b^{2}x^{2}\right)\right)b^{7}-\frac{\mathrm{S}\left(bx\right)}{7\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^8,x, algorithm="maxima")
```

```
[Out] -1/224*(pi^3*gamma(-3, 1/2*I*pi*b^2*x^2) + pi^3*gamma(-3, -1/2*I*pi*b^2*x^2
))*b^7 - 1/7*fresnel_sin(b*x)/x^7
```

Fricas [A]
time = 0.37, size = 98, normalized size = 0.96

$$-\frac{\pi^{3}b^{7}x^{7}\,\mathrm{Ci}\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)+\pi^{3}b^{7}x^{7}\,\mathrm{Ci}\left(-\frac{1}{2}\,\pi b^{2}x^{2}\right)+8\,\pi b^{3}x^{3}\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)-4\left(\pi^{2}b^{5}x^{5}-8\,bx\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)+192\,\mathrm{S}\left(bx\right)}{1344\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^8,x, algorithm="fricas")
```

```
[Out] -1/1344*(pi^3*b^7*x^7*cos_integral(1/2*pi*b^2*x^2) + pi^3*b^7*x^7*cos_integ
ral(-1/2*pi*b^2*x^2) + 8*pi*b^3*x^3*cos(1/2*pi*b^2*x^2) - 4*(pi^2*b^5*x^5 -
 8*b*x)*sin(1/2*pi*b^2*x^2) + 192*fresnel_sin(b*x))/x^7
```

Sympy [A]
time = 2.96, size = 68, normalized size = 0.67

$$\frac{\pi^5 b^{11} x^4 \Gamma\left(\frac{11}{4}\right) {}_3F_4\left(\begin{matrix} 1, 1, \frac{11}{4} \\ 2, 3, \frac{7}{2}, \frac{15}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{61440\Gamma\left(\frac{15}{4}\right)} - \frac{\pi^3 b^7 \log\left(b^4 x^4\right)}{1344} - \frac{\pi b^3}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**8,x)
```

```
[Out] pi**5*b**11*x**4*gamma(11/4)*hyper((1, 1, 11/4), (2, 3, 7/2, 15/4), -pi**2*
b**4*x**4/16)/(61440*gamma(15/4)) - pi**3*b**7*log(b**4*x**4)/1344 - pi*b**
3/(24*x**4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)/x^8, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}(b\,x)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^8,x)
```

```
[Out] int(FresnelS(b*x)/x^8, x)
```

3.17 $\int \frac{S(bx)}{x^9}\,dx$

Optimal. Leaf size=119

$$-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5}+\frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x}+\frac{1}{840}b^8\pi^4S(bx)-\frac{S(bx)}{8x^8}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3}$$

```
[Out] -1/280*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x^5+1/840*b^7*Pi^3*cos(1/2*b^2*Pi*x^2)/x+
1/840*b^8*Pi^4*FresnelS(b*x)-1/8*FresnelS(b*x)/x^8-1/56*b*sin(1/2*b^2*Pi*x^
2)/x^7+1/840*b^5*Pi^2*sin(1/2*b^2*Pi*x^2)/x^3
```

Rubi [A]
time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6561, 3468, 3469, 3432}

$$\frac{1}{840}\pi^4b^8S(bx)-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{56x^7}+\frac{\pi^3b^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{840x}+\frac{\pi^2b^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{840x^3}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{280x^5}-\frac{S(bx)}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^9,x]
```

```
[Out] -1/280*(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^5 + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2])/(8
40*x) + (b^8*Pi^4*FresnelS[b*x])/840 - FresnelS[b*x]/(8*x^8) - (b*Sin[(b^2*
Pi*x^2)/2])/(56*x^7) + (b^5*Pi^2*Sin[(b^2*Pi*x^2)/2])/(840*x^3)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)}{x^9}\,dx &= -\frac{S(bx)}{8x^8}+\frac{1}{8}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx\\
&= -\frac{S(bx)}{8x^8}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}+\frac{1}{56}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5}-\frac{S(bx)}{8x^8}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}-\frac{1}{280}\left(b^5\pi^2\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5}-\frac{S(bx)}{8x^8}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3}-\frac{1}{840}\left(b^7\pi^3\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi\right.}{x^2}\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5}+\frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x}-\frac{S(bx)}{8x^8}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3}+\frac{1}{84}\\
&= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5}+\frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x}+\frac{1}{840}b^8\pi^4S(bx)-\frac{S(bx)}{8x^8}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}+\frac{b^5\pi^2\,\mathrm{s}}{}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 84, normalized size = 0.71

$$\frac{b^3\pi x^3\left(-3+b^4\pi^2x^4\right)\cos\left(\frac{1}{2}b^2\pi x^2\right)+\left(-105+b^8\pi^4x^8\right)S(bx)+bx\left(-15+b^4\pi^2x^4\right)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^9,x]
```

```
[Out] (b^3*Pi*x^3*(-3 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8)
*FresnelS[b*x] + b*x*(-15 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(840*x^8)
```

Maple [A]
time = 0.32, size = 109, normalized size = 0.92

method	result	size
meijerg	$-\frac{\pi b^3\,\mathrm{hypergeom}\left(\left[-\frac{5}{4},\frac{3}{4}\right],\left[-\frac{1}{4},\frac{3}{2},\frac{7}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{30x^5}$	29

derivativedivides	$b^{8} \left(-\frac{\mathrm{S}(bx)}{8b^{8}x^{8}}-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{56b^{7}x^{7}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{5b^{5}x^{5}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{3b^{3}x^{3}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{bx}-\pi\,\mathrm{S}(bx)\right)}{3}\right)}{5}\right)}{56}\right)$	109
default	$b^{8} \left(-\frac{\mathrm{S}(bx)}{8b^{8}x^{8}}-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{56b^{7}x^{7}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{5b^{5}x^{5}}-\frac{\pi\left(-\frac{\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{3b^{3}x^{3}}+\frac{\pi\left(-\frac{\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{bx}-\pi\,\mathrm{S}(bx)\right)}{3}\right)}{5}\right)}{56}\right)$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] b^8*(-1/8*FresnelS(b*x)/b^8/x^8-1/56/b^7/x^7*sin(1/2*b^2*Pi*x^2)+1/56*Pi*(-
1/5/b^5/x^5*cos(1/2*b^2*Pi*x^2)-1/5*Pi*(-1/3/b^3/x^3*sin(1/2*b^2*Pi*x^2)+1/
3*Pi*(-1/b/x*cos(1/2*b^2*Pi*x^2)-Pi*FresnelS(b*x)))))
```

Maxima [C] Result contains complex when optimal does not.

time = 0.52, size = 61, normalized size = 0.51

$$-\frac{\sqrt{\frac{1}{2}}\,(\pi x^2)^{\frac{7}{2}}\left((i+1)\,\sqrt{2}\,\Gamma\left(-\frac{7}{2},\frac{1}{2}i\,\pi b^2x^2\right)-(i-1)\,\sqrt{2}\,\Gamma\left(-\frac{7}{2},-\frac{1}{2}i\,\pi b^2x^2\right)\right)b^8}{512\,x^7}-\frac{\mathrm{S}\,(bx)}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^9,x, algorithm="maxima")
```

```
[Out] -1/512*sqrt(1/2)*(pi*x^2)^(7/2)*((I + 1)*sqrt(2)*gamma(-7/2, 1/2*I*pi*b^2*x
^2) - (I - 1)*sqrt(2)*gamma(-7/2, -1/2*I*pi*b^2*x^2))*b^8/x^7 - 1/8*fresnel
_sin(b*x)/x^8
```

Fricas [A]
time = 0.35, size = 80, normalized size = 0.67

$$\frac{(\pi^3b^7x^7-3\,\pi b^3x^3)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)+(\pi^4b^8x^8-105)\,\mathrm{S}\,(bx)+(\pi^2b^5x^5-15\,bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{840\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^9,x, algorithm="fricas")
```

```
[Out] 1/840*((pi^3*b^7*x^7 - 3*pi*b^3*x^3)*cos(1/2*pi*b^2*x^2) + (pi^4*b^8*x^8 -
105)*fresnel_sin(b*x) + (pi^2*b^5*x^5 - 15*b*x)*sin(1/2*pi*b^2*x^2))/x^8
```

Sympy [A]
time = 1.82, size = 185, normalized size = 1.55

$$\frac{\pi^4b^8S(bx)\,\Gamma\left(-\frac{5}{4}\right)}{3584\Gamma\left(\frac{7}{4}\right)}+\frac{\pi^3b^7\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{5}{4}\right)}{3584x\Gamma\left(\frac{7}{4}\right)}+\frac{\pi^2b^5\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{5}{4}\right)}{3584x^3\Gamma\left(\frac{7}{4}\right)}-\frac{3\pi b^3\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{5}{4}\right)}{3584x^5\Gamma\left(\frac{7}{4}\right)}-\frac{15b\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{5}{4}\right)}{3584x^7\Gamma\left(\frac{7}{4}\right)}-\frac{15S(bx)\,\Gamma\left(-\frac{5}{4}\right)}{512x^8\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**9,x)
```

```
[Out] pi**4*b**8*fresnels(b*x)*gamma(-5/4)/(3584*gamma(7/4)) + pi**3*b**7*cos(pi*
b**2*x**2/2)*gamma(-5/4)/(3584*x*gamma(7/4)) + pi**2*b**5*sin(pi*b**2*x**2/
2)*gamma(-5/4)/(3584*x**3*gamma(7/4)) - 3*pi*b**3*cos(pi*b**2*x**2/2)*gamma
(-5/4)/(3584*x**5*gamma(7/4)) - 15*b*sin(pi*b**2*x**2/2)*gamma(-5/4)/(3584*
x**7*gamma(7/4)) - 15*fresnels(b*x)*gamma(-5/4)/(512*x**8*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^9,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)/x^9, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}(b\,x)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^9,x)
```

```
[Out] int(FresnelS(b*x)/x^9, x)
```

3.18 $\int \frac{S(bx)}{x^{10}}\,dx$

Optimal. Leaf size=127

$$-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}+\frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2}-\frac{S(bx)}{9x^9}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4}+\frac{b^9\pi^4\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)}{6912}$$

```
[Out] -1/432*b^3*Pi*cos(1/2*b^2*Pi*x^2)/x^6+1/3456*b^7*Pi^3*cos(1/2*b^2*Pi*x^2)/x
^2-1/9*FresnelS(b*x)/x^9+1/6912*b^9*Pi^4*Si(1/2*b^2*Pi*x^2)-1/72*b*sin(1/2*
b^2*Pi*x^2)/x^8+1/1728*b^5*Pi^2*sin(1/2*b^2*Pi*x^2)/x^4
```

Rubi [A]
time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6561, 3460, 3378, 3380}

$$-\frac{b\sin\left(\frac{1}{2}\pi b^2x^2\right)}{72x^8}+\frac{\pi^4b^9\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)}{6912}+\frac{\pi^3b^7\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3456x^2}+\frac{\pi^2b^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{1728x^4}-\frac{\pi b^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{432x^6}-\frac{S(bx)}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]/x^10,x]
```

```
[Out] -1/432*(b^3*Pi*Cos[(b^2*Pi*x^2)/2])/x^6 + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2])/(3
456*x^2) - FresnelS[b*x]/(9*x^9) - (b*Sin[(b^2*Pi*x^2)/2])/(72*x^8) + (b^5*
Pi^2*Sin[(b^2*Pi*x^2)/2])/(1728*x^4) + (b^9*Pi^4*SinIntegral[(b^2*Pi*x^2)/2
])/6912
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6561

```
Int[FresnelS[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelS[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*S
in[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{S(bx)}{x^{10}}\,dx &= -\frac{S(bx)}{9x^9}+\frac{1}{9}b\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx\\ &= -\frac{S(bx)}{9x^9}+\frac{1}{18}b\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^5}\,dx,x,x^2\right)\\ &= -\frac{S(bx)}{9x^9}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}+\frac{1}{144}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^4}\,dx,x,x^2\right)\\ &= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}-\frac{S(bx)}{9x^9}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}-\frac{1}{864}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx,x,x^2\right)\\ &= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}-\frac{S(bx)}{9x^9}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4}-\frac{\left(b^7\pi^3\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^2}\right.}{3456}\\ &= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}+\frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2}-\frac{S(bx)}{9x^9}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4}+\frac{(b^9}{}\\ &= -\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}+\frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2}-\frac{S(bx)}{9x^9}-\frac{b\sin\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}+\frac{b^5\pi^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4}+\frac{b^9\pi}{}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 96, normalized size = 0.76

$$\frac{\frac{2b^3\pi\left(-8+b^4\pi^2x^4\right)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}-\frac{768S(bx)}{x^9}+\frac{4b\left(-24+b^4\pi^2x^4\right)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}+b^9\pi^4\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)}{6912}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]/x^10,x]
```

```
[Out] ((2*b^3*Pi*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^6 - (768*FresnelS[b*x
])/x^9 + (4*b*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/x^8 + b^9*Pi^4*SinI
ntegral[(b^2*Pi*x^2)/2])/6912
```

Maple [A]
time = 0.38, size = 115, normalized size = 0.91

method	result
meijerg	$-\frac{\pi b^3 \operatorname{hypergeom}\left(\left[-\frac{3}{2}, \frac{3}{4}\right], \left[-\frac{1}{2}, \frac{3}{2}, \frac{7}{4}\right], -\frac{x^4 \pi^2 b^4}{16}\right)}{36 x^6}$
derivativedivides	$b^9 \left(-\frac{\mathrm{S}(bx)}{9 b^9 x^9} - \frac{\sin\left(\frac{b^2 \pi x^2}{2}\right)}{72 b^8 x^8} + \frac{\pi \left(-\frac{\cos\left(\frac{b^2 \pi x^2}{2}\right)}{6 b^6 x^6} - \frac{\pi \left(-\frac{\sin\left(\frac{b^2 \pi x^2}{2}\right)}{4 b^4 x^4} + \frac{\pi \left(-\frac{\cos\left(\frac{b^2 \pi x^2}{2}\right)}{2 b^2 x^2} - \frac{\pi \operatorname{sinIntegral}\left(\frac{b^2 \pi x^2}{2}\right)}{4}\right)}{4}\right)}{6}\right)}{72}\right)$
default	$b^9 \left(-\frac{\mathrm{S}(bx)}{9 b^9 x^9} - \frac{\sin\left(\frac{b^2 \pi x^2}{2}\right)}{72 b^8 x^8} + \frac{\pi \left(-\frac{\cos\left(\frac{b^2 \pi x^2}{2}\right)}{6 b^6 x^6} - \frac{\pi \left(-\frac{\sin\left(\frac{b^2 \pi x^2}{2}\right)}{4 b^4 x^4} + \frac{\pi \left(-\frac{\cos\left(\frac{b^2 \pi x^2}{2}\right)}{2 b^2 x^2} - \frac{\pi \operatorname{sinIntegral}\left(\frac{b^2 \pi x^2}{2}\right)}{4}\right)}{4}\right)}{6}\right)}{72}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] b^9*(-1/9*FresnelS(b*x)/b^9/x^9-1/72/b^8/x^8*sin(1/2*b^2*Pi*x^2)+1/72*Pi*(-
1/6/b^6/x^6*cos(1/2*b^2*Pi*x^2)-1/6*Pi*(-1/4/b^4/x^4*sin(1/2*b^2*Pi*x^2)+1/
4*Pi*(-1/2/b^2/x^2*cos(1/2*b^2*Pi*x^2)-1/4*Pi*Si(1/2*b^2*Pi*x^2)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.31, size = 48, normalized size = 0.38

$$-\frac{1}{576}\left(i\,\pi^4\Gamma\left(-4,\frac{1}{2}i\,\pi b^2x^2\right)-i\,\pi^4\Gamma\left(-4,-\frac{1}{2}i\,\pi b^2x^2\right)\right)b^9-\frac{\mathrm{S}\,(bx)}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^10,x, algorithm="maxima")
```

```
[Out] -1/576*(I*pi^4*gamma(-4, 1/2*I*pi*b^2*x^2) - I*pi^4*gamma(-4, -1/2*I*pi*b^2
*x^2))*b^9 - 1/9*fresnel_sin(b*x)/x^9
```

Fricas [A]
time = 0.37, size = 91, normalized size = 0.72

$$\frac{\pi^4b^9x^9\,\mathrm{Si}\left(\frac{1}{2}\,\pi b^2x^2\right)+2\left(\pi^3b^7x^7-8\,\pi b^3x^3\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)+4\left(\pi^2b^5x^5-24\,bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)-768\,\mathrm{S}\,(bx)}{6912\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^10,x, algorithm="fricas")
```

```
[Out] 1/6912*(pi^4*b^9*x^9*sin_integral(1/2*pi*b^2*x^2) + 2*(pi^3*b^7*x^7 - 8*pi*
b^3*x^3)*cos(1/2*pi*b^2*x^2) + 4*(pi^2*b^5*x^5 - 24*b*x)*sin(1/2*pi*b^2*x^2
) - 768*fresnel_sin(b*x))/x^9
```

Sympy [A]
time = 2.14, size = 48, normalized size = 0.38

$$-\frac{\pi b^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_3\left(\left.\begin{matrix}-\frac{3}{2},\frac{3}{4}\\-\frac{1}{2},\frac{3}{2},\frac{7}{4}\end{matrix}\right|-\frac{\pi^2b^4x^4}{16}\right)}{48x^6\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)/x**10,x)
```

```
[Out] -pi*b**3*gamma(3/4)*hyper((-3/2, 3/4), (-1/2, 3/2, 7/4), -pi**2*b**4*x**4/1
6)/(48*x**6*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)/x^10,x, algorithm="giac")
```

[Out] integrate(fresnel_sin(b*x)/x^10, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}(b\,x)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(FresnelS(b*x)/x^10,x)

[Out] int(FresnelS(b*x)/x^10, x)

3.19 $\int (c+dx)^3 S(a+bx)\,dx$

Optimal. Leaf size=296

$$\frac{(bc-ad)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi}+\frac{3d(bc-ad)^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi}+\frac{d^2(bc-ad)(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx\right.}{b^4\pi}$$

```
[Out] (-a*d+b*c)^3*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi+3/2*d*(-a*d+b*c)^2*(b*x+a)*cos(1/
2*Pi*(b*x+a)^2)/b^4/Pi+d^2*(-a*d+b*c)*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)/b^4/P
i+1/4*d^3*(b*x+a)^3*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi-3/2*d*(-a*d+b*c)^2*Fresnel
C(b*x+a)/b^4/Pi-1/4*(-a*d+b*c)^4*FresnelS(b*x+a)/b^4/d+3/4*d^3*FresnelS(b*x
+a)/b^4/Pi^2+1/4*(d*x+c)^4*FresnelS(b*x+a)/d-2*d^2*(-a*d+b*c)*sin(1/2*Pi*(b
*x+a)^2)/b^4/Pi^2-3/4*d^3*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi^2
```

Rubi [A]
time = 0.28, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6563, 3514, 3432, 3460, 2718, 3466, 3433, 3377, 2717, 3467}

$$-\frac{2d^2(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2b^4}+\frac{d^2(a+bx)^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}-\frac{3d(bc-ad)^2\text{FresnelC}(a+bx)}{2\pi b^4}-\frac{(bc-ad)^4S(a+bx)}{4b^4d}+\frac{(bc-ad)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}+\frac{3d(a+bx)(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^4}+\frac{3d^3S(a+bx)}{4\pi^2b^4}-\frac{3d^3(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4\pi^2b^4}+\frac{d^3(a+bx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4\pi b^4}+\frac{(c+dx)^4S(a+bx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*FresnelS[a + b*x],x]
```

```
[Out] ((b*c - a*d)^3*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi) + (3*d*(b*c - a*d)^2*(a +
b*x)*Cos[(Pi*(a + b*x)^2)/2])/(2*b^4*Pi) + (d^2*(b*c - a*d)*(a + b*x)^2*Cos
[(Pi*(a + b*x)^2)/2])/(b^4*Pi) + (d^3*(a + b*x)^3*Cos[(Pi*(a + b*x)^2)/2])/
(4*b^4*Pi) - (3*d*(b*c - a*d)^2*FresnelC[a + b*x])/(2*b^4*Pi) - ((b*c - a*d
)^4*FresnelS[a + b*x])/(4*b^4*d) + (3*d^3*FresnelS[a + b*x])/(4*b^4*Pi^2) +
 ((c + d*x)^4*FresnelS[a + b*x])/(4*d) - (2*d^2*(b*c - a*d)*Sin[(Pi*(a + b*
x)^2)/2])/(b^4*Pi^2) - (3*d^3*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^
2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6563

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelS[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
```

```
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^3 S(a+bx)\,dx &= \frac{(c+dx)^4 S(a+bx)}{4d} - \frac{b\int (c+dx)^4 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx}{4d} \\
&= \frac{(c+dx)^4 S(a+bx)}{4d} - \frac{\text{Subst}\Big(\int \Big(b^4c^4\Big(1+\frac{ad(-4b^3c^3+6ab^2c^2d-4a^2bcd^2+a^3d^3)}{b^4c^4}\Big)\sin\Big(\frac{\pi x^2}{2}}{} \\
&= \frac{(c+dx)^4 S(a+bx)}{4d} - \frac{d^3\text{Subst}\left(\int x^4\sin\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{4b^4} - \frac{(d^2(bc-ad))\,\text{Su}}{} \\
&= \frac{3d(bc-ad)^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} + \frac{d^3(a+bx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi} - \frac{(bc-}{} \\
&= \frac{(bc-ad)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{3d(bc-ad)^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} + \frac{d^2(bc-}{} \\
&= \frac{(bc-ad)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{3d(bc-ad)^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} + \frac{d^2(bc-}{}
\end{aligned}
$$

Mathematica [A]
time = 0.56, size = 424, normalized size = 1.43

$$
\frac{4b^3c^3\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right) - 6ab^2c^2d\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + 4a^2bcd^2\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right) - a^3d^3\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + 6b^3c^2d\pi x\cos\left(\frac{1}{2}\pi(a+bx)^2\right) - 4ab^2cd^2\pi x\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + a^2bd^3\pi x\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + 4b^3cd^2\pi x^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right) - ab^2d^3\pi x^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + b^3d^3\pi x^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right) - 6d(bc-ad)^2\pi\,\text{FresnelC}(a+bx) + \left(4b^3c^3\pi^2(a+bx) + 6b^2c^2d\pi^2(-a^2+b^2x^2) + 4bcd^2\pi^2(a^3+b^3x^3) + d^3(3-a^4\pi^2+b^4\pi^2x^4)\right)S(a+bx) - 8bcd^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right) + 5ad^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right) - 3bd^3x\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*FresnelS[a + b*x],x]
```

```
[Out] (4*b^3*c^3*Pi*Cos[(Pi*(a + b*x)^2)/2] - 6*a*b^2*c^2*d*Pi*Cos[(Pi*(a + b*x)^
2)/2] + 4*a^2*b*c*d^2*Pi*Cos[(Pi*(a + b*x)^2)/2] - a^3*d^3*Pi*Cos[(Pi*(a +
b*x)^2)/2] + 6*b^3*c^2*d*Pi*x*Cos[(Pi*(a + b*x)^2)/2] - 4*a*b^2*c*d^2*Pi*x*
Cos[(Pi*(a + b*x)^2)/2] + a^2*b*d^3*Pi*x*Cos[(Pi*(a + b*x)^2)/2] + 4*b^3*c*
d^2*Pi*x^2*Cos[(Pi*(a + b*x)^2)/2] - a*b^2*d^3*Pi*x^2*Cos[(Pi*(a + b*x)^2)/
2] + b^3*d^3*Pi*x^3*Cos[(Pi*(a + b*x)^2)/2] - 6*d*(b*c - a*d)^2*Pi*FresnelC
[a + b*x] + (4*b^3*c^3*Pi^2*(a + b*x) + 6*b^2*c^2*d*Pi^2*(-a^2 + b^2*x^2) +
 4*b*c*d^2*Pi^2*(a^3 + b^3*x^3) + d^3*(3 - a^4*Pi^2 + b^4*Pi^2*x^4))*Fresne
lS[a + b*x] - 8*b*c*d^2*Sin[(Pi*(a + b*x)^2)/2] + 5*a*d^3*Sin[(Pi*(a + b*x)
^2)/2] - 3*b*d^3*x*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

Maple [A]
time = 0.52, size = 401, normalized size = 1.35

method	result
derivativedivides	$\frac{\frac{S(bx+a)(ad-cb-d(bx+a))^4}{4b^3d} - \frac{-\frac{d^4(bx+a)^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{3d^4\left(\frac{(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} - \frac{S(bx+a)}{\pi}\right)}{\pi} - \frac{\left(-4a\,d^4+4bc\,d^3\right)(bx+a}{\pi} \cdots}{\cdots}}{\cdots}$
default	$\frac{\frac{S(bx+a)(ad-cb-d(bx+a))^4}{4b^3d} - \frac{-\frac{d^4(bx+a)^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{3d^4\left(\frac{(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} - \frac{S(bx+a)}{\pi}\right)}{\pi} - \frac{\left(-4a\,d^4+4bc\,d^3\right)(bx+a}{\pi} \cdots}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*FresnelS(b*x+a)*(a*d-c*b-d*(b*x+a))^4/b^3/d-1/4/b^3/d*(-d^4/Pi*(b*
x+a)^3*cos(1/2*Pi*(b*x+a)^2)+3*d^4/Pi*(1/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-1
/Pi*FresnelS(b*x+a))-(-4*a*d^4+4*b*c*d^3)/Pi*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2
)+2*(-4*a*d^4+4*b*c*d^3)/Pi^2*sin(1/2*Pi*(b*x+a)^2)-(6*a^2*d^4-12*a*b*c*d^3
+6*b^2*c^2*d^2)/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+(6*a^2*d^4-12*a*b*c*d^3+6*
b^2*c^2*d^2)/Pi*FresnelC(b*x+a)-(-4*a^3*d^4+12*a^2*b*c*d^3-12*a*b^2*c^2*d^2
+4*b^3*c^3*d)/Pi*cos(1/2*Pi*(b*x+a)^2)+a^4*d^4*FresnelS(b*x+a)-4*a^3*b*c*d^
3*FresnelS(b*x+a)+6*a^2*b^2*c^2*d^2*FresnelS(b*x+a)-4*a*b^3*c^3*d*FresnelS(
b*x+a)+b^4*c^4*FresnelS(b*x+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*fresnel_sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^3*fresnel_sin(b*x + a), x)
```

Fricas [A]
time = 0.36, size = 376, normalized size = 1.27

$$-\frac{6\,\pi(b^2c^2d - 2\,abcd^2 + a^2d^3)\sqrt{b^2}\,C\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right) - (\pi^2(4\,ab^3c^3 - 6\,a^2b^2c^2d + 4\,a^3bcd^2 - a^4d^3) + 3\,d^3)\sqrt{b^2}\,S\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right) - (\pi b^4d^3x^3 + \pi(4\,b^4cd^2 - ab^3d^3)x^2 + \pi(6\,b^4c^2d - 4\,ab^3cd^2 + a^2b^2d^3)x + \pi(4\,b^4c^3 - 6\,ab^3c^2d + 4\,a^2b^2cd^2 - a^3bd^3))\cos\left(\frac{1}{2}\,\pi b^2x^2 + \pi abx + \frac{1}{2}\,\pi a^2\right) - (\pi^2b^5d^3x^4 + 4\,\pi^2b^5cd^2x^3 + 6\,\pi^2b^5c^2dx^2 + 4\,\pi^2b^5c^3x)\,S\left(bx + a\right) + (3\,b^2d^3x + 8\,b^2cd^2 - 5\,abd^3)\sin\left(\frac{1}{2}\,\pi b^2x^2 + \pi abx + \frac{1}{2}\,\pi a^2\right)}{4\,\pi^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*fresnel_sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(6*pi*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(b^2)*fresnel_cos(sqrt(b
^2)*(b*x + a)/b) - (pi^2*(4*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 4*a^3*b*c*d^2 - a
```

```
^4*d^3) + 3*d^3)*sqrt(b^2)*fresnel_sin(sqrt(b^2)*(b*x + a)/b) - (pi*b^4*d^3
*x^3 + pi*(4*b^4*c*d^2 - a*b^3*d^3)*x^2 + pi*(6*b^4*c^2*d - 4*a*b^3*c*d^2 +
 a^2*b^2*d^3)*x + pi*(4*b^4*c^3 - 6*a*b^3*c^2*d + 4*a^2*b^2*c*d^2 - a^3*b*d
^3))*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) - (pi^2*b^5*d^3*x^4 + 4*pi
^2*b^5*c*d^2*x^3 + 6*pi^2*b^5*c^2*d*x^2 + 4*pi^2*b^5*c^3*x)*fresnel_sin(b*x
 + a) + (3*b^2*d^3*x + 8*b^2*c*d^2 - 5*a*b*d^3)*sin(1/2*pi*b^2*x^2 + pi*a*b
*x + 1/2*pi*a^2))/(pi^2*b^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 S(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*fresnels(b*x+a),x)
[Out] Integral((c + d*x)**3*fresnels(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*fresnel_sin(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^3*fresnel_sin(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{FresnelS}(a+b\,x)\,{\left(c+d\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)*(c + d*x)^3,x)
[Out] int(FresnelS(a + b*x)*(c + d*x)^3, x)
```

3.20 $\int (c+dx)^2 S(a+bx)\,dx$

Optimal. Leaf size=193

$$\frac{(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}+\frac{d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}+\frac{d^2(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi}-\frac{d(bc}{}$$

```
[Out] (-a*d+b*c)^2*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi+d*(-a*d+b*c)*(b*x+a)*cos(1/2*Pi*(
b*x+a)^2)/b^3/Pi+1/3*d^2*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi-d*(-a*d+b*c
)*FresnelC(b*x+a)/b^3/Pi-1/3*(-a*d+b*c)^3*FresnelS(b*x+a)/b^3/d+1/3*(d*x+c)
^3*FresnelS(b*x+a)/d-2/3*d^2*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi^2
```

Rubi [A]
time = 0.16, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6563, 3514, 3432, 3460, 2718, 3466, 3433, 3377, 2717}

$$-\frac{d(bc-ad)\text{FresnelC}(a+bx)}{\pi b^3}-\frac{(bc-ad)^3S(a+bx)}{3b^3d}+\frac{(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}+\frac{d(a+bx)(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}-\frac{2d^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi^2b^3}+\frac{d^2(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi b^3}+\frac{(c+dx)^3S(a+bx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*FresnelS[a + b*x],x]
```

```
[Out] ((b*c - a*d)^2*Cos[(Pi*(a + b*x)^2)/2])/(b^3*Pi) + (d*(b*c - a*d)*(a + b*x)
*Cos[(Pi*(a + b*x)^2)/2])/(b^3*Pi) + (d^2*(a + b*x)^2*Cos[(Pi*(a + b*x)^2)/
2])/(3*b^3*Pi) - (d*(b*c - a*d)*FresnelC[a + b*x])/(b^3*Pi) - ((b*c - a*d)^
3*FresnelS[a + b*x])/(3*b^3*d) + ((c + d*x)^3*FresnelS[a + b*x])/(3*d) - (2
*d^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6563

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelS[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2 S(a+bx)\,dx = \frac{(c+dx)^3 S(a+bx)}{3d} - \frac{b\int (c+dx)^3 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx}{3d}$$

$$= \frac{(c+dx)^3 S(a+bx)}{3d} - \frac{\text{Subst}\Big(\int \Big(b^3c^3\Big(1-\frac{ad(3b^2c^2-3abcd+a^2d^2)}{b^3c^3}\Big)\sin\left(\frac{\pi x^2}{2}\right)+3b^2c^2 \ldots}{\ldots}$$

$$= \frac{(c+dx)^3 S(a+bx)}{3d} - \frac{d^2\text{Subst}\Big(\int x^3\sin\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\Big)}{3b^3} - \frac{(d(bc-ad))\text{Su}\ldots}{\ldots}$$

$$= \frac{d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{(bc-ad)^3 S(a+bx)}{3b^3 d} + \frac{(c+dx)^3 S(a+\ldots}{3d}$$

$$= \frac{(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{d^2(a+\ldots}{\ldots}$$

$$= \frac{(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{d^2(a+\ldots}{\ldots}$$

Mathematica [A]
time = 0.30, size = 236, normalized size = 1.22

$$\frac{3b^2c^2\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right)-3abcd\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+a^2d^2\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+3b^2cd\pi x\cos\left(\frac{1}{2}\pi(a+bx)^2\right)-abd^2\pi x\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+b^2d^2\pi x^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+3d(-bc+ad)\pi\text{FresnelC}(a+bx)+\pi^2(3ab^2c^2-3a^2bcd+a^3d^2+b^3x(3c^2+3cdx+d^2x^2))S(a+bx)-2d^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*FresnelS[a + b*x],x]
```

```
[Out] (3*b^2*c^2*Pi*Cos[(Pi*(a + b*x)^2)/2] - 3*a*b*c*d*Pi*Cos[(Pi*(a + b*x)^2)/2
] + a^2*d^2*Pi*Cos[(Pi*(a + b*x)^2)/2] + 3*b^2*c*d*Pi*x*Cos[(Pi*(a + b*x)^2
)/2] - a*b*d^2*Pi*x*Cos[(Pi*(a + b*x)^2)/2] + b^2*d^2*Pi*x^2*Cos[(Pi*(a + b
*x)^2)/2] + 3*d*(-(b*c) + a*d)*Pi*FresnelC[a + b*x] + Pi^2*(3*a*b^2*c^2 - 3
*a^2*b*c*d + a^3*d^2 + b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2))*FresnelS[a + b*x]
 - 2*d^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Maple [A]
time = 0.51, size = 251, normalized size = 1.30

method	result
derivativedivides	$\frac{-\frac{S(bx+a)(ad-cb-d(bx+a))^3}{3b^2d}+\frac{\frac{d^3(bx+a)^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{2d^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi^2}-\frac{(3a\,d^3-3bc\,d^2)(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{(3a \ldots}{\ldots}}{\ldots}}{\ldots}$
default	$\frac{-\frac{S(bx+a)(ad-cb-d(bx+a))^3}{3b^2d}+\frac{\frac{d^3(bx+a)^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{2d^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi^2}-\frac{(3a\,d^3-3bc\,d^2)(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{(3a \ldots}{\ldots}}{\ldots}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/3*FresnelS(b*x+a)*(a*d-c*b-d*(b*x+a))^3/b^2/d+1/3/b^2/d*(d^3/Pi*(b*
x+a)^2*cos(1/2*Pi*(b*x+a)^2)-2*d^3/Pi^2*sin(1/2*Pi*(b*x+a)^2)-(3*a*d^3-3*b*
c*d^2)/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+(3*a*d^3-3*b*c*d^2)/Pi*FresnelC(b*x
+a)-(-3*a^2*d^3+6*a*b*c*d^2-3*b^2*c^2*d)/Pi*cos(1/2*Pi*(b*x+a)^2)+a^3*d^3*F
resnelS(b*x+a)-3*a^2*b*c*d^2*FresnelS(b*x+a)+3*a*b^2*c^2*d*FresnelS(b*x+a)-
b^3*c^3*FresnelS(b*x+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnel_sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2*fresnel_sin(b*x + a), x)
```

Fricas [A]
time = 0.37, size = 248, normalized size = 1.28

$$\frac{\pi^2(3\,ab^2c^2-3\,a^2bcd+a^3d^2)\sqrt{b^2}\,\mathrm{S}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)-2\,bd^2\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)-3\,\pi(bcd-ad^2)\sqrt{b^2}\,\mathrm{C}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)+(\pi b^3d^2x^2+\pi(3\,b^3cd-ab^2d^2)x+\pi(3\,b^3c^2-3\,ab^2cd+a^2bd^2))\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)+(\pi^2b^4d^2x^3+3\,\pi^2b^4cdx^2+3\,\pi^2b^4c^2x)\,\mathrm{S}\left(bx+a\right)}{3\,\pi^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnel_sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3*(pi^2*(3*a*b^2*c^2 - 3*a^2*b*c*d + a^3*d^2)*sqrt(b^2)*fresnel_sin(sqrt(
b^2)*(b*x + a)/b) - 2*b*d^2*sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) - 3
*pi*(b*c*d - a*d^2)*sqrt(b^2)*fresnel_cos(sqrt(b^2)*(b*x + a)/b) + (pi*b^3*
d^2*x^2 + pi*(3*b^3*c*d - a*b^2*d^2)*x + pi*(3*b^3*c^2 - 3*a*b^2*c*d + a^2*
b*d^2))*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) + (pi^2*b^4*d^2*x^3 + 3
*pi^2*b^4*c*d*x^2 + 3*pi^2*b^4*c^2*x)*fresnel_sin(b*x + a))/(pi^2*b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+dx\right)^2 S\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*fresnels(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*fresnels(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnel_sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*fresnel_sin(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{FresnelS}(a+b\,x)\;(c+d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)*(c + d*x)^2,x)
```

```
[Out] int(FresnelS(a + b*x)*(c + d*x)^2, x)
```

3.21 $\int (c+dx)S(a+bx)\,dx$

Optimal. Leaf size=121

$$\frac{(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi}+\frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}-\frac{d\text{FresnelC}(a+bx)}{2b^2\pi}-\frac{(bc-ad)^2S(a+bx)}{2b^2d}+\frac{(c+d}{}$$

```
[Out] (-a*d+b*c)*cos(1/2*Pi*(b*x+a)^2)/b^2/Pi+1/2*d*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)
/b^2/Pi-1/2*d*FresnelC(b*x+a)/b^2/Pi-1/2*(-a*d+b*c)^2*FresnelS(b*x+a)/b^2/d
+1/2*(d*x+c)^2*FresnelS(b*x+a)/d
```

Rubi [A]
time = 0.08, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6563, 3514, 3432, 3460, 2718, 3466, 3433}

$$-\frac{(bc-ad)^2S(a+bx)}{2b^2d}+\frac{(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}-\frac{d\text{FresnelC}(a+bx)}{2\pi b^2}+\frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{(c+dx)^2S(a+bx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*FresnelS[a + b*x],x]
```

```
[Out] ((b*c - a*d)*Cos[(Pi*(a + b*x)^2)/2])/(b^2*Pi) + (d*(a + b*x)*Cos[(Pi*(a +
b*x)^2)/2])/(2*b^2*Pi) - (d*FresnelC[a + b*x])/(2*b^2*Pi) - ((b*c - a*d)^2*
FresnelS[a + b*x])/(2*b^2*d) + ((c + d*x)^2*FresnelS[a + b*x])/(2*d)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6563

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelS[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int (c+dx)S(a+bx)\,dx = \frac{(c+dx)^2S(a+bx)}{2d} - \frac{b\int (c+dx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx}{2d}$$

$$= \frac{(c+dx)^2S(a+bx)}{2d} - \frac{\text{Subst}\left(\int\left(b^2c^2\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)\sin\left(\frac{\pi x^2}{2}\right)+2bcd\left(1-\frac{ad}{bc}\right)\right.\right.}{2b^2d}$$

$$= \frac{(c+dx)^2S(a+bx)}{2d} - \frac{d\text{Subst}\left(\int x^2\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{2b^2} - \frac{(bc-ad)\text{Subst}\left(\right.}{}$$

$$= \frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} - \frac{(bc-ad)^2S(a+bx)}{2b^2d} + \frac{(c+dx)^2S(a+bx)}{2d} - \frac{(bc}{}$$

$$= \frac{(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi} + \frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} - \frac{dC(a+bx)}{2b^2\pi} - \frac{(bc}{}$$

Mathematica [A]
time = 0.17, size = 61, normalized size = 0.50

$$\frac{-d\text{FresnelC}(a+bx)+(2bc-ad+bdx)\left(\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+\pi(a+bx)S(a+bx)\right)}{2b^2\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*FresnelS[a + b*x],x]
```

```
[Out] (-(d*FresnelC[a + b*x]) + (2*b*c - a*d + b*d*x)*(Cos[(Pi*(a + b*x)^2)/2] +
Pi*(a + b*x)*FresnelS[a + b*x]))/(2*b^2*Pi)
```

Maple [A]
time = 0.41, size = 109, normalized size = 0.90

method	result	size
derivativedivides	$\frac{-\frac{S(bx+a)\left(da(bx+a)-cb(bx+a)-\frac{d(bx+a)^2}{2}\right)}{b}+\frac{\frac{d(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{d\,\mathrm{FresnelC}(bx+a)}{\pi}-\frac{(2ad-2cb)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{2b}}{b}$	109
default	$\frac{-\frac{S(bx+a)\left(da(bx+a)-cb(bx+a)-\frac{d(bx+a)^2}{2}\right)}{b}+\frac{\frac{d(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{d\,\mathrm{FresnelC}(bx+a)}{\pi}-\frac{(2ad-2cb)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{2b}}{b}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-FresnelS(b*x+a)/b*(d*a*(b*x+a)-c*b*(b*x+a)-1/2*d*(b*x+a)^2)+1/2/b*(d/
Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)-d/Pi*FresnelC(b*x+a)-(2*a*d-2*b*c)/Pi*cos(
1/2*Pi*(b*x+a)^2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnel_sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*fresnel_sin(b*x + a), x)
```

Fricas [A]
time = 0.34, size = 132, normalized size = 1.09

$$\frac{\pi(2\,abc-a^2d)\sqrt{b^2}\,S\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)-\sqrt{b^2}\,d\,C\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)+(b^2dx+2\,b^2c-abd)\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)+(\pi b^3dx^2+2\,\pi b^3cx)\,S\,(bx+a)}{2\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnel_sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(pi*(2*a*b*c - a^2*d)*sqrt(b^2)*fresnel_sin(sqrt(b^2)*(b*x + a)/b) - sq
rt(b^2)*d*fresnel_cos(sqrt(b^2)*(b*x + a)/b) + (b^2*d*x + 2*b^2*c - a*b*d)*
```

```
cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) + (pi*b^3*d*x^2 + 2*pi*b^3*c*x)
*fresnel_sin(b*x + a))/(pi*b^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) S(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a),x)
[Out] Integral((c + d*x)*fresnels(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnel_sin(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)*fresnel_sin(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{FresnelS}(a + b\,x)\ (c + d\,x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)*(c + d*x),x)
[Out] int(FresnelS(a + b*x)*(c + d*x), x)
```

3.22 $\int S(a+bx)\,dx$

Optimal. Leaf size=36

$$\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}+\frac{(a+bx)S(a+bx)}{b}$$

```
[Out] cos(1/2*Pi*(b*x+a)^2)/b/Pi+(b*x+a)*FresnelS(b*x+a)/b
```

Rubi [A]
time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {6553}

$$\frac{(a+bx)S(a+bx)}{b}+\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[a + b*x],x]
```

```
[Out] Cos[(Pi*(a + b*x)^2)/2]/(b*Pi) + ((a + b*x)*FresnelS[a + b*x])/b
```

Rule 6553

```
Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelS[a +
 b*x]/b), x] + Simp[Cos[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int S(a+bx)\,dx=\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}+\frac{(a+bx)S(a+bx)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 89 vs. 2(36) = 72.
time = 0.02, size = 89, normalized size = 2.47

$$\frac{\cos\left(\frac{a^2\pi}{2}\right)\cos\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)}{b\pi}+\frac{aS(a+bx)}{b}+xS(a+bx)-\frac{\sin\left(\frac{a^2\pi}{2}\right)\sin\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[a + b*x],x]
```

```
[Out] (Cos[(a^2*Pi)/2]*Cos[a*b*Pi*x + (b^2*Pi*x^2)/2])/(b*Pi) + (a*FresnelS[a + b
*x])/b + x*FresnelS[a + b*x] - (Sin[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^2)
/2])/(b*Pi)
```

Maple [A]
time = 0.38, size = 33, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\mathrm{S}(bx+a)(bx+a)+\frac{\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	33
default	$\frac{\mathrm{S}(bx+a)(bx+a)+\frac{\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(FresnelS(b*x+a)*(b*x+a)+1/Pi*cos(1/2*Pi*(b*x+a)^2))
```

Maxima [A]
time = 0.27, size = 43, normalized size = 1.19

$$\frac{(bx+a)\,\mathrm{S}\,(bx+a)+\frac{\cos\left(\frac{1}{2}\pi b^2x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{\pi}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*x + a)*fresnel_sin(b*x + a) + cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^
2)/pi)/b
```

Fricas [A]
time = 0.35, size = 45, normalized size = 1.25

$$\frac{(\pi bx+\pi a)\,\mathrm{S}\,(bx+a)+\cos\left(\frac{1}{2}\pi b^2x^2+\pi abx+\frac{1}{2}\pi a^2\right)}{\pi b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a),x, algorithm="fricas")
```

```
[Out] ((pi*b*x + pi*a)*fresnel_sin(b*x + a) + cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2
*pi*a^2))/(pi*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int S(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a),x)
```

```
[Out] Integral(fresnels(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{FresnelS}(a + b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x),x)
```

```
[Out] int(FresnelS(a + b*x), x)
```

3.23 $\int \frac{S(a+bx)}{c+dx} \, dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{S(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x+a)/(d*x+c),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(a+bx)}{c+dx} \, dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelS[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][FresnelS[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{S(a+bx)}{c+dx} \, dx = \int \frac{S(a+bx)}{c+dx} \, dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{S(a+bx)}{c+dx} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelS[a + b*x]/(c + d*x),x]
```

```
[Out] Integrate[FresnelS[a + b*x]/(c + d*x), x]
```

Maple [A]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx+a)}{dx+c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)/(d*x+c),x)
```

```
[Out] int(FresnelS(b*x+a)/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x + a)/(d*x + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(fresnel_sin(b*x + a)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(fresnels(a + b*x)/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x + a)/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{FresnelS}(a+b\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)/(c + d*x),x)
[Out] int(FresnelS(a + b*x)/(c + d*x), x)
```

3.24 $\int \frac{S(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=17

$$\operatorname{Int}\left(\frac{S(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x+a)/(d*x+c)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelS[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Defer[Int][FresnelS[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{S(a+bx)}{(c+dx)^2}\,dx = \int \frac{S(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 3.26, size = 0, normalized size = 0.00

$$\int \frac{S(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelS[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Integrate[FresnelS[a + b*x]/(c + d*x)^2, x]
```

Maple [A]
time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)/(d*x+c)^2,x)
[Out] int(FresnelS(b*x+a)/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x + a)/(d*x + c)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S(a + bx)}{(c + dx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/(d*x+c)**2,x)
[Out] Integral(fresnels(a + b*x)/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x + a)/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{FresnelS}(a + b\,x)}{(c + d\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)/(c + d*x)^2,x)
```

```
[Out] int(FresnelS(a + b*x)/(c + d*x)^2, x)
```

3.25 $\int x^3 S(a+bx)\,dx$

Optimal. Leaf size=229

$$-\frac{a^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi}+\frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi}-\frac{a(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi}+\frac{(a+bx)^3\cos\left(\frac{1}{2}\pi(a\right.}{4b^4\pi}$$

```
[Out] -a^3*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi+3/2*a^2*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)/b^4
/Pi-a*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi+1/4*(b*x+a)^3*cos(1/2*Pi*(b*x+
a)^2)/b^4/Pi-3/2*a^2*FresnelC(b*x+a)/b^4/Pi-1/4*a^4*FresnelS(b*x+a)/b^4+3/4
*FresnelS(b*x+a)/b^4/Pi^2+1/4*x^4*FresnelS(b*x+a)+2*a*sin(1/2*Pi*(b*x+a)^2)
/b^4/Pi^2-3/4*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi^2
```

Rubi [A]
time = 0.13, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6563, 3514, 3432, 3460, 2718, 3466, 3433, 3377, 2717, 3467}

$$-\frac{a^4S(a+bx)}{4b^4}-\frac{a^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}-\frac{3a^2\text{FresnelC}(a+bx)}{2\pi b^4}+\frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^4}+\frac{3S(a+bx)}{4\pi^2b^4}+\frac{2a\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2b^4}-\frac{3(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4\pi^2b^4}-\frac{a(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}+\frac{(a+bx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4\pi b^4}+\frac{1}{4}x^4S(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelS[a + b*x],x]
```

```
[Out] -((a^3*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi)) + (3*a^2*(a + b*x)*Cos[(Pi*(a + b
*x)^2)/2])/(2*b^4*Pi) - (a*(a + b*x)^2*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi) +
((a + b*x)^3*Cos[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi) - (3*a^2*FresnelC[a + b*x]
)/(2*b^4*Pi) - (a^4*FresnelS[a + b*x])/(4*b^4) + (3*FresnelS[a + b*x])/(4*b
^4*Pi^2) + (x^4*FresnelS[a + b*x])/4 + (2*a*Sin[(Pi*(a + b*x)^2)/2])/(b^4*P
i^2) - (3*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6563

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelS[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^3 S(a+bx)\,dx &= \frac{1}{4}x^4 S(a+bx) - \frac{1}{4}b\int x^4 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx \\
&= \frac{1}{4}x^4 S(a+bx) - \frac{\text{Subst}\left(\int \left(a^4\sin\left(\frac{\pi x^2}{2}\right) - 4a^3x\sin\left(\frac{\pi x^2}{2}\right) + 6a^2x^2\sin\left(\frac{\pi x^2}{2}\right) - 4ax^3\sin\right.\right.}{4b^4} \\
&= \frac{1}{4}x^4 S(a+bx) - \frac{\text{Subst}\left(\int x^4\sin\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{4b^4} + \frac{a\text{Subst}\left(\int x^3\sin\left(\frac{\pi x^2}{2}\right)\,dx, x,\right.}{b^4} \\
&= \frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} + \frac{(a+bx)^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi} - \frac{a^4S(a+bx)}{4b^4} + \frac{1}{4}x^4S(\\
&= -\frac{a^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} - \frac{a(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right.}{b^4\pi} \\
&= -\frac{a^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi} + \frac{3a^2(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} - \frac{a(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right.}{b^4\pi}
\end{aligned}
$$

Mathematica [A]
time = 0.24, size = 166, normalized size = 0.72

$$
\frac{-a^3\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + a^2b\pi x\cos\left(\frac{1}{2}\pi(a+bx)^2\right) - ab^2\pi x^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + b^3\pi x^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right) - 6a^2\pi\text{FresnelC}(a+bx) + \left(3 - a^4\pi^2 + b^4\pi^2x^4\right)S(a+bx) + 5a\sin\left(\frac{1}{2}\pi(a+bx)^2\right) - 3bx\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelS[a + b*x],x]
```

```
[Out] (-(a^3*Pi*Cos[(Pi*(a + b*x)^2)/2]) + a^2*b*Pi*x*Cos[(Pi*(a + b*x)^2)/2] - a
*b^2*Pi*x^2*Cos[(Pi*(a + b*x)^2)/2] + b^3*Pi*x^3*Cos[(Pi*(a + b*x)^2)/2] -
6*a^2*Pi*FresnelC[a + b*x] + (3 - a^4*Pi^2 + b^4*Pi^2*x^4)*FresnelS[a + b*x
] + 5*a*Sin[(Pi*(a + b*x)^2)/2] - 3*b*x*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^
2)
```

Maple [A]
time = 0.40, size = 189, normalized size = 0.83

method	result
derivativedivides	$\frac{\frac{\mathrm{S}(bx+a)b^4x^4}{4} - \frac{a^4\mathrm{S}(bx+a)}{4} - \frac{a^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{3a^2(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi} - \frac{3a^2\,\mathrm{FresnelC}(bx+a)}{2\pi} - \frac{a(bx+a)^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b^4}$

default	$\frac{\frac{\mathrm{S}(bx+a)b^4x^4}{4}-\frac{a^4\mathrm{S}(bx+a)}{4}-\frac{a^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{3a^2(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}-\frac{3a^2\,\mathrm{FresnelC}(bx+a)}{2\pi}-\frac{a(bx+a)^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/4*FresnelS(b*x+a)*b^4*x^4-1/4*a^4*FresnelS(b*x+a)-a^3/Pi*cos(1/2*P
i*(b*x+a)^2)+3/2*a^2/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)-3/2*a^2/Pi*FresnelC(b
*x+a)-a/Pi*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)+2*a/Pi^2*sin(1/2*Pi*(b*x+a)^2)+1
/4/Pi*(b*x+a)^3*cos(1/2*Pi*(b*x+a)^2)-3/4/Pi*(1/Pi*(b*x+a)*sin(1/2*Pi*(b*x+
a)^2)-1/Pi*FresnelS(b*x+a)))
```

Maxima [**C**] Result contains complex when optimal does not.
time = 1.16, size = 503, normalized size = 2.20

$$\frac{1}{4}x^4\,\mathrm{S}(bx+a)-\frac{\left(16\left(\pi^2e^{\left(\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)}+\pi^2e^{\left(-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2\right)}\right)a^4+32\left(-i\pi\Gamma\left(2,\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)+i\pi\Gamma\left(2,-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2\right)\right)a^2+16\left(\left(\pi^2e^{\left(\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)}+\pi^2e^{\left(-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2\right)}\right)a^3+2\left(-i\pi\Gamma\left(2,\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)+i\pi\Gamma\left(2,-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2\right)\right)a\right)bx-\left(\left(-(i+1)\sqrt{2}\pi^{\frac{5}{2}}\left(\mathrm{erf}\left(\sqrt{\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2}\right)-1\right)+(i-1)\sqrt{2}\pi^{\frac{5}{2}}\left(\mathrm{erf}\left(\sqrt{-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2}\right)-1\right)\right)a^4-12\left((i-1)\sqrt{2}\pi\Gamma\left(\frac{3}{2},\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)-(i+1)\sqrt{2}\pi\Gamma\left(\frac{3}{2},-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2\right)\right)a^2-(4i+4)\sqrt{2}\Gamma\left(\frac{5}{2},\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)+(4i-4)\sqrt{2}\Gamma\left(\frac{5}{2},-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2\right)\right)\sqrt{2\pi b^2x^2+4\pi abx+2\pi a^2}\right)b}{32\left(\pi^3b^6x+\pi^3ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*fresnel_sin(b*x + a) - 1/32*(16*(pi^2*e^(1/2*I*pi*b^2*x^2 + I*pi*a*
b*x + 1/2*I*pi*a^2) + pi^2*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2
))*a^4 + 32*(-I*pi*gamma(2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) +
 I*pi*gamma(2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^2 + 16*((p
i^2*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + pi^2*e^(-1/2*I*pi*b^
2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^3 + 2*(-I*pi*gamma(2, 1/2*I*pi*b^2*x^
2 + I*pi*a*b*x + 1/2*I*pi*a^2) + I*pi*gamma(2, -1/2*I*pi*b^2*x^2 - I*pi*a*b
*x - 1/2*I*pi*a^2))*a)*b*x - ((-(I + 1)*sqrt(2)*pi^(5/2)*(erf(sqrt(1/2*I*pi
*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2)) - 1) + (I - 1)*sqrt(2)*pi^(5/2)*(erf
(sqrt(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)) - 1))*a^4 - 12*((I -
1)*sqrt(2)*pi*gamma(3/2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) - (I
 + 1)*sqrt(2)*pi*gamma(3/2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))
*a^2 - (4*I + 4)*sqrt(2)*gamma(5/2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*p
i*a^2) + (4*I - 4)*sqrt(2)*gamma(5/2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*
I*pi*a^2))*sqrt(2*pi*b^2*x^2 + 4*pi*a*b*x + 2*pi*a^2))*b/(pi^3*b^6*x + pi^3
*a*b^5)
```

Fricas [**A**]
time = 0.37, size = 175, normalized size = 0.76

$$\frac{\pi^2b^5x^4\,\mathrm{S}\left(bx+a\right)-6\,\pi a^2\sqrt{b^2}\,\mathrm{C}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)-\left(\pi^2a^4-3\right)\sqrt{b^2}\,\mathrm{S}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)+\left(\pi b^4x^3-\pi ab^3x^2+\pi a^2b^2x-\pi a^3b\right)\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)-\left(3\,b^2x-5\,ab\right)\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{4\,\pi^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(pi^2*b^5*x^4*fresnel_sin(b*x + a) - 6*pi*a^2*sqrt(b^2)*fresnel_cos(sqr
t(b^2)*(b*x + a)/b) - (pi^2*a^4 - 3)*sqrt(b^2)*fresnel_sin(sqrt(b^2)*(b*x +
 a)/b) + (pi*b^4*x^3 - pi*a*b^3*x^2 + pi*a^2*b^2*x - pi*a^3*b)*cos(1/2*pi*b
^2*x^2 + pi*a*b*x + 1/2*pi*a^2) - (3*b^2*x - 5*a*b)*sin(1/2*pi*b^2*x^2 + pi
*a*b*x + 1/2*pi*a^2))/(pi^2*b^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 S(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnels(b*x+a),x)
```

```
[Out] Integral(x**3*fresnels(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnel_sin(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\mathrm{FresnelS}(a + b\,x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(a + b*x),x)
```

```
[Out] int(x^3*FresnelS(a + b*x), x)
```

3.26 $\int x^2 S(a+bx)\,dx$

Optimal. Leaf size=147

$$\frac{a^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}-\frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}+\frac{(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi}+\frac{a\text{FresnelC}(a+bx)}{b^3\pi}+\frac{a^3S(a+bx)}{3b^3}$$

```
[Out] a^2*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi-a*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi+1/3
*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi+a*FresnelC(b*x+a)/b^3/Pi+1/3*a^3*Fr
esnelS(b*x+a)/b^3+1/3*x^3*FresnelS(b*x+a)-2/3*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi^
2
```

Rubi [A]
time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {6563, 3514, 3432, 3460, 2718, 3466, 3433, 3377, 2717}

$$\frac{a^3S(a+bx)}{3b^3}+\frac{a^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}+\frac{a\text{FresnelC}(a+bx)}{\pi b^3}-\frac{2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi^2b^3}-\frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}+\frac{(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi b^3}+\frac{1}{3}x^3S(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelS[a + b*x],x]
```

```
[Out] (a^2*Cos[(Pi*(a + b*x)^2)/2])/(b^3*Pi) - (a*(a + b*x)*Cos[(Pi*(a + b*x)^2)/
2])/(b^3*Pi) + ((a + b*x)^2*Cos[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi) + (a*Fresne
lC[a + b*x])/(b^3*Pi) + (a^3*FresnelS[a + b*x])/(3*b^3) + (x^3*FresnelS[a +
 b*x])/3 - (2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6563

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelS[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x^2 S(a+bx)\,dx = \frac{1}{3}x^3 S(a+bx) - \frac{1}{3}b\int x^3 \sin\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx$$

$$= \frac{1}{3}x^3 S(a+bx) - \frac{\text{Subst}\left(\int \left(-a^3\sin\left(\frac{\pi x^2}{2}\right) + 3a^2 x\sin\left(\frac{\pi x^2}{2}\right) - 3ax^2\sin\left(\frac{\pi x^2}{2}\right) + x^3\sin\left(\right.\right.\right.}{3b^3}$$

$$= \frac{1}{3}x^3 S(a+bx) - \frac{\text{Subst}\left(\int x^3\sin\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{3b^3} + \frac{a\text{Subst}\left(\int x^2\sin\left(\frac{\pi x^2}{2}\right)\,dx, x, a\right.}{b^3}$$

$$= -\frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{a^3 S(a+bx)}{3b^3} + \frac{1}{3}x^3 S(a+bx) - \frac{\text{Subst}\left(\int x\sin\left(\frac{\pi x}{2}\right)\,dx\right.}{6b^3}$$

$$= \frac{a^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi} + \frac{aC}{}$$

$$= \frac{a^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{a(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{(a+bx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi} + \frac{aC}{}$$

Mathematica [A]
time = 0.17, size = 115, normalized size = 0.78

$$\frac{a^2\pi\cos\left(\frac{1}{2}\pi(a+bx)^2\right) - ab\pi x\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + b^2\pi x^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + 3a\pi\text{FresnelC}(a+bx) + \pi^2(a^3+b^3x^3)\,S(a+bx) - 2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelS[a + b*x],x]
```

```
[Out] (a^2*Pi*Cos[(Pi*(a + b*x)^2)/2] - a*b*Pi*x*Cos[(Pi*(a + b*x)^2)/2] + b^2*Pi
*x^2*Cos[(Pi*(a + b*x)^2)/2] + 3*a*Pi*FresnelC[a + b*x] + Pi^2*(a^3 + b^3*x
^3)*FresnelS[a + b*x] - 2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Maple [A]
time = 0.40, size = 121, normalized size = 0.82

method	result
derivativedivides	$\frac{\frac{\mathrm{S}(bx+a)b^3x^3}{3} + \frac{a^3\mathrm{S}(bx+a)}{3} + \frac{a^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} - \frac{a(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{a\,\mathrm{FresnelC}(bx+a)}{\pi} + \frac{(bx+a)^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{3\pi} - \frac{2\sin\left(\right.}{}}{b^3}$
default	$\frac{\frac{\mathrm{S}(bx+a)b^3x^3}{3} + \frac{a^3\mathrm{S}(bx+a)}{3} + \frac{a^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} - \frac{a(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{a\,\mathrm{FresnelC}(bx+a)}{\pi} + \frac{(bx+a)^2\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{3\pi} - \frac{2\sin\left(\right.}{}}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/3*FresnelS(b*x+a)*b^3*x^3+1/3*a^3*FresnelS(b*x+a)+a^2/Pi*cos(1/2*P
i*(b*x+a)^2)-a/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+a/Pi*FresnelC(b*x+a)+1/3/Pi
*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)-2/3/Pi^2*sin(1/2*Pi*(b*x+a)^2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.91, size = 424, normalized size = 2.88

$$\frac{1}{3}x^3 \operatorname{S}(bx+a) + \frac{\left(12\left(\pi e^{\left(\frac{1}{2}i\pi b^2x^2 + i\pi abx + \frac{1}{2}i\pi a^2\right)} + \pi e^{\left(-\frac{1}{2}i\pi b^2x^2 - i\pi abx - \frac{1}{2}i\pi a^2\right)}\right)a^3 + 4\left(3\left(\pi e^{\left(\frac{1}{2}i\pi b^2x^2 + i\pi abx + \frac{1}{2}i\pi a^2\right)} + \pi e^{\left(-\frac{1}{2}i\pi b^2x^2 - i\pi abx - \frac{1}{2}i\pi a^2\right)}\right)a^2 - 2i\Gamma\left(2, \frac{1}{2}i\pi b^2x^2 + i\pi abx + \frac{1}{2}i\pi a^2\right) + 2i\Gamma\left(2, -\frac{1}{2}i\pi b^2x^2 - i\pi abx - \frac{1}{2}i\pi a^2\right)\right)bx + 8a\left(-i\Gamma\left(2, \frac{1}{2}i\pi b^2x^2 + i\pi abx + \frac{1}{2}i\pi a^2\right) + i\Gamma\left(2, -\frac{1}{2}i\pi b^2x^2 - i\pi abx - \frac{1}{2}i\pi a^2\right)\right) - \sqrt{2\pi b^2x^2 + 4\pi abx + 2\pi a^2}\left(\left(-(i+1)\sqrt{2}\pi^{\frac{3}{2}}\left(\operatorname{erf}\left(\sqrt{\frac{1}{2}i\pi b^2x^2 + i\pi abx + \frac{1}{2}i\pi a^2}\right) - 1\right) + (i-1)\sqrt{2}\pi^{\frac{3}{2}}\left(\operatorname{erf}\left(\sqrt{-\frac{1}{2}i\pi b^2x^2 - i\pi abx - \frac{1}{2}i\pi a^2}\right) - 1\right)\right)a^3 - 6\left((i-1)\sqrt{2}\Gamma\left(\frac{3}{2}, \frac{1}{2}i\pi b^2x^2 + i\pi abx + \frac{1}{2}i\pi a^2\right) - (i+1)\sqrt{2}\Gamma\left(\frac{3}{2}, -\frac{1}{2}i\pi b^2x^2 - i\pi abx - \frac{1}{2}i\pi a^2\right)\right)a\right)\right)b}{24\left(\pi^2b^5x + \pi^2ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*fresnel_sin(b*x + a) + 1/24*(12*(pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*
x + 1/2*I*pi*a^2) + pi*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a
^3 + 4*(3*(pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + pi*e^(-1/2
*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^2 - 2*I*gamma(2, 1/2*I*pi*b^2
*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + 2*I*gamma(2, -1/2*I*pi*b^2*x^2 - I*pi*a
*b*x - 1/2*I*pi*a^2))*b*x + 8*a*(-I*gamma(2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x
+ 1/2*I*pi*a^2) + I*gamma(2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)
) - sqrt(2*pi*b^2*x^2 + 4*pi*a*b*x + 2*pi*a^2)*((-(I + 1)*sqrt(2)*pi^(3/2)*
(erf(sqrt(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2)) - 1) + (I - 1)*sqr
t(2)*pi^(3/2)*(erf(sqrt(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)) - 1
))*a^3 - 6*((I - 1)*sqrt(2)*gamma(3/2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*
I*pi*a^2) - (I + 1)*sqrt(2)*gamma(3/2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2
*I*pi*a^2))*a))*b/(pi^2*b^5*x + pi^2*a*b^4)
```

Fricas [A]
time = 0.35, size = 147, normalized size = 1.00

$$\frac{\pi^2b^4x^3 \operatorname{S}(bx+a) + \pi^2a^3\sqrt{b^2}\operatorname{S}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + 3\pi a\sqrt{b^2}\operatorname{C}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right) + \left(\pi b^3x^2 - \pi ab^2x + \pi a^2b\right)\cos\left(\frac{1}{2}\pi b^2x^2 + \pi abx + \frac{1}{2}\pi a^2\right) - 2b\sin\left(\frac{1}{2}\pi b^2x^2 + \pi abx + \frac{1}{2}\pi a^2\right)}{3\pi^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3*(pi^2*b^4*x^3*fresnel_sin(b*x + a) + pi^2*a^3*sqrt(b^2)*fresnel_sin(sqr
t(b^2)*(b*x + a)/b) + 3*pi*a*sqrt(b^2)*fresnel_cos(sqrt(b^2)*(b*x + a)/b) +
 (pi*b^3*x^2 - pi*a*b^2*x + pi*a^2*b)*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*p
i*a^2) - 2*b*sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2))/(pi^2*b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 S(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnels(b*x+a),x)
```

```
[Out] Integral(x**2*fresnels(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_sin(b*x+a),x, algorithm="giac")
[Out] integrate(x^2*fresnel_sin(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{FresnelS}(a + b\,x)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(a + b*x),x)
[Out] int(x^2*FresnelS(a + b*x), x)
```

3.27 $\int xS(a+bx)\,dx$

Optimal. Leaf size=96

$$-\frac{a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi}+\frac{(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}-\frac{\text{FresnelC}(a+bx)}{2b^2\pi}-\frac{a^2S(a+bx)}{2b^2}+\frac{1}{2}x^2S(a+bx)$$

```
[Out] -a*cos(1/2*Pi*(b*x+a)^2)/b^2/Pi+1/2*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)/b^2/Pi-1/
2*FresnelC(b*x+a)/b^2/Pi-1/2*a^2*FresnelS(b*x+a)/b^2+1/2*x^2*FresnelS(b*x+a
)
```

Rubi [A]
time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6563, 3514, 3432, 3460, 2718, 3466, 3433}

$$-\frac{a^2S(a+bx)}{2b^2}-\frac{\text{FresnelC}(a+bx)}{2\pi b^2}-\frac{a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}+\frac{(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{1}{2}x^2S(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelS[a + b*x],x]
```

```
[Out] -((a*Cos[(Pi*(a + b*x)^2)/2])/(b^2*Pi)) + ((a + b*x)*Cos[(Pi*(a + b*x)^2)/2
])/(2*b^2*Pi) - FresnelC[a + b*x]/(2*b^2*Pi) - (a^2*FresnelS[a + b*x])/(2*b
^2) + (x^2*FresnelS[a + b*x])/2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3514

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Sin[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6563

```
Int[FresnelS[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelS[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Sin[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int xS(a+bx)\,dx &= \frac{1}{2}x^2S(a+bx)-\frac{1}{2}b\int x^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx\\
&= \frac{1}{2}x^2S(a+bx)-\frac{\text{Subst}\left(\int\left(a^2\sin\left(\frac{\pi x^2}{2}\right)-2ax\sin\left(\frac{\pi x^2}{2}\right)+x^2\sin\left(\frac{\pi x^2}{2}\right)\right)\,dx,x,a+bx\right)}{2b^2}\\
&= \frac{1}{2}x^2S(a+bx)-\frac{\text{Subst}\left(\int x^2\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{2b^2}+\frac{a\text{Subst}\left(\int x\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+\right.}{b^2}\\
&= \frac{(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}-\frac{a^2S(a+bx)}{2b^2}+\frac{1}{2}x^2S(a+bx)+\frac{a\text{Subst}\left(\int\sin\left(\frac{\pi x}{2}\right)\,dx,x,(\right.}{2b^2}\\
&= -\frac{a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi}+\frac{(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}-\frac{C(a+bx)}{2b^2\pi}-\frac{a^2S(a+bx)}{2b^2}+\frac{1}{2}x^2S
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 51, normalized size = 0.53

$$-\frac{\text{FresnelC}(a+bx)+(a-bx)\left(\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+\pi(a+bx)S(a+bx)\right)}{2b^2\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*FresnelS[a + b*x],x]
```

```
[Out] -1/2*(FresnelC[a + b*x] + (a - b*x)*(Cos[(Pi*(a + b*x)^2)/2] + Pi*(a + b*x)
*FresnelS[a + b*x]))/(b^2*Pi)
```

Maple [A]
time = 0.41, size = 80, normalized size = 0.83

method	result	size
derivativedivides	$\frac{\mathrm{S}(bx+a)\left(-a(bx+a)+\frac{(bx+a)^2}{2}\right)-\frac{a\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}-\frac{\mathrm{FresnelC}(bx+a)}{2\pi}}{b^2}$	80
default	$\frac{\mathrm{S}(bx+a)\left(-a(bx+a)+\frac{(bx+a)^2}{2}\right)-\frac{a\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}-\frac{\mathrm{FresnelC}(bx+a)}{2\pi}}{b^2}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(FresnelS(b*x+a)*(-a*(b*x+a)+1/2*(b*x+a)^2)-a/Pi*cos(1/2*Pi*(b*x+a)^2
)+1/2/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)-1/2/Pi*FresnelC(b*x+a))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.84, size = 307, normalized size = 3.20

$$\frac{1}{2}\,x^2\,\mathrm{S}\left(bx+a\right)-\frac{\left(8\left(\pi e^{\left(\frac{1}{2}i\,\pi b^2x^2+i\,\pi abx+\frac{1}{2}i\,\pi a^2\right)}+\pi e^{\left(-\frac{1}{2}i\,\pi b^2x^2-i\,\pi abx-\frac{1}{2}i\,\pi a^2\right)}\right)abx+8\left(\pi e^{\left(\frac{1}{2}i\,\pi b^2x^2+i\,\pi abx+\frac{1}{2}i\,\pi a^2\right)}+\pi e^{\left(-\frac{1}{2}i\,\pi b^2x^2-i\,\pi abx-\frac{1}{2}i\,\pi a^2\right)}\right)a^2-\sqrt{2\,\pi b^2x^2+4\,\pi abx+2\,\pi a^2}\left(\left(\left(-(i+1)\,\sqrt{2}\,\pi^{\frac{3}{2}}\left(\mathrm{erf}\left(\sqrt{\frac{1}{2}i\,\pi b^2x^2+i\,\pi abx+\frac{1}{2}i\,\pi a^2}\right)-1\right)+(i-1)\,\sqrt{2}\,\pi^{\frac{3}{2}}\left(\mathrm{erf}\left(\sqrt{-\frac{1}{2}i\,\pi b^2x^2-i\,\pi abx-\frac{1}{2}i\,\pi a^2}\right)-1\right)\right)a^2-(2i-2)\,\sqrt{2}\,\Gamma\left(\frac{3}{2},\frac{1}{2}i\,\pi b^2x^2+i\,\pi abx+\frac{1}{2}i\,\pi a^2\right)+(2i+2)\,\sqrt{2}\,\Gamma\left(\frac{3}{2},-\frac{1}{2}i\,\pi b^2x^2-i\,\pi abx-\frac{1}{2}i\,\pi a^2\right)\right)\right)b}{16\left(\pi^2b^4x+\pi^2ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*fresnel_sin(b*x + a) - 1/16*(8*(pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*x
 + 1/2*I*pi*a^2) + pi*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a*
b*x + 8*(pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + pi*e^(-1/2*I
*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^2 - sqrt(2*pi*b^2*x^2 + 4*pi*a*
b*x + 2*pi*a^2)*((-(I + 1)*sqrt(2)*pi^(3/2)*(erf(sqrt(1/2*I*pi*b^2*x^2 + I*
pi*a*b*x + 1/2*I*pi*a^2)) - 1) + (I - 1)*sqrt(2)*pi^(3/2)*(erf(sqrt(-1/2*I*
pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)) - 1))*a^2 - (2*I - 2)*sqrt(2)*gamm
a(3/2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + (2*I + 2)*sqrt(2)*ga
mma(3/2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)))*b/(pi^2*b^4*x + p
i^2*a*b^3)
```

Fricas [A]
time = 0.37, size = 104, normalized size = 1.08

$$\frac{\pi b^3x^2\,\mathrm{S}\left(bx+a\right)-\pi a^2\sqrt{b^2}\,\mathrm{S}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)+\left(b^2x-ab\right)\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)-\sqrt{b^2}\,\mathrm{C}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)}{2\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(pi*b^3*x^2*fresnel_sin(b*x + a) - pi*a^2*sqrt(b^2)*fresnel_sin(sqrt(b^
2)*(b*x + a)/b) + (b^2*x - a*b)*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2)
 - sqrt(b^2)*fresnel_cos(sqrt(b^2)*(b*x + a)/b))/(pi*b^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int xS(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x+a),x)
```

```
[Out] Integral(x*fresnels(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*fresnel_sin(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{FresnelS}(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(a + b*x),x)
```

```
[Out] int(x*FresnelS(a + b*x), x)
```

3.28 $\int S(a+bx)\,dx$

Optimal. Leaf size=36

$$\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}+\frac{(a+bx)S(a+bx)}{b}$$

```
[Out] cos(1/2*Pi*(b*x+a)^2)/b/Pi+(b*x+a)*FresnelS(b*x+a)/b
```

Rubi [A]
time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6553}

$$\frac{(a+bx)S(a+bx)}{b}+\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[a + b*x],x]
```

```
[Out] Cos[(Pi*(a + b*x)^2)/2]/(b*Pi) + ((a + b*x)*FresnelS[a + b*x])/b
```

Rule 6553

```
Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelS[a +
 b*x]/b), x] + Simp[Cos[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int S(a+bx)\,dx=\frac{\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}+\frac{(a+bx)S(a+bx)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 89 vs. 2(36) = 72.
time = 0.02, size = 89, normalized size = 2.47

$$\frac{\cos\left(\frac{a^2\pi}{2}\right)\cos\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)}{b\pi}+\frac{aS(a+bx)}{b}+xS(a+bx)-\frac{\sin\left(\frac{a^2\pi}{2}\right)\sin\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[a + b*x],x]
```

```
[Out] (Cos[(a^2*Pi)/2]*Cos[a*b*Pi*x + (b^2*Pi*x^2)/2])/(b*Pi) + (a*FresnelS[a + b
*x])/b + x*FresnelS[a + b*x] - (Sin[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^2)
/2])/(b*Pi)
```

Maple [A]
time = 0.33, size = 33, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\mathrm{S}(bx+a)(bx+a)+\frac{\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	33
default	$\frac{\mathrm{S}(bx+a)(bx+a)+\frac{\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(FresnelS(b*x+a)*(b*x+a)+1/Pi*cos(1/2*Pi*(b*x+a)^2))
```

Maxima [A]
time = 0.25, size = 43, normalized size = 1.19

$$\frac{(bx+a)\,\mathrm{S}\,(bx+a)+\frac{\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{\pi}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*x + a)*fresnel_sin(b*x + a) + cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^
2)/pi)/b
```

Fricas [A]
time = 0.34, size = 45, normalized size = 1.25

$$\frac{(\pi bx+\pi a)\,\mathrm{S}\,(bx+a)+\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{\pi b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a),x, algorithm="fricas")
```

```
[Out] ((pi*b*x + pi*a)*fresnel_sin(b*x + a) + cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2
*pi*a^2))/(pi*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int S(a+bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a),x)
```

```
[Out] Integral(fresnels(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{FresnelS}(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x),x)
```

```
[Out] int(FresnelS(a + b*x), x)
```

3.29 $\int \frac{S(a+bx)}{x}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{S(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x+a)/x,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(a+bx)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelS[a + b*x]/x,x]
```

```
[Out] Defer[Int][FresnelS[a + b*x]/x, x]
```

Rubi steps

$$\int \frac{S(a+bx)}{x}\,dx = \int \frac{S(a+bx)}{x}\,dx$$

Mathematica [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{S(a+bx)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelS[a + b*x]/x,x]
```

```
[Out] Integrate[FresnelS[a + b*x]/x, x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)/x,x)
```

```
[Out] int(FresnelS(b*x+a)/x,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x + a)/x, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(fresnel_sin(b*x + a)/x, x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S(a + bx)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x,x)
```

```
[Out] Integral(fresnels(a + b*x)/x, x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x + a)/x, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\mathrm{FresnelS}(a + b\,x)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)/x,x)
[Out] int(FresnelS(a + b*x)/x, x)
```

3.30 $\int \frac{S(a+bx)}{x^2} dx$

Optimal. Leaf size=13

$$\operatorname{Int}\left(\frac{S(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x+a)/x^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(a+bx)}{x^2} dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelS[a + b*x]/x^2,x]
```

```
[Out] Defer[Int][FresnelS[a + b*x]/x^2, x]
```

Rubi steps

$$\int \frac{S(a+bx)}{x^2} dx = \int \frac{S(a+bx)}{x^2} dx$$

Mathematica [A]
time = 2.17, size = 0, normalized size = 0.00

$$\int \frac{S(a+bx)}{x^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelS[a + b*x]/x^2,x]
```

```
[Out] Integrate[FresnelS[a + b*x]/x^2, x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx+a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)/x^2,x)
```

```
[Out] int(FresnelS(b*x+a)/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x + a)/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(fresnel_sin(b*x + a)/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)/x**2,x)
```

```
[Out] Integral(fresnels(a + b*x)/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x + a)/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\mathrm{FresnelS}(a+b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)/x^2,x)
```

```
[Out] int(FresnelS(a + b*x)/x^2, x)
```

3.31 $\int x^7 S(bx)^2 \, dx$

Optimal. Leaf size=253

$$-\frac{105x^2}{16b^6\pi^4}+\frac{7x^6}{48b^2\pi^2}-\frac{55x^2\cos(b^2\pi x^2)}{16b^6\pi^4}+\frac{x^6\cos(b^2\pi x^2)}{16b^2\pi^2}-\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b^5\pi^3}+\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b\pi}-\frac{105}{8}$$

```
[Out] -105/16*x^2/b^6/Pi^4+7/48*x^6/b^2/Pi^2-55/16*x^2*cos(b^2*Pi*x^2)/b^6/Pi^4+1
/16*x^6*cos(b^2*Pi*x^2)/b^2/Pi^2-35/4*x^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)
/b^5/Pi^3+1/4*x^7*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi-105/8*FresnelS(b*x
)^2/b^8/Pi^4+1/8*x^8*FresnelS(b*x)^2+105/4*x*FresnelS(b*x)*sin(1/2*b^2*Pi*x
^2)/b^7/Pi^4-7/4*x^5*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2+10*sin(b^2*
Pi*x^2)/b^8/Pi^5-5/8*x^4*sin(b^2*Pi*x^2)/b^4/Pi^3
```

Rubi [A]
time = 0.28, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6565, 6589, 6597, 3460, 3390, 30, 3377, 2717, 2714, 6575}

$$-\frac{105S(bx)^2}{8\pi^4b^8}-\frac{105x^2}{16\pi^4b^6}+\frac{x^7S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b}+\frac{7x^6}{48\pi^2b^2}+\frac{x^6\cos(\pi b^2x^2)}{16\pi^2b^2}+\frac{10\sin(\pi b^2x^2)}{\pi^5b^8}+\frac{105xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^4b^7}-\frac{55x^2\cos(\pi b^2x^2)}{16\pi^4b^6}-\frac{35x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^3b^5}-\frac{5x^4\sin(\pi b^2x^2)}{8\pi^3b^4}-\frac{7x^5S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^2b^3}+\frac{1}{8}x^8S(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^7*FresnelS[b*x]^2,x]
```

```
[Out] (-105*x^2)/(16*b^6*Pi^4) + (7*x^6)/(48*b^2*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])
/(16*b^6*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(16*b^2*Pi^2) - (35*x^3*Cos[(b^2*Pi*
x^2)/2]*FresnelS[b*x])/(4*b^5*Pi^3) + (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x
])/(4*b*Pi) - (105*FresnelS[b*x]^2)/(8*b^8*Pi^4) + (x^8*FresnelS[b*x]^2)/8
+ (105*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^7*Pi^4) - (7*x^5*FresnelS[
b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (10*Sin[b^2*Pi*x^2])/(b^8*Pi^5) -
(5*x^4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
```

1]

Rubi steps

$$\begin{aligned}\int x^7 S(bx)^2\,dx &= \frac{1}{8}x^8 S(bx)^2 - \frac{1}{4}b\int x^8 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\
&= \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b\pi} + \frac{1}{8}x^8 S(bx)^2 - \frac{\int x^7\sin\left(b^2\pi x^2\right)\,dx}{8\pi} - \frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{4b\pi}\\
&= \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b\pi} + \frac{1}{8}x^8 S(bx)^2 - \frac{7x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{35\int x^4 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2}\\
&= \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b\pi} + \frac{1}{8}x^8 S(bx)^2 - \frac{7x^5 S(b}{}\\
&= \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b\pi} + \frac{1}{8}x^8 S(bx)^2 + \frac{105xS(}{}\\
&= \frac{7x^6}{48b^2\pi^2} - \frac{41x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S}{4b\pi}\\
&= -\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} - \frac{55x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b^5\pi^3} + \frac{x^7\,\mathrm{co}}{}\\
&= -\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} - \frac{55x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b^5\pi^3} + \frac{x^7\,\mathrm{co}}{}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 253, normalized size = 1.00

$$-\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} - \frac{55x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b^5\pi^3} + \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4b\pi} - \frac{105S(bx)^2}{8b^8\pi^4} + \frac{1}{8}x^8S(bx)^2 + \frac{105xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^7\pi^4} - \frac{7x^5S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{10\sin\left(b^2\pi x^2\right)}{b^8\pi^5} - \frac{5x^4\sin\left(b^2\pi x^2\right)}{8b^4\pi^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*FresnelS[b*x]^2,x]
```

```
[Out] (-105*x^2)/(16*b^6*Pi^4) + (7*x^6)/(48*b^2*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])
/(16*b^6*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(16*b^2*Pi^2) - (35*x^3*Cos[(b^2*Pi*
x^2)/2]*FresnelS[b*x])/(4*b^5*Pi^3) + (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x
])/(4*b*Pi) - (105*FresnelS[b*x]^2)/(8*b^8*Pi^4) + (x^8*FresnelS[b*x]^2)/8
+ (105*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^7*Pi^4) - (7*x^5*FresnelS[
b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (10*Sin[b^2*Pi*x^2])/(b^8*Pi^5) -
(5*x^4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int x^7 \mathrm{S}(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelS(b*x)^2,x)
```

```
[Out] int(x^7*FresnelS(b*x)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnel_sin(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^7*fresnel_sin(b*x)^2, x)
```

Fricas [A]
time = 0.35, size = 183, normalized size = 0.72

$$\frac{2\,\pi^3 b^6 x^6 - 75\,\pi b^2 x^2 + 3\,(\pi^3 b^6 x^6 - 55\,\pi b^2 x^2)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 6\,(\pi^4 b^7 x^7 - 35\,\pi^2 b^3 x^3)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{S}\,(bx) - 3\,(105\,\pi - \pi^5 b^8 x^8)\,\mathrm{S}\,(bx)^2 - 6\,\left(5\,(\pi^2 b^4 x^4 - 16)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) + 7\,(\pi^3 b^5 x^5 - 15\,\pi bx)\,\mathrm{S}\,(bx)\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{24\,\pi^5 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnel_sin(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(2*pi^3*b^6*x^6 - 75*pi*b^2*x^2 + 3*(pi^3*b^6*x^6 - 55*pi*b^2*x^2)*cos
(1/2*pi*b^2*x^2)^2 + 6*(pi^4*b^7*x^7 - 35*pi^2*b^3*x^3)*cos(1/2*pi*b^2*x^2)
*fresnel_sin(b*x) - 3*(105*pi - pi^5*b^8*x^8)*fresnel_sin(b*x)^2 - 6*(5*(pi
^2*b^4*x^4 - 16)*cos(1/2*pi*b^2*x^2) + 7*(pi^3*b^5*x^5 - 15*pi*b*x)*fresnel
_sin(b*x))*sin(1/2*pi*b^2*x^2))/(pi^5*b^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 S^2(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*fresnels(b*x)**2,x)
```

```
[Out] Integral(x**7*fresnels(b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnel_sin(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^7*fresnel_sin(b*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^7 \operatorname{FresnelS}(b\,x)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelS(b*x)^2,x)
```

```
[Out] int(x^7*FresnelS(b*x)^2, x)
```

3.32 $\int x^6 S(bx)^2 \, dx$

Optimal. Leaf size=239

$$-\frac{48x}{7b^6\pi^4}+\frac{6x^5}{35b^2\pi^2}-\frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4}+\frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2}+\frac{531\text{FresnelC}\left(\sqrt{2}\,bx\right)}{56\sqrt{2}\,b^7\pi^4}-\frac{48x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7b^5\pi^3}+\frac{2x^6}{}$$

```
[Out] -48/7*x/b^6/Pi^4+6/35*x^5/b^2/Pi^2-21/8*x*cos(b^2*Pi*x^2)/b^6/Pi^4+1/14*x^5
*cos(b^2*Pi*x^2)/b^2/Pi^2-48/7*x^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^5/Pi
^3+2/7*x^6*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi+1/7*x^7*FresnelS(b*x)^2+9
6/7*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^7/Pi^4-12/7*x^4*FresnelS(b*x)*sin(1
/2*b^2*Pi*x^2)/b^3/Pi^2-17/28*x^3*sin(b^2*Pi*x^2)/b^4/Pi^3+531/112*FresnelC
(b*x*2^(1/2))/b^7/Pi^4*2^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6565, 6589, 6597, 3472, 30, 3467, 3466, 3433, 6595, 3438}

$$\frac{531\text{FresnelC}\left(\sqrt{2}\,bx\right)}{56\sqrt{2}\,\pi^4 b^7}-\frac{48x}{7\pi^4 b^6}+\frac{2x^6 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b}+\frac{6x^5}{35\pi^2 b^2}+\frac{x^5\cos\left(\pi b^2x^2\right)}{14\pi^2 b^2}+\frac{96S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4 b^7}-\frac{21x\cos\left(\pi b^2x^2\right)}{8\pi^4 b^6}-\frac{48x^2 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3 b^5}-\frac{17x^3\sin\left(\pi b^2x^2\right)}{28\pi^3 b^4}-\frac{12x^4 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2 b^3}+\frac{1}{7}x^7 S(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^6*FresnelS[b*x]^2,x]
```

```
[Out] (-48*x)/(7*b^6*Pi^4) + (6*x^5)/(35*b^2*Pi^2) - (21*x*Cos[b^2*Pi*x^2])/(8*b^
6*Pi^4) + (x^5*Cos[b^2*Pi*x^2])/(14*b^2*Pi^2) + (531*FresnelC[Sqrt[2]*b*x])
/(56*Sqrt[2]*b^7*Pi^4) - (48*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(7*b^5*
Pi^3) + (2*x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(7*b*Pi) + (x^7*FresnelS[
b*x]^2)/7 + (96*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) - (12*x^4*F
resnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*b^3*Pi^2) - (17*x^3*Sin[b^2*Pi*x^2])/(
28*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3472

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6595

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelS[b*x]/(2*d)), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
```

```
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$
\begin{aligned}
\int x^6 S(bx)^2\,dx &= \frac{1}{7}x^7 S(bx)^2 - \frac{1}{7}(2b)\int x^7 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{2x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7b\pi} + \frac{1}{7}x^7 S(bx)^2 - \frac{\int x^6\sin\left(b^2\pi x^2\right)\,dx}{7\pi} - \frac{12\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\cdots}{7b\pi} \\
&= \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{2x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7b\pi} + \frac{1}{7}x^7 S(bx)^2 - \frac{12x^4 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} + \frac{48\int \cdots}{\cdots} \\
&= \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} - \frac{48x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7b^5\pi^3} + \frac{2x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7b\pi} + \frac{1}{7}x^7 S(bx)^2 - \frac{12x^4\cdots}{\cdots} \\
&= \frac{6x^5}{35b^2\pi^2} - \frac{111x\cos\left(b^2\pi x^2\right)}{56b^6\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} - \frac{48x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7b^5\pi^3} + \frac{2x^6\cos\left(\frac{1}{2}b^2\pi x\cdots\right)}{7b\pi} \\
&= \frac{6x^5}{35b^2\pi^2} - \frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{15C\left(\sqrt{2}\,bx\right)}{56\sqrt{2}\,b^7\pi^4} + \frac{6\sqrt{2}\,C\left(\sqrt{2}\,bx\right)}{7b^7\pi^4} - \frac{48x^2\cdots}{\cdots} \\
&= -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} - \frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{51C\left(\sqrt{2}\,bx\right)}{56\sqrt{2}\,b^7\pi^4} + \frac{6\sqrt{2}\,C\left(\sqrt{2}\,bx\right)}{7b^7\pi^4}\cdots \\
&= -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} - \frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{51C\left(\sqrt{2}\,bx\right)}{56\sqrt{2}\,b^7\pi^4} + \frac{30\sqrt{2}\,C\left(\sqrt{2}\,b\cdots\right)}{7b^7\pi^4}\cdots
\end{aligned}
$$

Mathematica [A]
time = 0.21, size = 171, normalized size = 0.72

$$\frac{2655\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}\,bx\right)+80b^7\pi^4x^7S(bx)^2+160S(bx)\left(b^2\pi x^2\left(-24+b^4\pi^2x^4\right)\cos\left(\frac{1}{2}b^2\pi x^2\right)-6\left(-8+b^4\pi^2x^4\right)\sin\left(\frac{1}{2}b^2\pi x^2\right)\right)+2bx\left(5\left(-147+4b^4\pi^2x^4\right)\cos\left(b^2\pi x^2\right)-2\left(960-24b^4\pi^2x^4+85b^2\pi x^2\sin\left(b^2\pi x^2\right)\right)\right)}{560b^7\pi^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*FresnelS[b*x]^2,x]

[Out] (2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 80*b^7*Pi^4*x^7*FresnelS[b*x]^2 + 160
*FresnelS[b*x]*(b^2*Pi*x^2*(-24 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 6*(-8
 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*(5*(-147 + 4*b^4*Pi^2*x^4)*Co
s[b^2*Pi*x^2] - 2*(960 - 24*b^4*Pi^2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2])))
/(560*b^7*Pi^4)
```

Maple [A]
time = 0.65, size = 324, normalized size = 1.36

method	result
derivativedivides	$\frac{\frac{S(bx)^2 b^7 x^7}{7} - 2\,S(bx)\left(-\frac{b^6 x^6 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{7\pi} + \frac{\frac{6 b^4 x^4 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{7\pi} - \frac{24\left(-\frac{b^2 x^2 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{\pi} + \frac{2 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{\pi^2}\right)}{7\pi}}{\pi}\right) + \frac{\frac{6}{35}\pi^2 b^5 x^5 - \cdots}{\pi^4} \cdots}{\cdots}$
default	$\frac{\frac{S(bx)^2 b^7 x^7}{7} - 2\,S(bx)\left(-\frac{b^6 x^6 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{7\pi} + \frac{\frac{6 b^4 x^4 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{7\pi} - \frac{24\left(-\frac{b^2 x^2 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{\pi} + \frac{2 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{\pi^2}\right)}{7\pi}}{\pi}\right) + \frac{\frac{6}{35}\pi^2 b^5 x^5 - \cdots}{\pi^4} \cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelS(b*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^7*(1/7*FresnelS(b*x)^2*b^7*x^7-2*FresnelS(b*x)*(-1/7/Pi*b^6*x^6*cos(1/2
*b^2*Pi*x^2)+6/7/Pi*(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*c
os(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2))))+6/7/Pi^4*(1/5*Pi^2*b^5*x^5
-8*b*x)-6/7/Pi^4*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^
2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1
/2)))-1/7/Pi^3*(-1/2*Pi*b^5*x^5*cos(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(
b^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x
*2^(1/2))))+12/Pi*b*x*cos(b^2*Pi*x^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnel_sin(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^6*fresnel_sin(b*x)^2, x)
```

Fricas [A]
time = 0.38, size = 184, normalized size = 0.77

$$\frac{80\,\pi^4 b^8 x^7\,S(bx)^2 + 56\,\pi^2 b^6 x^5 - 2370\,b^2 x + 20\,(4\,\pi^2 b^6 x^5 - 147\,b^2 x)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 160\,(\pi^3 b^7 x^6 - 24\,\pi b^3 x^2)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)S(bx) + 2655\,\sqrt{2}\,\sqrt{b^2}\,C\left(\sqrt{2}\,\sqrt{b^2}\,x\right) - 40\,\left(17\,\pi b^4 x^3\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) + 24\,(\pi^2 b^5 x^4 - 8\,b)\,S(bx)\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{560\,\pi^4 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnel_sin(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/560*(80*pi^4*b^8*x^7*fresnel_sin(b*x)^2 + 56*pi^2*b^6*x^5 - 2370*b^2*x +
20*(4*pi^2*b^6*x^5 - 147*b^2*x)*cos(1/2*pi*b^2*x^2)^2 + 160*(pi^3*b^7*x^6 -
 24*pi*b^3*x^2)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + 2655*sqrt(2)*sqrt(b^
2)*fresnel_cos(sqrt(2)*sqrt(b^2)*x) - 40*(17*pi*b^4*x^3*cos(1/2*pi*b^2*x^2)
 + 24*(pi^2*b^5*x^4 - 8*b)*fresnel_sin(b*x))*sin(1/2*pi*b^2*x^2))/(pi^4*b^8
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 S^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnels(b*x)**2,x)
```

```
[Out] Integral(x**6*fresnels(b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnel_sin(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^6*fresnel_sin(b*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^6\,\mathrm{FresnelS}(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelS(b*x)^2,x)
```

```
[Out] int(x^6*FresnelS(b*x)^2, x)
```

3.33 $\int x^5 S(bx)^2 \, dx$

Optimal. Leaf size=265

$$\frac{5x^4}{24b^2\pi^2}-\frac{11\cos\left(b^2\pi x^2\right)}{6b^6\pi^4}+\frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2}-\frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^5\pi^3}+\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi}+\frac{5\text{FresnelC}(bx)S(bx}{2b^6\pi^3}$$

```
[Out] 5/24*x^4/b^2/Pi^2-11/6*cos(b^2*Pi*x^2)/b^6/Pi^4+1/12*x^4*cos(b^2*Pi*x^2)/b^
2/Pi^2-5*x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^5/Pi^3+1/3*x^5*cos(1/2*b^2*P
i*x^2)*FresnelS(b*x)/b/Pi+5/2*FresnelC(b*x)*FresnelS(b*x)/b^6/Pi^3+1/6*x^6*
FresnelS(b*x)^2-5/8*I*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^4/
Pi^3+5/8*I*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^4/Pi^3-5/3*x^3
*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2-7/12*x^2*sin(b^2*Pi*x^2)/b^4/Pi
^3
```

Rubi [A]
time = 0.20, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6565, 6589, 6597, 3460, 3390, 30, 3377, 2718, 6581}

$$-\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^4}+\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^4}+\frac{5\text{FresnelC}(bx)S(bx)}{2\pi^3b^6}+\frac{x^5S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}+\frac{5x^4}{24\pi^2b^2}+\frac{x^4\cos\left(\pi b^2x^2\right)}{12\pi^2b^2}-\frac{11\cos\left(\pi b^2x^2\right)}{6\pi^4b^6}-\frac{5xS(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^5}-\frac{7x^2\sin\left(\pi b^2x^2\right)}{12\pi^3b^4}-\frac{5x^3S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{1}{6}x^6S(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^5*FresnelS[b*x]^2,x]
```

```
[Out] (5*x^4)/(24*b^2*Pi^2) - (11*Cos[b^2*Pi*x^2])/(6*b^6*Pi^4) + (x^4*Cos[b^2*Pi
*x^2])/(12*b^2*Pi^2) - (5*x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^5*Pi^3) +
 (x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(3*b*Pi) + (5*FresnelC[b*x]*Fresne
lS[b*x])/(2*b^6*Pi^3) + (x^6*FresnelS[b*x]^2)/6 - (((5*I)/8)*x^2*Hypergeome
tricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/(b^4*Pi^3) + (((5*I)/8)*x^2
*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^4*Pi^3) - (5*x^3
*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*b^3*Pi^2) - (7*x^2*Sin[b^2*Pi*x^2])/
(12*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
```

1]

Rubi steps

$$
\begin{aligned}
\int x^5 S(bx)^2\,dx &= \frac{1}{6}x^6 S(bx)^2 - \frac{1}{3}b\int x^6 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{6}x^6 S(bx)^2 - \frac{\int x^5\sin\left(b^2\pi x^2\right)\,dx}{6\pi} - \frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{3b\pi} \\
&= \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{6}x^6 S(bx)^2 - \frac{5x^3 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} + \frac{5\int x^2 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,d}{b^3\pi^2} \\
&= \frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{6}x^6 S(bx)^2 - \frac{5x^3 S(bx)}{3} \\
&= \frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{5C(bx)S(bx)}{2b^6\pi^3} + \frac{1}{6}x^6 S(b \\
&= \frac{5x^4}{24b^2\pi^2} - \frac{17\cos\left(b^2\pi x^2\right)}{12b^6\pi^4} + \frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx}{3b\pi} \\
&= \frac{5x^4}{24b^2\pi^2} - \frac{11\cos\left(b^2\pi x^2\right)}{6b^6\pi^4} + \frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^5\pi^3} + \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx}{3b\pi}
\end{aligned}
$$

Mathematica [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int x^5 S(bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^5*FresnelS[b*x]^2,x]
[Out] Integrate[x^5*FresnelS[b*x]^2, x]
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int x^5 \mathrm{S}(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelS(b*x)^2,x)
[Out] int(x^5*FresnelS(b*x)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnel_sin(b*x)^2,x, algorithm="maxima")
[Out] integrate(x^5*fresnel_sin(b*x)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnel_sin(b*x)^2,x, algorithm="fricas")
[Out] integral(x^5*fresnel_sin(b*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 S^2(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*fresnels(b*x)**2,x)
[Out] Integral(x**5*fresnels(b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnel_sin(b*x)^2,x, algorithm="giac")
[Out] integrate(x^5*fresnel_sin(b*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\, \mathrm{FresnelS}(b\, x)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelS(b*x)^2,x)
[Out] int(x^5*FresnelS(b*x)^2, x)
```

3.34 $\int x^4 S(bx)^2\,dx$

Optimal. Leaf size=177

$$\frac{4x^3}{15b^2\pi^2}+\frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2}-\frac{16\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b^5\pi^3}+\frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b\pi}+\frac{1}{5}x^5S(bx)^2+\frac{43S\left(\sqrt{2}\,bx\right)}{20\sqrt{2}\,b^5\pi^3}-\frac{8x^2S(}{}$$

```
[Out] 4/15*x^3/b^2/Pi^2+1/10*x^3*cos(b^2*Pi*x^2)/b^2/Pi^2-16/5*cos(1/2*b^2*Pi*x^2
)*FresnelS(b*x)/b^5/Pi^3+2/5*x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi+1/5
*x^5*FresnelS(b*x)^2-8/5*x^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2-11/
20*x*sin(b^2*Pi*x^2)/b^4/Pi^3+43/40*FresnelS(b*x*2^(1/2))/b^5/Pi^3*2^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.900$, Rules used = {6565, 6589, 6597, 3472, 30, 3467, 3432, 6587, 3466}

$$\frac{43S\left(\sqrt{2}\,bx\right)}{20\sqrt{2}\,\pi^3b^5}+\frac{2x^4S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b}+\frac{4x^3}{15\pi^2b^2}+\frac{x^3\cos\left(\pi b^2x^2\right)}{10\pi^2b^2}-\frac{16S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5}-\frac{11x\sin\left(\pi b^2x^2\right)}{20\pi^3b^4}-\frac{8x^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3}+\frac{1}{5}x^5S(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^4*FresnelS[b*x]^2,x]
```

```
[Out] (4*x^3)/(15*b^2*Pi^2) + (x^3*Cos[b^2*Pi*x^2])/(10*b^2*Pi^2) - (16*Cos[(b^2*
Pi*x^2)/2]*FresnelS[b*x])/(5*b^5*Pi^3) + (2*x^4*Cos[(b^2*Pi*x^2)/2]*Fresnel
S[b*x])/(5*b*Pi) + (x^5*FresnelS[b*x]^2)/5 + (43*FresnelS[Sqrt[2]*b*x])/(20
*Sqrt[2]*b^5*Pi^3) - (8*x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2)
 - (11*x*Sin[b^2*Pi*x^2])/(20*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3472

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6587

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x]
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\int x^4 S(bx)^2\,dx = \frac{1}{5}x^5 S(bx)^2 - \frac{1}{5}(2b)\int x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$
$$= \frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b\pi} + \frac{1}{5}x^5 S(bx)^2 - \frac{\int x^4\sin\left(b^2\pi x^2\right)\,dx}{5\pi} - \frac{8\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{5b\pi}$$
$$= \frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} + \frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b\pi} + \frac{1}{5}x^5 S(bx)^2 - \frac{8x^2 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} + \frac{16\int xS(}{}$$
$$= \frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{16\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b^5\pi^3} + \frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b\pi} + \frac{1}{5}x^5 S(bx)^2 - \frac{8x^2 S(bx)}{}$$
$$= \frac{4x^3}{15b^2\pi^2} + \frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{16\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b^5\pi^3} + \frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b\pi} + \frac{1}{5}x^5 S(bx)^2 +$$
$$= \frac{4x^3}{15b^2\pi^2} + \frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{16\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b^5\pi^3} + \frac{2x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5b\pi} + \frac{1}{5}x^5 S(bx)^2 +$$

Mathematica [A]
time = 0.10, size = 137, normalized size = 0.77

$$\frac{32b^3\pi x^3 + 12b^3\pi x^3\cos\left(b^2\pi x^2\right) + 24b^5\pi^3x^5S(bx)^2 + 129\sqrt{2}\,S\left(\sqrt{2}\,bx\right) + 48S(bx)\left(\left(-8+b^4\pi^2x^4\right)\cos\left(\frac{1}{2}b^2\pi x^2\right) - 4b^2\pi x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)\right) - 66bx\sin\left(b^2\pi x^2\right)}{120b^5\pi^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*FresnelS[b*x]^2,x]
```

```
[Out] (32*b^3*Pi*x^3 + 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 24*b^5*Pi^3*x^5*FresnelS[b
*x]^2 + 129*Sqrt[2]*FresnelS[Sqrt[2]*b*x] + 48*FresnelS[b*x]*((-8 + b^4*Pi^
2*x^4)*Cos[(b^2*Pi*x^2)/2] - 4*b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) - 66*b*x*Sin
[b^2*Pi*x^2])/(120*b^5*Pi^3)
```

Maple [A]
time = 0.71, size = 208, normalized size = 1.18

method	result
derivativedivides	$\frac{\frac{\mathrm{S}(bx)^2 b^5 x^5}{5} - 2\,\mathrm{S}(bx)\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi} + \frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{5\pi} + \frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi^2}}{\pi}\right) + \frac{4b^3x^3}{15\pi^2} - \frac{4\left(\frac{bx\sin\left(b^2\pi x^2\right)}{2\pi} - \frac{\sqrt{2}\,\mathrm{S}\left(bx\sqrt{}\right.}{4\pi}\right.}{5\pi^2}}{b^5}$

default	$\frac{\frac{S(bx)^2 b^5 x^5}{5} - 2\,S(bx)\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi} + \frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{5\pi} + \frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi^2}}{\pi}\right) + \frac{4b^3x^3}{15\pi^2} - \frac{4\left(\frac{bx\sin\left(b^2\pi x^2\right)}{2\pi} - \frac{\sqrt{2}\,S\left(ba\ldots\right.}{4\pi}\right.}{5\pi^2}\ldots}{b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelS(b*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/5*FresnelS(b*x)^2*b^5*x^5-2*FresnelS(b*x)*(-1/5/Pi*b^4*x^4*cos(1/2
*b^2*Pi*x^2)+4/5/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi
*x^2)))+4/15/Pi^2*b^3*x^3-4/5/Pi^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/
2)*FresnelS(b*x*2^(1/2)))-1/5/Pi^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*
(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^(1/2)
*FresnelS(b*x*2^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnel_sin(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^4*fresnel_sin(b*x)^2, x)
```

Fricas [A]
time = 0.39, size = 149, normalized size = 0.84

$$\frac{24\,\pi^3 b^6 x^5\,S(bx)^2 + 24\,\pi b^4 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 20\,\pi b^4 x^3 + 48\left(\pi^2 b^5 x^4 - 8\,b\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)S(bx) + 129\,\sqrt{2}\,\sqrt{b^2}\,S\left(\sqrt{2}\,\sqrt{b^2}\,x\right) - 12\left(16\,\pi b^3 x^2\,S(bx) + 11\,b^2 x\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{120\,\pi^3 b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnel_sin(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/120*(24*pi^3*b^6*x^5*fresnel_sin(b*x)^2 + 24*pi*b^4*x^3*cos(1/2*pi*b^2*x^
2)^2 + 20*pi*b^4*x^3 + 48*(pi^2*b^5*x^4 - 8*b)*cos(1/2*pi*b^2*x^2)*fresnel_
sin(b*x) + 129*sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x) - 12*(16*
pi*b^3*x^2*fresnel_sin(b*x) + 11*b^2*x*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*
x^2))/(pi^3*b^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 S^2(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnels(b*x)**2,x)
```

```
[Out] Integral(x**4*fresnels(b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnel_sin(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnel_sin(b*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\mathrm{FresnelS}(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelS(b*x)^2,x)
```

```
[Out] int(x^4*FresnelS(b*x)^2, x)
```

3.35 $\int x^3 S(bx)^2\,dx$

Optimal. Leaf size=140

$$\frac{3x^2}{8b^2\pi^2}+\frac{x^2\cos\left(b^2\pi x^2\right)}{8b^2\pi^2}+\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2b\pi}+\frac{3S(bx)^2}{4b^4\pi^2}+\frac{1}{4}x^4S(bx)^2-\frac{3xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^3\pi^2}-\frac{\sin\left(b^2\pi x^2\right)}{2b^4\pi^3}$$

```
[Out] 3/8*x^2/b^2/Pi^2+1/8*x^2*cos(b^2*Pi*x^2)/b^2/Pi^2+1/2*x^3*cos(1/2*b^2*Pi*x^
2)*FresnelS(b*x)/b/Pi+3/4*FresnelS(b*x)^2/b^4/Pi^2+1/4*x^4*FresnelS(b*x)^2-
3/2*x*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^3/Pi^2-1/2*sin(b^2*Pi*x^2)/b^4/Pi
^3
```

Rubi [A]
time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6565, 6589, 6597, 3460, 2714, 6575, 30, 3377, 2717}

$$\frac{3S(bx)^2}{4\pi^2b^4}+\frac{x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b}+\frac{3x^2}{8\pi^2b^2}+\frac{x^2\cos\left(\pi b^2x^2\right)}{8\pi^2b^2}-\frac{\sin\left(\pi b^2x^2\right)}{2\pi^3b^4}-\frac{3xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^2b^3}+\frac{1}{4}x^4S(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelS[b*x]^2,x]
```

```
[Out] (3*x^2)/(8*b^2*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(8*b^2*Pi^2) + (x^3*Cos[(b^2*P
i*x^2)/2]*FresnelS[b*x])/(2*b*Pi) + (3*FresnelS[b*x]^2)/(4*b^4*Pi^2) + (x^4
*FresnelS[b*x]^2)/4 - (3*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*b^3*Pi^2)
- Sin[b^2*Pi*x^2]/(2*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\int x^3S(bx)^2\,dx = \frac{1}{4}x^4S(bx)^2 - \frac{1}{2}b\int x^4S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$
$$= \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2b\pi} + \frac{1}{4}x^4S(bx)^2 - \frac{\int x^3\sin\left(b^2\pi x^2\right)\,dx}{4\pi} - \frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{2b\pi}$$
$$= \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2b\pi} + \frac{1}{4}x^4S(bx)^2 - \frac{3xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^3\pi^2} + \frac{3\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b^3\pi^2}$$
$$= \frac{x^2\cos\left(b^2\pi x^2\right)}{8b^2\pi^2} + \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2b\pi} + \frac{1}{4}x^4S(bx)^2 - \frac{3xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^3\pi^2} + \frac{3\text{Subst}(\int$$
$$= \frac{3x^2}{8b^2\pi^2} + \frac{x^2\cos\left(b^2\pi x^2\right)}{8b^2\pi^2} + \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2b\pi} + \frac{3S(bx)^2}{4b^4\pi^2} + \frac{1}{4}x^4S(bx)^2 - \frac{3xS(bx)\sin(}{2b^3\pi^2}$$

Mathematica [A]
time = 0.01, size = 140, normalized size = 1.00

$$\frac{3x^2}{8b^2\pi^2} + \frac{x^2\cos\left(b^2\pi x^2\right)}{8b^2\pi^2} + \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2b\pi} + \frac{3S(bx)^2}{4b^4\pi^2} + \frac{1}{4}x^4S(bx)^2 - \frac{3xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^3\pi^2} - \frac{\sin\left(b^2\pi x^2\right)}{2b^4\pi^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelS[b*x]^2,x]
```

```
[Out] (3*x^2)/(8*b^2*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(8*b^2*Pi^2) + (x^3*Cos[(b^2*P
i*x^2)/2]*FresnelS[b*x])/(2*b*Pi) + (3*FresnelS[b*x]^2)/(4*b^4*Pi^2) + (x^4
*FresnelS[b*x]^2)/4 - (3*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*b^3*Pi^2)
- Sin[b^2*Pi*x^2]/(2*b^4*Pi^3)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int x^3\mathrm{S}(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(b*x)^2,x)
```

```
[Out] int(x^3*FresnelS(b*x)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_sin(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnel_sin(b*x)^2, x)
```

Fricas [A]
time = 0.37, size = 117, normalized size = 0.84

$$\frac{2\,\pi^2 b^3 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{S}\left(bx\right) + \pi b^2 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + \pi b^2 x^2 + \left(3\,\pi + \pi^3 b^4 x^4\right) \mathrm{S}\left(bx\right)^2 - 2\,\left(3\,\pi bx\,\mathrm{S}\left(bx\right) + 2\,\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\right) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{4\,\pi^3 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_sin(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*pi^2*b^3*x^3*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + pi*b^2*x^2*cos(1
/2*pi*b^2*x^2)^2 + pi*b^2*x^2 + (3*pi + pi^3*b^4*x^4)*fresnel_sin(b*x)^2 -
2*(3*pi*b*x*fresnel_sin(b*x) + 2*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/
(pi^3*b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 S^2(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnels(b*x)**2,x)
```

```
[Out] Integral(x**3*fresnels(b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_sin(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnel_sin(b*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{FresnelS}(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(b*x)^2,x)
```

```
[Out] int(x^3*FresnelS(b*x)^2, x)
```

3.36 $\int x^2 S(bx)^2\, dx$

Optimal. Leaf size=124

$$\frac{2x}{3b^2\pi^2}+\frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2}-\frac{5\text{FresnelC}\left(\sqrt{2}\,bx\right)}{6\sqrt{2}\,b^3\pi^2}+\frac{2x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi}+\frac{1}{3}x^3S(bx)^2-\frac{4S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2}$$

```
[Out] 2/3*x/b^2/Pi^2+1/6*x*cos(b^2*Pi*x^2)/b^2/Pi^2+2/3*x^2*cos(1/2*b^2*Pi*x^2)*F
resnelS(b*x)/b/Pi+1/3*x^3*FresnelS(b*x)^2-4/3*FresnelS(b*x)*sin(1/2*b^2*Pi*
x^2)/b^3/Pi^2-5/12*FresnelC(b*x*2^(1/2))/b^3/Pi^2*2^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6565, 6589, 6595, 3438, 3433, 3466}

$$-\frac{5\text{FresnelC}\left(\sqrt{2}\,bx\right)}{6\sqrt{2}\,\pi^2b^3}+\frac{2x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}+\frac{x\cos\left(\pi b^2x^2\right)}{6\pi^2b^2}+\frac{2x}{3\pi^2b^2}-\frac{4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{1}{3}x^3S(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelS[b*x]^2,x]
```

```
[Out] (2*x)/(3*b^2*Pi^2) + (x*Cos[b^2*Pi*x^2])/(6*b^2*Pi^2) - (5*FresnelC[Sqrt[2]
*b*x])/(6*Sqrt[2]*b^3*Pi^2) + (2*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(3*
b*Pi) + (x^3*FresnelS[b*x]^2)/3 - (4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*
b^3*Pi^2)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6595

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelS[b*x]/(2*d)), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$
\begin{aligned}
\int x^2 S(bx)^2\,dx &= \frac{1}{3}x^3 S(bx)^2 - \frac{1}{3}(2b)\int x^3 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{2x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{3}x^3S(bx)^2 - \frac{\int x^2\sin\left(b^2\pi x^2\right)\,dx}{3\pi} - \frac{4\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{3b\pi} \\
&= \frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} + \frac{2x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{3}x^3S(bx)^2 - \frac{4S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} - \frac{\int\cos\left(b^2\pi x\right.}{6b^2\pi^2} \\
&= \frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{C\left(\sqrt{2}\,bx\right)}{6\sqrt{2}\,b^3\pi^2} + \frac{2x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{3}x^3S(bx)^2 - \frac{4S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} \\
&= \frac{2x}{3b^2\pi^2} + \frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{C\left(\sqrt{2}\,bx\right)}{6\sqrt{2}\,b^3\pi^2} + \frac{2x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{3}x^3S(bx)^2 - \frac{4S(bx)\sin}{3b^3\pi} \\
&= \frac{2x}{3b^2\pi^2} + \frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{C\left(\sqrt{2}\,bx\right)}{6\sqrt{2}\,b^3\pi^2} - \frac{\sqrt{2}\,C\left(\sqrt{2}\,bx\right)}{3b^3\pi^2} + \frac{2x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3b\pi} + \frac{1}{3}x^3S(
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 100, normalized size = 0.81

$$\frac{2bx\left(4+\cos\left(b^2\pi x^2\right)\right) - 5\sqrt{2}\,\mathrm{FresnelC}\left(\sqrt{2}\,bx\right) + 4b^3\pi^2x^3S(bx)^2 + 8S(bx)\left(b^2\pi x^2\cos\left(\frac{1}{2}b^2\pi x^2\right) - 2\sin\left(\frac{1}{2}b^2\pi x^2\right)\right)}{12b^3\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelS[b*x]^2,x]
```

```
[Out] (2*b*x*(4 + Cos[b^2*Pi*x^2]) - 5*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 4*b^3*Pi^2
*x^3*FresnelS[b*x]^2 + 8*FresnelS[b*x]*(b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] - 2*
Sin[(b^2*Pi*x^2)/2]))/(12*b^3*Pi^2)
```

Maple [A]
time = 0.66, size = 122, normalized size = 0.98

method	result
derivativedivides	$\frac{\frac{S(bx)^2 b^3 x^3}{3} - 2\,S(bx)\left(-\frac{b^2 x^2 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{3\pi} + \frac{2\sin\left(\frac{b^2 \pi x^2}{2}\right)}{3\pi^2}\right) + \frac{2bx}{3\pi^2} - \frac{\sqrt{2}\ \mathrm{FresnelC}\left(bx\sqrt{2}\right)}{3\pi^2} - \frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{2\pi} + \frac{\sqrt{2}\ \mathrm{F}}{3\pi}}{}}{b^3}$
default	$\frac{\frac{S(bx)^2 b^3 x^3}{3} - 2\,S(bx)\left(-\frac{b^2 x^2 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{3\pi} + \frac{2\sin\left(\frac{b^2 \pi x^2}{2}\right)}{3\pi^2}\right) + \frac{2bx}{3\pi^2} - \frac{\sqrt{2}\ \mathrm{FresnelC}\left(bx\sqrt{2}\right)}{3\pi^2} - \frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{2\pi} + \frac{\sqrt{2}\ \mathrm{F}}{3\pi}}{}}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(b*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/3*FresnelS(b*x)^2*b^3*x^3-2*FresnelS(b*x)*(-1/3/Pi*b^2*x^2*cos(1/2
*b^2*Pi*x^2)+2/3/Pi^2*sin(1/2*b^2*Pi*x^2))+2/3*b*x/Pi^2-1/3/Pi^2*2^(1/2)*Fr
esnelC(b*x*2^(1/2))-1/3/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*Fres
nelC(b*x*2^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_sin(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnel_sin(b*x)^2, x)
```

Fricas [A]
time = 0.35, size = 111, normalized size = 0.90

$$\frac{4\,\pi^2 b^4 x^3\,\mathrm{S}\left(bx\right)^2 + 8\,\pi b^3 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{S}\left(bx\right) + 4\,b^2 x \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 6\,b^2 x - 16\,b\,\mathrm{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right) - 5\,\sqrt{2}\,\sqrt{b^2}\,\mathrm{C}\left(\sqrt{2}\,\sqrt{b^2}\,x\right)}{12\,\pi^2 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_sin(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(4*pi^2*b^4*x^3*fresnel_sin(b*x)^2 + 8*pi*b^3*x^2*cos(1/2*pi*b^2*x^2)*
fresnel_sin(b*x) + 4*b^2*x*cos(1/2*pi*b^2*x^2)^2 + 6*b^2*x - 16*b*fresnel_s
```

```
in(b*x)*sin(1/2*pi*b^2*x^2) - 5*sqrt(2)*sqrt(b^2)*fresnel_cos(sqrt(2)*sqrt(
b^2)*x))/(pi^2*b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 S^2(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnels(b*x)**2,x)
[Out] Integral(x**2*fresnels(b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_sin(b*x)^2,x, algorithm="giac")
[Out] integrate(x^2*fresnel_sin(b*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{FresnelS}(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(b*x)^2,x)
[Out] int(x^2*FresnelS(b*x)^2, x)
```

3.37 $\int xS(bx)^2\,dx$

Optimal. Leaf size=143

$$\frac{\cos(b^2\pi x^2)}{4b^2\pi^2}+\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b\pi}-\frac{\text{FresnelC}(bx)S(bx)}{2b^2\pi}+\frac{1}{2}x^2S(bx)^2+\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi}-\frac{ix^2\,{}_2F_2}{}$$

```
[Out] 1/4*cos(b^2*Pi*x^2)/b^2/Pi^2+x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi-1/2*F
resnelC(b*x)*FresnelS(b*x)/b^2/Pi+1/2*x^2*FresnelS(b*x)^2+1/8*I*x^2*hyperge
om([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/Pi-1/8*I*x^2*hypergeom([1, 1],[3/2, 2
],1/2*I*b^2*Pi*x^2)/Pi
```

Rubi [A]
time = 0.06, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6565, 6589, 6581, 3460, 2718}

$$\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi}-\frac{\text{FresnelC}(bx)S(bx)}{2\pi b^2}+\frac{xS(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+\frac{\cos(\pi b^2x^2)}{4\pi^2b^2}+\frac{1}{2}x^2S(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelS[b*x]^2,x]
```

```
[Out] Cos[b^2*Pi*x^2]/(4*b^2*Pi^2) + (x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b*Pi)
 - (FresnelC[b*x]*FresnelS[b*x])/(2*b^2*Pi) + (x^2*FresnelS[b*x]^2)/2 + ((I
/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/Pi - ((I/
8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/Pi
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rubi steps

$$\begin{aligned}
\int xS(bx)^2\,dx &= \frac{1}{2}x^2S(bx)^2 - b\int x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b\pi} + \frac{1}{2}x^2S(bx)^2 - \frac{\int x\sin\left(b^2\pi x^2\right)\,dx}{2\pi} - \frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b\pi} \\
&= \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b\pi} - \frac{C(bx)S(bx)}{2b^2\pi} + \frac{1}{2}x^2S(bx)^2 + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2\,{}_2F_2}{} \\
&= \frac{\cos\left(b^2\pi x^2\right)}{4b^2\pi^2} + \frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b\pi} - \frac{C(bx)S(bx)}{2b^2\pi} + \frac{1}{2}x^2S(bx)^2 + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi\right.}{8\pi}
\end{aligned}$$

Mathematica [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int xS(bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*FresnelS[b*x]^2,x]
```

```
[Out] Integrate[x*FresnelS[b*x]^2, x]
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int x\mathrm{S}(bx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(b*x)^2,x)
```

```
[Out] int(x*FresnelS(b*x)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_sin(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x*fresnel_sin(b*x)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_sin(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x*fresnel_sin(b*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int xS^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x)**2,x)
```

```
[Out] Integral(x*fresnels(b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_sin(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x*fresnel_sin(b*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{FresnelS}(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(b*x)^2,x)
```

```
[Out] int(x*FresnelS(b*x)^2, x)
```

3.38 $\int S(bx)^2\,dx$

Optimal. Leaf size=55

$$\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b\pi}+xS(bx)^2-\frac{S\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,b\pi}$$

```
[Out] 2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b/Pi+x*FresnelS(b*x)^2-1/2*FresnelS(b*x
*2^(1/2))/b/Pi*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6555, 12, 6587, 3432}

$$\frac{2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+xS(bx)^2-\frac{S\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]^2,x]
```

```
[Out] (2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b*Pi) + x*FresnelS[b*x]^2 - FresnelS
[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 6555

```
Int[FresnelS[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(FresnelS[a
 + b*x]^2/b), x] - Dist[2, Int[(a + b*x)*Sin[(Pi/2)*(a + b*x)^2]*FresnelS[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6587

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x]
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$
\begin{aligned}
\int S(bx)^2\,dx &= xS(bx)^2 - 2\int bxS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\
&= xS(bx)^2 - (2b)\int xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\
&= \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b\pi} + xS(bx)^2 - \frac{\int\sin\left(b^2\pi x^2\right)\,dx}{\pi}\\
&= \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b\pi} + xS(bx)^2 - \frac{S\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,b\pi}
\end{aligned}
$$

Mathematica [**A**]
time = 0.01, size = 55, normalized size = 1.00

$$\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b\pi} + xS(bx)^2 - \frac{S\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,b\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]^2,x]
```

```
[Out] (2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b*Pi) + x*FresnelS[b*x]^2 - FresnelS
[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi)
```

Maple [**A**]
time = 0.47, size = 49, normalized size = 0.89

method	result	size
derivativedivides	$\frac{\mathrm{S}(bx)^2 bx+\frac{2\,\mathrm{S}(bx)\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{\sqrt{2}\,\mathrm{S}\left(bx\sqrt{2}\right)}{2\pi}}{b}$	49
default	$\frac{\mathrm{S}(bx)^2 bx+\frac{2\,\mathrm{S}(bx)\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{\sqrt{2}\,\mathrm{S}\left(bx\sqrt{2}\right)}{2\pi}}{b}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(FresnelS(b*x)^2*b*x+2*FresnelS(b*x)/Pi*cos(1/2*b^2*Pi*x^2)-1/2/Pi*2^(1
/2)*FresnelS(b*x*2^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)^2, x)
```

Fricas [A]
time = 0.35, size = 60, normalized size = 1.09

$$\frac{2\,\pi b^2 x\,\mathrm{S}\left(bx\right)^2 + 4\,b\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{S}\left(bx\right) - \sqrt{2}\,\sqrt{b^2}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{b^2}\,x\right)}{2\,\pi b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*pi*b^2*x*fresnel_sin(b*x)^2 + 4*b*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*
x) - sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x))/(pi*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int S^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2,x)
```

```
[Out] Integral(fresnels(b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{FresnelS}(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2,x)
```

```
[Out] int(FresnelS(b*x)^2, x)
```

3.39 $\int \frac{S(bx)^2}{x} \, dx$

Optimal. Leaf size=13

$$\mathrm{Int}\left(\frac{S(bx)^2}{x},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x)^2/x,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(bx)^2}{x} \, dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x,x]
```

```
[Out] Defer[Int][FresnelS[b*x]^2/x, x]
```

Rubi steps

$$\int \frac{S(bx)^2}{x} \, dx = \int \frac{S(bx)^2}{x} \, dx$$

Mathematica [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{S(bx)^2}{x} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x,x]
```

```
[Out] Integrate[FresnelS[b*x]^2/x, x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx)^2}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x,x)
```

```
[Out] int(FresnelS(b*x)^2/x,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)^2/x, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(fresnel_sin(b*x)^2/x, x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^2(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x,x)
```

```
[Out] Integral(fresnels(b*x)**2/x, x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)^2/x, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\mathrm{FresnelS}(b\,x)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x,x)
```

```
[Out] int(FresnelS(b*x)^2/x, x)
```

3.40 $\int \frac{S(bx)^2}{x^2}\,dx$

Optimal. Leaf size=38

$$-\frac{S(bx)^2}{x}+2b\text{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)$$

```
[Out] -FresnelS(b*x)^2/x+2*b*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{S(bx)^2}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^2,x]
```

```
[Out] -(FresnelS[b*x]^2/x) + 2*b*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x
, x]
```

Rubi steps

$$\int \frac{S(bx)^2}{x^2}\,dx=-\frac{S(bx)^2}{x}+(2b)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{S(bx)^2}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^2,x]
```

```
[Out] Integrate[FresnelS[b*x]^2/x^2, x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^2,x)
```

```
[Out] int(FresnelS(b*x)^2/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)^2/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(fresnel_sin(b*x)^2/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^2(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**2,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)^2/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{FresnelS}(b\,x)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^2,x)
```

```
[Out] int(FresnelS(b*x)^2/x^2, x)
```

3.41 $\int \frac{S(bx)^2}{x^3} \, dx$

Optimal. Leaf size=39

$$-\frac{S(bx)^2}{2x^2}+b\text{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2},x\right)$$

```
[Out] -1/2*FresnelS(b*x)^2/x^2+b*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x
^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(bx)^2}{x^3} \, dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^3,x]
```

```
[Out] -1/2*FresnelS[b*x]^2/x^2 + b*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])
/x^2, x]
```

Rubi steps

$$\int \frac{S(bx)^2}{x^3} \, dx = -\frac{S(bx)^2}{2x^2}+b\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2} \, dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{S(bx)^2}{x^3} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^3,x]
```

```
[Out] Integrate[FresnelS[b*x]^2/x^3, x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx)^2}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^3,x)
```

```
[Out] int(FresnelS(b*x)^2/x^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)^2/x^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(fresnel_sin(b*x)^2/x^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^2(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**3,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)^2/x^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{FresnelS}(b\,x)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^3,x)
```

```
[Out] int(FresnelS(b*x)^2/x^3, x)
```

3.42 $\int \frac{S(bx)^2}{x^4} dx$

Optimal. Leaf size=120

$$-\frac{b^2}{6x}+\frac{b^2\cos\left(b^2\pi x^2\right)}{6x}-\frac{S(bx)^2}{3x^3}+\frac{b^3\pi S\left(\sqrt{2}\,bx\right)}{3\sqrt{2}}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^2}+\frac{1}{3}b^3\pi\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x},x\right)$$

```
[Out] -1/6*b^2/x+1/6*b^2*cos(b^2*Pi*x^2)/x-1/3*FresnelS(b*x)^2/x^3-1/3*b*FresnelS
(b*x)*sin(1/2*b^2*Pi*x^2)/x^2+1/6*b^3*Pi*FresnelS(b*x*2^(1/2))*2^(1/2)+1/3*
b^3*Pi*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(bx)^2}{x^4} dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^4,x]
```

```
[Out] -1/6*b^2/x + (b^2*Cos[b^2*Pi*x^2])/(6*x) - FresnelS[b*x]^2/(3*x^3) + (b^3*P
i*FresnelS[Sqrt[2]*b*x])/(3*Sqrt[2]) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2]
)/(3*x^2) + (b^3*Pi*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/3
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)^2}{x^4} dx &= -\frac{S(bx)^2}{3x^3}+\frac{1}{3}(2b)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3} dx \\
&= -\frac{b^2}{6x}-\frac{S(bx)^2}{3x^3}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^2}-\frac{1}{6}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2} dx+\frac{1}{3}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(
\\
&= -\frac{b^2}{6x}+\frac{b^2\cos\left(b^2\pi x^2\right)}{6x}-\frac{S(bx)^2}{3x^3}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^2}+\frac{1}{3}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x} dx \\
&= -\frac{b^2}{6x}+\frac{b^2\cos\left(b^2\pi x^2\right)}{6x}-\frac{S(bx)^2}{3x^3}+\frac{b^3\pi S\left(\sqrt{2}\,bx\right)}{3\sqrt{2}}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^2}+\frac{1}{3}\left(b^3\pi\right)\int \frac{\cos(
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{S(bx)^2}{x^4} dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^4,x]
```

```
[Out] Integrate[FresnelS[b*x]^2/x^4, x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^4,x)
```

```
[Out] int(FresnelS(b*x)^2/x^4,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)^2/x^4, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(fresnel_sin(b*x)^2/x^4, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^{2}(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**4,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**4, x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^4,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)^2/x^4, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}(b\,x)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^4,x)
[Out] int(FresnelS(b*x)^2/x^4, x)
```

3.43 $\int \frac{S(bx)^2}{x^5} \, dx$

Optimal. Leaf size=127

$$-\frac{b^2}{24x^2}+\frac{b^2\cos\left(b^2\pi x^2\right)}{24x^2}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{6x}-\frac{1}{12}b^4\pi^2S(bx)^2-\frac{S(bx)^2}{4x^4}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3}+\frac{1}{12}b^4\pi\text{Si}\left(b^2\pi x^2\right)$$

```
[Out] -1/24*b^2/x^2+1/24*b^2*cos(b^2*Pi*x^2)/x^2-1/6*b^3*Pi*cos(1/2*b^2*Pi*x^2)*F
resnelS(b*x)/x-1/12*b^4*Pi^2*FresnelS(b*x)^2-1/4*FresnelS(b*x)^2/x^4+1/12*b
^4*Pi*Si(b^2*Pi*x^2)-1/6*b*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3
```

Rubi [A]

time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6565, 6591, 6599, 6575, 30, 3456, 3461, 3378, 3380}

$$-\frac{1}{12}\pi^2b^4S(bx)^2-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x^3}-\frac{b^2}{24x^2}+\frac{b^2\cos\left(\pi b^2x^2\right)}{24x^2}+\frac{1}{12}\pi b^4\text{Si}\left(b^2\pi x^2\right)-\frac{\pi b^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x}-\frac{S(bx)^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]^2/x^5,x]
```

```
[Out] -1/24*b^2/x^2 + (b^2*Cos[b^2*Pi*x^2])/(24*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2
]*FresnelS[b*x])/(6*x) - (b^4*Pi^2*FresnelS[b*x]^2)/12 - FresnelS[b*x]^2/(4
*x^4) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x^3) + (b^4*Pi*SinIntegral
[b^2*Pi*x^2])/12
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6591

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[d*(x^(m + 2)/(Pi*b*(m + 1)*(m + 2))), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6599

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

Rubi steps

$$\int \frac{S(bx)^2}{x^5} dx = -\frac{S(bx)^2}{4x^4} + \frac{1}{2}b \int \frac{S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} dx$$
$$= -\frac{b^2}{24x^2} - \frac{S(bx)^2}{4x^4} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3} - \frac{1}{12}b^2 \int \frac{\cos\left(b^2\pi x^2\right)}{x^3} dx + \frac{1}{6}\left(b^3\pi\right) \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}$$
$$= -\frac{b^2}{24x^2} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{6x} - \frac{S(bx)^2}{4x^4} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3} - \frac{1}{24}b^2 \text{Subst}\left(\int \frac{\cos\left(b^2\pi\right.}{x^2}\right.$$
$$= -\frac{b^2}{24x^2} + \frac{b^2 \cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{6x} - \frac{S(bx)^2}{4x^4} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3} + \frac{1}{24}b^4\pi$$
$$= -\frac{b^2}{24x^2} + \frac{b^2 \cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{6x} - \frac{1}{12}b^4\pi^2 S(bx)^2 - \frac{S(bx)^2}{4x^4} - \frac{bS(bx) \sin\left(\frac{1}{2}\right.}{6x^3}$$

Mathematica [A]
time = 0.01, size = 127, normalized size = 1.00

$$-\frac{b^2}{24x^2} + \frac{b^2 \cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b^3\pi \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{6x} - \frac{1}{12}b^4\pi^2 S(bx)^2 - \frac{S(bx)^2}{4x^4} - \frac{bS(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^3} + \frac{1}{12}b^4\pi \text{Si}\left(b^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]^2/x^5,x]
```

```
[Out] -1/24*b^2/x^2 + (b^2*Cos[b^2*Pi*x^2])/(24*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2
]*FresnelS[b*x])/(6*x) - (b^4*Pi^2*FresnelS[b*x]^2)/12 - FresnelS[b*x]^2/(4
*x^4) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x^3) + (b^4*Pi*SinIntegral
[b^2*Pi*x^2])/12
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^5,x)
```

```
[Out] int(FresnelS(b*x)^2/x^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)^2/x^5, x)
```

Fricas [A]
time = 0.36, size = 111, normalized size = 0.87

$$\frac{\pi b^4x^4\,\mathrm{Si}\left(\pi b^2x^2\right)-2\,\pi b^3x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{S}\left(bx\right)+b^2x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2-b^2x^2-2\,bx\,\mathrm{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)-\left(\pi^2b^4x^4+3\right)\mathrm{S}\left(bx\right)^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(pi*b^4*x^4*sin_integral(pi*b^2*x^2) - 2*pi*b^3*x^3*cos(1/2*pi*b^2*x^2
)*fresnel_sin(b*x) + b^2*x^2*cos(1/2*pi*b^2*x^2)^2 - b^2*x^2 - 2*b*x*fresne
l_sin(b*x)*sin(1/2*pi*b^2*x^2) - (pi^2*b^4*x^4 + 3)*fresnel_sin(b*x)^2)/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{S^2(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**5,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)^2/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\mathrm{FresnelS}(b\,x)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^5,x)
```

```
[Out] int(FresnelS(b*x)^2/x^5, x)
```

3.44 $\int \frac{S(bx)^2}{x^6}\,dx$

Optimal. Leaf size=171

$$-\frac{b^2}{60x^3}+\frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3}+\frac{7b^5\pi^2\text{FresnelC}\left(\sqrt{2}\,bx\right)}{60\sqrt{2}}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{20x^2}-\frac{S(bx)^2}{5x^5}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4}-\frac{7\ldots}{}$$

```
[Out] -1/60*b^2/x^3+1/60*b^2*cos(b^2*Pi*x^2)/x^3-1/20*b^3*Pi*cos(1/2*b^2*Pi*x^2)*
FresnelS(b*x)/x^2-1/5*FresnelS(b*x)^2/x^5-1/10*b*FresnelS(b*x)*sin(1/2*b^2*
Pi*x^2)/x^4-7/120*b^4*Pi*sin(b^2*Pi*x^2)/x+7/120*b^5*Pi^2*FresnelC(b*x*2^(1
/2))*2^(1/2)-1/20*b^5*Pi^2*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x
,x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(bx)^2}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^6,x]
```

```
[Out] -1/60*b^2/x^3 + (b^2*Cos[b^2*Pi*x^2])/(60*x^3) + (7*b^5*Pi^2*FresnelC[Sqrt[
2]*b*x])/(60*Sqrt[2]) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(20*x^2)
 - FresnelS[b*x]^2/(5*x^5) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(10*x^4)
 - (7*b^4*Pi*Sin[b^2*Pi*x^2])/(120*x) - (b^5*Pi^2*Defer[Int][(FresnelS[b*x]
*Sin[(b^2*Pi*x^2)/2])/x, x])/20
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)^2}{x^6}\,dx &= -\frac{S(bx)^2}{5x^5}+\frac{1}{5}(2b)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx\\
&= -\frac{b^2}{60x^3}-\frac{S(bx)^2}{5x^5}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4}-\frac{1}{20}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx+\frac{1}{10}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\ldots}{x^3}\\
&= -\frac{b^2}{60x^3}+\frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{20x^2}-\frac{S(bx)^2}{5x^5}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4}+\frac{1}{40}\left(b^4\ldots\right)\\
&= -\frac{b^2}{60x^3}+\frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{20x^2}-\frac{S(bx)^2}{5x^5}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{10x^4}-\frac{7b^4\pi\,\text{si}\ldots}{1\ldots}\\
&= -\frac{b^2}{60x^3}+\frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3}+\frac{7b^5\pi^2C\left(\sqrt{2}\,bx\right)}{60\sqrt{2}}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{20x^2}-\frac{S(bx)^2}{5x^5}-\frac{bS(bx)\,\text{si}\ldots}{10\ldots}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{S(bx)^2}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^6,x]
```

```
[Out] Integrate[FresnelS[b*x]^2/x^6, x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^6,x)
```

```
[Out] int(FresnelS(b*x)^2/x^6,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^6,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)^2/x^6, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^6,x, algorithm="fricas")
```

```
[Out] integral(fresnel_sin(b*x)^2/x^6, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^2(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**6,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**6, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^6,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)^2/x^6, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}(b\,x)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^6,x)
```

```
[Out] int(FresnelS(b*x)^2/x^6, x)
```

3.45 $\int \frac{S(bx)^2}{x^7} dx$

Optimal. Leaf size=166

$$-\frac{b^2}{120x^4}+\frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4}+\frac{1}{72}b^6\pi^2\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{45x^3}-\frac{S(bx)^2}{6x^6}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5}$$

```
[Out] -1/120*b^2/x^4+1/72*b^6*Pi^2*Ci(b^2*Pi*x^2)+1/120*b^2*cos(b^2*Pi*x^2)/x^4-1
/45*b^3*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3-1/6*FresnelS(b*x)^2/x^6-1/
15*b*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^5-1/72*b^4*Pi*sin(b^2*Pi*x^2)/x^2-
1/45*b^5*Pi^2*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

Rubi [A]

time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(bx)^2}{x^7} dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^7,x]
```

```
[Out] -1/120*b^2/x^4 + (b^2*Cos[b^2*Pi*x^2])/(120*x^4) + (b^6*Pi^2*CosIntegral[b^
2*Pi*x^2])/72 - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(45*x^3) - Fresn
elS[b*x]^2/(6*x^6) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(15*x^5) - (b^4*
Pi*Sin[b^2*Pi*x^2])/(72*x^2) - (b^5*Pi^2*Defer[Int][(FresnelS[b*x]*Sin[(b^2
*Pi*x^2)/2])/x^2, x])/45
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)^2}{x^7} dx &= -\frac{S(bx)^2}{6x^6}+\frac{1}{3}b\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6} dx \\
&= -\frac{b^2}{120x^4}-\frac{S(bx)^2}{6x^6}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5}-\frac{1}{30}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^5} dx+\frac{1}{15}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi\right.}{x} \\
&= -\frac{b^2}{120x^4}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{45x^3}-\frac{S(bx)^2}{6x^6}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5}-\frac{1}{60}b^2\text{Subst}\left(\int \frac{\cos(b}{x}\right. \\
&= -\frac{b^2}{120x^4}+\frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{45x^3}-\frac{S(bx)^2}{6x^6}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5}+\frac{1}{180} \\
&= -\frac{b^2}{120x^4}+\frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{45x^3}-\frac{S(bx)^2}{6x^6}-\frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^5}-\frac{b^4\pi}{} \\
&= -\frac{b^2}{120x^4}+\frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4}+\frac{1}{72}b^6\pi^2\text{Ci}\left(b^2\pi x^2\right)-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{45x^3}-\frac{S(bx)^2}{6x^6}-\frac{bS(bx)}{}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{S(bx)^2}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^7,x]
[Out] Integrate[FresnelS[b*x]^2/x^7, x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^7,x)
[Out] int(FresnelS(b*x)^2/x^7,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^7,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)^2/x^7, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^7,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)^2/x^7, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^2(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**7,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**7, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^7,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)^2/x^7, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}(b\,x)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^7,x)
```

```
[Out] int(FresnelS(b*x)^2/x^7, x)
```

3.46 $\int \frac{S(bx)^2}{x^8}\,dx$

Optimal. Leaf size=259

$$-\frac{b^2}{210x^5}+\frac{b^6\pi^2}{336x}+\frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5}-\frac{67b^6\pi^2\cos\left(b^2\pi x^2\right)}{5040x}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{84x^4}-\frac{S(bx)^2}{7x^7}-\frac{b^7\pi^3S\left(\sqrt{2}\,bx\right)}{72\sqrt{2}}-\frac{2}{315}$$

```
[Out] -1/210*b^2/x^5+1/336*b^6*Pi^2/x+1/210*b^2*cos(b^2*Pi*x^2)/x^5-67/5040*b^6*P
i^2*cos(b^2*Pi*x^2)/x-1/84*b^3*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4-1/7
*FresnelS(b*x)^2/x^7-1/21*b*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^6+1/168*b^5
*Pi^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2-13/2520*b^4*Pi*sin(b^2*Pi*x^2)/
x^3-67/5040*b^7*Pi^3*FresnelS(b*x*2^(1/2))*2^(1/2)-1/168*b^7*Pi^3*Unintegra
ble(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Rubi [A]
time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(bx)^2}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^8,x]
```

```
[Out] -1/210*b^2/x^5 + (b^6*Pi^2)/(336*x) + (b^2*Cos[b^2*Pi*x^2])/(210*x^5) - (67
*b^6*Pi^2*Cos[b^2*Pi*x^2])/(5040*x) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[
b*x])/(84*x^4) - FresnelS[b*x]^2/(7*x^7) - (b^7*Pi^3*FresnelS[Sqrt[2]*b*x])
/(72*Sqrt[2]) - (2*Sqrt[2]*b^7*Pi^3*FresnelS[Sqrt[2]*b*x])/315 - (b*Fresnel
S[b*x]*Sin[(b^2*Pi*x^2)/2])/(21*x^6) + (b^5*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*
x^2)/2])/(168*x^2) - (13*b^4*Pi*Sin[b^2*Pi*x^2])/(2520*x^3) - (b^7*Pi^3*Def
er[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/168
```

Rubi steps

$$\int \frac{S(bx)^2}{x^8}\,dx = -\frac{S(bx)^2}{7x^7} + \frac{1}{7}(2b)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$
$$= -\frac{b^2}{210x^5} - \frac{S(bx)^2}{7x^7} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{21x^6} - \frac{1}{42}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}\,dx + \frac{1}{21}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi\right.}{x}$$
$$= -\frac{b^2}{210x^5} + \frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{84x^4} - \frac{S(bx)^2}{7x^7} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{21x^6} + \frac{1}{168}$$
$$= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} + \frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{84x^4} - \frac{S(bx)^2}{7x^7} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right.}{21x^6}$$
$$= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} + \frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5} - \frac{67b^6\pi^2\cos\left(b^2\pi x^2\right)}{5040x} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{84x^4} - \frac{S(bx)^2}{7x^7}$$
$$= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} + \frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5} - \frac{67b^6\pi^2\cos\left(b^2\pi x^2\right)}{5040x} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{84x^4} - \frac{S(bx)^2}{7x^7}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{S(bx)^2}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^8,x]
[Out] Integrate[FresnelS[b*x]^2/x^8, x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^8,x)
[Out] int(FresnelS(b*x)^2/x^8,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^8,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)^2/x^8, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^8,x, algorithm="fricas")
```

```
[Out] integral(fresnel_sin(b*x)^2/x^8, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^2(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**8,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**8, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)^2/x^8, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}(b\,x)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^8,x)
```

```
[Out] int(FresnelS(b*x)^2/x^8, x)
```

3.47 $\int \frac{S(bx)^2}{x^9}\,dx$

Optimal. Leaf size=242

$$-\frac{b^2}{336x^6}+\frac{b^6\pi^2}{1680x^2}+\frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6}-\frac{b^6\pi^2\cos\left(b^2\pi x^2\right)}{336x^2}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{140x^5}+\frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{420x}+\frac{1}{840}$$

```
[Out] -1/336*b^2/x^6+1/1680*b^6*Pi^2/x^2+1/336*b^2*cos(b^2*Pi*x^2)/x^6-1/336*b^6*
Pi^2*cos(b^2*Pi*x^2)/x^2-1/140*b^3*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5
+1/420*b^7*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x+1/840*b^8*Pi^4*FresnelS
(b*x)^2-1/8*FresnelS(b*x)^2/x^8-1/280*b^8*Pi^3*Si(b^2*Pi*x^2)-1/28*b*Fresne
lS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7+1/420*b^5*Pi^2*FresnelS(b*x)*sin(1/2*b^2*Pi
*x^2)/x^3-1/420*b^4*Pi*sin(b^2*Pi*x^2)/x^4
```

Rubi [A]
time = 0.27, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6565, 6591, 6599, 6575, 30, 3456, 3461, 3378, 3380, 3460}

$$\frac{1}{840}\pi^4 b^8 S(bx)^2+\frac{\pi^2 b^6}{1680x^2}-\frac{bS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{28x^7}-\frac{b^2}{336x^6}+\frac{b^2\cos\left(\pi b^2x^2\right)}{336x^6}-\frac{1}{280}\pi^3 b^8\text{Si}\left(b^2\pi x^2\right)+\frac{\pi^3 b^7 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{420x}-\frac{\pi^2 b^6\cos\left(\pi b^2x^2\right)}{336x^2}+\frac{\pi^2 b^5 S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{420x^3}-\frac{\pi b^4\sin\left(\pi b^2x^2\right)}{420x^4}-\frac{\pi b^3 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{140x^5}-\frac{S(bx)^2}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]^2/x^9,x]
```

```
[Out] -1/336*b^2/x^6 + (b^6*Pi^2)/(1680*x^2) + (b^2*Cos[b^2*Pi*x^2])/(336*x^6) -
(b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS
[b*x])/(140*x^5) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(420*x) + (
b^8*Pi^4*FresnelS[b*x]^2)/840 - FresnelS[b*x]^2/(8*x^8) - (b*FresnelS[b*x]*
Sin[(b^2*Pi*x^2)/2])/(28*x^7) + (b^5*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2]
)/(420*x^3) - (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) - (b^8*Pi^3*SinIntegral[b^
2*Pi*x^2])/280
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6591

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[d*(x^(m + 2)/(Pi*b*(m + 1)*(m + 2))), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6599

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

Rubi steps

$$\int \frac{S(bx)^2}{x^9}\,dx = -\frac{S(bx)^2}{8x^8} + \frac{1}{4}b\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$$

$$= -\frac{b^2}{336x^6} - \frac{S(bx)^2}{8x^8} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7} - \frac{1}{56}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^7}\,dx + \frac{1}{28}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi\right.}{x}$$

$$= -\frac{b^2}{336x^6} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{140x^5} - \frac{S(bx)^2}{8x^8} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7} - \frac{1}{112}b^2\text{Subst}\left(\int \frac{\cos(}{}\right.$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{140x^5} - \frac{S(bx)^2}{8x^8} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi\right.}{28x^7}$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{140x^5} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{420x} -$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} - \frac{b^6\pi^2\cos\left(b^2\pi x^2\right)}{336x^2} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{140x^5} + \frac{b^7\pi^3\cos}{}$$

$$= -\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} - \frac{b^6\pi^2\cos\left(b^2\pi x^2\right)}{336x^2} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{140x^5} + \frac{b^7\pi^3\cos}{}$$

Mathematica [A]
time = 0.01, size = 242, normalized size = 1.00

$$-\frac{b^2}{336x^6} + \frac{b^6\pi^2}{1680x^2} + \frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6} - \frac{b^6\pi^2\cos\left(b^2\pi x^2\right)}{336x^2} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{140x^5} + \frac{b^7\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{420x} + \frac{1}{840}b^8\pi^4S(bx)^2 - \frac{S(bx)^2}{8x^8} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{28x^7} + \frac{b^5\pi^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{420x^3} - \frac{b^4\pi\sin\left(b^2\pi x^2\right)}{420x^4} - \frac{1}{280}b^8\pi^3\text{Si}\left(b^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]^2/x^9,x]
```

```
[Out] -1/336*b^2/x^6 + (b^6*Pi^2)/(1680*x^2) + (b^2*Cos[b^2*Pi*x^2])/(336*x^6) -
(b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS
[b*x])/(140*x^5) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(420*x) + (
b^8*Pi^4*FresnelS[b*x]^2)/840 - FresnelS[b*x]^2/(8*x^8) - (b*FresnelS[b*x]*
Sin[(b^2*Pi*x^2)/2])/(28*x^7) + (b^5*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2]
)/(420*x^3) - (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) - (b^8*Pi^3*SinIntegral[b^
2*Pi*x^2])/280
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx)^2}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^9,x)
```

```
[Out] int(FresnelS(b*x)^2/x^9,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^9,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)^2/x^9, x)
```

Fricas [A]
time = 0.37, size = 187, normalized size = 0.77

$$-\frac{3\,\pi^{3}b^{8}x^{8}\,\mathrm{Si}\left(\pi b^{2}x^{2}\right) - 3\,\pi^{2}b^{6}x^{6} + 5\,b^{2}x^{2} + 5\left(\pi^{2}b^{6}x^{6} - b^{2}x^{2}\right)\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)^{2} - 2\left(\pi^{3}b^{7}x^{7} - 3\,\pi b^{3}x^{3}\right)\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)\mathrm{S}\left(bx\right) - \left(\pi^{4}b^{8}x^{8} - 105\right)\mathrm{S}\left(bx\right)^{2} + 2\left(2\,\pi b^{4}x^{4}\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right) - \left(\pi^{2}b^{5}x^{5} - 15\,bx\right)\mathrm{S}\left(bx\right)\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{840\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^9,x, algorithm="fricas")
```

```
[Out] -1/840*(3*pi^3*b^8*x^8*sin_integral(pi*b^2*x^2) - 3*pi^2*b^6*x^6 + 5*b^2*x^
2 + 5*(pi^2*b^6*x^6 - b^2*x^2)*cos(1/2*pi*b^2*x^2)^2 - 2*(pi^3*b^7*x^7 - 3*
pi*b^3*x^3)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) - (pi^4*b^8*x^8 - 105)*fre
snel_sin(b*x)^2 + 2*(2*pi*b^4*x^4*cos(1/2*pi*b^2*x^2) - (pi^2*b^5*x^5 - 15*
b*x)*fresnel_sin(b*x))*sin(1/2*pi*b^2*x^2))/x^8
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^{2}(bx)}{x^{9}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**9,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**9, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^9,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)^2/x^9, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}(b\,x)^2}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^9,x)
```

```
[Out] int(FresnelS(b*x)^2/x^9, x)
```

3.48 $\int \frac{S(bx)^2}{x^{10}}\,dx$

Optimal. Leaf size=286

$$-\frac{b^2}{504x^7}+\frac{b^6\pi^2}{5184x^3}+\frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7}-\frac{187b^6\pi^2\cos\left(b^2\pi x^2\right)}{181440x^3}-\frac{853b^9\pi^4\text{FresnelC}\left(\sqrt{2}\,bx\right)}{181440\sqrt{2}}-\frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{216x^6}$$

```
[Out] -1/504*b^2/x^7+1/5184*b^6*Pi^2/x^3+1/504*b^2*cos(b^2*Pi*x^2)/x^7-187/181440
*b^6*Pi^2*cos(b^2*Pi*x^2)/x^3-1/216*b^3*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x
)/x^6+1/1728*b^7*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2-1/9*FresnelS(b*
x)^2/x^9-1/36*b*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^8+1/864*b^5*Pi^2*Fresne
lS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4-19/15120*b^4*Pi*sin(b^2*Pi*x^2)/x^5+853/362
880*b^8*Pi^3*sin(b^2*Pi*x^2)/x-853/362880*b^9*Pi^4*FresnelC(b*x*2^(1/2))*2^
(1/2)+1/1728*b^9*Pi^4*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A]
time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(bx)^2}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelS[b*x]^2/x^10,x]
```

```
[Out] -1/504*b^2/x^7 + (b^6*Pi^2)/(5184*x^3) + (b^2*Cos[b^2*Pi*x^2])/(504*x^7) -
(187*b^6*Pi^2*Cos[b^2*Pi*x^2])/(181440*x^3) - (853*b^9*Pi^4*FresnelC[Sqrt[2
]*b*x])/(181440*Sqrt[2]) - (b^3*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(216*
x^6) + (b^7*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(1728*x^2) - FresnelS[b
*x]^2/(9*x^9) - (b*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(36*x^8) + (b^5*Pi^2*
FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(864*x^4) - (19*b^4*Pi*Sin[b^2*Pi*x^2])/
(15120*x^5) + (853*b^8*Pi^3*Sin[b^2*Pi*x^2])/(362880*x) + (b^9*Pi^4*Defer[I
nt][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/1728
```

Rubi steps

$$\int \frac{S(bx)^2}{x^{10}}\,dx = -\frac{S(bx)^2}{9x^9} + \frac{1}{9}(2b)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

$$= -\frac{b^2}{504x^7} - \frac{S(bx)^2}{9x^9} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{36x^8} - \frac{1}{72}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}\,dx + \frac{1}{36}\left(b^3\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi\right.}{x}$$

$$= -\frac{b^2}{504x^7} + \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{216x^6} - \frac{S(bx)^2}{9x^9} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{36x^8} + \frac{1}{432}$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{216x^6} - \frac{S(bx)^2}{9x^9} - \frac{bS(bx)\sin\left(\frac{1}{2}b^2\pi\right.}{36x^8}$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{187b^6\pi^2\cos\left(b^2\pi x^2\right)}{181440x^3} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{216x^6} + \frac{b^7\pi^3}{}$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{187b^6\pi^2\cos\left(b^2\pi x^2\right)}{181440x^3} - \frac{b^3\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{216x^6} + \frac{b^7\pi^3}{}$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} + \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{187b^6\pi^2\cos\left(b^2\pi x^2\right)}{181440x^3} - \frac{67b^9\pi^4C\left(\sqrt{2}\,bx\right)}{25920\sqrt{2}} - \frac{1}{945}\sqrt{2}\,b$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{S(bx)^2}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelS[b*x]^2/x^10,x]
```

```
[Out] Integrate[FresnelS[b*x]^2/x^10, x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^10,x)
```

```
[Out] int(FresnelS(b*x)^2/x^10,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^10,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)^2/x^10, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^10,x, algorithm="fricas")
```

```
[Out] integral(fresnel_sin(b*x)^2/x^10, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^2(bx)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2/x**10,x)
```

```
[Out] Integral(fresnels(b*x)**2/x**10, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2/x^10,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)^2/x^10, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}(b\,x)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2/x^10,x)
```

```
[Out] int(FresnelS(b*x)^2/x^10, x)
```

3.49 $\int (c+dx)^2 S(a+bx)^2 \, dx$

Optimal. Leaf size=497

$$\frac{2d^2x}{3b^2\pi^2}+\frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2}+\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}-\frac{5d^2\text{FresnelC}\left(\sqrt{2}\,(a+bx)\right)}{6\sqrt{2}\,b^3\pi^2}+\frac{2(bc-ad)}{}$$

```
[Out] 2/3*d^2*x/b^2/Pi^2+1/2*d*(-a*d+b*c)*cos(Pi*(b*x+a)^2)/b^3/Pi^2+1/6*d^2*(b*x
+a)*cos(Pi*(b*x+a)^2)/b^3/Pi^2+2*(-a*d+b*c)^2*cos(1/2*Pi*(b*x+a)^2)*Fresnel
S(b*x+a)/b^3/Pi+2*d*(-a*d+b*c)*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)*FresnelS(b*x+a
)/b^3/Pi+2/3*d^2*(b*x+a)^2*cos(1/2*Pi*(b*x+a)^2)*FresnelS(b*x+a)/b^3/Pi-d*(
-a*d+b*c)*FresnelC(b*x+a)*FresnelS(b*x+a)/b^3/Pi+(-a*d+b*c)^2*(b*x+a)*Fresn
elS(b*x+a)^2/b^3+d*(-a*d+b*c)*(b*x+a)^2*FresnelS(b*x+a)^2/b^3+1/3*d^2*(b*x+
a)^3*FresnelS(b*x+a)^2/b^3+1/4*I*d*(-a*d+b*c)*(b*x+a)^2*hypergeom([1, 1],[3
/2, 2],-1/2*I*Pi*(b*x+a)^2)/b^3/Pi-1/4*I*d*(-a*d+b*c)*(b*x+a)^2*hypergeom([
1, 1],[3/2, 2],1/2*I*Pi*(b*x+a)^2)/b^3/Pi-4/3*d^2*FresnelS(b*x+a)*sin(1/2*P
i*(b*x+a)^2)/b^3/Pi^2-5/12*d^2*FresnelC((b*x+a)*2^(1/2))/b^3/Pi^2*2^(1/2)-1
/2*(-a*d+b*c)^2*FresnelS((b*x+a)*2^(1/2))/b^3/Pi*2^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {6567, 6555, 6587, 3432, 6565, 6589, 6581, 3460, 2718, 6595, 3438, 3433, 3466}

$$\frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3}-\frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3}-\frac{d(bc-ad)\text{FresnelC}(a+bx)S(a+bx)}{\pi b^3}+\frac{d(a+bx)^2(bc-ad)S(a+bx)^2}{b^3}+\frac{(a+bx)(bc-ad)^2S(a+bx)^2}{b^3}-\frac{(bc-ad)^2S\left(\sqrt{2}\,(a+bx)\right)}{\sqrt{2}\,\pi b^3}+\frac{2d(a+bx)(bc-ad)S(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}+\frac{2(bc-ad)^2S(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}+\frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2\pi^2b^3}-\frac{5d^2\text{FresnelC}\left(\sqrt{2}\,(a+bx)\right)}{6\sqrt{2}\,\pi^2b^3}+\frac{d^2(a+bx)^3S(a+bx)^2}{3b^3}-\frac{4d^2S(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi^2b^3}+\frac{2d^2(a+bx)^2S(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi b^3}+\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6\pi^2b^3}+\frac{2d^2x}{3\pi^2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*FresnelS[a + b*x]^2,x]
```

```
[Out] (2*d^2*x)/(3*b^2*Pi^2) + (d*(b*c - a*d)*Cos[Pi*(a + b*x)^2])/(2*b^3*Pi^2) +
 (d^2*(a + b*x)*Cos[Pi*(a + b*x)^2])/(6*b^3*Pi^2) - (5*d^2*FresnelC[Sqrt[2]
*(a + b*x)])/(6*Sqrt[2]*b^3*Pi^2) + (2*(b*c - a*d)^2*Cos[(Pi*(a + b*x)^2)/2
]*FresnelS[a + b*x])/(b^3*Pi) + (2*d*(b*c - a*d)*(a + b*x)*Cos[(Pi*(a + b*x
)^2)/2]*FresnelS[a + b*x])/(b^3*Pi) + (2*d^2*(a + b*x)^2*Cos[(Pi*(a + b*x)^
2)/2]*FresnelS[a + b*x])/(3*b^3*Pi) - (d*(b*c - a*d)*FresnelC[a + b*x]*Fres
nelS[a + b*x])/(b^3*Pi) + ((b*c - a*d)^2*(a + b*x)*FresnelS[a + b*x]^2)/b^3
 + (d*(b*c - a*d)*(a + b*x)^2*FresnelS[a + b*x]^2)/b^3 + (d^2*(a + b*x)^3*F
resnelS[a + b*x]^2)/(3*b^3) - ((b*c - a*d)^2*FresnelS[Sqrt[2]*(a + b*x)])/(
Sqrt[2]*b^3*Pi) + ((I/4)*d*(b*c - a*d)*(a + b*x)^2*HypergeometricPFQ[{1, 1}
, {3/2, 2}, (-1/2*I)*Pi*(a + b*x)^2])/(b^3*Pi) - ((I/4)*d*(b*c - a*d)*(a +
b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^3*Pi)
- (4*d^2*FresnelS[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6555

```
Int[FresnelS[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(FresnelS[a
 + b*x]^2/b), x] - Dist[2, Int[(a + b*x)*Sin[(Pi/2)*(a + b*x)^2]*FresnelS[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6567

```
Int[FresnelS[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Dist[1/b^(m + 1), Subst[Int[ExpandIntegrand[FresnelS[x]^2, (b*c - a*d + d*x
)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rule 6587

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x]
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6595

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelS[b*x]/(2*d)), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\int (c+dx)^2S(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(b^2c^2\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)S(x)^2+2bcd\left(1-\frac{ad}{bc}\right)xS(x)^2+d^2x^2S(x)^2\right)\,dx,}{b^3}$$

$$= \frac{d^2\text{Subst}\left(\int x^2S(x)^2\,dx,x,a+bx\right)}{b^3}+\frac{(2d(bc-ad))\text{Subst}\left(\int xS(x)^2\,dx,x,a+bx\right)}{b^3}$$

$$= \frac{(bc-ad)^2(a+bx)S(a+bx)^2}{b^3}+\frac{d(bc-ad)(a+bx)^2S(a+bx)^2}{b^3}+\frac{d^2(a+bx)^3S}{3b^3}$$

$$= \frac{2(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^3\pi}+\frac{2d(bc-ad)(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right.}{b^3\pi}$$

$$= \frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}+\frac{2(bc-ad)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^3\pi}+\frac{2d(bc}{}$$

$$= \frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2}+\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}-\frac{d^2C\left(\sqrt{2}\,(a+bx)\right)}{6\sqrt{2}\,b^3\pi^2}$$

$$= \frac{2d^2x}{3b^2\pi^2}+\frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2}+\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}-\frac{d^2C\left(\sqrt{2}\right.}{6\sqrt{2}}$$

$$= \frac{2d^2x}{3b^2\pi^2}+\frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2}+\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}-\frac{d^2C\left(\sqrt{2}\right.}{6\sqrt{2}}$$

Mathematica [F]
time = 0.44, size = 0, normalized size = 0.00

$$\int (c+dx)^2S(a+bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^2*FresnelS[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^2*FresnelS[a + b*x]^2, x]
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int (dx+c)^2\,\mathrm{S}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*FresnelS(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^2*FresnelS(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnel_sin(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^2*fresnel_sin(b*x + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnel_sin(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*fresnel_sin(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 S^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*fresnels(b*x+a)**2,x)
[Out] Integral((c + d*x)**2*fresnels(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnel_sin(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^2*fresnel_sin(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{FresnelS}(a+b\,x)^2\,(c+d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)^2*(c + d*x)^2,x)
[Out] int(FresnelS(a + b*x)^2*(c + d*x)^2, x)
```

3.50 $\int (c+dx)S(a+bx)^2\,dx$

Optimal. Leaf size=279

$$\frac{d\cos\left(\pi(a+bx)^2\right)}{4b^2\pi^2}+\frac{2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^2\pi}+\frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^2\pi}-\frac{d\text{Fresn}}{}$$

```
[Out] 1/4*d*cos(Pi*(b*x+a)^2)/b^2/Pi^2+2*(-a*d+b*c)*cos(1/2*Pi*(b*x+a)^2)*Fresnel
S(b*x+a)/b^2/Pi+d*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)*FresnelS(b*x+a)/b^2/Pi-1/2*
d*FresnelC(b*x+a)*FresnelS(b*x+a)/b^2/Pi+(-a*d+b*c)*(b*x+a)*FresnelS(b*x+a)
^2/b^2+1/2*d*(b*x+a)^2*FresnelS(b*x+a)^2/b^2+1/8*I*d*(b*x+a)^2*hypergeom([1
, 1],[3/2, 2],-1/2*I*Pi*(b*x+a)^2)/b^2/Pi-1/8*I*d*(b*x+a)^2*hypergeom([1, 1
],[3/2, 2],1/2*I*Pi*(b*x+a)^2)/b^2/Pi-1/2*(-a*d+b*c)*FresnelS((b*x+a)*2^(1/
2))/b^2/Pi*2^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6567, 6555, 6587, 3432, 6565, 6589, 6581, 3460, 2718}

$$\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2}-\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2}+\frac{(a+bx)(bc-ad)S(a+bx)^2}{b^2}-\frac{(bc-ad)S\left(\sqrt{2}\,(a+bx)\right)}{\sqrt{2}\,\pi b^2}+\frac{2(bc-ad)S(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}-\frac{d\text{FresnelC}(a+bx)S(a+bx)}{2\pi b^2}+\frac{d(a+bx)^2S(a+bx)^2}{2b^2}+\frac{d(a+bx)S(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}+\frac{d\cos\left(\pi(a+bx)^2\right)}{4\pi^2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*FresnelS[a + b*x]^2,x]
```

```
[Out] (d*Cos[Pi*(a + b*x)^2])/(4*b^2*Pi^2) + (2*(b*c - a*d)*Cos[(Pi*(a + b*x)^2)/
2]*FresnelS[a + b*x])/(b^2*Pi) + (d*(a + b*x)*Cos[(Pi*(a + b*x)^2)/2]*Fresn
elS[a + b*x])/(b^2*Pi) - (d*FresnelC[a + b*x]*FresnelS[a + b*x])/(2*b^2*Pi)
 + ((b*c - a*d)*(a + b*x)*FresnelS[a + b*x]^2)/b^2 + (d*(a + b*x)^2*Fresnel
S[a + b*x]^2)/(2*b^2) - ((b*c - a*d)*FresnelS[Sqrt[2]*(a + b*x)])/(Sqrt[2]*
b^2*Pi) + ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)
*Pi*(a + b*x)^2])/(b^2*Pi) - ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^2*Pi)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6555

```
Int[FresnelS[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(FresnelS[a
 + b*x]^2/b), x] - Dist[2, Int[(a + b*x)*Sin[(Pi/2)*(a + b*x)^2]*FresnelS[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6565

```
Int[FresnelS[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
S[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Sin[(Pi/2)*b^2*x^
2]*FresnelS[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6567

```
Int[FresnelS[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Dist[1/b^(m + 1), Subst[Int[ExpandIntegrand[FresnelS[x]^2, (b*c - a*d + d*x
)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rule 6587

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x]
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)S(a+bx)^2\,dx &= \frac{\text{Subst}\left(\int \left(bc\left(1-\frac{ad}{bc}\right)S(x)^2+dxS(x)^2\right)\,dx,x,a+bx\right)}{b^2}\\
&= \frac{d\text{Subst}\left(\int xS(x)^2\,dx,x,a+bx\right)}{b^2}+\frac{(bc-ad)\text{Subst}\left(\int S(x)^2\,dx,x,a+bx\right)}{b^2}\\
&= \frac{(bc-ad)(a+bx)S(a+bx)^2}{b^2}+\frac{d(a+bx)^2S(a+bx)^2}{2b^2}-\frac{d\text{Subst}\left(\int x^2S(x)\sin\left(\frac{\pi}{}\right.\right.}{b^2}\\
&= \frac{2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^2\pi}+\frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^2\pi}\\
&= \frac{2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^2\pi}+\frac{d(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^2\pi}\\
&= \frac{d\cos\left(\pi(a+bx)^2\right)}{4b^2\pi^2}+\frac{2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b^2\pi}+\frac{d(a+bx)\cos\left(\frac{1}{2}\pi\right.}{}
\end{aligned}
$$

Mathematica [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int (c+dx)S(a+bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)*FresnelS[a + b*x]^2,x]
[Out] Integrate[(c + d*x)*FresnelS[a + b*x]^2, x]
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int (dx+c)\,\mathrm{S}(bx+a)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*FresnelS(b*x+a)^2,x)
[Out] int((d*x+c)*FresnelS(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnel_sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*fresnel_sin(b*x + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnel_sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*fresnel_sin(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) S^2(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnels(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*fresnels(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnel_sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*fresnel_sin(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{FresnelS}(a + b\,x)^2\,(c + d\,x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)^2*(c + d*x),x)
```

```
[Out] int(FresnelS(a + b*x)^2*(c + d*x), x)
```

3.51 $\int S(a+bx)^2\,dx$

Optimal. Leaf size=70

$$\frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b\pi}+\frac{(a+bx)S(a+bx)^2}{b}-\frac{S\left(\sqrt{2}\,(a+bx)\right)}{\sqrt{2}\,b\pi}$$

```
[Out] 2*cos(1/2*Pi*(b*x+a)^2)*FresnelS(b*x+a)/b/Pi+(b*x+a)*FresnelS(b*x+a)^2/b-1/
2*FresnelS((b*x+a)*2^(1/2))/b/Pi*2^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6555, 6587, 3432}

$$\frac{(a+bx)S(a+bx)^2}{b}-\frac{S\left(\sqrt{2}\,(a+bx)\right)}{\sqrt{2}\,\pi b}+\frac{2S(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[a + b*x]^2,x]
```

```
[Out] (2*Cos[(Pi*(a + b*x)^2)/2]*FresnelS[a + b*x])/(b*Pi) + ((a + b*x)*FresnelS[
a + b*x]^2)/b - FresnelS[Sqrt[2]*(a + b*x)]/(Sqrt[2]*b*Pi)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 6555

```
Int[FresnelS[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(FresnelS[a
 + b*x]^2/b), x] - Dist[2, Int[(a + b*x)*Sin[(Pi/2)*(a + b*x)^2]*FresnelS[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6587

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x]
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\int S(a+bx)^2\,dx &= \frac{(a+bx)S(a+bx)^2}{b} - 2\int (a+bx)S(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx\\ &= \frac{(a+bx)S(a+bx)^2}{b} - \frac{2\text{Subst}\left(\int xS(x)\sin\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{b}\\ &= \frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b\pi} + \frac{(a+bx)S(a+bx)^2}{b} - \frac{\text{Subst}\left(\int \sin\left(\pi x^2\right)\,dx,x,a+bx\right)}{b\pi}\\ &= \frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)}{b\pi} + \frac{(a+bx)S(a+bx)^2}{b} - \frac{S\left(\sqrt{2}\,(a+bx)\right)}{\sqrt{2}\,b\pi}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 67, normalized size = 0.96

$$\frac{4\cos\left(\frac{1}{2}\pi(a+bx)^2\right)S(a+bx)+2\pi(a+bx)S(a+bx)^2-\sqrt{2}\,S\left(\sqrt{2}\,(a+bx)\right)}{2b\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[a + b*x]^2,x]
```

```
[Out] (4*Cos[(Pi*(a + b*x)^2)/2]*FresnelS[a + b*x] + 2*Pi*(a + b*x)*FresnelS[a +
b*x]^2 - Sqrt[2]*FresnelS[Sqrt[2]*(a + b*x)])/(2*b*Pi)
```

Maple [A]
time = 0.52, size = 60, normalized size = 0.86

method	result	size
derivativedivides	$\frac{\mathrm{S}(bx+a)^2(bx+a)+\frac{2\,\mathrm{S}(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{\sqrt{2}\,\mathrm{S}\left((bx+a)\sqrt{2}\right)}{2\pi}}{b}$	60
default	$\frac{\mathrm{S}(bx+a)^2(bx+a)+\frac{2\,\mathrm{S}(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{\sqrt{2}\,\mathrm{S}\left((bx+a)\sqrt{2}\right)}{2\pi}}{b}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(FresnelS(b*x+a)^2*(b*x+a)+2*FresnelS(b*x+a)/Pi*cos(1/2*Pi*(b*x+a)^2)-1
/2/Pi*2^(1/2)*FresnelS((b*x+a)*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x + a)^2, x)
```

Fricas [A]
time = 0.39, size = 89, normalized size = 1.27

$$\frac{4\,b\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)\mathrm{S}\left(bx+a\right)+2\left(\pi b^2x+\pi ab\right)\mathrm{S}\left(bx+a\right)^2-\sqrt{2}\,\sqrt{b^2}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{b^2}\,(bx+a)}{b}\right)}{2\,\pi b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)^2,x, algorithm="fricas")
[Out] 1/2*(4*b*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2)*fresnel_sin(b*x + a) +
 2*(pi*b^2*x + pi*a*b)*fresnel_sin(b*x + a)^2 - sqrt(2)*sqrt(b^2)*fresnel_s
in(sqrt(2)*sqrt(b^2)*(b*x + a)/b))/(pi*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int S^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)**2,x)
[Out] Integral(fresnels(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)^2,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{FresnelS}(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)^2,x)
[Out] int(FresnelS(a + b*x)^2, x)
```

3.52 $\int \frac{S(a+bx)^2}{c+dx} dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{S(a+bx)^2}{c+dx},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x+a)^2/(d*x+c),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(a+bx)^2}{c+dx} \, dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelS[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][FresnelS[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{S(a+bx)^2}{c+dx} \, dx = \int \frac{S(a+bx)^2}{c+dx} \, dx$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{S(a+bx)^2}{c+dx} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelS[a + b*x]^2/(c + d*x),x]
```

```
[Out] Integrate[FresnelS[a + b*x]^2/(c + d*x), x]
```

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx+a)^2}{dx+c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(FresnelS(b*x+a)^2/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x + a)^2/(d*x + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(fresnel_sin(b*x + a)^2/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(fresnels(a + b*x)**2/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x + a)^2/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\mathrm{FresnelS}(a+b\,x)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)^2/(c + d*x),x)
[Out] int(FresnelS(a + b*x)^2/(c + d*x), x)
```

3.53 $\int \frac{S(a+bx)^2}{(c+dx)^2} dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{S(a+bx)^2}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x+a)^2/(d*x+c)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(a+bx)^2}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelS[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][FresnelS[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{S(a+bx)^2}{(c+dx)^2}\,dx = \int \frac{S(a+bx)^2}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{S(a+bx)^2}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelS[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Integrate[FresnelS[a + b*x]^2/(c + d*x)^2, x]
```

Maple [A]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(FresnelS(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x + a)^2/(d*x + c)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(fresnel_sin(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(fresnels(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\mathrm{FresnelS}(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(a + b*x)^2/(c + d*x)^2,x)
[Out] int(FresnelS(a + b*x)^2/(c + d*x)^2, x)
```

3.54 $\int x^2 S(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=231

$$\left(\frac{1}{12}-\frac{i}{12}\right)e^{-\frac{3a}{bn}+\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{-3/n}\,\mathrm{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)+\left(\frac{1}{12}-\frac{i}{12}\right)e^{-\frac{3a}{bn}-\frac{}{2b^2}}$$

```
[Out] (1/12-1/12*I)*exp(-3*a/b/n+9/2*I/b^2/d^2/n^2/Pi)*x^3*erf((1/2+1/2*I)*(3/n+I
*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(3/n))+(1/12-1/1
2*I)*exp(-3*a/b/n-9/2*I/b^2/d^2/n^2/Pi)*x^3*erfi((1/2+1/2*I)*(3/n-I*a*b*d^2
*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(3/n))+1/3*x^3*FresnelS(
d*(a+b*ln(c*x^n)))
```

Rubi [A]
time = 0.28, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$,
Rules used = {6606, 4713, 2314, 2308, 2266, 2235, 2236}

$$\left(\frac{1}{12}-\frac{i}{12}\right)x^3(cx^n)^{-3/n}e^{-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}}\mathrm{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{3}{n}\right)}{\sqrt{\pi}\,bd}\right)+\left(\frac{1}{12}-\frac{i}{12}\right)x^3(cx^n)^{-3/n}e^{-\frac{3a}{bn}-\frac{9i}{2\pi b^2d^2n^2}}\mathrm{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{3}{n}\right)}{\sqrt{\pi}\,bd}\right)+\frac{1}{3}x^3S(d(a+b\log(cx^n)))$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/12 - I/12)*E^((-3*a)/(b*n) + ((9*I)/2)/(b^2*d^2*n^2*Pi))*x^3*Erf[((1/2
+ I/2)*(3/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*
x^n)^(3/n) + ((1/12 - I/12)*E^((-3*a)/(b*n) - ((9*I)/2)/(b^2*d^2*n^2*Pi))*x
^3*Erfi[((1/2 + I/2)*(3/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*S
qrt[Pi])])/(c*x^n)^(3/n) + (x^3*FresnelS[d*(a + b*Log[c*x^n])])/3
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2314

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F])), Int[(d + e*x)^(m + 2*a*b*
f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a,
b, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 4713

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - D
ist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6606

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelS[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Dist[b*d*(n/(m + 1)), Int[(e*x)^m*Sin[(Pi/2)*(d*(a + b*Log[c*x^n
]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2 S(d(a+b\log(cx^n)))\,dx = \frac{1}{3}x^3 S(d(a+b\log(cx^n))) - \frac{1}{3}(bdn)\int x^2 \sin\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)\,dx$$
$$= \frac{1}{3}x^3 S(d(a+b\log(cx^n))) - \frac{1}{6}(ibdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}x^2\,dx + \frac{1}{6}(ibdn)$$
$$= \frac{1}{3}x^3 S(d(a+b\log(cx^n))) - \frac{1}{6}(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log(cx^n) -\right.$$
$$= \frac{1}{3}x^3 S(d(a+b\log(cx^n))) - \frac{1}{6}(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2(cx^n\right.$$
$$= \frac{1}{3}x^3 S(d(a+b\log(cx^n))) - \frac{1}{6}\left(ibdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi\right.$$
$$= \frac{1}{3}x^3 S(d(a+b\log(cx^n))) - \frac{1}{6}\left(ibdx^3(cx^n)^{-iabd^2\pi-\frac{3-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(\right.\right.$$
$$= \frac{1}{3}x^3 S(d(a+b\log(cx^n))) - \frac{1}{6}\left(ibde^{-\frac{3a}{bn}-\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{-iabd^2\pi-\frac{3-iabd^2n\pi}{n}}\right)\text{Su}$$
$$= \left(\frac{1}{12}-\frac{i}{12}\right)e^{-\frac{3a}{bn}+\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{-3/n}\,\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{n}+iabd^2\pi+ib^2d^2\pi\log\right.}{bd\sqrt{\pi}}\right.$$

Mathematica [A]
time = 4.49, size = 319, normalized size = 1.38

$$\frac{1}{12}x^3\left(4S(d(a+b\log(cx^n)))+\sqrt[4]{-1}\sqrt{2}\,e^{\frac{1}{2}\left(-\frac{6a}{bn}-\frac{9i}{b^2d^2n^2\pi}-ia^2d^2\pi+2iabd^2\pi(n\log(x)-\log(cx^n))-ib^2d^2\pi(-n\log(x)+\log(cx^n))^2\right)}(cx^n)^{-3/n}\left(e^{\frac{9i}{b^2d^2n^2\pi}}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-3i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)+i\text{Erfi}\left(\frac{(-1)^{3/4}(3i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)\right)\left(\cos\left(\frac{1}{2}d^2\pi(a-bn\log(x)+b\log(cx^n))^2\right)+i\sin\left(\frac{1}{2}d^2\pi(a-bn\log(x)+b\log(cx^n))^2\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^3*(4*FresnelS[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*Sqrt[2]*E^(((-6*a)/(b*
n) - (9*I)/(b^2*d^2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - L
og[c*x^n]) - I*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*(E^((9*I)/(b^2*d
^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-3*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n
]))/(b*d*n*Sqrt[Pi])] + I*Erfi[((-1)^(3/4)*(3*I + a*b*d^2*n*Pi + b^2*d^2*n*
Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c
*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(c*x^n
)^(3/n)))/12
```

Maple [F]
time = 0.48, size = 0, normalized size = 0.00

$$\int x^2 \mathrm{S}(d(a+b\ln(c\,x^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^2*FresnelS(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnel_sin((b*log(c*x^n) + a)*d), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(187) = 374.
time = 0.38, size = 448, normalized size = 1.94

$$\frac{1}{3}x^3\,\mathrm{S}\left(bd\log\left(cx^n\right)+ad\right)-\frac{1}{6}i\pi\sqrt{b^2d^2n^2}\,e^{\left(-\frac{3\log(c)}{n}-\frac{3a}{bn}-\frac{9i}{2\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+3i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+\frac{1}{6}i\pi\sqrt{b^2d^2n^2}\,e^{\left(-\frac{3\log(c)}{n}-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-3i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)-\frac{1}{6}\pi\sqrt{b^2d^2n^2}\,e^{\left(-\frac{3\log(c)}{n}-\frac{3a}{bn}-\frac{9i}{2\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n+3i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)-\frac{1}{6}\pi\sqrt{b^2d^2n^2}\,e^{\left(-\frac{3\log(c)}{n}-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{\left(\pi b^2d^2n^2\log\left(x\right)+\pi b^2d^2n\log\left(c\right)+\pi abd^2n-3i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*fresnel_sin(b*d*log(c*x^n) + a*d) - 1/6*I*pi*sqrt(b^2*d^2*n^2)*e^(-
3*log(c)/n - 3*a/(b*n) - 9/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^
2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + 3*I)*sqrt(b^2*d^2*n^2)/(pi*
b^2*d^2*n^2)) + 1/6*I*pi*sqrt(b^2*d^2*n^2)*e^(-3*log(c)/n - 3*a/(b*n) + 9/2
*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(
c) + pi*a*b*d^2*n - 3*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/6*pi*sqrt(
b^2*d^2*n^2)*e^(-3*log(c)/n - 3*a/(b*n) - 9/2*I/(pi*b^2*d^2*n^2))*fresnel_s
in((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + 3*I)*sqrt(
b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/6*pi*sqrt(b^2*d^2*n^2)*e^(-3*log(c)/n -
3*a/(b*n) + 9/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi
*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 3*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}S(ad+bd\log\left(cx^{n}\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnels(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x**2*fresnels(a*d + b*d*log(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnel_sin((b*log(c*x^n) + a)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{FresnelS}(d\,(a+b\,\ln\left(c\,x^n\right)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(d*(a + b*log(c*x^n))),x)
```

```
[Out] int(x^2*FresnelS(d*(a + b*log(c*x^n))), x)
```

3.55 $\int xS(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=227

$$\left(\frac{1}{8}-\frac{i}{8}\right)e^{\frac{2i-2abd^2n\pi}{b^2d^2n^2\pi}}x^2(cx^n)^{-2/n}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)+\left(\frac{1}{8}-\frac{i}{8}\right)e^{-\frac{2\left(i+abd^2n\pi\right)}{b^2d^2n^2\pi}}x^2(cx$$

```
[Out] (1/8-1/8*I)*exp((2*I-2*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*x^2*erf((1/2+1/2*I)*(2
/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(2/n))+(1/8-
1/8*I)*x^2*erfi((1/2+1/2*I)*(2/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/P
i^(1/2))/exp(2*(I+a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/((c*x^n)^(2/n))+1/2*x^2*Fre
snelS(d*(a+b*ln(c*x^n)))
```

Rubi [A]
time = 0.25, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {6606, 4713, 2314, 2308, 2266, 2235, 2236}

$$\left(\frac{1}{8}-\frac{i}{8}\right)x^2(cx^n)^{-2/n}e^{\frac{-2\pi abd^2n+2i}{\pi b^2d^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}\,bd}\right)+\left(\frac{1}{8}-\frac{i}{8}\right)x^2(cx^n)^{-2/n}e^{-\frac{2\left(\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}\,bd}\right)+\frac{1}{2}x^2S(d(a+b\log(cx^n)))$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/8 - I/8)*E^((2*I - 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*x^2*Erf[((1/2 + I/
2)*(2/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*x^n)
^(2/n) + ((1/8 - I/8)*x^2*Erfi[((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2
*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*(I + a*b*d^2*n*Pi))/(b^2*d^2*n^2*P
i))*(c*x^n)^(2/n)) + (x^2*FresnelS[d*(a + b*Log[c*x^n])])/2
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2314

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F])), Int[(d + e*x)^(m + 2*a*b*
f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a,
b, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 4713

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - D
ist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6606

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelS[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Dist[b*d*(n/(m + 1)), Int[(e*x)^m*Sin[(Pi/2)*(d*(a + b*Log[c*x^n
]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int xS(d(a+b\log(cx^n)))\,dx &= \frac{1}{2}x^2S(d(a+b\log(cx^n))) - \frac{1}{2}(bdn)\int x\sin\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)dx \\
&= \frac{1}{2}x^2S(d(a+b\log(cx^n))) - \frac{1}{4}(ibdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}x\,dx + \frac{1}{4}(ibdn)\int e \\
&= \frac{1}{2}x^2S(d(a+b\log(cx^n))) - \frac{1}{4}(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log(cx^n) - \right. \\
&= \frac{1}{2}x^2S(d(a+b\log(cx^n))) - \frac{1}{4}(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2(cx^n)\right) \\
&= \frac{1}{2}x^2S(d(a+b\log(cx^n))) - \frac{1}{4}\left(ibdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \right. \\
&= \frac{1}{2}x^2S(d(a+b\log(cx^n))) - \frac{1}{4}\left(ibdx^2(cx^n)^{-iabd^2\pi-\frac{2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(-\right.\right. \\
&= \frac{1}{2}x^2S(d(a+b\log(cx^n))) - \frac{1}{4}\left(ibde^{-\frac{2\left(i+abd^2n\pi\right)}{b^2d^2n^2\pi}}x^2(cx^n)^{-iabd^2\pi-\frac{2-iabd^2n\pi}{n}}\right)\text{Subst} \\
&= \left(\frac{1}{8}-\frac{i}{8}\right)e^{\frac{2i-2abd^2n\pi}{b^2d^2n^2\pi}}x^2(cx^n)^{-2/n}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right.
\end{aligned}
$$

Mathematica [A]
time = 4.38, size = 319, normalized size = 1.41

$$\frac{1}{8}x^2\left(4S(d(a+b\log(cx^n)))+\sqrt[4]{-1}\sqrt{2}e^{-\frac{2a}{bn}-\frac{2i}{b^2d^2n^2\pi}-\frac{1}{2}ia^2d^2\pi+iabd^2\pi(n\log(x)-\log(cx^n))-\frac{1}{2}ib^2d^2\pi(-n\log(x)+\log(cx^n))^2}(cx^n)^{-2/n}\left(e^{\frac{4i}{b^2d^2n^2\pi}}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-2i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)+i\text{Erfi}\left(\frac{(-1)^{3/4}(2i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)\right)\left(\cos\left(\frac{1}{2}d^2\pi(a-bn\log(x)+b\log(cx^n))^2\right)+i\sin\left(\frac{1}{2}d^2\pi(a-bn\log(x)+b\log(cx^n))^2\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^2*(4*FresnelS[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*Sqrt[2]*E^((-2*a)/(b*n
) - (2*I)/(b^2*d^2*n^2*Pi) - (I/2)*a^2*d^2*Pi + I*a*b*d^2*Pi*(n*Log[x] - Lo
g[c*x^n]) - (I/2)*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)*(E^((4*I)/(b^2*d
^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n
]))/(b*d*n*Sqrt[Pi])] + I*Erfi[((-1)^(3/4)*(2*I + a*b*d^2*n*Pi + b^2*d^2*n*
Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c
*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(c*x^n
)^(2/n)))/8
```

Maple [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int x\,\mathrm{S}(d(a+b\ln(c\,x^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x*FresnelS(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(x*fresnel_sin((b*log(c*x^n) + a)*d), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(187) = 374$.
time = 0.37, size = 448, normalized size = 1.97

$$-\frac{1}{4} i \pi \sqrt{b^2 d^2 n^2}\, e^{\left(-\frac{2 \log(c)}{n} - \frac{2a}{bn} - \frac{2i}{\pi b^2 d^2 n^2}\right)} \mathrm{C}\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + 2i)\sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) + \frac{1}{4} i \pi \sqrt{b^2 d^2 n^2}\, e^{\left(-\frac{2 \log(c)}{n} - \frac{2a}{bn} + \frac{2i}{\pi b^2 d^2 n^2}\right)} \mathrm{C}\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - 2i)\sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) - \frac{1}{4} \pi \sqrt{b^2 d^2 n^2}\, e^{\left(-\frac{2 \log(c)}{n} - \frac{2a}{bn} - \frac{2i}{\pi b^2 d^2 n^2}\right)} \mathrm{S}\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + 2i)\sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) - \frac{1}{4} \pi \sqrt{b^2 d^2 n^2}\, e^{\left(-\frac{2 \log(c)}{n} - \frac{2a}{bn} + \frac{2i}{\pi b^2 d^2 n^2}\right)} \mathrm{S}\left(\frac{(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - 2i)\sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) + \frac{1}{2} x^2 \mathrm{S}\left(bd \log(cx^n) + ad\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] -1/4*I*pi*sqrt(b^2*d^2*n^2)*e^(-2*log(c)/n - 2*a/(b*n) - 2*I/(pi*b^2*d^2*n^
2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n
 + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + 1/4*I*pi*sqrt(b^2*d^2*n^2)*e^
(-2*log(c)/n - 2*a/(b*n) + 2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^
2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*sqrt(b^2*d^2*n^2)/(pi*
b^2*d^2*n^2)) - 1/4*pi*sqrt(b^2*d^2*n^2)*e^(-2*log(c)/n - 2*a/(b*n) - 2*I/(
pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) +
 pi*a*b*d^2*n + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/4*pi*sqrt(b^2*
d^2*n^2)*e^(-2*log(c)/n - 2*a/(b*n) + 2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi
*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*sqrt(b^2*d^
2*n^2)/(pi*b^2*d^2*n^2)) + 1/2*x^2*fresnel_sin(b*d*log(c*x^n) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x S\left(ad + bd \log(cx^n)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x*fresnels(a*d + b*d*log(c*x**n)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(x*fresnel_sin((b*log(c*x^n) + a)*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{FresnelS}(d\,(a+b\,\ln{(c\,x^n)}))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(d*(a + b*log(c*x^n))),x)
[Out] int(x*FresnelS(d*(a + b*log(c*x^n))), x)
```

3.56 $\int S(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=214

$$\left(\frac{1}{4}-\frac{i}{4}\right)e^{-\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}x(cx^n)^{-1/n}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)+\left(\frac{1}{4}-\frac{i}{4}\right)e^{-\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}x(cx^n$$

```
[Out] (1/4-1/4*I)*x*erf((1/2+1/2*I)*(1/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d
/Pi^(1/2))/exp(1/2*(2*a*b*n-I/d^2/Pi)/b^2/n^2)/((c*x^n)^(1/n))+(1/4-1/4*I)*
x*erfi((1/2+1/2*I)*(1/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/
exp(1/2*(2*a*b*n+I/d^2/Pi)/b^2/n^2)/((c*x^n)^(1/n))+x*FresnelS(d*(a+b*ln(c*
x^n)))
```

Rubi [A]

time = 0.22, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {6603, 4711, 2312, 2308, 2266, 2235, 2236}

$$\left(\frac{1}{4}-\frac{i}{4}\right)x(cx^n)^{-1/n}e^{-\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}\,bd}\right)+\left(\frac{1}{4}-\frac{i}{4}\right)x(cx^n)^{-1/n}e^{-\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}\,bd}\right)+xS(d(a+b\log(cx^n)))$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/4 - I/4)*x*Erf[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c
*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^
(-1)) + ((1/4 - I/4)*x*Erfi[((1/2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2
*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(
c*x^n)^n^(-1)) + x*FresnelS[d*(a + b*Log[c*x^n])]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2312

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.)), x
_Symbol] :> Dist[(c*(d + e*x)^n)^(2*a*b*f*Log[F])/(d + e*x)^(2*a*b*f*n*Log[
F]), Int[(d + e*x)^(2*a*b*f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2
), x], x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] &&  !IntegerQ[2*a*b*f*Log[F
]]
```

Rule 4711

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)], x_Symbol] :> Dist[I
/2, Int[E^((-I)*d*(a + b*Log[c*x^n])^2), x], x] - Dist[I/2, Int[E^(I*d*(a +
 b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Rule 6603

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Sim
p[x*FresnelS[d*(a + b*Log[c*x^n])], x] - Dist[b*d*n, Int[Sin[(Pi/2)*(d*(a +
 b*Log[c*x^n]))^2], x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Rubi steps

$$\int S(d(a+b\log(cx^n)))\,dx = xS(d(a+b\log(cx^n))) - (bdn)\int \sin\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)dx$$

$$= xS(d(a+b\log(cx^n))) - \frac{1}{2}(ibdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}\,dx + \frac{1}{2}(ibdn)\int e^{\frac{1}{2}id^2\pi$$

$$= xS(d(a+b\log(cx^n))) - \frac{1}{2}(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log(cx^n) - \frac{1}{2}ib$$

$$= xS(d(a+b\log(cx^n))) - \frac{1}{2}(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2(cx^n)\right)(c$$

$$= xS(d(a+b\log(cx^n))) - \frac{1}{2}\left(ibdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}$$

$$= xS(d(a+b\log(cx^n))) - \frac{1}{2}\left(ibdx(cx^n)^{-iabd^2\pi-\frac{1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(-\frac{1}{2}ia$$

$$= xS(d(a+b\log(cx^n))) - \frac{1}{2}\left(ibde^{-\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}x(cx^n)^{-iabd^2\pi-\frac{1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int e$$

$$= \left(\frac{1}{4}-\frac{i}{4}\right)e^{-\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}x(cx^n)^{-1/n}\,\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}$$

Mathematica [A]
time = 4.35, size = 316, normalized size = 1.48

$$xS(d(a+b\log(cx^n))) + \frac{\sqrt[4]{-1}\,e^{\frac{1}{2}\left(-\frac{2a}{bn}-\frac{i}{b^2d^2n^2\pi}-ia^2d^2\pi+2iabd^2\pi(n\log(x)-\log(cx^n))-ib^2d^2\pi(-n\log(x)+\log(cx^n))^2\right)}x(cx^n)^{-1/n}\left(e^{\frac{i}{b^2d^2n^2\pi}}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)+i\text{Erfi}\left(\frac{(-1)^{3/4}(i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)\right)\left(\cos\left(\frac{1}{2}d^2\pi(a-bn\log(x)+b\log(cx^n))^2\right)+i\sin\left(\frac{1}{2}d^2\pi(a-bn\log(x)+b\log(cx^n))^2\right)\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x*FresnelS[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*E^(((-2*a)/(b*n) - I/(b^2*d^
2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - Log[c*x^n]) - I*b^2
*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*x*(E^(I/(b^2*d^2*n^2*Pi))*Erfi[((1
/2 + I/2)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])]
+ I*Erfi[((-1)^(3/4)*(I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*S
qrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2] + I*Sin[(d^
2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(2*Sqrt[2]*(c*x^n)^n^(-1))
```

Maple [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int \mathrm{S}(d(a+b\ln(c\,x^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(FresnelS(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin((b*log(c*x^n) + a)*d), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 445 vs. 2(176) = 352.
time = 0.38, size = 445, normalized size = 2.08

$$-\frac{1}{2} i \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{\log(c)}{n} - \frac{a}{bn} - \frac{i}{2 \pi b^2 d^2 n^2}\right)} \mathrm{C}\left(\frac{\left(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + i\right) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) + \frac{1}{2} i \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{\log(c)}{n} - \frac{a}{bn} + \frac{i}{2 \pi b^2 d^2 n^2}\right)} \mathrm{C}\left(\frac{\left(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - i\right) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) - \frac{1}{2} \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{\log(c)}{n} - \frac{a}{bn} - \frac{i}{2 \pi b^2 d^2 n^2}\right)} \mathrm{S}\left(\frac{\left(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n + i\right) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) - \frac{1}{2} \pi \sqrt{b^2 d^2 n^2} e^{\left(-\frac{\log(c)}{n} - \frac{a}{bn} + \frac{i}{2 \pi b^2 d^2 n^2}\right)} \mathrm{S}\left(\frac{\left(\pi b^2 d^2 n^2 \log(x) + \pi b^2 d^2 n \log(c) + \pi a b d^2 n - i\right) \sqrt{b^2 d^2 n^2}}{\pi b^2 d^2 n^2}\right) + x \mathrm{S}\left(b d \log(c x^n) + a d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] -1/2*I*pi*sqrt(b^2*d^2*n^2)*e^(-log(c)/n - a/(b*n) - 1/2*I/(pi*b^2*d^2*n^2)
)*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n +
 I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + 1/2*I*pi*sqrt(b^2*d^2*n^2)*e^(-lo
g(c)/n - a/(b*n) + 1/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(
x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*
n^2)) - 1/2*pi*sqrt(b^2*d^2*n^2)*e^(-log(c)/n - a/(b*n) - 1/2*I/(pi*b^2*d^2
*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^
2*n + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/2*pi*sqrt(b^2*d^2*n^2)*e^(
-log(c)/n - a/(b*n) + 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*l
og(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d
^2*n^2)) + x*fresnel_sin(b*d*log(c*x^n) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int S(d(a + b \log(cx^n)))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(fresnels(d*(a + b*log(c*x**n))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(fresnel_sin((b*log(c*x^n) + a)*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{FresnelS}(d\,(a+b\,\ln\left(c\,x^n\right)))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a + b*log(c*x^n))),x)
[Out] int(FresnelS(d*(a + b*log(c*x^n))), x)
```

3.57 $\int \frac{S(d(a+b\log(cx^n)))}{x}\,dx$

Optimal. Leaf size=65

$$\frac{\cos\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)}{bdn\pi}+\frac{S(d(a+b\log(cx^n)))\,(a+b\log(cx^n))}{bn}$$

```
[Out] cos(1/2*d^2*Pi*(a+b*ln(c*x^n))^2)/b/d/n/Pi+FresnelS(d*(a+b*ln(c*x^n)))*(a+b
*ln(c*x^n))/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {6553}

$$\frac{\cos\left(\frac{1}{2}\pi d^2(a+b\log(cx^n))^2\right)}{\pi bdn}+\frac{(a+b\log(cx^n))\,S(d(a+b\log(cx^n)))}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] Cos[(d^2*Pi*(a + b*Log[c*x^n])^2)/2]/(b*d*n*Pi) + (FresnelS[d*(a + b*Log[c*
x^n])]*(a + b*Log[c*x^n]))/(b*n)
```

Rule 6553

```
Int[FresnelS[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelS[a +
 b*x]/b), x] + Simp[Cos[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{S(d(a+b\log(cx^n)))}{x}\,dx &= \frac{\text{Subst}\left(\int S(d(a+bx))\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{\text{Subst}\left(\int S(x)\,dx, x, ad+bd\log(cx^n)\right)}{bdn}\\ &= \frac{\cos\left(\frac{1}{2}\pi(ad+bd\log(cx^n))^2\right)}{bdn\pi}+\frac{S(ad+bd\log(cx^n))\,(a+b\log(cx^n))}{bn}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 164 vs. 2(65) = 130.
time = 0.07, size = 164, normalized size = 2.52

$$\frac{\cos\left(\frac{1}{2}a^2d^2\pi\right)\cos\left(abd^2\pi\log(cx^n)+\frac{1}{2}b^2d^2\pi\log^2(cx^n)\right)}{bdn\pi}+\frac{aS(d(a+b\log(cx^n)))}{bn}+\frac{S(d(a+b\log(cx^n)))\log(cx^n)}{n}-\frac{\sin\left(\frac{1}{2}a^2d^2\pi\right)\sin\left(abd^2\pi\log(cx^n)+\frac{1}{2}b^2d^2\pi\log^2(cx^n)\right)}{bdn\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (Cos[(a^2*d^2*Pi)/2]*Cos[a*b*d^2*Pi*Log[c*x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/
2])/(b*d*n*Pi) + (a*FresnelS[d*(a + b*Log[c*x^n])])/(b*n) + (FresnelS[d*(a
+ b*Log[c*x^n])]*Log[c*x^n])/n - (Sin[(a^2*d^2*Pi)/2]*Sin[a*b*d^2*Pi*Log[c*
x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/2])/(b*d*n*Pi)
```

Maple [A]
time = 1.44, size = 63, normalized size = 0.97

method	result	size
derivativedivides	$\frac{\mathrm{S}(ad+bd\ln(c\,x^n))(ad+bd\ln(c\,x^n))+\frac{\cos\left(\frac{\pi(ad+bd\ln(c\,x^n))^2}{2}\right)}{\pi}}{nbd}$	63
default	$\frac{\mathrm{S}(ad+bd\ln(c\,x^n))(ad+bd\ln(c\,x^n))+\frac{\cos\left(\frac{\pi(ad+bd\ln(c\,x^n))^2}{2}\right)}{\pi}}{nbd}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a+b*ln(c*x^n)))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b/d*(FresnelS(a*d+b*d*ln(c*x^n))*(a*d+b*d*ln(c*x^n))+1/Pi*cos(1/2*Pi*(a
*d+b*d*ln(c*x^n))^2))
```

Maxima [A]
time = 0.26, size = 81, normalized size = 1.25

$$\frac{(b\log(cx^n)+a)d\,\mathrm{S}\left((b\log(cx^n)+a)d\right)+\frac{\cos\left(\frac{1}{2}\pi b^2d^2\log(cx^n)^2+\pi abd^2\log(cx^n)+\frac{1}{2}\pi a^2d^2\right)}{\pi}}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

```
[Out] ((b*log(c*x^n) + a)*d*fresnel_sin((b*log(c*x^n) + a)*d) + cos(1/2*pi*b^2*d^
2*log(c*x^n)^2 + pi*a*b*d^2*log(c*x^n) + 1/2*pi*a^2*d^2)/pi)/(b*d*n)
```

Fricas [A]
time = 0.37, size = 119, normalized size = 1.83

$$\frac{(\pi bdn\log(x)+\pi bd\log(c)+\pi ad)\,\mathrm{S}\left(bd\log(cx^n)+ad\right)+\cos\left(\frac{1}{2}\pi b^2d^2n^2\log(x)^2+\pi b^2d^2n\log(c)\log(x)+\frac{1}{2}\pi b^2d^2\log(c)^2+\pi abd^2n\log(x)+\pi abd^2\log(c)+\frac{1}{2}\pi a^2d^2\right)}{\pi bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

```
[Out] ((pi*b*d*n*log(x) + pi*b*d*log(c) + pi*a*d)*fresnel_sin(b*d*log(c*x^n) + a*
d) + cos(1/2*pi*b^2*d^2*n^2*log(x)^2 + pi*b^2*d^2*n*log(c)*log(x) + 1/2*pi*
```

```
b^2*d^2*log(c)^2 + pi*a*b*d^2*n*log(x) + pi*a*b*d^2*log(c) + 1/2*pi*a^2*d^2
))/(pi*b*d*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S(ad + bd \log{(cx^n)})}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*ln(c*x**n)))/x,x)
[Out] Integral(fresnels(a*d + b*d*log(c*x**n))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
[Out] integrate(fresnel_sin((b*log(c*x^n) + a)*d)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{FresnelS}(d\,(a + b\,\ln{(c\,x^n)}))}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a + b*log(c*x^n)))/x,x)
[Out] int(FresnelS(d*(a + b*log(c*x^n)))/x, x)
```

3.58 $\int \frac{S(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal. Leaf size=217

$$\frac{\left(\frac{1}{4}-\frac{i}{4}\right)e^{\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)e^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+i\right.}{bd\sqrt{\pi}}\right.}{x}$$

```
[Out] (1/4-1/4*I)*exp(1/2*(2*a*b*n+I/d^2/Pi)/b^2/n^2)*(c*x^n)^(1/n)*erf((1/2+1/2*
I)*(1/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x+(1/4-1/4*I)*ex
p(1/2*(2*a*b*n-I/d^2/Pi)/b^2/n^2)*(c*x^n)^(1/n)*erfi((1/2+1/2*I)*(1/n+I*a*b
*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x-FresnelS(d*(a+b*ln(c*x^n)))
/x
```

Rubi [A]
time = 0.26, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6606, 4713, 2314, 2308, 2266, 2235, 2236}

$$\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(cx^n)^{\frac{1}{n}}e^{\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}\,bd}\right)}{x}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(cx^n)^{\frac{1}{n}}e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}\,bd}\right)}{x}-\frac{S(d(a+b\log(cx^n)))}{x}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] ((1/4 - I/4)*E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*Erf[((1/
2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])]
)/x + ((1/4 - I/4)*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*Er
fi[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqr
t[Pi])])/x - FresnelS[d*(a + b*Log[c*x^n])]/x
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2314

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F])), Int[(d + e*x)^(m + 2*a*b*
f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a,
b, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 4713

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - D
ist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6606

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelS[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Dist[b*d*(n/(m + 1)), Int[(e*x)^m*Sin[(Pi/2)*(d*(a + b*Log[c*x^n
]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{S(d(a+b\log(cx^n)))}{x^2}\,dx = -\frac{S(d(a+b\log(cx^n)))}{x} + (bdn)\int \frac{\sin\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)}{x^2}\,dx$$

$$= -\frac{S(d(a+b\log(cx^n)))}{x} + \frac{1}{2}(ibdn)\int \frac{e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^2}\,dx - \frac{1}{2}(ibdn)\int \frac{e^{\frac{1}{2}i\ldots}}{\ldots}$$

$$= -\frac{S(d(a+b\log(cx^n)))}{x} + \frac{1}{2}(ibdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log(cx^n) - \frac{1}{2}ib\ldots\right.}{x^2}$$

$$= -\frac{S(d(a+b\log(cx^n)))}{x} + \frac{1}{2}(ibdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2(cx^n)\right)(c\ldots}{x^2}$$

$$= -\frac{S(d(a+b\log(cx^n)))}{x} + \frac{1}{2}\left(ibdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}\ldots\right.$$

$$= -\frac{S(d(a+b\log(cx^n)))}{x} + \frac{\left(ibd(cx^n)^{-iabd^2\pi-\frac{-1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{1}{2}ia^2d\ldots\right.\right.}{2x\ldots}$$

$$= -\frac{S(d(a+b\log(cx^n)))}{x} + \frac{\left(ibde^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{-iabd^2\pi-\frac{-1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\ldots\right.}{2x}$$

$$= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)e^{\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} + \frac{\left(\frac{1}{4}-\frac{i}{4}\right)e^{\frac{2abn-\ldots}{2b^2n^2}}\ldots}{\ldots}$$

Mathematica [A]
time = 2.58, size = 195, normalized size = 0.90

$$-\frac{\sqrt[4]{-1}\sqrt{2}\,e^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\left(i\text{Erfi}\left(\frac{(-1)^{3/4}(-i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)+e^{\frac{i}{b^2d^2n^2\pi}}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)\right)+4S(d(a+b\log(cx^n)))}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] -1/4*((-1)^(1/4)*Sqrt[2]*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(
-1)*(I*Erfi[((-1)^(3/4)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d
*n*Sqrt[2*Pi])] + E^(I/(b^2*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(I + a*b*d^2*n*P
i + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])]) + 4*FresnelS[d*(a + b*Log[
c*x^n])])/x
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(d(a+b\ln(c\,x^n)))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a+b*ln(c*x^n)))/x^2,x)
```

```
[Out] int(FresnelS(d*(a+b*ln(c*x^n)))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin((b*log(c*x^n) + a)*d)/x^2, x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(177) = 354$.
time = 0.37, size = 444, normalized size = 2.05

$$\frac{-i\,\pi\sqrt{b^2d^2n^2}\,xe^{\left(\frac{\log(c)}{n}+\frac{a}{bn}+\frac{i}{2\,\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+i\,\pi\sqrt{b^2d^2n^2}\,xe^{\left(\frac{\log(c)}{n}+\frac{a}{bn}-\frac{i}{2\,\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n-i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+\pi\sqrt{b^2d^2n^2}\,xe^{\left(\frac{\log(c)}{n}+\frac{a}{bn}+\frac{i}{2\,\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+\pi\sqrt{b^2d^2n^2}\,xe^{\left(\frac{\log(c)}{n}+\frac{a}{bn}-\frac{i}{2\,\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n-i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)-2\,\mathrm{S}\left(bd\log\left(cx^n\right)+ad\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(-I*pi*sqrt(b^2*d^2*n^2)*x*e^(log(c)/n + a/(b*n) + 1/2*I/(pi*b^2*d^2*n^
2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n
 + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + I*pi*sqrt(b^2*d^2*n^2)*x*e^(log
(c)/n + a/(b*n) - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x
) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n
^2)) + pi*sqrt(b^2*d^2*n^2)*x*e^(log(c)/n + a/(b*n) + 1/2*I/(pi*b^2*d^2*n^2
))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n
+ I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + pi*sqrt(b^2*d^2*n^2)*x*e^(log(c)
/n + a/(b*n) - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) +
 pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)
) - 2*fresnel_sin(b*d*log(c*x^n) + a*d))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S(ad + bd\log{(cx^n)})}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*ln(c*x**n)))/x**2,x)
```

```
[Out] Integral(fresnels(a*d + b*d*log(c*x**n))/x**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin((b*log(c*x^n) + a)*d)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}(d\,(a+b\,\ln\left(c\,x^n\right)))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int(FresnelS(d*(a + b*log(c*x^n)))/x^2, x)
```

3.59 $\int \frac{S(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal. Leaf size=228

$$\frac{\left(\frac{1}{8}-\frac{i}{8}\right)e^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}}(cx^n)^{2/n}\,\mathrm{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2}+\frac{\left(\frac{1}{8}-\frac{i}{8}\right)e^{-\frac{2\left(i-abd^2n\pi\right)}{b^2d^2n^2\pi}}(cx^n)^{2/n}\,\mathrm{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}+ia\right.}{}\right.}{x^2}$$

```
[Out] (1/8-1/8*I)*exp((2*I+2*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*(c*x^n)^(2/n)*erf((1/2
+1/2*I)*(2/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x^2+(1/8-1/
8*I)*(c*x^n)^(2/n)*erfi((1/2+1/2*I)*(2/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n
))/b/d/Pi^(1/2))/exp(2*(I-a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/x^2-1/2*FresnelS(d*
(a+b*ln(c*x^n)))/x^2
```

Rubi [A]

time = 0.26, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6606, 4713, 2314, 2308, 2266, 2235, 2236}

$$\frac{\left(\frac{1}{8}-\frac{i}{8}\right)(cx^n)^{2/n}e^{\frac{2\pi abd^2n+2i}{\pi b^2d^2n^2}}\,\mathrm{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}\,bd}\right)}{x^2}+\frac{\left(\frac{1}{8}-\frac{i}{8}\right)(cx^n)^{2/n}e^{-\frac{2\left(-\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\,\mathrm{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}\,bd}\right)}{x^2}-\frac{S(d(a+b\log(cx^n)))}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] ((1/8 - I/8)*E^((2*I + 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*(c*x^n)^(2/n)*Erf[
((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])
])/x^2 + ((1/8 - I/8)*(c*x^n)^(2/n)*Erfi[((1/2 + I/2)*(2/n + I*a*b*d^2*Pi +
 I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*(I - a*b*d^2*n*Pi))/(b^2
*d^2*n^2*Pi))*x^2) - FresnelS[d*(a + b*Log[c*x^n])]/(2*x^2)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2314

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F])), Int[(d + e*x)^(m + 2*a*b*
f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a,
b, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 4713

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - D
ist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6606

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelS[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Dist[b*d*(n/(m + 1)), Int[(e*x)^m*Sin[(Pi/2)*(d*(a + b*Log[c*x^n
]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{S(d(a+b\log(cx^n)))}{x^3}\,dx = -\frac{S(d(a+b\log(cx^n)))}{2x^2} + \frac{1}{2}(bdn)\int \frac{\sin\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)}{x^3}\,dx$$
$$= -\frac{S(d(a+b\log(cx^n)))}{2x^2} + \frac{1}{4}(ibdn)\int \frac{e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^3}\,dx - \frac{1}{4}(ibdn)\int \frac{e^{\frac{1}{2}id^2}}{}$$
$$= -\frac{S(d(a+b\log(cx^n)))}{2x^2} + \frac{1}{4}(ibdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log(cx^n) - \frac{1}{2}ib^2\right.}{x^3}$$
$$= -\frac{S(d(a+b\log(cx^n)))}{2x^2} + \frac{1}{4}(ibdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2(cx^n)\right)(cx^{}}{x^3}$$
$$= -\frac{S(d(a+b\log(cx^n)))}{2x^2} + \frac{1}{4}\left(ibdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}i\right.$$
$$= -\frac{S(d(a+b\log(cx^n)))}{2x^2} + \frac{\left(ibd(cx^n)^{-iabd^2\pi-\frac{-2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2d^2\right.\right.}{4x^2}$$
$$= -\frac{S(d(a+b\log(cx^n)))}{2x^2} + \frac{\left(ibde^{-\frac{2\left(i-abd^2n\pi\right)}{b^2d^2n^2\pi}}(cx^n)^{-iabd^2\pi-\frac{-2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \text{ex}\right.}{4x^2}$$
$$= \frac{\left(\frac{1}{8}-\frac{i}{8}\right)e^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}}(cx^n)^{2/n}\,\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2} + \frac{\left(\frac{1}{8}-\frac{i}{8}\right)e^{-\frac{2(i-}{b^2d}}}{}$$

Mathematica [A]
time = 2.59, size = 200, normalized size = 0.88

$$-\frac{\sqrt[4]{-1}\,e^{\frac{2\left(\frac{an}{b}-\frac{i}{b^2d^2\pi}+n(-n\log(x)+\log(cx^n))\right)}{n^2}}\left(i\text{Erfi}\left(\frac{(-1)^{3/4}(-2i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)+e^{\frac{4i}{b^2d^2n^2\pi}}\text{Erfi}\left(\frac{\sqrt[4]{-1}\,(2i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)\right)}{4\sqrt{2}} - \frac{S(d(a+b\log(cx^n)))}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] -1/4*((-1)^(1/4)*E^((2*((a*n)/b - I/(b^2*d^2*Pi) + n*(-(n*Log[x]) + Log[c*x
^n])))/n^2)*(I*Erfi[((-1)^(3/4)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[2*Pi])] + E^((4*I)/(b^2*d^2*n^2*Pi))*Erfi[((-1)^(1/4)*(2*
I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])]))/Sqrt[2]
- FresnelS[d*(a + b*Log[c*x^n])]/(2*x^2)
```

Maple [F]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(d(a+b\ln(c\,x^n)))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a+b*ln(c*x^n)))/x^3,x)
```

```
[Out] int(FresnelS(d*(a+b*ln(c*x^n)))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin((b*log(c*x^n) + a)*d)/x^3, x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 460 vs. 2(183) = 366.
time = 0.39, size = 460, normalized size = 2.02

$$\frac{-i\,\pi\sqrt{b^2d^2n^2}\,x^2e^{\left(\frac{2\,\log(c)}{n}+\frac{2\,a}{bn}+\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+2i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+i\,\pi\sqrt{b^2d^2n^2}\,x^2e^{\left(\frac{2\,\log(c)}{n}+\frac{2\,a}{bn}-\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n-2i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+\pi\sqrt{b^2d^2n^2}\,x^2e^{\left(\frac{2\,\log(c)}{n}+\frac{2\,a}{bn}+\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+2i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+\pi\sqrt{b^2d^2n^2}\,x^2e^{\left(\frac{2\,\log(c)}{n}+\frac{2\,a}{bn}-\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n-2i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)-2\,\mathrm{S}\left(bd\log\left(cx^n\right)+ad\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(-I*pi*sqrt(b^2*d^2*n^2)*x^2*e^(2*log(c)/n + 2*a/(b*n) + 2*I/(pi*b^2*d^
2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d
^2*n + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + I*pi*sqrt(b^2*d^2*n^2)*x^
2*e^(2*log(c)/n + 2*a/(b*n) - 2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2
*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*sqrt(b^2*d^2*n^2)/(
pi*b^2*d^2*n^2)) + pi*sqrt(b^2*d^2*n^2)*x^2*e^(2*log(c)/n + 2*a/(b*n) + 2*I
/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c)
 + pi*a*b*d^2*n + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + pi*sqrt(b^2*d^
2*n^2)*x^2*e^(2*log(c)/n + 2*a/(b*n) - 2*I/(pi*b^2*d^2*n^2))*fresnel_sin((p
i*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*sqrt(b^2*d
^2*n^2)/(pi*b^2*d^2*n^2)) - 2*fresnel_sin(b*d*log(c*x^n) + a*d))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{S(ad + bd\log{(cx^n)})}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(d*(a+b*ln(c*x**n)))/x**3,x)
```

```
[Out] Integral(fresnels(a*d + b*d*log(c*x**n))/x**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin((b*log(c*x^n) + a)*d)/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}(d\,(a+b\,\ln\left(c\,x^n\right)))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int(FresnelS(d*(a + b*log(c*x^n)))/x^3, x)
```

3.60 $\int (ex)^m S(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=280

$$\frac{\left(\frac{1}{4}-\frac{i}{4}\right)e^{\frac{i(1+m)\left(1+m+2iabd^2n\pi\right)}{2b^2d^2n^2\pi}}x(ex)^m\left(cx^n\right)^{-\frac{1+m}{n}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(1+m+iabd^2n\pi+ib^2d^2n\pi\log\left(cx^n\right)\right)}{bdn\sqrt{\pi}}\right)}{1+m}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)e^{-\frac{i(1+m)\left(1+m-\right.}{2b^2d^2\right.}}$$

```
[Out] (1/4-1/4*I)*exp(1/2*I*(1+m)*(1+m+2*I*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*x*(e*x)^
m*erf((1/2+1/2*I)*(1+m+I*a*b*d^2*n*Pi+I*b^2*d^2*n*Pi*ln(c*x^n))/b/d/n/Pi^(1
/2))/(1+m)/((c*x^n)^((1+m)/n))+(1/4-1/4*I)*x*(e*x)^m*erfi((1/2+1/2*I)*(1+m-
I*a*b*d^2*n*Pi-I*b^2*d^2*n*Pi*ln(c*x^n))/b/d/n/Pi^(1/2))/exp(1/2*I*(1+m)*(1
+m-2*I*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/(1+m)/((c*x^n)^((1+m)/n))+(e*x)^(1+m)*
FresnelS(d*(a+b*ln(c*x^n)))/e/(1+m)
```

Rubi [A]
time = 0.40, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6606, 4713, 2314, 2308, 2266, 2235, 2236}

$$\frac{\left(\frac{1}{4}-\frac{i}{4}\right)x(ex)^m\left(cx^n\right)^{-\frac{m+1}{n}}\exp\left(\frac{i(m+1)\left(2i\pi abd^2n+m+1\right)}{2\pi b^2d^2n^2}\right)\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2n+i\pi b^2d^2n\log\left(cx^n\right)+m+1\right)}{\sqrt{\pi}bdn}\right)}{m+1}+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)x(ex)^m\left(cx^n\right)^{-\frac{m+1}{n}}\exp\left(-\frac{i(m+1)\left(-2i\pi abd^2n+m+1\right)}{2\pi b^2d^2n^2}\right)\operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2n-i\pi b^2d^2n\log\left(cx^n\right)+m+1\right)}{\sqrt{\pi}bdn}\right)}{m+1}+\frac{(ex)^{m+1}S\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/4 - I/4)*E^(((I/2)*(1 + m)*(1 + m + (2*I)*a*b*d^2*n*Pi))/(b^2*d^2*n^2*P
i))*x*(e*x)^m*Erf[((1/2 + I/2)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log
[c*x^n]))/(b*d*n*Sqrt[Pi])])/((1 + m)*(c*x^n)^((1 + m)/n)) + ((1/4 - I/4)*x
*(e*x)^m*Erfi[((1/2 + I/2)*(1 + m - I*a*b*d^2*n*Pi - I*b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[Pi])])/(E^(((I/2)*(1 + m)*(1 + m - (2*I)*a*b*d^2*n*Pi))/(
b^2*d^2*n^2*Pi))*(1 + m)*(c*x^n)^((1 + m)/n)) + ((e*x)^(1 + m)*FresnelS[d*(
a + b*Log[c*x^n])])/(e*(1 + m))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2314

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F])), Int[(d + e*x)^(m + 2*a*b*
f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a,
b, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 4713

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)],
x_Symbol] :> Dist[I/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] - D
ist[I/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6606

```
Int[FresnelS[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelS[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Dist[b*d*(n/(m + 1)), Int[(e*x)^m*Sin[(Pi/2)*(d*(a + b*Log[c*x^n
]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int (ex)^m S(d(a+b\log(cx^n)))\,dx &= \frac{(ex)^{1+m}S(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(bdn)\int (ex)^m \sin\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))\right.}{1+m} \\
&= \frac{(ex)^{1+m}S(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(ibdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}(ex)^m\,dx}{2(1+m)} + \\
&= \frac{(ex)^{1+m}S(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log(c\right.}{2(1+} \\
&= \frac{(ex)^{1+m}S(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(ibdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2\right.}{2(1+m)} \\
&= \frac{(ex)^{1+m}S(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(ibdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia\right.}{2(} \\
&= \frac{(ex)^{1+m}S(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(ibdnx^{-m+iabd^2n\pi}(ex)^m(cx^n)^{-iabd^2\pi}\right)\int}{} \\
&= \frac{(ex)^{1+m}S(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(ibdx(ex)^m(cx^n)^{-iabd^2\pi-\frac{1+m-iabd^2n\pi}{n}}\right)\mathrm{S}}{} \\
&= \frac{(ex)^{1+m}S(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(ibd\exp\left(-\frac{i(1+m)(1+m-2iabd^2n\pi)}{2b^2d^2n^2\pi}\right)x(ex\right.}{} \\
&= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)\exp\left(\frac{i(1+m)(1+m+2iabd^2n\pi)}{2b^2d^2n^2\pi}\right)x(ex)^m(cx^n)^{-\frac{1+m}{n}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(1+m+iab}{b}\right.}{1+m}
\end{aligned}
$$

<u>Mathematica</u> [A]
time = 3.54, size = 244, normalized size = 0.87

$$
\frac{(ex)^m\left(-\sqrt[4]{-1}\sqrt{2}\,e^{-\frac{(1+m)\left(i+im+2abd^2n\pi+2b^2d^2n\pi(-n\log(x)+\log(cx^n))\right)}{2b^2d^2n^2\pi}}x^{-m}\left(\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(i+im+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)+e^{\frac{i(1+m)^2}{b^2d^2n^2\pi}}\operatorname{Erfi}\left(\frac{(-1)^{3/4}(1+m+iabd^2n\pi+ib^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)\right)+4xS(d(a+b\log(cx^n)))\right)}{4(1+m)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*FresnelS[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((e*x)^m*(-(((-1)^(1/4)*Sqrt[2]*(Erf[((1/2 + I/2)*(I + I*m + a*b*d^2*n*Pi +
 b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])] + E^((I*(1 + m)^2)/(b^2*d^2*n^
2*Pi))*Erfi[((-1)^(3/4)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log[c*x^n]
))/(b*d*n*Sqrt[2*Pi])]))/(E^(((1 + m)*(I + I*m + 2*a*b*d^2*n*Pi + 2*b^2*d^2
*n*Pi*(-(n*Log[x]) + Log[c*x^n])))/(2*b^2*d^2*n^2*Pi))*x^m)) + 4*x*FresnelS
[d*(a + b*Log[c*x^n])]))/(4*(1 + m))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int (ex)^m \operatorname{S}(d(a+b\ln(c\,x^n)))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*FresnelS(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*FresnelS(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m*fresnel_sin((b*log(c*x^n) + a)*d), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(312) = 624$.
time = 0.41, size = 676, normalized size = 2.41

$$\frac{-i\pi\sqrt{b^2d^2n^2}\,e^{\left(m-\frac{m\log(c)}{n}-\frac{am}{bn}-\frac{\log(c)}{n}-\frac{a}{bn}-\frac{im^2}{2\pi b^2d^2n^2}-\frac{im}{\pi b^2d^2n^2}-\frac{i}{2\pi b^2d^2n^2}\right)}\operatorname{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+im+i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+i\pi\sqrt{b^2d^2n^2}\,e^{\left(m-\frac{m\log(c)}{n}-\frac{am}{bn}-\frac{\log(c)}{n}-\frac{a}{bn}+\frac{im^2}{2\pi b^2d^2n^2}+\frac{im}{\pi b^2d^2n^2}+\frac{i}{2\pi b^2d^2n^2}\right)}\operatorname{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n-im-i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)-\pi\sqrt{b^2d^2n^2}\,e^{\left(m-\frac{m\log(c)}{n}-\frac{am}{bn}-\frac{\log(c)}{n}-\frac{a}{bn}-\frac{im^2}{2\pi b^2d^2n^2}-\frac{im}{\pi b^2d^2n^2}-\frac{i}{2\pi b^2d^2n^2}\right)}\operatorname{S}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+im+i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)-\pi\sqrt{b^2d^2n^2}\,e^{\left(m-\frac{m\log(c)}{n}-\frac{am}{bn}-\frac{\log(c)}{n}-\frac{a}{bn}+\frac{im^2}{2\pi b^2d^2n^2}+\frac{im}{\pi b^2d^2n^2}+\frac{i}{2\pi b^2d^2n^2}\right)}\operatorname{S}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n-im-i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+2\,xe^{(m\log(x)+m)}\operatorname{S}(bd\log(cx^n)+ad)}{2\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*pi*sqrt(b^2*d^2*n^2)*e^(m - m*log(c)/n - a*m/(b*n) - log(c)/n - a/(
b*n) - 1/2*I*m^2/(pi*b^2*d^2*n^2) - I*m/(pi*b^2*d^2*n^2) - 1/2*I/(pi*b^2*d^
2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d
^2*n + I*m + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + I*pi*sqrt(b^2*d^2*n^2
)*e^(m - m*log(c)/n - a*m/(b*n) - log(c)/n - a/(b*n) + 1/2*I*m^2/(pi*b^2*d^
2*n^2) + I*m/(pi*b^2*d^2*n^2) + 1/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2
*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I*m - I)*sqrt(b^2*d^
2*n^2)/(pi*b^2*d^2*n^2)) - pi*sqrt(b^2*d^2*n^2)*e^(m - m*log(c)/n - a*m/(b*
n) - log(c)/n - a/(b*n) - 1/2*I*m^2/(pi*b^2*d^2*n^2) - I*m/(pi*b^2*d^2*n^2)
 - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*
n*log(c) + pi*a*b*d^2*n + I*m + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - pi
*sqrt(b^2*d^2*n^2)*e^(m - m*log(c)/n - a*m/(b*n) - log(c)/n - a/(b*n) + 1/2
*I*m^2/(pi*b^2*d^2*n^2) + I*m/(pi*b^2*d^2*n^2) + 1/2*I/(pi*b^2*d^2*n^2))*fr
esnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I*m
 - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + 2*x*e^(m*log(x) + m)*fresnel_si
n(b*d*log(c*x^n) + a*d))/(m + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m S(ad + bd\log(cx^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*fresnels(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral((e*x)**m*fresnels(a*d + b*d*log(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*fresnel_sin(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*fresnel_sin((b*log(c*x^n) + a)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{FresnelS}(d\,(a + b\,\ln(c\,x^n)))\,(e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

```
[Out] int(FresnelS(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

3.61 $\int e^{c+\frac{1}{2}ib^2\pi x^2}S(bx)\,dx$

Optimal. Leaf size=64

$$-\frac{e^c\text{Erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)^2}{8b}+\frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

```
[Out] 1/8*exp(c)*erf((1/2-1/2*I)*b*x*Pi^(1/2))^2/b+1/4*I*b*exp(c)*x^2*hypergeom([
1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)
```

Rubi [A]
time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6571, 6511, 6510, 30}

$$\frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{e^c\text{Erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}\,bx\right)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + (I/2)*b^2*Pi*x^2)*FresnelS[b*x],x]
```

```
[Out] -1/8*(E^c*Erfi[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/b + (I/4)*b*E^c*x^2*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6510

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[E^c*
(Sqrt[Pi]/(2*b)), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 6511

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 6571

```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelS[(b_.)*(x_)], x_Symbol] :> Dist[(1 + I
)/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 + I)*b*x], x], x] + Dist[(1 - I)
/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 - I)*b*x], x], x] /; FreeQ[{b, c,
```

```
d}, x] && EqQ[d^2, (-Pi^2/4)*b^4]
```

Rubi steps

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}S(bx)\,dx = \left(-\frac{1}{4}-\frac{i}{4}\right)\int e^{c+\frac{1}{2}ib^2\pi x^2}\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)\,dx+\left(\frac{1}{4}+\frac{i}{4}\right)\int e^{c+\frac{1}{2}ib^2\pi x^2}\text{erf}\left(\left(\right.\right.$$

$$= \frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{e^c\text{Subst}\left(\int x\,dx,x,\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)\right)}{4b}$$

$$= -\frac{e^c\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)^2}{8b}+\frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}S(bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelS[b*x],x]
[Out] Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelS[b*x], x]
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int e^{c+\frac{ib^2\pi x^2}{2}}\mathrm{S}(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)
[Out] int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
[Out] integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
[Out] integral(e^(1/2*I*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{c}\int e^{\frac{i\pi b^{2}x^{2}}{2}}S(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b**2*pi*x**2)*fresnels(b*x),x)
[Out] exp(c)*Integral(exp(I*pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{\frac{1\mathrm{i}\,\Pi\,b^2\,x^2}{2}+c}\,\mathrm{FresnelS}(b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c + (Pi*b^2*x^2*1i)/2)*FresnelS(b*x),x)
[Out] int(exp(c + (Pi*b^2*x^2*1i)/2)*FresnelS(b*x), x)
```

3.62 $\int e^{c-\frac{1}{2}ib^2\pi x^2}S(bx)\,dx$

Optimal. Leaf size=64

$$\frac{e^c\text{Erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)^2}{8b}-\frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)$$

```
[Out] 1/8*exp(c)*erf((1/2+1/2*I)*b*x*Pi^(1/2))^2/b-1/4*I*b*exp(c)*x^2*hypergeom([
1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)
```

Rubi [A]
time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6571, 6508, 30, 6513}

$$\frac{e^c\text{Erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}\,bx\right)^2}{8b}-\frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(c - (I/2)*b^2*Pi*x^2)*FresnelS[b*x],x]
```

```
[Out] (E^c*Erf[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/(8*b) - (I/4)*b*E^c*x^2*Hypergeometri
cPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6508

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[E^c*(
Sqrt[Pi]/(2*b)), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 6513

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rule 6571

```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelS[(b_.)*(x_)], x_Symbol] :> Dist[(1 + I
)/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 + I)*b*x], x], x] + Dist[(1 - I)
/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 - I)*b*x], x], x] /; FreeQ[{b, c,
```

```
d}, x] && EqQ[d^2, (-Pi^2/4)*b^4]
```

Rubi steps

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}S(bx)\,dx = \left(-\frac{1}{4}-\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)\,dx+\left(\frac{1}{4}+\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\operatorname{erf}\left(\left(\frac{1}{2}\right.\right.$$

$$= -\frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{e^c\text{Subst}\left(\int x\,dx,x,\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)\right)}{4b}$$

$$= \frac{e^c\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)^2}{8b}-\frac{1}{4}ibe^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}S(bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelS[b*x],x]
[Out] Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelS[b*x], x]
```

Maple [**F**]
time = 0.14, size = 0, normalized size = 0.00

$$\int e^{c-\frac{ib^2\pi x^2}{2}}\mathrm{S}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)
[Out] int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
[Out] integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
[Out] integral(e^(-1/2*I*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{c}\int e^{-\frac{i\pi b^{2}x^{2}}{2}}S(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b**2*pi*x**2)*fresnels(b*x),x)
[Out] exp(c)*Integral(exp(-I*pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{c-\frac{\Pi\,b^2\,x^2\,1\mathrm{i}}{2}}\,\mathrm{FresnelS}(b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c - (Pi*b^2*x^2*1i)/2)*FresnelS(b*x),x)
[Out] int(exp(c - (Pi*b^2*x^2*1i)/2)*FresnelS(b*x), x)
```

3.63 $\int S(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=101

$$\frac{\cos(c)S(bx)^2}{2b}+\frac{\text{FresnelC}(bx)S(bx)\sin(c)}{2b}-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin(c)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi\right.$$

```
[Out] 1/2*cos(c)*FresnelS(b*x)^2/b+1/2*FresnelC(b*x)*FresnelS(b*x)*sin(c)/b-1/8*I
*b*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)*sin(c)+1/8*I*b*x^2*hype
rgeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)*sin(c)
```

Rubi [A]

time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {6577, 6581, 6575, 30}

$$-\frac{1}{8}ibx^2\sin(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\sin(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\sin(c)\text{FresnelC}(bx)S(bx)}{2b}+\frac{\cos(c)S(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]*Sin[c + (b^2*Pi*x^2)/2],x]
```

```
[Out] (Cos[c]*FresnelS[b*x]^2)/(2*b) + (FresnelC[b*x]*FresnelS[b*x]*Sin[c])/(2*b)
 - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2]*Sin
[c] + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]*Sin
[c]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6577

```
Int[FresnelS[(b_.)*(x_)]*Sin[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[Sin[c]
, Int[Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[Cos[c], Int[Sin[d*x^2]*Fresne
lS[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
```

```
{3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rubi steps

$$\int S(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx=\cos(c)\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx+\sin(c)\int\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx$$
$$=\frac{C(bx)S(bx)\sin(c)}{2b}-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin(c)+\frac{1}{8}ibx^2\,{}_2F_2\Big($$
$$=\frac{\cos(c)S(bx)^2}{2b}+\frac{C(bx)S(bx)\sin(c)}{2b}-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin($$

Mathematica [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int S(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelS[b*x]*Sin[c + (b^2*Pi*x^2)/2],x]
[Out] Integrate[FresnelS[b*x]*Sin[c + (b^2*Pi*x^2)/2], x]
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \mathrm{S}(bx)\sin\left(c+\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(c+1/2*b^2*Pi*x^2),x)
[Out] int(FresnelS(b*x)*sin(c+1/2*b^2*Pi*x^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(\frac{\pi b^2 x^2}{2} + c\right) S(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(c+1/2*b**2*pi*x**2),x)
[Out] Integral(sin(pi*b**2*x**2/2 + c)*fresnels(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(\frac{\Pi\, b^2\, x^2}{2} + c\right)\, \mathrm{FresnelS}(b\, x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + (Pi*b^2*x^2)/2)*FresnelS(b*x),x)
[Out] int(sin(c + (Pi*b^2*x^2)/2)*FresnelS(b*x), x)
```

3.64 $\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$

Optimal. Leaf size=101

$$\frac{\cos(c)\text{FresnelC}(bx)S(bx)}{2b}-\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-$$

```
[Out] 1/2*cos(c)*FresnelC(b*x)*FresnelS(b*x)/b-1/8*I*b*x^2*cos(c)*hypergeom([1, 1
],[3/2, 2],-1/2*I*b^2*Pi*x^2)+1/8*I*b*x^2*cos(c)*hypergeom([1, 1],[3/2, 2],
1/2*I*b^2*Pi*x^2)-1/2*FresnelS(b*x)^2*sin(c)/b
```

Rubi [A]
time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {6583, 6581, 6575, 30}

$$-\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\cos(c)\text{FresnelC}(bx)S(bx)}{2b}-\frac{\sin(c)S(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + (b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (Cos[c]*FresnelC[b*x]*FresnelS[b*x])/(2*b) - (I/8)*b*x^2*Cos[c]*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] + (I/8)*b*x^2*Cos[c]*Hypergeo
metricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2] - (FresnelS[b*x]^2*Sin[c])/(2
*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rule 6583

```
Int[Cos[(c_) + (d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Dist[Cos[c]
, Int[Cos[d*x^2]*FresnelS[b*x], x], x] - Dist[Sin[c], Int[Sin[d*x^2]*Fresne
lS[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx &= \cos(c)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx-\sin(c)\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\ &= \frac{\cos(c)C(bx)S(bx)}{2b}-\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\cos(c)\\ &= \frac{\cos(c)C(bx)S(bx)}{2b}-\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\cos(c)\end{aligned}$$

Mathematica [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelS[b*x],x]
[Out] Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

Maple [F]
time = 0.46, size = 0, normalized size = 0.00

$$\int \cos\left(c+\frac{b^2\pi\,x^2}{2}\right)\mathrm{S}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c+1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
[Out] int(cos(c+1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{\pi b^2x^2}{2} + c\right)S(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b**2*pi*x**2)*fresnels(b*x),x)
[Out] Integral(cos(pi*b**2*x**2/2 + c)*fresnels(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2 + c)*fresnel_sin(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(\frac{\Pi\,b^2\,x^2}{2} + c\right)\,\mathrm{FresnelS}(b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + (Pi*b^2*x^2)/2)*FresnelS(b*x),x)
[Out] int(cos(c + (Pi*b^2*x^2)/2)*FresnelS(b*x), x)
```

3.65 $\int S(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=13

$$\frac{S(bx)^3}{3b}$$

```
[Out] 1/3*FresnelS(b*x)^3/b
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6575, 30}

$$\frac{S(bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]^2*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] FresnelS[b*x]^3/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\int S(bx)^2 \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx &= \frac{\text{Subst}\left(\int x^2\,dx, x, S(bx)\right)}{b}\\ &= \frac{S(bx)^3}{3b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{S(bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]^2*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] FresnelS[b*x]^3/(3*b)
```

Maple [A]
time = 0.06, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\mathrm{S}(bx)^3}{3b}$	12
default	$\frac{\mathrm{S}(bx)^3}{3b}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*FresnelS(b*x)^3/b
```

Maxima [A]
time = 0.27, size = 11, normalized size = 0.85

$$\frac{\mathrm{S}\left(bx\right)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] 1/3*fresnel_sin(b*x)^3/b
```

Fricas [A]
time = 0.37, size = 11, normalized size = 0.85

$$\frac{\mathrm{S}\left(bx\right)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] 1/3*fresnel_sin(b*x)^3/b
```

Sympy [A]
time = 0.20, size = 10, normalized size = 0.77

$$\begin{cases} \frac{S^3(bx)}{3b} & \text{for}\ b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**2*sin(1/2*b**2*pi*x**2),x)
[Out] Piecewise((fresnels(b*x)**3/(3*b), Ne(b, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^2*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)^2*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \mathrm{FresnelS}(b\,x)^2\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^2*sin((Pi*b^2*x^2)/2),x)
[Out] int(FresnelS(b*x)^2*sin((Pi*b^2*x^2)/2), x)
```

3.66 $\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=13

$$\frac{S(bx)^2}{2b}$$

```
[Out] 1/2*FresnelS(b*x)^2/b
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6575, 30}

$$\frac{S(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] FresnelS[b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx &= \frac{\text{Subst}(\int x\,dx, x, S(bx))}{b}\\ &= \frac{S(bx)^2}{2b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{S(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] FresnelS[b*x]^2/(2*b)
```

Maple [A]
time = 0.06, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\mathrm{S}(bx)^2}{2b}$	12
default	$\frac{\mathrm{S}(bx)^2}{2b}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*FresnelS(b*x)^2/b
```

Maxima [A]
time = 0.26, size = 11, normalized size = 0.85

$$\frac{\mathrm{S}\left(bx\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*fresnel_sin(b*x)^2/b
```

Fricas [A]
time = 0.36, size = 11, normalized size = 0.85

$$\frac{\mathrm{S}\left(bx\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*fresnel_sin(b*x)^2/b
```

Sympy [A]
time = 0.10, size = 10, normalized size = 0.77

$$\begin{cases} \frac{S^2(bx)}{2b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((fresnels(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \mathrm{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
```

```
[Out] int(FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.67 $\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)}\,dx$

Optimal. Leaf size=9

$$\frac{\log(S(bx))}{b}$$

[Out] ln(FresnelS(b*x))/b

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6575, 29}

$$\frac{\log(S(bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x],x]

[Out] Log[FresnelS[b*x]]/b

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned} \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, S(bx)\right)}{b} \\ &= \frac{\log(S(bx))}{b} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 1.00

$$\frac{\log(S(bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x],x]
```

```
[Out] Log[FresnelS[b*x]]/b
```

Maple [A]
time = 0.12, size = 10, normalized size = 1.11

method	result	size
derivativedivides	$\frac{\ln(\mathrm{S}(bx))}{b}$	10
default	$\frac{\ln(\mathrm{S}(bx))}{b}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/2*b^2*Pi*x^2)/FresnelS(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(FresnelS(b*x))/b
```

Maxima [A]
time = 0.25, size = 9, normalized size = 1.00

$$\frac{\log\left(\mathrm{S}\left(bx\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] log(fresnel_sin(b*x))/b
```

Fricas [A]
time = 0.33, size = 9, normalized size = 1.00

$$\frac{\log\left(\mathrm{S}\left(bx\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x),x, algorithm="fricas")
```

```
[Out] log(fresnel_sin(b*x))/b
```

Sympy [A]
time = 0.10, size = 8, normalized size = 0.89

$$\begin{cases} \frac{\log\left(S\left(bx\right)\right)}{b} & \text{for}\: b \neq 0 \\ \mathrm{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b**2*pi*x**2)/fresnels(b*x),x)
[Out] Piecewise((log(fresnels(b*x))/b, Ne(b, 0)), (nan, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnel_sin(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{\mathrm{FresnelS}\left(b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x),x)
[Out] int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x), x)
```

3.68 $\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)^2}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{bS(bx)}$$

```
[Out] -1/b/FresnelS(b*x)
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6575, 30}

$$-\frac{1}{bS(bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^2,x]
```

```
[Out] -(1/(b*FresnelS[b*x]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned} \int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, S(bx)\right)}{b}\\ &= -\frac{1}{bS(bx)} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$-\frac{1}{bS(bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^2,x]
```

```
[Out] -(1/(b*FresnelS[b*x]))
```

Maple [A]
time = 0.06, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$-\frac{1}{b\,\mathrm{S}(bx)}$	12
default	$-\frac{1}{b\,\mathrm{S}(bx)}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/2*b^2*Pi*x^2)/FresnelS(b*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/FresnelS(b*x)
```

Maxima [A]
time = 0.26, size = 11, normalized size = 1.00

$$-\frac{1}{b\,\mathrm{S}\,(bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x)^2,x, algorithm="maxima")
```

```
[Out] -1/(b*fresnel_sin(b*x))
```

Fricas [A]
time = 0.34, size = 11, normalized size = 1.00

$$-\frac{1}{b\,\mathrm{S}\,(bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x)^2,x, algorithm="fricas")
```

```
[Out] -1/(b*fresnel_sin(b*x))
```

Sympy [A]
time = 0.26, size = 10, normalized size = 0.91

$$\begin{cases} -\frac{1}{bS(bx)} & \text{for}\: b \neq 0 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b**2*pi*x**2)/fresnels(b*x)**2,x)
```

```
[Out] Piecewise((-1/(b*fresnels(b*x)), Ne(b, 0)), (nan, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnel_sin(b*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{{\mathrm{FresnelS}\left(b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x)^2,x)
```

```
[Out] int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x)^2, x)
```

3.69 $\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)^3}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2bS(bx)^2}$$

```
[Out] -1/2/b/FresnelS(b*x)^2
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6575, 30}

$$-\frac{1}{2bS(bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^3,x]
```

```
[Out] -1/2*1/(b*FresnelS[b*x]^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{S(bx)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3}\,dx, x, S(bx)\right)}{b}\\ &= -\frac{1}{2bS(bx)^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$-\frac{1}{2bS(bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[(b^2*Pi*x^2)/2]/FresnelS[b*x]^3,x]
```

```
[Out] -1/2*1/(b*FresnelS[b*x]^2)
```

Maple [A]
time = 0.06, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$-\frac{1}{2b\mathrm{S}(bx)^{2}}$	12
default	$-\frac{1}{2b\mathrm{S}(bx)^{2}}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/2*b^2*Pi*x^2)/FresnelS(b*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/FresnelS(b*x)^2
```

Maxima [A]
time = 0.25, size = 11, normalized size = 0.85

$$-\frac{1}{2\,b\,\mathrm{S}\left(bx\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/(b*fresnel_sin(b*x)^2)
```

Fricas [A]
time = 0.35, size = 11, normalized size = 0.85

$$-\frac{1}{2\,b\,\mathrm{S}\left(bx\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2/(b*fresnel_sin(b*x)^2)
```

Sympy [A]
time = 0.44, size = 14, normalized size = 1.08

$$\begin{cases} -\frac{1}{2bS^{2}(bx)} & \text{for}\: b \neq 0 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b**2*pi*x**2)/fresnels(b*x)**3,x)
```

```
[Out] Piecewise((-1/(2*b*fresnels(b*x)**2), Ne(b, 0)), (nan, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/2*b^2*pi*x^2)/fresnel_sin(b*x)^3,x, algorithm="giac")
```

```
[Out] integrate(sin(1/2*pi*b^2*x^2)/fresnel_sin(b*x)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{\mathrm{FresnelS}\left(b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x)^3,x)
```

```
[Out] int(sin((Pi*b^2*x^2)/2)/FresnelS(b*x)^3, x)
```

3.70 $\int S(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=17

$$\frac{S(bx)^{1+n}}{b(1+n)}$$

```
[Out] FresnelS(b*x)^(1+n)/b/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6575, 30}

$$\frac{S(bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelS[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] FresnelS[b*x]^(1 + n)/(b*(1 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\int S(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx &= \frac{\text{Subst}\left(\int x^n \, dx, x, S(bx)\right)}{b}\\ &= \frac{S(bx)^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$\frac{S(bx)^{1+n}}{b(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] FresnelS[b*x]^(1 + n)/(b*(1 + n))
```

Maple [A]
time = 0.07, size = 18, normalized size = 1.06

method	result	size
derivativedivides	$\frac{\mathrm{S}(bx)^{1+n}}{b(1+n)}$	18
default	$\frac{\mathrm{S}(bx)^{1+n}}{b(1+n)}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^n*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] FresnelS(b*x)^(1+n)/b/(1+n)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(n>0)', see `assume?` for more detai
ls)Is n
```

Fricas [A]
time = 0.36, size = 18, normalized size = 1.06

$$\frac{\mathrm{S}\left(bx\right)^{n}\mathrm{S}\left(bx\right)}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] fresnel_sin(b*x)^n*fresnel_sin(b*x)/(b*n + b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.

time = 0.65, size = 31, normalized size = 1.82

$$\begin{cases} 0 & \text{for } b = 0 \wedge (b = 0 \vee n = -1) \\ \frac{\log(S(bx))}{b} & \text{for } n = -1 \\ \frac{S(bx)S^n(bx)}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)**n*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((0, Eq(b, 0) & (Eq(b, 0) | Eq(n, -1))), (log(fresnels(b*x))/b, Eq
(n, -1)), (fresnels(b*x)*fresnels(b*x)**n/(b*n + b), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \mathrm{FresnelS}(b\,x)^n\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^n*sin((Pi*b^2*x^2)/2),x)
```

```
[Out] int(FresnelS(b*x)^n*sin((Pi*b^2*x^2)/2), x)
```

3.71 $\int x^8 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=232

$$\frac{105x^2}{4b^7\pi^4}-\frac{7x^6}{12b^3\pi^2}+\frac{55x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4}-\frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{105S(b}{2b^9\pi}$$

```
[Out] 105/4*x^2/b^7/Pi^4-7/12*x^6/b^3/Pi^2+55/4*x^2*cos(b^2*Pi*x^2)/b^7/Pi^4-1/4*
x^6*cos(b^2*Pi*x^2)/b^3/Pi^2+35*x^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^6/P
i^3-x^7*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^2/Pi+105/2*FresnelS(b*x)^2/b^9/
Pi^4-105*x*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^8/Pi^4+7*x^5*FresnelS(b*x)*s
in(1/2*b^2*Pi*x^2)/b^4/Pi^2-40*sin(b^2*Pi*x^2)/b^9/Pi^5+5/2*x^4*sin(b^2*Pi*
x^2)/b^5/Pi^3
```

Rubi [A]
time = 0.27, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6589, 6597, 3460, 3390, 30, 3377, 2717, 2714, 6575}

$$\frac{105S(bx)^2}{2\pi^4b^9}+\frac{105x^2}{4\pi^4b^7}-\frac{7x^6}{12\pi^2b^3}-\frac{x^7S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{40\sin\left(\pi b^2x^2\right)}{\pi^5b^9}-\frac{105xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}+\frac{55x^2\cos\left(\pi b^2x^2\right)}{4\pi^4b^7}+\frac{35x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{5x^4\sin\left(\pi b^2x^2\right)}{2\pi^3b^5}+\frac{7x^5S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^6\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])/(
4*b^7*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (35*x^3*Cos[(b^2*Pi*x^2)
/2]*FresnelS[b*x])/(b^6*Pi^3) - (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^
2*Pi) + (105*FresnelS[b*x]^2)/(2*b^9*Pi^4) - (105*x*FresnelS[b*x]*Sin[(b^2*
Pi*x^2)/2])/(b^8*Pi^4) + (7*x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^
2) - (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) + (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\int x^8 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = -\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi} + \frac{\int x^7\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}$$

$$= -\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{7x^5S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{35\int x^4S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}$$

$$= -\frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3} - \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{7x^5S}{}$$

$$= -\frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3} - \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} - \frac{105x}{}$$

$$= -\frac{7x^6}{12b^3\pi^2} + \frac{41x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} - \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3} -$$

$$= \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} + \frac{55x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} - \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(}{b^6\pi^3}$$

$$= \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} + \frac{55x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} - \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(}{b^6\pi^3}$$

Mathematica [A]
time = 0.01, size = 232, normalized size = 1.00

$$\frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} + \frac{55x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} - \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3} - \frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{105S(bx)^2}{2b^9\pi^4} - \frac{105xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4} + \frac{7x^5S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{40\sin\left(b^2\pi x^2\right)}{b^9\pi^5} + \frac{5x^4\sin\left(b^2\pi x^2\right)}{2b^5\pi^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])/(
4*b^7*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (35*x^3*Cos[(b^2*Pi*x^2)
/2]*FresnelS[b*x])/(b^6*Pi^3) - (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^
2*Pi) + (105*FresnelS[b*x]^2)/(2*b^9*Pi^4) - (105*x*FresnelS[b*x]*Sin[(b^2*
Pi*x^2)/2])/(b^8*Pi^4) + (7*x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^
2) - (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) + (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^8 \mathrm{S}(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] int(x^8*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^8*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [A]
time = 0.35, size = 169, normalized size = 0.73

$$-\frac{2\,\pi^3b^6x^6 - 75\,\pi b^2x^2 + 3\,(\pi^3b^6x^6 - 55\,\pi b^2x^2)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2 + 6\,(\pi^4b^7x^7 - 35\,\pi^2b^3x^3)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{S}\left(bx\right) - 315\,\pi\,\mathrm{S}\left(bx\right)^2 - 6\,\left(5\,(\pi^2b^4x^4 - 16)\cos\left(\frac{1}{2}\,\pi b^2x^2\right) + 7\,(\pi^3b^5x^5 - 15\,\pi bx)\,\mathrm{S}\left(bx\right)\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{6\,\pi^5b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*pi^3*b^6*x^6 - 75*pi*b^2*x^2 + 3*(pi^3*b^6*x^6 - 55*pi*b^2*x^2)*cos
(1/2*pi*b^2*x^2)^2 + 6*(pi^4*b^7*x^7 - 35*pi^2*b^3*x^3)*cos(1/2*pi*b^2*x^2)
*fresnel_sin(b*x) - 315*pi*fresnel_sin(b*x)^2 - 6*(5*(pi^2*b^4*x^4 - 16)*co
s(1/2*pi*b^2*x^2) + 7*(pi^3*b^5*x^5 - 15*pi*b*x)*fresnel_sin(b*x))*sin(1/2*
pi*b^2*x^2))/(pi^5*b^9)
```

Sympy [A]
time = 16.72, size = 301, normalized size = 1.30

$$\begin{cases} -\frac{x^7\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi b^2} - \frac{x^6\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{3\pi^2b^3} - \frac{5x^6\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{6\pi^2b^3} + \frac{7x^5\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi^2b^4} + \frac{5x^4\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{\pi^3b^5} + \frac{35x^3\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi^3b^6} + \frac{25x^2\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^4b^7} + \frac{40x^2\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{\pi^4b^7} - \frac{105x\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi^4b^8} - \frac{80\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{\pi^5b^9} + \frac{105S^2(bx)}{2\pi^4b^9} & \text{for}\: b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((-x**7*cos(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) - x**6*sin(pi*
b**2*x**2/2)**2/(3*pi**2*b**3) - 5*x**6*cos(pi*b**2*x**2/2)**2/(6*pi**2*b**
3) + 7*x**5*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi**2*b**4) + 5*x**4*sin(pi*
b**2*x**2/2)*cos(pi*b**2*x**2/2)/(pi**3*b**5) + 35*x**3*cos(pi*b**2*x**2/2)
*fresnels(b*x)/(pi**3*b**6) + 25*x**2*sin(pi*b**2*x**2/2)**2/(2*pi**4*b**7)
 + 40*x**2*cos(pi*b**2*x**2/2)**2/(pi**4*b**7) - 105*x*sin(pi*b**2*x**2/2)*
fresnels(b*x)/(pi**4*b**8) - 80*sin(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/(pi
**5*b**9) + 105*fresnels(b*x)**2/(2*pi**4*b**9), Ne(b, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^8*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^8 \operatorname{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^8*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.72 $\int x^7 S(bx) \sin\left(\frac{1}{2} b^2 \pi x^2\right)\, dx$

Optimal. Leaf size=216

$$\frac{24x}{b^7\pi^4}-\frac{3x^5}{5b^3\pi^2}+\frac{147x\cos(b^2\pi x^2)}{16b^7\pi^4}-\frac{x^5\cos(b^2\pi x^2)}{4b^3\pi^2}-\frac{531\text{FresnelC}\left(\sqrt{2}\,bx\right)}{16\sqrt{2}\,b^8\pi^4}+\frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^6\cos\left(\right.}{}$$

```
[Out] 24*x/b^7/Pi^4-3/5*x^5/b^3/Pi^2+147/16*x*cos(b^2*Pi*x^2)/b^7/Pi^4-1/4*x^5*co
s(b^2*Pi*x^2)/b^3/Pi^2+24*x^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^6/Pi^3-x^
6*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^2/Pi-48*FresnelS(b*x)*sin(1/2*b^2*Pi*
x^2)/b^8/Pi^4+6*x^4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2+17/8*x^3*sin
(b^2*Pi*x^2)/b^5/Pi^3-531/32*FresnelC(b*x*2^(1/2))/b^8/Pi^4*2^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6589, 6597, 3472, 30, 3467, 3466, 3433, 6595, 3438}

$$-\frac{531\text{FresnelC}\left(\sqrt{2}\,bx\right)}{16\sqrt{2}\,\pi^4 b^8}+\frac{24x}{\pi^4 b^7}-\frac{3x^5}{5\pi^2 b^3}-\frac{x^6 S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{48S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4 b^8}+\frac{147x\cos\left(\pi b^2x^2\right)}{16\pi^4 b^7}+\frac{24x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3 b^6}+\frac{17x^3\sin\left(\pi b^2x^2\right)}{8\pi^3 b^5}+\frac{6x^4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2 b^4}-\frac{x^5\cos\left(\pi b^2x^2\right)}{4\pi^2 b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (24*x)/(b^7*Pi^4) - (3*x^5)/(5*b^3*Pi^2) + (147*x*Cos[b^2*Pi*x^2])/(16*b^7*
Pi^4) - (x^5*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (531*FresnelC[Sqrt[2]*b*x])/(1
6*Sqrt[2]*b^8*Pi^4) + (24*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^6*Pi^3)
 - (x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (48*FresnelS[b*x]*Sin
[(b^2*Pi*x^2)/2])/(b^8*Pi^4) + (6*x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b
^4*Pi^2) + (17*x^3*Sin[b^2*Pi*x^2])/(8*b^5*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3472

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6595

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelS[b*x]/(2*d)), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\begin{aligned}\int x^7 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx &= -\frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi} + \frac{\int x^6\sin\left(b^2\pi x^2\right)\ \cdots}{2b\pi} \\
&= -\frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{6x^4 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{24\int x\ \cdots}{\cdots} \\
&= -\frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3} - \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{6x^4\ \cdots}{\cdots} \\
&= -\frac{3x^5}{5b^3\pi^2} + \frac{111x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3} - \cdots \\
&= -\frac{3x^5}{5b^3\pi^2} + \frac{147x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{15C\left(\sqrt{2}\,bx\right)}{16\sqrt{2}\,b^8\pi^4} - \frac{3\sqrt{2}\,C\left(\sqrt{\cdots}\right.}{b^8\pi^4 \cdots} \\
&= \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} + \frac{147x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{51C\left(\sqrt{2}\,bx\right)}{16\sqrt{2}\,b^8\pi^4} - \frac{3\sqrt{2}\ \cdots}{\cdots} \\
&= \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} + \frac{147x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{51C\left(\sqrt{2}\,bx\right)}{16\sqrt{2}\,b^8\pi^4} - \frac{15\sqrt{\cdots}}{\cdots}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 153, normalized size = 0.71

$$\frac{-2655\sqrt{2}\,\mathrm{FresnelC}\left(\sqrt{2}\,bx\right) - 160S(bx)\left(b^2\pi x^2(-24+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right) - 6(-8+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)\right) + 2bx((735-20b^4\pi^2x^4)\cos\left(b^2\pi x^2\right) + 2(960-24b^4\pi^2x^4+85b^2\pi x^2\sin\left(b^2\pi x^2\right)))}{160b^8\pi^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 160*FresnelS[b*x]*(b^2*Pi*x^2*(-24 +
 b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 6*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)
/2]) + 2*b*x*((735 - 20*b^4*Pi^2*x^4)*Cos[b^2*Pi*x^2] + 2*(960 - 24*b^4*Pi^
2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2])))/(160*b^8*Pi^4)
```

Maple [A]
time = 0.59, size = 318, normalized size = 1.47

method	result

default	$\frac{\frac{\mathrm{S}(bx)\left(-\frac{b^6x^6\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{6b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{24\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}}{\pi}\right)}{b^7}-\frac{\frac{3}{5}\pi^2b^5x^5-24bx}{\pi^4}-\frac{3\left(\frac{\pi b^3x^3\sin\left(b^2\pi x^2\right)}{2}-\ldots\right.}{\ldots}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelS(b*x)/b^7*(-1/Pi*b^6*x^6*cos(1/2*b^2*Pi*x^2)+6/Pi*(1/Pi*b^4*x^4*si
n(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^
2*Pi*x^2))))-1/b^7*(3/Pi^4*(1/5*Pi^2*b^5*x^5-8*b*x)-3/Pi^4*(1/2*Pi*b^3*x^3*
sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC
(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1/2)))-1/2/Pi^3*(-1/2*Pi*b^5*x^5*c
os(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*c
os(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))+12/Pi*b*x*cos(b^2*Pi*
x^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^7*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [A]
time = 0.36, size = 167, normalized size = 0.77

$$-\frac{56\,\pi^2b^6x^5-2370\,b^2x+20\left(4\,\pi^2b^6x^5-147\,b^2x\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2+160\left(\pi^3b^7x^6-24\,\pi b^3x^2\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{S}\left(bx\right)+2655\,\sqrt{2}\,\sqrt{b^2}\,\mathrm{C}\left(\sqrt{2}\,\sqrt{b^2}\,x\right)-40\left(17\,\pi b^4x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)+24\left(\pi^2b^5x^4-8\,b\right)\mathrm{S}\left(bx\right)\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{160\,\pi^4b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] -1/160*(56*pi^2*b^6*x^5 - 2370*b^2*x + 20*(4*pi^2*b^6*x^5 - 147*b^2*x)*cos(
1/2*pi*b^2*x^2)^2 + 160*(pi^3*b^7*x^6 - 24*pi*b^3*x^2)*cos(1/2*pi*b^2*x^2)*
fresnel_sin(b*x) + 2655*sqrt(2)*sqrt(b^2)*fresnel_cos(sqrt(2)*sqrt(b^2)*x)
- 40*(17*pi*b^4*x^3*cos(1/2*pi*b^2*x^2) + 24*(pi^2*b^5*x^4 - 8*b)*fresnel_s
in(b*x))*sin(1/2*pi*b^2*x^2))/(pi^4*b^9)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x**7*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^7*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^7\, \mathrm{FresnelS}(b\, x)\, \sin\left(\frac{\Pi\, b^2\, x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^7*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.73 $\int x^6 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=248

$$-\frac{5x^4}{8b^3\pi^2}+\frac{11\cos(b^2\pi x^2)}{2b^7\pi^4}-\frac{x^4\cos(b^2\pi x^2)}{4b^3\pi^2}+\frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}-\frac{15\text{FresnelC}(bx)S}{2b^7\pi^3}$$

```
[Out] -5/8*x^4/b^3/Pi^2+11/2*cos(b^2*Pi*x^2)/b^7/Pi^4-1/4*x^4*cos(b^2*Pi*x^2)/b^3
/Pi^2+15*x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^6/Pi^3-x^5*cos(1/2*b^2*Pi*x^
2)*FresnelS(b*x)/b^2/Pi-15/2*FresnelC(b*x)*FresnelS(b*x)/b^7/Pi^3+15/8*I*x^
2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^5/Pi^3-15/8*I*x^2*hypergeo
m([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^5/Pi^3+5*x^3*FresnelS(b*x)*sin(1/2*b^
2*Pi*x^2)/b^4/Pi^2+7/4*x^2*sin(b^2*Pi*x^2)/b^5/Pi^3
```

Rubi [A]
time = 0.19, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6589, 6597, 3460, 3390, 30, 3377, 2718, 6581}

$$\frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^5}-\frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^5}-\frac{15\text{FresnelC}(bx)S(bx)}{2\pi^3b^7}-\frac{5x^4}{8\pi^2b^3}-\frac{x^5S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{11\cos(\pi b^2x^2)}{2\pi^4b^7}+\frac{15xS(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{7x^2\sin(\pi b^2x^2)}{4\pi^3b^5}+\frac{5x^3S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^4\cos(\pi b^2x^2)}{4\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-5*x^4)/(8*b^3*Pi^2) + (11*Cos[b^2*Pi*x^2])/(2*b^7*Pi^4) - (x^4*Cos[b^2*Pi
*x^2])/(4*b^3*Pi^2) + (15*x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^6*Pi^3) -
 (x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (15*FresnelC[b*x]*Fresn
elS[b*x])/(2*b^7*Pi^3) + (((15*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}
, (-1/2*I)*b^2*Pi*x^2])/(b^5*Pi^3) - (((15*I)/8)*x^2*HypergeometricPFQ[{1,
1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^5*Pi^3) + (5*x^3*FresnelS[b*x]*Sin[(b^2
*Pi*x^2)/2])/(b^4*Pi^2) + (7*x^2*Sin[b^2*Pi*x^2])/(4*b^5*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\int x^6 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = -\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi} + \frac{\int x^5\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}$$

$$= -\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{5x^3 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{15\int x^2 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}$$

$$= -\frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3} - \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + 5x^3S($$

$$= -\frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3} - \frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} - \frac{15C(b}{2b}$$

$$= -\frac{5x^4}{8b^3\pi^2} + \frac{17\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} - \frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3} - x^5 \text{co}$$

$$= -\frac{5x^4}{8b^3\pi^2} + \frac{11\cos\left(b^2\pi x^2\right)}{2b^7\pi^4} - \frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3} - x^5 \text{co}$$

Mathematica [F]
time = 0.34, size = 0, normalized size = 0.00

$$\int x^6 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^6*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] Integrate[x^6*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^6 \mathrm{S}(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] int(x^6*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^6*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(x^6*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x**6*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^6*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^6\, \mathrm{FresnelS}(b\, x)\, \sin\left(\frac{\Pi\, b^2\, x^2}{2}\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^6*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.74 $\int x^5 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\, dx$

Optimal. Leaf size=158

$$-\frac{2x^3}{3b^3\pi^2}-\frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3}-\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}-\frac{43S\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,b^6\pi^3}+\frac{4x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x\right.}{b^4\pi^2}$$

```
[Out] -2/3*x^3/b^3/Pi^2-1/4*x^3*cos(b^2*Pi*x^2)/b^3/Pi^2+8*cos(1/2*b^2*Pi*x^2)*Fr
esnelS(b*x)/b^6/Pi^3-x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^2/Pi+4*x^2*Fre
snelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2+11/8*x*sin(b^2*Pi*x^2)/b^5/Pi^3-43/
16*FresnelS(b*x*2^(1/2))/b^6/Pi^3*2^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$,
Rules used = {6589, 6597, 3472, 30, 3467, 3432, 6587, 3466}

$$-\frac{43S\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,\pi^3b^6}-\frac{2x^3}{3\pi^2b^3}-\frac{x^4S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{8S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{11x\sin\left(\pi b^2x^2\right)}{8\pi^3b^5}+\frac{4x^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^3\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-2*x^3)/(3*b^3*Pi^2) - (x^3*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (8*Cos[(b^2*Pi
*x^2)/2]*FresnelS[b*x])/(b^6*Pi^3) - (x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]
)/(b^2*Pi) - (43*FresnelS[Sqrt[2]*b*x])/(8*Sqrt[2]*b^6*Pi^3) + (4*x^2*Fresn
elS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (11*x*Sin[b^2*Pi*x^2])/(8*b^5*Pi
^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3472

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6587

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x]
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\int x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = -\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi} + \frac{\int x^4\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}$$

$$= -\frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{4x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{8\int xS(}{}$$

$$= -\frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} + \frac{4x^2S(bx)}{}$$

$$= -\frac{2x^3}{3b^3\pi^2} - \frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} -$$

$$= -\frac{2x^3}{3b^3\pi^2} - \frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi} -$$

Mathematica [A]
time = 0.12, size = 120, normalized size = 0.76

$$-\frac{32b^3\pi x^3 + 12b^3\pi x^3\cos\left(b^2\pi x^2\right) + 129\sqrt{2}\,S\left(\sqrt{2}\,bx\right) + 48S(bx)\left(\left(-8+b^4\pi^2x^4\right)\cos\left(\frac{1}{2}b^2\pi x^2\right) - 4b^2\pi x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)\right) - 66bx\sin\left(b^2\pi x^2\right)}{48b^6\pi^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] -1/48*(32*b^3*Pi*x^3 + 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 129*Sqrt[2]*FresnelS
[Sqrt[2]*b*x] + 48*FresnelS[b*x]*((-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] -
 4*b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) - 66*b*x*Sin[b^2*Pi*x^2])/(b^6*Pi^3)
```

Maple [A]
time = 0.58, size = 202, normalized size = 1.28

method	result
default	$\frac{\frac{S(bx)\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}}{\pi}\right)}{b^5}-\frac{\frac{2b^3x^3}{3\pi^2}-\frac{2\left(\frac{bx\sin\left(b^2\pi x^2\right)}{2\pi}-\frac{\sqrt{2}\,S\left(bx\sqrt{2}\right)}{4\pi}\right)}{\pi^2}-\frac{\pi b^3x^3\cos\left(b^2\pi x^2\right)}{2}}{b^5}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelS(b*x)/b^5*(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*si
n(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2)))-1/b^5*(2/3/Pi^2*b^3*x^3-2/Pi
^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-1/2/Pi
```

```
^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/
Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^5*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [A]
time = 0.35, size = 132, normalized size = 0.84

$$-\frac{24\,\pi b^4x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2+20\,\pi b^4x^3+48\,(\pi^2b^5x^4-8\,b)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{S}\,(bx)+129\,\sqrt{2}\,\sqrt{b^2}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{b^2}\,x\right)-12\,\left(16\,\pi b^3x^2\,\mathrm{S}\,(bx)+11\,b^2x\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{48\,\pi^3b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] -1/48*(24*pi*b^4*x^3*cos(1/2*pi*b^2*x^2)^2 + 20*pi*b^4*x^3 + 48*(pi^2*b^5*x
^4 - 8*b)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + 129*sqrt(2)*sqrt(b^2)*fres
nel_sin(sqrt(2)*sqrt(b^2)*x) - 12*(16*pi*b^3*x^2*fresnel_sin(b*x) + 11*b^2*
x*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/(pi^3*b^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{5}\sin\left(\frac{\pi b^{2}x^{2}}{2}\right)S(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x**5*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^5*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \,\mathrm{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^5*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.75 $\int x^4 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) \, dx$

Optimal. Leaf size=120

$$-\frac{3x^2}{4b^3\pi^2}-\frac{x^2\cos(b^2\pi x^2)}{4b^3\pi^2}-\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}-\frac{3S(bx)^2}{2b^5\pi^2}+\frac{3xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}+\frac{\sin(b^2\pi x^2)}{b^5\pi^3}$$

```
[Out] -3/4*x^2/b^3/Pi^2-1/4*x^2*cos(b^2*Pi*x^2)/b^3/Pi^2-x^3*cos(1/2*b^2*Pi*x^2)*
FresnelS(b*x)/b^2/Pi-3/2*FresnelS(b*x)^2/b^5/Pi^2+3*x*FresnelS(b*x)*sin(1/2
*b^2*Pi*x^2)/b^4/Pi^2+sin(b^2*Pi*x^2)/b^5/Pi^3
```

Rubi [A]
time = 0.08, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6589, 6597, 3460, 2714, 6575, 30, 3377, 2717}

$$-\frac{3S(bx)^2}{2\pi^2b^5}-\frac{3x^2}{4\pi^2b^3}-\frac{x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\sin(\pi b^2x^2)}{\pi^3b^5}+\frac{3xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x^2\cos(\pi b^2x^2)}{4\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-3*x^2)/(4*b^3*Pi^2) - (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (x^3*Cos[(b^2*
Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (3*FresnelS[b*x]^2)/(2*b^5*Pi^2) + (3*
x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + Sin[b^2*Pi*x^2]/(b^5*Pi^3
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\begin{aligned}\int x^4 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx &= -\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi}+\frac{\int x^3\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}\\ &= -\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{3xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}-\frac{3\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^4\pi^2}\\ &= -\frac{x^2\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{3xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}-3\text{Subst}(\\ &= -\frac{3x^2}{4b^3\pi^2}-\frac{x^2\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}-\frac{3S(bx)^2}{2b^5\pi^2}+\frac{3xS(bx)\sin}{b^4\pi^2}\end{aligned}$$

Mathematica [A]

time = 0.01, size = 120, normalized size = 1.00

$$-\frac{3x^2}{4b^3\pi^2}-\frac{x^2\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}-\frac{3S(bx)^2}{2b^5\pi^2}+\frac{3xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}+\frac{\sin\left(b^2\pi x^2\right)}{b^5\pi^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-3*x^2)/(4*b^3*Pi^2) - (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (x^3*Cos[(b^2*
Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi) - (3*FresnelS[b*x]^2)/(2*b^5*Pi^2) + (3*
x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + Sin[b^2*Pi*x^2]/(b^5*Pi^3
)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^4 \mathrm{S}(bx)\sin\left(\frac{b^2\pi\, x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] int(x^4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [A]
time = 0.35, size = 105, normalized size = 0.88

$$-\frac{2\,\pi^2b^3x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{S}\left(bx\right)+\pi b^2x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2+\pi b^2x^2+3\,\pi\,\mathrm{S}\left(bx\right)^2-2\left(3\,\pi bx\,\mathrm{S}\left(bx\right)+2\,\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{2\,\pi^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*pi^2*b^3*x^3*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + pi*b^2*x^2*cos(
1/2*pi*b^2*x^2)^2 + pi*b^2*x^2 + 3*pi*fresnel_sin(b*x)^2 - 2*(3*pi*b*x*fres
nel_sin(b*x) + 2*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/(pi^3*b^5)
```

Sympy [A]
time = 1.65, size = 151, normalized size = 1.26

$$\begin{cases} -\frac{x^3\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi b^2} - \frac{x^2\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^2b^3} - \frac{x^2\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{\pi^2b^3} + \frac{3x\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi^2b^4} + \frac{2\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{\pi^3b^5} - \frac{3S^2(bx)}{2\pi^2b^5} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((-x**3*cos(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) - x**2*sin(pi*
b**2*x**2/2)**2/(2*pi**2*b**3) - x**2*cos(pi*b**2*x**2/2)**2/(pi**2*b**3) +
 3*x*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi**2*b**4) + 2*sin(pi*b**2*x**2/2)
*cos(pi*b**2*x**2/2)/(pi**3*b**5) - 3*fresnels(b*x)**2/(2*pi**2*b**5), Ne(b
, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\mathrm{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^4*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.76 $\int x^3 S(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=105

$$-\frac{x}{b^3\pi^2}-\frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{5\text{FresnelC}\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,b^4\pi^2}-\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}$$

```
[Out] -x/b^3/Pi^2-1/4*x*cos(b^2*Pi*x^2)/b^3/Pi^2-x^2*cos(1/2*b^2*Pi*x^2)*FresnelS
(b*x)/b^2/Pi+2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2+5/8*FresnelC(b*x*
2^(1/2))/b^4/Pi^2*2^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {6589, 6595, 3438, 3433, 3466}

$$\frac{5\text{FresnelC}\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,\pi^2b^4}-\frac{x}{\pi^2b^3}-\frac{x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}-\frac{x\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] -(x/(b^3*Pi^2)) - (x*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (5*FresnelC[Sqrt[2]*b*
x])/(4*Sqrt[2]*b^4*Pi^2) - (x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi)
 + (2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6595

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelS[b*x]/(2*d)), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\int x^3S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx &= -\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi}+\frac{\int x^2\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}\\ &= -\frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}+\frac{\int\cos\left(b^2\pi\right.}{4b^3\pi}\\ &= -\frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{C\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,b^4\pi^2}-\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right.}{b^4\pi^2}\\ &= -\frac{x}{b^3\pi^2}-\frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{C\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,b^4\pi^2}-\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{2S(bx)\sin}{b^4\pi}\\ &= -\frac{x}{b^3\pi^2}-\frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{5C\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,b^4\pi^2}-\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{2S(bx)\sin}{b^4}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 83, normalized size = 0.79

$$\frac{-2bx\left(4+\cos\left(b^2\pi x^2\right)\right)+5\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}\,bx\right)-8S(bx)\left(b^2\pi x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)-2\sin\left(\frac{1}{2}b^2\pi x^2\right)\right)}{8b^4\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-2*b*x*(4 + Cos[b^2*Pi*x^2]) + 5*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 8*Fresnel
S[b*x]*(b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] - 2*Sin[(b^2*Pi*x^2)/2]))/(8*b^4*Pi^
2)
```

Maple [A]
time = 0.64, size = 115, normalized size = 1.10

method	result	size
default	$\frac{\frac{\mathrm{S}(bx)\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{b^3}-\frac{\frac{bx}{\pi^2}-\frac{\sqrt{2}\,\mathrm{FresnelC}\left(bx\sqrt{2}\right)}{2\pi^2}-\frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{2\pi}+\frac{\sqrt{2}\,\mathrm{FresnelC}\left(bx\sqrt{2}\right)}{4\pi}}{2\pi}}{b^3}}{b}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelS(b*x)/b^3*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi
*x^2))-1/b^3*(b*x/Pi^2-1/2/Pi^2*2^(1/2)*FresnelC(b*x*2^(1/2))-1/2/Pi*(-1/2/
Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [A]
time = 0.36, size = 94, normalized size = 0.90

$$-\frac{8\,\pi b^3x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{S}\left(bx\right)+4\,b^2x\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2+6\,b^2x-16\,b\,\mathrm{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)-5\,\sqrt{2}\,\sqrt{b^2}\,\mathrm{C}\left(\sqrt{2}\,\sqrt{b^2}\,x\right)}{8\,\pi^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] -1/8*(8*pi*b^3*x^2*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + 4*b^2*x*cos(1/2*p
i*b^2*x^2)^2 + 6*b^2*x - 16*b*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2) - 5*sqrt
(2)*sqrt(b^2)*fresnel_cos(sqrt(2)*sqrt(b^2)*x))/(pi^2*b^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x**3*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^3*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^3*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.77 $\int x^2 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=137

$$-\frac{\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{\text{FresnelC}(bx)S(bx)}{2b^3\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b\pi}+\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\right.}{8b\pi}$$

```
[Out] -1/4*cos(b^2*Pi*x^2)/b^3/Pi^2-x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^2/Pi+1/
2*FresnelC(b*x)*FresnelS(b*x)/b^3/Pi-1/8*I*x^2*hypergeom([1, 1],[3/2, 2],-1
/2*I*b^2*Pi*x^2)/b/Pi+1/8*I*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)
/b/Pi
```

Rubi [A]
time = 0.04, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6589, 6581, 3460, 2718}

$$-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi b}+\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi b}+\frac{\text{FresnelC}(bx)S(bx)}{2\pi b^3}-\frac{xS(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] -1/4*Cos[b^2*Pi*x^2]/(b^3*Pi^2) - (x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^
2*Pi) + (FresnelC[b*x]*FresnelS[b*x])/(2*b^3*Pi) - ((I/8)*x^2*Hypergeometri
cPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/(b*Pi) + ((I/8)*x^2*Hypergeome
tricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b*Pi)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
```

```
2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rubi steps

$$\begin{aligned}\int x^2 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx &= -\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx}{b^2\pi}+\frac{\int x\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}\\ &= -\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{C(bx)S(bx)}{2b^3\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b\pi}+\frac{ix^2\,{}_2\cdots}{\cdots}\\ &= -\frac{\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{C(bx)S(bx)}{2b^3\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\cdots\right)}{8b\pi}\end{aligned}$$

Mathematica [**F**]
time = 0.16, size = 0, normalized size = 0.00

$$\int x^2 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] Integrate[x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^2\mathrm{S}(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] int(x^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^2*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x**2*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\, \mathrm{FresnelS}(b\,x)\, \sin\left(\frac{\Pi\, b^2\, x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^2*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.78 $\int xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=49

$$-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{S\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,b^2\pi}$$

```
[Out] -cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^2/Pi+1/4*FresnelS(b*x*2^(1/2))/b^2/Pi*
2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {6587, 3432}

$$\frac{S\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,\pi b^2}-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] -((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^2*Pi)) + FresnelS[Sqrt[2]*b*x]/(2*
Sqrt[2]*b^2*Pi)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 6587

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x]
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\int xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx&=-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{\int\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}\\&=-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^2\pi}+\frac{S\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,b^2\pi}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.90

$$\frac{-4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)+\sqrt{2}\,S\left(\sqrt{2}\,bx\right)}{4b^2\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x] + Sqrt[2]*FresnelS[Sqrt[2]*b*x])/(4*b
^2*Pi)
```

Maple [A]
time = 0.43, size = 46, normalized size = 0.94

method	result	size
default	$\frac{-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\mathrm{S}(bx)}{\pi b}+\frac{\mathrm{S}\left(bx\sqrt{2}\right)\sqrt{2}}{4b\pi}}{b}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] (-cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/Pi/b+1/4*FresnelS(b*x*2^(1/2))/b/Pi*2^(
1/2))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [A]
time = 0.35, size = 47, normalized size = 0.96

$$-\frac{4\,b\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{S}\left(bx\right)-\sqrt{2}\,\sqrt{b^2}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{b^2}\,x\right)}{4\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) - sqrt(2)*sqrt(b^2)*fresnel_
sin(sqrt(2)*sqrt(b^2)*x))/(pi*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x*sin(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\, \mathrm{FresnelS}(b\, x)\, \sin\left(\frac{\Pi\, b^2\, x^2}{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x*FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.79 $\int S(bx) \sin \left(\frac{1}{2} b^2 \pi x^2\right) \, dx$

Optimal. Leaf size=13

$$\frac{S(bx)^2}{2b}$$

[Out] 1/2*FresnelS(b*x)^2/b

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6575, 30}

$$\frac{S(bx)^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]

[Out] FresnelS[b*x]^2/(2*b)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned} \int S(bx) \sin \left(\frac{1}{2} b^2 \pi x^2\right) \, dx &= \frac{\text{Subst}\left(\int x \, dx, x, S(bx)\right)}{b} \\ &= \frac{S(bx)^2}{2b} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{S(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] FresnelS[b*x]^2/(2*b)
```

Maple [A]
time = 0.07, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\mathrm{S}(bx)^2}{2b}$	12
default	$\frac{\mathrm{S}(bx)^2}{2b}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*FresnelS(b*x)^2/b
```

Maxima [A]
time = 0.25, size = 11, normalized size = 0.85

$$\frac{\mathrm{S}\left(bx\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*fresnel_sin(b*x)^2/b
```

Fricas [A]
time = 0.35, size = 11, normalized size = 0.85

$$\frac{\mathrm{S}\left(bx\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*fresnel_sin(b*x)^2/b
```

Sympy [A]
time = 0.10, size = 10, normalized size = 0.77

$$\begin{cases} \frac{S^2(bx)}{2b} & \text{for}\ b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((fresnels(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \mathrm{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin((Pi*b^2*x^2)/2),x)
```

```
[Out] int(FresnelS(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.80 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x,x]
```

```
[Out] Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]
```

Rubi steps

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x,x]
```

```
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x, x)
```

3.81 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2},x\right)$$

```
[Out] Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]
[Out] Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x]
```

Rubi steps

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx = \int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**2,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^2,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^2, x)
```

3.82 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$

Optimal. Leaf size=102

$$-\frac{b}{4x}+\frac{b\cos\left(b^2\pi x^2\right)}{4x}+\frac{b^2\pi S\left(\sqrt{2}\,bx\right)}{2\sqrt{2}}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}+\frac{1}{2}b^2\pi\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x},x\right)$$

```
[Out] -1/4*b/x+1/4*b*cos(b^2*Pi*x^2)/x-1/2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2+
1/4*b^2*Pi*FresnelS(b*x*2^(1/2))*2^(1/2)+1/2*b^2*Pi*Unintegrable(cos(1/2*b^
2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]
```

```
[Out] -1/4*b/x + (b*Cos[b^2*Pi*x^2])/(4*x) + (b^2*Pi*FresnelS[Sqrt[2]*b*x])/(2*Sq
rt[2]) - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*x^2) + (b^2*Pi*Defer[Int][(
Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/2
```

Rubi steps

$$\begin{aligned}\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx &= -\frac{b}{4x}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}-\frac{1}{4}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx+\frac{1}{2}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(b\\ &= -\frac{b}{4x}+\frac{b\cos\left(b^2\pi x^2\right)}{4x}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}+\frac{1}{2}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx+\\ &= -\frac{b}{4x}+\frac{b\cos\left(b^2\pi x^2\right)}{4x}+\frac{b^2\pi S\left(\sqrt{2}\,bx\right)}{2\sqrt{2}}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}+\frac{1}{2}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]
```

```
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**3,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^3,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^3, x)
```

3.83 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$

Optimal. Leaf size=109

$$-\frac{b}{12x^2}+\frac{b\cos(b^2\pi x^2)}{12x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x}-\frac{1}{6}b^3\pi^2 S(bx)^2-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}+\frac{1}{6}b^3\pi\mathrm{Si}\left(b^2\pi x^2\right)$$

```
[Out] -1/12*b/x^2+1/12*b*cos(b^2*Pi*x^2)/x^2-1/3*b^2*Pi*cos(1/2*b^2*Pi*x^2)*Fresn
elS(b*x)/x-1/6*b^3*Pi^2*FresnelS(b*x)^2+1/6*b^3*Pi*Si(b^2*Pi*x^2)-1/3*Fresn
elS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3
```

Rubi [A]
time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6591, 6599, 6575, 30, 3456, 3461, 3378, 3380}

$$-\frac{1}{6}\pi^2 b^3 S(bx)^2-\frac{\pi b^2 S(bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)}{3x}-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{3x^3}+\frac{b\cos\left(\pi b^2 x^2\right)}{12x^2}+\frac{1}{6}\pi b^3\mathrm{Si}\left(b^2\pi x^2\right)-\frac{b}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]
```

```
[Out] -1/12*b/x^2 + (b*Cos[b^2*Pi*x^2])/(12*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*Fr
esnelS[b*x])/(3*x) - (b^3*Pi^2*FresnelS[b*x]^2)/6 - (FresnelS[b*x]*Sin[(b^2
*Pi*x^2)/2])/(3*x^3) + (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6591

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[d*(x^(m + 2)/(Pi*b*(m + 1)*(m + 2))), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6599

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx &= -\frac{b}{12x^2}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}-\frac{1}{6}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx+\frac{1}{3}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S}{x^2} \\ &= -\frac{b}{12x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}-\frac{1}{12}b\text{Subst}\left(\int\frac{\cos\left(b^2\pi\right.}{x^2}\right. \\ &= -\frac{b}{12x^2}+\frac{b\cos\left(b^2\pi x^2\right)}{12x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}+\frac{1}{12}b^3\pi S \\ &= -\frac{b}{12x^2}+\frac{b\cos\left(b^2\pi x^2\right)}{12x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x}-\frac{1}{6}b^3\pi^2S(bx)^2-\frac{S(bx)\sin\left(\frac{1}{2}b\right.}{3x^3}\end{aligned}$$

Mathematica [A]

time = 0.01, size = 109, normalized size = 1.00

$$-\frac{b}{12x^2}+\frac{b\cos\left(b^2\pi x^2\right)}{12x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x}-\frac{1}{6}b^3\pi^2S(bx)^2-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}+\frac{1}{6}b^3\pi\mathrm{Si}\left(b^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]
```

```
[Out] -1/12*b/x^2 + (b*Cos[b^2*Pi*x^2])/(12*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*Fr
esnelS[b*x])/(3*x) - (b^3*Pi^2*FresnelS[b*x]^2)/6 - (FresnelS[b*x]*Sin[(b^2
*Pi*x^2)/2])/(3*x^3) + (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{S}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

Fricas [A]
time = 0.37, size = 98, normalized size = 0.90

$$-\frac{\pi^2b^3x^3\,\mathrm{S}\left(bx\right)^2-\pi b^3x^3\,\mathrm{Si}\left(\pi b^2x^2\right)+2\,\pi b^2x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{S}\left(bx\right)-bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2+bx+2\,\mathrm{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(pi^2*b^3*x^3*fresnel_sin(b*x)^2 - pi*b^3*x^3*sin_integral(pi*b^2*x^2)
 + 2*pi*b^2*x^2*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) - b*x*cos(1/2*pi*b^2*x
^2)^2 + b*x + 2*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**4,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^4,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^4, x)
```

3.84 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$

Optimal. Leaf size=153

$$-\frac{b}{24x^3}+\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}+\frac{7b^4\pi^2\text{FresnelC}\left(\sqrt{2}\,bx\right)}{24\sqrt{2}}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^2}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}-\frac{7b^3\pi\sin\left(b^2\right.}{48x}$$

```
[Out] -1/24*b/x^3+1/24*b*cos(b^2*Pi*x^2)/x^3-1/8*b^2*Pi*cos(1/2*b^2*Pi*x^2)*Fresn
elS(b*x)/x^2-1/4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4-7/48*b^3*Pi*sin(b^2*
Pi*x^2)/x+7/48*b^4*Pi^2*FresnelC(b*x*2^(1/2))*2^(1/2)-1/8*b^4*Pi^2*Unintegr
able(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]
```

```
[Out] -1/24*b/x^3 + (b*Cos[b^2*Pi*x^2])/(24*x^3) + (7*b^4*Pi^2*FresnelC[Sqrt[2]*b
*x])/(24*Sqrt[2]) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(8*x^2) - (F
resnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*x^4) - (7*b^3*Pi*Sin[b^2*Pi*x^2])/(48*
x) - (b^4*Pi^2*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/8
```

Rubi steps

$$\begin{aligned}\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx &= -\frac{b}{24x^3}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}-\frac{1}{8}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx+\frac{1}{4}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\\ &= -\frac{b}{24x^3}+\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^2}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}+\frac{1}{16}\left(b\right.\\ &= -\frac{b}{24x^3}+\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^2}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}-\frac{7b^3\pi}{}\\ &= -\frac{b}{24x^3}+\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}+\frac{7b^4\pi^2C\left(\sqrt{2}\,bx\right)}{24\sqrt{2}}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^2}-\frac{S(bx)\,\mathrm{s}}{}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]
```

```
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**5,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**5, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^5,x)
```

```
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^5, x)
```

3.85 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$

Optimal. Leaf size=148

$$-\frac{b}{40x^4}+\frac{b\cos\left(b^2\pi x^2\right)}{40x^4}+\frac{1}{24}b^5\pi^2\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x^3}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5}-\frac{b^3\pi\sin\left(b^2\right.}{24x^2}$$

```
[Out] -1/40*b/x^4+1/24*b^5*Pi^2*Ci(b^2*Pi*x^2)+1/40*b*cos(b^2*Pi*x^2)/x^4-1/15*b^
2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3-1/5*FresnelS(b*x)*sin(1/2*b^2*Pi
*x^2)/x^5-1/24*b^3*Pi*sin(b^2*Pi*x^2)/x^2-1/15*b^4*Pi^2*Unintegrable(Fresne
lS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
```

```
[Out] -1/40*b/x^4 + (b*Cos[b^2*Pi*x^2])/(40*x^4) + (b^5*Pi^2*CosIntegral[b^2*Pi*x
^2])/24 - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(15*x^3) - (FresnelS[b
*x]*Sin[(b^2*Pi*x^2)/2])/(5*x^5) - (b^3*Pi*Sin[b^2*Pi*x^2])/(24*x^2) - (b^4
*Pi^2*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x])/15
```

Rubi steps

$$\begin{aligned}
\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx &= -\frac{b}{40x^4}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5}-\frac{1}{10}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^5}\,dx+\frac{1}{5}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4} \\
&= -\frac{b}{40x^4}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x^3}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5}-\frac{1}{20}b\text{Subst}\left(\int\frac{\cos\left(b^2\pi\right.}{x^3}\right. \\
&= -\frac{b}{40x^4}+\frac{b\cos\left(b^2\pi x^2\right)}{40x^4}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x^3}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5}+\frac{1}{60}\left(b^3\pi\right. \\
&= -\frac{b}{40x^4}+\frac{b\cos\left(b^2\pi x^2\right)}{40x^4}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x^3}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5}-\frac{b^3\pi\sin}{2} \\
&= -\frac{b}{40x^4}+\frac{b\cos\left(b^2\pi x^2\right)}{40x^4}+\frac{1}{24}b^5\pi^2\text{Ci}\left(b^2\pi x^2\right)-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x^3}-\frac{S(bx)\sin}{5}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
```

```
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="fricas")
```

```
[Out] integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**6,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**6, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^6,x)
```

```
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^6, x)
```

3.86 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$

Optimal. Leaf size=241

$$-\frac{b}{60x^5}+\frac{b^5\pi^2}{96x}+\frac{b\cos\left(b^2\pi x^2\right)}{60x^5}-\frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{1440x}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{24x^4}-\frac{7b^6\pi^3S\left(\sqrt{2}\,bx\right)}{144\sqrt{2}}-\frac{1}{45}\sqrt{2}\,b^6\pi^3S$$

```
[Out] -1/60*b/x^5+1/96*b^5*Pi^2/x+1/60*b*cos(b^2*Pi*x^2)/x^5-67/1440*b^5*Pi^2*cos
(b^2*Pi*x^2)/x-1/24*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4-1/6*Fresne
lS(b*x)*sin(1/2*b^2*Pi*x^2)/x^6+1/48*b^4*Pi^2*FresnelS(b*x)*sin(1/2*b^2*Pi*
x^2)/x^2-13/720*b^3*Pi*sin(b^2*Pi*x^2)/x^3-67/1440*b^6*Pi^3*FresnelS(b*x*2^
(1/2))*2^(1/2)-1/48*b^6*Pi^3*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)
/x,x)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]
```

```
[Out] -1/60*b/x^5 + (b^5*Pi^2)/(96*x) + (b*Cos[b^2*Pi*x^2])/(60*x^5) - (67*b^5*Pi
^2*Cos[b^2*Pi*x^2])/(1440*x) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(
24*x^4) - (7*b^6*Pi^3*FresnelS[Sqrt[2]*b*x])/(144*Sqrt[2]) - (Sqrt[2]*b^6*P
i^3*FresnelS[Sqrt[2]*b*x])/45 - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x^6)
 + (b^4*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(48*x^2) - (13*b^3*Pi*Sin[b
^2*Pi*x^2])/(720*x^3) - (b^6*Pi^3*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[
b*x])/x, x])/48
```

Rubi steps

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = -\frac{b}{60x^5} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} - \frac{1}{12}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}\,dx + \frac{1}{6}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}$$

$$= -\frac{b}{60x^5} + \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{24x^4} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} + \frac{1}{48}\left(b^3\right.$$

$$= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} + \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{24x^4} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} +$$

$$= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} + \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} - \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{1440x} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{24x^4} -$$

$$= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} + \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} - \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{1440x} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{24x^4} -$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**7,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**7, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^7,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^7, x)
```

3.87 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$

Optimal. Leaf size=224

$$-\frac{b}{84x^6}+\frac{b^5\pi^2}{420x^2}+\frac{b\cos\left(b^2\pi x^2\right)}{84x^6}-\frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{35x^5}+\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{105x}+\frac{1}{210}b^7\pi^4$$

```
[Out] -1/84*b/x^6+1/420*b^5*Pi^2/x^2+1/84*b*cos(b^2*Pi*x^2)/x^6-1/84*b^5*Pi^2*cos
(b^2*Pi*x^2)/x^2-1/35*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5+1/105*b^
6*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x+1/210*b^7*Pi^4*FresnelS(b*x)^2-1
/70*b^7*Pi^3*Si(b^2*Pi*x^2)-1/7*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7+1/105
*b^4*Pi^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3-1/105*b^3*Pi*sin(b^2*Pi*x^2
)/x^4
```

Rubi [A]

time = 0.25, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6591, 6599, 6575, 30, 3456, 3461, 3378, 3380, 3460}

$$\frac{1}{210}\pi^4b^7S(bx)^2+\frac{\pi^2b^5}{420x^2}-\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}-\frac{\pi b^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{35x^5}+\frac{b\cos\left(\pi b^2x^2\right)}{84x^6}-\frac{1}{70}\pi^3b^7\text{Si}\left(b^2\pi x^2\right)+\frac{\pi^3b^6S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{105x}-\frac{\pi^2b^5\cos\left(\pi b^2x^2\right)}{84x^2}+\frac{\pi^2b^4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{105x^3}-\frac{\pi b^3\sin\left(\pi b^2x^2\right)}{105x^4}-\frac{b}{84x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]
```

```
[Out] -1/84*b/x^6 + (b^5*Pi^2)/(420*x^2) + (b*Cos[b^2*Pi*x^2])/(84*x^6) - (b^5*Pi
^2*Cos[b^2*Pi*x^2])/(84*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(
35*x^5) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(105*x) + (b^7*Pi^4*
FresnelS[b*x]^2)/210 - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*x^7) + (b^4*P
i^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x^3) - (b^3*Pi*Sin[b^2*Pi*x^2])
/(105*x^4) - (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6591

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[d*(x^(m + 2)/(Pi*b*(m + 1)*(m + 2))), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6599

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx &= -\frac{b}{84x^6}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7}-\frac{1}{14}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^7}\,dx+\frac{1}{7}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\cdots}{x^6}\cdots\\
&= -\frac{b}{84x^6}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{35x^5}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7}-\frac{1}{28}b\,\mathrm{Subst}\left(\int\frac{\cos\left(b^2\pi\cdots\right.}{x^4}\cdots\right.\\
&= -\frac{b}{84x^6}+\frac{b^5\pi^2}{420x^2}+\frac{b\cos\left(b^2\pi x^2\right)}{84x^6}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{35x^5}-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7}\cdots\\
&= -\frac{b}{84x^6}+\frac{b^5\pi^2}{420x^2}+\frac{b\cos\left(b^2\pi x^2\right)}{84x^6}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{35x^5}+\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S\cdots}{105x}\cdots\\
&= -\frac{b}{84x^6}+\frac{b^5\pi^2}{420x^2}+\frac{b\cos\left(b^2\pi x^2\right)}{84x^6}-\frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{35x^5}+\cdots\\
&= -\frac{b}{84x^6}+\frac{b^5\pi^2}{420x^2}+\frac{b\cos\left(b^2\pi x^2\right)}{84x^6}-\frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{35x^5}+\cdots
\end{aligned}
$$

Mathematica [**A**]
time = 0.01, size = 224, normalized size = 1.00

$$-\frac{b}{84x^6}+\frac{b^5\pi^2}{420x^2}+\frac{b\cos\left(b^2\pi x^2\right)}{84x^6}-\frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{35x^5}+\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{105x}+\frac{1}{210}b^7\pi^4S(bx)^2-\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7}+\frac{b^4\pi^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x^3}-\frac{b^3\pi\sin\left(b^2\pi x^2\right)}{105x^4}-\frac{1}{70}b^7\pi^3\mathrm{Si}\left(b^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]
```

```
[Out] -1/84*b/x^6 + (b^5*Pi^2)/(420*x^2) + (b*Cos[b^2*Pi*x^2])/(84*x^6) - (b^5*Pi
^2*Cos[b^2*Pi*x^2])/(84*x^2) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(
35*x^5) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(105*x) + (b^7*Pi^4*
FresnelS[b*x]^2)/210 - (FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*x^7) + (b^4*P
i^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x^3) - (b^3*Pi*Sin[b^2*Pi*x^2])
/(105*x^4) - (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)
```

```
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

Fricas [A]
time = 0.37, size = 172, normalized size = 0.77

$$\frac{\pi^4 b^7 x^7 \operatorname{S}(bx)^2 - 3\pi^3 b^7 x^7 \operatorname{Si}(\pi b^2 x^2) + 3\pi^2 b^5 x^5 - 5(\pi^2 b^5 x^5 - bx)\cos\left(\frac{1}{2}\pi b^2 x^2\right)^2 + 2(\pi^3 b^6 x^6 - 3\pi b^2 x^2)\cos\left(\frac{1}{2}\pi b^2 x^2\right)\operatorname{S}(bx) - 5bx - 2\left(2\pi b^3 x^3\cos\left(\frac{1}{2}\pi b^2 x^2\right) - (\pi^2 b^4 x^4 - 15)\operatorname{S}(bx)\right)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{210x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/210*(pi^4*b^7*x^7*fresnel_sin(b*x)^2 - 3*pi^3*b^7*x^7*sin_integral(pi*b^2
*x^2) + 3*pi^2*b^5*x^5 - 5*(pi^2*b^5*x^5 - b*x)*cos(1/2*pi*b^2*x^2)^2 + 2*(
pi^3*b^6*x^6 - 3*pi*b^2*x^2)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) - 5*b*x -
 2*(2*pi*b^3*x^3*cos(1/2*pi*b^2*x^2) - (pi^2*b^4*x^4 - 15)*fresnel_sin(b*x)
)*sin(1/2*pi*b^2*x^2))/x^7
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**8,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**8, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\text{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^8,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^8, x)
```

3.88 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$

Optimal. Leaf size=268

$$-\frac{b}{112x^7}+\frac{b^5\pi^2}{1152x^3}+\frac{b\cos\left(b^2\pi x^2\right)}{112x^7}-\frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3}-\frac{853b^8\pi^4\text{FresnelC}\left(\sqrt{2}\,bx\right)}{40320\sqrt{2}}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{48x^6}$$

```
[Out] -1/112*b/x^7+1/1152*b^5*Pi^2/x^3+1/112*b*cos(b^2*Pi*x^2)/x^7-187/40320*b^5*
Pi^2*cos(b^2*Pi*x^2)/x^3-1/48*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^6+
1/384*b^6*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2-1/8*FresnelS(b*x)*sin(
1/2*b^2*Pi*x^2)/x^8+1/192*b^4*Pi^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4-19
/3360*b^3*Pi*sin(b^2*Pi*x^2)/x^5+853/80640*b^7*Pi^3*sin(b^2*Pi*x^2)/x-853/8
0640*b^8*Pi^4*FresnelC(b*x*2^(1/2))*2^(1/2)+1/384*b^8*Pi^4*Unintegrable(Fre
snelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A]
time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]
```

```
[Out] -1/112*b/x^7 + (b^5*Pi^2)/(1152*x^3) + (b*Cos[b^2*Pi*x^2])/(112*x^7) - (187
*b^5*Pi^2*Cos[b^2*Pi*x^2])/(40320*x^3) - (853*b^8*Pi^4*FresnelC[Sqrt[2]*b*x
])/(40320*Sqrt[2]) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(48*x^6) +
(b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(384*x^2) - (FresnelS[b*x]*Sin
[(b^2*Pi*x^2)/2])/(8*x^8) + (b^4*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(1
92*x^4) - (19*b^3*Pi*Sin[b^2*Pi*x^2])/(3360*x^5) + (853*b^7*Pi^3*Sin[b^2*Pi
*x^2])/(80640*x) + (b^8*Pi^4*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])
/x, x])/384
```

Rubi steps

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = -\frac{b}{112x^7} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} - \frac{1}{16}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}\,dx + \frac{1}{8}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}$$

$$= -\frac{b}{112x^7} + \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{48x^6} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} + \frac{1}{96}(b^3$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} + \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{48x^6} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8}$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} + \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(b}{48x^6}$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} + \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(b}{48x^6}$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} + \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3} - \frac{853b^8\pi^4 C\left(\sqrt{2}\,bx\right)}{40320\sqrt{2}}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="maxima")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="fricas")
[Out] integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**9,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**9, x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="giac")
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^9,x)
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^9, x)
```

3.89 $\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$

Optimal. Leaf size=263

$$-\frac{b}{144x^8}+\frac{b^5\pi^2}{2520x^4}+\frac{b\cos(b^2\pi x^2)}{144x^8}-\frac{67b^5\pi^2\cos(b^2\pi x^2)}{30240x^4}-\frac{5b^9\pi^4\text{CosIntegral}(b^2\pi x^2)}{2016}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{63x^7}+\frac{b^6}{}$$

```
[Out] -1/144*b/x^8+1/2520*b^5*Pi^2/x^4-5/2016*b^9*Pi^4*Ci(b^2*Pi*x^2)+1/144*b*cos
(b^2*Pi*x^2)/x^8-67/30240*b^5*Pi^2*cos(b^2*Pi*x^2)/x^4-1/63*b^2*Pi*cos(1/2*
b^2*Pi*x^2)*FresnelS(b*x)/x^7+1/945*b^6*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b
*x)/x^3-1/9*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^9+1/315*b^4*Pi^2*FresnelS(b
*x)*sin(1/2*b^2*Pi*x^2)/x^5-11/3024*b^3*Pi*sin(b^2*Pi*x^2)/x^6+5/2016*b^7*P
i^3*sin(b^2*Pi*x^2)/x^2+1/945*b^8*Pi^4*Unintegrable(FresnelS(b*x)*sin(1/2*b
^2*Pi*x^2)/x^2,x)
```

Rubi [A]
time = 0.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]
```

```
[Out] -1/144*b/x^8 + (b^5*Pi^2)/(2520*x^4) + (b*Cos[b^2*Pi*x^2])/(144*x^8) - (67*
b^5*Pi^2*Cos[b^2*Pi*x^2])/(30240*x^4) - (5*b^9*Pi^4*CosIntegral[b^2*Pi*x^2]
)/2016 - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(63*x^7) + (b^6*Pi^3*Co
s[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(945*x^3) - (FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/(9*x^9) + (b^4*Pi^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(315*x^5) - (1
1*b^3*Pi*Sin[b^2*Pi*x^2])/(3024*x^6) + (5*b^7*Pi^3*Sin[b^2*Pi*x^2])/(2016*x
^2) + (b^8*Pi^4*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2, x])/945
```

Rubi steps

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx = -\frac{b}{144x^8} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} - \frac{1}{18}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^9}\,dx + \frac{1}{9}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right.}{x^8}$$

$$= -\frac{b}{144x^8} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{63x^7} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9} - \frac{1}{36}b\text{Subst}\left(\int \frac{\cos\left(b\right.}{x}\right.$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{63x^7} - \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi\right.}{9x^9}$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{63x^7} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x\right.}{945x^3}$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(b}{63x^7}$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)S(b}{63x^7}$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} + \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} - \frac{5b^9\pi^4\text{Ci}\left(b^2\pi x^2\right)}{2016} - \frac{b}{}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]
[Out] Integrate[(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{S}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
[Out] int(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="fricas")
```

```
[Out] integral(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnels(b*x)*sin(1/2*b**2*pi*x**2)/x**10,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnels(b*x)/x**10, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_sin(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^10,x)
```

```
[Out] int((FresnelS(b*x)*sin((Pi*b^2*x^2)/2))/x^10, x)
```

3.90 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)^n\,dx$

Optimal. Leaf size=22

$$\operatorname{Int}\left(\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)^n, x\right)$$

```
[Out] Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)^n,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n,x]
[Out] Defer[Int][Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n, x]
```

Rubi steps

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)^n\,dx = \int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)^n\,dx$$

Mathematica [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n,x]
[Out] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x]^n, x]
```

Maple [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{b^2\pi\,x^2}{2}\right) \mathrm{S}(bx)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)^n,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_sin(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)^n,x, algorithm="fricas")
```

```
[Out] integral(fresnel_sin(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{\pi b^2 x^2}{2}\right) S^n(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)**n,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)**n, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)^n,x, algorithm="giac")
```

```
[Out] integrate(fresnel_sin(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \mathrm{FresnelS}(b\,x)^n\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)^n*cos((Pi*b^2*x^2)/2),x)
[Out] int(FresnelS(b*x)^n*cos((Pi*b^2*x^2)/2), x)
```

3.91 $\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$

Optimal. Leaf size=307

$$\frac{35x^4}{8b^5\pi^3}-\frac{x^8}{16b\pi}-\frac{40\cos\left(b^2\pi x^2\right)}{b^9\pi^5}+\frac{5x^4\cos\left(b^2\pi x^2\right)}{2b^5\pi^3}-\frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^8\pi^4}+\frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2}+\frac{105\text{Fresn}}{}$$

```
[Out] 35/8*x^4/b^5/Pi^3-1/16*x^8/b/Pi-40*cos(b^2*Pi*x^2)/b^9/Pi^5+5/2*x^4*cos(b^2
*Pi*x^2)/b^5/Pi^3-105*x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^8/Pi^4+7*x^5*co
s(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^4/Pi^2+105/2*FresnelC(b*x)*FresnelS(b*x)/
b^9/Pi^4-105/8*I*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^7/Pi^4+
105/8*I*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^7/Pi^4-35*x^3*Fre
snelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^7*FresnelS(b*x)*sin(1/2*b^2*Pi*x^
2)/b^2/Pi-55/4*x^2*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^6*sin(b^2*Pi*x^2)/b^3/Pi^
2
```

Rubi [A]
time = 0.30, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6597, 3460, 3390, 30, 3377, 2718, 6589, 6581}

$$-\frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7}+\frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7}+\frac{105\text{FresnelC}(bx)S(bx)}{2\pi^4b^9}+\frac{35x^4}{8\pi^3b^5}+\frac{x^7S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{40\cos\left(\pi b^2x^2\right)}{\pi^5b^9}-\frac{105xS(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}-\frac{55x^2\sin\left(\pi b^2x^2\right)}{4\pi^4b^7}-\frac{35x^3S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{5x^4\cos\left(\pi b^2x^2\right)}{2\pi^3b^5}+\frac{7x^5S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x^6\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}-\frac{x^8}{16\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (35*x^4)/(8*b^5*Pi^3) - x^8/(16*b*Pi) - (40*Cos[b^2*Pi*x^2])/(b^9*Pi^5) + (
5*x^4*Cos[b^2*Pi*x^2])/(2*b^5*Pi^3) - (105*x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b
*x])/(b^8*Pi^4) + (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (1
05*FresnelC[b*x]*FresnelS[b*x])/(2*b^9*Pi^4) - (((105*I)/8)*x^2*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/(b^7*Pi^4) + (((105*I)/8)*x^
2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^7*Pi^4) - (35*x
^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^7*FresnelS[b*x]*Sin[(
b^2*Pi*x^2)/2])/(b^2*Pi) - (55*x^2*Sin[b^2*Pi*x^2])/(4*b^7*Pi^4) + (x^6*Sin
[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$
\begin{aligned}
\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx &= \frac{x^7 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{7\int x^6 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^7\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,d}{b\pi} \\
&= \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{x^7 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{35\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx}{b^4\pi^2} \\
&= \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{35x^3 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^7 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \\
&= -\frac{x^8}{16b\pi} + \frac{7x^4\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(b}{b^4\pi^2} \\
&= -\frac{x^8}{16b\pi} + \frac{5x^4\cos\left(b^2\pi x^2\right)}{2b^5\pi^3} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(b}{b^4\pi^2} \\
&= \frac{35x^4}{8b^5\pi^3} - \frac{x^8}{16b\pi} - \frac{119\cos\left(b^2\pi x^2\right)}{4b^9\pi^5} + \frac{5x^4\cos\left(b^2\pi x^2\right)}{2b^5\pi^3} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(b}{b^8\pi^4} \\
&= \frac{35x^4}{8b^5\pi^3} - \frac{x^8}{16b\pi} - \frac{40\cos\left(b^2\pi x^2\right)}{b^9\pi^5} + \frac{5x^4\cos\left(b^2\pi x^2\right)}{2b^5\pi^3} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx}{b^8\pi^4}
\end{aligned}
$$

Mathematica [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
[Out] Integrate[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int x^8 \cos\left(\frac{b^2\pi\, x^2}{2}\right) \mathrm{S}(bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
[Out] int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^8*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Integral(x**8*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^8\,\mathrm{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^8*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.92 $\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$

Optimal. Leaf size=217

$$\frac{4x^3}{b^5\pi^3}-\frac{x^7}{14b\pi}+\frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3}-\frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^8\pi^4}+\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2}+\frac{531S\left(\sqrt{2}\,bx\right)}{16\sqrt{2}\,b^8\pi^4}-\frac{24x^2S(bx)}{b}$$

```
[Out] 4*x^3/b^5/Pi^3-1/14*x^7/b/Pi+17/8*x^3*cos(b^2*Pi*x^2)/b^5/Pi^3-48*cos(1/2*b
^2*Pi*x^2)*FresnelS(b*x)/b^8/Pi^4+6*x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b
^4/Pi^2-24*x^2*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^6*FresnelS(b*x)
*sin(1/2*b^2*Pi*x^2)/b^2/Pi-147/16*x*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^5*sin(b
^2*Pi*x^2)/b^3/Pi^2+531/32*FresnelS(b*x*2^(1/2))/b^8/Pi^4*2^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6597, 3472, 30, 3467, 3466, 3432, 6589, 6587}

$$\frac{531S\left(\sqrt{2}\,bx\right)}{16\sqrt{2}\,\pi^4b^8}+\frac{4x^3}{\pi^3b^5}+\frac{x^6S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{48S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}-\frac{147x\sin\left(\pi b^2x^2\right)}{16\pi^4b^7}-\frac{24x^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{17x^3\cos\left(\pi b^2x^2\right)}{8\pi^3b^5}+\frac{6x^4S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x^5\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}-\frac{x^7}{14\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (4*x^3)/(b^5*Pi^3) - x^7/(14*b*Pi) + (17*x^3*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3)
- (48*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^8*Pi^4) + (6*x^4*Cos[(b^2*Pi*x^
2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (531*FresnelS[Sqrt[2]*b*x])/(16*Sqrt[2]*b
^8*Pi^4) - (24*x^2*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^6*Fre
snelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (147*x*Sin[b^2*Pi*x^2])/(16*b^7*
Pi^4) + (x^5*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
```

```
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3472

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6587

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x]
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx=\frac{x^6S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{6\int x^5S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi}-\frac{\int x^6\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,d}{b\pi}$$

$$=\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2}+\frac{x^6S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{24\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx}{b^4\pi^2}$$

$$=-\frac{x^7}{14b\pi}+\frac{3x^3\cos\left(b^2\pi x^2\right)}{2b^5\pi^3}+\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2}-\frac{24x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right.}{b^6\pi^3}$$

$$=-\frac{x^7}{14b\pi}+\frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3}-\frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^8\pi^4}+\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx}{b^4\pi^2}$$

$$=\frac{4x^3}{b^5\pi^3}-\frac{x^7}{14b\pi}+\frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3}-\frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^8\pi^4}+\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right.}{b^4\pi^2}$$

$$=\frac{4x^3}{b^5\pi^3}-\frac{x^7}{14b\pi}+\frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3}-\frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^8\pi^4}+\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right.}{b^4\pi^2}$$

Mathematica [A]
time = 0.13, size = 163, normalized size = 0.75

$$\frac{896b^3\pi x^3-16b^7\pi^3x^7+476b^3\pi x^3\cos\left(b^2\pi x^2\right)+3717\sqrt{2}\,S\left(\sqrt{2}\,bx\right)+224S(bx)\left(6(-8+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)+b^2\pi x^2(-24+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)\right)-2058bx\sin\left(b^2\pi x^2\right)+56b^5\pi^2x^5\sin\left(b^2\pi x^2\right)}{224b^8\pi^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (896*b^3*Pi*x^3 - 16*b^7*Pi^3*x^7 + 476*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 3717*S
qrt[2]*FresnelS[Sqrt[2]*b*x] + 224*FresnelS[b*x]*(6*(-8 + b^4*Pi^2*x^4)*Cos
[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) - 2
058*b*x*Sin[b^2*Pi*x^2] + 56*b^5*Pi^2*x^5*Sin[b^2*Pi*x^2])/(224*b^8*Pi^4)
```

Maple [A]
time = 0.77, size = 321, normalized size = 1.48

method	result
default	$\frac{\mathrm{S}(bx)\left(\frac{b^6x^6\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{6\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}}{\pi}\right)}{\pi}\right)-\frac{\frac{1}{7}\pi^2b^7x^7-8b^3x^3}{2\pi^3}+\frac{-\frac{3\pi b^3x^3\cos\left(b^2\pi x^2\right)}{2}+\frac{9\pi}{}}{}}{b^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelS(b*x)/b^7*(1/Pi*b^6*x^6*sin(1/2*b^2*Pi*x^2)-6/Pi*(-1/Pi*b^4*x^4*co
s(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2
*Pi*x^2))))-1/b^7*(1/2/Pi^3*(1/7*Pi^2*b^7*x^7-8*b^3*x^3)+3/Pi^4*(-1/2*Pi*b^
3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*Fre
snelS(b*x*2^(1/2)))-4*2^(1/2)*FresnelS(b*x*2^(1/2)))-1/2/Pi^3*(1/2*Pi*b^5*x
^5*sin(b^2*Pi*x^2)-5/2*Pi*(-1/2/Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2/Pi*(1/2/Pi*b
*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2))))-12/Pi*b*x*sin(b^2
*Pi*x^2)+6/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Fricas [A]
time = 0.36, size = 169, normalized size = 0.78

$$-\frac{16\,\pi^3 b^8 x^7 - 952\,\pi b^4 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 420\,\pi b^4 x^3 - 1344\,(\pi^2 b^5 x^4 - 8\,b)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{S}\left(bx\right) - 3717\,\sqrt{2}\,\sqrt{b^2}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{b^2}\,x\right) - 28\,\left((4\,\pi^2 b^6 x^5 - 147\,b^2 x)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) + 8\,(\pi^3 b^7 x^6 - 24\,\pi b^3 x^2)\,\mathrm{S}\left(bx\right)\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{224\,\pi^4 b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

```
[Out] -1/224*(16*pi^3*b^8*x^7 - 952*pi*b^4*x^3*cos(1/2*pi*b^2*x^2)^2 - 420*pi*b^4
*x^3 - 1344*(pi^2*b^5*x^4 - 8*b)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) - 371
7*sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x) - 28*((4*pi^2*b^6*x^5
- 147*b^2*x)*cos(1/2*pi*b^2*x^2) + 8*(pi^3*b^7*x^6 - 24*pi*b^3*x^2)*fresnel
_sin(b*x))*sin(1/2*pi*b^2*x^2))/(pi^4*b^9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Integral(x**7*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^7 \,\mathrm{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^7*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.93 $\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\, dx$

Optimal. Leaf size=184

$$\frac{15x^2}{4b^5\pi^3}-\frac{x^6}{12b\pi}+\frac{7x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3}+\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2}+\frac{15S(bx)^2}{2b^7\pi^3}-\frac{15xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3}+\frac{x^5S(bx)\sin\left(\frac{1}{2}\right.}{b^2\pi}$$

```
[Out] 15/4*x^2/b^5/Pi^3-1/12*x^6/b/Pi+7/4*x^2*cos(b^2*Pi*x^2)/b^5/Pi^3+5*x^3*cos(
1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^4/Pi^2+15/2*FresnelS(b*x)^2/b^7/Pi^3-15*x*F
resnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^5*FresnelS(b*x)*sin(1/2*b^2*Pi*
x^2)/b^2/Pi-11/2*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^4*sin(b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A]
time = 0.17, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6597, 3460, 3390, 30, 3377, 2717, 6589, 2714, 6575}

$$\frac{15S(bx)^2}{2\pi^3b^7}+\frac{15x^2}{4\pi^3b^5}+\frac{x^5S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{11\sin\left(\pi b^2x^2\right)}{2\pi^4b^7}-\frac{15xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{7x^2\cos\left(\pi b^2x^2\right)}{4\pi^3b^5}+\frac{5x^3S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x^4\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}-\frac{x^6}{12\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (15*x^2)/(4*b^5*Pi^3) - x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^3
) + (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (15*FresnelS[b*x
]^2)/(2*b^7*Pi^3) - (15*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (
x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (11*Sin[b^2*Pi*x^2])/(2*b
^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx = \frac{x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{5\int x^4 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^5 \sin^2\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$
$$= \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{15\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(b}{b^4\pi^2}$$
$$= \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{15xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} +$$
$$= -\frac{x^6}{12b\pi} + \frac{5x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{15xS(bx)\sin\left(\frac{1}{2}b^2\pi x}{b^6\pi^3}$$
$$= \frac{15x^2}{4b^5\pi^3} - \frac{x^6}{12b\pi} + \frac{7x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{15S(bx)^2}{2b^7\pi^3} - \frac{1}{}$$
$$= \frac{15x^2}{4b^5\pi^3} - \frac{x^6}{12b\pi} + \frac{7x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{15S(bx)^2}{2b^7\pi^3} - \frac{1}{}$$

Mathematica [**A**]
time = 0.01, size = 184, normalized size = 1.00

$$\frac{15x^2}{4b^5\pi^3} - \frac{x^6}{12b\pi} + \frac{7x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{15S(bx)^2}{2b^7\pi^3} - \frac{15xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^5 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{11\sin\left(b^2\pi x^2\right)}{2b^7\pi^4} + \frac{x^4\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (15*x^2)/(4*b^5*Pi^3) - x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^3
) + (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) + (15*FresnelS[b*x
]^2)/(2*b^7*Pi^3) - (15*x*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (
x^5*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (11*Sin[b^2*Pi*x^2])/(2*b
^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Maple [**F**]
time = 0.19, size = 0, normalized size = 0.00

$$\int x^6 \cos\left(\frac{b^2\pi\, x^2}{2}\right) \mathrm{S}(bx)\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

```
[Out] int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Fricas [A]
time = 0.34, size = 141, normalized size = 0.77

$$-\frac{\pi^{3} b^{6} x^{6} - 60 \pi^{2} b^{3} x^{3} \cos\left(\frac{1}{2} \pi b^{2} x^{2}\right) \mathrm{S}\left(b x\right) - 42 \pi b^{2} x^{2} \cos\left(\frac{1}{2} \pi b^{2} x^{2}\right)^{2} - 24 \pi b^{2} x^{2} - 90 \pi \mathrm{S}\left(b x\right)^{2} - 6 \left(\left(\pi^{2} b^{4} x^{4} - 22\right) \cos\left(\frac{1}{2} \pi b^{2} x^{2}\right) + 2 \left(\pi^{3} b^{5} x^{5} - 15 \pi b x\right) \mathrm{S}\left(b x\right)\right) \sin\left(\frac{1}{2} \pi b^{2} x^{2}\right)}{12 \pi^{4} b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

```
[Out] -1/12*(pi^3*b^6*x^6 - 60*pi^2*b^3*x^3*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)
- 42*pi*b^2*x^2*cos(1/2*pi*b^2*x^2)^2 - 24*pi*b^2*x^2 - 90*pi*fresnel_sin(b
*x)^2 - 6*((pi^2*b^4*x^4 - 22)*cos(1/2*pi*b^2*x^2) + 2*(pi^3*b^5*x^5 - 15*p
i*b*x)*fresnel_sin(b*x))*sin(1/2*pi*b^2*x^2))/(pi^4*b^7)
```

Sympy [A]
time = 5.46, size = 264, normalized size = 1.43

$$\begin{cases} - \frac{x^{6} \sin^{2}{\left(\frac{\pi b^{2} x^{2}}{2} \right)}}{12 \pi b} - \frac{x^{6} \cos^{2}{\left(\frac{\pi b^{2} x^{2}}{2} \right)}}{12 \pi b} + \frac{x^{5} \sin{\left(\frac{\pi b^{2} x^{2}}{2} \right)} S\left(b x\right)}{\pi b^{2}} + \frac{x^{4} \sin{\left(\frac{\pi b^{2} x^{2}}{2} \right)} \cos{\left(\frac{\pi b^{2} x^{2}}{2} \right)}}{2 \pi^{2} b^{3}} + \frac{5 x^{3} \cos{\left(\frac{\pi b^{2} x^{2}}{2} \right)} S\left(b x\right)}{\pi^{2} b^{4}} + \frac{2 x^{2} \sin^{2}{\left(\frac{\pi b^{2} x^{2}}{2} \right)}}{\pi^{3} b^{5}} + \frac{11 x^{2} \cos^{2}{\left(\frac{\pi b^{2} x^{2}}{2} \right)}}{2 \pi^{3} b^{5}} - \frac{15 x \sin{\left(\frac{\pi b^{2} x^{2}}{2} \right)} S\left(b x\right)}{\pi^{3} b^{6}} - \frac{11 \sin{\left(\frac{\pi b^{2} x^{2}}{2} \right)} \cos{\left(\frac{\pi b^{2} x^{2}}{2} \right)}}{\pi^{4} b^{7}} + \frac{15 S^{2}\left(b x\right)}{2 \pi^{3} b^{7}} & \text{for}\: b \neq 0 \\0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Piecewise((-x**6*sin(pi*b**2*x**2/2)**2/(12*pi*b) - x**6*cos(pi*b**2*x**2/2
)**2/(12*pi*b) + x**5*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) + x**4*si
n(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) + 5*x**3*cos(pi*b**2*x
**2/2)*fresnels(b*x)/(pi**2*b**4) + 2*x**2*sin(pi*b**2*x**2/2)**2/(pi**3*b*
*5) + 11*x**2*cos(pi*b**2*x**2/2)**2/(2*pi**3*b**5) - 15*x*sin(pi*b**2*x**2
/2)*fresnels(b*x)/(pi**3*b**6) - 11*sin(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)
/(pi**4*b**7) + 15*fresnels(b*x)**2/(2*pi**3*b**7), Ne(b, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^6 \operatorname{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^6*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.94 $\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\, dx$

Optimal. Leaf size=166

$$\frac{4x}{b^5\pi^3}-\frac{x^5}{10b\pi}+\frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3}-\frac{43\text{FresnelC}\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,b^6\pi^3}+\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2}-\frac{8S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3}+\frac{x^4S(bx}{}$$

```
[Out] 4*x/b^5/Pi^3-1/10*x^5/b/Pi+11/8*x*cos(b^2*Pi*x^2)/b^5/Pi^3+4*x^2*cos(1/2*b^
2*Pi*x^2)*FresnelS(b*x)/b^4/Pi^2-8*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi
^3+x^4*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi+1/4*x^3*sin(b^2*Pi*x^2)/b^3
/Pi^2-43/16*FresnelC(b*x*2^(1/2))/b^6/Pi^3*2^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6597, 3472, 30, 3467, 3466, 3433, 6589, 6595, 3438}

$$-\frac{43\text{FresnelC}\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,\pi^3b^6}+\frac{4x}{\pi^3b^5}+\frac{x^4S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{8S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{11x\cos\left(\pi b^2x^2\right)}{8\pi^3b^5}+\frac{4x^2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x^3\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}-\frac{x^5}{10\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (4*x)/(b^5*Pi^3) - x^5/(10*b*Pi) + (11*x*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3) - (4
3*FresnelC[Sqrt[2]*b*x])/(8*Sqrt[2]*b^6*Pi^3) + (4*x^2*Cos[(b^2*Pi*x^2)/2]*
FresnelS[b*x])/(b^4*Pi^2) - (8*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3
) + (x^4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) + (x^3*Sin[b^2*Pi*x^2]
)/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3472

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6595

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelS[b*x]/(2*d)), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx = \frac{x^4S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{4\int x^3S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^4\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,d}{b\pi}$$

$$= \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} + \frac{x^4S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{8\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,d}{b^4\pi^2}$$

$$= -\frac{x^5}{10b\pi} + \frac{x\cos\left(b^2\pi x^2\right)}{b^5\pi^3} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{8S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} +$$

$$= -\frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} - \frac{C\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,b^6\pi^3} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} - \frac{8S(bx)}{}$$

$$= \frac{4x}{b^5\pi^3} - \frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} - \frac{11C\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,b^6\pi^3} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2} -$$

$$= \frac{4x}{b^5\pi^3} - \frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} - \frac{11C\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,b^6\pi^3} - \frac{2\sqrt{2}\,C\left(\sqrt{2}\,bx\right)}{b^6\pi^3} + \frac{4x^2\,\mathrm{c}}{}$$

Mathematica [A]
time = 0.13, size = 126, normalized size = 0.76

$$\frac{-215\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}\,bx\right)+80S(bx)\left(4b^2\pi x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)+\left(-8+b^4\pi^2x^4\right)\sin\left(\frac{1}{2}b^2\pi x^2\right)\right)+2bx\left(160-4b^4\pi^2x^4+55\cos\left(b^2\pi x^2\right)+10b^2\pi x^2\sin\left(b^2\pi x^2\right)\right)}{80b^6\pi^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (-215*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 80*FresnelS[b*x]*(4*b^2*Pi*x^2*Cos[(b
^2*Pi*x^2)/2] + (-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*(160 - 4*b
^4*Pi^2*x^4 + 55*Cos[b^2*Pi*x^2] + 10*b^2*Pi*x^2*Sin[b^2*Pi*x^2]))/(80*b^6*
Pi^3)
```

Maple [A]
time = 0.82, size = 212, normalized size = 1.28

method	result
default	$\frac{\frac{S(bx)\left(\frac{b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{4\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}\right)}{b^5}-\frac{\frac{\frac{1}{5}\pi^2b^5x^5-8bx}{2\pi^3}+\frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{\pi}+\frac{\sqrt{2}\,\text{FresnelC}\left(bx\sqrt{2}\right)}{2\pi}}{\pi^2}-\frac{\pi b^3}{}}{}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelS(b*x)/b^5*(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*co
s(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2)))-1/b^5*(1/2/Pi^3*(1/5*Pi^2*b^
5*x^5-8*b*x)+2/Pi^2*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*
x*2^(1/2)))-1/2/Pi^3*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*co
s(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*
2^(1/2)))))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Fricas [A]
time = 0.37, size = 139, normalized size = 0.84

$$-\frac{8\,\pi^2 b^6 x^5 - 320\,\pi b^3 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{S}\left(bx\right) - 220\,b^2 x \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 210\,b^2 x + 215\,\sqrt{2}\,\sqrt{b^2}\,\mathrm{C}\left(\sqrt{2}\,\sqrt{b^2}\,x\right) - 40\,\left(\pi b^4 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) + 2\,\left(\pi^2 b^5 x^4 - 8\,b\right) \mathrm{S}\left(bx\right)\right) \sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{80\,\pi^3 b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

```
[Out] -1/80*(8*pi^2*b^6*x^5 - 320*pi*b^3*x^2*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)
 - 220*b^2*x*cos(1/2*pi*b^2*x^2)^2 - 210*b^2*x + 215*sqrt(2)*sqrt(b^2)*fres
nel_cos(sqrt(2)*sqrt(b^2)*x) - 40*(pi*b^4*x^3*cos(1/2*pi*b^2*x^2) + 2*(pi^2
*b^5*x^4 - 8*b)*fresnel_sin(b*x))*sin(1/2*pi*b^2*x^2))/(pi^3*b^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Integral(x**5*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \,\mathrm{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^5*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.95 $\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$

Optimal. Leaf size=195

$$-\frac{x^4}{8b\pi}+\frac{\cos\left(b^2\pi x^2\right)}{b^5\pi^3}+\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2}-\frac{3\text{FresnelC}(bx)S(bx)}{2b^5\pi^2}+\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2}-\frac{3ix^2\,{}_2F_2}{}$$

```
[Out] -1/8*x^4/b/Pi+cos(b^2*Pi*x^2)/b^5/Pi^3+3*x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x
)/b^4/Pi^2-3/2*FresnelC(b*x)*FresnelS(b*x)/b^5/Pi^2+3/8*I*x^2*hypergeom([1,
 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^3/Pi^2-3/8*I*x^2*hypergeom([1, 1],[3/2, 2
],1/2*I*b^2*Pi*x^2)/b^3/Pi^2+x^3*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi+1
/4*x^2*sin(b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A]
time = 0.11, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6597, 3460, 3390, 30, 3377, 2718, 6589, 6581}

$$\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2b^3}-\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2b^3}-\frac{3\text{FresnelC}(bx)S(bx)}{2\pi^2b^5}+\frac{x^3S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\cos\left(\pi b^2x^2\right)}{\pi^3b^5}+\frac{3xS(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x^2\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}-\frac{x^4}{8\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] -1/8*x^4/(b*Pi) + Cos[b^2*Pi*x^2]/(b^5*Pi^3) + (3*x*Cos[(b^2*Pi*x^2)/2]*Fre
snelS[b*x])/(b^4*Pi^2) - (3*FresnelC[b*x]*FresnelS[b*x])/(2*b^5*Pi^2) + (((
3*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/(b^3*
Pi^2) - (((3*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2
])/(b^3*Pi^2) + (x^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) + (x^2*Sin
[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rule 6589

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[1/(2*b*Pi), Int[x^(m - 1)*
Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ
[m, 1]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx = \frac{x^3 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{3\int x^2 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^3 \sin^2\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$
$$= \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} + \frac{x^3 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{3\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx}{b^4\pi^2}$$
$$= \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} - \frac{3C(bx)S(bx)}{2b^5\pi^2} + \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} -$$
$$= -\frac{x^4}{8b\pi} + \frac{3\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} + \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} - \frac{3C(bx)S(bx)}{2b^5\pi^2} + \frac{3ix^2\,{}_2F_2}{}$$
$$= -\frac{x^4}{8b\pi} + \frac{\cos\left(b^2\pi x^2\right)}{b^5\pi^3} + \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)}{b^4\pi^2} - \frac{3C(bx)S(bx)}{2b^5\pi^2} + \frac{3ix^2\,{}_2F_2(1}{}$$

__Mathematica__ [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
[Out] Integrate[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

__Maple__ [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int x^4 \cos\left(\frac{b^2\pi\,x^2}{2}\right) \mathrm{S}(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
[Out] int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

__Maxima__ [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
[Out] integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
[Out] integral(x^4*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
[Out] Integral(x**4*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\mathrm{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^4*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.96 $\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$

Optimal. Leaf size=108

$$-\frac{x^3}{6b\pi}+\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2}-\frac{5S\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,b^4\pi^2}+\frac{x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}+\frac{x\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

```
[Out] -1/6*x^3/b/Pi+2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/b^4/Pi^2+x^2*FresnelS(b*x
)*sin(1/2*b^2*Pi*x^2)/b^2/Pi+1/4*x*sin(b^2*Pi*x^2)/b^3/Pi^2-5/8*FresnelS(b*
x*2^(1/2))/b^4/Pi^2*2^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6597, 3472, 30, 3467, 3432, 6587}

$$-\frac{5S\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,\pi^2b^4}+\frac{x^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{2S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}-\frac{x^3}{6\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] -1/6*x^3/(b*Pi) + (2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(b^4*Pi^2) - (5*Fre
snelS[Sqrt[2]*b*x])/(4*Sqrt[2]*b^4*Pi^2) + (x^2*FresnelS[b*x]*Sin[(b^2*Pi*x
^2)/2])/(b^2*Pi) + (x*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3472

```
Int[(x_)^(m_.)*Sin[(a_.) + ((b_.)*(x_)^(n_))/2]^2, x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] - Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6587

```
Int[FresnelS[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelS[b*x]/(2*d)), x] + Dist[1/(2*b*Pi), Int[Sin[2*d*x^2], x], x]
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\begin{aligned}\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx &= \frac{x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{2\int xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi}-\frac{\int x^2\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi}\\ &= \frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2}+\frac{x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{\int\sin\left(b^2\pi x^2\right)\,dx}{b^3\pi^2}-\frac{\int x^2\,dx}{2b\pi}\\ &= -\frac{x^3}{6b\pi}+\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2}-\frac{S\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,b^4\pi^2}+\frac{x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}+\frac{x\sin}{4}\\ &= -\frac{x^3}{6b\pi}+\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{b^4\pi^2}-\frac{5S\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,b^4\pi^2}+\frac{x^2S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}+\frac{x\sin}{}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 90, normalized size = 0.83

$$\frac{-4b^3\pi x^3-15\sqrt{2}\,S\left(\sqrt{2}\,bx\right)+24S(bx)\left(2\cos\left(\frac{1}{2}b^2\pi x^2\right)+b^2\pi x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)\right)+6bx\sin\left(b^2\pi x^2\right)}{24b^4\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (-4*b^3*Pi*x^3 - 15*Sqrt[2]*FresnelS[Sqrt[2]*b*x] + 24*FresnelS[b*x]*(2*Cos
[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) + 6*b*x*Sin[b^2*Pi*x^2])
/(24*b^4*Pi^2)
```

Maple [A]
time = 0.86, size = 119, normalized size = 1.10

method	result	size
default	$\frac{\frac{\mathrm{S}(bx)\left(\frac{b^{2}x^{2}\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi}+\frac{2\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi^{2}}\right)}{b^{3}}-\frac{\frac{\sqrt{2}\,\mathrm{S}\left(bx\sqrt{2}\right)}{2\pi^{2}}+\frac{b^{3}x^{3}}{6\pi}-\frac{\frac{bx\sin\left(b^{2}\pi x^{2}\right)}{2\pi}-\frac{\sqrt{2}\,\mathrm{S}\left(bx\sqrt{2}\right)}{4\pi}}{2\pi}}{b^{3}}}{b}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelS(b*x)/b^3*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*
x^2))-1/b^3*(1/2/Pi^2*2^(1/2)*FresnelS(b*x*2^(1/2))+1/6*b^3*x^3/Pi-1/2/Pi*(
1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Fricas [A]
time = 0.35, size = 97, normalized size = 0.90

$$-\frac{4\,\pi b^{4}x^{3}-48\,b\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)\mathrm{S}\left(bx\right)+15\,\sqrt{2}\,\sqrt{b^{2}}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{b^{2}}\,x\right)-12\left(2\,\pi b^{3}x^{2}\,\mathrm{S}\left(bx\right)+b^{2}x\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{24\,\pi^{2}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

```
[Out] -1/24*(4*pi*b^4*x^3 - 48*b*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + 15*sqrt(2
)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x) - 12*(2*pi*b^3*x^2*fresnel_sin
(b*x) + b^2*x*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/(pi^2*b^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\cos\left(\frac{\pi b^{2}x^{2}}{2}\right)S(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Integral(x**3*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^3*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.97 $\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$

Optimal. Leaf size=73

$$-\frac{x^2}{4b\pi}-\frac{S(bx)^2}{2b^3\pi}+\frac{xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}+\frac{\sin(b^2\pi x^2)}{4b^3\pi^2}$$

```
[Out] -1/4*x^2/b/Pi-1/2*FresnelS(b*x)^2/b^3/Pi+x*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2
)/b^2/Pi+1/4*sin(b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6597, 3460, 2714, 6575, 30}

$$-\frac{S(bx)^2}{2\pi b^3}+\frac{xS(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\sin(\pi b^2x^2)}{4\pi^2b^3}-\frac{x^2}{4\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] -1/4*x^2/(b*Pi) - FresnelS[b*x]^2/(2*b^3*Pi) + (x*FresnelS[b*x]*Sin[(b^2*Pi
*x^2)/2])/(b^2*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6597

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelS[b*x]/(2*d)), x] + (-Dist[1/(Pi*b), Int[x^(m - 1
)*Sin[d*x^2]^2, x], x] - Dist[(m - 1)/(2*d), Int[x^(m - 2)*Sin[d*x^2]*Fresn
elS[b*x], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
1]
```

Rubi steps

$$\begin{aligned}\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx &= \frac{xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi}-\frac{\int x\sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi}\\ &= \frac{xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{\mathrm{Subst}(\int x\,dx,x,S(bx))}{b^3\pi}-\frac{\mathrm{Subst}\left(\int\sin^2\left(\frac{1}{2}b^2\pi x\right)\,dx,x\right.}{2b\pi}\\ &= -\frac{x^2}{4b\pi}-\frac{S(bx)^2}{2b^3\pi}+\frac{xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}+\frac{\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 73, normalized size = 1.00

$$-\frac{x^2}{4b\pi}-\frac{S(bx)^2}{2b^3\pi}+\frac{xS(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}+\frac{\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] -1/4*x^2/(b*Pi) - FresnelS[b*x]^2/(2*b^3*Pi) + (x*FresnelS[b*x]*Sin[(b^2*Pi
*x^2)/2])/(b^2*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int x^2\cos\left(\frac{b^2\pi\,x^2}{2}\right)\mathrm{S}(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

```
[Out] int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
[Out] integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Fricas [A]
time = 0.36, size = 58, normalized size = 0.79

$$-\frac{\pi b^2x^2 + 2\,\pi\,\mathrm{S}\,(bx)^2 - 2\left(2\,\pi bx\,\mathrm{S}\,(bx) + \cos\left(\frac{1}{2}\,\pi b^2x^2\right)\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{4\,\pi^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
[Out] -1/4*(pi*b^2*x^2 + 2*pi*fresnel_sin(b*x)^2 - 2*(2*pi*b*x*fresnel_sin(b*x) +
 cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/(pi^2*b^3)
```

Sympy [A]
time = 0.50, size = 114, normalized size = 1.56

$$\begin{cases} -\frac{x^2\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{4\pi b} - \frac{x^2\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{4\pi b} + \frac{x\sin\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{\pi b^2} + \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^2b^3} - \frac{S^2(bx)}{2\pi b^3} & \text{for}\ b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
[Out] Piecewise((-x**2*sin(pi*b**2*x**2/2)**2/(4*pi*b) - x**2*cos(pi*b**2*x**2/2)
**2/(4*pi*b) + x*sin(pi*b**2*x**2/2)*fresnels(b*x)/(pi*b**2) + sin(pi*b**2*
x**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) - fresnels(b*x)**2/(2*pi*b**3),
Ne(b, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^2*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.98 $\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$

Optimal. Leaf size=59

$$-\frac{x}{2b\pi}+\frac{\text{FresnelC}\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,b^2\pi}+\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}$$

```
[Out] -1/2*x/b/Pi+FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi+1/4*FresnelC(b*x*2^(1/
2))/b^2/Pi*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6595, 3438, 3433}

$$\frac{\text{FresnelC}\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,\pi b^2}+\frac{S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{x}{2\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] -1/2*x/(b*Pi) + FresnelC[Sqrt[2]*b*x]/(2*Sqrt[2]*b^2*Pi) + (FresnelS[b*x]*S
in[(b^2*Pi*x^2)/2])/(b^2*Pi)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3438

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Sin[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 6595

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelS[b*x]/(2*d)), x] - Dist[1/(Pi*b), Int[Sin[d*x^2]^2, x], x] /; F
reeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)\,dx = \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int \sin^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi}$$
$$= \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\int \left(\frac{1}{2}-\frac{1}{2}\cos\left(b^2\pi x^2\right)\right)\,dx}{b\pi}$$
$$= -\frac{x}{2b\pi} + \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{\int \cos\left(b^2\pi x^2\right)\,dx}{2b\pi}$$
$$= -\frac{x}{2b\pi} + \frac{C\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,b^2\pi} + \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.81

$$\frac{-2bx + \sqrt{2}\,\text{FresnelC}\left(\sqrt{2}\,bx\right) + 4S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^2\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (-2*b*x + Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 4*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/
2])/(4*b^2*Pi)
```

Maple [A]
time = 0.81, size = 52, normalized size = 0.88

method	result	size
default	$\frac{\frac{S(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{b\pi} - \frac{\frac{bx}{2} - \frac{\sqrt{2}\,\text{FresnelC}\left(bx\sqrt{2}\right)}{4}}{b\pi}}{b}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelS(b*x)/b/Pi*sin(1/2*b^2*Pi*x^2)-1/b/Pi*(1/2*b*x-1/4*2^(1/2)*Fresnel
C(b*x*2^(1/2))))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
[Out] integrate(x*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Fricas [A]
time = 0.37, size = 53, normalized size = 0.90

$$-\frac{2\,b^2x-4\,b\,\mathrm{S}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)-\sqrt{2}\,\sqrt{b^2}\,\,\mathrm{C}\left(\sqrt{2}\,\sqrt{b^2}\,x\right)}{4\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
[Out] -1/4*(2*b^2*x - 4*b*fresnel_sin(b*x)*sin(1/2*pi*b^2*x^2) - sqrt(2)*sqrt(b^2
)*fresnel_cos(sqrt(2)*sqrt(b^2)*x))/(pi*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
[Out] Integral(x*cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(x*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x*FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.99 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$

Optimal. Leaf size=80

$$\frac{\text{FresnelC}(bx)S(bx)}{2b}-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

```
[Out] 1/2*FresnelC(b*x)*FresnelS(b*x)/b-1/8*I*b*x^2*hypergeom([1, 1],[3/2, 2],-1/
2*I*b^2*Pi*x^2)+1/8*I*b*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {6581}

$$-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)S(bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] (FresnelC[b*x]*FresnelS[b*x])/(2*b) - (I/8)*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/
2, 2}, (I/2)*b^2*Pi*x^2]
```

Rule 6581

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)], x_Symbol] :> Simp[FresnelC[b*x]
*(FresnelS[b*x]/(2*b)), x] + (-Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-2^(-1))*I*b^2*Pi*x^2], x] + Simp[(1/8)*I*b*x^2*HypergeometricP
FQ[{1, 1}, {3/2, 2}, (1/2)*I*b^2*Pi*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^
2, (Pi^2/4)*b^4]
```

Rubi steps

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx=\frac{C(bx)S(bx)}{2b}-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{1}{2}b^2\pi x^2\right) S(bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x],x]
```

```
[Out] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x], x]
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{b^{2}\pi\,x^{2}}{2}\right) \mathrm{S}(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{\pi b^{2}x^{2}}{2}\right)S(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x),x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x),x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelS(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(FresnelS(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.100 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x},x\right)$$

```
[Out] Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x,x]
[Out] Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x]
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x]
```

Maple [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\mathrm{S}(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x,x)
```

```
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x, x)
```

3.101 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^2}\,dx$

Optimal. Leaf size=48

$$-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x}-\frac{1}{2}b\pi S(bx)^2+\frac{1}{4}b\,\mathrm{Si}\left(b^2\pi x^2\right)$$

```
[Out] -cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x-1/2*b*Pi*FresnelS(b*x)^2+1/4*b*Si(b^2*
Pi*x^2)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6599, 6575, 30, 3456}

$$-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\,\mathrm{Si}\left(b^2\pi x^2\right)-\frac{1}{2}\pi bS(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^2,x]
```

```
[Out] -((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x) - (b*Pi*FresnelS[b*x]^2)/2 + (b*Si
nIntegral[b^2*Pi*x^2])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6599

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
```

```
ILtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^2}\,dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x} + \frac{1}{2}b\int \frac{\sin\left(b^2\pi x^2\right)}{x}\,dx - \left(b^2\pi\right)\int S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$
$$= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x} + \frac{1}{4}b\mathrm{Si}\left(b^2\pi x^2\right) - (b\pi)\mathrm{Subst}\left(\int x\,dx, x, S(bx)\right)$$
$$= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x} - \frac{1}{2}b\pi S(bx)^2 + \frac{1}{4}b\mathrm{Si}\left(b^2\pi x^2\right)$$

Mathematica [**A**]
time = 0.01, size = 48, normalized size = 1.00

$$-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x} - \frac{1}{2}b\pi S(bx)^2 + \frac{1}{4}b\mathrm{Si}\left(b^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^2,x]
[Out] -((Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x) - (b*Pi*FresnelS[b*x]^2)/2 + (b*Si
nIntegral[b^2*Pi*x^2])/4
```

Maple [**F**]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\mathrm{S}(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^2, x)
```

Fricas [A]
time = 0.36, size = 46, normalized size = 0.96

$$-\frac{2\,\pi bx\,\mathrm{S}\left(bx\right)^{2}-bx\,\mathrm{Si}\left(\pi b^{2}x^{2}\right)+4\,\cos\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)\mathrm{S}\left(bx\right)}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*pi*b*x*fresnel_sin(b*x)^2 - b*x*sin_integral(pi*b^2*x^2) + 4*cos(1/
2*pi*b^2*x^2)*fresnel_sin(b*x))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(\frac{\pi b^{2}x^{2}}{2}\right)S(bx)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**2,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\mathrm{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^{2}\,x^{2}}{2}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^2,x)
```

```
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^2, x)
```

3.102 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^3}\,dx$

Optimal. Leaf size=94

$$\frac{b^2\pi\text{FresnelC}\left(\sqrt{2}\,bx\right)}{2\sqrt{2}}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{4x}-\frac{1}{2}b^2\pi\text{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)$$

```
[Out] -1/2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2-1/4*b*sin(b^2*Pi*x^2)/x+1/4*b^2*
Pi*FresnelC(b*x*2^(1/2))*2^(1/2)-1/2*b^2*Pi*Unintegrable(FresnelS(b*x)*sin(
1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^3,x]
```

```
[Out] (b^2*Pi*FresnelC[Sqrt[2]*b*x])/(2*Sqrt[2]) - (Cos[(b^2*Pi*x^2)/2]*FresnelS[
b*x])/(2*x^2) - (b*Sin[b^2*Pi*x^2])/(4*x) - (b^2*Pi*Defer[Int][(FresnelS[b*
x]*Sin[(b^2*Pi*x^2)/2])/x, x])/2
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^3}\,dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2x^2}+\frac{1}{4}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx-\frac{1}{2}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx\\ &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{4x}-\frac{1}{2}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx+\frac{1}{2}\left(b^3\right.\\ &= \frac{b^2\pi C\left(\sqrt{2}\,bx\right)}{2\sqrt{2}}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{2x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{4x}-\frac{1}{2}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\right.}{x}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^3,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^3, x]
```

Maple [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2 \pi x^2}{2}\right) \mathrm{S}(bx)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^3,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^3,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**3,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^3,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^3,x)
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^3, x)
```

3.103 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^4}\,dx$

Optimal. Leaf size=89

$$\frac{1}{12}b^3\pi\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x^3}-\frac{b\sin\left(b^2\pi x^2\right)}{12x^2}-\frac{1}{3}b^2\pi\text{Int}\left(\frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2},x\right)$$

```
[Out] 1/12*b^3*Pi*Ci(b^2*Pi*x^2)-1/3*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^3-1/12*b
*sin(b^2*Pi*x^2)/x^2-1/3*b^2*Pi*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x
^2)/x^2,x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^4,x]
```

```
[Out] (b^3*Pi*CosIntegral[b^2*Pi*x^2])/12 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(
3*x^3) - (b*Sin[b^2*Pi*x^2])/(12*x^2) - (b^2*Pi*Defer[Int][(FresnelS[b*x]*S
in[(b^2*Pi*x^2)/2])/x^2, x])/3
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^4}\,dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x^3}+\frac{1}{6}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^3}\,dx-\frac{1}{3}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,d\\ &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x^3}+\frac{1}{12}b\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^2}\,dx,x,x^2\right)-\frac{1}{3}\left(b^2\pi\right)\int \frac{S(bx)}{}\\ &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x^3}-\frac{b\sin\left(b^2\pi x^2\right)}{12x^2}-\frac{1}{3}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx+\frac{1}{12}(\\ &= \frac{1}{12}b^3\pi\text{Ci}\left(b^2\pi x^2\right)-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{3x^3}-\frac{b\sin\left(b^2\pi x^2\right)}{12x^2}-\frac{1}{3}\left(b^2\pi\right)\int \frac{S(bx)\sin(}{x^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^4,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^4, x]
```

Maple [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right) \mathrm{S}(bx)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^4,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^4, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^4,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^4, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**4,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**4, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^4,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^4,x)
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^4, x)
```

3.104 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^5}\,dx$

Optimal. Leaf size=156

$$\frac{b^3\pi}{16x}-\frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4x^4}-\frac{7b^4\pi^2S\left(\sqrt{2}\,bx\right)}{24\sqrt{2}}+\frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{24x^3}-\frac{1}{8}b^4\pi^2$$

```
[Out] 1/16*b^3*Pi/x-7/48*b^3*Pi*cos(b^2*Pi*x^2)/x-1/4*cos(1/2*b^2*Pi*x^2)*Fresnel
S(b*x)/x^4+1/8*b^2*Pi*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2-1/24*b*sin(b^2*
Pi*x^2)/x^3-7/48*b^4*Pi^2*FresnelS(b*x*2^(1/2))*2^(1/2)-1/8*b^4*Pi^2*Uninte
grable(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Rubi [A]

time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^5,x]
```

```
[Out] (b^3*Pi)/(16*x) - (7*b^3*Pi*Cos[b^2*Pi*x^2])/(48*x) - (Cos[(b^2*Pi*x^2)/2]*
FresnelS[b*x])/(4*x^4) - (7*b^4*Pi^2*FresnelS[Sqrt[2]*b*x])/(24*Sqrt[2]) +
(b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(8*x^2) - (b*Sin[b^2*Pi*x^2])/(2
4*x^3) - (b^4*Pi^2*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x, x])/8
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^5}\,dx &= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4x^4}+\frac{1}{8}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx-\frac{1}{4}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx \\
&= \frac{b^3\pi}{16x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4x^4}+\frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{24x^3}+\frac{1}{16}\left(b^3\pi\right)\int \\
&= \frac{b^3\pi}{16x}-\frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4x^4}+\frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2}-\frac{b\sin(}{24} \\
&= \frac{b^3\pi}{16x}-\frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{4x^4}-\frac{7b^4\pi^2S\left(\sqrt{2}\,bx\right)}{24\sqrt{2}}+\frac{b^2\pi S(bx)\sin}{8x}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^5,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^5, x]
```

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)\mathrm{S}(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^5,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^5, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^5,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^5, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)S(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**5,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**5, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^5, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^5,x)
```

```
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^5, x)
```

3.105 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^6}\,dx$

Optimal. Leaf size=163

$$\frac{b^3\pi}{60x^2}-\frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5x^5}+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x}+\frac{1}{30}b^5\pi^3S(bx)^2+\frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x\right.}{15x^3}$$

```
[Out] 1/60*b^3*Pi/x^2-1/24*b^3*Pi*cos(b^2*Pi*x^2)/x^2-1/5*cos(1/2*b^2*Pi*x^2)*Fre
snelS(b*x)/x^5+1/15*b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x+1/30*b^5*P
i^3*FresnelS(b*x)^2-7/120*b^5*Pi^2*Si(b^2*Pi*x^2)+1/15*b^2*Pi*FresnelS(b*x)
*sin(1/2*b^2*Pi*x^2)/x^3-1/40*b*sin(b^2*Pi*x^2)/x^4
```

Rubi [A]
time = 0.15, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6599, 6591, 6575, 30, 3456, 3461, 3378, 3380, 3460}

$$\frac{1}{30}\pi^3b^5S(bx)^2+\frac{\pi b^3}{60x^2}-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}+\frac{\pi b^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{15x^3}-\frac{b\sin\left(\pi b^2x^2\right)}{40x^4}-\frac{7}{120}\pi^2b^5\text{Si}\left(b^2\pi x^2\right)+\frac{\pi^2b^4S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{15x}-\frac{\pi b^3\cos\left(\pi b^2x^2\right)}{24x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^6,x]
```

```
[Out] (b^3*Pi)/(60*x^2) - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (Cos[(b^2*Pi*x^2)/2
]*FresnelS[b*x])/(5*x^5) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(15
*x) + (b^5*Pi^3*FresnelS[b*x]^2)/30 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/(15*x^3) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^2
*Pi*x^2])/120
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6591

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[d*(x^(m + 2)/(Pi*b*(m + 1)*(m + 2))), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6599

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
 ILtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^6}\,dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5x^5} + \frac{1}{10}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^5}\,dx - \frac{1}{5}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}$$

$$= \frac{b^3\pi}{60x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5x^5} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} + \frac{1}{20}b\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right.}{x^3}\right.$$

$$= \frac{b^3\pi}{60x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5x^5} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3}$$

$$= \frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5x^5} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x} + \frac{b^2\pi}{}$$

$$= \frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5x^5} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x} + \frac{1}{30}b$$

Mathematica [A]
time = 0.01, size = 163, normalized size = 1.00

$$\frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{5x^5} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{15x} + \frac{1}{30}b^5\pi^3 S(bx)^2 + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3} - \frac{b\sin\left(b^2\pi x^2\right)}{40x^4} - \frac{7}{120}b^5\pi^2\text{Si}\left(b^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^6,x]
```

```
[Out] (b^3*Pi)/(60*x^2) - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (Cos[(b^2*Pi*x^2)/2
]*FresnelS[b*x])/(5*x^5) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(15
*x) + (b^5*Pi^3*FresnelS[b*x]^2)/30 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/(15*x^3) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^2
*Pi*x^2])/120
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\mathrm{S}(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^6,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^6, x)
```

Fricas [A]
time = 0.36, size = 141, normalized size = 0.87

$$\frac{4\,\pi^3 b^5 x^5\,\mathrm{S}\left(bx\right)^2-7\,\pi^2 b^5 x^5\,\mathrm{Si}\left(\pi b^2 x^2\right)-10\,\pi b^3 x^3\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2+7\,\pi b^3 x^3+8\left(\pi^2 b^4 x^4-3\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{S}\left(bx\right)+2\left(4\,\pi b^2 x^2\,\mathrm{S}\left(bx\right)-3\,bx\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{120\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^6,x, algorithm="fricas")
```

```
[Out] 1/120*(4*pi^3*b^5*x^5*fresnel_sin(b*x)^2 - 7*pi^2*b^5*x^5*sin_integral(pi*b
^2*x^2) - 10*pi*b^3*x^3*cos(1/2*pi*b^2*x^2)^2 + 7*pi*b^3*x^3 + 8*(pi^2*b^4*
x^4 - 3)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) + 2*(4*pi*b^2*x^2*fresnel_sin
(b*x) - 3*b*x*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/x^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)S(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**6,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^6,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^6,x)
```

```
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^6, x)
```

3.106 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^7}\,dx$

Optimal. Leaf size=231

$$\frac{b^3\pi}{144x^3}-\frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3}-\frac{7b^6\pi^3\text{FresnelC}\left(\sqrt{2}\,bx\right)}{144\sqrt{2}}-\frac{1}{45}\sqrt{2}\,b^6\pi^3\text{FresnelC}\left(\sqrt{2}\,bx\right)-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{6x^6}+$$

```
[Out] 1/144*b^3*Pi/x^3-13/720*b^3*Pi*cos(b^2*Pi*x^2)/x^3-1/6*cos(1/2*b^2*Pi*x^2)*
FresnelS(b*x)/x^6+1/48*b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^2+1/24*
b^2*Pi*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^4-1/60*b*sin(b^2*Pi*x^2)/x^5+67/
1440*b^5*Pi^2*sin(b^2*Pi*x^2)/x-67/1440*b^6*Pi^3*FresnelC(b*x*2^(1/2))*2^(1
/2)+1/48*b^6*Pi^3*Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^7,x]
```

```
[Out] (b^3*Pi)/(144*x^3) - (13*b^3*Pi*Cos[b^2*Pi*x^2])/(720*x^3) - (7*b^6*Pi^3*Fr
esnelC[Sqrt[2]*b*x])/(144*Sqrt[2]) - (Sqrt[2]*b^6*Pi^3*FresnelC[Sqrt[2]*b*x
])/45 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(6*x^6) + (b^4*Pi^2*Cos[(b^2*Pi
*x^2)/2]*FresnelS[b*x])/(48*x^2) + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2
])/(24*x^4) - (b*Sin[b^2*Pi*x^2])/(60*x^5) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/
(1440*x) + (b^6*Pi^3*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/
48
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^7}\,dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{6x^6} + \frac{1}{12}b\int \frac{\sin(b^2\pi x^2)}{x^6}\,dx - \frac{1}{6}(b^2\pi)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

$$= \frac{b^3\pi}{144x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{6x^6} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{24x^4} - \frac{b\sin(b^2\pi x^2)}{60x^5} + \frac{1}{48}(b^3\pi)$$

$$= \frac{b^3\pi}{144x^3} - \frac{13b^3\pi\cos(b^2\pi x^2)}{720x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{6x^6} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{48x^2} + \frac{b}{}$$

$$= \frac{b^3\pi}{144x^3} - \frac{13b^3\pi\cos(b^2\pi x^2)}{720x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{6x^6} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{48x^2} + \frac{b}{}$$

$$= \frac{b^3\pi}{144x^3} - \frac{13b^3\pi\cos(b^2\pi x^2)}{720x^3} - \frac{7b^6\pi^3C\left(\sqrt{2}\,bx\right)}{144\sqrt{2}} - \frac{1}{45}\sqrt{2}\,b^6\pi^3C\left(\sqrt{2}\,bx\right) - \frac{\cos(}{}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^7,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^7, x]
```

Maple [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\mathrm{S}(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^7,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^7,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^7,x, algorithm="maxima")
```

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^7, x)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^7,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^7, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**7,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**7, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^7,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^7, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^7,x)
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^7, x)
```

3.107 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^8}\,dx$

Optimal. Leaf size=202

$$\frac{b^3\pi}{280x^4}-\frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4}-\frac{1}{84}b^7\pi^3\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7x^7}+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{105x^3}+\frac{b^2\pi S(b$$

```
[Out] 1/280*b^3*Pi/x^4-1/84*b^7*Pi^3*Ci(b^2*Pi*x^2)-1/105*b^3*Pi*cos(b^2*Pi*x^2)/
x^4-1/7*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^7+1/105*b^4*Pi^2*cos(1/2*b^2*Pi
*x^2)*FresnelS(b*x)/x^3+1/35*b^2*Pi*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^5-1
/84*b*sin(b^2*Pi*x^2)/x^6+1/84*b^5*Pi^2*sin(b^2*Pi*x^2)/x^2+1/105*b^6*Pi^3*
Unintegrable(FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

Rubi [A]

time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^8,x]
```

```
[Out] (b^3*Pi)/(280*x^4) - (b^3*Pi*Cos[b^2*Pi*x^2])/(105*x^4) - (b^7*Pi^3*CosInte
gral[b^2*Pi*x^2])/84 - (Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(7*x^7) + (b^4*P
i^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(105*x^3) + (b^2*Pi*FresnelS[b*x]*Si
n[(b^2*Pi*x^2)/2])/(35*x^5) - (b*Sin[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi^2*Sin[
b^2*Pi*x^2])/(84*x^2) + (b^6*Pi^3*Defer[Int][(FresnelS[b*x]*Sin[(b^2*Pi*x^2
)/2])/x^2, x])/105
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^8}\,dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7x^7} + \frac{1}{14}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^7}\,dx - \frac{1}{7}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}$$
$$= \frac{b^3\pi}{280x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7x^7} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5} + \frac{1}{28}b\text{Subst}\left(\int \frac{\sin\left(b^2\pi\right.}{x^4}\right.$$
$$= \frac{b^3\pi}{280x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7x^7} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{105x^3} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x\right.}{35x^5}$$
$$= \frac{b^3\pi}{280x^4} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7x^7} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{105x^3} + b^2$$
$$= \frac{b^3\pi}{280x^4} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7x^7} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{105x^3} + b^2$$
$$= \frac{b^3\pi}{280x^4} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4} - \frac{1}{84}b^7\pi^3\text{Ci}\left(b^2\pi x^2\right) - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{7x^7} + \frac{b^4\pi^2\cos}{}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^8,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^8, x]
```

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\mathrm{S}(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^8,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^8,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^8,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^8, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^8,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^8, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**8,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**8, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^8,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^8, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^8,x)
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^8, x)
```

3.108 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^9}\,dx$

Optimal. Leaf size=271

$$\frac{b^3\pi}{480x^5}-\frac{b^7\pi^3}{768x}-\frac{19b^3\pi\cos(b^2\pi x^2)}{3360x^5}+\frac{853b^7\pi^3\cos(b^2\pi x^2)}{80640x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^8}+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{192x^4}+\frac{853}{}$$

```
[Out] 1/480*b^3*Pi/x^5-1/768*b^7*Pi^3/x-19/3360*b^3*Pi*cos(b^2*Pi*x^2)/x^5+853/80
640*b^7*Pi^3*cos(b^2*Pi*x^2)/x-1/8*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^8+1/
192*b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^4+1/48*b^2*Pi*FresnelS(b*x
)*sin(1/2*b^2*Pi*x^2)/x^6-1/384*b^6*Pi^3*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/
x^2-1/112*b*sin(b^2*Pi*x^2)/x^7+187/40320*b^5*Pi^2*sin(b^2*Pi*x^2)/x^3+853/
80640*b^8*Pi^4*FresnelS(b*x*2^(1/2))*2^(1/2)+1/384*b^8*Pi^4*Unintegrable(co
s(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x,x)
```

Rubi [A]
time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^9,x]
```

```
[Out] (b^3*Pi)/(480*x^5) - (b^7*Pi^3)/(768*x) - (19*b^3*Pi*Cos[b^2*Pi*x^2])/(3360
*x^5) + (853*b^7*Pi^3*Cos[b^2*Pi*x^2])/(80640*x) - (Cos[(b^2*Pi*x^2)/2]*Fre
snelS[b*x])/(8*x^8) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(192*x^4
) + (853*b^8*Pi^4*FresnelS[Sqrt[2]*b*x])/(40320*Sqrt[2]) + (b^2*Pi*FresnelS
[b*x]*Sin[(b^2*Pi*x^2)/2])/(48*x^6) - (b^6*Pi^3*FresnelS[b*x]*Sin[(b^2*Pi*x
^2)/2])/(384*x^2) - (b*Sin[b^2*Pi*x^2])/(112*x^7) + (187*b^5*Pi^2*Sin[b^2*P
i*x^2])/(40320*x^3) + (b^8*Pi^4*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*
x])/x, x])/384
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^9}\,dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^8} + \frac{1}{16}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^8}\,dx - \frac{1}{8}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$
$$= \frac{b^3\pi}{480x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^8} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6} - \frac{b\sin\left(b^2\pi x^2\right)}{112x^7} + \frac{1}{96}\left(b^3\pi\right)$$
$$= \frac{b^3\pi}{480x^5} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^8} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{192x^4} + \frac{b}{}$$
$$= \frac{b^3\pi}{480x^5} - \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^8} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S}{192x^4}$$
$$= \frac{b^3\pi}{480x^5} - \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} + \frac{853b^7\pi^3\cos\left(b^2\pi x^2\right)}{80640x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^8}$$
$$= \frac{b^3\pi}{480x^5} - \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} + \frac{853b^7\pi^3\cos\left(b^2\pi x^2\right)}{80640x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{8x^8}$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^9,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^9, x]
```

Maple [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\mathrm{S}(bx)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^9,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^9,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^9,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^9, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^9,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^9, x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**9,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**9, x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^9,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^9, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^9,x)
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^9, x)
```

3.109 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^{10}}\,dx$

Optimal. Leaf size=278

$$\frac{b^3\pi}{756x^6}-\frac{b^7\pi^3}{3780x^2}-\frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6}+\frac{5b^7\pi^3\cos\left(b^2\pi x^2\right)}{2016x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9}+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{315x^5}-\frac{b^8\pi^4}{}$$

```
[Out] 1/756*b^3*Pi/x^6-1/3780*b^7*Pi^3/x^2-11/3024*b^3*Pi*cos(b^2*Pi*x^2)/x^6+5/2
016*b^7*Pi^3*cos(b^2*Pi*x^2)/x^2-1/9*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^9+
1/315*b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^5-1/945*b^8*Pi^4*cos(1/2
*b^2*Pi*x^2)*FresnelS(b*x)/x-1/1890*b^9*Pi^5*FresnelS(b*x)^2+83/30240*b^9*P
i^4*Si(b^2*Pi*x^2)+1/63*b^2*Pi*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^7-1/945*
b^6*Pi^3*FresnelS(b*x)*sin(1/2*b^2*Pi*x^2)/x^3-1/144*b*sin(b^2*Pi*x^2)/x^8+
67/30240*b^5*Pi^2*sin(b^2*Pi*x^2)/x^4
```

Rubi [A]
time = 0.34, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6599, 6591, 6575, 30, 3456, 3461, 3378, 3380, 3460}

$$-\frac{\pi^5b^9S(bx)^2}{1890}-\frac{\pi^3b^7}{3780x^2}+\frac{\pi b^3}{756x^6}-\frac{S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}+\frac{\pi b^2S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{63x^7}-\frac{b\sin\left(\pi b^2x^2\right)}{144x^8}+\frac{83\pi^4b^9\text{Si}\left(b^2\pi x^2\right)}{30240}-\frac{\pi^4b^8S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{945x}+\frac{5\pi^3b^7\cos\left(\pi b^2x^2\right)}{2016x^2}-\frac{\pi^3b^6S(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{945x^3}+\frac{67\pi^2b^5\sin\left(\pi b^2x^2\right)}{30240x^4}+\frac{\pi^2b^4S(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{315x^5}-\frac{11\pi b^3\cos\left(\pi b^2x^2\right)}{3024x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^10,x]
```

```
[Out] (b^3*Pi)/(756*x^6) - (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(3
024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (Cos[(b^2*Pi*x^2)/2]*F
resnelS[b*x])/(9*x^9) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(315*x
^5) - (b^8*Pi^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(945*x) - (b^9*Pi^5*Fres
nelS[b*x]^2)/1890 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(63*x^7) - (
b^6*Pi^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x^3) - (b*Sin[b^2*Pi*x^2])
/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinIn
tegral[b^2*Pi*x^2])/30240
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6575

```
Int[FresnelS[(b_.)*(x_)]^(n_.)*Sin[(d_.)*(x_)^2], x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelS[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6591

```
Int[FresnelS[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelS[b*x], x], x] + Dist[d/(Pi*b*(m + 1)), Int[x^(m
 + 1)*Cos[2*d*x^2], x], x] - Simp[d*(x^(m + 2)/(Pi*b*(m + 1)*(m + 2))), x])
 /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6599

```
Int[Cos[(d_.)*(x_)^2]*FresnelS[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelS[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelS[b*x], x], x] - Dist[d/(Pi*b*(m + 1)), Int[x^(m
+ 1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] &&
```

```
ILtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{x^{10}}\,dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9} + \frac{1}{18}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^9}\,dx - \frac{1}{9}\left(b^2\pi\right)\int \frac{S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$$

$$= \frac{b^3\pi}{756x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} + \frac{1}{36}b\text{Subst}\Bigg(\int \frac{\sin\left(b^2\pi x\right.}{x^5}$$

$$= \frac{b^3\pi}{756x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{315x^5} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7}$$

$$= \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5}$$

$$= \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5}$$

$$= \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} + \frac{5b^7\pi^3\cos\left(b^2\pi x^2\right)}{2016x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9}$$

$$= \frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} + \frac{5b^7\pi^3\cos\left(b^2\pi x^2\right)}{2016x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9}$$

Mathematica [A]
time = 0.01, size = 278, normalized size = 1.00

$$\frac{b^3\pi}{756x^6} - \frac{b^7\pi^3}{3780x^2} - \frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6} + \frac{5b^7\pi^3\cos\left(b^2\pi x^2\right)}{2016x^2} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{315x^5} - \frac{b^8\pi^4\cos\left(\frac{1}{2}b^2\pi x^2\right)S(bx)}{945x} - \frac{b^9\pi^5 S(bx)^2}{1890} + \frac{b^2\pi S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} - \frac{b^6\pi^3 S(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x^3} - \frac{b\sin\left(b^2\pi x^2\right)}{144x^8} + \frac{67b^5\pi^2\sin\left(b^2\pi x^2\right)}{30240x^4} + \frac{83b^9\pi^4\text{Si}\left(b^2\pi x^2\right)}{30240}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/x^10,x]
```

```
[Out] (b^3*Pi)/(756*x^6) - (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(3
024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (Cos[(b^2*Pi*x^2)/2]*F
resnelS[b*x])/(9*x^9) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(315*x
^5) - (b^8*Pi^4*Cos[(b^2*Pi*x^2)/2]*FresnelS[b*x])/(945*x) - (b^9*Pi^5*Fres
nelS[b*x]^2)/1890 + (b^2*Pi*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(63*x^7) - (
b^6*Pi^3*FresnelS[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x^3) - (b*Sin[b^2*Pi*x^2])
/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinIn
tegral[b^2*Pi*x^2])/30240
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{S}(bx)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^10,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelS(b*x)/x^10,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^10, x)
```

Fricas [A]
time = 0.37, size = 203, normalized size = 0.73

$$-\frac{16\,\pi^5 b^9 x^9\,\mathrm{S}\left(bx\right)^2 - 83\,\pi^4 b^9 x^9\,\mathrm{Si}\left(\pi b^2 x^2\right) + 83\,\pi^3 b^7 x^7 - 150\,\pi b^3 x^3 - 10\left(15\,\pi^3 b^7 x^7 - 22\,\pi b^3 x^3\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 32\left(\pi^4 b^8 x^8 - 3\,\pi^2 b^4 x^4 + 105\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{S}\left(bx\right) - 2\left(\left(67\,\pi^2 b^5 x^5 - 210\,bx\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - 16\left(\pi^3 b^6 x^6 - 15\,\pi b^2 x^2\right)\mathrm{S}\left(bx\right)\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{30240\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^10,x, algorithm="fricas")
```

```
[Out] -1/30240*(16*pi^5*b^9*x^9*fresnel_sin(b*x)^2 - 83*pi^4*b^9*x^9*sin_integral
(pi*b^2*x^2) + 83*pi^3*b^7*x^7 - 150*pi*b^3*x^3 - 10*(15*pi^3*b^7*x^7 - 22*
pi*b^3*x^3)*cos(1/2*pi*b^2*x^2)^2 + 32*(pi^4*b^8*x^8 - 3*pi^2*b^4*x^4 + 105
)*cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x) - 2*((67*pi^2*b^5*x^5 - 210*b*x)*cos
(1/2*pi*b^2*x^2) - 16*(pi^3*b^6*x^6 - 15*pi*b^2*x^2)*fresnel_sin(b*x))*sin(
1/2*pi*b^2*x^2))/x^9
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) S(bx)}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnels(b*x)/x**10,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnels(b*x)/x**10, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_sin(b*x)/x^10,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_sin(b*x)/x^10, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelS}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^10,x)
```

```
[Out] int((FresnelS(b*x)*cos((Pi*b^2*x^2)/2))/x^10, x)
```

3.110 $\int x^7 \text{FresnelC}(bx)\,dx$

Optimal. Leaf size=124

$$\frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4}-\frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2}-\frac{105\text{FresnelC}(bx)}{8b^8\pi^4}+\frac{1}{8}x^8\text{FresnelC}(bx)+\frac{35x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3}-\frac{x^7\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi}$$

```
[Out] 105/8*x*cos(1/2*b^2*Pi*x^2)/b^7/Pi^4-7/8*x^5*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2-1
05/8*FresnelC(b*x)/b^8/Pi^4+1/8*x^8*FresnelC(b*x)+35/8*x^3*sin(1/2*b^2*Pi*x
^2)/b^5/Pi^3-1/8*x^7*sin(1/2*b^2*Pi*x^2)/b/Pi
```

Rubi [A]
time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6562, 3467, 3466, 3433}

$$-\frac{105\text{FresnelC}(bx)}{8\pi^4b^8}-\frac{x^7\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi b}+\frac{105x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^4b^7}+\frac{35x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^3b^5}-\frac{7x^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{8\pi^2b^3}+\frac{1}{8}x^8\text{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^7*FresnelC[b*x],x]
```

```
[Out] (105*x*Cos[(b^2*Pi*x^2)/2])/(8*b^7*Pi^4) - (7*x^5*Cos[(b^2*Pi*x^2)/2])/(8*b
^3*Pi^2) - (105*FresnelC[b*x])/(8*b^8*Pi^4) + (x^8*FresnelC[b*x])/8 + (35*x
^3*Sin[(b^2*Pi*x^2)/2])/(8*b^5*Pi^3) - (x^7*Sin[(b^2*Pi*x^2)/2])/(8*b*Pi)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^7 C(bx)\,dx = \frac{1}{8}x^8 C(bx) - \frac{1}{8}b\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$
$$= \frac{1}{8}x^8 C(bx) - \frac{x^7\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{7\int x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{8b\pi}$$
$$= -\frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} + \frac{1}{8}x^8 C(bx) - \frac{x^7\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} + \frac{35\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{8b^3\pi^2}$$
$$= -\frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} + \frac{1}{8}x^8 C(bx) + \frac{35x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} - \frac{x^7\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi} - \frac{105\int x^2\sin\left(\frac{1}{2}b^2\pi\right.}{8b^5\pi^3}$$
$$= \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} + \frac{1}{8}x^8 C(bx) + \frac{35x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} - \frac{x^7\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b\pi}$$
$$= \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)}{8b^3\pi^2} - \frac{105C(bx)}{8b^8\pi^4} + \frac{1}{8}x^8 C(bx) + \frac{35x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^5\pi^3} - \frac{x^7}{}$$

Mathematica [A]
time = 0.05, size = 89, normalized size = 0.72

$$\frac{-7bx(-15+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right) + (-105+b^8\pi^4x^8)\,\text{FresnelC}(bx) + b^3\pi x^3(35-b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8b^8\pi^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*FresnelC[b*x],x]
[Out] (-7*b*x*(-15 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8)*Fr
esnelC[b*x] + b^3*Pi*x^3*(35 - b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(8*b^8*Pi
^4)
```

Maple [A]
time = 0.30, size = 123, normalized size = 0.99

method	result
meijerg	$\frac{b\,x^9\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{9}{4}\right],\left[\frac{1}{2},\frac{5}{4},\frac{13}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{9}$

derivativedivides	$\frac{\frac{\mathrm{FresnelC}(bx) b^8 x^8}{8} - \frac{b^7 x^7 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{8\pi} + \frac{-\frac{7 b^5 x^5 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{8\pi} + \frac{7\left(\frac{5 b^3 x^3 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{\pi} - \frac{15\left(-\frac{bx \cos\left(\frac{b^2 \pi x^2}{2}\right)}{\pi} + \frac{\mathrm{FresnelC}(bx)}{\pi}\right)}{\pi}\right)}{8\pi}}{\pi}}{b^8}$
default	$\frac{\frac{\mathrm{FresnelC}(bx) b^8 x^8}{8} - \frac{b^7 x^7 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{8\pi} + \frac{-\frac{7 b^5 x^5 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{8\pi} + \frac{7\left(\frac{5 b^3 x^3 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{\pi} - \frac{15\left(-\frac{bx \cos\left(\frac{b^2 \pi x^2}{2}\right)}{\pi} + \frac{\mathrm{FresnelC}(bx)}{\pi}\right)}{\pi}\right)}{8\pi}}{\pi}}{b^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^8*(1/8*FresnelC(b*x)*b^8*x^8-1/8/Pi*b^7*x^7*sin(1/2*b^2*Pi*x^2)+7/8/Pi*
(-1/Pi*b^5*x^5*cos(1/2*b^2*Pi*x^2)+5/Pi*(1/Pi*b^3*x^3*sin(1/2*b^2*Pi*x^2)-3
/Pi*(-1/Pi*b*x*cos(1/2*b^2*Pi*x^2)+1/Pi*FresnelC(b*x)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.49, size = 126, normalized size = 1.02

$$\frac{1}{8}x^8 \,\mathrm{C}(bx) - \frac{\sqrt{\frac{1}{2}}\left(-(105i-105)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi \operatorname{erf}\left(\sqrt{\frac{1}{2}i\pi}\,bx\right) + (105i+105)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi \operatorname{erf}\left(\sqrt{-\frac{1}{2}i\pi}\,bx\right) + 28\left(\sqrt{\frac{1}{2}}\pi^3 b^5 x^5 - 15\sqrt{\frac{1}{2}}\pi bx\right)\cos\left(\frac{1}{2}\pi b^2 x^2\right) + 4\left(\sqrt{\frac{1}{2}}\pi^4 b^7 x^7 - 35\sqrt{\frac{1}{2}}\pi^2 b^3 x^3\right)\sin\left(\frac{1}{2}\pi b^2 x^2\right)\right)}{16\,\pi^5 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] 1/8*x^8*fresnel_cos(b*x) - 1/16*sqrt(1/2)*(-(105*I - 105)*(1/4)^(1/4)*pi*er
f(sqrt(1/2*I*pi)*b*x) + (105*I + 105)*(1/4)^(1/4)*pi*erf(sqrt(-1/2*I*pi)*b*
x) + 28*(sqrt(1/2)*pi^3*b^5*x^5 - 15*sqrt(1/2)*pi*b*x)*cos(1/2*pi*b^2*x^2)
+ 4*(sqrt(1/2)*pi^4*b^7*x^7 - 35*sqrt(1/2)*pi^2*b^3*x^3)*sin(1/2*pi*b^2*x^2
))/(pi^5*b^8)
```

Fricas [A]
time = 0.36, size = 85, normalized size = 0.69

$$-\frac{7\left(\pi^2 b^5 x^5 - 15\,bx\right)\cos\left(\frac{1}{2}\pi b^2 x^2\right) - \left(\pi^4 b^8 x^8 - 105\right)\mathrm{C}(bx) + \left(\pi^3 b^7 x^7 - 35\,\pi b^3 x^3\right)\sin\left(\frac{1}{2}\pi b^2 x^2\right)}{8\,\pi^4 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] -1/8*(7*(pi^2*b^5*x^5 - 15*b*x)*cos(1/2*pi*b^2*x^2) - (pi^4*b^8*x^8 - 105)*
fresnel_cos(b*x) + (pi^3*b^7*x^7 - 35*pi*b^3*x^3)*sin(1/2*pi*b^2*x^2))/(pi^
4*b^8)
```

Sympy [A]
time = 1.25, size = 184, normalized size = 1.48

$$\frac{45x^8C(bx)\,\Gamma\left(\frac{1}{4}\right)}{512\Gamma\left(\frac{13}{4}\right)}-\frac{45x^7\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{512\pi b\Gamma\left(\frac{13}{4}\right)}-\frac{315x^5\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{512\pi^2b^3\Gamma\left(\frac{13}{4}\right)}+\frac{1575x^3\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{512\pi^3b^5\Gamma\left(\frac{13}{4}\right)}+\frac{4725x\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{512\pi^4b^7\Gamma\left(\frac{13}{4}\right)}-\frac{4725C(bx)\,\Gamma\left(\frac{1}{4}\right)}{512\pi^4b^8\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*fresnelc(b*x),x)
```

```
[Out] 45*x**8*fresnelc(b*x)*gamma(1/4)/(512*gamma(13/4)) - 45*x**7*sin(pi*b**2*x*
*2/2)*gamma(1/4)/(512*pi*b*gamma(13/4)) - 315*x**5*cos(pi*b**2*x**2/2)*gamm
a(1/4)/(512*pi**2*b**3*gamma(13/4)) + 1575*x**3*sin(pi*b**2*x**2/2)*gamma(1
/4)/(512*pi**3*b**5*gamma(13/4)) + 4725*x*cos(pi*b**2*x**2/2)*gamma(1/4)/(5
12*pi**4*b**7*gamma(13/4)) - 4725*fresnelc(b*x)*gamma(1/4)/(512*pi**4*b**8*
gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnel_cos(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^7*fresnel_cos(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^7\,\mathrm{FresnelC}(b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelC(b*x),x)
```

```
[Out] int(x^7*FresnelC(b*x), x)
```

3.111 $\int x^6\textbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=109

$$\frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4}-\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2}+\frac{1}{7}x^7\text{FresnelC}(bx)+\frac{24x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3}-\frac{x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi}$$

```
[Out] 48/7*cos(1/2*b^2*Pi*x^2)/b^7/Pi^4-6/7*x^4*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2+1/7*
x^7*FresnelC(b*x)+24/7*x^2*sin(1/2*b^2*Pi*x^2)/b^5/Pi^3-1/7*x^6*sin(1/2*b^2
*Pi*x^2)/b/Pi
```

Rubi [A]
time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6562, 3461, 3377, 2718}

$$-\frac{x^6\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b}+\frac{48\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7}+\frac{24x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5}-\frac{6x^4\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3}+\frac{1}{7}x^7\text{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^6*FresnelC[b*x],x]
```

```
[Out] (48*Cos[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) - (6*x^4*Cos[(b^2*Pi*x^2)/2])/(7*b^3*
Pi^2) + (x^7*FresnelC[b*x])/7 + (24*x^2*Sin[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) -
 (x^6*Sin[(b^2*Pi*x^2)/2])/(7*b*Pi)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^6 C(bx)\,dx &= \frac{1}{7}x^7C(bx)-\frac{1}{7}b\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\ &= \frac{1}{7}x^7C(bx)-\frac{1}{14}b\text{Subst}\left(\int x^3\cos\left(\frac{1}{2}b^2\pi x\right)\,dx,x,x^2\right)\\ &= \frac{1}{7}x^7C(bx)-\frac{x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi}+\frac{3\text{Subst}\left(\int x^2\sin\left(\frac{1}{2}b^2\pi x\right)\,dx,x,x^2\right)}{7b\pi}\\ &= -\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2}+\frac{1}{7}x^7C(bx)-\frac{x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi}+\frac{12\text{Subst}\left(\int x\cos\left(\frac{1}{2}b^2\pi x\right)\,dx,x,x^2\right)}{7b^3\pi^2}\\ &= -\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2}+\frac{1}{7}x^7C(bx)+\frac{24x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3}-\frac{x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi}-\frac{24\text{Subst}\left(\int\sin\left(\frac{1}{2}\right.\right.}{7b^5}\\ &= \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4}-\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2}+\frac{1}{7}x^7C(bx)+\frac{24x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3}-\frac{x^6\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 83, normalized size = 0.76

$$-\frac{6(-8+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^7\pi^4}+\frac{1}{7}x^7\text{FresnelC}(bx)-\frac{x^2(-24+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*FresnelC[b*x],x]
```

```
[Out] (-6*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/(7*b^7*Pi^4) + (x^7*FresnelC[b
*x])/7 - (x^2*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3)
```

Maple [A]
time = 0.31, size = 107, normalized size = 0.98

method	result	size
meijerg	$\frac{b\,x^8\,\text{hypergeom}\left(\left[\frac{1}{4},2\right],\left[\frac{1}{2},\frac{5}{4},3\right],-\frac{x^4\pi^2b^4}{16}\right)}{8}$	26
derivativedivides	$\frac{\frac{\text{FresnelC}(bx)b^7x^7}{7}-\frac{b^6x^6\sin\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}+\frac{-\frac{6b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}+\frac{6\left(\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{7\pi}}{\pi}}{b^7}$	107

default	$\frac{\frac{\mathrm{FresnelC}(bx)b^7x^7}{7}-\frac{b^6x^6\sin\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}+\frac{-\frac{6b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{7\pi}+\frac{6\left(\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{7\pi}}{\pi}}{b^7}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^7*(1/7*FresnelC(b*x)*b^7*x^7-1/7/Pi*b^6*x^6*sin(1/2*b^2*Pi*x^2)+6/7/Pi*
(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2
/Pi^2*cos(1/2*b^2*Pi*x^2))))
```

Maxima [A]
time = 0.27, size = 74, normalized size = 0.68

$$\frac{1}{7}\,x^7\,\mathrm{C}\left(bx\right)-\frac{6\left(\pi^2b^4x^4-8\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)+\left(\pi^3b^6x^6-24\,\pi b^2x^2\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{7\,\pi^4b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] 1/7*x^7*fresnel_cos(b*x) - 1/7*(6*(pi^2*b^4*x^4 - 8)*cos(1/2*pi*b^2*x^2) +
(pi^3*b^6*x^6 - 24*pi*b^2*x^2)*sin(1/2*pi*b^2*x^2))/(pi^4*b^7)
```

Fricas [A]
time = 0.37, size = 79, normalized size = 0.72

$$\frac{\pi^4b^7x^7\,\mathrm{C}\left(bx\right)-6\left(\pi^2b^4x^4-8\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)-\left(\pi^3b^6x^6-24\,\pi b^2x^2\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{7\,\pi^4b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] 1/7*(pi^4*b^7*x^7*fresnel_cos(b*x) - 6*(pi^2*b^4*x^4 - 8)*cos(1/2*pi*b^2*x^
2) - (pi^3*b^6*x^6 - 24*pi*b^2*x^2)*sin(1/2*pi*b^2*x^2))/(pi^4*b^7)
```

Sympy [A]
time = 1.18, size = 153, normalized size = 1.40

$$\frac{x^7C(bx)\,\Gamma\left(\frac{1}{4}\right)}{28\Gamma\left(\frac{5}{4}\right)}-\frac{x^6\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{28\pi b\Gamma\left(\frac{5}{4}\right)}-\frac{3x^4\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{14\pi^2b^3\Gamma\left(\frac{5}{4}\right)}+\frac{6x^2\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{7\pi^3b^5\Gamma\left(\frac{5}{4}\right)}+\frac{12\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{7\pi^4b^7\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnelc(b*x),x)
```

```
[Out] x**7*fresnelc(b*x)*gamma(1/4)/(28*gamma(5/4)) - x**6*sin(pi*b**2*x**2/2)*ga
mma(1/4)/(28*pi*b*gamma(5/4)) - 3*x**4*cos(pi*b**2*x**2/2)*gamma(1/4)/(14*p
```

```
i**2*b**3*gamma(5/4)) + 6*x**2*sin(pi*b**2*x**2/2)*gamma(1/4)/(7*pi**3*b**5
*gamma(5/4)) + 12*cos(pi*b**2*x**2/2)*gamma(1/4)/(7*pi**4*b**7*gamma(5/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnel_cos(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^6*fresnel_cos(b*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\,\mathrm{FresnelC}(b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelC(b*x),x)
```

```
[Out] int(x^6*FresnelC(b*x), x)
```

3.112 $\int x^5 \mathbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=99

$$-\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2}+\frac{1}{6}x^6\text{FresnelC}(bx)-\frac{5S(bx)}{2b^6\pi^3}+\frac{5x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3}-\frac{x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi}$$

```
[Out] -5/6*x^3*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2+1/6*x^6*FresnelC(b*x)-5/2*FresnelS(b*
x)/b^6/Pi^3+5/2*x*sin(1/2*b^2*Pi*x^2)/b^5/Pi^3-1/6*x^5*sin(1/2*b^2*Pi*x^2)/
b/Pi
```

Rubi [A]
time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6562, 3467, 3466, 3432}

$$-\frac{5S(bx)}{2\pi^3b^6}-\frac{x^5\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi b}+\frac{5x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^3b^5}-\frac{5x^3\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6\pi^2b^3}+\frac{1}{6}x^6\text{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*FresnelC[b*x],x]
```

```
[Out] (-5*x^3*Cos[(b^2*Pi*x^2)/2])/(6*b^3*Pi^2) + (x^6*FresnelC[b*x])/6 - (5*Fres
nelS[b*x])/(2*b^6*Pi^3) + (5*x*Sin[(b^2*Pi*x^2)/2])/(2*b^5*Pi^3) - (x^5*Sin
[(b^2*Pi*x^2)/2])/(6*b*Pi)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^5 C(bx)\,dx &= \frac{1}{6}x^6 C(bx) - \frac{1}{6}b\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\ &= \frac{1}{6}x^6 C(bx) - \frac{x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{5\int x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{6b\pi}\\ &= -\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{1}{6}x^6 C(bx) - \frac{x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} + \frac{5\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b^3\pi^2}\\ &= -\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{1}{6}x^6 C(bx) + \frac{5x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3} - \frac{x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi} - \frac{5\int \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b^5\pi^3}\\ &= -\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6b^3\pi^2} + \frac{1}{6}x^6 C(bx) - \frac{5S(bx)}{2b^6\pi^3} + \frac{5x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b^5\pi^3} - \frac{x^5\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b\pi}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 80, normalized size = 0.81

$$\frac{-5b^3\pi x^3\cos\left(\frac{1}{2}b^2\pi x^2\right) + b^6\pi^3x^6\text{FresnelC}(bx) - 15S(bx) + bx(15 - b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6b^6\pi^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*FresnelC[b*x],x]
[Out] (-5*b^3*Pi*x^3*Cos[(b^2*Pi*x^2)/2] + b^6*Pi^3*x^6*FresnelC[b*x] - 15*Fresne
lS[b*x] + b*x*(15 - b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(6*b^6*Pi^3)
```

Maple [A]
time = 0.32, size = 97, normalized size = 0.98

method	result	size
meijerg	$\frac{b\,x^7\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{7}{4}\right],\left[\frac{1}{2},\frac{5}{4},\frac{11}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{7}$	26
derivativedivides	$\frac{\frac{\mathrm{FresnelC}(bx)b^6x^6}{6}-\frac{b^5x^5\sin\left(\frac{b^2\pi x^2}{2}\right)}{6\pi}+\frac{-\frac{5b^3x^3\cos\left(\frac{b^2\pi x^2}{2}\right)}{6\pi}+\frac{5\left(\frac{3bx\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{3\,\mathrm{S}(bx)}{\pi}\right)}{6\pi}}{\pi}}{b^6}$	97

default	$\frac{\frac{\text{FresnelC}(bx)b^6x^6}{6}-\frac{b^5x^5\sin\left(\frac{b^2\pi x^2}{2}\right)}{6\pi}+\frac{-\frac{5b^3x^3\cos\left(\frac{b^2\pi x^2}{2}\right)}{6\pi}+\frac{5\left(\frac{3bx\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{3\,\mathrm{S}(bx)}{\pi}\right)}{6\pi}}{\pi}}{b^6}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^6*(1/6*FresnelC(b*x)*b^6*x^6-1/6/Pi*b^5*x^5*sin(1/2*b^2*Pi*x^2)+5/6/Pi*
(-1/Pi*b^3*x^3*cos(1/2*b^2*Pi*x^2)+3/Pi*(1/Pi*b*x*sin(1/2*b^2*Pi*x^2)-1/Pi*
FresnelS(b*x))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.48, size = 110, normalized size = 1.11

$$\frac{1}{6}\,x^6\,\mathrm{C}\left(bx\right)-\frac{\sqrt{\frac{1}{2}}\left(20\sqrt{\frac{1}{2}}\,\pi^2b^3x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)+\left(15i+15\right)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{\frac{1}{2}i\,\pi}\,bx\right)-\left(15i-15\right)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\mathrm{erf}\left(\sqrt{-\frac{1}{2}i\,\pi}\,bx\right)+4\left(\sqrt{\frac{1}{2}}\,\pi^3b^5x^5-15\sqrt{\frac{1}{2}}\,\pi bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)\right)}{12\,\pi^4b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] 1/6*x^6*fresnel_cos(b*x) - 1/12*sqrt(1/2)*(20*sqrt(1/2)*pi^2*b^3*x^3*cos(1/
2*pi*b^2*x^2) + (15*I + 15)*(1/4)^(1/4)*pi*erf(sqrt(1/2*I*pi)*b*x) - (15*I
- 15)*(1/4)^(1/4)*pi*erf(sqrt(-1/2*I*pi)*b*x) + 4*(sqrt(1/2)*pi^3*b^5*x^5 -
 15*sqrt(1/2)*pi*b*x)*sin(1/2*pi*b^2*x^2))/(pi^4*b^6)
```

Fricas [A]
time = 0.35, size = 86, normalized size = 0.87

$$\frac{\pi^3b^7x^6\,\mathrm{C}\left(bx\right)-5\,\pi b^4x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)-\left(\pi^2b^6x^5-15\,b^2x\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)-15\,\sqrt{b^2}\,\mathrm{S}\left(\sqrt{b^2}\,x\right)}{6\,\pi^3b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] 1/6*(pi^3*b^7*x^6*fresnel_cos(b*x) - 5*pi*b^4*x^3*cos(1/2*pi*b^2*x^2) - (pi
^2*b^6*x^5 - 15*b^2*x)*sin(1/2*pi*b^2*x^2) - 15*sqrt(b^2)*fresnel_sin(sqrt(
b^2)*x))/(pi^3*b^7)
```

Sympy [A]
time = 0.53, size = 49, normalized size = 0.49

$$\frac{bx^7\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{7}{4}\right){}_2F_3\left(\left.\begin{matrix}\frac{1}{4},\frac{7}{4}\\\frac{1}{2},\frac{5}{4},\frac{11}{4}\end{matrix}\right|-\frac{\pi^2b^4x^4}{16}\right)}{16\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*fresnelc(b*x),x)
```

```
[Out] b*x**7*gamma(1/4)*gamma(7/4)*hyper((1/4, 7/4), (1/2, 5/4, 11/4), -pi**2*b**
4*x**4/16)/(16*gamma(5/4)*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnel_cos(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^5*fresnel_cos(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \operatorname{FresnelC}(b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelC(b*x),x)
```

```
[Out] int(x^5*FresnelC(b*x), x)
```

3.113 $\int x^4 \textbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=84

$$-\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2}+\frac{1}{5}x^5\text{FresnelC}(bx)+\frac{8\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3}-\frac{x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi}$$

```
[Out] -4/5*x^2*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2+1/5*x^5*FresnelC(b*x)+8/5*sin(1/2*b^2
*Pi*x^2)/b^5/Pi^3-1/5*x^4*sin(1/2*b^2*Pi*x^2)/b/Pi
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {6562, 3461, 3377, 2717}

$$-\frac{x^4\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b}+\frac{8\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5}-\frac{4x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3}+\frac{1}{5}x^5\text{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*FresnelC[b*x],x]
```

```
[Out] (-4*x^2*Cos[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2) + (x^5*FresnelC[b*x])/5 + (8*Sin[
(b^2*Pi*x^2)/2])/(5*b^5*Pi^3) - (x^4*Sin[(b^2*Pi*x^2)/2])/(5*b*Pi)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
```

```
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^4 C(bx)\,dx &= \frac{1}{5}x^5 C(bx) - \frac{1}{5}b\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{1}{5}x^5 C(bx) - \frac{1}{10}b\text{Subst}\left(\int x^2\cos\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right) \\
&= \frac{1}{5}x^5 C(bx) - \frac{x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{2\text{Subst}\left(\int x\sin\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{5b\pi} \\
&= -\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} + \frac{1}{5}x^5 C(bx) - \frac{x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{4\text{Subst}\left(\int\cos\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{5b^3\pi^2} \\
&= -\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} + \frac{1}{5}x^5 C(bx) + \frac{8\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} - \frac{x^4\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 71, normalized size = 0.85

$$-\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{5b^3\pi^2} + \frac{1}{5}x^5\text{FresnelC}(bx) - \frac{(-8+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*FresnelC[b*x],x]
[Out] (-4*x^2*Cos[(b^2*Pi*x^2)/2])/(5*b^3*Pi^2) + (x^5*FresnelC[b*x])/5 - ((-8 +
b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(5*b^5*Pi^3)
```

Maple [A]
time = 0.33, size = 81, normalized size = 0.96

method	result	size
meijerg	$\frac{b\,x^6\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{1}{2},\frac{5}{4},\frac{5}{2}\right],-\frac{x^4\pi^2b^4}{16}\right)}{6}$	26
derivativedivides	$\frac{\frac{\text{FresnelC}(bx)b^5x^5}{5}-\frac{b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{5\pi}+\frac{-\frac{4b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi}+\frac{8\sin\left(\frac{b^2\pi x^2}{2}\right)}{5\pi^2}}{\pi}}{b^5}$	81
default	$\frac{\frac{\text{FresnelC}(bx)b^5x^5}{5}-\frac{b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{5\pi}+\frac{-\frac{4b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{5\pi}+\frac{8\sin\left(\frac{b^2\pi x^2}{2}\right)}{5\pi^2}}{\pi}}{b^5}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/5*FresnelC(b*x)*b^5*x^5-1/5/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)+4/5/Pi*
(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2)))
```

Maxima [A]
time = 0.28, size = 61, normalized size = 0.73

$$\frac{1}{5}\,x^5\,\mathrm{C}\left(bx\right)-\frac{4\,\pi b^2x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)+\left(\pi^2b^4x^4-8\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{5\,\pi^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*fresnel_cos(b*x) - 1/5*(4*pi*b^2*x^2*cos(1/2*pi*b^2*x^2) + (pi^2*b^
4*x^4 - 8)*sin(1/2*pi*b^2*x^2))/(pi^3*b^5)
```

Fricas [A]
time = 0.35, size = 66, normalized size = 0.79

$$\frac{\pi^3b^5x^5\,\mathrm{C}\left(bx\right)-4\,\pi b^2x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)-\left(\pi^2b^4x^4-8\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{5\,\pi^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] 1/5*(pi^3*b^5*x^5*fresnel_cos(b*x) - 4*pi*b^2*x^2*cos(1/2*pi*b^2*x^2) - (pi
^2*b^4*x^4 - 8)*sin(1/2*pi*b^2*x^2))/(pi^3*b^5)
```

Sympy [A]
time = 0.62, size = 116, normalized size = 1.38

$$\frac{x^5C(bx)\,\Gamma\left(\frac{1}{4}\right)}{20\Gamma\left(\frac{5}{4}\right)}-\frac{x^4\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{20\pi b\Gamma\left(\frac{5}{4}\right)}-\frac{x^2\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{5\pi^2b^3\Gamma\left(\frac{5}{4}\right)}+\frac{2\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{5\pi^3b^5\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnelc(b*x),x)
```

```
[Out] x**5*fresnelc(b*x)*gamma(1/4)/(20*gamma(5/4)) - x**4*sin(pi*b**2*x**2/2)*ga
mma(1/4)/(20*pi*b*gamma(5/4)) - x**2*cos(pi*b**2*x**2/2)*gamma(1/4)/(5*pi**
2*b**3*gamma(5/4)) + 2*sin(pi*b**2*x**2/2)*gamma(1/4)/(5*pi**3*b**5*gamma(5
/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnel_cos(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnel_cos(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \operatorname{FresnelC}(b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelC(b*x),x)
```

```
[Out] int(x^4*FresnelC(b*x), x)
```

3.114 $\int x^3\text{FresnelC}(bx)\,dx$

Optimal. Leaf size=74

$$-\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2}+\frac{3\text{FresnelC}(bx)}{4b^4\pi^2}+\frac{1}{4}x^4\text{FresnelC}(bx)-\frac{x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi}$$

```
[Out] -3/4*x*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2+3/4*FresnelC(b*x)/b^4/Pi^2+1/4*x^4*Fres
nelC(b*x)-1/4*x^3*sin(1/2*b^2*Pi*x^2)/b/Pi
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6562, 3467, 3466, 3433}

$$\frac{3\text{FresnelC}(bx)}{4\pi^2b^4}-\frac{x^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b}-\frac{3x\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^2b^3}+\frac{1}{4}x^4\text{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelC[b*x],x]
```

```
[Out] (-3*x*Cos[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (3*FresnelC[b*x])/(4*b^4*Pi^2) +
(x^4*FresnelC[b*x])/4 - (x^3*Sin[(b^2*Pi*x^2)/2])/(4*b*Pi)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
```

```
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3 C(bx)\,dx &= \frac{1}{4}x^4 C(bx) - \frac{1}{4}b\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx \\
&= \frac{1}{4}x^4 C(bx) - \frac{x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{3\int x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{4b\pi} \\
&= -\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{1}{4}x^4 C(bx) - \frac{x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{3\int\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{4b^3\pi^2} \\
&= -\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3C(bx)}{4b^4\pi^2} + \frac{1}{4}x^4 C(bx) - \frac{x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 74, normalized size = 1.00

$$-\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3\text{FresnelC}(bx)}{4b^4\pi^2} + \frac{1}{4}x^4\text{FresnelC}(bx) - \frac{x^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelC[b*x],x]
```

```
[Out] (-3*x*Cos[(b^2*Pi*x^2)/2])/(4*b^3*Pi^2) + (3*FresnelC[b*x])/(4*b^4*Pi^2) +
(x^4*FresnelC[b*x])/4 - (x^3*Sin[(b^2*Pi*x^2)/2])/(4*b*Pi)
```

Maple [A]
time = 0.30, size = 70, normalized size = 0.95

method	result	size
meijerg	$\frac{-\frac{3\cos\left(\frac{b^2\pi x^2}{2}\right)bx}{4}-\frac{\pi b^3x^3\sin\left(\frac{b^2\pi x^2}{2}\right)}{4}+\frac{\left(5x^4\pi^2b^4+15\right)\text{FresnelC}(bx)}{20}}{b^4\pi^2}$	62
derivativedivides	$\frac{\frac{\text{FresnelC}(bx)b^4x^4}{4}-\frac{b^3x^3\sin\left(\frac{b^2\pi x^2}{2}\right)}{4\pi}+\frac{-\frac{3bx\cos\left(\frac{b^2\pi x^2}{2}\right)}{4\pi}+\frac{3\,\text{FresnelC}(bx)}{4\pi}}{\pi}}{b^4}$	70
default	$\frac{\frac{\text{FresnelC}(bx)b^4x^4}{4}-\frac{b^3x^3\sin\left(\frac{b^2\pi x^2}{2}\right)}{4\pi}+\frac{-\frac{3bx\cos\left(\frac{b^2\pi x^2}{2}\right)}{4\pi}+\frac{3\,\text{FresnelC}(bx)}{4\pi}}{\pi}}{b^4}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/4*FresnelC(b*x)*b^4*x^4-1/4/Pi*b^3*x^3*sin(1/2*b^2*Pi*x^2)+3/4/Pi*
(-1/Pi*b*x*cos(1/2*b^2*Pi*x^2)+1/Pi*FresnelC(b*x)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.48, size = 94, normalized size = 1.27

$$\frac{1}{4}\,x^4\,\mathrm{C}\left(bx\right)-\frac{\sqrt{\frac{1}{2}}\left(4\,\sqrt{\frac{1}{2}}\,\pi^2b^3x^3\sin\left(\frac{1}{2}\,\pi b^2x^2\right)+12\,\sqrt{\frac{1}{2}}\,\pi bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)+\left(3i-3\right)\,\left(\frac{1}{4}\right)^{\frac{1}{4}}\,\pi\,\mathrm{erf}\left(\sqrt{\frac{1}{2}i\pi}\,bx\right)-\left(3i+3\right)\,\left(\frac{1}{4}\right)^{\frac{1}{4}}\,\pi\,\mathrm{erf}\left(\sqrt{-\frac{1}{2}i\pi}\,bx\right)\right)}{8\,\pi^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*fresnel_cos(b*x) - 1/8*sqrt(1/2)*(4*sqrt(1/2)*pi^2*b^3*x^3*sin(1/2*
pi*b^2*x^2) + 12*sqrt(1/2)*pi*b*x*cos(1/2*pi*b^2*x^2) + (3*I - 3)*(1/4)^(1/
4)*pi*erf(sqrt(1/2*I*pi)*b*x) - (3*I + 3)*(1/4)^(1/4)*pi*erf(sqrt(-1/2*I*pi
)*b*x))/(pi^3*b^4)
```

Fricas [A]
time = 0.33, size = 59, normalized size = 0.80

$$-\frac{\pi b^3x^3\sin\left(\frac{1}{2}\,\pi b^2x^2\right)+3\,bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)-\left(\pi^2b^4x^4+3\right)\mathrm{C}\left(bx\right)}{4\,\pi^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] -1/4*(pi*b^3*x^3*sin(1/2*pi*b^2*x^2) + 3*b*x*cos(1/2*pi*b^2*x^2) - (pi^2*b^
4*x^4 + 3)*fresnel_cos(b*x))/(pi^2*b^4)
```

Sympy [A]
time = 0.51, size = 112, normalized size = 1.51

$$\frac{5x^4C(bx)\,\Gamma\left(\frac{1}{4}\right)}{64\Gamma\left(\frac{9}{4}\right)}-\frac{5x^3\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{64\pi b\Gamma\left(\frac{9}{4}\right)}-\frac{15x\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{64\pi^2b^3\Gamma\left(\frac{9}{4}\right)}+\frac{15C(bx)\,\Gamma\left(\frac{1}{4}\right)}{64\pi^2b^4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnelc(b*x),x)
```

```
[Out] 5*x**4*fresnelc(b*x)*gamma(1/4)/(64*gamma(9/4)) - 5*x**3*sin(pi*b**2*x**2/2
)*gamma(1/4)/(64*pi*b*gamma(9/4)) - 15*x*cos(pi*b**2*x**2/2)*gamma(1/4)/(64
*pi**2*b**3*gamma(9/4)) + 15*fresnelc(b*x)*gamma(1/4)/(64*pi**2*b**4*gamma(
9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_cos(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnel_cos(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{FresnelC}(b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x),x)
```

```
[Out] int(x^3*FresnelC(b*x), x)
```

3.115 $\int x^2 \mathbf{FresnelC}(bx)\, dx$

Optimal. Leaf size=59

$$-\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2}+\frac{1}{3}x^3\text{FresnelC}(bx)-\frac{x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}$$

[Out] -2/3*cos(1/2*b^2*Pi*x^2)/b^3/Pi^2+1/3*x^3*FresnelC(b*x)-1/3*x^2*sin(1/2*b^2*Pi*x^2)/b/Pi

Rubi [A]
time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6562, 3461, 3377, 2718}

$$-\frac{x^2\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}-\frac{2\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{1}{3}x^3\text{FresnelC}(bx)$$

Antiderivative was successfully verified.

[In] Int[x^2*FresnelC[b*x],x]

[Out] (-2*Cos[(b^2*Pi*x^2)/2])/(3*b^3*Pi^2) + (x^3*FresnelC[b*x])/3 - (x^2*Sin[(b^2*Pi*x^2)/2])/(3*b*Pi)

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
```

```
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2 C(bx)\,dx &= \frac{1}{3}x^3 C(bx) - \frac{1}{3}b\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\ &= \frac{1}{3}x^3 C(bx) - \frac{1}{6}b\text{Subst}\left(\int x\cos\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)\\ &= \frac{1}{3}x^3 C(bx) - \frac{x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{\text{Subst}\left(\int \sin\left(\frac{1}{2}b^2\pi x\right)\,dx, x, x^2\right)}{3b\pi}\\ &= -\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} + \frac{1}{3}x^3 C(bx) - \frac{x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 1.00

$$-\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{3b^3\pi^2} + \frac{1}{3}x^3\text{FresnelC}(bx) - \frac{x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelC[b*x],x]
[Out] (-2*Cos[(b^2*Pi*x^2)/2])/(3*b^3*Pi^2) + (x^3*FresnelC[b*x])/3 - (x^2*Sin[(b
^2*Pi*x^2)/2])/(3*b*Pi)
```

Maple [A]
time = 0.30, size = 54, normalized size = 0.92

method	result	size
meijerg	$\frac{b\,x^4\,\text{hypergeom}\left(\left[\frac{1}{4},1\right],\left[\frac{1}{2},\frac{5}{4},2\right],-\frac{x^4\pi^2 b^4}{16}\right)}{4}$	26
derivativedivides	$\frac{\frac{\text{FresnelC}(bx)b^3x^3}{3}-\frac{b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\pi}-\frac{2\cos\left(\frac{b^2\pi x^2}{2}\right)}{3\pi^2}}{b^3}$	54
default	$\frac{\frac{\text{FresnelC}(bx)b^3x^3}{3}-\frac{b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\pi}-\frac{2\cos\left(\frac{b^2\pi x^2}{2}\right)}{3\pi^2}}{b^3}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(b*x),x,method=_RETURNVERBOSE)
[Out] 1/b^3*(1/3*FresnelC(b*x)*b^3*x^3-1/3/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)-2/3/Pi^
2*cos(1/2*b^2*Pi*x^2))
```

Maxima [A]
time = 0.26, size = 49, normalized size = 0.83

$$\frac{1}{3}\,x^3\,\mathrm{C}\left(bx\right) - \frac{\pi b^2x^2\sin\left(\frac{1}{2}\,\pi b^2x^2\right) + 2\,\cos\left(\frac{1}{2}\,\pi b^2x^2\right)}{3\,\pi^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*fresnel_cos(b*x) - 1/3*(pi*b^2*x^2*sin(1/2*pi*b^2*x^2) + 2*cos(1/2*
pi*b^2*x^2))/(pi^2*b^3)
```

Fricas [A]
time = 0.35, size = 54, normalized size = 0.92

$$\frac{\pi^2b^3x^3\,\mathrm{C}\left(bx\right) - \pi b^2x^2\sin\left(\frac{1}{2}\,\pi b^2x^2\right) - 2\,\cos\left(\frac{1}{2}\,\pi b^2x^2\right)}{3\,\pi^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] 1/3*(pi^2*b^3*x^3*fresnel_cos(b*x) - pi*b^2*x^2*sin(1/2*pi*b^2*x^2) - 2*cos
(1/2*pi*b^2*x^2))/(pi^2*b^3)
```

Sympy [A]
time = 0.54, size = 80, normalized size = 1.36

$$\frac{x^3C(bx)\,\Gamma\left(\frac{1}{4}\right)}{12\Gamma\left(\frac{5}{4}\right)} - \frac{x^2\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi b\Gamma\left(\frac{5}{4}\right)} - \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(\frac{1}{4}\right)}{6\pi^2b^3\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnelc(b*x),x)
```

```
[Out] x**3*fresnelc(b*x)*gamma(1/4)/(12*gamma(5/4)) - x**2*sin(pi*b**2*x**2/2)*ga
mma(1/4)/(12*pi*b*gamma(5/4)) - cos(pi*b**2*x**2/2)*gamma(1/4)/(6*pi**2*b**
3*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_cos(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*fresnel_cos(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2 \operatorname{FresnelC}(b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(b*x),x)
[Out] int(x^2*FresnelC(b*x), x)
```

3.116 $\int x\text{FresnelC}(bx)\,dx$

Optimal. Leaf size=49

$$\frac{1}{2}x^2\text{FresnelC}(bx)+\frac{S(bx)}{2b^2\pi}-\frac{x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi}$$

```
[Out] 1/2*x^2*FresnelC(b*x)+1/2*FresnelS(b*x)/b^2/Pi-1/2*x*sin(1/2*b^2*Pi*x^2)/b/
Pi
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6562, 3467, 3432}

$$\frac{S(bx)}{2\pi b^2}-\frac{x\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b}+\frac{1}{2}x^2\text{FresnelC}(bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelC[b*x],x]
```

```
[Out] (x^2*FresnelC[b*x])/2 + FresnelS[b*x]/(2*b^2*Pi) - (x*Sin[(b^2*Pi*x^2)/2])/
(2*b*Pi)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int xC(bx)\,dx &= \frac{1}{2}x^2C(bx) - \frac{1}{2}b\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\ &= \frac{1}{2}x^2C(bx) - \frac{x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{\int \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b\pi}\\ &= \frac{1}{2}x^2C(bx) + \frac{S(bx)}{2b^2\pi} - \frac{x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 1.00

$$\frac{1}{2}x^2\text{FresnelC}(bx) + \frac{S(bx)}{2b^2\pi} - \frac{x\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*FresnelC[b*x],x]
```

```
[Out] (x^2*FresnelC[b*x])/2 + FresnelS[b*x]/(2*b^2*Pi) - (x*Sin[(b^2*Pi*x^2)/2])/
(2*b*Pi)
```

Maple [A]
time = 0.30, size = 44, normalized size = 0.90

method	result	size
meijerg	$\frac{b\,x^3\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{1}{2},\frac{5}{4},\frac{7}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{3}$	26
derivativedivides	$\frac{\frac{\text{FresnelC}(bx)b^2x^2}{2}-\frac{bx\sin\left(\frac{b^2\pi x^2}{2}\right)}{2\pi}+\frac{\text{S}(bx)}{2\pi}}{b^2}$	44
default	$\frac{\frac{\text{FresnelC}(bx)b^2x^2}{2}-\frac{bx\sin\left(\frac{b^2\pi x^2}{2}\right)}{2\pi}+\frac{\text{S}(bx)}{2\pi}}{b^2}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*FresnelC(b*x)*b^2*x^2-1/2/Pi*b*x*sin(1/2*b^2*Pi*x^2)+1/2/Pi*Fres
nelS(b*x))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.48, size = 70, normalized size = 1.43

$$\frac{1}{2}x^2\,\text{C}(bx) - \frac{\sqrt{\frac{1}{2}}\left(4\sqrt{\frac{1}{2}}\,\pi bx\sin\left(\frac{1}{2}\pi b^2x^2\right) - (i+1)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\text{erf}\left(\sqrt{\frac{1}{2}i\pi}\,bx\right) + (i-1)\left(\frac{1}{4}\right)^{\frac{1}{4}}\pi\,\text{erf}\left(\sqrt{-\frac{1}{2}i\pi}\,bx\right)\right)}{4\pi^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*fresnel_cos(b*x) - 1/4*sqrt(1/2)*(4*sqrt(1/2)*pi*b*x*sin(1/2*pi*b^2
*x^2) - (I + 1)*(1/4)^(1/4)*pi*erf(sqrt(1/2*I*pi)*b*x) + (I - 1)*(1/4)^(1/4
)*pi*erf(sqrt(-1/2*I*pi)*b*x))/(pi^2*b^2)
```

Fricas [A]
time = 0.36, size = 51, normalized size = 1.04

$$\frac{\pi b^3x^2\,\mathrm{C}\,(bx)-b^2x\sin\left(\frac{1}{2}\,\pi b^2x^2\right)+\sqrt{b^2}\,\mathrm{S}\left(\sqrt{b^2}\,x\right)}{2\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] 1/2*(pi*b^3*x^2*fresnel_cos(b*x) - b^2*x*sin(1/2*pi*b^2*x^2) + sqrt(b^2)*fr
esnel_sin(sqrt(b^2)*x))/(pi*b^3)
```

Sympy [A]
time = 0.30, size = 49, normalized size = 1.00

$$\frac{bx^3\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{16\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x),x)
```

```
[Out] b*x**3*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -pi**2*b**4
*x**4/16)/(16*gamma(5/4)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_cos(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*fresnel_cos(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{FresnelC}(b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(b*x),x)
[Out] int(x*FresnelC(b*x), x)
```

3.117 $\int \text{FresnelC}(bx)\,dx$

Optimal. Leaf size=27

$$x\text{FresnelC}(bx)-\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

[Out] x*FresnelC(b*x)-sin(1/2*b^2*Pi*x^2)/b/Pi

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6554}

$$x\text{FresnelC}(bx)-\frac{\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[b*x],x]

[Out] x*FresnelC[b*x] - Sin[(b^2*Pi*x^2)/2]/(b*Pi)

Rule 6554

```
Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelC[a +
 b*x]/b), x] - Simp[Sin[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int C(bx)\,dx=xC(bx)-\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$x\text{FresnelC}(bx)-\frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

Antiderivative was successfully verified.

[In] Integrate[FresnelC[b*x],x]

[Out] x*FresnelC[b*x] - Sin[(b^2*Pi*x^2)/2]/(b*Pi)

Maple [A]
time = 0.39, size = 28, normalized size = 1.04

method	result	size
derivativedivides	$\frac{\mathrm{FresnelC}(bx)bx-\frac{\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{\pi}}{b}$	28
default	$\frac{\mathrm{FresnelC}(bx)bx-\frac{\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{\pi}}{b}$	28
meijerg	$\frac{-\frac{4\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{\sqrt{\pi}}+4\sqrt{\pi}\,bx\,\mathrm{FresnelC}(bx)}{4\sqrt{\pi}\,b}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x),x,method=_RETURNVERBOSE)
[Out] 1/b*(FresnelC(b*x)*b*x-1/Pi*sin(1/2*b^2*Pi*x^2))
```

Maxima [A]
time = 0.27, size = 27, normalized size = 1.00

$$\frac{bx\,\mathrm{C}\,(bx)-\frac{\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{\pi}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x),x, algorithm="maxima")
[Out] (b*x*fresnel_cos(b*x) - sin(1/2*pi*b^2*x^2)/pi)/b
```

Fricas [A]
time = 0.37, size = 28, normalized size = 1.04

$$\frac{\pi bx\,\mathrm{C}\,(bx)-\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{\pi b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x),x, algorithm="fricas")
[Out] (pi*b*x*fresnel_cos(b*x) - sin(1/2*pi*b^2*x^2))/(pi*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(20) = 40$.
time = 0.36, size = 44, normalized size = 1.63

$$\frac{xC(bx)\,\Gamma\left(\frac{1}{4}\right)}{4\Gamma\left(\frac{5}{4}\right)}-\frac{\sin\left(\frac{\pi b^{2}x^{2}}{2}\right)\Gamma\left(\frac{1}{4}\right)}{4\pi b\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x),x)
```

```
[Out] x*fresnelc(b*x)*gamma(1/4)/(4*gamma(5/4)) - sin(pi*b**2*x**2/2)*gamma(1/4)/
(4*pi*b*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x),x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \mathrm{FresnelC}(b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x),x)
```

```
[Out] int(FresnelC(b*x), x)
```

3.118 $\int \frac{\text{FresnelC}(bx)}{x} \, dx$

Optimal. Leaf size=69

$$\frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)$$

```
[Out] 1/2*b*x*hypergeom([1/2, 1/2],[3/2, 3/2],-1/2*I*b^2*Pi*x^2)+1/2*b*x*hypergeo
m([1/2, 1/2],[3/2, 3/2],1/2*I*b^2*Pi*x^2)
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6560, 6493, 6495}

$$\frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x,x]
```

```
[Out] (b*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (-1/2*I)*b^2*Pi*x^2])/2 + (b
*x*HypergeometricPFQ[{1/2, 1/2}, {3/2, 3/2}, (I/2)*b^2*Pi*x^2])/2
```

Rule 6493

```
Int[Erf[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[2*b*(x/Sqrt[Pi])*Hypergeometric
PFQ[{1/2, 1/2}, {3/2, 3/2}, (-b^2)*x^2], x] /; FreeQ[b, x]
```

Rule 6495

```
Int[Erfi[(b_.)*(x_)]/(x_), x_Symbol] :> Simp[2*b*(x/Sqrt[Pi])*Hypergeometri
cPFQ[{1/2, 1/2}, {3/2, 3/2}, b^2*x^2], x] /; FreeQ[b, x]
```

Rule 6560

```
Int[FresnelC[(b_.)*(x_)]/(x_), x_Symbol] :> Dist[(1 - I)/4, Int[Erf[(Sqrt[P
i]/2)*(1 + I)*b*x]/x, x], x] + Dist[(1 + I)/4, Int[Erf[(Sqrt[Pi]/2)*(1 - I)
*b*x]/x, x], x] /; FreeQ[b, x]
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)}{x}\,dx &= \left(\frac{1}{4}-\frac{i}{4}\right)\int \frac{\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)}{x}\,dx+\left(\frac{1}{4}-\frac{i}{4}\right)\int \frac{\operatorname{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)}{x}\,dx\\ &= \frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{2}bx\,{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{1}{2}ib^2\pi x^2\right)\end{aligned}$$

Mathematica [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelC[b*x]/x,x]
[Out] Integrate[FresnelC[b*x]/x, x]
```

Maple [A]
time = 0.38, size = 23, normalized size = 0.33

method	result	size
meijerg	$bx\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{4}\right],\left[\frac{1}{2},\frac{5}{4},\frac{5}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x,x,method=_RETURNVERBOSE)
[Out] b*x*hypergeom([1/4,1/4],[1/2,5/4,5/4],-1/16*x^4*Pi^2*b^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)/x, x)
```

Sympy [A]
time = 0.29, size = 41, normalized size = 0.59

$$\frac{bx\Gamma^2\left(\frac{1}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{5}{4},\frac{5}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{16\Gamma^2\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x,x)
```

```
[Out] b*x*gamma(1/4)**2*hyper((1/4, 1/4), (1/2, 5/4, 5/4), -pi**2*b**4*x**4/16)/(
16*gamma(5/4)**2)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}(b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x,x)
```

```
[Out] int(FresnelC(b*x)/x, x)
```

3.119 $\int \frac{\mathbf{FresnelC}(bx)}{x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}b\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{\text{FresnelC}(bx)}{x}$$

```
[Out] 1/2*b*Ci(1/2*b^2*Pi*x^2)-FresnelC(b*x)/x
```

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6562, 3457}

$$\frac{1}{2}b\text{CosIntegral}\left(\frac{1}{2}\pi b^2x^2\right)-\frac{\text{FresnelC}(bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^2,x]
```

```
[Out] (b*CosIntegral[(b^2*Pi*x^2)/2])/2 - FresnelC[b*x]/x
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)}{x^2}\,dx &= -\frac{C(bx)}{x}+b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx\\ &= \frac{1}{2}b\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{C(bx)}{x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.00

$$\frac{1}{2}b\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{\text{FresnelC}(bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^2,x]
```

```
[Out] (b*CosIntegral[(b^2*Pi*x^2)/2])/2 - FresnelC[b*x]/x
```

Maple [A]
time = 0.36, size = 28, normalized size = 1.04

method	result	size
derivativedivides	$b\left(-\frac{\operatorname{FresnelC}(bx)}{bx}+\frac{\operatorname{cosineIntegral}\left(\frac{b^2\pi x^2}{2}\right)}{2}\right)$	28
default	$b\left(-\frac{\operatorname{FresnelC}(bx)}{bx}+\frac{\operatorname{cosineIntegral}\left(\frac{b^2\pi x^2}{2}\right)}{2}\right)$	28
meijerg	$\frac{b\sqrt{\pi}\left(-\frac{\pi^{\frac{3}{2}}x^4b^4\operatorname{hypergeom}\left(\left[1,1,\frac{5}{4}\right],\left[\frac{3}{2},2,2,\frac{9}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{10}+\frac{8\gamma-8\ln(2)-16+16\ln(x)+8\ln(\pi)+16\ln(b)}{\sqrt{\pi}}\right)}{16}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-FresnelC(b*x)/b/x+1/2*Ci(1/2*b^2*Pi*x^2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.35, size = 34, normalized size = 1.26

$$\frac{1}{4}\,b\left(\operatorname{Ei}\left(\frac{1}{2}i\,\pi b^2x^2\right)+\operatorname{Ei}\left(-\frac{1}{2}i\,\pi b^2x^2\right)\right)-\frac{\operatorname{C}(bx)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*(Ei(1/2*I*pi*b^2*x^2) + Ei(-1/2*I*pi*b^2*x^2)) - fresnel_cos(b*x)/x
```

Fricas [A]
time = 0.35, size = 38, normalized size = 1.41

$$\frac{bx\operatorname{Ci}\left(\frac{1}{2}\,\pi b^2x^2\right)+bx\operatorname{Ci}\left(-\frac{1}{2}\,\pi b^2x^2\right)-4\operatorname{C}(bx)}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b*x*cos_integral(1/2*pi*b^2*x^2) + b*x*cos_integral(-1/2*pi*b^2*x^2) -
 4*fresnel_cos(b*x))/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(20) = 40$.
time = 0.50, size = 53, normalized size = 1.96

$$-\frac{\pi^2 b^5 x^4 \Gamma\left(\frac{5}{4}\right) {}_3F_4\left(\begin{matrix} 1, 1, \frac{5}{4} \\ \frac{3}{2}, 2, 2, \frac{9}{4} \end{matrix}\middle| -\frac{\pi^2 b^4 x^4}{16}\right)}{128\Gamma\left(\frac{9}{4}\right)} + \frac{b \log\left(b^4 x^4\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**2,x)
```

```
[Out] -pi**2*b**5*x**4*gamma(5/4)*hyper((1, 1, 5/4), (3/2, 2, 2, 9/4), -pi**2*b**
4*x**4/16)/(128*gamma(9/4)) + b*log(b**4*x**4)/4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{FresnelC}(b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^2,x)
```

```
[Out] int(FresnelC(b*x)/x^2, x)
```

3.120 $\int \frac{\text{FresnelC}(bx)}{x^3}\,dx$

Optimal. Leaf size=44

$$-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2x}-\frac{\text{FresnelC}(bx)}{2x^2}-\frac{1}{2}b^2\pi S(bx)$$

```
[Out] -1/2*b*cos(1/2*b^2*Pi*x^2)/x-1/2*FresnelC(b*x)/x^2-1/2*b^2*Pi*FresnelS(b*x)
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6562, 3469, 3432}

$$-\frac{1}{2}\pi b^2 S(bx)-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2x}-\frac{\text{FresnelC}(bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^3,x]
```

```
[Out] -1/2*(b*Cos[(b^2*Pi*x^2)/2])/x - FresnelC[b*x]/(2*x^2) - (b^2*Pi*FresnelS[b
*x])/2
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)}{x^3}\,dx &= -\frac{C(bx)}{2x^2} + \frac{1}{2}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2x} - \frac{C(bx)}{2x^2} - \frac{1}{2}\left(b^3\pi\right)\int \sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2x} - \frac{C(bx)}{2x^2} - \frac{1}{2}b^2\pi S(bx)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 1.00

$$-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{2x} - \frac{\text{FresnelC}(bx)}{2x^2} - \frac{1}{2}b^2\pi S(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^3,x]
```

```
[Out] -1/2*(b*Cos[(b^2*Pi*x^2)/2])/x - FresnelC[b*x]/(2*x^2) - (b^2*Pi*FresnelS[b
*x])/2
```

Maple [A]
time = 0.31, size = 43, normalized size = 0.98

method	result	size
meijerg	$-\frac{b\,\text{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4}\right],\left[\frac{1}{2},\frac{3}{4},\frac{5}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{x}$	26
derivativedivides	$b^2\left(-\frac{\text{FresnelC}(bx)}{2b^2x^2}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2bx}-\frac{\pi\,\text{S}(bx)}{2}\right)$	43
default	$b^2\left(-\frac{\text{FresnelC}(bx)}{2b^2x^2}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{2bx}-\frac{\pi\,\text{S}(bx)}{2}\right)$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/2*FresnelC(b*x)/b^2/x^2-1/2/b/x*cos(1/2*b^2*Pi*x^2)-1/2*Pi*FresnelS
(b*x))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 61, normalized size = 1.39

$$-\frac{\sqrt{\frac{1}{2}}\,\sqrt{\pi x^2}\,\left(\left(i+1\right)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},\frac{1}{2}i\,\pi b^2x^2\right)-\left(i-1\right)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},-\frac{1}{2}i\,\pi b^2x^2\right)\right)b^2}{16\,x} - \frac{\text{C}\,(bx)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(1/2)*sqrt(pi*x^2)*((I + 1)*sqrt(2)*gamma(-1/2, 1/2*I*pi*b^2*x^2)
 - (I - 1)*sqrt(2)*gamma(-1/2, -1/2*I*pi*b^2*x^2))*b^2/x - 1/2*fresnel_cos(
b*x)/x^2
```

Fricas [A]
time = 0.38, size = 42, normalized size = 0.95

$$-\frac{\pi\sqrt{b^2}\,bx^2\,\mathrm{S}\left(\sqrt{b^2}\,x\right)+bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)+\mathrm{C}\left(bx\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(pi*sqrt(b^2)*b*x^2*fresnel_sin(sqrt(b^2)*x) + b*x*cos(1/2*pi*b^2*x^2)
 + fresnel_cos(b*x))/x^2
```

Sympy [A]
time = 0.36, size = 51, normalized size = 1.16

$$\frac{b\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_{2}F_{3}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{3}{4},\frac{5}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{16x\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**3,x)
```

```
[Out] b*gamma(-1/4)*gamma(1/4)*hyper((-1/4, 1/4), (1/2, 3/4, 5/4), -pi**2*b**4*x**
*4/16)/(16*x*gamma(3/4)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{FresnelC}(b\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^3,x)
```

```
[Out] int(FresnelC(b*x)/x^3, x)
```

3.121 $\int \frac{\textbf{FresnelC}(bx)}{x^4} dx$

Optimal. Leaf size=52

$$-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2}-\frac{\text{FresnelC}(bx)}{3x^3}-\frac{1}{12}b^3\pi\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

```
[Out] -1/6*b*cos(1/2*b^2*Pi*x^2)/x^2-1/3*FresnelC(b*x)/x^3-1/12*b^3*Pi*Si(1/2*b^2
*Pi*x^2)
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6562, 3461, 3378, 3380}

$$-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^2}-\frac{1}{12}\pi b^3\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{\text{FresnelC}(bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^4,x]
```

```
[Out] -1/6*(b*Cos[(b^2*Pi*x^2)/2])/x^2 - FresnelC[b*x]/(3*x^3) - (b^3*Pi*SinInteg
ral[(b^2*Pi*x^2)/2])/12
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)}{x^4}\,dx &= -\frac{C(bx)}{3x^3}+\frac{1}{3}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx\\ &= -\frac{C(bx)}{3x^3}+\frac{1}{6}b\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx,x,x^2\right)\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2}-\frac{C(bx)}{3x^3}-\frac{1}{12}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx,x,x^2\right)\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2}-\frac{C(bx)}{3x^3}-\frac{1}{12}b^3\pi\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 1.00

$$-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{6x^2}-\frac{\text{FresnelC}(bx)}{3x^3}-\frac{1}{12}b^3\pi\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^4,x]
[Out] -1/6*(b*Cos[(b^2*Pi*x^2)/2])/x^2 - FresnelC[b*x]/(3*x^3) - (b^3*Pi*SinInteg
ral[(b^2*Pi*x^2)/2])/12
```

Maple [A]
time = 0.32, size = 49, normalized size = 0.94

method	result	size
meijerg	$-\frac{b\,\text{hypergeom}\left(\left[-\frac{1}{2},\frac{1}{4}\right],\left[\frac{1}{2},\frac{1}{2},\frac{5}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{2x^2}$	26
derivativedivides	$b^3\left(-\frac{\text{FresnelC}(bx)}{3b^3x^3}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{6b^2x^2}-\frac{\pi\,\text{sinIntegral}\left(\frac{b^2\pi x^2}{2}\right)}{12}\right)$	49
default	$b^3\left(-\frac{\text{FresnelC}(bx)}{3b^3x^3}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{6b^2x^2}-\frac{\pi\,\text{sinIntegral}\left(\frac{b^2\pi x^2}{2}\right)}{12}\right)$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*(-1/3*FresnelC(b*x)/b^3/x^3-1/6/b^2/x^2*cos(1/2*b^2*Pi*x^2)-1/12*Pi*Si(
1/2*b^2*Pi*x^2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.32, size = 44, normalized size = 0.85

$$-\frac{1}{24}\left(i\,\pi\Gamma\left(-1,\frac{1}{2}i\,\pi b^2x^2\right)-i\,\pi\Gamma\left(-1,-\frac{1}{2}i\,\pi b^2x^2\right)\right)b^3-\frac{\mathrm{C}\,(bx)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/24*(I*pi*gamma(-1, 1/2*I*pi*b^2*x^2) - I*pi*gamma(-1, -1/2*I*pi*b^2*x^2)
)*b^3 - 1/3*fresnel_cos(b*x)/x^3
```

Fricas [A]
time = 0.37, size = 44, normalized size = 0.85

$$-\frac{\pi b^3x^3\,\mathrm{Si}\left(\frac{1}{2}\,\pi b^2x^2\right)+2\,bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)+4\,\mathrm{C}\,(bx)}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(pi*b^3*x^3*sin_integral(1/2*pi*b^2*x^2) + 2*b*x*cos(1/2*pi*b^2*x^2)
+ 4*fresnel_cos(b*x))/x^3
```

Sympy [A]
time = 0.40, size = 42, normalized size = 0.81

$$-\frac{b\Gamma\left(\frac{1}{4}\right){}_{2}F_{3}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{1}{2},\frac{1}{2},\frac{5}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{8x^2\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**4,x)
```

```
[Out] -b*gamma(1/4)*hyper((-1/2, 1/4), (1/2, 1/2, 5/4), -pi**2*b**4*x**4/16)/(8*x
**2*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{FresnelC}(b\,x)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^4,x)
```

```
[Out] int(FresnelC(b*x)/x^4, x)
```

3.122 $\int \frac{\text{FresnelC}(bx)}{x^5} dx$

Optimal. Leaf size=69

$$-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}-\frac{1}{12}b^4\pi^2\text{FresnelC}(bx)-\frac{\text{FresnelC}(bx)}{4x^4}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x}$$

```
[Out] -1/12*b*cos(1/2*b^2*Pi*x^2)/x^3-1/12*b^4*Pi^2*FresnelC(b*x)-1/4*FresnelC(b*
x)/x^4+1/12*b^3*Pi*sin(1/2*b^2*Pi*x^2)/x
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6562, 3469, 3468, 3433}

$$-\frac{1}{12}\pi^2b^4\text{FresnelC}(bx)-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{12x^3}+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{12x}-\frac{\text{FresnelC}(bx)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^5,x]
```

```
[Out] -1/12*(b*Cos[(b^2*Pi*x^2)/2])/x^3 - (b^4*Pi^2*FresnelC[b*x])/12 - FresnelC[
b*x]/(4*x^4) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(12*x)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
```

```
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{C(bx)}{x^5}\,dx &= -\frac{C(bx)}{4x^4}+\frac{1}{4}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}-\frac{C(bx)}{4x^4}-\frac{1}{12}\left(b^3\pi\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}-\frac{C(bx)}{4x^4}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x}-\frac{1}{12}\left(b^5\pi^2\right)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)\,dx\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}-\frac{1}{12}b^4\pi^2C(bx)-\frac{C(bx)}{4x^4}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 69, normalized size = 1.00

$$
-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{12x^3}-\frac{1}{12}b^4\pi^2\text{FresnelC}(bx)-\frac{\text{FresnelC}(bx)}{4x^4}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{12x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^5,x]
```

```
[Out] -1/12*(b*Cos[(b^2*Pi*x^2)/2])/x^3 - (b^4*Pi^2*FresnelC[b*x])/12 - FresnelC[
b*x]/(4*x^4) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(12*x)
```

Maple [A]
time = 0.30, size = 64, normalized size = 0.93

method	result	size
derivativedivides	$b^4\left(-\frac{\text{FresnelC}(bx)}{4b^4x^4}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{12b^3x^3}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{bx}+\pi\,\text{FresnelC}(bx)\right)}{12}\right)$	64
default	$b^4\left(-\frac{\text{FresnelC}(bx)}{4b^4x^4}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{12b^3x^3}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{bx}+\pi\,\text{FresnelC}(bx)\right)}{12}\right)$	64
meijerg	$\frac{\pi^2b^4\left(-\frac{32\cos\left(\frac{b^2\pi x^2}{2}\right)}{3\pi^2x^3b^3}+\frac{32\sin\left(\frac{b^2\pi x^2}{2}\right)}{3\pi xb}-\frac{32\left(x^4\pi^2b^4+3\right)\text{FresnelC}(bx)}{3\pi^2x^4b^4}\right)}{128}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] b^4*(-1/4*FresnelC(b*x)/b^4/x^4-1/12/b^3/x^3*cos(1/2*b^2*Pi*x^2)-1/12*Pi*(-
1/b/x*sin(1/2*b^2*Pi*x^2)+Pi*FresnelC(b*x)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 61, normalized size = 0.88

$$-\frac{\sqrt{\frac{1}{2}}\,(\pi x^2)^{\frac{3}{2}}\left((i-1)\,\sqrt{2}\,\Gamma\left(-\frac{3}{2},\frac{1}{2}i\,\pi b^2x^2\right)-(i+1)\,\sqrt{2}\,\Gamma\left(-\frac{3}{2},-\frac{1}{2}i\,\pi b^2x^2\right)\right)b^4}{64\,x^3}-\frac{\mathrm{C}\,(bx)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/64*sqrt(1/2)*(pi*x^2)^(3/2)*((I - 1)*sqrt(2)*gamma(-3/2, 1/2*I*pi*b^2*x^
2) - (I + 1)*sqrt(2)*gamma(-3/2, -1/2*I*pi*b^2*x^2))*b^4/x^3 - 1/4*fresnel_
cos(b*x)/x^4
```

Fricas [A]
time = 0.36, size = 56, normalized size = 0.81

$$\frac{\pi b^3x^3\sin\left(\frac{1}{2}\,\pi b^2x^2\right)-bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)-(\pi^2b^4x^4+3)\,\mathrm{C}\,(bx)}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(pi*b^3*x^3*sin(1/2*pi*b^2*x^2) - b*x*cos(1/2*pi*b^2*x^2) - (pi^2*b^4*
x^4 + 3)*fresnel_cos(b*x))/x^4
```

Sympy [A]
time = 0.61, size = 110, normalized size = 1.59

$$\frac{\pi^2b^4C(bx)\,\Gamma\left(-\frac{3}{4}\right)}{64\Gamma\left(\frac{5}{4}\right)}-\frac{\pi b^3\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{3}{4}\right)}{64x\Gamma\left(\frac{5}{4}\right)}+\frac{b\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{3}{4}\right)}{64x^3\Gamma\left(\frac{5}{4}\right)}+\frac{3C(bx)\,\Gamma\left(-\frac{3}{4}\right)}{64x^4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**5,x)
```

```
[Out] pi**2*b**4*fresnelc(b*x)*gamma(-3/4)/(64*gamma(5/4)) - pi*b**3*sin(pi*b**2*
x**2/2)*gamma(-3/4)/(64*x*gamma(5/4)) + b*cos(pi*b**2*x**2/2)*gamma(-3/4)/(
64*x**3*gamma(5/4)) + 3*fresnelc(b*x)*gamma(-3/4)/(64*x**4*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}(b\,x)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^5,x)
```

```
[Out] int(FresnelC(b*x)/x^5, x)
```

3.123 $\int \frac{\mathbf{FresnelC}(bx)}{x^6}\,dx$

Optimal. Leaf size=77

$$-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4}-\frac{1}{80}b^5\pi^2\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{\text{FresnelC}(bx)}{5x^5}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}$$

```
[Out] -1/80*b^5*Pi^2*Ci(1/2*b^2*Pi*x^2)-1/20*b*cos(1/2*b^2*Pi*x^2)/x^4-1/5*Fresne
lC(b*x)/x^5+1/40*b^3*Pi*sin(1/2*b^2*Pi*x^2)/x^2
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6562, 3461, 3378, 3383}

$$-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{20x^4}-\frac{1}{80}\pi^2b^5\text{CosIntegral}\left(\frac{1}{2}\pi b^2x^2\right)+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{40x^2}-\frac{\text{FresnelC}(bx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^6,x]
```

```
[Out] -1/20*(b*Cos[(b^2*Pi*x^2)/2])/x^4 - (b^5*Pi^2*CosIntegral[(b^2*Pi*x^2)/2])/
80 - FresnelC[b*x]/(5*x^5) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(40*x^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)}{x^6}\,dx &= -\frac{C(bx)}{5x^5}+\frac{1}{5}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx\\ &= -\frac{C(bx)}{5x^5}+\frac{1}{10}b\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx,x,x^2\right)\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4}-\frac{C(bx)}{5x^5}-\frac{1}{40}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx,x,x^2\right)\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4}-\frac{C(bx)}{5x^5}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}-\frac{1}{80}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x}\,dx,x,x^2\right)\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4}-\frac{1}{80}b^5\pi^2\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{C(bx)}{5x^5}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 77, normalized size = 1.00

$$-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{20x^4}-\frac{1}{80}b^5\pi^2\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{\text{FresnelC}(bx)}{5x^5}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{40x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^6,x]
```

```
[Out] -1/20*(b*Cos[(b^2*Pi*x^2)/2])/x^4 - (b^5*Pi^2*CosIntegral[(b^2*Pi*x^2)/2])/
80 - FresnelC[b*x]/(5*x^5) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(40*x^2)
```

Maple [A]
time = 0.38, size = 71, normalized size = 0.92

method	result	size
derivativedivides	$b^5\left(-\frac{\text{FresnelC}(bx)}{5b^5x^5}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{20b^4x^4}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}+\frac{\pi\,\text{cosineIntegral}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{20}\right)$	71

default	$b^5\left(-\frac{\mathrm{FresnelC}(bx)}{5b^5x^5}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{20b^4x^4}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}+\frac{\pi\,\mathrm{cosineIntegral}\left(\frac{b^2\pi x^2}{2}\right)}{4}\right)}{20}\right)$	71
meijerg	$\frac{\pi^{\frac{5}{2}}b^5\left(\frac{\pi^{\frac{3}{2}}x^4b^4\,\mathrm{hypergeom}\left(\left[1,1,\frac{9}{4}\right],\left[2,\frac{5}{2},3,\frac{13}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{54}-\frac{8\left(-\frac{19}{5}+2\gamma-2\ln(2)+4\ln(x)+2\ln(\pi)+4\ln(b)\right)}{5\sqrt{\pi}}-\frac{64}{\pi^{\frac{5}{2}}x^4b^4}\right)}{256}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] b^5*(-1/5*FresnelC(b*x)/b^5/x^5-1/20/b^4/x^4*cos(1/2*b^2*Pi*x^2)-1/20*Pi*(-
1/2/b^2/x^2*sin(1/2*b^2*Pi*x^2)+1/4*Pi*Ci(1/2*b^2*Pi*x^2)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.31, size = 46, normalized size = 0.60

$$\frac{1}{80}\left(\pi^2\Gamma\left(-2,\frac{1}{2}i\,\pi b^2x^2\right)+\pi^2\Gamma\left(-2,-\frac{1}{2}i\,\pi b^2x^2\right)\right)b^5-\frac{\mathrm{C}\left(bx\right)}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^6,x, algorithm="maxima")
```

```
[Out] 1/80*(pi^2*gamma(-2, 1/2*I*pi*b^2*x^2) + pi^2*gamma(-2, -1/2*I*pi*b^2*x^2))
*b^5 - 1/5*fresnel_cos(b*x)/x^5
```

Fricas [A]
time = 0.36, size = 85, normalized size = 1.10

$$-\frac{\pi^2b^5x^5\,\mathrm{Ci}\left(\frac{1}{2}\,\pi b^2x^2\right)+\pi^2b^5x^5\,\mathrm{Ci}\left(-\frac{1}{2}\,\pi b^2x^2\right)-4\,\pi b^3x^3\sin\left(\frac{1}{2}\,\pi b^2x^2\right)+8\,bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)+32\,\mathrm{C}\left(bx\right)}{160\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^6,x, algorithm="fricas")
```

```
[Out] -1/160*(pi^2*b^5*x^5*cos_integral(1/2*pi*b^2*x^2) + pi^2*b^5*x^5*cos_integr
al(-1/2*pi*b^2*x^2) - 4*pi*b^3*x^3*sin(1/2*pi*b^2*x^2) + 8*b*x*cos(1/2*pi*b
^2*x^2) + 32*fresnel_cos(b*x))/x^5
```

Sympy [A]
time = 1.34, size = 65, normalized size = 0.84

$$\frac{\pi^4b^9x^4\Gamma\left(\frac{9}{4}\right){}_3F_4\left(\begin{matrix}1,1,\frac{9}{4}\\2,\frac{5}{2},3,\frac{13}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{6144\Gamma\left(\frac{13}{4}\right)}-\frac{\pi^2b^5\log\left(b^4x^4\right)}{160}-\frac{b}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**6,x)
```

```
[Out] pi**4*b**9*x**4*gamma(9/4)*hyper((1, 1, 9/4), (2, 5/2, 3, 13/4), -pi**2*b**
4*x**4/16)/(6144*gamma(13/4)) - pi**2*b**5*log(b**4*x**4)/160 - b/(4*x**4)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^6,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)/x^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}(b\,x)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^6,x)
```

```
[Out] int(FresnelC(b*x)/x^6, x)
```

3.124 $\int \frac{\text{FresnelC}(bx)}{x^7} dx$

Optimal. Leaf size=94

$$-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x}-\frac{\text{FresnelC}(bx)}{6x^6}+\frac{1}{90}b^6\pi^3S(bx)+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}$$

```
[Out] -1/30*b*cos(1/2*b^2*Pi*x^2)/x^5+1/90*b^5*Pi^2*cos(1/2*b^2*Pi*x^2)/x-1/6*Fre
snelC(b*x)/x^6+1/90*b^6*Pi^3*FresnelS(b*x)+1/90*b^3*Pi*sin(1/2*b^2*Pi*x^2)/
x^3
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6562, 3469, 3468, 3432}

$$\frac{1}{90}\pi^3b^6S(bx)-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{30x^5}+\frac{\pi^2b^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{90x}+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{90x^3}-\frac{\text{FresnelC}(bx)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^7,x]
```

```
[Out] -1/30*(b*Cos[(b^2*Pi*x^2)/2])/x^5 + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2])/(90*x) -
 FresnelC[b*x]/(6*x^6) + (b^6*Pi^3*FresnelS[b*x])/90 + (b^3*Pi*Sin[(b^2*Pi*
x^2)/2])/(90*x^3)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)}{x^7}\,dx &= -\frac{C(bx)}{6x^6}+\frac{1}{6}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}-\frac{C(bx)}{6x^6}-\frac{1}{30}\left(b^3\pi\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}-\frac{C(bx)}{6x^6}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}-\frac{1}{90}\left(b^5\pi^2\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x}-\frac{C(bx)}{6x^6}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}+\frac{1}{90}\left(b^7\pi^3\right)\int \sin\left(\frac{1}{2}b^2\pi\right.\\ &= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{30x^5}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{90x}-\frac{C(bx)}{6x^6}+\frac{1}{90}b^6\pi^3S(bx)+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{90x^3}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 74, normalized size = 0.79

$$\frac{1}{90}\left(\frac{b(-3+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}-\frac{15\text{FresnelC}(bx)}{x^6}+b^6\pi^3S(bx)+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^7,x]
```

```
[Out] ((b*(-3 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^5 - (15*FresnelC[b*x])/x^6 +
 b^6*Pi^3*FresnelS[b*x] + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/x^3)/90
```

Maple [A]
time = 0.32, size = 87, normalized size = 0.93

method	result	size
meijerg	$-\frac{b\,\mathrm{hypergeom}\left(\left[-\frac{5}{4},\frac{1}{4}\right],\left[-\frac{1}{4},\frac{1}{2},\frac{5}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{5x^5}$	26

derivativedivides	$b^6\left(-\frac{\text{FresnelC}(bx)}{6b^6x^6}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{30b^5x^5}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{3b^3x^3}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{bx}-\pi\,\mathrm{S}(bx)\right)}{3}\right)}{30}\right)$	87
default	$b^6\left(-\frac{\text{FresnelC}(bx)}{6b^6x^6}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{30b^5x^5}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{3b^3x^3}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{bx}-\pi\,\mathrm{S}(bx)\right)}{3}\right)}{30}\right)$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] b^6*(-1/6*FresnelC(b*x)/b^6/x^6-1/30/b^5/x^5*cos(1/2*b^2*Pi*x^2)-1/30*Pi*(-
1/3/b^3/x^3*sin(1/2*b^2*Pi*x^2)+1/3*Pi*(-1/b/x*cos(1/2*b^2*Pi*x^2)-Pi*Fresn
elS(b*x))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 61, normalized size = 0.65

$$-\frac{\sqrt{\frac{1}{2}}\,(\pi x^2)^{\frac{5}{2}}\left(-(i+1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},\frac{1}{2}i\,\pi b^2x^2\right)+(i-1)\sqrt{2}\,\Gamma\left(-\frac{5}{2},-\frac{1}{2}i\,\pi b^2x^2\right)\right)b^6}{192\,x^5}-\frac{\mathrm{C}\,(bx)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^7,x, algorithm="maxima")
```

```
[Out] -1/192*sqrt(1/2)*(pi*x^2)^(5/2)*(-(I + 1)*sqrt(2)*gamma(-5/2, 1/2*I*pi*b^2*
x^2) + (I - 1)*sqrt(2)*gamma(-5/2, -1/2*I*pi*b^2*x^2))*b^6/x^5 - 1/6*fresne
l_cos(b*x)/x^6
```

Fricas [A]
time = 0.36, size = 79, normalized size = 0.84

$$\frac{\pi^3\sqrt{b^2}\,b^5x^6\,\mathrm{S}\left(\sqrt{b^2}\,x\right)+\pi b^3x^3\sin\left(\frac{1}{2}\,\pi b^2x^2\right)+\left(\pi^2b^5x^5-3\,bx\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)-15\,\mathrm{C}\,(bx)}{90\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^7,x, algorithm="fricas")
```

```
[Out] 1/90*(pi^3*sqrt(b^2)*b^5*x^6*fresnel_sin(sqrt(b^2)*x) + pi*b^3*x^3*sin(1/2*
pi*b^2*x^2) + (pi^2*b^5*x^5 - 3*b*x)*cos(1/2*pi*b^2*x^2) - 15*fresnel_cos(b
*x))/x^6
```

Sympy [A]
time = 0.81, size = 56, normalized size = 0.60

$$\frac{b\Gamma\left(-\frac{5}{4}\right)\Gamma\left(\frac{1}{4}\right){}_{2}F_{3}\left(\begin{matrix}-\frac{5}{4},\frac{1}{4}\\-\frac{1}{4},\frac{1}{2},\frac{5}{4}\end{matrix}\middle|-\frac{\pi^{2}b^{4}x^{4}}{16}\right)}{16x^{5}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**7,x)
```

```
[Out] b*gamma(-5/4)*gamma(1/4)*hyper((-5/4, 1/4), (-1/4, 1/2, 5/4), -pi**2*b**4*x
**4/16)/(16*x**5*gamma(-1/4)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^7,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)/x^7, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}(b\,x)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^7,x)
```

```
[Out] int(FresnelC(b*x)/x^7, x)
```

3.125 $\int \frac{\text{FresnelC}(bx)}{x^8}\,dx$

Optimal. Leaf size=102

$$-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2}-\frac{\text{FresnelC}(bx)}{7x^7}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4}+\frac{1}{672}b^7\pi^3\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)$$

```
[Out] -1/42*b*cos(1/2*b^2*Pi*x^2)/x^6+1/336*b^5*Pi^2*cos(1/2*b^2*Pi*x^2)/x^2-1/7*
FresnelC(b*x)/x^7+1/672*b^7*Pi^3*Si(1/2*b^2*Pi*x^2)+1/168*b^3*Pi*sin(1/2*b^
2*Pi*x^2)/x^4
```

Rubi [A]
time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6562, 3461, 3378, 3380}

$$-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{42x^6}+\frac{1}{672}\pi^3b^7\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)+\frac{\pi^2b^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{336x^2}+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{168x^4}-\frac{\text{FresnelC}(bx)}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^8,x]
```

```
[Out] -1/42*(b*Cos[(b^2*Pi*x^2)/2])/x^6 + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2])/(336*x^2
) - FresnelC[b*x]/(7*x^7) + (b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(168*x^4) + (b^7*P
i^3*SinIntegral[(b^2*Pi*x^2)/2])/672
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)}{x^8}\,dx &= -\frac{C(bx)}{7x^7}+\frac{1}{7}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx\\
&= -\frac{C(bx)}{7x^7}+\frac{1}{14}b\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^4}\,dx,x,x^2\right)\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6}-\frac{C(bx)}{7x^7}-\frac{1}{84}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx,x,x^2\right)\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6}-\frac{C(bx)}{7x^7}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4}-\frac{1}{336}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^2}\,dx,x,x^2\right.\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2}-\frac{C(bx)}{7x^7}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4}+\frac{1}{672}\left(b^7\pi^3\right)\text{Subst}\left(\int \right.\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{42x^6}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{336x^2}-\frac{C(bx)}{7x^7}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{168x^4}+\frac{1}{672}b^7\pi^3\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 84, normalized size = 0.82

$$\frac{1}{672}\left(\frac{2b(-8+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}-\frac{96\text{FresnelC}(bx)}{x^7}+\frac{4b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}+b^7\pi^3\text{Si}\left(\frac{1}{2}b^2\pi x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^8,x]
```

```
[Out] ((2*b*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^6 - (96*FresnelC[b*x])/x^7
 + (4*b^3*Pi*Sin[(b^2*Pi*x^2)/2])/x^4 + b^7*Pi^3*SinIntegral[(b^2*Pi*x^2)/2
])/672
```

Maple [A]
time = 0.32, size = 93, normalized size = 0.91

method	result	size

meijerg	$-\frac{b\,\text{hypergeom}\left(\left[-\frac{3}{2},\frac{1}{4}\right],\left[-\frac{1}{2},\frac{1}{2},\frac{5}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{6x^6}$	26
derivativedivides	$b^7\left(-\frac{\text{FresnelC}(bx)}{7b^7x^7}-\frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{42b^6x^6}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{4b^4x^4}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{2b^2x^2}-\frac{\pi\,\text{sinIntegral}\left(\frac{b^2\pi\,x^2}{2}\right)}{4}\right)}{4}\right)}{42}\right)$	93
default	$b^7\left(-\frac{\text{FresnelC}(bx)}{7b^7x^7}-\frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{42b^6x^6}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{4b^4x^4}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)}{2b^2x^2}-\frac{\pi\,\text{sinIntegral}\left(\frac{b^2\pi\,x^2}{2}\right)}{4}\right)}{4}\right)}{42}\right)$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] b^7*(-1/7*FresnelC(b*x)/b^7/x^7-1/42/b^6/x^6*cos(1/2*b^2*Pi*x^2)-1/42*Pi*(-
1/4/b^4/x^4*sin(1/2*b^2*Pi*x^2)+1/4*Pi*(-1/2/b^2/x^2*cos(1/2*b^2*Pi*x^2)-1/
4*Pi*Si(1/2*b^2*Pi*x^2))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.33, size = 48, normalized size = 0.47

$$-\frac{1}{224}\left(-i\,\pi^3\Gamma\left(-3,\frac{1}{2}i\,\pi b^2x^2\right)+i\,\pi^3\Gamma\left(-3,-\frac{1}{2}i\,\pi b^2x^2\right)\right)b^7-\frac{\mathrm{C}\,(bx)}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^8,x, algorithm="maxima")
```

```
[Out] -1/224*(-I*pi^3*gamma(-3, 1/2*I*pi*b^2*x^2) + I*pi^3*gamma(-3, -1/2*I*pi*b^
2*x^2))*b^7 - 1/7*fresnel_cos(b*x)/x^7
```

Fricas [A]
time = 0.36, size = 78, normalized size = 0.76

$$\frac{\pi^3b^7x^7\,\mathrm{Si}\left(\frac{1}{2}\,\pi b^2x^2\right)+4\,\pi b^3x^3\sin\left(\frac{1}{2}\,\pi b^2x^2\right)+2\left(\pi^2b^5x^5-8\,bx\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)-96\,\mathrm{C}\,(bx)}{672\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^8,x, algorithm="fricas")
```

```
[Out] 1/672*(pi^3*b^7*x^7*sin_integral(1/2*pi*b^2*x^2) + 4*pi*b^3*x^3*sin(1/2*pi*
b^2*x^2) + 2*(pi^2*b^5*x^5 - 8*b*x)*cos(1/2*pi*b^2*x^2) - 96*fresnel_cos(b*
x))/x^7
```

Sympy [A]
time = 1.12, size = 44, normalized size = 0.43

$$-\frac{b\Gamma\left(\frac{1}{4}\right){}_{2}F_{3}\left(\begin{matrix}-\frac{3}{2},\frac{1}{4}\\-\frac{1}{2},\frac{1}{2},\frac{5}{4}\end{matrix}\middle|-\frac{\pi^2b^4x^4}{16}\right)}{24x^6\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**8,x)
```

```
[Out] -b*gamma(1/4)*hyper((-3/2, 1/4), (-1/2, 1/2, 5/4), -pi**2*b**4*x**4/16)/(24
*x**6*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)/x^8, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}(b\,x)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^8,x)
```

```
[Out] int(FresnelC(b*x)/x^8, x)
```

3.126 $\int \frac{\text{FresnelC}(bx)}{x^9} dx$

Optimal. Leaf size=119

$$-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3}+\frac{1}{840}b^8\pi^4\text{FresnelC}(bx)-\frac{\text{FresnelC}(bx)}{8x^8}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5}-\frac{b^7\pi^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x}$$

```
[Out] -1/56*b*cos(1/2*b^2*Pi*x^2)/x^7+1/840*b^5*Pi^2*cos(1/2*b^2*Pi*x^2)/x^3+1/84
0*b^8*Pi^4*FresnelC(b*x)-1/8*FresnelC(b*x)/x^8+1/280*b^3*Pi*sin(1/2*b^2*Pi*
x^2)/x^5-1/840*b^7*Pi^3*sin(1/2*b^2*Pi*x^2)/x
```

Rubi [A]
time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6562, 3469, 3468, 3433}

$$\frac{1}{840}\pi^4 b^8\text{FresnelC}(bx)-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{56x^7}-\frac{\pi^3b^7\sin\left(\frac{1}{2}\pi b^2x^2\right)}{840x}+\frac{\pi^2b^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{840x^3}+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{280x^5}-\frac{\text{FresnelC}(bx)}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^9,x]
```

```
[Out] -1/56*(b*Cos[(b^2*Pi*x^2)/2])/x^7 + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2])/(840*x^3
) + (b^8*Pi^4*FresnelC[b*x])/840 - FresnelC[b*x]/(8*x^8) + (b^3*Pi*Sin[(b^2
*Pi*x^2)/2])/(280*x^5) - (b^7*Pi^3*Sin[(b^2*Pi*x^2)/2])/(840*x)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3468

```
Int[((e_.)*(x_))^(m_)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e*x)
^(m + 1)*(Sin[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)}{x^9}\,dx &= -\frac{C(bx)}{8x^8}+\frac{1}{8}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}-\frac{C(bx)}{8x^8}-\frac{1}{56}\left(b^3\pi\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}-\frac{C(bx)}{8x^8}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5}-\frac{1}{280}\left(b^5\pi^2\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3}-\frac{C(bx)}{8x^8}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5}+\frac{1}{840}\left(b^7\pi^3\right)\int \frac{\sin\left(\frac{1}{2}b^2\pi\right.}{x^2}\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3}-\frac{C(bx)}{8x^8}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5}-\frac{b^7\pi^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x}+\frac{1}{84}\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{56x^7}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{840x^3}+\frac{1}{840}b^8\pi^4C(bx)-\frac{C(bx)}{8x^8}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{280x^5}-\frac{b^7\pi^3}{}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 85, normalized size = 0.71

$$\frac{bx(-15+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right)+(-105+b^8\pi^4x^8)\,\mathrm{FresnelC}(bx)+b^3\pi x^3(3-b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{840x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^9,x]
```

```
[Out] (b*x*(-15 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + (-105 + b^8*Pi^4*x^8)*Fresn
elC[b*x] + b^3*Pi*x^3*(3 - b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2])/(840*x^8)
```

Maple [A]
time = 0.32, size = 108, normalized size = 0.91

method	result
meijerg	$-\frac{b\,\mathrm{hypergeom}\left(\left[-\frac{7}{4},\frac{1}{4}\right],\left[-\frac{3}{4},\frac{1}{2},\frac{5}{4}\right],-\frac{x^4\pi^2b^4}{16}\right)}{7x^7}$

derivativedivides	$b^8\left(-\frac{\text{FresnelC}(bx)}{8b^8x^8}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{56b^7x^7}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{5b^5x^5}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{3b^3x^3}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{bx}+\pi\,\text{FresnelC}(bx)\right)}{3}\right)}{5}\right)}{56}\right)$
default	$b^8\left(-\frac{\text{FresnelC}(bx)}{8b^8x^8}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{56b^7x^7}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{5b^5x^5}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{3b^3x^3}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{bx}+\pi\,\text{FresnelC}(bx)\right)}{3}\right)}{5}\right)}{56}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] b^8*(-1/8*FresnelC(b*x)/b^8/x^8-1/56/b^7/x^7*cos(1/2*b^2*Pi*x^2)-1/56*Pi*(-
1/5/b^5/x^5*sin(1/2*b^2*Pi*x^2)+1/5*Pi*(-1/3/b^3/x^3*cos(1/2*b^2*Pi*x^2)-1/
3*Pi*(-1/b/x*sin(1/2*b^2*Pi*x^2)+Pi*FresnelC(b*x)))))
```

Maxima [C] Result contains complex when optimal does not.

time = 0.54, size = 61, normalized size = 0.51

$$-\frac{\sqrt{\frac{1}{2}}\,(\pi x^2)^{\frac{7}{2}}\left(-(i-1)\,\sqrt{2}\,\Gamma\left(-\frac{7}{2},\frac{1}{2}i\,\pi b^2x^2\right)+(i+1)\,\sqrt{2}\,\Gamma\left(-\frac{7}{2},-\frac{1}{2}i\,\pi b^2x^2\right)\right)b^8}{512\,x^7}-\frac{\mathrm{C}\,(bx)}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^9,x, algorithm="maxima")
```

```
[Out] -1/512*sqrt(1/2)*(pi*x^2)^(7/2)*(-(I - 1)*sqrt(2)*gamma(-7/2, 1/2*I*pi*b^2*
x^2) + (I + 1)*sqrt(2)*gamma(-7/2, -1/2*I*pi*b^2*x^2))*b^8/x^7 - 1/8*fresne
l_cos(b*x)/x^8
```

Fricas [A]
time = 0.36, size = 81, normalized size = 0.68

$$\frac{(\pi^2b^5x^5-15\,bx)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)+(\pi^4b^8x^8-105)\,\mathrm{C}\,(bx)-(\pi^3b^7x^7-3\,\pi b^3x^3)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{840\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^9,x, algorithm="fricas")
```

```
[Out] 1/840*((pi^2*b^5*x^5 - 15*b*x)*cos(1/2*pi*b^2*x^2) + (pi^4*b^8*x^8 - 105)*f
resnel_cos(b*x) - (pi^3*b^7*x^7 - 3*pi*b^3*x^3)*sin(1/2*pi*b^2*x^2))/x^8
```

Sympy [A]
time = 1.84, size = 185, normalized size = 1.55

$$\frac{\pi^4b^8C(bx)\,\Gamma\left(-\frac{7}{4}\right)}{2560\Gamma\left(\frac{5}{4}\right)}-\frac{\pi^3b^7\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{7}{4}\right)}{2560x\Gamma\left(\frac{5}{4}\right)}+\frac{\pi^2b^5\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{7}{4}\right)}{2560x^3\Gamma\left(\frac{5}{4}\right)}+\frac{3\pi b^3\sin\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{7}{4}\right)}{2560x^5\Gamma\left(\frac{5}{4}\right)}-\frac{3b\cos\left(\frac{\pi b^2x^2}{2}\right)\Gamma\left(-\frac{7}{4}\right)}{512x^7\Gamma\left(\frac{5}{4}\right)}-\frac{21C(bx)\,\Gamma\left(-\frac{7}{4}\right)}{512x^8\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**9,x)
```

```
[Out] pi**4*b**8*fresnelc(b*x)*gamma(-7/4)/(2560*gamma(5/4)) - pi**3*b**7*sin(pi*
b**2*x**2/2)*gamma(-7/4)/(2560*x*gamma(5/4)) + pi**2*b**5*cos(pi*b**2*x**2/
2)*gamma(-7/4)/(2560*x**3*gamma(5/4)) + 3*pi*b**3*sin(pi*b**2*x**2/2)*gamma
(-7/4)/(2560*x**5*gamma(5/4)) - 3*b*cos(pi*b**2*x**2/2)*gamma(-7/4)/(512*x*
*7*gamma(5/4)) - 21*fresnelc(b*x)*gamma(-7/4)/(512*x**8*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^9,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)/x^9, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}(b\,x)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^9,x)
```

```
[Out] int(FresnelC(b*x)/x^9, x)
```

3.127 $\int \frac{\textbf{FresnelC}(bx)}{x^{10}}\,dx$

Optimal. Leaf size=127

$$-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4}+\frac{b^9\pi^4\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right)}{6912}-\frac{\text{FresnelC}(bx)}{9x^9}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}-\frac{b^7\pi^3\sin}{34}$$

```
[Out] 1/6912*b^9*Pi^4*Ci(1/2*b^2*Pi*x^2)-1/72*b*cos(1/2*b^2*Pi*x^2)/x^8+1/1728*b^
5*Pi^2*cos(1/2*b^2*Pi*x^2)/x^4-1/9*FresnelC(b*x)/x^9+1/432*b^3*Pi*sin(1/2*b
^2*Pi*x^2)/x^6-1/3456*b^7*Pi^3*sin(1/2*b^2*Pi*x^2)/x^2
```

Rubi [A]
time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6562, 3461, 3378, 3383}

$$-\frac{b\cos\left(\frac{1}{2}\pi b^2x^2\right)}{72x^8}+\frac{\pi^4b^9\text{CosIntegral}\left(\frac{1}{2}\pi b^2x^2\right)}{6912}-\frac{\pi^3b^7\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3456x^2}+\frac{\pi^2b^5\cos\left(\frac{1}{2}\pi b^2x^2\right)}{1728x^4}+\frac{\pi b^3\sin\left(\frac{1}{2}\pi b^2x^2\right)}{432x^6}-\frac{\text{FresnelC}(bx)}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]/x^10,x]
```

```
[Out] -1/72*(b*Cos[(b^2*Pi*x^2)/2])/x^8 + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2])/(1728*x^
4) + (b^9*Pi^4*CosIntegral[(b^2*Pi*x^2)/2])/6912 - FresnelC[b*x]/(9*x^9) +
(b^3*Pi*Sin[(b^2*Pi*x^2)/2])/(432*x^6) - (b^7*Pi^3*Sin[(b^2*Pi*x^2)/2])/(34
56*x^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rule 6562

```
Int[FresnelC[(b_.)*(x_)]*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1
)*(FresnelC[b*x]/(d*(m + 1))), x] - Dist[b/(d*(m + 1)), Int[(d*x)^(m + 1)*C
os[(Pi/2)*b^2*x^2], x], x] /; FreeQ[{b, d, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)}{x^{10}}\,dx &= -\frac{C(bx)}{9x^9}+\frac{1}{9}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx\\
&= -\frac{C(bx)}{9x^9}+\frac{1}{18}b\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^5}\,dx,x,x^2\right)\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}-\frac{C(bx)}{9x^9}-\frac{1}{144}\left(b^3\pi\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^4}\,dx,x,x^2\right)\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}-\frac{C(bx)}{9x^9}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}-\frac{1}{864}\left(b^5\pi^2\right)\text{Subst}\left(\int \frac{\cos\left(\frac{1}{2}b^2\pi x\right)}{x^3}\,dx,x,x^2\right)\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4}-\frac{C(bx)}{9x^9}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}+\frac{\left(b^7\pi^3\right)\text{Subst}\left(\int \frac{\sin\left(\frac{1}{2}b^2\pi x\right)}{x^2}\right.}{3456}\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4}-\frac{C(bx)}{9x^9}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}-\frac{b^7\pi^3\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3456x^2}+\frac{(b^9}{}\\
&= -\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)}{72x^8}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{1728x^4}+\frac{b^9\pi^4\text{Ci}\left(\frac{1}{2}b^2\pi x^2\right)}{6912}-\frac{C(bx)}{9x^9}+\frac{b^3\pi\sin\left(\frac{1}{2}b^2\pi x^2\right)}{432x^6}-\frac{b^7\pi^3}{}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 96, normalized size = 0.76

$$\frac{\frac{4b\left(-24+b^4\pi^2x^4\right)\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}+b^9\pi^4\text{CosIntegral}\left(\frac{1}{2}b^2\pi x^2\right)-\frac{768\text{FresnelC}(bx)}{x^9}-\frac{2b^3\pi\left(-8+b^4\pi^2x^4\right)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}}{6912}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]/x^10,x]
```

```
[Out] ((4*b*(-24 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2])/x^8 + b^9*Pi^4*CosIntegral[
(b^2*Pi*x^2)/2] - (768*FresnelC[b*x])/x^9 - (2*b^3*Pi*(-8 + b^4*Pi^2*x^4)*S
in[(b^2*Pi*x^2)/2])/x^6)/6912
```

Maple [A]
time = 0.42, size = 115, normalized size = 0.91

method	result
meijerg	$\frac{\pi^{\frac{9}{2}} b^9 \left(-\frac{\pi^{\frac{3}{2}} x^4 b^4 \,\text{hypergeom}\left(\left[1,1,\frac{13}{4}\right],\left[2,\frac{7}{2},4,\frac{17}{4}\right],-\frac{x^4\pi^2 b^4}{16}\right)}{585}+\frac{-\frac{332}{243}+\frac{16\gamma}{27}-\frac{16\ln(2)}{27}+\frac{32\ln(x)}{27}+\frac{16\ln(\pi)}{27}+\frac{32\ln(b)}{27}}{\sqrt{\pi}}-\frac{512}{\pi^{\frac{9}{2}} x^8 b^8}+\right.}{4096}$
derivativedivides	$b^9\left(-\frac{\text{FresnelC}(bx)}{9b^9x^9}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{72b^8x^8}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{6b^6x^6}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{4b^4x^4}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}+\frac{\pi\,\text{cosineIntegral}\left(\frac{b^2}{}\right.}{4}\right.}{4}\right.}{6}\right.}{72}\right.$
default	$b^9\left(-\frac{\text{FresnelC}(bx)}{9b^9x^9}-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{72b^8x^8}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{6b^6x^6}+\frac{\pi\left(-\frac{\cos\left(\frac{b^2\pi x^2}{2}\right)}{4b^4x^4}-\frac{\pi\left(-\frac{\sin\left(\frac{b^2\pi x^2}{2}\right)}{2b^2x^2}+\frac{\pi\,\text{cosineIntegral}\left(\frac{b^2}{}\right.}{4}\right.}{4}\right.}{6}\right.}{72}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] b^9*(-1/9*FresnelC(b*x)/b^9/x^9-1/72/b^8/x^8*cos(1/2*b^2*Pi*x^2)-1/72*Pi*(-
1/6/b^6/x^6*sin(1/2*b^2*Pi*x^2)+1/6*Pi*(-1/4/b^4/x^4*cos(1/2*b^2*Pi*x^2)-1/
4*Pi*(-1/2/b^2/x^2*sin(1/2*b^2*Pi*x^2)+1/4*Pi*Ci(1/2*b^2*Pi*x^2)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.31, size = 46, normalized size = 0.36

$$-\frac{1}{576}\left(\pi^4\Gamma\left(-4,\frac{1}{2}i\,\pi b^2x^2\right)+\pi^4\Gamma\left(-4,-\frac{1}{2}i\,\pi b^2x^2\right)\right)b^9-\frac{\mathrm{C}\left(bx\right)}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^10,x, algorithm="maxima")
```

```
[Out] -1/576*(pi^4*gamma(-4, 1/2*I*pi*b^2*x^2) + pi^4*gamma(-4, -1/2*I*pi*b^2*x^2
))*b^9 - 1/9*fresnel_cos(b*x)/x^9
```

Fricas [A]
time = 0.36, size = 111, normalized size = 0.87

$$\frac{\pi^4b^9x^9\,\mathrm{Ci}\left(\frac{1}{2}\,\pi b^2x^2\right)+\pi^4b^9x^9\,\mathrm{Ci}\left(-\frac{1}{2}\,\pi b^2x^2\right)+8\left(\pi^2b^5x^5-24\,bx\right)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)-4\left(\pi^3b^7x^7-8\,\pi b^3x^3\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)-1536\,\mathrm{C}\left(bx\right)}{13824\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^10,x, algorithm="fricas")
```

```
[Out] 1/13824*(pi^4*b^9*x^9*cos_integral(1/2*pi*b^2*x^2) + pi^4*b^9*x^9*cos_integ
ral(-1/2*pi*b^2*x^2) + 8*(pi^2*b^5*x^5 - 24*b*x)*cos(1/2*pi*b^2*x^2) - 4*(p
i^3*b^7*x^7 - 8*pi*b^3*x^3)*sin(1/2*pi*b^2*x^2) - 1536*fresnel_cos(b*x))/x^
9
```

Sympy [A]
time = 7.39, size = 76, normalized size = 0.60

$$-\frac{\pi^6b^{13}x^4\Gamma\left(\frac{13}{4}\right){}_3F_4\left(\left.\begin{matrix}1,1,\frac{13}{4}\\2,\frac{7}{2},4,\frac{17}{4}\end{matrix}\right|-\frac{\pi^2b^4x^4}{16}\right)}{737280\Gamma\left(\frac{17}{4}\right)}+\frac{\pi^4b^9\log\left(b^4x^4\right)}{13824}+\frac{\pi^2b^5}{160x^4}-\frac{b}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)/x**10,x)
```

```
[Out] -pi**6*b**13*x**4*gamma(13/4)*hyper((1, 1, 13/4), (2, 7/2, 4, 17/4), -pi**2
*b**4*x**4/16)/(737280*gamma(17/4)) + pi**4*b**9*log(b**4*x**4)/13824 + pi*
*2*b**5/(160*x**4) - b/(8*x**8)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)/x^10,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)/x^10, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}(b\,x)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)/x^10,x)
```

```
[Out] int(FresnelC(b*x)/x^10, x)
```

3.128 $\int (c+dx)^3 \textbf{FresnelC}(a+bx)\,dx$

Optimal. Leaf size=298

$$-\frac{2d^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi^2}-\frac{3d^3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2}-\frac{(bc-ad)^4\text{FresnelC}(a+bx)}{4b^4d}+\frac{3d^3\text{FresnelC}}{4b^4\pi}$$

```
[Out] -2*d^2*(-a*d+b*c)*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi^2-3/4*d^3*(b*x+a)*cos(1/2*Pi
*(b*x+a)^2)/b^4/Pi^2-1/4*(-a*d+b*c)^4*FresnelC(b*x+a)/b^4/d+3/4*d^3*Fresnel
C(b*x+a)/b^4/Pi^2+1/4*(d*x+c)^4*FresnelC(b*x+a)/d+3/2*d*(-a*d+b*c)^2*Fresne
lS(b*x+a)/b^4/Pi-(-a*d+b*c)^3*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi-3/2*d*(-a*d+b*c)
^2*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi-d^2*(-a*d+b*c)*(b*x+a)^2*sin(1/2*Pi
*(b*x+a)^2)/b^4/Pi-1/4*d^3*(b*x+a)^3*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi
```

Rubi [A]
time = 0.26, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6564, 3515, 3433, 3461, 2717, 3467, 3432, 3377, 2718, 3466}

$$-\frac{d^2(a+bx)^2(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}-\frac{2d^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2b^4}-\frac{(bc-ad)^4\text{FresnelC}(a+bx)}{4b^4d}+\frac{3d(bc-ad)^2S(a+bx)}{2\pi b^4}-\frac{(bc-ad)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}-\frac{3d(a+bx)(bc-ad)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^4}+\frac{3d^3\text{FresnelC}(a+bx)}{4\pi^2b^4}-\frac{d^3(a+bx)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4\pi b^4}-\frac{3d^3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4\pi^2b^4}+\frac{(c+dx)^4\text{FresnelC}(a+bx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*FresnelC[a + b*x],x]
```

```
[Out] (-2*d^2*(b*c - a*d)*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi^2) - (3*d^3*(a + b*x)*
Cos[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2) - ((b*c - a*d)^4*FresnelC[a + b*x])/(
4*b^4*d) + (3*d^3*FresnelC[a + b*x])/(4*b^4*Pi^2) + ((c + d*x)^4*FresnelC[a
 + b*x])/(4*d) + (3*d*(b*c - a*d)^2*FresnelS[a + b*x])/(2*b^4*Pi) - ((b*c -
 a*d)^3*Sin[(Pi*(a + b*x)^2)/2])/(b^4*Pi) - (3*d*(b*c - a*d)^2*(a + b*x)*Si
n[(Pi*(a + b*x)^2)/2])/(2*b^4*Pi) - (d^2*(b*c - a*d)*(a + b*x)^2*Sin[(Pi*(a
 + b*x)^2)/2])/(b^4*Pi) - (d^3*(a + b*x)^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*
Pi)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3515

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6564

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelC[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
```

```
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 C(a+bx)\,dx = \frac{(c+dx)^4 C(a+bx)}{4d} - \frac{b\int (c+dx)^4 \cos\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx}{4d}$$

$$= \frac{(c+dx)^4 C(a+bx)}{4d} - \frac{\mathrm{Subst}\Big(\int \Big(b^4c^4\Big(1+\frac{ad(-4b^3c^3+6ab^2c^2d-4a^2bcd^2+a^3d^3)}{b^4c^4}\Big)\cos\Big(\frac{\pi x^2}{2}$$

$$= \frac{(c+dx)^4 C(a+bx)}{4d} - \frac{d^3\mathrm{Subst}\left(\int x^4\cos\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{4b^4} - \frac{(d^2(bc-ad))\,\mathrm{Su}}{}$$

$$= -\frac{(bc-ad)^4 C(a+bx)}{4b^4 d} + \frac{(c+dx)^4 C(a+bx)}{4d} - \frac{3d(bc-ad)^2(a+bx)\sin\left(\frac{1}{2}\pi(a-\right.}{2b^4\pi}$$

$$= -\frac{3d^3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} - \frac{(bc-ad)^4 C(a+bx)}{4b^4 d} + \frac{(c+dx)^4 C(a+bx)}{4d} +$$

$$= -\frac{2d^2(bc-ad)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi^2} - \frac{3d^3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} - \frac{(bc-ad)^4 C}{4b^4}$$

Mathematica [A]
time = 0.53, size = 424, normalized size = 1.42

$$\frac{-8bcd^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+5ad^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)-3bd^3x\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+\left(4b^3c^3\pi^2(a+bx)+6b^2c^2d\pi^2(-a^2+b^2x^2)+4bcd^2\pi^2(a^3+b^3x^3)+d^3(3-a^4\pi^2+b^4\pi^2x^4)\right)\mathrm{FresnelC}(a+bx)+6d(bc-ad)^2\pi S(a+bx)-4b^3c^3\pi\sin\left(\frac{1}{2}\pi(a+bx)^2\right)+6ab^2c^2d\pi\sin\left(\frac{1}{2}\pi(a+bx)^2\right)-4a^2bcd^2\pi\sin\left(\frac{1}{2}\pi(a+bx)^2\right)+a^3d^3\pi\sin\left(\frac{1}{2}\pi(a+bx)^2\right)-6b^3c^2d\pi x\sin\left(\frac{1}{2}\pi(a+bx)^2\right)+4ab^2cd^2\pi x\sin\left(\frac{1}{2}\pi(a+bx)^2\right)-a^2bd^3\pi x\sin\left(\frac{1}{2}\pi(a+bx)^2\right)-4b^3cd^2\pi x^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)+ab^2d^3\pi x^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)-b^3d^3\pi x^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*FresnelC[a + b*x],x]
```

```
[Out] (-8*b*c*d^2*Cos[(Pi*(a + b*x)^2)/2] + 5*a*d^3*Cos[(Pi*(a + b*x)^2)/2] - 3*b
*d^3*x*Cos[(Pi*(a + b*x)^2)/2] + (4*b^3*c^3*Pi^2*(a + b*x) + 6*b^2*c^2*d*Pi
^2*(-a^2 + b^2*x^2) + 4*b*c*d^2*Pi^2*(a^3 + b^3*x^3) + d^3*(3 - a^4*Pi^2 +
b^4*Pi^2*x^4))*FresnelC[a + b*x] + 6*d*(b*c - a*d)^2*Pi*FresnelS[a + b*x] -
 4*b^3*c^3*Pi*Sin[(Pi*(a + b*x)^2)/2] + 6*a*b^2*c^2*d*Pi*Sin[(Pi*(a + b*x)^
2)/2] - 4*a^2*b*c*d^2*Pi*Sin[(Pi*(a + b*x)^2)/2] + a^3*d^3*Pi*Sin[(Pi*(a +
b*x)^2)/2] - 6*b^3*c^2*d*Pi*x*Sin[(Pi*(a + b*x)^2)/2] + 4*a*b^2*c*d^2*Pi*x*
Sin[(Pi*(a + b*x)^2)/2] - a^2*b*d^3*Pi*x*Sin[(Pi*(a + b*x)^2)/2] - 4*b^3*c*
d^2*Pi*x^2*Sin[(Pi*(a + b*x)^2)/2] + a*b^2*d^3*Pi*x^2*Sin[(Pi*(a + b*x)^2)/
2] - b^3*d^3*Pi*x^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

Maple [A]
time = 0.49, size = 398, normalized size = 1.34

method	result
derivativedivides	$\frac{\frac{\text{FresnelC}(bx+a)(ad-cb-d(bx+a))^4}{4b^3d}-\frac{\frac{d^4(bx+a)^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{3d^4\left(-\frac{(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{\text{FresnelC}(bx+a)}{\pi}\right)}{\pi}+\frac{\left(-4a\,d^4\cdots\right.}{\cdots}}{\cdots}}{\cdots}$
default	$\frac{\frac{\text{FresnelC}(bx+a)(ad-cb-d(bx+a))^4}{4b^3d}-\frac{\frac{d^4(bx+a)^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{3d^4\left(-\frac{(bx+a)\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{\text{FresnelC}(bx+a)}{\pi}\right)}{\pi}+\frac{\left(-4a\,d^4\cdots\right.}{\cdots}}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*FresnelC(b*x+a)*(a*d-c*b-d*(b*x+a))^4/b^3/d-1/4/b^3/d*(d^4/Pi*(b*x
+a)^3*sin(1/2*Pi*(b*x+a)^2)-3*d^4/Pi*(-1/Pi*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)+1
/Pi*FresnelC(b*x+a))+(-4*a*d^4+4*b*c*d^3)/Pi*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2
)+2*(-4*a*d^4+4*b*c*d^3)/Pi^2*cos(1/2*Pi*(b*x+a)^2)+(6*a^2*d^4-12*a*b*c*d^3
+6*b^2*c^2*d^2)/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-(6*a^2*d^4-12*a*b*c*d^3+6*
b^2*c^2*d^2)/Pi*FresnelS(b*x+a)+(-4*a^3*d^4+12*a^2*b*c*d^3-12*a*b^2*c^2*d^2
+4*b^3*c^3*d)/Pi*sin(1/2*Pi*(b*x+a)^2)+a^4*d^4*FresnelC(b*x+a)-4*a^3*b*c*d^
3*FresnelC(b*x+a)+6*a^2*b^2*c^2*d^2*FresnelC(b*x+a)-4*a*b^3*c^3*d*FresnelC(
b*x+a)+b^4*c^4*FresnelC(b*x+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*fresnel_cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^3*fresnel_cos(b*x + a), x)
```

Fricas [A]
time = 0.36, size = 375, normalized size = 1.26

$$\frac{6\,\pi(b^2c^2d-2\,abcd^2+a^2d^3)\sqrt{b^2}\,\mathrm{S}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)+(\pi^2(4\,ab^3c^3-6\,a^2b^2c^2d+4\,a^3bcd^2-a^4d^3)+3\,d^3)\sqrt{b^2}\,\mathrm{C}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)-(3\,b^2d^3x+8\,b^2cd^2-5\,abd^3)\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)+(\pi^2b^5d^3x^4+4\,\pi^2b^5cd^2x^3+6\,\pi^2b^5c^2dx^2+4\,\pi^2b^5c^3x)\,\mathrm{C}\,(bx+a)-(\pi b^4d^3x^3+\pi(4\,b^4cd^2-ab^3d^3)x^2+\pi(6\,b^4c^2d-4\,ab^3cd^2+a^2b^2d^3)x+\pi(4\,b^4c^3-6\,ab^3c^2d+4\,a^2b^2cd^2-a^3bd^3))\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{4\,\pi^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*fresnel_cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(6*pi*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(b^2)*fresnel_sin(sqrt(b^
2)*(b*x + a)/b) + (pi^2*(4*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 4*a^3*b*c*d^2 - a^
```

```
4*d^3) + 3*d^3)*sqrt(b^2)*fresnel_cos(sqrt(b^2)*(b*x + a)/b) - (3*b^2*d^3*x
 + 8*b^2*c*d^2 - 5*a*b*d^3)*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) + (
pi^2*b^5*d^3*x^4 + 4*pi^2*b^5*c*d^2*x^3 + 6*pi^2*b^5*c^2*d*x^2 + 4*pi^2*b^5
*c^3*x)*fresnel_cos(b*x + a) - (pi*b^4*d^3*x^3 + pi*(4*b^4*c*d^2 - a*b^3*d^
3)*x^2 + pi*(6*b^4*c^2*d - 4*a*b^3*c*d^2 + a^2*b^2*d^3)*x + pi*(4*b^4*c^3 -
 6*a*b^3*c^2*d + 4*a^2*b^2*c*d^2 - a^3*b*d^3))*sin(1/2*pi*b^2*x^2 + pi*a*b*
x + 1/2*pi*a^2))/(pi^2*b^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 C(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*fresnelc(b*x+a),x)
[Out] Integral((c + d*x)**3*fresnelc(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*fresnel_cos(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^3*fresnel_cos(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{FresnelC}(a+b\,x)\,{(c+d\,x)}^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)*(c + d*x)^3,x)
[Out] int(FresnelC(a + b*x)*(c + d*x)^3, x)
```

3.129 $\int (c+dx)^2 \textbf{FresnelC}(a+bx)\,dx$

Optimal. Leaf size=194

$$-\frac{2d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}-\frac{(bc-ad)^3\text{FresnelC}(a+bx)}{3b^3d}+\frac{(c+dx)^3\text{FresnelC}(a+bx)}{3d}+\frac{d(bc-ad)S(a+bx)}{b^3\pi}-$$

```
[Out] -2/3*d^2*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi^2-1/3*(-a*d+b*c)^3*FresnelC(b*x+a)/b^
3/d+1/3*(d*x+c)^3*FresnelC(b*x+a)/d+d*(-a*d+b*c)*FresnelS(b*x+a)/b^3/Pi-(-a
*d+b*c)^2*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi-d*(-a*d+b*c)*(b*x+a)*sin(1/2*Pi*(b*x
+a)^2)/b^3/Pi-1/3*d^2*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi
```

Rubi [A]
time = 0.15, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6564, 3515, 3433, 3461, 2717, 3467, 3432, 3377, 2718}

$$-\frac{(bc-ad)^3\text{FresnelC}(a+bx)}{3b^3d}+\frac{d(bc-ad)S(a+bx)}{\pi b^3}-\frac{(bc-ad)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}-\frac{d(a+bx)(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}-\frac{d^2(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi b^3}-\frac{2d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi^2b^3}+\frac{(c+dx)^3\text{FresnelC}(a+bx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*FresnelC[a + b*x],x]
```

```
[Out] (-2*d^2*Cos[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2) - ((b*c - a*d)^3*FresnelC[a +
 b*x])/(3*b^3*d) + ((c + d*x)^3*FresnelC[a + b*x])/(3*d) + (d*(b*c - a*d)*F
resnelS[a + b*x])/(b^3*Pi) - ((b*c - a*d)^2*Sin[(Pi*(a + b*x)^2)/2])/(b^3*P
i) - (d*(b*c - a*d)*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi) - (d^2*(a +
 b*x)^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3515

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6564

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelC[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2 C(a+bx)\,dx = \frac{(c+dx)^3 C(a+bx)}{3d} - \frac{b\int (c+dx)^3 \cos\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx}{3d}$$

$$= \frac{(c+dx)^3 C(a+bx)}{3d} - \frac{\text{Subst}\Big(\int \Big(b^3c^3\Big(1-\frac{ad(3b^2c^2-3abcd+a^2d^2)}{b^3c^3}\Big)\cos\Big(\frac{\pi x^2}{2}\Big)+3b^2c \cdots}{\cdots}$$

$$= \frac{(c+dx)^3 C(a+bx)}{3d} - \frac{d^2\text{Subst}\Big(\int x^3\cos\Big(\frac{\pi x^2}{2}\Big)\,dx,x,a+bx\Big)}{3b^3} - \frac{(d(bc-ad))\text{Su}\cdots}{\cdots}$$

$$= -\frac{(bc-ad)^3 C(a+bx)}{3b^3d} + \frac{(c+dx)^3 C(a+bx)}{3d} - \frac{d(bc-ad)(a+bx)\sin\left(\frac{1}{2}\pi(a+\cdots\right.}{b^3\pi}$$

$$= -\frac{(bc-ad)^3 C(a+bx)}{3b^3d} + \frac{(c+dx)^3 C(a+bx)}{3d} + \frac{d(bc-ad)S(a+bx)}{b^3\pi} - \frac{(bc-a\cdots}{\cdots}$$

$$= -\frac{2d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2} - \frac{(bc-ad)^3 C(a+bx)}{3b^3d} + \frac{(c+dx)^3 C(a+bx)}{3d} + \frac{d(bc-\cdots}{\cdots}$$

Mathematica [A]
time = 0.31, size = 237, normalized size = 1.22

$$\frac{-2d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)+\pi^2(3ab^2c^2-3a^2bcd+a^3d^2+b^3x(3c^2+3cdx+d^2x^2))\,\text{FresnelC}(a+bx)+3d(bc-ad)\pi S(a+bx)-3b^2c^2\pi\sin\left(\frac{1}{2}\pi(a+bx)^2\right)+3abcd\pi\sin\left(\frac{1}{2}\pi(a+bx)^2\right)-a^2d^2\pi\sin\left(\frac{1}{2}\pi(a+bx)^2\right)-3b^2cd\pi x\sin\left(\frac{1}{2}\pi(a+bx)^2\right)+abd^2\pi x\sin\left(\frac{1}{2}\pi(a+bx)^2\right)-b^2d^2\pi x^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*FresnelC[a + b*x],x]
[Out] (-2*d^2*Cos[(Pi*(a + b*x)^2)/2] + Pi^2*(3*a*b^2*c^2 - 3*a^2*b*c*d + a^3*d^2
 + b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2))*FresnelC[a + b*x] + 3*d*(b*c - a*d)*P
i*FresnelS[a + b*x] - 3*b^2*c^2*Pi*Sin[(Pi*(a + b*x)^2)/2] + 3*a*b*c*d*Pi*S
in[(Pi*(a + b*x)^2)/2] - a^2*d^2*Pi*Sin[(Pi*(a + b*x)^2)/2] - 3*b^2*c*d*Pi*
x*Sin[(Pi*(a + b*x)^2)/2] + a*b*d^2*Pi*x*Sin[(Pi*(a + b*x)^2)/2] - b^2*d^2*
Pi*x^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2)
```

Maple [A]
time = 0.49, size = 251, normalized size = 1.29

method	result
derivativedivides	$\frac{-\frac{\text{FresnelC}(bx+a)(ad-cb-d(bx+a))^3}{3b^2d}+\frac{-\frac{d^3(bx+a)^2\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{2d^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi^2}+\frac{\left(3a\,d^3-3bc\,d^2\right)(bx+a)\sin\left(\frac{\pi(bx+a)}{2}\cdots\right.}{\pi}\cdots}{\cdots}}{\cdots}$
default	$\frac{-\frac{\text{FresnelC}(bx+a)(ad-cb-d(bx+a))^3}{3b^2d}+\frac{-\frac{d^3(bx+a)^2\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{2d^3\cos\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi^2}+\frac{\left(3a\,d^3-3bc\,d^2\right)(bx+a)\sin\left(\frac{\pi(bx+a)}{2}\cdots\right.}{\pi}\cdots}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/3*FresnelC(b*x+a)*(a*d-c*b-d*(b*x+a))^3/b^2/d+1/3/b^2/d*(-d^3/Pi*(b
*x+a)^2*sin(1/2*Pi*(b*x+a)^2)-2*d^3/Pi^2*cos(1/2*Pi*(b*x+a)^2)+(3*a*d^3-3*b
*c*d^2)/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-(3*a*d^3-3*b*c*d^2)/Pi*FresnelS(b*
x+a)+(-3*a^2*d^3+6*a*b*c*d^2-3*b^2*c^2*d)/Pi*sin(1/2*Pi*(b*x+a)^2)+a^3*d^3*
FresnelC(b*x+a)-3*a^2*b*c*d^2*FresnelC(b*x+a)+3*a*b^2*c^2*d*FresnelC(b*x+a)
-b^3*c^3*FresnelC(b*x+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnel_cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2*fresnel_cos(b*x + a), x)
```

Fricas [A]
time = 0.36, size = 249, normalized size = 1.28

$$\frac{\pi^2(3\,ab^2c^2-3\,a^2bcd+a^3d^2)\sqrt{b^2}\ \mathrm{C}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)-2\,bd^2\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)+3\,\pi(bcd-ad^2)\sqrt{b^2}\ \mathrm{S}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)+(\pi^2b^4d^2x^3+3\,\pi^2b^4cdx^2+3\,\pi^2b^4c^2x)\,\mathrm{C}\,(bx+a)-(\pi b^3d^2x^2+\pi(3\,b^3cd-ab^2d^2)x+\pi(3\,b^3c^2-3\,ab^2cd+a^2bd^2))\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{3\,\pi^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnel_cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3*(pi^2*(3*a*b^2*c^2 - 3*a^2*b*c*d + a^3*d^2)*sqrt(b^2)*fresnel_cos(sqrt(
b^2)*(b*x + a)/b) - 2*b*d^2*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) + 3
*pi*(b*c*d - a*d^2)*sqrt(b^2)*fresnel_sin(sqrt(b^2)*(b*x + a)/b) + (pi^2*b^
4*d^2*x^3 + 3*pi^2*b^4*c*d*x^2 + 3*pi^2*b^4*c^2*x)*fresnel_cos(b*x + a) - (
pi*b^3*d^2*x^2 + pi*(3*b^3*c*d - a*b^2*d^2)*x + pi*(3*b^3*c^2 - 3*a*b^2*c*d
 + a^2*b*d^2))*sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2))/(pi^2*b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2\,C(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*fresnelc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*fresnelc(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnel_cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*fresnel_cos(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{FresnelC}(a+b\,x)\,\left(c+d\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)*(c + d*x)^2,x)
```

```
[Out] int(FresnelC(a + b*x)*(c + d*x)^2, x)
```

3.130 $\int (c+dx)\textbf{FresnelC}(a+bx)\,dx$

Optimal. Leaf size=122

$$-\frac{(bc-ad)^2\text{FresnelC}(a+bx)}{2b^2d}+\frac{(c+dx)^2\text{FresnelC}(a+bx)}{2d}+\frac{dS(a+bx)}{2b^2\pi}-\frac{(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi}-\frac{d(a+$$

```
[Out] -1/2*(-a*d+b*c)^2*FresnelC(b*x+a)/b^2/d+1/2*(d*x+c)^2*FresnelC(b*x+a)/d+1/2
*d*FresnelS(b*x+a)/b^2/Pi-(-a*d+b*c)*sin(1/2*Pi*(b*x+a)^2)/b^2/Pi-1/2*d*(b*
x+a)*sin(1/2*Pi*(b*x+a)^2)/b^2/Pi
```

Rubi [A]
time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6564, 3515, 3433, 3461, 2717, 3467, 3432}

$$-\frac{(bc-ad)^2\text{FresnelC}(a+bx)}{2b^2d}-\frac{(bc-ad)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}+\frac{dS(a+bx)}{2\pi b^2}-\frac{d(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{(c+dx)^2\text{FresnelC}(a+bx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*FresnelC[a + b*x],x]
```

```
[Out] -1/2*((b*c - a*d)^2*FresnelC[a + b*x])/(b^2*d) + ((c + d*x)^2*FresnelC[a +
b*x])/(2*d) + (d*FresnelS[a + b*x])/(2*b^2*Pi) - ((b*c - a*d)*Sin[(Pi*(a +
b*x)^2)/2])/(b^2*Pi) - (d*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(2*b^2*Pi)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3515

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6564

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelC[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int (c+dx)C(a+bx)\,dx = \frac{(c+dx)^2C(a+bx)}{2d} - \frac{b\int (c+dx)^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx}{2d}$$

$$= \frac{(c+dx)^2C(a+bx)}{2d} - \frac{\text{Subst}\left(\int \left(b^2c^2\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)\cos\left(\frac{\pi x^2}{2}\right) + 2bcd\left(1-\frac{ad}{bc}\right.\right.\right.}{2b^2d}$$

$$= \frac{(c+dx)^2C(a+bx)}{2d} - \frac{d\text{Subst}\left(\int x^2\cos\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{2b^2} - \frac{(bc-ad)\text{Subst}\left(\right.}{}$$

$$= -\frac{(bc-ad)^2C(a+bx)}{2b^2d} + \frac{(c+dx)^2C(a+bx)}{2d} - \frac{d(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi} - $$

$$= -\frac{(bc-ad)^2C(a+bx)}{2b^2d} + \frac{(c+dx)^2C(a+bx)}{2d} + \frac{dS(a+bx)}{2b^2\pi} - \frac{(bc-ad)\sin\left(\frac{1}{2}\pi\right.}{b^2\pi}$$

Mathematica [A]
time = 0.17, size = 74, normalized size = 0.61

$$\frac{-\pi(a+bx)(ad-b(2c+dx))\text{FresnelC}(a+bx) + dS(a+bx) + (-2bc+ad-bdx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*FresnelC[a + b*x],x]
```

```
[Out] (-(Pi*(a + b*x)*(a*d - b*(2*c + d*x))*FresnelC[a + b*x]) + d*FresnelS[a + b
*x] + (-2*b*c + a*d - b*d*x)*Sin[(Pi*(a + b*x)^2)/2])/(2*b^2*Pi)
```

Maple [A]
time = 0.44, size = 108, normalized size = 0.89

method	result	siz
derivativedivides	$\frac{-\frac{\mathrm{FresnelC}(bx+a)\left(da(bx+a)-cb(bx+a)-\frac{d(bx+a)^2}{2}\right)}{b}+\frac{-\frac{d(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{d\,\mathrm{S}(bx+a)}{\pi}+\frac{(2ad-2cb)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{2b}}{b}$	10
default	$\frac{-\frac{\mathrm{FresnelC}(bx+a)\left(da(bx+a)-cb(bx+a)-\frac{d(bx+a)^2}{2}\right)}{b}+\frac{-\frac{d(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}+\frac{d\,\mathrm{S}(bx+a)}{\pi}+\frac{(2ad-2cb)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{2b}}{b}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-FresnelC(b*x+a)/b*(d*a*(b*x+a)-c*b*(b*x+a)-1/2*d*(b*x+a)^2)+1/2/b*(-d
/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)+d/Pi*FresnelS(b*x+a)+(2*a*d-2*b*c)/Pi*sin
(1/2*Pi*(b*x+a)^2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnel_cos(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*fresnel_cos(b*x + a), x)
```

Fricas [A]
time = 0.37, size = 132, normalized size = 1.08

$$\frac{\pi(2\,abc-a^2d)\sqrt{b^2}\;\mathrm{C}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)+\sqrt{b^2}\,d\,\mathrm{S}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)+(\pi b^3dx^2+2\,\pi b^3cx)\,\mathrm{C}\,(bx+a)-(b^2dx+2\,b^2c-abd)\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{2\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnel_cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(pi*(2*a*b*c - a^2*d)*sqrt(b^2)*fresnel_cos(sqrt(b^2)*(b*x + a)/b) + sq
rt(b^2)*d*fresnel_sin(sqrt(b^2)*(b*x + a)/b) + (pi*b^3*d*x^2 + 2*pi*b^3*c*x
```

```
)*fresnel_cos(b*x + a) - (b^2*d*x + 2*b^2*c - a*b*d)*sin(1/2*pi*b^2*x^2 + p
i*a*b*x + 1/2*pi*a^2))/(pi*b^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)\, C(a + bx)\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a),x)
[Out] Integral((c + d*x)*fresnelc(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnel_cos(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)*fresnel_cos(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{FresnelC}(a + b\,x)\;(c + d\,x)\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)*(c + d*x),x)
[Out] int(FresnelC(a + b*x)*(c + d*x), x)
```

3.131 $\int \text{FresnelC}(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{(a+bx)\text{FresnelC}(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}$$

```
[Out] (b*x+a)*FresnelC(b*x+a)/b-sin(1/2*Pi*(b*x+a)^2)/b/Pi
```

Rubi [A]
time = 0.00, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{6554\}$

$$\frac{(a+bx)\text{FresnelC}(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[a + b*x],x]
[Out] ((a + b*x)*FresnelC[a + b*x])/b - Sin[(Pi*(a + b*x)^2)/2]/(b*Pi)
```

Rule 6554

```
Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelC[a +
 b*x]/b), x] - Simp[Sin[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int C(a+bx)\,dx=\frac{(a+bx)C(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 90 vs. 2(37) = 74.
time = 0.02, size = 90, normalized size = 2.43

$$\frac{a\text{FresnelC}(a+bx)}{b}+x\text{FresnelC}(a+bx)-\frac{\cos\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)\sin\left(\frac{a^2\pi}{2}\right)}{b\pi}-\frac{\cos\left(\frac{a^2\pi}{2}\right)\sin\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[a + b*x],x]
[Out] (a*FresnelC[a + b*x])/b + x*FresnelC[a + b*x] - (Cos[a*b*Pi*x + (b^2*Pi*x^2
)/2]*Sin[(a^2*Pi)/2])/(b*Pi) - (Cos[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^2)
/2])/(b*Pi)
```

Maple [A]
time = 0.38, size = 34, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\mathrm{FresnelC}(bx+a)(bx+a)-\frac{\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	34
default	$\frac{\mathrm{FresnelC}(bx+a)(bx+a)-\frac{\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}}{b}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(FresnelC(b*x+a)*(b*x+a)-1/Pi*sin(1/2*Pi*(b*x+a)^2))
```

Maxima [A]
time = 0.26, size = 44, normalized size = 1.19

$$\frac{(bx+a)\,\mathrm{C}\,(bx+a)-\frac{\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{\pi}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*x + a)*fresnel_cos(b*x + a) - sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^
2)/pi)/b
```

Fricas [A]
time = 0.34, size = 47, normalized size = 1.27

$$\frac{(\pi bx+\pi a)\,\mathrm{C}\,(bx+a)-\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{\pi b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a),x, algorithm="fricas")
```

```
[Out] ((pi*b*x + pi*a)*fresnel_cos(b*x + a) - sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2
*pi*a^2))/(pi*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int C(a+bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a),x)
```

```
[Out] Integral(fresnelc(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{FresnelC}(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x),x)
```

```
[Out] int(FresnelC(a + b*x), x)
```

3.132 $\int \frac{\text{FresnelC}(a+bx)}{c+dx} \, dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{\text{FresnelC}(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(FresnelC(b*x+a)/(d*x+c),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx} \, dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelC[a + b*x]/(c + d*x),x]
[Out] Defer[Int][FresnelC[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{C(a+bx)}{c+dx} \, dx = \int \frac{C(a+bx)}{c+dx} \, dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(a+bx)}{c+dx} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelC[a + b*x]/(c + d*x),x]
[Out] Integrate[FresnelC[a + b*x]/(c + d*x), x]
```

Maple [A]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\left(bx+a\right)}{dx+c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a)/(d*x+c),x)
[Out] int(FresnelC(b*x+a)/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x + a)/(d*x + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x + a)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/(d*x+c),x)
[Out] Integral(fresnelc(a + b*x)/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x + a)/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{FresnelC}(a+b\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)/(c + d*x),x)
[Out] int(FresnelC(a + b*x)/(c + d*x), x)
```

3.133 $\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{\text{FresnelC}(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(FresnelC(b*x+a)/(d*x+c)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelC[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Defer[Int][FresnelC[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{C(a+bx)}{(c+dx)^2} dx = \int \frac{C(a+bx)}{(c+dx)^2} dx$$

Mathematica [A]
time = 1.59, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(a+bx)}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelC[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Integrate[FresnelC[a + b*x]/(c + d*x)^2, x]
```

Maple [A]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\left(bx+a\right)}{\left(dx+c\right)^{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(FresnelC(b*x+a)/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x + a)/(d*x + c)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(fresnel_cos(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C(a + bx)}{(c + dx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(fresnelc(a + b*x)/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x + a)/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{FresnelC}(a + b\,x)}{{(c + d\,x)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)/(c + d*x)^2,x)
[Out] int(FresnelC(a + b*x)/(c + d*x)^2, x)
```

3.134 $\int x^3\text{FresnelC}(a+bx)\,dx$

Optimal. Leaf size=227

$$\frac{2a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi^2}-\frac{3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2}-\frac{a^4\text{FresnelC}(a+bx)}{4b^4}+\frac{3\text{FresnelC}(a+bx)}{4b^4\pi^2}+\frac{1}{4}x^4\text{FresnelC}$$

```
[Out] 2*a*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi^2-3/4*(b*x+a)*cos(1/2*Pi*(b*x+a)^2)/b^4/Pi
^2-1/4*a^4*FresnelC(b*x+a)/b^4+3/4*FresnelC(b*x+a)/b^4/Pi^2+1/4*x^4*Fresnel
C(b*x+a)+3/2*a^2*FresnelS(b*x+a)/b^4/Pi+a^3*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi-3/
2*a^2*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi+a*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2
)/b^4/Pi-1/4*(b*x+a)^3*sin(1/2*Pi*(b*x+a)^2)/b^4/Pi
```

Rubi [A]
time = 0.12, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6564, 3515, 3433, 3461, 2717, 3467, 3432, 3377, 2718, 3466}

$$-\frac{a^4\text{FresnelC}(a+bx)}{4b^4}+\frac{a^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}+\frac{3a^2S(a+bx)}{2\pi b^4}-\frac{3a^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^4}+\frac{3\text{FresnelC}(a+bx)}{4\pi^2b^4}+\frac{a(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^4}-\frac{(a+bx)^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4\pi b^4}+\frac{2a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi^2b^4}-\frac{3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4\pi^2b^4}+\frac{1}{4}x^4\text{FresnelC}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelC[a + b*x],x]
```

```
[Out] (2*a*Cos[(Pi*(a + b*x)^2)/2])/(b^4*Pi^2) - (3*(a + b*x)*Cos[(Pi*(a + b*x)^2
)/2])/(4*b^4*Pi^2) - (a^4*FresnelC[a + b*x])/(4*b^4) + (3*FresnelC[a + b*x]
)/(4*b^4*Pi^2) + (x^4*FresnelC[a + b*x])/4 + (3*a^2*FresnelS[a + b*x])/(2*b
^4*Pi) + (a^3*Sin[(Pi*(a + b*x)^2)/2])/(b^4*Pi) - (3*a^2*(a + b*x)*Sin[(Pi*
(a + b*x)^2)/2])/(2*b^4*Pi) + (a*(a + b*x)^2*Sin[(Pi*(a + b*x)^2)/2])/(b^4*
Pi) - ((a + b*x)^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3515

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6564

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelC[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x^3 C(a+bx)\,dx = \frac{1}{4}x^4 C(a+bx) - \frac{1}{4}b\int x^4\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx$$

$$= \frac{1}{4}x^4 C(a+bx) - \frac{\text{Subst}\left(\int \left(a^4\cos\left(\frac{\pi x^2}{2}\right) - 4a^3x\cos\left(\frac{\pi x^2}{2}\right) + 6a^2x^2\cos\left(\frac{\pi x^2}{2}\right) - 4ax^3\text{ c}\right.}{4b^4}$$

$$= \frac{1}{4}x^4 C(a+bx) - \frac{\text{Subst}\left(\int x^4\cos\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{4b^4} + \frac{a\text{Subst}\left(\int x^3\cos\left(\frac{\pi x^2}{2}\right)\,dx, x\right.}{b^4}$$

$$= -\frac{a^4 C(a+bx)}{4b^4} + \frac{1}{4}x^4 C(a+bx) - \frac{3a^2(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^4\pi} - \frac{(a+bx)^3\sin\left(\frac{1}{2}\pi(\right.}{4b^4\pi}$$

$$= -\frac{3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} - \frac{a^4 C(a+bx)}{4b^4} + \frac{1}{4}x^4 C(a+bx) + \frac{3a^2 S(a+bx)}{2b^4\pi} + \frac{a^3\text{ s}}{}$$

$$= \frac{2a\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^4\pi^2} - \frac{3(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2} - \frac{a^4 C(a+bx)}{4b^4} + \frac{3C(a+bx)}{4b^4\pi^2} +$$

Mathematica [A]
time = 0.21, size = 166, normalized size = 0.73

$$\frac{5a\cos\left(\frac{1}{2}\pi(a+bx)^2\right) - 3bx\cos\left(\frac{1}{2}\pi(a+bx)^2\right) + (3 - a^4\pi^2 + b^4\pi^2x^4)\,\text{FresnelC}(a+bx) + 6a^2\pi S(a+bx) + a^3\pi\sin\left(\frac{1}{2}\pi(a+bx)^2\right) - a^2b\pi x\sin\left(\frac{1}{2}\pi(a+bx)^2\right) + ab^2\pi x^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right) - b^3\pi x^3\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{4b^4\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelC[a + b*x],x]
```

```
[Out] (5*a*Cos[(Pi*(a + b*x)^2)/2] - 3*b*x*Cos[(Pi*(a + b*x)^2)/2] + (3 - a^4*Pi^
2 + b^4*Pi^2*x^4)*FresnelC[a + b*x] + 6*a^2*Pi*FresnelS[a + b*x] + a^3*Pi*S
in[(Pi*(a + b*x)^2)/2] - a^2*b*Pi*x*Sin[(Pi*(a + b*x)^2)/2] + a*b^2*Pi*x^2*
Sin[(Pi*(a + b*x)^2)/2] - b^3*Pi*x^3*Sin[(Pi*(a + b*x)^2)/2])/(4*b^4*Pi^2)
```

Maple [A]
time = 0.40, size = 187, normalized size = 0.82

method	result
derivativedivides	$\frac{\frac{\text{FresnelC}(bx+a)b^4x^4}{4} - \frac{a^4\,\text{FresnelC}(bx+a)}{4} + \frac{a^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} - \frac{3a^2(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi} + \frac{3a^2\text{S}(bx+a)}{2\pi} + \frac{a(bx+a)^2\sin\left(\frac{\pi(bx\right.}{\pi}}{b^4}$
default	$\frac{\frac{\text{FresnelC}(bx+a)b^4x^4}{4} - \frac{a^4\,\text{FresnelC}(bx+a)}{4} + \frac{a^3\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} - \frac{3a^2(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi} + \frac{3a^2\text{S}(bx+a)}{2\pi} + \frac{a(bx+a)^2\sin\left(\frac{\pi(bx\right.}{\pi}}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/4*FresnelC(b*x+a)*b^4*x^4-1/4*a^4*FresnelC(b*x+a)+a^3/Pi*sin(1/2*P
i*(b*x+a)^2)-3/2*a^2/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)+3/2*a^2/Pi*FresnelS(b
*x+a)+a/Pi*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2)+2*a/Pi^2*cos(1/2*Pi*(b*x+a)^2)-1
/4/Pi*(b*x+a)^3*sin(1/2*Pi*(b*x+a)^2)+3/4/Pi*(-1/Pi*(b*x+a)*cos(1/2*Pi*(b*x
+a)^2)+1/Pi*FresnelC(b*x+a)))
```

Maxima [C] Result contains complex when optimal does not.
time = 1.18, size = 502, normalized size = 2.21

$$\frac{1}{4}x^4\,\mathrm{C}\left(bx+a\right)+\frac{\left(16\left(-i\,\pi^2e^{\left(\frac{1}{2}i\,\pi b^2x^2+i\,\pi abx+\frac{1}{2}i\,\pi a^2\right)}+i\,\pi^2e^{\left(-\frac{1}{2}i\,\pi b^2x^2-i\,\pi abx-\frac{1}{2}i\,\pi a^2\right)}\right)a^4+32\left(\pi\Gamma\left(2,\frac{1}{2}i\,\pi b^2x^2+i\,\pi abx+\frac{1}{2}i\,\pi a^2\right)+\pi\Gamma\left(2,-\frac{1}{2}i\,\pi b^2x^2-i\,\pi abx-\frac{1}{2}i\,\pi a^2\right)\right)a^2+16\left(\left(-i\,\pi^2e^{\left(\frac{1}{2}i\,\pi b^2x^2+i\,\pi abx+\frac{1}{2}i\,\pi a^2\right)}+i\,\pi^2e^{\left(-\frac{1}{2}i\,\pi b^2x^2-i\,\pi abx-\frac{1}{2}i\,\pi a^2\right)}\right)a^3+2\left(\pi\Gamma\left(2,\frac{1}{2}i\,\pi b^2x^2+i\,\pi abx+\frac{1}{2}i\,\pi a^2\right)+\pi\Gamma\left(2,-\frac{1}{2}i\,\pi b^2x^2-i\,\pi abx-\frac{1}{2}i\,\pi a^2\right)\right)a\right)bx+\left(\left(\left(i-1\right)\sqrt{2}\,\pi^{\frac{5}{2}}\left(\mathrm{erf}\left(\sqrt{\frac{1}{2}i\,\pi b^2x^2+i\,\pi abx+\frac{1}{2}i\,\pi a^2}\right)-1\right)-\left(i+1\right)\sqrt{2}\,\pi^{\frac{5}{2}}\left(\mathrm{erf}\left(\sqrt{-\frac{1}{2}i\,\pi b^2x^2-i\,\pi abx-\frac{1}{2}i\,\pi a^2}\right)-1\right)\right)a^4+12\left(-\left(i+1\right)\sqrt{2}\,\pi\Gamma\left(\frac{3}{2},\frac{1}{2}i\,\pi b^2x^2+i\,\pi abx+\frac{1}{2}i\,\pi a^2\right)+\left(i-1\right)\sqrt{2}\,\pi\Gamma\left(\frac{3}{2},-\frac{1}{2}i\,\pi b^2x^2-i\,\pi abx-\frac{1}{2}i\,\pi a^2\right)\right)a^2+\left(4i-4\right)\sqrt{2}\,\Gamma\left(\frac{5}{2},\frac{1}{2}i\,\pi b^2x^2+i\,\pi abx+\frac{1}{2}i\,\pi a^2\right)-\left(4i+4\right)\sqrt{2}\,\Gamma\left(\frac{5}{2},-\frac{1}{2}i\,\pi b^2x^2-i\,\pi abx-\frac{1}{2}i\,\pi a^2\right)\right)\sqrt{2\,\pi b^2x^2+4\,\pi abx+2\,\pi a^2}\right)b}{32\left(\pi^3b^6x+\pi^3ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*fresnel_cos(b*x + a) + 1/32*(16*(-I*pi^2*e^(1/2*I*pi*b^2*x^2 + I*pi
*a*b*x + 1/2*I*pi*a^2) + I*pi^2*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*p
i*a^2))*a^4 + 32*(pi*gamma(2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2)
 + pi*gamma(2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^2 + 16*((-
I*pi^2*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + I*pi^2*e^(-1/2*I*
pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^3 + 2*(pi*gamma(2, 1/2*I*pi*b^2*
x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + pi*gamma(2, -1/2*I*pi*b^2*x^2 - I*pi*a*b
*x - 1/2*I*pi*a^2))*a)*b*x + (((I - 1)*sqrt(2)*pi^(5/2)*(erf(sqrt(1/2*I*pi*
b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2)) - 1) - (I + 1)*sqrt(2)*pi^(5/2)*(erf(
sqrt(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)) - 1))*a^4 + 12*(-(I +
1)*sqrt(2)*pi*gamma(3/2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + (I
 - 1)*sqrt(2)*pi*gamma(3/2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))
*a^2 + (4*I - 4)*sqrt(2)*gamma(5/2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*p
i*a^2) - (4*I + 4)*sqrt(2)*gamma(5/2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*
I*pi*a^2))*sqrt(2*pi*b^2*x^2 + 4*pi*a*b*x + 2*pi*a^2))*b/(pi^3*b^6*x + pi^3
*a*b^5)
```

Fricas [A]
time = 0.39, size = 176, normalized size = 0.78

$$\frac{\pi^2b^5x^4\,\mathrm{C}\left(bx+a\right)+6\,\pi a^2\sqrt{b^2}\,\mathrm{S}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)-\left(\pi^2a^4-3\right)\sqrt{b^2}\,\mathrm{C}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)-\left(3\,b^2x-5\,ab\right)\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)-\left(\pi b^4x^3-\pi ab^3x^2+\pi a^2b^2x-\pi a^3b\right)\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{4\,\pi^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(pi^2*b^5*x^4*fresnel_cos(b*x + a) + 6*pi*a^2*sqrt(b^2)*fresnel_sin(sqr
t(b^2)*(b*x + a)/b) - (pi^2*a^4 - 3)*sqrt(b^2)*fresnel_cos(sqrt(b^2)*(b*x +
 a)/b) - (3*b^2*x - 5*a*b)*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) - (p
i*b^4*x^3 - pi*a*b^3*x^2 + pi*a^2*b^2*x - pi*a^3*b)*sin(1/2*pi*b^2*x^2 + pi
*a*b*x + 1/2*pi*a^2))/(pi^2*b^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 C(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnelc(b*x+a),x)
```

```
[Out] Integral(x**3*fresnelc(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnel_cos(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\, \mathrm{FresnelC}(a + b\, x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(a + b*x),x)
```

```
[Out] int(x^3*FresnelC(a + b*x), x)
```

3.135 $\int x^2 \textbf{FresnelC}(a + bx)\,dx$

Optimal. Leaf size=148

$$-\frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}+\frac{a^3\text{FresnelC}(a+bx)}{3b^3}+\frac{1}{3}x^3\text{FresnelC}(a+bx)-\frac{aS(a+bx)}{b^3\pi}-\frac{a^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi}+\frac{a(a+}{}$$

```
[Out] -2/3*cos(1/2*Pi*(b*x+a)^2)/b^3/Pi^2+1/3*a^3*FresnelC(b*x+a)/b^3+1/3*x^3*Fre
snelC(b*x+a)-a*FresnelS(b*x+a)/b^3/Pi-a^2*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi+a*(b
*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi-1/3*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2)/b^3/
Pi
```

Rubi [A]

time = 0.09, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6564, 3515, 3433, 3461, 2717, 3467, 3432, 3377, 2718}

$$\frac{a^3\text{FresnelC}(a+bx)}{3b^3}-\frac{a^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}-\frac{aS(a+bx)}{\pi b^3}+\frac{a(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}-\frac{(a+bx)^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi b^3}-\frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi^2b^3}+\frac{1}{3}x^3\text{FresnelC}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelC[a + b*x],x]
```

```
[Out] (-2*Cos[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi^2) + (a^3*FresnelC[a + b*x])/(3*b^3)
 + (x^3*FresnelC[a + b*x])/3 - (a*FresnelS[a + b*x])/(b^3*Pi) - (a^2*Sin[(P
i*(a + b*x)^2)/2])/(b^3*Pi) + (a*(a + b*x)*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi
) - ((a + b*x)^2*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3515

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6564

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelC[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int x^2 C(a+bx)\,dx = \frac{1}{3}x^3 C(a+bx) - \frac{1}{3}b\int x^3\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx$$

$$= \frac{1}{3}x^3 C(a+bx) - \frac{\text{Subst}\left(\int \left(-a^3\cos\left(\frac{\pi x^2}{2}\right) + 3a^2x\cos\left(\frac{\pi x^2}{2}\right) - 3ax^2\cos\left(\frac{\pi x^2}{2}\right) + x^3\cos\right.\right.}{3b^3}$$

$$= \frac{1}{3}x^3 C(a+bx) - \frac{\text{Subst}\left(\int x^3\cos\left(\frac{\pi x^2}{2}\right)\,dx, x, a+bx\right)}{3b^3} + \frac{a\text{Subst}\left(\int x^2\cos\left(\frac{\pi x^2}{2}\right)\,dx, x,\right.}{b^3}$$

$$= \frac{a^3 C(a+bx)}{3b^3} + \frac{1}{3}x^3 C(a+bx) + \frac{a(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} - \frac{\text{Subst}\left(\int x\cos\left(\frac{\pi x}{2}\right)\,dx,\right.}{6b^3}$$

$$= \frac{a^3 C(a+bx)}{3b^3} + \frac{1}{3}x^3 C(a+bx) - \frac{aS(a+bx)}{b^3\pi} - \frac{a^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^3\pi} + \frac{a(a+bx)\sin\left(\frac{1}{2}\right.}{b^3\pi}$$

$$= -\frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2} + \frac{a^3 C(a+bx)}{3b^3} + \frac{1}{3}x^3 C(a+bx) - \frac{aS(a+bx)}{b^3\pi} - \frac{a^2\sin\left(\frac{1}{2}\pi(a+\right.}{b^3\pi}$$

Mathematica [A]
time = 0.20, size = 116, normalized size = 0.78

$$-\frac{2\cos\left(\frac{1}{2}\pi(a+bx)^2\right) - \pi^2(a^3+b^3x^3)\,\text{FresnelC}(a+bx) + 3a\pi S(a+bx) + a^2\pi\sin\left(\frac{1}{2}\pi(a+bx)^2\right) - ab\pi x\sin\left(\frac{1}{2}\pi(a+bx)^2\right) + b^2\pi x^2\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3b^3\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelC[a + b*x],x]
```

```
[Out] -1/3*(2*Cos[(Pi*(a + b*x)^2)/2] - Pi^2*(a^3 + b^3*x^3)*FresnelC[a + b*x] +
3*a*Pi*FresnelS[a + b*x] + a^2*Pi*Sin[(Pi*(a + b*x)^2)/2] - a*b*Pi*x*Sin[(P
i*(a + b*x)^2)/2] + b^2*Pi*x^2*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi^2)
```

Maple [A]
time = 0.38, size = 122, normalized size = 0.82

method	result
derivativedivides	$\frac{\frac{\text{FresnelC}(bx+a)b^3x^3}{3} + \frac{a^3\,\text{FresnelC}(bx+a)}{3} - \frac{a^2\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{a(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} - \frac{a\,\text{S}(bx+a)}{\pi} - \frac{(bx+a)^2\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{3\pi}}{b^3}$
default	$\frac{\frac{\text{FresnelC}(bx+a)b^3x^3}{3} + \frac{a^3\,\text{FresnelC}(bx+a)}{3} - \frac{a^2\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{a(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} - \frac{a\,\text{S}(bx+a)}{\pi} - \frac{(bx+a)^2\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{3\pi}}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/3*FresnelC(b*x+a)*b^3*x^3+1/3*a^3*FresnelC(b*x+a)-a^2/Pi*sin(1/2*P
i*(b*x+a)^2)+a/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)-a/Pi*FresnelS(b*x+a)-1/3/Pi
*(b*x+a)^2*sin(1/2*Pi*(b*x+a)^2)-2/3/Pi^2*cos(1/2*Pi*(b*x+a)^2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.90, size = 423, normalized size = 2.86

$$\frac{1}{3}x^3\,\mathrm{C}\,(bx+a)-\frac{\left(12\left(-i\pi e^{\left(\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)}+i\pi e^{\left(-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2\right)}\right)a^3+4\left(3\left(-i\pi e^{\left(\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)}+i\pi e^{\left(-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2\right)}\right)a^2+2\,\Gamma\left(2,\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)+2\,\Gamma\left(2,-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2\right)\right)bx+8\,a\left(\Gamma\left(2,\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)+\Gamma\left(2,-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2\right)\right)+\sqrt{2\pi b^2x^2+4\pi abx+2\pi a^2}\left(\left((i-1)\sqrt{2}\pi^{\frac{3}{2}}\left(\operatorname{erf}\left(\sqrt{\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2}\right)-1\right)-(i+1)\sqrt{2}\pi^{\frac{3}{2}}\left(\operatorname{erf}\left(\sqrt{-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2}\right)-1\right)\right)a^3+6\left(-(i+1)\sqrt{2}\,\Gamma\left(\frac{3}{2},\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)+(i-1)\sqrt{2}\,\Gamma\left(\frac{3}{2},-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2\right)\right)a\right)\right)b}{24\left(\pi^2b^5x+\pi^2ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*fresnel_cos(b*x + a) - 1/24*(12*(-I*pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a
*b*x + 1/2*I*pi*a^2) + I*pi*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^
2))*a^3 + 4*(3*(-I*pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + I*
pi*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^2 + 2*gamma(2, 1/2*
I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + 2*gamma(2, -1/2*I*pi*b^2*x^2 -
I*pi*a*b*x - 1/2*I*pi*a^2))*b*x + 8*a*(gamma(2, 1/2*I*pi*b^2*x^2 + I*pi*a*b
*x + 1/2*I*pi*a^2) + gamma(2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2
)) + sqrt(2*pi*b^2*x^2 + 4*pi*a*b*x + 2*pi*a^2)*(((I - 1)*sqrt(2)*pi^(3/2)*
(erf(sqrt(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2)) - 1) - (I + 1)*sqr
t(2)*pi^(3/2)*(erf(sqrt(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)) - 1
))*a^3 + 6*(-(I + 1)*sqrt(2)*gamma(3/2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2
*I*pi*a^2) + (I - 1)*sqrt(2)*gamma(3/2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/
2*I*pi*a^2))*a))*b/(pi^2*b^5*x + pi^2*a*b^4)
```

Fricas [A]
time = 0.37, size = 148, normalized size = 1.00

$$\frac{\pi^2b^4x^3\,\mathrm{C}\,(bx+a)+\pi^2a^3\sqrt{b^2}\,\mathrm{C}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)-3\,\pi a\sqrt{b^2}\,\mathrm{S}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)-2\,b\cos\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)-\left(\pi b^3x^2-\pi ab^2x+\pi a^2b\right)\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{3\,\pi^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3*(pi^2*b^4*x^3*fresnel_cos(b*x + a) + pi^2*a^3*sqrt(b^2)*fresnel_cos(sqr
t(b^2)*(b*x + a)/b) - 3*pi*a*sqrt(b^2)*fresnel_sin(sqrt(b^2)*(b*x + a)/b) -
 2*b*cos(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) - (pi*b^3*x^2 - pi*a*b^2*x
 + pi*a^2*b)*sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2))/(pi^2*b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2C(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnel_c(b*x+a),x)
```

```
[Out] Integral(x**2*fresnelc(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_cos(b*x+a),x, algorithm="giac")
[Out] integrate(x^2*fresnel_cos(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{FresnelC}(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(a + b*x),x)
[Out] int(x^2*FresnelC(a + b*x), x)
```

3.136 $\int x\textbf{FresnelC}(a+bx)\,dx$

Optimal. Leaf size=95

$$-\frac{a^2\text{FresnelC}(a+bx)}{2b^2}+\frac{1}{2}x^2\text{FresnelC}(a+bx)+\frac{S(a+bx)}{2b^2\pi}+\frac{a\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi}-\frac{(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}$$

```
[Out] -1/2*a^2*FresnelC(b*x+a)/b^2+1/2*x^2*FresnelC(b*x+a)+1/2*FresnelS(b*x+a)/b^
2/Pi+a*sin(1/2*Pi*(b*x+a)^2)/b^2/Pi-1/2*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^2/P
i
```

Rubi [A]

time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {6564, 3515, 3433, 3461, 2717, 3467, 3432}

$$-\frac{a^2\text{FresnelC}(a+bx)}{2b^2}+\frac{S(a+bx)}{2\pi b^2}+\frac{a\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}-\frac{(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2\pi b^2}+\frac{1}{2}x^2\text{FresnelC}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelC[a + b*x],x]
```

```
[Out] -1/2*(a^2*FresnelC[a + b*x])/b^2 + (x^2*FresnelC[a + b*x])/2 + FresnelS[a +
 b*x]/(2*b^2*Pi) + (a*Sin[(Pi*(a + b*x)^2)/2])/(b^2*Pi) - ((a + b*x)*Sin[(P
i*(a + b*x)^2)/2])/(2*b^2*Pi)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3515

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 6564

```
Int[FresnelC[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> S
imp[(c + d*x)^(m + 1)*(FresnelC[a + b*x]/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[(c + d*x)^(m + 1)*Cos[(Pi/2)*(a + b*x)^2], x], x] /; FreeQ[{a, b,
c, d}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int xC(a+bx)\,dx &= \frac{1}{2}x^2C(a+bx)-\frac{1}{2}b\int x^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\,dx\\
&= \frac{1}{2}x^2C(a+bx)-\frac{\text{Subst}\left(\int\left(a^2\cos\left(\frac{\pi x^2}{2}\right)-2ax\cos\left(\frac{\pi x^2}{2}\right)+x^2\cos\left(\frac{\pi x^2}{2}\right)\right)\,dx,x,a+bx\right)}{2b^2}\\
&= \frac{1}{2}x^2C(a+bx)-\frac{\text{Subst}\left(\int x^2\cos\left(\frac{\pi x^2}{2}\right)\,dx,x,a+bx\right)}{2b^2}+\frac{a\text{Subst}\left(\int x\cos\left(\frac{\pi x^2}{2}\right)\,dx,x,a\cdots\right.}{b^2}\\
&= -\frac{a^2C(a+bx)}{2b^2}+\frac{1}{2}x^2C(a+bx)-\frac{(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}+\frac{a\text{Subst}\left(\int\cos\left(\frac{\pi x}{2}\right)\,dx,x\cdots\right.}{2b^2}\\
&= -\frac{a^2C(a+bx)}{2b^2}+\frac{1}{2}x^2C(a+bx)+\frac{S(a+bx)}{2b^2\pi}+\frac{a\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b^2\pi}-\frac{(a+bx)\sin\left(\frac{1}{2}\pi(a\cdots\right.}{2b^2\pi}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 59, normalized size = 0.62

$$
\frac{(-a^2\pi+b^2\pi x^2)\,\text{FresnelC}(a+bx)+S(a+bx)+(a-bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b^2\pi}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*FresnelC[a + b*x],x]
```

```
[Out] ((-(a^2*Pi) + b^2*Pi*x^2)*FresnelC[a + b*x] + FresnelS[a + b*x] + (a - b*x)
*Sin[(Pi*(a + b*x)^2)/2])/(2*b^2*Pi)
```

Maple [A]
time = 0.38, size = 79, normalized size = 0.83

method	result	size
derivativedivides	$\frac{\operatorname{FresnelC}(bx+a)\left(-a(bx+a)+\frac{(bx+a)^2}{2}\right)+\frac{a\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}+\frac{\mathrm{S}(bx+a)}{2\pi}}{b^2}$	79
default	$\frac{\operatorname{FresnelC}(bx+a)\left(-a(bx+a)+\frac{(bx+a)^2}{2}\right)+\frac{a\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi}-\frac{(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{2\pi}+\frac{\mathrm{S}(bx+a)}{2\pi}}{b^2}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(FresnelC(b*x+a)*(-a*(b*x+a)+1/2*(b*x+a)^2)+a/Pi*sin(1/2*Pi*(b*x+a)^2
)-1/2/Pi*(b*x+a)*sin(1/2*Pi*(b*x+a)^2)+1/2/Pi*FresnelS(b*x+a))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.82, size = 311, normalized size = 3.27

$$\frac{1}{2}x^2 C(bx+a) + \frac{\left(8\left(-i\pi e^{\left(\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)} + i\pi e^{\left(-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2\right)}\right)abx + 8\left(-i\pi e^{\left(\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)} + i\pi e^{\left(-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2\right)}\right)a^2 - \sqrt{2\pi b^2x^2+4\pi abx+2\pi a^2}\left(\left(-(i-1)\sqrt{2}\pi^{\frac{3}{2}}\left(\operatorname{erf}\left(\sqrt{\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2}\right)-1\right)+(i+1)\sqrt{2}\pi^{\frac{3}{2}}\left(\operatorname{erf}\left(\sqrt{-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2}\right)-1\right)\right)a^2+(2i+2)\sqrt{2}\Gamma\left(\frac{3}{2},\frac{1}{2}i\pi b^2x^2+i\pi abx+\frac{1}{2}i\pi a^2\right)-(2i-2)\sqrt{2}\Gamma\left(\frac{3}{2},-\frac{1}{2}i\pi b^2x^2-i\pi abx-\frac{1}{2}i\pi a^2\right)\right)\right)b}{16\left(\pi^2b^4x+\pi^2ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*fresnel_cos(b*x + a) + 1/16*(8*(-I*pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a*
b*x + 1/2*I*pi*a^2) + I*pi*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2
))*a*b*x + 8*(-I*pi*e^(1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) + I*pi
*e^(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2))*a^2 - sqrt(2*pi*b^2*x^2
 + 4*pi*a*b*x + 2*pi*a^2)*((-(I - 1)*sqrt(2)*pi^(3/2)*(erf(sqrt(1/2*I*pi*b^
2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2)) - 1) + (I + 1)*sqrt(2)*pi^(3/2)*(erf(sq
rt(-1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)) - 1))*a^2 + (2*I + 2)*sq
rt(2)*gamma(3/2, 1/2*I*pi*b^2*x^2 + I*pi*a*b*x + 1/2*I*pi*a^2) - (2*I - 2)*
sqrt(2)*gamma(3/2, -1/2*I*pi*b^2*x^2 - I*pi*a*b*x - 1/2*I*pi*a^2)))*b/(pi^2
*b^4*x + pi^2*a*b^3)
```

Fricas [A]
time = 0.36, size = 104, normalized size = 1.09

$$\frac{\pi b^3x^2\,C\left(bx+a\right) - \pi a^2\sqrt{b^2}\,C\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right) - \left(b^2x-ab\right)\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right) + \sqrt{b^2}\,S\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)}{2\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(pi*b^3*x^2*fresnel_cos(b*x + a) - pi*a^2*sqrt(b^2)*fresnel_cos(sqrt(b^
2)*(b*x + a)/b) - (b^2*x - a*b)*sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2)
 + sqrt(b^2)*fresnel_sin(sqrt(b^2)*(b*x + a)/b))/(pi*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int xC(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x+a),x)
```

```
[Out] Integral(x*fresnelc(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*fresnel_cos(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{FresnelC}(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(a + b*x),x)
```

```
[Out] int(x*FresnelC(a + b*x), x)
```

3.137 $\int \text{FresnelC}(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{(a+bx)\text{FresnelC}(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}$$

```
[Out] (b*x+a)*FresnelC(b*x+a)/b-sin(1/2*Pi*(b*x+a)^2)/b/Pi
```

Rubi [A]
time = 0.00, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6554}

$$\frac{(a+bx)\text{FresnelC}(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[a + b*x],x]
```

```
[Out] ((a + b*x)*FresnelC[a + b*x])/b - Sin[(Pi*(a + b*x)^2)/2]/(b*Pi)
```

Rule 6554

```
Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelC[a +
 b*x]/b), x] - Simp[Sin[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int C(a+bx)\,dx=\frac{(a+bx)C(a+bx)}{b}-\frac{\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 90 vs. 2(37) = 74.
time = 0.02, size = 90, normalized size = 2.43

$$\frac{a\text{FresnelC}(a+bx)}{b}+x\text{FresnelC}(a+bx)-\frac{\cos\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)\sin\left(\frac{a^2\pi}{2}\right)}{b\pi}-\frac{\cos\left(\frac{a^2\pi}{2}\right)\sin\left(ab\pi x+\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[a + b*x],x]
```

```
[Out] (a*FresnelC[a + b*x])/b + x*FresnelC[a + b*x] - (Cos[a*b*Pi*x + (b^2*Pi*x^2
)/2]*Sin[(a^2*Pi)/2])/(b*Pi) - (Cos[(a^2*Pi)/2]*Sin[a*b*Pi*x + (b^2*Pi*x^2)
/2])/(b*Pi)
```

Maple [A]
time = 0.24, size = 34, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\mathrm{FresnelC}(bx+a)(bx+a)-\frac{\sin\left(\frac{\pi(bx+a)^{2}}{2}\right)}{\pi}}{b}$	34
default	$\frac{\mathrm{FresnelC}(bx+a)(bx+a)-\frac{\sin\left(\frac{\pi(bx+a)^{2}}{2}\right)}{\pi}}{b}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(FresnelC(b*x+a)*(b*x+a)-1/Pi*sin(1/2*Pi*(b*x+a)^2))
```

Maxima [A]
time = 0.26, size = 44, normalized size = 1.19

$$\frac{(bx+a)\,\mathrm{C}\,(bx+a)-\frac{\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{\pi}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a),x, algorithm="maxima")
[Out] ((b*x + a)*fresnel_cos(b*x + a) - sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^
2)/pi)/b
```

Fricas [A]
time = 0.36, size = 47, normalized size = 1.27

$$\frac{(\pi bx+\pi a)\,\mathrm{C}\,(bx+a)-\sin\left(\frac{1}{2}\,\pi b^2x^2+\pi abx+\frac{1}{2}\,\pi a^2\right)}{\pi b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a),x, algorithm="fricas")
[Out] ((pi*b*x + pi*a)*fresnel_cos(b*x + a) - sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2
*pi*a^2))/(pi*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int C(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a),x)
```

```
[Out] Integral(fresnelc(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{FresnelC}(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x),x)
```

```
[Out] int(FresnelC(a + b*x), x)
```

3.138 $\int \frac{\text{FresnelC}(a+bx)}{x}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{\text{FresnelC}(a+bx)}{x},x\right)$$

```
[Out] Unintegrable(FresnelC(b*x+a)/x,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelC[a + b*x]/x,x]
[Out] Defer[Int][FresnelC[a + b*x]/x, x]
```

Rubi steps

$$\int \frac{C(a+bx)}{x}\,dx = \int \frac{C(a+bx)}{x}\,dx$$

Mathematica [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(a+bx)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelC[a + b*x]/x,x]
[Out] Integrate[FresnelC[a + b*x]/x, x]
```

Maple [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\left(bx+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a)/x,x)
```

```
[Out] int(FresnelC(b*x+a)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x + a)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(fresnel_cos(b*x + a)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x,x)
```

```
[Out] Integral(fresnelc(a + b*x)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x + a)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\mathrm{FresnelC}(a+b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)/x,x)
[Out] int(FresnelC(a + b*x)/x, x)
```

3.139 $\int \frac{\text{FresnelC}(a+bx)}{x^2} dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{\text{FresnelC}(a+bx)}{x^2},x\right)$$

```
[Out] Unintegrable(FresnelC(b*x+a)/x^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)}{x^2} dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelC[a + b*x]/x^2,x]
```

```
[Out] Defer[Int][FresnelC[a + b*x]/x^2, x]
```

Rubi steps

$$\int \frac{C(a+bx)}{x^2} dx = \int \frac{C(a+bx)}{x^2} dx$$

Mathematica [A]
time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(a+bx)}{x^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelC[a + b*x]/x^2,x]
```

```
[Out] Integrate[FresnelC[a + b*x]/x^2, x]
```

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\left(bx+a\right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a)/x^2,x)
[Out] int(FresnelC(b*x+a)/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)/x^2,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x + a)/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)/x^2,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x + a)/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)/x**2,x)
[Out] Integral(fresnelc(a + b*x)/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)/x^2,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x + a)/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\mathrm{FresnelC}(a+b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)/x^2,x)
[Out] int(FresnelC(a + b*x)/x^2, x)
```

3.140 $\int x^7 \textbf{FresnelC}(bx)^2\,dx$

Optimal. Leaf size=253

$$-\frac{105x^2}{16b^6\pi^4}+\frac{7x^6}{48b^2\pi^2}+\frac{55x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4}-\frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2}+\frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{4b^7\pi^4}-\frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{Fre}}{4b^3\pi^2}$$

```
[Out] -105/16*x^2/b^6/Pi^4+7/48*x^6/b^2/Pi^2+55/16*x^2*cos(b^2*Pi*x^2)/b^6/Pi^4-1
/16*x^6*cos(b^2*Pi*x^2)/b^2/Pi^2+105/4*x*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/
b^7/Pi^4-7/4*x^5*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^3/Pi^2-105/8*FresnelC(
b*x)^2/b^8/Pi^4+1/8*x^8*FresnelC(b*x)^2+35/4*x^3*FresnelC(b*x)*sin(1/2*b^2*
Pi*x^2)/b^5/Pi^3-1/4*x^7*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi-10*sin(b^2*
Pi*x^2)/b^8/Pi^5+5/8*x^4*sin(b^2*Pi*x^2)/b^4/Pi^3
```

Rubi [A]
time = 0.29, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {6566, 6590, 6598, 6576, 30, 3461, 2714, 3460, 3377, 2717, 3390}

$$-\frac{105\text{FresnelC}(bx)^2}{8\pi^4b^8}-\frac{105x^2}{16\pi^4b^6}-\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi b}+\frac{7x^6}{48\pi^2b^2}-\frac{x^6\cos\left(\pi b^2x^2\right)}{16\pi^2b^2}-\frac{10\sin\left(\pi b^2x^2\right)}{\pi^5b^8}+\frac{105x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^4b^7}+\frac{55x^2\cos\left(\pi b^2x^2\right)}{16\pi^4b^6}+\frac{35x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^3b^5}+\frac{5x^4\sin\left(\pi b^2x^2\right)}{8\pi^3b^4}-\frac{7x^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{4\pi^2b^3}+\frac{1}{8}x^8\text{FresnelC}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^7*FresnelC[b*x]^2,x]
```

```
[Out] (-105*x^2)/(16*b^6*Pi^4) + (7*x^6)/(48*b^2*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])
/(16*b^6*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(16*b^2*Pi^2) + (105*x*Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/(4*b^7*Pi^4) - (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*
x])/(4*b^3*Pi^2) - (105*FresnelC[b*x]^2)/(8*b^8*Pi^4) + (x^8*FresnelC[b*x]^
2)/8 + (35*x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^5*Pi^3) - (x^7*Fresn
elC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b*Pi) - (10*Sin[b^2*Pi*x^2])/(b^8*Pi^5) +
(5*x^4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
```

```
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$
\begin{aligned}
\int x^7 C(bx)^2\,dx &= \frac{1}{8}x^8 C(bx)^2 - \frac{1}{4}b\int x^8\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx \\
&= \frac{1}{8}x^8 C(bx)^2 - \frac{x^7 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{\int x^7\sin\left(b^2\pi x^2\right)\,dx}{8\pi} + \frac{7\int x^6 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{4b\pi} \\
&= -\frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^3\pi^2} + \frac{1}{8}x^8 C(bx)^2 - \frac{x^7 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} + \frac{35\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(}{4b^3\pi^2} \\
&= -\frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^3\pi^2} + \frac{1}{8}x^8 C(bx)^2 + \frac{35x^3 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^5\pi^3} - \frac{x^7 C}{} \\
&= -\frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} + \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^3\pi^2} + \frac{1}{8}x^8 C(bx)^2 + \frac{35x}{} \\
&= \frac{7x^6}{48b^2\pi^2} + \frac{41x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4} - \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} + \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right.}{4b^3\pi^2} \\
&= -\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} + \frac{55x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4} - \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} + \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^7\pi^4} - \frac{7x^5}{} \\
&= -\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} + \frac{55x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4} - \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} + \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4b^7\pi^4} - \frac{7x^5}{}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 253, normalized size = 1.00

$$
-\frac{105x^2}{16b^6\pi^4} + \frac{7x^6}{48b^2\pi^2} + \frac{55x^2\cos\left(b^2\pi x^2\right)}{16b^6\pi^4} - \frac{x^6\cos\left(b^2\pi x^2\right)}{16b^2\pi^2} + \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{4b^7\pi^4} - \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{4b^3\pi^2} - \frac{105\text{FresnelC}(bx)^2}{8b^8\pi^4} + \frac{1}{8}x^8\text{FresnelC}(bx)^2 + \frac{35x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^5\pi^3} - \frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b\pi} - \frac{10\sin\left(b^2\pi x^2\right)}{b^8\pi^5} + \frac{5x^4\sin\left(b^2\pi x^2\right)}{8b^4\pi^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*FresnelC[b*x]^2,x]
```

```
[Out] (-105*x^2)/(16*b^6*Pi^4) + (7*x^6)/(48*b^2*Pi^2) + (55*x^2*Cos[b^2*Pi*x^2])
/(16*b^6*Pi^4) - (x^6*Cos[b^2*Pi*x^2])/(16*b^2*Pi^2) + (105*x*Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/(4*b^7*Pi^4) - (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*
x])/(4*b^3*Pi^2) - (105*FresnelC[b*x]^2)/(8*b^8*Pi^4) + (x^8*FresnelC[b*x]^
```

```
2)/8 + (35*x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b^5*Pi^3) - (x^7*Fresn
elC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*b*Pi) - (10*Sin[b^2*Pi*x^2])/(b^8*Pi^5) +
(5*x^4*Sin[b^2*Pi*x^2])/(8*b^4*Pi^3)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int x^7 \operatorname{FresnelC}(bx)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelC(b*x)^2,x)
```

```
[Out] int(x^7*FresnelC(b*x)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnel_cos(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^7*fresnel_cos(b*x)^2, x)
```

Fricas [A]
time = 0.36, size = 183, normalized size = 0.72

$$\frac{5\,\pi^3 b^6 x^6 - 240\,\pi b^2 x^2 - 3\,(\pi^3 b^6 x^6 - 55\,\pi b^2 x^2)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 42\,(\pi^3 b^5 x^5 - 15\,\pi b x)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \operatorname{C}(bx) - 3\,(105\,\pi - \pi^5 b^8 x^8)\operatorname{C}(bx)^2 + 6\,\left(5\,(\pi^2 b^4 x^4 - 16)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - (\pi^4 b^7 x^7 - 35\,\pi^2 b^3 x^3)\operatorname{C}(bx)\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{24\,\pi^5 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(5*pi^3*b^6*x^6 - 240*pi*b^2*x^2 - 3*(pi^3*b^6*x^6 - 55*pi*b^2*x^2)*co
s(1/2*pi*b^2*x^2)^2 - 42*(pi^3*b^5*x^5 - 15*pi*b*x)*cos(1/2*pi*b^2*x^2)*fre
snel_cos(b*x) - 3*(105*pi - pi^5*b^8*x^8)*fresnel_cos(b*x)^2 + 6*(5*(pi^2*b
^4*x^4 - 16)*cos(1/2*pi*b^2*x^2) - (pi^4*b^7*x^7 - 35*pi^2*b^3*x^3)*fresnel
_cos(b*x))*sin(1/2*pi*b^2*x^2))/(pi^5*b^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 C^2(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**7*fresnelc(b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnel_cos(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^7*fresnel_cos(b*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^7 \operatorname{FresnelC}(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelC(b*x)^2,x)
```

```
[Out] int(x^7*FresnelC(b*x)^2, x)
```

3.141 $\int x^6 \mathbf{FresnelC}(bx)^2\,dx$

Optimal. Leaf size=239

$$-\frac{48x}{7b^6\pi^4}+\frac{6x^5}{35b^2\pi^2}+\frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4}-\frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2}+\frac{96\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7b^7\pi^4}-\frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{Fresn}}{7b^3\pi^2}$$

```
[Out] -48/7*x/b^6/Pi^4+6/35*x^5/b^2/Pi^2+21/8*x*cos(b^2*Pi*x^2)/b^6/Pi^4-1/14*x^5
*cos(b^2*Pi*x^2)/b^2/Pi^2+96/7*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^7/Pi^4-1
2/7*x^4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^3/Pi^2+1/7*x^7*FresnelC(b*x)^2+
48/7*x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^5/Pi^3-2/7*x^6*FresnelC(b*x)*s
in(1/2*b^2*Pi*x^2)/b/Pi+17/28*x^3*sin(b^2*Pi*x^2)/b^4/Pi^3-531/112*FresnelC
(b*x*2^(1/2))/b^7/Pi^4*2^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6566, 6590, 6598, 6596, 3439, 3433, 3466, 3473, 30, 3467}

$$-\frac{531\text{FresnelC}\left(\sqrt{2}\,bx\right)}{56\sqrt{2}\,\pi^4b^7}-\frac{48x}{7\pi^4b^6}-\frac{2x^6\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi b}+\frac{6x^5}{35\pi^2b^2}-\frac{x^5\cos\left(\pi b^2x^2\right)}{14\pi^2b^2}+\frac{96\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^4b^7}+\frac{21x\cos\left(\pi b^2x^2\right)}{8\pi^4b^6}+\frac{48x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^3b^5}+\frac{17x^3\sin\left(\pi b^2x^2\right)}{28\pi^3b^4}-\frac{12x^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7\pi^2b^3}+\frac{1}{7}x^7\text{FresnelC}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^6*FresnelC[b*x]^2,x]
```

```
[Out] (-48*x)/(7*b^6*Pi^4) + (6*x^5)/(35*b^2*Pi^2) + (21*x*Cos[b^2*Pi*x^2])/(8*b^
6*Pi^4) - (x^5*Cos[b^2*Pi*x^2])/(14*b^2*Pi^2) + (96*Cos[(b^2*Pi*x^2)/2]*Fre
snelC[b*x])/(7*b^7*Pi^4) - (12*x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(7*b^
3*Pi^2) + (x^7*FresnelC[b*x]^2)/7 - (531*FresnelC[Sqrt[2]*b*x])/(56*Sqrt[2]
*b^7*Pi^4) + (48*x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*b^5*Pi^3) - (2*x
^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*b*Pi) + (17*x^3*Sin[b^2*Pi*x^2])/(
28*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3439

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3473

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6596

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelC[b*x]/(2*d)), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /;
 FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
```

```
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}
\int x^6 C(bx)^2\,dx &= \frac{1}{7}x^7C(bx)^2 - \frac{1}{7}(2b)\int x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx \\
&= \frac{1}{7}x^7C(bx)^2 - \frac{2x^6C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \frac{\int x^6\sin\left(b^2\pi x^2\right)\,dx}{7\pi} + \frac{12\int x^5C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} \\
&= -\frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} - \frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7b^3\pi^2} + \frac{1}{7}x^7C(bx)^2 - \frac{2x^6C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b\pi} + \\
&= -\frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} - \frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7b^3\pi^2} + \frac{1}{7}x^7C(bx)^2 + \frac{48x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7b^5\pi^3} - \\
&= \frac{6x^5}{35b^2\pi^2} + \frac{111x\cos\left(b^2\pi x^2\right)}{56b^6\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{96\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7b^7\pi^4} - \\
&= \frac{6x^5}{35b^2\pi^2} + \frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{96\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7b^7\pi^4} - \frac{12x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)}{7b^3\pi^2} \\
&= -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} + \frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{96\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7b^7\pi^4} - \\
&= -\frac{48x}{7b^6\pi^4} + \frac{6x^5}{35b^2\pi^2} + \frac{21x\cos\left(b^2\pi x^2\right)}{8b^6\pi^4} - \frac{x^5\cos\left(b^2\pi x^2\right)}{14b^2\pi^2} + \frac{96\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7b^7\pi^4} -
\end{aligned}$$

Mathematica [**A**]
time = 0.17, size = 170, normalized size = 0.71

$$\frac{80b^7\pi^4x^7\text{FresnelC}(bx)^2 - 2655\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}\,bx\right) - 160\text{FresnelC}(bx)\left(6(-8+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right) + b^2\pi x^2(-24+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)\right) + 2bx\left((735-20b^4\pi^2x^4)\cos\left(b^2\pi x^2\right) + 2(-960+24b^4\pi^2x^4+85b^2\pi x^2\sin\left(b^2\pi x^2\right))\right)}{560b^7\pi^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*FresnelC[b*x]^2,x]
```

```
[Out] (80*b^7*Pi^4*x^7*FresnelC[b*x]^2 - 2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 160
*FresnelC[b*x]*(6*(-8 + b^4*Pi^2*x^4)*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*(-24
 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*((735 - 20*b^4*Pi^2*x^4)*Cos[
b^2*Pi*x^2] + 2*(-960 + 24*b^4*Pi^2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2])))/
(560*b^7*Pi^4)
```

Maple [**A**]
time = 0.80, size = 324, normalized size = 1.36

method	result
derivativedivides	$\frac{\frac{\text{FresnelC}(bx)^2 b^7 x^7}{7} - 2\,\text{FresnelC}(bx)\left(\frac{b^6 x^6 \sin\left(\frac{b^2\pi x^2}{2}\right)}{7\pi} - \frac{6\left(-\frac{b^4 x^4 \cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{4\left(\frac{b^2 x^2 \sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}\right)}{7\pi}\right) + \cdots}{\cdots}$
default	$\frac{\frac{\text{FresnelC}(bx)^2 b^7 x^7}{7} - 2\,\text{FresnelC}(bx)\left(\frac{b^6 x^6 \sin\left(\frac{b^2\pi x^2}{2}\right)}{7\pi} - \frac{6\left(-\frac{b^4 x^4 \cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{4\left(\frac{b^2 x^2 \sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}\right)}{7\pi}\right) + \cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelC(b*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^7*(1/7*FresnelC(b*x)^2*b^7*x^7-2*FresnelC(b*x)*(1/7/Pi*b^6*x^6*sin(1/2*
b^2*Pi*x^2)-6/7/Pi*(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*si
n(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2))))+6/7/Pi^4*(1/5*Pi^2*b^5*x^5-
8*b*x)+6/7/Pi^4*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2
*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1/
2)))+1/7/Pi^3*(-1/2*Pi*b^5*x^5*cos(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(b
^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*
2^(1/2))))+12/Pi*b*x*cos(b^2*Pi*x^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnel_cos(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^6*fresnel_cos(b*x)^2, x)
```

Fricas [A]
time = 0.37, size = 184, normalized size = 0.77

$$\frac{80\,\pi^4 b^8 x^7\,\mathrm{C}(bx)^2 + 136\,\pi^2 b^6 x^5 - 5310\,b^2 x - 20\,(4\,\pi^2 b^6 x^5 - 147\,b^2 x)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 960\,(\pi^2 b^5 x^4 - 8\,b)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}(bx) - 2655\,\sqrt{2}\,\sqrt{b^2}\,\mathrm{C}\left(\sqrt{2}\,\sqrt{b^2}\,x\right) + 40\,(17\,\pi b^4 x^3\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - 4\,(\pi^3 b^7 x^6 - 24\,\pi b^3 x^2)\,\mathrm{C}(bx))\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{560\,\pi^4 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/560*(80*pi^4*b^8*x^7*fresnel_cos(b*x)^2 + 136*pi^2*b^6*x^5 - 5310*b^2*x -
 20*(4*pi^2*b^6*x^5 - 147*b^2*x)*cos(1/2*pi*b^2*x^2)^2 - 960*(pi^2*b^5*x^4
- 8*b)*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) - 2655*sqrt(2)*sqrt(b^2)*fresne
l_cos(sqrt(2)*sqrt(b^2)*x) + 40*(17*pi*b^4*x^3*cos(1/2*pi*b^2*x^2) - 4*(pi^
3*b^7*x^6 - 24*pi*b^3*x^2)*fresnel_cos(b*x))*sin(1/2*pi*b^2*x^2))/(pi^4*b^8
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 C^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**6*fresnelc(b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnel_cos(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^6*fresnel_cos(b*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^6\,\mathrm{FresnelC}(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelC(b*x)^2,x)
```

```
[Out] int(x^6*FresnelC(b*x)^2, x)
```

3.142 $\int x^5 \textbf{FresnelC}(bx)^2 \, dx$

Optimal. Leaf size=265

$$\frac{5x^4}{24b^2\pi^2}+\frac{11\cos(b^2\pi x^2)}{6b^6\pi^4}-\frac{x^4\cos(b^2\pi x^2)}{12b^2\pi^2}-\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3b^3\pi^2}+\frac{1}{6}x^6\text{FresnelC}(bx)^2-\frac{5\text{FresnelC}(bx)S}{2b^6\pi^3}$$

```
[Out] 5/24*x^4/b^2/Pi^2+11/6*cos(b^2*Pi*x^2)/b^6/Pi^4-1/12*x^4*cos(b^2*Pi*x^2)/b^
2/Pi^2-5/3*x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^3/Pi^2+1/6*x^6*FresnelC(
b*x)^2-5/2*FresnelC(b*x)*FresnelS(b*x)/b^6/Pi^3-5/8*I*x^2*hypergeom([1, 1],
[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^4/Pi^3+5/8*I*x^2*hypergeom([1, 1],[3/2, 2],1/
2*I*b^2*Pi*x^2)/b^4/Pi^3+5*x*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^5/Pi^3-1/3
*x^5*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi+7/12*x^2*sin(b^2*Pi*x^2)/b^4/Pi
^3
```

Rubi [A]
time = 0.21, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6566, 6590, 6598, 6582, 3460, 2718, 3461, 3390, 30, 3377}

$$-\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^4}+\frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^4}-\frac{5\text{FresnelC}(bx)S(bx)}{2\pi^3b^6}-\frac{x^5\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}+\frac{5x^4}{24\pi^2b^2}-\frac{x^4\cos(\pi b^2x^2)}{12\pi^2b^2}+\frac{11\cos(\pi b^2x^2)}{6\pi^4b^6}+\frac{5x\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^5}+\frac{7x^2\sin(\pi b^2x^2)}{12\pi^3b^4}-\frac{5x^3\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{1}{6}x^6\text{FresnelC}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^5*FresnelC[b*x]^2,x]
```

```
[Out] (5*x^4)/(24*b^2*Pi^2) + (11*Cos[b^2*Pi*x^2])/(6*b^6*Pi^4) - (x^4*Cos[b^2*Pi
*x^2])/(12*b^2*Pi^2) - (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(3*b^3*Pi^
2) + (x^6*FresnelC[b*x]^2)/6 - (5*FresnelC[b*x]*FresnelS[b*x])/(2*b^6*Pi^3)
 - (((5*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])
/(b^4*Pi^3) + (((5*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*
Pi*x^2])/(b^4*Pi^3) + (5*x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^5*Pi^3) -
(x^5*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(3*b*Pi) + (7*x^2*Sin[b^2*Pi*x^2])/
(12*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}
\int x^5 C(bx)^2\,dx &= \frac{1}{6}x^6 C(bx)^2 - \frac{1}{3}b\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx\\
&= \frac{1}{6}x^6 C(bx)^2 - \frac{x^5 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{\int x^5\sin\left(b^2\pi x^2\right)\,dx}{6\pi} + \frac{5\int x^4 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{3b\pi}\\
&= -\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{6}x^6 C(bx)^2 - \frac{x^5 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{5\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(b
\end{aligned}$$

$$\begin{aligned}
&= -\frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{6}x^6 C(bx)^2 + \frac{5xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^5\pi^3} - \frac{x^5 C(b
\end{aligned}$$

$$\begin{aligned}
&= -\frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{6}x^6 C(bx)^2 - \frac{5C(bx)S(bx)}{2b^6\pi^3} - \frac{5ix^2\,{}_2F_2\left(1,1;\frac{3}{2}\right.}{8b^4
\end{aligned}$$

$$\begin{aligned}
&= \frac{5x^4}{24b^2\pi^2} + \frac{17\cos\left(b^2\pi x^2\right)}{12b^6\pi^4} - \frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{6}x^6 C(bx)^2 - \frac{5C(b}{2b\\
&= \frac{5x^4}{24b^2\pi^2} + \frac{11\cos\left(b^2\pi x^2\right)}{6b^6\pi^4} - \frac{x^4\cos\left(b^2\pi x^2\right)}{12b^2\pi^2} - \frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{6}x^6 C(bx)^2 - \frac{5C(b}{2b
\end{aligned}$$

Mathematica [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int x^5\,\mathrm{FresnelC}(bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^5*FresnelC[b*x]^2,x]
[Out] Integrate[x^5*FresnelC[b*x]^2, x]
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int x^5\,\mathrm{FresnelC}\left(bx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelC(b*x)^2,x)
```

```
[Out] int(x^5*FresnelC(b*x)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnel_cos(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^5*fresnel_cos(b*x)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^5*fresnel_cos(b*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 C^2(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**5*fresnelc(b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnel_cos(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5*fresnel_cos(b*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\, \mathrm{FresnelC}(b\,x)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelC(b*x)^2,x)
```

```
[Out] int(x^5*FresnelC(b*x)^2, x)
```

3.143 $\int x^4 \mathbf{FresnelC}(bx)^2\, dx$

Optimal. Leaf size=177

$$\frac{4x^3}{15b^2\pi^2}-\frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2}-\frac{8x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{5b^3\pi^2}+\frac{1}{5}x^5\text{FresnelC}(bx)^2-\frac{43S\left(\sqrt{2}\,bx\right)}{20\sqrt{2}\,b^5\pi^3}+\frac{16\text{FresnelC}(b}{5b}$$

```
[Out] 4/15*x^3/b^2/Pi^2-1/10*x^3*cos(b^2*Pi*x^2)/b^2/Pi^2-8/5*x^2*cos(1/2*b^2*Pi*
x^2)*FresnelC(b*x)/b^3/Pi^2+1/5*x^5*FresnelC(b*x)^2+16/5*FresnelC(b*x)*sin(
1/2*b^2*Pi*x^2)/b^5/Pi^3-2/5*x^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi+11/
20*x*sin(b^2*Pi*x^2)/b^4/Pi^3-43/40*FresnelS(b*x*2^(1/2))/b^5/Pi^3*2^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6566, 6590, 6598, 6588, 3432, 3473, 30, 3467, 3466}

$$-\frac{43S\left(\sqrt{2}\,bx\right)}{20\sqrt{2}\,\pi^3b^5}-\frac{2x^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi b}+\frac{4x^3}{15\pi^2b^2}-\frac{x^3\cos\left(\pi b^2x^2\right)}{10\pi^2b^2}+\frac{16\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^3b^5}+\frac{11x\sin\left(\pi b^2x^2\right)}{20\pi^3b^4}-\frac{8x^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{5\pi^2b^3}+\frac{1}{5}x^5\text{FresnelC}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^4*FresnelC[b*x]^2,x]
```

```
[Out] (4*x^3)/(15*b^2*Pi^2) - (x^3*Cos[b^2*Pi*x^2])/(10*b^2*Pi^2) - (8*x^2*Cos[(b
^2*Pi*x^2)/2]*FresnelC[b*x])/(5*b^3*Pi^2) + (x^5*FresnelC[b*x]^2)/5 - (43*F
resnelS[Sqrt[2]*b*x])/(20*Sqrt[2]*b^5*Pi^3) + (16*FresnelC[b*x]*Sin[(b^2*Pi
*x^2)/2])/(5*b^5*Pi^3) - (2*x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*b*Pi)
 + (11*x*Sin[b^2*Pi*x^2])/(20*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3473

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6588

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\int x^4 C(bx)^2\,dx = \frac{1}{5}x^5C(bx)^2 - \frac{1}{5}(2b)\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx$$
$$= \frac{1}{5}x^5C(bx)^2 - \frac{2x^4C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{\int x^4\sin\left(b^2\pi x^2\right)\,dx}{5\pi} + \frac{8\int x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,d}{5b\pi}$$
$$= -\frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{8x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5b^3\pi^2} + \frac{1}{5}x^5C(bx)^2 - \frac{2x^4C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b\pi} + \frac{16\int}{}$$
$$= -\frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{8x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5b^3\pi^2} + \frac{1}{5}x^5C(bx)^2 + \frac{16C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5b^5\pi^3} - \frac{2x^4C}{}$$
$$= \frac{4x^3}{15b^2\pi^2} - \frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{8x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5b^3\pi^2} + \frac{1}{5}x^5C(bx)^2 - \frac{3S\left(\sqrt{2}\,bx\right)}{20\sqrt{2}\,b^5\pi^3} - \frac{4\sqrt{2}}{}$$
$$= \frac{4x^3}{15b^2\pi^2} - \frac{x^3\cos\left(b^2\pi x^2\right)}{10b^2\pi^2} - \frac{8x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5b^3\pi^2} + \frac{1}{5}x^5C(bx)^2 - \frac{3S\left(\sqrt{2}\,bx\right)}{20\sqrt{2}\,b^5\pi^3} - \frac{\sqrt{2}\,S}{}$$

Mathematica [A]
time = 0.10, size = 137, normalized size = 0.77

$$\frac{32b^3\pi x^3 - 12b^3\pi x^3\cos\left(b^2\pi x^2\right) + 24b^5\pi^3x^5\text{FresnelC}(bx)^2 - 129\sqrt{2}\,S\left(\sqrt{2}\,bx\right) - 48\text{FresnelC}(bx)\left(4b^2\pi x^2\cos\left(\frac{1}{2}b^2\pi x^2\right) + \left(-8 + b^4\pi^2x^4\right)\sin\left(\frac{1}{2}b^2\pi x^2\right)\right) + 66bx\sin\left(b^2\pi x^2\right)}{120b^5\pi^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*FresnelC[b*x]^2,x]
```

```
[Out] (32*b^3*Pi*x^3 - 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 24*b^5*Pi^3*x^5*FresnelC[b
*x]^2 - 129*Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 48*FresnelC[b*x]*(4*b^2*Pi*x^2*
Cos[(b^2*Pi*x^2)/2] + (-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) + 66*b*x*Sin
[b^2*Pi*x^2])/(120*b^5*Pi^3)
```

Maple [A]
time = 0.80, size = 209, normalized size = 1.18

method	result
derivativedivides	$\frac{\frac{\text{FresnelC}(bx)^2b^5x^5}{5} - 2\,\text{FresnelC}(bx)\left(\frac{b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{5\pi} - \frac{4\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{5\pi}\right) + \frac{4b^3x^3}{15\pi^2} + \frac{2bx\sin\left(b^2\pi\right.}{5\pi}}{b^5}$

default	$\dfrac{\frac{\mathrm{FresnelC}(bx)^2 b^5 x^5}{5} - 2\,\mathrm{FresnelC}(bx)\left(\frac{b^4 x^4 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{5\pi} - \frac{4\left(-\frac{b^2 x^2 \cos\left(\frac{b^2 \pi x^2}{2}\right)}{\pi} + \frac{2 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{\pi^2}\right)}{5\pi}\right) + \frac{4b^3 x^3}{15\pi^2} + \frac{2bx \sin\left(b^2 \pi x^2\right.}{5\pi}}{b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelC(b*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/5*FresnelC(b*x)^2*b^5*x^5-2*FresnelC(b*x)*(1/5/Pi*b^4*x^4*sin(1/2*
b^2*Pi*x^2)-4/5/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi
*x^2)))+4/15/Pi^2*b^3*x^3+4/5/Pi^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/
2)*FresnelS(b*x*2^(1/2)))+1/5/Pi^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*
(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^(1/2)
*FresnelS(b*x*2^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnel_cos(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^4*fresnel_cos(b*x)^2, x)
```

Fricas [A]
time = 0.38, size = 149, normalized size = 0.84

$$\frac{24\,\pi^3 b^6 x^5\,\mathrm{C}\left(bx\right)^2 - 24\,\pi b^4 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 44\,\pi b^4 x^3 - 192\,\pi b^3 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}\left(bx\right) - 129\,\sqrt{2}\,\sqrt{b^2}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{b^2}\,x\right) + 12\left(11\,b^2 x \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - 4\left(\pi^2 b^5 x^4 - 8\,b\right)\mathrm{C}\left(bx\right)\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{120\,\pi^3 b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/120*(24*pi^3*b^6*x^5*fresnel_cos(b*x)^2 - 24*pi*b^4*x^3*cos(1/2*pi*b^2*x^
2)^2 + 44*pi*b^4*x^3 - 192*pi*b^3*x^2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)
- 129*sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x) + 12*(11*b^2*x*cos
(1/2*pi*b^2*x^2) - 4*(pi^2*b^5*x^4 - 8*b)*fresnel_cos(b*x))*sin(1/2*pi*b^2*
x^2))/(pi^3*b^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 C^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**4*fresnelc(b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnel_cos(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnel_cos(b*x)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\mathrm{FresnelC}(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelC(b*x)^2,x)
```

```
[Out] int(x^4*FresnelC(b*x)^2, x)
```

3.144 $\int x^3 \mathbf{FresnelC}(bx)^2\,dx$

Optimal. Leaf size=140

$$\frac{3x^2}{8b^2\pi^2}-\frac{x^2\cos\left(b^2\pi x^2\right)}{8b^2\pi^2}-\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{2b^3\pi^2}+\frac{3\text{FresnelC}(bx)^2}{4b^4\pi^2}+\frac{1}{4}x^4\text{FresnelC}(bx)^2-\frac{x^3\text{FresnelC}(bx)}{2b\pi}$$

```
[Out] 3/8*x^2/b^2/Pi^2-1/8*x^2*cos(b^2*Pi*x^2)/b^2/Pi^2-3/2*x*cos(1/2*b^2*Pi*x^2)
*FresnelC(b*x)/b^3/Pi^2+3/4*FresnelC(b*x)^2/b^4/Pi^2+1/4*x^4*FresnelC(b*x)^
2-1/2*x^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi+1/2*sin(b^2*Pi*x^2)/b^4/Pi
^3
```

Rubi [A]
time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6566, 6590, 6598, 6576, 30, 3461, 2714, 3460, 3377, 2717}

$$\frac{3\text{FresnelC}(bx)^2}{4\pi^2b^4}-\frac{x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi b}+\frac{3x^2}{8\pi^2b^2}-\frac{x^2\cos\left(\pi b^2x^2\right)}{8\pi^2b^2}+\frac{\sin\left(\pi b^2x^2\right)}{2\pi^3b^4}-\frac{3x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^2b^3}+\frac{1}{4}x^4\text{FresnelC}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelC[b*x]^2,x]
```

```
[Out] (3*x^2)/(8*b^2*Pi^2) - (x^2*Cos[b^2*Pi*x^2])/(8*b^2*Pi^2) - (3*x*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(2*b^3*Pi^2) + (3*FresnelC[b*x]^2)/(4*b^4*Pi^2) +
(x^4*FresnelC[b*x]^2)/4 - (x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*b*Pi)
+ Sin[b^2*Pi*x^2]/(2*b^4*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\int x^3C(bx)^2\,dx = \frac{1}{4}x^4C(bx)^2 - \frac{1}{2}b\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx$$
$$= \frac{1}{4}x^4C(bx)^2 - \frac{x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{\int x^3\sin\left(b^2\pi x^2\right)\,dx}{4\pi} + \frac{3\int x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{2b\pi}$$
$$= -\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{2b^3\pi^2} + \frac{1}{4}x^4C(bx)^2 - \frac{x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{3\int\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,d}{2b^3\pi^2}$$
$$= -\frac{x^2\cos\left(b^2\pi x^2\right)}{8b^2\pi^2} - \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{2b^3\pi^2} + \frac{1}{4}x^4C(bx)^2 - \frac{x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{3\mathrm{Subst}}{}$$
$$= \frac{3x^2}{8b^2\pi^2} - \frac{x^2\cos\left(b^2\pi x^2\right)}{8b^2\pi^2} - \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{2b^3\pi^2} + \frac{3C(bx)^2}{4b^4\pi^2} + \frac{1}{4}x^4C(bx)^2 - \frac{x^3C(bx)\sin\left(\right.}{2b\pi}$$

Mathematica [**A**]
time = 0.01, size = 140, normalized size = 1.00

$$\frac{3x^2}{8b^2\pi^2} - \frac{x^2\cos\left(b^2\pi x^2\right)}{8b^2\pi^2} - \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)}{2b^3\pi^2} + \frac{3\mathrm{FresnelC}(bx)^2}{4b^4\pi^2} + \frac{1}{4}x^4\mathrm{FresnelC}(bx)^2 - \frac{x^3\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2b\pi} + \frac{\sin\left(b^2\pi x^2\right)}{2b^4\pi^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelC[b*x]^2,x]
```

```
[Out] (3*x^2)/(8*b^2*Pi^2) - (x^2*Cos[b^2*Pi*x^2])/(8*b^2*Pi^2) - (3*x*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(2*b^3*Pi^2) + (3*FresnelC[b*x]^2)/(4*b^4*Pi^2) +
(x^4*FresnelC[b*x]^2)/4 - (x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(2*b*Pi)
+ Sin[b^2*Pi*x^2]/(2*b^4*Pi^3)
```

Maple [**F**]
time = 0.13, size = 0, normalized size = 0.00

$$\int x^3\,\mathrm{FresnelC}\left(bx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x)^2,x)
```

```
[Out] int(x^3*FresnelC(b*x)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_cos(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnel_cos(b*x)^2, x)
```

Fricas [A]
time = 0.35, size = 118, normalized size = 0.84

$$-\frac{\pi b^2x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2-2\,\pi b^2x^2+6\,\pi bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{C}\left(bx\right)-\left(3\,\pi+\pi^3b^4x^4\right)\mathrm{C}\left(bx\right)^2+2\,\left(\pi^2b^3x^3\,\mathrm{C}\left(bx\right)-2\,\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{4\,\pi^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(pi*b^2*x^2*cos(1/2*pi*b^2*x^2)^2 - 2*pi*b^2*x^2 + 6*pi*b*x*cos(1/2*pi
*b^2*x^2)*fresnel_cos(b*x) - (3*pi + pi^3*b^4*x^4)*fresnel_cos(b*x)^2 + 2*(
pi^2*b^3*x^3*fresnel_cos(b*x) - 2*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))
/(pi^3*b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3C^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnelc(b*x)**2,x)
```

```
[Out] Integral(x**3*fresnelc(b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_cos(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*fresnel_cos(b*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{FresnelC}(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x)^2,x)
```

```
[Out] int(x^3*FresnelC(b*x)^2, x)
```

3.145 $\int x^2 \mathbf{FresnelC}(bx)^2 \, dx$

Optimal. Leaf size=124

$$\frac{2x}{3b^2\pi^2}-\frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2}-\frac{4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3b^3\pi^2}+\frac{1}{3}x^3\text{FresnelC}(bx)^2+\frac{5\text{FresnelC}\left(\sqrt{2}\,bx\right)}{6\sqrt{2}\,b^3\pi^2}-\frac{2x^2\text{FresnelC}(}{3}$$

```
[Out] 2/3*x/b^2/Pi^2-1/6*x*cos(b^2*Pi*x^2)/b^2/Pi^2-4/3*cos(1/2*b^2*Pi*x^2)*Fresn
elC(b*x)/b^3/Pi^2+1/3*x^3*FresnelC(b*x)^2-2/3*x^2*FresnelC(b*x)*sin(1/2*b^2
*Pi*x^2)/b/Pi+5/12*FresnelC(b*x*2^(1/2))/b^3/Pi^2*2^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6566, 6590, 6596, 3439, 3433, 3466}

$$\frac{5\text{FresnelC}\left(\sqrt{2}\,bx\right)}{6\sqrt{2}\,\pi^2b^3}-\frac{2x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi b}-\frac{x\cos\left(\pi b^2x^2\right)}{6\pi^2b^2}+\frac{2x}{3\pi^2b^2}-\frac{4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3\pi^2b^3}+\frac{1}{3}x^3\text{FresnelC}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelC[b*x]^2,x]
```

```
[Out] (2*x)/(3*b^2*Pi^2) - (x*Cos[b^2*Pi*x^2])/(6*b^2*Pi^2) - (4*Cos[(b^2*Pi*x^2)
/2]*FresnelC[b*x])/(3*b^3*Pi^2) + (x^3*FresnelC[b*x]^2)/3 + (5*FresnelC[Sqr
t[2]*b*x])/(6*Sqrt[2]*b^3*Pi^2) - (2*x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])
/(3*b*Pi)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3439

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6596

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelC[b*x]/(2*d)), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /;
 FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}
\int x^2 C(bx)^2\,dx &= \frac{1}{3}x^3C(bx)^2 - \frac{1}{3}(2b)\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx \\
&= \frac{1}{3}x^3C(bx)^2 - \frac{2x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{\int x^2\sin\left(b^2\pi x^2\right)\,dx}{3\pi} + \frac{4\int xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{3b\pi} \\
&= -\frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{3}x^3C(bx)^2 - \frac{2x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3b\pi} + \frac{\int\cos\left(\right.}{6} \\
&= -\frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{3}x^3C(bx)^2 + \frac{C\left(\sqrt{2}\,bx\right)}{6\sqrt{2}\,b^3\pi^2} - \frac{2x^2C(bx)\sin\left(\frac{1}{2}b^2\right.}{3b\pi} \\
&= \frac{2x}{3b^2\pi^2} - \frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{3}x^3C(bx)^2 + \frac{C\left(\sqrt{2}\,bx\right)}{6\sqrt{2}\,b^3\pi^2} - \frac{2x^2C(bx)\,\mathrm{s}}{3b} \\
&= \frac{2x}{3b^2\pi^2} - \frac{x\cos\left(b^2\pi x^2\right)}{6b^2\pi^2} - \frac{4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3b^3\pi^2} + \frac{1}{3}x^3C(bx)^2 + \frac{C\left(\sqrt{2}\,bx\right)}{6\sqrt{2}\,b^3\pi^2} + \frac{\sqrt{2}\,C\left(\sqrt{2}\right.}{3b^3\pi^2}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 100, normalized size = 0.81

$$\frac{-2bx\left(-4+\cos\left(b^2\pi x^2\right)\right)+4b^3\pi^2x^3\mathrm{FresnelC}(bx)^2+5\sqrt{2}\,\mathrm{FresnelC}\left(\sqrt{2}\,bx\right)-8\mathrm{FresnelC}(bx)\left(2\cos\left(\frac{1}{2}b^2\pi x^2\right)+b^2\pi x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)\right)}{12b^3\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelC[b*x]^2,x]
```

```
[Out] (-2*b*x*(-4 + Cos[b^2*Pi*x^2]) + 4*b^3*Pi^2*x^3*FresnelC[b*x]^2 + 5*Sqrt[2]
*FresnelC[Sqrt[2]*b*x] - 8*FresnelC[b*x]*(2*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^
2*Sin[(b^2*Pi*x^2)/2]))/(12*b^3*Pi^2)
```

Maple [A]
time = 0.75, size = 122, normalized size = 0.98

method	result
derivativedivides	$\frac{\frac{\text{FresnelC}(bx)^2 b^3 x^3}{3} - 2\,\text{FresnelC}(bx)\left(\frac{b^2 x^2 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{3\pi} + \frac{2\cos\left(\frac{b^2 \pi x^2}{2}\right)}{3\pi^2}\right) + \frac{2bx}{3\pi^2} + \frac{\sqrt{2}\ \text{FresnelC}\left(bx\sqrt{2}\right)}{3\pi^2} + \frac{-\frac{bx\cos\left(b^2 \pi x^2\right)}{2\pi}}{}}{b^3}$
default	$\frac{\frac{\text{FresnelC}(bx)^2 b^3 x^3}{3} - 2\,\text{FresnelC}(bx)\left(\frac{b^2 x^2 \sin\left(\frac{b^2 \pi x^2}{2}\right)}{3\pi} + \frac{2\cos\left(\frac{b^2 \pi x^2}{2}\right)}{3\pi^2}\right) + \frac{2bx}{3\pi^2} + \frac{\sqrt{2}\ \text{FresnelC}\left(bx\sqrt{2}\right)}{3\pi^2} + \frac{-\frac{bx\cos\left(b^2 \pi x^2\right)}{2\pi}}{}}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(b*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/3*FresnelC(b*x)^2*b^3*x^3-2*FresnelC(b*x)*(1/3/Pi*b^2*x^2*sin(1/2*
b^2*Pi*x^2)+2/3/Pi^2*cos(1/2*b^2*Pi*x^2))+2/3*b*x/Pi^2+1/3/Pi^2*2^(1/2)*Fre
snelC(b*x*2^(1/2))+1/3/Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*Fresn
elC(b*x*2^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_cos(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnel_cos(b*x)^2, x)
```

Fricas [A]
time = 0.35, size = 111, normalized size = 0.90

$$\frac{4\,\pi^2 b^4 x^3\,C\left(bx\right)^2 - 8\,\pi b^3 x^2\,C\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right) - 4\,b^2 x\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 10\,b^2 x - 16\,b\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)C\left(bx\right) + 5\,\sqrt{2}\,\sqrt{b^2}\ C\left(\sqrt{2}\,\sqrt{b^2}\,x\right)}{12\,\pi^2 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(4*pi^2*b^4*x^3*fresnel_cos(b*x)^2 - 8*pi*b^3*x^2*fresnel_cos(b*x)*sin
(1/2*pi*b^2*x^2) - 4*b^2*x*cos(1/2*pi*b^2*x^2)^2 + 10*b^2*x - 16*b*cos(1/2*
```

```
pi*b^2*x^2)*fresnel_cos(b*x) + 5*sqrt(2)*sqrt(b^2)*fresnel_cos(sqrt(2)*sqrt
(b^2)*x))/(pi^2*b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 C^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnelc(b*x)**2,x)
[Out] Integral(x**2*fresnelc(b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_cos(b*x)^2,x, algorithm="giac")
[Out] integrate(x^2*fresnel_cos(b*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{FresnelC}(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(b*x)^2,x)
[Out] int(x^2*FresnelC(b*x)^2, x)
```

3.146 $\int x\mathbf{FresnelC}(bx)^2\,dx$

Optimal. Leaf size=144

$$-\frac{\cos\left(b^2\pi x^2\right)}{4b^2\pi^2}+\frac{1}{2}x^2\text{FresnelC}(bx)^2+\frac{\text{FresnelC}(bx)S(bx)}{2b^2\pi}+\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi}$$

```
[Out] -1/4*cos(b^2*Pi*x^2)/b^2/Pi^2+1/2*x^2*FresnelC(b*x)^2+1/2*FresnelC(b*x)*Fre
snelS(b*x)/b^2/Pi+1/8*I*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/Pi
-1/8*I*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/Pi-x*FresnelC(b*x)*s
in(1/2*b^2*Pi*x^2)/b/Pi
```

Rubi [A]
time = 0.06, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6566, 6590, 6582, 3460, 2718}

$$\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi}+\frac{\text{FresnelC}(bx)S(bx)}{2\pi b^2}-\frac{x\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}-\frac{\cos\left(\pi b^2x^2\right)}{4\pi^2b^2}+\frac{1}{2}x^2\text{FresnelC}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelC[b*x]^2,x]
```

```
[Out] -1/4*Cos[b^2*Pi*x^2]/(b^2*Pi^2) + (x^2*FresnelC[b*x]^2)/2 + (FresnelC[b*x]*
FresnelS[b*x])/(2*b^2*Pi) + ((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2},
(-1/2*I)*b^2*Pi*x^2])/Pi - ((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (
I/2)*b^2*Pi*x^2])/Pi - (x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b*Pi)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}\int xC(bx)^2\,dx &= \frac{1}{2}x^2C(bx)^2 - b\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx\\ &= \frac{1}{2}x^2C(bx)^2 - \frac{xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi} + \frac{\int x\sin\left(b^2\pi x^2\right)\,dx}{2\pi} + \frac{\int C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi}\\ &= \frac{1}{2}x^2C(bx)^2 + \frac{C(bx)S(bx)}{2b^2\pi} + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi} -\\ &= -\frac{\cos\left(b^2\pi x^2\right)}{4b^2\pi^2} + \frac{1}{2}x^2C(bx)^2 + \frac{C(bx)S(bx)}{2b^2\pi} + \frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi} - \frac{ix^2\,{}_2F_2\left(1,1;\right.}{8}\end{aligned}$$

Mathematica [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int x\text{FresnelC}(bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*FresnelC[b*x]^2,x]
[Out] Integrate[x*FresnelC[b*x]^2, x]
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int x\,\text{FresnelC}\left(bx\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(b*x)^2,x)
[Out] int(x*FresnelC(b*x)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_cos(b*x)^2,x, algorithm="maxima")
[Out] integrate(x*fresnel_cos(b*x)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_cos(b*x)^2,x, algorithm="fricas")
[Out] integral(x*fresnel_cos(b*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int xC^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x)**2,x)
[Out] Integral(x*fresnelc(b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_cos(b*x)^2,x, algorithm="giac")
[Out] integrate(x*fresnel_cos(b*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{FresnelC}(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(b*x)^2,x)
```

```
[Out] int(x*FresnelC(b*x)^2, x)
```

3.147 $\int \text{FresnelC}(bx)^2 \, dx$

Optimal. Leaf size=54

$$x\text{FresnelC}(bx)^2+\frac{S\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,b\pi}-\frac{2\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

```
[Out] x*FresnelC(b*x)^2-2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b/Pi+1/2*FresnelS(b*x
*2^(1/2))/b/Pi*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6556, 12, 6588, 3432}

$$-\frac{2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b}+x\text{FresnelC}(bx)^2+\frac{S\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]^2,x]
```

```
[Out] x*FresnelC[b*x]^2 + FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi) - (2*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(b*Pi)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 6556

```
Int[FresnelC[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(FresnelC[a
 + b*x]^2/b), x] - Dist[2, Int[(a + b*x)*Cos[(Pi/2)*(a + b*x)^2]*FresnelC[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6588

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$
\begin{aligned}
\int C(bx)^2\,dx &= xC(bx)^2 - 2\int bx\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx\\
&= xC(bx)^2 - (2b)\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx\\
&= xC(bx)^2 - \frac{2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi} + \frac{\int \sin\left(b^2\pi x^2\right)\,dx}{\pi}\\
&= xC(bx)^2 + \frac{S\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,b\pi} - \frac{2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 1.00

$$x\text{FresnelC}(bx)^2 + \frac{S\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,b\pi} - \frac{2\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]^2,x]
```

```
[Out] x*FresnelC[b*x]^2 + FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b*Pi) - (2*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(b*Pi)
```

Maple [A]
time = 0.40, size = 49, normalized size = 0.91

method	result	size
derivativedivides	$\frac{\text{FresnelC}(bx)^2 bx - \frac{2\,\text{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{\sqrt{2}\,\mathrm{S}\left(bx\sqrt{2}\right)}{2\pi}}{b}$	49
default	$\frac{\text{FresnelC}(bx)^2 bx - \frac{2\,\text{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{\sqrt{2}\,\mathrm{S}\left(bx\sqrt{2}\right)}{2\pi}}{b}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(FresnelC(b*x)^2*b*x-2*FresnelC(b*x)/Pi*sin(1/2*b^2*Pi*x^2)+1/2/Pi*2^(1
/2)*FresnelS(b*x*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)^2, x)
```

Fricas [A]
time = 0.36, size = 59, normalized size = 1.09

$$\frac{2\,\pi b^2 x\,\mathrm{C}\left(bx\right)^2-4\,b\,\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)+\sqrt{2}\,\sqrt{b^2}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{b^2}\,x\right)}{2\,\pi b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*pi*b^2*x*fresnel_cos(b*x)^2 - 4*b*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^
2) + sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x))/(pi*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int C^2(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2,x)
```

```
[Out] Integral(fresnelc(b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{FresnelC}(b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2,x)
```

```
[Out] int(FresnelC(b*x)^2, x)
```

3.148 $\int \frac{\text{FresnelC}(bx)^2}{x} \, dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{\text{FresnelC}(bx)^2}{x},x\right)$$

```
[Out] Unintegrable(FresnelC(b*x)^2/x,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x} \, dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x,x]
```

```
[Out] Defer[Int][FresnelC[b*x]^2/x, x]
```

Rubi steps

$$\int \frac{C(bx)^2}{x} \, dx = \int \frac{C(bx)^2}{x} \, dx$$

Mathematica [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)^2}{x} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x,x]
```

```
[Out] Integrate[FresnelC[b*x]^2/x, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\left(bx\right)^2}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x,x)
```

```
[Out] int(FresnelC(b*x)^2/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)^2/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(fresnel_cos(b*x)^2/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2(bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)^2/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\mathrm{FresnelC}(b\,x)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x,x)
```

```
[Out] int(FresnelC(b*x)^2/x, x)
```

3.149 $\int \frac{\text{FresnelC}(bx)^2}{x^2} dx$

Optimal. Leaf size=38

$$-\frac{\text{FresnelC}(bx)^2}{x} + 2b\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}, x\right)$$

```
[Out] -FresnelC(b*x)^2/x+2*b*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^2} dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^2,x]
[Out] -(FresnelC[b*x]^2/x) + 2*b*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x
, x]
```

Rubi steps

$$\int \frac{C(bx)^2}{x^2} dx = -\frac{C(bx)^2}{x} + (2b)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x} dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^2,x]
[Out] Integrate[FresnelC[b*x]^2/x^2, x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^2,x)
```

```
[Out] int(FresnelC(b*x)^2/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)^2/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(fresnel_cos(b*x)^2/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2(bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**2,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)^2/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{FresnelC}(b\,x)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^2,x)
```

```
[Out] int(FresnelC(b*x)^2/x^2, x)
```

3.150 $\int \frac{\text{FresnelC}(bx)^2}{x^3}\,dx$

Optimal. Leaf size=39

$$-\frac{\text{FresnelC}(bx)^2}{2x^2}+b\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2},x\right)$$

```
[Out] -1/2*FresnelC(b*x)^2/x^2+b*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x
^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^3,x]
[Out] -1/2*FresnelC[b*x]^2/x^2 + b*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])
/x^2, x]
```

Rubi steps

$$\int \frac{C(bx)^2}{x^3}\,dx=-\frac{C(bx)^2}{2x^2}+b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2}\,dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^3,x]
[Out] Integrate[FresnelC[b*x]^2/x^3, x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\left(bx\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^3,x)
```

```
[Out] int(FresnelC(b*x)^2/x^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)^2/x^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(fresnel_cos(b*x)^2/x^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2(bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**3,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)^2/x^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{FresnelC}(b\,x)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^3,x)
```

```
[Out] int(FresnelC(b*x)^2/x^3, x)
```

3.151 $\int \frac{\mathbf{FresnelC}(bx)^2}{x^4}\,dx$

Optimal. Leaf size=120

$$-\frac{b^2}{6x}-\frac{b^2\cos\left(b^2\pi x^2\right)}{6x}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3x^2}-\frac{\text{FresnelC}(bx)^2}{3x^3}-\frac{b^3\pi S\left(\sqrt{2}\,bx\right)}{3\sqrt{2}}-\frac{1}{3}b^3\pi\text{Int}\left(\frac{\text{FresnelC}(bx)}{x}\right.$$

```
[Out] -1/6*b^2/x-1/6*b^2*cos(b^2*Pi*x^2)/x-1/3*b*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x
)/x^2-1/3*FresnelC(b*x)^2/x^3-1/6*b^3*Pi*FresnelS(b*x*2^(1/2))*2^(1/2)-1/3*
b^3*Pi*Unintegrable(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^4,x]
```

```
[Out] -1/6*b^2/x - (b^2*Cos[b^2*Pi*x^2])/(6*x) - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[
b*x])/(3*x^2) - FresnelC[b*x]^2/(3*x^3) - (b^3*Pi*FresnelS[Sqrt[2]*b*x])/(3
*Sqrt[2]) - (b^3*Pi*Defer[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/3
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)^2}{x^4}\,dx &= -\frac{C(bx)^2}{3x^3}+\frac{1}{3}(2b)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^3}\,dx\\ &= -\frac{b^2}{6x}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^2}-\frac{C(bx)^2}{3x^3}+\frac{1}{6}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx-\frac{1}{3}\left(b^3\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi\right.}{x}\\ &= -\frac{b^2}{6x}-\frac{b^2\cos\left(b^2\pi x^2\right)}{6x}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^2}-\frac{C(bx)^2}{3x^3}-\frac{1}{3}\left(b^3\pi\right)\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx\\ &= -\frac{b^2}{6x}-\frac{b^2\cos\left(b^2\pi x^2\right)}{6x}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^2}-\frac{C(bx)^2}{3x^3}-\frac{b^3\pi S\left(\sqrt{2}\,bx\right)}{3\sqrt{2}}-\frac{1}{3}\left(b^3\pi\right)\int \frac{C(ba}{}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^4,x]
```

```
[Out] Integrate[FresnelC[b*x]^2/x^4, x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{FresnelC}\,(bx)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^4,x)
```

```
[Out] int(FresnelC(b*x)^2/x^4,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)^2/x^4, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(fresnel_cos(b*x)^2/x^4, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2(bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**4,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**4, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)^2/x^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}(b\,x)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^4,x)
```

```
[Out] int(FresnelC(b*x)^2/x^4, x)
```

3.152 $\int \frac{\text{FresnelC}(bx)^2}{x^5}\,dx$

Optimal. Leaf size=127

$$-\frac{b^2}{24x^2}-\frac{b^2\cos\left(b^2\pi x^2\right)}{24x^2}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{6x^3}-\frac{1}{12}b^4\pi^2\text{FresnelC}(bx)^2-\frac{\text{FresnelC}(bx)^2}{4x^4}+\frac{b^3\pi\text{FresnelC}(}{}$$

```
[Out] -1/24*b^2/x^2-1/24*b^2*cos(b^2*Pi*x^2)/x^2-1/6*b*cos(1/2*b^2*Pi*x^2)*Fresne
lC(b*x)/x^3-1/12*b^4*Pi^2*FresnelC(b*x)^2-1/4*FresnelC(b*x)^2/x^4-1/12*b^4*
Pi*Si(b^2*Pi*x^2)+1/6*b^3*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x
```

Rubi [A]
time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {6566, 6592, 6600, 6576, 30, 3456, 3461, 3378, 3380}

$$-\frac{1}{12}\pi^2b^4\text{FresnelC}(bx)^2-\frac{b\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{6x^3}-\frac{b^2}{24x^2}-\frac{b^2\cos\left(\pi b^2x^2\right)}{24x^2}-\frac{1}{12}\pi b^4\text{Si}\left(b^2\pi x^2\right)+\frac{\pi b^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{6x}-\frac{\text{FresnelC}(bx)^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]^2/x^5,x]
```

```
[Out] -1/24*b^2/x^2 - (b^2*Cos[b^2*Pi*x^2])/(24*x^2) - (b*Cos[(b^2*Pi*x^2)/2]*Fre
snelC[b*x])/(6*x^3) - (b^4*Pi^2*FresnelC[b*x]^2)/12 - FresnelC[b*x]^2/(4*x^
4) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x) - (b^4*Pi*SinIntegral
[b^2*Pi*x^2])/12
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6592

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[b*(x^(m + 2)/(2*(m + 1)*(m + 2))), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6600

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && I
LtQ[m, -1]
```

Rubi steps

$$\int \frac{C(bx)^2}{x^5}\,dx = -\frac{C(bx)^2}{4x^4} + \frac{1}{2}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^4}\,dx$$
$$= -\frac{b^2}{24x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^3} - \frac{C(bx)^2}{4x^4} + \frac{1}{12}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx - \frac{1}{6}\left(b^3\pi\right)\int \frac{C(bx)\sin(}{x^2}$$
$$= -\frac{b^2}{24x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^3} - \frac{C(bx)^2}{4x^4} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x} + \frac{1}{24}b^2\text{Subst}\left(\int \frac{\cos(b}{x}\right.$$
$$= -\frac{b^2}{24x^2} - \frac{b^2\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^3} - \frac{C(bx)^2}{4x^4} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x} - \frac{1}{24}b$$
$$= -\frac{b^2}{24x^2} - \frac{b^2\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^3} - \frac{1}{12}b^4\pi^2C(bx)^2 - \frac{C(bx)^2}{4x^4} + \frac{b^3\pi C(bx)\sin}{6x}$$

Mathematica [A]
time = 0.01, size = 127, normalized size = 1.00

$$-\frac{b^2}{24x^2} - \frac{b^2\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{6x^3} - \frac{1}{12}b^4\pi^2\text{FresnelC}(bx)^2 - \frac{\text{FresnelC}(bx)^2}{4x^4} + \frac{b^3\pi\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x} - \frac{1}{12}b^4\pi\text{Si}\left(b^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]^2/x^5,x]
```

```
[Out] -1/24*b^2/x^2 - (b^2*Cos[b^2*Pi*x^2])/(24*x^2) - (b*Cos[(b^2*Pi*x^2)/2]*Fre
snelC[b*x])/(6*x^3) - (b^4*Pi^2*FresnelC[b*x]^2)/12 - FresnelC[b*x]^2/(4*x^
4) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(6*x) - (b^4*Pi*SinIntegral
[b^2*Pi*x^2])/12
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\left(bx\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^5,x)
```

```
[Out] int(FresnelC(b*x)^2/x^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)^2/x^5, x)
```

Fricas [A]
time = 0.37, size = 102, normalized size = 0.80

$$-\frac{\pi b^4x^4\operatorname{Si}\left(\pi b^2x^2\right)-2\,\pi b^3x^3\operatorname{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)+b^2x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2+2\,bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\operatorname{C}\left(bx\right)+\left(\pi^2b^4x^4+3\right)\operatorname{C}\left(bx\right)^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(pi*b^4*x^4*sin_integral(pi*b^2*x^2) - 2*pi*b^3*x^3*fresnel_cos(b*x)*
sin(1/2*pi*b^2*x^2) + b^2*x^2*cos(1/2*pi*b^2*x^2)^2 + 2*b*x*cos(1/2*pi*b^2*
x^2)*fresnel_cos(b*x) + (pi^2*b^4*x^4 + 3)*fresnel_cos(b*x)^2)/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{C^2(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**5,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)^2/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\mathrm{FresnelC}(b\,x)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^5,x)
```

```
[Out] int(FresnelC(b*x)^2/x^5, x)
```

3.153 $\int \frac{\text{FresnelC}(bx)^2}{x^6} \, dx$

Optimal. Leaf size=171

$$-\frac{b^2}{60x^3}-\frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{10x^4}-\frac{\text{FresnelC}(bx)^2}{5x^5}-\frac{7b^5\pi^2\text{FresnelC}\left(\sqrt{2}\,bx\right)}{60\sqrt{2}}+\frac{b^3\pi\text{Fresne}}{}$$

```
[Out] -1/60*b^2/x^3-1/60*b^2*cos(b^2*Pi*x^2)/x^3-1/10*b*cos(1/2*b^2*Pi*x^2)*Fresn
elC(b*x)/x^4-1/5*FresnelC(b*x)^2/x^5+1/20*b^3*Pi*FresnelC(b*x)*sin(1/2*b^2*
Pi*x^2)/x^2+7/120*b^4*Pi*sin(b^2*Pi*x^2)/x-7/120*b^5*Pi^2*FresnelC(b*x*2^(1
/2))*2^(1/2)-1/20*b^5*Pi^2*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x
,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^6} \, dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^6,x]
```

```
[Out] -1/60*b^2/x^3 - (b^2*Cos[b^2*Pi*x^2])/(60*x^3) - (b*Cos[(b^2*Pi*x^2)/2]*Fre
snelC[b*x])/(10*x^4) - FresnelC[b*x]^2/(5*x^5) - (7*b^5*Pi^2*FresnelC[Sqrt[
2]*b*x])/(60*Sqrt[2]) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(20*x^2)
 + (7*b^4*Pi*Sin[b^2*Pi*x^2])/(120*x) - (b^5*Pi^2*Defer[Int][(Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/x, x])/20
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)^2}{x^6} \, dx &= -\frac{C(bx)^2}{5x^5}+\frac{1}{5}(2b)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^5} \, dx \\
&= -\frac{b^2}{60x^3}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{10x^4}-\frac{C(bx)^2}{5x^5}+\frac{1}{20}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^4} \, dx-\frac{1}{10}\left(b^3\pi\right)\int \frac{C(bx)\sin}{x} \\
&= -\frac{b^2}{60x^3}-\frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{10x^4}-\frac{C(bx)^2}{5x^5}+\frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^2}-\frac{1}{40}(\\
&= -\frac{b^2}{60x^3}-\frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{10x^4}-\frac{C(bx)^2}{5x^5}+\frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{20x^2}+\frac{7b^4}{} \\
&= -\frac{b^2}{60x^3}-\frac{b^2\cos\left(b^2\pi x^2\right)}{60x^3}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{10x^4}-\frac{C(bx)^2}{5x^5}-\frac{7b^5\pi^2C\left(\sqrt{2}\,bx\right)}{60\sqrt{2}}+\frac{b^3\pi C(bx)}{}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^6,x]
```

```
[Out] Integrate[FresnelC[b*x]^2/x^6, x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\left(bx\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^6,x)
```

```
[Out] int(FresnelC(b*x)^2/x^6,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^6,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)^2/x^6, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^6,x, algorithm="fricas")
```

```
[Out] integral(fresnel_cos(b*x)^2/x^6, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**6,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**6, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^6,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)^2/x^6, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}(b\,x)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^6,x)
```

```
[Out] int(FresnelC(b*x)^2/x^6, x)
```

3.154 $\int \frac{\textbf{FresnelC}(bx)^2}{x^7}\,dx$

Optimal. Leaf size=166

$$-\frac{b^2}{120x^4}-\frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4}-\frac{1}{72}b^6\pi^2\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{15x^5}-\frac{\text{FresnelC}(bx)^2}{6x^6}+\frac{b^3\pi\text{Fres}}{}$$

```
[Out] -1/120*b^2/x^4-1/72*b^6*Pi^2*Ci(b^2*Pi*x^2)-1/120*b^2*cos(b^2*Pi*x^2)/x^4-1
/15*b*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5-1/6*FresnelC(b*x)^2/x^6+1/45*b^
3*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3+1/72*b^4*Pi*sin(b^2*Pi*x^2)/x^2-
1/45*b^5*Pi^2*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

Rubi [A]
time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^7,x]
```

```
[Out] -1/120*b^2/x^4 - (b^2*Cos[b^2*Pi*x^2])/(120*x^4) - (b^6*Pi^2*CosIntegral[b^
2*Pi*x^2])/72 - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(15*x^5) - FresnelC[b
*x]^2/(6*x^6) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(45*x^3) + (b^4*
Pi*Sin[b^2*Pi*x^2])/(72*x^2) - (b^5*Pi^2*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*Fr
esnelC[b*x])/x^2, x])/45
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)^2}{x^7}\,dx &= -\frac{C(bx)^2}{6x^6}+\frac{1}{3}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^6}\,dx\\
&= -\frac{b^2}{120x^4}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^5}-\frac{C(bx)^2}{6x^6}+\frac{1}{30}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^5}\,dx-\frac{1}{15}\left(b^3\pi\right)\int \frac{C(bx)\sin}{x}\\
&= -\frac{b^2}{120x^4}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^5}-\frac{C(bx)^2}{6x^6}+\frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{45x^3}+\frac{1}{60}b^2\text{Subst}\left(\int \frac{\cos(b}{x^3}\right.\\
&= -\frac{b^2}{120x^4}-\frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^5}-\frac{C(bx)^2}{6x^6}+\frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{45x^3}-\frac{1}{180}\\
&= -\frac{b^2}{120x^4}-\frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^5}-\frac{C(bx)^2}{6x^6}+\frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{45x^3}+\frac{b^4\pi}{}\\
&= -\frac{b^2}{120x^4}-\frac{b^2\cos\left(b^2\pi x^2\right)}{120x^4}-\frac{1}{72}b^6\pi^2\text{Ci}\left(b^2\pi x^2\right)-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^5}-\frac{C(bx)^2}{6x^6}+\frac{b^3\pi C(bx)}{4}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^7,x]
```

```
[Out] Integrate[FresnelC[b*x]^2/x^7, x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\left(bx\right)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^7,x)
```

```
[Out] int(FresnelC(b*x)^2/x^7,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^7,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)^2/x^7, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^7,x, algorithm="fricas")
```

```
[Out] integral(fresnel_cos(b*x)^2/x^7, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**7,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**7, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^7,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)^2/x^7, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}(b\,x)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^7,x)
```

```
[Out] int(FresnelC(b*x)^2/x^7, x)
```

3.155 $\int \frac{\text{FresnelC}(bx)^2}{x^8} dx$

Optimal. Leaf size=259

$$-\frac{b^2}{210x^5}+\frac{b^6\pi^2}{336x}-\frac{b^2\cos\left(b^2\pi x^2\right)}{210x^5}+\frac{67b^6\pi^2\cos\left(b^2\pi x^2\right)}{5040x}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{21x^6}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{Fresn}}{168x^2}$$

```
[Out] -1/210*b^2/x^5+1/336*b^6*Pi^2/x-1/210*b^2*cos(b^2*Pi*x^2)/x^5+67/5040*b^6*P
i^2*cos(b^2*Pi*x^2)/x-1/21*b*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6+1/168*b^
5*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2-1/7*FresnelC(b*x)^2/x^7+1/84*b
^3*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4+13/2520*b^4*Pi*sin(b^2*Pi*x^2)/
x^3+67/5040*b^7*Pi^3*FresnelS(b*x*2^(1/2))*2^(1/2)+1/168*b^7*Pi^3*Unintegra
ble(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A]
time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^8} dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^8,x]
```

```
[Out] -1/210*b^2/x^5 + (b^6*Pi^2)/(336*x) - (b^2*Cos[b^2*Pi*x^2])/(210*x^5) + (67
*b^6*Pi^2*Cos[b^2*Pi*x^2])/(5040*x) - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])
/(21*x^6) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(168*x^2) - Fresne
lC[b*x]^2/(7*x^7) + (b^7*Pi^3*FresnelS[Sqrt[2]*b*x])/(72*Sqrt[2]) + (2*Sqrt
[2]*b^7*Pi^3*FresnelS[Sqrt[2]*b*x])/315 + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi
*x^2)/2])/(84*x^4) + (13*b^4*Pi*Sin[b^2*Pi*x^2])/(2520*x^3) + (b^7*Pi^3*Def
er[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/168
```

Rubi steps

$$\int \frac{C(bx)^2}{x^8}\,dx = -\frac{C(bx)^2}{7x^7} + \frac{1}{7}(2b)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^7}\,dx$$

$$= -\frac{b^2}{210x^5} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{21x^6} - \frac{C(bx)^2}{7x^7} + \frac{1}{42}b^2\int \frac{\cos(b^2\pi x^2)}{x^6}\,dx - \frac{1}{21}(b^3\pi)\int \frac{C(bx)\sin}{x}$$

$$= -\frac{b^2}{210x^5} - \frac{b^2\cos(b^2\pi x^2)}{210x^5} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{21x^6} - \frac{C(bx)^2}{7x^7} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{84x^4} - \frac{1}{168}$$

$$= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} - \frac{b^2\cos(b^2\pi x^2)}{210x^5} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{21x^6} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{168x^2} - \frac{C(bx}{7x}$$

$$= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} - \frac{b^2\cos(b^2\pi x^2)}{210x^5} + \frac{67b^6\pi^2\cos(b^2\pi x^2)}{5040x} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{21x^6} + \frac{b^5\pi^2\cos\left(\frac{1}{2}\right.}{1}$$

$$= -\frac{b^2}{210x^5} + \frac{b^6\pi^2}{336x} - \frac{b^2\cos(b^2\pi x^2)}{210x^5} + \frac{67b^6\pi^2\cos(b^2\pi x^2)}{5040x} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{21x^6} + \frac{b^5\pi^2\cos\left(\frac{1}{2}\right.}{1}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^8,x]
```

```
[Out] Integrate[FresnelC[b*x]^2/x^8, x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^8,x)
```

```
[Out] int(FresnelC(b*x)^2/x^8,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^8,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)^2/x^8, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^8,x, algorithm="fricas")
```

```
[Out] integral(fresnel_cos(b*x)^2/x^8, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2(bx)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**8,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**8, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)^2/x^8, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelC}(b\,x)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^8,x)
```

```
[Out] int(FresnelC(b*x)^2/x^8, x)
```

3.156 $\int \frac{\textbf{FresnelC}(bx)^2}{x^9} dx$

Optimal. Leaf size=242

$$-\frac{b^2}{336x^6}+\frac{b^6\pi^2}{1680x^2}-\frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6}+\frac{b^6\pi^2\cos\left(b^2\pi x^2\right)}{336x^2}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{28x^7}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{Fresnel}}{420x^3}$$

```
[Out] -1/336*b^2/x^6+1/1680*b^6*Pi^2/x^2-1/336*b^2*cos(b^2*Pi*x^2)/x^6+1/336*b^6*
Pi^2*cos(b^2*Pi*x^2)/x^2-1/28*b*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^7+1/420
*b^5*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3+1/840*b^8*Pi^4*FresnelC(b*x
)^2-1/8*FresnelC(b*x)^2/x^8+1/280*b^8*Pi^3*Si(b^2*Pi*x^2)+1/140*b^3*Pi*Fres
nelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5-1/420*b^7*Pi^3*FresnelC(b*x)*sin(1/2*b^2*
Pi*x^2)/x+1/420*b^4*Pi*sin(b^2*Pi*x^2)/x^4
```

Rubi [A]
time = 0.27, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6566, 6592, 6600, 6576, 30, 3456, 3461, 3378, 3380, 3460}

$$\frac{1}{840}\pi^4b^8\text{FresnelC}(bx)^2+\frac{\pi^2b^6}{1680x^2}-\frac{b\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{28x^7}-\frac{b^2}{336x^6}-\frac{b^2\cos\left(\pi b^2x^2\right)}{336x^6}+\frac{1}{280}\pi^3b^8\text{Si}\left(b^2\pi x^2\right)-\frac{\pi^3b^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{420x}+\frac{\pi^2b^6\cos\left(\pi b^2x^2\right)}{336x^2}+\frac{\pi^2b^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{420x^3}+\frac{\pi b^4\sin\left(\pi b^2x^2\right)}{420x^4}+\frac{\pi b^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{140x^5}-\frac{\text{FresnelC}(bx)^2}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]^2/x^9,x]
```

```
[Out] -1/336*b^2/x^6 + (b^6*Pi^2)/(1680*x^2) - (b^2*Cos[b^2*Pi*x^2])/(336*x^6) +
(b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]
)/(28*x^7) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(420*x^3) + (b^8*
Pi^4*FresnelC[b*x]^2)/840 - FresnelC[b*x]^2/(8*x^8) + (b^3*Pi*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(140*x^5) - (b^7*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/
2])/(420*x) + (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) + (b^8*Pi^3*SinIntegral[b^
2*Pi*x^2])/280
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6592

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[b*(x^(m + 2)/(2*(m + 1)*(m + 2))), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6600

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && I
LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)^2}{x^9}\,dx &= -\frac{C(bx)^2}{8x^8}+\frac{1}{4}b\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^8}\,dx \\
&= -\frac{b^2}{336x^6}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7}-\frac{C(bx)^2}{8x^8}+\frac{1}{56}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^7}\,dx-\frac{1}{28}\left(b^3\pi\right)\int \frac{C(bx)\sin}{x} \\
&= -\frac{b^2}{336x^6}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7}-\frac{C(bx)^2}{8x^8}+\frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{140x^5}+\frac{1}{112}b^2\text{Subst}\left(\int \frac{\cos(}{x}\right. \\
&= -\frac{b^2}{336x^6}+\frac{b^6\pi^2}{1680x^2}-\frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{420x^3}-\frac{C(}{8} \\
&= -\frac{b^2}{336x^6}+\frac{b^6\pi^2}{1680x^2}-\frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{420x^3}-\frac{C(}{8} \\
&= -\frac{b^2}{336x^6}+\frac{b^6\pi^2}{1680x^2}-\frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6}+\frac{b^6\pi^2\cos\left(b^2\pi x^2\right)}{336x^2}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7}+\frac{b^5\pi^2\cos\left(\frac{1}{2}\right.}{42} \\
&= -\frac{b^2}{336x^6}+\frac{b^6\pi^2}{1680x^2}-\frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6}+\frac{b^6\pi^2\cos\left(b^2\pi x^2\right)}{336x^2}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{28x^7}+\frac{b^5\pi^2\cos\left(\frac{1}{2}\right.}{42}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 242, normalized size = 1.00

$$-\frac{b^2}{336x^6}+\frac{b^6\pi^2}{1680x^2}-\frac{b^2\cos\left(b^2\pi x^2\right)}{336x^6}+\frac{b^6\pi^2\cos\left(b^2\pi x^2\right)}{336x^2}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{28x^7}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{420x^3}+\frac{1}{840}b^8\pi^4\text{FresnelC}(bx)^2-\frac{\text{FresnelC}(bx)^2}{8x^8}+\frac{b^3\pi\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{140x^5}-\frac{b^7\pi^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{420x}+\frac{b^4\pi\sin\left(b^2\pi x^2\right)}{420x^4}+\frac{1}{280}b^8\pi^3\text{Si}\left(b^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[b*x]^2/x^9,x]
```

```
[Out] -1/336*b^2/x^6 + (b^6*Pi^2)/(1680*x^2) - (b^2*Cos[b^2*Pi*x^2])/(336*x^6) +
(b^6*Pi^2*Cos[b^2*Pi*x^2])/(336*x^2) - (b*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]
)/(28*x^7) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(420*x^3) + (b^8*
Pi^4*FresnelC[b*x]^2)/840 - FresnelC[b*x]^2/(8*x^8) + (b^3*Pi*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(140*x^5) - (b^7*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/
2])/(420*x) + (b^4*Pi*Sin[b^2*Pi*x^2])/(420*x^4) + (b^8*Pi^3*SinIntegral[b^
2*Pi*x^2])/280
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{FresnelC}(bx)^2}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^9,x)
```

```
[Out] int(FresnelC(b*x)^2/x^9,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^9,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)^2/x^9, x)
```

Fricas [A]
time = 0.38, size = 178, normalized size = 0.74

$$\frac{3\,\pi^3 b^8 x^8 \operatorname{Si}\left(\pi b^2 x^2\right) - 2\,\pi^2 b^6 x^6 + 5\left(\pi^2 b^6 x^6 - b^2 x^2\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 2\left(\pi^2 b^5 x^5 - 15\,bx\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{C}\left(bx\right) + \left(\pi^4 b^8 x^8 - 105\right)\mathrm{C}\left(bx\right)^2 + 2\left(2\,\pi b^4 x^4 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - \left(\pi^3 b^7 x^7 - 3\,\pi b^3 x^3\right)\mathrm{C}\left(bx\right)\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{840\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^9,x, algorithm="fricas")
```

```
[Out] 1/840*(3*pi^3*b^8*x^8*sin_integral(pi*b^2*x^2) - 2*pi^2*b^6*x^6 + 5*(pi^2*b
^6*x^6 - b^2*x^2)*cos(1/2*pi*b^2*x^2)^2 + 2*(pi^2*b^5*x^5 - 15*b*x)*cos(1/2
*pi*b^2*x^2)*fresnel_cos(b*x) + (pi^4*b^8*x^8 - 105)*fresnel_cos(b*x)^2 + 2
*(2*pi*b^4*x^4*cos(1/2*pi*b^2*x^2) - (pi^3*b^7*x^7 - 3*pi*b^3*x^3)*fresnel_
cos(b*x))*sin(1/2*pi*b^2*x^2))/x^8
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2(bx)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**9,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**9, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^9,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)^2/x^9, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelC}(b\,x)^2}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^9,x)
```

```
[Out] int(FresnelC(b*x)^2/x^9, x)
```

3.157 $\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx$

Optimal. Leaf size=286

$$-\frac{b^2}{504x^7}+\frac{b^6\pi^2}{5184x^3}-\frac{b^2\cos(b^2\pi x^2)}{504x^7}+\frac{187b^6\pi^2\cos(b^2\pi x^2)}{181440x^3}-\frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{36x^8}+\frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{Fr}}{864x^4}$$

```
[Out] -1/504*b^2/x^7+1/5184*b^6*Pi^2/x^3-1/504*b^2*cos(b^2*Pi*x^2)/x^7+187/181440
*b^6*Pi^2*cos(b^2*Pi*x^2)/x^3-1/36*b*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^8+
1/864*b^5*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^4-1/9*FresnelC(b*x)^2/x^
9+1/216*b^3*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^6-1/1728*b^7*Pi^3*Fresne
lC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2+19/15120*b^4*Pi*sin(b^2*Pi*x^2)/x^5-853/362
880*b^8*Pi^3*sin(b^2*Pi*x^2)/x+853/362880*b^9*Pi^4*FresnelC(b*x*2^(1/2))*2^
(1/2)+1/1728*b^9*Pi^4*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

Rubi [A]
time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelC[b*x]^2/x^10,x]
```

```
[Out] -1/504*b^2/x^7 + (b^6*Pi^2)/(5184*x^3) - (b^2*Cos[b^2*Pi*x^2])/(504*x^7) +
(187*b^6*Pi^2*Cos[b^2*Pi*x^2])/(181440*x^3) - (b*Cos[(b^2*Pi*x^2)/2]*Fresne
lC[b*x])/(36*x^8) + (b^5*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(864*x^4)
- FresnelC[b*x]^2/(9*x^9) + (853*b^9*Pi^4*FresnelC[Sqrt[2]*b*x])/(181440*Sq
rt[2]) + (b^3*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(216*x^6) - (b^7*Pi^3*F
resnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(1728*x^2) + (19*b^4*Pi*Sin[b^2*Pi*x^2])/
(15120*x^5) - (853*b^8*Pi^3*Sin[b^2*Pi*x^2])/(362880*x) + (b^9*Pi^4*Defer[I
nt][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x])/1728
```

Rubi steps

$$\int \frac{C(bx)^2}{x^{10}}\,dx = -\frac{C(bx)^2}{9x^9} + \frac{1}{9}(2b)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^9}\,dx$$

$$= -\frac{b^2}{504x^7} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{36x^8} - \frac{C(bx)^2}{9x^9} + \frac{1}{72}b^2\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}\,dx - \frac{1}{36}\left(b^3\pi\right)\int \frac{C(bx)\sin}{x}$$

$$= -\frac{b^2}{504x^7} - \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{36x^8} - \frac{C(bx)^2}{9x^9} + \frac{b^3\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{216x^6} - \frac{1}{432}$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} - \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{36x^8} + \frac{b^5\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{864x^4} - \frac{C(}{9}$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} - \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} + \frac{187b^6\pi^2\cos\left(b^2\pi x^2\right)}{181440x^3} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{36x^8} + \frac{b^5\pi^2\cos}{}$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} - \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} + \frac{187b^6\pi^2\cos\left(b^2\pi x^2\right)}{181440x^3} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{36x^8} + \frac{b^5\pi^2\cos}{}$$

$$= -\frac{b^2}{504x^7} + \frac{b^6\pi^2}{5184x^3} - \frac{b^2\cos\left(b^2\pi x^2\right)}{504x^7} + \frac{187b^6\pi^2\cos\left(b^2\pi x^2\right)}{181440x^3} - \frac{b\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{36x^8} + \frac{b^5\pi^2\cos}{}$$

Mathematica [**A**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)^2}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^2/x^10,x]
```

```
[Out] Integrate[FresnelC[b*x]^2/x^10, x]
```

Maple [**A**]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\left(bx\right)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^10,x)
```

```
[Out] int(FresnelC(b*x)^2/x^10,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^10,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)^2/x^10, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^10,x, algorithm="fricas")
```

```
[Out] integral(fresnel_cos(b*x)^2/x^10, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2(bx)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**2/x**10,x)
```

```
[Out] Integral(fresnelc(b*x)**2/x**10, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^2/x^10,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)^2/x^10, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelC}(b\,x)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2/x^10,x)
```

```
[Out] int(FresnelC(b*x)^2/x^10, x)
```

3.158 $\int (c+dx)^2 \textbf{FresnelC}(a+bx)^2\,dx$

Optimal. Leaf size=495

$$\frac{2d^2x}{3b^2\pi^2}-\frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2}-\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}-\frac{4d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)\text{FresnelC}(a+bx)}{3b^3\pi^2}+$$

```
[Out] 2/3*d^2*x/b^2/Pi^2-1/2*d*(-a*d+b*c)*cos(Pi*(b*x+a)^2)/b^3/Pi^2-1/6*d^2*(b*x
+a)*cos(Pi*(b*x+a)^2)/b^3/Pi^2-4/3*d^2*cos(1/2*Pi*(b*x+a)^2)*FresnelC(b*x+a
)/b^3/Pi^2+(-a*d+b*c)^2*(b*x+a)*FresnelC(b*x+a)^2/b^3+d*(-a*d+b*c)*(b*x+a)^
2*FresnelC(b*x+a)^2/b^3+1/3*d^2*(b*x+a)^3*FresnelC(b*x+a)^2/b^3+d*(-a*d+b*c
)*FresnelC(b*x+a)*FresnelS(b*x+a)/b^3/Pi+1/4*I*d*(-a*d+b*c)*(b*x+a)^2*hyper
geom([1, 1],[3/2, 2],-1/2*I*Pi*(b*x+a)^2)/b^3/Pi-1/4*I*d*(-a*d+b*c)*(b*x+a)
^2*hypergeom([1, 1],[3/2, 2],1/2*I*Pi*(b*x+a)^2)/b^3/Pi-2*(-a*d+b*c)^2*Fres
nelC(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi-2*d*(-a*d+b*c)*(b*x+a)*FresnelC(b*
x+a)*sin(1/2*Pi*(b*x+a)^2)/b^3/Pi-2/3*d^2*(b*x+a)^2*FresnelC(b*x+a)*sin(1/2
*Pi*(b*x+a)^2)/b^3/Pi+5/12*d^2*FresnelC((b*x+a)*2^(1/2))/b^3/Pi^2*2^(1/2)+1
/2*(-a*d+b*c)^2*FresnelS((b*x+a)*2^(1/2))/b^3/Pi*2^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {6568, 6556, 6588, 3432, 6566, 6590, 6582, 3460, 2718, 6596, 3439, 3433, 3466}

$$\frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3}-\frac{id(a+bx)^2(bc-ad)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{4\pi b^3}+\frac{d(bc-ad)\text{FresnelC}(a+bx)S(a+bx)}{\pi b^3}+\frac{d(a+bx)^2(bc-ad)\text{FresnelC}(a+bx)^2}{b^3}+\frac{(a+bx)(bc-ad)^2\text{FresnelC}(a+bx)^2}{b^3}-\frac{2d(a+bx)(bc-ad)\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}-\frac{2(bc-ad)^2\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^3}+\frac{(bc-ad)^2S\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}\pi b^3}-\frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2\pi^2b^3}+\frac{d^2(a+bx)^3\text{FresnelC}(a+bx)^2}{3b^3}+\frac{5d^2\text{FresnelC}\left(\sqrt{2}(a+bx)\right)}{6\sqrt{2}\pi^2b^3}-\frac{2d^2(a+bx)^2\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi b^3}-\frac{4d^2\text{FresnelC}(a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)}{3\pi^2b^3}-\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6\pi^2b^3}+\frac{2d^2x}{3\pi^2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*FresnelC[a + b*x]^2,x]
```

```
[Out] (2*d^2*x)/(3*b^2*Pi^2) - (d*(b*c - a*d)*Cos[Pi*(a + b*x)^2])/(2*b^3*Pi^2) -
 (d^2*(a + b*x)*Cos[Pi*(a + b*x)^2])/(6*b^3*Pi^2) - (4*d^2*Cos[(Pi*(a + b*x
)^2)/2]*FresnelC[a + b*x])/(3*b^3*Pi^2) + ((b*c - a*d)^2*(a + b*x)*FresnelC
[a + b*x]^2)/b^3 + (d*(b*c - a*d)*(a + b*x)^2*FresnelC[a + b*x]^2)/b^3 + (d
^2*(a + b*x)^3*FresnelC[a + b*x]^2)/(3*b^3) + (5*d^2*FresnelC[Sqrt[2]*(a +
b*x)])/(6*Sqrt[2]*b^3*Pi^2) + (d*(b*c - a*d)*FresnelC[a + b*x]*FresnelS[a +
 b*x])/(b^3*Pi) + ((b*c - a*d)^2*FresnelS[Sqrt[2]*(a + b*x)])/(Sqrt[2]*b^3*
Pi) + ((I/4)*d*(b*c - a*d)*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {3/2, 2},
(-1/2*I)*Pi*(a + b*x)^2])/(b^3*Pi) - ((I/4)*d*(b*c - a*d)*(a + b*x)^2*Hyper
geometricPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^3*Pi) - (2*(b*c -
a*d)^2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi) - (2*d*(b*c - a*
d)*(a + b*x)*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b^3*Pi) - (2*d^2*(
a + b*x)^2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(3*b^3*Pi)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3439

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6556

```
Int[FresnelC[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(FresnelC[a
 + b*x]^2/b), x] - Dist[2, Int[(a + b*x)*Cos[(Pi/2)*(a + b*x)^2]*FresnelC[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6568

```
Int[FresnelC[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Dist[1/b^(m + 1), Subst[Int[ExpandIntegrand[FresnelC[x]^2, (b*c - a*d + d*x
)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6588

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6596

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelC[b*x]/(2*d)), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /;
 FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\int (c+dx)^2 C(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(b^2c^2\left(1+\frac{ad(-2bc+ad)}{b^2c^2}\right)C(x)^2+2bcd\left(1-\frac{ad}{bc}\right)xC(x)^2+d^2x^2C(x)^2\right)\right.}{b^3}$$

$$= \frac{d^2\text{Subst}\left(\int x^2C(x)^2\,dx,x,a+bx\right)}{b^3}+\frac{(2d(bc-ad))\text{Subst}\left(\int xC(x)^2\,dx,x,a+b\right.}{b^3}$$

$$= \frac{(bc-ad)^2(a+bx)C(a+bx)^2}{b^3}+\frac{d(bc-ad)(a+bx)^2C(a+bx)^2}{b^3}+\frac{d^2(a+bx)}{3}$$

$$= \frac{(bc-ad)^2(a+bx)C(a+bx)^2}{b^3}+\frac{d(bc-ad)(a+bx)^2C(a+bx)^2}{b^3}+\frac{d^2(a+bx)}{3}$$

$$= -\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}-\frac{4d^2\cos\left(\frac{1}{2}\pi(a+bx)^2\right)C(a+bx)}{3b^3\pi^2}+\frac{(bc-ad)^2}{}$$

$$= -\frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2}-\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}-\frac{4d^2\cos\left(\frac{1}{2}\pi(a+\right.}{3b}$$

$$= \frac{2d^2x}{3b^2\pi^2}-\frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2}-\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}-\frac{4d^2\cos(}{}$$

$$= \frac{2d^2x}{3b^2\pi^2}-\frac{d(bc-ad)\cos\left(\pi(a+bx)^2\right)}{2b^3\pi^2}-\frac{d^2(a+bx)\cos\left(\pi(a+bx)^2\right)}{6b^3\pi^2}-\frac{4d^2\cos(}{}$$

Mathematica [F]
time = 0.44, size = 0, normalized size = 0.00

$$\int (c+dx)^2\text{FresnelC}(a+bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^2*FresnelC[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^2*FresnelC[a + b*x]^2, x]
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int (dx+c)^2\,\text{FresnelC}\,(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*FresnelC(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^2*FresnelC(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnel_cos(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^2*fresnel_cos(b*x + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnel_cos(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*fresnel_cos(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 C^2(a+bx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*fresnelc(b*x+a)**2,x)
[Out] Integral((c + d*x)**2*fresnelc(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*fresnel_cos(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^2*fresnel_cos(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{FresnelC}(a+b\,x)^2\,(c+d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)^2*(c + d*x)^2,x)
[Out] int(FresnelC(a + b*x)^2*(c + d*x)^2, x)
```

3.159 $\int (c+dx)\text{FresnelC}(a+bx)^2\,dx$

Optimal. Leaf size=279

$$-\frac{d\cos\left(\pi(a+bx)^2\right)}{4b^2\pi^2}+\frac{(bc-ad)(a+bx)\text{FresnelC}(a+bx)^2}{b^2}+\frac{d(a+bx)^2\text{FresnelC}(a+bx)^2}{2b^2}+\frac{d\text{FresnelC}(a+b}{2b^2\pi}$$

```
[Out] -1/4*d*cos(Pi*(b*x+a)^2)/b^2/Pi^2+(-a*d+b*c)*(b*x+a)*FresnelC(b*x+a)^2/b^2+
1/2*d*(b*x+a)^2*FresnelC(b*x+a)^2/b^2+1/2*d*FresnelC(b*x+a)*FresnelS(b*x+a)
/b^2/Pi+1/8*I*d*(b*x+a)^2*hypergeom([1, 1],[3/2, 2],-1/2*I*Pi*(b*x+a)^2)/b^
2/Pi-1/8*I*d*(b*x+a)^2*hypergeom([1, 1],[3/2, 2],1/2*I*Pi*(b*x+a)^2)/b^2/Pi
-2*(-a*d+b*c)*FresnelC(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^2/Pi-d*(b*x+a)*Fresne
lC(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b^2/Pi+1/2*(-a*d+b*c)*FresnelS((b*x+a)*2^(1
/2))/b^2/Pi*2^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6568, 6556, 6588, 3432, 6566, 6590, 6582, 3460, 2718}

$$\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2}-\frac{id(a+bx)^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}i\pi(a+bx)^2\right)}{8\pi b^2}+\frac{(a+bx)(bc-ad)\text{FresnelC}(a+bx)^2}{b^2}-\frac{2(bc-ad)\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}+\frac{(bc-ad)S\left(\sqrt{2}(a+bx)\right)}{\sqrt{2}\pi b^2}+\frac{d\text{FresnelC}(a+bx)S(a+bx)}{2\pi b^2}+\frac{d(a+bx)^2\text{FresnelC}(a+bx)^2}{2b^2}-\frac{d(a+bx)\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b^2}-\frac{d\cos\left(\pi(a+bx)^2\right)}{4\pi^2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*FresnelC[a + b*x]^2,x]
```

```
[Out] -1/4*(d*Cos[Pi*(a + b*x)^2])/(b^2*Pi^2) + ((b*c - a*d)*(a + b*x)*FresnelC[a
 + b*x]^2)/b^2 + (d*(a + b*x)^2*FresnelC[a + b*x]^2)/(2*b^2) + (d*FresnelC[
a + b*x]*FresnelS[a + b*x])/(2*b^2*Pi) + ((b*c - a*d)*FresnelS[Sqrt[2]*(a +
 b*x)])/(Sqrt[2]*b^2*Pi) + ((I/8)*d*(a + b*x)^2*HypergeometricPFQ[{1, 1}, {
3/2, 2}, (-1/2*I)*Pi*(a + b*x)^2])/(b^2*Pi) - ((I/8)*d*(a + b*x)^2*Hypergeo
metricPFQ[{1, 1}, {3/2, 2}, (I/2)*Pi*(a + b*x)^2])/(b^2*Pi) - (2*(b*c - a*d
)*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b^2*Pi) - (d*(a + b*x)*Fresne
lC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b^2*Pi)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6556

```
Int[FresnelC[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(FresnelC[a
 + b*x]^2/b), x] - Dist[2, Int[(a + b*x)*Cos[(Pi/2)*(a + b*x)^2]*FresnelC[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6566

```
Int[FresnelC[(b_.)*(x_)]^2*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(Fresnel
C[b*x]^2/(m + 1)), x] - Dist[2*(b/(m + 1)), Int[x^(m + 1)*Cos[(Pi/2)*b^2*x^
2]*FresnelC[b*x], x], x] /; FreeQ[b, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6568

```
Int[FresnelC[(a_) + (b_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Dist[1/b^(m + 1), Subst[Int[ExpandIntegrand[FresnelC[x]^2, (b*c - a*d + d*x
)^m, x], x], x, a + b*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0]
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6588

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rubi steps

$$\int (c+dx)C(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(bc\left(1-\frac{ad}{bc}\right)C(x)^2+dxC(x)^2\right)\,dx,x,a+bx\right)}{b^2}$$
$$= \frac{d\text{Subst}\left(\int xC(x)^2\,dx,x,a+bx\right)}{b^2}+\frac{(bc-ad)\text{Subst}\left(\int C(x)^2\,dx,x,a+bx\right)}{b^2}$$
$$= \frac{(bc-ad)(a+bx)C(a+bx)^2}{b^2}+\frac{d(a+bx)^2C(a+bx)^2}{2b^2}-\frac{d\text{Subst}\left(\int x^2\cos\left(\frac{\pi x^2}{2}\right)\right.}{b^2}$$
$$= \frac{(bc-ad)(a+bx)C(a+bx)^2}{b^2}+\frac{d(a+bx)^2C(a+bx)^2}{2b^2}-\frac{2(bc-ad)C(a+bx)\text{s}}{b^2\pi}$$
$$= \frac{(bc-ad)(a+bx)C(a+bx)^2}{b^2}+\frac{d(a+bx)^2C(a+bx)^2}{2b^2}+\frac{dC(a+bx)S(a+bx)}{2b^2\pi}$$
$$= -\frac{d\cos\left(\pi(a+bx)^2\right)}{4b^2\pi^2}+\frac{(bc-ad)(a+bx)C(a+bx)^2}{b^2}+\frac{d(a+bx)^2C(a+bx)^2}{2b^2}+$$

Mathematica [F]
time = 0.39, size = 0, normalized size = 0.00

$$\int (c+dx)\text{FresnelC}(a+bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)*FresnelC[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)*FresnelC[a + b*x]^2, x]
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (dx+c)\,\text{FresnelC}\,(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*FresnelC(b*x+a)^2,x)
```

```
[Out] int((d*x+c)*FresnelC(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnel_cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*fresnel_cos(b*x + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnel_cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*fresnel_cos(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) C^2(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnelc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*fresnelc(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*fresnel_cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*fresnel_cos(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{FresnelC}(a + b\,x)^2 \,(c + d\,x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)^2*(c + d*x),x)
```

```
[Out] int(FresnelC(a + b*x)^2*(c + d*x), x)
```

3.160 $\int \text{FresnelC}(a+bx)^2\,dx$

Optimal. Leaf size=69

$$\frac{(a+bx)\text{FresnelC}(a+bx)^2}{b}+\frac{S\left(\sqrt{2}\,(a+bx)\right)}{\sqrt{2}\,b\pi}-\frac{2\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}$$

[Out] (b*x+a)*FresnelC(b*x+a)^2/b-2*FresnelC(b*x+a)*sin(1/2*Pi*(b*x+a)^2)/b/Pi+1/2*FresnelS((b*x+a)*2^(1/2))/b/Pi*2^(1/2)

Rubi [A]
time = 0.11, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.375$, Rules used = {6556, 6588, 3432}

$$\frac{(a+bx)\text{FresnelC}(a+bx)^2}{b}-\frac{2\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{\pi b}+\frac{S\left(\sqrt{2}\,(a+bx)\right)}{\sqrt{2}\,\pi b}$$

Antiderivative was successfully verified.

[In] Int[FresnelC[a + b*x]^2,x]

[Out] ((a + b*x)*FresnelC[a + b*x]^2)/b + FresnelS[Sqrt[2]*(a + b*x)]/(Sqrt[2]*b*Pi) - (2*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(b*Pi)

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 6556

```
Int[FresnelC[(a_.) + (b_.)*(x_)]^2, x_Symbol] :> Simp[(a + b*x)*(FresnelC[a
 + b*x]^2/b), x] - Dist[2, Int[(a + b*x)*Cos[(Pi/2)*(a + b*x)^2]*FresnelC[a
 + b*x], x], x] /; FreeQ[{a, b}, x]
```

Rule 6588

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\int C(a+bx)^2\,dx = \frac{(a+bx)C(a+bx)^2}{b} - 2\int (a+bx)\cos\left(\frac{1}{2}\pi(a+bx)^2\right)C(a+bx)\,dx$$
$$= \frac{(a+bx)C(a+bx)^2}{b} - \frac{2\text{Subst}\left(\int x\cos\left(\frac{\pi x^2}{2}\right)C(x)\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)C(a+bx)^2}{b} - \frac{2C(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi} + \frac{\text{Subst}\left(\int \sin(\pi x^2)\,dx, x, a+bx\right)}{b\pi}$$
$$= \frac{(a+bx)C(a+bx)^2}{b} + \frac{S\left(\sqrt{2}\,(a+bx)\right)}{\sqrt{2}\,b\pi} - \frac{2C(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{b\pi}$$

Mathematica [A]
time = 0.01, size = 66, normalized size = 0.96

$$\frac{2\pi(a+bx)\text{FresnelC}(a+bx)^2 + \sqrt{2}\,S\left(\sqrt{2}\,(a+bx)\right) - 4\text{FresnelC}(a+bx)\sin\left(\frac{1}{2}\pi(a+bx)^2\right)}{2b\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[a + b*x]^2,x]
```

```
[Out] (2*Pi*(a + b*x)*FresnelC[a + b*x]^2 + Sqrt[2]*FresnelS[Sqrt[2]*(a + b*x)] -
 4*FresnelC[a + b*x]*Sin[(Pi*(a + b*x)^2)/2])/(2*b*Pi)
```

Maple [A]
time = 0.48, size = 60, normalized size = 0.87

method	result	size
derivativedivides	$\frac{\text{FresnelC}(bx+a)^2(bx+a) - \frac{2\,\text{FresnelC}(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{\sqrt{2}\,\text{S}\left((bx+a)\sqrt{2}\right)}{2\pi}}{b}$	60
default	$\frac{\text{FresnelC}(bx+a)^2(bx+a) - \frac{2\,\text{FresnelC}(bx+a)\sin\left(\frac{\pi(bx+a)^2}{2}\right)}{\pi} + \frac{\sqrt{2}\,\text{S}\left((bx+a)\sqrt{2}\right)}{2\pi}}{b}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(FresnelC(b*x+a)^2*(b*x+a)-2*FresnelC(b*x+a)/Pi*sin(1/2*Pi*(b*x+a)^2)+1
/2/Pi*2^(1/2)*FresnelS((b*x+a)*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x + a)^2, x)
```

Fricas [A]
time = 0.35, size = 88, normalized size = 1.28

$$\frac{2\left(\pi b^{2}x+\pi ab\right)\mathrm{C}\left(bx+a\right)^{2}-4\,b\,\mathrm{C}\left(bx+a\right)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}+\pi abx+\frac{1}{2}\,\pi a^{2}\right)+\sqrt{2}\,\sqrt{b^{2}}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{b^{2}}\,(bx+a)}{b}\right)}{2\,\pi b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(pi*b^2*x + pi*a*b)*fresnel_cos(b*x + a)^2 - 4*b*fresnel_cos(b*x + a
)*sin(1/2*pi*b^2*x^2 + pi*a*b*x + 1/2*pi*a^2) + sqrt(2)*sqrt(b^2)*fresnel_s
in(sqrt(2)*sqrt(b^2)*(b*x + a)/b))/(pi*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int C^{2}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)**2,x)
```

```
[Out] Integral(fresnelc(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{FresnelC}(a+b\,x)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)^2,x)
```

```
[Out] int(FresnelC(a + b*x)^2, x)
```

3.161 $\int \frac{\text{FresnelC}(a+bx)^2}{c+dx} dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\text{FresnelC}(a+bx)^2}{c+dx}, x\right)$$

```
[Out] Unintegrable(FresnelC(b*x+a)^2/(d*x+c),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx} dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelC[a + b*x]^2/(c + d*x),x]
[Out] Defer[Int][FresnelC[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{C(a+bx)^2}{c+dx} dx = \int \frac{C(a+bx)^2}{c+dx} dx$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(a+bx)^2}{c+dx} dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelC[a + b*x]^2/(c + d*x),x]
[Out] Integrate[FresnelC[a + b*x]^2/(c + d*x), x]
```

Maple [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\left(bx+a\right)^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a)^2/(d*x+c),x)
[Out] int(FresnelC(b*x+a)^2/(d*x+c),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)^2/(d*x+c),x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x + a)^2/(d*x + c), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)^2/(d*x+c),x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x + a)^2/(d*x + c), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)**2/(d*x+c),x)
[Out] Integral(fresnelc(a + b*x)**2/(c + d*x), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x + a)^2/(d*x + c), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{{\mathrm{FresnelC}\left(a+b\,x\right)}^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)^2/(c + d*x),x)
[Out] int(FresnelC(a + b*x)^2/(c + d*x), x)
```

3.162 $\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2} dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2}, x\right)$$

```
[Out] Unintegrable(FresnelC(b*x+a)^2/(d*x+c)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelC[a + b*x]^2/(c + d*x)^2,x]
[Out] Defer[Int][FresnelC[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{C(a+bx)^2}{(c+dx)^2} dx = \int \frac{C(a+bx)^2}{(c+dx)^2} dx$$

Mathematica [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(a+bx)^2}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelC[a + b*x]^2/(c + d*x)^2,x]
[Out] Integrate[FresnelC[a + b*x]^2/(c + d*x)^2, x]
```

Maple [A]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\left(bx+a\right)^2}{\left(dx+c\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(FresnelC(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x + a)^2/(d*x + c)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(fresnel_cos(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C^2(a+bx)}{(c+dx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(fresnelc(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\mathrm{FresnelC}(a+b\,x)^2}{(c+d\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(a + b*x)^2/(c + d*x)^2,x)
```

```
[Out] int(FresnelC(a + b*x)^2/(c + d*x)^2, x)
```

3.163 $\int x^2 \mathbf{FresnelC}(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=231

$$\left(\frac{1}{12}+\frac{i}{12}\right)e^{-\frac{3a}{bn}+\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{-3/n}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)-\left(\frac{1}{12}+\frac{i}{12}\right)e^{-\frac{3a}{bn}-\frac{9}{2b^2d^2}}$$

```
[Out] (1/12+1/12*I)*exp(-3*a/b/n+9/2*I/b^2/d^2/n^2/Pi)*x^3*erf((1/2+1/2*I)*(3/n+I
*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(3/n))-(1/12+1/1
2*I)*exp(-3*a/b/n-9/2*I/b^2/d^2/n^2/Pi)*x^3*erfi((1/2+1/2*I)*(3/n-I*a*b*d^2
*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(3/n))+1/3*x^3*FresnelC(
d*(a+b*ln(c*x^n)))
```

Rubi [A]
time = 0.26, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6607, 4714, 2314, 2308, 2266, 2235, 2236}

$$\left(\frac{1}{12}+\frac{i}{12}\right)x^3(cx^n)^{-3/n}e^{-\frac{3a}{bn}+\frac{9i}{2\pi b^2d^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{3}{n}\right)}{\sqrt{\pi}bd}\right)-\left(\frac{1}{12}+\frac{i}{12}\right)x^3(cx^n)^{-3/n}e^{-\frac{3a}{bn}-\frac{9i}{2\pi b^2d^2n^2}}\operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{3}{n}\right)}{\sqrt{\pi}bd}\right)+\frac{1}{3}x^3\text{FresnelC}(d(a+b\log(cx^n)))$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/12 + I/12)*E^((-3*a)/(b*n) + ((9*I)/2)/(b^2*d^2*n^2*Pi))*x^3*Erf[((1/2
+ I/2)*(3/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*
x^n)^(3/n) - ((1/12 + I/12)*E^((-3*a)/(b*n) - ((9*I)/2)/(b^2*d^2*n^2*Pi))*x
^3*Erfi[((1/2 + I/2)*(3/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*S
qrt[Pi])])/(c*x^n)^(3/n) + (x^3*FresnelC[d*(a + b*Log[c*x^n])])/3
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2314

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F])), Int[(d + e*x)^(m + 2*a*b*
f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a,
b, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 4714

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6607

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelC[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Dist[b*d*(n/(m + 1)), Int[(e*x)^m*Cos[(Pi/2)*(d*(a + b*Log[c*x^n
]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2 C(d(a+b\log(cx^n)))\,dx = \frac{1}{3}x^3 C(d(a+b\log(cx^n))) - \frac{1}{3}(bdn)\int x^2\cos\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)\,dx$$

$$= \frac{1}{3}x^3 C(d(a+b\log(cx^n))) - \frac{1}{6}(bdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}x^2\,dx - \frac{1}{6}(bdn)\int e$$

$$= \frac{1}{3}x^3 C(d(a+b\log(cx^n))) - \frac{1}{6}(bdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log(cx^n) -\right.$$

$$= \frac{1}{3}x^3 C(d(a+b\log(cx^n))) - \frac{1}{6}(bdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2(cx^n)\right)$$

$$= \frac{1}{3}x^3 C(d(a+b\log(cx^n))) - \frac{1}{6}\left(bdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi -\right.$$

$$= \frac{1}{3}x^3 C(d(a+b\log(cx^n))) - \frac{1}{6}\left(bdx^3(cx^n)^{-iabd^2\pi-\frac{3-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(-\right.\right.$$

$$= \frac{1}{3}x^3 C(d(a+b\log(cx^n))) - \frac{1}{6}\left(bde^{-\frac{3a}{bn}-\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{-iabd^2\pi-\frac{3-iabd^2n\pi}{n}}\right)\text{Subs}$$

$$= \left(\frac{1}{12}+\frac{i}{12}\right)e^{-\frac{3a}{bn}+\frac{9i}{2b^2d^2n^2\pi}}x^3(cx^n)^{-3/n}\,\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{3}{n}+iabd^2\pi+ib^2d^2\pi\log\right.}{bd\sqrt{\pi}}\right.$$

Mathematica [A]
time = 4.46, size = 318, normalized size = 1.38

$$\frac{1}{12}x^3\left(4\text{FresnelC}(d(a+b\log(cx^n)))+\sqrt[4]{-1}\sqrt{2}\,e^{\frac{1}{2}\left(-\frac{6a}{bn}-\frac{9i}{b^2d^2n^2\pi}-ia^2d^2\pi+2iabd^2\pi(n\log(x)-\log(cx^n))-ib^2d^2\pi(-n\log(x)+\log(cx^n))^2\right)}(cx^n)^{-3/n}\left(ie^{\frac{9i}{b^2d^2n^2\pi}}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-3i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)+\text{Erfi}\left(\frac{(-1)^{3/4}(3i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)\right)\left(\cos\left(\frac{1}{2}d^2\pi(a-bn\log(x)+b\log(cx^n))^2\right)+i\sin\left(\frac{1}{2}d^2\pi(a-bn\log(x)+b\log(cx^n))^2\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^3*(4*FresnelC[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*Sqrt[2]*E^(((-6*a)/(b*
n) - (9*I)/(b^2*d^2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - L
og[c*x^n]) - I*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*(I*E^((9*I)/(b^2
*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-3*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[Pi])] + Erfi[((-1)^(3/4)*(3*I + a*b*d^2*n*Pi + b^2*d^2*n*
Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c
*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(c*x^n
)^(3/n)))/12
```

Maple [F]
time = 0.54, size = 0, normalized size = 0.00

$$\int x^2\,\text{FresnelC}\left(d(a+b\ln(c\,x^n))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^2*FresnelC(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(x^2*fresnel_cos((b*log(c*x^n) + a)*d), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(187) = 374.
time = 0.37, size = 448, normalized size = 1.94

$$\frac{1}{3}\,x^{3}\,\mathrm{C}\left(bd\log\left(cx^{n}\right)+ad\right)-\frac{1}{6}\,\pi\sqrt{b^{2}d^{2}n^{2}}\,e^{\left(-\frac{3\,\log\left(c\right)}{n}-\frac{3\,a}{bn}-\frac{9\,i}{2\,\pi b^{2}d^{2}n^{2}}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^{2}d^{2}n^{2}\log\left(x\right)+\pi b^{2}d^{2}n\log\left(c\right)+\pi abd^{2}n+3\,i\right)\sqrt{b^{2}d^{2}n^{2}}}{\pi b^{2}d^{2}n^{2}}\right)-\frac{1}{6}\,\pi\sqrt{b^{2}d^{2}n^{2}}\,e^{\left(-\frac{3\,\log\left(c\right)}{n}-\frac{3\,a}{bn}+\frac{9\,i}{2\,\pi b^{2}d^{2}n^{2}}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^{2}d^{2}n^{2}\log\left(x\right)+\pi b^{2}d^{2}n\log\left(c\right)+\pi abd^{2}n-3\,i\right)\sqrt{b^{2}d^{2}n^{2}}}{\pi b^{2}d^{2}n^{2}}\right)+\frac{1}{6}\,i\,\pi\sqrt{b^{2}d^{2}n^{2}}\,e^{\left(-\frac{3\,\log\left(c\right)}{n}-\frac{3\,a}{bn}-\frac{9\,i}{2\,\pi b^{2}d^{2}n^{2}}\right)}\,\mathrm{S}\left(\frac{\left(\pi b^{2}d^{2}n^{2}\log\left(x\right)+\pi b^{2}d^{2}n\log\left(c\right)+\pi abd^{2}n+3\,i\right)\sqrt{b^{2}d^{2}n^{2}}}{\pi b^{2}d^{2}n^{2}}\right)-\frac{1}{6}\,i\,\pi\sqrt{b^{2}d^{2}n^{2}}\,e^{\left(-\frac{3\,\log\left(c\right)}{n}-\frac{3\,a}{bn}+\frac{9\,i}{2\,\pi b^{2}d^{2}n^{2}}\right)}\,\mathrm{S}\left(\frac{\left(\pi b^{2}d^{2}n^{2}\log\left(x\right)+\pi b^{2}d^{2}n\log\left(c\right)+\pi abd^{2}n-3\,i\right)\sqrt{b^{2}d^{2}n^{2}}}{\pi b^{2}d^{2}n^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*fresnel_cos(b*d*log(c*x^n) + a*d) - 1/6*pi*sqrt(b^2*d^2*n^2)*e^(-3*
log(c)/n - 3*a/(b*n) - 9/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*
log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + 3*I)*sqrt(b^2*d^2*n^2)/(pi*b^
2*d^2*n^2)) - 1/6*pi*sqrt(b^2*d^2*n^2)*e^(-3*log(c)/n - 3*a/(b*n) + 9/2*I/(
pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) +
 pi*a*b*d^2*n - 3*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + 1/6*I*pi*sqrt(b^
2*d^2*n^2)*e^(-3*log(c)/n - 3*a/(b*n) - 9/2*I/(pi*b^2*d^2*n^2))*fresnel_sin
((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + 3*I)*sqrt(b^
2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/6*I*pi*sqrt(b^2*d^2*n^2)*e^(-3*log(c)/n -
3*a/(b*n) + 9/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi
*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 3*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}C\left(ad+bd\log\left(cx^{n}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnelc(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x**2*fresnelc(a*d + b*d*log(c*x**n)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(x^2*fresnel_cos((b*log(c*x^n) + a)*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \,\mathrm{FresnelC}(d\,(a+b\,\ln\left(c\,x^n\right)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(d*(a + b*log(c*x^n))),x)
[Out] int(x^2*FresnelC(d*(a + b*log(c*x^n))), x)
```

3.164 $\int x\text{FresnelC}(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=227

$$\left(\frac{1}{8}+\frac{i}{8}\right)e^{\frac{2i-2abd^2n\pi}{b^2d^2n^2\pi}}x^2(cx^n)^{-2/n}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)-\left(\frac{1}{8}+\frac{i}{8}\right)e^{-\frac{2\left(i+abd^2n\pi\right)}{b^2d^2n^2\pi}}x^2(c$$

```
[Out] (1/8+1/8*I)*exp((2*I-2*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*x^2*erf((1/2+1/2*I)*(2
/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/((c*x^n)^(2/n))-(1/8+
1/8*I)*x^2*erfi((1/2+1/2*I)*(2/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/P
i^(1/2))/exp(2*(I+a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/((c*x^n)^(2/n))+1/2*x^2*Fre
snelC(d*(a+b*ln(c*x^n)))
```

Rubi [A]
time = 0.23, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {6607, 4714, 2314, 2308, 2266, 2235, 2236}

$$\left(\frac{1}{8}+\frac{i}{8}\right)x^2(cx^n)^{-2/n}e^{\frac{-2\pi abd^2n+2i}{\pi b^2d^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)-\left(\frac{1}{8}+\frac{i}{8}\right)x^2(cx^n)^{-2/n}e^{-\frac{2\left(\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}bd}\right)+\frac{1}{2}x^2\text{FresnelC}(d(a+b\log(cx^n)))$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/8 + I/8)*E^((2*I - 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*x^2*Erf[((1/2 + I/
2)*(2/n + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(c*x^n)
^(2/n) - ((1/8 + I/8)*x^2*Erfi[((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2
*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*(I + a*b*d^2*n*Pi))/(b^2*d^2*n^2*P
i))*(c*x^n)^(2/n)) + (x^2*FresnelC[d*(a + b*Log[c*x^n])])/2
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2314

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F])), Int[(d + e*x)^(m + 2*a*b*
f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a,
b, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 4714

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6607

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelC[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Dist[b*d*(n/(m + 1)), Int[(e*x)^m*Cos[(Pi/2)*(d*(a + b*Log[c*x^n
]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int xC(d(a+b\log(cx^n)))\,dx = \frac{1}{2}x^2C(d(a+b\log(cx^n))) - \frac{1}{2}(bdn)\int x\cos\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)\,dx$$

$$= \frac{1}{2}x^2C(d(a+b\log(cx^n))) - \frac{1}{4}(bdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}x\,dx - \frac{1}{4}(bdn)\int e$$

$$= \frac{1}{2}x^2C(d(a+b\log(cx^n))) - \frac{1}{4}(bdn)\int \exp\Big(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log(cx^n) -$$

$$= \frac{1}{2}x^2C(d(a+b\log(cx^n))) - \frac{1}{4}(bdn)\int \exp\Big(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2(cx^n)$$

$$= \frac{1}{2}x^2C(d(a+b\log(cx^n))) - \frac{1}{4}\left(bdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right)\int \exp\Big(-\frac{1}{2}ia^2d^2\pi$$

$$= \frac{1}{2}x^2C(d(a+b\log(cx^n))) - \frac{1}{4}\left(bdx^2(cx^n)^{-iabd^2\pi-\frac{2-iabd^2n\pi}{n}}\right)\text{Subst}\Big(\int \exp\Big($$

$$= \frac{1}{2}x^2C(d(a+b\log(cx^n))) - \frac{1}{4}\left(bde^{-\frac{2\left(i+abd^2n\pi\right)}{b^2d^2n^2\pi}}x^2(cx^n)^{-iabd^2\pi-\frac{2-iabd^2n\pi}{n}}\right)\text{Subs}$$

$$= \left(\frac{1}{8}+\frac{i}{8}\right)e^{\frac{2i-2abd^2n\pi}{b^2d^2n^2\pi}}x^2(cx^n)^{-2/n}\,\text{erf}\Big(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right.}{bd\sqrt{\pi}}$$

Mathematica [A]
time = 4.34, size = 318, normalized size = 1.40

$$\frac{1}{8}x^2\left(4\text{FresnelC}(d(a+b\log(cx^n))) + \sqrt[4]{-1}\sqrt{2}\,e^{-\frac{2a}{bn}-\frac{2i}{b^2d^2n^2\pi}-\frac{1}{2}ia^2d^2\pi+iabd^2\pi(n\log(x)-\log(cx^n))-\frac{1}{2}ib^2d^2\pi(-n\log(x)+\log(cx^n))^2}(cx^n)^{-2/n}\left(ie^{\frac{4i}{b^2d^2n^2\pi}}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-2i+abd^2n\pi+b^2d^2n\pi\log(cx^n)\right)}{bdn\sqrt{\pi}}\right)+\text{Erfi}\left(\frac{(-1)^{3/4}\left(2i+abd^2n\pi+b^2d^2n\pi\log(cx^n)\right)}{bdn\sqrt{2\pi}}\right)\right)\left(\cos\left(\frac{1}{2}d^2\pi(a-bn\log(x)+b\log(cx^n))^2\right)+i\sin\left(\frac{1}{2}d^2\pi(a-bn\log(x)+b\log(cx^n))^2\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^2*(4*FresnelC[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*Sqrt[2]*E^((-2*a)/(b*n
) - (2*I)/(b^2*d^2*n^2*Pi) - (I/2)*a^2*d^2*Pi + I*a*b*d^2*Pi*(n*Log[x] - Lo
g[c*x^n]) - (I/2)*b^2*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)*(I*E^((4*I)/(b^2
*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[Pi])] + Erfi[((-1)^(3/4)*(2*I + a*b*d^2*n*Pi + b^2*d^2*n*
Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c
*x^n])^2)/2] + I*Sin[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(c*x^n
)^(2/n)))/8
```

Maple [F]
time = 0.47, size = 0, normalized size = 0.00

$$\int x\,\text{FresnelC}\left(d(a+b\ln(c\,x^n))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x*FresnelC(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(x*fresnel_cos((b*log(c*x^n) + a)*d), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(187) = 374$.
time = 0.40, size = 448, normalized size = 1.97

$$-\frac{1}{4}\pi\sqrt{b^2d^2n^2}\,e^{\left(-\frac{2\log(c)}{n}-\frac{2a}{bn}-\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+2i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)-\frac{1}{4}\pi\sqrt{b^2d^2n^2}\,e^{\left(-\frac{2\log(c)}{n}-\frac{2a}{bn}+\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n-2i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+\frac{1}{4}i\pi\sqrt{b^2d^2n^2}\,e^{\left(-\frac{2\log(c)}{n}-\frac{2a}{bn}-\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+2i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)-\frac{1}{4}i\pi\sqrt{b^2d^2n^2}\,e^{\left(-\frac{2\log(c)}{n}-\frac{2a}{bn}+\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n-2i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+\frac{1}{2}x^2\,\mathrm{C}\left(bd\log\left(cx^n\right)+ad\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] -1/4*pi*sqrt(b^2*d^2*n^2)*e^(-2*log(c)/n - 2*a/(b*n) - 2*I/(pi*b^2*d^2*n^2)
)*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n +
 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/4*pi*sqrt(b^2*d^2*n^2)*e^(-2*
log(c)/n - 2*a/(b*n) + 2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*lo
g(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*
d^2*n^2)) + 1/4*I*pi*sqrt(b^2*d^2*n^2)*e^(-2*log(c)/n - 2*a/(b*n) - 2*I/(pi
*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + p
i*a*b*d^2*n + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/4*I*pi*sqrt(b^2*
d^2*n^2)*e^(-2*log(c)/n - 2*a/(b*n) + 2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi
*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*sqrt(b^2*d^
2*n^2)/(pi*b^2*d^2*n^2)) + 1/2*x^2*fresnel_cos(b*d*log(c*x^n) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int xC(ad + bd\log{(cx^{n})})\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x*fresnelc(a*d + b*d*log(c*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(x*fresnel_cos((b*log(c*x^n) + a)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{FresnelC}(d\,(a+b\,\ln{(c\,x^n)}))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(d*(a + b*log(c*x^n))),x)
[Out] int(x*FresnelC(d*(a + b*log(c*x^n))), x)
```

3.165 $\int \textbf{FresnelC}(d(a + b\log(cx^n)))\,dx$

Optimal. Leaf size=214

$$\left(\frac{1}{4}+\frac{i}{4}\right)e^{-\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}x(cx^n)^{-1/n}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)-\left(\frac{1}{4}+\frac{i}{4}\right)e^{-\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}x(cx^n)^{-}$$

```
[Out] (1/4+1/4*I)*x*erf((1/2+1/2*I)*(1/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d
/Pi^(1/2))/exp(1/2*(2*a*b*n-I/d^2/Pi)/b^2/n^2)/((c*x^n)^(1/n))-(1/4+1/4*I)*
x*erfi((1/2+1/2*I)*(1/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/
exp(1/2*(2*a*b*n+I/d^2/Pi)/b^2/n^2)/((c*x^n)^(1/n))+x*FresnelC(d*(a+b*ln(c*
x^n)))
```

Rubi [A]
time = 0.19, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {6604, 4712, 2312, 2308, 2266, 2235, 2236}

$$\left(\frac{1}{4}+\frac{i}{4}\right)x(cx^n)^{-1/n}e^{-\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}\,bd}\right)-\left(\frac{1}{4}+\frac{i}{4}\right)x(cx^n)^{-1/n}e^{-\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}\,bd}\right)+x\operatorname{FresnelC}(d(a+b\log(cx^n)))$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/4 + I/4)*x*Erf[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c
*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^
(-1)) - ((1/4 + I/4)*x*Erfi[((1/2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2
*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(
c*x^n)^n^(-1)) + x*FresnelC[d*(a + b*Log[c*x^n])]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2312

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.)), x
_Symbol] :> Dist[(c*(d + e*x)^n)^(2*a*b*f*Log[F])/(d + e*x)^(2*a*b*f*n*Log[
F]), Int[(d + e*x)^(2*a*b*f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2
), x], x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] &&  !IntegerQ[2*a*b*f*Log[F
]]
```

Rule 4712

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)], x_Symbol] :> Dist[1
/2, Int[E^((-I)*d*(a + b*Log[c*x^n])^2), x], x] + Dist[1/2, Int[E^(I*d*(a +
 b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Rule 6604

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Sim
p[x*FresnelC[d*(a + b*Log[c*x^n])], x] - Dist[b*d*n, Int[Cos[(Pi/2)*(d*(a +
 b*Log[c*x^n]))^2], x], x] /; FreeQ[{a, b, c, d, n}, x]
```

Rubi steps

$$\int C(d(a+b\log(cx^n)))\,dx = xC(d(a+b\log(cx^n))) - (bdn)\int \cos\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)\,dx$$

$$= xC(d(a+b\log(cx^n))) - \frac{1}{2}(bdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}\,dx - \frac{1}{2}(bdn)\int e^{\frac{1}{2}id^2\pi(a\cdots}$$

$$= xC(d(a+b\log(cx^n))) - \frac{1}{2}(bdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log(cx^n) - \frac{1}{2}ib^2\cdots\right.$$

$$= xC(d(a+b\log(cx^n))) - \frac{1}{2}(bdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2(cx^n)\right)(cx\cdots$$

$$= xC(d(a+b\log(cx^n))) - \frac{1}{2}\left(bdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib\cdots\right.$$

$$= xC(d(a+b\log(cx^n))) - \frac{1}{2}\left(bdx(cx^n)^{-iabd^2\pi-\frac{1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2\cdots\right.\right.$$

$$= xC(d(a+b\log(cx^n))) - \frac{1}{2}\left(bde^{-\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}x(cx^n)^{-iabd^2\pi-\frac{1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \text{ex}\cdots\right.$$

$$= \left(\frac{1}{4}+\frac{i}{4}\right)e^{-\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}x(cx^n)^{-1/n}\,\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)\cdots$$

Mathematica [A]
time = 4.32, size = 315, normalized size = 1.47

$$x\text{FresnelC}(d(a+b\log(cx^n))) + \frac{\sqrt[4]{-1}\,e^{\frac{1}{2}\left(-\frac{2a}{bn}-\frac{i}{b^2d^2n^2\pi}-ia^2d^2\pi+2iabd^2\pi(n\log(x)-\log(cx^n))-ib^2d^2\pi(-n\log(x)+\log(cx^n))^2\right)}x(cx^n)^{-1/n}\left(ie^{\frac{i}{b^2d^2n^2\pi}}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)+\text{Erfi}\left(\frac{(-1)^{3/4}(i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)\right)\left(\cos\left(\frac{1}{2}d^2\pi(a-bn\log(x)+b\log(cx^n))^2\right)+i\sin\left(\frac{1}{2}d^2\pi(a-bn\log(x)+b\log(cx^n))^2\right)\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x*FresnelC[d*(a + b*Log[c*x^n])] + ((-1)^(1/4)*E^(((-2*a)/(b*n) - I/(b^2*d^
2*n^2*Pi) - I*a^2*d^2*Pi + (2*I)*a*b*d^2*Pi*(n*Log[x] - Log[c*x^n]) - I*b^2
*d^2*Pi*(-(n*Log[x]) + Log[c*x^n])^2)/2)*x*(I*E^(I/(b^2*d^2*n^2*Pi))*Erfi[(
(1/2 + I/2)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])
] + Erfi[((-1)^(3/4)*(I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*S
qrt[2*Pi])])*(Cos[(d^2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2] + I*Sin[(d^
2*Pi*(a - b*n*Log[x] + b*Log[c*x^n])^2)/2]))/(2*Sqrt[2]*(c*x^n)^n^(-1))
```

Maple [F]
time = 0.54, size = 0, normalized size = 0.00

$$\int \text{FresnelC}\left(d(a+b\ln(c\,x^n))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(FresnelC(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos((b*log(c*x^n) + a)*d), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(176) = 352$.
time = 0.39, size = 445, normalized size = 2.08

$$-\frac{1}{2}\pi\sqrt{b^2d^2n^2}\,e^{\left(-\frac{\log(c)}{n}-\frac{a}{bn}-\frac{i}{2\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)-\frac{1}{2}\pi\sqrt{b^2d^2n^2}\,e^{\left(-\frac{\log(c)}{n}-\frac{a}{bn}+\frac{i}{2\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{\left(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n-i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+\frac{1}{2}i\pi\sqrt{b^2d^2n^2}\,e^{\left(-\frac{\log(c)}{n}-\frac{a}{bn}-\frac{i}{2\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{\left(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)-\frac{1}{2}i\pi\sqrt{b^2d^2n^2}\,e^{\left(-\frac{\log(c)}{n}-\frac{a}{bn}+\frac{i}{2\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{\left(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n-i\right)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+x\,\mathrm{C}\left(bd\log\left(cx^n\right)+ad\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] -1/2*pi*sqrt(b^2*d^2*n^2)*e^(-log(c)/n - a/(b*n) - 1/2*I/(pi*b^2*d^2*n^2))*
fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + I
)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/2*pi*sqrt(b^2*d^2*n^2)*e^(-log(c)
/n - a/(b*n) + 1/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) +
 pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)
) + 1/2*I*pi*sqrt(b^2*d^2*n^2)*e^(-log(c)/n - a/(b*n) - 1/2*I/(pi*b^2*d^2*n
^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*
n + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 1/2*I*pi*sqrt(b^2*d^2*n^2)*e^(
-log(c)/n - a/(b*n) + 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*l
og(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d
^2*n^2)) + x*fresnel_cos(b*d*log(c*x^n) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int C\left(d\left(a+b\log\left(cx^{n}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(fresnelc(d*(a + b*log(c*x**n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(fresnel_cos((b*log(c*x^n) + a)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{FresnelC}(d\,(a+b\,\ln\left(c\,x^n\right)))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a + b*log(c*x^n))),x)
[Out] int(FresnelC(d*(a + b*log(c*x^n))), x)
```

3.166 $\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x} \, dx$

Optimal. Leaf size=66

$$\frac{\text{FresnelC}(d(a+b\log(cx^n)))\,(a+b\log(cx^n))}{bn} - \frac{\sin\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)}{bdn\pi}$$

```
[Out] FresnelC(d*(a+b*ln(c*x^n)))*(a+b*ln(c*x^n))/b/n-sin(1/2*d^2*Pi*(a+b*ln(c*x^
n))^2)/b/d/n/Pi
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{6554\}$

$$\frac{(a+b\log(cx^n))\,\text{FresnelC}(d(a+b\log(cx^n)))}{bn} - \frac{\sin\left(\frac{1}{2}\pi d^2(a+b\log(cx^n))^2\right)}{\pi bdn}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (FresnelC[d*(a + b*Log[c*x^n])]*(a + b*Log[c*x^n]))/(b*n) - Sin[(d^2*Pi*(a
+ b*Log[c*x^n])^2)/2]/(b*d*n*Pi)
```

Rule 6554

```
Int[FresnelC[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(FresnelC[a +
 b*x]/b), x] - Simp[Sin[(Pi/2)*(a + b*x)^2]/(b*Pi), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{C(d(a+b\log(cx^n)))}{x}\,dx &= \frac{\text{Subst}\left(\int C(d(a+bx))\,dx, x, \log(cx^n)\right)}{n} \\
&= \frac{\text{Subst}\left(\int C(x)\,dx, x, ad+bd\log(cx^n)\right)}{bdn} \\
&= \frac{C(ad+bd\log(cx^n))\,(a+b\log(cx^n))}{bn} - \frac{\sin\left(\frac{1}{2}\pi(ad+bd\log(cx^n))^2\right)}{bdn\pi}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 165 vs. 2(66) = 132.
time = 0.07, size = 165, normalized size = 2.50

$$\frac{a\text{FresnelC}(d(a+b\log(cx^n)))}{bn} + \frac{\text{FresnelC}(d(a+b\log(cx^n)))\log(cx^n)}{n} - \frac{\cos\left(abd^2\pi\log(cx^n)+\frac{1}{2}b^2d^2\pi\log^2(cx^n)\right)\sin\left(\frac{1}{2}a^2d^2\pi\right)}{bdn\pi} - \frac{\cos\left(\frac{1}{2}a^2d^2\pi\right)\sin\left(abd^2\pi\log(cx^n)+\frac{1}{2}b^2d^2\pi\log^2(cx^n)\right)}{bdn\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (a*FresnelC[d*(a + b*Log[c*x^n])])/(b*n) + (FresnelC[d*(a + b*Log[c*x^n])]*
Log[c*x^n])/n - (Cos[a*b*d^2*Pi*Log[c*x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/2]*S
in[(a^2*d^2*Pi)/2])/(b*d*n*Pi) - (Cos[(a^2*d^2*Pi)/2]*Sin[a*b*d^2*Pi*Log[c*
x^n] + (b^2*d^2*Pi*Log[c*x^n]^2)/2])/(b*d*n*Pi)
```

Maple [A]
time = 1.47, size = 64, normalized size = 0.97

method	result	size
derivativedivides	$\frac{\text{FresnelC}(ad+bd\ln(c\,x^{n}))(ad+bd\ln(c\,x^{n}))-\frac{\sin\left(\frac{\pi(ad+bd\ln(c\,x^{n}))^{2}}{2}\right)}{\pi}}{nbd}$	64
default	$\frac{\text{FresnelC}(ad+bd\ln(c\,x^{n}))(ad+bd\ln(c\,x^{n}))-\frac{\sin\left(\frac{\pi(ad+bd\ln(c\,x^{n}))^{2}}{2}\right)}{\pi}}{nbd}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a+b*ln(c*x^n)))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b/d*(FresnelC(a*d+b*d*ln(c*x^n))*(a*d+b*d*ln(c*x^n))-1/Pi*sin(1/2*Pi*(a
*d+b*d*ln(c*x^n))^2))
```

Maxima [A]
time = 0.26, size = 82, normalized size = 1.24

$$\frac{(b\log(cx^n)+a)d\,\mathrm{C}\left((b\log(cx^n)+a)d\right)-\frac{\sin\left(\frac{1}{2}\pi b^2d^2\log(cx^n)^2+\pi abd^2\log(cx^n)+\frac{1}{2}\pi a^2d^2\right)}{\pi}}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

```
[Out] ((b*log(c*x^n) + a)*d*fresnel_cos((b*log(c*x^n) + a)*d) - sin(1/2*pi*b^2*d^
2*log(c*x^n)^2 + pi*a*b*d^2*log(c*x^n) + 1/2*pi*a^2*d^2)/pi)/(b*d*n)
```

Fricas [A]
time = 0.37, size = 121, normalized size = 1.83

$$\frac{(\pi bdn\log(x)+\pi bd\log(c)+\pi ad)\,\mathrm{C}\left(bd\log(cx^n)+ad\right)-\sin\left(\frac{1}{2}\pi b^2d^2n^2\log(x)^2+\pi b^2d^2n\log(c)\log(x)+\frac{1}{2}\pi b^2d^2\log(c)^2+\pi abd^2n\log(x)+\pi abd^2\log(c)+\frac{1}{2}\pi a^2d^2\right)}{\pi bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

```
[Out] ((pi*b*d*n*log(x) + pi*b*d*log(c) + pi*a*d)*fresnel_cos(b*d*log(c*x^n) + a*
d) - sin(1/2*pi*b^2*d^2*n^2*log(x)^2 + pi*b^2*d^2*n*log(c)*log(x) + 1/2*pi*
```

```
b^2*d^2*log(c)^2 + pi*a*b*d^2*n*log(x) + pi*a*b*d^2*log(c) + 1/2*pi*a^2*d^2
))/(pi*b*d*n)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C(ad + bd\log{(cx^n)})}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*ln(c*x**n)))/x,x)
[Out] Integral(fresnelc(a*d + b*d*log(c*x**n))/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
[Out] integrate(fresnel_cos((b*log(c*x^n) + a)*d)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{FresnelC}(d\,(a + b\,\ln{(c\,x^n)}))}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a + b*log(c*x^n)))/x,x)
[Out] int(FresnelC(d*(a + b*log(c*x^n)))/x, x)
```

3.167 $\int \frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal. Leaf size=217

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}+iabd^2\pi+ib^2\right.}{bd\sqrt{\pi}}\right.}{x}$$

```
[Out] (1/4+1/4*I)*exp(1/2*(2*a*b*n+I/d^2/Pi)/b^2/n^2)*(c*x^n)^(1/n)*erf((1/2+1/2*
I)*(1/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x-(1/4+1/4*I)*ex
p(1/2*(2*a*b*n-I/d^2/Pi)/b^2/n^2)*(c*x^n)^(1/n)*erfi((1/2+1/2*I)*(1/n+I*a*b
*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x-FresnelC(d*(a+b*ln(c*x^n)))
/x
```

Rubi [A]
time = 0.26, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6607, 4714, 2314, 2308, 2266, 2235, 2236}

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(cx^n)^{\frac{1}{n}}e^{\frac{2abn+\frac{i}{\pi d^2}}{2b^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}\,bd}\right)}{x}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(cx^n)^{\frac{1}{n}}e^{\frac{2abn-\frac{i}{\pi d^2}}{2b^2n^2}}\operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{1}{n}\right)}{\sqrt{\pi}\,bd}\right)}{x}-\frac{\text{FresnelC}(d(a+b\log(cx^n)))}{x}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] ((1/4 + I/4)*E^((2*a*b*n + I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*Erf[((1/
2 + I/2)*(n^(-1) - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])]
)/x - ((1/4 + I/4)*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(-1)*Er
fi[((1/2 + I/2)*(n^(-1) + I*a*b*d^2*Pi + I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqr
t[Pi])])/x - FresnelC[d*(a + b*Log[c*x^n])]/x
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2314

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F])), Int[(d + e*x)^(m + 2*a*b*
f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a,
b, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 4714

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6607

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelC[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Dist[b*d*(n/(m + 1)), Int[(e*x)^m*Cos[(Pi/2)*(d*(a + b*Log[c*x^n
]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{C(d(a+b\log(cx^n)))}{x^2}\,dx = -\frac{C(d(a+b\log(cx^n)))}{x} + (bdn)\int \frac{\cos\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)}{x^2}\,dx$$
$$= -\frac{C(d(a+b\log(cx^n)))}{x} + \frac{1}{2}(bdn)\int \frac{e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^2}\,dx + \frac{1}{2}(bdn)\int \frac{e^{\frac{1}{2}id^2\pi(}}{}$$
$$= -\frac{C(d(a+b\log(cx^n)))}{x} + \frac{1}{2}(bdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log(cx^n) - \frac{1}{2}ib^2d\right.}{x^2}$$
$$= -\frac{C(d(a+b\log(cx^n)))}{x} + \frac{1}{2}(bdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2(cx^n)\right)(cx^n}{x^2}$$
$$= -\frac{C(d(a+b\log(cx^n)))}{x} + \frac{1}{2}\left(bdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}i\right.$$
$$= -\frac{C(d(a+b\log(cx^n)))}{x} + \frac{\left(bd(cx^n)^{-iabd^2\pi - \frac{-1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\left(-\frac{1}{2}ia^2d^2\right.\right.}{2x}$$
$$= -\frac{C(d(a+b\log(cx^n)))}{x} + \frac{\left(bde^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{-iabd^2\pi - \frac{-1-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int\exp\right.}{2x}$$
$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{\frac{2abn+\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\text{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x} - \frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}}{}$$

Mathematica [A]
time = 2.58, size = 194, normalized size = 0.89

$$-\frac{\sqrt[4]{-1}\sqrt{2}e^{\frac{2abn-\frac{i}{d^2\pi}}{2b^2n^2}}(cx^n)^{\frac{1}{n}}\left(\text{Erfi}\left(\frac{(-1)^{3/4}(-i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)+ie^{\frac{i}{b^2d^2n^2\pi}}\text{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)\right)+4\text{FresnelC}(d(a+b\log(cx^n)))}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] -1/4*((-1)^(1/4)*Sqrt[2]*E^((2*a*b*n - I/(d^2*Pi))/(2*b^2*n^2))*(c*x^n)^n^(
-1)*(Erfi[((-1)^(3/4)*(-I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n
*Sqrt[2*Pi])] + I*E^(I/(b^2*d^2*n^2*Pi))*Erfi[((1/2 + I/2)*(I + a*b*d^2*n*P
i + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])]) + 4*FresnelC[d*(a + b*Log[
c*x^n])])/x
```

Maple [F]
time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\left(d(a+b\ln(c\,x^n))\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a+b*ln(c*x^n)))/x^2,x)
```

```
[Out] int(FresnelC(d*(a+b*ln(c*x^n)))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos((b*log(c*x^n) + a)*d)/x^2, x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(177) = 354.
time = 0.38, size = 444, normalized size = 2.05

$$\frac{\pi \sqrt{b^2d^2n^2}\, xe^{\left(\frac{\log(c)}{n}+\frac{a}{bn}+\frac{i}{2\,\pi b^2d^2n^2}\right)} \mathrm{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right) + \pi \sqrt{b^2d^2n^2}\, xe^{\left(\frac{\log(c)}{n}+\frac{a}{bn}-\frac{i}{2\,\pi b^2d^2n^2}\right)} \mathrm{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n-i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right) + i\,\pi \sqrt{b^2d^2n^2}\, xe^{\left(\frac{\log(c)}{n}+\frac{a}{bn}+\frac{i}{2\,\pi b^2d^2n^2}\right)} \mathrm{S}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right) - i\,\pi \sqrt{b^2d^2n^2}\, xe^{\left(\frac{\log(c)}{n}+\frac{a}{bn}-\frac{i}{2\,\pi b^2d^2n^2}\right)} \mathrm{S}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n-i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right) - 2\,\mathrm{C}\left(bd\log\left(cx^n\right)+ad\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(pi*sqrt(b^2*d^2*n^2)*x*e^(log(c)/n + a/(b*n) + 1/2*I/(pi*b^2*d^2*n^2))
*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n +
I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + pi*sqrt(b^2*d^2*n^2)*x*e^(log(c)/n
 + a/(b*n) - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + p
i*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2))
+ I*pi*sqrt(b^2*d^2*n^2)*x*e^(log(c)/n + a/(b*n) + 1/2*I/(pi*b^2*d^2*n^2))*
fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n + I
)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - I*pi*sqrt(b^2*d^2*n^2)*x*e^(log(c)/
n + a/(b*n) - 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) +
pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2))
 - 2*fresnel_cos(b*d*log(c*x^n) + a*d))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C(ad + bd\log(cx^n))}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*ln(c*x**n)))/x**2,x)
```

```
[Out] Integral(fresnelc(a*d + b*d*log(c*x**n))/x**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos((b*log(c*x^n) + a)*d)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelC}(d\,(a+b\,\ln\left(c\,x^n\right)))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int(FresnelC(d*(a + b*log(c*x^n)))/x^2, x)
```

3.168 $\int \frac{\textbf{FresnelC}(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal. Leaf size=228

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)e^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}}(cx^n)^{2/n}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2}-\frac{\left(\frac{1}{8}+\frac{i}{8}\right)e^{-\frac{2\left(i-abd^2n\pi\right)}{b^2d^2n^2\pi}}(cx^n)^{2/n}\operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}+\right.}{}\right.}{x^2}$$

```
[Out] (1/8+1/8*I)*exp((2*I+2*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*(c*x^n)^(2/n)*erf((1/2
+1/2*I)*(2/n-I*a*b*d^2*Pi-I*b^2*d^2*Pi*ln(c*x^n))/b/d/Pi^(1/2))/x^2-(1/8+1/
8*I)*(c*x^n)^(2/n)*erfi((1/2+1/2*I)*(2/n+I*a*b*d^2*Pi+I*b^2*d^2*Pi*ln(c*x^n
))/b/d/Pi^(1/2))/exp(2*(I-a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/x^2-1/2*FresnelC(d*
(a+b*ln(c*x^n)))/x^2
```

Rubi [A]
time = 0.25, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6607, 4714, 2314, 2308, 2266, 2235, 2236}

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)(cx^n)^{2/n}e^{\frac{2\pi abd^2n+2i}{\pi b^2d^2n^2}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2-i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}\,bd}\right)}{x^2}-\frac{\left(\frac{1}{8}+\frac{i}{8}\right)(cx^n)^{2/n}e^{-\frac{2\left(-\pi abd^2n+i\right)}{\pi b^2d^2n^2}}\operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2+i\pi b^2d^2\log(cx^n)+\frac{2}{n}\right)}{\sqrt{\pi}\,bd}\right)}{x^2}-\frac{\text{FresnelC}(d(a+b\log(cx^n)))}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] ((1/8 + I/8)*E^((2*I + 2*a*b*d^2*n*Pi)/(b^2*d^2*n^2*Pi))*(c*x^n)^(2/n)*Erf[
((1/2 + I/2)*(2/n - I*a*b*d^2*Pi - I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])
])/x^2 - ((1/8 + I/8)*(c*x^n)^(2/n)*Erfi[((1/2 + I/2)*(2/n + I*a*b*d^2*Pi +
 I*b^2*d^2*Pi*Log[c*x^n]))/(b*d*Sqrt[Pi])])/(E^((2*(I - a*b*d^2*n*Pi))/(b^2
*d^2*n^2*Pi))*x^2) - FresnelC[d*(a + b*Log[c*x^n])]/(2*x^2)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2314

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F])), Int[(d + e*x)^(m + 2*a*b*
f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a,
b, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 4714

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6607

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelC[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Dist[b*d*(n/(m + 1)), Int[(e*x)^m*Cos[(Pi/2)*(d*(a + b*Log[c*x^n
]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{C(d(a+b\log(cx^n)))}{x^3}\,dx &= -\frac{C(d(a+b\log(cx^n)))}{2x^2}+\frac{1}{2}(bdn)\int \frac{\cos\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))^2\right)}{x^3}\,dx \\
&= -\frac{C(d(a+b\log(cx^n)))}{2x^2}+\frac{1}{4}(bdn)\int \frac{e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}}{x^3}\,dx+\frac{1}{4}(bdn)\int \frac{e^{\frac{1}{2}id^2}}{} \\
&= -\frac{C(d(a+b\log(cx^n)))}{2x^2}+\frac{1}{4}(bdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi-iabd^2\pi\log(cx^n)-\frac{1}{2}ib\right.}{x^3} \\
&= -\frac{C(d(a+b\log(cx^n)))}{2x^2}+\frac{1}{4}(bdn)\int \frac{\exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}ib^2d^2\pi\log^2(cx^n)\right)(cx}{x^3} \\
&= -\frac{C(d(a+b\log(cx^n)))}{2x^2}+\frac{1}{4}\left(bdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2d^2\pi-\frac{1}{2}\right. \\
&= -\frac{C(d(a+b\log(cx^n)))}{2x^2}+\frac{\left(bd(cx^n)^{-iabd^2\pi-\frac{-2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int \exp\left(-\frac{1}{2}ia^2d\right.\right.}{4x} \\
&= -\frac{C(d(a+b\log(cx^n)))}{2x^2}+\frac{\left(bde^{-\frac{2\left(i-abd^2n\pi\right)}{b^2d^2n^2\pi}}(cx^n)^{-iabd^2\pi-\frac{-2-iabd^2n\pi}{n}}\right)\text{Subst}\left(\int e\right.}{4x^2} \\
&= \frac{\left(\frac{1}{8}+\frac{i}{8}\right)e^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}}(cx^n)^{2/n}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2}{n}-iabd^2\pi-ib^2d^2\pi\log(cx^n)\right)}{bd\sqrt{\pi}}\right)}{x^2}-\frac{\left(\frac{1}{8}+\frac{i}{8}\right)e^{-\frac{2(}{}}}{}
\end{aligned}
$$

Mathematica [A]
time = 2.58, size = 199, normalized size = 0.87

$$
-\frac{\sqrt[4]{-1}\,e^{\frac{2\left(\frac{an}{b}-\frac{i}{b^2d^2\pi}+n(-n\log(x)+\log(cx^n))\right)}{n^2}}\left(\operatorname{Erfi}\left(\frac{(-1)^{3/4}(-2i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)+ie^{\frac{4i}{b^2d^2n^2\pi}}\operatorname{Erfi}\left(\frac{\sqrt[4]{-1}\,(2i+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)\right)}{4\sqrt{2}}-\frac{\text{FresnelC}(d(a+b\log(cx^n)))}{2x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[FresnelC[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] -1/4*((-1)^(1/4)*E^((2*((a*n)/b - I/(b^2*d^2*Pi) + n*(-(n*Log[x]) + Log[c*x
^n])))/n^2)*(Erfi[((-1)^(3/4)*(-2*I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n
]))/(b*d*n*Sqrt[2*Pi])] + I*E^((4*I)/(b^2*d^2*n^2*Pi))*Erfi[((-1)^(1/4)*(2*
I + a*b*d^2*n*Pi + b^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[2*Pi])]))/Sqrt[2]
- FresnelC[d*(a + b*Log[c*x^n])]/(2*x^2)
```

Maple [F]
time = 0.50, size = 0, normalized size = 0.00

$$
\int \frac{\text{FresnelC}\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)}{x^3}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a+b*ln(c*x^n)))/x^3,x)
```

```
[Out] int(FresnelC(d*(a+b*ln(c*x^n)))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos((b*log(c*x^n) + a)*d)/x^3, x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 460 vs. 2(183) = 366.
time = 0.35, size = 460, normalized size = 2.02

$$\frac{\pi\sqrt{b^2d^2n^2}\,x^2e^{\left(\frac{2\log(c)}{n}+\frac{2a}{bn}+\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+2i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+\pi\sqrt{b^2d^2n^2}\,x^2e^{\left(\frac{2\log(c)}{n}+\frac{2a}{bn}-\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n-2i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+i\pi\sqrt{b^2d^2n^2}\,x^2e^{\left(\frac{2\log(c)}{n}+\frac{2a}{bn}+\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+2i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)-i\pi\sqrt{b^2d^2n^2}\,x^2e^{\left(\frac{2\log(c)}{n}+\frac{2a}{bn}-\frac{2i}{\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n-2i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)-2\,\mathrm{C}\left(bd\log\left(cx^n\right)+ad\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(pi*sqrt(b^2*d^2*n^2)*x^2*e^(2*log(c)/n + 2*a/(b*n) + 2*I/(pi*b^2*d^2*n
^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*
n + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + pi*sqrt(b^2*d^2*n^2)*x^2*e^(
2*log(c)/n + 2*a/(b*n) - 2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2*n^2*
log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^
2*d^2*n^2)) + I*pi*sqrt(b^2*d^2*n^2)*x^2*e^(2*log(c)/n + 2*a/(b*n) + 2*I/(p
i*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) +
pi*a*b*d^2*n + 2*I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - I*pi*sqrt(b^2*d^2
*n^2)*x^2*e^(2*log(c)/n + 2*a/(b*n) - 2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi
*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - 2*I)*sqrt(b^2*d^
2*n^2)/(pi*b^2*d^2*n^2)) - 2*fresnel_cos(b*d*log(c*x^n) + a*d))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{C(ad+bd\log(cx^n))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(d*(a+b*ln(c*x**n)))/x**3,x)
```

```
[Out] Integral(fresnelc(a*d + b*d*log(c*x**n))/x**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
[Out] integrate(fresnel_cos((b*log(c*x^n) + a)*d)/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelC}(d\,(a+b\,\ln\left(c\,x^n\right)))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a + b*log(c*x^n)))/x^3,x)
[Out] int(FresnelC(d*(a + b*log(c*x^n)))/x^3, x)
```

3.169 $\int (ex)^m \mathbf{FresnelC}(d(a+b\log(cx^n)))\,dx$

Optimal. Leaf size=280

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{\frac{i(1+m)\left(1+m+2iabd^2n\pi\right)}{2b^2d^2n^2\pi}}x(ex)^m\left(cx^n\right)^{-\frac{1+m}{n}}\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(1+m+iabd^2n\pi+ib^2d^2n\pi\log(cx^n)\right)}{bdn\sqrt{\pi}}\right)}{1+m}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{-\frac{i(1+m)\left(1+m-2\right.}{2b^2d^2n^2}}}{}$$

```
[Out] (1/4+1/4*I)*exp(1/2*I*(1+m)*(1+m+2*I*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)*x*(e*x)^
m*erf((1/2+1/2*I)*(1+m+I*a*b*d^2*n*Pi+I*b^2*d^2*n*Pi*ln(c*x^n))/b/d/n/Pi^(1
/2))/(1+m)/((c*x^n)^((1+m)/n))-(1/4+1/4*I)*x*(e*x)^m*erfi((1/2+1/2*I)*(1+m-
I*a*b*d^2*n*Pi-I*b^2*d^2*n*Pi*ln(c*x^n))/b/d/n/Pi^(1/2))/exp(1/2*I*(1+m)*(1
+m-2*I*a*b*d^2*n*Pi)/b^2/d^2/n^2/Pi)/(1+m)/((c*x^n)^((1+m)/n))+(e*x)^(1+m)*
FresnelC(d*(a+b*ln(c*x^n)))/e/(1+m)
```

Rubi [A]
time = 0.37, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$,
Rules used = {6607, 4714, 2314, 2308, 2266, 2235, 2236}

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)x(ex)^m\left(cx^n\right)^{-\frac{m+1}{n}}\exp\left(\frac{i(m+1)\left(2i\pi abd^2n+m+1\right)}{2\pi b^2d^2n^2}\right)\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(i\pi abd^2n+i\pi b^2d^2n\log(cx^n)+m+1\right)}{\sqrt{\pi}\,bdn}\right)}{m+1}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)x(ex)^m\left(cx^n\right)^{-\frac{m+1}{n}}\exp\left(-\frac{i(m+1)\left(-2i\pi abd^2n+m+1\right)}{2\pi b^2d^2n^2}\right)\operatorname{Erfi}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i\pi abd^2n-i\pi b^2d^2n\log(cx^n)+m+1\right)}{\sqrt{\pi}\,bdn}\right)}{m+1}+\frac{(ex)^{m+1}\operatorname{FresnelC}(d(a+b\log(cx^n)))}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((1/4 + I/4)*E^(((I/2)*(1 + m)*(1 + m + (2*I)*a*b*d^2*n*Pi))/(b^2*d^2*n^2*P
i))*x*(e*x)^m*Erf[((1/2 + I/2)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log
[c*x^n]))/(b*d*n*Sqrt[Pi])])/((1 + m)*(c*x^n)^((1 + m)/n)) - ((1/4 + I/4)*x
*(e*x)^m*Erfi[((1/2 + I/2)*(1 + m - I*a*b*d^2*n*Pi - I*b^2*d^2*n*Pi*Log[c*x
^n]))/(b*d*n*Sqrt[Pi])])/(E^(((I/2)*(1 + m)*(1 + m - (2*I)*a*b*d^2*n*Pi))/(
b^2*d^2*n^2*Pi))*(1 + m)*(c*x^n)^((1 + m)/n)) + ((e*x)^(1 + m)*FresnelC[d*(
a + b*Log[c*x^n])])/(e*(1 + m))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2308

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n)), Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]*
x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m
, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 2314

```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F])), Int[(d + e*x)^(m + 2*a*b*
f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a,
b, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Rule 4714

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^2*(d_.)]*((e_.)*(x_))^(m_.),
x_Symbol] :> Dist[1/2, Int[(e*x)^m/E^(I*d*(a + b*Log[c*x^n])^2), x], x] + D
ist[1/2, Int[(e*x)^m*E^(I*d*(a + b*Log[c*x^n])^2), x], x] /; FreeQ[{a, b, c
, d, e, m, n}, x]
```

Rule 6607

```
Int[FresnelC[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e*x)^(m + 1)*(FresnelC[d*(a + b*Log[c*x^n])]/(e*(m +
1))), x] - Dist[b*d*(n/(m + 1)), Int[(e*x)^m*Cos[(Pi/2)*(d*(a + b*Log[c*x^n
]))^2], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int (ex)^m C(d(a+b\log(cx^n)))\,dx &= \frac{(ex)^{1+m}C(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(bdn)\int (ex)^m \cos\left(\frac{1}{2}d^2\pi(a+b\log(cx^n))\right)^2}{1+m} \\
&= \frac{(ex)^{1+m}C(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(bdn)\int e^{-\frac{1}{2}id^2\pi(a+b\log(cx^n))^2}(ex)^m\,dx}{2(1+m)} - \frac{(\ldots}{} \\
&= \frac{(ex)^{1+m}C(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(bdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - iabd^2\pi\log(cx\ldots\right.}{2(1+m\ldots} \\
&= \frac{(ex)^{1+m}C(d(a+b\log(cx^n)))}{e(1+m)} - \frac{(bdn)\int \exp\left(-\frac{1}{2}ia^2d^2\pi - \frac{1}{2}ib^2d^2\pi\log^2(c\ldots\right.}{2(1+m)} \\
&= \frac{(ex)^{1+m}C(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(bdnx^{iabd^2n\pi}(cx^n)^{-iabd^2\pi}\right)\int \exp\left(-\frac{1}{2}ia^2\ldots\right.}{2(1\ldots} \\
&= \frac{(ex)^{1+m}C(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(bdnx^{-m+iabd^2n\pi}(ex)^m(cx^n)^{-iabd^2\pi}\right)\int \mathrm{ex}\ldots}{\ldots} \\
&= \frac{(ex)^{1+m}C(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(bdx(ex)^m(cx^n)^{-iabd^2\pi-\frac{1+m-iabd^2n\pi}{n}}\right)\mathrm{Sub}\ldots}{\ldots} \\
&= \frac{(ex)^{1+m}C(d(a+b\log(cx^n)))}{e(1+m)} - \frac{\left(bd\exp\left(-\frac{i(1+m)(1+m-2iabd^2n\pi)}{2b^2d^2n^2\pi}\right)x(ex)^m\ldots\right.}{\ldots} \\
&= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\exp\left(\frac{i(1+m)(1+m+2iabd^2n\pi)}{2b^2d^2n^2\pi}\right)x(ex)^m(cx^n)^{-\frac{1+m}{n}}\operatorname{erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(1+m+iabd\ldots}{bdn\ldots}\right.}{1+m}
\end{aligned}
$$

Mathematica [A]
time = 3.50, size = 244, normalized size = 0.87

$$
\frac{(ex)^m\left((-1)^{3/4}\sqrt{2}\,e^{-\frac{(1+m)\left(i+im+2abd^2n\pi+2b^2d^2n\pi(-n\log(x)+\log(cx^n))\right)}{2b^2d^2n^2\pi}}x^{-m}\left(\operatorname{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(i+im+abd^2n\pi+b^2d^2n\pi\log(cx^n))}{bdn\sqrt{\pi}}\right)-e^{\frac{i(1+m)^2}{b^2d^2n^2\pi}}\operatorname{Erfi}\left(\frac{(-1)^{3/4}(1+m+iabd^2n\pi+ib^2d^2n\pi\log(cx^n))}{bdn\sqrt{2\pi}}\right)\right)+4x\operatorname{FresnelC}(d(a+b\log(cx^n)))\right)}{4(1+m)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*FresnelC[d*(a + b*Log[c*x^n])],x]
```

```
[Out] ((e*x)^m*(((-1)^(3/4)*Sqrt[2]*(Erf[((1/2 + I/2)*(I + I*m + a*b*d^2*n*Pi + b
^2*d^2*n*Pi*Log[c*x^n]))/(b*d*n*Sqrt[Pi])] - E^((I*(1 + m)^2)/(b^2*d^2*n^2*
Pi))*Erfi[((-1)^(3/4)*(1 + m + I*a*b*d^2*n*Pi + I*b^2*d^2*n*Pi*Log[c*x^n]))
/(b*d*n*Sqrt[2*Pi])]))/(E^(((1 + m)*(I + I*m + 2*a*b*d^2*n*Pi + 2*b^2*d^2*n
*Pi*(-(n*Log[x]) + Log[c*x^n])))/(2*b^2*d^2*n^2*Pi))*x^m) + 4*x*FresnelC[d*
(a + b*Log[c*x^n])]))/(4*(1 + m))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (ex)^m \,\mathrm{FresnelC}\left(d(a+b\ln\left(c\,x^n\right))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*FresnelC(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*FresnelC(d*(a+b*ln(c*x^n))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate((x*e)^m*fresnel_cos((b*log(c*x^n) + a)*d), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 674 vs. 2(312) = 624.
time = 0.39, size = 674, normalized size = 2.41

$$-\frac{\pi\sqrt{b^2d^2n^2}\,e^{\left(m-\frac{m\log(c)}{n}-\frac{am}{bn}-\frac{\log(c)}{n}-\frac{a}{bn}-\frac{i\,m^2}{2\,\pi b^2d^2n^2}-\frac{i\,m}{\pi b^2d^2n^2}-\frac{i}{2\,\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+i\,m+i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+\pi\sqrt{b^2d^2n^2}\,e^{\left(m-\frac{m\log(c)}{n}-\frac{am}{bn}-\frac{\log(c)}{n}-\frac{a}{bn}+\frac{i\,m^2}{2\,\pi b^2d^2n^2}+\frac{i\,m}{\pi b^2d^2n^2}+\frac{i}{2\,\pi b^2d^2n^2}\right)}\,\mathrm{C}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n-i\,m-i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)-i\,\pi\sqrt{b^2d^2n^2}\,e^{\left(m-\frac{m\log(c)}{n}-\frac{am}{bn}-\frac{\log(c)}{n}-\frac{a}{bn}-\frac{i\,m^2}{2\,\pi b^2d^2n^2}-\frac{i\,m}{\pi b^2d^2n^2}-\frac{i}{2\,\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n+i\,m+i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)+i\,\pi\sqrt{b^2d^2n^2}\,e^{\left(m-\frac{m\log(c)}{n}-\frac{am}{bn}-\frac{\log(c)}{n}-\frac{a}{bn}+\frac{i\,m^2}{2\,\pi b^2d^2n^2}+\frac{i\,m}{\pi b^2d^2n^2}+\frac{i}{2\,\pi b^2d^2n^2}\right)}\,\mathrm{S}\left(\frac{(\pi b^2d^2n^2\log(x)+\pi b^2d^2n\log(c)+\pi abd^2n-i\,m-i)\sqrt{b^2d^2n^2}}{\pi b^2d^2n^2}\right)-2\,xe^{(m\log(x)+m)}\,\mathrm{C}\left(bd\log\left(cx^n\right)+ad\right)}{2\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] -1/2*(pi*sqrt(b^2*d^2*n^2)*e^(m - m*log(c)/n - a*m/(b*n) - log(c)/n - a/(b*
n) - 1/2*I*m^2/(pi*b^2*d^2*n^2) - I*m/(pi*b^2*d^2*n^2) - 1/2*I/(pi*b^2*d^2*
n^2))*fresnel_cos((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2
*n + I*m + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + pi*sqrt(b^2*d^2*n^2)*e^
(m - m*log(c)/n - a*m/(b*n) - log(c)/n - a/(b*n) + 1/2*I*m^2/(pi*b^2*d^2*n^
2) + I*m/(pi*b^2*d^2*n^2) + 1/2*I/(pi*b^2*d^2*n^2))*fresnel_cos((pi*b^2*d^2
*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I*m - I)*sqrt(b^2*d^2*n^
2)/(pi*b^2*d^2*n^2)) - I*pi*sqrt(b^2*d^2*n^2)*e^(m - m*log(c)/n - a*m/(b*n)
 - log(c)/n - a/(b*n) - 1/2*I*m^2/(pi*b^2*d^2*n^2) - I*m/(pi*b^2*d^2*n^2) -
 1/2*I/(pi*b^2*d^2*n^2))*fresnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*
log(c) + pi*a*b*d^2*n + I*m + I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) + I*pi
*sqrt(b^2*d^2*n^2)*e^(m - m*log(c)/n - a*m/(b*n) - log(c)/n - a/(b*n) + 1/2
*I*m^2/(pi*b^2*d^2*n^2) + I*m/(pi*b^2*d^2*n^2) + 1/2*I/(pi*b^2*d^2*n^2))*fr
esnel_sin((pi*b^2*d^2*n^2*log(x) + pi*b^2*d^2*n*log(c) + pi*a*b*d^2*n - I*m
 - I)*sqrt(b^2*d^2*n^2)/(pi*b^2*d^2*n^2)) - 2*x*e^(m*log(x) + m)*fresnel_co
s(b*d*log(c*x^n) + a*d))/(m + 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m C(ad + bd \log (cx^n)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*fresnelc(d*(a+b*ln(c*x**n))),x)
[Out] Integral((e*x)**m*fresnelc(a*d + b*d*log(c*x**n)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*fresnel_cos(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate((e*x)^m*fresnel_cos((b*log(c*x^n) + a)*d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{FresnelC}(d\,(a + b\,\ln(c\,x^n)))\,(e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(d*(a + b*log(c*x^n)))*(e*x)^m,x)
[Out] int(FresnelC(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

3.170 $\int e^{c+\frac{1}{2}ib^2\pi x^2}\text{FresnelC}(bx)\,dx$

Optimal. Leaf size=64

$$-\frac{ie^c\text{Erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)^2}{8b}+\frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

```
[Out] 1/8*I*exp(c)*erf((1/2-1/2*I)*b*x*Pi^(1/2))^2/b+1/4*b*exp(c)*x^2*hypergeom([
1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)
```

Rubi [A]
time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6572, 6511, 6510, 30}

$$\frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{ie^c\text{Erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}\,bx\right)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c + (I/2)*b^2*Pi*x^2)*FresnelC[b*x],x]
```

```
[Out] ((-1/8*I)*E^c*Erfi[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/b + (b*E^c*x^2*Hypergeometr
icPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6510

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[E^c*
(Sqrt[Pi]/(2*b)), Subst[Int[x^n, x], x, Erfi[b*x]], x] /; FreeQ[{b, c, d, n
}, x] && EqQ[d, b^2]
```

Rule 6511

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2/
Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, b^2*x^2], x] /; FreeQ[{b, c,
d}, x] && EqQ[d, b^2]
```

Rule 6572

```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelC[(b_.)*(x_)], x_Symbol] :> Dist[(1 - I
)/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 + I)*b*x], x], x] + Dist[(1 + I)
/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 - I)*b*x], x], x] /; FreeQ[{b, c,
```

```
d}, x] && EqQ[d^2, (-Pi^2/4)*b^4]
```

Rubi steps

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}C(bx)\,dx = \left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c+\frac{1}{2}ib^2\pi x^2}\text{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)\,dx + \left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c+\frac{1}{2}ib^2\pi x^2}\text{erfi}\left(\left(\frac{1}{2}+\right.\right.$$
$$= \frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right) - \frac{(ie^c)\,\text{Subst}\left(\int x\,dx, x, \text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)\right)}{4b}$$
$$= -\frac{ie^c\text{erfi}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)^2}{8b} + \frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int e^{c+\frac{1}{2}ib^2\pi x^2}\text{FresnelC}(bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelC[b*x],x]
[Out] Integrate[E^(c + (I/2)*b^2*Pi*x^2)*FresnelC[b*x], x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int e^{c+\frac{ib^2\pi x^2}{2}}\,\text{FresnelC}\,(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)
[Out] int(exp(c+1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
[Out] integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
[Out] integral(e^(1/2*I*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{c}\int e^{\frac{i\pi b^{2}x^{2}}{2}}C\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] exp(c)*Integral(exp(I*pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c+1/2*I*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(e^(1/2*I*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{\frac{1\mathrm{i}\,\Pi\,b^2\,x^2}{2}+c}\,\mathrm{FresnelC}\left(b\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c + (Pi*b^2*x^2*1i)/2)*FresnelC(b*x),x)
[Out] int(exp(c + (Pi*b^2*x^2*1i)/2)*FresnelC(b*x), x)
```

3.171 $\int e^{c-\frac{1}{2}ib^2\pi x^2}\text{FresnelC}(bx)\,dx$

Optimal. Leaf size=64

$$-\frac{ie^c\text{Erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)^2}{8b}+\frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)$$

```
[Out] -1/8*I*exp(c)*erf((1/2+1/2*I)*b*x*Pi^(1/2))^2/b+1/4*b*exp(c)*x^2*hypergeom(
[1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)
```

Rubi [A]
time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6572, 6508, 30, 6513}

$$\frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{ie^c\text{Erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\pi}\,bx\right)^2}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c - (I/2)*b^2*Pi*x^2)*FresnelC[b*x],x]
```

```
[Out] ((-1/8*I)*E^c*Erf[(1/2 + I/2)*b*Sqrt[Pi]*x]^2)/b + (b*E^c*x^2*Hypergeometri
cPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6508

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erf[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[E^c*(
Sqrt[Pi]/(2*b)), Subst[Int[x^n, x], x, Erf[b*x]], x] /; FreeQ[{b, c, d, n},
 x] && EqQ[d, -b^2]
```

Rule 6513

```
Int[E^((c_.) + (d_.)*(x_)^2)*Erfi[(b_.)*(x_)], x_Symbol] :> Simp[b*E^c*(x^2
/Sqrt[Pi])*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-b^2)*x^2], x] /; FreeQ[{b,
 c, d}, x] && EqQ[d, -b^2]
```

Rule 6572

```
Int[E^((c_.) + (d_.)*(x_)^2)*FresnelC[(b_.)*(x_)], x_Symbol] :> Dist[(1 - I
)/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 + I)*b*x], x], x] + Dist[(1 + I)
/4, Int[E^(c + d*x^2)*Erf[(Sqrt[Pi]/2)*(1 - I)*b*x], x], x] /; FreeQ[{b, c,
```

```
d}, x] && EqQ[d^2, (-Pi^2/4)*b^4]
```

Rubi steps

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}C(bx)\,dx = \left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)dx+\left(\frac{1}{4}-\frac{i}{4}\right)\int e^{c-\frac{1}{2}ib^2\pi x^2}\operatorname{erfi}\left(\left(\frac{1}{2}\right.\right.$$
$$= \frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{(ie^c)\operatorname{Subst}\left(\int x\,dx,x,\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)\right)}{4b}$$
$$= -\frac{ie^c\operatorname{erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)b\sqrt{\pi}\,x\right)^2}{8b}+\frac{1}{4}be^cx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int e^{c-\frac{1}{2}ib^2\pi x^2}\operatorname{FresnelC}(bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelC[b*x],x]
[Out] Integrate[E^(c - (I/2)*b^2*Pi*x^2)*FresnelC[b*x], x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int e^{c-\frac{ib^2\pi x^2}{2}}\operatorname{FresnelC}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)
[Out] int(exp(c-1/2*I*b^2*Pi*x^2)*FresnelC(b*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
[Out] integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
[Out] integral(e^(-1/2*I*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$e^{c}\int e^{-\frac{i\pi b^{2}x^{2}}{2}}C(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] exp(c)*Integral(exp(-I*pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c-1/2*I*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(e^(-1/2*I*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{c-\frac{\Pi\,b^2\,x^2\,1{}\mathrm{i}}{2}}\,\mathrm{FresnelC}(b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c - (Pi*b^2*x^2*1i)/2)*FresnelC(b*x),x)
[Out] int(exp(c - (Pi*b^2*x^2*1i)/2)*FresnelC(b*x), x)
```

3.172 $\int \textbf{FresnelC}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=101

$$\frac{\cos(c)\text{FresnelC}(bx)S(bx)}{2b}+\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+$$

```
[Out] 1/2*cos(c)*FresnelC(b*x)*FresnelS(b*x)/b+1/8*I*b*x^2*cos(c)*hypergeom([1, 1
],[3/2, 2],-1/2*I*b^2*Pi*x^2)-1/8*I*b*x^2*cos(c)*hypergeom([1, 1],[3/2, 2],
1/2*I*b^2*Pi*x^2)+1/2*FresnelC(b*x)^2*sin(c)/b
```

Rubi [A]

time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {6584, 6576, 30, 6582}

$$\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\cos(c)\text{FresnelC}(bx)S(bx)}{2b}+\frac{\sin(c)\text{FresnelC}(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]*Sin[c + (b^2*Pi*x^2)/2],x]
```

```
[Out] (Cos[c]*FresnelC[b*x]*FresnelS[b*x])/(2*b) + (I/8)*b*x^2*Cos[c]*Hypergeomet
ricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] - (I/8)*b*x^2*Cos[c]*Hypergeo
metricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2] + (FresnelC[b*x]^2*Sin[c])/(2
*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6584

```
Int[FresnelC[(b_.)*(x_)]*Sin[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[Sin[c]
, Int[Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[Cos[c], Int[Sin[d*x^2]*Fresne
lC[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\int C(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx=\cos(c)\int C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx+\sin(c)\int\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx$$
$$=\frac{\cos(c)C(bx)S(bx)}{2b}+\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\cos(c)$$
$$=\frac{\cos(c)C(bx)S(bx)}{2b}+\frac{1}{8}ibx^2\cos(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\cos(c)$$

Mathematica [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int\text{FresnelC}(bx)\sin\left(c+\frac{1}{2}b^2\pi x^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelC[b*x]*Sin[c + (b^2*Pi*x^2)/2],x]
[Out] Integrate[FresnelC[b*x]*Sin[c + (b^2*Pi*x^2)/2], x]
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int\text{FresnelC}\left(bx\right)\sin\left(c+\frac{b^2\pi\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(c+1/2*b^2*Pi*x^2),x)
[Out] int(FresnelC(b*x)*sin(c+1/2*b^2*Pi*x^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(\frac{\pi b^2x^2}{2}+c\right)C(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(c+1/2*b**2*pi*x**2),x)
[Out] Integral(sin(pi*b**2*x**2/2 + c)*fresnelc(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(c+1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(\frac{\Pi\,b^2\,x^2}{2}+c\right)\,\mathrm{FresnelC}(b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + (Pi*b^2*x^2)/2)*FresnelC(b*x),x)
[Out] int(sin(c + (Pi*b^2*x^2)/2)*FresnelC(b*x), x)
```

3.173 $\int \cos \left(c+\frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=101

$$\frac{\cos(c)\text{FresnelC}(bx)^2}{2b}-\frac{\text{FresnelC}(bx)S(bx)\sin(c)}{2b}-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin(c)+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},\right.$$

```
[Out] 1/2*cos(c)*FresnelC(b*x)^2/b-1/2*FresnelC(b*x)*FresnelS(b*x)*sin(c)/b-1/8*I
*b*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)*sin(c)+1/8*I*b*x^2*hype
rgeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)*sin(c)
```

Rubi [A]
time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {6578, 6576, 30, 6582}

$$-\frac{1}{8}ibx^2\sin(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)+\frac{1}{8}ibx^2\sin(c)\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)-\frac{\sin(c)\text{FresnelC}(bx)S(bx)}{2b}+\frac{\cos(c)\text{FresnelC}(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + (b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (Cos[c]*FresnelC[b*x]^2)/(2*b) - (FresnelC[b*x]*FresnelS[b*x]*Sin[c])/(2*b)
 - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2]*Sin
[c] + (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2]*Sin
[c]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6578

```
Int[Cos[(c_) + (d_.)*(x_)^2]*FresnelC[(b_.)*(x_)], x_Symbol] :> Dist[Cos[c]
, Int[Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[Sin[c], Int[Sin[d*x^2]*Fresne
lC[b*x], x], x] /; FreeQ[{b, c, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
```

```
1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx = \cos(c)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx - \sin(c)\int C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$
$$= -\frac{C(bx)S(bx)\sin(c)}{2b} - \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin(c) + \frac{1}{8}ibx^2\,{}_2F$$
$$= \frac{\cos(c)C(bx)^2}{2b} - \frac{C(bx)S(bx)\sin(c)}{2b} - \frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)\sin$$

Mathematica [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \cos\left(c+\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelC[b*x],x]
[Out] Integrate[Cos[c + (b^2*Pi*x^2)/2]*FresnelC[b*x], x]
```

Maple [F]
time = 0.39, size = 0, normalized size = 0.00

$$\int \cos\left(c+\frac{b^2\pi\,x^2}{2}\right)\text{FresnelC}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c+1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
[Out] int(cos(c+1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{\pi b^2x^2}{2} + c\right)C(bx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] Integral(cos(pi*b**2*x**2/2 + c)*fresnelc(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(c+1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2 + c)*fresnel_cos(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(\frac{\Pi\,b^2\,x^2}{2} + c\right)\,\mathrm{FresnelC}(b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + (Pi*b^2*x^2)/2)*FresnelC(b*x),x)
[Out] int(cos(c + (Pi*b^2*x^2)/2)*FresnelC(b*x), x)
```

3.174 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \textbf{FresnelC}(bx)^2\, dx$

Optimal. Leaf size=13

$$\frac{\text{FresnelC}(bx)^3}{3b}$$

```
[Out] 1/3*FresnelC(b*x)^3/b
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6576, 30}

$$\frac{\text{FresnelC}(bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^2,x]
```

```
[Out] FresnelC[b*x]^3/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)^2\, dx &= \frac{\text{Subst}\left(\int x^2\, dx, x, C(bx)\right)}{b}\\ &= \frac{C(bx)^3}{3b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{\text{FresnelC}(bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^2,x]
```

```
[Out] FresnelC[b*x]^3/(3*b)
```

Maple [A]
time = 0.17, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\mathrm{FresnelC}(bx)^3}{3b}$	12
default	$\frac{\mathrm{FresnelC}(bx)^3}{3b}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*FresnelC(b*x)^3/b
```

Maxima [A]
time = 0.26, size = 11, normalized size = 0.85

$$\frac{\mathrm{C}\left(bx\right)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*fresnel_cos(b*x)^3/b
```

Fricas [A]
time = 0.35, size = 11, normalized size = 0.85

$$\frac{\mathrm{C}\left(bx\right)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*fresnel_cos(b*x)^3/b
```

Sympy [A]
time = 0.20, size = 10, normalized size = 0.77

$$\begin{cases} \frac{C^{3}(bx)}{3b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)**2,x)
[Out] Piecewise((fresnelc(b*x)**3/(3*b), Ne(b, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)^2,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \mathrm{FresnelC}(b\,x)^2\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^2*cos((Pi*b^2*x^2)/2),x)
[Out] int(FresnelC(b*x)^2*cos((Pi*b^2*x^2)/2), x)
```

3.175 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=13

$$\frac{\text{FresnelC}(bx)^2}{2b}$$

```
[Out] 1/2*FresnelC(b*x)^2/b
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6576, 30}

$$\frac{\text{FresnelC}(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] FresnelC[b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\int \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\,dx &= \frac{\text{Subst}(\int x\,dx, x, C(bx))}{b}\\ &= \frac{C(bx)^2}{2b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{\text{FresnelC}(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] FresnelC[b*x]^2/(2*b)
```

Maple [A]
time = 0.19, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\mathrm{FresnelC}(bx)^2}{2b}$	12
default	$\frac{\mathrm{FresnelC}(bx)^2}{2b}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*FresnelC(b*x)^2/b
```

Maxima [A]
time = 0.26, size = 11, normalized size = 0.85

$$\frac{\mathrm{C}\,(bx)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] 1/2*fresnel_cos(b*x)^2/b
```

Fricas [A]
time = 0.37, size = 11, normalized size = 0.85

$$\frac{\mathrm{C}\,(bx)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] 1/2*fresnel_cos(b*x)^2/b
```

Sympy [A]
time = 0.10, size = 10, normalized size = 0.77

$$\begin{cases} \frac{C^2(bx)}{2b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Piecewise((fresnelc(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \mathrm{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
```

```
[Out] int(FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.176 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\mathbf{FresnelC}(bx)}\,dx$

Optimal. Leaf size=9

$$\frac{\log(\text{FresnelC}(bx))}{b}$$

```
[Out] ln(FresnelC(b*x))/b
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6576, 29}

$$\frac{\log(\text{FresnelC}(bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x],x]
```

```
[Out] Log[FresnelC[b*x]]/b
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{C(bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, C(bx)\right)}{b}\\ &= \frac{\log(C(bx))}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 1.00

$$\frac{\log(\text{FresnelC}(bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x],x]
```

```
[Out] Log[FresnelC[b*x]]/b
```

Maple [A]
time = 0.23, size = 10, normalized size = 1.11

method	result	size
derivativedivides	$\frac{\ln(\mathrm{FresnelC}(bx))}{b}$	10
default	$\frac{\ln(\mathrm{FresnelC}(bx))}{b}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)/FresnelC(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(FresnelC(b*x))/b
```

Maxima [A]
time = 0.26, size = 9, normalized size = 1.00

$$\frac{\log\left(\mathrm{C}\left(bx\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] log(fresnel_cos(b*x))/b
```

Fricas [A]
time = 0.33, size = 9, normalized size = 1.00

$$\frac{\log\left(\mathrm{C}\left(bx\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] log(fresnel_cos(b*x))/b
```

Sympy [A]
time = 0.10, size = 10, normalized size = 1.11

$$\begin{cases} \frac{\log\left(C\left(bx\right)\right)}{b} & \text{for}\ b \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)/fresnelc(b*x),x)
[Out] Piecewise((log(fresnelc(b*x))/b, Ne(b, 0)), (zoo*x, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnel_cos(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{\mathrm{FresnelC}\left(b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x),x)
[Out] int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x), x)
```

3.177 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\text{FresnelC}(bx)^2}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{b\text{FresnelC}(bx)}$$

```
[Out] -1/b/FresnelC(b*x)
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6576, 30}

$$-\frac{1}{b\text{FresnelC}(bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^2,x]
```

```
[Out] -(1/(b*FresnelC[b*x]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{C(bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, C(bx)\right)}{b}\\ &= -\frac{1}{bC(bx)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$-\frac{1}{b\text{FresnelC}(bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^2,x]
```

```
[Out] -(1/(b*FresnelC[b*x]))
```

Maple [A]
time = 0.16, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$-\frac{1}{b\,\mathrm{FresnelC}(bx)}$	12
default	$-\frac{1}{b\,\mathrm{FresnelC}(bx)}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)/FresnelC(b*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/FresnelC(b*x)
```

Maxima [A]
time = 0.26, size = 11, normalized size = 1.00

$$-\frac{1}{b\,\mathrm{C}\,(bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x)^2,x, algorithm="maxima")
```

```
[Out] -1/(b*fresnel_cos(b*x))
```

Fricas [A]
time = 0.32, size = 11, normalized size = 1.00

$$-\frac{1}{b\,\mathrm{C}\,(bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x)^2,x, algorithm="fricas")
```

```
[Out] -1/(b*fresnel_cos(b*x))
```

Sympy [A]
time = 0.26, size = 12, normalized size = 1.09

$$\begin{cases} -\frac{1}{bC(bx)} & \text{for}\ b \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)/fresnelc(b*x)**2,x)
```

```
[Out] Piecewise((-1/(b*fresnelc(b*x)), Ne(b, 0)), (zoo*x, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnel_cos(b*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{{\mathrm{FresnelC}\left(b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x)^2,x)
```

```
[Out] int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x)^2, x)
```

3.178 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{\mathbf{FresnelC}(bx)^3}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2b\text{FresnelC}(bx)^2}$$

```
[Out] -1/2/b/FresnelC(b*x)^2
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6576, 30}

$$-\frac{1}{2b\text{FresnelC}(bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^3,x]
```

```
[Out] -1/2*1/(b*FresnelC[b*x]^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{C(bx)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3}\,dx, x, C(bx)\right)}{b} \\ &= -\frac{1}{2bC(bx)^2} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$-\frac{1}{2b\text{FresnelC}(bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]/FresnelC[b*x]^3,x]
```

```
[Out] -1/2*1/(b*FresnelC[b*x]^2)
```

Maple [A]
time = 0.19, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$-\frac{1}{2b\,\mathrm{FresnelC}(bx)^{2}}$	12
default	$-\frac{1}{2b\,\mathrm{FresnelC}(bx)^{2}}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)/FresnelC(b*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/FresnelC(b*x)^2
```

Maxima [A]
time = 0.25, size = 11, normalized size = 0.85

$$-\frac{1}{2\,b\,\mathrm{C}\,(bx)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/(b*fresnel_cos(b*x)^2)
```

Fricas [A]
time = 0.32, size = 11, normalized size = 0.85

$$-\frac{1}{2\,b\,\mathrm{C}\,(bx)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2/(b*fresnel_cos(b*x)^2)
```

Sympy [A]
time = 0.44, size = 15, normalized size = 1.15

$$\begin{cases} -\frac{1}{2bC^{2}(bx)} & \text{for } b \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)/fresnelc(b*x)**3,x)
```

```
[Out] Piecewise((-1/(2*b*fresnelc(b*x)**2), Ne(b, 0)), (zoo*x, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)/fresnel_cos(b*x)^3,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)/fresnel_cos(b*x)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{\mathrm{FresnelC}\left(b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x)^3,x)
```

```
[Out] int(cos((Pi*b^2*x^2)/2)/FresnelC(b*x)^3, x)
```

3.179 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right) \textbf{FresnelC}(bx)^n\,dx$

Optimal. Leaf size=17

$$\frac{\text{FresnelC}(bx)^{1+n}}{b(1+n)}$$

```
[Out] FresnelC(b*x)^(1+n)/b/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6576, 30}

$$\frac{\text{FresnelC}(bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^n,x]
```

```
[Out] FresnelC[b*x]^(1 + n)/(b*(1 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\int \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)^n\,dx &= \frac{\text{Subst}\left(\int x^n\,dx, x, C(bx)\right)}{b}\\ &= \frac{C(bx)^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$\frac{\text{FresnelC}(bx)^{1+n}}{b(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]^n,x]
```

```
[Out] FresnelC[b*x]^(1 + n)/(b*(1 + n))
```

Maple [A]
time = 0.19, size = 18, normalized size = 1.06

method	result	size
derivativedivides	$\frac{\mathrm{FresnelC}(bx)^{1+n}}{b(1+n)}$	18
default	$\frac{\mathrm{FresnelC}(bx)^{1+n}}{b(1+n)}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)^n,x,method=_RETURNVERBOSE)
```

```
[Out] FresnelC(b*x)^(1+n)/b/(1+n)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(n>0)', see `assume?` for more detai
ls)Is n
```

Fricas [A]
time = 0.34, size = 18, normalized size = 1.06

$$\frac{\mathrm{C}\left(bx\right)^{n}\mathrm{C}\left(bx\right)}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)^n,x, algorithm="fricas")
```

```
[Out] fresnel_cos(b*x)^n*fresnel_cos(b*x)/(b*n + b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(12) = 24$.

time = 0.61, size = 34, normalized size = 2.00

$$\begin{cases} \tilde{\infty}x & \text{for } b = 0 \wedge n = -1 \\ 0^n x & \text{for } b = 0 \\ \frac{\log(C(bx))}{b} & \text{for } n = -1 \\ \frac{C(bx)C^n(bx)}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)**n,x)
[Out] Piecewise((zoo*x, Eq(b, 0) & Eq(n, -1)), (0**n*x, Eq(b, 0)), (log(fresnelc(
b*x))/b, Eq(n, -1)), (fresnelc(b*x)*fresnelc(b*x)**n/(b*n + b), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)^n,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)^n*cos(1/2*pi*b^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \mathrm{FresnelC}(b\,x)^n\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^n*cos((Pi*b^2*x^2)/2),x)
[Out] int(FresnelC(b*x)^n*cos((Pi*b^2*x^2)/2), x)
```

3.180 $\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx)\, dx$

Optimal. Leaf size=231

$$\frac{105x^2}{4b^7\pi^4}-\frac{7x^6}{12b^3\pi^2}-\frac{55x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4}+\frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^8\pi^4}+\frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{Fres}}{b^4\pi^2}$$

```
[Out] 105/4*x^2/b^7/Pi^4-7/12*x^6/b^3/Pi^2-55/4*x^2*cos(b^2*Pi*x^2)/b^7/Pi^4+1/4*
x^6*cos(b^2*Pi*x^2)/b^3/Pi^2-105*x*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^8/Pi
^4+7*x^5*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^4/Pi^2+105/2*FresnelC(b*x)^2/b
^9/Pi^4-35*x^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^7*FresnelC(b*x)
*sin(1/2*b^2*Pi*x^2)/b^2/Pi+40*sin(b^2*Pi*x^2)/b^9/Pi^5-5/2*x^4*sin(b^2*Pi*
x^2)/b^5/Pi^3
```

Rubi [A]
time = 0.28, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6590, 6598, 6576, 30, 3461, 2714, 3460, 3377, 2717, 3390}

$$\frac{105\text{FresnelC}(bx)^2}{2\pi^4b^9}+\frac{105x^2}{4\pi^4b^7}-\frac{7x^6}{12\pi^2b^3}+\frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{40\sin\left(\pi b^2x^2\right)}{\pi^5b^9}-\frac{105x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}-\frac{55x^2\cos\left(\pi b^2x^2\right)}{4\pi^4b^7}-\frac{35x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}-\frac{5x^4\sin\left(\pi b^2x^2\right)}{2\pi^3b^5}+\frac{7x^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x^6\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])/(
4*b^7*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (105*x*Cos[(b^2*Pi*x^2)/
2]*FresnelC[b*x])/(b^8*Pi^4) + (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b
^4*Pi^2) + (105*FresnelC[b*x]^2)/(2*b^9*Pi^4) - (35*x^3*FresnelC[b*x]*Sin[(
b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*P
i) + (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) - (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
```

```
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\int x^8 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\,dx = \frac{x^7 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{7\int x^6 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^7 \sin\left(b^2\pi x^2\right)\,dx}{2b\pi}$$

$$= \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{x^7 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{35\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(}{b^4\pi^2}$$

$$= \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - \frac{35x^3 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^7 C}{}$$

$$= \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - 35x$$

$$= -\frac{7x^6}{12b^3\pi^2} - \frac{41x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^8\pi^4}$$

$$= \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} - \frac{55x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4}$$

$$= \frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} - \frac{55x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)}{b^8\pi^4}$$

Mathematica [A]
time = 0.01, size = 231, normalized size = 1.00

$$\frac{105x^2}{4b^7\pi^4} - \frac{7x^6}{12b^3\pi^2} - \frac{55x^2\cos\left(b^2\pi x^2\right)}{4b^7\pi^4} + \frac{x^6\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{105x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^8\pi^4} + \frac{7x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} + \frac{105\text{FresnelC}(bx)^2}{2b^9\pi^4} - \frac{35x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^7\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} + \frac{40\sin\left(b^2\pi x^2\right)}{b^9\pi^5} - \frac{5x^4\sin\left(b^2\pi x^2\right)}{2b^5\pi^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]

[Out] (105*x^2)/(4*b^7*Pi^4) - (7*x^6)/(12*b^3*Pi^2) - (55*x^2*Cos[b^2*Pi*x^2])/(
4*b^7*Pi^4) + (x^6*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (105*x*Cos[(b^2*Pi*x^2)/
2]*FresnelC[b*x])/(b^8*Pi^4) + (7*x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b
^4*Pi^2) + (105*FresnelC[b*x]^2)/(2*b^9*Pi^4) - (35*x^3*FresnelC[b*x]*Sin[(
b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*P
i) + (40*Sin[b^2*Pi*x^2])/(b^9*Pi^5) - (5*x^4*Sin[b^2*Pi*x^2])/(2*b^5*Pi^3)
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int x^8\cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelC}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
[Out] int(x^8*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
[Out] integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Fricas [A]
time = 0.35, size = 169, normalized size = 0.73

$$-\frac{5\,\pi^3 b^6 x^6 - 240\,\pi b^2 x^2 - 3\,(\pi^3 b^6 x^6 - 55\,\pi b^2 x^2)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 42\,(\pi^3 b^5 x^5 - 15\,\pi b x)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}\left(bx\right) - 315\,\pi\,\mathrm{C}\left(bx\right)^2 + 6\,\left(5\,(\pi^2 b^4 x^4 - 16)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - (\pi^4 b^7 x^7 - 35\,\pi^2 b^3 x^3)\,\mathrm{C}\left(bx\right)\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{6\,\pi^5 b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
[Out] -1/6*(5*pi^3*b^6*x^6 - 240*pi*b^2*x^2 - 3*(pi^3*b^6*x^6 - 55*pi*b^2*x^2)*co
s(1/2*pi*b^2*x^2)^2 - 42*(pi^3*b^5*x^5 - 15*pi*b*x)*cos(1/2*pi*b^2*x^2)*fre
snel_cos(b*x) - 315*pi*fresnel_cos(b*x)^2 + 6*(5*(pi^2*b^4*x^4 - 16)*cos(1/
2*pi*b^2*x^2) - (pi^4*b^7*x^7 - 35*pi^2*b^3*x^3)*fresnel_cos(b*x))*sin(1/2*
pi*b^2*x^2))/(pi^5*b^9)
```

Sympy [A]
time = 16.70, size = 301, normalized size = 1.30

$$\begin{cases} \frac{x^7\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{\pi b^2} - \frac{5x^6\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{6\pi^2b^3} - \frac{x^6\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{3\pi^2b^3} + \frac{7x^5\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{\pi^2b^4} - \frac{5x^4\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{\pi^3b^5} - \frac{35x^3\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{\pi^3b^6} + \frac{40x^2\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{\pi^4b^7} + \frac{25x^2\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^4b^7} - \frac{105x\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{\pi^4b^8} + \frac{80\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{\pi^5b^9} + \frac{105C^2(bx)}{2\pi^4b^9} & \text{for}\ b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] Piecewise((x**7*sin(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) - 5*x**6*sin(pi
*b**2*x**2/2)**2/(6*pi**2*b**3) - x**6*cos(pi*b**2*x**2/2)**2/(3*pi**2*b**3
) + 7*x**5*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi**2*b**4) - 5*x**4*sin(pi*b
**2*x**2/2)*cos(pi*b**2*x**2/2)/(pi**3*b**5) - 35*x**3*sin(pi*b**2*x**2/2)*
fresnelc(b*x)/(pi**3*b**6) + 40*x**2*sin(pi*b**2*x**2/2)**2/(pi**4*b**7) +
25*x**2*cos(pi*b**2*x**2/2)**2/(2*pi**4*b**7) - 105*x*cos(pi*b**2*x**2/2)*f
resnelc(b*x)/(pi**4*b**8) + 80*sin(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/(pi*
*5*b**9) + 105*fresnelc(b*x)**2/(2*pi**4*b**9), Ne(b, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^8*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^8 \operatorname{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^8*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.181 $\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) \mathbf{FresnelC}(bx)\, dx$

Optimal. Leaf size=215

$$\frac{24x}{b^7\pi^4}-\frac{3x^5}{5b^3\pi^2}-\frac{147x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4}+\frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^8\pi^4}+\frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(b}{b^4\pi^2}$$

```
[Out] 24*x/b^7/Pi^4-3/5*x^5/b^3/Pi^2-147/16*x*cos(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^5*co
s(b^2*Pi*x^2)/b^3/Pi^2-48*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^8/Pi^4+6*x^4*
cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^4/Pi^2-24*x^2*FresnelC(b*x)*sin(1/2*b^2
*Pi*x^2)/b^6/Pi^3+x^6*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi-17/8*x^3*sin
(b^2*Pi*x^2)/b^5/Pi^3+531/32*FresnelC(b*x*2^(1/2))/b^8/Pi^4*2^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6590, 6598, 6596, 3439, 3433, 3466, 3473, 30, 3467}

$$\frac{531\text{FresnelC}\left(\sqrt{2}\,bx\right)}{16\sqrt{2}\,\pi^4b^8}+\frac{24x}{\pi^4b^7}-\frac{3x^5}{5\pi^2b^3}+\frac{x^6\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{48\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}-\frac{147x\cos\left(\pi b^2x^2\right)}{16\pi^4b^7}-\frac{24x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}-\frac{17x^3\sin\left(\pi b^2x^2\right)}{8\pi^3b^5}+\frac{6x^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x^5\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (24*x)/(b^7*Pi^4) - (3*x^5)/(5*b^3*Pi^2) - (147*x*Cos[b^2*Pi*x^2])/(16*b^7*
Pi^4) + (x^5*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) - (48*Cos[(b^2*Pi*x^2)/2]*Fresne
lC[b*x])/(b^8*Pi^4) + (6*x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2)
+ (531*FresnelC[Sqrt[2]*b*x])/(16*Sqrt[2]*b^8*Pi^4) - (24*x^2*FresnelC[b*x]
*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/
(b^2*Pi) - (17*x^3*Sin[b^2*Pi*x^2])/(8*b^5*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3439

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3473

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6596

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelC[b*x]/(2*d)), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /;
 FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\int x^7 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\,dx = \frac{x^6 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{6\int x^5 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^6\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}$$

$$= \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{x^6 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{24\int x^3 \cdots}{}$$

$$= \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{6x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - \frac{24x^2 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^6 C(\cdots}{}$$

$$= -\frac{3x^5}{5b^3\pi^2} - \frac{111x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^8\pi^4} + \frac{6x^4 \cdots}{}$$

$$= -\frac{3x^5}{5b^3\pi^2} - \frac{147x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^8\pi^4} + \frac{6x^4 \cdots}{}$$

$$= \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} - \frac{147x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^8\pi^4}$$

$$= \frac{24x}{b^7\pi^4} - \frac{3x^5}{5b^3\pi^2} - \frac{147x\cos\left(b^2\pi x^2\right)}{16b^7\pi^4} + \frac{x^5\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} - \frac{48\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^8\pi^4}$$

Mathematica [A]
time = 0.17, size = 154, normalized size = 0.72

$$\frac{2655\sqrt{2}\,\text{FresnelC}\left(\sqrt{2}\,bx\right) + 160\text{FresnelC}(bx)\left(6(-8+b^4\pi^2x^4)\cos\left(\frac{1}{2}b^2\pi x^2\right) + b^2\pi x^2(-24+b^4\pi^2x^4)\sin\left(\frac{1}{2}b^2\pi x^2\right)\right) + 2bx(5(-147+4b^4\pi^2x^4)\cos\left(b^2\pi x^2\right) - 2(-960+24b^4\pi^2x^4+85b^2\pi x^2\sin\left(b^2\pi x^2\right)))}{160b^8\pi^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (2655*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 160*FresnelC[b*x]*(6*(-8 + b^4*Pi^2*x
^4)*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*(-24 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/
2]) + 2*b*x*(5*(-147 + 4*b^4*Pi^2*x^4)*Cos[b^2*Pi*x^2] - 2*(-960 + 24*b^4*P
i^2*x^4 + 85*b^2*Pi*x^2*Sin[b^2*Pi*x^2])))/(160*b^8*Pi^4)
```

Maple [A]
time = 0.90, size = 317, normalized size = 1.47

method	result

default	$\frac{\text{FresnelC}(bx)\left(\frac{b^6x^6\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{6\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}}{\pi}\right)}{\pi}\right)}{b^7}-\frac{\frac{\frac{3}{5}\pi^2b^5x^5-24bx}{\pi^4}+\frac{3\pi b^3x^3\sin\left(b^2\pi x^2\right)}{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelC(b*x)/b^7*(1/Pi*b^6*x^6*sin(1/2*b^2*Pi*x^2)-6/Pi*(-1/Pi*b^4*x^4*co
s(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2
*Pi*x^2))))-1/b^7*(3/Pi^4*(1/5*Pi^2*b^5*x^5-8*b*x)+3/Pi^4*(1/2*Pi*b^3*x^3*s
in(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(
b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x*2^(1/2)))+1/2/Pi^3*(-1/2*Pi*b^5*x^5*co
s(b^2*Pi*x^2)+5/2*Pi*(1/2/Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2/Pi*(-1/2/Pi*b*x*co
s(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2))))+12/Pi*b*x*cos(b^2*Pi*x
^2)-6/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Fricas [A]
time = 0.37, size = 167, normalized size = 0.78

$$-\frac{136\,\pi^2b^6x^5-5310\,b^2x-20\,(4\,\pi^2b^6x^5-147\,b^2x)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2-960\,(\pi^2b^5x^4-8\,b)\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{C}\,(bx)-2655\,\sqrt{2}\,\sqrt{b^2}\,\mathrm{C}\left(\sqrt{2}\,\sqrt{b^2}\,x\right)+40\,\left(17\,\pi b^4x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)-4\,(\pi^3b^7x^6-24\,\pi b^3x^2)\,\mathrm{C}\,(bx)\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{160\,\pi^4b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] -1/160*(136*pi^2*b^6*x^5 - 5310*b^2*x - 20*(4*pi^2*b^6*x^5 - 147*b^2*x)*cos
(1/2*pi*b^2*x^2)^2 - 960*(pi^2*b^5*x^4 - 8*b)*cos(1/2*pi*b^2*x^2)*fresnel_c
os(b*x) - 2655*sqrt(2)*sqrt(b^2)*fresnel_cos(sqrt(2)*sqrt(b^2)*x) + 40*(17*
pi*b^4*x^3*cos(1/2*pi*b^2*x^2) - 4*(pi^3*b^7*x^6 - 24*pi*b^3*x^2)*fresnel_c
os(b*x))*sin(1/2*pi*b^2*x^2))/(pi^4*b^9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{7}\cos\left(\frac{\pi b^{2}x^{2}}{2}\right)C(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] Integral(x**7*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(x^7*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{7}\,\mathrm{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^{2}\,x^{2}}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^7*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.182 $\int x^6 \cos\left(\frac{1}{2}b^2\pi x^2\right) \textbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=247

$$-\frac{5x^4}{8b^3\pi^2}-\frac{11\cos\left(b^2\pi x^2\right)}{2b^7\pi^4}+\frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2}+\frac{15\text{FresnelC}(bx)S(bx)}{2b^7\pi^3}+\frac{15ix^2\,{}_2F_2$$

```
[Out] -5/8*x^4/b^3/Pi^2-11/2*cos(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^4*cos(b^2*Pi*x^2)/b^3
/Pi^2+5*x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^4/Pi^2+15/2*FresnelC(b*x)*F
resnelS(b*x)/b^7/Pi^3+15/8*I*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^
2)/b^5/Pi^3-15/8*I*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^5/Pi^3
-15*x*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^5*FresnelC(b*x)*sin(1/2*
b^2*Pi*x^2)/b^2/Pi-7/4*x^2*sin(b^2*Pi*x^2)/b^5/Pi^3
```

Rubi [A]
time = 0.18, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6590, 6598, 6582, 3460, 2718, 3461, 3390, 30, 3377}

$$\frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^5}-\frac{15ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^3b^5}+\frac{15\text{FresnelC}(bx)S(bx)}{2\pi^3b^7}-\frac{5x^4}{8\pi^2b^3}+\frac{x^5\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{11\cos\left(\pi b^2x^2\right)}{2\pi^4b^7}-\frac{15x\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}-\frac{7x^2\sin\left(\pi b^2x^2\right)}{4\pi^3b^5}+\frac{5x^3\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x^4\cos\left(\pi b^2x^2\right)}{4\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (-5*x^4)/(8*b^3*Pi^2) - (11*Cos[b^2*Pi*x^2])/(2*b^7*Pi^4) + (x^4*Cos[b^2*Pi
*x^2])/(4*b^3*Pi^2) + (5*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2)
+ (15*FresnelC[b*x]*FresnelS[b*x])/(2*b^7*Pi^3) + (((15*I)/8)*x^2*Hypergeom
etricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/(b^5*Pi^3) - (((15*I)/8)*x
^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^5*Pi^3) - (15*
x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^5*FresnelC[b*x]*Sin[(b
^2*Pi*x^2)/2])/(b^2*Pi) - (7*x^2*Sin[b^2*Pi*x^2])/(4*b^5*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx=\frac{x^5C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{5\int x^4C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi}-\frac{\int x^5\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}$$
$$=\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2}+\frac{x^5C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{15\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(}{b^4\pi^2}$$
$$=\frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2}-\frac{15xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3}+\frac{x^5C(}{}$$
$$=\frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2}+\frac{15C(bx)S(bx)}{2b^7\pi^3}+\frac{15ix^2\,{}_2F_2(1,1}{8}$$
$$=-\frac{5x^4}{8b^3\pi^2}-\frac{17\cos\left(b^2\pi x^2\right)}{4b^7\pi^4}+\frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2}+\frac{15C}{}$$
$$=-\frac{5x^4}{8b^3\pi^2}-\frac{11\cos\left(b^2\pi x^2\right)}{2b^7\pi^4}+\frac{x^4\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{5x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2}+\frac{15C}{}$$

Mathematica [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
[Out] Integrate[x^6*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x], x]
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int x^6\cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelC}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
[Out] int(x^6*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
[Out] integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
[Out] integral(x^6*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] Integral(x**6*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(x^6*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^6\, \mathrm{FresnelC}(b\,x)\, \cos\left(\frac{\Pi\, b^2\, x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^6*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.183 $\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

Optimal. Leaf size=157

$$-\frac{2x^3}{3b^3\pi^2}+\frac{x^3\cos(b^2\pi x^2)}{4b^3\pi^2}+\frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2}+\frac{43S\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,b^6\pi^3}-\frac{8\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3}+\frac{x^4\text{F}}{}$$

[Out] -2/3*x^3/b^3/Pi^2+1/4*x^3*cos(b^2*Pi*x^2)/b^3/Pi^2+4*x^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^4/Pi^2-8*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^6/Pi^3+x^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi-11/8*x*sin(b^2*Pi*x^2)/b^5/Pi^3+43/16*FresnelS(b*x*2^(1/2))/b^6/Pi^3*2^(1/2)

Rubi [A]
time = 0.11, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6590, 6598, 6588, 3432, 3473, 30, 3467, 3466}

$$\frac{43S\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,\pi^3b^6}-\frac{2x^3}{3\pi^2b^3}+\frac{x^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{8\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}-\frac{11x\sin(\pi b^2x^2)}{8\pi^3b^5}+\frac{4x^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x^3\cos(\pi b^2x^2)}{4\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (-2*x^3)/(3*b^3*Pi^2) + (x^3*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (4*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) + (43*FresnelS[Sqrt[2]*b*x])/(8*Sqrt[2]*b^6*Pi^3) - (8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^6*Pi^3) + (x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - (11*x*Sin[b^2*Pi*x^2])/(8*b^5*Pi^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n + 1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3473

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6588

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\int x^5 \cos\left(\frac{1}{2}b^2\pi x^2\right) C(bx)\,dx = \frac{x^4 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{4\int x^3 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^4 \sin\left(b^2\pi x^2\right)\,dx}{2b\pi}$$

$$= \frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{x^4 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{8\int x\,\mathrm{c}}{}$$

$$= \frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - \frac{8C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^6\pi^3} + \frac{x^4 C(bx}{}$$

$$= -\frac{2x^3}{3b^3\pi^2} + \frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{3S\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,b^6\pi^3} + \frac{2\sqrt{2}}{}$$

$$= -\frac{2x^3}{3b^3\pi^2} + \frac{x^3\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{4x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{11S\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,b^6\pi^3} + \frac{2\sqrt{}}{}$$

Mathematica [A]
time = 0.09, size = 120, normalized size = 0.76

$$\frac{-32b^3\pi x^3 + 12b^3\pi x^3\cos\left(b^2\pi x^2\right) + 129\sqrt{2}\,S\left(\sqrt{2}\,bx\right) + 48\mathrm{FresnelC}(bx)\left(4b^2\pi x^2\cos\left(\frac{1}{2}b^2\pi x^2\right) + \left(-8 + b^4\pi^2x^4\right)\sin\left(\frac{1}{2}b^2\pi x^2\right)\right) - 66bx\sin\left(b^2\pi x^2\right)}{48b^6\pi^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (-32*b^3*Pi*x^3 + 12*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 129*Sqrt[2]*FresnelS[Sqrt
[2]*b*x] + 48*FresnelC[b*x]*(4*b^2*Pi*x^2*Cos[(b^2*Pi*x^2)/2] + (-8 + b^4*P
i^2*x^4)*Sin[(b^2*Pi*x^2)/2]) - 66*b*x*Sin[b^2*Pi*x^2])/(48*b^6*Pi^3)
```

Maple [A]
time = 0.95, size = 202, normalized size = 1.29

method	result
default	$\frac{\frac{\mathrm{FresnelC}(bx)\left(\frac{b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi} - \frac{4\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi} + \frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}\right)}{b^5} - \frac{\frac{2b^3x^3}{3\pi^2} + \frac{\frac{bx\sin\left(b^2\pi x^2\right)}{\pi} - \frac{\sqrt{2}\,\mathrm{S}\left(bx\sqrt{2}\right)}{2\pi}}{\pi^2} + \frac{-\frac{\pi b^3x^3\cos}{2}}{}}{}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelC(b*x)/b^5*(1/Pi*b^4*x^4*sin(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*co
s(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi*x^2)))-1/b^5*(2/3/Pi^2*b^3*x^3+2/Pi
^2*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))+1/2/Pi
```

```
^3*(-1/2*Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/
Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))-4*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
[Out] integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Fricas [A]
time = 0.37, size = 132, normalized size = 0.84

$$\frac{24\,\pi b^4x^3\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2-44\,\pi b^4x^3+192\,\pi b^3x^2\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{C}\left(bx\right)+129\,\sqrt{2}\,\sqrt{b^2}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{b^2}\,x\right)-12\left(11\,b^2x\cos\left(\frac{1}{2}\,\pi b^2x^2\right)-4\left(\pi^2b^5x^4-8\,b\right)\mathrm{C}\left(bx\right)\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{48\,\pi^3b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
[Out] 1/48*(24*pi*b^4*x^3*cos(1/2*pi*b^2*x^2)^2 - 44*pi*b^4*x^3 + 192*pi*b^3*x^2*
cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) + 129*sqrt(2)*sqrt(b^2)*fresnel_sin(sq
rt(2)*sqrt(b^2)*x) - 12*(11*b^2*x*cos(1/2*pi*b^2*x^2) - 4*(pi^2*b^5*x^4 - 8
*b)*fresnel_cos(b*x))*sin(1/2*pi*b^2*x^2))/(pi^3*b^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
[Out] Integral(x**5*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(x^5*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \,\mathrm{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^5*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.184 $\int x^4 \cos\left(\frac{1}{2}b^2\pi x^2\right) \textbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=120

$$-\frac{3x^2}{4b^3\pi^2}+\frac{x^2\cos(b^2\pi x^2)}{4b^3\pi^2}+\frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2}-\frac{3\text{FresnelC}(bx)^2}{2b^5\pi^2}+\frac{x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{\sin}{}$$

```
[Out] -3/4*x^2/b^3/Pi^2+1/4*x^2*cos(b^2*Pi*x^2)/b^3/Pi^2+3*x*cos(1/2*b^2*Pi*x^2)*
FresnelC(b*x)/b^4/Pi^2-3/2*FresnelC(b*x)^2/b^5/Pi^2+x^3*FresnelC(b*x)*sin(1
/2*b^2*Pi*x^2)/b^2/Pi-sin(b^2*Pi*x^2)/b^5/Pi^3
```

Rubi [A]

time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6590, 6598, 6576, 30, 3461, 2714, 3460, 3377, 2717}

$$-\frac{3\text{FresnelC}(bx)^2}{2\pi^2b^5}-\frac{3x^2}{4\pi^2b^3}+\frac{x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{\sin(\pi b^2x^2)}{\pi^3b^5}+\frac{3x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x^2\cos(\pi b^2x^2)}{4\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (-3*x^2)/(4*b^3*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (3*x*Cos[(b^2*
Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) - (3*FresnelC[b*x]^2)/(2*b^5*Pi^2) + (
x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - Sin[b^2*Pi*x^2]/(b^5*Pi^3
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx = \frac{x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{3\int x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi} - \frac{\int x^3\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}$$
$$= \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{3\int\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^4\pi^2}$$
$$= \frac{x^2\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} + \frac{x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{3\text{Subst}(\int}{}$$
$$= -\frac{3x^2}{4b^3\pi^2} + \frac{x^2\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2} - \frac{3C(bx)^2}{2b^5\pi^2} + \frac{x^3C(bx)\sin}{b^2}$$

Mathematica [A]
time = 0.01, size = 120, normalized size = 1.00

$$-\frac{3x^2}{4b^3\pi^2} + \frac{x^2\cos\left(b^2\pi x^2\right)}{4b^3\pi^2} + \frac{3x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2} - \frac{3\text{FresnelC}(bx)^2}{2b^5\pi^2} + \frac{x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi} - \frac{\sin\left(b^2\pi x^2\right)}{b^5\pi^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
[Out] (-3*x^2)/(4*b^3*Pi^2) + (x^2*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (3*x*Cos[(b^2*
Pi*x^2)/2]*FresnelC[b*x])/(b^4*Pi^2) - (3*FresnelC[b*x]^2)/(2*b^5*Pi^2) + (
x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi) - Sin[b^2*Pi*x^2]/(b^5*Pi^3
)
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int x^4\cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelC}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
[Out] int(x^4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
[Out] integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Fricas [A]
time = 0.36, size = 105, normalized size = 0.88

$$\frac{\pi b^2x^2\cos\left(\frac{1}{2}\pi b^2x^2\right)^2-2\pi b^2x^2+6\pi bx\cos\left(\frac{1}{2}\pi b^2x^2\right)\mathrm{C}\left(bx\right)-3\pi\,\mathrm{C}\left(bx\right)^2+2\left(\pi^2b^3x^3\,\mathrm{C}\left(bx\right)-2\cos\left(\frac{1}{2}\pi b^2x^2\right)\right)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{2\pi^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] 1/2*(pi*b^2*x^2*cos(1/2*pi*b^2*x^2)^2 - 2*pi*b^2*x^2 + 6*pi*b*x*cos(1/2*pi*
b^2*x^2)*fresnel_cos(b*x) - 3*pi*fresnel_cos(b*x)^2 + 2*(pi^2*b^3*x^3*fresn
el_cos(b*x) - 2*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/(pi^3*b^5)
```

Sympy [A]
time = 1.61, size = 151, normalized size = 1.26

$$\begin{cases}\frac{x^3\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{\pi b^2}-\frac{x^2\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{\pi^2b^3}-\frac{x^2\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^2b^3}+\frac{3x\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{\pi^2b^4}-\frac{2\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{\pi^3b^5}-\frac{3C^2(bx)}{2\pi^2b^5} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Piecewise((x**3*sin(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) - x**2*sin(pi*b
**2*x**2/2)**2/(pi**2*b**3) - x**2*cos(pi*b**2*x**2/2)**2/(2*pi**2*b**3) +
3*x*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi**2*b**4) - 2*sin(pi*b**2*x**2/2)*
cos(pi*b**2*x**2/2)/(pi**3*b**5) - 3*fresnelc(b*x)**2/(2*pi**2*b**5), Ne(b,
 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\mathrm{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^4*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.185 $\int x^3 \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

Optimal. Leaf size=104

$$-\frac{x}{b^3\pi^2}+\frac{x\cos(b^2\pi x^2)}{4b^3\pi^2}+\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^4\pi^2}-\frac{5\text{FresnelC}\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,b^4\pi^2}+\frac{x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}$$

```
[Out] -x/b^3/Pi^2+1/4*x*cos(b^2*Pi*x^2)/b^3/Pi^2+2*cos(1/2*b^2*Pi*x^2)*FresnelC(b
*x)/b^4/Pi^2+x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi-5/8*FresnelC(b*x*
2^(1/2))/b^4/Pi^2*2^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6590, 6596, 3439, 3433, 3466}

$$-\frac{5\text{FresnelC}\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,\pi^2 b^4}-\frac{x}{\pi^2 b^3}+\frac{x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2 b^4}+\frac{x\cos(\pi b^2x^2)}{4\pi^2 b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] -(x/(b^3*Pi^2)) + (x*Cos[b^2*Pi*x^2])/(4*b^3*Pi^2) + (2*Cos[(b^2*Pi*x^2)/2]
*FresnelC[b*x])/(b^4*Pi^2) - (5*FresnelC[Sqrt[2]*b*x])/(4*Sqrt[2]*b^4*Pi^2)
 + (x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3439

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6596

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelC[b*x]/(2*d)), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /;
 FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx &= \frac{x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{2\int xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi}-\frac{\int x^2\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}\\
&= \frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2}+\frac{x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{\int\cos\left(b^2\pi\right.}{4b^3\pi}\\
&= \frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2}-\frac{C\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,b^4\pi^2}+\frac{x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right.}{b^2\pi}\\
&= -\frac{x}{b^3\pi^2}+\frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2}-\frac{C\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,b^4\pi^2}+\frac{x^2C(bx)\,\mathrm{si}}{b}\\
&= -\frac{x}{b^3\pi^2}+\frac{x\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}+\frac{2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^4\pi^2}-\frac{5C\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,b^4\pi^2}+\frac{x^2C(bx)}{}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 83, normalized size = 0.80

$$\frac{2bx\left(-4+\cos\left(b^2\pi x^2\right)\right)-5\sqrt{2}\,\mathrm{FresnelC}\left(\sqrt{2}\,bx\right)+8\mathrm{FresnelC}(bx)\left(2\cos\left(\frac{1}{2}b^2\pi x^2\right)+b^2\pi x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)\right)}{8b^4\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] (2*b*x*(-4 + Cos[b^2*Pi*x^2]) - 5*Sqrt[2]*FresnelC[Sqrt[2]*b*x] + 8*Fresnel
C[b*x]*(2*Cos[(b^2*Pi*x^2)/2] + b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]))/(8*b^4*Pi^
2)
```

Maple [A]
time = 0.89, size = 114, normalized size = 1.10

method	result	size
default	$\frac{\frac{\mathrm{FresnelC}(bx)\left(\frac{b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{b^3}-\frac{\frac{bx}{\pi^2}+\frac{\sqrt{2}\,\mathrm{FresnelC}\left(bx\sqrt{2}\right)}{2\pi^2}+\frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{2\pi}+\frac{\sqrt{2}\,\mathrm{FresnelC}\left(bx\sqrt{2}\right)}{4\pi}}{2\pi}}{b^3}}{b}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelC(b*x)/b^3*(1/Pi*b^2*x^2*sin(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*
x^2))-1/b^3*(b*x/Pi^2+1/2/Pi^2*2^(1/2)*FresnelC(b*x*2^(1/2))+1/2/Pi*(-1/2/P
i*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Fricas [A]
time = 0.36, size = 94, normalized size = 0.90

$$\frac{8\,\pi b^3x^2\,\mathrm{C}\,(bx)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)+4\,b^2x\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2-10\,b^2x+16\,b\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{C}\,(bx)-5\,\sqrt{2}\,\sqrt{b^2}\,\mathrm{C}\left(\sqrt{2}\,\sqrt{b^2}\,x\right)}{8\,\pi^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] 1/8*(8*pi*b^3*x^2*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2) + 4*b^2*x*cos(1/2*pi
*b^2*x^2)^2 - 10*b^2*x + 16*b*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) - 5*sqrt
(2)*sqrt(b^2)*fresnel_cos(sqrt(2)*sqrt(b^2)*x))/(pi^2*b^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Integral(x**3*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
[Out] integrate(x^3*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
[Out] int(x^3*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.186 $\int x^2 \cos\left(\frac{1}{2}b^2\pi x^2\right) \textbf{FresnelC}(bx)\,dx$

Optimal. Leaf size=136

$$\frac{\cos(b^2\pi x^2)}{4b^3\pi^2}-\frac{\text{FresnelC}(bx)S(bx)}{2b^3\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b\pi}+\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b\pi}+\frac{x\text{FresnelC}(bx)\,s}{b^2\pi}$$

```
[Out] 1/4*cos(b^2*Pi*x^2)/b^3/Pi^2-1/2*FresnelC(b*x)*FresnelS(b*x)/b^3/Pi-1/8*I*x
^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b/Pi+1/8*I*x^2*hypergeom([1
, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b/Pi+x*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^
2/Pi
```

Rubi [A]

time = 0.05, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6590, 6582, 3460, 2718}

$$-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi b}+\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi b}-\frac{\text{FresnelC}(bx)S(bx)}{2\pi b^3}+\frac{x\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\cos(\pi b^2x^2)}{4\pi^2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] Cos[b^2*Pi*x^2]/(4*b^3*Pi^2) - (FresnelC[b*x]*FresnelS[b*x])/(2*b^3*Pi) - (
(I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/(b*Pi)
+ ((I/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b*Pi)
+ (x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^2*Pi)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx &= \frac{xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{\int C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b^2\pi}-\frac{\int x\sin\left(b^2\pi x^2\right)\,dx}{2b\pi}\\ &= -\frac{C(bx)S(bx)}{2b^3\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b\pi}+\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8b\pi}\\ &= \frac{\cos\left(b^2\pi x^2\right)}{4b^3\pi^2}-\frac{C(bx)S(bx)}{2b^3\pi}-\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b\pi}+\frac{ix^2\,{}_2F_2\left(1,1;\frac{3}{2}\right.}{8b}\end{aligned}$$

Mathematica [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)\,dx$$

Verification is not applicable to the result.

[In] Integrate[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]

[Out] Integrate[x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x], x]

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int x^2\cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelC}\left(bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)

[Out] int(x^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] integral(x^2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Integral(x**2*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\, \mathrm{FresnelC}(b\, x)\, \cos\left(\frac{\Pi\, b^2\, x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^2*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.187 $\int x \cos\left(\frac{1}{2}b^2\pi x^2\right) \text{FresnelC}(bx)\,dx$

Optimal. Leaf size=48

$$-\frac{S\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,b^2\pi}+\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}$$

```
[Out] FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^2/Pi-1/4*FresnelS(b*x*2^(1/2))/b^2/Pi*2
^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6588, 3432}

$$\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{S\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,\pi b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] -1/2*FresnelS[Sqrt[2]*b*x]/(Sqrt[2]*b^2*Pi) + (FresnelC[b*x]*Sin[(b^2*Pi*x^
2)/2])/(b^2*Pi)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 6588

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx&=\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}-\frac{\int \sin\left(b^2\pi x^2\right)\,dx}{2b\pi}\\&=-\frac{S\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,b^2\pi}+\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^2\pi}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.92

$$-\frac{\sqrt{2}\,S\left(\sqrt{2}\,bx\right)-4\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4b^2\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] -1/4*(Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/
(b^2*Pi)
```

Maple [A]
time = 0.61, size = 45, normalized size = 0.94

method	result	size
default	$\frac{\frac{\text{FresnelC}(bx)\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi b}-\frac{\text{S}\left(bx\sqrt{2}\right)\sqrt{2}}{4b\pi}}{b}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/Pi/b-1/4*FresnelS(b*x*2^(1/2))/b/Pi*2^(1
/2))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Fricas [A]
time = 0.35, size = 47, normalized size = 0.98

$$\frac{4\,b\,\text{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)-\sqrt{2}\,\sqrt{b^2}\,\text{S}\left(\sqrt{2}\,\sqrt{b^2}\,x\right)}{4\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2) - sqrt(2)*sqrt(b^2)*fresnel_s
in(sqrt(2)*sqrt(b^2)*x))/(pi*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Integral(x*cos(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
```

```
[Out] integrate(x*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x*FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.188 $\int \cos\left(\frac{1}{2}b^2\pi x^2\right)$ **FresnelC**$(bx)\,dx$

Optimal. Leaf size=13

$$\frac{\text{FresnelC}(bx)^2}{2b}$$

```
[Out] 1/2*FresnelC(b*x)^2/b
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6576, 30}

$$\frac{\text{FresnelC}(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] FresnelC[b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\begin{aligned}\int \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx &= \frac{\text{Subst}(\int x\,dx, x, C(bx))}{b}\\ &= \frac{C(bx)^2}{2b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{\text{FresnelC}(bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x],x]
```

```
[Out] FresnelC[b*x]^2/(2*b)
```

Maple [A]
time = 0.16, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\mathrm{FresnelC}(bx)^2}{2b}$	12
default	$\frac{\mathrm{FresnelC}(bx)^2}{2b}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*FresnelC(b*x)^2/b
```

Maxima [A]
time = 0.26, size = 11, normalized size = 0.85

$$\frac{\mathrm{C}\,(bx)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="maxima")
```

```
[Out] 1/2*fresnel_cos(b*x)^2/b
```

Fricas [A]
time = 0.36, size = 11, normalized size = 0.85

$$\frac{\mathrm{C}\,(bx)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="fricas")
```

```
[Out] 1/2*fresnel_cos(b*x)^2/b
```

Sympy [A]
time = 0.10, size = 10, normalized size = 0.77

$$\begin{cases} \frac{C^2(bx)}{2b} & \text{for}\ b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x),x)
```

```
[Out] Piecewise((fresnelc(b*x)**2/(2*b), Ne(b, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \mathrm{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*cos((Pi*b^2*x^2)/2),x)
```

```
[Out] int(FresnelC(b*x)*cos((Pi*b^2*x^2)/2), x)
```

3.189 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x},x\right)$$

[Out] Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx$$

Verification is not applicable to the result.

[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x,x]

[Out] Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x]

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x}\,dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x,x]

[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x]

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}\left(bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x,x)
```

```
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x, x)
```

3.190 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathbf{FresnelC}(bx)}{x^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2},x\right)$$

```
[Out] Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2,x]
[Out] Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x]
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2}\,dx = \int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2}\,dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x]
```

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}\left(bx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**2,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^2,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^2, x)
```

3.191 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx$

Optimal. Leaf size=102

$$-\frac{b}{4x}-\frac{b\cos\left(b^2\pi x^2\right)}{4x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{2x^2}-\frac{b^2\pi S\left(\sqrt{2}\,bx\right)}{2\sqrt{2}}-\frac{1}{2}b^2\pi\text{Int}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)$$

```
[Out] -1/4*b/x-1/4*b*cos(b^2*Pi*x^2)/x-1/2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2-
1/4*b^2*Pi*FresnelS(b*x*2^(1/2))*2^(1/2)-1/2*b^2*Pi*Unintegrable(FresnelC(b
*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^3,x]
```

```
[Out] -1/4*b/x - (b*Cos[b^2*Pi*x^2])/(4*x) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/
(2*x^2) - (b^2*Pi*FresnelS[Sqrt[2]*b*x])/(2*Sqrt[2]) - (b^2*Pi*Defer[Int][(
FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/2
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^3}\,dx &= -\frac{b}{4x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{2x^2}+\frac{1}{4}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^2}\,dx-\frac{1}{2}\left(b^2\pi\right)\int\frac{C(bx)\sin\left(\frac{1}{2}b^2\right.}{x}\\ &= -\frac{b}{4x}-\frac{b\cos\left(b^2\pi x^2\right)}{4x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{2x^2}-\frac{1}{2}\left(b^2\pi\right)\int\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx\\ &= -\frac{b}{4x}-\frac{b\cos\left(b^2\pi x^2\right)}{4x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{2x^2}-\frac{b^2\pi S\left(\sqrt{2}\,bx\right)}{2\sqrt{2}}-\frac{1}{2}\left(b^2\pi\right)\int\frac{C(b}{}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^3,x]
```

```
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^3, x]
```

Maple [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)\mathrm{FresnelC}\left(bx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)C\left(bx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**3,x)
```

[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**3, x)

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^3,x, algorithm="giac")

[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^3, x)

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^3,x)

[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^3, x)

3.192 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\textbf{FresnelC}(bx)}{x^4}\,dx$

Optimal. Leaf size=109

$$-\frac{b}{12x^2}-\frac{b\cos\left(b^2\pi x^2\right)}{12x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3x^3}-\frac{1}{6}b^3\pi^2\text{FresnelC}(bx)^2+\frac{b^2\pi\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x}-\frac{1}{6}b$$

```
[Out] -1/12*b/x^2-1/12*b*cos(b^2*Pi*x^2)/x^2-1/3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x
)/x^3-1/6*b^3*Pi^2*FresnelC(b*x)^2-1/6*b^3*Pi*Si(b^2*Pi*x^2)+1/3*b^2*Pi*Fre
snelC(b*x)*sin(1/2*b^2*Pi*x^2)/x
```

Rubi [A]
time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6592, 6600, 6576, 30, 3456, 3461, 3378, 3380}

$$-\frac{1}{6}\pi^2b^3\text{FresnelC}(bx)^2+\frac{\pi b^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{3x}-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{3x^3}-\frac{b\cos\left(\pi b^2x^2\right)}{12x^2}-\frac{1}{6}\pi b^3\text{Si}\left(b^2\pi x^2\right)-\frac{b}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^4,x]
```

```
[Out] -1/12*b/x^2 - (b*Cos[b^2*Pi*x^2])/(12*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[
b*x])/(3*x^3) - (b^3*Pi^2*FresnelC[b*x]^2)/6 + (b^2*Pi*FresnelC[b*x]*Sin[(b
^2*Pi*x^2)/2])/(3*x) - (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6592

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[b*(x^(m + 2)/(2*(m + 1)*(m + 2))), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6600

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && I
LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^4}\,dx &= -\frac{b}{12x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^3}+\frac{1}{6}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^3}\,dx-\frac{1}{3}\left(b^2\pi\right)\int\frac{C(bx)\sin\left(\frac{1}{2}\right.}{x^2}\\ &= -\frac{b}{12x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^3}+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x}+\frac{1}{12}b\text{Subst}\left(\int\frac{\cos\left(b^2\right.}{x^2}\right.\\ &= -\frac{b}{12x^2}-\frac{b\cos\left(b^2\pi x^2\right)}{12x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^3}+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x}-\frac{1}{12}b^3\\ &= -\frac{b}{12x^2}-\frac{b\cos\left(b^2\pi x^2\right)}{12x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{3x^3}-\frac{1}{6}b^3\pi^2C(bx)^2+\frac{b^2\pi C(bx)\sin\left(\right.}{3x}\end{aligned}$$

Mathematica [A]

time = 0.01, size = 109, normalized size = 1.00

$$-\frac{b}{12x^2}-\frac{b\cos\left(b^2\pi x^2\right)}{12x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{3x^3}-\frac{1}{6}b^3\pi^2\text{FresnelC}(bx)^2+\frac{b^2\pi\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x}-\frac{1}{6}b^3\pi\text{Si}\left(b^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^4,x]
```

```
[Out] -1/12*b/x^2 - (b*Cos[b^2*Pi*x^2])/(12*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[
b*x])/(3*x^3) - (b^3*Pi^2*FresnelC[b*x]^2)/6 + (b^2*Pi*FresnelC[b*x]*Sin[(b
^2*Pi*x^2)/2])/(3*x) - (b^3*Pi*SinIntegral[b^2*Pi*x^2])/6
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}\left(bx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^4,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^4, x)
```

Fricas [A]
time = 0.37, size = 93, normalized size = 0.85

$$-\frac{\pi^2b^3x^3\,\mathrm{C}\left(bx\right)^2+\pi b^3x^3\,\mathrm{Si}\left(\pi b^2x^2\right)-2\,\pi b^2x^2\,\mathrm{C}\left(bx\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)+bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)^2+2\,\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{C}\left(bx\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(pi^2*b^3*x^3*fresnel_cos(b*x)^2 + pi*b^3*x^3*sin_integral(pi*b^2*x^2)
 - 2*pi*b^2*x^2*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2) + b*x*cos(1/2*pi*b^2*x
^2)^2 + 2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**4,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^4,x)
```

```
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^4, x)
```

3.193 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\textbf{FresnelC}(bx)}{x^5}\,dx$

Optimal. Leaf size=153

$$-\frac{b}{24x^3}-\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{4x^4}-\frac{7b^4\pi^2\text{FresnelC}\left(\sqrt{2}\,bx\right)}{24\sqrt{2}}+\frac{b^2\pi\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2}$$

```
[Out] -1/24*b/x^3-1/24*b*cos(b^2*Pi*x^2)/x^3-1/4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x
)/x^4+1/8*b^2*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2+7/48*b^3*Pi*sin(b^2*
Pi*x^2)/x-7/48*b^4*Pi^2*FresnelC(b*x*2^(1/2))*2^(1/2)-1/8*b^4*Pi^2*Unintegr
able(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^5,x]
```

```
[Out] -1/24*b/x^3 - (b*Cos[b^2*Pi*x^2])/(24*x^3) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[
b*x])/(4*x^4) - (7*b^4*Pi^2*FresnelC[Sqrt[2]*b*x])/(24*Sqrt[2]) + (b^2*Pi*F
resnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(8*x^2) + (7*b^3*Pi*Sin[b^2*Pi*x^2])/(48*
x) - (b^4*Pi^2*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x])/8
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^5}\,dx &= -\frac{b}{24x^3}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4x^4}+\frac{1}{8}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^4}\,dx-\frac{1}{4}\left(b^2\pi\right)\int\frac{C(bx)\sin\left(\frac{1}{2}b^2\right.}{x^3}\\ &= -\frac{b}{24x^3}-\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4x^4}+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2}-\frac{1}{16}\left(b^3\right.\\ &= -\frac{b}{24x^3}-\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4x^4}+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^2}+\frac{7b^3\pi\,\mathrm{s}}{}\\ &= -\frac{b}{24x^3}-\frac{b\cos\left(b^2\pi x^2\right)}{24x^3}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{4x^4}-\frac{7b^4\pi^2C\left(\sqrt{2}\,bx\right)}{24\sqrt{2}}+\frac{b^2\pi C(bx)\,\mathrm{si}}{8x}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^5,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^5, x]
```

Maple [**A**]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}\left(bx\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^5,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^5, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^5,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^5, x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right)C(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**5,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**5, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^5, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^5,x)
```

```
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^5, x)
```

3.194 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\textbf{FresnelC}(bx)}{x^6}\,dx$

Optimal. Leaf size=148

$$-\frac{b}{40x^4}-\frac{b\cos\left(b^2\pi x^2\right)}{40x^4}-\frac{1}{24}b^5\pi^2\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{5x^5}+\frac{b^2\pi\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi\right.}{15x^3}$$

```
[Out] -1/40*b/x^4-1/24*b^5*Pi^2*Ci(b^2*Pi*x^2)-1/40*b*cos(b^2*Pi*x^2)/x^4-1/5*cos
(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5+1/15*b^2*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi
*x^2)/x^3+1/24*b^3*Pi*sin(b^2*Pi*x^2)/x^2-1/15*b^4*Pi^2*Unintegrable(cos(1/
2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^6,x]
```

```
[Out] -1/40*b/x^4 - (b*Cos[b^2*Pi*x^2])/(40*x^4) - (b^5*Pi^2*CosIntegral[b^2*Pi*x
^2])/24 - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(5*x^5) + (b^2*Pi*FresnelC[b*
x]*Sin[(b^2*Pi*x^2)/2])/(15*x^3) + (b^3*Pi*Sin[b^2*Pi*x^2])/(24*x^2) - (b^4
*Pi^2*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x])/15
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^6}\,dx = -\frac{b}{40x^4}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5x^5}+\frac{1}{10}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^5}\,dx-\frac{1}{5}\left(b^2\pi\right)\int\frac{C(bx)\sin\left(\right.}{x^4}$$

$$= -\frac{b}{40x^4}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5x^5}+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3}+\frac{1}{20}b\text{Subst}\left(\int\frac{\cos\left(b^2\right.}{x^3}\right.$$

$$= -\frac{b}{40x^4}-\frac{b\cos\left(b^2\pi x^2\right)}{40x^4}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5x^5}+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3}-\frac{1}{60}\left(b\right.$$

$$= -\frac{b}{40x^4}-\frac{b\cos\left(b^2\pi x^2\right)}{40x^4}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5x^5}+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x^3}+\frac{b^3\pi\,\mathrm{s}}{}$$

$$= -\frac{b}{40x^4}-\frac{b\cos\left(b^2\pi x^2\right)}{40x^4}-\frac{1}{24}b^5\pi^2\text{Ci}\left(b^2\pi x^2\right)-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{5x^5}+\frac{b^2\pi C(bx)}{1}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^6,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^6, x]
```

Maple [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi\,x^2}{2}\right)\text{FresnelC}\left(bx\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^6,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^6, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^6,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^6, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**6,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**6, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^6,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^6, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^6,x)
```

```
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^6, x)
```

3.195 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\textbf{FresnelC}(bx)}{x^7}\,dx$

Optimal. Leaf size=241

$$-\frac{b}{60x^5}+\frac{b^5\pi^2}{96x}-\frac{b\cos\left(b^2\pi x^2\right)}{60x^5}+\frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{1440x}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{6x^6}+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(b$$

```
[Out] -1/60*b/x^5+1/96*b^5*Pi^2/x-1/60*b*cos(b^2*Pi*x^2)/x^5+67/1440*b^5*Pi^2*cos
(b^2*Pi*x^2)/x-1/6*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6+1/48*b^4*Pi^2*cos(
1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2+1/24*b^2*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*
x^2)/x^4+13/720*b^3*Pi*sin(b^2*Pi*x^2)/x^3+67/1440*b^6*Pi^3*FresnelS(b*x*2^
(1/2))*2^(1/2)+1/48*b^6*Pi^3*Unintegrable(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)
/x,x)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^7,x]
```

```
[Out] -1/60*b/x^5 + (b^5*Pi^2)/(96*x) - (b*Cos[b^2*Pi*x^2])/(60*x^5) + (67*b^5*Pi
^2*Cos[b^2*Pi*x^2])/(1440*x) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(6*x^6)
+ (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(48*x^2) + (7*b^6*Pi^3*Fresn
elS[Sqrt[2]*b*x])/(144*Sqrt[2]) + (Sqrt[2]*b^6*Pi^3*FresnelS[Sqrt[2]*b*x])/
45 + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(24*x^4) + (13*b^3*Pi*Sin[b
^2*Pi*x^2])/(720*x^3) + (b^6*Pi^3*Defer[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2
)/2])/x, x])/48
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^7}\,dx = -\frac{b}{60x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^6} + \frac{1}{12}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^6}\,dx - \frac{1}{6}\left(b^2\pi\right)\int \frac{C(bx)\sin\left(\right.}{x^5}$$

$$= -\frac{b}{60x^5} - \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^6} + \frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{24x^4} - \frac{1}{48}(b$$

$$= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} - \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^6} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx}{48x^2}$$

$$= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} - \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} + \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{1440x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^6} + \frac{b}{}$$

$$= -\frac{b}{60x^5} + \frac{b^5\pi^2}{96x} - \frac{b\cos\left(b^2\pi x^2\right)}{60x^5} + \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{1440x} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{6x^6} + \frac{b}{}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^7,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^7, x]
```

Maple [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}\left(bx\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^7,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^7,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^7,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^7, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^7,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^7, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**7,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**7, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^7,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^7, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^7,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^7, x)
```

3.196 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathbf{FresnelC}(bx)}{x^8}\,dx$

Optimal. Leaf size=224

$$-\frac{b}{84x^6}+\frac{b^5\pi^2}{420x^2}-\frac{b\cos\left(b^2\pi x^2\right)}{84x^6}+\frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{7x^7}+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(b}{105x^3}$$

```
[Out] -1/84*b/x^6+1/420*b^5*Pi^2/x^2-1/84*b*cos(b^2*Pi*x^2)/x^6+1/84*b^5*Pi^2*cos
(b^2*Pi*x^2)/x^2-1/7*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^7+1/105*b^4*Pi^2*c
os(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3+1/210*b^7*Pi^4*FresnelC(b*x)^2+1/70*b^
7*Pi^3*Si(b^2*Pi*x^2)+1/35*b^2*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5-1/1
05*b^6*Pi^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x+1/105*b^3*Pi*sin(b^2*Pi*x^2
)/x^4
```

Rubi [A]
time = 0.26, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6592, 6600, 6576, 30, 3456, 3461, 3378, 3380, 3460}

$$\frac{1}{210}\pi^4 b^7\text{FresnelC}(bx)^2+\frac{\pi^2 b^5}{420x^2}-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{7x^7}+\frac{\pi b^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{35x^5}-\frac{b\cos\left(\pi b^2x^2\right)}{84x^6}+\frac{1}{70}\pi^3b^7\text{Si}\left(b^2\pi x^2\right)-\frac{\pi^3b^6\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{105x}+\frac{\pi^2b^5\cos\left(\pi b^2x^2\right)}{84x^2}+\frac{\pi^2b^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{105x^3}+\frac{\pi b^3\sin\left(\pi b^2x^2\right)}{105x^4}-\frac{b}{84x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^8,x]
```

```
[Out] -1/84*b/x^6 + (b^5*Pi^2)/(420*x^2) - (b*Cos[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi
^2*Cos[b^2*Pi*x^2])/(84*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(7*x^7)
+ (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(105*x^3) + (b^7*Pi^4*Fresne
lC[b*x]^2)/210 + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(35*x^5) - (b^6
*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x) + (b^3*Pi*Sin[b^2*Pi*x^2])
/(105*x^4) + (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6592

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[b*(x^(m + 2)/(2*(m + 1)*(m + 2))), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6600

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && I
LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^8}\,dx &= -\frac{b}{84x^6}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7}+\frac{1}{14}b\int\frac{\cos\left(b^2\pi x^2\right)}{x^7}\,dx-\frac{1}{7}\left(b^2\pi\right)\int\frac{C(bx)\sin\left(\right.}{x^6}\\ &= -\frac{b}{84x^6}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7}+\frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5}+\frac{1}{28}b\mathrm{Subst}\Bigg(\int\frac{\cos\left(b^2\right.}{x^4}\\ &= -\frac{b}{84x^6}+\frac{b^5\pi^2}{420x^2}-\frac{b\cos\left(b^2\pi x^2\right)}{84x^6}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7}+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(}{105x^3}\\ &= -\frac{b}{84x^6}+\frac{b^5\pi^2}{420x^2}-\frac{b\cos\left(b^2\pi x^2\right)}{84x^6}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7}+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(}{105x^3}\\ &= -\frac{b}{84x^6}+\frac{b^5\pi^2}{420x^2}-\frac{b\cos\left(b^2\pi x^2\right)}{84x^6}+\frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7}+\frac{b^4}{}\\ &= -\frac{b}{84x^6}+\frac{b^5\pi^2}{420x^2}-\frac{b\cos\left(b^2\pi x^2\right)}{84x^6}+\frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{7x^7}+\frac{b^4}{}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 224, normalized size = 1.00

$$-\frac{b}{84x^6}+\frac{b^5\pi^2}{420x^2}-\frac{b\cos\left(b^2\pi x^2\right)}{84x^6}+\frac{b^5\pi^2\cos\left(b^2\pi x^2\right)}{84x^2}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)}{7x^7}+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\mathrm{FresnelC}(bx)}{105x^3}+\frac{1}{210}b^7\pi^4\mathrm{FresnelC}(bx)^2+\frac{b^2\pi\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{35x^5}-\frac{b^6\pi^3\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{105x}+\frac{b^3\pi\sin\left(b^2\pi x^2\right)}{105x^4}+\frac{1}{70}b^7\pi^3\mathrm{Si}\left(b^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^8,x]
```

```
[Out] -1/84*b/x^6 + (b^5*Pi^2)/(420*x^2) - (b*Cos[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi
^2*Cos[b^2*Pi*x^2])/(84*x^2) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(7*x^7)
+ (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(105*x^3) + (b^7*Pi^4*Fresne
lC[b*x]^2)/210 + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(35*x^5) - (b^6
*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(105*x) + (b^3*Pi*Sin[b^2*Pi*x^2])
/(105*x^4) + (b^7*Pi^3*SinIntegral[b^2*Pi*x^2])/70
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\mathrm{FresnelC}\left(bx\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^8,x)
```

```
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^8,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^8, x)
```

Fricas [A]
time = 0.39, size = 168, normalized size = 0.75

$$\frac{\pi^4 b^7 x^7\,\mathrm{C}(bx)^2 + 3\,\pi^3 b^7 x^7\,\mathrm{Si}\left(\pi b^2 x^2\right) - 2\,\pi^2 b^5 x^5 + 5\left(\pi^2 b^5 x^5 - bx\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 2\left(\pi^2 b^4 x^4 - 15\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}(bx) + 2\left(2\,\pi b^3 x^3\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - \left(\pi^3 b^6 x^6 - 3\,\pi b^2 x^2\right)\mathrm{C}(bx)\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{210\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^8,x, algorithm="fricas")
```

```
[Out] 1/210*(pi^4*b^7*x^7*fresnel_cos(b*x)^2 + 3*pi^3*b^7*x^7*sin_integral(pi*b^2
*x^2) - 2*pi^2*b^5*x^5 + 5*(pi^2*b^5*x^5 - b*x)*cos(1/2*pi*b^2*x^2)^2 + 2*(
pi^2*b^4*x^4 - 15)*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) + 2*(2*pi*b^3*x^3*c
os(1/2*pi*b^2*x^2) - (pi^3*b^6*x^6 - 3*pi*b^2*x^2)*fresnel_cos(b*x))*sin(1/
2*pi*b^2*x^2))/x^7
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**8,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**8, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^8,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^8, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^8,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^8, x)
```

3.197 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx$

Optimal. Leaf size=268

$$-\frac{b}{112x^7}+\frac{b^5\pi^2}{1152x^3}-\frac{b\cos\left(b^2\pi x^2\right)}{112x^7}+\frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{8x^8}+\frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{Fresne}}{192x^4}$$

```
[Out] -1/112*b/x^7+1/1152*b^5*Pi^2/x^3-1/112*b*cos(b^2*Pi*x^2)/x^7+187/40320*b^5*
Pi^2*cos(b^2*Pi*x^2)/x^3-1/8*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^8+1/192*b^
4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^4+1/48*b^2*Pi*FresnelC(b*x)*sin(
1/2*b^2*Pi*x^2)/x^6-1/384*b^6*Pi^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2+19
/3360*b^3*Pi*sin(b^2*Pi*x^2)/x^5-853/80640*b^7*Pi^3*sin(b^2*Pi*x^2)/x+853/8
0640*b^8*Pi^4*FresnelC(b*x*2^(1/2))*2^(1/2)+1/384*b^8*Pi^4*Unintegrable(cos
(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

Rubi [A]
time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^9,x]
```

```
[Out] -1/112*b/x^7 + (b^5*Pi^2)/(1152*x^3) - (b*Cos[b^2*Pi*x^2])/(112*x^7) + (187
*b^5*Pi^2*Cos[b^2*Pi*x^2])/(40320*x^3) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x]
)/(8*x^8) + (b^4*Pi^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(192*x^4) + (853*b
^8*Pi^4*FresnelC[Sqrt[2]*b*x])/(40320*Sqrt[2]) + (b^2*Pi*FresnelC[b*x]*Sin[
(b^2*Pi*x^2)/2])/(48*x^6) - (b^6*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(3
84*x^2) + (19*b^3*Pi*Sin[b^2*Pi*x^2])/(3360*x^5) - (853*b^7*Pi^3*Sin[b^2*Pi
*x^2])/(80640*x) + (b^8*Pi^4*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])
/x, x])/384
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^9}\,dx = -\frac{b}{112x^7} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8} + \frac{1}{16}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^8}\,dx - \frac{1}{8}\left(b^2\pi\right)\int \frac{C(bx)\sin}{x}$$

$$= -\frac{b}{112x^7} - \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8} + \frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6} - \frac{1}{96}$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} - \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^8} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)}{192x^4}$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} - \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} + \frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx}{8x^8}$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} - \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} + \frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx}{8x^8}$$

$$= -\frac{b}{112x^7} + \frac{b^5\pi^2}{1152x^3} - \frac{b\cos\left(b^2\pi x^2\right)}{112x^7} + \frac{187b^5\pi^2\cos\left(b^2\pi x^2\right)}{40320x^3} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx}{8x^8}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^9,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^9, x]
```

Maple [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}\left(bx\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^9,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^9,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^9,x, algorithm="maxima")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^9, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^9,x, algorithm="fricas")
```

```
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^9, x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**9,x)
```

```
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**9, x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^9,x, algorithm="giac")
```

```
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^9, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^9,x)
```

```
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^9, x)
```

3.198 $\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\textbf{FresnelC}(bx)}{x^{10}}\,dx$

Optimal. Leaf size=263

$$-\frac{b}{144x^8}+\frac{b^5\pi^2}{2520x^4}-\frac{b\cos\left(b^2\pi x^2\right)}{144x^8}+\frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4}+\frac{5b^9\pi^4\text{CosIntegral}(b^2\pi x^2)}{2016}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx}{9x^9}$$

```
[Out] -1/144*b/x^8+1/2520*b^5*Pi^2/x^4+5/2016*b^9*Pi^4*Ci(b^2*Pi*x^2)-1/144*b*cos
(b^2*Pi*x^2)/x^8+67/30240*b^5*Pi^2*cos(b^2*Pi*x^2)/x^4-1/9*cos(1/2*b^2*Pi*x
^2)*FresnelC(b*x)/x^9+1/315*b^4*Pi^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5+
1/63*b^2*Pi*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^7-1/945*b^6*Pi^3*FresnelC(b
*x)*sin(1/2*b^2*Pi*x^2)/x^3+11/3024*b^3*Pi*sin(b^2*Pi*x^2)/x^6-5/2016*b^7*P
i^3*sin(b^2*Pi*x^2)/x^2+1/945*b^8*Pi^4*Unintegrable(cos(1/2*b^2*Pi*x^2)*Fre
snelC(b*x)/x^2,x)
```

Rubi [A]
time = 0.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^10,x]
```

```
[Out] -1/144*b/x^8 + (b^5*Pi^2)/(2520*x^4) - (b*Cos[b^2*Pi*x^2])/(144*x^8) + (67*
b^5*Pi^2*Cos[b^2*Pi*x^2])/(30240*x^4) + (5*b^9*Pi^4*CosIntegral[b^2*Pi*x^2]
)/2016 - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(9*x^9) + (b^4*Pi^2*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(315*x^5) + (b^2*Pi*FresnelC[b*x]*Sin[(b^2*Pi*x^2)
/2])/(63*x^7) - (b^6*Pi^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x^3) + (1
1*b^3*Pi*Sin[b^2*Pi*x^2])/(3024*x^6) - (5*b^7*Pi^3*Sin[b^2*Pi*x^2])/(2016*x
^2) + (b^8*Pi^4*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^2, x])/945
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^{10}}\,dx = -\frac{b}{144x^8} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{9x^9} + \frac{1}{18}b\int \frac{\cos\left(b^2\pi x^2\right)}{x^9}\,dx - \frac{1}{9}\left(b^2\pi\right)\int \frac{C(bx)\sin\left(\right.}{x^8}$$

$$= -\frac{b}{144x^8} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{9x^9} + \frac{b^2\pi C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} + \frac{1}{36}b\text{Subst}\left(\int \frac{\cos\left(b^2\right.}{x^5}\right.$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C}{315x^5}$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{9x^9} + \frac{b^4\pi^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C}{315x^5}$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} + \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{9x^9} +$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} + \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{9x^9} +$$

$$= -\frac{b}{144x^8} + \frac{b^5\pi^2}{2520x^4} - \frac{b\cos\left(b^2\pi x^2\right)}{144x^8} + \frac{67b^5\pi^2\cos\left(b^2\pi x^2\right)}{30240x^4} + \frac{5b^9\pi^4\text{Ci}(b^2\pi x^2)}{2016} - \text{co}$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^10,x]
[Out] Integrate[(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x^10, x]
```

Maple [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{b^2\pi x^2}{2}\right)\text{FresnelC}\left(bx\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^10,x)
[Out] int(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^10,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^10,x, algorithm="maxima")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^10, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^10,x, algorithm="fricas")
[Out] integral(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^10, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b**2*pi*x**2)*fresnelc(b*x)/x**10,x)
[Out] Integral(cos(pi*b**2*x**2/2)*fresnelc(b*x)/x**10, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*b^2*pi*x^2)*fresnel_cos(b*x)/x^10,x, algorithm="giac")
[Out] integrate(cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x)/x^10, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelC}(b\,x)\,\cos\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^10,x)
[Out] int((FresnelC(b*x)*cos((Pi*b^2*x^2)/2))/x^10, x)
```

3.199 $\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=22

$$\text{Int}\left(\text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right), x\right)$$

```
[Out] Unintegrable(FresnelC(b*x)^n*sin(1/2*b^2*Pi*x^2),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Verification is not applicable to the result.

```
[In] Int[FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]
[Out] Defer[Int][FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2], x]
```

Rubi steps

$$\int C(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx = \int C(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Mathematica [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \text{FresnelC}(bx)^n \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2],x]
[Out] Integrate[FresnelC[b*x]^n*Sin[(b^2*Pi*x^2)/2], x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \text{FresnelC}\left(bx\right)^n \sin\left(\frac{b^2\pi\, x^2}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^n*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] int(FresnelC(b*x)^n*sin(1/2*b^2*Pi*x^2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(fresnel_cos(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(\frac{\pi b^2 x^2}{2}\right) C^n(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)**n*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)**n, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)^n*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)^n*sin(1/2*pi*b^2*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \mathrm{FresnelC}(b\,x)^n\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)^n*sin((Pi*b^2*x^2)/2),x)
[Out] int(FresnelC(b*x)^n*sin((Pi*b^2*x^2)/2), x)
```

3.200 $\int x^8 \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right)\, dx$

Optimal. Leaf size=308

$$-\frac{35x^4}{8b^5\pi^3}+\frac{x^8}{16b\pi}-\frac{40\cos\left(b^2\pi x^2\right)}{b^9\pi^5}+\frac{5x^4\cos\left(b^2\pi x^2\right)}{2b^5\pi^3}+\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^6\pi^3}-\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{Fresnel}}{b^2\pi}$$

```
[Out] -35/8*x^4/b^5/Pi^3+1/16*x^8/b/Pi-40*cos(b^2*Pi*x^2)/b^9/Pi^5+5/2*x^4*cos(b^
2*Pi*x^2)/b^5/Pi^3+35*x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^6/Pi^3-x^7*co
s(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^2/Pi+105/2*FresnelC(b*x)*FresnelS(b*x)/b^
9/Pi^4+105/8*I*x^2*hypergeom([1, 1],[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^7/Pi^4-10
5/8*I*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)/b^7/Pi^4-105*x*Fresne
lC(b*x)*sin(1/2*b^2*Pi*x^2)/b^8/Pi^4+7*x^5*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2
)/b^4/Pi^2-55/4*x^2*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^6*sin(b^2*Pi*x^2)/b^3/Pi
^2
```

Rubi [A]
time = 0.31, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6598, 6590, 6582, 3460, 2718, 3461, 3390, 30, 3377}

$$\frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7}-\frac{105ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^4b^7}+\frac{105\text{FresnelC}(bx)S(bx)}{2\pi^4b^9}-\frac{35x^4}{8\pi^3b^5}-\frac{x^7\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{40\cos\left(\pi b^2x^2\right)}{\pi^5b^9}-\frac{105x\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}-\frac{55x^2\sin\left(\pi b^2x^2\right)}{4\pi^4b^7}+\frac{35x^3\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{5x^4\cos\left(\pi b^2x^2\right)}{2\pi^3b^5}+\frac{7x^5\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x^6\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}+\frac{x^8}{16\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-35*x^4)/(8*b^5*Pi^3) + x^8/(16*b*Pi) - (40*Cos[b^2*Pi*x^2])/(b^9*Pi^5) +
(5*x^4*Cos[b^2*Pi*x^2])/(2*b^5*Pi^3) + (35*x^3*Cos[(b^2*Pi*x^2)/2]*FresnelC
[b*x])/(b^6*Pi^3) - (x^7*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + (105
*FresnelC[b*x]*FresnelS[b*x])/(2*b^9*Pi^4) + (((105*I)/8)*x^2*Hypergeometri
cPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/(b^7*Pi^4) - (((105*I)/8)*x^2*
HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b^7*Pi^4) - (105*x*
FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^8*Pi^4) + (7*x^5*FresnelC[b*x]*Sin[(b
^2*Pi*x^2)/2])/(b^4*Pi^2) - (55*x^2*Sin[b^2*Pi*x^2])/(4*b^7*Pi^4) + (x^6*Si
n[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
```

```
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$
\begin{aligned}
\int x^8 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx &= -\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{7\int x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^2\pi}+\frac{\int x^7\cos^2\left(\frac{1}{2}b^2\pi x\right.}{b\pi}\\
&= -\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{7x^5C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}-\frac{35\int x^4C(bx)\sin\left(\frac{1}{2}b^2\right.}{b^4\pi^2}\\
&= \frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3}-\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{7x^5C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}\\
&= \frac{x^8}{16b\pi}+\frac{7x^4\cos\left(b^2\pi x^2\right)}{4b^5\pi^3}+\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3}-\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx}{b^2\pi}\\
&= \frac{x^8}{16b\pi}+\frac{5x^4\cos\left(b^2\pi x^2\right)}{2b^5\pi^3}+\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3}-\frac{x^7\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx}{b^2\pi}\\
&= -\frac{35x^4}{8b^5\pi^3}+\frac{x^8}{16b\pi}-\frac{119\cos\left(b^2\pi x^2\right)}{4b^9\pi^5}+\frac{5x^4\cos\left(b^2\pi x^2\right)}{2b^5\pi^3}+\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C}{b^6\pi^3}\\
&= -\frac{35x^4}{8b^5\pi^3}+\frac{x^8}{16b\pi}-\frac{40\cos\left(b^2\pi x^2\right)}{b^9\pi^5}+\frac{5x^4\cos\left(b^2\pi x^2\right)}{2b^5\pi^3}+\frac{35x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C}{b^6\pi^3}
\end{aligned}
$$

Mathematica [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^8\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] Integrate[x^8*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^8\,\text{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] int(x^8*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^8*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(x^8*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x**8*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^8*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^8\,\mathrm{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^8*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.201 $\int x^7 \textbf{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=218

$$-\frac{4x^3}{b^5\pi^3}+\frac{x^7}{14b\pi}+\frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3}+\frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^6\pi^3}-\frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi}+\frac{531S\left(\sqrt{}\right.}{16\sqrt{2}}$$

```
[Out] -4*x^3/b^5/Pi^3+1/14*x^7/b/Pi+17/8*x^3*cos(b^2*Pi*x^2)/b^5/Pi^3+24*x^2*cos(
1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^6/Pi^3-x^6*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x
)/b^2/Pi-48*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^8/Pi^4+6*x^4*FresnelC(b*x)*
sin(1/2*b^2*Pi*x^2)/b^4/Pi^2-147/16*x*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^5*sin(
b^2*Pi*x^2)/b^3/Pi^2+531/32*FresnelS(b*x*2^(1/2))/b^8/Pi^4*2^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6598, 6590, 6588, 3432, 3473, 30, 3467, 3466}

$$\frac{531S\left(\sqrt{2}\,bx\right)}{16\sqrt{2}\,\pi^4b^8}-\frac{4x^3}{\pi^3b^5}-\frac{x^6\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{48\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^4b^8}-\frac{147x\sin\left(\pi b^2x^2\right)}{16\pi^4b^7}+\frac{24x^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{17x^3\cos\left(\pi b^2x^2\right)}{8\pi^3b^5}+\frac{6x^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x^5\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}+\frac{x^7}{14\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-4*x^3)/(b^5*Pi^3) + x^7/(14*b*Pi) + (17*x^3*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3)
 + (24*x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^6*Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + (531*FresnelS[Sqrt[2]*b*x])/(16*Sqrt[2]*
b^8*Pi^4) - (48*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^8*Pi^4) + (6*x^4*Fres
nelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (147*x*Sin[b^2*Pi*x^2])/(16*b^7
*Pi^4) + (x^5*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
```

```
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3473

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6588

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$
\begin{aligned}
\int x^7 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)dx &= -\frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{6\int x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^2\pi} + \frac{\int x^6\cos^2\left(\frac{1}{2}b^2\pi x\right.}{b\pi} \\
&= -\frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{6x^4C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{24\int x^3C(bx)\sin\left(\frac{1}{2}b^2\right.}{b^4\pi^2} \\
&= \frac{x^7}{14b\pi} + \frac{3x^3\cos\left(b^2\pi x^2\right)}{2b^5\pi^3} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx}{b^2\pi} \\
&= \frac{x^7}{14b\pi} + \frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^6\cos\left(\frac{1}{2}b^2\pi x^2\right)C(b}{b^2\pi} \\
&= -\frac{4x^3}{b^5\pi^3} + \frac{x^7}{14b\pi} + \frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^6\cos\left(\frac{1}{2}b\right.}{b} \\
&= -\frac{4x^3}{b^5\pi^3} + \frac{x^7}{14b\pi} + \frac{17x^3\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{24x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^6\cos\left(\frac{1}{2}b\right.}{b}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 163, normalized size = 0.75

$$
\frac{-896b^3\pi x^3+16b^7\pi^3x^7+476b^3\pi x^3\cos\left(b^2\pi x^2\right)+3717\sqrt{2}\,S\left(\sqrt{2}\,bx\right)-224\text{FresnelC}(bx)\left(b^2\pi x^2\left(-24+b^4\pi^2x^4\right)\cos\left(\frac{1}{2}b^2\pi x^2\right)-6\left(-8+b^4\pi^2x^4\right)\sin\left(\frac{1}{2}b^2\pi x^2\right)\right)-2058bx\sin\left(b^2\pi x^2\right)+56b^5\pi^2x^5\sin\left(b^2\pi x^2\right)}{224b^8\pi^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-896*b^3*Pi*x^3 + 16*b^7*Pi^3*x^7 + 476*b^3*Pi*x^3*Cos[b^2*Pi*x^2] + 3717*
Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 224*FresnelC[b*x]*(b^2*Pi*x^2*(-24 + b^4*Pi
^2*x^4)*Cos[(b^2*Pi*x^2)/2] - 6*(-8 + b^4*Pi^2*x^4)*Sin[(b^2*Pi*x^2)/2]) -
2058*b*x*Sin[b^2*Pi*x^2] + 56*b^5*Pi^2*x^5*Sin[b^2*Pi*x^2])/(224*b^8*Pi^4)
```

Maple [A]
time = 0.85, size = 322, normalized size = 1.48

method	result
default	$\frac{\frac{\text{FresnelC}(bx)\left(-\frac{b^6x^6\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{6b^4x^4\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}-\frac{24\left(-\frac{b^2x^2\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}\right)}{\pi}\right)}{b^7}-\frac{\frac{\frac{1}{7}\pi^2b^7x^7-8b^3x^3}{2\pi^3}+\frac{-\frac{3\pi b^3x^3\cos}{2}}{}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelC(b*x)/b^7*(-1/Pi*b^6*x^6*cos(1/2*b^2*Pi*x^2)+6/Pi*(1/Pi*b^4*x^4*si
n(1/2*b^2*Pi*x^2)-4/Pi*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^
2*Pi*x^2))))-1/b^7*(-1/2/Pi^3*(1/7*Pi^2*b^7*x^7-8*b^3*x^3)+3/Pi^4*(-1/2*Pi*
b^3*x^3*cos(b^2*Pi*x^2)+3/2*Pi*(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*F
resnelS(b*x*2^(1/2)))-4*2^(1/2)*FresnelS(b*x*2^(1/2)))-1/2/Pi^3*(1/2*Pi*b^5
*x^5*sin(b^2*Pi*x^2)-5/2*Pi*(-1/2/Pi*b^3*x^3*cos(b^2*Pi*x^2)+3/2/Pi*(1/2/Pi
*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2))))-12/Pi*b*x*sin(b
^2*Pi*x^2)+6/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^7*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [**A**]
time = 0.37, size = 169, normalized size = 0.78

$$\frac{16\,\pi^3 b^8 x^7 + 952\,\pi b^4 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 1372\,\pi b^4 x^3 - 224\,\left(\pi^3 b^7 x^6 - 24\,\pi b^3 x^2\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) \mathrm{C}\left(bx\right) + 3717\,\sqrt{2}\,\sqrt{b^2}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{b^2}\,x\right) + 28\,\left(\left(4\,\pi^2 b^6 x^5 - 147\,b^2 x\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) + 48\,\left(\pi^2 b^5 x^4 - 8\,b\right)\mathrm{C}\left(bx\right)\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{224\,\pi^4 b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] 1/224*(16*pi^3*b^8*x^7 + 952*pi*b^4*x^3*cos(1/2*pi*b^2*x^2)^2 - 1372*pi*b^4
*x^3 - 224*(pi^3*b^7*x^6 - 24*pi*b^3*x^2)*cos(1/2*pi*b^2*x^2)*fresnel_cos(b
*x) + 3717*sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x) + 28*((4*pi^2
*b^6*x^5 - 147*b^2*x)*cos(1/2*pi*b^2*x^2) + 48*(pi^2*b^5*x^4 - 8*b)*fresnel
_cos(b*x))*sin(1/2*pi*b^2*x^2))/(pi^4*b^9)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x**7*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^7*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^7 \operatorname{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^7*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.202 $\int x^6 \text{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=185

$$-\frac{15x^2}{4b^5\pi^3}+\frac{x^6}{12b\pi}+\frac{7x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3}+\frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^6\pi^3}-\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi}-\frac{15\text{FresnelC}}{2b^7\pi^3}$$

```
[Out] -15/4*x^2/b^5/Pi^3+1/12*x^6/b/Pi+7/4*x^2*cos(b^2*Pi*x^2)/b^5/Pi^3+15*x*cos(
1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^6/Pi^3-x^5*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x
)/b^2/Pi-15/2*FresnelC(b*x)^2/b^7/Pi^3+5*x^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x
^2)/b^4/Pi^2-11/2*sin(b^2*Pi*x^2)/b^7/Pi^4+1/4*x^4*sin(b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A]
time = 0.17, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6598, 6590, 6576, 30, 3461, 2714, 3460, 3377, 2717, 3390}

$$-\frac{15\text{FresnelC}(bx)^2}{2\pi^3b^7}-\frac{15x^2}{4\pi^3b^5}-\frac{x^5\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}-\frac{11\sin\left(\pi b^2x^2\right)}{2\pi^4b^7}+\frac{15x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{7x^2\cos\left(\pi b^2x^2\right)}{4\pi^3b^5}+\frac{5x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x^4\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}+\frac{x^6}{12\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-15*x^2)/(4*b^5*Pi^3) + x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^
3) + (15*x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^5*Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/(b^2*Pi) - (15*FresnelC[b*x]^2)/(2*b^7*Pi^3) + (5*x
^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (11*Sin[b^2*Pi*x^2])/(2*
b^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$
\begin{aligned}
\int x^6 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx &= -\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{5\int x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^2\pi}+\frac{\int x^5\cos^2\left(\frac{1}{2}b^2\pi x^2\right.}{b\pi} \\
&= -\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{5x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}-\frac{15\int x^2C(bx)\sin\left(\frac{1}{2}b^2\pi\right.}{b^4\pi^2} \\
&= \frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3}-\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{5x^3C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}- \\
&= \frac{x^6}{12b\pi}+\frac{5x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3}+\frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3}-\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} \\
&= -\frac{15x^2}{4b^5\pi^3}+\frac{x^6}{12b\pi}+\frac{7x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3}+\frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3}-\frac{x^5\cos\left(\frac{1}{2}b^2\pi\right.}{b^2\pi} \\
&= -\frac{15x^2}{4b^5\pi^3}+\frac{x^6}{12b\pi}+\frac{7x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3}+\frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3}-\frac{x^5\cos\left(\frac{1}{2}b^2\pi\right.}{b^2\pi}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 185, normalized size = 1.00

$$-\frac{15x^2}{4b^5\pi^3}+\frac{x^6}{12b\pi}+\frac{7x^2\cos\left(b^2\pi x^2\right)}{4b^5\pi^3}+\frac{15x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^6\pi^3}-\frac{x^5\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi}-\frac{15\text{FresnelC}(bx)^2}{2b^7\pi^3}+\frac{5x^3\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}-\frac{11\sin\left(b^2\pi x^2\right)}{2b^7\pi^4}+\frac{x^4\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-15*x^2)/(4*b^5*Pi^3) + x^6/(12*b*Pi) + (7*x^2*Cos[b^2*Pi*x^2])/(4*b^5*Pi^
3) + (15*x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^5*Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/(b^2*Pi) - (15*FresnelC[b*x]^2)/(2*b^7*Pi^3) + (5*x
^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) - (11*Sin[b^2*Pi*x^2])/(2*
b^7*Pi^4) + (x^4*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^6\,\text{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] int(x^6*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^6*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [A]
time = 0.37, size = 141, normalized size = 0.76

$$\frac{\pi^3 b^6 x^6 + 42\,\pi b^2 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 66\,\pi b^2 x^2 - 12\left(\pi^3 b^5 x^5 - 15\,\pi b x\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}\left(bx\right) - 90\,\pi\,\mathrm{C}\left(bx\right)^2 + 6\left(10\,\pi^2 b^3 x^3\,\mathrm{C}\left(bx\right) + \left(\pi^2 b^4 x^4 - 22\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{12\,\pi^4 b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(pi^3*b^6*x^6 + 42*pi*b^2*x^2*cos(1/2*pi*b^2*x^2)^2 - 66*pi*b^2*x^2 -
12*(pi^3*b^5*x^5 - 15*pi*b*x)*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) - 90*pi*
fresnel_cos(b*x)^2 + 6*(10*pi^2*b^3*x^3*fresnel_cos(b*x) + (pi^2*b^4*x^4 -
22)*cos(1/2*pi*b^2*x^2))*sin(1/2*pi*b^2*x^2))/(pi^4*b^7)
```

Sympy [A]
time = 5.64, size = 264, normalized size = 1.43

$$\begin{cases} \frac{x^6 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{12\pi b} + \frac{x^6 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{12\pi b} - \frac{x^5 \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi b^2} + \frac{x^4 \sin\left(\frac{\pi b^2 x^2}{2}\right)\cos\left(\frac{\pi b^2 x^2}{2}\right)}{2\pi^2 b^3} + \frac{5x^3 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi^2 b^4} - \frac{11x^2 \sin^2\left(\frac{\pi b^2 x^2}{2}\right)}{2\pi^3 b^5} - \frac{2x^2 \cos^2\left(\frac{\pi b^2 x^2}{2}\right)}{\pi^3 b^5} + \frac{15x \cos\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{\pi^3 b^6} - \frac{11 \sin\left(\frac{\pi b^2 x^2}{2}\right)\cos\left(\frac{\pi b^2 x^2}{2}\right)}{\pi^4 b^7} - \frac{15 C^2(bx)}{2\pi^3 b^7} & \text{for}\ b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Piecewise((x**6*sin(pi*b**2*x**2/2)**2/(12*pi*b) + x**6*cos(pi*b**2*x**2/2)
**2/(12*pi*b) - x**5*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) + x**4*sin
(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) + 5*x**3*sin(pi*b**2*x*
*2/2)*fresnelc(b*x)/(pi**2*b**4) - 11*x**2*sin(pi*b**2*x**2/2)**2/(2*pi**3*
b**5) - 2*x**2*cos(pi*b**2*x**2/2)**2/(pi**3*b**5) + 15*x*cos(pi*b**2*x**2/
2)*fresnelc(b*x)/(pi**3*b**6) - 11*sin(pi*b**2*x**2/2)*cos(pi*b**2*x**2/2)/
(pi**4*b**7) - 15*fresnelc(b*x)**2/(2*pi**3*b**7), Ne(b, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^6*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\,\mathrm{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^6*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.203 $\int x^5\mathbf{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=167

$$-\frac{4x}{b^5\pi^3}+\frac{x^5}{10b\pi}+\frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3}+\frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^6\pi^3}-\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi}-\frac{43\text{FresnelC}\left(\right.}{8\sqrt{2}\,b^6}$$

```
[Out] -4*x/b^5/Pi^3+1/10*x^5/b/Pi+11/8*x*cos(b^2*Pi*x^2)/b^5/Pi^3+8*cos(1/2*b^2*P
i*x^2)*FresnelC(b*x)/b^6/Pi^3-x^4*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^2/Pi+
4*x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2+1/4*x^3*sin(b^2*Pi*x^2)/b^
3/Pi^2-43/16*FresnelC(b*x*2^(1/2))/b^6/Pi^3*2^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6598, 6590, 6596, 3439, 3433, 3466, 3473, 30, 3467}

$$-\frac{43\text{FresnelC}\left(\sqrt{2}\,bx\right)}{8\sqrt{2}\,\pi^3b^6}-\frac{4x}{\pi^3b^5}-\frac{x^4\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{8\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^3b^6}+\frac{11x\cos\left(\pi b^2x^2\right)}{8\pi^3b^5}+\frac{4x^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x^3\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}+\frac{x^5}{10\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-4*x)/(b^5*Pi^3) + x^5/(10*b*Pi) + (11*x*Cos[b^2*Pi*x^2])/(8*b^5*Pi^3) + (
8*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^6*Pi^3) - (x^4*Cos[(b^2*Pi*x^2)/2]*
FresnelC[b*x])/(b^2*Pi) - (43*FresnelC[Sqrt[2]*b*x])/(8*Sqrt[2]*b^6*Pi^3) +
 (4*x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (x^3*Sin[b^2*Pi*x^2
])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3439

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3466

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(-e^
(n - 1))*(e*x)^(m - n + 1)*(Cos[c + d*x^n]/(d*n)), x] + Dist[e^n*((m - n +
1)/(d*n)), Int[(e*x)^(m - n)*Cos[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3473

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
 m, n}, x]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6596

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelC[b*x]/(2*d)), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /;
 FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$
\begin{aligned}
\int x^5 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx &= -\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{4\int x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^2\pi} + \frac{\int x^4\cos^2\left(\frac{1}{2}b^2\pi x \cdots\right.}{b\pi} \cdots \\
&= -\frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{4x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{8\int xC(bx)\sin\left(\frac{1}{2}b^2\pi x \cdots\right.}{b^4\pi^2} \cdots \\
&= \frac{x^5}{10b\pi} + \frac{x\cos\left(b^2\pi x^2\right)}{b^5\pi^3} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{4 \cdots}{\cdots} \\
&= \frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} - \cdots \\
&= -\frac{4x}{b^5\pi^3} + \frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2 \cdots\right.}{b^2\pi} \cdots \\
&= -\frac{4x}{b^5\pi^3} + \frac{x^5}{10b\pi} + \frac{11x\cos\left(b^2\pi x^2\right)}{8b^5\pi^3} + \frac{8\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^6\pi^3} - \frac{x^4\cos\left(\frac{1}{2}b^2\pi x^2 \cdots\right.}{b^2\pi} \cdots
\end{aligned}
$$

Mathematica [**A**]
time = 0.11, size = 126, normalized size = 0.75

$$
\frac{-215\sqrt{2}\,\mathrm{FresnelC}\left(\sqrt{2}\,bx\right) - 80\mathrm{FresnelC}(bx)\left(\left(-8+b^4\pi^2x^4\right)\cos\left(\frac{1}{2}b^2\pi x^2\right) - 4b^2\pi x^2\sin\left(\frac{1}{2}b^2\pi x^2\right)\right) + 2bx\left(-160+4b^4\pi^2x^4+55\cos\left(b^2\pi x^2\right)+10b^2\pi x^2\sin\left(b^2\pi x^2\right)\right)}{80b^6\pi^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (-215*Sqrt[2]*FresnelC[Sqrt[2]*b*x] - 80*FresnelC[b*x]*((-8 + b^4*Pi^2*x^4)
*Cos[(b^2*Pi*x^2)/2] - 4*b^2*Pi*x^2*Sin[(b^2*Pi*x^2)/2]) + 2*b*x*(-160 + 4*
b^4*Pi^2*x^4 + 55*Cos[b^2*Pi*x^2] + 10*b^2*Pi*x^2*Sin[b^2*Pi*x^2]))/(80*b^6
*Pi^3)
```

Maple [**A**]
time = 0.82, size = 212, normalized size = 1.27

method	result
default	$\frac{\frac{\mathrm{FresnelC}(bx)\left(-\frac{b^4x^4\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{\frac{4b^2x^2\sin\left(\frac{b^2\pi x^2}{2}\right)}{\pi}+\frac{8\cos\left(\frac{b^2\pi x^2}{2}\right)}{\pi^2}}{\pi}\right)}{b^5}-\frac{-\frac{\frac{1}{5}\pi^2b^5x^5-8bx}{2\pi^3}+\frac{-\frac{bx\cos\left(b^2\pi x^2\right)}{\pi}+\frac{\sqrt{2}\,\mathrm{FresnelC}\left(bx\sqrt{2}\right) \cdots}{2\pi}}{\pi^2} \cdots}{\cdots}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelC(b*x)/b^5*(-1/Pi*b^4*x^4*cos(1/2*b^2*Pi*x^2)+4/Pi*(1/Pi*b^2*x^2*si
n(1/2*b^2*Pi*x^2)+2/Pi^2*cos(1/2*b^2*Pi*x^2)))-1/b^5*(-1/2/Pi^3*(1/5*Pi^2*b
^5*x^5-8*b*x)+2/Pi^2*(-1/2/Pi*b*x*cos(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b
*x*2^(1/2)))-1/2/Pi^3*(1/2*Pi*b^3*x^3*sin(b^2*Pi*x^2)-3/2*Pi*(-1/2/Pi*b*x*c
os(b^2*Pi*x^2)+1/4/Pi*2^(1/2)*FresnelC(b*x*2^(1/2)))-4*2^(1/2)*FresnelC(b*x
*2^(1/2)))))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^5*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [A]
time = 0.36, size = 139, normalized size = 0.83

$$\frac{8\,\pi^2 b^6 x^5 + 220\,b^2 x \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 430\,b^2 x - 80\,\left(\pi^2 b^5 x^4 - 8\,b\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}\left(bx\right) - 215\,\sqrt{2}\,\sqrt{b^2}\;\mathrm{C}\left(\sqrt{2}\,\sqrt{b^2}\,x\right) + 40\,\left(\pi b^4 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right) + 8\,\pi b^3 x^2\,\mathrm{C}\left(bx\right)\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{80\,\pi^3 b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] 1/80*(8*pi^2*b^6*x^5 + 220*b^2*x*cos(1/2*pi*b^2*x^2)^2 - 430*b^2*x - 80*(pi
^2*b^5*x^4 - 8*b)*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) - 215*sqrt(2)*sqrt(b
^2)*fresnel_cos(sqrt(2)*sqrt(b^2)*x) + 40*(pi*b^4*x^3*cos(1/2*pi*b^2*x^2) +
 8*pi*b^3*x^2*fresnel_cos(b*x))*sin(1/2*pi*b^2*x^2))/(pi^3*b^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{5} \sin{\left(\frac{\pi b^{2} x^{2}}{2} \right)} C\left(b x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x**5*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^5*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,\mathrm{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^5*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.204 $\int x^4\textbf{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=196

$$\frac{x^4}{8b\pi}+\frac{\cos(b^2\pi x^2)}{b^5\pi^3}-\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi}-\frac{3\text{FresnelC}(bx)S(bx)}{2b^5\pi^2}-\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2}+\frac{3ix^2}{}$$

```
[Out] 1/8*x^4/b/Pi+cos(b^2*Pi*x^2)/b^5/Pi^3-x^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)
/b^2/Pi-3/2*FresnelC(b*x)*FresnelS(b*x)/b^5/Pi^2-3/8*I*x^2*hypergeom([1, 1]
,[3/2, 2],-1/2*I*b^2*Pi*x^2)/b^3/Pi^2+3/8*I*x^2*hypergeom([1, 1],[3/2, 2],1
/2*I*b^2*Pi*x^2)/b^3/Pi^2+3*x*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/b^4/Pi^2+1/
4*x^2*sin(b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A]
time = 0.11, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6598, 6590, 6582, 3460, 2718, 3461, 3390, 30, 3377}

$$-\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2b^3}+\frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)}{8\pi^2b^3}-\frac{3\text{FresnelC}(bx)S(bx)}{2\pi^2b^5}-\frac{x^3\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\cos(\pi b^2x^2)}{\pi^3b^5}+\frac{3x\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4}+\frac{x^2\sin(\pi b^2x^2)}{4\pi^2b^3}+\frac{x^4}{8\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] x^4/(8*b*Pi) + Cos[b^2*Pi*x^2]/(b^5*Pi^3) - (x^3*Cos[(b^2*Pi*x^2)/2]*Fresne
lC[b*x])/(b^2*Pi) - (3*FresnelC[b*x]*FresnelS[b*x])/(2*b^5*Pi^2) - (((3*I)/
8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (-1/2*I)*b^2*Pi*x^2])/(b^3*Pi^2)
 + (((3*I)/8)*x^2*HypergeometricPFQ[{1, 1}, {3/2, 2}, (I/2)*b^2*Pi*x^2])/(b
^3*Pi^2) + (3*x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(b^4*Pi^2) + (x^2*Sin[b^
2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3390

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + ((f_.)*(x_))/2]^2, x_Symbol] :>
Dist[1/2, Int[(c + d*x)^m, x], x] - Dist[1/2, Int[(c + d*x)^m*Cos[2*e + f*x
], x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6590

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m - 1)*Sin[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (-Dist[(m - 1)/(2*d), Int[x^(
m - 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(4*d), Int[x^(m - 1)*Sin[2
*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m, 1
]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\int x^4 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = -\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{3\int x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^2\pi} + \frac{\int x^3\cos^2\left(\frac{1}{2}b^2\pi x^2\right.}{b\pi}$$
$$= -\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{3xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} - \frac{3\int C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,d}{b^4\pi^2}$$
$$= -\frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} - \frac{3C(bx)S(bx)}{2b^5\pi^2} - \frac{3ix^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)}{8b^3\pi^2} +$$
$$= \frac{x^4}{8b\pi} + \frac{3\cos\left(b^2\pi x^2\right)}{4b^5\pi^3} - \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} - \frac{3C(bx)S(bx)}{2b^5\pi^2} - \frac{3ix^2\,{}_2F_2\left(1,\right.}{}$$
$$= \frac{x^4}{8b\pi} + \frac{\cos\left(b^2\pi x^2\right)}{b^5\pi^3} - \frac{x^3\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} - \frac{3C(bx)S(bx)}{2b^5\pi^2} - \frac{3ix^2\,{}_2F_2\left(1,1;\right.}{8}$$

Mathematica [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] Integrate[x^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^4\,\text{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi\,x^2}{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] int(x^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^4*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^4*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x**4*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^4*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\mathrm{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
```

```
[Out] int(x^4*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.205 $\int x^3 \textbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) dx$

Optimal. Leaf size=109

$$\frac{x^3}{6b\pi} - \frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi} - \frac{5S\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,b^4\pi^2} + \frac{2\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2} + \frac{x\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

```
[Out] 1/6*x^3/b/Pi-x^2*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^2/Pi+2*FresnelC(b*x)*s
in(1/2*b^2*Pi*x^2)/b^4/Pi^2+1/4*x*sin(b^2*Pi*x^2)/b^3/Pi^2-5/8*FresnelS(b*x
*2^(1/2))/b^4/Pi^2*2^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6598, 6588, 3432, 3473, 30, 3467}

$$-\frac{5S\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,\pi^2b^4} - \frac{x^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2} + \frac{2\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{\pi^2b^4} + \frac{x\sin\left(\pi b^2x^2\right)}{4\pi^2b^3} + \frac{x^3}{6\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] x^3/(6*b*Pi) - (x^2*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) - (5*Fresne
lS[Sqrt[2]*b*x])/(4*Sqrt[2]*b^4*Pi^2) + (2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2
])/(b^4*Pi^2) + (x*Sin[b^2*Pi*x^2])/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3473

```
Int[Cos[(a_.) + ((b_.)*(x_)^(n_))/2]^2*(x_)^(m_.), x_Symbol] :> Dist[1/2, I
nt[x^m, x], x] + Dist[1/2, Int[x^m*Cos[2*a + b*x^n], x], x] /; FreeQ[{a, b,
```

```
m, n}, x]
```

Rule 6588

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_), x_Symbol] :> Simp[Sin[d*x^
2]*(FresnelC[b*x]/(2*d)), x] - Dist[b/(4*d), Int[Sin[2*d*x^2], x], x] /; Fr
eeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}\int x^3 C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx &= -\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{2\int x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^2\pi}+\frac{\int x^2\cos^2\left(\frac{1}{2}b^2\pi x^2\right)}{b\pi}\\
&= -\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}-\frac{\int\sin\left(b^2\pi x^2\right)\,dx}{b^3\pi^2}+\frac{\int x^2}{2b}\\
&= \frac{x^3}{6b\pi}-\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}-\frac{S\left(\sqrt{2}\,bx\right)}{\sqrt{2}\,b^4\pi^2}+\frac{2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}+\frac{x\sin}{4}\\
&= \frac{x^3}{6b\pi}-\frac{x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}-\frac{5S\left(\sqrt{2}\,bx\right)}{4\sqrt{2}\,b^4\pi^2}+\frac{2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{b^4\pi^2}+\frac{x\sin}{}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 90, normalized size = 0.83

$$\frac{4b^3\pi x^3-15\sqrt{2}\,S\left(\sqrt{2}\,bx\right)-24\text{FresnelC}(bx)\left(b^2\pi x^2\cos\left(\frac{1}{2}b^2\pi x^2\right)-2\sin\left(\frac{1}{2}b^2\pi x^2\right)\right)+6bx\sin\left(b^2\pi x^2\right)}{24b^4\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (4*b^3*Pi*x^3 - 15*Sqrt[2]*FresnelS[Sqrt[2]*b*x] - 24*FresnelC[b*x]*(b^2*Pi
*x^2*Cos[(b^2*Pi*x^2)/2] - 2*Sin[(b^2*Pi*x^2)/2]) + 6*b*x*Sin[b^2*Pi*x^2])/
(24*b^4*Pi^2)
```

Maple [A]
time = 0.80, size = 120, normalized size = 1.10

method	result	size
default	$\frac{\frac{\mathrm{FresnelC}(bx)\left(-\frac{b^{2}x^{2}\cos\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi}+\frac{2\sin\left(\frac{b^{2}\pi x^{2}}{2}\right)}{\pi^{2}}\right)}{b^{3}}-\frac{\frac{\sqrt{2}\,\mathrm{S}\left(bx\sqrt{2}\right)}{2\pi^{2}}-\frac{b^{3}x^{3}}{6\pi}-\frac{\frac{bx\sin\left(b^{2}\pi x^{2}\right)}{2\pi}-\frac{\sqrt{2}\,\mathrm{S}\left(bx\sqrt{2}\right)}{4\pi}}{2\pi}}{b^{3}}}{b}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] (FresnelC(b*x)/b^3*(-1/Pi*b^2*x^2*cos(1/2*b^2*Pi*x^2)+2/Pi^2*sin(1/2*b^2*Pi
*x^2))-1/b^3*(1/2/Pi^2*2^(1/2)*FresnelS(b*x*2^(1/2))-1/6*b^3*x^3/Pi-1/2/Pi*
(1/2/Pi*b*x*sin(b^2*Pi*x^2)-1/4/Pi*2^(1/2)*FresnelS(b*x*2^(1/2)))))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [A]
time = 0.37, size = 97, normalized size = 0.89

$$\frac{4\,\pi b^{4}x^{3}-24\,\pi b^{3}x^{2}\cos\left(\tfrac{1}{2}\,\pi b^{2}x^{2}\right)\mathrm{C}\left(bx\right)-15\,\sqrt{2}\,\sqrt{b^{2}}\,\mathrm{S}\left(\sqrt{2}\,\sqrt{b^{2}}\,x\right)+12\left(b^{2}x\cos\left(\tfrac{1}{2}\,\pi b^{2}x^{2}\right)+4\,b\,\mathrm{C}\left(bx\right)\right)\sin\left(\tfrac{1}{2}\,\pi b^{2}x^{2}\right)}{24\,\pi^{2}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
```

```
[Out] 1/24*(4*pi*b^4*x^3 - 24*pi*b^3*x^2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) - 1
5*sqrt(2)*sqrt(b^2)*fresnel_sin(sqrt(2)*sqrt(b^2)*x) + 12*(b^2*x*cos(1/2*pi
*b^2*x^2) + 4*b*fresnel_cos(b*x))*sin(1/2*pi*b^2*x^2))/(pi^2*b^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\sin\left(\frac{\pi b^{2}x^{2}}{2}\right)C(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
```

```
[Out] Integral(x**3*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^3*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^3*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.206 $\int x^2 \mathbf{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=74

$$\frac{x^2}{4b\pi}-\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi}+\frac{\text{FresnelC}(bx)^2}{2b^3\pi}+\frac{\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

```
[Out] 1/4*x^2/b/Pi-x*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^2/Pi+1/2*FresnelC(b*x)^2
/b^3/Pi+1/4*sin(b^2*Pi*x^2)/b^3/Pi^2
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6598, 6576, 30, 3461, 2714}

$$\frac{\text{FresnelC}(bx)^2}{2\pi b^3}-\frac{x\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\sin\left(\pi b^2x^2\right)}{4\pi^2b^3}+\frac{x^2}{4\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] x^2/(4*b*Pi) - (x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + FresnelC[b*
x]^2/(2*b^3*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2714

```
Int[sin[(c_.) + ((d_.)*(x_))/2]^2, x_Symbol] :> Simp[x/2, x] - Simp[Sin[2*c
 + d*x]/(2*d), x] /; FreeQ[{c, d}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6598

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-x
^(m - 1))*Cos[d*x^2]*(FresnelC[b*x]/(2*d)), x] + (Dist[(m - 1)/(2*d), Int[x
^(m - 2)*Cos[d*x^2]*FresnelC[b*x], x], x] + Dist[b/(2*d), Int[x^(m - 1)*Cos
[d*x^2]^2, x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && IGtQ[m,
 1]
```

Rubi steps

$$\begin{aligned}\int x^2C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx &= -\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{\int\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx}{b^2\pi}+\frac{\int x\cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi}\\ &= -\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{\text{Subst}(\int x\,dx,x,C(bx))}{b^3\pi}+\frac{\text{Subst}(\int\cos^2\left(\frac{1}{2}b^2\pi x\right)}{2b\pi}\\ &= \frac{x^2}{4b\pi}-\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi}+\frac{C(bx)^2}{2b^3\pi}+\frac{\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 74, normalized size = 1.00

$$\frac{x^2}{4b\pi}-\frac{x\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi}+\frac{\text{FresnelC}(bx)^2}{2b^3\pi}+\frac{\sin\left(b^2\pi x^2\right)}{4b^3\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] x^2/(4*b*Pi) - (x*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + FresnelC[b*
x]^2/(2*b^3*Pi) + Sin[b^2*Pi*x^2]/(4*b^3*Pi^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^2\,\text{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

```
[Out] int(x^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x^2*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [A]
time = 0.36, size = 67, normalized size = 0.91

$$\frac{\pi b^2x^2 - 4\,\pi bx\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{C}\left(bx\right) + 2\,\pi\,\mathrm{C}\left(bx\right)^2 + 2\,\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\sin\left(\frac{1}{2}\,\pi b^2x^2\right)}{4\,\pi^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] 1/4*(pi*b^2*x^2 - 4*pi*b*x*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) + 2*pi*fres
nel_cos(b*x)^2 + 2*cos(1/2*pi*b^2*x^2)*sin(1/2*pi*b^2*x^2))/(pi^2*b^3)
```

Sympy [A]
time = 0.44, size = 114, normalized size = 1.54

$$\begin{cases} \frac{x^2\sin^2\left(\frac{\pi b^2x^2}{2}\right)}{4\pi b} + \frac{x^2\cos^2\left(\frac{\pi b^2x^2}{2}\right)}{4\pi b} - \frac{x\cos\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{\pi b^2} + \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)\cos\left(\frac{\pi b^2x^2}{2}\right)}{2\pi^2b^3} + \frac{C^2(bx)}{2\pi b^3} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Piecewise((x**2*sin(pi*b**2*x**2/2)**2/(4*pi*b) + x**2*cos(pi*b**2*x**2/2)*
*2/(4*pi*b) - x*cos(pi*b**2*x**2/2)*fresnelc(b*x)/(pi*b**2) + sin(pi*b**2*x
**2/2)*cos(pi*b**2*x**2/2)/(2*pi**2*b**3) + fresnelc(b*x)**2/(2*pi*b**3), N
e(b, 0)), (0, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x^2*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x^2*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.207 $\int x\textbf{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$

Optimal. Leaf size=60

$$\frac{x}{2b\pi}-\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{b^2\pi}+\frac{\text{FresnelC}\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,b^2\pi}$$

```
[Out] 1/2*x/b/Pi-cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/b^2/Pi+1/4*FresnelC(b*x*2^(1/2
))/b^2/Pi*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6596, 3439, 3433}

$$-\frac{\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{\pi b^2}+\frac{\text{FresnelC}\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,\pi b^2}+\frac{x}{2\pi b}$$

Antiderivative was successfully verified.

```
[In] Int[x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] x/(2*b*Pi) - (Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(b^2*Pi) + FresnelC[Sqrt[2
]*b*x]/(2*Sqrt[2]*b^2*Pi)
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3439

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 6596

```
Int[FresnelC[(b_.)*(x_)]*(x_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[(-Cos[d*
x^2])*(FresnelC[b*x]/(2*d)), x] + Dist[b/(2*d), Int[Cos[d*x^2]^2, x], x] /;
 FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\int xC(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx = -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{\int \cos^2\left(\frac{1}{2}b^2\pi x^2\right)\,dx}{b\pi}$$
$$= -\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{\int\left(\frac{1}{2}+\frac{1}{2}\cos\left(b^2\pi x^2\right)\right)\,dx}{b\pi}$$
$$= \frac{x}{2b\pi} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{\int \cos\left(b^2\pi x^2\right)\,dx}{2b\pi}$$
$$= \frac{x}{2b\pi} - \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{b^2\pi} + \frac{C\left(\sqrt{2}\,bx\right)}{2\sqrt{2}\,b^2\pi}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.80

$$\frac{2bx - 4\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx) + \sqrt{2}\,\text{FresnelC}\left(\sqrt{2}\,bx\right)}{4b^2\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] (2*b*x - 4*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x] + Sqrt[2]*FresnelC[Sqrt[2]*b*x
])/(4*b^2*Pi)
```

Maple [A]
time = 0.62, size = 52, normalized size = 0.87

method	result	size
default	$\frac{-\frac{\text{FresnelC}(bx)\cos\left(\frac{b^2\pi x^2}{2}\right)}{b\pi}+\frac{\frac{bx}{2}+\frac{\sqrt{2}\,\text{FresnelC}\left(bx\sqrt{2}\right)}{4}}{b\pi}}{b}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x,method=_RETURNVERBOSE)
[Out] (-FresnelC(b*x)/b/Pi*cos(1/2*b^2*Pi*x^2)+1/b/Pi*(1/2*b*x+1/4*2^(1/2)*Fresne
lC(b*x*2^(1/2))))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(x*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [A]
time = 0.36, size = 52, normalized size = 0.87

$$\frac{2\,b^2x-4\,b\cos\left(\frac{1}{2}\,\pi b^2x^2\right)\mathrm{C}\left(bx\right)+\sqrt{2}\,\sqrt{b^2}\,\mathrm{C}\left(\sqrt{2}\,\sqrt{b^2}\,x\right)}{4\,\pi b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] 1/4*(2*b^2*x - 4*b*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) + sqrt(2)*sqrt(b^2)
*fresnel_cos(sqrt(2)*sqrt(b^2)*x))/(pi*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(x*sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
[Out] integrate(x*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
[Out] int(x*FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.208 $\int \mathbf{FresnelC}(bx) \sin\left(\frac{1}{2}b^2\pi x^2\right) \, dx$

Optimal. Leaf size=80

$$\frac{\text{FresnelC}(bx)S(bx)}{2b}+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

```
[Out] 1/2*FresnelC(b*x)*FresnelS(b*x)/b+1/8*I*b*x^2*hypergeom([1, 1],[3/2, 2],-1/
2*I*b^2*Pi*x^2)-1/8*I*b*x^2*hypergeom([1, 1],[3/2, 2],1/2*I*b^2*Pi*x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {6582}

$$\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)+\frac{\text{FresnelC}(bx)S(bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
```

```
[Out] (FresnelC[b*x]*FresnelS[b*x])/(2*b) + (I/8)*b*x^2*HypergeometricPFQ[{1, 1},
 {3/2, 2}, (-1/2*I)*b^2*Pi*x^2] - (I/8)*b*x^2*HypergeometricPFQ[{1, 1}, {3/
2, 2}, (I/2)*b^2*Pi*x^2]
```

Rule 6582

```
Int[FresnelC[(b_.)*(x_)]*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[b*Pi*FresnelC
[b*x]*(FresnelS[b*x]/(4*d)), x] + (Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1,
 1}, {3/2, 2}, (-I)*d*x^2], x] - Simp[(1/8)*I*b*x^2*HypergeometricPFQ[{1, 1
}, {3/2, 2}, I*d*x^2], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4]
```

Rubi steps

$$\int C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx=\frac{C(bx)S(bx)}{2b}+\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;-\frac{1}{2}ib^2\pi x^2\right)-\frac{1}{8}ibx^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{1}{2}ib^2\pi x^2\right)$$

Mathematica [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2],x]
[Out] Integrate[FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2], x]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \mathrm{FresnelC}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2),x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2),x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin((Pi*b^2*x^2)/2),x)
```

```
[Out] int(FresnelC(b*x)*sin((Pi*b^2*x^2)/2), x)
```

3.209 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x},x\right)$$

```
[Out] Unintegrable(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x,x]
[Out] Defer[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]
```

Rubi steps

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx = \int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x,x]
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x,x)
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x, x)
```

3.210 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx$

Optimal. Leaf size=48

$$\frac{1}{2}b\pi\text{FresnelC}(bx)^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)$$

```
[Out] 1/2*b*Pi*FresnelC(b*x)^2+1/4*b*Si(b^2*Pi*x^2)-FresnelC(b*x)*sin(1/2*b^2*Pi*
x^2)/x
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6600, 6576, 30, 3456}

$$-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{x}+\frac{1}{4}b\text{Si}\left(b^2\pi x^2\right)+\frac{1}{2}\pi b\text{FresnelC}(bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]
```

```
[Out] (b*Pi*FresnelC[b*x]^2)/2 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x + (b*SinIn
tegral[b^2*Pi*x^2])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6600

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && I
```

```
LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\,dx &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}+\frac{1}{2}b\int \frac{\sin\left(b^2\pi x^2\right)}{x}\,dx+\left(b^2\pi\right)\int \cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)\,dx\\ &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}+\frac{1}{4}b\mathrm{Si}\left(b^2\pi x^2\right)+(b\pi)\mathrm{Subst}\left(\int x\,dx,x,C(bx)\right)\\ &= \frac{1}{2}b\pi C(bx)^2-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}+\frac{1}{4}b\mathrm{Si}\left(b^2\pi x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 1.00

$$\frac{1}{2}b\pi\mathrm{FresnelC}(bx)^2-\frac{\mathrm{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x}+\frac{1}{4}b\mathrm{Si}\left(b^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^2,x]
[Out] (b*Pi*FresnelC[b*x]^2)/2 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x + (b*SinIn
tegral[b^2*Pi*x^2])/4
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="maxima")
```

[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)

Fricas [A]
time = 0.40, size = 45, normalized size = 0.94

$$\frac{2\,\pi bx\,\mathrm{C}\,(bx)^{2}+bx\,\mathrm{Si}\,(\pi b^{2}x^{2})-4\,\mathrm{C}\,(bx)\sin\left(\frac{1}{2}\,\pi b^{2}x^{2}\right)}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="fricas")

[Out] 1/4*(2*pi*b*x*fresnel_cos(b*x)^2 + b*x*sin_integral(pi*b^2*x^2) - 4*fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2))/x

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(\frac{\pi b^{2}x^{2}}{2}\right)C(bx)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**2,x)

[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**2, x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^2,x, algorithm="giac")

[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^2, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\mathrm{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^{2}\,x^{2}}{2}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^2,x)

[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^2, x)

3.211 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$

Optimal. Leaf size=94

$$\frac{b^2\pi\text{FresnelC}\left(\sqrt{2}\,bx\right)}{2\sqrt{2}}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{4x}+\frac{1}{2}b^2\pi\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x},x\right)$$

```
[Out] -1/2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2-1/4*b*sin(b^2*Pi*x^2)/x+1/4*b^2*
Pi*FresnelC(b*x*2^(1/2))*2^(1/2)+1/2*b^2*Pi*Unintegrable(cos(1/2*b^2*Pi*x^2
)*FresnelC(b*x)/x,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]
```

```
[Out] (b^2*Pi*FresnelC[Sqrt[2]*b*x])/(2*Sqrt[2]) - (FresnelC[b*x]*Sin[(b^2*Pi*x^2
)/2])/(2*x^2) - (b*Sin[b^2*Pi*x^2])/(4*x) + (b^2*Pi*Defer[Int][(Cos[(b^2*Pi
*x^2)/2]*FresnelC[b*x])/x, x])/2
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}+\frac{1}{4}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^2}\,dx+\frac{1}{2}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x}\,d\\ &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{4x}+\frac{1}{2}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x}\,dx+\frac{1}{2}(\\ &= \frac{b^2\pi C\left(\sqrt{2}\,bx\right)}{2\sqrt{2}}-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{2x^2}-\frac{b\sin\left(b^2\pi x^2\right)}{4x}+\frac{1}{2}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3,x]
```

```
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^3, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{FresnelC}\left(bx\right)\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^{2}x^{2}}{2}\right)C\left(bx\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**3,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^3,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^3,x)
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^3, x)
```

3.212 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$

Optimal. Leaf size=89

$$\frac{1}{12}b^3\pi\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}-\frac{b\sin\left(b^2\pi x^2\right)}{12x^2}+\frac{1}{3}b^2\pi\text{Int}\left(\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{x^2},\right.$$

```
[Out] 1/12*b^3*Pi*Ci(b^2*Pi*x^2)-1/3*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3-1/12*b
*sin(b^2*Pi*x^2)/x^2+1/3*b^2*Pi*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b
*x)/x^2,x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]
```

```
[Out] (b^3*Pi*CosIntegral[b^2*Pi*x^2])/12 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(
3*x^3) - (b*Sin[b^2*Pi*x^2])/(12*x^2) + (b^2*Pi*Defer[Int][(Cos[(b^2*Pi*x^2
)/2]*FresnelC[b*x])/x^2, x])/3
```

Rubi steps

$$\begin{aligned}\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}+\frac{1}{6}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^3}\,dx+\frac{1}{3}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2}\,dx\\ &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}+\frac{1}{12}b\text{Subst}\left(\int \frac{\sin\left(b^2\pi x\right)}{x^2}\,dx,x,x^2\right)+\frac{1}{3}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\right.}{}\\ &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}-\frac{b\sin\left(b^2\pi x^2\right)}{12x^2}+\frac{1}{3}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^2}\,dx+\frac{1}{12}(b\\ &= \frac{1}{12}b^3\pi\text{Ci}\left(b^2\pi x^2\right)-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{3x^3}-\frac{b\sin\left(b^2\pi x^2\right)}{12x^2}+\frac{1}{3}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)}{x^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4,x]
```

```
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^4, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{FresnelC}\left(bx\right)\sin\left(\frac{b^{2}\pi\,x^{2}}{2}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^{2}x^{2}}{2}\right)C\left(bx\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**4,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**4, x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^4,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^4, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^4,x)
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^4, x)
```

3.213 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$

Optimal. Leaf size=156

$$-\frac{b^3\pi}{16x}-\frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{8x^2}-\frac{7b^4\pi^2S\left(\sqrt{2}\,bx\right)}{24\sqrt{2}}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}-b$$

```
[Out] -1/16*b^3*Pi/x-7/48*b^3*Pi*cos(b^2*Pi*x^2)/x-1/8*b^2*Pi*cos(1/2*b^2*Pi*x^2)
*FresnelC(b*x)/x^2-1/4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4-1/24*b*sin(b^2
*Pi*x^2)/x^3-7/48*b^4*Pi^2*FresnelS(b*x*2^(1/2))*2^(1/2)-1/8*b^4*Pi^2*Unint
egrable(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]
```

```
[Out] -1/16*(b^3*Pi)/x - (7*b^3*Pi*Cos[b^2*Pi*x^2])/(48*x) - (b^2*Pi*Cos[(b^2*Pi*
x^2)/2]*FresnelC[b*x])/(8*x^2) - (7*b^4*Pi^2*FresnelS[Sqrt[2]*b*x])/(24*Sqr
t[2]) - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(4*x^4) - (b*Sin[b^2*Pi*x^2])/(
24*x^3) - (b^4*Pi^2*Defer[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/8
```

Rubi steps

$$\begin{aligned}
\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}+\frac{1}{8}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^4}\,dx+\frac{1}{4}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^3}\,d \\
&= -\frac{b^3\pi}{16x}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^2}-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}-\frac{b\sin\left(b^2\pi x^2\right)}{24x^3}+\frac{1}{16}(b^3 \\
&= -\frac{b^3\pi}{16x}-\frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^2}-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{4x^4}-b \\
&= -\frac{b^3\pi}{16x}-\frac{7b^3\pi\cos\left(b^2\pi x^2\right)}{48x}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{8x^2}-\frac{7b^4\pi^2S\left(\sqrt{2}\,bx\right)}{24\sqrt{2}}-C(b
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5,x]
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^5, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2x^2}{2}\right)C(bx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**5,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**5, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^5,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^5, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^5,x)
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^5, x)
```

3.214 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx$

Optimal. Leaf size=163

$$-\frac{b^3\pi}{60x^2}-\frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{15x^3}-\frac{1}{30}b^5\pi^3\text{FresnelC}(bx)^2-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5}$$

```
[Out] -1/60*b^3*Pi/x^2-1/24*b^3*Pi*cos(b^2*Pi*x^2)/x^2-1/15*b^2*Pi*cos(1/2*b^2*Pi
*x^2)*FresnelC(b*x)/x^3-1/30*b^5*Pi^3*FresnelC(b*x)^2-7/120*b^5*Pi^2*Si(b^2
*Pi*x^2)-1/5*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5+1/15*b^4*Pi^2*FresnelC(b
*x)*sin(1/2*b^2*Pi*x^2)/x-1/40*b*sin(b^2*Pi*x^2)/x^4
```

Rubi [A]
time = 0.15, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6600, 6592, 6576, 30, 3456, 3461, 3378, 3380, 3460}

$$-\frac{1}{30}\pi^3b^5\text{FresnelC}(bx)^2-\frac{\pi b^3}{60x^2}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{5x^5}-\frac{\pi b^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{15x^3}-\frac{b\sin\left(\pi b^2x^2\right)}{40x^4}-\frac{7}{120}\pi^2b^5\text{Si}\left(b^2\pi x^2\right)+\frac{\pi^2b^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{15x}-\frac{\pi b^3\cos\left(\pi b^2x^2\right)}{24x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
```

```
[Out] -1/60*(b^3*Pi)/x^2 - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (b^2*Pi*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(15*x^3) - (b^5*Pi^3*FresnelC[b*x]^2)/30 - (Fresne
lC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*x^5) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*
x^2)/2])/(15*x) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^
2*Pi*x^2])/120
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6592

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[b*(x^(m + 2)/(2*(m + 1)*(m + 2))), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6600

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && I
LtQ[m, -1]
```

Rubi steps

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^6}\,dx = -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{1}{10}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^5}\,dx + \frac{1}{5}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^4}\,d$$

$$= -\frac{b^3\pi}{60x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^3} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{1}{20}b\text{Subst}\left(\int \frac{\sin\left(b^2\pi\right.}{x^3}\right.$$

$$= -\frac{b^3\pi}{60x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^3} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{b^4\pi^2C(bx)\sin\left(\frac{1}{2}b^2\pi x\right.}{15x}$$

$$= -\frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^3} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{b^4\pi}{}$$

$$= -\frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{15x^3} - \frac{1}{30}b^5\pi^3C(bx)^2 - \frac{C(bx)\sin}{5}$$

Mathematica [A]
time = 0.01, size = 163, normalized size = 1.00

$$-\frac{b^3\pi}{60x^2} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{24x^2} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{15x^3} - \frac{1}{30}b^5\pi^3\text{FresnelC}(bx)^2 - \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{5x^5} + \frac{b^4\pi^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{15x} - \frac{b\sin\left(b^2\pi x^2\right)}{40x^4} - \frac{7}{120}b^5\pi^2\text{Si}\left(b^2\pi x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^6,x]
```

```
[Out] -1/60*(b^3*Pi)/x^2 - (b^3*Pi*Cos[b^2*Pi*x^2])/(24*x^2) - (b^2*Pi*Cos[(b^2*P
i*x^2)/2]*FresnelC[b*x])/(15*x^3) - (b^5*Pi^3*FresnelC[b*x]^2)/30 - (Fresne
lC[b*x]*Sin[(b^2*Pi*x^2)/2])/(5*x^5) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*
x^2)/2])/(15*x) - (b*Sin[b^2*Pi*x^2])/(40*x^4) - (7*b^5*Pi^2*SinIntegral[b^
2*Pi*x^2])/120
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\,(bx)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

Fricas [A]
time = 0.34, size = 141, normalized size = 0.87

$$-\frac{4\,\pi^3 b^5 x^5\,\mathrm{C}\left(bx\right)^2 + 7\,\pi^2 b^5 x^5\,\mathrm{Si}\left(\pi b^2 x^2\right) + 10\,\pi b^3 x^3 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 - 3\,\pi b^3 x^3 + 8\,\pi b^2 x^2 \cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}\left(bx\right) + 2\,\left(3\,bx\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - 4\,\left(\pi^2 b^4 x^4 - 3\right)\mathrm{C}\left(bx\right)\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{120\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/120*(4*pi^3*b^5*x^5*fresnel_cos(b*x)^2 + 7*pi^2*b^5*x^5*sin_integral(pi*
b^2*x^2) + 10*pi*b^3*x^3*cos(1/2*pi*b^2*x^2)^2 - 3*pi*b^3*x^3 + 8*pi*b^2*x^
2*cos(1/2*pi*b^2*x^2)*fresnel_cos(b*x) + 2*(3*b*x*cos(1/2*pi*b^2*x^2) - 4*(
pi^2*b^4*x^4 - 3)*fresnel_cos(b*x))*sin(1/2*pi*b^2*x^2))/x^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**6,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^6,x)
```

```
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^6, x)
```

3.215 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$

Optimal. Leaf size=231

$$-\frac{b^3\pi}{144x^3}-\frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{24x^4}-\frac{7b^6\pi^3\text{FresnelC}\left(\sqrt{2}\,bx\right)}{144\sqrt{2}}-\frac{1}{45}\sqrt{2}\,b^6\pi^3\text{FresnelC}$$

```
[Out] -1/144*b^3*Pi/x^3-13/720*b^3*Pi*cos(b^2*Pi*x^2)/x^3-1/24*b^2*Pi*cos(1/2*b^2
*Pi*x^2)*FresnelC(b*x)/x^4-1/6*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^6+1/48*b
^4*Pi^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^2-1/60*b*sin(b^2*Pi*x^2)/x^5+67
/1440*b^5*Pi^2*sin(b^2*Pi*x^2)/x-67/1440*b^6*Pi^3*FresnelC(b*x*2^(1/2))*2^(
1/2)-1/48*b^6*Pi^3*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x,x)
```

Rubi [A]
time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]
```

```
[Out] -1/144*(b^3*Pi)/x^3 - (13*b^3*Pi*Cos[b^2*Pi*x^2])/(720*x^3) - (b^2*Pi*Cos[(
b^2*Pi*x^2)/2]*FresnelC[b*x])/(24*x^4) - (7*b^6*Pi^3*FresnelC[Sqrt[2]*b*x])
/(144*Sqrt[2]) - (Sqrt[2]*b^6*Pi^3*FresnelC[Sqrt[2]*b*x])/45 - (FresnelC[b*
x]*Sin[(b^2*Pi*x^2)/2])/(6*x^6) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/
2])/(48*x^2) - (b*Sin[b^2*Pi*x^2])/(60*x^5) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])
/(1440*x) - (b^6*Pi^3*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/x, x])
/48
```

Rubi steps

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx = -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} + \frac{1}{12}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^6}\,dx + \frac{1}{6}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^5}$$

$$= -\frac{b^3\pi}{144x^3} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{24x^4} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} - \frac{b\sin\left(b^2\pi x^2\right)}{60x^5} + \frac{1}{48}$$

$$= -\frac{b^3\pi}{144x^3} - \frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{24x^4} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} -$$

$$= -\frac{b^3\pi}{144x^3} - \frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{24x^4} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{6x^6} -$$

$$= -\frac{b^3\pi}{144x^3} - \frac{13b^3\pi\cos\left(b^2\pi x^2\right)}{720x^3} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{24x^4} - \frac{7b^6\pi^3C\left(\sqrt{2}\,bx\right)}{144\sqrt{2}} -$$

Mathematica [**A**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7,x]
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^7, x]
```

Maple [**A**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^7,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="fricas")
```

```
[Out] integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**7,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**7, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^7, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^7,x)
```

```
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^7, x)
```

3.216 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$

Optimal. Leaf size=202

$$-\frac{b^3\pi}{280x^4}-\frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4}-\frac{1}{84}b^7\pi^3\text{CosIntegral}\left(b^2\pi x^2\right)-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{35x^5}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\right.}{7x^7}$$

```
[Out] -1/280*b^3*Pi/x^4-1/84*b^7*Pi^3*Ci(b^2*Pi*x^2)-1/105*b^3*Pi*cos(b^2*Pi*x^2)
/x^4-1/35*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^5-1/7*FresnelC(b*x)*si
n(1/2*b^2*Pi*x^2)/x^7+1/105*b^4*Pi^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^3-
1/84*b*sin(b^2*Pi*x^2)/x^6+1/84*b^5*Pi^2*sin(b^2*Pi*x^2)/x^2-1/105*b^6*Pi^3
*Unintegrable(cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2,x)
```

Rubi [A]
time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]
```

```
[Out] -1/280*(b^3*Pi)/x^4 - (b^3*Pi*Cos[b^2*Pi*x^2])/(105*x^4) - (b^7*Pi^3*CosInt
egral[b^2*Pi*x^2])/84 - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(35*x^5)
 - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(7*x^7) + (b^4*Pi^2*FresnelC[b*x]*Si
n[(b^2*Pi*x^2)/2])/(105*x^3) - (b*Sin[b^2*Pi*x^2])/(84*x^6) + (b^5*Pi^2*Sin
[b^2*Pi*x^2])/(84*x^2) - (b^6*Pi^3*Defer[Int][(Cos[(b^2*Pi*x^2)/2]*FresnelC
[b*x])/x^2, x])/105
```

Rubi steps

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx = -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{1}{14}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^7}\,dx + \frac{1}{7}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^6}\,d$$

$$= -\frac{b^3\pi}{280x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{35x^5} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{1}{28}b\text{Subst}\left(\int \frac{\sin\left(b^2\right.}{x^4}\right.$$

$$= -\frac{b^3\pi}{280x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{35x^5} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{b^4\pi^2C(bx)\sin\left(\frac{1}{2}b^2\pi\right.}{105x^3}$$

$$= -\frac{b^3\pi}{280x^4} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{35x^5} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{b^4}{}$$

$$= -\frac{b^3\pi}{280x^4} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{35x^5} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{7x^7} + \frac{b^4}{}$$

$$= -\frac{b^3\pi}{280x^4} - \frac{b^3\pi\cos\left(b^2\pi x^2\right)}{105x^4} - \frac{1}{84}b^7\pi^3\text{Ci}\left(b^2\pi x^2\right) - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{35x^5} - \frac{C(b}{}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8,x]
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^8, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi x^2}{2}\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^8,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="fricas")
```

```
[Out] integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**8,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**8, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^8,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^8, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^8,x)
```

```
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^8, x)
```

3.217 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$

Optimal. Leaf size=271

$$-\frac{b^3\pi}{480x^5}+\frac{b^7\pi^3}{768x}-\frac{19b^3\pi\cos(b^2\pi x^2)}{3360x^5}+\frac{853b^7\pi^3\cos(b^2\pi x^2)}{80640x}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{48x^6}+\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)}{384x^2}$$

[Out] -1/480*b^3*Pi/x^5+1/768*b^7*Pi^3/x-19/3360*b^3*Pi*cos(b^2*Pi*x^2)/x^5+853/80640*b^7*Pi^3*cos(b^2*Pi*x^2)/x-1/48*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^6+1/384*b^6*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^2-1/8*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^8+1/192*b^4*Pi^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^4-1/112*b*sin(b^2*Pi*x^2)/x^7+187/40320*b^5*Pi^2*sin(b^2*Pi*x^2)/x^3+853/80640*b^8*Pi^4*FresnelS(b*x*2^(1/2))*2^(1/2)+1/384*b^8*Pi^4*Unintegrable(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x,x)

Rubi [A]
time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

Verification is not applicable to the result.

[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]

[Out] -1/480*(b^3*Pi)/x^5 + (b^7*Pi^3)/(768*x) - (19*b^3*Pi*Cos[b^2*Pi*x^2])/(3360*x^5) + (853*b^7*Pi^3*Cos[b^2*Pi*x^2])/(80640*x) - (b^2*Pi*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(48*x^6) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x])/(384*x^2) + (853*b^8*Pi^4*FresnelS[Sqrt[2]*b*x])/(40320*Sqrt[2]) - (FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(8*x^8) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(192*x^4) - (b*Sin[b^2*Pi*x^2])/(112*x^7) + (187*b^5*Pi^2*Sin[b^2*Pi*x^2])/(40320*x^3) + (b^8*Pi^4*Defer[Int][(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x, x])/384

Rubi steps

$$\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx = -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} + \frac{1}{16}b\int \frac{\sin\left(b^2\pi x^2\right)}{x^8}\,dx + \frac{1}{8}\left(b^2\pi\right)\int \frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^7}$$

$$= -\frac{b^3\pi}{480x^5} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{48x^6} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8} - \frac{b\sin\left(b^2\pi x^2\right)}{112x^7} + \frac{1}{96}$$

$$= -\frac{b^3\pi}{480x^5} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{48x^6} - \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{8x^8}$$

$$= -\frac{b^3\pi}{480x^5} + \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{48x^6} + \frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\right.}{384}$$

$$= -\frac{b^3\pi}{480x^5} + \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} + \frac{853b^7\pi^3\cos\left(b^2\pi x^2\right)}{80640x} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6}$$

$$= -\frac{b^3\pi}{480x^5} + \frac{b^7\pi^3}{768x} - \frac{19b^3\pi\cos\left(b^2\pi x^2\right)}{3360x^5} + \frac{853b^7\pi^3\cos\left(b^2\pi x^2\right)}{80640x} - \frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{48x^6}$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9,x]
[Out] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^9, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^9,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="maxima")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="fricas")
[Out] integral(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**9,x)
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**9, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^9,x, algorithm="giac")
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^9, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^9,x)
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^9, x)
```

3.218 $\int \frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx$

Optimal. Leaf size=278

$$-\frac{b^3\pi}{756x^6}+\frac{b^7\pi^3}{3780x^2}-\frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6}+\frac{5b^7\pi^3\cos\left(b^2\pi x^2\right)}{2016x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{63x^7}+\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right.}{945}$$

```
[Out] -1/756*b^3*Pi/x^6+1/3780*b^7*Pi^3/x^2-11/3024*b^3*Pi*cos(b^2*Pi*x^2)/x^6+5/
2016*b^7*Pi^3*cos(b^2*Pi*x^2)/x^2-1/63*b^2*Pi*cos(1/2*b^2*Pi*x^2)*FresnelC(
b*x)/x^7+1/945*b^6*Pi^3*cos(1/2*b^2*Pi*x^2)*FresnelC(b*x)/x^3+1/1890*b^9*Pi
^5*FresnelC(b*x)^2+83/30240*b^9*Pi^4*Si(b^2*Pi*x^2)-1/9*FresnelC(b*x)*sin(1
/2*b^2*Pi*x^2)/x^9+1/315*b^4*Pi^2*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^5-1/9
45*b^8*Pi^4*FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x-1/144*b*sin(b^2*Pi*x^2)/x^8
+67/30240*b^5*Pi^2*sin(b^2*Pi*x^2)/x^4
```

Rubi [A]
time = 0.34, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6600, 6592, 6576, 30, 3456, 3461, 3378, 3380, 3460}

$$\frac{\pi^5 b^9 \text{FresnelC}(bx)^2}{1890}+\frac{\pi^3 b^7}{3780x^2}-\frac{\pi b^3}{756x^6}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{9x^9}-\frac{\pi b^2\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{63x^7}-\frac{b\sin\left(\pi b^2x^2\right)}{144x^8}+\frac{83\pi^4 b^9\text{Si}\left(b^2\pi x^2\right)}{30240}-\frac{\pi^4 b^8\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{945x}+\frac{5\pi^3 b^7\cos\left(\pi b^2x^2\right)}{2016x^2}+\frac{\pi^3 b^6\text{FresnelC}(bx)\cos\left(\frac{1}{2}\pi b^2x^2\right)}{945x^3}+\frac{67\pi^2 b^5\sin\left(\pi b^2x^2\right)}{30240x^4}+\frac{\pi^2 b^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}\pi b^2x^2\right)}{315x^5}-\frac{11\pi b^3\cos\left(\pi b^2x^2\right)}{3024x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]
```

```
[Out] -1/756*(b^3*Pi)/x^6 + (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(
3024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (b^2*Pi*Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/(63*x^7) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x
])/(945*x^3) + (b^9*Pi^5*FresnelC[b*x]^2)/1890 - (FresnelC[b*x]*Sin[(b^2*Pi
*x^2)/2])/(9*x^9) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(315*x^5)
- (b^8*Pi^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x) - (b*Sin[b^2*Pi*x^2]
)/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinI
ntegral[b^2*Pi*x^2])/30240
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 6576

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[Pi*(b/(
2*d)), Subst[Int[x^n, x], x, FresnelC[b*x]], x] /; FreeQ[{b, d, n}, x] && E
qQ[d^2, (Pi^2/4)*b^4]
```

Rule 6592

```
Int[Cos[(d_.)*(x_)^2]*FresnelC[(b_.)*(x_)]*(x_)^(m_), x_Symbol] :> Simp[x^(
m + 1)*Cos[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (Dist[2*(d/(m + 1)), Int[x^
(m + 2)*Sin[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m + 1
)*Cos[2*d*x^2], x], x] - Simp[b*(x^(m + 2)/(2*(m + 1)*(m + 2))), x]) /; Fre
eQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && ILtQ[m, -2]
```

Rule 6600

```
Int[FresnelC[(b_.)*(x_)]*(x_)^(m_)*Sin[(d_.)*(x_)^2], x_Symbol] :> Simp[x^(
m + 1)*Sin[d*x^2]*(FresnelC[b*x]/(m + 1)), x] + (-Dist[2*(d/(m + 1)), Int[x
^(m + 2)*Cos[d*x^2]*FresnelC[b*x], x], x] - Dist[b/(2*(m + 1)), Int[x^(m +
1)*Sin[2*d*x^2], x], x]) /; FreeQ[{b, d}, x] && EqQ[d^2, (Pi^2/4)*b^4] && I
```

```
LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{x^{10}}\,dx &= -\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9}+\frac{1}{18}b\int\frac{\sin\left(b^2\pi x^2\right)}{x^9}\,dx+\frac{1}{9}\left(b^2\pi\right)\int\frac{\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{x^8} \\
&= -\frac{b^3\pi}{756x^6}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{63x^7}-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9}+\frac{1}{36}b\text{Subst}\left(\int\frac{\sin(}{x}\right. \\
&= -\frac{b^3\pi}{756x^6}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{63x^7}-\frac{C(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9}+\frac{b^4\pi^2C(bx)\sin\left(\frac{1}{2}b^2\right.}{315x^5} \\
&= -\frac{b^3\pi}{756x^6}+\frac{b^7\pi^3}{3780x^2}-\frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{63x^7}+\frac{b^6\pi^3\cos\left(\frac{1}{2}\right.}{9} \\
&= -\frac{b^3\pi}{756x^6}+\frac{b^7\pi^3}{3780x^2}-\frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)C(bx)}{63x^7}+\frac{b^6\pi^3\cos\left(\frac{1}{2}\right.}{9} \\
&= -\frac{b^3\pi}{756x^6}+\frac{b^7\pi^3}{3780x^2}-\frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6}+\frac{5b^7\pi^3\cos\left(b^2\pi x^2\right)}{2016x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7} \\
&= -\frac{b^3\pi}{756x^6}+\frac{b^7\pi^3}{3780x^2}-\frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6}+\frac{5b^7\pi^3\cos\left(b^2\pi x^2\right)}{2016x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)}{63x^7}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 278, normalized size = 1.00

$$-\frac{b^3\pi}{756x^6}+\frac{b^7\pi^3}{3780x^2}-\frac{11b^3\pi\cos\left(b^2\pi x^2\right)}{3024x^6}+\frac{5b^7\pi^3\cos\left(b^2\pi x^2\right)}{2016x^2}-\frac{b^2\pi\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{63x^7}+\frac{b^6\pi^3\cos\left(\frac{1}{2}b^2\pi x^2\right)\text{FresnelC}(bx)}{945x^3}+\frac{b^9\pi^5\text{FresnelC}(bx)^2}{1890}-\frac{\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{9x^9}+\frac{b^4\pi^2\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{315x^5}-\frac{b^8\pi^4\text{FresnelC}(bx)\sin\left(\frac{1}{2}b^2\pi x^2\right)}{945x}-\frac{b\sin\left(b^2\pi x^2\right)}{144x^8}+\frac{67b^5\pi^2\sin\left(b^2\pi x^2\right)}{30240x^4}+\frac{83b^9\pi^4\text{Si}\left(b^2\pi x^2\right)}{30240}$$

Antiderivative was successfully verified.

```
[In] Integrate[(FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/x^10,x]
[Out] -1/756*(b^3*Pi)/x^6 + (b^7*Pi^3)/(3780*x^2) - (11*b^3*Pi*Cos[b^2*Pi*x^2])/(
3024*x^6) + (5*b^7*Pi^3*Cos[b^2*Pi*x^2])/(2016*x^2) - (b^2*Pi*Cos[(b^2*Pi*x
^2)/2]*FresnelC[b*x])/(63*x^7) + (b^6*Pi^3*Cos[(b^2*Pi*x^2)/2]*FresnelC[b*x
])/(945*x^3) + (b^9*Pi^5*FresnelC[b*x]^2)/1890 - (FresnelC[b*x]*Sin[(b^2*Pi
*x^2)/2])/(9*x^9) + (b^4*Pi^2*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(315*x^5)
- (b^8*Pi^4*FresnelC[b*x]*Sin[(b^2*Pi*x^2)/2])/(945*x) - (b*Sin[b^2*Pi*x^2]
)/(144*x^8) + (67*b^5*Pi^2*Sin[b^2*Pi*x^2])/(30240*x^4) + (83*b^9*Pi^4*SinI
ntegral[b^2*Pi*x^2])/30240
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{FresnelC}\left(bx\right)\sin\left(\frac{b^2\pi\,x^2}{2}\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
```

```
[Out] int(FresnelC(b*x)*sin(1/2*b^2*Pi*x^2)/x^10,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="maxima")
```

```
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

Fricas [A]
time = 0.37, size = 203, normalized size = 0.73

$$\frac{16\,\pi^5 b^9 x^9\,\mathrm{C}\left(bx\right)^2 + 83\,\pi^4 b^9 x^9\,\mathrm{Si}\left(\pi b^2 x^2\right) - 67\,\pi^3 b^7 x^7 + 70\,\pi b^3 x^3 + 10\left(15\,\pi^3 b^7 x^7 - 22\,\pi b^3 x^3\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)^2 + 32\left(\pi^3 b^6 x^6 - 15\,\pi b^2 x^2\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right)\mathrm{C}\left(bx\right) + 2\left(\left(67\,\pi^2 b^5 x^5 - 210\,bx\right)\cos\left(\frac{1}{2}\,\pi b^2 x^2\right) - 16\left(\pi^4 b^8 x^8 - 3\,\pi^2 b^4 x^4 + 105\right)\mathrm{C}\left(bx\right)\right)\sin\left(\frac{1}{2}\,\pi b^2 x^2\right)}{30240\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/30240*(16*pi^5*b^9*x^9*fresnel_cos(b*x)^2 + 83*pi^4*b^9*x^9*sin_integral(
pi*b^2*x^2) - 67*pi^3*b^7*x^7 + 70*pi*b^3*x^3 + 10*(15*pi^3*b^7*x^7 - 22*pi
*b^3*x^3)*cos(1/2*pi*b^2*x^2)^2 + 32*(pi^3*b^6*x^6 - 15*pi*b^2*x^2)*cos(1/2
*pi*b^2*x^2)*fresnel_cos(b*x) + 2*((67*pi^2*b^5*x^5 - 210*b*x)*cos(1/2*pi*b
^2*x^2) - 16*(pi^4*b^8*x^8 - 3*pi^2*b^4*x^4 + 105)*fresnel_cos(b*x))*sin(1/
2*pi*b^2*x^2))/x^9
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\pi b^2 x^2}{2}\right) C(bx)}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnelc(b*x)*sin(1/2*b**2*pi*x**2)/x**10,x)
```

```
[Out] Integral(sin(pi*b**2*x**2/2)*fresnelc(b*x)/x**10, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(fresnel_cos(b*x)*sin(1/2*b^2*pi*x^2)/x^10,x, algorithm="giac")
```

```
[Out] integrate(fresnel_cos(b*x)*sin(1/2*pi*b^2*x^2)/x^10, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{FresnelC}(b\,x)\,\sin\left(\frac{\Pi\,b^2\,x^2}{2}\right)}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^10,x)
```

```
[Out] int((FresnelC(b*x)*sin((Pi*b^2*x^2)/2))/x^10, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(4,m1)  #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(5,m1)  #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(6,m1)  #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```