Computer algebra independent integration tests

Summer 2022 edition

7-Inverse-hyperbolic-functions/7.5-Inverse-hyperbolic-secant/201-7.5.2-Inverse-hyperbolic-secant-functions

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [100]. This is test number [201].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (100)	0.00 (0)
Mathematica	96.00 (96)	4.00 (4)
Maple	74.00 (74)	26.00 (26)
Fricas	73.00 (73)	27.00 (27)
Mupad	56.00 (56)	44.00 (44)
Maxima	21.00 (21)	79.00 (79)
Giac	4.00 (4)	96.00 (96)
Sympy	2.00 (2)	98.00 (98)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

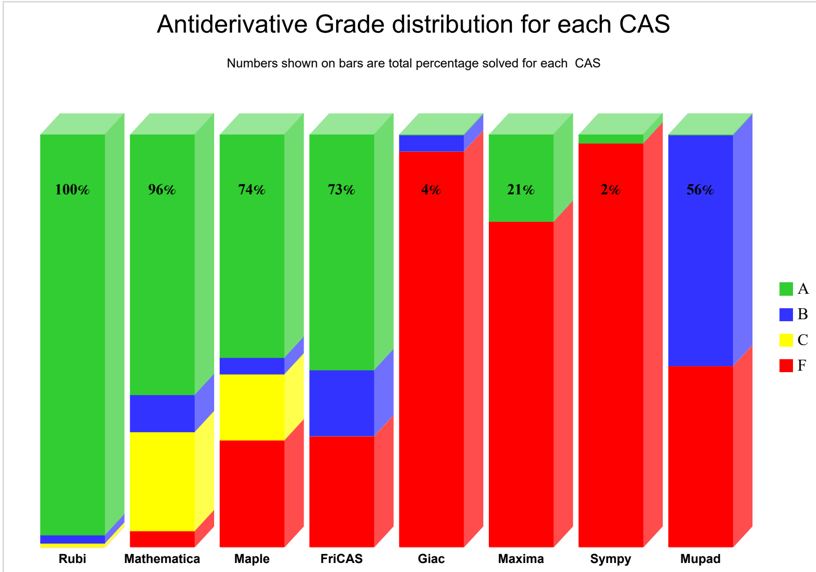
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

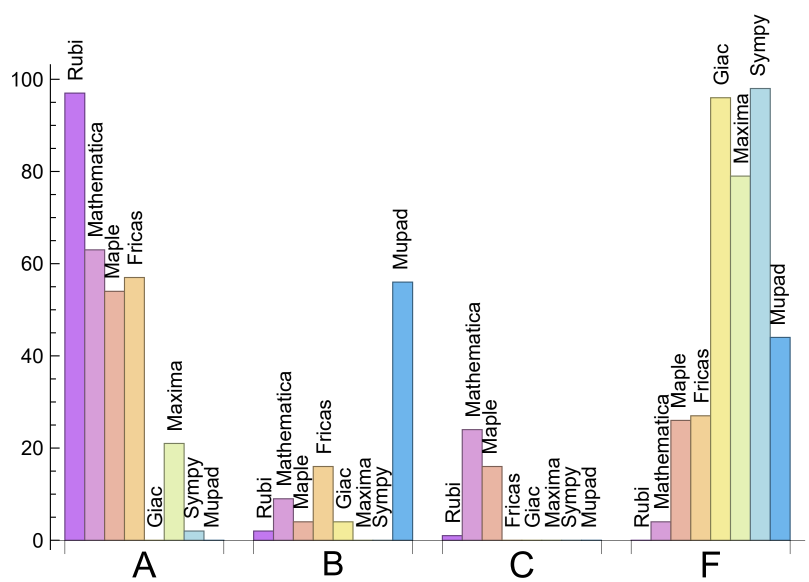
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.00	2.00	1.00	0.00
Mathematica	63.00	9.00	24.00	4.00
Fricas	57.00	16.00	0.00	27.00
Maple	54.00	4.00	16.00	26.00
Maxima	21.00	0.00	0.00	79.00
Sympy	2.00	0.00	0.00	98.00
Mupad	N/A	56.00	0.00	44.00
Giac	0.00	4.00	0.00	96.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	4	75.00 %	25.00 %	0.00 %
Maple	26	100.00 %	0.00 %	0.00 %
Fricas	27	77.78 %	0.00 %	22.22 %
Giac	96	89.58 %	0.00 %	10.42 %
Maxima	79	93.67 %	0.00 %	6.33 %
Sympy	98	98.98 %	1.02 %	0.00 %
Mupad	44	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

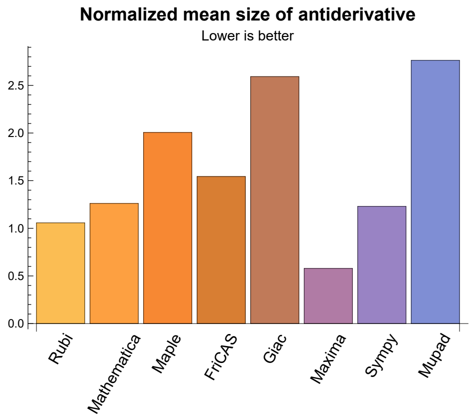
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

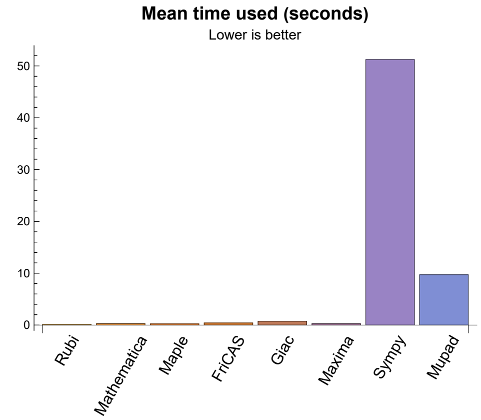
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.15	135.47	1.06	107.00	1.00
Mathematica	0.28	140.55	1.26	105.00	1.10
Maple	0.24	194.65	2.01	107.50	1.23
Maxima	0.27	49.76	0.58	46.00	0.67
Fricas	0.42	135.45	1.54	85.00	1.05
Sympy	51.22	82.00	1.23	82.00	1.23
Giac	0.74	194.75	2.59	197.50	2.62
Mupad	9.71	329.71	2.76	88.50	1.74

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0.0
0.5
1.0
1.5
2.0
2.5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
10
20
30
40
50
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {32, 34, 36, 37, 38, 39, 47, 53, 63, 97}

Mathematica {5, 12, 17, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 72, 73, 74, 75, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 92, 93, 94, 95, 96, 97, 98}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

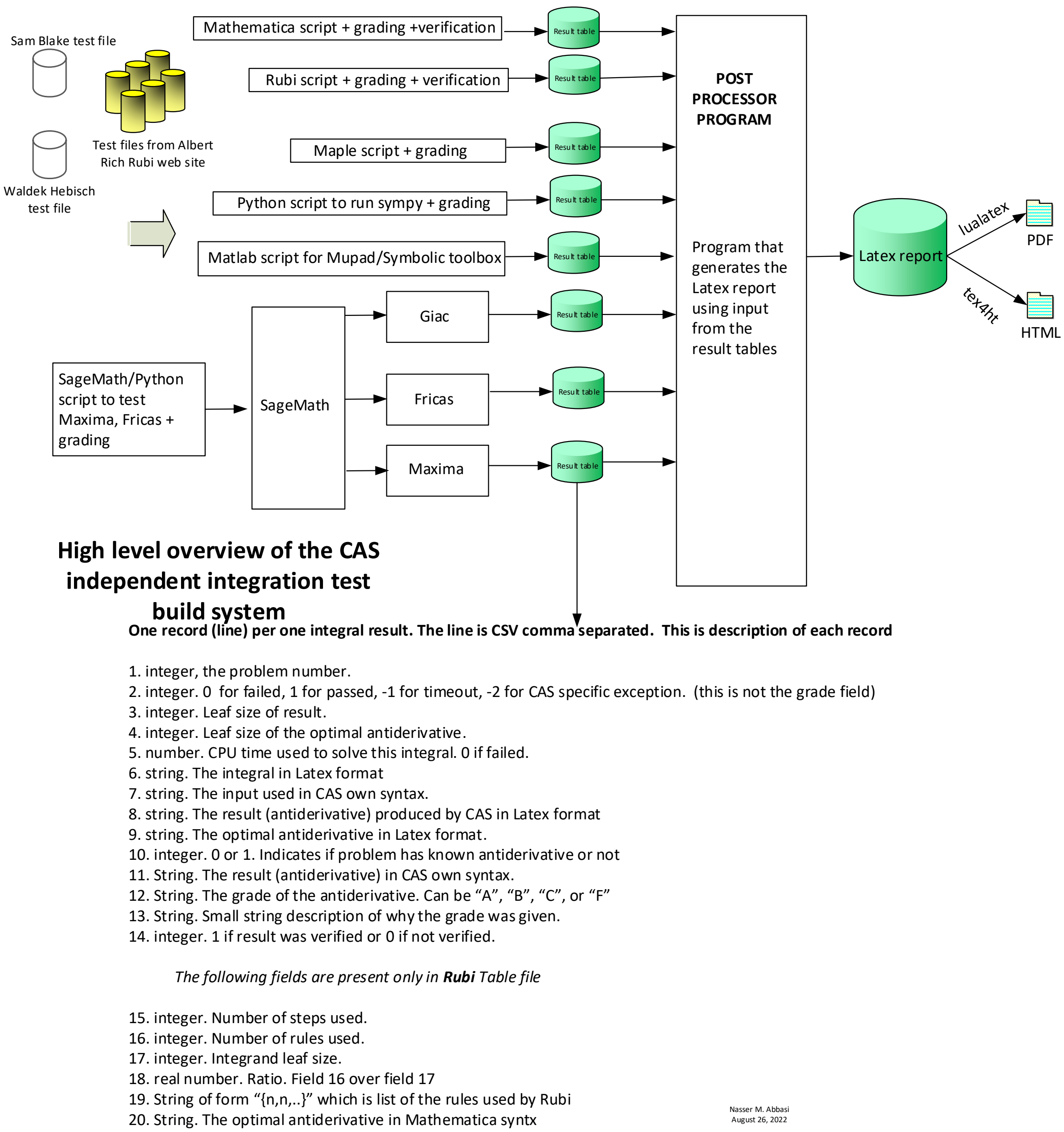
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100 }

B grade: { 38, 97 }

C grade: { 39 }

F grade: { }

2.1.2 Mathematica

A grade: { 8, 9, 10, 11, 12, 15, 16, 17, 20, 21, 22, 24, 25, 26, 27, 28, 30, 32, 34, 39, 40, 41, 42, 43, 44, 47, 51, 53, 55, 56, 57, 59, 60, 61, 62, 64, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 80, 82, 83, 84, 85, 86, 87, 89, 91, 93, 94, 95, 96, 98, 99, 100 }

B grade: { 4, 6, 7, 23, 29, 31, 36, 38, 97 }

C grade: { 1, 2, 3, 5, 13, 14, 33, 35, 37, 45, 46, 48, 49, 50, 52, 54, 63, 65, 67, 77, 79, 81, 90, 92 }

F grade: { 18, 19, 58, 88 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 9, 10, 11, 13, 14, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 66, 68, 70, 71, 72, 73, 74, 75, 89, 91, 93, 95, 97, 98, 99 }

B grade: { 6, 7, 8, 80 }

C grade: { 5, 33, 35, 37, 51, 55, 63, 65, 67, 69, 76, 78, 90, 92, 94, 96 }

F grade: { 12, 15, 16, 17, 18, 19, 30, 56, 57, 58, 59, 60, 61, 62, 64, 77, 79, 81, 82, 83, 84, 85, 86, 87, 88, 100 }

2.1.4 Maxima

A grade: { 4, 20, 21, 22, 23, 25, 26, 27, 28, 32, 34, 39, 41, 43, 47, 66, 71, 73, 75, 99, 100 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 24, 29, 30, 31, 33, 35, 36, 37, 38, 40, 42, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 72, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

2.1.5 FriCAS

A grade: { 1, 20, 21, 22, 23, 25, 26, 27, 28, 32, 33, 34, 35, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 55, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 91, 94, 96, 97 }

B grade: { 2, 3, 4, 6, 7, 8, 36, 38, 51, 53, 90, 92, 93, 95, 99, 100 }

C grade: { }

F grade: { 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 24, 29, 30, 31, 52, 56, 57, 58, 59, 60, 61, 62, 64, 88, 98 }

2.1.6 Sympy

A grade: { 80, 93 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100 }

2.1.7 Giac

A grade: { }

B grade: { 45, 47, 49, 53 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 48, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100 }

2.1.8 Mupad

A grade: { }

B grade: { 4, 25, 28, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 49, 51, 53, 55, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100 }

C grade: { }

F grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 29, 30, 31, 46, 48, 50, 52, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 88, 98 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	225	250	0	345	0	0	-1
N.S.	1	1.00	1.11	1.23	0.00	1.70	0.00	0.00	-0.00
time (sec)	N/A	0.109	0.352	0.324	0.000	0.483	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	200	189	0	327	0	0	-1
N.S.	1	1.00	1.31	1.24	0.00	2.14	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.170	0.315	0.000	0.498	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	176	111	0	308	0	0	-1
N.S.	1	1.00	1.64	1.04	0.00	2.88	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.106	0.306	0.000	0.389	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	97	44	31	253	0	0	43
N.S.	1	1.00	2.20	1.00	0.70	5.75	0.00	0.00	0.98
time (sec)	N/A	0.038	0.180	0.044	0.251	0.437	0.000	0.000	2.161

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	332	882	0	0	0	0	-1
N.S.	1	1.00	1.95	5.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.207	0.147	0.891	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	244	171	0	651	0	0	-1
N.S.	1	1.00	3.49	2.44	0.00	9.30	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.171	0.444	0.000	0.400	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	315	483	0	865	0	0	-1
N.S.	1	1.00	2.37	3.63	0.00	6.50	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.681	0.460	0.000	0.443	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	368	1027	0	987	0	0	-1
N.S.	1	1.00	1.87	5.21	0.00	5.01	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.274	0.452	0.000	0.489	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	305	599	0	0	0	0	-1
N.S.	1	1.00	1.09	2.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.165	1.394	1.294	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	172	331	0	0	0	0	-1
N.S.	1	1.00	1.15	2.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.304	1.128	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	105	192	0	0	0	0	-1
N.S.	1	1.00	1.31	2.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.146	0.578	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	280	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.313	0.213	0.421	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	678	362	0	0	0	0	-1
N.S.	1	1.00	3.03	1.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.277	1.830	0.994	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	537	537	1439	982	0	0	0	0	-1
N.S.	1	1.00	2.68	1.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.524	4.726	1.398	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	254	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.183	0.374	0.990	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	153	0	0	0	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.102	0.191	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	384	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.353	0.226	0.369	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.397	180.003	0.536	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	965	965	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.905	7.463	0.944	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	84	54	58	57	0	0	-1
N.S.	1	1.00	0.51	0.33	0.35	0.35	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.042	0.190	0.250	0.382	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	72	49	46	52	0	0	-1
N.S.	1	1.00	0.57	0.39	0.37	0.41	0.00	0.00	-0.01
time (sec)	N/A	0.018	0.023	0.189	0.264	0.391	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	56	42	34	45	0	0	-1
N.S.	1	1.00	0.64	0.48	0.39	0.51	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.021	0.193	0.247	0.544	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	118	36	19	39	0	0	-1
N.S.	1	1.00	2.74	0.84	0.44	0.91	0.00	0.00	-0.02
time (sec)	N/A	0.005	0.097	0.189	0.261	0.488	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	65	0	0	0	0	-1
N.S.	1	1.00	0.98	1.41	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.060	0.032	0.299	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	111	64	65	45	0	0	40
N.S.	1	1.00	1.13	0.65	0.66	0.46	0.00	0.00	0.41
time (sec)	N/A	0.016	0.053	0.174	0.265	0.467	0.000	0.000	2.150

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	125	79	92	54	0	0	-1
N.S.	1	1.00	0.92	0.58	0.68	0.40	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.078	0.186	0.250	0.498	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	140	91	116	60	0	0	-1
N.S.	1	1.00	0.81	0.53	0.67	0.35	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.097	0.214	0.264	0.365	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	25	29	16	22	0	0	17
N.S.	1	1.00	1.19	1.38	0.76	1.05	0.00	0.00	0.81
time (sec)	N/A	0.005	0.010	0.167	0.269	0.504	0.000	0.000	1.349

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	219	111	0	0	0	0	-1
N.S.	1	1.00	3.59	1.82	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.071	0.731	0.599	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	49	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.066	0.033	0.072	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	249	135	0	0	0	0	-1
N.S.	1	1.00	3.23	1.75	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.984	0.598	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	83	65	64	47	65	0	0	75
N.S.	1	1.30	1.02	1.00	0.73	1.02	0.00	0.00	1.17
time (sec)	N/A	0.025	0.063	0.033	0.279	0.398	0.000	0.000	1.469

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	97	118	0	95	0	0	521
N.S.	1	1.00	1.15	1.40	0.00	1.13	0.00	0.00	6.20
time (sec)	N/A	0.021	0.090	0.033	0.000	0.428	0.000	0.000	11.925

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	38	52	48	54	38	54	0	0	55
N.S.	1	1.37	1.26	1.42	1.00	1.42	0.00	0.00	1.45
time (sec)	N/A	0.015	0.045	0.028	0.269	0.379	0.000	0.000	1.433

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	75	92	0	79	0	0	303
N.S.	1	1.00	1.42	1.74	0.00	1.49	0.00	0.00	5.72
time (sec)	N/A	0.012	0.050	0.031	0.000	0.393	0.000	0.000	6.959

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	24	39	79	80	0	115	0	0	182
N.S.	1	1.62	3.29	3.33	0.00	4.79	0.00	0.00	7.58
time (sec)	N/A	0.081	0.029	0.065	0.000	0.387	0.000	0.000	2.983

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	64	75	92	0	77	0	0	184
N.S.	1	1.33	1.56	1.92	0.00	1.60	0.00	0.00	3.83
time (sec)	N/A	0.020	0.041	0.031	0.000	0.404	0.000	0.000	3.104

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	A	F	B	F	F	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	35	99	93	91	0	128	0	0	71
N.S.	1	2.83	2.66	2.60	0.00	3.66	0.00	0.00	2.03
time (sec)	N/A	0.027	0.044	0.032	0.000	0.407	0.000	0.000	1.842

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	A	A	A	F	F	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	84	43	53	43	52	0	0	58
N.S.	1	1.53	0.78	0.96	0.78	0.95	0.00	0.00	1.05
time (sec)	N/A	0.024	0.030	0.032	0.266	0.415	0.000	0.000	1.473

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	110	0	138	0	0	602
N.S.	1	1.00	0.83	0.83	0.00	1.05	0.00	0.00	4.56
time (sec)	N/A	0.038	0.052	0.033	0.000	0.473	0.000	0.000	13.421

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	60	63	51	60	0	0	76
N.S.	1	1.00	0.52	0.55	0.44	0.52	0.00	0.00	0.66
time (sec)	N/A	0.035	0.046	0.032	0.279	0.362	0.000	0.000	1.562

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	129	132	0	148	0	0	878
N.S.	1	1.00	0.79	0.81	0.00	0.91	0.00	0.00	5.39
time (sec)	N/A	0.051	0.077	0.036	0.000	0.402	0.000	0.000	34.076

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	76	71	60	69	0	0	95
N.S.	1	1.00	0.52	0.49	0.41	0.47	0.00	0.00	0.65
time (sec)	N/A	0.050	0.059	0.036	0.298	0.450	0.000	0.000	1.675

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	145	152	0	156	0	0	1155
N.S.	1	1.00	0.75	0.78	0.00	0.80	0.00	0.00	5.95
time (sec)	N/A	0.069	0.084	0.048	0.000	0.429	0.000	0.000	38.560

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	111	137	0	116	0	205	521
N.S.	1	1.00	1.00	1.23	0.00	1.05	0.00	1.85	4.69
time (sec)	N/A	0.042	0.125	0.105	0.000	0.415	0.000	0.438	14.445

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	139	114	0	61	0	0	-1
N.S.	1	1.00	1.21	0.99	0.00	0.53	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.212	0.032	0.000	0.078	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	58	71	56	60	42	60	0	190	57
N.S.	1	1.22	0.97	1.03	0.72	1.03	0.00	3.28	0.98
time (sec)	N/A	0.028	0.070	0.026	0.285	0.457	0.000	0.424	1.638

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	140	136	0	58	0	0	-1
N.S.	1	1.00	1.25	1.21	0.00	0.52	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.301	0.033	0.000	0.085	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	92	112	0	102	0	132	306
N.S.	1	1.00	1.46	1.78	0.00	1.62	0.00	2.10	4.86
time (sec)	N/A	0.025	0.073	0.068	0.000	0.593	0.000	0.422	7.139

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	116	102	0	47	0	0	-1
N.S.	1	1.00	1.73	1.52	0.00	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.136	0.031	0.000	0.134	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	100	127	0	133	0	0	182
N.S.	1	1.00	1.47	1.87	0.00	1.96	0.00	0.00	2.68
time (sec)	N/A	0.029	0.047	0.099	0.000	0.362	0.000	0.000	3.202

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	135	132	0	0	0	0	-1
N.S.	1	1.00	0.92	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.198	0.098	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	93	113	103	0	102	0	252	185
N.S.	1	1.16	1.41	1.29	0.00	1.28	0.00	3.15	2.31
time (sec)	N/A	0.038	0.101	0.074	0.000	0.423	0.000	1.685	3.996

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	123	104	0	63	0	0	-1
N.S.	1	1.00	1.07	0.90	0.00	0.55	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.137	0.034	0.000	0.077	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	105	129	0	146	0	0	71
N.S.	1	1.00	0.89	1.09	0.00	1.24	0.00	0.00	0.60
time (sec)	N/A	0.042	0.194	0.076	0.000	0.435	0.000	0.000	2.017

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	214	0	0	0	0	0	-1
N.S.	1	1.00	1.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	1.807	0.040	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	214	0	0	0	0	0	-1
N.S.	1	1.00	2.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	1.716	0.023	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.606	0.022	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	147	0	0	0	0	0	-1
N.S.	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.621	0.201	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	212	0	0	0	0	0	-1
N.S.	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	3.784	0.422	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	159	0	0	0	0	0	-1
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.205	0.411	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	139	0	0	0	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.284	0.420	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	106	96	116	0	102	0	0	-1
N.S.	1	1.22	1.10	1.33	0.00	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.110	0.618	0.000	0.610	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	156	0	0	0	0	0	-1
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.199	0.461	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	105	136	0	103	0	0	808
N.S.	1	1.00	0.52	0.67	0.00	0.51	0.00	0.00	3.98
time (sec)	N/A	0.480	0.102	0.043	0.000	0.496	0.000	0.000	19.727

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	52	72	42	62	0	0	63
N.S.	1	1.00	0.44	0.62	0.36	0.53	0.00	0.00	0.54
time (sec)	N/A	0.367	0.050	0.027	0.269	0.390	0.000	0.000	1.751

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	86	105	0	87	0	0	420
N.S.	1	1.00	0.51	0.62	0.00	0.51	0.00	0.00	2.49
time (sec)	N/A	0.309	0.051	0.033	0.000	0.433	0.000	0.000	8.956

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	89	98	0	124	0	0	56
N.S.	1	1.00	1.05	1.15	0.00	1.46	0.00	0.00	0.66
time (sec)	N/A	0.282	0.043	0.036	0.000	0.451	0.000	0.000	3.692

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	75	111	0	85	0	0	162
N.S.	1	1.00	1.32	1.95	0.00	1.49	0.00	0.00	2.84
time (sec)	N/A	0.110	0.069	0.040	0.000	0.481	0.000	0.000	4.636

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	110	0	138	0	0	323
N.S.	1	1.00	1.00	1.28	0.00	1.60	0.00	0.00	3.76
time (sec)	N/A	0.300	0.040	0.035	0.000	0.354	0.000	0.000	11.205

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	52	73	46	61	0	0	67
N.S.	1	1.00	0.91	1.28	0.81	1.07	0.00	0.00	1.18
time (sec)	N/A	0.257	0.046	0.033	0.272	0.347	0.000	0.000	1.796

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	121	131	0	146	0	0	885
N.S.	1	1.00	0.82	0.89	0.00	0.99	0.00	0.00	6.02
time (sec)	N/A	0.297	0.085	0.035	0.000	0.539	0.000	0.000	46.991

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	69	84	56	69	0	0	86
N.S.	1	1.00	0.38	0.46	0.31	0.38	0.00	0.00	0.47
time (sec)	N/A	0.334	0.057	0.033	0.273	0.424	0.000	0.000	1.938

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	137	153	0	156	0	0	2480
N.S.	1	1.00	0.51	0.57	0.00	0.58	0.00	0.00	9.29
time (sec)	N/A	0.370	0.103	0.041	0.000	0.440	0.000	0.000	65.189

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	85	92	65	78	0	0	105
N.S.	1	1.00	0.28	0.31	0.22	0.26	0.00	0.00	0.35
time (sec)	N/A	0.378	0.069	0.037	0.268	0.362	0.000	0.000	2.125

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	65	531	0	65	0	0	73
N.S.	1	1.00	0.44	3.61	0.00	0.44	0.00	0.00	0.50
time (sec)	N/A	0.404	0.062	0.216	0.000	0.406	0.000	0.000	2.138

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	97	0	0	95	0	0	795
N.S.	1	1.00	0.60	0.00	0.00	0.58	0.00	0.00	4.88
time (sec)	N/A	0.362	0.084	180.000	0.000	0.389	0.000	0.000	19.825

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	48	269	0	54	0	0	57
N.S.	1	1.00	0.64	3.59	0.00	0.72	0.00	0.00	0.76
time (sec)	N/A	0.318	0.042	0.173	0.000	0.369	0.000	0.000	2.063

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	75	0	0	79	0	0	407
N.S.	1	1.00	0.80	0.00	0.00	0.84	0.00	0.00	4.33
time (sec)	N/A	0.194	0.049	180.000	0.000	0.366	0.000	0.000	9.223

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	72	2616	0	115	116	0	47
N.S.	1	1.00	1.11	40.25	0.00	1.77	1.78	0.00	0.72
time (sec)	N/A	0.098	0.026	0.149	0.000	0.358	92.997	0.000	4.169

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	74	0	0	76	0	0	184
N.S.	1	1.00	1.61	0.00	0.00	1.65	0.00	0.00	4.00
time (sec)	N/A	0.244	0.035	180.000	0.000	0.460	0.000	0.000	3.845

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	92	0	0	128	0	0	323
N.S.	1	1.00	1.28	0.00	0.00	1.78	0.00	0.00	4.49
time (sec)	N/A	0.249	0.039	180.000	0.000	0.430	0.000	0.000	12.053

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	43	0	0	52	0	0	58
N.S.	1	1.00	0.37	0.00	0.00	0.45	0.00	0.00	0.50
time (sec)	N/A	0.276	0.037	180.000	0.000	0.368	0.000	0.000	2.137

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	110	0	0	138	0	0	1511
N.S.	1	1.00	0.55	0.00	0.00	0.69	0.00	0.00	7.56
time (sec)	N/A	0.326	0.071	180.000	0.000	0.430	0.000	0.000	43.714

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	60	0	0	60	0	0	75
N.S.	1	1.00	0.26	0.00	0.00	0.26	0.00	0.00	0.32
time (sec)	N/A	0.339	0.053	180.000	0.000	0.387	0.000	0.000	2.366

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	129	0	0	148	0	0	2479
N.S.	1	1.00	0.40	0.00	0.00	0.46	0.00	0.00	7.75
time (sec)	N/A	0.382	0.099	180.000	0.000	0.500	0.000	0.000	65.404

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	76	0	0	69	0	0	91
N.S.	1	1.00	0.22	0.00	0.00	0.20	0.00	0.00	0.26
time (sec)	N/A	0.405	0.067	180.000	0.000	0.348	0.000	0.000	2.593

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.453	0.111	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	69	74	0	69	0	0	76
N.S.	1	1.00	0.78	0.84	0.00	0.78	0.00	0.00	0.86
time (sec)	N/A	0.117	0.137	0.296	0.000	0.340	0.000	0.000	2.603

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	110	122	0	91	0	0	340
N.S.	1	1.00	1.47	1.63	0.00	1.21	0.00	0.00	4.53
time (sec)	N/A	0.107	0.133	0.322	0.000	0.444	0.000	0.000	8.085

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	52	0	49	0	0	44
N.S.	1	1.00	0.98	1.16	0.00	1.09	0.00	0.00	0.98
time (sec)	N/A	0.083	0.081	0.310	0.000	0.395	0.000	0.000	2.700

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	68	97	0	53	0	0	84
N.S.	1	1.00	1.84	2.62	0.00	1.43	0.00	0.00	2.27
time (sec)	N/A	0.052	0.036	0.291	0.000	0.528	0.000	0.000	3.567

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	73	82	0	92	48	0	59
N.S.	1	1.00	1.03	1.15	0.00	1.30	0.68	0.00	0.83
time (sec)	N/A	0.079	0.034	0.304	0.000	0.369	9.444	0.000	2.918

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	59	65	0	62	0	0	37
N.S.	1	1.00	1.40	1.55	0.00	1.48	0.00	0.00	0.88
time (sec)	N/A	0.095	0.127	0.296	0.000	0.557	0.000	0.000	2.528

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	119	0	156	0	0	331
N.S.	1	1.00	1.00	1.10	0.00	1.44	0.00	0.00	3.06
time (sec)	N/A	0.121	0.110	0.294	0.000	0.565	0.000	0.000	14.525

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	90	90	0	89	0	0	75
N.S.	1	1.00	1.06	1.06	0.00	1.05	0.00	0.00	0.88
time (sec)	N/A	0.109	0.183	0.339	0.000	0.579	0.000	0.000	2.505

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	A	F	A	F	F	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	12	26	28	36	0	35	0	0	76
N.S.	1	2.17	2.33	3.00	0.00	2.92	0.00	0.00	6.33
time (sec)	N/A	0.739	0.165	0.305	0.000	0.507	0.000	0.000	2.962

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	52	111	0	0	0	0	-1
N.S.	1	1.00	0.85	1.82	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.042	0.488	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	106	53	38	283	0	0	56
N.S.	1	1.00	1.86	0.93	0.67	4.96	0.00	0.00	0.98
time (sec)	N/A	0.074	0.118	0.068	0.256	0.401	0.000	0.000	2.992

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	106	0	40	385	0	0	54
N.S.	1	1.00	1.83	0.00	0.69	6.64	0.00	0.00	0.93
time (sec)	N/A	0.079	0.149	0.050	0.252	0.418	0.000	0.000	2.349

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [97] had the largest ratio of [25]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	7	1.00	10	0.700
2	A	7	6	1.00	10	0.600
3	A	6	6	1.00	8	0.750
4	A	4	4	1.00	6	0.667
5	A	14	8	1.00	10	0.800
6	A	5	5	1.00	10	0.500
7	A	7	7	1.00	10	0.700
8	A	8	8	1.00	10	0.800
9	A	17	9	1.00	12	0.750
10	A	11	8	1.00	10	0.800
11	A	8	6	1.00	8	0.750
12	A	17	9	1.00	12	0.750
13	A	12	8	1.00	12	0.667
14	A	23	11	1.00	12	0.917
15	A	16	12	1.00	10	1.200
16	A	10	7	1.00	8	0.875
17	A	20	10	1.00	12	0.833
18	A	14	9	1.00	12	0.750
19	A	32	13	1.00	12	1.083
20	A	4	3	1.00	10	0.300
21	A	4	3	1.00	10	0.300
22	A	4	3	1.00	8	0.375
23	A	3	3	1.00	6	0.500
24	A	7	6	1.00	10	0.600
25	A	5	5	1.00	10	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	6	5	1.00	10	0.500
27	A	7	5	1.00	10	0.500
28	A	3	3	1.00	4	0.750
29	A	7	6	1.00	10	0.600
30	A	7	6	1.00	10	0.600
31	A	7	7	1.00	10	0.700
32	A	5	5	1.30	10	0.500
33	A	5	5	1.00	10	0.500
34	A	3	3	1.37	10	0.300
35	A	4	4	1.00	8	0.500
36	A	3	3	1.62	6	0.500
37	A	5	5	1.33	10	0.500
38	B	6	6	2.83	10	0.600
39	C	5	5	1.53	10	0.500
40	A	8	6	1.00	10	0.600
41	A	7	5	1.00	10	0.500
42	A	10	6	1.00	10	0.600
43	A	9	5	1.00	10	0.500
44	A	12	6	1.00	10	0.600
45	A	6	6	1.00	12	0.500
46	A	5	5	1.00	12	0.417
47	A	4	4	1.22	12	0.333
48	A	7	7	1.00	12	0.583
49	A	5	5	1.00	12	0.417
50	A	4	4	1.00	12	0.333
51	A	6	6	1.00	10	0.600
52	A	8	8	1.00	8	1.000
53	A	5	5	1.16	12	0.417
54	A	5	5	1.00	12	0.417
55	A	7	7	1.00	12	0.583
56	A	4	4	1.00	12	0.333
57	A	4	4	1.00	12	0.333
58	A	4	4	1.00	10	0.400
59	A	5	5	1.00	12	0.417
60	A	4	4	1.00	12	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	4	4	1.00	10	0.400
62	A	4	4	1.00	8	0.500
63	A	6	6	1.22	12	0.500
64	A	4	4	1.00	12	0.333
65	A	9	6	1.00	12	0.500
66	A	8	6	1.00	12	0.500
67	A	7	6	1.00	12	0.500
68	A	8	5	1.00	10	0.500
69	A	7	5	1.00	8	0.625
70	A	5	3	1.00	12	0.250
71	A	4	2	1.00	12	0.167
72	A	5	3	1.00	12	0.250
73	A	4	2	1.00	12	0.167
74	A	5	3	1.00	12	0.250
75	A	4	2	1.00	12	0.167
76	A	8	5	1.00	12	0.417
77	A	7	5	1.00	12	0.417
78	A	6	5	1.00	12	0.417
79	A	5	4	1.00	10	0.400
80	A	6	4	1.00	8	0.500
81	A	5	3	1.00	12	0.250
82	A	5	3	1.00	12	0.250
83	A	4	2	1.00	12	0.167
84	A	5	3	1.00	12	0.250
85	A	4	2	1.00	12	0.167
86	A	5	3	1.00	12	0.250
87	A	4	2	1.00	12	0.167
88	A	5	4	1.00	24	0.167
89	A	8	7	1.00	22	0.318
90	A	7	7	1.00	22	0.318
91	A	4	4	1.00	22	0.182
92	A	5	5	1.00	20	0.250
93	A	8	8	1.00	19	0.421
94	A	5	5	1.00	22	0.227
95	A	9	8	1.00	22	0.364

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	8	7	1.00	22	0.318
97	B	7	5	2.17	25	0.200
98	A	8	8	1.00	19	0.421
99	A	5	5	1.00	12	0.417
100	A	5	5	1.00	14	0.357

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^3 \operatorname{sech}^{-1}(a+bx)\,dx$

Optimal. Leaf size=203

$$-\frac{(2+17a^2)\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{12b^4}-\frac{x^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{12b^2}+\frac{a(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+}{3b^4}$$

```
[Out] -1/4*a^4*arcsech(b*x+a)/b^4+1/4*x^4*arcsech(b*x+a)+1/2*a*(2*a^2+1)*arctan((
b*x+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/(b*x+a))/b^4-1/12*(17*a^2+2)*(b*x+a+1
)*((-b*x-a+1)/(b*x+a+1))^(1/2)/b^4-1/12*x^2*(b*x+a+1)*((-b*x-a+1)/(b*x+a+1)
)^(1/2)/b^2+1/3*a*(b*x+a)*(b*x+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/b^4
```

Rubi [A]
time = 0.11, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6456, 5576, 3867, 4133, 3855, 3852, 8}

$$-\frac{a^4\operatorname{sech}^{-1}(a+bx)}{4b^4}+\frac{(2a^2+1)\,a\operatorname{ArcTan}\left(\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{a+bx}\right)}{2b^4}-\frac{(17a^2+2)\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{12b^4}+\frac{a(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{3b^4}-\frac{x^2\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{12b^2}+\frac{1}{4}x^4\operatorname{sech}^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSech[a + b*x],x]
```

```
[Out] -1/12*((2 + 17*a^2)*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/b^4 -
(x^2*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(12*b^2) + (a*(a + b*
x)*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(3*b^4) - (a^4*ArcSech[
a + b*x])/(4*b^4) + (x^4*ArcSech[a + b*x])/4 + (a*(1 + 2*a^2)*ArcTan[(Sqrt[
(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(a + b*x)])/(2*b^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4133

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A
 + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x]
```

Rule 5576

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b
*d*(n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6456

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int x^3 \operatorname{sech}^{-1}(a+bx)\,dx = -\frac{\text{Subst}\left(\int x\operatorname{sech}(x)(-a+\operatorname{sech}(x))^3\tanh(x)\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b^4}$$

$$= \frac{1}{4}x^4\operatorname{sech}^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\operatorname{sech}(x))^4\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{4b^4}$$

$$= -\frac{x^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{12b^2} + \frac{1}{4}x^4\operatorname{sech}^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\operatorname{sech}(x))\right.\ \cdots}{\cdots}$$

$$= -\frac{x^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{12b^2} + \frac{a(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{3b^4} + \frac{1}{4}x^4\operatorname{sec}\cdots$$

$$= -\frac{x^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{12b^2} + \frac{a(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{3b^4} - \frac{a^4\operatorname{sech}\cdots}{\cdots}$$

$$= -\frac{x^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{12b^2} + \frac{a(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{3b^4} - \frac{a^4\operatorname{sech}\cdots}{\cdots}$$

$$= -\frac{(2+17a^2)\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{12b^4} - \frac{x^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{12b^2} + \frac{a(a+\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.35, size = 225, normalized size = 1.11

$$-\frac{\sqrt{-\frac{-1+a+bx}{1+a+bx}}\left(2+2a+13a^2+13a^3+(2-4a+9a^2)bx+(1-3a)b^2x^2+b^3x^3\right)-3b^4x^4\operatorname{sech}^{-1}(a+bx)-3a^4\log(a+bx)+3a^4\log\left(1+\sqrt{-\frac{-1+a+bx}{1+a+bx}}+a\sqrt{-\frac{-1+a+bx}{1+a+bx}}+bx\sqrt{-\frac{-1+a+bx}{1+a+bx}}\right)+6ia(1+2a^2)\log\left(-2i(a+bx)+2\sqrt{-\frac{-1+a+bx}{1+a+bx}}\,(1+a+bx)\right)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSech[a + b*x],x]
```

```
[Out] -1/12*(Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(2 + 2*a + 13*a^2 + 13*a^3 + (
2 - 4*a + 9*a^2)*b*x + (1 - 3*a)*b^2*x^2 + b^3*x^3) - 3*b^4*x^4*ArcSech[a +
 b*x] - 3*a^4*Log[a + b*x] + 3*a^4*Log[1 + Sqrt[-((-1 + a + b*x)/(1 + a + b
*x))] + a*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + b*x*Sqrt[-((-1 + a + b*x)
/(1 + a + b*x))]] + (6*I)*a*(1 + 2*a^2)*Log[(-2*I)*(a + b*x) + 2*Sqrt[-((-1
 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)])/b^4
```

Maple [A]
time = 0.32, size = 250, normalized size = 1.23

method	result
derivativedivides	$\frac{\frac{\operatorname{arcsech}(bx+a)a^4}{4}-\operatorname{arcsech}(bx+a)a^3(bx+a)+\frac{3\operatorname{arcsech}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arcsech}(bx+a)a(bx+a)^3+\frac{\operatorname{arcsech}(bx+a)(bx+a)^4}{4}-\cdots}{\cdots}$
default	$\frac{\frac{\operatorname{arcsech}(bx+a)a^4}{4}-\operatorname{arcsech}(bx+a)a^3(bx+a)+\frac{3\operatorname{arcsech}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arcsech}(bx+a)a(bx+a)^3+\frac{\operatorname{arcsech}(bx+a)(bx+a)^4}{4}-\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsech(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/4*arcsech(b*x+a)*a^4-arcsech(b*x+a)*a^3*(b*x+a)+3/2*arcsech(b*x+a)
*a^2*(b*x+a)^2-arcsech(b*x+a)*a*(b*x+a)^3+1/4*arcsech(b*x+a)*(b*x+a)^4-1/12
*(-(b*x+a-1)/(b*x+a))^(1/2)*(b*x+a)*((b*x+a+1)/(b*x+a))^(1/2)*(3*a^4*arctan
h(1/(-(b*x+a)^2+1)^(1/2))+12*a^3*arcsin(b*x+a)+18*a^2*(-(b*x+a)^2+1)^(1/2)-
6*a*(b*x+a)*(-(b*x+a)^2+1)^(1/2)+(-(b*x+a)^2+1)^(1/2)*(b*x+a)^2+6*a*arcsin(
b*x+a)+2*(-(b*x+a)^2+1)^(1/2))/(-(b*x+a)^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(2*b^4*x^4*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a
+ 1)*sqrt(-b*x - a + 1)*a + b*x + a) - 2*b^4*x^4*log(b*x + a) - b^2*x^2 + 6
*a*b*x - (a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*log(b*x + a + 1) - 2*(b^4*x^4 - a^
4)*log(b*x + a) - (a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*log(-b*x - a + 1))/b^4 +
integrate(1/4*(b^2*x^5 + a*b*x^4)/(b^2*x^2 + 2*a*b*x + a^2 + (b^2*x^2 + 2*a
*b*x + a^2 - 1)*e^(1/2*log(b*x + a + 1) + 1/2*log(-b*x - a + 1)) - 1), x)
```

Fricas [A]
time = 0.48, size = 345, normalized size = 1.70

$$\frac{6\,b^4x^4\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{bx+a}\right)-3\,a^4\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{x}\right)+3\,a^4\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}-1}{x}\right)+12\,(2\,a^3+a)\arctan\left(\frac{(b^2x^2+2\,abx+a^2)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}}{b^2x^2+2\,abx+a^2-1}\right)-2\,(b^3x^3-3\,ab^2x^2+13\,a^3+(9\,a^2+2)bx+2\,a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/24*(6*b^4*x^4*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2
 + 2*a*b*x + a^2)) + 1)/(b*x + a)) - 3*a^4*log(((b*x + a)*sqrt(-(b^2*x^2 +
2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) + 3*a^4*log(((b*x + a
)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) +
12*(2*a^3 + a)*arctan((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-(b^2*x^2 + 2*a*b*x +
a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - 2*(b^3
*x^3 - 3*a*b^2*x^2 + 13*a^3 + (9*a^2 + 2)*b*x + 2*a)*sqrt(-(b^2*x^2 + 2*a*b
*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)))/b^4
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{asech}(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asech(b*x+a),x)
```

```
[Out] Integral(x**3*asech(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*arcsech(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \operatorname{acosh}\left(\frac{1}{a + b\,x}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acosh(1/(a + b*x)),x)
```

```
[Out] int(x^3*acosh(1/(a + b*x)), x)
```

3.2 $\int x^2 \text{sech}^{-1}(a+bx)\,dx$

Optimal. Leaf size=153

$$\frac{5a\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{6b^3}-\frac{x\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{6b^2}+\frac{a^3\text{sech}^{-1}(a+bx)}{3b^3}+\frac{1}{3}x^3\text{sech}^{-1}(a+bx)-\frac{(1+6a}{}$$

```
[Out] 1/3*a^3*arcsech(b*x+a)/b^3+1/3*x^3*arcsech(b*x+a)-1/6*(6*a^2+1)*arctan((b*x
+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/(b*x+a))/b^3+5/6*a*(b*x+a+1)*((-b*x-a+1)
/(b*x+a+1))^(1/2)/b^3-1/6*x*(b*x+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/b^2
```

Rubi [A]
time = 0.08, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$,
Rules used = {6456, 5576, 3867, 3855, 3852, 8}

$$\frac{a^3\text{sech}^{-1}(a+bx)}{3b^3}-\frac{(6a^2+1)\,\text{ArcTan}\left(\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{a+bx}\right)}{6b^3}+\frac{5a\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{6b^3}-\frac{x\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{6b^2}+\frac{1}{3}x^3\text{sech}^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSech[a + b*x],x]
```

```
[Out] (5*a*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(6*b^3) - (x*Sqrt[(1
- a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(6*b^2) + (a^3*ArcSech[a + b*x])/(
3*b^3) + (x^3*ArcSech[a + b*x])/3 - ((1 + 6*a^2)*ArcTan[(Sqrt[(1 - a - b*x)
/(1 + a + b*x)]*(1 + a + b*x))/(a + b*x)])/(6*b^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 5576

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b
*d*(n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6456

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int x^2\text{sech}^{-1}(a+bx)\,dx = -\frac{\text{Subst}\left(\int x\text{sech}(x)(-a+\text{sech}(x))^2\tanh(x)\,dx, x, \text{sech}^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{1}{3}x^3\text{sech}^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\text{sech}(x))^3\,dx, x, \text{sech}^{-1}(a+bx)\right)}{3b^3}$$

$$= -\frac{x\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6b^2} + \frac{1}{3}x^3\text{sech}^{-1}(a+bx) - \frac{\text{Subst}\left(\int\left(-2a^3+(1+6a^2)\right.\right.}{}$$

$$= -\frac{x\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6b^2} + \frac{a^3\text{sech}^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\text{sech}^{-1}(a+bx) + \frac{(5a)\text{Su}}{}$$

$$= -\frac{x\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6b^2} + \frac{a^3\text{sech}^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\text{sech}^{-1}(a+bx) - \frac{(1+6a}{}$$

$$= \frac{5a\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6b^3} - \frac{x\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{6b^2} + \frac{a^3\text{sech}^{-1}(a+bx)}{3b^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.17, size = 200, normalized size = 1.31

$$\frac{\sqrt{-\frac{-1+a+bx}{1+a+bx}}\left(5a^2-bx(1+bx)+a(5+4bx)\right)+2b^3x^3\text{sech}^{-1}(a+bx)-2a^3\log(a+bx)+2a^3\log\left(1+\sqrt{-\frac{-1+a+bx}{1+a+bx}}+a\sqrt{-\frac{-1+a+bx}{1+a+bx}}+bx\sqrt{-\frac{-1+a+bx}{1+a+bx}}\right)+i(1+6a^2)\log\left(-2i(a+bx)+2\sqrt{-\frac{-1+a+bx}{1+a+bx}}(1+a+bx)\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSech[a + b*x],x]
```

```
[Out] (Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(5*a^2 - b*x*(1 + b*x) + a*(5 + 4*b*
x)) + 2*b^3*x^3*ArcSech[a + b*x] - 2*a^3*Log[a + b*x] + 2*a^3*Log[1 + Sqrt[
-((-1 + a + b*x)/(1 + a + b*x))] + a*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]
+ b*x*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]] + I*(1 + 6*a^2)*Log[(-2*I)*(a
+ b*x) + 2*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)])/(6*b^3)
```

Maple [A]
time = 0.32, size = 189, normalized size = 1.24

method	result

derivativedivides	$-\frac{\operatorname{arcsech}(bx+a)a^3}{3}+\operatorname{arcsech}(bx+a)a^2(bx+a)-\operatorname{arcsech}(bx+a)a(bx+a)^2+\frac{\operatorname{arcsech}(bx+a)(bx+a)^3}{3}+\frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{}}{}$
default	$-\frac{\operatorname{arcsech}(bx+a)a^3}{3}+\operatorname{arcsech}(bx+a)a^2(bx+a)-\operatorname{arcsech}(bx+a)a(bx+a)^2+\frac{\operatorname{arcsech}(bx+a)(bx+a)^3}{3}+\frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsech(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(-1/3*arcsech(b*x+a)*a^3+arcsech(b*x+a)*a^2*(b*x+a)-arcsech(b*x+a)*a*
(b*x+a)^2+1/3*arcsech(b*x+a)*(b*x+a)^3+1/6*(-(b*x+a-1)/(b*x+a))^(1/2)*(b*x+
a)*((b*x+a+1)/(b*x+a))^(1/2)*(2*a^3*arctanh(1/(-(b*x+a)^2+1)^(1/2))+6*a^2*a
rcsin(b*x+a)+6*a*(-(b*x+a)^2+1)^(1/2)-(b*x+a)*(-(b*x+a)^2+1)^(1/2)+arcsin(b
*x+a))/(-(b*x+a)^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*x^3*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a
+ 1)*sqrt(-b*x - a + 1)*a + b*x + a) - 2*b^3*x^3*log(b*x + a) - 2*b*x + (a^
3 + 3*a^2 + 3*a + 1)*log(b*x + a + 1) - 2*(b^3*x^3 + a^3)*log(b*x + a) + (a
^3 - 3*a^2 + 3*a - 1)*log(-b*x - a + 1))/b^3 + integrate(1/3*(b^2*x^4 + a*b
*x^3)/(b^2*x^2 + 2*a*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(1/2*log(b
*x + a + 1) + 1/2*log(-b*x - a + 1)) - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(131) = 262$.
time = 0.50, size = 327, normalized size = 2.14

$$\frac{2\,b^3x^3\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{bx+a}\right)+a^3\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{x}\right)-a^3\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}-1}{x}\right)-(6\,a^2+1)\arctan\left(\frac{(b^2x^2+2\,abx+a^2)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}}{b^2x^2+2\,abx+a^2-1}\right)-(b^2x^2-4\,abx-5\,a^2)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2
+ 2*a*b*x + a^2)) + 1)/(b*x + a)) + a^3*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a
*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) - a^3*log(((b*x + a)*sqr
t(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) - (6*a^
2 + 1)*arctan((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)
/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - (b^2*x^2 - 4*a
*b*x - 5*a^2)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)
))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{asech}(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asech(b*x+a),x)
```

```
[Out] Integral(x**2*asech(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsech(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acosh}\left(\frac{1}{a + b\,x}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acosh(1/(a + b*x)),x)
```

```
[Out] int(x^2*acosh(1/(a + b*x)), x)
```

3.3 $\int x\text{sech}^{-1}(a+bx)\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{2b^2}-\frac{a^2\text{sech}^{-1}(a+bx)}{2b^2}+\frac{1}{2}x^2\text{sech}^{-1}(a+bx)+\frac{a\text{ArcTan}\left(\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{a+bx}\right)}{b^2}$$

```
[Out] -1/2*a^2*arcsech(b*x+a)/b^2+1/2*x^2*arcsech(b*x+a)+a*arctan((b*x+a+1)*((-b*
x-a+1)/(b*x+a+1))^(1/2)/(b*x+a))/b^2-1/2*(b*x+a+1)*((-b*x-a+1)/(b*x+a+1))^(
1/2)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6456, 5576, 3858, 3855, 3852, 8}

$$-\frac{a^2\text{sech}^{-1}(a+bx)}{2b^2}+\frac{a\text{ArcTan}\left(\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{a+bx}\right)}{b^2}-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{2b^2}+\frac{1}{2}x^2\text{sech}^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSech[a + b*x],x]
```

```
[Out] -1/2*(Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/b^2 - (a^2*ArcSech[a
 + b*x])/(2*b^2) + (x^2*ArcSech[a + b*x])/2 + (a*ArcTan[(Sqrt[(1 - a - b*x)
/(1 + a + b*x)]*(1 + a + b*x))/(a + b*x)])/b^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 5576

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b
*d*(n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6456

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x\operatorname{sech}^{-1}(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int x\operatorname{sech}(x)(-a+\operatorname{sech}(x))\tanh(x)\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b^2} \\
&= \frac{1}{2}x^2\operatorname{sech}^{-1}(a+bx) - \frac{\operatorname{Subst}\left(\int(-a+\operatorname{sech}(x))^2\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{2b^2} \\
&= -\frac{a^2\operatorname{sech}^{-1}(a+bx)}{2b^2} + \frac{1}{2}x^2\operatorname{sech}^{-1}(a+bx) - \frac{\operatorname{Subst}\left(\int \operatorname{sech}^2(x)\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{2b^2} \\
&= -\frac{a^2\operatorname{sech}^{-1}(a+bx)}{2b^2} + \frac{1}{2}x^2\operatorname{sech}^{-1}(a+bx) + \frac{a\tan^{-1}\left(\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{a+bx}\right)}{b^2} - \cdots \\
&= -\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{2b^2} - \frac{a^2\operatorname{sech}^{-1}(a+bx)}{2b^2} + \frac{1}{2}x^2\operatorname{sech}^{-1}(a+bx) + \frac{a\tan^{-1}\left(\cdots\right)}{\cdots}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 176, normalized size = 1.64

$$\frac{-\sqrt{-\frac{-1+a+bx}{1+a+bx}}(1+a+bx)+b^2x^2\operatorname{sech}^{-1}(a+bx)+a^2\log(a+bx)-a^2\log\left(1+\sqrt{-\frac{-1+a+bx}{1+a+bx}}+a\sqrt{-\frac{-1+a+bx}{1+a+bx}}+bx\sqrt{-\frac{-1+a+bx}{1+a+bx}}\right)-2ia\log\left(-2i(a+bx)+2\sqrt{-\frac{-1+a+bx}{1+a+bx}}(1+a+bx)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSech[a + b*x],x]
```

```
[Out] (-(Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)) + b^2*x^2*ArcSech[a
 + b*x] + a^2*Log[a + b*x] - a^2*Log[1 + Sqrt[-((-1 + a + b*x)/(1 + a + b*x
))] + a*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + b*x*Sqrt[-((-1 + a + b*x)/(
1 + a + b*x))]] - (2*I)*a*Log[(-2*I)*(a + b*x) + 2*Sqrt[-((-1 + a + b*x)/(1
 + a + b*x))]*(1 + a + b*x)])/(2*b^2)
```

Maple [A]
time = 0.31, size = 111, normalized size = 1.04

method	result
derivativedivides	$\frac{-\operatorname{arcsech}(bx+a)a(bx+a)+\frac{\operatorname{arcsech}(bx+a)(bx+a)^2}{2}-\frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+1}{bx+a}}\left(2a\arcsin(bx+a)+\sqrt{-(bx+}\right.}{2\sqrt{-(bx+a)^2+1}}}{b^2}$
default	$\frac{-\operatorname{arcsech}(bx+a)a(bx+a)+\frac{\operatorname{arcsech}(bx+a)(bx+a)^2}{2}-\frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+1}{bx+a}}\left(2a\arcsin(bx+a)+\sqrt{-(bx+}\right.}{2\sqrt{-(bx+a)^2+1}}}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsech(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(-arcsech(b*x+a)*a*(b*x+a)+1/2*arcsech(b*x+a)*(b*x+a)^2-1/2*(-(b*x+a-
1)/(b*x+a))^(1/2)*(b*x+a)*((b*x+a+1)/(b*x+a))^(1/2)*(2*a*arcsin(b*x+a)+(-(b
*x+a)^2+1)^(1/2))/(-(b*x+a)^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b^2*x^2*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a
+ 1)*sqrt(-b*x - a + 1)*a + b*x + a) - 2*b^2*x^2*log(b*x + a) - (a^2 + 2*a
+ 1)*log(b*x + a + 1) - 2*(b^2*x^2 - a^2)*log(b*x + a) - (a^2 - 2*a + 1)*lo
g(-b*x - a + 1))/b^2 + integrate(1/2*(b^2*x^3 + a*b*x^2)/(b^2*x^2 + 2*a*b*x
 + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(1/2*log(b*x + a + 1) + 1/2*log(-b
*x - a + 1)) - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(93) = 186$.
time = 0.39, size = 308, normalized size = 2.88

$$\frac{2\,b^{2}x^{2}\log\left(\frac{(bx+a)\sqrt{-\frac{b^{2}x^{2}+2\,abx+a^{2}-1}{b^{2}x^{2}+2\,abx+a^{2}}}+1}{bx+a}\right)-a^{2}\log\left(\frac{(bx+a)\sqrt{-\frac{b^{2}x^{2}+2\,abx+a^{2}-1}{b^{2}x^{2}+2\,abx+a^{2}}}+1}{x}\right)+a^{2}\log\left(\frac{(bx+a)\sqrt{-\frac{b^{2}x^{2}+2\,abx+a^{2}-1}{b^{2}x^{2}+2\,abx+a^{2}}}-1}{x}\right)+4\,a\arctan\left(\frac{(b^{2}x^{2}+2\,abx+a^{2})\sqrt{-\frac{b^{2}x^{2}+2\,abx+a^{2}-1}{b^{2}x^{2}+2\,abx+a^{2}}}}{b^{2}x^{2}+2\,abx+a^{2}-1}\right)-2\,(bx+a)\sqrt{-\frac{b^{2}x^{2}+2\,abx+a^{2}-1}{b^{2}x^{2}+2\,abx+a^{2}}}}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2
+ 2*a*b*x + a^2)) + 1)/(b*x + a)) - a^2*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a
*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) + a^2*log(((b*x + a)*sqr
t(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) + 4*a*a
rctan((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^
2 + 2*a*b*x + a^2))/(b^2*x^2 + 2*a*b*x + a^2 - 1)) - 2*(b*x + a)*sqrt(-(b^2
*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asech}(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asech(b*x+a),x)
```

```
[Out] Integral(x*asech(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*arcsech(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acosh}\left(\frac{1}{a + b\,x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acosh(1/(a + b*x)),x)
```

```
[Out] int(x*acosh(1/(a + b*x)), x)
```

3.4 $\int \mathrm{sech}^{-1}(a+bx)\,dx$

Optimal. Leaf size=44

$$\frac{(a+bx)\mathrm{sech}^{-1}(a+bx)}{b}-\frac{2\mathrm{ArcTan}\left(\sqrt{\frac{1-a-bx}{1+a+bx}}\right)}{b}$$

```
[Out] (b*x+a)*arcsech(b*x+a)/b-2*arctan(((-b*x-a+1)/(b*x+a+1))^(1/2))/b
```

Rubi [A]
time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6448, 1983, 12, 209}

$$\frac{(a+bx)\mathrm{sech}^{-1}(a+bx)}{b}-\frac{2\mathrm{ArcTan}\left(\sqrt{\frac{-a-bx+1}{a+bx+1}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x],x]
```

```
[Out] ((a + b*x)*ArcSech[a + b*x])/b - (2*ArcTan[Sqrt[(1 - a - b*x)/(1 + a + b*x)
]])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1983

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n),
Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(
b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1/
n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rule 6448

```
Int[ArcSech[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcSech[c + d*
x]/d), x] + Int[Sqrt[(1 - c - d*x)/(1 + c + d*x)]/(1 - c - d*x), x] /; Free
Q[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^{-1}(a+bx)\,dx &= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)}{b}+\int \frac{\sqrt{\frac{1-a-bx}{1+a+bx}}}{1-a-bx}\,dx\\ &= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)}{b}-(4b)\text{Subst}\left(\int \frac{1}{2b^2\left(1+x^2\right)}\,dx,x,\sqrt{\frac{1-a-bx}{1+a+bx}}\right)\\ &= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)}{b}-\frac{2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{\frac{1-a-bx}{1+a+bx}}\right)}{b}\\ &= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)}{b}-\frac{2\tan^{-1}\left(\sqrt{\frac{1-a-bx}{1+a+bx}}\right)}{b}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 97 vs. 2(44) = 88.
time = 0.18, size = 97, normalized size = 2.20

$$x\operatorname{sech}^{-1}(a+bx)-\frac{2\sqrt{-\frac{-1+a+bx}{1+a+bx}}\left(-a\text{ArcTan}\left(\sqrt{\frac{-1+a+bx}{1+a+bx}}\right)+\tanh^{-1}\left(\sqrt{\frac{-1+a+bx}{1+a+bx}}\right)\right)}{b\sqrt{\frac{-1+a+bx}{1+a+bx}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a + b*x],x]
```

```
[Out] x*ArcSech[a + b*x] - (2*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(-(a*ArcTan[S
qrt[(-1 + a + b*x)/(1 + a + b*x)]]) + ArcTanh[Sqrt[(-1 + a + b*x)/(1 + a +
b*x)]]))/(b*Sqrt[(-1 + a + b*x)/(1 + a + b*x)])
```

Maple [**A**]
time = 0.04, size = 44, normalized size = 1.00

method	result	size

derivativedivides	$\frac{(bx+a)\text{arcsech}(bx+a)-\arctan\left(\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)}{b}$	44
default	$\frac{(bx+a)\text{arcsech}(bx+a)-\arctan\left(\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)}{b}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*((b*x+a)*arcsech(b*x+a)-arctan((1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))
)
```

Maxima [A]
time = 0.25, size = 31, normalized size = 0.70

$$\frac{(bx+a)\,\text{arsech}\,(bx+a)-\arctan\left(\sqrt{\frac{1}{(bx+a)^2}-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a),x, algorithm="maxima")
[Out] ((b*x + a)*arcsech(b*x + a) - arctan(sqrt(1/(b*x + a)^2 - 1)))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(40) = 80.
time = 0.44, size = 253, normalized size = 5.75

$$\frac{2\,bx\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{bx+a}\right)+a\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}+1}{x}\right)-a\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}-1}{x}\right)-2\arctan\left(\frac{(b^2x^2+2\,abx+a^2)\sqrt{-\frac{b^2x^2+2\,abx+a^2-1}{b^2x^2+2\,abx+a^2}}}{b^2x^2+2\,abx+a^2-1}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a),x, algorithm="fricas")
[Out] 1/2*(2*b*x*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*
a*b*x + a^2)) + 1)/(b*x + a)) + a*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x +
 a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) - a*log(((b*x + a)*sqrt(-(b^2*
x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) - 2*arctan((b^2
*x^2 + 2*a*b*x + a^2)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*
x + a^2))/(b^2*x^2 + 2*a*b*x + a^2 - 1)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \text{asech}\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a),x)
```

```
[Out] Integral(asech(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a), x)
```

Mupad [B]
time = 2.16, size = 43, normalized size = 0.98

$$\frac{\operatorname{atan}\left(\frac{1}{\sqrt{\frac{1}{a+b\,x}-1}\,\sqrt{\frac{1}{a+b\,x}+1}}\right)+\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)\,\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x)),x)
```

```
[Out] (atan(1/((1/(a + b*x) - 1)^(1/2)*(1/(a + b*x) + 1)^(1/2))) + acosh(1/(a + b
*x))*(a + b*x))/b
```

3.5 $\int \frac{\operatorname{sech}^{-1}(a+bx)}{x}\,dx$

Optimal. Leaf size=170

$$\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)-\operatorname{sech}^{-1}(a+bx)\log\left(1+e\right.$$

```
[Out] -arcsech(b*x+a)*ln(1+(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)
+arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/
(1-(-a^2+1)^(1/2)))+arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1
/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))-1/2*polylog(2,-(1/(b*x+a)+(1/(b*x+a)
-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)+polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2
)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))+polylog(2,a*(1/(b*x+a)+(1/(b*x+a
)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))
```

Rubi [A]
time = 0.21, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6456, 5714, 5689, 3799, 2221, 2317, 2438, 5681}

$$\operatorname{Li}_2\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\operatorname{Li}_2\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)-\frac{1}{2}\operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(a+bx)}\right)-\operatorname{sech}^{-1}(a+bx)\log\left(e^{2\operatorname{sech}^{-1}(a+bx)}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]/x,x]
```

```
[Out] ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + ArcS
ech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - ArcSech[
a + b*x]*Log[1 + E^(2*ArcSech[a + b*x])] + PolyLog[2, (a*E^ArcSech[a + b*x]
)/(1 - Sqrt[1 - a^2])] + PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^
2])] - PolyLog[2, -E^(2*ArcSech[a + b*x])]/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5689

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/(Cosh[(c_.)
 + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Tanh[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Sinh[c + d*x]*(Tanh[c + d*x]^
(n - 1)/(a + b*Cosh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5714

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sech[(c_.) + (d_.)*(x_)]), x_Symbol] :> I
nt[(e + f*x)^m*Cosh[c + d*x]*F[c + d*x]^n*(G[c + d*x]^p/(b + a*Cosh[c + d*x
])), x] /; FreeQ[{a, b, c, d, e, f}, x] && HyperbolicQ[F] && HyperbolicQ[G]
 && IntegersQ[m, n, p]
```

Rule 6456

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(a+bx)}{x}\,dx &= -\text{Subst}\left(\int \frac{x\operatorname{sech}(x)\tanh(x)}{-a+\operatorname{sech}(x)}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
&= -\text{Subst}\left(\int \frac{x\tanh(x)}{1-a\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
&= -\left(a\text{Subst}\left(\int \frac{x\sinh(x)}{1-a\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right) - \text{Subst}\left(\int x\tanh(x)\,dx, x,\right. \\
&= -\left(2\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right) - a\text{Subst}\left(\int \frac{e^{x}x}{1-\sqrt{1-a^2}-ae^{x}}\right. \\
&= \operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) \\
&= \operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) \\
&= \operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.15, size = 332, normalized size = 1.95

$$-4i\text{ArcSin}\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\tanh^{-1}\left(\frac{(1+a)\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(a+bx)\right)}{\sqrt{1-a^2}}\right) - \operatorname{sech}^{-1}(a+bx)\log\left(1+e^{-2\operatorname{sech}^{-1}(a+bx)}\right) + \operatorname{sech}^{-1}(a+bx)\log\left(1+\frac{\left(-1+\sqrt{1-a^2}\right)e^{-\operatorname{sech}^{-1}(a+bx)}}{a}\right) + 2i\text{ArcSin}\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\log\left(1+\frac{\left(-1+\sqrt{1-a^2}\right)e^{-\operatorname{sech}^{-1}(a+bx)}}{a}\right) + \operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{\left(1+\sqrt{1-a^2}\right)e^{-\operatorname{sech}^{-1}(a+bx)}}{a}\right) - 2i\text{ArcSin}\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\log\left(1-\frac{\left(1+\sqrt{1-a^2}\right)e^{-\operatorname{sech}^{-1}(a+bx)}}{a}\right) + \frac{1}{2}\text{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(a+bx)}\right) - \text{PolyLog}\left(2,-\frac{\left(-1+\sqrt{1-a^2}\right)e^{-\operatorname{sech}^{-1}(a+bx)}}{a}\right) - \text{PolyLog}\left(2,\frac{\left(1+\sqrt{1-a^2}\right)e^{-\operatorname{sech}^{-1}(a+bx)}}{a}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSech[a + b*x]/x,x]
```

```
[Out] (-4*I)*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*ArcTanh[((1 + a)*Tanh[ArcSech[a + b
*x]/2])/Sqrt[1 - a^2]] - ArcSech[a + b*x]*Log[1 + E^(-2*ArcSech[a + b*x])]
+ ArcSech[a + b*x]*Log[1 + (-1 + Sqrt[1 - a^2])/(a*E^ArcSech[a + b*x])] + (
2*I)*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Log[1 + (-1 + Sqrt[1 - a^2])/(a*E^Arc
Sech[a + b*x])] + ArcSech[a + b*x]*Log[1 - (1 + Sqrt[1 - a^2])/(a*E^ArcSech
[a + b*x])] - (2*I)*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Log[1 - (1 + Sqrt[1 -
a^2])/(a*E^ArcSech[a + b*x])] + PolyLog[2, -E^(-2*ArcSech[a + b*x])]/2 - Po
lyLog[2, -((-1 + Sqrt[1 - a^2])/(a*E^ArcSech[a + b*x]))] - PolyLog[2, (1 +
Sqrt[1 - a^2])/(a*E^ArcSech[a + b*x])]
```

Maple [C] Result contains complex when optimal does not.
time = 0.89, size = 882, normalized size = 5.19

method	result

derivativedivides	$-\operatorname{arcsech}(bx+a)\ln\left(1+i\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)\right)-\operatorname{arcsech}(bx+a)\ln\left(1-\right.$
default	$-\operatorname{arcsech}(bx+a)\ln\left(1+i\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)\right)-\operatorname{arcsech}(bx+a)\ln\left(1-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -arcsech(b*x+a)*ln(1+I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))
-arcsech(b*x+a)*ln(1-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))
-dilog(1+I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-dilog(1-I*(
1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))+1/2*arcsech(b*x+a)*ln((
-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1
+(-a^2+1)^(1/2)))+1/2*arcsech(b*x+a)*ln((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(
1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))-1/2*(-a^2+1)^(1/
2)/(a^2-1)*arcsech(b*x+a)*ln((-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+
1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))+1/2*(-a^2+1)^(1/2)/(a^2-1)*
arcsech(b*x+a)*ln((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-
a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))+dilog((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2
)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))+dilog((-a*(1/
(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2
+1)^(1/2)))+1/2*(a^2+(-a^2+1)^(1/2)-1)*arcsech(b*x+a)*(ln((-a*(1/(b*x+a)+(1
/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2))
)*a^2+ln((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1
/2)-1)/(-1+(-a^2+1)^(1/2)))*a^2-2*ln((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(
b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))-2*ln((a*(1/(b*x+a)+
(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/
2)))*(-a^2+1)^(1/2))/a^2/(a^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsech(b*x + a)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsech(b*x + a)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}(a+bx)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)/x,x)
```

```
[Out] Integral(asech(a + b*x)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))/x,x)
```

```
[Out] int(acosh(1/(a + b*x))/x, x)
```

3.6 $\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^2} dx$

Optimal. Leaf size=70

$$-\frac{b\operatorname{sech}^{-1}(a+bx)}{a}-\frac{\operatorname{sech}^{-1}(a+bx)}{x}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{1+a}\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}$$

```
[Out] -b*arcsech(b*x+a)/a-arcsech(b*x+a)/x+2*b*arctanh((1+a)^(1/2)*tanh(1/2*arcse
ch(b*x+a))/(1-a)^(1/2))/a/(-a^2+1)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6456, 5576, 3868, 2738, 214}

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{a+1}\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}-\frac{b\operatorname{sech}^{-1}(a+bx)}{a}-\frac{\operatorname{sech}^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]/x^2,x]
```

```
[Out] -((b*ArcSech[a + b*x])/a) - ArcSech[a + b*x]/x + (2*b*ArcTanh[(Sqrt[1 + a]*
Tanh[ArcSech[a + b*x]/2])/Sqrt[1 - a]])/(a*Sqrt[1 - a^2])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 5576

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b
*d*(n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6456

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{sech}^{-1}(a+bx)}{x^2}\,dx &= -\left(b\text{Subst}\left(\int \frac{x\text{sech}(x)\tanh(x)}{(-a+\text{sech}(x))^2}\,dx, x, \text{sech}^{-1}(a+bx)\right)\right)\\
&= -\frac{\text{sech}^{-1}(a+bx)}{x} + b\text{Subst}\left(\int \frac{1}{-a+\text{sech}(x)}\,dx, x, \text{sech}^{-1}(a+bx)\right)\\
&= -\frac{b\text{sech}^{-1}(a+bx)}{a} - \frac{\text{sech}^{-1}(a+bx)}{x} + \frac{b\text{Subst}\left(\int \frac{1}{1-a\cosh(x)}\,dx, x, \text{sech}^{-1}(a+bx)\right)}{a}\\
&= -\frac{b\text{sech}^{-1}(a+bx)}{a} - \frac{\text{sech}^{-1}(a+bx)}{x} + \frac{(2b)\text{Subst}\left(\int \frac{1}{1-a-(1+a)x^2}\,dx, x, \tanh\left(\frac{1}{2}\text{sech}^{-1}\right.\right.}{a}\\
&= -\frac{b\text{sech}^{-1}(a+bx)}{a} - \frac{\text{sech}^{-1}(a+bx)}{x} + \frac{2b\tanh^{-1}\left(\frac{\sqrt{1+a}\tanh\left(\frac{1}{2}\text{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 244 vs. 2(70) = 140.
time = 0.17, size = 244, normalized size = 3.49

$$
-\frac{\text{sech}^{-1}(a+bx)}{x} + \frac{b\left(-\log(x) + \sqrt{1-a^2}\log(a+bx) - \sqrt{1-a^2}\log\left(1 + \sqrt{-\frac{-1+a+bx}{1+a+bx}} + a\sqrt{-\frac{-1+a+bx}{1+a+bx}} + bx\sqrt{-\frac{-1+a+bx}{1+a+bx}}\right) + \log\left(1 - a^2 - abx + \sqrt{1-a^2}\sqrt{-\frac{-1+a+bx}{1+a+bx}} + a\sqrt{1-a^2}\sqrt{-\frac{-1+a+bx}{1+a+bx}} + \sqrt{1-a^2}\,bx\sqrt{-\frac{-1+a+bx}{1+a+bx}}\right)\right)}{a\sqrt{1-a^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a + b*x]/x^2,x]

[Out] -(ArcSech[a + b*x]/x) + (b*(-Log[x] + Sqrt[1 - a^2]*Log[a + b*x] - Sqrt[1 -
 a^2]*Log[1 + Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + a*Sqrt[-((-1 + a + b*
x)/(1 + a + b*x))] + b*x*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]] + Log[1 - a
```

```
^2 - a*b*x + Sqrt[1 - a^2]*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + a*Sqrt[1
 - a^2]*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + Sqrt[1 - a^2]*b*x*Sqrt[-((-
1 + a + b*x)/(1 + a + b*x))]]))/(a*Sqrt[1 - a^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(62) = 124$.
time = 0.44, size = 171, normalized size = 2.44

method	result
derivativedivides	$b\left(-\frac{\operatorname{arcsech}(bx+a)}{bx}-\frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+1}{bx+a}}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-(bx+a)^2+1}}\right)a^2+\sqrt{-a^2+\cdots}\right.}{\sqrt{-(bx+\cdots}}\right.$
default	$b\left(-\frac{\operatorname{arcsech}(bx+a)}{bx}-\frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+1}{bx+a}}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-(bx+a)^2+1}}\right)a^2+\sqrt{-a^2+\cdots}\right.}{\sqrt{-(bx+\cdots}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-1/b/x*arcsech(b*x+a)-(-(b*x+a-1)/(b*x+a))^(1/2)*(b*x+a)*((b*x+a+1)/(b*x
+a))^(1/2)*(arctanh(1/(-(b*x+a)^2+1)^(1/2))*a^2+(-a^2+1)^(1/2)*ln(2*((-a^2+
1)^(1/2)*(-(b*x+a)^2+1)^(1/2)-a*(b*x+a)+1)/b/x)-arctanh(1/(-(b*x+a)^2+1)^(1
/2)))/(-(b*x+a)^2+1)^(1/2)/a/(-1+a)/(1+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] b*log(x)/(a^3 - a) - 1/2*((a^2*b - a*b)*x*log(b*x + a + 1) + (a^2*b + a*b)*
x*log(-b*x - a + 1) + 2*(a^3 - a)*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*
b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a) - 2*(a^3 + (a^2*b -
 b)*x - a)*log(b*x + a) - 2*(a^3 - a)*log(b*x + a))/((a^3 - a)*x) - integra
te((b^2*x + a*b)/(b^2*x^3 + 2*a*b*x^2 + (a^2 - 1)*x + (b^2*x^3 + 2*a*b*x^2
+ (a^2 - 1)*x)*e^(1/2*log(b*x + a + 1) + 1/2*log(-b*x - a + 1))), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(62) = 124$.
time = 0.40, size = 651, normalized size = 9.30

$$\left[-\frac{(a^2-1)bx\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+1}{x}\right)-(a^2-1)bx\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}-1}{x}\right)+\sqrt{-a^2+1}\,bx\log\left(\frac{(2a^2-1)b^2x^2+2a^4+4(a^3-a)bx-4a^2-2(ab^2x^2+a^3+(2a^2-1)bx-a)\sqrt{-a^2+1}\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+2}{x^2}\right)+2(a^3-a)\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+1}{bx+a}\right)}{2(a^3-a)x}, -\frac{(a^2-1)bx\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+1}{x}\right)-(a^2-1)bx\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}-1}{x}\right)+2\sqrt{a^2-1}\,bx\arctan\left(\frac{(ab^2x^2+a^3+(2a^2-1)bx-a)\sqrt{a^2-1}\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}}{(a^2-1)b^2x^2+a^4+2(a^3-a)bx-2a^2+1}\right)+2(a^3-a)\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+1}{bx+a}\right)}{2(a^3-a)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2 - 1)*b*x*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^
2*x^2 + 2*a*b*x + a^2)) + 1)/x) - (a^2 - 1)*b*x*log(((b*x + a)*sqrt(-(b^2*x
^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) + sqrt(-a^2 + 1)
*b*x*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*b*x - 4*a^2 - 2*(a*b^2*
x^2 + a^3 + (2*a^2 - 1)*b*x - a)*sqrt(-a^2 + 1)*sqrt(-(b^2*x^2 + 2*a*b*x +
a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 2)/x^2) + 2*(a^3 - a)*log(((b*x + a)*
sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/(b*x +
a)))/((a^3 - a)*x), -1/2*((a^2 - 1)*b*x*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a
*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) - (a^2 - 1)*b*x*log(((b*
x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/
x) + 2*sqrt(a^2 - 1)*b*x*arctan((a*b^2*x^2 + a^3 + (2*a^2 - 1)*b*x - a)*sqr
t(a^2 - 1)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2))/(
(a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) + 2*(a^3 - a)*log((
(b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) +
1)/(b*x + a)))/((a^3 - a)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}(a + bx)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)/x**2,x)
```

```
[Out] Integral(asech(a + b*x)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))/x^2,x)
```

```
[Out] int(acosh(1/(a + b*x))/x^2, x)
```

3.7 $\int \frac{\text{sech}^{-1}(a+bx)}{x^3}\,dx$

Optimal. Leaf size=133

$$\frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{2a\left(1-a^2\right)x}+\frac{b^2\text{sech}^{-1}(a+bx)}{2a^2}-\frac{\text{sech}^{-1}(a+bx)}{2x^2}-\frac{\left(1-2a^2\right)b^2\tanh^{-1}\left(\frac{\sqrt{1+a}\,\tanh\left(\frac{1}{2}\text{sech}\right)}{\sqrt{1-a}}\right)}{a^2\left(1-a^2\right)^{3/2}}$$

```
[Out] 1/2*b^2*arcsech(b*x+a)/a^2-1/2*arcsech(b*x+a)/x^2-(-2*a^2+1)*b^2*arctanh((1
+a)^(1/2)*tanh(1/2*arcsech(b*x+a))/(1-a)^(1/2))/a^2/(-a^2+1)^(3/2)+1/2*b*(b
*x+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/a/(-a^2+1)/x
```

Rubi [A]
time = 0.15, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6456, 5576, 3870, 4004, 3916, 2738, 214}

$$\frac{b^2\text{sech}^{-1}(a+bx)}{2a^2}-\frac{\left(1-2a^2\right)b^2\tanh^{-1}\left(\frac{\sqrt{a+1}\,\tanh\left(\frac{1}{2}\text{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a^2\left(1-a^2\right)^{3/2}}+\frac{b\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{2a\left(1-a^2\right)x}-\frac{\text{sech}^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]/x^3,x]
```

```
[Out] (b*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(2*a*(1 - a^2)*x) + (b^
2*ArcSech[a + b*x])/(2*a^2) - ArcSech[a + b*x]/(2*x^2) - ((1 - 2*a^2)*b^2*A
rcTanh[(Sqrt[1 + a]*Tanh[ArcSech[a + b*x]/2])/Sqrt[1 - a]])/(a^2*(1 - a^2)^
(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
```

```
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 5576

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b
*d*(n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6456

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\text{sech}^{-1}(a+bx)}{x^3}\,dx = -\left(b^2\text{Subst}\left(\int \frac{x\,\text{sech}(x)\tanh(x)}{(-a+\text{sech}(x))^3}\,dx, x, \text{sech}^{-1}(a+bx)\right)\right)$$

$$= -\frac{\text{sech}^{-1}(a+bx)}{2x^2} + \frac{1}{2}b^2\text{Subst}\left(\int \frac{1}{(-a+\text{sech}(x))^2}\,dx, x, \text{sech}^{-1}(a+bx)\right)$$

$$= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{2a(1-a^2)x} - \frac{\text{sech}^{-1}(a+bx)}{2x^2} - \frac{b^2\text{Subst}\left(\int \frac{1-a^2-a\,\text{sech}(x)}{-a+\text{sech}(x)}\,dx, x, \text{sec}}{2a(1-a^2)}$$

$$= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{2a(1-a^2)x} + \frac{b^2\text{sech}^{-1}(a+bx)}{2a^2} - \frac{\text{sech}^{-1}(a+bx)}{2x^2} - \frac{((1-2a^2)b^2}{}$$

$$= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{2a(1-a^2)x} + \frac{b^2\text{sech}^{-1}(a+bx)}{2a^2} - \frac{\text{sech}^{-1}(a+bx)}{2x^2} - \frac{((1-2a^2)b^2}{}$$

$$= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{2a(1-a^2)x} + \frac{b^2\text{sech}^{-1}(a+bx)}{2a^2} - \frac{\text{sech}^{-1}(a+bx)}{2x^2} - \frac{((1-2a^2)b^2}{}$$

$$= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}(1+a+bx)}{2a(1-a^2)x} + \frac{b^2\text{sech}^{-1}(a+bx)}{2a^2} - \frac{\text{sech}^{-1}(a+bx)}{2x^2} - \frac{(1-2a^2)b^2\,\text{t}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 315 vs. 2(133) = 266.

time = 0.68, size = 315, normalized size = 2.37

$$\frac{1}{2}\left(-\frac{b\sqrt{-\frac{-1+a+bx}{1+a+bx}}(1+a+bx)}{(-1+a)a(1+a)x} - \frac{\text{sech}^{-1}(a+bx)}{x^2} - \frac{(-1+2a^2)b^2\log(x)}{a^2(1-a^2)^{3/2}} - \frac{b^2\log(a+bx)}{a^2} + \frac{b^2\log\left(1+\sqrt{-\frac{-1+a+bx}{1+a+bx}}+a\sqrt{-\frac{-1+a+bx}{1+a+bx}}+bx\sqrt{-\frac{-1+a+bx}{1+a+bx}}\right)}{a^2} + \frac{(-1+2a^2)b^2\log\left(1-a^2-abx+\sqrt{1-a^2}\sqrt{-\frac{-1+a+bx}{1+a+bx}}+a\sqrt{1-a^2}\sqrt{-\frac{-1+a+bx}{1+a+bx}}+\sqrt{1-a^2}\,bx\sqrt{-\frac{-1+a+bx}{1+a+bx}}\right)}{a^2(1-a^2)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a + b*x]/x^3,x]
```

```
[Out] (-((b*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x))/((-1 + a)*a*(1 +
 a)*x)) - ArcSech[a + b*x]/x^2 - ((-1 + 2*a^2)*b^2*Log[x])/(a^2*(1 - a^2)^(
3/2)) - (b^2*Log[a + b*x])/a^2 + (b^2*Log[1 + Sqrt[-((-1 + a + b*x)/(1 + a
+ b*x))] + a*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + b*x*Sqrt[-((-1 + a + b
*x)/(1 + a + b*x))]])/a^2 + ((-1 + 2*a^2)*b^2*Log[1 - a^2 - a*b*x + Sqrt[1
- a^2]*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + a*Sqrt[1 - a^2]*Sqrt[-((-1 +
 a + b*x)/(1 + a + b*x))] + Sqrt[1 - a^2]*b*x*Sqrt[-((-1 + a + b*x)/(1 + a
+ b*x))]])/(a^2*(1 - a^2)^(3/2)))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(117) = 234.

time = 0.46, size = 483, normalized size = 3.63

method	result
derivativedivides	$b^2 \left(-\frac{\mathrm{arcsech}(bx+a)}{2b^2x^2} - \frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+1}{bx+a}}\left(\mathrm{arctanh}\left(\frac{1}{\sqrt{-(bx+a)^2+1}}\right)a^5 - \mathrm{arctanh}\left(\right.\right.}{}\right.$
default	$b^2 \left(-\frac{\mathrm{arcsech}(bx+a)}{2b^2x^2} - \frac{\sqrt{-\frac{bx+a-1}{bx+a}}\,(bx+a)\sqrt{\frac{bx+a+1}{bx+a}}\left(\mathrm{arctanh}\left(\frac{1}{\sqrt{-(bx+a)^2+1}}\right)a^5 - \mathrm{arctanh}\left(\right.\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/2/b^2/x^2*arcsech(b*x+a)-1/2*(-(b*x+a-1)/(b*x+a))^(1/2)*(b*x+a)*((b
*x+a+1)/(b*x+a))^(1/2)*(arctanh(1/(-(b*x+a)^2+1)^(1/2))*a^5-arctanh(1/(-(b*
x+a)^2+1)^(1/2))*a^4*(b*x+a)+2*(-a^2+1)^(1/2)*ln(2*((-a^2+1)^(1/2)*(-(b*x+a
)^2+1)^(1/2)-a*(b*x+a)+1)/b/x)*a^3-2*(-a^2+1)^(1/2)*ln(2*((-a^2+1)^(1/2)*(-
(b*x+a)^2+1)^(1/2)-a*(b*x+a)+1)/b/x)*a^2*(b*x+a)-2*a^3*arctanh(1/(-(b*x+a)^
2+1)^(1/2))+2*arctanh(1/(-(b*x+a)^2+1)^(1/2))*a^2*(b*x+a)+(-(b*x+a)^2+1)^(1
/2)*a^3-(-a^2+1)^(1/2)*ln(2*((-a^2+1)^(1/2)*(-(b*x+a)^2+1)^(1/2)-a*(b*x+a)+
1)/b/x)*a+(-a^2+1)^(1/2)*ln(2*((-a^2+1)^(1/2)*(-(b*x+a)^2+1)^(1/2)-a*(b*x+a
)+1)/b/x)*(b*x+a)+arctanh(1/(-(b*x+a)^2+1)^(1/2))*a-arctanh(1/(-(b*x+a)^2+1
)^(1/2))*(b*x+a)-a*(-(b*x+a)^2+1)^(1/2))/b/x/(a^2-1)/(1+a)/a^2/(-1+a)/(-(b*
x+a)^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(3*a^2*b^2 - b^2)*log(x)/(a^6 - 2*a^4 + a^2) + 1/4*((a^4*b^2 - 2*a^3*b
^2 + a^2*b^2)*x^2*log(b*x + a + 1) + (a^4*b^2 + 2*a^3*b^2 + a^2*b^2)*x^2*lo
g(-b*x - a + 1) - 2*(a^3*b - a*b)*x - 2*(a^6 - 2*a^4 + a^2)*log(sqrt(b*x +
a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*
x + a) + 2*(a^6 - 2*a^4 - (a^4*b^2 - 2*a^2*b^2 + b^2)*x^2 + a^2)*log(b*x +
```

```
a) + 2*(a^6 - 2*a^4 + a^2)*log(b*x + a))/((a^6 - 2*a^4 + a^2)*x^2) - integr
ate(1/2*(b^2*x + a*b)/(b^2*x^4 + 2*a*b*x^3 + (a^2 - 1)*x^2 + (b^2*x^4 + 2*a
*b*x^3 + (a^2 - 1)*x^2)*e^(1/2*log(b*x + a + 1) + 1/2*log(-b*x - a + 1))),
x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(112) = 224$.
time = 0.44, size = 865, normalized size = 6.50

$$\left[-\frac{(2a^2-1)\sqrt{-a^2+1}\,b^2x^2\log\left(\frac{(2a^2-1)b^2x^2+2a^4+4(a^3-a)bx-4a^2+2(ab^2x^2+a^3+(2a^2-1)bx-a)\sqrt{-a^2+1}\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+2}{x^2}\right)-(a^4-2a^2+1)b^2x^2\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+1}{x}\right)+(a^4-2a^2+1)b^2x^2\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}-1}{x}\right)+2(a^6-2a^4+a^2)\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+1}{bx+a}\right)+2((a^3-a)b^2x^2+(a^4-a^2)bx)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}}{4(a^6-2a^4+a^2)x^2},\ \frac{2(2a^2-1)\sqrt{a^2-1}\,b^2x^2\arctan\left(\frac{(ab^2x^2+a^3+(2a^2-1)bx-a)\sqrt{a^2-1}\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}}{(a^2-1)b^2x^2+a^4+2(a^3-a)bx-2a^2+1}\right)+(a^4-2a^2+1)b^2x^2\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+1}{x}\right)-(a^4-2a^2+1)b^2x^2\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}-1}{x}\right)-2(a^6-2a^4+a^2)\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+1}{bx+a}\right)-2((a^3-a)b^2x^2+(a^4-a^2)bx)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}}{4(a^6-2a^4+a^2)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/4*((2*a^2 - 1)*sqrt(-a^2 + 1)*b^2*x^2*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4
+ 4*(a^3 - a)*b*x - 4*a^2 + 2*(a*b^2*x^2 + a^3 + (2*a^2 - 1)*b*x - a)*sqrt(
-a^2 + 1)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) +
2)/x^2) - (a^4 - 2*a^2 + 1)*b^2*x^2*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x
 + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) + (a^4 - 2*a^2 + 1)*b^2*x^2*
log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2
)) - 1)/x) + 2*(a^6 - 2*a^4 + a^2)*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x
+ a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/(b*x + a)) + 2*((a^3 - a)*b^2*x^
2 + (a^4 - a^2)*b*x)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x
 + a^2)))/((a^6 - 2*a^4 + a^2)*x^2), 1/4*(2*(2*a^2 - 1)*sqrt(a^2 - 1)*b^2*x
^2*arctan((a*b^2*x^2 + a^3 + (2*a^2 - 1)*b*x - a)*sqrt(a^2 - 1)*sqrt(-(b^2*
x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2))/((a^2 - 1)*b^2*x^2 + a^
4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) + (a^4 - 2*a^2 + 1)*b^2*x^2*log(((b*x + a
)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) -
(a^4 - 2*a^2 + 1)*b^2*x^2*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1
)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) - 2*(a^6 - 2*a^4 + a^2)*log(((b*x + a)
*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/(b*x +
 a)) - 2*((a^3 - a)*b^2*x^2 + (a^4 - a^2)*b*x)*sqrt(-(b^2*x^2 + 2*a*b*x + a
^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)))/((a^6 - 2*a^4 + a^2)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)/x**3,x)
```

```
[Out] Integral(asech(a + b*x)/x**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a)/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acosh}\left(\frac{1}{a+b\,x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))/x^3,x)
```

```
[Out] int(acosh(1/(a + b*x))/x^3, x)
```

3.8 $\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^4}\,dx$

Optimal. Leaf size=197

$$\frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{6a\,(1-a^2)\,x^2}-\frac{(2-5a^2)\,b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{6a^2\,(1-a^2)^2\,x}-\frac{b^3\operatorname{sech}^{-1}(a+bx)}{3a^3}-\frac{\operatorname{sech}^{-1}(a+bx)}{3x^3}+$$

```
[Out] -1/3*b^3*arcsech(b*x+a)/a^3-1/3*arcsech(b*x+a)/x^3+1/3*(6*a^4-5*a^2+2)*b^3*
arctanh((1+a)^(1/2)*tanh(1/2*arcsech(b*x+a))/(1-a)^(1/2))/a^3/(-a^2+1)^(5/2
)+1/6*b*(b*x+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/a/(-a^2+1)/x^2-1/6*(-5*a^2+2
)*b^2*(b*x+a+1)*((-b*x-a+1)/(b*x+a+1))^(1/2)/a^2/(-a^2+1)^2/x
```

Rubi [A]
time = 0.22, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6456, 5576, 3870, 4145, 4004, 3916, 2738, 214}

$$-\frac{b^3\operatorname{sech}^{-1}(a+bx)}{3a^3}-\frac{(2-5a^2)\,b^2\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{6a^2\,(1-a^2)^2\,x}+\frac{b\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)}{6a\,(1-a^2)\,x^2}+\frac{(6a^4-5a^2+2)\,b^3\tanh^{-1}\left(\frac{\sqrt{a+1}\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{3a^3\,(1-a^2)^{5/2}}-\frac{\operatorname{sech}^{-1}(a+bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]/x^4,x]
```

```
[Out] (b*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(6*a*(1 - a^2)*x^2) - (
(2 - 5*a^2)*b^2*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x))/(6*a^2*(1
- a^2)^2*x) - (b^3*ArcSech[a + b*x])/(3*a^3) - ArcSech[a + b*x]/(3*x^3) + (
(2 - 5*a^2 + 6*a^4)*b^3*ArcTanh[(Sqrt[1 + a]*Tanh[ArcSech[a + b*x]/2])/Sqrt
[1 - a]])/(3*a^3*(1 - a^2)^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
```

```
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 5576

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b
*d*(n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6456

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(a+bx)}{x^4}\,dx &= -\left(b^3\operatorname{Subst}\left(\int \frac{x\operatorname{sech}(x)\tanh(x)}{(-a+\operatorname{sech}(x))^4}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right) \\
&= -\frac{\operatorname{sech}^{-1}(a+bx)}{3x^3} + \frac{1}{3}b^3\operatorname{Subst}\left(\int \frac{1}{(-a+\operatorname{sech}(x))^3}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right) \\
&= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{6a\,(1-a^2)\,x^2} - \frac{\operatorname{sech}^{-1}(a+bx)}{3x^3} - \frac{b^3\operatorname{Subst}\left(\int \frac{2(1-a^2)-2a\operatorname{sech}(x)-\operatorname{sech}\cdots}{(-a+\operatorname{sech}(x))^2}\cdots\right.}{6a\,(1-a\cdots} \\
&= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{6a\,(1-a^2)\,x^2} - \frac{(2-5a^2)\,b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{6a^2\,(1-a^2)^2\,x} - \frac{\operatorname{sech}^{-1}(a\cdots}{3x^3\cdots} \\
&= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{6a\,(1-a^2)\,x^2} - \frac{(2-5a^2)\,b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{6a^2\,(1-a^2)^2\,x} - \frac{b^3\operatorname{sech}^{-1}(\cdots}{3a\cdots} \\
&= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{6a\,(1-a^2)\,x^2} - \frac{(2-5a^2)\,b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{6a^2\,(1-a^2)^2\,x} - \frac{b^3\operatorname{sech}^{-1}(\cdots}{3a\cdots} \\
&= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{6a\,(1-a^2)\,x^2} - \frac{(2-5a^2)\,b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{6a^2\,(1-a^2)^2\,x} - \frac{b^3\operatorname{sech}^{-1}(\cdots}{3a\cdots} \\
&= \frac{b\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{6a\,(1-a^2)\,x^2} - \frac{(2-5a^2)\,b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)}{6a^2\,(1-a^2)^2\,x} - \frac{b^3\operatorname{sech}^{-1}(\cdots}{3a\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.27, size = 368, normalized size = 1.87

$$\frac{1}{6}\left(\frac{b\sqrt{-\frac{-1+a+bx}{1+a+bx}}\,(a-a^4-abx-2bx(1+bx)+a^3(-1+4bx)+a^2(1+5bx+5b^2x^2))}{(-1+a)^2a^2(1+a)^2x^2} - \frac{2\operatorname{sech}^{-1}(a+bx)}{x^3} - \frac{(2-5a^2+6a^4)\,b^3\log(x)}{a^3\,(1-a^2)^{5/2}} + \frac{2b^3\log(a+bx)}{a^3} - \frac{2b^3\log\left(1+\sqrt{-\frac{-1+a+bx}{1+a+bx}}+a\sqrt{-\frac{-1+a+bx}{1+a+bx}}+bx\sqrt{-\frac{-1+a+bx}{1+a+bx}}\right)}{a^3} + \frac{(2-5a^2+6a^4)\,b^3\log\left(1-a^2-abx+\sqrt{1-a^2}\sqrt{-\frac{-1+a+bx}{1+a+bx}}+a\sqrt{1-a^2}\sqrt{-\frac{-1+a+bx}{1+a+bx}}+\sqrt{1-a^2}\,bx\sqrt{-\frac{-1+a+bx}{1+a+bx}}\right)}{a^3\,(1-a^2)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a + b*x]/x^4,x]
```

```
[Out] ((b*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(a - a^4 - a*b*x - 2*b*x*(1 + b*x
) + a^3*(-1 + 4*b*x) + a^2*(1 + 5*b*x + 5*b^2*x^2)))/((-1 + a)^2*a^2*(1 + a
)^2*x^2) - (2*ArcSech[a + b*x])/x^3 - ((2 - 5*a^2 + 6*a^4)*b^3*Log[x])/(a^3
*(1 - a^2)^(5/2)) + (2*b^3*Log[a + b*x])/a^3 - (2*b^3*Log[1 + Sqrt[-((-1 +
a + b*x)/(1 + a + b*x))] + a*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + b*x*Sq
rt[-((-1 + a + b*x)/(1 + a + b*x))]])/a^3 + ((2 - 5*a^2 + 6*a^4)*b^3*Log[1
- a^2 - a*b*x + Sqrt[1 - a^2]*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + a*Sqr
```

```
t[1 - a^2]*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))] + Sqrt[1 - a^2]*b*x*Sqrt[-
((-1 + a + b*x)/(1 + a + b*x))]])/(a^3*(1 - a^2)^(5/2)))/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1026 vs. $2(175) = 350$.
time = 0.45, size = 1027, normalized size = 5.21 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*(-1/3/b^3/x^3*arcsech(b*x+a)-1/6*(-(b*x+a-1)/(b*x+a))^(1/2)*(b*x+a)*((b
*x+a+1)/(b*x+a))^(1/2)*(6*a^4*arctanh(1/(-(b*x+a)^2+1)^(1/2))+3*a^2*(-(b*x+
a)^2+1)^(1/2)-5*(-a^2+1)^(1/2)*ln(2*((-a^2+1)^(1/2)*(-(b*x+a)^2+1)^(1/2)-a*
(b*x+a)+1)/b/x)*a^4-12*arctanh(1/(-(b*x+a)^2+1)^(1/2))*a^3*(b*x+a)+6*arctan
h(1/(-(b*x+a)^2+1)^(1/2))*a^2*(b*x+a)^2+2*(-a^2+1)^(1/2)*ln(2*((-a^2+1)^(1/
2)*(-(b*x+a)^2+1)^(1/2)-a*(b*x+a)+1)/b/x)*a^2+2*(-a^2+1)^(1/2)*ln(2*((-a^2+
1)^(1/2)*(-(b*x+a)^2+1)^(1/2)-a*(b*x+a)+1)/b/x)*(b*x+a)^2+4*arctanh(1/(-(b*
x+a)^2+1)^(1/2))*a*(b*x+a)+7*(-(b*x+a)^2+1)^(1/2)*a^3*(b*x+a)+2*arctanh(1/(
-(b*x+a)^2+1)^(1/2))*a^6*(b*x+a)^2-4*arctanh(1/(-(b*x+a)^2+1)^(1/2))*a^7*(b
*x+a)+6*(-a^2+1)^(1/2)*ln(2*((-a^2+1)^(1/2)*(-(b*x+a)^2+1)^(1/2)-a*(b*x+a)+
1)/b/x)*a^6-5*(-(b*x+a)^2+1)^(1/2)*a^5*(b*x+a)+12*arctanh(1/(-(b*x+a)^2+1)^
(1/2))*a^5*(b*x+a)-6*arctanh(1/(-(b*x+a)^2+1)^(1/2))*a^4*(b*x+a)^2-2*a*(b*x
+a)*(-(b*x+a)^2+1)^(1/2)-12*(-a^2+1)^(1/2)*ln(2*((-a^2+1)^(1/2)*(-(b*x+a)^2
+1)^(1/2)-a*(b*x+a)+1)/b/x)*a^5*(b*x+a)+6*(-a^2+1)^(1/2)*ln(2*((-a^2+1)^(1/
2)*(-(b*x+a)^2+1)^(1/2)-a*(b*x+a)+1)/b/x)*a^4*(b*x+a)^2+10*(-a^2+1)^(1/2)*l
n(2*((-a^2+1)^(1/2)*(-(b*x+a)^2+1)^(1/2)-a*(b*x+a)+1)/b/x)*a^3*(b*x+a)-5*(-
a^2+1)^(1/2)*ln(2*((-a^2+1)^(1/2)*(-(b*x+a)^2+1)^(1/2)-a*(b*x+a)+1)/b/x)*a^
2*(b*x+a)^2-4*(-a^2+1)^(1/2)*ln(2*((-a^2+1)^(1/2)*(-(b*x+a)^2+1)^(1/2)-a*(b
*x+a)+1)/b/x)*a*(b*x+a)-2*arctanh(1/(-(b*x+a)^2+1)^(1/2))*a^2+2*arctanh(1/(
-(b*x+a)^2+1)^(1/2))*a^8+6*(-(b*x+a)^2+1)^(1/2)*a^6-6*arctanh(1/(-(b*x+a)^2
+1)^(1/2))*a^6-9*(-(b*x+a)^2+1)^(1/2)*a^4-2*arctanh(1/(-(b*x+a)^2+1)^(1/2))
*(b*x+a)^2)/b^2/x^2/(a^2-1)^2/(1+a)/(-1+a)/(-(b*x+a)^2+1)^(1/2)/a^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(6*a^4*b^3 - 3*a^2*b^3 + b^3)*log(x)/(a^9 - 3*a^7 + 3*a^5 - a^3) - 1/6*
((a^6*b^3 - 3*a^5*b^3 + 3*a^4*b^3 - a^3*b^3)*x^3*log(b*x + a + 1) + (a^6*b^
3 + 3*a^5*b^3 + 3*a^4*b^3 + a^3*b^3)*x^3*log(-b*x - a + 1) - 2*(3*a^5*b^2 -
 4*a^3*b^2 + a*b^2)*x^2 + (a^6*b - 2*a^4*b + a^2*b)*x + 2*(a^9 - 3*a^7 + 3*
a^5 - a^3)*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)
```

```
*sqrt(-b*x - a + 1)*a + b*x + a) - 2*(a^9 - 3*a^7 + 3*a^5 + (a^6*b^3 - 3*a^
4*b^3 + 3*a^2*b^3 - b^3)*x^3 - a^3)*log(b*x + a) - 2*(a^9 - 3*a^7 + 3*a^5 -
 a^3)*log(b*x + a))/((a^9 - 3*a^7 + 3*a^5 - a^3)*x^3) - integrate(1/3*(b^2*
x + a*b)/(b^2*x^5 + 2*a*b*x^4 + (a^2 - 1)*x^3 + (b^2*x^5 + 2*a*b*x^4 + (a^2
 - 1)*x^3)*e^(1/2*log(b*x + a + 1) + 1/2*log(-b*x - a + 1))), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(167) = 334$.
time = 0.49, size = 987, normalized size = 5.01

$$\left[-\frac{(6a^4-5a^2+2)\sqrt{-a^2+1}\,b^3x^3\log\left(\frac{(2a^2-1)b^2x^2+2a^4+4(a^3-a)bx-4a^2-2(ab^2x^2+a^3+(2a^2-1)bx-a)\sqrt{-a^2+1}\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+2}{x^2}\right)+2(a^6-3a^4+3a^2-1)b^3x^3\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+1}{x}\right)-2(a^6-3a^4+3a^2-1)b^3x^3\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}-1}{x}\right)+4(a^9-3a^7+3a^5-a^3)\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+1}{bx+a}\right)-2((5a^5-7a^3+2a)b^3x^3+(4a^6-5a^4+a^2)b^2x^2-(a^7-2a^5+a^3)bx)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}}{12(a^9-3a^7+3a^5-a^3)x^3},\ -\frac{2(6a^4-5a^2+2)\sqrt{a^2-1}\,b^3x^3\arctan\left(\frac{(ab^2x^2+a^3+(2a^2-1)bx-a)\sqrt{a^2-1}\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}}{(a^2-1)b^2x^2+a^4+2(a^3-a)bx-2a^2+1}\right)+(a^6-3a^4+3a^2-1)b^3x^3\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+1}{x}\right)-(a^6-3a^4+3a^2-1)b^3x^3\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}-1}{x}\right)+2(a^9-3a^7+3a^5-a^3)\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}+1}{bx+a}\right)-((5a^5-7a^3+2a)b^3x^3+(4a^6-5a^4+a^2)b^2x^2-(a^7-2a^5+a^3)bx)\sqrt{-\frac{b^2x^2+2abx+a^2-1}{b^2x^2+2abx+a^2}}}{6(a^9-3a^7+3a^5-a^3)x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/12*((6*a^4 - 5*a^2 + 2)*sqrt(-a^2 + 1)*b^3*x^3*log(((2*a^2 - 1)*b^2*x^2
 + 2*a^4 + 4*(a^3 - a)*b*x - 4*a^2 - 2*(a*b^2*x^2 + a^3 + (2*a^2 - 1)*b*x -
 a)*sqrt(-a^2 + 1)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x +
 a^2)) + 2)/x^2) + 2*(a^6 - 3*a^4 + 3*a^2 - 1)*b^3*x^3*log(((b*x + a)*sqrt(
-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) - 2*(a^6
- 3*a^4 + 3*a^2 - 1)*b^3*x^3*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2
- 1)/(b^2*x^2 + 2*a*b*x + a^2)) - 1)/x) + 4*(a^9 - 3*a^7 + 3*a^5 - a^3)*log
(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2))
+ 1)/(b*x + a)) - 2*((5*a^5 - 7*a^3 + 2*a)*b^3*x^3 + (4*a^6 - 5*a^4 + a^2)*
b^2*x^2 - (a^7 - 2*a^5 + a^3)*b*x)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2
*x^2 + 2*a*b*x + a^2)))/((a^9 - 3*a^7 + 3*a^5 - a^3)*x^3), -1/6*((6*a^4 - 5
*a^2 + 2)*sqrt(a^2 - 1)*b^3*x^3*arctan((a*b^2*x^2 + a^3 + (2*a^2 - 1)*b*x -
 a)*sqrt(a^2 - 1)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x +
a^2))/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) + (a^6 - 3*a
^4 + 3*a^2 - 1)*b^3*x^3*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/
(b^2*x^2 + 2*a*b*x + a^2)) + 1)/x) - (a^6 - 3*a^4 + 3*a^2 - 1)*b^3*x^3*log(
((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) -
 1)/x) + 2*(a^9 - 3*a^7 + 3*a^5 - a^3)*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*
b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)) + 1)/(b*x + a)) - ((5*a^5 - 7*a^3
 + 2*a)*b^3*x^3 + (4*a^6 - 5*a^4 + a^2)*b^2*x^2 - (a^7 - 2*a^5 + a^3)*b*x)*
sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - 1)/(b^2*x^2 + 2*a*b*x + a^2)))/((a^9 - 3*a
^7 + 3*a^5 - a^3)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)/x**4,x)
```

```
[Out] Integral(asech(a + b*x)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))/x^4,x)
```

```
[Out] int(acosh(1/(a + b*x))/x^4, x)
```

3.9 $\int x^2 \text{sech}^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=279

$$-\frac{x}{3b^2}+\frac{2a\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)}{b^3}-\frac{(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)}{3b^3}+\frac{a}{}$$

```
[Out] -1/3*x/b^2+1/3*a^3*arcsech(b*x+a)^2/b^3+1/3*x^3*arcsech(b*x+a)^2-2/3*arcsec
h(b*x+a)*arctan(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/b^3-4*a^
2*arcsech(b*x+a)*arctan(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/
b^3+2*a*ln(b*x+a)/b^3+1/3*I*polylog(2,-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/
(b*x+a)+1)^(1/2)))/b^3+2*I*a^2*polylog(2,-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*
(1/(b*x+a)+1)^(1/2)))/b^3-1/3*I*polylog(2,I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*
(1/(b*x+a)+1)^(1/2)))/b^3-2*I*a^2*polylog(2,I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2
)*(1/(b*x+a)+1)^(1/2)))/b^3+2*a*(b*x+a+1)*arcsech(b*x+a)*((-b*x-a+1)/(b*x+a
+1))^(1/2)/b^3-1/3*(b*x+a)*(b*x+a+1)*arcsech(b*x+a)*((-b*x-a+1)/(b*x+a+1))^
(1/2)/b^3
```

Rubi [A]

time = 0.17, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6456, 5576, 4275, 4265, 2317, 2438, 4269, 3556, 4270}

$$\frac{a^3\text{sech}^{-1}(a+bx)^2}{3b^3}-\frac{4a^2\text{sech}^{-1}(a+bx)\text{ArcTan}\left(e^{\text{sech}^{-1}(a+bx)}\right)}{b^3}+\frac{2ia^2\text{Li}_2\left(-ie^{\text{sech}^{-1}(a+bx)}\right)}{b^3}-\frac{2ia^2\text{Li}_2\left(ie^{\text{sech}^{-1}(a+bx)}\right)}{b^3}-\frac{2\text{sech}^{-1}(a+bx)\text{ArcTan}\left(e^{\text{sech}^{-1}(a+bx)}\right)}{3b^3}+\frac{i\text{Li}_2\left(-ie^{\text{sech}^{-1}(a+bx)}\right)}{3b^3}-\frac{i\text{Li}_2\left(ie^{\text{sech}^{-1}(a+bx)}\right)}{3b^3}+\frac{2a\log(a+bx)}{b^3}+\frac{2a\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)\text{sech}^{-1}(a+bx)}{b^3}-\frac{(a+bx)\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)\text{sech}^{-1}(a+bx)}{3b^3}+\frac{1}{3}x^3\text{sech}^{-1}(a+bx)^2-\frac{x}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSech[a + b*x]^2,x]
```

```
[Out] -1/3*x/b^2 + (2*a*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*ArcSech[a
 + b*x])/b^3 - ((a + b*x)*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*A
rcSech[a + b*x])/(3*b^3) + (a^3*ArcSech[a + b*x]^2)/(3*b^3) + (x^3*ArcSech[
a + b*x]^2)/3 - (2*ArcSech[a + b*x]*ArcTan[E^ArcSech[a + b*x]])/(3*b^3) - (
4*a^2*ArcSech[a + b*x]*ArcTan[E^ArcSech[a + b*x]])/b^3 + (2*a*Log[a + b*x])
/b^3 + ((I/3)*PolyLog[2, (-I)*E^ArcSech[a + b*x]])/b^3 + ((2*I)*a^2*PolyLog
[2, (-I)*E^ArcSech[a + b*x]])/b^3 - ((I/3)*PolyLog[2, I*E^ArcSech[a + b*x]]
)/b^3 - ((2*I)*a^2*PolyLog[2, I*E^ArcSech[a + b*x]])/b^3
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5576

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b
*d*(n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6456

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int x^2\text{sech}^{-1}(a+bx)^2\,dx = -\frac{\text{Subst}\left(\int x^2\text{sech}(x)(-a+\text{sech}(x))^2\tanh(x)\,dx,x,\text{sech}^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{1}{3}x^3\text{sech}^{-1}(a+bx)^2 - \frac{2\text{Subst}\left(\int x(-a+\text{sech}(x))^3\,dx,x,\text{sech}^{-1}(a+bx)\right)}{3b^3}$$

$$= \frac{1}{3}x^3\text{sech}^{-1}(a+bx)^2 - \frac{2\text{Subst}\left(\int \left(-a^3x+3a^2x\text{sech}(x)-3ax\text{sech}^2(x)+x\text{sech}^3(\right.\right.}{3b^3}$$

$$= \frac{a^3\text{sech}^{-1}(a+bx)^2}{3b^3} + \frac{1}{3}x^3\text{sech}^{-1}(a+bx)^2 - \frac{2\text{Subst}\left(\int x\text{sech}^3(x)\,dx,x,\text{sech}^{-1}(a\right.}{3b^3}$$

$$= -\frac{x}{3b^2} + \frac{2a\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)}{b^3} - \frac{(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}\,(}{3}$$

$$= -\frac{x}{3b^2} + \frac{2a\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)}{b^3} - \frac{(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}\,(}{3}$$

$$= -\frac{x}{3b^2} + \frac{2a\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)}{b^3} - \frac{(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}\,(}{3}$$

$$= -\frac{x}{3b^2} + \frac{2a\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)}{b^3} - \frac{(a+bx)\sqrt{\frac{1-a-bx}{1+a+bx}}\,(}{3}$$

Mathematica [A]
time = 1.39, size = 305, normalized size = 1.09

$$-\frac{2(a+bx)\sqrt{-\frac{-1+a+bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)+6a(a+bx)^2\text{sech}^{-1}(a+bx)^2-2(a+bx)^3\text{sech}^{-1}(a+bx)^2+2\left(a+bx-6a\sqrt{-\frac{-1+a+bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)-3a^2(a+bx)\text{sech}^{-1}(a+bx)^2\right)+12a\log\left(\frac{1}{a+bx}\right)-(1+6a^2)\left(\pi\log\left(1-ie^{\text{sech}^{-1}(a+bx)}\right)-2i\text{sech}^{-1}(a+bx)\log\left(1-ie^{\text{sech}^{-1}(a+bx)}\right)-\pi\log\left(1+ie^{\text{sech}^{-1}(a+bx)}\right)+2i\text{sech}^{-1}(a+bx)\log\left(1+ie^{\text{sech}^{-1}(a+bx)}\right)-\pi\log\left(\cot\left(\frac{1}{4}\left(\pi+2i\text{sech}^{-1}(a+bx)\right)\right)\right)+2i\text{PolyLog}\left(2,-ie^{\text{sech}^{-1}(a+bx)}\right)-2i\text{PolyLog}\left(2,ie^{\text{sech}^{-1}(a+bx)}\right)\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSech[a + b*x]^2,x]

[Out] -1/6*(2*(a + b*x)*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)*ArcSe
ch[a + b*x] + 6*a*(a + b*x)^2*ArcSech[a + b*x]^2 - 2*(a + b*x)^3*ArcSech[a
+ b*x]^2 + 2*(a + b*x - 6*a*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a +
b*x)*ArcSech[a + b*x] - 3*a^2*(a + b*x)*ArcSech[a + b*x]^2) + 12*a*Log[(a +
 b*x)^(-1)] - (1 + 6*a^2)*(Pi*Log[1 - I*E^ArcSech[a + b*x]] - (2*I)*ArcSech
```

```
[a + b*x]*Log[1 - I*E^ArcSech[a + b*x]] - Pi*Log[1 + I*E^ArcSech[a + b*x]]
+ (2*I)*ArcSech[a + b*x]*Log[1 + I*E^ArcSech[a + b*x]] - Pi*Log[Cot[(Pi + (
2*I)*ArcSech[a + b*x])/4]] + (2*I)*PolyLog[2, (-I)*E^ArcSech[a + b*x]] - (2
*I)*PolyLog[2, I*E^ArcSech[a + b*x]]))/b^3
```

Maple [A]
time = 1.29, size = 599, normalized size = 2.15

method	result
derivativedivides	$\frac{\operatorname{arcsech}(bx+a)^2 a^2 (bx+a) - \operatorname{arcsech}(bx+a)^2 a (bx+a)^2 + \frac{\operatorname{arcsech}(bx+a)^2 (bx+a)^3}{3} + 2 \operatorname{arcsech}(bx+a) \sqrt{\frac{bx+a+1}{bx+a}} \sqrt{-\frac{bx+a}{bx+}}}{}$
default	$\frac{\operatorname{arcsech}(bx+a)^2 a^2 (bx+a) - \operatorname{arcsech}(bx+a)^2 a (bx+a)^2 + \frac{\operatorname{arcsech}(bx+a)^2 (bx+a)^3}{3} + 2 \operatorname{arcsech}(bx+a) \sqrt{\frac{bx+a+1}{bx+a}} \sqrt{-\frac{bx+a}{bx+}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(arcsech(b*x+a)^2*a^2*(b*x+a)-arcsech(b*x+a)^2*a*(b*x+a)^2+1/3*arcsec
h(b*x+a)^2*(b*x+a)^3+2*arcsech(b*x+a)*((b*x+a+1)/(b*x+a))^(1/2)*(-(b*x+a-1)
/(b*x+a))^(1/2)*a*(b*x+a)-1/3*arcsech(b*x+a)*((b*x+a+1)/(b*x+a))^(1/2)*(-(b
*x+a-1)/(b*x+a))^(1/2)*(b*x+a)^2-2*a*arcsech(b*x+a)-1/3*b*x-1/3*a-1/3*I*dil
og(1-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-1/3*I*arcsech(b
*x+a)*ln(1-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))+1/3*I*arc
sech(b*x+a)*ln(1+I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-2*I
*a^2*dilog(1-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-2*a*ln(
1+(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)+4*a*ln(1/(b*x+a)+(
1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+1/3*I*dilog(1+I*(1/(b*x+a)+(1/(b*x+
a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-2*I*a^2*arcsech(b*x+a)*ln(1-I*(1/(b*x+a)+
(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))+2*I*a^2*dilog(1+I*(1/(b*x+a)+(1/(
b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))+2*I*a^2*arcsech(b*x+a)*ln(1+I*(1/(b*x
+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sq
rt(-b*x - a + 1)*a + b*x + a)^2 - integrate(-2/3*(6*(b^3*x^5 + 3*a*b^2*x^4
```

```
+ (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*l
og(b*x + a)^2 + 6*(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^
2)*log(b*x + a)^2 - (b^3*x^5 + 2*a*b^2*x^4 + (a^2*b - b)*x^3 + 6*(b^3*x^5 +
 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*log(b*x + a) + (3*(b^3*x^
5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*sqrt(b*x + a + 1)*log(
b*x + a) + (2*b^3*x^5 + 4*a*b^2*x^4 + (2*a^2*b - b)*x^3 + 3*(b^3*x^5 + 3*a*
b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*log(b*x + a))*sqrt(b*x + a + 1
))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(
b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^3 + 3*a*b^2*x^2 + a^3
+ (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqr
t(-b*x - a + 1) + (3*a^2*b - b)*x - a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^2*arcsech(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{asech}^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asech(b*x+a)**2,x)
[Out] Integral(x**2*asech(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^2*arcsech(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{acosh}\left(\frac{1}{a+b\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acosh(1/(a + b*x))^2,x)
[Out] int(x^2*acosh(1/(a + b*x))^2, x)
```

3.10 $\int x\text{sech}^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=149

$$-\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)}{b^2}-\frac{a^2\text{sech}^{-1}(a+bx)^2}{2b^2}+\frac{1}{2}x^2\text{sech}^{-1}(a+bx)^2+\frac{4a\text{sech}^{-1}(a+bx)\text{Ar}}{b^2}$$

```
[Out] -1/2*a^2*arcsech(b*x+a)^2/b^2+1/2*x^2*arcsech(b*x+a)^2+4*a*arcsech(b*x+a)*a
rctan(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/b^2-ln(b*x+a)/b^2-
2*I*a*polylog(2,-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b^2
+2*I*a*polylog(2,I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b^2
-(b*x+a+1)*arcsech(b*x+a)*((-b*x-a+1)/(b*x+a+1))^(1/2)/b^2
```

Rubi [A]

time = 0.10, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6456, 5576, 4275, 4265, 2317, 2438, 4269, 3556}

$$-\frac{a^2\text{sech}^{-1}(a+bx)^2}{2b^2}+\frac{4a\text{sech}^{-1}(a+bx)\text{ArcTan}\left(e^{\text{sech}^{-1}(a+bx)}\right)}{b^2}-\frac{2ia\text{Li}_2\left(-ie^{\text{sech}^{-1}(a+bx)}\right)}{b^2}+\frac{2ia\text{Li}_2\left(ie^{\text{sech}^{-1}(a+bx)}\right)}{b^2}-\frac{\log(a+bx)}{b^2}-\frac{\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)\text{sech}^{-1}(a+bx)}{b^2}+\frac{1}{2}x^2\text{sech}^{-1}(a+bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSech[a + b*x]^2,x]
```

```
[Out] -((Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*ArcSech[a + b*x])/b^2) -
 (a^2*ArcSech[a + b*x]^2)/(2*b^2) + (x^2*ArcSech[a + b*x]^2)/2 + (4*a*ArcSe
ch[a + b*x]*ArcTan[E^ArcSech[a + b*x]])/b^2 - Log[a + b*x]/b^2 - ((2*I)*a*P
olyLog[2, (-I)*E^ArcSech[a + b*x]])/b^2 + ((2*I)*a*PolyLog[2, I*E^ArcSech[a
 + b*x]])/b^2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5576

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b
*d*(n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6456

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int x\operatorname{sech}^{-1}(a+bx)^2\,dx &= -\frac{\operatorname{Subst}\left(\int x^2\operatorname{sech}(x)(-a+\operatorname{sech}(x))\tanh(x)\,dx,x,\operatorname{sech}^{-1}(a+bx)\right)}{b^2} \\
&= \frac{1}{2}x^2\operatorname{sech}^{-1}(a+bx)^2 - \frac{\operatorname{Subst}\left(\int x(-a+\operatorname{sech}(x))^2\,dx,x,\operatorname{sech}^{-1}(a+bx)\right)}{b^2} \\
&= \frac{1}{2}x^2\operatorname{sech}^{-1}(a+bx)^2 - \frac{\operatorname{Subst}\left(\int \left(a^2x-2ax\operatorname{sech}(x)+x\operatorname{sech}^2(x)\right)\,dx,x,\operatorname{sech}^{-1}(a-\right.}{b^2} \\
&= -\frac{a^2\operatorname{sech}^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\operatorname{sech}^{-1}(a+bx)^2 - \frac{\operatorname{Subst}\left(\int x\operatorname{sech}^2(x)\,dx,x,\operatorname{sech}^{-1}(a-\right.}{b^2} \\
&= -\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{b^2} - \frac{a^2\operatorname{sech}^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\operatorname{sech}^{-1}(a \\
&= -\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{b^2} - \frac{a^2\operatorname{sech}^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\operatorname{sech}^{-1}(a \\
&= -\frac{\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{b^2} - \frac{a^2\operatorname{sech}^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\operatorname{sech}^{-1}(a
\end{aligned}
$$

Mathematica [A]
time = 0.30, size = 172, normalized size = 1.15

$$
\frac{-2\sqrt{-\frac{-1+a+bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)-2a(a+bx)\operatorname{sech}^{-1}(a+bx)^2+(a+bx)^2\operatorname{sech}^{-1}(a+bx)^2-4ia\operatorname{sech}^{-1}(a+bx)\left(\log\left(1-ie^{-\operatorname{sech}^{-1}(a+bx)}\right)-\log\left(1+ie^{-\operatorname{sech}^{-1}(a+bx)}\right)\right)+2\log\left(\frac{1}{a+bx}\right)-4ia\left(\operatorname{PolyLog}\left(2,-ie^{-\operatorname{sech}^{-1}(a+bx)}\right)-\operatorname{PolyLog}\left(2,ie^{-\operatorname{sech}^{-1}(a+bx)}\right)\right)}{2b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSech[a + b*x]^2,x]
```

```
[Out] (-2*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)*ArcSech[a + b*x] -
2*a*(a + b*x)*ArcSech[a + b*x]^2 + (a + b*x)^2*ArcSech[a + b*x]^2 - (4*I)*a
*ArcSech[a + b*x]*(Log[1 - I/E^ArcSech[a + b*x]] - Log[1 + I/E^ArcSech[a +
b*x]]) + 2*Log[(a + b*x)^(-1)] - (4*I)*a*(PolyLog[2, (-I)/E^ArcSech[a + b*x
]] - PolyLog[2, I/E^ArcSech[a + b*x]]))/(2*b^2)
```

Maple [A]
time = 1.13, size = 331, normalized size = 2.22

method	result
derivativedivides	$\frac{-\frac{\operatorname{arcsech}(bx+a)\left(2\operatorname{arcsech}(bx+a)a(bx+a)-\operatorname{arcsech}(bx+a)(bx+a)^2+2\sqrt{-\frac{bx+a-1}{bx+a}}\sqrt{\frac{bx+a+1}{bx+a}}(bx+a)-2\right)}{2}+\ln\left(1+\left(\frac{1}{bx+\right.\right.}{}$

default	$-\frac{\operatorname{arcsech}(bx+a)\left(2\operatorname{arcsech}(bx+a)a(bx+a)-\operatorname{arcsech}(bx+a)(bx+a)^2+2\sqrt{-\frac{bx+a-1}{bx+a}}\sqrt{\frac{bx+a+1}{bx+a}}(bx+a)-2\right)}{2}+\ln\left(1+\left(\frac{1}{bx+a}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(-1/2*arcsech(b*x+a)*(2*arcsech(b*x+a)*a*(b*x+a)-arcsech(b*x+a)*(b*x+
a)^2+2*(-(b*x+a-1)/(b*x+a))^(1/2)*((b*x+a+1)/(b*x+a))^(1/2)*(b*x+a)-2)+ln(1
+(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)-2*ln(1/(b*x+a)+(1/(
b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))-2*I*a*arcsech(b*x+a)*ln(1+I*(1/(b*x+a)
+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))+2*I*a*arcsech(b*x+a)*ln(1-I*(1/(
b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-2*I*a*dilog(1+I*(1/(b*x+a)
+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))+2*I*a*dilog(1-I*(1/(b*x+a)+(1/(b
*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sq
rt(-b*x - a + 1)*a + b*x + a)^2 - integrate(-(4*(b^3*x^4 + 3*a*b^2*x^3 + (3
*a^2*b - b)*x^2 + (a^3 - a)*x)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x
 + a)^2 + 4*(b^3*x^4 + 3*a*b^2*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*log(b
*x + a)^2 - (b^3*x^4 + 2*a*b^2*x^3 + (a^2*b - b)*x^2 + 4*(b^3*x^4 + 3*a*b^2
*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*log(b*x + a) + (2*(b^3*x^4 + 3*a*b^
2*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*sqrt(b*x + a + 1)*log(b*x + a) + (
2*b^3*x^4 + 4*a*b^2*x^3 + (2*a^2*b - b)*x^2 + 2*(b^3*x^4 + 3*a*b^2*x^3 + (3
*a^2*b - b)*x^2 + (a^3 - a)*x)*log(b*x + a))*sqrt(b*x + a + 1))*sqrt(-b*x -
 a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*s
qrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (b^3*x^3 + 3
*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1
) + (3*a^2*b - b)*x - a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x*arcsech(b*x + a)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\text{asech}^2\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asech(b*x+a)**2,x)
[Out] Integral(x*asech(a + b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x*arcsech(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\text{acosh}\left(\frac{1}{a+b\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acosh(1/(a + b*x))^2,x)
[Out] int(x*acosh(1/(a + b*x))^2, x)
```

3.11 $\int \operatorname{sech}^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=80

$$\frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{b}-\frac{4\operatorname{sech}^{-1}(a+bx)\operatorname{ArcTan}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}+\frac{2i\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}-\frac{2i\operatorname{PolyLo}}{}$$

```
[Out] (b*x+a)*arcsech(b*x+a)^2/b-4*arcsech(b*x+a)*arctan(1/(b*x+a)+(1/(b*x+a)-1)^
(1/2)*(1/(b*x+a)+1)^(1/2))/b+2*I*polylog(2,-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2
)*(1/(b*x+a)+1)^(1/2)))/b-2*I*polylog(2,I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1
/(b*x+a)+1)^(1/2)))/b
```

Rubi [A]
time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6450, 6414, 5526, 4265, 2317, 2438}

$$-\frac{4\operatorname{sech}^{-1}(a+bx)\operatorname{ArcTan}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}+\frac{2i\operatorname{Li}_2\left(-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}-\frac{2i\operatorname{Li}_2\left(ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}+\frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]^2,x]
```

```
[Out] ((a + b*x)*ArcSech[a + b*x]^2)/b - (4*ArcSech[a + b*x]*ArcTan[E^ArcSech[a +
 b*x]])/b + ((2*I)*PolyLog[2, (-I)*E^ArcSech[a + b*x]])/b - ((2*I)*PolyLog[
2, I*E^ArcSech[a + b*x]])/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rule 6414

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-c^(-1), Su
bst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a,
b, c, n}, x] && IGtQ[n, 0]
```

Rule 6450

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSech[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \operatorname{sech}^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \operatorname{sech}^{-1}(x)^2\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\text{Subst}\left(\int x^2\operatorname{sech}(x)\tanh(x)\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b}$$

$$= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{b} - \frac{2\text{Subst}\left(\int x\operatorname{sech}(x)\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b}$$

$$= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{b} - \frac{4\operatorname{sech}^{-1}(a+bx)\tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{(2i)\text{Subst}\left(\int \text{lo}\right.}{}$$

$$= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{b} - \frac{4\operatorname{sech}^{-1}(a+bx)\tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{(2i)\text{Subst}\left(\int \text{lo}\right.}{}$$

$$= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^2}{b} - \frac{4\operatorname{sech}^{-1}(a+bx)\tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{2i\text{Li}_2\left(-ie^{\operatorname{sech}^{-}}\right.}{b}$$

Mathematica [A]
time = 0.15, size = 105, normalized size = 1.31

$$\frac{i\left(\operatorname{sech}^{-1}(a+bx)\left(-i(a+bx)\operatorname{sech}^{-1}(a+bx)+2\log\left(1-ie^{-\operatorname{sech}^{-1}(a+bx)}\right)-2\log\left(1+ie^{-\operatorname{sech}^{-1}(a+bx)}\right)\right)+2\text{PolyLog}\left(2,-ie^{-\operatorname{sech}^{-1}(a+bx)}\right)-2\text{PolyLog}\left(2,ie^{-\operatorname{sech}^{-1}(a+bx)}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a + b*x]^2,x]
```

```
[Out] (I*(ArcSech[a + b*x]*((-I)*(a + b*x)*ArcSech[a + b*x] + 2*Log[1 - I/E^ArcSe
ch[a + b*x]] - 2*Log[1 + I/E^ArcSech[a + b*x]]) + 2*PolyLog[2, (-I)/E^ArcSe
ch[a + b*x]] - 2*PolyLog[2, I/E^ArcSech[a + b*x]]))/b
```

Maple [A]
time = 0.58, size = 192, normalized size = 2.40

method	result
derivativedivides	$\frac{\operatorname{arcsech}(bx+a)^2(bx+a)+2i\operatorname{arcsech}(bx+a)\ln\left(1+i\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)\right)-2i\operatorname{arcsech}(bx+a)\ln\left(1-\right.}{}$
default	$\frac{\operatorname{arcsech}(bx+a)^2(bx+a)+2i\operatorname{arcsech}(bx+a)\ln\left(1+i\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)\right)-2i\operatorname{arcsech}(bx+a)\ln\left(1-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arcsech(b*x+a)^2*(b*x+a)+2*I*arcsech(b*x+a)*ln(1+I*(1/(b*x+a)+(1/(b*x+
a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))-2*I*arcsech(b*x+a)*ln(1-I*(1/(b*x+a)+(1/(
b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))+2*I*dilog(1+I*(1/(b*x+a)+(1/(b*x+a)-1
)^(1/2)*(1/(b*x+a)+1)^(1/2)))-2*I*dilog(1-I*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*
(1/(b*x+a)+1)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*
x - a + 1)*a + b*x + a)^2 - integrate(-2*(2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 +
(3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^2 +
2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^2 - (b^3
*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^
2*b - b)*x - a)*log(b*x + a) + ((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b
)*x - a)*sqrt(b*x + a + 1)*log(b*x + a) + (2*b^3*x^3 + 4*a*b^2*x^2 + (2*a^2
*b - b)*x + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a
))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x -
 a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^3 +
 3*a*b^2*x^2 + a^3 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sq
rt(b*x + a + 1)*sqrt(-b*x - a + 1) + (3*a^2*b - b)*x - a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2,x, algorithm="fricas")
[Out] integral(arcsech(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)**2,x)
[Out] Integral(asech(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2,x, algorithm="giac")
[Out] integrate(arcsech(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}\left(\frac{1}{a+b\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))^2,x)
[Out] int(acosh(1/(a + b*x))^2, x)
```

3.12 $\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x}\,dx$

Optimal. Leaf size=274

$$\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)-\operatorname{sech}^{-1}(a+bx)^2\log\left(1+\right.$$

```
[Out] -arcsech(b*x+a)^2*ln(1+(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^
2)+arcsech(b*x+a)^2*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/
2))/(1-(-a^2+1)^(1/2)))+arcsech(b*x+a)^2*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1
/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))-arcsech(b*x+a)*polylog(2,-(1/(
b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)+2*arcsech(b*x+a)*polylog
(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2))
)+2*arcsech(b*x+a)*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)
^(1/2))/(1+(-a^2+1)^(1/2)))+1/2*polylog(3,-(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(
1/(b*x+a)+1)^(1/2))^2)-2*polylog(3,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x
+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))-2*polylog(3,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1
/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))
```

Rubi [A]
time = 0.31, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6456, 5714, 5689, 3799, 2221, 2611, 2320, 6724, 5681}

$$2\operatorname{sech}^{-1}(a+bx)\operatorname{Li}_2\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+2\operatorname{sech}^{-1}(a+bx)\operatorname{Li}_2\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)-2\operatorname{Li}_3\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-2\operatorname{Li}_3\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)-\operatorname{sech}^{-1}(a+bx)\operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(a+bx)}\right)+\frac{1}{2}\operatorname{Li}_3\left(-e^{2\operatorname{sech}^{-1}(a+bx)}\right)-\operatorname{sech}^{-1}(a+bx)^2\log\left(e^{2\operatorname{sech}^{-1}(a+bx)}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]^2/x,x]
```

```
[Out] ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + Ar
cSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - ArcS
ech[a + b*x]^2*Log[1 + E^(2*ArcSech[a + b*x])] + 2*ArcSech[a + b*x]*PolyLog
[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + 2*ArcSech[a + b*x]*PolyLo
g[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - ArcSech[a + b*x]*PolyLog
[2, -E^(2*ArcSech[a + b*x])] - 2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 - Sqr
t[1 - a^2])] - 2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] + P
olyLog[3, -E^(2*ArcSech[a + b*x])]/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5689

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/(Cosh[(c_.)
 + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Tanh[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Sinh[c + d*x]*(Tanh[c + d*x]^
(n - 1)/(a + b*Cosh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5714

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sech[(c_.) + (d_.)*(x_)]), x_Symbol] :> I
nt[(e + f*x)^m*Cosh[c + d*x]*F[c + d*x]^n*(G[c + d*x]^p/(b + a*Cosh[c + d*x
])), x] /; FreeQ[{a, b, c, d, e, f}, x] && HyperbolicQ[F] && HyperbolicQ[G]
 && IntegersQ[m, n, p]
```

Rule 6456

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\text{sech}^{-1}(a+bx)^2}{x}\,dx = -\text{Subst}\left(\int \frac{x^2\text{sech}(x)\tanh(x)}{-a+\text{sech}(x)}\,dx, x, \text{sech}^{-1}(a+bx)\right)$$

$$= -\text{Subst}\left(\int \frac{x^2\tanh(x)}{1-a\cosh(x)}\,dx, x, \text{sech}^{-1}(a+bx)\right)$$

$$= -\left(a\text{Subst}\left(\int \frac{x^2\sinh(x)}{1-a\cosh(x)}\,dx, x, \text{sech}^{-1}(a+bx)\right)\right) - \text{Subst}\left(\int x^2\tanh(x)\,dx, x,\right.$$

$$= -\left(2\text{Subst}\left(\int \frac{e^{2x}x^2}{1+e^{2x}}\,dx, x, \text{sech}^{-1}(a+bx)\right)\right) - a\text{Subst}\left(\int \frac{e^x x^2}{1-\sqrt{1-a^2}-ae^x}\,d\right.$$

$$= \text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right.$$

$$= \text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right.$$

$$= \text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right.$$

$$= \text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right.$$

Mathematica [A]
time = 0.21, size = 280, normalized size = 1.02

$$-\frac{2}{3}\text{sech}^{-1}(a+bx)^3-\text{sech}^{-1}(a+bx)^2\log\left(1+e^{-2\text{sech}^{-1}(a+bx)}\right)+\text{sech}^{-1}(a+bx)^2\log\left(1+\frac{ae^{\text{sech}^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right)+\text{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)+\text{sech}^{-1}(a+bx)\text{PolyLog}\left(2,-e^{-2\text{sech}^{-1}(a+bx)}\right)+2\text{sech}^{-1}(a+bx)\text{PolyLog}\left(2,-\frac{ae^{\text{sech}^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right)+2\text{sech}^{-1}(a+bx)\text{PolyLog}\left(2,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)+\frac{1}{2}\text{PolyLog}\left(3,-e^{-2\text{sech}^{-1}(a+bx)}\right)-2\text{PolyLog}\left(3,-\frac{ae^{\text{sech}^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right)-2\text{PolyLog}\left(3,\frac{ae^{\text{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSech[a + b*x]^2/x,x]
```

```
[Out] (-2*ArcSech[a + b*x]^3)/3 - ArcSech[a + b*x]^2*Log[1 + E^(-2*ArcSech[a + b*
x])] + ArcSech[a + b*x]^2*Log[1 + (a*E^ArcSech[a + b*x])/(-1 + Sqrt[1 - a^2
])] + ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])
] + ArcSech[a + b*x]*PolyLog[2, -E^(-2*ArcSech[a + b*x])] + 2*ArcSech[a + b
*x]*PolyLog[2, -((a*E^ArcSech[a + b*x])/(-1 + Sqrt[1 - a^2]))] + 2*ArcSech[
a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] + PolyLog[3
, -E^(-2*ArcSech[a + b*x])]/2 - 2*PolyLog[3, -((a*E^ArcSech[a + b*x])/(-1 +
 Sqrt[1 - a^2]))] - 2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])
]
```

Maple [**F**]
time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsech}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)^2/x,x)
```

```
[Out] int(arcsech(b*x+a)^2/x,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsech(b*x + a)^2/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arcsech(b*x + a)^2/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)**2/x,x)
```

```
[Out] Integral(asech(a + b*x)**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))^2/x,x)
```

```
[Out] int(acosh(1/(a + b*x))^2/x, x)
```

3.13 $\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^2} dx$

Optimal. Leaf size=224

$$-\frac{b\operatorname{sech}^{-1}(a+bx)^2}{a}-\frac{\operatorname{sech}^{-1}(a+bx)^2}{x}+\frac{2b\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}-\frac{2b\operatorname{sech}^{-1}(a+bx)\log\left(1-\right.}{a\sqrt{1-a^2}}$$

```
[Out] -b*arcsech(b*x+a)^2/a-arcsech(b*x+a)^2/x+2*b*arcsech(b*x+a)*ln(1-a*(1/(b*x+
a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2+1)^
(1/2)-2*b*arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1
)^(1/2))/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)+2*b*polylog(2,a*(1/(b*x+a)+(1
/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)
-2*b*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a
^2+1)^(1/2)))/a/(-a^2+1)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6456, 5576, 4276, 3401, 2296, 2221, 2317, 2438}

$$\frac{2b\operatorname{Li}_2\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}-\frac{2b\operatorname{Li}_2\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}+\frac{2b\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}-\frac{2b\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}-\frac{b\operatorname{sech}^{-1}(a+bx)^2}{a}-\frac{\operatorname{sech}^{-1}(a+bx)^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]^2/x^2,x]
```

```
[Out] -((b*ArcSech[a + b*x]^2)/a) - ArcSech[a + b*x]^2/x + (2*b*ArcSech[a + b*x]*
Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) - (2
*b*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a
*Sqrt[1 - a^2]) + (2*b*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2]
)])/(a*Sqrt[1 - a^2]) - (2*b*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1
- a^2])])/(a*Sqrt[1 - a^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5576

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b
*d*(n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6456

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^2}\,dx = -\left(b\text{Subst}\left(\int \frac{x^2\operatorname{sech}(x)\tanh(x)}{(-a+\operatorname{sech}(x))^2}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right)$$

$$= -\frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + (2b)\text{Subst}\left(\int \frac{x}{-a+\operatorname{sech}(x)}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)$$

$$= -\frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + (2b)\text{Subst}\left(\int \left(-\frac{x}{a} + \frac{x}{a(1-a\cosh(x))}\right)\,dx, x, \operatorname{sech}^{-1}(a+b$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^2}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + \frac{(2b)\text{Subst}\left(\int \frac{x}{1-a\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(a+}{a}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^2}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + \frac{(4b)\text{Subst}\left(\int \frac{e^x x}{-a+2e^x-ae^{2x}}\,dx, x, \operatorname{sech}^{-1}(a-}{a}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^2}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} - \frac{(4b)\text{Subst}\left(\int \frac{e^x x}{2-2\sqrt{1-a^2}-2ae^x}\,dx, x, \text{sec}}{\sqrt{1-a^2}}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^2}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + \frac{2b\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^2}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + \frac{2b\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^2}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^2}{x} + \frac{2b\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.83, size = 678, normalized size = 3.03

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a + b*x]^2/x^2,x]
```

```
[Out] (-(((a + b*x)*ArcSech[a + b*x]^2)/x) + (2*b*(2*ArcSech[a + b*x]*ArcTan[((-1
 + a)*Coth[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] - (2*I)*ArcCos[a^(-1)]*ArcT
an[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] + (ArcCos[a^(-1)] + 2
*(ArcTan[((-1 + a)*Coth[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] + ArcTan[((1 +
 a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]]))*Log[Sqrt[-1 + a^2]/(Sqrt[2]
*Sqrt[a]*E^(ArcSech[a + b*x]/2)*Sqrt[-((b*x)/(a + b*x))])] + (ArcCos[a^(-1)
```

```
] - 2*(ArcTan[((-1 + a)*Coth[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] + ArcTan[
((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]]))*Log[(Sqrt[-1 + a^2]*E^
(ArcSech[a + b*x]/2))/(Sqrt[2]*Sqrt[a]*Sqrt[-((b*x)/(a + b*x))])] - (ArcCos
[a^(-1)] + 2*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]])*Log
[-(((-1 + a)*(1 + a - I*Sqrt[-1 + a^2])*(-1 + Tanh[ArcSech[a + b*x]/2]))/(a
*(-1 + a + I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2])))] - (ArcCos[a^(-1)]
- 2*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]])*Log[((-1 + a
)*(1 + a + I*Sqrt[-1 + a^2])*(1 + Tanh[ArcSech[a + b*x]/2]))/(a*(-1 + a + I
*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))] + I*(PolyLog[2, ((-1 - I*Sqrt[-
1 + a^2])*(-1 + a - I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))/(a*(-1 + a
+ I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))] - PolyLog[2, ((I + Sqrt[-1 +
 a^2])*(-1 + a - I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))/(a*((-I)*(-1 +
 a) + Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))])))/Sqrt[-1 + a^2])/a
```

Maple [A]
time = 0.99, size = 362, normalized size = 1.62

method	result
derivativedivides	$b\left(-\frac{(bx+a)\operatorname{arcsech}(bx+a)^2}{abx}+\frac{2\sqrt{-a^2+1}\,\operatorname{arcsech}(bx+a)\ln\left(\frac{-a\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)+\sqrt{}}{1+\sqrt{-a^2+1}}\right)}{a(a^2-1)}\right.$
default	$b\left(-\frac{(bx+a)\operatorname{arcsech}(bx+a)^2}{abx}+\frac{2\sqrt{-a^2+1}\,\operatorname{arcsech}(bx+a)\ln\left(\frac{-a\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)+\sqrt{}}{1+\sqrt{-a^2+1}}\right)}{a(a^2-1)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-(b*x+a)*arcsech(b*x+a)^2/a/b/x+2*(-a^2+1)^(1/2)/a/(a^2-1)*arcsech(b*x+a
)*ln((-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)
+1)/(1+(-a^2+1)^(1/2)))-2*(-a^2+1)^(1/2)/a/(a^2-1)*arcsech(b*x+a)*ln((a*(1/
(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^
2+1)^(1/2)))+2*(-a^2+1)^(1/2)/a/(a^2-1)*dilog((-a*(1/(b*x+a)+(1/(b*x+a)-1)^
(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))-2*(-a^2+1)
^(1/2)/a/(a^2-1)*dilog((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2
))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x
 - a + 1)*a + b*x + a)^2/x - integrate(-2*(2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 +
 (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^2 +
 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^2 + (b^
3*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x - 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a
^2*b - b)*x - a)*log(b*x + a) - ((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b -
b)*x - a)*sqrt(b*x + a + 1)*log(b*x + a) - (2*b^3*x^3 + 4*a*b^2*x^2 + (2*a^
2*b - b)*x - (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x +
a))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x
- a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^5
+ 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 + (a^3 - a)*x^2 + (b^3*x^5 + 3*a*b^2*x^4
+ (3*a^2*b - b)*x^3 + (a^3 - a)*x^2)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsech(b*x + a)^2/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)**2/x**2,x)
```

```
[Out] Integral(asech(a + b*x)**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))^2/x^2,x)
```

```
[Out] int(acosh(1/(a + b*x))^2/x^2, x)
```

3.14 $\int \frac{\operatorname{sech}^{-1}(a+bx)^2}{x^3}\,dx$

Optimal. Leaf size=537

$$\frac{b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)}{a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)}+\frac{b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2}-\frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2}+\frac{b^2\operatorname{sech}^{-1}(a+bx)\log\Big(}{a^2\left(1-a\right.}$$

[Out] 1/2*b^2*arcsech(b*x+a)^2/a^2-1/2*arcsech(b*x+a)^2/x^2+b^2*ln(x/(b*x+a))/a^2/(-a^2+1)+b^2*arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)-b^2*arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)+b^2*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)-b^2*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)-2*b^2*arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(1/2)+2*b^2*arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(1/2)-2*b^2*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(1/2)+2*b^2*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(1/2)+b^2*(b*x+a+1)*arcsech(b*x+a)*((-b*x-a+1)/(b*x+a+1))^(1/2)/a/(-a^2+1)/(b*x+a)/(1-a/(b*x+a))

Rubi [A]
time = 0.52, antiderivative size = 537, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {6456, 5576, 4276, 3405, 3401, 2296, 2221, 2317, 2438, 2747, 31}

$$-\frac{2b^2\operatorname{Li}_2\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2\sqrt{1-a^2}}+\frac{b^2\operatorname{Li}_2\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2\left(1-a^2\right)^{3/2}}+\frac{2b^2\operatorname{Li}_2\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a^2\sqrt{1-a^2}}-\frac{b^2\operatorname{Li}_2\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a^2\left(1-a^2\right)^{3/2}}+\frac{b^2\log\left(\frac{x}{a+bx}\right)}{a^2\left(1-a^2\right)}+\frac{b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2}+\frac{b^2\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)\operatorname{sech}^{-1}(a+bx)}{a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)}-\frac{2b^2\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2\sqrt{1-a^2}}+\frac{b^2\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a^2\left(1-a^2\right)^{3/2}}+\frac{2b^2\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a^2\sqrt{1-a^2}}-\frac{b^2\operatorname{sech}^{-1}(a+bx)\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a^2\left(1-a^2\right)^{3/2}}-\frac{\operatorname{sech}^{-1}(a+bx)^2}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[ArcSech[a + b*x]^2/x^3,x]

[Out] (b^2*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 + a + b*x)*ArcSech[a + b*x])/(a*(1 - a^2)*(a + b*x)*(1 - a/(a + b*x))) + (b^2*ArcSech[a + b*x]^2)/(2*a^2) - ArcSech[a + b*x]^2/(2*x^2) + (b^2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*(1 - a^2)^(3/2)) - (2*b^2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2]) - (b^2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*(1 - a^2)^(3/2)) + (2*b^2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2]) + (b^2*Log[x/(a + b*x)])/(a^2*(1 - a^2)) + (b^2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*(1 - a^2)^(3/2)) - (2*b^2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqr

```
t[1 - a^2])])/(a^2*Sqrt[1 - a^2]) - (b^2*PolyLog[2, (a*E^ArcSech[a + b*x])/
(1 + Sqrt[1 - a^2])])/(a^2*(1 - a^2)^(3/2)) + (2*b^2*PolyLog[2, (a*E^ArcSec
h[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
```

```
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5576

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b
*d*(n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6456

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\text{sech}^{-1}(a+bx)^2}{x^3}\,dx = -\left(b^2\text{Subst}\left(\int \frac{x^2\text{sech}(x)\tanh(x)}{(-a+\text{sech}(x))^3}\,dx, x, \text{sech}^{-1}(a+bx)\right)\right)$$

$$= -\frac{\text{sech}^{-1}(a+bx)^2}{2x^2} + b^2\text{Subst}\left(\int \frac{x}{(-a+\text{sech}(x))^2}\,dx, x, \text{sech}^{-1}(a+bx)\right)$$

$$= -\frac{\text{sech}^{-1}(a+bx)^2}{2x^2} + b^2\text{Subst}\left(\int \left(\frac{x}{a^2} + \frac{x}{a^2(-1+a\cosh(x))^2} + \frac{2x}{a^2(-1+a\cosh(x))}\right.\right.$$

$$= \frac{b^2\text{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\text{sech}^{-1}(a+bx)^2}{2x^2} + \frac{b^2\text{Subst}\left(\int \frac{x}{(-1+a\cosh(x))^2}\,dx, x, \text{sech}^{-1}(a+b\right.}{a^2}$$

$$= \frac{b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)}{a\,(1-a^2)\,(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\text{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\text{sech}^{-1}(a+bx)}{2x^2}$$

$$= \frac{b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)}{a\,(1-a^2)\,(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\text{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\text{sech}^{-1}(a+bx)}{2x^2}$$

$$= \frac{b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)}{a\,(1-a^2)\,(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\text{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\text{sech}^{-1}(a+bx)}{2x^2}$$

$$= \frac{b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)}{a\,(1-a^2)\,(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\text{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\text{sech}^{-1}(a+bx)}{2x^2}$$

$$= \frac{b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)}{a\,(1-a^2)\,(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\text{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\text{sech}^{-1}(a+bx)}{2x^2}$$

$$= \frac{b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)}{a\,(1-a^2)\,(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\text{sech}^{-1}(a+bx)^2}{2a^2} - \frac{\text{sech}^{-1}(a+bx)}{2x^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.73, size = 1439, normalized size = 2.68

Too large to display

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a + b*x]^2/x^3,x]
```

```
[Out] -1/2*((a + b*x)^2*ArcSech[a + b*x]^2)/(a^2*x^2) + (b*ArcSech[a + b*x]*(-(a*
Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)) + (-1 + a^2)*(a + b*x)
*ArcSech[a + b*x]))/((-1 + a)*a^2*(1 + a)*x) + (b^2*Log[(b*x)/(a + b*x)])/(
a^2 - a^4) - (2*b^2*(2*ArcSech[a + b*x]*ArcTan[((-1 + a)*Coth[ArcSech[a + b
*x]/2])/Sqrt[-1 + a^2]] - (2*I)*ArcCos[a^(-1)]*ArcTan[((1 + a)*Tanh[ArcSech
[a + b*x]/2])/Sqrt[-1 + a^2]] + (ArcCos[a^(-1)] + 2*(ArcTan[((-1 + a)*Coth[
ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] + ArcTan[((1 + a)*Tanh[ArcSech[a + b*x
]/2])/Sqrt[-1 + a^2]]))*Log[Sqrt[-1 + a^2]/(Sqrt[2]*Sqrt[a]*E^(ArcSech[a +
b*x]/2)*Sqrt[-((b*x)/(a + b*x))])] + (ArcCos[a^(-1)] - 2*(ArcTan[((-1 + a)*
Coth[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] + ArcTan[((1 + a)*Tanh[ArcSech[a
+ b*x]/2])/Sqrt[-1 + a^2]]))*Log[(Sqrt[-1 + a^2]*E^(ArcSech[a + b*x]/2))/(S
qrt[2]*Sqrt[a]*Sqrt[-((b*x)/(a + b*x))])] - (ArcCos[a^(-1)] + 2*ArcTan[((1
+ a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]])*Log[-(((-1 + a)*(1 + a - I*
Sqrt[-1 + a^2])*(-1 + Tanh[ArcSech[a + b*x]/2]))/(a*(-1 + a + I*Sqrt[-1 + a
^2]*Tanh[ArcSech[a + b*x]/2])))] - (ArcCos[a^(-1)] - 2*ArcTan[((1 + a)*Tanh
[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]])*Log[((-1 + a)*(1 + a + I*Sqrt[-1 + a
^2])*(1 + Tanh[ArcSech[a + b*x]/2]))/(a*(-1 + a + I*Sqrt[-1 + a^2]*Tanh[Arc
Sech[a + b*x]/2]))] + I*(PolyLog[2, ((-1 - I*Sqrt[-1 + a^2])*(-1 + a - I*Sq
rt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))/(a*(-1 + a + I*Sqrt[-1 + a^2]*Tanh[
ArcSech[a + b*x]/2]))] - PolyLog[2, ((I + Sqrt[-1 + a^2])*(-1 + a - I*Sqrt[
-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))/(a*((-I)*(-1 + a) + Sqrt[-1 + a^2]*Tan
h[ArcSech[a + b*x]/2]))])))/(-1 + a^2)^(3/2) + (b^2*(2*ArcSech[a + b*x]*Arc
Tan[((-1 + a)*Coth[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] - (2*I)*ArcCos[a^(-
1)]*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] + (ArcCos[a^(
-1)] + 2*(ArcTan[((-1 + a)*Coth[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] + ArcT
an[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]]))*Log[Sqrt[-1 + a^2]/
(Sqrt[2]*Sqrt[a]*E^(ArcSech[a + b*x]/2)*Sqrt[-((b*x)/(a + b*x))])] + (ArcCo
s[a^(-1)] - 2*(ArcTan[((-1 + a)*Coth[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]] +
 ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]]))*Log[(Sqrt[-1 +
 a^2]*E^(ArcSech[a + b*x]/2))/(Sqrt[2]*Sqrt[a]*Sqrt[-((b*x)/(a + b*x))])] -
 (ArcCos[a^(-1)] + 2*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^
2]])*Log[-(((-1 + a)*(1 + a - I*Sqrt[-1 + a^2])*(-1 + Tanh[ArcSech[a + b*x]
/2]))/(a*(-1 + a + I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2])))] - (ArcCos[
a^(-1)] - 2*ArcTan[((1 + a)*Tanh[ArcSech[a + b*x]/2])/Sqrt[-1 + a^2]])*Log[
((-1 + a)*(1 + a + I*Sqrt[-1 + a^2])*(1 + Tanh[ArcSech[a + b*x]/2]))/(a*(-1
 + a + I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))] + I*(PolyLog[2, ((-1 -
I*Sqrt[-1 + a^2])*(-1 + a - I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))/(a*
(-1 + a + I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))] - PolyLog[2, ((I + S
qrt[-1 + a^2])*(-1 + a - I*Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))/(a*((-
I)*(-1 + a) + Sqrt[-1 + a^2]*Tanh[ArcSech[a + b*x]/2]))])))/(a^2*(-1 + a^2)
^(3/2))
```

Maple [A]

time = 1.40, size = 982, normalized size = 1.83 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/2*arcsech(b*x+a)*(2*arcsech(b*x+a)*a^3*(b*x+a)-arcsech(b*x+a)*a^2*(
b*x+a)^2-2*((b*x+a+1)/(b*x+a))^(1/2)*(-(b*x+a-1)/(b*x+a))^(1/2)*a^2*(b*x+a)
+2*((b*x+a+1)/(b*x+a))^(1/2)*(-(b*x+a-1)/(b*x+a))^(1/2)*a*(b*x+a)^2-2*arcse
ch(b*x+a)*a*(b*x+a)+arcsech(b*x+a)*(b*x+a)^2+2*a^2-4*a*(b*x+a)+2*(b*x+a)^2)
/a^2/(a^2-1)/b^2/x^2+2/a^2/(a^2-1)*ln(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x
+a)+1)^(1/2))-1/a^2/(a^2-1)*ln(a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+
1)^(1/2))^2+a-2/(b*x+a)-2*(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)
^(1/2)/a^2/(a^2-1)^2*arcsech(b*x+a)*ln((-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(
1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))-(-a^2+1)^(1/2)/a^
2/(a^2-1)^2*arcsech(b*x+a)*ln((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+
1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))+(-a^2+1)^(1/2)/a^2/(a^2-1)
^2*dilog((-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(
1/2)+1)/(1+(-a^2+1)^(1/2)))-(-a^2+1)^(1/2)/a^2/(a^2-1)^2*dilog((a*(1/(b*x+a
)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(
1/2)))-2*(-a^2+1)^(1/2)/(a^2-1)^2*arcsech(b*x+a)*ln((-a*(1/(b*x+a)+(1/(b*x+
a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))+2*(-
a^2+1)^(1/2)/(a^2-1)^2*arcsech(b*x+a)*ln((a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*
(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))-2*(-a^2+1)^(1/2
)/(a^2-1)^2*dilog((-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(
-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))+2*(-a^2+1)^(1/2)/(a^2-1)^2*dilog((a*(1
/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a
^2+1)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(
-b*x - a + 1)*a + b*x + a)^2/x^2 - integrate(-(4*(b^3*x^3 + 3*a*b^2*x^2 + a
^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)
^2 + 4*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^2 +
 (b^3*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x - 4*(b^3*x^3 + 3*a*b^2*x^2 + a^3 +
(3*a^2*b - b)*x - a)*log(b*x + a) - (2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^
2*b - b)*x - a)*sqrt(b*x + a + 1)*log(b*x + a) - (2*b^3*x^3 + 4*a*b^2*x^2 +
 (2*a^2*b - b)*x - 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*lo
g(b*x + a))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sq
rt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(
b^3*x^6 + 3*a*b^2*x^5 + (3*a^2*b - b)*x^4 + (a^3 - a)*x^3 + (b^3*x^6 + 3*a*
b^2*x^5 + (3*a^2*b - b)*x^4 + (a^3 - a)*x^3)*sqrt(b*x + a + 1)*sqrt(-b*x -
a + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2/x^3,x, algorithm="fricas")
[Out] integral(arcsech(b*x + a)^2/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}^{2}(a+bx)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)**2/x**3,x)
[Out] Integral(asech(a + b*x)**2/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^2/x^3,x, algorithm="giac")
[Out] integrate(arcsech(b*x + a)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{acosh}\left(\frac{1}{a+b\,x}\right)}^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))^2/x^3,x)
[Out] int(acosh(1/(a + b*x))^2/x^3, x)
```

3.15 $\int x\text{sech}^{-1}(a+bx)^3\,dx$

Optimal. Leaf size=260

$$-\frac{3\text{sech}^{-1}(a+bx)^2}{2b^2}-\frac{3\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\text{sech}^{-1}(a+bx)^2}{2b^2}-\frac{a^2\text{sech}^{-1}(a+bx)^3}{2b^2}+\frac{1}{2}x^2\text{sech}^{-1}(a+bx)^3+$$

```
[Out] -3/2*arcsech(b*x+a)^2/b^2-1/2*a^2*arcsech(b*x+a)^3/b^2+1/2*x^2*arcsech(b*x+
a)^3+6*a*arcsech(b*x+a)^2*arctan(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1
)^(1/2))/b^2+3*arcsech(b*x+a)*ln(1+(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a
)+1)^(1/2))^2)/b^2-6*I*a*arcsech(b*x+a)*polylog(2,-I*(1/(b*x+a)+(1/(b*x+a)-
1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b^2+6*I*a*arcsech(b*x+a)*polylog(2,I*(1/(b*x
+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b^2+3/2*polylog(2,-(1/(b*x+a)
+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)/b^2+6*I*a*polylog(3,-I*(1/(b*x
+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b^2-6*I*a*polylog(3,I*(1/(b*x
+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b^2-3/2*(b*x+a+1)*arcsech(b*x
+a)^2*((-b*x-a+1)/(b*x+a+1))^(1/2)/b^2
```

Rubi [A]
time = 0.18, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {6456, 5576, 4275, 4265, 2611, 2320, 6724, 4269, 3799, 2221, 2317, 2438}

$$-\frac{a^2\text{sech}^{-1}(a+bx)^3}{2b^2}+\frac{6a\text{sech}^{-1}(a+bx)^2\text{ArcTan}\left(e^{\text{sech}^{-1}(a+bx)}\right)}{b^2}-\frac{6ia\text{sech}^{-1}(a+bx)\text{Li}_2\left(-ie^{\text{sech}^{-1}(a+bx)}\right)}{b^2}+\frac{6ia\text{sech}^{-1}(a+bx)\text{Li}_2\left(ie^{\text{sech}^{-1}(a+bx)}\right)}{b^2}+\frac{3\text{Li}_2\left(-e^{2\text{sech}^{-1}(a+bx)}\right)}{2b^2}+\frac{6ia\text{Li}_3\left(-ie^{\text{sech}^{-1}(a+bx)}\right)}{b^2}-\frac{6ia\text{Li}_3\left(ie^{\text{sech}^{-1}(a+bx)}\right)}{b^2}-\frac{3\sqrt{\frac{-a-bx+1}{a+bx+1}}\,(a+bx+1)\text{sech}^{-1}(a+bx)^2}{2b^2}-\frac{3\text{sech}^{-1}(a+bx)^2}{2b^2}+\frac{3\text{sech}^{-1}(a+bx)\log\left(e^{2\text{sech}^{-1}(a+bx)}+1\right)}{b^2}+\frac{1}{2}x^2\text{sech}^{-1}(a+bx)^3$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSech[a + b*x]^3,x]
```

```
[Out] (-3*ArcSech[a + b*x]^2)/(2*b^2) - (3*Sqrt[(1 - a - b*x)/(1 + a + b*x)]*(1 +
 a + b*x)*ArcSech[a + b*x]^2)/(2*b^2) - (a^2*ArcSech[a + b*x]^3)/(2*b^2) +
(x^2*ArcSech[a + b*x]^3)/2 + (6*a*ArcSech[a + b*x]^2*ArcTan[E^ArcSech[a + b
*x]])/b^2 + (3*ArcSech[a + b*x]*Log[1 + E^(2*ArcSech[a + b*x])])/b^2 - ((6*
I)*a*ArcSech[a + b*x]*PolyLog[2, (-I)*E^ArcSech[a + b*x]])/b^2 + ((6*I)*a*A
rcSech[a + b*x]*PolyLog[2, I*E^ArcSech[a + b*x]])/b^2 + (3*PolyLog[2, -E^(2
*ArcSech[a + b*x])])/(2*b^2) + ((6*I)*a*PolyLog[3, (-I)*E^ArcSech[a + b*x]]
)/b^2 - ((6*I)*a*PolyLog[3, I*E^ArcSech[a + b*x]])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
```

```
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5576

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b
*d*(n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6456

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\operatorname{sech}^{-1}(a+bx)^3\,dx = -\frac{\operatorname{Subst}\left(\int x^3\operatorname{sech}(x)(-a+\operatorname{sech}(x))\tanh(x)\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b^2}$$

$$= \frac{1}{2}x^2\operatorname{sech}^{-1}(a+bx)^3 - \frac{3\operatorname{Subst}\left(\int x^2(-a+\operatorname{sech}(x))^2\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{2b^2}$$

$$= \frac{1}{2}x^2\operatorname{sech}^{-1}(a+bx)^3 - \frac{3\operatorname{Subst}\left(\int \left(a^2x^2 - 2ax^2\operatorname{sech}(x) + x^2\operatorname{sech}^2(x)\right)\,dx, x, \operatorname{sech}^{-1}\right.}{2b^2}$$

$$= -\frac{a^2\operatorname{sech}^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2\operatorname{sech}^{-1}(a+bx)^3 - \frac{3\operatorname{Subst}\left(\int x^2\operatorname{sech}^2(x)\,dx, x, \operatorname{sech}^{-1}(\right.}{2b^2}$$

$$= -\frac{3\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2b^2} - \frac{a^2\operatorname{sech}^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2\operatorname{sech}^{-1}$$

$$= -\frac{3\operatorname{sech}^{-1}(a+bx)^2}{2b^2} - \frac{3\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2b^2} - \frac{a^2\operatorname{sech}^{-1}(a}{2b^2}$$

$$= -\frac{3\operatorname{sech}^{-1}(a+bx)^2}{2b^2} - \frac{3\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2b^2} - \frac{a^2\operatorname{sech}^{-1}(a}{2b^2}$$

$$= -\frac{3\operatorname{sech}^{-1}(a+bx)^2}{2b^2} - \frac{3\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2b^2} - \frac{a^2\operatorname{sech}^{-1}(a}{2b^2}$$

$$= -\frac{3\operatorname{sech}^{-1}(a+bx)^2}{2b^2} - \frac{3\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2b^2} - \frac{a^2\operatorname{sech}^{-1}(a}{2b^2}$$

Mathematica [A]
time = 0.37, size = 254, normalized size = 0.98

$$\frac{-3\sqrt{-\frac{-1+a+bx}{1+a+bx}}(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2 - 2a(a+bx)\operatorname{sech}^{-1}(a+bx)^3 + (a+bx)^2\operatorname{sech}^{-1}(a+bx)^3 + 3\operatorname{sech}^{-1}(a+bx)\left(\operatorname{sech}^{-1}(a+bx) + 2\log\left(1+e^{-2\operatorname{sech}^{-1}(a+bx)}\right)\right) - 3\operatorname{PolyLog}\left(2, -e^{-2\operatorname{sech}^{-1}(a+bx)}\right) + 6ia\left(-\operatorname{sech}^{-1}(a+bx)^2\left(\log\left(1-ie^{-\operatorname{sech}^{-1}(a+bx)}\right) - \log\left(1+ie^{-\operatorname{sech}^{-1}(a+bx)}\right)\right) - 2\operatorname{sech}^{-1}(a+bx)\left(\operatorname{PolyLog}\left(2, -ie^{-\operatorname{sech}^{-1}(a+bx)}\right) - \operatorname{PolyLog}\left(2, ie^{-\operatorname{sech}^{-1}(a+bx)}\right)\right) - 2\operatorname{PolyLog}\left(3, -ie^{-\operatorname{sech}^{-1}(a+bx)}\right) + 2\operatorname{PolyLog}\left(3, ie^{-\operatorname{sech}^{-1}(a+bx)}\right)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSech[a + b*x]^3,x]
```

```
[Out] (-3*Sqrt[-((-1 + a + b*x)/(1 + a + b*x))]*(1 + a + b*x)*ArcSech[a + b*x]^2
- 2*a*(a + b*x)*ArcSech[a + b*x]^3 + (a + b*x)^2*ArcSech[a + b*x]^3 + 3*Arc
Sech[a + b*x]*(ArcSech[a + b*x] + 2*Log[1 + E^(-2*ArcSech[a + b*x])]) - 3*P
olyLog[2, -E^(-2*ArcSech[a + b*x])] + (6*I)*a*(-(ArcSech[a + b*x]^2*(Log[1
- I/E^ArcSech[a + b*x]] - Log[1 + I/E^ArcSech[a + b*x]])) - 2*ArcSech[a + b
*x]*(PolyLog[2, (-I)/E^ArcSech[a + b*x]] - PolyLog[2, I/E^ArcSech[a + b*x]]
) - 2*PolyLog[3, (-I)/E^ArcSech[a + b*x]] + 2*PolyLog[3, I/E^ArcSech[a + b*
x]]))/(2*b^2)
```

Maple [F]
time = 0.99, size = 0, normalized size = 0.00

$$\int x \mathrm{arcsech}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsech(b*x+a)^3,x)
```

```
[Out] int(x*arcsech(b*x+a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sq
rt(-b*x - a + 1)*a + b*x + a)^3 - integrate(1/2*(16*(b^3*x^4 + 3*a*b^2*x^3
+ (3*a^2*b - b)*x^2 + (a^3 - a)*x)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log
(b*x + a)^3 + 16*(b^3*x^4 + 3*a*b^2*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*
log(b*x + a)^3 + 3*(b^3*x^4 + 2*a*b^2*x^3 + (a^2*b - b)*x^2 + 4*(b^3*x^4 +
3*a*b^2*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*log(b*x + a) + (2*(b^3*x^4 +
 3*a*b^2*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*sqrt(b*x + a + 1)*log(b*x +
 a) + (2*b^3*x^4 + 4*a*b^2*x^3 + (2*a^2*b - b)*x^2 + 2*(b^3*x^4 + 3*a*b^2*x
^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*log(b*x + a))*sqrt(b*x + a + 1))*sqrt
(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a
 + 1)*sqrt(-b*x - a + 1)*a + b*x + a)^2 - 24*((b^3*x^4 + 3*a*b^2*x^3 + (3*a
^2*b - b)*x^2 + (a^3 - a)*x)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x +
 a)^2 + (b^3*x^4 + 3*a*b^2*x^3 + (3*a^2*b - b)*x^2 + (a^3 - a)*x)*log(b*x +
 a)^2)*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqr
t(-b*x - a + 1)*a + b*x + a))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (b^3*x^3 + 3*a
*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)
+ (3*a^2*b - b)*x - a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x*arcsech(b*x + a)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asech(b*x+a)**3,x)
[Out] Integral(x*asech(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x*arcsech(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{acosh}\left(\frac{1}{a+b\,x}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acosh(1/(a + b*x))^3,x)
[Out] int(x*acosh(1/(a + b*x))^3, x)
```

3.16 $\int \operatorname{sech}^{-1}(a+bx)^3\,dx$

Optimal. Leaf size=136

$$\frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^3}{b}-\frac{6\operatorname{sech}^{-1}(a+bx)^2\operatorname{ArcTan}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}+\frac{6i\operatorname{sech}^{-1}(a+bx)\operatorname{PolyLog}\left(2,-ie^{\operatorname{sech}^{-1}(a+}\right.}{b}$$

```
[Out] (b*x+a)*arcsech(b*x+a)^3/b-6*arcsech(b*x+a)^2*arctan(1/(b*x+a)+(1/(b*x+a)-1
)^(1/2)*(1/(b*x+a)+1)^(1/2))/b+6*I*arcsech(b*x+a)*polylog(2,-I*(1/(b*x+a)+(
1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b-6*I*arcsech(b*x+a)*polylog(2,I*(
1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b-6*I*polylog(3,-I*(1/(
b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b+6*I*polylog(3,I*(1/(b*x+
a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2)))/b
```

Rubi [A]

time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {6450, 6414, 5526, 4265, 2611, 2320, 6724}

$$-\frac{6\operatorname{sech}^{-1}(a+bx)^2\operatorname{ArcTan}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}+\frac{6i\operatorname{sech}^{-1}(a+bx)\operatorname{Li}_2\left(-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}-\frac{6i\operatorname{sech}^{-1}(a+bx)\operatorname{Li}_2\left(ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}-\frac{6i\operatorname{Li}_3\left(-ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}+\frac{6i\operatorname{Li}_3\left(ie^{\operatorname{sech}^{-1}(a+bx)}\right)}{b}+\frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^3}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]^3,x]
```

```
[Out] ((a + b*x)*ArcSech[a + b*x]^3)/b - (6*ArcSech[a + b*x]^2*ArcTan[E^ArcSech[a
 + b*x]])/b + ((6*I)*ArcSech[a + b*x]*PolyLog[2, (-I)*E^ArcSech[a + b*x]])/
b - ((6*I)*ArcSech[a + b*x]*PolyLog[2, I*E^ArcSech[a + b*x]])/b - ((6*I)*Po
lyLog[3, (-I)*E^ArcSech[a + b*x]])/b + ((6*I)*PolyLog[3, I*E^ArcSech[a + b*
x]])/b
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5526

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Sech[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rule 6414

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-c^(-1), Su
bst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a,
b, c, n}, x] && IGtQ[n, 0]
```

Rule 6450

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSech[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \operatorname{sech}^{-1}(a+bx)^3\,dx &= \frac{\text{Subst}\left(\int \operatorname{sech}^{-1}(x)^3\,dx, x, a+bx\right)}{b} \\
&= -\frac{\text{Subst}\left(\int x^3\operatorname{sech}(x)\tanh(x)\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b} \\
&= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^3}{b} - \frac{3\text{Subst}\left(\int x^2\operatorname{sech}(x)\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)}{b} \\
&= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^3}{b} - \frac{6\operatorname{sech}^{-1}(a+bx)^2\tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{(6i)\text{Subst}\left(\int x\,\text{l}\right.}{} \\
&= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^3}{b} - \frac{6\operatorname{sech}^{-1}(a+bx)^2\tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{6i\operatorname{sech}^{-1}(a+b}{} \\
&= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^3}{b} - \frac{6\operatorname{sech}^{-1}(a+bx)^2\tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{6i\operatorname{sech}^{-1}(a+b}{} \\
&= \frac{(a+bx)\operatorname{sech}^{-1}(a+bx)^3}{b} - \frac{6\operatorname{sech}^{-1}(a+bx)^2\tan^{-1}\left(e^{\operatorname{sech}^{-1}(a+bx)}\right)}{b} + \frac{6i\operatorname{sech}^{-1}(a+b}{}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 153, normalized size = 1.12

$$
-\frac{-\left((a+bx)\operatorname{sech}^{-1}(a+bx)^3\right)+3i\left(-\operatorname{sech}^{-1}(a+bx)^2\left(\log\left(1-ie^{-\operatorname{sech}^{-1}(a+bx)}\right)-\log\left(1+ie^{-\operatorname{sech}^{-1}(a+bx)}\right)\right)-2\operatorname{sech}^{-1}(a+bx)\left(\text{PolyLog}\left(2,-ie^{-\operatorname{sech}^{-1}(a+bx)}\right)-\text{PolyLog}\left(2,ie^{-\operatorname{sech}^{-1}(a+bx)}\right)\right)-2\left(\text{PolyLog}\left(3,-ie^{-\operatorname{sech}^{-1}(a+bx)}\right)-\text{PolyLog}\left(3,ie^{-\operatorname{sech}^{-1}(a+bx)}\right)\right)\right)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a + b*x]^3,x]
```

```
[Out] -((-((a + b*x)*ArcSech[a + b*x]^3) + (3*I)*(-(ArcSech[a + b*x]^2*(Log[1 - I
/E^ArcSech[a + b*x]] - Log[1 + I/E^ArcSech[a + b*x]])) - 2*ArcSech[a + b*x]
*(PolyLog[2, (-I)/E^ArcSech[a + b*x]] - PolyLog[2, I/E^ArcSech[a + b*x]]) -
 2*(PolyLog[3, (-I)/E^ArcSech[a + b*x]] - PolyLog[3, I/E^ArcSech[a + b*x]])
))/b)
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \operatorname{arcsech}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)^3,x)
```

```
[Out] int(arcsech(b*x+a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*
x - a + 1)*a + b*x + a)^3 - integrate((8*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*
a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^3 + 8*(
b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^3 + 3*(b^3*
x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2
*b - b)*x - a)*log(b*x + a) + ((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)
*x - a)*sqrt(b*x + a + 1)*log(b*x + a) + (2*b^3*x^3 + 4*a*b^2*x^2 + (2*a^2*
b - b)*x + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)
)*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x -
a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a)^2 - 12*((b^3
*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x
 - a + 1)*log(b*x + a)^2 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x -
 a)*log(b*x + a)^2)*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x
 + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (
b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-
b*x - a + 1) + (3*a^2*b - b)*x - a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(arcsech(b*x + a)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asech}^3(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)**3,x)
```

```
[Out] Integral(asech(a + b*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3,x, algorithm="giac")
[Out] integrate(arcsech(b*x + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}\left(\frac{1}{a+b\,x}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))^3,x)
[Out] int(acosh(1/(a + b*x))^3, x)
```

3.17 $\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x}\,dx$

Optimal. Leaf size=378

$$\operatorname{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\operatorname{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)-\operatorname{sech}^{-1}(a+bx)^3\log\left(1+\right.$$

```
[Out] -arcsech(b*x+a)^3*ln(1+(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^
2)+arcsech(b*x+a)^3*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/
2))/(1-(-a^2+1)^(1/2)))+arcsech(b*x+a)^3*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1
/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))-3/2*arcsech(b*x+a)^2*polylog(2
,-(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)+3*arcsech(b*x+a)^2
*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1
)^(1/2)))+3*arcsech(b*x+a)^2*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/
(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))+3/2*arcsech(b*x+a)*polylog(3,-(1/(b*x
+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)-6*arcsech(b*x+a)*polylog(3,
a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))-6
*arcsech(b*x+a)*polylog(3,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1
/2))/(1+(-a^2+1)^(1/2)))-3/4*polylog(4,-(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(
b*x+a)+1)^(1/2))^2)+6*polylog(4,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)
+1)^(1/2))/(1-(-a^2+1)^(1/2)))+6*polylog(4,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)
*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))
```

Rubi [A]
time = 0.35, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6456, 5714, 5689, 3799, 2221, 2611, 6744, 2320, 6724, 5681}

$$3\operatorname{sech}^{-1}(a+bx)^2\operatorname{Li}_2\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+3\operatorname{sech}^{-1}(a+bx)^2\operatorname{Li}_2\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)-6\operatorname{sech}^{-1}(a+bx)\operatorname{Li}_3\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-6\operatorname{sech}^{-1}(a+bx)\operatorname{Li}_3\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+6\operatorname{Li}_4\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+6\operatorname{Li}_4\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+\operatorname{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\operatorname{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)-\frac{3}{2}\operatorname{sech}^{-1}(a+bx)^2\operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(a+bx)}\right)+\frac{3}{2}\operatorname{sech}^{-1}(a+bx)\operatorname{Li}_3\left(-e^{2\operatorname{sech}^{-1}(a+bx)}\right)-\frac{3}{4}\operatorname{Li}_4\left(-e^{2\operatorname{sech}^{-1}(a+bx)}\right)-\operatorname{sech}^{-1}(a+bx)^3\log\left(e^{2\operatorname{sech}^{-1}(a+bx)}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]^3/x,x]
```

```
[Out] ArcSech[a + b*x]^3*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + Ar
cSech[a + b*x]^3*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - ArcS
ech[a + b*x]^3*Log[1 + E^(2*ArcSech[a + b*x])] + 3*ArcSech[a + b*x]^2*PolyL
og[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] + 3*ArcSech[a + b*x]^2*Po
lyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - (3*ArcSech[a + b*x]^
2*PolyLog[2, -E^(2*ArcSech[a + b*x])])/2 - 6*ArcSech[a + b*x]*PolyLog[3, (a
*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])] - 6*ArcSech[a + b*x]*PolyLog[3, (
a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] + (3*ArcSech[a + b*x]*PolyLog[3,
 -E^(2*ArcSech[a + b*x])])/2 + 6*PolyLog[4, (a*E^ArcSech[a + b*x])/(1 - Sqr
t[1 - a^2])] + 6*PolyLog[4, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] - (
3*PolyLog[4, -E^(2*ArcSech[a + b*x])])/4
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5689

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/(Cosh[(c_.)
 + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Tanh[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Sinh[c + d*x]*(Tanh[c + d*x]^
(n - 1)/(a + b*Cosh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5714

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sech[(c_.) + (d_.)*(x_)]), x_Symbol] :> I
nt[(e + f*x)^m*Cosh[c + d*x]*F[c + d*x]^n*(G[c + d*x]^p/(b + a*Cosh[c + d*x
])), x] /; FreeQ[{a, b, c, d, e, f}, x] && HyperbolicQ[F] && HyperbolicQ[G]
 && IntegersQ[m, n, p]
```

Rule 6456

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x}\,dx = -\mathrm{Subst}\left(\int \frac{x^3\operatorname{sech}(x)\tanh(x)}{-a+\operatorname{sech}(x)}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)$$

$$= -\mathrm{Subst}\left(\int \frac{x^3\tanh(x)}{1-a\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)$$

$$= -\left(a\mathrm{Subst}\left(\int \frac{x^3\sinh(x)}{1-a\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right) - \mathrm{Subst}\left(\int x^3\tanh(x)\,dx, x,\right.$$

$$= -\left(2\mathrm{Subst}\left(\int \frac{e^{2x}x^3}{1+e^{2x}}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right) - a\mathrm{Subst}\left(\int \frac{e^x x^3}{1-\sqrt{1-a^2}-ae^x}\right.$$

$$= \operatorname{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right.$$

$$= \operatorname{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right.$$

$$= \operatorname{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right.$$

$$= \operatorname{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right.$$

$$= \operatorname{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right.$$

Mathematica [A]
time = 0.23, size = 384, normalized size = 1.02

$$-\frac{1}{2}\operatorname{sech}^{-1}(a+bx)^4 - \operatorname{sech}^{-1}(a+bx)^3\log\left(1+e^{-2\operatorname{sech}^{-1}(a+bx)}\right) + \operatorname{sech}^{-1}(a+bx)^3\log\left(1+\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right) + \operatorname{sech}^{-1}(a+bx)^3\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) + \frac{3}{2}\operatorname{sech}^{-1}(a+bx)^2\mathrm{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(a+bx)}\right) + 3\operatorname{sech}^{-1}(a+bx)^2\mathrm{PolyLog}\left(2,-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right) + 3\operatorname{sech}^{-1}(a+bx)^2\mathrm{PolyLog}\left(2,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) + \frac{3}{2}\operatorname{sech}^{-1}(a+bx)\mathrm{PolyLog}\left(3,-e^{-2\operatorname{sech}^{-1}(a+bx)}\right) - 6\operatorname{sech}^{-1}(a+bx)\mathrm{PolyLog}\left(3,-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right) - 6\operatorname{sech}^{-1}(a+bx)\mathrm{PolyLog}\left(3,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) + \frac{3}{4}\mathrm{PolyLog}\left(4,-e^{-2\operatorname{sech}^{-1}(a+bx)}\right) + 6\mathrm{PolyLog}\left(4,-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{-1+\sqrt{1-a^2}}\right) + 6\mathrm{PolyLog}\left(4,\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSech[a + b*x]^3/x,x]
```

```
[Out] -1/2*ArcSech[a + b*x]^4 - ArcSech[a + b*x]^3*Log[1 + E^(-2*ArcSech[a + b*x]
)] + ArcSech[a + b*x]^3*Log[1 + (a*E^ArcSech[a + b*x])/(-1 + Sqrt[1 - a^2])
] + ArcSech[a + b*x]^3*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])]
+ (3*ArcSech[a + b*x]^2*PolyLog[2, -E^(-2*ArcSech[a + b*x])])/2 + 3*ArcSech
[a + b*x]^2*PolyLog[2, -((a*E^ArcSech[a + b*x])/(-1 + Sqrt[1 - a^2]))] + 3*
ArcSech[a + b*x]^2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] +
 (3*ArcSech[a + b*x]*PolyLog[3, -E^(-2*ArcSech[a + b*x])])/2 - 6*ArcSech[a
+ b*x]*PolyLog[3, -((a*E^ArcSech[a + b*x])/(-1 + Sqrt[1 - a^2]))] - 6*ArcSe
```

```
ch[a + b*x]*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])] + (3*Pol
yLog[4, -E^(-2*ArcSech[a + b*x])])/4 + 6*PolyLog[4, -((a*E^ArcSech[a + b*x]
)/(-1 + Sqrt[1 - a^2]))] + 6*PolyLog[4, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1
- a^2])]
```

Maple [**F**]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsech}(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)^3/x,x)
[Out] int(arcsech(b*x+a)^3/x,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3/x,x, algorithm="maxima")
[Out] integrate(arcsech(b*x + a)^3/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3/x,x, algorithm="fricas")
[Out] integral(arcsech(b*x + a)^3/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)**3/x,x)
[Out] Integral(asech(a + b*x)**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3/x,x, algorithm="giac")
[Out] integrate(arcsech(b*x + a)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acosh}\left(\frac{1}{a+b\,x}\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))^3/x,x)
[Out] int(acosh(1/(a + b*x))^3/x, x)
```

3.18 $\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^2}\,dx$

Optimal. Leaf size=330

$$-\frac{b\operatorname{sech}^{-1}(a+bx)^3}{a}-\frac{\operatorname{sech}^{-1}(a+bx)^3}{x}+\frac{3b\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}-\frac{3b\operatorname{sech}^{-1}(a+bx)^2\log\left(\right.}{a\sqrt{1-a}}$$

```
[Out] -b*arcsech(b*x+a)^3/a-arcsech(b*x+a)^3/x+3*b*arcsech(b*x+a)^2*ln(1-a*(1/(b*
x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2+1
)^(1/2)-3*b*arcsech(b*x+a)^2*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+
a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)+6*b*arcsech(b*x+a)*polylo
g(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)
))/a/(-a^2+1)^(1/2)-6*b*arcsech(b*x+a)*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)
^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)-6*b*polylo
g(3,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)
))/a/(-a^2+1)^(1/2)+6*b*polylog(3,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+
a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6456, 5576, 4276, 3401, 2296, 2221, 2611, 2320, 6724}

$$\frac{6b\operatorname{sech}^{-1}(a+bx)\operatorname{Li}_2\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}-\frac{6b\operatorname{sech}^{-1}(a+bx)\operatorname{Li}_2\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}-\frac{6b\operatorname{Li}_3\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{6b\operatorname{Li}_3\left(\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}+\frac{3b\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}-\frac{3b\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}-\frac{b\operatorname{sech}^{-1}(a+bx)^3}{a}-\frac{\operatorname{sech}^{-1}(a+bx)^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]^3/x^2,x]
```

```
[Out] -((b*ArcSech[a + b*x]^3)/a) - ArcSech[a + b*x]^3/x + (3*b*ArcSech[a + b*x]^
2*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) -
(3*b*ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])]
)/(a*Sqrt[1 - a^2]) + (6*b*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x
])/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) - (6*b*ArcSech[a + b*x]*PolyLog[
2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) - (6*b*Po
lyLog[3, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) + (
6*b*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2
])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5576

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b
```

```
*d*(n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6456

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^2}\,dx = -\left(b\text{Subst}\left(\int \frac{x^3\operatorname{sech}(x)\tanh(x)}{(-a+\operatorname{sech}(x))^2}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right)$$

$$= -\frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + (3b)\text{Subst}\left(\int \frac{x^2}{-a+\operatorname{sech}(x)}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)$$

$$= -\frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + (3b)\text{Subst}\left(\int \left(-\frac{x^2}{a} + \frac{x^2}{a(1-a\cosh(x))}\right)\,dx, x, \operatorname{sech}^{-1}(a+bx\right.$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + \frac{(3b)\text{Subst}\left(\int \frac{x^2}{1-a\cosh(x)}\,dx, x, \operatorname{sech}^{-1}(a+bx\right.}{a}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + \frac{(6b)\text{Subst}\left(\int \frac{e^x x^2}{-a+2e^x-ae^{2x}}\,dx, x, \operatorname{sech}^{-1}(a+\right.}{a}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} - \frac{(6b)\text{Subst}\left(\int \frac{e^x x^2}{2-2\sqrt{1-a^2}-2ae^x}\,dx, x, \operatorname{sech}\right.}{\sqrt{1-a^2}}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + \frac{3b\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + \frac{3b\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + \frac{3b\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$

$$= -\frac{b\operatorname{sech}^{-1}(a+bx)^3}{a} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{x} + \frac{3b\operatorname{sech}^{-1}(a+bx)^2\log\left(1-\frac{ae^{\operatorname{sech}^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] Integrate[ArcSech[a + b*x]^3/x^2,x]
[Out] $Aborted
```

Maple [F]
time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsech}(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)^3/x^2,x)
```

```
[Out] int(arcsech(b*x+a)^3/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x
 - a + 1)*a + b*x + a)^3/x - integrate((8*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3
*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^3 + 8*
(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a)^3 - 3*(b^3
*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x - 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^
2*b - b)*x - a)*log(b*x + a) - ((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b
)*x - a)*sqrt(b*x + a + 1)*log(b*x + a) - (2*b^3*x^3 + 4*a*b^2*x^2 + (2*a^2
*b - b)*x - (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a
))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a + 1)*sqrt(-b*x -
 a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a)^2 - 12*((b^
3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*
x - a + 1)*log(b*x + a)^2 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x
- a)*log(b*x + a)^2)*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*
x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2
*b - b)*x^3 + (a^3 - a)*x^2 + (b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b - b)*x^3 +
(a^3 - a)*x^2)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsech(b*x + a)^3/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)**3/x**2,x)
[Out] Integral(asech(a + b*x)**3/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3/x^2,x, algorithm="giac")
[Out] integrate(arcsech(b*x + a)^3/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))^3/x^2,x)
[Out] int(acosh(1/(a + b*x))^3/x^2, x)
```

3.19 $\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^3} \, dx$

Optimal. Leaf size=965

$$-\frac{3b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2\left(1-a^2\right)}+\frac{3b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)}+\frac{b^2\operatorname{sech}^{-1}(a+bx)^3}{2a^2}-\frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2}$$

```
[Out] -3/2*b^2*arcsech(b*x+a)^2/a^2/(-a^2+1)+1/2*b^2*arcsech(b*x+a)^3/a^2-1/2*arc
sech(b*x+a)^3/x^2+3*b^2*arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2
)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)+3/2*b^2*arcsech(b*x
+a)^2*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1
)^(1/2)))/a^2/(-a^2+1)^(3/2)+3*b^2*arcsech(b*x+a)*ln(1-a*(1/(b*x+a)+(1/(b*x
+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)-3/2*b^2*
arcsech(b*x+a)^2*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))
/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)+3*b^2*polylog(2,a*(1/(b*x+a)+(1/(b*
x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)+3*b^2*a
rcsech(b*x+a)*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2
))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)+3*b^2*polylog(2,a*(1/(b*x+a)+(1/(
b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)-3*b^2
*arcsech(b*x+a)*polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1
/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)-3*b^2*polylog(3,a*(1/(b*x+a)+(1
/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/
2)+3*b^2*polylog(3,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1
+(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(3/2)-3*b^2*arcsech(b*x+a)^2*ln(1-a*(1/(b*x+
a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)^(1/2)))/a^2/(-a^2+1
)^(1/2)+3*b^2*arcsech(b*x+a)^2*ln(1-a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*
x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(1/2)-6*b^2*arcsech(b*x+a)*
polylog(2,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-(-a^2+1)
^(1/2)))/a^2/(-a^2+1)^(1/2)+6*b^2*arcsech(b*x+a)*polylog(2,a*(1/(b*x+a)+(1/
(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(1/2
)+6*b^2*polylog(3,a*(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1-
(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(1/2)-6*b^2*polylog(3,a*(1/(b*x+a)+(1/(b*x+a)
-1)^(1/2)*(1/(b*x+a)+1)^(1/2))/(1+(-a^2+1)^(1/2)))/a^2/(-a^2+1)^(1/2)+3/2*b
^2*(b*x+a+1)*arcsech(b*x+a)^2*((-b*x-a+1)/(b*x+a+1))^(1/2)/a/(-a^2+1)/(b*x+
a)/(1-a/(b*x+a))
```

Rubi [A]

time = 0.91, antiderivative size = 965, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 13, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {6456, 5576, 4276, 3405, 3401, 2296, 2221, 2611, 2320, 6724, 5681, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]^3/x^3,x]
```

```
[Out] (-3*b^2*ArcSech[a + b*x]^2)/(2*a^2*(1 - a^2)) + (3*b^2*Sqrt[(1 - a - b*x)/(
1 + a + b*x)]*(1 + a + b*x)*ArcSech[a + b*x]^2)/(2*a*(1 - a^2)*(a + b*x)*(1
 - a/(a + b*x))) + (b^2*ArcSech[a + b*x]^3)/(2*a^2) - ArcSech[a + b*x]^3/(2
*x^2) + (3*b^2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1
- a^2])])/(a^2*(1 - a^2)) + (3*b^2*ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech[
a + b*x])/(1 - Sqrt[1 - a^2])])/(2*a^2*(1 - a^2)^(3/2)) - (3*b^2*ArcSech[a
+ b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*Sqrt[1 -
 a^2]) + (3*b^2*ArcSech[a + b*x]*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1
 - a^2])])/(a^2*(1 - a^2)) - (3*b^2*ArcSech[a + b*x]^2*Log[1 - (a*E^ArcSech
[a + b*x])/(1 + Sqrt[1 - a^2])])/(2*a^2*(1 - a^2)^(3/2)) + (3*b^2*ArcSech[a
 + b*x]^2*Log[1 - (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*Sqrt[1
- a^2]) + (3*b^2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a
^2*(1 - a^2)) + (3*b^2*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(
1 - Sqrt[1 - a^2])])/(a^2*(1 - a^2)^(3/2)) - (6*b^2*ArcSech[a + b*x]*PolyLo
g[2, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2]) + (3*
b^2*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*(1 - a^2))
 - (3*b^2*ArcSech[a + b*x]*PolyLog[2, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 -
a^2])])/(a^2*(1 - a^2)^(3/2)) + (6*b^2*ArcSech[a + b*x]*PolyLog[2, (a*E^Arc
Sech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^2]) - (3*b^2*PolyLog[3
, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*(1 - a^2)^(3/2)) + (6*b
^2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 - Sqrt[1 - a^2])])/(a^2*Sqrt[1 - a^
2]) + (3*b^2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^2])])/(a^2*(
1 - a^2)^(3/2)) - (6*b^2*PolyLog[3, (a*E^ArcSech[a + b*x])/(1 + Sqrt[1 - a^
2])])/(a^2*Sqrt[1 - a^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5576

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sech[
(c_.) + (d_.)*(x_)])^(n_.)*Tanh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e
 + f*x)^m)*((a + b*Sech[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b
*d*(n + 1))), Int[(e + f*x)^(m - 1)*(a + b*Sech[c + d*x])^(n + 1), x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 6456

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[-(d^(m + 1))^(-1), Subst[Int[(a + b*x)^p*Sech[x]*Ta
nh[x]*(d*e - c*f + f*Sech[x])^m, x], x, ArcSech[c + d*x]], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^3}\,dx = -\left(b^2\text{Subst}\left(\int \frac{x^3\operatorname{sech}(x)\tanh(x)}{(-a+\operatorname{sech}(x))^3}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)\right)$$

$$= -\frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2} + \frac{1}{2}\left(3b^2\right)\text{Subst}\left(\int \frac{x^2}{(-a+\operatorname{sech}(x))^2}\,dx, x, \operatorname{sech}^{-1}(a+bx)\right)$$

$$= -\frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2} + \frac{1}{2}\left(3b^2\right)\text{Subst}\left(\int \left(\frac{x^2}{a^2} + \frac{x^2}{a^2(-1+a\cosh(x))^2} + \frac{2x^2}{a^2(-1+a\,\mathrm{c}}\right.\right.$$

$$= \frac{b^2\operatorname{sech}^{-1}(a+bx)^3}{2a^2} - \frac{\operatorname{sech}^{-1}(a+bx)^3}{2x^2} + \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{x^2}{(-1+a\cosh(x))^2}\,dx, x, \operatorname{sech}^{-1}(\right.}{2a^2}$$

$$= \frac{3b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\operatorname{sech}^{-1}(a+bx)^3}{2a^2} - \frac{\operatorname{sech}^{-1}(a+}{2x^2}$$

$$= -\frac{3b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2\left(1-a^2\right)} + \frac{3b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\operatorname{sech}^{-1}}{2}$$

$$= -\frac{3b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2\left(1-a^2\right)} + \frac{3b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\operatorname{sech}^{-1}}{2}$$

$$= -\frac{3b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2\left(1-a^2\right)} + \frac{3b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\operatorname{sech}^{-1}}{2}$$

$$= -\frac{3b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2\left(1-a^2\right)} + \frac{3b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\operatorname{sech}^{-1}}{2}$$

$$= -\frac{3b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2\left(1-a^2\right)} + \frac{3b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\operatorname{sech}^{-1}}{2}$$

$$= -\frac{3b^2\operatorname{sech}^{-1}(a+bx)^2}{2a^2\left(1-a^2\right)} + \frac{3b^2\sqrt{\frac{1-a-bx}{1+a+bx}}\,(1+a+bx)\operatorname{sech}^{-1}(a+bx)^2}{2a\left(1-a^2\right)(a+bx)\left(1-\frac{a}{a+bx}\right)} + \frac{b^2\operatorname{sech}^{-1}}{2}$$

Mathematica [F]

time = 7.46, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcSech[a + b*x]^3/x^3,x]
```

```
[Out] Integrate[ArcSech[a + b*x]^3/x^3, x]
```

Maple [F]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsech}(bx+a)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)^3/x^3,x)
```

```
[Out] int(arcsech(b*x+a)^3/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(
-b*x - a + 1)*a + b*x + a)^3/x^2 - integrate(1/2*(16*(b^3*x^3 + 3*a*b^2*x^2
 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*log(b*x
+ a)^3 + 16*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*log(b*x + a
)^3 - 3*(b^3*x^3 + 2*a*b^2*x^2 + (a^2*b - b)*x - 4*(b^3*x^3 + 3*a*b^2*x^2 +
 a^3 + (3*a^2*b - b)*x - a)*log(b*x + a) - (2*(b^3*x^3 + 3*a*b^2*x^2 + a^3
+ (3*a^2*b - b)*x - a)*sqrt(b*x + a + 1)*log(b*x + a) - (2*b^3*x^3 + 4*a*b^
2*x^2 + (2*a^2*b - b)*x - 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x
- a)*log(b*x + a))*sqrt(b*x + a + 1))*sqrt(-b*x - a + 1))*log(sqrt(b*x + a
+ 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x
+ a)^2 - 24*((b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b - b)*x - a)*sqrt(b*x +
 a + 1)*sqrt(-b*x - a + 1)*log(b*x + a)^2 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 +
(3*a^2*b - b)*x - a)*log(b*x + a)^2)*log(sqrt(b*x + a + 1)*sqrt(-b*x - a +
1)*b*x + sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*a + b*x + a))/(b^3*x^6 + 3*a*
b^2*x^5 + (3*a^2*b - b)*x^4 + (a^3 - a)*x^3 + (b^3*x^6 + 3*a*b^2*x^5 + (3*a
^2*b - b)*x^4 + (a^3 - a)*x^3)*sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3/x^3,x, algorithm="fricas")
[Out] integral(arcsech(b*x + a)^3/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}^{3}(a + bx)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)**3/x**3,x)
[Out] Integral(asech(a + b*x)**3/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)^3/x^3,x, algorithm="giac")
[Out] integrate(arcsech(b*x + a)^3/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a+b\,x}\right)^3}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))^3/x^3,x)
[Out] int(acosh(1/(a + b*x))^3/x^3, x)
```

3.20 $\int x^3\mathrm{sech}^{-1}(\sqrt{x})\,dx$

Optimal. Leaf size=164

$$-\frac{1-x}{4\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}+\frac{(1-x)^2}{4\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}-\frac{3(1-x)^3}{20\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}+\frac{}{28}$$

```
[Out] 1/4*x^4*arcsech(x^(1/2))+1/4*(-1+x)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/
2))^(1/2)+1/4*(1-x)^2/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)-3/20
*(1-x)^3/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)+1/28*(1-x)^4/x^(1
/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6480, 12, 45}

$$\frac{1}{4}x^4\mathrm{sech}^{-1}(\sqrt{x})+\frac{(1-x)^4}{28\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{3(1-x)^3}{20\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}+\frac{(1-x)^2}{4\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{1-x}{4\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSech[Sqrt[x]],x]
```

```
[Out] -1/4*(1 - x)/(Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) + (1 - x)^2
/(4*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) - (3*(1 - x)^3)/(20*S
qrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) + (1 - x)^4/(28*Sqrt[-1 +
1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) + (x^4*ArcSech[Sqrt[x]])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6480

```
Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSech[u])/(d*(m + 1))), x] + Dist[b*(Sqrt[1
- u^2]/(d*(m + 1)*u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u])), Int[SimplifyIntegrand[(
```

```
c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[1 - u^2])), x], x], x] /; FreeQ[{a, b, c,
 d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c
+ d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\int x^3\text{sech}^{-1}(\sqrt{x})\,dx = \frac{1}{4}x^4\text{sech}^{-1}(\sqrt{x}) + \frac{\sqrt{1-x}\int\frac{x^3}{2\sqrt{1-x}}\,dx}{4\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$
$$= \frac{1}{4}x^4\text{sech}^{-1}(\sqrt{x}) + \frac{\sqrt{1-x}\int\frac{x^3}{\sqrt{1-x}}\,dx}{8\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$
$$= \frac{1}{4}x^4\text{sech}^{-1}(\sqrt{x}) + \frac{\sqrt{1-x}\int\left(\frac{1}{\sqrt{1-x}}-3\sqrt{1-x}+3(1-x)^{3/2}-(1-x)^{5/2}\right)}{8\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$
$$= -\frac{1-x}{4\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} + \frac{(1-x)^2}{4\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} - \frac{}{20\sqrt{-1+}}$$

Mathematica [A]
time = 0.04, size = 84, normalized size = 0.51

$$-\frac{1}{140}\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\left(16+16\sqrt{x}+8x+8x^{3/2}+6x^2+6x^{5/2}+5x^3+5x^{7/2}\right)+\frac{1}{4}x^4\text{sech}^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSech[Sqrt[x]],x]
```

```
[Out] -1/140*(Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(16 + 16*Sqrt[x] + 8*x + 8*x^(3/2
) + 6*x^2 + 6*x^(5/2) + 5*x^3 + 5*x^(7/2))) + (x^4*ArcSech[Sqrt[x]])/4
```

Maple [A]
time = 0.19, size = 54, normalized size = 0.33

method	result	size

derivativedivides	$\frac{x^4 \operatorname{arcsech}\left(\sqrt{x}\right)}{4}-\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{x}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,\left(5x^3+6x^2+8x+16\right)}{140}$	54
default	$\frac{x^4 \operatorname{arcsech}\left(\sqrt{x}\right)}{4}-\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{x}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,\left(5x^3+6x^2+8x+16\right)}{140}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsech(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*arcsech(x^(1/2))-1/140*(-(x^(1/2)-1)/x^(1/2))^(1/2)*x^(1/2)*((x^(1/
2)+1)/x^(1/2))^(1/2)*(5*x^3+6*x^2+8*x+16)
```

Maxima [A]
time = 0.25, size = 58, normalized size = 0.35

$$\frac{1}{28}\,x^{\frac{7}{2}}\left(\frac{1}{x}-1\right)^{\frac{7}{2}}-\frac{3}{20}\,x^{\frac{5}{2}}\left(\frac{1}{x}-1\right)^{\frac{5}{2}}+\frac{1}{4}\,x^4\operatorname{arsech}\left(\sqrt{x}\right)+\frac{1}{4}\,x^{\frac{3}{2}}\left(\frac{1}{x}-1\right)^{\frac{3}{2}}-\frac{1}{4}\,\sqrt{x}\,\sqrt{\frac{1}{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/28*x^(7/2)*(1/x - 1)^(7/2) - 3/20*x^(5/2)*(1/x - 1)^(5/2) + 1/4*x^4*arcse
ch(sqrt(x)) + 1/4*x^(3/2)*(1/x - 1)^(3/2) - 1/4*sqrt(x)*sqrt(1/x - 1)
```

Fricas [A]
time = 0.38, size = 57, normalized size = 0.35

$$\frac{1}{4}\,x^4\log\left(\frac{x\sqrt{-\frac{x-1}{x}}+\sqrt{x}}{x}\right)-\frac{1}{140}\left(5\,x^3+6\,x^2+8\,x+16\right)\sqrt{x}\,\sqrt{-\frac{x-1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*log((x*sqrt(-(x - 1)/x) + sqrt(x))/x) - 1/140*(5*x^3 + 6*x^2 + 8*x
+ 16)*sqrt(x)*sqrt(-(x - 1)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\operatorname{asech}\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asech(x**(1/2)),x)
```

```
[Out] Integral(x**3*asech(sqrt(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^3*arcsech(sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{acosh}\left(\frac{1}{\sqrt{x}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acosh(1/x^(1/2)),x)
```

```
[Out] int(x^3*acosh(1/x^(1/2)), x)
```

3.21 $\int x^2 \text{sech}^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=126

$$-\frac{1-x}{3\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}+\frac{2(1-x)^2}{9\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}-\frac{(1-x)^3}{15\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}+\frac{1}{3}x$$

```
[Out] 1/3*x^3*arcsech(x^(1/2))+1/3*(-1+x)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/
2))^(1/2)+2/9*(1-x)^2/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)-1/15
*(1-x)^3/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6480, 12, 45}

$$\frac{1}{3}x^3\text{sech}^{-1}\left(\sqrt{x}\right)-\frac{(1-x)^3}{15\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}+\frac{2(1-x)^2}{9\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{1-x}{3\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSech[Sqrt[x]],x]
```

```
[Out] -1/3*(1 - x)/(Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) + (2*(1 - x
)^2)/(9*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) - (1 - x)^3/(15*S
qrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) + (x^3*ArcSech[Sqrt[x]])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6480

```
Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSech[u])/(d*(m + 1))), x] + Dist[b*(Sqrt[1
- u^2]/(d*(m + 1)*u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u])), Int[SimplifyIntegrand[(
c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[1 - u^2])), x], x], x] /; FreeQ[{a, b, c,
```

```
d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c
+ d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\int x^2\operatorname{sech}^{-1}(\sqrt{x})\,dx=\frac{1}{3}x^3\operatorname{sech}^{-1}(\sqrt{x})+\frac{\sqrt{1-x}\int\frac{x^2}{2\sqrt{1-x}}\,dx}{3\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$=\frac{1}{3}x^3\operatorname{sech}^{-1}(\sqrt{x})+\frac{\sqrt{1-x}\int\frac{x^2}{\sqrt{1-x}}\,dx}{6\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$=\frac{1}{3}x^3\operatorname{sech}^{-1}(\sqrt{x})+\frac{\sqrt{1-x}\int\left(\frac{1}{\sqrt{1-x}}-2\sqrt{1-x}+(1-x)^{3/2}\right)dx}{6\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$=-\frac{1-x}{3\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}+\frac{2(1-x)^2}{9\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}-\frac{}{15\sqrt{-1+}}$$

Mathematica [**A**]
time = 0.02, size = 72, normalized size = 0.57

$$-\frac{1}{45}\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\left(8+8\sqrt{x}+4x+4x^{3/2}+3x^2+3x^{5/2}\right)+\frac{1}{3}x^3\operatorname{sech}^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSech[Sqrt[x]],x]
```

```
[Out] -1/45*(Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(8 + 8*Sqrt[x] + 4*x + 4*x^(3/2) +
 3*x^2 + 3*x^(5/2))) + (x^3*ArcSech[Sqrt[x]])/3
```

Maple [**A**]
time = 0.19, size = 49, normalized size = 0.39

method	result	size

derivativedivides	$\frac{x^3 \operatorname{arcsech}\left(\sqrt{x}\right)}{3}-\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{x}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,(3x^2+4x+8)}{45}$	49
default	$\frac{x^3 \operatorname{arcsech}\left(\sqrt{x}\right)}{3}-\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{x}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,(3x^2+4x+8)}{45}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsech(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*arcsech(x^(1/2))-1/45*(-(x^(1/2)-1)/x^(1/2))^(1/2)*x^(1/2)*((x^(1/2
)+1)/x^(1/2))^(1/2)*(3*x^2+4*x+8)
```

Maxima [A]
time = 0.26, size = 46, normalized size = 0.37

$$-\frac{1}{15}\,x^{\frac{5}{2}}\left(\frac{1}{x}-1\right)^{\frac{5}{2}}+\frac{1}{3}\,x^3\operatorname{arsech}\left(\sqrt{x}\right)+\frac{2}{9}\,x^{\frac{3}{2}}\left(\frac{1}{x}-1\right)^{\frac{3}{2}}-\frac{1}{3}\,\sqrt{x}\,\sqrt{\frac{1}{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(x^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/15*x^(5/2)*(1/x - 1)^(5/2) + 1/3*x^3*arcsech(sqrt(x)) + 2/9*x^(3/2)*(1/x
 - 1)^(3/2) - 1/3*sqrt(x)*sqrt(1/x - 1)
```

Fricas [A]
time = 0.39, size = 52, normalized size = 0.41

$$\frac{1}{3}\,x^3\log\left(\frac{x\sqrt{-\frac{x-1}{x}}+\sqrt{x}}{x}\right)-\frac{1}{45}\left(3\,x^2+4\,x+8\right)\sqrt{x}\,\sqrt{-\frac{x-1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*log((x*sqrt(-(x - 1)/x) + sqrt(x))/x) - 1/45*(3*x^2 + 4*x + 8)*sqrt
(x)*sqrt(-(x - 1)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{asech}\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asech(x**(1/2)),x)
```

```
[Out] Integral(x**2*asech(sqrt(x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsech(x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsech(sqrt(x)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{acosh}\left(\frac{1}{\sqrt{x}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acosh(1/x^(1/2)),x)
```

```
[Out] int(x^2*acosh(1/x^(1/2)), x)
```

3.22 $\int x\text{sech}^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=88

$$-\frac{1-x}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}+\frac{(1-x)^2}{6\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}+\frac{1}{2}x^2\text{sech}^{-1}\left(\sqrt{x}\right)$$

```
[Out] 1/2*x^2*arcsech(x^(1/2))+1/2*(-1+x)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/
2))^(1/2)+1/6*(1-x)^2/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6480, 12, 45}

$$\frac{1}{2}x^2\text{sech}^{-1}\left(\sqrt{x}\right)+\frac{(1-x)^2}{6\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{1-x}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSech[Sqrt[x]],x]
```

```
[Out] -1/2*(1 - x)/(Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) + (1 - x)^2
/(6*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x]) + (x^2*ArcSech[Sqrt[x
]])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6480

```
Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSech[u]))/(d*(m + 1))), x] + Dist[b*(Sqrt[1
 - u^2]/(d*(m + 1)*u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u])), Int[SimplifyIntegrand[(
c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[1 - u^2])), x], x], x] /; FreeQ[{a, b, c,
```

```
 d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c
+ d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int x\operatorname{sech}^{-1}(\sqrt{x})\,dx &= \frac{1}{2}x^2\operatorname{sech}^{-1}(\sqrt{x}) + \frac{\sqrt{1-x}\int \frac{x}{2\sqrt{1-x}}\,dx}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} \\ &= \frac{1}{2}x^2\operatorname{sech}^{-1}(\sqrt{x}) + \frac{\sqrt{1-x}\int \frac{x}{\sqrt{1-x}}\,dx}{4\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} \\ &= \frac{1}{2}x^2\operatorname{sech}^{-1}(\sqrt{x}) + \frac{\sqrt{1-x}\int \left(\frac{1}{\sqrt{1-x}}-\sqrt{1-x}\right)dx}{4\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} \\ &= -\frac{1-x}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} + \frac{(1-x)^2}{6\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} + \frac{1}{2}x^2\operatorname{sech}^{-1}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 0.64

$$-\frac{1}{6}\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\left(2+2\sqrt{x}+x+x^{3/2}\right)+\frac{1}{2}x^2\operatorname{sech}^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSech[Sqrt[x]],x]
```

```
[Out] -1/6*(Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(2 + 2*Sqrt[x] + x + x^(3/2))) + (x
^2*ArcSech[Sqrt[x]])/2
```

Maple [A]
time = 0.19, size = 42, normalized size = 0.48

method	result	size

derivativedivides	$\frac{x^2 \operatorname{arcsech}\left(\sqrt{x}\right)}{2}-\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{x}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,(x+2)}{6}$	42
default	$\frac{x^2 \operatorname{arcsech}\left(\sqrt{x}\right)}{2}-\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{x}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,(x+2)}{6}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsech(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*arcsech(x^(1/2))-1/6*(-(x^(1/2)-1)/x^(1/2))^(1/2)*x^(1/2)*((x^(1/2)
+1)/x^(1/2))^(1/2)*(x+2)
```

Maxima [A]
time = 0.25, size = 34, normalized size = 0.39

$$\frac{1}{6}\,x^{\frac{3}{2}}\left(\frac{1}{x}-1\right)^{\frac{3}{2}}+\frac{1}{2}\,x^2\,\operatorname{arsech}\left(\sqrt{x}\right)-\frac{1}{2}\,\sqrt{x}\,\sqrt{\frac{1}{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/6*x^(3/2)*(1/x - 1)^(3/2) + 1/2*x^2*arcsech(sqrt(x)) - 1/2*sqrt(x)*sqrt(1
/x - 1)
```

Fricas [A]
time = 0.54, size = 45, normalized size = 0.51

$$\frac{1}{2}\,x^2\log\left(\frac{x\sqrt{-\frac{x-1}{x}}+\sqrt{x}}{x}\right)-\frac{1}{6}\,(x+2)\sqrt{x}\,\sqrt{-\frac{x-1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log((x*sqrt(-(x - 1)/x) + sqrt(x))/x) - 1/6*(x + 2)*sqrt(x)*sqrt(-(
x - 1)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\operatorname{asech}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asech(x**(1/2)),x)
```

```
[Out] Integral(x*asech(sqrt(x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsech(x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x*arcsech(sqrt(x)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{acosh}\left(\frac{1}{\sqrt{x}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acosh(1/x^(1/2)),x)
```

```
[Out] int(x*acosh(1/x^(1/2)), x)
```

3.23 $\int \operatorname{sech}^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=43

$$-\frac{1-x}{\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}+x\operatorname{sech}^{-1}\left(\sqrt{x}\right)$$

```
[Out] x*arcsech(x^(1/2))+(-1+x)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6478, 12, 32}

$$x\operatorname{sech}^{-1}\left(\sqrt{x}\right)-\frac{1-x}{\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[Sqrt[x]],x]
```

```
[Out] -((1 - x)/(Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x])) + x*ArcSech[S
qrt[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6478

```
Int[ArcSech[u_], x_Symbol] :> Simp[x*ArcSech[u], x] + Dist[Sqrt[1 - u^2]/(u
*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u]), Int[SimplifyIntegrand[x*(D[u, x]/(u*Sqrt[1
- u^2])), x], x], x] /; InverseFunctionFreeQ[u, x] &&  !FunctionOfExponenti
alQ[u, x]
```

Rubi steps

$$\int \operatorname{sech}^{-1}\left(\sqrt{x}\right)\,dx = x\operatorname{sech}^{-1}\left(\sqrt{x}\right) + \frac{\sqrt{1-x}\,\int \frac{1}{2\sqrt{1-x}}\,dx}{\sqrt{-1+\frac{1}{\sqrt{x}}}\,\sqrt{1+\frac{1}{\sqrt{x}}}\,\sqrt{x}}$$
$$= x\operatorname{sech}^{-1}\left(\sqrt{x}\right) + \frac{\sqrt{1-x}\,\int \frac{1}{\sqrt{1-x}}\,dx}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\,\sqrt{1+\frac{1}{\sqrt{x}}}\,\sqrt{x}}$$
$$= -\frac{1-x}{\sqrt{-1+\frac{1}{\sqrt{x}}}\,\sqrt{1+\frac{1}{\sqrt{x}}}\,\sqrt{x}} + x\operatorname{sech}^{-1}\left(\sqrt{x}\right)$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 118 vs. 2(43) = 86.
time = 0.10, size = 118, normalized size = 2.74

$$-\frac{2\left(-1+\sqrt{1-\sqrt{x}}\right)^2\left(-1+\sqrt{1+\sqrt{x}}\right)^2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,\sqrt{1+\sqrt{x}}}{\left(-2+\sqrt{1-\sqrt{x}}+\sqrt{1+\sqrt{x}}\right)^2\sqrt{1-\sqrt{x}}} + x\operatorname{sech}^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[Sqrt[x]],x]
```

```
[Out] (-2*(-1 + Sqrt[1 - Sqrt[x]])^2*(-1 + Sqrt[1 + Sqrt[x]])^2*Sqrt[(1 - Sqrt[x]
)/(1 + Sqrt[x])]*Sqrt[1 + Sqrt[x]])/((-2 + Sqrt[1 - Sqrt[x]] + Sqrt[1 + Sqr
t[x]])^2*Sqrt[1 - Sqrt[x]]) + x*ArcSech[Sqrt[x]]
```

Maple [**A**]
time = 0.19, size = 36, normalized size = 0.84

method	result	size
derivativedivides	$x\,\mathrm{arcsech}\left(\sqrt{x}\right) - \sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{x}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}$	36
default	$x\,\mathrm{arcsech}\left(\sqrt{x}\right) - \sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{x}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*arcsech(x^(1/2))-(-(x^(1/2)-1)/x^(1/2))^(1/2)*x^(1/2)*((x^(1/2)+1)/x^(1/2
))^(1/2)
```

Maxima [A]
time = 0.26, size = 19, normalized size = 0.44

$$x \operatorname{arsech}\left(\sqrt{x}\right) - \sqrt{x}\sqrt{\frac{1}{x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2)),x, algorithm="maxima")
[Out] x*arcsech(sqrt(x)) - sqrt(x)*sqrt(1/x - 1)
```

Fricas [A]
time = 0.49, size = 39, normalized size = 0.91

$$x \log\left(\frac{x\sqrt{-\frac{x-1}{x}} + \sqrt{x}}{x}\right) - \sqrt{x}\sqrt{-\frac{x-1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2)),x, algorithm="fricas")
[Out] x*log((x*sqrt(-(x - 1)/x) + sqrt(x))/x) - sqrt(x)*sqrt(-(x - 1)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asech}\left(\sqrt{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(x**(1/2)),x)
[Out] Integral(asech(sqrt(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2)),x, algorithm="giac")
[Out] integrate(arcsech(sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/x^(1/2)),x)
[Out] int(acosh(1/x^(1/2)), x)
```

3.24 $\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x} \, dx$

Optimal. Leaf size=46

$$\operatorname{sech}^{-1}(\sqrt{x})^2 - 2\operatorname{sech}^{-1}(\sqrt{x}) \log\left(1 + e^{2\operatorname{sech}^{-1}(\sqrt{x})}\right) - \operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(\sqrt{x})}\right)$$

```
[Out] arcsech(x^(1/2))^2-2*arcsech(x^(1/2))*ln(1+(1/x^(1/2)+(-1+1/x^(1/2))^(1/2)*
(1+1/x^(1/2))^(1/2))^2)-polylog(2,-(1/x^(1/2)+(-1+1/x^(1/2))^(1/2)*(1+1/x^(
1/2))^(1/2))^2)
```

Rubi [A]
time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6416, 5882, 3799, 2221, 2317, 2438}

$$-\operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(\sqrt{x})}\right) + \operatorname{sech}^{-1}(\sqrt{x})^2 - 2\operatorname{sech}^{-1}(\sqrt{x}) \log\left(e^{2\operatorname{sech}^{-1}(\sqrt{x})} + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[Sqrt[x]]/x,x]
```

```
[Out] ArcSech[Sqrt[x]]^2 - 2*ArcSech[Sqrt[x]]*Log[1 + E^(2*ArcSech[Sqrt[x]])] - P
olyLog[2, -E^(2*ArcSech[Sqrt[x]])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
```

```
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6416

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x}\,dx &= 2\text{Subst}\left(\int \frac{\operatorname{sech}^{-1}(x)}{x}\,dx, x, \sqrt{x}\right) \\
&= -\left(2\text{Subst}\left(\int \frac{\cosh^{-1}(x)}{x}\,dx, x, \frac{1}{\sqrt{x}}\right)\right) \\
&= -\left(2\text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)\right)\right) \\
&= \cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)^2 - 4\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)\right) \\
&= \cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)^2 - 2\cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)}\right) + 2\text{Subst}\left(\int \log\left(1+\right.\right. \\
&= \cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)^2 - 2\cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)}\right) + \text{Subst}\left(\int \frac{\log(1+x}{x}\right. \\
&= \cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)^2 - 2\cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)}\right) - \text{Li}_2\left(-e^{2\cosh^{-1}\left(\frac{1}{\sqrt{x}}\right)}\right.
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 45, normalized size = 0.98

$$-\operatorname{sech}^{-1}(\sqrt{x})\left(\operatorname{sech}^{-1}(\sqrt{x}) + 2\log\left(1+e^{-2\operatorname{sech}^{-1}(\sqrt{x})}\right)\right) + \text{PolyLog}\left(2, -e^{-2\operatorname{sech}^{-1}(\sqrt{x})}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[Sqrt[x]]/x,x]
```

```
[Out] -(ArcSech[Sqrt[x]]*(ArcSech[Sqrt[x]] + 2*Log[1 + E^(-2*ArcSech[Sqrt[x]])]))
 + PolyLog[2, -E^(-2*ArcSech[Sqrt[x]])]
```

Maple [A]
time = 0.30, size = 65, normalized size = 1.41

method	result
derivativedivides	$\mathrm{arcsech}\left(\sqrt{x}\,\right)^{2}-2\,\mathrm{arcsech}\left(\sqrt{x}\,\right)\ln\left(1+\left(\frac{1}{\sqrt{x}}+\sqrt{-1+\frac{1}{\sqrt{x}}}\,\sqrt{1+\frac{1}{\sqrt{x}}}\right)^{2}\right)-\mathrm{po}$
default	$\mathrm{arcsech}\left(\sqrt{x}\,\right)^{2}-2\,\mathrm{arcsech}\left(\sqrt{x}\,\right)\ln\left(1+\left(\frac{1}{\sqrt{x}}+\sqrt{-1+\frac{1}{\sqrt{x}}}\,\sqrt{1+\frac{1}{\sqrt{x}}}\right)^{2}\right)-\mathrm{po}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(x^(1/2))/x,x,method=_RETURNVERBOSE)
[Out] arcsech(x^(1/2))^2-2*arcsech(x^(1/2))*ln(1+(1/x^(1/2)+(-1+1/x^(1/2))^(1/2)*
(1+1/x^(1/2))^(1/2))^2)-polylog(2,-(1/x^(1/2)+(-1+1/x^(1/2))^(1/2)*(1+1/x^(
1/2))^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x,x, algorithm="maxima")
[Out] -1/4*log(x)^2 + log(x)*log(sqrt(sqrt(x) + 1)*sqrt(-sqrt(x) + 1) + 1) - log(
sqrt(x) + 1)*log(sqrt(x)) - log(sqrt(x))*log(-sqrt(x) + 1) - dilog(-sqrt(x)
) - dilog(sqrt(x)) + integrate(1/2*log(x)/((x - 1)*e^(1/2*log(sqrt(x) + 1)
+ 1/2*log(-sqrt(x) + 1)) + x - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x,x, algorithm="fricas")
[Out] integral(arcsech(sqrt(x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}\left(\sqrt{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(x**(1/2))/x,x)
```

```
[Out] Integral(asech(sqrt(x))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(arcsech(sqrt(x))/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acosh}\left(\frac{1}{\sqrt{x}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/x^(1/2))/x,x)
```

```
[Out] int(acosh(1/x^(1/2))/x, x)
```

3.25 $\int \frac{\text{sech}^{-1}(\sqrt{x})}{x^2} dx$

Optimal. Leaf size=98

$$\frac{1-x}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{3/2}}-\frac{\text{sech}^{-1}(\sqrt{x})}{x}+\frac{\sqrt{1-x}\,\tanh^{-1}\left(\sqrt{1-x}\right)}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

```
[Out] -arcsech(x^(1/2))/x+1/2*(1-x)/x^(3/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1
/2)+1/2*arctanh((1-x)^(1/2))*(1-x)^(1/2)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/
x^(1/2))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6480, 12, 44, 65, 212}

$$\frac{1-x}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,x^{3/2}}+\frac{\sqrt{1-x}\,\tanh^{-1}\left(\sqrt{1-x}\right)}{2\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}-\frac{\text{sech}^{-1}(\sqrt{x})}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[Sqrt[x]]/x^2,x]
```

```
[Out] (1 - x)/(2*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*x^(3/2)) - ArcSech[Sqrt
[x]]/x + (Sqrt[1 - x]*ArcTanh[Sqrt[1 - x]])/(2*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1
+ 1/Sqrt[x]]*Sqrt[x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6480

```
Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSech[u])/(d*(m + 1))), x] + Dist[b*(Sqrt[1
- u^2]/(d*(m + 1)*u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u])), Int[SimplifyIntegrand[(
c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[1 - u^2])), x], x], x] /; FreeQ[{a, b, c,
 d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c
+ d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{x^2}\,dx &= -\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{x} - \frac{\sqrt{1-x}\int \frac{1}{2\sqrt{1-x}\,x^2}\,dx}{\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} \\
&= -\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{x} - \frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x}\,x^2}\,dx}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} \\
&= \frac{1-x}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{3/2}} - \frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{x} - \frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x}\,x}\,dx}{4\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} \\
&= \frac{1-x}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{3/2}} - \frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{x} + \frac{\sqrt{1-x}\,\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x}\right)}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}} \\
&= \frac{1-x}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{3/2}} - \frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{x} + \frac{\sqrt{1-x}\,\tanh^{-1}\left(\sqrt{1-x}\right)}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 111, normalized size = 1.13

$$\frac{\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\left(1+\sqrt{x}\right)-2\mathrm{sech}^{-1}\left(\sqrt{x}\right)+x\log\left(1+\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\sqrt{x}\right)-\frac{1}{2}x\log(x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[Sqrt[x]]/x^2,x]
```

```
[Out] (Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(1 + Sqrt[x]) - 2*ArcSech[Sqrt[x]] + x*L
og[1 + Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])] + Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])
]*Sqrt[x]] - (x*Log[x])/2)/(2*x)
```

Maple [A]
time = 0.17, size = 64, normalized size = 0.65

method	result	size
derivativedivides	$-\frac{\mathrm{arcsech}\left(\sqrt{x}\right)}{x}+\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\left(\mathrm{arctanh}\left(\frac{1}{\sqrt{1-x}}\right)x+\sqrt{1-x}\right)}{2\sqrt{x}\sqrt{1-x}}$	64
default	$-\frac{\mathrm{arcsech}\left(\sqrt{x}\right)}{x}+\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\left(\mathrm{arctanh}\left(\frac{1}{\sqrt{1-x}}\right)x+\sqrt{1-x}\right)}{2\sqrt{x}\sqrt{1-x}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -arcsech(x^(1/2))/x+1/2*(-(x^(1/2)-1)/x^(1/2))^(1/2)/x^(1/2)*((x^(1/2)+1)/x
^(1/2))^(1/2)*(arctanh(1/(1-x)^(1/2))*x+(1-x)^(1/2))/(1-x)^(1/2)
```

Maxima [A]
time = 0.27, size = 65, normalized size = 0.66

$$-\frac{\sqrt{x}\sqrt{\frac{1}{x}-1}}{2\left(x\left(\frac{1}{x}-1\right)-1\right)}-\frac{\mathrm{arsech}\left(\sqrt{x}\right)}{x}+\frac{1}{4}\log\left(\sqrt{x}\sqrt{\frac{1}{x}-1}+1\right)-\frac{1}{4}\log\left(\sqrt{x}\sqrt{\frac{1}{x}-1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(x)*sqrt(1/x - 1)/(x*(1/x - 1) - 1) - arcsech(sqrt(x))/x + 1/4*log
(sqrt(x)*sqrt(1/x - 1) + 1) - 1/4*log(sqrt(x)*sqrt(1/x - 1) - 1)
```

Fricas [A]
time = 0.47, size = 45, normalized size = 0.46

$$\frac{(x-2)\log\left(\frac{x\sqrt{-\frac{x-1}{x}}+\sqrt{x}}{x}\right)+\sqrt{x}\sqrt{-\frac{x-1}{x}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^2,x, algorithm="fricas")
[Out] 1/2*((x - 2)*log((x*sqrt(-(x - 1)/x) + sqrt(x))/x) + sqrt(x)*sqrt(-(x - 1)/
x))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}\left(\sqrt{x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(x**(1/2))/x**2,x)
[Out] Integral(asech(sqrt(x))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^2,x, algorithm="giac")
[Out] integrate(arcsech(sqrt(x))/x^2, x)
```

Mupad [B]
time = 2.15, size = 40, normalized size = 0.41

$$\frac{\sqrt{\frac{1}{\sqrt{x}}-1}\,\sqrt{\frac{1}{\sqrt{x}}+1}}{2\,\sqrt{x}}-\frac{2\,\mathrm{acosh}\left(\frac{1}{\sqrt{x}}\right)\,\left(\frac{1}{2\,\sqrt{x}}-\frac{\sqrt{x}}{4}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/x^(1/2))/x^2,x)
[Out] ((1/x^(1/2) - 1)^(1/2)*(1/x^(1/2) + 1)^(1/2))/(2*x^(1/2)) - (2*acosh(1/x^(1
/2))*(1/(2*x^(1/2)) - x^(1/2)/4))/x^(1/2)
```

3.26 $\int \frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{x^3}\,dx$

Optimal. Leaf size=136

$$\frac{1-x}{8\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{5/2}}+\frac{3(1-x)}{16\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{3/2}}-\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{3\sqrt{1-x}\,\tanh^{-1}\left(\sqrt{1\right.}{16\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+}$$

```
[Out] -1/2*arcsech(x^(1/2))/x^2+1/8*(1-x)/x^(5/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/
2))^(1/2)+3/16*(1-x)/x^(3/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)+3/16*
arctanh((1-x)^(1/2))*(1-x)^(1/2)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))
^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6480, 12, 44, 65, 212}

$$\frac{3(1-x)}{16\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,x^{3/2}}+\frac{1-x}{8\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,x^{5/2}}-\frac{\operatorname{sech}^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{3\sqrt{1-x}\,\tanh^{-1}\left(\sqrt{1-x}\right)}{16\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[Sqrt[x]]/x^3,x]
```

```
[Out] (1 - x)/(8*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*x^(5/2)) + (3*(1 - x))/
(16*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*x^(3/2)) - ArcSech[Sqrt[x]]/(2
*x^2) + (3*Sqrt[1 - x]*ArcTanh[Sqrt[1 - x]])/(16*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[
1 + 1/Sqrt[x]]*Sqrt[x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6480

```
Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSech[u])/(d*(m + 1))), x] + Dist[b*(Sqrt[1
- u^2]/(d*(m + 1)*u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u])), Int[SimplifyIntegrand[(
c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[1 - u^2])), x], x], x] /; FreeQ[{a, b, c,
 d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c
+ d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^3}\,dx = -\frac{\operatorname{sech}^{-1}(\sqrt{x})}{2x^2} - \frac{\sqrt{1-x}\int \frac{1}{2\sqrt{1-x}\,x^3}\,dx}{2\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$= -\frac{\operatorname{sech}^{-1}(\sqrt{x})}{2x^2} - \frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x}\,x^3}\,dx}{4\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$= \frac{1-x}{8\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{5/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{2x^2} - \frac{\left(3\sqrt{1-x}\right)\int \frac{1}{\sqrt{1-x}\,x^2}\,dx}{16\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$= \frac{1-x}{8\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{5/2}} + \frac{3(1-x)}{16\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{3/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{2x^2}$$

$$= \frac{1-x}{8\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{5/2}} + \frac{3(1-x)}{16\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{3/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{2x^2}$$

$$= \frac{1-x}{8\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{5/2}} + \frac{3(1-x)}{16\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{3/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{2x^2}$$

Mathematica [A]
time = 0.08, size = 125, normalized size = 0.92

$$\frac{1}{16}\left(\frac{\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\left(2+2\sqrt{x}+3x+3x^{3/2}\right)}{x^2} - \frac{8\operatorname{sech}^{-1}(\sqrt{x})}{x^2} + 3\log\left(1+\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\sqrt{x}\right) - \frac{3\log(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[Sqrt[x]]/x^3,x]
```

```
[Out] ((Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(2 + 2*Sqrt[x] + 3*x + 3*x^(3/2)))/x^2
- (8*ArcSech[Sqrt[x]])/x^2 + 3*Log[1 + Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])] +
Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]] - (3*Log[x])/2)/16
```

Maple [A]
time = 0.19, size = 79, normalized size = 0.58

method	result
derivativedivides	$-\frac{\operatorname{arcsech}\left(\sqrt{x}\right)}{2x^2}+\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,\left(3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{1-x}}\right)x^2+3\sqrt{1-x}\,x+2\sqrt{1-x}\right)}{16x^{\frac{3}{2}}\sqrt{1-x}}$
default	$-\frac{\operatorname{arcsech}\left(\sqrt{x}\right)}{2x^2}+\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,\left(3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{1-x}}\right)x^2+3\sqrt{1-x}\,x+2\sqrt{1-x}\right)}{16x^{\frac{3}{2}}\sqrt{1-x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arcsech(x^(1/2))/x^2+1/16*(-(x^(1/2)-1)/x^(1/2))^(1/2)/x^(3/2)*((x^(1/
2)+1)/x^(1/2))^(1/2)*(3*arctanh(1/(1-x)^(1/2))*x^2+3*(1-x)^(1/2)*x+2*(1-x)^
(1/2))/(1-x)^(1/2)
```

Maxima [A]
time = 0.25, size = 92, normalized size = 0.68

$$-\frac{3\,x^{\frac{3}{2}}\left(\frac{1}{x}-1\right)^{\frac{3}{2}}-5\,\sqrt{x}\,\sqrt{\frac{1}{x}-1}}{16\,\left(x^2\left(\frac{1}{x}-1\right)^2-2\,x\left(\frac{1}{x}-1\right)+1\right)}-\frac{\operatorname{arsech}\left(\sqrt{x}\right)}{2\,x^2}+\frac{3}{32}\,\log\left(\sqrt{x}\,\sqrt{\frac{1}{x}-1}+1\right)-\frac{3}{32}\,\log\left(\sqrt{x}\,\sqrt{\frac{1}{x}-1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(3*x^(3/2)*(1/x - 1)^(3/2) - 5*sqrt(x)*sqrt(1/x - 1))/(x^2*(1/x - 1)^
2 - 2*x*(1/x - 1) + 1) - 1/2*arcsech(sqrt(x))/x^2 + 3/32*log(sqrt(x)*sqrt(1
/x - 1) + 1) - 3/32*log(sqrt(x)*sqrt(1/x - 1) - 1)
```

Fricas [A]
time = 0.50, size = 54, normalized size = 0.40

$$\frac{(3\,x+2)\sqrt{x}\,\sqrt{-\frac{x-1}{x}}+(3\,x^2-8)\log\left(\frac{x\sqrt{-\frac{x-1}{x}}+\sqrt{x}}{x}\right)}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/16*((3*x + 2)*sqrt(x)*sqrt(-(x - 1)/x) + (3*x^2 - 8)*log((x*sqrt(-(x - 1)
/x) + sqrt(x))/x))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}\left(\sqrt{x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(x**(1/2))/x**3,x)
```

```
[Out] Integral(asech(sqrt(x))/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] integrate(arcsech(sqrt(x))/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/x^(1/2))/x^3,x)
```

```
[Out] int(acosh(1/x^(1/2))/x^3, x)
```

3.27 $\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^4}\,dx$

Optimal. Leaf size=172

$$\frac{1-x}{18\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{7/2}}+\frac{5(1-x)}{72\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{5/2}}+\frac{5(1-x)}{48\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{3/2}}-\frac{\mathrm{se}}{}$$

```
[Out] -1/3*arcsech(x^(1/2))/x^3+1/18*(1-x)/x^(7/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1
/2))^(1/2)+5/72*(1-x)/x^(5/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)+5/48
*(1-x)/x^(3/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)+5/48*arctanh((1-x)^
(1/2))*(1-x)^(1/2)/x^(1/2)/(-1+1/x^(1/2))^(1/2)/(1+1/x^(1/2))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6480, 12, 44, 65, 212}

$$\frac{5(1-x)}{48\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,x^{3/2}}+\frac{5(1-x)}{72\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,x^{5/2}}+\frac{1-x}{18\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\,x^{7/2}}-\frac{\operatorname{sech}^{-1}(\sqrt{x})}{3x^3}+\frac{5\sqrt{1-x}\tanh^{-1}\left(\sqrt{1-x}\right)}{48\sqrt{\frac{1}{\sqrt{x}}-1}\sqrt{\frac{1}{\sqrt{x}}+1}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[Sqrt[x]]/x^4,x]
```

```
[Out] (1 - x)/(18*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*x^(7/2)) + (5*(1 - x))
/(72*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*x^(5/2)) + (5*(1 - x))/(48*Sq
rt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/Sqrt[x]]*x^(3/2)) - ArcSech[Sqrt[x]]/(3*x^3)
+ (5*Sqrt[1 - x]*ArcTanh[Sqrt[1 - x]])/(48*Sqrt[-1 + 1/Sqrt[x]]*Sqrt[1 + 1/
Sqrt[x]]*Sqrt[x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6480

```
Int[((a_.) + ArcSech[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSech[u])/(d*(m + 1))), x] + Dist[b*(Sqrt[1
- u^2]/(d*(m + 1)*u*Sqrt[-1 + 1/u]*Sqrt[1 + 1/u])), Int[SimplifyIntegrand[(
c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[1 - u^2])), x], x], x] /; FreeQ[{a, b, c,
 d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&  !FunctionOfQ[(c
+ d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{-1}(\sqrt{x})}{x^4}\,dx = -\frac{\operatorname{sech}^{-1}(\sqrt{x})}{3x^3} - \frac{\sqrt{1-x}\int \frac{1}{2\sqrt{1-x}\,x^4}\,dx}{3\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$= -\frac{\operatorname{sech}^{-1}(\sqrt{x})}{3x^3} - \frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x}\,x^4}\,dx}{6\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$= \frac{1-x}{18\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{7/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{3x^3} - \frac{\left(5\sqrt{1-x}\right)\int \frac{1}{\sqrt{1-x}\,x^3}\,dx}{36\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}$$

$$= \frac{1-x}{18\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{7/2}} + \frac{5(1-x)}{72\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{5/2}} - \frac{\operatorname{sech}^{-1}(\sqrt{x})}{3x^3}$$

$$= \frac{1-x}{18\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{7/2}} + \frac{5(1-x)}{72\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{5/2}} + \frac{}{48\sqrt{-1+}}$$

$$= \frac{1-x}{18\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{7/2}} + \frac{5(1-x)}{72\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{5/2}} + \frac{}{48\sqrt{-1+}}$$

$$= \frac{1-x}{18\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{7/2}} + \frac{5(1-x)}{72\sqrt{-1+\frac{1}{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}\,x^{5/2}} + \frac{}{48\sqrt{-1+}}$$

<u>Mathematica</u> [A]
time = 0.10, size = 140, normalized size = 0.81

$$\frac{\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\left(8+8\sqrt{x}+10x+10x^{3/2}+15x^2+15x^{5/2}\right)-48\operatorname{sech}^{-1}(\sqrt{x})+15x^3\log\left(1+\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\sqrt{x}\right)-\frac{15}{2}x^3\log(x)}{144x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[Sqrt[x]]/x^4,x]
[Out] (Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*(8 + 8*Sqrt[x] + 10*x + 10*x^(3/2) + 15*
x^2 + 15*x^(5/2)) - 48*ArcSech[Sqrt[x]] + 15*x^3*Log[1 + Sqrt[(1 - Sqrt[x])
```

```
/(1 + Sqrt[x])] + Sqrt[(1 - Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]] - (15*x^3*Log[
x])/2)/(144*x^3)
```

Maple [A]
time = 0.21, size = 91, normalized size = 0.53

method	result
derivativedivides	$-\frac{\mathrm{arcsech}\left(\sqrt{x}\right)}{3x^{3}}+\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\left(15\,\mathrm{arctanh}\left(\frac{1}{\sqrt{1-x}}\right)x^{3}+15\sqrt{1-x}\,x^{2}+10\sqrt{1-x}\,x\right.}{144x^{\frac{5}{2}}\sqrt{1-x}}$
default	$-\frac{\mathrm{arcsech}\left(\sqrt{x}\right)}{3x^{3}}+\frac{\sqrt{-\frac{\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\left(15\,\mathrm{arctanh}\left(\frac{1}{\sqrt{1-x}}\right)x^{3}+15\sqrt{1-x}\,x^{2}+10\sqrt{1-x}\,x\right.}{144x^{\frac{5}{2}}\sqrt{1-x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(x^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arcsech(x^(1/2))/x^3+1/144*(-(x^(1/2)-1)/x^(1/2))^(1/2)/x^(5/2)*((x^(1
/2)+1)/x^(1/2))^(1/2)*(15*arctanh(1/(1-x)^(1/2))*x^3+15*(1-x)^(1/2)*x^2+10*
(1-x)^(1/2)*x+8*(1-x)^(1/2))/(1-x)^(1/2)
```

Maxima [A]
time = 0.26, size = 116, normalized size = 0.67

$$-\frac{15\,x^{\frac{5}{2}}\left(\frac{1}{x}-1\right)^{\frac{5}{2}}-40\,x^{\frac{3}{2}}\left(\frac{1}{x}-1\right)^{\frac{3}{2}}+33\,\sqrt{x}\,\sqrt{\frac{1}{x}-1}}{144\left(x^{3}\left(\frac{1}{x}-1\right)^{3}-3\,x^{2}\left(\frac{1}{x}-1\right)^{2}+3\,x\left(\frac{1}{x}-1\right)-1\right)}-\frac{\mathrm{arsech}\left(\sqrt{x}\right)}{3\,x^{3}}+\frac{5}{96}\,\log\left(\sqrt{x}\,\sqrt{\frac{1}{x}-1}+1\right)-\frac{5}{96}\,\log\left(\sqrt{x}\,\sqrt{\frac{1}{x}-1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] -1/144*(15*x^(5/2)*(1/x - 1)^(5/2) - 40*x^(3/2)*(1/x - 1)^(3/2) + 33*sqrt(x
)*sqrt(1/x - 1))/(x^3*(1/x - 1)^3 - 3*x^2*(1/x - 1)^2 + 3*x*(1/x - 1) - 1)
- 1/3*arcsech(sqrt(x))/x^3 + 5/96*log(sqrt(x)*sqrt(1/x - 1) + 1) - 5/96*log
(sqrt(x)*sqrt(1/x - 1) - 1)
```

Fricas [A]
time = 0.36, size = 60, normalized size = 0.35

$$\frac{\left(15\,x^{2}+10\,x+8\right)\sqrt{x}\,\sqrt{-\frac{x-1}{x}}+3\left(5\,x^{3}-16\right)\log\left(\frac{x\sqrt{-\frac{x-1}{x}}+\sqrt{x}}{x}\right)}{144\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] 1/144*((15*x^2 + 10*x + 8)*sqrt(x)*sqrt(-(x - 1)/x) + 3*(5*x^3 - 16)*log((x
*sqrt(-(x - 1)/x) + sqrt(x))/x))/x^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}\left(\sqrt{x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(x**(1/2))/x**4,x)
```

```
[Out] Integral(asech(sqrt(x))/x**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(x^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsech(sqrt(x))/x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/x^(1/2))/x^4,x)
```

```
[Out] int(acosh(1/x^(1/2))/x^4, x)
```

3.28 $\int \operatorname{sech}^{-1}\left(\frac{1}{x}\right)\,dx$

Optimal. Leaf size=21

$$-\sqrt{-1+x}\,\sqrt{1+x}\,+x\cosh^{-1}(x)$$

```
[Out] x*arccosh(x)-(-1+x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6462, 5879, 75}

$$x\cosh^{-1}(x)-\sqrt{x-1}\,\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[x^(-1)],x]
```

```
[Out] -(Sqrt[-1 + x]*Sqrt[1 + x]) + x*ArcCosh[x]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 6462

```
Int[ArcSech[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCosh[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^{-1}\left(\frac{1}{x}\right)\,dx &= \int \cosh^{-1}(x)\,dx\\
&= x\cosh^{-1}(x)-\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx\\
&= -\sqrt{-1+x}\,\sqrt{1+x}\,+x\cosh^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.19

$$-\sqrt{\frac{-1+x}{1+x}}\,(1+x)+x\mathrm{sech}^{-1}\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[x^(-1)],x]
[Out] -(Sqrt[(-1 + x)/(1 + x)]*(1 + x)) + x*ArcSech[x^(-1)]
```

Maple [A]
time = 0.17, size = 29, normalized size = 1.38

method	result	size
derivativedivides	$x \,\mathrm{arcsech}\left(\frac{1}{x}\right)-\sqrt{-\left(-1+\frac{1}{x}\right) x}\,\sqrt{\left(1+\frac{1}{x}\right) x}$	29
default	$x \,\mathrm{arcsech}\left(\frac{1}{x}\right)-\sqrt{-\left(-1+\frac{1}{x}\right) x}\,\sqrt{\left(1+\frac{1}{x}\right) x}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(1/x),x,method=_RETURNVERBOSE)
[Out] x*arcsech(1/x)-(-(-1+1/x)*x)^(1/2)*((1+1/x)*x)^(1/2)
```

Maxima [A]
time = 0.27, size = 16, normalized size = 0.76

$$x\,\mathrm{arsech}\left(\frac{1}{x}\right)-\sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(1/x),x, algorithm="maxima")
[Out] x*arcsech(1/x) - sqrt(x^2 - 1)
```

Fricas [A]
time = 0.50, size = 22, normalized size = 1.05

$$x\log\left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(1/x),x, algorithm="fricas")
[Out] x*log(x + sqrt(x^2 - 1)) - sqrt(x^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asech}\left(\frac{1}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(1/x),x)
```

```
[Out] Integral(asech(1/x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(1/x),x, algorithm="giac")
```

```
[Out] integrate(arcsech(1/x), x)
```

Mupad [B]
time = 1.35, size = 17, normalized size = 0.81

$$x\operatorname{acosh}(x)-\sqrt{x-1}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x),x)
```

```
[Out] x*acosh(x) - (x - 1)^(1/2)*(x + 1)^(1/2)
```

3.29 $\int \frac{\operatorname{sech}^{-1}(ax^n)}{x}\,dx$

Optimal. Leaf size=61

$$\frac{\operatorname{sech}^{-1}(ax^n)^2}{2n}-\frac{\operatorname{sech}^{-1}(ax^n)\log\left(1+e^{2\operatorname{sech}^{-1}(ax^n)}\right)}{n}-\frac{\operatorname{PolyLog}\left(2,-e^{2\operatorname{sech}^{-1}(ax^n)}\right)}{2n}$$

```
[Out] 1/2*arcsech(a*x^n)^2/n-arcsech(a*x^n)*ln(1+(1/a/(x^n)+(1/a/(x^n)-1)^(1/2)*(
1/a/(x^n)+1)^(1/2))^2)/n-1/2*polylog(2,-(1/a/(x^n)+(1/a/(x^n)-1)^(1/2)*(1/a
/(x^n)+1)^(1/2))^2)/n
```

Rubi [A]
time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6416, 5882, 3799, 2221, 2317, 2438}

$$-\frac{\operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(ax^n)}\right)}{2n}+\frac{\operatorname{sech}^{-1}(ax^n)^2}{2n}-\frac{\operatorname{sech}^{-1}(ax^n)\log\left(e^{2\operatorname{sech}^{-1}(ax^n)}+1\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a*x^n]/x,x]
```

```
[Out] ArcSech[a*x^n]^2/(2*n) - (ArcSech[a*x^n]*Log[1 + E^(2*ArcSech[a*x^n])])/n -
 PolyLog[2, -E^(2*ArcSech[a*x^n])]/(2*n)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6416

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{-1}(ax^n)}{x}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\operatorname{sech}^{-1}(ax)}{x}\,dx, x, x^n\right)}{n}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)}{x}\,dx, x, x^{-n}\right)}{n}$$

$$= -\frac{\operatorname{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}\left(\frac{x^{-n}}{a}\right)\right)}{n}$$

$$= \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{2\operatorname{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(\frac{x^{-n}}{a}\right)\right)}{n}$$

$$= \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{n} + \frac{\operatorname{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x\right.}{n}$$

$$= \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{n} + \frac{\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\cos}\right.}{2n}$$

$$= \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\cosh^{-1}\left(\frac{x^{-n}}{a}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{n} - \frac{\operatorname{Li}_2\left(-e^{2\cosh^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{2n}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 219 vs. 2(61) = 122.

time = 0.73, size = 219, normalized size = 3.59

$$\operatorname{sech}^{-1}(ax^n)\log(x)+\frac{\sqrt{\frac{1-ax^n}{1+ax^n}}\left(4\sqrt{-1+a^2x^{2n}}\operatorname{ArcTan}\left(\sqrt{-1+a^2x^{2n}}\right)(2n\log(x)-\log(a^2x^{2n}))+\sqrt{1-a^2x^{2n}}\left(\log^2(a^2x^{2n})-4\log(a^2x^{2n})\log\left(\frac{1}{2}\left(1+\sqrt{1-a^2x^{2n}}\right)\right)+2\log^2\left(\frac{1}{2}\left(1+\sqrt{1-a^2x^{2n}}\right)\right)-4\text{PolyLog}\left(2,\frac{1}{2}-\frac{1}{2}\sqrt{1-a^2x^{2n}}\right)\right)\right)}{8(n-anx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a*x^n]/x,x]
```

```
[Out] ArcSech[a*x^n]*Log[x] + (Sqrt[(1 - a*x^n)/(1 + a*x^n)]*(4*Sqrt[-1 + a^2*x^(
2*n)]*ArcTan[Sqrt[-1 + a^2*x^(2*n)]]*(2*n*Log[x] - Log[a^2*x^(2*n)]) + Sqrt
[1 - a^2*x^(2*n)]*(Log[a^2*x^(2*n)]^2 - 4*Log[a^2*x^(2*n)]*Log[(1 + Sqrt[1
- a^2*x^(2*n)])/2] + 2*Log[(1 + Sqrt[1 - a^2*x^(2*n)])/2]^2 - 4*PolyLog[2,
1/2 - Sqrt[1 - a^2*x^(2*n)]/2])))/(8*(n - a*n*x^n))
```

Maple [A]
time = 0.60, size = 111, normalized size = 1.82

method	result
derivativedivides	$\frac{\frac{\operatorname{arcsech}(a\,x^n)^2}{2}-\operatorname{arcsech}(a\,x^n)\ln\left(1+\left(\frac{x^{-n}}{a}+\sqrt{\frac{x^{-n}}{a}-1}\,\sqrt{\frac{x^{-n}}{a}+1}\right)^2\right)-\frac{\operatorname{polylog}\left(2,-\left(\frac{x^{-n}}{a}+\sqrt{\frac{x^{-n}}{a}-1}\right.\right.}{2}}{n}$
default	$\frac{\frac{\operatorname{arcsech}(a\,x^n)^2}{2}-\operatorname{arcsech}(a\,x^n)\ln\left(1+\left(\frac{x^{-n}}{a}+\sqrt{\frac{x^{-n}}{a}-1}\,\sqrt{\frac{x^{-n}}{a}+1}\right)^2\right)-\frac{\operatorname{polylog}\left(2,-\left(\frac{x^{-n}}{a}+\sqrt{\frac{x^{-n}}{a}-1}\right.\right.}{2}}{n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(a*x^n)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(1/2*arcsech(a*x^n)^2-arcsech(a*x^n)*ln(1+(1/a/(x^n)+(1/a/(x^n)-1)^(1/2
)*(1/a/(x^n)+1)^(1/2))^2)-1/2*polylog(2,-(1/a/(x^n)+(1/a/(x^n)-1)^(1/2)*(1/
a/(x^n)+1)^(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] a^2*n*integrate(x^(2*n)*log(x)/(a^2*x*x^(2*n) + (a^2*x*x^(2*n) - x)*sqrt(a*
x^n + 1)*sqrt(-a*x^n + 1) - x), x) + n*integrate(1/2*log(x)/(a*x*x^n + x),
x) - n*integrate(1/2*log(x)/(a*x*x^n - x), x) + log(sqrt(a*x^n + 1)*sqrt(-a
*x^n + 1) + 1)*log(x) - log(a)*log(x) - log(x)*log(x^n)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(a*x**n)/x,x)
```

```
[Out] Integral(asech(a*x**n)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsech(a*x^n)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}\left(\frac{1}{a\,x^n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a*x^n))/x,x)
```

```
[Out] int(acosh(1/(a*x^n))/x, x)
```

3.30 $\int \frac{\text{sech}^{-1}(ax^5)}{x}\,dx$

Optimal. Leaf size=54

$$\frac{1}{10}\text{sech}^{-1}(ax^5)^2-\frac{1}{5}\text{sech}^{-1}(ax^5)\log\left(1+e^{2\text{sech}^{-1}(ax^5)}\right)-\frac{1}{10}\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}(ax^5)}\right)$$

```
[Out] 1/10*arcsech(a*x^5)^2-1/5*arcsech(a*x^5)*ln(1+(1/a/x^5+(1/a/x^5-1)^(1/2)*(1
/a/x^5+1)^(1/2))^2)-1/10*polylog(2,-(1/a/x^5+(1/a/x^5-1)^(1/2)*(1/a/x^5+1)^
(1/2))^2)
```

Rubi [A]
time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.600$, Rules used = {6416, 5882, 3799, 2221, 2317, 2438}

$$-\frac{1}{10}\text{Li}_2\left(-e^{2\text{sech}^{-1}(ax^5)}\right)+\frac{1}{10}\text{sech}^{-1}(ax^5)^2-\frac{1}{5}\text{sech}^{-1}(ax^5)\log\left(e^{2\text{sech}^{-1}(ax^5)}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a*x^5]/x,x]
```

```
[Out] ArcSech[a*x^5]^2/10 - (ArcSech[a*x^5]*Log[1 + E^(2*ArcSech[a*x^5])])/5 - Po
lyLog[2, -E^(2*ArcSech[a*x^5])]/10
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
```

```
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6416

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(ax^5)}{x}\,dx &= \frac{1}{5}\text{Subst}\left(\int \frac{\operatorname{sech}^{-1}(ax)}{x}\,dx, x, x^5\right)\\
&= -\left(\frac{1}{5}\text{Subst}\left(\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)}{x}\,dx, x, \frac{1}{x^5}\right)\right)\\
&= -\left(\frac{1}{5}\text{Subst}\left(\int x\tanh(x)\,dx, x, \operatorname{sech}^{-1}(ax^5)\right)\right)\\
&= \frac{1}{10}\operatorname{sech}^{-1}(ax^5)^2 - \frac{2}{5}\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \operatorname{sech}^{-1}(ax^5)\right)\\
&= \frac{1}{10}\operatorname{sech}^{-1}(ax^5)^2 - \frac{1}{5}\operatorname{sech}^{-1}(ax^5)\log\left(1+e^{2\operatorname{sech}^{-1}(ax^5)}\right) + \frac{1}{5}\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx,\right.\\
&= \frac{1}{10}\operatorname{sech}^{-1}(ax^5)^2 - \frac{1}{5}\operatorname{sech}^{-1}(ax^5)\log\left(1+e^{2\operatorname{sech}^{-1}(ax^5)}\right) + \frac{1}{10}\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x\right.\\
&= \frac{1}{10}\operatorname{sech}^{-1}(ax^5)^2 - \frac{1}{5}\operatorname{sech}^{-1}(ax^5)\log\left(1+e^{2\operatorname{sech}^{-1}(ax^5)}\right) - \frac{1}{10}\text{Li}_2\left(-e^{2\operatorname{sech}^{-1}(ax^5)}\right)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 0.91

$$\frac{1}{10}\left(-\operatorname{sech}^{-1}(ax^5)\left(\operatorname{sech}^{-1}(ax^5)+2\log\left(1+e^{-2\operatorname{sech}^{-1}(ax^5)}\right)\right)+\text{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(ax^5)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a*x^5]/x,x]
```

```
[Out] (-(ArcSech[a*x^5]*(ArcSech[a*x^5] + 2*Log[1 + E^(-2*ArcSech[a*x^5])])) + Po
lyLog[2, -E^(-2*ArcSech[a*x^5])])/10
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsech}(a\,x^{5})}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(a*x^5)/x,x)
```

```
[Out] int(arcsech(a*x^5)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x^5)/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsech(a*x^5)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsech(a*x^5)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asech}\left(ax^{5}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(a*x**5)/x,x)
```

```
[Out] Integral(asech(a*x**5)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x^5)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsech(a*x^5)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acosh}\left(\frac{1}{a\,x^5}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a*x^5))/x,x)
```

```
[Out] int(acosh(1/(a*x^5))/x, x)
```

3.31 $\int \text{sech}^{-1}\left(ce^{a+bx}\right) dx$

Optimal. Leaf size=77

$$\frac{\text{sech}^{-1}\left(ce^{a+bx}\right)^2}{2b} - \frac{\text{sech}^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2\text{sech}^{-1}\left(ce^{a+bx}\right)}\right)}{b} - \frac{\text{PolyLog}\left(2,-e^{2\text{sech}^{-1}\left(ce^{a+bx}\right)}\right)}{2b}$$

```
[Out] 1/2*arcsech(c*exp(b*x+a))^2/b-arcsech(c*exp(b*x+a))*ln(1+(1/c/exp(b*x+a)+(1
/c/exp(b*x+a)-1)^(1/2)*(1/c/exp(b*x+a)+1)^(1/2))^2)/b-1/2*polylog(2,-(1/c/e
xp(b*x+a)+(1/c/exp(b*x+a)-1)^(1/2)*(1/c/exp(b*x+a)+1)^(1/2))^2)/b
```

Rubi [A]
time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {2320, 6416, 5882, 3799, 2221, 2317, 2438}

$$-\frac{\text{Li}_2\left(-e^{2\text{sech}^{-1}\left(ce^{a+bx}\right)}\right)}{2b} + \frac{\text{sech}^{-1}\left(ce^{a+bx}\right)^2}{2b} - \frac{\text{sech}^{-1}\left(ce^{a+bx}\right)\log\left(e^{2\text{sech}^{-1}\left(ce^{a+bx}\right)}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[c*E^(a + b*x)],x]
```

```
[Out] ArcSech[c*E^(a + b*x)]^2/(2*b) - (ArcSech[c*E^(a + b*x)]*Log[1 + E^(2*ArcSe
ch[c*E^(a + b*x)])])/b - PolyLog[2, -E^(2*ArcSech[c*E^(a + b*x)])]/(2*b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6416

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \operatorname{sech}^{-1}\left(ce^{a+bx}\right) dx = \frac{\text{Subst}\left(\int \frac{\operatorname{sech}^{-1}(cx)}{x} dx, x, e^{a+bx}\right)}{b}$$

$$= -\frac{\text{Subst}\left(\int \frac{\cosh^{-1}\left(\frac{x}{c}\right)}{x} dx, x, e^{-a-bx}\right)}{b}$$

$$= -\frac{\text{Subst}\left(\int x \tanh(x) \, dx, x, \cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)\right)}{b}$$

$$= \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{2\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}} dx, x, \cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)\right)}{b}$$

$$= \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right) \log\left(1+e^{2\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} + \frac{\text{Subst}\left(\int \log (1 -\right.}{}$$

$$= \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right) \log\left(1+e^{2\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} + \frac{\text{Subst}\left(\int \frac{\log(1+x}{x}\right.}{}$$

$$= \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right) \log\left(1+e^{2\cosh^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} - \frac{\text{Li}_2\left(-e^{2\cosh^{-1}\left(\right.}\right.}{2b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 249 vs. $2(77) = 154$.

time = 0.98, size = 249, normalized size = 3.23

$$x\operatorname{sech}^{-1}\left(ce^{a+bx}\right) - \frac{\sqrt{\frac{1-ce^{a+bx}}{1+ce^{a+bx}}}\sqrt{1+ce^{a+bx}}\left(\tanh^{-1}\left(\sqrt{1-c^2e^{2(a+bx)}}\right)\left(8bx-4\log\left(c^2e^{2(a+bx)}\right)\right) - \log^2\left(c^2e^{2(a+bx)}\right) + 4\log\left(c^2e^{2(a+bx)}\right)\log\left(\frac{1}{2}\left(1+\sqrt{1-c^2e^{2(a+bx)}}\right)\right) - 2\log^2\left(\frac{1}{2}\left(1+\sqrt{1-c^2e^{2(a+bx)}}\right)\right) + 4\text{PolyLog}\left(2,\frac{1}{2}\left(1-\sqrt{1-c^2e^{2(a+bx)}}\right)\right)\right)}{8b\sqrt{1-ce^{a+bx}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[c*E^(a + b*x)],x]
```

```
[Out] x*ArcSech[c*E^(a + b*x)] - (Sqrt[(1 - c*E^(a + b*x))/(1 + c*E^(a + b*x))]*S
qrt[1 + c*E^(a + b*x)]*(ArcTanh[Sqrt[1 - c^2*E^(2*(a + b*x))]]*(8*b*x - 4*L
og[c^2*E^(2*(a + b*x))]) - Log[c^2*E^(2*(a + b*x))]^2 + 4*Log[c^2*E^(2*(a +
 b*x))]*Log[(1 + Sqrt[1 - c^2*E^(2*(a + b*x))])/2] - 2*Log[(1 + Sqrt[1 - c^
2*E^(2*(a + b*x))])/2]^2 + 4*PolyLog[2, (1 - Sqrt[1 - c^2*E^(2*(a + b*x))])
/2]))/(8*b*Sqrt[1 - c*E^(a + b*x)])
```

Maple [A]

time = 0.60, size = 135, normalized size = 1.75

method	result

derivativedivides	$\frac{\frac{\operatorname{arcsech}\left(e^{bx+a}c\right)^2}{2}-\operatorname{arcsech}\left(e^{bx+a}c\right)\ln\left(1+\left(\frac{e^{-bx-a}}{c}+\sqrt{\frac{e^{-bx-a}}{c}-1}\,\sqrt{\frac{e^{-bx-a}}{c}+1}\right)^2\right)-\frac{\operatorname{polylog}\left(2,-\left(\frac{e^{-bx-a}}{c}\right.\right.}{}}{b}$
default	$\frac{\frac{\operatorname{arcsech}\left(e^{bx+a}c\right)^2}{2}-\operatorname{arcsech}\left(e^{bx+a}c\right)\ln\left(1+\left(\frac{e^{-bx-a}}{c}+\sqrt{\frac{e^{-bx-a}}{c}-1}\,\sqrt{\frac{e^{-bx-a}}{c}+1}\right)^2\right)-\frac{\operatorname{polylog}\left(2,-\left(\frac{e^{-bx-a}}{c}\right.\right.}{}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(exp(b*x+a)*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*arcsech(exp(b*x+a)*c)^2-arcsech(exp(b*x+a)*c)*ln(1+(1/exp(b*x+a)/c
+(1/exp(b*x+a)/c-1)^(1/2)*(1/exp(b*x+a)/c+1)^(1/2))^2)-1/2*polylog(2,-(1/ex
p(b*x+a)/c+(1/exp(b*x+a)/c-1)^(1/2)*(1/exp(b*x+a)/c+1)^(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(c*exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] b*c^2*integrate(x*e^(2*b*x + 2*a)/(c^2*e^(2*b*x + 2*a) + (c^2*e^(2*b*x + 2*
a) - 1)*e^(1/2*log(c*e^(b*x + a) + 1) + 1/2*log(-c*e^(b*x + a) + 1)) - 1),
x) - 1/2*b*x^2 - (a + log(c))*x + x*log(sqrt(c*e^(b*x + a) + 1)*sqrt(-c*e^(
b*x + a) + 1) + 1) - 1/2*(b*x*log(c*e^(b*x + a) + 1) + dilog(-c*e^(b*x + a)
))/b - 1/2*(b*x*log(-c*e^(b*x + a) + 1) + dilog(c*e^(b*x + a)))/b
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(c*exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asech}\left(ce^{a+bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(c*exp(b*x+a)),x)
```

```
[Out] Integral(asech(c*exp(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(c*exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arcsech(c*e^(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acosh}\left(\frac{\mathrm{e}^{-a-b\,x}}{c}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(exp(- a - b*x)/c),x)
```

```
[Out] int(acosh(exp(- a - b*x)/c), x)
```

3.32 $\int e^{\text{sech}^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=64

$$-\frac{2e^{\text{sech}^{-1}(ax)}x}{15a^4}+\frac{x^2}{15a^3}-\frac{e^{\text{sech}^{-1}(ax)}x^3}{15a^2}+\frac{x^4}{20a}+\frac{1}{5}e^{\text{sech}^{-1}(ax)}x^5$$

```
[Out] -2/15*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x/a^4+1/15*x^2/a^3-1/15*(1/a/
x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^3/a^2+1/20*x^4/a+1/5*(1/a/x+(1/a/x-1)^
(1/2)*(1+1/a/x)^(1/2))*x^5
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {6470, 30, 102, 12, 75}

$$-\frac{2\sqrt{1-ax}}{15a^5\sqrt{\frac{1}{ax+1}}}-\frac{x^2\sqrt{1-ax}}{15a^3\sqrt{\frac{1}{ax+1}}}+\frac{1}{5}x^5e^{\text{sech}^{-1}(ax)}+\frac{x^4}{20a}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcSech[a*x]*x^4,x]
```

```
[Out] x^4/(20*a) + (E^ArcSech[a*x]*x^5)/5 - (2*Sqrt[1 - a*x])/(15*a^5*Sqrt[(1 + a
*x)^(-1)]) - (x^2*Sqrt[1 - a*x])/(15*a^3*Sqrt[(1 + a*x)^(-1)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int e^{\operatorname{sech}^{-1}(ax)}x^4\,dx = \frac{1}{5}e^{\operatorname{sech}^{-1}(ax)}x^5 + \frac{\int x^3\,dx}{5a} + \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{x^3}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{5a}$$

$$= \frac{x^4}{20a} + \frac{1}{5}e^{\operatorname{sech}^{-1}(ax)}x^5 - \frac{x^2\sqrt{1-ax}}{15a^3\sqrt{\frac{1}{1+ax}}} - \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int -\frac{2x}{\sqrt{1-ax}\sqrt{1+\cdots}}}{15a^3}$$

$$= \frac{x^4}{20a} + \frac{1}{5}e^{\operatorname{sech}^{-1}(ax)}x^5 - \frac{x^2\sqrt{1-ax}}{15a^3\sqrt{\frac{1}{1+ax}}} + \frac{\left(2\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{x}{\sqrt{1-ax}\sqrt{1+\cdots}}}{15a^3}$$

$$= \frac{x^4}{20a} + \frac{1}{5}e^{\operatorname{sech}^{-1}(ax)}x^5 - \frac{2\sqrt{1-ax}}{15a^5\sqrt{\frac{1}{1+ax}}} - \frac{x^2\sqrt{1-ax}}{15a^3\sqrt{\frac{1}{1+ax}}}$$

Mathematica [A]
time = 0.06, size = 65, normalized size = 1.02

$$\frac{15a^4x^4 + 4\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^2\left(-2+2ax-3a^2x^2+3a^3x^3\right)}{60a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]*x^4,x]
```

```
[Out] (15*a^4*x^4 + 4*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-2 + 2*a*x - 3*a^2*x
^2 + 3*a^3*x^3))/(60*a^5)
```

Maple [A]
time = 0.03, size = 64, normalized size = 1.00

method	result	size
default	$\frac{\sqrt{\frac{ax+1}{ax}}\, x \sqrt{-\frac{ax-1}{ax}}\, (a^2x^2-1)(3a^2x^2+2)}{15a^4}+\frac{x^4}{4a}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*((a*x+1)/a/x)^(1/2)*x*(-(a*x-1)/a/x)^(1/2)*(a^2*x^2-1)*(3*a^2*x^2+2)/a
^4+1/4*x^4/a
```

Maxima [A]
time = 0.28, size = 47, normalized size = 0.73

$$\frac{x^4}{4\,a}+\frac{(3\,a^4x^4-a^2x^2-2)\sqrt{ax+1}\,\sqrt{-ax+1}}{15\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^4,x, algorithm="maxima"
)
```

```
[Out] 1/4*x^4/a + 1/15*(3*a^4*x^4 - a^2*x^2 - 2)*sqrt(a*x + 1)*sqrt(-a*x + 1)/a^5
```

Fricas [A]
time = 0.40, size = 65, normalized size = 1.02

$$\frac{15\,a^3x^4+4\,(3\,a^4x^5-a^2x^3-2\,x)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}}{60\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^4,x, algorithm="fricas"
)
```

```
[Out] 1/60*(15*a^3*x^4 + 4*(3*a^4*x^5 - a^2*x^3 - 2*x)*sqrt((a*x + 1)/(a*x))*sqrt
(-(a*x - 1)/(a*x)))/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int x^3\,dx+\int ax^4\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x**4,x)
```

```
[Out] (Integral(x**3, x) + Integral(a*x**4*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)),
x))/a
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.47, size = 75, normalized size = 1.17

$$\frac{x^4}{4\,a}-\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{2\,x\,\sqrt{\frac{1}{a\,x}+1}}{15\,a^4}-\frac{x^5\,\sqrt{\frac{1}{a\,x}+1}}{5}+\frac{x^3\,\sqrt{\frac{1}{a\,x}+1}}{15\,a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

```
[Out] x^4/(4*a) - (1/(a*x) - 1)^(1/2)*((2*x*(1/(a*x) + 1)^(1/2))/(15*a^4) - (x^5*
(1/(a*x) + 1)^(1/2))/5 + (x^3*(1/(a*x) + 1)^(1/2))/(15*a^2))
```

3.33 $\int e^{\text{sech}^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=84

$$\frac{x^3}{12a}+\frac{1}{4}e^{\text{sech}^{-1}(ax)}x^4-\frac{x\sqrt{1-ax}}{8a^3\sqrt{\frac{1}{1+ax}}}+\frac{\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\,\text{ArcSin}(ax)}{8a^4}$$

```
[Out] 1/12*x^3/a+1/4*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^4-1/8*x*(-a*x+1)^(
1/2)/a^3/(1/(a*x+1))^(1/2)+1/8*arcsin(a*x)*(1/(a*x+1))^(1/2)*(a*x+1)^(1/2)/
a^4
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {6470, 30, 92, 41, 222}

$$\frac{\sqrt{\frac{1}{ax+1}}\,\sqrt{ax+1}\,\text{ArcSin}(ax)}{8a^4}-\frac{x\sqrt{1-ax}}{8a^3\sqrt{\frac{1}{ax+1}}}+\frac{1}{4}x^4e^{\text{sech}^{-1}(ax)}+\frac{x^3}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x]*x^3,x]
```

```
[Out] x^3/(12*a) + (E^ArcSech[a*x]*x^4)/4 - (x*Sqrt[1 - a*x])/(8*a^3*Sqrt[(1 + a*
x)^(-1)]) + (Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x]*ArcSin[a*x])/(8*a^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
```

```
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$
\begin{aligned}
\int e^{\operatorname{sech}^{-1}(ax)}x^3\,dx &= \frac{1}{4}e^{\operatorname{sech}^{-1}(ax)}x^4+\frac{\int x^2\,dx}{4a}+\frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int\frac{x^2}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{4a}\\
&= \frac{x^3}{12a}+\frac{1}{4}e^{\operatorname{sech}^{-1}(ax)}x^4-\frac{x\sqrt{1-ax}}{8a^3\sqrt{\frac{1}{1+ax}}}+\frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int\frac{1}{\sqrt{1-ax}\sqrt{1+ax}}}{8a^3}\\
&= \frac{x^3}{12a}+\frac{1}{4}e^{\operatorname{sech}^{-1}(ax)}x^4-\frac{x\sqrt{1-ax}}{8a^3\sqrt{\frac{1}{1+ax}}}+\frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{8a^3}\\
&= \frac{x^3}{12a}+\frac{1}{4}e^{\operatorname{sech}^{-1}(ax)}x^4-\frac{x\sqrt{1-ax}}{8a^3\sqrt{\frac{1}{1+ax}}}+\frac{\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\sin^{-1}(ax)}{8a^4}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.09, size = 97, normalized size = 1.15

$$
\frac{8a^3x^3-3a\sqrt{\frac{1-ax}{1+ax}}\left(x+ax^2-2a^2x^3-2a^3x^4\right)+3i\log\left(-2iax+2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\right)}{24a^4}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]*x^3,x]
```

```
[Out] (8*a^3*x^3 - 3*a*Sqrt[(1 - a*x)/(1 + a*x)]*(x + a*x^2 - 2*a^2*x^3 - 2*a^3*x
^4) + (3*I)*Log[(-2*I)*a*x + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)])/(24*a^
4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 118, normalized size = 1.40

method	result
default	$\frac{\sqrt{\frac{ax+1}{ax}}\,x\sqrt{-\frac{ax-1}{ax}}\left(2\,\mathrm{csgn}(a)a^3x^3\sqrt{-a^2x^2+1}-x\sqrt{-a^2x^2+1}\,\mathrm{csgn}(a)a+\arctan\left(\frac{\mathrm{csgn}(a)ax}{\sqrt{-a^2x^2+1}}\right)\right)\mathrm{csgn}(a)}{8\sqrt{-a^2x^2+1}\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*((a*x+1)/a/x)^(1/2)*x*(-(a*x-1)/a/x)^(1/2)*(2*csgn(a)*a^3*x^3*(-a^2*x^2
+1)^(1/2)-x*(-a^2*x^2+1)^(1/2)*csgn(a)*a+arctan(csgn(a)*a*x/(-a^2*x^2+1)^(1
/2)))*csgn(a)/(-a^2*x^2+1)^(1/2)/a^3+1/3*x^3/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^3,x, algorithm="maxima"
)
```

```
[Out] 1/3*x^3/a + integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x^2, x)/a
```

Fricas [A]
time = 0.43, size = 95, normalized size = 1.13

$$\frac{8\,a^3x^3+3\left(2\,a^4x^4-a^2x^2\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-3\,\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^3,x, algorithm="fricas"
)
```

```
[Out] 1/24*(8*a^3*x^3 + 3*(2*a^4*x^4 - a^2*x^2)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x
- 1)/(a*x)) - 3*arctan(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))))/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int x^2\,dx+\int ax^3\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x**3,x)
```

```
[Out] (Integral(x**2, x) + Integral(a*x**3*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)),
x))/a
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 11.93, size = 521, normalized size = 6.20

$$\frac{\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1\right)1\mathrm{i}}{8\,a^4}-\frac{\frac{1\mathrm{i}}{1024\,a^4}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2 1\mathrm{i}}{128\,a^4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4 11\mathrm{i}}{512\,a^4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6 7\mathrm{i}}{256\,a^4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8 239\mathrm{i}}{1024\,a^4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{10} 1\mathrm{i}}{256\,a^4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{10}}}{\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{6\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}+\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{10}}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{12}}}-\frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)1\mathrm{i}}{8\,a^4}+\frac{x^3}{3\,a}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2 1\mathrm{i}}{256\,a^4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4 1\mathrm{i}}{1024\,a^4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

```
[Out] (log(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1)*1i)/(8*a
^4) - (1i/(1024*a^4) + (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(128*a^4*((1/(a*x)
 + 1)^(1/2) - 1)^2) + (((1/(a*x) - 1)^(1/2) - 1i)^4*11i)/(512*a^4*((1/(a*x)
 + 1)^(1/2) - 1)^4) + (((1/(a*x) - 1)^(1/2) - 1i)^6*7i)/(256*a^4*((1/(a*x)
+ 1)^(1/2) - 1)^6) - (((1/(a*x) - 1)^(1/2) - 1i)^8*239i)/(1024*a^4*((1/(a*x
) + 1)^(1/2) - 1)^8) + (((1/(a*x) - 1)^(1/2) - 1i)^10*1i)/(256*a^4*((1/(a*x
) + 1)^(1/2) - 1)^10))/((((1/(a*x) - 1)^(1/2) - 1i)^4/((1/(a*x) + 1)^(1/2) -
 1)^4 + (4*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (6*(
(1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8 + (4*((1/(a*x) - 1
```

```
)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1)^10 + ((1/(a*x) - 1)^(1/2) - 1i)
^12/((1/(a*x) + 1)^(1/2) - 1)^12) - (log(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*
x) + 1)^(1/2) - 1))*1i)/(8*a^4) + x^3/(3*a) - (((1/(a*x) - 1)^(1/2) - 1i)^2
*1i)/(256*a^4*((1/(a*x) + 1)^(1/2) - 1)^2) - (((1/(a*x) - 1)^(1/2) - 1i)^4*
1i)/(1024*a^4*((1/(a*x) + 1)^(1/2) - 1)^4)
```

3.34 $\int e^{\text{sech}^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=38

$$-\frac{e^{\text{sech}^{-1}(ax)}x}{3a^2}+\frac{x^2}{6a}+\frac{1}{3}e^{\text{sech}^{-1}(ax)}x^3$$

```
[Out] -1/3*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x/a^2+1/6*x^2/a+1/3*(1/a/x+(1/
a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^3
```

Rubi [A]
time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.37, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {6470, 30, 75}

$$-\frac{\sqrt{1-ax}}{3a^3\sqrt{\frac{1}{ax+1}}}+\frac{1}{3}x^3e^{\text{sech}^{-1}(ax)}+\frac{x^2}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcSech[a*x]*x^2,x]
```

```
[Out] x^2/(6*a) + (E^ArcSech[a*x]*x^3)/3 - Sqrt[1 - a*x]/(3*a^3*Sqrt[(1 + a*x)^(-
1)])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int e^{\operatorname{sech}^{-1}(ax)}x^2\,dx = \frac{1}{3}e^{\operatorname{sech}^{-1}(ax)}x^3 + \frac{\int x\,dx}{3a} + \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{x}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{3a}$$
$$= \frac{x^2}{6a} + \frac{1}{3}e^{\operatorname{sech}^{-1}(ax)}x^3 - \frac{\sqrt{1-ax}}{3a^3\sqrt{\frac{1}{1+ax}}}$$

Mathematica [A]
time = 0.05, size = 48, normalized size = 1.26

$$\frac{3a^2x^2 + 2(-1+ax)\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^2}{6a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]*x^2,x]
[Out] (3*a^2*x^2 + 2*(-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(6*a^3)
```

Maple [A]
time = 0.03, size = 54, normalized size = 1.42

method	result	size
default	$\frac{\sqrt{\frac{ax+1}{ax}}\,x\sqrt{-\frac{ax-1}{ax}}\,(a^2x^2-1)}{3a^2}+\frac{x^2}{2a}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^2,x,method=_RETURNVERBOSE)
[Out] 1/3*((a*x+1)/a/x)^(1/2)*x*(-(a*x-1)/a/x)^(1/2)*(a^2*x^2-1)/a^2+1/2*x^2/a
```

Maxima [A]
time = 0.27, size = 38, normalized size = 1.00

$$\frac{x^2}{2\,a} + \frac{(a^2x^2-1)\sqrt{ax+1}\,\sqrt{-ax+1}}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^2,x, algorithm="maxima"
)
```

```
[Out] 1/2*x^2/a + 1/3*(a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1)/a^3
```

Fricas [A]
time = 0.38, size = 54, normalized size = 1.42

$$\frac{3\,ax^2 + 2\,(a^2x^3 - x)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^2,x, algorithm="fricas"
)
```

```
[Out] 1/6*(3*a*x^2 + 2*(a^2*x^3 - x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))
)/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int x\,dx + \int ax^2\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x**2,x)
```

```
[Out] (Integral(x, x) + Integral(a*x**2*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)), x))
/a
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.43, size = 55, normalized size = 1.45

$$\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{x^3\,\sqrt{\frac{1}{a\,x}+1}}{3}-\frac{x\,\sqrt{\frac{1}{a\,x}+1}}{3\,a^2}\right)+\frac{x^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

```
[Out] (1/(a*x) - 1)^(1/2)*((x^3*(1/(a*x) + 1)^(1/2))/3 - (x*(1/(a*x) + 1)^(1/2))/
(3*a^2)) + x^2/(2*a)
```

3.35 $\int e^{\operatorname{sech}^{-1}(ax)} x \, dx$

Optimal. Leaf size=53

$$\frac{x}{2a}+\frac{1}{2}e^{\operatorname{sech}^{-1}(ax)}x^2+\frac{\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\operatorname{ArcSin}(ax)}{2a^2}$$

```
[Out] 1/2*x/a+1/2*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^2+1/2*arcsin(a*x)*(1/
(a*x+1))^(1/2)*(a*x+1)^(1/2)/a^2
```

Rubi [A]

time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6470, 8, 41, 222}

$$\frac{\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\operatorname{ArcSin}(ax)}{2a^2}+\frac{1}{2}x^2e^{\operatorname{sech}^{-1}(ax)}+\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x]*x,x]
```

```
[Out] x/(2*a) + (E^ArcSech[a*x]*x^2)/2 + (Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x]*ArcS
in[a*x])/(2*a^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
```

]

Rubi steps

$$\int e^{\text{sech}^{-1}(ax)}x\,dx = \frac{1}{2}e^{\text{sech}^{-1}(ax)}x^2 + \frac{\int 1\,dx}{2a} + \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2a}$$
$$= \frac{x}{2a} + \frac{1}{2}e^{\text{sech}^{-1}(ax)}x^2 + \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a}$$
$$= \frac{x}{2a} + \frac{1}{2}e^{\text{sech}^{-1}(ax)}x^2 + \frac{\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\,\sin^{-1}(ax)}{2a^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.05, size = 75, normalized size = 1.42

$$\frac{2ax + ax\sqrt{\frac{1-ax}{1+ax}}(1+ax) + i\log\left(-2iax + 2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]*x,x]
```

```
[Out] (2*a*x + a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) + I*Log[(-2*I)*a*x + 2*Sqr
t[(1 - a*x)/(1 + a*x)]*(1 + a*x)])/(2*a^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 92, normalized size = 1.74

method	result	size
default	$\frac{\sqrt{\frac{ax+1}{ax}}\,x\sqrt{-\frac{ax-1}{ax}}\left(x\sqrt{-a^2x^2+1}\,\text{csgn}(a)a+\arctan\left(\frac{\text{csgn}(a)ax}{\sqrt{-a^2x^2+1}}\right)\right)\text{csgn}(a)}{2\sqrt{-a^2x^2+1}\,a} + \frac{x}{a}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x+1)/a/x)^(1/2)*x*(-(a*x-1)/a/x)^(1/2)*(x*(-a^2*x^2+1)^(1/2)*csgn(a
)*a+arctan(csgn(a)*a*x/(-a^2*x^2+1)^(1/2)))/(-a^2*x^2+1)^(1/2)*csgn(a)/a+x/
a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x,x, algorithm="maxima")
[Out] x/a + integrate(sqrt(a*x + 1)*sqrt(-a*x + 1), x)/a
```

Fricas [A]
time = 0.39, size = 79, normalized size = 1.49

$$\frac{a^2x^2\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+2\,ax-\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x,x, algorithm="fricas")
[Out] 1/2*(a^2*x^2*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 2*a*x - arctan(
sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int 1\,dx+\int ax\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x,x)
[Out] (Integral(1, x) + Integral(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x,x, algorithm="giac")
[Out] integrate(x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

Mupad [B]
time = 6.96, size = 303, normalized size = 5.72

$$\frac{\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1\right)\,1\mathrm{i}}{2\,a^2}-\frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\,1\mathrm{i}}{2\,a^2}+\frac{\frac{1\mathrm{i}}{32\,a^2}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{16\,a^2\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4\,15\mathrm{i}}{32\,a^2\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}}{\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+\frac{2\,\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}}+\frac{x}{a}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{32\,a^2\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

```
[Out] (log(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1)*1i)/(2*a
^2) - (log(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i)/(2*a^2
) + (1i/(32*a^2) + (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(16*a^2*((1/(a*x) + 1)
^(1/2) - 1)^2) - (((1/(a*x) - 1)^(1/2) - 1i)^4*15i)/(32*a^2*((1/(a*x) + 1)^
(1/2) - 1)^4))/((((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 +
(2*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 + ((1/(a*x) -
1)^(1/2) - 1i)^6/((1/(a*x) + 1)^(1/2) - 1)^6) + x/a + (((1/(a*x) - 1)^(1/2)
 - 1i)^2*1i)/(32*a^2*((1/(a*x) + 1)^(1/2) - 1)^2)
```

3.36 $\int e^{\text{sech}^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$e^{\text{sech}^{-1}(ax)}x-\frac{\text{sech}^{-1}(ax)}{a}+\frac{\log(x)}{a}$$

```
[Out] (1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x-arcsech(a*x)/a+ln(x)/a
```

Rubi [A]
time = 0.08, antiderivative size = 39, normalized size of antiderivative = 1.62, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {6464, 1984, 214}

$$\frac{\log(x)}{a}-\frac{2\tanh^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{a}+xe^{\text{sech}^{-1}(ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcSech[a*x],x]
```

```
[Out] E^ArcSech[a*x]*x - (2*ArcTanh[Sqrt[(1 - a*x)/(1 + a*x)]])/a + Log[x]/a
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1984

```
Int[(u_)^(r_.)*(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)
*(x_)^(n_.)))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c
- a*d)/n), Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^
((m + 1)/n - 1)/(b*e - d*x^q)^((m + 1)/n + 1))*(u /. x -> ((-a)*e + c*x^q)^
(1/n)/(b*e - d*x^q)^(1/n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q
)], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] &
& IntegerQ[1/n] && IntegersQ[m, r]
```

Rule 6464

```
Int[E^ArcSech[(a_.)*(x_)], x_Symbol] :> Simp[x*E^ArcSech[a*x], x] + (Dist[1
/a, Int[(1/(x*(1 - a*x)))*Sqrt[(1 - a*x)/(1 + a*x)], x], x] + Simp[Log[x]/a
, x]) /; FreeQ[a, x]
```

Rubi steps

$$\int e^{\operatorname{sech}^{-1}(ax)}\,dx = e^{\operatorname{sech}^{-1}(ax)}x + \frac{\log(x)}{a} + \frac{\int \frac{\sqrt{\frac{1-ax}{1+ax}}}{x(1-ax)}\,dx}{a}$$
$$= e^{\operatorname{sech}^{-1}(ax)}x + \frac{\log(x)}{a} - 4\text{Subst}\left(\int \frac{1}{2a-2ax^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)$$
$$= e^{\operatorname{sech}^{-1}(ax)}x - \frac{2\tanh^{-1}\left(\sqrt{\frac{1-ax}{1+ax}}\right)}{a} + \frac{\log(x)}{a}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 79 vs. 2(24) = 48.
time = 0.03, size = 79, normalized size = 3.29

$$\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax) + 2\log(ax) - \log\left(1+\sqrt{\frac{1-ax}{1+ax}} + ax\sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x],x]
```

```
[Out] (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) + 2*Log[a*x] - Log[1 + Sqrt[(1 - a*x)/
(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/a
```

Maple [**A**]
time = 0.06, size = 80, normalized size = 3.33

method	result	size
default	$\frac{\ln(x)}{a} - \frac{\sqrt{\frac{ax+1}{ax}}\,x\sqrt{-\frac{ax-1}{ax}}\left(-\sqrt{-a^2x^2+1}+\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)}{\sqrt{-a^2x^2+1}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)/a-((a*x+1)/a/x)^(1/2)*x*(-(a*x-1)/a/x)^(1/2)*(-(-a^2*x^2+1)^(1/2)+arc
tanh(1/(-a^2*x^2+1)^(1/2)))/(-a^2*x^2+1)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(49) = 98$.
time = 0.39, size = 115, normalized size = 4.79

$$\frac{2\,ax\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}} - \log\left(ax\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}+1\right) + \log\left(ax\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}-1\right) + 2\,\log\left(x\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - log(a*x*sqrt((a*x
 + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) + log(a*x*sqrt((a*x + 1)/(a*x))*sq
rt(-(a*x - 1)/(a*x)) - 1) + 2*log(x))/a
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x}\,dx + \int a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2),x)
```

```
[Out] (Integral(1/x, x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)), x))/a
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x), x)
```

Mupad [**B**]

time = 2.98, size = 182, normalized size = 7.58

$$\frac{\ln\left(x\right)}{a}-\frac{4\,\mathrm{atanh}\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)}{a}+\frac{\frac{5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1}{\frac{4\,a\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}{\sqrt{\frac{1}{a\,x}+1}-1}+\frac{4\,a\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^3}}+\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{4\,a\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x),x)
```

```
[Out] log(x)/a - (4*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)))/
a + ((5*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 + 1)/((4*
a*((1/(a*x) - 1)^(1/2) - 1i))/((1/(a*x) + 1)^(1/2) - 1) + (4*a*((1/(a*x) -
1)^(1/2) - 1i)^3)/((1/(a*x) + 1)^(1/2) - 1)^3) + ((1/(a*x) - 1)^(1/2) - 1i)
/(4*a*((1/(a*x) + 1)^(1/2) - 1))
```

3.37 $\int \frac{e^{\text{sech}^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=48

$$-\frac{2}{1-\sqrt{\frac{1-ax}{1+ax}}}+2\text{ArcTan}\left(\sqrt{\frac{1-ax}{1+ax}}\right)$$

```
[Out] 2*arctan(((-a*x+1)/(a*x+1))^(1/2))-2/(1-((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6469, 99, 12, 41, 222}

$$-\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\,\text{ArcSin}(ax)-\frac{\sqrt{1-ax}}{ax\sqrt{\frac{1}{ax+1}}}-\frac{1}{ax}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcSech[a*x]/x,x]
```

```
[Out] -(1/(a*x)) - Sqrt[1 - a*x]/(a*x*Sqrt[(1 + a*x)^(-1)]) - Sqrt[(1 + a*x)^(-1)
]*Sqrt[1 + a*x]*ArcSin[a*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6469

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]/(x_), x_Symbol] :> -Simp[(a*p*x^p)^(-1), x]
 + Dist[(Sqrt[1 + a*x^p]/a)*Sqrt[1/(1 + a*x^p)], Int[Sqrt[1 + a*x^p]*(Sqrt[
1 - a*x^p]/x^(p + 1)), x], x] /; FreeQ[{a, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x}\,dx &= -\frac{1}{ax}+\frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{\sqrt{1-ax}\sqrt{1+ax}}{x^2}\,dx}{a}\\
&= -\frac{1}{ax}-\frac{\sqrt{1-ax}}{ax\sqrt{\frac{1}{1+ax}}}-\frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{a^2}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a}\\
&= -\frac{1}{ax}-\frac{\sqrt{1-ax}}{ax\sqrt{\frac{1}{1+ax}}}-\left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}}\,dx\\
&= -\frac{1}{ax}-\frac{\sqrt{1-ax}}{ax\sqrt{\frac{1}{1+ax}}}-\left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{1}{ax}-\frac{\sqrt{1-ax}}{ax\sqrt{\frac{1}{1+ax}}}-\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\,\sin^{-1}(ax)
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 75, normalized size = 1.56

$$-\frac{1}{ax}+\left(-1-\frac{1}{ax}\right)\sqrt{\frac{1-ax}{1+ax}}-i\log\left(-2iax+2\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]/x,x]
```

```
[Out] -(1/(a*x)) + (-1 - 1/(a*x))*Sqrt[(1 - a*x)/(1 + a*x)] - I*Log[(-2*I)*a*x +
2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 92, normalized size = 1.92

method	result	size
default	$-\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,\left(\arctan\left(\frac{\operatorname{csgn}(a)ax}{\sqrt{-a^2x^2+1}}\right)ax+\operatorname{csgn}(a)\sqrt{-a^2x^2+1}\right)\operatorname{csgn}(a)}{\sqrt{-a^2x^2+1}}-\frac{1}{ax}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x,method=_RETURNVERBOSE)
[Out] -((a*x+1)/a/x)^(1/2)*(-(a*x-1)/a/x)^(1/2)*(arctan(csgn(a)*a*x/(-a^2*x^2+1)^
(1/2))*a*x+csgn(a)*(-a^2*x^2+1)^(1/2))*csgn(a)/(-a^2*x^2+1)^(1/2)-1/a/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="maxima")
[Out] integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^2, x)/a - 1/(a*x)
```

Fricas [A]
time = 0.40, size = 77, normalized size = 1.60

$$-\frac{ax\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}-ax\arctan\left(\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\right)+1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="fricas")
[Out] -(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - a*x*arctan(sqrt((a*x +
 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))) + 1)/(a*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^2}\,dx+\int \frac{a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x,x)
```

```
[Out] (Integral(x**(-2), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x,
x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x, x)
```

Mupad [B]
time = 3.10, size = 184, normalized size = 3.83

$$-\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1\right)\,1\mathrm{i}+\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\,1\mathrm{i}-\frac{1}{a\,x}+\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^2\,8\mathrm{i}}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^2\,\left(1+\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^4}-\frac{2\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x,x)
```

```
[Out] log(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i - log(((1/(a*x
) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1)*1i - 1/(a*x) + (((1/(
a*x) - 1)^(1/2) - 1i)^2*8i)/(((1/(a*x) + 1)^(1/2) - 1)^2*(((1/(a*x) - 1)^(1
/2) - 1i)^4/((1/(a*x) + 1)^(1/2) - 1)^4 - (2*((1/(a*x) - 1)^(1/2) - 1i)^2)/
((1/(a*x) + 1)^(1/2) - 1)^2 + 1))
```

3.38 $\int \frac{e^{\text{sech}^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=35

$$-\frac{e^{\text{sech}^{-1}(ax)}}{2x}+a\tanh^{-1}\left(\sqrt{\frac{1-ax}{1+ax}}\right)$$

```
[Out] -1/2*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x+a*arctanh(((-a*x+1)/(a*x+1))
^(1/2))
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 99 vs. $2(35) = 70$.
time = 0.03, antiderivative size = 99, normalized size of antiderivative = 2.83, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6470, 30, 105, 12, 94, 214}

$$\frac{\sqrt{1-ax}}{2ax^2\sqrt{\frac{1}{ax+1}}}+\frac{1}{2ax^2}+\frac{1}{2}a\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)-\frac{e^{\text{sech}^{-1}(ax)}}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcSech[a*x]/x^2,x]
```

```
[Out] 1/(2*a*x^2) - E^ArcSech[a*x]/x + Sqrt[1 - a*x]/(2*a*x^2*Sqrt[(1 + a*x)^(-1)
]) + (a*Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x]*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a
*x]])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^2}\,dx = -\frac{e^{\operatorname{sech}^{-1}(ax)}}{x} - \frac{\int \frac{1}{x^3}\,dx}{a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int \frac{1}{x^3\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{a}$$

$$= \frac{1}{2ax^2} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{x} + \frac{\sqrt{1-ax}}{2ax^2\sqrt{\frac{1}{1+ax}}} - \frac{\left(\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int \frac{a^2}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{2a}$$

$$= \frac{1}{2ax^2} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{x} + \frac{\sqrt{1-ax}}{2ax^2\sqrt{\frac{1}{1+ax}}} - \frac{1}{2}\left(a\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int \frac{1}{x\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{1}{2ax^2} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{x} + \frac{\sqrt{1-ax}}{2ax^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{2}\left(a^2\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\operatorname{Subst}\left(\int \frac{1}{a-ax^2}\,dx,\right.$$

$$= \frac{1}{2ax^2} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{x} + \frac{\sqrt{1-ax}}{2ax^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{2}a\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 93 vs. 2(35) = 70.
time = 0.04, size = 93, normalized size = 2.66

$$\frac{1}{2}\left(-\frac{1}{ax^2}-\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{ax^2}-a\log(x)+a\log\left(1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]/x^2,x]
```

```
[Out] (-(1/(a*x^2)) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(a*x^2) - a*Log[x] +
a*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/2
```

Maple [A]
time = 0.03, size = 91, normalized size = 2.60

method	result	size
default	$\frac{\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\left(a^2x^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\sqrt{-a^2x^2+1}\right)}{2x\sqrt{-a^2x^2+1}}-\frac{1}{2a\,x^2}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x+1)/a/x)^(1/2)/x*(-(a*x-1)/a/x)^(1/2)*(a^2*x^2*arctanh(1/(-a^2*x^2
+1)^(1/2))-(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2)-1/2/a/x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^3, x)/a - 1/2/(a*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(56) = 112.
time = 0.41, size = 128, normalized size = 3.66

$$\frac{a^2x^2\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-a^2x^2\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)-2\,ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-2}{4\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="fricas"
)
```

```
[Out] 1/4*(a^2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - a^
2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) - 2*a*x*sqr
t((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 2)/(a*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^3}\,dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**2,x)
```

```
[Out] (Integral(x**(-3), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**
2, x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^2, x)
```

Mupad [B]
time = 1.84, size = 71, normalized size = 2.03

$$\frac{a\,\ln\left(\sqrt{\frac{1}{a\,x}-1}\,\sqrt{\frac{1}{a\,x}+1}+\frac{1}{a\,x}\right)}{2}-\frac{1}{2\,a\,x^2}-\frac{\sqrt{\frac{1}{a\,x}-1}\,\sqrt{\frac{1}{a\,x}+1}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^2,x)
```

```
[Out] (a*log((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)))/2 - 1/(2*a*x^2)
- ((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2))/(2*x)
```

3.39 $\int \frac{e^{\text{sech}^{-1}(ax)}}{x^3} \, dx$

Optimal. Leaf size=55

$$-\frac{1}{3ax^3}-\frac{8a^2\left(\frac{1-ax}{1+ax}\right)^{3/2}}{3\left(1-\frac{1-ax}{1+ax}\right)^3}$$

```
[Out] -1/3/a/x^3-8/3*a^2*((-a*x+1)/(a*x+1))^(3/2)/(1+(a*x-1)/(a*x+1))^3
```

Rubi [C] Result contains higher order function than in optimal. Order 3 vs. order 2 in optimal.
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.53, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6470, 30, 105, 12, 97}

$$\frac{\sqrt{1-ax}}{6ax^3\sqrt{\frac{1}{ax+1}}}+\frac{1}{6ax^3}-\frac{e^{\text{sech}^{-1}(ax)}}{2x^2}+\frac{a\sqrt{1-ax}}{3x\sqrt{\frac{1}{ax+1}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcSech[a*x]/x^3,x]
```

```
[Out] 1/(6*a*x^3) - E^ArcSech[a*x]/(2*x^2) + Sqrt[1 - a*x]/(6*a*x^3*Sqrt[(1 + a*x
)^(-1)]) + (a*Sqrt[1 - a*x])/(3*x*Sqrt[(1 + a*x)^(-1)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^3}\,dx = -\frac{e^{\operatorname{sech}^{-1}(ax)}}{2x^2} - \frac{\int \frac{1}{x^4}\,dx}{2a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{x^4\sqrt{1-ax}\sqrt{1+ax}}\,dx}{2a}$$

$$= \frac{1}{6ax^3} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{2x^2} + \frac{\sqrt{1-ax}}{6ax^3\sqrt{\frac{1}{1+ax}}} + \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int -\frac{2a^2}{x^2\sqrt{1-ax}\sqrt{1+ax}}\,dx}{6a}$$

$$= \frac{1}{6ax^3} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{2x^2} + \frac{\sqrt{1-ax}}{6ax^3\sqrt{\frac{1}{1+ax}}} - \frac{1}{3}\left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{x^2\sqrt{1-ax}\sqrt{1+ax}}\,dx$$

$$= \frac{1}{6ax^3} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{2x^2} + \frac{\sqrt{1-ax}}{6ax^3\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{3x\sqrt{\frac{1}{1+ax}}}$$

Mathematica [A]
time = 0.03, size = 43, normalized size = 0.78

$$\frac{-1+(-1+ax)\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^2}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSech[a*x]/x^3,x]
```

```
[Out] (-1 + (-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(3*a*x^3)
```

Maple [A]
time = 0.03, size = 53, normalized size = 0.96

method	result	size
default	$\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,(a^2x^2-1)}{3x^2}-\frac{1}{3a\,x^3}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((a*x+1)/a/x)^(1/2)/x^2*(-(a*x-1)/a/x)^(1/2)*(a^2*x^2-1)-1/3/a/x^3
```

Maxima [A]
time = 0.27, size = 43, normalized size = 0.78

$$\frac{(a^2x^3-x)\sqrt{ax+1}\,\sqrt{-ax+1}}{3\,ax^4}-\frac{1}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="maxima"
)
```

```
[Out] 1/3*(a^2*x^3 - x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a*x^4) - 1/3/(a*x^3)
```

Fricas [A]
time = 0.42, size = 52, normalized size = 0.95

$$\frac{(a^3x^3-ax)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}-1}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="fricas"
)
```

```
[Out] 1/3*((a^3*x^3 - a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1)/(a*x
^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^4}\,dx+\int \frac{a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^3}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**3,x)
```

```
[Out] (Integral(x**(-4), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**
3, x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^3, x)
```

Mupad [B]
time = 1.47, size = 58, normalized size = 1.05

$$-\frac{1}{3\,a\,x^3}-\frac{\left(\frac{\sqrt{\frac{1}{a\,x}+1}}{3}-\frac{a^2\,x^2\,\sqrt{\frac{1}{a\,x}+1}}{3}\right)\,\sqrt{\frac{1}{a\,x}-1}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^3,x)
```

```
[Out] - 1/(3*a*x^3) - (((1/(a*x) + 1)^(1/2)/3 - (a^2*x^2*(1/(a*x) + 1)^(1/2))/3)*
(1/(a*x) - 1)^(1/2))/x^2
```

3.40 $\int \frac{e^{\text{sech}^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=132

$$\frac{1}{12ax^4}-\frac{e^{\text{sech}^{-1}(ax)}}{3x^3}+\frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{1+ax}}}+\frac{a\sqrt{1-ax}}{8x^2\sqrt{\frac{1}{1+ax}}}+\frac{1}{8}a^3\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right.$$

```
[Out] 1/12/a/x^4-1/3*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3+1/12*(-a*x+1)^(1
/2)/a/x^4/(1/(a*x+1))^(1/2)+1/8*a*(-a*x+1)^(1/2)/x^2/(1/(a*x+1))^(1/2)+1/8*
a^3*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(1/(a*x+1))^(1/2)*(a*x+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6470, 30, 105, 12, 94, 214}

$$\frac{1}{8}a^3\sqrt{\frac{1}{ax+1}}\,\sqrt{ax+1}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)+\frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{ax+1}}}+\frac{1}{12ax^4}-\frac{e^{\text{sech}^{-1}(ax)}}{3x^3}+\frac{a\sqrt{1-ax}}{8x^2\sqrt{\frac{1}{ax+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x]/x^4,x]
```

```
[Out] 1/(12*a*x^4) - E^ArcSech[a*x]/(3*x^3) + Sqrt[1 - a*x]/(12*a*x^4*Sqrt[(1 + a
*x)^(-1)]) + (a*Sqrt[1 - a*x])/(8*x^2*Sqrt[(1 + a*x)^(-1)]) + (a^3*Sqrt[(1
+ a*x)^(-1)]*Sqrt[1 + a*x]*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^4}\,dx = -\frac{e^{\text{sech}^{-1}(ax)}}{3x^3} - \frac{\int \frac{1}{x^5}\,dx}{3a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{x^5\sqrt{1-ax}\sqrt{1+ax}}\,dx}{3a}$$

$$= \frac{1}{12ax^4} - \frac{e^{\text{sech}^{-1}(ax)}}{3x^3} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{1+ax}}} + \frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int -\frac{3a^2}{x^3\sqrt{1-ax}\sqrt{1+}}}{12a}$$

$$= \frac{1}{12ax^4} - \frac{e^{\text{sech}^{-1}(ax)}}{3x^3} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{1+ax}}} - \frac{1}{4}\left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{1}{x^3\sqrt{1-ax}\sqrt{1+}}$$

$$= \frac{1}{12ax^4} - \frac{e^{\text{sech}^{-1}(ax)}}{3x^3} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{8x^2\sqrt{\frac{1}{1+ax}}} - \frac{1}{8}\left(a\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int$$

$$= \frac{1}{12ax^4} - \frac{e^{\text{sech}^{-1}(ax)}}{3x^3} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{8x^2\sqrt{\frac{1}{1+ax}}} - \frac{1}{8}\left(a^3\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)$$

$$= \frac{1}{12ax^4} - \frac{e^{\text{sech}^{-1}(ax)}}{3x^3} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{8x^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{8}\left(a^4\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)$$

$$= \frac{1}{12ax^4} - \frac{e^{\text{sech}^{-1}(ax)}}{3x^3} + \frac{\sqrt{1-ax}}{12ax^4\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{8x^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{8}a^3\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\,\tanh$$

Mathematica [A]
time = 0.05, size = 110, normalized size = 0.83

$$\frac{-2+\sqrt{\frac{1-ax}{1+ax}}\left(-2-2ax+a^2x^2+a^3x^3\right)-a^4x^4\log(x)+a^4x^4\log\left(1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)}{8ax^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]/x^4,x]
```

```
[Out] (-2 + Sqrt[(1 - a*x)/(1 + a*x)]*(-2 - 2*a*x + a^2*x^2 + a^3*x^3) - a^4*x^4*
Log[x] + a^4*x^4*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1
+ a*x)]])/(8*a*x^4)
```

Maple [A]
time = 0.03, size = 110, normalized size = 0.83

method	result	size
default	$\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)a^4x^4+a^2x^2\sqrt{-a^2x^2+1}-2\sqrt{-a^2x^2+1}\right)}{8x^3\sqrt{-a^2x^2+1}}-\frac{1}{4a\,x^4}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*((a*x+1)/a/x)^(1/2)/x^3*(-(a*x-1)/a/x)^(1/2)*(arctanh(1/(-a^2*x^2+1)^(1
/2))*a^4*x^4+a^2*x^2*(-a^2*x^2+1)^(1/2)-2*(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^
(1/2)-1/4/a/x^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^5, x)/a - 1/4/(a*x^4)
```

Fricas [A]
time = 0.47, size = 138, normalized size = 1.05

$$\frac{a^4x^4\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-a^4x^4\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)+2\left(a^3x^3-2\,ax\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-4}{16\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="fricas"
)
```

```
[Out] 1/16*(a^4*x^4*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - a
^4*x^4*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 2*(a^3*x
^3 - 2*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 4)/(a*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^5}\,dx+\int \frac{a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^4}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**4,x)
```

```
[Out] (Integral(x**(-5), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**
4, x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^4, x)
```

Mupad [B]
time = 13.42, size = 602, normalized size = 4.56

$$\frac{a^3\,\mathrm{atanh}\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)}{2}-\frac{\frac{35\,a^3\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^3}{2\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^3}+\frac{273\,a^3\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^5}{2\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^5}+\frac{715\,a^3\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^7}{2\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^7}+\frac{715\,a^3\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^9}{2\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^9}+\frac{273\,a^3\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^{11}}{2\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^{11}}+\frac{35\,a^3\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^{13}}{2\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^{13}}+\frac{a^3\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^{15}}{2\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^{15}}+\frac{a^3\,\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}{2\,\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}}{1+\frac{28\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^4}-\frac{56\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^6}+\frac{70\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^8}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^8}-\frac{56\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^{10}}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^{10}}+\frac{28\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^{12}}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^{12}}-\frac{8\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^{14}}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^{14}}+\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^{16}}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^{16}}-\frac{8\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^2}}-\frac{1}{4\,a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^4,x)
```

```
[Out] (a^3*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)))/2 - ((35*
a^3*((1/(a*x) - 1)^(1/2) - 1i)^3)/(2*((1/(a*x) + 1)^(1/2) - 1)^3) + (273*a^
3*((1/(a*x) - 1)^(1/2) - 1i)^5)/(2*((1/(a*x) + 1)^(1/2) - 1)^5) + (715*a^3*
((1/(a*x) - 1)^(1/2) - 1i)^7)/(2*((1/(a*x) + 1)^(1/2) - 1)^7) + (715*a^3*((
1/(a*x) - 1)^(1/2) - 1i)^9)/(2*((1/(a*x) + 1)^(1/2) - 1)^9) + (273*a^3*((1/
(a*x) - 1)^(1/2) - 1i)^11)/(2*((1/(a*x) + 1)^(1/2) - 1)^11) + (35*a^3*((1/(
a*x) - 1)^(1/2) - 1i)^13)/(2*((1/(a*x) + 1)^(1/2) - 1)^13) + (a^3*((1/(a*x)
 - 1)^(1/2) - 1i)^15)/(2*((1/(a*x) + 1)^(1/2) - 1)^15) + (a^3*((1/(a*x) - 1
)^(1/2) - 1i))/(2*((1/(a*x) + 1)^(1/2) - 1)))/((28*((1/(a*x) - 1)^(1/2) - 1
i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (8*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a
*x) + 1)^(1/2) - 1)^2 - (56*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1
/2) - 1)^6 + (70*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8
- (56*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1)^10 + (28*((1
/(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x) + 1)^(1/2) - 1)^12 - (8*((1/(a*x) - 1
)^(1/2) - 1i)^14)/((1/(a*x) + 1)^(1/2) - 1)^14 + ((1/(a*x) - 1)^(1/2) - 1i)
^16/((1/(a*x) + 1)^(1/2) - 1)^16 + 1) - 1/(4*a*x^4)
```

3.41 $\int \frac{e^{\mathbf{sech}^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=115

$$\frac{1}{20ax^5}-\frac{e^{\operatorname{sech}^{-1}(ax)}}{4x^4}+\frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{1+ax}}}+\frac{a\sqrt{1-ax}}{15x^3\sqrt{\frac{1}{1+ax}}}+\frac{2a^3\sqrt{1-ax}}{15x\sqrt{\frac{1}{1+ax}}}$$

```
[Out] 1/20/a/x^5-1/4*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4+1/20*(-a*x+1)^(1
/2)/a/x^5/(1/(a*x+1))^(1/2)+1/15*a*(-a*x+1)^(1/2)/x^3/(1/(a*x+1))^(1/2)+2/1
5*a^3*(-a*x+1)^(1/2)/x/(1/(a*x+1))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6470, 30, 105, 12, 97}

$$\frac{2a^3\sqrt{1-ax}}{15x\sqrt{\frac{1}{ax+1}}}+\frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{ax+1}}}+\frac{1}{20ax^5}-\frac{e^{\operatorname{sech}^{-1}(ax)}}{4x^4}+\frac{a\sqrt{1-ax}}{15x^3\sqrt{\frac{1}{ax+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x]/x^5,x]
```

```
[Out] 1/(20*a*x^5) - E^ArcSech[a*x]/(4*x^4) + Sqrt[1 - a*x]/(20*a*x^5*Sqrt[(1 + a
*x)^(-1)]) + (a*Sqrt[1 - a*x])/(15*x^3*Sqrt[(1 + a*x)^(-1)]) + (2*a^3*Sqrt[
1 - a*x])/(15*x*Sqrt[(1 + a*x)^(-1)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{e^{\text{sech}^{-1}(ax)}}{x^5}\,dx = -\frac{e^{\text{sech}^{-1}(ax)}}{4x^4} - \frac{\int \frac{1}{x^6}\,dx}{4a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int \frac{1}{x^6\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{4a}$$

$$= \frac{1}{20ax^5} - \frac{e^{\text{sech}^{-1}(ax)}}{4x^4} + \frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{1+ax}}} + \frac{\left(\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int -\frac{4a^2}{x^4\sqrt{1-ax}\,\sqrt{1+}}}{20a}$$

$$= \frac{1}{20ax^5} - \frac{e^{\text{sech}^{-1}(ax)}}{4x^4} + \frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{1+ax}}} - \frac{1}{5}\left(a\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int \frac{1}{x^4\sqrt{1-ax}\,\sqrt{1}}$$

$$= \frac{1}{20ax^5} - \frac{e^{\text{sech}^{-1}(ax)}}{4x^4} + \frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{15x^3\sqrt{\frac{1}{1+ax}}} + \frac{1}{15}\left(a\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)$$

$$= \frac{1}{20ax^5} - \frac{e^{\text{sech}^{-1}(ax)}}{4x^4} + \frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{15x^3\sqrt{\frac{1}{1+ax}}} - \frac{1}{15}\left(2a^3\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right.$$

$$= \frac{1}{20ax^5} - \frac{e^{\text{sech}^{-1}(ax)}}{4x^4} + \frac{\sqrt{1-ax}}{20ax^5\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{15x^3\sqrt{\frac{1}{1+ax}}} + \frac{2a^3\sqrt{1-ax}}{15x\sqrt{\frac{1}{1+ax}}}$$

Mathematica [A]
time = 0.05, size = 60, normalized size = 0.52

$$\frac{-3+\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^2\,(-3+3ax-2a^2x^2+2a^3x^3)}{15ax^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]/x^5,x]
```

```
[Out] (-3 + Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-3 + 3*a*x - 2*a^2*x^2 + 2*a^3
*x^3))/(15*a*x^5)
```

Maple [A]
time = 0.03, size = 63, normalized size = 0.55

method	result	size
default	$\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,(a^2x^2-1)\,(2a^2x^2+3)}{15x^4}-\frac{1}{5a\,x^5}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*((a*x+1)/a/x)^(1/2)/x^4*(-(a*x-1)/a/x)^(1/2)*(a^2*x^2-1)*(2*a^2*x^2+3)
-1/5/a/x^5
```

Maxima [A]
time = 0.28, size = 51, normalized size = 0.44

$$\frac{(2\,a^4x^5+a^2x^3-3\,x)\sqrt{ax+1}\,\sqrt{-ax+1}}{15\,ax^6}-\frac{1}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="maxima"
)
```

```
[Out] 1/15*(2*a^4*x^5 + a^2*x^3 - 3*x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a*x^6) - 1/5
/(a*x^5)
```

Fricas [A]
time = 0.36, size = 60, normalized size = 0.52

$$\frac{(2\,a^5x^5+a^3x^3-3\,ax)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}-3}{15\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="fricas"
)
```

```
[Out] 1/15*((2*a^5*x^5 + a^3*x^3 - 3*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(
a*x)) - 3)/(a*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^6}\,dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^5}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**5,x)
```

```
[Out] (Integral(x**(-6), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**
5, x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^5, x)
```

Mupad [B]
time = 1.56, size = 76, normalized size = 0.66

$$\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{a^2\,x^2\,\sqrt{\frac{1}{a\,x}+1}}{15}-\frac{\sqrt{\frac{1}{a\,x}+1}}{5}+\frac{2\,a^4\,x^4\,\sqrt{\frac{1}{a\,x}+1}}{15}\right)}{x^4}-\frac{1}{5\,a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^5,x)
```

```
[Out] ((1/(a*x) - 1)^(1/2)*((a^2*x^2*(1/(a*x) + 1)^(1/2))/15 - (1/(a*x) + 1)^(1/2
)/5 + (2*a^4*x^4*(1/(a*x) + 1)^(1/2))/15))/x^4 - 1/(5*a*x^5)
```

3.42 $\int \frac{e^{\text{sech}^{-1}(ax)}}{x^6}\,dx$

Optimal. Leaf size=163

$$\frac{1}{30ax^6}-\frac{e^{\text{sech}^{-1}(ax)}}{5x^5}+\frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}}+\frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{1+ax}}}+\frac{a^3\sqrt{1-ax}}{16x^2\sqrt{\frac{1}{1+ax}}}+\frac{1}{16}a^5\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\tanh^{-1}\left(\right.$$

```
[Out] 1/30/a/x^6-1/5*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5+1/30*(-a*x+1)^(1
/2)/a/x^6/(1/(a*x+1))^(1/2)+1/24*a*(-a*x+1)^(1/2)/x^4/(1/(a*x+1))^(1/2)+1/1
6*a^3*(-a*x+1)^(1/2)/x^2/(1/(a*x+1))^(1/2)+1/16*a^5*arctanh((-a*x+1)^(1/2)*
(a*x+1)^(1/2))*(1/(a*x+1))^(1/2)*(a*x+1)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6470, 30, 105, 12, 94, 214}

$$\frac{1}{16}a^5\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)+\frac{a^3\sqrt{1-ax}}{16x^2\sqrt{\frac{1}{ax+1}}}+\frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{ax+1}}}+\frac{1}{30ax^6}-\frac{e^{\text{sech}^{-1}(ax)}}{5x^5}+\frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{ax+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x]/x^6,x]
```

```
[Out] 1/(30*a*x^6) - E^ArcSech[a*x]/(5*x^5) + Sqrt[1 - a*x]/(30*a*x^6*Sqrt[(1 + a
*x)^(-1)]) + (a*Sqrt[1 - a*x])/(24*x^4*Sqrt[(1 + a*x)^(-1)]) + (a^3*Sqrt[1
- a*x])/(16*x^2*Sqrt[(1 + a*x)^(-1)]) + (a^5*Sqrt[(1 + a*x)^(-1)]*Sqrt[1 +
a*x]*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/16
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^6}\,dx = -\frac{e^{\operatorname{sech}^{-1}(ax)}}{5x^5} - \frac{\int \frac{1}{x^7}\,dx}{5a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int \frac{1}{x^7\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{5a}$$

$$= \frac{1}{30ax^6} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{\left(\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int -\frac{5a^2}{x^5\sqrt{1-ax}\,\sqrt{1+a}}}{30a}$$

$$= \frac{1}{30ax^6} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} - \frac{1}{6}\left(a\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int \frac{1}{x^5\sqrt{1-ax}\,\sqrt{1+}}$$

$$= \frac{1}{30ax^6} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{1+ax}}} + \frac{1}{24}\left(a\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int$$

$$= \frac{1}{30ax^6} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{1+ax}}} - \frac{1}{8}\left(a^3\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int$$

$$= \frac{1}{30ax^6} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{1+ax}}} + \frac{a^3\sqrt{1-ax}}{16x^2\sqrt{\frac{1}{1+ax}}} - \frac{1}{16}\left(a^3\sqrt{\frac{}{1}}\right.$$

$$= \frac{1}{30ax^6} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{1+ax}}} + \frac{a^3\sqrt{1-ax}}{16x^2\sqrt{\frac{1}{1+ax}}} - \frac{1}{16}\left(a^5\sqrt{\frac{}{1}}\right.$$

$$= \frac{1}{30ax^6} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{1+ax}}} + \frac{a^3\sqrt{1-ax}}{16x^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{16}\left(a^6\sqrt{\frac{}{1}}\right.$$

$$= \frac{1}{30ax^6} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{5x^5} + \frac{\sqrt{1-ax}}{30ax^6\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{24x^4\sqrt{\frac{1}{1+ax}}} + \frac{a^3\sqrt{1-ax}}{16x^2\sqrt{\frac{1}{1+ax}}} + \frac{1}{16}a^5\sqrt{\frac{}{1+}}$$

Mathematica [A]
time = 0.08, size = 129, normalized size = 0.79

$$\frac{-8+\sqrt{\frac{1-ax}{1+ax}}\left(-8-8ax+2a^2x^2+2a^3x^3+3a^4x^4+3a^5x^5\right)-3a^6x^6\log(x)+3a^6x^6\log\left(1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)}{48ax^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]/x^6,x]
```

```
[Out] (-8 + Sqrt[(1 - a*x)/(1 + a*x)]*(-8 - 8*a*x + 2*a^2*x^2 + 2*a^3*x^3 + 3*a^4
*x^4 + 3*a^5*x^5) - 3*a^6*x^6*Log[x] + 3*a^6*x^6*Log[1 + Sqrt[(1 - a*x)/(1
+ a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/(48*a*x^6)
```

Maple [A]
time = 0.04, size = 132, normalized size = 0.81

method	result
default	$\frac{\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\left(3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)a^6x^6+3\sqrt{-a^2x^2+1}\,a^4x^4+2a^2x^2\sqrt{-a^2x^2+1}-8\sqrt{-a^2x^2}\right.}{48x^5\sqrt{-a^2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*((a*x+1)/a/x)^(1/2)/x^5*(-(a*x-1)/a/x)^(1/2)*(3*arctanh(1/(-a^2*x^2+1)
^(1/2))*a^6*x^6+3*(-a^2*x^2+1)^(1/2)*a^4*x^4+2*a^2*x^2*(-a^2*x^2+1)^(1/2)-8
*(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2)-1/6/a/x^6
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^7, x)/a - 1/6/(a*x^6)
```

Fricas [A]
time = 0.40, size = 148, normalized size = 0.91

$$\frac{3\,a^6x^6\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-3\,a^6x^6\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)+2\left(3\,a^5x^5+2\,a^3x^3-8\,ax\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-16}{96\,ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="fricas"
)
```

```
[Out] 1/96*(3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) -
 3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 2*(3
*a^5*x^5 + 2*a^3*x^3 - 8*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))
- 16)/(a*x^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^7}\, dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^6}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**6,x)
```

```
[Out] (Integral(x**(-7), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**
6, x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^6, x)
```

Mupad [B]
time = 34.08, size = 878, normalized size = 5.39

$$\frac{\frac{35\,a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^3}{12\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^3}+\frac{757\,a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^5}{4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^5}+\frac{7339\,a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^7}{4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^7}+\frac{41929\,a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^9}{6\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^9}+\frac{25661\,a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{11}}{2\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{11}}+\frac{25661\,a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{13}}{2\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{13}}+\frac{41929\,a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{15}}{6\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{15}}+\frac{7339\,a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{17}}{4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{17}}+\frac{757\,a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{19}}{4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{19}}+\frac{35\,a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{21}}{12\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{21}}-\frac{a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{23}}{4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{23}}-\frac{a^5\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}{4\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}}{1+\frac{66\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}-\frac{220\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{495\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}-\frac{792\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{10}}+\frac{924\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{12}}-\frac{792\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{14}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{14}}+\frac{495\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{16}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{16}}-\frac{220\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{18}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{18}}+\frac{66\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{20}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{20}}-\frac{12\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{22}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{22}}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{24}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{24}}-\frac{12\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}}+\frac{a^5\,\mathrm{atanh}\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)}{4}-\frac{1}{6\,a\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^6,x)
```

```
[Out] ((35*a^5*((1/(a*x) - 1)^(1/2) - 1i)^3)/(12*((1/(a*x) + 1)^(1/2) - 1)^3) + (
757*a^5*((1/(a*x) - 1)^(1/2) - 1i)^5)/(4*((1/(a*x) + 1)^(1/2) - 1)^5) + (73
39*a^5*((1/(a*x) - 1)^(1/2) - 1i)^7)/(4*((1/(a*x) + 1)^(1/2) - 1)^7) + (419
29*a^5*((1/(a*x) - 1)^(1/2) - 1i)^9)/(6*((1/(a*x) + 1)^(1/2) - 1)^9) + (256
61*a^5*((1/(a*x) - 1)^(1/2) - 1i)^11)/(2*((1/(a*x) + 1)^(1/2) - 1)^11) + (2
5661*a^5*((1/(a*x) - 1)^(1/2) - 1i)^13)/(2*((1/(a*x) + 1)^(1/2) - 1)^13) +
(41929*a^5*((1/(a*x) - 1)^(1/2) - 1i)^15)/(6*((1/(a*x) + 1)^(1/2) - 1)^15)
+ (7339*a^5*((1/(a*x) - 1)^(1/2) - 1i)^17)/(4*((1/(a*x) + 1)^(1/2) - 1)^17)
 + (757*a^5*((1/(a*x) - 1)^(1/2) - 1i)^19)/(4*((1/(a*x) + 1)^(1/2) - 1)^19)
 + (35*a^5*((1/(a*x) - 1)^(1/2) - 1i)^21)/(12*((1/(a*x) + 1)^(1/2) - 1)^21)
 - (a^5*((1/(a*x) - 1)^(1/2) - 1i)^23)/(4*((1/(a*x) + 1)^(1/2) - 1)^23) - (
a^5*((1/(a*x) - 1)^(1/2) - 1i))/(4*((1/(a*x) + 1)^(1/2) - 1)))/((66*((1/(a*
```

```
x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (12*((1/(a*x) - 1)^(1/
2) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (220*((1/(a*x) - 1)^(1/2) - 1i)^6
)/((1/(a*x) + 1)^(1/2) - 1)^6 + (495*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x
) + 1)^(1/2) - 1)^8 - (792*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1
/2) - 1)^10 + (924*((1/(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x) + 1)^(1/2) - 1)
^12 - (792*((1/(a*x) - 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^(1/2) - 1)^14 + (4
95*((1/(a*x) - 1)^(1/2) - 1i)^16)/((1/(a*x) + 1)^(1/2) - 1)^16 - (220*((1/(
a*x) - 1)^(1/2) - 1i)^18)/((1/(a*x) + 1)^(1/2) - 1)^18 + (66*((1/(a*x) - 1)
^(1/2) - 1i)^20)/((1/(a*x) + 1)^(1/2) - 1)^20 - (12*((1/(a*x) - 1)^(1/2) -
1i)^22)/((1/(a*x) + 1)^(1/2) - 1)^22 + ((1/(a*x) - 1)^(1/2) - 1i)^24/((1/(a
*x) + 1)^(1/2) - 1)^24 + 1) + (a^5*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*
x) + 1)^(1/2) - 1)))/4 - 1/(6*a*x^6)
```

3.43 $\int \frac{e^{\text{sech}^{-1}(ax)}}{x^7} \, dx$

Optimal. Leaf size=146

$$\frac{1}{42ax^7} - \frac{e^{\text{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{1+ax}}} + \frac{4a^3\sqrt{1-ax}}{105x^3\sqrt{\frac{1}{1+ax}}} + \frac{8a^5\sqrt{1-ax}}{105x\sqrt{\frac{1}{1+ax}}}$$

```
[Out] 1/42/a/x^7-1/6*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6+1/42*(-a*x+1)^(1
/2)/a/x^7/(1/(a*x+1))^(1/2)+1/35*a*(-a*x+1)^(1/2)/x^5/(1/(a*x+1))^(1/2)+4/1
05*a^3*(-a*x+1)^(1/2)/x^3/(1/(a*x+1))^(1/2)+8/105*a^5*(-a*x+1)^(1/2)/x/(1/(
a*x+1))^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6470, 30, 105, 12, 97}

$$\frac{8a^5\sqrt{1-ax}}{105x\sqrt{\frac{1}{ax+1}}} + \frac{4a^3\sqrt{1-ax}}{105x^3\sqrt{\frac{1}{ax+1}}} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{ax+1}}} + \frac{1}{42ax^7} - \frac{e^{\text{sech}^{-1}(ax)}}{6x^6} + \frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{ax+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x]/x^7,x]
```

```
[Out] 1/(42*a*x^7) - E^ArcSech[a*x]/(6*x^6) + Sqrt[1 - a*x]/(42*a*x^7*Sqrt[(1 + a
*x)^(-1)]) + (a*Sqrt[1 - a*x])/(35*x^5*Sqrt[(1 + a*x)^(-1)]) + (4*a^3*Sqrt[
1 - a*x])/(105*x^3*Sqrt[(1 + a*x)^(-1)]) + (8*a^5*Sqrt[1 - a*x])/(105*x*Sqr
t[(1 + a*x)^(-1)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
```

```
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^7}\,dx = -\frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6} - \frac{\int \frac{1}{x^8}\,dx}{6a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int \frac{1}{x^8\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{6a}$$

$$= \frac{1}{42ax^7} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} + \frac{\left(\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int -\frac{6a^2}{x^6\sqrt{1-ax}\,\sqrt{1+a}}}{42a}$$

$$= \frac{1}{42ax^7} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} - \frac{1}{7}\left(a\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int \frac{1}{x^6\sqrt{1-ax}\,\sqrt{1+}}$$

$$= \frac{1}{42ax^7} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{1+ax}}} + \frac{1}{35}\left(a\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int$$

$$= \frac{1}{42ax^7} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{1+ax}}} - \frac{1}{35}\left(4a^3\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)$$

$$= \frac{1}{42ax^7} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{1+ax}}} + \frac{4a^3\sqrt{1-ax}}{105x^3\sqrt{\frac{1}{1+ax}}} + \frac{1}{105}\left(4a^3\right.$$

$$= \frac{1}{42ax^7} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{1+ax}}} + \frac{4a^3\sqrt{1-ax}}{105x^3\sqrt{\frac{1}{1+ax}}} - \frac{1}{105}\left(8a^5\right.$$

$$= \frac{1}{42ax^7} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{6x^6} + \frac{\sqrt{1-ax}}{42ax^7\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{35x^5\sqrt{\frac{1}{1+ax}}} + \frac{4a^3\sqrt{1-ax}}{105x^3\sqrt{\frac{1}{1+ax}}} + \frac{8a^5\sqrt{1-}}{105x\sqrt{\frac{}{1}}}$$

__Mathematica__ [A]
time = 0.06, size = 76, normalized size = 0.52

$$\frac{-15+\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^2\left(-15+15ax-12a^2x^2+12a^3x^3-8a^4x^4+8a^5x^5\right)}{105ax^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]/x^7,x]
```

```
[Out] (-15 + Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-15 + 15*a*x - 12*a^2*x^2 + 1
2*a^3*x^3 - 8*a^4*x^4 + 8*a^5*x^5))/(105*a*x^7)
```

Maple [A]
time = 0.04, size = 71, normalized size = 0.49

method	result	size
default	$\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,(a^2x^2-1)(8a^4x^4+12a^2x^2+15)}{105x^6}-\frac{1}{7a\,x^7}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*((a*x+1)/a/x)^(1/2)/x^6*(-(a*x-1)/a/x)^(1/2)*(a^2*x^2-1)*(8*a^4*x^4+1
2*a^2*x^2+15)-1/7/a/x^7
```

Maxima [A]
time = 0.30, size = 60, normalized size = 0.41

$$\frac{(8\,a^6x^7+4\,a^4x^5+3\,a^2x^3-15\,x)\sqrt{ax+1}\,\sqrt{-ax+1}}{105\,ax^8}-\frac{1}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="maxima"
)
```

```
[Out] 1/105*(8*a^6*x^7 + 4*a^4*x^5 + 3*a^2*x^3 - 15*x)*sqrt(a*x + 1)*sqrt(-a*x +
1)/(a*x^8) - 1/7/(a*x^7)
```

Fricas [A]
time = 0.45, size = 69, normalized size = 0.47

$$\frac{(8\,a^7x^7+4\,a^5x^5+3\,a^3x^3-15\,ax)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}-15}{105\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="fricas"
)
```

```
[Out] 1/105*((8*a^7*x^7 + 4*a^5*x^5 + 3*a^3*x^3 - 15*a*x)*sqrt((a*x + 1)/(a*x))*s
qrt(-(a*x - 1)/(a*x)) - 15)/(a*x^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^8}\,dx+\int \frac{a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^7}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**7,x)
```

```
[Out] (Integral(x**(-8), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**
7, x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^7, x)
```

Mupad [B]
time = 1.68, size = 95, normalized size = 0.65

$$\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{a^2\,x^2\,\sqrt{\frac{1}{a\,x}+1}}{35}-\frac{\sqrt{\frac{1}{a\,x}+1}}{7}+\frac{4\,a^4\,x^4\,\sqrt{\frac{1}{a\,x}+1}}{105}+\frac{8\,a^6\,x^6\,\sqrt{\frac{1}{a\,x}+1}}{105}\right)}{x^6}-\frac{1}{7\,a\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^7,x)
```

```
[Out] ((1/(a*x) - 1)^(1/2)*((a^2*x^2*(1/(a*x) + 1)^(1/2))/35 - (1/(a*x) + 1)^(1/2
)/7 + (4*a^4*x^4*(1/(a*x) + 1)^(1/2))/105 + (8*a^6*x^6*(1/(a*x) + 1)^(1/2))
/105))/x^6 - 1/(7*a*x^7)
```

3.44 $\int \frac{e^{\text{sech}^{-1}(ax)}}{x^8}\,dx$

Optimal. Leaf size=194

$$\frac{1}{56ax^8}-\frac{e^{\text{sech}^{-1}(ax)}}{7x^7}+\frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}}+\frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}}+\frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{1+ax}}}+\frac{5a^5\sqrt{1-ax}}{128x^2\sqrt{\frac{1}{1+ax}}}+\frac{5}{128}a^7\sqrt{\frac{1}{1+a}}$$

```
[Out] 1/56/a/x^8-1/7*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7+1/56*(-a*x+1)^(1
/2)/a/x^8/(1/(a*x+1))^(1/2)+1/48*a*(-a*x+1)^(1/2)/x^6/(1/(a*x+1))^(1/2)+5/1
92*a^3*(-a*x+1)^(1/2)/x^4/(1/(a*x+1))^(1/2)+5/128*a^5*(-a*x+1)^(1/2)/x^2/(1
/(a*x+1))^(1/2)+5/128*a^7*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(1/(a*x+1))
^(1/2)*(a*x+1)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6470, 30, 105, 12, 94, 214}

$$\frac{5}{128}a^7\sqrt{\frac{1}{ax+1}}\sqrt{ax+1}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)+\frac{5a^5\sqrt{1-ax}}{128x^2\sqrt{\frac{1}{ax+1}}}+\frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{ax+1}}}+\frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{ax+1}}}+\frac{1}{56ax^8}-\frac{e^{\text{sech}^{-1}(ax)}}{7x^7}+\frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{ax+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x]/x^8,x]
```

```
[Out] 1/(56*a*x^8) - E^ArcSech[a*x]/(7*x^7) + Sqrt[1 - a*x]/(56*a*x^8*Sqrt[(1 + a
*x)^(-1)]) + (a*Sqrt[1 - a*x])/(48*x^6*Sqrt[(1 + a*x)^(-1)]) + (5*a^3*Sqrt[
1 - a*x])/(192*x^4*Sqrt[(1 + a*x)^(-1)]) + (5*a^5*Sqrt[1 - a*x])/(128*x^2*S
qrt[(1 + a*x)^(-1)]) + (5*a^7*Sqrt[(1 + a*x)^(-1)]*Sqrt[1 + a*x]*ArcTanh[Sq
rt[1 - a*x]*Sqrt[1 + a*x]])/128
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
```

```
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{e^{\operatorname{sech}^{-1}(ax)}}{x^8}\,dx = -\frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} - \frac{\int \frac{1}{x^9}\,dx}{7a} - \frac{\left(\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int \frac{1}{x^9\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{7a}$$

$$= \frac{1}{56ax^8} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{\left(\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int -\frac{7a^2}{x^7\sqrt{1-ax}\,\sqrt{1+\cdots}}}{56a}$$

$$= \frac{1}{56ax^8} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} - \frac{1}{8}\left(a\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\int \frac{1}{x^7\sqrt{1-ax}\,\sqrt{1\cdots}}$$

$$= \frac{1}{56ax^8} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{1}{48}\left(a\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\right)\cdots$$

$$= \frac{1}{56ax^8} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} - \frac{1}{48}\left(5a^3\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\cdots\right.$$

$$= \frac{1}{56ax^8} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{1+ax}}} + \frac{1}{192}\left(5a\cdots\right.$$

$$= \frac{1}{56ax^8} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{1+ax}}} - \frac{1}{64}\left(5a^5\cdots\right.$$

$$= \frac{1}{56ax^8} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{1+ax}}} + \frac{5a^5\sqrt{1\cdots}}{128x^2\sqrt{\cdots}}$$

$$= \frac{1}{56ax^8} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{1+ax}}} + \frac{5a^5\sqrt{1\cdots}}{128x^2\sqrt{\cdots}}$$

$$= \frac{1}{56ax^8} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{1+ax}}} + \frac{5a^5\sqrt{1\cdots}}{128x^2\sqrt{\cdots}}$$

$$= \frac{1}{56ax^8} - \frac{e^{\operatorname{sech}^{-1}(ax)}}{7x^7} + \frac{\sqrt{1-ax}}{56ax^8\sqrt{\frac{1}{1+ax}}} + \frac{a\sqrt{1-ax}}{48x^6\sqrt{\frac{1}{1+ax}}} + \frac{5a^3\sqrt{1-ax}}{192x^4\sqrt{\frac{1}{1+ax}}} + \frac{5a^5\sqrt{1\cdots}}{128x^2\sqrt{\cdots}}$$

Mathematica [A]
time = 0.08, size = 145, normalized size = 0.75

$$\frac{-48+\sqrt{\frac{1-ax}{1+ax}}\,(-48-48ax+8a^2x^2+8a^3x^3+10a^4x^4+10a^5x^5+15a^6x^6+15a^7x^7)-15a^8x^8\log(x)+15a^8x^8\log\left(1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)}{384ax^8}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x]/x^8,x]
```

```
[Out] (-48 + Sqrt[(1 - a*x)/(1 + a*x)]*(-48 - 48*a*x + 8*a^2*x^2 + 8*a^3*x^3 + 10
*a^4*x^4 + 10*a^5*x^5 + 15*a^6*x^6 + 15*a^7*x^7) - 15*a^8*x^8*Log[x] + 15*a
^8*x^8*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/
(384*a*x^8)
```

Maple [A]
time = 0.05, size = 152, normalized size = 0.78

method	result
default	$\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,\left(15\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)a^8x^8+15\sqrt{-a^2x^2+1}\,a^6x^6+10\sqrt{-a^2x^2+1}\,a^4x^4+8a^2x^2\sqrt{-\right.}{384x^7\sqrt{-a^2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*((a*x+1)/a/x)^(1/2)/x^7*(-(a*x-1)/a/x)^(1/2)*(15*arctanh(1/(-a^2*x^2+
1)^(1/2))*a^8*x^8+15*(-a^2*x^2+1)^(1/2)*a^6*x^6+10*(-a^2*x^2+1)^(1/2)*a^4*x
^4+8*a^2*x^2*(-a^2*x^2+1)^(1/2)-48*(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2)-1
/8/a/x^8
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^8,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^9, x)/a - 1/8/(a*x^8)
```

Fricas [A]
time = 0.43, size = 156, normalized size = 0.80

$$\frac{15\,a^8x^8\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-15\,a^8x^8\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)+2\,(15\,a^7x^7+10\,a^5x^5+8\,a^3x^3-48\,ax)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-96}{768\,ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^8,x, algorithm="fricas"
)
```

```
[Out] 1/768*(15*a^8*x^8*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1)
 - 15*a^8*x^8*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 2
*(15*a^7*x^7 + 10*a^5*x^5 + 8*a^3*x^3 - 48*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(
-(a*x - 1)/(a*x)) - 96)/(a*x^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^9}\,dx + \int \frac{a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^8}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**8,x)
```

```
[Out] (Integral(x**(-9), x) + Integral(a*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/x**
8, x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^8,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))/x^8, x)
```

Mupad [B]
time = 38.56, size = 1155, normalized size = 5.95

$$-\frac{-\frac{235\,a^7\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^3}{96\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^3}+\frac{1723\,a^7\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^5}{96\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^5}+\frac{72283\,a^7\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^7}{32\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^7}+\frac{848801\,a^7\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^9}{32\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^9}+\frac{4181067\,a^7\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{11}}{32\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{11}}+\frac{10994181\,a^7\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{13}}{32\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{13}}+\frac{17457599\,a^7\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{15}}{32\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{15}}+\frac{17457599\,a^7\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{17}}{32\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{17}}+\frac{10994181\,a^7\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{19}}{32\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{19}}+\frac{4181067\,a^7\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{21}}{32\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{21}}+\frac{848801\,a^7\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{23}}{32\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{23}}+\frac{72283\,a^7\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{25}}{32\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{25}}+\frac{1723\,a^7\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{27}}{96\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{27}}-\frac{235\,a^7\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{29}}{96\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{29}}+\frac{5\,a^7\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{31}}{32\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{31}}+\frac{5\,a^7\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}{32\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}}{1+\frac{120\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}-\frac{560\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{1820\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}-\frac{4368\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{10}}+\frac{8008\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{12}}-\frac{11440\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{14}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{14}}+\frac{12870\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{16}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{16}}-\frac{11440\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{18}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{18}}+\frac{8008\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{20}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{20}}-\frac{4368\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{22}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{22}}+\frac{1820\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{24}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{24}}-\frac{560\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{26}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{26}}+\frac{120\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{28}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{28}}-\frac{16\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{30}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{30}}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^{32}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^{32}}-\frac{16\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}}+\frac{5\,a^7\,\mathrm{atanh}\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)}{32}-\frac{1}{8\,a\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))/x^8,x)
```

```
[Out] (5*a^7*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)))/32 - ((
1723*a^7*((1/(a*x) - 1)^(1/2) - 1i)^5)/(96*((1/(a*x) + 1)^(1/2) - 1)^5) - (
235*a^7*((1/(a*x) - 1)^(1/2) - 1i)^3)/(96*((1/(a*x) + 1)^(1/2) - 1)^3) + (7
2283*a^7*((1/(a*x) - 1)^(1/2) - 1i)^7)/(32*((1/(a*x) + 1)^(1/2) - 1)^7) + (
```

```
848801*a^7*((1/(a*x) - 1)^(1/2) - 1i)^9)/(32*((1/(a*x) + 1)^(1/2) - 1)^9) +
 (4181067*a^7*((1/(a*x) - 1)^(1/2) - 1i)^11)/(32*((1/(a*x) + 1)^(1/2) - 1)^
11) + (10994181*a^7*((1/(a*x) - 1)^(1/2) - 1i)^13)/(32*((1/(a*x) + 1)^(1/2)
 - 1)^13) + (17457599*a^7*((1/(a*x) - 1)^(1/2) - 1i)^15)/(32*((1/(a*x) + 1)
^(1/2) - 1)^15) + (17457599*a^7*((1/(a*x) - 1)^(1/2) - 1i)^17)/(32*((1/(a*x
) + 1)^(1/2) - 1)^17) + (10994181*a^7*((1/(a*x) - 1)^(1/2) - 1i)^19)/(32*((
1/(a*x) + 1)^(1/2) - 1)^19) + (4181067*a^7*((1/(a*x) - 1)^(1/2) - 1i)^21)/(
32*((1/(a*x) + 1)^(1/2) - 1)^21) + (848801*a^7*((1/(a*x) - 1)^(1/2) - 1i)^2
3)/(32*((1/(a*x) + 1)^(1/2) - 1)^23) + (72283*a^7*((1/(a*x) - 1)^(1/2) - 1i
)^25)/(32*((1/(a*x) + 1)^(1/2) - 1)^25) + (1723*a^7*((1/(a*x) - 1)^(1/2) -
1i)^27)/(96*((1/(a*x) + 1)^(1/2) - 1)^27) - (235*a^7*((1/(a*x) - 1)^(1/2) -
 1i)^29)/(96*((1/(a*x) + 1)^(1/2) - 1)^29) + (5*a^7*((1/(a*x) - 1)^(1/2) -
1i)^31)/(32*((1/(a*x) + 1)^(1/2) - 1)^31) + (5*a^7*((1/(a*x) - 1)^(1/2) - 1
i))/(32*((1/(a*x) + 1)^(1/2) - 1)))/((120*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1
/(a*x) + 1)^(1/2) - 1)^4 - (16*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)
^(1/2) - 1)^2 - (560*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1
)^6 + (1820*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8 - (43
68*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1)^10 + (8008*((1/
(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x) + 1)^(1/2) - 1)^12 - (11440*((1/(a*x)
- 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^(1/2) - 1)^14 + (12870*((1/(a*x) - 1)^(
1/2) - 1i)^16)/((1/(a*x) + 1)^(1/2) - 1)^16 - (11440*((1/(a*x) - 1)^(1/2) -
 1i)^18)/((1/(a*x) + 1)^(1/2) - 1)^18 + (8008*((1/(a*x) - 1)^(1/2) - 1i)^20
)/((1/(a*x) + 1)^(1/2) - 1)^20 - (4368*((1/(a*x) - 1)^(1/2) - 1i)^22)/((1/(
a*x) + 1)^(1/2) - 1)^22 + (1820*((1/(a*x) - 1)^(1/2) - 1i)^24)/((1/(a*x) +
1)^(1/2) - 1)^24 - (560*((1/(a*x) - 1)^(1/2) - 1i)^26)/((1/(a*x) + 1)^(1/2)
 - 1)^26 + (120*((1/(a*x) - 1)^(1/2) - 1i)^28)/((1/(a*x) + 1)^(1/2) - 1)^28
 - (16*((1/(a*x) - 1)^(1/2) - 1i)^30)/((1/(a*x) + 1)^(1/2) - 1)^30 + ((1/(a
*x) - 1)^(1/2) - 1i)^32/((1/(a*x) + 1)^(1/2) - 1)^32 + 1) - 1/(8*a*x^8)
```

3.45 $\int e^{\text{sech}^{-1}(ax^2)} x^7 \, dx$

Optimal. Leaf size=111

$$\frac{x^6}{24a}+\frac{1}{8}e^{\text{sech}^{-1}(ax^2)}x^8-\frac{x^2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{16a^3}+\frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,\text{ArcSin}(ax^2)}{16a^4}$$

```
[Out] 1/24*x^6/a+1/8*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^8+1/16*arcsi
n(a*x^2)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^4-1/16*x^2*(1/(a*x^2+1))^(1/
2)*(a*x^2+1)^(1/2)*(-a^2*x^4+1)^(1/2)/a^3
```

Rubi [A]
time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6470, 30, 265, 281, 327, 222}

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,\text{ArcSin}(ax^2)}{16a^4}-\frac{x^2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{16a^3}+\frac{x^6}{24a}+\frac{1}{8}x^8e^{\text{sech}^{-1}(ax^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2]*x^7,x]
```

```
[Out] x^6/(24*a) + (E^ArcSech[a*x^2]*x^8)/8 - (x^2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1
+ a*x^2]*Sqrt[1 - a^2*x^4])/(16*a^3) + (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x
^2]*ArcSin[a*x^2])/(16*a^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}
\int e^{\operatorname{sech}^{-1}(ax^2)}x^7\,dx &= \frac{1}{8}e^{\operatorname{sech}^{-1}(ax^2)}x^8+\frac{\int x^5\,dx}{4a}+\frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int\frac{x^5}{\sqrt{1-ax^2}\sqrt{1+ax^2}}\,dx}{4a}\\
&= \frac{x^6}{24a}+\frac{1}{8}e^{\operatorname{sech}^{-1}(ax^2)}x^8+\frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int\frac{x^5}{\sqrt{1-a^2x^4}}\,dx}{4a}\\
&= \frac{x^6}{24a}+\frac{1}{8}e^{\operatorname{sech}^{-1}(ax^2)}x^8+\frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\operatorname{Subst}\left(\int\frac{x^2}{\sqrt{1-a^2x^2}}\,dx,x,x^2\right)}{8a}\\
&= \frac{x^6}{24a}+\frac{1}{8}e^{\operatorname{sech}^{-1}(ax^2)}x^8-\frac{x^2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{16a^3}+\frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right.}{}\\
&= \frac{x^6}{24a}+\frac{1}{8}e^{\operatorname{sech}^{-1}(ax^2)}x^8-\frac{x^2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{16a^3}+\frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,\mathrm{s}}{16a^4}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.13, size = 111, normalized size = 1.00

$$\frac{8a^3x^6 - 3a\sqrt{\frac{1-ax^2}{1+ax^2}}\,(x^2 + ax^4 - 2a^2x^6 - 2a^3x^8) + 3i\log\left(-2iax^2 + 2\sqrt{\frac{1-ax^2}{1+ax^2}}\,(1+ax^2)\right)}{48a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]*x^7,x]
```

```
[Out] (8*a^3*x^6 - 3*a*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(x^2 + a*x^4 - 2*a^2*x^6 - 2
*a^3*x^8) + (3*I)*Log[(-2*I)*a*x^2 + 2*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(1 + a
*x^2)])/(48*a^4)
```

Maple [A]
time = 0.10, size = 137, normalized size = 1.23

method	result	size
default	$\frac{\sqrt{-\frac{a\,x^2-1}{a\,x^2}}\,x^2\sqrt{\frac{a\,x^2+1}{a\,x^2}}\left(2x^6\sqrt{-\frac{a^2x^4-1}{a^2}}\,a^4-x^2\sqrt{-\frac{a^2x^4-1}{a^2}}\,a^2+\arctan\left(\frac{x^2}{\sqrt{-\frac{a^2x^4-1}{a^2}}}\right)\right)}{16\sqrt{-\frac{a^2x^4-1}{a^2}}\,a^4}+\frac{x^6}{6a}$	137

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^7,x,method=_RETURNVERBO
SE)
```

```
[Out] 1/16*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(2*x^6*(-(a^2*x^4
-1)/a^2)^(1/2)*a^4-x^2*(-(a^2*x^4-1)/a^2)^(1/2)*a^2+arctan(x^2/(-(a^2*x^4-1
)/a^2)^(1/2)))/(-(a^2*x^4-1)/a^2)^(1/2)/a^4+1/6/a*x^6
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^7,x, algorithm="m
axima")
```

```
[Out] 1/6*x^6/a + integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)*x^5, x)/a
```

Fricas [A]
time = 0.42, size = 116, normalized size = 1.05

$$\frac{8\,a^3x^6 + 3\,(2\,a^4x^8 - a^2x^4)\sqrt{\frac{ax^2+1}{ax^2}}\,\sqrt{-\frac{ax^2-1}{ax^2}} - 6\,\arctan\left(\frac{ax^2\sqrt{\frac{ax^2+1}{ax^2}}\,\sqrt{-\frac{ax^2-1}{ax^2}} - 1}{ax^2}\right)}{48\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^7,x, algorithm="f
ricas")
```

```
[Out] 1/48*(8*a^3*x^6 + 3*(2*a^4*x^8 - a^2*x^4)*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(
a*x^2 - 1)/(a*x^2)) - 6*arctan((a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^
2 - 1)/(a*x^2)) - 1)/(a*x^2)))/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int x^5\,dx + \int a x^7 \sqrt{-1+\frac{1}{ax^2}}\,\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**7,x)
```

```
[Out] (Integral(x**5, x) + Integral(a*x**7*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x*
*2)), x))/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(76) = 152.
time = 0.44, size = 205, normalized size = 1.85

$$\frac{8\,a^2x^6 + 4\sqrt{a^2x^2+a}\,\sqrt{-a^2x^2+a}\left((a^2x^2+a)\left(\frac{2\,(a^2x^2+a)}{a^4}-\frac{7}{a^3}\right)+\frac{9}{a^2}\right) + \left(\sqrt{a^2x^2+a}\,\sqrt{-a^2x^2+a}\left((a^2x^2+a)\left(2\,(a^2x^2+a)\left(\frac{3\,(a^2x^2+a)}{a^6}-\frac{13}{a^5}\right)+\frac{43}{a^4}\right)-\frac{39}{a^3}\right)-\frac{18\arcsin\left(\frac{\sqrt{2}\,\sqrt{a^2x^2+a}}{2\,\sqrt{a}}\right)}{a^2}\right)a + \frac{24\arcsin\left(\frac{\sqrt{2}\,\sqrt{a^2x^2+a}}{2\,\sqrt{a}}\right)}{a}}{48\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^7,x, algorithm="g
iac")
```

```
[Out] 1/48*(8*a^2*x^6 + 4*sqrt(a^2*x^2 + a)*sqrt(-a^2*x^2 + a)*((a^2*x^2 + a)*(2*
(a^2*x^2 + a)/a^4 - 7/a^3) + 9/a^2) + (sqrt(a^2*x^2 + a)*sqrt(-a^2*x^2 + a)
*((a^2*x^2 + a)*(2*(a^2*x^2 + a)*(3*(a^2*x^2 + a)/a^6 - 13/a^5) + 43/a^4) -
 39/a^3) - 18*arcsin(1/2*sqrt(2)*sqrt(a^2*x^2 + a)/sqrt(a))/a^2)*a + 24*arc
sin(1/2*sqrt(2)*sqrt(a^2*x^2 + a)/sqrt(a))/a)/a^3
```

Mupad [B]
time = 14.45, size = 521, normalized size = 4.69

$$\frac{\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}+1\right)1\mathrm{i}}{16\,a^4} - \frac{\frac{1\mathrm{i}}{2048\,a^4}+\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2 1\mathrm{i}}{256\,a^4\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}+\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^4 11\mathrm{i}}{1024\,a^4\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^6 7\mathrm{i}}{512\,a^4\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^6}-\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^8 239\mathrm{i}}{2048\,a^4\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^8}+\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^{10} 1\mathrm{i}}{512\,a^4\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^{10}}}{\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^4}+\frac{4\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^6}+\frac{6\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^8}+\frac{4\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^{10}}+\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^{12}}} - \frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x^2}+1}-1}\right)1\mathrm{i}}{16\,a^4} + \frac{x^6}{6\,a} - \frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2 1\mathrm{i}}{512\,a^4\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2} - \frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^4 1\mathrm{i}}{2048\,a^4\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] (log((((1/(a*x^2) - 1)^(1/2) - 1i)^2/((1/(a*x^2) + 1)^(1/2) - 1)^2 + 1)*1i)/
(16*a^4) - (1i/(2048*a^4) + (((1/(a*x^2) - 1)^(1/2) - 1i)^2*1i)/(256*a^4*((
1/(a*x^2) + 1)^(1/2) - 1)^2) + (((1/(a*x^2) - 1)^(1/2) - 1i)^4*11i)/(1024*a
^4*((1/(a*x^2) + 1)^(1/2) - 1)^4) + (((1/(a*x^2) - 1)^(1/2) - 1i)^6*7i)/(51
2*a^4*((1/(a*x^2) + 1)^(1/2) - 1)^6) - (((1/(a*x^2) - 1)^(1/2) - 1i)^8*239i
)/(2048*a^4*((1/(a*x^2) + 1)^(1/2) - 1)^8) + (((1/(a*x^2) - 1)^(1/2) - 1i)^
10*1i)/(512*a^4*((1/(a*x^2) + 1)^(1/2) - 1)^10))/((((1/(a*x^2) - 1)^(1/2) -
1i)^4/((1/(a*x^2) + 1)^(1/2) - 1)^4 + (4*((1/(a*x^2) - 1)^(1/2) - 1i)^6)/((
1/(a*x^2) + 1)^(1/2) - 1)^6 + (6*((1/(a*x^2) - 1)^(1/2) - 1i)^8)/((1/(a*x^2
) + 1)^(1/2) - 1)^8 + (4*((1/(a*x^2) - 1)^(1/2) - 1i)^10)/((1/(a*x^2) + 1)^
(1/2) - 1)^10 + ((1/(a*x^2) - 1)^(1/2) - 1i)^12/((1/(a*x^2) + 1)^(1/2) - 1)
^12) - (log(((1/(a*x^2) - 1)^(1/2) - 1i)/((1/(a*x^2) + 1)^(1/2) - 1))*1i)/(
16*a^4) + x^6/(6*a) - (((1/(a*x^2) - 1)^(1/2) - 1i)^2*1i)/(512*a^4*((1/(a*x
^2) + 1)^(1/2) - 1)^2) - (((1/(a*x^2) - 1)^(1/2) - 1i)^4*1i)/(2048*a^4*((1/
(a*x^2) + 1)^(1/2) - 1)^4)
```

3.46 $\int e^{\text{sech}^{-1}(ax^2)} x^6 \, dx$

Optimal. Leaf size=115

$$\frac{2x^5}{35a}+\frac{1}{7}e^{\text{sech}^{-1}(ax^2)}x^7-\frac{2x\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{21a^3}+\frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}F\left(\text{ArcSin}\left(\sqrt{a}\,x\right)\middle|-1\right)}{21a^{7/2}}$$

```
[Out] 2/35*x^5/a+1/7*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^7+2/21*Ellip
ticF(x*a^(1/2),I)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^(7/2)-2/21*x*(1/(a*
x^2+1))^(1/2)*(a*x^2+1)^(1/2)*(-a^2*x^4+1)^(1/2)/a^3
```

Rubi [A]
time = 0.03, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6470, 30, 265, 327, 227}

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}F\left(\text{ArcSin}\left(\sqrt{a}\,x\right)\middle|-1\right)}{21a^{7/2}}-\frac{2x\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{21a^3}+\frac{2x^5}{35a}+\frac{1}{7}x^7e^{\text{sech}^{-1}(ax^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2]*x^6,x]
```

```
[Out] (2*x^5)/(35*a) + (E^ArcSech[a*x^2]*x^7)/7 - (2*x*Sqrt[(1 + a*x^2)^(-1)]*Sqr
t[1 + a*x^2]*Sqrt[1 - a^2*x^4])/(21*a^3) + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1
 + a*x^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/(21*a^(7/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^6\,dx = \frac{1}{7}e^{\operatorname{sech}^{-1}(ax^2)}x^7 + \frac{2\int x^4\,dx}{7a} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int\frac{x^4}{\sqrt{1-ax^2}\sqrt{1+ax^2}}\,dx}{7a}$$

$$= \frac{2x^5}{35a} + \frac{1}{7}e^{\operatorname{sech}^{-1}(ax^2)}x^7 + \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int\frac{x^4}{\sqrt{1-a^2x^4}}\,dx}{7a}$$

$$= \frac{2x^5}{35a} + \frac{1}{7}e^{\operatorname{sech}^{-1}(ax^2)}x^7 - \frac{2x\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{21a^3} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+\ldots\right.}{2\ldots}$$

$$= \frac{2x^5}{35a} + \frac{1}{7}e^{\operatorname{sech}^{-1}(ax^2)}x^7 - \frac{2x\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{21a^3} + \frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax\ldots}}{2\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 139, normalized size = 1.21

$$\frac{x^5}{5a} + \frac{x\sqrt{\frac{1-ax^2}{1+ax^2}}\left(-2-2ax^2+3a^2x^4+3a^3x^6\right)}{21a^3} - \frac{2i\sqrt{\frac{1-ax^2}{1+ax^2}}\sqrt{1-a^2x^4}\,F\left(i\sinh^{-1}\left(\sqrt{-a}\,x\right)\middle|-1\right)}{21(-a)^{7/2}\left(-1+ax^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]*x^6,x]
```

```
[Out] x^5/(5*a) + (x*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(-2 - 2*a*x^2 + 3*a^2*x^4 + 3*
a^3*x^6))/(21*a^3) - (((2*I)/21)*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*Sqrt[1 - a^2
*x^4]*EllipticF[I*ArcSinh[Sqrt[-a]*x], -1])/((-a)^(7/2)*(-1 + a*x^2))
```

Maple [A]
time = 0.03, size = 114, normalized size = 0.99

method	result	size
default	$\frac{\sqrt{-\frac{a\,x^{2}-1}{a\,x^{2}}}\, x^{2} \sqrt{\frac{a\,x^{2}+1}{a\,x^{2}}}\, \left(3 a^{\frac{9}{2}} x^{9}-5 a^{\frac{5}{2}} x^{5}-2 \,\mathrm{EllipticF}\left(x \sqrt{a}, i\right) \sqrt{-a\,x^{2}+1}\, \sqrt{a\,x^{2}+1}+2 x \sqrt{a}\right)}{21 a^{\frac{5}{2}} \left(a^{2} x^{4}-1\right)}+\frac{x^{5}}{5 a}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^6,x,method=_RETURNVERBO
SE)
```

```
[Out] 1/21*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(3*a^(9/2)*x^9-5*
a^(5/2)*x^5-2*EllipticF(x*a^(1/2),I)*(-a*x^2+1)^(1/2)*(a*x^2+1)^(1/2)+2*x*a
^(1/2))/a^(5/2)/(a^2*x^4-1)+1/5*x^5/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^6,x, algorithm="m
axima")
```

```
[Out] 1/5*x^5/a + integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)*x^4, x)/a
```

Fricas [A]
time = 0.08, size = 61, normalized size = 0.53

$$\frac{21\,ax^{5}+5\,(3\,a^{2}x^{7}-2\,x^{3})\sqrt{\frac{ax^{2}+1}{ax^{2}}}\,\sqrt{-\frac{ax^{2}-1}{ax^{2}}}}{105\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^6,x, algorithm="f
ricas")
```

```
[Out] 1/105*(21*a*x^5 + 5*(3*a^2*x^7 - 2*x^3)*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*
x^2 - 1)/(a*x^2)))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int x^{4}\,dx+\int ax^{6}\sqrt{-1+\frac{1}{ax^{2}}}\,\sqrt{1+\frac{1}{ax^{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**6,x)
```

```
[Out] (Integral(x**4, x) + Integral(a*x**6*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x*
*2)), x))/a
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^6,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\left(\sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}+\frac{1}{a\,x^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] int(x^6*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)), x)
```

3.47 $\int e^{\text{sech}^{-1}(ax^2)} x^5 \, dx$

Optimal. Leaf size=58

$$\frac{x^4}{12a} + \frac{1}{6} e^{\text{sech}^{-1}(ax^2)} x^6 - \frac{\sqrt{1-ax^2}}{6a^3 \sqrt{\frac{1}{1+ax^2}}}$$

```
[Out] 1/12*x^4/a+1/6*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^6-1/6*(-a*x^
2+1)^(1/2)/a^3/(1/(a*x^2+1))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6470, 30, 265, 267}

$$-\frac{\sqrt{\frac{1}{ax^2+1}} \sqrt{ax^2+1} \sqrt{1-a^2x^4}}{6a^3} + \frac{x^4}{12a} + \frac{1}{6} x^6 e^{\text{sech}^{-1}(ax^2)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcSech[a*x^2]*x^5,x]
```

```
[Out] x^4/(12*a) + (E^ArcSech[a*x^2]*x^6)/6 - (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*
x^2]*Sqrt[1 - a^2*x^4])/(6*a^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}
\int e^{\text{sech}^{-1}(ax^2)}x^5\,dx &= \frac{1}{6}e^{\text{sech}^{-1}(ax^2)}x^6+\frac{\int x^3\,dx}{3a}+\frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int\frac{x^3}{\sqrt{1-ax^2}\sqrt{1+ax^2}}\,dx}{3a}\\
&= \frac{x^4}{12a}+\frac{1}{6}e^{\text{sech}^{-1}(ax^2)}x^6+\frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int\frac{x^3}{\sqrt{1-a^2x^4}}\,dx}{3a}\\
&= \frac{x^4}{12a}+\frac{1}{6}e^{\text{sech}^{-1}(ax^2)}x^6-\frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{6a^3}
\end{aligned}$$

Mathematica [**A**]
time = 0.07, size = 56, normalized size = 0.97

$$\frac{x^4}{4a}+\frac{(-1+ax^2)\sqrt{\frac{1-ax^2}{1+ax^2}}\,(1+ax^2)^2}{6a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]*x^5,x]
[Out] x^4/(4*a) + ((-1 + a*x^2)*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2)^2)/(6*a
^3)
```

Maple [**A**]
time = 0.03, size = 60, normalized size = 1.03

method	result	size
default	$\frac{\sqrt{-\frac{a\,x^2-1}{a\,x^2}}\,x^2\sqrt{\frac{a\,x^2+1}{a\,x^2}}\,(a^2x^4-1)}{6a^2}+\frac{x^4}{4a}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^5,x,method=_RETURNVERBO
SE)
```

```
[Out] 1/6*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(a^2*x^4-1)/a^2+1/
4*x^4/a
```

Maxima [A]
time = 0.29, size = 42, normalized size = 0.72

$$\frac{x^4}{4\,a}+\frac{\left(a^2x^4-1\right)\sqrt{ax^2+1}\,\sqrt{-ax^2+1}}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^5,x, algorithm="m
axima")
```

```
[Out] 1/4*x^4/a + 1/6*(a^2*x^4 - 1)*sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/a^3
```

Fricas [A]
time = 0.46, size = 60, normalized size = 1.03

$$\frac{3\,ax^4+2\left(a^2x^6-x^2\right)\sqrt{\frac{ax^2+1}{ax^2}}\,\sqrt{-\frac{ax^2-1}{ax^2}}}{12\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^5,x, algorithm="f
ricas")
```

```
[Out] 1/12*(3*a*x^4 + 2*(a^2*x^6 - x^2)*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 -
1)/(a*x^2)))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int x^3\,dx+\int ax^5\sqrt{-1+\frac{1}{ax^2}}\,\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**5,x)
```

```
[Out] (Integral(x**3, x) + Integral(a*x**5*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x*
*2)), x))/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(69) = 138.
time = 0.42, size = 190, normalized size = 3.28

$$\frac{\left(\sqrt{a^2x^2+a}\,\sqrt{-a^2x^2+a}\,\left(\left(a^2x^2+a\right)\left(\frac{2\left(a^2x^2+a\right)}{a^4}-\frac{7}{a^3}\right)+\frac{9}{a^2}\right)+\frac{6\arcsin\left(\frac{\sqrt{2}\,\sqrt{a^2x^2+a}}{2\sqrt{a}}\right)}{a}\right)a-\frac{3\left(2\,a^2\arcsin\left(\frac{\sqrt{2}\,\sqrt{a^2x^2+a}}{2\sqrt{a}}\right)-\sqrt{a^2x^2+a}\,\left(a^2x^2-2\,a\right)\sqrt{-a^2x^2+a}\right)}{a^2}+\frac{3\left(\left(a^2x^2+a\right)^2-2\left(a^2x^2+a\right)a\right)}{a^2}}{12\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^5,x, algorithm="g
iac")
```

```
[Out] 1/12*((sqrt(a^2*x^2 + a)*sqrt(-a^2*x^2 + a)*((a^2*x^2 + a)*(2*(a^2*x^2 + a)
/a^4 - 7/a^3) + 9/a^2) + 6*arcsin(1/2*sqrt(2)*sqrt(a^2*x^2 + a)/sqrt(a))/a)
*a - 3*(2*a^2*arcsin(1/2*sqrt(2)*sqrt(a^2*x^2 + a)/sqrt(a)) - sqrt(a^2*x^2
+ a)*(a^2*x^2 - 2*a)*sqrt(-a^2*x^2 + a))/a^2 + 3*((a^2*x^2 + a)^2 - 2*(a^2*
x^2 + a)*a)/a^2)/a^3
```

Mupad [B]
time = 1.64, size = 57, normalized size = 0.98

$$\sqrt{\frac{1}{a\,x^2}-1}\,\left(\frac{x^6\,\sqrt{\frac{1}{a\,x^2}+1}}{6}-\frac{x^2\,\sqrt{\frac{1}{a\,x^2}+1}}{6\,a^2}\right)+\frac{x^4}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] (1/(a*x^2) - 1)^(1/2)*((x^6*(1/(a*x^2) + 1)^(1/2))/6 - (x^2*(1/(a*x^2) + 1)
^(1/2))/(6*a^2)) + x^4/(4*a)
```

3.48 $\int e^{\text{sech}^{-1}(ax^2)}x^4\,dx$

Optimal. Leaf size=112

$$\frac{2x^3}{15a}+\frac{1}{5}e^{\text{sech}^{-1}(ax^2)}x^5+\frac{2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\,E(\text{ArcSin}(\sqrt{a}\,x)|-1)}{5a^{5/2}}-\frac{2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\,F(\text{ArcSin}(\sqrt{}}{5a^{5/2}}$$

```
[Out] 2/15*x^3/a+1/5*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^5+2/5*Ellipt
icE(x*a^(1/2),I)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^(5/2)-2/5*EllipticF(
x*a^(1/2),I)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^(5/2)
```

Rubi [A]
time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$,
Rules used = {6470, 30, 265, 313, 227, 1213, 435}

$$-\frac{2\sqrt{\frac{1}{ax^2+1}}\,\sqrt{ax^2+1}\,F(\text{ArcSin}(\sqrt{a}\,x)|-1)}{5a^{5/2}}+\frac{2\sqrt{\frac{1}{ax^2+1}}\,\sqrt{ax^2+1}\,E(\text{ArcSin}(\sqrt{a}\,x)|-1)}{5a^{5/2}}+\frac{2x^3}{15a}+\frac{1}{5}x^5e^{\text{sech}^{-1}(ax^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2]*x^4,x]
```

```
[Out] (2*x^3)/(15*a) + (E^ArcSech[a*x^2]*x^5)/5 + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[
1 + a*x^2]*EllipticE[ArcSin[Sqrt[a]*x], -1])/(5*a^(5/2)) - (2*Sqrt[(1 + a*x
^2)^(-1)]*Sqrt[1 + a*x^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/(5*a^(5/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int e^{\operatorname{sech}^{-1}(ax^2)}x^4\,dx = \frac{1}{5}e^{\operatorname{sech}^{-1}(ax^2)}x^5 + \frac{2\int x^2\,dx}{5a} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{x^2}{\sqrt{1-ax^2}\sqrt{1+ax^2}}\,dx}{5a}$$

$$= \frac{2x^3}{15a} + \frac{1}{5}e^{\operatorname{sech}^{-1}(ax^2)}x^5 + \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{x^2}{\sqrt{1-a^2x^4}}\,dx}{5a}$$

$$= \frac{2x^3}{15a} + \frac{1}{5}e^{\operatorname{sech}^{-1}(ax^2)}x^5 - \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{1}{\sqrt{1-a^2x^4}}\,dx}{5a^2} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\right.}{}$$

$$= \frac{2x^3}{15a} + \frac{1}{5}e^{\operatorname{sech}^{-1}(ax^2)}x^5 - \frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,F\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-1\right)}{5a^{5/2}} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\right.}{}$$

$$= \frac{2x^3}{15a} + \frac{1}{5}e^{\operatorname{sech}^{-1}(ax^2)}x^5 + \frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,E\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-1\right)}{5a^{5/2}} - \frac{2\sqrt{\frac{1}{1+ax^2}}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.30, size = 140, normalized size = 1.25

$$\frac{1}{15}\left(\frac{5x^3}{a}+\frac{3\sqrt{\frac{1-ax^2}{1+ax^2}}\,(x^3+ax^5)}{a}+\frac{6i\sqrt{\frac{1-ax^2}{1+ax^2}}\,\sqrt{1-a^2x^4}\,\left(E\left(i\sinh^{-1}\left(\sqrt{-a}\,x\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\sqrt{-a}\,x\right)\middle|-1\right)\right)}{(-a)^{5/2}\,(-1+ax^2)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]*x^4,x]
```

```
[Out] ((5*x^3)/a + (3*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(x^3 + a*x^5))/a + ((6*I)*Sqr
t[(1 - a*x^2)/(1 + a*x^2)]*Sqrt[1 - a^2*x^4]*(EllipticE[I*ArcSinh[Sqrt[-a]*
x], -1] - EllipticF[I*ArcSinh[Sqrt[-a]*x], -1]))/((-a)^(5/2)*(-1 + a*x^2)))
/15
```

Maple [A]
time = 0.03, size = 136, normalized size = 1.21

method	result
default	$\frac{\sqrt{-\frac{a x^2-1}{a x^2}}\, x^2 \sqrt{\frac{a x^2+1}{a x^2}}\left(a^{\frac{7}{2}} x^7-x^3 a^{\frac{3}{2}}+2\,\mathrm{EllipticF}\left(x\sqrt{a},i\right)\sqrt{-a x^2+1}\,\sqrt{a x^2+1}-2\sqrt{-a x^2+1}\,\sqrt{a x^2+}\right.}{5(a^2x^4-1)a^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^4,x,method=_RETURNVERBO
SE)
```

```
[Out] 1/5*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(a^(7/2)*x^7-x^3*a
^(3/2)+2*EllipticF(x*a^(1/2),I)*(-a*x^2+1)^(1/2)*(a*x^2+1)^(1/2)-2*(-a*x^2+
1)^(1/2)*(a*x^2+1)^(1/2)*EllipticE(x*a^(1/2),I))/(a^2*x^4-1)/a^(3/2)+1/3*x^
3/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^4,x, algorithm="m
axima")
```

```
[Out] 1/3*x^3/a + integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)*x^2, x)/a
```

Fricas [A]
time = 0.09, size = 58, normalized size = 0.52

$$\frac{5\,ax^3+3\,(a^2x^5-2\,x)\sqrt{\frac{ax^2+1}{ax^2}}\,\sqrt{-\frac{ax^2-1}{ax^2}}}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^4,x, algorithm="f
ricas")
```

```
[Out] 1/15*(5*a*x^3 + 3*(a^2*x^5 - 2*x)*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 -
1)/(a*x^2)))/a^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int x^2\,dx + \int ax^4\sqrt{-1+\frac{1}{ax^2}}\,\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**4,x)
```

```
[Out] (Integral(x**2, x) + Integral(a*x**4*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x*
*2)), x))/a
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^4,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(\sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}+\frac{1}{a\,x^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] int(x^4*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)), x)
```

3.49 $\int e^{\text{sech}^{-1}(ax^2)}x^3\,dx$

Optimal. Leaf size=63

$$\frac{x^2}{4a}+\frac{1}{4}e^{\text{sech}^{-1}(ax^2)}x^4+\frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,\text{ArcSin}(ax^2)}{4a^2}$$

```
[Out] 1/4*x^2/a+1/4*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^4+1/4*arcsin(
a*x^2)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6470, 30, 265, 281, 222}

$$\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,\text{ArcSin}(ax^2)}{4a^2}+\frac{x^2}{4a}+\frac{1}{4}x^4e^{\text{sech}^{-1}(ax^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2]*x^3,x]
```

```
[Out] x^2/(4*a) + (E^ArcSech[a*x^2]*x^4)/4 + (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x
^2]*ArcSin[a*x^2])/(4*a^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}
\int e^{\operatorname{sech}^{-1}(ax^2)}x^3\,dx &= \frac{1}{4}e^{\operatorname{sech}^{-1}(ax^2)}x^4+\frac{\int x\,dx}{2a}+\frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int\frac{x}{\sqrt{1-ax^2}\sqrt{1+ax^2}}\,dx}{2a}\\
&= \frac{x^2}{4a}+\frac{1}{4}e^{\operatorname{sech}^{-1}(ax^2)}x^4+\frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int\frac{x}{\sqrt{1-a^2x^4}}\,dx}{2a}\\
&= \frac{x^2}{4a}+\frac{1}{4}e^{\operatorname{sech}^{-1}(ax^2)}x^4+\frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-a^2x^2}}\,dx,x,x^2\right)}{4a}\\
&= \frac{x^2}{4a}+\frac{1}{4}e^{\operatorname{sech}^{-1}(ax^2)}x^4+\frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sin^{-1}(ax^2)}{4a^2}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 92, normalized size = 1.46

$$\frac{2ax^2+a\sqrt{\frac{1-ax^2}{1+ax^2}}\,(x^2+ax^4)+i\log\left(-2iax^2+2\sqrt{\frac{1-ax^2}{1+ax^2}}\,(1+ax^2)\right)}{4a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]*x^3,x]
```

```
[Out] (2*a*x^2 + a*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(x^2 + a*x^4) + I*Log[(-2*I)*a*x
^2 + 2*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2)])/(4*a^2)
```

Maple [A]
time = 0.07, size = 112, normalized size = 1.78

method	result	size
default	$\frac{\sqrt{-\frac{a\,x^{2}-1}{a\,x^{2}}}\,x^{2}\sqrt{\frac{a\,x^{2}+1}{a\,x^{2}}}\,\left(x^{2}\sqrt{-\frac{a^{2}x^{4}-1}{a^{2}}}\,a^{2}+\arctan\left(\frac{x^{2}}{\sqrt{-\frac{a^{2}x^{4}-1}{a^{2}}}}\right)\right)}{4\sqrt{-\frac{a^{2}x^{4}-1}{a^{2}}}\,a^{2}}+\frac{x^{2}}{2a}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^3,x,method=_RETURNVERBO
SE)
```

```
[Out] 1/4*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(x^2*(-(a^2*x^4-1)
/a^2)^(1/2)*a^2+arctan(x^2/(-(a^2*x^4-1)/a^2)^(1/2)))/(-(a^2*x^4-1)/a^2)^(1
/2)/a^2+1/2*x^2/a
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^3,x, algorithm="m
axima")
```

```
[Out] 1/2*x^2/a + integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)*x, x)/a
```

Fricas [**A**]
time = 0.59, size = 102, normalized size = 1.62

$$\frac{a^{2}x^{4}\sqrt{\frac{ax^{2}+1}{ax^{2}}}\sqrt{-\frac{ax^{2}-1}{ax^{2}}}+2\,ax^{2}-2\,\arctan\left(\frac{ax^{2}\sqrt{\frac{ax^{2}+1}{ax^{2}}}\sqrt{-\frac{ax^{2}-1}{ax^{2}}}-1}{ax^{2}}\right)}{4\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^3,x, algorithm="f
ricas")
```

```
[Out] 1/4*(a^2*x^4*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + 2*a*x^2
 - 2*arctan((a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - 1
)/(a*x^2)))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int x\,dx + \int ax^3\sqrt{-1+\frac{1}{ax^2}}\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**3,x)
```

```
[Out] (Integral(x, x) + Integral(a*x**3*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2)
), x))/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(56) = 112.
time = 0.42, size = 132, normalized size = 2.10

$$\frac{2\,a^2x^2+4\,a\arcsin\left(\frac{\sqrt{2}\,\sqrt{a^2x^2+a}}{2\,\sqrt{a}}\right)+2\,\sqrt{a^2x^2+a}\,\sqrt{-a^2x^2+a}+2\,a-\frac{2\,a^2\arcsin\left(\frac{\sqrt{2}\,\sqrt{a^2x^2+a}}{2\,\sqrt{a}}\right)-\sqrt{a^2x^2+a}\,(a^2x^2-2\,a)\sqrt{-a^2x^2+a}}{a}}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^3,x, algorithm="g
iac")
```

```
[Out] 1/4*(2*a^2*x^2 + 4*a*arcsin(1/2*sqrt(2)*sqrt(a^2*x^2 + a)/sqrt(a)) + 2*sqrt
(a^2*x^2 + a)*sqrt(-a^2*x^2 + a) + 2*a - (2*a^2*arcsin(1/2*sqrt(2)*sqrt(a^2
*x^2 + a)/sqrt(a)) - sqrt(a^2*x^2 + a)*(a^2*x^2 - 2*a)*sqrt(-a^2*x^2 + a))/
a)/a^3
```

Mupad [B]
time = 7.14, size = 306, normalized size = 4.86

$$\frac{\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}+1\right)\,1\mathrm{i}}{4\,a^2}-\frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x^2}+1}-1}\right)\,1\mathrm{i}}{4\,a^2}+\frac{\frac{1\mathrm{i}}{64\,a^2}+\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{32\,a^2\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^4\,15\mathrm{i}}{64\,a^2\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^4}}{\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}+\frac{2\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^6}}+\frac{x^2}{2\,a}+\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{64\,a^2\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] (log(((1/(a*x^2) - 1)^(1/2) - 1i)^2/((1/(a*x^2) + 1)^(1/2) - 1)^2 + 1)*1i)/
(4*a^2) - (log(((1/(a*x^2) - 1)^(1/2) - 1i)/((1/(a*x^2) + 1)^(1/2) - 1))*1i
)/(4*a^2) + (1i/(64*a^2) + (((1/(a*x^2) - 1)^(1/2) - 1i)^2*1i)/(32*a^2*((1/
```

```
(a*x^2) + 1)^(1/2) - 1)^2) - (((1/(a*x^2) - 1)^(1/2) - 1i)^4*15i)/(64*a^2*(
(1/(a*x^2) + 1)^(1/2) - 1)^4))/(((1/(a*x^2) - 1)^(1/2) - 1i)^2/((1/(a*x^2)
+ 1)^(1/2) - 1)^2 + (2*((1/(a*x^2) - 1)^(1/2) - 1i)^4)/((1/(a*x^2) + 1)^(1/
2) - 1)^4 + ((1/(a*x^2) - 1)^(1/2) - 1i)^6/((1/(a*x^2) + 1)^(1/2) - 1)^6) +
 x^2/(2*a) + (((1/(a*x^2) - 1)^(1/2) - 1i)^2*1i)/(64*a^2*((1/(a*x^2) + 1)^(
1/2) - 1)^2)
```

3.50 $\int e^{\text{sech}^{-1}(ax^2)}x^2\,dx$

Optimal. Leaf size=67

$$\frac{2x}{3a}+\frac{1}{3}e^{\text{sech}^{-1}(ax^2)}x^3+\frac{2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\,F\left(\text{ArcSin}\left(\sqrt{a}\,x\right)\middle|-1\right)}{3a^{3/2}}$$

```
[Out] 2/3*x/a+1/3*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^3+2/3*EllipticF
(x*a^(1/2),I)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6470, 8, 254, 227}

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\,\sqrt{ax^2+1}\,F\left(\text{ArcSin}\left(\sqrt{a}\,x\right)\middle|-1\right)}{3a^{3/2}}+\frac{1}{3}x^3e^{\text{sech}^{-1}(ax^2)}+\frac{2x}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2]*x^2,x]
```

```
[Out] (2*x)/(3*a) + (E^ArcSech[a*x^2]*x^3)/3 + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 +
 a*x^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/(3*a^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
```

```
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}\int e^{\operatorname{sech}^{-1}(ax^2)}x^2\,dx &= \frac{1}{3}e^{\operatorname{sech}^{-1}(ax^2)}x^3+\frac{2\int 1\,dx}{3a}+\frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int\frac{1}{\sqrt{1-ax^2}\sqrt{1+ax^2}}\,dx}{3a}\\ &= \frac{2x}{3a}+\frac{1}{3}e^{\operatorname{sech}^{-1}(ax^2)}x^3+\frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int\frac{1}{\sqrt{1-a^2x^4}}\,dx}{3a}\\ &= \frac{2x}{3a}+\frac{1}{3}e^{\operatorname{sech}^{-1}(ax^2)}x^3+\frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,F\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-1\right)}{3a^{3/2}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.14, size = 116, normalized size = 1.73

$$\frac{x}{a}+\frac{\sqrt{\frac{1-ax^2}{1+ax^2}}\,(x+ax^3)}{3a}-\frac{2i\sqrt{\frac{1-ax^2}{1+ax^2}}\sqrt{1-a^2x^4}\,F\left(i\sinh^{-1}\left(\sqrt{-a}\,x\right)\middle|-1\right)}{3(-a)^{3/2}\left(-1+ax^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]*x^2,x]
```

```
[Out] x/a + (Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(x + a*x^3))/(3*a) - (((2*I)/3)*Sqrt[(
1 - a*x^2)/(1 + a*x^2)]*Sqrt[1 - a^2*x^4]*EllipticF[I*ArcSinh[Sqrt[-a]*x],
-1])/((-a)^(3/2)*(-1 + a*x^2))
```

Maple [A]
time = 0.03, size = 102, normalized size = 1.52

method	result	size
default	$\frac{\sqrt{-\frac{a\,x^2-1}{a\,x^2}}\,x^2\sqrt{\frac{a\,x^2+1}{a\,x^2}}\left(a^{\frac{5}{2}}x^5-2\,\mathrm{EllipticF}\left(x\sqrt{a}\,,i\right)\sqrt{-a\,x^2+1}\,\sqrt{a\,x^2+1}-x\sqrt{a}\right)}{3(a^2x^4-1)\sqrt{a}}+\frac{x}{a}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^2,x,method=_RETURNVERBO
SE)
```

```
[Out] 1/3*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(a^(5/2)*x^5-2*Ell
ipticF(x*a^(1/2),I)*(-a*x^2+1)^(1/2)*(a*x^2+1)^(1/2)-x*a^(1/2))/(a^2*x^4-1)
/a^(1/2)+x/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^2,x, algorithm="m
axima")
```

```
[Out] x/a + integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1), x)/a
```

Fricas [A]
time = 0.13, size = 47, normalized size = 0.70

$$\frac{ax^3\sqrt{\frac{ax^2+1}{ax^2}}\sqrt{-\frac{ax^2-1}{ax^2}}+3\,x}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^2,x, algorithm="f
ricas")
```

```
[Out] 1/3*(a*x^3*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + 3*x)/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int 1\,dx+\int ax^2\sqrt{-1+\frac{1}{ax^2}}\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**2,x)
```

```
[Out] (Integral(1, x) + Integral(a*x**2*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2)
), x))/a
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^2,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(\sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}+\frac{1}{a\,x^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] int(x^2*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)), x)
```

3.51 $\int e^{\text{sech}^{-1}(ax^2)}x\,dx$

Optimal. Leaf size=68

$$\frac{1}{2}e^{\text{sech}^{-1}(ax^2)}x^2-\frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\tanh^{-1}\left(\sqrt{1-a^2x^4}\right)}{2a}+\frac{\log(x)}{a}$$

```
[Out] 1/2*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^2+ln(x)/a-1/2*arctanh((
-a^2*x^4+1)^(1/2))*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6470, 29, 265, 272, 65, 214}

$$-\frac{\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\tanh^{-1}\left(\sqrt{1-a^2x^4}\right)}{2a}+\frac{1}{2}x^2e^{\text{sech}^{-1}(ax^2)}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2]*x,x]
```

```
[Out] (E^ArcSech[a*x^2]*x^2)/2 - (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*ArcTanh[
Sqrt[1 - a^2*x^4]])/(2*a) + Log[x]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
```

```
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}
\int e^{\operatorname{sech}^{-1}(ax^2)} x\,dx &= \frac{1}{2} e^{\operatorname{sech}^{-1}(ax^2)} x^2 + \frac{\int \frac{1}{x}\,dx}{a} + \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{1}{x\sqrt{1-ax^2}\sqrt{1+ax^2}}\,dx}{a} \\
&= \frac{1}{2} e^{\operatorname{sech}^{-1}(ax^2)} x^2 + \frac{\log(x)}{a} + \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{1}{x\sqrt{1-a^2x^4}}\,dx}{a} \\
&= \frac{1}{2} e^{\operatorname{sech}^{-1}(ax^2)} x^2 + \frac{\log(x)}{a} + \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^4\right)}{4a} \\
&= \frac{1}{2} e^{\operatorname{sech}^{-1}(ax^2)} x^2 + \frac{\log(x)}{a} - \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^4}\right)}{2a^3} \\
&= \frac{1}{2} e^{\operatorname{sech}^{-1}(ax^2)} x^2 - \frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\tanh^{-1}\left(\sqrt{1-a^2x^4}\right)}{2a} + \frac{\log(x)}{a}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 100, normalized size = 1.47

$$\frac{\sqrt{\frac{1-ax^2}{1+ax^2}}\left(1+ax^2\right)+2\log\left(ax^2\right)-\log\left(1+\sqrt{\frac{1-ax^2}{1+ax^2}}+ax^2\sqrt{\frac{1-ax^2}{1+ax^2}}\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]*x,x]
```

```
[Out] (Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2) + 2*Log[a*x^2] - Log[1 + Sqrt[(1
 - a*x^2)/(1 + a*x^2)] + a*x^2*Sqrt[(1 - a*x^2)/(1 + a*x^2)]])/(2*a)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.10, size = 127, normalized size = 1.87

method	result	size
default	$\frac{\sqrt{-\frac{a\,x^{2}-1}{a\,x^{2}}}\,x^{2}\sqrt{\frac{a\,x^{2}+1}{a\,x^{2}}}\,\left(\mathrm{csgn}\left(\frac{1}{a}\right)a\sqrt{-\frac{a^{2}x^{4}-1}{a^{2}}}-\ln\left(\frac{2\,\mathrm{csgn}\left(\frac{1}{a}\right)a\sqrt{-\frac{a^{2}x^{4}-1}{a^{2}}}+2}{a^{2}x^{2}}\right)\right)\mathrm{csgn}\left(\frac{1}{a}\right)}{2a\sqrt{-\frac{a^{2}x^{4}-1}{a^{2}}}}+\frac{\ln\left(x\right)}{a}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/2*(-(a*x^2-1)/a/x^2)^(1/2)*x^2*((a*x^2+1)/a/x^2)^(1/2)*(csgn(1/a)*a*(-(a^
2*x^4-1)/a^2)^(1/2)-ln(2*(csgn(1/a)*a*(-(a^2*x^4-1)/a^2)^(1/2)+1)/a^2/x^2))
*csgn(1/a)/a/(-(a^2*x^4-1)/a^2)^(1/2)+ln(x)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/x, x)/a + log(x)/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(61) = 122.
time = 0.36, size = 133, normalized size = 1.96

$$\frac{2\,ax^{2}\sqrt{\frac{ax^{2}+1}{ax^{2}}}\,\sqrt{-\frac{ax^{2}-1}{ax^{2}}}-\log\left(ax^{2}\sqrt{\frac{ax^{2}+1}{ax^{2}}}\,\sqrt{-\frac{ax^{2}-1}{ax^{2}}}+1\right)+\log\left(ax^{2}\sqrt{\frac{ax^{2}+1}{ax^{2}}}\,\sqrt{-\frac{ax^{2}-1}{ax^{2}}}-1\right)+4\,\log\left(x\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x,x, algorithm="fri
cas")
```

```
[Out] 1/4*(2*a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - log(a*x
^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + 1) + log(a*x^2*sq
rt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - 1) + 4*log(x))/a
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x}\,dx + \int ax\sqrt{-1+\frac{1}{ax^2}}\,\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x,x)
```

```
[Out] (Integral(1/x, x) + Integral(a*x*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2))
, x))/a
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x,x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 3.20, size = 182, normalized size = 2.68

$$\frac{\ln(x)}{a}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x^2}+1}-1}\right)}{a}+\frac{\frac{5\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}+1}{\frac{8\,a\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)}{\sqrt{\frac{1}{a\,x^2}+1}-1}+\frac{8\,a\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^3}}+\frac{\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}}{8\,a\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] log(x)/a - (2*atanh(((1/(a*x^2) - 1)^(1/2) - 1i)/((1/(a*x^2) + 1)^(1/2) - 1
)))/a + ((5*((1/(a*x^2) - 1)^(1/2) - 1i)^2)/((1/(a*x^2) + 1)^(1/2) - 1)^2 +
 1)/((8*a*((1/(a*x^2) - 1)^(1/2) - 1i))/((1/(a*x^2) + 1)^(1/2) - 1) + (8*a*
((1/(a*x^2) - 1)^(1/2) - 1i)^3)/((1/(a*x^2) + 1)^(1/2) - 1)^3) + ((1/(a*x^2
) - 1)^(1/2) - 1i)/(8*a*((1/(a*x^2) + 1)^(1/2) - 1))
```

3.52 $\int e^{\text{sech}^{-1}(ax^2)}\,dx$

Optimal. Leaf size=147

$$-\frac{2}{ax}+e^{\text{sech}^{-1}(ax^2)}x-\frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{ax}-\frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}E\left(\text{ArcSin}\left(\sqrt{a}\,x\right)\middle|-1\right)}{\sqrt{a}}+\frac{2}{}$$

```
[Out] -2/a/x+(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x-2*EllipticE(x*a^(1/2
),I)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^(1/2)+2*EllipticF(x*a^(1/2),I)*(
1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)/a^(1/2)-2*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^
(1/2)*(-a^2*x^4+1)^(1/2)/a/x
```

Rubi [A]

time = 0.05, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6465, 30, 265, 331, 313, 227, 1213, 435}

$$-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{ax}+\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}F\left(\text{ArcSin}\left(\sqrt{a}\,x\right)\middle|-1\right)}{\sqrt{a}}-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}E\left(\text{ArcSin}\left(\sqrt{a}\,x\right)\middle|-1\right)}{\sqrt{a}}+xe^{\text{sech}^{-1}(ax^2)}-\frac{2}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2],x]
```

```
[Out] -2/(a*x) + E^ArcSech[a*x^2]*x - (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*S
qrt[1 - a^2*x^4])/(a*x) - (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*Ellipti
cE[ArcSin[Sqrt[a]*x], -1])/Sqrt[a] + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x
^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/Sqrt[a]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 6465

```
Int[E^ArcSech[(a_.)*(x_)^(p_)], x_Symbol] :> Simp[x*E^ArcSech[a*x^p], x] +
(Dist[p/a, Int[1/x^p, x], x] + Dist[p*(Sqrt[1 + a*x^p]/a)*Sqrt[1/(1 + a*x^p
)], Int[1/(x^p*Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, p}, x
]
```

Rubi steps

$$\int e^{\operatorname{sech}^{-1}(ax^2)}\,dx = e^{\operatorname{sech}^{-1}(ax^2)}x + \frac{2\int \frac{1}{x^2}\,dx}{a} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\right)\int \frac{1}{x^2\sqrt{1-ax^2}\,\sqrt{1+ax^2}}\,dx}{a}$$

$$= -\frac{2}{ax} + e^{\operatorname{sech}^{-1}(ax^2)}x + \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\right)\int \frac{1}{x^2\sqrt{1-a^2x^4}}\,dx}{a}$$

$$= -\frac{2}{ax} + e^{\operatorname{sech}^{-1}(ax^2)}x - \frac{2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\,\sqrt{1-a^2x^4}}{ax} - \left(2a\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\right)$$

$$= -\frac{2}{ax} + e^{\operatorname{sech}^{-1}(ax^2)}x - \frac{2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\,\sqrt{1-a^2x^4}}{ax} + \left(2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\right)\int$$

$$= -\frac{2}{ax} + e^{\operatorname{sech}^{-1}(ax^2)}x - \frac{2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\,\sqrt{1-a^2x^4}}{ax} + \frac{2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\,F(\mathrm{si}}{\sqrt{a}}$$

$$= -\frac{2}{ax} + e^{\operatorname{sech}^{-1}(ax^2)}x - \frac{2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\,\sqrt{1-a^2x^4}}{ax} - \frac{2\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\,E(\mathrm{si}}{\sqrt{a}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.20, size = 135, normalized size = 0.92

$$-\frac{1}{ax} + \left(-\frac{1}{ax} - x\right)\sqrt{\frac{1-ax^2}{1+ax^2}} - \frac{2i\sqrt{\frac{1-ax^2}{1+ax^2}}\,\sqrt{1-a^2x^4}\,\left(E\left(i\sinh^{-1}\left(\sqrt{-a}\,x\right)\middle|-1\right) - F\left(i\sinh^{-1}\left(\sqrt{-a}\,x\right)\middle|-1\right)\right)}{\sqrt{-a}\,(-1+ax^2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2],x]
```

```
[Out] -(1/(a*x)) + (-(1/(a*x)) - x)*Sqrt[(1 - a*x^2)/(1 + a*x^2)] - ((2*I)*Sqrt[(
1 - a*x^2)/(1 + a*x^2)]*Sqrt[1 - a^2*x^4]*(EllipticE[I*ArcSinh[Sqrt[-a]*x],
 -1] - EllipticF[I*ArcSinh[Sqrt[-a]*x], -1]))/(Sqrt[-a]*(-1 + a*x^2))
```

Maple [A]
time = 0.10, size = 132, normalized size = 0.90

method	result

default	$-\frac{1}{ax}-\frac{\sqrt{-\frac{a\,x^2-1}{a\,x^2}}\,x\sqrt{\frac{a\,x^2+1}{a\,x^2}}\,\left(a^2x^4+2\sqrt{-a\,x^2+1}\,\sqrt{a\,x^2+1}\,x\,\mathrm{EllipticF}\left(x\sqrt{a},i\right)\sqrt{a}-2\sqrt{-a\,x^2+1}\right.}{a^2x^4-1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/x-(-(a*x^2-1)/a/x^2)^(1/2)*x*((a*x^2+1)/a/x^2)^(1/2)*(a^2*x^4+2*(-a*x^
2+1)^(1/2)*(a*x^2+1)^(1/2)*x*EllipticF(x*a^(1/2),I)*a^(1/2)-2*(-a*x^2+1)^(1
/2)*(a*x^2+1)^(1/2)*x*EllipticE(x*a^(1/2),I)*a^(1/2)-1)/(a^2*x^4-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/x^2, x)/a - 1/(a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + 1)/(
a*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^2}\,dx+\int a\sqrt{-1+\frac{1}{ax^2}}\,\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2),x)
```

```
[Out] (Integral(x**(-2), x) + Integral(a*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2
)), x))/a
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(1/(a*x^2) + 1)*sqrt(1/(a*x^2) - 1) + 1/(a*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}+\frac{1}{a\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2),x)
[Out] int((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2), x)
```

3.53 $\int \frac{e^{\operatorname{sech}^{-1}\left(ax^2\right)}}{x}\,dx$

Optimal. Leaf size=80

$$-\frac{1}{2ax^2}-\frac{\sqrt{1-ax^2}}{2ax^2\sqrt{\frac{1}{1+ax^2}}}-\frac{1}{2}\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\,\mathrm{ArcSin}\left(ax^2\right)$$

[Out] -1/2/a/x^2-1/2*(-a*x^2+1)^(1/2)/a/x^2/(1/(a*x^2+1))^(1/2)-1/2*arcsin(a*x^2)
*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)

Rubi [A]

time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6469, 265, 281, 283, 222}

$$-\frac{\sqrt{\frac{1}{ax^2+1}}\,\sqrt{ax^2+1}\,\sqrt{1-a^2x^4}}{2ax^2}-\frac{1}{2}\sqrt{\frac{1}{ax^2+1}}\,\sqrt{ax^2+1}\,\mathrm{ArcSin}\left(ax^2\right)-\frac{1}{2ax^2}$$

Warning: Unable to verify antiderivative.

[In] Int[E^ArcSech[a*x^2]/x,x]

[Out] -1/2*1/(a*x^2) - (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*Sqrt[1 - a^2*x^4])
/(2*a*x^2) - (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*ArcSin[a*x^2])/2

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6469

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]/(x_), x_Symbol] :> -Simp[(a*p*x^p)^(-1), x]
 + Dist[(Sqrt[1 + a*x^p]/a)*Sqrt[1/(1 + a*x^p)], Int[Sqrt[1 + a*x^p]*(Sqrt[
1 - a*x^p]/x^(p + 1)), x], x] /; FreeQ[{a, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x}\,dx &= -\frac{1}{2ax^2}+\frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{\sqrt{1-ax^2}\sqrt{1+ax^2}}{x^3}\,dx}{a}\\
&= -\frac{1}{2ax^2}+\frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{\sqrt{1-a^2x^4}}{x^3}\,dx}{a}\\
&= -\frac{1}{2ax^2}+\frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1-a^2x^2}}{x^2}\,dx,x,x^2\right)}{2a}\\
&= -\frac{1}{2ax^2}-\frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{2ax^2}-\frac{1}{2}\left(a\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\operatorname{Subst}\left(\int \frac{}{\sqrt{}}\right.\\
&= -\frac{1}{2ax^2}-\frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{2ax^2}-\frac{1}{2}\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sin^{-1}\left(ax^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 113, normalized size = 1.41

$$-\frac{1}{2ax^2}+\left(-\frac{1}{2}-\frac{1}{2ax^2}\right)\sqrt{\frac{1-ax^2}{1+ax^2}}+\frac{\sqrt{\frac{1-ax^2}{1+ax^2}}\,(1+ax^2)\tanh^{-1}\left(\frac{ax^2}{\sqrt{-1+a^2x^4}}\right)}{2\sqrt{-1+a^2x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]/x,x]
```

```
[Out] -1/2*1/(a*x^2) + (-1/2 - 1/(2*a*x^2))*Sqrt[(1 - a*x^2)/(1 + a*x^2)] + (Sqrt
[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2)*ArcTanh[(a*x^2)/Sqrt[-1 + a^2*x^4]])/
(2*Sqrt[-1 + a^2*x^4])
```

Maple [A]
time = 0.07, size = 103, normalized size = 1.29

method	result	size
default	$-\frac{\sqrt{-\frac{a\,x^{2}-1}{a\,x^{2}}}\,\sqrt{\frac{a\,x^{2}+1}{a\,x^{2}}}\,\left(\arctan\left(\frac{x^{2}}{\sqrt{-\frac{a^{2}x^{4}-1}{a^{2}}}}\right)x^{2}+\sqrt{-\frac{a^{2}x^{4}-1}{a^{2}}}\right)}{2\sqrt{-\frac{a^{2}x^{4}-1}{a^{2}}}}-\frac{1}{2a\,x^{2}}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/2*(-(a*x^2-1)/a/x^2)^(1/2)*((a*x^2+1)/a/x^2)^(1/2)*(arctan(x^2/(-(a^2*x^
4-1)/a^2)^(1/2))*x^2+(-(a^2*x^4-1)/a^2)^(1/2))/(-(a^2*x^4-1)/a^2)^(1/2)-1/2
/a/x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/x^3, x)/a - 1/2/(a*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(44) = 88$.
time = 0.42, size = 102, normalized size = 1.28

$$-\frac{ax^{2}\sqrt{\frac{ax^{2}+1}{ax^{2}}}\,\sqrt{-\frac{ax^{2}-1}{ax^{2}}}-2\,ax^{2}\arctan\left(\frac{ax^{2}\sqrt{\frac{ax^{2}+1}{ax^{2}}}\,\sqrt{-\frac{ax^{2}-1}{ax^{2}}}-1}{ax^{2}}\right)+1}{2\,ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x,x, algorithm="fri
cas")
```

```
[Out] -1/2*(a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - 2*a*x^2*
arctan((a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - 1)/(a*
x^2)) + 1)/(a*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^3}\,dx + \int \frac{a\sqrt{-1+\frac{1}{ax^2}}\,\sqrt{1+\frac{1}{ax^2}}}{x}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))/x,x)
```

```
[Out] (Integral(x**(-3), x) + Integral(a*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2
))/x, x))/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(44) = 88.
time = 1.69, size = 252, normalized size = 3.15

$$-\frac{\left(\pi+2\,\arctan\left(\frac{\sqrt{a^2x^2+a}\left(\frac{\left(\sqrt{2}\,\sqrt{a}-\sqrt{-a^2x^2+a}\right)^2}{a^2x^2+a}-1\right)}{2\left(\sqrt{2}\,\sqrt{a}-\sqrt{-a^2x^2+a}\right)}\right)\right)a^3+\frac{4\,a^3\left(\frac{\sqrt{2}\,\sqrt{a}-\sqrt{-a^2x^2+a}}{\sqrt{a^2x^2+a}}-\frac{\sqrt{a^2x^2+a}}{\sqrt{2}\,\sqrt{a}-\sqrt{-a^2x^2+a}}\right)}{\left(\frac{\sqrt{2}\,\sqrt{a}-\sqrt{-a^2x^2+a}}{\sqrt{a^2x^2+a}}-\frac{\sqrt{a^2x^2+a}}{\sqrt{2}\,\sqrt{a}-\sqrt{-a^2x^2+a}}\right)^2-4}+\frac{a^2}{x^2}}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x,x, algorithm="gia
c")
```

```
[Out] -1/2*((pi + 2*arctan(1/2*sqrt(a^2*x^2 + a)*((sqrt(2)*sqrt(a) - sqrt(-a^2*x^
2 + a))^2/(a^2*x^2 + a) - 1)/(sqrt(2)*sqrt(a) - sqrt(-a^2*x^2 + a))))*a^3 +
 4*a^3*((sqrt(2)*sqrt(a) - sqrt(-a^2*x^2 + a))/sqrt(a^2*x^2 + a) - sqrt(a^2
*x^2 + a)/(sqrt(2)*sqrt(a) - sqrt(-a^2*x^2 + a)))/(((sqrt(2)*sqrt(a) - sqrt
(-a^2*x^2 + a))/sqrt(a^2*x^2 + a) - sqrt(a^2*x^2 + a)/(sqrt(2)*sqrt(a) - sq
rt(-a^2*x^2 + a)))^2 - 4) + a^2/x^2)/a^3
```

Mupad [B]
time = 4.00, size = 185, normalized size = 2.31

$$-\frac{\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}+1\right)\,1\mathrm{i}}{2}+\frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x^2}+1}-1}\right)\,1\mathrm{i}}{2}-\frac{1}{2\,a\,x^2}+\frac{\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2\,8\mathrm{i}}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2\left(2+\frac{2\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^4}-\frac{4\left(\sqrt{\frac{1}{a\,x^2}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x^2}+1}-1\right)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2))/x,x)
```

```
[Out] (log(((1/(a*x^2) - 1)^(1/2) - 1i)/((1/(a*x^2) + 1)^(1/2) - 1))*1i)/2 - (log
(((1/(a*x^2) - 1)^(1/2) - 1i)^2/((1/(a*x^2) + 1)^(1/2) - 1)^2 + 1)*1i)/2 -
1/(2*a*x^2) + (((1/(a*x^2) - 1)^(1/2) - 1i)^2*8i)/(((1/(a*x^2) + 1)^(1/2) -
 1)^2*((2*((1/(a*x^2) - 1)^(1/2) - 1i)^4)/((1/(a*x^2) + 1)^(1/2) - 1)^4 - (
4*((1/(a*x^2) - 1)^(1/2) - 1i)^2)/((1/(a*x^2) + 1)^(1/2) - 1)^2 + 2))
```

3.54 $\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x^2} dx$

Optimal. Leaf size=115

$$\frac{2}{3ax^3} - \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x} + \frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{3ax^3} - \frac{2}{3}\sqrt{a}\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,F\big(\text{ArcSin}(\sqrt{a}\,x)\big| -1\big)$$

```
[Out] 2/3/a/x^3-(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x-2/3*EllipticF(x*a
^(1/2),I)*a^(1/2)*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)+2/3*(1/(a*x^2+1))^(1/
2)*(a*x^2+1)^(1/2)*(-a^2*x^4+1)^(1/2)/a/x^3
```

Rubi [A]
time = 0.03, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6470, 30, 265, 331, 227}

$$\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\sqrt{1-a^2x^4}}{3ax^3} - \frac{2}{3}\sqrt{a}\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,F\big(\text{ArcSin}(\sqrt{a}\,x)\big| -1\big) + \frac{2}{3ax^3} - \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2]/x^2,x]
```

```
[Out] 2/(3*a*x^3) - E^ArcSech[a*x^2]/x + (2*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2
]*Sqrt[1 - a^2*x^4])/(3*a*x^3) - (2*Sqrt[a]*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 +
 a*x^2]*EllipticF[ArcSin[Sqrt[a]*x], -1])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\text{sech}^{-1}(ax^2)}}{x^2}\,dx &= -\frac{e^{\text{sech}^{-1}(ax^2)}}{x}-\frac{2\int \frac{1}{x^4}\,dx}{a}-\frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{1}{x^4\sqrt{1-ax^2}\sqrt{1+ax^2}}\,dx}{a}\\
&= \frac{2}{3ax^3}-\frac{e^{\text{sech}^{-1}(ax^2)}}{x}-\frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{1}{x^4\sqrt{1-a^2x^4}}\,dx}{a}\\
&= \frac{2}{3ax^3}-\frac{e^{\text{sech}^{-1}(ax^2)}}{x}+\frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{3ax^3}-\frac{1}{3}\left(2a\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right.\\
&= \frac{2}{3ax^3}-\frac{e^{\text{sech}^{-1}(ax^2)}}{x}+\frac{2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{3ax^3}-\frac{2}{3}\sqrt{a}\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.14, size = 123, normalized size = 1.07

$$-\frac{1}{3ax^3}-\frac{\sqrt{\frac{1-ax^2}{1+ax^2}}\,(1+ax^2)}{3ax^3}+\frac{2i\sqrt{-a}\sqrt{\frac{1-ax^2}{1+ax^2}}\sqrt{1-a^2x^4}\,F\left(i\sinh^{-1}\left(\sqrt{-a}\,x\right)\middle|-1\right)}{-3+3ax^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]/x^2,x]
```

```
[Out] -1/3*1/(a*x^3) - (Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2))/(3*a*x^3) + ((
2*I)*Sqrt[-a]*Sqrt[(1 - a*x^2)/(1 + a*x^2)]*Sqrt[1 - a^2*x^4]*EllipticF[I*A
rcSinh[Sqrt[-a]*x], -1])/(-3 + 3*a*x^2)
```

Maple [A]
time = 0.03, size = 104, normalized size = 0.90

method	result	size
default	$\frac{\sqrt{-\frac{a\,x^{2}-1}{a\,x^{2}}}\,\sqrt{\frac{a\,x^{2}+1}{a\,x^{2}}}\,\left(2\sqrt{-a\,x^{2}+1}\,\sqrt{a\,x^{2}+1}\,\mathrm{EllipticF}\left(x\sqrt{a}\,,i\right)x^{3}a^{\frac{3}{2}}-a^{2}x^{4}+1\right)}{3x\left(a^{2}x^{4}-1\right)}-\frac{1}{3a\,x^{3}}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^2,x,method=_RETURNVERBO
SE)
```

```
[Out] 1/3*(-(a*x^2-1)/a/x^2)^(1/2)/x*((a*x^2+1)/a/x^2)^(1/2)*(2*(-a*x^2+1)^(1/2)*
(a*x^2+1)^(1/2)*EllipticF(x*a^(1/2),I)*x^3*a^(3/2)-a^2*x^4+1)/(a^2*x^4-1)-1
/3/a/x^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/x^4, x)/a - 1/3/(a*x^3)
```

Fricas [A]
time = 0.08, size = 63, normalized size = 0.55

$$-\frac{2\,a^{\frac{3}{2}}x^{3}\mathrm{ellipticF}\left(\sqrt{a}\,x,-1\right)+ax^{2}\sqrt{\frac{ax^{2}+1}{ax^{2}}}\,\sqrt{-\frac{ax^{2}-1}{ax^{2}}}+1}{3\,ax^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^2,x, algorithm="f
ricas")
```

```
[Out] -1/3*(2*a^(3/2)*x^3*ellipticF(sqrt(a)*x, -1) + a*x^2*sqrt((a*x^2 + 1)/(a*x^
2))*sqrt(-(a*x^2 - 1)/(a*x^2)) + 1)/(a*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^4}\,dx + \int \frac{a\sqrt{-1+\frac{1}{ax^2}}\sqrt{1+\frac{1}{ax^2}}}{x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))/x**2,x)
```

```
[Out] (Integral(x**(-4), x) + Integral(a*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2
))/x**2, x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^2,x, algorithm="g
iac")
```

```
[Out] integrate((sqrt(1/(a*x^2) + 1)*sqrt(1/(a*x^2) - 1) + 1/(a*x^2))/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}+\frac{1}{a\,x^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2))/x^2,x)
```

```
[Out] int(((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2))/x^2, x)
```

3.55 $\int \frac{e^{\text{sech}^{-1}\left(ax^2\right)}}{x^3}\,dx$

Optimal. Leaf size=118

$$\frac{1}{4ax^4}-\frac{e^{\text{sech}^{-1}\left(ax^2\right)}}{2x^2}+\frac{\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\,\sqrt{1-a^2x^4}}{4ax^4}+\frac{1}{4}a\sqrt{\frac{1}{1+ax^2}}\,\sqrt{1+ax^2}\,\tanh^{-1}\left(\sqrt{1-a^2x^4}\right)$$

```
[Out] 1/4/a/x^4-1/2*(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^2+1/4*a*arcta
nh((-a^2*x^4+1)^(1/2))*(1/(a*x^2+1))^(1/2)*(a*x^2+1)^(1/2)+1/4*(1/(a*x^2+1)
)^(1/2)*(a*x^2+1)^(1/2)*(-a^2*x^4+1)^(1/2)/a/x^4
```

Rubi [A]
time = 0.04, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6470, 30, 265, 272, 44, 65, 214}

$$\frac{\sqrt{\frac{1}{ax^2+1}}\,\sqrt{ax^2+1}\,\sqrt{1-a^2x^4}}{4ax^4}+\frac{1}{4}a\sqrt{\frac{1}{ax^2+1}}\,\sqrt{ax^2+1}\,\tanh^{-1}\left(\sqrt{1-a^2x^4}\right)+\frac{1}{4ax^4}-\frac{e^{\text{sech}^{-1}\left(ax^2\right)}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2]/x^3,x]
```

```
[Out] 1/(4*a*x^4) - E^ArcSech[a*x^2]/(2*x^2) + (Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a
*x^2]*Sqrt[1 - a^2*x^4])/(4*a*x^4) + (a*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x
^2]*ArcTanh[Sqrt[1 - a^2*x^4]])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{x^3}\,dx = -\frac{e^{\operatorname{sech}^{-1}(ax^2)}}{2x^2} - \frac{\int \frac{1}{x^5}\,dx}{a} - \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{1}{x^5\sqrt{1-ax^2}\sqrt{1+ax^2}}\,dx}{a}$$

$$= \frac{1}{4ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{2x^2} - \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{1}{x^5\sqrt{1-a^2x^4}}\,dx}{a}$$

$$= \frac{1}{4ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{2x^2} - \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1-a^2x}}\,dx, x, x^4\right)}{4a}$$

$$= \frac{1}{4ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{2x^2} + \frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{4ax^4} - \frac{1}{8}\left(a\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right) \mathrm{S}$$

$$= \frac{1}{4ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{2x^2} + \frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{4ax^4} + \frac{\left(\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\mathrm{Sul}}{}$$

$$= \frac{1}{4ax^4} - \frac{e^{\operatorname{sech}^{-1}(ax^2)}}{2x^2} + \frac{\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\sqrt{1-a^2x^4}}{4ax^4} + \frac{1}{4}a\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,\mathrm{tanh}$$

Mathematica [A]
time = 0.19, size = 105, normalized size = 0.89

$$-\frac{\frac{1}{x^4} + \frac{\sqrt{\frac{1-ax^2}{1+ax^2}}\,(1+ax^2)}{x^4} - \frac{a^2\sqrt{\frac{1-ax^2}{1+ax^2}}\,(1+ax^2)\operatorname{ArcTan}\left(\sqrt{-1+a^2x^4}\right)}{\sqrt{-1+a^2x^4}}}{4a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]/x^3,x]
```

```
[Out] -1/4*(x^(-4) + (Sqrt[(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2))/x^4 - (a^2*Sqrt[
(1 - a*x^2)/(1 + a*x^2)]*(1 + a*x^2)*ArcTan[Sqrt[-1 + a^2*x^4]])/Sqrt[-1 +
a^2*x^4])/a
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.08, size = 129, normalized size = 1.09

method	result	size

default	$-\frac{\sqrt{-\frac{a\,x^{2}-1}{a\,x^{2}}}\,\sqrt{\frac{a\,x^{2}+1}{a\,x^{2}}}\,\left(-\ln\left(\frac{2\,\mathrm{csgn}\left(\frac{1}{a}\right)a\sqrt{-\frac{a^{2}x^{4}-1}{a^{2}}}+2}{a^{2}x^{2}}\right)x^{4}a+\sqrt{-\frac{a^{2}x^{4}-1}{a^{2}}}\,\mathrm{csgn}\left(\frac{1}{a}\right)\right)\mathrm{csgn}\left(\frac{1}{a}\right)}{4x^{2}\sqrt{-\frac{a^{2}x^{4}-1}{a^{2}}}}-\frac{1}{4a\,x^{4}}$	129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^3,x,method=_RETURNVERBO
SE)
```

```
[Out] -1/4*(-(a*x^2-1)/a/x^2)^(1/2)/x^2*((a*x^2+1)/a/x^2)^(1/2)*(-ln(2*(csgn(1/a)
*a*(-(a^2*x^4-1)/a^2)^(1/2)+1)/a^2/x^2)*x^4*a+(-(a^2*x^4-1)/a^2)^(1/2)*csgn
(1/a))*csgn(1/a)/(-(a^2*x^4-1)/a^2)^(1/2)-1/4/a/x^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^3,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1)/x^5, x)/a - 1/4/(a*x^4)
```

Fricas [A]
time = 0.43, size = 146, normalized size = 1.24

$$\frac{a^{2}x^{4}\log\left(ax^{2}\sqrt{\frac{ax^{2}+1}{ax^{2}}}\sqrt{-\frac{ax^{2}-1}{ax^{2}}}+1\right)-a^{2}x^{4}\log\left(ax^{2}\sqrt{\frac{ax^{2}+1}{ax^{2}}}\sqrt{-\frac{ax^{2}-1}{ax^{2}}}-1\right)-2\,ax^{2}\sqrt{\frac{ax^{2}+1}{ax^{2}}}\sqrt{-\frac{ax^{2}-1}{ax^{2}}}-2}{8\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^3,x, algorithm="f
ricas")
```

```
[Out] 1/8*(a^2*x^4*log(a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2))
 + 1) - a^2*x^4*log(a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^
2)) - 1) - 2*a*x^2*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) - 2
)/(a*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^{5}}\,dx+\int \frac{a\sqrt{-1+\frac{1}{ax^{2}}}\sqrt{1+\frac{1}{ax^{2}}}}{x^{3}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))/x**3,x)
```

```
[Out] (Integral(x**(-5), x) + Integral(a*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/(a*x**2
))/x**3, x))/a
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))/x^3,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 2.02, size = 71, normalized size = 0.60

$$\frac{a\ln\left(\sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}+\frac{1}{a\,x^2}\right)}{4}-\frac{1}{4\,a\,x^4}-\frac{\sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2))/x^3,x)
```

```
[Out] (a*log((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)))/4 - 1/(4*a
*x^4) - ((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2))/(4*x^2)
```

3.56 $\int e^{\text{sech}^{-1}(ax^3)} x^m \, dx$

Optimal. Leaf size=109

$$-\frac{3x^{-2+m}}{a(2+m-m^2)}+\frac{e^{\text{sech}^{-1}(ax^3)}x^{1+m}}{1+m}-\frac{3x^{-2+m}\sqrt{\frac{1}{1+ax^3}}\sqrt{1+ax^3}\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-2+m);\frac{4+m}{6};a^2x^6\right)}{a(2+m-m^2)}$$

```
[Out] -3*x^(-2+m)/a/(-m^2+m+2)+(1/a/x^3+(1/a/x^3-1)^(1/2)*(1/a/x^3+1)^(1/2))*x^(1
+m)/(1+m)-3*x^(-2+m)*hypergeom([1/2, -1/3+1/6*m],[2/3+1/6*m],a^2*x^6)*(1/(a
*x^3+1))^(1/2)*(a*x^3+1)^(1/2)/a/(-m^2+m+2)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6470, 30, 265, 371}

$$-\frac{3\sqrt{\frac{1}{ax^3+1}}\sqrt{ax^3+1}\,x^{m-2}\,{}_2F_1\left(\frac{1}{2},\frac{m-2}{6};\frac{m+4}{6};a^2x^6\right)}{a(-m^2+m+2)}-\frac{3x^{m-2}}{a(-m^2+m+2)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax^3)}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^3]*x^m,x]
```

```
[Out] (-3*x^(-2 + m))/(a*(2 + m - m^2)) + (E^ArcSech[a*x^3]*x^(1 + m))/(1 + m) -
(3*x^(-2 + m)*Sqrt[(1 + a*x^3)^(-1)]*Sqrt[1 + a*x^3]*Hypergeometric2F1[1/2,
 (-2 + m)/6, (4 + m)/6, a^2*x^6])/(a*(2 + m - m^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int e^{\operatorname{sech}^{-1}(ax^3)}x^m\,dx = \frac{e^{\operatorname{sech}^{-1}(ax^3)}x^{1+m}}{1+m} + \frac{3\int x^{-3+m}\,dx}{a(1+m)} + \frac{\left(3\sqrt{\frac{1}{1+ax^3}}\,\sqrt{1+ax^3}\right)\int \frac{x^{-3+m}}{\sqrt{1-ax^3}\,\sqrt{1+ax^3}}}{a(1+m)}$$

$$= -\frac{3x^{-2+m}}{a\left(2+m-m^2\right)} + \frac{e^{\operatorname{sech}^{-1}(ax^3)}x^{1+m}}{1+m} + \frac{\left(3\sqrt{\frac{1}{1+ax^3}}\,\sqrt{1+ax^3}\right)\int \frac{x^{-3+m}}{\sqrt{1-a^2x^6}}\,dx}{a(1+m)}$$

$$= -\frac{3x^{-2+m}}{a\left(2+m-m^2\right)} + \frac{e^{\operatorname{sech}^{-1}(ax^3)}x^{1+m}}{1+m} - \frac{3x^{-2+m}\sqrt{\frac{1}{1+ax^3}}\,\sqrt{1+ax^3}\;{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-2+n\right.}{a\left(2+m-m^2\right)}$$

Mathematica [A]
time = 1.81, size = 214, normalized size = 1.96

$$\frac{2^{\frac{1+m}{3}}e^{-\frac{1}{3}(1+m)\operatorname{sech}^{-1}(ax^3)}\left(\frac{e^{\operatorname{sech}^{-1}(ax^3)}}{1+e^{2\operatorname{sech}^{-1}(ax^3)}}\right)^{\frac{1+m}{3}}\left(1+e^{2\operatorname{sech}^{-1}(ax^3)}\right)^{\frac{1+m}{3}}x^{1+m}(ax^3)^{\frac{1}{3}(-1-m)}\left(e^{\frac{1}{3}(4+m)\operatorname{sech}^{-1}(ax^3)}(10+m)\,{}_2F_1\left(\frac{4+m}{6},\frac{4+m}{3};\frac{10+m}{6};-e^{2\operatorname{sech}^{-1}(ax^3)}\right)-e^{\frac{1}{3}(10+m)\operatorname{sech}^{-1}(ax^3)}(4+m)\,{}_2F_1\left(\frac{4+m}{3},\frac{10+m}{6};\frac{16+m}{6};-e^{2\operatorname{sech}^{-1}(ax^3)}\right)\right)}{(4+m)(10+m)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^3]*x^m,x]
```

```
[Out] (2^((1 + m)/3)*(E^ArcSech[a*x^3]/(1 + E^(2*ArcSech[a*x^3])))^((1 + m)/3)*(1
 + E^(2*ArcSech[a*x^3]))^((1 + m)/3)*x^(1 + m)*(a*x^3)^((-1 - m)/3)*(E^(((4
 + m)*ArcSech[a*x^3])/3)*(10 + m)*Hypergeometric2F1[(4 + m)/6, (4 + m)/3, (
10 + m)/6, -E^(2*ArcSech[a*x^3])] - E^(((10 + m)*ArcSech[a*x^3])/3)*(4 + m)
*Hypergeometric2F1[(4 + m)/3, (10 + m)/6, (16 + m)/6, -E^(2*ArcSech[a*x^3])
]))/(E^(((1 + m)*ArcSech[a*x^3])/3)*(4 + m)*(10 + m))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(\frac{1}{a\,x^3}+\sqrt{\frac{1}{a\,x^3}-1}\,\sqrt{\frac{1}{a\,x^3}+1}\right)x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^3+(1/a/x^3-1)^(1/2)*(1/a/x^3+1)^(1/2))*x^m,x)
```

```
[Out] int((1/a/x^3+(1/a/x^3-1)^(1/2)*(1/a/x^3+1)^(1/2))*x^m,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^3+(1/a/x^3-1)^(1/2)*(1/a/x^3+1)^(1/2))*x^m,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(m-3>0)', see `assume?` for more det
ails)Is
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^3+(1/a/x^3-1)^(1/2)*(1/a/x^3+1)^(1/2))*x^m,x, algorithm="f
ricas")
```

```
[Out] integral((a*x^3*x^m*sqrt((a*x^3 + 1)/(a*x^3))*sqrt(-(a*x^3 - 1)/(a*x^3)) +
x^m)/(a*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m}{x^3}\,dx + \int ax^m\sqrt{-1+\frac{1}{ax^3}}\,\sqrt{1+\frac{1}{ax^3}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**3+(1/a/x**3-1)**(1/2)*(1/a/x**3+1)**(1/2))*x**m,x)
```

```
[Out] (Integral(x**m/x**3, x) + Integral(a*x**m*sqrt(-1 + 1/(a*x**3))*sqrt(1 + 1/
(a*x**3)), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^3+(1/a/x^3-1)^(1/2)*(1/a/x^3+1)^(1/2))*x^m,x, algorithm="g
iac")
```

```
[Out] integrate(x^m*(sqrt(1/(a*x^3) + 1)*sqrt(1/(a*x^3) - 1) + 1/(a*x^3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \left(\sqrt{\frac{1}{a\,x^3}-1}\,\sqrt{\frac{1}{a\,x^3}+1}+\frac{1}{a\,x^3}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((1/(a*x^3) - 1)^(1/2)*(1/(a*x^3) + 1)^(1/2) + 1/(a*x^3)),x)
```

```
[Out] int(x^m*((1/(a*x^3) - 1)^(1/2)*(1/(a*x^3) + 1)^(1/2) + 1/(a*x^3)), x)
```

3.57 $\int e^{\text{sech}^{-1}(ax^2)} x^m \, dx$

Optimal. Leaf size=107

$$-\frac{2x^{-1+m}}{a(1-m^2)}+\frac{e^{\text{sech}^{-1}(ax^2)}x^{1+m}}{1+m}-\frac{2x^{-1+m}\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1+m);\frac{3+m}{4};a^2x^4\right)}{a(1-m^2)}$$

```
[Out] -2*x^(-1+m)/a/(-m^2+1)+(1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^(1+m
)/(1+m)-2*x^(-1+m)*hypergeom([1/2, -1/4+1/4*m],[3/4+1/4*m],a^2*x^4)*(1/(a*x
^2+1))^(1/2)*(a*x^2+1)^(1/2)/a/(-m^2+1)
```

Rubi [A]

time = 0.04, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6470, 30, 265, 371}

$$-\frac{2\sqrt{\frac{1}{ax^2+1}}\sqrt{ax^2+1}\,x^{m-1}\,{}_2F_1\left(\frac{1}{2},\frac{m-1}{4};\frac{m+3}{4};a^2x^4\right)}{a(1-m^2)}-\frac{2x^{m-1}}{a(1-m^2)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax^2)}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^2]*x^m,x]
```

```
[Out] (-2*x^(-1 + m))/(a*(1 - m^2)) + (E^ArcSech[a*x^2]*x^(1 + m))/(1 + m) - (2*x
^(-1 + m)*Sqrt[(1 + a*x^2)^(-1)]*Sqrt[1 + a*x^2]*Hypergeometric2F1[1/2, (-1
 + m)/4, (3 + m)/4, a^2*x^4])/(a*(1 - m^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int e^{\text{sech}^{-1}(ax^2)}x^m\,dx = \frac{e^{\text{sech}^{-1}(ax^2)}x^{1+m}}{1+m} + \frac{2\int x^{-2+m}\,dx}{a(1+m)} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{x^{-2+m}}{\sqrt{1-ax^2}\sqrt{1+ax^2}}}{a(1+m)}$$

$$= -\frac{2x^{-1+m}}{a\left(1-m^2\right)} + \frac{e^{\text{sech}^{-1}(ax^2)}x^{1+m}}{1+m} + \frac{\left(2\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\right)\int \frac{x^{-2+m}}{\sqrt{1-a^2x^4}}\,dx}{a(1+m)}$$

$$= -\frac{2x^{-1+m}}{a\left(1-m^2\right)} + \frac{e^{\text{sech}^{-1}(ax^2)}x^{1+m}}{1+m} - \frac{2x^{-1+m}\sqrt{\frac{1}{1+ax^2}}\sqrt{1+ax^2}\;{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1+m);\frac{3+}{}\right.}{a\left(1-m^2\right)}$$

Mathematica [A]
time = 1.72, size = 214, normalized size = 2.00

$$\frac{2^{\frac{1+m}{2}}e^{-\frac{1}{2}(1+m)\text{sech}^{-1}(ax^2)}\left(\frac{e^{\text{sech}^{-1}(ax^2)}}{1+e^{2\text{sech}^{-1}(ax^2)}}\right)^{\frac{1+m}{2}}\left(1+e^{2\text{sech}^{-1}(ax^2)}\right)^{\frac{1+m}{2}}x^{1+m}(ax^2)^{\frac{1}{2}(-1-m)}\left(e^{\frac{1}{2}(3+m)\text{sech}^{-1}(ax^2)}(7+m)\,{}_2F_1\left(\frac{3+m}{4},\frac{3+m}{2};\frac{7+m}{4};-e^{2\text{sech}^{-1}(ax^2)}\right)-e^{\frac{1}{2}(7+m)\text{sech}^{-1}(ax^2)}(3+m)\,{}_2F_1\left(\frac{3+m}{2},\frac{7+m}{4};\frac{11+m}{4};-e^{2\text{sech}^{-1}(ax^2)}\right)\right)}{(3+m)(7+m)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^2]*x^m,x]
```

```
[Out] (2^((1 + m)/2)*(E^ArcSech[a*x^2]/(1 + E^(2*ArcSech[a*x^2])))^((1 + m)/2)*(1
 + E^(2*ArcSech[a*x^2]))^((1 + m)/2)*x^(1 + m)*(a*x^2)^((-1 - m)/2)*(E^(((3
 + m)*ArcSech[a*x^2])/2)*(7 + m)*Hypergeometric2F1[(3 + m)/4, (3 + m)/2, (7
 + m)/4, -E^(2*ArcSech[a*x^2])] - E^(((7 + m)*ArcSech[a*x^2])/2)*(3 + m)*Hy
pergeometric2F1[(3 + m)/2, (7 + m)/4, (11 + m)/4, -E^(2*ArcSech[a*x^2])]))/
(E^(((1 + m)*ArcSech[a*x^2])/2)*(3 + m)*(7 + m))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(\frac{1}{a\,x^2}+\sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}\right)x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^m,x)
```

```
[Out] int((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^m,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^m,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(m-2>0)', see `assume?` for more det
ails)Is
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^m,x, algorithm="f
ricas")
```

```
[Out] integral((a*x^2*x^m*sqrt((a*x^2 + 1)/(a*x^2))*sqrt(-(a*x^2 - 1)/(a*x^2)) +
x^m)/(a*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m}{x^2}\,dx + \int ax^m\sqrt{-1+\frac{1}{ax^2}}\,\sqrt{1+\frac{1}{ax^2}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x**2+(1/a/x**2-1)**(1/2)*(1/a/x**2+1)**(1/2))*x**m,x)
```

```
[Out] (Integral(x**m/x**2, x) + Integral(a*x**m*sqrt(-1 + 1/(a*x**2))*sqrt(1 + 1/
(a*x**2)), x))/a
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x^2+(1/a/x^2-1)^(1/2)*(1/a/x^2+1)^(1/2))*x^m,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \left(\sqrt{\frac{1}{a\,x^2}-1}\,\sqrt{\frac{1}{a\,x^2}+1}+\frac{1}{a\,x^2}\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)),x)
```

```
[Out] int(x^m*((1/(a*x^2) - 1)^(1/2)*(1/(a*x^2) + 1)^(1/2) + 1/(a*x^2)), x)
```

3.58 $\int e^{\text{sech}^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=91

$$\frac{x^m}{am(1+m)}+\frac{e^{\text{sech}^{-1}(ax)}x^{1+m}}{1+m}+\frac{x^m\sqrt{\frac{1}{1+ax}}\,\sqrt{1+ax}\;{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};a^2x^2\right)}{am(1+m)}$$

```
[Out] x^m/a/m/(1+m)+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^(1+m)/(1+m)+x^m*hyp
ergeom([1/2, 1/2*m],[1+1/2*m],a^2*x^2)*(1/(a*x+1))^(1/2)*(a*x+1)^(1/2)/a/m/
(1+m)
```

Rubi [A]
time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6470, 30, 126, 371}

$$\frac{\sqrt{\frac{1}{ax+1}}\,\sqrt{ax+1}\,x^m\;{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};a^2x^2\right)}{am(m+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax)}}{m+1}+\frac{x^m}{am(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x]*x^m,x]
```

```
[Out] x^m/(a*m*(1 + m)) + (E^ArcSech[a*x]*x^(1 + m))/(1 + m) + (x^m*Sqrt[(1 + a*x
)^(-1)]*Sqrt[1 + a*x]*Hypergeometric2F1[1/2, m/2, (2 + m)/2, a^2*x^2])/(a*m
*(1 + m))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f,
m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && GtQ[a, 0] && GtQ[c, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}\int e^{\operatorname{sech}^{-1}(ax)}x^m\,dx &= \frac{e^{\operatorname{sech}^{-1}(ax)}x^{1+m}}{1+m}+\frac{\int x^{-1+m}\,dx}{a(1+m)}+\frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int\frac{x^{-1+m}}{\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a(1+m)}\\ &= \frac{x^m}{am(1+m)}+\frac{e^{\operatorname{sech}^{-1}(ax)}x^{1+m}}{1+m}+\frac{\left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int\frac{x^{-1+m}}{\sqrt{1-a^2x^2}}\,dx}{a(1+m)}\\ &= \frac{x^m}{am(1+m)}+\frac{e^{\operatorname{sech}^{-1}(ax)}x^{1+m}}{1+m}+\frac{x^m\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};a^2x^2\right)}{am(1+m)}\end{aligned}$$

Mathematica [F]
time = 0.61, size = 0, normalized size = 0.00

$$\int e^{\operatorname{sech}^{-1}(ax)}x^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^ArcSech[a*x]*x^m,x]
```

```
[Out] Integrate[E^ArcSech[a*x]*x^m, x]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^m,x)
```

```
[Out] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^m,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x^m/x, x)/a + x^m/(a*m)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^m,x, algorithm="fricas"
)
```

```
[Out] integral((a*x*x^m*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + x^m)/(a*x)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^m}{x}\,dx + \int ax^m\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x**m,x)
```

```
[Out] (Integral(x**m/x, x) + Integral(a*x**m*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))
, x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\left(\sqrt{\frac{1}{a\,x}-1}\,\sqrt{\frac{1}{a\,x}+1}+\frac{1}{a\,x}\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
[Out] int(x^m*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)), x)
```

3.59 $\int e^{\text{sech}^{-1}\left(\frac{a}{x}\right)} x^m \, dx$

Optimal. Leaf size=109

$$\frac{e^{\text{sech}^{-1}\left(\frac{a}{x}\right)} x^{1+m}}{1+m} - \frac{x^{2+m}}{a\left(2+3m+m^2\right)} - \frac{\sqrt{\frac{1}{1+\frac{a}{x}}}\sqrt{1+\frac{a}{x}}\, x^{2+m}\, {}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2-m);-\frac{m}{2};\frac{a^2}{x^2}\right)}{a\left(2+3m+m^2\right)}$$

```
[Out] (x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^(1+m)/(1+m)-x^(2+m)/a/(m^2+3*m+2)-x^(2
+m)*hypergeom([1/2, -1-1/2*m],[-1/2*m],a^2/x^2)*(1/(1+a/x))^(1/2)*(1+a/x)^(
1/2)/a/(m^2+3*m+2)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6470, 30, 265, 346, 371}

$$-\frac{\sqrt{\frac{1}{\frac{a}{x}+1}}\sqrt{\frac{a}{x}+1}\, x^{m+2}\, {}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-2);-\frac{m}{2};\frac{a^2}{x^2}\right)}{a\left(m^2+3m+2\right)} - \frac{x^{m+2}}{a\left(m^2+3m+2\right)} + \frac{x^{m+1} e^{\text{sech}^{-1}\left(\frac{a}{x}\right)}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a/x]*x^m,x]
```

```
[Out] (E^ArcSech[a/x]*x^(1 + m))/(1 + m) - x^(2 + m)/(a*(2 + 3*m + m^2)) - (Sqrt[
(1 + a/x)^(-1)]*Sqrt[1 + a/x]*x^(2 + m)*Hypergeometric2F1[1/2, (-2 - m)/2,
-1/2*m, a^2/x^2])/(a*(2 + 3*m + m^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 346

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1), Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x
, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)} x^m \, dx = \frac{e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)} x^{1+m}}{1+m} - \frac{\int x^{1+m} \, dx}{a(1+m)} - \frac{\left(\sqrt{\frac{1}{1+\frac{a}{x}}} \sqrt{1+\frac{a}{x}}\right) \int \frac{x^{1+m}}{\sqrt{1-\frac{a}{x}} \sqrt{1+\frac{a}{x}}} \, dx}{a(1+m)}$$

$$= \frac{e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)} x^{1+m}}{1+m} - \frac{x^{2+m}}{a\left(2+3m+m^2\right)} - \frac{\left(\sqrt{\frac{1}{1+\frac{a}{x}}} \sqrt{1+\frac{a}{x}}\right) \int \frac{x^{1+m}}{\sqrt{1-\frac{a^2}{x^2}}} \, dx}{a(1+m)}$$

$$= \frac{e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)} x^{1+m}}{1+m} - \frac{x^{2+m}}{a\left(2+3m+m^2\right)} + \frac{\left(\sqrt{\frac{1}{1+\frac{a}{x}}} \sqrt{1+\frac{a}{x}} \left(\frac{1}{x}\right)^m x^m\right) \operatorname{Subst}\left(\int \frac{x^{-3-m}}{\sqrt{1-a^2}} \right.}{a(1+m)}$$

$$= \frac{e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)} x^{1+m}}{1+m} - \frac{x^{2+m}}{a\left(2+3m+m^2\right)} - \frac{\sqrt{\frac{1}{1+\frac{a}{x}}} \sqrt{1+\frac{a}{x}} \, x^{2+m} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-2-m); -\frac{m}{2}; \frac{a}{x} \right.}{a\left(2+3m+m^2\right)}$$

Mathematica [A]
time = 0.62, size = 147, normalized size = 1.35

$$-\frac{2^{-1-m} a \left(\frac{e^{\operatorname{sech}^{-1}\left(\frac{a}{x}\right)}}{1+e^{2\operatorname{sech}^{-1}\left(\frac{a}{x}\right)}}\right)^{-m} \left(1+e^{2\operatorname{sech}^{-1}\left(\frac{a}{x}\right)}\right)^{-m} \left(\frac{a}{x}\right)^m x^m \left(e^{2\operatorname{sech}^{-1}\left(\frac{a}{x}\right)} m \, {}_2F_1\left(1-\frac{m}{2},-m;2-\frac{m}{2};-e^{2\operatorname{sech}^{-1}\left(\frac{a}{x}\right)}\right)-(-2+m) \, {}_2F_1\left(-m,-\frac{m}{2};1-\frac{m}{2};-e^{2\operatorname{sech}^{-1}\left(\frac{a}{x}\right)}\right)\right)}{(-2+m) m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a/x]*x^m,x]
```

```
[Out] -((2^(-1 - m)*a*(a/x)^m*x^m*(E^(2*ArcSech[a/x])*m*Hypergeometric2F1[1 - m/2
, -m, 2 - m/2, -E^(2*ArcSech[a/x])] - (-2 + m)*Hypergeometric2F1[-m, -1/2*m
, 1 - m/2, -E^(2*ArcSech[a/x])]))/((E^ArcSech[a/x]/(1 + E^(2*ArcSech[a/x]))
)^m*(1 + E^(2*ArcSech[a/x]))^m*(-2 + m)*m))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \left(\frac{x}{a}+\sqrt{\frac{x}{a}-1}\, \sqrt{\frac{x}{a}+1}\right) x^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/a+(x/a-1)^(1/2)*(x/a+1)^(1/2))*x^m,x)
```

```
[Out] int((x/a+(x/a-1)^(1/2)*(x/a+1)^(1/2))*x^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^m,x, algorithm="maxima")
```

```
[Out] x^2*x^m/(a*(m + 2)) + integrate(sqrt(a + x)*sqrt(-a + x)*x^m, x)/a
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^m,x, algorithm="fricas")
```

```
[Out] integral((a*x^m*sqrt((a + x)/a)*sqrt(-(a - x)/a) + x*x^m)/a, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int xx^{m}\, dx + \int ax^{m} \sqrt{-1+\frac{x}{a}}\, \sqrt{1+\frac{x}{a}}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/a+(-1+x/a)**(1/2)*(1+x/a)**(1/2))*x***m,x)
```

```
[Out] (Integral(x*x**m, x) + Integral(a*x**m*sqrt(-1 + x/a)*sqrt(1 + x/a), x))/a
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/a+(-1+x/a)^(1/2)*(1+x/a)^(1/2))*x^m,x, algorithm="giac")
[Out] integrate(x^m*(sqrt(x/a + 1)*sqrt(x/a - 1) + x/a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \left(\sqrt{\frac{x}{a}-1}\,\sqrt{\frac{x}{a}+1}+\frac{x}{a}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((x/a - 1)^(1/2)*(x/a + 1)^(1/2) + x/a),x)
[Out] int(x^m*((x/a - 1)^(1/2)*(x/a + 1)^(1/2) + x/a), x)
```

3.60 $\int e^{\text{sech}^{-1}(ax^p)}x^m\,dx$

Optimal. Leaf size=133

$$\frac{e^{\text{sech}^{-1}(ax^p)}x^{1+m}}{1+m}+\frac{px^{1+m-p}}{a(1+m)(1+m-p)}+\frac{px^{1+m-p}\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\,{}_2F_1\left(\frac{1}{2},\frac{1+m-p}{2p};\frac{1+m+p}{2p};a^2x^{2p}\right)}{a(1+m)(1+m-p)}$$

```
[Out] (1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^(1+m)/(1+m)+p*x^(1+m-
p)/a/(1+m)/(1+m-p)+p*x^(1+m-p)*hypergeom([1/2, 1/2*(1+m-p)/p],[1/2*(1+m+p)/
p],a^2*x^(2*p))*(1/(1+a*x^p))^(1/2)*(1+a*x^p)^(1/2)/a/(1+m)/(1+m-p)
```

Rubi [A]
time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6470, 30, 265, 371}

$$\frac{p\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\,x^{m-p+1}\,{}_2F_1\left(\frac{1}{2},\frac{m-p+1}{2p};\frac{m+p+1}{2p};a^2x^{2p}\right)}{a(m+1)(m-p+1)}+\frac{px^{m-p+1}}{a(m+1)(m-p+1)}+\frac{x^{m+1}e^{\text{sech}^{-1}(ax^p)}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^p]*x^m,x]
```

```
[Out] (E^ArcSech[a*x^p]*x^(1 + m))/(1 + m) + (p*x^(1 + m - p))/(a*(1 + m)*(1 + m
- p)) + (p*x^(1 + m - p)*Sqrt[(1 + a*x^p)^(-1)]*Sqrt[1 + a*x^p]*Hypergeomet
ric2F1[1/2, (1 + m - p)/(2*p), (1 + m + p)/(2*p), a^2*x^(2*p)])/(a*(1 + m)*
(1 + m - p))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int e^{\operatorname{sech}^{-1}(ax^p)}x^m\,dx = \frac{e^{\operatorname{sech}^{-1}(ax^p)}x^{1+m}}{1+m} + \frac{p\int x^{m-p}\,dx}{a(1+m)} + \frac{\left(p\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\int \frac{x^{m-p}}{\sqrt{1-ax^p}\sqrt{1+ax^p}}\,d}{a(1+m)}$$

$$= \frac{e^{\operatorname{sech}^{-1}(ax^p)}x^{1+m}}{1+m} + \frac{px^{1+m-p}}{a(1+m)(1+m-p)} + \frac{\left(p\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\int \frac{x^{m-p}}{\sqrt{1-a^2x^{2p}}}}{a(1+m)}$$

$$= \frac{e^{\operatorname{sech}^{-1}(ax^p)}x^{1+m}}{1+m} + \frac{px^{1+m-p}}{a(1+m)(1+m-p)} + \frac{px^{1+m-p}\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\,{}_2F_1\left(\frac{1}{2},\frac{1+m-}{2p}\right.}{a(1+m)(1+m-p)}$$

Mathematica [A]
time = 3.78, size = 212, normalized size = 1.59

$$\frac{2^{\frac{1+m}{p}}\left(\frac{e^{\operatorname{sech}^{-1}(ax^p)}}{1+e^{2\operatorname{sech}^{-1}(ax^p)}}\right)^{\frac{1+m+p}{p}}\left(1+e^{2\operatorname{sech}^{-1}(ax^p)}\right)^{\frac{1+m+p}{p}}x^{1+m}(ax^p)^{-\frac{1+m}{p}}\left((1+m+3p)\,{}_2F_1\left(\frac{1+m+p}{2p},\frac{1+m+p}{p};\frac{1+m+3p}{2p};-e^{2\operatorname{sech}^{-1}(ax^p)}\right)-e^{2\operatorname{sech}^{-1}(ax^p)}(1+m+p)\,{}_2F_1\left(\frac{1+m+p}{p},\frac{1+m+3p}{2p};\frac{1+m+5p}{2p};-e^{2\operatorname{sech}^{-1}(ax^p)}\right)\right)}{(1+m+p)(1+m+3p)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^p]*x^m,x]
```

```
[Out] (2^((1 + m)/p)*(E^ArcSech[a*x^p]/(1 + E^(2*ArcSech[a*x^p])))^((1 + m + p)/p
)*(1 + E^(2*ArcSech[a*x^p]))^((1 + m + p)/p)*x^(1 + m)*((1 + m + 3*p)*Hyper
geometric2F1[(1 + m + p)/(2*p), (1 + m + p)/p, (1 + m + 3*p)/(2*p), -E^(2*A
rcSech[a*x^p])] - E^(2*ArcSech[a*x^p])*(1 + m + p)*Hypergeometric2F1[(1 + m
 + p)/p, (1 + m + 3*p)/(2*p), (1 + m + 5*p)/(2*p), -E^(2*ArcSech[a*x^p])]))
/((1 + m + p)*(1 + m + 3*p)*(a*x^p)^((1 + m)/p))
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int \left(\frac{x^{-p}}{a}+\sqrt{\frac{x^{-p}}{a}-1}\,\sqrt{\frac{x^{-p}}{a}+1}\right)x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^m,x)
```

```
[Out] int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^m,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^m,x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(m-p>0)', see `assume?` for more det
ails)Is
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^m,x, algori
thm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int x^m x^{-p}\,dx + \int a x^m \sqrt{-1 + \frac{x^{-p}}{a}}\sqrt{1 + \frac{x^{-p}}{a}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x**p)+(1/a/(x**p)-1)**(1/2)*(1/a/(x**p)+1)**(1/2))*x**m,x)
```

```
[Out] (Integral(x**m/x**p, x) + Integral(a*x**m*sqrt(-1 + 1/(a*x**p))*sqrt(1 + 1/
(a*x**p)), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^m,x, algori
thm="giac")
```

```
[Out] integrate(x^m*(sqrt(1/(a*x^p) + 1)*sqrt(1/(a*x^p) - 1) + 1/(a*x^p)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \left(\sqrt{\frac{1}{a\,x^p}-1}\,\sqrt{\frac{1}{a\,x^p}+1}+\frac{1}{a\,x^p}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p)),x)
```

```
[Out] int(x^m*((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p)), x)
```

3.61 $\int e^{\text{sech}^{-1}(ax^p)}x\,dx$

Optimal. Leaf size=119

$$\frac{1}{2}e^{\text{sech}^{-1}(ax^p)}x^2+\frac{px^{2-p}}{2a(2-p)}+\frac{px^{2-p}\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\ {}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(-1+\frac{2}{p}\right);\frac{1}{2}\left(1+\frac{2}{p}\right);a^2x^{2p}\right)}{2a(2-p)}$$

```
[Out] 1/2*(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x^2+1/2*p*x^(2-p)/a
/(2-p)+1/2*p*x^(2-p)*hypergeom([1/2, -1/2+1/p],[1/2+1/p],a^2*x^(2*p))*(1/(1
+a*x^p))^(1/2)*(1+a*x^p)^(1/2)/a/(2-p)
```

Rubi [A]
time = 0.04, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6470, 30, 265, 371}

$$\frac{px^{2-p}\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\ {}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(\frac{2}{p}-1\right);\frac{1}{2}\left(1+\frac{2}{p}\right);a^2x^{2p}\right)}{2a(2-p)}+\frac{px^{2-p}}{2a(2-p)}+\frac{1}{2}x^2e^{\text{sech}^{-1}(ax^p)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^p]*x,x]
```

```
[Out] (E^ArcSech[a*x^p]*x^2)/2 + (p*x^(2 - p))/(2*a*(2 - p)) + (p*x^(2 - p)*Sqrt[
(1 + a*x^p)^(-1)]*Sqrt[1 + a*x^p]*Hypergeometric2F1[1/2, (-1 + 2/p)/2, (1 +
 2/p)/2, a^2*x^(2*p)])/(2*a*(2 - p))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\int e^{\operatorname{sech}^{-1}(ax^p)}x\,dx=\frac{1}{2}e^{\operatorname{sech}^{-1}(ax^p)}x^2+\frac{p\int x^{1-p}\,dx}{2a}+\frac{\left(p\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\int\frac{x^{1-p}}{\sqrt{1-ax^p}\sqrt{1+ax^p}}\,dx}{2a}$$

$$=\frac{1}{2}e^{\operatorname{sech}^{-1}(ax^p)}x^2+\frac{px^{2-p}}{2a(2-p)}+\frac{\left(p\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\int\frac{x^{1-p}}{\sqrt{1-a^2x^{2p}}}\,dx}{2a}$$

$$=\frac{1}{2}e^{\operatorname{sech}^{-1}(ax^p)}x^2+\frac{px^{2-p}}{2a(2-p)}+\frac{px^{2-p}\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(-1+\frac{2}{p}\right);\frac{1}{2}\left(1+\frac{2}{p}\right)\right.}{2a(2-p)}$$

Mathematica [A]
time = 0.21, size = 159, normalized size = 1.34

$$\frac{x^{2-p}\left(-1-\sqrt{\frac{1-ax^p}{1+ax^p}}-ax^p\sqrt{\frac{1-ax^p}{1+ax^p}}+\frac{a^2px^{2p}\sqrt{\frac{1-ax^p}{1+ax^p}}\sqrt{1-a^2x^{2p}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+\frac{1}{p};\frac{3}{2}+\frac{1}{p};a^2x^{2p}\right)}{(2+p)(-1+ax^p)}\right)}{a(-2+p)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^p]*x,x]
```

```
[Out] (x^(2 - p)*(-1 - Sqrt[(1 - a*x^p)/(1 + a*x^p)] - a*x^p*Sqrt[(1 - a*x^p)/(1
+ a*x^p)] + (a^2*p*x^(2*p)*Sqrt[(1 - a*x^p)/(1 + a*x^p)]*Sqrt[1 - a^2*x^(2*
p)]*Hypergeometric2F1[1/2, 1/2 + p^(-1), 3/2 + p^(-1), a^2*x^(2*p)])/((2 +
p)*(-1 + a*x^p))))/(a*(-2 + p))
```

Maple [F]
time = 0.41, size = 0, normalized size = 0.00

$$\int\left(\frac{x^{-p}}{a}+\sqrt{\frac{x^{-p}}{a}-1}\,\sqrt{\frac{x^{-p}}{a}+1}\right)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x,x)
```

```
[Out] int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x,x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-p>0)', see `assume?` for more det
ails)Is
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x,x, algorith
m="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int xx^{-p}\,dx + \int ax\sqrt{-1 + \frac{x^{-p}}{a}}\sqrt{1 + \frac{x^{-p}}{a}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x**p)+(1/a/(x**p)-1)**(1/2)*(1/a/(x**p)+1)**(1/2))*x,x)
```

```
[Out] (Integral(x/x**p, x) + Integral(a*x*sqrt(-1 + 1/(a*x**p))*sqrt(1 + 1/(a*x**
p)), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x,x, algorith
m="giac")
```

```
[Out] integrate(x*(sqrt(1/(a*x^p) + 1)*sqrt(1/(a*x^p) - 1) + 1/(a*x^p)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(\sqrt{\frac{1}{a\,x^p}-1}\,\sqrt{\frac{1}{a\,x^p}+1}+\frac{1}{a\,x^p}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p)),x)
```

```
[Out] int(x*((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p)), x)
```

3.62 $\int e^{\mathrm{sech}^{-1}(ax^p)}\,dx$

Optimal. Leaf size=105

$$e^{\mathrm{sech}^{-1}(ax^p)}x+\frac{px^{1-p}}{a(1-p)}+\frac{px^{1-p}\sqrt{\frac{1}{1+ax^p}}\,\sqrt{1+ax^p}\;{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(-1+\frac{1}{p}\right);\frac{1+p}{2p};a^2x^{2p}\right)}{a(1-p)}$$

```
[Out] (1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))*x+p*x^(1-p)/a/(1-p)+p*x
^(1-p)*hypergeom([1/2, -1/2+1/2/p],[1/2*(1+p)/p],a^2*x^(2*p))*(1/(1+a*x^p))
^(1/2)*(1+a*x^p)^(1/2)/a/(1-p)
```

Rubi [A]
time = 0.03, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {6465, 30, 265, 371}

$$\frac{px^{1-p}\sqrt{\frac{1}{ax^p+1}}\,\sqrt{ax^p+1}\;{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(\frac{1}{p}-1\right);\frac{p+1}{2p};a^2x^{2p}\right)}{a(1-p)}+\frac{px^{1-p}}{a(1-p)}+xe^{\mathrm{sech}^{-1}(ax^p)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^p],x]
```

```
[Out] E^ArcSech[a*x^p]*x + (p*x^(1 - p))/(a*(1 - p)) + (p*x^(1 - p)*Sqrt[(1 + a*x
^p)^(-1)]*Sqrt[1 + a*x^p]*Hypergeometric2F1[1/2, (-1 + p^(-1))/2, (1 + p)/(
2*p), a^2*x^(2*p)])/(a*(1 - p))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6465

```
Int[E^ArcSech[(a_.)*(x_)^(p_)], x_Symbol] :> Simp[x*E^ArcSech[a*x^p], x] +
(Dist[p/a, Int[1/x^p, x], x] + Dist[p*(Sqrt[1 + a*x^p]/a)*Sqrt[1/(1 + a*x^p
)], Int[1/(x^p*Sqrt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, p}, x
]
```

Rubi steps

$$\begin{aligned}\int e^{\operatorname{sech}^{-1}(ax^p)}\,dx &= e^{\operatorname{sech}^{-1}(ax^p)}x+\frac{p\int x^{-p}\,dx}{a}+\frac{\left(p\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\int\frac{x^{-p}}{\sqrt{1-ax^p}\sqrt{1+ax^p}}\,dx}{a}\\ &= e^{\operatorname{sech}^{-1}(ax^p)}x+\frac{px^{1-p}}{a(1-p)}+\frac{\left(p\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\int\frac{x^{-p}}{\sqrt{1-a^2x^{2p}}}\,dx}{a}\\ &= e^{\operatorname{sech}^{-1}(ax^p)}x+\frac{px^{1-p}}{a(1-p)}+\frac{px^{1-p}\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(-1+\frac{1}{p}\right);\frac{1+p}{2p};a^2x^{2p}\right)}{a(1-p)}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 139, normalized size = 1.32

$$\frac{x\left(x^{-p}+(a+x^{-p})\sqrt{\frac{1-ax^p}{1+ax^p}}-\frac{a^2px^p\sqrt{\frac{1-ax^p}{1+ax^p}}\sqrt{1-a^2x^{2p}}\,{}_2F_1\left(\frac{1}{2},\frac{1+p}{2p};\frac{1}{2}\left(3+\frac{1}{p}\right);a^2x^{2p}\right)}{(1+p)(-1+ax^p)}\right)}{a-ap}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^p],x]
```

```
[Out] (x*(x^(-p) + (a + x^(-p))*Sqrt[(1 - a*x^p)/(1 + a*x^p)] - (a^2*p*x^p*Sqrt[(
1 - a*x^p)/(1 + a*x^p)]*Sqrt[1 - a^2*x^(2*p)]*Hypergeometric2F1[1/2, (1 + p
)/(2*p), (3 + p^(-1))/2, a^2*x^(2*p)])/((1 + p)*(-1 + a*x^p))))/(a - a*p)
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int\frac{x^{-p}}{a}+\sqrt{\frac{x^{-p}}{a}-1}\,\sqrt{\frac{x^{-p}}{a}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2),x)
```

```
[Out] int(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-p>0)', see `assume?` for more deta
ils)Is
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int x^{-p}\,dx + \int a\sqrt{-1+\frac{x^{-p}}{a}}\,\sqrt{1+\frac{x^{-p}}{a}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/(x**p)+(1/a/(x**p)-1)**(1/2)*(1/a/(x**p)+1)**(1/2),x)
```

```
[Out] (Integral(x**(-p), x) + Integral(a*sqrt(-1 + 1/(a*x**p))*sqrt(1 + 1/(a*x**p
)), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(1/(a*x^p) + 1)*sqrt(1/(a*x^p) - 1) + 1/(a*x^p), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{1}{a\,x^p}-1}\,\sqrt{\frac{1}{a\,x^p}+1}+\frac{1}{a\,x^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p),x)
```

```
[Out] int((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p), x)
```

3.63 $\int \frac{e^{\text{sech}^{-1}(ax^p)}}{x}\,dx$

Optimal. Leaf size=87

$$-\frac{x^{-p}}{ap}-\frac{x^{-p}\sqrt{1-ax^p}}{ap\sqrt{\frac{1}{1+ax^p}}}-\frac{\sqrt{\frac{1}{1+ax^p}}\,\sqrt{1+ax^p}\,\text{ArcSin}(ax^p)}{p}$$

```
[Out] -1/a/p/(x^p)-(1-a*x^p)^(1/2)/a/p/(x^p)/(1/(1+a*x^p))^(1/2)-arcsin(a*x^p)*(1
/(1+a*x^p))^(1/2)*(1+a*x^p)^(1/2)/p
```

Rubi [A]
time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6469, 265, 352, 248, 283, 222}

$$-\frac{x^{-p}\sqrt{\frac{1}{ax^p+1}}\,\sqrt{ax^p+1}\,\sqrt{1-a^2x^{2p}}}{ap}-\frac{x^{-p}}{ap}-\frac{\sqrt{\frac{1}{ax^p+1}}\,\sqrt{ax^p+1}\,\csc^{-1}\left(\frac{x^{-p}}{a}\right)}{p}$$

Warning: Unable to verify antiderivative.

```
[In] Int[E^ArcSech[a*x^p]/x,x]
```

```
[Out] -(1/(a*p*x^p)) - (Sqrt[(1 + a*x^p)^(-1)]*Sqrt[1 + a*x^p]*Sqrt[1 - a^2*x^(2*
p)])/(a*p*x^p) - (Sqrt[(1 + a*x^p)^(-1)]*Sqrt[1 + a*x^p]*ArcCsc[1/(a*x^p)])
/p
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 352

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 6469

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]/(x_), x_Symbol] :> -Simp[(a*p*x^p)^(-1), x]
 + Dist[(Sqrt[1 + a*x^p]/a)*Sqrt[1/(1 + a*x^p)], Int[Sqrt[1 + a*x^p]*(Sqrt[
1 - a*x^p]/x^(p + 1)), x], x] /; FreeQ[{a, p}, x]
```

Rubi steps

$$\int \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x}\,dx = -\frac{x^{-p}}{ap} + \frac{\left(\sqrt{\frac{1}{1+ax^p}}\,\sqrt{1+ax^p}\right)\int x^{-1-p}\sqrt{1-ax^p}\,\sqrt{1+ax^p}\,dx}{a}$$

$$= -\frac{x^{-p}}{ap} + \frac{\left(\sqrt{\frac{1}{1+ax^p}}\,\sqrt{1+ax^p}\right)\int x^{-1-p}\sqrt{1-a^2x^{2p}}\,dx}{a}$$

$$= -\frac{x^{-p}}{ap} - \frac{\left(\sqrt{\frac{1}{1+ax^p}}\,\sqrt{1+ax^p}\right)\text{Subst}\left(\int \sqrt{1-\frac{a^2}{x^2}}\,dx, x, x^{-p}\right)}{ap}$$

$$= -\frac{x^{-p}}{ap} + \frac{\left(\sqrt{\frac{1}{1+ax^p}}\,\sqrt{1+ax^p}\right)\text{Subst}\left(\int \frac{\sqrt{1-a^2x^2}}{x^2}\,dx, x, x^p\right)}{ap}$$

$$= -\frac{x^{-p}}{ap} - \frac{x^{-p}\sqrt{\frac{1}{1+ax^p}}\,\sqrt{1+ax^p}\,\sqrt{1-a^2x^{2p}}}{ap} - \frac{\left(a\sqrt{\frac{1}{1+ax^p}}\,\sqrt{1+ax^p}\right)\text{Subst}\left(\int$$

$$= -\frac{x^{-p}}{ap} - \frac{x^{-p}\sqrt{\frac{1}{1+ax^p}}\,\sqrt{1+ax^p}\,\sqrt{1-a^2x^{2p}}}{ap} - \frac{\sqrt{\frac{1}{1+ax^p}}\,\sqrt{1+ax^p}\,\sin^{-1}(ax^p)}{p}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 96, normalized size = 1.10

$$-\frac{i\left(-ix^{-p}-i(a+x^{-p})\sqrt{\frac{1-ax^p}{1+ax^p}}+a\log\left(-2iax^p+2\sqrt{\frac{1-ax^p}{1+ax^p}}\,(1+ax^p)\right)\right)}{ap}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^p]/x,x]
```

```
[Out] ((-I)*((-I)/x^p - I*(a + x^(-p))*Sqrt[(1 - a*x^p)/(1 + a*x^p)] + a*Log[(-2*
I)*a*x^p + 2*Sqrt[(1 - a*x^p)/(1 + a*x^p)]*(1 + a*x^p)]))/(a*p)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.62, size = 116, normalized size = 1.33

method	result	s
derivativedivides	$\frac{-\frac{\sqrt{\frac{(1+a\,x^p)x^{-p}}{a}}\,\sqrt{-\frac{(a\,x^p-1)x^{-p}}{a}}\,\left(\arctan\left(\frac{\mathrm{csgn}(a)a\,x^p}{\sqrt{1-a^2x^{2p}}}\right)a\,x^p+\mathrm{csgn}(a)\sqrt{1-a^2x^{2p}}\right)\mathrm{csgn}(a)}{\sqrt{1-a^2x^{2p}}}-\frac{x^{-p}}{a}}{p}$	1
default	$\frac{-\frac{\sqrt{\frac{(1+a\,x^p)x^{-p}}{a}}\,\sqrt{-\frac{(a\,x^p-1)x^{-p}}{a}}\,\left(\arctan\left(\frac{\mathrm{csgn}(a)a\,x^p}{\sqrt{1-a^2x^{2p}}}\right)a\,x^p+\mathrm{csgn}(a)\sqrt{1-a^2x^{2p}}\right)\mathrm{csgn}(a)}{\sqrt{1-a^2x^{2p}}}-\frac{x^{-p}}{a}}{p}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x,x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/p*(-((1+a*x^p)/a/(x^p))^(1/2)*(-(a*x^p-1)/a/(x^p))^(1/2)*(arctan(csgn(a)*
a*x^p/(-(x^p)^2*a^2+1)^(1/2))*a*x^p+csgn(a)*(-(x^p)^2*a^2+1)^(1/2))*csgn(a)
/(-(x^p)^2*a^2+1)^(1/2)-1/a/(x^p))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x,x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(a*x^p + 1)*sqrt(-a*x^p + 1)/(x*x^p), x)/a - 1/(a*p*x^p)
```

Fricas [A]
time = 0.61, size = 102, normalized size = 1.17

$$-\frac{ax^p\sqrt{\frac{ax^p+1}{ax^p}}\sqrt{-\frac{ax^p-1}{ax^p}}-ax^p\arctan\left(\sqrt{\frac{ax^p+1}{ax^p}}\sqrt{-\frac{ax^p-1}{ax^p}}\right)+1}{apx^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x,x, algorith
m="fricas")
```

```
[Out] -(a*x^p*sqrt((a*x^p + 1)/(a*x^p))*sqrt(-(a*x^p - 1)/(a*x^p)) - a*x^p*arctan
(sqrt((a*x^p + 1)/(a*x^p))*sqrt(-(a*x^p - 1)/(a*x^p))) + 1)/(a*p*x^p)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{-p}}{x}\,dx+\int \frac{a\sqrt{-1+\frac{x^{-p}}{a}}\sqrt{1+\frac{x^{-p}}{a}}}{x}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x**p)+(1/a/(x**p)-1)**(1/2)*(1/a/(x**p)+1)**(1/2))/x,x)
```

```
[Out] (Integral(1/(x*x**p), x) + Integral(a*sqrt(-1 + 1/(a*x**p))*sqrt(1 + 1/(a*x
**p))/x, x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x,x, algorith
m="giac")
```

```
[Out] integrate((sqrt(1/(a*x^p) + 1)*sqrt(1/(a*x^p) - 1) + 1/(a*x^p))/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{a\,x^p}-1}\,\sqrt{\frac{1}{a\,x^p}+1}+\frac{1}{a\,x^p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p))/x,x)
```

```
[Out] int(((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p))/x, x)
```

3.64 $\int \frac{e^{\text{sech}^{-1}(ax^p)}}{x^2} \, dx$

Optimal. Leaf size=107

$$-\frac{e^{\text{sech}^{-1}(ax^p)}}{x}+\frac{px^{-1-p}}{a(1+p)}+\frac{px^{-1-p}\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\,{}_2F_1\left(\frac{1}{2},-\frac{1+p}{2p};-\frac{1-p}{2p};a^2x^{2p}\right)}{a(1+p)}$$

```
[Out] -(1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x+p*x^(-1-p)/a/(1+p)+p
*x^(-1-p)*hypergeom([1/2, 1/2*(-1-p)/p],[1/2*(-1+p)/p],a^2*x^(2*p))*(1/(1+a
*x^p))^(1/2)*(1+a*x^p)^(1/2)/a/(1+p)
```

Rubi [A]
time = 0.04, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6470, 30, 265, 371}

$$\frac{px^{-p-1}\sqrt{\frac{1}{ax^p+1}}\sqrt{ax^p+1}\,{}_2F_1\left(\frac{1}{2},-\frac{p+1}{2p};-\frac{1-p}{2p};a^2x^{2p}\right)}{a(p+1)}+\frac{px^{-p-1}}{a(p+1)}-\frac{e^{\text{sech}^{-1}(ax^p)}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[a*x^p]/x^2,x]
```

```
[Out] -(E^ArcSech[a*x^p]/x) + (p*x^(-1 - p))/(a*(1 + p)) + (p*x^(-1 - p)*Sqrt[(1
+ a*x^p)^(-1)]*Sqrt[1 + a*x^p]*Hypergeometric2F1[1/2, -1/2*(1 + p)/p, -1/2*
(1 - p)/p, a^2*x^(2*p)])/(a*(1 + p))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Int[(c*x)^m*(a1*a2 + b1*b2*x^(2*n))^p, x] /; Free
Q[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p]
 || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 6470

```
Int[E^ArcSech[(a_.)*(x_)^(p_.)]*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(E^
ArcSech[a*x^p]/(m + 1)), x] + (Dist[p/(a*(m + 1)), Int[x^(m - p), x], x] +
Dist[p*(Sqrt[1 + a*x^p]/(a*(m + 1)))*Sqrt[1/(1 + a*x^p)], Int[x^(m - p)/(Sq
rt[1 + a*x^p]*Sqrt[1 - a*x^p]), x], x]) /; FreeQ[{a, m, p}, x] && NeQ[m, -1
]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x^2}\,dx &= -\frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x} - \frac{p\int x^{-2-p}\,dx}{a} - \frac{\left(p\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\int \frac{x^{-2-p}}{\sqrt{1-ax^p}\sqrt{1+ax^p}}\,dx}{a}\\
&= -\frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x} + \frac{px^{-1-p}}{a(1+p)} - \frac{\left(p\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\right)\int \frac{x^{-2-p}}{\sqrt{1-a^2x^{2p}}}\,dx}{a}\\
&= -\frac{e^{\operatorname{sech}^{-1}(ax^p)}}{x} + \frac{px^{-1-p}}{a(1+p)} + \frac{px^{-1-p}\sqrt{\frac{1}{1+ax^p}}\sqrt{1+ax^p}\,{}_2F_1\left(\frac{1}{2},-\frac{1+p}{2p};-\frac{1-p}{2p};a^2x^{2p}\right)}{a(1+p)}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 156, normalized size = 1.46

$$x^{-1-p}\left(-\frac{1}{a+ap} - \frac{\sqrt{\frac{1-ax^p}{1+ax^p}}\,(1+ax^p)}{a(1+p)} + \frac{apx^{2p}\sqrt{\frac{1-ax^p}{1+ax^p}}\sqrt{1-a^2x^{2p}}\,{}_2F_1\left(\frac{1}{2},\frac{-1+p}{2p};\frac{3}{2}-\frac{1}{2p};a^2x^{2p}\right)}{(-1+p)(1+p)(-1+ax^p)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[a*x^p]/x^2,x]
```

```
[Out] x^(-1 - p)*(-(a + a*p)^(-1) - (Sqrt[(1 - a*x^p)/(1 + a*x^p)]*(1 + a*x^p))/(
a*(1 + p)) + (a*p*x^(2*p)*Sqrt[(1 - a*x^p)/(1 + a*x^p)]*Sqrt[1 - a^2*x^(2*p
)]*Hypergeometric2F1[1/2, (-1 + p)/(2*p), 3/2 - 1/(2*p), a^2*x^(2*p)])/((-1
 + p)*(1 + p)*(-1 + a*x^p)))
```

Maple [F]
time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\frac{x^{-p}}{a}+\sqrt{\frac{x^{-p}}{a}-1}\,\sqrt{\frac{x^{-p}}{a}+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x^2,x)
```

```
[Out] int((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x^2,x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(a*x^p + 1)*sqrt(-a*x^p + 1)/(x^2*x^p), x)/a - x^(-p - 1)/(a*
(p + 1))
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x^2,x, algori
thm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x^{-p}}{x^2}\,dx + \int \frac{a\sqrt{-1+\frac{x^{-p}}{a}}\sqrt{1+\frac{x^{-p}}{a}}}{x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x**p)+(1/a/(x**p)-1)**(1/2)*(1/a/(x**p)+1)**(1/2))/x**2,x)
```

```
[Out] (Integral(1/(x**2*x**p), x) + Integral(a*sqrt(-1 + 1/(a*x**p))*sqrt(1 + 1/(
a*x**p))/x**2, x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/(x^p)+(1/a/(x^p)-1)^(1/2)*(1/a/(x^p)+1)^(1/2))/x^2,x, algori
thm="giac")
```

```
[Out] integrate((sqrt(1/(a*x^p) + 1)*sqrt(1/(a*x^p) - 1) + 1/(a*x^p))/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{a\,x^p}-1}\,\sqrt{\frac{1}{a\,x^p}+1}+\frac{1}{a\,x^p}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p))/x^2,x)
```

```
[Out] int(((1/(a*x^p) - 1)^(1/2)*(1/(a*x^p) + 1)^(1/2) + 1/(a*x^p))/x^2, x)
```

3.65 $\int e^{2\text{sech}^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=203

$$\frac{5\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^2}{4a^5}+\frac{(1-ax)(1+ax)^4}{5a^5}+\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^4\left(5-6\sqrt{\frac{1-ax}{1+ax}}\right)}{10a^5}+\frac{(1+ax)\left(4-\sqrt{\frac{1-}{1+}}\right.}{4a^5}$$

```
[Out] 1/5*(-a*x+1)*(a*x+1)^4/a^5-1/2*arctan(((-a*x+1)/(a*x+1))^(1/2))/a^5+1/4*(a*
x+1)*(4-((-a*x+1)/(a*x+1))^(1/2))/a^5+5/4*(a*x+1)^2*((-a*x+1)/(a*x+1))^(1/2
)/a^5+1/10*(a*x+1)^4*(5-6*((-a*x+1)/(a*x+1))^(1/2))*((-a*x+1)/(a*x+1))^(1/2
)/a^5-1/30*(a*x+1)^3*(4+45*((-a*x+1)/(a*x+1))^(1/2))/a^5
```

Rubi [A]
time = 0.48, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6472, 1818, 1825, 1828, 653, 209}

$$-\frac{\text{ArcTan}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{2a^5}+\frac{(1-ax)(ax+1)^4}{5a^5}+\frac{\sqrt{\frac{1-ax}{ax+1}}\left(5-6\sqrt{\frac{1-ax}{ax+1}}\right)(ax+1)^4}{10a^5}-\frac{\left(45\sqrt{\frac{1-ax}{ax+1}}+4\right)(ax+1)^3}{30a^5}+\frac{5\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)^2}{4a^5}+\frac{\left(4-\sqrt{\frac{1-ax}{ax+1}}\right)(ax+1)}{4a^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x])*x^4,x]
```

```
[Out] (5*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(4*a^5) + ((1 - a*x)*(1 + a*x)^4)
/(5*a^5) + (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^4*(5 - 6*Sqrt[(1 - a*x)/(1
+ a*x)]))/(10*a^5) + ((1 + a*x)*(4 - Sqrt[(1 - a*x)/(1 + a*x)]))/(4*a^5) -
((1 + a*x)^3*(4 + 45*Sqrt[(1 - a*x)/(1 + a*x)]))/(30*a^5) - ArcTan[Sqrt[(1
- a*x)/(1 + a*x)]]/(2*a^5)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
```

```
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1825

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int e^{2\text{sech}^{-1}(ax)}x^4\,dx = \int x^4\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2 dx$$

$$= -\frac{4\text{Subst}\left(\int \frac{(-1+x)^2x(1+x)^6}{(1+x^2)^6}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^5}$$

$$= \frac{(1-ax)(1+ax)^4}{5a^5}+\frac{2\text{Subst}\left(\int \frac{-42x-40x^2+130x^3+80x^4-30x^5-40x^6-10x^7}{(1+x^2)^5}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{5a^5}$$

$$= \frac{(1-ax)(1+ax)^4}{5a^5}+\frac{2\text{Subst}\left(\int \frac{x(-42-40x+130x^2+80x^3-30x^4-40x^5-10x^6)}{(1+x^2)^5}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{5a^5}$$

$$= \frac{(1-ax)(1+ax)^4}{5a^5}+\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^4\left(5-6\sqrt{\frac{1-ax}{1+ax}}\right)}{10a^5}-\frac{\text{Subst}\left(\int \frac{160-48x-960}{}\right.}{}$$

$$= \frac{(1-ax)(1+ax)^4}{5a^5}+\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^4\left(5-6\sqrt{\frac{1-ax}{1+ax}}\right)}{10a^5}-\frac{(1+ax)^3\left(4+45\sqrt{}\right.}{30a^5}$$

$$= \frac{5\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2}{4a^5}+\frac{(1-ax)(1+ax)^4}{5a^5}+\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^4\left(5-6\sqrt{\frac{1-ax}{1+ax}}\right)}{10a^5}$$

$$= \frac{5\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2}{4a^5}+\frac{(1-ax)(1+ax)^4}{5a^5}+\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^4\left(5-6\sqrt{\frac{1-ax}{1+ax}}\right)}{10a^5}$$

$$= \frac{5\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2}{4a^5}+\frac{(1-ax)(1+ax)^4}{5a^5}+\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^4\left(5-6\sqrt{\frac{1-ax}{1+ax}}\right)}{10a^5}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 105, normalized size = 0.52

$$\frac{40a^3x^3-12a^5x^5-15a\sqrt{\frac{1-ax}{1+ax}}(x+ax^2-2a^2x^3-2a^3x^4)+15i\log\left(-2iax+2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\right)}{60a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcSech[a*x])*x^4,x]
```

```
[Out] (40*a^3*x^3 - 12*a^5*x^5 - 15*a*Sqrt[(1 - a*x)/(1 + a*x)]*(x + a*x^2 - 2*a^
2*x^3 - 2*a^3*x^4) + (15*I)*Log[(-2*I)*a*x + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1
 + a*x)])/(60*a^5)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 136, normalized size = 0.67

method	result
default	$\frac{-\frac{1}{5}a^2x^5+\frac{1}{3}x^3}{a^2}+\frac{\sqrt{\frac{ax+1}{ax}}\,x\sqrt{-\frac{ax-1}{ax}}\left(2\,\mathrm{csgn}(a)a^3x^3\sqrt{-a^2x^2+1}-x\sqrt{-a^2x^2+1}\,\mathrm{csgn}(a)a+\arctan\left(\frac{\mathrm{csgn}(a)a\cdots}{\sqrt{-a^2x^2\cdots}}\right.\cdots}{4a^4\sqrt{-a^2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/5*a^2*x^5+1/3*x^3)+1/4/a^4*((a*x+1)/a/x)^(1/2)*x*(-(a*x-1)/a/x)^(
1/2)*(2*csgn(a)*a^3*x^3*(-a^2*x^2+1)^(1/2)-x*(-a^2*x^2+1)^(1/2)*csgn(a)*a+a
rctan(csgn(a)*a*x/(-a^2*x^2+1)^(1/2)))*csgn(a)/(-a^2*x^2+1)^(1/2)+1/3*x^3/a
^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^4,x, algorithm="maxim
a")
```

```
[Out] 2/3*x^3/a^2 + 2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x^2, x)/a^2 - integr
ate(x^4, x)
```

Fricas [A]
time = 0.50, size = 103, normalized size = 0.51

$$-\frac{12\,a^5x^5-40\,a^3x^3-15\,(2\,a^4x^4-a^2x^2)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+15\,\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right)}{60\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^4,x, algorithm="frica
s")
```

```
[Out] -1/60*(12*a^5*x^5 - 40*a^3*x^3 - 15*(2*a^4*x^4 - a^2*x^2)*sqrt((a*x + 1)/(a
*x))*sqrt(-(a*x - 1)/(a*x)) + 15*arctan(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x -
1)/(a*x))))/a^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int 2x^2\,dx + \int \left(-a^2x^4\right)\,dx + \int 2ax^3\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2*x**4,x)
```

```
[Out] (Integral(2*x**2, x) + Integral(-a**2*x**4, x) + Integral(2*a*x**3*sqrt(-1
+ 1/(a*x))*sqrt(1 + 1/(a*x)), x))/a**2
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^4,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 19.73, size = 808, normalized size = 3.98

$$-\frac{\frac{1\mathrm{i}}{512\,a^5}-\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^2\,3\mathrm{i}}{64\,a^5\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^2}-\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^4\,53\mathrm{i}}{256\,a^5\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^4}+\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^6\,87\mathrm{i}}{128\,a^5\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^6}+\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^8\,657\mathrm{i}}{512\,a^5\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^8}+\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^{10}\,121\mathrm{i}}{128\,a^5\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^{10}}}{\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^4}+\frac{4\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^6}+\frac{6\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^8}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^8}+\frac{4\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^{10}}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^{10}}+\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^{12}}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^{12}}}-\frac{\frac{1\mathrm{i}}{16\,a^5}+\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{8\,a^5\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^2}-\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^4\,15\mathrm{i}}{16\,a^5\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^4}}{\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^2}+\frac{2\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^4}+\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^6}}-\frac{x^5\,\left(\frac{a^2}{5}-\frac{2}{3\,x^2}\right)}{a^2}-\frac{\ln\left(\frac{a\,\sqrt{\frac{1}{a\,x}+1}-\frac{1}{x}+a\,\sqrt{\frac{1}{a\,x}-1}\,1\mathrm{i}}{2\,a-2\,a\,\sqrt{\frac{1}{a\,x}+1}+\frac{1}{x}}\right)\,3\mathrm{i}}{4\,a^5}-\frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\,1\mathrm{i}}{4\,a^5}+\frac{\ln\left(\frac{2\,a\,\sqrt{\frac{a+\frac{1}{x}}{a}}-\frac{2}{x}+a\,\sqrt{-\frac{a-\frac{1}{x}}{a}}\,2\mathrm{i}}{2\,a+\frac{1}{x}-2\,a\,\sqrt{\frac{a+\frac{1}{x}}{a}}}\right)\,1\mathrm{i}}{a^5}-\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{128\,a^5\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^2}-\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^4\,1\mathrm{i}}{512\,a^5\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2,x)
```

```
[Out] (log((a*(-(a - 1/x)/a)^(1/2)*2i - 2/x + 2*a*((a + 1/x)/a)^(1/2))/(2*a + 1/x
 - 2*a*((a + 1/x)/a)^(1/2)))*1i)/a^5 - (1i/(512*a^5) - (((1/(a*x) - 1)^(1/2
) - 1i)^2*3i)/(64*a^5*((1/(a*x) + 1)^(1/2) - 1)^2) - (((1/(a*x) - 1)^(1/2)
- 1i)^4*53i)/(256*a^5*((1/(a*x) + 1)^(1/2) - 1)^4) + (((1/(a*x) - 1)^(1/2)
- 1i)^6*87i)/(128*a^5*((1/(a*x) + 1)^(1/2) - 1)^6) + (((1/(a*x) - 1)^(1/2)
- 1i)^8*657i)/(512*a^5*((1/(a*x) + 1)^(1/2) - 1)^8) + (((1/(a*x) - 1)^(1/2)
 - 1i)^10*121i)/(128*a^5*((1/(a*x) + 1)^(1/2) - 1)^10))/((((1/(a*x) - 1)^(1/
2) - 1i)^4/((1/(a*x) + 1)^(1/2) - 1)^4 + (4*((1/(a*x) - 1)^(1/2) - 1i)^6)/(
(1/(a*x) + 1)^(1/2) - 1)^6 + (6*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1
```

```
)^(1/2) - 1)^8 + (4*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1
)^10 + ((1/(a*x) - 1)^(1/2) - 1i)^12/((1/(a*x) + 1)^(1/2) - 1)^12) - (log((
(1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i)/(4*a^5) - (1i/(16*
a^5) + (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(8*a^5*((1/(a*x) + 1)^(1/2) - 1)^2
) - (((1/(a*x) - 1)^(1/2) - 1i)^4*15i)/(16*a^5*((1/(a*x) + 1)^(1/2) - 1)^4)
)/(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + (2*((1/(a*x)
- 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 + ((1/(a*x) - 1)^(1/2) - 1i
)^6/((1/(a*x) + 1)^(1/2) - 1)^6) - (log((a*(1/(a*x) - 1)^(1/2)*1i + a*(1/(a
*x) + 1)^(1/2) - 1/x)/(2*a - 2*a*(1/(a*x) + 1)^(1/2) + 1/x))*3i)/(4*a^5) -
(((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(128*a^5*((1/(a*x) + 1)^(1/2) - 1)^2) - (
((1/(a*x) - 1)^(1/2) - 1i)^4*1i)/(512*a^5*((1/(a*x) + 1)^(1/2) - 1)^4) - (x
^5*(a^2/5 - 2/(3*x^2)))/a^2
```

3.66 $\int e^{2\text{sech}^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=117

$$-\frac{x}{a^3}+\frac{(1-ax)(1+ax)^3}{4a^4}+\frac{(1+ax)^2\left(3-8\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^4}+\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^3\left(4-3\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^4}$$

```
[Out] -x/a^3+1/4*(-a*x+1)*(a*x+1)^3/a^4+1/6*(a*x+1)^2*(3-8*((-a*x+1)/(a*x+1))^(1/
2))/a^4+1/6*(a*x+1)^3*(4-3*((-a*x+1)/(a*x+1))^(1/2))*((-a*x+1)/(a*x+1))^(1/
2)/a^4
```

Rubi [A]
time = 0.37, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6472, 1818, 1825, 1828, 12, 267}

$$\frac{(1-ax)(ax+1)^3}{4a^4}+\frac{\sqrt{\frac{1-ax}{ax+1}}\left(4-3\sqrt{\frac{1-ax}{ax+1}}\right)(ax+1)^3}{6a^4}+\frac{\left(3-8\sqrt{\frac{1-ax}{ax+1}}\right)(ax+1)^2}{6a^4}-\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x])*x^3,x]
```

```
[Out] -(x/a^3) + ((1 - a*x)*(1 + a*x)^3)/(4*a^4) + ((1 + a*x)^2*(3 - 8*Sqrt[(1 -
a*x)/(1 + a*x)]))/(6*a^4) + (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^3*(4 - 3*S
qrt[(1 - a*x)/(1 + a*x)]))/(6*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
```

```
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1825

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int e^{2\text{sech}^{-1}(ax)}x^3\,dx = \int x^3\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2 dx$$

$$= \frac{4\text{Subst}\left(\int \frac{(-1+x)x(1+x)^5}{(1+x^2)^5}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^4}$$

$$= \frac{(1-ax)(1+ax)^3}{4a^4} - \frac{\text{Subst}\left(\int \frac{24x+32x^2-32x^3-32x^4-8x^5}{(1+x^2)^4}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{2a^4}$$

$$= \frac{(1-ax)(1+ax)^3}{4a^4} - \frac{\text{Subst}\left(\int \frac{x(24+32x-32x^2-32x^3-8x^4)}{(1+x^2)^4}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{2a^4}$$

$$= \frac{(1-ax)(1+ax)^3}{4a^4} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^3\left(4-3\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^4} + \frac{\text{Subst}\left(\int \frac{-64-48x+192}{(1+x^2)}\right.}{}$$

$$= \frac{(1-ax)(1+ax)^3}{4a^4} + \frac{(1+ax)^2\left(3-8\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^4} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^3\left(4-3\sqrt{\frac{1}{1}}\right.}{6a^4}$$

$$= \frac{(1-ax)(1+ax)^3}{4a^4} + \frac{(1+ax)^2\left(3-8\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^4} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^3\left(4-3\sqrt{\frac{1}{1}}\right.}{6a^4}$$

$$= -\frac{x}{a^3} + \frac{(1-ax)(1+ax)^3}{4a^4} + \frac{(1+ax)^2\left(3-8\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^4} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^3\left(4\right.}{6a^4}$$

Mathematica [A]
time = 0.05, size = 52, normalized size = 0.44

$$\frac{x^2}{a^2} - \frac{x^4}{4} + \frac{2(-1+ax)\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^2}{3a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcSech[a*x])*x^3,x]
```

```
[Out] x^2/a^2 - x^4/4 + (2*(-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(3*a
^4)
```

Maple [A]
time = 0.03, size = 72, normalized size = 0.62

method	result	size
default	$\frac{-\frac{1}{4}a^2x^4+\frac{1}{2}x^2}{a^2}+\frac{2\sqrt{\frac{ax+1}{ax}}\,x\sqrt{-\frac{ax-1}{ax}}\,(a^2x^2-1)}{3a^3}+\frac{x^2}{2a^2}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/4*a^2*x^4+1/2*x^2)+2/3/a^3*((a*x+1)/a/x)^(1/2)*x*(-(a*x-1)/a/x)^(
1/2)*(a^2*x^2-1)+1/2*x^2/a^2
```

Maxima [A]
time = 0.27, size = 42, normalized size = 0.36

$$-\frac{1}{4}x^4+\frac{x^2}{a^2}+\frac{2\,(a^2x^2-1)\sqrt{ax+1}\,\sqrt{-ax+1}}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^3,x, algorithm="maxim
a")
```

```
[Out] -1/4*x^4 + x^2/a^2 + 2/3*(a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1)/a^4
```

Fricas [A]
time = 0.39, size = 62, normalized size = 0.53

$$-\frac{3\,a^3x^4-12\,ax^2-8\,(a^2x^3-x)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}}{12\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^3,x, algorithm="frica
s")
```

```
[Out] -1/12*(3*a^3*x^4 - 12*a*x^2 - 8*(a^2*x^3 - x)*sqrt((a*x + 1)/(a*x))*sqrt(-(
a*x - 1)/(a*x)))/a^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int 2x\,dx+\int\left(-a^2x^3\right)dx+\int 2ax^2\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2*x**3,x)
```

```
[Out] (Integral(2*x, x) + Integral(-a**2*x**3, x) + Integral(2*a*x**2*sqrt(-1 + 1
/(a*x))*sqrt(1 + 1/(a*x)), x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^3,x, algorithm="giac"
)
```

```
[Out] integrate(x^3*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2, x)
```

Mupad [B]
time = 1.75, size = 63, normalized size = 0.54

$$\frac{x^2}{a^2} - \frac{x^4}{4} - \sqrt{\frac{1}{a\,x} - 1}\,\left(\frac{2\,x\,\sqrt{\frac{1}{a\,x} + 1}}{3\,a^3} - \frac{2\,x^3\,\sqrt{\frac{1}{a\,x} + 1}}{3\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2,x)
```

```
[Out] x^2/a^2 - x^4/4 - (1/(a*x) - 1)^(1/2)*((2*x*(1/(a*x) + 1)^(1/2))/(3*a^3) -
(2*x^3*(1/(a*x) + 1)^(1/2))/(3*a))
```

3.67 $\int e^{2\text{sech}^{-1}(ax)} x^2\,dx$

Optimal. Leaf size=169

$$\frac{(1+ax)\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}{2a^3}-\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^2\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^3}{6a^3}+\frac{(1+ax)^3\left(1+\sqrt{\ldots}\right.}{12a^3}$$

```
[Out] -2*arctan(((-a*x+1)/(a*x+1))^(1/2))/a^3+1/2*(a*x+1)*(1-((-a*x+1)/(a*x+1))^(
1/2))*(1+((-a*x+1)/(a*x+1))^(1/2))/a^3-1/6*(a*x+1)^2*((-a*x+1)/(a*x+1))^(1/
2)*(1+((-a*x+1)/(a*x+1))^(1/2))^3/a^3+1/12*(a*x+1)^3*(1+((-a*x+1)/(a*x+1))^
(1/2))^4/a^3
```

Rubi [A]

time = 0.31, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6472, 835, 12, 743, 737, 209}

$$-\frac{2\text{ArcTan}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{a^3}+\frac{(ax+1)^3\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}{12a^3}-\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)^2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}{6a^3}+\frac{(ax+1)\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x])*x^2,x]
```

```
[Out] ((1 + a*x)*(1 - Sqrt[(1 - a*x)/(1 + a*x)])*(1 + Sqrt[(1 - a*x)/(1 + a*x)]))
/(2*a^3) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(1 + Sqrt[(1 - a*x)/(1 +
a*x)])^3)/(6*a^3) + ((1 + a*x)^3*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^4)/(12*a^3
) - (2*ArcTan[Sqrt[(1 - a*x)/(1 + a*x)]])/a^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[(2*p + 3)*((c*d^2 + a*e^2)/(2*a*c*(p + 1))), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
```

```
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^m)*(2*c*x)*((a + c*x^2)^(p + 1)/(4*a*c*(p + 1))), x] - Dist[m*(
(2*c*d)/(4*a*c*(p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x]
 /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p +
3, 0] && LtQ[p, -1]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\text{sech}^{-1}(ax)}x^2\,dx &= \int x^2\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2 dx \\
&= -\frac{4\text{Subst}\left(\int \frac{x(1+x)^4}{(1+x^2)^4}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^3} \\
&= \frac{(1+ax)^3\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^4}{12a^3} - \frac{2\text{Subst}\left(\int \frac{4(1+x)^3}{(1+x^2)^3}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{3a^3} \\
&= \frac{(1+ax)^3\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^4}{12a^3} - \frac{8\text{Subst}\left(\int \frac{(1+x)^3}{(1+x^2)^3}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{3a^3} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^3}{6a^3} + \frac{(1+ax)^3\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^4}{12a^3} - \frac{2\text{Subst}\Big(}{} \\
&= \frac{(1+ax)\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}{2a^3} - \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2\Big(1+\sqrt{\frac{1-ax}{1+ax}}}{6a^3} \\
&= \frac{(1+ax)\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}{2a^3} - \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)^2\Big(1+\sqrt{\frac{1-ax}{1+ax}}}{6a^3}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.05, size = 86, normalized size = 0.51

$$
\frac{2x}{a^2}-\frac{x^3}{3}+\sqrt{\frac{1-ax}{1+ax}}\left(\frac{x}{a^2}+\frac{x^2}{a}\right)+\frac{i\log\left(-2iax+2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\right)}{a^3}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcSech[a*x])*x^2,x]
```

```
[Out] (2*x)/a^2 - x^3/3 + Sqrt[(1 - a*x)/(1 + a*x)]*(x/a^2 + x^2/a) + (I*Log[(-2*
I)*a*x + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)])/a^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 105, normalized size = 0.62

method	result	size
default	$\frac{-\frac{1}{3}a^2x^3+x}{a^2}+\frac{\sqrt{\frac{ax+1}{ax}}\, x\sqrt{-\frac{ax-1}{ax}}\,\left(x\sqrt{-a^2x^2+1}\,\mathrm{csgn}(a)a+\arctan\left(\frac{\mathrm{csgn}(a)ax}{\sqrt{-a^2x^2+1}}\right)\right)\mathrm{csgn}(a)}{a^2\sqrt{-a^2x^2+1}}+\frac{x}{a^2}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/3*a^2*x^3+x)+1/a^2*((a*x+1)/a/x)^(1/2)*x*(-(a*x-1)/a/x)^(1/2)*(x*
(-a^2*x^2+1)^(1/2)*csgn(a)*a+arctan(csgn(a)*a*x/(-a^2*x^2+1)^(1/2)))/(-a^2*
x^2+1)^(1/2)*csgn(a)+x/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^2,x, algorithm="maxim
a")
```

```
[Out] 2*x/a^2 + 2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1), x)/a^2 - integrate(x^2,
 x)
```

Fricas [A]
time = 0.43, size = 87, normalized size = 0.51

$$-\frac{a^3x^3-3\,a^2x^2\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}-6\,ax+3\,\arctan\left(\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^2,x, algorithm="frica
s")
```

```
[Out] -1/3*(a^3*x^3 - 3*a^2*x^2*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 6*
a*x + 3*arctan(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))))/a^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int 2\,dx+\int\left(-a^2x^2\right)\,dx+\int 2ax\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2*x**2,x)
```

```
[Out] (Integral(2, x) + Integral(-a**2*x**2, x) + Integral(2*a*x*sqrt(-1 + 1/(a*x
))*sqrt(1 + 1/(a*x)), x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x^2,x, algorithm="giac"
)
```

```
[Out] integrate(x^2*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2, x)
```

Mupad [B]
time = 8.96, size = 420, normalized size = 2.49

$$\frac{\frac{1i}{16a^3}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-i\right)^2\,1i}{8a^3\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-i\right)^4\,15i}{16a^3\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}}{\frac{\left(\sqrt{\frac{1}{a\,x}-1}-i\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+\frac{2\left(\sqrt{\frac{1}{a\,x}-1}-i\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-i\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}}-\frac{x^3\left(\frac{a^2}{3}-\frac{2}{x^2}\right)}{a^2}+\frac{\left(\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x}-1}-i\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-i}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\right)2i}{a^3}+\frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-i}{\sqrt{\frac{1}{a\,x}+1}-1}\right)1i}{a^3}-\frac{\ln\left(\frac{2a\sqrt{\frac{a+\frac{1}{x}}{a}}-\frac{2}{x}+a\sqrt{-\frac{a-\frac{1}{x}}{a}}\,2i}{2a+\frac{1}{x}-2a\sqrt{\frac{a+\frac{1}{x}}{a}}}\right)1i}{a^3}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-i\right)^2\,1i}{16a^3\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2,x)
```

```
[Out] ((log(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1) - log((
(1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)))*2i)/a^3 + (log(((1/(a
*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i)/a^3 + (1i/(16*a^3) + ((
(1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(8*a^3*((1/(a*x) + 1)^(1/2) - 1)^2) - (((1/
(a*x) - 1)^(1/2) - 1i)^4*15i)/(16*a^3*((1/(a*x) + 1)^(1/2) - 1)^4))/(((1/(a
*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + (2*((1/(a*x) - 1)^(1/2
) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 + ((1/(a*x) - 1)^(1/2) - 1i)^6/((1/(
a*x) + 1)^(1/2) - 1)^6) - (log((a*(-(a - 1/x)/a)^(1/2)*2i - 2/x + 2*a*((a +
 1/x)/a)^(1/2))/(2*a + 1/x - 2*a*((a + 1/x)/a)^(1/2)))*1i)/a^3 + (((1/(a*x)
 - 1)^(1/2) - 1i)^2*1i)/(16*a^3*((1/(a*x) + 1)^(1/2) - 1)^2) - (x^3*(a^2/3
- 2/x^2))/a^2
```

3.68 $\int e^{2\text{sech}^{-1}(ax)} x \, dx$

Optimal. Leaf size=85

$$-\frac{(1+ax)^2}{2a^2}+\frac{(1+ax)\left(1+2\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}+\frac{2\log(1+ax)}{a^2}+\frac{4\log\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}$$

```
[Out] -1/2*(a*x+1)^2/a^2+2*ln(a*x+1)/a^2+4*ln(1-((-a*x+1)/(a*x+1))^(1/2))/a^2+(a*
x+1)*(1+2*((-a*x+1)/(a*x+1))^(1/2))/a^2
```

Rubi [A]
time = 0.28, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6472, 1661, 1607, 815, 266}

$$-\frac{(ax+1)^2}{2a^2}+\frac{\left(2\sqrt{\frac{1-ax}{ax+1}}+1\right)(ax+1)}{a^2}+\frac{2\log(ax+1)}{a^2}+\frac{4\log\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x])*x,x]
```

```
[Out] -1/2*(1 + a*x)^2/a^2 + ((1 + a*x)*(1 + 2*Sqrt[(1 - a*x)/(1 + a*x)]))/a^2 +
(2*Log[1 + a*x])/a^2 + (4*Log[1 - Sqrt[(1 - a*x)/(1 + a*x)]])/a^2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
```

```
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int e^{2\text{sech}^{-1}(ax)}x\,dx = \int x\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2 dx$$

$$= \frac{4\text{Subst}\left(\int \frac{x(1+x)^3}{(-1+x)(1+x^2)^3}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}$$

$$= -\frac{(1+ax)^2}{2a^2} - \frac{\text{Subst}\left(\int \frac{-12x-4x^2}{(-1+x)(1+x^2)^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}$$

$$= -\frac{(1+ax)^2}{2a^2} - \frac{\text{Subst}\left(\int \frac{(-12-4x)x}{(-1+x)(1+x^2)^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}$$

$$= -\frac{(1+ax)^2}{2a^2} + \frac{(1+ax)\left(1+2\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2} + \frac{\text{Subst}\left(\int \frac{8+8x}{(-1+x)(1+x^2)}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{2a^2}$$

$$= -\frac{(1+ax)^2}{2a^2} + \frac{(1+ax)\left(1+2\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2} + \frac{\text{Subst}\left(\int \left(\frac{8}{-1+x}-\frac{8x}{1+x^2}\right)dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{2a^2}$$

$$= -\frac{(1+ax)^2}{2a^2} + \frac{(1+ax)\left(1+2\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2} + \frac{4\log\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2} - \frac{4\text{Subst}\left(\int \frac{}{1+}\right.}{}$$

$$= -\frac{(1+ax)^2}{2a^2} + \frac{(1+ax)\left(1+2\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2} + \frac{2\log(1+ax)}{a^2} + \frac{4\log\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}$$

Mathematica [A]
time = 0.04, size = 89, normalized size = 1.05

$$\frac{-a^2x^2+4\sqrt{\frac{1-ax}{1+ax}}(1+ax)+8\log(x)-4\log\left(1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcSech[a*x])*x,x]
```

```
[Out] (-(a^2*x^2) + 4*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) + 8*Log[x] - 4*Log[1 +
Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/(2*a^2)
```

Maple [A]
time = 0.04, size = 98, normalized size = 1.15

method	result	size
default	$\frac{-\frac{a^{2} x^{2}}{2}+\ln \left(x \right)}{a^{2}}-\frac{2 \sqrt{\frac{a x +1}{a x}}\, x \sqrt{-\frac{a x -1}{a x}}\, \left(-\sqrt{-a^{2} x^{2}+1}+\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^{2} x^{2}+1}}\right)\right)}{a \sqrt{-a^{2} x^{2}+1}}+\frac{\ln \left(x \right)}{a^{2}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/2*a^2*x^2+ln(x))-2/a*((a*x+1)/a/x)^(1/2)*x*(-(a*x-1)/a/x)^(1/2)*(
-(-a^2*x^2+1)^(1/2)+arctanh(1/(-a^2*x^2+1)^(1/2)))/(-a^2*x^2+1)^(1/2)+ln(x)
/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x,x, algorithm="maxima"
)
```

```
[Out] integrate(x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2, x)
```

Fricas [A]
time = 0.45, size = 124, normalized size = 1.46

$$-\frac{a^{2} x^{2} - 4 \, a x \sqrt{\frac{a x + 1}{a x}} \sqrt{-\frac{a x - 1}{a x}} + 2 \, \log\left(a x \sqrt{\frac{a x + 1}{a x}} \sqrt{-\frac{a x - 1}{a x}} + 1\right) - 2 \, \log\left(a x \sqrt{\frac{a x + 1}{a x}} \sqrt{-\frac{a x - 1}{a x}} - 1\right) - 4 \, \log\left(x\right)}{2 \, a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x,x, algorithm="fricas"
)
```

```
[Out] -1/2*(a^2*x^2 - 4*a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 2*log(
a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - 2*log(a*x*sqrt((a*x
 + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) - 4*log(x))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{2}{x}\, dx + \int \left(- a^{2} x\right)\, dx + \int 2 a \sqrt{-1 + \frac{1}{a x}} \sqrt{1 + \frac{1}{a x}}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2*x,x)
```

```
[Out] (Integral(2/x, x) + Integral(-a**2*x, x) + Integral(2*a*sqrt(-1 + 1/(a*x))*
sqrt(1 + 1/(a*x)), x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2*x,x, algorithm="giac")
```

```
[Out] integrate(x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2, x)
```

Mupad [B]
time = 3.69, size = 56, normalized size = 0.66

$$\frac{2\,x\,\sqrt{\frac{1}{a\,x}-1}\,\sqrt{\frac{1}{a\,x}+1}}{a}-\frac{2\,\mathrm{acosh}\left(\frac{1}{a\,x}\right)}{a^2}-\frac{x^2}{2}-\frac{2\,\ln\left(\frac{1}{x}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2,x)
```

```
[Out] (2*x*(1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2))/a - (2*acosh(1/(a*x)))/a^2 -
x^2/2 - (2*log(1/x))/a^2
```

3.69 $\int e^{2\text{sech}^{-1}(ax)}\,dx$

Optimal. Leaf size=57

$$-x-\frac{4}{a\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}+\frac{4\text{ArcTan}\left(\sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

```
[Out] -x+4*arctan(((-a*x+1)/(a*x+1))^(1/2))/a-4/a/(1-((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A]
time = 0.11, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6467, 1661, 12, 815, 209}

$$\frac{4\text{ArcTan}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{a}-\frac{4}{a\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x]),x]
```

```
[Out] -x - 4/(a*(1 - Sqrt[(1 - a*x)/(1 + a*x)])) + (4*ArcTan[Sqrt[(1 - a*x)/(1 +
a*x)]])/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6467

```
Int[E^(ArcSech[u_]*(n_.)), x_Symbol] :> Int[(1/u + Sqrt[(1 - u)/(1 + u)] +
(1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; IntegerQ[n]
```

Rubi steps

$$\int e^{2\text{sech}^{-1}(ax)}\,dx = \int \left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2 dx$$

$$= -\frac{4\text{Subst}\left(\int \frac{x(1+x)^2}{(-1+x)^2(1+x^2)^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

$$= -x+\frac{2\text{Subst}\left(\int -\frac{4x}{(-1+x)^2(1+x^2)}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

$$= -x-\frac{8\text{Subst}\left(\int \frac{x}{(-1+x)^2(1+x^2)}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

$$= -x-\frac{8\text{Subst}\left(\int \left(\frac{1}{2(-1+x)^2}-\frac{1}{2(1+x^2)}\right)dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

$$= -x-\frac{4}{a\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}+\frac{4\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

$$= -x-\frac{4}{a\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}+\frac{4\tan^{-1}\left(\sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

<u>Mathematica</u> [A]
time = 0.07, size = 75, normalized size = 1.32

$$-\frac{2+a^2x^2+2\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)+2ax\text{ArcTan}\left(\frac{ax}{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}\right)}{a^2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcSech[a*x]),x]
```

[Out] -((2 + a^2*x^2 + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) + 2*a*x*ArcTan[(a*x)/(Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))])/(a^2*x))

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 111, normalized size = 1.95

method	result	size
default	$\frac{-a^2x-\frac{1}{x}}{a^2}-\frac{2\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,\left(\arctan\left(\frac{\mathrm{csgn}(a)ax}{\sqrt{-a^2x^2+1}}\right)ax+\mathrm{csgn}(a)\sqrt{-a^2x^2+1}\right)\mathrm{csgn}(a)}{a\sqrt{-a^2x^2+1}}-\frac{1}{a^2x}$	111

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2,x,method=_RETURNVERBOSE)

[Out] 1/a^2*(-a^2*x-1/x)-2/a*((a*x+1)/a/x)^(1/2)*(-(a*x-1)/a/x)^(1/2)*(arctan(csgn(a)*a*x/(-a^2*x^2+1)^(1/2))*a*x+csgn(a)*(-a^2*x^2+1)^(1/2))*csgn(a)/(-a^2*x^2+1)^(1/2)-1/a^2/x

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2,x, algorithm="maxima")

[Out] -x + 2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^2, x)/a^2 + integrate(x^(-2), x)/a^2 - 1/(a^2*x)

Fricas [A]
time = 0.48, size = 85, normalized size = 1.49

$$-\frac{a^2x^2+2\,ax\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}-2\,ax\arctan\left(\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\right)+2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2,x, algorithm="fricas")

[Out] -(a^2*x^2 + 2*a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 2*a*x*arctan(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))) + 2)/(a^2*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\left(-a^2\right)\,dx+\int\frac{2}{x^2}\,dx+\int\frac{2a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2,x)
```

```
[Out] (Integral(-a**2, x) + Integral(2/x**2, x) + Integral(2*a*sqrt(-1 + 1/(a*x))
*sqrt(1 + 1/(a*x))/x, x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2, x)
```

Mupad [B]
time = 4.64, size = 162, normalized size = 2.84

$$-x-\frac{\left(\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\right)2\mathrm{i}}{a}-\frac{2}{a^2\,x}+\frac{\left(1+\sqrt{-\frac{a-\frac{1}{x}}{a}}\,1\mathrm{i}\right)^2\left(\sqrt{\frac{a+\frac{1}{x}}{a}}-1\right)^2 4\mathrm{i}}{a\left(\sqrt{\frac{a+\frac{1}{x}}{a}}\,1\mathrm{i}+\sqrt{-\frac{a-\frac{1}{x}}{a}}-2\mathrm{i}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2,x)
```

```
[Out] (((-(a - 1/x)/a)^(1/2)*1i + 1)^2*(((a + 1/x)/a)^(1/2) - 1)^2*4i)/(a*(((a +
1/x)/a)^(1/2)*1i + (-(a - 1/x)/a)^(1/2) - 2i)^2) - ((log(((1/(a*x) - 1)^(1/
2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1) - log(((1/(a*x) - 1)^(1/2) - 1i
)/((1/(a*x) + 1)^(1/2) - 1)))*2i)/a - 2/(a^2*x) - x
```

3.70 $\int \frac{e^{2\text{sech}^{-1}(ax)}}{x} \, dx$

Optimal. Leaf size=86

$$-\frac{2}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{2}{1-\sqrt{\frac{1-ax}{1+ax}}}-\log(1+ax)-2\log\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)$$

```
[Out] -ln(a*x+1)-2*ln(1-((-a*x+1)/(a*x+1))^(1/2))-2/(1-((-a*x+1)/(a*x+1))^(1/2))^
2+2/(1-((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A]
time = 0.30, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6472, 1643, 266}

$$\frac{2}{1-\sqrt{\frac{1-ax}{ax+1}}}-\frac{2}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}-\log(ax+1)-2\log\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x])/x,x]
```

```
[Out] -2/(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2 + 2/(1 - Sqrt[(1 - a*x)/(1 + a*x)]) -
Log[1 + a*x] - 2*Log[1 - Sqrt[(1 - a*x)/(1 + a*x)]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x}\,dx = \int \frac{\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2}{x}\,dx$$

$$= 4\text{Subst}\left(\int \frac{x(1+x)}{(-1+x)^3\,(1+x^2)}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)$$

$$= 4\text{Subst}\left(\int \left(\frac{1}{(-1+x)^3}+\frac{1}{2(-1+x)^2}-\frac{1}{2(-1+x)}+\frac{x}{2\,(1+x^2)}\right)\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)$$

$$= -\frac{2}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{2}{1-\sqrt{\frac{1-ax}{1+ax}}}-2\log\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)+2\text{Subst}\left(\int \frac{x}{1+x^2}\,d$$

$$= -\frac{2}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{2}{1-\sqrt{\frac{1-ax}{1+ax}}}-\log(1+ax)-2\log\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)$$

Mathematica [A]
time = 0.04, size = 86, normalized size = 1.00

$$-\frac{1}{a^2x^2}-\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{a^2x^2}-2\log(x)+\log\left(1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcSech[a*x])/x,x]
```

```
[Out] -(1/(a^2*x^2)) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(a^2*x^2) - 2*Log[x]
 + Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]]
```

Maple [A]
time = 0.04, size = 110, normalized size = 1.28

method	result	size
default	$\frac{-\frac{1}{2x^2}-a^2\ln(x)}{a^2}-\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,\left(-a^2x^2\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+\sqrt{-a^2x^2+1}\right)}{ax\sqrt{-a^2x^2+1}}-\frac{1}{2a^2x^2}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/2/x^2-a^2*ln(x))-1/a*((a*x+1)/a/x)^(1/2)/x*(-(a*x-1)/a/x)^(1/2)*(
-a^2*x^2*arctanh(1/(-a^2*x^2+1)^(1/2))+(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/
2)-1/2/a^2/x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x,x, algorithm="maxima"
)
```

```
[Out] 2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^3, x)/a^2 - 1/(a^2*x^2) - integr
ate(1/x, x)
```

Fricas [A]
time = 0.35, size = 138, normalized size = 1.60

$$\frac{a^2x^2\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-a^2x^2\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)-2\,a^2x^2\log\left(x\right)-2\,ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-2}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x,x, algorithm="fricas"
)
```

```
[Out] 1/2*(a^2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - a^
2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) - 2*a^2*x^2
*log(x) - 2*a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 2)/(a^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{2}{x^3}\,dx+\int\left(-\frac{a^2}{x}\right)\,dx+\int\frac{2a\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{x^2}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x,x)
```

```
[Out] (Integral(2/x**3, x) + Integral(-a**2/x, x) + Integral(2*a*sqrt(-1 + 1/(a*x
))*sqrt(1 + 1/(a*x))/x**2, x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x,x, algorithm="giac")
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x, x)
```

Mupad [B]
time = 11.21, size = 323, normalized size = 3.76

$$\ln\left(\frac{1}{x}\right)-4\,\mathrm{atanh}\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)+2\,\mathrm{acosh}\left(\frac{1}{a\,x}\right)+\frac{\frac{28\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^3}+\frac{28\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^5}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^5}+\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^7}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^7}+\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}{\sqrt{\frac{1}{a\,x}+1}-1}}{1+\frac{6\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}-\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^6}+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^8}-\frac{4\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}}-\frac{1}{a^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2/x,x)
```

```
[Out] log(1/x) - 4*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)) +
2*acosh(1/(a*x)) + ((28*((1/(a*x) - 1)^(1/2) - 1i)^3)/((1/(a*x) + 1)^(1/2)
- 1)^3 + (28*((1/(a*x) - 1)^(1/2) - 1i)^5)/((1/(a*x) + 1)^(1/2) - 1)^5 + (4
*((1/(a*x) - 1)^(1/2) - 1i)^7)/((1/(a*x) + 1)^(1/2) - 1)^7 + (4*((1/(a*x) -
 1)^(1/2) - 1i))/((1/(a*x) + 1)^(1/2) - 1))/((6*((1/(a*x) - 1)^(1/2) - 1i)^
4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (4*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x)
 + 1)^(1/2) - 1)^2 - (4*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2)
- 1)^6 + ((1/(a*x) - 1)^(1/2) - 1i)^8/((1/(a*x) + 1)^(1/2) - 1)^8 + 1) - 1/
(a^2*x^2)
```

3.71 $\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=57

$$-\frac{4a}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}+\frac{2a}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}$$

```
[Out] -4/3*a/(1-((-a*x+1)/(a*x+1))^(1/2))^3+2*a/(1-((-a*x+1)/(a*x+1))^(1/2))^2
```

Rubi [A]
time = 0.26, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6472, 45}

$$\frac{2a}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}-\frac{4a}{3\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x])/x^2,x]
```

```
[Out] (-4*a)/(3*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^3) + (2*a)/(1 - Sqrt[(1 - a*x)/(1
 + a*x)])^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^2}\,dx = \int \frac{\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2}{x^2}\,dx$$
$$= -\left((4a)\text{Subst}\left(\int \frac{x}{(-1+x)^4}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\right)$$
$$= -\left((4a)\text{Subst}\left(\int \left(\frac{1}{(-1+x)^4}+\frac{1}{(-1+x)^3}\right)\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\right)$$
$$= -\frac{4a}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}+\frac{2a}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}$$

Mathematica [A]
time = 0.05, size = 52, normalized size = 0.91

$$\frac{-2+3a^2x^2+2(-1+ax)\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^2}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcSech[a*x])/x^2,x]
```

```
[Out] (-2 + 3*a^2*x^2 + 2*(-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(3*a^
2*x^3)
```

Maple [A]
time = 0.03, size = 73, normalized size = 1.28

method	result	size
default	$\frac{\frac{a^2}{x}-\frac{1}{3x^3}}{a^2}+\frac{2\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,(a^2x^2-1)}{3a\,x^2}-\frac{1}{3a^2x^3}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(a^2/x-1/3/x^3)+2/3/a*((a*x+1)/a/x)^(1/2)/x^2*(-(a*x-1)/a/x)^(1/2)*(a
^2*x^2-1)-1/3/a^2/x^3
```

Maxima [A]
time = 0.27, size = 46, normalized size = 0.81

$$\frac{1}{x}+\frac{2\,(a^2x^3-x)\sqrt{ax+1}\,\sqrt{-ax+1}}{3\,a^2x^4}-\frac{2}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^2,x, algorithm="maxim
a")
```

```
[Out] 1/x + 2/3*(a^2*x^3 - x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a^2*x^4) - 2/3/(a^2*x
^3)
```

Fricas [A]
time = 0.35, size = 61, normalized size = 1.07

$$\frac{3\,a^2x^2+2\,(a^3x^3-ax)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}-2}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^2,x, algorithm="frica
s")
```

```
[Out] 1/3*(3*a^2*x^2 + 2*(a^3*x^3 - a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a
*x)) - 2)/(a^2*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{2}{x^4}\,dx+\int\left(-\frac{a^2}{x^2}\right)\,dx+\int\frac{2a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^3}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x**2,x)
```

```
[Out] (Integral(2/x**4, x) + Integral(-a**2/x**2, x) + Integral(2*a*sqrt(-1 + 1/(
a*x))*sqrt(1 + 1/(a*x))/x**3, x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^2,x, algorithm="giac"
)
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x^2, x)
```

Mupad [B]
time = 1.80, size = 67, normalized size = 1.18

$$\frac{a^2\,x^2-\frac{2}{3}}{a^2\,x^3}-\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{2\,\sqrt{\frac{1}{a\,x}+1}}{3\,a}-\frac{2\,a\,x^2\,\sqrt{\frac{1}{a\,x}+1}}{3}\right)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2/x^2,x)
```

```
[Out] (a^2*x^2 - 2/3)/(a^2*x^3) - ((1/(a*x) - 1)^(1/2)*((2*(1/(a*x) + 1)^(1/2))/(
3*a) - (2*a*x^2*(1/(a*x) + 1)^(1/2))/3))/x^2
```

3.72 $\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=147

$$-\frac{a^2}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}+\frac{2a^2}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}-\frac{3a^2}{2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a^2}{2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}+\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{\right.$$

```
[Out] 1/2*a^2*arctanh(((-a*x+1)/(a*x+1))^(1/2))-a^2/(1-((-a*x+1)/(a*x+1))^(1/2))^
4+2*a^2/(1-((-a*x+1)/(a*x+1))^(1/2))^3-3/2*a^2/(1-((-a*x+1)/(a*x+1))^(1/2))
^2+1/2*a^2/(1-((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A]
time = 0.30, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6472, 1626, 213}

$$\frac{a^2}{2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{3a^2}{2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}+\frac{2a^2}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}-\frac{a^2}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^4}+\frac{1}{2}a^2\tanh^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x])/x^3,x]
```

```
[Out] -(a^2/(1 - Sqrt[(1 - a*x)/(1 + a*x)])^4) + (2*a^2)/(1 - Sqrt[(1 - a*x)/(1 +
 a*x)])^3 - (3*a^2)/(2*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2) + a^2/(2*(1 - Sqr
t[(1 - a*x)/(1 + a*x)])) + (a^2*ArcTanh[Sqrt[(1 - a*x)/(1 + a*x)]])/2
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
```

`Q[n]`

Rubi steps

$$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^3}\,dx = \int \frac{\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2}{x^3}\,dx$$

$$= \left(4a^2\right)\operatorname{Subst}\left(\int \frac{x(1+x^2)}{(-1+x)^5(1+x)}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)$$

$$= \left(4a^2\right)\operatorname{Subst}\left(\int \left(\frac{1}{(-1+x)^5}+\frac{3}{2(-1+x)^4}+\frac{3}{4(-1+x)^3}+\frac{1}{8(-1+x)^2}-\frac{1}{8\,(-1+x^2)}\right)\right.$$

$$= -\frac{a^2}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}+\frac{2a^2}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}-\frac{3a^2}{2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a^2}{2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right.}$$

$$= -\frac{a^2}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}+\frac{2a^2}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}-\frac{3a^2}{2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a^2}{2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right.}$$

Mathematica [A]
time = 0.09, size = 121, normalized size = 0.82

$$\frac{\frac{(1+ax)\left(-2+2ax-2\sqrt{\frac{1-ax}{1+ax}}+a^2x^2\sqrt{\frac{1-ax}{1+ax}}\right)}{x^4}-a^4\log(x)+a^4\log\left(1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)}{4a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcSech[a*x])/x^3,x]
```

```
[Out] (((1 + a*x)*(-2 + 2*a*x - 2*Sqrt[(1 - a*x)/(1 + a*x)] + a^2*x^2*Sqrt[(1 - a
*x)/(1 + a*x)]))/x^4 - a^4*Log[x] + a^4*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] +
 a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/(4*a^2)
```

Maple [A]
time = 0.04, size = 131, normalized size = 0.89

method	result
default	$\frac{-\frac{1}{4x^4}+\frac{a^2}{2x^2}}{a^2}+\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)a^4x^4+a^2x^2\sqrt{-a^2x^2+1}-2\sqrt{-a^2x^2+1}\right)}{4a\,x^3\sqrt{-a^2x^2+1}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/4/x^4+1/2*a^2/x^2)+1/4/a*((a*x+1)/a/x)^(1/2)/x^3*(-(a*x-1)/a/x)^(
1/2)*(arctanh(1/(-a^2*x^2+1)^(1/2))*a^4*x^4+a^2*x^2*(-a^2*x^2+1)^(1/2)-2*(-
a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2)-1/4/a^2/x^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^3,x, algorithm="maxim
a")
```

```
[Out] 2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^5, x)/a^2 - 1/2/(a^2*x^4) - inte
grate(x^(-3), x)
```

Fricas [A]
time = 0.54, size = 146, normalized size = 0.99

$$\frac{a^4x^4\log\left(ax\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}+1\right)-a^4x^4\log\left(ax\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}-1\right)+4\,a^2x^2+2\,(a^3x^3-2\,ax)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}-4}{8\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^3,x, algorithm="frica
s")
```

```
[Out] 1/8*(a^4*x^4*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - a^
4*x^4*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 4*a^2*x^2
 + 2*(a^3*x^3 - 2*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 4)/(a
^2*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{2}{x^5}\,dx+\int\left(-\frac{a^2}{x^3}\right)dx+\int\frac{2a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^4}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x**3,x)
```

```
[Out] (Integral(2/x**5, x) + Integral(-a**2/x**3, x) + Integral(2*a*sqrt(-1 + 1/(
a*x))*sqrt(1 + 1/(a*x))/x**4, x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^3,x, algorithm="giac"
)
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x^3, x)
```

Mupad [B]
time = 46.99, size = 885, normalized size = 6.02

$$a^2\,\mathrm{atanh}\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)-\frac{\frac{28\,a^2\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^3}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^3}+\frac{28\,a^2\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^5}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^5}+\frac{4\,a^2\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^7}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^7}+\frac{4\,a^2\,\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}{\sqrt{\frac{1}{a\,x}+1}-1}}{1+\frac{6\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^4}-\frac{4\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^6}+\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^8}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^8}-\frac{4\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^2}}-\frac{\frac{23\,a^2\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^3}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^3}+\frac{333\,a^2\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^5}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^5}+\frac{671\,a^2\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^7}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^7}+\frac{671\,a^2\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^9}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^9}+\frac{333\,a^2\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^{11}}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^{11}}+\frac{23\,a^2\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^{13}}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^{13}}-\frac{3\,a^2\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^{15}}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^{15}}-\frac{3\,a^2\,\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}{\sqrt{\frac{1}{a\,x}+1}-1}}{1+\frac{28\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^4}-\frac{56\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^6}+\frac{70\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^8}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^8}-\frac{56\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^{10}}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^{10}}+\frac{28\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^{12}}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^{12}}-\frac{8\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^{14}}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^{14}}+\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^{16}}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^{16}}-\frac{8\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^2}}+\frac{1}{2\,x^2}-\frac{1}{2\,a^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2/x^3,x)
```

```
[Out] a^2*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)) - ((28*a^2*
((1/(a*x) - 1)^(1/2) - 1i)^3)/((1/(a*x) + 1)^(1/2) - 1)^3 + (28*a^2*((1/(a*
x) - 1)^(1/2) - 1i)^5)/((1/(a*x) + 1)^(1/2) - 1)^5 + (4*a^2*((1/(a*x) - 1)^
(1/2) - 1i)^7)/((1/(a*x) + 1)^(1/2) - 1)^7 + (4*a^2*((1/(a*x) - 1)^(1/2) -
1i))/((1/(a*x) + 1)^(1/2) - 1))/((6*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x)
 + 1)^(1/2) - 1)^4 - (4*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2)
- 1)^2 - (4*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + ((1
/(a*x) - 1)^(1/2) - 1i)^8/((1/(a*x) + 1)^(1/2) - 1)^8 + 1) - ((23*a^2*((1/(
a*x) - 1)^(1/2) - 1i)^3)/((1/(a*x) + 1)^(1/2) - 1)^3 + (333*a^2*((1/(a*x) -
 1)^(1/2) - 1i)^5)/((1/(a*x) + 1)^(1/2) - 1)^5 + (671*a^2*((1/(a*x) - 1)^(1
/2) - 1i)^7)/((1/(a*x) + 1)^(1/2) - 1)^7 + (671*a^2*((1/(a*x) - 1)^(1/2) -
1i)^9)/((1/(a*x) + 1)^(1/2) - 1)^9 + (333*a^2*((1/(a*x) - 1)^(1/2) - 1i)^11
)/((1/(a*x) + 1)^(1/2) - 1)^11 + (23*a^2*((1/(a*x) - 1)^(1/2) - 1i)^13)/((1
/(a*x) + 1)^(1/2) - 1)^13 - (3*a^2*((1/(a*x) - 1)^(1/2) - 1i)^15)/((1/(a*x)
 + 1)^(1/2) - 1)^15 - (3*a^2*((1/(a*x) - 1)^(1/2) - 1i))/((1/(a*x) + 1)^(1/
2) - 1))/((28*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (
8*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (56*((1/(a*x)
 - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (70*((1/(a*x) - 1)^(1/2)
 - 1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8 - (56*((1/(a*x) - 1)^(1/2) - 1i)^10)/
```

```
((1/(a*x) + 1)^(1/2) - 1)^10 + (28*((1/(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x)
 + 1)^(1/2) - 1)^12 - (8*((1/(a*x) - 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^(1/2
) - 1)^14 + ((1/(a*x) - 1)^(1/2) - 1i)^16/((1/(a*x) + 1)^(1/2) - 1)^16 + 1)
 + 1/(2*x^2) - 1/(2*a^2*x^4)
```

3.73 $\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=183

$$-\frac{4a^3}{5\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5}+\frac{2a^3}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}-\frac{7a^3}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}+\frac{3a^3}{2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}-\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}$$

```
[Out] -4/5*a^3/(1-((-a*x+1)/(a*x+1))^(1/2))^5+2*a^3/(1-((-a*x+1)/(a*x+1))^(1/2))^
4-7/3*a^3/(1-((-a*x+1)/(a*x+1))^(1/2))^3+3/2*a^3/(1-((-a*x+1)/(a*x+1))^(1/2
))^2-1/4*a^3/(1-((-a*x+1)/(a*x+1))^(1/2))-1/4*a^3/(1+((-a*x+1)/(a*x+1))^(1/
2))
```

Rubi [A]
time = 0.33, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6472, 1626}

$$-\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{a^3}{4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}+\frac{3a^3}{2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}-\frac{7a^3}{3\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}+\frac{2a^3}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^4}-\frac{4a^3}{5\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x])/x^4,x]
```

```
[Out] (-4*a^3)/(5*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^5) + (2*a^3)/(1 - Sqrt[(1 - a*x
)/(1 + a*x)])^4 - (7*a^3)/(3*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^3) + (3*a^3)/(
2*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2) - a^3/(4*(1 - Sqrt[(1 - a*x)/(1 + a*x)
])) - a^3/(4*(1 + Sqrt[(1 - a*x)/(1 + a*x)]))
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^4}\,dx = \int \frac{\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2}{x^4}\,dx$$
$$= -\left((4a)\text{Subst}\left(\int \frac{x(a+ax^2)^2}{(-1+x)^6(1+x)^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\right)$$
$$= -\left((4a)\text{Subst}\left(\int \left(\frac{a^2}{(-1+x)^6}+\frac{2a^2}{(-1+x)^5}+\frac{7a^2}{4(-1+x)^4}+\frac{3a^2}{4(-1+x)^3}+\frac{a^2}{16(-1+x\right.\right.\right.$$
$$= -\frac{4a^3}{5\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5}+\frac{2a^3}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}-\frac{7a^3}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}+\frac{3a^3}{2\left(1-\sqrt{\frac{1-}{1+}}\right.}$$

Mathematica [A]
time = 0.06, size = 69, normalized size = 0.38

$$\frac{-6+5a^2x^2+2\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^2\,(-3+3ax-2a^2x^2+2a^3x^3)}{15a^2x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcSech[a*x])/x^4,x]
```

```
[Out] (-6 + 5*a^2*x^2 + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-3 + 3*a*x - 2*a
^2*x^2 + 2*a^3*x^3))/(15*a^2*x^5)
```

Maple [A]
time = 0.03, size = 84, normalized size = 0.46

method	result	size
default	$\frac{\frac{a^2}{3x^3}-\frac{1}{5x^5}}{a^2}+\frac{2\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\,(a^2x^2-1)\,(2a^2x^2+3)}{15a\,x^4}-\frac{1}{5a^2x^5}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/3*a^2/x^3-1/5/x^5)+2/15/a*((a*x+1)/a/x)^(1/2)/x^4*(-(a*x-1)/a/x)^(
1/2)*(a^2*x^2-1)*(2*a^2*x^2+3)-1/5/a^2/x^5
```

Maxima [A]
time = 0.27, size = 56, normalized size = 0.31

$$\frac{1}{3\,x^3} + \frac{2\,(2\,a^4x^5 + a^2x^3 - 3\,x)\sqrt{ax+1}\,\sqrt{-ax+1}}{15\,a^2x^6} - \frac{2}{5\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^4,x, algorithm="maxim
a")
```

```
[Out] 1/3/x^3 + 2/15*(2*a^4*x^5 + a^2*x^3 - 3*x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/(a^
2*x^6) - 2/5/(a^2*x^5)
```

Fricas [A]
time = 0.42, size = 69, normalized size = 0.38

$$\frac{5\,a^2x^2 + 2\,(2\,a^5x^5 + a^3x^3 - 3\,ax)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}} - 6}{15\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^4,x, algorithm="frica
s")
```

```
[Out] 1/15*(5*a^2*x^2 + 2*(2*a^5*x^5 + a^3*x^3 - 3*a*x)*sqrt((a*x + 1)/(a*x))*sqr
t(-(a*x - 1)/(a*x)) - 6)/(a^2*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{2}{x^6}\,dx + \int \left(-\frac{a^2}{x^4}\right)\,dx + \int \frac{2a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^5}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x**4,x)
```

```
[Out] (Integral(2/x**6, x) + Integral(-a**2/x**4, x) + Integral(2*a*sqrt(-1 + 1/(
a*x))*sqrt(1 + 1/(a*x))/x**5, x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^4,x, algorithm="giac"
)
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x^4, x)
```

Mupad [B]
time = 1.94, size = 86, normalized size = 0.47

$$\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{2\,a\,x^2\,\sqrt{\frac{1}{a\,x}+1}}{15}-\frac{2\,\sqrt{\frac{1}{a\,x}+1}}{5\,a}+\frac{4\,a^3\,x^4\,\sqrt{\frac{1}{a\,x}+1}}{15}\right)}{x^4}+\frac{\frac{a^2\,x^2}{3}-\frac{2}{5}}{a^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2/x^4,x)
```

```
[Out] ((1/(a*x) - 1)^(1/2)*((2*a*x^2*(1/(a*x) + 1)^(1/2))/15 - (2*(1/(a*x) + 1)^(
1/2))/(5*a) + (4*a^3*x^4*(1/(a*x) + 1)^(1/2))/15))/x^4 + ((a^2*x^2)/3 - 2/5
)/(a^2*x^5)
```

3.74 $\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=267

$$-\frac{2a^4}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^6}+\frac{2a^4}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5}-\frac{3a^4}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}+\frac{8a^4}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}-\frac{11a^4}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right.}$$

```
[Out] 1/4*a^4*arctanh(((-a*x+1)/(a*x+1))^(1/2))-2/3*a^4/(1-((-a*x+1)/(a*x+1))^(1/
2))^6+2*a^4/(1-((-a*x+1)/(a*x+1))^(1/2))^5-3*a^4/(1-((-a*x+1)/(a*x+1))^(1/2
))^4+8/3*a^4/(1-((-a*x+1)/(a*x+1))^(1/2))^3-11/8*a^4/(1-((-a*x+1)/(a*x+1))^
(1/2))^2+3/8*a^4/(1-((-a*x+1)/(a*x+1))^(1/2))-1/8*a^4/(1+((-a*x+1)/(a*x+1))
^(1/2))^2+1/8*a^4/(1+((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A]
time = 0.37, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6472, 1626, 213}

$$\frac{3a^4}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}+\frac{a^4}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}-\frac{11a^4}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}-\frac{a^4}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}+\frac{8a^4}{3\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}-\frac{3a^4}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^4}+\frac{2a^4}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^5}-\frac{2a^4}{3\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^6}+\frac{1}{4}a^4\tanh^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x])/x^5,x]
```

```
[Out] (-2*a^4)/(3*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^6) + (2*a^4)/(1 - Sqrt[(1 - a*x
)/(1 + a*x)])^5 - (3*a^4)/(1 - Sqrt[(1 - a*x)/(1 + a*x)])^4 + (8*a^4)/(3*(1
 - Sqrt[(1 - a*x)/(1 + a*x)])^3) - (11*a^4)/(8*(1 - Sqrt[(1 - a*x)/(1 + a*x
)])^2) + (3*a^4)/(8*(1 - Sqrt[(1 - a*x)/(1 + a*x)])) - a^4/(8*(1 + Sqrt[(1
- a*x)/(1 + a*x)])^2) + a^4/(8*(1 + Sqrt[(1 - a*x)/(1 + a*x)])) + (a^4*ArcT
anh[Sqrt[(1 - a*x)/(1 + a*x)]])/4
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^5}\,dx = \int \frac{\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2}{x^5}\,dx$$

$$= (4a)\text{Subst}\left(\int \frac{x\left(a+ax^2\right)^3}{(-1+x)^7(1+x)^3}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)$$

$$= (4a)\text{Subst}\left(\int \left(\frac{a^3}{(-1+x)^7}+\frac{5a^3}{2(-1+x)^6}+\frac{3a^3}{(-1+x)^5}+\frac{2a^3}{(-1+x)^4}+\frac{11a^3}{16(-1+x)^3}+\right.\right.$$

$$= -\frac{2a^4}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^6}+\frac{2a^4}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5}-\frac{3a^4}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}+\frac{8a^4}{3\left(1-\sqrt{\frac{1-a}{1+a}}\right.}$$

$$= -\frac{2a^4}{3\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^6}+\frac{2a^4}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5}-\frac{3a^4}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}+\frac{8a^4}{3\left(1-\sqrt{\frac{1-a}{1+a}}\right.}$$

Mathematica [A]
time = 0.10, size = 137, normalized size = 0.51

$$\frac{-8+6a^2x^2+\sqrt{\frac{1-ax}{1+ax}}\left(-8-8ax+2a^2x^2+2a^3x^3+3a^4x^4+3a^5x^5\right)-3a^6x^6\log(x)+3a^6x^6\log\left(1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)}{24a^2x^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcSech[a*x])/x^5,x]
```

```
[Out] (-8 + 6*a^2*x^2 + Sqrt[(1 - a*x)/(1 + a*x)]*(-8 - 8*a*x + 2*a^2*x^2 + 2*a^3
*x^3 + 3*a^4*x^4 + 3*a^5*x^5) - 3*a^6*x^6*Log[x] + 3*a^6*x^6*Log[1 + Sqrt[(
1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/(24*a^2*x^6)
```

Maple [A]
time = 0.04, size = 153, normalized size = 0.57

method	result
default	$\frac{\frac{a^2}{4x^4}-\frac{1}{6x^6}}{a^2}+\frac{\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\,\left(3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)a^6x^6+3\sqrt{-a^2x^2+1}\,a^4x^4+2a^2x^2\sqrt{-a^2x^2+1}-8$
	$24a\,x^5\sqrt{-a^2x^2+1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/4*a^2/x^4-1/6/x^6)+1/24/a*((a*x+1)/a/x)^(1/2)/x^5*(-(a*x-1)/a/x)^(
1/2)*(3*arctanh(1/(-a^2*x^2+1)^(1/2))*a^6*x^6+3*(-a^2*x^2+1)^(1/2)*a^4*x^4+
2*a^2*x^2*(-a^2*x^2+1)^(1/2)-8*(-a^2*x^2+1)^(1/2))/(-a^2*x^2+1)^(1/2)-1/6/a
^2/x^6
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^5,x, algorithm="maxim
a")
```

```
[Out] 2*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)/x^7, x)/a^2 - 1/3/(a^2*x^6) - inte
grate(x^(-5), x)
```

Fricas [A]
time = 0.44, size = 156, normalized size = 0.58

$$\frac{3\,a^6x^6\log\left(ax\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}+1\right)-3\,a^6x^6\log\left(ax\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}-1\right)+12\,a^2x^2+2\,(3\,a^5x^5+2\,a^3x^3-8\,ax)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}-16}{48\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^5,x, algorithm="frica
s")
```

```
[Out] 1/48*(3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) -
 3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 12*a
^2*x^2 + 2*(3*a^5*x^5 + 2*a^3*x^3 - 8*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x
 - 1)/(a*x)) - 16)/(a^2*x^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{2}{x^7}\,dx+\int\left(-\frac{a^2}{x^5}\right)\,dx+\int\frac{2a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^6}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x**5,x)
```

```
[Out] (Integral(2/x**7, x) + Integral(-a**2/x**5, x) + Integral(2*a*sqrt(-1 + 1/(
a*x))*sqrt(1 + 1/(a*x))/x**6, x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^5,x, algorithm="giac"
)
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x^5, x)
```

Mupad [B]
time = 65.19, size = 2480, normalized size = 9.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2/x^5,x)
```

```
[Out] ((311*a^4*((1/(a*x) - 1)^(1/2) - 1i)^5)/(2*((1/(a*x) + 1)^(1/2) - 1)^5) - (
175*a^4*((1/(a*x) - 1)^(1/2) - 1i)^3)/(6*((1/(a*x) + 1)^(1/2) - 1)^3) + (83
61*a^4*((1/(a*x) - 1)^(1/2) - 1i)^7)/(2*((1/(a*x) + 1)^(1/2) - 1)^7) + (422
59*a^4*((1/(a*x) - 1)^(1/2) - 1i)^9)/(3*((1/(a*x) + 1)^(1/2) - 1)^9) + (252
95*a^4*((1/(a*x) - 1)^(1/2) - 1i)^11)/((1/(a*x) + 1)^(1/2) - 1)^11 + (25295
*a^4*((1/(a*x) - 1)^(1/2) - 1i)^13)/((1/(a*x) + 1)^(1/2) - 1)^13 + (42259*a
^4*((1/(a*x) - 1)^(1/2) - 1i)^15)/(3*((1/(a*x) + 1)^(1/2) - 1)^15) + (8361*
a^4*((1/(a*x) - 1)^(1/2) - 1i)^17)/(2*((1/(a*x) + 1)^(1/2) - 1)^17) + (311*
a^4*((1/(a*x) - 1)^(1/2) - 1i)^19)/(2*((1/(a*x) + 1)^(1/2) - 1)^19) - (175*
a^4*((1/(a*x) - 1)^(1/2) - 1i)^21)/(6*((1/(a*x) + 1)^(1/2) - 1)^21) + (5*a^
4*((1/(a*x) - 1)^(1/2) - 1i)^23)/(2*((1/(a*x) + 1)^(1/2) - 1)^23) + (5*a^4*
((1/(a*x) - 1)^(1/2) - 1i))/(2*((1/(a*x) + 1)^(1/2) - 1)))/((66*((1/(a*x) -
 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (12*((1/(a*x) - 1)^(1/2) -
 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (220*((1/(a*x) - 1)^(1/2) - 1i)^6)/((
1/(a*x) + 1)^(1/2) - 1)^6 + (495*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) +
1)^(1/2) - 1)^8 - (792*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2)
- 1)^10 + (924*((1/(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x) + 1)^(1/2) - 1)^12
- (792*((1/(a*x) - 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^(1/2) - 1)^14 + (495*(
(1/(a*x) - 1)^(1/2) - 1i)^16)/((1/(a*x) + 1)^(1/2) - 1)^16 - (220*((1/(a*x)
 - 1)^(1/2) - 1i)^18)/((1/(a*x) + 1)^(1/2) - 1)^18 + (66*((1/(a*x) - 1)^(1/
```

```
2) - 1i)^20)/((1/(a*x) + 1)^(1/2) - 1)^20 - (12*((1/(a*x) - 1)^(1/2) - 1i)^
22)/((1/(a*x) + 1)^(1/2) - 1)^22 + ((1/(a*x) - 1)^(1/2) - 1i)^24/((1/(a*x)
+ 1)^(1/2) - 1)^24 + 1) + (a^4*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) +
 1)^(1/2) - 1)))/2 - ((a^4*((1/(a*x) - 1)^(1/2) - 1i)^6*4096i)/(3*((1/(a*x)
 + 1)^(1/2) - 1)^6) + (a^4*((1/(a*x) - 1)^(1/2) - 1i)^8*8192i)/(3*((1/(a*x)
 + 1)^(1/2) - 1)^8) + (a^4*((1/(a*x) - 1)^(1/2) - 1i)^10*24576i)/(5*((1/(a*
x) + 1)^(1/2) - 1)^10) + (a^4*((1/(a*x) - 1)^(1/2) - 1i)^12*8192i)/(3*((1/(
a*x) + 1)^(1/2) - 1)^12) + (a^4*((1/(a*x) - 1)^(1/2) - 1i)^14*4096i)/(3*((1
/(a*x) + 1)^(1/2) - 1)^14))/((45*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) +
1)^(1/2) - 1)^4 - (10*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2) -
1)^2 - (120*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (21
0*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8 - (252*((1/(a*x
) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1)^10 + (210*((1/(a*x) - 1)^(
1/2) - 1i)^12)/((1/(a*x) + 1)^(1/2) - 1)^12 - (120*((1/(a*x) - 1)^(1/2) - 1
i)^14)/((1/(a*x) + 1)^(1/2) - 1)^14 + (45*((1/(a*x) - 1)^(1/2) - 1i)^16)/((
1/(a*x) + 1)^(1/2) - 1)^16 - (10*((1/(a*x) - 1)^(1/2) - 1i)^18)/((1/(a*x) +
 1)^(1/2) - 1)^18 + ((1/(a*x) - 1)^(1/2) - 1i)^20/((1/(a*x) + 1)^(1/2) - 1)
^20 + 1) + ((a^4*((1/(a*x) - 1)^(1/2) - 1i)^6*20480i)/((1/(a*x) + 1)^(1/2)
- 1)^6 + (a^4*((1/(a*x) - 1)^(1/2) - 1i)^8*40960i)/((1/(a*x) + 1)^(1/2) - 1
)^8 + (a^4*((1/(a*x) - 1)^(1/2) - 1i)^10*73728i)/((1/(a*x) + 1)^(1/2) - 1)^
10 + (a^4*((1/(a*x) - 1)^(1/2) - 1i)^12*40960i)/((1/(a*x) + 1)^(1/2) - 1)^1
2 + (a^4*((1/(a*x) - 1)^(1/2) - 1i)^14*20480i)/((1/(a*x) + 1)^(1/2) - 1)^14
)/(15*((45*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (10*
((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (120*((1/(a*x)
- 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (210*((1/(a*x) - 1)^(1/2)
 - 1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8 - (252*((1/(a*x) - 1)^(1/2) - 1i)^10)
/((1/(a*x) + 1)^(1/2) - 1)^10 + (210*((1/(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*
x) + 1)^(1/2) - 1)^12 - (120*((1/(a*x) - 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^
(1/2) - 1)^14 + (45*((1/(a*x) - 1)^(1/2) - 1i)^16)/((1/(a*x) + 1)^(1/2) - 1
)^16 - (10*((1/(a*x) - 1)^(1/2) - 1i)^18)/((1/(a*x) + 1)^(1/2) - 1)^18 + ((
1/(a*x) - 1)^(1/2) - 1i)^20/((1/(a*x) + 1)^(1/2) - 1)^20 + 1)) + ((23*a^4*(
(1/(a*x) - 1)^(1/2) - 1i)^3)/((1/(a*x) + 1)^(1/2) - 1)^3 + (333*a^4*((1/(a*
x) - 1)^(1/2) - 1i)^5)/((1/(a*x) + 1)^(1/2) - 1)^5 + (671*a^4*((1/(a*x) - 1
)^(1/2) - 1i)^7)/((1/(a*x) + 1)^(1/2) - 1)^7 + (671*a^4*((1/(a*x) - 1)^(1/2
) - 1i)^9)/((1/(a*x) + 1)^(1/2) - 1)^9 + (333*a^4*((1/(a*x) - 1)^(1/2) - 1i
)^11)/((1/(a*x) + 1)^(1/2) - 1)^11 + (23*a^4*((1/(a*x) - 1)^(1/2) - 1i)^13)
/((1/(a*x) + 1)^(1/2) - 1)^13 - (3*a^4*((1/(a*x) - 1)^(1/2) - 1i)^15)/((1/(
a*x) + 1)^(1/2) - 1)^15 - (3*a^4*((1/(a*x) - 1)^(1/2) - 1i))/((1/(a*x) + 1)
^(1/2) - 1))/((28*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4
 - (8*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (56*((1/(
a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (70*((1/(a*x) - 1)^(
1/2) - 1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8 - (56*((1/(a*x) - 1)^(1/2) - 1i)^
10)/((1/(a*x) + 1)^(1/2) - 1)^10 + (28*((1/(a*x) - 1)^(1/2) - 1i)^12)/((1/(
a*x) + 1)^(1/2) - 1)^12 - (8*((1/(a*x) - 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^
(1/2) - 1)^14 + ((1/(a*x) - 1)^(1/2) - 1i)^16/((1/(a*x) + 1)^(1/2) - 1)^16
```

```
+ 1) + 1/(4*x^4) - 1/(3*a^2*x^6)
```

3.75 $\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^6}\,dx$

Optimal. Leaf size=301

$$-\frac{4a^5}{7\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^7}+\frac{2a^5}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^6}-\frac{18a^5}{5\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5}+\frac{4a^5}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}-\frac{35a^5}{12\left(1-\sqrt{\frac{1-ax}{1+ax}}\right.}$$

```
[Out] -4/7*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))^7+2*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))^
6-18/5*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))^5+4*a^5/(1-((-a*x+1)/(a*x+1))^(1/2)
)^4-35/12*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))^3+11/8*a^5/(1-((-a*x+1)/(a*x+1))
^(1/2))^2-1/4*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))-1/12*a^5/(1+((-a*x+1)/(a*x+1
))^(1/2))^3+1/8*a^5/(1+((-a*x+1)/(a*x+1))^(1/2))^2-1/4*a^5/(1+((-a*x+1)/(a*
x+1))^(1/2))
```

Rubi [A]

time = 0.38, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6472, 1626}

$$-\frac{a^5}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{a^5}{4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}+\frac{11a^5}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}+\frac{a^5}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}-\frac{35a^5}{12\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}-\frac{a^5}{12\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}+\frac{4a^5}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^4}-\frac{18a^5}{5\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^5}+\frac{2a^5}{\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^6}-\frac{4a^5}{7\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcSech[a*x])/x^6,x]
```

```
[Out] (-4*a^5)/(7*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^7) + (2*a^5)/(1 - Sqrt[(1 - a*x
)/(1 + a*x)])^6 - (18*a^5)/(5*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^5) + (4*a^5)/
(1 - Sqrt[(1 - a*x)/(1 + a*x)])^4 - (35*a^5)/(12*(1 - Sqrt[(1 - a*x)/(1 + a
*x)])^3) + (11*a^5)/(8*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2) - a^5/(4*(1 - Sqr
t[(1 - a*x)/(1 + a*x)])) - a^5/(12*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^3) + a^5
/(8*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^2) - a^5/(4*(1 + Sqrt[(1 - a*x)/(1 + a*
x)]))
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
```

`Q[n]`

Rubi steps

$$\int \frac{e^{2\text{sech}^{-1}(ax)}}{x^6}\,dx = \int \frac{\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)^2}{x^6}\,dx$$
$$= -\left((4a)\text{Subst}\left(\int \frac{x(a+ax^2)^4}{(-1+x)^8(1+x)^4}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\right)$$
$$= -\left((4a)\text{Subst}\left(\int \left(\frac{a^4}{(-1+x)^8}+\frac{3a^4}{(-1+x)^7}+\frac{9a^4}{2(-1+x)^6}+\frac{4a^4}{(-1+x)^5}+\frac{35a^4}{16(-1+x)^4}\right.\right.\right.$$
$$= -\frac{4a^5}{7\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^7}+\frac{2a^5}{\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^6}-\frac{18a^5}{5\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5}+\frac{4a^5}{\left(1-\sqrt{\frac{1-a}{1+a}}\right.}$$

Mathematica [A]
time = 0.07, size = 85, normalized size = 0.28

$$\frac{-30+21a^2x^2+2\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^2\left(-15+15ax-12a^2x^2+12a^3x^3-8a^4x^4+8a^5x^5\right)}{105a^2x^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcSech[a*x])/x^6,x]
```

```
[Out] (-30 + 21*a^2*x^2 + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-15 + 15*a*x -
 12*a^2*x^2 + 12*a^3*x^3 - 8*a^4*x^4 + 8*a^5*x^5))/(105*a^2*x^7)
```

Maple [A]
time = 0.04, size = 92, normalized size = 0.31

method	result	size
default	$\frac{-\frac{1}{7x^7}+\frac{a^2}{5x^5}}{a^2}+\frac{2\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\,(a^2x^2-1)(8a^4x^4+12a^2x^2+15)}{105a\,x^6}-\frac{1}{7a^2x^7}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/7/x^7+1/5*a^2/x^5)+2/105/a*((a*x+1)/a/x)^(1/2)/x^6*(-(a*x-1)/a/x)
^(1/2)*(a^2*x^2-1)*(8*a^4*x^4+12*a^2*x^2+15)-1/7/a^2/x^7
```

Maxima [A]
time = 0.27, size = 65, normalized size = 0.22

$$\frac{1}{5\,x^5}+\frac{2\,(8\,a^6x^7+4\,a^4x^5+3\,a^2x^3-15\,x)\sqrt{ax+1}\,\sqrt{-ax+1}}{105\,a^2x^8}-\frac{2}{7\,a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^6,x, algorithm="maxim
a")
```

```
[Out] 1/5/x^5 + 2/105*(8*a^6*x^7 + 4*a^4*x^5 + 3*a^2*x^3 - 15*x)*sqrt(a*x + 1)*sq
rt(-a*x + 1)/(a^2*x^8) - 2/7/(a^2*x^7)
```

Fricas [A]
time = 0.36, size = 78, normalized size = 0.26

$$\frac{21\,a^2x^2+2\,(8\,a^7x^7+4\,a^5x^5+3\,a^3x^3-15\,ax)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}-30}{105\,a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^6,x, algorithm="frica
s")
```

```
[Out] 1/105*(21*a^2*x^2 + 2*(8*a^7*x^7 + 4*a^5*x^5 + 3*a^3*x^3 - 15*a*x)*sqrt((a*
x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 30)/(a^2*x^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{2}{x^8}\,dx+\int\left(-\frac{a^2}{x^6}\right)\,dx+\int\frac{2a\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{x^7}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))**2/x**6,x)
```

```
[Out] (Integral(2/x**8, x) + Integral(-a**2/x**6, x) + Integral(2*a*sqrt(-1 + 1/(
a*x))*sqrt(1 + 1/(a*x))/x**7, x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2/x^6,x, algorithm="giac"
)
```

```
[Out] integrate((sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))^2/x^6, x)
```

Mupad [B]
time = 2.12, size = 105, normalized size = 0.35

$$\frac{\frac{a^2\,x^2}{5}-\frac{2}{7}}{a^2\,x^7}+\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{2\,a\,x^2\,\sqrt{\frac{1}{a\,x}+1}}{35}-\frac{2\,\sqrt{\frac{1}{a\,x}+1}}{7\,a}+\frac{8\,a^3\,x^4\,\sqrt{\frac{1}{a\,x}+1}}{105}+\frac{16\,a^5\,x^6\,\sqrt{\frac{1}{a\,x}+1}}{105}\right)}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))^2/x^6,x)
```

```
[Out] ((a^2*x^2)/5 - 2/7)/(a^2*x^7) + ((1/(a*x) - 1)^(1/2)*((2*a*x^2*(1/(a*x) + 1
)^(1/2))/35 - (2*(1/(a*x) + 1)^(1/2))/(7*a) + (8*a^3*x^4*(1/(a*x) + 1)^(1/2
))/105 + (16*a^5*x^6*(1/(a*x) + 1)^(1/2))/105))/x^6
```

3.76 $\int e^{-\text{sech}^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=147

$$-\frac{x}{a^4}-\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^5}{5a^5}+\frac{(1+ax)^2\left(9+4\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^5}+\frac{(1+ax)^4\left(5+16\sqrt{\frac{1-ax}{1+ax}}\right)}{20a^5}-\frac{(1+ax)^3\left(15\right.}{1}$$

```
[Out] -x/a^4-1/5*(a*x+1)^5*((-a*x+1)/(a*x+1))^(1/2)/a^5+1/6*(a*x+1)^2*(9+4*((-a*x
+1)/(a*x+1))^(1/2))/a^5+1/20*(a*x+1)^4*(5+16*((-a*x+1)/(a*x+1))^(1/2))/a^5-
1/15*(a*x+1)^3*(15+17*((-a*x+1)/(a*x+1))^(1/2))/a^5
```

Rubi [A]
time = 0.40, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6472, 1818, 1828, 12, 267}

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)^5}{5a^5}+\frac{\left(16\sqrt{\frac{1-ax}{ax+1}}+5\right)(ax+1)^4}{20a^5}-\frac{\left(17\sqrt{\frac{1-ax}{ax+1}}+15\right)(ax+1)^3}{15a^5}+\frac{\left(4\sqrt{\frac{1-ax}{ax+1}}+9\right)(ax+1)^2}{6a^5}-\frac{x}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/E^ArcSech[a*x],x]
```

```
[Out] -(x/a^4) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^5)/(5*a^5) + ((1 + a*x)^2*(
9 + 4*Sqrt[(1 - a*x)/(1 + a*x)]))/(6*a^5) + ((1 + a*x)^4*(5 + 16*Sqrt[(1 -
a*x)/(1 + a*x)]))/(20*a^5) - ((1 + a*x)^3*(15 + 17*Sqrt[(1 - a*x)/(1 + a*x)
]))/(15*a^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
```

```
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int e^{-\text{sech}^{-1}(ax)}x^4\,dx = \int \frac{x^4}{\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}}\,dx$$

$$= \frac{4\text{Subst}\left(\int \frac{(-1+x)^5x(1+x)^3}{(1+x^2)^6}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^5}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^5}{5a^5} - \frac{2\text{Subst}\left(\int \frac{-16+10x+140x^2-30x^3-80x^4+30x^5+20x^6-10x^7}{(1+x^2)^5}\,dx, x, \sqrt{\frac{1-}{1+}}\right.}{5a^5}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^5}{5a^5} + \frac{(1+ax)^4\left(5+16\sqrt{\frac{1-ax}{1+ax}}\right)}{20a^5} + \frac{\text{Subst}\left(\int \frac{-128+560x+800x^2-32}{(1+x^2)}\right.}{}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^5}{5a^5} + \frac{(1+ax)^4\left(5+16\sqrt{\frac{1-ax}{1+ax}}\right)}{20a^5} - \frac{(1+ax)^3\left(15+17\sqrt{\frac{1-a}{1+a}}\right.}{15a^5}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^5}{5a^5} + \frac{(1+ax)^2\left(9+4\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^5} + \frac{(1+ax)^4\left(5+16\sqrt{\frac{1-ax}{1+ax}}\right.}{20a^5}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^5}{5a^5} + \frac{(1+ax)^2\left(9+4\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^5} + \frac{(1+ax)^4\left(5+16\sqrt{\frac{1-ax}{1+ax}}\right.}{20a^5}$$

$$= -\frac{x}{a^4} - \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^5}{5a^5} + \frac{(1+ax)^2\left(9+4\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^5} + \frac{(1+ax)^4\left(5+16\sqrt{\frac{1}{1}}\right.}{20a^5}$$

Mathematica [A]
time = 0.06, size = 65, normalized size = 0.44

$$\frac{15a^4x^4-4\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^2(-2+2ax-3a^2x^2+3a^3x^3)}{60a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/E^ArcSech[a*x],x]
```

```
[Out] (15*a^4*x^4 - 4*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-2 + 2*a*x - 3*a^2*x
^2 + 3*a^3*x^3))/(60*a^5)
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.22, size = 531, normalized size = 3.61

method	result
default	$\frac{(ax+1)\left(15a^{10}x^{10}\left(-\frac{ax-1}{ax}\right)^{\frac{7}{2}}\left(\frac{ax+1}{ax}\right)^{\frac{5}{2}}+30a^{8}x^{8}\left(-\frac{ax-1}{ax}\right)^{\frac{7}{2}}\left(\frac{ax+1}{ax}\right)^{\frac{5}{2}}-30\left(-\frac{ax-1}{ax}\right)^{\frac{7}{2}}\left(\frac{ax+1}{ax}\right)^{\frac{3}{2}}a^{8}x^{8}+30x^{6}\ln\left(a^{2}x^{2}\right)\left(-\frac{ax-1}{ax}\right)^{\frac{7}{2}}\left(\frac{ax+}{ax}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*(a*x+1)/x^7*(15*a^10*x^10*(-(a*x-1)/a/x)^(7/2)*((a*x+1)/a/x)^(5/2)+30*
a^8*x^8*(-(a*x-1)/a/x)^(7/2)*((a*x+1)/a/x)^(5/2)-30*(-(a*x-1)/a/x)^(7/2)*((
a*x+1)/a/x)^(3/2)*a^8*x^8+30*x^6*ln(a^2*x^2)*(-(a*x-1)/a/x)^(7/2)*((a*x+1)/
a/x)^(5/2)*a^6-30*a^7*x^7*(-(a*x-1)/a/x)^(7/2)*((a*x+1)/a/x)^(3/2)-60*(-(a*
x-1)/a/x)^(7/2)*((a*x+1)/a/x)^(3/2)*ln(a^2*x^2)*a^6*x^6-12*a^11*x^11-60*x^5
*ln(a^2*x^2)*(-(a*x-1)/a/x)^(7/2)*((a*x+1)/a/x)^(3/2)*a^5+30*(-(a*x-1)/a/x)
^(7/2)*((a*x+1)/a/x)^(1/2)*ln(a^2*x^2)*a^6*x^6+12*a^10*x^10+60*(-(a*x-1)/a/
x)^(7/2)*((a*x+1)/a/x)^(1/2)*ln(a^2*x^2)*a^5*x^5+40*a^9*x^9+30*x^4*ln(a^2*x
^2)*(-(a*x-1)/a/x)^(7/2)*((a*x+1)/a/x)^(1/2)*a^4-40*a^8*x^8-40*a^7*x^7+40*a
^6*x^6+20*a^3*x^3-20*a^2*x^2-8*a*x+8)/a^12/(-(a*x-1)/a/x)^(7/2)/((a*x+1)/a/
x)^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="maxima"
)
```

```
[Out] integrate(x^4/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

Fricas [A]
time = 0.41, size = 65, normalized size = 0.44

$$\frac{15\,a^{3}x^{4}-4\,(3\,a^{4}x^{5}-a^{2}x^{3}-2\,x)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}}{60\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="fricas"
)
```

```
[Out] 1/60*(15*a^3*x^4 - 4*(3*a^4*x^5 - a^2*x^3 - 2*x)*sqrt((a*x + 1)/(a*x))*sqrt
(-(a*x - 1)/(a*x)))/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\int \frac{x^5}{ax\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2)),x)
[Out] a*Integral(x**5/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="giac")
[Out] integrate(x^4/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

Mupad [B]
time = 2.14, size = 73, normalized size = 0.50

$$\frac{x^4}{4\,a}+\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{2\,x}{15\,a^4}+\frac{2}{15\,a^5}-\frac{x^5}{5}-\frac{x^4}{5\,a}+\frac{x^3}{15\,a^2}+\frac{x^2}{15\,a^3}\right)}{\sqrt{\frac{1}{a\,x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
[Out] x^4/(4*a) + ((1/(a*x) - 1)^(1/2)*((2*x)/(15*a^4) + 2/(15*a^5) - x^5/5 - x^4
/(5*a) + x^3/(15*a^2) + x^2/(15*a^3)))/(1/(a*x) + 1)^(1/2)
```

3.77 $\int e^{-\text{sech}^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=163

$$-\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^4}{4a^4}+\frac{(1+ax)\left(8+\sqrt{\frac{1-ax}{1+ax}}\right)}{8a^4}-\frac{(1+ax)^2\left(8+5\sqrt{\frac{1-ax}{1+ax}}\right)}{8a^4}+\frac{(1+ax)^3\left(4+9\sqrt{\frac{1}{1}}\right.}{12a^4}$$

```
[Out] 1/4*arctan(((-a*x+1)/(a*x+1))^(1/2))/a^4-1/4*(a*x+1)^4*((-a*x+1)/(a*x+1))^(
1/2)/a^4+1/8*(a*x+1)*(8+((-a*x+1)/(a*x+1))^(1/2))/a^4-1/8*(a*x+1)^2*(8+5*((
-a*x+1)/(a*x+1))^(1/2))/a^4+1/12*(a*x+1)^3*(4+9*((-a*x+1)/(a*x+1))^(1/2))/a
^4
```

Rubi [A]
time = 0.36, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6472, 1818, 1828, 653, 209}

$$\frac{\text{ArcTan}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{4a^4}-\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)^4}{4a^4}+\frac{\left(9\sqrt{\frac{1-ax}{ax+1}}+4\right)(ax+1)^3}{12a^4}-\frac{\left(5\sqrt{\frac{1-ax}{ax+1}}+8\right)(ax+1)^2}{8a^4}+\frac{\left(\sqrt{\frac{1-ax}{ax+1}}+8\right)(ax+1)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^ArcSech[a*x],x]
```

```
[Out] -1/4*(Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^4)/a^4 + ((1 + a*x)*(8 + Sqrt[(1
- a*x)/(1 + a*x)]))/(8*a^4) - ((1 + a*x)^2*(8 + 5*Sqrt[(1 - a*x)/(1 + a*x)]
))/(8*a^4) + ((1 + a*x)^3*(4 + 9*Sqrt[(1 - a*x)/(1 + a*x)]))/(12*a^4) + Arc
Tan[Sqrt[(1 - a*x)/(1 + a*x)]]/(4*a^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
```

```
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int e^{-\operatorname{sech}^{-1}(ax)}x^3\,dx = \int \frac{x^3}{\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}}\,dx$$

$$= -\frac{4\text{Subst}\left(\int \frac{(-1+x)^4x(1+x)^2}{(1+x^2)^5}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^4}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^4}{4a^4}+\frac{\text{Subst}\left(\int \frac{8-8x-48x^2+16x^3+16x^4-8x^5}{(1+x^2)^4}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{2a^4}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^4}{4a^4}+\frac{(1+ax)^3\left(4+9\sqrt{\frac{1-ax}{1+ax}}\right)}{12a^4}-\frac{\text{Subst}\left(\int \frac{24-144x-96x^2+48x^3}{(1+x^2)^3}\right.}{12a^4}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^4}{4a^4}-\frac{(1+ax)^2\left(8+5\sqrt{\frac{1-ax}{1+ax}}\right)}{8a^4}+\frac{(1+ax)^3\left(4+9\sqrt{\frac{1-ax}{1+ax}}\right.}{12a^4}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^4}{4a^4}+\frac{(1+ax)\left(8+\sqrt{\frac{1-ax}{1+ax}}\right)}{8a^4}-\frac{(1+ax)^2\left(8+5\sqrt{\frac{1-ax}{1+ax}}\right)}{8a^4}$$

$$= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^4}{4a^4}+\frac{(1+ax)\left(8+\sqrt{\frac{1-ax}{1+ax}}\right)}{8a^4}-\frac{(1+ax)^2\left(8+5\sqrt{\frac{1-ax}{1+ax}}\right)}{8a^4}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.08, size = 97, normalized size = 0.60

$$\frac{8a^3x^3+3a\sqrt{\frac{1-ax}{1+ax}}\,(x+ax^2-2a^2x^3-2a^3x^4)-3i\log\left(-2iax+2\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\right)}{24a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^ArcSech[a*x],x]
```

```
[Out] (8*a^3*x^3 + 3*a*Sqrt[(1 - a*x)/(1 + a*x)]*(x + a*x^2 - 2*a^2*x^3 - 2*a^3*x
^4) - (3*I)*Log[(-2*I)*a*x + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)])/(24*a^
4)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x)
[Out] int(x^3/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="maxima"
)
[Out] integrate(x^3/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

Fricas [A]
time = 0.39, size = 95, normalized size = 0.58

$$\frac{8\,a^3x^3-3\,(2\,a^4x^4-a^2x^2)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}+3\,\arctan\left(\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="fricas"
)
[Out] 1/24*(8*a^3*x^3 - 3*(2*a^4*x^4 - a^2*x^2)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x
- 1)/(a*x)) + 3*arctan(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))))/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\int \frac{x^4}{ax\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2)),x)
```

```
[Out] a*Integral(x**4/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^3/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

Mupad [B]
time = 19.83, size = 795, normalized size = 4.88

$$\frac{\ln\left(\frac{a\sqrt{\frac{1}{ax}-1}\,1i+a\sqrt{\frac{1}{ax}+1}-\frac{1}{x}}{2a-2a\sqrt{\frac{1}{ax}+1}+\frac{1}{x}}\right)3i}{8a^4}+\frac{\frac{1i}{1024a^4}-\frac{\left(\sqrt{\frac{1}{ax}-1}-1i\right)^2 3i}{128a^4\left(\sqrt{\frac{1}{ax}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{ax}-1}-1i\right)^4 53i}{512a^4\left(\sqrt{\frac{1}{ax}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{ax}-1}-1i\right)^6 87i}{256a^4\left(\sqrt{\frac{1}{ax}+1}-1\right)^6}+\frac{\left(\sqrt{\frac{1}{ax}-1}-1i\right)^8 657i}{1024a^4\left(\sqrt{\frac{1}{ax}+1}-1\right)^8}+\frac{\left(\sqrt{\frac{1}{ax}-1}-1i\right)^{10} 121i}{256a^4\left(\sqrt{\frac{1}{ax}+1}-1\right)^{10}}}{\frac{\left(\sqrt{\frac{1}{ax}-1}-1i\right)^4}{\left(\sqrt{\frac{1}{ax}+1}-1\right)^4}+\frac{4\left(\sqrt{\frac{1}{ax}-1}-1i\right)^6}{\left(\sqrt{\frac{1}{ax}+1}-1\right)^6}+\frac{6\left(\sqrt{\frac{1}{ax}-1}-1i\right)^8}{\left(\sqrt{\frac{1}{ax}+1}-1\right)^8}+\frac{4\left(\sqrt{\frac{1}{ax}-1}-1i\right)^{10}}{\left(\sqrt{\frac{1}{ax}+1}-1\right)^{10}}+\frac{\left(\sqrt{\frac{1}{ax}-1}-1i\right)^{12}}{\left(\sqrt{\frac{1}{ax}+1}-1\right)^{12}}}+\frac{\ln\left(\frac{\sqrt{\frac{1}{ax}-1}-1i}{\sqrt{\frac{1}{ax}+1}-1}\right)1i}{8a^4}+\frac{\frac{1i}{32a^4}+\frac{\left(\sqrt{\frac{1}{ax}-1}-1i\right)^2 1i}{16a^4\left(\sqrt{\frac{1}{ax}+1}-1\right)^2}-\frac{\left(\sqrt{\frac{1}{ax}-1}-1i\right)^4 15i}{32a^4\left(\sqrt{\frac{1}{ax}+1}-1\right)^4}}{\frac{\left(\sqrt{\frac{1}{ax}-1}-1i\right)^2}{\left(\sqrt{\frac{1}{ax}+1}-1\right)^2}+\frac{2\left(\sqrt{\frac{1}{ax}-1}-1i\right)^4}{\left(\sqrt{\frac{1}{ax}+1}-1\right)^4}+\frac{\left(\sqrt{\frac{1}{ax}-1}-1i\right)^6}{\left(\sqrt{\frac{1}{ax}+1}-1\right)^6}}-\frac{\ln\left(\frac{2a\sqrt{-\frac{a-\frac{1}{x}}{a}}\,2i-\frac{2}{x}+2a\sqrt{\frac{a+\frac{1}{x}}{a}}}{2a+\frac{1}{x}-2a\sqrt{\frac{a+\frac{1}{x}}{a}}}\right)1i}{2a^4}+\frac{x^3}{3a}+\frac{\left(\sqrt{\frac{1}{ax}-1}-1i\right)^2 1i}{256a^4\left(\sqrt{\frac{1}{ax}+1}-1\right)^2}+\frac{\left(\sqrt{\frac{1}{ax}-1}-1i\right)^4 1i}{1024a^4\left(\sqrt{\frac{1}{ax}+1}-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

```
[Out] (log((a*(1/(a*x) - 1)^(1/2)*1i + a*(1/(a*x) + 1)^(1/2) - 1/x)/(2*a - 2*a*(1
/(a*x) + 1)^(1/2) + 1/x))*3i)/(8*a^4) + (1i/(1024*a^4) - (((1/(a*x) - 1)^(1
/2) - 1i)^2*3i)/(128*a^4*((1/(a*x) + 1)^(1/2) - 1)^2) - (((1/(a*x) - 1)^(1/
2) - 1i)^4*53i)/(512*a^4*((1/(a*x) + 1)^(1/2) - 1)^4) + (((1/(a*x) - 1)^(1/
2) - 1i)^6*87i)/(256*a^4*((1/(a*x) + 1)^(1/2) - 1)^6) + (((1/(a*x) - 1)^(1/
2) - 1i)^8*657i)/(1024*a^4*((1/(a*x) + 1)^(1/2) - 1)^8) + (((1/(a*x) - 1)^(
1/2) - 1i)^10*121i)/(256*a^4*((1/(a*x) + 1)^(1/2) - 1)^10))/(((1/(a*x) - 1)
^(1/2) - 1i)^4/((1/(a*x) + 1)^(1/2) - 1)^4 + (4*((1/(a*x) - 1)^(1/2) - 1i)^
6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (6*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x)
 + 1)^(1/2) - 1)^8 + (4*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2)
 - 1)^10 + ((1/(a*x) - 1)^(1/2) - 1i)^12/((1/(a*x) + 1)^(1/2) - 1)^12) + (l
og(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i)/(8*a^4) + (1i/
(32*a^4) + (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(16*a^4*((1/(a*x) + 1)^(1/2) -
 1)^2) - (((1/(a*x) - 1)^(1/2) - 1i)^4*15i)/(32*a^4*((1/(a*x) + 1)^(1/2) -
1)^4))/(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + (2*((1/(
a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 + ((1/(a*x) - 1)^(1/2)
 - 1i)^6/((1/(a*x) + 1)^(1/2) - 1)^6) - (log((a*(-(a - 1/x)/a)^(1/2)*2i - 2
/x + 2*a*((a + 1/x)/a)^(1/2))/(2*a + 1/x - 2*a*((a + 1/x)/a)^(1/2)))*1i)/(2
*a^4) + x^3/(3*a) + (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(256*a^4*((1/(a*x) +
1)^(1/2) - 1)^2) + (((1/(a*x) - 1)^(1/2) - 1i)^4*1i)/(1024*a^4*((1/(a*x) +
1)^(1/2) - 1)^4)
```

3.78 $\int e^{-\text{sech}^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=75

$$-\frac{x}{a^2}-\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^3}{3a^3}+\frac{(1+ax)^2\left(3+4\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^3}$$

```
[Out] -x/a^2-1/3*(a*x+1)^3*((-a*x+1)/(a*x+1))^(1/2)/a^3+1/6*(a*x+1)^2*(3+4*((-a*x
+1)/(a*x+1))^(1/2))/a^3
```

Rubi [A]
time = 0.32, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6472, 1818, 1828, 12, 267}

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}\,(ax+1)^3}{3a^3}+\frac{\left(4\sqrt{\frac{1-ax}{ax+1}}+3\right)(ax+1)^2}{6a^3}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^ArcSech[a*x],x]
```

```
[Out] -(x/a^2) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^3)/(3*a^3) + ((1 + a*x)^2*(
3 + 4*Sqrt[(1 - a*x)/(1 + a*x)]))/(6*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
```

```
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\begin{aligned}
\int e^{-\operatorname{sech}^{-1}(ax)}x^2\,dx &= \int \frac{x^2}{\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}}\,dx \\
&= \frac{4\text{Subst}\left(\int \frac{(-1+x)^3x(1+x)}{(1+x^2)^4}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^3} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^3}{3a^3} - \frac{2\text{Subst}\left(\int \frac{-4+6x+12x^2-6x^3}{(1+x^2)^3}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{3a^3} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^3}{3a^3} + \frac{(1+ax)^2\left(3+4\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^3} + \frac{\text{Subst}\left(\int \frac{24x}{(1+x^2)^2}\,dx, x, \sqrt{\frac{1}{1}}\right.}{6a^3} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^3}{3a^3} + \frac{(1+ax)^2\left(3+4\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^3} + \frac{4\text{Subst}\left(\int \frac{x}{(1+x^2)^2}\,dx, x, \sqrt{\right.}{a^3} \\
&= -\frac{x}{a^2} - \frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^3}{3a^3} + \frac{(1+ax)^2\left(3+4\sqrt{\frac{1-ax}{1+ax}}\right)}{6a^3}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 48, normalized size = 0.64

$$\frac{3a^2x^2 - 2(-1+ax)\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^2}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^ArcSech[a*x],x]
```

```
[Out] (3*a^2*x^2 - 2*(-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(6*a^3)
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.17, size = 269, normalized size = 3.59

method	result
default	$\frac{(ax+1)\left(3a^6x^6\left(-\frac{ax-1}{ax}\right)^{\frac{5}{2}}\left(\frac{ax+1}{ax}\right)^{\frac{3}{2}}+3x^4\ln(a^2x^2)\left(-\frac{ax-1}{ax}\right)^{\frac{5}{2}}\left(\frac{ax+1}{ax}\right)^{\frac{3}{2}}a^4-3\left(-\frac{ax-1}{ax}\right)^{\frac{5}{2}}\sqrt{\frac{ax+1}{ax}}\,\ln(a^2x^2)a^4x^4+2a^7x^7-3x^3\ln(a^2\right.}{6x^5a^8\left(-\frac{ax-1}{ax}\right)^{\frac{5}{2}}\left(\frac{ax+1}{ax}\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(a*x+1)/x^5*(3*a^6*x^6*(-(a*x-1)/a/x)^(5/2)*((a*x+1)/a/x)^(3/2)+3*x^4*l
n(a^2*x^2)*(-(a*x-1)/a/x)^(5/2)*((a*x+1)/a/x)^(3/2)*a^4-3*(-(a*x-1)/a/x)^(5
/2)*((a*x+1)/a/x)^(1/2)*ln(a^2*x^2)*a^4*x^4+2*a^7*x^7-3*x^3*ln(a^2*x^2)*(-(
a*x-1)/a/x)^(5/2)*((a*x+1)/a/x)^(1/2)*a^3-2*a^6*x^6-6*a^5*x^5+6*a^4*x^4+6*a
^3*x^3-6*a^2*x^2-2*a*x+2)/a^8/(-(a*x-1)/a/x)^(5/2)/((a*x+1)/a/x)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="maxima"
)
```

```
[Out] integrate(x^2/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

Fricas [A]
time = 0.37, size = 54, normalized size = 0.72

$$\frac{3\,ax^2 - 2\,(a^2x^3 - x)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="fricas"
)
```

```
[Out] 1/6*(3*a*x^2 - 2*(a^2*x^3 - x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))
)/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\int\frac{x^3}{ax\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2)),x)
```

```
[Out] a*Integral(x**3/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

Mupad [B]
time = 2.06, size = 57, normalized size = 0.76

$$\frac{x^2}{2\,a}+\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{x}{3\,a^2}+\frac{1}{3\,a^3}-\frac{x^3}{3}-\frac{x^2}{3\,a}\right)}{\sqrt{\frac{1}{a\,x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

```
[Out] x^2/(2*a) + ((1/(a*x) - 1)^(1/2)*(x/(3*a^2) + 1/(3*a^3) - x^3/3 - x^2/(3*a)
))/(1/(a*x) + 1)^(1/2)
```

3.79 $\int e^{-\mathbf{sech}^{-1}(ax)}x\,dx$

Optimal. Leaf size=94

$$\frac{(1+ax)^2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}{4a^2}+\frac{(1+ax)\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}{2a^2}+\frac{\mathrm{ArcTan}\left(\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}$$

```
[Out] arctan(((-a*x+1)/(a*x+1))^(1/2))/a^2+1/4*(a*x+1)^2*(1-((-a*x+1)/(a*x+1))^(1
/2))^2/a^2+1/2*(a*x+1)*(1+((-a*x+1)/(a*x+1))^(1/2))/a^2
```

Rubi [A]
time = 0.19, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6472, 833, 653, 209}

$$\frac{\mathrm{ArcTan}\left(\sqrt{\frac{1-ax}{ax+1}}\right)}{a^2}+\frac{(ax+1)^2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}{4a^2}+\frac{(ax+1)\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^ArcSech[a*x],x]
```

```
[Out] ((1 + a*x)^2*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2)/(4*a^2) + ((1 + a*x)*(1 + S
qrt[(1 - a*x)/(1 + a*x)]))/(2*a^2) + ArcTan[Sqrt[(1 - a*x)/(1 + a*x)]]/a^2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
```

```
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\begin{aligned}
\int e^{-\operatorname{sech}^{-1}(ax)}x\,dx &= \int \frac{x}{\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}}\,dx \\
&= -\frac{4\text{Subst}\left(\int \frac{(-1+x)^2x}{(1+x^2)^3}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^2} \\
&= \frac{(1+ax)^2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}{4a^2} - \frac{\text{Subst}\left(\int \frac{-2+2x}{(1+x^2)^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a^2} \\
&= \frac{(1+ax)^2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}{4a^2} + \frac{(1+ax)\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}{2a^2} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\right.}{a^2} \\
&= \frac{(1+ax)^2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}{4a^2} + \frac{(1+ax)\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}{2a^2} + \frac{\tan^{-1}\left(\sqrt{\frac{1-ax}{1+ax}}\right)}{a^2}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.05, size = 75, normalized size = 0.80

$$-\frac{-2ax+ax\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)+i\log\left(-2iax+2\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^ArcSech[a*x],x]
```

```
[Out] -1/2*(-2*a*x + a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) + I*Log[(-2*I)*a*x +
 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)])/a^2
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x)
[Out] int(x/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="maxima")
[Out] integrate(x/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

Fricas [A]
time = 0.37, size = 79, normalized size = 0.84

$$-\frac{a^{2}x^{2}\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}-2\,ax-\arctan\left(\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}\right)}{2\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="fricas")
[Out] -1/2*(a^2*x^2*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 2*a*x - arctan
(sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\int \frac{x^{2}}{ax\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2)),x)
[Out] a*Integral(x**2/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="giac")
[Out] integrate(x/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

Mupad [B]
time = 9.22, size = 407, normalized size = 4.33

$$\frac{x}{a}-\frac{\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\,1\mathrm{i}}{2\,a^2}-\frac{\frac{1\mathrm{i}}{32\,a^2}+\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{16\,a^2\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^2}-\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^4\,15\mathrm{i}}{32\,a^2\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^4}}{\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^2}+\frac{2\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^4}+\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^6}}-\frac{\left(\ln\left(\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^2}+1\right)-\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\right)\,1\mathrm{i}}{a^2}+\frac{\ln\left(\frac{2\,a\,\sqrt{\frac{a+\frac{1}{x}}{a}}-\frac{2}{x}+a\,\sqrt{-\frac{a-\frac{1}{x}}{a}}\,2\mathrm{i}}{2\,a+\frac{1}{x}-2\,a\,\sqrt{\frac{a+\frac{1}{x}}{a}}}\right)\,1\mathrm{i}}{2\,a^2}-\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{32\,a^2\,{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
[Out] x/a - (log(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i)/(2*a^2
) - (1i/(32*a^2) + (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(16*a^2*((1/(a*x) + 1)
^(1/2) - 1)^2) - (((1/(a*x) - 1)^(1/2) - 1i)^4*15i)/(32*a^2*((1/(a*x) + 1)^
(1/2) - 1)^4))/(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 +
(2*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 + ((1/(a*x) -
1)^(1/2) - 1i)^6/((1/(a*x) + 1)^(1/2) - 1)^6) - ((log(((1/(a*x) - 1)^(1/2)
- 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1) - log(((1/(a*x) - 1)^(1/2) - 1i)/(
(1/(a*x) + 1)^(1/2) - 1)))*1i)/a^2 + (log((a*(-(a - 1/x)/a)^(1/2)*2i - 2/x
+ 2*a*((a + 1/x)/a)^(1/2))/(2*a + 1/x - 2*a*((a + 1/x)/a)^(1/2)))*1i)/(2*a^
2) - (((1/(a*x) - 1)^(1/2) - 1i)^2*1i)/(32*a^2*((1/(a*x) + 1)^(1/2) - 1)^2)
```

3.80 $\int e^{-\text{sech}^{-1}(ax)} dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{a}+\frac{\log(1+ax)}{a}+\frac{2\log\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

[Out] ln(a*x+1)/a+2*ln(1+((-a*x+1)/(a*x+1))^(1/2))/a-(a*x+1)*((-a*x+1)/(a*x+1))^(1/2)/a

Rubi [A]
time = 0.10, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6467, 1661, 815, 266}

$$-\frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{a}+\frac{\log(ax+1)}{a}+\frac{2\log\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[E^(-ArcSech[a*x]),x]

[Out] -((Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/a) + Log[1 + a*x]/a + (2*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)]])/a

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
```

```
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6467

```
Int[E^(ArcSech[u_]*(n_.)), x_Symbol] :> Int[(1/u + Sqrt[(1 - u)/(1 + u)] +
(1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int e^{-\operatorname{sech}^{-1}(ax)}\,dx &= \int \frac{1}{\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}}\,dx\\
&= \frac{4\operatorname{Subst}\left(\int \frac{(-1+x)x}{(1+x)(1+x^2)^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a}\\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{a} - \frac{2\operatorname{Subst}\left(\int \frac{-1+x}{(1+x)(1+x^2)}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a}\\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{a} - \frac{2\operatorname{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{x}{1+x^2}\right)\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a}\\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{a} + \frac{2\log\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}{a} - \frac{2\operatorname{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)}{a}\\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{a} + \frac{\log(1+ax)}{a} + \frac{2\log\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}{a}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 72, normalized size = 1.11

$$\frac{-\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)+\log\left(1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(-ArcSech[a*x]),x]
```

```
[Out] (-(Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)) + Log[1 + Sqrt[(1 - a*x)/(1 + a*x)]
 + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2615 vs. $2(61) = 122$.
time = 0.15, size = 2616, normalized size = 40.25

method	result	size
default	Expression too large to display	2616

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a*x+1)/x^3*(x^2*ln(a^2*x^2)*(-(a*x-1)/a/x)^(3/2)*((a*x+1)/a/x)^(1/2)*(
-a^2*x^2+1)^(1/2)*a^2+ln(2*(((-a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/
2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*(a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+
1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*a^2/(a*x-1)/x^2/(a*x+1)
)^(1/2)*x+(-a^2*x^2+1)^(1/2)+1)*a^2/(a^2*x+((-a*x^2*(-(a*x-1)/a/x)^(1/2)*((
a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*(a*x^2*(-(a*x-1)/a/
x)^(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*a^2/(a*x
-1)/x^2/(a*x+1))^(1/2)))*a^3*x^3+ln(2*(((-a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+
1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*(a*x^2*(-(a*x-1)/a/x)^(
1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*a^2/(a*x-1)/
x^2/(a*x+1))^(1/2)*x-(-a^2*x^2+1)^(1/2)-1)*a^2/(-a^2*x+((-a*x^2*(-(a*x-1)/a
/x)^(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*(a*x^2*
(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/
2))*a^2/(a*x-1)/x^2/(a*x+1))^(1/2)))*a^3*x^3-2*(-a^2*x^2+1)^(1/2)*a^3*x^3-x
^2*ln(2*(((-a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/
2)*(x*(a*x+1))^(1/2))*(a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*
x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*a^2/(a*x-1)/x^2/(a*x+1))^(1/2)*x+(-a^2*x^2
+1)^(1/2)+1)*a^2/(a^2*x+((-a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+(
-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*(a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a
/x)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*a^2/(a*x-1)/x^2/(a*x+1))^(1
/2)))*a^2-x^2*ln(2*(((-a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*
x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*(a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^
(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*a^2/(a*x-1)/x^2/(a*x+1))^(1/2)*
x-(-a^2*x^2+1)^(1/2)-1)*a^2/(-a^2*x+((-a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/
a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*(a*x^2*(-(a*x-1)/a/x)^(1/2
)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*a^2/(a*x-1)/x^2
/(a*x+1))^(1/2)))*a^2+2*a^2*x^2*(-a^2*x^2+1)^(1/2)-x*ln(2*(((-a*x^2*(-(a*x-
1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*(a*
x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))
^(1/2))*a^2/(a*x-1)/x^2/(a*x+1))^(1/2)*x+(-a^2*x^2+1)^(1/2)+1)*a^2/(a^2*x+(
(-a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x
+1))^(1/2))*(a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1
```

```
/2)*(x*(a*x+1))^(1/2))*a^2/(a*x-1)/x^2/(a*x+1))^(1/2)))*a-x*ln(2*(((-a*x^2*
(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/
2))*(a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(
a*x+1))^(1/2))*a^2/(a*x-1)/x^2/(a*x+1))^(1/2)*x-(-a^2*x^2+1)^(1/2)-1)*a^2/(
-a^2*x+((-a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)
*(x*(a*x+1))^(1/2))*(a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-
1)*x)^(1/2)*(x*(a*x+1))^(1/2))*a^2/(a*x-1)/x^2/(a*x+1))^(1/2)))*a+2*(-a^2*x
^2+1)^(1/2)*a*x+ln(2*(((-a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+(-(
a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*(a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x
)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*a^2/(a*x-1)/x^2/(a*x+1))^(1/2
)*x+(-a^2*x^2+1)^(1/2)+1)*a^2/(a^2*x+((-a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)
/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*(a*x^2*(-(a*x-1)/a/x)^(1/
2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*a^2/(a*x-1)/x^
2/(a*x+1))^(1/2)))+ln(2*(((-a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+
(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*(a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/
a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*a^2/(a*x-1)/x^2/(a*x+1))^(
1/2)*x-(-a^2*x^2+1)^(1/2)-1)*a^2/(-a^2*x+((-a*x^2*(-(a*x-1)/a/x)^(1/2)*((a*
x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*(a*x^2*(-(a*x-1)/a/x)
^(1/2)*((a*x+1)/a/x)^(1/2)+(-(a*x-1)*x)^(1/2)*(x*(a*x+1))^(1/2))*a^2/(a*x-1
)/x^2/(a*x+1))^(1/2)))-2*(-a^2*x^2+1)^(1/2))/a^4/(-(a*x-1)/a/x)^(3/2)/((a*x
+1)/a/x)^(3/2)/(-a^2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

Fricas [A]
time = 0.36, size = 115, normalized size = 1.77

$$-\frac{2\,ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - \log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right) + \log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right) - 2\,\log\left(x\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - log(a*x*sqrt((a*
x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) + log(a*x*sqrt((a*x + 1)/(a*x))*s
qrt(-(a*x - 1)/(a*x)) - 1) - 2*log(x))/a
```

Sympy [A]
time = 93.00, size = 116, normalized size = 1.78

$$-2a^2\left(\begin{cases}\frac{2\left(\frac{\sqrt{-1+\frac{1}{ax}}}{2\sqrt{1+\frac{1}{ax}}\left(\frac{-1+\frac{1}{ax}}{1+\frac{1}{ax}}+1\right)}-\frac{\log\left(\frac{\sqrt{-1+\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}+1\right)}{2}+\frac{\log\left(\frac{-1+\frac{1}{ax}}{1+\frac{1}{ax}}+1\right)}{4}\right)}{a^3} & \text{for } \sqrt{1+\frac{1}{ax}}>-\sqrt{2}\wedge\sqrt{1+\frac{1}{ax}}<\sqrt{2}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2)),x)
```

```
[Out] -2*a**2*Piecewise((2*(sqrt(-1 + 1/(a*x))/(2*sqrt(1 + 1/(a*x))*((-1 + 1/(a*x
))/(1 + 1/(a*x)) + 1)) - log(sqrt(-1 + 1/(a*x))/sqrt(1 + 1/(a*x)) + 1)/2 +
log((-1 + 1/(a*x))/(1 + 1/(a*x)) + 1)/4)/a**3, (sqrt(1 + 1/(a*x)) < sqrt(2)
) & (sqrt(1 + 1/(a*x)) > -sqrt(2))))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)), x)
```

Mupad [B]
time = 4.17, size = 47, normalized size = 0.72

$$\frac{\mathrm{acosh}\left(\frac{1}{a\,x}\right)}{a}-\frac{\ln\left(\frac{1}{x}\right)}{a}-x\,\sqrt{\frac{1}{a\,x}-1}\,\sqrt{\frac{1}{a\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)),x)
```

```
[Out] acosh(1/(a*x))/a - log(1/x)/a - x*(1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2)
```

3.81 $\int \frac{e^{-\text{sech}^{-1}(ax)}}{x} \, dx$

Optimal. Leaf size=46

$$-\frac{2}{1+\sqrt{\frac{1-ax}{1+ax}}}-2\text{ArcTan}\left(\sqrt{\frac{1-ax}{1+ax}}\right)$$

```
[Out] -2*arctan(((-a*x+1)/(a*x+1))^(1/2))-2/(1+((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A]
time = 0.24, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6472, 815, 209}

$$-2\text{ArcTan}\left(\sqrt{\frac{1-ax}{ax+1}}\right)-\frac{2}{\sqrt{\frac{1-ax}{ax+1}}+1}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcSech[a*x]*x),x]
[Out] -2/(1 + Sqrt[(1 - a*x)/(1 + a*x)]) - 2*ArcTan[Sqrt[(1 - a*x)/(1 + a*x)]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)}\,dx$$

$$= -\left(4\text{Subst}\left(\int \frac{x}{(1+x)^2\,(1+x^2)}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\right)$$

$$= -\left(4\text{Subst}\left(\int \left(-\frac{1}{2(1+x)^2}+\frac{1}{2\,(1+x^2)}\right)\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\right)$$

$$= -\frac{2}{1+\sqrt{\frac{1-ax}{1+ax}}} - 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)$$

$$= -\frac{2}{1+\sqrt{\frac{1-ax}{1+ax}}} - 2\tan^{-1}\left(\sqrt{\frac{1-ax}{1+ax}}\right)$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.03, size = 74, normalized size = 1.61

$$-\frac{1}{ax}+\left(1+\frac{1}{ax}\right)\sqrt{\frac{1-ax}{1+ax}}+i\log\left(-2iax+2\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcSech[a*x]*x),x]
```

```
[Out] -(1/(a*x)) + (1 + 1/(a*x))*Sqrt[(1 - a*x)/(1 + a*x)] + I*Log[(-2*I)*a*x + 2
*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)]
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x)
```

```
[Out] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="maxima"
)
```

```
[Out] integrate(1/(x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

Fricas [A]
time = 0.46, size = 76, normalized size = 1.65

$$\frac{ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} - ax\arctan\left(\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}\right) - 1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="fricas"
)
```

```
[Out] (a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - a*x*arctan(sqrt((a*x +
1)/(a*x))*sqrt(-(a*x - 1)/(a*x))) - 1)/(a*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\int\frac{1}{ax\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x,x)
```

```
[Out] a*Integral(1/(a*x*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(1/(x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

Mupad [B]
time = 3.85, size = 184, normalized size = 4.00

$$\ln\left(\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}+1\right)\,1\mathrm{i}-\ln\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)\,1\mathrm{i}-\frac{1}{a\,x}-\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2\,8\mathrm{i}}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2\,\left(1+\frac{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^4}-\frac{2\,\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

```
[Out] log(((1/(a*x) - 1)^(1/2) - 1i)^2/((1/(a*x) + 1)^(1/2) - 1)^2 + 1)*1i - log(
((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1))*1i - 1/(a*x) - (((1/(
a*x) - 1)^(1/2) - 1i)^2*8i)/(((1/(a*x) + 1)^(1/2) - 1)^2*(((1/(a*x) - 1)^(1
/2) - 1i)^4/((1/(a*x) + 1)^(1/2) - 1)^4 - (2*((1/(a*x) - 1)^(1/2) - 1i)^2)/
((1/(a*x) + 1)^(1/2) - 1)^2 + 1))
```

3.82 $\int \frac{e^{-\mathbf{sech}^{-1}(ax)}}{x^2} \, dx$

Optimal. Leaf size=72

$$-\frac{a}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a}{1+\sqrt{\frac{1-ax}{1+ax}}}-a\tanh^{-1}\left(\sqrt{\frac{1-ax}{1+ax}}\right)$$

```
[Out] -a*arctanh(((-a*x+1)/(a*x+1))^(1/2))-a/(1+((-a*x+1)/(a*x+1))^(1/2))^2+a/(1+
((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A]
time = 0.25, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6472, 78, 213}

$$\frac{a}{\sqrt{\frac{1-ax}{ax+1}}+1}-\frac{a}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}+a\left(-\tanh^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcSech[a*x]*x^2),x]
```

```
[Out] -(a/(1 + Sqrt[(1 - a*x)/(1 + a*x)])^2) + a/(1 + Sqrt[(1 - a*x)/(1 + a*x)])
- a*ArcTanh[Sqrt[(1 - a*x)/(1 + a*x)]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
```

`Q[n]`

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^2}\,dx &= \int \frac{1}{x^2\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)}\,dx\\
&= (4a)\text{Subst}\left(\int \frac{x}{(-1+x)(1+x)^3}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\\
&= (4a)\text{Subst}\left(\int \left(\frac{1}{2(1+x)^3}-\frac{1}{4(1+x)^2}+\frac{1}{4\left(-1+x^2\right)}\right)\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\\
&= -\frac{a}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a}{1+\sqrt{\frac{1-ax}{1+ax}}}+a\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\\
&= -\frac{a}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a}{1+\sqrt{\frac{1-ax}{1+ax}}}-a\tanh^{-1}\left(\sqrt{\frac{1-ax}{1+ax}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 92, normalized size = 1.28

$$
\frac{1}{2}\left(-\frac{1}{ax^2}+\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{ax^2}+a\log(x)-a\log\left(1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcSech[a*x]*x^2),x]
```

```
[Out] (-(1/(a*x^2)) + (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(a*x^2) + a*Log[x] -
a*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/2
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x)
```

```
[Out] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(1/(x^2*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

Fricas [A]
time = 0.43, size = 128, normalized size = 1.78

$$-\frac{a^2x^2\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-a^2x^2\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)-2\,ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+2}{4\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="frica
s")
```

```
[Out] -1/4*(a^2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) - a
^2*x^2*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) - 2*a*x*sq
rt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 2)/(a*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\int\frac{1}{ax^2\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**2,x)
```

```
[Out] a*Integral(1/(a*x**2*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^2,x, algorithm="giac"
)
```

```
[Out] integrate(1/(x^2*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

Mupad [B]
time = 12.05, size = 323, normalized size = 4.49

$$2\,a\,\mathrm{atanh}\left(\frac{\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{a\,x}+1}-1}\right)-a\,\mathrm{acosh}\left(\frac{1}{a\,x}\right)-\frac{1}{2\,a\,x^2}-\frac{a\,\left(\frac{14\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^3}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^3}+\frac{14\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^5}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^5}+\frac{2\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^7}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^7}+\frac{2\,\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}{\sqrt{\frac{1}{a\,x}+1}-1}\right)}{1+\frac{6\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^4}-\frac{4\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^6}+\frac{{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^8}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^8}-\frac{4\,{\left(\sqrt{\frac{1}{a\,x}-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{\frac{1}{a\,x}+1}-1\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

```
[Out] 2*a*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)) - a*acosh(1
/(a*x)) - 1/(2*a*x^2) - (a*((14*((1/(a*x) - 1)^(1/2) - 1i)^3)/((1/(a*x) + 1
)^(1/2) - 1)^3 + (14*((1/(a*x) - 1)^(1/2) - 1i)^5)/((1/(a*x) + 1)^(1/2) - 1
)^5 + (2*((1/(a*x) - 1)^(1/2) - 1i)^7)/((1/(a*x) + 1)^(1/2) - 1)^7 + (2*((1
/(a*x) - 1)^(1/2) - 1i))/((1/(a*x) + 1)^(1/2) - 1)))/((6*((1/(a*x) - 1)^(1/
2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (4*((1/(a*x) - 1)^(1/2) - 1i)^2)/
((1/(a*x) + 1)^(1/2) - 1)^2 - (4*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) +
1)^(1/2) - 1)^6 + ((1/(a*x) - 1)^(1/2) - 1i)^8/((1/(a*x) + 1)^(1/2) - 1)^8
+ 1)
```

3.83 $\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=116

$$-\frac{a^2}{2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}-\frac{2a^2}{3\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^3}+\frac{a^2}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^2}-\frac{a^2}{2\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)}$$

```
[Out] -1/2*a^2/(1-((-a*x+1)/(a*x+1))^(1/2))-2/3*a^2/(1+((-a*x+1)/(a*x+1))^(1/2))^
3+a^2/(1+((-a*x+1)/(a*x+1))^(1/2))^2-1/2*a^2/(1+((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A]
time = 0.28, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6472, 1626}

$$-\frac{a^2}{2\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{a^2}{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}+\frac{a^2}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}-\frac{2a^2}{3\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcSech[a*x]*x^3),x]
```

```
[Out] -1/2*a^2/(1 - Sqrt[(1 - a*x)/(1 + a*x)]) - (2*a^2)/(3*(1 + Sqrt[(1 - a*x)/(
1 + a*x)])^3) + a^2/(1 + Sqrt[(1 - a*x)/(1 + a*x)])^2 - a^2/(2*(1 + Sqrt[(1
 - a*x)/(1 + a*x)]))
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)}\,dx$$
$$= -\left((4a^2)\,\text{Subst}\left(\int \frac{x(1+x^2)}{(-1+x)^2(1+x)^4}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\right)$$
$$= -\left((4a^2)\,\text{Subst}\left(\int \left(\frac{1}{8(-1+x)^2}-\frac{1}{2(1+x)^4}+\frac{1}{2(1+x)^3}-\frac{1}{8(1+x)^2}\right)dx, x, \sqrt{\frac{1-a}{1+a}}\right.\right.$$
$$= -\frac{a^2}{2\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)} - \frac{2a^2}{3\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^3} + \frac{a^2}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^2} - \frac{a^2}{2\left(1+\sqrt{\frac{1-a}{1+a}}\right.}$$

Mathematica [A]
time = 0.04, size = 43, normalized size = 0.37

$$-\frac{1+(-1+ax)\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^2}{3ax^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcSech[a*x]*x^3),x]
[Out] -1/3*(1 + (-1 + a*x)*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2)/(a*x^3)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x)
[Out] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="maxim
a")
```

```
[Out] integrate(1/(x^3*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

Fricas [A]
time = 0.37, size = 52, normalized size = 0.45

$$-\frac{(a^3x^3 - ax)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}} + 1}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="frica
s")
```

```
[Out] -1/3*((a^3*x^3 - a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1)/(a*
x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\int\frac{1}{ax^3\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}} + x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**3,x)
```

```
[Out] a*Integral(1/(a*x**3*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^3,x, algorithm="giac"
)
```

```
[Out] integrate(1/(x^3*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

Mupad [B]
time = 2.14, size = 58, normalized size = 0.50

$$\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{x}{3}-\frac{a\,x^2}{3}+\frac{1}{3\,a}-\frac{a^2\,x^3}{3}\right)}{x^3\,\sqrt{\frac{1}{a\,x}+1}}-\frac{1}{3\,a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

```
[Out] ((1/(a*x) - 1)^(1/2)*(x/3 - (a*x^2)/3 + 1/(3*a) - (a^2*x^3)/3))/(x^3*(1/(a*
x) + 1)^(1/2)) - 1/(3*a*x^3)
```

3.84 $\int \frac{e^{-\mathbf{sech}^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=200

$$-\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}-\frac{a^3}{2\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^4}+\frac{a^3}{\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^3}-\frac{a^3}{\left(1+\sqrt{\frac{1-a}{1+a}}\right.}$$

```
[Out] -1/4*a^3*arctanh(((-a*x+1)/(a*x+1))^(1/2))-1/4*a^3/(1-((-a*x+1)/(a*x+1))^(1
/2))^2+1/4*a^3/(1-((-a*x+1)/(a*x+1))^(1/2))-1/2*a^3/(1+((-a*x+1)/(a*x+1))^(
1/2))^4+a^3/(1+((-a*x+1)/(a*x+1))^(1/2))^3-a^3/(1+((-a*x+1)/(a*x+1))^(1/2))
^2+1/2*a^3/(1+((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A]
time = 0.33, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6472, 1626, 213}

$$\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}+\frac{a^3}{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}-\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}-\frac{a^3}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}+\frac{a^3}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}-\frac{a^3}{2\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}-\frac{1}{4}a^3\tanh^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcSech[a*x]*x^4),x]
```

```
[Out] -1/4*a^3/(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2 + a^3/(4*(1 - Sqrt[(1 - a*x)/(1
+ a*x)])) - a^3/(2*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^4) + a^3/(1 + Sqrt[(1 -
a*x)/(1 + a*x)])^3 - a^3/(1 + Sqrt[(1 - a*x)/(1 + a*x)])^2 + a^3/(2*(1 + Sq
rt[(1 - a*x)/(1 + a*x)])) - (a^3*ArcTanh[Sqrt[(1 - a*x)/(1 + a*x)]])/4
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
```

Q[n]

Rubi steps

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)}\,dx$$

$$= (4a)\text{Subst}\left(\int \frac{x(a+ax^2)^2}{(-1+x)^3(1+x)^5}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)$$

$$= (4a)\text{Subst}\left(\int \left(\frac{a^2}{8(-1+x)^3}+\frac{a^2}{16(-1+x)^2}+\frac{a^2}{2(1+x)^5}-\frac{3a^2}{4(1+x)^4}+\frac{a^2}{2(1+x)^3}-\frac{}{8(1\right.\right.$$

$$= -\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}-\frac{a^3}{2\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^4}+\frac{a^3}{\left(1+\sqrt{\frac{1-a}{1+a}}\right.}$$

$$= -\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a^3}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}-\frac{a^3}{2\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^4}+\frac{a^3}{\left(1+\sqrt{\frac{1-a}{1+a}}\right.}$$

Mathematica [A]
time = 0.07, size = 110, normalized size = 0.55

$$-\frac{2+\sqrt{\frac{1-ax}{1+ax}}\,(-2-2ax+a^2x^2+a^3x^3)-a^4x^4\log(x)+a^4x^4\log\left(1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)}{8ax^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcSech[a*x]*x^4),x]
```

```
[Out] -1/8*(2 + Sqrt[(1 - a*x)/(1 + a*x)]*(-2 - 2*a*x + a^2*x^2 + a^3*x^3) - a^4*
x^4*Log[x] + a^4*x^4*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)
/(1 + a*x)]])/(a*x^4)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x)
```

```
[Out] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="maxim
a")
```

```
[Out] integrate(1/(x^4*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

Fricas [A]
time = 0.43, size = 138, normalized size = 0.69

$$-\frac{a^4x^4\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+1\right)-a^4x^4\log\left(ax\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}-1\right)+2\left(a^3x^3-2\,ax\right)\sqrt{\frac{ax+1}{ax}}\sqrt{-\frac{ax-1}{ax}}+4}{16\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="frica
s")
```

```
[Out] -1/16*(a^4*x^4*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1) -
a^4*x^4*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 2*(a^3*
x^3 - 2*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 4)/(a*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\int\frac{1}{ax^4\sqrt{-1+\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}+x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**4,x)
```

```
[Out] a*Integral(1/(a*x**4*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^4,x, algorithm="giac"
)
```

```
[Out] integrate(1/(x^4*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

Mupad [B]
time = 43.71, size = 1511, normalized size = 7.56

$$-\frac{\frac{a^3\left(\sqrt{\frac{1}{a x}-1}-i\right)^4 64i}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^4}+\frac{a^3\left(\sqrt{\frac{1}{a x}-1}-i\right)^6 128i}{3\left(\sqrt{\frac{1}{a x}+1}-1\right)^6}+\frac{a^3\left(\sqrt{\frac{1}{a x}-1}-i\right)^8 64i}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^8}}{1+\frac{15\left(\sqrt{\frac{1}{a x}-1}-i\right)^4}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^4}-\frac{20\left(\sqrt{\frac{1}{a x}-1}-i\right)^6}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^6}+\frac{15\left(\sqrt{\frac{1}{a x}-1}-i\right)^8}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^8}-\frac{6\left(\sqrt{\frac{1}{a x}-1}-i\right)^{10}}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^{10}}+\frac{\left(\sqrt{\frac{1}{a x}-1}-i\right)^{12}}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^{12}}-\frac{6\left(\sqrt{\frac{1}{a x}-1}-i\right)^2}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^2}}+\frac{\frac{a^3\left(\sqrt{\frac{1}{a x}-1}-i\right)^4 192i}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^4}+\frac{a^3\left(\sqrt{\frac{1}{a x}-1}-i\right)^6 128i}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^6}+\frac{a^3\left(\sqrt{\frac{1}{a x}-1}-i\right)^8 192i}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^8}}{3\left(1+\frac{15\left(\sqrt{\frac{1}{a x}-1}-i\right)^4}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^4}-\frac{20\left(\sqrt{\frac{1}{a x}-1}-i\right)^6}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^6}+\frac{15\left(\sqrt{\frac{1}{a x}-1}-i\right)^8}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^8}-\frac{6\left(\sqrt{\frac{1}{a x}-1}-i\right)^{10}}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^{10}}+\frac{\left(\sqrt{\frac{1}{a x}-1}-i\right)^{12}}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^{12}}-\frac{6\left(\sqrt{\frac{1}{a x}-1}-i\right)^2}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^2}\right)}-\frac{a^3\,\mathrm{atanh}\left(\frac{\sqrt{\frac{1}{a x}-1}-i}{\sqrt{\frac{1}{a x}+1}-1}\right)}{2}+\frac{\frac{14a^3\left(\sqrt{\frac{1}{a x}-1}-i\right)^3}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^3}+\frac{14a^3\left(\sqrt{\frac{1}{a x}-1}-i\right)^5}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^5}+\frac{2a^3\left(\sqrt{\frac{1}{a x}-1}-i\right)^7}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^7}+\frac{2a^3\left(\sqrt{\frac{1}{a x}-1}-i\right)}{\sqrt{\frac{1}{a x}+1}-1}}{1+\frac{6\left(\sqrt{\frac{1}{a x}-1}-i\right)^4}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^4}-\frac{4\left(\sqrt{\frac{1}{a x}-1}-i\right)^6}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^6}+\frac{\left(\sqrt{\frac{1}{a x}-1}-i\right)^8}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^8}-\frac{4\left(\sqrt{\frac{1}{a x}-1}-i\right)^2}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^2}}+\frac{\frac{23a^3\left(\sqrt{\frac{1}{a x}-1}-i\right)^3}{2\left(\sqrt{\frac{1}{a x}+1}-1\right)^3}+\frac{333a^3\left(\sqrt{\frac{1}{a x}-1}-i\right)^5}{2\left(\sqrt{\frac{1}{a x}+1}-1\right)^5}+\frac{671a^3\left(\sqrt{\frac{1}{a x}-1}-i\right)^7}{2\left(\sqrt{\frac{1}{a x}+1}-1\right)^7}+\frac{671a^3\left(\sqrt{\frac{1}{a x}-1}-i\right)^9}{2\left(\sqrt{\frac{1}{a x}+1}-1\right)^9}+\frac{333a^3\left(\sqrt{\frac{1}{a x}-1}-i\right)^{11}}{2\left(\sqrt{\frac{1}{a x}+1}-1\right)^{11}}+\frac{23a^3\left(\sqrt{\frac{1}{a x}-1}-i\right)^{13}}{2\left(\sqrt{\frac{1}{a x}+1}-1\right)^{13}}-\frac{3a^3\left(\sqrt{\frac{1}{a x}-1}-i\right)^{15}}{2\left(\sqrt{\frac{1}{a x}+1}-1\right)^{15}}-\frac{3a^3\left(\sqrt{\frac{1}{a x}-1}-i\right)}{2\left(\sqrt{\frac{1}{a x}+1}-1\right)}}{1+\frac{28\left(\sqrt{\frac{1}{a x}-1}-i\right)^4}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^4}-\frac{56\left(\sqrt{\frac{1}{a x}-1}-i\right)^6}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^6}+\frac{70\left(\sqrt{\frac{1}{a x}-1}-i\right)^8}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^8}-\frac{56\left(\sqrt{\frac{1}{a x}-1}-i\right)^{10}}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^{10}}+\frac{28\left(\sqrt{\frac{1}{a x}-1}-i\right)^{12}}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^{12}}-\frac{8\left(\sqrt{\frac{1}{a x}-1}-i\right)^{14}}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^{14}}+\frac{\left(\sqrt{\frac{1}{a x}-1}-i\right)^{16}}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^{16}}-\frac{8\left(\sqrt{\frac{1}{a x}-1}-i\right)^2}{\left(\sqrt{\frac{1}{a x}+1}-1\right)^2}}-\frac{1}{4 a x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

```
[Out] ((a^3*((1/(a*x) - 1)^(1/2) - 1i)^4*192i)/((1/(a*x) + 1)^(1/2) - 1)^4 + (a^3
*((1/(a*x) - 1)^(1/2) - 1i)^6*128i)/((1/(a*x) + 1)^(1/2) - 1)^6 + (a^3*((1/
(a*x) - 1)^(1/2) - 1i)^8*192i)/((1/(a*x) + 1)^(1/2) - 1)^8)/(3*((15*((1/(a*
x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (6*((1/(a*x) - 1)^(1/2
) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (20*((1/(a*x) - 1)^(1/2) - 1i)^6)/
((1/(a*x) + 1)^(1/2) - 1)^6 + (15*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) +
 1)^(1/2) - 1)^8 - (6*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) -
 1)^10 + ((1/(a*x) - 1)^(1/2) - 1i)^12/((1/(a*x) + 1)^(1/2) - 1)^12 + 1)) -
 ((a^3*((1/(a*x) - 1)^(1/2) - 1i)^4*64i)/((1/(a*x) + 1)^(1/2) - 1)^4 + (a^3
*((1/(a*x) - 1)^(1/2) - 1i)^6*128i)/(3*((1/(a*x) + 1)^(1/2) - 1)^6) + (a^3*
((1/(a*x) - 1)^(1/2) - 1i)^8*64i)/((1/(a*x) + 1)^(1/2) - 1)^8)/((15*((1/(a*
x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (6*((1/(a*x) - 1)^(1/2
) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (20*((1/(a*x) - 1)^(1/2) - 1i)^6)/
((1/(a*x) + 1)^(1/2) - 1)^6 + (15*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) +
 1)^(1/2) - 1)^8 - (6*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) -
 1)^10 + ((1/(a*x) - 1)^(1/2) - 1i)^12/((1/(a*x) + 1)^(1/2) - 1)^12 + 1) -
(a^3*atanh(((1/(a*x) - 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)))/2 + ((14*
a^3*((1/(a*x) - 1)^(1/2) - 1i)^3)/((1/(a*x) + 1)^(1/2) - 1)^3 + (14*a^3*((1
/(a*x) - 1)^(1/2) - 1i)^5)/((1/(a*x) + 1)^(1/2) - 1)^5 + (2*a^3*((1/(a*x) -
 1)^(1/2) - 1i)^7)/((1/(a*x) + 1)^(1/2) - 1)^7 + (2*a^3*((1/(a*x) - 1)^(1/2
) - 1i))/((1/(a*x) + 1)^(1/2) - 1))/((6*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(
a*x) + 1)^(1/2) - 1)^4 - (4*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1
/2) - 1)^2 - (4*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 +
 ((1/(a*x) - 1)^(1/2) - 1i)^8/((1/(a*x) + 1)^(1/2) - 1)^8 + 1) + ((23*a^3*(
(1/(a*x) - 1)^(1/2) - 1i)^3)/(2*((1/(a*x) + 1)^(1/2) - 1)^3) + (333*a^3*((1
/(a*x) - 1)^(1/2) - 1i)^5)/(2*((1/(a*x) + 1)^(1/2) - 1)^5) + (671*a^3*((1/(
a*x) - 1)^(1/2) - 1i)^7)/(2*((1/(a*x) + 1)^(1/2) - 1)^7) + (671*a^3*((1/(a*
x) - 1)^(1/2) - 1i)^9)/(2*((1/(a*x) + 1)^(1/2) - 1)^9) + (333*a^3*((1/(a*x)
 - 1)^(1/2) - 1i)^11)/(2*((1/(a*x) + 1)^(1/2) - 1)^11) + (23*a^3*((1/(a*x)
- 1)^(1/2) - 1i)^13)/(2*((1/(a*x) + 1)^(1/2) - 1)^13) - (3*a^3*((1/(a*x) -
1)^(1/2) - 1i)^15)/(2*((1/(a*x) + 1)^(1/2) - 1)^15) - (3*a^3*((1/(a*x) - 1)
```

```
^(1/2) - 1i))/(2*((1/(a*x) + 1)^(1/2) - 1)))/((28*((1/(a*x) - 1)^(1/2) - 1i
)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (8*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*
x) + 1)^(1/2) - 1)^2 - (56*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/
2) - 1)^6 + (70*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8 -
 (56*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1)^10 + (28*((1/
(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x) + 1)^(1/2) - 1)^12 - (8*((1/(a*x) - 1)
^(1/2) - 1i)^14)/((1/(a*x) + 1)^(1/2) - 1)^14 + ((1/(a*x) - 1)^(1/2) - 1i)^
16/((1/(a*x) + 1)^(1/2) - 1)^16 + 1) - 1/(4*a*x^4)
```

3.85 $\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=233

$$-\frac{a^4}{6\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}+\frac{a^4}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}-\frac{3a^4}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}-\frac{2a^4}{5\left(1+\sqrt{\frac{1-ax}{1+ax}}\right)^5}+\frac{a^4}{\left(1+\sqrt{\frac{1-a}{1+a}}\right.}$$

```
[Out] -1/6*a^4/(1-((-a*x+1)/(a*x+1))^(1/2))^3+1/4*a^4/(1-((-a*x+1)/(a*x+1))^(1/2)
)^2-3/8*a^4/(1-((-a*x+1)/(a*x+1))^(1/2))-2/5*a^4/(1+((-a*x+1)/(a*x+1))^(1/2
))^5+a^4/(1+((-a*x+1)/(a*x+1))^(1/2))^4-4/3*a^4/(1+((-a*x+1)/(a*x+1))^(1/2)
)^3+a^4/(1+((-a*x+1)/(a*x+1))^(1/2))^2-3/8*a^4/(1+((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A]
time = 0.34, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6472, 1626}

$$-\frac{3a^4}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{3a^4}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}+\frac{a^4}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}+\frac{a^4}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}-\frac{a^4}{6\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}-\frac{4a^4}{3\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}+\frac{a^4}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}-\frac{2a^4}{5\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcSech[a*x]*x^5),x]
```

```
[Out] -1/6*a^4/(1 - Sqrt[(1 - a*x)/(1 + a*x)])^3 + a^4/(4*(1 - Sqrt[(1 - a*x)/(1
+ a*x)])^2) - (3*a^4)/(8*(1 - Sqrt[(1 - a*x)/(1 + a*x)])) - (2*a^4)/(5*(1 +
 Sqrt[(1 - a*x)/(1 + a*x)])^5) + a^4/(1 + Sqrt[(1 - a*x)/(1 + a*x)])^4 - (4
*a^4)/(3*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^3) + a^4/(1 + Sqrt[(1 - a*x)/(1 +
a*x)])^2 - (3*a^4)/(8*(1 + Sqrt[(1 - a*x)/(1 + a*x)]))
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^5}\,dx = \int \frac{1}{x^5\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)}\,dx$$

$$= -\left((4a)\text{Subst}\left(\int \frac{x(a+ax^2)^3}{(-1+x)^4(1+x)^6}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\right)$$

$$= -\left((4a)\text{Subst}\left(\int \left(\frac{a^3}{8(-1+x)^4}+\frac{a^3}{8(-1+x)^3}+\frac{3a^3}{32(-1+x)^2}-\frac{a^3}{2(1+x)^6}+\frac{a^3}{(1+x)^5}\right.\right.\right.$$

$$= -\frac{a^4}{6\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}+\frac{a^4}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}-\frac{3a^4}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}-\frac{2a^4}{5\left(1+\sqrt{\frac{1-}{1-}}\right.}$$

Mathematica [A]
time = 0.05, size = 60, normalized size = 0.26

$$-\frac{3+\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^2\,(-3+3ax-2a^2x^2+2a^3x^3)}{15ax^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcSech[a*x]*x^5),x]
```

```
[Out] -1/15*(3 + Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-3 + 3*a*x - 2*a^2*x^2 +
2*a^3*x^3))/(a*x^5)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x)
```

```
[Out] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="maxim
a")
```

```
[Out] integrate(1/(x^5*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

Fricas [A]
time = 0.39, size = 60, normalized size = 0.26

$$-\frac{(2\,a^5x^5+a^3x^3-3\,ax)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}+3}{15\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="frica
s")
```

```
[Out] -1/15*((2*a^5*x^5 + a^3*x^3 - 3*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/
(a*x)) + 3)/(a*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\int\frac{1}{ax^5\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}+x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**5,x)
```

```
[Out] a*Integral(1/(a*x**5*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^5,x, algorithm="giac"
)
```

```
[Out] integrate(1/(x^5*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

Mupad [B]
time = 2.37, size = 75, normalized size = 0.32

$$-\frac{1}{5\,a\,x^5}-\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{a\,x^2}{15}-\frac{x}{5}-\frac{1}{5\,a}+\frac{a^2\,x^3}{15}+\frac{2\,a^3\,x^4}{15}+\frac{2\,a^4\,x^5}{15}\right)}{x^5\,\sqrt{\frac{1}{a\,x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

```
[Out] - 1/(5*a*x^5) - ((1/(a*x) - 1)^(1/2)*((a*x^2)/15 - x/5 - 1/(5*a) + (a^2*x^3
)/15 + (2*a^3*x^4)/15 + (2*a^4*x^5)/15))/(x^5*(1/(a*x) + 1)^(1/2))
```

3.86 $\int \frac{e^{-\text{sech}^{-1}(ax)}}{x^6}\,dx$

Optimal. Leaf size=320

$$-\frac{a^5}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}+\frac{a^5}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}-\frac{3a^5}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a^5}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)}-\frac{a^5}{3\left(1+\sqrt{\frac{1-}{1+}}\right.}$$

```
[Out] -1/8*a^5*arctanh(((-a*x+1)/(a*x+1))^(1/2))-1/8*a^5/(1-((-a*x+1)/(a*x+1))^(1
/2))^4+1/4*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))^3-3/8*a^5/(1-((-a*x+1)/(a*x+1))
^(1/2))^2+1/4*a^5/(1-((-a*x+1)/(a*x+1))^(1/2))-1/3*a^5/(1+((-a*x+1)/(a*x+1)
)^(1/2))^6+a^5/(1+((-a*x+1)/(a*x+1))^(1/2))^5-13/8*a^5/(1+((-a*x+1)/(a*x+1)
)^(1/2))^4+19/12*a^5/(1+((-a*x+1)/(a*x+1))^(1/2))^3-a^5/(1+((-a*x+1)/(a*x+1
))^(1/2))^2+3/8*a^5/(1+((-a*x+1)/(a*x+1))^(1/2))
```

Rubi [A]
time = 0.38, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6472, 1626, 213}

$$\frac{a^5}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}+\frac{3a^5}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}-\frac{3a^5}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}-\frac{a^5}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}+\frac{a^5}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}+\frac{19a^5}{12\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}-\frac{a^5}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^4}-\frac{13a^5}{8\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}+\frac{a^5}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^5}-\frac{a^5}{3\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^6}-\frac{1}{8}a^5\tanh^{-1}\left(\sqrt{\frac{1-ax}{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcSech[a*x]*x^6),x]
```

```
[Out] -1/8*a^5/(1 - Sqrt[(1 - a*x)/(1 + a*x)])^4 + a^5/(4*(1 - Sqrt[(1 - a*x)/(1
+ a*x)])^3) - (3*a^5)/(8*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2) + a^5/(4*(1 - S
qrt[(1 - a*x)/(1 + a*x)])) - a^5/(3*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^6) + a^
5/(1 + Sqrt[(1 - a*x)/(1 + a*x)])^5 - (13*a^5)/(8*(1 + Sqrt[(1 - a*x)/(1 +
a*x)])^4) + (19*a^5)/(12*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^3) - a^5/(1 + Sqrt
[(1 - a*x)/(1 + a*x)])^2 + (3*a^5)/(8*(1 + Sqrt[(1 - a*x)/(1 + a*x)])) - (a
^5*ArcTanh[Sqrt[(1 - a*x)/(1 + a*x)]])/8
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
```

```
x] && IntegersQ[m, n]
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^6}\,dx = \int \frac{1}{x^6\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)}\,dx$$

$$= (4a)\text{Subst}\left(\int \frac{x(a+ax^2)^4}{(-1+x)^5(1+x)^7}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)$$

$$= (4a)\text{Subst}\left(\int \left(\frac{a^4}{8(-1+x)^5}+\frac{3a^4}{16(-1+x)^4}+\frac{3a^4}{16(-1+x)^3}+\frac{a^4}{16(-1+x)^2}+\frac{a^4}{2(1+x)}\right.\right.$$

$$= -\frac{a^5}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}+\frac{a^5}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}-\frac{3a^5}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a^5}{4\left(1-\sqrt{\frac{1}{1}}\right.}$$

$$= -\frac{a^5}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}+\frac{a^5}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}-\frac{3a^5}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}+\frac{a^5}{4\left(1-\sqrt{\frac{1}{1}}\right.}$$

Mathematica [A]
time = 0.10, size = 129, normalized size = 0.40

$$-\frac{8+\sqrt{\frac{1-ax}{1+ax}}\left(-8-8ax+2a^2x^2+2a^3x^3+3a^4x^4+3a^5x^5\right)-3a^6x^6\log(x)+3a^6x^6\log\left(1+\sqrt{\frac{1-ax}{1+ax}}+ax\sqrt{\frac{1-ax}{1+ax}}\right)}{48ax^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcSech[a*x]*x^6),x]
```

```
[Out] -1/48*(8 + Sqrt[(1 - a*x)/(1 + a*x)]*(-8 - 8*a*x + 2*a^2*x^2 + 2*a^3*x^3 +
3*a^4*x^4 + 3*a^5*x^5) - 3*a^6*x^6*Log[x] + 3*a^6*x^6*Log[1 + Sqrt[(1 - a*x
)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x)]])/(a*x^6)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)x^{6}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x)
```

```
[Out] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="maxim
a")
```

```
[Out] integrate(1/(x^6*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

Fricas [A]
time = 0.50, size = 148, normalized size = 0.46

$$-\frac{3\,a^{6}x^{6}\log\left(ax\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}+1\right)-3\,a^{6}x^{6}\log\left(ax\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}-1\right)+2\,(3\,a^{5}x^{5}+2\,a^{3}x^{3}-8\,ax)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}+16}{96\,ax^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="frica
s")
```

```
[Out] -1/96*(3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) + 1)
- 3*a^6*x^6*log(a*x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x)) - 1) + 2*(
3*a^5*x^5 + 2*a^3*x^3 - 8*a*x)*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))
 + 16)/(a*x^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\int \frac{1}{ax^{6}\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}+x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**6,x)
```

```
[Out] a*Integral(1/(a*x**6*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x**5), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^6,x, algorithm="giac"
)
```

```
[Out] integrate(1/(x^6*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

Mupad [B]
time = 65.40, size = 2479, normalized size = 7.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

```
[Out] ((a^5*((1/(a*x) - 1)^(1/2) - 1i)^6*10240i)/((1/(a*x) + 1)^(1/2) - 1)^6 + (a
^5*((1/(a*x) - 1)^(1/2) - 1i)^8*20480i)/((1/(a*x) + 1)^(1/2) - 1)^8 + (a^5*
((1/(a*x) - 1)^(1/2) - 1i)^10*36864i)/((1/(a*x) + 1)^(1/2) - 1)^10 + (a^5*(
(1/(a*x) - 1)^(1/2) - 1i)^12*20480i)/((1/(a*x) + 1)^(1/2) - 1)^12 + (a^5*((
1/(a*x) - 1)^(1/2) - 1i)^14*10240i)/((1/(a*x) + 1)^(1/2) - 1)^14)/(15*((45*
((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (10*((1/(a*x) -
 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (120*((1/(a*x) - 1)^(1/2)
- 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6 + (210*((1/(a*x) - 1)^(1/2) - 1i)^8)/(
(1/(a*x) + 1)^(1/2) - 1)^8 - (252*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x)
+ 1)^(1/2) - 1)^10 + (210*((1/(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x) + 1)^(1/
2) - 1)^12 - (120*((1/(a*x) - 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^(1/2) - 1)^
14 + (45*((1/(a*x) - 1)^(1/2) - 1i)^16)/((1/(a*x) + 1)^(1/2) - 1)^16 - (10*
((1/(a*x) - 1)^(1/2) - 1i)^18)/((1/(a*x) + 1)^(1/2) - 1)^18 + ((1/(a*x) - 1
)^(1/2) - 1i)^20/((1/(a*x) + 1)^(1/2) - 1)^20 + 1)) - (a^5*atanh(((1/(a*x)
- 1)^(1/2) - 1i)/((1/(a*x) + 1)^(1/2) - 1)))/4 - ((a^5*((1/(a*x) - 1)^(1/2)
 - 1i)^6*2048i)/(3*((1/(a*x) + 1)^(1/2) - 1)^6) + (a^5*((1/(a*x) - 1)^(1/2)
 - 1i)^8*4096i)/(3*((1/(a*x) + 1)^(1/2) - 1)^8) + (a^5*((1/(a*x) - 1)^(1/2)
 - 1i)^10*12288i)/(5*((1/(a*x) + 1)^(1/2) - 1)^10) + (a^5*((1/(a*x) - 1)^(1
/2) - 1i)^12*4096i)/(3*((1/(a*x) + 1)^(1/2) - 1)^12) + (a^5*((1/(a*x) - 1)^
(1/2) - 1i)^14*2048i)/(3*((1/(a*x) + 1)^(1/2) - 1)^14))/((45*((1/(a*x) - 1)
^(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (10*((1/(a*x) - 1)^(1/2) - 1i
)^2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (120*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(
a*x) + 1)^(1/2) - 1)^6 + (210*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1)^
```

```
(1/2) - 1)^8 - (252*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1
)^10 + (210*((1/(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x) + 1)^(1/2) - 1)^12 - (
120*((1/(a*x) - 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^(1/2) - 1)^14 + (45*((1/(
a*x) - 1)^(1/2) - 1i)^16)/((1/(a*x) + 1)^(1/2) - 1)^16 - (10*((1/(a*x) - 1)
^(1/2) - 1i)^18)/((1/(a*x) + 1)^(1/2) - 1)^18 + ((1/(a*x) - 1)^(1/2) - 1i)^
20/((1/(a*x) + 1)^(1/2) - 1)^20 + 1) - ((311*a^5*((1/(a*x) - 1)^(1/2) - 1i)
^5)/(4*((1/(a*x) + 1)^(1/2) - 1)^5) - (175*a^5*((1/(a*x) - 1)^(1/2) - 1i)^3
)/(12*((1/(a*x) + 1)^(1/2) - 1)^3) + (8361*a^5*((1/(a*x) - 1)^(1/2) - 1i)^7
)/(4*((1/(a*x) + 1)^(1/2) - 1)^7) + (42259*a^5*((1/(a*x) - 1)^(1/2) - 1i)^9
)/(6*((1/(a*x) + 1)^(1/2) - 1)^9) + (25295*a^5*((1/(a*x) - 1)^(1/2) - 1i)^1
1)/(2*((1/(a*x) + 1)^(1/2) - 1)^11) + (25295*a^5*((1/(a*x) - 1)^(1/2) - 1i)
^13)/(2*((1/(a*x) + 1)^(1/2) - 1)^13) + (42259*a^5*((1/(a*x) - 1)^(1/2) - 1
i)^15)/(6*((1/(a*x) + 1)^(1/2) - 1)^15) + (8361*a^5*((1/(a*x) - 1)^(1/2) -
1i)^17)/(4*((1/(a*x) + 1)^(1/2) - 1)^17) + (311*a^5*((1/(a*x) - 1)^(1/2) -
1i)^19)/(4*((1/(a*x) + 1)^(1/2) - 1)^19) - (175*a^5*((1/(a*x) - 1)^(1/2) -
1i)^21)/(12*((1/(a*x) + 1)^(1/2) - 1)^21) + (5*a^5*((1/(a*x) - 1)^(1/2) - 1
i)^23)/(4*((1/(a*x) + 1)^(1/2) - 1)^23) + (5*a^5*((1/(a*x) - 1)^(1/2) - 1i)
)/(4*((1/(a*x) + 1)^(1/2) - 1)))/((66*((1/(a*x) - 1)^(1/2) - 1i)^4)/((1/(a*
x) + 1)^(1/2) - 1)^4 - (12*((1/(a*x) - 1)^(1/2) - 1i)^2)/((1/(a*x) + 1)^(1/
2) - 1)^2 - (220*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x) + 1)^(1/2) - 1)^6
+ (495*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1)^(1/2) - 1)^8 - (792*((1
/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1)^10 + (924*((1/(a*x) -
 1)^(1/2) - 1i)^12)/((1/(a*x) + 1)^(1/2) - 1)^12 - (792*((1/(a*x) - 1)^(1/2
) - 1i)^14)/((1/(a*x) + 1)^(1/2) - 1)^14 + (495*((1/(a*x) - 1)^(1/2) - 1i)^
16)/((1/(a*x) + 1)^(1/2) - 1)^16 - (220*((1/(a*x) - 1)^(1/2) - 1i)^18)/((1/
(a*x) + 1)^(1/2) - 1)^18 + (66*((1/(a*x) - 1)^(1/2) - 1i)^20)/((1/(a*x) + 1
)^(1/2) - 1)^20 - (12*((1/(a*x) - 1)^(1/2) - 1i)^22)/((1/(a*x) + 1)^(1/2) -
 1)^22 + ((1/(a*x) - 1)^(1/2) - 1i)^24/((1/(a*x) + 1)^(1/2) - 1)^24 + 1) -
((23*a^5*((1/(a*x) - 1)^(1/2) - 1i)^3)/(2*((1/(a*x) + 1)^(1/2) - 1)^3) + (3
33*a^5*((1/(a*x) - 1)^(1/2) - 1i)^5)/(2*((1/(a*x) + 1)^(1/2) - 1)^5) + (671
*a^5*((1/(a*x) - 1)^(1/2) - 1i)^7)/(2*((1/(a*x) + 1)^(1/2) - 1)^7) + (671*a
^5*((1/(a*x) - 1)^(1/2) - 1i)^9)/(2*((1/(a*x) + 1)^(1/2) - 1)^9) + (333*a^5
*((1/(a*x) - 1)^(1/2) - 1i)^11)/(2*((1/(a*x) + 1)^(1/2) - 1)^11) + (23*a^5*
((1/(a*x) - 1)^(1/2) - 1i)^13)/(2*((1/(a*x) + 1)^(1/2) - 1)^13) - (3*a^5*((
1/(a*x) - 1)^(1/2) - 1i)^15)/(2*((1/(a*x) + 1)^(1/2) - 1)^15) - (3*a^5*((1/
(a*x) - 1)^(1/2) - 1i))/(2*((1/(a*x) + 1)^(1/2) - 1)))/((28*((1/(a*x) - 1)^
(1/2) - 1i)^4)/((1/(a*x) + 1)^(1/2) - 1)^4 - (8*((1/(a*x) - 1)^(1/2) - 1i)^
2)/((1/(a*x) + 1)^(1/2) - 1)^2 - (56*((1/(a*x) - 1)^(1/2) - 1i)^6)/((1/(a*x
) + 1)^(1/2) - 1)^6 + (70*((1/(a*x) - 1)^(1/2) - 1i)^8)/((1/(a*x) + 1)^(1/2
) - 1)^8 - (56*((1/(a*x) - 1)^(1/2) - 1i)^10)/((1/(a*x) + 1)^(1/2) - 1)^10
+ (28*((1/(a*x) - 1)^(1/2) - 1i)^12)/((1/(a*x) + 1)^(1/2) - 1)^12 - (8*((1/
(a*x) - 1)^(1/2) - 1i)^14)/((1/(a*x) + 1)^(1/2) - 1)^14 + ((1/(a*x) - 1)^(1
/2) - 1i)^16/((1/(a*x) + 1)^(1/2) - 1)^16 + 1) - 1/(6*a*x^6)
```

3.87 $\int \frac{e^{-\mathbf{sech}^{-1}(ax)}}{x^7}\,dx$

Optimal. Leaf size=353

$$-\frac{a^6}{10\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5}+\frac{a^6}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}-\frac{5a^6}{12\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}+\frac{3a^6}{8\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^2}-\frac{5}{16\left(1-\right.}$$

```
[Out] -1/10*a^6/(1-((-a*x+1)/(a*x+1))^(1/2))^5+1/4*a^6/(1-((-a*x+1)/(a*x+1))^(1/2
))^4-5/12*a^6/(1-((-a*x+1)/(a*x+1))^(1/2))^3+3/8*a^6/(1-((-a*x+1)/(a*x+1))^
(1/2))^2-5/16*a^6/(1-((-a*x+1)/(a*x+1))^(1/2))-2/7*a^6/(1+((-a*x+1)/(a*x+1)
)^(1/2))^7+a^6/(1+((-a*x+1)/(a*x+1))^(1/2))^6-19/10*a^6/(1+((-a*x+1)/(a*x+1
))^(1/2))^5+9/4*a^6/(1+((-a*x+1)/(a*x+1))^(1/2))^4-11/6*a^6/(1+((-a*x+1)/(a
*x+1))^(1/2))^3+a^6/(1+((-a*x+1)/(a*x+1))^(1/2))^2-5/16*a^6/(1+((-a*x+1)/(a
*x+1))^(1/2))
```

Rubi [A]
time = 0.41, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6472, 1626}

$$-\frac{5a^6}{16\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)}-\frac{5a^6}{16\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)}+\frac{3a^6}{8\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^2}+\frac{a^6}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^2}-\frac{5a^6}{12\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^3}-\frac{11a^6}{6\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^3}+\frac{a^6}{4\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^4}+\frac{9a^6}{4\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^4}-\frac{a^6}{10\left(1-\sqrt{\frac{1-ax}{ax+1}}\right)^5}-\frac{19a^6}{10\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^5}+\frac{a^6}{\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^6}-\frac{2a^6}{7\left(\sqrt{\frac{1-ax}{ax+1}}+1\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcSech[a*x]*x^7),x]
```

```
[Out] -1/10*a^6/(1 - Sqrt[(1 - a*x)/(1 + a*x)])^5 + a^6/(4*(1 - Sqrt[(1 - a*x)/(1
 + a*x)])^4) - (5*a^6)/(12*(1 - Sqrt[(1 - a*x)/(1 + a*x)])^3) + (3*a^6)/(8*
(1 - Sqrt[(1 - a*x)/(1 + a*x)])^2) - (5*a^6)/(16*(1 - Sqrt[(1 - a*x)/(1 + a
*x)])) - (2*a^6)/(7*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^7) + a^6/(1 + Sqrt[(1 -
 a*x)/(1 + a*x)])^6 - (19*a^6)/(10*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^5) + (9*
a^6)/(4*(1 + Sqrt[(1 - a*x)/(1 + a*x)])^4) - (11*a^6)/(6*(1 + Sqrt[(1 - a*x
)/(1 + a*x)])^3) + a^6/(1 + Sqrt[(1 - a*x)/(1 + a*x)])^2 - (5*a^6)/(16*(1 +
 Sqrt[(1 - a*x)/(1 + a*x)]))
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rule 6472

```
Int[E^(ArcSech[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(1/u + Sqrt[(1 -
 u)/(1 + u)] + (1/u)*Sqrt[(1 - u)/(1 + u)])^n, x] /; FreeQ[m, x] && Integer
Q[n]
```

Rubi steps

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^7}\,dx = \int \frac{1}{x^7\left(\frac{1}{ax}+\sqrt{\frac{1-ax}{1+ax}}+\frac{\sqrt{\frac{1-ax}{1+ax}}}{ax}\right)}\,dx$$
$$= -\left((4a)\text{Subst}\left(\int \frac{x\left(a+ax^2\right)^5}{(-1+x)^6(1+x)^8}\,dx, x, \sqrt{\frac{1-ax}{1+ax}}\right)\right)$$
$$= -\left((4a)\text{Subst}\left(\int \left(\frac{a^5}{8(-1+x)^6}+\frac{a^5}{4(-1+x)^5}+\frac{5a^5}{16(-1+x)^4}+\frac{3a^5}{16(-1+x)^3}+\frac{5a^5}{64(-1\right.\right.\right.$$
$$= -\frac{a^6}{10\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^5}+\frac{a^6}{4\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^4}-\frac{5a^6}{12\left(1-\sqrt{\frac{1-ax}{1+ax}}\right)^3}+\frac{3a}{8\left(1-\sqrt{\right.}}$$

Mathematica [A]
time = 0.07, size = 76, normalized size = 0.22

$$-\frac{15+\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)^2\left(-15+15ax-12a^2x^2+12a^3x^3-8a^4x^4+8a^5x^5\right)}{105ax^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcSech[a*x]*x^7),x]
```

```
[Out] -1/105*(15 + Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)^2*(-15 + 15*a*x - 12*a^2*x
^2 + 12*a^3*x^3 - 8*a^4*x^4 + 8*a^5*x^5))/(a*x^7)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{1}{ax}+\sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x)
```

```
[Out] int(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="maxim
a")
```

```
[Out] integrate(1/(x^7*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

Fricas [A]
time = 0.35, size = 69, normalized size = 0.20

$$-\frac{\left(8\,a^{7}x^{7}+4\,a^{5}x^{5}+3\,a^{3}x^{3}-15\,ax\right)\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}+15}{105\,ax^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="frica
s")
```

```
[Out] -1/105*((8*a^7*x^7 + 4*a^5*x^5 + 3*a^3*x^3 - 15*a*x)*sqrt((a*x + 1)/(a*x))*
sqrt(-(a*x - 1)/(a*x)) + 15)/(a*x^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\int\frac{1}{ax^{7}\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}+x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))/x**7,x)
```

```
[Out] a*Integral(1/(a*x**7*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x)) + x**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))/x^7,x, algorithm="giac"
)
```

```
[Out] integrate(1/(x^7*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x))), x)
```

Mupad [B]
time = 2.59, size = 91, normalized size = 0.26

$$-\frac{1}{7\,a\,x^7}-\frac{\sqrt{\frac{1}{a\,x}-1}\,\left(\frac{a\,x^2}{35}-\frac{x}{7}-\frac{1}{7\,a}+\frac{a^2\,x^3}{35}+\frac{4\,a^3\,x^4}{105}+\frac{4\,a^4\,x^5}{105}+\frac{8\,a^5\,x^6}{105}+\frac{8\,a^6\,x^7}{105}\right)}{x^7\,\sqrt{\frac{1}{a\,x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x))),x)
```

```
[Out] - 1/(7*a*x^7) - ((1/(a*x) - 1)^(1/2)*((a*x^2)/35 - x/7 - 1/(7*a) + (a^2*x^3
)/35 + (4*a^3*x^4)/105 + (4*a^4*x^5)/105 + (8*a^5*x^6)/105 + (8*a^6*x^7)/10
5))/(x^7*(1/(a*x) + 1)^(1/2))
```

3.88 $\int \frac{e^{\text{sech}^{-1}(cx)}(dx)^m}{1-c^2x^2}\,dx$

Optimal. Leaf size=89

$$\frac{(dx)^m\sqrt{\frac{1}{1+cx}}\,\sqrt{1+cx}\;{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};c^2x^2\right)}{cm}+\frac{(dx)^m\,{}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};c^2x^2\right)}{cm}$$

```
[Out] (d*x)^m*hypergeom([1, 1/2*m],[1+1/2*m],c^2*x^2)/c/m+(d*x)^m*hypergeom([1/2,
 1/2*m],[1+1/2*m],c^2*x^2)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c/m
```

Rubi [A]
time = 0.12, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6476, 1972, 126, 371}

$$\frac{\sqrt{\frac{1}{cx+1}}\,\sqrt{cx+1}\,(dx)^m\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};c^2x^2\right)}{cm}+\frac{(dx)^m\,{}_2F_1\left(1,\frac{m}{2};\frac{m+2}{2};c^2x^2\right)}{cm}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcSech[c*x]*(d*x)^m)/(1 - c^2*x^2),x]
```

```
[Out] ((d*x)^m*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Hypergeometric2F1[1/2, m/2, (2
+ m)/2, c^2*x^2])/(c*m) + ((d*x)^m*Hypergeometric2F1[1, m/2, (2 + m)/2, c^2
*x^2])/(c*m)
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f,
m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && GtQ[a, 0] && GtQ[c, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1972

```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)], Int[u*(a + b*x^n)^(p*q), x], x]
 /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

Rule 6476

```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d/(a*c), Int[(d*x)^(m - 1)*(Sqrt[1/(1 + c*x)]/Sqrt[1 - c*x]),
x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d, m}, x] && EqQ[b + a*c^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(cx)}(dx)^m}{1-c^2x^2}\,dx &= \frac{d\int \frac{(dx)^{-1+m}\sqrt{\frac{1}{1+cx}}}{\sqrt{1-cx}}\,dx}{c}+\frac{d\int \frac{(dx)^{-1+m}}{1-c^2x^2}\,dx}{c}\\
&= \frac{(dx)^m\,{}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};c^2x^2\right)}{cm}+\frac{\left(d\sqrt{\frac{1}{1+cx}}\,\sqrt{1+cx}\right)\int \frac{(dx)^{-1+m}}{\sqrt{1-cx}\,\sqrt{1+cx}}\,dx}{c}\\
&= \frac{(dx)^m\,{}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};c^2x^2\right)}{cm}+\frac{\left(d\sqrt{\frac{1}{1+cx}}\,\sqrt{1+cx}\right)\int \frac{(dx)^{-1+m}}{\sqrt{1-c^2x^2}}\,dx}{c}\\
&= \frac{(dx)^m\sqrt{\frac{1}{1+cx}}\,\sqrt{1+cx}\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};c^2x^2\right)}{cm}+\frac{(dx)^m\,{}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};c^2x^2\right)}{cm}
\end{aligned}
$$

Mathematica [F]
time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}(dx)^m}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(E^ArcSech[c*x]*(d*x)^m)/(1 - c^2*x^2),x]
[Out] Integrate[(E^ArcSech[c*x]*(d*x)^m)/(1 - c^2*x^2), x]
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{1}{cx}+\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}\right)(dx)^m}{-c^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(d*x)^m/(-c^2*x^2+1),x)
```

```
[Out] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(d*x)^m/(-c^2*x^2+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(d*x)^m/(-c^2*x^2+1),x,
algorithm="maxima")
```

```
[Out] -d^m*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m/(c^3*x^3 - c*x), x) - d^m*i
ntegrate(1/2*x^m/(c*x + 1), x) - d^m*integrate(1/2*x^m/(c*x - 1), x) + d^m*
x^m/(c*m)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(d*x)^m/(-c^2*x^2+1),x,
algorithm="fricas")
```

```
[Out] integral(-((d*x)^m*c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) + (d*x)
^m)/(c^3*x^3 - c*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(dx)^m}{c^2x^3-x}\,dx + \int \frac{cx(dx)^m\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{c^2x^3-x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))*(d*x)**m/(-c**2*x**2+1
),x)
```

```
[Out] -(Integral((d*x)**m/(c**2*x**3 - x), x) + Integral(c*x*(d*x)**m*sqrt(-1 + 1
/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**3 - x), x))/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*(d*x)^m/(-c^2*x^2+1),x,
algorithm="giac")
```

```
[Out] integrate(-(d*x)^m*(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2
 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}+\frac{1}{c\,x}\right)\,{\left(d\,x\right)}^m}{c^2\,x^2-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x))*(d*x)^m)/(c^2*x^2
 - 1),x)
```

```
[Out] -int((((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x))*(d*x)^m)/(c^2*x^2
 - 1), x)
```

3.89 $\int \frac{e^{\text{sech}^{-1}(cx)}x^4}{1-c^2x^2}\,dx$

Optimal. Leaf size=88

$$-\frac{x^2}{2c^3}-\frac{2\sqrt{1-cx}}{3c^5\sqrt{\frac{1}{1+cx}}}-\frac{x^2\sqrt{1-cx}}{3c^3\sqrt{\frac{1}{1+cx}}}-\frac{\log\left(1-c^2x^2\right)}{2c^5}$$

```
[Out] -1/2*x^2/c^3-1/2*ln(-c^2*x^2+1)/c^5-2/3*(-c*x+1)^(1/2)/c^5/(1/(c*x+1))^(1/2
)-1/3*x^2*(-c*x+1)^(1/2)/c^3/(1/(c*x+1))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6476, 1972, 102, 12, 75, 272, 45}

$$-\frac{2\sqrt{1-cx}}{3c^5\sqrt{\frac{1}{cx+1}}}-\frac{x^2\sqrt{1-cx}}{3c^3\sqrt{\frac{1}{cx+1}}}-\frac{x^2}{2c^3}-\frac{\log\left(1-c^2x^2\right)}{2c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcSech[c*x]*x^4)/(1 - c^2*x^2),x]
```

```
[Out] -1/2*x^2/c^3 - (2*Sqrt[1 - c*x])/(3*c^5*Sqrt[(1 + c*x)^(-1)]) - (x^2*Sqrt[1
 - c*x])/(3*c^3*Sqrt[(1 + c*x)^(-1)]) - Log[1 - c^2*x^2]/(2*c^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1972

```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)], Int[u*(a + b*x^n)^(p*q), x], x]
 /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

Rule 6476

```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d/(a*c), Int[(d*x)^(m - 1)*(Sqrt[1/(1 + c*x)]/Sqrt[1 - c*x]),
x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d, m}, x] && EqQ[b + a*c^2, 0]
```

Rubi steps

$$\int \frac{e^{\text{sech}^{-1}(cx)}x^4}{1-c^2x^2}\,dx = \frac{\int \frac{x^3\sqrt{\frac{1}{1+cx}}}{\sqrt{1-cx}}\,dx}{c} + \frac{\int \frac{x^3}{1-c^2x^2}\,dx}{c}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{1-c^2x}\,dx, x, x^2\right)}{2c} + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{x^3}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{c}$$

$$= -\frac{x^2\sqrt{1-cx}}{3c^3\sqrt{\frac{1}{1+cx}}} + \frac{\text{Subst}\left(\int \left(-\frac{1}{c^2}-\frac{1}{c^2(-1+c^2x)}\right)\,dx, x, x^2\right)}{2c} - \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int}{3c}$$

$$= -\frac{x^2}{2c^3} - \frac{x^2\sqrt{1-cx}}{3c^3\sqrt{\frac{1}{1+cx}}} - \frac{\log(1-c^2x^2)}{2c^5} + \frac{\left(2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{x}{\sqrt{1-cx}\sqrt{1+c}}}{3c^3}$$

$$= -\frac{x^2}{2c^3} - \frac{2\sqrt{1-cx}}{3c^5\sqrt{\frac{1}{1+cx}}} - \frac{x^2\sqrt{1-cx}}{3c^3\sqrt{\frac{1}{1+cx}}} - \frac{\log(1-c^2x^2)}{2c^5}$$

Mathematica [**A**]
time = 0.14, size = 69, normalized size = 0.78

$$-\frac{3c^2x^2+2\sqrt{\frac{1-cx}{1+cx}}\,(2+2cx+c^2x^2+c^3x^3)+3\log(1-c^2x^2)}{6c^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcSech[c*x]*x^4)/(1 - c^2*x^2),x]
```

```
[Out] -1/6*(3*c^2*x^2 + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(2 + 2*c*x + c^2*x^2 + c^3*x^
3) + 3*Log[1 - c^2*x^2])/c^5
```

Maple [**A**]
time = 0.30, size = 74, normalized size = 0.84

method	result	size
default	$-\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}\,(c^2x^2+2)}{3c^4}+\frac{-\frac{x^2}{2c^2}-\frac{\ln(c^2x^2-1)}{2c^4}}{c}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^4/(-c^2*x^2+1),x,method=_RET
URNVERBOSE)
```

```
[Out] -1/3*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*(c^2*x^2+2)/c^4+1/c*(-1/2*x
^2/c^2-1/2/c^4*ln(c^2*x^2-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^4/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] -integrate(x, x)/c^3 - 1/2*log(c*x + 1)/c^5 - 1/2*log(c*x - 1)/c^5 - integr
ate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^3/(c^3*x^2 - c), x)
```

Fricas [A]
time = 0.34, size = 69, normalized size = 0.78

$$-\frac{3\,c^2x^2 + 2\,(c^3x^3 + 2\,cx)\sqrt{\frac{cx+1}{cx}}\,\sqrt{-\frac{cx-1}{cx}} + 3\,\log\left(c^2x^2 - 1\right)}{6\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^4/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] -1/6*(3*c^2*x^2 + 2*(c^3*x^3 + 2*c*x)*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)
/(c*x)) + 3*log(c^2*x^2 - 1))/c^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^3}{c^2x^2-1}\,dx + \int \frac{cx^4\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{c^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))*x**4/(-c**2*x**2+1),x)
```

```
[Out] -(Integral(x**3/(c**2*x**2 - 1), x) + Integral(c*x**4*sqrt(-1 + 1/(c*x))*sq
rt(1 + 1/(c*x))/(c**2*x**2 - 1), x))/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^4/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-x^4*(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 - 1
), x)
```

Mupad [B]
time = 2.60, size = 76, normalized size = 0.86

$$-\frac{\ln\left(c^2\,x^2-1\right)+c^2\,x^2}{2\,c^5}-x^3\,\sqrt{\frac{1}{c\,x}-1}\,\left(\frac{\sqrt{\frac{1}{c\,x}+1}}{3\,c^2}+\frac{2\,\sqrt{\frac{1}{c\,x}+1}}{3\,c^4\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 - 1
),x)
```

```
[Out] - (log(c^2*x^2 - 1) + c^2*x^2)/(2*c^5) - x^3*(1/(c*x) - 1)^(1/2)*((1/(c*x)
+ 1)^(1/2)/(3*c^2) + (2*(1/(c*x) + 1)^(1/2))/(3*c^4*x^2))
```

3.90 $\int \frac{e^{\text{sech}^{-1}(cx)}x^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=75

$$-\frac{x}{c^3}-\frac{x\sqrt{1-cx}}{2c^3\sqrt{\frac{1}{1+cx}}}+\frac{\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{ArcSin}(cx)}{2c^4}+\frac{\tanh^{-1}(cx)}{c^4}$$

```
[Out] -x/c^3+arctanh(c*x)/c^4-1/2*x*(-c*x+1)^(1/2)/c^3/(1/(c*x+1))^(1/2)+1/2*arcs
in(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^4
```

Rubi [A]
time = 0.11, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6476, 1972, 92, 41, 222, 327, 212}

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\text{ArcSin}(cx)}{2c^4}+\frac{\tanh^{-1}(cx)}{c^4}-\frac{x\sqrt{1-cx}}{2c^3\sqrt{\frac{1}{cx+1}}}-\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcSech[c*x]*x^3)/(1 - c^2*x^2),x]
```

```
[Out] -(x/c^3) - (x*Sqrt[1 - c*x])/(2*c^3*Sqrt[(1 + c*x)^(-1)]) + (Sqrt[(1 + c*x)
^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/(2*c^4) + ArcTanh[c*x]/c^4
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1972

```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)], Int[u*(a + b*x^n)^(p*q), x], x]
 /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

Rule 6476

```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d/(a*c), Int[(d*x)^(m - 1)*(Sqrt[1/(1 + c*x)]/Sqrt[1 - c*x]),
x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d, m}, x] && EqQ[b + a*c^2, 0]
```

Rubi steps

$$\int \frac{e^{\text{sech}^{-1}(cx)}x^3}{1-c^2x^2}\,dx = \frac{\int \frac{x^2\sqrt{\frac{1}{1+cx}}}{\sqrt{1-cx}}\,dx}{c} + \frac{\int \frac{x^2}{1-c^2x^2}\,dx}{c}$$

$$= -\frac{x}{c^3} + \frac{\int \frac{1}{1-c^2x^2}\,dx}{c^3} + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{x^2}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{c}$$

$$= -\frac{x}{c^3} - \frac{x\sqrt{1-cx}}{2c^3\sqrt{\frac{1}{1+cx}}} + \frac{\tanh^{-1}(cx)}{c^4} + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{2c^3}$$

$$= -\frac{x}{c^3} - \frac{x\sqrt{1-cx}}{2c^3\sqrt{\frac{1}{1+cx}}} + \frac{\tanh^{-1}(cx)}{c^4} + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-c^2x^2}}\,dx}{2c^3}$$

$$= -\frac{x}{c^3} - \frac{x\sqrt{1-cx}}{2c^3\sqrt{\frac{1}{1+cx}}} + \frac{\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sin^{-1}(cx)}{2c^4} + \frac{\tanh^{-1}(cx)}{c^4}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.13, size = 110, normalized size = 1.47

$$-\frac{2cx + cx\sqrt{\frac{1-cx}{1+cx}} + c^2x^2\sqrt{\frac{1-cx}{1+cx}} + \log(1-cx) - \log(1+cx) - i\log\left(-2icx + 2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right)}{2c^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcSech[c*x]*x^3)/(1 - c^2*x^2),x]
```

```
[Out] -1/2*(2*c*x + c*x*Sqrt[(1 - c*x)/(1 + c*x)] + c^2*x^2*Sqrt[(1 - c*x)/(1 + c
*x)] + Log[1 - c*x] - Log[1 + c*x] - I*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1
 + c*x)]*(1 + c*x)])/c^4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.32, size = 122, normalized size = 1.63

method	result	si

default	$-\frac{\sqrt{-\frac{cx-1}{cx}}\, x \sqrt{\frac{cx+1}{cx}}\, \left(x\sqrt{-c^2x^2+1}\, \mathrm{csgn}(c) c-\arctan\left(\frac{\mathrm{csgn}(c) c x}{\sqrt{-c^2x^2+1}}\right)\right) \mathrm{csgn}(c)}{2c^3\sqrt{-c^2x^2+1}}+\frac{-\frac{x}{c^2}+\frac{\ln(cx+1)}{2c^3}-\frac{\ln(cx-1)}{2c^3}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^3/(-c^2*x^2+1),x,method=_RET
URNVERBOSE)
```

```
[Out] -1/2*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/c^3*(x*(-c^2*x^2+1)^(1/2)*c
sgn(c)*c-arctan(csgn(c)*c*x/(-c^2*x^2+1)^(1/2)))/(-c^2*x^2+1)^(1/2)*csgn(c)
+1/c*(-x/c^2+1/2/c^3*ln(c*x+1)-1/2/c^3*ln(c*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^3/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] -x/c^3 + 1/2*log(c*x + 1)/c^4 - 1/2*log(c*x - 1)/c^4 - integrate(sqrt(c*x +
 1)*sqrt(-c*x + 1)*x^2/(c^3*x^2 - c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(45) = 90.
time = 0.44, size = 91, normalized size = 1.21

$$-\frac{c^2x^2\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}+2\,cx+\arctan\left(\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}\right)-\log\left(cx+1\right)+\log\left(cx-1\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^3/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] -1/2*(c^2*x^2*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) + 2*c*x + arctan
(sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x))) - log(c*x + 1) + log(c*x - 1
))/c^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^2}{c^2x^2-1}\, dx+\int \frac{cx^3\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{c^2x^2-1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))*x**3/(-c**2*x**2+1),x)
```

```
[Out] -(Integral(x**2/(c**2*x**2 - 1), x) + Integral(c*x**3*sqrt(-1 + 1/(c*x))*sq
rt(1 + 1/(c*x))/(c**2*x**2 - 1), x))/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^3/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-x^3*(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 - 1
), x)
```

Mupad [B]
time = 8.08, size = 340, normalized size = 4.53

$$\frac{\mathrm{atanh}(c\,x)-c\,x}{c^4}-\frac{\ln\left(\frac{\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{c\,x}+1}-1}\right)\,1{}\mathrm{i}}{2\,c^4}-\frac{\frac{1{}\mathrm{i}}{32\,c^4}+\frac{{\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)}^2\,1{}\mathrm{i}}{16\,c^4\,{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)}^2}-\frac{{\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)}^4\,15{}\mathrm{i}}{32\,c^4\,{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)}^4}}{\frac{{\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)}^2}+\frac{2\,{\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)}^4}+\frac{{\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)}^6}}+\frac{\ln\left(\frac{2\,c\,\sqrt{\frac{c+\frac{1}{x}}{c}}-\frac{2}{x}+c\,\sqrt{-\frac{c-\frac{1}{x}}{c}}\,2{}\mathrm{i}}{2\,c+\frac{1}{x}-2\,c\,\sqrt{\frac{c+\frac{1}{x}}{c}}}\right)\,1{}\mathrm{i}}{2\,c^4}-\frac{{\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)}^2\,1{}\mathrm{i}}{32\,c^4\,{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 - 1
),x)
```

```
[Out] (atanh(c*x) - c*x)/c^4 - (log(((1/(c*x) - 1)^(1/2) - 1i)/((1/(c*x) + 1)^(1/
2) - 1))*1i)/(2*c^4) - (1i/(32*c^4) + (((1/(c*x) - 1)^(1/2) - 1i)^2*1i)/(16
*c^4*((1/(c*x) + 1)^(1/2) - 1)^2) - (((1/(c*x) - 1)^(1/2) - 1i)^4*15i)/(32*
c^4*((1/(c*x) + 1)^(1/2) - 1)^4))/((((1/(c*x) - 1)^(1/2) - 1i)^2/((1/(c*x) +
 1)^(1/2) - 1)^2 + (2*((1/(c*x) - 1)^(1/2) - 1i)^4)/((1/(c*x) + 1)^(1/2) -
1)^4 + ((1/(c*x) - 1)^(1/2) - 1i)^6/((1/(c*x) + 1)^(1/2) - 1)^6) + (log((c*
(-(c - 1/x)/c)^(1/2)*2i - 2/x + 2*c*((c + 1/x)/c)^(1/2))/(2*c + 1/x - 2*c*(
(c + 1/x)/c)^(1/2)))*1i)/(2*c^4) - (((1/(c*x) - 1)^(1/2) - 1i)^2*1i)/(32*c^
4*((1/(c*x) + 1)^(1/2) - 1)^2)
```

3.91 $\int \frac{e^{\text{sech}^{-1}(cx)}x^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=45

$$-\frac{\sqrt{1-cx}}{c^3\sqrt{\frac{1}{1+cx}}}-\frac{\log\left(1-c^2x^2\right)}{2c^3}$$

```
[Out] -1/2*ln(-c^2*x^2+1)/c^3-(-c*x+1)^(1/2)/c^3/(1/(c*x+1))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6476, 1972, 75, 266}

$$-\frac{\sqrt{1-cx}}{c^3\sqrt{\frac{1}{cx+1}}}-\frac{\log\left(1-c^2x^2\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcSech[c*x]*x^2)/(1 - c^2*x^2),x]
```

```
[Out] -(Sqrt[1 - c*x]/(c^3*Sqrt[(1 + c*x)^(-1)])) - Log[1 - c^2*x^2]/(2*c^3)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1972

```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)], Int[u*(a + b*x^n)^(p*q), x], x]
 /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

Rule 6476

```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d/(a*c), Int[(d*x)^(m - 1)*(Sqrt[1/(1 + c*x)]/Sqrt[1 - c*x]),
x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
```

```
d, m}, x] && EqQ[b + a*c^2, 0]
```

Rubi steps

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}x^2}{1-c^2x^2}\,dx = \frac{\int \frac{x\sqrt{\frac{1}{1+cx}}}{\sqrt{1-cx}}\,dx}{c} + \frac{\int \frac{x}{1-c^2x^2}\,dx}{c}$$
$$= -\frac{\log(1-c^2x^2)}{2c^3} + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{x}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{c}$$
$$= -\frac{\sqrt{1-cx}}{c^3\sqrt{\frac{1}{1+cx}}} - \frac{\log(1-c^2x^2)}{2c^3}$$

Mathematica [A]
time = 0.08, size = 44, normalized size = 0.98

$$-\frac{2\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)+\log(1-c^2x^2)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcSech[c*x]*x^2)/(1 - c^2*x^2),x]
```

```
[Out] -1/2*(2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + Log[1 - c^2*x^2])/c^3
```

Maple [A]
time = 0.31, size = 52, normalized size = 1.16

method	result	size
default	$-\frac{\sqrt{-\frac{cx-1}{cx}}\,x\sqrt{\frac{cx+1}{cx}}}{c^2}-\frac{\ln(c^2x^2-1)}{2c^3}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^2/(-c^2*x^2+1),x,method=_RET
URNVERBOSE)
```

```
[Out] -(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/c^2-1/2/c^3*ln(c^2*x^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^2/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] -1/2*log(c*x + 1)/c^3 - 1/2*log(c*x - 1)/c^3 - integrate(sqrt(c*x + 1)*sqrt
(-c*x + 1)*x/(c^3*x^2 - c), x)
```

Fricas [A]
time = 0.39, size = 49, normalized size = 1.09

$$-\frac{2\,cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}+\log\left(c^2x^2-1\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^2/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] -1/2*(2*c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) + log(c^2*x^2 - 1)
)/c^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{x}{c^2x^2-1}\,dx+\int\frac{cx^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{c^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))*x**2/(-c**2*x**2+1),x)
```

```
[Out] -(Integral(x/(c**2*x**2 - 1), x) + Integral(c*x**2*sqrt(-1 + 1/(c*x))*sqrt(
1 + 1/(c*x))/(c**2*x**2 - 1), x))/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x^2/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-x^2*(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 - 1
), x)
```

Mupad [B]
time = 2.70, size = 44, normalized size = 0.98

$$-\frac{\ln\left(c^2\,x^2-1\right)}{2\,c^3}-\frac{x\,\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 - 1
),x)
```

```
[Out] - log(c^2*x^2 - 1)/(2*c^3) - (x*(1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2))/c^
2
```

3.92 $\int \frac{e^{\text{sech}^{-1}(cx)}x}{1-c^2x^2}\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\text{ArcSin}(cx)}{c^2}+\frac{\tanh^{-1}(cx)}{c^2}$$

[Out] arctanh(c*x)/c^2+arcsin(c*x)*(1/(c*x+1))^(1/2)*(c*x+1)^(1/2)/c^2

Rubi [A]
time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6476, 1972, 41, 222, 212}

$$\frac{\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\,\text{ArcSin}(cx)}{c^2}+\frac{\tanh^{-1}(cx)}{c^2}$$

Antiderivative was successfully verified.

[In] Int[(E^ArcSech[c*x]*x)/(1 - c^2*x^2),x]

[Out] (Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/c^2 + ArcTanh[c*x]/c^2

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1972

```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)], Int[u*(a + b*x^n)^(p*q), x], x]
 /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

Rule 6476

```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d/(a*c), Int[(d*x)^(m - 1)*(Sqrt[1/(1 + c*x)]/Sqrt[1 - c*x]),
x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d, m}, x] && EqQ[b + a*c^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(cx)}x}{1-c^2x^2}\,dx &= \frac{\int \frac{\sqrt{\frac{1}{1+cx}}}{\sqrt{1-cx}}\,dx}{c}+\frac{\int \frac{1}{1-c^2x^2}\,dx}{c}\\
&= \frac{\tanh^{-1}(cx)}{c^2}+\frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-cx}\sqrt{1+cx}}\,dx}{c}\\
&= \frac{\tanh^{-1}(cx)}{c^2}+\frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{1-c^2x^2}}\,dx}{c}\\
&= \frac{\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\,\sin^{-1}(cx)}{c^2}+\frac{\tanh^{-1}(cx)}{c^2}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 68, normalized size = 1.84

$$-\frac{\log(1-cx)}{2c^2}+\frac{\log(1+cx)}{2c^2}+\frac{i\log\left(-2icx+2\sqrt{\frac{1-cx}{1+cx}}\,(1+cx)\right)}{c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcSech[c*x]*x)/(1 - c^2*x^2),x]
```

```
[Out] -1/2*Log[1 - c*x]/c^2 + Log[1 + c*x]/(2*c^2) + (I*Log[(-2*I)*c*x + 2*Sqrt[(
1 - c*x)/(1 + c*x)]*(1 + c*x)])/c^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.29, size = 97, normalized size = 2.62

method	result	size

default	$\frac{\sqrt{-\frac{cx-1}{cx}}\, x \sqrt{\frac{cx+1}{cx}}\, \arctan\left(\frac{\mathrm{csgn}(c)cx}{\sqrt{-(cx-1)(cx+1)}}\right)\mathrm{csgn}(c)}{\sqrt{-c^2x^2+1}\, c} + \frac{\frac{\ln(cx+1)}{2c}-\frac{\ln(cx-1)}{2c}}{c}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x/(-c^2*x^2+1),x,method=_RETUR
NVERBOSE)
```

```
[Out] (-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*arctan(csgn(c)*c*x/(-(c*x-1)*(c*
x+1))^(1/2))/(-c^2*x^2+1)^(1/2)*csgn(c)/c+1/c*(1/2/c*ln(c*x+1)-1/2/c*ln(c*x
-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x/(-c^2*x^2+1),x, algori
thm="maxima")
```

```
[Out] 1/2*log(c*x + 1)/c^2 - 1/2*log(c*x - 1)/c^2 - integrate(sqrt(c*x + 1)*sqrt(
-c*x + 1)/(c^3*x^2 - c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(17) = 34$.
time = 0.53, size = 53, normalized size = 1.43

$$-\frac{2\arctan\left(\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}\right)-\log(cx+1)+\log(cx-1)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x/(-c^2*x^2+1),x, algori
thm="fricas")
```

```
[Out] -1/2*(2*arctan(sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x))) - log(c*x + 1)
 + log(c*x - 1))/c^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{cx\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}}{c^2x^2-1}\,dx+\int\frac{1}{c^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))*x/(-c**2*x**2+1),x)
```

```
[Out] -(Integral(c*x*sqrt(-1 + 1/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**2 - 1), x) + I
ntegral(1/(c**2*x**2 - 1), x))/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*x/(-c^2*x^2+1),x, algori
thm="giac")
```

```
[Out] integrate(-x*(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 - 1),
 x)
```

Mupad [B]
time = 3.57, size = 84, normalized size = 2.27

$$\frac{\operatorname{atanh}(c\,x)}{c^2}+\frac{\left(\ln\left(\frac{{\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)}^2}+1\right)-\ln\left(\frac{\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{c\,x}+1}-1}\right)\right)\,1{}\mathrm{i}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x)))/(c^2*x^2 - 1),
x)
```

```
[Out] ((log(((1/(c*x) - 1)^(1/2) - 1i)^2/((1/(c*x) + 1)^(1/2) - 1)^2 + 1) - log((
(1/(c*x) - 1)^(1/2) - 1i)/((1/(c*x) + 1)^(1/2) - 1)))*1i)/c^2 + atanh(c*x)/
c^2
```

3.93 $\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{1-c^2x^2}\,dx$

Optimal. Leaf size=71

$$-\frac{\sqrt{\frac{1}{1+cx}}\,\sqrt{1+cx}\,\tanh^{-1}\left(\sqrt{1-cx}\,\sqrt{1+cx}\right)}{c}+\frac{\log(x)}{c}-\frac{\log\left(1-c^2x^2\right)}{2c}$$

```
[Out] ln(x)/c-1/2*ln(-c^2*x^2+1)/c-arctanh((-c*x+1)^(1/2)*(c*x+1)^(1/2))*(1/(c*x+
1))^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A]
time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6474, 1972, 94, 214, 272, 36, 29, 31}

$$-\frac{\log\left(1-c^2x^2\right)}{2c}+\frac{\log(x)}{c}-\frac{\sqrt{\frac{1}{cx+1}}\,\sqrt{cx+1}\,\tanh^{-1}\left(\sqrt{1-cx}\,\sqrt{cx+1}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[c*x]/(1 - c^2*x^2),x]
```

```
[Out] -((Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c*x]*Sqrt[1 + c*x]])
/c) + Log[x]/c - Log[1 - c^2*x^2]/(2*c)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
```

```
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1972

```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)], Int[u*(a + b*x^n)^(p*q), x], x]
 /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

Rule 6474

```
Int[E^ArcSech[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] :> Dist[1/(a*c),
 Int[Sqrt[1/(1 + c*x)]/(x*Sqrt[1 - c*x]), x], x] + Dist[1/c, Int[1/(x*(a +
b*x^2)), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b + a*c^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{1-c^2x^2}\,dx &= \frac{\int \frac{\sqrt{\frac{1}{1+cx}}}{x\sqrt{1-cx}}\,dx}{c}+\frac{\int \frac{1}{x(1-c^2x^2)}\,dx}{c}\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x(1-c^2x)}\,dx,x,x^2\right)}{2c}+\frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}}\,dx}{c}\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2c}+\frac{1}{2}c\operatorname{Subst}\left(\int \frac{1}{1-c^2x}\,dx,x,x^2\right)-\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\Bigg(\\ &= -\frac{\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{1+cx}\right)}{c}+\frac{\log(x)}{c}-\frac{\log(1-c^2x^2)}{2c}\end{aligned}$$

Mathematica [A]

time = 0.03, size = 73, normalized size = 1.03

$$\frac{2\log(x)}{c}-\frac{\log\left(1-c^2x^2\right)}{2c}-\frac{\log\left(1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right)}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[c*x]/(1 - c^2*x^2),x]
```

```
[Out] (2*Log[x])/c - Log[1 - c^2*x^2]/(2*c) - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] +
 c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/c
```

Maple [A]
time = 0.30, size = 82, normalized size = 1.15

method	result	size
default	$-\frac{\sqrt{-\frac{cx-1}{cx}}\, x\sqrt{\frac{cx+1}{cx}}\, \operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{\sqrt{-c^2x^2+1}}+\frac{-\frac{\ln(cx+1)}{2}+\ln(x)-\frac{\ln(cx-1)}{2}}{c}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/(-c^2*x^2+1),x,method=_RETURNV
ERBOSE)
```

```
[Out] -(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))/(
-c^2*x^2+1)^(1/2)+1/c*(-1/2*ln(c*x+1)+ln(x)-1/2*ln(c*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/(-c^2*x^2+1),x, algorith
m="maxima")
```

```
[Out] integrate(1/x, x)/c - 1/2*log(c*x + 1)/c - 1/2*log(c*x - 1)/c - integrate(s
qrt(c*x + 1)*sqrt(-c*x + 1)/(c^3*x^3 - c*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(45) = 90.
time = 0.37, size = 92, normalized size = 1.30

$$-\frac{\log\left(c^2x^2-1\right)+\log\left(cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}+1\right)-\log\left(cx\sqrt{\frac{cx+1}{cx}}\sqrt{-\frac{cx-1}{cx}}-1\right)-2\log\left(x\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/(-c^2*x^2+1),x, algorith
m="fricas")
```

```
[Out] -1/2*(log(c^2*x^2 - 1) + log(c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x
)) + 1) - log(c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) - 1) - 2*log
(x))/c
```

Sympy [A]
time = 9.44, size = 48, normalized size = 0.68

$$-\frac{\log\left(-1+\frac{1}{cx}\right)}{2c}-\frac{\log\left(\sqrt{1+\frac{1}{cx}}\right)}{c}-\frac{2\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{1+\frac{1}{cx}}}{2}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))/(-c**2*x**2+1),x)
```

```
[Out] -log(-1 + 1/(c*x))/(2*c) - log(sqrt(1 + 1/(c*x)))/c - 2*acosh(sqrt(2)*sqrt(
1 + 1/(c*x))/2)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/(-c^2*x^2+1),x, algorith
m="giac")
```

```
[Out] integrate(-(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(c^2*x^2 - 1), x
)
```

Mupad [B]
time = 2.92, size = 59, normalized size = 0.83

$$\frac{\ln(x)}{c}-\frac{4\,\mathrm{atanh}\left(\frac{\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{c\,x}+1}-1}\right)}{c}-\frac{\ln\left(3\,c^2\,x^2-3\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x))/(c^2*x^2 - 1),x)
```

```
[Out] log(x)/c - (4*atanh(((1/(c*x) - 1)^(1/2) - 1i)/((1/(c*x) + 1)^(1/2) - 1)))/
c - log(3*c^2*x^2 - 3)/(2*c)
```

3.94 $\int \frac{e^{\mathrm{sech}^{-1}(cx)}}{x(1-c^2x^2)}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{cx}-\frac{\sqrt{1-cx}}{cx\sqrt{\frac{1}{1+cx}}}+\tanh^{-1}(cx)$$

```
[Out] -1/c/x+arctanh(c*x)-(-c*x+1)^(1/2)/c/x/(1/(c*x+1))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6476, 1972, 97, 331, 212}

$$-\frac{\sqrt{1-cx}}{cx\sqrt{\frac{1}{cx+1}}}-\frac{1}{cx}+\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[c*x]/(x*(1 - c^2*x^2)),x]
```

```
[Out] -(1/(c*x)) - Sqrt[1 - c*x]/(c*x*Sqrt[(1 + c*x)^(-1)]) + ArcTanh[c*x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1972

```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)], Int[u*(a + b*x^n)^(p*q), x], x]
 /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

Rule 6476

```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d/(a*c), Int[(d*x)^(m - 1)*(Sqrt[1/(1 + c*x)]/Sqrt[1 - c*x]),
x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d, m}, x] && EqQ[b + a*c^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x(1-c^2x^2)}\,dx &= \frac{\int \frac{\sqrt{\frac{1}{1+cx}}}{x^2\sqrt{1-cx}}\,dx}{c} + \frac{\int \frac{1}{x^2(1-c^2x^2)}\,dx}{c} \\
&= -\frac{1}{cx} + c\int \frac{1}{1-c^2x^2}\,dx + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^2\sqrt{1-cx}\sqrt{1+cx}}\,dx}{c} \\
&= -\frac{1}{cx} - \frac{\sqrt{1-cx}}{cx\sqrt{\frac{1}{1+cx}}} + \tanh^{-1}(cx)
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 59, normalized size = 1.40

$$
-\frac{1}{cx} - \left(1+\frac{1}{cx}\right)\sqrt{\frac{1-cx}{1+cx}} - \frac{1}{2}\log(1-cx) + \frac{1}{2}\log(1+cx)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[c*x]/(x*(1 - c^2*x^2)),x]
```

```
[Out] -(1/(c*x)) - (1 + 1/(c*x))*Sqrt[(1 - c*x)/(1 + c*x)] - Log[1 - c*x]/2 + Log
[1 + c*x]/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.30, size = 65, normalized size = 1.55

method	result	size
default	$-\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\operatorname{csgn}(c)^{2}+\frac{\frac{c\ln(cx+1)}{2}-\frac{1}{x}-\frac{c\ln(cx-1)}{2}}{c}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x/(-c^2*x^2+1),x,method=_RETUR
NVERBOSE)
```

```
[Out] -(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*csgn(c)^2+1/c*(1/2*c*ln(c*x+1)-1/
x-1/2*c*ln(c*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x/(-c^2*x^2+1),x, algori
thm="maxima")
```

```
[Out] integrate(x^(-2), x)/c - integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^3*x^4 -
c*x^2), x) + 1/2*log(c*x + 1) - 1/2*log(c*x - 1)
```

Fricas [A]
time = 0.56, size = 62, normalized size = 1.48

$$-\frac{2\,cx\sqrt{\frac{cx+1}{cx}}\,\sqrt{-\frac{cx-1}{cx}}-cx\log\left(cx+1\right)+cx\log\left(cx-1\right)+2}{2\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x/(-c^2*x^2+1),x, algori
thm="fricas")
```

```
[Out] -1/2*(2*c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) - c*x*log(c*x + 1)
 + c*x*log(c*x - 1) + 2)/(c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{cx\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{c^{2}x^{4}-x^{2}}\,dx+\int \frac{1}{c^{2}x^{4}-x^{2}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))/x/(-c**2*x**2+1),x)
```

```
[Out] -(Integral(c*x*sqrt(-1 + 1/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**4 - x**2), x)
+ Integral(1/(c**2*x**4 - x**2), x))/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x/(-c^2*x^2+1),x, algori
thm="giac")
```

```
[Out] integrate(-(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/((c^2*x^2 - 1)*x
), x)
```

Mupad [B]
time = 2.53, size = 37, normalized size = 0.88

$$\mathrm{atanh}(c\,x)-\sqrt{\frac{1}{c\,x}-1}\,\sqrt{\frac{1}{c\,x}+1}-\frac{1}{c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x))/(x*(c^2*x^2 - 1)),
x)
```

```
[Out] atanh(c*x) - (1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) - 1/(c*x)
```

3.95 $\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^2(1-c^2x^2)}\,dx$

Optimal. Leaf size=108

$$-\frac{1}{2cx^2}-\frac{\sqrt{1-cx}}{2cx^2\sqrt{\frac{1}{1+cx}}}-\frac{1}{2}c\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{1+cx}\right)+c\log(x)-\frac{1}{2}c\log\left(1-c^2x^2\right)$$

```
[Out] -1/2/c/x^2+c*ln(x)-1/2*c*ln(-c^2*x^2+1)-1/2*(-c*x+1)^(1/2)/c/x^2/(1/(c*x+1)
)^(1/2)-1/2*c*arctanh((-c*x+1)^(1/2)*(c*x+1)^(1/2))*(1/(c*x+1))^(1/2)*(c*x+
1)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6476, 1972, 105, 12, 94, 214, 272, 46}

$$-\frac{1}{2}c\log\left(1-c^2x^2\right)-\frac{\sqrt{1-cx}}{2cx^2\sqrt{\frac{1}{cx+1}}}-\frac{1}{2cx^2}+c\log(x)-\frac{1}{2}c\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{cx+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[c*x]/(x^2*(1 - c^2*x^2)),x]
```

```
[Out] -1/2*1/(c*x^2) - Sqrt[1 - c*x]/(2*c*x^2*Sqrt[(1 + c*x)^(-1)]) - (c*Sqrt[(1
+ c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c*x]*Sqrt[1 + c*x]])/2 + c*Log[
x] - (c*Log[1 - c^2*x^2])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
```

```
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1972

```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)], Int[u*(a + b*x^n)^(p*q), x], x]
 /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

Rule 6476

```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d/(a*c), Int[(d*x)^(m - 1)*(Sqrt[1/(1 + c*x)]/Sqrt[1 - c*x]),
x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d, m}, x] && EqQ[b + a*c^2, 0]
```

Rubi steps

$$\int \frac{e^{\text{sech}^{-1}(cx)}}{x^2(1-c^2x^2)}\,dx = \frac{\int \frac{\sqrt{\frac{1}{1+cx}}}{x^3\sqrt{1-cx}}\,dx}{c} + \frac{\int \frac{1}{x^3(1-c^2x^2)}\,dx}{c}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1-c^2x)}\,dx, x, x^2\right)}{2c} + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^3\sqrt{1-cx}\sqrt{1+cx}}\,dx}{c}$$
$$= -\frac{\sqrt{1-cx}}{2cx^2\sqrt{\frac{1}{1+cx}}} + \frac{\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{c^2}{x}-\frac{c^4}{-1+c^2x}\right)\,dx, x, x^2\right)}{2c} + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right.}{}$$
$$= -\frac{1}{2cx^2} - \frac{\sqrt{1-cx}}{2cx^2\sqrt{\frac{1}{1+cx}}} + c\log(x) - \frac{1}{2}c\log\left(1-c^2x^2\right) + \frac{1}{2}\left(c\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)$$
$$= -\frac{1}{2cx^2} - \frac{\sqrt{1-cx}}{2cx^2\sqrt{\frac{1}{1+cx}}} + c\log(x) - \frac{1}{2}c\log\left(1-c^2x^2\right) - \frac{1}{2}\left(c^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)$$
$$= -\frac{1}{2cx^2} - \frac{\sqrt{1-cx}}{2cx^2\sqrt{\frac{1}{1+cx}}} - \frac{1}{2}c\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{1+cx}\right) + c\,l$$

Mathematica [A]
time = 0.11, size = 108, normalized size = 1.00

$$\frac{1}{2}\left(-\frac{1}{cx^2} - \frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{cx^2} + 3c\log(x) - c\log\left(1-c^2x^2\right) - c\log\left(1+\sqrt{\frac{1-cx}{1+cx}} + cx\sqrt{\frac{1-cx}{1+cx}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[c*x]/(x^2*(1 - c^2*x^2)),x]
```

```
[Out] (-(1/(c*x^2)) - (Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(c*x^2) + 3*c*Log[x]
- c*Log[1 - c^2*x^2] - c*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 -
c*x)/(1 + c*x)]])/2
```

Maple [A]
time = 0.29, size = 119, normalized size = 1.10

method	result
default	$-\frac{\sqrt{-\frac{cx-1}{cx}}\,\sqrt{\frac{cx+1}{cx}}\,\left(c^{2}x^{2}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-c^{2}x^{2}+1}}\right)+\sqrt{-c^{2}x^{2}+1}\right)}{2x\sqrt{-c^{2}x^{2}+1}}+\frac{-\frac{c^{2}\ln(cx+1)}{2}-\frac{1}{2x^{2}}+c^{2}\ln(x)-\frac{c^{2}\ln(cx-1)}{2}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^2/(-c^2*x^2+1),x,method=_RET
URNVERBOSE)
```

```
[Out] -1/2*(-(c*x-1)/c/x)^(1/2)/x*((c*x+1)/c/x)^(1/2)*(c^2*x^2*arctanh(1/(-c^2*x^
2+1)^(1/2))+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+1/c*(-1/2*c^2*ln(c*x+1)-
1/2/x^2+c^2*ln(x)-1/2*c^2*ln(c*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^2/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] c*integrate(1/x, x) - 1/2*c*log(c*x + 1) - 1/2*c*log(c*x - 1) + integrate(x
^(-3), x)/c - integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^3*x^5 - c*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(70) = 140$.
time = 0.57, size = 156, normalized size = 1.44

$$-\frac{2\,c^{2}x^{2}\log\left(c^{2}x^{2}-1\right)+c^{2}x^{2}\log\left(cx\sqrt{\frac{cx+1}{cx}}\,\sqrt{-\frac{cx-1}{cx}}+1\right)-c^{2}x^{2}\log\left(cx\sqrt{\frac{cx+1}{cx}}\,\sqrt{-\frac{cx-1}{cx}}-1\right)-4\,c^{2}x^{2}\log\left(x\right)+2\,cx\sqrt{\frac{cx+1}{cx}}\,\sqrt{-\frac{cx-1}{cx}}+2}{4\,cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^2/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] -1/4*(2*c^2*x^2*log(c^2*x^2 - 1) + c^2*x^2*log(c*x*sqrt((c*x + 1)/(c*x))*sq
rt(-(c*x - 1)/(c*x)) + 1) - c^2*x^2*log(c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(c*
x - 1)/(c*x)) - 1) - 4*c^2*x^2*log(x) + 2*c*x*sqrt((c*x + 1)/(c*x))*sqrt(-(
c*x - 1)/(c*x)) + 2)/(c*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{cx\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{c^{2}x^{5}-x^{3}}\,dx+\int \frac{1}{c^{2}x^{5}-x^{3}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))/x**2/(-c**2*x**2+1),x)
```

```
[Out] -(Integral(c*x*sqrt(-1 + 1/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**5 - x**3), x)
+ Integral(1/(c**2*x**5 - x**3), x))/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^2/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/((c^2*x^2 - 1)*x
^2), x)
```

Mupad [B]
time = 14.53, size = 331, normalized size = 3.06

$$c\ln(x)+\frac{\frac{2c\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)}{\sqrt{\frac{1}{c\,x}+1}-1}+\frac{14c\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)^3}+\frac{14c\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)^5}{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)^5}+\frac{2c\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)^7}{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)^7}}{1+\frac{6\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)^4}-\frac{4\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)^6}+\frac{\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)^8}-\frac{4\left(\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\frac{1}{c\,x}+1}-1\right)^2}}-\frac{c\ln\left(c^2x^2-1\right)}{2}-2c\,\mathrm{atanh}\left(\frac{\sqrt{\frac{1}{c\,x}-1}-\mathrm{i}}{\sqrt{\frac{1}{c\,x}+1}-1}\right)-\frac{1}{2\,c\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x))/(x^2*(c^2*x^2 - 1)
),x)
```

```
[Out] ((2*c*((1/(c*x) - 1)^(1/2) - 1i))/((1/(c*x) + 1)^(1/2) - 1) + (14*c*((1/(c*
x) - 1)^(1/2) - 1i)^3)/((1/(c*x) + 1)^(1/2) - 1)^3 + (14*c*((1/(c*x) - 1)^(
1/2) - 1i)^5)/((1/(c*x) + 1)^(1/2) - 1)^5 + (2*c*((1/(c*x) - 1)^(1/2) - 1i)
^7)/((1/(c*x) + 1)^(1/2) - 1)^7)/((6*((1/(c*x) - 1)^(1/2) - 1i)^4)/((1/(c*x
) + 1)^(1/2) - 1)^4 - (4*((1/(c*x) - 1)^(1/2) - 1i)^2)/((1/(c*x) + 1)^(1/2)
 - 1)^2 - (4*((1/(c*x) - 1)^(1/2) - 1i)^6)/((1/(c*x) + 1)^(1/2) - 1)^6 + ((
1/(c*x) - 1)^(1/2) - 1i)^8/((1/(c*x) + 1)^(1/2) - 1)^8 + 1) - 2*c*atanh(((1
/(c*x) - 1)^(1/2) - 1i)/((1/(c*x) + 1)^(1/2) - 1)) - (c*log(c^2*x^2 - 1))/2
 + c*log(x) - 1/(2*c*x^2)
```

3.96 $\int \frac{e^{\text{sech}^{-1}(cx)}}{x^3(1-c^2x^2)}\,dx$

Optimal. Leaf size=85

$$-\frac{1}{3cx^3}-\frac{c}{x}-\frac{\sqrt{1-cx}}{3cx^3\sqrt{\frac{1}{1+cx}}}-\frac{2c\sqrt{1-cx}}{3x\sqrt{\frac{1}{1+cx}}}+c^2\tanh^{-1}(cx)$$

```
[Out] -1/3/c/x^3-c/x+c^2*arctanh(c*x)-1/3*(-c*x+1)^(1/2)/c/x^3/(1/(c*x+1))^(1/2)-
2/3*c*(-c*x+1)^(1/2)/x/(1/(c*x+1))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6476, 1972, 105, 12, 97, 331, 212}

$$c^2\tanh^{-1}(cx)-\frac{\sqrt{1-cx}}{3cx^3\sqrt{\frac{1}{cx+1}}}-\frac{1}{3cx^3}-\frac{2c\sqrt{1-cx}}{3x\sqrt{\frac{1}{cx+1}}}-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSech[c*x]/(x^3*(1 - c^2*x^2)),x]
```

```
[Out] -1/3*1/(c*x^3) - c/x - Sqrt[1 - c*x]/(3*c*x^3*Sqrt[(1 + c*x)^(-1)]) - (2*c*
Sqrt[1 - c*x])/(3*x*Sqrt[(1 + c*x)^(-1)]) + c^2*ArcTanh[c*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1972

```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)], Int[u*(a + b*x^n)^(p*q), x], x]
 /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

Rule 6476

```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d/(a*c), Int[(d*x)^(m - 1)*(Sqrt[1/(1 + c*x)]/Sqrt[1 - c*x]),
x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d, m}, x] && EqQ[b + a*c^2, 0]
```

Rubi steps

$$\int \frac{e^{\operatorname{sech}^{-1}(cx)}}{x^3(1-c^2x^2)}\,dx = \frac{\int \frac{\sqrt{\frac{1}{1+cx}}}{x^4\sqrt{1-cx}}\,dx}{c} + \frac{\int \frac{1}{x^4(1-c^2x^2)}\,dx}{c}$$

$$= -\frac{1}{3cx^3} + c\int \frac{1}{x^2(1-c^2x^2)}\,dx + \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{x^4\sqrt{1-cx}\sqrt{1+cx}}\,dx}{c}$$

$$= -\frac{1}{3cx^3} - \frac{c}{x} - \frac{\sqrt{1-cx}}{3cx^3\sqrt{\frac{1}{1+cx}}} + c^3\int \frac{1}{1-c^2x^2}\,dx - \frac{\left(\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int -\frac{}{x^2\sqrt{1}}}{3c}$$

$$= -\frac{1}{3cx^3} - \frac{c}{x} - \frac{\sqrt{1-cx}}{3cx^3\sqrt{\frac{1}{1+cx}}} + c^2\tanh^{-1}(cx) + \frac{1}{3}\left(2c\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{}{x^2\sqrt{1}}$$

$$= -\frac{1}{3cx^3} - \frac{c}{x} - \frac{\sqrt{1-cx}}{3cx^3\sqrt{\frac{1}{1+cx}}} - \frac{2c\sqrt{1-cx}}{3x\sqrt{\frac{1}{1+cx}}} + c^2\tanh^{-1}(cx)$$

Mathematica [A]
time = 0.18, size = 90, normalized size = 1.06

$$-\frac{2+6c^2x^2+2\sqrt{\frac{1-cx}{1+cx}}\,(1+cx+2c^2x^2+2c^3x^3)+3c^3x^3\log(1-cx)-3c^3x^3\log(1+cx)}{6cx^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSech[c*x]/(x^3*(1 - c^2*x^2)),x]
```

```
[Out] -1/6*(2 + 6*c^2*x^2 + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x + 2*c^2*x^2 + 2*
c^3*x^3) + 3*c^3*x^3*Log[1 - c*x] - 3*c^3*x^3*Log[1 + c*x])/(c*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.34, size = 90, normalized size = 1.06

method	result	size
default	$-\frac{\sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\operatorname{csgn}(c)^2(2c^2x^2+1)}{3x^2}+\frac{\frac{c^3\ln(cx+1)}{2}-\frac{1}{3x^3}-\frac{c^2}{x}-\frac{c^3\ln(cx-1)}{2}}{c}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^3/(-c^2*x^2+1),x,method=_RET
URNVERBOSE)
```

```
[Out] -1/3*(-(c*x-1)/c/x)^(1/2)/x^2*((c*x+1)/c/x)^(1/2)*csgn(c)^2*(2*c^2*x^2+1)+1
/c*(1/2*c^3*ln(c*x+1)-1/3/x^3-c^2/x-1/2*c^3*ln(c*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^3/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] 1/2*c^2*log(c*x + 1) - 1/2*c^2*log(c*x - 1) + c*integrate(x^(-2), x) + inte
grate(x^(-4), x)/c - integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^3*x^6 - c*x^
4), x)
```

Fricas [A]
time = 0.58, size = 89, normalized size = 1.05

$$\frac{3\,c^3x^3\log\left(cx+1\right)-3\,c^3x^3\log\left(cx-1\right)-6\,c^2x^2-2\left(2\,c^3x^3+cx\right)\sqrt{\frac{cx+1}{cx}}\,\sqrt{-\frac{cx-1}{cx}}-2}{6\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^3/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] 1/6*(3*c^3*x^3*log(c*x + 1) - 3*c^3*x^3*log(c*x - 1) - 6*c^2*x^2 - 2*(2*c^3
*x^3 + c*x)*sqrt((c*x + 1)/(c*x))*sqrt(-(c*x - 1)/(c*x)) - 2)/(c*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{cx\sqrt{-1+\frac{1}{cx}}\,\sqrt{1+\frac{1}{cx}}}{c^2x^6-x^4}\,dx+\int\frac{1}{c^2x^6-x^4}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)**(1/2)*(1+1/c/x)**(1/2))/x**3/(-c**2*x**2+1),x)
```

```
[Out] -(Integral(c*x*sqrt(-1 + 1/(c*x))*sqrt(1 + 1/(c*x))/(c**2*x**6 - x**4), x)
+ Integral(1/(c**2*x**6 - x**4), x))/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/x^3/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/((c^2*x^2 - 1)*x
^3), x)
```

Mupad [B]
time = 2.51, size = 75, normalized size = 0.88

$$c^2\,\mathrm{atanh}(c\,x)-\frac{\left(\frac{\sqrt{\frac{1}{c\,x}+1}}{3}+\frac{2\,c^2\,x^2\,\sqrt{\frac{1}{c\,x}+1}}{3}\right)\,\sqrt{\frac{1}{c\,x}-1}}{x^2}-\frac{c^2\,x^2+\frac{1}{3}}{c\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((1/(c*x) - 1)^(1/2)*(1/(c*x) + 1)^(1/2) + 1/(c*x))/(x^3*(c^2*x^2 - 1)
),x)
```

```
[Out] c^2*atanh(c*x) - (((1/(c*x) + 1)^(1/2)/3 + (2*c^2*x^2*(1/(c*x) + 1)^(1/2))/
3)*(1/(c*x) - 1)^(1/2))/x^2 - (c^2*x^2 + 1/3)/(c*x^3)
```

3.97 $\int \frac{x\left(-1+ae^{\mathbf{sech}^{-1}(ax)}x\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=12

$$-\frac{e^{\operatorname{sech}^{-1}(ax)}x}{a}$$

```
[Out] -(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x/a
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.74, antiderivative size = 26, normalized size of antiderivative = 2.17, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$,
Rules used = {6857, 266, 6476, 1972, 75}

$$-\frac{\sqrt{1-ax}}{a^2\sqrt{\frac{1}{ax+1}}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x*(-1 + a*E^ArcSech[a*x]*x))/(1 - a^2*x^2),x]
```

```
[Out] -(Sqrt[1 - a*x]/(a^2*Sqrt[(1 + a*x)^(-1)]))
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1972

```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)], Int[u*(a + b*x^n)^(p*q), x], x]
 /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

Rule 6476

```
Int[(E^ArcSech[(c_.)*(x_)]*((d_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^2), x_Sym
bol] :> Dist[d/(a*c), Int[(d*x)^(m - 1)*(Sqrt[1/(1 + c*x)]/Sqrt[1 - c*x]),
x], x] + Dist[d/c, Int[(d*x)^(m - 1)/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
```

```
d, m}, x] && EqQ[b + a*c^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(-1+ae^{\operatorname{sech}^{-1}(ax)}x\right)}{1-a^2x^2}\,dx &= \int \left(\frac{x}{-1+a^2x^2}-\frac{ae^{\operatorname{sech}^{-1}(ax)}x^2}{-1+a^2x^2}\right)dx\\
&= -\left(a\int \frac{e^{\operatorname{sech}^{-1}(ax)}x^2}{-1+a^2x^2}\,dx\right)+\int \frac{x}{-1+a^2x^2}\,dx\\
&= \frac{\log\left(1-a^2x^2\right)}{2a^2}+\int \frac{x\sqrt{\frac{1}{1+ax}}}{\sqrt{1-ax}}\,dx-\int \frac{x}{-1+a^2x^2}\,dx\\
&= \left(\sqrt{\frac{1}{1+ax}}\sqrt{1+ax}\right)\int \frac{x}{\sqrt{1-ax}\sqrt{1+ax}}\,dx\\
&= -\frac{\sqrt{1-ax}}{a^2\sqrt{\frac{1}{1+ax}}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 28 vs. 2(12) = 24.
time = 0.17, size = 28, normalized size = 2.33

$$-\frac{\sqrt{\frac{1-ax}{1+ax}}\,(1+ax)}{a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(-1 + a*E^ArcSech[a*x]*x))/(1 - a^2*x^2),x]
```

```
[Out] -((Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/a^2)
```

Maple [A]
time = 0.30, size = 36, normalized size = 3.00

method	result	size
gosper	$-\frac{x\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}}{a}$	36
default	$-\frac{x\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}}{a}$	36
risch	$-\frac{x\sqrt{-\frac{ax-1}{ax}}\,\sqrt{\frac{ax+1}{ax}}}{a}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-1+a*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x)/(-a^2*x^2+1),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/a*x*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-1+a*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x)/(-a^2*x^2+1),x
, algorithm="maxima")
```

```
[Out] -integrate((a*x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)) - 1)*x/(a^2
*x^2 - 1), x)
```

Fricas [A]
time = 0.51, size = 35, normalized size = 2.92

$$-\frac{x\sqrt{\frac{ax+1}{ax}}\,\sqrt{-\frac{ax-1}{ax}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-1+a*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x)/(-a^2*x^2+1),x
, algorithm="fricas")
```

```
[Out] -x*sqrt((a*x + 1)/(a*x))*sqrt(-(a*x - 1)/(a*x))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a\int\frac{x^2\sqrt{-1+\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-1+a*(1/a/x+(1/a/x-1)**(1/2)*(1+1/a/x)**(1/2))*x)/(-a**2*x**2+
1),x)
```

```
[Out] -a*Integral(x**2*sqrt(-1 + 1/(a*x))*sqrt(1 + 1/(a*x))/(a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-1+a*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))*x)/(-a^2*x^2+1),x
, algorithm="giac")
```

```
[Out] integrate(-(a*x*(sqrt(1/(a*x) + 1)*sqrt(1/(a*x) - 1) + 1/(a*x)) - 1)*x/(a^2
*x^2 - 1), x)
```

Mupad [B]
time = 2.96, size = 76, normalized size = 6.33

$$\frac{\ln\left(\frac{1}{x}\right)}{a^2}-\frac{\ln\left(a+\frac{1}{x}\right)}{2\,a^2}-\frac{\ln\left(\frac{1}{x}-a\right)}{2\,a^2}+\frac{\ln\left(a^2\,x^2-1\right)}{2\,a^2}-\frac{x\,\sqrt{\frac{1}{a\,x}-1}\,\sqrt{\frac{1}{a\,x}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a*x*((1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2) + 1/(a*x)) - 1))/(a^2
*x^2 - 1),x)
```

```
[Out] log(1/x)/a^2 - log(a + 1/x)/(2*a^2) - log(1/x - a)/(2*a^2) + log(a^2*x^2 -
1)/(2*a^2) - (x*(1/(a*x) - 1)^(1/2)*(1/(a*x) + 1)^(1/2))/a
```

3.98 $\int \frac{\operatorname{sech}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal. Leaf size=61

$$\frac{\operatorname{sech}^{-1}(a+bx)^2}{2d}-\frac{\operatorname{sech}^{-1}(a+bx)\log\left(1+e^{2\operatorname{sech}^{-1}(a+bx)}\right)}{d}-\frac{\operatorname{PolyLog}\left(2,-e^{2\operatorname{sech}^{-1}(a+bx)}\right)}{2d}$$

```
[Out] 1/2*arcsech(b*x+a)^2/d-arcsech(b*x+a)*ln(1+(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(
1/(b*x+a)+1)^(1/2))^2)/d-1/2*polylog(2,-(1/(b*x+a)+(1/(b*x+a)-1)^(1/2)*(1/(
b*x+a)+1)^(1/2))^2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {6454, 12, 6416, 5882, 3799, 2221, 2317, 2438}

$$-\frac{\operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(a+bx)}\right)}{2d}+\frac{\operatorname{sech}^{-1}(a+bx)^2}{2d}-\frac{\operatorname{sech}^{-1}(a+bx)\log\left(e^{2\operatorname{sech}^{-1}(a+bx)}+1\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] ArcSech[a + b*x]^2/(2*d) - (ArcSech[a + b*x]*Log[1 + E^(2*ArcSech[a + b*x])
])/d - PolyLog[2, -E^(2*ArcSech[a + b*x])]/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6416

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rule 6454

```
Int[((a_.) + ArcSech[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcSech[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{\text{Subst}\left(\int \frac{b\operatorname{sech}^{-1}(x)}{dx}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\operatorname{sech}^{-1}(x)}{x}\,dx, x, a+bx\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{\cosh^{-1}(x)}{x}\,dx, x, \frac{1}{a+bx}\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}\left(\frac{1}{a+bx}\right)\right)}{d}$$
$$= \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{2\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(\frac{1}{a+bx}\right)\right)}{d}$$
$$= \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{1}{a+bx}\right)}\right)}{d} + \frac{\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,d\right.}{d}$$
$$= \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{1}{a+bx}\right)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x,\right.}{2d}$$
$$= \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\cosh^{-1}\left(\frac{1}{a+bx}\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{1}{a+bx}\right)}\right)}{d} - \frac{\text{Li}_2\left(-e^{2\cosh^{-1}\left(\frac{1}{a+bx}\right)}\right)}{2d}$$

Mathematica [A]
time = 0.04, size = 52, normalized size = 0.85

$$\frac{-\operatorname{sech}^{-1}(a+bx)\left(\operatorname{sech}^{-1}(a+bx)+2\log\left(1+e^{-2\operatorname{sech}^{-1}(a+bx)}\right)\right)+\text{PolyLog}\left(2,-e^{-2\operatorname{sech}^{-1}(a+bx)}\right)}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSech[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] (-(ArcSech[a + b*x]*(ArcSech[a + b*x] + 2*Log[1 + E^(-2*ArcSech[a + b*x])])
) + PolyLog[2, -E^(-2*ArcSech[a + b*x])])/(2*d)
```

Maple [A]
time = 0.49, size = 111, normalized size = 1.82

method	result

derivativedivides	$\frac{\frac{b\,\mathrm{arcsech}(bx+a)^2}{2d}-\frac{b\,\mathrm{arcsech}(bx+a)\ln\left(1+\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)^2\right)}{d}-\frac{b\,\mathrm{polylog}\left(2,-\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-\right.\right.}{2d}}{b}$
default	$\frac{\frac{b\,\mathrm{arcsech}(bx+a)^2}{2d}-\frac{b\,\mathrm{arcsech}(bx+a)\ln\left(1+\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{\frac{1}{bx+a}+1}\right)^2\right)}{d}-\frac{b\,\mathrm{polylog}\left(2,-\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-\right.\right.}{2d}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*b/d*arcsech(b*x+a)^2-b/d*arcsech(b*x+a)*ln(1+(1/(b*x+a)+(1/(b*x+a)
-1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2)-1/2*b/d*polylog(2,-(1/(b*x+a)+(1/(b*x+a)-
1)^(1/2)*(1/(b*x+a)+1)^(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] 1/2*(2*log(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*b*x + sqrt(b*x + a + 1)*sqr
t(-b*x - a + 1)*a + b*x + a)*log(b*x + a) - 3*log(b*x + a)^2)/d - 1/2*(log(
b*x + a + 1)*log(b*x + a) + dilog(-b*x - a))/d - 1/2*(log(b*x + a)*log(-b*x
 - a + 1) + dilog(b*x + a))/d + integrate((b^2*x + a*b)*log(b*x + a)/(b^2*d
*x^2 + 2*a*b*d*x + a^2*d + (b^2*d*x^2 + 2*a*b*d*x + a^2*d - d)*sqrt(b*x + a
 + 1)*sqrt(-b*x - a + 1) - d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arcsech(b*x + a)/(b*d*x + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int\frac{\operatorname{asech}(a+bx)}{a+bx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] b*Integral(asech(a + b*x)/(a + b*x), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] integrate(arcsech(b*x + a)/(d*x + a*d/b), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acosh}\left(\frac{1}{a+b\,x}\right)}{d\,x+\frac{a\,d}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/(a + b*x))/(d*x + (a*d)/b),x)
```

```
[Out] int(acosh(1/(a + b*x))/(d*x + (a*d)/b), x)
```

3.99 $\int x^3 \textbf{sech}^{-1}(a + bx^4)\, dx$

Optimal. Leaf size=57

$$\frac{(a+bx^4)\,\mathrm{sech}^{-1}(a+bx^4)}{4b}-\frac{\mathrm{ArcTan}\left(\sqrt{\frac{1-a-bx^4}{1+a+bx^4}}\right)}{2b}$$

```
[Out] 1/4*(b*x^4+a)*arcsech(b*x^4+a)/b-1/2*arctan(((-b*x^4-a+1)/(b*x^4+a+1))^(1/2
))/b
```

Rubi [A]
time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6847, 6448, 1983, 12, 209}

$$\frac{(a+bx^4)\,\mathrm{sech}^{-1}(a+bx^4)}{4b}-\frac{\mathrm{ArcTan}\left(\sqrt{\frac{-a-bx^4+1}{a+bx^4+1}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSech[a + b*x^4],x]
```

```
[Out] ((a + b*x^4)*ArcSech[a + b*x^4])/(4*b) - ArcTan[Sqrt[(1 - a - b*x^4)/(1 + a
 + b*x^4)]]/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1983

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n),
Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(
b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1/
n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
```

```
gerQ[r]
```

Rule 6448

```
Int[ArcSech[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcSech[c + d*
x]/d), x] + Int[Sqrt[(1 - c - d*x)/(1 + c + d*x)]/(1 - c - d*x), x] /; Free
Q[{c, d}, x]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\int x^3\operatorname{sech}^{-1}\left(a+bx^4\right)\,dx &= \frac{1}{4}\text{Subst}\left(\int \operatorname{sech}^{-1}(a+bx)\,dx,x,x^4\right)\\ &= \frac{(a+bx^4)\operatorname{sech}^{-1}(a+bx^4)}{4b}+\frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{\frac{1-a-bx}{1+a+bx}}}{1-a-bx}\,dx,x,x^4\right)\\ &= \frac{(a+bx^4)\operatorname{sech}^{-1}(a+bx^4)}{4b}-b\text{Subst}\left(\int \frac{1}{2b^2\left(1+x^2\right)}\,dx,x,\sqrt{\frac{1-a-bx^4}{1+a+bx^4}}\right)\\ &= \frac{(a+bx^4)\operatorname{sech}^{-1}(a+bx^4)}{4b}-\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{\frac{1-a-bx^4}{1+a+bx^4}}\right)}{2b}\\ &= \frac{(a+bx^4)\operatorname{sech}^{-1}(a+bx^4)}{4b}-\frac{\tan^{-1}\left(\sqrt{\frac{1-a-bx^4}{1+a+bx^4}}\right)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 106, normalized size = 1.86

$$\frac{(a+bx^4)\operatorname{sech}^{-1}(a+bx^4)+\frac{2\sqrt{-\frac{-1+a+bx^4}{1+a+bx^4}}\sqrt{1-(a+bx^4)^2}\,\text{ArcTan}\left(\frac{\sqrt{1-a-bx^4}}{\sqrt{1+a+bx^4}}\right)}{-1+a+bx^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSech[a + b*x^4],x]
```

```
[Out] ((a + b*x^4)*ArcSech[a + b*x^4] + (2*Sqrt[-((-1 + a + b*x^4)/(1 + a + b*x^4
))]*Sqrt[1 - (a + b*x^4)^2]*ArcTan[Sqrt[1 - a - b*x^4]/Sqrt[1 + a + b*x^4]]
)/(-1 + a + b*x^4))/(4*b)
```

Maple [A]
time = 0.07, size = 53, normalized size = 0.93

method	result	size
derivativedivides	$\frac{(b\,x^4+a)\operatorname{arcsech}(b\,x^4+a)-\arctan\left(\sqrt{\frac{1}{b\,x^4+a}-1}\,\sqrt{\frac{1}{b\,x^4+a}+1}\right)}{4b}$	53
default	$\frac{(b\,x^4+a)\operatorname{arcsech}(b\,x^4+a)-\arctan\left(\sqrt{\frac{1}{b\,x^4+a}-1}\,\sqrt{\frac{1}{b\,x^4+a}+1}\right)}{4b}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsech(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*((b*x^4+a)*arcsech(b*x^4+a)-arctan((1/(b*x^4+a)-1)^(1/2)*(1/(b*x^4+a)
+1)^(1/2)))
```

Maxima [A]
time = 0.26, size = 38, normalized size = 0.67

$$\frac{(bx^4+a)\operatorname{arsech}(bx^4+a)-\arctan\left(\sqrt{\frac{1}{(bx^4+a)^2}-1}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*((b*x^4 + a)*arcsech(b*x^4 + a) - arctan(sqrt(1/(b*x^4 + a)^2 - 1)))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(49) = 98.
time = 0.40, size = 283, normalized size = 4.96

$$\frac{2\,bx^4\log\left(\frac{(bx^4+a)\sqrt{-\frac{b^2x^8+2\,abx^4+a^2-1}{b^2x^8+2\,abx^4+a^2}}+1}{bx^4+a}\right)+a\log\left(\frac{(bx^4+a)\sqrt{-\frac{b^2x^8+2\,abx^4+a^2-1}{b^2x^8+2\,abx^4+a^2}}+1}{x^4}\right)-a\log\left(\frac{(bx^4+a)\sqrt{-\frac{b^2x^8+2\,abx^4+a^2-1}{b^2x^8+2\,abx^4+a^2}}-1}{x^4}\right)-2\arctan\left(\frac{(b^2x^8+2\,abx^4+a^2)\sqrt{-\frac{b^2x^8+2\,abx^4+a^2-1}{b^2x^8+2\,abx^4+a^2}}}{b^2x^8+2\,abx^4+a^2-1}\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/8*(2*b*x^4*log(((b*x^4 + a)*sqrt(-(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)/(b^2*x^
8 + 2*a*b*x^4 + a^2)) + 1)/(b*x^4 + a)) + a*log(((b*x^4 + a)*sqrt(-(b^2*x^8
```

```
 + 2*a*b*x^4 + a^2 - 1)/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 1)/x^4) - a*log(((b*
x^4 + a)*sqrt(-(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)/(b^2*x^8 + 2*a*b*x^4 + a^2))
 - 1)/x^4) - 2*arctan((b^2*x^8 + 2*a*b*x^4 + a^2)*sqrt(-(b^2*x^8 + 2*a*b*x^
4 + a^2 - 1)/(b^2*x^8 + 2*a*b*x^4 + a^2))/(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)))
/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{asech}\left(a + bx^4\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asech(b*x**4+a),x)
```

```
[Out] Integral(x**3*asech(a + b*x**4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsech(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*arcsech(b*x^4 + a), x)
```

Mupad [**B**]
time = 2.99, size = 56, normalized size = 0.98

$$\frac{\operatorname{atan}\left(\frac{1}{\sqrt{\frac{1}{b\,x^4+a}-1}\,\sqrt{\frac{1}{b\,x^4+a}+1}}\right)}{4\,b}+\frac{\operatorname{acosh}\left(\frac{1}{b\,x^4+a}\right)\,\left(b\,x^4+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acosh(1/(a + b*x^4)),x)
```

```
[Out] atan(1/((1/(a + b*x^4) - 1)^(1/2)*(1/(a + b*x^4) + 1)^(1/2)))/(4*b) + (acos
h(1/(a + b*x^4))*(a + b*x^4))/(4*b)
```

3.100 $\int x^{-1+n}\text{sech}^{-1}(a+bx^n)\,dx$

Optimal. Leaf size=58

$$\frac{(a+bx^n)\,\text{sech}^{-1}(a+bx^n)}{bn}-\frac{2\text{ArcTan}\left(\sqrt{\frac{1-a-bx^n}{1+a+bx^n}}\right)}{bn}$$

```
[Out] (a+b*x^n)*arcsech(a+b*x^n)/b/n-2*arctan(((1-a-b*x^n)/(1+a+b*x^n))^(1/2))/b/
n
```

Rubi [A]
time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6847, 6448, 1983, 12, 209}

$$\frac{(a+bx^n)\,\text{sech}^{-1}(a+bx^n)}{bn}-\frac{2\text{ArcTan}\left(\sqrt{\frac{-a-bx^n+1}{a+bx^n+1}}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*ArcSech[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)*ArcSech[a + b*x^n])/(b*n) - (2*ArcTan[Sqrt[(1 - a - b*x^n)/(1
+ a + b*x^n)]])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1983

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n),
Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(
b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1/
n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
```

```
gerQ[r]
```

Rule 6448

```
Int[ArcSech[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcSech[c + d*
x]/d), x] + Int[Sqrt[(1 - c - d*x)/(1 + c + d*x)]/(1 - c - d*x), x] /; Free
Q[{c, d}, x]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}\operatorname{sech}^{-1}(a+bx^n)\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{sech}^{-1}(a+bx)\,dx, x, x^n\right)}{n}\\
&= \frac{(a+bx^n)\operatorname{sech}^{-1}(a+bx^n)}{bn}+\frac{\operatorname{Subst}\left(\int \frac{\sqrt{\frac{1-a-bx}{1+a+bx}}}{1-a-bx}\,dx, x, x^n\right)}{n}\\
&= \frac{(a+bx^n)\operatorname{sech}^{-1}(a+bx^n)}{bn}-\frac{(4b)\operatorname{Subst}\left(\int \frac{1}{2b^2(1+x^2)}\,dx, x, \sqrt{\frac{1-a-bx^n}{1+a+bx^n}}\right)}{n}\\
&= \frac{(a+bx^n)\operatorname{sech}^{-1}(a+bx^n)}{bn}-\frac{2\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{1-a-bx^n}{1+a+bx^n}}\right)}{bn}\\
&= \frac{(a+bx^n)\operatorname{sech}^{-1}(a+bx^n)}{bn}-\frac{2\tan^{-1}\left(\sqrt{\frac{1-a-bx^n}{1+a+bx^n}}\right)}{bn}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 106, normalized size = 1.83

$$\frac{(a+bx^n)\operatorname{sech}^{-1}(a+bx^n)+\frac{2\sqrt{-\frac{-1+a+bx^n}{1+a+bx^n}}\sqrt{1-(a+bx^n)^2}\operatorname{ArcTan}\left(\frac{\sqrt{1-a-bx^n}}{\sqrt{1+a+bx^n}}\right)}{-1+a+bx^n}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcSech[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)*ArcSech[a + b*x^n] + (2*Sqrt[-((-1 + a + b*x^n)/(1 + a + b*x^n
))]*Sqrt[1 - (a + b*x^n)^2]*ArcTan[Sqrt[1 - a - b*x^n]/Sqrt[1 + a + b*x^n]]
)/(-1 + a + b*x^n))/(b*n)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int x^{-1+n} \operatorname{arcsech}(a + b\, x^{n})\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*arcsech(a+b*x^n),x)
```

```
[Out] int(x^(-1+n)*arcsech(a+b*x^n),x)
```

Maxima [A]
time = 0.25, size = 40, normalized size = 0.69

$$\frac{(bx^n + a)\operatorname{arsech}(bx^n + a) - \arctan\left(\sqrt{\frac{1}{(bx^n + a)^2} - 1}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsech(a+b*x^n),x, algorithm="maxima")
```

```
[Out] ((b*x^n + a)*arcsech(b*x^n + a) - arctan(sqrt(1/(b*x^n + a)^2 - 1)))/(b*n)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(54) = 108.
time = 0.42, size = 385, normalized size = 6.64

$$\frac{2\left(b\cosh\left(n\log\left(x\right)\right) + b\sinh\left(n\log\left(x\right)\right)\right)\log\left(\frac{\sqrt{-\frac{2\,ab + (a^2 + b^2 - 1)\cosh\left(n\log\left(x\right)\right) - (a^2 - b^2 - 1)\sinh\left(n\log\left(x\right)\right)}{\cosh\left(n\log\left(x\right)\right) - \sinh\left(n\log\left(x\right)\right)}} + 1}{b\cosh(n\log(x)) + b\sinh(n\log(x)) + a}\right) + a\log\left(\frac{\sqrt{-\frac{2\,ab + (a^2 + b^2 - 1)\cosh\left(n\log\left(x\right)\right) - (a^2 - b^2 - 1)\sinh\left(n\log\left(x\right)\right)}{\cosh\left(n\log\left(x\right)\right) - \sinh\left(n\log\left(x\right)\right)}} + 1}{\cosh(n\log(x)) + \sinh(n\log(x))}\right) - a\log\left(\frac{\sqrt{-\frac{2\,ab + (a^2 + b^2 - 1)\cosh\left(n\log\left(x\right)\right) - (a^2 - b^2 - 1)\sinh\left(n\log\left(x\right)\right)}{\cosh\left(n\log\left(x\right)\right) - \sinh\left(n\log\left(x\right)\right)}} - 1}{\cosh(n\log(x)) + \sinh(n\log(x))}\right) - 2\arctan\left(\frac{(b\cosh(n\log(x)) + b\sinh(n\log(x)) + a)\sqrt{-\frac{2\,ab + (a^2 + b^2 - 1)\cosh\left(n\log\left(x\right)\right) - (a^2 - b^2 - 1)\sinh\left(n\log\left(x\right)\right)}{\cosh\left(n\log\left(x\right)\right) - \sinh\left(n\log\left(x\right)\right)}}}{b^2\cosh(n\log(x))^2 + b^2\sinh(n\log(x))^2 + 2\,ab\cosh(n\log(x)) + a^2 + 2\,(b^2\cosh(n\log(x)) + ab)\sinh(n\log(x)) - 1}\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsech(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b*cosh(n*log(x)) + b*sinh(n*log(x)))*log((sqrt(-(2*a*b + (a^2 + b^2
 - 1)*cosh(n*log(x)) - (a^2 - b^2 - 1)*sinh(n*log(x)))/(cosh(n*log(x)) - si
nh(n*log(x)))) + 1)/(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)) + a*log((sqr
t(-(2*a*b + (a^2 + b^2 - 1)*cosh(n*log(x)) - (a^2 - b^2 - 1)*sinh(n*log(x))
)/(cosh(n*log(x)) - sinh(n*log(x)))) + 1)/(cosh(n*log(x)) + sinh(n*log(x)))
) - a*log((sqrt(-(2*a*b + (a^2 + b^2 - 1)*cosh(n*log(x)) - (a^2 - b^2 - 1)*
```

```
sinh(n*log(x)))/(cosh(n*log(x)) - sinh(n*log(x)))) - 1)/(cosh(n*log(x)) + s
inh(n*log(x)))) - 2*arctan((b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)*sqrt(-
(2*a*b + (a^2 + b^2 - 1)*cosh(n*log(x)) - (a^2 - b^2 - 1)*sinh(n*log(x)))/(
cosh(n*log(x)) - sinh(n*log(x))))/(b^2*cosh(n*log(x))^2 + b^2*sinh(n*log(x)
)^2 + 2*a*b*cosh(n*log(x)) + a^2 + 2*(b^2*cosh(n*log(x)) + a*b)*sinh(n*log(
x)) - 1)))/(b*n)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*asech(a+b*x**n),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsech(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(n - 1)*arcsech(b*x^n + a), x)
```

Mupad [**B**]
time = 2.35, size = 54, normalized size = 0.93

$$\frac{\operatorname{atan}\left(\frac{1}{\sqrt{\frac{1}{a+b\,x^n}-1}\,\sqrt{\frac{1}{a+b\,x^n}+1}}\right)+\operatorname{acosh}\left(\frac{1}{a+b\,x^n}\right)\,\left(a+b\,x^n\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*acosh(1/(a + b*x^n)),x)
```

```
[Out] (atan(1/((1/(a + b*x^n) - 1)^(1/2)*(1/(a + b*x^n) + 1)^(1/2))) + acosh(1/(a
 + b*x^n))*(a + b*x^n))/(b*n)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstar
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```